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Universal Nonadiabatic Geometric Gates in Two-Qubit Decoherence-Free Subspaces

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Geometric quantum computation in decoherence-free subspaces is of great practical importance because it can protect quantum information from both control errors and collective dephasing. However, previous proposed schemes have either states leakage or four-body interactions problems. Here, we propose a feasible scheme without these two problems. Our scheme is realized in two-qubit decoherence-free subspaces. Since the Hamiltonian we use is generic, our scheme looks promising to be demonstrated experimentally in different systems, including superconducting charge qubits.

Decoherence has been regarded as one main practical obstacle in building a quantum computer. It makes the desired coherence of the system collapse and hence reduces the efficiency of quantum computation. So far, three main strategies, namely, quantum error corrections, decoherence-free subspaces (DFSs) and dynamical decoupling, have been proposed to combat decoherence. As one promising way to avoid quantum decoherence, DFSs can be constructed if the interaction between the system and its environment has some symmetry structures[1]. The basic idea of DFSs is to encode information in a subspace of the system, over which the dynamics is unitary. DFSs have been experimentally realized in many physical systems[2–11].

Besides the requirement of coherence protection, one also needs to achieve highly accurate control to enact quantum computation. The requirement of highly accurate control has been regarded as another main practical obstacle in building a quantum computer. Fortunately, this requirement can be relaxed by using quantum computing paradigms with built-in fault tolerance. Geometric quantum computation (GQC) is one such quantum computing paradigm. It exploits different types of quantum holonomies and provides a resilient way of information processing through all-geometric control. Until now, GQC has attracted much attention and many efforts, both in theory and in practice[12–34].

To overcome both decoherence and control errors, the schemes of GQC in DFSs have been proposed[27–34]. Although impressive progresses have been made in this field, two problems still persist. One problem is the states leakage problem. This problem happens in two different ways. In one way, the evolutions generated by the geometric gates can drive the logical states out of the DFSs and hence spoil the predicted protection against decoherence[28,29]. In another way, when extra degrees of freedom, like center-of-mass vibrational mode of ions[30,31] or cavity mode[32,33], are used to generate quantum holonomies, although the logical states are always protected by the DFSs, the extra degrees of freedom are affected by the environment. So, geometric gates generated in this way can only be partially protected by the DFSs. Another problem is the four-body interactions problem. There exist GQC-DFS schemes[27,34] without the states leakage problem. However, four-body interactions are used to build the logical entangling gates in these schemes. Considering four-body interactions are very challenge in experiment, this kind of interactions should be avoided.

In this article, we propose a universal set of nonadiabatic geometric gates without the states leakage and four-body interactions problems. Specifically, we use the tunable *XXZ* Hamiltonian to realize a universal set of nonadiabatic geometric gates in two-qubit DFSs. This is the major merit of our scheme. We also investigate how to make our scheme resilient to arbitrary collective decoherence. Since the *XXZ* Hamiltonian is generic, our scheme looks promising to be demonstrated experimentally in different systems and we use superconducting charge qubits as an illustration.

Results

The *XXZ* model. Before proceeding further, we explain how nonadiabatic quantum holonomies arise. Consider a M-dimensional subspace $\mathcal{S}(0)$ spanned by the orthonormal vectors $\{|\phi_\mu(0)\rangle\}$. The evolution operator is a holonomic matrix acting on $\mathcal{S}(0)$ if $|\phi_\mu(t)\rangle$ satisfy the following conditions[21,34]: (i) $\sum_{\mu=1}^{M}|\phi_\mu(T_0)\rangle\langle\phi_\mu(T_0)| = \sum_{\mu=1}^{M}|\phi_\mu(0)\rangle\langle\phi_\mu(0)|$; (ii) $\langle\phi_\mu(t)|\mathcal{H}(t)|\phi_\nu(t)\rangle=0$, μ, ν = 1, …, M. In the above, $\mathcal{H}(t)$ is the Hamiltonian of the system, T_0 is the evolution period, and $|\phi_\mu(t)\rangle=\mathbf{T}\exp\left(-i\int_0^t\mathcal{H}(t_1)dt_1\right)|\phi_\mu(0)\rangle$, where $\mathbf{T}$ is the time ordering operator. While condition (i) means the subspace $\mathcal{S}(0)$ completes a cyclic evolution, condition (ii) means this cyclic evolution is purely geometric.

