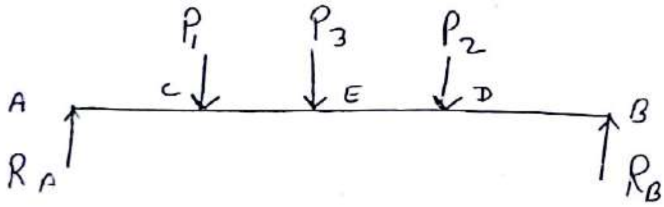
Bending Moment :-
The bending Moment at a Section through a Structural element may be defind as 'the Sum of moments about that Section of all external forces acting to one Side of that Section'



Bending Moment at E $= BM_E = R_B \times BE - P_2 \times ED$

or

$BM_E = R_A \times AE - P_1 \times CE$

its Unit is N.m.

BEAM :-
Beam is a horizontal member spanning a distance between one or more supports and carrying vertical Loads across its longitudinal axi

Types of Beam :-
There are following types of beam -

1. Simply Supported Beam.
2. Cantilever Beam
3. Fixed Beam
4. Over hanging Beam
5. Continuous Beam.

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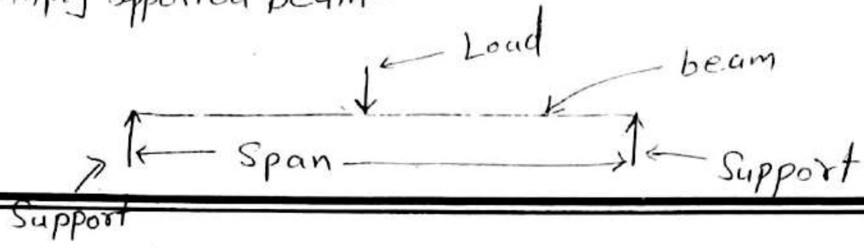
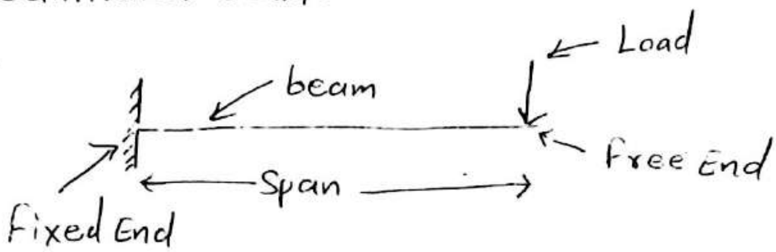
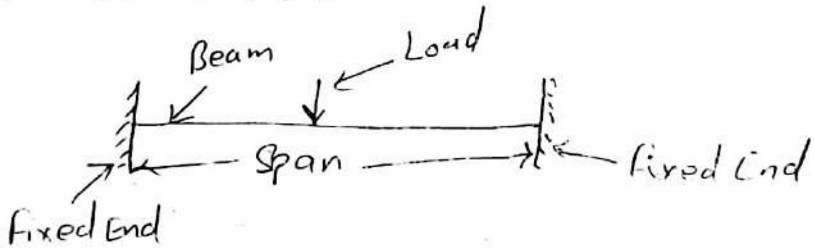
1 Simply Supported Beam -



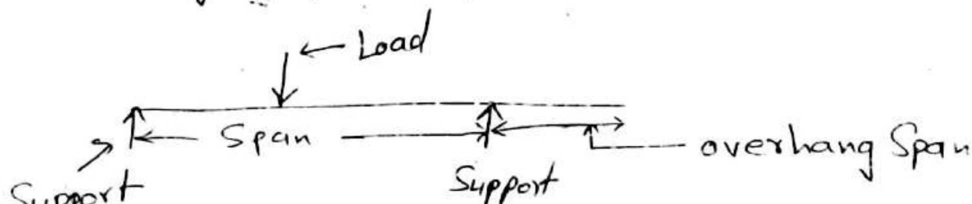
2 Cantilever Beam -



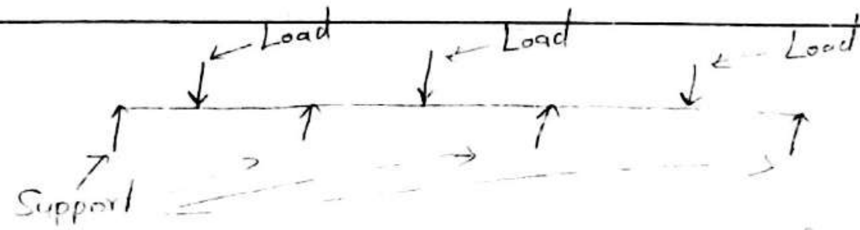
3 Fixed Beam -



4 overhanging beam →



5 Continuous Beam →



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Types of Supports →: There are following types of Supports :-

1. Simply Support

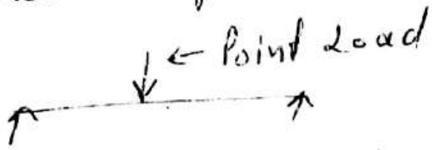
2. Roller Support

3. Fixed Support

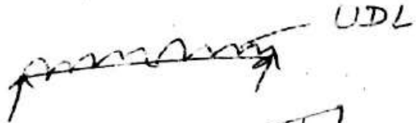
4. Hinged Support

Types of Loads →: There are following types of Loads :-

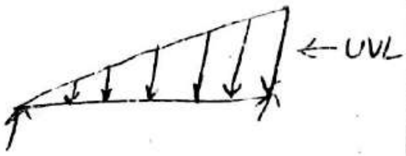
1. Point Load



2. Uniformly Distributed Load

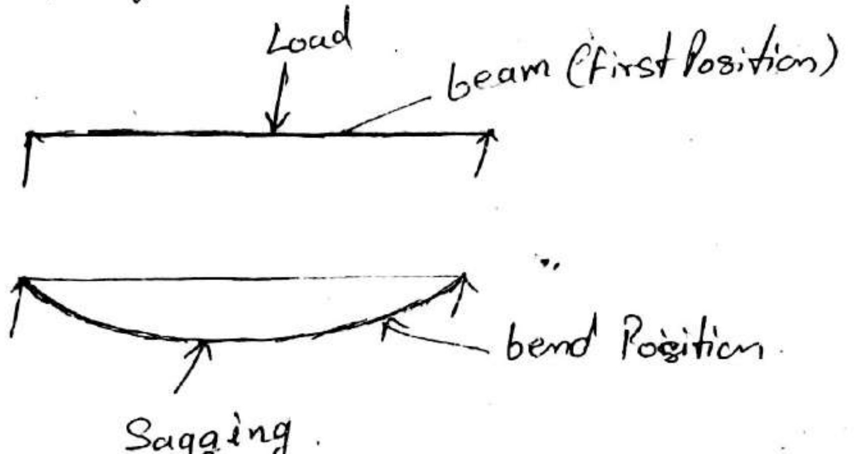


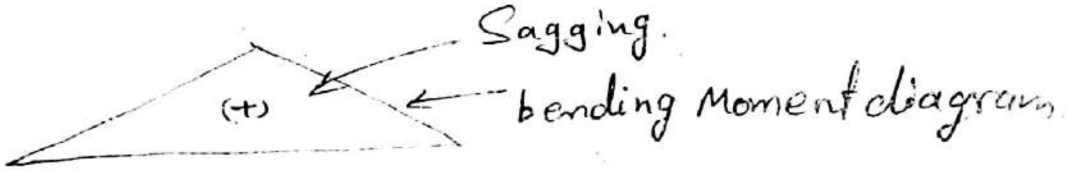
3. Uniformly Varying Load (UVL)



Sagging :-

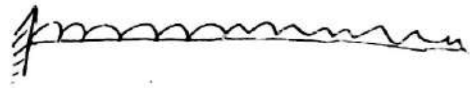
The Positive diagram of Bending Moment is Called Sagging or downward bending of a beam called Sagging.

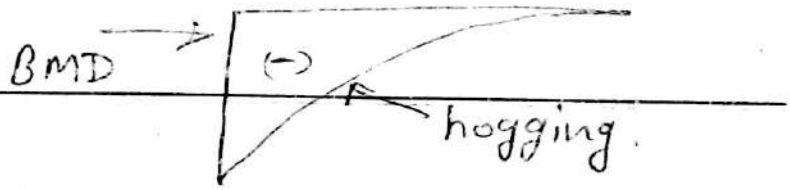




Hogging :-

The Negative Diagram of bending Moment is Called Hogging or upward bending of a beam Called hogging.





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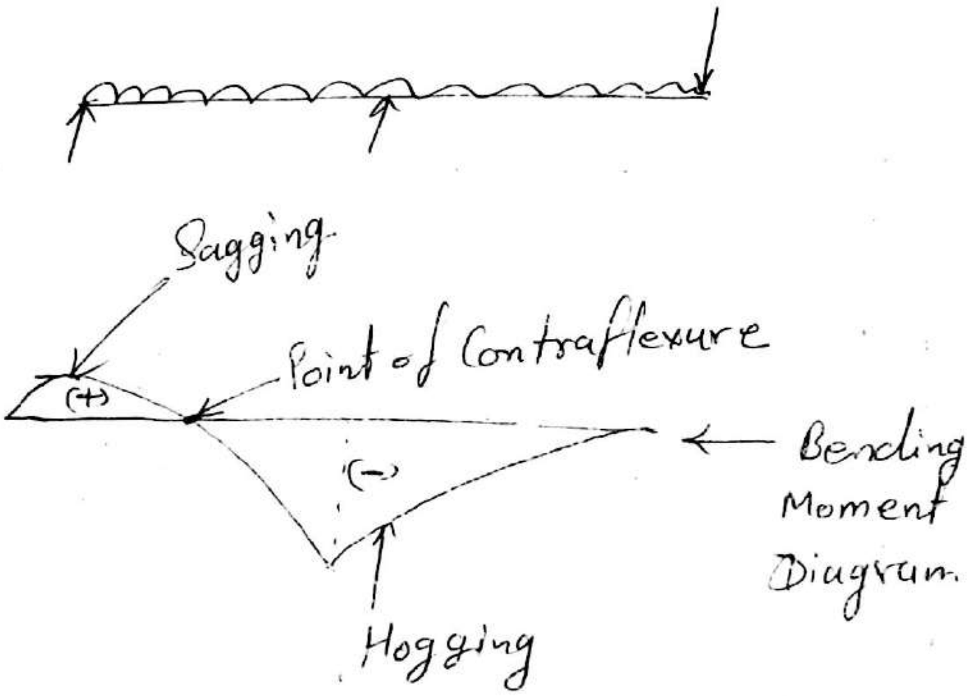
Point of Contraflexure → it is the zero bending Moment Point at which the bending Moment Change Sign. Positive to Negative or Negative to Positive.



Note:- ① Shear force Diagram 0 Line या Base Line से Start होकर Base Line पर ही सम्पूर्ण होता है तभी Beam सन्तुलन की अवस्था में होगी।

② Bending Moment Diagram भी zero line या Base line से Start होकर Base line पर ही सम्पूर्ण होता है तभी Beam सन्तुलन की अवस्था में होगी।

③ Beam की गणना के लिये उसे पूर्ण सन्तुलन की अवस्था में माना जाता है।

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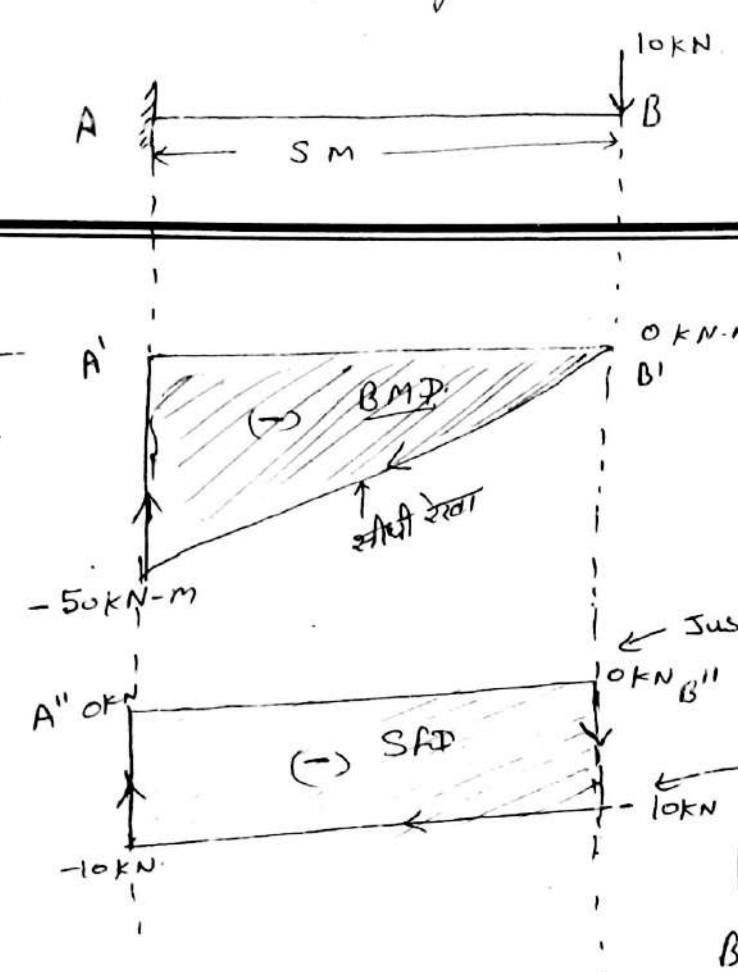
SFD & BMD Examples for Cantilever Beam

⇒ EXP. 1



Let (+↺)

BMD.

SFD.

← Just before for B Point

← Just after for B Point

Calculation

For Shear Force Diagram

$\overleftarrow{SF}_B = 0\ KN$ (Just before)

$\overrightarrow{SF} = -10\ KN$ (Just after)

$\overleftarrow{SF}_A = -10\ KN$

$\overrightarrow{SF}_A = 0\ KN.$

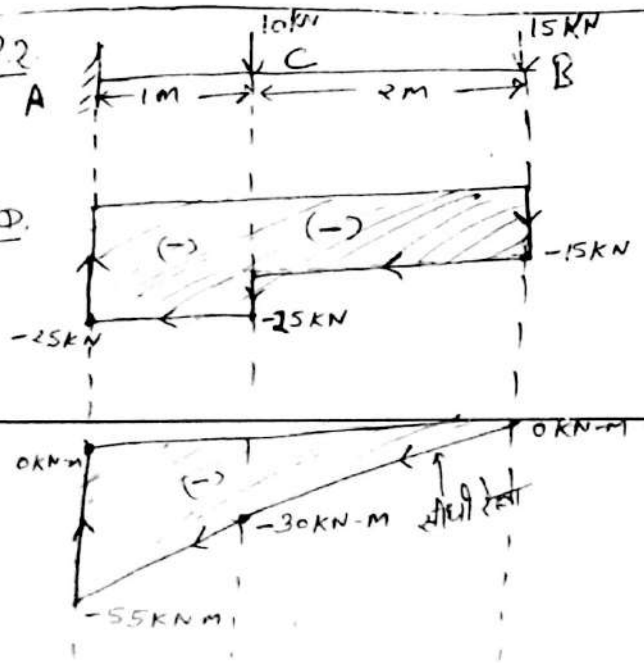
For Bending Moment Diagram

$BM_B = 0\ KN\text{-}M$

$\overleftarrow{BM}_A = -10 \times 5 = -50\ KN\text{-}M$

$\overrightarrow{BM}_A = 0\ KN\text{-}M.$

⇒ EXP. 2



SFD.

Calculation

For SFD. $\overleftarrow{SF}_B = 0\ KN$, $\overrightarrow{SF}_B = -15\ KN$

$\overleftarrow{SF}_C = -15\ KN$, $\overrightarrow{SF}_C = -25\ KN$

$\overleftarrow{SF}_A = -25\ KN$, $\overrightarrow{SF}_A = 0\ KN$

For BMD.

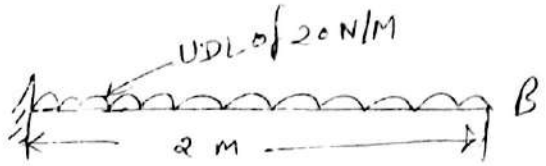
$BM_B = 0\ KN\text{-}M$

$BM_C = -30\ KN\text{-}M = 15 \times 2$

$\overleftarrow{BM}_A = -15 \times 3 - 10 \times 1 = -55\ KN\text{-}M$

$\overrightarrow{BM}_A = 0\ KN\text{-}M$

⇒ EXP.3 :-



Calculation.

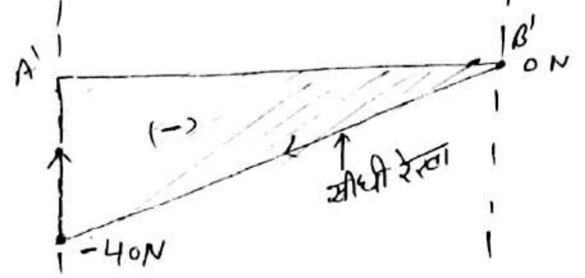
For SFD.

$\overleftarrow{SF_B} = 0N$

$\overleftarrow{SF_A} = -40N$

$\overrightarrow{SF_A} = 0N$

SFD.



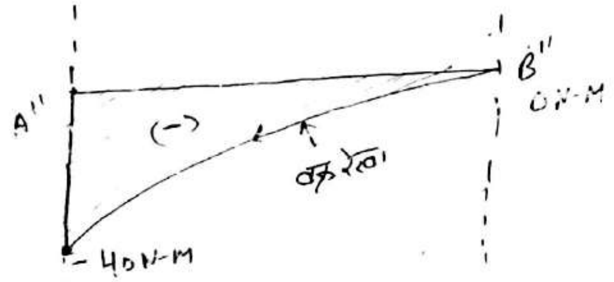
For BMD.

$BM_B = 0$

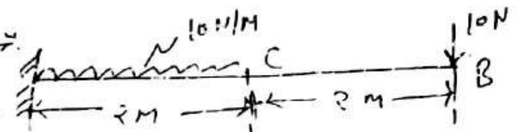
$\overleftarrow{BM_A} = -20 \times 2 \times \frac{2}{2}$

$= -40\ N\text{-}M.$

$\overrightarrow{BM_A} = 0\ N\text{-}M.$



⇒ EXP.4 :-



Calculation.

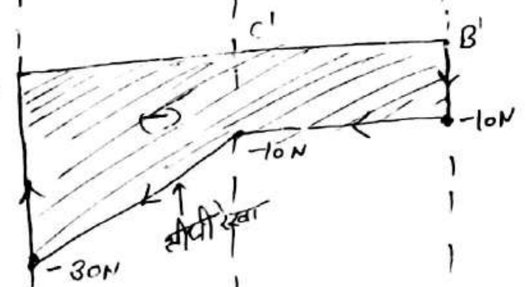
For SFD.

$\overleftarrow{SF_B} = 0$, $\overrightarrow{SF_D} = -10N$

$\overleftarrow{SF_C} = -10N$, $\overrightarrow{SF_C} = -10N$

$\overleftarrow{SF_A} = -30N$, $\overrightarrow{SF_A} = 0N$.

SFD.



