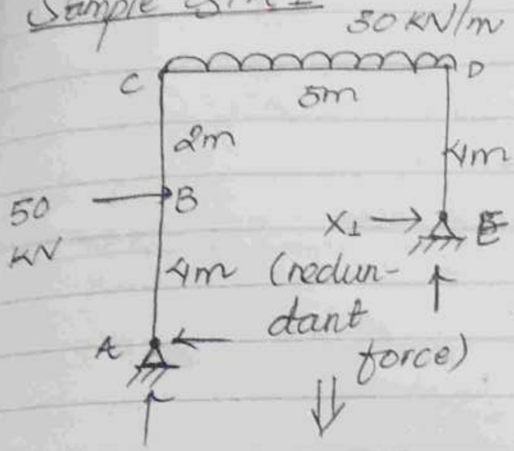
By PK Sir

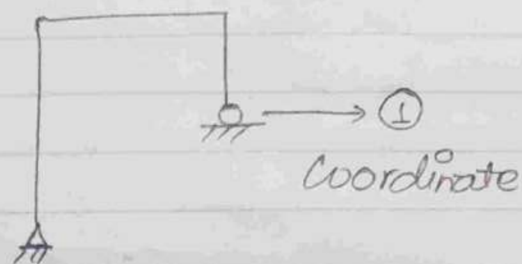
Sample Q/A 1

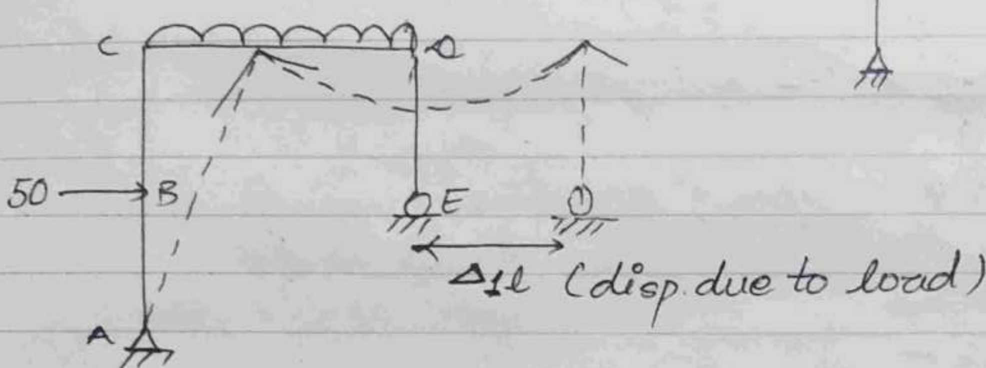


Degree of static indeterminacy (DOSI) = No. of unknown − No. of known

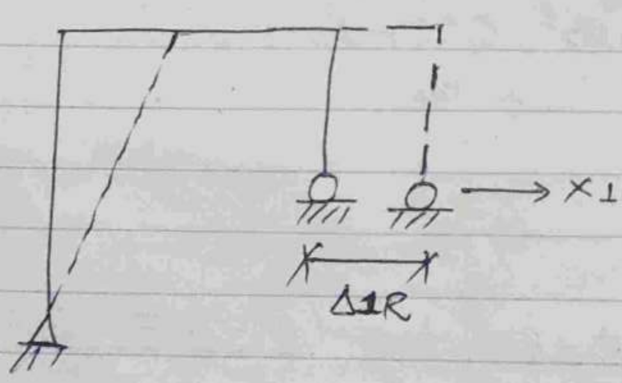
= 4 − 3 = 1

→ One extra equation required.

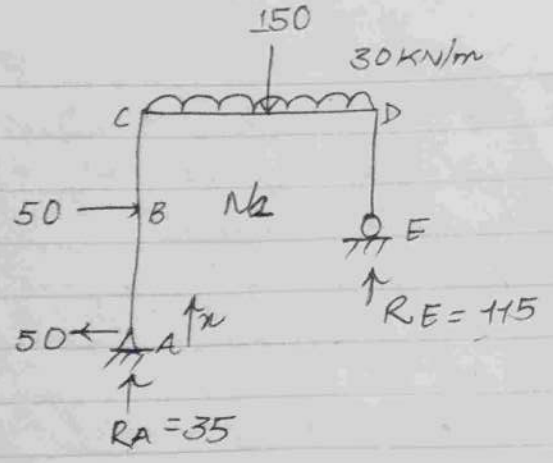




primary structure: structure formed by removing restraints from statically indeterminate structure so that the resulting structure is determinate.



$$\Delta_{1L} + \Delta_{1R} = \Delta_1 = 0$$



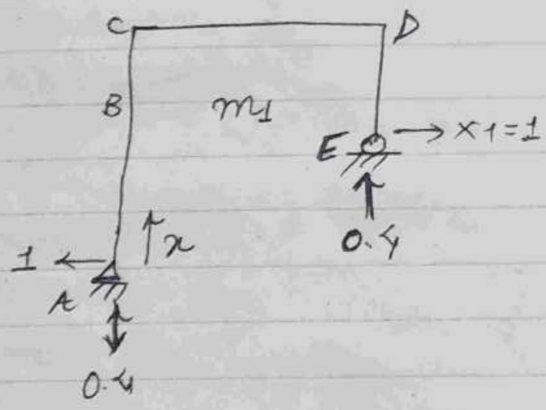
$$R_E \times 5 - 150 \times 2.5 - 50 \times 4 = 0$$

$$\therefore R_E = \cancel{35}\ 115$$

$$R_A + R_E = 150$$

$$\Rightarrow R_A = \cancel{115}\ 35$$

~~RAx5~~



Portion	Origin	Limit	ML	m1
AB	A	0-4	$50x$	x
BC	B	0-2	200	$x+4$
CD	D	0-5	$115x - 15x^2$	$0.4x + 4$
DE	E	0-4	0	x

$$\Delta_{1L} = \Sigma \int \frac{M_L \cdot m_1\, dx}{EI}$$

$$= \int_0^4 \frac{50x^2\, dx}{EI} + \int_0^2 \frac{(200x + 800)\, dx}{EI} + \int_0^5 \frac{(115x - 15x^2)(0.4x + 4)\, dx}{EI}$$

$$+ \int_0^4 \frac{0\, dx}{EI}$$

$$= \left[\frac{50x^3}{3EI}\right]_0^4 + \left[\frac{200x^2}{2} + 800x\right]_0^2 + \left[\frac{46x^3}{3EI} - \frac{6x^4}{4EI} + \frac{460x^2}{2EI} - \frac{60x^3}{3EI}\right]_0^5 = \frac{7295.8}{EI}$$

$$\delta_{11} = \int \frac{m_1^2 dx}{EI} = \int_0^4 \frac{x^2 dx}{EI} + \int_0^2 \frac{(x^2+8x+16)dx}{EI} + \int_0^4 \frac{x^2 dx}{EI} + \int_0^5 \frac{(0.4x+4)^2 dx}{EI}$$

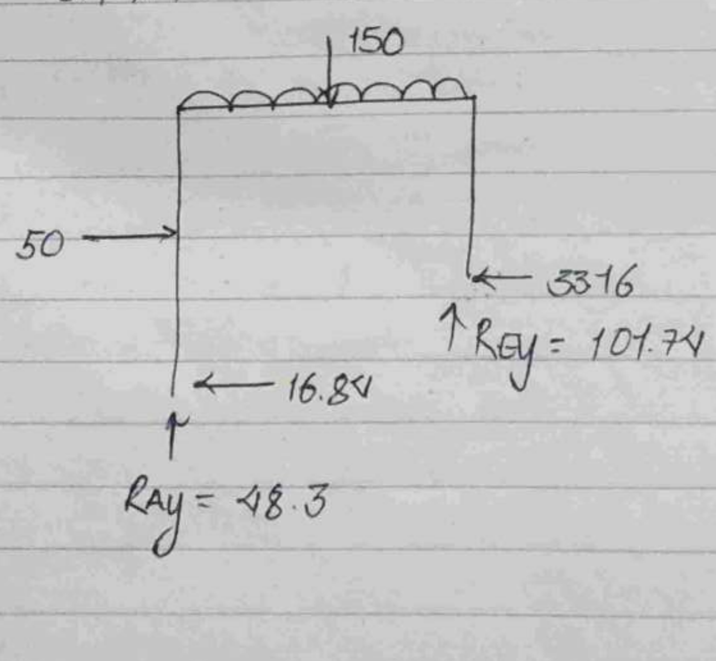
$$= \frac{220}{EI}$$

$\Delta_1 L + \Delta_1 R = 0$ → δ_{11} is due to unit load but X_1 is the redundant force acting so multiply by X_1.

or, $\Delta_1 L + \delta_{11} . X_1 = 0$

or, $\frac{7295.8}{EI} + \frac{220 \times X_1}{EI} = 0$

or, $X_1 = -33.1627$ N

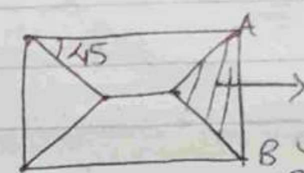


Flexible

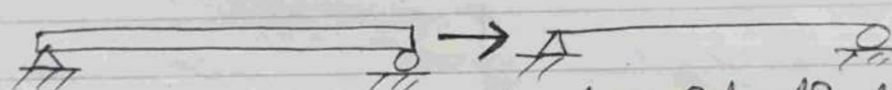
↳ Conditions/Requirements of structural system
→ Strength, stiffness & Stability

↳ Conditions and equations in structural analysis
→ Static, Compatibility & physical

↳ Partial restraint: Spring  Spring, allows partial movement

↳ Structure idealization : 

Load taken as UDL, shaded portion taken by AB



structure idealization, breadth & height ignored.
→ Making some modifications in structure to make subsequent calculations easier.

↳ Global coordinate system : ~~On~~ Single origin point for entire structure

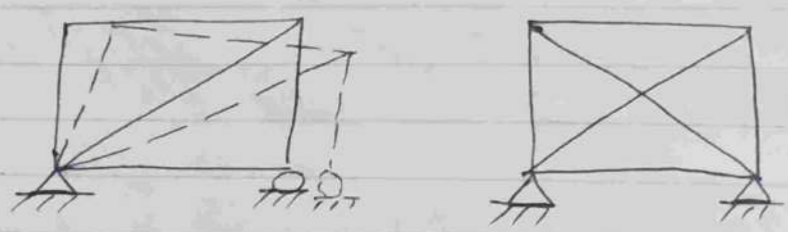
↳ Local coordinate system: Individual origin points for each member of structure.

↳ Indeterminancy of structural system, its physical meaning and types

↳ Solvable with static eqb^{m} eq^{ns} → determinate

If a structural system can be analyzed by using equations of static eqb^{m}, then the system is said to be statically determinate.

Physical meaning: Indeterminate structures are more rigid than determinate structures. Indeterminate str are capable of bearing of more load and more restrained.



Statically determinate   Statically inderminate

Types:

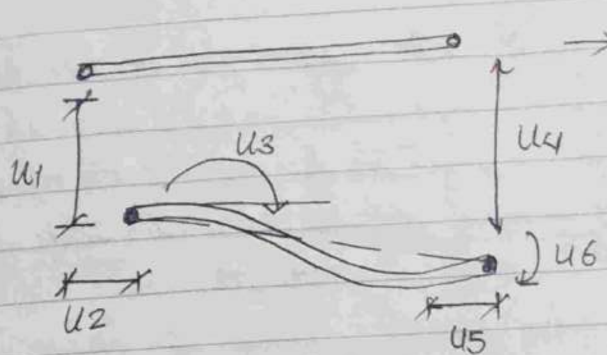
1) Static indeterminancy: DSI = No. of (unknown − known) (forces) eq

2) Kinematic indeterminancy: DKI = No. of unknown displacement at joints − No. of known eq^{n}

Reactions and forces are variable

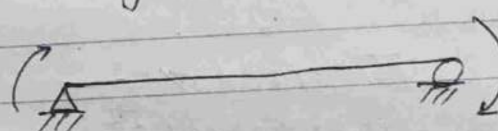
Displacement of joints are variable.
(Slope and displacement)

DSI : Degree of static indeterminancy
DKI : Degree of kinematic indeterminancy



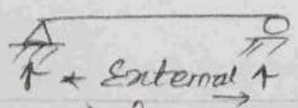
→ freely suspended bar in space

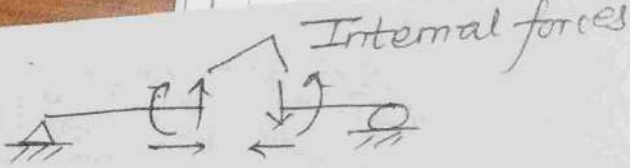
Three displacement possible at a joint



→ Only 3 displacement possible in total

So, $DKI = 3j - r$





Chpt 1.8 in book : A) degree of static indeterminancy : a) External
b) Internal

↳ Beam

In beam, external indeterminancy only exists.

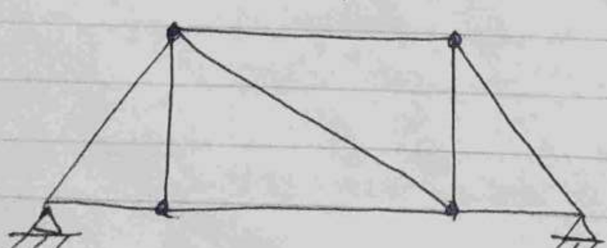
⇒ [External DSI : indeterminancy due to external forces → Rxⁿ at supports
Internal DSI : " " internal " → Internal rxnˢ in member (SF) eq.

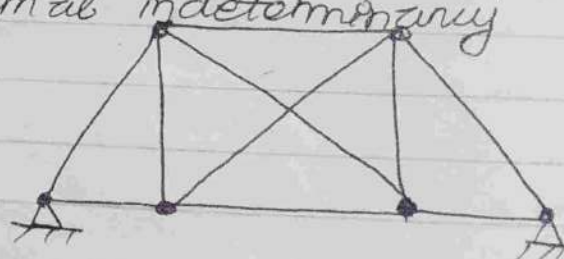
DSI : r − 3

DSI = r − (3 + c)   c = condition due to internal hinge
Unknown − Known

Truss

Member cross & overlap
internal indeterminancy





DSI = (m + r) − 2j

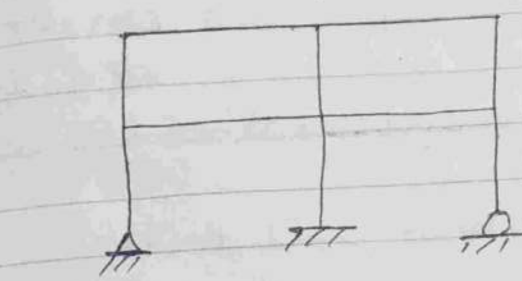
$(DSI)_{ext} = 4 - 3 = 1$
(4 unknown rxⁿ − 3 eqⁿ)

2j known for truss because in each joint, two eqⁿs are available
$\Sigma F_x = 0 ; \Sigma F_y = 0$

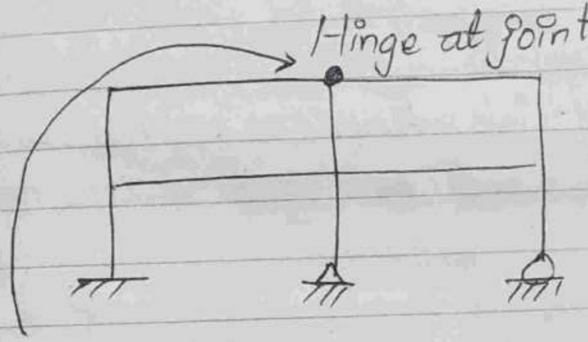
$(DSI)_{tot} = m + r - 2j$
$= 10 + 4 - 12 = 2$
$(DSI)_{int} = 2 - 1 = 1$

Frame

$3j$ because 3 eq^{ns} are known $\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M = 0$



$$DSI = (3m+r) - 3j$$
$$= 30 \cancel{27} + 6 - 27 = 9$$
$$(DSI)_{ext} = r - 3$$
$$= 6 - 3 = 3$$
$$(DSI)_{int} = 9 - 3 = 6$$



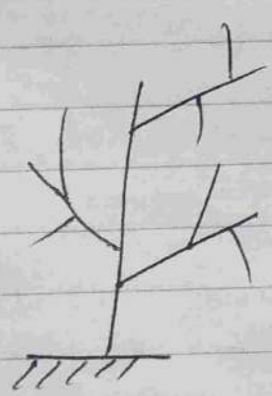
$$(DSI)_{ext} = r - 3 = 6 - 3 = 3$$
$$(DSI)_{int} = 3 \times C' - \Sigma(m-1)$$
$$= 3 \times 2 - 2$$
$$= 4$$
$$(DSI)_{tot} = 7$$

No. of member at joint = one less

$$DSI = (3m+r) - (3j+c)$$
$$= (3 \times 10 + 3) - (3 \times 9 + 2) = 7$$

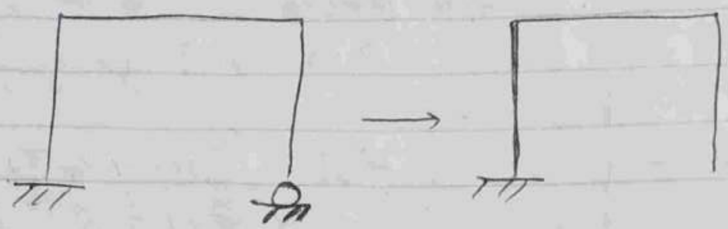
At joint, hinge → two condition eq^n

In between members, hinge → one condition eq^n



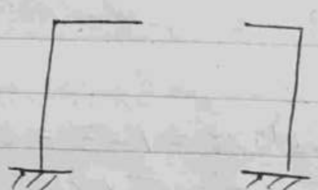
Tree config. always determinate

No. of restraints to be removed to attain this config. = Degree of indeterminacy



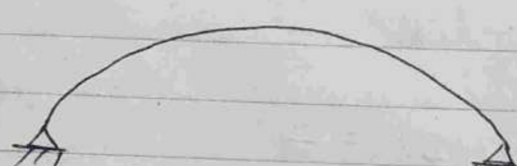
One restraint removed
DsI = 1

When restraint removed, structure should be stable.



DsI = 3
3 restraints (BM, SF, AF) removed.

<u>Arch</u> (Two Hinged)



DsI = 4 − 3 = 1

→ Count no of independent joint displacements that are free to occur.

B) Degree of Kinematic Indeterminancy

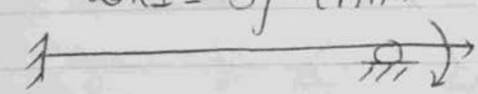
↳ No. of unknown displacement at joints is degree of kinetic indeterminancy.

— No. of independent joint displacements – both joint rotations and translations in structure are termed as degree of kinematic indeterminacy.

DKI = 3j − r

If members are inextensible,

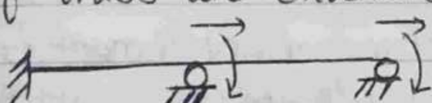
DKI = 3j − (r+m)



DKI = 3×2 − 4 = 2

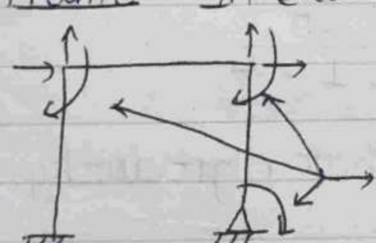
But if inextensible, DKI = 3×2 − (4+1) = 1

Generally, members of beam, frame are inextensible and that of truss are extensible.



- If extensible, DKI = 3×3 − 5 = 4
- If inextensible, DKI = 3×3 − (5+2) = 2

Frame: In exam inextensible must be considered if not given.

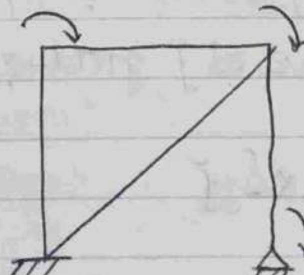


Extensible: DKI = 3×4 − 5 = 7

Inextensible: DKI = 3×4 − (5+3) = 4

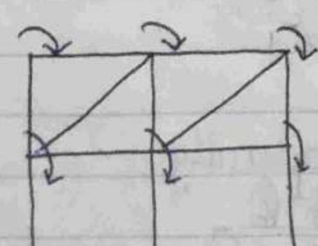
Side sway

If bracing is done,

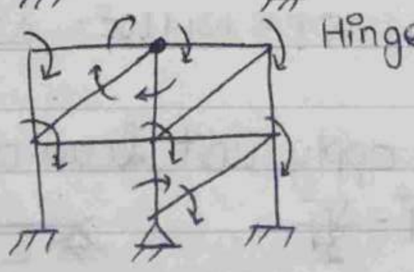


Inextensible:

DKI = 3×4 − (r+m) − bracing
= 12 − (5+3) − 1
= 3



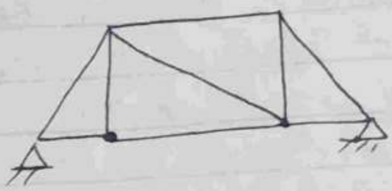
DKI = 7

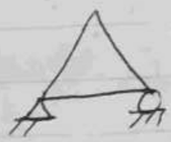


DSI = 11

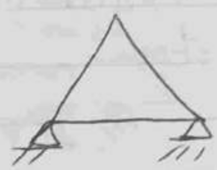
Truss: Member को rotation हुँदैन।

$\alpha KI = 2j - r$
$= 2\times 6 - 4 = 8$





$\alpha KI = 2\times 3 - 3 = 3$

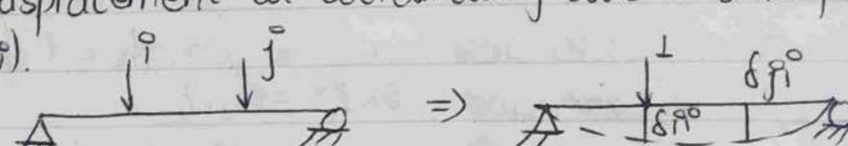


$\alpha KI = 2$

Betti's law and Maxwell's reciprocal Theorem

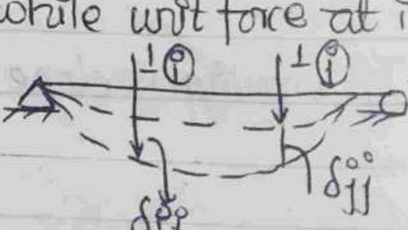
A) MRT

Theorem: In a linearly elastic structure, displacement at coordinate (i) due to unit force at coordinate (j) i.e δ_{ij} is equal to displacement at coordinate j due to unit force at coordinate i (δ_{ji}).



Proof: let us apply unit force at coordinate i gradually
workdone, $W_1 = \frac{1}{2}\times 1\times \delta_{ii} = \delta_{ii}/2$

Again, apply unit force at j gradually while unit force at i is already present

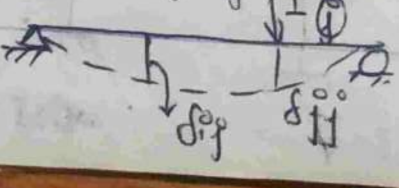


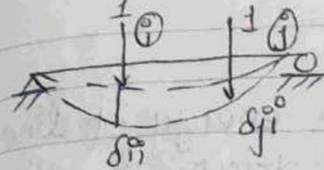
$W_2 = 1.\,\delta_{ij} + \frac{1}{2}\times 1\times \delta_{jj}$

Total workdone: $W_T = W_1 + W_2 = \frac{\delta_{ii} + \delta_{jj}}{2} + \delta_{ij}$

Again, let us apply unit load at j gradually.

$W_1' = \frac{1}{2}\times 1\times \delta_{jj} = \frac{\delta_{jj}}{2}$





$$W_e' = \delta_{ji} + \delta_{ii}/2$$
$$W_T' = \delta_{ji} + \frac{\delta_{ii}+\delta_{jj}}{2}$$

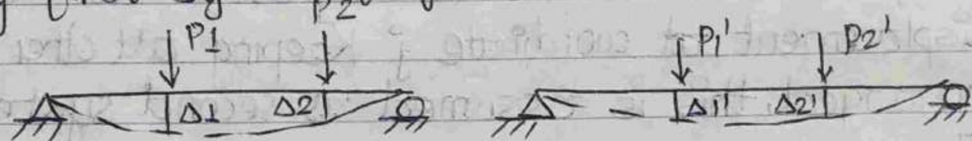
For linearly elastic structure, $W_T = W_T'$

or, $\delta_{ii}+\delta_{jj}+2\delta_{ij} = 2\delta_{ji}+\delta_{ii}+\delta_{jj}$

$\Rightarrow \boxed{\delta_{ji} = \delta_{ij}}$

B) Betti's law

Theorem: In a linearly elastic structure, virtual workdone by system of forces in undergoing displacements caused by another system of forces is equal to virtual workdone by second system of forces undergoing displacements caused by first system of forces.



$\Delta_1 = \delta_{11}P_1 + \delta_{12}P_2, \quad \Delta_2 = \delta_{21}P_1 + \delta_{22}P_2$

$\Delta_1' = \delta_{11}P_1' + \delta_{12}P_2', \quad \Delta_2' = \delta_{21}P_1' + \delta_{22}P_2'$

Virtual workdone by 1st system of forces while undergoing displacement caused by 2nd system of forces:

$$\begin{aligned}\Sigma P\Delta' &= P_1\Delta_1' + P_2\Delta_2' \\ &= P_1(\delta_{11}P_1' + \delta_{12}P_2') + P_2(\delta_{21}P_1' + \delta_{22}P_2') \\ &= P_1P_1'\delta_{11} + P_1P_2'\delta_{12} + P_1'P_2\delta_{21} + P_2P_2'\delta_{22}\end{aligned}$$

Again, Virtual workdone by 2nd system of forces while undergoing displacements caused by 1st system of forces

$$\begin{aligned}\Sigma P'\Delta &= P_1'\Delta_1 + P_2'\Delta_2 \\ &= P_1'(\delta_{11}P_1 + \delta_{12}P_2) + P_2'(\delta_{21}P_1 + \delta_{22}P_2) \\ &= P_1'P_1\delta_{11} + P_1'P_2\delta_{12} + P_1P_2'\delta_{21} + P_2'P_2\delta_{22}\end{aligned}$$

Using Maxwell's Reciprocal theorem,

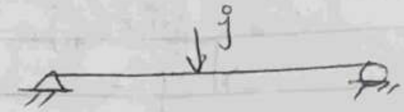
$\delta_{12} = \delta_{21}$, we get $\boxed{\Sigma P\Delta' = \Sigma P'\Delta}$

Compatibility Eq^n:

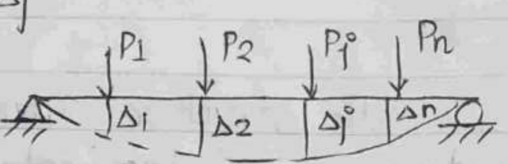
Castigliano's Theorem

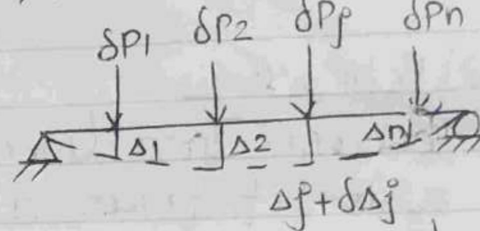
Theorem I: The partial derivative of strain energy of linearly elastic structure expressed in terms of displacement with respect to any displacement Δ_j is equal to force P_j at coordinate j.

$$\frac{\partial u}{\partial \Delta_j} = P_j$$



Proof:





Consider several forces $P_1, P_2 \ldots P_j \ldots P_n$ are acting on structure producing displacements $\Delta_1, \Delta_2 \ldots \Delta_j \ldots \Delta_n$. This is assumed as 1st system for Betti's theorem. Now, impose small increment $\delta\Delta_j$ to displacement at coordinate j. Keeping all other displacements unchanged, this is assumed as second system for Betti's Theorem.

Applying Betti's Theorem,

$$P_1.0 + P_2.0 + P_j.\delta\Delta_j + P_n.0 = \delta P_1\Delta_1 + \delta P_2\Delta_2 + \delta P_j\Delta_j + \delta P_n\Delta_n$$

$$\Rightarrow P_j\,\delta\Delta_j = \delta U$$

$$\Rightarrow \frac{\delta U}{\delta \Delta_j} = P_j$$

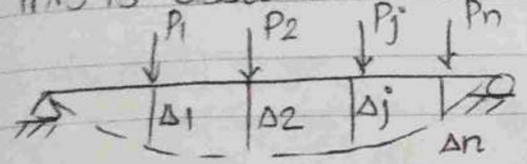
or, $$\lim_{\delta\Delta_j \to 0} \frac{\delta U}{\delta \Delta_j} = \frac{\partial U}{\partial \Delta_j} = P_j$$

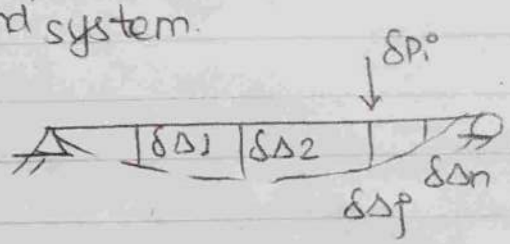
Theorem II: The partial derivative of strain energy of linearly elastic system expressed in terms of forces with respect to any force P_j is equal to displacement Δ_j at coordinate j.

Mathematically, $$\frac{\partial U}{\partial P_j} = \Delta_j$$

To apply castigliano's theorem to find redundant force, member should be determinate after removing redundant support.

<u>Proof</u>: Consider several forces $P_1, P_2, P_j \dots P_n$ are acting on structure at coordinate $1, 2, j \dots n$ producing displacements $\Delta_1, \Delta_2, \Delta_j \dots \Delta_n$. This is assumed as first system. Now impose small increment δP_j to force P_j, keeping all other forces unchanged. As a result, increments in displacements are $\delta\Delta_1, \delta\Delta_2, \delta\Delta_j \dots \delta\Delta_n$. This is assumed to be as 2nd system.





Applying Betti's law,

$$P_1\delta\Delta_1 + P_2\delta\Delta_2 + \dots + P_j\delta\Delta_j + \dots + P_n\delta\Delta_n = 0 \times \Delta_1 + 0 \times \Delta_2 + \delta P_j \times \Delta_j + 0$$

$$\Rightarrow \delta U = \delta P_j \Delta_j$$

$$\Rightarrow \frac{\delta U}{\delta P_j} = \Delta_j \quad \text{or,} \quad \lim_{\delta P_j \to 0} \frac{\delta U}{\delta P_j} = \frac{dU}{\delta P_j} = \Delta_j$$

<u>Uses</u>: 1) To find displacement (slope, deflections) in statically determinate structure

2) To find redundant force in statically indeterminate structure

<u>Beam (Flexure Member)</u>: Strain Energy $(U) = \Sigma \int \frac{M^2 dx}{2EI}$

$$\text{or,} \quad \frac{\delta U}{\delta P} = \Sigma \int \frac{2M}{2EI} \times \frac{\partial M}{\partial P} \times \partial x$$

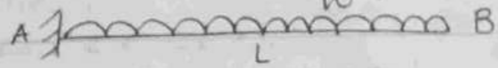
$$\therefore \Delta = \Sigma \int M . \frac{\partial M}{\partial P} \frac{dx}{EI}$$

Truss (Axial Member)

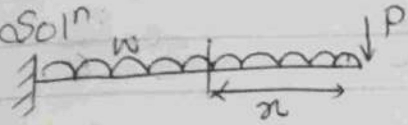
$$U = \Sigma \frac{P^2 L}{2AE}$$

or, $\frac{\partial U}{\partial P'} = \sum \frac{2PL}{2AE} \cdot \frac{\partial P}{\partial P'}$ $\Rightarrow \Delta = \Sigma P \left(\frac{\partial P}{\partial P'}\right) \frac{L}{AE}$ → dummy load (P')

Eg. Determine the vertical deflection & rotation at B.



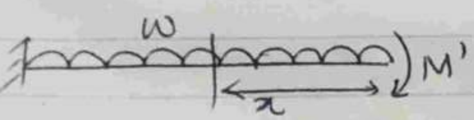
Soln



$M(x) = -Px - wx^2/2$

$\frac{\partial M}{\partial P} = -x$

$\Delta_{Bv} = \int M \cdot \frac{\partial M}{\partial P} \cdot \frac{dx}{EI} = \int_0^L \left(-\frac{wx^2}{2}\right)(-x)\frac{dx}{EI} = \frac{wL^4}{8EI}$

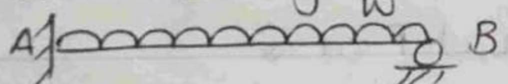


$M(x) = -M' - wx^2/2$

$\frac{\partial M}{\partial M'} = -1$

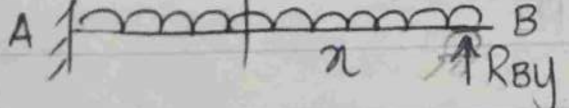
$\theta_B = \int M \cdot \frac{\partial M}{\partial M'} \frac{dx}{EI} = \int_0^L \left(-\frac{wx^2}{2}\right) \times (-1)\frac{dx}{EI} = \frac{wL^3}{6EI}$

Calculate R_{By}



$M(x) = R_{By} \times x - \frac{wx^2}{2}$

$\Delta_{By} = 0$



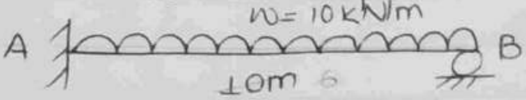
or, $\int M \cdot \frac{\partial M}{\partial R_{By}} \cdot \frac{dx}{EI} = 0$

or, $\int_0^L \left(R_{By} \times x - \frac{wx^2}{2}\right) \times x \frac{dx}{EI} = 0$

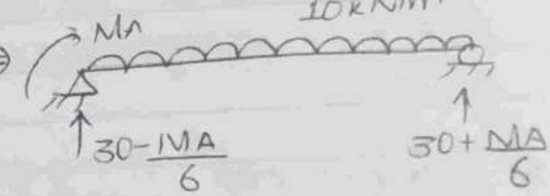
or, $R_{By} \times \frac{L^3}{3EI} - \frac{wL^4}{8EI} = 0$

or, $R_{By} = \frac{3wL}{8}$

Find moment at fixed end



$\Rightarrow$



$\theta_A = 0$

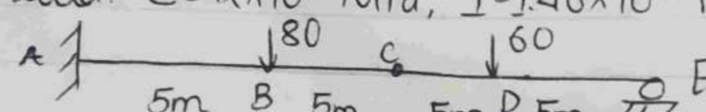
or, $\int Mx \cdot \frac{\partial Mx}{\partial MA} \frac{dx}{EI} = 0$

or, $\int_0^6 \left(30x + \frac{MAx}{6} - 5x^2\right)\left(\frac{x}{6}\right)\frac{dx}{EI} = 0$

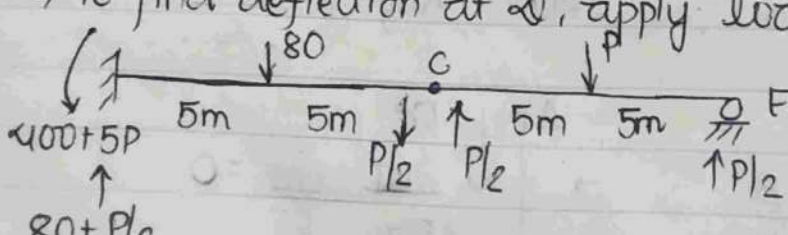
or, $\frac{1}{EI}\left[\frac{5x^3}{3} + \frac{MAx^3}{36 \times 3} - \frac{5x^4}{24}\right]_0^6 = 0$

$\Rightarrow MA = -45$ KNm

Using Castigliano's theorem, find slope & deflection at 60 KN load. $E = 2 \times 10^5$ MPa, $I = 1.46 \times 10^9$ mm^4

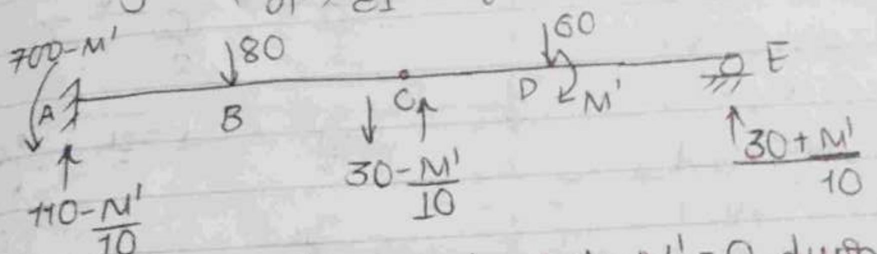


↳ To find deflection at D, apply load P at D



Portion	Origin	Limit	Mx	∂M/∂P	Mx, P=60
AB	A	0-5	$-400 - 5P + 80x + Px/2$	$-5 + x/2$	$-700 + 110x$
BC	C	0-5	$-Px/2$	$-x/2$	$-30x$
CD	C	0-5	$Px/2$	$x/2$	$30x$
DE	E	0-5	$Px/2$	$x/2$	$30x$

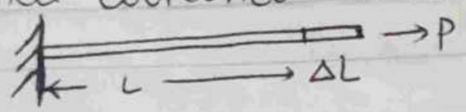
$$\Delta_D = \int M_m\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^5 (-700+110x)(-5+x/2)\frac{dx}{EI} = \frac{10416.67}{EI}$$



Similar as above, only put $M'=0$ during integration

Flexibility & stiffness

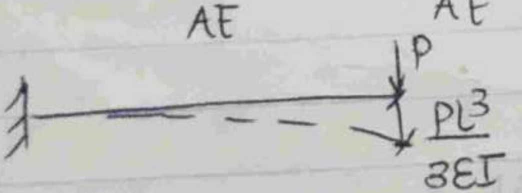
1) The displacement at any coordinate due to unit force at that coordinate is called flexibility (δ).



$\Delta L = \frac{PL}{AE} \Rightarrow \delta = \frac{1}{AE}$

The force required to produce unit displacement is called stiffness (k).

