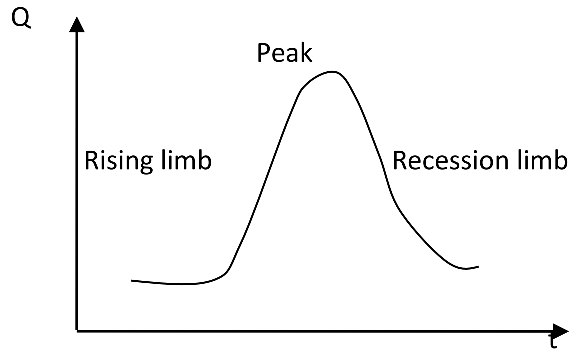
Chapter 5: Hydrograph analysis

5.1 Hydrograph

Hydrograph is a graphical plot of discharge (Q) of a river at a given location over time.
It is the output or total response of a basin.

Components of hydrograph



Hydrograph

1. Rising limb

It is ascending portion of hydrograph. It is influenced by storm and basin characteristics. The rising limb rises slowly in the early stage of flood but more rapidly towards the end portion. This is because in the initial stage the losses are high. The flow begins to build up in the channel as the storm duration increases. It gradually reaches the peak when maximum area contributes.

2. Peak or crest segment

It is the part which contains peak flow, which is of interest to hydrologists. Peak of hydrograph occurs when all portions of basins contribute at the outlet simultaneously at the maximum rate. Depending upon the rainfall-basin characteristics, the peak may be sharp, flat or may have several well defined peaks.

3. Recession limb

Recession limb represents withdrawal of water from the storage built up in the basin during the earlier phase of the hydrograph. It extends from the point of inflection at the end of the crest to the beginning of natural groundwater flow. The recession limb is affected by basin characteristics only and independent of the storm.

Equation for recession curve

$$Q_t = Q_0 K_r^t$$

imp

Q_0: initial discharge
Q_t: discharge at a time interval of t days
K_r: recession constant

Alternative form

$$Q_t = Q_0 e^{-at}$$

Where a = $-\ln K_r$

Terms

Time to peak: time lapse between starting of the rising limb to the peak

Time lag: time interval between centre of mass of rainfall hyetograph to the centre of mass of runoff hydrograph.

Time of concentration: time taken by a drop of water to travel from the remotest part to the outlet

Time base of hydrograph: time between starting of runoff hydrograph to the end of direct runoff due to storm.

5.2 Direct runoff and base flow

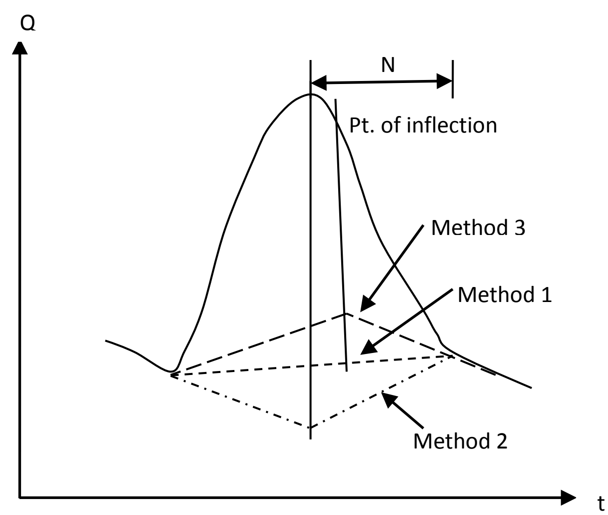
Direct runoff

It is the part of precipitation which appears quickly as flow in the river. (direct runoff = surface +subsurface)

Base flow

The part of runoff which receives water from the groundwater storage is called base flow.

Base flow separation



Base flow separation methods

1. Straight line method

Join the beginning of surface runoff to a point on the recession limb representing the end of direct runoff.

