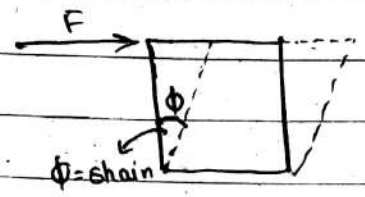
Fluid Mechanics

If applied sheershess is directly proportional to ~~a~~ observed shain. then it is called solid.

i.e.



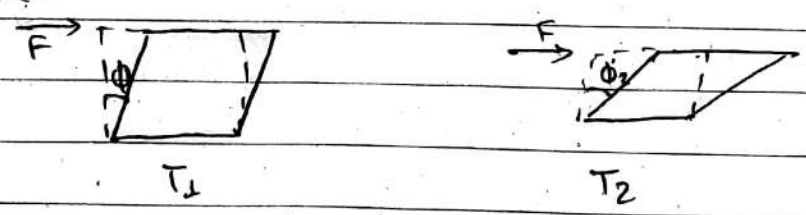
if F is constantly applied then ϕ remains constant.

i.e. $\phi \propto \tau$ where $\tau = \frac{F}{A}$ (sheer shess)

If the sheer shain continuously ~~de~~ changes (i.e. the substance deforms continuosly) ~~whale~~ under the action of sheer force (sheer shess) whatever small the magnitude is.

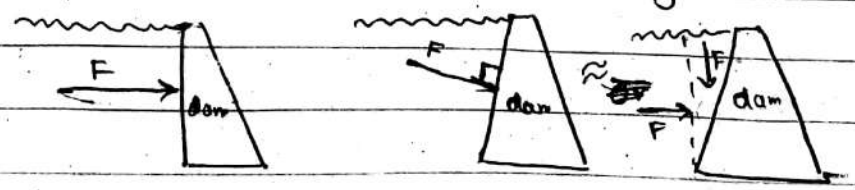
i.e. $\phi \propto \frac{\tau}{t}$

i.e.



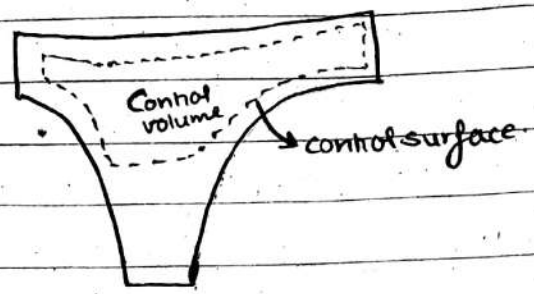
$\phi_2 > \phi_1$.

In case of static ~~force~~, fluid (fluid ~~in~~ inside closed boundary). eg. bucket, dam. there will only be normal force, no tangential (sheer) force.



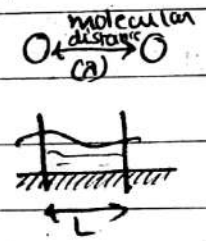
Control volume concept

Arbitrary volume in the space through which the fluid flows is K/A control volume. Geometric boundary of control volume is control surface.



Concept of continuum

Knudsen no. (Kn) $= \dfrac{\text{mean free path } (\lambda)}{\text{char. length } (L)}$



Civil eng. → $Ku < 0.001$ (no slip condition)

$0.01 < Ku < 10$ = slip flow

aerospace eng. → $Ku > 10$ = free molecular flow

engineering problems usually concerned with bulk behavior of the fluid which consist of millions of particles, so for simplicity, fluid is considered to be hypothetical continuous substance called continuum.

Properties of the fluid.

mass density (ρ)

$$\frac{\text{Mass}}{\text{volume}} = \frac{kg}{m^3}$$

relative desity (specific gravity)

$$SG = \frac{\text{density of substance}}{\text{density of water at } 4^\circ c}$$

for gas $= \frac{\text{density of gas}}{\text{density of air at STP}}$

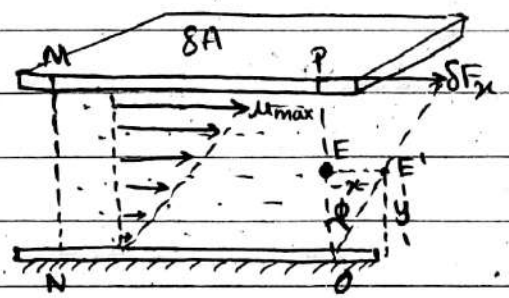
specific wt. (γ)

$$\frac{\text{Weight}}{V} = \frac{mg}{V} = \rho . g$$

Specific volume

$$\frac{1}{\rho} = \text{specific volume}$$

Viscosity



Viscosity is the property that opposes the flow of one layer of the fluid to the next consecutive layer.

$$\phi_{small} = \frac{x}{y}$$

$$\text{shear stress} = \frac{\delta F_x}{\delta A}$$

for fluid

$$\tau \propto \frac{\phi}{t}$$

$$\tau = k\frac{\phi}{t} = k\frac{x}{yt}$$

$$= \frac{k\mu}{y}$$

$$\boxed{\therefore \tau = k\frac{\mu}{y}}$$

$$\boxed{\tau = \mu\frac{du}{dy}}$$ Newton's law of viscosity

μ = coefficient of dynamic viscosity

$kg\,m^{-1}s^{-1}$, Ns/m^2, poise

for water $\boxed{\mu_w = 1.14 \times 10^{-3}\ Ns/m^2}$

10 poise (P) = 1 kg/ms

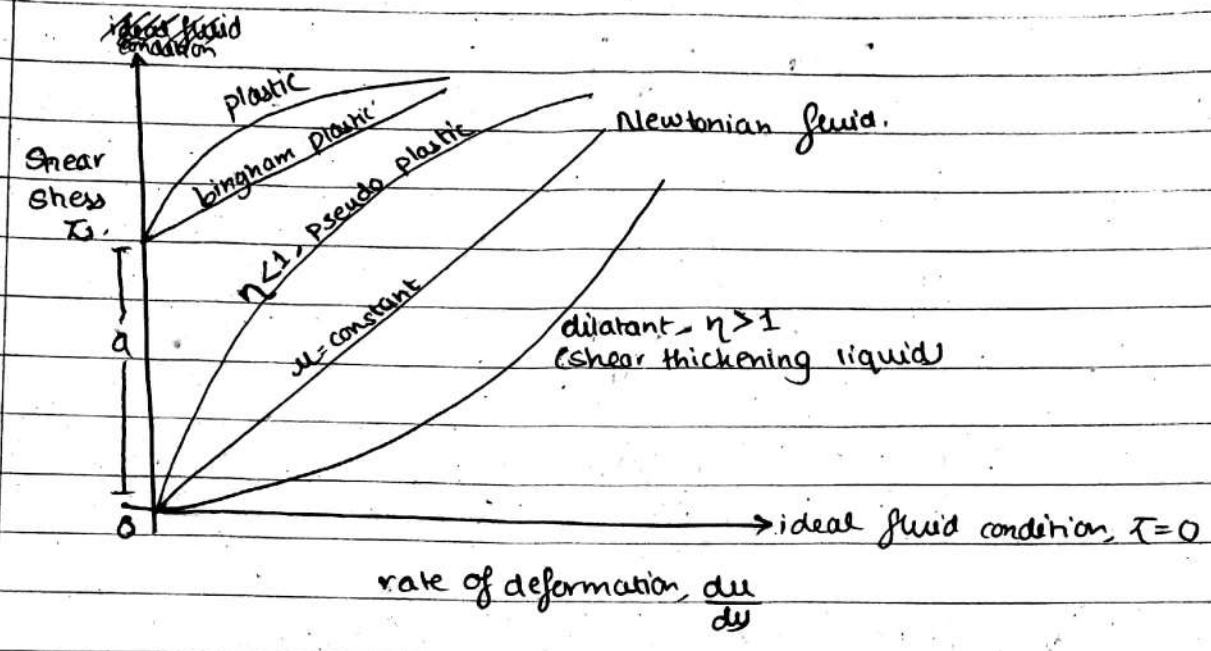
<u>kinematic viscosity</u> "nu" $(\nu) = \frac{\mu}{\rho}$ (m^2/sec)

$$\nu_{water} = 1.14 \times 10^{-6}\ m^2/sec$$

unit is m^2/sec, stokes (St)

$$10^4\ St = 1\ m^2/s$$

Newtonian fluid vs Non-newtonian fluid.



* The fluid which obeys Newton's law of viscosity is Newtonian fluid
* " " " doesn't obey " " " " " non-newtonian fluid.

generally, $\tau = k\left(\frac{du}{dy}\right)^n$.

if $k=\mu$ & $n=1$ - newtonian fluid.

$$\tau = k\left(\frac{du}{dy}\right)^{n-1} \cdot \frac{du}{dy}$$

$$= \eta \cdot \frac{du}{dy}$$

η = apparent viscosity for non-newtonian fluid.

(sheer thinning fluids)
pseudo plastic : η decreases with increasing deformation rate.

dilatant : η increases when deformation rate increases.

Plastic: fluid behave as solid until a minimum yield stress and then flow.

$$\tau = a + B\left(\frac{du}{dy}\right)$$

if $B = \mu$

then,

$$\tau = a + \mu\left(\frac{du}{dy}\right)$$

it is called bingham plastic.

example (sewage sludge)

~~Thix~~ **Thixotropic**: decrease in apparent viscosity with time under constant applied stress

example: paints, yougohort, peanut butter etc

can also be called and time dependent pseudo plastic.

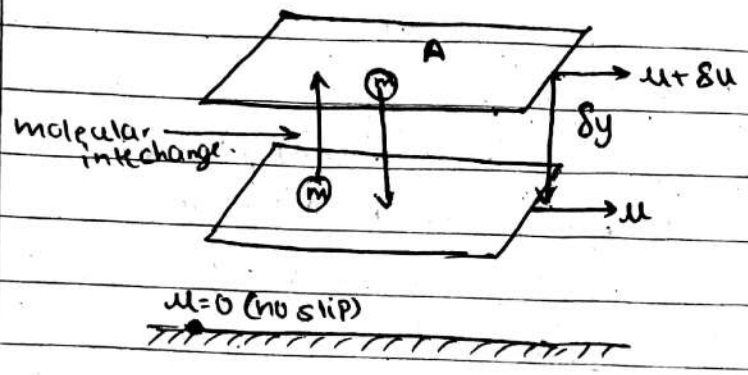
~~Rheotopic~~

Rheopectic: increase in viscosity with time.

example: printer ink.

visco-elastic: lubricants and whipped cream.

Causes of viscosity in gases and in liquid



When gases flow on a solid bondary. the molecules attached with the solid boundary becomes zero and velocity gradient develops. The slower layer try to drag the upper fast layer and upper fast layer develops accelerating force on lower layer, ~~Due to~~ molecular mass interchange (m/t) depends on area.

$$\text{i.e. } \frac{m}{t} \propto A.$$

$$\text{and } \frac{m}{t} \propto \frac{1}{\delta y}$$

$$\text{i.e. } \frac{m}{t} = k.\frac{A}{\delta y} \quad \ldots (i)$$

$$F = \frac{m}{t}.\Delta u. = \frac{kA}{\delta y}.\delta u$$

$$\Rightarrow \boxed{\frac{F}{A} = \tau = k.\frac{\delta u}{\delta y}}$$ Newton's law of viscosity.

So, molecular mass Interchange is the cause of viscosity in gas, when temp is increased, viscosity increases due to more molecular mass interchange.

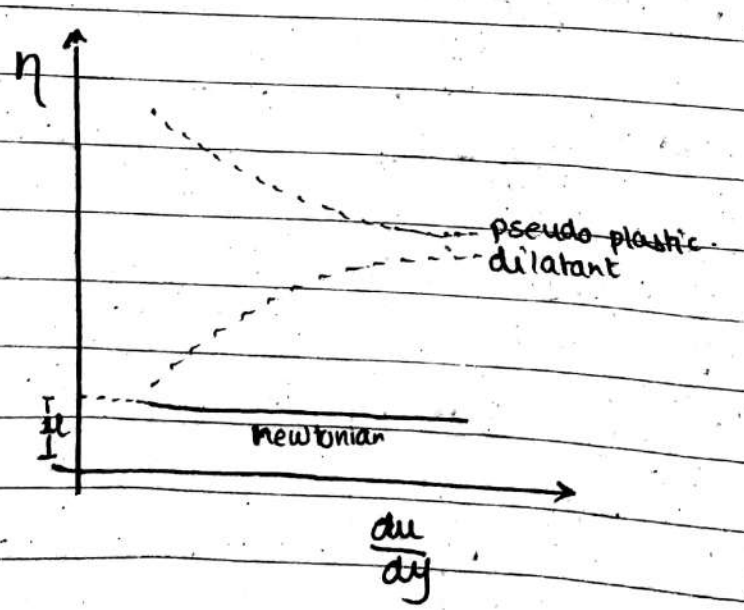
in liquid both molecular ~~force~~ mass interchange and cohesive force cause the viscosity of the liquid.
& when temperature increases, cohesive force weaken and tends to decrease viscosity but when temperature increase, molecular mass interchange ~~decrease~~ increases and viscosity increases. But cohesive force is more prominient in the liquid. So, net effect is decrease in viscosity.

~~Due to~~ pressure (↑) in liquid, viscosity (↑).

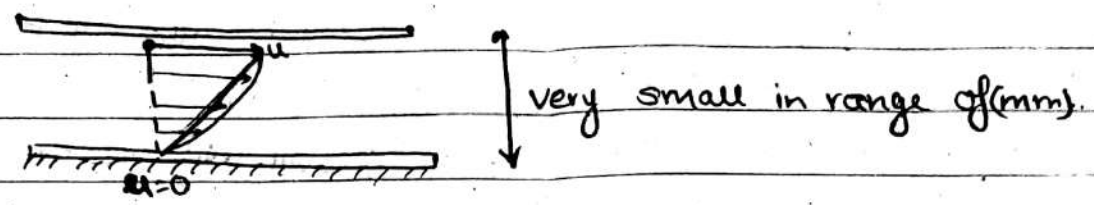
- <u>viscosity at temperature change:</u>

$$\mu_T = \frac{\mu_0}{1 + A_1 t + B_1 t^2}$$

$$\mu_P = \mu_0 e^{c(P-P_0)}$$ (viscosity pressure relation)



linearization of Newton's law of viscosity:



then,

$$\tau = \mu \frac{du}{dy} \quad \text{(Newton's law of viscosity)}$$

$$\Rightarrow \tau = \mu \frac{u-0}{y-0} = \mu \frac{u}{y}$$

because when, y is very small, the velocity gradient which is curvilinear ~~will~~ can be assumed to be a straight line. and du becomes u-0 and dy becomes y-0

Numericals on viscosity:

* Assume the viscosity distribution is given by $v = 5\sin(5\pi y)$ for $y \leq 0.1m$. Calculate shear stress at wall. Assume dynamic viscosity to be 5 poise.

→ Solution: $v = 5\sin(5\pi y)$

$$\frac{dv}{dy} = 25\pi \cos 5\pi y$$

at wall, $y=0$. so $\frac{dv}{dy} = 25\pi \cos 0 = 25\pi$

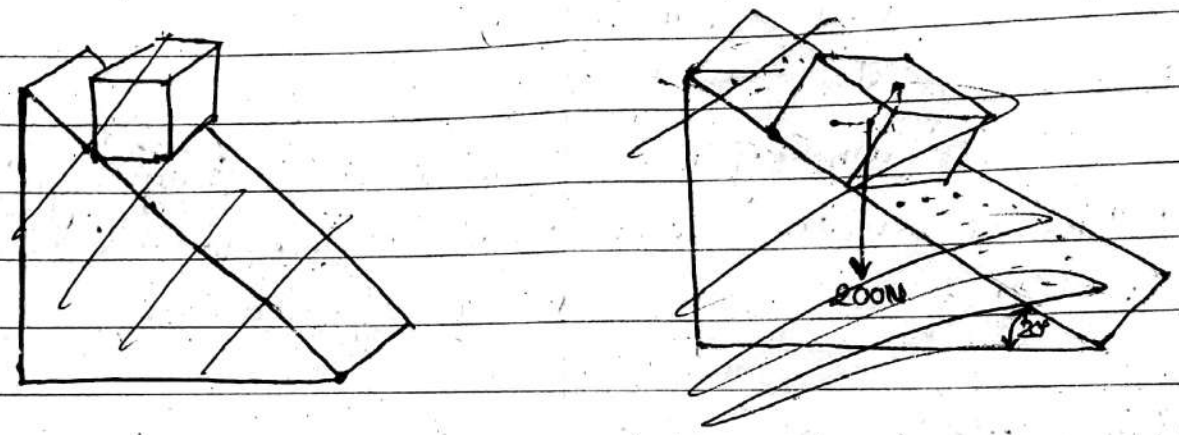
given $\mu = 5$ poise (10 poise = 1 kg/ms)
$= 0.5$ kg/ms

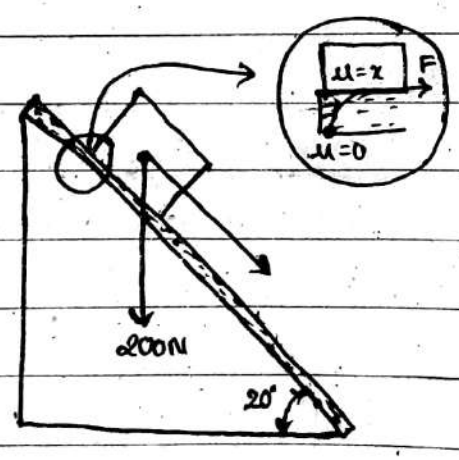
so, $\tau = \mu \frac{dv}{dy} = 12.5\pi$ //

~~Laplace~~ 1

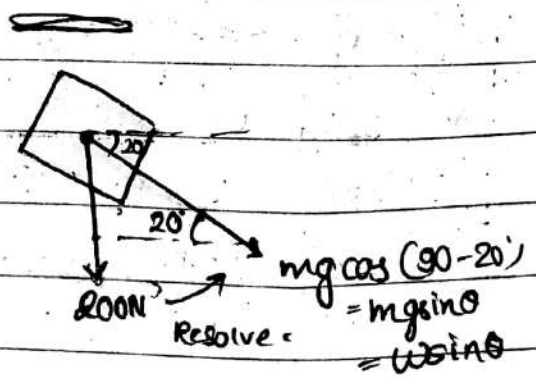
* A cubical block of 20cm edge and 200N is allowed to slide down a plane inclined at 20° to the horizontal on which there is a thin film of oil with $\mu = 2.2\times10^{-3}$ Nm/s^2. What terminal velocity will be acquired by the block if thickness of the film is 0.025mm?

→





FBD of the block:



$F = W\sin\theta = W\sin 20°$
$= 200\sin 20°$

now,

shearing stress (τ) = F × surface area in contact.
$= 200\sin 20° \times (0.2\times0.2)\, m^2$
$= 2.74\ Nm^2$

but,

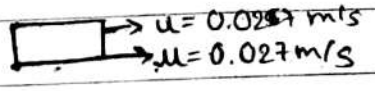
$$\text{shearing stress } (\tau) = \mu \cdot \frac{du}{dy}$$

$$= 2.2 \times 10^{-3} \times \frac{u}{0.025/1000} \quad \text{(linearization)}$$

$$\Rightarrow \frac{2.74\ Nm^2 \times 0.025}{2.2} = u$$

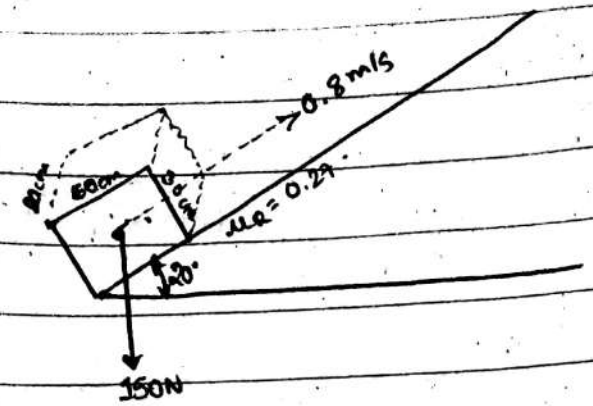
$$\Rightarrow u = 0.027\ m/s$$

why, velocity of oil in contact is velocity of block?
- because the surface in contact with that of the block is of the same velocity as the oil because of No slip condition.

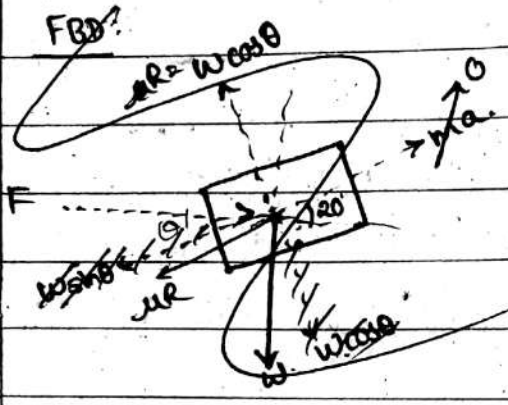


relative velocity = 0 m/s. No slip condition.

* A 50 cm x 30 cm x 20 cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with coefficient of friction 0.27.
  i) Determine the force F needed to apply in horizontal direction.
  ii) If a 0.4 mm thick oil film with viscosity 0.012 pas pascal second is applied between block and inclined surface. Determine % reduction in req^d forces.



FBD:



a = 0.

So, ma = 0

So,

$\sum F_x = 0$

$\Rightarrow F - \mu R\cos\theta = 0.$

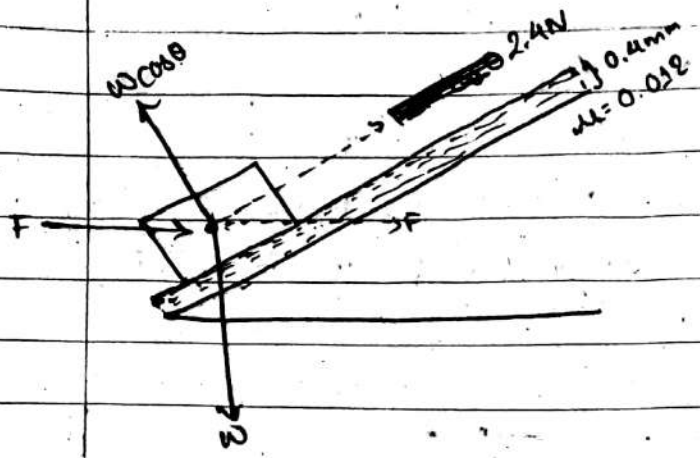
$\sum F_y = 0$

$\Rightarrow W\cos\theta = W + \mu R\sin\theta$

$F - \mu R\cos\theta - W\cos\theta\sin\theta = 0$

$\Rightarrow F = 83.97\ N.$

when oil is added.



Shearing stress that should be provided by the external push is $\tau = \mu \frac{du}{dy}$

$= \mu \frac{u}{y}$ (linearization)

$= 0.012 \times \frac{0.8}{0.4} \frac{N}{m^2}$

$= \cancel{0.024 \text{ Pas} \times 10^3}$ 24Pa

so force which can achieve that value is $24 \times 0.5 \times 0.2$
$= 2.4$ N.

i.e.

$F + 2.4\cos 20^\circ = (W\cos\theta)\sin\theta$.

$\Rightarrow F = 45.9538$ N.

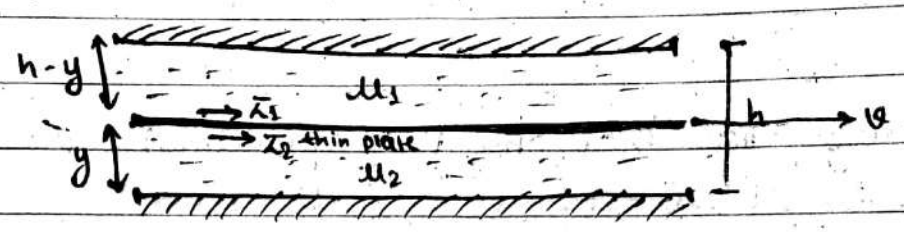
% reduction in force $= \frac{83.97 - 45.953}{83.97} \times 100$ %

$= 45.27$ %

* Through a narrow gap of height 'h', a thin plate of large extent is pulled at a velocity v. One side of the plate is oil of viscosity μ_1 and other side is fluid of viscosity μ_2. Calculate the position of the plate such that:

i) Shear forces of the two sides of the plates are equal.

ii) The pull required to drage the plate is minimum.



i) $\tau_1 = \tau_2$ (question)

$\Rightarrow \mu_1 \times \frac{v}{h-y} = \mu_2 \cdot \frac{v}{y}$ (assume $h << $) (linearization)

$\Rightarrow \mu_1 y = \mu_2 h - \mu_2 y$

$\Rightarrow (\mu_1 + \mu_2) y = +\mu_2 h$

$\therefore \cancel{\boxed{y = \left(\frac{\mu_1 - \mu_2}{\mu_2}\right) \times h}}$ $\quad y = \left(\frac{\mu_2}{\mu_1 + \mu_2}\right) \times h$

here, force should be minimum.

i.e. force due to change in position is minimum

we have:

~~$\tau_1 + \tau_2$ = total shear~~

$F_1 + F_2$ = total force due to individual sum of shearing stress

$\Rightarrow \tau_1 \times A + \tau_2 \times A = F$

$\Rightarrow \mu_1 \cdot \frac{v}{h-y} \times A + \mu_2 \times \frac{v}{y} \times A = F$

diff. w.r.t y.

$$-\frac{\mu_1 \cdot V \cdot A}{(h-y)^2} \times (-1) \mp \frac{\mu_2 \cdot V \cdot A}{y^2} = \frac{dF}{dy}.$$

for force due to change in position to be minimum,

$$\frac{dF}{dy} = 0.$$

$$\Rightarrow \quad \frac{\mu_1 VA}{(h-y)^2} = \frac{\mu_2 VA}{y^2}.$$

$$\Rightarrow \quad \mu_1 VA \cdot y^2 = \mu_2 VAh^2 - 2\mu_2 VAhy + \mu_2 VAy^2.$$

$$\Rightarrow \quad y^2(\mu_1 VA - \mu_2 VA) + 2\mu_2 VAhy - \mu_2 VAh^2 = 0.$$

$$\cancel{\Rightarrow \quad y^2 \times VA \cdot (\mu_1 - \mu_2) + 2VA}$$

$$y = \frac{-2\mu_2 VA \pm \sqrt{4\mu_2^2 V^2 A^2 + 4V^2 A^2 (\mu_1 - \mu_2)\mu_2 VA}}{2(\mu_1 - \mu_2)VA}.$$

$$y = \frac{-2\mu_2 VA \pm 2\sqrt{\mu_2^2 + (\mu_1 - \mu_2)\mu_2}\ 2VA}{2(\mu_1 - \mu_2)VA}.$$

$$y = \frac{-\mu_2 \pm \sqrt{\mu_2^2 + (\mu_1 - \mu_2)\mu_2}}{(\mu_1 - \mu_2)}$$

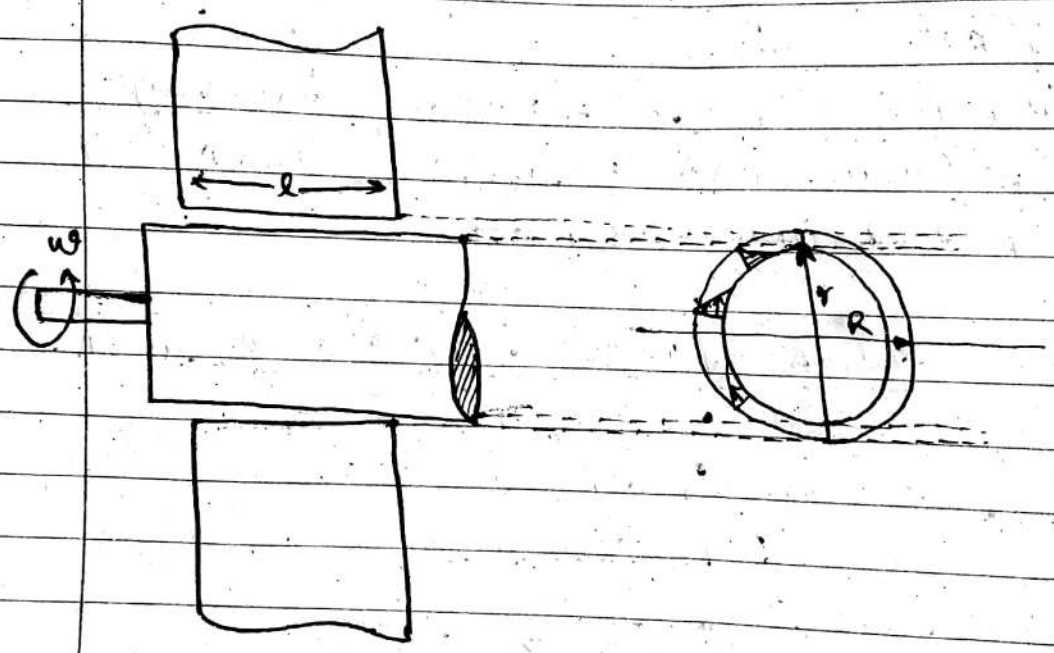
$$\boxed{y = \frac{-\mu_2 \pm \sqrt{\mu_1 \mu_2}}{(\mu_1 - \mu_2)}}$$

is the reqd condition.

Rotational cases of bearings:

Case-I

If the following configuration of shaft and hole is shown and ω be the angular velocity of the shaft, 'R' be the radius of hole, 'r' be the radius of shaft and a fluid of viscosity μ is kept in between the clearance, what will be the Torque ~~developed and~~ required and energy (heat) generated.



~~F = F×~~ $v = \omega r$.

$$\tau = \mu \cdot \frac{v}{(R-r)} = \mu \cdot \frac{\omega r}{(R-r)}$$

$$F = \tau \times A$$

$$= \mu \cdot \frac{\omega r}{(R-r)} \times 2\pi r l.$$

$$= \frac{\mu \omega 2\pi r^2 l}{(R-r)}$$

now, torque = F×r.

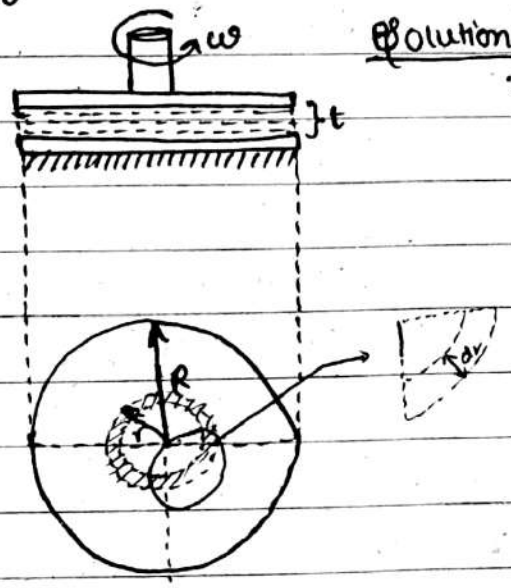
$$\boxed{\text{torque req}^{d}\ (T) = \frac{2\pi r^3 \mu \omega l}{(R-r)}}$$

power dissipated = T.ω.

$$\boxed{P = \frac{2\pi r^3 \mu \omega^2 l}{(R-r)}}$$

Case-II

* A uniform film of oil 0.1mm thick separates two disks of 400mm in diameter mounted co-axially. Ignoring the side effect, calculate the torque required to rotate, one disk relative to other at 10 rev/sec if μ_{oil} = 0.13 pascal sec.



Solution: Shearing stress $(\tau) = \mu \cdot \frac{du}{dy}$

but du varies with radius as.

$v = \omega r$

$dT = dF \times r$

$= d\tau \times dA \times r$

$= \mu \cdot \frac{du}{dy} \times dA \times r$

but dA = area of small shaded region

$= \pi(r+dr)^2 - \pi r^2$

$= \cancel{\pi r^2} + 2\pi r dr + \cancel{\pi dr^2} - \cancel{\pi r^2}$ (0)

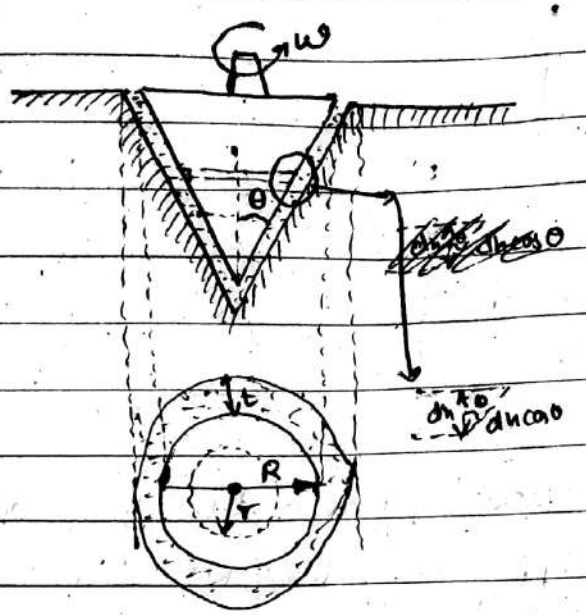
$= 2\pi r dr.$

So, $dT = \frac{\mu \cdot \omega \cdot r}{t} \times 2\pi r\, dr \times r = \frac{2\pi r^3 \mu \omega\, dr}{t}$

$\therefore T = \int_0^R \frac{2\pi r^3 \mu \omega\, dr}{t} = \frac{2\pi \mu \omega}{t}\int_0^R r^3 dr = 2\pi \mu \omega \frac{R^4}{4} = \frac{\pi \mu R^4 \omega}{2}$

So for above question: $T = \frac{\pi \times 0.13 \times 0.2^4 \times 10}{2}$

case III :



* A conical thrust bearing idealized as cone of semi-angle 30°, maximum cone diameter, 20cm. Rests and revolves over a uniform fluid layer of thickness 0.1cm at 600 rpm. If μ=1 poise. Calculate heat dissipated by the bearing (Ans : $4\pi^3$ watt)

Here,

$$dT = dF \times r$$

$$= d\tau \times dA \times r$$

$$= \mu \cdot \frac{du}{dy} \times dA \times r$$

$$= \mu \times \frac{V}{t} \times dA \times r$$

$$= \mu \times \frac{\omega r}{t} \times dA \times r$$

$$= \mu \times \frac{\omega r}{t} \times 2\pi r \times dh \times r \cos\theta$$

$$= \frac{\pi \mu \omega r^3 \cdot dh}{t} \cos\theta$$

$$= \frac{\pi \mu \omega h^3 \cdot \tan^3\theta \cdot dh}{t} \cos\theta$$

but: $\tan\theta = r/h$.

$\therefore h \tan\theta = r$.

$\Rightarrow \frac{dh}{dr} \times \tan\theta = 1$. (since, θ is constant)

$\Rightarrow dh \tan\theta = dr$.

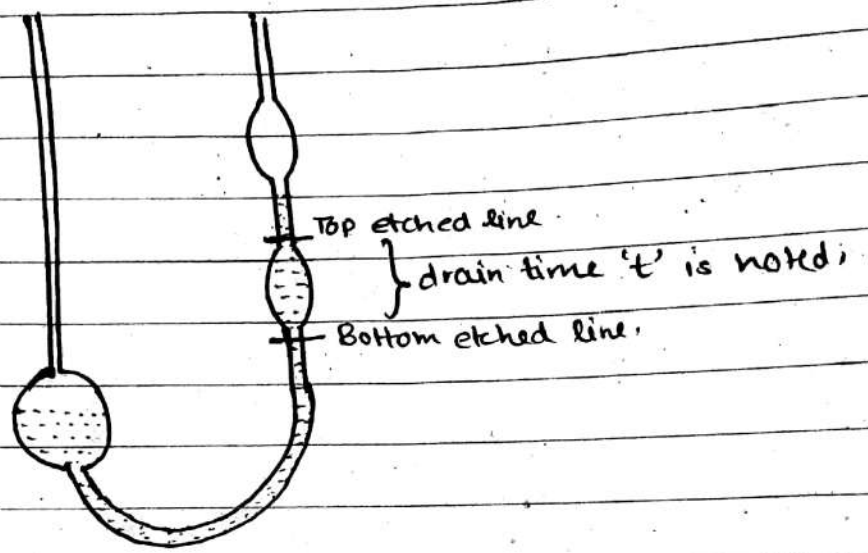
$\Rightarrow dh = dr/\tan\theta$.

$$T = \int_0^H \frac{\pi \mu \omega h^3 \tan^3\theta \cos\theta \cdot dh}{t} = \frac{\pi \mu \omega \tan^3\theta \cos\theta}{t} \int_0^H H^3 dH$$

$$= \frac{\pi \mu \omega \tan^3\theta \cos\theta \cdot H^4}{4t}$$

Capillary tube viscometer

Used to measure the viscosity.



For this device, liquid to be tested is drawn into the tube to a level above top-etched line. The time is then obtained for liquid to drain to bottom etched line. The kinematic viscosity of the liquid is obtained as linearly proportional to $R^4 t$. Where R is the radius (mm) of capillary tube and t is drain time in s. $\boxed{\nu \propto R^4 T}$ $(\nu = \mu/\rho)$

When glycerine $(1.19 \times 10^{-3}\ m^2/s)$ is used as a calibration fluid in a particular viscometer, the drain time is 1.43 sec.

When liquid of density $970\ kg/m^3$ is tested in such viscometer, drain time is 900 sec. Determine, dynamic viscosity of the liquid.

Here,

$$V = k.R^4.t \quad \ldots\ldots (i)$$

$$\Rightarrow \underset{\text{(viscometer Property)}}{kR^4} = \frac{V}{t} = \frac{1.19 \times 10^{-3}}{1.43} = 8.32 \times 10^{-4}.$$

(caliberated using glycerine)

now, for liquid:

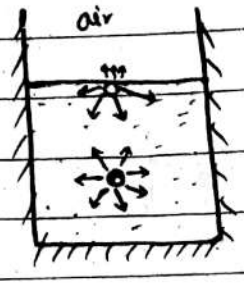
$$V = k.R^4.t$$

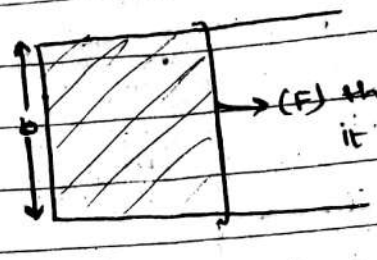
$$= 8.32 \times 10^{-4} \times 900s$$

$$= 0.748$$

$$\text{so, } \mu = \frac{V}{S} = 7.72 \times 10^{-4}.$$

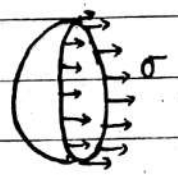
Surface tension



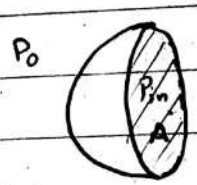


then, surface tension = F/b.

a) when a bubble of liquid is cut; there will be two forces.
(radius R)



force due to surface tension



force due to pressure change.

i.e.

in this case:

$$\sigma \times 2\pi R = \Delta P \times A$$

$$\Rightarrow \sigma \times 2\pi R = (P_o - P_{in}) \times A.$$

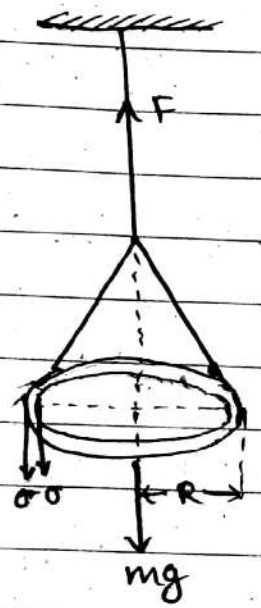
$$\therefore \sigma = \frac{\Delta P_{drop} \times \pi R^2}{2\pi R}$$

$$= \frac{\Delta P_{drop} \times R}{2}$$

$$\boxed{\therefore \Delta P_{drop} = \frac{2\sigma}{R}}$$

b) Ring placed in a liquid

A ring of radius 'R' and mass 'm' is kept at rest on the surface of liquid of surface tension σ by a string tied as shown in figure. Find the force required to lift the ring above the liquid.



$F = mg + 2\sigma \times$ perimeter of ring

$F = mg + 2\sigma \times 2\pi R$. (If R is similar for both outer and forces

$\boxed{\therefore F = mg + 4\pi\sigma R}$

c) For soap bubble, since it is hollow

$F = \sigma \times 2 \times 2\pi R$

due to double layer

Wet, no wet condition

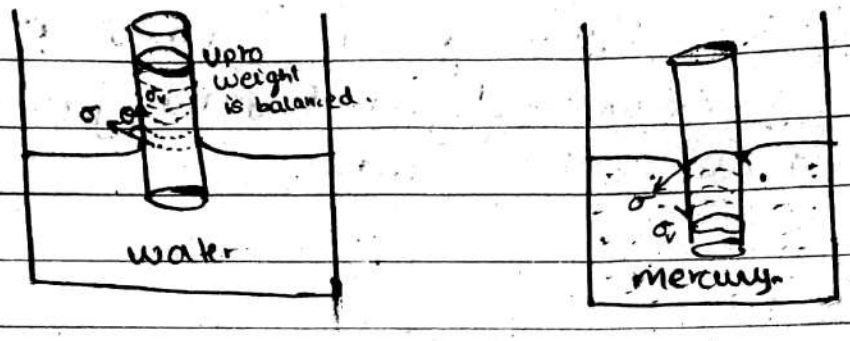
$\phi < 90°$
(wet)

$\phi > 90°$
(no wet)

$\boxed{\phi = \text{contact angle}}$

Civil लाई खासै चाहिन surface tension
but micro measurement
मा affect गर्छ ।

Capillary action



$F_{up} = F_{down}$.

$\Rightarrow \sigma_v = mg$

$\Rightarrow \sigma\cos\theta \times 2\pi R = mg$.

$\Rightarrow \sigma\cos\theta \times 2\pi r = \rho . V . g$.

$\Rightarrow \sigma\cos\theta \times 2\pi r = \rho . \pi r^2 \times h \times g$.

$\Rightarrow h = \dfrac{2\sigma\cos\theta}{\rho g r}$

$$\boxed{\therefore h = \frac{2\sigma\cos\theta}{\rho g r}}$$

* Two glass tubes of bore 1mm and 2mm are introduced inside the liquid. The capillary rise of the liquid in tube indicates a difference in hight between two miniscus of 1.3cm. If mean density of liquid is 800 kg/m³. Estimate surface tension of the liquid. Take angle of contact of 0°.

=> Solution:

$$h_1 = \frac{2.\sigma.\cos\theta}{800 \times 9.81 \times 10^{-3}} \quad \text{.... (i)}$$

$$h_2 = \frac{2\sigma}{800 \times 9.81 \times 2 \times 10^{-3}} \quad \text{..... (ii)}$$

also, $h_1 - h_2 = 1.3 \times 10^{-2}\,m$ ..... (iii)

So

$$\frac{(i)}{(ii)} = 2.$$

i.e. $\frac{h_1}{h_2} = 2.$

$\therefore h_1 = 2h_2.$

$\Rightarrow 2h_2 = 1.3 \times 10^{-2}\,m.$

and $h_1 = 2.6 \times 10^{-2}\,m.$

So,

$\sigma = 0.102\,N/m.$

Compressibility (β)

$$\beta = -\frac{dv/v}{dp} = -\frac{\text{volumetric strain}}{\text{compressive stress}} \quad \text{--- (i)}$$

-ve sign indicates that volume decreases on increase in pressure

Law of conservation of mass:

$$m = \text{constant}$$

$$\Rightarrow \rho V = \text{constant}$$

$$\Rightarrow \rho dV + V d\rho = 0 \quad \text{(diff w.r.t anything)}$$

$$\Rightarrow \frac{dV}{V} = \frac{-d\rho}{\rho}$$

$$\therefore \frac{-dV}{V} = \frac{d\rho}{\rho}$$

so,

$$\beta = \frac{d\rho/\rho}{dp}$$

$$\boxed{\therefore \beta = \frac{1}{\rho}\frac{d\rho}{dp}}$$

here,

$$\beta = f(\rho)$$

$$\rho = f(P)$$

if: $\frac{d\rho}{dp} \neq 0$

$$\beta \neq 0$$

the it is called compressible fluid.

if. $\rho \neq f(P)$.

$\Rightarrow \frac{d\rho}{dp} = 0$.

$\beta = 0$ {is compressible}.

reciprocal of compressibility is called the bulk modulus of elasticity.

$$\underset{\text{(bulk modulus)}}{K} = \frac{1}{\beta} = \frac{dP}{-dV/V} = \rho\frac{dP}{d\rho}$$

$$\boxed{\frac{d\rho}{\rho} = \frac{dP}{K} = \frac{d\gamma}{\gamma}}$$ where, γ = specific weigh. $= \rho g$.

* At the ocean surface, the specific weight is 10056 KN/m³. calculate the specific weight and specific volume at a depth of 12km if the pressure here is 102660 KN/m² and average bulk modulus of sea water is $K = 2.345 \times 10$ KN/

~~$P_{12km} = \rho g h$~~

~~$= 1000 \times 9.81 \times 12000$~~

~~$= 117720$ K~~

$$\frac{\gamma_1}{\gamma_2} = \frac{102660 \text{ KN/m}^2}{2.345 \times 10^6}$$

$$\frac{d\gamma}{10056} = \frac{102660}{2.345 \times 10^6}$$

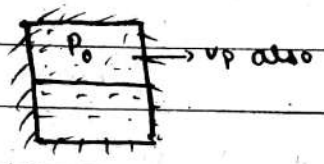
$\therefore d\gamma =$ ~~440.28 KN/m³. 54.2895~~ 517.423 KN

$\therefore \gamma_{12KM} =$ ~~10496.2841 KN/m³.~~ 10573.42 KN/m³

Vapour pressure: (V_p)

Pressure exerted by pressure.

When V_p < pressure acting in the surface P_o = evaporation
V_p = pressure " " " " " = Boiling.
V_p > " " " " " " = Vaporization (phase change)

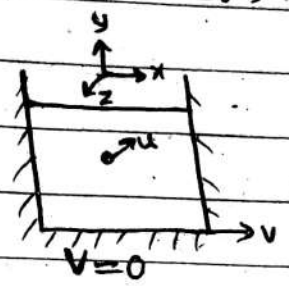


Cavitation

Cavitation is the erosion process in the solid surface due to pressurized vapour bubble formed because of the pressure at the point is less the vapour pressure.

Fluid Statics

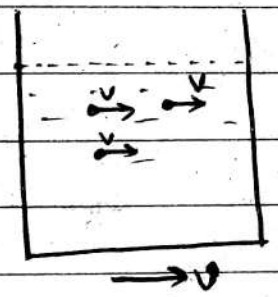
1) Fluid is at rest, its study.



$u = f(x, y, z)$.

if $u \neq f(x, y, z)$
$V = 0$.
$u = 0$, w.r.to time.
} absolute rest.

2) Study of fluid in rigid body motion.



relative velocity $= v - v = 0$

so, no particles move w.r.t each other

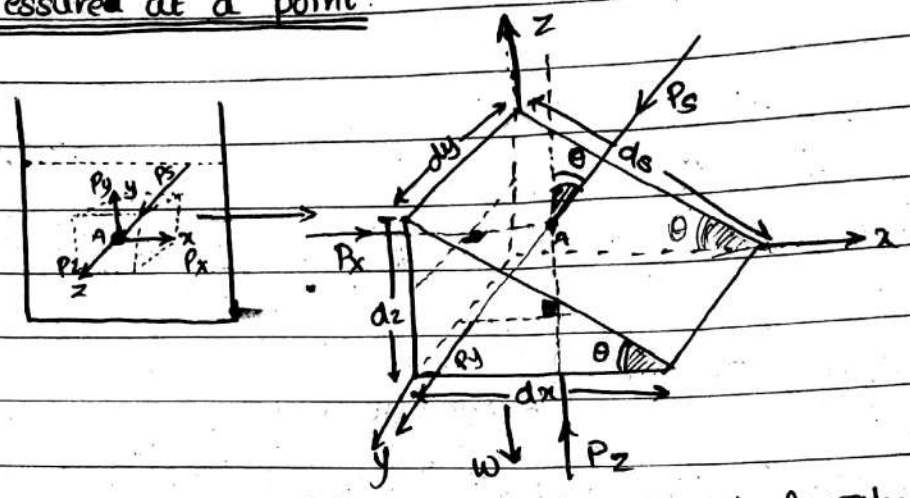
only force acting on the fluid at rest is normal force.

$p = N/m^2$ अब Pressure कहलाता, Intensity pressure कहलाता, Pascal, bar, torr, 1 bar = 10^5 Pascal., torr = 1mm/Hg

$P = p \times A = N$ अब force होगी pressure force पता कहलाता.

Pascal's law:

i) Pressure at a point:



Let us consider a consider a control volume over a point A.

Since the fluid is in equilibrium and inside rigid body, there are no shear forces.

$$W = \rho g V$$

for equilibrium condition:

$$\sum F = 0 = m\cancel{a}^{0}$$

$\Rightarrow$

$$\sum F_x = 0$$

$$\Rightarrow P_x \times dy\,dz - P_s \times (ds\,dy)\sin\theta = 0$$

$$\Rightarrow P_x \times dy\,dz - P_s \times (ds\,dy) \times \frac{dz}{ds} = 0.$$

$$\Rightarrow P_x\,dy\,dz - P_s\,dz\,dy = 0.$$

$$\Rightarrow dy\,dz\,(P_x - P_s) = 0.$$

since, $dy\,dz \neq 0$.

$$P_x = P_s \quad \ldots (i)$$

i.e pressure in P_x = Pressure in P_y.

Partial and total derivative:

$$dP = \frac{\partial P}{\partial x}\cdot dx + \frac{\partial P}{\partial y}\times dy + \frac{\partial P}{\partial z}\cdot dz$$

similarly in y direction,

$$P_y = P_s \quad \text{--- (ii)}$$

now,

$+\uparrow \Sigma F_z = 0$

$\Rightarrow P_z \cdot dx \times dy - w - P_s \cos\theta \times ds\, dy = 0.$

$\Rightarrow P_z \cdot dx\, dy - \rho g V - P_s\, dx\, dy = 0.$

$\Rightarrow dx\, dy\,(P_z - P_s) - \rho g V = 0.$

$\Rightarrow dx\, dy\,(P_z - P_s) - \rho g \cdot \frac{dx \times dz \times dy}{2} = 0.$

Since volume is very small $dx \times dz \times dy \ll$ less.

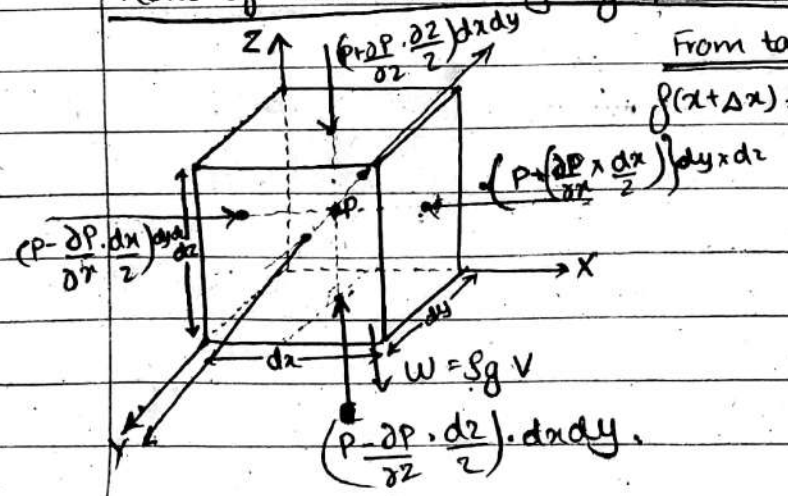
so,

$$P_z = P_s \quad \text{---- (iii)}$$

$$\boxed{P_x = P_y = P_z = P_s}$$

Statement: Pressure at a point in the fluid at rest is same in all direction.

Law of transmissibility of pressure:



From taylor series

$$f(x+\Delta x) = f(x) + \frac{\partial}{\partial x} f(x)\, dx + \frac{\partial^2}{\partial x} f(x)\, dx + \ldots\ldots$$

$\rightarrow \Sigma F_x = 0$

$$\Rightarrow \left(P - \frac{\partial P}{\partial x} \cdot \frac{dx}{2}\right) dy\,dz - \left(P + \frac{\partial P}{\partial x} \cdot \frac{\partial x}{2}\right) \partial y \cdot \partial z = 0.$$

$$\Rightarrow \frac{\partial P}{\partial x} = 0.$$

$\therefore$ No change in pressure in x-direction

$\Sigma F_y = 0.$

$$\Rightarrow \frac{\partial P}{\partial y} = 0.$$

$\therefore$ No change in pressure in y-direction.

$\uparrow \Sigma F_z = 0$

$$\Rightarrow \left(P - \frac{\partial P}{\partial z} \cdot \frac{\partial z}{2}\right) \cdot \partial x \cdot \partial y - \left(P + \frac{\partial P}{\partial z} \cdot \frac{\partial z}{2}\right) dx \cdot dy - \rho g (dx\,dy\,dz) = 0.$$

or, $$\left[-\frac{\partial P}{\partial z} - \rho g\right](dx\,dy\,dz) = 0,$$

now,

$$\frac{\partial P}{\partial z} = -\rho g$$ (Pressure ~~mathi~~ tala badcha, tala ~~jada~~ mathi ghatcha (river

now, $$dp = \frac{\partial P}{\partial x} \cdot dx + \frac{\partial P}{\partial y} \cdot dy + \frac{\partial P}{\partial z} \cdot dz.$$

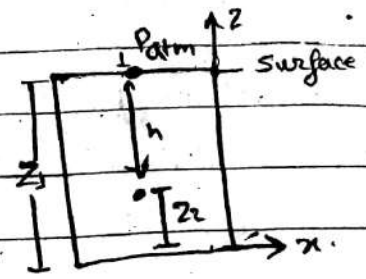
$$= 0 + 0 - \rho g\,dz.$$

So, $$\frac{dP}{dz} = -\rho g.$$

integrating

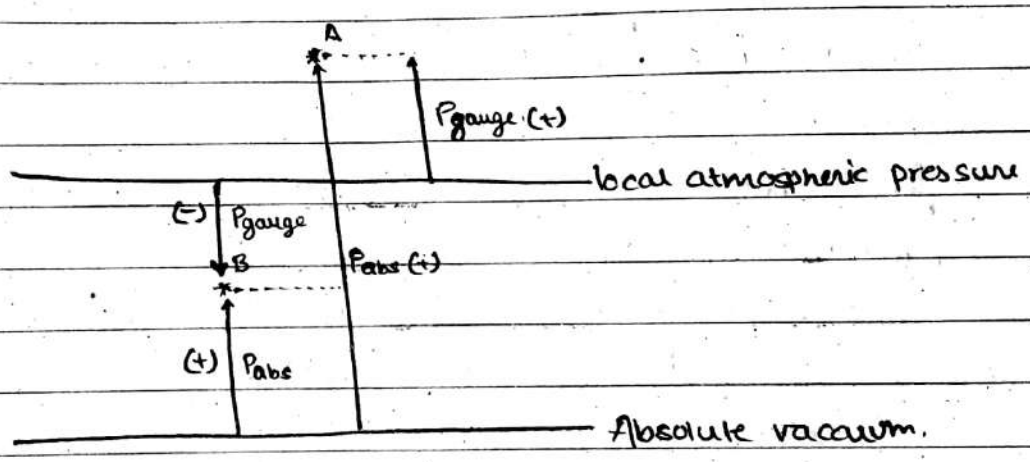
$$\int dp = -\int \rho g\,dz. \Rightarrow \boxed{P = -\rho g z + \text{constant}}$$

$$\int_1^2 dP = \int_1^2 -\rho g \, dz.$$



$$P_2 - P_1 = -\rho g (z_2 - z_1)$$
$$= \rho g h \quad (z_2 - z_1 = -h)$$

$$P_2 = P_1 + \rho g h.$$
$$(P_2 = P_{atm} + \rho g h).$$



$$\boxed{P_{abs} = P_{atm} \pm P_{gauge}}$$

<u>Pressure head :</u>

$$P_2 = \cancel{P_{atm}}^{\,0} + \rho g h.$$

$$\boxed{P_{gauge} = \rho g h}$$

$$\boxed{h = \frac{P}{\rho g}} \rightarrow \text{Pressure head.}$$

* Convert the pressure of 350 KN/m² into head of water and mercury. (S.g = 13.6)

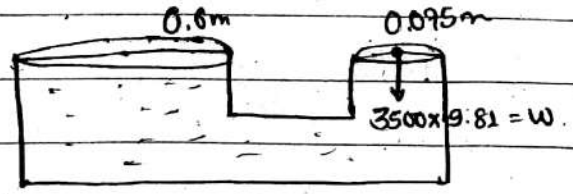
⇒ head $_{water}$ = $\frac{P}{\rho g}$ = $\frac{350000}{1000 \times 9.8}$ = 35.71 m of water

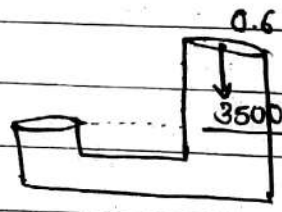
head $_{mercury}$ = $\frac{350000}{13600 \times 9.8}$ = 2.62 m of Hg.

* A hydraulic press has a diameter ratio between two piston of 8:1, the diameter of the larger piston is 600mm and it is required to suppat of mass of 3500 kg. The press is filled with hydraulic fluid of (s.g. 0.8). Calculate force required on the smaller piston to provide the required force:

a) When 2 pistons are in level.

b) When smaller piston is 2.6m below the larger piston.





a) $\frac{F}{0.075^2} = \frac{3500 \times 9.81}{0.6^2}$

~~F = 2197.44 kN~~

$\boxed{F = 536.48\ N}$

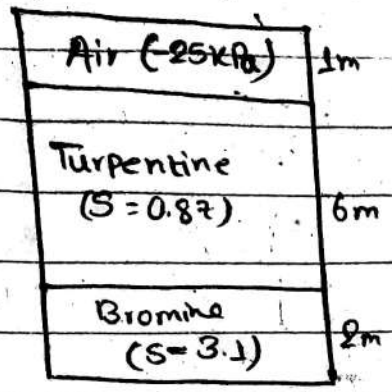
Total W = ~~36553.96972~~ 40104.321

~~3500~~

$\frac{F}{0.075^2} = \frac{40104.321}{0.6^2}$

F = ~~571.15 N~~ $\boxed{626.63}$

*



Find the absolute pressure at the bottom of the tank in KPa and in cm of mercury. Take $P_{atm} = 101.3$ KPa and Neglect density of the air.

$$P_{bottom} = \rho_T g h_1 + \rho_B g h_2 + Air.$$
$$= 870 \times 9.81 \times 6 + 3100 \times 9.81 \times 2 - 25000.$$
$$= \cancel{32290.4\ Pa.}\ \cancel{93112.4}.\ 87030.2$$

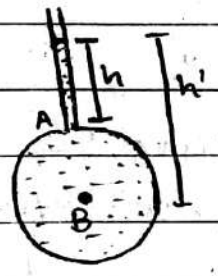
$$P_{abs} = P_{bottom} + P_{atm}$$
$$= \cancel{322904}\ \cancel{93112.4} + 101.3\ KPa.$$
$$= \cancel{133.5904\ KPa.}\ 188.3302\ KPa$$

$$h = \frac{P}{\rho g} = \frac{\cancel{188.590.4}\ 188.330.2}{13600 \times 9.81} = \cancel{1.0013\ m.}\ 1.4116 = \cancel{1000.13\ cm.}\ 141.16\ cm$$

Pressure measurements:

- Manometer
  - Simple manometer
    - Piezometer
    - U-tube manometer
    - single column or or well type or micromanometer
    - inclined U-tube manometer
  - Differential manometer
    - Inverted U-tube manometer
    - Differential U-tube manometer
    - inclined U-tube manometer.
- Mechanical gauges
  - Bourdan tube.
  - pressure transducers.
  - Blow pressure gauge.
  - diaphragm pressure gauge.

1) Piezometer



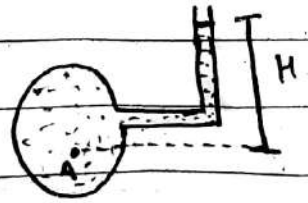
P at A, $P_A = \cancel{P_{atm}}^{0} + \rho g h$.

$= \rho g h$ (gauge pressure)

P at B, $P_B = \cancel{P_{atm}}^{0} + \rho g h'$

$= \rho g h'$ (gauge pressure).

also,



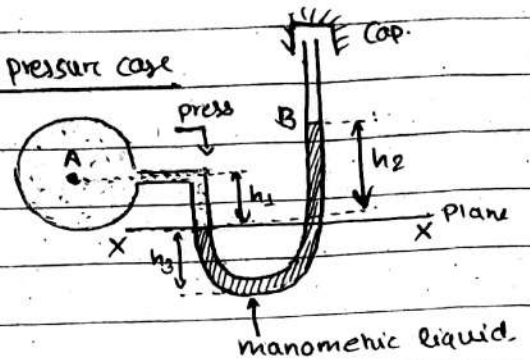
P at A, $P_A = P_{atm} + \rho g h$

$= \rho g h$ (gauge pressure.

demerits: Not applicable for gas, h can be very high sometimes, and negative pressure cannot be measured.

