

OPEN Exploration of multiphoton entangled states by using weak nonlinearities

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We propose a fruitful scheme for exploring multiphoton entangled states based on linear optics and weak nonlinearities. Compared with the previous schemes the present method is more feasible because there are only small phase shifts instead of a series of related functions of photon numbers in the process of interaction with Kerr nonlinearities. In the absence of decoherence we analyze the error probabilities induced by homodyne measurement and show that the maximal error probability can be made small enough even when the number of photons is large. This implies that the present scheme is quite tractable and it is possible to produce entangled states involving a large number of photons.

Undoubtedly, entanglement¹⁻⁴ is one of the most crucial elements in quantum information processing, e.g. quantum cryptography⁵, quantum teleportation⁶, quantum secure direct communication^{7,8}, etc. In recent years, quantum entanglement has been extensively investigated in various candidate physical systems^{9–16}, in particular, one can prepare and manipulate multipartite entanglement in optical systems 17-28

Generally, a spontaneous parametric down-conversion (PDC) source^{29,30} is capable of emitting pairs of strongly time-correlated photons in two spatial modes. As extensions of interest, with linear optics and nonlinear optical materials several schemes for creating multiphoton entangled states have been proposed^{31–37}. For a large number of photons, however, there are some technological challenges such as probabilistic emission of PDC sources and imperfect detectors. A feasible approach is to use the simple single-photon sources, instead of waiting the successive pairs, and quantum nondemolition (QND) measurement^{38–45} with weak Kerr nonlinearities. Note that the Kerr nonlinearities $^{46-48}$ are extremely weak and the order of magnitude of them is only 10^{-2} even by using electromagnetically induced transparency^{49,50}. More recently, Shapiro et al.^{51,52} showed that the causality-induced phase noise will preclude high-fidelity π -radian conditional phase shifts created by the cross-Kerr effect. In these cases, with the increase of the number of photons it is usually more and more difficult to study multiphoton entanglement in the regime of weak nonlinearities.

In this paper, we focus on the exploration of multiphoton entangled states with linear optics and weak nonlinearities. We show a quantum circuit to evolve multimode signal photons fed by a group of arbitrary single-photon states and the coherent probe beam. Particularly, there are only two specified but small phase shifts induced in the process of interaction with weak nonlinearities. This fruitful architecture allows us to explore multiphoton entangled states with a large number of photons but still in the regime of weak nonlinearities.

Kerr nonlinearities

Before describing the proposed scheme, let us first give a brief introduction of the Kerr nonlinearities. The nonlinear Kerr media can be used to induce a cross phase modulation with Hamiltonian of the form $H = \hbar \chi a_s^{\dagger} a_s a_p^{\dagger} a_p$, where χ is the coupling constant and $a_s(a_p)$ represents the annihilation operator for photons in the signal (probé) mode. If we assume that the signal mode is initially described by the state $|\psi\rangle_c = c_0|0\rangle_c + c_1|1\rangle_c$ and the coherent probe beam is $|\alpha\rangle_{p}$, then after the Kerr interaction the whole system evolves as

$$e^{iHt/\hbar}|\psi\rangle_{s}|\alpha\rangle_{p} = c_{0}|0\rangle_{s}|\alpha\rangle_{p} + c_{1}|1\rangle_{s}|\alpha e^{i\theta}\rangle_{p}, \tag{1}$$

¹College of Physics Science and Information Engineering, Hebei Normal University, Shijiazhuang 050024, China. ²Department of Basic Curriculum, North China Institute of Science and Technology, Beijing 101601, China. ³College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang 050024, China. Correspondence and requests for materials should be addressed to F.-L.Y. (email: flyan@hebtu.edu.cn) or T. G. (email: gaoting@ hebtu.edu.cn)

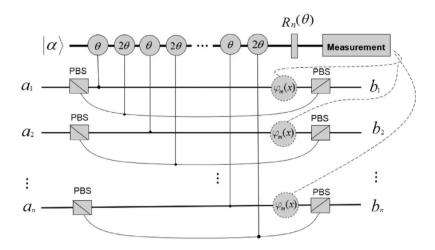


Figure 1. The schematic diagram of creating multiphoton entangled states with linear optics and weak nonlinearities. $a_i, i=1,2,\cdots,n$ are input ports and each port is supplied with an arbitrary single-photon state, while $b_i, i=1,2,\cdots,n$ are the corresponding outputs, respectively. θ and 2θ represent phase shifts in the coherent probe beam $|\alpha\rangle$ induced by Kerr interaction between photons. $R_n(\theta)$ is a phase shift gate. Each $\varphi_m(x)$ represents a phase shift on the signal photons based on classical feed-forward information.