For BMD :-

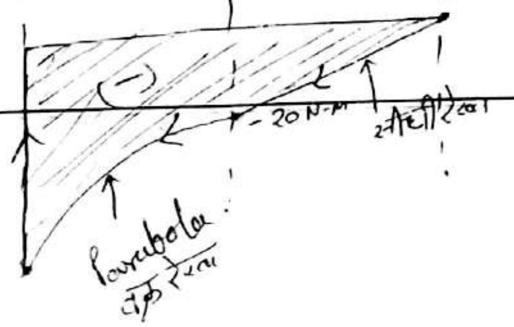
$BM_B = 0$

$BM_C = -20\ N\text{-}M$

$\overleftarrow{BM_A} = -10 \times 4 - 10 \times 2 \times \frac{2}{2}$

$= -60\ N\text{-}M.$

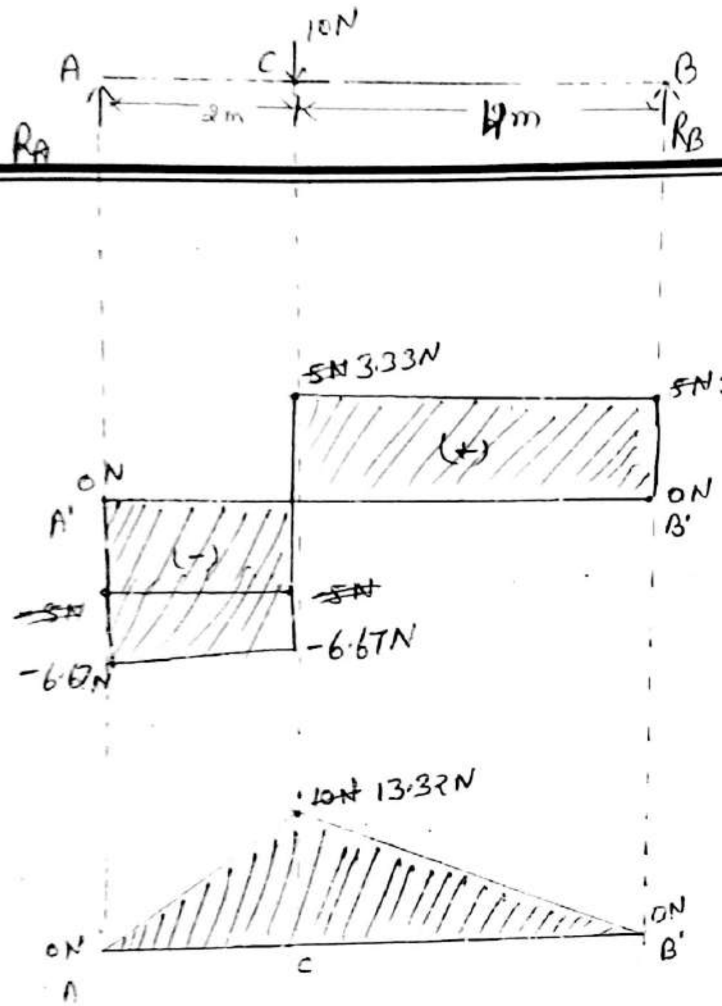
$\overrightarrow{BM_A} = 0\ N\text{-}M.$



Ex.5→

S.F.D.

B.M.D



Calculation:-

$R_A + R_B = 10N$ —— (1)

Taking moment about 'A'

$R_B = \cancel{5N}\ 3.33N$

$R_A = \cancel{5N}\ 6.67N$

For SFD:-

$\overleftarrow{SF_A} = 0N$

$\overrightarrow{SF_B} = \cancel{5N}\ 3.33N$

$\overleftarrow{SF_C} = \cancel{5N}\ 3.33N$

$\overrightarrow{SF_C} = \cancel{5N}\ -6.67N$

$\overrightarrow{SF_A} = \cancel{5N}\ -6.67N$

$\overleftarrow{SF_A} = 0N$

For B.M.D:-

$BM_B = 0N\cdot m \quad BM_A = 0\ N\cdot m$

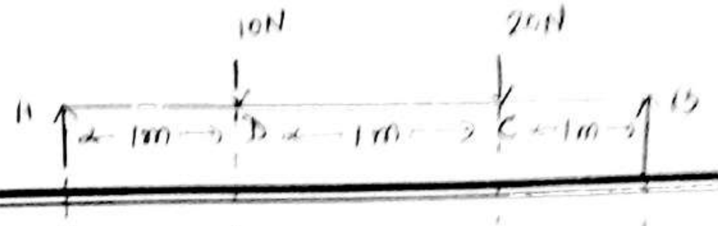
$BM_C = \cancel{5\times2}\ 3.33 \times 4 = 13.32\ N\cdot m$

$= \cancel{10Nm}$

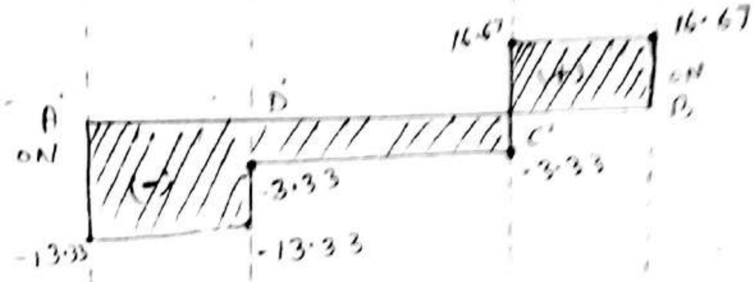
$BM_A = \cancel{5\times4 - 10\times2}$

$= 0N$

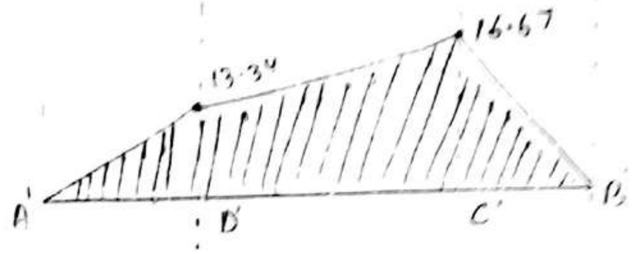
Ex 6:



SFD:-



BMD:



Calculation:-

$R_A + R_B = 30\,N$

$R_B \times 3 - 20 \times 2 - 10 \times 1 = 0$

$R_B \times 3 - 40 - 10 = 0$

$R_B = 50/3 = 16.67\,N$

$R_A + R_B = 30N \qquad R_A = 13.33N$

For SFD:-

$\overleftarrow{SF_B} = 0N$

$\overrightarrow{SF_B} = 16.67N$

$\overleftarrow{SF_C} = -3.33N$

$\overrightarrow{SF_C} = -3.33N$

$\overleftarrow{SF_D} = -3.33N$

$\overrightarrow{SF_D} = -13.33N$

$\overleftarrow{SF_A} = -13.33N$

$\overrightarrow{SF_A} = 0N$

For BMD:-

$BM_B = 0N\cdot m$

$BM_C = 16.67\,Nm$

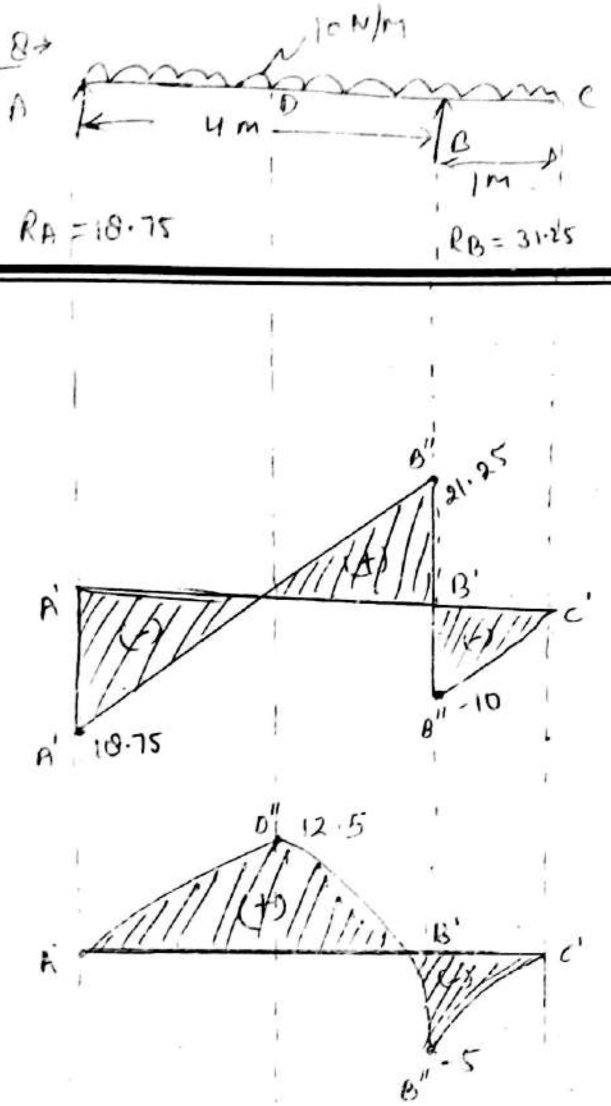
$BM_D = 16.67 \times 2 - 20 \times 1$

$= 13.34\,N\text{-}m$

$BM_A = 16.67 \times 3 - 20 \times 2 - 10 \times 1$

$= 0$

EXP 8→



Calculation:-

$R_A + R_B = 50$

$R_B \times 4 - 10 \times 5 \times 5/2$

$R_B = 31.25$

$R_A = 50 - 31.25$

$= 18.75$

BMD:-

$BM_B = -10 \times 1 \times \frac{1}{2}$

$= -5$

$BM_D = 10 \times 3 \times \frac{3}{2}$

$= +31.25 \times 2$

$= 17.5\ N$

$BM_A = 0\ N$

SFD:-

$\overleftarrow{SF_C} = 0N$

$\overrightarrow{SF_C} = 0N$

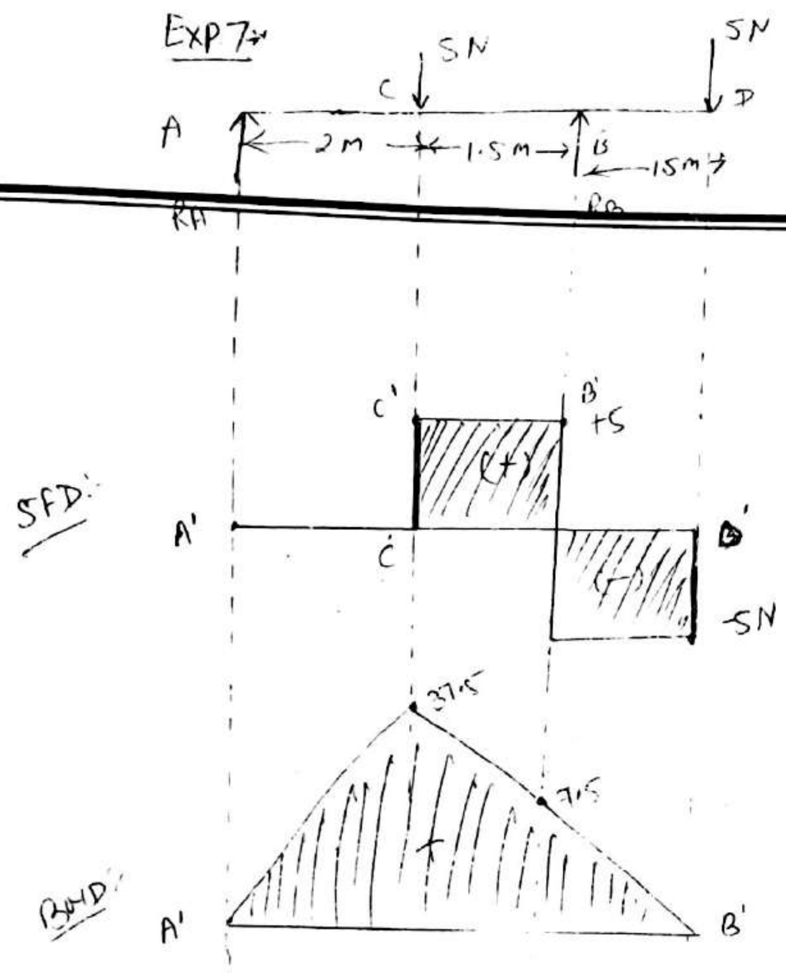
$\overleftarrow{SF_B} = -10N$

$\overrightarrow{SF_B} = 21.25\ N$

$\overleftarrow{SF_A} = -18.75N$

$\overrightarrow{SF_A} = 0$

Exp 7 :-



Calculation:- $R_A + R_B = 10N$

Taking moment about "A"

$$R_B \times 3.5 - 5 \times 5 - 5 \times 2 = 0$$

$$R_B = \frac{35}{3.5} = 10 \quad \therefore R_A = 0N$$

SF_D → $\overleftarrow{SF_D} = 0$ $\quad \overrightarrow{SF_D} = -5N$

$\overleftarrow{SF_C} = -5N$ $\quad \overrightarrow{SF_C} = +5N$

$\overleftarrow{SF_A} = 0$

BM_D:-

$BM_D = 0$

~~$BM_B = 5 \times 1.5$~~

$= 7.5$

$BM_C = 5 \times 3 + 10 \times$

$= 3c$

$BM_A = 0$

UNIT-2

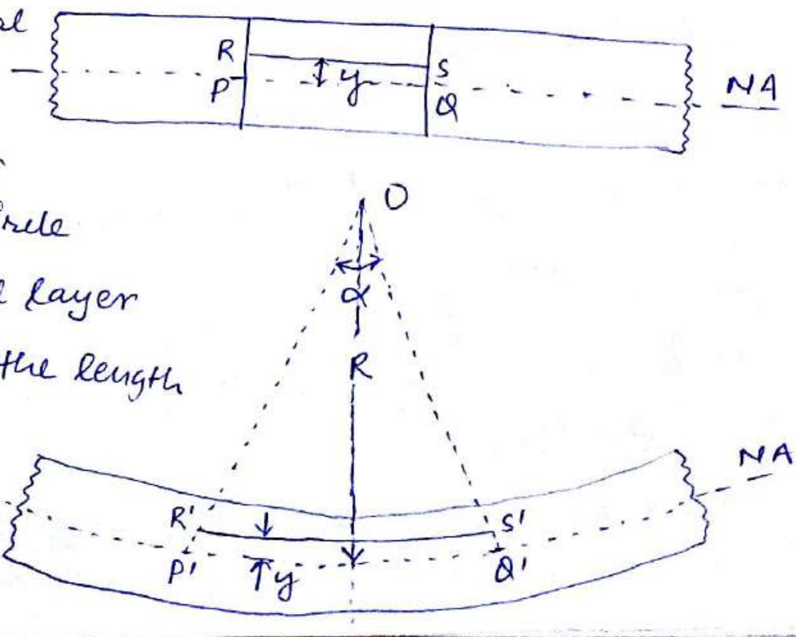
Bending Stresses

Auss- Assumptions in 'Theory of simple bending':-

1. Material of beam is perfectly homogeneous throughout.
2. The stresses induced is proportional to the main strain and at no place the stress exceeds the elastic limit.
3. The value of modulus of elasticity (E) is same, for the fibre of the beam under compression or under tension.
4. The transverse section of the beam, which is plane before bending, remains plane after bending,
5. There is no resultant pull or push on the cross section of the beam.
6. The loads are applied in the plane of bending,
7. The transverse section of the beam is symmetrical about a line passing through the centre of gravity in the plane of bending.
8. The radius of curvature of the beam before bending is very large in comparison to its transverse section.

#Bending Equation:-

Figure shows a longitudinal section of a beam. the neutral layer (NA) being bent to form an arc of circle of radius R. The neutral layer is then, before bending the length PQ which after bending becomes P'Q'.



Consider some layer RS at some distance y from PQ, which after bending becomes R'S'. Let P'Q' substend an angle α at the centre of curvature.

$$P'Q' = R\alpha$$

$$R'S' = (R-y)\alpha$$

Initially – ∴ $PQ = RS$

Since, there is no stress at neutral layer.

$$P'Q' = PQ$$

$$\text{Strain in } RS = \frac{RS - R'S'}{RS} \qquad RS = PQ = P'Q'$$

$$\epsilon = \frac{P'Q' - R'S'}{RS} = \frac{P'Q' - R'S'}{P'Q'}$$

$$\epsilon = \frac{R\alpha - (R-y)\alpha}{R\alpha} = \frac{y\alpha}{R\alpha}$$

$$\epsilon = \frac{y}{R} \quad \text{—— (1)}$$

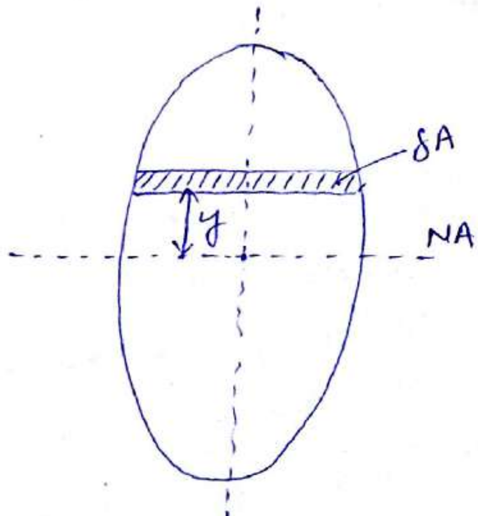
Now, - Strain, $\epsilon = \frac{\sigma}{E}$ —— (2)

from eqn (1) and (2).

$$\frac{\sigma}{E} = \frac{y}{R} \quad \text{—— (3)}$$

Now if transverse section is considered.

Let a strip of area δA, lie at a distance y from N.A.



Normal Force, $F = \sigma \cdot \delta A$

$$= \frac{E}{R} y \cdot \delta A$$

Now moment of this ~~area~~ Force about N.A –

$$\delta M = \frac{E}{R} y \cdot \delta A \cdot y = \frac{E}{R} \cdot y^2 \cdot \delta A$$

This is the resisting moment of the material caused by the stress produced.

The total resisting Moment, $M = \sum \frac{E}{R} y^2 \cdot \delta A$

$$M = \frac{E}{R} \sum y^2 \delta A$$

$\sum y^2 \delta A = I_{NA}$ = second moment of area about N.A.

$$M = \frac{E}{R} \cdot I$$

$$\frac{M}{I} = \frac{E}{R} \quad \text{—— (4)}$$

From eqn (3) and (4).

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

This equation is known as bending equation.

Centre of Gravity:- The centre of gravity of the body may be defined as the point through which the whole weight of a body may be assumed to act.

The position of C.G. depends upon shape of the body and this may ~~not~~ or may not necessarily be within the boundary of the body.

centroid:- The centroid or centre of area is defined as the point where the whole area of the figure is assumed to be concentrated. Thus centroid can be taken as quite analogous to centre of gravity of a body when bodies have area only and not weight.

