Structure and Interpretation of Classical Mechanics
6.946J/8.351J/12.620J Project 2

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1. Exercise 2.21: Rotation of Mercury.

The code for evolving Mercury can be found in project2.scm.

2. A.

The Lagrange equation (2.105) in terms of $\theta(t)$:
$D^2\theta(t) = -\frac{n^2\epsilon^2}{2}\left(\frac{a}{R(t)}\right)^3 \sin 2(\theta(t) - f(t))$

Changing variables for $\tau = nt$, we get:
$Df(\tau) = \sqrt{1-e^2}\left(\frac{a}{R(t)}\right)^2$

$\frac{a}{R(\tau)} = \frac{1+e\cos f(\tau)}{1-e^2}$

$D^2\theta(\tau) = -\frac{\epsilon^2}{2}\left(\frac{a}{R(\tau)}\right)^3 \sin 2(\theta(\tau) - f(\tau))$

The state is conceptualized as $(\tau, \theta, \dot{\theta}, f)$, and its derivative becomes $(1, \dot{\theta}, \ddot{\theta}, \dot{f})$.

3. B.

Set up a window over a long period of time (2000 τs), and choose a relatively large integration step (t=0.25).

Before evolving the state determine the initial conditions. Let's start with Mercury at the end of the semi-major axis, so $f(t=0) = \theta(t=0) = 0$. Choose $D\theta(t=0) = 1.5$ as a stable orbit. (A quick reasoning is that since the angle is $\theta - 3/2\tau$, the rate of change becomes $\dot{\theta} - 3/2$, and this is zero when $\dot{\theta} = 1.5$.) If we evolve the state with this value, we see that $\theta(\tau) - 3/2\tau$ evolves as a straight (constant) line.

Nudging the initial $\dot{\theta}$ by a small amount, use 1.51 for the state evolution, to get oscillations.

4. C.

If we exchange for τ instead of t in (2.122), we get a range of $\dot{\theta} - 3/2$ being between 0.021753160, or $\dot{\theta}$ being found between 1.47824684 and 1.52175316. We tighten the interval by investigation.

To eight digits, the range of initial angular velocities for which the spin-orbit resonance is stable seems to be between 1.52093252 and 1.47937647. Outside of this range the angle $\dot{\theta} - 3/2$ does not oscillate.

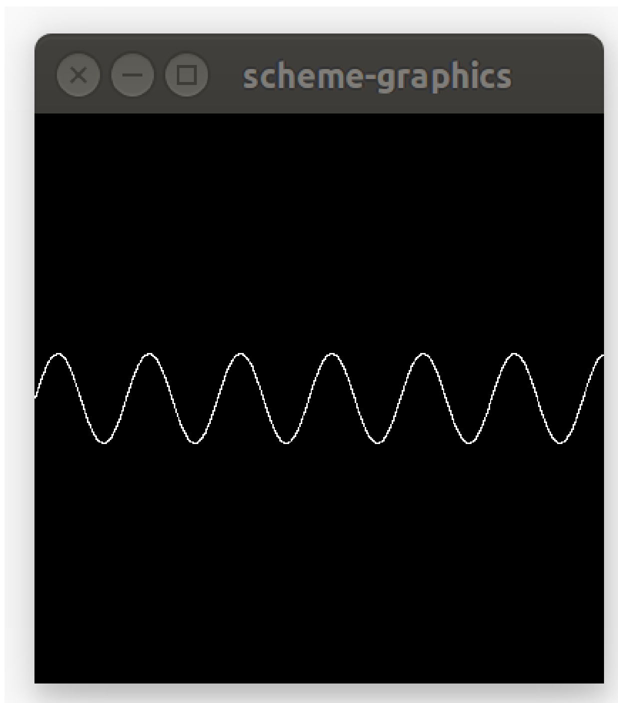
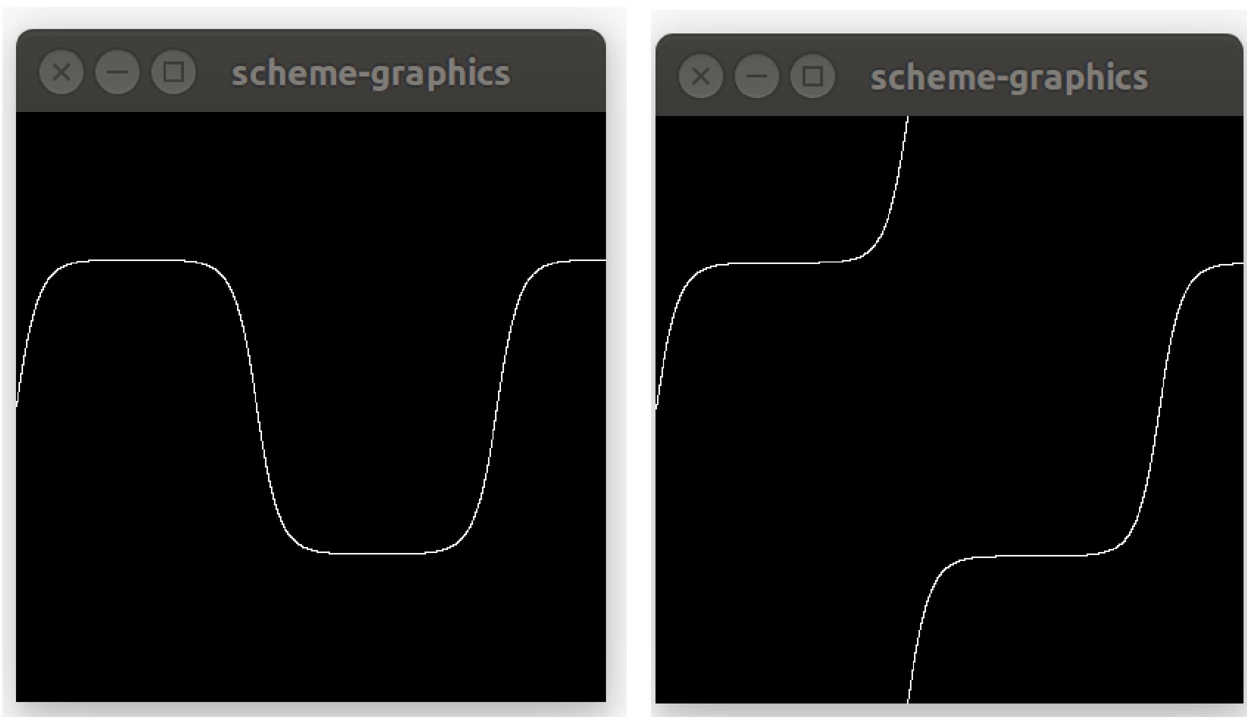


Figure 1: Plot of $\theta(\tau) - 3/2\tau$, for $0 \leq \tau \leq 2000$, with integration step $\triangle t = 0.25$. Initial angular velocity $D\theta(t = 0) = 1.51$. Ordinate axis spans from -π and π.



(a) (b)

Figure 2: Plot of $\theta(\tau) - 3/2\tau$, for $0 \leq \tau \leq 2000$, with integration step $\triangle t = 0.25$. The initial angular velocities are 1.52093252 on the left, and 1.52093253 on the right. Ordinate axis spans from -π and π.

(a) (b)

Figure 3: Plot of $\theta(\tau) - 3/2\tau$, for $0 \leq \tau \leq 2000$, with integration step $\triangle t = 0.25$. The initial angular velocities are 1.47937646 on the left, and 1.47937647 on the right. Ordinate axis spans from -π and π.