where $\theta=\chi t$ with interaction time t. In order to distinguish different cases, one may perform a homodyne measurement³⁹ on the probe beam with quadrature operator $\hat{x}(\phi)=a_p\mathrm{e}^{\mathrm{i}\phi}+a_p^\dagger\mathrm{e}^{-\mathrm{i}\phi}$, where ϕ is a real constant. Especially for $\phi=0$, this operation is conventionally referred to as X homodyne measurement; while for $\phi=\pi/2$, it is called P homodyne measurement.

Creation of multiphoton entangled states with linear optics and weak nonlinearities

Let a_i , $i=1,2,\cdots,n$ represent input ports with respective spatial modes, namely signal modes, and $|\alpha\rangle$ is a coherent beam in probe mode. The setup of creating multiphoton entangled states is shown in Fig. 1.

Without loss of generality we may suppose that each input port is supplied with an arbitrary single-photon state. Then, the total input state reads

$$\begin{split} |\Psi_{\mathrm{in}}\rangle|\alpha\rangle &= (a_{00\cdots00}|H\rangle^{\otimes n} + \sum_{\mathrm{Perm}} a_{100\cdots0}|H\rangle^{\otimes (n-1)}|V\rangle \\ &+ \sum_{\mathrm{Perm}} a_{110\cdots0}|H\rangle^{\otimes (n-2)}|V\rangle^{\otimes 2} + \cdots \\ &+ \sum_{\mathrm{Perm}} a_{1\cdots100}|H\rangle^{\otimes 2}|V\rangle^{\otimes (n-2)} \\ &+ \sum_{\mathrm{Perm}} a_{1\cdots110}|H\rangle|V\rangle^{\otimes (n-1)} + a_{11\cdots11}|V\rangle^{\otimes n})|\alpha\rangle, \end{split}$$

where $a_{i_1i_2\cdots i_n}$, i_1 , i_2 , \cdots , $i_n \in \{0,1\}$, are complex coefficients satisfying the normalization condition $\sum_{i_1,i_2,\dots,i_n=0}^1 |a_{i_1i_2\cdots i_n}|^2 = 1$ and \sum_{Perm} denotes the sum over all possible permutations of the signal modes, for example,

$$\begin{split} \sum_{\text{Perm}} a_{100\cdots 0} |H\rangle^{\otimes (n-1)} |V\rangle &\equiv a_{100\cdots 0} |V\rangle_{a_{1}} |H\rangle_{a_{2},\cdots,a_{n}}^{\otimes (n-1)} + a_{010\cdots 0} |H\rangle_{a_{1}} |V\rangle_{a_{2}} |H\rangle_{a_{3},\cdots,a_{n}}^{\otimes (n-2)} + \cdots \\ &+ a_{0\cdots 010} |H\rangle_{a_{1},\cdots,a_{n-2}}^{\otimes (n-2)} |V\rangle_{a_{n-1}} |H\rangle_{a_{n}} + a_{0\cdots 001} |H\rangle_{a_{1},\cdots,a_{n-1}}^{\otimes (n-1)} |V\rangle_{a_{n}}. \end{split} \tag{3}$$

Each polarizing beam splitter (PBS) is used to transmit $|H\rangle$ polarization photons and reflect $|V\rangle$ polarization photons. When the signal photons travel to the PBSs, they will be individually split into two spatial modes and then interact with the nonlinear media so that pairs of phase shifts θ and 2θ are induced on the coherent probe beam, respectively. We here introduce a single phase gate $R_n(\theta) = -3n\theta/2$ so as to implement the next X homodyne measurement on the probe beam. $\varphi_m(x), m=1,2,\cdots,n/2$ for even n and $m=1/2,3/2,\cdots,n/2$ for odd n, are phase shifts on the signal photons based on the measured values of x via the classical feed-forward information. At last, at the ports b_i , $i=1,2,\cdots,n$ one may obtain n/2+1 output states for even n or (n+1)/2 states for odd n.