2) U-tube manometer:

i) Positive pressure case



$\rho_{man} > \rho$.
generally, mercury is used.

writing manomatic equation:

same plane so can be removed directly.

$$P_A + \rho g h_1 + \rho g h_3 - \rho_m g h_3 - \rho_m g h_2 = P_B$$

$$\Rightarrow P_A + \rho g h_1 = \rho_m g h_2 = P_B$$

also:

Using pascals law:

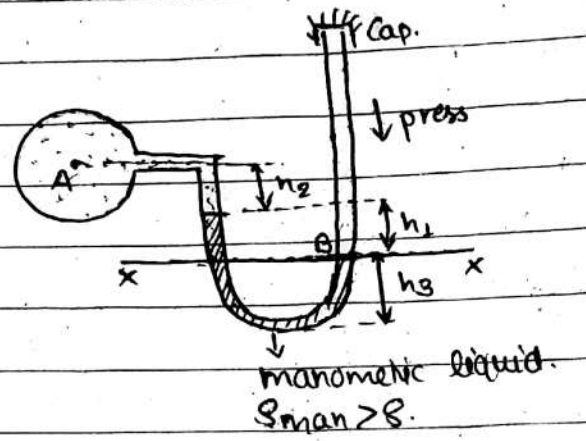
$$P_{xx}\ \text{left} = P_{xx}\ \text{right}$$

$$\Rightarrow P_A + \rho g h_1 = P_B + \rho_m g h_1$$

$$\therefore P_A + \rho g h_1 - \rho_m g h_1 = P_B$$

when cap is inserted, P_B persists but when cap is removed $P_B = P_{atm}$ and can be assumed to be zero to calculate the gauge pressure of P_A (If P_{atm} is kept than, absolute pressure of A is known).

ii) Negative pressure case:



Writing manometric equation:

$$P_A + \rho g h_2 + \rho_m g h_1 + \overbrace{\rho_m g h_3 - \rho_m g h_3}^{\text{removed}} = P_B$$

$$\Rightarrow P_A + \rho g h_2 + \rho_m g h_1 = P_B.$$

using pascal's law:

$$P_{xx}\ \text{left} = P_{xx}\ \text{right}.$$

$$\Rightarrow P_A + \rho g h_2 + \rho_m g h_1 = P_B.$$

when cap is inserted, P_B persists but when when cap is removed, $P_B = P_{atm}$ and can be assumed to be zero to calculate the gauge pressure. (If P_{atm} is kept, than absolute pressure of A is known)

3) Single colum or well type or micromanometer:

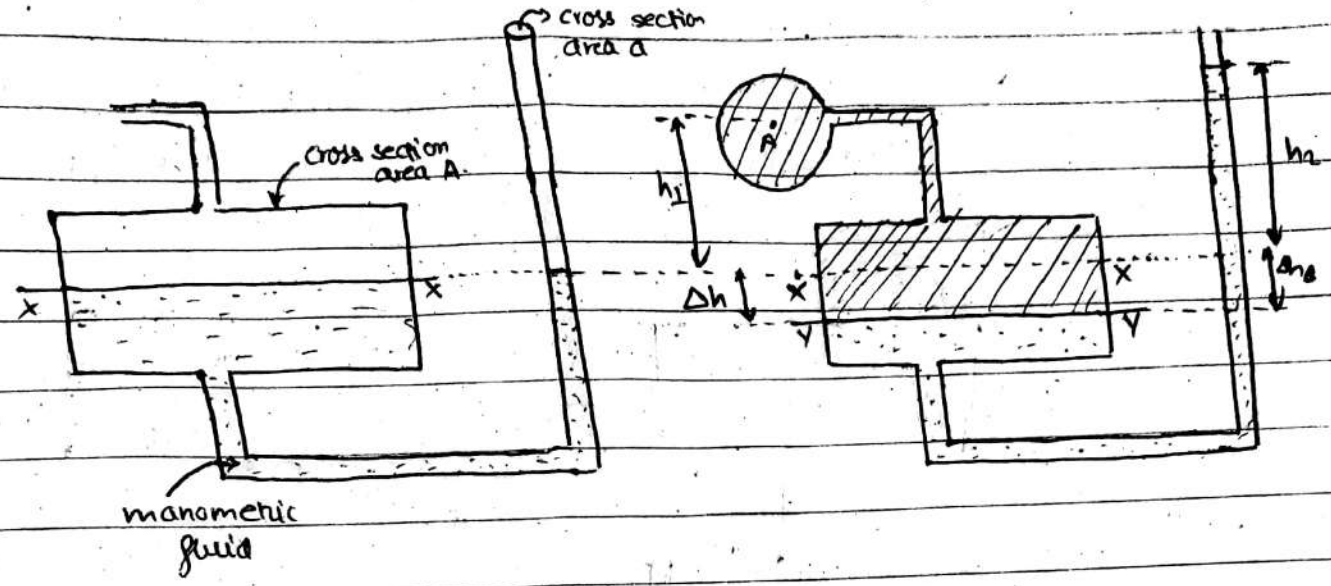


Fig: Before connection.

From continuity, displace volume in the left limb = rise in volume in right limb.

$\therefore \Delta h \times A = a \times h_2$

$\Rightarrow \Delta h = \frac{a}{A} \cdot h_2$ ------ (i)

$P_A + \rho g (h_1 + \Delta h) - \rho_m g (h_2 + \Delta h) = 0$

$\Rightarrow P_A = \rho_m g (h_2 + \Delta h) - \rho g (h_1 + \Delta h)$

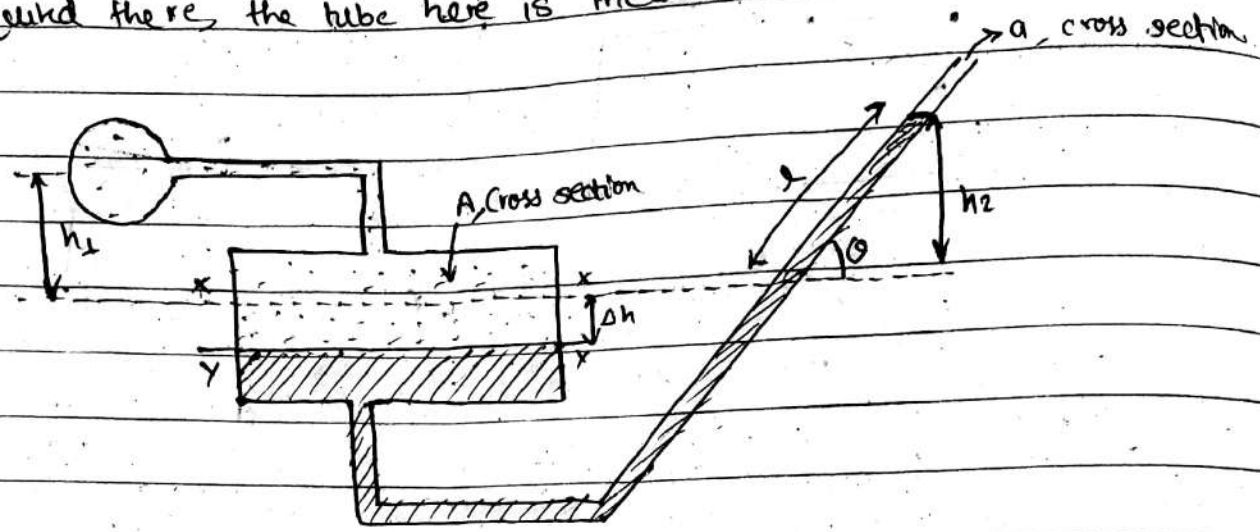
$$\boxed{\therefore P_A = \frac{a}{A} h_2 g (\rho_m - \rho) + \rho_m g h_2 - \rho g h_1}$$

if $A >>> a$, then $\frac{a}{A}$ is negligible.

$$\boxed{\therefore P_A = \rho_m g h_2 - \rho g h_1}$$

4) Inclined U-tube manometer

Same as micromanometer but since long tube might be required there, the tube here is inclined at certain angle θ.



l is measured.

since, this manometer is based on same principle as the micromanometer so its equation is:

$$P_A = \frac{a}{A} l h_2 g (\rho_m - \rho) + \rho_m h_2 g - \rho g h_1$$

but $h_2 = l \sin\theta$.

$$\boxed{P_A = \frac{a}{A} l \sin\theta (\rho_m - \rho) + \rho_m l \sin\theta g - \rho g h_1}$$

if $A >>> a$,

$$\boxed{P_A = \rho_m l \sin g - \rho g h_1}$$

Question:

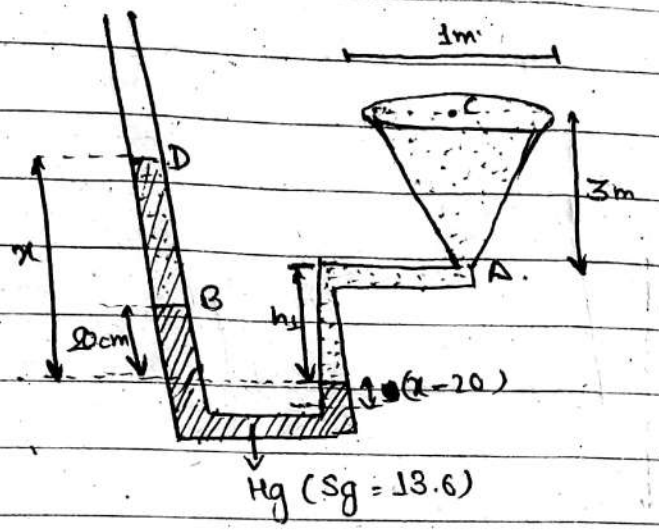


Figure shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer is as shown, when the vessel is empty. Find the reading of the manometer, when the vessel is completely filled with water.

=> Solution:

initially.

$$P_A = P_B = P_{atm}.$$

using manometric eq^n:

$$P_B + \rho_m g(0.2) - \rho_w g h_1 = P_A.$$

$$\Rightarrow \rho_m g(0.2) = \rho_w g h_1$$

$$\Rightarrow \frac{13.6 \times 9.81 \times 0.2}{9.81} = h_1$$

$$\Rightarrow \boxed{h_1 = 2.72m. \quad \therefore h_1 = 272\,cm}$$

afterwards, when the vessel is filled,

$$P_D + x \times \rho_m \times g - (2.72+x+0.20) \times g \times \rho_w - \rho_w \times 3 \times g = P_C.$$

but $P_D = P_C = P_{atm}$ so,

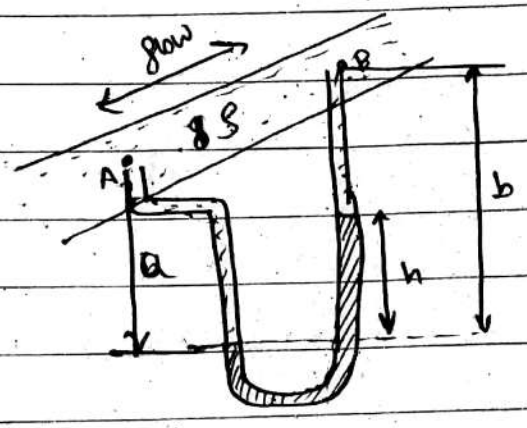
$$\cancel{x \times 13.6 \times 9.81 - 2.72 \times 9.81 - 3 \times 9.81 = 0} \therefore x$$

$$\Rightarrow x \times 13.6 \times 9.81 - 2.52 \times 9.81 - x \times 9.81 - 3 \times 9.81 = 0$$

$$\Rightarrow x = 0.43.80cm$$

Differential manometer:

It is a device used to measure pressure difference between two points.



now using manometric equation,

$$P_A + \rho g a - \rho_m g h - \rho g (b-h) = P_B.$$

$$\boxed{\therefore P_A - P_B = \rho_m g h + \rho g (b-h) - \rho g a}$$

$$\boxed{= \rho g (b-a) + (\rho_m - \rho) g h}$$

* if the pipe is horizontal, $b = a$.

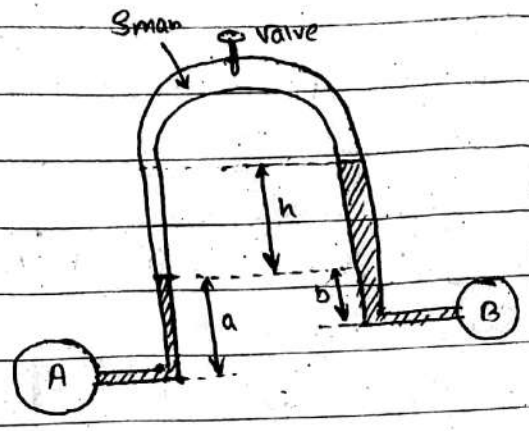
and $\boxed{P_A - P_B = (\rho_m - \rho) g h}$

if static $h = 0$, and $P_A - P_B = 0$

but in dynamic condition, $P_A - P_B$ persists.

$$\boxed{1\,kgf = 9.81\ N}$$

Inverted U-tube manometer



Writing manometric equation:

$$P_A - g\rho a - \rho_m g h + \rho g h + \rho g b = P_B$$

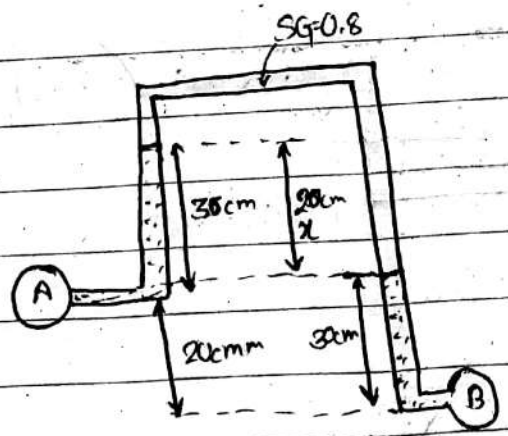
$$\Rightarrow P_A - P_B = \rho_m g h + \rho g (a - b - h)$$

$$\Rightarrow P_A - P_B = \rho_m g h - \rho g h + \rho g (a-b)$$

$$\Rightarrow P_A - P_B = -gh(\rho - \rho_m) + \rho g (a-b)$$

$$\Rightarrow \boxed{P_A - P_B = -(\rho - \rho_m) g h + \rho g (a-b)}$$

Question:



we have: $x = 30 - (30-20)$

$= 20\,cm$

$= 0.20\,m$

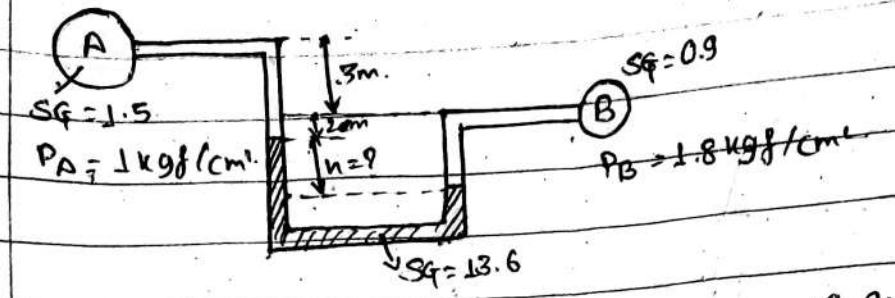
So using manometric equation,

$$P_A - \rho_w \times 0.35 \times g + \rho \times 0.20 \times 9.81 + 9.81 \times 0.3 \times \rho_w = P_B$$

$$\Rightarrow P_A - P_B = \cancel{1.4715} - 1.0791$$

$$\boxed{\therefore P_B - P_A = \cancel{1.4715}\ 1.0791}$$

*



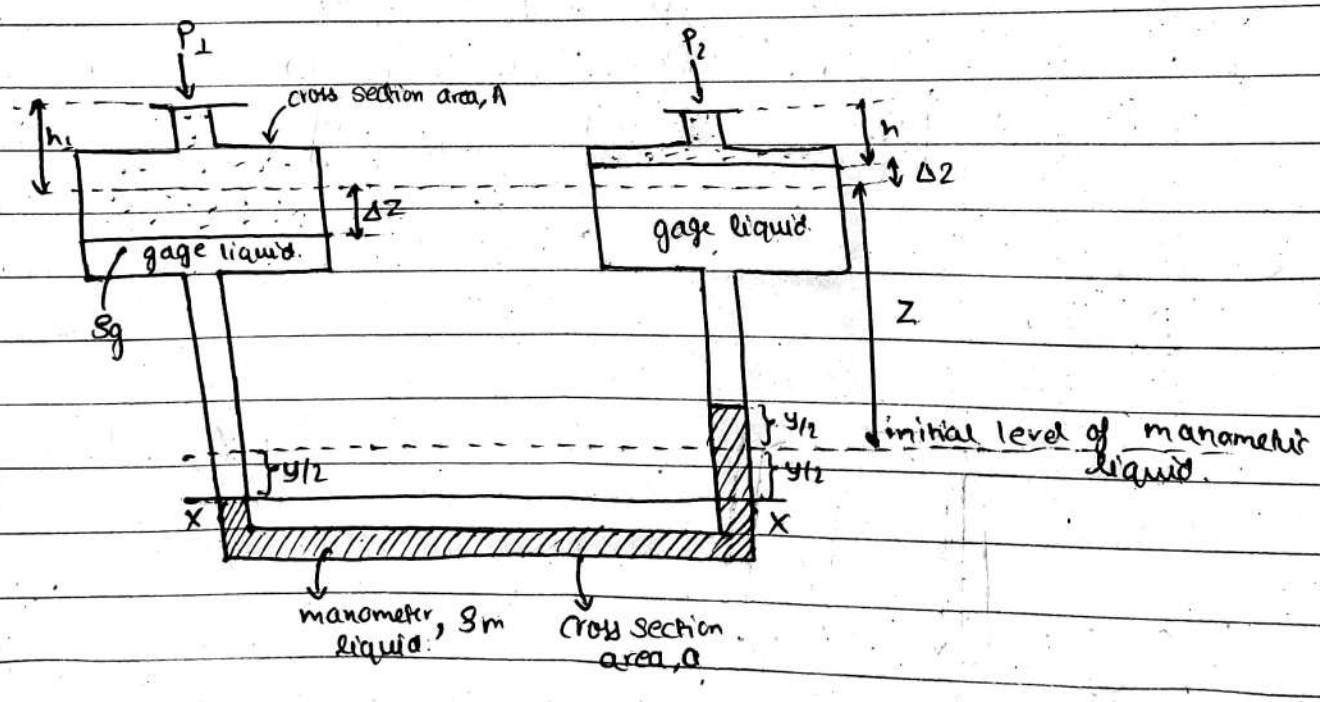
$P_A + 3\times1.5\times g + 2\times1.5\times g + h\times g\times13.6 - 0.9\times(h+2)\times9.81 = P_B.$

$\Rightarrow P_A - P_B = 73.575 + 124.587h - 17.658$

$\Rightarrow 9.81\times10^{-4} - 1.7658\times10^{-3} = 73.575 + 17.658 = 124.587h$

$\Rightarrow h = 0.448\ m$

Differential type micromanometer



according to pascals law in plane XX:

P_{xx} left = P_{xx} right

$\Rightarrow P_1 + \rho_w g(h_1+\Delta z) + \rho_g(z - \Delta z + y/2) = P_2 + \rho_w g h + \rho_g(z+\Delta z - y/2) + \rho_m g(y)$

$$\Rightarrow P_1 - P_2 = \rho_w g h + \rho_g (z + \Delta z - y/2) + \rho_m g (y) - \rho_w g (h_1 + \Delta z) + \rho_g (z - \Delta z + y/2)$$

$$\Rightarrow P_1 - P_2 = \rho_w g (h - h_1 - \Delta z) + \rho_g (z + \cancel{\Delta z} - \cancel{y/2} + z - \cancel{\Delta z} + \cancel{y/2}) + \rho_m g y$$

$$P_1 - P_2 = \rho_m g y + 2\rho_g g \Delta z - 2\rho_g g\, y/2 - 2\rho_w g\, \Delta z \quad \text{--- (i)}$$

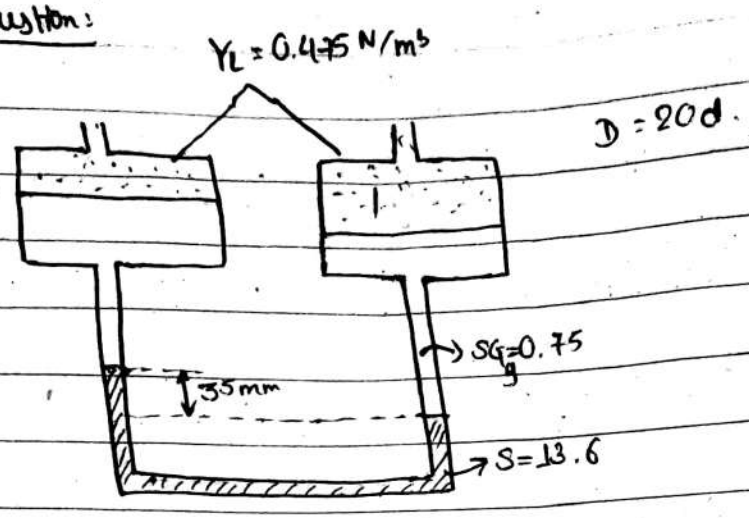
From continuity, volume dropped in the oil = volume moved in the tube:

$$A . \Delta z = a \times y/2$$

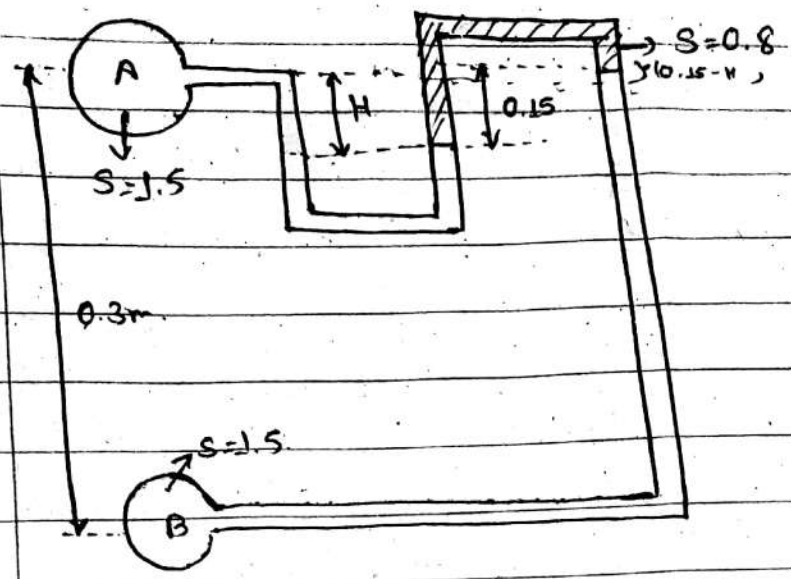
$$\Rightarrow \Delta z = \frac{a}{A} \frac{y}{2} \quad \text{--- (ii)}$$

$$\boxed{\therefore P_1 - P_2 = g y \left\{ \rho_m - \rho_g \left(1 - \frac{a}{A}\right) - \rho_w . \frac{a}{A} \right\}}$$

Question:



*



$$P_A + \rho g H - \rho_0 g\, 0.15 + \rho g(0.3 + 0.15 - H) = P_B$$

$$\Rightarrow P_A - P_B = -\cancel{\rho g H} + \rho_0 g \times 0.15 \cancel{- 0.3\rho g} - 0.45\rho g + \cancel{\rho g H}$$

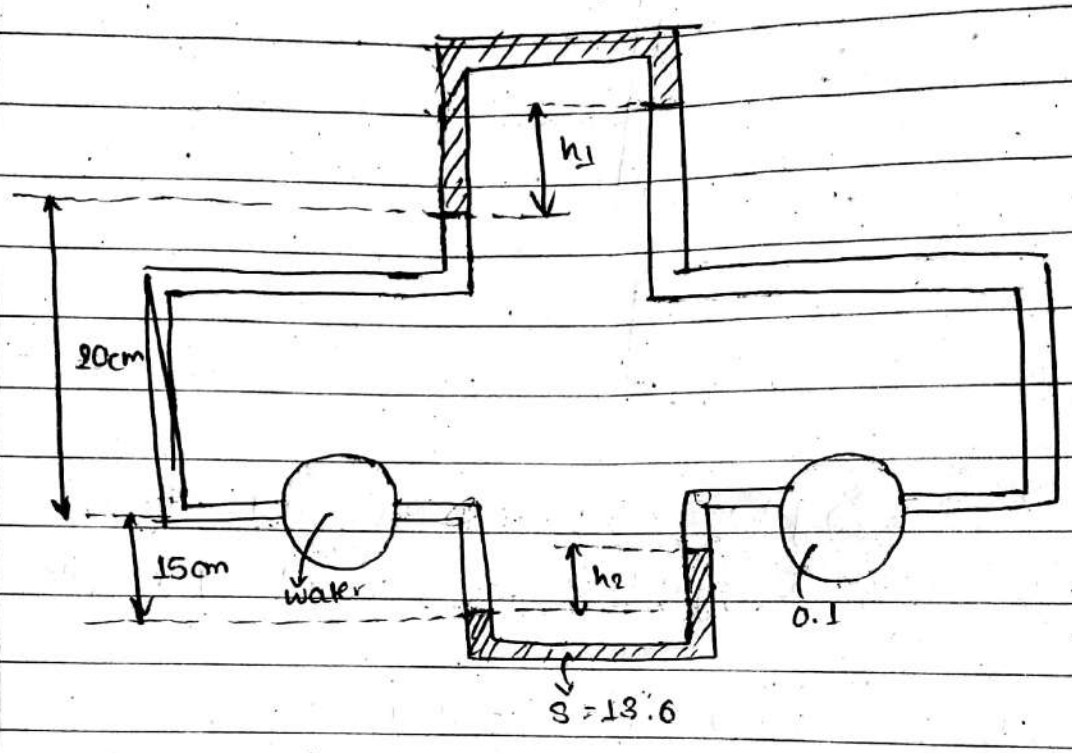
$$\Rightarrow P_A - P_B = 0.8 \times 9.81 \times 0.15 - 0.45 \times 1.5 \times 9.81$$

$$= -5.4445 \times 1000$$

$$\boxed{\therefore P_B - P_A = 5,4445}$$

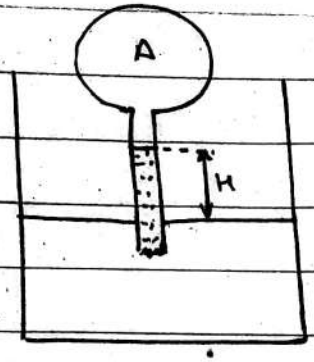
*

* **Question :**



$h_1 = 5cm$, what is h_2?

*



For given figure, if the pressure of gas in the bulb A is 50 cm of Hg Vacuum and P_{atm} = 76 cm of Hg. What will be the height of column H

<u>manometric equation</u>

$0 \; \cancel{P_{atm}} - \rho_m \times g H + P_{atm} = P_A + \cancel{P_{atm}}$

$\Rightarrow \cancel{\rho_m g (76} - H) = -\rho_m g (50)$

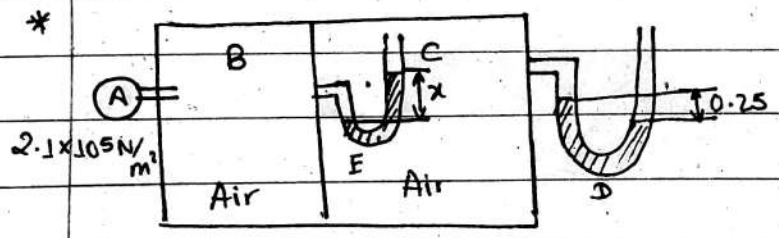
$\Rightarrow \cancel{\rho_m g (76} + 50 - H) = \cancel{50} \; 0.$

$\Rightarrow$ since, $\rho_m g \neq 0$,

$\cancel{76} + 50 = H$

$\cancel{\Rightarrow H = 126 cm}$ $H = 50 cm.$

*



If barometer reading in 99.572 KN/m², and if readings of A and D are as shown in figure. Calculate the value of x.

$P_A + P_{atm} + P_B.$

$\Rightarrow (P_A + P_{atm}) = P_B.$ $\cancel{= (i)}$ $= \cancel{110429 \; N/m^2}$ $309579 \; N/m^2$

now,

$P_C + 0.25 \times \rho_m \times g = P_{atm}.$

$\Rightarrow P_C = 99.571 \times 1000 - 0.25 \times 13600 \times 9.81$

$= \cancel{-33254.429 \; KN/m^2}$ $6621.4 \; N/m^2.$

Note: Gauge measures (displaye) gauge pressure only.

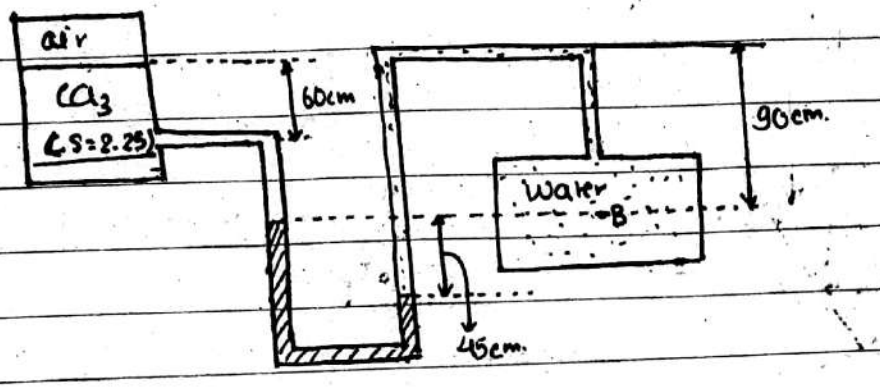
now,

$P_B - \rho_m g x = P_C$

$\Rightarrow$ ~~110472~~ 309572 $- 13600 \times 9.81 \times x = 66217$

$x =$ ~~0.331m~~ $1.824m$

*



$P_{air} + \rho_c 0.6 \times g + 0.3 \rho_c \times g + \rho_m \times 0.45 \times g - \rho_w \times g \times 0.45 = P_B$

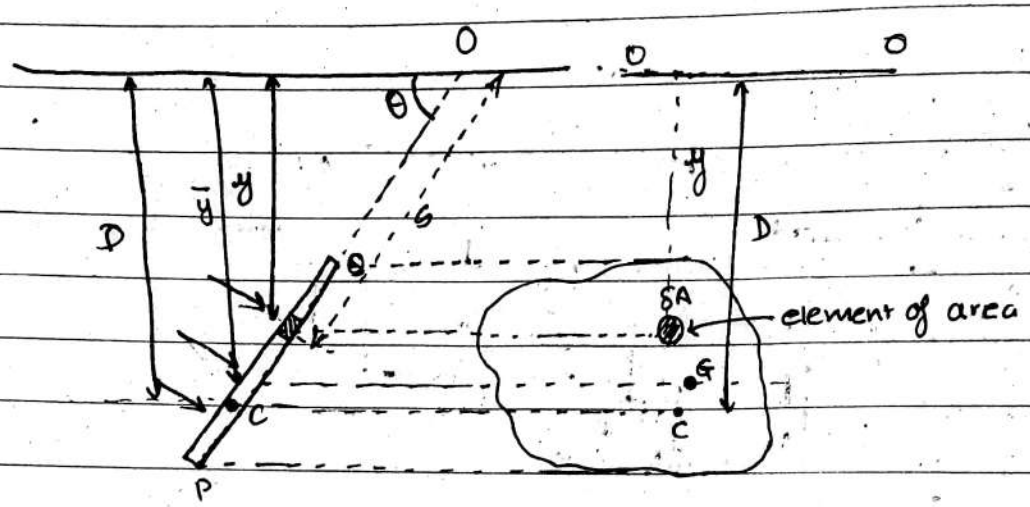
$\Rightarrow P_{air} + 2250 \times 0.6 \times 9.81 + 0.3 \times 2250 \times 9.81 + 13600 \times 0.45 \times 9.81 - 1000 \times 0.45 \times 9.81$

$= 117720$

$\Rightarrow P_{air} = 42.23$ KPa

Action of Fluid pressure on surface.

resultant force and centre of pressure on a plane surface immersed in a liquid.



intensity of pressure at elemental area = $\rho g y$ (P)
(where ρ is the density of liquid where it is immersed)

so, pressure force at elemental area (F), $dP = \rho g y \times dA$

since all the forces acting on the surface (such elemental areas) are parallel to each other. So

$$R = \Sigma dP = \Sigma \rho g y \, dA$$
$$= \rho g \Sigma y \, dA.$$

here summation y da; $\Sigma y dA$ = first moment of area of the surface on PQ about free surface of liquid.

So,

$$\Sigma y dA = A.\bar{y}$$

where A is the total area of the surface and $\bar{y}$ is the vertical distance of the centroid of the surface from the free surface of liquid

$$\boxed{\therefore R = \rho g A\bar{y}}$$

Location of resultant:

Centre of pressure (COP):

Let C be the point where the resultant acts upon whose depth is D.

using varignon's theorem.

moment of R about O = sum of moments of forces of all elements of area

~~$\Rightarrow \rho g A\bar{y}$~~ so Force of elemental area = $\rho g y \cdot dA$.

we know,

$$y = s\sin\theta.$$

so,

$$dP = dF = \rho g y s\sin\theta . dA.$$

now,

moment about of this force = $\rho g s \sin\theta . dA \times s$.

$$= \rho g s^2 \sin\theta . dA.$$

Sum of the moment = $\sum \rho g s^2 \sin\theta . dA$

$$= \rho g \sin\theta \sum s^2 dA.$$

Here, $\sum s^2 dA$ = Second moment of area of immersed surface about an axis in free surface. through 'O'. i.e.

$\sum s^2 dA = I_O$ (moment of inertia abt O)

$\therefore$ Sum of the moment = $\rho g \sin\theta . I_O$

So,

$$\frac{R}{\cancel{D}} \times \frac{D}{\sin\theta} = \rho g \sin\theta I_0.$$

$$\Rightarrow \cancel{\rho g} A\bar{y} \times \frac{D}{\sin\theta} = \cancel{\rho g} \sin\theta I_0.$$

$$\Rightarrow \boxed{D = \frac{I_0 \sin^2\theta}{A\bar{y}}}$$

according to parallel axis theorem,

$$I_0 = I_G + A \times \left(\frac{\bar{y}}{\sin\theta}\right)^2$$

So,

$$D = \cancel{I_G} + \frac{\cancel{A}\bar{y}^{\cancel{2}}}{\cancel{\sin^2\theta} \times \cancel{\bar{y}} \times \cancel{A}} \times \cancel{\sin^2\theta} + \frac{I_G \sin^2\theta}{A\bar{y}}$$

$$\boxed{D = \bar{y} + \frac{I_G \sin^2\theta}{A\bar{y}}}$$

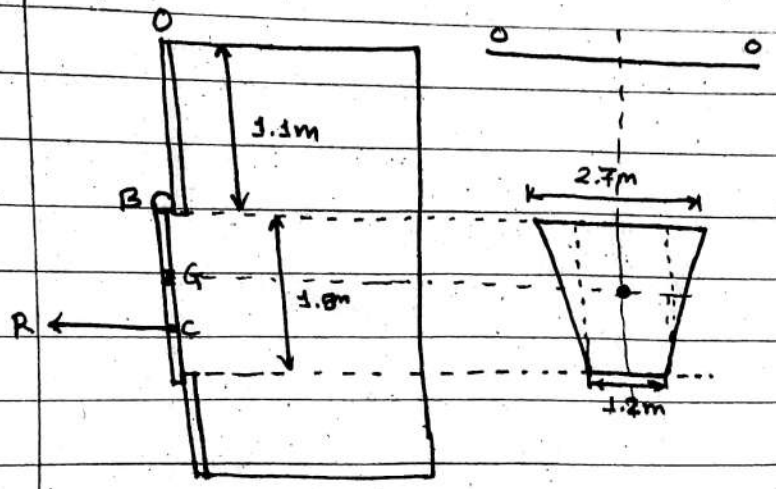
Since, I_G, A, $\bar{y}$, $\sin^2\theta$, never negative,

$D = \bar{y} +$ something > 0.

So, the resultant acts at maximum at $\bar{y}$

or in greater depth than $\bar{y}$.

* **Question**: (Trapezoidal sluice gate)



Ans: 38 kNm

Calculate the moment about the hinge B to kee the plate closed.

$R = \rho g A \bar{y}$

$= 1000 \times 9.81 \times 0.75 \times 1.8 \times \bar{y}$

$= 13243.5 \times \bar{y}$

$$\bar{y} = \frac{\left(\frac{2}{3} \times 1.8\right) \times \frac{1}{2} \times 0.75 \times 1.8 + \frac{1}{2} \times 0.75 \times 1.8 \times \left(\frac{2}{3} \times 1.8\right) + 0.9 \times 1.2 \times 1.8}{(0.75 \times 1.8) + 0.9 \times 1.2 \times 1.8}$$

$= (1.0819\ m + 1.1) = 2.1819$

$\therefore R = 13243.5 \times 2.1819$

$= 28896.$

$D_c = \frac{I_g}{A\bar{y}} \times \sin^2 90°$

$= \frac{I_g}{2.1819 \times 3.294}$

$I_g =$

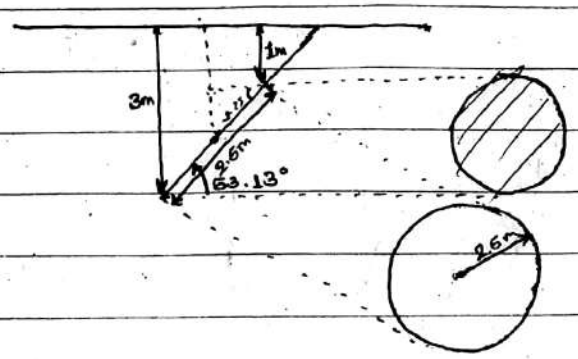
$R = \rho g A \bar{y}$

$D = \bar{y} + \frac{I_G}{A\bar{y}} \cdot \sin^2\theta$

Najra

* A circular plate, 2.5m in diameter is immersed in water. It's greatest and least depths, below the free surface being 3m and 1m respectively. Find, total pressure on one face of the plate and location of pressure:

→ Solution:



$\bar{y}$ is located from centre.

$\therefore \bar{y} = 1 + 1.25 \sin 53.13$

$= 2m$

$\therefore R = \rho g A \bar{y}$

$= \rho g \frac{\pi d^2}{4} \times 2$

$= 1000 \times 9.81 \times \frac{\pi (2.5)^2}{4} \times 2$

$= 96309.45\ N$

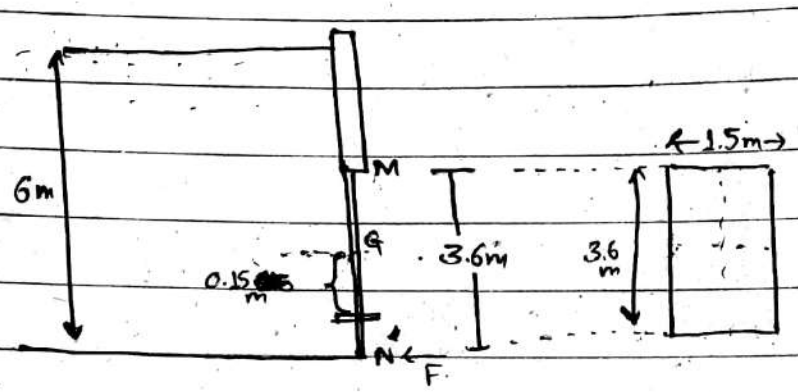
$D = \bar{y} + \frac{I_G}{A\bar{y}} \times \sin^2\theta$

$= 2 + \frac{\pi d^4 \times 4}{32 \times \pi d^2 \times 2} \times \sin^2(53.13°)$ $= 2 + \frac{1}{32} \times \sin^2(53.13)$

$= 2.0199\ m$

it is acted at the depth of 2.0199m

* A 3.6m ~~by~~ x 1.5m wide rectangular gate is vertical and is hinged at point 0.15m below centre of Gravity of the gate. Total depth of the water is 6m. what horizontal force must be applied at the bottom of the gate, to keep the gate closed.



$\bar{y} = 4.2m.$

$R = \rho g A \bar{y}$

$= (1000 \times 9.81 \times 3.6 \times 1.5 \times 4.2) N$

$= 222490.8 N.$

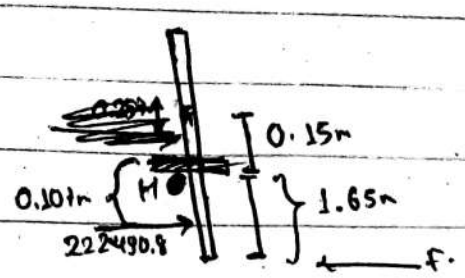
So,

$I_G = \frac{db^3}{12} = \frac{1.5 \times 3.6^3}{12} = 5.832 m$

$D = \bar{y} + \frac{I_G}{A\bar{y}}$

$= 4.2m + \frac{5.832}{3.6 \times 1.5 \times 4.2}$

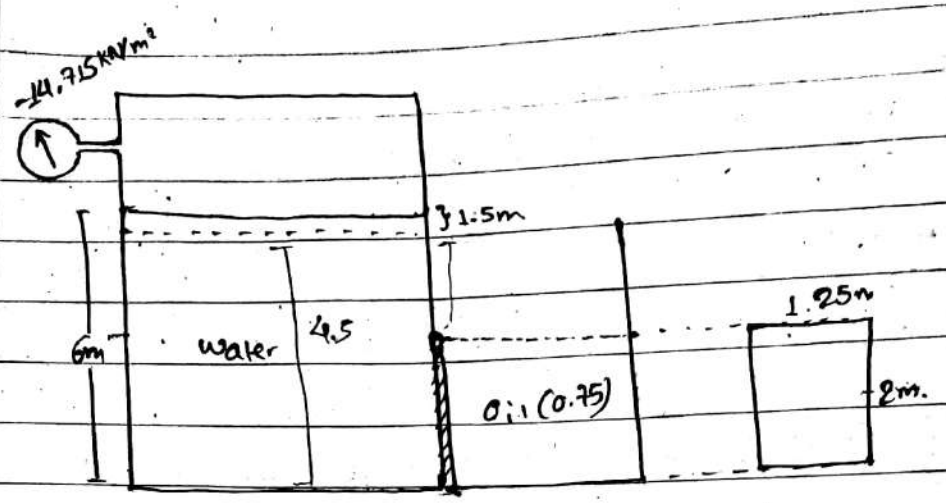
$= 4.457$



$\circlearrowleft + \sum M_{hinge} = 0$

$\Rightarrow$ ~~0.251 ×~~ $F \times 1.65 - 0.107 \times 222490.8 = 0$

$\Rightarrow F = 14428.19127\ N$ //



What horizontal force must be applied at Q to keep the gate closed.

$\bar{y} = 5m.$

$R = (\rho g \bar{y} + P_{air}) * A$

$= (1000 \times 9.81 \times 5 - (14.715 \times 1000)) * (1.25 * 2)$

$= 85837.5 N.$

OR,

equivalent level of water (replacing the air pressure)

or, $h = \frac{P}{\rho g} = -1.5m$

Same $\bar{y} = 5m - 1.5m = 3.5m$

$\therefore R = (\rho g \bar{y} A)$

$= 85837.5 N$

$D = \bar{y} + \frac{I_G}{A\bar{y}} \cdot \sin^2 90$

$= 3.5 + \frac{1.25 \times 2^3}{12 \times (1.25 \times 2) \times 3.5}$

$= 3.5952 m$

Due to oil,

$\bar{Y}_{oil} = 1m.$

$R = \rho g \bar{Y}_{oil} \times A$

$= 0.95 \times 1000 \times 9.81 \times 1 \times 1.25 \times 2$

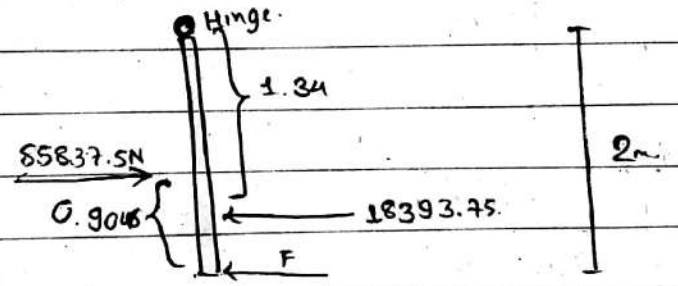
$= 18393.75\ N.$

$D = \bar{Y} + \frac{I_G}{A\bar{y}} \cdot \sin^2 90$

$= 1 + \frac{1.25 \times 2^3}{12 \times (1.25 \times 2) \times 1}$

$= 1.34m.$

FB.D:

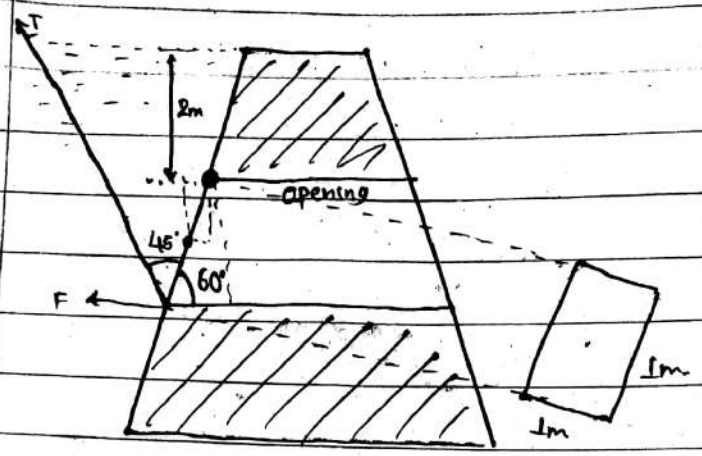


$\Rightarrow \circlearrowleft \sum M_{Hinge} = 0$

$\Rightarrow +18393.75 \times 1.34 - 85837.5(1.0952m) + F \times 2 = 0$

$\Rightarrow F = 34680.8025\ N$

*



An opening in a dam is closed by a plate 1m×1m which is hinged at upper horizontal edge as shown. Weight of the plate is 1.962 KN. Calculate tension T required to open the gate:

$$\bar{Y} = 2 + \sqrt{3}/4 = 2.433\,m$$

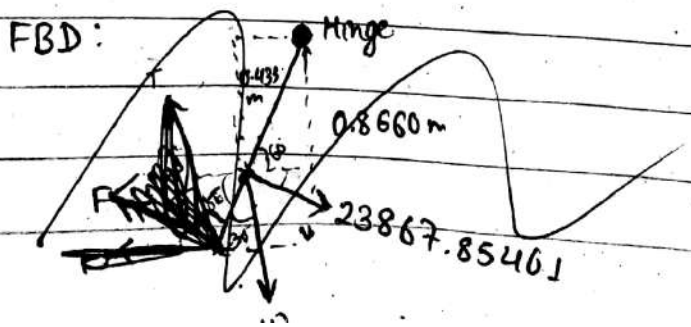
$$R = \rho g A \bar{y}$$
$$= 1000 \times 9.81 \times 1 \times 2.433$$
$$= 23867.85461\ N.$$

$$D = \bar{Y} + \frac{I_G}{A\bar{y}} \times \sin^2 60$$

$$= 2.433 + \frac{1 \times 1^3}{12 \times 2.433 \times 1 \times 4}$$

$$= 2.458\ m.$$

FBD:



$\circlearrowleft \Sigma M_{hinge} = 0$

$\Rightarrow$ ~~$-23867.8546 \times 0.433m + F \times 0.8660 = 0$~~

$\Rightarrow$ ~~$F = 11933.9273$~~.

$\Rightarrow -0.5 \times 23867.854 + F \times 1 - W \times \sin 30^\circ \times 0.5$

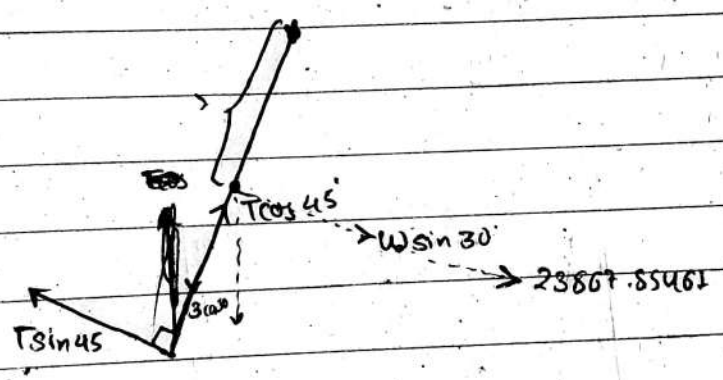
~~$F = 11$~~ $F = 13895.927m$

So, $F\cos 45$ is required.

i.e.

$F = 9825.90N$

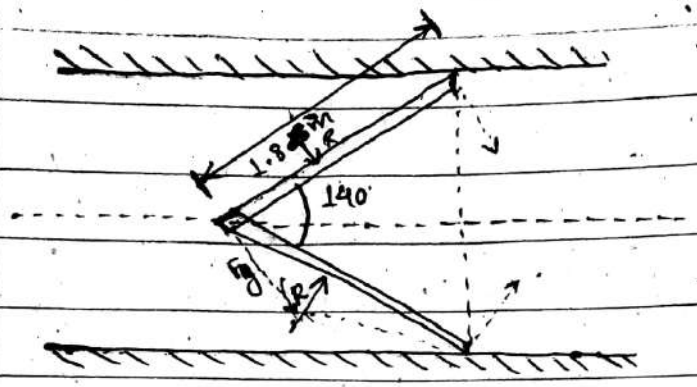
FBD :

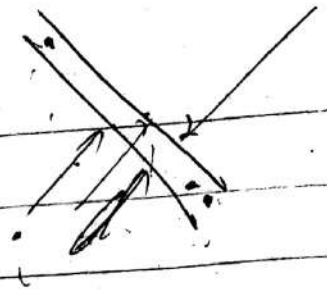


$-0.5 \times 23867.854 + T\sin 45 - W \times \sin 30^\circ \times 0.5 = 0$

$\therefore T = 19.65KN$

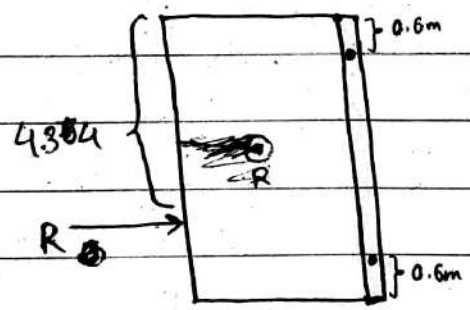
*





lock gates

The angle between the pair of log gates is 140° and each gate is 6m high and 1.8m wide. Supported on the hinges 0.6m from the top and bottom of the gate. If the depths of the water on upstream and downsheam sides are 5m and 1.5m respectively. Estimate reactions at the top and bottom hinges.



$R = \rho g \bar{Y} \times A$

$= 1000 \times 9.81 \times 2.5\,m \times 9$

~~317844 N = 204870N~~ 220500N

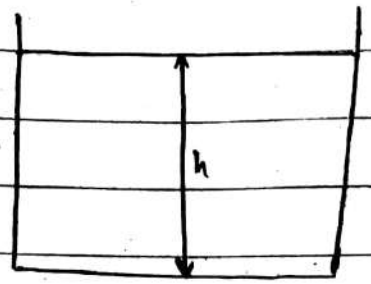
~~for two surface~~ $D = \bar{Y} + \frac{I_g}{\bar{Y} \cdot A} \times \sin^2\theta$

$= 2.5 + \frac{1.8 \times 53}{12 \times 2.5 \times (5 \times 1.8)}$

$= 3.34m$

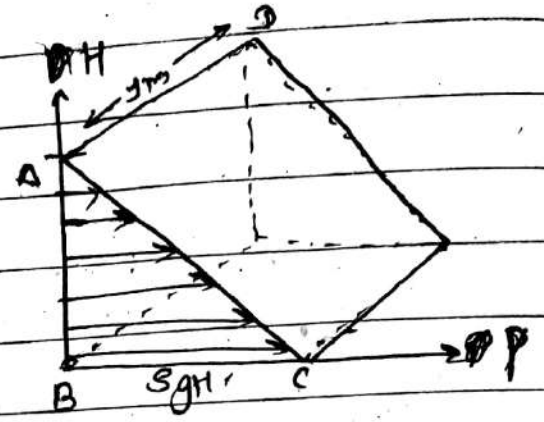
i.e. 4.34m from the top.

* Pressure diagram (method)



$P = \rho g h$

$\Rightarrow P \propto h$



It is the plot of intensity of pressure to the height.

resultant pressure force on the wall (R) = Area of ΔABC

$= \frac{1}{2} \rho g H^2$ (unit N/m)

if x m is AD, then, $\boxed{\frac{1}{2} \rho g H^2 \times x}$

centre of pressure D = centroid of pressure diagram

= centroid of ΔABC.

= $\frac{2}{3}H$ from free surface.

$\therefore R = \frac{\rho g H^2}{2} \times x$

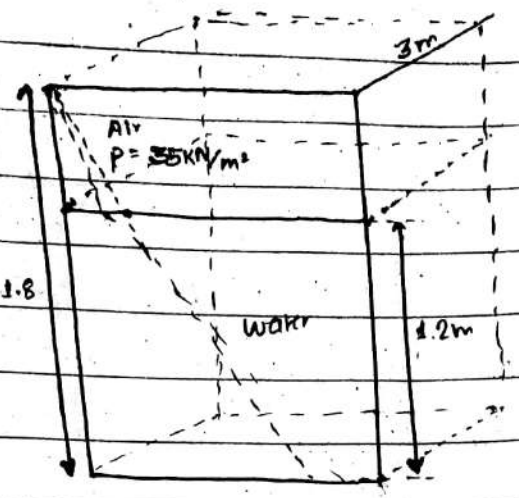
Proved

$D = \bar{y} + \frac{I_G}{\bar{y} \times A} \cdot \sin\theta$

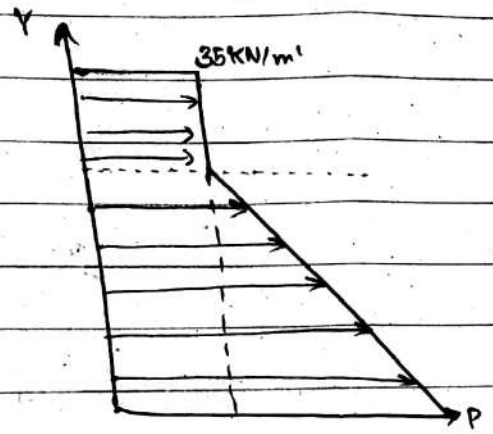
$= \frac{H}{2} + \frac{\frac{1 \cdot H^3}{12}}{(H \times 1)(H/2)}$

$= \frac{2}{3} H$.

*



using pressure force



intensity of ~~force~~ pressure force (R_{H_2O}) = Area of ~~ex~~ diagram

$= \frac{1}{2} \times \rho g H^2 \times 3m$

$= \frac{1}{2} \times 1000 \times 9.81 \times (1.2)^2 \times 3$

$= 21189.6$ N

pressure force (R_{air}) = $35 \times 0.6 \times 1000 \times 3$

= ~~630000N~~ 189000 N.

R_{air} position: centroid of rectangle

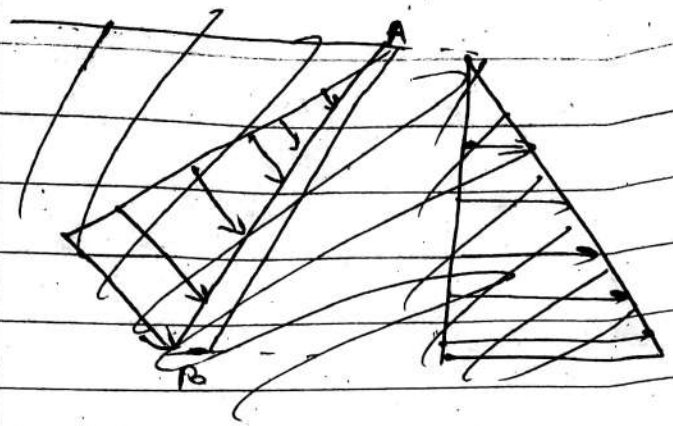
= 0.9 from free surface.

R_{water} position: 1.4 from free surface.

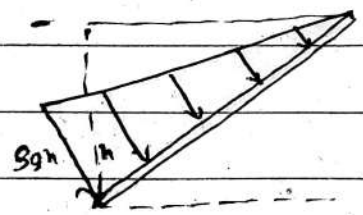
$R_{total} = R_{air} + R_{water}$

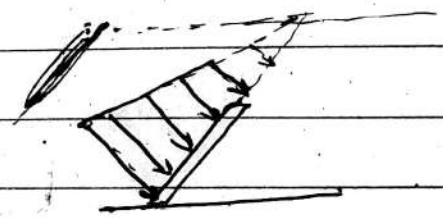
$= 210189.6$ N

$R \times x = R_{air} \times 0.9 + R_{H_2O} \times 0.4$, $\therefore x = 0.8m$ from bottom.

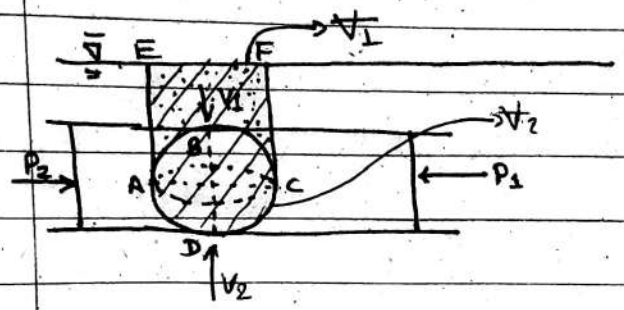


For inclined surfaces:





Buoyancy and upthrust



$P_1 = P_2$ but $V_1 < V_2$

this imbalance in vertical force is known as the buyancy and this force acts on the centroid (CG) of the object ~~di~~ submerged.

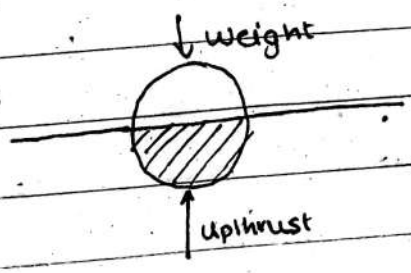
$$\boxed{V_2 - V_1 = F_{upthrust}}$$

$$V_2 = \rho g \forall_2$$
$$V_1 = \rho g \forall_1$$
$$\Rightarrow V_2 - V_1 = \rho g (\forall_2 - \forall_1)$$
$$= \rho g (\forall_{object})$$

$$\boxed{\therefore \rho g (\forall_{object}) = F_{upthrust}}$$

$$\boxed{\therefore \text{Weight of object} = F_{upthrust}}$$

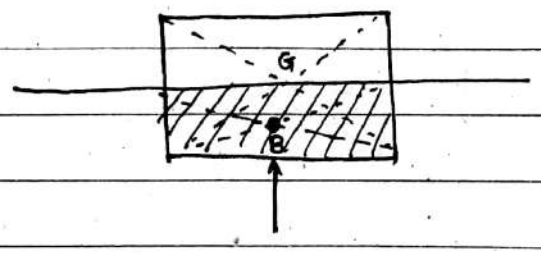
diagrametically:



equilibrium i.e. Weight of submerged object = upthrust

upthrust in body = upward force on lower surface ~~ABC~~ ADC
- downward force on upper surface ABC.
= weight of volume of fluid ADCFEA
- weight of volume of fluid ABCFEA
= weight of volume of ABCD.

∴ Upthrust = Weight of fluid displaced by the body and it acts through centroid of volume of fluid displaced. known as centre of buoyancy.



$R = \rho_{fluid} \times g \times V_{displaced}$ ------ (i)

and acts through centre of buoyancy B.

and $R = \rho_{object} \times g \times V_{object}$ ----- (ii)

$\Rightarrow \quad \rho_{fluid} \times g \times V_{displaced} = \rho_{object} \times g \times V_{object}$

$\Rightarrow \quad \frac{\rho_{fluid}}{\rho_{object}} = \frac{V_{object}}{V_{displaced}}$.

<u>Archimedes principle</u>: Upthrust = weight of liquid displaced.

weight of the displaced fluid = wt. of object in vaccuum - weight of object in fluid.

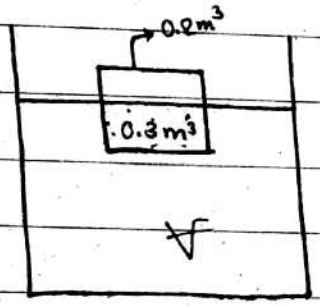
So, Buoyancy reduces apparent wt of the object that have sunk completely.

apparent ~~inc~~ immersed wt = wt of object - wt. of displaced fluid.

<u>note:</u>

In the case of floatation, weight of the object = wt. of the fluid displaced by the floating body = upthrust.

#



Volume of ~~water~~ fluid displaced = $0.3m^3$.

weight of volume of fluid displaced = $0.3 \times \rho_{fluid} \times g$

$= 0.3\rho_{fluid} \times g$.

this is equal to wt. of object (since it is upthrust)

~~Since~~ $V_{object} \times \rho_{object} \times g = 0.3\rho_{fluid} \times g$.

$\Rightarrow \frac{0.5}{0.3} = \frac{\rho_{fluid}}{\rho_{object}}$

$\Rightarrow \boxed{0.6 \times \rho_{fluid} = \rho_{object}}$

now, when the ice melts, its ~~volume~~ mass becomes:

~~0.5 x~~ ~~0.5~~ mass = $\frac{0.5}{600}$

~~when ice in~~ but mass is constant, so Volume of melted water = $\frac{0.5}{600} \times 1000 = \frac{1}{120 \times 10^{-5}}$

Volume of fluid only = V ~~0.3m³~~

Volume of ~~fluid ice~~ fluid with ice = $V + 0.3m^3$

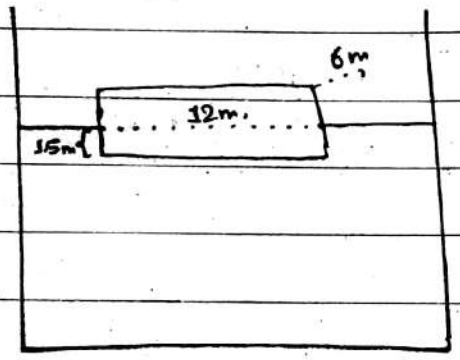
Volume of fluid with ice melting = $V + 5/9\ m^3$

A rectangular Pantoon has a width B = 6m, L = 12m and draught, D = 1.5m in fresh water (ρ = 1000 kg/m³). Calculate :

i) weight of the Pantoon.

ii) Its draught is sea water (1025 kg/m³).

iii) Load in KN, that can be supported by the pantoon in fresh water if maximum permissible draught is 2m.



volume of submerged part = 1.5 × 12 × 6.

