MASSACHVSETTS INSTITVTE OF TECHNOLOGY
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Classical Mechanics: A Computational Approach

Problem Set 3—Fall 2020

Issued: 16 September 2020 Due: 25 September 2020

Note: Project 1 is also due on Friday, 25 September 2020

Reading: SICM2 Chapter 1 through section 1.9

Introduction

This problem set concerned with the initial-value problem. We evolve a system from an inital state by integration of the equations of motion. We can algebraically derive a "state derivative" from a Lagrangian for the system. The numerical techniques for accurate and stable evolution are very complicated and beyond the scope of this class.

We also learned about how continuous symmetries of the Lagrangian give us conserved quantities. A component of momentum is conserved when the corresponding coordinate does not appear in the Lagrangian. Energy is conserved when time does not explicitly appear in the Lagrangian. This is generalized, by Noether's Theorem, to any continuous symmetry of the Lagrangian.

This problem set is shorter than usual because Project 1 is due at the same time as this problem set.

Exercises

- Exercise 1.28: Identifying total time derivatives. SICM2 page 68
- Exercise 1.30: Orbits in a central potential. SICM2 page 78
- Exercise 1.36: Noether integral. SICM2 page 94
- Exercise: Velocity-dependence continued

  In problem set 2 we saw how Coriolis and centrifugal forces arise from observation of the motion of a free particle in a rotating coordinate system, and we observed how such forces can be derived from velocity-dependent terms in a Lagrangian.

  An important special case of this arises in the restricted three-body problem (Section 1.8.4 in SICM2.) We generalized this to an arbitrary rotating potential energy in problem set 2.

  The formal Lagrangian for this potential energy is just the `L0` seen on page 87 of SICM2 with `V` the rotating potential energy you computed in problem set 2.

  1. Find the time-independent Lagrangian in rotating coordinates analogous to Equation 1.150 on page 88 and identify the interesting terms.
  2. Compute the conserved "energy" for this time-independent Lagrangian.