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Quantum Clock Synchronization with a Single Qudit

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Clock synchronization for nonfaulty processes in multiprocess networks is indispensable for a variety of technologies. A reliable system must be able to resynchronize the nonfaulty processes upon some components failing causing the distribution of incorrect or conflicting information in the network. The task of synchronizing such networks is related to Byzantine agreement (BA), which can classically be solved using recursive algorithms if and only if less than one-third of the processes are faulty. Here we introduce a nonrecursive quantum algorithm, based on a quantum solution of the detectable BA, which achieves clock synchronization in the presence of arbitrary many faulty processes by using only a single quantum system.

n many multiprocess networks, including data transfer networks, telecommunications networks, the global positioning system, and long baseline interferometry, the individual processes need to have clocks that must be synchronized with one another^{1,2}. To this purpose, individual processes' clocks must periodically be resynchronized. This motivates the need for clock synchronization algorithms which work despite the faulty behavior by some of the processes. Faulty behavior can occur due to a variety of causes, including crashing, transmission failure, and distribution of incorrect or inconsistent information in the network³. A clock synchronization algorithm should achieve the following tasks: C1) For any given instant, the time of all nonfaulty processes' clocks must be the same. This is necessary, but not sufficient, since simply stopping all clocks at zero satisfies C1. We therefore need to assume that a process' logical clock also keeps the rate of its corresponding physical clock. In addition, synchronizing may cause further errors, so we require that: C2) There is a small bound on the amount that a process' clock is changed during synchronization⁴.

Reliable clock synchronization algorithms can be complicated. To simplify the problem we shall work under the following assumptions⁴: A1) Initially, all clocks are synchronized to the same value. Physical clocks typically do not keep perfect time but drift with respect to one another. This motivates the following assumption: A2) All nonfaulty processes' clocks run at one second in clock time per second in real time. A general problem arises from the clocks continuously changing during the synchronization procedure. Unless the synchronization algorithm is very fast, this will cause problems. This motivates our last assumption: A3) A nonfaulty process can read the time difference between the clock of another process and its own.

A method to achieve synchronization is to use interactive consistency algorithms (ICAs) in which all nonfaulty processes reach a mutual agreement about all the clocks⁴. An ICA should satisfy that, for every process p: (1) Any two nonfaulty processes obtain the same value of process p's clock, even if p is faulty. (2) If p is nonfaulty, then every nonfaulty process obtains the value of p's clock. The synchronization problem can classically be solved using recursive algorithms if and only if less than one-third of the clock are faulty.

The conditions for an ICAs are similar to the ones of the problem of Byzantine Agreement (BA) in the case of which: (i) All nonfaulty processes obtain the same value and (ii) if process p is nonfaulty, then all nonfaulty processes obtain the value it sends^{4,5}. Nevertheless, it has been shown that even quantum methods cannot solve the BA if one-third or more of processes are faulty⁶.

However, for most applications, including clock synchronization, it is sufficient to consider a scenario called detectable Byzantine agreement (DBA) or detectable broadcast^{7,8}. In this case, conditions (i) and (ii) are replaced with: (i') either all nonfaulty processes obtain the same value or all abort, and (ii') if process p is nonfaulty, then either every nonfaulty process obtains the same value or aborts. By "abort" we mean treating the value as undefined and exiting the protocol.

Classical ICAs can only achieve DBA if less than one-third of the processes are faulty⁴ and agreement is achieved by majority voting using a recursive algorithm, called OM(n), where n is the number of faulty processes. The OM(n) algorithm works as follows. We label the processes as P_k , with k = 1, 2, ..., m. If n = 0, then P_1



distributes its value to every other process. Every process uses the value received from P_1 and, in case no value is obtained, uses 0. If n > 0, then P_1 distributes its value to every other process. For k = 2, ..., m, let x_k denote the value obtained by P_k from P_1 . If P_k receives no message, then let $x_k = 0$. P_k acts as P_1 in algorithm OM(n-1) by distributing x_k to the remaining m-2 processes. For every k and for all $j \neq k$, let x_j be the value received by P_k from P_j using OM(n-1), and in case no value was received $x_j = 0$. P_k decides on the value obtained from the median of $(x_1, ..., x_m)$. Thus, OM(n) requires $O(m^{n+1})$ transmitted messages to solve the task.