We describe our method in details. For n is even, after the interaction between the photons with Kerr media and followed by the action of the phase gate, the combined system evolves as

$$\begin{split} |\Psi_{\mathrm{ck}}\rangle &= a_{00\cdots00}|H\rangle^{\otimes n}|\alpha\mathrm{e}^{-(n/2)\mathrm{i}\theta}\rangle + (\sum_{\mathrm{Perm}} a_{100\cdots0}|H\rangle^{\otimes (n-1)}|V\rangle)|\alpha\mathrm{e}^{-(n/2-1)\mathrm{i}\theta}\rangle \\ &+ (\sum_{\mathrm{Perm}} a_{110\cdots0}|H\rangle^{\otimes (n-2)}|V\rangle^{\otimes 2})|\alpha\mathrm{e}^{-(n/2-2)\mathrm{i}\theta}\rangle \\ &+ \cdots + (\sum_{\mathrm{Perm}} a_{11\cdots1}\underbrace{00\cdots0}_{n/2}|H\rangle^{\otimes n/2}|V\rangle^{\otimes n/2})|\alpha\rangle \\ &+ \cdots + (\sum_{\mathrm{Perm}} a_{1\cdots100}|H\rangle^{\otimes 2}|V\rangle^{\otimes (n-2)})|\alpha\mathrm{e}^{(n/2-2)\mathrm{i}\theta}\rangle \\ &+ (\sum_{\mathrm{Perm}} a_{11\cdots10}|H\rangle|V\rangle^{\otimes (n-1)})|\alpha\mathrm{e}^{(n/2-1)\mathrm{i}\theta}\rangle \\ &+ a_{11\cdots11}|V\rangle^{\otimes n}|\alpha\mathrm{e}^{(n/2)\mathrm{i}\theta}\rangle. \end{split} \tag{4}$$

In order to create the desired multiphoton entangled states, we here perform an X homodyne measurement ^{38,45} on the probe beam. If the value x of the X homodyne measurement is obtained, then the signal photons become

$$\begin{split} |\Psi_{x}\rangle &= f(x,\alpha) \times \sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{n/2} \underbrace{00\cdots 0}_{n/2}} |H\rangle^{\otimes n/2} |V\rangle^{\otimes n/2} \\ &+ f(x,\alpha\cos\theta) \times (\mathrm{e}^{-\mathrm{i}\phi_{1}(x)} \sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{n/2-1} \underbrace{00\cdots 0}_{n/2-1}} |H\rangle^{\otimes (n/2+1)} |V\rangle^{\otimes (n/2-1)} \\ &+ \mathrm{e}^{\mathrm{i}\phi_{1}(x)} \sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{n/2-1} \underbrace{00\cdots 0}} |H\rangle^{\otimes (n/2-1)} |V\rangle^{\otimes (n/2+1)}) + \cdots \\ &+ f(x,\alpha\cos[(n/2-1)\theta]) \times (\mathrm{e}^{-\mathrm{i}\phi_{n/2-1}(x)} \sum_{\text{Perm}} a_{100\cdots 0} |H\rangle^{\otimes (n-1)} |V\rangle \\ &+ \mathrm{e}^{\mathrm{i}\phi_{n/2-1}(x)} \sum_{\text{Perm}} a_{11\cdots 10} |H\rangle |V\rangle^{\otimes (n-1)}) \\ &+ f(x,\alpha\cos(n\theta/2)) \times (\mathrm{e}^{-\mathrm{i}\phi_{n/2}(x)} a_{00\cdots 00} |H\rangle^{\otimes n} + \mathrm{e}^{\mathrm{i}\phi_{n/2}(x)} a_{11\cdots 11} |V\rangle^{\otimes n}), \end{split}$$
 (5)

where

$$f(x, \alpha \cos(m\theta)) = (2\pi)^{-1/4} e^{-[x-2\alpha \cos(m\theta)]^2/4},$$
(6)

 $m = 0, 1, 2, \dots, n/2$, are respectively Gaussian curves which are associated with the probability amplitudes of the outputs, and

$$\phi_m(x) = \alpha \sin(m\theta)[x - 2\alpha \cos(m\theta)] \mod 2\pi, \tag{7}$$

 $m=1,2,\cdots,n/2$, are respectively phase factors based on the values of the X homodyne measurement. Note that the peaks of these Gaussian distributions locate at $2\alpha \cos(m\theta)$. Thus, the midpoints between two neighboring peaks $x_{m_k}=2\alpha \cos(\theta/2)\cos[(k-1/2)\theta]$ and the distances of two nearby peaks $x_{d_k}=2\alpha\{\cos[(k-1)\theta]-\cos(k\theta)\}$ with $k=1,2,\cdots,n/2$. Obviously, with these n/2 midpoint values x_{m_k} there exist n/2+1 intervals and each interval corresponds to an output state.