= 108 m³.

~~weight~~ mass of liquid displaced = 108 × 1000

= 108000 kg.

∴ weight of object = weight of liquid displaced

= 1059480 N.

= 1059.48 KN.

now,

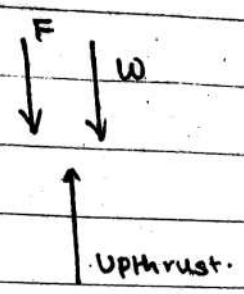
in sea level,

weight of pantoon = weight of liquid displaced

⇒ 1059480 = $V_{\text{pantoon displaced}}$ × 1025 × 9.81

⇒ 1059480 = D × 12 × 6 × 1025 × 9.81

∴ D = 1.46m



$\Rightarrow \quad 1059480 + F = (2 \times 12 \times 6) \times \rho_{fresh} \times 9.81$

$\Rightarrow \quad \therefore F = 353.16 \text{ KN}$

Determine density and volume of object that weight 3N in water and 4N in oil of SG 0.83.

$\Rightarrow$ Solution:

Apparent weight in water = 3N.

Actual weight - Upthrust = Apparent weight.

$\Rightarrow \quad W - V \times 1000 \times 9.81 = 3N. \quad \cdots (i)$

also,

$W - V \times 830 \times 9.81 = 4N \quad \cdots (ii)$

Solving these equations we get:

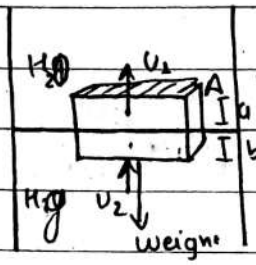
$W = 8.88 \text{ N}$

$V = 5.99 \times 10^{-4} m^3$

$\boxed{\therefore W = 8.88 \text{ N}}$

$\boxed{\rho = 1511.18 \text{ Kg/m}^3}$

A block of steel floats at the mercury water interface. What will be the ratio of a and b if sp. gravity of steel block is 7.85.



$\Rightarrow$ Weight of steel block $= U_1 + U_2$

~~$\Rightarrow$ weight of steel~~

$\Rightarrow (a+b) \times \cancel{A} \times S_{steel} \times \cancel{g} = \cancel{A} \times (a \times S_{H_2O} \times \cancel{g}$

$+ \cancel{A} \times b \times S_{Hg} \times \cancel{g}$.

$\Rightarrow (a+b)\, 7.85 = a + b \times 13.6$.

$\Rightarrow 7.85a + 7.85b = a + 13.6b$

$\Rightarrow 5.75b = 6.85a$.

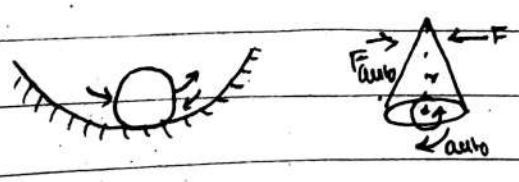
$\Rightarrow \boxed{\frac{a}{b} = 0.839}$ //

Types of Equilibrium

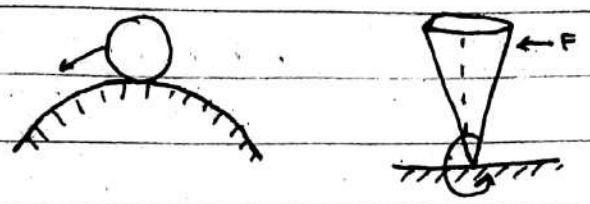
i) Stable equilibrium

ii) Unstable equilibrium

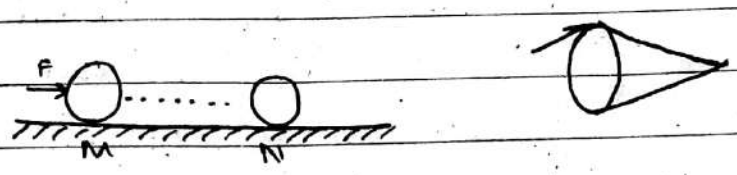
iii) Neutral equilibrium.



A small displacement from the equilibrium position generates a force producing a righting moment bending to restore the body to its equilibrium position. is called stable equilibrium.



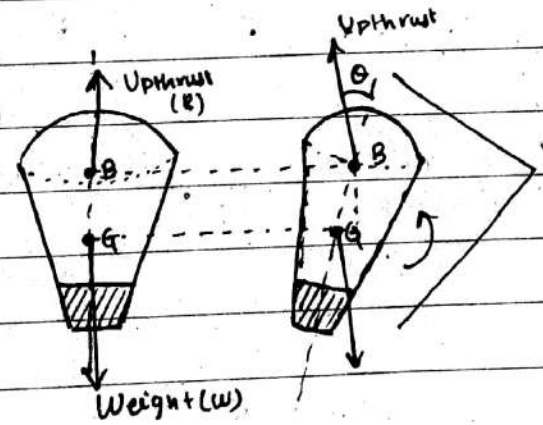
A small displacement produces an overturning moment tending to displace the body further from its equilibrium position is called unstable equilibrium.



Body remains in rest at any position to which it is displaced is called neutral equilibrium

Stability of submerged body:

(Hot air balloon, submarine)



This couple neutralizes the Hot air balloon in its original state.

but if B was down and G was up, the overturning occurs.

Condition for stability of submerged body:

i) The centre of gravity must be below the centre of buoyancy.

For body totally immersed in the fluid, weight $W = mg$ acts through CG of the body while upthrust acts through centroid of the body B (i.e centre of buoyancy). Whatever be the orientation of the body, these two points remain in same position relative to the body. So, from the figure, A small angular displacement,

θ from the equilibrium position will generate a moment

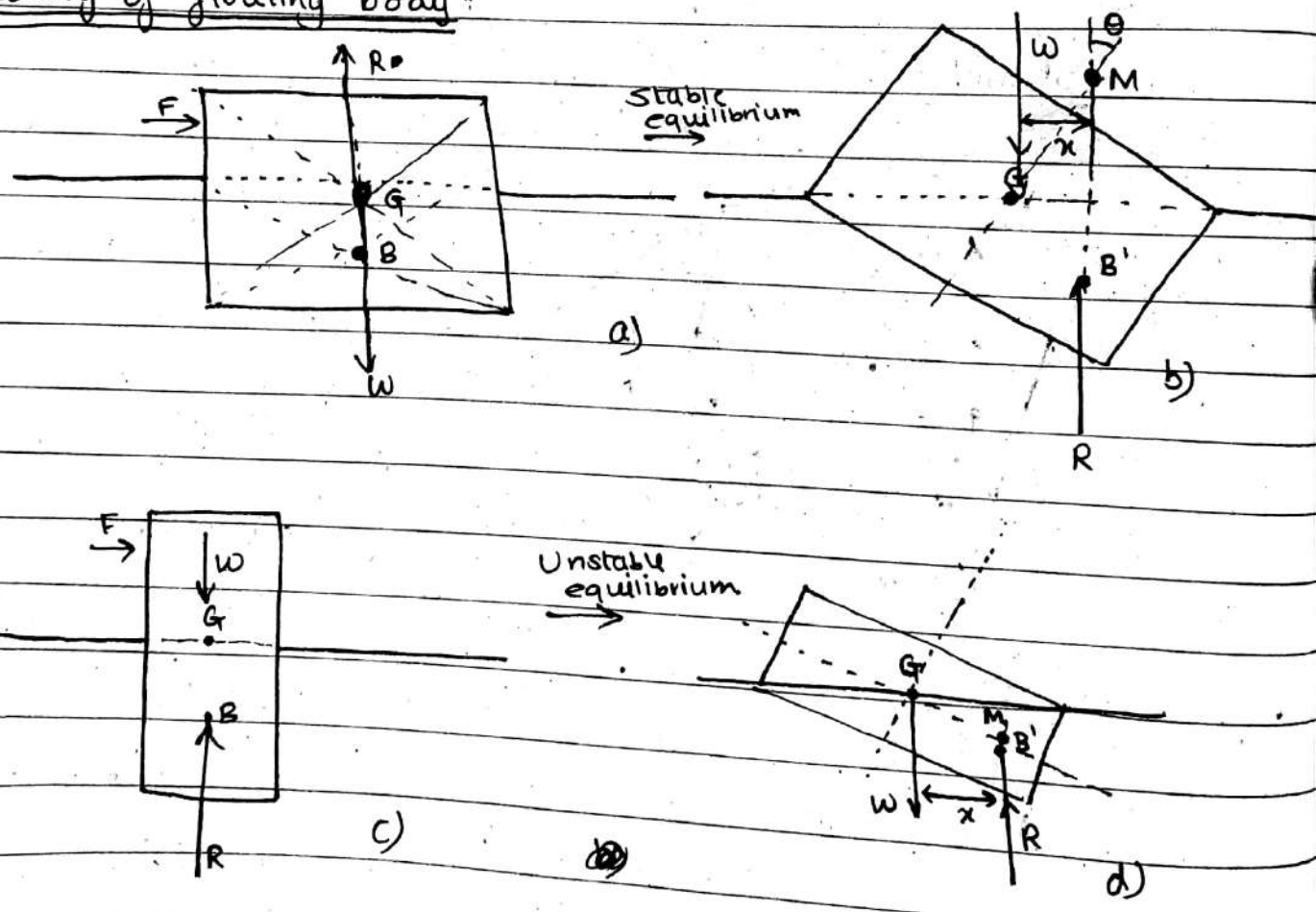
$W * BG * \theta$ (for small angle of tilt, $\sin\theta \approx \tan\theta \approx \theta$)

So,

i) If centre of gravity is below the centre of buoyancy, this moment will be righting moment and body will return to its equilibrium position i.e. stable equilibrium case.

ii) If CG is above the centre of buoyancy, the moment will be negative and a overturning moment of same magnitude will be produced and the body is in unstable equilibrium.

Stability of floating body:



When the body is displaced through an angle 'θ', W continues to act through G, the volume of liquid remains constant, since $R = W$ but shape of this volume changes and its centre of buoyancy B moves relative to body from B to B'. Since, R and W are no longer in straight line, a turning point proportional to, $W \times \theta$ is produced which is in figure b, a righting moment and figure d is an overturning moment. If M is a point at which the line of action of upthrust R cuts the original vertical through CG G then $x = GM \times \theta$

(provided that angle of tilt is very small)

this point M is called **METACENTER** and $\overline{GM}$ is known as **METACENTRIC HEIGHT** and $\overline{BM}$ is **METACENTRIC RADIUS**.

i) If M lies above the G, figure (b), a righting moment $W \times \overline{GM} \times \theta$ is produced, equilibrium is stable. $\overline{GM}$ = positive.

ii) If M lies below the G, an overturning moment, $W \times \overline{GM} \times \theta$ is produced, equilibrium is unstable and $\overline{GM}$ is negative.

iii) If M coincides with G, the equilibrium is neutral and $\overline{GM} = 0$

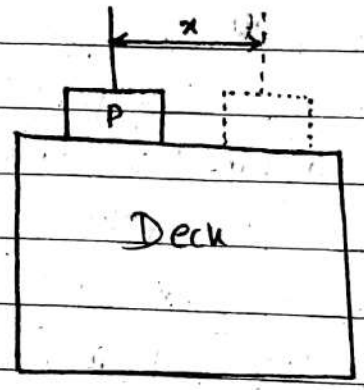
$\therefore$ For stability, $GM \geq 0$.

$\Rightarrow BM - BG \geq 0$

$\Rightarrow BM \geq BG$.

Determination of metacentric height

i) **By experimental method:**



If angle of tilt 'θ' caused by moving a load 'p' a known distance 'x' across the deck is measured.

Overturning moment due to movement of load 'p' equals $p \times x$ ---- (i)

Similarly, if $\overline{GM}$ is metacentric height

righting moment produced for equilibrium = $W \times \overline{GM} \times \theta$ --- (ii)

for equilibrium,

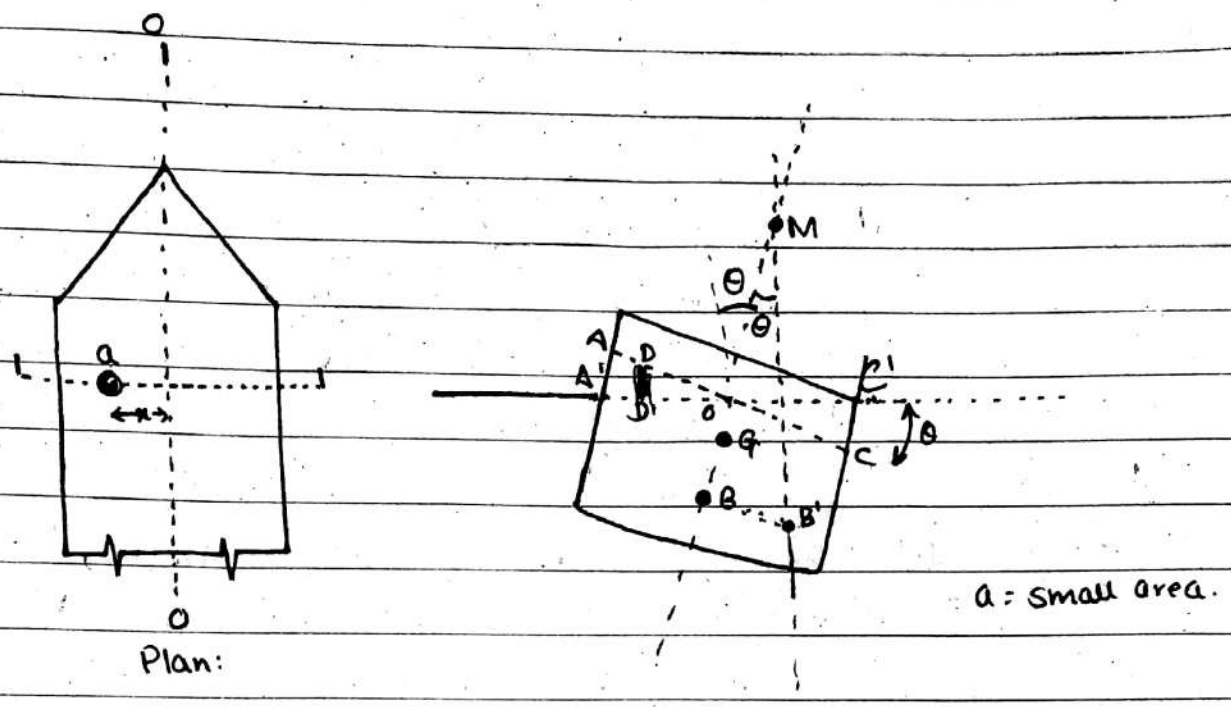
$$p \times x = W \times \overline{GM} \times \theta.$$

$$\therefore \overline{GM} = \frac{Px}{W\theta}$$

W = weight of the deck + load.

the true metacentric height is value of GM when $\theta \to 0$

ii) By analytical method:



i) AC is original plane of water line. If vessel is tilted through an small angle θ, centre of buoyancy moves from B to B' as a result ~~in~~ of alteration in shape of volume of displaced fluid. Now, A'C' is the water line plane in the displaced ~~direction~~ position. So, for small angle of tilt. $BM = \frac{BB'}{\theta}$ .... (i)

ii) Movement of B to B' is the result of removal of volume of fluid corresponding to wedge AOA' and addition of wedge COC'.

Now, wt. of small ~~AOA'~~ volume A. $= \rho g \forall_{AOA'}$

$= \rho g (DD' \times a) = \rho g (x\theta . a) = \rho g \theta . ax.$

so, total weight of AOA' $= \sum_{0}^{AO} \rho g \theta . ax = \rho g \theta \sum_{x=0}^{AO} ax.$

total weight of COC' $= \sum_{x=0}^{CO} \rho g \theta . ax = \rho g \theta \sum_{x=0}^{CO} ax.$

and

$$\rho g \theta \sum_{x=0}^{AO} ax = \rho g \theta \sum_{x=0}^{CO} ax.$$

$$\Rightarrow \sum ax = 0$$

so here $\sum ax$ = first moment of area and first moment of area about waterline plane about OO' is zero i.e ~~if~~ OO' axis must pass through the centroid of waterline plane.

iii) Couple produced by movement of wedge AOA' to COC'
= Couple produced due to movement of R from B to B'.

moment about OO' of the weight of the fluid swept by area a.
$= (\rho g \theta \, ax) \cdot x$.

total moment $= \sum \rho g \theta ax^2 = \rho g \theta \sum ax^2$

$\sum ax^2$ = Second moment of area ~~abt~~ of waterline plane about OO' axis.

moment due to movement of R = $R \times BB'$

$= \rho_{fluid} g \, V_{displaced} \times BB'$

So, $\rho g \theta \sum ax^2 = \rho_{fluid} g \, V_{displaced} \times BB'$

$$\boxed{\therefore BB' = \frac{\theta I}{V_{displaced}}} \quad \text{--- (ii)}$$

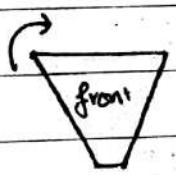
Substituting this value in ①,

$$\boxed{BM = \frac{BB'}{\theta} = \frac{I_{OO}}{\forall_{Displaced}}}$$

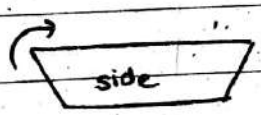
$$GM = BM - BG$$

$$\boxed{GM = \frac{I_{OO}}{\forall_{displaced}} - BG}$$

Question ?

i) Ship stable 

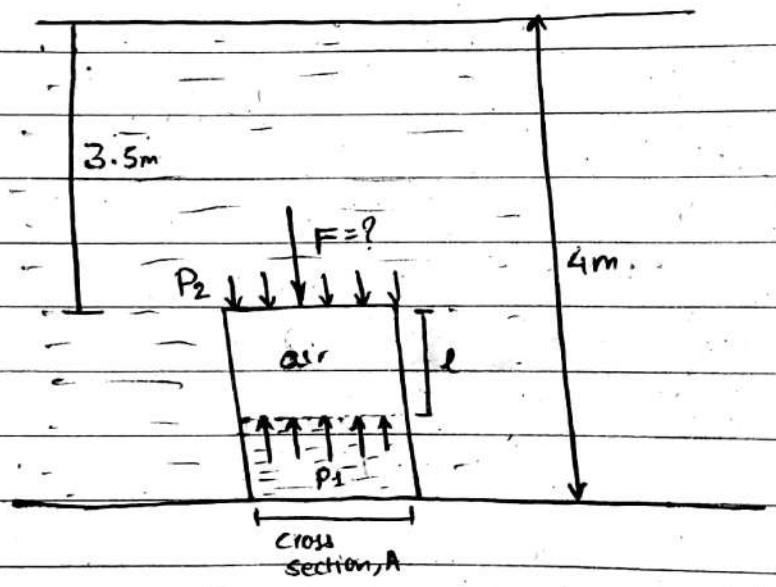
less

ship stable 

more, why?

* An empty cylindrical bucket 30cm in diameter and 50cm long whose wall thickness and weight can be considered as negligible is forced open in first into water until its lower edge is 4m below the surface. What force will be required to maintain this position. Assuming trapped air to remains at constant temperature during the entire ~~proc~~ process.

→ Solution



$P_1 A - P_2 A = F$ ..... (i)

everything is known except P_1,

$P_1 = \rho g (3.5 + l)$

so l is the thing that we need.

$P_2 = \rho g\, 3.5 = 1000 \times 9.81 \times 3.5 = 34335$ Pa.

$A = \pi r^2 = \pi \times (0.15^2) = 0.07068$

Since, the air here is happed in constant pressure,

$$P_A V_A = P_B V_B$$

initially,

Pressure = P_{atm}.

Volume of air = volume of a bucket.

$= 0.07068 \times 0.5$

$= \frac{9}{800}\pi$.

finally,

Pressure = $(P_{atm} + P_1)$

$= (P_{atm} + \rho g(3.5+l))$

Volume = $0.07068 \times l$.

So,

$$P_{atm} \times \frac{9}{800}\pi = [P_{atm} + \rho g(3.5+l)] \times 0.07068\, l$$

$$\Rightarrow 101000 \times \frac{9}{800}\pi = [101000 + 1000 \times 9.81 (3.5+l)] \times 0.07068\, l$$

$l = 0.03635$ m

So,

from eq^n ①,

$P_1 - P_2 (A) = F$

$\Rightarrow (37900.935 - 34335)(0.07068) = F$

$\therefore F = 252.04$ N

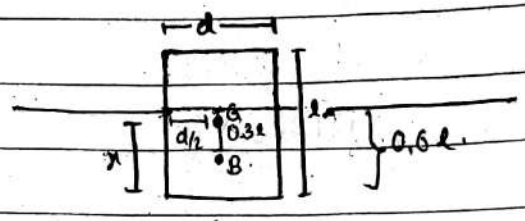
OR,

upthrust = weight of the water displaced by the object

= volume of air $\times \rho$ water $\times 9.81$.

= 252.04 N

* A uniform wooden cylinder has RD = 0.6. Determine ratio of diameter to length so that it will just float upright in the water.



Volume of water displaced = $\frac{\pi d^2}{4} \times x$.

weight of same volume of water = $\frac{\pi d^2}{4} \times x \times 1000 \times 9.81$

but,

$$\cancel{\frac{\pi d^2}{4}} \times l \times 0.6 \times \cancel{9.81} = \cancel{\frac{\pi d^2}{4}} \times x \times 1 \times \cancel{9.81}$$

$$\therefore \quad 0.6l = x$$

So,

Centre of buyoncy = centroid of volume of wood submerged

$$\cancel{\left(\frac{0.6l}{2}, \frac{d}{2}\right)} \quad \left(\frac{d}{2}, 0.3l\right)$$

cetre of gravity = centroid of volume of whole wood

$$= \left(\frac{d}{2}, l/2\right)$$

Since,

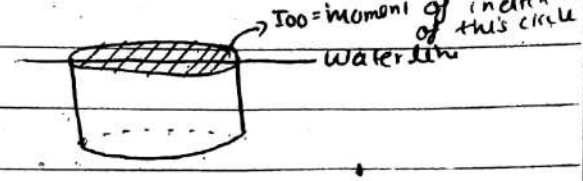
$$BG = l/2 - 0.3l$$

$$= 0.2l$$

$$BM = \frac{I_{OO}}{\forall_{displaced}}$$ (moment of inertia about water line of the plane)

$$= \frac{\cancel{\pi} d^2 \times u}{64 \times (0.6\ell) \times \cancel{\pi} d^2}$$

$$= \frac{d^2}{9.6\ell}$$



For stability,

$$BM - BG \geq 0 \quad (\because GM \geq 0)$$

$$\Rightarrow \frac{d^2}{9.6\ell} - 0.2\ell = 0$$

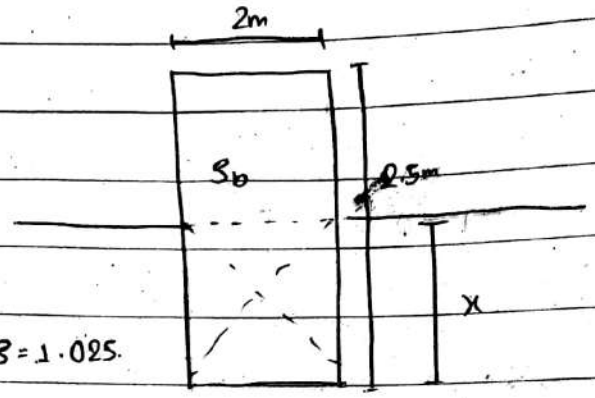
$$\Rightarrow \frac{d^2}{9.6\ell^2} = 0.2\ell$$

$$\Rightarrow \frac{d^2}{\ell^2} = 1.92$$

$$\Rightarrow \boxed{\frac{d}{\ell} = 1.3856}$$

* A cylindrical buoy 2m in diameter, 2.5m high, weight 18kN. Examine wheather the buoy will or will not float with its axis vertical is sea water having sp. gravity 1.025.

→ Solution.



$$\rho_b = \frac{18000 \times 4}{9.81 \times \pi \times 4 \times 2.5} = 233.6219 \text{ kg/m}^3$$

now,

volume of liquid displaced $= \frac{\pi \times 4}{4} \times x = \pi x$.

weight of liquid of same volume $= \pi x \times 1025 \times 9.81$.

$= 31589.499x$.

if float, $31589.499x = 18000$ $= 0.5698$ m.

centre of buoyancy $(B) = (1, x/2)$

centre of gravity $(G) = (1, 1.25)$

so,

$$BG = 1.25 - x/2 = \frac{2.5 - x}{2}$$

$$BM = \frac{I_{00}}{V_{displaced}} = \frac{\pi \times 2^4 \times 4}{64 \times \pi \times 4 \times x} = \frac{1}{16x}$$

So,

$GM \geq 0$ ∴ $BM - BG > 0$

is the condition reqᵃ for floatation

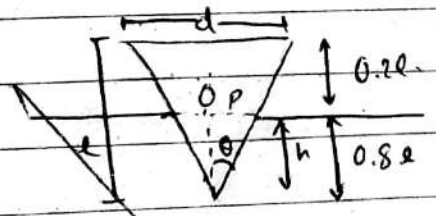
But,

$$BM - BG = \frac{I}{V_x} - \frac{2.5 - x}{2}$$

$$= -0.8554 < 0$$

∴ the body doesn't float.

* A cone floats in water with vertex downward. If sp. gravity of cone material is 0.8. Find the least apex angle for cone for stable equilibrium.



$\sin\theta = P/h$

Solution,

centre of buoyancy $= \frac{1}{3} \times h$ from O

$= h/3 = \frac{0.8l}{3} = 0.928\,l$

centre of gravity $= \frac{1}{3} l = 0.07\,l$

$$BG = \frac{l}{3} - \frac{0.8l}{3} = \frac{0.5l}{3}$$

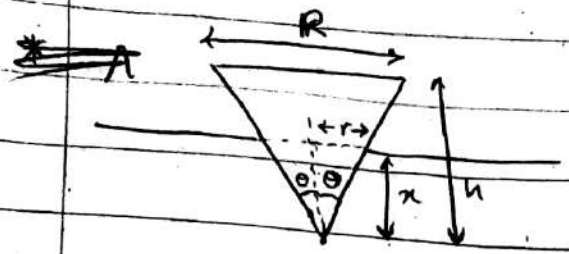
$$BM = \frac{I_{oo}}{V_{displaced}} = \frac{1}{3}\pi(h\sin\theta) = 0.8\,l\sin\theta =$$

$$\frac{1}{3}\pi \times (h\sin\theta)^2 \times h \times 0.8 \times 9.81$$

$$= \pi \times 9.81 \times \frac{1}{3}\pi l^2 \times (l\sin\theta)$$

$$\Rightarrow h^3\sin^2\theta \times 0.5 = l^3\sin^2\theta$$

$$h = 0.5\,l$$



Let the figure shown be considered.

volume of liquid displaced $= \frac{1}{3}\pi \times r^2 \times x$.

but $r = x\sin\theta$. So,

$$\forall_{displaced} = \frac{1}{3}\pi x^3 \sin^2\theta$$

Also, for floatation,

Weight = Upthrust = weight of liquid displaced

$$\Rightarrow \frac{1}{3}\pi R^2 h \times 0.8 \times 9.81 = \frac{1}{3}\pi x^3 \sin^2\theta \times 1 \times 9.81$$

$$\Rightarrow \cancel{\frac{1}{3}\pi} h^3 \cancel{\sin^2\theta} \times 0.8 \times \cancel{9.81} = \cancel{\frac{1}{3}\pi} x^3 \cancel{\sin^2\theta} \times \cancel{9.81}$$

$$\Rightarrow h^3 \times 0.8 = x^3.$$

minimum condition, $\boxed{\therefore x = \sqrt[3]{0.8}\, h}$

For stable equilibrium of a floating body = GM = 0

$$\Rightarrow BM - BG = 0 \quad \text{--- (i)}$$

here, B is located in $\frac{2}{3}x$ from the ~~water line~~ bottom i.e. $\frac{2}{3}(0.8)^{1/3}h$

G is located in $\frac{2}{3}h$ from the bottom line So,

$$BG = \frac{2}{3}h - \frac{2}{3} \times (0.8)^{1/3} h$$

$$= 0.023894h \times 2$$

$$BM = \frac{I_{oo}}{\forall_{displaced}} = \frac{\pi r^4}{4} \times \frac{1}{(\frac{1}{3}\pi x^3 \sin^2\theta)}$$

$$= \frac{x x^4 \sin^4\theta \times 3}{4 \pi x^3 \sin^2\theta}$$

$$= \frac{3x^2 \sin^2\theta}{4} = \frac{3x^2 \sin^2\theta}{4}$$

$$= \frac{3}{4}(0.8)^{2/3} h^2 \sin^2\theta$$

$$= \frac{3}{4}(0.8)^{1/3} h \sin^2\theta$$

putting these values in eq^n (i),

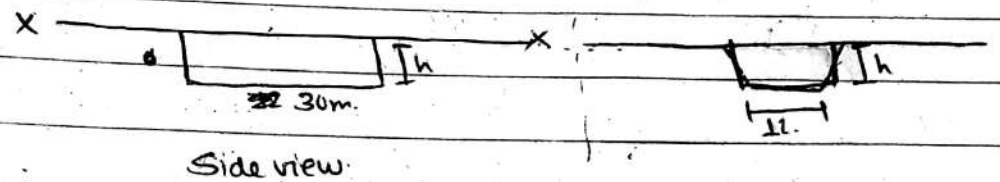
we get:

$$\frac{3}{4}(0.8)^{1/3} h \sin^2\theta = \frac{1}{3}h\left(1 - (0.8)^{1/3}\right).$$

$$\theta = 15.1880^\circ$$

$\therefore$ apex angle $= 2\theta = 30.376^\circ$

* A ship 12m wide at 30m long at the water line has a displacement of 6000 KN. The centre of gravity is 40 cm below the water line. And centre of bugoncy is 150 cm below the ~~water line~~ G. Determine metacentric height for rolling and pitching axis of the ship. Comment on your result.

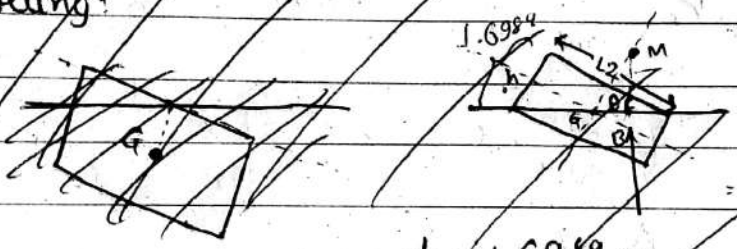


Side view.

~~now,~~

~~BG = 0.4 (1.5 - 0.4) = 1.1m~~

~~On rolling:~~



~~$\Rightarrow \theta = 8.1892°$,~~

~~$h = 1.6989$ m,~~

Solution, $BG = 150$ cm $\Rightarrow h/2 = 150$ cm $\Rightarrow h = 300$ cm

$$BM \text{ for } \cancel{\text{rolling}}\ \text{pitching} = \frac{I_{oo}}{\forall_{displaced}} = \frac{30 \times 10^3}{12 \times (12 \times 30 \times 3)} = \cancel{2.3148}\ 4$$

again,

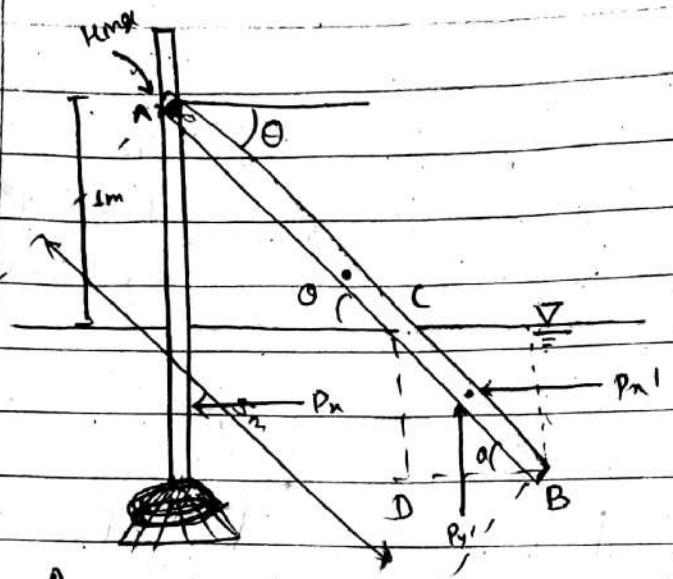
$$BM \text{ for } \cancel{\text{rolling}}\ \text{pitching} \Rightarrow \frac{I_{oo}}{\forall_{displaced}} = \frac{12 \times 30^3}{12(12 \times 30 \times 3)} = 25.$$

So, $GM = BM - BG = 4 - 1.5 = 2.5$ m,

& $25 - 1.5 = 23.5$ m.

So ship is more stable while ~~rolling~~ pitching than in rolling because the value of metacentric height is greater in pitching due to the dimensions of the ship.

*



A woodem beam 200mm x 200mm in cross section and 5m long is hinged at one end and floats on water in equilibrium as shown. Find the inclination θ if the SG of the wood is 0.6.

~~$\frac{1}{\sin\theta} = AC$~~

~~$\left(5 - \frac{1}{\sin\theta}\right) = CB$~~

~~$CD = \left(5 - \frac{1}{\sin\theta}\right)\sin\theta$~~

~~Area =~~

$AC = 1/\sin\theta$

So, $BC = \left(5 - 1/\sin\theta\right)$

Wt of beam $= 0.6 \times 9.81 \times 1000 \times (0.2 \times 0.2 \times 5)$

$= 11772 N$

Upthrust $= \left[0.2 \times 0.2 \times \left(5 - 1/\sin\theta\right)\right] \times 1000 \times 9.81$

$= 392.4(5 - \operatorname{cosec}\theta)$

So,

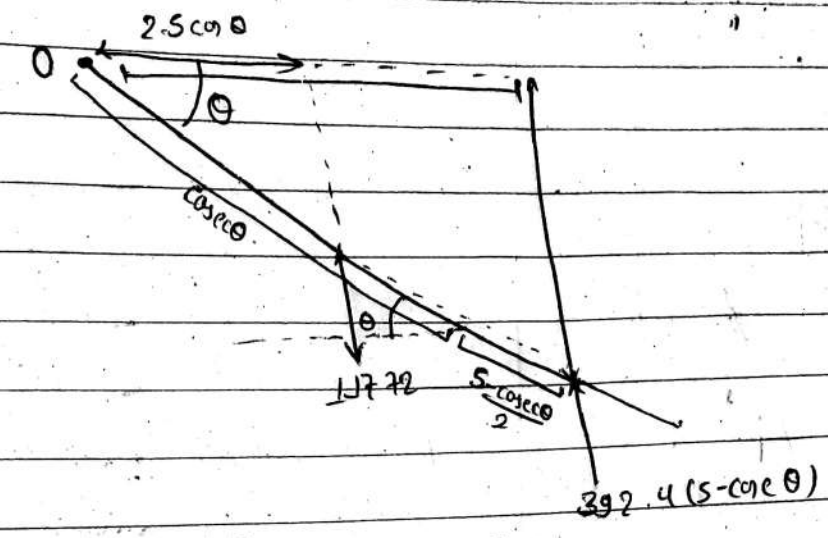
application of upthrust $= \frac{(5 - \operatorname{cosec}\theta)}{2}$ along the beam from free surface

~~$= \frac{5 - \operatorname{cosec}\theta}{2}$~~

application of weight $= 2.5m$ along the beam

~~$= 2.5\sin\theta \perp^r$~~ $= 2.5\cos\theta \perp^r$.

So, FBD.



$\circlearrowleft + \; \sum M_O = 0$ [For equilibrium]

$\Rightarrow 117.72 \times 2.5\cos\theta = 392.4\,(5-\operatorname{cosec}\theta) * \left(\frac{\operatorname{cosec}\theta - 5 - \operatorname{cosec}\theta}{2}\right)\cos\theta$

$\Rightarrow 117.72 \times 2.5\cos\theta = 392.4\,(5-\operatorname{cosec}\theta) * \frac{(3\operatorname{cosec}\theta - 5)\cos\theta}{2}$

$\Rightarrow 1.5\,\cancel{\cos\theta} = \left(5 - \frac{1}{\sin\theta}\right) * (3 - 5\sin\theta)\frac{\cancel{\cos\theta}}{\sin\theta}$

$\Rightarrow 1.5 = \frac{(5\sin\theta - 1)(3 - 5\sin\theta)}{\sin^2\theta}$

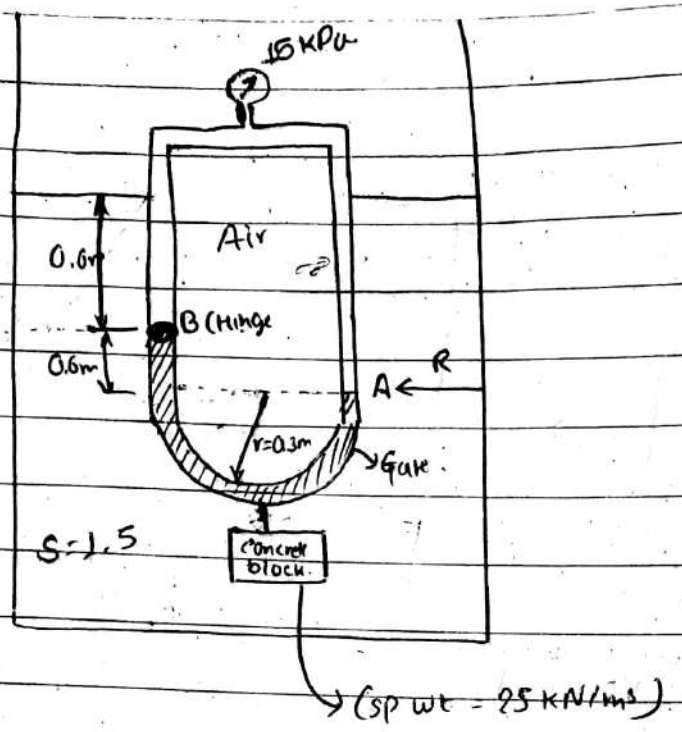
$\Rightarrow 1.5\sin^2\theta = 15\sin\theta - 25\sin^2\theta - 3 + 5\sin\theta$

$\Rightarrow 26.5\sin^2\theta - 20\sin\theta + 3 = 0$

$\sin\theta = 0.54821 \Rightarrow \theta = 33.244°$

or, $\sin\theta = 0.20650 \Rightarrow \theta = 11.9173°$

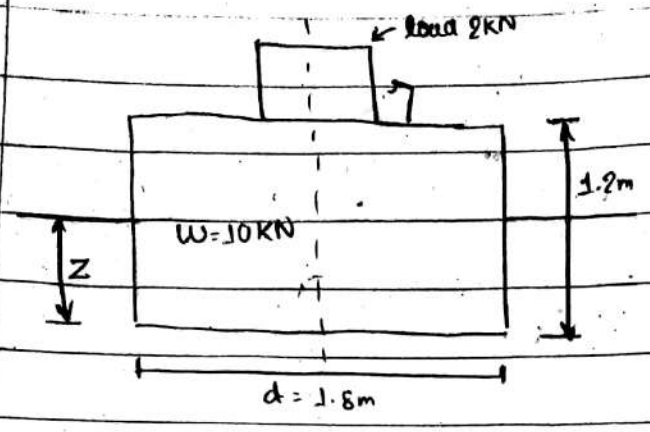
*



Determine required volume of the concrete block so that reaction of the gate on partition A is zero. The gate is 0.75 m wide with negligible weight and hinge is smooth.

($\forall = 0.3358 m^3$)

*



A cylindrical buoy 1.8m in diameter and 1.2m high and weighing 10KN floats in salt water of density 1025 kg/m³. It's CG, 0.45 m from the bottom. If a load of 2KN is placed on the top, find maximum height of CG of this load above the bottom if buoy is to remain in stable equilibrium.

Solution:

Weight of the buoy = weight of the salt water displaced

$\Rightarrow 12000 = 1025 \times 9.81 \times \frac{\pi \times 1.8^2}{4} \times z.$

$z = 0.4689 m$

~~BG~~ centre of buyoncy (B) = $\frac{0.4689}{2}$ m.

$= 0.23448$ m.

~~Cetre of gravity (CG) (G)~~ =

For stable equilibrium

$BM = BG' = \frac{I_{00}}{V_{displaced}}$

$\Rightarrow \frac{\pi \times 1.8^4 \times 4}{16 \times \pi \times 1.8^2 \times (0.4689)} = 0.4318$

but,

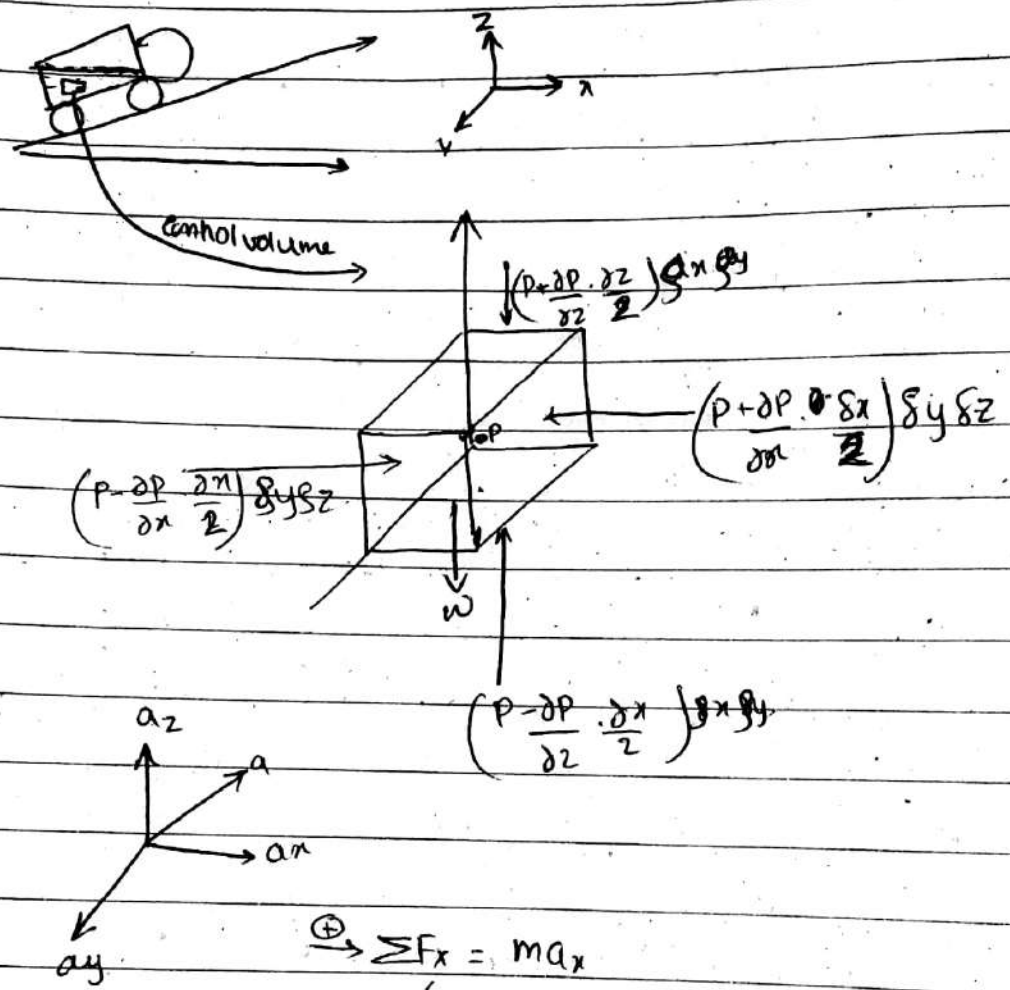
$$\left(0.45 \times \frac{10000}{9.81}\right) + \left(\frac{2000}{9.81} \times x\right) = \left(0.4318 + 2/2\right) \times \frac{12000}{9.81}$$

$$\Rightarrow \quad 458.715 + 203.87x = 7995/9.81$$

$$x = 1.747m$$

Chapter:

Liquid in relative equilibrium



$$\xrightarrow{\oplus} \sum F_x = m a_x$$

$$\Rightarrow -\left(p+\frac{\partial p}{\partial x}\cdot\frac{\delta x}{2}\right)\delta y\,\delta z + \left(p-\frac{\partial p}{\partial x}\,\frac{\partial x}{2}\right)\delta y\,\delta z = m a_x$$

$$\Rightarrow \cancel{-\frac{\partial p}{\partial x}\,\frac{\delta x}{2} - \frac{\partial p}{\partial x}\,\frac{\delta x}{2} \quad \frac{\partial p}{\partial x}}$$

$$\Rightarrow -\frac{\partial p}{\partial x}(dx\,dy\,dz) = \rho\,(dx\,dy\,dz)\,a_x$$

$$\boxed{\therefore \frac{\partial p}{\partial x} = -\rho a_x}$$

Similarly in y direction,

$$\frac{\partial p}{\partial y} = -\rho a_y$$

Similarly in z direction:

$$\left(p - \frac{\partial p}{\partial z}\cdot\frac{dz}{2}\right) dx\,dy - \left(p + \frac{\partial p}{\partial z}\cdot\frac{dz}{2}\right) dx\,dy - \rho g\,dx\,dy\,dz = \rho \times dx\,dy\,dz\,a_z$$

$$\Rightarrow \frac{-\partial p}{\partial z}\cancel{(dx\,dy\,dz)} - \rho\,\cancel{(dx\,dy\,dz)} = \rho\,\cancel{dx\,dy\,dz}\,a_z$$

$$\Rightarrow \frac{-\partial p}{\partial z} - \rho g = \rho a_z$$

$$\Rightarrow \frac{\partial p}{\partial z} = \rho g - \rho a_z$$

$$= \rho(g - a_z)$$

$$\boxed{\therefore \frac{\partial p}{\partial z} = \rho(g - a_z)}$$

we get:

$$\boxed{\frac{\partial p}{\partial z} = -\rho(g + a_z)}$$

considering xz plane only.

$$dp = \frac{\partial p}{\partial x}\cdot dx + \frac{\partial p}{\partial z}\,dz$$

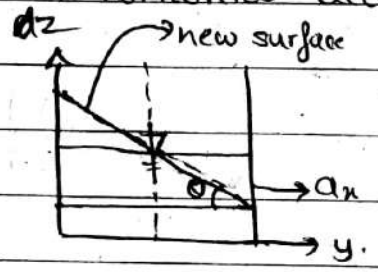
$$\boxed{dp = -\rho a_x\,dx - \rho(g + a_z)\,dz} \quad \cdots \text{(ii)}$$

~~case (1).~~

~~for ρ horizontal~~

Case-I

For horizontal acceleration only:



i.e. $a_z = 0$

$a_x = a$

then,

$$\gamma p = -\rho dx - \rho g\, dz$$

for surface of constant pressure, $dp = 0$
(Patm)

So,

$$-\rho a_x \cdot dx \cancel{-\rho(g+a_z)\,dz} = 0$$

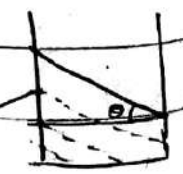
$$\Rightarrow -\rho a dx - \rho g\, dz = 0.$$

$$\Rightarrow \frac{dz}{dx} = \frac{-a_x}{g} = \tan\theta$$

$$\boxed{\therefore \theta = \tan^{-1}\left(\frac{a_x}{g}\right)}$$

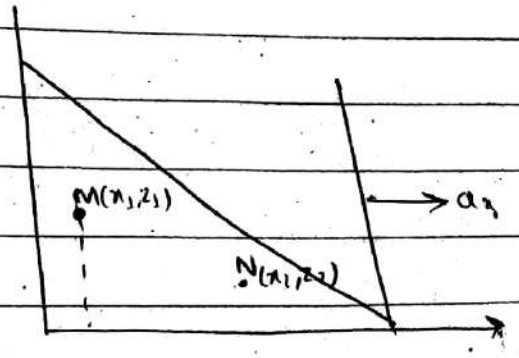
the new surface has a same pressure of Patm.

The another layer where pressure is same is →



these parallel lines are called isobars.

for a arbitrary point

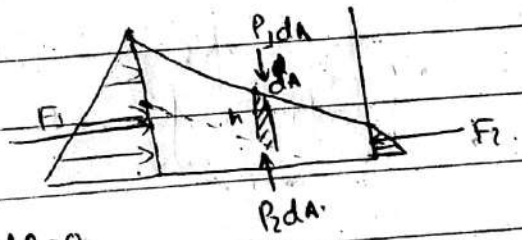


pressure difference between M or N

is

$$\int_N^M dp = \int_N^M -\rho a_x dx - \int_N^M \rho (g+a_z)\, dz$$

$$P_M - P_N = -\rho a_x (x_1 - x_2) - \rho (g+a_x)(z_1 - z_2).$$



$$P_2 dA - P_1 dA - \gamma h \cdot dA = 0$$

$$P_2 = (P_1 + \gamma h)$$

also, $F_1 - F_2 = m \cdot a_x$.

$$= \boxed{\tfrac{1}{2}\gamma h_1^2 B - \tfrac{1}{2}\gamma h_2^2 B = m a_x}$$

case(II)

vertical acceleration only!

$a_x = 0,\ a_z = 0$

$$dp = -\rho(g + a_z)dz - \rho a_x dx \quad \cdots (1)$$

For free surface:

$dp = 0$

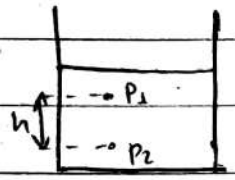
$$\frac{dz}{dx} = -\frac{\rho a_x}{\rho(g + a_z)}$$

but $a_x = 0$

so, the surface is a horizontal plane.

eq^n (1) becomes

$$dp = -\rho(g + a_z)dz$$



$$dp = -\rho(g + a_z)dz$$

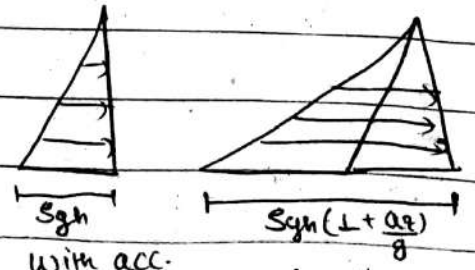
$$\int_{P_1}^{P_2} dp = \int_{z_1}^{z_2} -\rho(g + a_z)dz$$

$$P_2 - P_1 = \rho(g + a_z)h$$

$$\boxed{P_2 - P_1 = \rho g h\left(1 + \frac{a_z}{g}\right)} \quad \cdots (2)$$

this $\left(1 + \frac{a_z}{g}\right)$ increased the pressure more than just $\rho g h$.

So, the pressure diagram becomes:



with acc. (Static) (vertical acc.)

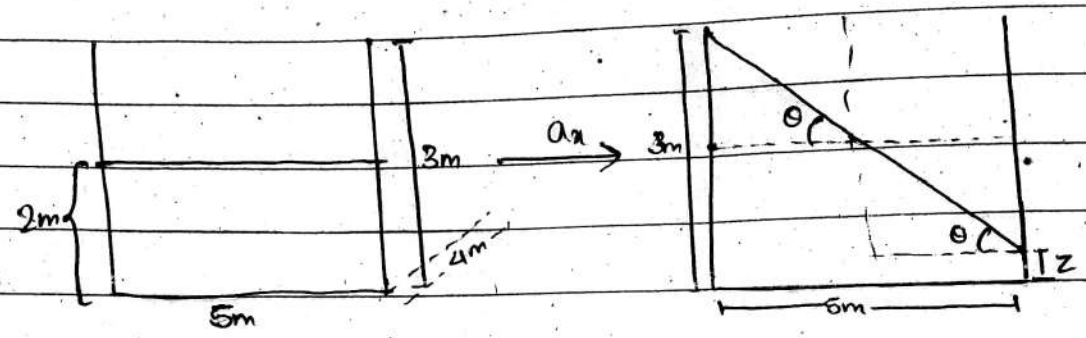
in case the vertical acceleration is downward.

eq^n (2) becomes:

$$\boxed{P_2 - P_1 = \rho g h \left(1 - \frac{a_z}{g}\right)}$$

Question:

* An open rectangular tank $(5 \times 4 \times 3)m$ containing water upto the height of 2m is accelerated horizontally along longer side. Determine maximum acceleration that can be given without spilling the water and also calculate % of water spill over if acceleration is increased by 20%.



$$4 \times 2 \times 5 = \left[(2 \times 5) + \tfrac{1}{2} \times (3-2) \times 5\right] \times 4.$$

$$\Rightarrow 10 = 52 + 7.5 - 2.52.$$

$$\Rightarrow \quad Z = 1m$$

$$\therefore \tan\theta = \frac{2}{5} =$$

$$\therefore \theta = 21.80^\circ$$

Method I:

but $\tan\theta = \dfrac{a_x}{g}$

$$a_x = 3.923.$$

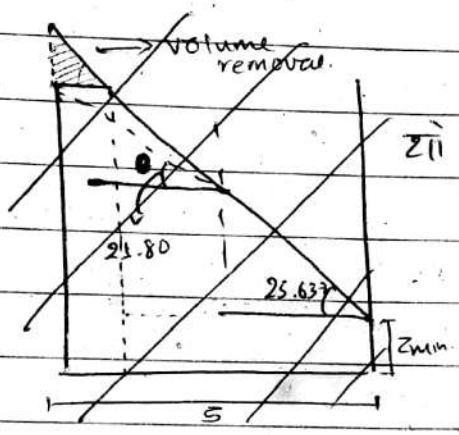
Solution

new $a_x = 4.708\ m/s^2$

so,

$$\text{new } \theta = \tan^{-1}\left(\frac{4.708}{9.81}\right)$$

$$= 25.637^\circ$$



यो हुँदैन किन भने पानि पोरिन सक्छ यहाँ 25.637° huney हो।

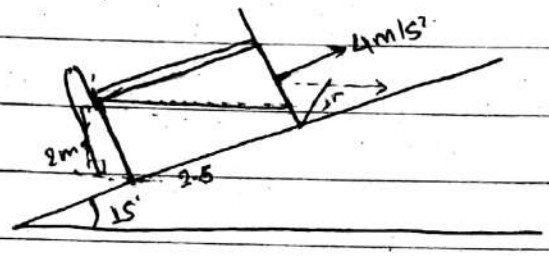
$$Z = 3 - 5\tan 25.637$$

$$= 0.600\ m$$

* An open rectangular container 2.5 x 1m base and 2m height is half filled with water is accelerated at 4m/s² up a 15° inclined along its length.

i) Will the water ~~smit~~ spill.

ii) A what acc. will the water actually begin to spill.



$a_x = 4\cos 15 = 3.8637\ m/s^2$

$a_z = 4\sin 15 = 1.0352\ m/s^2$

~~maximum~~ angle $\theta = \tan^{-1}\left(\frac{a_x}{g+a_z}\right) = \frac{4\cos 15}{9+4\sin 15}$

$\therefore \theta = 19.6°$

$\therefore$ ~~maximum~~ due to slope $\alpha = 15°$

$\therefore$ total angle $= 19.5 + 5$

$= 34.6°$

maximum angle : $\tan^{-1}\left(\frac{2}{2.5}\right) = 38.66°$

So the water wont spill.

angle at which spill begins $= 38.66 - 15 = 23.66°$

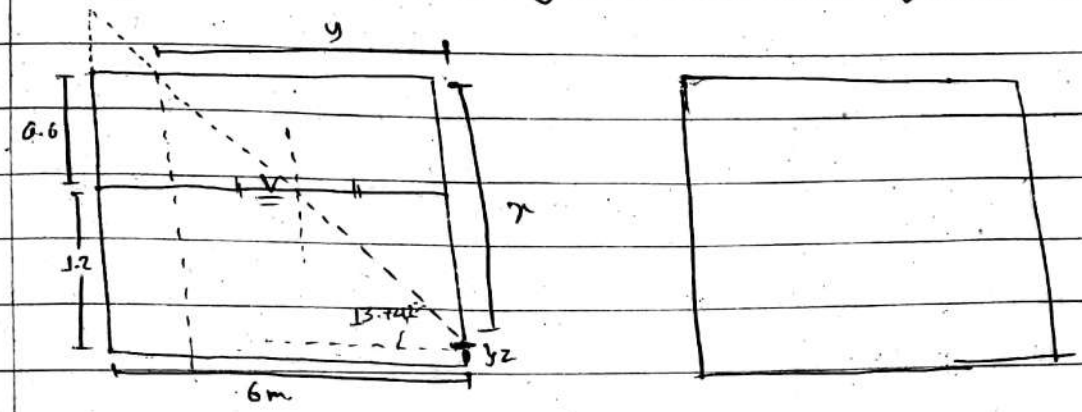
now,

$\theta = \tan^{-1}\left(\frac{a_x}{g+a_z}\right) = \tan 23.66 = \frac{a\cos 15}{9 + a\sin 15}$

$\Rightarrow 0.438 \times (9.81 + a\sin 15) = a\cos 15$

$a = 5.039$

* A closed tank 6m long, 2m wide and 1.8m deep initially contains water to a depth of 1.2m. The top has an opening infront part to have air space at atmospheric pressure. If the tank is given horizontal acc. at 2.4m/s² along its length. Calculate total force on the top of the tank.



$$\tan\theta = \frac{a}{g} = \frac{2.4}{9.81}$$

$$\therefore \theta = 13.7472$$

$$\frac{1}{2}xy = 0.6\times 6 \cdots \Rightarrow xy = 7.2 \cdots (1)$$

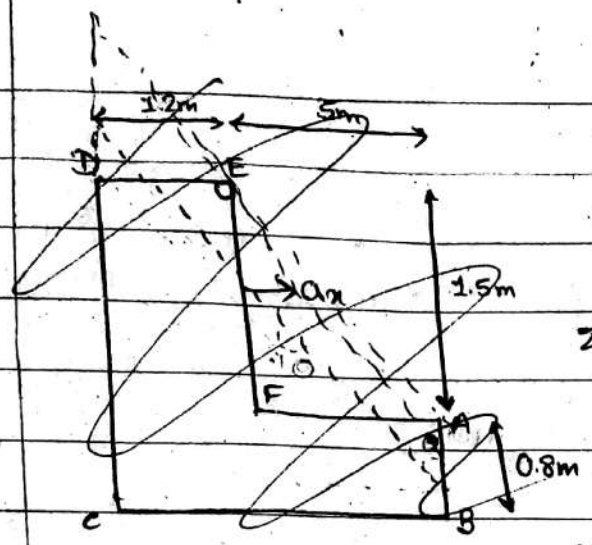
also $\tan\theta = \frac{x}{y}$ $\Rightarrow$ $\frac{x}{y} = \cancel{23.5444}$ 0.2446

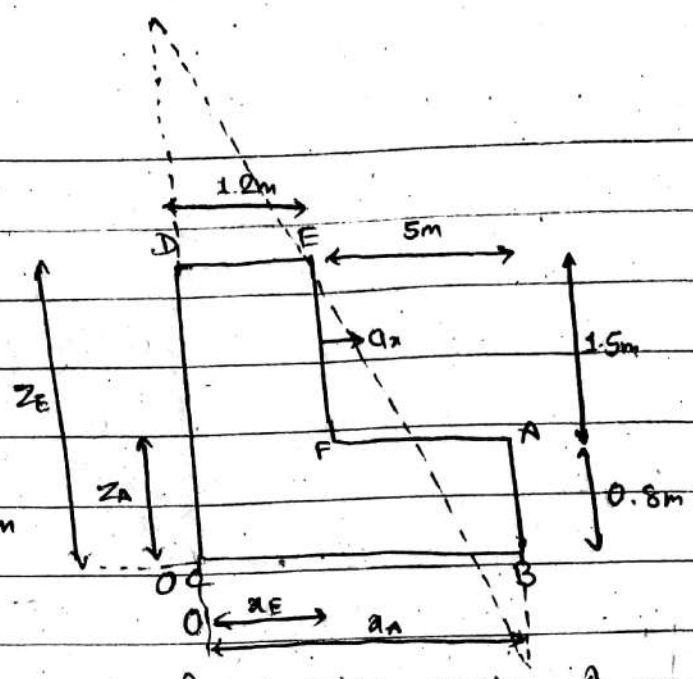
$\Rightarrow x = \cancel{23.5444}$ 0.2446y.

$\Rightarrow$ ~~y = 0.553~~ y = 5.424 m

~~and x = 13.019~~ x = 1.327m.

*





A tank ABCDEF as shown, is filled with water. A small opening at E keeps the pressure at E atmospheric.

i) Calculate acceleration a_x required to cause onset of cavitation at A.

ii) What would be the pressure at that acceleration, at B, C, D and F.