The DBA is an example of a communication task for which quantum resources can provide a solution, while classical tools cannot. Nevertheless, the special case of DBA in a three process network with one faulty process, has been solved using quantum methods based on three-qutrit singlet states^{7,9}, four-qubit entangled states¹⁰, and three⁸ or two¹² pairwise quantum key distribution (QKD) channels, and experimentally demonstrated using four photon-polarization entangled state¹¹.

Interestingly, later works have shown that there are quantum solutions for certain communication complexity problems and secret sharing tasks which do not require entanglement, but, instead, sequential communication of a single quantum system^{13,14}. These protocols have been shown to be much more resistant to noise and imperfections, and significantly more scalable than protocols based on entanglement.

In this article, we introduce a quantum algorithm that solves the DBA and achieves clock synchronization in the presence of an arbitrary number of faulty processes, with only one single round of message passing per process independently of the number of faulty processes, utilizing only a single quantum system.

In order to solve the DBA problem, the m processes need to share data in the form of lists l_k , of numbers subject to specific correlations, and the distribution must be such that the list l_k held by process P_k is known only by P_k . Quantum mechanics provides methods to generate and securely distribute such data, here we shall seek for one which is simple, efficient, and easily extendible to an arbitrary number of processes. We assume that all processes can communicate with one another with oral messages by pairwise authenticated error-free classical channels and pairwise authenticated quantum channels.

Correlated lists and their use

The initial stage of the quantum protocol is to distribute lists l_k , for k=1,...,m, each of them available only to process P_k . All lists have to be of the same length L and are required to satisfy the property that if N=0 (or 1) is at position j in l_1 , then 0 (respectively, 1) is at position j in lists l_k for k=2,...,m (i.e., they are perfectly correlated). However, if $N \in \{2,...,m-1\}$ is at position j in l_1 , then the sum of numbers at positions j in lists l_k for k=2,...,m equals m-N, and all elements in these lists are either 0 or 1. Given an N, all the possible combinations of binary numbers satisfying the condition are uniformly probable.

Note that, on one hand, P_1 has information about at which positions the lists of all other processes the values are perfectly correlated, and at which positions they are random bits, with the property that their sum is anticorrelated with the value, $N \ge 1$, in l_k . On the other hand, the holder of one the lists l_k , with k = 2, ..., m, has no information whatsoever on whether the lists are correlated at a given position or not.

Once the processes have these lists, they can use them to achieve mutual agreement and solve the DBA by applying the algorithmic part of the protocol, which we shall call QB(n, m). The special case, QB(1, 3), reproduces the protocol in¹¹.

(1) P_1 sends bit-valued messages to all processes. The message sent to process P_k will be denoted by $m_{1,k}$. Together with each message, P_1 sends a list $l_{1,k}$ of all of the positions in l_1 in which the value $m_{1,k}$ appears. If P_1 is nonfaulty all lists and messages are

- identical. The full information which P_k receives from P_1 will be denoted by $\{m_{1,k}, l_{1,k}\}$.
- (2) The receiving processes P_k analyze (singlehandedly) the obtained lists and messages. If the analysis of P_k shows that $l_{1,k}$ is of appropriate length (i.e., about L/m) and $\{m_{1,k}, l_{1,k}\}$ is consistent with l_k at all positions, then if P_k is nonfaulty, it conveys $\{m_{1,k}, l_{1,k}\}$ to all other processes $P_{k\neq 1}$. A faulty process sends a flipped bit value of the message with whatever list it chooses. The full information which P_j receives from P_k will be denoted by $\{m_{k,j}, l_{k,j}\}$.

A nonfaulty P_k will also decide on the final bit value it adopts V_k . This is $m_{1,k}$, unless messages from the other processes force it to decide that P_1 is faulty. However, if $\{m_{1,k}, l_{1,k}\}$ is not consistent with l_k , then P_k immediately ascertains that P_1 is faulty and relays to other processes neither 0 nor 1 but \bot , meaning "I have received inconsistent data."

(3) Once all messages have been exchanged between P₂, ..., P_m, each process considers the obtained data and acts according to the instructions in Table 1. The overall aim is, if P₁ is nonfaulty, to have the same value of V_k for all nonfaulty processes, or all of them aborting.

