The emergence of 3+1D Einstein gravity from topological gravity theory

Zheng-Cheng Gu¹

¹Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, China (Dated: April 11, 2019)

Quantum field theory successfully explains the origin of all fundamental forces except gravity due to the renormalizability problem. In this paper, we proposed a topological scenario to understand this puzzle. First, we proposed a 3+1D topological (quantum) gravity theory which is renormalizable, and it can be regarded as a straightforward generalization of Edward Witten's Chern-Simons theory approach to 2+1D topological gravity. Then, we showed that the (vacuum) Einstein-Cartan equation and classical space-time naturally emerge from topological (quantum) gravity via loop condensation. The second step is a unique feature in 3+1D and it might even naturally explain why our space-time is four dimensional.

Introduction – Recent discovery of gravitational wave by LIGO[1] verifies Einstein's theory of gravity and brings back the old paradox between general relativity and quantum mechanics. Naively, it has been argued that the absolute time in quantum mechanics is intrinsically inconsistent with diffeomorphism invariance and that it is impossible to construct a gravitational theory that can be consistently quantized.

On the other hand, the modern perspective of continuum field theory based on the concept of renomalization group (RG) suggests that any meaningful continuum field theory must emerge as an effective theory from an underlying RG fixed point, hence it must be a renormalizable quantum field theory(QFT). Indeed, the well known standard model is controlled by a conformal field theory (CFT) fixed point in the asymptotic freedom limit. Therefore, the essential task of defining a quantum theory of gravity becomes defining new types of RG fixed point that can reproduce Einstein's gravity in the semi-classical limit. Recent development in ADS/CFT correspondence conjecture provides us a novel example of defining d+1dimensional ADS-space quantum gravity in terms of ddimensional CFT[2, 3]. Nevertheless, the ADS/CFT correspondence does not work in De Sitter space. Hence a much more general physical concept and mathematical framework of understanding quantum gravity are very desired. The so-called loop quantum gravity(LQG) is a very interesting attempt along this direction [4–8].

Three decades ago, Edward Witten proposed to use Chern-Simons theory to reformulate 2 + 1D Einstein's gravity[9] and a consistent quantum gravity theory can be defined (at least perturbatively)in the absence of matter fields(with or without cosmological constant term). Although 2 + 1D gravity is somewhat trivial due to the absence of propagating gravitational wave and vanishing of space-time curvature, it still provides us a concrete example of understanding quantum gravity in terms of a topological quantum field theory(TQFT). Moreover, according to the correspondence between Chern-Simons theory and CFT, the ADS3/CFT2 correspondence conjecture can be understood in a very natural way[10].

Nevertheless, the TQFT approach can not be easily

generalized into 3+1D due to the following difficulties. (a) Einstein's gravity in 3+1D contains propagating mode – the gravitational wave, therefore it is obviously not a TQFT in the usual sense. (b) Our knowledge of higher dimensional TQFT is very limited and there is no Chern-Simons like action in 3+1D.

Thanks to the recent development of the classification of topological phases of quantum matter in higher dimensions[13-16], new types of TQFT have been discovered in 3 + 1D to describe the so-called three-loopbraiding statistics. In this paper, we argued that such a TQFT is closely related to Einstein gravity. In particular, we conjectured that gravitational wave will disappear at an extremely high energy scale and 3 + 1Dquantum gravity is indeed controlled by a TQFT RG fixed point. At an intermediate energy scale, Einstein gravity and classical space-time can emerge via loop(flux lines) condensation. In the loop condensed phase, the quantum fluctuation is controlled by a small parameter θ and the theory is still power-counting renormalizable. In the semi-classical limit, we can derive the same equation of motion as Einstein-Cartan equation (in the absence of matter fields). Furthermore, our theory predicts the noncommutative geometry between spin connection ω and curvature tensor R.