End point: by expert judgment or empirical equation

Empirical equation to find end of direct runoff

$N = 0.83\ A^{0.2}$

N = time interval from the peak to the end of direct runoff

A = Basin area

2. Extend the base flow curve prior to the commencement of surface runoff till it intersects the ordinate drawn at the peak point. Join this point to the end point of direct runoff

3. Extend the base flow recession curve backwards after the depletion of flood water till it intersects the ordinate at the point of inflection. Join this point to the beginning of the surface runoff by smooth curve.

Direct runoff hydrograph: the surface runoff hydrograph obtained after separating base flow

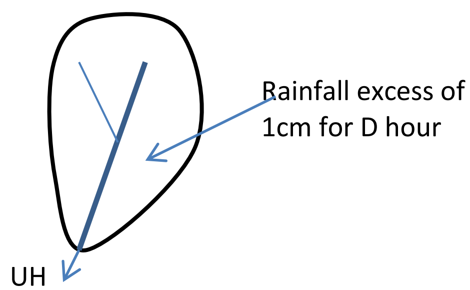
Types of stream

- Perennial: always carry flow
- Intermittent: limited contribution from groundwater
- Ephemeral: no base flow

Yield: total quantity of water that can be expected from a stream in a given period.

5.3 Unit Hydrograph

A unit hydrograph (UH) of a basin is defined as a direct runoff hydrograph (DRH) resulting from one unit depth of rainfall excess generated uniformly over the basin at a constant rate for an effective duration (D). The term unit refers to a unit depth of rainfall excess which is 1cm in SI unit and 1 inch in FPS unit. (Rainfall excess/effective rainfall = rainfall-loss)



Duration of unit hydrograph (D-hour UH): duration of rainfall excess

Assumptions

- Constant intensity of excess rainfall within the effective
- Uniform distribution of excess rainfall over the basin
- Constant base time of the DRH for excess rainfall of given duration
- Linear model: principle of superposition and proportionality holds
- Principle of time invariance holds
  -Given excess rainfall will always produce the same DRH whatever may be the season of the year (unchanging basin characteristics)

Principles applied in UH

I. Linearity principles

Linear relationship means output varies linearly with input. This principle is expressed by convolution theorem.

If $I(\tau)$ is intensity of input at time τ and $u(t-\tau)$ is the unit response after time t, then total response is given by $Q(t) = \int_0^t I(\tau)\, u(t-\tau)d\tau$. This is convolution integral.

There are two principles of linearity.
a. Principle of proportionality: If a solution y is multiplied by a constant c, the resulting function cy is also a solution.
r_e = excess rainfall, UH = Unit hydrograph (solution)
Output (DRH) = r_e*UH
b. Principle of superposition: If two solutions y1 and y2 of the equation are added, the resulting function y1+y2 is also a solution of the equation.
$r_{e1,}$ r_{e1} = excess rainfall at t hr interval, UH = Unit hydrograph (solution)
Output (DRH) = (r_{e1}*UH)+ (r_{e2}*UH lagged by t hr)

II. Principle of time invariance: Given excess rainfall will always produce the same DRH whatever may be the season of the year (unchanging basin characteristics)

Features
- Rainfall excess (r_e) =1cm, runoff depth (r_d) = 1cm

  Continuity: Total depth of rainfall excess = total depth of direct runoff
- Runoff volume (V_d) = Basin area(A) x r_d = Ax1cm
- Rainfall intensity: 1/D in cm/h
- Lumped response: catchment as a single unit
- Initial loss absorbed by basin, no effect of antecedent storm condition

Applications of UH
- Computation of flood hydrograph for the design of hydraulic structures
- Extension of flow records at a site
- Flood forecasting
- Comparing the basin characteristics

Limitations of UH
- Minimum basin size> 2km^2, Maximum basin size up to 5000 km^2
- Not suitable for very long basins
- Applicable for short duration
- Not very suitable for basins having large snow cover
- UH is not applicable for basins having large storages
- UH is not applicable for basins having high variation of rainfall intensity.