Let us now elucidate our physical model. Suppose we can handle the following Hamiltonian

$$H=\sum_k \vec{f}_k\cdot\vec{\sigma}_k+\sum_{k<l}\left(J_{kl}^{xy}\sigma_{kl}^{xy}+J_{kl}^{z}\sigma_k^z\sigma_l^z\right), \tag{1}$$

where $\vec{f}_k$ is the effective local field applied to the kth physical qubit, J_{kl}^{xy} and J_{kl}^{z} are coupling parameters, σ_α^β is the Pauli β operator acting on the αth physical qubit and $\sigma_{kl}^{xy}=\sigma_k^x\sigma_l^x+\sigma_k^y\sigma_l^y$. The Hamiltonian in Eq. (1) is the tunable *XXZ* Hamiltonian and can be realized in different systems. For example, the physical qubit can be a superconducting island which is coupled to a ring by two symmetric Josephson junctions and the states $|0\rangle$ and $|1\rangle$ are respectively the two charge states near the charging energy degeneracy point.

For the *XXZ* model, the major source of decoherence is dephasing. So, if two physical qubits are put close, the interaction Hamiltonian between these two physical qubits and its environment can be described by

$$H_I=(\sigma^z\otimes I+I\otimes\sigma^z)\otimes B, \tag{2}$$

where σ^z and I are respectively the Pauli Z and identity operators acting on the corresponding physical qubit and B is an arbitrary environment operator. The symmetry of the interaction implies there exists a two-dimensional DFS

$$\mathcal{S}=\mathrm{Span}\{|0\rangle\otimes|1\rangle,\ |1\rangle\otimes|0\rangle\}, \tag{3}$$

where $|0\rangle$ and $|1\rangle$ are the eigenstates of the Pauli Z operator of the corresponding physical qubit. We encode a logical qubit in the subspace $\mathcal{S}$ and respectively denote the logical states $|0\rangle_L$ and $|1\rangle_L$ as

$$|0\rangle_L=|01\rangle,\quad |1\rangle_L=|10\rangle. \tag{4}$$

For convenience, we respectively write $|0\rangle\otimes|1\rangle$ and $|1\rangle\otimes|0\rangle$ as $|01\rangle$ and $|10\rangle$. In the whole article, we use the subscript L to denote that the states and the operators are respectively logical states and logical operators. In the following paragraphs, we will show how to use the tunable *XXZ* Hamiltonian to realize a universal set of non-adiabatic geometric gates in two-qubit DFSs.

One-logical-qubit gates. As is well known, to achieve a universal set of quantum logical gates, one needs to realize two noncommutative one-logical-qubit gates (not two fixed one-logical-qubit gates but two classes of one-logical-qubit gates) and one nontrivial entangling logical gate. Firstly, we demonstrate how to realize two non-commutative one-logical-qubit gates. For convenience, the corresponding two physical qubits are respectively denoted as 1 and 2. We consider the following one-logical-qubit gate

$$U_1(T_1,0)=e^{-i\int_{\tau_1}^{T_1}H'_1(t)dt}e^{-i\int_0^{\tau_1}H_1(t)dt}. \tag{5}$$