Quantum protocol for distributing lists lk

All processes are equipped with devices which can unitarily transform qudits. In addition, P_1 has a source of *single qudits of dimension* m and the last process, P_m , has *additionally* a measurement device. The protocol runs as follows (for an illustration, see Fig. 1):

(I) P_1 prepares the state

$$|\psi_0\rangle = \frac{1}{\sqrt{m}} \sum_{j=0}^{m-1} |j\rangle. \tag{1}$$

(II) P_1 randomly chooses the "encoding basis" from m different options $U_0,...,U_{m-1}$ and labels the choice c_1 . Having chosen the c_1 'st encoding basis, process P_1 applies the following unitary transformation to the qudit:

$$U_{c_1} = |0\rangle\langle 0| + \sum_{k=1}^{m-1} \omega^{c_1} |k\rangle\langle k|, \qquad (2)$$

where $\omega = e^{i\frac{2\pi}{m}}$. From the interferometric point of view, applying U_{c_1} introduces a phase-shift of $-2\pi c_1/m$ in the first beam.

(III) After that, P_1 randomly chooses a value N_1 in the set $\{0, 1, ..., m-1\}$ and encodes N_1 , by applying the following unitary transformation:

$$U(N_1) = \sum_{j=0}^{m-1} \omega^{jN_1} |j\rangle\langle j|.$$
 (3)

Afterwards, the qudit is sent to P_2 .

- (IV) P_2 , in the same manner as P_1 , choses a $c_2 \in \{0,...,m-1\}$ and applies the unitary U_{c_2} corresponding to choice of encoding
- (V) Next, P_2 randomly chooses a value N_2 in the set $\{0, 1\}$. If $N_2 = 0$, no action is taken, i.e., P_2 applies the transformation $U(N_2 = 0) = 1$. If $N_2 = 1$, then P_2 applies $U(N_2 = 1)$ and then sends the qudit to P_3 .
- (VI) $P_3, ..., P_m$ consecutively repeat the same procedure as P_2 with independent choices of basis and encoding their respective random values $N_3, ..., N_m$.
- (VII) In addition, P_m measures the qudit using a device which distinguishes the state $|\psi_0\rangle$ from any set of states orthogonal to it



Table 1 | Once P_k receives all messages and lists from all other processes, it will study the obtained lists and messages and compare to its own list I_k . Depending on the consistency between the obtained and private data, P_k will act according to table below. Notation $\{m_{j,k}, l_{j,k}\} \cong l_k$ means that $m_{j,k}$ and $l_{j,k}$ are found to be consistent with l_k whereas \ncong means "inconsistent with." The symbol \bot means "I have received inconsistent data." By \mathbb{M}_k we denote some nonempty subset of $\{1, \ldots, m\} \setminus \{k\}$

	local analysis of all data received by P_k	decision of P_k on the value V_k
(iia) (iib)	$\forall j \in \mathbb{N}_m \setminus \{k\}, \ \left\{m_{j,k}, l_{j,k}\right\} \cong l_k \text{ and all messages are equal} \ \forall j \in \mathbb{N}_m \setminus \{k\}, \ \left\{m_{j,k}, l_{j,k}\right\} \cong l_k \text{ and } not \text{ all messages are equal}$	$V_k = m_{1,k}$, no faulty process as P_1 is faulty, $V_k = abort$
(iic) (iid)	$\forall j \in \mathbb{M}_k, \ \left\{m_{j,k}, l_{j,k}\right\} \not\cong l_k \ \text{and} \ \forall j \not\in \mathbb{M}_k, \ \left\{m_{j,k}, l_{j,k}\right\} \cong l_k \ \forall j \in \mathbb{M}_k, \ \left\{m_{j,k}, l_{j,k}\right\} \cong l_k \ \text{and} \ \bot \forall j \not\in \mathbb{M}_k$	$V_k = m_{j,k}$, for $j \notin \mathbb{M}_k$, as the other P_j 's are faulty $V_k = m_{1,k}$, although P_1 could be faulty
(iie)	$\forall j \in \mathbb{M}_k, \ \{m_{j,k}, l_{j,k}\} \cong l_k \text{ but with unequal messages, and } \perp \text{ from } \forall j \notin \mathbb{M}_k$	$V_k = abort$, at least P_1 is faulty

(VIII) If P_m obtains $|\psi_0\rangle$, then the processes consecutively reveal their encoding bases (but not their values N_k) in reverse order: First P_m and last P_1 . If it turns out that the sum of the basis choices modulo m equals zero, then the run is treated as a valid distribution of the numbers N_k at the same position in the private lists l_k .