5.4 Derivation of unit hydrograph

Selection criteria for flood hydrograph
- Selection of isolated storms occurring individually
- Fairly uniform rainfall over the entire basin
- Duration of rainfall: 1/5 to 1/3 of basin lag
- Range of rainfall excess: 1 to 4 cm

1. Derivation of UH for single storm
Given: streamflow data (Q) and basin area (A)
Single storm: all of the rainfall excess occurs at a reasonably uniform rate over a fairly short time period

- Separate baseflow (BF).
- DRH=Q-BF
- Volume of DRH (V_d) = $\sum DRH * \Delta t$
- CRunoff depth (r_d) = V_d/A
- UH= DRH/r_d

Effective duration of UH = Duration of excess rainfall.
Check whether total depth of runoff = total rainfall excess

2. Derivation of UH for multiple storms
Multiple storms: relatively long and varying intensities if rainfall
Storms: divided into number of equal periods and fairly constant rate of rainfall for each period
Duration of UH = Duration of period of each storm

De-convolution method
Given: DRH data and rainfall excess data
(If DRH is not given, compute base flow and compute DRH by subtracting baseflow from streamflow data)
Convolution Equation in discrete form
$Q_n = \sum_{m=1}^{n \leq m} P_m U_{n-m+1}$
n = number of runoff ordinates
m = number of periods of rainfall excess
Q_n = Direct runoff
P_m= Excess rainfall
U_{n-m+1} = UH ordinate

Use above equation for computing ordinate of UH with excess rainfall and direct runoff data.
For complex multi-peaked hydrograph: solution of above equation by least square regression.

5.5 Computation of runoff from given UH

1. Single storm

- DRH =UH * rainfall excess.
- Total runoff=DRH+BF

2. Multiple storms

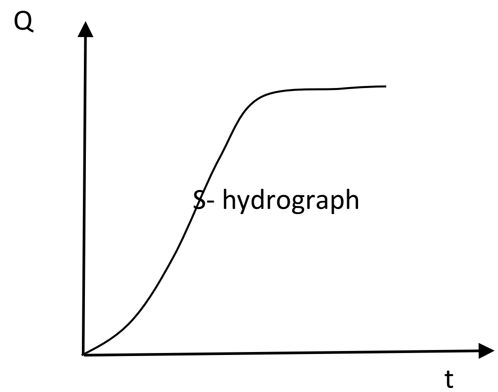
- Given: UH and effective rainfall for multiple durations

Use principle of proportionality and superposition

- DRH1=UH * first rainfall excess.
- DRH2=UH*second rainfall excess lagged by duration of first and second rainfall
  DRH3 = UH*third rainfall excess lagged by duration of first and third rainfall
  So on….
- DRH= DRH1+DHR2+DHR3+……
- Total runoff=DRH+BF

5.6 S-Hydrograph

S Hydrograph is a hydrograph resulting from a continuous excess rainfall at a constant rate of 1cm/h for an indefinite period. It is a theoretical concept. The curve is named S hydrograph as it looks like deformed S shape. The curve is obtained by adding a series of D-h unit hydrographs spaced at D-h apart.



The S-curve reaches a maximum equilibrium discharge at a time equal to the time base of the first unit hydrograph.
Unit rainfall excess = 1 cm in D hr
Rainfall intensity = 1/D in cm/hr
If A = basin area in km^2 and D is in hour, then
Equilibrium discharge $(Q_s) = \left(\frac{1}{D} x \frac{1}{3600x100}\right)(Ax10^6) = 2.778\frac{A}{D}$ m^3/s

Construction of S-curve
U(t) = S(t) – S(t-D)
S(t) = U(t)+ S(t-D)
where D = Duration of UH, S(t) = ordinate of S-curve at t, U(t) = ordinate of UH at t, S(t-D) = ordinate of S-curve at t-D

In other words,
Ordinate of S-curve at t = ordinate of D-hr UH at t + S-curve addition at time t
For $t \leq D$, S(t-D) = 0.

