# AdS/CFT Correspondence and Symmetry Breaking

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#### Abstract

We study, using the dual AdS description, the vacua of field theories where some of the gauge symmetry is broken by expectation values of scalar fields. In such vacua, operators built out of the scalar fields acquire expectation values, and we show how to calculate them from the behavior of perturbations to the AdS background near the boundary. Specific examples include the  $\mathcal{N}=4$  SYM theory, and theories on D3 branes placed on orbifolds and conifolds. We also clarify some subtleties of the AdS/CFT correspondence that arise in this analysis. In particular, we explain how scalar fields in AdS space of sufficiently negative mass-squared can be associated with CFT operators of two possible dimensions. All dimensions are bounded from below by (d-2)/2; this is the unitarity bound for scalar operators in d-dimensional field theory. We further argue that the generating functional for correlators in the theory with one choice of operator dimension is a Legendre transform of the generating functional in the theory with the other choice.

## 1. Introduction

The AdS/CFT correspondence [1,2,3] may be motivated by comparing stacks of elementary branes with corresponding gravitational backgrounds in string or M-theory. For example, the correspondence [4] between a large number N of coincident D3-branes and the 3-brane classical solution leads, after an appropriate low-energy limit is taken, to the duality between  $\mathcal{N}=4$  supersymmetric SU(N) gauge theory and Type IIB strings on  $AdS_5 \times S^5$  [1,2,3]. This construction gives an explicit realization to the ideas of gauge theory strings [5,6].

In order to construct the Type IIB duals of other 4-dimensional CFT's, one may place the D3-branes at appropriate conical singularities [7,8,9,10,11]. Then the background dual to the CFT on the D3-branes is  $AdS_5 \times X_5$  where  $X_5$  is the Einstein manifold which is the base of the cone. Indeed, the metric of a 6-dimensional cone  $Y_6$  has the general form

$$ds_{\text{cone}}^2 = dr^2 + r^2 ds_5^2 \ . \tag{1.1}$$

Here  $Y_6$  is a cone over a five-manifold  $X_5$ , and  $ds_5^2$  is a metric on  $X_5$ . If a large number N of D3-branes is placed at the apex of the cone, that is at r=0, then the resulting geometry has the metric

$$ds^{2} = H^{-1/2}(-dt^{2} + dx_{1}^{2} + dx_{2}^{2} + dx_{3}^{2}) + H^{1/2}ds_{\text{cone}}^{2},$$
(1.2)

where

$$H = 1 + \frac{L^4}{r^4}$$
,  $L^4 \sim g_{\rm st} N(\alpha')^2$ .

In the near-horizon limit the constant term in H may be ignored and the geometry becomes  $AdS_5 \times X_5$  where  $X_5$  is the base of the cone. Type IIB string theory in this background is then conjectured to be dual to the infrared limit of the field theory on the stack of D3-branes. Some explicit examples of such duality were exhibited in [12,13,9,11].

In this paper we study in some detail the vacuum states of these CFT's in which some of the gauge symmetry is broken by expectation values of scalar fields. In terms of the AdS description, such vacua arise either by moving the threebranes away from the conical singularity or from each other, or from the dynamics of the manifold  $Y_6$ , whose singularity might be either resolved or deformed. These cases are all somewhat similar as, whether the threebranes are moved or  $Y_6$  is resolved or deformed, the threebranes tend to end up at a smooth point in  $Y_6$ . In fact, vacua obtained by resolution, deformation, and threebrane motion can all be described in an AdS language by using metrics that look like  $AdS_5 \times X_5$  only near infinity.

These vacua also fit in a common framework in the description via boundary conformal field theory. They are all obtained by symmetry breaking, that is by giving expectation values to various scalar fields.

Perhaps the simplest example of such gauge symmetry breaking arises in the  $\mathcal{N}=4$  SYM theory, by turning on scalar fields such that the gauge group SU(N) is broken down to  $S(U(N_1) \times U(N_2) \dots \times U(N_k))$ . In the language of D3-branes, this corresponds to separating them into k parallel stacks. The appropriate geometry is the k-center threebrane solution [1,14], and one may once again take a scaling limit which amounts to dropping the constant term in the Green's function H. Following [14], we will put the interpretation of the k-center solution via gauge symmetry breaking on a more precise and systematic basis by using the general principles of the AdS/CFT correspondence to compute expectation values of gauge-invariant order parameters in vacua described by the k-center solution.

A wider range of examples comes from considering threebranes near a conical singularity. The most elementary examples are the "orbifold" CFT's, where the six-dimensional cone is  $\mathbf{R}^6/\Gamma$ , with  $\Gamma$  a discrete subgroup of SO(6) [7,12,13]. We denote the elements of  $\Gamma$  as  $\omega_i$ . If the D3-branes are displaced away from the orbifold singularity to a transverse position  $\vec{y}_0$ , then the metric is given by

$$ds^{2} = H^{-1/2}dx^{2} + H^{1/2}(d\vec{y})^{2}, \qquad (1.3)$$

where the Green's function is

$$H = L^4 \sum_{i=1}^{n} \frac{1}{|\vec{y} - \omega_i \vec{y_0}|^4} ,$$

and the resulting space is subsequently divided by  $\Gamma$ . (Here we are denoting the four coordinates that parametrize the brane world-volumes as x and the six normal coordinates as y.) Intuitively, if  $y_0$  is displaced from all of the orbifold fixed points, then this metric has the same singular structure as that obtained from N D3-branes at a smooth point on  $\mathbf{R}^6$ . This suggests that the metric describes the flow (via a Higgs effect) from an orbifold field theory [12,13] at short distances to an SU(N) theory with  $\mathcal{N}=4$  supersymmetry at long distances. (For instance, if  $\Gamma=\mathbf{Z}_n$ , the orbifold theory has gauge group  $S(U(N)^n)$ , which can be broken to a diagonal SU(N) by scalar expectation values.) We will aim to put this interpretation on a precise and systematic basis by computing the expectation values of the natural gauge-invariant order parameters.

Less elementary is the case that the conical manifold  $Y_6$  is not simply an orbifold. A simple case is that  $Y_6$  may be the conifold singularity in complex dimension three, described in terms of complex variables  $w_1, \ldots, w_4$  by the equation

$$\sum_{a=1}^{4} w_a^2 = 0 \ .$$

The conifold admits a conical Calabi-Yau metric of the form

$$ds^2 = dr^2 + r^2 d\Omega^2 .$$

Here

$$d\Omega^{2} = \frac{1}{9} \left( d\psi + \cos \theta_{1} d\phi_{1} + \cos \theta_{2} d\phi_{2} \right)^{2} + \frac{1}{6} \sum_{a=1}^{2} \left( d\theta_{a}^{2} + \sin^{2} \theta_{a} d\phi_{a}^{2} \right)$$
(1.4)

is the metric on the base of the cone [15], which is  $T^{1,1} = (SU(2) \times SU(2))/U(1)$ . The  $\mathcal{N} = 1$  superconformal field theory with gauge group  $SU(N) \times SU(N)$  that results when the D3-branes are placed at r = 0, and is dual to  $AdS_5 \times T^{1,1}$ , was discussed in detail in [9,11].

In this example, as anticipated above, symmetry breaking can take several forms. One may move the threebranes away from r=0 to a smooth point, or one may resolve the singularity of  $Y_6$  to get a smooth manifold  $Y_6''$  that looks like  $Y_6$  near infinity (in which case the threebranes are necessarily at a smooth point). In either case, assuming the threebranes are all at the same point, the low energy theory will be the  $\mathcal{N}=4$  SU(N) gauge theory. Thus, the model analyzed in [9,11] can flow in the infrared to one of these vacua. We will analyze the geometries that are relevant to these flows, and compute the expectation values of chiral superfields in these vacua, getting results that are in agreement with field theory analysis [9].

Section 2 of this paper is devoted to some details of the AdS/CFT correspondence that will arise in our analysis. In particular, we explain how scalar fields in AdS space of sufficiently negative mass-squared can be associated with CFT operators of two possible dimensions. This subtlety is important for the conifold model, because it contains such fields in its spectrum. We also formulate, following similar ideas in [16,17,14,18,19], the general procedure for computing the expectation value of an operator in a quantum vacuum that is related to a given classical solution. This will be important for our applications to symmetry breaking. Those applications are presented in section 3 for the  $\mathcal{N}=4$  theory and orbifolds, and in section 4 for the conifold. Some technical details of the spectrum for the conifold are given in an appendix.