In the above equation, τ_1 and T_1 are respectively an arbitrary intermediate time and the evolution period, the Hamiltonians $H_1(t)$ and $H'_1(t)$ respectively read

$$\begin{aligned}H_1(t)&=J_1(t)\left(\cos\phi\sigma_{12}^{xy}+\sin\phi\sigma_1^z\right),\\ H'_1(t)&=J'_1(t)\sigma_{12}^{xy},\end{aligned} \tag{6}$$

where ϕ is an arbitrary parameter, $J_1(t)$ and $J'_1(t)$ are controllable coupling parameters and satisfy the conditions

$$\int_0^{\tau_1}J_1(t)dt=\int_{\tau_1}^{T_1}J'_1(t)dt=\frac{\pi}{2}. \tag{7}$$

If one ignores the global phase, the logical gate $U(T_1, 0)$ can be written as

$$U_1(T_1,0)=e^{-i\phi Y_L}, \tag{8}$$

where Y_L is the logical Pauli Y operator and can be written as $Y_L = -i|0\rangle_L\langle 1|_L + i|1\rangle_L\langle 0|_L$.

One can verify that $U_1(T_1, 0)$ has both decoherence-free and geometric properties. $U_1(T_1, 0)$ has decoherence-free property because $\mathcal{S}$ is an invariant subspace of the evolution operator $U_1(t, 0)$. The geometric property of $U_1(T_1, 0)$ can be verified by using the holonomic conditions (i) and (ii). Consider the eigenstates of the logical operator Y_L. According to Eq. (8), the two one-dimensional subspaces respectively complete cyclic evolutions and the evolution operators are phases. Then, condition (i) is satisfied. By calculating the matrix elements of $H_1(t)$ and $H'_1(t)$ in the basis of the eigenstates of Y_L, one can verify condition (ii) is satisfied . Since both conditions (i) and (ii) are satisfied, the accumulated phases ϕ and $-\phi$ are geometric and hence $U_1(T_1, 0)$ is a geometric gate. The geometric property of $U_1(T_1, 0)$ can also be illustrated by Fig. 1 in which the Hamiltonians $H_1(t)$ and $H'_1(t)$ drive the eigenstates of Y_L from one starting pole to the opposite pole and then back to the initial pole. Since these trajectories are connected geodesics, the dynamical phases are zero and the logical gate $U_1(T_1, 0)$ is geometric.

Next, we illustrate how to realize the second one-logical-qubit gate. One can see that the logical states $|0\rangle_L$ and $|1\rangle_L$ (the eigenstates

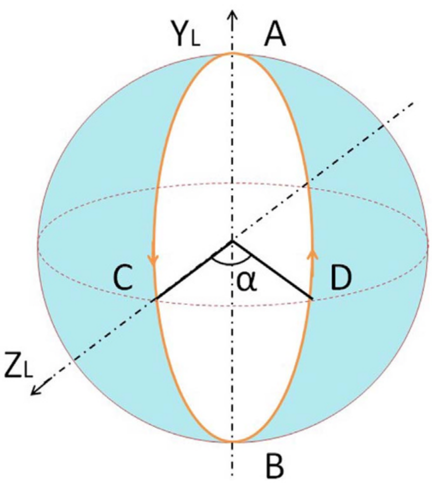

Figure 1 | Evolutions of $U_1(T_1, 0)$ and $U_2(T_2, 0)$ in logical Bloch sphere. For logical gate $U_1(T_1, 0)$, the $+1$ eigenstate of Y_L represented by point A completes a cyclic evolution on $ACBDA$ and the accumulated geometric phase is proportional to the solid angle α; for logical gate $U_2(T_2, 0)$, the $+1$ eigenstate of Z_L represented by point C completes a cyclic evolution on $CBDAC$ and the accumulated geometric phase is also proportional to the solid angle α. Similar evolutions exist for the -1 eigenstates of Y_L and Z_L.