We now consider the phase shifts $\varphi_m(x)$. The signal photon evolves as $\hat{b}_i^{\dagger} = \mathrm{e}^{\mathrm{i}\varphi_m(x)}\hat{a}_i^{\dagger}$, $i=1,2,\cdots,n$. A straightforward calculation shows that

$$\varphi_m(x) = \alpha \sin(m\theta)[x - 2\alpha \cos(m\theta)]/m \mod 2\pi.$$
 (8)

After these feed-forward phase shifts have been implemented and the signal photons pass through the PBSs, one can obtain the desired states as follows. Clearly, for $x < x_{m_{n/2}}$ we have

$$a_{00...00}|H\rangle^{\otimes n} + a_{11...1}|V\rangle^{\otimes n};$$
 (9)

for $x_{m_{k+1}} < x < x_{m_k}$, $k = 1, 2, \dots, n/2 - 1$, we obtain the states

$$\sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{n/2-k}}\underbrace{00\cdots 0}_{n/2-k} |H\rangle^{\otimes (n/2+k)} |V\rangle^{\otimes (n/2-k)} + \sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{n/2-k}} \underbrace{00\cdots 0}_{n/2-k} |H\rangle^{\otimes (n/2-k)} |V\rangle^{\otimes (n/2+k)}; \tag{10}$$

and for $x > x_{m_1}$ we obtain the state

$$\sum_{\text{Perm}} a_{\underbrace{11...1}_{n/2}} \underbrace{00...0}_{n/2} |H)^{\otimes n/2} |V\rangle^{\otimes n/2}.$$
(11)

Similarly, for odd *n*, we have

$$\begin{split} |\Psi_{\rm ck}\rangle &= a_{00\cdots00}|H\rangle^{\otimes n}|\alpha {\rm e}^{-(n/2){\rm i}\theta}\rangle + (\sum_{\rm Perm} a_{100\cdots0}|H\rangle^{\otimes (n-1)}|V\rangle)|\alpha {\rm e}^{-(n/2-1){\rm i}\theta}\rangle \\ &+ (\sum_{\rm Perm} a_{110\cdots0}|H\rangle^{\otimes (n-2)}|V\rangle^{\otimes 2})|\alpha {\rm e}^{-(n/2-2){\rm i}\theta}\rangle + \cdots \\ &+ (\sum_{\rm Perm} a_{\underline{11\cdots1}} \underline{00\cdots0}_{(n-1)/2(n+1)/2}|H\rangle^{\otimes (n+1)/2}|V\rangle^{\otimes (n-1)/2})|\alpha {\rm e}^{-1/2{\rm i}\theta}\rangle \\ &+ (\sum_{\rm Perm} a_{\underline{11\cdots1}} \underline{00\cdots0}_{(n+1)/2(n-1)/2}|H\rangle^{\otimes (n-1)/2}|V\rangle^{\otimes (n+1)/2})|\alpha {\rm e}^{1/2{\rm i}\theta}\rangle + \cdots \\ &+ (\sum_{\rm Perm} a_{11\cdots100}|H\rangle^{\otimes 2}|V\rangle^{\otimes (n-2)})|\alpha {\rm e}^{(n/2-2){\rm i}\theta}\rangle \\ &+ (\sum_{\rm Perm} a_{11\cdots10}|H\rangle|V\rangle^{\otimes (n-1)})|\alpha {\rm e}^{(n/2-1){\rm i}\theta}\rangle + a_{11\cdots11}|V\rangle^{\otimes n}|\alpha {\rm e}^{(n/2){\rm i}\theta}\rangle. \end{split} \tag{12}$$