$\Delta L = \frac{PL}{AE} \Rightarrow 1 = \frac{kL}{AE}$ or, $K = \frac{AE}{L}$ (stiffness axial)



If $P=1$, $\delta = L^3/3EI$

if $\delta = 1$, $K = \frac{3EI}{L^3}$

Flexibility matrix

$\delta_{11}X_1+\delta_{12}X_2+\delta_{13}X_3+\Delta_{1L}=\Delta_1$

$\delta_{21}X_1+\delta_{22}X_2+\delta_{23}X_3+\Delta_{2L}=\Delta_2$

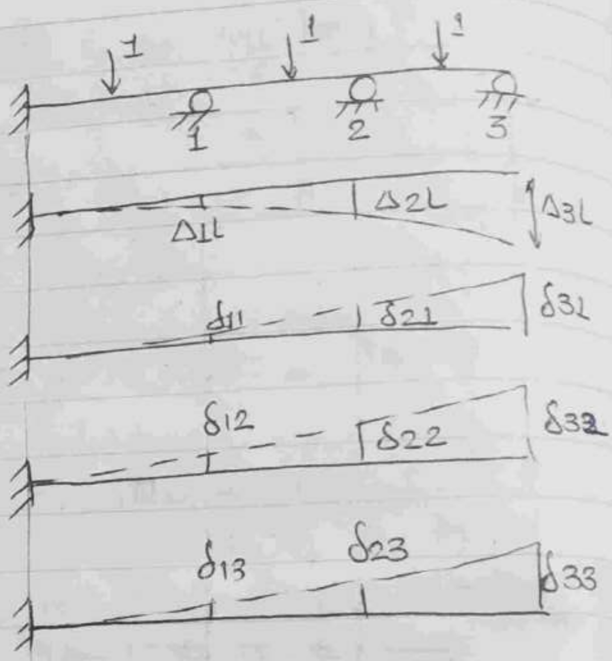
$\delta_{31}X_1+\delta_{32}X_2+\delta_{33}X_3+\Delta_{3L}=\Delta_3$

These eq^n can be expressed as

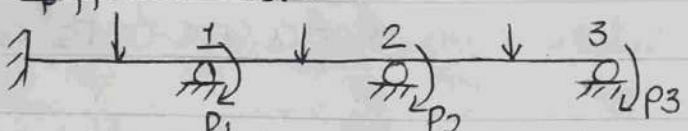
$$\begin{bmatrix}\delta_{11} & \delta_{12} & \delta_{13}\\ \delta_{21} & \delta_{22} & \delta_{23}\\ \delta_{31} & \delta_{32} & \delta_{33}\end{bmatrix}\begin{bmatrix}X_1\\X_2\\X_3\end{bmatrix}+\begin{bmatrix}\Delta_{1L}\\ \Delta_{2L}\\ \Delta_{3L}\end{bmatrix}=\begin{bmatrix}\Delta_1\\ \Delta_2\\ \Delta_3\end{bmatrix}$$

$[\delta][X]=[\Delta]$

∴ This is flexibility matrix.



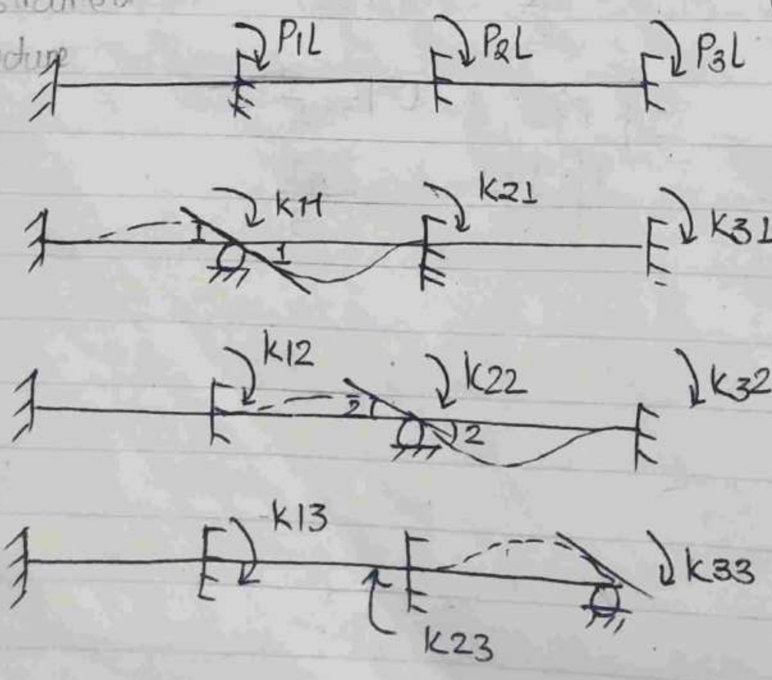
Stiffness Matrix



$D_{KI}=3j-(r+m)$
$=12-(6+3)=3$

Unknown $=\Delta_1,\Delta_2,\Delta_3$

Restrained Structure

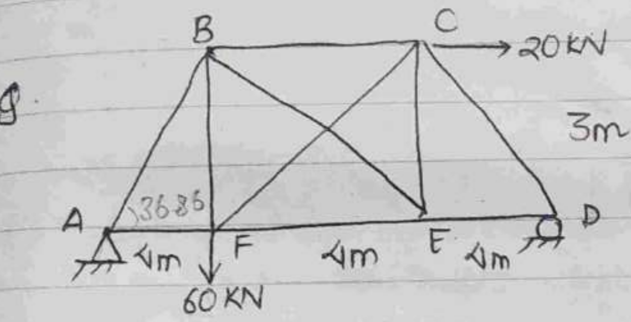


Eqbm eqⁿ at coordinate 1, 2 & 3 are:

$$P_1L + k_{11}\Delta_1 + k_{12}\Delta_2 + k_{13}\Delta_3 = P_1$$
$$P_2L + k_{21}\Delta_1 + k_{22}\Delta_2 + k_{23}\Delta_3 = P_2$$
$$P_3L + k_{31}\Delta_1 + k_{32}\Delta_2 + k_{33}\Delta_3 = P_3$$

$$\Rightarrow \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} + \begin{bmatrix} P_1L \\ P_2L \\ P_3L \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Q.

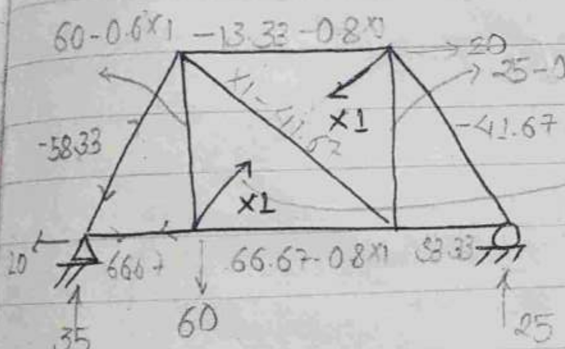


$AF - 20 - 58.33\cos 36.86 = 0$

$AF = 66.67$

$-AB\sin 36.86 + 35 = 0$

$AB = -58.34$



This member should be considered as well

$-X_1\cos 36.86 + 66.67 + FE = 0$

$FE = 66.67 - 0.8X_1$

CD	5	41.67	0	0
ED	4	33.33	0	0

Member	length	P	$\partial P/\partial X_1$	$P(\partial P/\partial X_1)\,L/AE$
AB	5	-58.33	0	0
AF	4	66.67	0	0
BF	3	$60-0.6X_1$	-0.6	$-1.8(60-0.6X_1)$
BC	4	$-13.33-0.8X_1$	-0.8	$-3.2(-13.33-0.8X_1)$
BE	5	$X_1-41.67$	1	$5(X_1-41.67)$
FE	4	$66.67-0.8X_1$	-0.8	$-3.2(66.67-0.8X_1)$
CE	3	$25-0.6X_1$	-0.6	$-1.8(25-0.6X_1)$
FC	5	X_1	1	$5X_1$

~~Sum = 0~~

Sum = 0 $\Rightarrow X_1 = 43.326$ kN

29.448 kN

Relation between flexibility & stiffness matrix

$$[\Delta] = [\delta][P]$$

Multiplying by $[\delta]^{-1}$

$$[\delta]^{-1}[\Delta] = [\delta^{-1}][\delta][P]$$

or, $[\delta]^{-1}[\Delta] = [P]$ — (i)

Also, $[P] = [k][\Delta]$ — (ii)

from (i) & (ii),

$$[\delta]^{-1} = [k]$$

Multiplying by $[\delta]$,

$$[\delta][\delta]^{-1} = [\delta][k]$$

or, $\boxed{[I] = [\delta][k]}$

Hence, $[\delta]$ and $[k]$ are inverse of each other.

If product of 2 matrices is identity matrix they are inverse of each other.

Chapter 2 Force Method / flexibility Matrix Method

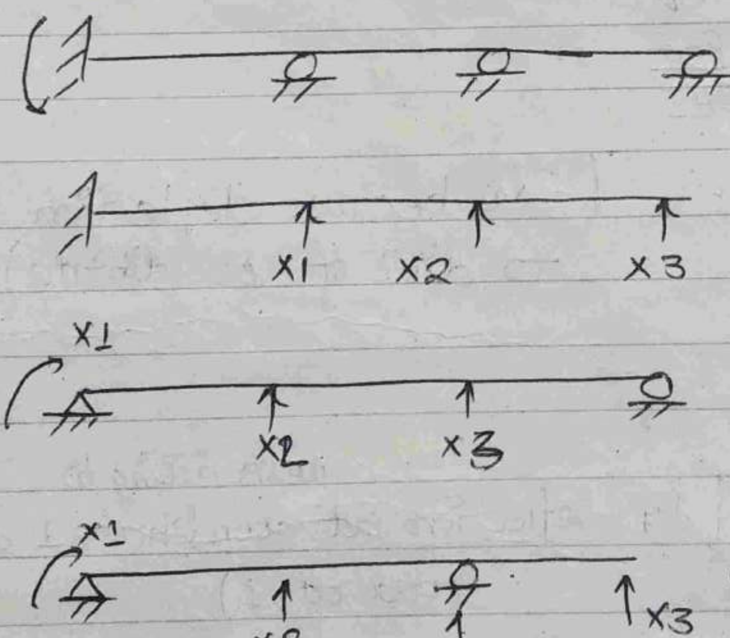
Method of consistent deformation

Definitions

1) It is a method of analysis of statically indeterminate structures in which force quantities are considered as unknown quantities and they are determined by solving compatibility equations.

Speciality: Unknown forces are calculated directly by using compatibility equation and it is independent of any other method.

Limitations: There are many choices to obtain primary structure which results in many flexibility equations. So, it is difficult to program its sol^{n} procedure in computer



& so on.

Remember

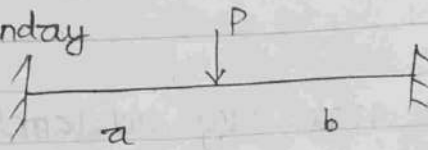
Arrow towards beam & fig in compression side

+ve BM ↓ +ve SF ↓

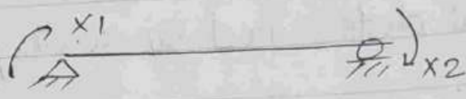
−ve BM ↑ −ve SF ↑

Sunday

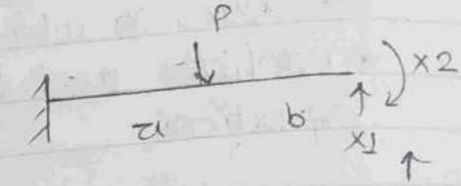
Q/A



Choices :

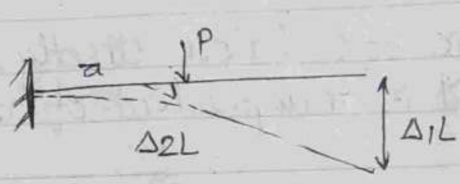


OR

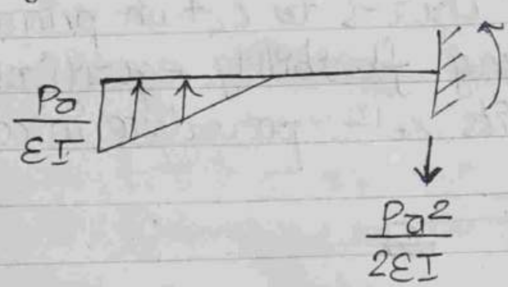


Coordinates

(Reactions if restraints hadn't been removed)



Conjugate beam :



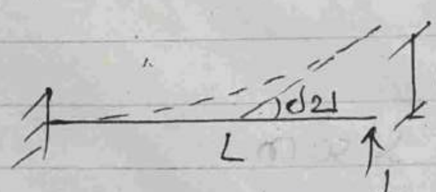
$\frac{1}{2} \times \frac{Pa}{EI} \times a \left(\frac{3b+2a}{3} \right) = \frac{Pa^2}{6EI}(2l+b)$

$\Delta_{1L} = -\frac{Pa^2}{6EI}(2L+b)$ (−ve because deflection is opposite to dirⁿ of coordinate)

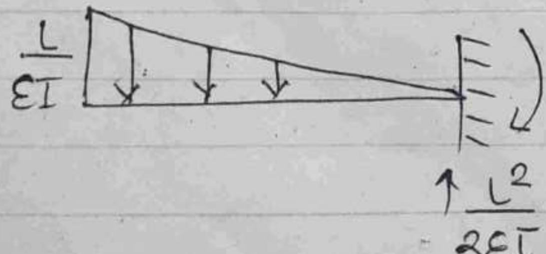
(Deflection)

$\Delta_{2L} = \frac{Pa^2}{2EI}$

(Slope)

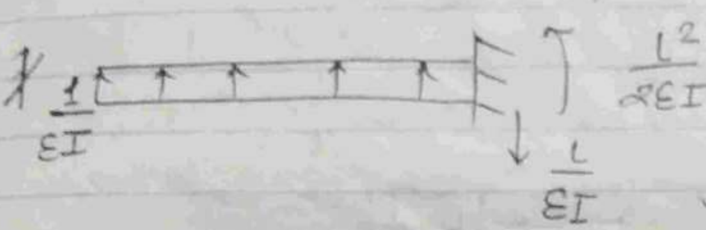


δ_{11} (Deflection ~~at~~ corresponding to coordinate 1 due to load at 1)



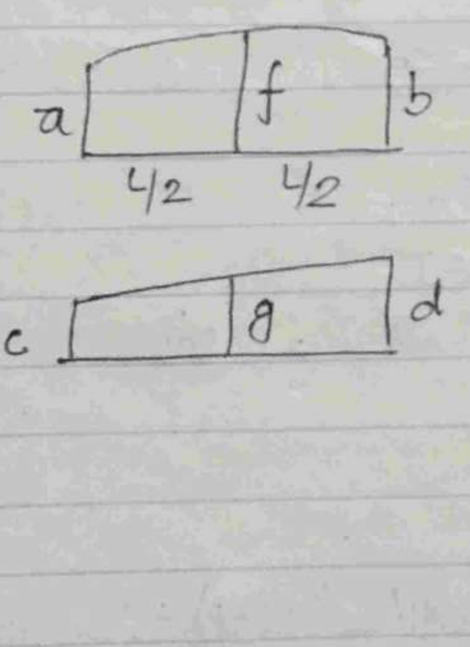
$\frac{1}{3} \frac{L^3}{EI}$

$\delta_{11} = \frac{L^3}{3EI}$, $\delta_{21} = -\frac{L^2}{2EI}$

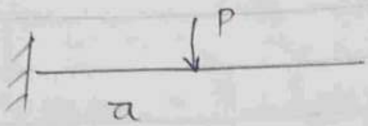


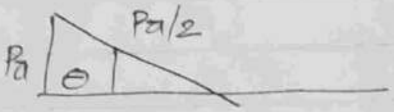
$\delta_{12} = -\frac{L^2}{2EI}$, $\delta_{22} = \frac{L}{EI}$

Note: $\Delta_{1L} = \int M_L \, m_1 \cdot \frac{dx}{EI}$



$\Delta_{1L} = \frac{L}{6EI}\left[ac + 4fg + bd\right]$

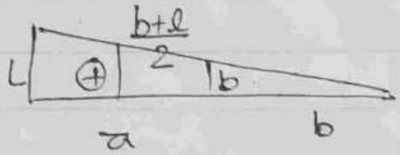


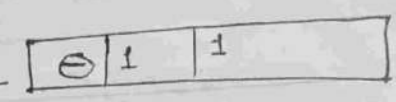


ML

m1

m2





$$\Delta_{1L} = \frac{a}{6EI} \times \left[(-Pa) \times L + 4 \times \left(-\frac{Pa}{2}\right) \times \left(\frac{b+L}{2}\right) + 0\right]$$

$$= \frac{a}{6EI}\left[-Pal - Pab - Pal\right]$$

$$= -\frac{Pa^2}{6EI}\left[2l + b\right]$$

$$\Delta_{2L} = \frac{a}{6EI}\left[(-Pa) \times (-1) + 4 \times \left(-\frac{Pa}{2}\right) \times (-1) + 0\right]$$

$$= \frac{a}{6EI}\left[Pa + 2Pa\right] = \frac{Pa^2}{2EI}$$

$$\delta_{11} = \frac{L}{6EI}\left[L \times L + 4\left(\frac{L}{2}\right)\left(\frac{L}{2}\right) + 0\right] = \frac{L^3}{3EI}$$

$$\delta_{12} = \delta_{21} = \int m_1 m_2 \frac{dx}{EI}$$

$$= \frac{L}{6EI}\left[L \times (-1) + 4 \times \left(\frac{L}{2}\right) \times (-1) + 0\right]$$

$$= -\frac{L^2}{2EI}$$

$\Delta_{1L} \rightarrow$ 1 represents location of deflection, L represents cause of deflection
$\delta_{11} \rightarrow$ first letter $\rightarrow$ location of deflection
second letter $\rightarrow$ location of load causing deflection

$$\delta_{22} = \frac{L}{6EI}\left[(-1)\times(-1) + 4(-1)\times(-1) + (-1)\times(-1)\right] = \frac{L}{EI}$$

$$\Delta_1 = 0$$

or, $\Delta_{1L} + \delta_{11}.X_1 + \delta_{12}\times X_2 = 0$

or, $\dfrac{L^3}{3EI}X_1 - \dfrac{L^2}{2EI}X_2 = \dfrac{Pa^2}{6EI}(2l+b)$

or, $\dfrac{L^2}{\cancel{6EI}}\left[2X_1L - 3X_2\right] = \dfrac{Pa^2}{\cancel{6EI}}(2L+b)$

or, $2X_1L - 3X_2 = \dfrac{Pa^2}{L^2}(2L+b)$

$$\Delta_2 = 0$$

or, $\Delta_{2L} + \delta_{21}X_1 + \delta_{22}.X_2 = 0$

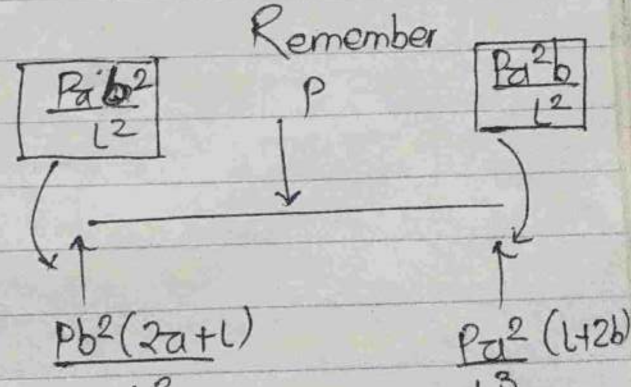
or, $\dfrac{Pa^2}{2EI} + \left(-\dfrac{L^2}{2EI}\right)\times X_1 + \dfrac{L}{EI}\times X_2 = 0$

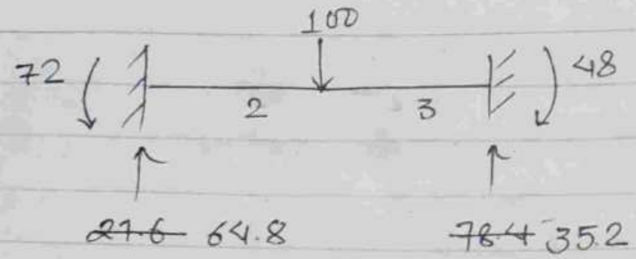
or, $\dfrac{L}{\cancel{2EI}}\left[LX_1 - 2X_2\right] = \dfrac{Pa^2}{\cancel{2EI}}$

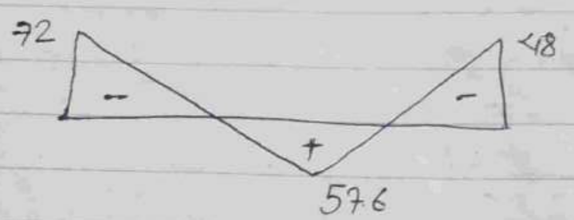
or, $LX_1 - 2X_2 = \dfrac{Pa^2}{L}$

Solving, $X_1 = \dfrac{Pa^2}{L^3}\left[2b+L\right]$

$X_2 = \dfrac{Pa^2b}{L^2}$







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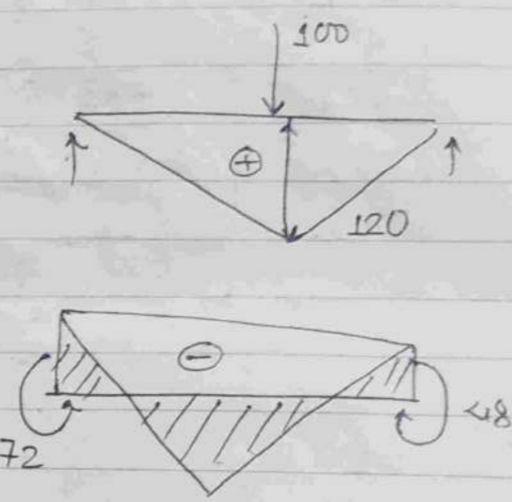
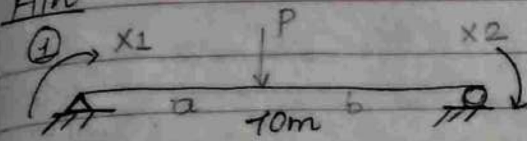


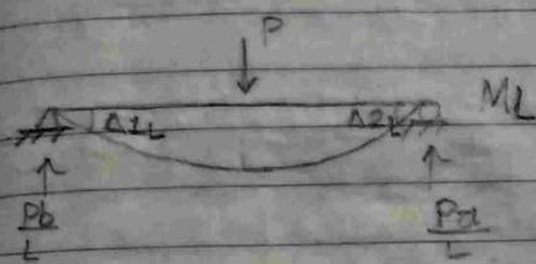
H/W



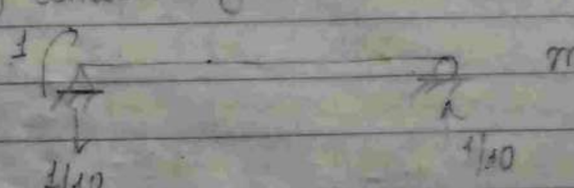
Flexibility matrix = ?

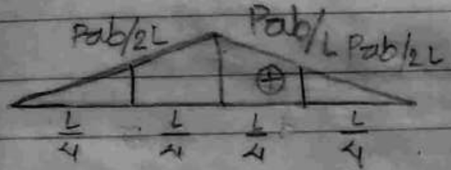
$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

1) Under action of load



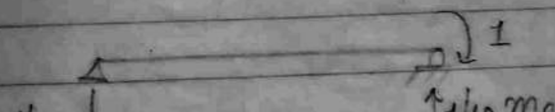
a) Corresponding to coordinate 1





$\Delta_{1L} = \int_0^a \frac{Pbx\,dx}{L\,EI} = \frac{Pa^2b}{2LEI}$ $\quad \Delta_{2L} = \int_0^b \frac{-Pax\,dx}{L} = \frac{-Pab^2}{2L\,EI}$

b) Corresponding to coordinate 2



2) Under action of redundant

$\delta_{11} = \int \frac{m_1^2\,dx}{EI} = \frac{10}{6EI}\left[1\times1 + 4\times\frac{1}{2}\times\frac{1}{2} + 0\right] = \frac{10}{3EI}$

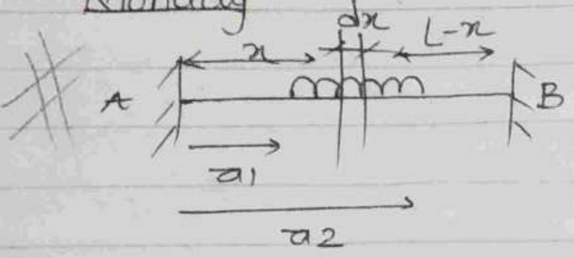
$\delta_{12} = \delta_{21} = \frac{10}{6EI}\left[1\times0 + 4\times\frac{1}{2}\times\left(\frac{-1}{2}\right) + 0\right] = \frac{-5}{3EI}$

$\delta_{22} = \frac{10}{6EI}\left[1 + 1 + 0\right] = \frac{10}{3EI}$

Don't miss

8/16

Monday



$$M_{AB} = \int_{a_1}^{a_2} \frac{w.\,dx\,(L-x)^2.x}{L^2}$$

For whole span, $M_{AB} = \int_0^l \frac{w}{L^2}(xL^2 - 2x^2L + x^3)\,dx$

$$= \frac{w}{L^2}\left[\frac{L^2x^2}{2} - \frac{2x^3L}{3} + \frac{x^4}{4}\right]_0^l$$

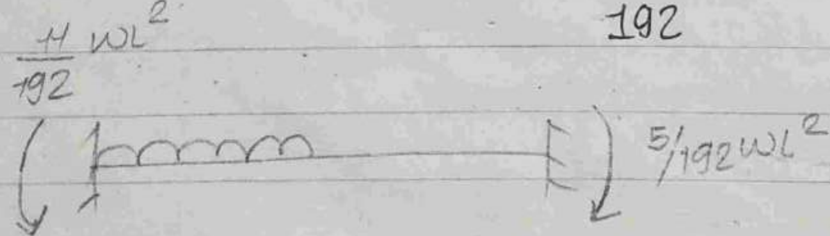
$$= \frac{w}{L^2}\left[\frac{L^4}{2} - \frac{2L^4}{3} + \frac{L^4}{4}\right]$$

$$= \frac{w}{12L^2} \times L^4 = \frac{wL^2}{12}$$

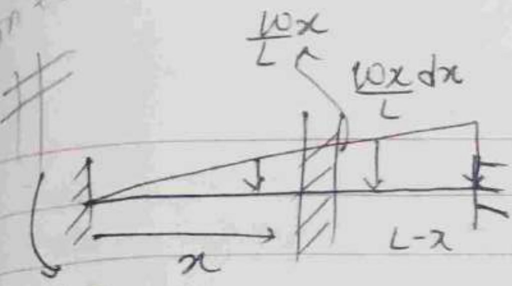
For half span, $M_{AB} = \int_0^{l/2} \frac{w}{L^2}(xL^2 - 2x^2L + x^3)\,dx$

$$= \frac{w}{L^2}\left[\frac{L^2}{2} \times \frac{L^2}{4} - \frac{2}{3}L \times \frac{L^3}{8} + \frac{L^4}{64}\right)$$

$$= \frac{11}{192}wL^2$$



Don't miss



MAB

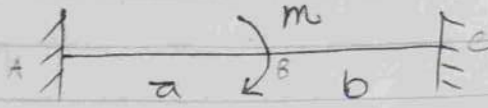
For whole span,

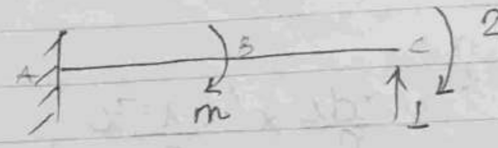
$$M_{AB} = \int_0^{l} \frac{wx\,dx}{l} \times \frac{(l-x)^2 x}{l^2}$$

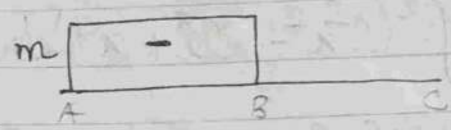
$$= \frac{w}{l^3}\int_0^{l} (l^2x^2 - 2lx^3 + x^4)\,dx$$

$$= \frac{w}{l^3}\left[\frac{l^2x^3}{3} - 2l\frac{x^4}{4} + \frac{x^5}{5}\right]_0^{l}$$

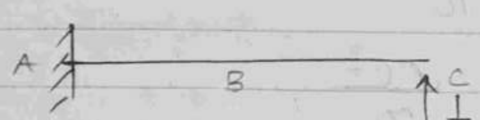
$$= \frac{w}{l^3}\left[\frac{l^5}{3} - \frac{2l^5}{4} + \frac{l^5}{5}\right] = \frac{wl^2}{30}$$

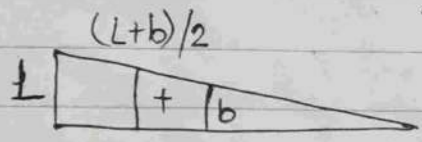


Primary structure : 

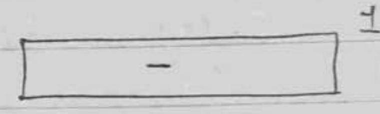


ML





m1



m2

$$\Delta_{1L} = \frac{a}{6EI}\left[(-m)\times L + 4\times(-m)\times\left(\frac{L+b}{2}\right) + (-m)\times b\right]$$

$$= \frac{am}{6EI}\left[-L - 2L - 2b - b\right] = -\frac{ma}{6EI}\left[3L+3b\right]$$

$$= -\frac{ma}{2EI}\left[L+b\right]$$

$$\Delta_{2L} = \frac{a}{6EI}\left[(-m)\times(-1) + 4\times(-m)\times(-1) + (-m)\times(-1)\right]$$

$$= \frac{+a}{6EI}\times 6m = \frac{+am}{EI}$$

$$\delta_{11} = \frac{L}{6EI}\left[L\times L + 4\times\frac{L}{2}\times\frac{L}{2} + 0\right] = \frac{L}{6EI}\times 2L^2 = \frac{L^3}{3EI}$$

$$\delta_{21} = \delta_{12} = \frac{L}{6EI}\left[L\times(-1) + 4\times\frac{L}{2}\times(-1) + 0\right] = -\frac{L^2}{2EI}$$

$$\delta_{22} = \frac{L}{6EI}\left[1+4+1\right] = \frac{L}{EI}$$

Compatibility eq^n.

$\Delta_1 = 0$

or, $\Delta_{1L} + X_1\cdot\delta_{11} + X_2\cdot\delta_{12} = 0$

or, $-\frac{ma}{2EI}\left[L+b\right] + \frac{L^3}{3EI}X_1 - \frac{L^2}{2EI}X_2 = 0$

or, $\frac{L^2}{6EI}\left[2X_1L - 3X_2\right] = \frac{ma}{2EI}\left[L+b\right]$

or, $2LX_1 - 3X_2 = \frac{3ma}{L^2}\left[L+b\right]$ — (i)

$\Delta_{2L} + X_2\,\delta_{22} + X_1\,\delta_{21} = 0 = \Delta_2$

or, $-\frac{L^2}{2EI}X_1 + \frac{L}{EI}X_2 + \frac{ma}{EI} = 0$

or, $\frac{L}{2EI}\left[LX_1 - 2X_2\right] = \frac{ma}{EI}$

or, $LX_1 - 2X_2 = \frac{2ma}{L}$ — (ii)

Solving, $-3X_2 + 4X_2 = \frac{3ma}{L} + \frac{3mab}{L^2} - \frac{4ma}{L}$

or, $X_2 = \frac{3mab}{L^2} - \frac{ma}{L} = \frac{ma}{L^2}\left[3b - L\right]$

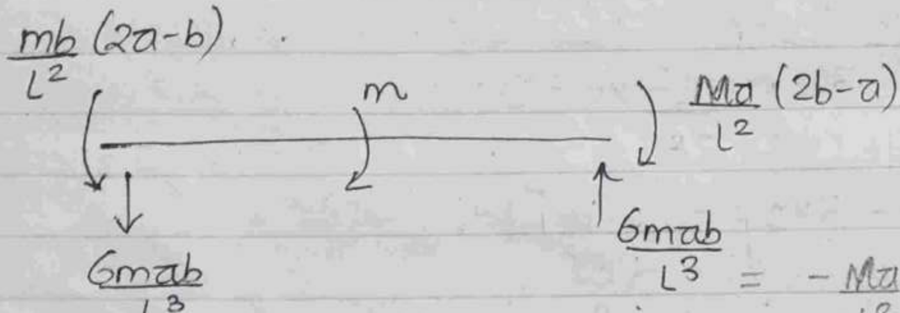
$= \frac{ma}{L^2}\left[2b - a\right]$

$$LX_1 - \frac{2ma}{L^2}(2b-a) = \frac{2ma}{L}$$

or, $LX_1 = \frac{4mab}{L^2} - \frac{2ma^2}{L^2} + \frac{2ma}{L}$

or, $LX_1 = \frac{2ma}{L^2}\left[2b - a + L\right]$

or, $X_1 = \frac{2ma}{L^3}\left[2b - \cancel{a} + \cancel{a} + b\right] = \frac{6mab}{L^3}$



$$= -\frac{Ma}{L^2}(2b-a) - m + \frac{6mab}{L^3} \times L$$

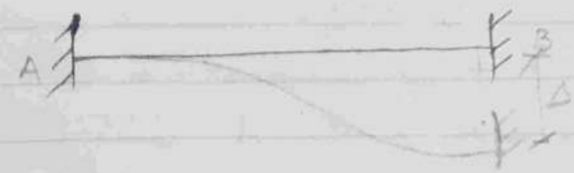
$$\frac{m}{L^2}\left[-L^2 - a(2b-a) + 6ab\right]$$

$$\frac{m}{L^2}\left[-\cancel{a^2} - 2ab - b^2 - 2ab + \cancel{a^2} + 6ab\right]$$

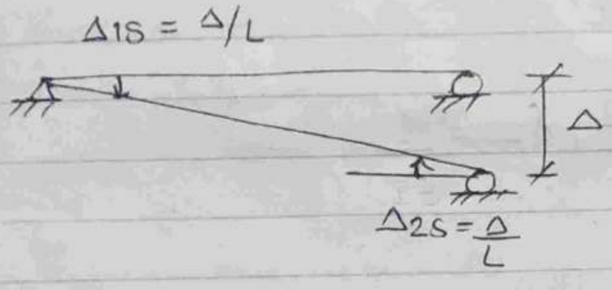
$$\frac{m}{L^2}\left[2ab - b^2\right]$$

$$\frac{mb}{L^2}\left[2a - b\right]$$

Effect of settlement of support and rotation of support and temperature variation



primary structure



$\delta_{11} = \frac{L}{3EI}$, $\delta_{12} = \frac{-L}{6EI} = \delta_{21}$, $\delta_{22} = \frac{L}{3EI}$

$\delta_{11}X_1 + \delta_{12}X_2 + \Delta_{1S} = \Delta_1 = 0$

or, $\frac{L}{3EI}X_1 - \frac{L}{6EI}X_2 = -\frac{\Delta}{L}$

or, $\frac{L}{6EI}\left[2X_1 - X_2\right] = -\frac{\Delta}{L}$ $\Rightarrow 2X_1 - X_2 = \frac{-\Delta \times 6EI}{L^2}$

$\delta_{22}X_2 + \delta_{21}X_1 + \Delta_{2S} = 0$

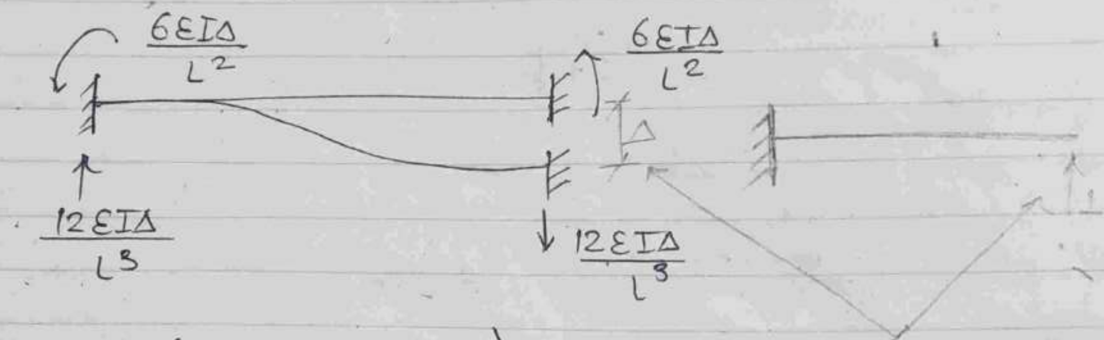
or, $-\frac{L}{6EI}X_1 + \frac{L}{3EI}X_2 + \frac{\Delta}{L} = 0$

or, $X_1 - 2X_2 = 6EI \times \frac{\Delta}{L}$

Solving, $-X_1 + 4X_1 = -\frac{6EI\Delta}{L^2} - \frac{12EI\Delta}{L^2}$

or, $+3X_1 = -18EI\Delta/L^2$ $\Rightarrow X_1 = -6EI\Delta/L^2$

$X_2 = -\frac{6EI\Delta}{L^2}$



settlement & coordinate's effect coincides

OR,

$\delta_{11} = \frac{L^3}{3EI}$, $\delta_{12} = \delta_{21} = -\frac{L^2}{2EI}$, $\delta_{22} = \frac{L}{EI}$

Compatibility eq^n (small change)

$\delta_{11} X_1 + \delta_{12} X_2 = \Delta_1 = -\Delta$ ⇒ -ve becoz settlement opp to dir^n of coordinate

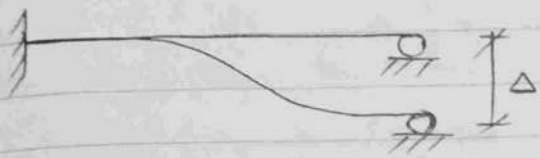
$\delta_{21} X_1 + \delta_{22} X_2 = \Delta_2 = 0$