## 2. The Mass Spectrum And Operator Dimensions

### 2.1. Two Theories From The Same Lagrangian

The AdS/CFT correspondence gives the following relation between the mass m of a scalar in  $AdS_{d+1}$  and the dimension  $\Delta$  of the corresponding operator [2,3],

$$\Delta(\Delta - d) = m^2 \ . \tag{2.1}$$

There are two solutions,

and it is often assumed that only  $\Delta_+$  is admissible. If true, this would imply that dimensions of scalar operators are bounded from below by d/2, which is more stringent than the unitarity bound (d-2)/2.

In various explicit examples of the AdS/CFT duality, however, the field theory side contains operators of dimension less than d/2. Examples of this include the large N (2,0) theory dual to M theory on  $AdS_4 \times S^7$  where one finds operators of dimension 1 [20], the F-theory constructions of  $AdS_5$  duals [21] where one finds dimensions 6/5, 4/3, 3/2, and the D3-branes on the conifold [9] where there are operators of dimension 3/2. There are also operators in the D1-D5 system with arbitrarily low dimensions [22,23]. In all these examples the supersymmetry unambiguously requires the presence of these low dimensions, all of which are consistent with the unitarity bound but are smaller than d/2. Therefore, if the AdS/CFT correspondence is correct, then there must be a loophole in the conclusion that only  $\Delta_+$  is admissible. This issue was raised and discussed in [16], where the relevance of old work by Breitenlohner and Freedman [24] was also suggested.

Breitenlohner and Freedman considered a free scalar field of mass m in AdS space, and showed that, while for  $m^2 > -\frac{d^2}{4} + 1$  there is a unique admissible boundary condition for such a field that is invariant under the symmetries of AdS space, leading to a unique AdS-invariant quantization, for

$$-\frac{d^2}{4} < m^2 < -\frac{d^2}{4} + 1 \tag{2.3}$$

there are two possible quantizations. These two possibilities correspond to the fall-off of scalar wave functions as  $z^{\Delta_+}$  and  $z^{\Delta_-}$  near z=0, where the  $AdS_{d+1}$  metric is written as

$$ds^{2} = \frac{1}{z^{2}} \left( dz^{2} + \sum_{i=1}^{d} (dx^{i})^{2} \right) , \qquad (2.4)$$

and we set L=1. Breitenlohner and Freedman formulated their arguments in Hamiltonian terms and looked for boundary conditions that make the energy finite. Instead of repeating their argument, we will give a heuristic derivation of the result in Euclidean space, by requiring finiteness of the action.

The conventional expression for the Euclidean action of a massive scalar field in  $AdS_{d+1}$  is

$$\frac{1}{2} \int d^{d+1}x \sqrt{g} \left[ g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + m^2 \phi^2 \right] = \frac{1}{2} \int d^d x dz z^{-d+1} \left[ (\partial_z \phi)^2 + (\partial_i \phi)^2 + \frac{m^2}{z^2} \phi^2 \right] . \tag{2.5}$$

Solutions of the classical equations of motion of this theory behave near z = 0 – that is, near the boundary of AdS space – as

$$\phi(z, \vec{x}) \to z^{\Delta}(A(\vec{x}) + O(z^2)), \tag{2.6}$$

where  $\Delta$  can be either  $\Delta_+$  or  $\Delta_-$ . Any boundary condition on the field must set to zero half of the modes of the field near the boundary. It is natural and completely AdS-invariant to pick a particular root,  $\Delta = \Delta_+$  or  $\Delta = \Delta_-$ , and require that  $\phi$  behave as in (2.6) near the boundary. (Of course, we do not require that  $\phi$  obey the classical equations of motion in the interior of AdS space.) With this asymptotic condition, the action (2.5) is finite for  $\Delta > d/2$ . But the bound on  $\Delta$  can be relaxed by adding appropriate boundary terms to the action. By integrating by parts and discarding the boundary term, we can replace the action by

$$\frac{1}{2} \int d^{d+1}x \sqrt{g} \ \phi(-\nabla^2 + m^2)\phi \ . \tag{2.7}$$

The boundary term in this integration by parts is nonzero (and in fact divergent) if  $\Delta \leq d/2$ , so in writing the action (2.7), we are modifying the definition of the action. The modified action integral is convergent if

$$\Delta > \frac{d-2}{2}.\tag{2.8}$$

This is precisely the unitarity bound on the dimension of a scalar operator in d dimensions, so in particular we cannot expect by any further device to get even smaller  $\Delta$ 's. In the mass range (2.3), this condition allows  $\Delta = \Delta_{-}$  as well as  $\Delta_{+}$ , while for larger  $m^{2}$  only  $\Delta = \Delta_{+}$  is allowed.

Though convergent for fields that obey the boundary conditions, the action (2.7) is not manifestly positive definite. However, it is positive definite, because the operator  $-\nabla^2 + m^2$  is positive definite for the range of  $m^2$  of interest, namely  $m^2 > -d^2/4$ .

Thus, as pointed out in [24], there are two different AdS-invariant quantizations of the scalar field with  $m^2$  in the range (2.3). One Lagrangian – in this case that of a scalar field of given  $m^2$  – can give rise to two different quantum field theories in AdS space, depending on the choice of boundary condition. According to the general AdS/CFT correspondence, any quantum field theory in AdS space is equivalent to a conformal field theory on the boundary. The two different AdS theories with a given  $m^2$  will correspond to two different CFT's, one with an operator of dimension  $\Delta_+$  and the other with an operator of dimension  $\Delta_-$ . In many examples, one of the two theories is much more readily studied than the other because one is supersymmetric and the other is not. But both exist in principle.

<sup>&</sup>lt;sup>1</sup> Of course, treating the AdS scalar field as a free field of mass  $m^2$  can never be precisely right, since this field always interacts at least with gravity. There will in general be mass renormalization, in which case the dimensions of the operators will not be precisely  $\Delta_+$  and  $\Delta_-$ .

### 2.2. Correlation Functions

Our main remaining goal will be to define the correlation functions from the AdS/CFT correspondence for both choices of the theory. In the process we will also give an important formula for the expectation values of operators. We will look for the definition of the Euclidean action which generates properly normalized correlation functions.

To compute correlation functions, one must relax the boundary condition (2.6), so we have to exercise additional care in defining the action. Indeed, there is a subtlety with the normalization of the two-point function in the AdS/CFT correspondence. In [25], it was shown that an extra factor of  $(2\Delta - d)/d$ , not coming in an obvious way from evaluating the classical action, is needed for consistency with the Ward identities. This factor was then derived by imposing the boundary condition at  $z = \epsilon$ , as advocated in [2,26,25], and taking the  $\epsilon \to 0$  limit at the end of the calculation. We will present a different way of obtaining this factor which involves adding an appropriate boundary term to the action.

In calculating correlation functions of vertex operators from the AdS/CFT correspondence, the first problem is to reconstruct an on-shell field in  $AdS_{d+1}$  from its boundary behavior. If  $\Delta$  is one of the roots of (2.1), then one requires that for small z

$$\phi(z, \vec{x}) \to z^{d-\Delta} [\phi_0(\vec{x}) + O(z^2)] + z^{\Delta} [A(\vec{x}) + O(z^2)],$$
 (2.9)

where  $\phi_0(\vec{x})$  is a prescribed "source" function and  $A(\vec{x})$  describes a physical fluctuation that will be determined from the source by solving the classical equations. In our discussion so far, we only considered the physical fluctuation  $A(\vec{x})$ .