Also,

$$\begin{split} |\Psi_{x}\rangle &= f(x, \alpha \cos(\theta/2)) (\mathrm{e}^{-\mathrm{i}\phi_{1/2}(x)} \sum_{\mathrm{Perm}} a_{\underbrace{11\cdots 1}_{(n-1)/2} \underbrace{00\cdots 0}_{(n-1)/2} |H\rangle^{\otimes (n+1)/2} |V\rangle^{\otimes (n-1)/2} \\ &+ \mathrm{e}^{\mathrm{i}\phi_{1/2}(x)} \sum_{\mathrm{Perm}} a_{\underbrace{11\cdots 1}_{(n+1)/2} \underbrace{00\cdots 0}_{(n-1)/2} |V\rangle^{\otimes (n+1)/2}) \\ &+ f(x, \alpha\cos(3\theta/2)) (\mathrm{e}^{-\mathrm{i}\phi_{3/2}(x)} \sum_{\mathrm{Perm}} a_{\underbrace{11\cdots 1}_{(n-3)/2} \underbrace{00\cdots 0}_{(n-3)/2} |H\rangle^{\otimes (n+3)/2} |V\rangle^{\otimes (n-3)/2} \\ &+ \mathrm{e}^{\mathrm{i}\phi_{3/2}(x)} \sum_{\mathrm{Perm}} a_{\underbrace{11\cdots 1}_{(n+3)/2} \underbrace{00\cdots 0}_{(n-3)/2} |V\rangle^{\otimes (n+3)/2}) \\ &+ \cdots + f(x, \alpha\cos((n/2-1)\theta)) (\mathrm{e}^{-\mathrm{i}\phi_{n/2-1}(x)} \sum_{\mathrm{Perm}} a_{100\cdots 0} |H\rangle^{\otimes (n-1)} |V\rangle \\ &+ \mathrm{e}^{\mathrm{i}\phi_{n/2-1}(x)} \sum_{\mathrm{Perm}} a_{11\cdots 10} |H\rangle |V\rangle^{\otimes (n-1)}) \\ &+ f(x, \alpha\cos(n\theta/2)) (\mathrm{e}^{-\mathrm{i}\phi_{n/2}(x)} a_{00\cdots 00} |H\rangle^{\otimes n} + \mathrm{e}^{\mathrm{i}\phi_{n/2}(x)} a_{11\cdots 11} |V\rangle^{\otimes n}). \end{split} \tag{13}$$

Here, the functions $f(x, \alpha \cos(m\theta))$, phase shifts $\phi_m(x)$ and $\varphi_m(x)$ are approximately the same as those described for even n, except for $m=1/2,3/2,\cdots,n/2$, and then the similar results hold for the midpoints x_{m_k} and the distances x_{d_k} with $k=3/2,5/2,\cdots,n/2$. Of course, with (n-1)/2 midpoint values x_{m_k} there may be (n+1)/2 output states; that is, one can obtain the states $a_{00\cdots 00}|H\rangle^{\otimes n}+a_{11\cdots 11}|V\rangle^{\otimes n}$ for

$$x < x_{m_{n/2}}, \sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{n/2-k}} \underbrace{00\cdots 0}_{n/2+k} |H\rangle^{\otimes (n/2+k)} |V\rangle^{\otimes (n/2-k)} + \sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{n/2+k}} \underbrace{00\cdots 0}_{n/2-k} |H\rangle^{\otimes (n/2-k)} |V\rangle^{\otimes (n/2+k)} \tag{14}$$

for

$$x_{m_{k+1}} < x < x_{m_k}, k = 3/2, \dots, n/2 - 1,$$
 (15)

and

$$\sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{(n-1)/2}\underbrace{00\cdots 0}_{(n-1)/2}} |H\rangle^{\otimes (n+1)/2} |V\rangle^{\otimes (n-1)/2} + \sum_{\text{Perm}} a_{\underbrace{11\cdots 1}_{(n+1)/2}\underbrace{00\cdots 0}_{(n+1)/2}} |H\rangle^{\otimes (n-1)/2} |V\rangle^{\otimes (n+1)/2} \text{ for } x > x_{m_{3/2}}. \tag{16}$$

As an example of the applications of interest for the present scheme, we introduce a class of remarkable multipartite entangled states

$$|\Psi_n^k\rangle = \frac{1}{\sqrt{2}}(|\psi_n^k\rangle + |\tilde{\psi}_n^k\rangle),\tag{17}$$

where

$$|\psi_n^k\rangle = \frac{1}{\sqrt{\binom{n}{n/2 - k}}} \sum_{\text{Perm}} |0\rangle^{\otimes (n/2 + k)} |1\rangle^{\otimes (n/2 - k)}$$
(18)

and

$$|\tilde{\psi}_{n}^{k}\rangle = \frac{1}{\sqrt{\binom{n}{(n/2-k)}}} \sum_{\text{Perm}} |0\rangle^{\otimes (n/2-k)} |1\rangle^{\otimes (n/2+k)}$$
(19)