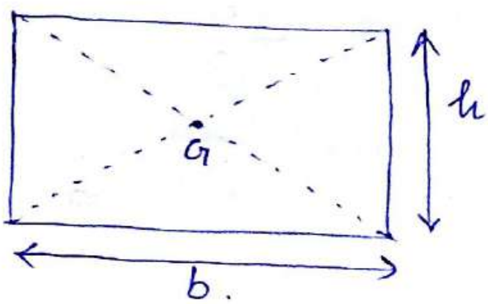
Positions of centroids of plane geometrical figures:-

(1) Rectangle:-

Area = bh

$\bar{x} = \frac{b}{2}$

$\bar{y} = \frac{h}{2}$

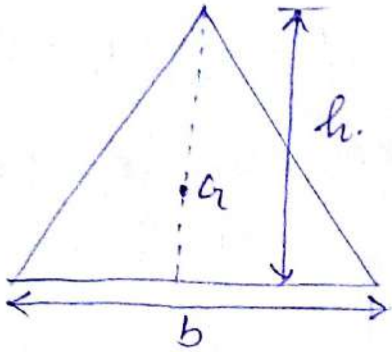


② Triangle:-

Area $= \frac{bh}{2}$

$\bar{x} = b/3$

$\bar{y} = h/3$

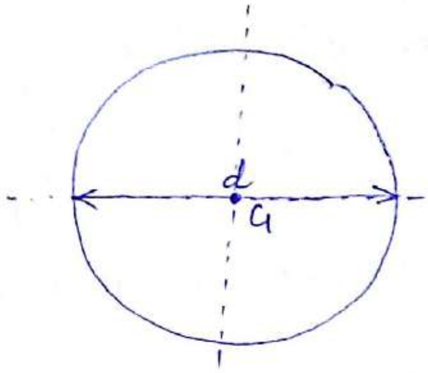


③ Circle:-

Area $= \frac{\pi}{4} d^2$

$\bar{x} = \frac{d}{2}$

$\bar{y} = \frac{d}{2}$

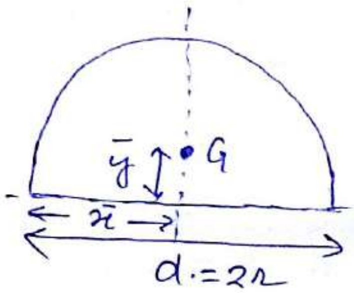


④ Semicircle:-

$A = \frac{\pi}{8} d^2$

$\bar{x} = \frac{d}{2}$

$\bar{y} = \frac{4r}{3\pi}$

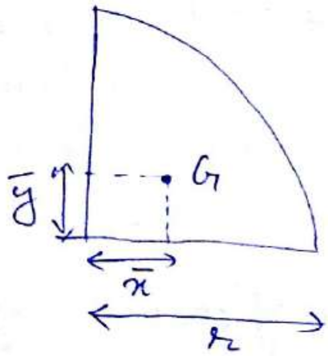


⑤ Quadrant:-

Area $= \frac{\pi}{16} d^2$

$\bar{x} = \frac{4r}{3\pi}$

$\bar{y} = \frac{4r}{3\pi}$

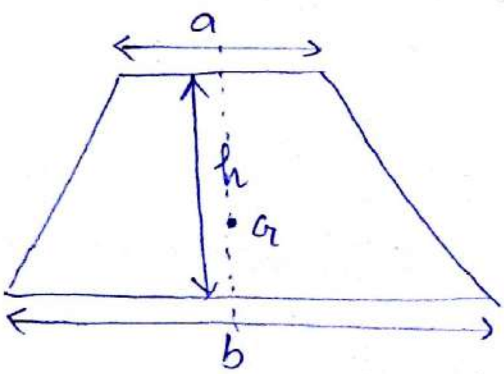


⑥ Trapezium:-

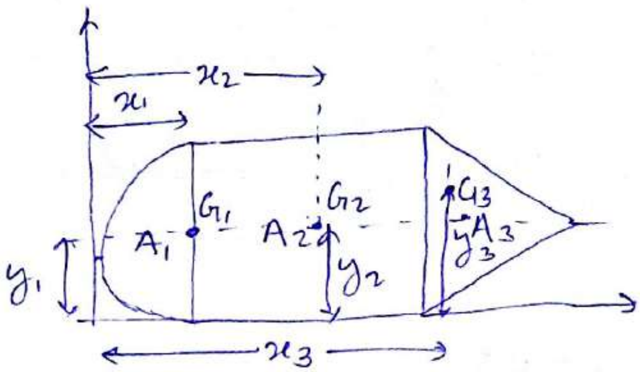
Area $= (a+b)\frac{h}{2}$

$\bar{x} = \frac{a^2+b^2+ab}{3(a+b)}$

$\bar{y} = \frac{(2a+b)}{(a+b)} \times \frac{h}{3}$



Centroids of composite Areas:-



$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

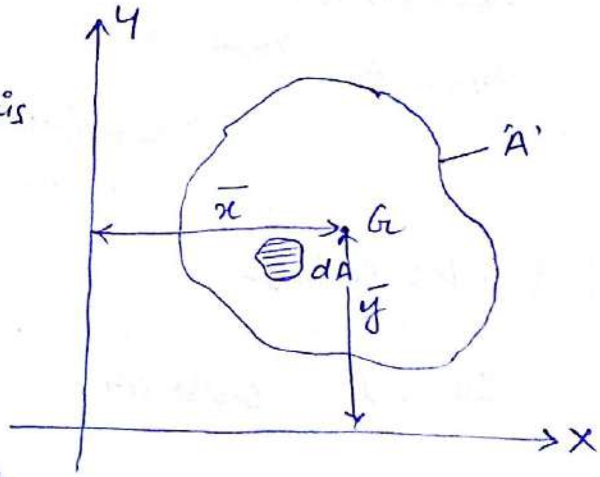
$$\bar{x} = \frac{\Sigma ax}{\Sigma a}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

Moment of Inertia (Second moment of area):-

Moment of Inertia of any plane area 'A' is the second moment of ~~area~~ all the small areas 'dA' comprising area 'A' about any axis in the plane of area A.



I_{yy} = Moment of inertia about y-axis

$I_{yy} = \Sigma (dA \cdot x)x$

where $(dA \cdot x)$ is the first moment of area dA about yy axis and $(dA \cdot x)x$ is the moment of first moment (second moment) of area dA about the same axis y-y.

$$I_{yy} = \Sigma dA \cdot x^2 = \int_A dA \cdot x^2$$

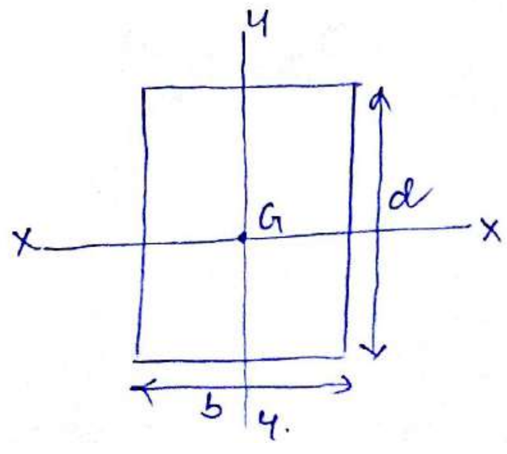
$$I_{xx} = \Sigma dA \cdot y^2 = \int_A dA \cdot y^2$$

Moment of Inertia for simple areas:-

① Rectangle:-

$$I_{xx} = \frac{bd^3}{12}$$

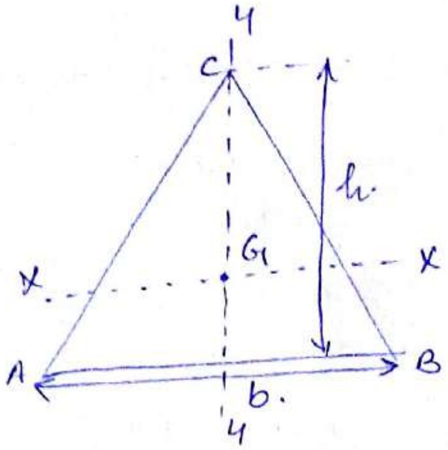
$$I_{yy} = \frac{db^3}{12}$$



② Triangle:-

$$I_{xx} = \frac{bh^3}{36}$$

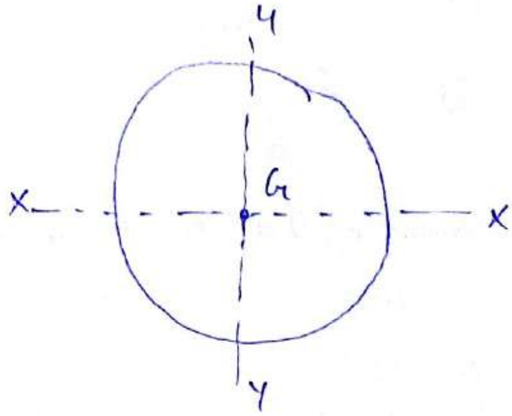
$$I_{AB} = \frac{bh^3}{12} \quad \text{(about base)}$$



③ Circle:-

$$I_{xx} = I_{yy} = \frac{\pi}{4} r^4 = \frac{\pi}{64} d^4$$

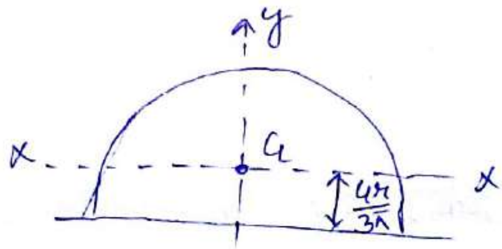
$$I_P = \frac{\pi}{32} d^4 \quad \text{(Polar Moment of Inertia)}$$



④ Semicircle:-

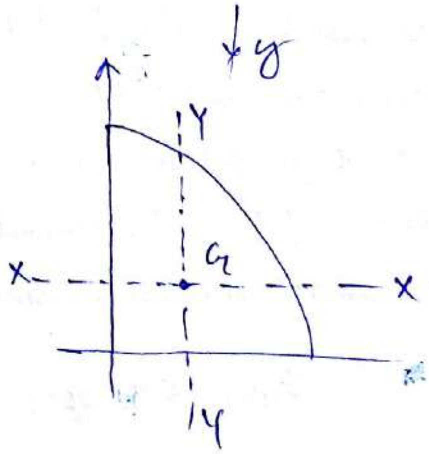
$$I_{xx} = 0.11 r^4$$

$$I_{yy} = \frac{\pi}{8} r^4$$



⑤ Quarter Circle:-

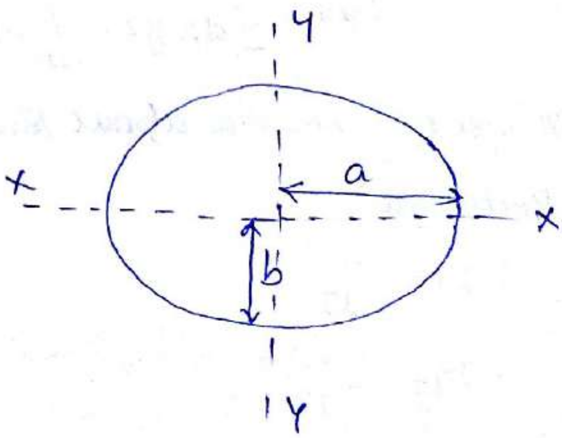
$$I_{xx} = I_{yy} = 0.055 r^4$$



⑥ Ellipse:-

$$I_{xx} = \frac{\pi a b^3}{4}$$

$$I_{yy} = \frac{\pi b a^3}{4}$$



→ Radius of Gyration:-

Radius of Gyration is the property of cross-section which influences the structural behaviour of the member. ~~is~~

Radius of Gyration is defined as the square root of the ratio of Moment of Inertia to the area of cross section.

$$I = AK^2$$

$$K = \sqrt{\frac{I}{A}}$$

K = radius of Gyration

The load at which member will buckle is proportional to the square of radius of gyration.

Theorem of Parallel axis:-

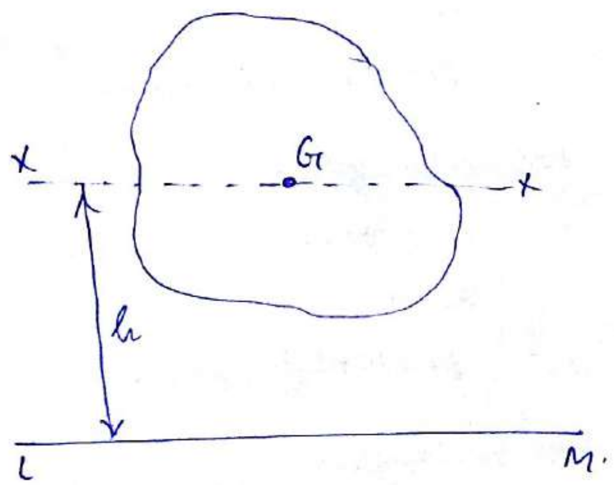
The theorem of parallel axis states that -

" The moment of inertia of a lamina about any axis in the plane of lamina equals the sum of moment of inertia about a parallel centroidal axis in the plane of lamina and the product of the area of lamina and square of distance b/w the two axis."

$$I_{LM} = I_{xx} + A \cdot h^2$$

$$\boxed{I_{LM} = I_{cg} + Ah^2}$$

$$I_{xx} = I_{cg}$$

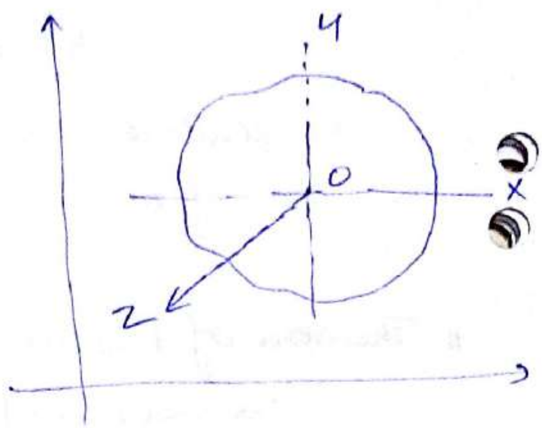


Theorem of Perpendicular axes:-

The theorem of perpendicular axes states that -

" If OX and OY be the moments of inertia of a lamina about mutually perpendicular axes OX and OY in the plane of lamina and OZ be the moment of Inertia of lamina about an axis (OZ) normal to lamina and passing through the point of intersection of the axes OX and OY then-

$$\boxed{I_{oz} = I_{ox} + I_{oy}}$$



Ques:- Find the moment of Inertia about the horizontal and vertical axis through the C.G. of the section shown in figure.

Soln:- for rectangle ①.

$A_1 = 60 \times 10 = 600\ mm^2$

$x_1 = 0$

$y_1 = 10 + 100 + 5 = 115\ mm$

For rectangle ② -

$A_2 = 100 \times 10 = 1000\ mm^2$

$x_2 = 0$

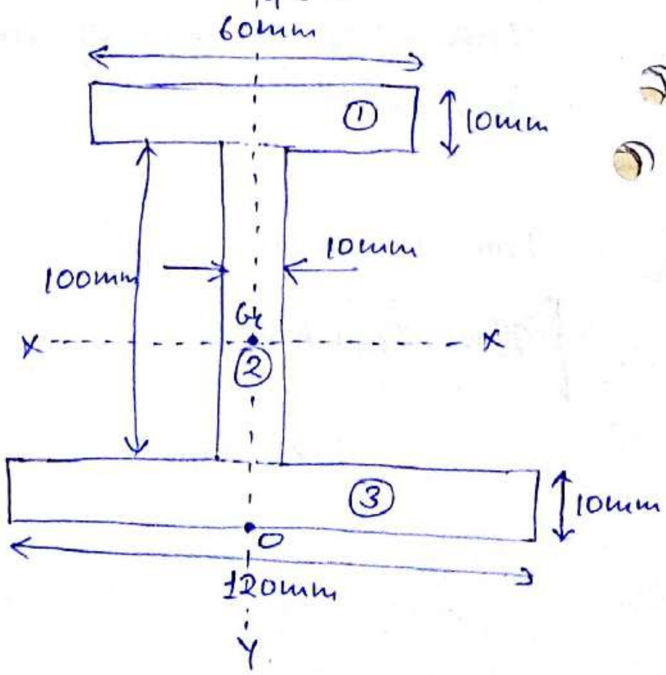
$y_2 = 10 + \frac{100}{2} = 60\ mm$

For rectangle ③ -

$A_3 = 120 \times 10 = 1200\ mm^2$

$x_3 = 0$

$y_3 = \frac{10}{2} = 5\ mm$



$\bar{x} = 0$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1+A_2+A_3}$$

$$\bar{y} = \frac{600\times115 + 1000\times60 + 1200\times5}{600+1000+1200}$$

$$\bar{y} = \frac{135000}{2800} = 48.21\text{mm}$$

$$I_{xx} = \left[\frac{60\times10^3}{12} + 600(115-48.21)^2\right] + \left[\frac{100\times10^3}{12} + 1000(60-48.21)^2\right] + \left[\frac{120\times10^3}{12} + 1200(48.21-5)^2\right]$$

$I_{xx} = 2681542.46 + 972337.43 + 2250524.92$

$I_{xx} = 5904404.81\text{ mm}^4$ Ans.

$$I_{yy} = \frac{10\times60^3}{12} + \frac{100\times10^3}{12} + \frac{10\times120^3}{12}$$

$= 180,000 + 8333.33 + 1440,000$

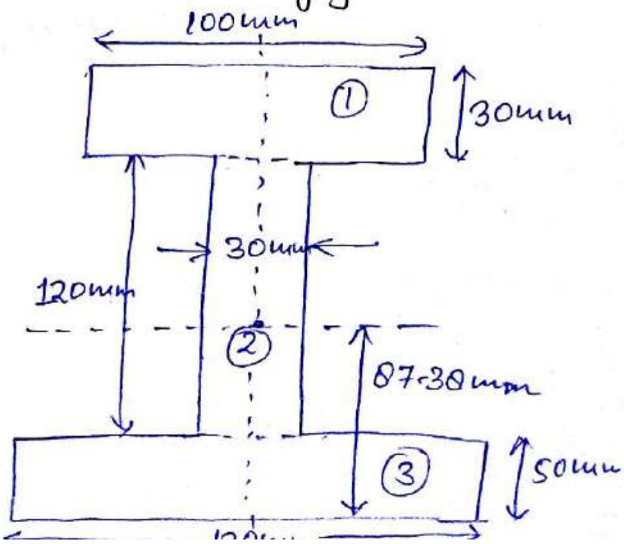
$I_{yy} = 1628333.33\text{ mm}^4$ Ans

$I_p = I_{zz} = I_{xx} + I_{yy}$

$= 5904404.81 + 1628333.33$

$= 7532738.14\text{ mm}^4$ Ans.

Que:- Find the moment of Inertia ~~of~~ about the horizontal and vertical axis through C.G. of section shown in figure.



Sol^n:- Rectangle (1).

$A_1 = 100\times30 = 3000\text{ mm}^2$

$y_1 = 50+120+\frac{30}{2} = 185\text{ mm}$

Rectangle (2).