We begin with the usual case  $\Delta = \Delta_+$ . In this case, the first term in (2.9) dominates over the second near z = 0, and the construction of  $\phi(z, \vec{x})$  from  $\phi_0(\vec{x})$  is usually accomplished with the help of the bulk-to-boundary propagator [3,25],

$$K_{\Delta}(z, \vec{x}, \vec{x}') = \pi^{-d/2} \frac{\Gamma(\Delta)}{\Gamma(\Delta - (d/2))} \frac{z^{\Delta}}{(z^2 + (\vec{x} - \vec{x}')^2)^{\Delta}} , \qquad (2.10)$$

so that

$$\phi(z, \vec{x}) = \int d^d x' K_{\Delta}(z, \vec{x}, \vec{x}') \phi_0(\vec{x}') . \qquad (2.11)$$

The normalization in (2.10) is chosen so that (2.9) is satisfied. We note that

$$A(\vec{x}) = \pi^{-d/2} \frac{\Gamma(\Delta)}{\Gamma(\Delta - (d/2))} \int d^d x' \phi_0(\vec{x}') |\vec{x} - \vec{x}'|^{-2\Delta} .$$

For extended sources this is a formal expression because it diverges for  $\Delta > d/2$ , but it will be useful after appropriate regularization is taken into account. We may also consider a localized source,  $\phi_0(\vec{x}) = \delta^d(\vec{x} - \vec{x}')$ . Then

$$A(\vec{x}) = \pi^{-d/2} \frac{\Gamma(\Delta)}{\Gamma(\Delta - (d/2))} |\vec{x} - \vec{x}'|^{-2\Delta}$$

$$(2.12)$$

and it was observed in [17] that, up to a normalization factor, this is the two-point function  $\langle \mathcal{O}(\vec{x})\mathcal{O}(\vec{x}')\rangle$ . This suggests that  $A(\vec{x})$  has the interpretation of the expectation value of the operator  $\mathcal{O}(\vec{x})$  in the theory where another operator  $\mathcal{O}$  is inserted at  $\vec{x}'$ . We will see that the precise relation is

$$A(\vec{x}) = \frac{1}{2\Delta - d} \langle \mathcal{O}(\vec{x}) \rangle \tag{2.13}$$

Up to normalization this is the same relation as the one advocated in [16,17,14,18,19]. The precise factor is related to the normalization of the two-point function first found in [25]. We will be able to show that this relation holds beyond the linearized approximation.

In order to define the value of action on the solution (2.11), it is convenient to introduce another field,  $\chi$ , through

$$\phi(z, \vec{x}) = z^{d-\Delta} \chi(z, \vec{x}) .$$

After integrating by parts and discarding an appropriate boundary term, the action assumes the following form:

$$I = \frac{1}{2} \int d^d x dz \ z^{d+1-2\Delta} \left[ (\partial_z \chi)^2 + (\partial_i \chi)^2 \right] \ . \tag{2.14}$$

We propose to define the two-point function for  $\Delta = \Delta_+$  using this action. It differs from the original action (2.5) in that the leading small z divergence has been discarded. The action integral in (2.14) is convergent if  $d/2 + 1 > \Delta > d/2$ . For  $\Delta \geq d/2 + 1$ , a more complicated subtraction of boundary divergences is needed to get a well-defined action. This corresponds to the fact that the conformal field theory generating functional that we will compute has additional short distance singularities if  $\Delta \geq d/2 + 1$ . We will not explicitly make the additional regularization of the action that is needed for  $\Delta$  in this range.

Now we are ready to calculate the two-point function from the AdS/CFT correspondence. We need to evaluate the improved action I in terms of  $\phi_0(\vec{x})$ ; that is, we need to evaluate I for a classical solution (2.11) with given  $\phi_0(\vec{x})$ . Integrating (2.14) by parts we find

$$I = -\lim_{z \to 0} z^{d+1-2\Delta} \int d^d \vec{x} \frac{1}{2} \chi \partial_z \chi . \qquad (2.15)$$

In evaluating this expression, the  $\phi_0 \cdot \phi_0$  terms vanish if  $\Delta < d/2 + 1$ , and the  $A \cdot A$  terms vanish if  $\Delta > d/2$ . To be more precise, to evaluate (2.15), we can replace  $\chi$  by  $\phi_0$  and  $\partial_z \chi$  by  $(2\Delta - d)z^{2\Delta - d - 1}A(\vec{x})$  for small z. (Terms with A coming from  $\chi$  and  $\phi_0$  from  $\partial_z \chi$  vanish using the property emphasized in the last footnote.) So we find that

$$I(\phi_0) = -(\Delta - (d/2))\pi^{-d/2} \frac{\Gamma(\Delta)}{\Gamma(\Delta - (d/2))} \int d^d \vec{x} \int d^d \vec{x}' \frac{\phi_0(\vec{x})\phi_0(\vec{x}')}{|\vec{x} - \vec{x}'|^{2\Delta}}.$$
 (2.16)

In particular, as expected,  $\Delta$  is the dimension of the operator  $\mathcal{O}$  that couples to the source  $\phi_0$  in the boundary conformal field theory. Because of the divergence for  $\vec{x}$  near  $\vec{x}'$  (2.16) has to be understood in an appropriately regularized sense. For example, the corresponding expression in momentum space is

$$I(\phi_0) = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \phi_0(k) \phi_0(-k) f_+(|k|) , \qquad (2.17)$$

where

$$f_{+}(|k|) = -2\nu \left(\frac{|k|}{2}\right)^{2\nu} \frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}$$
 (2.18)

is the Fourier transform of the two-point function (we have defined  $\nu = \sqrt{m^2 + d^2/4} = \Delta_+ - d/2$ ). (2.17) is finite for  $\phi_0(k)$  that fall off sufficiently fast for large k. Note that the Fourier transform of  $|\vec{x}|^{-2\Delta_+}$  is actually UV divergent and when appropriately defined has the negative coefficient that is indicated.

We will not attempt a similar derivation in detail for  $\Delta > d/2 + 1$ . However, we claim that in this range, after the additional subtractions of boundary terms that are needed to make I finite, the  $\phi_0 \cdot \phi_0$  terms vanish and the  $\phi_0 \cdot A$  terms can be evaluated to give the same formula as (2.16).

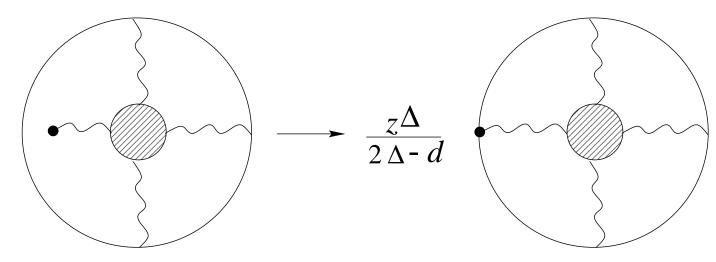
The overall minus sign in (2.16) is crucial: since  $\phi_0$  is interpreted as the source coupling to the CFT operator  $\mathcal{O}$ , this is the correct sign to insure the positivity of the two-point function. Indeed,  $\exp(-I)$  is interpreted in the boundary field theory as  $\langle \exp(\int \phi_0 \mathcal{O}) \rangle$ . So negativity of (2.16) is needed for positivity of the correlation function  $\langle \mathcal{O}(\vec{x})\mathcal{O}(\vec{x}')\rangle$ .

#### One-Point Function In Presence Of Sources

The prefactor  $\Delta - (d/2)$  in (2.16) is also important: this is the factor advocated in [25]. Due to the presence of this factor, we see, on comparing (2.12) to (2.16), that the

<sup>&</sup>lt;sup>2</sup> To prove this, one uses the fact that the correction to the  $\phi_0$  term in a classical solution (2.9) is of order  $z^2$ .

relation  $\langle \mathcal{O}(\vec{x}) \rangle = (2\Delta - d)A(\vec{x})$  holds to linear order in the sources. Let us show that this relation holds to all orders.



The diagrammatic relation between the term  $A(\vec{x})z^{\Delta}$  in the asymptotic behavior of the field in an arbitrary correlation function or physical process (left) and the same process with an extra insertion of an operator  $\mathcal{O}(\vec{x})$  (right). The solid black dot represents on the left a point in AdS near the boundary where a field is measured and on the right a point on the boundary at which an operator is inserted. As the black dot approaches the boundary, the two figures are related by the indicated factor.

