# STRONG COUPLING EXPANSION OF CALABI-YAU COMPACTIFICATION

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In a certain strong coupling limit, compactification of the  $E_8 \times E_8$  heterotic string on a Calabi-Yau manifold X can be described by an eleven-dimensional theory compactified on  $X \times \mathbf{S}^1/\mathbf{Z}_2$ . In this limit, the usual relations among low energy gauge couplings hold, but the usual (problematic) prediction for Newton's constant does not. In this paper, the equations for unbroken supersymmetry are expanded to the first non-trivial order, near this limit, verifying the consistency of the description and showing how, in some cases, if one tries to make Newton's constant too small, strong coupling develops in one of the two  $E_8$ 's. The lower bound on Newton's constant (beyond which strong coupling develops) is estimated and is relatively close to the actual value.

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### 1. Introduction

This paper will consist mainly of a technical calculation in eleven dimensions, which could be of conceivable relevance to grand unified model building. To explain this, I will begin with an extended introduction focusing on the model building, postponing most of the eleven-dimensional details to section two.

The models of particle physics that are most straightforwardly derived from the  $E_8 \times E_8$  heterotic string – with an assumption of small string coupling constant at all scales and low energy N=1 supersymmetry in four dimensions – are in many ways strikingly attractive. For example, one naturally gets the right gauge groups and fermion quantum numbers and recovers the usual, successful super-GUT prediction for low energy gauge couplings, even though other aspects of the GUT package are substantially modified (for instance, the usual and not obviously desireable GUT relations among Yukawa couplings do not arise; there are generally [1] unconfined particles with fractional electric charge with GUT masses but not fitting in GUT multiplets).

A major difficulty of the framework (as exhibited in early computations of the unification scale [2-5]) has been the following. In contrast to usual GUT models, which do not incorporate gravity and so make no prediction for Newton's constant, these perturbative string models do make a definite prediction for the gravitational coupling strength, a prediction that comes out too large. If expressed as a prediction for Newton's constant, the error is typically a factor of 400; if expressed (as it naturally arises) as a prediction for the logarithm of the Planck mass, the error is about 6% (but given the accuracy with which the low energy gauge couplings are now known, the discrepancy is six or seven standard deviations). Various proposals have been put forward for dealing with this problem in the context of perturbative string theory, but none of them is obviously compelling. For discussions of some of the possible scenarios, which include large threshold corrections, extra matter fields, higher or exotic Kac-Moody levels, GUT models more strictly embedded in strings, and anisotropic Calabi-Yau manifolds, see [6-16]; the last reference contains an extensive review and references.

As an alternative to approaches that depend on weak coupling, one might wonder whether the problem has a natural solution in a region of *large* string coupling constant. Certainly when the string coupling constant is large, there might be large corrections to the predicted value of Newton's constant, but at the cost of possibly ruining the successful weak coupling predictions for gauge couplings. General arguments have been given [17]

for why there might exist a strong coupling region in which the usual predictions would hold for gauge couplings but not for Newton's constant, but these arguments require some assumptions about wave function renormalization.

With the understanding of the strong coupling behavior of string theory that has been obtained of late, it seems appropriate to revisit this question: Is there a regime of strongly coupled string theory in which the four-dimensional gauge couplings are small and obey the usual GUT relations, but Newton's constant is significantly smaller than usual? The answer, as we will see, is "yes"; the desired properties hold in a regime in which the tendimensional string coupling constant is large, and the volume of the Calabi-Yau is also large, in such a proportion that the four-dimensional effective gauge couplings are small.

To analyze this region, first note that if the Calabi-Yau volume goes to infinity fast enough compared to the string coupling constant, the strong coupling behavior can be deduced from what happens in ten dimensions for strong coupling. Here the behavior is completely different depending on whether one considers the SO(32) or  $E_8 \times E_8$  heterotic string. For SO(32) the strong coupling limit is described by a weakly coupled tendimensional Type I superstring theory [18-21], while for  $E_8 \times E_8$  the strong coupling limit involves an eleven-dimensional description [22]. In one case, the gauge fields propagate on the boundary of the world-sheet, and in the other case they propagate on the boundary of space-time. In either case, as we will see, in the limit of large Calabi-Yau volume and large string coupling, one can naturally preserve the usual predictions for gauge coupling constants while making Newton's constant smaller.

Since neither the length scale of string theory  $\sqrt{\alpha'}$  nor the volume V of the Calabi-Yau manifold X (measured in the string metric) nor the expectation value of the dilaton field  $\phi$  is directly known from experiment, one might think that by adjusting  $\alpha'$ , V, and  $\langle \phi \rangle$  one can fit to any desired values of Newton's constant, the GUT scale  $M_{GUT}$ , and the GUT coupling constant  $\alpha_{GUT}$ , thus imitating the situation that prevails in conventional GUT theories. Let us first recall why things do not work out that way for the weakly coupled heterotic string. In ten dimensions, the low energy effective action looks like<sup>2</sup>

$$L_{eff} = -\int d^{10}x \sqrt{g} e^{-2\phi} \left( \frac{4}{(\alpha')^4} R + \frac{1}{(\alpha')^3} \text{tr} F^2 + \dots \right).$$
 (1.1)

<sup>&</sup>lt;sup>2</sup> Slightly varying conventions concerning the weak coupling formulas that appear in the present paragraph (and will not be used later in the paper) can be found in the literature.

After compactification on a Calabi-Yau manifold X of volume V (in the string metric), one gets a four-dimensional effective action that looks like

$$L_{eff} = -\int d^4x \sqrt{g} e^{-2\phi} V\left(\frac{4}{(\alpha')^4} R + \frac{1}{(\alpha')^3} \operatorname{tr} F^2 + \ldots\right). \tag{1.2}$$

The important point is that the same function  $Ve^{-2\phi}$  multiplies both R and  ${\rm tr} F^2$ . Because of T-duality one can assume

$$V \ge (\alpha')^3 \tag{1.3}$$

in order of magnitude; for  $V \sim (\alpha')^3$  one must use conformal field theory (and not classical geometry) to compute the effective V to be used in (1.1). From (1.2), we get at tree level

$$G_N = \frac{e^{2\phi}(\alpha')^4}{64\pi V}$$

$$\alpha_{GUT} = \frac{e^{2\phi}(\alpha')^3}{16\pi V},$$
(1.4)

so that

$$G_N = \frac{\alpha_{GUT}\alpha'}{4}. (1.5)$$

This means that the string scale, controlled by  $\alpha'$ , is known in terms of  $G_N$  and  $\alpha_{GUT}$ , and in particular the string mass scale is not much below the Planck scale. On the other hand, for weak coupling the GUT scale cannot be much smaller. If we suppose that  $e^{2\phi} \leq 1$  so that the ten-dimensional string theory is not strongly coupled, we get

$$V \le \frac{(\alpha')^3}{\alpha_{GUT}}. (1.6)$$

For a more or less isotropic Calabi-Yau,  $V \sim M_{GUT}^{-6}$ , and then the upper bound (1.6) on V translates into

$$G_N \ge \frac{\alpha_{GUT}^{4/3}}{M_{GUT}^2},\tag{1.7}$$

which is too large.<sup>3</sup>

Note that the problem might be ameliorated by considering an anisotropic Calabi-Yau, for instance one with a scale  $\sqrt{\alpha'}$  in d directions and  $1/M_{GUT}$  in 6-d directions (with some fairly severe restrictions on d and the Calabi-Yau manifold X to ensure that it is the large dimensions in X that control the GUT breaking), so that  $V \sim (\alpha')^{d/2}/M_{GUT}^{6-d}$ . The amelioration obtained this way, if too small, could possibly be combined with the strong coupling effect considered below.

Rather than pursuing one of the possible solutions of this problem in weak coupling, we will here suppose that the ten-dimensional string coupling constant is *large*. There are different strong coupling limits one could consider, depending, for instance, on what happens to V while  $\phi \to \infty$ . If V and  $\phi$  are both suitably large, then the theory can be analyzed using a knowledge of the *ten-dimensional* strong coupling behavior. That is the region we will consider in this paper.

First we consider the SO(32) heterotic string, which involves less novelty because the strong coupling limit of this theory is simply another string theory – the Type I superstring theory in ten dimensions. We repeat the above discussion, using the Type I dilaton  $\phi_I$ , metric  $g_I$ , and scalar curvature  $R_I$ . The analog of (1.1) is

$$L_{eff} = -\int d^{10}x \sqrt{g_I} \left( e^{-2\phi_I} \frac{4}{(\alpha')^4} R_I + e^{-\phi_I} \frac{1}{(\alpha')^3} \text{tr} F^2 + \dots \right).$$
 (1.8)

The point is that in contrast to the heterotic string, the gravitational and gauge actions multiply different functions of  $\phi_I$ , namely  $e^{-2\phi_I}$  and  $e^{-\phi_I}$ , respectively, since one is generated by a world-sheet path integral on a sphere and one on a disc. The analog of (1.2) is then

$$L_{eff} = -\int d^4x \sqrt{g}_I V_I \left( \frac{4e^{-2\phi_I}}{(\alpha')^4} R + \frac{e^{-\phi_I}}{(\alpha')^3} \text{tr} F^2 + \dots \right)$$
(1.9)

 $(V_I$  is the Calabi-Yau volume measured in the Type I metric), and the couplings become

$$G_N = \frac{e^{2\phi_I} (\alpha')^4}{64\pi V_I}$$

$$\alpha_{GUT} = \frac{e^{\phi_I} (\alpha')^3}{16\pi V_I}.$$
(1.10)

Hence

$$G_N = \frac{e^{\phi_I} \alpha_{GUT} \alpha'}{4},\tag{1.11}$$

showing that after taking  $\alpha_{GUT}$  from experiment and adjusting  $\alpha'$  so that the string scale is comparable to the experimentally inferred GUT scale, one can make  $G_N$  as small as one wishes simply by taking  $e^{\phi_I}$  to be small, that is, by taking the Type I superstring to be weakly coupled. Of course, when the Type I coupling is weak, the usual tree level relations among gauge couplings will hold (in a suitable class of string vacua).