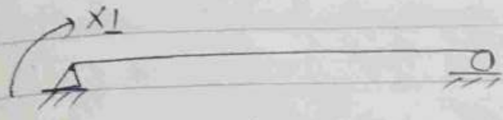
(effect of settlement/rotation)

If settlement/rotation coincides with coordinate, it is written on RHS of compatibility eq^n else written in LHS as

$\delta_{11} X_1 + \delta_{12} X_2 + \Delta_{1S} = 0$

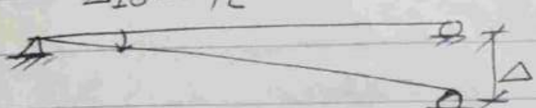
Solving, $X_1 = -\frac{12EI\Delta}{L^3}$, $X_2 = -\frac{6EI\Delta}{L^2}$





$$\delta_{11} = \frac{L}{3EI}, \quad \Delta_{1S} = \frac{\Delta}{L}$$

$\Delta_{1S} = \Delta / L$

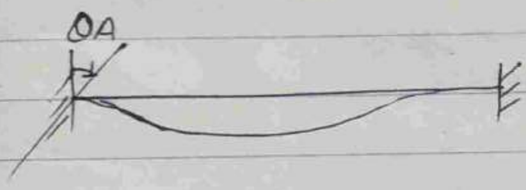


$$\delta_{11} X_1 + \Delta_{1S} = 0$$

$$\text{or, } \frac{L}{3EI} \times X_1 = -\frac{\Delta}{L}$$

Δ_{1S} = effect of Δ at 1
= effect of load at coordinate settlement

$$\text{or, } X_1 = -\frac{3\Delta EI}{L^2}$$



$$\delta_{11} = \frac{L}{3EI}, \quad \delta_{12} = \delta_{21} = -\frac{L}{6EI}$$

$$\delta_{22} = \frac{L}{3EI}$$

$$\delta_{11} X_1 + \delta_{12} X_2 = \theta_A$$

$$\text{or, } \frac{L}{3EI} X_1 - \frac{L}{6EI} X_2 = \theta_A$$

$$\text{or, } \frac{L}{6EI}\left[2X_1 - X_2\right] = \theta_A$$

$$\text{or, } 2X_1 - X_2 = \frac{6\theta_A EI}{L}$$

$$\delta_{21} X_1 + \delta_{22} X_2 = 0$$

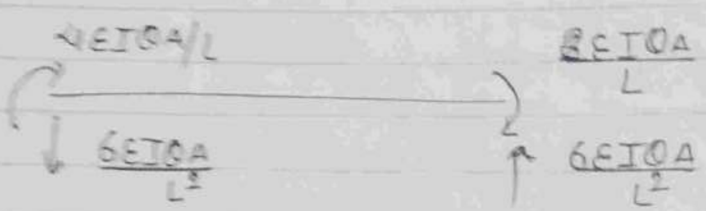
$$\text{or, } -\frac{L}{6EI} X_1 + \frac{L}{3EI} X_2 = 0$$

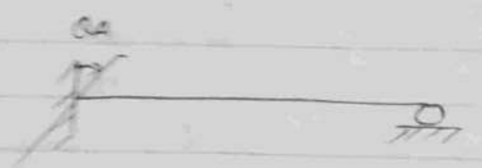
$$\text{or, } \cancel{2}X_1 - 2X_2 = 0$$

$$\text{or, } \cancel{2}X_1 = 2X_2$$

$$\text{Solving, } 4X_2 - X_2 = \frac{6\theta_A EI}{L}$$

$$\therefore X_2 = \frac{2\theta_A EI}{L}, \quad X_1 = \frac{4\theta_A EI}{L}$$





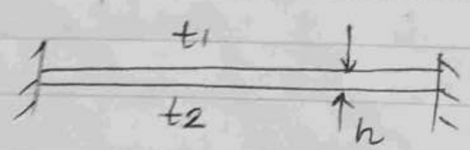
$\delta_{11} = \dfrac{L}{3EI}$,

$\delta_{11} x_1 = \theta_A$

or, $x_1 = \dfrac{3EI\theta_A}{L}$

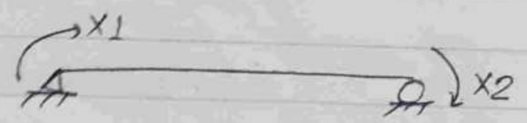
Tuesday 1/8/17

Temperature Variation

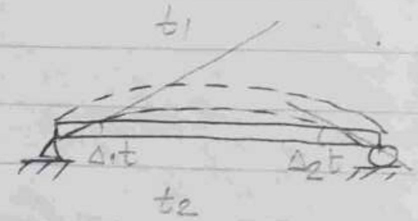


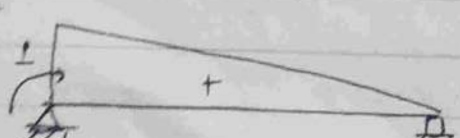
$t_1 > t_2$

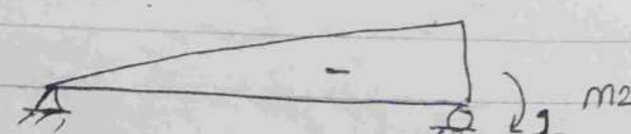
primary structure



$\Rightarrow$





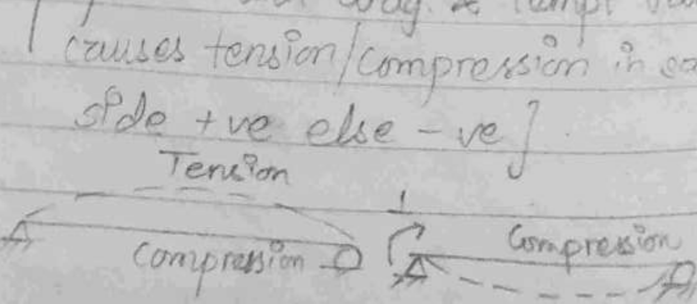


$\delta_{11} = \dfrac{L}{3EI}$, $\delta_{12} = \delta_{21} = -\dfrac{L}{6EI}$, $\delta_{22} = \dfrac{L}{3EI}$

$\Delta_1 t = -\alpha\left(\dfrac{t_1 - t_2}{h}\right) \times \dfrac{L}{2}$

$\Delta_2 t = \alpha\left(\dfrac{t_1 - t_2}{h}\right) \times \dfrac{L}{2}$

[If both unit diag. & tempt var causes tension/compression in same side +ve else -ve]



$\delta_{11}X_1 + \delta_{12}X_2 + \Delta_1 t = 0$

or, $\frac{L}{3EI}X_1 - \frac{L}{6EI}X_2 = \alpha\left(\frac{t_1-t_2}{h}\right)\times\frac{L}{2}$

or, $\frac{L}{6EI}\left[2X_1 - X_2\right] = \frac{\alpha L}{2}\left(\frac{t_1-t_2}{h}\right)$

or, $2X_1 - X_2 = 3EI\alpha\frac{(t_1-t_2)}{h}$

$\delta_{21}X_1 + \delta_{22}X_2 + \Delta_2 t = 0$

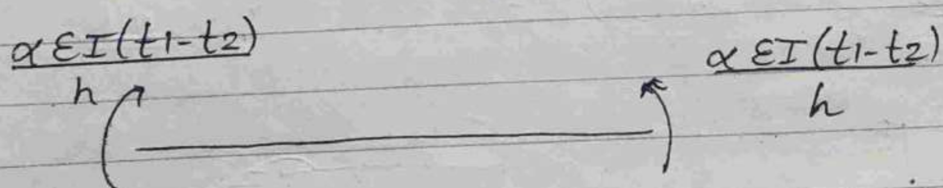
or, $-\frac{L}{6EI}X_1 + \frac{L}{3EI}X_2 = -\alpha\left(\frac{t_1-t_2}{h}\right)\times\frac{L}{2}$

or, $\frac{L}{6EI}(\cancel{2}X_1 - 2X_2) = \frac{\alpha L}{2}\left(\frac{t_1-t_2}{h}\right)$

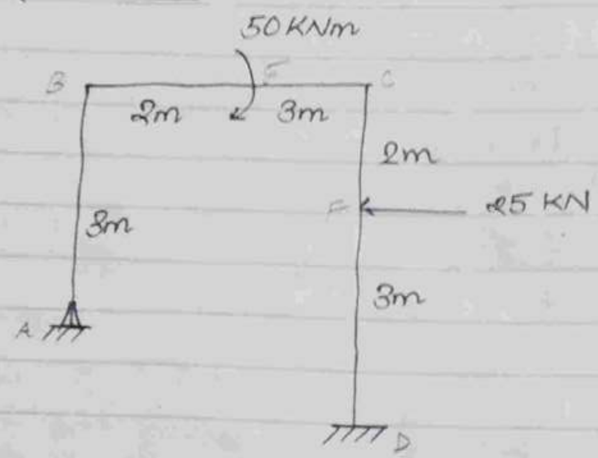
or, $X_1 - 2X_2 = 3EI\alpha\left(\frac{t_1-t_2}{h}\right)$

Solving, $3X_2 = -3EI\alpha\frac{(t_1-t_2)}{h}$ $\therefore X_2 = -\alpha EI\frac{(t_1-t_2)}{h}$

$X_1 = \alpha EI\frac{(t_1-t_2)}{h}$

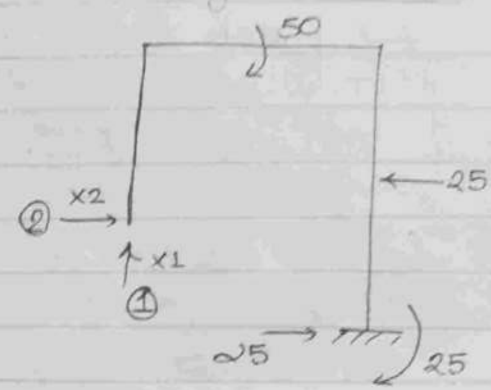


Numerical

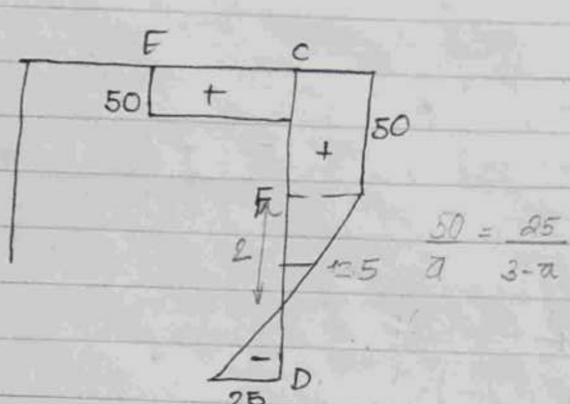


DSI = 2

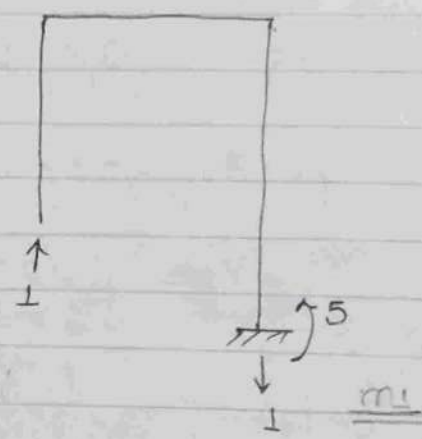
Primary structure



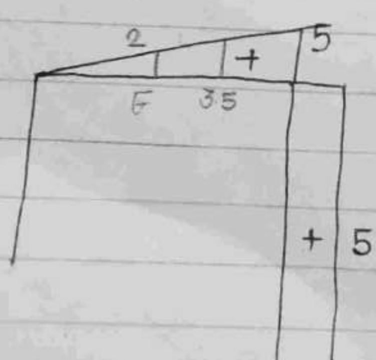
ML



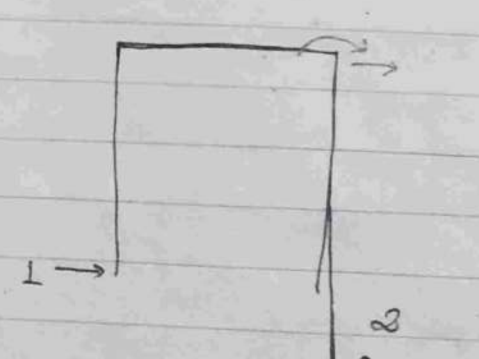
ML (BMD)



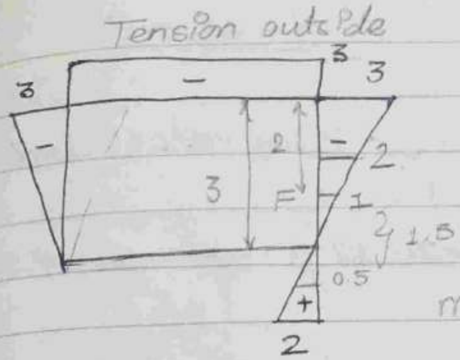
m1



m1 (BMD)



m2



By method of integration

Portion	Origin	Limit	M_L	m_1	m_2
AB	A	0-3	0	0	$-x$
BE	B	0-2	0	x	-3
EC	E	0-3	50	$2+x$	-3
CF	F	0-2	$25(x+3)-25-25x=50$	5	$-x-1$
DF	D	0-3	$-25+25x$	5	$2-x$

$$\Delta_{1L} = \Sigma\int \frac{M_L m_1 dx}{EI} = \frac{1}{EI}\left[\int_0^3 (100+50x)dx + \int_0^2 250\,dx + \int_0^3 (-125+125x)dx\right]$$

$$= \frac{1}{EI}\left[525 + 500 + 187.5\right] = \frac{1212.5}{EI}$$

By graphical,

$$\Delta_{1L} = \frac{3}{6EI}\left[50\times2 + 4\times50\times3.5 + 50\times5\right] + \frac{2}{6EI}\left[50\times5 + 4\times50\times5 + 50\times5\right]$$

$$+ \frac{3}{6EI}\left[50\times5 + 4\times12.5\times5 - 25\times5\right]$$

$$= \frac{525}{EI} + \frac{500}{EI} + \frac{187.5}{EI} = \frac{1212.5}{EI}$$

$$\Delta_{2L} = \frac{1}{EI}\left[\int_0^3 -150\,dx + \int_0^2 (-50x-50)dx + \int_0^3 (-25+25x)(2-x)dx\right]$$

$$= -\frac{\cancel{912.5}\ 687.5}{EI}$$

Graphical

$$\Delta_{2L} = \frac{3}{6EI}\left[-150 + 4\times -150 - 150\right] + \frac{2}{6EI}\left[-150 + 4\times 50\times(-2) + 50\times(-1)\right] +$$

$$\frac{3}{6EI}\left[50\times(-1) + 4\times 12.5\times(0.5) + (-25)\times 2\right]$$

$$= -\frac{450}{EI} - \frac{200}{EI} - \frac{37.5}{EI} = -\frac{687.5}{EI}$$

$$\delta_{11} = \int m_1^2 \frac{dx}{EI} = \int_0^2 \frac{x^2 dx}{EI} + \int_0^3 (2+x)^2 \frac{dx}{EI} + \int_0^5 5^2 dx$$

$$= \frac{166.67}{E}$$

$$\delta_{12} = \delta_{21} = \cancel{\frac{12.5}{EI}} \quad -\frac{50}{EI}$$

$$\delta_{22} = \frac{197}{8EI}$$

Matrix sol^n :

Actual $\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} - \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \end{bmatrix}$

$\Delta_1 = \Delta_2 = 0$

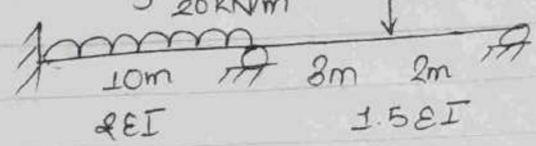
$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \begin{bmatrix} -\Delta_{1L} \\ -\Delta_{2L} \end{bmatrix}$$

$$\Rightarrow X_1 = \cancel{-6.741} \quad -5.358$$

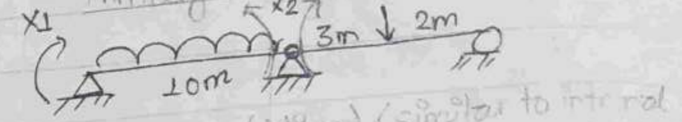
$$X_2 = \cancel{1.778} \quad 6.389$$

If fixed support is replaced by hinge support, EI is still 1 ... take middle support as hinge so that moment is released there

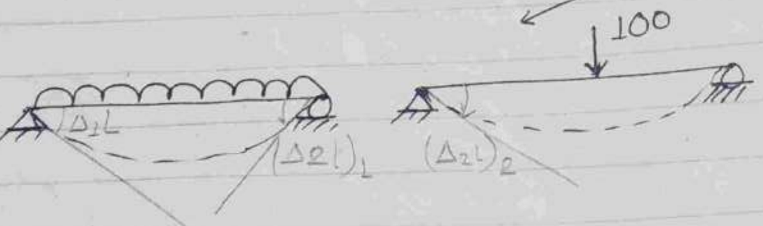
Sunday



Primary structure

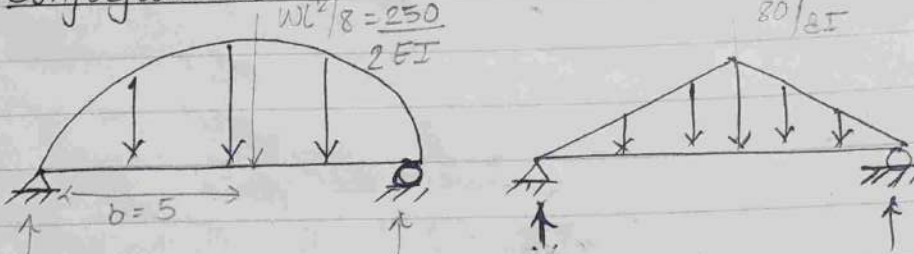


(Hinge) (similar to internal hinge)



(Analyzing by separating at hinge)

Conjugate beam



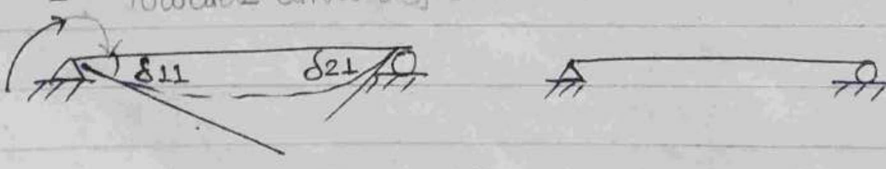
$\frac{2}{3}bh = \frac{2}{3} \times 5 \times \frac{250}{2EI}$ $\quad \frac{1250}{3EI}$ $\quad \frac{280}{3EI}$ $\quad 320/3EI$

$= \frac{1250}{3EI}$

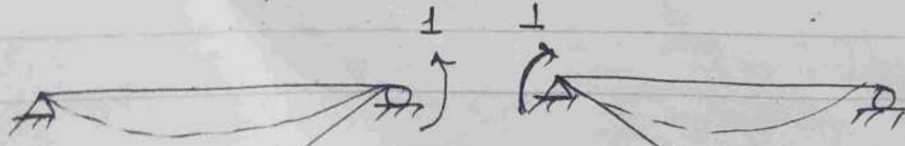
$\Delta 1l = \frac{1250}{3EI}$

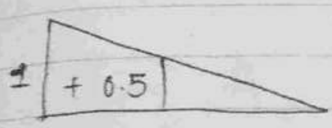
$\Delta 2l = \frac{1250}{3EI} + \frac{280}{3EI} = \frac{510}{EI}$

1 Towards arrow deflection



m1





BMD for m1

BMD for m2

$$\delta_{11} = \frac{10}{6 \times 2EI}\left[1 + 4 \times 0.5^2\right] = \frac{5}{3EI}$$

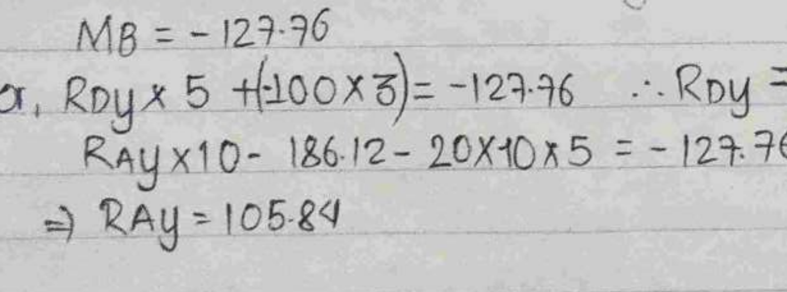
$$\delta_{12} = \frac{10}{12EI}\left[0 + 4 \times 0.5^2 + 0\right] + 0 = \cancel{\frac{5EI}{6}}\ \frac{5}{6EI}$$

$$\delta_{22} = \frac{10}{12EI}\left[1 + 4 \times 0.5^2\right] + \frac{5}{6 \times 1.5EI}\left[1 + 4 \times 0.5^2\right] = \frac{25}{9EI}$$

$$\delta_{11}X_1 + \delta_{12}X_2 + \Delta_{1L} = \Delta_1 = 0$$

and $\delta_{22}X_2 + \delta_{21}X_1 + \Delta_{2L} = \Delta_2 = 0$

Solving, $X_1 = -186.12$, $X_2 = -127.76$



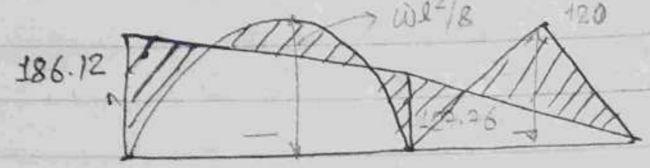
$M_B = -127.76$

or, $R_{Dy} \times 5 + (-100 \times 3) = -127.76$ $\therefore R_{Dy} = \cancel{-85.552}\ 34.45$

$R_{Ay} \times 10 - 186.12 - 20 \times 10 \times 5 = -127.76$

$\Rightarrow R_{Ay} = 105.84$

$\Delta = \int M q N_P \frac{dx}{EI}$

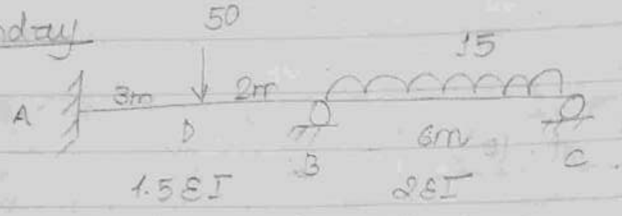


BMD

Settlement

Sunday

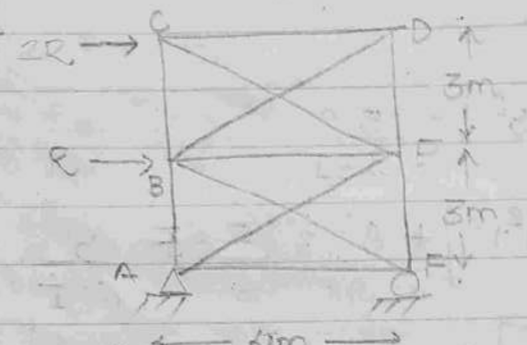
HW



$\Delta_B = \frac{100}{EI}$ (↓)

$\Delta_C = \frac{200}{EI}$ (↓)

H/W Monday

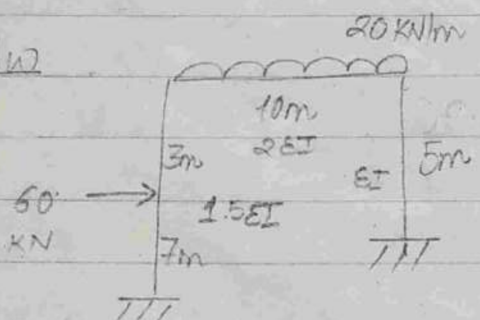


AE = constant

Mem L P p1 p2 $P p_1 L$ $P p_2 L$ $p_1^2 L$ $p_2^2 L$ $p_1 p_2 L$

$P' = P + p_1 x_1 + p_2 x_2$

H/W

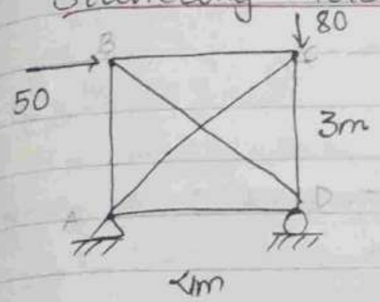


Deflection. $\Delta = F_q \times \Delta L_p$

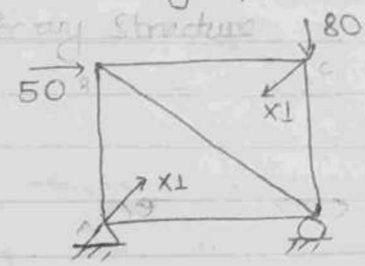
$F_q \rightarrow$ internal forces due to ...

$\Delta L_p \rightarrow$ deformation of member due to ...

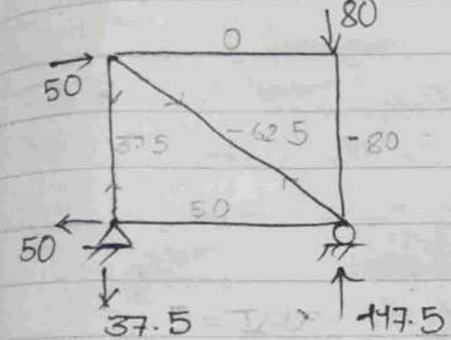
Statically indeterminate truss by force method



Primary Structure



AF → constant if not given



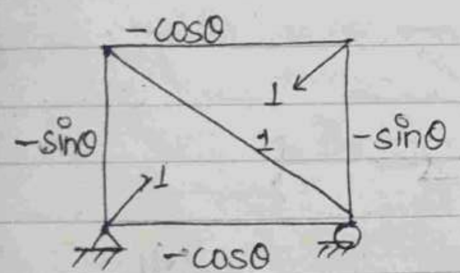
$\Sigma F_y = 0$

or, $-37.5 - F_{BD} \sin\theta = 0$

$\Rightarrow F_{BD} = -62.5$

P

If fabrication effect and temperature effect



p

$p \times \left(PL/AE + \alpha \Delta T L + \Delta L_{fab}\right)$

$p \times \frac{PL}{AE}$

Mem.	Length	P	p	PpL	p^2L	$P' = P + p_1 X_1$
AB	3	37.5	−0.6	−67.5	1.08	~~433.56~~ 23.75
AD	4	50	−0.8	−160	2.56	
AC	5	—	1	—	5	
BC	4	0	−0.8	0	2.56	
CD	3	−80	−0.6	144	1.08	
BD	5	−62.5	1	−312.5	5	

$\Delta_{1L} = -396 \quad \delta_{11} = 17.28$

$\delta_{11} X_1 + \Delta_{1L} = 0$

$\Rightarrow X_1 = 22.92$

$M_1 = M_{AB}$

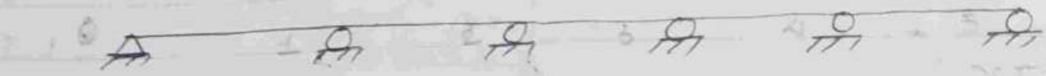
$M_2 = M_{BC}$ & so on

The calculated moments are end moment

Monday | 076 | 7 | 23

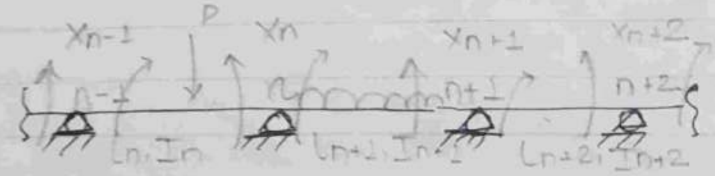
Three moment equation

Sign convention clockwise +ve

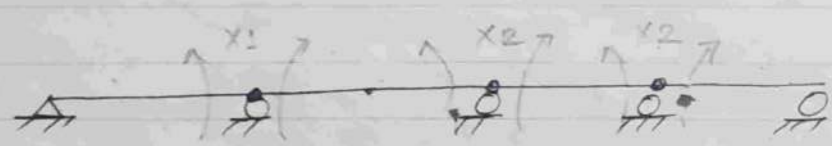


(numbering)

→ Start from 0. If last value is k, DSI = k − 1

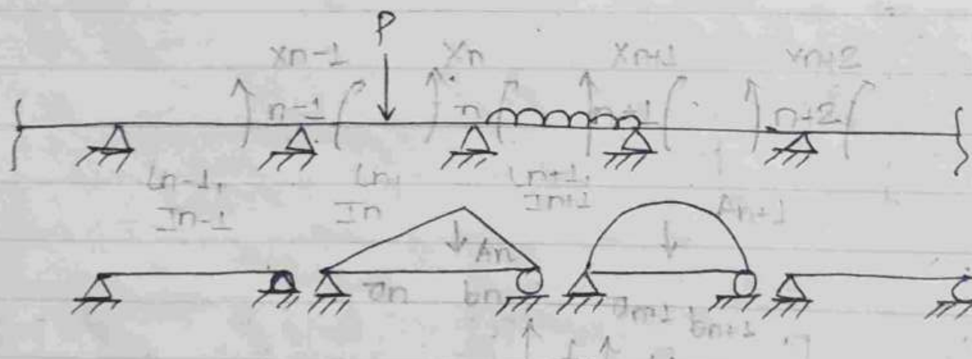


Calculate external moment.

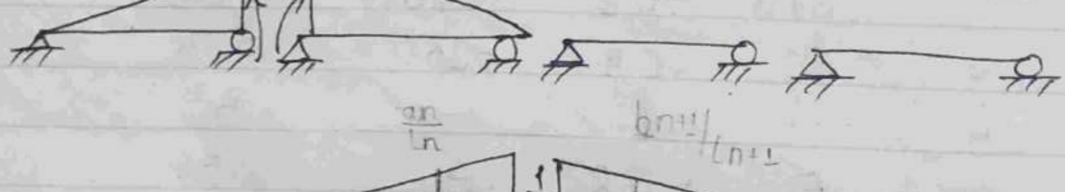


$\alpha DSI = 3$

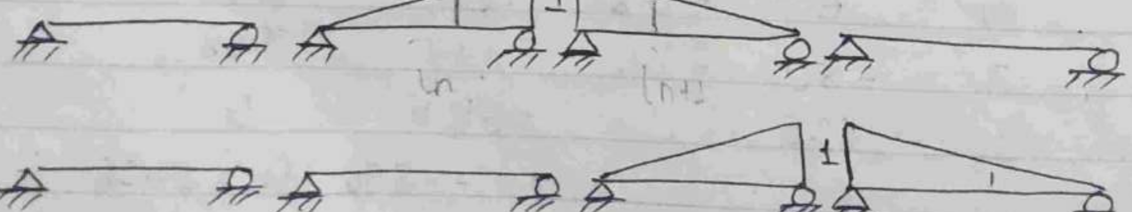
Add hinges at 1, 2, 3 (internal)



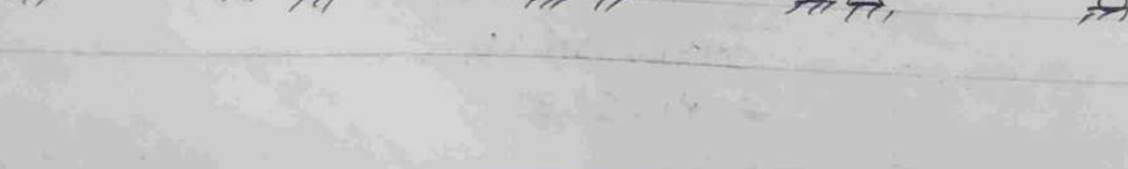
M_L



m_{n-1}



m_n



m_{n+1}

Δ = Centroid of BMD diagram in loading & corresponding coordinate in ... diagram $\times$ Area of BMD of loading

Compatibility equation

$$\delta_{11}X_1 + \delta_{12}X_2 + \dots + \delta_{1n-1}X_{n-1} + \delta_{1n}X_n + \delta_{1n+1}X_{n+1} + \dots$$
$$+ \delta_{1k-1}X_{k-1} + \Delta_{1L} = 0$$

$\vdots$

$$\delta_{n1}X_1 + \delta_{n2}X_2 + \dots + \delta_{n(n-1)}X_{n-1} + \delta_{nn}X_n + \delta_{n(n+1)}X_{n+1} + \dots$$
$$+ \delta_{n(k-1)}X_{k-1} + \Delta_{nL} = 0 \quad \text{—(i)}$$

→ In eqⁿ (i), all other coefficients except, $\delta_{n(n-1)}$, δ_{nn} & $\delta_{n(n+1)}$ and Δ_{nL} are equal to 0.

$$\delta_{n(n-1)} = \frac{L_n}{6EI_n}\left[0+1+0\right] = \frac{L_n}{6EI_n}$$

$$\delta_{nn} = \frac{L_n}{6EI_n}\left[0+1+1\right] + \frac{L_{n+1}}{6EI_{n+1}}\left[0+1+1\right]$$

$$= \frac{L_n}{3EI_n} + \frac{L_{n+1}}{3EI_{n+1}}$$

$$\delta_{n(n+1)} = \frac{L_{n+1}}{6EI_{n+1}}$$

$$\Delta_{nL} = \frac{A_n}{EI_n} \times \frac{a_n}{L_n} + \frac{A_{n+1}}{EI_{n+1}} \times \frac{b_{n+1}}{L_{n+1}}$$

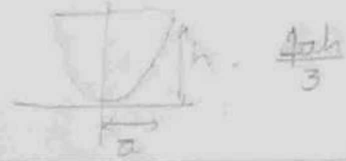
Then, $$\frac{L_n}{6EI_n}X_{n-1} + \left(\frac{L_n}{3EI_n} + \frac{L_{n+1}}{3EI_{n+1}}\right)X_n + \frac{L_{n+1}}{6EI_{n+1}} \times X_{n+1} +$$

$$\frac{A_n a_n}{EI_nL_n} + \frac{(A_{n+1})(b_{n+1})}{E(I_{n+1})(L_{n+1})} = 0$$

or, $$\frac{L_nX_{n-1}}{I_n} + \cancel{\frac{2X_nL_n}{I_n}} + 2X_n\left[\frac{L_n}{I_n} + \frac{L_{n+1}}{I_{n+1}}\right] + \frac{L_{n+1}}{I_{n+1}} \times X_{n+1} =$$

$$-\frac{6A_na_n}{I_nL_n} - \frac{6A_{n+1}b_{n+1}}{I_{n+1} \cdot L_{n+1}}$$

This is three moment equation in general form.



f = fictitious

$$= -6\left[\frac{R^{fr}_{n}}{I_n} + \frac{R^{fr}_{n+1}}{I_{n+1}}\right]$$

where $R^{fr}_{n} = \frac{A_n \bar{a}_n}{L_n}$ = reaction at fictitious beam

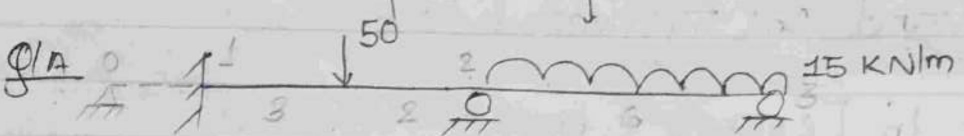
R^{fr}_{n} → reaction at right support of n^{th} span

→ Clausius Clapeyron eqⁿ

→ If moment of inertia for all span is equal,

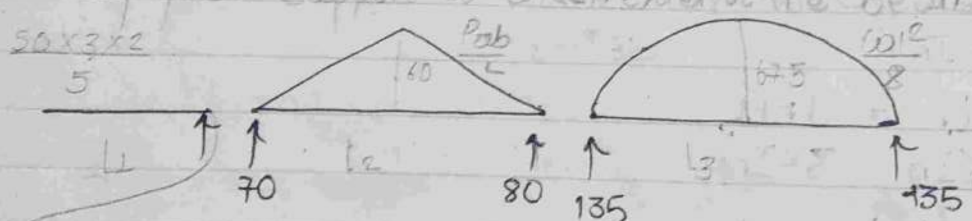
$$L_n x_{n-1} + 2x_n\left[L_n + L_{n+1}\right] + L_{n+1} x_{n+1} = -6\left[R^{fr}_{n} + R^{fL}_{n+1}\right]$$

Q/A



Clockwise (+ve)

→ If fixed support at ends, extend the beam & add hinge



fixed support removed & internal hinge added

→ writing 3 moment eqⁿ for support 1,

$n = 1$

$$M_0 \cancel{\frac{L_1}{I_1}}^{0} + 2M_1\left[\frac{L_1}{I_1} + \frac{L_2}{I_2}\right] + \frac{L_2}{I_2}\times M_2 = -6\left[\frac{80}{I_1}\,0 + \frac{70}{I_2}\right]$$

or, $2M_1\left[0 + \frac{5}{1.5I}\right] + M_2\times\frac{5}{1.5I} = -6\left[\frac{70}{1.5I}\right]$

or, $6.67M_1 + 8.33M_2 = -280$ ——(i)

→ Writing 3 moment eqⁿ for support 2, $n = 2$

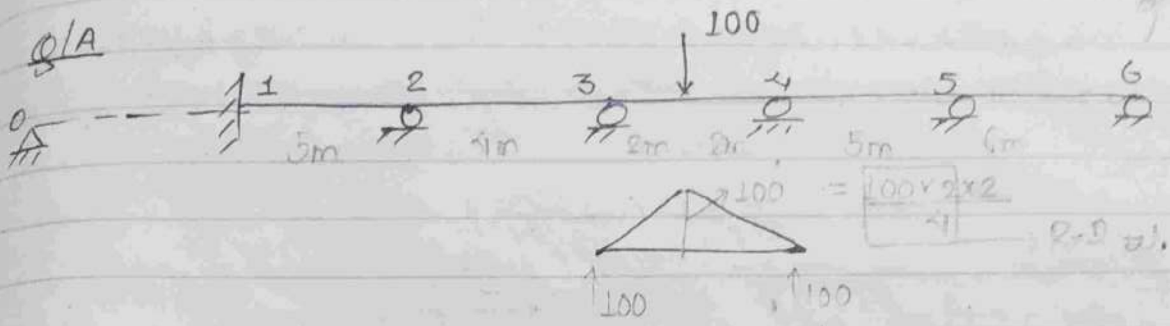
$$M_1\frac{L_2}{I_2} + 2M_2\left[\frac{L_2}{I_2} + \frac{L_3}{I_3}\right] + \frac{L_3}{I_3}\times M_3 = -6\left[\frac{80}{1.5I} + \frac{135}{2I}\right]$$

or, $3.33M_1 + 12.667M_2 + 3M_3 = -725$ — (ii)

$M_3 = 0$ because at 3 no moment. If overhanging, moment can appear.