The expectation value of  $\mathcal{O}(\vec{x})$  is given by the sum over diagrams where a bulk-to-boundary propagator  $K_{\Delta}(\vec{x}; z', \vec{x}')$  connects the point  $\vec{x}$  to the rest of the diagram with source points located at the boundary. The classical field  $\phi(z, \vec{x})$  is given by summing the same diagrams, except the bulk-to-boundary propagator is replaced by the bulk-to-bulk propagator leading to the point  $(z, \vec{x})$ . The normalized expression for the bulk-to-bulk propagator is given, for instance, in [27]:

$$G_{\Delta}(z, \vec{x}; z', \vec{x}') = \frac{\Gamma(\Delta)\Gamma(\Delta - \frac{d}{2} + \frac{1}{2})}{(4\pi)^{(d+1)/2}\Gamma(2\Delta - d + 1)} (2u^{-1})^{\Delta} F(\Delta, \Delta - \frac{d}{2} + \frac{1}{2}; 2\Delta - d + 1; -2u^{-1}) ,$$

where F is the hypergeometric function, and  $u=[(z-z')^2+(\vec{x}-\vec{x}')^2]/2zz'$ . We note that, as  $z\to 0$  away from the source points,  $\phi(z,\vec{x})\to z^\Delta A(\vec{x})$ . In this limit

$$G_{\Delta}(z, \vec{x}; z', \vec{x}') \rightarrow z^{\Delta} \frac{K_{\Delta}(\vec{x}; z', \vec{x}')}{2\Delta - d}$$
.

This property of the normalized Green's functions, first emphasized in [28], thus provides a general explanation of the relation (2.13).

# Extension To $\Delta < d/2$

The derivations presented so far apply to the theory with  $\Delta = \Delta_+$ . As we have explained, for masses in the range (2.3) it should be possible to define a different theory where the operator corresponding to the scalar field in  $AdS_5$  has dimension  $\Delta_-$ . The  $\Delta_-$  theory is not independent from the  $\Delta_+$  theory but is, in fact, related to it by a canonical transformation that interchanges the roles of  $\phi_0(\vec{x})$  and  $A(\vec{x})$ . The fact that  $\phi_0$  and A are conjugate variables is also suggested by the group-theoretic analysis in [29]. If from the point of view of the  $\Delta_+$  theory  $\phi_0$  is "the source" and  $(2\Delta - d)A$  is "the field," then the opposite is true for the  $\Delta_-$  theory. This strongly suggests that the generator of connected correlators of the  $\Delta_+$  theory is obtained by Legendre transforming the generator of connected correlators of the  $\Delta_+$  theory. This type of relationship is familiar from the Liouville theory where it has been suggested that theories with two different branches of gravitational dressing of a given operator are related by a Legendre transform [30].

To see how this works for the two-point functions, it is convenient to use Fourier space. The quadratic part of the action is given in (2.17), where  $f_+(|k|)$  is the Fourier transform of the two-point function in the  $\Delta_+$  theory. The Legendre transform is carried out by first setting

$$J(\phi_0, A) = I(\phi_0) + (2\Delta - d) \int \frac{d^4k}{(2\pi)^4} \phi_0(k) A(-k).$$

We have included a factor of  $(2\Delta - d)$  based on the idea that the conjugate of  $\phi_0$  is actually  $(2\Delta - d)A$ . As we will see, this factor, though we have not justified it precisely, gives the nicest normalization for the two point function of the transformed theory. The Legendre transformed functional  $\tilde{I}(A)$  is the minimum of  $J(\phi_0, A)$  with respect to  $\phi_0$  (for fixed A), and is explicitly

$$\tilde{I}(A) = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} A(k) A(-k) f_-(|k|)$$

with

$$f_{-}(|k|) = -\frac{(2\Delta - d)^2}{f_{+}(|k|)}. {(2.19)}$$

Substituting (2.18) we find

$$f_{-}(|k|) = 2\nu \left(\frac{|k|}{2}\right)^{-2\nu} \frac{\Gamma(1+\nu)}{\Gamma(1-\nu)} ,$$

which is related to  $f_+(|k|)$  by  $\nu \to -\nu$ . Fourier transforming back to position space (via an integral which now converges, so that there is no additional flip of sign) and recalling

that the two-point function is minus the second derivative of the effective action, we find that in the  $\Delta_{-}$  theory

$$\langle \mathcal{O}(\vec{x})\mathcal{O}(\vec{x}')\rangle = \frac{(2\Delta_{-} - d)\Gamma(\Delta_{-})}{\pi^{d/2}\Gamma(\Delta_{-} - (d/2))} \frac{1}{|\vec{x} - \vec{x}'|^{2\Delta_{-}}}.$$
 (2.20)

Happily, this function is indeed positive for all dimensions  $\Delta_{-}$  above the unitarity bound. Note also that (2.20) is exactly the same formula as the one we would find by extrapolating (2.16) to  $(d-2)/2 < \Delta < d/2$ .

One may be puzzled by the double zero of the two-point function at  $\Delta = d/2$ . In fact, value of  $\Delta$ , which corresponds to  $m^2 = -d^2/4$  is the special case where  $\Delta_+ = \Delta_-$ . Here the two possible small z behaviors of a classical solution are  $z^{d/2} \ln(z/z_0)$  and  $z^{d/2}$ . Now, only one conformally-invariant boundary condition is possible. We can ask that physical fluctuations behave as  $z^{d/2}$  with no  $z^{d/2} \ln(z/z_0)$  term. But it does not make sense to ask that they behave as  $z^{d/2} \ln(z/z_0)$  with no  $z^{d/2}$  term; such a condition would depend on the choice of  $z_0$ , violating conformal invariance. There is therefore likewise only one natural way to incorporate an external source  $\phi_0$ ; we require that  $\phi(z,\vec{x})$  approaches  $\phi_0(\vec{x})z^{d/2}\ln(z/z_0)$  for small z.<sup>3</sup> The bulk field as defined by (2.11), however, behaves for  $\Delta = d/2$  as  $z^{d/2}\phi_0(\vec{x})$ . To remove this discrepancy we may simply divide the operator  $\mathcal{O}$  by  $\Delta - (d/2)$ , in which case we should divide and multiply  $\phi_0$  and A by the same factor. With this perhaps more useful normalization, the AdS two-point function becomes

$$\langle \mathcal{O}(\vec{x})\mathcal{O}(\vec{x}')\rangle = 2\pi^{-d/2} \frac{\Gamma(\Delta)}{\Gamma(\Delta - (d/2) + 1)} \frac{1}{|\vec{x} - \vec{x}'|^{2\Delta}} , \qquad (2.21)$$

which does not vanish until the dimension approaches the unitarity bound. This field renormalization has a similar effect on the three-point functions: the leg factors  $\frac{1}{\Gamma(\Delta-(d/2))}$  that appear in the results of [25] are changed into  $\frac{1}{\Gamma(\Delta-(d/2)+1)}$  so that the zeroes at  $\Delta=d/2$  are eliminated. This is in accord with the field theory results which give non-vanishing correlators of dimension d/2 operators, such as those found in the  $\mathcal{N}=4$  SYM theory [31]. In general, one should keep in mind that the AdS results agree with field theory calculations only after certain dimension dependent field normalization factors are included [31].

Once such a source is included, the  $\ln(z/z_0)$  term gives a violation of conformal invariance in defining the expectation value  $\langle \mathcal{O} \rangle$ . This is the correct answer from the point of view of conformal field theory. If  $\mathcal{O}$  has dimension d/2, there is a logarithmic divergence in the two point function  $\int d^dx e^{i\vec{p}\cdot\vec{x}} \langle \mathcal{O}(\vec{x})\mathcal{O}(0) \rangle$ , as a result of which if one computes  $\langle \mathcal{O} \rangle$  in the presence of a source, there is a logarithmic violation of conformal invariance.

To summarize the present section, in the range of scalar masses (2.3), there are two possible theories. One of them associates to the scalar field an operator of dimension  $\Delta_+$ , while the other associates an operator of dimension  $\Delta_-$ . The two different definitions of the theory correspond to interchanging the source  $\phi_0(\vec{x})$  and the expectation value  $(2\Delta - d)A(\vec{x})$  defined by the boundary behavior (2.9). Thus, the generating functionals of correlation functions are related by a Legendre transform. This is analogous to the situation found in Liouville theory where the generating functional corresponding to the theory with one branch of gravitational dressing is the Legendre transform of the generating functional corresponding to the other branch [30].

## 3. Examples of symmetry breaking

In this section we discuss perhaps the simplest example of the AdS/CFT duality in presence of symmetry breaking. This example was discussed previously in [1,14]: the gauge symmetry present on coincident D3-branes in flat space may be broken simply by separating them into several parallel stacks. Below we will discuss a simple case of breaking SU(N) down to  $S(U(N_1) \times U(N_2))$  by separating N coincident D3-branes into two parallel stacks containing  $N_1$  and  $N_2$  coincident D3-branes. Its generalizations to more complicated breaking patterns will then be immediate.