5.7 Computation of Unit hydrograph of different durations

In the computation of flood hydrograph, if the duration (D) of given UH and the duration (D’) of excess rainfall is different, then the UH of D hour should be converted to UH of D’ hour.

Given: UH of duration D
To compute: UH of duration D’
n = D’/D
If n is integer, use superposition method or S-curve method.
If n is real, use S-curve method.

a. Superposition method

- Lag the UH ordinate by D, 2D, ….(n-1)D.
- U1=Sum of the ordinates of all UHs.
- Ordinate of D'-hour UH = U1/n

b. S Hydrograph method

- Compute S-curve addition (=S(t-D)).
- Compute the ordinate of S-curve.
  S1 = UH(t)+ S(t-D)
- Lag the ordinates of S1 hydrograph by the duration D'. This is S2.
- Ordinate of D'-hour UH= (S1-S2)/n

In case of D'<D and the time interval of data is not equal to D', first plot the given UH and read the values with time interval equal to D'. Then follow above steps.

If the ordinates of UH becomes negative or shows fluctuations in the tail part, then manually smoothen the tail part.

Basic Numericals of Unit Hydrograph

<u>Derivation of UH</u>

Single storm

Given below are the observed flows from a storm of 4hr duration on a stream with a catchment area of 613 km^2. Derive 4hr unit hydrograph. Make suitable assumptions regarding base flow.

Time (hr)	0	4	8	12	16	20	24	28	32	36	40	44	48
Observed flow (m3/s)	10	110	225	180	130	100	70	60	50	35	25	15	10

Solution:

Catchment area (A) = 613 km^2

Assume base flow (BF) = 10 m^3/s

Direct runoff (Q_{dr}) = Q-BF

Volume of runoff (V) = $\sum Q_{dr}\, \Delta t$

Runoff depth (r_d) = V/A

Divide Q_{dr} by r_d to get UH ordinate.

Δt is same for each runoff ordinate.

Δt = 4 hour = 4x3600 s

V = $\sum Q_{dr}\, \Delta t = \Delta t \sum Q_{dr}$ = 890x4x3600

$r_d = \frac{V}{A} = \frac{890 \times 4 \times 3600}{613 \times 10^6}$ = 0.02m =2cm

Time (hr)	0	4	8	12	16	20	24	28	32	36	40	44	48
Q (m^3/s)	10	110	225	180	130	100	70	60	50	35	25	15	10
BF (m^3/s)	10	10	10	10	10	10	10	10	10	10	10	10	10
Qdr (m^3/s)	0	100	215	170	120	90	60	50	40	25	15	5	0
UH (m^3/s)	0	50	108	85	60	45	30	25	20	13	7.5	2.5	0

The ordinates of a hydrograph of a surface runoff (DRH) resulting from 4.5cm of rainfall excess of duration 8hr in a catchment are as follows:

Time (hr)	0	5	13	21	28	32	35	41	45	55	61	91	98	115	138
Discharge (m^3/s)	0	40	210	400	600	820	1150	1440	1510	1420	1190	650	520	290	0

Derive the ordinates of 8hr-unit hydrograph.

Solution:

Direct runoff = Q

Rainfall excess (R_e) = 4.5cm

For single storm, UH ordinate = Q/R_e

Time (hr)	0	5	13	21	28	32	35	41	45	55	61	91	98	115	138

Discharge (m^3/s)	0	40	210	400	600	820	1150	1440	1510	1420	1190	650	520	290	0
UH	0	8.9	46.7	88.9	133.3	182.2	256	320	335.6	316	264.4	144	116	64.44	0

<u>Multiple storm</u>

The following table gives the ordinates of a DRH resulting from two successive 3-hour durations of rainfall excess value of 2 cm and 4 cm respectively.

t (hr)	0	3	6	9	12	15	18	21	24	27	30
DRH (m^3/s)	0	120	480	660	460	260	160	100	50	20	0

Derive the ordinates of 3-hr UH.