$A_2 = 120\times30 = 3600\text{ mm}^2$

$y_2 = 50+\frac{120}{2} = 110\text{ mm}$

Rectangle (3) –

$$A_3 = 120 \times 50 = 6000 \text{ mm}^2$$

$$y_3 = \frac{50}{2} = 25 \text{ mm}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{3000 \times 185 + 3600 \times 110 + 6000 \times 25}{3000 + 3600 + 6000}$$

$$\bar{y} = \frac{1101000}{12600} = 87.38 \text{ mm}$$

$$I_{xx} = \left[\frac{100 \times 30^3}{12} + 3000(185 - 87.38)^2\right] + \left[\frac{30 \times 120^3}{12} + 3600(110 - 87.38)^2\right] + \left[\frac{120 \times 50^3}{12} + 6000(87.38 - 25)^2\right]$$

$$I_{xx} = 28813993.2 + 6161991.84 + 24597586.4$$

$$I_{xx} = 59573571.44 \text{ mm}^4$$

$$I_{yy} = \frac{30 \times 100^3}{12} + \frac{120 \times 30^3}{12} + \frac{50 \times 120^3}{12}$$

$$I_{yy} = 2500{,}000 + 270{,}000 + 7200{,}000$$

$$I_{yy} = 9970{,}000 \text{ mm}^4$$

Polar Moment of Inertia –

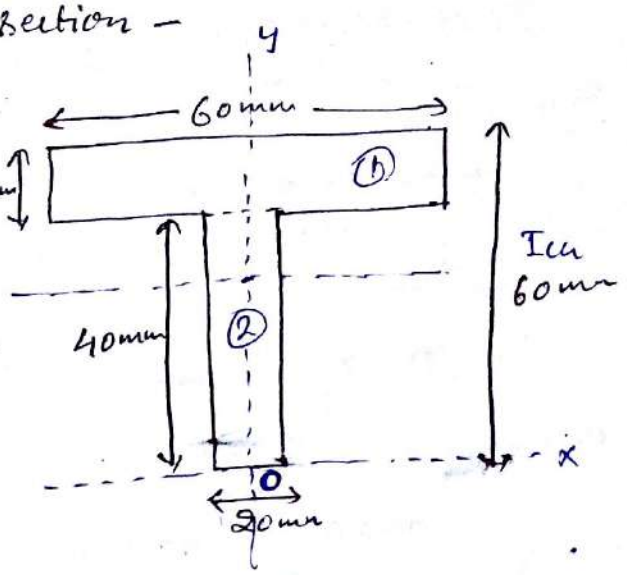
$$I_P = I_{xx} + I_{yy}$$

$$= 59573571.44 + 9970{,}000$$

$$= 69543571.44 \text{ mm}^4 \text{ Ans}$$

Moment of Inertia ~~about~~ of T-section:-

Ques:- Calculate the moment of inertia about horizontal and vertical gravity axis (I_{xx} and I_{yy}) of the section -



Rectangle ① -

$A_1 = 60 \times 20 = 1200\ mm^2$

$y_1 = 40 + 10 = 50m$

Rectangle ② -

$A_2 = 40 \times 20 = 800\ mm^2$

$y_2 = \frac{40}{2} = 20mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$\bar{y} = \frac{1200 \times 50 + 800 \times 20}{1200 + 800}$$

$$\bar{y} = \frac{60000 + 16000}{2000} = 38\ mm$$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= \left[\frac{60 \times 20^3}{12} + 60 \times 20 (50-38)^2\right] + \left[\frac{20 \times 40^3}{12} + 40 \times 20 (38-20)^2\right]$$

$I_{xx} = 212800 + 365866 = 578666\ mm^4$ Ans

$$I_{yy} = \left[\frac{20 \times 60^3}{12}\right] + \left[\frac{40 \times 20^3}{12}\right]$$

$= 360000 + 26666$

$= 386666\ mm^4$ Ans

Polar Moment of Inertia -

$I_p = I_{xx} + I_{yy}$

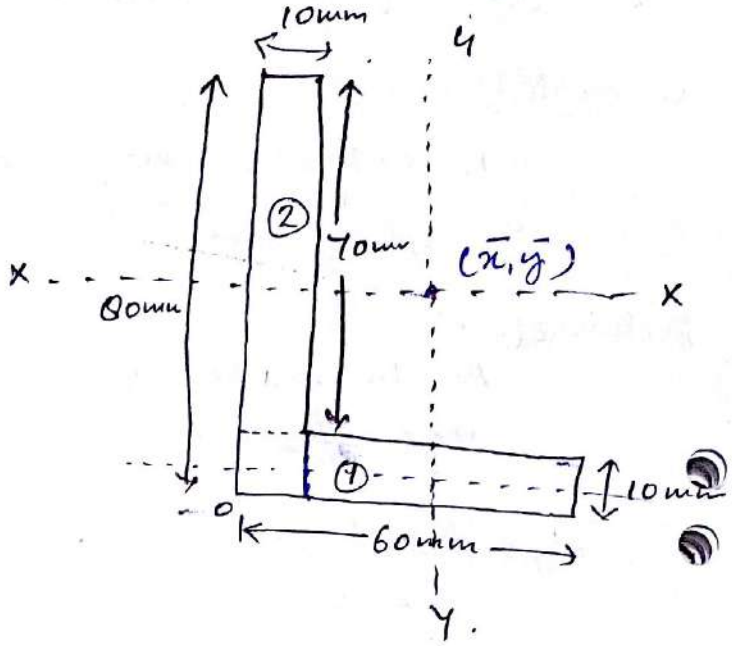
$= 578666 + 386666$

$I_p = 965332\ mm^4$ Ans

→ Moment of Inertia of angle (L-section):-

Ques:- Find the moment of inertia about the centroidal axis XX and YY of the section -

Sol^n



Rectangle ①.

$$A_1 = 50\times10 = 500\ mm^2$$

$$x_1 = 10+25 = 35\ mm$$

$$y_1 = \frac{10}{2} = 5\ mm$$

Rectangle ② -

$$A_2 = 80\times10 = 800\ mm^2$$

$$x_2 = \frac{10}{2} = 5\ mm$$

$$y_2 = \frac{80}{2} = 40\ mm$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1+A_2}$$

$$\bar{x} = \frac{500\times35 + 800\times5}{500+800}$$

$$\bar{x} = 16.5\ mm$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2}$$

$$\bar{y} = \frac{500\times5 + 800\times40}{500+800}$$

$$\bar{y} = 26.5\ mm$$

Using Parallel axis theorem -

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$I_{xx} = \left[\frac{50\times10^3}{12} + 50\times10\,(26.5-5)^2\right] + \left[\frac{10\times80^3}{12} + 80\times10\times(40-26.5)\right.$$

$$I_{xx} = 235291 + 572466$$

$$I_{xx} = 807757\ mm^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$I_{yy} = \left[\frac{10\times50^3}{12} + 10\times50\times(35-16.5)^2\right] + \left[\frac{80\times10^3}{12} + 80\times10\,(16.5-5)\right.$$

$$I_{yy} = 275291 + 112466$$

$$= 387757\ mm^4 \text{ Ans}$$

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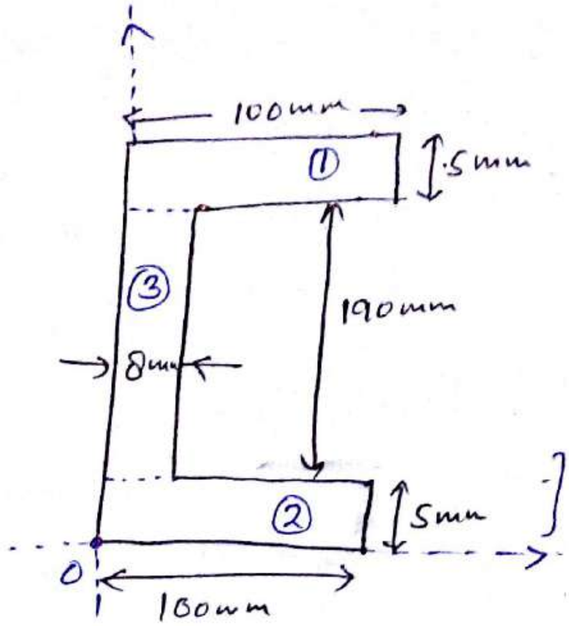
Moment of Inertia of Channel Section:-



Rectangle ①.

$A_1 = 5 \times 100 = 500\,mm^2$

$x_1 = \frac{100}{2} = 50\ mm$

$y_1 = 5 + 190 + \frac{5}{2} = 197.5\ mm$

Rectangle ②.

$A_2 = 5 \times 100 = 500\,mm^2$

$x_2 = \frac{100}{2} = 50\,mm$

$y_2 = \frac{5}{2} = 2.5\ mm$

Rectangle ③.

$A_3 = 8 \times 190 = 1520\ mm^2$

$y_3 = 5 + \frac{190}{2} = 5 + 95 = 100\,mm$

$x_3 = \frac{8}{2} = 4\,mm$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$\bar{x} = \frac{500\times50 + 500\times50 + 1520\times4}{500+500+1520}$$

$$\bar{x} = \frac{56080}{2520}$$

$$\bar{x} = 22.25\ mm$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1+A_2+A_3}$$

$$\bar{y} = \frac{500\times197.5 + 500\times2.5 + 1520\times100}{500+500+1520}$$

$$\bar{y} = \frac{252000}{2520}$$

$$\bar{y} = 100\ mm$$

Moment of Inertia -

$$I_{xx} = \left[\frac{100\times5^3}{12} + 500(197.5-100)^3\right] + \left[\frac{8\times190^3}{12} + 1520(100-100)^2\right] + \left[\frac{100\times5^3}{12} + 500(100-2.5)^2\right]$$

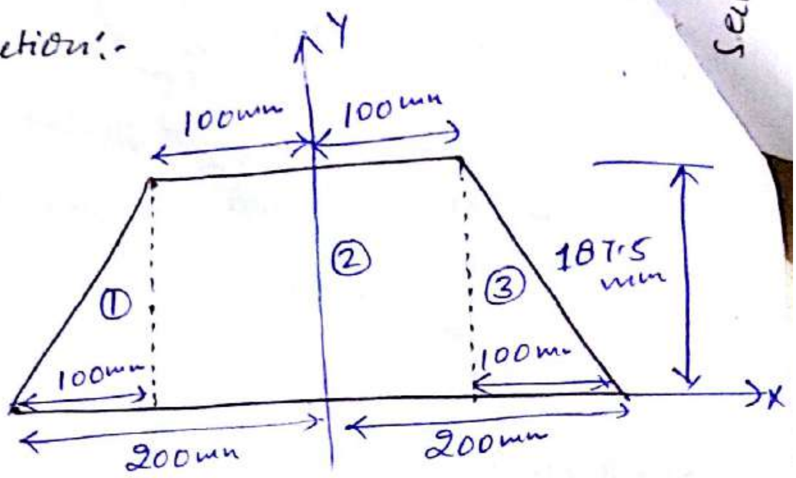
$I_{xx} = 14081000\ mm^4$ Ans;

$$I_{yy} = \left[\frac{5\times100^3}{12} + 500(50-22.25)^2\right] + \left[\frac{190\times8^3}{12} + 1520(22.25-4)^2\right] + \left[\frac{5\times100^3}{12} + 500(50-22.25)^2\right]$$

$= 801697.92 + 514361.67 + 801697.92$

$= 2117757.51\ mm^4$ Ans;

→ Moment of Inertia of Trapezium section:-



Triangle ①.

$A_1 = \frac{1}{2} \times 100 \times 187.5 = 9375\ mm^2$

$y_1 = \frac{187.5}{3} = 62.5\ mm$

Triangle ③.

$A_3 = A_1 = 9375\ mm^2$

$y_3 = \frac{187.5}{3} = 62.5\ mm$

$x_1 = x_3 = 100 + \frac{100}{3} = 133.33$

Rectangle ②.

$A_2 = 200 \times 187.5 = 37500\ mm^2$

$y_2 = \frac{187.5}{2} = 93.75\ mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{9375 \times 62.5 + 37500 \times 93.75 + 9375 \times 62.5}{9375 + 37500 + 9375}$$

$$\bar{y} = \frac{585937 + 3515625 + 585937}{56250} = \frac{4687499}{56250} = 83.33\ mm.$$

Moment of Inertia -

$$I_{xx} = \left[\frac{100 \times 187.5^3}{36} + 9375(83.33 - 62.5)^2\right] + \left[\frac{200 \times 187.5^3}{12} + 37500(83.33 - 93.75)^2\right] + \left[\frac{100 \times 187.5^3}{36} + 9375(83.33 - 62.5)^2\right]$$

$I_{xx} = 22378255.31 + 113934896.3 + 22378255.31$

$= 158691406.9\ mm^4$ Ans;

$$I_{yy} = \left[\frac{187.5 \times 100^3}{36} + 9375(133.33 - 0)^2\right] + \left[\frac{187.5 \times 200^3}{12}\right] + \left[\frac{187.5 \times 100^3}{36} + 9375(133.33 - 0)^2\right]$$

$= 171866666.8 + 1250,000,000 + 171866666.8$

$= 1593733334\ mm^4$ Ans;

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Section Modulus:-

From bending equation.

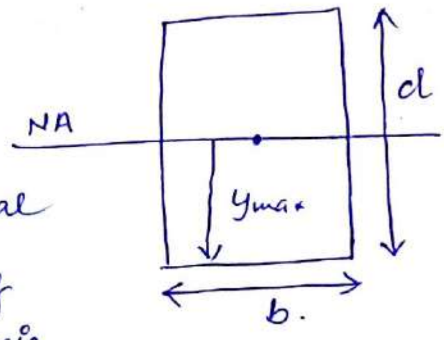
$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Permissible bending stress ← $\sigma = \frac{My}{I} = \frac{M}{I/y} = \frac{M}{Z}$ —(1)

Z = section modulus $= \frac{I}{y_{max}}$.

→ The strength of beam section mainly depends on the section modulus.

(1) Rectangular Section:-



Section modulus, $Z = \frac{I}{y_{max}}$

I = moment of ~~out~~ inertia about Neutral-axis

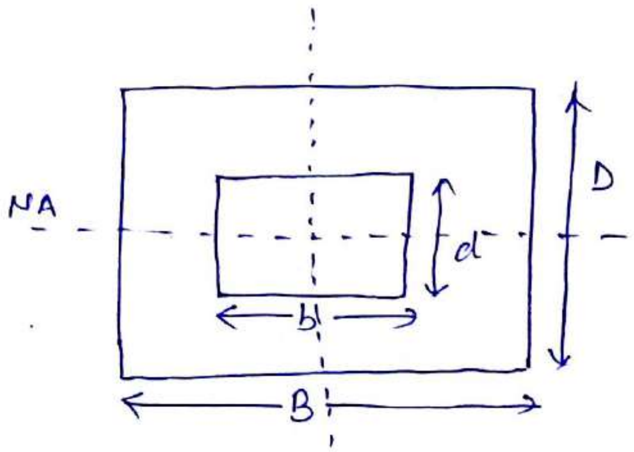
y_{max} = distance of most distant point of the section from the neutral axis

$$I = \frac{bd^3}{12} \qquad y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{bd^{\cancel{3}}/12}{\cancel{d}/2} = \frac{bd^2}{6}$$

Permissible bending stress, $\sigma = \frac{M}{Z} = \frac{M}{bd^2/6} = \frac{6M}{bd^2}$ Ans

(2) Hollow Rectangular Section:-



$$I_{NA} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}}$$

$$Z = \frac{\frac{1}{12}(BD^3 - bd^3)}{D/2}$$

$$Z = \frac{BD^3 - bd^3}{6D}$$

Permissible bending stress $\sigma = \frac{M}{Z} = \frac{M \times 6D}{(BD^3 - bd^3)}$ Ans

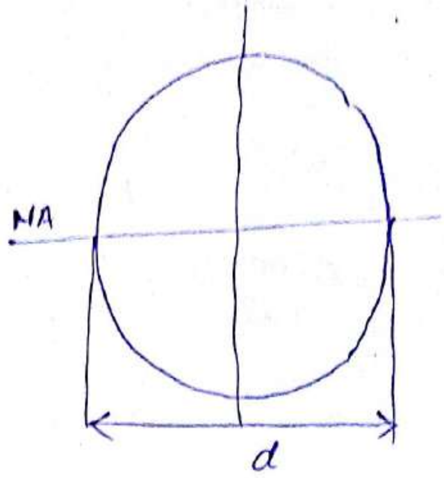
(3) Circular Section:-

$$I_{NA} = \frac{\pi d^4}{64}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\pi d^4/64}{d/2}$$

$$Z = \frac{\pi d^3}{32}$$



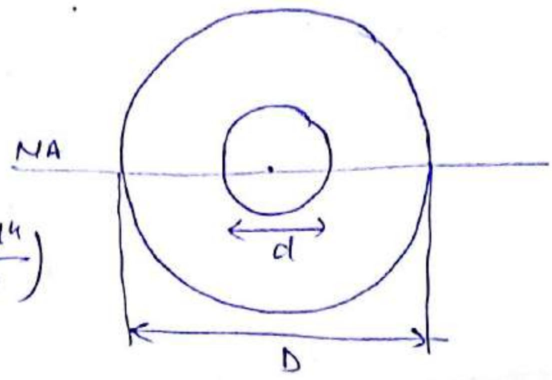
Permissible Bending Stress, $\sigma_b = \frac{M}{Z} = \frac{32M}{\pi d^3}$ Ans

(4) Hollow Circular Section:-

$$I_{NA} = \frac{\pi}{64}(D^4 - d^4)$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}(D^4 - d^4)}{D/2} = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$$



Ques:- A 250mm (depth) x 150mm (width) rectangular beam is subjected to maximum bending moment of 750KN.m. Determine the maximum stress in the beam.

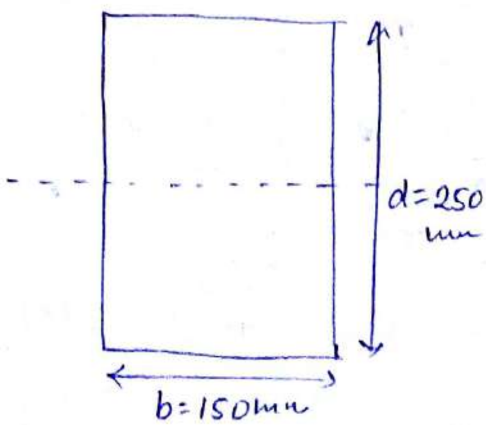
(ii) For E = 200 GN/m² Find out the radius of curvature for that portion of beam where the bending is maximum.

Soln:- Given. $b = 150mm$

$d = 250 mm$

$M = 750 KN.m$

$E = 200 GN/m^2$



$$I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12} \times 10^{-12}$$

$$= 195.31 \times 10^{-6} m^4$$

$$y_{max} = \frac{d}{2} = \frac{250}{2} = 125 mm = 0.125 m$$

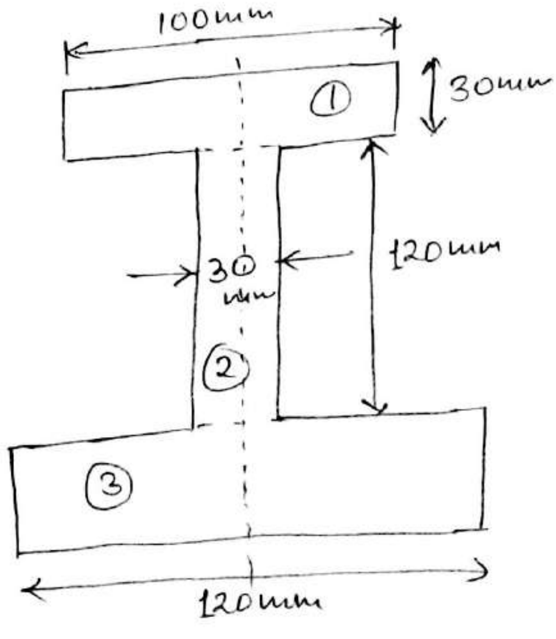
$$\sigma_b = \frac{My}{I} = \frac{750 \times 0.125}{195.33 \times 10^{-6}} = 479.95 \times 10^3 \ KN/m^2$$

$$\sigma_b = 479.95 \ MN/m^2 \ \text{Ans}$$

(ii)
$$\frac{M}{I} = \frac{E}{R}$$

$$R = \frac{EI}{M} = \frac{200 \times 10^9 \times 195.33 \times 10^{-6}}{750 \times 10^3} = 52.08 \ m \ \text{Ans}$$

Ques:- A beam simply supported at ends and having cross-section as shown in figure is loaded with udl over whole of its span. If beam is 8m long, find the UDL, if max^m permissible bending stress in tension is limited to 30 MN/m2 and in compression to 45 MN/m2. What are the actual maximum bending stresses set up in the section.