The two-stack configuration of branes corresponds to Higgsing of the  $\mathcal{N}=4$  gauge theory by scalar fields

$$\vec{X} = \begin{pmatrix} \vec{d_1} \cdot I_1 & 0\\ 0 & \vec{d_2} \cdot I_2 \end{pmatrix} \tag{3.1}$$

where  $\vec{d_i}$  is the position of the *i*-th brane stack and  $I_i$  is the  $N_i \times N_i$  identity matrix. Such a Higgsing gives expectation values to the chiral fields  $\mathcal{O}_{i_1 i_2 \dots i_n}^{(n)} = \operatorname{Tr} X_{i_1} X_{i_2} \dots X_{i_n}$  trace terms:

$$\langle \mathcal{O}_{i_1 i_2 \dots i_n}^{(n)} \rangle \sim N_1[(d_1)_{i_1}(d_1)_{i_2} \dots (d_1)_{i_n} - \text{traces}] + N_2[(d_2)_{i_1}(d_2)_{i_2} \dots (d_2)_{i_n} - \text{traces}] .$$
 (3.2)

We wish to see how these order parameters emerge in the AdS description.

To obtain that description, we must find the appropriate analog of the  $AdS_5 \times S^5$  metric. For this, we proceed along lines described in the introduction. The Green's function with two separated sources is

$$H = L^4 \left( \frac{a_1}{|\vec{y} - \vec{d_1}|^4} + \frac{a_2}{|\vec{y} - \vec{d_2}|^4} \right) , \qquad (3.3)$$

where  $y_1, \ldots, y_6$  are the coordinates normal to the brane and

$$a_i = \frac{N_i}{N} \ . \tag{3.4}$$

It is always possible, by a shift of the coordinates, to choose the origin at the "center of mass" so that  $a_1\vec{d}_1 + a_2\vec{d}_2 = 0$ . Adopting this choice we find that the Green's function (3.3) can be given the following Taylor series expansion at large  $\vec{y}$   $(r = |\vec{y}|)$ :

$$H = \frac{L^4}{r^4} \left( 1 + \sum_{n=2}^{\infty} 2^n \frac{(n+1)d_{i_1 i_2 \dots i_n}^{(n)} y^{i_1} y^{i_2} \dots y^{i_n}}{r^{2n}} \right), \tag{3.5}$$

with

$$d_{i_1 i_2 \dots i_n}^{(n)} = a_1[(d_1)_{i_1}(d_1)_{i_2} \dots (d_1)_{i_n} - \text{traces}] + a_2[(d_2)_{i_1}(d_2)_{i_2} \dots (d_2)_{i_n} - \text{traces}] . \quad (3.6)$$

The trace terms are precisely such that  $d_{i_1 i_2 \dots i_n}^{(n)}$  is a traceless symmetric tensor.

Given the Green's function, the corresponding spacetime metric is, as in (1.2),

$$ds^{2} = H^{-1/2}(-dt^{2} + dx_{1}^{2} + dx_{2}^{2} + dx_{3}^{2}) + H^{1/2}\sum_{j=1}^{6} dy_{j}^{2}.$$
 (3.7)

The large r behavior can be worked out using (3.5). In particular, if we simply set  $H = L^4/r^4$ , we get the familiar  $AdS_5 \times S^5$  metric, with as usual r combining with  $t, x_1, x_2, x_3$  to make the five  $AdS_5$  coordinates. In this description, the boundary of  $AdS_5$  is at  $r = \infty$ , and r is related to the parameter z of section 2 by

$$r = \frac{L^2}{z}. ag{3.8}$$

Expanding  $H^{1/2}$  to linear order in  $a_i$  we find that for every  $n=2,3,\ldots$ , there is in the metric a correction term proportional to

$$\frac{d_{i_1...i_n}^{(n)} \hat{y}^{i_1} \dots \hat{y}^{i_n}}{r^n} \ . \tag{3.9}$$

where, in general,  $d_{i_1...i_n}^{(n)}$  are the tensors that appear in the partial wave expansion of the Green's function H.

As we have explained in eqn. (2.13), the existence of a correction to the metric proportional to  $r^{-n} = z^n$  means that a conformal field of dimension n has an expectation value. Given the structure of (3.9), this conformal field clearly transforms under the SO(6) symmetry of the  $y^i$  as  $d_{i_1...i_n}^{(n)}$ . The field in question is precisely  $\mathcal{O}_{i_1i_2...i_n}^{(n)}$  and, up

to proportionality constants that we have not checked, the  $d_{i_1...i_n}^{(n)}$  given in (3.6) agree with the expectation values (3.2) calculated in field theory. In fact, in the AdS/CFT correspondence,  $\mathcal{O}_{i_1i_2...i_n}^{(n)}$  is mapped to conformal fluctuations in the metrics of  $AdS_5$  and  $S^5$  (and a related fluctuation in the four-form gauge potential); in the present context, such a fluctuation comes from the  $r^{-n}$  correction term in H.

In a more general multi-stack example, with

$$H = L^4 \sum_{i} \frac{a_i}{|\vec{y} - \vec{d_i}|^4}, \qquad \sum_{i} a_i = 1,$$
 (3.10)

we can always eliminate the spin one or "dipole" harmonic in the expansion of H by adding a constant to  $\vec{y}$  to transform to a "center of mass" frame with  $\sum_i a_i \vec{d_i} = 0$ . The higher harmonics cannot be so eliminated and obey no general restrictions, so in general we get expectation values of the chiral fields  $\mathcal{O}^{(n)}$  for all  $n \geq 2$ . The vanishing of  $\mathcal{O}^{(1)}$  means that the classical supergravity solutions of this kind, modulo coordinate transformations, are in natural correspondence with the vacua of an SU(N), rather than U(N), gauge theory.

So far we have discussed the terms in the metric coming from expanding  $H^{1/2}$  to linear order in the "charges"  $a_i$ . However, as pointed out in [14], starting with n=4 the coefficients of  $r^{-n}$  terms in the metric also have corrections containing higher powers of  $a_i$ . Since  $a_i = N_i/N$  the structure of these terms is suggestive of expectation values of operators containing more than a single trace [14]. For example, for n=4, in addition to  $\langle \mathcal{O}^{(4)} \rangle$  we also seem to find  $\frac{1}{N} \langle \mathcal{O}^{(2)} \mathcal{O}^{(2)} \rangle$ . We postpone investigation of these extra terms for the future.

## Orbifolds

Orbifolds obtained by dividing the space  $\mathbf{R}^6$  transverse to the threebranes by a finite subgroup  $\Gamma$  of SO(6) can be discussed in a very similar fashion. If all branes are at the origin in  $\mathbf{R}^6$ , we get a theory whose infrared limit is described by  $AdS_5 \times S^5/\Gamma$ . For example, in some much-studied cases with  $\Gamma = \mathbf{Z}_n$  [12,13], the  $AdS_5 \times S^5/\Gamma$  geometry is dual to a gauge theory with gauge group  $S(U(N)^n)$ .

Higgsing of these theories will be described in the AdS/CFT correspondence by replacing  $AdS_5 \times S^5/\Gamma$  by a solution that looks like that near the boundary but is different in the interior. For instance, Higgsing of the gauge theory to a diagonal SU(N) is described in terms of branes by placing all branes at the same smooth point in  $\mathbf{R}^6/\Gamma$ , away from all orbifold singularities. The corresponding Green's function is

$$H = L^4 \sum_{i=1}^n \frac{1}{|\vec{y} - \omega_i \vec{y_0}|^4},\tag{3.11}$$

with generic  $\vec{y}_0$  and with  $\omega_i$  the elements of  $\Gamma$ . Again, H can be expanded in spherical harmonics. The leading term at long distances is  $H = nL^4/r^4$ , and gives back an  $AdS_5 \times S^5/\Gamma$  metric near the boundary; the corrections vanish as higher powers of r and correspond to expectation values of chiral fields in the gauge theory. Other symmetry-breaking vacua are described, in the AdS language, by taking H to have sources at fixed points of some of the elements of  $\Gamma$  or to have sources consisting of more than a single  $\Gamma$  orbit.

