COMP1378 Assignment_2

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In this exercise, we are going to compare two methods used in comparing the mean of two independent populations - two-sample t-test and Welch's t-test.

Two-sample t-test

The two-sample t-test is used to determine if the means of two sets of data are significantly different from each other or not [1].

For two-sample t-test [2], the following assumptions should be met:

- The sample means being compared for two populations follow normal distribution.
- The two samples are from populations with equal/same variances.

We will use the below example for illustration purposes.

Let A1 denote a set obtained by drawing a random sample of six measurements:

A1 = {30.02, 29.99, 30.11, 29.97, 30.01, 29.99}

and let A2 denote a second set obtained similarly:

A2 = {29.89, 29.93, 29.97, 29.98, 30.02, 29.98}

A1 and A2 are the weights of screws that were produced by two different machines.

Descriptive statistics of A1 and A2

Sample	Sample size	Mean	Std
A1	$n_{A1} = 6$	$\bar{x}_{A1} = 30.015$	$s_{A1} = 0.049699$
A2	$n_{A2} = 6$	$\bar{x}_{A1} = 29.96167$	$s_{A2} = 0.04535$

Hypothesis

- Null hypothesis: There is no difference between the average weights of screws produced by two different machines A1 and A2.

  $H_0 : \mu_{A1} = \mu_{A2}$

- Alternate hypothesis, Ha: There is a difference between the average weights of screws produced by two different machines A1 and A2

  $H_a : \mu_{A1} \neq \mu_{A2}$

Pooled Variance

In two sample t-test, we use pooled variance because we assume that the two samples are from populations with equal variances [2].

$$s_p = \sqrt{\frac{(n_{A1}-1)s_{A1}^2 + (n_{A2}-1)s_{A2}^2}{n_{A1}+n_{A2}-2}}.$$

$$s_p = \sqrt{\frac{(6-1)0.049699^2 + (6-1)0.04535^2}{6+6-2}} = 0.0475741$$

Test statistics and degrees of freedom

$df = n_{A2} + n_{A2} - 2 = 6 + 6 - 2 = 10$

$$t_{obs} = \frac{\bar{x}_{A1} - \bar{x}_{A2}}{s_p\sqrt{1/n_{A1} + 1/n_{A2}}} \sim t_{10}.$$

$$t_{obs} = \frac{30.015 - 29.96167}{0.0475741\sqrt{1/6 + 1/6}} = 1.941608$$

Critical value

The critical value can be manually extracted using the statistical table where df=10 at 5% significance level or qt() function from r package can be used.

$$t_{crit} = 2.228$$

P value

P value can be calculated using pt() function from r by specifying t statistics and df.

$$p - value = 0.08085294$$

Confidence intervals

At 5% significance level

$$CI = \bar{x}_{A1} - \bar{x}_{A2} \pm tcrit * s_p\sqrt{1/n_{A1} + 1/n_{A2}}$$

$$CI = 30.015 - 29.96167 \pm 2.228 * 0.0475741\sqrt{1/6 + 1/6} = (-0.007866298, 0.1145263)$$

Conducting two sample t-test in R

We can conduct two sample t-test in R using t.test() function. We can pass the A1 and A2 sets and specify var.equal=TRUE as we assume equal variances between our two populations.

The below output shows the result of two sample t-test in R.

```
##
##  Two Sample t-test
##
## data:  A1 and A2
## t = 1.9417, df = 10, p-value = 0.08085
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.007867295  0.114533962
## sample estimates:
## mean of x mean of y
##  30.01500  29.96167
```

Comparing manually vs r calculated two sample t-test results

We can see from above sections, that the manually vs r calculated two sample t-test results are similar, where

$$t_{obs} = 1.9417, df = 10, p - value = 0.08085, CI = (-0.007867, 0.1145)$$

Decision of two sample t-test

At significance level of 0.05, we fail to reject the null hypothesis because p-value is = 0.08085, which is not less than 0.05.

Conclusion of two sample t-test

There is no statistically significant evidence to suggest that average weights of screws produced by machine A1 differs from screws produced by machine A2.

Welsh t-test

The Welsh t-test is used to determine if the means of two sets of data are significantly different from each other or not [4]. However, Welch's t-test is designed for unequal population variances, i.e. used when the assumption that the two populations have equal variances seems unreasonable [4].