Solving, $M_1 = -15.429, -53.179 = M_2$

Q/A



$$M_0 \frac{L_1}{I_1} + 2M_1\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_2 \frac{L_2}{I_2} = 0 \longrightarrow \text{support } 1$$

or, $10M_1 + 5M_2 = 0$ — (i) $\Rightarrow 2M_1 = M_2$

$$M_1 \frac{L_2}{I_2} + 2M_2\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + \frac{M_3 L_3}{I_3} = 0 \longrightarrow S2$$

or, $5M_1 + 18M_2 + 4M_3 = 0$ — (ii)

$$M_2 \frac{L_3}{I_3} + 2M_3\left(\frac{L_3}{I_3} + \frac{L_4}{I_4}\right) + M_4 \frac{L_4}{I_4} = -6\left[0 + \frac{100}{I_4}\right] \longrightarrow S_3$$

or, $4M_2 + 16M_3 + 4M_4 = -600$ — (iii)

$$M_3 \frac{L_4}{I_4} + 2M_4\left(\frac{L_4}{I_4} + \frac{L_5}{I_5}\right) + M_5 \frac{L_5}{I_5} = -6\left[\frac{100}{I_4} + 0\right] \longrightarrow S_4$$

or, $4M_3 + 18M_4 + 5M_5 = -600$ — (iv)

$$M_4 \frac{L_5}{I_5} + 2M_5\left[\frac{L_5}{I_5} + \frac{L_6}{I_6}\right] + \frac{M_6 L_6}{I_6} = 0 \longrightarrow S5$$

or, $5M_4 + 22M_5 + \cancel{6M_6}^{0} = 0$ ($M_6 = 0$, end roller support)

$\Rightarrow M_4 = -4.4M_5$ — (v)

Then, $4M_3 - 74.2M_5 = -600$ — (vi) (in (iv))

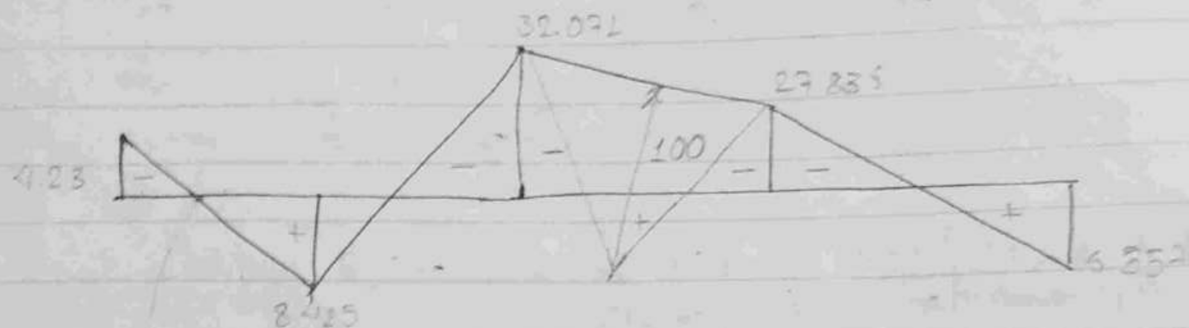
$\cancel{5M_1}$ $41M_1 + 4M_3 = 0$ — (vii) (in (ii))

In (iii), $\cancel{8M_1 + (-41 \times 4)M_1 +}$

$4 \times 2 \times \frac{(-4)}{41} M3 + 16 M3 + 4 \times (-4.4) M5 = -600 \quad - (viii)$

$\Rightarrow M3 = -32.071, \; M5 = 6.357, \; M1 = -4.23, \; M2 = 8.425,$

$M4 = -27.836$

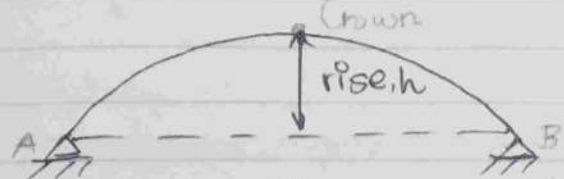


focal point (point of contraflexure)

focal point ratio $= \frac{4.23}{8.425}$

Two hinged Arch

Tuesday 1076/9/24

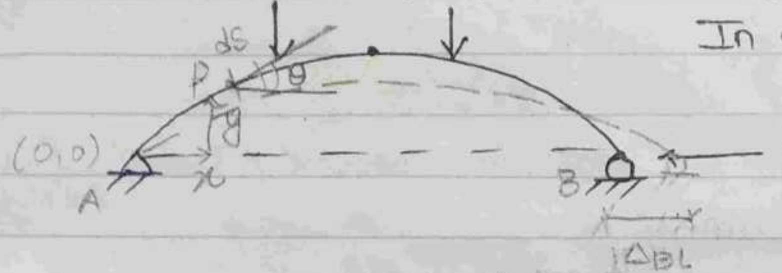


$\Delta SI = 4 - 3 = 1$

I_c = MOI at crown

Crown (I_c) $\Rightarrow$ I at any point $= I_c \sec\theta$

In eg. $I_P = I_c \sec\theta$



M

$\Rightarrow H.\delta_{BB} = \Delta_{BL}$

m Moment due to unit force. $m = 1 \times y = y$ (at any pt.)

$$\Delta_{BL} = \int \frac{M_L . m\, dx}{EI} = \sum M_{beam} . y \frac{ds}{EI}$$

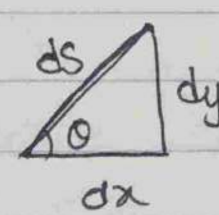
(Note: $M_L = M_{beam} - Hy$ but $H = 0$ because we replace hinge with roller which doesn't give hor. rxn)

$$\delta_{BB} = \delta_{11} = \int \frac{m_1^2 ds}{EI} = \int \frac{y^2 ds}{EI}$$

$$\Delta_{BL} = \delta_{11} . H \Rightarrow H \times \int \frac{y^2 ds}{EI} = \int M_{beam} . y \frac{ds}{EI}$$

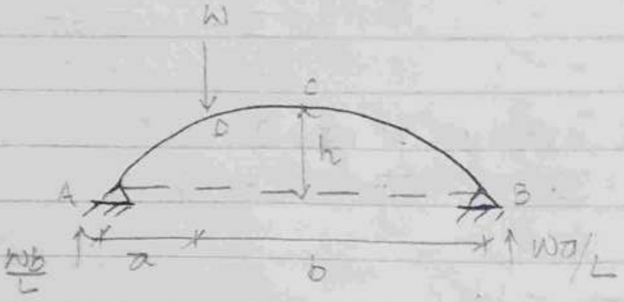
$$\therefore H = \frac{\int M_{beam} . y\, ds / EI}{\int y^2 ds / EI}$$

$$= \frac{\int M_{beam} . y . \frac{dx \sec\theta}{EI_c \sec\theta}}{\int y^2 \frac{dx \sec\theta}{EI_c \sec\theta}}$$



$\sec\theta = \frac{ds}{dx} \Rightarrow ds = dx \sec\theta$

$$\boxed{\therefore\ H = \frac{\int M_{beam} \cdot y\,dx}{\int y^2\,dx}}$$



M_{beam} = Moment of equivalent simple supported beam

Numerator only. $\int M_{beam} \cdot y\,dx$

$$= \int_0^a \frac{wb}{l} \times x \times y\,dx + \int_0^{l-a} \frac{wa}{l} \times x \times y\,dx$$

$$= \int_0^a \frac{wb}{l} \times x \times \frac{4hx}{l^2}(l-x)\,dx + \int_0^{l-a} \frac{wax}{l} \times \frac{4hx}{l^2}(l-x)\,dx$$

$$= \frac{4hw(l-a)}{l^3}\left[\int_0^a (lx^2 - x^3)\,dx\right] + \left[\int_0^{l-a} (lx^2 - x^3)\,dx\right] \times \frac{4hwa}{l^3}$$

$$= \frac{4hw}{l^3}\left[(l-a)\left[\frac{lx^3}{3} - \frac{x^4}{4}\right]_0^a + a\left[\frac{lx^3}{3} - \frac{x^4}{4}\right]_0^{l-a}\right]$$

$$= \frac{4hw}{l^3}\left[(l-a)\left[\frac{la^3}{3} - \frac{a^4}{4}\right] + \frac{la^4}{3} - \frac{a^5}{4}\right]$$

$$= \frac{4hw}{l^3}\left[\frac{l^2a^3}{3} - \frac{la^4}{4} - \cancel{\frac{la^4}{3}} + \cancel{\frac{a^5}{4}} + \cancel{\frac{la^4}{3}} - \cancel{\frac{a^5}{4}}\right]$$

$$= \frac{4hwa^3}{l^2}\left[\frac{l}{3} - \frac{a}{4}\right] = \frac{hwa^3(4l-3a)}{3l^2}$$

$$= \frac{hwa^3(a+3b)}{3l^2}$$

$$= \frac{4hw}{l^3}\left[(l-a)\left[\frac{la^3}{3}-\frac{a^4}{4}\right]+a\left[\frac{l(l-a)^3}{3}-\frac{(l-a)^4}{4}\right]\right]$$

$$= \frac{4hw}{l^3}\left[(l-a)\frac{(4la^3-3a^4)}{12}+a\left(\frac{4l(l-a)^3-3(l-a)^4}{12}\right)\right]$$

$$= \frac{hw}{3l^3}(l-a)\left[4la^3-3a^4+4al(l-a)^2-3a(l-a)^3\right]$$

$$= \frac{hw(l-a)}{3l^3}\left[4a^3l-3a^4+4al^3-8a^2l^2+4a^3l-3la^3+9l^2a^2-9la^3+3a^4\right]$$

$$= \frac{wha(l-a)}{3l^2}(l^2+al-a^2)$$

Denominator : $\int_0^l y^2 dx = \int_0^l \left(\frac{4hx}{l^2}\right)^2 (l-x)^2 dx$

$$= \frac{16h^2}{l^4}\int_0^l x^2(l^2-2lx+x^2)dx = \frac{8h^2l}{15}$$

$$\Rightarrow H = \frac{\frac{wha(l-a)(l^2+al-a^2)}{3l^2}}{\frac{8h^2l}{15}}$$

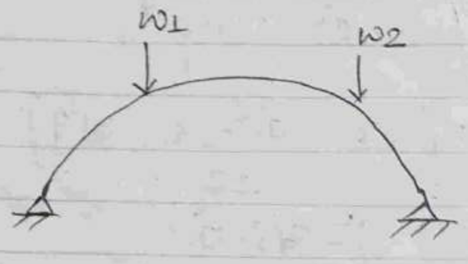
$$= \frac{5wa}{8hl^3}(l^2+al-a^2)(l-a)$$

If $\boxed{a=nl}$

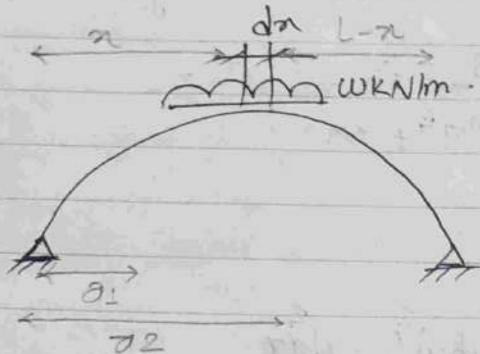
$$= \frac{5wnl}{8hl^3}(l^2+nl^2-n^2l^2)(l-nl)$$

$$= \frac{5wnl}{8h}(1+n-n^2)(1-n)$$

$$\boxed{\therefore H = \frac{-5wnl}{8h}(1-2n^2+n^3)}$$



→ Calculate separately
we superpose



$$\int dH = \int_{a_1}^{a_2} \frac{5(w\,dx)}{8hl^3}\, x(l-x)(l^2+xl-a^2)$$

↓ $a = x$ as well

If for whole span,

$$H = \int_0^l \frac{5w}{8hl^3}\, x(l-x)(l^2+xl-a^2)\,dx$$

$$= \frac{5w}{8hl^3}\int_0^l (lx-x^2)(l^2+lx-a^2)\,dx$$

$$= \frac{5w}{8hl^3}\int_0^l \left(l^3x + \cancel{l^2x^2} - a^2lx - \cancel{l^2x^2} - lx^3 + a^2x^2\right)dx$$

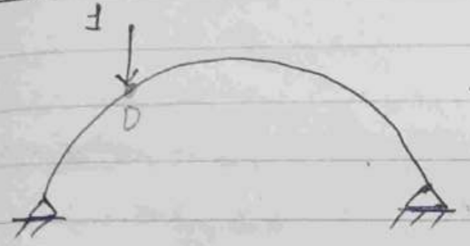
$$= \frac{5w}{8hl^3}\left[\frac{l^5}{2} + \cancel{\frac{l^5}{3}} - \frac{a^2l^3}{2} - \cancel{\frac{l^5}{3}} - \frac{l^5}{4} + \frac{a^2l^3}{3}\right]$$

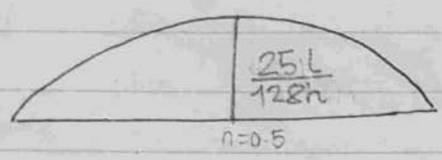
$$= \frac{5w}{8hl}\left[\frac{l^3}{2} - \frac{a^2l}{2} - \frac{l^3}{4} + \frac{a^2l}{3}\right] \quad [\text{Here, } a = x]$$

$$= \frac{wl^2}{8h}$$

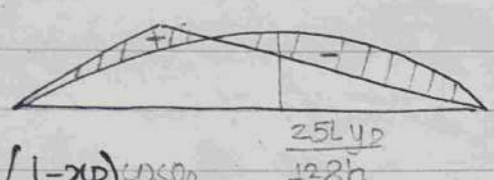
Half span $= \frac{wl^2}{16h}$

ILD



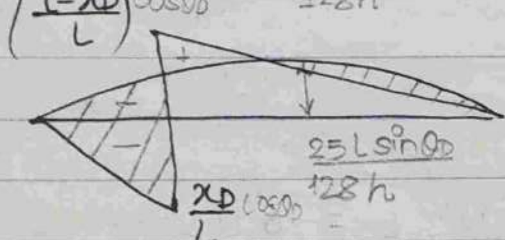


ILD for H



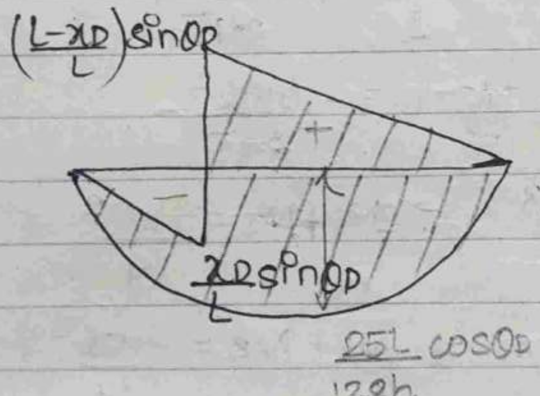
ILD for M_D

$M_D = M_D\text{beam} - Hy_D$



ILD for Q_D (radial shear)

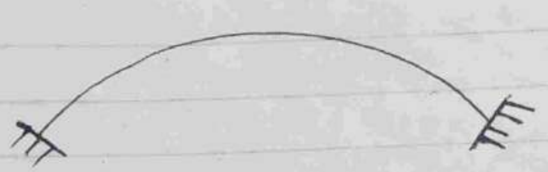
$Q_D = V_D\cos\theta_D - H\sin\theta_D$

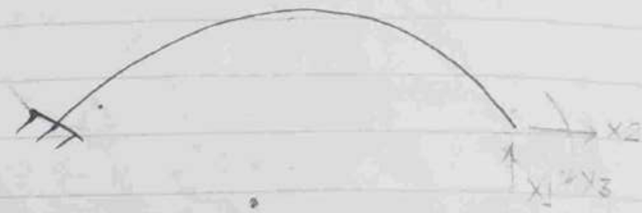


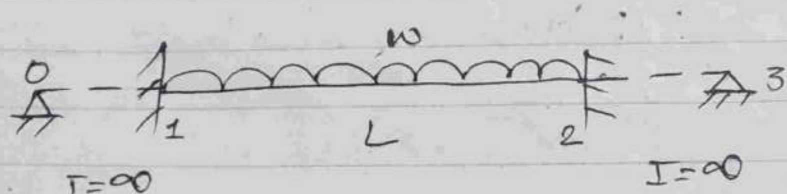
ILD for N_D (normal thrust)

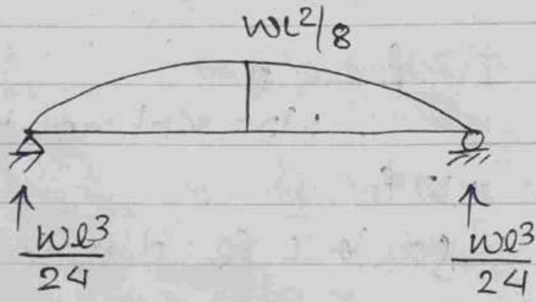
$N_D = V_D\sin\theta_D + H\cos\theta_D$

Fixed arch (Hingeless arch)









For 1, $M_0 \cdot \frac{L_1}{\infty} + 2M_1 \left(\frac{L_1}{\infty} + \frac{L}{I} \right) + M_2 \cdot \frac{L}{I} = -6 \frac{wl^3}{24I}$

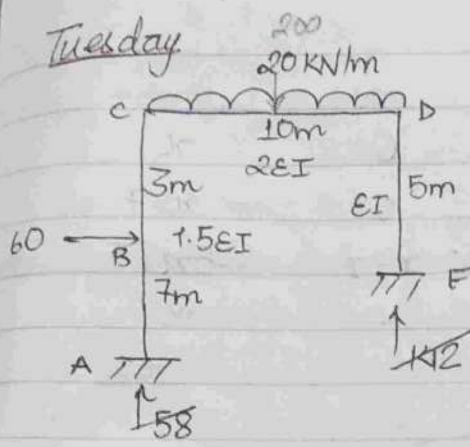
For 2, $M_1 \cdot \frac{L}{I} + 2M_2 \left(\frac{L}{I} + \frac{L_3}{\infty} \right) + \frac{M_3}{\infty} \cdot L = -\frac{6wl^3}{24I}$

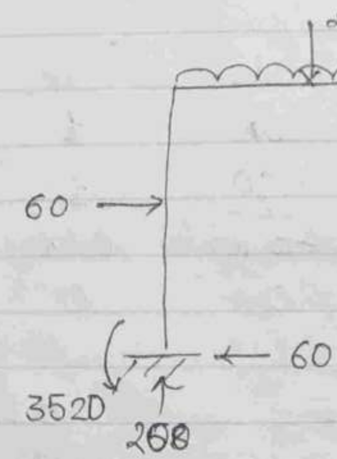
or, $LM_1 + 2LM_2 = -\frac{6wl^3}{24} \Rightarrow 2M_1 + M_2 = -\frac{wL^2}{4}$

or, $LM_1 + 2LM_2 = -\frac{wl^2}{4} \Rightarrow M_1 + M_2 = -\frac{wl^2}{4}$

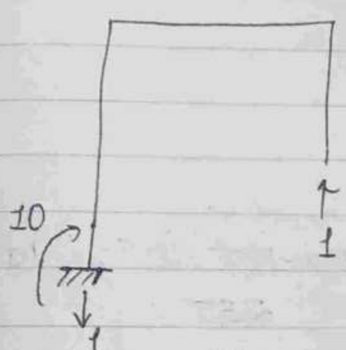
Solving, $M_1 = M_2 = \frac{wL^2}{12}$

Tuesday

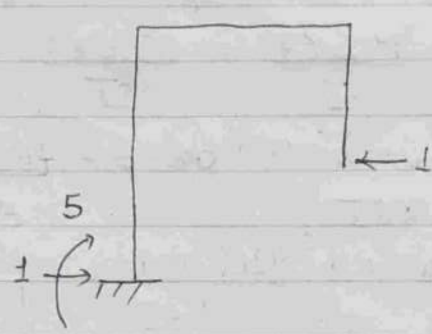




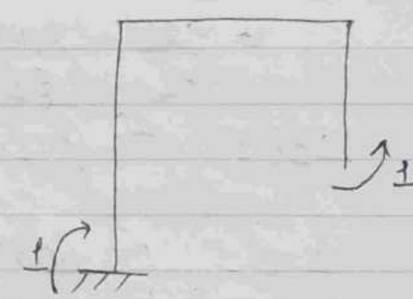
primary structure



m1

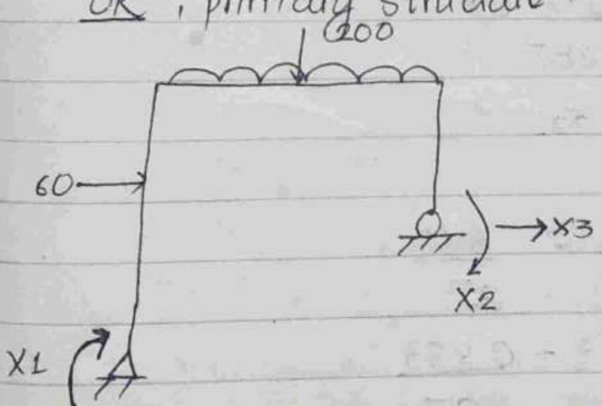


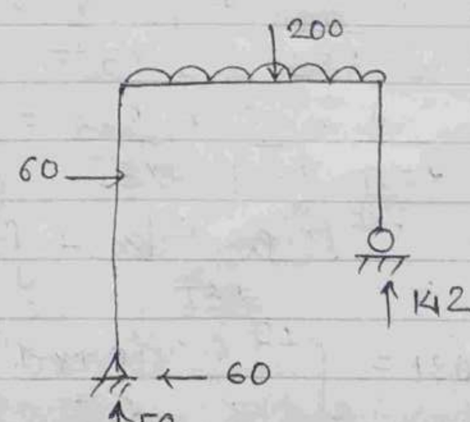
m2

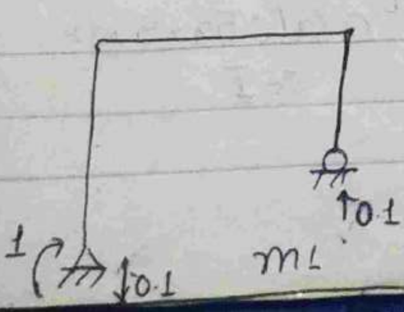


m3

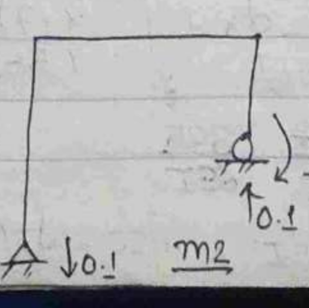
OR, primary structure



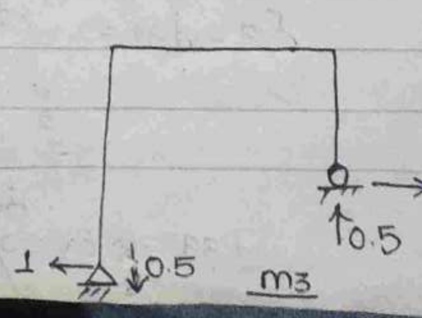




m1



m2



m3

Portion	Origin	Limit	M_L	m_1	m_2	m_3
(1.5EI) AB	A	0-7	$60x$	1	0	x
(1.5EI) BC	B	0-3	420	1	0	$x+7$
(2EI) CD	D	0-10	$\cancel{58x}$ $142x-10x^2$	$0.1x$	$0.1x-1$	$0.5x+5$
(EI) DE	E	0-5	0	0	-1	x

$$\Delta_{1L} = \Sigma \int \frac{M_L m_1 dx}{EI}$$

$$= \int_0^7 \frac{60x\,dx}{1.5EI} + \int_0^3 \frac{420\,dx}{1.5EI} + \int_0^{10} \frac{(142x-10x^2)\times(0.1x)\,dx}{2EI}$$

$$= \frac{980}{EI} + \frac{840}{EI} + \frac{1116.67}{EI} = \frac{2936.67}{EI}$$

$$\Delta_{2L} = \int_0^{10} \frac{(142x-10x^2)(0.1x-1)\,dx}{2EI} = -\frac{766.67}{EI}$$

$$\Delta_{3L} = \int_0^7 \frac{60x^2 dx}{1.5EI} + \int_0^3 \frac{420(x+7)\,dx}{1.5EI} + \int_0^{10} \frac{(142x-10x^2)(0.5x+5)\,dx}{2EI}$$

$$= \frac{4573.33}{EI} + \frac{7140}{EI} + \frac{15000}{EI} = \frac{26713.33}{EI}$$

$$\delta_{11} = \int_0^7 \frac{dx}{1.5EI} + \int_0^3 \frac{dx}{1.5EI} + \int_0^{10} \frac{(0.1x)^2 dx}{2EI}$$

$$= \frac{7}{1.5EI} + \frac{3}{1.5EI} + \frac{3.33}{2\times EI} = \frac{\cancel{10}\ 8.33}{EI}$$

$$\delta_{22} = \int_0^{10} \frac{(0.1x-1)^2 dx}{2EI} + \int_0^5 \frac{dx}{EI} = \frac{6.67}{EI}$$

$$\delta_{12} = \delta_{21} = \int_0^{10} \frac{0.1x(0.1x-1)\,dx}{2EI} = -\frac{0.833}{EI}$$

$$\delta_{13} = \delta_{31} = \int_0^7 \frac{x\,dx}{1.5EI} + \int_0^3 \frac{(x+7)\,dx}{1.5EI} + \int_0^{10} \frac{0.1x(0.5x+5)\,dx}{2EI}$$

$$= \frac{\cancel{188}}{\cancel{3EI}} \quad \frac{325}{6EI}$$

$$\delta_{33} = 5000/9EI, \quad \delta_{23} = \delta_{32} = -\frac{175}{6EI}$$

Now, $x_1 = -36.846$

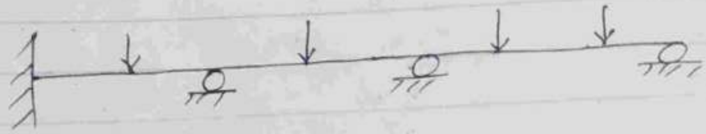
$x_2 = -109.305$

$x_3 = -50.229$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix}^{-1} \begin{bmatrix} \Delta_1 - \Delta_{1L} \\ \Delta_2 - \Delta_{2L} \\ \Delta_3 - \Delta_{3L} \end{bmatrix}$$

Ch. 3 Displacement Method | Stiffness | Eqbm method

stiffness method



$k_{11}\Delta_1 + k_{12}\delta_2 + P_{1L} = P_1$

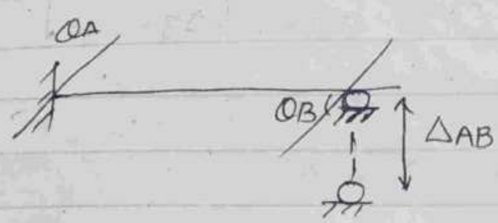
$k_{21}\Delta_1 + k_{22}\delta_2 + P_{2L} = P_2$

$$M_{AB} = FEM_{AB} + \frac{2EI_{AB}}{L_{AB}}\left(2\theta_A + \theta_B - \frac{3\Delta_{AB}}{L_{AB}}\right)$$

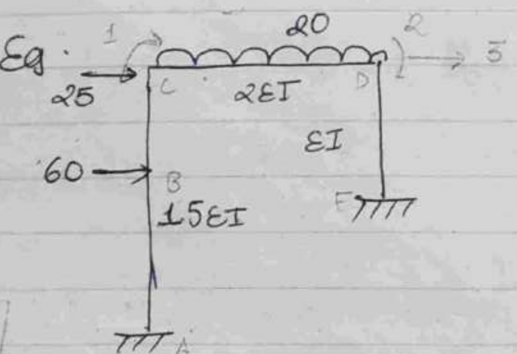
(fixed end moment)

$$\begin{bmatrix}\Delta_1 \\ \Delta_2\end{bmatrix}\begin{bmatrix}k_{11} & k_{12} \\ k_{21} & k_{22}\end{bmatrix} + \begin{bmatrix}P_{1L} \\ P_{2L}\end{bmatrix} = \begin{bmatrix}P_1 \\ P_2\end{bmatrix}$$

$$= \begin{bmatrix}P_1 - P_{1L} \\ P_2 - P_{2L}\end{bmatrix}$$



Eg.

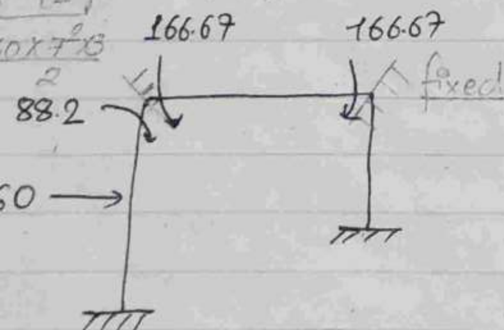


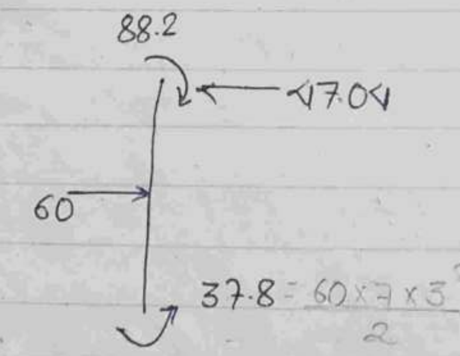
$2DKI = 3j - (r+m) = 3\times4 - (6+3)$
$= 3$

dirⁿ +ve if same as coordinate.

$\frac{wl^2}{12}$

$\frac{60\times7^2\times3}{2}$



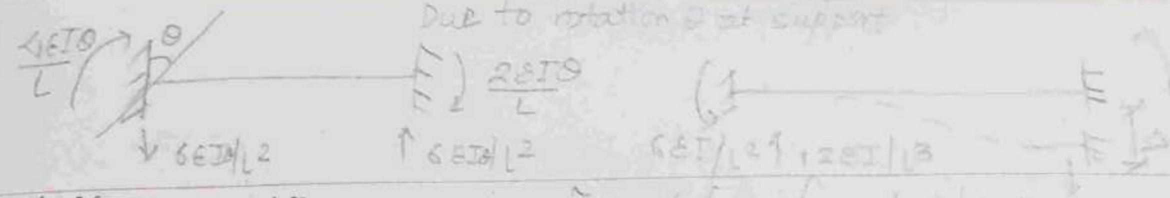


$P_1L = 88.2 - 166.67 = -78.47$

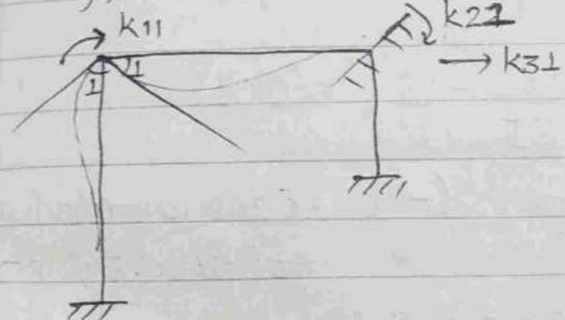
$P_2L = 166.67$

$P_3L = -47.04$ (in opposite dirⁿ to coordinate)

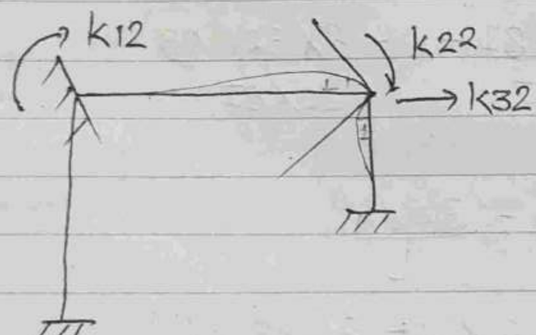
$P_1 = 0$, $P_2 = 0$, $P_3 = 25$ (same dirⁿ to coordinate)



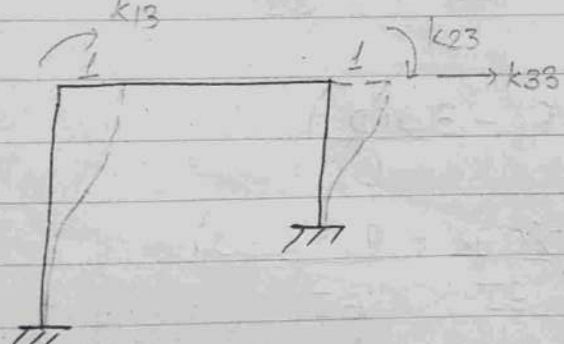
Stiffness matrix :



$$k_{11} = \frac{4\times(1.5EI)}{10} + \frac{4\times(2EI)}{10}$$
$$= 1.4EI$$
$$k_{21} = \frac{2\times(2EI)}{10} = 0.4EI$$
$$k_{31} = -\frac{6(1.5EI)}{L^2} = -0.09EI$$



$$k_{12} = \frac{2\times(2EI)}{10} = 0.4EI$$
$$k_{22} = \frac{4\times(2EI)}{10} + \frac{4\times EI}{5} = 1.6EI$$
$$k_{32} = -\frac{6EI}{5^2} = -0.24EI$$



$$k_{13} = -\frac{6\times 1.5EI}{10^2} = -0.09EI$$
$$k_{23} = -\frac{6\times(EI)}{5^2} = -0.24EI$$
$$\boxed{k_{33} = \frac{12EI\times 1.5}{10^3} + \frac{12EI}{5^3}}$$
$$= 0.114EI$$

$$\begin{bmatrix}\Delta_1\\ \Delta_2\\ \Delta_3\end{bmatrix} = \begin{bmatrix}k_{11} & k_{12} & k_{13}\\ k_{21} & k_{22} & k_{23}\\ k_{31} & k_{32} & k_{33}\end{bmatrix}^{-1}\left\{\begin{bmatrix}P_1\\ P_2\\ P_3\end{bmatrix} - \begin{bmatrix}P_{1L}\\ P_{2L}\\ P_{3L}\end{bmatrix}\right\}$$

Solving. $\Delta_1 = +99.82/EI$ ← $\frac{1}{EI}\begin{bmatrix}1.4 & 0.4 & -0.09\\ 0.4 & 1.6 & -0.24\\ -0.09 & -0.24 & 0.114\end{bmatrix}^{-1}\begin{bmatrix}78.47\\ -166.67\\ 47.64\end{bmatrix}$

$\Delta_2 = -80.981/EI$

$\Delta_3 = +320.9/EI$

Δ_1 is θ_C, Δ_2 is θ_D, $\Delta_3 = \Delta_{xc} = \Delta_{DE}$

P=25 ignored

$M_{AB} = FEM_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_C - \frac{3\Delta_{AC}}{L}\right)$

$= -37.8 + \frac{2(1.5EI)}{10}\left(2\times 0 + \frac{99.82}{EI} - \frac{3}{10}\times\frac{320.95}{EI}\right)$

anticlockwise

$= \cancel{-36.9255\ KNm}\ -36.73\ KNm$ (-ve means anticlockwise for fixed end momen

~~$M_{DE} = FEM_D$~~

$M_{CA} = FEM_{CA} + \frac{2EI}{L}\left(2\theta_C + \theta_A - \frac{3\Delta_{CA}}{L}\right)$

$= +88.2 + \frac{2\times(1.5EI)}{10}\left(2\times\frac{99.82}{EI} + 0 - \frac{3\times 320.95}{10\times EI}\right)$

clockwise

$= 119.21\ KNm$

$M_{CD} = FEM_{CD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\Delta_{CD}}{L}\right)$

$= -166.67 + \frac{2\times 2EI}{10}\left(2\times\frac{99.82}{EI} - \frac{80.98}{EI} - 0\right)$

$= -119.206\ KNm$

$M_{DC} = FEM_{DC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - 3\frac{\Delta_{CD}}{L}\right)$

$= 166.67 + \frac{2\times 2EI}{10}\left(2\times -\frac{80.98}{EI} + \frac{99.82}{EI} - 0\right)$

$= 141.814\ KNm$

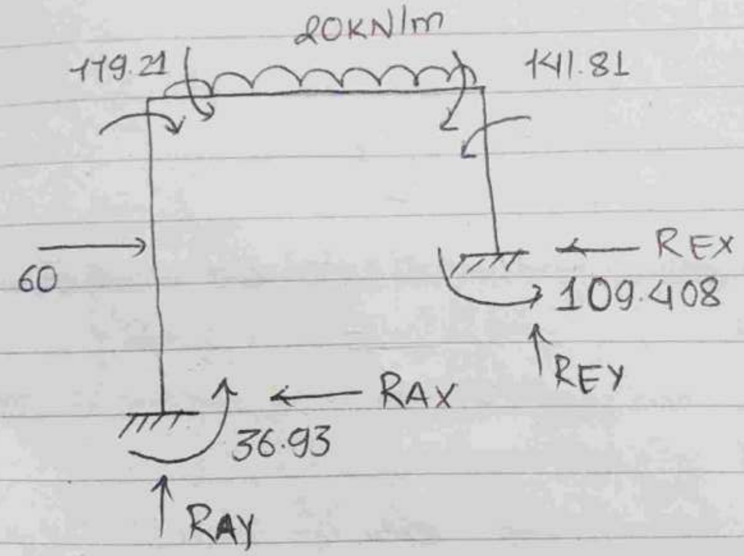
$M_{DE} = FEM_{DE} + \frac{2EI}{L}\left(2\theta_D + \theta_E - \frac{3\Delta_{DE}}{L}\right)$