Note that this discussion does not require a knowledge that the Type I superstring is equivalent to the strong coupling limit of the SO(32) heterotic string. We have simply shown directly that for the weakly coupled Type I superstring the usual contradiction with

measured GUT parameters does not arise (and by the same token, just as in standard GUTs, one gets no prediction for the value of  $G_N$  in terms of known quantities). This assertion is not essentially new, though it has not been much used in attempts at string phenomenology.

We will now argue that the  $E_8 \times E_8$  heterotic string has an analogous strong coupling behavior: one keeps the standard GUT relations among the gauge couplings, but loses the prediction for Newton's constant, which can be considerably smaller than the weak coupling bound. There is, however, one important difference from SO(32). Except under a certain topological restriction that will be explained, we cannot make  $G_N$  arbitrarily small in the strongly coupled  $E_8 \times E_8$  heterotic string (keeping other moduli fixed) without losing control on the discussion. There is a lower bound, whose order of magnitude we will estimate, on how small  $G_N$  can be in such a model.

The ten-dimensional  $E_8 \times E_8$  heterotic string has for its strong coupling limit M-theory on  $\mathbf{R}^{10} \times \mathbf{S}^1/\mathbf{Z}_2$  [22]. The gravitational field propagates in bulk over  $\mathbf{R}^{10} \times \mathbf{S}^1/\mathbf{Z}_2$ , while the  $E_8 \times E_8$  gauge fields propagate only at the  $\mathbf{Z}_2$  fixed points. We write  $M^{11}$  for  $\mathbf{R}^{10} \times \mathbf{S}^1$  and  $M_i^{10}$ , i = 1, 2 for the two components of the fixed point set. The gauge and gravitational kinetic energies take the form

$$L = -\frac{1}{2\kappa^2} \int_{M^{11}} d^{11}x \sqrt{g}R - \sum_i \frac{1}{8\pi (4\pi\kappa^2)^{2/3}} \int_{M_i^{10}} d^{10}x \sqrt{g} \operatorname{tr} F_i^2.$$
 (1.12)

A few points about this formula need to be explained. First,  $\kappa$  is here the elevendimensional gravitational coupling; implicit in (1.12) is a relation of the ten-dimensional gauge coupling to  $\kappa$  that will be obtained elsewhere [23]. Second, as written, the Einstein action in (1.12) involves an integral over  $M^{11} = \mathbf{R}^{10} \times \mathbf{S}^1$ , it being understood that the fields are  $\mathbf{Z}_2$ -invariant; if one wishes the integral to run over  $M^{11}/\mathbf{Z}_2$ , one must multiply the gravitational action by a factor of two. Finally,  $F_i$ , for i = 1, 2, is the field strength of the  $i^{th}$   $E_8$ , which propagates according to [22] on the  $i^{th}$  component of the fixed point set, that is on  $M_i^{10}$ .

Now compactify to four dimensions on a Calabi-Yau manifold X whose volume (in the eleven-dimensional metric) is V.<sup>4</sup> Let the  $\mathbf{S}^1$  have radius  $\rho$  or circumference  $2\pi\rho$ . Upon

<sup>&</sup>lt;sup>4</sup> This is a change in notation, as earlier V was the Calabi-Yau volume in the ten-dimensional string metric. In the remainder of the paper we use always the metric of the eleven-dimensional theory.

reducing (1.12) to four dimensions, one learns that the four-dimensional  $G_N$  and  $\alpha_{GUT}$  are

$$G_N = \frac{\kappa^2}{16\pi^2 V \rho}$$

$$\alpha_{GUT} = \frac{(4\pi\kappa^2)^{2/3}}{2V}.$$
(1.13)

In so far as experiment shows that  $\alpha_{GUT} << 1$ , the second formula shows that the dimensionless number  $\kappa^{4/3}/V$  is small. That is just as well, because the eleven-dimensional description is only valid under that restriction. The fact that  $\kappa^{4/3}/V$  is small also means that the fields propagating on  $M_i^{10}$  are weakly coupled so that, if the vacuum is determined by an  $E_8 \times E_8$  gauge bundle of the sort usually used in string theory compactification (for instance, the standard embedding of the spin connection in the gauge group or one of the usual generalizations) then the standard GUT relations among gauge couplings will hold. After picking  $\kappa$  and V to get the experimentally inferred value of  $\alpha_{GUT}$ , and also choosing V so that  $V^{-1/6}$  agrees with the experimentally inferred GUT mass scale  $M_{GUT}$ , the first equation in (1.13) shows that we may also make  $G_N$  small by making  $\rho$  large. That is again a favorable result, since for  $\rho$  to be large compared to the eleven-dimensional Planck scale is a necessary condition for validity of the eleven-dimensional description.

In fact, the validity of the eleven-dimensional description certainly requires that  $\rho$  and V should be large compared to the eleven-dimensional Planck length and Planck volume. To the extent that those are the only restrictions, the above formulas show that while keeping  $\alpha_{GUT}$  and  $M_{GUT}$  fixed and making  $\rho$  sufficiently big, we can make  $G_N$  as small as we wish. That is the actual state of affairs under a certain topological restriction, which is roughly that the vacuum gauge bundle has the same instanton number in each  $E_8$ . More generally (for instance if one considers the standard embedding of the spin connection in the gauge group, for which the instanton number vanishes in one of the two  $E_8$ 's), we will find that the validity of the eleven-dimensional description requires a further condition which in order of magnitude is  $\rho \leq V^{2/3}/\kappa^{2/3}$ . (When this does not hold, one is driven to strong coupling in one of the two  $E_8$ 's by a mechanism analogous to that in [21].) In order of magnitude, this translates into  $G_N \geq \kappa^{8/3}/V^{5/3} = \alpha_{GUT}^2 V^{1/3}$ . Setting  $V^{1/3} = M_{GUT}^{-2}$ , the lower bound on Newton's constant becomes in order of magnitude

$$G_N \ge \frac{\alpha_{GUT}^2}{M_{GUT}^2}. (1.14)$$

(As in the weak coupling case discussed in a footnote after equation (1.7), this bound can be made smaller by considering an anisotropic Calabi-Yau.)

Comparing this to the bound  $G_N \ge \alpha_{GUT}^{4/3}/M_{GUT}^2$  of weakly coupled string theory, we see that the effect of going to strong coupling is to make the lower bound on  $G_N$  smaller by a factor of  $\alpha_{GUT}^{2/3}$  (or to remove the lower bound entirely if the instanton numbers are equal in the two  $E_8$ 's). We will evaluate the bound somewhat more precisely in the next section, and in the approximation considered there, we get the critical value of Newton's constant (at which strong coupling develops in one of the  $E_8$ 's) to be

$$G_N^{crit} = \frac{\alpha_{GUT}^2}{16\pi^2} \left| \int_X \omega \wedge \frac{\text{tr}F \wedge F - \frac{1}{2}\text{tr}R \wedge R}{8\pi^2} \right|, \tag{1.15}$$

with  $\omega$  the Kahler form of X and F the field strength of either of the  $E_8$ 's. The integral on the right hand side of (1.15) is plausibly  $M_{GUT}^{-2}$  times a number of order one.  $\alpha_{GUT}$  is this formula is the coupling in the  $E_8$  that is not strongly coupled. Because of the factor of  $16\pi^2$  in the denominator, (1.15) may be small enough to agree with experiment, for reasonable values of the integral. Our calculation in section two is, however, only carried out in a linearized approximation, and it is not clear to what extent it might be modified by the nonlinear terms. (We will in section three compute the nonlinear terms for compactifications to six dimensions.)

# Perturbative Expansion

The rest of this paper will consist primarily of a rather detailed calculation expanding the eleven-dimensional vacuum to lowest order in 1/V and  $1/\rho$ . This mainly involves showing how the four-form field strength G of the eleven-dimensional theory is turned on without breaking supersymmetry. A non-zero G field appears in any compactification, as we will see.

By performing this calculation, we will get an interesting test of the consistency of the sort of vacua considered here and get some intuition about their structure. We will show explicitly that supersymmetry is not spontaneously broken in the limit we will consider; since instanton effects are turned off in the limit of large V and  $\rho$ , this follows from holomorphy of the superpotential and the decoupling of the pseudoscalar partners of V and  $\rho$  at zero momentum, but it is nice to have an explicit check. Also, in our computation, we will derive the lower bound on  $G_N$  stated above. The computation is a sort of strong coupling analog of a weak coupling, large radius expansion of (0, 2) models that was carried out in [24].

Our computation is relevant to other problems in which it is important to consider turning on G in a supersymmetric way. Such a problem is M-theory compactification on

 $\mathbf{T}^5/\mathbf{Z}_2$ , where five-branes<sup>5</sup> – which are magnetic sources of G – appear in the vacuum [25]; this compactification has also been studied by Dasgupta and Mukhi [26]. For this and for other supersymmetric compactifications to six dimensions, we will obtain more precise results than we get for compactification to four dimensions on Calabi-Yau threefolds; in compactification to six dimensions the nonlinear structure is much simpler. Among other things, we will get a verification that the five-brane configuration considered in [25] in compactification on  $\mathbf{T}^5/\mathbf{Z}_2$  is compatible with supersymmetry, adding support to the proposal made in that paper for the strong coupling behavior of Type IIB superstrings on K3. We will also describe how to incorporate the five-branes in Calabi-Yau compactification.

While the strong coupling region we will study has the potential to ameliorate the usual puzzle about the low energy couplings, we will not obtain any clear insight about the perhaps related puzzle, stressed in [27] and revisited in [17,28], of why supersymmetry breaking in a region where physics is computable does not result in a runaway to weak coupling. Thus, we will not obtain any particular insight about why there would be a stable vacuum with broken supersymmetry (and no runaway to zero  $\alpha_{GUT}$ ) in a regime that can be approximated by the description worked out below. Perhaps the occurrence of strong coupling in the second  $E_8$  when Newton's constant reaches its lower bound is a clue.

### 2. Long Wavelength Expansion In Eleven Dimensions

Our conventions in eleven dimensions will be those of [29]. For example, the signature of the space-time manifold  $M^{11}$  is  $-+++\ldots+$ ; gamma matrices obey  $\{\Gamma_I,\Gamma_J\}=2g_{IJ}$  and  $\Gamma_1\Gamma_2\ldots\Gamma_{11}=1$ . One also defines  $\Gamma_{I_1I_2\ldots I_n}=(1/n!)\,(\Gamma_{I_1}\Gamma_{I_2}\ldots\Gamma_{I_n}\pm\text{permutations})$ .