3+1D Topological gravity - In Edward Witten's pioneer work for 2 + 1D quantum gravity, he pointed out that the Einstein-Cartan action in 2 + 1D can be regarded as a Chern-Simons action $\int Tr[A \wedge (dA + A \wedge A)]$ where A is the gauge connection of Poincare group ISO(2,1)(SO(3,1) or SO(2,2) for nonzero cosmologicalconstant case). However, he further argued that since there is no $\int Tr[A \wedge A \wedge (dA + A \wedge A)]$ type topological quantum field theory (TQFT) in 3 + 1D, the corresponding Einstein-Cartan action can not be regarded as a TQFT and in fact it is even not a well defined renormalizable QFT. The tremendous efforts on super gravity theory[11, 12] and ADS/CFT correspondence conjecture all aim to developed a well defined QFT description for gravity. (We hesitate to mention super string theory here since its relevant part to physics still relies on super gravity and ADS/CFT correspondence.)

Instead of using super symmetry (SUSY) and ADS/CFT correspondence to define and understand quantum gravity, here we attempt to consider the problem from a different angle. We would like to ask: Is there any TQFT in 3+1D that is closely related to the Einstein gravity, e.g., can we realize Einstein gravity through a proper phase transition from a TQFT? There are several advantages in TQFT approach to quantum gravity, as having already been demonstrated in the 2+1D case. First, it is manifested renormalizable and

super symmetry is not necessary. Second, it can handle general cases with or without cosmological constant.

Surprisingly, recent development in condensed matter physics indicates that $\int Tr[A \wedge A \wedge (dA + A \wedge A)] + \int Tr(B \wedge F)$ type actions[17–19] do exist and might serve as the most general 3 + 1D TQFT that describes nontrivial three-loop-braiding statistics[20, 21]. For discrete gauge group, they are known as Dijkgraaf-Witten theories[22]. Now let us generalize the above action into Poincare group and define the following topological gauge theory:

$$S_{\text{Top}} = \frac{1}{2} \int \epsilon_{abcd} e^a \wedge e^b \wedge R^{cd} + \int B_{ab} \wedge R^{ab} + \int \tilde{B}_a \wedge T^a$$

$$= \frac{1}{4} \int d^4 x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} e_\mu{}^a e_\nu{}^b R_{\rho\sigma}{}^{cd} + \frac{1}{4} \int d^4 x \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu ab} R_{\rho\sigma}{}^{ab} + \frac{1}{4} \int d^4 x \epsilon^{\mu\nu\rho\sigma} \tilde{B}_{\mu\nu a} T_{\rho\sigma}{}^a$$

$$(1)$$

Here B, \tilde{B} are 2-form gauge fields which were first introduced in usual topological BF theory[23–25], and R, T are the usual curvature and torsion tensors:

$$R_{\rho\sigma}^{cd} = \partial_{\rho}\omega_{\sigma}^{cd} - \partial_{\sigma}\omega_{\rho}^{cd} + \omega_{\rho}^{ce}\omega_{\sigma e}^{d} - \omega_{\sigma}^{ce}\omega_{\rho e}^{d},$$

$$T_{\mu\nu}^{a} = \partial_{\mu}e_{\nu}^{a} - \partial_{\nu}e_{\mu}^{a} + \omega_{\mu}^{ab}e_{\nu b} - \omega_{\nu}^{ab}e_{\mu b}$$
(2)

The first term in the above action is the usual Einstein-Cartan action. It is easy to verify that such a topological action is invariant under local Lorentz symmetry transformation. Interestingly, the total action is actually invariant under the whole local Poincare symmetry transformation, if we properly define the gauge transformation of translational symmetry for 2-form gauge fields:

$$e_{\mu}{}^{a} \rightarrow e_{\mu}{}^{a} + D_{\mu}f^{a} \equiv e_{\mu}{}^{a} + \partial_{\mu}f^{a} + \omega_{\mu}{}^{ab}f_{b}$$

$$\tilde{B}_{\mu\nu a} \rightarrow \tilde{B}_{\mu\nu a} - \epsilon_{abcd}f^{b}R_{\mu\nu}{}^{cd}$$

$$B_{\mu\nu ab} \rightarrow B_{\mu\nu ab} - \frac{1}{2}(\tilde{B}_{\mu\nu a}f_{b} - \tilde{B}_{\mu\nu b}f_{a})$$
(3)