Sol^n:- Given- $l = 8m$

$(\sigma_b)_t = 30 MN/m^2$

$(\sigma_B)_c = 45 MN/m^2$

Rectangle (I) -

$A_1 = 100 \times 30 = 3000 \ mm^2$

$y_1 = 50 + 120 + 15 = 185 \ mm$

$A_2 = 120 \times 30 = 3600 \ mm^2$

$y_2 = 50 + 60 = 110 \ mm$

$A_3 = 120 \times 50 = 6000 \ mm^2$

$y_3 = \frac{50}{2} = 25 \ mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{3000 \times 185 + 3600 \times 110 + 6000 \times 25}{3000 + 3600 + 6000} = 87.38 \ mm$$

$$I_{xx} = \left[\frac{100\times30^3}{12} + 3000(185-87.30)^2\right] + \left[\frac{30\times120^3}{12} + 3600(110-87.30)^2\right]$$
$$+\left[\frac{120\times50^3}{12} + 6000(87.30-25)^2\right]$$

$$I_{xx} = 59573571.44 \text{ mm}^4$$
$$= 5957.35\times10^{-8} \text{ m}^4$$

$$M = \frac{wl^2}{8} = \frac{w\times8^2}{8} = 8w$$

Tension side-

$$\frac{M}{I} = \frac{\sigma_t}{y_t}$$

$$M = I\times\frac{\sigma_t}{y_t} = \frac{5957\times10^{-8}\times30\times10^6}{(200-87.30)\times10^{-3}}$$

$$= 15860.40 \text{ N}$$
$$M_t = 15.860 \text{ KN}.$$

Compression Side-

$$M = I\cdot\frac{\sigma_c}{y_c} = \frac{5957\times10^{-8}\times45\times10^6}{87.30\times10^{-3}}$$

$$= 30670.07 \text{ N}.$$
$$M_c = 30.670 \text{ KN}.$$

Take minimum value of moment of Resistance-

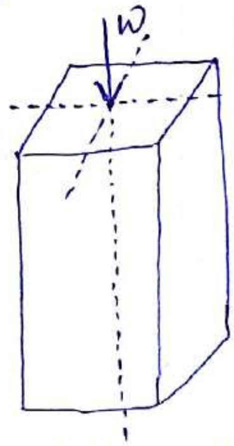
$$M = 15.860 \text{ KN}.$$

Actual Max^m stress in the top most fibre of beam-

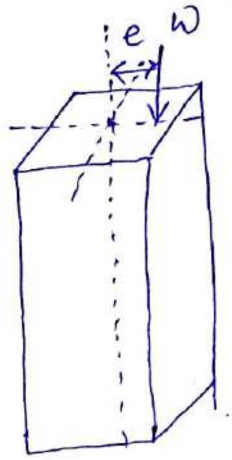
$$\sigma_{actual} = \frac{M}{I}\times y_e$$
$$= \frac{15.860}{5957\times10^{-8}}\times112.62\times10^{-3}$$
$$= 29.99\times10^3 \text{ KN/m}^2$$
$$= 30 \text{ MN/m}^2$$ Ans, (~~Tensile~~) compressive-

Combined Direct and Bending Stresses.

(1) Concentric Load:- If a load acts along the axis of symmetry of a cross section, then the load is known as concentric load. In case of concentric loading the member is subjected to only direct stresses.



Concentric Load



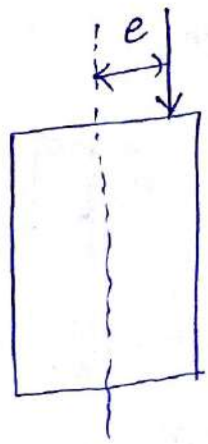
Eccentric Load.

(2) Eccentric Load:- If a load acts on a cross section along a line which is parallel to the axis of symmetry (geometrical axis) at a distance 'e' (eccentricity) from the centroid of the section. Then the load is known as eccentric load.

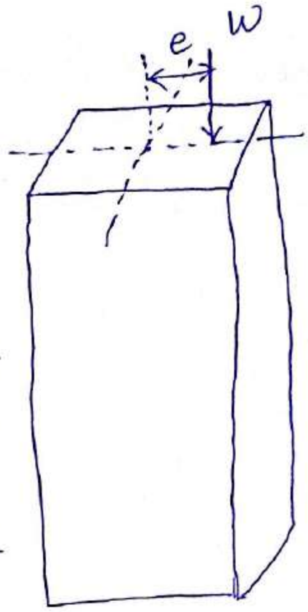
In case of eccentric loading, the member is subjected to both direct stresses and bending stresses.

Eccentricity:- In case of eccentric loading.

Eccentricity is the distance b/w the line of action of eccentric load and the line of axis of symmetry (geometric axis) passing threw the centroid of the cross section.



Effect of eccentric load on the section:-



consider a short column subjected to a direct load W, the line of action of which is parallel to the axis of the column and intersects an axis of symmetry (ie geometric axis) at a distance 'e' (eccentricity) from the centroid of section.

There will be two induced stresses.

① Direct stress
② Bending stress.

$\sigma_d = \frac{W}{A}$ → compressive

$\sigma_b = \frac{My}{I} = \frac{(W.e)y}{I}$

Bending stress is of tensile nature if y is measured to the left of the N.A and is of compressive nature if y is measured to the right of the N.A..

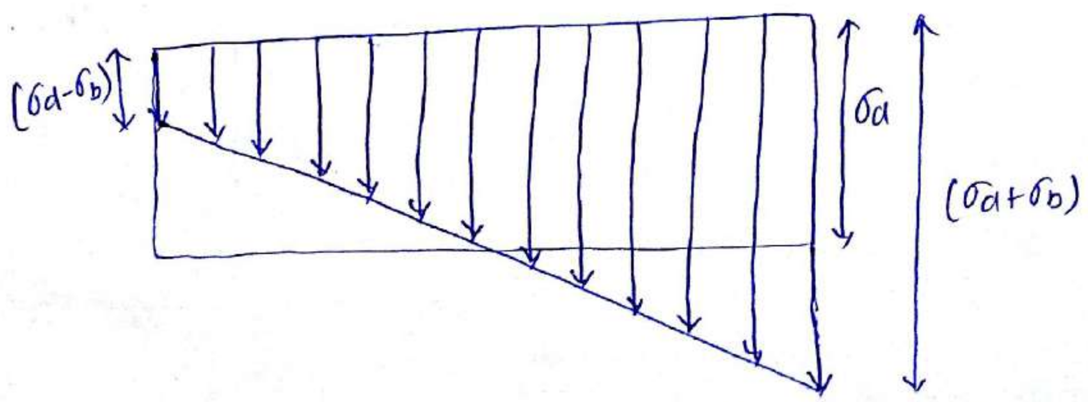
Resultant stress-

$\sigma_{max} = \sigma_d + \sigma_b = \frac{W}{A} + \frac{(W.e)y}{I}$ (when σ_b is compressive)

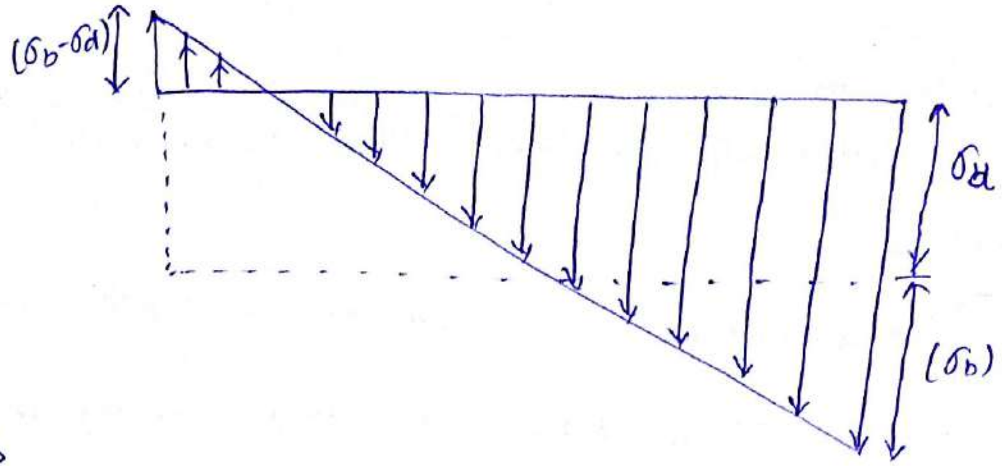
$\sigma_{min} = \sigma_d - \sigma_b = \frac{W}{A} - \frac{(W.e)y}{I}$ (when σ_b is tensile)

There can be three possibilities of stress distribution across the section-

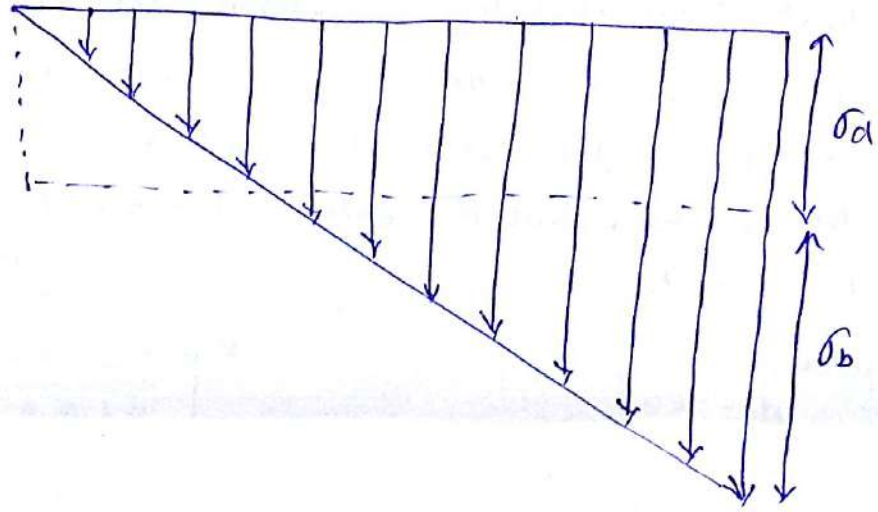
(i) $\sigma_d > \sigma_b$



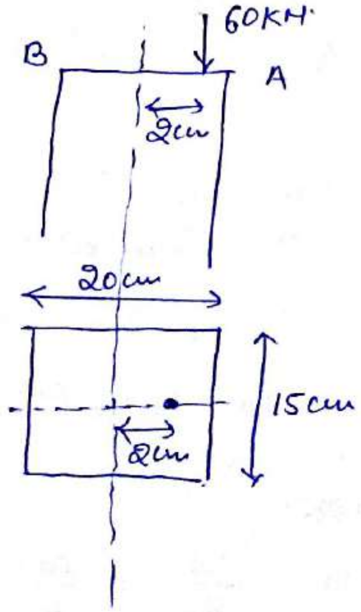
(ii) $\sigma_d < \sigma_b$



(iii) $\sigma_d = \sigma_b$



Ques:- A rectangular strut is 20cm wide and 15 cm thick. It carries a load of 60 kN at an eccentricity of 2 cm in a plane bisecting the thickness. Find the maximum and minimum intensities of stress in the section.



Sol^n:-
$b = 20cm$
$d = 15cm$
$e = 2cm$
$W = 60kN.$

direct stress-

$$\sigma_d = \frac{W}{A} = \frac{60}{0.2 \times 0.15}$$

$$\sigma_d = 2000 \ kN/m^2$$
$$= 2 \ MN/m^2$$

Bending Stress.

$$\sigma_b = \frac{My}{I} = \frac{(We)y}{I}$$

(4)

$$\sigma_b = \frac{60 \times 0.02 \times 0.10}{\frac{db^3}{12}}$$

$$\sigma_b = \frac{60 \times 0.02 \times 0.10}{0.15 \times 0.2^3} \times 12 = 1200 \ KN/m^2 = 1.2 \ MN/m^2$$

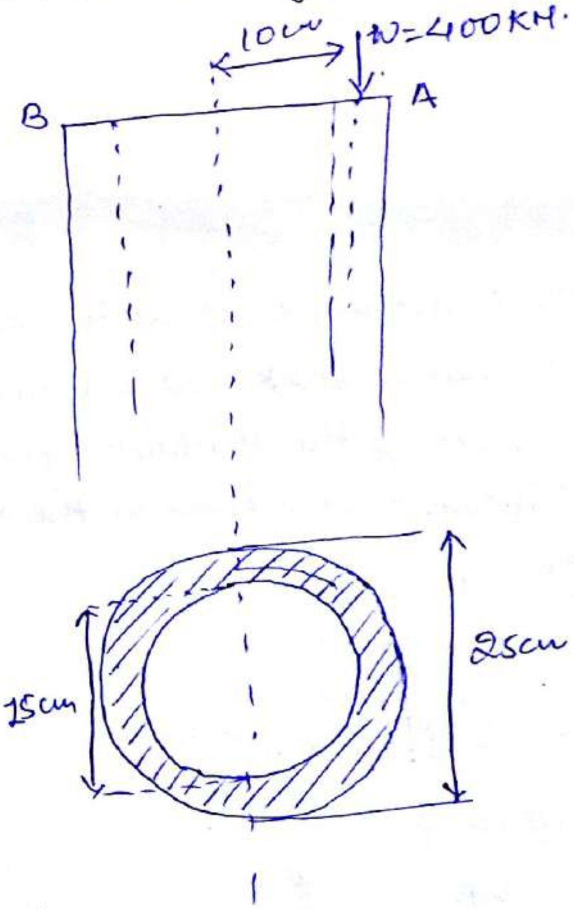
Maximum stress.—

$$\sigma_{max}\big|_A = \sigma_d + \sigma_b = 2 + 1.2 = 3.2 \ MN/m^2 \ (compressive)$$

Minimum stress.

$$\sigma_{min}\big|_B = \sigma_d - \sigma_b = 2 - 1.2 = 0.8 \ MN/m^2 \ (compressive) \quad Ans$$

Ques:- A short column of hollow cylindrical section 25 cm outside diameter and 15 cm inside diameter carries a vertical load of 400 KN along one of the diameter planes 10 cm away from the axis of the column. Find the extreme intensities of stresses and state their nature



Sol^n:- Given-

$d_o = 25 cm$

$d_i = 15 cm$

$e = 10 cm$

$W = 400 KN.$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(25^2 - 15^2)$$

$$A = 314.2 \ cm^2$$

$$A = 314.2 \times 10^{-4} \ m^2$$

Direct stress.

$$\sigma_d = \frac{W}{A} = \frac{400}{314.2 \times 10^{-4}}$$

$$\sigma_d = 12.73 \times 10^3 \ KN/m^2$$

$$\sigma_d = 12.73 \ MN/m^2$$

Bending stress-

$$\sigma_b = \frac{My}{I} = \frac{M}{Z}$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4). \qquad Z = \frac{I}{y} = \frac{\frac{\pi}{64}(25^4 - 15^4)}{25/2} = 1335 \ cm^3$$

$$= 1335 \times 10^{-6} \ m^3$$

$$\sigma_b = \frac{M}{y} = \frac{4000 \times 0.10}{1335 \times 10^{-6}} = 29.96 \text{ MN/m}^2$$

Maximum Stress. $\sigma_{max} = \sigma_d + \sigma_b$

$= 12.73 + 29.96 = 42.69$ MN/m² (Compressive)

Minimum stress, $\sigma_{min} = \sigma_d - \sigma_b$

$= 12.73 - 29.96$

$= -17.23$ MN/m²

$= 17.23$ MN/m² (Tensile) Ans;

Condition for no tension in the section:-

$\sigma_d \geq \sigma_b \geq \frac{M}{Z}$ (Limited)

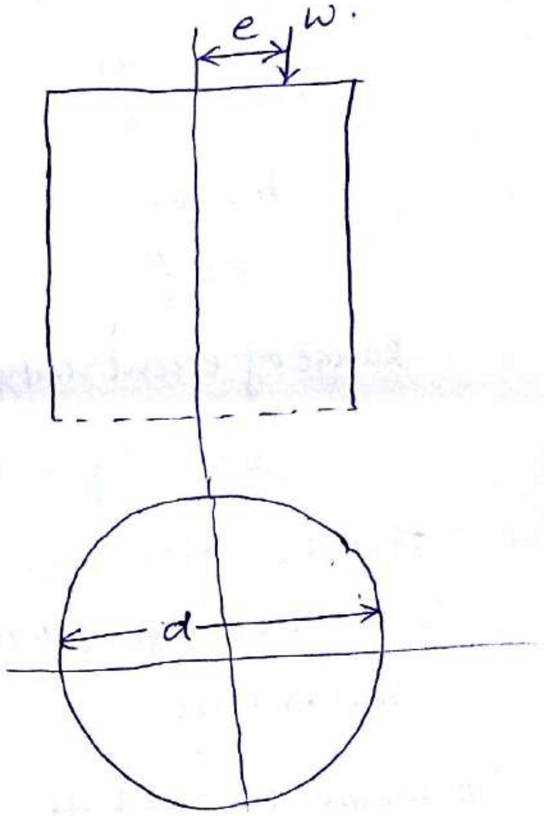
$\sigma_d \geq \sigma_b$.

$$\frac{W}{A} \geq \frac{My}{I}$$

$$\frac{\cancel{W}}{\cancel{A}} \geq \frac{\cancel{W} \cdot e \times d/2}{\cancel{A} \cdot k^2}$$

$$\frac{2K^2}{d} \geq e$$

$$\boxed{e \leq \frac{2K^2}{d}}$$



Since massonery is not capable of taking tension, so we have to ensure that no face of masonry structure develops tension to avoid failure due to cracking. These limits the eccentricity (e) to a certain value, So that no reverse tension (stress) is developed.

middle Third Rule:- (Rectangular Section):-

$\sigma_d \geq \sigma_b$.

$I_{yy} = \frac{db^3}{12}$ $A = bd$.

$y = \frac{b}{2}$

$\sigma_d \geq \sigma_b$.

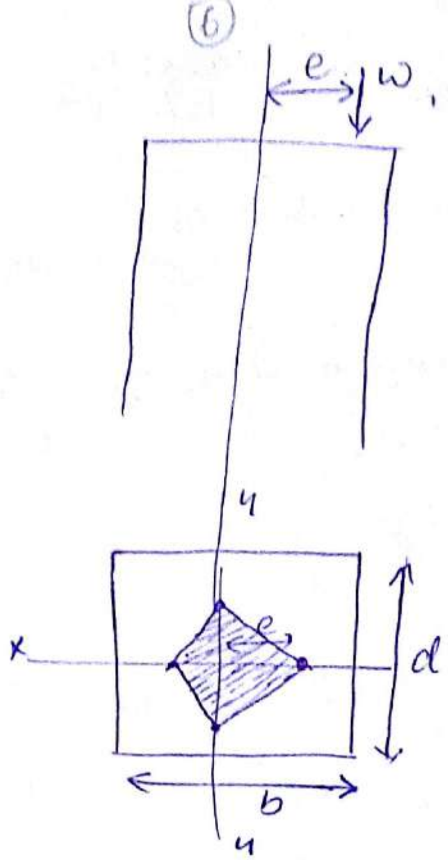
$\frac{W}{A} \geq \frac{My}{I}$

$\frac{W}{A} \geq \frac{W.e \times b/2}{db^3/12_6}$

$\frac{1}{bd} \geq \frac{6 \times e}{b^2 d}$

$b \geq 6e$

$e \leq \frac{b}{6}$



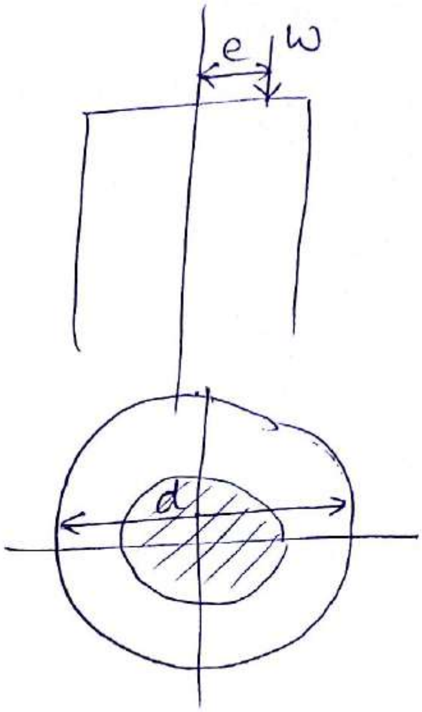
Range of eccentricity $e_x = \frac{b}{6} + \frac{b}{6} = \frac{b}{3}$

Similarly. $e_y = \frac{d}{3}$

Thus the stress will be of same sign throughout the section if the load lying within the middle third of the section. This is known as middle third rule.