The protocol distributes the numbers in the required way because all the unitary operators are diagonal and, therefore, commute.

Additionally, if
$$\sum_{k=1}^{m} c_k = 0 \mod m$$
 then

$$\prod_{k=1}^{m} U_{c_k} = 1, \tag{4}$$

and, if $\sum_{k=1}^{m} N_k = 0$, modulo m, then

$$\prod_{k=1}^{m} U(N_k) = 1.$$
 (5)

Whenever this condition is not satisfied, the final state of the system is orthogonal to $|\psi_0\rangle$ and will therefore never be an outcome of P_m 's measurement.

Clock synchronization

Now, we will show how to apply our method for solving the DBA to achieve fault tolerant clock synchronization. However, a problem

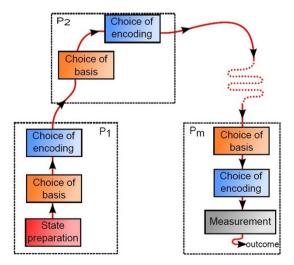


Figure 1 | Scheme of the quantum protocol for the distribution of the correlated lists. P_1 prepares a uniform d-level superposition state, makes a choice of basis and encoding, and forwards the qudit to P_2 which applies a choice a basis and encoding and forwards the qudit to P_3 . Processes P_3 , ..., P_m act in analogy with P_2 . Finally P_m projects the state onto the initial state prepared by P_1 and, if the outcome is 1, the processes reveal their bases and, if all bases are the same, the round is treated as valid.

arises from clocks ticking during the synchronization procedure. This is solved by exploiting assumption A3: Instead of sending a number, the processes send their clock differences to each other. In the classical case, we achieve clock synchronization by running the algorithm OM(1) m times, sending clock differences instead of the binary values, and analogously for $OM(n)^4$. In analogy with the classical case, the processes send clock differences also in the quantum case, exploiting the fact that the clock differences can be decomposed into binary strings up to arbitrary accuracy agreed upon in advance. We run QB(n, m) m times in such a way that for each run a new processes takes the roll of P_1 in QB(n, m). More explicitly, P_v reads the clock difference Δ_{xy} between its own clock and the clock of P_x . If P_y is nonfaulty it will relay Δ_{xy} to P_z but if P_y is a faulty process, it can arbitrarily change Δ_{xy} before sending it. If P_y relays the value obtained from P_x to P_z , then P_z knows the time difference between P_x and P_y . Also, since QB(n, m) is ran m times, P_z will also obtain Δ_{yz} from P_v and thus P_z knows that P_v is claiming that the time difference between P_x and P_z is $\Delta_{xy} + \Delta_{yz}$, which can then be compared to Δ_{xz} obtained directly from P_x .

Comparison with the other solutions

The correlated lists needed for achieving DBA can be distributed by other means than with the single-qudit protocol. Successful distribution can be achieved by the process P_m sharing a QKD channel with every other process. P_m uses a QKD protocol, e.g., BB84¹⁷ to distribute numbers such that (1) P_m and P_1 share a string $K_{1,m} = k_{1,m}^1 \dots k_{1,m}^L$, where $k_{1,m}^j \in \{0, \dots, m-1\}$. (2) For every l = 2, ..., m-1, P_m and P_l share a string $K_{l,m}=k_{l,m}^1\ldots k_{l,m}^L$ such that $k_{l,m}^{j} \in \{0,1\}$. (3) For a given j, the lists satisfy $\left(\sum_{l=1}^{m} k_{l,m}^{j}\right)_{\text{mod m}} = 0$. (4) None of $P_2, ..., P_{m-1}$ have any information about a particular list element of any other process. (5) Whenever P_1 receives an element $k_{1,m}^j \ge 2$, P_1 has no information on the bit value of $k_{l,m}^{j}$ for l = 2, ..., m, and whenever P_1 receives $k_{1,m}^j = p \in \{0,1\}$, P_1 knows that $k_{l,m}^j = p$ for all l=2,...,m. All QKD channels except that shared between P_1 and P_m transmit bit values. In order to transmit elements of $\{0, ..., m-1\}$ to P_1 , the numbers must be encoded into $\lceil \log_2(m) \rceil$ qubits. One additional requirement that has to be made for solving the DBA using the QKD distributed lists is that P_m is not required to convey any lists. This is necessary since P_m has full knowledge about the lists of all other processes and therefore easily could cheat. Instead, P_m may announce the message it received from P_1 , and if any inconsistency is noted by $P_2, ..., P_{m-1}$, then P_m will change its final value if the other processes convince P_m of them being nonfaulty.