We note that the usual first order Einstein-Cartan action is not invariant under the above gauge transformation, and that's why it is not a well defined TQFT in 3 + 1D. In addition, we can also define the following gauge transformation for 2-form gauge fields $\tilde{B}_{\mu\nu a}$ and $B_{\mu\nu ab}$:

$$\begin{split} \tilde{B}_{\mu\nu a} \; &\rightarrow \; \tilde{B}_{\mu\nu a} + \partial_{\mu}\tilde{\xi}_{\nu a} - \partial_{\nu}\tilde{\xi}_{\mu a} + \omega_{\mu a}{}^{b}\tilde{\xi}_{\nu b} - \omega_{\nu a}{}^{b}\tilde{\xi}_{\mu b} \\ B_{\mu\nu ab} \; &\rightarrow \; B_{\mu\nu ab} - \frac{1}{2}[(\tilde{\xi}_{\mu a}e_{\nu b} - \tilde{\xi}_{\nu a}e_{\mu b}) - (\tilde{\xi}_{\mu b}e_{\nu a} - \tilde{\xi}_{\nu b}e_{\mu a})] \end{split}$$

and

$$B_{\mu\nu ab} \rightarrow B_{\mu\nu ab} + D_{\mu}\xi_{\nu ab} - D_{\nu}\xi_{\mu ab}$$

where the covariant derivative D_{μ} is defined as:

$$D_{\mu}\xi_{\nu ab} \equiv \partial_{\mu}\xi_{\nu ab} + \omega_{\mu a}{}^{c}\xi_{\nu cb} + \omega_{\mu b}{}^{c}\xi_{\nu ac} \tag{4}$$

Therefore the above action can be regarded as the 3+1D generalization of 2+1D topological gravity. Apparently, the beta function vanishes for $S_{\rm Top}$ and it is renormalizable. The argument is exactly the same as the 2+1D case, where the counterterms, if any, are integrals of local gauge invariant functional and can not renormalize the above action. Another straightforward argument is that for compact gauge groups, all the terms in the above actions are actually quantized and the beta function must vanish[17]. Similar to the 2+1D case, e, ω are dimension one operators while B is dimension two operator. Finally, the equation of motion implies the vanishing of curvature and torsion tensors.

$$R^{ab} = 0, \quad T^a = 0, \quad \nabla \tilde{B}_a = -\epsilon_{abcd} e^b \wedge R^{cd} = 0,$$

$$\nabla B_{ab} + \frac{1}{2} (\tilde{B}_a \wedge e_b - \tilde{B}_b \wedge e_a) = -\epsilon_{abcd} T^c \wedge e^d = 0$$

Quantization of topological gravity – Before discussing the possible connection with 3+1D Einstein gravity, let us proceed the standard canonical quantization for the above topological gravity and explain its underlying physics. The Lagrangian density reads:

$$\mathcal{L}_{\text{Top}} = \Pi^{i}{}_{ab}\partial_{0}\omega_{i}{}^{ab} + \pi^{i}{}_{a}\partial_{0}e_{i}{}^{a} + \frac{1}{2}\epsilon^{ijk}B_{0iab}R_{jk}{}^{ab} + \frac{1}{2}\epsilon^{ijk}\tilde{B}_{0ia}T_{jk}{}^{a} + e_{0}{}^{a}(\nabla_{i}\pi^{i}{}_{a} + \frac{1}{2}\epsilon^{ijk}\epsilon_{abcd}e_{i}{}^{b}R_{jk}{}^{cd}) + \omega_{0}{}^{ab}\left[\nabla_{i}\Pi^{i}{}_{ab} + \frac{1}{2}(\pi^{i}{}_{a}e_{i}{}^{b} - \pi^{i}{}_{b}e_{i}{}^{a})\right]$$
(5)

where the canonical momentums of $\omega_i{}^{ab}$ and $e_i{}^a$ are defined as $\Pi^i{}_{ab} = \frac{1}{2} \epsilon^{ijk} \epsilon_{abcd} e_j{}^c e_k{}^d + \frac{1}{2} \epsilon^{ijk} B_{jkab}$ and

