University of Connecticut
ECE1101
Spring 2021

Homework 2 – Calculating Arc lengths and integrals
Due February 18, 2021 End of Day

Write a program that can calculate the arc length of a curve. For any function f, the arc length of the curve between points $(a, f(a)$ and $(b, f(b)$ is defined as:

$$\int_a^b \sqrt{1 + (f'(x))^2}dx$$

where $f'(x)$ is the derivative of $f(x)$.

If you haven't taken Calculus 2, a definite integral is defined as follows:

$$\int_a^b g(x)dx = G(b) - G(a)$$

where G is the antiderivative of g – i.e. $g = \frac{dG}{dx}$. If the function g is simple, it's possible to find the antiderivative. However, for the arc length, the function $g(x) = \sqrt{1 + (f'(x))^2}$, is not simple, and it is very difficult to find the antiderivative.

In this case, we can use the following approximation:

$$\int_a^b g(x)dx = \sum_{i=0}^{i=N-1} \frac{g(a + i\Delta x) + g(a + (i+1)\Delta x)}{2} \Delta x$$

Where $N = \frac{b-a}{\Delta x}$. This is equivalent to summing up $\left(\frac{g(x)+g(x+\Delta x)}{2}\Delta x\right)$ as x goes from a to b in steps of Δx. Since, we want to find the arc length of $f(x)$, we substitute $g(x) = \sqrt{1 + (f'(x))^2}$ and get $\left(\frac{\sqrt{1+(f\prime(x))^2}+\sqrt{1+(f\prime(x+\Delta x))^2}}{2}\Delta x\right)$

Running the program

Your program should take 4 arguments: *function a b deltax* where *function* is 1, 2, or 3. If *function* is 1, $f(x) = x$. If *function* is 2, $f(x) = e^x$. If *function* is 3, $f(x) = \sqrt{1 - x^2}$).

So, to calculate the length of the e^x curve from 0 to 1 with $\Delta x = 0.0001$, you should run the program as follows:

```
$ ./arclen 2 0 1 0.0001
arclen value = 2.003787
```

The exact values that you get may be off a little bit, depending on how you calculate the functions.

To calculate the length of the $\sqrt{1-x^2}$ curve from 0 to $\frac{\sqrt{2}}{2}$, do the following:

```
$ ./arclen 3 0 0.707107 0.0001
arclen value = 0.785530
```

Writing the code

You will need to input the 4 arguments. The first argument (*function*) is an integer, but the remaining 3 arguments are all floating-point arguments, so use the `double` type and the `atof` function instead of `atoi`.

You will need to write a loop that increments x in steps of Δx from a to b to sum up the integral. Inside the loop, you will need to check the *function* variable to select the right *f* function and *f'* derivative. You won't need to evaluate $f(x)$ in your code - just $f'(x)$. You can use your Calc 1 math to figure out the derivative of the 3 functions. For example, when $f(x) = x$, $f'(x) = 1$, and $g(x) = \sqrt{2}$. Note, that even though it is possible to calculate the integral ahead of time for some of the functions, you should always use the summation method that is shown above.

To calculate the square root, you will need to use the `sqrt` function which is part of the built-in math library. Make sure you add `#include <math.h>` at the beginning of your file, so that your code knows about the `sqrt` function.

For the e^x function, you can use the code from the class exercise or use the built-in `exp` function in the math library.

At the terminal in Cygwin or on a Mac, you can type "`man exp`" to get more information about how to use the `exp` function. You can use `man` for any of the library functions.

There is no square operator in C – i.e you can't do `x^2`. Instead, you will need to do `x*x`.

If you're having trouble figuring out why you're getting wrong answers, put in `printf`'s in the code to print out important values and see if it is what you expected. You can also use the GDB debugger to help debug the code. Often times, this is more efficient and quicker than using `printf`'s. There is a video in the resources folder that explains how to use the GDB debugger.

Submit your `.c` file on HuskyCT. Use comments to explain what your code does.