of Z_L) are respectively evolved to the logical states $\frac{1}{\sqrt{2}}(|0\rangle_L - i|1\rangle_L)$ and $\frac{1}{\sqrt{2}}(|0\rangle_L + i|1\rangle_L)$ (the eigenstates of Y_L) under the action of the operation $\exp\left(-i\frac{\pi}{4}\sigma_{12}^{xy}\right)$ while the inverse evolution can be realized by using the operation $\exp\left(i\frac{\pi}{4}\sigma_{12}^{xy}\right)$. So, we realize the second one-logical-qubit gate by first using the operation $\exp\left(-i\frac{\pi}{4}\sigma_{12}^{xy}\right)$ to evolve the eigenstates of Z_L to the eigenstates of Y_L, and then using the operation $U_1(T_1, 0)$ to evolve the eigenstates of Y_L cyclicly, and then using the operation $\exp\left(i\frac{\pi}{4}\sigma_{12}^{xy}\right)$ to evolve the eigenstates of Y_L back to the eigenstates of Z_L. Specifically, we realize the following one-logical-qubit gate

$$U_2(T_2,0) = e^{-i\int_{\tau_2'}^{T_2} H_1'(t)dt} e^{-i\int_{\tau_2}^{\tau_2'} H_1(t)dt} e^{-i\int_0^{\tau_2} H_1'(t)dt}, \tag{9}$$

where τ_2, τ_2' and T_2 are respectively arbitrary intermediate time parameters and the evolution period, the Hamiltonians $H_1(t)$ and $H_1'(t)$ are described by Eq. (6). Here, the control coupling parameters $J_1(t)$ and $J_1'(t)$ need to satisfy the conditions

$$\int_0^{\tau_2} J_1'(t)dt = \int_{\tau_2'}^{T_2} J_1'(t)dt = \frac{\pi}{4}, \quad \int_{\tau_2}^{\tau_2'} J_1(t)dt = \frac{\pi}{2}. \tag{10}$$

If one ignores the global phase, the second one-logical-qubit gate can be written as

$$U_2(T_2,0) = e^{i\phi Z_L}, \tag{11}$$

where Z_L is the logical Pauli Z operator and can be written as $Z_L = |0\rangle_L\langle 0|_L - |1\rangle_L\langle 1|_L$.

The illustration of the decoherence-free and geometric properties of $U_2(T_2, 0)$ is similar to that of $U_1(T_1, 0)$. The logical gate $U_2(T_2, 0)$ is always protected by the subspace $\mathcal{S}$ because $\mathcal{S}$ is an invariant subspace of the evolution operator $U_2(t, 0)$. By using the holonomic conditions (i) and (ii), one can verify that the accumulated phases of the logical states $|0\rangle_L$ and $|1\rangle_L$ are geometric phases and hence the logical gate $U_2(t, 0)$ is geometric gate. The evolution of the logical gate $U_2(T_2, 0)$ can also be illustrated by Fig. 1 in which the logical eigenstates of Z_L respectively act as the starting points of the evolution and the accumulate geometric phases are proportional to the solid angle α.

In the above, we have realized two two noncommutative one-logical-qubit gates $U_1(T_1,0) = e^{-i\phi Y_L}$ and $U_2(T_2,0) = e^{i\phi Z_L}$. It is well known that arbitrary rotations around two orthogonal axes are sufficient to realize any one-logical-qubit rotation. Since the rotation axes Y and Z are orthogonal and the parameter ϕ can be chosen arbitrarily, our scheme is sufficient to realize any one-logical-qubit rotation.

Two-logical-qubit gate. We now demonstrate how to realize a nontrivial entangling logical gate. For convenience, the corresponding four physical qubits are respectively denoted as 1, 2, 3, and 4. The DFS of this four-qubit system reads

$$\mathcal{S}' = \mathrm{Span}\{|00\rangle_L, |01\rangle_L, |10\rangle_L, |11\rangle_L\}. \tag{12}$$