 $\pi^{i}{}_{a} = \frac{1}{2} \epsilon^{ijk} \tilde{B}_{jka}$. Canonical quantization requires:

$$[\omega_{j}^{cd}(\mathbf{y}), \Pi^{i}{}_{ab}(\mathbf{x})] = i\delta^{i}_{j}\delta^{cd}_{ab}\delta(\mathbf{x} - \mathbf{y}),$$

$$[e_{j}^{b}(\mathbf{y}), \pi^{i}{}_{a}(\mathbf{x})] = i\delta^{i}_{j}\delta^{b}_{a}\delta(\mathbf{x} - \mathbf{y})$$
all other commutators = 0 (6)

with the following flat-connection constraints:

$$\frac{1}{2} \epsilon^{ijk} R_{jk}{}^{ab} = 0, \quad \frac{1}{2} \epsilon^{ijk} T_{jk}{}^{a} = 0$$

$$\nabla_{i} \pi^{i}{}_{a} = -\frac{1}{2} \epsilon^{ijk} \epsilon_{abcd} e_{i}{}^{b} R_{jk}{}^{cd} = 0$$

$$\nabla_{i} \Pi^{i}{}_{ab} + \frac{1}{2} (\pi^{i}{}_{a} e_{i}{}^{b} - \pi^{i}{}_{b} e_{i}{}^{a}) = 0$$
(7)

Similar to the 2+1D topological gravity, the phase-space to be quantized is exactly the solutions of above constraints divided by the group of gauge transformations generated by the constraints. The quantum Hilbert space is the flat connections of Poincare group modulo gauge transformations.(If we regard e and ω as coordinates while π and Π as momentums.)Of course, in order to define an ultraviolet(UV) complete theory, it is much better to use the algebraic framework of tensor 2-category theory[26, 27](It is well known that the 2+1D Chern-Simons theory can be described by the algebraic tensor category theory.)

In fact, the above constraints are exactly the same as the usual BF theory of Poincare group, and the subtle difference only arises from the definition of physical observable corresponding to loop like excitation, namely, the Wilson surface operator. Let us rewrite the commutation relations in terms of B, \tilde{B}, e, ω :

$$[\omega_{i}^{ab}(\mathbf{x}), B_{jkcd}(\mathbf{y})] = i\epsilon_{ijk}\delta_{cd}^{ab}\delta(\mathbf{x} - \mathbf{y}),$$

$$[e_{i}^{a}(\mathbf{x}), \tilde{B}_{jkb}(\mathbf{y})] = i\epsilon_{ijk}\delta_{b}^{a}\delta(\mathbf{x} - \mathbf{y}),$$

$$[B_{ijab}(\mathbf{x}), \tilde{B}_{klc}(\mathbf{y})] = i\epsilon_{abcd}(e_{i}^{d}\epsilon_{jkl} - e_{j}^{d}\epsilon_{ikl})\delta(\mathbf{x} - \mathbf{y}),$$
all other commutators = 0. (8)

In recent works, it has been pointed out that such modified commutation relations actually imply the nontrivial three-loop-braiding [17–19] statistics among flux lines of gauge fields, which makes it different from the usual BF theory of Poincare group with trivial three-loop-braiding statistics.

Loop condensation and the emergence of Einstein gravity – To this point, one may wonder why we are interested in the 3+1D topological gravity theory which is somewhat trivial. Here we conjecture that quantum gravity is actually controlled by a topological gravity fixed point and the classical space-time vanishes at extremely high energy scale. Therefore it is quite natural to expect the

vanishing of curvature and torsion at that scale. Mathematically, $3+1\mathrm{D}$ TQFT can be described and classified by tensor 2-category theory and a possible way to generate interesting dynamics is condensing loops(flux lines in the context of topological gauge theory). If we further assume that the condensed loop carries a nontrivial linking Berry phase[28–30], a $\int Tr(B \wedge B)$ type term can be induced. Let us consider the following term:

$$S_{\theta} = -\frac{\theta}{2} \int B_{ab} \wedge B^{ab} \tag{9}$$

This term breaks the 2-form gauge symmetry as well as the translational gauge symmetry explicitly. A microscopic derivation of the above term from loop condensation is beyond the scope of this paper. Here we just introduce such a term phenomenologically to describe low energy dynamics and ignore all the microscopic details of loop dynamics, which is the analog of using massive gauge boson to describe Abelian Higgs phase and considering the infinite massive limit for Higgs boson. (More precisely, one can assume that the total action S is consisting of two terms S_{Top} and S_{Loop} at UV scale. S_{Loop} describes the dynamics of closed loop and it can be approximated by S_{θ} in the loop condensed phase after taking the infinite massive limit for loop.) Remarkably, for small θ , the total action $S = S_{\text{Top}} + S_{\theta}$ is still powercounting renormalizable since S_{θ} only contains dimension four operators. A detailed calculation of beta functions will be presented elsewhere.