$= 0 + \frac{2EI}{5}\left(2\times\left(\frac{-80.98}{EI}\right) + 0 - \frac{3\times 320.9}{5EI}\right)$

$= -141.8\ KNm$

$M_{ED} = FEM_{ED} + \frac{2EI}{L}\left(2\theta_E + \theta_D - \frac{3\Delta_{DE}}{L}\right)$

$= 0 + \frac{2EI}{5}\left(2\times 0 - \frac{80.98}{EI} - \frac{3\times 320.9}{5EI}\right) = -109.408$



$M_C = -119.21$ $\quad$ (+ve rule)

or, $-36.93 + R_{AX} \times 10 - 60 \times 3 = 0 - 119.21$

$\therefore R_{AX} = 9.772$ KN

$\Sigma F_x = 0$

$\therefore R_{EX} + R_{Ax} - 60 = 0$

$\therefore R_{Ex} = 50.228$ KN

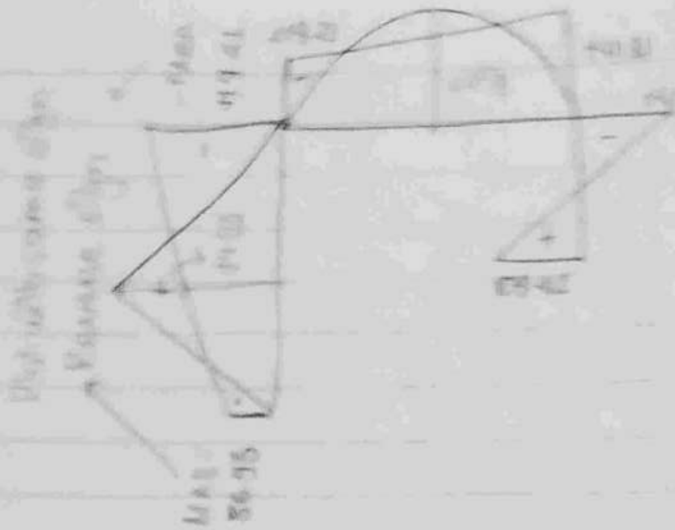
$M_C = -119.21$ (from right)

or, $109.408 + 50.228 \times 5 + 200 \times 5 + R_{EY} \times 10) = -119.21$

$\therefore R_{EY} = \cancel{102.25 \text{ KN}}\ \cancel{126.09 \text{ KN}}\ 102.25$ KN

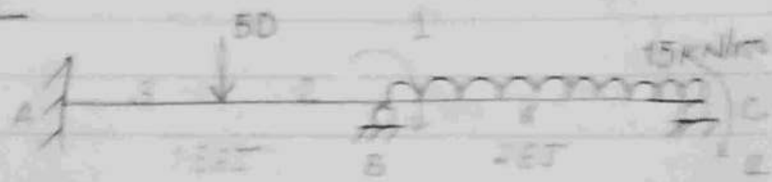
$\Sigma F_y = 0$

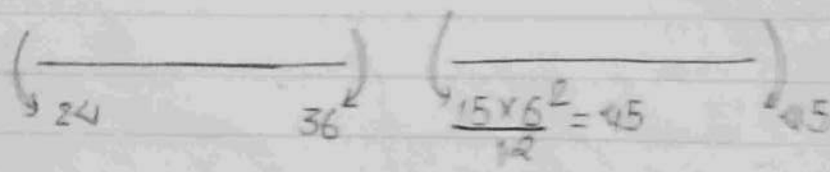
$\therefore R_{Ay} + R_{Ey} = 200$ $\quad \therefore R_{Ay} = 97.75$ KN



Sishal Shrestha
074 BCE

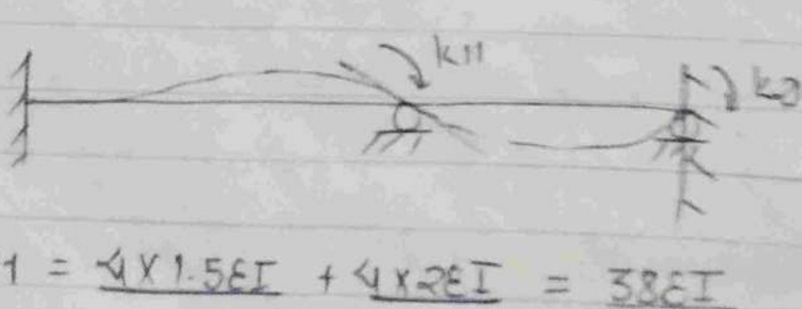
Question
9.4





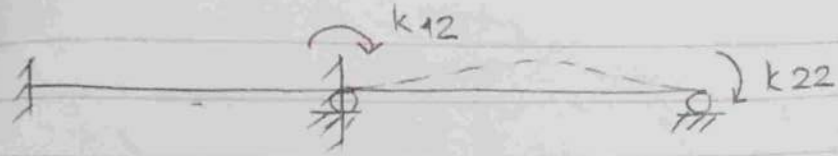
$P_{1L} = 36 - 45 = -9$ $\qquad P_1 = 0$

$P_{2L} = 45$ $\qquad P_2 = 0$



$$k_{11} = \frac{4 \times 1.5EI}{5} + \frac{4 \times 2EI}{6} = \frac{38EI}{15}$$

$$k_{21} = \frac{2 \times 2EI}{6} = \frac{2EI}{3}$$



$$K_{12} = \frac{2\times 2EI}{6} = \frac{2EI}{3}$$

$$K_{22} = \frac{4\times 2EI}{6} = \frac{4EI}{3}$$

$$\begin{bmatrix} \Delta 1 \\ \Delta 2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} - \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} \right\}$$

Solving, $\Delta 1 = 14.318/EI \Rightarrow \theta_B$ 14.318

$\Delta 2 = -40.9/EI \Rightarrow \theta_C$ -40.9

$$M_{AB} = FEM_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta_{AB}}{L}\right)$$

$$= -24 + \frac{2\times 1.5EI}{5}\left(0\times 2 + \frac{14.32}{EI} - 0\right) = \cancel{20.5184}\ KNm$$

$$\cancel{32.592}\ -15.386\ KNm$$

$$M_{BA} = FEM_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta_{AB}}{L}\right)$$

$$= 36 + \frac{2\times 1.5EI}{5}\left(2\times \frac{14.32}{EI} + 0 - 0\right) = 53.184\ KNm$$

$$M_{BC} = FEM_{BC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta_{BC}}{L}\right)$$

$$= -45 + \frac{2\times 2EI}{6}\left(2\times \frac{14.318}{EI} + \frac{(-40.9)}{EI} - 0\right)$$

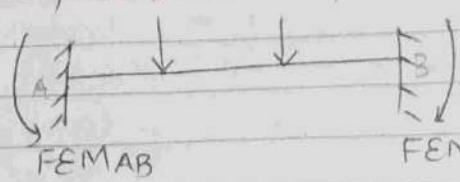
$$= -53.176\ KNm$$

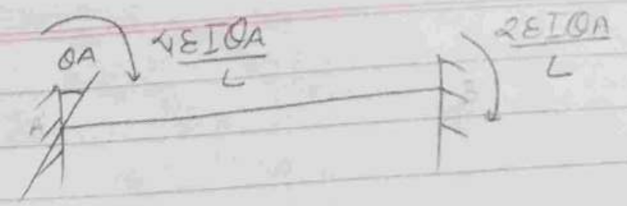
$$M_{CB} = FEM_{CB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta_{CB}}{L}\right)$$

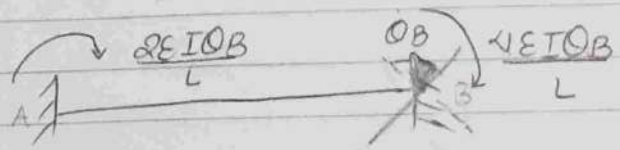
$$= 45 + \frac{2\times 2EI}{6}\left(\frac{2\times(-40.9)}{EI} + \frac{14.32}{EI}\right) = 0.01\ KNm$$

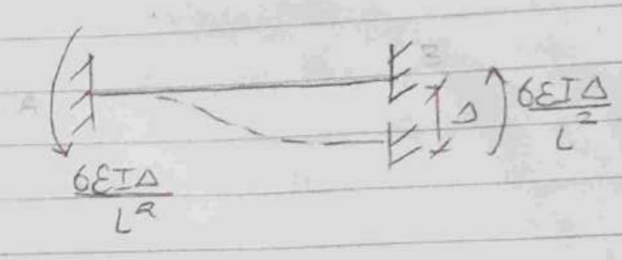
* Sign Convention: Clockwise +ve

Slope deflection equation









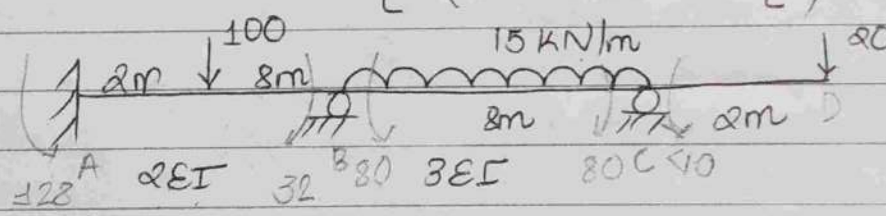
$$M_{AB} = FEM_{AB} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2}$$

$$= FEM_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{BA} = FEM_{BA} + \frac{2EI\theta_A}{L} + \frac{4EI\theta_B}{L} - \frac{6EI\Delta}{L^2}$$

$$= FEM_{BA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\Delta}{L}\right)$$

Q.



* Writing slope deflection equations,

$$M_{AB} = FEM_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -128 + \frac{2\times 2EI}{10}\left(2\times 0 + \theta_B - 0\right) = -128 + \frac{2\theta_B EI}{5}$$

$$M_{BA} = FEM_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 32 + \frac{2\times 2EI}{10}\left(2\theta_B + 0\right) = 32 + 0.8EI\theta_B$$

$\cancel{FEMBC}$ $M_{BC} = -80 + \frac{2\times3EI}{8}(2\theta_B + \theta_C - 0) = -80 + 1.5\theta_B EI + 0.75\theta_C EI$

$M_{CB} = 80 + \frac{2\times3EI}{8}(2\theta_C + \theta_B - 0) = 80 + 1.5\theta_C EI + 0.75\theta_B EI$

Writing equilibrium equation,

$M_{BA} + M_{BC} = 0$

or, $2.3EI\theta_B + 0.75EI\theta_C = 48$

$M_{CB} + M_{CD} = 0$

or, $M_{CB} = +40$

or, $0.75EI\theta_B + 1.5EI\theta_C = \cancel{120} -40$

$\Rightarrow \theta_B = \frac{35.32}{EI}, \theta_C = \frac{-44.329}{EI}$

$M_{AB} = \cancel{-105.56}\ -113.872$

$M_{BA} = \cancel{76.88}\ 60.256$

$M_{BC} = \cancel{-76.88}\ -60.266$

$M_{CB} = \cancel{-40}\ 39.996$

By displacement method

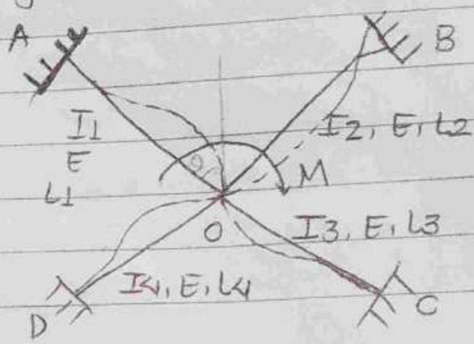
$\cancel{d}KI = 2$

Sign convention. Clockwise +ve

Moment Distribution Method

Stiffness of member at joint (rigid)

$M_{OA} = \frac{4EI_1}{L_1}\theta = k_{OA}\theta$



$M_{OB} = \frac{3EI_2}{L_2}\theta = \frac{3}{4}\left(\frac{4EI_2}{L_2}\right)\theta$

$M_{OB} = \frac{3}{4}k_{OB}\theta$

$M_{OC} = \frac{4EI_3}{L_3}\theta = k_{OC}\theta$

$M_{OD} = \frac{4EI_4}{L_4}\theta = k_{OD}\theta$

For eqbLm, $M_{OA} + M_{OB} + M_{OC} + M_{OD} = M$

or, $\left(k_{OA} + \frac{3}{4}k_{OB} + k_{OC} + k_{OD}\right)\theta = M$

or, $\theta \times \Sigma K = M \quad \therefore \theta = \frac{M}{\Sigma K}$

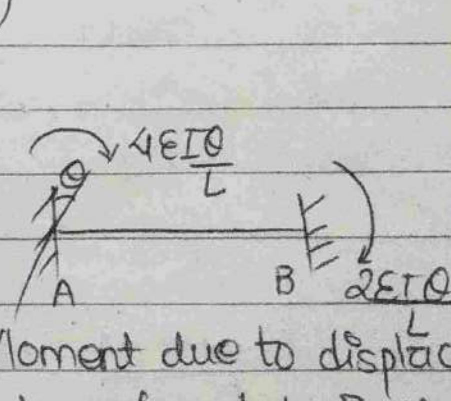
$\therefore M_{OA} = k_{OA}\frac{M}{\Sigma k} = \frac{k_{OA}}{\Sigma k}(M) = (\text{Distr. factor})_{OA} \cdot M$

Absolute stiffness $\frac{4EI}{L}$ (when $\theta = 1$)

Relative stiffness $= \frac{I}{L}$ (Obtd. from $\frac{k}{\Sigma K}$)

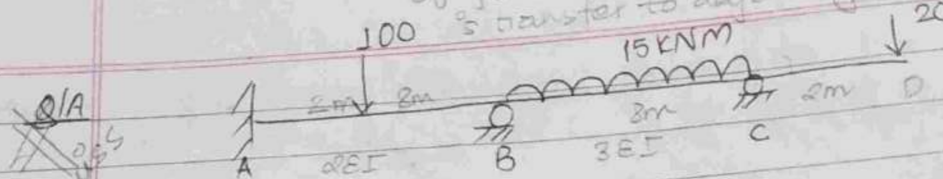
Distribution factor, $DF = \frac{k}{\Sigma K}$

Carry over factor: $\frac{M_{OB}}{M_{OA}}$ = CO factor $= \frac{1}{2}$



Moment due to displacement at A is transfered to B as well.

Joint C can't take unbalanced moment as member CD doesn't take any moment since stability of joint is ensured only when ... is transfer to adjoining members ... of one ... take ... other ... unable to take unbalanced moment ...



Calculate DF

$$D_{BA} = \frac{\frac{2I}{10}}{\frac{2I}{10} + \frac{3I}{8} \times \frac{3}{4}} = 0.4155 = \frac{32}{77}$$

→ C is end support multiplying ... moment can't transfer ... support C.

$$D_{BC} = \frac{3/4 \times 3I/8}{\frac{2I}{10} + \frac{3}{4} \times \frac{3I}{8}} = \frac{45}{77}$$

If end support isn't fixed multiply by 3/4

Joint	A	B		C	
Member	AB	BA	BC	CB	CD
D.F	–	32/77	45/77	–	–
FEM	−128	~~128~~ 32	−80	80	−40
Bal.	–	$+48 \times \frac{32}{77}$ = 19.948	$+48 \times \frac{45}{77}$ = 28.052	−40	–
CO	$\frac{19.948}{2}$ = 9.974	–	−20	–	
Bal.		+8.312	+11.688		
CO	4.156	–	–		
	−128 + 9.974 + 4.156 = −113.87	60.266	−60.26	40	−40

Assume all joint fixed then

sum zero

Simply add, reverse sign of result ...

total should be zero because unbalanced moment doesn't exist at C

→ final end moments

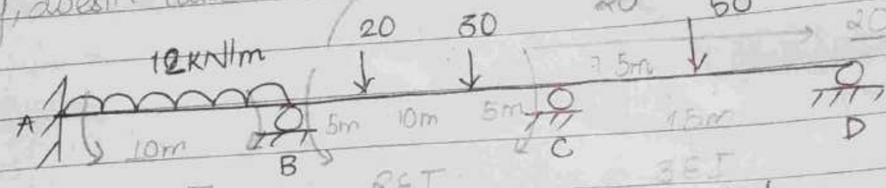
CK1

(fixed)
Here, support A takes moment only while support B gives moment only, doesn't take it.

$\frac{20 \times 15^2 \times 5}{20^2} + \frac{30 \times 5^2 \times 15}{20^2}$

$\frac{20 \times 5 \times 15^2}{20^2} + \frac{30 \times 15 \times 5^2}{20^2}$

Q/A



$DBA = \frac{1.5I/10}{1.5I/10 + 2I/20} = \frac{3}{5}$, $DCB = \frac{2I/20}{2I/20 + 3I/15} = \frac{1}{3}$

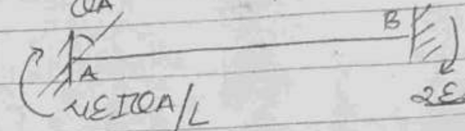
$DBC = \frac{2}{5}$, $DCD = \cancel{0.6}\ 2/3$

checking method of not multiply the 3/4 at end support

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	3/5	2/5	1/3	2/3	–
FEM	-100	+100	-84.375	103.125	-93.75	93.75
Bal	–	-9.375	-6.250	-3.125	-6.250	-93.75
CO	-4.6875	–	-1.5625	-3.125	-46.875	-3.125
Bal	–	+0.9375	0.625	16.667	33.33	+3.125
CO	+0.469	–	8.333	0.3125	1.5625	16.67
Bal	–	-5	-3.333	-0.625	-1.25	-16.67
CO	-2.5	–	-0.3125	-1.667	-8.333	-0.625
Bal	–	+0.1875	+0.125	+3.333	+6.667	+0.625
CO	0.09375	–	1.6605	0.0625	0.3125	3.335
Bal	–	+0.9999	-0.6666	-0.125	+0.25	-3.335
CO	0.4999	–	-0.0625	-0.333	-1.667	0.125
Bal	–	0.0375	+0.025	-0.667	+0.667	-0.125
CO	0.01875	–	-0.3335	0.0125	-0.0625	0.335
~~Bal~~	–	~~+0.2001~~	~~+0.1334~~			

Moment distribution method

1) Absolute stiffness, $\bar{K}$: Moment required to produce unit rotation, $\bar{K} = \frac{4EI}{L}$

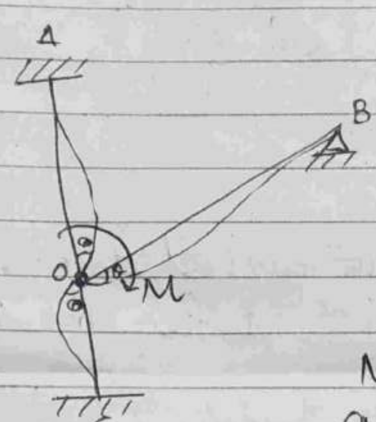


2) Relative stiffness, K

$$K = \frac{\bar{K}}{4E}$$

$$M_A = \frac{4EI\theta_A}{L}$$

$$\theta_A = 1,\ M_A = K$$



$$M_{OA} = \frac{4E_1I_1\theta_1}{L_1} = \bar{K}_{OA}\theta$$

$$M_{OB} = \frac{3E_2I_2\theta}{L_2} = \frac{3}{4}\times\frac{4E_2I_2}{L_2}\theta = \frac{3}{4}\bar{K}_{OB}\theta$$

$$M_{OC} = \frac{4E_3I_3\theta}{L_3} = \theta\bar{K}_{OC}$$

$$M_{OA} + M_{OB} + M_{OC} = M$$

or, $\theta\left(\bar{K}_{OA} + \frac{3}{4}\bar{K}_{OB} + \bar{K}_{OC}\right) = M$

or, $\theta = \frac{M}{\Sigma\bar{K}}$

Again, $\theta = \frac{M_{OA}}{\bar{K}_{OA}}$ Then, $M_{OA} = M\times\frac{\bar{K}_{OA}}{\Sigma\bar{K}}$ or, $\frac{M_{OA}}{M} = \frac{\bar{K}_{OA}}{\Sigma\bar{K}}$

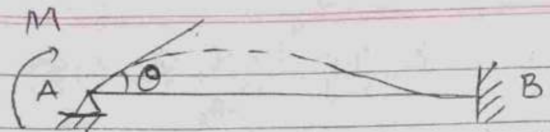
Here, $\frac{\bar{K}_{OA}}{\Sigma\bar{K}}$ is known as distribution factor. This determines what amount of unbalanced moment M developed at joint is distributed to or resisted by member OA.

Now, if modulus of elasticity (E) is constant,

$$DF = \frac{K_{OA}}{\Sigma K}$$

DF = I/L if far end is fixed

$\frac{3}{4}I/L$ if far end is hinged



By slope deflection method,

$$M_{BA} = \cancel{FEM_{BA}}^{0} + \frac{2EI}{L}\left(2\cancel{\theta_B}^{0} + \theta_A - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$\text{or, } M_{BA} = \frac{2EI}{L} \times \theta_A = \frac{2EI\theta_A}{L}$$

$$M_{AB} = \cancel{FEM_{AB}}^{0} + \frac{2EI}{L}\left(2\theta_A + \cancel{\theta_B}^{0} - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$\text{or, } M = \frac{4EI\theta_A}{L}$$

It seems that half of moment applied at hinge support A is transfered to fixed support at far end. This value is known as carry over factor.

$CF = \frac{1}{2}$ if far end is fixed

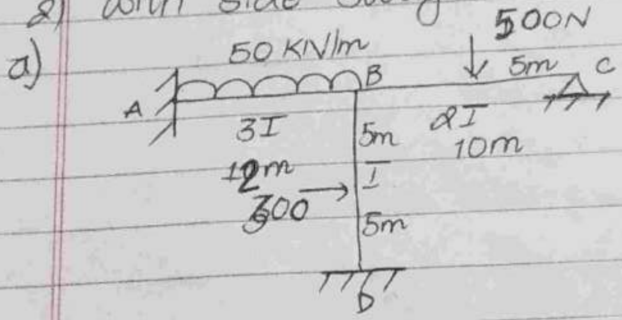
$\quad\;\; 0$ if far end is hinged

DF fixed end ⇒ doesn't exist
DF hinge support ⇒ 1
DF overhanging ⇒ 0

don't forget

Frame
1) without side sway
2) with side sway

a)

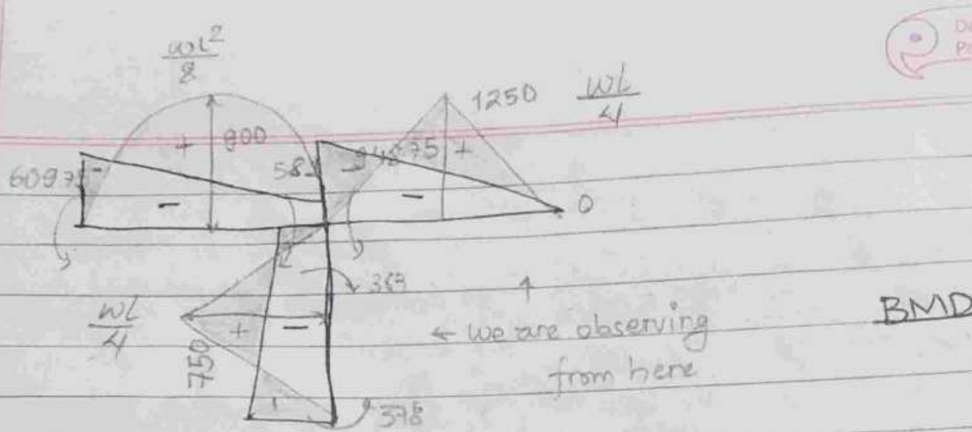


$$DF_{BA} = \frac{3/12\, I}{(3I/12 + I/10 + 2I/10 \times 3/4)} = 0.5$$

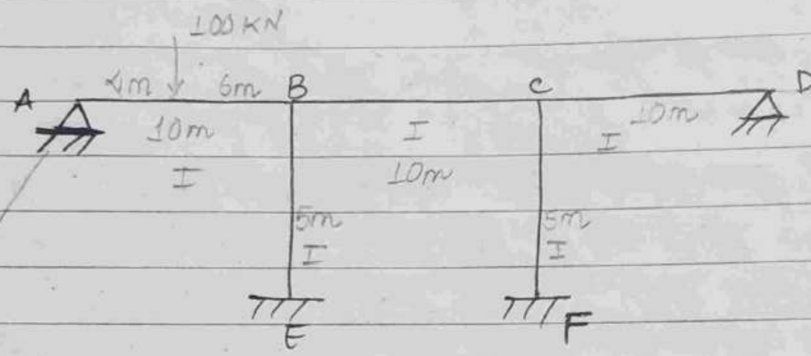
$$DF_{BC} = \frac{2I/10 \times 3/4}{''} = 0.3$$

$$DF_{BD} = 0.2$$

Joint	A	B			C	D
Member	AB	BA	BC	BD	CB	DB
DF	–	0.5	0.3	0.2	–	–
FEM	-600	600	-625	375	625	-375
Bal	–	-175	-105	-70	-625	—
CO	-87.5	–	-312.5	–	–	-35
Bal	—	156.25	93.75	62.5	–	—
CO	78.125	–	–	–	–	31.25
Moment =	-609.375	581.25	-948.75	367.5	0	-378.75



b)



If hinge

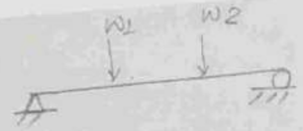
$$D_{BA} = \frac{I/10 \times 3/4}{\frac{3}{4} \times I/10 + I/10 + I/5} = \frac{1}{5} = 0.2$$

$$D_{BC} = \frac{I/10}{''} = 0.267$$

$$D_{BE} = 0.533$$

$$D_{CB} = \frac{I/10}{3/4 \times I/10 + I/5 + I/10} = 0.267$$

$$D_{CF} = 0.533, \quad D_{CD} = 0.2$$

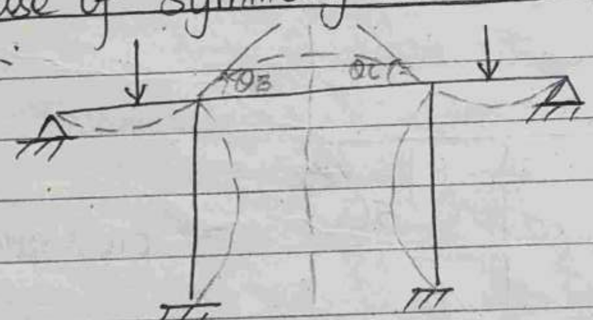


Date
Page

Joint	A	B				C		E	F	D
Member	AB	BA	BC	BE	CB	CD	CF	EB	FC	DC
DF	–	0.2	0.267	0.533	0.267	0.2	0.533	–	–	–
FEM	–144	96	0	0	0	0	0	0	0	0
Bal	+ ~~144~~	–19.2	–25.632	–51.168	–	–	–	–	–	–
CO	–	72	–	–	–12.82	–	–	–25.58	–	–
Bal	–	–14.4	–19.224	–38.376	3.423	+2.564	+6.83	–	–	–
CO	–	–	1.712	–	–9.612	–	–	–19.188	+3.415	–
Bal	–	0.342	–0.457	0.912	2.566	1.922	5.123	–	–	–
CO	–	–	1.283	–	–0.229	–	–	0.456	2.562	–
Bal	–	0.257	–0.343	0.683	0.061	0.046	0.122	–	–	–
CO	–	–	0.031	–	–0.172	–	–	0.342	0.061	–
Bal	–	0.006	–0.008	0.017	+0.046	–0.034	–0.092	–	–	–
CO	–	–	0.023	–	–0.004	–	0.009	0.008	–0.046	–
Total	0	135	–42.62	–87.93	–16.74	~~0.6~~ 4.498	~~–1.688~~ 11.992	–43.96	~~–0.84~~ 5992	0

Case of symmetry and antisymmetry

Symmetry :-



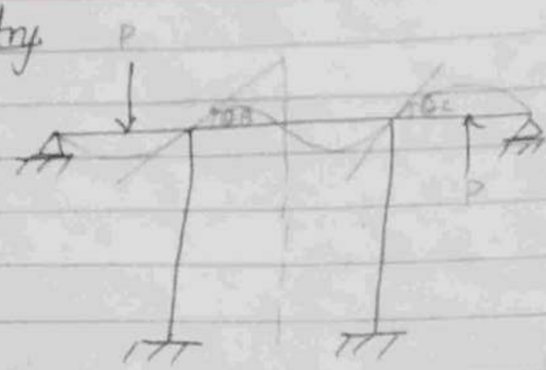
$\theta_B = -\theta_C$ (dir^n opposite)

$$M_{BC} = 0 + \frac{2EI}{L}(2\theta_B - \theta_B) = \frac{2EI\theta_B}{L} = \frac{1}{2}\left(\frac{4EI\theta}{L}\right)$$

→ for member on axis of symmetry, I/L value should be halved

Now, take only half structure & analyze.

Antisymmetry



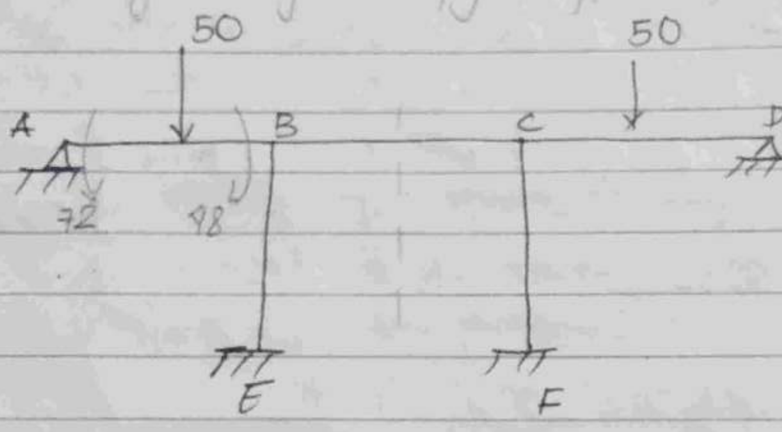
$\theta_B = \theta_C$

Δ same

$$M_{BC} = 0 + \frac{2EI}{L}(2\theta_B + \theta_B) = \frac{6EI\theta_B}{L} = \frac{3}{2}\left(\frac{4EI\theta_B}{L}\right)$$

For antisymmetry, multiply stiffness factor by 3/2.

Q/A



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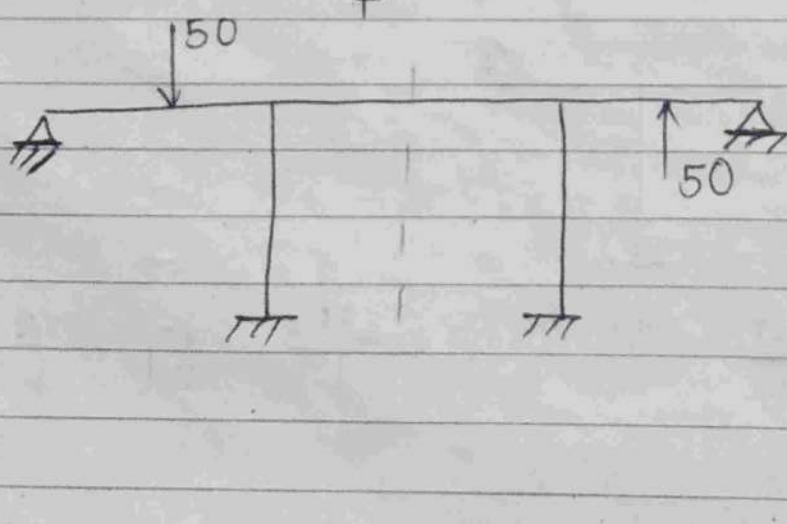
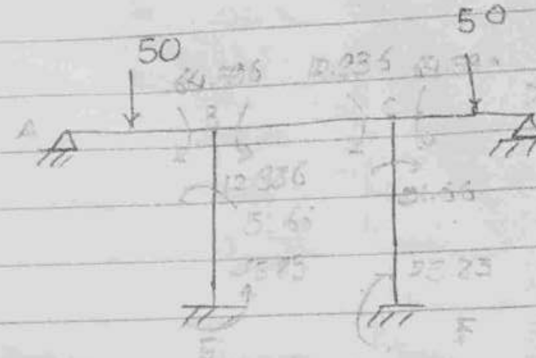


= prev. ques

$DF_{BA} = \dfrac{3/4 \times I/10}{3/4 \times I/10 + I/5 + 1/2 \times I/10} = 0.231$

$DF_{BC} = 0.154, \quad DF_{BE} = 0.615$

DF	-	0.231	0.154	0.615	-
Joint	A	B			E
Member	AB	BA	BC	BE	EB
FEM	-72	48	0	0	0
Bal	+72	-11.088	-7.392	-29.52	-
CO	-	36	-	-	-14.76
Bal	-	-8.316	-5.544	-22.14	-
CO	-	-	-	-	-11.07
Mom.	0	64.596	-12.936	~~-7.38~~	~~-3.69~~
Final				-51.66	-25.83

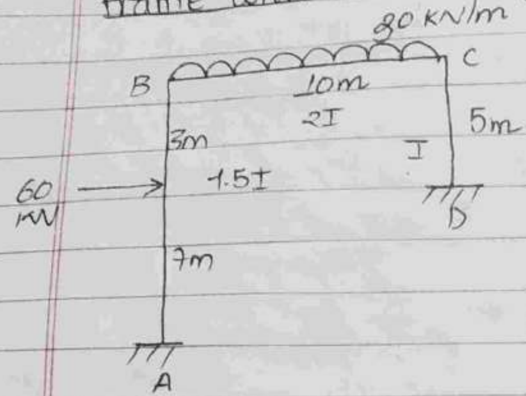


$DF_{BA} = \dfrac{3/4 \times I/10}{3/4 \times I/10 + 3/2 \times I/10 + I/5} = 0.176$

$DF_{BC} = 0.353, \quad DF_{BE} = 0.471$

Joint	A		B		E
Member	AB	BA	BC	BE	EB
~~DF~~ FEM	-72	48	0	0	0
DF ~~FEM~~	-	0.176	0.353	0.471	-
Bal	+72	-8.448	-16.944	-22.608	-
CO	-	36	-	-	-11.304
Bal	-	-6.336	-12.708	-16.956	-
CO	-	-	-	-	-8.478
Mom.	0	69.216	-29.652	-39.564	-19.782
Total	0	133.812	-~~39.~~42.588	-91.224	-45.612

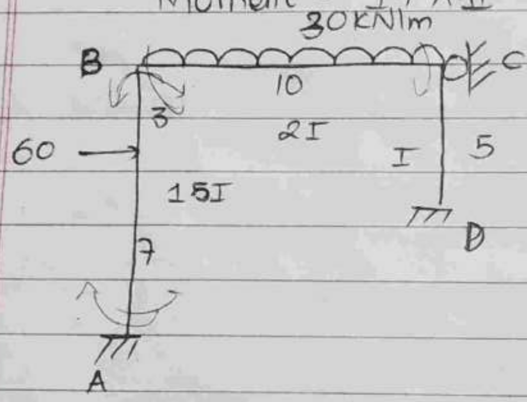
Frame with side sway



I) Distribution without side sway

II) Distribution with only side sway

Moment = I + λ.II



$$DBA = \frac{1.5I/10}{1.5I/10 + 2I/10} = 0.429$$

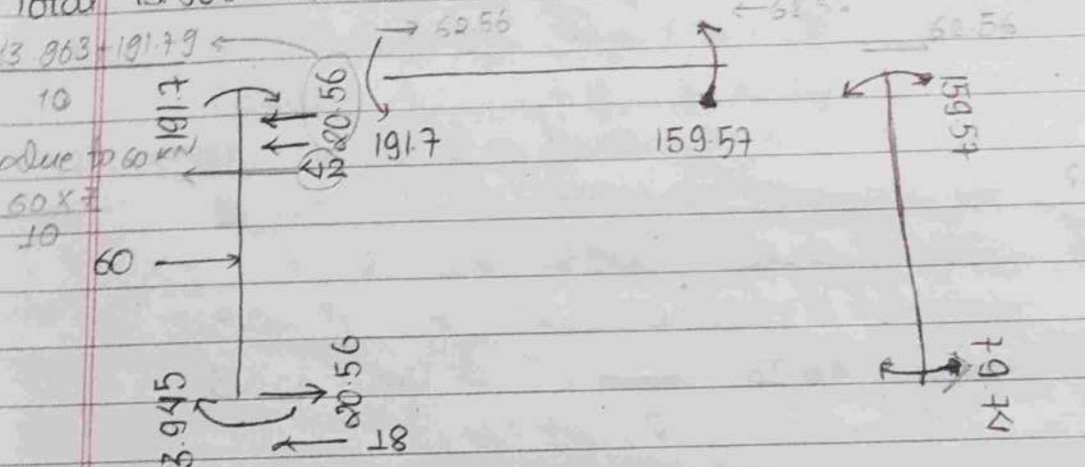
$$DBC = \frac{2I/10}{1.5I/10 + 2I/10} = 0.571$$

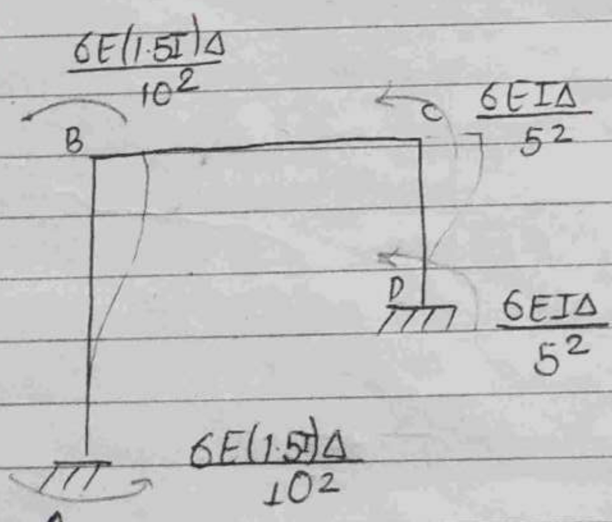
$$DCB = \frac{2I/10}{2I/10 + I/5} = 0.5$$

$$DCD = 0.5$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	-	0.429	0.571	0.5	0.5	-
~~FEM~~	~~-88.2~~	~~37.8~~	~~-166.67~~	~~166.67~~	~~0~~	~~0~~
~~Bal~~ FEM	-37.8	88.2	-250	250	0	0
Bal	-	69.41	92.38	-125	-125	0
CO	34.672	-	-62.5	46.288	-	-62.5
Bal	-	26.81	35.69	-23.14	-23.14	-
CO	13.405	-	-11.57	17.85	-	-11.57
Bal	-	4.963	6.606	-8.925	-8.925	-

