

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

6.946J, 8.351J, 12.620J

Classical Mechanics: A Computational Approach

Problem Set 8—Fall 2020

Issued: 28 October 2020

Due: 30 October 2020

Note: Project 3 is also due on Friday, 30 October 2020, so this problem set is short.

Note: Project 4 announcement is at the end of this problem set.

Reading: SICM2 Chapter 3 through end of chapter, Chapter 4 through Section 4.2.

Introduction

The nature of the neighborhood of a particular trajectory in phase space is revealed by consideration of the variational equations. These are linear differential equations driven by the trajectory being studied. The neighborhood is distorted. It is stretched in some directions and it is compressed in others. Chaotic behavior is characterized by an exponential distortion, quantified by the *Lyapunov exponent*. More globally, any tube of trajectories in the phase space has a striking property. The phase-space volume of states at any subsequent moment is the same as the volume of initial states. This is the *Liouville Theorem*.

Exercises

- Exercise: 3.13: Fun with Henon's quadratic map SICM2 page 280
- Exercise: 4.3: Standard Map eigenvalues and eigenvectors SICM2 page 297
- Exercise: More fun with the Standard Map.

Reproduce figures 3.27 and 3.28. The two-line code to do this is on page 279.

Project 4

Project 4 will be due on 20 November 2020. There will also be problem set 10 due on that date, but it will be shorter. Project 4 will be more about the Poincaré–Birkhoff construction.

- Exercise 4.9: Secondary Islands SICM2 page 333