The bosonic fields in eleven-dimensional supergravity are the metric g and a threeform A; the fermions are the spin 3/2 gravitino  $\psi_{I\alpha}$ ,  $\alpha$  being a spinor index. The field

<sup>&</sup>lt;sup>5</sup> To avoid confusion, let me stress that five-branes in this paper will always be the five-branes of eleven-dimensional M-theory. Thus, for instance in compactification on  $M^{10} \times \mathbf{S}^1/\mathbf{Z}_2$ , the five-brane propagates in bulk on the eleven-manifold, and is distinct from  $E_8 \times E_8$  instantons, also often called five-branes, which occur on the boundaries. The two kinds of five-brane have some couplings in common, but are definitely different as one has a tensor multiplet on the world volume and one does not.

strength of A is  $G_{IJKL} = \partial_I A_{JKL} \pm 23$  more terms. The supersymmetry transformation law for  $\psi$  is

$$\delta\psi_I = D_I \eta + \frac{\sqrt{2}}{288} (\Gamma_{IJKLM} - 8g_{IJ} \Gamma_{KLM}) G^{JKLM} \eta. \tag{2.1}$$

The condition for a spinor field  $\eta$  to generate an unbroken supersymmetry is that the right hand side of (2.1) vanishes,

$$D_I \eta + \frac{\sqrt{2}}{288} (\Gamma_{IJKLM} - 8g_{IJ} \Gamma_{KLM}) G^{JKLM} \eta = 0.$$
 (2.2)

The most obvious way to obey this is to set G = 0 and pick on  $M^{11}$  a metric that admits a covariantly constant spinor field,  $D_I \eta = 0$ . For example, in compactifying from eleven to four dimensions on  $X \times \mathbf{S}^1$ , X being a Calabi-Yau manifold, one can take G = 0 and then the covariantly constant spinors on X give unbroken supersymmetries.

Compactification on  $X \times \mathbf{S}^1/\mathbf{Z}_2$  is more subtle because one may not take G = 0. The reason for this is that there is a magnetic source of the G field at the fixed points in  $\mathbf{S}^1/\mathbf{Z}_2$ . In fact, while the Bianchi identity for G on a smooth eleven-manifold without five-branes or other singularities states that dG = 0 (where  $(dG)_{IJKLM} = \partial_I G_{JKLM} \pm$  cyclic permutations), there are several types of "impurity" that give contributions to dG. For instance, dG receives a delta function contribution at the location of five-branes, and also [25] at certain codimension five singularities. More directly relevant for us is that in compactification on  $\mathbf{S}^1/\mathbf{Z}_2$ , there is a delta function contribution to dG, supported at the  $\mathbf{Z}_2$  fixed points. If we consider the fixed point set to be at  $x^{11} = 0$ , then the non-vanishing part of the fixed point contribution to dG is

$$(dG)_{11IJKL} = -\frac{3\sqrt{2}\delta(x^{11})}{2\pi} \left(\frac{\kappa}{4\pi}\right)^{2/3} \left(\text{tr}F_{[IJ}F_{KL]} - \frac{1}{2}\text{tr}R_{[IJ}R_{KL]}\right). \tag{2.3}$$

Only one of the two  $E_8$ 's appears in (2.3), namely the one propagating at  $x^{11}=0$ ; that is the reason for the 1/2 multiplying  $\operatorname{tr} R \wedge R$ . A formula such as the above is needed to reproduce in eleven dimensions the effects of the string theory relation  $dH = -\sum_i \operatorname{tr} F_i \wedge F_i + \operatorname{tr} R \wedge R$ ; the precise coefficient in (2.3) will be obtained elsewhere [23]. In (2.3),  $\operatorname{tr} F \wedge F$  is as usual 1/30 times the trace in the adjoint representation of  $E_8$ , and  $\operatorname{tr} R \wedge R$  is the trace in the vector representation of SO(1,10); also,  $\operatorname{tr} F_{[IJ}F_{KL]} = (1/24)\operatorname{tr} F_{IJ}F_{KL} \pm \operatorname{permutations}$ . It is apparently impossible in Calabi-Yau compactification to find a vacuum with the property that  $\operatorname{tr} F \wedge F - (1/2)\operatorname{tr} R \wedge R = 0$  pointwise (the standard embedding of

the spin connection in the gauge group is a natural way to get  $\operatorname{tr} F \wedge F - \operatorname{tr} R \wedge R = 0$ , so (2.3) implies generically and perhaps always that  $G \neq 0$  in Calabi-Yau compactification.

Note that in studying physics on an orbifold, one can either work "upstairs," in this case on  $M^{11} = \mathbf{R}^4 \times X \times \mathbf{S}^1$  with X a Calabi-Yau manifold, and require invariance under the orbifolding group, in this case a  $\mathbf{Z}_2$  that acts only on  $\mathbf{S}^1$ , or "downstairs," directly on the quotient, in this case  $M^{11}/\mathbf{Z}_2 = \mathbf{R}^4 \times X \times \mathbf{S}^1/\mathbf{Z}_2$ . (2.3) has been written in the "upstairs" version, G being a four-form on  $M^{11}$  that is odd under the  $\mathbf{Z}_2$  (G is odd because it changes sign under orientation reversal); in the analogous "downstairs" version, explained in [23], (2.3) is replaced by a boundary condition that has the same effect and in particular forces  $G \neq 0$ .

Our goal, then, is to show in the first non-trivial order how G can be turned on to obey (2.3) and preserve supersymmetry. To be more specific, we work on  $M^{11}/\mathbb{Z}_2 = \mathbb{R}^4 \times X \times \mathbb{S}^1/\mathbb{Z}_2$ . According to (2.3), one can take G to be of order  $\kappa^{2/3}$ , so to lowest order in  $\kappa$  (or in other words to lowest order in an expansion in the inverse of the length scale of  $X \times \mathbb{S}^1/\mathbb{Z}_2$ ), we can set G = 0. For the starting point, then, we take the metric on  $M^{11}/\mathbb{Z}_2$  to be the product of a flat metric on  $\mathbb{R}^4 \times \mathbb{S}^1/\mathbb{Z}_2$  with a Calabi-Yau metric on X. The unbroken supersymmetries come from covariantly constant spinor fields on X. Then in order  $\kappa^{2/3}$ , we must pick holomorphic  $E_8$  bundles on the  $\mathbb{Z}_2$  fixed points (note from (1.12) that the gauge kinetic energy is of order  $\kappa^{2/3}$  compared to the gravitational kinetic energy, so that as in the study of Type I superstrings the choice of a gauge bundle can be considered a kind of higher order correction relative to the basic choice of gravitational vacuum). Also in order  $\kappa^{2/3}$ , we find a solution of the equations of motion for G, including the source term (2.3). Then modifying also the metric on  $M^{11}$  and the spinor field  $\eta$ , we aim to obey the condition (2.2) of unbroken supersymmetry in order  $\kappa^{2/3}$ .

The first step is then to find a solution of the equations of motion for G together with the Bianchi identity. The equation of motion contains a term involving  $G \wedge G$ , but this will vanish in the situations we will consider, since (given the four-dimensional Poincaré invariance and the vanishing of  $G_{1234}$ ), there is no "room" for a non-zero eight-form  $G \wedge G$ . The equation of motion for G then reduces simply to

$$D^I G_{IJKL} = 0, (2.4)$$

and the Bianchi identity reads

$$D_I G_{JKLM} \pm \text{permutations} = \text{sources},$$
 (2.5)

where the "sources" are delta functions supported at  $\mathbb{Z}_2$  fixed points or at five-branes. The condition that the Bianchi identity has a solution is that the source terms add up to zero cohomologically. If no five-branes are present, this condition simply amounts to the standard string theory constraint on the total  $E_8 \times E_8$  instanton number (thus,  $\sum_{i=1,2} \operatorname{tr} F_i \wedge F_i - \operatorname{tr} R \wedge R$  must vanish cohomologically, as in perturbative string theory where this expression equals -dH). (2.4) and (2.5) then have a solution which becomes unique if one asks that G comes from a four-form on  $X \times S^1$  that is odd under the  $\mathbb{Z}_2$  action on  $\mathbb{S}^1$  and cohomologically trivial. With this G as a starting point, we will verify that the conditions for unbroken supersymmetry have a unique solution up to order  $\kappa^{2/3}$ .

It is interesting to consider the general case in which the source terms in the Bianchi identity come from five-branes as well as  $\mathbb{Z}_2$  fixed points. This generalization involves issues that are less familiar from the string theory viewpoint. To preserve supersymmetry (roughly as in [31]), take the  $\alpha^{th}$  five-brane (whose world-volume should be a submanifold of  $M^{11}$  of codimension five) to be located at the product of a holomorphic curve  $C_{\alpha} \subset X$  times a point  $P_{\alpha} \in \mathbb{S}^1/\mathbb{Z}_2$ . One must then include the five-brane source terms in the Bianchi identity for G. The condition for the Bianchi identity to have a solution is now that (cohomologically)

$$\frac{\sum_{i} \operatorname{tr} F_{i} \wedge F_{i} - \operatorname{tr} R \wedge R}{8\pi^{2}} + \sum_{\alpha} [C_{\alpha}] = 0.$$
 (2.6)

with  $[C_{\alpha}]$  the Poincaré dual cohomology class to  $C_{\alpha}$ . (The fact that the coefficient of  $[C_{\alpha}]$  in this formula is precisely +1 follows from the observation in [30] that the five-brane and Yang-Mills instanton make the same contribution to the irreducible part of the gravitational anomaly.)

