EECS 844 – Fall 2017

Exam 3 Cover page*

Each student is expected to complete the exam individually using only course notes, the book, and technical literature, and without aid from other outside sources.

Aside from the most general conversation of the exam material, I assert that I have neither provided help nor accepted help from another student in completing this exam. As such, the work herein is mine and mine alone.

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Signature	Date
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Name (printed)	Student ID #

*** Attach as cover page to completed exam.**

EECS 844 Exam 3 (Due November 20)

Data sets can be found at http://www.ittc.ku.edu/~sdblunt/844/EECS844_Exam3

Provide: Complete and concise answers to all questions
Matlab code with solutions as appropriate
All solution material (including discussion and figures) for a given problem together (*i.e.* don't put all the plots or code at the end)
Email final Matlab code to me in a zip file (all together in 1 email)

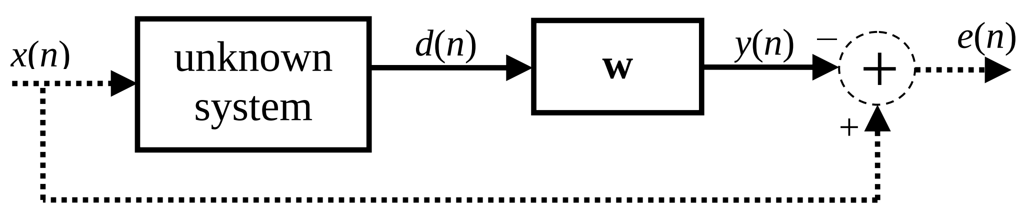
** All data time sequences are column vectors with **increasing** time index as one traverses down the vector. (you will need to properly orient the data into "snapshots")
** Provide figures (magnitude and phase) of all solutions.

1. In P1.mat the output of a hypothesized unknown linear system is the observed signal $d(n)$, to which we shall apply a whitening filter via linear prediction (the solid lines in the figure). In contrast, the dashed lines represent the inverse filtering formulation that we could perform if the hypothesized "driving signal" $x(n)$ were actually known (in practice it is not available).

a) Determine $\mathbf{w}$ as the linear prediction error (LPE) filter using only $d(n)$ when the linear prediction component is $M = 19$ (so the LPE filter is length 20).

b) Alternatively, determine $\mathbf{w}$ as the inverse system identification Wiener filter using both $d(n)$ and $x(n)$ for $M = 20$.

c) For each filter, plot the time-domain magnitude/phase and frequency response (use 'freqz' with 'whole'). Comment on how they are related/different.



2. Repeat Problem 1 using the data set P2.mat. How do the linear prediction and Wiener filter solutions compare for this data set? Explain what you observe and demonstrate (using signal processing of the data) why your explanation is correct.

3. We wish to derive and implement the steepest-descent algorithm for the cost function $J\left(\mathbf{w}(n)\right) = \mathbf{w}^H(n)\,\mathbf{R}\,\mathbf{w}(n) + \left|\mathbf{w}^H(n)\,\mathbf{s}_1(\theta) - 1\right|^2 + \left|\mathbf{w}^H(n)\,\mathbf{s}_2(\theta)\right|^2$. *{Note: this is* *not* *a LCMV or GSC formulation as these are not actually constraints}*

a) Analytically derive the optimum closed-form solution $\mathbf{w}_o$.

b) Using $\mathbf{R}$ from P3.mat, with $\mathbf{s}_1(\theta)$ and $\mathbf{s}_2(\theta)$ length N = 10 steering vectors pointed at electrical angles −20° and +20°, respectively, calculate $\mathbf{w}_o$ and plot the beampattern in terms of electrical angle.

c) For 3000 iterations and $\mathbf{w} = \mathbf{0}_{N\times 1}$ as initialization, now implement the steepest-descent algorithm. For five decimal precision, what is the largest constant step-size μ (denoted as $\mu_{\max}$) that still provides convergence. *{Hint: you can use trial and error, or determine the bound like in class}*

d) Using $\mu_{\max}$, plot the convergence of steepest descent in terms of the *squared deviation*, which is defined as $\|\mathbf{w}_o - \mathbf{w}(n)\|^2$ (plot versus iteration index *n*).