are two orthonormal states, namely Dicke states^{53,54}. In view of its "catness", the state $|\Psi_n^k\rangle$ can be referred to as *cat-like state*, and especially for k=n/2 it can be expressed as the canonical n-partite Greenberger-Horne-Zeilinger (GHZ) state. In the present scheme, obviously, for $a_{i_1i_2\cdots i_n}=1/\sqrt{2^n}, i_1, i_2, \cdots, i_n=0, 1$, we can obtain these cat-like states with $k=0,1,\cdots,n/2$ for even n and $k=1/2,3/2,\cdots,n/2$ for odd n, where the qubits are encoded with the polarization modes $|H\rangle\equiv|0\rangle$ and $|V\rangle\equiv|1\rangle$. Of course, more generally, we may project out a group of multiphoton entangled states involving generalized Dicke states.

Discussion

There are two models commonly employed in the process of Kerr interaction, single-mode model and continuous-time multi-mode model⁵¹. The former implies that one may ignore the temporal behavior of the optical pulses but the latter is causal, non-instantaneous model involving phase noise. It has been shown that⁵² this causality-induced phase noise will preclude the possibility of high-fidelity CPHASE gates created by the cross-Kerr effect in optical fiber. To solve this problem, one may need to find an optimum response function for the available medium, or to exploit more favorable systems, such as cavitylike systems⁵⁵. After all, the ultimate possible performance of Kerr interaction with a larger system is an interesting open issue. More recently, we note that Feizpour *et al.*⁵⁶ showed the first direct measurement of the cross-phase shift due to single photons. It may be possible to open a door for future studies of nonlinear optics in quantum information processing. In the present scheme, we restrict ourselves to ignoring the phase noise and concentrate mainly on showing a method for exploring multiphoton entangled states in the regime of weak cross-Kerr nonlinearities, i.e. $\theta \ll \pi$.

It is worth noting that, there are only small phase shifts θ and 2θ instead of a series of related functions of the number of photons in the process of interaction with Kerr nonlinearities. This implies that the present scheme is quite tractable especially for creating entangled states with a larger number of photons. In addition, the error probabilities ε_k are $\mathrm{erfc} \left(x_{d_k} / 2 \sqrt{2} \right) / 2$, which come from small overlaps between two neighboring curves. Considering the distances of two nearby peaks $x_{d_k} \approx (2k-1)\alpha\theta^2$ with $k=1,2,\cdots,n/2$ for even n and $k=3/2,5/2,\cdots,n/2$ for odd n, the maximal error probability $\varepsilon_{\max} = \mathrm{erfc} \left(\alpha\theta^2/2\sqrt{2}\right) / 2$, which is exactly the result described by Nemoto and Munro in Obviously, the error probabilities in our scheme are no more than that one even when the number of photons is large. Therefore, by choosing an appropriate coherent probe beam the error probability can be reduced to as low a level as desired and then the present scheme may be realized in a nearly deterministic manner.

In summary, based on linear optics and weak nonlinearities we have shown a fruitful method for exploring a class of multiphoton entangled states, the *generalized cat-like states*. Evidently, three aspects are noteworthy in the present framework. First, since there are no large phase shifts in the interacting process with weak Kerr nonlinearities, our scheme is more feasible compared with the previous schemes. Second, the system is measured only once with a small error probability and it means that the present scheme might be realized near deterministically. Finally, the fruitful architecture allows us to explore a group of multiphoton entangled states involving a large number of photons, i.e., to produce entangled states approaching the macroscopic domain.