The classical equation of motion for the total action S reads:

$$B^{ab} = \frac{1}{\theta} R^{ab}, \quad T^a = 0, \quad \epsilon_{abcd} e^b \wedge R^{cd} = -\nabla \tilde{B}_a,$$

$$\epsilon_{abcd} T^c \wedge e^d + \frac{1}{2} (\tilde{B}_a \wedge e_b - \tilde{B}_b \wedge e_a) = -\nabla B_{ab} \quad (10)$$

Insert the first two equations into the last equation, we have:

$$\frac{1}{2}(\tilde{B}_a \wedge e_b - \tilde{B}_b \wedge e_a) = -\frac{1}{\theta} \nabla R_{ab} = 0 \tag{11}$$

The above equation can be rewritten in a compact form as $\epsilon^{abcd}\tilde{B}_a \wedge e_b = 0$, which further implies $\tilde{B}^a = 0$. Thus, we eventually derive the vacuum Einstein-Cartan equation:

$$\epsilon_{abcd}e^b \wedge R^{cd} = 0. {12}$$

Einstein gravity as a non-commutative geometry — Now let us proceed the canonical quantization for the total action S. The total Lagrangian density reads:

$$\mathcal{L} = \Pi^{i}{}_{ab}\partial_{0}\omega_{i}{}^{ab} + \pi^{i}{}_{a}\partial_{0}e_{i}{}^{a} + \frac{1}{2}\epsilon^{ijk}B_{0iab}(R_{jk}{}^{ab} - \theta B_{jk}{}^{ab})
+ \frac{1}{2}\epsilon^{ijk}\tilde{B}_{0ia}T_{jk}{}^{a} + e_{0}{}^{a}(\nabla_{i}\pi^{i}{}_{a} + \frac{1}{2}\epsilon^{ijk}\epsilon_{abcd}e_{i}{}^{b}R_{jk}{}^{cd}) + \omega_{0}{}^{ab}\left[\nabla_{i}\Pi^{i}{}_{ab} + \frac{1}{2}(\pi^{i}{}_{a}e_{i}{}^{b} - \pi^{i}{}_{b}e_{i}{}^{a})\right]$$
(13)

where the canonical momentum Π_{ab}^i , π_a^i have the same definition as in 3+1D topological gravity. By integrating out B_{0iab} and \tilde{B}_{0ia} , we derive the following constraints:

$$B_{ij}^{ab} = \frac{1}{\theta} R_{ij}^{ab}, \quad T_{ij}^{a} = 0$$
 (14)

We note that the torsion free condition arises as a quantum constraint instead of equation of motion here. This feature is very different from the usual Einstein-Cartan theory. The canonical quantization conditions Eq.(8) imply the following noncommutative geometry:

$$[\omega_{i}^{ab}(\mathbf{x}), R_{jkcd}(\mathbf{y})] = i\theta\epsilon_{ijk}\delta_{cd}^{ab}\delta(\mathbf{x} - \mathbf{y}),$$

$$[e_{i}^{a}(\mathbf{x}), \tilde{B}_{jkb}(\mathbf{y})] = i\epsilon_{ijk}\delta_{b}^{a}\delta(\mathbf{x} - \mathbf{y}),$$

$$[R_{ijab}(\mathbf{x}), \tilde{B}_{klc}(\mathbf{y})] = i\theta\epsilon_{abcd}(e_{i}^{d}\epsilon_{jkl} - e_{j}^{d}\epsilon_{ikl})\delta(\mathbf{x} - \mathbf{y}),$$