e) In general, now show analytically that the optimum adaptive step-size is $\mu_o(n) = \dfrac{\mathbf{g}^H(n)\,\mathbf{g}(n)}{\mathbf{g}^H(n)\,(\mathbf{R} + \mathbf{s}_1(\theta)\mathbf{s}_1^H(\theta) + \mathbf{s}_2(\theta)\mathbf{s}_2^H(\theta))\,\mathbf{g}(n)}$ by minimizing the modified cost function $J\big(\mathbf{w}(n) - 0.5\mu\,\mathbf{g}(n)\big)$ with respect to μ. You can assume $\mathbf{R} = \mathbf{R}^H$.

f) Implement the steepest-descent algorithm using $\mu_o(n)$ and plot the squared deviation for 3000 iterations using $\mathbf{w} = \mathbf{0}_{N\times 1}$ as initialization.

g) How many iterations are required for the squared deviation to surpass –30 dB when using $\mu_{\max}$ and $\mu_o(n)$, respectively?

4. Data set P4.mat contains a known signal $\mathbf{x}$ of length M = 100 that we wish to deconvolve from other unknown signals.

a) Generate the *normalized matched filter* as $\mathbf{h}_{\text{NMF}} = \mathbf{x}^{B*} / (\mathbf{x}^H \mathbf{x})$ and plot the convolution of this filter with the signal $\mathbf{x}$. Note that a filter output is an amplitude when plotting in dB.

b) Implement the Least-Squares *mismatched filter* $\mathbf{h}_{\text{MMF}}$ with a length of 2*M* and place the '1' in the elementary vector in element *m* = 1.5*M*. Once determined, normalize the MMF as

$$\mathbf{h}_{\text{NMMF}} = \frac{\mathbf{h}_{\text{MMF}}}{(\mathbf{h}_{\text{MMF}}^H \mathbf{h}_{\text{MMF}})^{1/2} (\mathbf{x}^H \mathbf{x})^{1/2}}$$

to allow for determination of the loss in SNR. Plot (in dB) the convolution of the normalized MMF with the signal $\mathbf{x}$. Comment on what you observe. *{Hint: the 'toeplitz' command is useful for constructing the matrix* $\mathbf{A}$ *comprised of delay shifted versions of* $\mathbf{x}$*}*

c) Repeat part *b)* except modify the matrix $\mathbf{A}$ by replacing the all values in the $(m-2)$, $(m-1)$, $(m+1)$, and $(m+2)$ rows with zeros. Comment on what you observe.

d) Repeat parts *b*) and *c*) except incorporate a diagonal load term that is 20 dB less than largest eigenvalue of the matrix $\mathbf{A}^H\mathbf{A}$.

e) For each of the four normalized mismatched filters obtained in the above steps, compute

$$\text{mismatch loss} = -20\log_{10}\left(\frac{\max|\mathbf{h}_{\text{NMMF}} * \mathbf{x}|}{\max|\mathbf{h}_{\text{NMF}} * \mathbf{x}|}\right)\text{dB},$$

for * representing convolution. Comment on what you observe.

5. Data set P4.mat also contains the received signal $y(n)$ that is the result of convolving the known signal $\mathbf{x}$ from Prob. 4 with some unknown system. Apply each of the 5 filters (MF and 4 MMF) from Prob. 4 to perform deconvolution to estimate the unknown system. Plot the results (in dB) and comment on what you observe.

6. Data set P6.m contains time samples from five different M = 20 element uniform linear arrays (ULAs) with half-wavelength element spacing. Each array has a different degree of calibration error with respect to the idealized array manifold. None of the incident source signals are temporally correlated (i.e. no multipath effects).

a) Plot the MVDR power spectrum for each array using both the normal estimate of the correlation matrix and the forward/backward averaging estimate of the correlation matrix (see Appendix A). Comment on what you observe.
b) For each array, plot the eigenvalues of the two correlation matrix estimates and comment on what you observe.
c) Based on the assumptions involved with forward/backward averaging and the relation to mismatch effects, use your previous observations to rank the five arrays in terms of the severity of calibration error.

Appendix A – Forward-Backward Averaging

One way to form the forward-backward averaged covariance matrix given the $M \times L$ data matrix $\mathbf{X}$ for a uniform linear array is

$$\mathbf{R}_{\text{FB}} = \frac{1}{2L}(\mathbf{X}\mathbf{X}^H + \mathbf{J}\mathbf{X}^*\mathbf{X}^T\mathbf{J})$$

where $\mathbf{J}$ is the $M \times M$ reflection matrix (looks like a reversed identity matrix) defined as

$$\mathbf{J} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & \iddots & 1 & 0 \\ 0 & \iddots & \iddots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}.$$