CO	2.482	–	-4.463	3.303	–	-4.463
Bal	-	1.915	2.548	-1.652	-1.652	–
CO	0.958	-	-0.826	1.274	–	-0.826
Bal	–	0.354	0.472	-0.637	-0.637	-
CO	0.177	-	-0.319	0.236	–	-0.319
Bal	-	0.137	0.182	-0.118	-0.118	-
CO	0.0685	-	-0.059	0.091	-	-0.059
Total	13.963	191.79	-191.86	159.57	-159.47	-79.737



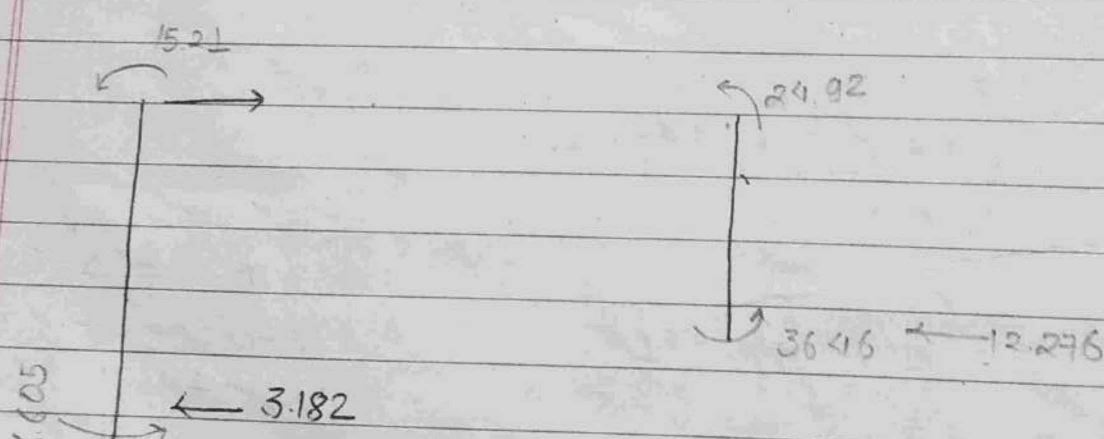


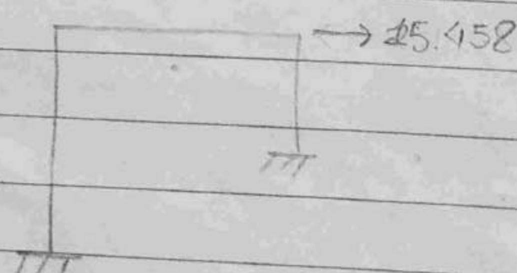
Assume $EI\Delta = 200$

$$\frac{6\times1.5\times200}{10^2} = 18$$

$$\frac{6\times200}{25} = 48$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	3/7	4/7	0.5	0.5	–
FEM	−18	−18	–	–	−48	−48
Bal	–	7.714	10.286	24	24	–
CO	3.857	–	12	5.143	–	12
Bal	–	−5.143	−6.857	−2.572	−2.572	–
CO	−2.572	–	−1.286	−3.429	–	−1.286
Bal	–	−0.551	0.735	1.715	1.715	–
CO	−0.276	–	0.858	0.368	–	0.858
Bal	–	−0.368	0.490	−0.184	−0.184	–
CO	−0.184	–	−0.092	0.245	–	0.092
Bal	–	−0.039	0.053	−0.123	−0.123	–
CO	−0.0195	–	−0.0615	0.0265	–	−0.0615
Total	−17.195	~~51.981~~ −16.387	16.23	~~26.9~~ 25.189	−25.164	−36.43





$X + \lambda \bar{X} = 0$ or, 14.64 + 15.458

or, $\lambda = \frac{X}{\bar{X}} = \frac{14.64}{} = 0.947$

$$M_{AB} = 13.963 + 0.947 \times - 17.195 = -\cancel{12.32} \text{ KN-m}$$

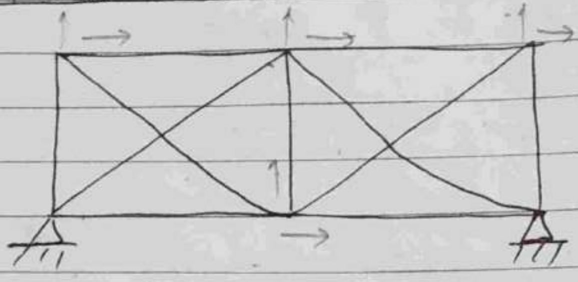
$$-2.32$$

Gs Pandit

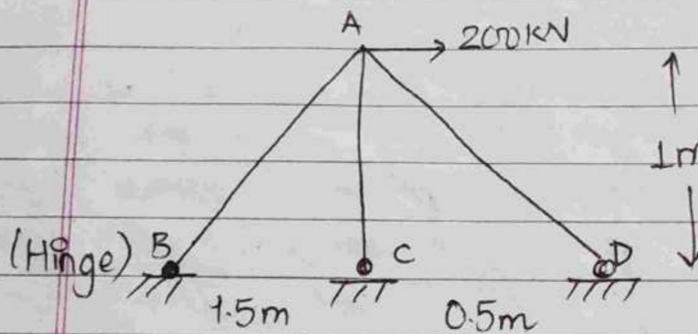
Date
Page

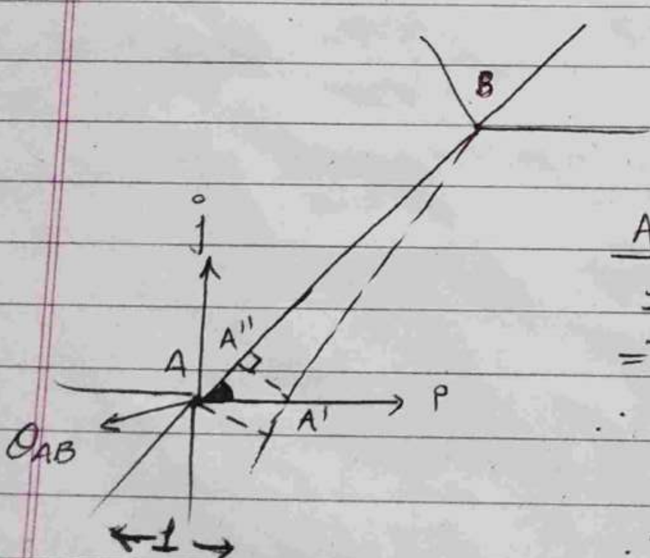
Truss

Stiffness method



$D_{KI} = 2j - r$
$= 12 - 4 = 8$





$$\frac{AA''}{1} = \cos\theta_{AB}$$

$\Rightarrow \cos\theta_{AB} = AA'' = \Delta L$

$\therefore \Delta L = \dfrac{PL}{AE}$

$\therefore F_{AB} = \dfrac{AE}{L}\cos\theta_{AB}$

$$k_{ii} = \frac{AE}{L}\cos\theta_{AB} \times \cos\theta_{AB}$$

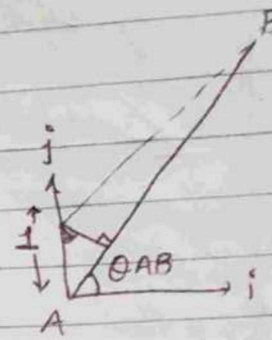
$$k_{ji} = \frac{AE}{L}\cos\theta_{AB} \times \sin\theta_{AB}$$

(force in j dirn due to displacement in i dirn)

Similarly for unit displacement in y dirⁿ ($\hat{j}$ dirⁿ)

$$k_{ij} = \frac{AE}{L} \sin\theta_{AB} \times \cos\theta_{AB}$$

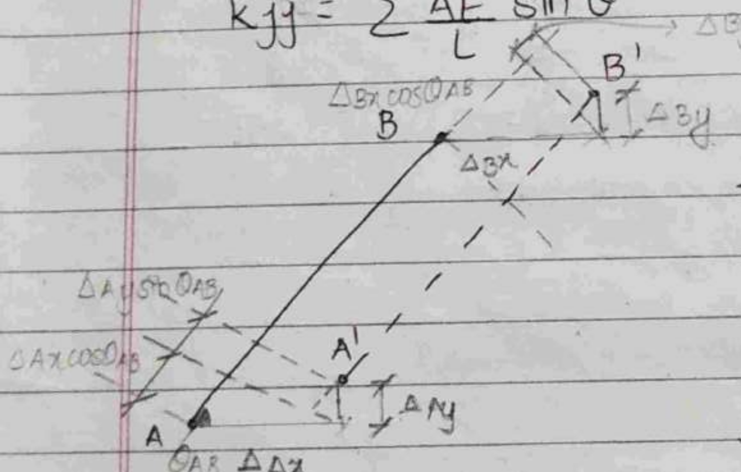
$$k_{jj} = \frac{AE}{L} \sin\theta_{AB} \times \sin\theta_{AB}$$



$$\therefore k_{ii} = \sum \frac{AE}{L} \cos^2\theta$$

$$k_{ij} = k_{ji} = \sum \frac{AE}{L} \sin\theta\cos\theta$$

$$k_{jj} = \sum \frac{AE}{L} \sin^2\theta$$



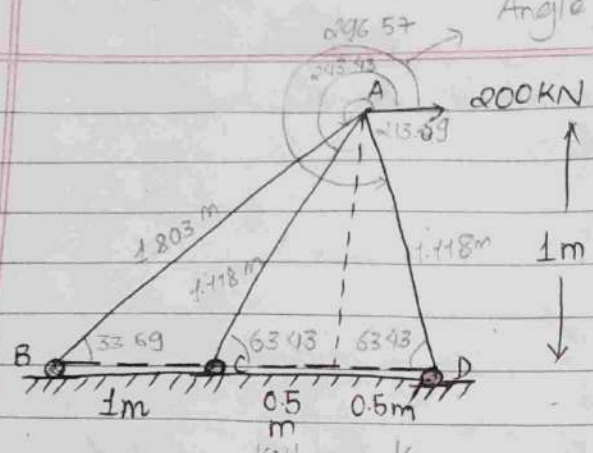
Net elongation. $\Delta L = \Delta B_x \cos\theta_{AB} + \Delta B_y \sin\theta_{AB} - \Delta A_x \cos\theta_{AB} - \Delta A_y \sin\theta_{AB}$

$$= (\Delta B_x - \Delta A_x)\cos\theta_{AB} + (\Delta B_y - \Delta A_y)\sin\theta_{AB}$$

$$\therefore \text{Force} = \frac{AE}{L}\left[\quad '' \qquad\qquad '' \quad \right]$$

θ is measured from horizontal to required member at pt. where load acts.

Date
Page



Angle from horizontal upto the member in counterclockwise direction

$A = 800 mm^2$
$E = 200 KN/mm^2$

KN/mm k

Member	θ	AE/L	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\cos\theta\cdot\sin\theta$	$\frac{AE}{L}\sin^2\theta$
AB	213.69	88.89	61.539	41.025	27.35
AC	243.43	143.11	28.630	57.251	114.48
AD	296.57	143.11	28.632	-57.251	114.48
			$k_{ii} = 118.803$	$k_{ij} = k_{ji} = 41.025$	$k_{jj} = 256.31$

$k_{11} = 118.803$

$k_{12} = k_{21} = 41.025$

$k_{22} = 256.31$

Now, $\Delta = [K]^{-1}[P]$

$$= \begin{bmatrix} 118.803 & 41.025 \\ 41.025 & 256.31 \end{bmatrix}^{-1} \begin{bmatrix} 200 \\ 0 \end{bmatrix}$$

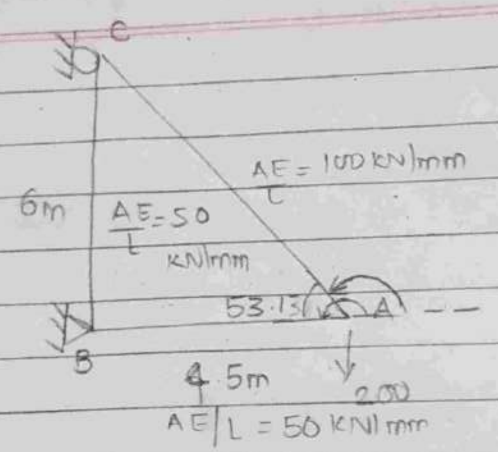
$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} 1.782 \\ -0.285 \end{bmatrix} \begin{matrix} \Rightarrow \Delta A x \text{ (in mm)} \\ \Rightarrow \Delta A y \text{ (in mm)} \end{matrix}$$

~~PAEB~~ $P_{AB} = \frac{AE}{L}[(0 - 1.782)\cos 213.69 + (0 + 0.285)\sin 213.69]$

88.89 ←

$= 117.746 KN$

+ve force tension (away from joint)
-ve force compression (towards joint)



Solve the frame at pt. where load acts and only for those members through which load acts.

Here AB & AC.

Member	θ	AE/L	$AE/L\cos^2\theta$	$AE/L\sin\theta\cos\theta$	$AE/L\sin^2\theta$
AB	180	50	50	0	0
AC	126.8699	100	36	-48	64

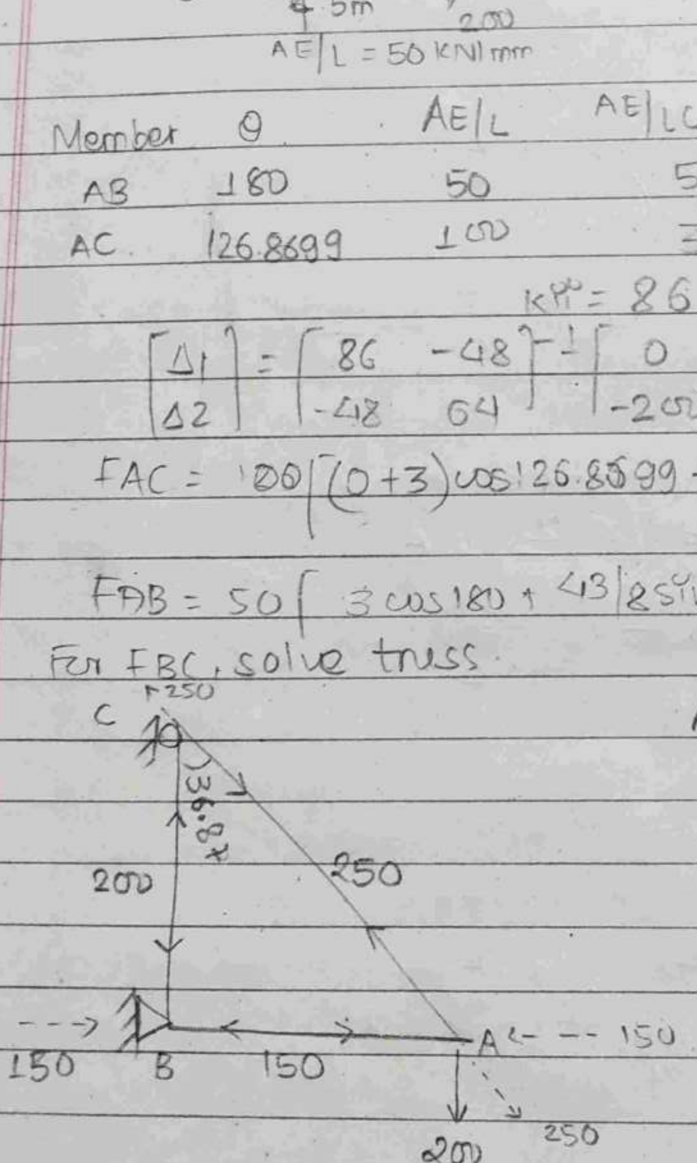
$k_{ii} = 86$ $\quad k_{ji} = k_{ij} = -48$ $\quad k_{jj} = 64$

$$\begin{bmatrix}\Delta_1\\ \Delta_2\end{bmatrix} = \begin{bmatrix}86 & -48\\ -48 & 64\end{bmatrix}^{-1}\begin{bmatrix}0\\ -200\end{bmatrix} = \begin{bmatrix}-3\\ -43/8\end{bmatrix}$$

$$F_{AC} = 100\left[(0+3)\cos 126.8699 + \frac{43}{8}\times\sin 126.8699\right] = 250\ (T)$$

$$F_{AB} = 50\left[3\cos 180 + 43/8\sin 180\right] = -150\ (C)$$

For F_{BC}, solve truss.

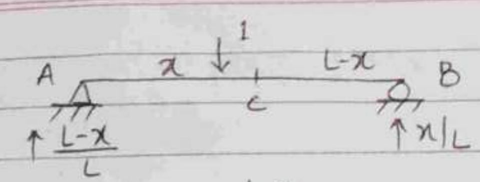


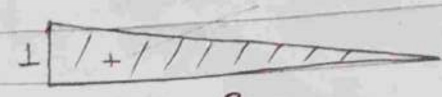
At C,

$250\cos 36.87 + CB = 0$

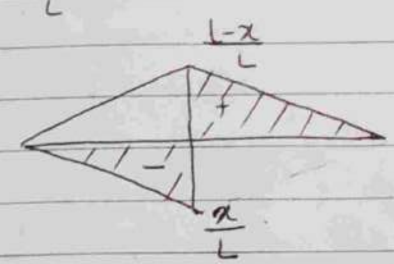
$CB = -200\ (C)$

Ch. 4 Influence Line Diagram (Statically Indeterminate Structure)

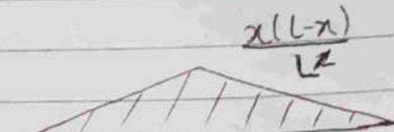




R_A



SF_C

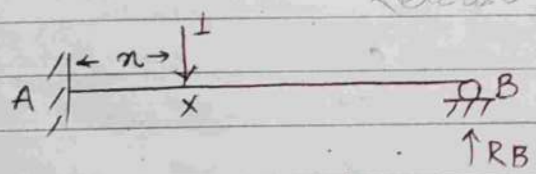


BM

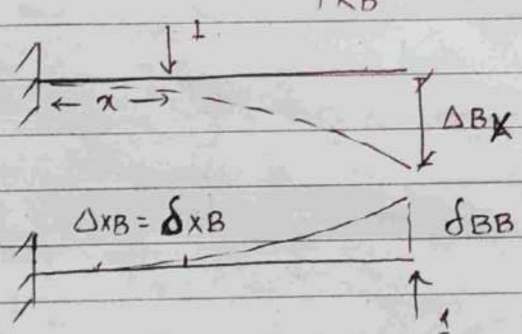
Release the func^n whose ILD is to be drawn

Propped cantilever beam

Eg. 1)



Primary Structure



Maxwell's Reciprocal Theorem

$$R_B \times \delta_{BB} + \Delta_{Bx} = 0$$

$$\therefore R_B = \frac{-\Delta_{Bx}}{\delta_{BB}} = \frac{-\Delta_{xB}}{\delta_{BB}}$$

δ_{BB} = deflection

If for moment,

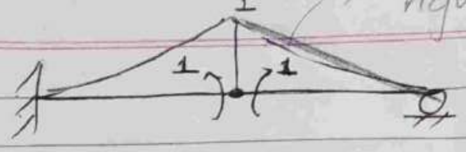
$$M_P = \frac{\Delta_{XP}}{\delta_{PP}}$$

δ_{PP} = slope

→ Muller-Breslau Principle

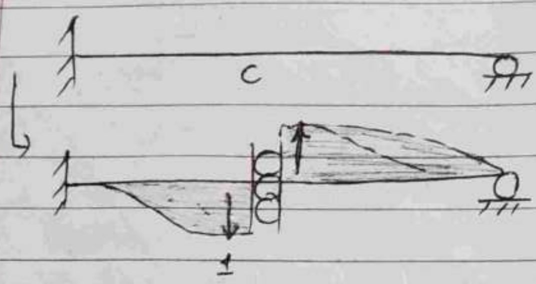
If a func^n is released & str. is displaced by unit magnitude of that func^n, resulting shape gives ILD of func^n

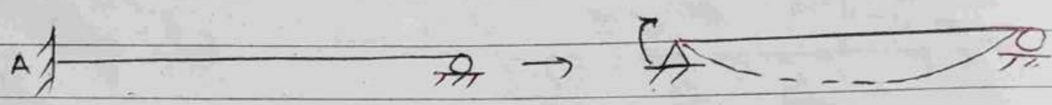
$$\text{ILD coordinate} = \frac{\text{Deflection due to moving unit load at pt. where our func}^n\text{ acts}}{\text{Slope/Deflection due to unit magnitude of function at pt. where it acts}}$$



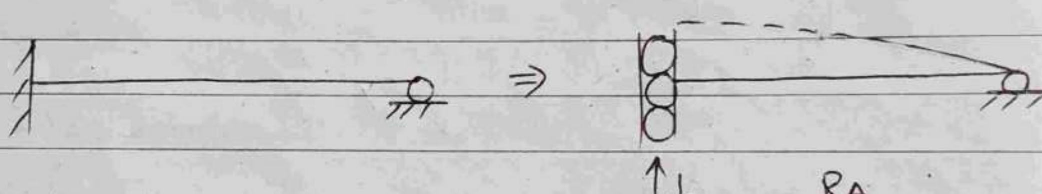
Always the func^n on the right is the actual internal magnitude & sign of that function. Value on left is to balance

Releasing SF at C

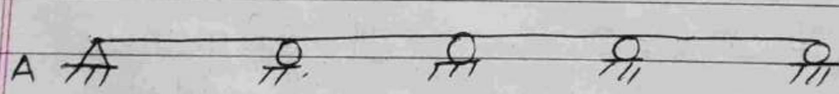


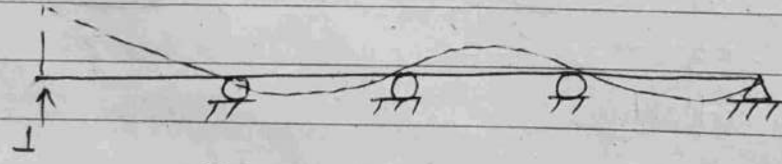


MA

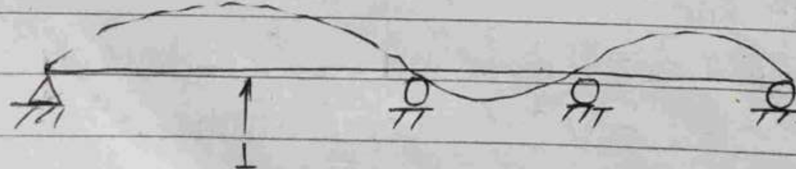


RA



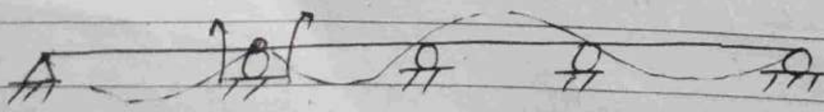


RA



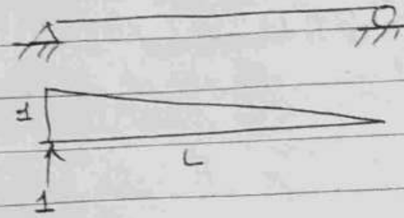
RB

MB

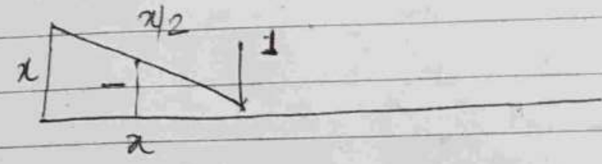


Date
Page

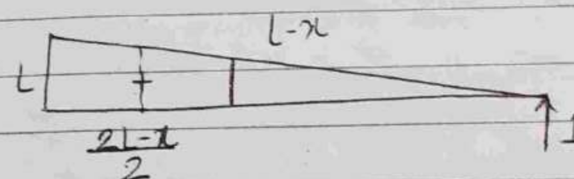
→ Parasitic



Contd.
Eg 1



ML



mL

$$\Delta_{Bx} = \Delta_{1L} = \frac{x}{6EI}\left[-xL + 4\times\left(\frac{-x}{2}\right)\times\frac{2L-x}{2}\right]$$

$$= \frac{x}{6EI}\left[-xL + 2xL + (-x^2)\right]$$

$$= \frac{x}{6EI}\left[-x^2 + xL\right] = \frac{x^2(-x+L)}{6EI}$$

$$= \frac{x}{6EI}\left[-xL + 4\times\left(\frac{-x}{2}\right)\times\frac{2L-x}{2}\right]$$

$$= \frac{x}{6EI}\left[-3xL + x^2\right]$$

$$= \frac{x^2(x-3L)}{6EI}$$

$$\delta_{BB} = \frac{L}{6EI}\left[L^2 + \cancel{4\times(L-x)\times(L-x)}\ \frac{L}{2}\times\frac{L}{2}\times 4\right]$$

$$= \frac{L}{6EI}\left[L^2 + 4\times\frac{L^2}{4}\right] = \frac{L^3}{3EI}$$

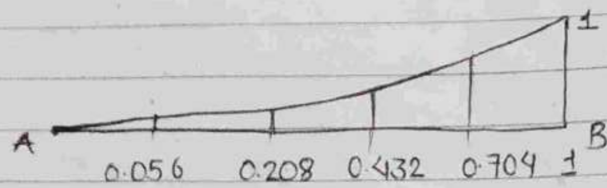
Conjugate beam is formed for structure after fun^c is released

Conjugate beam
Shear force ⇒ Rotation
BM ⇒ Deflection (Δ) of real beam

$$R_B = \frac{-x^2/6EI\,(x-3L)}{L^3/3EI}$$ (because conjugate beam is reverse of real beam)

$$= -\frac{x^2(x-3L)}{2L^3}$$

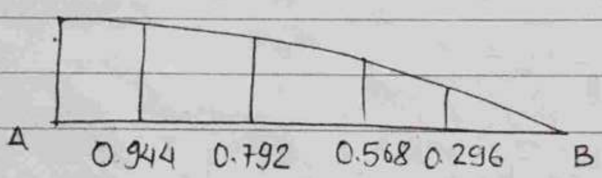
(Take L = 10 m)
Plot at 2m interval.



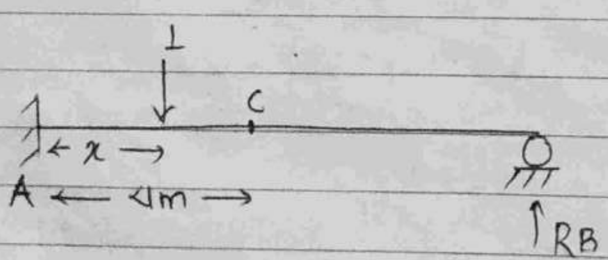
RB

$R_A + R_B = 1$

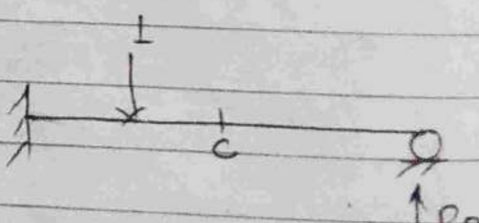
$$\therefore R_A = 1 + \frac{x^2(x-3L)}{2L^3}$$

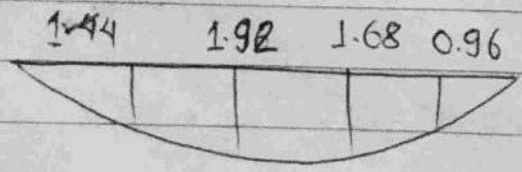


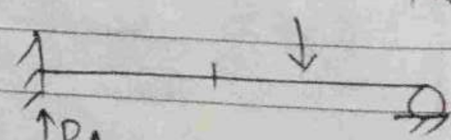
RA

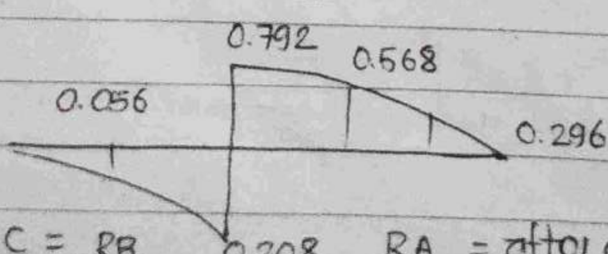


MA = RB×L−x





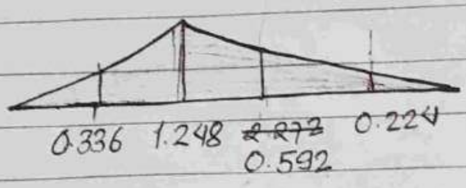




SFC

before C = RB   RA = after C

If replaced func. is force, divide by Moment at that position of conjugate beam.
If replaced func. is moment, divide by SF at that position of conjugate beam.



$\underline{M_C}$

When 1 kN left of C,

$M_C = R_B \times 6$

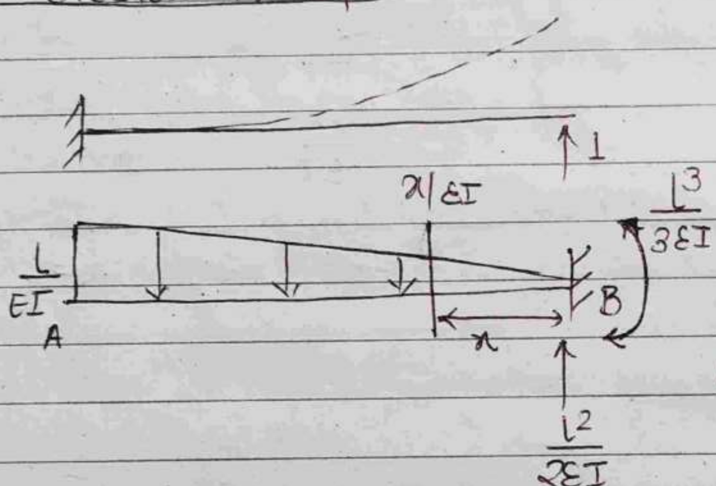
After 1 kN crosses C,

$M_C = R_A \times 4 + M_A$

At $x = 4$, $M_C = 0.792 \times 4 - 1.92$

Muller- Breslau Principle

$\underline{R_B}$



$M_B = \frac{L}{EI} \times \frac{L}{2} \times \frac{2L}{3}$

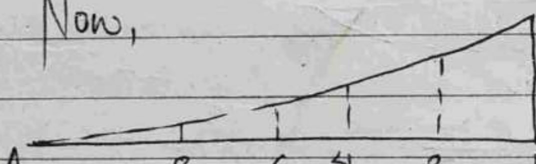
$= \frac{L^3}{3EI}$

$M_x = -\frac{L^3}{3EI} + \frac{L^2}{2EI}x - \frac{1}{2} \times \frac{x}{EI} \times x \times \frac{x}{3}$

$= -\frac{L^3}{3EI} + \frac{L^2 x}{2EI} - \frac{x^3}{6EI}$

Now,

$\underline{R_B}$



$\frac{L^3}{3EI} \times \frac{3EI}{L^3} = 1$

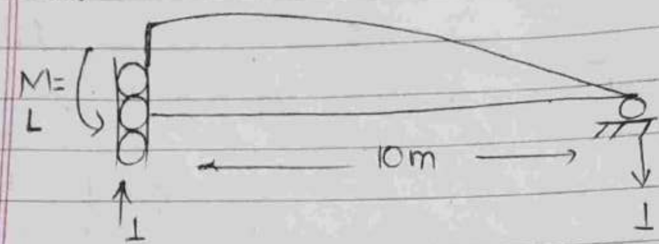
$M_2 =$

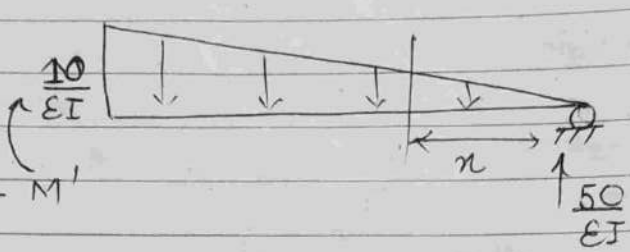
$M_4 =$

$M_6 =$

$M_8 =$

R_A





Moment to make structure stable

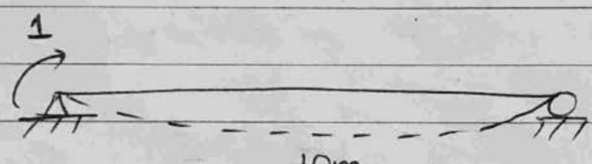
$$M' - \frac{1}{2}\times 10\times \frac{10}{EI}\times\frac{20}{3}=0$$

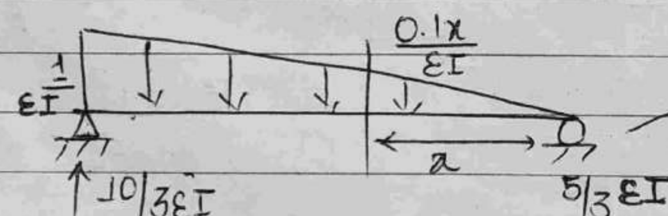
or, $M' = \frac{1000}{3EI}$

$M_x =$

M_A

Release func^n whose ILD is required & apply unit of func^n



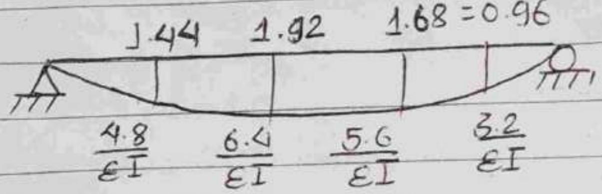


$$\frac{1}{2}\times\frac{1}{EI}\times 10\times\frac{10}{3}\times\frac{1}{10}$$

$$M_x = \frac{5}{3EI}\times x - \frac{1}{2}\times\frac{0.1x^2}{EI}\times\frac{x}{3}$$

dist	M_x	dist	M_x
2	3.2/EI	6	6.4/EI
4	5.6/EI	8	4.8/EI

$$\frac{\cancel{4.8}\;3.2}{EI} \times \frac{3EI}{10}$$



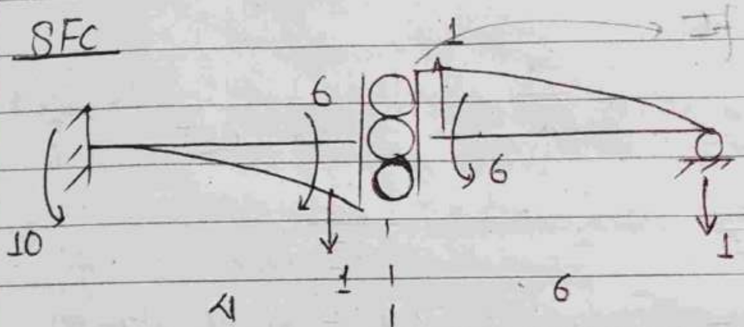
BM of conjugate beam

$$M_A = \frac{\Delta_{xA}}{\delta_{AA}}$$

where $\Delta_{xA} = \frac{3.2}{EI}, \frac{6.4}{EI}$

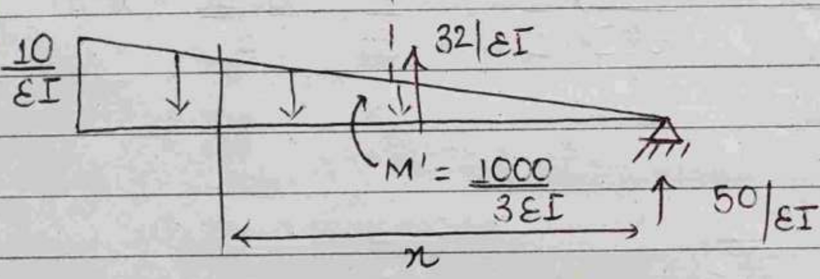
$$\delta_{AA} = Q_A = \frac{10}{3EI} =$$

SFC



If such internal roller support is placed, stabilizing moment must be applied.

SF of conjugate beam

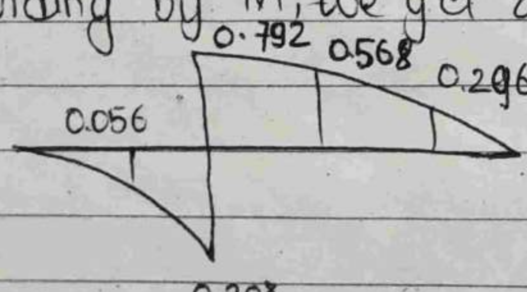


$$M' - \frac{1}{2} \times \frac{10}{EI} \times 10 \times \frac{20}{3} = 0$$

$$M_x = \frac{50}{EI} \times x - \frac{1}{2} \times \frac{x}{EI} \times x \times \frac{x}{3} - M'$$

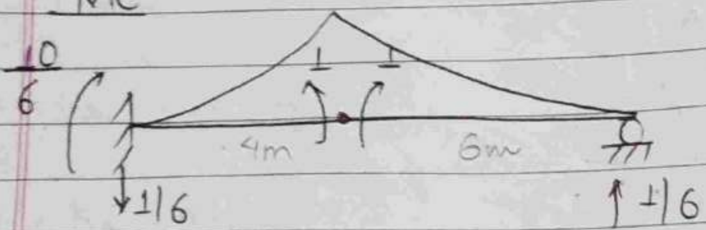
Dist	M_x	Dist	M_x	Dist	M_x
2	98.67/EI	6 (R)	264/EI	8	−18.67/EI
4	189.3/EI	6 (L)	−69.33/EI		

Dividing by M', we get ans.



