COMP1378 Assignment_2

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Investigating the below statement

Welsh t-test's Wikipedia page states that, "For unequal variances, Student's t-test gave a low p-value when the smaller sample had a larger variance and a high p-value when the larger sample had a larger variance. For unequal variances, Welch's t-test gave p-values close to simulated p-values." by checking the rejection proportion against pre-specified significance level.

Simulation study

We have generated 10,000 samples in 3 different settings. For each setting, we have simulated samples from the appropriate normal distributions and ran both the two-sample t-test and the Welsh's t-test and collected their p-values.

– Setting 1 (equal sample sizes, unequal but near variances):

$$\mu_1 = 20, \mu_2 = 22, \sigma_1^2 = 7.9, \sigma_2^2 = 3.8, N_1 = N_2 = 15$$

– Setting 2 (unequal sample sizes, unequal variances, smaller sample has the larger variance):

$$\mu_1 = 20, \mu_2 = 22, \sigma_1^2 = 9.0, \sigma_2^2 = 0.9, N_1 = 10, N_2 = 20$$

– Setting 3 (unequal sample sizes, unequal variances, larger sample has the larger variance):

$$\mu_1 = 20, \mu_2 = 22, \sigma_1^2 = 1.4, \sigma_2^2 = 17.1, N_1 = 10, N_2 = 20$$

We have compared the rejection proportion based on three significance levels: 10%, 5% and 1%.

At 10% significance level

```
##     Setting  Student_p Student_p_sim    Welch_p Welch_p_sim
## 1 Setting 1 0.37985485    0.11045660 0.01516998   0.1115807
## 2 Setting 2 0.05890985    0.08811487 0.07601055   0.1523441
## 3 Setting 3 0.71567020    0.22669251 0.02848265   0.1508272
```

From the above output, we can see that the result at 10% significance level agrees with the statement, "For unequal variances, Student's t-test gave a low p-value when the smaller sample had a larger variance and a high p-value when the larger sample had a larger variance." However, for unequal variances, Welch's t-test gave p-values different to simulated p-values.

At 5% significance level

```
##     Setting    Student_p Student_p_sim      Welch_p Welch_p_sim
## 1 Setting 1 7.609194e-03    0.11121162 0.0004853037   0.1104528
## 2 Setting 2 3.876405e-06    0.08949945 0.1239852199   0.1521240
## 3 Setting 3 1.503099e-01    0.22590273 0.0050940577   0.1533088
```

From the above output, we can see that the result at 5% significance level agrees with the statement, "For unequal variances, Student's t-test gave a low p-value when the smaller sample had a larger variance and a high p-value when the larger sample had a larger variance." However, for unequal variances, Welch's t-test gave p-values different to simulated p-values.

At 1% significance level

```
##     Setting   Student_p Student_p_sim      Welch_p Welch_p_sim
## 1 Setting 1 0.006098105    0.10818602 0.0517236615   0.1099200
## 2 Setting 2 0.001793543    0.08706747 0.0042526337   0.1503388
## 3 Setting 3 0.212399368    0.22850199 0.0006849773   0.1486977
```

From the above output, we can see that the result at 1% significance level does not agree with the statement from the Welsh t-test's Wikipedia page. As for unequal variances, Student's t-test gave a high p-value when the smaller sample had a larger variance and a low p-value when the larger sample had a larger variance.For unequal variances, Welch's t-test gave p-values different to simulated p-values.

Comparing the rejection proportion based on three significance levels:10%, 5% and 1%

At 10% significance level,

- Student's t-test: For Setting 2, we reject the null hypothesis but not for Setting 1 and 3
- Welch's test: For Setting 1 and 2, we reject the null hypothesis but not for Setting 3

At 5% significance level,

- Student's t-test: For Setting 2, we reject the null hypothesis but not for Setting 1 and 3
- Welch's test: For Setting 1 and 2, we reject the null hypothesis but not for Setting 3

At 1% significance level,

- Student's t-test: For Setting 1 and 2, we reject the null hypothesis but not for Setting 3
- Welch's test: For Setting 1, we reject the null hypothesis but not for Setting 2 and 3

The dplyr package is used in this section[1].

References

[1] H. Wickham, R. Francois, L. Henry, and K. Muller. *dplyr: A Grammar of Data Manipulation*, 2022. R package version 1.0.9.