When the two samples do not assume same variance like in two-sample t-test, we use Welsh t-test[2].

For Welsh t-test, the following assumption should be met [2]:

- The sample means being compared for two populations follow normal distribution.

We will use the same above example for illustration purposes where A1 and A2 are the weights of screws that were produced by two different machines.

A1 = {30.02, 29.99, 30.11, 29.97, 30.01, 29.99} A2 = {29.89, 29.93, 29.97, 29.98, 30.02, 29.98}

Hypothesis

- Null hypothesis: There is no difference between the average weights of screws produced by two different machines A1 and A2.

  $H_0 : \mu_{A1} = \mu_{A2}$

- Alternate hypothesis, Ha: There is a difference between the average weights of screws produced by two different machines A1 and A2

  $H_a : \mu_{A1} \neq \mu_{A2}$

Test statistics and degrees of freedom

$$t = \frac{\bar{x}_{A1} - \bar{x}_{A2}}{\sqrt{s_{A1}^2/n_{A1} + s_{A2}^2/n_{A2}}}$$

$$t = \frac{30.015 - 29.96167}{\sqrt{0.049699^2/6 + 0.04535^2/6}} = 1.941603$$

$$df = \frac{\left(\dfrac{s_{A1}^2}{n_{A1}} + \dfrac{s_{A2}^2}{n_{A2}}\right)^2}{\dfrac{(s_{A1}^2/n_{A1})^2}{n_{A1} - 1} + \dfrac{(s_{A2}^2/n_{A2})^2}{n_{A2} - 1}}$$

$$df = \frac{\left(\frac{0.049699^2}{6} + \frac{0.04535^2}{6}\right)^2}{\frac{(0.049699^2/6)^2}{6-1} + \frac{(0.04535^2/6)^2}{6-1}} = 9.917297$$

Critical value

The critical value can be manually extracted using the statistical table where df=10 at 5% significance level or qt() function from r package can be used.

$$t_{crit} = 2.262$$

P value

P value can be calculated using pt() function from r by specifying t statistics and df.

$$p - value = 0.08110804$$

Confidence intervals

At 5% significance level

$$CI = \bar{x}_{A1} - \bar{x}_{A2} \pm tcrit * \sqrt{s_{A1}/n_{A1} + s_{A2}/n_{A2}}$$

$$CI = 30.015 - 29.96167 \pm 2.262 * \sqrt{0.049699^2/6 + 0.04535^2/6} = -0.008800331, 0.1154603$$

Conducting Welsh's t-test in R

We can conduct Welch's t-test in R using t.test() function. We can pass the A1 and A2 sets and specify var.equal=FALSE as we do not assume equal variances between our two populations.

The below output shows the result of Welsh's t-test in R.

```
##
##  Welch Two Sample t-test
##
## data:  A1 and A2
## t = 1.9417, df = 9.9173, p-value = 0.08109
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.007936532  0.114603199
## sample estimates:
## mean of x mean of y
##  30.01500  29.96167
```

Comparing manually vs r calculated Welsh's t-test results

We can see from above sections, that the manually vs r calculated Welsh'st-test results are similar, where

$$t_{obs} = 1.9417, df = 9.9173, p - value = 0.0811, CI = (-0.008, 0.115)$$

Decision of Welsh's t test

At significance level of 0.05, we fail to reject the null hypothesis because p-value is = 0.0811, which is not less than 0.05.

Conclusion of Welsh's t test

There is no statistically significant evidence to suggest that average weights of screws produced by machine A1 differs from screws produced by machine A2.

The dplyr package is used in this section[3].

References

[1] S. Boslaugh. *Statistics in a Nutshell.* O'Reilly Media, 2nd edition, 2012.

[2] A. Ross and V. Willson. *Basic and Advanced Statistical Tests.* Sense Publishers, 1st edition, 2017.

[3] H. Wickham, R. Francois, L. Henry, and K. Muller. *dplyr: A Grammar of Data Manipulation*, 2022. R package version 1.0.9.

[4] Z. Xuemao and M. Zoe. Using r as a simulation tool in teaching introductory statistics. *International electronic journal of Mathematics Education*, 14(3):59–61, 2019.