$$[R_{ijab}(\mathbf{x}), \tilde{B}_{klc}(\mathbf{y})] = i\theta\epsilon_{abcd}(e_{i}^{d}\epsilon_{jkl} - e_{j}^{d}\epsilon_{ikl})\delta(\mathbf{x} - \mathbf{y}),$$

$$[R_{ijab}(\mathbf{x}), \tilde{B}_{ijkl}(\mathbf{y})] = i\theta\epsilon_{abcd}(e_{i}^{d}\epsilon_{jkl} - e_{j}^{d}\epsilon_{ikl})\delta(\mathbf{x} - \mathbf{y}),$$

$$[R_{ijab}(\mathbf{x}), \tilde{B}_{ijkl}(\mathbf{y})] = i\theta\epsilon_{abcd}(e_{i}^{d}\epsilon_{ijkl} - e_{j}^{d}\epsilon_{ikl})\delta(\mathbf{x} - \mathbf{y}),$$

$$[R_{ijab}(\mathbf{x}), \tilde{B}_{ijkl}(\mathbf{y})] = i\theta\epsilon_{abcd}(e_{i}^{d}\epsilon_{ijkl} - e_{j}^{d}\epsilon_{ikl})\delta(\mathbf{x} - \mathbf{y}),$$

In the semi-classical limit with, both R and e fields have weak quantum fluctuations while ω and \tilde{B} fields have strong quantum fluctuations. A very interesting observation is that the small parameter θ enter the commutation relation and it will be very interesting to understand the relationship between θ and planck constant \hbar in our future work.

To this end, we see that the nature of quantum gravity is the emergence of non-commutative geometry via loop condensation from an underlying topological gravity theory. We stress that S_{Top} with vanishing R and T describes the absolute vacuum of our universe in the absence of classical space time, and it might provide a new route towards resolving the black hole singularity as well as the big bang singularity.

Cosmological constant term – Our construction for topological gravity action in 3 + 1D can be easily generalized into the case with cosmological constant term:

$$S'_{\text{Top}} = S_{\text{Top}} + \frac{\Lambda}{4!} \int \epsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d$$
$$= S_{\text{Top}} + \frac{\Lambda}{4!} \int d^4 x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} e^a_\mu e^b_\nu e^c_\rho e^d_\sigma \quad (16)$$

We only need to properly redefine the gauge transformation of translational symmetry:

$$e_{\mu}{}^{a} \rightarrow e_{\mu}{}^{a} + D_{\mu}f^{a} \equiv e_{\mu}{}^{a} + \partial_{\mu}f^{a} + \omega_{\mu}{}^{ab}f_{b}$$

$$\tilde{B}_{\mu\nu a} \rightarrow \tilde{B}_{\mu\nu a} - \epsilon_{abcd}f^{b}R_{\mu\nu}{}^{cd} - \frac{\Lambda}{2}\epsilon_{abcd}(e_{\mu}^{b}e_{\nu}^{c} - e_{\nu}^{b}e_{\mu}^{c})f^{d}$$

$$B_{\mu\nu ab} \rightarrow B_{\mu\nu ab} - \frac{1}{2}(\tilde{B}_{\mu\nu a}f_{b} - \tilde{B}_{\mu\nu b}f_{a})$$

$$(17)$$

Similar to the case without cosmological constant term, loop condensation will lead to Einstein-Cartan action with cosmological constant term, and the whole theory remains to be power-counting renormalizable.

Super symmetric generalization – Finally, let us discuss the SUSY generalization of 3+1D topological gravity. Similar to the 2+1D topological gravity theory, we just need to introduce the gauge connection of super Poincare group and write the action as $\int sTr[A \wedge A \wedge (dA + A \wedge A)] + \int sTr(B \wedge F)$. For example, for the N=1 case, we can just express A, B and F as:

$$A_{\mu} \equiv \frac{1}{2} \omega_{\mu}^{ab} M_{ab} + e_{\mu}^{a} P_{a} + \bar{\psi}_{\mu\alpha} Q^{\alpha}$$

$$B_{\mu\nu} \equiv \frac{1}{2} B_{\mu\nu}^{ab} M_{ab} + \tilde{B}_{\mu\nu}^{a} P_{a} + \mathfrak{B}_{\mu\nu\alpha} Q^{\alpha}$$

$$F_{\mu\nu} \equiv \frac{1}{2} R_{\mu\nu}^{ab} M_{ab} + T_{\mu\nu}^{a} P_{a} + \bar{R}_{\mu\nu\alpha} Q^{\alpha} \qquad (18)$$