(Assume ~~to~~ local atm pressure head = 10m of water and vapour pressure head = 0.5m absolute of water)

⇒ Vapour pressure $= \frac{0.5 \times 9.81 \times 1000}{9.81 \times 1000}$ ~~5.096×10^{-5} Pa~~. 4905 Pa

$P_{atm} = \frac{10 \times 9.81 \times 1000}{9.81 \times 1000}$ ~~1.019×10^{-3} Pa~~. 98100 Pa

$\tan\theta = \frac{a_x}{a_z + g}$ but $a_z = 0$

So $\tan\theta = \frac{a_x}{g}$

$a_x = g\tan\theta$

we know:

i) $$dP = -\rho a_x dx - \rho(g + a_z)\,dz.$$

but ~~dz~~ $a_z = 0$

so,

$$dP = -\rho a_x dx - \rho g\,dz.$$

integrating from A to E.

$$\int_A^E dP = \int_A^E -\rho a_x dx - \rho(g + a_z)\,dz.$$

$$(P_E - P_A) = -\rho a_x (x_E - x_A) - \rho g (z_E - z_A).$$

$$\Rightarrow -\left[\frac{P_E - P_A + \rho g (z_E - z_A)}{\rho (x_E - x_A)}\right] = a_x.$$

$$a_x = 21.582\ m/s^2.$$

ii) $$\int_B^E dP = \int_B^E -\rho a_x dx - \rho g\,dz.$$

$$P_E - P_B = -\rho a_x (x_E - x_B) - \rho g (z_E - z_B)$$

$$\Rightarrow \boxed{P_E + \rho a_x (x_E - x_B) + \rho g (z_E - z_B) = P_B}$$

$$\Rightarrow P_B = 12753\ Pa\ (abs)$$

$$\int_C^E dP = \int_C^E -\rho a_x dx - \rho g\,dz$$

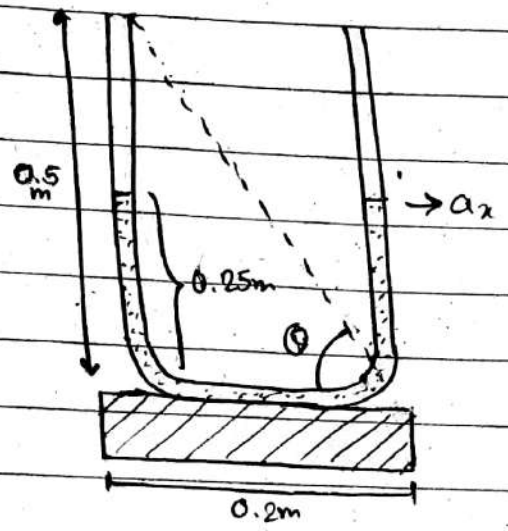
$$\Rightarrow \cancel{P_E =} \quad P_C = P_E + \rho a_x (x_E - x_C) + \rho g (z_E - z_C)$$

$$= 98100 + 1000 \times 21.528 \times (1.2 - 0) + 1000 \times 9.81 \times (2.3)$$

$$= 146496.6\ Pa\ (abs)$$

and so on // $P_F = \cancel{105948}\ 112815\ Pa\ (abs)$

Accelerometer:

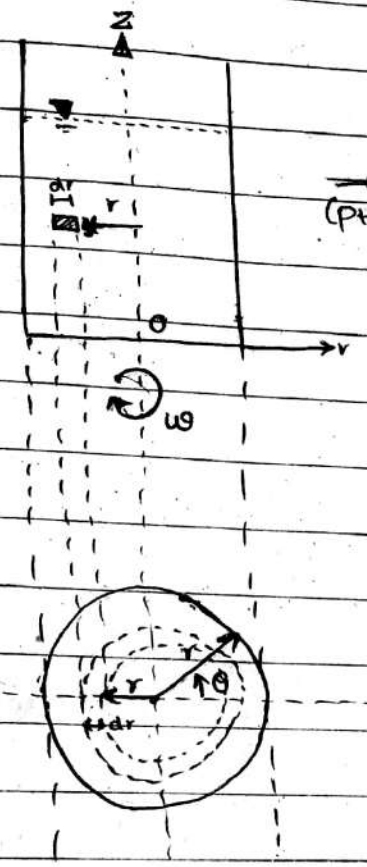


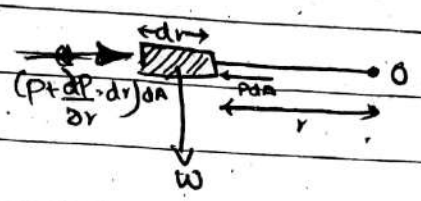
$$\tan\theta = \frac{a_x}{a_z + g} = \frac{a_x}{g}$$

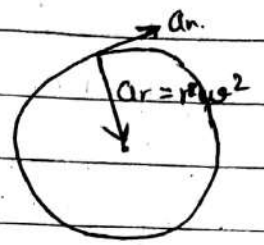
$$\Rightarrow \frac{0.5}{0.2} = \frac{a_x}{g}$$

$$\Rightarrow a_x = 24.525 \ m/s^2$$

Rigid body motion in rotation







So, $\sum_r F = m.a_{radial}$

$$\Rightarrow \left(P + \frac{\partial P}{\partial r} dr\right) dA - P dA = \frac{W}{g} \times r\omega^2$$

$$\Rightarrow \frac{\partial P}{\partial r} \cancel{dr.dA} = \frac{\rho \cancel{g}.\cancel{dA.dr}}{\cancel{g}} \omega^2 r$$

$$\Rightarrow \frac{\partial P}{\partial r} = \omega^2 r \rho \quad \text{-----} \ (i)$$

also, $\frac{\partial P}{\partial z} = -\gamma$ ----- (ii)

So, $dp = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$

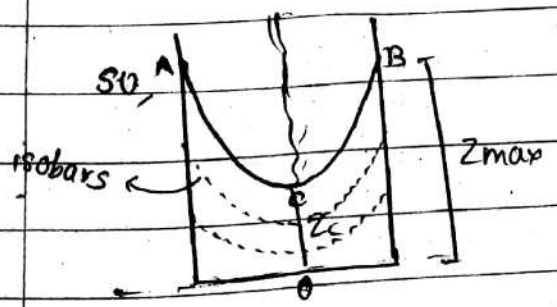
$$\Rightarrow \boxed{dP = \rho\omega^2 r dr - \gamma dz} \quad \text{---} \ (iii)$$

for shape of the surface at free surface $dP = 0$.

~~and~~ So,

$$\rho\omega^2 r dr = \gamma dz.$$

$$\Rightarrow dz = \frac{\omega^2 r}{g} dr.$$

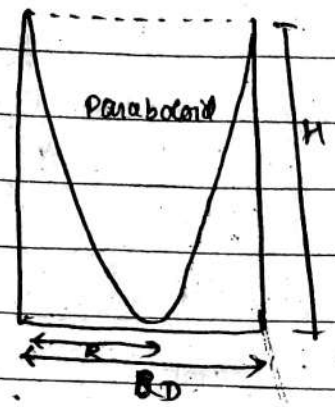


$$\int_{z_c}^{z} dz = \int_{b}^{R} \frac{\omega^2 r}{g} dr$$

$$\Rightarrow \boxed{z = \frac{\omega^2 r^2}{2g} + c} \text{ parabolic.}$$

when, $r = b$, $z = z_c$
when, $r = R$, $z = z_{max}$

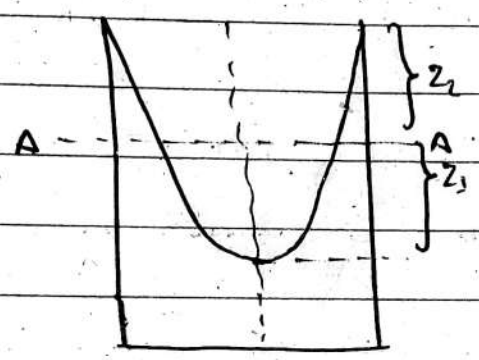
Volume of a paraboloid



$$= \frac{1}{2} \times \pi R^2 H$$

$$\text{or, } \frac{\pi D^2 H}{8}$$

incase of no liquid loss and just touching the rim.



AA is the initial water level than,

$$\pi R^2 z_2 = \frac{1}{2} \pi R^2 (z_1 + z_2)$$

$$\Rightarrow \frac{1}{2} \pi R^2 z_1 = \frac{1}{2} \pi R^2 z_2$$

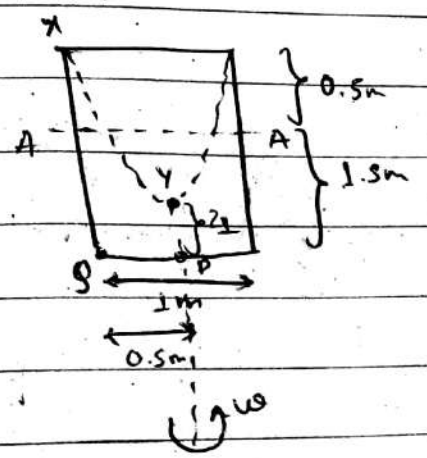
$$\Rightarrow \boxed{z_1 = z_2}$$

So, rise in water level = drop in water level in centre.

* An open cylindrical tank 2m high and 1m diameter contains, 1.5m of water. If the cylinder rotates about its geometrical axis.

i) What constant angular velocity can be attained without spilling the water.

ii) What is the pressure intensity at the bottom of the tank at centre and at walls when $\omega = 6 rad/s$.



we know,

$$dP = \rho \omega^2 r dr - \gamma dz.$$

at x and y,

$$dP = 0.$$

$$\Rightarrow 0 = \rho \omega^2 r dr - \gamma dz.$$

$$\Rightarrow \rho \omega^2 r dr = \gamma dz.$$

$$\Rightarrow \int_0^{0.5} \rho \omega^2 r dr = \int_{z_y}^{z_x} \gamma dz.$$

$$\Rightarrow \cancel{\rho} \omega^2 \frac{(0.5)^2}{2} = \cancel{\rho} g (z_x - z_y)$$

now,

~~volume~~ at no spill condition, $z_x = z_y$ from AA.

so,

$$\omega^2 \frac{(0.5)^2}{2} = g (2 \times 0.5)$$

$$\Rightarrow \omega^2 = \frac{g(1)}{(0.5^2)} \times 2.$$

$$\omega = 8.85 \text{ rad/s}.$$

again:

$$dP = \rho\omega^2 r\,dr - \gamma\,dz$$

$$\int_{0y}^{P} dP = \int_0^0 \omega^2 r\,dr - \int_{z_1}^{0} \gamma\,dz$$

$$\Rightarrow P_p - P_{atm} = -\int \gamma(0 - z_1)$$

$$\Rightarrow P_p - P_{atm} = \gamma z_1$$

$$\Rightarrow P_p = \boxed{\gamma z_1 + P_{atm}}$$

also,

$$dP = 0$$

at $\rho\omega^2 \frac{(r_2 - r_1)^2}{2} = \gamma(z_2 - z_1)$

$$\rho\omega^2 \frac{R^2}{2} = \gamma(z_2 - z_1)$$

$$\Rightarrow 6^2 \times \frac{R^2}{2} = g(z_2 - z_1)$$

$$\Rightarrow \frac{6^2 \times 0.5^2}{2 \times 9.81} = z_2 - z_1$$

$$z_2 - z_1 = 0.458$$

$$\Rightarrow 2z_1 = 0.458$$

$$= 0.2293$$

$$\therefore z_y = 1.27$$

$$\therefore P_0 = \gamma z_y = 9.81 \times 1000 \times 1.27$$

$$= 12465\ Pa \text{ (gauge)}$$

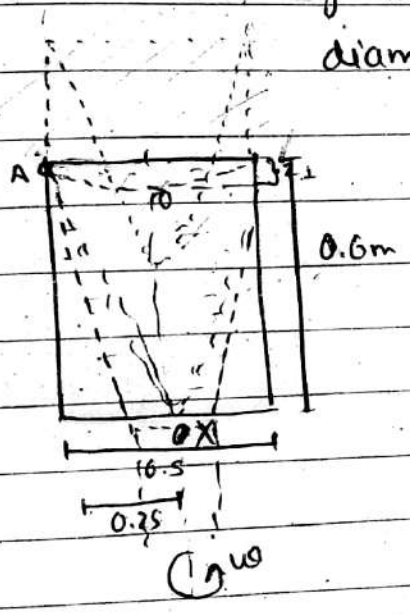
$P_B = \gamma(z_y + 0.2293 \times 2) = 9.81 \times 1000 \times$

$= 16964.433$ Pa (gauge)

*

A cylindrical tank 0.6m high and 0.5m in diameter is filled completely with water. If it is rotated at 120rpm, how many lites of water will be spilled. Also determine speed at which i) ~~wha~~ water surface will just touch the top rim and centre bottom of the tank.

ii) water surface will just touch top and ~~to~~ bottom of the tank having unovered circular area of diameter 0.2m.



$dp = \rho\omega^2 r\,dr - \gamma dz.$

$\Rightarrow \int_0^R \rho\omega^2 r\,dr = \int_0^a \gamma\,dz.$

$\Rightarrow \frac{\rho\omega^2}{\rho g} \cdot \frac{R^2}{2} = z_1 = \frac{R^2\omega^2}{2g}.$

$\Rightarrow \boxed{z_1 = 0.505}$

So volume spilled = volume of paraboloid

$= \frac{\pi R^2 z_1}{2} = \frac{\pi \times 0.25^2}{2 \times 0.505}$

$= 49.57$ lite.

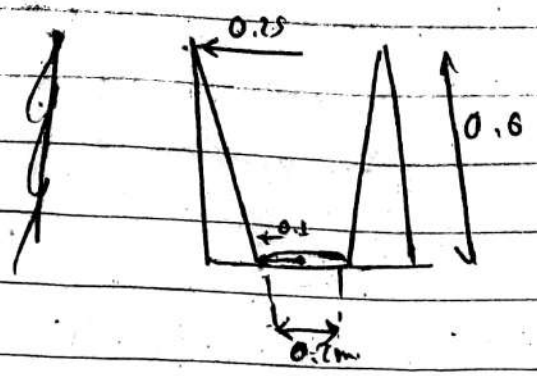
at A ~~0.6~~ and x

$dP = \rho\omega^2 r\,dr - \gamma dz.$

at A and x $\Rightarrow dP = 0.$

$\Rightarrow (0.6) = \frac{\omega^2 R^2}{2g}$

$\Rightarrow \sqrt{\frac{0.6 \times 2g}{R^2}} = \omega \Rightarrow \omega = 13.72$ rad/s.



$dP = 0$

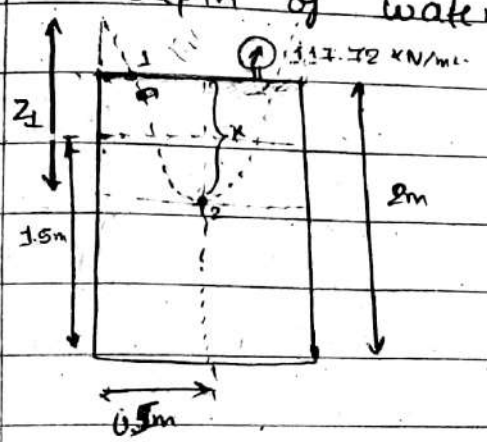
$\Rightarrow \rho\omega^2 \frac{(r_2 - r_1)}{2} = \gamma(0.6)$

$\Rightarrow \omega = \sqrt{\frac{0.6 \times 2}{(r_2 - r_1)}}$

$\rho\omega^2 \times (0.25 - 0.1) = \gamma(0.6 - 0) \times 2$

$\Rightarrow \omega = \sqrt{\frac{0.6 \times 2}{(0.25 - 0.1)}}$

* A closed tank 2m high and 1m diameter contains 1.5 m of water. The air in the space above the water is subjected to a pressure of 117.72 KN/m². If the tank is rotated, at an angular velocity of 12 rad/sec. What are the pressure intensities at centre of bottom and near the wall at the bottom of the tank. Also find the speed at which the tank should be rotated in order that the centre of the bottom has zero depth of water.



$\Rightarrow \frac{\pi r^2 \times 0.5}{2} = \frac{\pi r^2 \times x}{2}$.

$\Rightarrow$ 1.57 m³ = Volume of paraboloid.

now, $\boxed{0.25 = r^2 x}$ --- (ii)

at ① and ②, dP = 0.

$\Rightarrow 0 = \frac{\rho \omega^2 (r^2)}{2} - \gamma(2-(2-x))$

$\Rightarrow \frac{\omega^2 r^2}{2} = g x$

$\Rightarrow \frac{12^2 r^2}{2} = g x$

$\Rightarrow \boxed{r^2 = 0.13625 x}$ --- (i)

From (ii),

$\Rightarrow \frac{0.25}{x} = 0.13625 x$.

~~x = 2.205 m~~ $x = 1.354$ m

So, $r = 0.429$ m

at centre $= \gamma \times (2-x) + 117.720$

$= 124.057$ kPa

~~at wall = γx~~

$dP = 0$, for z_1 and (2)

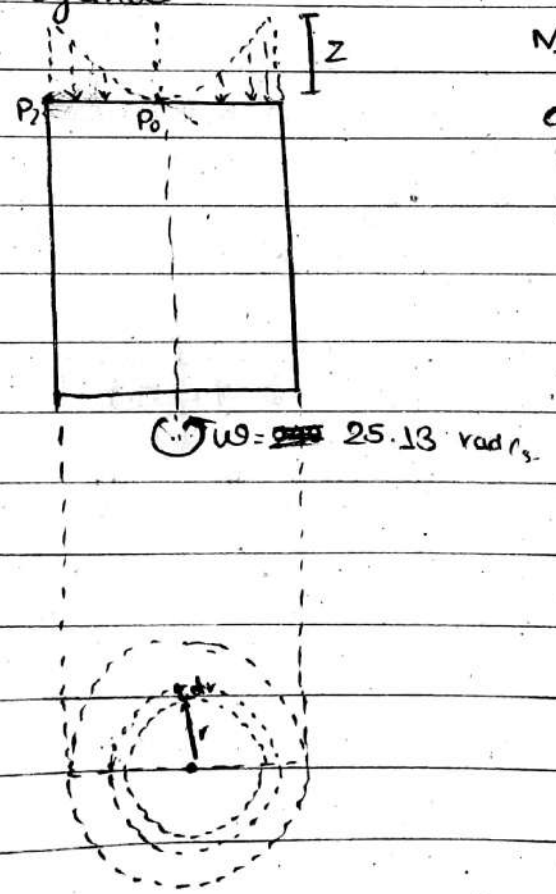
$$\frac{\rho \omega^2 (0.5)^2}{2} = \gamma \times z_1$$

$$\Rightarrow \frac{\omega^2 \cdot (0.5^2)}{2 \cdot g} = z_1$$

$$\Rightarrow \frac{144 \times 0.5^2}{2 \times g} = z_1$$

$$\Rightarrow z_1 = 1.834 \text{ m}$$

* A closed cylinder of radius 10 cm, height 30cm is filled with water if cylinder is rotated about vertical axis at 240 rpm. Calculate force exerted at the top and bottom covers of the cylinder.



Method I:

$dP = \rho \omega^2 r dr - \gamma dz$.

~~for chang~~ for the surface,

$dz = 0$.

and $dP = \rho \omega^2 r dr$.

~~so $\int_0^R dP = \int_0^R \rho \omega^2 r dr$.~~

~~$\Rightarrow (P_2 - P_0) = \rho \omega^2 (R)$~~

~~but~~

but $dF = dP \times dA$

$= \rho \omega^2 r dr . 2\pi r dr$

$= 2\pi r^2 \omega^2 \rho (dr)^2$

Method II:

We know

$$z = \frac{\omega^2 r^2}{2g}$$

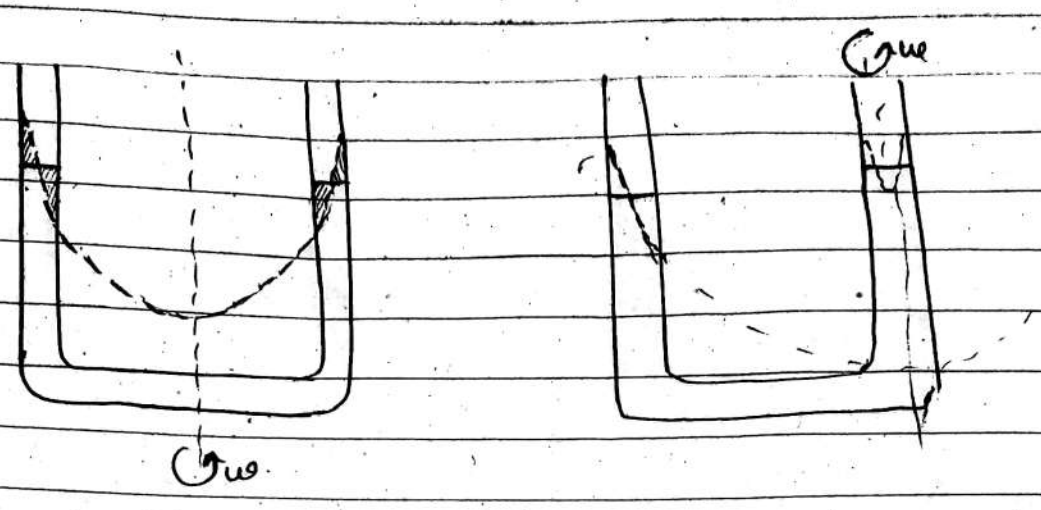
so, $dP = \rho 2\pi r dr . \gamma$

$$= \frac{\omega^2 r^2 . \gamma . 2\pi r}{2g} dr$$

so, $\int dP = \int_0^R \frac{\gamma \omega^2 r^2}{2g} 2\pi r dr$

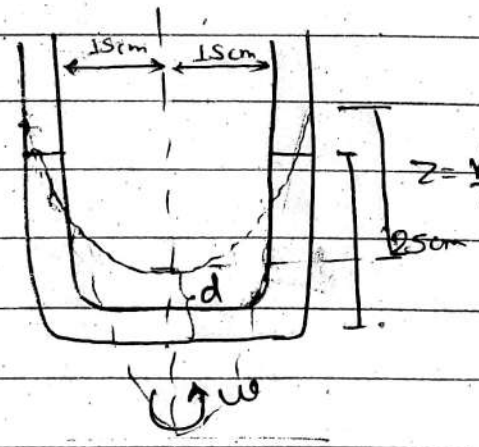
$$P = \frac{\gamma \pi \omega^2 R^4}{4g}$$

*



* A U-tube contains a liquid of RD = 1.25 to a height of 25cm in both the column. If it is rotated about a symmetrical vertical axis which is 15cm from both the limb. Calculate pressure at the bottom horizontal connecting tubes at ~~MNS~~ M,NS when i) ω = 120 rpm
ii) ω = 240 rpm.

Assume diameter of the tube is very small.



$z = \frac{r^2\omega^2}{2g} = 0.181m$

now,

~~$\gamma.(d-0) = \cancel{\gamma d}\ S_g d$~~

$d = 0.25 - 0.181 = 0.069m$

at 240 rpm,

$z = 72.4\,cm.$

So,

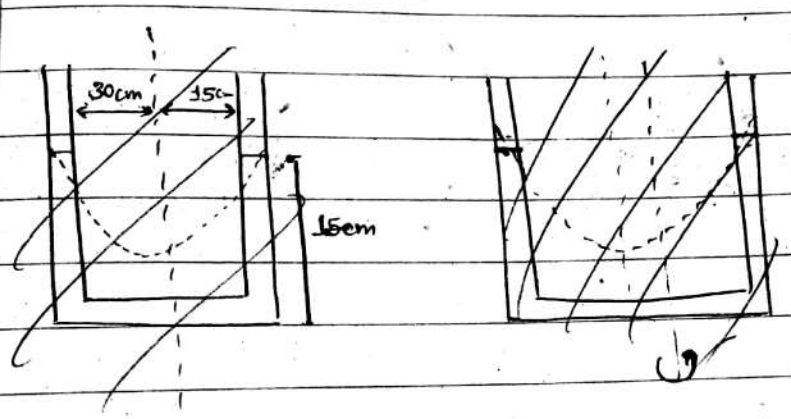
$d = -47.4.$

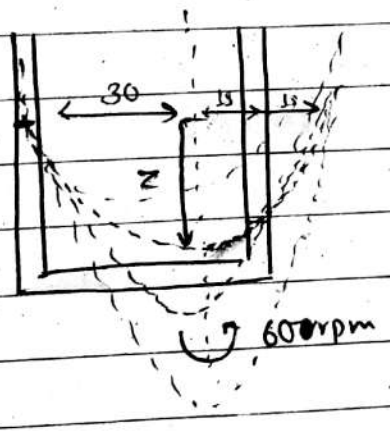
So,

$P = \rho g(-47.4) \rightarrow$ convert to m,

$= -464.994$ (Gauge)

*





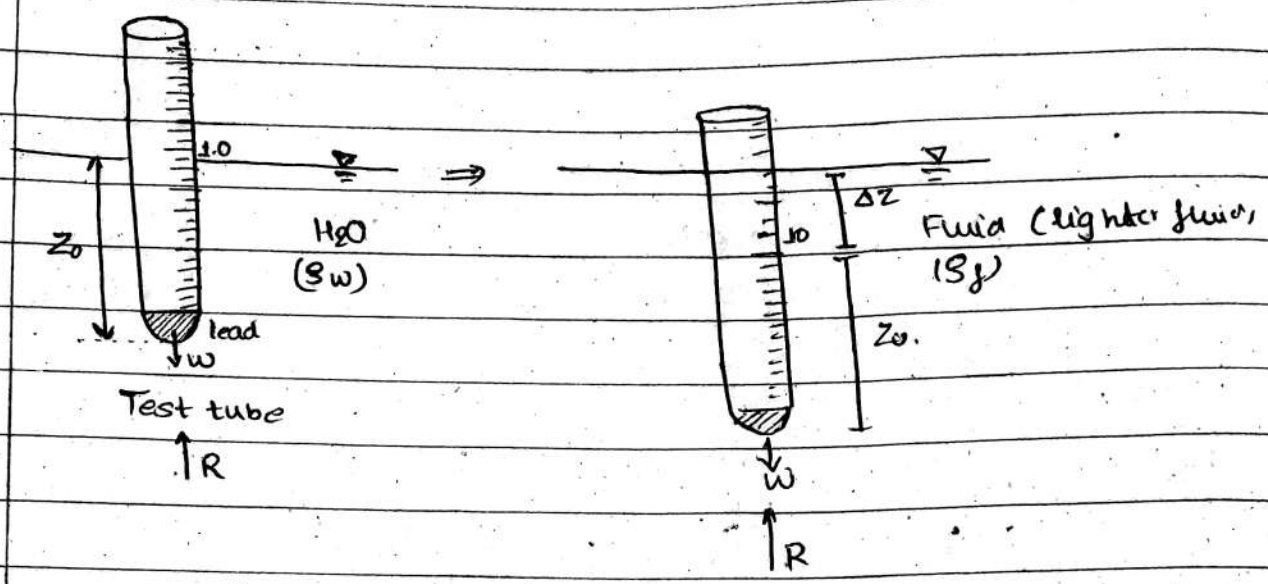
$z = \frac{r^2\omega^2}{2g} = 0.18m$

$= 18.1m$

$\rho\omega^2 \cdot \frac{(30^2 - 15)^2}{2} = \gamma(18.1 - d)$

$\Rightarrow$

Hydrometer: (To find specific gravity of a fluid S_f)



$W = \gamma \cdot \forall_{sub} = R$.

$W_{Test\ lead} = S_w g A z_0$ ----- (1)

$S_f < S_w$

$W_{Tt+lead} = \gamma \forall_{sub} = S_f g A (z_0 + \Delta z)$ --- (2)

So,

(1) = (2) So,

$S_w \cdot g A z_0 = f_0 g A (z_0 + \Delta z)$

$SG = \boxed{\therefore \frac{S_f}{S_w} = \frac{z_0}{z_0 + \Delta z}}$

if $R_f > R_w$ if $R_f < R_w$

$\Delta z = -ve$ $\Delta z = +ve$.

* A hydrometer weighs 0.2N and has a stem diameter of 5mm. Find distance between the marking corresponding to relative densities of a) 1.0 and 0.95. (Ans: ~~[illegible]~~ 54.76mm
b) 1.0 and 1.25. 49.54mm)

$$\cancel{\frac{\rho_{f1}}{\rho_w} = \frac{z_0}{z_0+\Delta z} \quad \text{and} \quad \frac{\rho_{f2}}{\rho_w} = \frac{z_0}{z_0+\Delta z}}$$

$$0.95 = \frac{z_0}{z_0+\Delta z}$$

$$\Rightarrow \quad 0.95\Delta z = 0.05 z$$

$$\Rightarrow \quad \Delta z = \frac{z}{19}$$

also, $0.2 = \cancel{1000} \; 1000 \times 9.81 \times V_{sub}$

$0.2 = \cancel{1000} \; 950 \times 9.81 \times V_{sub}$ ~~+ Volume above~~ below

$$\Rightarrow \quad 0.2 = 1000 \times 9.81 \times \pi (\cancel{0.05 \times} \; 5\times10^{-3})^2 \times z$$

Fluid dynamics:

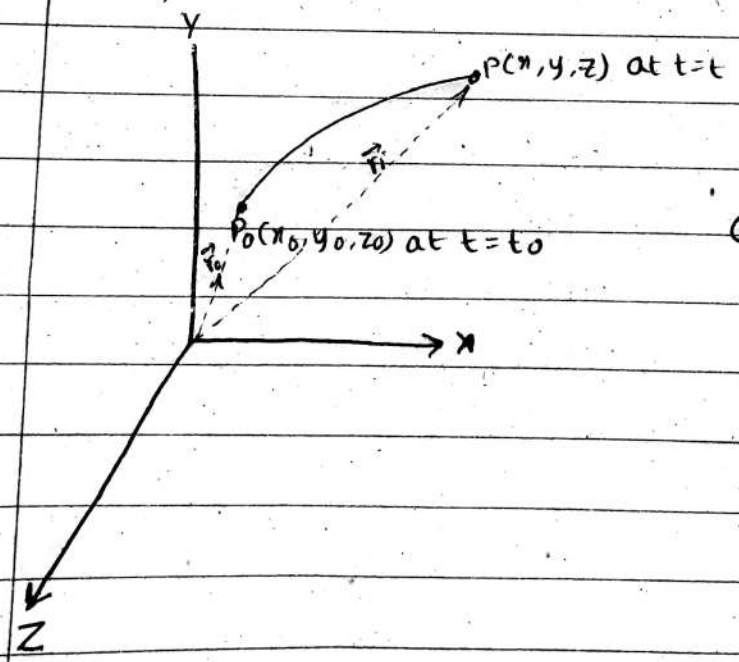
- → Kinematics
- → Kinetics

Kinematics- Study of fluids in motion i.e. space time relations of velocity and acceleration without considering the force associated with them.

Analysis of flow patterns and their dependency on boundary geometry. It can be useful for derivation of powerful tools like stream and velocity potential function and flow net.

Methods for describing the motion:

1) Lagrangian method:



In this method, the motion of individual particle of fluid is analyzed to obtain velocity component of fluid at any desired location. Position of fluid particles is completely defined if (x,y,z) are obtained as a function of $x_0 y_0$ and z_0.

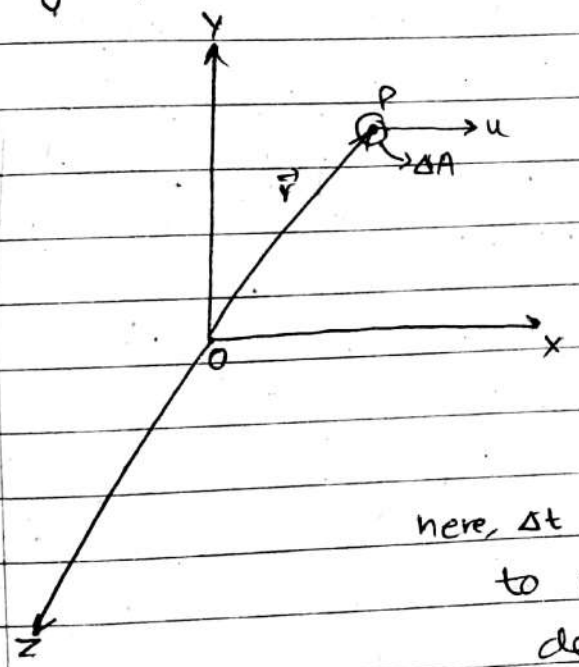
$$x = f(x_0, y_0, z_0)$$
$$y = f(x_0, y_0, z_0)$$
$$z = f(x_0, y_0, z_0)$$

Once, x, y, z are computed,

$$u = \frac{\partial x}{\partial t}, \quad v = \frac{\partial y}{\partial t}, \quad w = \frac{\partial z}{\partial t}$$

<u>merit</u>: History of all fluid particles can be traced.
<u>demerit</u>: Resulting equations of motion ~~are~~ maybe non-linear in nature and hence, tedious to solve.

2) <u>Eulerian method</u>: Velocity components are obtained by considering finite areas instead of examining the path of ~~ie~~ individual fluid particles.



Here, ΔA = small area around P which is oriented in such a way that normal to the area is parallel to $-OX$. So, velocity component in x-direction,

$$u = \frac{\text{Volume flux entering } \Delta A}{\Delta A \cdot \Delta t}$$

here, Δt is the time interval for the volume to pass through ΔA. Let this velocity be denoted by: $u = f(x, y, z, t)$

$v = f(x, y, z, t)$

$w = f(x, y, z, t)$

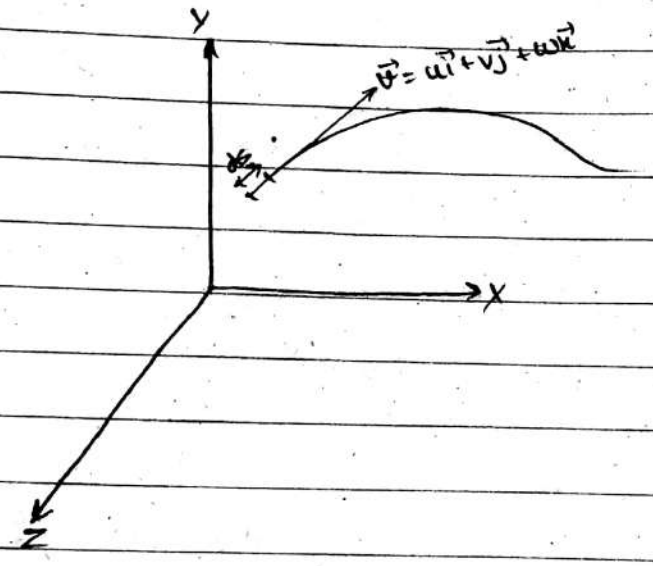
this concept is the eulerian concept

<u>demerit</u>: Background information of individual fluid particle is not known.

merit: Eq^n of motion obtained by this method is easily linearized using acceptable approximation.

The basic defination of fluid kinematics:

1) **Streamline:** It is an instantaneous line drawn in a flow field in such a way that all the fluid particles falling on the line will have velocity vectors along the tangents drawn to the line a particle location at that instant.

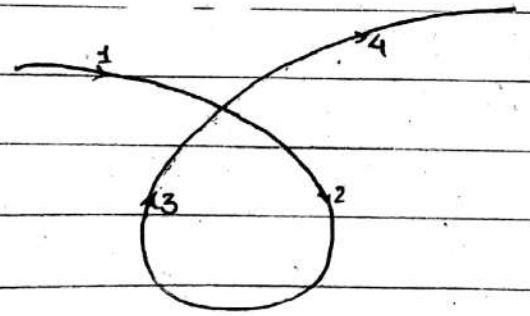


$ds = dx\vec{i} + dy\vec{j} + dz\vec{k}$

since the tangent vector at ds and ds is parallel,

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

2) **Path line:**



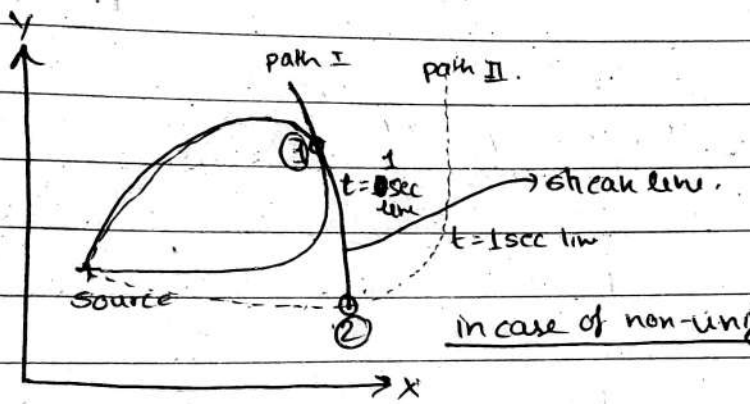
The line drawn through the path traced by a single fluid particle over a period of time. Example: Path taken by small observation float in a river or canal ~~in a~~ is a path line in water surface.

equation of path line is:

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}$$

$$\Rightarrow \underbrace{x = \int u\,dt, \quad y = \int v\,dt, \quad z = \int w\,dt}_{\text{together is a path line.}}$$

3) Streak line:



in case of non-uniform flow

The line which at a given instant connects the temporary location of all the fluid particles ~~thave~~ that have traversed a fixed point in a flow field is called streakline.
example: line traced by a die, Injected in a liquid is an example of streak line.

equation of streakline is:

$$\left.\begin{aligned} x &= f_1(x_0, y_0, z_0, t). \\ y &= f_2(x_0, y_0, z_0, t). \\ z &= f_3(x_0, y_0, z_0, t). \end{aligned}\right\} \text{equations of streak line.}$$

(−0.625, 1.875)

* A steady incompressible two dimensional flow field is defined by:

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

where x and y are in m.

a stagnation point is defined as a point in the field where the velocity is zero.

So, determine if there are any stagnation points in this ~~at several~~ flow field. If so, where. Sketch the velocity vectors at several locations in the domain. $x = -2$ to $+2$ m and $y = 0$ to $+5$m.

Qualitatively discuss the flow field.

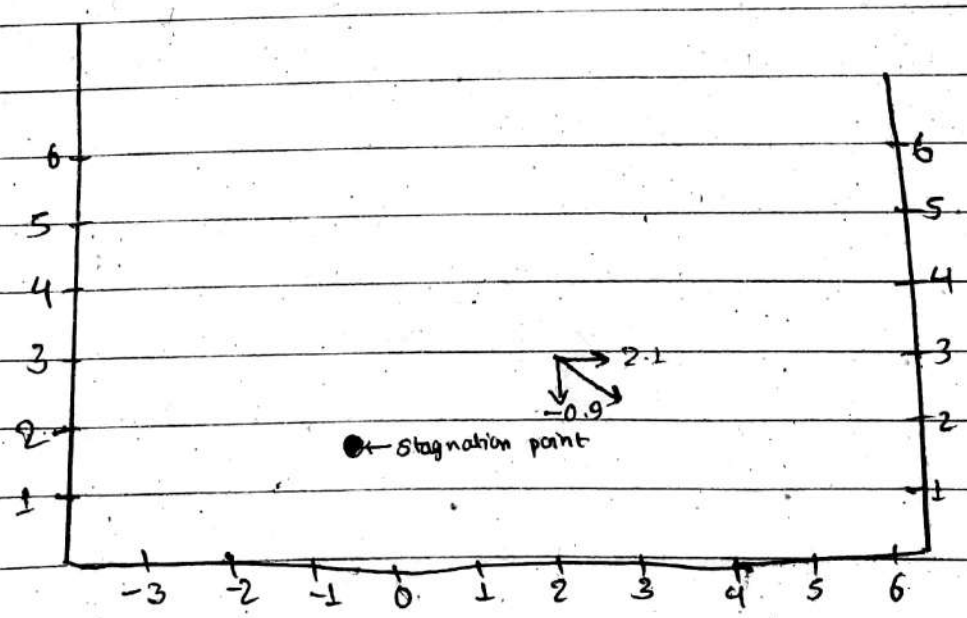
→

for stagnation point,

$u = 0$ i.e. $0.5 + 0.8x = 0 \Rightarrow x = -0.625$m

& $v = 0$ $1.5 - 0.8y = 0 \Rightarrow y = 1.875$m

Flow field:



Sketch stream lines:

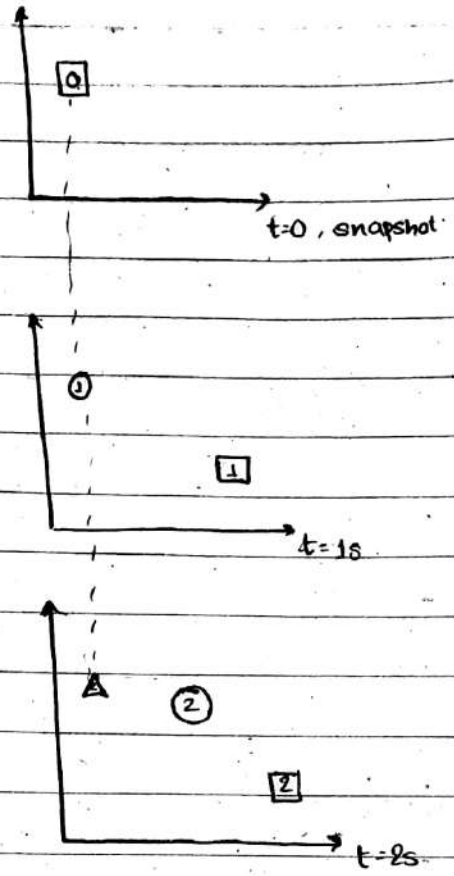
eq^n of stream line is:

$$\frac{dx}{u} = \frac{dy}{v}$$

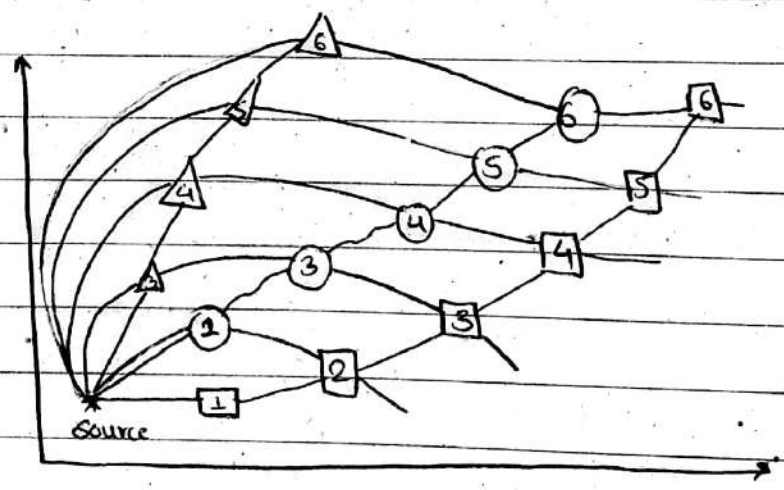
$$\Rightarrow \int \frac{dx}{0.5+0.8x} = \int \frac{dy}{0.1\cdot 5-0.8y}$$

$$y = \frac{c}{0.8(0.5+0.8x)} + 1.875$$

For steady flow, all the flowlines: streamline, streakline and path line are same.



pathline:



□ - particle releases at t=0.

[1] - particle that released at 0s moved at 1 sec.

O - particle releases at 1s.

Δ - particle releases at 2s.

* An unsteady incompressible two dimensional velocity field is given by :

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 + 2.5\sin\omega t - 0.8y)\vec{j}$$

where,

$\omega = 2\pi$ rad/s.

under complete cycle from $t=0$ to $t=2$sec, draw streamline at $t=2$s. pathline from $0<t<2$ and streak line between $0<t<2$s.

* A hypothetical 2 dimensional unsteady flow is given by:

$$\left.\begin{array}{l} u=0 \\ v=1\,m/s \\ w=0 \end{array}\right\} \text{for all } x, y, z \qquad 0 \le t \le 1 \text{ sec.}$$

$$\left.\begin{array}{l} u=-1 \\ v=0 \\ w=0 \end{array}\right\} \text{for all } x, y, z \qquad 1 < t \le 2 \text{ sec}$$

draw sheamline path line and sheanline

* The velocity distribution in the three dimensional flow is given by $u = -x$, $v = 2y$, $w = 3-z$. Find the equation of sheamline passing through $(1,1,1)$

$u = -x$ | $v = 2y$ | $w = 3-z$

$\Rightarrow du = -dx$ | $\Rightarrow dv = 2dy$ | $\Rightarrow dw = -dz$

so, $-\frac{du}{u} = \frac{dv}{2v} = \frac{-dw}{w}$

eq^n of sheamline is: $\frac{dx}{-x} = \frac{dy}{2y} = \frac{dz}{3-z}$

integrating

$$-\log_e x = \tfrac{1}{2}\log_e y + A, \quad A = 0.$$

$$\therefore \log_e x = \log_e y^{-1/2} \Rightarrow x = 1/\sqrt{y}.$$

Similarly,

$$\frac{dx}{-x} = \frac{dz}{3-z}$$

$$\Rightarrow -\log_e x = -\log_e(3-z) + B$$

at $(1,1,1)$.

$$\Rightarrow \cancel{-\log_e(1)} \quad B = \log_e(2)$$

So,

$$-\log_e x = -\log_e(3-z) + \log_e(2)$$

$$\Rightarrow x = \frac{3-z}{2} \quad \text{--- (ii)}$$

$$x = \frac{1}{\sqrt{y}} = \frac{3-z}{2}$$

* A fluid particles moves in the following flow field starting from $(2,1,0)$ at $t=0$, $u = \frac{t^2}{2x}$, $v = \frac{ty^2}{18}$, $w = \frac{z}{2t}$. Determine location of fluid particle at $t = 2s$.

$$\Rightarrow \frac{dx}{dt} = \frac{t^2}{2x}$$

$$\Rightarrow 2x\,dx = t^2 dt$$

$$\Rightarrow \frac{2x^2}{2} = \frac{t^3}{3} + A$$

$$\Rightarrow \frac{\cancel{2}x^2}{\cancel{2}} = \frac{t^3}{3} + 4$$

$$\Rightarrow x^2 = 13$$

$$x = \sqrt{13}$$

$$\frac{dy}{dt} = \frac{ty^2}{18}$$

$$\Rightarrow \frac{1}{y^2}\,dy = \frac{t\,dt}{18}$$

$$\Rightarrow \frac{-1}{y} = \frac{t^2}{2\times 18} + B$$

$$\Rightarrow B = -1$$

so,

$$\frac{-1}{y} = \frac{t^2}{36} - 1$$

at 3s. $-1/y = 1/4 - 1$

$$\Rightarrow 1/y = 1 - 1/4$$

$$\Rightarrow 1/y = 3/4$$

$$\boxed{\therefore y = 4/3}$$

$$w = \frac{z}{2t}$$

$$\Rightarrow \frac{dz}{z} = \frac{dt}{2t}$$

$$\Rightarrow \log_e z = \frac{1}{2}\log_e t + C$$

$\Rightarrow C =$ ~~indermina~~ 0 (ধরাসমা Indetermmant)

$\Rightarrow$ ~~$z = t^{1/2} + C$~~. so,

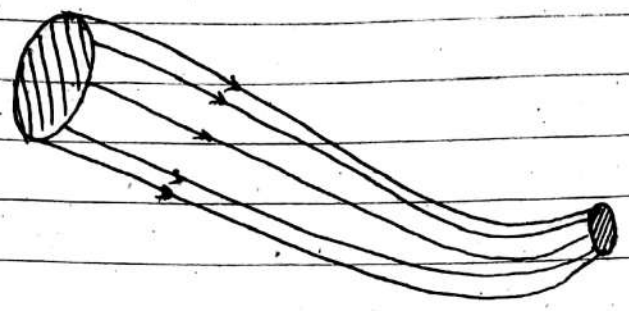
$$\log_e z = \frac{1}{2}\log_e t$$

$$\Rightarrow z = \sqrt{t}$$

at 3s.

$$z = \sqrt{3}$$

Stream tube:

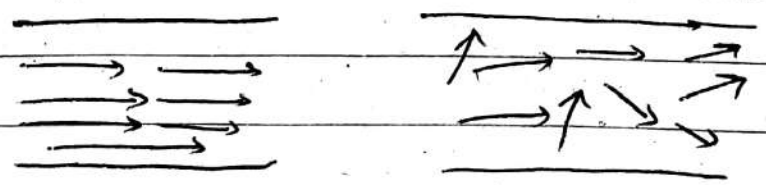


From the defination of streamline, no velocity normal to streamline at any point is present. In three dimension, streamline becomes stream surface and the stream surface in the form of ~~stream tub~~ tube is called stream tube. Flow can either enter or leave of a streamtube only through its cross-section and not from sideways.

Stream tube can have regular or irregular shape. It is generally assumed to have small cross section area so that, velocity across the section could be considered uniform.

Classification of fluid flow:

i) Laminar flow and turbulant flow. (By Reynold)



laminar flow Turbulat flow.

It is defined by Reynold number $Re = \frac{\rho V D}{\mu} = < 2000$ laminar.
> 4000 turbulant

ii) <u>Steady and unsteady flow:</u>

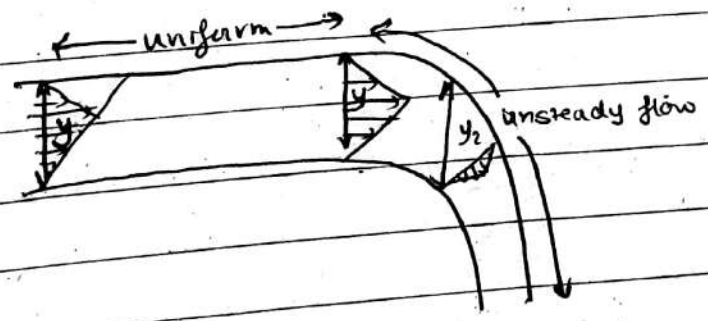
If $\frac{\partial V}{\partial t} = 0$, $\frac{\partial P}{\partial t} = 0$. i.e its ~~properties~~ flow parameter are time independent

So known as steady flow and each particle haves the same path that the previous particle.

else, if there ~~partic~~ parameter depend ~~to~~ on time, then is called unsteady flow.

iii) <u>Uniform and non-uniform flow:</u>

If $\frac{\partial V}{\partial S}$, $\frac{\partial H}{\partial S}$ (depth) (distance), $\frac{\partial P}{\partial S} = 0$. i.e if parameter are distance independent then uniform, else non-uniform.

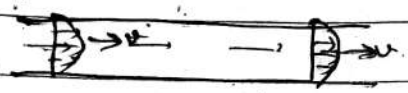


four combinations of flow types are:

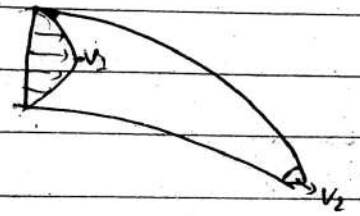
i) Steady uniform flow:

$$\frac{\partial \text{Parameter}}{\partial t}, \frac{\partial \text{Parameter}}{\partial s} = 0$$

eg: flow through prismatic pipe or channel with constant discharge



ii) Steady non-uniform flow:



flow characteristics remain invariant with time but vary from location to location.

iii) Unsteady uniform flow:

It is an impractical flow. The flow characteristics changes with time but uniformity requires changes of characteristics of all points at same time.

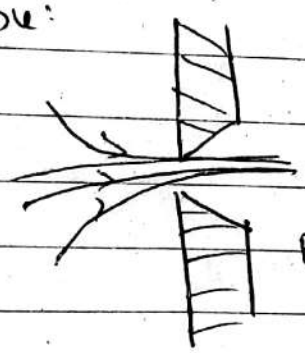
iv) Unsteady non-uniform flow:

both time and distance dependent Breaking of dam

One, two and 3 dimensional flow:

if: $u = f_1(x, y, z, t)$
$v = f_2(x, y, z, t)$
$w = f_3(x, y, z, t)$
} 3 ~~time~~ dimensional flow.

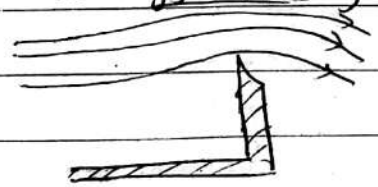
example:



Flow through orifice.

if $u = f_1(x, y, t)$
$v = f_2(x, y, t)$
$w = \cancel{f_3(x, y, t)}$ 0
} 2 dimensional flow

eg:



Flow through rectangular weir.

if $u = f_1(x, t)$
$v = 0$
$w = 0$
} 1 dimensional.

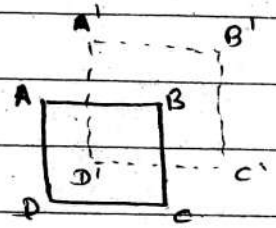
eg: central streamline flow in pipe where velocity is averaged.

Rotational and irrotational flow:

Flow in which each element of moving fluid undergoes no net rotation from instant to instant with reference to given frame of reference is known as irrotational flow.

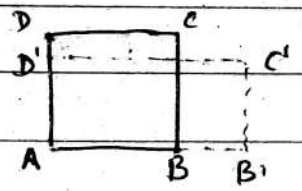
The flow is called rotational if the fluid element undergo rotation.

4 types of movement of ~~bt~~ fluid elements:

i) 

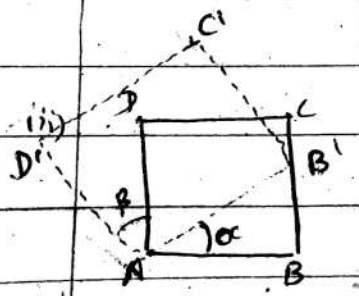
example: Mere shift of location of fluid elements. It occurs in all types of fluid flow.

a) Translation.

ii) 

example: It involves volume changes and only occurs in compressible flow.

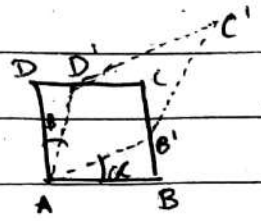
b) linear deformation

iii) 

example: Two ⊥r sides AB and AD of element rotates in same direction through angles α & β. If $\alpha = \beta$, it is uniform rotation

c) Rotation

iv)



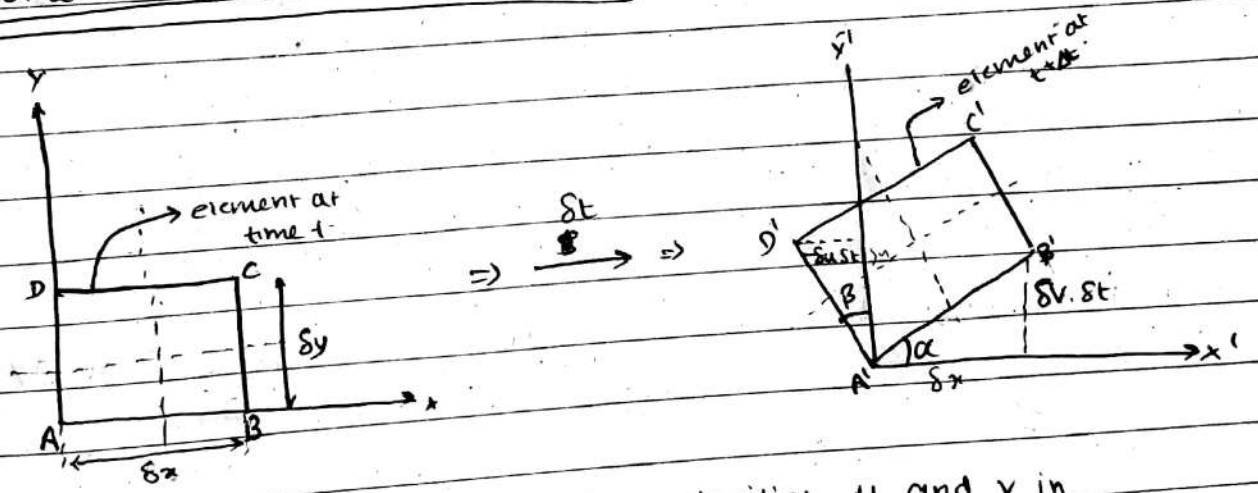
example:

Two ⊥r sides of element AB & AD turn through opposite direction.

d) angular rotation

Non-uniform rotations and angular deformations are caused by sheer shesses in the fluid flow. Since, sheer shess is associated with viscosity, it is evident that these two types of movement are present in all real fluid flows.

Mathematical derivation of rotation:



Let δu and δv be the change in velocities u and v in time δt. W.r.to axis A'x', the vertical displace of B is δv.δt. Horizontal displacement of D with respect to A'y' is $-\delta u.\delta t$

assuming small angles in radian,

$$\alpha = \frac{\delta v.\delta t}{\delta x}, \quad \beta = \frac{-\delta u \delta t}{\delta y}$$

in the limit, when the elements shrinks to the point,

$$\alpha = \frac{\partial v}{\partial x} \cdot \delta t$$

$$\& \ \beta = -\frac{\partial u}{\partial y} \cdot \delta t$$

since, rate of angular deformation is the rotation,

rotation of AB $= \frac{\alpha}{\delta t} = \frac{\partial v}{\partial x}$

rotation of AD $= \frac{\beta}{\delta t} = -\frac{\partial u}{\partial y}$

∴ rotation of entire fluid element about z axis is usually taken as average rotation of two 1ʳ axis.

So, rotation about z axis $= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$$\boxed{= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$

twice the rotation is known as VORTICITY (ξ) (xi)

z दिर $= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

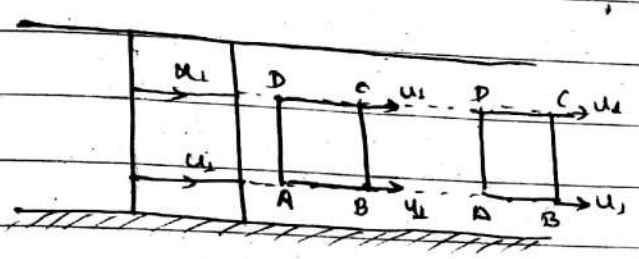
if the flow is 3-dimensional, two other components of vorticity exists,

x दिर, η (eta) $= \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$

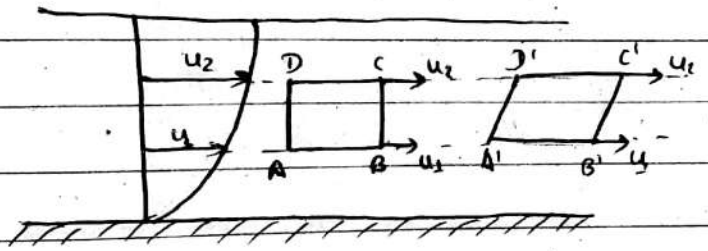
y दिर, ζ (zeta) $= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$

condition of irrotationality :

$$\left.\begin{aligned} \frac{\partial v}{\partial x} &= \frac{\partial u}{\partial y} \\ \frac{\partial w}{\partial y} &= \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} &= \frac{\partial w}{\partial x} \end{aligned}\right\}$$



uniform vel. dist (ideal fluid) no deformation.



uniform vel. dist (real fluids deformation.

y

effect of fluid viscosity in producing sheer stress is limited to narrow region closed to the boundary but in the vast region of fluid flow, away from the boundary, rotation is week and fluid behaves like ideal fluid.

* Examine: Whether the following velocity component represents rotational or irrotation. If flow is rotational, calculate vorticity at point (2,1)

$u = y^2 + 5x - \log_e x$

$v = 4y + x^2 - y/x$

so,

$\frac{\partial v}{\partial x} = 5 - \frac{1}{x}$

$\frac{\partial u}{\partial y} = 2y$

$\frac{\partial v}{\partial x} = 2x + \frac{y}{x^2}$

$\frac{\partial v}{\partial y} = 4 - 1/x$

at (2,1)

$= 5 - 1/2 = 4.5$

$= 2$

$= \cancel{3.75}\ 4.25$

$= 3.5$

Mean velocity and discharge:

↳ Q (denoted by)

Discharge (Q) – Rate of flow:

i) volume rate of flow. $Q (m^3/sec)$ – Cumecs. ⇒ Generally for liquid.

ii) Mass rate of flow $Q_m (kg/sec)$

iii) Weight rate of flow, $W (N/sec)$

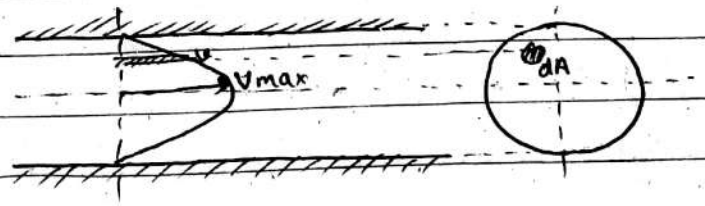
} ⇒ Generally for gases.

Mean velocity

Mean velocity across any cross section is defined as ratio of volume rate of flow to area of cross section.

$$V = \frac{Q}{A} = \frac{m^3/sec}{m^2} = (m/s)$$

If Q cannot be measured, then we need to find out velocity distribution.



dQ through dQ = VdA

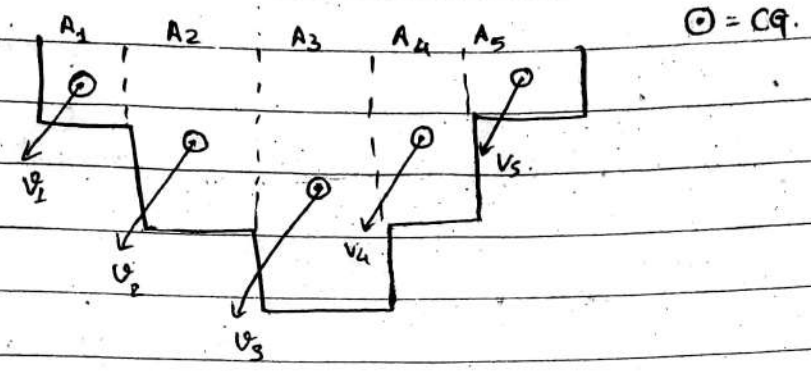
V = fluid velocity normal to area (if not take component)

$$Q = \int_A dQ = \int_A V.dA.$$

$$\therefore V_{av} = \frac{Q}{A} = \frac{1}{A}\int_A V.dA.$$

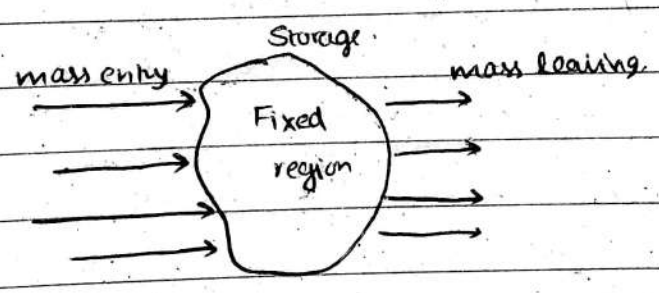
Or,

$$V = \frac{\sum v \cdot dA}{A}$$



$$V = \frac{V_1 A_1 + V_2 A_2 + V_3 A_3 + \ldots}{A_1 + A_2 + \ldots}$$

Principle of conservation of mass:



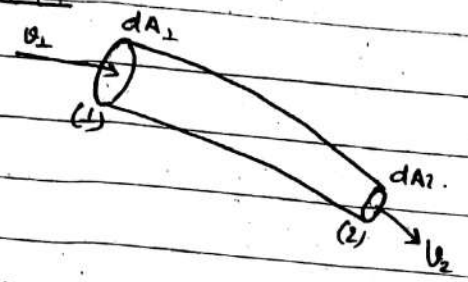
mass entering = mass leaving + mass stored

for steady case:

mass entering = mass leaving.