Note that if the group  $\Gamma$  acts on  $\mathbf{R}^6$  with no invariant vectors, then a "dipole" term, of order  $1/r^5$ , is always absent in the expansion of H in spherical harmonics, as this term is annihilated by the sum over  $\omega_i$ . Even if there are invariants in the action of  $\Gamma$  on  $\mathbf{R}^6$ , the dipole term can be eliminated by shifting to "center of mass" coordinates in the directions on which  $\Gamma$  acts trivially. Once this is done, there is no  $1/r^5$  term in H, so we never get an expectation value for a chiral field of dimension 1. This is just as well, for in a unitary quantum field theory in four dimensions, a scalar field of dimension 1 must be free and so cannot be described by the dynamics in the bulk of AdS space. Such a field can only enter the AdS/CFT correspondence as a "singleton" field supported at infinity. All assertions in this paragraph remain true if the configuration of sources in (3.11) is generalized to a more general  $\Gamma$ -invariant configuration.

# $Global\ Structure$

To conclude this section, we will make a few remarks about the global structure of the supergravity solutions that we have discussed.

We recall the form of the  $AdS_5 \times S^5$  solution:

$$ds^{2} = \frac{L^{2}}{r^{2}}dr^{2} + \frac{r^{2}}{L^{2}}\sum_{i=1}^{4}dx_{i}^{2} + L^{2}d\Omega_{5}^{2}.$$
(3.12)

Here  $\Omega_5$  is the metric on a round five-sphere, and  $x_i$ , i = 1, ..., 4, are coordinates on the four dimensions parallel to the threebranes.

Roughly speaking, the boundary of  $AdS_5$  is at  $r = \infty$  (or z = 0 in the notation of section 2, the relation being  $z = L^2/r$ ). The boundary thus appears to be a copy of  $\mathbf{R}^4$ , parametrized by the  $x_i$ . However, this is not the whole story, because the coordinate system used in (3.12) behaves badly at r = 0. In fact, r = 0 contributes one more point to the boundary of  $AdS_5$  (heuristically, because the coefficient of  $dx_i^2$  vanishes at r = 0, r = 0 is just a single point on the boundary). When we add this point, the boundary of  $AdS_5$  is compactified from  $\mathbf{R}^4$  to  $S^4$ .

The compactness of  $S^4$  means that, as long as we only consider perturbations that admit this same global structure at infinity, we cannot encounter infrared divergences.

Moreover, the symmetry breaking or Higgsing phenomena that we have studied above cannot occur if the boundary is  $S^4$ . One way to explain this uses the positivity of the scalar curvature R of  $S^4$ . The scalar fields X of the  $\mathcal{N}=4$  theory have a conformally invariant quadratic term in the action,

$$\int d^4x \sqrt{g} \left( \text{Tr} (dX)^2 + \frac{R}{6} \text{Tr} X^2 \right) , \qquad (3.13)$$

that is strictly positive definite and prevents X from acquiring an expectation value, as long as one only considers perturbations that make sense when the boundary is  $S^4$ . (In particular, a constant X field on  $\mathbf{R}^4$  has a singularity at the "point at infinity" if one tries to compactify to  $S^4$ .)

Because of all these facts, the theory on  $S^4$  is completely stable under sufficiently small perturbations, and there is in fact a well-defined procedure for computing its correlation functions. If we want to study symmetry-breaking, we cannot achieve this degree of "safety"; we have to allow perturbations that cannot be naturally interpreted on  $S^4$  and for which the metric will, in fact, have dangerous and difficult to interpret regions. To see how this works, we rewrite (3.12) in the form (3.7) with

$$H = \frac{L^4}{r^4}. ag{3.14}$$

This metric appears to be badly behaved near r = 0, but as we have discussed, this is part of the one-point compactification that converts the boundary from  $\mathbf{R}^4$  to  $S^4$ . Now, however, consider one of the symmetry-breaking choices like

$$H = L^4 \left( \frac{a_1}{|\vec{y} - \vec{d_1}|^4} + \frac{a_2}{|\vec{y} - \vec{d_2}|^4} \right). \tag{3.15}$$

There are now two dangerous points, at  $\vec{y} = \vec{d_1}$ ,  $\vec{d_2}$ . Either of these looks in this coordinate system just like the r = 0 point in the case (3.14). With two singularities, it is not possible to absorb them as part of the "boundary," since there is no reasonable compactification of  $\mathbf{R}^4$  by adding two points. Thus, for the symmetry-breaking vacua, we are committed to thinking of the boundary as  $\mathbf{R}^4$ . This is just another manifestation of the fact that positivity of (3.13) makes symmetry breaking impossible if the boundary is  $S^4$ .

With the boundary, on which the dual four-dimensional field theory is formulated, being  $\mathbf{R}^4$ , it is possible to have various types of infrared instabilities if relevant perturbations are added. In the AdS description, this is related to the fact that the bad behavior of the metric at the dangerous points  $\vec{y} = \vec{d_1}$ ,  $\vec{d_2}$  may be unstable against small perturbations. To compute correlation functions in these symmetry-breaking vacua, care will be needed in specifying the desired behavior of perturbations near the bad points of the metric; the crucial clue is presumably that when there is only one such bad point, we know (via compactification of the boundary) how to treat it. In any event, we know from the case of just one bad point that a "singularity" of this type signals flow in the infrared to a non-trivial renormalization group fixed point, the  $\mathcal{N}=4$  Yang-Mills theory with gauge groups determined by the coefficients of the singularities in H.

#### 4. Threebranes on the Conifold

In this section, we will consider symmetry breaking in a more complex example – threebranes near the conifold singularity. We recall that the conifold can be described in terms of complex variables  $w_a$ , a = 1, ..., 4 by the equation

$$\sum_{a} w_a^2 = 0. (4.1)$$

Alternatively, one can introduce a  $2 \times 2$  matrix of complex variables  $m_{ij}$ , i, j = 1, 2, and write

$$0 = \det m = m_{11}m_{22} - m_{12}m_{21}. (4.2)$$

The two descriptions are related by an obvious linear change of variables. We denote the manifold described by either of these equations as  $Y_6$ ; it is a cone over a five-manifold  $X_5$ , also denoted as  $T^{1,1}$ .  $T^{1,1}$  is a homogeneous space  $(SU(2) \times SU(2))/U(1)$ , where the U(1) is a diagonal subgroup of  $SU(2) \times SU(2)$  [32].

The near horizon geometry of a system of N Type IIB threebranes at the conifold singularity is  $AdS_5 \times T^{1,1}$ . In [9], we considered a conformal field theory that is dual to Type IIB on that spacetime. In this conformal field theory, the gauge group is  $SU(N) \times SU(N)$  and there are chiral superfields  $A_i$ ,  $B_j$ , i, j = 1, 2, with  $A_i$  transforming as  $(\overline{\mathbf{N}}, \overline{\mathbf{N}})$  and  $B_j$  transforming as  $(\overline{\mathbf{N}}, \mathbf{N})$  under  $SU(N) \times SU(N)$ . There is also a superpotential  $W = \epsilon^{ij} \epsilon^{kl} \text{Tr} A_i B_k A_j B_l$ .

In [9], we considered only the vacuum of the gauge theory with zero expectation value for  $A_i$  and  $B_j$ , and only the  $AdS_5 \times T^{1,1}$  solution on the string theory side. Our intent here is to consider more general vacua and solutions.

On the supergravity/string theory side, we can, to begin with, consider the following three operations:

- 1) Deformation of the singularity.
- 2) Resolution of the singularity.
- 3) Moving the branes away from the singularity.

We will want to interpret all of these operations – to the extent that they can occur – in tems of Higgsing of the gauge theory.

Deformation of the singularity means merely that an additional term is added to the equation describing the conifold. It becomes

$$\sum_{a} w_a^2 = \epsilon. \tag{4.3}$$

Resolution of the singularity is more subtle to describe.<sup>4</sup> One "solves" the equation (4.2) by introducing complex variables  $a_i$ ,  $b_j$ , i, j = 1, 2, and writing  $m_{ij} = a_i b_j$ . Then one imposes the constraint

$$\sum_{i} |a_{i}|^{2} - \sum_{j} |b_{j}|^{2} = \delta, \tag{4.4}$$

with  $\delta$  a constant. If  $\delta = 0$ , then one gets a description of the conifold by imposing also the equivalence relation

$$a_i \to e^{i\theta} a_i, \quad b_j \to e^{-i\theta} b_j,$$
 (4.5)

to remove the redundancy in the "solution"  $m_{ij} = a_i b_j$ . The resolution of the conifold is described by taking  $\delta \neq 0$ , and still imposing the equivalence relation.