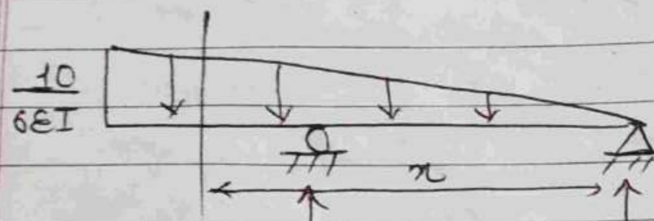
$$= \frac{98.67}{EI} \times \frac{3EI}{1000} =$$

Mc



$A\times 6-1=0$

$A=1/6$



$$-\frac{1}{2}\times\frac{10}{6EI}\times 10\times\frac{20}{3}+R_C'\times 6=0$$

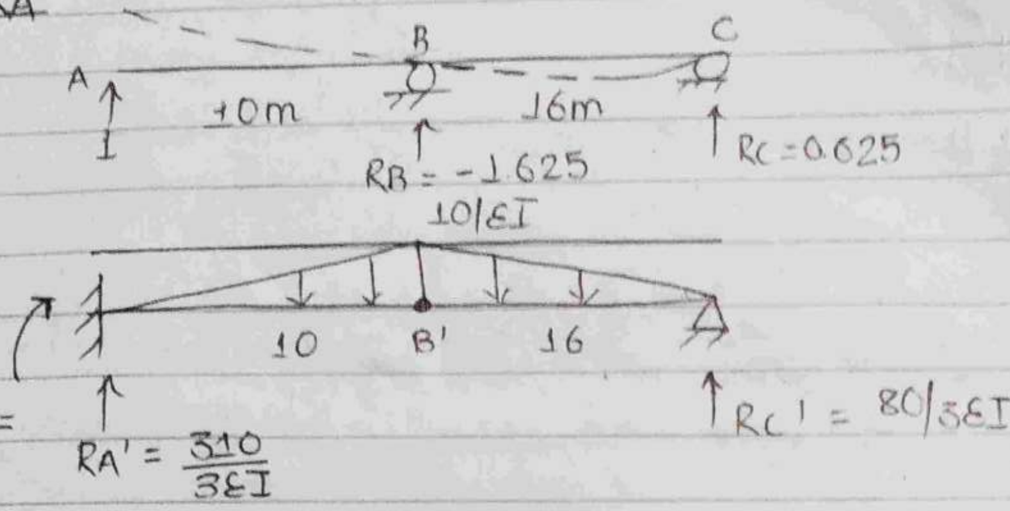
$R_C'=\frac{9.26}{EI}$ $\qquad R_B'=-\frac{0.927}{EI}$

$$M_x=-\frac{0.927x}{EI}-\frac{x}{6EI}\times\frac{1}{2}\times x\times\frac{x}{3}+\frac{9.26}{EI}\times(x-6)$$

Dist	M_x
2	
4	
6	

Divide by R_C' at end

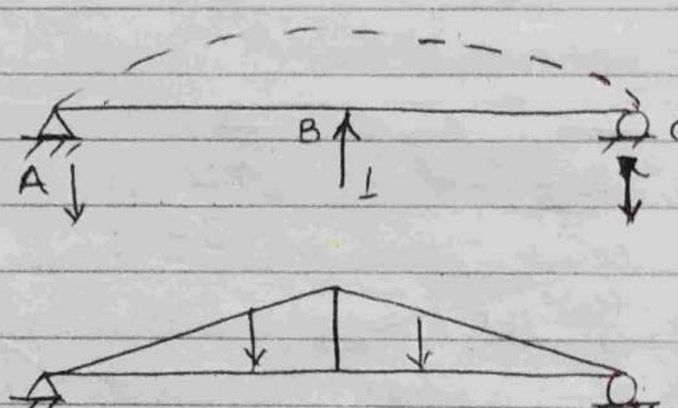
$\underline{R_A}$



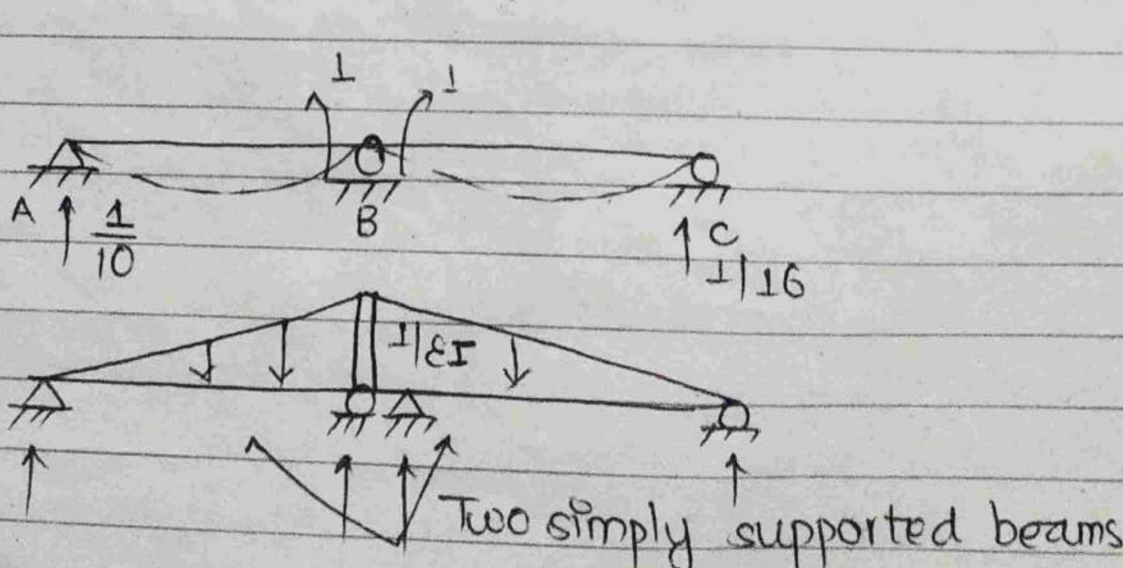
$-\frac{2600}{3EI}$

$$M_A' + \frac{310}{3EI} \times 10 - \frac{1}{2} \times 10 \times \frac{10}{EI} \times \frac{10}{3} = 0 \Rightarrow M_A' = -\frac{2600}{3EI}$$

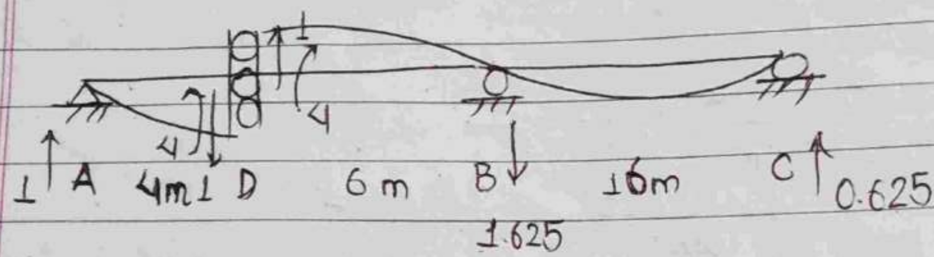
$\underline{R_B}$

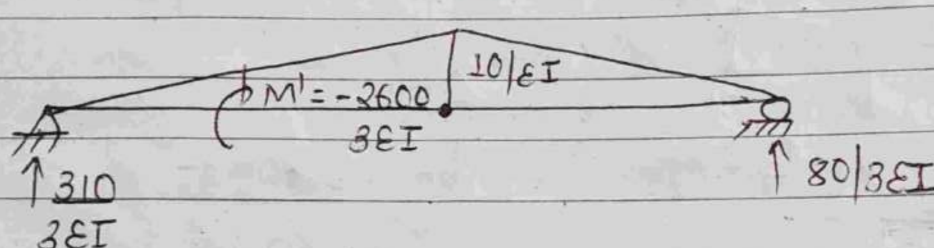


$\underline{M_B}$

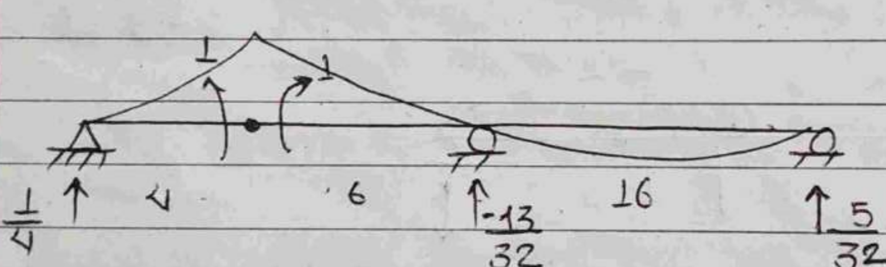


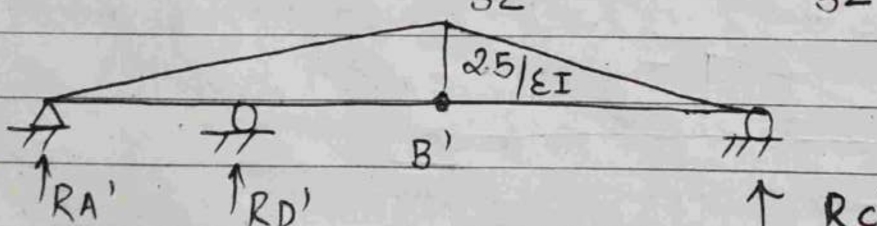
SF at D


1
4
4
1
A
4m
1
D
6 m
B
16m
C
0.625
1.625

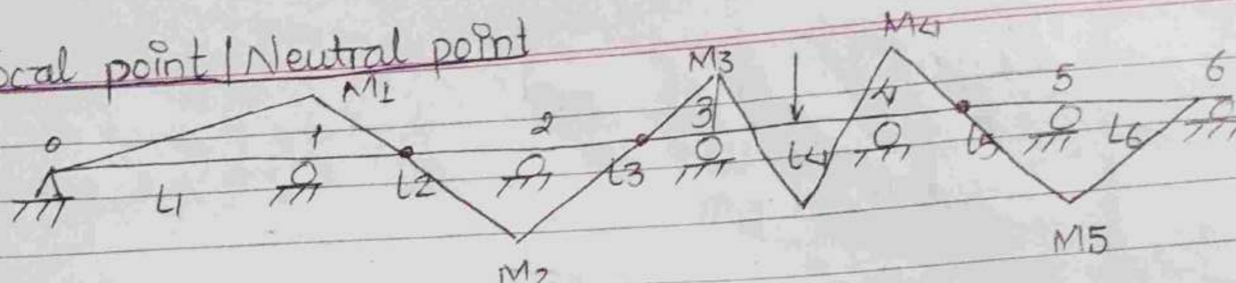

10/EI
M' = -2600 / 3EI
310 / 3EI
80/3EI

MD


1
1
1/4
4
6
16
-13/32
5/32


25/EI
B'
RA'
RD'
RC'

Focal point / Neutral point



Left focal point ratio, $k_n = -\dfrac{M_n}{M_{n-1}}$

$k_1 = -\dfrac{M_1}{M_0}$, $k_2 = -\dfrac{M_2}{M_1}$ . . . .

Right focal point ratio, $k_n' = -\dfrac{M_{n-1}}{M_n}$

$k_6' = -\dfrac{M_5}{M_6}$, $k_5' = -\dfrac{M_4}{M_5}$ . . .

(0 BM)

In continuous beam, there are points of contraflexure on unloaded spans whose positions are unchanged even if loads on loaded span change (in magnitude or position). These points are called focal points. Focal points divide span in fixed ratio which is called focal point ratio. If loaded span is on right side, focal point ratios of unloaded span on left are called left focal point ratio, denoted by k_n.

$$k_n = -\frac{M_n}{M_{n-1}}$$

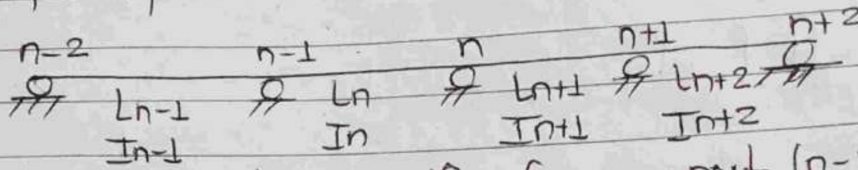
If loaded span is on left side, focal point ratios of unloaded span on right are called right focal point ratio, denoted by k_n'.

$$k_n' = -\frac{M_{n-1}}{M_n}$$

Exam Determination of focal point ratio

Derivation of recurrent formula for focal point ~~two~~ ratio

Left focal point ratio



Writing 3 moment equation for support (n-1),

$$M_{n-2}\cdot\frac{L_{n-1}}{I_{n-1}} + 2\frac{M_{n-1}}{\cancel{x}}\left[\frac{L_{n-1}}{I_{n-1}}+\frac{L_n}{I_n}\right] + M_n\frac{L_n}{I_n} = 0 \text{ (Unloaded)}$$

$$\text{or, } \frac{M_{n-2}}{M_{n-1}}L'_{n-1} + 2(L'_{n-1}+L'_n) + \frac{M_n}{M_{n-1}}L'_n = 0$$

$$k_n = \frac{-M_n}{M_{n-1}}, \quad k_{n-1} = \frac{-M_{n-1}}{M_{n-2}}$$

$$\text{or, } -\frac{1}{k_{n-1}}L'_{n-1} + 2L'_{n-1} + 2L'_n - k_nL'_n = 0$$

$$\text{or, } k_n = 2 + \frac{L'_{n-1}}{L'_n}\left[2-\frac{1}{k_{n-1}}\right]$$

Writing 3 moment eq^n for support n,

$$M_{n-1}\frac{L_n}{I_n} + 2M_n\left[\frac{L_{n+1}}{I_{n+1}}+\frac{L_n}{I_n}\right] + M_{n+1}\cdot\frac{L_{n+2}}{I_{n+2}} = 0$$

$$\text{or, } \frac{M_{n-1}}{M_n}L'_n + 2L'_{n+1} + 2L'_n + \frac{M_{n+1}}{M_n}L'_{n+2} = 0$$

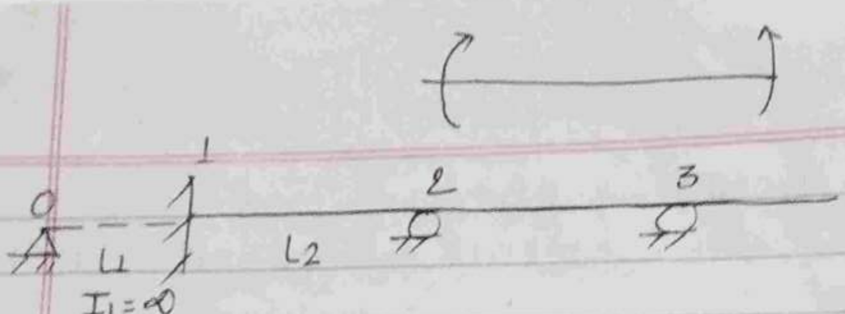
$$\text{Now, } k'_n = -\frac{M_{n-1}}{M_n}, \quad k'_{n+1} = \frac{-M_n}{M_{n+1}}$$

$$\text{or, } -k'_nL'_n + 2L'_{n+1} + 2L'_n - \frac{1}{k'_{n+1}}L'_{n+2} = 0$$

$$\text{or, } k'_n = 2 + \frac{L'_{n+1}}{L'_n}\left[2-\frac{1}{k'_{n+1}}\right]$$

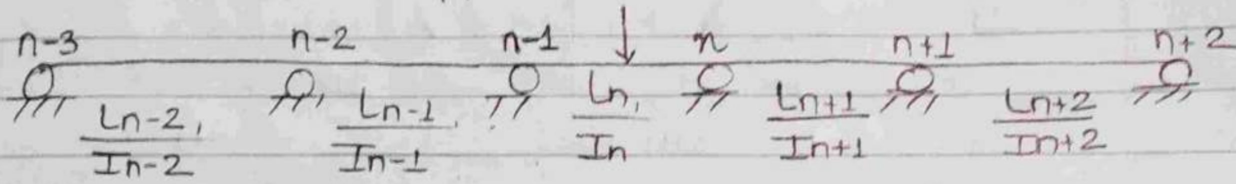
Assumed dirⁿ of moment

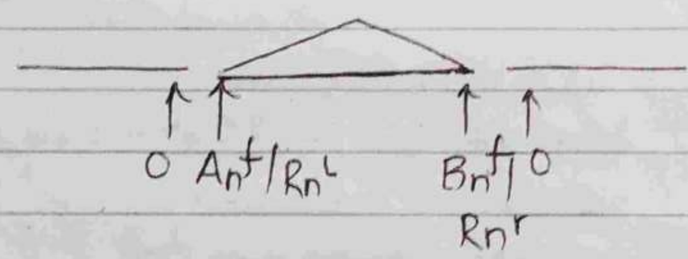
Date ____ Page ____



$$k_1 = \frac{-M_1}{M_0} = \infty, \quad k_2 = 2 + L_1'\left[2 - \frac{1}{\infty}\right] = 2$$

Determination of support moment on loaded span





Writing three moment equation for support (n-1),

$$M_{n-2} L_{n-1}' + 2M_{n-1}[L_{n-1}' + L_n'] + M_n L_n' = -6\left[0 + \frac{R_n^L}{I_n}\right]$$

$$k_n = \frac{-M_n}{M_{n-1}}, \quad k_{n-1} = -\frac{M_{n-1}}{M_{n-2}}$$

or, $-\frac{M_{n-1}}{k_{n-1}} L_{n-1}' + 2M_{n-1} L_{n-1}' + 2M_{n-1} L_n' + M_n L_n' = -\frac{6R_n^L}{I_n}$

or, $\cancel{M_{n-1}}\left[\cancel{-\frac{L_{n-1}'}{k_{n-1} \cdot L_n'}} + \cancel{\frac{2L_{n-1}'}{L_n'}} + 2\right]$ ← Dividing by L_n' and taking M_{n-1} common.

or, $k_n \cdot M_{n-1} + M_n = -\frac{6R_n^L}{I_n \cdot L_n'}$ —(i)

$$= -\frac{6R_n^L}{I_n}$$

Writing three moment equation for support n.

$$M_{n-1}L_n' + 2M_n(L_n' + L_{n+1}') + M_{n+1}L_{n+1}' = -6\left[0 + \frac{R_n^r}{I_n}\right]$$

$$k_n' = -\frac{M_{n-1}}{M_n}, \quad k_{n+1}' = -\frac{M_{n-2}}{M_{n+1}}$$

or, $M_{n-1}L_n' + 2M_n(L_n' + L_{n+1}') - \frac{M_n}{k_{n+1}'}L_{n+1}' = -\frac{6R_n^r}{I_n}$

or, $M_{n-1}\frac{\cancel{L_n'}}{\cancel{L_n'}} + M_n\left[2 + \cancel{2}\frac{L_{n+1}'}{L_n}\left[2 - \frac{1}{k_{n+1}'}\right]\right] = \frac{-6R_n^r}{I_nL_n'}$

or, $M_{n-1} + M_nk_n' = \frac{-6R_n^r}{I_nL_n'}$ — (ii)
$= -6R_n^r/L_n$

Solving (i) & (ii),

$$M_{n-1} + k_n'\left[-\frac{6R_n^L}{L_n} - k_nM_{n-1}\right] = -\frac{6R_n^r}{L_n}$$

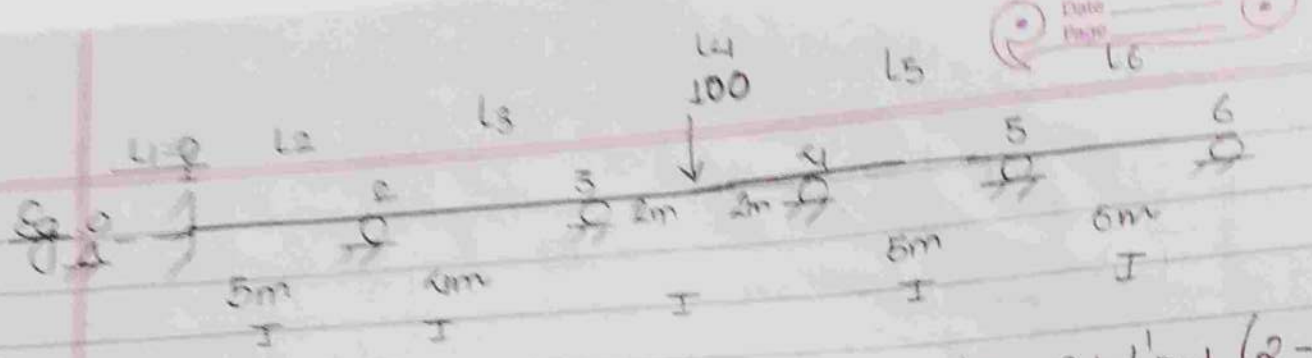
or, $M_{n-1} + (-k_n'k_nM_{n-1}) = \frac{6R_n^Lk_n'}{L_n} - \frac{6R_n^r}{L_n}$

(At $n=4$)

~~Moment at~~ support left of load $\rightarrow$ $M_{n-1} = \frac{6}{L_n}\left[R_n^Lk_n' - R_n^r\right] \times \frac{1}{(1 + (-k_n'k_n))}$

$$= \frac{-6}{L_n} \times \frac{(-R_n^r + R_n^Lk_n')}{(k_n'k_n - 1)}$$

~~Moment at~~ support right of load $\rightarrow$ $M_n = \frac{-6}{L_n}\left[\frac{R_n^rk_n - R_n^L}{k_nk_n' - 1}\right]$



Span	k_n	k_n'
2	2	3.387
3	3.875	3.763
4	3.742	4.216
5	3.386	4.4
6	3.421	∞

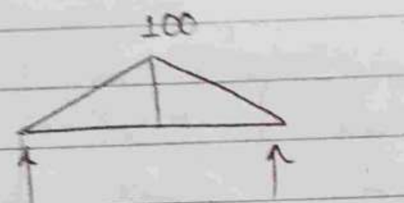
$$k_n = 2 + \frac{L'_{n-1}}{L_n'}\left(2 - \frac{1}{k_{n-1}}\right)$$

$$k_3 = 2 + \frac{L_2}{L_3}\left(2 - \frac{1}{2}\right)$$

$$= 2 + 5/4 \times 3/2$$

$$= 3.875$$

$$k_n' = 2 + \frac{L'_{n+1}}{L_n'}\left[2 - \frac{1}{k'_{n+1}}\right]$$



$R_4^l = 100 \qquad R_4^r = 100$

(n=4) $$M_{n-1} = M_3 = -\frac{6}{4}\left[\frac{100 \times 4.216 - 100}{4.216 \times 3.742 - 1}\right] = -32.647 \text{ KNm}$$

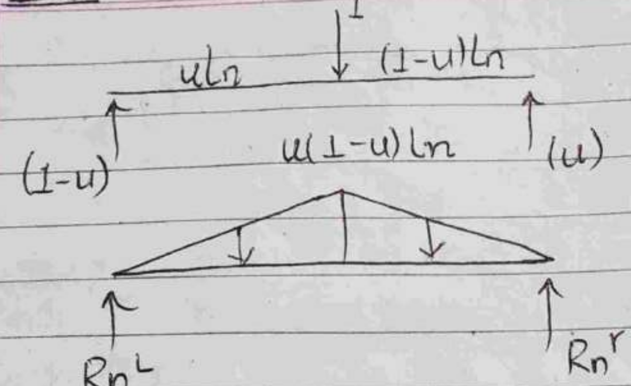
$$M_4 = M_n = -\frac{6}{4}\left[\frac{100 \times \cancel{4.216}^{3.742} - 100}{4.216 \times 3.742 - 1}\right] = \cancel{-32.647}\ -27.835 \text{ KNm}$$

$$k_3 = -\frac{M_3}{M_2} \Rightarrow M_2 = 8.425 \text{ KNm}$$

$$M_1 = -\frac{M_2}{k_2} = -4.213 \text{ KNm}$$

$$M_5 = -\frac{M_4}{k_5'} = 6.326 \text{ KNm}$$

ILD



$= u(1-u)(2-u)\frac{Ln^2}{6}$ $\quad = \frac{Ln^2 u(1-u^2)}{6}$

$= \alpha(u)\frac{Ln^2}{6}$ $\quad = \beta(u)\frac{Ln^2}{6}$

$Rn^r \times \cancel{L}n - \frac{u}{2}(1-u)\cancel{L}n \times Ln \times (1-u)$

$\left[uLn + \frac{1}{3}(1-u)Ln\right] -$

$\frac{1}{2} \times u(1-u)Ln \times u\cancel{L}n \times \frac{2}{3}uLn = 0$

or, $Rn^r = \frac{u}{2}(1-u)^2 Ln\left[\frac{2uLn + Ln}{3}\right]$

$+ \frac{2}{6}u^3 Ln^2(1-u) = 0$

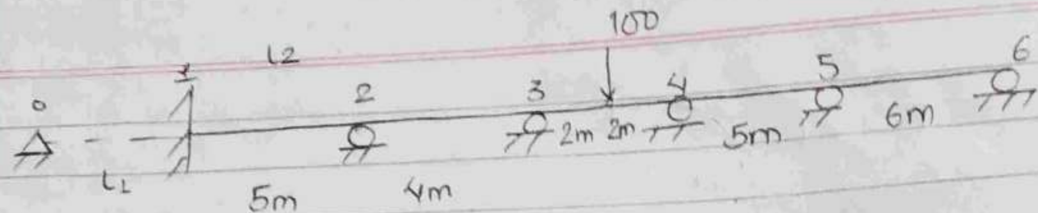
$= uLn^2(1-u)\left[\frac{1-u}{2} \times \frac{2u+1}{3} + \frac{2u^2}{6}\right]$

$= \frac{uLn^2(1-u)6(u+1)}{6}$

$= \frac{uLn^2(1-u^2)}{6}$

Now,

u	$\alpha(u)$	$\beta(u)$
0.1	0.171	0.099
0.2	0.288	0.192
0.3	0.357	0.273
0.4	0.384	0.336
0.5	0.375	0.375
0.6	0.336	0.384
0.7	0.273	0.357
0.8	0.192	0.288
0.9	0.099	0.171



ILD for M_1

When unit load is on span 2

For 2nd span, $M_1 = M_{n-1} = \cancel{-\frac{L_n^2}{6}} - \frac{6}{L_n}\left[\frac{R_n^L k_n' - R_n^r}{k_n k_n' - 1}\right]$ $(n = 2)$

$$= -\frac{\cancel{6}}{\cancel{L_2}}\left[\frac{\alpha(u)\times 3.387 - \beta(u)}{2\times 3.387 - 1}\right]\times\frac{L_2^{\cancel{2}}}{\cancel{6}}$$

$$= -L_2\left[\frac{3.387\alpha(u) - \beta(u)}{5.774}\right] = -0.865\left[3.387\alpha(u) - \beta(u)\right]$$

For calculating moment at 1 m ~~interval~~, for span of 5m, we take u as 0.2, for 2m, we take u = 0.4 & so on.

Load Position	Moment, M_1
1	−0.678
2	−0.835
3	−0.653
4	−0.314
~~5~~	

Again,

$M_2 = M_{n-1} = -\frac{6}{L_n}\left(\frac{R_n^L k_n' - R_n^r}{k_n k_n' - 1}\right)$ $(n=3)$

(For 3rd span) $= -\frac{6}{4}\left[\frac{\alpha(u)\times 3.763 - \beta(u)}{3.875\times 3.763 - 1}\right]\times\frac{4^2}{6}$

$$= -0.2945\left(3.763\alpha(u) - \beta(u)\right)$$

load At middle of span, $u = 0.5$

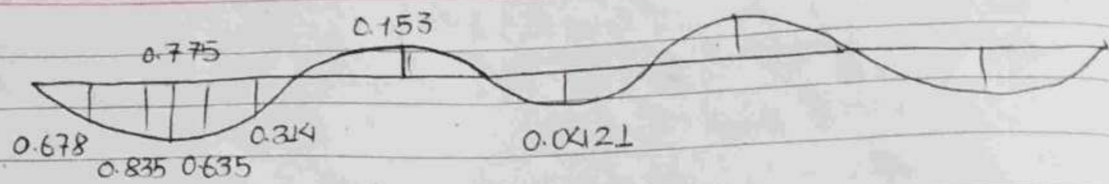
$$\therefore M_2 = -0.305$$

$$k_2 = -\frac{M_2}{M_1} \quad \left[k_n = \frac{-M_n}{M_{n-1}}\right]$$

$$\therefore M_1 = \frac{0.305}{2} = 0.153$$

Date
Page

(n=3)



For 4th span, n=4

$$M_{n-1} = M_3 = \frac{-6}{4}\left[\frac{\alpha(u)\times 4.216 - \beta(u)}{4.216\times 3.742-1}\right]\times\frac{4^2}{6} = -0.271(4.216\,\alpha(u)-\beta(u))$$

Load at middle of span, $u = 0.5$

$\therefore M_3 = -0.3268$

$$\therefore M_1 = \frac{+M_3}{k_2 k_3} = -0.0421$$

ILD for M_2

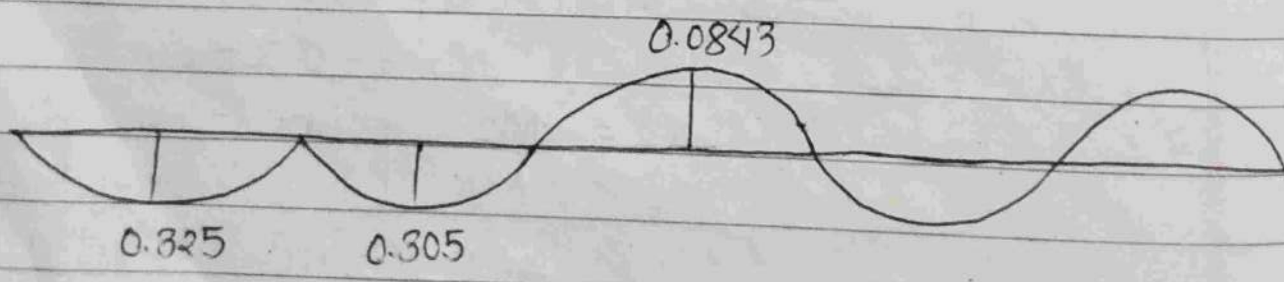
-> When unit load is on span 2,

$$M_n = M_2 = \frac{-6}{l_n}\left[\frac{R_n' k_n - R_n^L}{k_n' k_{n-1}}\right]$$

$$= \frac{-6}{5}\left[\frac{\beta(u)\times \cancel{3.387}\,2 - \alpha(u)}{3.387\times 2 - 1}\right]\times\frac{5^2}{6}$$

$$= -0.865(\cancel{3.387}\,2\,\beta(u) - \alpha(u))$$

At $u=0.5$, $M_2 = -0.325$



Note — Propped cantilever beam

$$(n=3)\quad M_{n-1} = M_2 = -\frac{6}{4}\left[\frac{3.763\,\alpha(u)-\beta(u)}{3.763\times 3.875-1}\right]\times\frac{4^2}{6} = -0.295\,(3.763\,\alpha(u)-\beta(u))$$

$$\therefore M_2 = -0.305$$

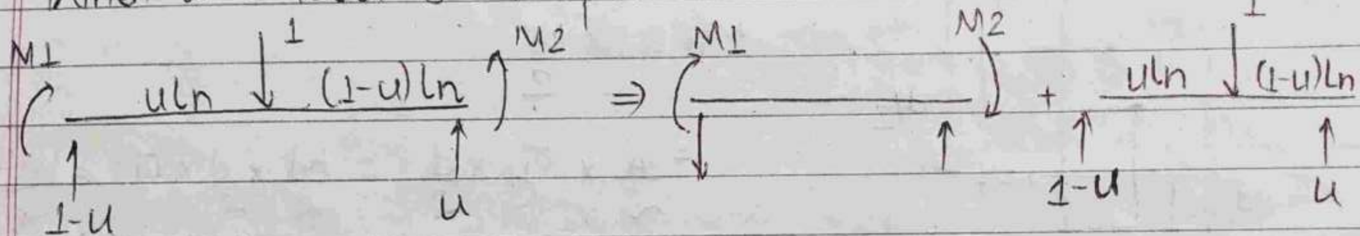
Similarly, load on span 4, $M_3 = -0.3268$

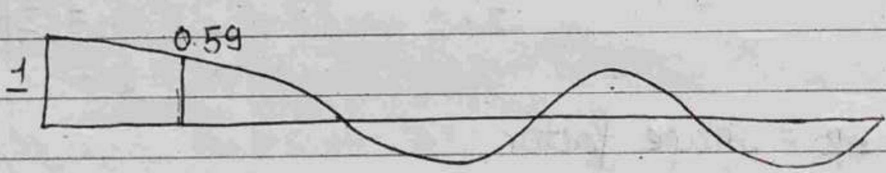
$$\therefore M_2 = \frac{0.3268}{3.875} = 0.0843$$

(we use k_n to divide because load is on right and we transfer moment towards left so left focal pt. ratio.)