~~Middle Fourth Rule~~:- If the four points of the middle third distance ~~of~~ on the section xx and yy are joined, a rhombus of diamond shape is obtained which is known as the core or Kernel of the section. If the load is placed anywhere inside the rhombus. The reverse stress will not occur in any part of the entire rectangular section.

Middle Fourth Rule (Circular Section):-



$$\sigma_d \geq \sigma_b$$

$$A = \frac{\pi}{4} d^2$$

$$I_{yy} = \frac{\pi}{64} d^4$$

$$y = d/2$$

$$\frac{\cancel{W}}{A} \geq \frac{\cancel{W} \cdot e \times y}{I_{yy}}$$

$$\frac{1}{A} \geq \frac{e \times d/2}{\frac{\pi}{64} d^4}$$

$$\frac{4}{\cancel{\pi} d^{\cancel{2}}} \geq \frac{{}^{16}\cancel{64}\, e \times \cancel{d}}{2 \times d^{\cancel{4}} \times \cancel{\pi}}$$

$$e \leq \frac{d}{8}$$

diameter of kernel $= 2e = 2 \times \frac{d}{8} = \frac{d}{4}$

Thus in order to avoid tensile stress in massonry, the load must fall within the middle fourth of the section. This is known as middle fourth rule.

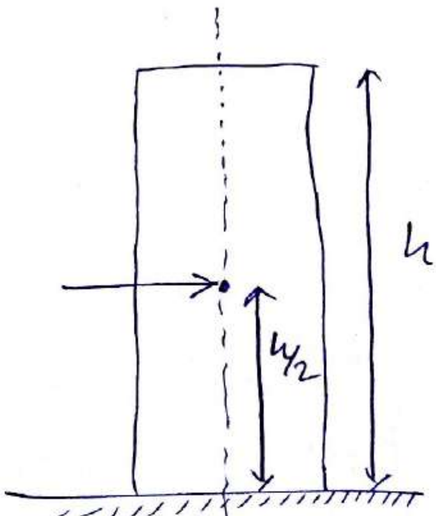
Chimney :- Wind Pressure on chimney.

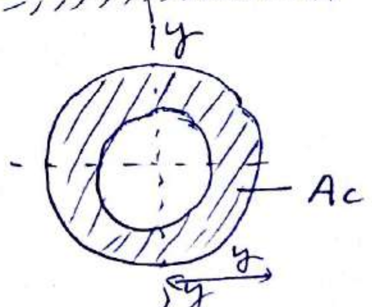
- $w \rightarrow$ specific weight of chimney.
- $A_c \rightarrow$ Area of cross section.

Volume, $V = A_c \times h$

Weight of chimney. $W = w \times \text{Volume}$

$W = w \times A_c \times h$





Area for wind Pressure -

$A = K_p \times A_p$.

A_p = Projected area.

$K_p = \frac{2}{3} \rightarrow$ circular cross section

$K_p = 1 \rightarrow$ rectangular x-section

Pressure force, F_p = Pressure × Area

$$F_p = P \times K_p \times A_p$$

This pressure force will act at $h/2$ from base.

$$\text{Moment}, M = F_p \times h/2$$

$$I = I_{yy}.$$

direct stress, $\sigma_d = \frac{W}{A_c}$

bending stress, $\sigma_b = \frac{My}{I}$

Que:- A masonry chimney 24 m high of uniform circular section 3·5 metre external diameter and 2 metres internal diameter is subjected to a horizontal wind pressure of 1 KN/m2 of projected area. Find the maximum and minimum stress intensities at the base, if the specific weight of masonry is 22 KN/m3.

Solⁿ:- Given. $d_o = 3.5\,m$

$d_i = 2\,m$

$h = 24\,m$

$w = 22\,KN/m^3$

$$A_c = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(3.5^2 - 2^2)$$

$$A_c = 6.48\,m^2$$

wind pressure, $P = 1\,KN/m^2$

Projected area = $A_p = d_o \times h = 3.5 \times 24 = 84\,m^2$

load, $W = w \times V = 22 \times A_c \times h$

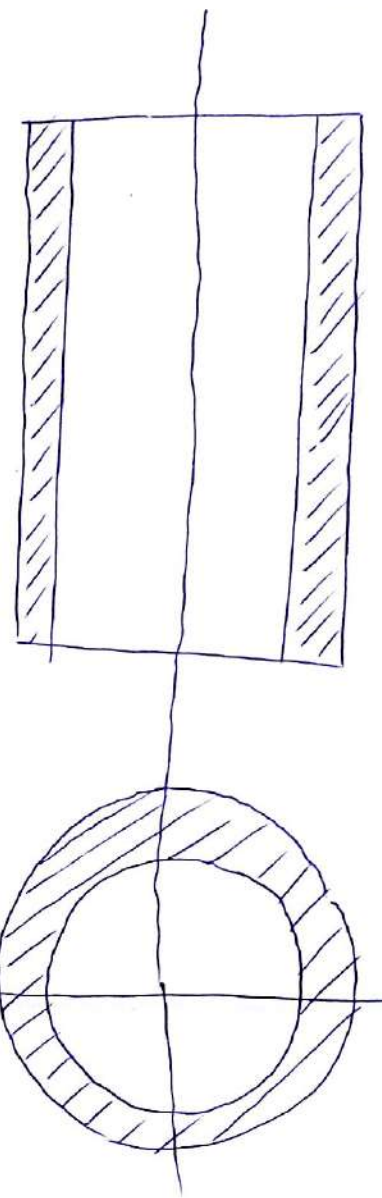
$= 22 \times 6.48 \times 24$

$W = 3421.44\,KN.$

Direct Stress, $\sigma_d = \frac{W}{A_c}$

$$\sigma_d = \frac{3421.44}{6.48}$$

$$\sigma_d = 528\ KN/m^2$$



Moment of Inertia -

$$I_{yy} = \frac{\pi}{64}(d_o^4 - d_i^4)$$

$$= \frac{\pi}{64}(3.5^4 - 2^4)$$

$$I_{yy} = 6.58\ m^4$$

Given, pressure -

$$P = 1\ KN/m^2$$

Horizontal Pressure force.

$$F_P = K_P \times A_P \times Pressure$$

$$= \frac{2}{3} \times 24 \times 3.5 \times 1$$

$$F_P = 56\ KN.$$

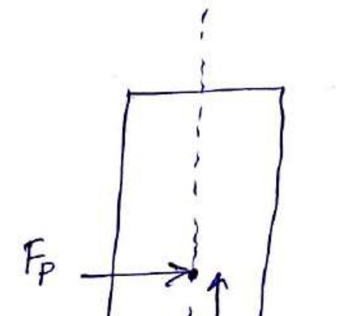
[$K_P = \frac{2}{3}$ = for cylindrical chimney]

Bending moment -

$$M = F_P \times \frac{h}{2}$$

$$M = 56 \times \frac{24}{2}$$

$$M = 672\ KN.m$$



Pressure force acts at $h/2$ distance from base.

Earth

Bending Stress -

$$\sigma_b = \frac{My}{I}$$

$$y = \frac{d_o}{2} = \frac{3.5}{2} = 1.75\,m$$

$$\sigma_b = \frac{672 \times 1.75}{6.58} = 178.7\ KN/m^2$$

direct Stress -

$$\sigma_d = \frac{W}{A_c}$$

$$\sigma_d = \frac{3421.44}{6.48} = 528\ KN/m^2 \text{ (compressive)}$$

$$\sigma_{max} = \sigma_d + \sigma_b$$
$$= 528 + 178.7$$
$$= 706.7\ KN/m^2 \text{ (compressive)}$$

$$\sigma_{min} = \sigma_d - \sigma_b$$
$$= 528 - 178.7$$
$$= 349.3\ KN/m^2 \text{ (compressive)}.$$

Ans;

Earth Pressure on retaining walls (Dams):-

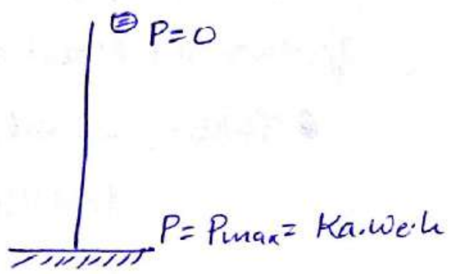
In case of dams ~~earth~~ pressure (intensity of pressure) varies linearly from zero at the top to $Ka.We.h$ at bottom.

Ka = coefficient of active pressure

We = Density of earth

h = height of earth retained.

→ $Ka = \frac{1-\sin\phi}{1+\sin\phi}$



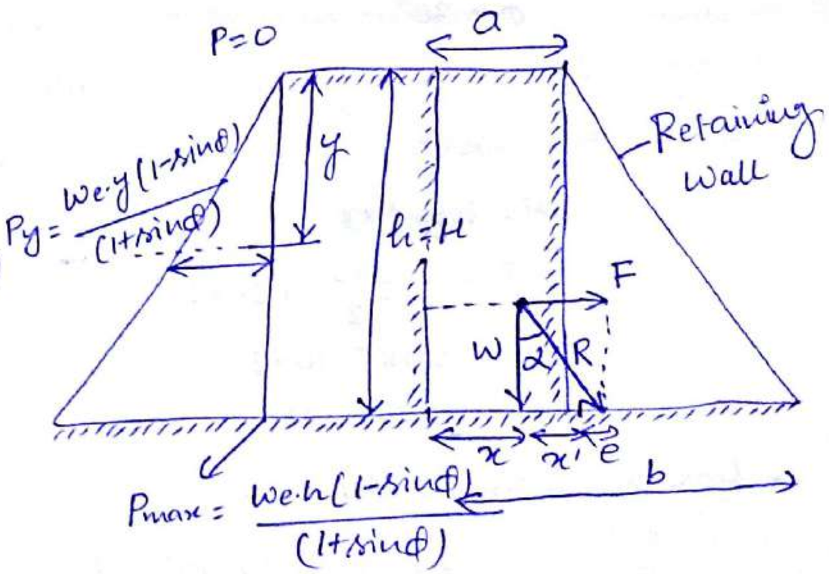
ϕ = angle of repose for earth.

→ For 1 m length of retaining wall-

Horizontal Thrust Force = $F = \frac{We.h^2}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$ ⟶ Rankine Formula

→ Max^m earth pressure occurs at the bottom of the retaining wall and is equal to. $We.h\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$

→ The C.G of the earth pressure diagram (triangle) lies at $h/3$ from the bottom and so the total horizontal or lateral pressure acts at this point.



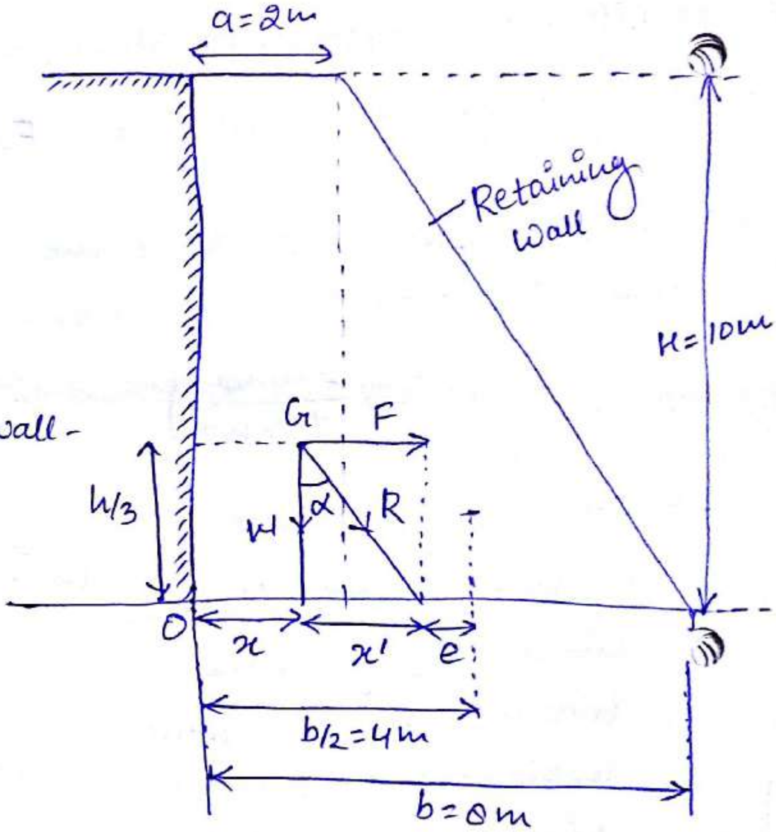
→ Average Pressure on the retaining wall.

$$P_{av} = \frac{We.h}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

Ques:- A masonry retaining wall of trapezoidal section is 10m high and retains earth which is level upto the top. The width at the top is 2m and at the bottom is 8m and the exposed face is vertical. Find the maximum and minimum intensities of normal stress at the base.

- Take- density of earth = 16 KN/m2
  density of masonry = 24 KN/m2
  angle of repose of earth = 30°

Solⁿ:- Given-



$a = 2m$

$b = 8m$

$H = 10m$

$w_e = 16\ KN/m^2$

$w_m = 24\ KN/m^2$

$\phi = 30°$

→ Consider 1m length of retaining wall-

Weight of masonry —

$$W = w_m \times A \times l.$$

$$= 24 \times \left(\frac{8+2}{2}\right) \times 10 \times 1$$

$$= 24 \times 5 \times 10 \times 1$$

$$W = 1200 KN.$$

→ Horizontal Thrust force. —

$$F = \frac{w_e \cdot H^2}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

$$F = \frac{16 \times 10^2}{2}\left(\frac{1-\sin 30°}{1+\sin 30°}\right) = 266.67 KN.$$

let x - be the distance of C.G from vertical face -

Rectangle. $A_1 = 2 \times 10 = 20 m^2$ $\quad x_1 = \frac{2}{2} = 1m$

Triangle - $A_2 = \frac{1}{2} \times 6 \times 10 = 30 m^2$ $\quad x_2 = 2 + \frac{6}{3} = 2 + 2 = 4m$

$$x = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$x = \frac{20\times1 + 30\times4}{20+30}$$

$$x = \frac{20+120}{50} = \frac{140}{50} = \frac{140}{50} = 2.8\,m$$

$$x = 2.8\,m$$

$$\tan\alpha = \frac{F}{W} = \frac{266.67}{1200}$$

$$\tan\alpha = \frac{x'}{h/3}$$

$$\frac{x'}{10/3} = \frac{266.67}{1200}$$

$$x' = \frac{266.67\times10}{1200\times3} = 0.74\,m$$

$$x + x' = 2.8 + 0.74 = 3.54\,m$$

Eccentricity, $e = 4 - 3.54 = 0.46\,m$

Bending Moment due to eccentricity.

$$M = W.e = 1200\times0.46 = 552\ KN.m$$

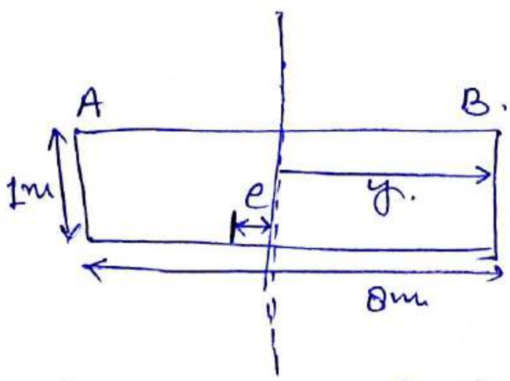
→ Bending stress-

$$\sigma_b = \frac{My}{I}$$

$$y = \frac{8}{2} = 4\,m$$

$$I = \frac{1\times8^3}{12} = \frac{512}{12}$$

$$\sigma_b = \frac{552\times4\times12}{512} = 51.75\ KN/m^2$$



Top view

direct stress, $\sigma_d = \frac{W}{A} = \frac{1200}{8\times1} = 150\ KN/m^2$

$\sigma_{max} = \sigma_d + \sigma_b = 150 + 51.75 = 201.75\ KN/m^2$ (compressive)

$\sigma_{min} = \sigma_d - \sigma_b = 150 - 51.75 = 98.25\ KN/m^2$ (compressive).

Ans

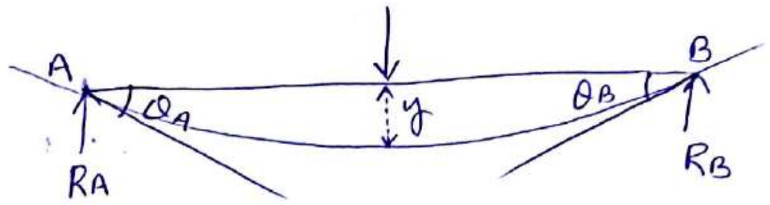
Unit-4

Slopes and Deflection

A beam is designed on the basis of two properties –

① Strength:- It is the property of a beam to resist shear force and bending moment.

② Stiffness:- It is the property of a beam material to resist deflection.



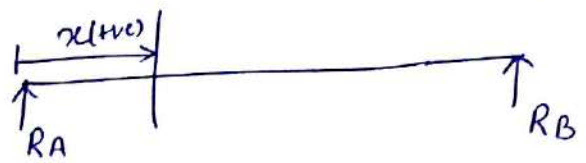
Deflection:- The deflection at a point in a beam can be defined as the vertical distance between the point on the undeflected axis and the corresponding point on the deflected axis.

or Deflection of a beam is the lateral displacement from the initial horizontal axis after the application of load.

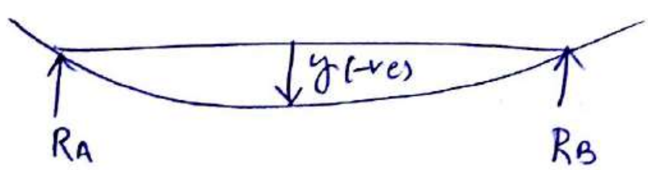
Slope:- The slope of a beam axis at a point is defined as the angle between the tangent to the elastic curve at that point and the undeflected beam axis.

Sign Conventions:-

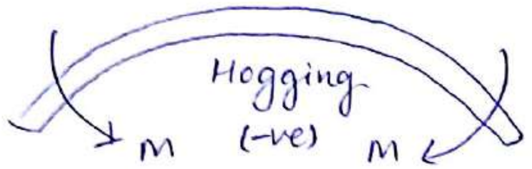
① x is positive when measured towards right.