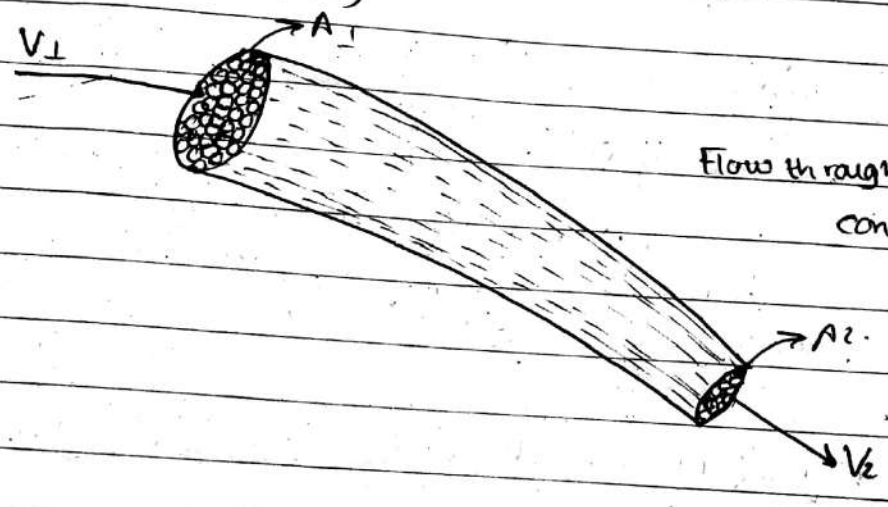
1-dimensional continuity equation:

Small pipe



flow through stream tube.

~~Sm~~ pipe
(composite of small pipes)



Flow through non-uniform conduit.

mass of fluid entering section 1 of the stream tube in unit time.

$$\frac{m}{t} = \rho\frac{V}{t} = \rho Q$$

$$= \rho Q = \rho_1 v_1 dA_1$$

mass of fluid leaving the section 2 per unit time.

$$= \rho_2 v_2 dA_2$$

assuming steady flow there will be no accumulation of fluid within the tube. So, according to continuity equation,

$$\rho_1 v_1 dA_1 = \rho_2 v_2 dA_2.$$

For non uniform conduit i.e cluster of stream tubes

$$\int_{A_1} \rho_1 V_1 dA_1 = \int_{A_2} \rho_2 V_2 dA_2$$

for incompressible fluid, $\rho_1 = \rho_2$

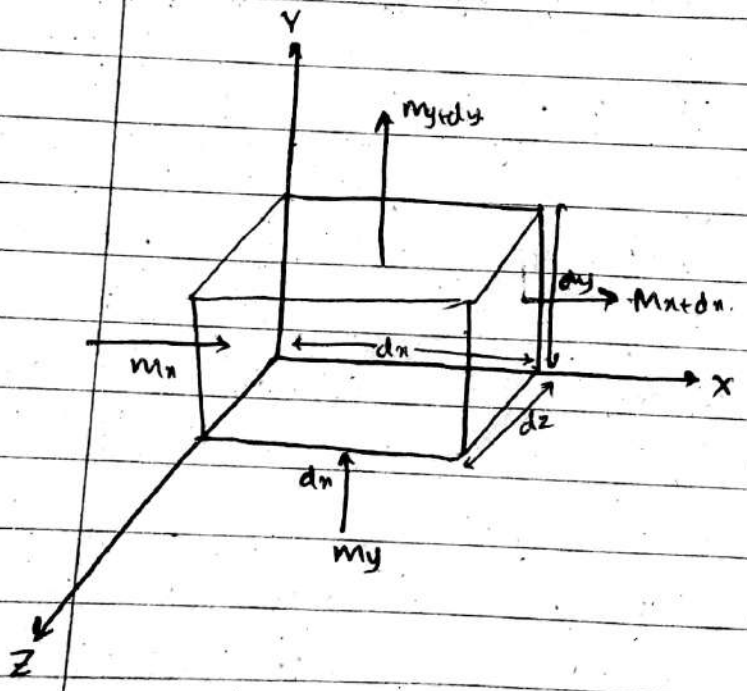
$$\Rightarrow \int_{A_1} V_1 dA_1 = \int_{A_2} V_2 dA_2$$

$$\boxed{A_1 V_1 = A_2 V_2 = \text{constant}}$$

$$\Rightarrow \boxed{Q = \text{constant}}$$

volume rate of flow is constant at any section.

Continuity equation in three dimension for cartesian co-ordinate system:



$$m_x + m_y + m_z = m_{x+dx} + m_{y+dy} + m_{z+dz} + m_{storage}$$

$$\Rightarrow \rho\, dz\,dy \cdot u + \rho\, dx\,dz \cdot v + \rho\, dy\,dx\, w$$

$$= \left(\rho + \frac{\partial \rho}{\partial x} \cdot dx\right)\left(u + \frac{\partial v}{\partial x} \cdot dx\right) dy\,dz$$

$$+ \left(\rho + \frac{\partial \rho}{\partial y} \cdot dy\right)\left(u + \frac{\partial v}{\partial y} \cdot dy\right) dx\,dz$$

$$+ \left(\rho + \frac{\partial \rho}{\partial z} \cdot dz\right)\left(w + \frac{\partial w}{\partial z} \cdot dz\right) dy\,dx$$

$dx^2 = 0$ (neglected) we get

in x-direction,

$$m_x = \rho u dy dz$$

$$m_{x+dx} = \rho u dy\, dz + \frac{\partial(\rho u)}{\partial x} . d\forall$$

so, net inflow of mass in x direction per unit time $= m_x - m_{x+dx}$

$$= -\frac{\partial(\rho u)}{\partial x} . d\forall$$

similarly,

net inflow of mass in y per unit time $= m_y - m_{y+dy}$

$$= -\frac{\partial(\rho v)}{\partial y} . d\forall$$

net inflow " " " z " " " " $= -\frac{\partial(\rho w)}{\partial z} . d\forall$

now,

$$m_{storage} = \frac{\partial m}{\partial t} = \frac{\partial(\rho d\forall)}{\partial t}$$

so,

$$-\frac{\partial}{\partial x}(\rho u) . d\forall - \frac{\partial(\rho v)}{\partial y} d\forall - \frac{\partial(\rho w)}{\partial z} d\forall = \frac{\partial \rho}{\partial t} . d\forall$$

$$\Rightarrow \boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial(\rho v)}{\partial y} + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0}$$

this continuity eq^n for compressible and unsteady flow.

for incompressible & steady flow,

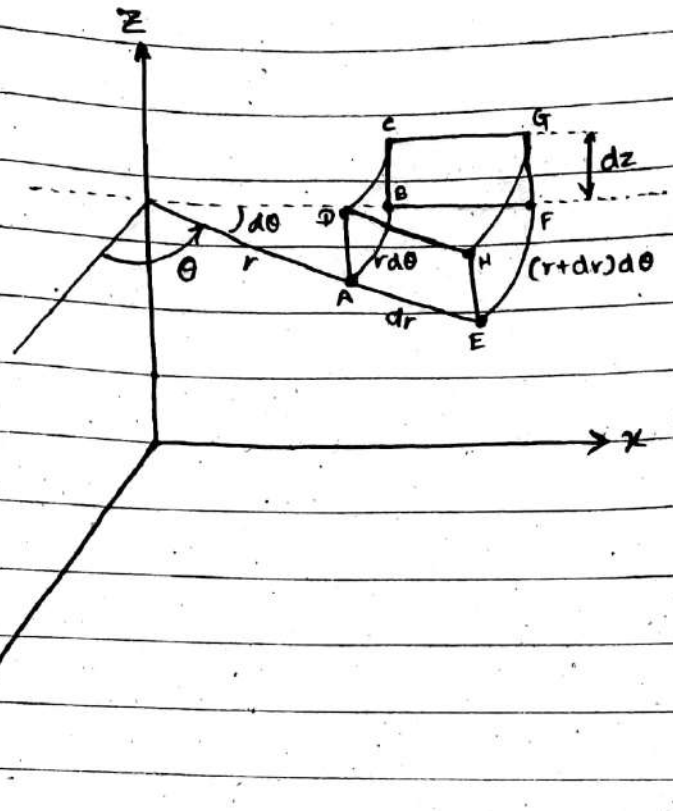
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

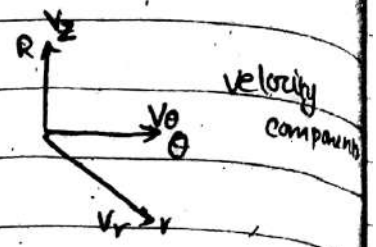
for 1 dimensional,

$$A\frac{\partial u}{\partial x} = 0 \times A \quad \text{(multiplying by A)}$$

$$\Rightarrow \frac{\partial(Au)}{\partial x} = 0 \Rightarrow Au = \text{Constant}$$

Continuity equation in cylindrical co-ordinate system





$$m_r + m_\theta + m_z = m_{r+dr} + m_{\theta+d\theta} + m_{z+dz} + m_{storage}$$

$$m_r = \rho . v_r (r d\theta . dz)$$

$$m_{r+dr} = \left(\rho v_r (r d\theta . dz)\right) + \frac{\partial}{\partial r}\left(\rho v_r (r d\theta . dz)\right) . dr$$

net inflow in r direction per unit time $= m_r - m_{r+dr}$

$$= -\frac{\partial}{\partial r}\left(\rho v_r (r d\theta . dz)\right) dr$$

similarly in $\theta = m_\theta - m_{\theta+d\theta}$

$$= \frac{-\partial}{\partial \theta}\left(\rho v_\theta (dr . dz)\right) d\theta$$

similarly in $z = m_z - m_{z+dz}$

$$= \frac{-\partial}{\partial z}\left(\rho v_z (r d\theta . dr)\right) dz$$

mass ~~storage~~ $= \frac{\partial}{\partial t}(\rho r d\theta \cdot dz \cdot dr)$.

so,

$$+\frac{\partial(\rho V_r \cdot r)}{\partial r} + \frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z \cdot r) + \frac{\partial \rho (r)}{\partial t} = 0.$$

$$\Rightarrow \boxed{\frac{1}{r}\frac{\partial}{\partial r}(\rho V_r \cdot r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) + \frac{\partial \rho}{\partial t} = 0}$$

this is
for ~~an~~ compressible, unsteady flow.

For incompressible and steady flow:

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}(r V_r) + \frac{\partial}{r\partial \theta}(V_\theta) + \frac{\partial(\rho V_z)}{\partial z} = 0}$$

* Prove that the velocity of the field

$$\vec{V} = 2ax(3y^2 - x^2)\vec{i} + 2ay(3x^2 - y^2)\vec{j}.$$

represents possible incompressible fluid flow:

$\Rightarrow$ If it is possible, it must follow the continuity eqn in 2-dimensional flow ~~is~~ for incompressible fluid i.e.

ρ is constant and:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\cancel{\Rightarrow 2ax3y^2 - 4ax^2 + 12axy - 6ay^2 = 0.}$$

$$\cancel{\Rightarrow 6ay^2 - 6ay^2 = 6ax^2}$$

which is true so, it is possible and fisible.

Stream function / Velocity potential function:

Stream function (Ψ, psi)

We must obtain solution of three partial differential equation to calculate unknowns i.e. u, v and P.(pressure) so, due to complexity of the sol^n, the partial derivatives are reduced to a single function capable of describing two velocity components of flow field.

If $\psi_2 > \psi_1$, Q is positive & flow is left to right & vice-versa.

<u>Velocity potential function</u> (VPH) (ϕ)

A velocity potential function for steady irrotational flow in the x-y plane is defined as function of (x, y, z) such that the partial derivative of ϕ with respect to displacement in any choosen direction = velocity in that direction.

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{d\phi}{dz}$$

Checking with the continuity equation.

i.e.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial z}\right) = 0$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

now, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\Rightarrow \boxed{\nabla^2 \phi = 0}$ this is known as the Laplace equation

now,

$$\zeta \text{ (zeta) (vorticity)} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

1) potential flow satisfies the condition of irrotationality i.e. existance of velocity potential implies that flow is irrotational.

2) Difference between ϕ and ψ functions lies in the fact that ϕ function exists only for irrotational flow while ψ function are not restricted to irrotational flow.

3) Velocity potential can be defined for general 3-d flow while stream function is restricted to 2-d only.

<u>Alternatively:</u>

* $u = \frac{\partial \psi}{\partial y}$, $v = \frac{\partial \psi}{\partial x}$

$\Rightarrow \frac{\partial u}{\partial y} = \frac{\partial^2 \psi}{\partial y^2}$, $\frac{\partial v}{\partial x} = \frac{\partial^2 \psi}{\partial x^2}$

$$\Rightarrow \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \underbrace{\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} = 0}_{\text{Laplace eqn.}}$$

Laplace equation which must be satisfied is flow is to be potential flow.

* assume $\phi(x,y) = \text{constant}$

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy = 0$$

$$\Rightarrow u\,dx + v\,dy = 0.$$

$$\Rightarrow \frac{dy}{dx} = -\frac{u}{v} \quad \cdots\cdots \text{(i)}$$

* if $\psi(x,y) = \text{constant}$

$$d\psi = \frac{\partial \psi}{\partial y} \times dy - \frac{\partial \psi}{\partial x} \times dx = 0$$

$$= \cancel{udy - vdy + udx}$$

$$\Rightarrow v\,dx - u\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = +\frac{v}{u}$$

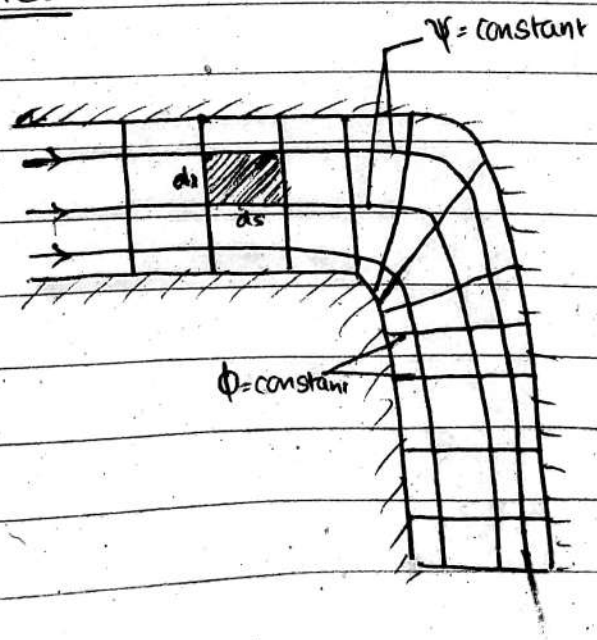
slope of $\psi(x,y)$ and $\phi(x,y)$ when multiplied, gives -1.

so ψ and ϕ are $\perp^r$.

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

Flow net:



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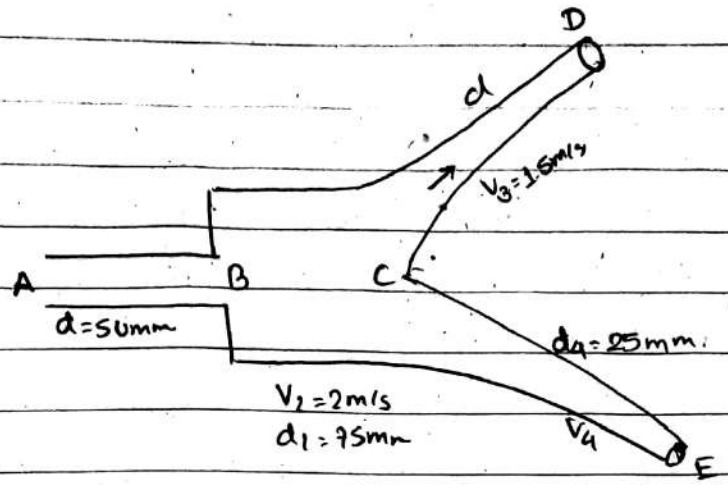


If discharge through CD is twice the discharge through CE, calculate,

① Discharge in pipe AB

② Velocity in AB.

③ Discharge in CD.

④ Velocity in CE

⑤ Diameter of CD.

$Q_{AB} = Q_{BC} = Q_{CD} + Q_{CE}$.

$\Rightarrow \pi d_A^2 \times V_A = \pi d_2^2 \times 2$.

$\Rightarrow \cancel{\pi} \times 50^2 \times V_A = \cancel{\pi} \times 75^2 \times 2$

$V_A = 4.5 m/s$.

$\Rightarrow \pi \times 50^2 \times 4.5 = \pi \times 25^2 \times V_4 + \pi \times d^2 \times 1.5$.

$\Rightarrow 11250 = 625 V_4 + 1.5 d^2$. ---- ①

also,

$\cancel{\pi \times 75^2 \times 2 = \pi \times 25^2 \times V_4 + \pi \times d^2 \times 1.5}$

$\Rightarrow$

also, $\pi d^2 \times V_3 = 2 \pi \times 25^2 \times V_4$.

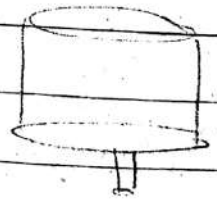
$\Rightarrow \pi d^2 \times 1.5 = 2\pi 25^2 \times V_4$

$\Rightarrow d^2 = \frac{2500}{3} V_4$ --- ⓘ

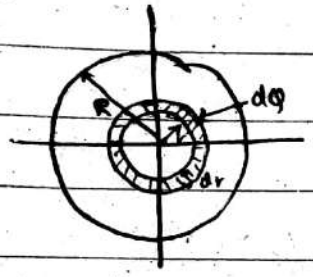
so, $11250 = 1875 V_4 \Rightarrow \therefore V_4 = 6 m/s$

so, $d = 70.710 m/s$

* Oil is flowing through a large tank having a diameter of 4m, the velocity of ~~water~~ oil relative to tank is given by $v = 5 - r^2$ m/s. where 'r' is the radius of tank. Calculate, the velocity of the oil leaving by the pipe, having inside diameter of 1.5m.



~~$2.75 \times \pi \times 1.5^2 \times 800 = \pi \times 1.5^2 \times v \times$~~



$dQ = 2\pi r dr \times (5 - r^2)$

$\Rightarrow Q = \int_0^R 10\pi r\, dr - \int_0^R 2\pi r^3 dr$

$= 10\pi \left[\frac{(r^2)}{2}\right]_0^2 - 2\pi \left(\frac{r^4}{4}\right)_0^2$

$= 10\pi \times 2 - 2\pi\, 4.$

$= 12\pi$

$= 37.7\ m^3/s.$

$Q = A v_{oil}$

$\Rightarrow \frac{37.7}{\pi \times 0.75^2} = v_{oil}.$

$v_{oil} = 21.33$ m/s.

* Show that $u = \frac{-2xyz}{(x^2+y^2)^2}$

$v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}$

$w = \frac{2xy}{(x^2+y^2)^2}$.

are velocity components of possible fluid flow.

→ continuity must be valid.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{--- (1)}$$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^2)^2(-2yz) + 2xyz\,[2(x^2+y^2).(2x)]}{(x^2+y^2)^4}$$

$$= \frac{-2yz(x^2+y^2)^2 + 8x^2yz\,(x^2+y^2)}{(x^2+y^2)^4}$$

$$= \frac{-2yz(x^2+y^2) + 8x^2yz}{(x^2+y^2)^3}$$

$$= \frac{-2x^2yz - 2y^3z + 8x^2yz}{(x^2+y^2)^3} = \frac{6x^2yz - 2y^3z}{(x^2+y^2)^3}.$$

$$\frac{\partial v}{\partial y} = \frac{(x^2+y^2)^2.(-2yz) - (x^2-y^2)z\,[2(x^2+y^2)(2y)]}{(x^2+y^2)^4}$$

$$= \frac{-2yz\,(x^2+y^2)^2 - 4y(x^2-y^2)(x^2+y^2)}{(x^2+y^2)^4}$$

$$= \frac{-2yz(x^2+y^2) - 4yz\,(x^2-y)}{(x^2+y^2)^3}$$

$$= \frac{-2yz\,(x^2+y^2-2x^2+2y^2)}{(x^2+y^2)^3} = \frac{-2yz\,(3y^2-x^2)}{(x^2+y^2)^3}$$

* A two dimensional flow can be described by:

$$u = \frac{-y}{b^2}, \quad v = \frac{x}{a^2}$$

verify that, this is a flow of an incompressible fluid and that the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a stream line.

→ using continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\Rightarrow 0 + 0 = 0$$

So is a incompressible flow.

equation of stream line is:

$$\frac{dy}{dx} = \frac{v}{u}$$

so,

$$\frac{dy}{dx} = \frac{x/a^2}{-y/b^2}$$

$$\Rightarrow \frac{-y}{b^2}dy = \frac{x}{a^2}dx$$

$\int$ integrate

$$= \frac{x^2}{a^2} + \frac{y^2}{b^2} = K.$$

for $K=1$, it is a streamline.

* Velocity component for two dimensional incompressible flow is given by: $u = 2x+y$, $v = x-2x$.

i) Is continuity satisfied, if so, find stream function.

ii) Is this flow irrotational, if so, find potential function

→ Here,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

so flow is possible.

$$u = \frac{\partial \psi}{\partial y}$$

$$\Rightarrow (2x+y)\partial y = \partial \psi$$

$$\Rightarrow \psi = 2xy + \frac{y^2}{2} + f(x)$$

diff w.r.to x.

$$\frac{\partial \psi}{\partial x} = 2y + f'(x)$$

also, $\cancel{v = -\frac{\partial \psi}{\partial x}}$

$\cancel{\Rightarrow v = 2y + f'(x)}$

$$\therefore \psi = 2xy + \frac{y^2}{2} + \frac{x^2}{2} + c$$

$$\Rightarrow x + 2y = 2y + f'(x) \Rightarrow x = f'(x) \Rightarrow f(x) = x^2/2 + c$$

for irrotationality:

check for ~~ζ~~ vorticity,

$\zeta =$

* A fluid flow is characterized by a streamfunction,
$\psi = y^3 x$. Determine velocity potential function ϕ.

$\Rightarrow$ Solution

$$-\frac{\partial \psi}{\partial x} = v$$

$$\Rightarrow -y^3 = v$$

and, $\frac{\partial \psi}{\partial y} = u$

$$\Rightarrow 3xy^2 = u$$

check for irrotationality,

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\Rightarrow -0 - 6xy$$

$$\neq 0.$$

so, potential function ϕ doesn't exist.

flow exists but is not ~~irrotion~~ irrotational.

* Given stream function $\psi = \frac{y^3}{3} - x^2 y$.

i) What is the velocity vector which describes the flow
ii) Show that flow is laplacian.
iii) What is ϕ.

$$v = -\frac{\partial \psi}{\partial x} = +2xy.$$

$$u = \frac{\partial \psi}{\partial y} = y^2 - x^2.$$

now, check for irrotionality,

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 2y - 2y = 0$$

so, ϕ exists.

So the velocity vector is

$$u = 2xy\vec{j} + (y^2 - x^2)\vec{i}$$

Laplacian check.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial^2 y} = 0$$

$\Rightarrow$

now,

$$\frac{\partial \phi}{\partial x} = u$$

$$\Rightarrow \partial \phi = (y^2 - x^2)\partial x.$$

$$\Rightarrow \phi = y^2 x - \frac{x^3}{3} + f(y)$$

$$\Rightarrow \frac{\partial \phi}{\partial y} = 2yx + f'(y)$$

$$\Rightarrow 2xy = 2xy + f'(y) \Rightarrow 0 \quad \text{so, } f(y) = C$$

so,

$$\boxed{\phi = y^2 x - \frac{x^3}{3} + c}$$

* Given:

$\vec{V} = Ax\vec{i} - Ay\vec{j}$

when,

$A = 0.3 s^{-1}$.

Determine stream function ψ and plot in the first and second Quadrant and interpret the results.

here,

$u = 0.3x$.

$v = -0.3y$.

$\frac{\partial u}{\partial x}$ at $0.3s = 0.3$.

$\frac{\partial u}{\partial y}$ at $0.3s = -0.3$.

~~so, $\phi = \partial u$ $\frac{\partial \phi}{\partial x} = \phi . u$.~~

~~$\Rightarrow \partial \phi = u \partial x$.~~

~~$\Rightarrow \phi = \frac{0.3x^2}{2} + f(y)$~~

$\frac{\partial \psi}{\partial y} = u$

$\Rightarrow \partial \psi = 0.3x \partial y$.

$\Rightarrow \psi = 0.3xy + f(x)$

$\Rightarrow -\frac{\partial \psi}{\partial x} = -0.3y - f'(x)$

$\Rightarrow \cancel{0.3y} = -0.3y - f'(x)$

~~$\Rightarrow -(0.3y + 0.3xy) = f'(x)$~~ $f'(x) = 0$

Acceleration of fluid particles

$$\vec{v} = f(x, y, z, t)$$

$$\Rightarrow d\vec{v} = \frac{\partial \vec{v}}{\partial x} \cdot dx + \frac{\partial \vec{v}}{\partial y} \cdot dy + \frac{\partial \vec{v}}{\partial z} \cdot dz + \frac{\partial \vec{v}}{\partial t} \cdot dt$$

$$\Rightarrow \frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial \vec{v}}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial \vec{v}}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial \vec{v}}{\partial t} \cdot \frac{dt}{dt}$$

$$\Rightarrow \boxed{\vec{a} = u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z} + \frac{\partial \vec{v}}{\partial t}}$$

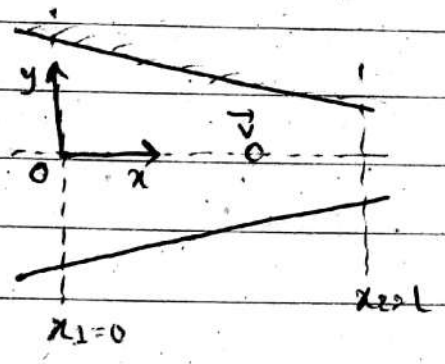
this is the real acceleration or (TOTAL ACCELERATION)

where,

total acceleration = convective acceleration + local acceleration

$$= \left(u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z}\right) + \left(\frac{\partial \vec{v}}{\partial t}\right)$$

*



If $\vec{v} = V_1 \times (1 + x/L)\,\vec{i}$ on x-axis

So, calculate

i) acceleration of the particle, P moving along the x-axis.

ii) for particle located at $x = 0$ at $t = 0$, express its position as a function of time

iii) X-component of acceleration

as a function of time

$$\vec{v} = V_1 (1 + x/L)\, i$$

now,

$$\vec{a} = \frac{\partial \vec{v}}{\partial x} \times \frac{dx}{\partial t} + \cancel{\frac{\partial \vec{v}}{\partial y} \times \frac{dy}{\partial t}}^{0} + \cancel{\frac{\partial \vec{v}}{\partial z} \times \frac{dz}{\partial t}}^{0} + \cancel{\frac{\partial \vec{v}}{\partial t}}^{0}$$

$$= \frac{V_1}{L} \cdot V_1 (1 + x/L)\, \vec{i}$$

$$= \frac{V_1^2}{L} (1 + x/L)\, \vec{i}$$ this is eulerian acceleration.

we know,

$$\vec{v} = \frac{dx}{dt} = V_1 (1 + x/L)$$

$$\Rightarrow \frac{L\,dx}{L + x} = V_1\, dt$$

$$\Rightarrow L \log(L + x) = V_1 t$$

$$\Rightarrow \log(L + x) = \frac{V_1 t}{L}$$

$$L + x = e^{V_1 t/L}$$

$$\boxed{\therefore x = e^{V_1 t/L} - L}$$

again, acceleration.

$$\vec{a} = \frac{\partial^2 x}{\partial t^2} = \frac{d}{dx} \frac{V_1}{L} e^t = \frac{V_1}{L} e^t$$ this is lagrangian

KINETICS

$\boxed{\Sigma F = \text{Reynolds eq}^n}$

$$d\vec{F} = dm \cdot \frac{d\vec{v}}{dt}$$ → total acceleration

$$\Rightarrow d\vec{F} = dm \left[u \frac{\partial \vec{v}}{\partial x} + v \frac{\partial \vec{v}}{\partial y} + w \frac{\partial \vec{v}}{\partial z} + \frac{\partial \vec{v}}{\partial t} \right] \quad \text{.....} \quad (i)$$

forces

Newtons equation

Forces
- → Body forces
  - → gravity
  - → Electric / Magnetic forces
- → Surface forces
  - → shear forces
  - → pressure
- → linear forces
  - → surface tension

$\Sigma F = F_g + F_p + F_v + F_{ST} + F_{turbulance}$

is the reynold's eq^n.

but F_{ST} and $F_{turbulance}$ is very small in engineering projects.

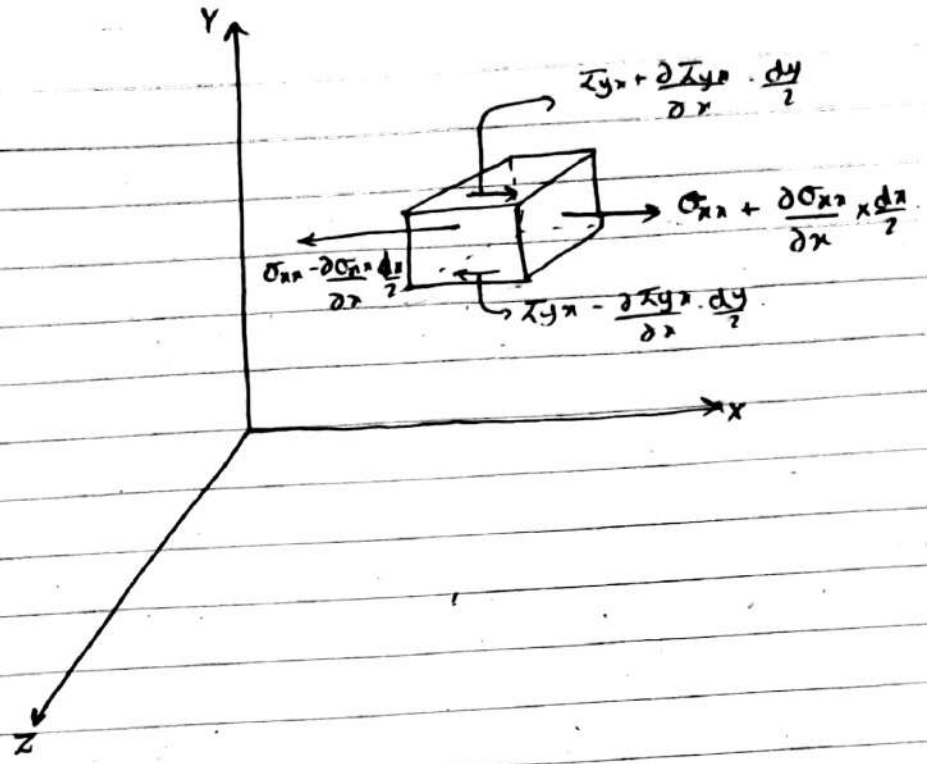
but:

$$\boxed{F = F_g + F_p + F_{Vis}}$$

↳ ~~stoke~~ Navier stokes eq^n.

$$\boxed{F = F_g + F_p}$$

↳ Euler's eq^n.



Net force in x-direction:

$$dF_{sx} = \left(\sigma_{xx} + \frac{\partial}{\partial x}\sigma_{xx} \cdot \frac{dx}{2}\right) dy\,dz - \left(\sigma_{xx} - \frac{\partial}{\partial x}\sigma_{xx} \frac{dx}{2}\right) dy\,dz$$

$$+ \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial x} \cdot \frac{dy}{2}\right) dx\,dz - \left(\tau_{yx} - \frac{\partial \tau_{yx}}{\partial x} \cdot \frac{dy}{2}\right) dx\,dz$$

$$+ \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \cdot \frac{dz}{2}\right) dx\,dy - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \frac{dz}{2}\right) dx\,dy$$

$$\Rightarrow dF_{sx} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) dx\,dy\,dz$$

$$dF_{gx} = \rho g_x (dx\,dy\,dz).$$

$$\cancel{dF_x = dF_{sx} + dF_{gx} = (\rho}$$

$$dF_x = dF_{sx} + dF_{gx}$$

$$= \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) dx\,dy\,dz$$

$$dF_y = \left(\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) dx\,dy\,dz$$

$$dF_z = \left(\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) dx\,dy\,dz$$

in x-direction.

$$dF_x = dm \cdot \left[\frac{\partial u}{\partial t} + \frac{u\,\partial u}{\partial x} + \frac{v\,\partial u}{\partial y} + \frac{w\,\partial u}{\partial z} \right]$$

$$\Rightarrow \rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho \left[\frac{\partial u}{\partial t} + \frac{u\,\partial u}{\partial x} + \frac{v\,\partial u}{\partial y} + \frac{w\,\partial u}{\partial z} \right]$$

newtonian, $\tau_{yx} \propto \frac{du}{dy}$

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

$$\sigma_{xx} = -p - \tfrac{2}{3}\mu \nabla \cdot \vec{V} + 2\mu \frac{\partial u}{\partial x}$$

so,

$$\boxed{\rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\mu \left(2\frac{\partial u}{\partial x} - \tfrac{2}{3} \nabla \cdot \vec{V} \right) \right] + \frac{\partial}{\partial y}\left[\mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] + \frac{\partial}{\partial z}\left[\mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right]}$$

nebier stoke eq^n in x-direction

So, for incompressible flow with constant viscosity, the nebier stoke eq^n becomes,

In x-axis:

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

In y-axis:

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

In z-axis:

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

vectorially:

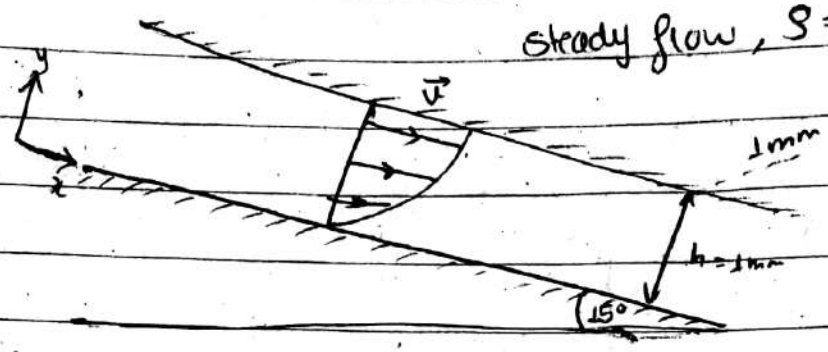
total acceleration

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla P + \mu\nabla^2\vec{V}$$

Do you have exchange 4 5067

NO

Steady flow, ρ = constant (Incompressible)



i) continuity and nebier stoke equation simplified to this model.

ii) velocity profile.

iii) Shear shers distribution

iv) Discharge per unit depth

v) Average flow velocity.

Continuity eqn

$$\cancel{\frac{\partial u}{\partial x}}^{0} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}}^{0} = 0$$

$$\Rightarrow \frac{\partial v}{\partial y} = 0$$

Now,

Nebier stoke equation:

$$\rho\left[\cancel{\frac{\partial u}{\partial t}}^{0} + \cancel{u\frac{\partial u}{\partial x}}^{0} + \cancel{v\frac{\partial u}{\partial y}}^{0} + \cancel{w\frac{\partial u}{\partial z}}^{0}\right] = \rho g_x - \cancel{\frac{\partial p}{\partial x}}^{0} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 v}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{0}\right)$$

$$\rho g_x - \mu\frac{\partial^2 v}{\partial y^2} = 0 \quad \text{--- (i)}$$

In y-axis:

$$0 = \rho g_y - \frac{\partial P}{\partial y} + 0$$

$$\Rightarrow \frac{\partial P}{\partial y} = \rho g_y \quad \ldots\ldots \text{(ii)}$$

From (i):

$$\frac{\partial^2 u}{\partial y^2} = \frac{-\rho g_x}{\mu} = -\frac{\rho g \sin\theta}{\mu}$$

$$\cancel{\Rightarrow \frac{\partial u}{\partial y} = -\rho g_x \, dy'}$$

$$\frac{du}{dy} = -\rho g \frac{\sin\theta}{\mu} y + C_1$$

$$u = -\rho g \frac{\sin\theta}{\mu} \frac{y^2}{2} + C_1 y + C_2$$

$x = 0, u = 0$ $y = h, \frac{du}{dy} = 0$

$$0 = -\rho g \frac{\sin\theta}{\mu} \cdot h + C_1 \Rightarrow C_1 = \frac{\rho g \sin\theta h}{\mu}$$

$$\therefore u = \rho g \frac{\sin\theta}{\mu} \left(hy - y^2/2\right)$$

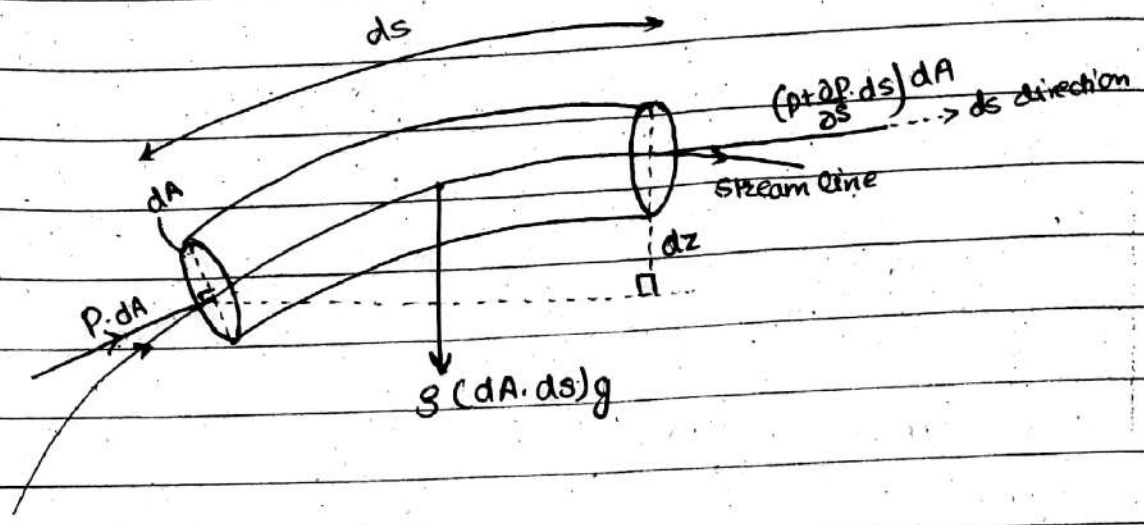
$$\tau = \mu \frac{du}{dy} = \mu \times \left(\frac{\rho g \sin\theta h}{\mu} - \frac{\rho g \sin\theta}{\cancel{2}\mu} \times \cancel{2} y\right)$$

$$= \cancel{\mu} \frac{\rho g \sin\theta}{\cancel{\mu}} (h - y)$$

$$= \rho g \sin\theta (h - y)$$

$$Q = \int u \cdot dA = \frac{\rho g \sin\theta}{\mu} \cdot \frac{bh^3}{3} \qquad \bar{V} = \frac{Q}{A} = \frac{Q}{bh}$$

Euler's equation in one dimensional flow:



Flow of an ideal incompressible fluid along a streamline is considered

$$\sum F_{s\text{-direction}} = m \times a_{s\text{-direction}}$$

$$\Rightarrow \quad p.dA - \left(p + \frac{\partial p}{\partial s}.ds\right)dA + \rho.dA.ds.S = \rho(dA.ds)a_{total}$$

S = body force per unit mass i.e. $\rho(dA.ds)g$ को cos component.

$$\boxed{a_t = -\frac{1}{\rho}\frac{\partial p}{\partial s} + S}$$

$$\Rightarrow \quad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} = S - \frac{1}{\rho}\frac{\partial p}{\partial s}$$

euler's eqn is 1-dimension flow

force due to local acceleration per unit mass

force due to convective acceleration per unit mass

pressure force per unit mass

body force per units mass

steady flow:

$$\frac{\partial v}{\partial t} = 0.$$

$$\Rightarrow \frac{1}{\rho}\frac{dP}{ds} + v\frac{dv}{ds} - S = 0.$$

$$\Rightarrow \rho(dA.ds).S = -\rho g(dA\,ds)\cos\theta.$$

$$\therefore S = -g\cos\theta$$

$$= -g\frac{dz}{ds}$$

So,

$$\boxed{\frac{1}{\rho}\frac{dP}{ds} + v\frac{dv}{ds} + g\frac{dz}{ds} = 0}$$

this is one dimensional euler's eq^n for steady flow:

Bernoulli's equation from euler's equation

integrating euler's equation,

$$\frac{1}{\rho}\int\frac{dP}{ds}ds + \int v\frac{dv}{ds}ds + \int g\frac{dz}{ds}ds = \text{constant}.$$

$$\Rightarrow \frac{1}{\rho}\int dP + \frac{v^2}{2} + g\int dz = \text{constant}.$$

$$\Rightarrow \frac{P}{\rho} + \frac{v^2}{2} + gz = \text{constant}.$$

$$\Rightarrow \boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}}$$

flow energy per unit weight (pressure head)

kinetic energy per unit weight (velocity head)

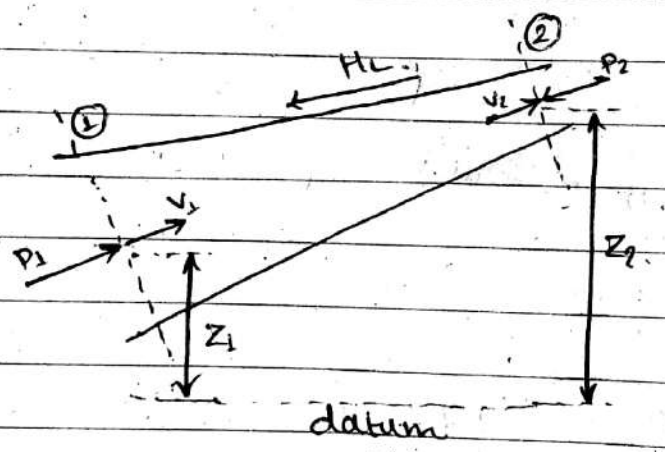
potential energy per unit ~~mass~~ weight (datum head or elevation head)

C = bernoulli's constant a total head. It is ~~not~~ NOT DIMENSIONLESS

Statement " In an ideal and incompressible fluid, when the flow is steady, and continuous the sum of the ~~potential head~~, pressure head, velocity head and elevation head (sum of flow energy, KE & PE per ~~at~~ unit weight) of the fluid is constant at all points in the flow system

Application of bernoulli's equation to the real fluid problems:

a)



a) loss of head in real fluid system.

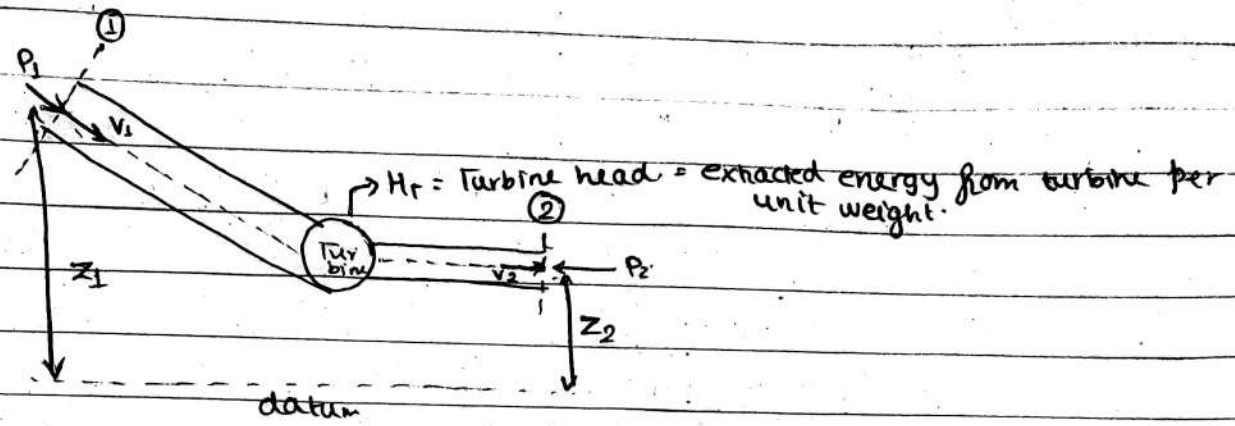
applying bernoulli's equation between section ① and ②,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 - H_L = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + H_L \rightarrow \text{Head loss.}$$

Head loss - energy is wasted in real fluids due to frictional effects per unit weight of fluid i.e called loss of head due to friction.

b)



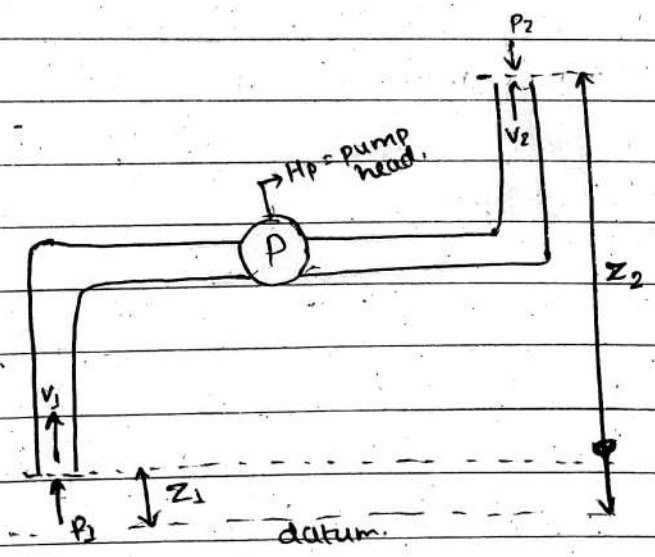
b) Turbine in flow system.

now, applying bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 - H_T \overset{-H_L}{=} \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + H_L + H_T$$

c)



H_P = added energy per unit weight by pump.

c) Pump in flow section.

applying bernoullis equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + H_P = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + H_L$$

combinining:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + H_p = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + H_L + H_T$$

Power produced by pump or turbine:

$$\Rightarrow \quad P = \gamma Q H$$

$$\Rightarrow \quad \boxed{P = \eta \rho g Q H_{P/T}} \text{ watt}$$

η = efficiency.

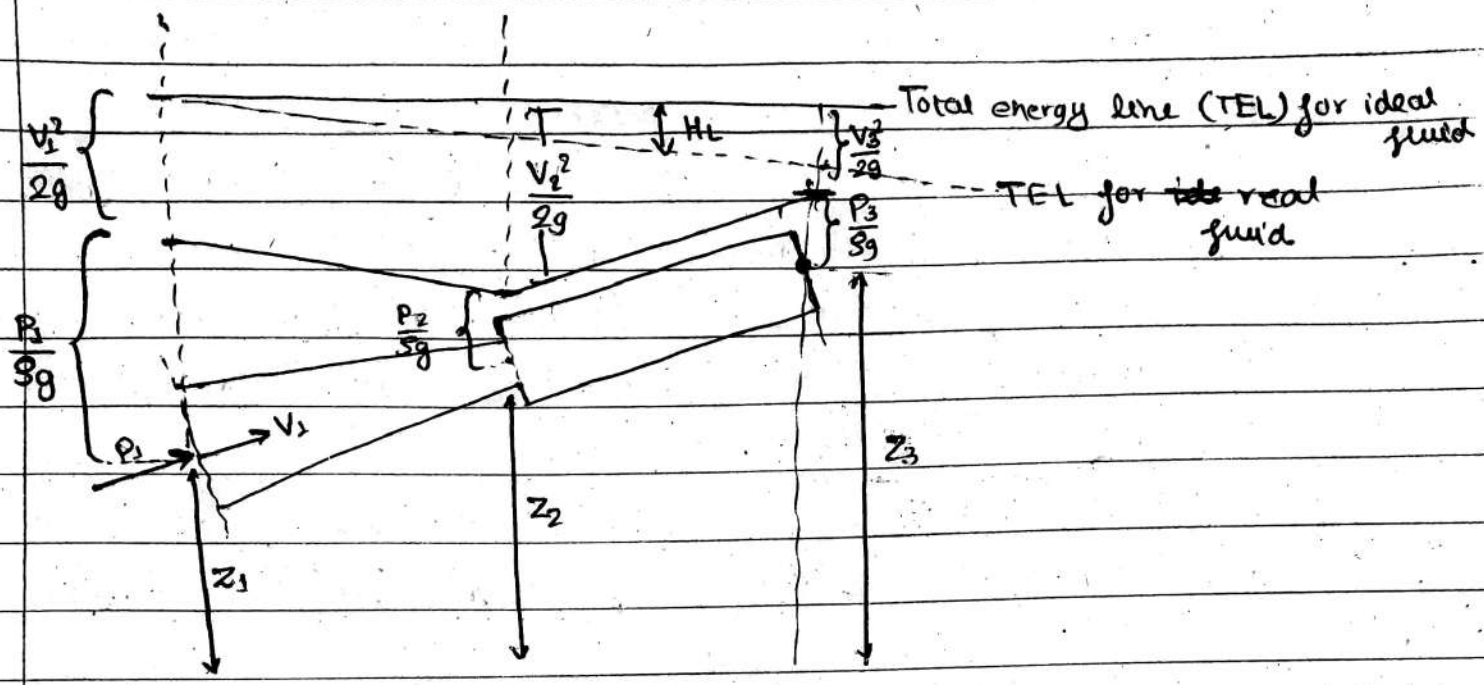
ρ = density

g = acc due to gravity.

Q = discharge.

$H_{P/T}$ = Pump head or Turbine head

Total energy line (TEL) / Hydraulic Gradient line (HGL)



$\frac{P}{\rho g} + z$ = Piezometric head. $\longrightarrow$ Hydraulic Gradient line (HGL)

$\left(\frac{P}{\rho g} + z\right) + \frac{V^2}{2g}$ = Total head $\rightarrow$ Total energy line (TEL)

~~1000~~ = ~~100~~ 1 m^3

Numerical:

* A pipe carying crude oil of SG = 0.84 is of 25cm diameter at section (1) and 50cm diameter at section (2). The rate of flow is 450 l/s. Section 1 is at an elevation of 25m above (2). If the pressure of the fluid at two section are 55 kPa at (1) and 320 kPa at (2) respectively. Find the direction of the flow and head lost between the section.

→ For section (1).

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z$$

$$= \frac{P}{840 \times 9.81} + \frac{2.291^2}{2 \times 9.81} + 0.25$$

$$= \frac{55000}{840 \times 9.81} + \frac{2.291^2}{2 \times 9.81} + 0.25$$

$$= 6.9913 \text{ J/N}$$

$$\frac{V}{t} = 450 \text{ l/s}$$

$$A.V = 450 \text{ l/s}$$

$$= \frac{0.45 m^3}{s}$$

$$\Rightarrow \pi \times 0.25^2 \times V = 0.47$$

$$V = 2.291 \text{ m/s}$$

for section (2).

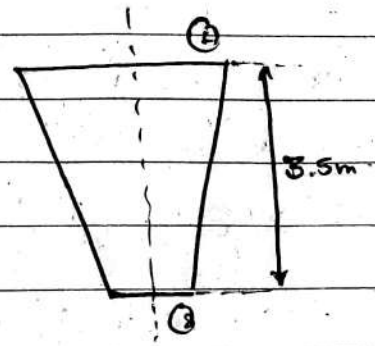
$$\frac{320000}{840 \times 9.81} + \frac{0.5729^2}{2 \times 9.81} + 0$$

$$= 38.84979 \text{ J/N}$$

$$0.45 \frac{m^3}{s} = \pi \times 0.5^2 \times V$$

$$\Rightarrow V = 0.5729$$

* Flow of water occurs in downward direction through a vertical tapering pipe as shown in figure. The height of the taper is 3.5m and diameter of section (1) is 20cm. The rate of flow is 10000 lpm. What should be diameter d, if $P_1 = P_2$, neglect loss of head. What would be the value of d if the flow is upward. Does, the result change if any liquid other than water flows through the pipe.



$H_1 = H_2$

$$\Rightarrow \cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} + 3.5 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + 0$$

$$\Rightarrow \frac{V_1^2}{2g} + 3.5 = \frac{V_2^2}{2g} \quad \text{--- (1)}$$

now,

$$Q = 10000 \, l/m = \frac{10}{6} m^3/s$$

$Q = AV_1 = AV_2$ similarly,

$\Rightarrow Q = 0.3141 \cdot V_1$ $\qquad V_2 = \frac{4}{6 \times \pi d^2}$

$\Rightarrow V_1 = 5.3 \, m/s$ $\qquad = \frac{2}{3\pi d^2}$

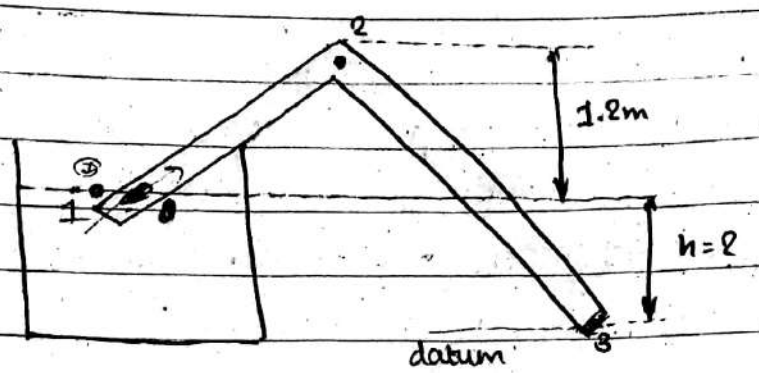
putting in (1),

$$\frac{5.3^2}{2 \times 9.81} + 3.5 = \frac{4}{9\pi^2 d^4 \times 2 \times 9.81}$$

$$d = 14.68 \, cm$$

if $d < 14.68$, then, flow is upward

so, $V_2 = 9.84 \times 10^{-4} \, m/s$



A liquid of sp. gravity 1.52, discharges from a tank through a pipe, whose summit point is 1.2m above water level in the tank. Atmospheric pressure = 101 KPa. Vapour pressure of liquid = 28 KPa, (abs). How far below the liquid level in the tank, the outlet be ~~seno~~ safely located. What is the maximum discharge? Neglect loss of head.

($\phi = 10$ cm)

applying bernoulis equation in ② and ③.

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + (1.2+h) = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + 0$$

$$\Rightarrow \frac{28000}{\rho g} + (1.2+h) = \frac{101000}{\rho g} + 0$$

$\Rightarrow$ $h = 3.695$ m. this is the safe maximum limit

again,

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}}^{0} + 3.695 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + 0.$$

$$\Rightarrow \cancel{\frac{101000}{1520 \times 9.81}} + 3.695 = \cancel{\frac{101000}{1520 \times 9.81}} + \frac{V_3^2}{2 \times 9.81}$$

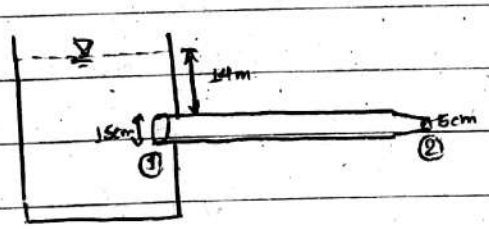
$V = 8.5144$ m/s

So,

$$Q_{max} = \frac{\pi\, 0.1^2}{4} \times 8.5144$$

$$= 0.667\ m^3/s$$

* A 15cm diameter horizontal pipe of length 16 m takes off from a reservoir in which water level is 14m above pipe centre. A nozzle of exit diameter 5cm is fitted to the pipe outlet. The nozzle discharges water into the atmosphere. Determine power in the water jet issuing from the nozzle and pressure of water at the base of the nozzle. Assume that loss of head in the pipe is $2.2\frac{V^2}{2g}$ and that in the nozzle is $0.06\frac{v^2}{2g}$, where V and v are velocities through the pipe and nozzle respectively.