The reason that there is no difficulty in including fivebranes in our computation is really the following. Let us decompose the space of complex-valued p forms on  $X \times \mathbf{S}^1$  into forms of type (a, b, c), where a+b+c=p, and a counts the number of holomorphic indices tangent to X, b counts the number of antiholomorphic indices tangent to X, and c is 1 or 0 depending on whether a factor  $dx^{11}$  is present or absent. Then in our computation, the only important fact about (2.3) will be that the source term for dG is supported at singularities and is a (2, 2, 1) form. Inclusion of five-branes causes no trouble if the five-brane source for

<sup>&</sup>lt;sup>6</sup> A cohomologically non-trivial and  $\mathbb{Z}_2$  odd addition to G would be of the form  $H \wedge dx^{11}$ , with H a harmonic form on X. This addition breaks supersymmetry, just as turning on a cohomologically non-trivial three-form field H in the weakly coupled heterotic string breaks supersymmetry.

dG is likewise of type (2,2,1), which we ensure by locating the  $\alpha^{th}$  five-brane at  $C_{\alpha} \times P_{\alpha}$  with  $C_{\alpha}$  and  $P_{\alpha}$  as above. Given this restriction, we need not mention whether five-branes are present or not (except in quantitatively estimating the lower bound on  $G_N$ ).

Incidentally, this formalism makes it manifest that (as partly verified in [30]) the vacua with five-branes present are free of space-time anomalies, as long as (2.6) holds. Since there are no gravitational anomalies on a smooth eleven-manifold, the anomalies in eleven dimensions are localized at five-branes and singularities, but the anomaly on the five-brane world-volume cancels [32,25], and the same is true at the  $\mathbb{Z}_2$  orbifold singularities [23]. The space-time anomaly cancellation can thus be understood locally in eleven dimensions, with no need to verify low energy details.

# 2.1. The Computation

In performing the computation on  $M^{11} = \mathbf{R}^4 \times X \times \mathbf{S}^1$ , we adopt the following conventions. Indices  $I, J, K, \ldots$  from the middle of the alphabet run from 1 to 11 and are tangent to  $M^{11}$ . Indices  $X, Y, Z, \ldots$  from the end of the alphabet run from 5 to 11 and are tangent to  $X \times \mathbf{S}^1$ . Indices  $A, B, C, \ldots$  from the beginning of the alphabet are tangent to X; we also use  $a, b, c, \ldots = 1 \ldots 3$  for holomorphic indices tangent to X, and  $\overline{a}, \overline{b}, \overline{c}, \ldots = 1 \ldots 3$  for analogous antiholomorphic indices. The index 11 is tangent to  $\mathbf{S}^1$ . Meanwhile,  $\mu, \nu, \lambda, \ldots = 1 \ldots 4$  will be tangent to  $\mathbf{R}^4$ .

We take the metric on  $\mathbf{R}^4$  to be the Minkowski metric  $\eta_{\mu\nu}dx^{\mu}dx^{\nu}$  and the metric on  $\mathbf{S}^1$  to be  $(dx^{11})^2$ . (The  $\mathbf{S}^1$  has circumference  $2\pi\rho$ .) The Calabi-Yau manifold X has a metric tensor  $g_{AB}$  with non-zero components  $g_{a\overline{b}}=g_{\overline{b}a}$  and a Kahler form  $\omega_{AB}$  with non-zero components  $\omega_{a\overline{b}}=-ig_{a\overline{b}}=-\omega_{\overline{b}a}$ . The gamma matrices tangent to X are  $\Gamma^a$  and  $\Gamma^{\overline{b}}$  with  $\{\Gamma^a,\Gamma^{\overline{b}}\}=2g^{a\overline{b}}$  and other anticommutators vanishing. We will look for a solution of the supersymmetry condition (2.2) that obeys  $\Gamma^{11}\eta=\eta$  (so as to be invariant under the  $\mathbf{Z}_2$  projection), and  $\Gamma^a\eta=0$ ; the complex conjugate of  $\eta$  would obey  $\Gamma^{11}\eta'=\eta'$  and  $\Gamma^{\overline{b}}\eta'=0$ .

We begin, as explained above, with the cohomologically trivial solution  $G_{IJKL}$  of the Bianchi identity dG = "sources" and equations of motion  $D^I G_{IJKL} = 0$ . Because the source term in the Bianchi identity is of type (2, 2, 1), the non-zero components of G are

 $G_{ABCD}$ , of type (2, 2, 0), and  $G_{ABC 11}$ , a mixture of forms of types (2, 1, 1) and (1, 2, 1). <sup>7</sup> It is convenient to introduce

$$\beta_A = \omega^{BC} G_{ABC 11}$$

$$\theta_{AB} = G_{ABCD} \omega^{CD}$$

$$\alpha = \omega^{AB} \theta_{AB} = \omega^{AB} \omega^{CD} G_{ABCD}.$$
(2.7)

These objects obey various identities that follow from the equations of motion and Bianchi identities of G. We will write these equations as they hold away from the singularities and five-branes, so we can ignore the source terms in the Bianchi identities. The (1,3,1) part of the Bianchi identity gives

$$D_{\overline{a}}G_{a\overline{b}\overline{c}\,11} + D_{\overline{b}}G_{a\overline{c}\overline{a}\,11} + D_{\overline{c}}G_{a\overline{a}\overline{b}\,11} = 0. \tag{2.8}$$

The equation of motion  $D^I G_{I\overline{a}\overline{b}\,11} = 0$ , when combined with the vanishing of the (0,3,1) part of G, gives

$$0 = \omega^{a\overline{a}} D_{\overline{a}} G_{a\overline{b}\overline{c}\,11} = 0. \tag{2.9}$$

If one contracts (2.8) with  $\omega^{a\overline{a}}$  and uses (2.9), one geta

$$D_{\overline{a}}\beta_{\overline{b}} - D_{\overline{b}}\beta_{\overline{a}} = 0. (2.10)$$

The (2,3,0) part of the Bianchi identity, that is the condition  $\partial_{\overline{a}}G_{b\overline{b}c\overline{c}} + \partial_{\overline{b}}G_{b\overline{c}c\overline{a}} + \partial_{\overline{c}}G_{b\overline{a}c\overline{b}} = 0$ , can if contracted with  $g^{c\overline{c}}$ , be reduced after using the equations of motion to

$$0 = -\frac{i}{2} \left( \partial_{\overline{a}} \theta_{b\overline{b}} - \partial_{\overline{b}} \theta_{b\overline{a}} \right) + D^{11} G_{b\overline{a}\overline{b} 11}. \tag{2.11}$$

If this is contracted with  $\omega^{b\overline{b}}$  one gets

$$0 = -\frac{i}{2}D^A\theta_{A\overline{b}} + \frac{1}{4}D_{\overline{b}}\alpha - \frac{i}{2}D_{11}\beta_{\overline{b}}.$$
 (2.12)

One component of the equation of motion is  $D^{11}G_{a\overline{a}\overline{b}11} + D^bG_{a\overline{a}\overline{b}b} = 0$ ; if this is contracted with  $\omega^{a\overline{a}}$  one gets

$$0 = D_{11}\beta_{\overline{b}} - D^b\theta_{b\overline{b}}. (2.13)$$

 $<sup>\</sup>overline{\phantom{a}}^7$  A (3,0,1) part of G would be of the form  $H \wedge dx^{11}$ , with H a (3,0) form on X. Given that the sources are of type (2,2,1), the Bianchi identity dG = sources implies that the (3,1,1) part of dG vanishes so that  $\overline{\partial}H=0$ . The equation of motion similarly implies that H is independent of  $x^{11}$ . Since G is cohomologically trivial, the  $x^{11}$ -independent holomorphic three-form H must vanish. The (0,3,1) part of G vanishes likewise.

Combining (2.12) and (2.13), we obtain

$$D_{11}\beta_{\overline{a}} = -\frac{i}{4}\partial_{\overline{a}}\alpha. \tag{2.14}$$

Also, upon starting with the equation of motion  $D^A G_{ABC \, 11} = 0$  and contracting with  $\omega^{BC}$ , one learns

$$D^A \beta_A = 0. (2.15)$$

Finally, starting with the component  $dG_{ABCD\,11}=0$  of the Bianchi identity and contracting with  $\omega^{AB}\omega^{CD}$ , one learns that  $\partial_{11}\alpha$  is a total derivative with respect to  $x^A$ , as a result of which

$$0 = \frac{\partial}{\partial x^{11}} \int_{X} \alpha \sqrt{g} d^{6}x. \tag{2.16}$$

To evaluate the  $\Gamma G \eta$  terms in the supersymmetry condition (2.2), one needs the following identities:

$$\Gamma^{XYZW}G_{XYZW}\eta = \left(-3\alpha - 12i\beta_{\overline{b}}\Gamma^{\overline{b}}\right)\eta$$

$$\Gamma^{XYZ}G_{11\,XYZ}\eta = 3i\Gamma^{\overline{b}}\beta_{\overline{b}}\eta$$

$$\Gamma^{XYZ}G_{aXYZ} = \left(-3i\beta_a + 3G_{a\overline{b}\overline{c}\,11}\Gamma^{\overline{b}\overline{c}} - 3i\theta_{a\overline{b}}\Gamma^{\overline{b}}\right)\eta$$

$$\Gamma^{XYZ}G_{\overline{b}XYZ} = -3i\overline{\beta}_{\overline{b}}\eta.$$
(2.17)

These identities use  $\Gamma^{11}\eta = \eta$ ,  $\Gamma^b\eta = 0$ . Using these identities, one can compute that the  $\Gamma G\eta$  terms in (2.2) are

$$\frac{\sqrt{2} dx^{I}}{288} \left( \Gamma_{IJKLM} - 8g_{IJ}\Gamma_{KLM} \right) G^{JKLM} \eta = \frac{\sqrt{2}}{288} \left( dx^{11} \left( -3\alpha - 24i\beta_{\overline{b}}\Gamma^{\overline{b}} \right) + dx^{\overline{b}} \cdot 12i\beta_{\overline{b}} + dx^{a} \left( 36i\beta_{a} + \left( 36i\theta_{a\overline{b}} - 3\alpha g_{a\overline{b}} \right) \Gamma^{\overline{b}} + \left( -36G_{a\overline{a}\overline{b}}_{11} - 6i(g_{a\overline{a}}\beta_{\overline{b}} - g_{a\overline{b}}\beta_{\overline{a}}) \Gamma^{\overline{a}\overline{b}} \right) + dx^{\mu}\Gamma_{\mu} \left( -3\alpha - 12i\beta_{\overline{b}}\Gamma^{\overline{b}} \right) \right) \eta.$$
(2.18)

To evaluate (2.2), we also need to look at terms that come from a change in the metric on  $M^{11}$ . A careful study of the equations shows that one can take the perturbation in the metric to be block diagonal and to obey certain additional restrictions, so that the perturbed line element is

$$ds^{2} = \eta_{\mu\nu} dx^{\mu} dx^{\nu} (1+b) + 2(g_{a\overline{b}} + h_{a\overline{b}}) dx^{a} dx^{\overline{b}} + (1+\gamma)(dx^{11})^{2} + \dots,$$
 (2.19)

where b, h, and  $\gamma$  will be of order G, that is of order  $\kappa^{2/3}$ , and the ... are terms of order  $\kappa^{4/3}$ . To preserve four-dimensional Lorentz invariance, b, h, and  $\gamma$  are functions only of the  $x^Y$ ,  $Y \geq 5$ . Note, for example, that the metric on X is still hermitian, but will no longer be Kahler as we will see. We also replace the spinor field  $\eta$  with

$$\widetilde{\eta} = e^{-\psi} \eta, \tag{2.20}$$

 $\psi$  being again of order  $\kappa^{2/3}$ .