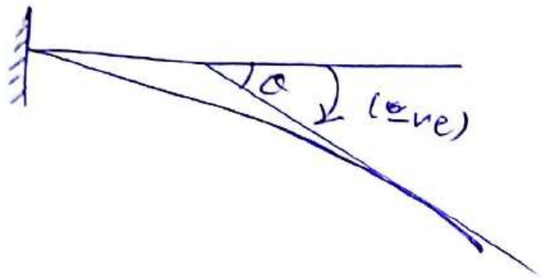
② y is negative when measured downwards.



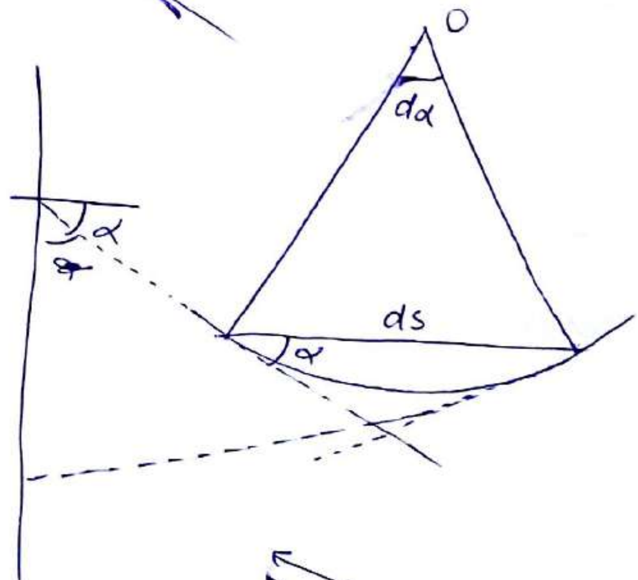
③ Bending Moment is negative when hogging.



④ Slope is negative when rotation is clockwise.



Circular Bending:-



Arc = Angle × Radius

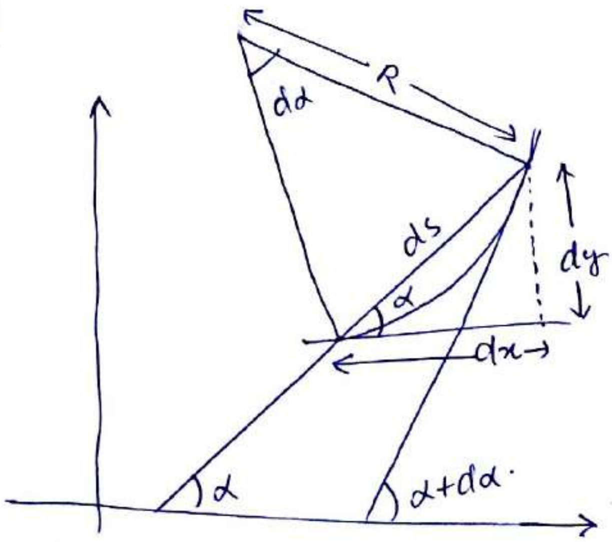
$ds = d\alpha \cdot R$

$ds = R \cdot d\alpha$

$ds \simeq dx$

$dx = R \cdot d\alpha$.

$$\frac{d\alpha}{dx} = \frac{1}{R} \quad \text{——— ①}$$



$$\tan\alpha = \frac{dy}{dx}$$

since α is very small.

$\tan\alpha \simeq \alpha$.

$$\alpha = \frac{dy}{dx} \quad \text{——— ②}$$

from eq^n ① & ②.

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{R}$$

$$\frac{d^2y}{dx^2} = \frac{1}{R} \quad \text{——— ③}$$

From bending equation.

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R}$$

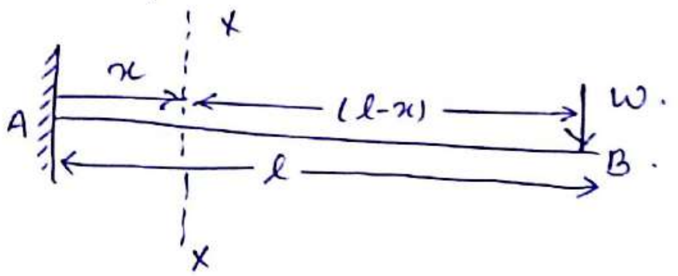
From eq^n (3).

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\boxed{M = EI \cdot \frac{d^2y}{dx^2}}$$

Find out the slope and deflection :-

(1) Cantilever beam with ~~con~~ point load at free end :-



$$\text{Moment} = -W(l-x)$$

$$M = EI \cdot \frac{d^2y}{dx^2}$$

$$EI \cdot \frac{d^2y}{dx^2} = -W(l-x)$$

Integrating w.r. to x -

$$EI \cdot \frac{dy}{dx} = -W\left(lx - \frac{x^2}{2}\right) + C_1 \quad \text{—— (1)}$$

at $x = 0$, $\frac{dy}{dx} = 0$

$$C_1 = 0$$

$$EI \cdot \frac{dy}{dx} = -W\left(lx - \frac{x^2}{2}\right) \quad \text{—— (2)}$$

Again Integrating eq^n (2).

$$EI \cdot y = -W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$$

at $x = 0$, $y = 0$ $\quad C_2 = 0$

$$EI \cdot y = -W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \quad \text{—— (3)}$$

Slope and deflection at free end.

at point B. $x = l$.

$$EI\left(\frac{dy}{dx}\right)_B = -W\left(l^2 - \frac{l^2}{2}\right)$$

Slope.
$$\boxed{\theta_B = \frac{-Wl^2}{2EI}}$$

Deflection-

$$EI \cdot y_B = -W\left(\frac{l^3}{2} - \frac{l^3}{6}\right)$$

$$= -W\left(\frac{3l^3 - l^3}{6}\right)$$

$$EI \cdot y_B = \frac{-Wl^3}{3}$$

$$\boxed{y_B = \frac{-Wl^3}{3EI}}$$

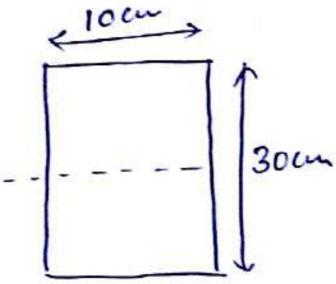
Ques:- Given.

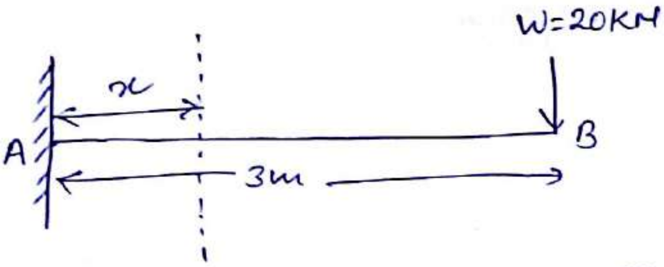
$W = 20KN$.

$E = 210 GN/m^2$

$\theta_B = ?$

$y_B = ?$





$$M = -W(l-x)$$

$$M = -20(3-x)$$

$$I = \frac{0.1 \times 0.3^3}{12} = 2.25\times10^{-4}\, m^4$$

$$EI \cdot \frac{d^2y}{dx^2} = -20(3-x)$$

Integrating w.r.to x-

$$EI \cdot \frac{dy}{dx} = -20\left(3x - \frac{x^2}{2}\right) + C_1$$

at $x = 0$, $\frac{dy}{dx} = 0$, $C_1 = 0$

$$EI \cdot \frac{dy}{dx} = -20\left(3x - \frac{x^2}{2}\right) \quad \text{——— (1)}$$

at point B- $x = 3$

at Point B $x = l = 3m$

$$EI\left(\frac{dy}{dx}\right)_B = -20\left(3\times3 - \frac{3^2}{2}\right)$$

$$\left(\frac{dy}{dx}\right)_B = \frac{-20\times 9/2}{EI} = \frac{-20\times9}{2\times210\times10^6\times2.25\times10^{-4}}$$

$$\theta_B = -1.9\times10^{-3} \text{ radian Ans;}$$

Again Integrating eqn ①.

$$EI.y = -20\left[\frac{3x^2}{2} - \frac{x^3}{6}\right] + C_2$$

at $x=0$, $y=0$, $C_2 = 0$

$$EI.y = -20\left[\frac{3x^2}{2} - \frac{x^3}{6}\right] \quad \text{—} (2)$$

at point B, $x = l = 3m$

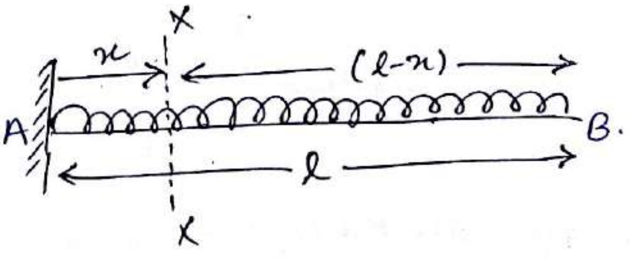
$$EI.y_B = -20\left[\frac{3\times3^2}{2} - \frac{3^3}{6}\right]$$

$$EI.y_B = -20\times\frac{3^{\cancel{3}}}{\cancel{3}}$$

$$y_B = \frac{-20\times9}{210\times10^6\times2.25\times10^{-4}} = -3.8\times10^{-3} m$$

$$= -3.8 \text{ mm Ans;}$$

Case-II :- cantilever with UDL over entire span:-



$$M = -w\frac{(l-x)^2}{2}$$

$$EI\frac{d^2y}{dx^2} = -w\frac{(l-x)^2}{2}$$

Integrating w.r.to x-

$$EI.\frac{dy}{dx} = \frac{+w}{3\times2}(l-x)^3 + C_1 \quad \text{—} (1)$$

at $x=0$, $\frac{dy}{dx} = 0$

$$C_1 = \frac{-wl^3}{6}$$

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From equation (1).

$$EI.\frac{dy}{dx} = \frac{w}{6}(l-x)^3 - \frac{wl^3}{6} \quad \text{——} (2)$$

At point B. $x = l$.

$$EI.\left(\frac{dy}{dx}\right)_B = 0 - \frac{wl^3}{6}$$

$$\boxed{\theta_B = -\frac{wl^3}{6EI}}$$

Again integrating eqn (2).

$$EI.y = -\frac{w}{4\times 6}(l-x)^4 - \frac{wl^3}{6}x + c_2 \quad \text{——} (3)$$

at $x=0$, $y=0$

$$0 = -\frac{wl^4}{24} - 0 + c_2$$

$$c_2 = \frac{wl^4}{24}$$

from equation (3).

$$EI.y = -\frac{w}{24}(l-x)^4 - \frac{wl^3}{6}x + \frac{wl^4}{24} \quad \text{——} (4)$$

at point B. $x = l$.

$$EI.y_B = 0 - \frac{wl^4}{6} + \frac{wl^4}{24}$$

$$EI.y_B = -\frac{wl^4}{8}$$

$$\boxed{y_B = -\frac{wl^4}{8EI}}$$

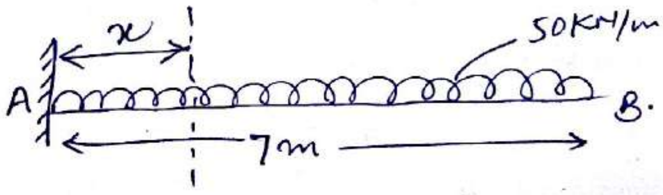
Ques:- Calculate the deflection and slope at free end of a steel cantilever 7m long when it carries uniformly distributed load of 50 KN/m.

Take $E = 2\times10^8$ KN/m^2 and $I = 0.0009$ m^4 $= 9\times10^{-4}$ m^4

Soln:- $l = 7m$

$w = 50$ KN/m

$\theta_B = ?$ $y_B = ?$



$$M_x = -\frac{w(l-x)^2}{2}$$

$$M_x = -\frac{50}{2}(7-x)^2 = -25(7-x)^2$$

$$EI.\frac{d^2y}{dx^2} = -25(7-x)^2$$

Integrating w.r.to x -

$$EI.\frac{dy}{dx} = +25\frac{(7-x)^3}{3} + C_1 \quad \text{——} (1)$$

at $x=0$, $\frac{dy}{dx}=0$

$$0 = \frac{25\times 7^3}{3} + C_1$$

$$C_1 = -\frac{25\times 7^3}{3} = -2858.33$$

from eq^n (1).

$$EI.\frac{dy}{dx} = \frac{25}{3}(7-x)^3 - 2858.33 \quad \text{——} (2)$$

at point B, $x = 7$

$$EI.\left(\frac{dy}{dx}\right)_B = 0 - 2858.33$$

$$\theta_B = \frac{-2858.33}{EI}$$

$$\theta_B = \frac{-2858.33}{2\times10^8 \times 9\times10^{-4}}$$

$$\theta_B = -158.79\times10^{-4} \text{ radian}$$

$$\theta_B = -0.015879 \text{ radian} \quad \text{Ans;}$$

Again Integrating eq^n (2) -

$$EI.\cancel{\frac{dy}{dx}}\,y = -\frac{25}{3}\times\frac{(7-x)^4}{4} - 2858.33x + C_2 \quad \text{——} (3)$$

at $x=0$, $y=0$

$$0 = -\frac{25}{12}\times 7^4 - 0 + C_2$$

$$C_2 = \frac{25\times 7^4}{12} = 5002.08$$

gain

From eqⁿ (3).

$$EI \cdot y = \frac{-25}{12}(7-x)^4 - 2850.33x + 5002.08$$

at point B. $x = 7\,m$

$$EI \cdot y = 0 - 2850.33 \times 7 + 5002.08$$

$$y = -\frac{15006.23}{EI}$$

$$y = \frac{-15006.23}{2\times10^8 \times 9\times10^{-4}} = -0833.68\times10^{-4}\,m$$

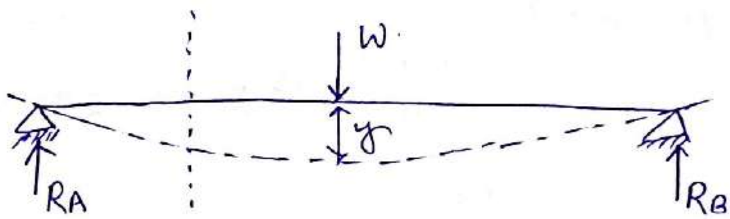
$$= -83.368\times10^{-3}\,m$$

$$= -83.36\,mm \text{ Ans,}$$

Case-III :- Simply supported beam of span l carrying a point load at mid span.

Solⁿ :-

$$R_A = R_B = \frac{W}{2}$$



$$M_x = \frac{W}{2}x$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{W}{2}x$$

Integrating w.r.to x -

$$EI \cdot \frac{dy}{dx} = \frac{Wx^2}{2\times2} + C_1 \quad \text{—— (1)}$$

at $x = l/2$, $\frac{dy}{dx} = 0$

$$0 = \frac{Wl^2}{2\times8} + C_1$$

$$C_1 = \frac{-Wl^2}{8\times2} = \frac{-Wl^2}{16}$$

from eqⁿ (1).

$$EI \cdot \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} \quad \text{—— (2)}$$

at point A, $x = 0$

$$EI \cdot \left(\frac{dy}{dx}\right)_A = 0 - \frac{Wl^2}{16}$$

$$\boxed{\theta_A = -\frac{Wl^2}{16EI}}$$

gain integrating equ (2).

$$EI \cdot y = \frac{wx^3}{12} - \frac{wl^2}{16}x + C_2 \quad \text{--- (3)}$$

at $x=0,\ y=0$

$$0 = 0 - 0 + C_2 \Rightarrow C_2 = 0$$

from equ (3).

$$EI \cdot y = \frac{wx^3}{12} - \frac{wl^2}{16}x \quad \text{--- (4)}$$

at mid point $x = l/2$

$$EI \cdot y = \frac{w(l/2)^3}{12} - \frac{wl^2}{16} \times \frac{l}{2}$$

$$EI \cdot y = \frac{wl^3}{96} - \frac{wl^3}{32}$$

$$EI \cdot y = \frac{(1-3)wl^3}{96}$$

$$EI \cdot y = \frac{-2}{96} wl^3$$

$$EI \cdot y = \frac{-wl^3}{48}$$

$$\boxed{y_{max} = \frac{-wl^3}{48EI}}$$

Ques:- A Girder of uniform section and constant depth is freely supported over a span of 3 metre. If the point load at mid span is 30 KN and $I_{xx} = 15.614 \times 10^{-6}\ m^4$. Calculate the central deflection and slopes at the ends of the beam. Take $E = 200\ GN/m^2$

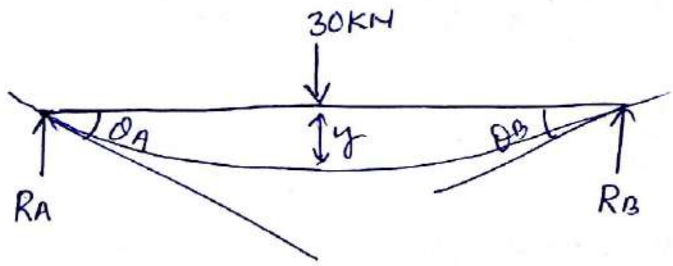
Solⁿ:-

Length, $l = 3m$

$W = 30KN$.

$I_{xx} = 15.614 \times 10^{-6}\ m^4$

$E = 200\ GN/m^2$



iii) central deflection.

$$y_{max} = -\frac{Wl^3}{48EI}$$

$$= -\frac{30\times3^3}{48\times200\times10^6\times15.614\times10^{-6}}$$

$$= -5.4\times10^{-3}\,m$$

$$= -5.4\,mm \quad Ans$$

(ii) Slopes at the ends of the beam.

$$\theta_A = -\frac{Wl^2}{16EI}$$

$$\theta_A = -\frac{30\times3^2}{16\times200\times10^6\times15.614\times10^{-6}}$$

$$\theta_A = -0.0054 \text{ radian}$$

$$= -0.0054\times\frac{180}{\pi}$$

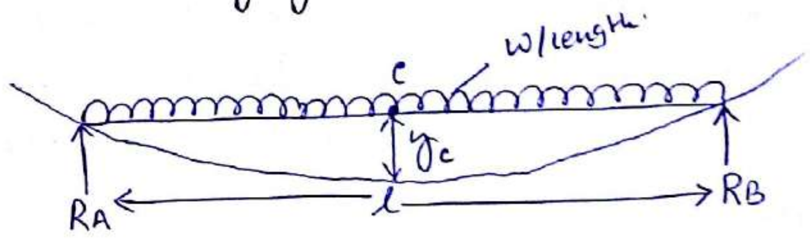
$$\theta_A = -0.309^\circ$$

$$\theta_B = -\theta_A = +0.0054 \text{ radian}$$

$$= +0.309^\circ \quad Ans.$$

Case(IV): Simply supported beam carrying UDL over the whole span:-

$$R_A = R_B = \frac{wl}{2}$$



$$M_x = R_A x - \frac{wx^2}{2}$$

$$M_x = \frac{wl}{2}x - \frac{wx^2}{2}$$

$$EI\frac{d^2y}{dx^2} = \frac{wl}{2}x - \frac{wx^2}{2}$$

Integrating w.r.to x.

$$EI\frac{dy}{dx} = \frac{wl}{2}\cdot\frac{x^2}{2} - \frac{wx^3}{6} + C_1 \quad \text{——— (7)}$$

at $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$

$$0 = \frac{wl}{2\times2}\times(l/2)^2 - \frac{w}{6}(l/2)^3 + C_1$$

$$0 = \frac{wl^3}{16} - \frac{wl^3}{48} + C_1$$

$$C_1 = -\frac{wl^3}{16} + \frac{wl^3}{48} = -\frac{wl^3}{24}$$

from eq^n (1) -

$$EI\cdot\frac{dy}{dx} = \frac{wl}{4}x^2 - \frac{wx^3}{6} - \frac{wl^3}{24} \quad \text{—— (2)}$$

again integrate -

$$EI\cdot y = \frac{wl}{4}\times\frac{x^3}{3} - \frac{w}{6}\times\frac{x^4}{4} - \frac{wl^3}{24}x + C_2 \quad \text{—— (3)}$$

at $x=0$, $y=0$

$$0 = 0 - 0 - 0 + C_2 \Rightarrow C_2 = 0$$

$$EI\cdot y = \frac{wl}{12}x^3 - \frac{wx^4}{24} - \frac{wl^3}{24}x \quad \text{—— (4)}$$

y is maximum at $x = l/2$

$$EI\cdot y_{max} = \frac{wl}{12}\times\frac{l^3}{8} - \frac{wl^4}{24\times16} - \frac{wl^3}{48}$$

$$EI\cdot y_{max} = \frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^3}{48}$$

$$EI\cdot y_{max} = \frac{(4-1-8)wl^4}{384}$$

$$\boxed{y_{max} = \frac{-5wl^4}{384EI}}$$

Slope is maximum at end points. at A, $x=0$

from eq^n (2).