Solution:
a. Effective rainfall, R_1 = 2 cm and R_2 = 4cm
It is a case of multiple storms. We have to use discrete time convolution equation to compute UH ordinate. The equation is

$$Q_n = \sum_{m=1}^{n\leq m} R_m U_{n-m+1}$$

Q = Direct runoff, R = Excess rainfall, U = UH ordinate
Here, total no. of runoff ordinates (n) = 9
Total number of rainfall excess values (m) =2

For n =1, m =1
$Q_1 = R_1U_1$
$U_1 = Q_1/R_1$ =120/2 = 60
For n = 2, m=1, 2
$Q_2 = R_1U_2+R_2U_1$
$U_2 = (Q_2-R_2U_1)/R_1$ = (480-4x60)/2 = 120
For n = 3 onwards, m= 1, 2. So, we can use the similar expression as that of U_2 for n = 3 onwards.
$U_n = (Q_n-R_2U_{n-1})/R_1$

$U_3 = (Q_3-R_2U_2)/R_1$ = (660-4x120)/2 = 90
$U_4 = (Q_4-R_2U_3)/R_1$ = (460-4x90)/2 = 50
$U_5 = (Q_5-R_2U_4)/R_1$ = (260-4x50)/2 = 30
$U_6 = (Q_6-R_2U_5)/R_1$ = (160-4x30)/2 = 20
$U_7 = (Q_7-R_2U_6)/R_1$ = (100-4x20)/2 = 10
$U_8 = (Q_8-R_2U_7)/R_1$ = (50-4x10)/2 = 5
$U_9= (Q_9-R_2U_8)/R_1$ = (20-4x5)/2 = 0

Resulting UH

t (hr)	0	3	6	9	12	15	18	21	24	27	30
UH(m^3/s)	0	60	120	90	50	30	20	10	5	0	0

UH to flood hydrograph

The ordinate of a 4-h UH of a catchment of area 1000km^2 are given below. Calculate flood hydrograph resulting from two successive 4-h storms having rainfall of 1.5cm each. Assume uniform base flow of 10 m^3/s and ϕ-index equal to 0.10 cm/hr.

t(hr)	0	4	8	12	16	20	24	28	32	36	40	44
4hr UH (m^3/s)	0	20	60	150	120	90	66	50	32	20	10	0

Solution:

ϕ-index (infiltration loss) = 0.1 cm/hr

For 4 hour, loss (L) = 4x0.1 =0.4 cm

Rainfall values, R1 = 1.5 cm and R2 = 1.5 cm

Rainfall excess (R_{e1}) = R1-L = 1.5-0.4 =1.1cm

Rainfall excess (R_{e2}) = R2-L = 1.5-0.4 =1.1cm

DHR1 = UHxR_{e1}

DRH2 =UHxR_{e2} (lagged by 4 hour)

DRH = DRH1+DRH2

Q = DRH + BF

Computation of flood hydrograph

t(h)	4 hr UH (m^3/s)	DRH1 (m^3/s)	DRH2 (m^3/s)	DRH (m^3/s)	BF (m^3/s)	Q (m^3/s)
0	0	0		0	10	10
4	20	22	0	22	10	32
8	60	66	22	88	10	98
12	150	165	66	231	10	241
16	120	132	165	297	10	307
20	90	99	132	231	10	241
24	66	72.6	99	171.6	10	181.6
28	50	55	72.6	127.6	10	137.6
32	32	35.2	55	90.2	10	100.2
36	20	22	35.2	57.2	10	67.2
40	10	11	22	33	10	43
44	0	0	11	11	10	21
(48)			0	0	10	10

UH of different durations

The ordinates of a 4 hour UH of a basin of area 25 km^2 are given below.

t (hr)	0	4	8	12	16	20	24	28	32	36	40	44	48	52
UH(m^3/s)	0	30	55	90	130	170	180	160	110	60	35	20	8	0

Calculate the following.
a) 4-hr DRH for a rainfall of 3.25cm with ϕ-index of 0.25cm.
b) a 12-hr UH by using the method of superposition.
c) a 12-hr UH by using the S-curve method.