The nontrivial entangling logical gate we realize reads

$$U(T,0) = e^{-i\int_\tau^T H'(t)dt} e^{-i\int_0^\tau H(t)dt}. \tag{13}$$

In the above, τ and T respectively being an arbitrary intermediate time and the evolution period, the Hamiltonians $H(t)$ and $H'(t)$ respectively read

$$\begin{aligned} H(t) &= J(t)\left(\cos\theta\sigma_{12}^{xy} + \sin\theta\sigma_2^z\sigma_3^z\right), \\ H'(t) &= J'(t)\sigma_{12}^{xy}, \end{aligned} \tag{14}$$

where θ is an arbitrary parameter, $J(t)$ and $J'(t)$ are controllable coupling parameters and satisfy the conditions

$$\int_0^\tau J(t)dt = \int_\tau^T J'(t)dt = \frac{\pi}{2}. \tag{15}$$

If one ignores the global phase, the logical gate generated by the Hamiltonians $H(t)$ and $H'(t)$ can be written as

$$U(T,0) = e^{i\theta Y_L \otimes Z_L}. \tag{16}$$

One can see that $U(T, 0)$ is a nontrivial entangling logical gate if $\sin\theta$ and $\cos\theta$ are nonzero.

The decoherence-free and geometric properties of $U(T, 0)$ can be illustrated as follows. The logical gate $U(T, 0)$ is always protected by the subspace $\mathcal{S}'$ because $\mathcal{S}'$ is an invariant subspace of the evolution operators $U(t, 0)$. In other words, the logical gate $U(T, 0)$ has decoherence-free property. To ensure $U(T, 0)$ is a geometric gate, one needs to use the holonomic conditions (i) and (ii). Letting $U_L = [I_L - i(X_L + Y_L - Z_L)]/2$, $|\bar{0}\rangle_L = U_L|0\rangle_L$ and $|\bar{1}\rangle_L = U_L|1\rangle_L$. In the basis $\{|\bar{0}\bar{0}\rangle_L, |\bar{0}\bar{1}\rangle_L, |\bar{1}\bar{0}\rangle_L, |\bar{1}\bar{1}\rangle_L\}$, the entangling logical gate $U(T, 0)$ has the following form

$$U(T,0) = \begin{pmatrix} U_1 & \mathbb{O} \\ \mathbb{O} & U_2 \end{pmatrix}, \tag{17}$$

where $\mathbb{O}$ is a two-dimensional zero matrix, U_1 and U_2 respectively read

$$U_1 = \begin{pmatrix} -\cos\theta & -i\sin\theta \\ -i\sin\theta & -\cos\theta \end{pmatrix}, \quad U_2 = \begin{pmatrix} -\cos\theta & i\sin\theta \\ i\sin\theta & -\cos\theta \end{pmatrix}. \tag{18}$$

We are splitting $\mathcal{S}'$ into the following two orthogonal subspaces

$$\mathcal{S}_1 = \mathrm{Span}\{|\bar{0}\bar{0}\rangle_L, |\bar{0}\bar{1}\rangle_L\}, \mathcal{S}_2 = \mathrm{Span}\{|\bar{1}\bar{0}\rangle_L, |\bar{1}\bar{1}\rangle_L\}. \tag{19}$$

According to Eq. (17), the subspaces $\mathcal{S}_1$ and $\mathcal{S}_2$ undergo cyclic evolutions and the evolution operators are U_1 and U_2, respectively. Thus, the matrices U_1 and U_2 satisfy condition (i). The second condition of U_1 can be reduced to $\langle\bar{0}\bar{k}|_L H(t)|\bar{0}\bar{k}'\rangle_L = 0$ and $\langle\bar{1}\bar{k}|_L H'(t)|\bar{1}\bar{k}'\rangle_L = 0$ because $H(t)$ and $H'(t)$ respectively commute with their evolution operators and $\exp\left(-i\int_0^\tau H(t)dt\right)\mathcal{S}_1 = \mathcal{S}_2$, where k, $k' \in \{0, 1\}$. So, the matrix U_1 satisfies condition (ii). Similarly, one can verify that the matrix U_2 also satisfies condition (ii). Since the matrices U_1 and U_2 satisfy both conditions (i) and (ii), they are holonomic matrices. Observing that $U(T, 0)$ is direct sum of U_1 and U_2, the logical gate $U(T, 0)$ has the required geometric property. The geometric property of $U(T, 0)$ may also be illustrated by Fig. 1 if one replaces the semi-great circles by two-dimensional holonomies.