References

- 1. Horodecki, R., Horodecki, P., Horodecki, M. & Horodecki, K. Quantum entanglement. Rev. Mod. Phys. 81, 865–942 (2009).
- 2. Gühne, O. & Tóth, G. Entanglement detection. Phys. Rep. 474, 1-75 (2009).
- 3. Yan, F. L., Gao, T. & Chitambar, E. Two local observables are sufficient to characterize maximally entangled states of N qubits. Phys. Rev. A 83, 022319 (2011).
- 4. Gao, T., Yan, F. L. & van Enk, S. J. Permutationally invariant part of a density matrix and nonseparability of N-qubit states. Phys. Rev. Lett. 112, 180501 (2014).
- 5. Ekert, A. K. Quantum cryptography based on Bell's theorem. Phys. Rev. Lett. 67, 661-663 (1991).
- 6. Bennett, C. H. et al. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. 70, 1895–1899 (1993).
- 7. Long, G. L. & Liu, X. S. Theoretically efficient high-capacity quantum-key-distribution scheme. Phys. Rev. A 65, 032302 (2002).
- 8. Deng, F. G., Long, G. L. & Liu, X. S. Two-step quantum direct communication protocol using the Einstein-Podolsky-Rosen pair block. *Phys. Rev. A* 68, 042317 (2003).
- 9. Nielsen, M. A. & Chuang, I. L. Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).
- Waks, E. & Monroe, C. Protocol for hybrid entanglement between a trapped atom and a quantum dot. Phys. Rev. A 80, 062330 (2009).
- 11. Gonzalez-Tudela, A. et al. Entanglement of two qubits mediated by one-dimensional plasmonic waveguides. Phys. Rev. Lett. 106, 020501 (2011).
- 12. Shwa, D., Cohen, R. D., Retzker, A. & Katz, N. Heralded generation of Bell states using atomic ensembles. *Phys. Rev. A* 88, 063844 (2013).
- 13. Casabone, B. et al. Heralded entanglement of two ions in an optical cavity. Phys. Rev. Lett. 111, 100505 (2013).
- 14. Feng, G. R., Long, G. L. & Laflamme, R. Experimental simulation of anyonic fractional statistics with an NMR quantum-information processor. *Phys. Rev. A* 88, 022305 (2013).
- Sheng, Y. B., Deng, F. G. & Long, G. L. Complete hyperentangled-Bell-state analysis for quantum communication. Phys. Rev. A 82, 032318 (2010).
- 16. Liu, S. P., Li, J. H., Yu, R. & Wu, Y. Achieving three-dimensional entanglement between two spatially separated atoms by using the quantum Zeno effect. *Phys. Rev. A* 87, 062316 (2013).
- 17. Scully, M. O. & Zubairy, M. S. Quantum Optics (Cambridge University Press, Cambridge, 1997).
- 18. Knill, E., Laflamme, R. & Milburn, G. J. A scheme for efficient quantum computation with linear optics. Nature 409, 46–52 (2001).
- 19. Kok, P. et al. Linear optical quantum computing with photonic qubits. Rev. Mod. Phys. 79, 135-174 (2007).
- 20. Pan, J. W. et al. Multiphoton entanglement and interferometry. Rev. Mod. Phys. 84, 777-838 (2012).
- 21. Li, J. H. *et al.* Optical-frequency-comb generation and entanglement with low-power optical input in a photonic molecule. *Phys. Rev.* A **90**, 033830 (2014).
- 22. Lü, X. Y. et al. Quantum-criticality-induced strong Kerr nonlinearities in optomechanical systems. Sci. Rep. 3, 2943 (2013).
- 23. Ren, B. C., Wang, G. Y. & Deng, F. G. Universal hyperparallel hybrid photonic quantum gates with dipole-induced transparency in the weak-coupling regime. *Phys. Rev. A* **91**, 032328 (2015).

