

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

6.946J, 8.351J, 12.620J

Classical Mechanics: A Computational Approach

Problem Set 1—Fall 2020

Issued: 2 September 2020

Due: 11 September 2020

Reading: SICM2 Chapter 1 through section 1.6.1

Warning: There is an egregious typo on page 25, in equation (1.16). Here is the bad version:

$$\begin{aligned} Dg(0) &= \int_{t_1}^{t_2} (\partial_1 L(t, q(t), Dq(t))\eta(t)) dt \\ &\quad + \partial_2 L(t, q(t), Dq(t))\eta(t)|_{t_1}^{t_2} \\ &\quad - \int_{t_1}^{t_2} \frac{d}{dt} (\partial_2 L(t, q(t), Dq(t))) \eta(t) dt. \end{aligned}$$

This equation should read:

$$\begin{aligned} Dg(0) &= \int_{t_1}^{t_2} (\partial_1 L(t, q(t), Dq(t))\eta(t)) dt \\ &\quad + \partial_2 L(t, q(t), Dq(t))\eta(t)|_{t_1}^{t_2} \\ &\quad - \int_{t_1}^{t_2} \frac{d}{dt} (\partial_2 L(t, q(t), Dq(t))) \eta(t) dt. \end{aligned}$$

General Instructions

A problem set for this class generally asks you to solve some mechanics problems. Some problems involve simple programming. We expect that you will produce a document describing the way you solved the problems. If you used the computer we want you to include the code you wrote and the results you obtained. This should not be a long story: if your solution includes pages of numbers or long algebraic expressions we don't want to see that. We may also ask for short essays explaining an idea. Your answers to these questions must be clear and concise: good logic expressed in good English is required. Thank you.

You need not type in the general procedures that appear in the book, they are provided by the system. However, you will have to define procedures that are specific to each problem.

Introduction

In this problem set we are introducing the variational way to think about mechanics. Important concepts are configuration space and ways to make coordinates. The problem is to distinguish realizable trajectories through the configuration space from others. We invent a function, the action, that is stationary on the realizable trajectories. This action is locally defined in terms of a Lagrangian function. We can derive a set of differential equations, the Lagrange-Euler equations that are satisfied by realizable trajectories. For many systems an appropriate Lagrangian is the difference between the kinetic energy and the potential energy.

Exercises

- Exercise 1.1: Degrees of freedom. SICM2 page 5
- Exercise 1.2: Generalized coordinates. SICM2 page 8
- Exercise 1.5: Solution process. SICM2 page 23
you need not hand in anything for this exercise, but we want you to observe the result.
- Exercise 1.8: Implementation of δ . SICM2 page 28
This is a programming problem. It is not hard, but it requires you to understand the construction of high-order functional procedures. No limits are needed: equation (1.22) is the essential idea.
- Exercise 1.9: Lagrange's equations. SICM2 page 32
- Exercise 1.12: Computing Lagrange equations. SICM2 page 36
Note: You do not have to type in the definition of the `Lagrange-equations` procedure. It is provided in the system.