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2} + H_L + H_N$$

$$\Rightarrow \frac{\rho g \times 14}{\rho g} + \frac{V_1^2}{2g} = \frac{0}{\rho g} + \frac{V_2^2}{2g} + 2.2\frac{V_1^2}{2g} + 0.06\frac{V_2^2}{2g}$$

$$\Rightarrow 14 + \frac{(1-2.2)V_1^2}{2g} = \frac{V_2^2}{2g}(1+0.06)$$

$$\Rightarrow 14 - 1.2\frac{V_1^2}{2g} = \frac{V_2^2}{2g} \times 1.06 \quad \text{--- (i)}$$

$V_1 = 1.751\ m/s$, $V_2 = 15.759\ m/s$.

also, $A_1V_1 = A_2V_2$

$\Rightarrow \cancel{\pi} \times \frac{15^2}{\cancel{4}} \times V_1 = \cancel{\pi} \times \frac{5^2}{\cancel{4}} \times V_2$

$\Rightarrow 9V_1 = V_2$.

now,

$$Q = \pi \times \frac{0.05^2}{4} \times 15.759$$

$$Q = 0.0309 \ m^3/s$$

$$\text{Power} = Q.H_N \times \rho \times g$$

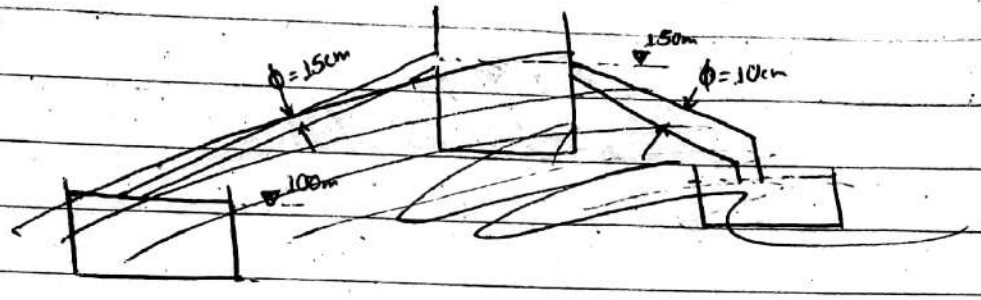
$$= 1000 \times 9.81 \times 0.0309 \times \left(\frac{0 + 0 + 15.759^2}{2 \times 9.81} \right)$$

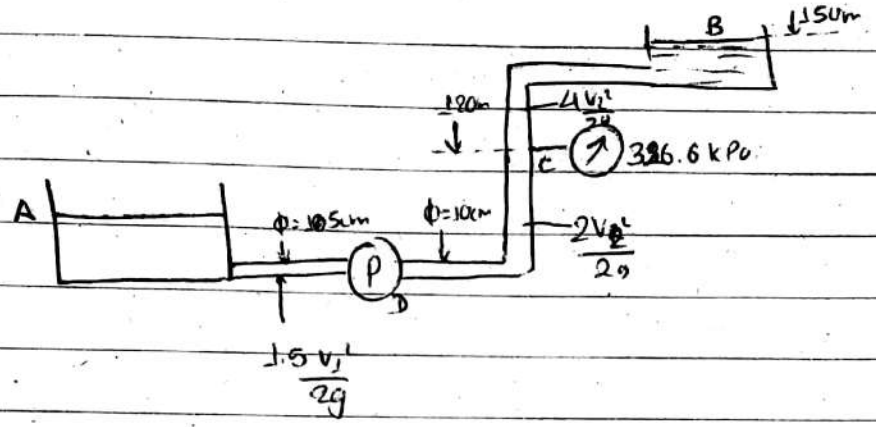
$$= \frac{230.216 \text{ watt}}{0.06}$$

$$= 3945 \text{ watt}$$

$P_{at\ C} = 137.34$ KPa (gauge)

* Water is pumped from reservoir A at an elevation of 10cm to reservoir B at an elevation of 150m. The suction pipe (pipe from reservoir A to the pump is 15 cm diameter and delivery pipe (from pump to resoirvoir B is 10cm diameter. The delivery pipe passes through a point C which is at an elevation of 120m and has a gauge pressure of 326.6 KPa. Head loss in the suction pipe is $1.5\ V_1^2/2g$, pump upto C is $\frac{2V_2^2}{2g}$ and from C to reservoir B is $\frac{4V_2^2}{2g}$ where V_1 is velocity in suction pipe & V_2 is velocity in delivery pipe. Determine discharge and the power supplied by the pump.





applying bernoulli's eqⁿ ⓪ in CB.

$$\frac{P_c}{\rho g} + \frac{V_2^2}{2g} + 120 - \frac{4V_2^2}{2g} = 0 + 0 + 150$$

$$\Rightarrow \cancel{326.60} \quad 33.29 + 120 - 150 = \frac{3V^2}{2g}$$

$$V_2 = 4.64 \text{ m/s}$$

so,

$$V_1 = \frac{A_2 V_2}{A_1} = \frac{10^2}{15^2} \times 4.64 = 2.062 \text{ m/s}$$

total discharge (Q) = $A_2 V_2 = 0.0364 \text{ m}^3/\text{s}$.

applying bernoulli's equation in A to B.

$$0 + 0 + 100 + H_p - \frac{1.5V_1^2}{2g} - \frac{2V_2^2}{2g} - \frac{4V_2^2}{2g} = 0 + 0 + 150.$$

$$H_p = \cancel{50} \; 56.90$$

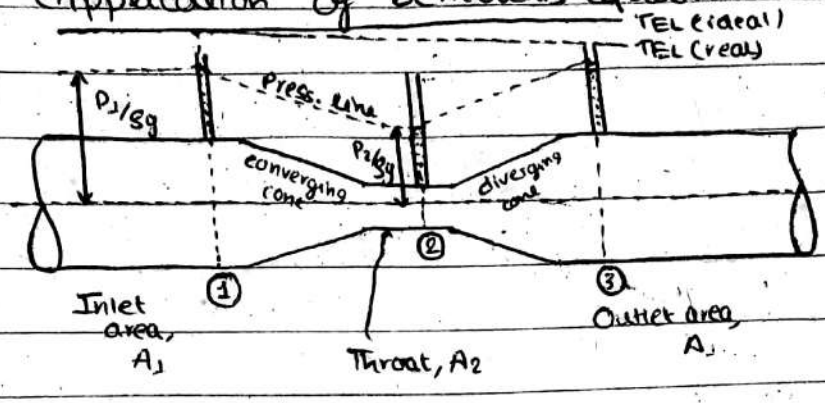
now,

Power pump = $Q H_p \times \rho \times g$

= 20318.0796 Watts

Discharge measurement instrument

(Application of bernoulli's equation)



A pressure difference related to the rate of flow can be created in a pipe by deliberately reducing its area of cross section. Let inlet diameter d_1 (same as pipe) and throat diameter d_2. Then,

$$\frac{d_2}{d_1} = \text{throat ratio}$$

(practically, $d_2 \approx (1/3 \text{ to } 3/4 \text{ of } d_1)$)

for ideal fluid case, losses are neglected, so, applying BE in section 1 and 2.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} \quad (\text{but } z_1 = z_2)$$

$$\frac{(P_1 - P_2)}{\gamma} = \frac{(V_2^2 - V_1^2)}{2g} = h = \cancel{\text{piezometric head}} \text{ difference in piezometric head between 1 and 2.} \quad \text{------ (1)}$$

so,

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = h$$

From continuity, $A_1 V_1 = A_2 V_2 \Rightarrow V_1 = \frac{A_2 V_2}{A_1}$

so,

$$\frac{V_2^2}{2g} - \frac{A_2^2 V_2^2}{A_1^2 \, 2g} = h.$$

$$\Rightarrow \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right) = h$$

$$\Rightarrow V_2 = \sqrt{\frac{2gh}{\left(1 - \frac{A_2^2}{A_1^2}\right)}}$$

$$= A_1\sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}$$

since, losses are not considered, this is the theoretical velocity:

$$V_{2th} = \frac{A_1\sqrt{2gh}}{\sqrt{A_2^2 - A_1^2}}.$$

$$\boxed{Q_{theoretical} = A_2 \cdot V_{2th} = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

<u>Coefficient of discharge</u>

Let C_D = coeff of discharge

$$= \frac{\text{actual discharge}}{\text{Theoretical discharge}}.$$

now, $Q_{actual} = C_D \times Q_{theoretical}$

$$= \frac{C_D A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = K\sqrt{h} \quad \text{---} (1)$$

where, k = meter constant.

$$\boxed{Q \propto \sqrt{h}}$$

For real fluid, there will be head loss.

applying bernoulli's equation between section ① and ②.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_f.$$

$$\frac{V_2 - V_1^2}{2g} = \left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) - h_f = (h - h_f)$$

$$Q = \frac{A_1 A_2 \sqrt{2g(h-h_f)}}{\sqrt{A_1^2 - A_2^2}} \quad \text{------} \quad (2)$$

comparing (1) and (2),

$$C_d\sqrt{h} = \sqrt{h - h_f}.$$

$$\therefore C_d = \frac{\sqrt{h - h_f}}{\sqrt{h}}$$

and $C_d^2 h = h - h_f.$

$$\boxed{\therefore h_f = h - C_d^2 h}$$

$$\boxed{\therefore h_f = h(1 - C_d^2)} \quad \text{---} \quad (3).$$

from (1),

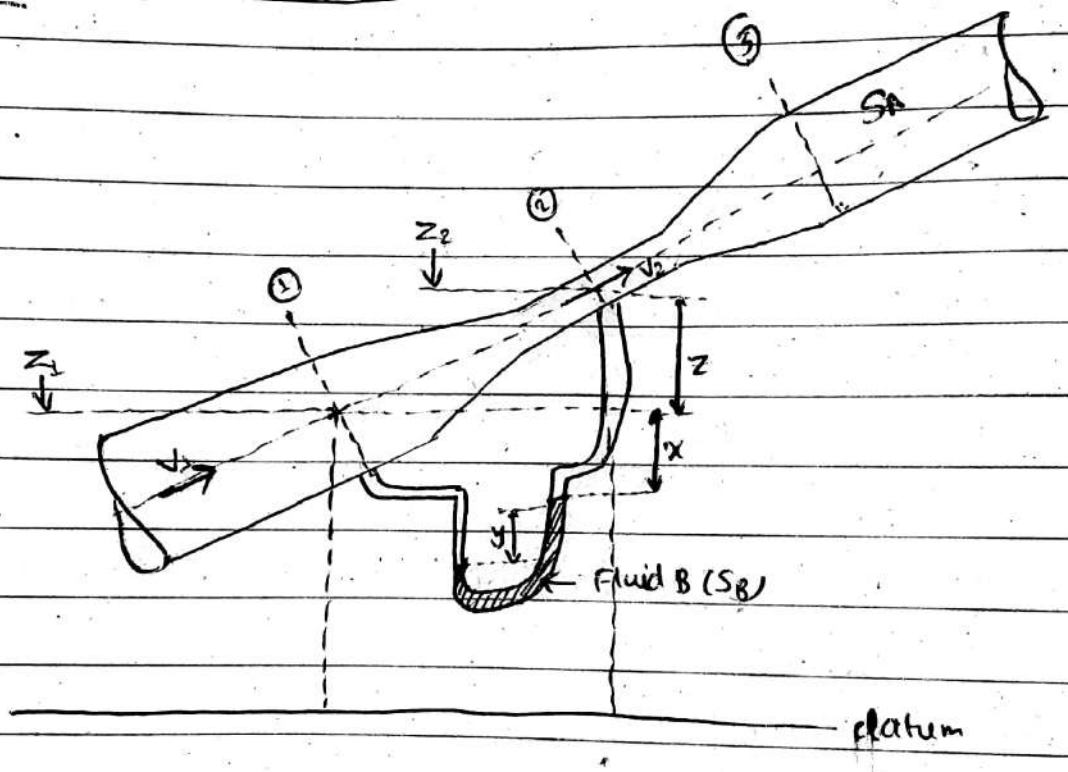
$$h = \frac{Q^2}{C_d^2 \cdot 2g}\left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right) = \frac{1}{C_d^2}\left(\frac{V_2^2 - V_1^2}{2g}\right)$$

$$\therefore h_f = \frac{1}{C_d^2}\left(\frac{V_2^2 - V_1^2}{2g}\right)(1 - C_d^2) = \left(\frac{1}{C_d^2} - 1\right)\left(\frac{V_2^2 - V_1^2}{2g}\right)$$

$$= \left(\frac{1}{C_d^2} - 1\right) \frac{Q^2}{2g} \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

$$= \left(\frac{1}{C_d^2} - 1\right) \left[1 - \left(\frac{d_2}{d_1}\right)^4\right] \frac{V_2^2}{2g}$$

Inclined venturimeter:



$$Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{(A_1^2 - A_2^2)}}$$

applying bernoulli's equation in ① and ②,

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2.$$

$$\Rightarrow \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = h \quad \text{--- (i)}$$

applying ~~manome~~ manometric eq^n:

$$P_1 + \rho g(x+y) - \rho_B \cdot g y - \rho g(x+z) = P_2$$

$$\cancel{\Rightarrow P_1 + \rho g x + \rho g y - \rho_B g y - \rho g}$$

$$= P_1 + \cancel{S_A \gamma_w x} + S_A \gamma_w \cdot y - S_B \cdot \gamma_w \cdot y - \cancel{S_A \gamma_w x} - S_A \gamma_w z = P_2$$

$$\Rightarrow \frac{P_1}{\gamma_w} + \frac{S_A \cancel{\gamma_w} y}{\cancel{\gamma_w}} - \frac{S_B \cancel{\gamma_w} y}{\cancel{\gamma_w}} - \frac{S_A \cancel{\gamma_w} z}{\cancel{\gamma_w}} = \frac{P_2}{\gamma_w}$$

$$\Rightarrow \frac{P_1}{\gamma_w} - \frac{P_2}{\gamma_w} - z S_A = y(S_B - S_A)$$

$$\Rightarrow \frac{P_1}{S_A \gamma_w} - \frac{P_2}{S_A \gamma_w} - z = y\frac{(S_B - S_A)}{S_A}$$

$$\Rightarrow \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2 = y\left(\frac{S_B}{S_A} - 1\right)$$

$$\Rightarrow \left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right) = y\left(\frac{S_B}{S_A} - 1\right)$$

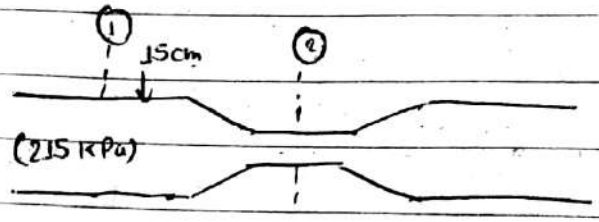
So,

in ①,

$$\boxed{y\left(\frac{S_B}{S_A} - 1\right) = h}$$

* Water flows through a horizontal venturimeter of inlet diameter 15cm and inlet pressure 215 KPa (abs). Find the minimum throat diameter of the meter to pass a discharge of 150 l/s without causing cavitation. Assume sat vapour pressure of water (-80KPa) (gauge). Atmospheric pressure = 76 cm of Hg and Cd of the meter 0.978.

→ Here,



$Q = 150 \times 10^{-3} m^3/s$

now, applying BE:

$$\frac{215000}{1000 \times 9.81} + \frac{V_1^2}{2g} + \cancel{h_1} = 10.336 + \frac{V_2^2}{2g} + \cancel{h_1}$$

⇒ piezometric height difference $= 11.58 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

now,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \pi \times \frac{0.15^2}{4} \times V_1 = \pi \times \frac{d^2}{4} \times V_2$$

$$\Rightarrow \boxed{V_2 = \frac{0.0225\, V_1}{d^2}}$$

So, $Q_{act} = 150 \times 10^{-3} m^3/s$.

⇒ ~~$Q_{real} = 0.978$~~.

⇒ $Q_{the} = \frac{Q_{real}}{Cd} = 0.153374$

now,

$$11.58 = \frac{\left(\frac{0.0225^2}{d^4} - 1\right) V_1^2}{2 \times 9.81}$$

$$\sqrt{\frac{227.1996}{\left(\frac{0.0225^2}{d^4} - 1\right)}} = V_1$$

now,

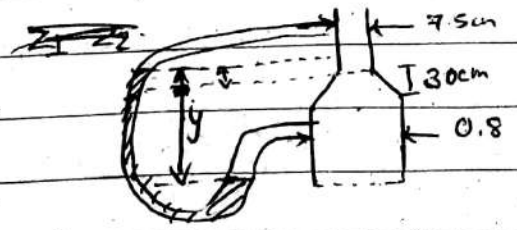
$$A_1 V_1 = Q_{th}$$

$$\Rightarrow \frac{\pi \times 0.15^2}{4} \times \sqrt{\frac{227.1996}{\left(\frac{0.0225^2}{d^4} - 1\right)}} = 0.153374$$

$$d = 0.1059 \text{ m}$$

$$\Rightarrow d = 10.59 \text{ cm}$$

* 500 l/s of petrol of density 0.8 kg/lt flows through a 15cm diameter by 7.5cm vertical venturimeter. The flow is in upward direction And Cd = 0.991. The length of the converging cone of the meter is 30cm. Compute pressure difference between inlet and the throat and the deflection that would record by a mercury differential youtube manometer connected between those section.



$Q_{actual} = 500 \times 10^{-3} \, m^3/s$

applying BE:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow (P_1 - P_2) = \left[0.3m + \frac{(V_2^2 - V_1^2)}{2g}\right] \times 7848$$

$$\Rightarrow P_1 - P_2 = 2354.4 + \frac{(V_2^2 - V_1^2)}{400}$$

also,

$$A_1 V_1 = A_2 V_2 = \frac{Q_{act}}{C_d} = \frac{500 \times 10^{-3}}{0.991}$$

$$\Rightarrow$$

$$= 0.5045.$$

$V_1 = \cancel{64.21 m/s}$ 28.53 m/s.

$V_2 = 114.14 m/s$

so, $P_1 - P_2 = \cancel{2354.4 +}$ 71891.47863 //

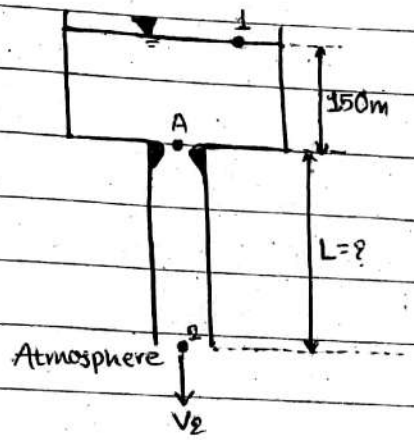
$\cancel{= 2378.668 Pa}$

$\cancel{P_1 - \rho_m \times g \times y + \rho_w \times g \times 0.3 = P_2}$

$\cancel{\Rightarrow 71891.47863 = 13600}$

Numericals on Bernoulli's equation

1) The outlet at the bottom of a tank is so formed that the velocity of water at point A is 2.0 times the mean velocity within the outlet pipe. What is the greatest length of pipe L which may be used without producing cavitation? Atmospheric pressure = 95.480 KPa (abs) and vapour pressure = 4.00 KPa (abs). Neglect all other losses.



→ Solution:

It is given that, $V_2 = \frac{1}{2} V_A$

applying bernoulli's equation in (1) and (A).

$$\frac{P_1}{\rho_w \times g}^{0} + \frac{V_1^2}{2g}^{0} + (L + 150m) = \frac{P_A}{\rho_w \times g} + \frac{V_A^2}{2g} + (L)$$

$$\Rightarrow (L+150) = \frac{(4 - 95.480) \times 1000}{1000 \times 9.81} + \frac{4V_2^2}{2 \times 9.81} + (L)$$

$$\Rightarrow (L+150) + 9.325 = 0.2038 V_2^2 + L$$

$$\Rightarrow (L + 159.325) = 0.2038 V_2^2 \quad \text{---- (i)}$$

$$\Rightarrow \frac{159.325}{0.2038} = V_2^2 \Rightarrow V_2 = 27.96$$

$$V_A = 55.920$$

applying bernoulli's equation in (A) and (2),

$$\frac{P_A}{\rho_w g} + \frac{V_A^2}{2g} + L = \cancel{\frac{P_2}{\rho_w g}}^{\,0} + \frac{V_2^2}{2g} + 0$$

$$\Rightarrow \left(\frac{4 - 95.48}{9.81}\right) + \frac{55.92^2}{2\times 9.81} + L = \frac{27.96^2}{2\times 9.81}$$

$$\Rightarrow L = -110.21 \text{ ??}$$

~~that practically wrong~~

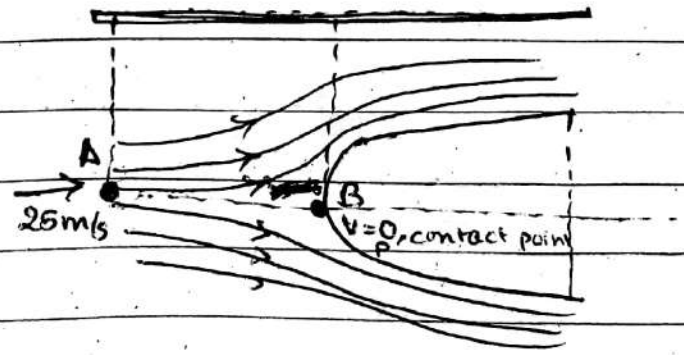
2) A torpedo is found to move in a sea at a speed of 25m/s when submerged at the depth of 15m. What is the pressure at the nose of the torpedo (Take the density of sea ~~of~~ water as 1025 kg/m³)

→ What is torpedo?

- self firing underwater misiles

note:

If torpedo ~~flo~~ moves in a stagnant seawater, the flow becomes unsteady but this motion can be relooked by keeping the torpedo at a constant velocity and moving the water in the same velocity opposite to the ~~flow~~ movement of torpedo to form a steady flow.



applying bernoulli's equation in (A) and (B).

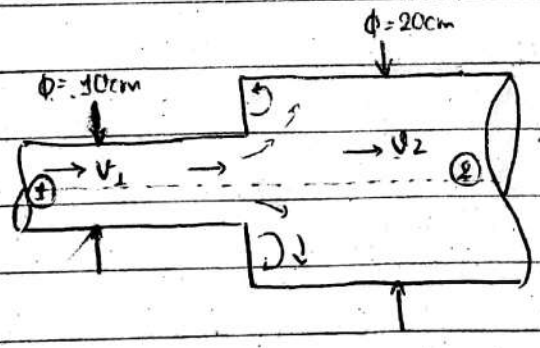
$$\frac{P_A}{\rho_w g} + \frac{V_A}{2g} + \cancel{z_1} = \frac{P_B}{\rho_w g} + \cancel{\frac{V_B}{2g}}^{0} + \cancel{z_2} \quad (z_1 = z_2 \text{ for horizontal motion})$$

$$\Rightarrow \frac{\cancel{\rho_w g} h}{\cancel{\rho_w g}} + \frac{V_A}{2g} = \frac{P_B}{\rho_w g}$$

$$\Rightarrow \left(15m + \frac{25^2}{2 \times 9.81}\right) \times 1025 \times 9.81 = P_B$$

$$\boxed{\therefore P_B = 471141.25 \text{ Pa}}$$

3) The loss of head in sudden expansion of pipe is given by $H_L = \frac{(V_1 - V_2)^2}{2g}$ where suffixes 1 and 2 refer to sections before and after the expansion. A 10cm pipe carrying oil (RD = 0.85) undergoes a sudden expansion to 20cm diameter in a horizontal sketch of the pipeline. If the pressure difference between sections 2 and 1 is recorded as 660 Pa, estimate the discharge through the pipe and sketch the energy line and piezometric head line at the expansion, by considering the pressure at section 1, $P_1 = 3.0$ kPa.



→ applying bernoulli's equation in (1) and (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_1 + \underbrace{\frac{V_1^2 + V_2^2}{2g} - \frac{2V_1V_2}{2g}}_{\text{Head loss}}$$

$$\Rightarrow \frac{3000}{850 \times 9.81} = \frac{3660}{850 \times 9.81} + \frac{V_2^2 - 2V_1V_2}{2g} \quad \text{--- (i)}$$

using continuity equation,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \frac{d_1^2}{d_2} = \frac{V_2}{V_1}$$

$$\Rightarrow V_1 = 4V_2$$

so, eq^n (1) becomes,

$$0.71955 = \frac{2 \times 4V_2^2 - V_2^2}{2 \times 9.81}$$

$$V_2 = \cancel{1.412\ m/s}\ 0.471\ m/s$$

so,

$$V_1 = 1.884\ m/s$$

now,

$$Q = A_1 \times V_1$$

$$= 1.884 \times \pi \times \frac{0.1^2}{4}$$

$$= 0.0147\ m^3/s$$

pressure head of 1.

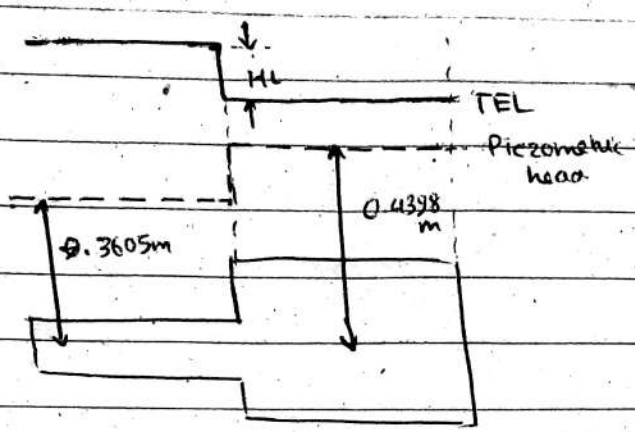
$$= \frac{P_1}{\gamma} = 0.3605\ m$$

pressure head of 2.

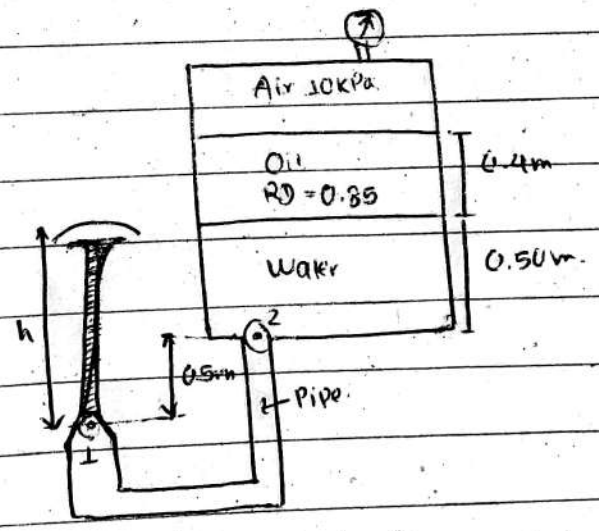
$$= \frac{P_2}{\gamma} = \frac{3665}{9.81 \times 1000} = 0.4398\ m$$

Head loss $= \frac{(V_1 - V_2)^2}{2g} = 0.10170\ m$

So,

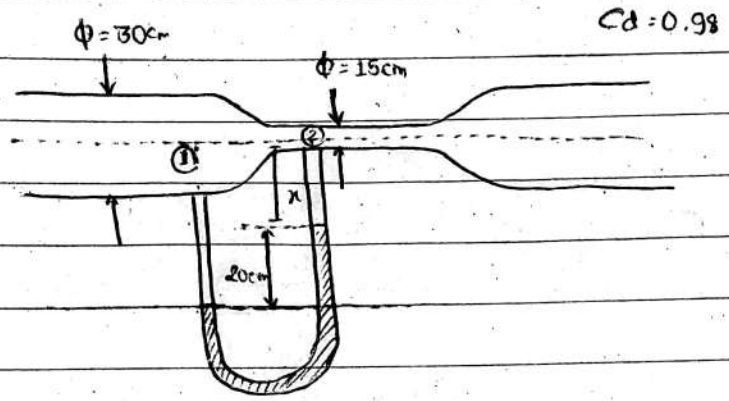


4) For the system shown, find the height h to which the jet from the nozzle would rise. If the nozzle and the pipe were to have diameters of 10cm and 20cm respectively, calculate the discharge and velocity in the pipe. Neglect losses.



Applying bernoulli's eq^n in ① and ②,

5) A horizontal venturimeter with inlet and throat diameters of 30cm and 15cm respectively is used to measure the flow of water. The differential manometer reading connecting the the inlet and the throat is 20cm of mercury. Determine the rat of flow. Take $C_d = 0.98$



Applying bernoulli's equation in 1 and 2

$$\frac{P_1}{\rho_w g} + \frac{V_1^2}{2xg} + \cancel{Z_1} = \frac{P_2}{\rho_w g} + \frac{V_2^2}{2xg} + \cancel{Z_2} \quad (\because Z_1 = Z_2, \text{ horizontal})$$

$$\Rightarrow \frac{P_1 - P_2}{\rho_w \cancel{g}} = \frac{V_2^2 - V_1^2}{2\cancel{g}}$$

but,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \frac{d_1^2}{d_2^2} = \frac{V_2}{V_1}$$

$$\Rightarrow \frac{0.3^2}{0.1^2} = \frac{V_2}{V_1}$$

$$\Rightarrow 9V_1 = V_2$$

So,

$$\frac{P_1 - P_2}{\rho_w} = 4V_1^2$$

now, using manometric eq^n:

$$P_1 + (x+0.2)1000 \times 9.81 - 0.2 \times 13600 \times 9.81 - x \times 1000 \times 9.81 = P_2$$

$$\Rightarrow P_1 - P_2 = 24721.2 \text{ Pa}$$

so,

$$\frac{24721.2}{1000} = 4V_1^2$$

$$\Rightarrow V_1 = 2.486$$

and, $Q_{theoretical} = V_1 \times A_1$

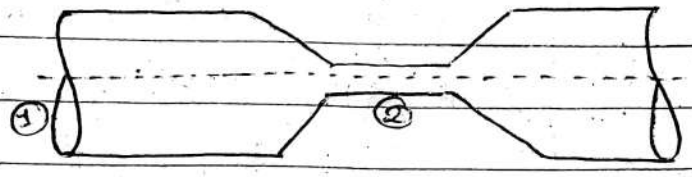
$$= 2.486 \times \pi \times \frac{0.3^2}{4}$$

$$= 0.1757 \text{ m}^3/\text{s}$$

so, $Q_{actual} = 0.1757 \times 0.98$

$$= 0.1722 \text{ m}^3/\text{s}$$

6) The coefficient of discharge for a venturimeter used for measuring the flow of an incompressible fluid was found to be constant when the rate of flow a exceeded a certain value. Show that under these conditions, the loss of head h_f in the convergent portion of the venturimeter can be expressed as $K_1 Q^2$ & K_1 is constant.



→ applying bernoulli's equation 1 and 2.

$$\frac{P}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_f$$

or, $\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = h$

or, $h - h_f = h' = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$ ---- 1

If a_1 & a_2 are cross sectional areas of inlet & throat sections

$$Q = a_1 v_1 = a_2 v_2$$

$$h' = \frac{Q^2}{2g}\left[\frac{1}{a_2^2} - \frac{1}{a_1^2}\right]$$

$$Q = \frac{a_1 a_2 \sqrt{2gh'}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q = C\sqrt{h'} \quad \text{where, } C = \frac{a_1 a_2 \sqrt{2g}}{\sqrt{a_1^2 - a_2^2}}$$

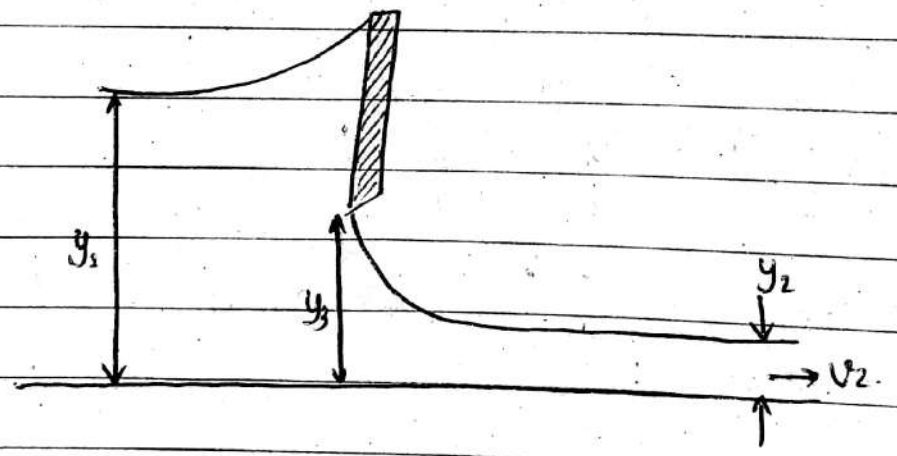
also,

$Q_a = C_d C \sqrt{h}$ where C_d is coeff of discharge

$$\therefore h - h' = \frac{Q^2}{C^2 C_d^2} - \frac{Q^2}{C^2} = \frac{Q^2}{C^2}\left[\frac{1}{C_d^2} - 1\right]$$

$\therefore h_f = K_1 Q^2$ where $K_1 = \frac{1}{C^2}\left[\frac{1}{K^2} - 1\right]$ is constant

7) Water is issuing through a sluice gate as shown in figure below. Suppose the gate opening is set so the depth downstream is $Y_2 = 0.7m$. Find the depth at the upstream under these condition if the flow rate is $4.24\ m^3/s$ per meter width of channel assuming no head loss.



Solution: Assuming no head loss

applying bernoulli's eq^n between ① and ②

$$\cancel{\frac{P_1}{\gamma}} + \frac{V_1^2}{2g} + Z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + Z_2$$

$$\text{or, } \frac{V_1^2}{2g} + Y_1 = \frac{V_2^2}{2g} + Y_2$$

also, $Q = A_1 V_1 = A_2 V_2$

$\therefore V_1 = \frac{Q}{A_1} = \frac{4.24}{Y \times 1} = \frac{4.24}{y_1}$ and

$$V_2 = \frac{Q}{A_2} = \frac{4.24}{Y_2 \times 1} = \frac{4.24}{0.7} = 6.06 \, m/s$$

$$\therefore y_1 + \frac{\left(\frac{4.24}{y_1}\right)^2}{2 \times 9.81} = 0.7 \times \frac{(6.06)^2}{2 \times 9.81}$$

$$y_1 + \frac{0.916}{y_1^2} = 2.54$$

$$\Rightarrow y_1 = 0.70, 2.41 m$$

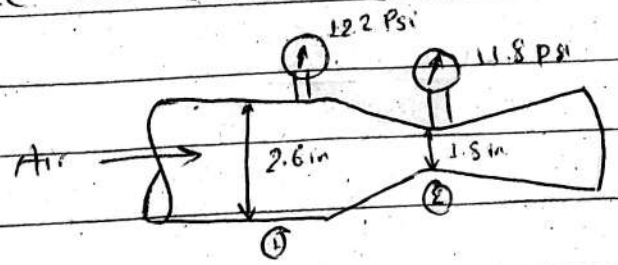
$\therefore$ up stream depth $= 2.41 m$

5) Air is flowing through a venturimeter whose diameter is 2.6 in at the enhance part and 1.8 in at the throat. The gage pressure ~~in~~ is measured to be 12.2 psi at the enhance and 11.8 psi at the throat. Neglecting frictional effects, show that the volume flow rate can be expressed as:

$$V = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho (1 - A_2^2/A_1^2)}}$$

and determine the flow rate of air.

Take the air density to be 0.075 lbm/ft3.



Solution.

Applying bernoulli's equation wt section (1) and (2).

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$$\text{or, } P_1 - P_2 = \rho \frac{V_2^2 - V_1^2}{2} \quad \text{---- (1)}$$

$$A_1 V_1 = A_2 V_2 = V$$

$$V_1 = \frac{V}{A_1} \quad \& \quad V_2 = \frac{V}{A_2}$$

$$P_1 - P_2 = \frac{\rho V^2}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

$$= \frac{\rho V^2}{2A_2^2}\left(1 - \frac{A_2^2}{A_1^2}\right)$$

$$V = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}} \quad \text{---- (2)}$$

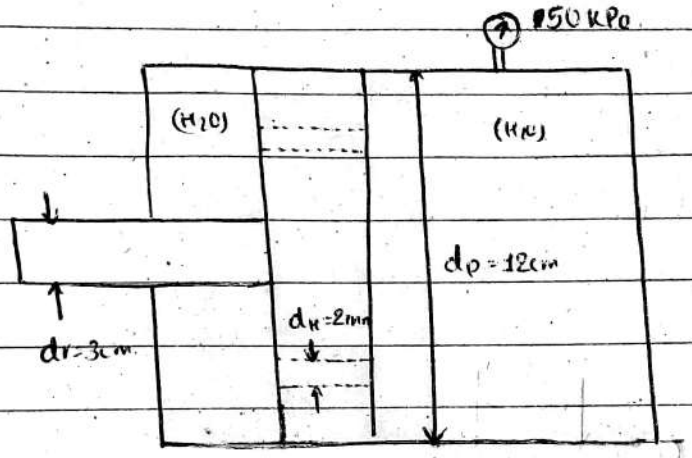
putting given values

$$V = \frac{\pi D^2}{4} \sqrt{\frac{2(P_1 - P_2)}{\rho\left[1 - \left(\frac{D_2}{D_1}\right)^2\right]}}$$

$$= \frac{\pi (1.5/12 \text{ ft})^2}{4} \sqrt{\frac{2 \times 12 \times 2 - 11.51}{0.075\left(1 - \left(\frac{1.5}{3.6}\right)^4\right)} \cdot \left(\frac{144}{1}\right) \cdot \left(\frac{32.2}{1}\right)}$$

$$= 4.48 \text{ ft}^3/\text{s}$$

9) A well fitting piston with 4 small holes in a sealed, water filled cylinder is pushed to right at constant speed of 4 mm/s. The pressure in compartment remains constant at 50 kPa gauge. Diregarding fuctional effects determine the force F that needs to be applied to the piston to maintain this motion.

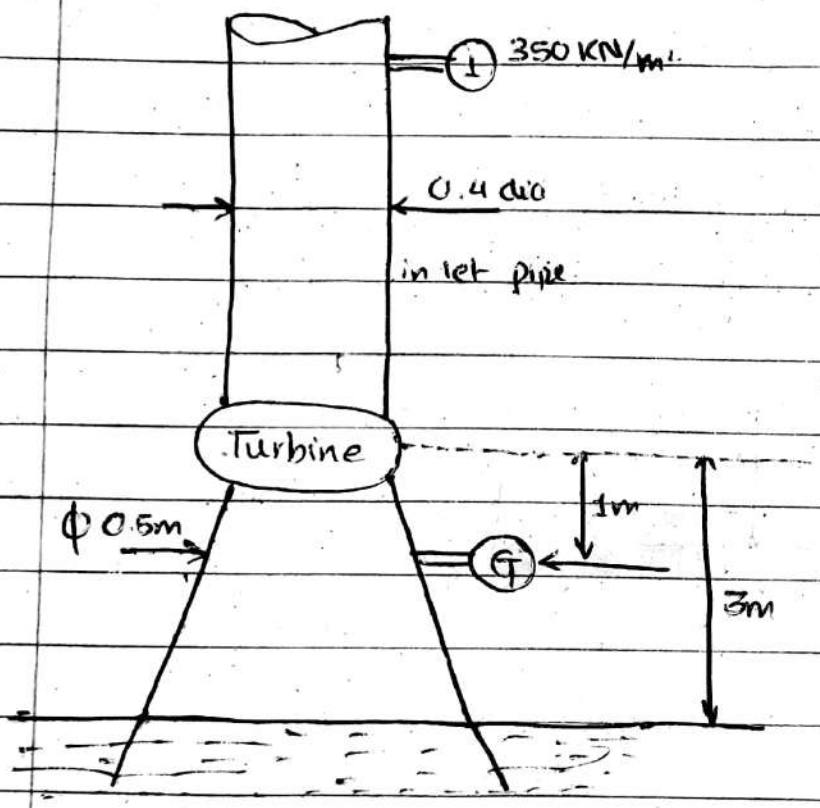


10) In figure shown a turbine with inlet pipe and draft tube. If the efficiency of turbine is 80 % and discharge is 1000 lit s^{-1}. Find

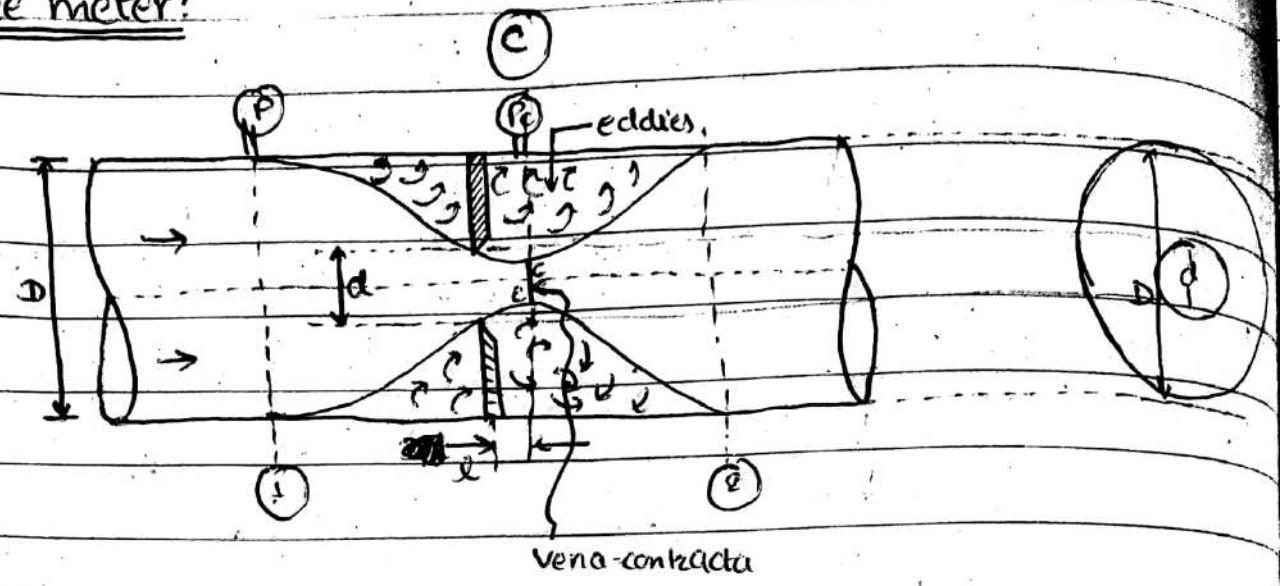
i) Power developed by turbine

&

ii) reading of the gauge G.



2) Orifice meter:



Orifice:

Generally $l = D/2$

and distance between C and 2, ~~the~~ is $(4-8) \times D$

So

loss in eddies before orifice is negligible

neglecting small loss of head in the jet contraction.
Applying bernoulli's eq^n between section 1-1 and C-C.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_c}{\gamma} + \frac{V_c^2}{2g} + Z_c$$

$$\therefore h = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_c}{\gamma} + Z_c\right) = \frac{V_c^2 - V_1^2}{2g} \quad \text{--- (ii)}$$

$\therefore C_c$: coefficient of contraction $= \dfrac{\text{jet area at C-C}}{\text{orifice area}}$

$$= \frac{a_c}{a}$$

now, continuity equation becomes

$$V_1 A = V_c a_c = V_c \times (a \cdot C_c)$$

$$V_1 = V_c \left(\frac{a}{A}\right) C_c$$

so,

$$h = \frac{V_c^2}{2g}\left[1 - \left(\frac{C_c \cdot a}{A}\right)^2\right]$$

$$\therefore V_c = \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{C_c a}{A}\right)^2}} = \frac{\sqrt{2gh}}{\sqrt{1 - C_c^2 \frac{d^4}{D^4}}} \Rightarrow \boxed{V_c = \frac{D^2\sqrt{2gh}}{\sqrt{1 - C_c^2 d^4}}}$$

C_v = coefficient of velocity $= \cancel{\dfrac{Q_{act}}{Q}}$ $\dfrac{V_{act}}{V_{theo}}$

$$\boxed{\therefore V_{c_{actual}} = \frac{C_v D^2 \sqrt{2gh}}{\sqrt{1 - C_c^2 d^4}}}$$

Q_{actual} = Area of contraction $\times V_c$

$$= C_c \cdot a \, \frac{C_v D^2 \sqrt{2gh}}{\sqrt{1 - C_c^2 d^4}}$$

~~C_c~~ dropping C_c,

$$Q_{act} = \frac{C_c \, C_v \, a \sqrt{2gh}}{\sqrt{1 - (d/D)^4}}$$

Let, $C_c \cdot C_v = C_d$.

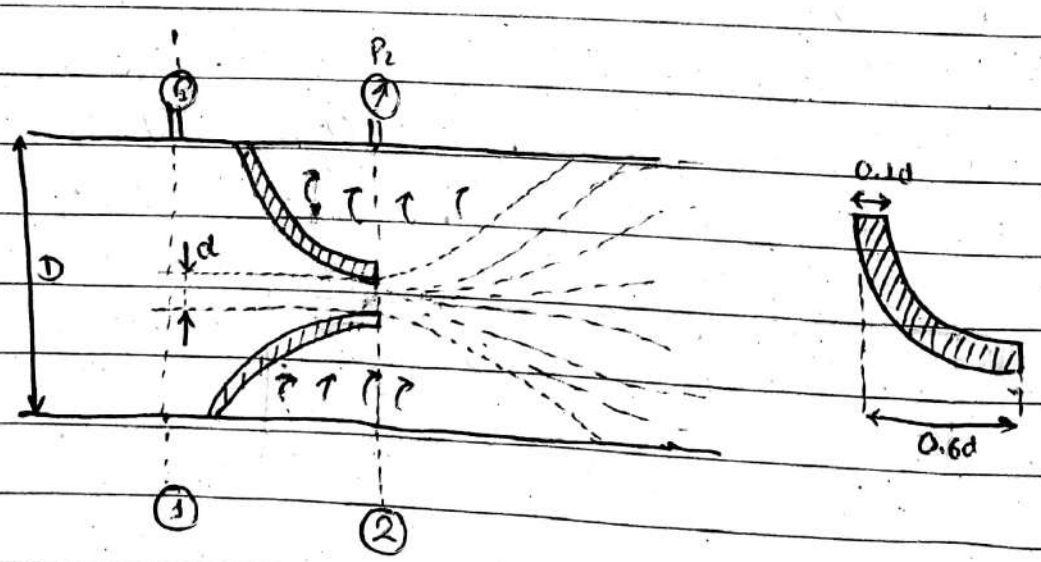
So,

$$Q_{ac} = \frac{C_d a \sqrt{2gh}}{\sqrt{1-(d/D)^4}}$$

$$\boxed{Q_{act} = E \cdot C_d \cdot \frac{\pi d^2}{4} \sqrt{2gh}}$$

$$E = \frac{1}{\sqrt{1-(d/D)^4}} = \text{approach factor.}$$

Flow nozzle meter

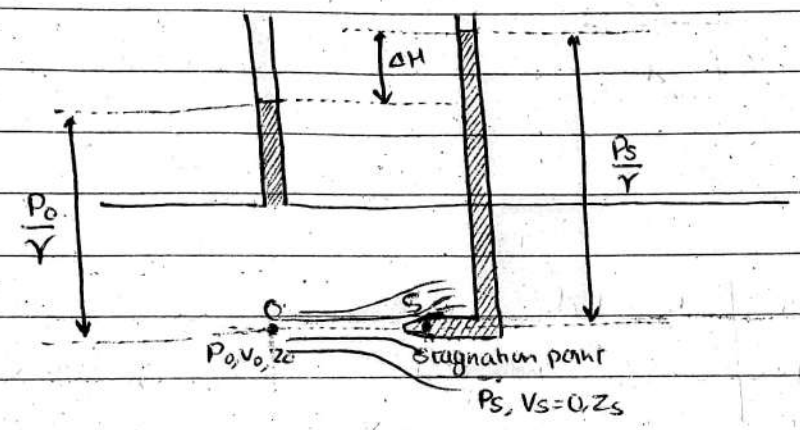


discharge equation of same as venturimeter.

Coefficient of discharge:

Venturimeter → $C_d \approx (0.977 \sim 0.994)$
Nozzle meter → $C_d \approx (0.93 \sim 0.98)$
Orifice meter → $C_d \approx (0.6 \sim 0.62)$

Pitot tube



If velocity of flow at particular point is reduced to zero, i.e. stagnation point, pressure there is increased due to conversion of kinetic energy into ~~potential~~ pressure energy and thereby measuring increase in Pressure energy at this point; velocity of flow is determined

applying bernoulli's eq^n between stagnation point and O.

$$\frac{P_s}{\gamma} + \cancel{\frac{V_s^2}{2g}}^{0} + \cancel{Z_s} = \frac{P_o}{\gamma} + \frac{V_o^2}{2g} + \cancel{Z_o}$$

$$\Rightarrow \frac{P_s}{\gamma} = \frac{P_o}{\gamma} + \frac{V_o^2}{2g} \quad \cdots\cdots (1)$$

$\frac{V_0^2}{2g} = \Delta h$, because all the velocity head goes to increase the height of mercury.

So,

$$\frac{V_0^2}{2g} = \frac{P_s}{\gamma} - \frac{P_0}{\gamma} = \frac{\Delta P}{\gamma} = \Delta h.$$

$$\therefore V_0 = \sqrt{2g\left(\frac{P_s - P_0}{\gamma}\right)}$$

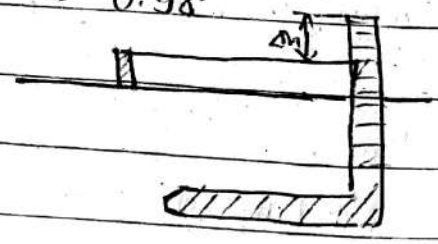
$$\boxed{\therefore V_0 = \sqrt{2g\Delta h}}$$

$$\boxed{V_0 = C\sqrt{2g\Delta h}}$$ where C is the instrument constant (0.98 ~ 1)

Numerical:

* A pitot tube is inserted ~~is~~ in a pipe of 30cm diameter. The static pressure of the tube is 10cm of Hg (vacuum). The pressure at the centre line of the pipe recorded by the pitot tube is 1N/cm². Calculate discharge through the pipe if mean ~~ave~~ velocity is 0.85 times centre line velocity. Assume coefficient of the tube is 0.98.

→ Solution.



$$\Delta h = \frac{P_s}{\gamma} - \frac{P_0}{\gamma}$$

$$\Rightarrow \Delta h = \frac{10000}{1000 \times 9.81} + \frac{0.1 \times 13600 \times 9.81}{1000 \times 9.81}$$

$$= 2.37m$$

now,

velocity $= \sqrt{2g\Delta h}$

$= \cancel{5.0812\ m/s}$ $6.682\ m/s$

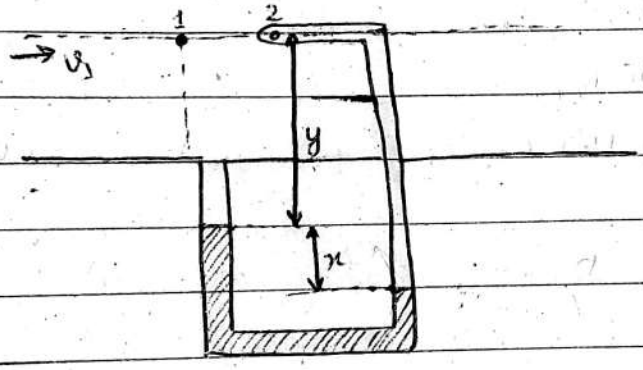
So,

discharge $= Q = V.A$

$= \cancel{5.0812}\ 6.682 \times \pi \times \frac{0.3^2}{4}$

$= \cancel{0.359\ m^3/s}$ $0.47\ m^3/s$

*



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}}^{0} + \cancel{z_2}$$

$$\Rightarrow \cancel{P_1 - P_2} \quad P_2 - P_1 = \frac{V_1^2}{2\cancel{g}} \times \rho\cancel{g}$$

$$P_2 - P_1 = \frac{V_1^2 \rho}{2}$$

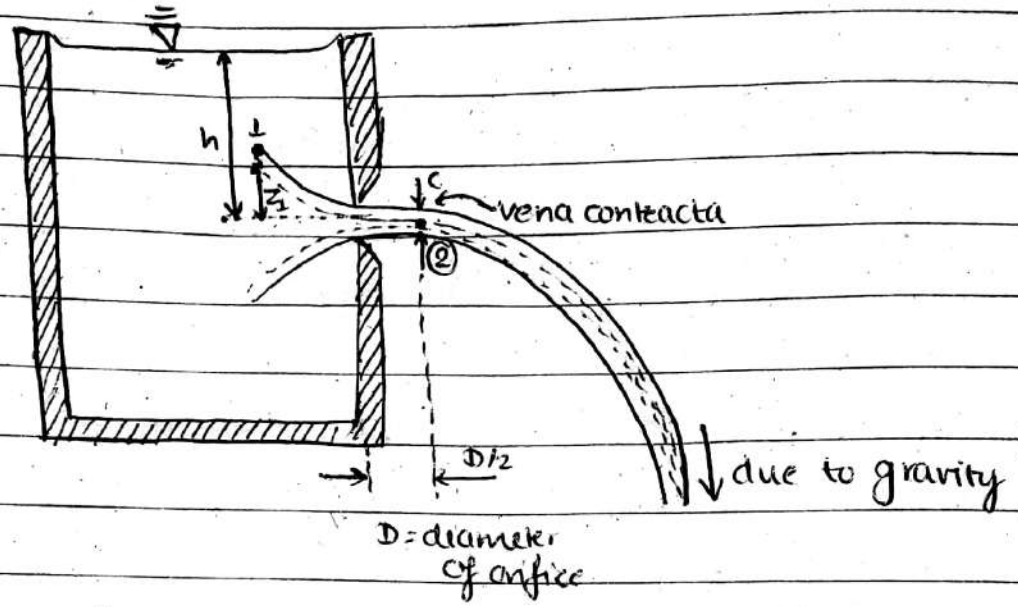
now,

$$P_2 - \rho_m \times g \times x = P_1$$

$$\Rightarrow \frac{V_1^2 \rho}{2} = \rho_m g x$$

$$\therefore V_1 = \sqrt{\frac{2\rho_m g x}{\rho}} = 16.334\sqrt{x}$$

FLOW THROUGH ORIFICES



V_1 = approach velocity

For a particular streamline applying bernoulli's equation between 1 and 2.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2}^{\,0} \quad \text{.....} \quad (1)$$

hydrostatic condition still present so, $P_1 = P_{atm} + \rho g(h - z_1)$

$P_2 = P_{atm}$

So

$$\frac{\rho g(h - \cancel{z_1})}{\cancel{\gamma}} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{V_1^2}{2g} + h = \frac{V_2^2}{2g} \quad \text{....} \quad (2)$$

V_1 constant across section of reservoir at point 1.

and V_2 = constant across vena contracta since orifice is small compared to h.

then continuity equation $\Rightarrow$ $A_1 V_1 = A_2 V_2 = Q$

A_1 → reserfoir cross section

A_2 → ~~orifice~~ jet cross section

$$\therefore V_2 = \sqrt{\frac{2gh}{1-\left(\frac{a_2}{a_1}\right)^2}}$$

If reservoir is sufficiently large and point ① is sufficiently far from the orifice, then V_1 is small compared to V_2. Then from eq^n ②.

$$\frac{V_2^2}{2g} = h$$

so, $\boxed{V_2 = \sqrt{2gh}}$

this formula is known as Toricelli formula

Velocity at jet (V_2) is always greater than average velocity of jet at a section in a plane of orifice.

2

Hydraulic coefficients

1) Coeff of velocity $(C_v) = \frac{V_{act}}{V_{theo}} = \frac{V_{act}}{\sqrt{2gh}}$

$\therefore V_{act} = C_v\sqrt{2gh}$

↳ due to friction

So,

$$\boxed{V_{act} = C_v\sqrt{2gh}} \quad \text{------ (i)}$$

$C_v \approx 0.95 - 0.99$ (assume either 0.97 or 0.98)

2) Coeff. of contraction $(C_c) = \frac{\text{Area of jet at vena contracta } (a_c)}{\text{Area of orifice. } (a)}$

$$\boxed{\therefore a_c = C_c a} \quad \text{------ (ii)}$$

$\boxed{C_c = \frac{\pi}{\pi+2}}$, practically, $0.69 \sim 0.61$.

(assume 0.64 or 0.65)

If bell-mouth opening, $C_c = 1$

3) Coeff of discharge $(C_d) = \frac{Q_{act}}{Q_{theo}}$

$\Rightarrow Q_{act} = C_d \cdot Q_{theo}$

$= C_d \cdot a_c \cdot V$

$= C_c \cdot a \cdot C_v \cdot \sqrt{2gh}$

$= a C_c C_v \sqrt{2gh}$

also, $Q_{theo} = a \times \sqrt{2gh}$.

now,

$$Q_{act} = C_d \cdot Q_{theo}$$

$$\Rightarrow a C_c C_v \sqrt{2gh} = a \times \sqrt{2gh} \times C_d$$

$$\boxed{\therefore C_d = C_c C_v}$$

4) Coefficient of resistance (C_r) = $\dfrac{\text{loss of KE as liquid passes through orifice}}{\text{KE possesed by the fluid flow}}$

theoretical KE/wt = $\dfrac{V_{th}^2}{2g} = \dfrac{(\sqrt{2gh})^2}{2g} = h$

actual KE/wt = $\dfrac{V_{act}^2}{2g} = \dfrac{C_v^2(\sqrt{2gh})^2}{2g} = hC_v^2$

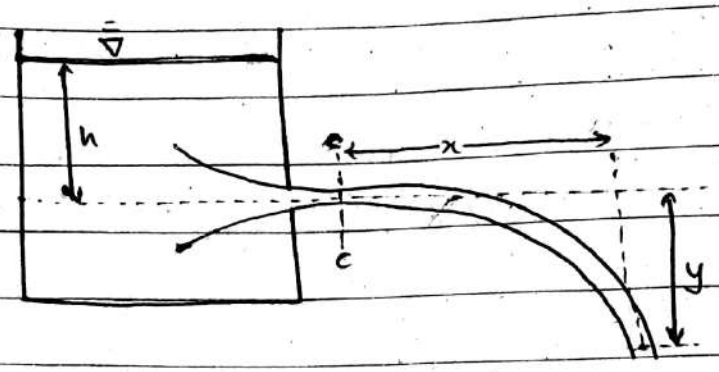
loss = $h(1-C_v^2)$

$$\therefore C_r = \frac{h(1-C_v^2)}{hC_v^2} = \left(\frac{1}{C_v^2} - 1\right)$$

$$\boxed{\therefore C_r = \left(\frac{1}{C_v^2} - 1\right)}$$

Experimental determination of hydraulic coefficients:

adjust inflow so that h remains constant.



If air resistance is neglected and horizontal component of velocity v remains unchanged. Then, distance travelled

$$x = v.t \quad \text{------} \; (i)$$

In vertical direction, since there is uniform downward acceleration g and as vertical component of velocity is initially zero for horizontal discharge:

then,

$$y = \frac{1}{2} g t^2 \quad \text{------} \; (ii)$$

$$\Rightarrow \quad y = \frac{1}{2} g \frac{x^2}{V^2}$$

$$\Rightarrow \quad V^2 = \frac{1}{2} g \frac{x^2}{y}$$

$$\Rightarrow \quad V = x\sqrt{\frac{g}{2y}}$$

So the path of the liquid particle moving along a free jet is parabolic with vertex of parabola at vena contracta (C-C)

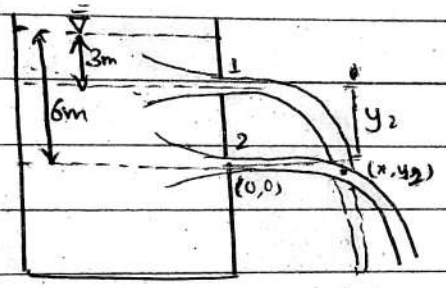
So,

$$V_{th} = \sqrt{2gh}.$$

$$C_v = \frac{V}{V_{th}} = \frac{x\sqrt{g/2y}}{\sqrt{2gh}} = \frac{x}{\sqrt{4hy}} =$$

$$\boxed{\therefore C_v = \frac{x}{2\sqrt{hy}}}$$

* A tank has two identical orifices provided in one of its side at $h_1 = 3m$ and $h_2 = 6m$ below the free surface. Determine point of intersection of two jets if $C_v = 0.98$ for both the orifices.



for Orifice one,

$$V_{th} = \sqrt{2gh}$$
$$= \sqrt{2 \times 9.81 \times 3}$$
$$= 7.672 \text{ m/s}$$

$$V_{act} = 0.98 \times 7.672.$$
$$= 7.51856 \text{ m/s}.$$

for orifice 2,

$$V_{th} = \sqrt{2gh}$$
$$= 10.849 \text{ m/s}.$$

$$V_{act} = 0.98 \times 10.849$$
$$= 10.633 \text{ m/s}.$$

$$7.51856 = x\sqrt{\frac{9.81}{2 \times y_1}}$$

$$\Rightarrow 2.214x = 7.51856\sqrt{y_1}.$$

$$x^2 = 11.524\, y_1 \text{ ---- (i)}$$

also,

~~10.633x~~ =

$$2.214x = 10.633\sqrt{y_2}.$$

$$\Rightarrow x^2 = 23.065\, y_2 \text{ --- (ii)}$$

So,

$$\frac{y_1}{y_2} = 2.0014$$

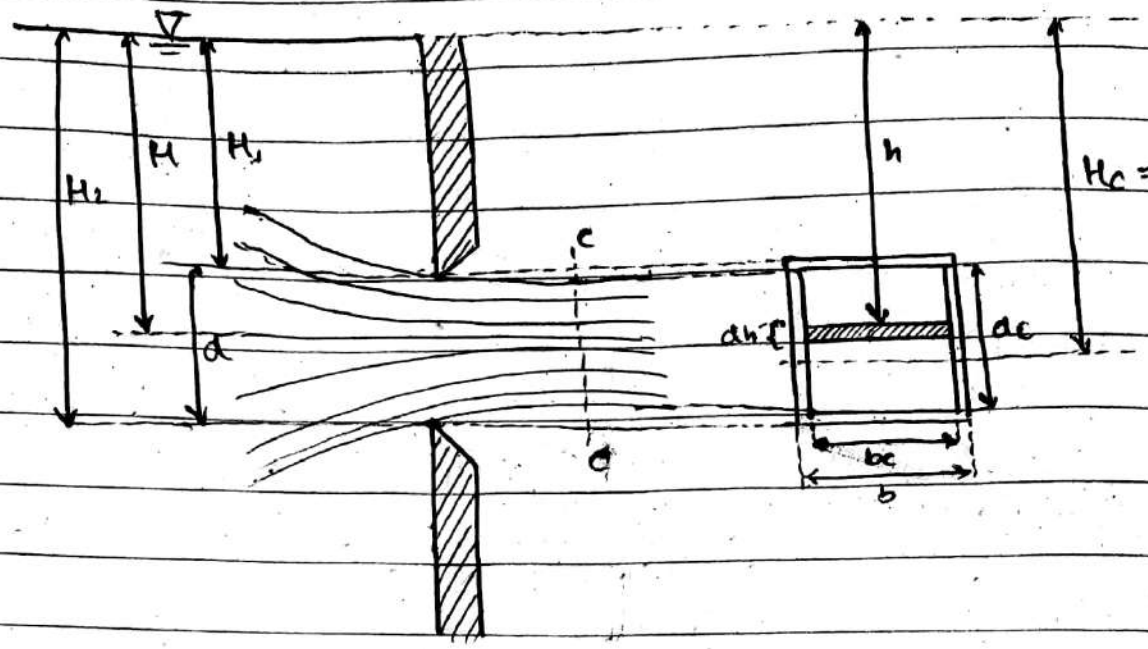
$$y_2 - y_1 = 6 - 3$$ ~~$y_1 = 2.995$~~

$$\Rightarrow y_2 - y_1 = 3$$

$$x = 8.315$$

Solve $y_1 = 3$, $y_1 = 6$

Flow through large orifices



$$dQ = dh \cdot b_c \times \sqrt{2gh} \cdot C_v$$

$$dQ = C_v b_c \sqrt{2gh}\, dh$$

C_v → coeff of velocity for orifice.

$$\therefore Q = C_v b_c \sqrt{2g} \int_{H_c - \frac{d_c}{2}}^{H_c + d_c/2} h^{1/2}\, dh$$

$$= C_v b_c \sqrt{2g} \left[\frac{2h^{3/2}}{3}\right]_{H_c - d_c/2}^{H_c + d_c/2}$$

$$= \frac{2}{3} C_v b_c \sqrt{2g} \cdot \left[(H_c + d_c/2)^{3/2} - (H_c - d_c/2)^{3/2}\right]$$

b_c, d_c & H_c are unknown.

=> So,

b_c, d_c & H_c is replaced by, b, d, H and C_c.

So,

$$Q = \frac{2}{3} C_c . C_v . b\sqrt{2g} \left\{ (H+d/2)^{3/2} - (H-d/2)^{3/2} \right\}$$

$$Q = \frac{2}{3} C_d b\sqrt{2g} \left\{ (H+d/2)^{3/2} - (H-d/2)^{3/2} \right\} \text{ ----- (i)}$$

$$Q = \frac{2}{3} C_d b\sqrt{2g} \left\{ H_2^{3/2} - H_1^{3/2} \right\} \text{ ----- (ii)}$$

now,

If this orifice is taken as small orifice,

then

$$Q = C_d\, b \times d \sqrt{2gH} \text{ --- (iii)}$$

if H=d,

$$Q = \frac{2}{3} C_d . b\sqrt{2g} \left[\left(\frac{3}{2} H \right)^{3/2} - \left(\frac{1}{2} H \right)^{3/2} \right]$$

$$= \frac{2}{3} C_d b \sqrt{2g} \left[\left(\frac{1}{2} \right)^{3/2} H . \sqrt{H} \left(3^{3/2} - 1 \right) \right]$$

$$= C_d b . H . \sqrt{2gh} \times \sqrt{\frac{1}{2}} \times \left(3^{3/2} - 1 \right)$$

$$= 0.99\, C_d\, (b \times d) \sqrt{2gh}.$$

So, 1% error if H=d, if small orifice is replaced in large orifice,

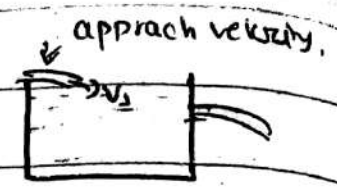
if $H > d$, error becomes less than 1%.

So, large orifice means, if head is greater than d

i.e. $\boxed{h > d}$ //

if $H < d$, large orifice

if $H > d$, treat as small orifice.



If velocity of approach V_1 is also considered, then:

$$dQ = (b_c . dh) . C_v \sqrt{2g\,(h + V_1^2/2g)}$$

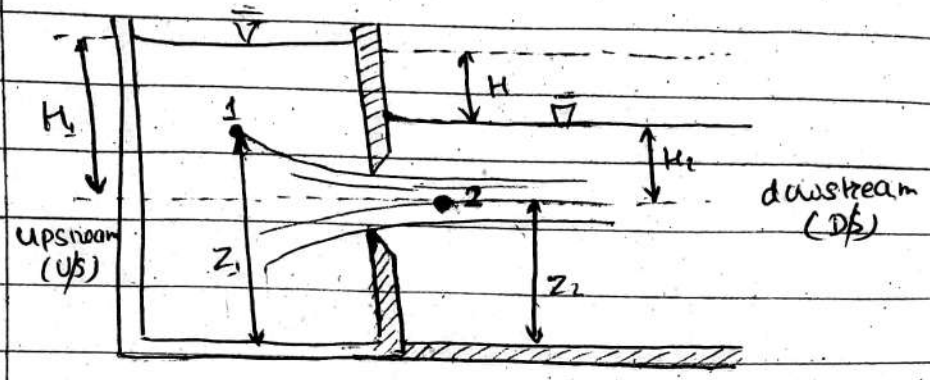
So,

$$Q = 2/3\, C_d b . \sqrt{2g} \left\{ \left(H + d/2 + \frac{V_1^2}{2g} \right)^{3/2} - \left(H - d/2 + \frac{V_1^2}{2g} \right)^{3/2} \right\}$$

$$\Rightarrow Q = 2/3\, C_d b . \sqrt{2g} \left\{ \left(H_2 + \frac{V_1^2}{2g} \right)^{3/2} - \left(H_1 + \frac{V_1^2}{2g} \right)^{3/2} \right\}$$

2) Submerged / Drowned Orifice:

a) Totally submerged.