- 24. Sheng, Y. B. & Deng, F. G. Deterministic entanglement purification and complete nonlocal Bell-state analysis with hyperentanglement. *Phys. Rev. A* 81, 032307 (2010).
- 25. Sheng, Y. B., Zhou, L., Zhao, S. M. & Zheng, B. Y. Efficient single-photon-assisted entanglement concentration for partially entangled photon pairs. *Phys. Rev. A* 85, 012307 (2012).
- Deng, F. G. Optimal nonlocal multipartite entanglement concentration based on projection measurements. Phys. Rev. A 85, 022311 (2012).
- 27. Sheng, Y. B. & Zhou, L. Deterministic entanglement distillation for secure double-server blind quantum computation. Sci. Rep. 5, 7815 (2015).
- 28. Sheng, Y. B. & Zhou, L. Two-step complete polarization logic Bell-state analysis. Sci. Rep. 5, 13453 (2015).
- 29. Kwiat, P. G. et al. New high-intensity source of polarization-entangled photon pairs. Phys. Rev. Lett. 75, 4337-4341 (1995).
- 30. Simon, C. & Bouwmeester, D. Theory of an entanglement laser. Phys. Rev. Lett. 91, 053601 (2003).
- 31. Bouwmeester, D. et al. Observation of three-photon Greenberger-Horne-Zeilinger entanglement. Phys. Rev. Lett. 82, 1345–1349 (1999)
- 32. Pan, J. W. et al. Experimental demonstration of four-photon entanglement and high-fidelity teleportation. Phys. Rev. Lett. 86, 4435–4438 (2001).
- 33. Eisenberg, H. S. et al. Quantum entanglement of a large number of photon. Phys. Rev. Lett. 93, 193901 (2004).
- 34. Walther, P., Aspelmeyer, M. & Zeilinger, A. Heralded generation of multiphoton entanglement. Phys. Rev. A 75, 012313 (2007).
- 35. Wei, T. C., Lavoie, J. & Kaltenbaek, R. Creating multiphoton-polarization bound entangled states. Phys. Rev. A 83, 033839 (2011).
- 36. Tichy, M. C., Mintert, F. & Buchleitner, A. Limits to multipartite entanglement generation with bosons and fermions. *Phys. Rev. A* 87, 022319 (2013).
- 37. He, Y. Q., Ding, D., Yan, F. L. & Gao, T. Preparation and purification of four-photon Greenberger-Horne-Zeilinger state. *J. Phys. B: At. Mol. Opt. Phys.* 48, 055501 (2015).
- 38. Barrett, S. D. *et al.* Symmetry analyzer for nondestructive Bell-state detection using weak nonlinearities. *Phys. Rev. A* **71**, 060302 (2005).
- Munro, W. J., Nemoto, K., Beausoleil, R. G. & Spiller, T. P. High-efficiency quantum-nondemolition single-photon-number-resolving detector. Phys. Rev. A 71, 033819 (2005).
- 40. Nemoto, K. & Munro, W. J. Nearly deterministic linear optical controlled-NOT gate. Phys. Rev. Lett. 93, 250502 (2004).
- 41. Munro, W. J., Nemoto, K. & Spiller, T. P. Weak nonlinearities: a new route to optical quantum computation. New J. Phys. 7, 137 (2005).
- 42. Kok, P. Effects of self-phase-modulation on weak nonlinear optical quantum gates. Phys. Rev. A 77, 013808 (2008).
- 43. Ding, D., Yan, F. L. & Gao, T. Preparation of km-photon concatenated Greenberger-Horne-Zeilinger states for observing distinctive quantum effects at macroscopic scales. J. Opt. Soc. Am. B 30, 3075–3078 (2013).
- 44. Ding, D., Yan, F. L. & Gao, T. Entangler and analyzer for multiphoton Greenberger-Horne-Zeilinger states using weak nonlinearities. *Sci. China-Phys. Mech. Astron.* **57**, 2098–2103 (2014).
- 45. He, Y. Q., Ding, D., Yan, F. L. & Gao, T. Exploration of photon-number entangled states using weak nonlinearities. *Opt. Express* 23, 21671–21677 (2015).
- 46. Imoto, N., Haus, H. A. & Yamamoto, Y. P. Quantum nondemolition measurement of the photon number via the optical Kerr effect. *Phys. Rev. A* 32, 2287–2292 (1985).
- 47. Rokhsari, H. & Vahala, K. J. Observation of Kerr nonlinearity in microcavities at room temperature. Opt. Lett. 30, 427-429 (2005).
- 48. Matsuda, N. et al. Observation of optical-fibre Kerr nonlinearity at the single-photon level. Nature Photon. 3, 95-98 (2009).
- Schmidt, H. & Imamoğlu, A. Giant Kerr nonlinearities obtained by electromagnetically induced transparency. Opt. Lett. 21, 1936–1938 (1996).
- 50. Lukin, D. & Imamoğlu, A. Controlling photons using electromagnetically induced transparency. Nature 413, 273-276 (2001).
- 51. Shapiro, J. H. & Razavi, M. Continuous-time cross-phase modulation and quantum computation. New J. Phys. 9, 16 (2007).
- Dove, J., Chudzicki, C. & Shapiro, J. H. Phase-noise limitations on single-photon cross-phase modulation with differing group velocities. Phys. Rev. A 90, 062314 (2014).
- 53. Dicke, R. H. Coherence in spontaneous radiation processes. Phys. Rev. 93, 99-110 (1954).
- 54. Thiel, C. *et al.* Generation of symmetric Dicke states of remote qubits with linear optics. *Phys. Rev. Lett.* **99**, 193602 (2007).
- 55. Chudzicki, C., Chuang, I. L. & Shapiro, J. H. Deterministic and cascadable conditional phase gate for photonic qubits. *Phys. Rev. A* 87, 042325 (2013).
- 56. Feizpour, A., Hallaji, M., Dmochowski, G. & Steinberg, A. M. Observation of the nonlinear phase shift due to single post-selected photons. *Nature Phys.* 11, 905–909 (2015).

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Author Contributions

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Additional Information

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