Here $\bar{R}_{\mu\nu\alpha}$ is the super curvature tensor defined as $\bar{R}_{\mu\nu\alpha} = D_{\mu}\bar{\psi}_{\nu\alpha} - D_{\nu}\bar{\psi}_{\mu\alpha}$ where D_{μ} is the covariant derivative for spinon fields. However, as fermionic loops(flux lines) can not be condensed, super symmetry breaking already happens at very high energy scale when bosonic loops condense and classical space-time emerges. Thus, the super curvature $\bar{R}_{\mu\nu\alpha}$ always vanishes and the semiclassical limit of 3+1D quantum gravity can still be described by S at low energy. However, the SUSY generalization might provide us a natural way to extend the our model to include fermionic matter fields.

Topological gravity in arbitrary dimensions and the emergence of 3+1D space-time – Before conclusion, let us generalize topological gravity theory into arbitrary dimensions with the following gauge invariant action:

$$S_{\text{Top}} = \frac{1}{n-2} \int \epsilon_{aba_3...a_n} R^{ab} \wedge e^{a_3} \wedge \dots \wedge e^{a_n} + \int C_{ab} \wedge R^{ab} + \int \tilde{C}_a \wedge T^a$$
(19)

where C and \tilde{C} are n-2 forms. Interestingly, we see that it is only possible to introduce $\int C \wedge C$ type term for four dimensional space-time. Thus, we may start with a model describing topological gravity in all dimensions(e.g. topological nonlinear sigma model of the Poincare group classifying space) and condense the loop, only the four dimensional vielbein field admits a semi-classical limit that defines the classical space-time!

Conclusions and discussions – In conclusion, we propose a topological paradigm to understand 3 + 1D quantum gravity. In particular, we generalize Edward Witten's 2 + 1D topological gravity theory into arbitrary dimensions. In 3 + 1D, by condensing loops(flux lines), we find a semi-classical limit where the Einstein-Cartan equation emerges.(In the absence of matter fields.) Our approach can be generalized into the case with cosmological constant term. In fact, it is well known that starting from a topological BF theory of gauge group G with action $S_{\text{top}} = \int Tr(B \wedge F)$, condensing the loops by introducing a mass term $S_g = g \int d^4x Tr(B^2)$ (we consider flat space-time background here for simplicity) is another way to derive a gauge theory with a Maxwell term $\frac{1}{a} \int d^4x Tr(F^2)$ (by integrating out the B fields and regarding g as the coupling constant). Thus, we argue that the concept of condensing loops from a topological gauge theory (which can be rigorously defined as topological nonlinear sigma model in classifying space of the corresponding gauge group[22]) might provide us a unified description for both gauge theory and gravity.

Acknowledgments – We thank S T Yau for invitation to visit Center of Mathematical Sciences and Applications at Harvard University where this work was initialized. We also thank Dvide Gaiotto and Kevin Costello for critical reading and useful comments. We acknowledge start-up support from Department of Physics, The Chinese University of Hong Kong, Direct Grant No. 4053224 from The Chinese University of Hong Kong and the funding from RGC(No.2191110).

- BP Abbott, et al., Observation of gravitational waves from a binary black hole merger, Phys. Rev. Lett. 116, 061102 (2016).
- [2] J Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2, 231 (1998).
- [3] E Witten, Anti de Sitter space and holography, Adv. Theor. Math. Phys. 2, 253 (1998).
- [4] C Rovelli, and L Smolin, Knot theory and quantum gravity, Physical Review Letters 61, 1155 (1988).
- [5] C Rovelli, and L Smolin, Loop space representation of quantum general relativity, Nuclear Physics B 331, 80 (1990).
- [6] C Rovelli, and L Smolin, Spin networks and quantum gravity, Physical Review D 52, 5743 (1995).
- [7] L Smolin, and A Starodubtsev, General relativity with a topological phase: an action principle, arXiv:0311163
- [8] L Freidel, and A Starodubtsev, Quantum gravity in terms of topological observables, arXiv:0501191
- [9] E Witten, 2+ 1 dimensional gravity as an exactly soluble system, Nuclear Physics B **311**, 46 (1988).
- [10] Edward Witten, Three-Dimensional Gravity Revisited, arXiv:0706.3359
- [11] Daniel Freedman, Peter van Nieuwenhuizen, and Sergio Ferrara, *Progress toward a theory of supergravity*, Phys.

