Structure and Interpretation of Classical Mechanics
6.946J/8.351J/12.620J Project 1

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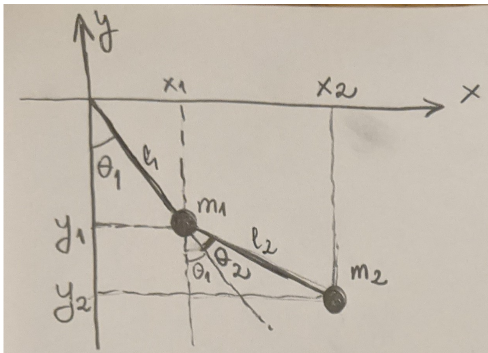
1. 1.44: Double pendulum Behavior

The code can be found in project1.scm

a.



- $x_1 = l_1 \sin\theta_1$ $\qquad$ $y_1 = -l_1 \cos\theta_1$

  $v_{1x} = l_1 \cos\theta_1 \dot{\theta}_1$ $\qquad$ $v_{1y} = l_1 \sin\theta_1 \dot{\theta}_1$

  $v_1 = l_1 \dot{\theta}_1$

- $x_2 = x_1 + l_2 \sin(\theta_1 + \theta_2)$ $\qquad$ $y_2 = y_1 - l_2 \cos(\theta_1 + \theta_2)$

  $v_{2x} = v_{1x} + l_2 \cos(\theta_1+\theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$ $\qquad$ $v_{2y} = v_{1y} + l_2 \sin(\theta_1+\theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$

  $$v_2^2 = l_1^2\dot{\theta}_1^2 + 2l_2(\dot{\theta}_1+\dot{\theta}_2)(l_1\dot{\theta}_1\cos(\theta_1+\theta_2)\cos\theta_1 + l_1\dot{\theta}_1\sin(\theta_1+\theta_2)\sin\theta_1) + l_2^2(\dot{\theta}_1+\dot{\theta}_2)^2$$

  $$v_2^2 = l_1^2\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1+\dot{\theta}_2)^2 + 2l_1l_2\dot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2)\cos\theta_2$$

- $$T = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\left(l_1^2\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1+\dot{\theta}_2)^2 + 2l_1l_2\dot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2)\cos\theta_2\right)$$

  $$V = m_1 g y_1 + m_2 g y_2$$
  $$= -m_1 g l_1\cos\theta_1 - m_2 g(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2))$$

- $L = T - V$

b.

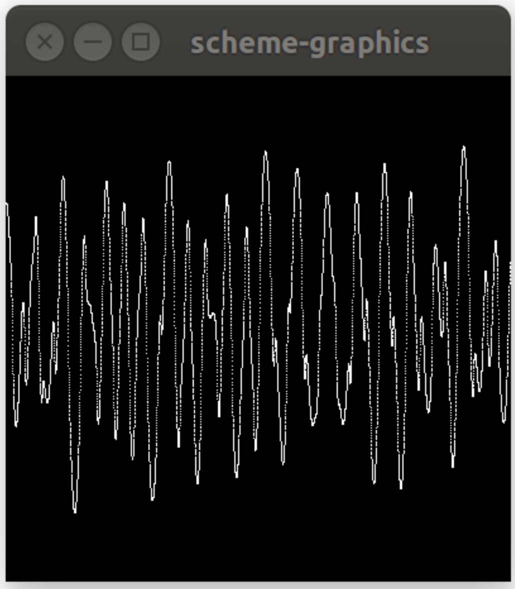


Figure 1: Trajectory of bob1 over a 50 second experiment, with time-step 0.005.

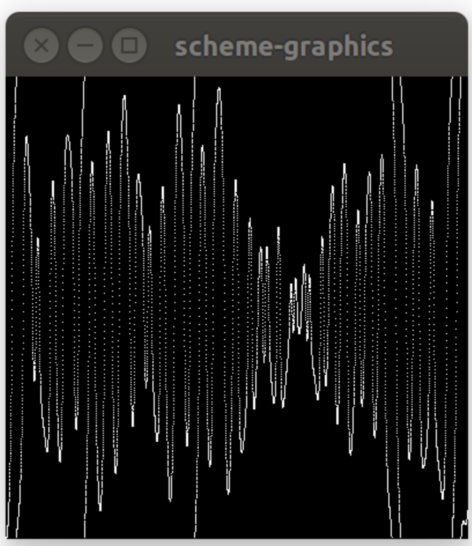


Figure 2: Trajectory of bob2 over a 50 second experiment, with time-step 0.005.

c.
The absolute error is less than 10^{-10} Joules, so the conservation of energy is nearly maintained.

Figure 3: Total energy of the double-pendulum system over a 50 second experiment, with time-step 0.005.

d.