In general, under a perturbation of the metric  $g_{IJ} \to g_{IJ} + h_{IJ}$ , to first order in h, the covariant derivative of a spinor changes by  $D_I \eta \to D_I \eta - \frac{1}{8} (D_J h_{KI} - D_K h_{JI}) \Gamma^{JK} \eta$ . Using this formula, one finds that, with  $\eta$  covariantly constant with respect to the unperturbed metric, one has to first order in the perturbation

$$\begin{split} dx^I D_I \widetilde{\eta} &= \left( -dx^Y \partial_Y \psi + dx^\mu \Gamma_\mu \left( \frac{1}{4} \partial_{11} b + \frac{1}{4} \partial_{\overline{a}} b \Gamma^{\overline{a}} \right) \right. \\ &\left. - dx^a \left( -\frac{1}{4} \partial_{11} h_{\overline{a}a} \Gamma^{\overline{a}} + \frac{1}{8} \left( \partial_{\overline{a}} h_{\overline{b}a} - \partial_{\overline{b}} h_{\overline{a}a} \right) \Gamma^{\overline{a}\overline{b}} + \frac{1}{4} g^{b\overline{b}} \partial_b h_{a\overline{b}} \right) \\ &\left. - dx^{\overline{a}} \left( -\frac{1}{4} g^{a\overline{b}} \partial_{\overline{b}} h_{a\overline{a}} \right) - \frac{dx^{11}}{4} \partial_{\overline{a}} \gamma \Gamma^{\overline{a}} \right) \widetilde{\eta}. \end{split} \tag{2.21}$$

The condition

$$D_I \widetilde{\eta} + \frac{\sqrt{2}}{288} (\Gamma_{IJKLM} - 8g_{IJ} \Gamma_{KLM}) G^{JKLM} \widetilde{\eta} = 0$$
 (2.22)

can now be evaluated. The terms proportional to  $\Gamma^{\overline{ab}}\eta$  give

$$\partial_{\overline{a}}h_{b\overline{b}} - \partial_{\overline{b}}h_{b\overline{a}} = -\sqrt{2}\left(G_{b\overline{a}\overline{b}\,11} + \frac{i}{6}\left(g_{b\overline{a}}\beta_{\overline{b}} - g_{b\overline{b}}\beta_{\overline{a}}\right)\right). \tag{2.23}$$

The terms proportional to  $\Gamma^{\overline{a}}\eta$  give two equations:

$$\beta_{\overline{a}} = \frac{3i}{\sqrt{2}} \partial_{\overline{a}} \gamma$$

$$\partial_{11} h_{a\overline{b}} = -\frac{1}{\sqrt{2}} \left( i\theta_{a\overline{b}} - \frac{1}{12} \alpha g_{a\overline{b}} \right). \tag{2.24}$$

The terms involving  $\eta$  multiplied by a function, without gamma matrices, give

$$0 = -dx^{Y} \partial_{Y} \psi + dx^{a} \left( -\frac{1}{4} g^{b\overline{b}} \partial_{b} h_{a\overline{b}} \right) + dx^{\overline{a}} \left( \frac{1}{4} g^{a\overline{b}} \partial_{\overline{b}} h_{a\overline{a}} \right)$$

$$+ \frac{\sqrt{2}}{288} \left( dx^{11} (-3\alpha) + dx^{\overline{b}} \cdot 12i\beta_{\overline{b}} + dx^{a} \cdot 36i\beta_{a} \right).$$

$$(2.25)$$

The imaginary part of this equation gives

$$g^{b\overline{b}}\partial_b h_{a\overline{b}} = \frac{\sqrt{2}i}{3}\beta_a. \tag{2.26}$$

The real part gives

$$\partial_{11}\psi + \frac{\sqrt{2}}{96}\alpha = 0$$

$$\partial_{\overline{a}}\psi + \frac{\sqrt{2}i}{24}\beta_{\overline{a}} = 0.$$
(2.27)

which determine  $\psi$  in terms of  $\alpha$  and  $\beta$ , that is in terms of G, up to an irrelevant additive constant. However, we need to verify that (with a suitable choice of the constant) the solution  $\psi$  of those equations is real. To prove that  $\operatorname{Im} \psi$  is a constant, it suffices to prove that  $g^{a\overline{b}}\partial_a\partial_{\overline{b}}\operatorname{Im}\psi=0$ . In fact, using the second equation in (2.27) (and its complex conjugate) together with (2.15), one gets

$$g^{a\overline{b}}\partial_a\partial_{\overline{b}}\operatorname{Im}\psi = g^{a\overline{b}}\partial_{\overline{b}}\partial_a\left(\frac{\psi - \overline{\psi}}{2i}\right) = \frac{1}{24\sqrt{2}}D^A\beta_A = 0.$$
 (2.28)

Furthermore, comparison of the first equation in (2.24) to the second in (2.27) shows that

$$\gamma = 8\psi + f(x^{11}). \tag{2.29}$$

It is impossible to determine f, as f can be changed arbitrarily by a reparametrization of  $x^{11}$ , which has not yet been fixed. One natural coordinate condition would be f = 0; another would be

$$\int_{X} \gamma \sqrt{g} d^6 x = 0. \tag{2.30}$$

Finally, the terms proportional to  $\Gamma_{\mu}$  give

$$\beta_{\overline{a}} = -\frac{6i}{\sqrt{2}} \partial_{\overline{a}} b$$

$$\partial_{11} b = \frac{\sqrt{2}}{24} \alpha.$$
(2.31)

Comparing to the above this implies

$$b = -4\psi \tag{2.32}$$

up to an irrelevant additive constant.

What remains is to determine h. The three equations that we have not yet used in determining  $\psi$ ,  $\gamma$ , and b may be collected as follows

$$\partial_{\overline{a}}h_{b\overline{b}} - \partial_{\overline{b}}h_{b\overline{a}} = -\sqrt{2}\left(G_{b\overline{a}\overline{b}}_{11} + \frac{i}{6}\left(g_{b\overline{a}}\beta_{\overline{b}} - g_{b\overline{b}}\beta_{\overline{a}}\right)\right)$$

$$g^{\overline{b}b}\partial_{\overline{b}}h_{b\overline{a}} = -\frac{\sqrt{2}i}{3}\beta_{\overline{a}}$$

$$\partial_{11}h_{a\overline{b}} = -\frac{1}{\sqrt{2}}\left(i\theta_{a\overline{b}} - \frac{1}{12}\alpha g_{a\overline{b}}\right)$$

$$(2.33)$$

and must serve to determine h. The integrability condition for the first equation in (2.33) is that the right hand side should be annihilated by  $\overline{\partial}$ . This is true according to (2.8) and (2.10). Given this, according to standard Hodge theory, at fixed  $x^{11}$ , the first two equations in (2.33) have a common solution which is unique up to the possibility of adding to h a harmonic (1,1) form on X. The  $x^{11}$  dependence is then to be determined from the last equation in (2.33). This will determine h uniquely up to the possibility of adding an  $x^{11}$ -independent harmonic (1,1) form, which is the expected ambiguity corresponding to a displacement of the Kahler moduli of the vacuum. The only remaining point is that the last equation in (2.33) is compatible with the first two. Compatibility of the first and third equations in (2.33) is the statement that  $\partial_{11}$  of the right hand side of the first equation equals  $\overline{\partial}$  of the right hand side of the third. This can be verified using (2.11), (2.13), and (2.14). Compatibility of the second and third equations in (2.33) can likewise be verified using (2.13) and (2.14).

### 2.2. Lower Bound On Newton's Constant

Now, as anticipated in the introduction, let us try to estimate the lower bound on Newton's constant subject to the validity of the calculation that we have performed. We let  $v(x^{11}) = \int_X \sqrt{g} d^6x$  be the volume of X at given  $x^{11}$ . As explained in the introduction, the GUT coupling is  $\alpha_{GUT} = (4\pi\kappa^2)^{2/3}/2V$ , where V is the value of v at the  $\mathbb{Z}_2$  fixed point. However, v will have different values at the two fixed points  $x^{11} = 0$  and  $x^{11} = \pi \rho$ . We set V = v(0) and undertake to compute  $v(\pi \rho)$ .

The starting point is simply that

$$\frac{\partial}{\partial x^{11}}v = \int_{X} \frac{1}{2}g^{AB}\partial_{11}h_{AB}\sqrt{g}d^{6}x = \int_{X} g^{a\overline{b}}\partial_{11}h_{a\overline{b}}\sqrt{g}d^{6}x. \tag{2.34}$$

So, using the last equation in (2.33),

$$\frac{\partial}{\partial x^{11}}v = -\frac{1}{4\sqrt{2}} \int_X \alpha \sqrt{g} d^6x. \tag{2.35}$$

According to (2.16), the right hand side is independent of  $x^{11}$ , so v will vary linearly in  $x^{11}$ . Generically,  $\alpha$  will be non-zero. If, for example,  $\alpha$  is positive, then  $v(0) > v(\pi \rho)$ . The  $E_8$  which is located at  $x^{11} = 0$  is therefore the more weakly coupled of the two. If we understand  $\alpha_{GUT}$  to be the coupling of the more weakly coupled of the two  $E_8$ 's, then if one keeps  $\alpha_{GUT}$  fixed and increases  $\rho$ , the coupling of the second  $E_8$  will diverge at a finite value of  $\rho$ , where v = 0. Newton's constant, meanwhile, decreases with increasing  $\rho$ . The smallest possible value of Newton's constant, in a regime in which the eleven-dimensional calculation is valid, is the value at which v = 0 and the coupling in the second  $E_8$  diverges. The occurrence at this point of "infinite bare coupling" in one  $E_8$  is extremely interesting; it is reminiscent of an unexpected failure of perturbation theory found in [21], and also is related to the phase transition at finite heterotic string coupling noted in [30] (which arises in just the same way in K3 ×  $\mathbf{S}^1/\mathbf{Z}_2$  compactification, as will be clear in the next section).