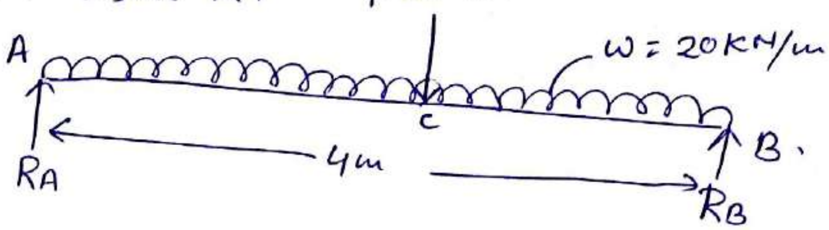
$$EI\left(\frac{dy}{dx}\right)_A = 0 - 0 - \frac{wl^3}{24}$$

$$\boxed{\theta_A = -\frac{wl^3}{24EI}}$$

Ques:- A simply supported beam of 4m span carries a UDL of 20KN/m on the whole span and in addition carries a point load of 40KN at the centre of span. Calculate the slope at the ends and maximum deflection of the beam. Take -

$E = 200 GN/m^2$ and $I = 5000 cm^4$

Solⁿ:- $I = 5000 cm^4 = 5000\times10^{-8} = 5\times10^{-5} m^4$



$R_A = R_B = \frac{40}{2} + \frac{20\times4}{2}$

$= 20 + 40$

$R_A = R_B = 60 KN.$

Slopes at ends -

$\theta_A = -\frac{Pl^2}{16EI} - \frac{wl^3}{24EI}$

$\theta_A = \frac{-40\times4^2}{16\times200\times10^6\times5\times10^{-5}} - \frac{20\times4^3}{24\times200\times10^6\times5\times10^{-5}}$

$\theta_A = -4\times10^{-3} - 2.67\times10^{-3}\times2$

$\theta_A = -\cancel{6.67\times10^{-3}}$ radian

$\theta_A = -4\times10^{-3} - 5.33\times10^{-3}$

$\theta_A = -9.33\times10^{-3}$ radian

$\theta_A = -0.00933$ radian Ans.

$\theta_B = +0.00933$ radian.

Maximum Deflection -

$y_{max} = \frac{-Pl^3}{48EI} - \frac{5wl^4}{384EI}$

$y_{max} = \frac{-40\times4^3}{48\times200\times10^6\times5\times10^{-5}} - \frac{5\times20\times4^4}{384\times200\times10^6\times5\times10^{-5}}$

$y_{max} = -5.33\times10^{-3} - 6.67\times10^{-3}$

$= -12\times10^{-3} m$

$= -12 mm$ Ans.

Unit-5

Columns and Struts:.

Strut:- A member of structure or bar which carries an axial compressive load is called the strut.

Column:- If the strut is vertical ie inclined at 90° to the horizontal is known as column or pillar.

A column is a long vertical slender bar or vertical member, subjected to an axial compressive load and fixed rigidly or both ends.

Strut:- A strut is a slender bar or ~~column~~ member in any position other than vertical, subjected to a ~~radius of gyration~~ compressive load and fixed rigidly or pin jointed at one or both the ends.

Slenderness Ratio (k):- It is the ratio of unsupported length of the column to the minimum radius of gyration of the cross sectional ends of the column. It has no unit.

$$\text{Slenderness Ratio}(k) = \frac{\text{unsupported length}}{\text{min. radius of gyration}}$$

Buckling Load:- The maximum limiting load at which the column tends to have lateral displacement or tends to buckle is called buckling load. The buckling takes place about the axis having minimum radius of gyration.

Safe Load:- It is the load to which a column is actually subjected to and is well below the buckling load. It is obtained by dividing the buckling load by a suitable factor of safety

$$\text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}}$$

Classification of columns:-

Depending upon the slenderness ratio or length to diameter ratio, column can be divided into three classes.

1. Short Column:-

Columns which have lengths less than 8 times their respective diameters or slenderness ratio less than 32 are called short columns. When short columns are subjected to compressive loads. Their buckling is generally negligible and as such the buckling stresses are very small as compared with direct compressive stresses.

Therefore it is assumed that short column are always subjected to direct compressive stresses only.

$\boxed{\frac{l}{d} < 8}$ $\boxed{K < 32}$

2. Long Columns:-

The columns having their lengths more than 30 times their respective diameter or slenderness ratio more than 120 are called long columns.

$\boxed{\frac{l}{d} > 30}$ $\boxed{K > 120}$

Long columns are usually subjected to buckling stress only. Direct compressive stress is very small as compared with buckling stress and hence it is neglected.

3. Medium size columns:-

The columns which have their lengths varying from 8 times their diameters to 30 times their respective diameters or their slenderness ratio lying between 32 and 120 are called medium sized columns or intermediate columns.

$\boxed{8 < \frac{l}{d} < 30}$ $\boxed{32 < K < 120}$

In medium sized columns both the buckling as well as direct stresses are of significant values. Therefore in the design of intermediate columns both these stresses are taken into consideration.

Equivalent Lengths:-

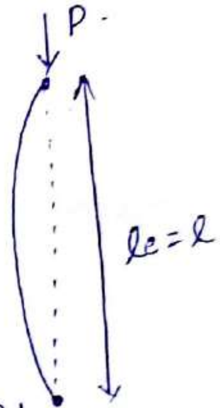
The distance b/w adjacent points of inflexion is called equivalent length or effective length of a column.

The effective length of a beam can vary according to the end conditions of a beam -

(i) Both ends hinged:-

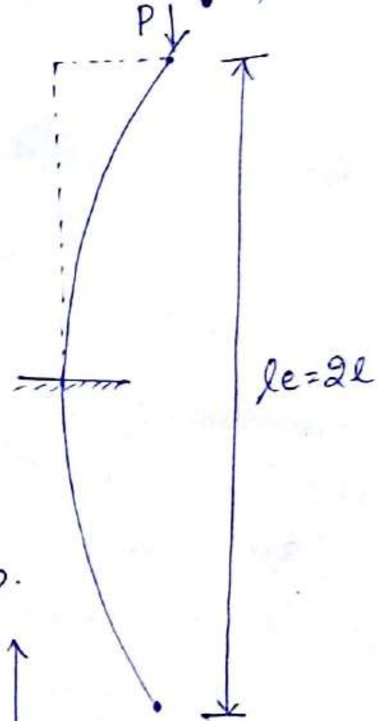
Equivalent length = actual length.

$$\boxed{l_e = l}$$



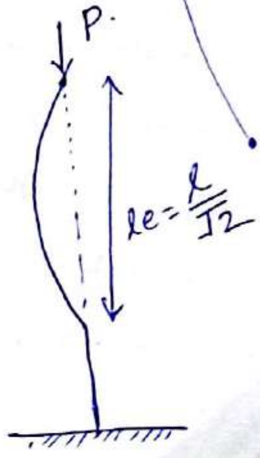
(ii) One end fixed and other end free:-

$$\boxed{l_e = 2l}$$



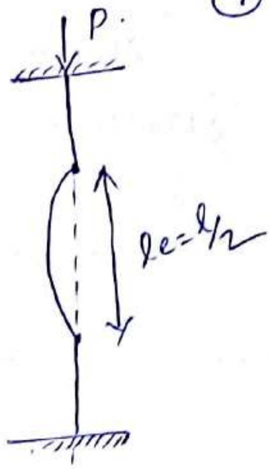
(iii) One end fixed and other end pin jointed:-

$$\boxed{l_e = \frac{l}{\sqrt{2}}}$$



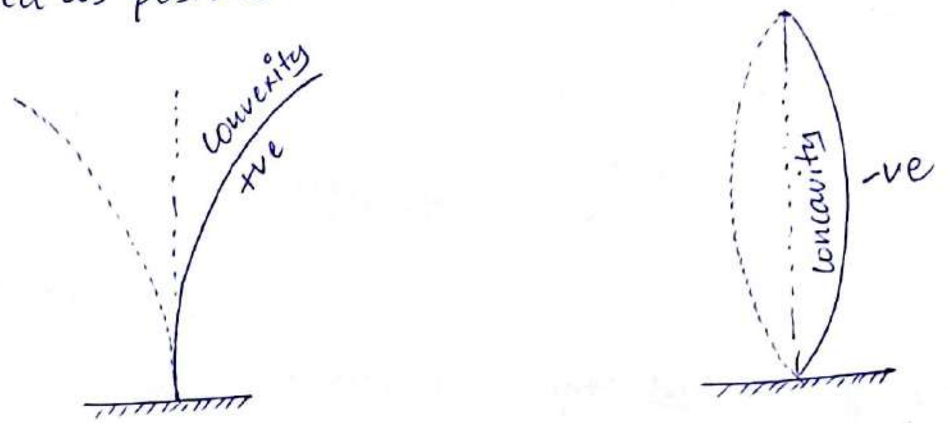
(iv) Both ends fixed :-

$$\boxed{l_e = \frac{l}{2}}$$



Sign conventions for bending moment :-

A bending moment which bends the column so as to present convexity towards the initial centre line of the member will be regarded as positive.



A bending moment which bends the column so as to ~~present~~ present concavity towards the initial centre line of the member will be regarded as negative.

Euler's Theory (For long column):-

Assumptions:-

1. The column is initially straight and of uniform lateral dimension.
2. The compressive load is exactly axial and it passes through the centroid of the column section.
3. The material of the column is exactly (perfectly) homogeneous and isotropic.
4. Pin joints are frictionless and fixed ends are perfectly rigid.

5. The weight of the column itself is neglected.
6. The columns fails by buckling only
7. Limit of proportionality is not exceeded.

Euler's Formula:-

Euler's formula is used for calculating the critical load for a long column or strut -

$$\boxed{P_{Euler} = \frac{\pi^2 E I_{least}}{l_e^2}}$$

P = critical load.

E = Young's mod. of elasticity

I_{least} = least moment of inertia of the section of column

l_e = Equivalent length

Ques:- A solid round bar 60 mm in diameter and 2.5 m long is used as a strut. One end of the strut is fixed while its other end is hinged. Find the safe compressive load for this strut using Euler's formula.

Assume, $E = 200\ GN/m^2$ and FOS = 3

Sol^n:- Given-

$d = 60\ mm = 0.06\ m$

$l = 2.5\ m$

$E = 200\ GN/m^2$

FOS = 3

Given- One end fixed and other end hinged-

$$l_e = \frac{l}{\sqrt{2}} = \frac{2.5}{\sqrt{2}} = 1.768\ m$$

Euler's critical load -

$$P_E = \frac{\pi^2 E I_{least}}{l_e^2} \qquad I = \frac{\pi}{64} d^4 = 6.358 \times 10^{-7}\ m^4$$

$$P_E = \frac{\pi^2 \times 200 \times 10^6 \times 6.358 \times 10^{-7}}{(1.768)^2} = 401.7\ KN.$$

Safe compressive load = $\frac{P_E}{FOS} = \frac{401.7}{3} = 133.9$ KN. Ans;

Ques:- A bar of length 4 m when used as a simply supported beam and subjected to a udl of 30 KN/m over the whole span, deflects 15mm at the centre. Determine the crippling load when it is used as a column with following end conditions-

(i) Both ends pin-jointed
(ii) One end fixed and other end hinged.
(iii) Both ends fixed.

Solⁿ:- Given:- $l = 4m$

simply supported beam-

udl, $w = 30$ KN/m

$\delta = 15$ mm → deflection (centre)

$$\delta = \frac{5wl^4}{384EI}$$

$$0.015 = \frac{5 \times 30 \times 4^4}{384\ EI}$$

$$EI = 6.66 \times 10^6\ N.m^2$$

(i) Both ends pin jointed-

$$l_e = l = 4m.$$

$$P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 6.66 \times 10^6}{(4)^2} = 4108 \text{ KN Ans;}$$

(ii) One end fixed and other end hinged -

$$l_e = \frac{l}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ m$$

$$P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 6.66 \times 10^6}{(2\sqrt{2})^2} = 8216 \text{ KN Ans,}$$

(iii) Both ends fixed -

$$l_e = \frac{l}{2} = \frac{4}{2} = 2m$$

$$P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 6.66 \times 10^6}{(2)^2} = 16432 \text{ KN Ans.}$$

Rankine Formula:-

In case of short columns - failure occurs mainly due to direct compressive stress only.

Critical load. $P_c = \sigma_c \cdot A$.

In case of long columns failure occurs mainly due to buckling stresses -

Critical load. $P_E = \frac{\pi^2 EI}{le^2}$

But in case of intermediate columns both direct compressive stress and buckling stress considered when failure occures -

Critical load. $\rightarrow P_R$ — (Rankine Critical load)

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_E}$$

$$\frac{1}{P_R} = \frac{1}{\sigma_c \cdot A} + \frac{le^2}{\pi^2 EI}$$

$$\frac{1}{P_R} = \frac{1}{\sigma_c \cdot A} + \frac{le^2}{\pi^2 E \cdot A k^2}$$

$$\frac{1}{P_R} = \frac{1}{\sigma_c \cdot A} + \frac{1}{\pi^2 EA \left(\frac{K}{le}\right)^2}$$

$$\frac{A}{P_R} = \frac{1}{\sigma_c} + \frac{1}{\pi^2 E \left(\frac{K}{le}\right)^2}$$

$$\frac{A}{P_R} = \frac{1}{\sigma_c}\left[1 + \frac{(le/K)^2}{\frac{\pi^2 E}{\sigma_c}}\right]$$

$$\frac{P_R}{A} = \frac{\sigma_c}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{le}{K}\right)^2}$$

$$\boxed{P_R = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{le}{K}\right)^2}}$$

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a\left(\frac{le}{K}\right)^2}$$

$a = \frac{\sigma_c}{\pi^2 E}$ = Rankine coeff.

Ques.- A 1.5 m long C.I column has a circular cross section of 5 cm diameter. One end of column is fixed in direction and position and other is free. Taking factor of safety as 3. Calculate the safe load using-

(i) Rankine-Gordon formula, take yield stress 560 MN/m² and $a = \frac{1}{1600}$ for pinned ends.

(ii) Euler's Formula-, youngs mod. for. C.I = 120 GN/m²

Solⁿ:- Given-

$l = 1.5 m$

$d = 5 cm = 0.05 m$

$FOS = 3$

→ one end fixed and other end free.

$l_e = 2l = 2 \times 1.5 = 3 m$

$\sigma_c = 560 MN/m^2$

$a = \frac{1}{1600}$

$E = 120 GN/m^2$

$K^2 = \frac{I}{A} = \frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2}$

$K^2 = 1.5625 cm^2$
$= 1.5625 \times 10^{-4} m^2$

(i)

Rankine critical load -

$$P_R = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{K}\right)^2}$$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.05)^2$

$A = 1.9625 \times 10^{-3} m^2$

$$P_R = \frac{560 \times 10^3 \times 1.9625 \times 10^{-3}}{1 + \frac{1}{1600} \times \frac{3^2}{1.5625 \times 10^{-4}}}$$

$P_R = 29.72 KN.$

$$\text{Safe load} = \frac{P_R}{FOS} = \frac{29.72}{3} = 9.9 KN \text{ Ans.}$$

(ii) Euler's Critical load-

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}(0.05)^4 = 4.9 \times 10^{-6} m^4$$

$$P_E = \frac{\pi^2 \times 120 \times 10^6 \times 4.9 \times 10^{-6}}{(3)^2}$$

$P_E = 40.4$ KN.

safe load $= \frac{P_E}{3} = \frac{40.4}{3} = 13.47$ KN Ans.

Que:- A hollow C.I column whose outside diameter is 200mm has a thickness of 20 mm. It is 4.5 m long and is fixed at both ends. Calculate the safe load by Rankine-Gordon formula using a factor of safety 4.

Take, $\sigma_c = 550$ MN/m², $a = \frac{1}{1600}$

Solⁿ:- Given-

outside diameter, $d_o = 200$mm

thickness, $t = 20$mm

inside diameter $d_i = d_o - 2t$

$= 200 - 40 = 160$mm

length, $l = 4.5$m

→ Fixed at both ends. $l_e = \frac{l}{2} = \frac{4.5}{2} = 2.25$m

FOS = 4.

$\sigma_c = 550$ MN/m²

$a = \frac{1}{1600}$

Rankine critical load -

$$P_R = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{K}\right)^2}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}\left((0.2)^2 - (0.16)^2\right)$$

$A = 0.0113$ m²

$$K^2 = \frac{I}{A} = \frac{\frac{\pi}{64}\left[(0.20)^4 - (0.16)^4\right]}{\frac{\pi}{4}\left[(0.20)^2 - 0.16^2\right]} = 0.0041 \text{ m}^2$$

$$P_E = \frac{\pi^2 \times 120\times10^6 \times 4.9\times10^{-6}}{(3)^2}$$

$P_E = 40.4$ KN.

safe load $= \frac{P_E}{3} = \frac{40.4}{3} = 13.47$ KN Ans.

Que:- A hollow C.I column whose outside diameter is 200mm has a thickness of 20 mm. It is 4.5 m long and is fixed at both ends. Calculate the safe load by Rankine-Gordon formula using a factor of safety 4.

Take, $\sigma_c = 550$ MN/m², $a = \frac{1}{1600}$

Solⁿ:- Given-

outside diameter, $d_o = 200$mm

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$= 200 - 40 = 160$mm

length, $l = 4.5$m

→ Fixed at both ends. $l_e = \frac{l}{2} = \frac{4.5}{2} = 2.25$m

FOS $= 4$.

$\sigma_c = 550$ MN/m²

$a = \frac{1}{1600}$

Rankine critical load -

$$P_R = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{K}\right)^2}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}\left((0.2)^2 - (0.16)^2\right)$$

$$A = 0.0113\ m^2$$

$$K^2 = \frac{I}{A} = \frac{\frac{\pi}{64}\left[(0.20)^4 - (0.16)^4\right]}{\frac{\pi}{4}\left[(0.20)^2 - 0.16^2\right]} = 0.0041\ m^2$$

$$P_R = \frac{550\times10^3 \times 0.0113}{1+\frac{1}{1600}\times\frac{(2.25)^2}{0.0041}}$$

$P_R = 3510\,KN.$

$$\text{Safe load} = \frac{P_R}{FOS} = \frac{3510}{4} = 877\ KN \text{ Ans}$$