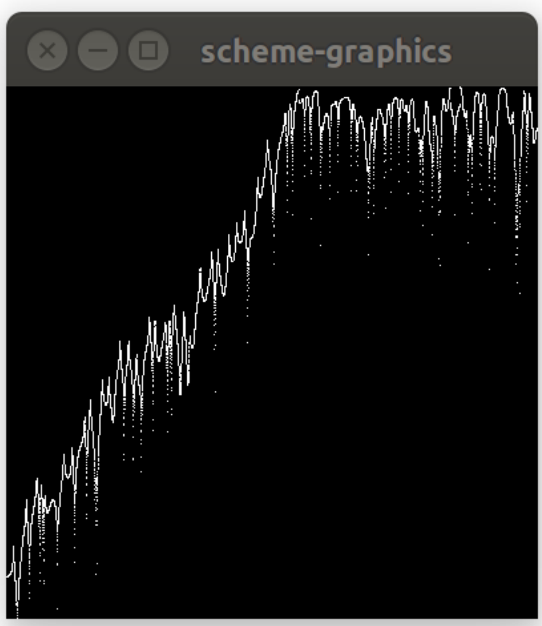


Figure 4: Logarithmic plot of squared differences of the positions (in terms of θ_2 values) between the two bobs2, over a 50 second experiment, with time-step 0.005.

The log of differences between the theta2 angles increases roughly linearly with time, until reaching maximum error, at about 30 seconds. The system is in its chaotic regime, where small disturbances to the initial conditions produce large discrepancies quickly. For example, in just 30 seconds the system's behavior becomes completely chaotic. Since we're in a log-display, it means the divergence between the two bobs at this point will be almost exponential.

e.

With the new set of initial conditions, there's little difference in the resulting motion, since the difference between the positions of the bobs m2 are small and do not seem to grow with time. We're in the stable region of the dual pendulum behavior.

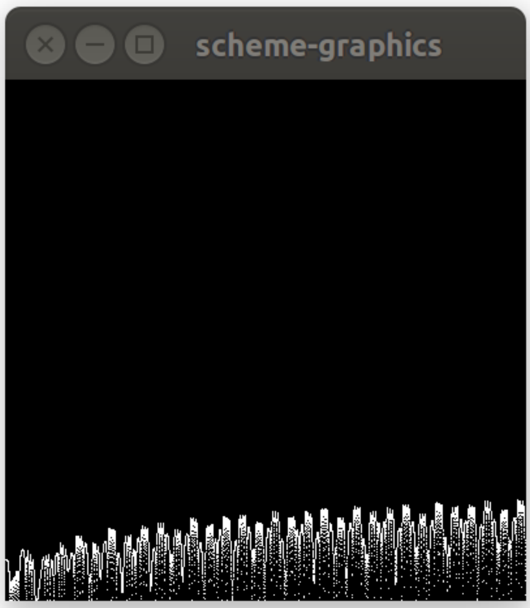


Figure 5: Logarithmic plot of squared differences of the positions (in terms of θ_2 values) between the two bobs2, over a 50 second experiment, with time-step 0.005, and new initial conditions.

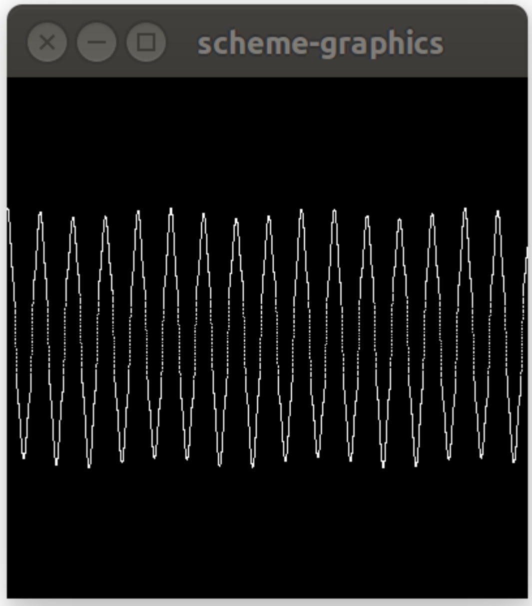


Figure 6: Trajectory of bob1 over a 50 second experiment, with time-step 0.005, and new initial conditions, for pendulum 1.

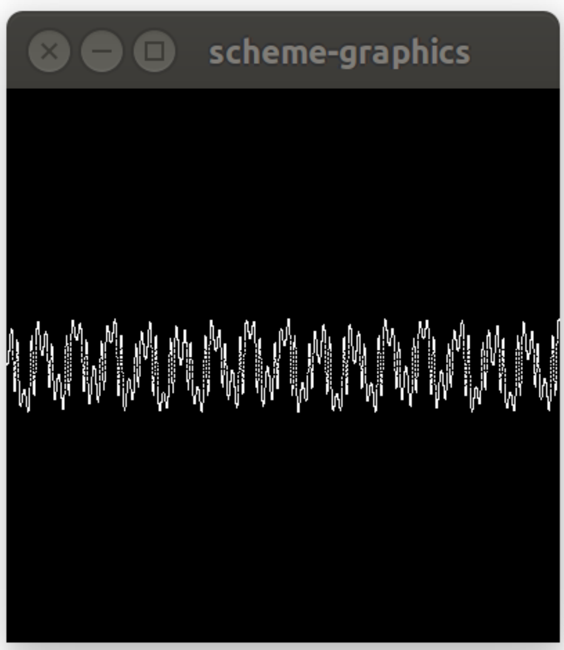


Figure 7: Trajectory of bob2 over a 50 second experiment, with time-step 0.005, and new initial conditions, for pendulum 1.