Since  $\alpha$  is independent of  $x^{11}$ , we can determine it by looking at the limit as  $x^{11}$  approaches zero from above. According to [23], the limiting value of  $G_{ABCD}$  is

$$G_{ABCD} = -\frac{3}{2\pi\sqrt{2}} \left(\frac{\kappa}{4\pi}\right)^{2/3} \left(F_{[AB}F_{CD]} - \frac{1}{2}R_{[AB}R_{CD]}\right). \tag{2.36}$$

This implies that

$$\sqrt{g}\alpha = \sqrt{g}\omega^{AB}\omega^{CD}G_{ABCD} = -\frac{2}{\pi\sqrt{2}}\left(\frac{\kappa}{4\pi}\right)^{2/3}\omega\wedge\left(\operatorname{tr}F\wedge F - \frac{1}{2}\operatorname{tr}R\wedge R\right). \tag{2.37}$$

So

$$\frac{\partial v}{\partial x^{11}} = 2\pi \left(\frac{\kappa}{4\pi}\right)^{2/3} \int_{X} \omega \wedge \frac{\operatorname{tr} F \wedge F - \frac{1}{2} \operatorname{tr} R \wedge R}{8\pi^{2}}$$
 (2.38)

and hence in this approximation

$$V' = v(\pi\rho) = V + 2\pi^2 \rho \left(\frac{\kappa}{4\pi}\right)^{2/3} \int_{Y} \omega \wedge \frac{\operatorname{tr} F \wedge F - \frac{1}{2} \operatorname{tr} R \wedge R}{8\pi^2}.$$
 (2.39)

Note that, if the "instanton number" is larger at  $x^{11} = 0$  than at  $\pi \rho$ , the integral on the right hand side of (2.39) is negative; in fact it follows from the supersymmetric relation

 $\omega^{AB}F_{AB}=0$  that  $\omega \wedge \mathrm{tr} F \wedge F$  is negative. The critical value of  $\rho$ , at which V'=0, is thus in this approximation

$$\rho_c = \frac{V}{2\pi^2 \left(\frac{\kappa}{4\pi}\right)^{2/3} \left| \int_X \omega \wedge \frac{\operatorname{tr} F \wedge F - \frac{1}{2} \operatorname{tr} R \wedge R}{8\pi^2} \right|}.$$
 (2.40)

Let W be the volume of  $X \times \mathbf{S}^1$ . Newton's constant is

$$G_N = \kappa^2 / 8\pi W \tag{2.41}$$

(this formula is given in (1.13), using the tree level expression  $W = 2\pi\rho V$ ). In the approximation of taking v to vary linearly (strictly valid only to lowest non-trivial order in  $\kappa^{2/3}$ ), the volume at the critical point is  $W_c = \pi V \rho_c$ . (This comes by multiplying the circumference,  $2\pi\rho$ , of the  $\mathbf{S}^1$  times the average value V/2 of the K3 volume.) This upper bound on W, when inserted in (2.41), gives the lower bound on  $G_N$  claimed in (1.15).

Since we treated the supergravity only in a linearized approximation, the precise critical values we estimated for  $\rho$ , W, and  $G_N$  are uncertain to within factors of order one. However, it is possible, by looking at the gauge couplings, to be more precise about the fact that there is a breakdown of the low energy supergravity, at roughly the point found above. The inverse of the  $E_8$  gauge coupling at  $x^{11} = 0$  is, from (1.13),

$$\frac{1}{\alpha_{GUT}} = \frac{2V}{(4\pi\kappa^2)^{2/3}}. (2.42)$$

The inverse of the second  $E_8$  gauge coupling  $\tilde{\alpha}$  is given by a similar formula with V replaced by V', so from (2.39)

$$\frac{1}{\widetilde{\alpha}} = \frac{2(V - \rho \sum_{a} c_a \omega_a)}{(4\pi\kappa^2)^{2/3}},\tag{2.43}$$

with  $\omega_a$  the periods of  $\omega$  and  $c_a$  certain constants that can be found from (2.39). The number of independent periods of  $\omega$  is  $h = \dim H^{1,1}(X)$ . The pseudoscalar partners of V and the  $\omega_a$  are "axions" that decouple at zero momentum in the approximation of elevendimensional supergravity (where one ignores Yang-Mills instantons and also the membrane instantons that are related to world-sheet instantons of string theory). From V and the  $\omega_a$  one can make h + 1 functions  $r_\lambda$ ,  $\lambda = 0, \ldots, h$ , which (up to linear transformations and addition of constants) are determined uniquely by the following conditions: they are real parts of chiral superfields and are invariant under shifts of the axions. The usual constraints of holomorphy together with the axion decoupling imply that  $1/\tilde{\alpha}$  must be a

linear combination of these functions. From (2.43), we see that in the approximation that we have considered, the  $r_{\lambda}$  are  $r_0 = V$  and  $r_a = \rho \omega_a$ , a = 1, ..., h. (2.43) suffices to determine the coefficients when  $1/\tilde{\alpha}$  is expressed as a linear combination of the  $r_{\lambda}$ , and shows that in the supergravity approximation  $1/\tilde{\alpha}$  goes to zero at a certain finite value of the  $r_{\lambda}$ . At that point, the low energy supergravity approximation breaks down, and instantons in the second  $E_8$  must be taken into account.

# 3. Compactification To Six Dimensions

In this section, we will apply similar ideas to supersymmetric compactifications from eleven to six dimensions with non-vanishing G field. The main examples are compactification on  $K3 \times S^1/\mathbb{Z}_2$ , related to the heterotic string on K3, compactification on  $\mathbb{T}^5/\mathbb{Z}_2$ , related [33,34] to Type IIB on K3, and various other  $\mathbb{Z}_2$  orbifolds of  $K3 \times S^1$  [35], some apparently related to K3 orientifolds discussed in [36] and F-theory compactifications [37]. On  $K3 \times S^1/\mathbb{Z}_2$ , five-branes can be included if one wishes [30], and in the other examples five-branes must be included [34] to neutralize the overall magnetic charge.

There are three types of source for the G field. A five-brane is a delta function source of strength 1, a codimension five  $\mathbb{Z}_2$  orbifold singularity is a source of strength -1/2 [34], and a codimension one  $\mathbb{Z}_2$  orbifold singularity contributes a source term, proportional to  $\operatorname{tr} F \wedge F - (1/2)\operatorname{tr} R \wedge R$ , which figured in the last section.

Despite the diversity of examples, they can all be treated together. Moreover, the results are much simpler than for compactification to four dimensions, and it is straightforward to go beyond the linearized approximation used in the last section and find the exact supersymmetric solution of the supergravity theory to all orders in  $\kappa$ . This gives a description that is valid even when the corrections from turning on G are big, as long as all the relevant length scales are large compared to the eleven-dimensional Planck length. That condition breaks down under certain conditions, but treating the non-linear terms will enable us to get a better understanding of how it breaks down and thus a better understanding of how the strong coupling conundrum explained in [30] appears in this framework.

We will work on  $M^{11} = \mathbf{R}^6 \times K$ , where K may be  $K3 \times \mathbf{S}^1$  or  $\mathbf{T}^5$ . We really want a  $\mathbf{Z}_2$  orbifold of  $\mathbf{R}^6 \times K$  (with the  $\mathbf{Z}_2$  acting on K only), which we describe by giving a

 $\mathbf{Z}_2$ -invariant configuration on  $\mathbf{R}^6 \times \mathbf{K}$ . We write the starting line element on  $M^{11}$  (valid to lowest order in  $\kappa$ , before turning on G) as

$$ds_{(0)}^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu} + g_{AB} dx^A dx^B, \tag{3.1}$$

where  $x^{\mu}$ ,  $\mu = 1, ..., 6$  are the coordinates of  $\mathbf{R}^6$  and  $x^A$  are local coordinates on K; the first term is the Minkowski metric on  $\mathbf{R}^6$ , and the second is a standard metric on K (that is, the metric on K is the product of a hyper-kahler metric on K3 times a flat metric on  $\mathbf{S}^1$  if  $\mathbf{K} = \mathbf{K}3 \times \mathbf{S}^1$ , or is a flat metric on K if  $\mathbf{K} = \mathbf{T}^5$ ). Indices  $\mu, \nu$  will run from 1 to 6, A, B, C... from 5 to 11, and I, J, K, ... from 1 to 11. (3.1) is the metric on  $M^{11}$  to lowest order in  $\kappa$ , with the G field ignored. With G turned on, it will turn out that the metric is of the form

$$ds^{2} = e^{b} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + e^{f} g_{AB} dx^{A} dx^{B}, \tag{3.2}$$

where b and f are functions on K that will be determined.

