

MAT165 ASSIGNMENT 1

MANJIL SAIKIA

Last Date of Submission. 26 October, 2025 **before** the lecture starts.

Instructions.

- You can discuss the problems with any of your class-mates. In fact, I encourage you to talk to your friends and come up with the solutions together.
- Avoid using AI or web search to arrive at the solutions. This way would be easy, but you will learn very little.
- Write down the solutions in A4 sized sheets of paper (either blank or dotted), and staple them before submission. Use either black or blue ink for writing the solutions.
- It is NOT important to write a perfect solution for this assignment; I am more interested in seeing how you approach a particular problem. You can submit any number of false starts or wrong solutions you may have arrived at. I am happy to give you feedback on those as well.

Questions.

- (1) Give an example (not from the notes) of how abstraction can simplify a real-world problem.
- (2) In Chapter 2 of the notes, the author shows that simple statements can be rewritten symbolically to make reasoning clearer. Take this sentence: “If a number is divisible by 4, then it is even.” Express this statement using letters and logical symbols.
- (3) How would you prove the identity $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$? Try to give a logical proof using geometrical ideas.
- (4) Prove that the difference between consecutive perfect squares always forms an odd number.
- (5) Compute the sum of the first few odd numbers:

$$1, \quad 1 + 3, \quad 1 + 3 + 5, \quad 1 + 3 + 5 + 7.$$

- (a) Observe the results and make a **conjecture** about the pattern you see.
 - (b) **Prove** your conjecture using mathematical reasoning.¹
- (6) You notice that:

$$1 \times 1 = 1, \quad 11 \times 11 = 121, \quad 111 \times 111 = 12321.$$

- (a) Predict the result of 1111×1111 .
 - (b) Describe the **structural pattern** you observe in these results.
- (7) Translate the following English statement into symbolic logic and simplify:

Date: 15 October 2025.

¹For those who know *mathematical induction*, try not to use it to prove your conjecture.

“If it is raining and cold, then I will stay home.”

Let R represent “it is raining,” C represent “it is cold,” and H represent “I stay home.”

- (a) Express the statement symbolically.
 - (b) Write its contrapositive.
- (8) Prove that for any integer n , the number $n^3 - n$ is divisible by 6.
- (9) Determine whether $n^2 - 1$ is always divisible by 8 when n is odd. To do this, you can follow the following steps.
- (a) Write $n = 2k + 1$ for some integer k .
 - (b) Simplify $n^2 - 1$ and decide if it is a multiple of 8.
 - (c) Provide a short proof or counterexample.
- (10) We wish to compute the sum

$$1^2 + 2^2 + 3^2 + \cdots + n^2.$$

- (a) Calculate this sum for $n = 1, 2, 3, 4, 5$.
- (b) Make a conjecture for a general formula in terms of n .
- (c) Verify your conjecture for at least one larger value of n .
- (d) Can you prove this conjecture using the ideas we have discussed so far in the lectures?