 $AdS_5 \times T^{1,1}$  is the same topologically as  $\mathbf{R}^4 \times \mathbf{R} \times T^{1,1}$ , where  $\mathbf{R} \times T^{1,1}$  is the cone  $Y_6$  over  $T^{1,1}$  with the singularity of the cone omitted. The usual metric on  $AdS_5 \times T^{1,1}$  is

$$ds^{2} = H^{-1/2} \sum_{i} dx_{i}^{2} + H^{1/2} \left( dr^{2} + r^{2} d\Omega^{2} \right), \tag{4.6}$$

where H is the standard Green's function  $H = L^4/r^4$ ,  $d\Omega^2$  is the Einstein metric (1.4) on  $T^{1,1}$ , and  $\sum_i dx_i^2$  is the standard flat metric on  $\mathbf{R}^4$ . In (4.6),  $dr^2 + r^2 d\Omega^2$  should be interpreted as a Calabi-Yau metric on  $\mathbf{R} \times T^{1,1}$ . In the spirit of the present paper, we will incorporate symmetry breaking by replacing H by a more general Green's function to account for motion of the threebranes, and by replacing  $dr^2 + r^2 d\Omega^2$  by a Calabi-Yau metric on the deformation  $Y_6'$  of the conifold, or its resolution  $Y_6''$ . (These Calabi-Yau metrics have been discussed in [15].) Our goal will then be to match this more general geometry to a symmetry breaking vacuum in the gauge theory.

The two operations of deforming and resolving the conifold are usually described in string theory in terms of motion in complex structure or Kähler moduli space. In the case of the conifold, these motions are mutually incompatible; one can do either one but not both. (There can be under suitable conditions a phase transition between the two branches [33].) In the present context, only the resolution of the conifold is generically allowed, in the following sense. The manifold  $T^{1,1}$  has second Betti number 1, and as a result, in string theory on  $AdS_5 \times T^{1,1}$  there are RR and NS-NS B-fields. These combine together into a complex parameter which is interpreted in terms of the gauge couplings of the  $SU(N) \times SU(N)$  theory. On the other hand, the deformation  $Y_6'$  of the conifold is

<sup>&</sup>lt;sup>4</sup> The resolution of interest to us is a "small resolution" that preserves the Calabi-Yau condition. If one did not wish to preserve the Calabi-Yau condition, many more general resolutions would be possible.

topologically  $T^*S^3$  (the cotangent bundle of the three-sphere) and in particular its second Betti number vanishes. Hence, in string theory on  $\mathbb{R}^4 \times Y_6'$ , any flat B-field can be gauged away. If there are nonzero theta angles at infinity (that is, on  ${f R}^4 imes T^{1,1}$  – we recall that  $Y_6'$  looks like  $T^{1,1}$  at infinity) – then there must be non-zero curvatures H=dB in the interior of  $Y_6'$ . Non-zero values of the H-fields break supersymmetry, so one would not get supersymmetric vacua in this way. Moreover, the nonzero H would provide a source for the dilaton, so the fields with non-trivial B-fields at infinity may not give  $\mathbf{R}^4 \times Y_6'$ solutions at all. We conclude, then, that the deformation of the conifold is possible (or at least, accessible) only if the B-fields vanish at infinity, and thus only for special values of the gauge couplings of the dual conformal field theory. A phenomenon that can occur only for special values of the gauge couplings is beyond our present understanding of the gauge theory, and thus we will not attempt to further analyze the deformation of the conifold in the present paper. Furthermore, this particular case may turn out to be the hardest to analyze because for B=0 one of the two SU(N) gauge couplings is expected to blow up [11]. This is natural from the point of view of flowing to the conifold field theory from an orbifold field theory. Also, in the "T-dual" description in terms of the NS5-branes and D4-branes, this is the point in the moduli space where the distance between the two NS5-branes vanishes [34], presumably giving rise to tensionless strings. So the locus for which deformation of the conifold is possible is probably quite subtle to describe.

On the other hand, the resolution  $Y_6''$  is topologically an  $\mathbb{R}^4$  bundle over  $S^2$ ; its second Betti number is 1. Thus, the flat B-fields on  $T^{1,1}$  extend over  $Y_6''$ , and hence should make sense for generic values of the gauge couplings. Consequently, in the rest of this section we concentrate on matching the resolution of the conifold and the motion of the branes with phenomena in the gauge theory.

#### 4.1. First Look At The Gauge Theory

For simplicity, we will look at vacua in which  $SU(N) \times SU(N)$  is broken to a diagonal SU(N). Once these vacua are understood, the extension to more general cases is apparent.

To have an unbroken diagonal SU(N), the  $N \times N$  matrices  $A_i$  and  $B_j$  of chiral superfields must be (perhaps after a gauge transformation) multiples of the identity. So to get vacua of this type, we set  $A_i = a_i$ ,  $B_j = b_j$ , with complex numbers  $a_i$  and  $b_j$ . The order parameters  $m_{ij} = \langle \operatorname{Tr} A_i B_j \rangle / N$  are thus equal to  $a_i b_j$ . As we have already mentioned, equation (4.2) is an immediate consequence of  $m_{ij} = a_i b_j$ . On the other hand, there is no restriction in the gauge theory on the value of  $\delta = \sum_i |a_i|^2 - \sum_j |b_j|^2$ . The gauge theory has both vacua with  $\delta = 0$  and vacua with  $\delta \neq 0$ . We interpret this to mean that the dynamics of the gauge theory includes a description of the resolution of the conifold.

The gauge theory also lacks the equivalence relation (4.5). We interpret this to mean that a pseudoscalar mode in  $\mathbf{R}^4 \times Y_6''$  that is related by supersymmetry to the resolution of the conifold is likewise described by the gauge theory dynamics. The mode in question can be described as follows: it is a mode of the four-form potential that transforms as a two-form on  $\mathbf{R}^4$  times a two-form on  $Y_6''$ . We recall that a two-form on  $\mathbf{R}^4$  is dual to a scalar.

In short, we propose that the resolution (but of course, not the deformation) of the conifold is described by the dynamics of the dual  $SU(N) \times SU(N)$  gauge theory. According to this proposal, the resolution of the conifold is described by the choice of vacuum in a fixed gauge theory with fixed coupling constants, rather than by a change in the coupling constants of the theory.

Here is a bit of topological evidence for this proposal. The  $SU(N) \times SU(N)$  gauge theory with the chiral superfields  $A_i$ ,  $B_j$ , has a "baryon number" global symmetry  $A_i \rightarrow e^{i\theta}A_i$ ,  $B_j \rightarrow e^{-i\theta}B_j$ . This symmetry might be called "baryon number" because typical order parameters are the baryonic, or dibaryonic, operators det  $A_i$  and det  $B_j$ . According to [35], the baryon number is mapped in the dual  $AdS_5 \times T^{1,1}$  description to the wrapping number of threebranes on a three-cycle in  $T^{1,1}$ . Such a wrapping number exists because the third Betti number of  $T^{1,1}$  is 1. (Topologically  $T^{1,1}$  is  $S^3 \times S^2$  [15].) Now, when we replace  $AdS_5 \times T^{1,1}$  by  $\mathbf{R}^4 \times Y_6$ ", this conserved threebrane wrapping number no longer exists, because the third Betti number of  $Y_6$ " is zero. (A wrapped threebrane at infinity in  $\mathbf{R}^4 \times Y_6$ " can move into the interior and annihilate.) Thus,  $\mathbf{R}^4 \times Y_6$ " must correspond in the  $SU(N) \times SU(N)$  gauge theory to vacua in which the baryonic charge is not conserved. Indeed, in vacua with  $\delta \neq 0$ , the  $a_i$  and  $b_j$  cannot all vanish, and hence the expectation values of operators det  $A_i$  and det  $B_j$  carrying baryon number are likewise not all zero.