Discussion

We have succeeded in constructing two noncommutative one-logical-qubit gates and one nontrivial entangling logical gate by using the tunable *XXZ* Hamiltonian. Since two-qubit DFSs (or the direct product of two-qubit DFSs) are invariant subspaces of the proposed geometric gates, these logical gates are always protected by the two-qubits DFSs. The one-logical-qubit gates $U_1(T_1, 0)$ and $U_2(T_2, 0)$ are conditional geometric gates. Specifically, for each logical gate, two orthogonal logical states respectively complete cyclic evolutions and each gets a geometric phase. The entangling logical gate $U(T, 0)$ can be seen as a non-Abelian generalization of the conditional geometric gate. The reason is that when realizing $U(T, 0)$, two orthogonal two-dimensional subspaces respectively complete cyclic evolutions and each subspace gets a nontrivial non-Abelian holonomy.

Since dephasing is the major source of decoherence, the Hamiltonian in Eq. (2) describes the interaction between the system and its environment quite well. To make our scheme more robust, we here discuss how to make our scheme resilient to arbitrary collective decoherence. As is well known that arbitrary collective decoherence includes collective dephasing and collective dissipation. Since collective dephasing can be overcome by using DFSs, one only need to consider how to reduce collective dissipation. To do this, one can combine our GQC-DFS scheme with dynamical decoupling. Specifically, one can use the simultaneous Pauli Z pulse $\sigma^z \otimes \sigma^z$ to protect one logical qubit against dissipation. It should be noted that the decoupling sequences for different logical qubits do not need to be simultaneous. Since the simultaneous Pauli Z pulse commutes with the Hamiltonians of our scheme and preserves the two-qubit DFSs, the advantages of quantum holonomies, DFSs and dynamical decoupling can be usefully combined. In other words, our scheme can be resilient to both control errors and arbitrary collective decoherence.

The XXZ Hamiltonian can be demonstrated in different systems and we here briefly illustrate how to use the superconducting charge qubits (SCQs) to realize our scheme. For each SCQ, a superconducting island (Cooper-pair box) is coupled to a ring by two symmetric Josephson junctions characterized by coupling energy E_{J0} and capacitance C_J. We operate the system in the charging regime, then the extra Cooper-pairs n in the box is a good quantum number. Near the charging energy degeneracy point, only two charge states ($n = 0, 1$) play a dominant role and we use this two charge states as the physical qubit states[35].

For the kth SCQ, a control gate voltage V_{gk} is applied to the box through a capacitance C_k and an external magnetic flux Φ_k is used to modulate the Josephson coupling energy. The Hamiltonian of the kth SCQ reads

$$H_k = \varepsilon_k \sigma_k^z - E_{Jk} \sigma_k^x, \tag{20}$$

where the charging energy is $\varepsilon_k = E_{ck}(1 - 2n_{gk})/2$ and the effective Josephson coupling energy is $E_{Jk} = E_{J0}\cos(\pi\Phi_k/\Phi_0)$, with the charging energy scale being $E_{ck} = 2e^2/(C_k + 2C_J)$, the corresponding gate charge being $n_{gk} = C_k V_{gk}/2e$ and the fluxon being $\Phi_0 = h/2e$.

We realize a logical qubit by connecting two SCQs k and k' with a superconducting quantum interference device (SQUID), see Fig. 2(a). The SQUID is pierced by a magnetic flux $\Phi_{kk'}$ which can be used to modulate the Josephson coupling. Choosing small junction capacitances of the SQUID, the electrostatic energy between boxes k and k' is much smaller than the corresponding Josephson energy, and hence the effect of electrostatic coupling energy can be ignored[36]. The interaction Hamiltonian can be written as

$$H_{kk'} = -E_{Jkk'} \sigma_{kk'}^{xy}, \tag{21}$$

where the tunable Josephson coupling is $E_{Jkk'} = E_{Jkk'}^{(0)} \cos(\Phi_{kk'}/\Phi_0)/2$.