There are also other proposed solutions to the DBA considering three processes where one is faulty. The first one, proposed in Ref. 7, relies on the three qutrit entangled Aharonov state. The goal is to distribute lists given by all permutations of the elements of the set $\{0, 1, 2\}$, i.e., $\{0-1-2, 0-2-1, 1-0-2, 1-2-0, 2-0-1, \text{ and } 2-1-0\}$.



Generalization to *m* parties along the lines of Ref. 7 would require the usage of multipartite *m*-level entanglement, provided by the state

$$|\kappa_m\rangle = \frac{1}{\sqrt{m!}} \sum_{\bar{i}=\sigma(S_m)} (-1)^{N(\sigma(S_m))} |i_1,\dots,i_m\rangle, \tag{6}$$

where $\bar{i} = \{i_1, \dots, i_n\}$, $S_m = \{0, \dots, m-1\}$ and $N(\sigma(S_m))$ is the parity of the permutation of S_m . Already for the simplest case of m = 3, this approach requires the preparation of a very complex state which, to our knowledge, has not yet been experimentally realized. However, for the three process case, it has been pointed out in Ref. 12 that the distribution of the lists can be realized without the state (6), by utilizing two separated OKD channels. With small modification for the m process setting, distribution of the lists is achieved with m-1 QKD channels. However, to encode the entire space provided by S_m , the QKD requires $\lceil \log_2(m) \rceil$ qubits. If the efficiency of a detector η is not perfect and the QKD is performed with single qubits using von Neuman measurements, successful distribution occurs only with probability $n^{(m-1)\lceil \log_2(m) \rceil}$. Typically, the classical part of the protocol in Ref. 7 and its possible generalizations scale rapidly with the number of processes. It is required that m! different types of lists are distributed. However, a solution to the three party DBA exploiting four-qubit entanglement provides a simpler classical part of the protocol: the number of different lists is lowered from six to

The general m process protocol presented in this paper generalizes the protocol in Ref. 11 and requires 2^{m-1} different types of lists. As emphasized earlier, the distribution of the required lists can be achieved both with single-qudit and with m-1 QKD channels. Using QKD channels, only one channel needs to transmit all elements in S_m while the remaining m-2 channels only transmit bit values. In the presence of nonperfect detectors, successful distribution occurs with probability $\eta^{m-2+\lceil \log_2(m) \rceil}$. However, in the single-qudit approach only one single detection is needed and, therefore, successful distribution of the lists occur with probability η independently of m. The single-qudit protocol is highly scalable, both in terms of success probability with inefficient detectors and requirements on the classical lists.

Conclusions

We have presented a single-qudit protocol which provides an efficient solution to an important multiparty communication problem: It solves DBA and achieves clock synchronization in the presence of arbitrary many faulty clocks. In principle, our quantum algorithm is not limited to the case of clock synchronization, it can with small adaptation be used for other tasks requiring oral message interactive consistency. Interestingly, our algorithm works by transmitting a single qudit among the parties rather than by distributing a quantum entangled state among them. This makes the protocol much more practical, as single qudits can be experimentally realized easily in many ways. For example, using unbiased multiport beamsplitters¹⁵ or time-bin¹⁶. Compared to schemes based on several QKD channels, the single-qubit protocol is more scalable and robust against detection inefficiencies. This results shows that single-qudit quantum information protocols are interesting beyond QKD^{18,19} and random number generation^{20,21}, and should stimulate experimental implementations and further research in quantum information protocols.

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Author contributions

A.C., M.Z. and M.B. proposed and initiated the project. A.T. performed the analysis and the extension for n parties. All authors discussed the results, agreed on the conclusions, and wrote the manuscript.

Additional information

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