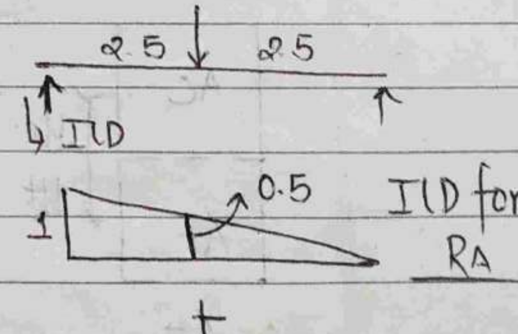
ILD for R_1

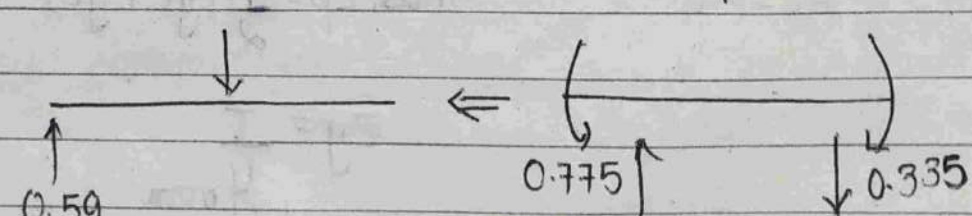
When unit load is on span 2,





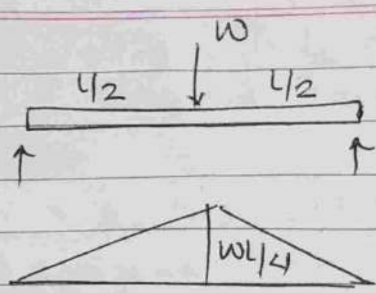
At middle of span 2

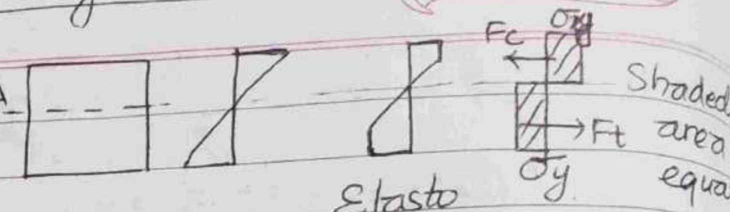




$$0.09 = \frac{0.775-0.335}{5}$$

Plastic Analysis





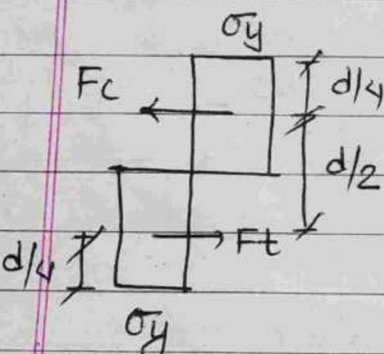
$\sigma = \frac{My}{I}$

Elasto plastic M_y, W_y

M_p, W_p

$$M_y = \sigma_y \times \frac{I}{y} \quad \left[\frac{I}{y} = Z, \text{ Section modulus}\right]$$

$$= \sigma_y \times Z_y = \frac{bd^2}{6} \times \sigma_y = Z_y \sigma_y$$



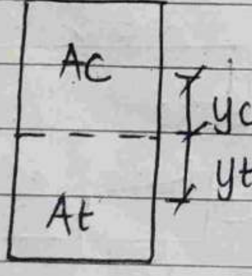
$$M_p = F_c \times \frac{d}{2}$$

$$= \frac{A}{2} \times \sigma_y \times \frac{d}{2} = \frac{bd}{2} \times \frac{d}{2} \times \sigma_y = \frac{bd^2}{4} \times \sigma_y$$

$$= Z_p \sigma_y$$

$$\boxed{\therefore \frac{M_p}{M_y} = \frac{Z_p}{Z_y} = \text{Shape factor}}$$

Rectangular, Shape factor = 1.5



$$M_p = F_c \cdot y_c + F_t \cdot y_t$$

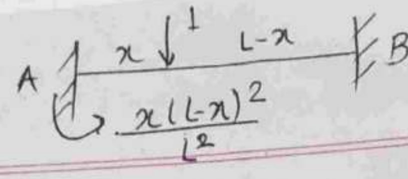
$$= A_c \cdot \sigma_y \cdot y_c + A_t \cdot \sigma_y \cdot y_t$$

$$= \frac{A}{2}(y_c + y_t)\sigma_y$$

$A_c = A_t$

Thus, $Z_p = \frac{A}{2}(y_c + y_t)$

$$Z_y = \frac{I}{y_{max}}$$



Circular section

$$Z_y = \frac{I}{y_{max}} = \frac{\pi D^4/64}{D/2} = \frac{\pi D^3}{32}$$

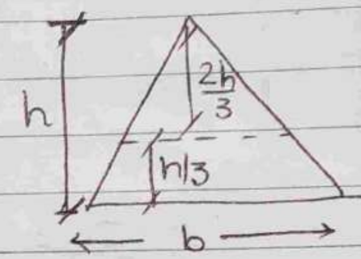
$$Z_p = \frac{A}{2}(y_c + y_t) = \frac{\pi D^2}{8}\left(\frac{2D}{3\pi} + \frac{2D}{3\pi}\right)$$

$$\therefore S_{\odot} = 1.69 \approx 1.7$$

Triangular section

$$I = bh^3/36, \; y_{max} = 2h/3$$

$$Z_y = \frac{bh^2}{24}$$

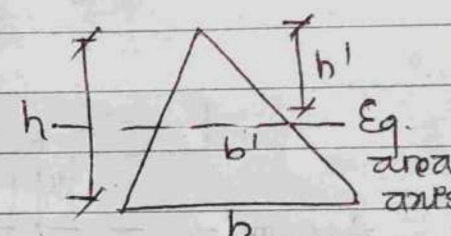


$$Z_p = \frac{bh}{4}\left(\frac{h'}{3} + \left(\frac{h-h'}{3}\right)\left(\frac{b'+2b}{b'+b}\right)\right)$$

$$= \frac{bh}{4}\left(\frac{h}{3\sqrt{2}} + \left(\frac{2-\sqrt{2}}{6}\right)h \times (3-\sqrt{2})\cancel{b}\right)$$

$$= 0.0976\, bh^2$$

$$\therefore S_{\Delta} = 2.343$$



$$\frac{1}{\cancel{2}} b'h' = \left(\frac{1}{2}bh\right) \times \frac{1}{\cancel{2}}$$

$$\Rightarrow b' = \frac{bh}{2h'}$$

$$\frac{b'}{h'} = \frac{b}{h} \Rightarrow b' = \frac{bh'}{h}$$

$$\Rightarrow \frac{\cancel{b}h'}{h} = \frac{\cancel{b}h}{2h'}$$

$$\Rightarrow h'^2 = h^2/2$$

$$\Rightarrow h' = h/\sqrt{2}$$

$$b' = b/\sqrt{2}$$

A = Area of crosssection

y_c, y_t = distance of centroid of areas above and below equal area axis from equal area axis

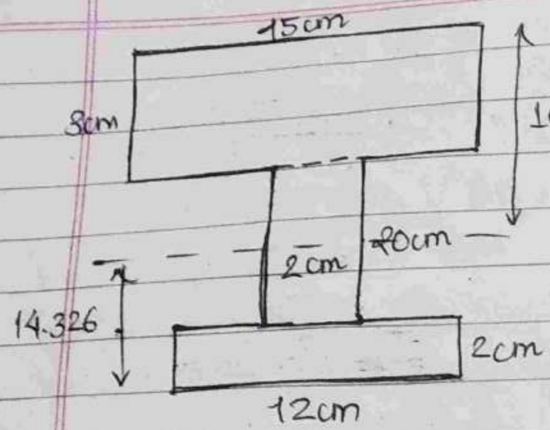
Rectangular

$$Z_p = \frac{bd}{2}\left(\frac{d}{4} + \frac{d}{4}\right) = \frac{bd^2}{4}$$

$$Z_y = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}$$

$$\frac{Z_p}{Z_y} = \frac{6}{4} = 1.5$$

→ largest bacteria, 16-45 µm diam.
→ parasitic smallest bacteria



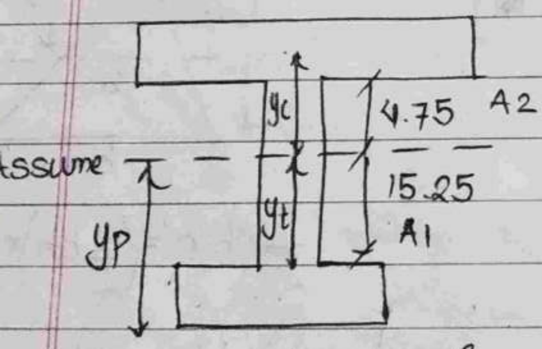
$$\bar{y} = \frac{24\times1 + 40\times12 + 45\times23.5}{24+40+45}$$

$$= 14.326\text{ cm}$$

$$I = \frac{12\times2^3}{12} + 24(14.326-1)^2 + \frac{2\times20^3}{12} + 40(14.326-12)^2 + \frac{15\times3^2}{12} + 45\times(23.5-14.326)^2$$

$$= 9637.27$$

$$\therefore Z_y = \frac{I}{y_{max}} = 672.712\text{ cm}^3$$



$$A_1 = A_2 = A/2$$

$$\text{or, } 12\times2 + (y_p-2)\times2 = \frac{109}{2}$$

$\therefore y_p = 17.25\text{ cm}$ ~~> 14.326 cm so~~ < 22 cm correct (assumption is right)

Total area = 109 cm^2

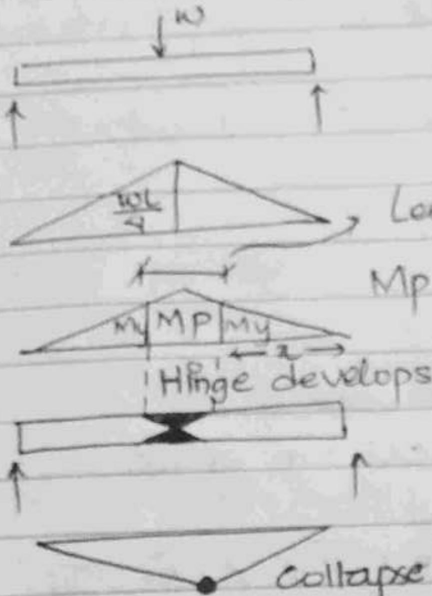
$$y_c = \frac{15\times3\times6.25 + 4.75\times2\times4.75/2}{15\times3 + 4.75\times2} = 5.575\text{ cm}$$

$$\text{Similarly, } y_t = \frac{12\times2\times(2+15.25) + 15.25^2\times2/2}{24+15.25\times2} = \cancel{11.864}\ 11.423\text{ cm}$$

$$\therefore Z_p = \frac{109}{2}(5.575 + \cancel{11.864}\ 11.423) = \cancel{950.4}\ 926.4\text{ cm}^3$$

$$\therefore S = \frac{Z_p}{Z_y} = \cancel{1.413}\ 1.377$$

Determination of collapse load



Length of plastic hinge

$M_p = \frac{w_c L}{4} \Rightarrow \boxed{w_c = \frac{4M_p}{L}}$

$\frac{M_p}{M_y} = \frac{L/2}{x}$ (for rectangular)

or, $1.5 = \frac{L}{2x} \Rightarrow L = 3x$

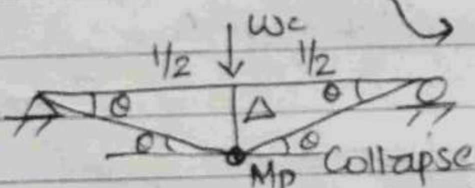
$\Rightarrow x = L/3$

Length of plastic hinge $= L - \frac{2L}{3} = \frac{L}{3}$

Based on cond^n before collapse

→ Static theorem : If $w \leq w_c$, structure is safe
↳ Safe condition

→ Kinematic theorem : ↳ Based on condition after collapse



If $w \geq w_c$,
→ Collapse condition

We = Wi (Virtual work)

or, $w_c \times \Delta = M_p \times \theta + M_p \times \theta$

or, $w_c \times \frac{L}{2} \times \cancel{\theta} = 2M_p \cancel{\theta}$

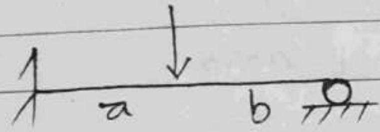
or, $\boxed{w_c = \frac{4M_p}{L}}$

m → parasitic smallest bacteria

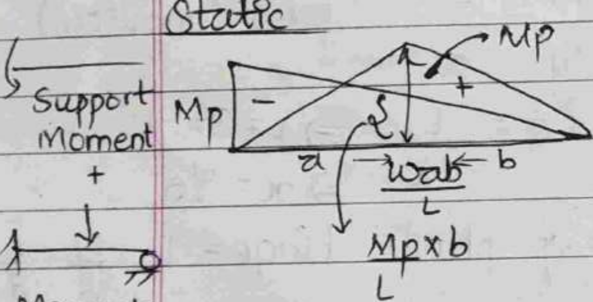
Date ______
Page ______

→ Hinge forms at i) fixed support (position of max^m BM) ii) position of pt. load
iii) cross section change iv) joint (frames)

$\boxed{\text{No. of hinges required for collapse condition} = DSI + 1}$

1) 

Static



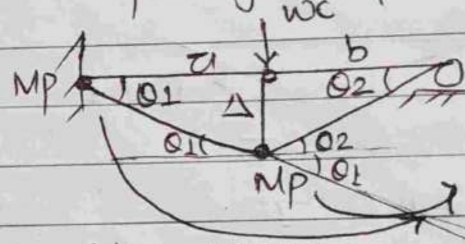
$$\frac{W_c ab}{L} - \frac{Mp\,b}{L} = Mp$$

or, $W_c ab = Mp(L+b)$

or, $\boxed{W_c = \frac{Mp(L+b)}{ab}}$

Kinematic

No. of hinges req. $= 1+1 = 2$



$W_e = W_c \times \Delta$

$W_i = Mp(\theta_1+\theta_2) + Mp\,\theta_1$

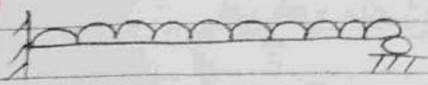
$= Mp\left(\frac{\Delta}{a}\right) + Mp\left(\frac{\Delta}{a} + \frac{\Delta}{b}\right) = \frac{Mp(a+b)\Delta + Mp\,b\,\Delta}{ab}$

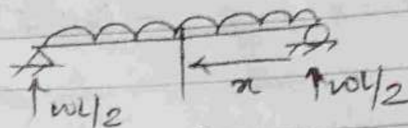
$W_e = W_i$

or, $W_c \times \cancel{\Delta} = \frac{Mp(a+b+b)\cancel{\Delta}}{ab}$

$\boxed{\therefore W_c = \frac{Mp(L+b)}{ab}}$

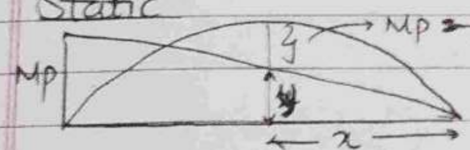
2)





$M_x = \frac{wLx}{2} - \frac{wx^2}{2}$

Static



$\to M_p = \cancel{\frac{w_cL^2}{8}}\ \frac{wLx}{2} - \frac{wx^2}{2} - \frac{M_p x}{L}$

$$y = \frac{M_p \cdot x}{L}$$

$$(M_{max})_x^+ = \frac{wLx}{2} - \frac{wx^2}{2} - \frac{M_p x}{L}$$

$$\frac{dM_x^+}{\partial x} = 0 \quad \text{or, } \frac{wL}{2} - wx - \frac{M_p}{L} = 0$$

$$\text{or, } x = \left(\frac{wL}{2} - \frac{M_p}{L}\right) \times \frac{1}{w} = \frac{L}{2} - \frac{M_p}{wL}$$

$$\Rightarrow M_p = wL\left(\frac{L}{2} - x\right)$$

For collapse condition, $\frac{w_cL}{2}x - \frac{w_cx^2}{2} - \frac{M_p x}{L} = M_p$

$$\text{or, } \frac{w_cL}{2}x - \frac{w_cx^2}{2} = \left(\frac{x}{L} + 1\right) \times w_cL\left(\frac{L}{2} - x\right)$$

$$\text{or, } \frac{L\cancel{x}}{\cancel{2}} - \frac{x^2}{2} = \frac{\cancel{Lx}}{\cancel{2}} - x^2 + \frac{L^2}{2} - Lx$$

$$\text{or, } x^2 + 2Lx - L^2 = 0$$

$$\text{or, } x = \frac{-2L \pm \sqrt{4L^2 - 4 \times (-L^2)}}{2}$$

$$= \frac{-2L \pm \sqrt{4L^2 + 4L^2}}{2}$$

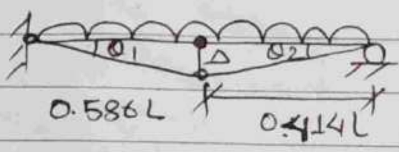
$$= -L \pm \sqrt{2L^2}$$

$$\therefore x = \underline{0.414L} \text{ or } -2.414L$$

$$M_p = w_c L\left(\frac{L}{2} - 0.414L\right) = 0.086 w_c L^2$$

Or, $$\boxed{w_c = \frac{11.65 M_p}{L^2}}$$

Kinematic



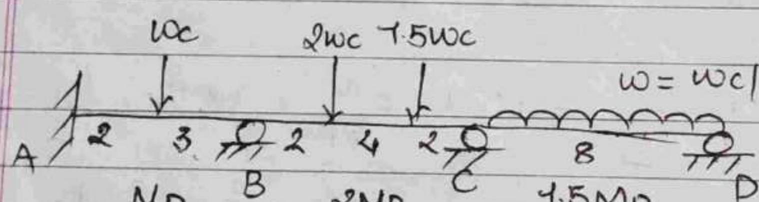
$W_e = W_i$

Or, $\frac{w_c l}{2} \times \Delta = M_p\theta_1 + M_p(\theta_1+\theta_2)$

Or, $\frac{w_c L}{2} \times \cancel{\Delta} = M_p\left[\frac{2\Delta}{0.586L} + \frac{\Delta}{0.414L}\right]$

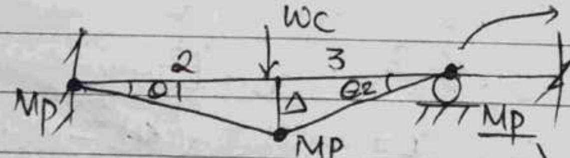
$\therefore$ $$\boxed{w_c = \frac{11.65 M_p}{L^2}}$$

3)



For determining W_c, lowest value is ans
For M_c, highest value is ans

For AB



Hinge should be formed here because it is continuous beam.

Take smaller value of Mp for support at junc^n

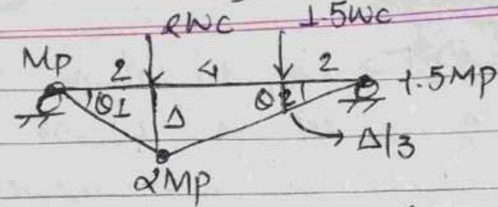
$w_c \times \Delta = M_p[2\theta_1 + \theta_2] + M_p\theta_2$

or, $w_c = M_p\left[2 \times \frac{\Delta}{2} + \frac{\Delta}{3}\right] \times \frac{1}{\Delta} + \frac{\Delta}{3} \times \frac{1}{\Delta} \times M_p$

$= \frac{5}{3}M_p$

Extern

For BC

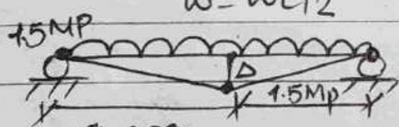


$$\frac{1.5W_c \times \Delta}{3} + 2W_c \times \Delta = M_p\theta_1 + 2M_p(\theta_1+\theta_2) + 1.5M_p\theta_2$$

or, $$\frac{5}{2}W_c\Delta = M_p\left[\frac{\Delta}{2} + 2\times\left(\frac{\Delta}{2}+\frac{\Delta}{6}\right) + 1.5\times\frac{\Delta}{6}\right]$$

$$\therefore W_c = \frac{25}{12}M_p \times \frac{2}{5} = \frac{5}{6}M_p$$

For CD

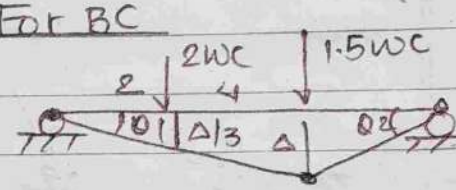


$$\frac{W_c\times 8}{4}\times\Delta = 1.5M_p\theta_1 + 1.5M_p(\theta_1+\theta_2)$$

or, $$2W_c\Delta = 1.5M_p\left[\frac{2\Delta}{4.688}+\frac{\Delta}{3.312}\right]$$

$$\therefore W_c = \underline{0.546M_p} \Rightarrow \text{Ans.}$$

For BC

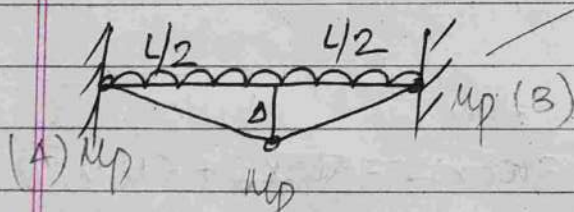


$$\frac{2W_c\times\Delta}{3} + 1.5W_c\times\Delta = \frac{25}{12}M_p$$

$$\therefore W_c = \frac{25}{26}M_p$$

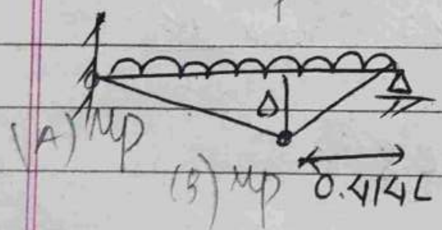
1.038Mp

Note:



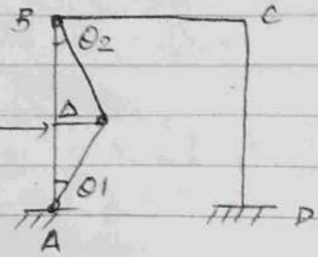
As long as (A = B), no need to calulate

Any other values of Mp given we need to calulate value of $\underline{x}$.



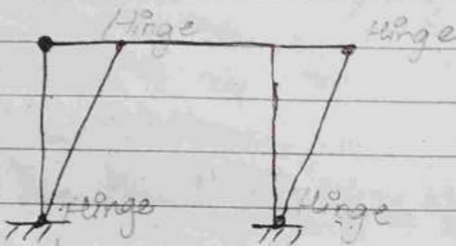
As long as A = B, no need to calulate

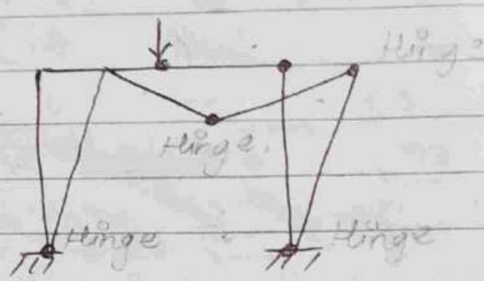
Frame



1) Beam mechanism
2) Sway mechanism
3) Combined mechanism

Beam mechanism

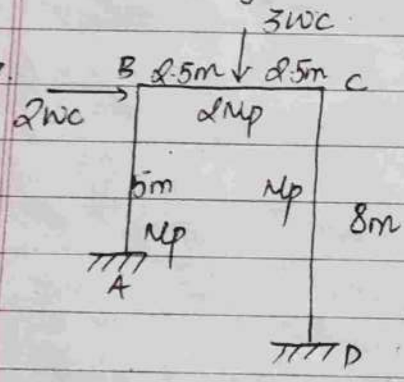




Sway mechanism

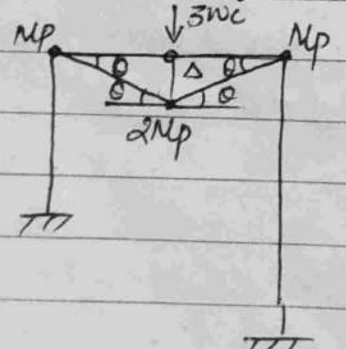
Combined mechanism

Eg.



Beam mechanism (Beam BC)

Length equal on two sides of load



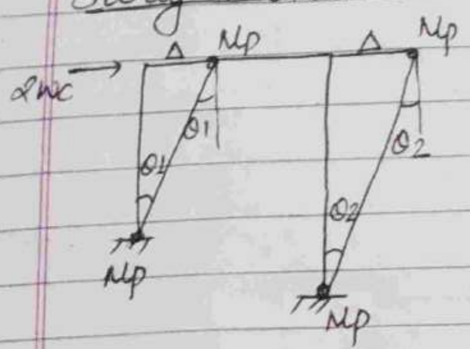
$W_e = W_i$

or, $3w_c \times \Delta = M_p \times \theta + 2M_p \times 2\theta + M_p \times \theta$

or, $3w_c \times \cancel{\Delta} = 6M_p \times \dfrac{\cancel{\Delta}}{2.5}$

$\therefore w_c = 0.8 M_p$

Sway mechanism



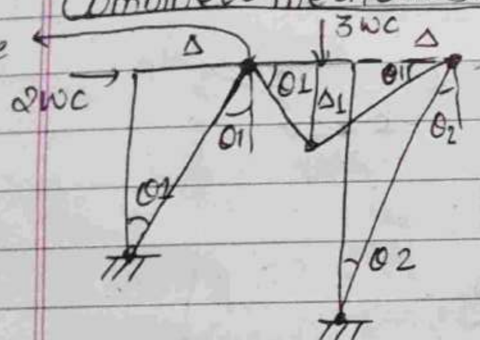
$2wc \times \Delta = M_p\theta_1 + M_p\theta_1 + 2M_p\theta_2$

or, $2wc \times \Delta = 2M_p\left[\frac{\Delta}{5} + \frac{\Delta}{8}\right]$

$\therefore wc = 0.325 M_p$

Combined mechanism

No hinge here



$2wc \times \Delta + 3wc \times \Delta_1 = M_p \times \theta_1 + M_p \times \theta_2 + [M_p \times \theta_2 + M_p \times \theta_1] + 2M_p \times 2\theta_1$

or, $2wc \times \Delta + 3wc \times \cancel{2\theta_1 \times}\, \Delta_1 =$

$M_p \times \frac{\Delta}{5} + \cancel{\frac{M_p \times \Delta \times 2}{8}} + M_p \times \frac{\Delta}{8} +$

$M_p\left(\frac{\Delta_1}{2.5} + \frac{\Delta}{8}\right) + 4M_p \times \frac{\Delta_1}{2.5}$

$\theta_1 = \frac{\Delta_1}{2.5}$

$\theta_1 = \frac{\Delta}{5}$

$\Rightarrow \frac{\Delta_1}{2.5} = \frac{\Delta}{5}$

$\therefore \Delta = 2\Delta_1$

or, $2wc \times \Delta + 3wc \times \frac{\Delta}{2} = M_p \times \frac{\Delta}{5} + M_p \times \frac{\Delta}{8} +$

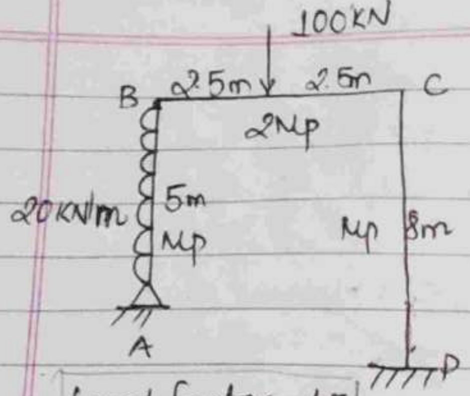
$M_p\left(\frac{\Delta}{5} + \frac{\Delta}{8}\right) + 4M_p \times \frac{\Delta}{2.5 \times 2}$

or, $\frac{7}{2} wc = \frac{29}{20} M_p$

$\therefore wc = \cancel{0.642}\, M_p \quad (0.414)$

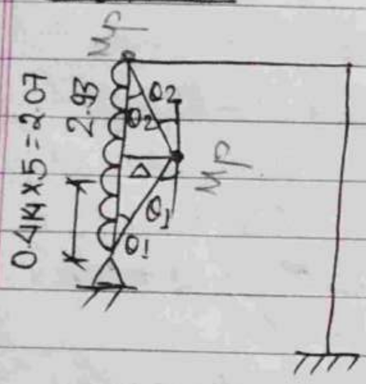
0.414

Ans: $wc = 0.325 M_p$ (Minimum)

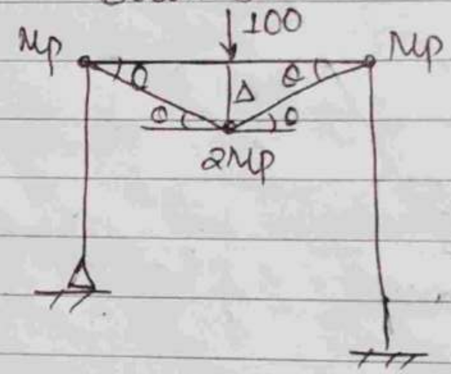


$\boxed{\text{Load factor} = 1.5}$

Beam AB



Beam BC



Avg. Δ

$$(1.5 \times 20 \times 5) \times \frac{\Delta}{2} = M_p(\theta_1 + \theta_2) + M_p\theta_2$$

$$\text{or, } 75\Delta = M_p\left[\frac{\Delta}{2.07} + 2 \times \frac{\Delta}{2.93}\right]$$

$$\therefore M_p = 64.34 \text{ KNm}$$

[Load factor x w x Length of UDL]

where $w = \frac{wc}{2}$

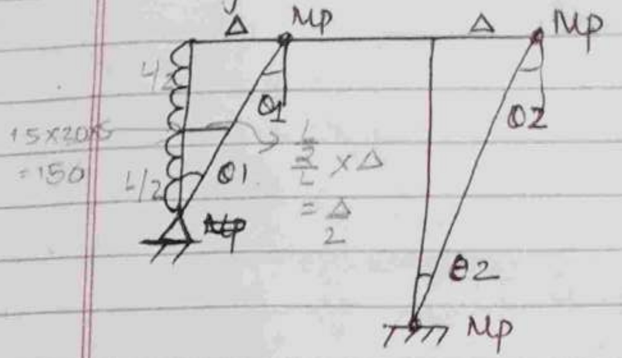
$$1.5 \times 100 \times \Delta = M_p \times \theta + 2M_p \times 2\theta + M_p \times \theta$$

$$\text{or, } 150\Delta = M_p \times 6 \times \frac{\Delta}{2.5}$$

$$\therefore M_p = 62.5 \text{ KNm}$$

$\frac{\text{Load factor x load intensity}}{2}$ x Length of UDL

Sway

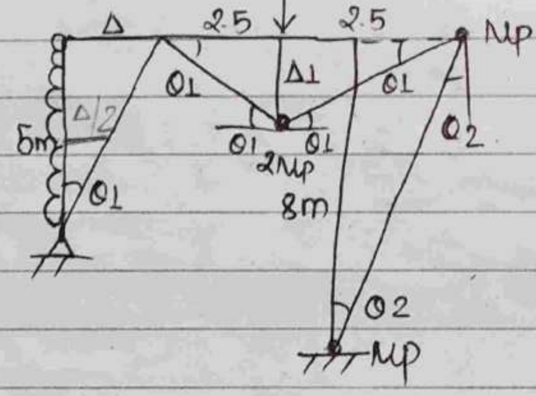


$$1.5\times 20\times 5\times \frac{\Delta}{2} = M_p\theta_1 + M_p\theta_2 + M_p\theta_2$$

or, $75\Delta = M_p\left[\frac{\Delta}{5} + 2\times\frac{\Delta}{8}\right]$

$\therefore M_p = 166.67$ KNm

Combined



$$1.5\times 20\times 5\times\frac{\Delta}{2} + 100\times\frac{\Delta}{2}\times 1.5 =$$

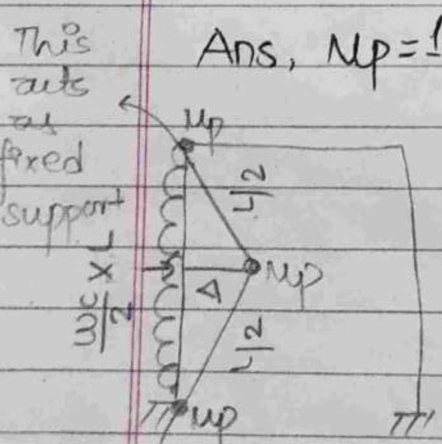
$$2M_p\times 2\theta_1 + M_p(\theta_1+\theta_2) + M_p\theta_2 + M_p\theta_1$$

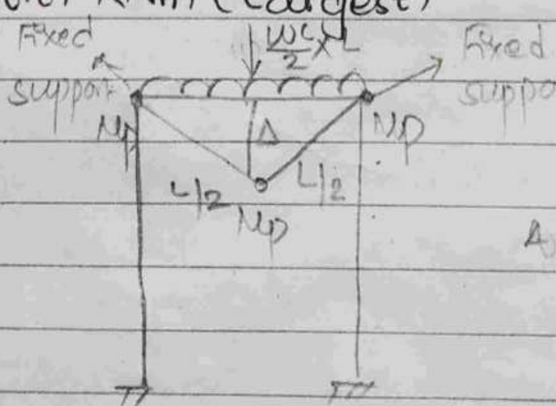
or, $150\Delta = M_p\left[4\times\frac{\Delta}{5} + \frac{\Delta}{5} + \frac{\Delta}{8} + \frac{\Delta}{8}\right]$

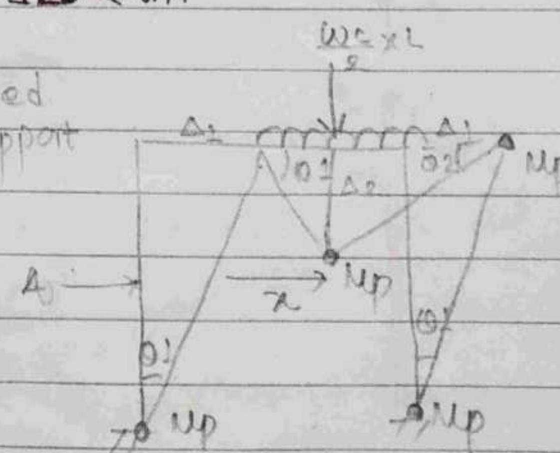
$\therefore M_p = 120$ KNm

$\frac{\Delta}{5} = \frac{\Delta_1}{2.5} \Rightarrow \Delta = 2\Delta_1$

Ans, $M_p = 166.67$ KNm (Largest)



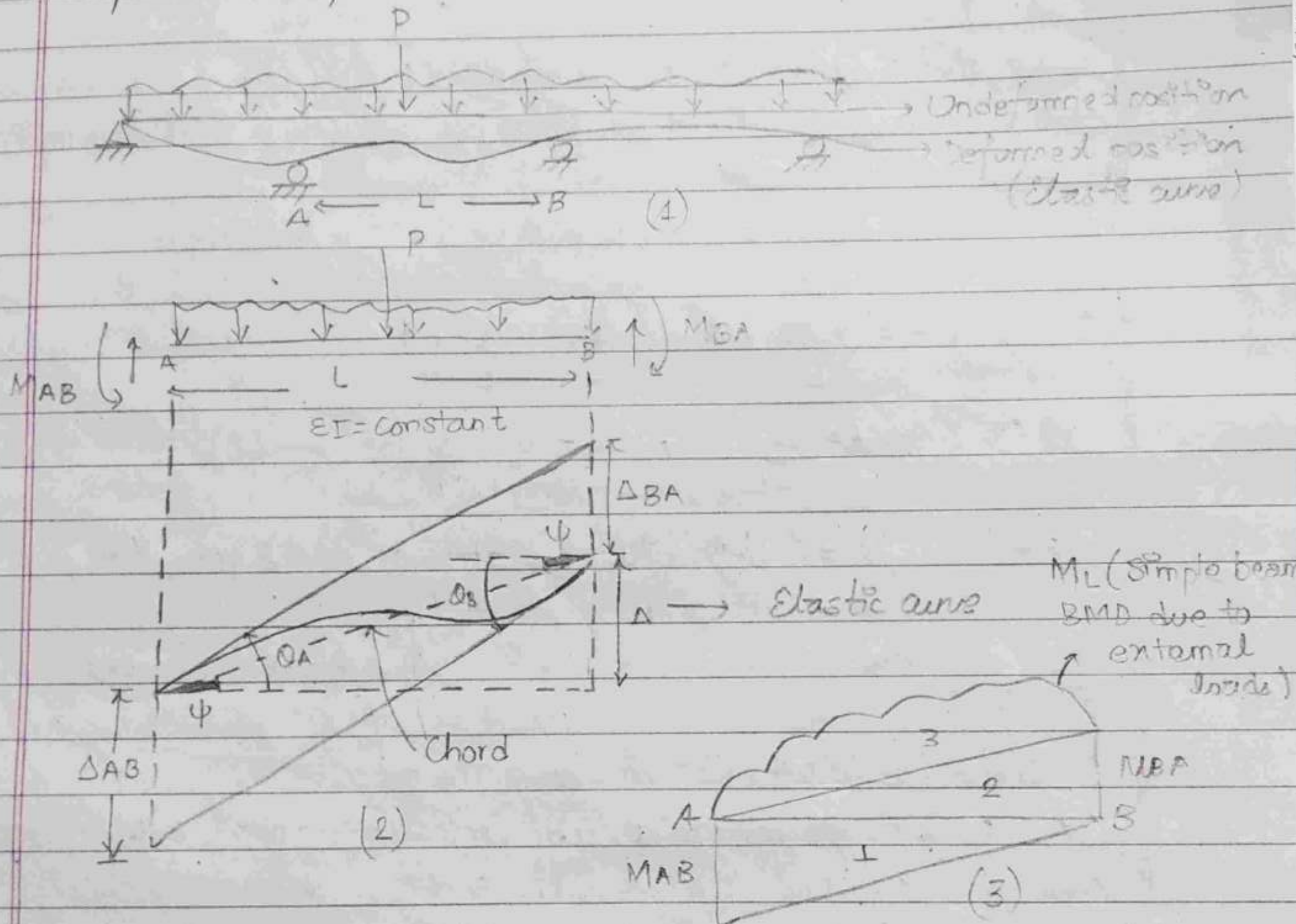




Calculate x

Derivation of Slope-Deflection Equation

When a continuous beam or frame is subjected to external loads, internal moments develop at ends of its members. This moment can be related to rotations. & ~~displacements~~ of Similarly, the external moments applied can be related to displacement of ends.



Take an arbitrary member AB of continuous beam.

Fig(1) shows deformation AB under support settlement & external loads

Fig(2) shows elastic curve of AB.

θ_A and θ_B denote rotation of end A & B wrt horizontal position of member

$\Delta \Rightarrow$ relative translation between two ends of member in dir^n $\perp^r$

to undeformed axis of member

ψ - rotation of member's chord (chord is st. line connecting deformed positions of member ends) due to relative translation.

Since deformation is small, $\psi = \Delta / L$ $\therefore$ $\Delta = \psi . L$

Sign convention: All moments and rotations are +ve when anticlockwise.

Slope-deflection eq^n can be derived by relating end moments to end rotations & chord rotation by applying 2nd moment area theorem.

$$\theta A = \frac{\Delta BA + \Delta}{L}, \quad \theta B = \frac{\Delta AB + \Delta}{L} \quad \text{[from fig]}$$

$$\text{or, } \theta A = \frac{\Delta BA}{L} + \psi, \quad \theta B = \frac{\Delta AB}{L} + \psi$$

$$\text{or } \theta A - \psi = \frac{\Delta BA}{L}, \quad \theta B - \psi = \frac{\Delta AB}{L} \quad \text{— (i)}$$

where ΔBA = tangential deviation of end B from tangent to elastic curve at A

ΔAB = " " " end A " "
" " at B.

According to 2nd moment area theorem,

ΔAB : Sum of moment, abt. end A, of area under M/EI diag

ΔBA = " " " end B, " " "

$$\text{ie } \Delta BA = \frac{1}{EI}\left[\left(\frac{1}{2} MAB . L\right) \times \left(\frac{2L}{3}\right) - \left(\frac{1}{2} MBA . L\right) \times \frac{L}{3} - gB\right.$$

$$\text{or, } \Delta BA = \frac{MAB L^2}{3EI} - \frac{MBA L^2}{6EI} - \frac{gB}{EI}$$

$$\Delta AB = \frac{1}{EI}\left[-\left(\frac{1}{2} MAB . L\right) \times \frac{L}{3} + \left(\frac{MBA . L}{2}\right)\left(\frac{2L}{3}\right) + gA\right]$$

$$\Delta AB = -\frac{MAB L^2}{6EI} + \frac{MBA L^2}{3EI} + \frac{gA}{EI}$$

g_B and g_A are moments abt. end B & A resp. of area under simple beam BMD due to external loading.

Then. in (i),

$$\theta_A - \psi = \frac{M_{AB} \cdot L}{3EI} - \frac{M_{BA} L}{6EI} - \frac{g_B}{EIL} \quad \text{—(ii)}$$

$$\theta_B - \psi = -\frac{M_{AB} L}{6EI} + \frac{M_{BA} L}{3EI} + \frac{g_A}{EIL} \quad \text{—(iii)}$$

Eqn (ii) can be written as:

$$2 \cdot \frac{M_{BA} \cdot L}{6EI} = 2\left[\frac{M_{AB} \cdot L}{3EI} - \frac{g_B}{EIL} - (\theta_A - \psi)\right]$$

$$\text{or, } \frac{M_{BA} \cdot L}{3EI} = \frac{2M_{AB} \cdot L}{3EI} - \frac{2g_B}{EIL} - 2(\theta_A - \psi)$$

Substituting in (iii),

$$\theta_B - \psi = -\frac{M_{AB} \cdot L}{6EI} + \frac{g_A}{EIL} + \frac{2M_{AB} \cdot L}{3EI} - \frac{2g_B}{EIL} - 2(\theta_A - \psi)$$

$$\text{or, } \frac{3M_{AB} \cdot L}{6EI} = \theta_B - \psi + \frac{2g_B}{EIL} + 2\theta_A - 2\psi - \frac{g_A}{EIL}$$

$$\text{or, } M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi) + \frac{2}{L^2}\left[2g_B - g_A\right]$$

Similarly, $M_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A - 3\psi) + \frac{2}{L^2}\left(g_B - 2g_A\right]$

If member AB is considered as isolated beam with both ends fixed, $\theta_A = \theta_B = \psi = 0$ then moments at end of such beam is called FEM.

Then, $M_{AB} = FEM_{AB} = 2/L^2\left[2g_B - g_A\right]$

$M_{BA} = FEM_{BA} = 2/L^2\left[g_B - 2g_A\right]$

$$\therefore M_{AB} = FEM_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi)$$

$$M_{BA} = FEM_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A - 3\psi) \cdot \quad \psi = \frac{\Delta}{L}$$