Solution:
a) Rainfall (R) = 3.25 cm, ϕ-index = 0.25cm
Rainfall excess (re) = 3.25-0.25 = 3cm
DRH = UH xre

Computation of DRH

t (hr)	0	4	8	12	16	20	24	28	32	36	40	44	48	52
UH(m^3/s)	0	30	55	90	130	170	180	160	110	60	35	20	8	0
DRH (m^3/s)	0	90	165	270	390	510	540	480	330	180	105	60	24	0

b) Required duration of UH (D') =12 hr
Given duration (D) =4 hr
n = D'/D = 3 (integer)

UHa = UH lagged by 4 hr, UHb = UH lagged by 8 hour
UH1 = UH + UHa+ UHb
12hr-UH = UH1/(D'/D) = UH1/3

Computation of 12-hr UH using method of superposition

t (hr)	UH	Uha	Uhb	UH1	12-hr UH (m^3/s)
0	0			0	0
4	30	0		30	10
8	55	30	0	85	28.3
12	90	55	30	175	58.3
16	130	90	55	275	91.7
20	170	130	90	390	130
24	180	170	130	480	160
28	160	180	170	510	170
32	110	160	180	450	150
36	60	110	160	330	110
40	35	60	110	205	68.3
44	20	35	60	115	38.3
48	8	20	35	63	21
52	0	8	20	28	9.3

56		0	8	8	2.7
60			0	0	0

c. S-curve addition = Ordinate of S-curve at (t-D)
Ordinate of S curve (S1) = ordinate of UH+ S-curve addition
S 2 = S1 lagged by 12 hr hour
12hr-UH = (S1-S2)/(D'/D) = (S1-S2)/3

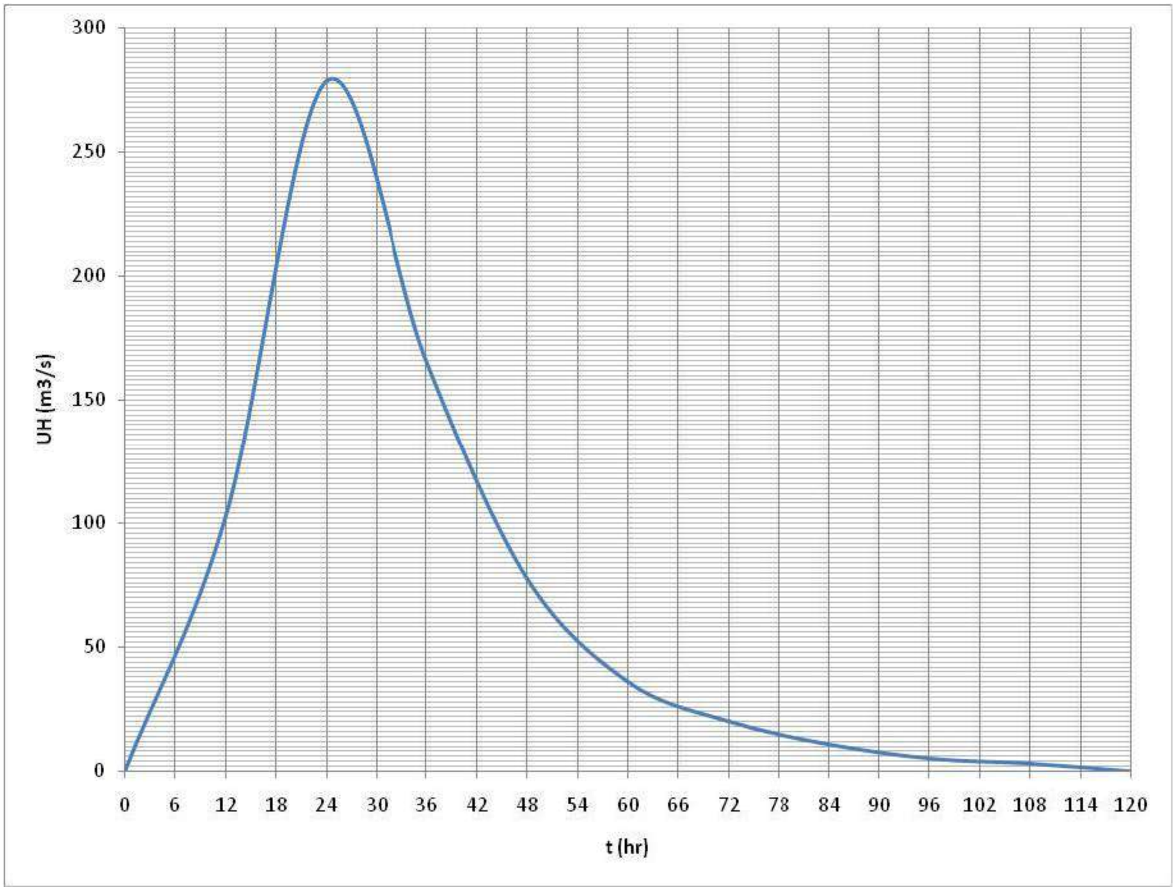
Computation of 12-hr UH using S-Curve method

