EECS 844 – Fall 2017

Exam 1 Cover page*

Each student is expected to complete the exam individually using only course notes, the book, and technical literature, and without aid from other outside sources.

Aside from the most general conversation of the exam material, I assert that I have neither provided help nor accepted help from another student in completing this exam. As such, the work herein is mine and mine alone.

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Signature Date

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Name (printed) Student ID #

*** Attach as cover page to completed exam.**

EECS 844 Exam 1 (Due September 29)

Data sets can be found at http://www.ittc.ku.edu/~sdblunt/844/EECS844_Exam1

Provide: a) Complete and concise answers to all questions
b) Matlab code with solutions as appropriate
c) All solution material (including discussion, figures, and code) for each problem together (*i.e.* don't put all the plots or code at the end)
d) Email final Matlab code to me in a zip file (all together in 1 email)

** All data time sequences are column vectors with **increasing** time index as one traverses down the vector. (you will need to properly orient the data into "snapshots")

1. For the cost functions below, determine the derivative with respect to $\mathbf{w}^*$. You can assume that $\mathbf{R}$ is PDH as needed. (*Note: the chain rule still holds*)

a) $J(\mathbf{w}) = \tan\left(\mathbf{w}^H \mathbf{R} \mathbf{w}\right)$

b) $J(\mathbf{w}) = \|\mathbf{w}\|_1$

c) $J(\mathbf{w}) = \left|\mathbf{b}^H \mathbf{w}\right|^2$

d) $J(\mathbf{w}) = \dfrac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\left|\mathbf{w}^H \mathbf{a}\right|^2}$

2. For the length K time-series data in P2.mat, use Appendix A to form the $M \times N$ delay-shifted snapshot matrix $\mathbf{X}$ for M = 20 (length of each snapshot and also the number of rows in $\mathbf{X}$) and $N = K - M + 1$. Starting with the 1st column in $\mathbf{X}$, estimate the correlation matrix using different numbers of snapshots as

$$\mathbf{R}_n = \left(\frac{1}{n}\right) \sum_{\ell=1}^{n} \mathbf{x}(M+\ell-1)\, \mathbf{x}^H(M+\ell-1) \quad \text{for} \quad n = 1, 2, \cdots, N\,.$$

For each of these correlation matrices, determine the eigenvalues having the maximum and minimum absolute values. As a function of n, plot (in dB) the maximum eigenvalues. Likewise plot (in dB) the minimum eigenvalues as a function of n. Note that eigenvalues are in units of power in this case.

What can we infer regarding the condition number and invertibility of the correlation matrix as a function of the number of snapshots used to estimate it?

3. Repeat problem 2 using "diagonally loading" of each correlation matrix as

$$\mathbf{R}_n = \left[\left(\frac{1}{n}\right) \sum_{\ell=1}^{n} \mathbf{x}(M+\ell-1)\, \mathbf{x}^H(M+\ell-1) \right] + \sigma^2 \mathbf{I} \quad \text{for} \quad n = 1,\, 2, \cdots, N$$

where $\mathbf{I}$ is an $M \times M$ identity matrix and setting $\sigma^2 = 1$ (in one case) and $\sigma^2 = 10$ (in another case), and $\sigma^2 = 100$ (in the last case). Like problem 2, plot the maximum and minimum eigenvalue magnitudes (in dB) for these three cases along with the previous unloaded case. What do you observe in comparison to the Problem 2 results? What are the implications to the condition number and matrix invertibility as a function of the number of snapshots when diagonal loading is used? What is the impact of different values of the loading factor σ^2?

4. Again using the data from Problem 2, for $n = N$ (all the snapshots) plot the complete set of eigenvalues (there are $M = 20$) for each of the four different estimates of the correlation matrix (i.e. unloaded from Prob. 2 and for each of the three different loading factors from Prob. 3). What do you observe occurring as the loading factor is increased?

5. The dataset P5.mat contains $N = 200$ time samples collected from an $M = 15$ element antenna array (this is spatial data). This data is already collected into snapshot form, with each snapshot (of length M) corresponding to the samples obtained from the antenna elements at a single time instant. Using the complete set of snapshots, estimate the spatial correlation matrix $\mathbf{R}$ and plot its eigenvalues (in dB). What do you observe about the eigenvalue structure?

6. The dataset P6.mat contains two length $M = 14$ antenna array filters (i.e. beamformers) for a uniform linear array whose elements are separated by a half-wavelength. The filter 'w_non_adap' is a non-adaptive filter while the filter 'w_adap' is an adaptive filter. Using Appendices B and C, plot the beampatterns of these two filters in terms of electrical angle and spatial angle (plot in dB). Discuss what you observe.

7. For each of the cost functions in Problem 1, apply the linear constraint $\mathbf{w}^H \mathbf{s} = 1$ and solve for the complex Lagrange multiplier (you do not need to solve for the filter). Assume all matrices are PDH. (*In each case, pre-multiply by* $\mathbf{s}^H$ *when solving*)

Appendix A: Generating a matrix of time-series "snapshot" vectors

Given a vector of time samples defined as $\mathbf{x} = [x(1)\; x(2)\; x(3)\; \ldots\; x(K)]^T$, construct the matrix as

$$\mathbf{X} = \left[\mathbf{x}(M)\; \mathbf{x}(M+1)\; \cdots\; \mathbf{x}(K-1)\; \mathbf{x}(K)\right]$$
$$= \begin{bmatrix} x(M) & x(M+1) & \cdots & x(K-1) & x(K) \\ x(M-1) & x(M) & \cdots & x(K-2) & x(K-1) \\ \vdots & \vdots & & \vdots & \vdots \\ x(2) & x(3) & \cdots & x(K-M+1) & x(K-M+2) \\ x(1) & x(2) & \cdots & x(K-M) & x(K-M+1) \end{bmatrix}.$$

Appendix B: Generating a steering vector matrix for a uniform linear array (ULA)

Generate a matrix $\mathbf{S}_\theta$ where each column $\mathbf{s}(\theta) = [1\;\; e^{-j\theta}\;\; e^{-j2\theta}\;\; \cdots\;\; e^{-j(M-1)\theta}]^T$ is a spatial steering vector in terms of the electrical angle $-\pi \leq \theta \leq \pi$ where the number of samples in angle is much greater than M (*e.g.* $10M$). Use the matlab command 'linspace' to get equally-spaced sample values over $-\pi \leq \theta \leq \pi$. This matrix may alternatively be parameterized in terms of spatial angle by first defining the equal-spaced sampling over $-\pi/2 \leq \phi \leq \pi/2$ and then converting via $\theta = \pi \sin(\phi)$ for half-wavelength spacing.

Appendix C: Computing the beampattern (or frequency response) of the filter $\mathbf{w}$

Using the steering vector matrix from Appendix B, the resulting vector ($\mathbf{S}_\theta^H\, \mathbf{w}$) is the beampattern (or frequency response) for a fixed filter $\mathbf{w}$ (we generally plot the magnitude of this response in dB). Note that this result is over-sampled (relative to the DFT) to provide better visibility. The steering vector matrix $\mathbf{S}_\theta$ is also used for other signal processing algorithms such as MVDR, MUSIC, or model-based approaches.