To implement nonlocal operations, different logical qubits are coupled by a variable electrostatic transformer C_m, see Fig. 2(b). C_{ml} ($C_{ml'}$) is one part of C_m, E_c and E_J are the charging and Josephson coupling energies, and V_c is the voltage[37]. If we respectively denote the two logical qubits as a and b, the corresponding four physical qubits as a_1, a_2, b_1 and b_2, the interaction Hamiltonian reads

$$H_{ab} = \frac{\Delta_m}{2}\cos(2\pi q_0)\sigma_{a_2}^z \sigma_{b_1}^z, \tag{22}$$

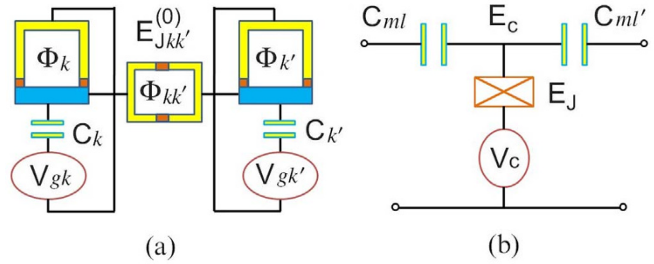

Figure 2 | The demonstration using SCQs. (a) A sketch of a logical qubit which consist of two SCQs. The two SCQs are connected with a SQUID. (b) The equivalent circuit of the variable electrostatic transformer C_m.

where Δ_m is the characteristic energy gap of the transformer junction and the induced charge is $q_0 = q_c + \left[\left(n_{ga_2} - 1/2\right) + \left(n_{gb_1} - 1/2\right)\right]/4$, with q_c being $3V_c(C_{ma_2} + C_{mb_1})/8e$. It is noting that we have assumed C_m has a symmetric structure and the capacitance ratio $c = C_{ma_2}/C_{\Sigma\, a_2} = C_{mb_1}/C_{\Sigma\, b_1} = 1/4$, where the total capacitance is $C_{\Sigma\, a_2(\Sigma\, b_1)} = 2C_j + C_{a_2(b_1)} + C_{ma_2(mb_1)}$. Under this assumption, the charging energy shift is zero.

By modulating the Hamiltonian operators H_k, $H_{kk'}$ and H_{ab}, our scheme can be readily realized. We always set $\Phi_k = \Phi_0/2$ to ensure the term $-E_{Jk}\sigma_k^x$ is zero. To realize the one-logical-qubit gates $U_1(T_1, 0)$ and $U_2(T_2, 0)$, the coupling between logical qubits and the charging energy of the second physical qubit are respectively switched off by setting $q_0 = 1/4$ and $n_{g^2} = 1/2$. Then the Hamiltonians $H_1(t)$ and $H_1'(t)$ can be realized by adjusting the voltage V_{g^1} and the magnetic flux Φ_{12}. To realize the entangling logical gate $U(T, 0)$, one needs to set $n_{g^1} = n_{g^2} = n_{g^3} = n_{g^4} = 1/2$, $\Phi_{34} = \Phi_0/2$ and then adjust q_0 and Φ_{12}.

In summary, we have put forward a universal set of nonadiabatic geometric gates by using only two-qubit DFSs. Specifically, we have used the tunable XXZ Hamiltonian to realize two noncommutative one-logical-qubit geometric gates and one nontrivial logical entangling geometric gate and made the evolutions of the gates fall entirely into the encoded two-qubit DFSs. Comparing with previous proposed GQC-DFS schemes, the major merit of our scheme is that our GQC-DFS scheme avoids the states leakage and the four-body interactions problems. We also have investigated how to use dynamical decoupling to make our scheme resilient to arbitrary collective decoherence. Since the Hamiltonian operators in our scheme are generic, our scheme seems promising in different experimental implementations and we have illustrated the realization of our scheme in the superconducting charge qubits. We hope our scheme can shed light on the applications of GQC in DFSs.

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Additional information

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