- Rev. D 13, 3214 (1976).
- [12] E Cremmer, B Julia, and J Scherk, Supergravity in theory in 11 dimensions, Physics Letters B 76, 409 (1978).
- [13] X. Chen, Z.-C. Gu, Z.-X. Liu, and X.-G. Wen, Symmetry protected topological orders and the group cohomology of their symmetry group, Phys. Rev. B 87, 155114 (2013).
- [14] X. Chen, Z.-C. Gu, Z.-X. Liu, X.-G. Wen, Symmetry-Protected Topological Orders in Interacting Bosonic Systems, Science 338, 1604 (2012).
- [15] A. Kapustin, Symmetry Protected Topological Phases, Anomalies, and Cobordisms: Beyond Group Cohomology, arXiv:1403.1467; A. Kapustin, Bosonic Topological Insulators and Paramagnets: a view from cobordisms, arXiv:1404.6659.
- [16] X.-G. Wen, Construction of bosonic symmetry-protectedtrivial states and their topological invariants via $G \times SO(\infty)$ non-linear σ -models, Phys. Rev. B **91**, 205101 (2015).
- [17] Peng Ye and Zheng-Cheng Gu, Topological quantum field theory of three-dimensional bosonic Abelian-symmetryprotected topological phases, Phys. Rev. B 93, 205157 (2016).
- [18] Juven Wang, Xiao-Gang Wen, and Shing-Tung Yau, Quantum Statistics and Spacetime Surgery, arXiv:1602.05951
- [19] Pavel Putrov, Juven Wang, and Shing-Tung Yau, Braiding Statistics and Link Invariants of Bosonic/Fermionic Topological Quantum Matter in 2+1 and 3+1 dimensions, Annals of Physics 384, 254 (2017).
- [20] Chenjie Wang and Michael Levin, Braiding statistics of loop excitations in three dimensions Phys. Rev. Lett. 113, 080403 (2014).
- [21] Shenghan Jiang, Andrej Mesaros, and Ying Ran, Generalized modular transformations in 3+1D topologically ordered phases and triple linking invariant of loop braiding, Phys. Rev. X 4, 031048 (2014).
- [22] R. Dijkgraaf and E. Witten, Topological gauge theories and group cohomology, Commun. Math. Phys. 129, 393 (1990).
- [23] M. Blau and G. Thompson, Topological Gauge Theories of Antisymmetric Tensor Fields, Ann. Phys. 205, 130 (1991).
- [24] M. Bergeron, G. W. Semenoff, and R. J. Szabo, Canonical BF-type topological field theory and fractional statistics of strings, Nucl. Phys. B 437, 695 (1995).
- [25] R. J. Szabo, String holonomy and extrinsic geometry in four-dimensional topological gauge theory, Nucl. Phys. B 531, 525 (1998).
- [26] Liang Kong and Xiao-Gang Wen, Braided fusion categories, gravitational anomalies, and the mathematical framework for topological orders in any dimensions, arXiv:1405.5858
- [27] Tian Lan, Liang Kong, and Xiao-Gang Wen, A classification of 3+1D bosonic topological orders (I): the case when point-like excitations are all bosons, arXiv:1704.04221
- [28] Kapustin A. and Seiberg N. Coupling a QFT to a TQFT and duality, J. High Energ. Phys. (2014) 2014: 1
- [29] Peng Ye and Zheng-Cheng Gu, Vortex-line condensation in three dimensions: A physical mechanism for bosonic topological insulatorsPhys. Rev. X 5, 021029 (2015).
- [30] Davide Gaiotto, Anton Kapustin, Nathan Seiberg, and Brian Willett, Generalized Global Symmetries, J. High Energ. Phys. (2015) 2015: 172