As in the last section, the starting point is to determine  $G_{ABCD}$  from the equations

$$D^{A}G_{ABCD} = 0$$

$$dG = \text{sources.}$$
(3.3)

There is a subtlety here which is the key to eventually obtaining a simple solution. The first equation in (3.3) depends on the metric (3.2) on  $M^{11}$ , which we do not yet know; we only know the starting metric (3.1). It turns out, however, that if we determine  $G_{ABCD}$  (as a differential form, that is with all indices down!) using the starting metric, then the same G will solve (3.3) in the exact metric. This depends on the following circumstance. Solving (3.3) in the starting metric is equivalent to finding a function w such that

$$G_{ABCD} = -\epsilon_{ABCDE}^{(0)} \partial^E w \tag{3.4}$$

– with  $\epsilon^{(0)}$  the completely antisymmetric tensor in the metric  $ds_{(0)}^2$  – such that

$$\nabla_0 w = \text{sources},$$
 (3.5)

where  $\nabla_0$  is the Laplacian in the original metric, and the "sources" in (3.5) are derived from the sources in the Bianchi identity for G, that is the second equation in (3.3). In particular, (3.5) determines w uniquely up to an irrelevant additive constant. Changing the metric on K will not affect the Bianchi identity, which is completely metric-independent. It will, however, change the equations of motion. In the corrected metric (3.2), the relation of G to w is

$$G_{ABCD} = -e^{-3f/2} \epsilon_{ABCDE} \partial^E w \tag{3.6}$$

where now  $\epsilon$  is the  $\epsilon$  tensor in the corrected metric. The equation of motion for G – that is, the first equation in (3.3) – then becomes

$$d(e^{-3f/2}dw) = 0. (3.7)$$

For general f, this would not hold, but when we actually solve for f, we will find that f = F(w) for some function F, which ensures that (3.7) holds. To recapitulate then, we first solve (3.3) and determine w; then we find the corrected metric, determining b and f, which will be such that (3.3) holds also in the new metric.

For any of the compactifications of interest, the unbroken supersymmetries, in the limit of ignoring G, are generated by covariantly constant spinor fields  $\eta$  that obey

$$\Gamma_{ABCDE}\eta = \epsilon_{ABCDE}\eta. \tag{3.8}$$

(For the case of compactification on K3 ×  $\mathbf{S}^1/\mathbf{Z}_2$ , one also has  $\Gamma_{11}\eta = \eta$ , but we will not need to use this.) As in section two, we first compute

$$\frac{\sqrt{2}}{288}dx^{I}\left(\Gamma_{IJKLM} - 8g_{IJ}\Gamma_{KLM}\right)G^{JKLM}\eta = \frac{\sqrt{2}e^{-3f/2}}{288}\left(-24dx^{A}\partial_{A}w - 24dx^{\mu}\Gamma_{\mu}\Gamma^{A}\partial_{A}w + 48dx^{A}\Gamma_{AB}\partial^{B}w\right)\eta.$$
(3.9)

Then, introducing a rescaled spinor field  $\tilde{\eta} = e^{-\psi} \eta$ , we compute that the covariant derivative of  $\tilde{\eta}$  in the corrected metric (3.2) is

$$dx^{I}D_{I}\widetilde{\eta} = \left(-dx^{A}\partial_{A}\psi - \frac{1}{8}dx^{\mu}\Gamma_{\mu}\Gamma^{A}\partial_{A}b - \frac{1}{8}dx^{A}\partial^{B}f\Gamma_{AB}\right)\widetilde{\eta}.$$
 (3.10)

The condition (2.2) of unbroken supersymmetry then gives

$$-dx^{A}\partial_{A}\psi - \frac{1}{8}dx^{\mu}\Gamma_{\mu}\Gamma^{A}\partial_{A}b - \frac{1}{8}dx^{A}\partial^{B}f\Gamma_{AB}$$

$$+ \frac{\sqrt{2}e^{-3f/2}}{12} \left( -dx^{A}\partial_{A}w - dx^{\mu}\Gamma_{\mu}\Gamma^{A}\partial_{A}w + 2dx^{A}\Gamma_{AB}\partial^{B}w \right) \eta = 0.$$

$$(3.11)$$

Setting successive terms to zero, one can readily determine f, b, and  $\psi$ . In particular, one gets

$$\frac{1}{8}\partial_B f = \frac{\sqrt{2}e^{-3f/2}}{6}\partial_B w,\tag{3.12}$$

so that

$$e^{3f/2} = c + 2\sqrt{2}w\tag{3.13}$$

with c a constant. (In particular, f is as promised above a function of w, so that the differential form G is uncorrected from the solution found with the unperturbed metric.) Moreover, one gets

$$\frac{1}{8}\partial_A b = -\frac{e^{-3f/2}\sqrt{2}}{12}\partial_A w = -\frac{\sqrt{2}}{12}\frac{\partial_A w}{c + 2\sqrt{2}w}.$$
 (3.14)

so that up to an irrelevant additive constant,

$$b = -\frac{1}{3}\ln(c + 2\sqrt{2}w). \tag{3.15}$$

The metric (3.2) thus turns out to be

$$ds^{2} = (c + 2\sqrt{2}w)^{-1/3}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + (c + 2\sqrt{2}w)^{2/3}g_{AB}dx^{A}dx^{B}.$$
 (3.16)

As a check, this can be compared to the special case of the eleven-dimensional extreme five-brane solution, as obtained by Guven [38]. In this case, K is replaced by  $\mathbf{R}^5$ , and the "unperturbed" metric on  $\mathbf{R}^{11} = \mathbf{R}^6 \times \mathbf{K}$  (before turning on the G field) is simply the flat metric  $\eta_{\mu\nu}dx^{\mu}dx^{\nu} + \delta_{AB}dx^Adx^B$ . G is then taken to be the magnetic field due to a point charge at  $x^A = 0$ , so that  $w = q/R^3$ , where  $R = \sqrt{x_A x^A}$  and q is the charge. By scaling one may set c = 1. If we let

$$\Delta^{-1} = c + 2\sqrt{2}w = 1 + \frac{2\sqrt{2}q}{R^3},\tag{3.17}$$

then the metric (3.16) becomes

$$ds^{2} = \Delta^{1/3} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + \Delta^{-2/3} \delta_{AB} dx^{A} dx^{B}, \qquad (3.18)$$

which is a standard form of the five-brane solution. Similarly, one may take

$$w = \sum_{i} \frac{q_i}{|\vec{x} - \vec{x}_i|^3} \tag{3.19}$$

and recover a solution with parallel five-branes in  $\mathbf{R}^{11}$ .

Note that as long as q > 0, which is the correct sign of the charge for a five-brane that obeys the supersymmetry condition (3.8) (anti-fivebranes with q < 0 would require a supersymmetry condition obtained by changing the sign on the right hand side of (3.8)),

the function  $\Delta^{-1} = c + 2\sqrt{2}w$  is positive definite throughout  $\mathbf{R}^5 - \{0\}$ , so the metric is completely sensible outside of the origin. (It can in fact also be continued past R = 0 [39].) That is compatible with the fact that (by considering a superposition of many parallel five-branes that become coincident at the origin) one could obtain arbitrarily big q, so that the description by the long wavelength equations we have been using would be valid arbitrarily close to R = 0.

An individual five-brane has a charge quantum  $q = q_0$  with  $q_0$  of order  $\kappa^{2/3}$ . For such an individual five-brane, the solution given above in terms of classical supergravity is only valid at  $R >> \kappa^{2/9}$ , that is, it breaks down at a Planck length from R = 0.

There is one important situation in which – keeping the supersymmetry condition precisely as in (3.8) – one does meet an object very similar to a five-brane but with q < 0. This is the case of a codimension five  $\mathbb{Z}_2$  fixed point, which carries [34] a five-brane charge  $q = -q_0/2$ . In this case, positivity fails at R of order  $|q|^{1/3}$ , that is at R of order the Planck length, and the description breaks down there. A macroscopic failure of the description would occur if one could get a macroscopic negative q. One cannot, however, use  $\mathbb{Z}_2$  orbifold singularities to obtain a fivebrane charge more negative than  $-q_0/2$  except by making the singularities meet, obtaining a worse singularity of space-time that would need a separate analysis.

Now we consider the other case in which the sources are codimension one  $\mathbb{Z}_2$  fixed points. This arises for  $K = K3 \times S^1/\mathbb{Z}_2$ . For brevity, we will omit the five-branes; their inclusion would not greatly change things.

We start with a bare metric  $ds_{(0)}^2$  on  $K3 \times S^1$  such that the K3 has volume  $V_0$  and the  $S^1$  has circumference  $\rho_0$ . (More specifically, take the metric on the  $S^1$  to be simply  $(dx^{11})^2$  where  $x^{11}$  runs from 0 to  $2\pi\rho_0$ .) Then we determine w by the equation  $\nabla_0 w$  = "sources." This only determines w up to an additive constant. To fix the constant, we may proceed as follows. On the interior of  $K3 \times S^1/\mathbb{Z}_2$ , w is harmonic and so cannot have a minimum. (This is still true if five-branes are present, as  $w \to +\infty$  near a five-brane.) The minimum of w is thus automatically at one of the  $\mathbb{Z}_2$  fixed points, and there is no essential loss in assuming that this is at  $x^{11} = \pi\rho_0$ . By adding a constant to w, one can suppose that w vanishes at its minimum. The solution (3.16) for the exact metric is thus regular as long as c > 0, and develops a singularity on the boundary precisely at c = 0. This is the singularity encountered in [30] and also in section two of the present paper; we would like to understand it a little better.

When the dimensionless number  $\rho_0/V_0^{1/4}$  is of order one, by the time one reaches c=0, the volume of K3 ×  $\mathbf{S}^1/\mathbf{Z}_2$  is Planckian and our approximations are no longer valid. The interesting case to look at is  $\rho_0/V_0^{1/4} >> 1$ . In this case, the qualitative behavior of w is as follows.

Define a function of  $x^{11}$  by

$$z(x^{11}) = \int_{K3} w\sqrt{g_{(0)}} d^4x, \qquad (3.20)$$

with the integral taken at fixed  $x^{11}$ . The equation  $\nabla_{(0)}w=0$  implies that

$$\frac{\partial^2 z}{\partial (x^{11})^2} = 0 \tag{3.21}$$

so that  $\partial z/\partial x^{11}$  is constant. The value of the constant follows from (2.3):

$$\frac{\partial z}{\partial x^{11}} = -6\pi\sqrt{2} \left(\frac{\kappa}{4\pi}\right)^{2/3} (k-12),\tag{3.22}$$

where k is the instanton number at  $x^{11}=0$ . (The instanton number at  $x^{11}=\pi\rho_0$  is 24-k so that the Bianchi identity has a solution.) Thus for k=12, z is constant and the volume of the K3 remains bounded away from zero even for  $\rho\to\infty$ ; this is the case studied in [30], which leads to the most straightforward string-string duality. We want to look at the case  $k\neq 12$  where a singularity will develop at finite  $\rho$ . Up to a change of coordinates  $x^{11}\to\pi\rho_0-x^{11}$ , we can assume that k>12; in fact, assuming as above that the minimum of w is at  $x^{11}=\pi\rho_0$  (rather than  $x^{11}=0$ ) is equivalent to taking k>12.