Considering no losses, applying bernoulli equation between Section 1 and 2,

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}}^{\,0} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow (H_1 + \cancel{Z_1} - \cancel{Z_1}) + 0 + Z_1 = H_2 + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$$\Rightarrow (H_1 - H_2) = \frac{V_2^2}{2g}$$

$$\Rightarrow V_2 = \sqrt{2g(H_1 - H_2)}$$

So torecelli's formula still applicable.

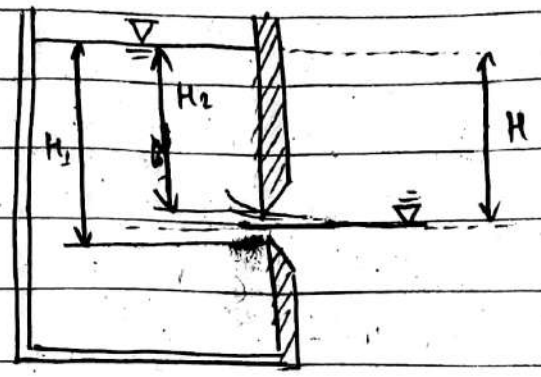
provided that h is replaced by $H_2 - H_1$

now,

$$Q_{theo} = A \times V_2$$
$$= a\sqrt{2g(H_1 - H_2)}$$

$$Q_{act} = C_d \cdot a\sqrt{2g(H_1 - H_2)}$$

b) Partial submerged:



so, upper portion (orifice discharging free),
lower portion (submerged orifice).

so,

$$Q_{total} = Q_1 + Q_1$$

$$\cancel{= a\sqrt{2gh}.Cd + a\sqrt{2g.(H_1-H)}.Cd}$$

$$Q_1 = 2/3\, Cd_1 b\,[H^{3/2} - H_2^{3/2}]\sqrt{2g}.$$

where,

Cd_1 = coeff of discharge for freely discharging orifice.

similarly,

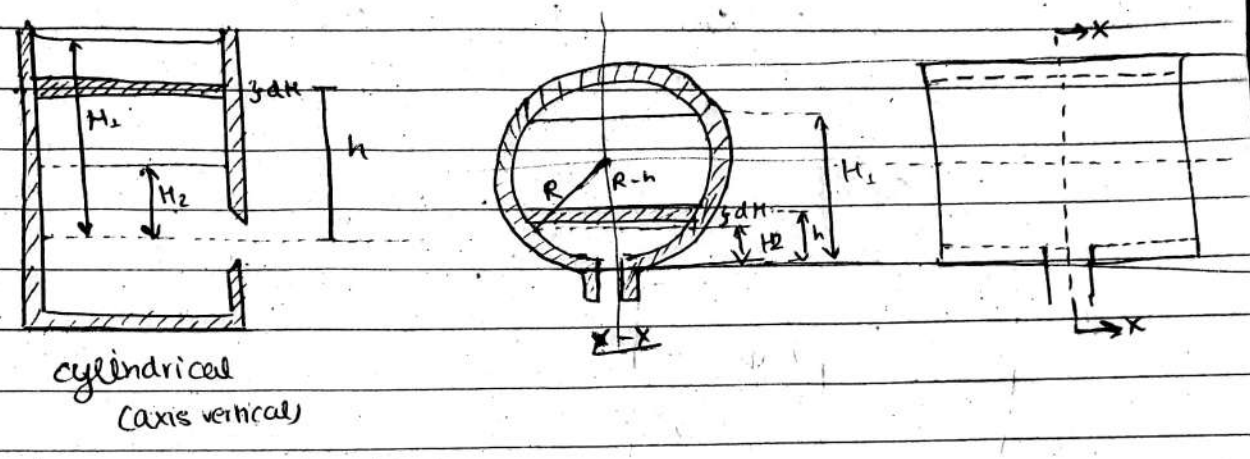
$$Q_2 = Cd_2 . b . (H_1 - H) . \sqrt{2g\, H}$$

so,

$$Q = 2/3\, Cd_1 b\,[H^{3/2} - H_2^{3/2}]\sqrt{2g} + Cd_2 b\,(H_1 - H)\sqrt{2gH}.$$

Flow through the orifice under variable head.

1) Time of emptying the tank with no inflow:



cylindrical
(axis vertical)

Let at any instance, the liquid surface be at heigh 'h' above the orifice and let the surface falls by dh in dt time.

Let A = horizontal cross sectional area of the tank.

than,

volume of liquid living the tank in dt

$= A \times dh$ ... (i) (− if volume decrease, + if volume increase)

if at this instant, discharge through the orifice = Q.

so,

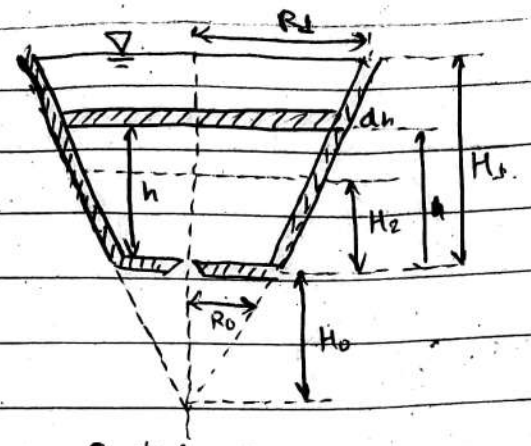
volume of liquid leaving during $dt = Q.dt$ ... (ii)

so,

$Q dt = A \cdot dh$.

$\Rightarrow Q dt = -A dh$.

$\Rightarrow dt = \frac{-A dh}{Q} = \frac{-A dh}{C_d . a \sqrt{2gh}}$.



Conical.

So,

$$\boxed{t = -\int_{H_2}^{H_2} \frac{A\,dh}{C_d \cdot a\sqrt{2gh}}}$$

For cylindrical tank with axis vertical,

$$A = \text{constant} = \pi r^2.$$

So,

$$t = -\int_{H_1}^{H_2} \frac{\pi r^2\,dh}{C_d \cdot a\sqrt{2gh}}$$

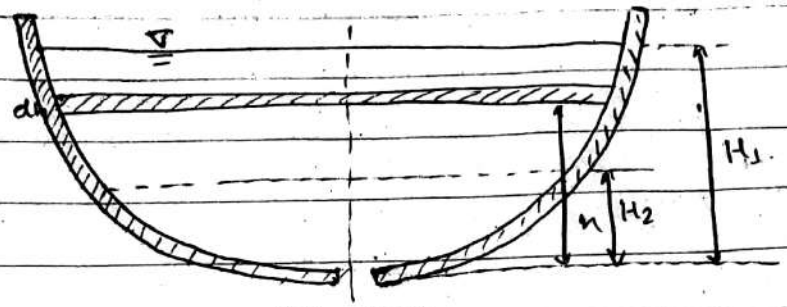
$$= \frac{2\pi r^2 \sqrt{h}}{C_d \cdot a\sqrt{2g}}$$

$$= \frac{-2\pi r^2 [\sqrt{h}]_{H_1}^{H_2}}{C_d \cdot a\sqrt{2g}}$$

for completely emptying $H_2 = 0$

So,

$$t = \frac{2\pi r^2\sqrt{H}}{C_d\, a\sqrt{2g}}$$



(hemispherical)

<u>Case II:</u>

Cylindrical tank with horizontal axis:

$$t = -\int_{H_1}^{H_2} \frac{A\,dH}{C_d a\sqrt{2gh}}$$

$$= \frac{-1}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \frac{A\,dH}{\sqrt{H}}$$

$$= \frac{-1}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \frac{\left[2\sqrt{R^2+(R-h)^2} \times l\right] dH}{\sqrt{H}}$$

$$= \frac{-2l}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \frac{\sqrt{2RH-H^2}}{\sqrt{H}}\, dH$$

$$= \frac{-2l}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \sqrt{2R-H}\, dH$$

$$= \frac{-2l}{C_d a\sqrt{2g}} \left[\frac{(2R-H)^{3/2}}{(3/2)\times(-1)}\right]_{H_1}^{H_2}$$

$$= \frac{4l}{3C_d a\sqrt{2g}} \left[(2R-H)^{3/2}\right]_{H_2}^{H_1}$$

for emptying:

$$H_2 = 0.$$

So,

$$t = \frac{4l}{3C_d a\sqrt{2g}} \left[(2R)^{3/2} - (2R - H_1)^{3/2}\right].$$

Case III:

(For conical tank).

$$t = \int \frac{A.dh}{C_d.a.\sqrt{2gh}}$$

$$= \int \frac{\pi \left(\frac{R_1(h+H_0)}{(H_1+H_0)}\right)^2 . dh}{C_d.a\sqrt{2gh}}$$

$$= \cancel{\frac{\pi R_1^2}{C_d a}} \int \frac{\pi R_1^2}{\sqrt{2g}\, C_d.a(H_1+H_0)^2} \int \frac{(h+H_0)^2 . dh}{\sqrt{h}}$$

$$= \frac{\pi R_1^2}{\sqrt{2g}\, C_d.a(H_1+H_0)^2} \int_{H_1}^{H_2} \frac{h^2 + 2hH_0 + H_0}{\sqrt{h}} dh$$

$$= \frac{\pi R_1^2}{\sqrt{2g}\, C_d a(H_1+H_0)^2} \int_{H_1}^{H_2} h^{3/2} + 2h^{1/2}H_0 + H_0 h^{-1/2} dh$$

* solve//

Case IV :

hemisphere :

$$t = \int_{H_2}^{H_1} \frac{\pi \left(R^2 - (R-h)^2\right) \times dh}{C_d \cdot a\sqrt{2gh}}$$

$$= \frac{\pi}{C_d \cdot a} \int_{H_2}^{H_1} \frac{R^2 - R^2 + 2Rh - h^2 \times dh}{\sqrt{2gh}}$$

$$= \frac{\pi}{C_d \cdot a\sqrt{2g}} \int_{H_2}^{H_1} \frac{2Rh - h^2}{\sqrt{h}} dh$$

$$= \frac{\pi}{C_d \cdot a\sqrt{2g}} \int_{H_2}^{H_1} 2Rh^{1/2} - h^{3/2} dh$$

solve

* A tank of constant cross sectional area of $2.8\,m^2$. has 2 orifices each $9.3\times10^{-4}\,m^2$ in one of its vertical sides at height 6m and 1.5m respectively above the bottom of the tank. Calculate time taken to lower the water level from 9m to 3.6m above the bottom of the tank. Assume $C_d = 0.625$.

$$t_1 = -\int_{7.5}^{4.5} \frac{2.8\,dh}{0.625\times9.3\times10^{-4}\times\sqrt{2g\times h}}$$

$$= -1087.54\int_{9}^{6}\frac{1}{\sqrt{h}}\,dh$$

$$= 1342.660\,s.$$

$$t_2 = \qquad (Q_1+Q_2)dt = 2.8\times dh$$

$$\Rightarrow \frac{0.625\times9.3\times10^{-4}\times\sqrt{2g}\,(\sqrt{h}+\sqrt{h+4.5})}{2.8}\,dt = dh$$

$$\Rightarrow dt = \frac{1087.54}{(\sqrt{h}+\sqrt{h+4.5})}\,dh$$

$$\text{So, } t = -\int_{7.5}^{4.5}\frac{1087.54}{(\sqrt{h}+(\sqrt{h+4.5})}\,dh$$

$$= 576.08\,s \approx 9\,min\,0\,s$$

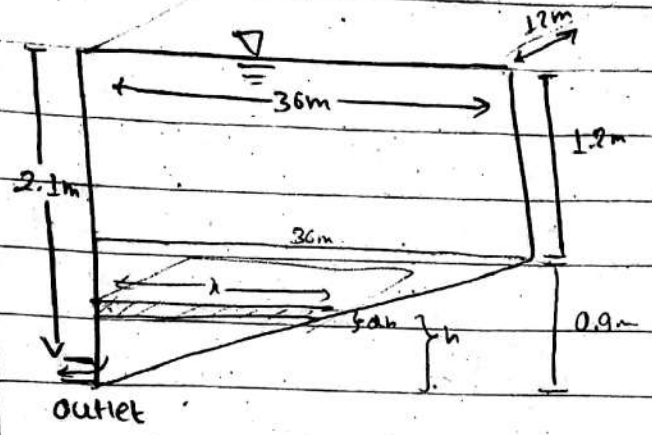
$$Q\,dt = 2.8\times dh$$

$$\Rightarrow \cancel{\frac{0.625\times9.3\times10^{-4}\times\sqrt{2g}\,h}{2.8}} \qquad t = \int_{2.1}^{4.5}\frac{1087.54}{\sqrt{h}}\,dh$$

$$= 1461.051\,s.$$

So, total $t = 2038.131 = 33\,min\,96\,s$

*



$$\frac{36}{0.9} = \frac{x}{h}$$

$$\Rightarrow x = \frac{36 \times h}{0.9} \times 12 \qquad A = 480h$$

$$\cancel{A = \frac{1}{2} \times h \times \frac{18h^2}{0.9}}$$

A swimming pool ~~has~~ is 36m long and 12m wide. There are two outlets at the bottom of deeper end. One is orifice 0.225m dia. and other is small pipe 0.75m long fitted on outside of 0.225m dia. opening. Find the time required to empty the ~~sm~~ swimming pool if Cd of orifice 0.62 and Cd of mouthpiece (pipe) 0.85. (Ans: 2724s)

upto 1.2m:

$$(Q_1 + Q_2)\,dt = (36 \times 12)\,dh$$

$$dt = \frac{432\,dh}{C_{d_1} a_1 \sqrt{2gh} + C_{d_2} a_2 \sqrt{2gh}}$$

$$= \frac{432\,dh}{0.62 \times 0.0397 \times \sqrt{2 \times g \cdot h} + 0.85 \times 0.0397 \times \sqrt{2gh}}$$

$$= \cancel{\frac{432\,dh}{0.1758}}$$

$$= 2456.652 - \frac{dh}{(0.62 + 0.85)\sqrt{h}}$$

$$dt = -1671.192215\,\frac{dh}{\sqrt{h}}$$

$$\therefore t = -1671.192215 \times \int_{2.1}^{0.9} \frac{1}{\sqrt{h}}\,dh$$

$$= \cancel{1182.1765\,s} \quad 1670.3391704\,s$$

$$dt = \cancel{20h^2\,dh} \quad \cancel{108.318}\,\frac{h^2}{\sqrt{h}}\,dh$$

$$\Rightarrow dt = -108.318 \int_{0.9}^{0} h^{3/2}\,dh$$

$$t = 33.29$$

$$Q \cdot dt = A \cdot dh$$

$$\Rightarrow dt = \frac{A\,dh}{Q}$$

$$= \frac{480h\,dh}{C_{d_1} a\sqrt{2gh} + C_{d_2} a\sqrt{2gh}}$$

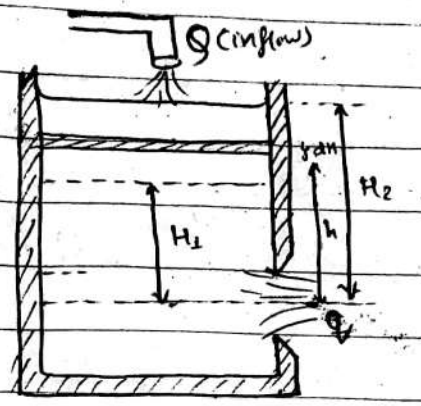
$$dt = 1856.88 \times \frac{h}{\sqrt{h}}\,dh$$

$$dt = 1856.88 \times h^{1/2}\,dh$$

$$t = 1056.978\ s$$

$$t = \cancel{2039.155},\ 2725.\ \text{something}\ s$$

Time of emptying a tank with inflow:



$$q_t = C_d a\sqrt{2gh} = k\sqrt{h}$$

Continuity equation,

$$(Q-q)dt = (A.dh)$$

$$\Rightarrow Q - k\sqrt{h}\,dt = A\,dh$$

$$\int dt = \int_{H_1}^{H_2} \frac{A\,dh}{Q-k\sqrt{h}}$$

$$= A\int_{H_2}^{H_1} \frac{dH}{Q-k\sqrt{h}}$$

Let $Q - k\sqrt{h} = z$.

$$\Rightarrow \cancel{Q - \frac{-kh}{\sqrt{h}}\,dh = dz} \qquad \frac{(Q-z)^2}{k^2} = h$$

$$\Rightarrow \qquad \Rightarrow dh = \frac{2(Q-z)}{k^2} \times dz.$$

$$t = \frac{-2A}{k^2}\int \frac{Q-z}{z}\,dz$$

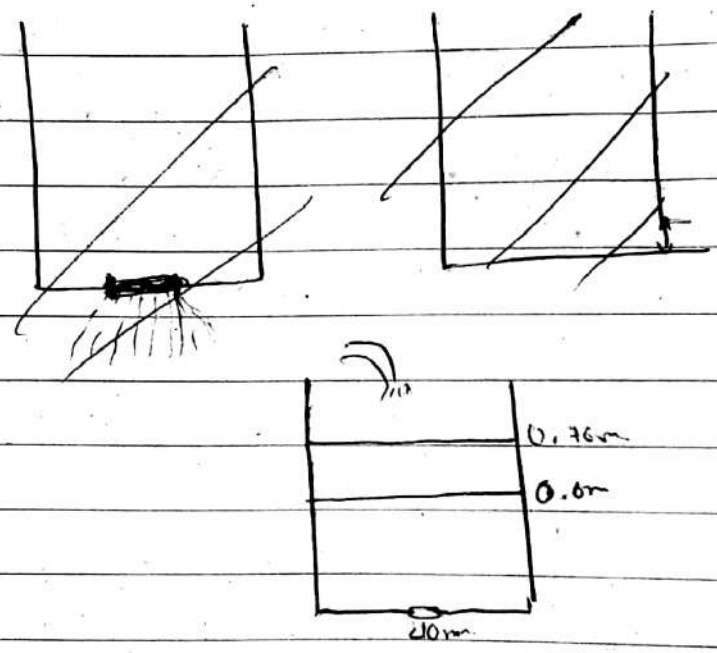
$$= \frac{-2A}{k^2}\left[Q\log z - z\right]_{h_2}^{h_1}$$

$$= -\frac{2A}{k^2}\left[Q\log(Q-k\sqrt{h}) - (Q-k\sqrt{h})\right]_{h_2}^{h_1}$$

$$= \frac{-2A}{Q^2}\left[Q\log_e \frac{Q-k\sqrt{H_2}}{Q-k\sqrt{H_1}} + k(\sqrt{H_2}-\sqrt{H_1})\right]$$

* A cylindrical tank is placed with its axis vertical and is provided with a circular orifice 40mm diameter in its bottom. Water flows into the tank at a uniform rate and is discharged through the orifice. It is found that, it takes 107 s for the heads in the tank to rise from 0.6m to 0.76m and takes 120s for its rise from 1.2m to 1.28m. Find, rate of uniform ~~flow~~ inflow and cross sectional area of the tank. Take $C_d = 0.6$

=)



$$107 = \frac{-2A}{2C_d^2 a^2 g}\left[\cancel{Q \log_e \frac{Q = C_d a\sqrt{2gH_2}}{Q = C_d a\sqrt{2gH_1}} + C_d a\sqrt{2g(H_1 - H_2)}}\right]$$

$0.16 \times A$

$$\frac{}{107s} = Q - C_d \times \frac{(40\times10^{-3})^2}{4} \times \sqrt{2g \times} \; 0.168$$

$$\frac{0.08A}{120} = Q - C_d \times \frac{(40\times10^{-3})^2}{4} \times \sqrt{2g \times} \; 1.2$$

$$\frac{0.16A}{107} = \varphi - 2.8458 \times 10^{-3}$$

$$\frac{0.08A}{120} = \varphi - 1.22324 \times 10^{-3}$$

$$A = 1.958$$

$$\varphi = 8.212 \times 10^{-5}$$

Notches and Weirs

Notches - Opening provided in the side of the tank such that the liquid surface in the tank is below top edge of the opening. It is ~~mo~~ made of metallic plate and used in laboratories.

Weir - Weir is same given to concrete to masonary structure structure built across a river or canal in order to raise the water level in upstream side and to allow excess water to flow over its entire length to downstream site.

Classification

Bicep, rectangular, triangular (V-notch), trapezoidal, parabolic

by effects of sides :

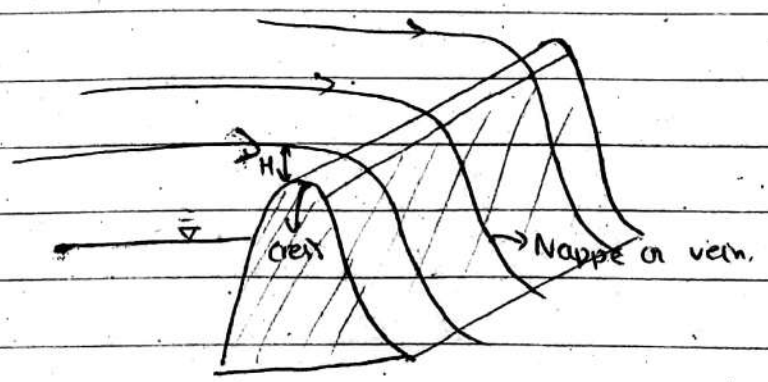
i) weir with end contraction

ii) weir without end contraction (suppressed notch)

Discharge condition

i) Free discharging weir.

ii) Submerged weir.

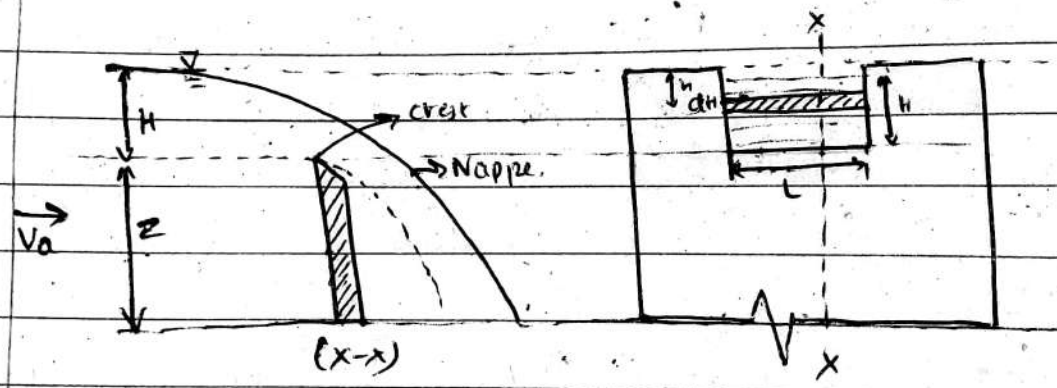


ogee's curve

Shape of the crest:

i) Sharp crested weir or notch.
ii) Broad crested
iii) Ogee shaped.

Discharge over rectangular sharpe crested weir:



$$dQ = C_d \cdot L\, dh \sqrt{2gh}$$

$$Q = \int_0^H C_d L\, dh \sqrt{2gh}$$

$$= \sqrt{2g}\, C_d L \int_0^H \sqrt{h}\, dh$$

$$= \sqrt{2g}\, C_d L \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2} \quad \text{---} (1)$$

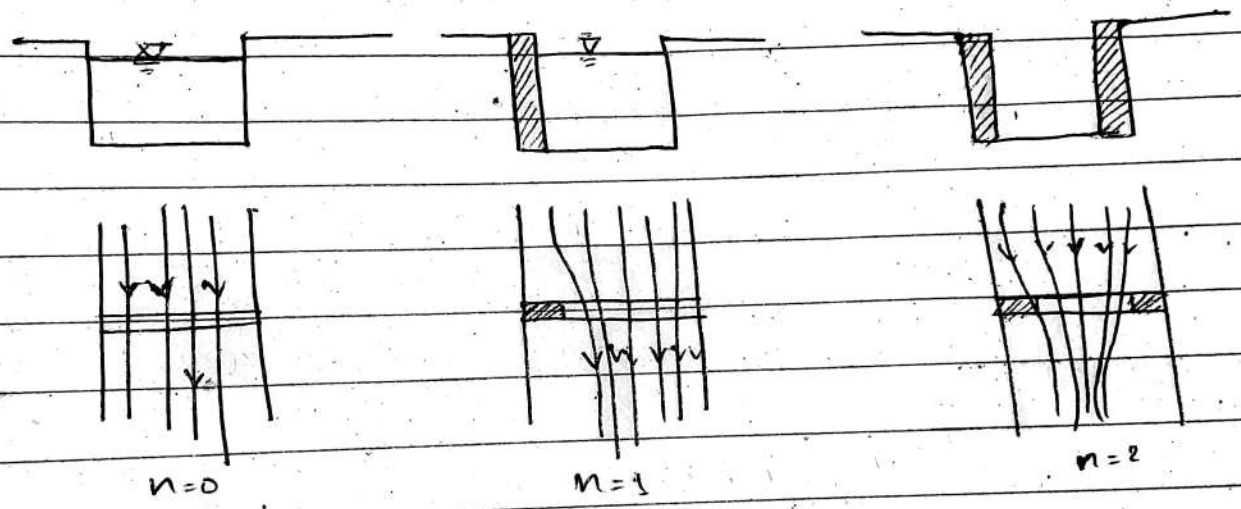
if approach velocity, $\vec{V_a}$ is consider,

$$h_a = \frac{V_a^2}{2g}$$

$$\cancel{Q = \frac{2}{3} C_d L \sqrt{2g}}$$

$$Q = \int_{h_a}^{H+h_a} C_d L \cdot \sqrt{2gh}\, dh = \frac{2}{3} C_d \sqrt{2g}\, L \left[(H+h_a)^{3/2} - h_a^{3/2} \right]$$

This is for with no end contraction case.



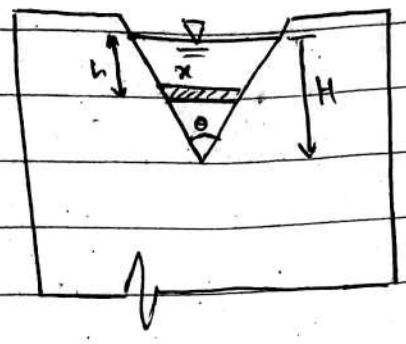
$$L = (L - 0.1nH)$$

So,

$$Q = \frac{2}{3} C_d \sqrt{2g}\, (L - 0.1nH_1)\, (H_1^{3/2} - h_a^{3/2})$$

$$V_A = \frac{Q}{(2+H) \times L}$$

Flow over triangular notch or V-notch



$$\frac{x}{2(H-h)} = \tan\theta/2$$

$$\therefore x = 2(H-h)\tan\theta/2 \quad \text{---- (i)}$$

$$dQ = C_d \cdot 2(H-h)\tan\theta/2 \, x \, dh \sqrt{2gh}$$

$$Q = \int_0^H C_d \, 2(H-h)\tan\theta/2 \, dh \sqrt{2gh}$$

$$\Rightarrow 2C_d \tan\theta/2 \sqrt{2g} \frac{H^{3/2}}{3/2} - C_d \tan\theta/2 \sqrt{2g} \frac{H^{5/2}}{5/2}$$

$$\Rightarrow \boxed{Q = \frac{8}{15} C_d \sqrt{2g} \cdot \tan\theta/2 \cdot H^{5/2}}$$

if $\theta = 90°$

$$\boxed{Q = \frac{8}{15} C_d \sqrt{2g} \, H^{5/2}}$$

if velocity of approach is included,

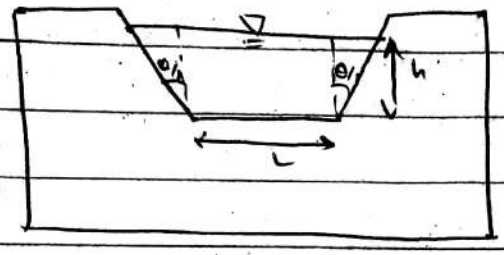
$$h_a = \frac{V_a^2}{2g}$$

$$Q = \int_H^{H+h_a} C_d \, 2(H-h)\tan\theta/2 \, dh \sqrt{2gh}$$

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\theta/2 \left[(H+h_a)^{5/2} - h_a^{5/2} \right]$$

~~Rectangle~~

Flow over haperoidal notch.



$Q = Q_1 + Q_2 \longrightarrow$ V notch with angle θ.

Q_1 = rectangular with crest length L

so,

$$Q = Q_1 + Q_2$$

$$= \frac{8}{15} C_{d_2} \sqrt{2g}\, H^{5/2} \tan\theta/2 + \frac{2}{3} C_{d_1} \sqrt{2g}\, H^{3/2}$$

if C_{d_1} and C_{d_2} is replaced by C_d

$$Q = C_d \sqrt{2g}\, H^{3/2} \left[\frac{2}{3} L + \frac{8}{15} H \tan\theta/2 \right]$$

Cippolleti weir (Italian engineer)

loss due to end contraction :

$$Q = \frac{2}{3} C_d \sqrt{2g}\, (L - 0.1nH)\, H^{3/2}$$

$$= \frac{2}{3} C_d \sqrt{2g}\, L H^{3/2} - \boxed{\frac{2}{3} C_d \sqrt{2g} \cdot 0.1nH^{5/2}} \text{ loss}$$

it is compensated by slanting the both sides by angle $\theta/2$ to form a haperoidal shape.

i.e. $(n=2)$,

$$\frac{2}{3} C_d \sqrt{2g}\, 0.1nH^{5/2} = \frac{8}{15} C_d \sqrt{2g}\, H^{5/2} \tan\theta/2$$

$$\Rightarrow \tan\theta/2 = 1/4$$

$$\Rightarrow \theta/2 = 14.036 \quad \therefore \theta = 28.07^\circ$$

~~Error due to measurement~~

Error in discharge due to error in head measurement:

For rectangular weir:

$$Q = C_d H^{3/2} \sqrt{2g} \cdot L \cdot \frac{2}{3}$$

$$= K H^{3/2}$$

where, $K = \frac{2}{3}\sqrt{2g}\, C_d$.

$$\frac{dQ}{dH} = \frac{3}{2} K H^{1/2}$$

$$\Rightarrow dQ = \frac{3}{2} K H^{1/2} dH$$

$$\Rightarrow \frac{dQ}{Q} = \frac{dH}{H} \times \frac{3}{2}$$

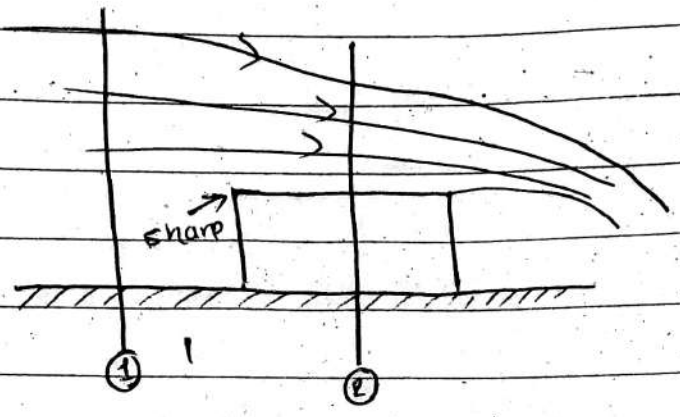
For V-notch:

$$Q = C_d H^{5/2} \sqrt{2g} \cdot \frac{8}{15} \tan\theta/2$$

$$\frac{dQ}{dh} = \frac{5}{2} K h^{3/2}$$

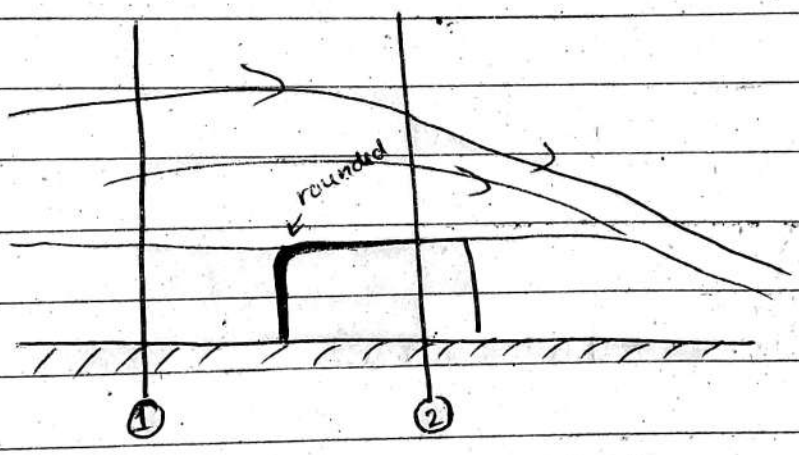
So, $\frac{dQ}{Q} = \frac{5}{2} \frac{dH}{H}$

Broad-crested weir

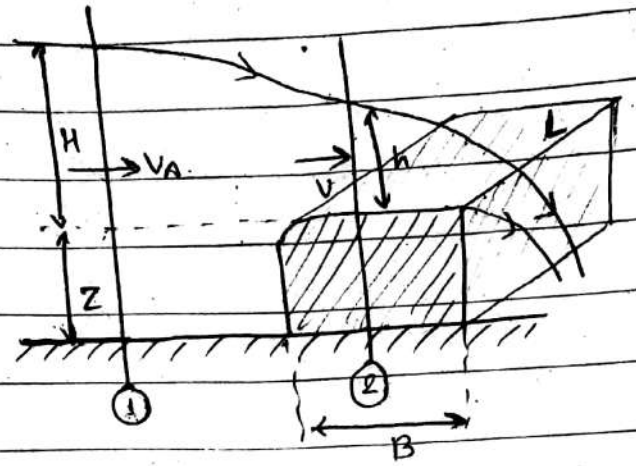


in section 1 and 2 Headloss cannot be neglected.

in order to neguct, we ~~d~~ make



now it can be neglected.



* $B < 0.625H \left(\frac{H}{B} > 1.6\right)$ - thin plate.

* $B \geq 0.625$ & $\leq 2.5H$

  $\left[0.4 \leq \frac{H}{B} \leq 1.6\right]$ - narrow crest

* $B \geq 2.5H$, $\leq 10H$

  $\left[0.1 < \frac{H}{B} \leq 0.4\right)$ - Broad crested weir

applying bernoulli's equation between section 1 and 2

$$H = h + \frac{v^2}{2g}$$ [Approach velocity neglected.

$$\therefore v = \sqrt{2g(H-h)}$$

$$\therefore Q = Lh\sqrt{2g(H-h)}$$

$$Q_{act} = C_d L h \sqrt{2g(H-h)} \quad \text{----} \quad (2)$$

For measuring maximus discharge, $\frac{dQ}{dh} = 0$.

$$\Rightarrow \frac{dQ}{dh} = 0$$

$$\Rightarrow C_d L \sqrt{2g} \frac{d}{dh}\left(h\sqrt{H-h} \right) = 0$$

$$\Rightarrow C_d L \sqrt{2g} \left(1\sqrt{H-h} + \frac{1}{2\sqrt{H-h}} (-1)\, h \right) = 0$$

$$\Rightarrow C_d L \sqrt{2g} \left(\frac{2H-2h-h}{2\sqrt{H-h}} \right) = 0$$

$$\Rightarrow C_d L \sqrt{2g} \frac{(2H-3h)}{2\sqrt{H-h}} = 0$$

So,

$$2H - 3h = 0$$

$$\Rightarrow h = \frac{2}{3} H$$

So,

$$Q_{act} = C_d L \frac{2}{3} H \sqrt{2g(H - 2/3 H)}$$

$$\boxed{Q_{act} = 1.70\, C_d . L H^{3/2}}$$

$$\therefore C_d = 0.85 \sim 1.$$

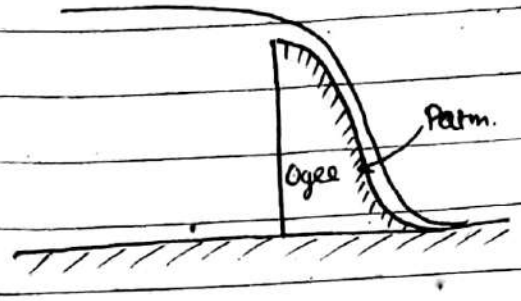
if velocity of approach is considered.

$$H_1 = H + H_A \xrightarrow{\text{replace}} H \quad (H_A = V_A^2/2g)$$

So,

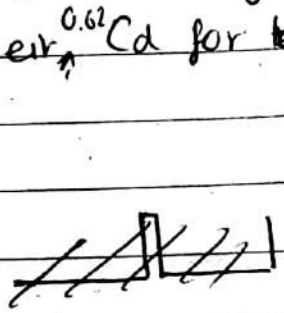
$$\boxed{Q_{act} = 1.70\, C_d . L (H + H_A)^{3/2}}$$

Ogee shaped weir:



* Water flows over a retangular weir 1m wide and depth of 0.15m and afterwards passes through the triangular right angled weir. Find the depth of the water through triangular weir. Take Cd for rectangular weir 0.62, Cd for triangular = 0.59)

Solution,



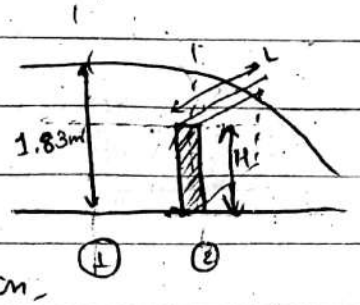
discharge through rectangular weir = discharge from triangular weir

$$\frac{2}{3} C_d L H^{3/2} \sqrt{2g} = \frac{8}{15} C_d \sqrt{2g}\, H^{5/2} \tan \theta/2$$

$$\Rightarrow 1 \times \frac{2}{3} \times 0.62 \times 0.15^{3/2} = \frac{8}{15} \times 0.59 \times H^{5/2}$$

$$\Rightarrow 0.3573 \text{ m}$$

* A suppressed rectangular weir 7.62 m long is to discharge 10.6 m^3/s into a channel. The weir factor $K = 1.85$ $[Q = KLH^{3/2}]$. To what height may the weir be built if the water behind the weir must not exceed 1.83 m in deep.



→ Solution.

Consider velocity of approach also.

$$Q = KL\left[(H+H_A)^{3/2} - H_A^{3/2}\right]$$

$$10.6 = KL\left[\left(H + \frac{V_a^2}{2g}\right)^{3/2} - H_A^{3/2}\right]$$

$$\Rightarrow 10.6 = KL\left(H + \frac{\left[Q/(1.83 \times L)\right]^2}{2g}\right)^{3/2}$$

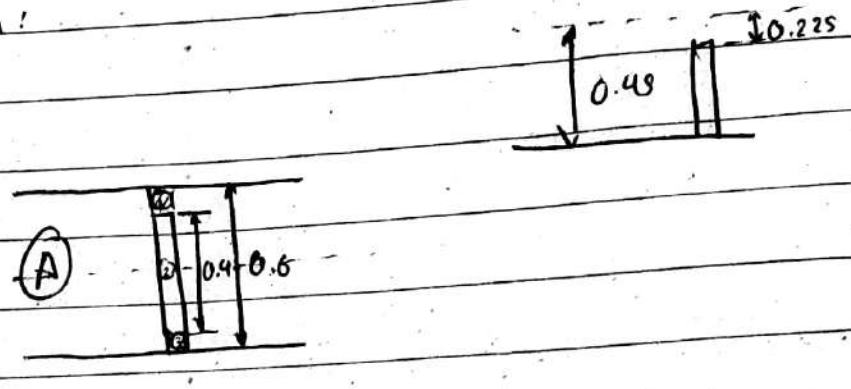
$$\Rightarrow \frac{10.6}{1.85 \times 7.62} = \left(H + \frac{\frac{10.6}{(1.83 \times 7.62)}}{2 \times 9.81}\right)^{3/2}$$

$H = 0.788\,m$

$$\frac{10.6}{1.85 \times 7.62} + \left[\frac{\left(\frac{10.6}{1.83 \times 7.62}\right)}{2 \times 9.81}\right]^{3/2} = \left(H + \frac{\left(\frac{10.6}{1.83 \times 7.62}\right)}{2 \times 9.81}\right)^{3/2}$$

$H = 1.028\,m$

* A rectangular notch of crest width 0.4m is used to measure the flow of water in a rectangular channel 0.6 m wide and 0.45m deep. If water level in the channel is 0.225 m above the weir crest. Find discharge in the channel if $C_d = 0.63$. Take V_A in consideration:



discharge at A = ~~discharge area~~ at ① + discharge at ②
+ discharge at ③

$\Rightarrow$ ~~$V_A \times 0.6 \times 0.45$~~ =

$$\Rightarrow Q = \frac{2}{3} C_d \sqrt{2g} \left(L - 0.1 \times 2\right) \left(H^{3/2} - H_a^{3/2}\right)$$

neglect approach velocity:

$$Q = 2/3\, C_d \sqrt{2g}\, (L - 0.2H)\, (H^{3/2})$$

$$\Rightarrow = 2/3\, (0.63 \sqrt{2 \times 9.81} \times (0.4 - 0.2 \times 0.225)\, (0.225^{3/2})$$

$$= 0.07048\ m^3/s$$

So, $V_A = 0.261\ m/s$

$H_A = 3.472 \times 10^{-3}\ m^3/s$

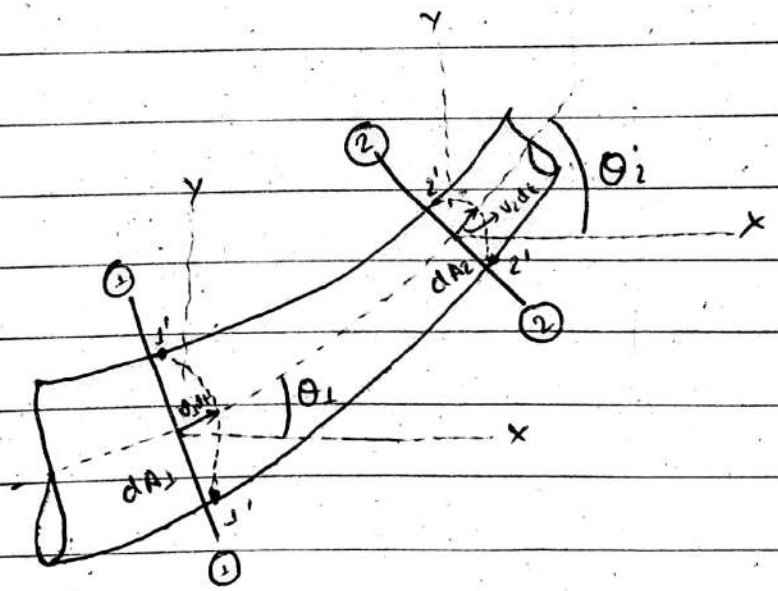
So, $Q = 0.07062\ m^3/s$

MOMENTUM EQUATION:

$$F_x = \frac{dM_x}{dt}, \quad \boxed{F_x dt = dM_x} \quad \cdots\cdots (1)$$

this equation is known as impulse-momentum principle.

However, this equation is applicable to finite or discreak bodies for which action of any force may take place and completed in finite period of time. On the other hand, steady flow of fluid involves a motion which is continuous and is not completed in finite period of time. so, this equation must be expressed suited to fluid flow problem



Let us consider a stream tube.

Let 1-1, 2-2 at time t.

1'-1', 2'-2' at time t+dt.

For steady flow, the following continuity equation holds:

Fluid mass within section 1-1 and 1'-1' = Fluid mass within section 2-2 and 2'-2'. ..... (i)

In x-direction,

apply impulse momentum theorem:

eq^n (2) $\rightarrow$ $\left[\begin{array}{c}\text{Change in momentum of fluid} \\ \text{mass during } dt\end{array}\right]_x = \left[\begin{array}{c}\text{momentum at } t+dt \\ \text{of fluid } 1'-1' \text{ and} \\ 2'-2' \text{ in } x\end{array}\right]_x - \left[\begin{array}{c}\text{momentum of} \\ \text{fluid } 1-1 \text{ and} \\ 2-2 \text{ in } x\end{array}\right]_x$

but,

$$\left[\begin{array}{c}\text{momentum at } t+dt \\ \text{of flow } 1'-1' \,\&\, 2'-2'\end{array}\right]_x = \left[\begin{array}{c}\text{momentum at } t+dt \\ \text{of flow } 1'-1' \,\&\, 2-2\end{array}\right]_x + \left[\begin{array}{c}\text{momentum at } t+dt \\ \text{of flow } 2-2 \,\&\, 2'-2'\end{array}\right]$$

and,

$$\left[\begin{array}{c}\text{momentum at } t \text{ of} \\ \text{flow } 1-1 \,\&\, 2-2\end{array}\right]_x = \left[\begin{array}{c}\text{momentum at } t \text{ of} \\ \text{flow } 1-1 \,\&\, 1'-1'\end{array}\right]_x + \left[\begin{array}{c}\text{momentum at } t \\ \text{of flow in} \\ 1'-1' \,\&\, 2-2\end{array}\right]_x$$

$\Rightarrow$ So from eq^n (i),

$$\left[\begin{array}{c}\text{Change in momentum of} \\ \text{fluid mass during } dt\end{array}\right]_x = \left[\begin{array}{c}\text{momentum at } t+dt \\ \text{of flow } 2-2 \,\&\, 2'-2'\end{array}\right]_x - \left[\begin{array}{c}\text{momentum at } t \text{ of} \\ \text{flow } 1-1 \,\&\, 1'-1'\end{array}\right]$$

$$\Sigma d(mV_x) = \int_{A_2} \rho_2 V_2\, dt \cdot dA_2 \cdot V_{2x} - \int_{A_1} \rho_1 V_1\, dt\, dA_1\, V_{1x}$$

How? $\dfrac{\text{momentum}}{t} = \rho \times \dfrac{\forall}{t} \cdot v$

$= \rho\, Q \cdot v$

$= \rho\, (Av)\, v$.

so,

$$\Sigma F_x = \int_{A_2} \rho_2 V_2 \, dA_2 \cdot (V_2 \cos\theta_2) - \int_{A_1} \rho_1 V_1 \, dA_1 (V_1 \cos\theta_1) \quad \text{----- (ii)}$$

if velocity is taken as average V.

then,

$$\boxed{\Sigma F_x = \rho_2 A_2 \cos\theta_2 V_2^2 - \rho_1 A_1 \cos\theta_1 V_1^2}$$

if,

$$Q = A_1 V_1 = A_2 V_2.$$
$$\rho_1 = \rho_2 = \rho.$$

then,

$$\boxed{\begin{aligned} &\Sigma F_x = \rho Q (V_2 \cos\theta_2 - V_1 \cos\theta_1). \\ &\Rightarrow \Sigma F_x = \dot{m} (V_{2x} - V_{1x}) \end{aligned}}$$

similarly in ~~x~~ y-direction

$$\boxed{\begin{aligned} &\Sigma F_y = \rho Q (V_2 \sin\theta_2 - V_1 \sin\theta_1) \\ &\Rightarrow \Sigma F_y = \dot{m} (V_{2y} - V_{1y}). \end{aligned}}$$

in summary,

total force exerted <u>on the fluid</u> in a control volume in a given direction = rate of change of momentum in a given direction of flow passing through the control volume. So, $\boxed{F = \dot{m}(V_{out} - V_{in})}$

F is +ve in direction in which velocity is assumed positive

The F can be further divided:

F_1 = Force exerted in the given direction on the fluid in control volume by any solid body within control volume (CV) or coinciding with the boundaries of control volume

F_2 = Force exerted in given direction on the fluid in the control volume by body forces such as gravity.

F_3 = Force exerted in the given direction on the fluid in the control volume by fluid outside the control volume.
(So, F_3 = pressure force)

So,

$$\boxed{F_1 + F_2 + F_3 = \dot{m}\,(V_{out} - V_{in})} \quad \text{.....} \quad (i)$$

So,

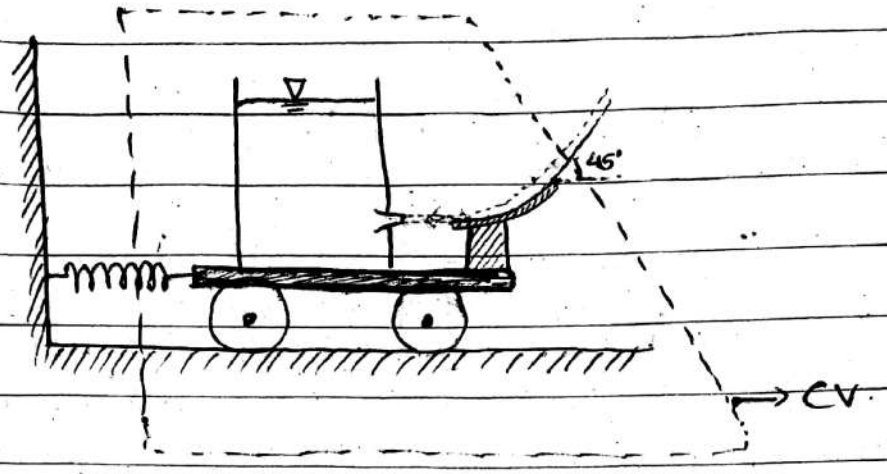
R exerted by the fluid on the solid boundary ~~of the control volum~~ inside, outside or coinciding with control volume in the given direction will be equal and opposite to F_1 so,

$$R = -F_1$$

So,

$$R = (F_2 + F_3) - \dot{m}\,(V_{out} - V_{in})$$

Application of momentum equation



50mm dia. nozzle at 5m/s.

Find compression of spring if stiffness of spring is 2kN/m.

$$Q = A.v$$

$$\Rightarrow \frac{V}{t} = \pi \times \frac{(50\times10^{-3})^2}{4} \times 5 m/s.$$

$$\Rightarrow \frac{m}{t} = \pi \times \frac{(50\times10^{-3})^2}{4} \times 5 \times \rho$$

$$\therefore \dot{m} = \frac{25\pi}{8}$$

now,

$$F_1 + F_2 + F_3 = \dot{m}(V_{out} - V_{in})$$

only external force acting is spring reaction.

F_1 = force on the fluid by spring.

$F_2 = 0$.

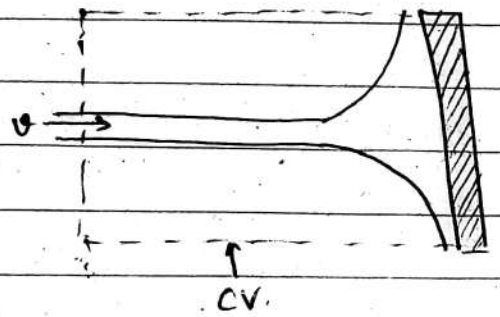
$F_3 = 0$

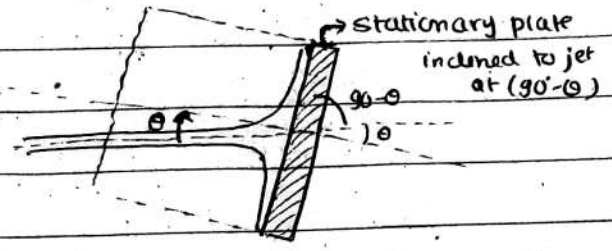
So,

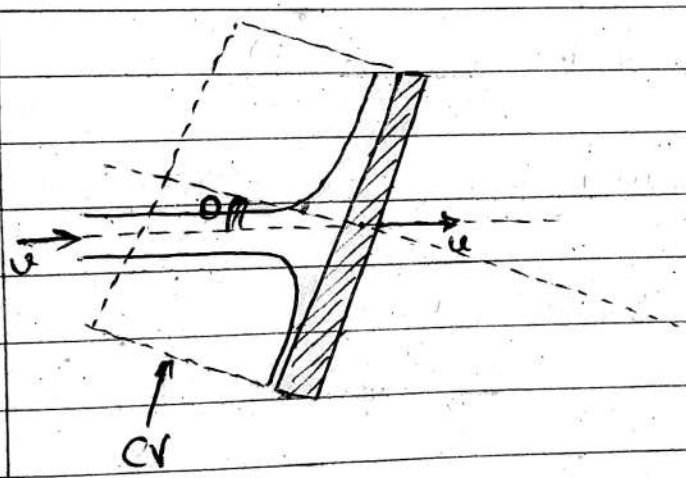
$$-Kx = \frac{25\pi}{8}(5 \cos 45 - 0)$$

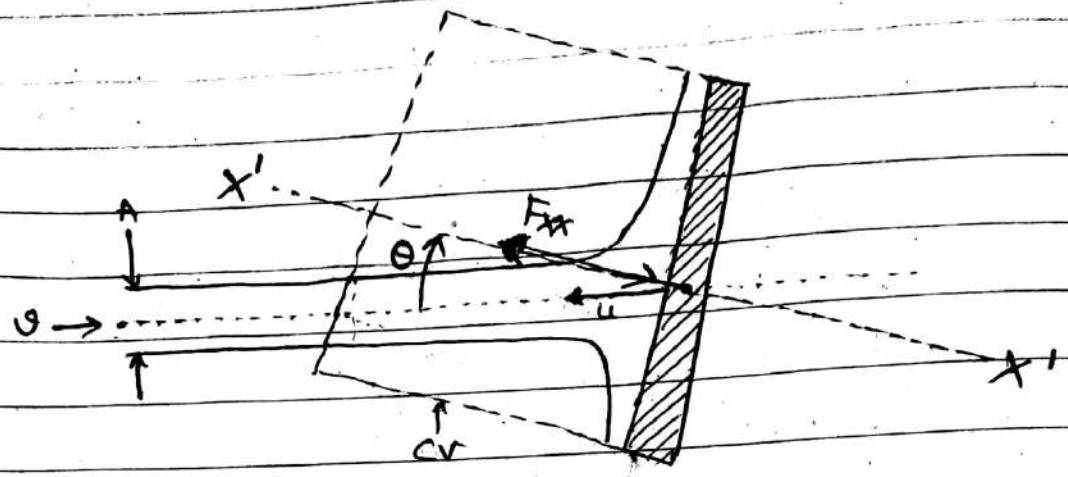
$x = 0.017 m$

2) Force exerted by a jet striking a flat plate









* sign convention is +ve in the direction of initial jet velocity.

* It is helpful to consider component of force perpendicular and parallel to surface of plate.

(Analysis in x'-x' direction)

$\dot{m}$ = mass flow entering the CV = $\rho A(v-u)$

Force normal to the plate by the jet = $\dot{m}[0-(v-u)\cos\theta]$

$$\boxed{\begin{aligned} \therefore F_{x'x'} &= \rho A (v-u)^2 \cos\theta \\ &= \rho A (v-u)(v-u)\cos\theta \end{aligned}}$$

For case B:

$u=0$,

$$\boxed{F_{x'x'} = \rho A v^2 \cos\theta}$$

For case C:

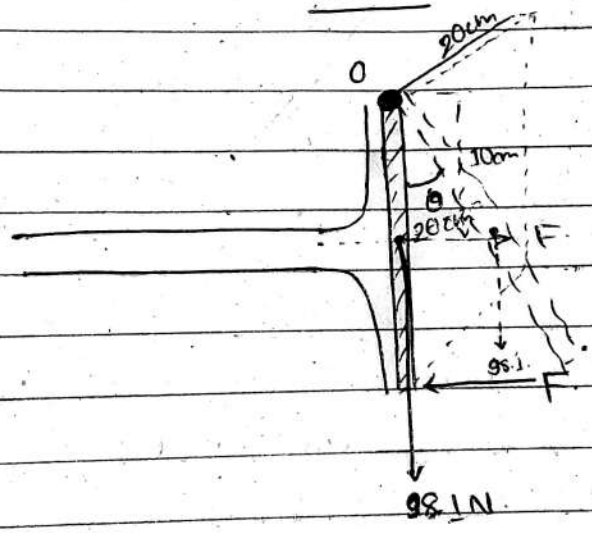
$u=0$, $\theta=0$

so, x'-x' becomes x-x

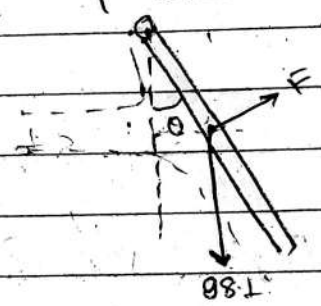
so, $\boxed{F_{xx} = F_{x'x'} = \rho A v^2}$

* A jet of water of diameter 25mm strikes a 20cm x 20cm² square plate of uniform thickness with a velocity of 10m/s at the centre of the plate which is suspended vertically by a hinge on its top horizontal edge. The weight of the plate is 98.1N. The jet strikes normal to the plate. What force must be applied at the lower edge of the plate so that the plate is always kept vertical. If the plate is allowed to deflect freely what will be the inclination of the plate with vertical due to force exerted by the jet of water.

Case I



Case II



now,

$$F_{xx} = \rho a v^2$$

$$= 1000 \times \pi \times \frac{(25\times10^{-3})^2}{4} \times 10^2$$

$$= 49.087\ N.$$

So,

$$\Sigma M_0 = 0$$

$$\Rightarrow 49.087 \times 0.1 = F \times 0.2$$

$$F = 24.5436\ N$$

$$F = \rho A v^2 \cos\theta$$

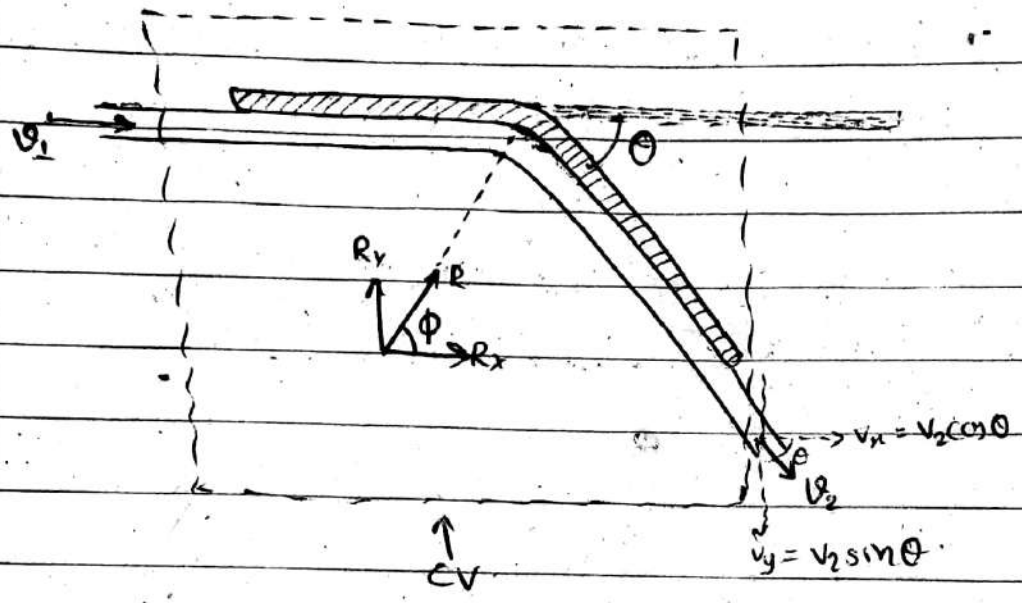
$$= 49.087 \cos\theta$$

$$\Sigma M_0 = 0$$

$$\Rightarrow 98.1 \times 0.1 \sin\theta - 49.087 \cos^2\theta \times 0.1 \cos\theta - 49.087 \cos\theta \sin\theta \times 0.1 \sin\theta = 0.$$

$$\theta = 26.58^\circ$$

Force due to deflection of the jet by a curved vein



A jet of the water from a nozzle is deflected through an angle 60° from its original direction by a curved vein which enters tangentially without shock with a mean velocity of 30m/s and leaves with 25m/s. If discharge through the nozzle is 0.8kg/s. Calculate magnitude and direction of resultant force on the vein if vein is stationary.

$$F_1 + \cancel{F_2}^{0} + \cancel{F_3}^{0} = \dot{m}\,(V_{out} - V_{in})$$

but

$$R = -F_1$$
$$= -\dot{m}\,(V_{out} - V_{in})$$
$$= \dot{m}\,(V_{in} - V_{out})$$

$R_x = \dot{m}\,(v_{in} - v_{out})_x$

$R_y = \dot{m}\,(v_{in} - v_{out})_y$

$\Rightarrow \quad R_x = 0.8\,(V_1 - V_2\cos\theta) = 14$

$R_y = 0.8\,(V_2\sin\theta) = 10\sqrt{3}$

$\Rightarrow$ ~~R_x~~ so, $R = \sqrt{(10\sqrt{3})^2 + 14^2}$

$= 22.91\ N$

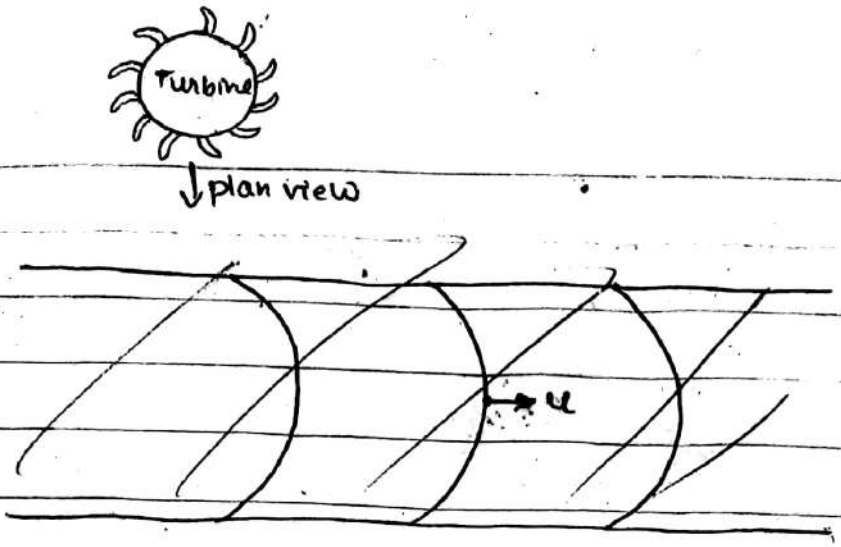
so, $\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

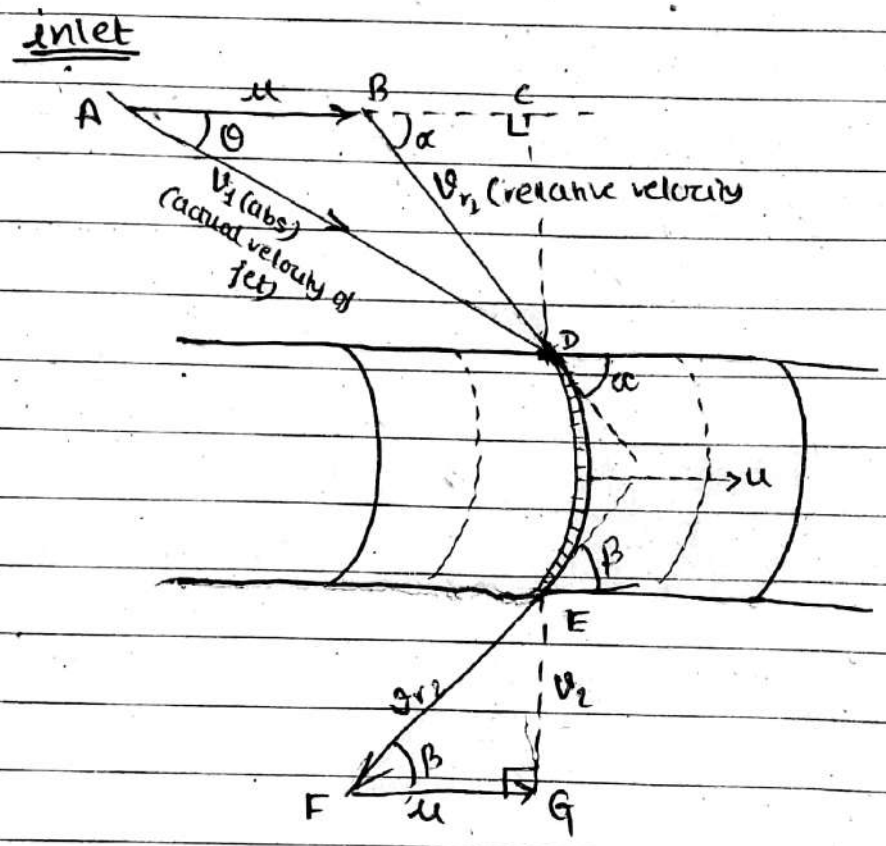
$= 51.05^\circ$

$= 51^\circ\ 3'$

<u>Force exerted when a jet is deflected by a moving curved vane:</u>

PTO





A jet of water 100mm in diameter leaves a nozzle with 36 m/s and is deflected by a series of veins moving with 15 m/s in a direction at 30° to the direction of the jet so that it leaves the vein with absolute mean velocity which is right angled to the direction of motion of vein. Due to the friction, the velocity of fluid relative to the vein.

at the outlet is 0.85 times relative velocity at the inlet. Calculate, inlet angle α and outlet angle β of the vein. ii) Force exerted on the series of veins in the direction of motion of vein.