t (hr)	UH	S-Curve addition	S-curve (S1)	S2	12-hr UH (m^3/s)
0	0		0		0
4	30	0	30		10
8	55	30	85		28.3
12	90	85	175	0	58.3
16	130	175	305	30	91.7
20	170	305	475	85	130
24	180	475	655	175	160
28	160	655	815	305	170
32	110	815	925	475	150
36	60	925	985	655	110
40	35	985	1020	815	68.3
44	20	1020	1040	925	38.3
48	8	1040	1048	985	21
52	0	1048	1048	1020	9.3
56			1048	1040	2.7
60			1048	1048	0
64			1048	1048	0

Given below is a 12-hr UH. Derive 6-hr UH.

t (hr)	0	12	24	36	48	60	72	84	96	108	120
UH(m^3/s)	0	103	279	165	78	36	20	11	5	3	0

Solution:
Required duration of UH (D') =6 hr
Given duration (D) =12 hr
n = D'/D = 0.5 (real)
Here, D'<D. To derive UH of 6 hr, the interval of ordinates of given UH should be at least 6hour.
Plot given UH versus t on a graph paper and get the values of UH at 6 hour interval.



S-curve addition = Ordinate of S-curve at (t-D)
Ordinate of S curve (S1) = ordinate of UH+ S-curve addition
S2 = S1 lagged by 6 hour
6-hr UH = (S1-S2)/(D’/D) = (S1-S2)/0.5

Computation of 6-hr UH

t (hr)	UH(m3/s)	S curve addition	S1	S2	6-hr UH	6-hr UH (corrected)
0	0		0		0	0
6	48		48	0	96	96
12	103	0	103	48	110	110
18	191	48	239	103	272	272
24	279	103	382	239	286	286
30	238	239	477	382	190	190
36	165	382	547	477	140	140
42	117	477	594	547	94	94
48	78	547	625	594	62	62
54	53	594	647	625	44	44
60	36	625	661	647	28	28

66	27	647	674	661	26	26
72	20	661	681	674	14	14
78	15	674	689	681	16	10
84	11	681	692	689	6	6
90	8	689	697	692	10	4
96	5	692	697	697	0	0
102	4	697	701	697	8	0
108	3	697	700	701	-2	0
114	2	701	703	700	6	0
120	0	700	700	703	-6	0
126		703	700	700	0	0

The UH of 6 hour should be corrected manually from 90 hour onwards to make it smooth.