The case in which our approximations say something interesting about the singularity is the case in which  $\rho_0^4/V_0 >> 1$ . In this limit, K becomes a long tube, and one can to good approximation ignore the dependence of w on the "small" K3 directions, so w is almost a function of  $x^{11}$  only. In that approximation, the Laplace equation for w reduces to  $\partial^2 w/\partial (x^{11})^2 = 0$ , with solution

$$w = \frac{\pi \rho_0 - x^{11}}{V_0} 6\pi \sqrt{2} \left(\frac{\kappa}{4\pi}\right)^{2/3} (k - 12). \tag{3.23}$$

The error in this formula is of order

$$\overline{w} = \frac{\kappa^{2/3}}{V_0^{3/4}}. (3.24)$$

(This is estimated as follows. If one studies the equation for w near  $x^{11}=0$  or  $x^{11}=\pi\rho$ , one sees that the deviation from (3.23) is determined by a linear equation and a boundary condition that are independent of  $\rho$  for  $\rho \to \infty$ , so the correction to (3.23) is  $\rho$ -independent for large  $\rho$ ; (3.24) then follows by dimensional analysis given that the source term in the equation is proportional to  $\kappa^{2/3}$ .)  $\overline{w}$  can also serve as an estimate for the order of magnitude of w at  $x^{11}=\pi\rho_0$ .

The volume V of the K3 at  $x^{11} = 0$  (in the exact metric), is

$$V = V_0(c + 2\sqrt{2}w(x^{11} = 0))^{4/3}. (3.25)$$

At the critical point, c = 0, this volume is

$$V = \left(24\pi^2 \left(\frac{\kappa}{4\pi}\right)^{2/3} (k-12)\right)^{4/3} \left(\frac{\rho_0^4}{V_0}\right)^{1/3}.$$
 (3.26)

The critical value of  $\rho$  is  $\rho = \frac{1}{\pi} \int_0^{\pi \rho_0} dx^{11} e^{f/2}$  or

$$\rho = \frac{3}{4} \left(\frac{\rho_0^4}{V_0}\right)^{1/3} \left(24\pi^2 \left(\frac{\kappa}{4\pi}\right)^{2/3} (k-12)\right)^{1/3}.$$
 (3.27)

So the critical value of  $V/\rho$  is

$$\frac{V}{\rho} = 32\pi^2 (k - 12) \left(\frac{\kappa}{4\pi}\right)^{2/3}.$$
 (3.28)

What characterizes the critical point is that, while the gauge coupling is small in one  $E_8$ , it becomes strong in the second  $E_8$ . In fact, for sufficiently big  $\rho_0^4/V_0$ , the critical value of the volume V in (3.26) can be as big as one wants, the coupling in the  $E_8$  that is supported at  $x^{11} = 0$  then being of order 1/V. On the other hand, using (3.24), the volume in the second  $E_8$  at the critical point is of order  $\overline{w}^{4/3}V_0 = \kappa^{8/9}$ , that is of order the Planck volume, independent of  $\rho_0/V_0$ . The gauge coupling in the second  $E_8$  is thus apparently of order one at the critical point.

In [30], it appeared that the gauge coupling in the second  $E_8$  is infinite at the critical point, corresponding to the K3 at  $x^{11} = \pi \rho_0$  having zero volume there. In the present approach, the volume appears to be Planckian rather than zero (and therefore the gauge coupling appears to be of order one rather than infinite). But as our approximations in the present discussion break down in any case when the volume is Planckian, it does not seem that we have any real evidence for a phase transition occurring while the gauge coupling is still finite.

## References

- [1] X.-G. Wen and E. Witten, "Electric And Magnetic Charges In Superstring Models," Nucl. Phys. **B261** (1985) 651.
- [2] P. Ginsparg, "Gauge and Gravitational Couplings in Four-Dimensional String Theories", Phys. Lett. B197 (1987) 139.
- [3] V.S. Kaplunovsky, "One Loop Threshold Effects In String Unification", Nucl. Phys. B307 (1988) 145, erratum Nucl. Phys. B382 (1992) 436.
- [4] I. Antoniadis, J. Ellis, R. Lacaze, and D.V. Nanopoulos, "String Threshold Corrections and Flipped SU(5)", Phys. Lett. B268 (1991) 188; S. Kalara, J. L. Lopez, and D. V. Nanopoulos, "Threshold Corrections And Modular Invariance In Free Fermionic Strings," Phys. Lett. B269 (1991) 84.
- [5] P. Mayr, H.P. Nilles, and S. Stieberger, "String Unification and Threshold Corrections", Phys. Lett. B317 (1993) 53.
- [6] D.C. Lewellen, "Embedding Higher-Level Kac-Moody Algebras in Heterotic String Models", Nucl. Phys. B337 (1990) 61.
- [7] A. Font, L.E. Ibanez, and F. Quevedo, "Higher-Level Kac-Moody String Models and Their Phenomenological Implications", Nucl. Phys. B345 (1990) 389.
- [8] G. Aldazabal, A. Font, L.E. Ibanez, A.M. Uranga, "String GUT's", Nucl. Phys. B452 (1995) 3, "Building GUTS From Strings," hep-th/9508033.
- [9] S. Chaudhuri, S.-W. Chung, G. Hockney, and J. Lykken, "String Consistency For Unified Model Building"; S. Chaudhuri, G. Hockney, and J. Lykken, "Three Generations In The Fermionic Construction."
- [10] D. Finnell, "Grand Unification with Three Generations in Free Fermionic String Models," hep-th/9508073.
- [11] L.E. Ibanez, "Gauge Coupling Unification: Strings vs. SUSY GUT's", Phys. Lett. B318 (1993) 73.
- [12] K. R. Dienes, A. E. Faraggi, and J. March-Russell, "String Unification, Higher-Level Gauge Symmetries, and Exotic Hypercharge Normalizations," hep-th/9510223.
- [13] L.J. Dixon, V.S. Kaplunovsky, and J. Louis, "Moduli Dependence of String Loop Corrections to Gauge Coupling Constants", Nucl. Phys. B355 (1991) 649.
- [14] K. R. Dienes and A. E. Faraggi, "Making Ends Meet: String Unification And Low-Energy Data," Phys. Rev. Lett. 75 (1995) 2646, "Gauge Coupling Unification In Realistic Free-Fermionic String Models," Nucl. Phys. B457 (1995) 409.
- [15] J. L. Lopez, D. V. Nanopoulos, and K. Yuan, "The Search For A Realistic Flipped SU(5) Standard Model"; J. L. Lopez and D. V. Nanopoulos, "A New Scenario For String Unification," hep-ph/9511426.
- [16] K. R. Dienes, "String Theory And The Path To Unification: A Review Of Recent Developments," hep-th/9602045.

- [17] T. Banks and M. Dine, "Coping With Strongly Coupled String Theory," Phys. Rev. D50 (1994) 7454; M. Dine, "Coming To Terms With Strongly Coupled Strings," hep-th/9508085.
- [18] E. Witten, "String Theory Dynamics In Various Dimensions," Nucl. Phys. **B433** (1995) 85.
- [19] A. Dabholkar, "Ten Dimensional Heterotic String As A Soliton," Phys. Lett. B357 (1995) 307.
- [20] C. M. Hull, "String-String Duality In Ten Dimensions," Phys. Lett. **B357** (1995) 545.
- [21] J. Polchinski and E. Witten, "Evidence For Heterotic Type I String Duality," hep-th/9510169, to appear in Nucl. Phys. **B460** (1996) 525.
- [22] P. Horava and E. Witten, "Heterotic And Type I String Dynamics From Eleven Dimensions," hep-th/9510209, to appear in Nucl. Phys. **B460** (1996) 506.
- [23] P. Horava and E. Witten, to appear.
- [24] L. Witten and E. Witten, "Large Radius Expansion Of Superstring Compactifications," Nucl. Phys. **B281** (1987) 109.
- [25] E. Witten, "Five-Branes and M-Theory On An Orbifold," hep-th/9512219.
- [26] K. Dasgupta and S. Mukhi, "Orbifolds Of M-Theory," hepth/9512196.
- [27] M. Dine and N. Seiberg, "Is The Superstring Weakly Coupled?" Phys. Lett. 162B (1985) 299, "Is The Superstring Semiclassical?" in *Unified String Theories*, eds. M. B. Green and D. J. Gross (World Scientific, 1986).
- [28] M. Dine and Y. Shirman, "Truly Strong Couplings And Large Radius In String Theory" to appear.
- [29] E. Bergshoeff, M. de Roo, B. De Wit, and P. van Nieuwenhuysen, "Ten-Dimensional Maxwell-Einstein Supergravity, Its Currents, And The Issue Of Its Auxiliary Fields," Nucl. Phys. B195 (1982) 97.
- [30] M. Duff, R. Minasian, and E. Witten, "Evidence For Heterotic/Heterotic Duality," hep-th/9601036.
- [31] K. Becker, M. Becker, and A. Strominger, "Five-Branes, Membranes, And Nonperturbative String Theory," Nucl. Phys. **B456** (1995) 130.
- [32] M. J. Duff, R. Minasian, and J. T. Liu, "Eleven-Dimensional Origin Of String/String Duality," Nucl. Phys. **B452** (1995) 261.
- [33] K. Dasgupta and S. Mukhi, "Orbifolds Of M Theory," hep-th/9512196.
- [34] E. Witten, "Five-branes And M Theory On An Orbifold," hep-th/9512219.
- [35] A. Sen, "M-Theory On K3  $\times$  S<sup>1</sup>/ $\mathbb{Z}_2$ ," to appear.
- [36] A. Dabholkar and J. Park, "An Orientifold Of Type IIB Theory On K3," hepth/9602030.
- [37] C. Vafa, "Evidence For F-Theory," hepth/9602022
- [38] R. Guven, Phys. Lett. **B276** (1992) 49.
- [39] M. J. Duff, G. W. Gibbons, and P. K. Townsend, Phys. Lett. **B332** (1994) 321.