→ Solution.

From inlet velocity triangle.

$Q = \frac{V}{t}$

$= \frac{m}{\rho t}$

$Q \times \rho = \dot{m}$

$\cancel{\tan\alpha = \frac{CD}{BC}}$ $\cancel{\sin\alpha = \frac{CD}{BD} = \frac{V_1 \sin 30^\circ}{\sqrt{u^2 + V_1^2}}}$

$\cancel{= 0.4615}$

$\cancel{\alpha = 27.4864^\circ}$

$$\tan\alpha = \frac{CD}{BC} = \frac{V_1 \sin 30}{V_1 \cos 30^\circ - u}$$

$$\alpha = 48.05^\circ.$$

$$\cos\beta = \frac{u}{V_{r_2}} = \frac{15}{0.85 V_{r_1}} = \frac{15}{0.85 \times 24.20}$$

$$\beta = 43.181^\circ$$

now,

$$F_1 + \cancel{F_2}^{0} + \cancel{F_3}^{0} = \dot{m}(V_{out} - V_{in})$$

$$-R = \dot{m}(V_{out} - V_{in})$$

$$\Rightarrow -R_x = \dot{m}(V_{out} - V_{in})_x$$

$$\Rightarrow R_x = \dot{m}(V_{in} - V_{out})_x$$

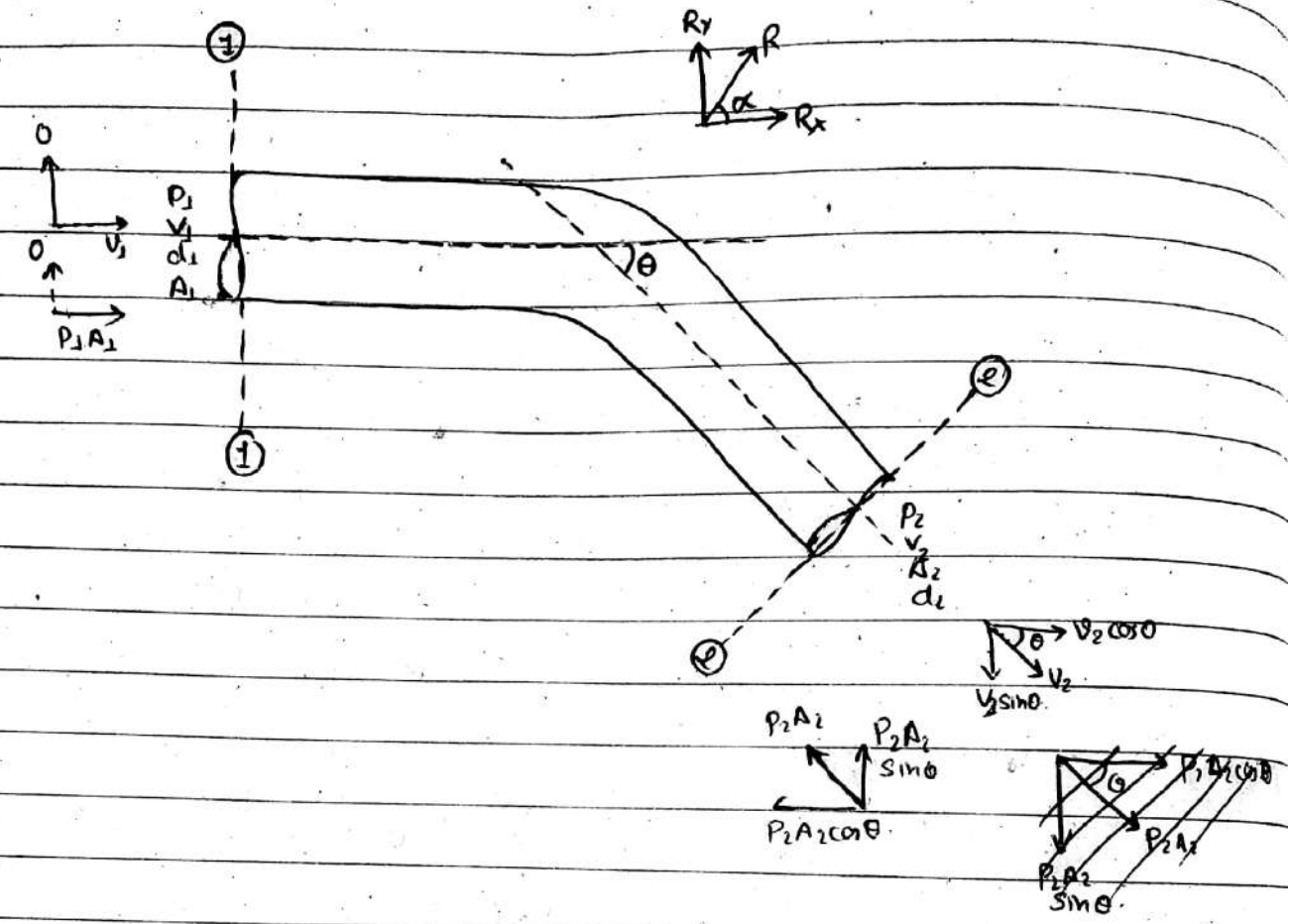
$$= Q \times \rho (V_{in} - V_{out})_x$$

$$= \frac{\pi d^2}{4} \times 36 \times \rho (V_{in} - V_{out})_x$$

$$\Rightarrow 90\pi (V_{in} - V_{out})_x = 90\pi (V_1 \cos 30^\circ + V_2 \cos 90^\circ)$$

$$= 8815.06 \text{ N}$$

Forces on pipe bends and closed conduits:



* A pipe bend tapers from 500mm at inlet to 250mm at the outlet and turns the flow through an angle 45°. $P_1 = 40 KN/m^2$, $P_2 = 23 KN/m^2$. If pipe is conveying oil of density 850 kg/m³. Calculate magnitude and direction of resultant force on the bend when flow rate is 0.45m³/s. The bend is in horizontal plane.

$$R_x = -F_1x$$

$$(F_1 + \cancel{F_2} + F_3)_x = \dot{m}(V_{out} - V_{in})_x$$

$$\Rightarrow R_x = F_{3x} + m(V_{in} - V_{out})_x$$

$$= (P_1A_1 - P_2A\cos\theta) + \rho \times 0.45 \times 0.45\left(\frac{1}{A_1} - \frac{1}{A_2}\right)\cos\theta$$

$$= 2500\pi - \cancel{1194.0986}\ 798.33 + 172.125\left(\frac{16}{\pi} - \frac{64}{\pi}\cos 45\right)$$

$= 5443.493\ N.$

Similarly,

$$R_y = F_{3y} + \dot{m}(V_{in} - V_{out})_y$$

$$= + P_2 A_2 \sin\theta + 0.45 \times 850 \times \left(0 + \frac{0.45 \times 4 \sin 45}{\pi \times 250^2 \times 10^{-6}}\right)$$

$$= 3277.80\ N$$

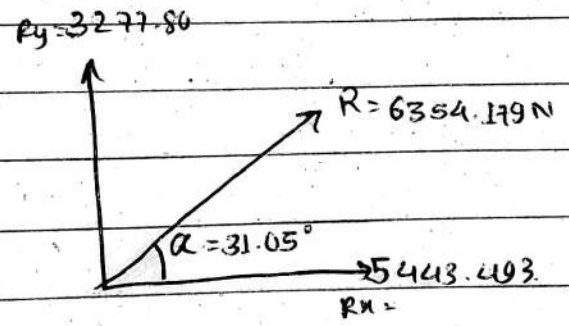
So,

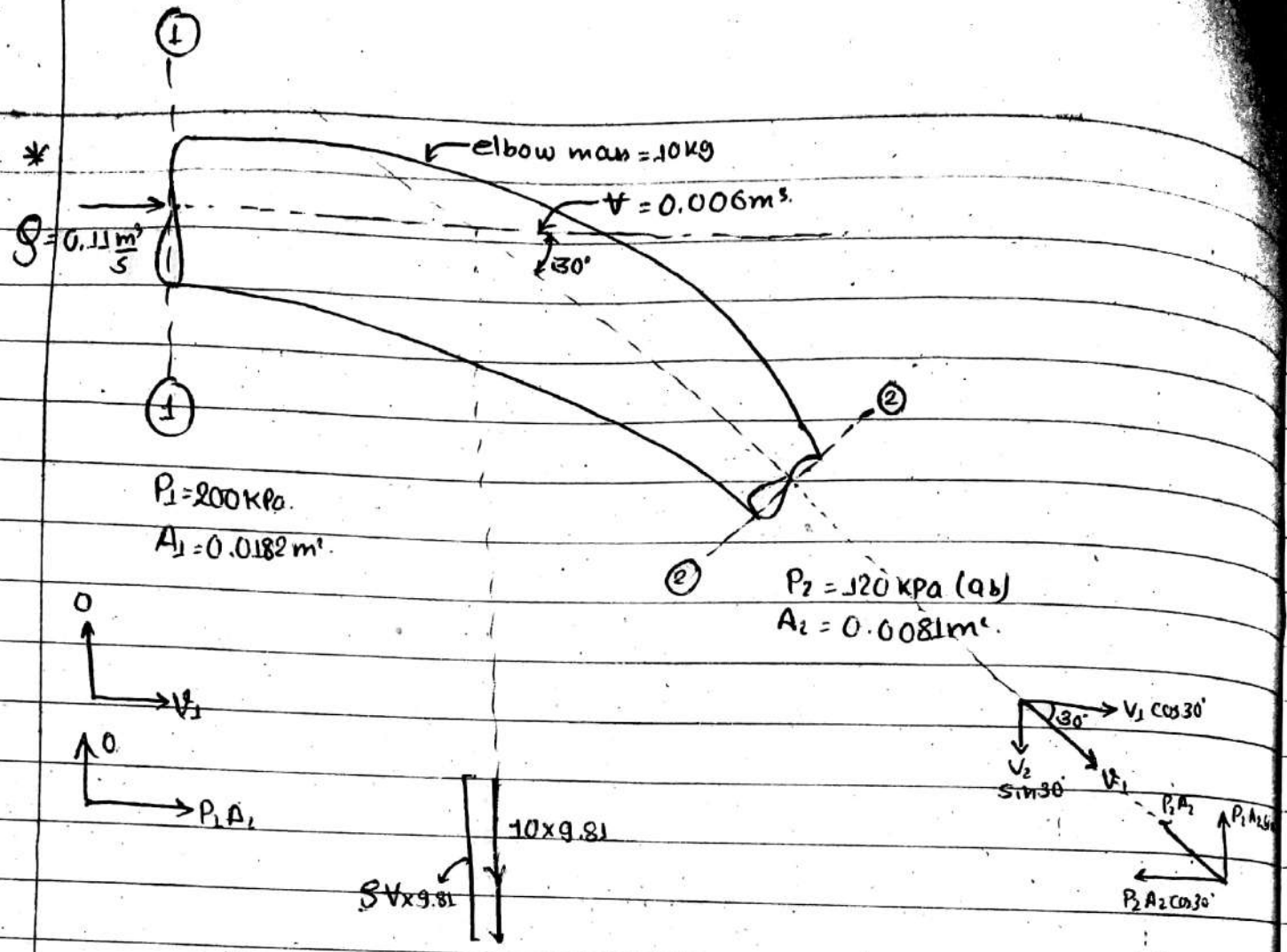
$$R = \sqrt{R_x^2 + R_y^2}$$

$$= 6354.179\ N$$

and,

$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 31.05^\circ$$





now, using principle of momentum:

$$F_1 + F_2 + F_3 = \dot{m}\,(V_{out} - V_{in})$$

in x-direction

$$(F_1 + F_2 + F_3)_x = \dot{m}\,(V_{out} - V_{in})$$

$$\Rightarrow R_x = \cancel{F_{2x}}^{\,0} + F_{3x} + \dot{m}\,(V_{in} - V_{out})$$

$$= P_1A_1 - P_2A_2\cos 30^\circ + Q.\rho\left(\frac{Q}{A_1} - \frac{Q}{A_2}\cos 30^\circ\right)$$

$$R_y = F_{2y} + F_{3y} + \dot{m}\,(V_{in} - V_{out})_y$$

$$= -(10 * 9.81 + \rho \forall * 9.81) + \rho Q \left(0 + \frac{Q}{A_2} \sin 30^\circ\right)$$

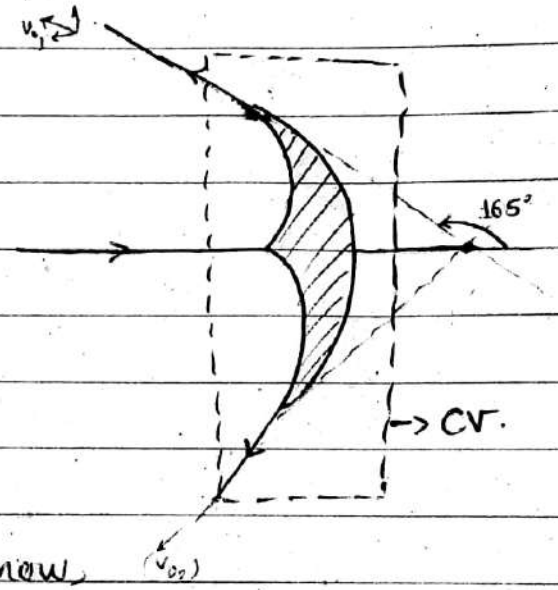
$$+ \cancel{P_2 A_2} \; 0 + P_2 \cancel{A_2} \; A_2 \cos 30^\circ$$

Numericals:

* A jet of water 15mm in diameter impinges on a fixed cup which deflects the jet by 165° as shown in figure. If the reaction of the cup was found to be 25.5N when Q = 980N per minute, calculate the ratio of:

a) Actual to theoretical force of the jet

b) Velocity at outlet to velocity at inlet



we know,

From momentum equation:

$$F_1 + F_2 + F_3 = \dot{m}(V_{out} - V_{in})$$

here,

F_2 and F_3 are neglected and F_1 = on the fluid.

R = by the fluid so, R = -F_1

so, $-R = \dot{m}(V_{out} - V_{in})$

$\Rightarrow R = \dot{m}(V_{in} - V_{out})$

given,

$$Q = 980\frac{N}{m} = \frac{980}{9.81 \times 1000 \times 60} = 1.6649 \times 10^{-3}\ m^3/s$$

$$= \frac{980}{9.81 \times 60 \times 1000} = 1664.96$$

So, $\dot{m} = 1.66496$ kg/s.

So,

$$R_x = 1.66496 \left[V_{in} - (-V_{in} \cos 15^\circ) \right]$$

now, $V_{in} = \frac{Q}{A_1} = \frac{1.66496 \times 10^{-3} \times 4}{\pi \times 15^2 \times 10^{-6}} = 9.421$ m/s.

So,

$$R_x = 1.66496 \left[9.421 + 9.421 \cos 15 \right]$$
$$= 30.83914$$

given,

~~theoretical~~ observed actual force = 26.5N

theoretical force = 30.83914 N.

So,

$$a) = \frac{26.5}{30.83914} = 0.8621$$

now,

$$V_0 = \sqrt{V_{0x}^2 + V_{0y}^2} \quad \text{--- (1)}$$

now

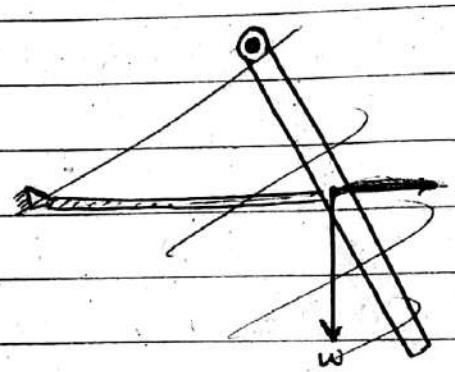
$$V_{0x} = V_1 \cos 15^\circ = 9.0998 \text{ m/s}$$

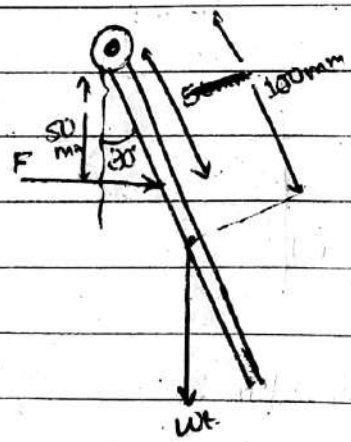
$$V_{0y} = V_0 \sin 15$$

So,

$$V_0^2 = 82.80976 + V_0^2 \times (0.066987)$$

* A metal plate of 10 mm thickness and 200 mm square is hung so that it can swing freely about the upper horizontal edge. A horizontal jet of water of 20mm diameter impinges with its axis 1' and 50 mm below the edge of the hinge and keeps it steadily inclined at 30° to the vertical. Find the velocity of the jet if the sp. wt of the metal is 75.54 KN/m³.





given:

$$\text{sp.wt} = 75540 \frac{N}{m^3}$$

so,

$$wt = 75540 \times (10 \times 20 \times 20 \times 10^{-9})$$

$$= 0.30216 = 30.22N$$

now, Area of jet $= \frac{\pi \times 20^2 \times 10^{-6}}{4} = 3.1415 \times 10$

$$F = \left(\frac{mv^2}{g}\right) \sin\theta$$

$$= 0.272 V^2.$$

taking moment,

$$20.22\left(\frac{100}{1000}\right)\sin 30 = 0.272\left(\frac{50}{1000}\right)\sec 30$$

$$\therefore V = 9.81 \text{ m/s}.$$

* A jet of water 75mm diameter having a velocity of 20 m/s strikes normally a flat smooth plate Determine the thrust on the plate (a) if the plate is at rest b) If the plate is moving in same direction as jet with a velocity of 5m/s. Also find the work done per se on the plate in each case and the efficiency of the jet when the plate is moving.

→ Solution,

a) $F = \frac{wav^2}{g}$

area of jet, $a = \frac{\pi}{4}(75^2 \times 10^{-6}) = 0.00442 m^2$

velocity = 20 m/s.

$$F = \frac{9810 \times (0.00442) \times 20^2}{9.81} = 1768 N.$$

in case of rest $u = 0$.

work done = $F \times u = 0$.

b) again,

$$F = \frac{wa(v-u)^2}{g}$$

$$= 994.5 N$$

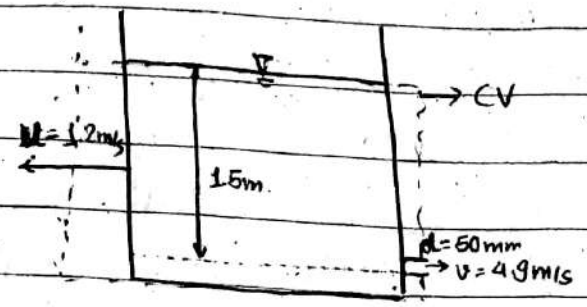
So, work done per sec = $F \times u = 994.5 \times 5 = 4972.5$ Nm.

So in case (b) output = 4972.5 J/s.

input = $\frac{1}{2}\left(\frac{mv}{g}\right)v^2 = 17680$ Nm/s.

$\therefore \eta$ of jet = 28.13%

Reaction on Jet:



Calculate reaction of the jet on the tank

1) If it is stationary.

2) When it is moving in opposite direction with velocity 1.2m/s

$m = \rho v = \frac{m}{s}$

$Q = \frac{V}{t} = \frac{m}{t\rho}$

QS =

$$F_1 + \cancel{F_2}^{0} + \cancel{F_3}^{0} = \dot{m}(V_{out} - V_{in})$$

$$\Rightarrow -R = \dot{m}(V_{out} - V_{in})$$

$$\begin{bmatrix}\text{reaction force in the} \\ \text{direction opposite to} \\ \text{jet}\end{bmatrix} = \begin{bmatrix}\text{mass discharge} \\ \text{per unit time}\end{bmatrix} * \begin{bmatrix}\text{increased velocity} \\ \text{in the direction of jet}\end{bmatrix}$$

$$\dot{m} = \frac{\pi d^2}{4} \times v \times \rho$$

increase in velocity = $(-4.9 - 0) = -4.9$.

$$\Rightarrow R = \frac{\pi d^2 \times v \times \rho}{4} \times +4.9$$

$$= 9.621 \text{ N}$$

b) increase in velocity = ~~(-4.9 i + j.2)~~

$v_{out} = (-4.9\,\hat{i} + j.2)$

$v_{in} = \hat{i} + j.2)$

so $v_{out} - v_{in} = -4.9$

so, the ~~this~~ Force is independent of velocity of the tank.

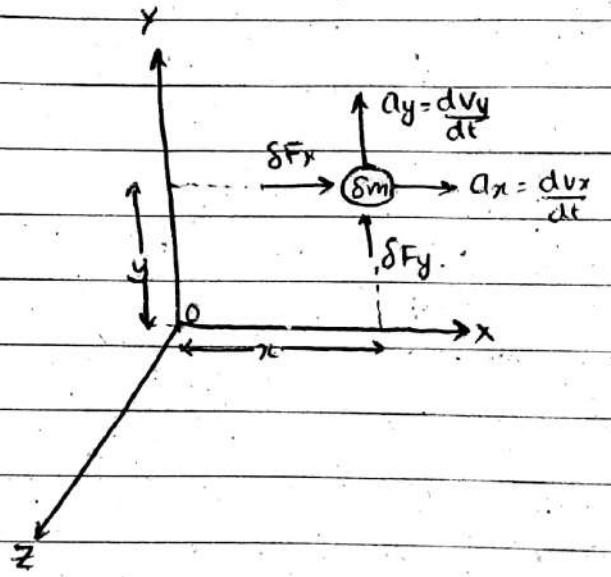
Angular momentum principle:

OR.

Moment of momentum principle:

A torque exerted on any body is equal to the rate of change of angular momentum. The torque is defined as moment of force. and the angular momentum is defined as moment of momentum.

Moment being taken about axis of rotation.



consider a mass δm rotating about z-axis. Then.

$$\delta F_x = \frac{dv_x}{dt}\cdot\delta m, \quad \delta F_y = \frac{dv_y}{dt}\cdot\delta m$$

moment of external force about z axis (+) i.e.

$$\text{torque } (T) = \delta T_z = x\,\delta F_y - y\,\delta F_x$$

$$= x\frac{dv_y}{dt}\delta m - y\frac{dv_x}{dt}\cdot\delta m$$

$$= \delta m\left[x\frac{dv_y}{dt} - y\frac{dv_x}{dt}\right]$$

From rule of differentiation,

$$\frac{d}{dt}(xV_y - yV_x) = \frac{dx}{dt}V_y - \frac{dy}{dt}V_x + x\frac{dV_y}{dt} - y\cdot\frac{dV_x}{dt}$$

$$= \cancel{V_x V_y} - \cancel{V_y V_x} + x\frac{dV_y}{dt} - y\frac{dV_x}{dt}$$

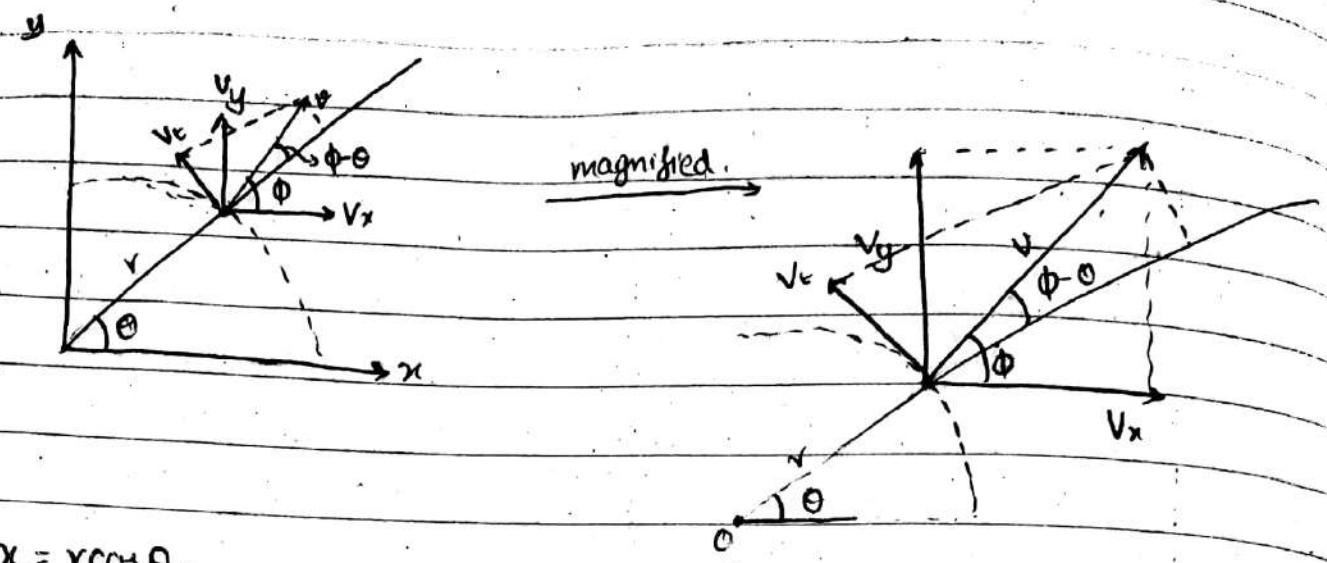
so,

$$\delta T_z = \delta m \frac{d}{dt}(xV_y - yV_x)$$

$$= \frac{d}{dt}\left[(xV_y - yV_x)\delta m\right]$$

$$= \frac{d}{dt}\left[(xV_y - yV_x)\delta m\right] \quad \text{----} \quad (1)$$

$$= \frac{d}{dt}(\text{change in angular momentum}) \quad \cancel{\text{----}} \quad \cancel{(1)}$$



$x = r\cos\theta$

$y = r\sin\theta$.

$V_x = V\cos\phi$

$V_y = V\sin\phi$

in equation 6,

$$x V_y - y V_x = r\cos\theta . V\sin\phi - r\sin\theta . V\cos\phi$$
$$= rV\sin(\phi-\theta)$$
$$= rV_t$$

$$\therefore \delta T_z = \frac{d}{dt}[r V_t . \delta m]$$

$$T_z = \dot{m}(r_2 V_{t_2} - r_1 V_{t_1})$$

$$\boxed{T_z = Q . \rho (r_2 V_{t_2} - r_1 V_{t_1})}$$

$$\Rightarrow \boxed{T_z - \rho Q r_2 V_{t_2} + \rho Q r_1 V_{t_1} = 0} \rightarrow T_z = \text{external force,}$$

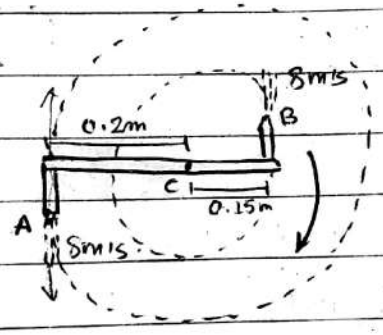
moment of momentum of fluid measured with respect to that axis remains constant if, no external torque is developed i.e $T_z = 0$.

This principle is known as law of conservation of moment of principle.

i.e.

Law of conservation of angular momentum

* A water sprinkler has 10 mm diameter nozzles at either end of rotating arm each of which is discharging water in opposite direction at right angles to the rotating arm at a velocity of 8 m/s. If the axis of rotation is at a distance of 0.15 m from one end and 0.2 m from other, determine the torque required to hold the arm stationary. If friction is neglected determine constant angular speed of the arm.



torque exerted by a water coming through the nozzle A, $T_A = \rho r_A V_A$

$= 1000 \times \frac{\pi d^2}{4} \times 0.2 \times V_A^2$

$= 1.0053$ Nm

Torque exerted by a water coming through the nozzle B, $T_B = \rho r_B V_B$

$= 1000 \times \frac{\pi \times d^2}{4} \times 0.15 \times 8$

$= 0.75398$ Nm.

So, Total torque $= 0.7398 + 1.0053$

$= 1.75928$ So anti-clockwise.

now,

the force developed opposite to B and A are equal but B is is small distance, to that of A so it rotates in clockwise direction.

absolute velocity of water at A = $V_1 = V_A - r_A\omega$

$= 8 - 0.2\omega$

$V_2 = V_B - r_B\omega$

$= 8 - 0.15\omega$

torque at A, $T_A = \rho Q r_A V_1$

$= 0.12566 V_1$

torque at B, $T_B = \rho Q r_B V_2$

$= 0.09424 V_2$

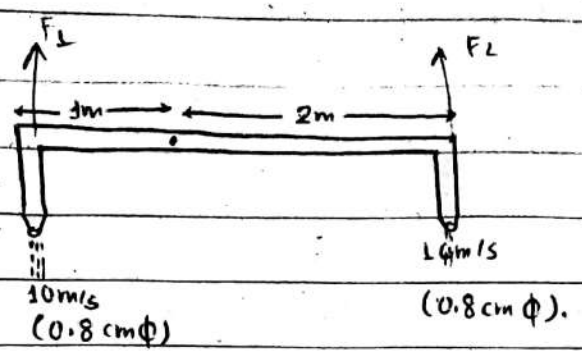
now,

$T_A + T_B = 0$

$\Rightarrow 0.12566(8 - 0.2\omega) + 0.09424(8 - 0.15\omega) = 0$

$\Rightarrow \omega = 44.8 \text{ rad/s}$

*



$\Rightarrow \quad F_1 = F_2$

Q. So, $T_1 = F_1 \times 1$ (+)

$T_2 = F_2 \times 2$ (+)

$T = T_1 + T_2$

$= F_1 - 2F_1$

$= -F_1$

$\therefore$ it rotates anti-clockwise.

$T = \dot{m}(10-0)$

$= \rho Q (10)$

$= -\pi \times \frac{(0.8 \times 10^{-2})^2}{4} \times 10^2 \times 1000$

$= 5.02$ Nm (+)

$V_1 = V_{rotation} + V_{actual}$

$\Rightarrow V_{actual} = V_1 - V_{rotation}$

A. $= 10 - (-1.\omega)$

$= 10 + \omega$

$V_{actual} = V_B - V_{rotation}$

B $= 10 - (-2\omega)$

$= 10 - 2\omega$

Torque due to this actual velocity $\Rightarrow \rho Q r_1 V_1 + \rho Q r_2 V_2$

$\Rightarrow 0.502(10+\omega) + 1.004(10 - 2\omega) = 0$

$\omega = 10$

Dimensional Analysis and Similarity:

Methods of dimensional analysis:

1) **Rayliegh Method:**

In this method, a functional relationship of some variables is expressed in the form of exponential equation which must be dimensionally homogeneous. If x is some function of variables $x_1, x_2, x_3 \cdots x_n$, the functional equation can be written as:

$$\underbrace{x}_{\text{dependent variable}} = f\,[\underbrace{x_1, x_2, x_3 \cdots x_n}_{\text{Independent variable}}]$$

This equation may be expressed as,

$$x = c\,(x_1^a . x_2^b . x_3^c \cdots x_n^{\cdot n})$$

where c is a dimensionless constant.

The exponent $a, b, c \cdots n$ are then evaluated on the basis that the equation is dimensionally homogenous.

* Consider the problem of fluid flow through small orifice discharge into the atmosphere at a constant head.

→ Let,

Q be the discharge.

d be the diameter of the orifice under constant head H. Let ρ be the density, viscosity of the liquid, ~~grav~~ acc. due to gravity 'g'.

$Q = f(a, H, \rho, g, \mu)$

by Rayleigh's method,

$$Q = k\mu^a \rho^b d^c H^e g^f$$

$$\Rightarrow \cancel{m^3/s} = [L^3T^{-1}] = k\,[ML^{-1}T^{-1}]^a [ML^{-3}]^b [L]^c [L]^e [LT^{-2}]^f.$$

$$\Rightarrow [L^3T^{-1}] = [M^0L^0T^0][ML^{-1}T^{-1}]^a [ML^{-3}]^b [L]^c [L^e][LT^{-2}]^f$$

for dimensionally homogeneous,

exponents must be identified.

For M,

$$0 = a + b \quad \text{---- (i)}$$

For, L, $3 = -a - 3b + c + e + f$ --- (ii)

For, T, $-1 = -a - 2f$ ---- (iii)

three of the unknown can be expressed interms of other 2,

$\Rightarrow b = -a$,

$$f = \frac{1-a}{2}$$

$$c = 3 + a + 3b - e - f.$$

$$= 3 + a - 3a - e - \frac{1-a}{2}$$

$$= \frac{6 - 4a - 2e - 1 + a}{2}$$

$$= \frac{5 - 3a - 2e}{2}$$

$$\Rightarrow Q = k\mu^a \rho^b d^c H^e g^f$$

$$\Rightarrow Q = k\mu^a \rho^{-a} d^{(5/2 - 3/2 a - e)} H^e g^{(1-a)/2}$$

$$= \cancel{k\,\frac{\mu\, d^{5/2} H^e \sqrt{g^{1-a}}}{\rho\, d^{3/2 a} \cdot d^e}} \quad k\left[d^{5/2} g^{1/2} \left(\mu^a \rho^{-a} d^{-3a/2} g^{-a/2}\right)\left(H^e d^{-e}\right)\right]$$

$$Q = k\left[d^2 \cdot d^{1/2} \cdot g^{1/2} \left(\frac{\mu}{\rho d^{3/2} g^{1/2}}\right)^a \left(\frac{H}{d}\right)^{-e}\right]$$

$$Q = k\left(\frac{\mu}{\rho d^{3/2} g^{1/2}}\right)^a \left(\frac{H}{d}\right)^{e-1/2} d^2 H^{1/2} g^{1/2}$$

$$= \frac{k}{\pi/4\sqrt{2}} \left(\frac{\mu}{\rho d^{3/2} g^{1/2}}\right)\left(\frac{H}{d}\right)^{d-1/2} \frac{\pi}{4} d^2 \sqrt{2gH}.$$

$$Q = a\sqrt{2gH} \cdot f_1\left[\frac{\mu}{\rho d^{3/2} g^{1/2}}, \frac{H}{d}\right]$$

$$\boxed{Q = C_d\, a\sqrt{2gH}}$$

C_d is a ~~fn~~ constant which is a function of dimensionless,

$$C_d = f\left(\frac{\mu}{\rho d^{3/2} g^{1/2}}, H/d\right)$$

* If resistance R to the motion of sphere of diameter 'D' moving with uniform velocity V through a fluid having density ρ and viscosity μ. Derive the formula for resistance using Rayleigh's method and also show that the equation can be reduced to $R = K\mu VD$. when the motion is through viscous fluid at low velocity. 'K' is dimensionless quantity Then find the viscosity in poise of a liquid through which a steel ball of diameter 1mm falls with a uniform velocity 20mm/s. The sp. gravity of liquid 0.91 and steel 7.8 given constant $K = 3\pi$.

→ Solution,

Let $R = f(V, D, \rho, \mu)$

- by rayleigh's method:

$$R = KV^a D^b \rho^c \mu^d$$

or, $[MLT^{-2}] = [M^0L^0T^0][LT^{-1}]^a [L]^b [ML^{-3}]^c [ML^{-1}T^{-1}]^d$

So,

$M \Rightarrow 1 = c + d$ ... (i)

$L \Rightarrow 1 = -a + b - 3c - d$ ... (ii)

$T \Rightarrow -2 = -a - d$ ... (iii)

$\Rightarrow c = 1 - d$

$a = 2 - d$

$b = 1 + (2-d) + 3(1-d) + d$

$= 1 + 2 - d - 3d + d$ $= 1 + 2 - d + 3 - 3d + d$

$= 3 - 3d$ $= 6 - 3d$

So,

$$R = KV^{2-d} D^{6-3d} \rho^{-d} \mu^d$$

$$= K\frac{V^2}{V^d} \cdot \frac{D^6}{D^{3d}} \frac{\mu^d}{\rho^d} = KV^2D^3\left(\frac{\mu^d}{V^d D^{3d} \rho^d}\right)$$

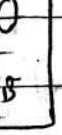
$$R = K \rho V^2 D^2 \phi\left(\frac{\mu}{\rho D V}\right)^d$$

for low velocity high viscous fluid,

taking $d = 1$:

$$R = K \mu V D.$$

$$\mu = \frac{R}{KVD}$$



$$R = (W - U)$$

$$= \frac{4}{3}\pi R^3 (7.8 - 0.91) \times 9810.$$

$$= \cancel{283124.1}\ 35390.5148$$

So,

$$\mu = \frac{\quad}{3\pi \times 20 \times}$$

$$= 187.75$$

* The thrust F of a propeller depends upon its diameter 'd'. Speed of advance v, revolution per sec 'N'. Fluid density ρ and viscosity μ. Find the expression of F in terms of other quantities.

* Variables controlling the motion of a floating vessel through the surrounding fluid are drag force 'F', vessel speed 'v', It's length 'l', density 'ρ' and viscosity 'μ' and acc. due to gravity 'g'. Derive an expression for f using Rayleigh's method.

Buckingham - π method:

It states that if there are n-dimensional variable involved in a phenomena which can be completely described by 'm' fundmental quantities or dimension such as MLT. and are related by a dimensionally homogenous eq^n then the relationship among the ^{n} quantities can always be expressed in terms (n-m) dimensionless and independent π terms.

If variable Q_1 depends upon ~~it~~ independent variable $\rightarrow Q_2, Q_3, Q_4 \cdots Q_n$.

$$Q_1 = f_1 (Q_2, Q_3, Q_4 \cdots Q_n).$$

$$f_1 [Q_1, Q_2, Q_3 \cdots Q_n] = C.$$

C is dimensionless constant

example:

$$x = f(y) = y^2 + C.$$

$$x - y^2 = f(y) = C$$

now.

$$f_2 (\pi_1, \pi_2, \pi_3 \cdots \pi_{n-m}) = C_1.$$

$$\pi_1 = Q_1^{a_1} Q_2^{b_1} Q_3^{c_1} \cdots\cdots Q_m^{m_1} \cdot Q_{m+1}$$

$$\pi_2 = Q_1^{a_2} Q_2^{b_2} Q_3^{c_2} \cdots\cdots Q_m^{m_2} \cdot Q_{m+2}.$$

$$\pi_{n-m} = Q_1^{a_{n-m}} Q_2^{b_{n-m}} \cdot Q_3^{c_{n-m}} \cdots Q_m^{m_{n-m}} \cdot Q_n$$

then,

$\pi_1 = f_1(\pi_2, \pi_3, \pi_4 \ldots \pi_{n-m})$

$\pi_2 = f_1(\pi_1, \pi_3, \pi_4 \ldots \pi_{n-m})$.

$Q = f(d, H, g, \mu, \rho)$

Step 1:

$f_1(\underbrace{Q, d, H, g, \mu, \rho}_{n=6}) = C$.

nos of π terms $= n-m = 6-3 = 3$.

so, $f_2(\pi_1, \pi_2, \pi_3) = C$.

Step 2:

choosing repeating variables:

we have to chose 3 repeating variables since $M=3$

It should collectively contain all 3 fundamental dimension

they are: ρ, d and g.

so,

$\pi_1 = \rho^{a_1} d^{b_1} g^{c_1} . Q$ ---- (i)

$\pi_2 = \rho^{a_2} d^{b_2} g^{c_2} . \mu$ ---- (ii)

$\pi_3 = \rho^{a_3} d^{b_3} g^{c_3} . H$ ---- (iii)

Step-4:

for π_1; $\Rightarrow M^0 L^0 T^0 = [ML^{-3}]^{a_1} [L]^{b_1} [LT^{-2}]^{c_1} . [L^3 T^{-1}]$

$\Rightarrow 0 = a_1$

$0 = -3a_1 + b_1 + c_1 + 3$.

$\Rightarrow 0 = b_1 + c_1 + 3$. $\Rightarrow b_1 = -5/2$.

$\Rightarrow 0 = -2c_1 - 1$.

$\Rightarrow c_1 = -1/2$

so, $\pi_1 = d^{-5/2} . g^{-1/2} . Q$.

Simarly for π_2.

$$[M^0L^0T^0] = [ML^{-3}]^{a_2}[L]^{b_2}[LT^{-2}]^{c_2}[ML^{-1}T^{-1}]$$

$\Rightarrow 0 = a_2 + 1$

$\Rightarrow a_2 = -1$

$\Rightarrow 0 = -3a_2 + b_2 + c_2 - 1 \Rightarrow b_2 = 3/2$

$\Rightarrow 0 = -2c_2 - 1$

$\Rightarrow c_2 = -1/2$

So,

$$\pi_2 = \rho^{-1} \cdot d^{3/2} \cdot g^{-1/2} \cdot \mu \quad \text{---(ii)}$$

Similarly for π_3.

$$[M^0L^0T^0] = [ML^{-3}]^{a_3}[L]^{b_2}[LT^{-2}]^{c_2} L.$$

$a_3 = 0$

$0 = -3a_3 + b_2 + c_2 + 1$

$\Rightarrow b_2 + c_2 + 1 = 0 \Rightarrow b_2 = -1$.

$c_2 = 0$.

So,

$$\pi_3 = d^{-1} \cdot H.$$

Procedure for Buckingham π-method

1) List all the n physical quantities or variables involved in the phenomena. so there will be $(n-m)\pi$ terms where m = no. of fundamental dimensions. Select m variables as a repeating variables.

2) Guidelines for repeating variables.
   - These variable should be such that non of them is dimensionless.
   - No two variables should have same dimension.
   - All the m-fundamental dimensions are included collectively in them
   - As far as possible, the dependant variable should not be taken as repeating variable.
   - In the fluid flow problem usually, characteristic linear dimension, characteristic velocity and characteristic fluid property are choosen and repeating variable (generally 3)

3) Write the general equation for different π-terms. These may be expressed as the product of repeating variable each raised to an arbitrary exponent and one of the remaining variables taken in turn with known power.

4) Write the dimensional equation of the π-terms from step 3.

5) Compute the value of unknown/arbitrary exponent by equating the exponent of the respective fundamental dimension and write the final general equation in terms of π-terms.

6) Guidelines for grouping of π-terms.

- Any π-terms may be replaced by any power of that term including negative as well as fractional power.
- Any π-term may be replaced by multiplying it by a numerical constant
- Any π-term may be replaced by multiplying or dividing it by another π-terms.

Matrix method

(Capillary rise)

	h	r	σ	γ
M	0	0	1	1
L	1	1	0	-2
T	0	0	-2	-2

$\rightarrow A_2 = -2 = 2$

$A_3 = 0$

So rank 2.

So no of π terms $= 4-2 = 2$

The resistance R to the motion of supersonic aircraft of length L, moving with velocity V in the air of density ρ depends on viscosity μ and bulk modulus of ~~air~~ elasticity K of air. Obtain the expression for R using Buckingham π method

$$R = f(L, V, \rho, \mu, K)$$

$$f(R, L, V, \rho, \mu, K) = \text{constant}$$

no. of π terms $= 6-3 = 3$

So,

$$\pi_1 = [L]^a [V]^b [\rho]^c [R]$$
$$\pi_2 = [L]^{a'} [V]^{b'} [\rho]^{c'} [\mu]$$
$$\pi_3 = [L]^{a''} [V]^{b''} [\rho]^{c''} [K]$$

now,

$\pi_1 \rightarrow$

$$[M^0 L^0 T^0] = [M^0 L^1 T^0]^a [M^0 L^1 T^{-1}]^b [M^1 L^{-3} T^0]^c [M^1 L^1 T^{-2}]$$

$\Rightarrow 0 = c+1$	$0 = a+b-3c+1$	$0 = -b-2$
$\Rightarrow \underline{c = -1}$	$0 = a-2+3+1$	$\Rightarrow \underline{b = -2}$
	$\Rightarrow \underline{a = -2}$	

π_2

$$[M^0 L^0 T^0] = [M^0 L^1 T^0]^{a'} [M^0 L^1 T^{-1}]^{b'} [M^1 L^{-3} T^0]^{c'} [M L^{-1} T^{-1}]$$

$\Rightarrow 0 = c'+1$	$0 = a'+b'-3c'-1$	$0 = -b'-1$
$\Rightarrow \underline{c' = -1}$	$\Rightarrow \underline{a' = -1}$	$\Rightarrow \underline{b' = -1}$

π_3

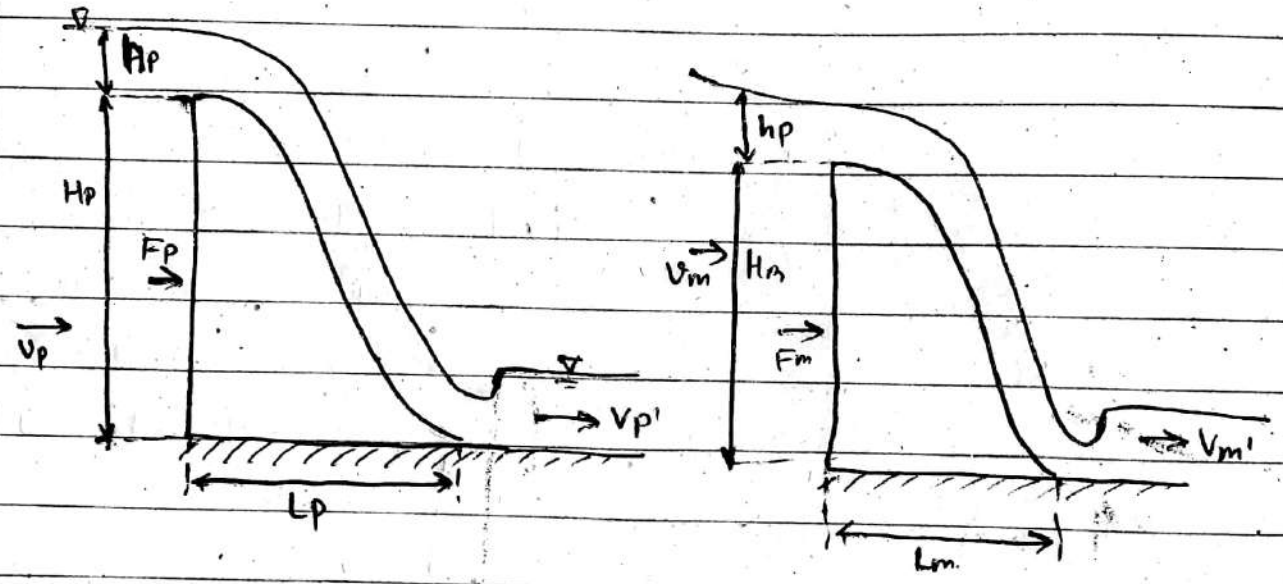
$$[M^0 L^0 T^0] = [M^0 L^1 T^0]^{a''} [M^0 L^1 T^{-1}]^{b''} [M^1 L^{-3} T^0]^{c''} [M L^{-3} T^{-1}]$$

$\underline{c'' = -1}$	$0 = a''+b''-3c''-3$	$\underline{b'' = -1}$
	$\Rightarrow \underline{a'' = 1}$	

$$\pi_1 = L^{-2} V^{-2} \rho^{-1} R$$
$$\pi_2 = L^{-1} V^{-1} \rho^{-1} \mu$$
$$\pi_3 = L^{1} V^{-1} \rho^{-1} K.$$

$$\cancel{f(\pi_1, \pi_2, \pi_3) = c}$$

$$\pi_1 = f(\pi_2, \pi_3)$$
$$\Rightarrow \frac{R}{L^2 V^2 \rho} = f\left(\frac{\mu}{LV\rho}, \frac{KL}{V\rho^2}\right)$$



prototype (Spillway) — Model

The small scale replica of the actual structure is known as its model. While the actual structure is called prototype.

Types of similarities:

i) Geometric similarities:

Geometrical similarities exist between the model and the prototype if the ratios of corresponding length dimension in the model and the prototype are equal. Such a ratio is called length ratio.

$$\text{Length scale ratio} = L_r = \frac{L_m}{L_p} = \frac{b_m}{b_p} = \frac{d_m}{d_p} \text{ etc.}$$

$$\text{Area scale ratio} = A_r = \frac{A_m}{A_p} = \left(\frac{L_m \times b_m}{L_p \times b_p}\right) = L_r^2$$

$$\text{Volume scale ratio} = V_r = \frac{V_m}{V_p} = \left(\frac{L_m \times b_m \times d_m}{L_p \times b_p \times d_p}\right) = L_r^3$$

ii) Kinematic similarities:

Kinematic similarity exist between the model and prototype if

i) The path of the moving particles are geometrically similar.

ii) Ratios of the velocities as well as acceleration of the particles are equal.

iii) Time scale ratio.

$$T_r = \frac{T_m}{T_p}$$

iv) Velocity scale ratio

$$V_r = \frac{V_m}{V_p} = \frac{L_m/T_m}{L_p/T_p} = \frac{L_r}{T_r}$$

v) Acceleration scale ratio

$$A_r = \frac{A_m}{A_p} = \frac{L_m/T_m^2}{L_p/T_p^2} = \frac{L_r}{T_r^2}$$

vi) Dischage scale ratio

$$Q_r = \frac{Q_m}{Q_p} = \frac{L_r^2 \times L_r/T_r}{} = \frac{L_r^3}{T_r}$$

vii) Dynamic similarities

Dynamic similarities exist between the model and prototype which are geometrically and kinematically similar if the ratio of all the forces acting at homologous point in the two system (i.e. model and prototype) are equal

a) Inertia force, F_i.
b) Viscous force, F_v.
c) Pressure force, F_p
d) Gravity - F_g
e) Elastic force, F_e.
f) Surface tension force, F_s.

$$\frac{(\Sigma F)_m}{(\Sigma F)_p} = \frac{(F_v + F_g + F_p + F_e + F_s)_m}{(F_v + F_g + F_p + F_e + F_s)_p} = \frac{(ma)_m}{(ma)_p}$$

also,

$$\frac{(Ma)_m}{(Ma)_p} = \frac{(F_v)_m}{(F_v)_p}$$

$$\frac{(Ma)_m}{(Ma)_p} = \frac{(F_g)_m}{(F_g)_p}$$

⋮

Force ratios (dimensionless numbers)

i) Inertia force ratio

$$IF = m \times a$$

$$= \rho \times \text{Volume} \times \frac{\text{Vel}}{\text{time}}$$

$$= \rho \frac{\text{vol}}{\text{time}} \times \text{vel}$$

$$= \rho A v \times v$$

$$= \rho L^2 v^2 \quad (A \rightarrow L^2)$$

now,

$$(F_i)_r = \frac{(F_i)_m}{(F_i)_p} = \frac{\rho_m L_m^2 V_m^2}{\rho_p L_p^2 V_p^2} = \rho_r L_r^2 v_r^2.$$

ii) Inertia viscous force ratio

$$F_v = \tau \cdot A$$

$$= \mu \cdot \frac{du}{dy} \cdot A = \mu \frac{v}{L} A = \mu \frac{v}{L} \cdot L^2 = \mu V L$$

$$\boxed{\frac{F_i}{F_v} = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho V L}{\mu}} \rightarrow \text{reynold's no.}$$

iii) Inertia gravity force ratio:

$$F_g = m \times g$$

$$= \rho \times \text{vol} \times g$$

$$= \rho L^3 \times g$$

$$\frac{F_i}{F_g} = \frac{\rho L^2 V^2}{\rho L^3 g} = \frac{V^3}{Lg}, \quad \frac{V}{\sqrt{Lg}} = \text{Froude no.}$$

4) Inertia and pressure force ratio

$F_p = P \times A$

$= P \times L^2$

$\therefore \frac{F_i}{F_p} = \frac{\rho L^2 V^2}{P L^2} = \left(\frac{\rho V^2}{P}\right) \rightarrow \left[\frac{V^2}{P/\rho}\right] = \text{euler no.}$

$\rightarrow \left[\frac{V}{\sqrt{P/\rho}}\right]$

$\frac{1}{\text{Euler no.}} = \text{Newton no.}$

5) Inertia elastic force ratio:

$F_e = K \cdot A$

$= K L^2$

$\frac{F_i}{F_e} = \frac{\rho L^2 V^2}{K L^2} = \left(\frac{\rho V^2}{K}\right) \rightarrow \left(\frac{V^2}{K/\rho} \Leftrightarrow \frac{V^2}{c^2}\right) \rightarrow$ cauchy no.

$c = \text{velocity} = \sqrt{K/\rho}$

also, $\frac{V}{\sqrt{K/\rho}} = \text{Mach no (Ma)}$

6) Inertia surface tension force

$F_s = \sigma \times L$

$\frac{F_i}{F_s} = \frac{\rho L^2 V^2}{\sigma L} = \frac{V^2}{(\sigma/\rho L)} \Leftrightarrow \left(\frac{V}{\sqrt{\sigma/\rho L}}\right) \rightarrow$ weber no.

Model laws:

Reignnold model law:

For flow where in addition to inertia force, viscous force is predominant

Reynold no. in model = Reynold no. in prototype.

$(Re)_{model} = (Re)_{prototype}$

$\Rightarrow \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$

$\Rightarrow \frac{\rho_r . V_r . L_r}{\mu_r} = 1 \Leftrightarrow \frac{L_r L_r}{\nu_r} = 1$

Length $\Rightarrow L_r$

Velocity $\Rightarrow V_r = \frac{\mu_r}{L_r . \rho_r}$

time $\Rightarrow T_r = \frac{L_r}{V_r} = \frac{L_r^2 . \rho_r}{\mu_r}$

acceleration $\Rightarrow a_r = \frac{\mu_r^2}{L_r^3 \rho_r^2}$

discharge $\Rightarrow Q_r = \frac{L_r \mu_r}{\rho_r}$

~~Force~~ Power $\Rightarrow P_r = \frac{\mu_r^3}{\rho_r^2 L_r}$

work energy $\Rightarrow Q_r = \frac{\mu_r^2 L_r}{\rho_r}$

Reynold's model law is applicable to:

i) Fluid in closed pipe.
ii) Motion of submarines underwater
iii) Motion of airplanes
iv) Flow around the structure.

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Froude model law:

$$(F_r)_m = (F_r)_p$$

$$\Rightarrow \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

$$\Rightarrow \frac{V_r}{\sqrt{g_r L_r}} = 1$$

velocity $\therefore V_r = \sqrt{g_r \cdot L_r}$

$V_r = \sqrt{L_r}$ (since, g_r is generally 1).

time: $T_r = \frac{L_r}{V_r} = \sqrt{L_r}$

acc: $a_r = \frac{V_r}{T_r} = 1$.

discharge: $Q_r = A_r \cdot V_r = L_r^2 \cdot \sqrt{L_r}$

$= L_r^{5/2}$.

~~Power~~: Power: $P_r = L_r^{7/2} \rho_r$.

Work energy: $W_r = L_r^4 \rho_r$.

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* Water at 15°c flows at 4m/s in 150mm dia. pipe. At what velocity must oil at 30°c in 75mm pipe flow for the two flow to be dynamically similar. Take ν of water $= 1.145 \times 10^{-6}$ m²/s and $\nu_{oil} = 3 \times 10^{-6}$ m²/s

→ Solution

this is a pipe flow and viscosity is dominant here.

So, we apply reynold law.

$$(Re)_m = (Re)_p$$

$$\Rightarrow \frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\Rightarrow \frac{V_r L_r}{\nu_r} = 1$$

given,

$$L_r = \frac{75}{150} = \frac{1}{2}$$

$$\nu_r = \frac{1.145 \times 10^{-6}}{3 \times 10^{-6}} = \cancel{0.38167}, \quad 2.6200$$

So,

$$V_r \text{ must be } \frac{1 \times \cancel{0.3816}\ 2.62}{0.5} = \cancel{0.763}$$

So $\frac{V_p}{V_m} \quad \frac{V_m}{V_p} = \cancel{0.763} \quad \frac{2.62}{0.5}$

$\therefore V_m = 20.96$ m/s.

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A solid sphere of dia. 100 mm moves in a water at 5 m/s, it experience a drag of magnitude 19.62 N, ~~which~~ what would be the velocity of 5m dia sphere moving in air in order to ensure similarities what will be the drag experienced by it? Take: ρ of water = 1000 kg/m³

ρ of air = 1.2 kg/m³

$\nu_{air} = 13\,\nu_{water}$

→ Solution.

$F = \rho a^2 V^2$.

Model	Prototype
$d_m = 100$ mm	$d_p = 5$m
$V_m = 5$ m/s	$V_p = ?$
$F_{dm} = 19.62$	$F_{dp} = ?$
$\rho_m = 1000$	$\rho_p = 1.2$
$\nu_m = \nu_{water}$	$\nu_p = 13\,\nu_{water}$

this is the case of submerged flow so Reynold's no. is used

So,

$$\frac{V_r L_r}{\nu_r} = 1$$

$$\Rightarrow \frac{5/x \times 1/50}{1/13} = 1$$

$$v = 1.3 \text{ m/s}$$

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* A 1:10 scale of submarine moving far below the surface of water is tested in a water tunnel. If the speed of the prototype is 8m/s. Determine corresponding velocity ~~of~~ in water tunnel. Also determine the ratio of the drag for the model & the prototype. $\nu_{sea\ water} = 1.121 \times 10^{-6}\ m^2/s$

$\nu_{water} = 1 \times 10^{-6}\ m^2/s$.

$\rho_{sea\ water} = 1027\ kg/m^3$.

$\rho_{water} = 1000\ kg/m^3$.

Would this result be true if the prototype were to move close to the surface. Explain.

→ $\nu_r = 1/1.121$

$\rho_r = \frac{1000}{1027}$

$l_r = 1/10$

$V_r = \frac{x}{8}$

$\frac{V_r\, l_r}{\nu_r} = 1$.

$\Rightarrow \frac{x/8 \times 1/10}{1/1.121} = 1$

$x = 71.364\ m/s$

$F_r = \frac{F_m}{F_p} = \rho_r V_r^2 L_r^2$.

$= \frac{1000}{1027} \times \left(\frac{71.36}{8}\right)^2 \times \frac{1}{100}$

$= 0.77484$

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The performance of a spillway of a power project is to be studied by means of model constructed to a scale of 1:9. Neglecting the viscous & surface tension effect. Determine rate of flow in the model for a prototype discharge of $1000 m^3/s$. Dissipation of energy in prototype hydraulic jump if the model studies dissipate 294 watt.

→ Solution;

$L_r = 1/9$

$Q_r = \frac{Q_m}{Q_P} = \frac{Q_m}{1000}$

$P_r = \frac{P_m}{P_P} = \frac{294}{P_P}$

⇒ from Froude model,

$\frac{V_r}{\sqrt{g_m l_r}} = 1$

$\Rightarrow Q_r = A_r \cdot V_r$

$= L_r^2 \cdot \sqrt{g_m \cdot L_r}$

$= L_r^{5/2} g_r^{1/2}$

So, generally $g_r = 1$.

So,

$\frac{Q_m}{Q_P} = \frac{Q_m}{1000} = L_r^{5/2}$

$Q_m = 4.115 \, m^3/s$

$P_r = \rho_r L_r^2 V_r^2 \times V_r$

$= \rho_r L_r^2 V_r^3$

$= \rho_r L_r^2 \sqrt{g_m^3 \cdot L_r^3}$

$= \rho_r \cdot L_r^{7/2} = 4.5724 \times 10^{-4}$

$\Rightarrow 294/P_P$

$\Rightarrow P_P = 642978$ Watt