At the cost of jumping slightly ahead of our story, we can also consider the case that the gauge symmetry is broken, with  $m_{ij} \neq 0$ , but  $\delta = 0$  and the conifold singularity is not resolved. Also in this case, some of the baryonic order parameters det  $A_i$  and det  $B_j$  of the field theory are nonzero, so baryon number should not be conserved. On the AdS side, we will interpret these vacua in terms of string theory on  $\mathbf{R}^4 \times Y_6$ , with a Green's function H whose singularity is not at the conical singularity of  $Y_6$ . It follows that in this case, the conifold singularity is at a finite distance in spacetime, and a wrapped threebrane can presumably disappear by collapsing to the conifold singularity. This contrasts with the  $AdS_5 \times T^{1,1}$  spacetime, which is dual to a vacuum with unbroken symmetries; here the conifold singularity has disappeared "to infinity" in spacetime, and there is no way for a wrapped threebrane to annihilate.

## 4.2. Quantitative Treatment

We now wish to describe these vacua somewhat more quantitatively. The first step is to find the appropriate Calabi-Yau metric  $ds_6^2$  on  $Y_6$  or  $Y_6''$ . Then one finds the appropriate Green's function H on  $Y_6$  or  $Y_6''$ , with sources at the desired positions of the threebranes, and the spacetime metric takes the familiar form:

$$ds^{2} = H^{-1/2} \sum_{i} dx_{i}^{2} + H^{1/2} ds_{6}^{2}.$$
 (4.7)

This program can be described most explicitly for the case of  $Y_6$  – moving the three-branes away from the conifold singularity without resolving it – because the Calabi-Yau metric on  $Y_6$  is particularly elementary. In terms of the description of the conifold via an equation  $\sum_{a=1}^4 w_a^2 = 0$ , set

$$\sum_{a=1}^{4} \overline{w}_a w_a = \rho^2. \tag{4.8}$$

Then the  $AdS_5$  radial coordinate is  $r = \sqrt{3/2}\rho^{2/3}$ . The  $Y_6$  metric has the familiar conical form  $ds_{\text{cone}}^2 = dr^2 + r^2 ds_5^2$ . The Laplace equation for the Green's function away from the source is

$$-\frac{1}{r^5}\frac{\partial}{\partial r}\left(r^5\frac{\partial H}{\partial r}\right) + \frac{E}{r^2}H = 0,\tag{4.9}$$

where E is the angular Laplacian on  $T^{1,1}$ .

Because  $T^{1,1}$  is a homogeneous space, the spectrum of the angular Laplacian can be worked out via group theory [36,37].  $T^{1,1}$  has symmetry group  $SU(2)\times SU(2)\times U(1)$ , where the  $w_a$  transform as (1/2,1/2) under  $SU(2)\times SU(2)$  and with charge 1 under U(1). (The U(1) is an R-symmetry group.) The spherical harmonics that are relevant for studying expectation values of chiral superfields are the modes that transform as (k/2, k/2) with U(1) charge  $\pm k$ . The corresponding wavefunctions are simply

$$\widehat{w}_{a_1 a_2 \dots a_k} = \frac{w_{a_1} w_{a_2} \dots w_{a_k}}{|\sum_b \overline{w}_b w_b|^{k/2}},\tag{4.10}$$

or the complex conjugate of this to reverse the sign of the U(1) charge. (The reason that these are the relevant modes is roughly that modes with both w's and  $\overline{w}$ 's in the numerator would have a larger eigenvalue of the Laplacian for given U(1) charge and would ultimately lead to nonchiral operators.) The eigenvalue of the Laplacian for these modes is [36,37]

$$E(k) = 3\left(k(k+2) - \frac{k^2}{4}\right). \tag{4.11}$$

If we look for a contribution to H of the form  $r^{c(k)}\widehat{w}_{a_1a_2...a_k}$ , we find that the equation (4.9) implies

$$c(k)(c(k) + 4) = E(k). (4.12)$$

We want the negative root, since we want H to behave as  $r^{-4}$  at infinity. So

$$c(k) = -2 - \sqrt{E(k) + 4}. (4.13)$$

With the given values of E(k), this gives the attractively simple result

$$c(k) = -4 - 3k/2. (4.14)$$

Given this, it follows that the relevant terms in H take the form

$$H = \frac{L^4}{r^4} \left( 1 + \sum_{k=1}^{\infty} \frac{(f_{a_1 \dots a_k} \widehat{w}_{a_1 \dots a_k} + c.c.)}{r^{3k/2}} \right)$$
(4.15)

for some f's.

The series of correction terms to H of relative order  $r^{-3k/2}$  give corrections to the  $AdS_5 \times T^{1,1}$  metric that vanish like  $r^{-3k/2}$  or  $z^{3k/2}$  near the boundary. According to eqn. (2.13), such corrections imply that an operator of dimension 3k/2 has an expectation value. For the harmonics described above, the U(1) or R-charge is k. A field of R-charge k and dimension 3k/2 is a chiral superfield. In this case, the chiral superfields in question transform like (k/2, k/2) under  $SU(2) \times SU(2)$ . These are the quantum numbers of the chiral superfields discussed in [9], namely  $\operatorname{Tr} A_{i_1} B_{j_1} A_{i_2} B_{j_2} \dots A_{i_k} B_{j_k} + \operatorname{permutations}$  of indices  $i_1, \dots, i_k$  and  $j_1, \dots, j_k$ .

Notice that for k = 1, we have here an expectation value of an operator of dimension 3/2, which is above the unitarity bound (which is 1 in d = 4 dimensions), but below the naive AdS bound of d/2 = 2. The formalism for studying in AdS space a scalar operator of dimension in this range was discussed in section 2. For our present purposes, the net effect is simply that the term of relative order  $r^{-3k/2}$  in the metric is due to the expectation value of a dimension 3k/2 operator both for k = 1 and for k > 1.

So far, we have made no assumption about the nature of the source terms for H: we have merely assumed that near  $r=\infty$ , H obeys the Laplace equation and vanishes as  $r^{-4}$ . We can, if we wish, require further that all threebranes are located at a point  $w_a = \epsilon_a$  on  $Y_6$  away from the conifold singularity. In this case, H will be invariant under the subgroup of  $SU(2) \times SU(2) \times U(1)$  that leaves fixed the point  $w_a = \epsilon_a$ . This implies that  $f_{a_1 a_2 \dots a_k}$  in (4.15) is a multiple of  $\overline{\epsilon}_{a_1} \overline{\epsilon}_{a_2} \dots \overline{\epsilon}_{a_k}$ . In particular, for k=1, we have  $\sum_a f_a^2 = 0$ , and f is subject to no other restriction. Translated into the language of the gauge theory,

this amounts to the statement that for a vacuum with an unbroken diagonal SU(N), the order parameters  $m_{ij} = \langle \operatorname{Tr} A_i B_j \rangle$  are coordinates of a point on  $Y_6$ . This is the familiar statement that the equation (4.2) of the conifold is a consequence of  $m_{ij} = a_i b_j$ . Moreover, as explained in [9], for these Higgsed vacua with unbroken diagonal SU(N), the low energy theory has  $\mathcal{N} = 4$  supersymmetry (even though the microscopic theory has  $\mathcal{N} = 1$ ). This is the result expected on the gravitational side, since with  $\epsilon_a \neq 0$  the metric singularity is that of N threebranes at a smooth point.

One can make a similar comparison between gauge theory and gravity for the expectation values of chiral superfields with k > 1. For single trace operators, which correspond to linear terms in the expansion of  $H^{1/2}$ , these additional comparisons, to the extent that they can be made without further detailed computations, yield little that is really new since the results are nearly determined by the symmetries. Both the expectation values of the chiral fields and the coefficients in the expansion of H are determined, up to a k-dependent normalization constant that depends on quantities we have not computed (such as the precise constants in (4.15) if H has only a single delta function source term at  $w_i = \epsilon_i$ ), by the unbroken symmetries. However, similarly to the example of section 3, for k > 1 one finds that the coefficient of the  $r^{-3k/2}$  term in the metric is corrected by non-linear terms in the expansion of  $H^{1/2}$ . Their structure once again seems to correspond to multiple-trace operators in the gauge theory, and the meaning of this requires further investigation.

## The Resolution

Threebranes on the resolved manifold  $Y_6''$  can be described similarly, the main difference being that the Calabi-Yau metric of  $Y_6''$  is known less explicitly [15], and the description of the Green's function H will be correspondingly less explicit.

The resolution of the conifold can be interpreted, in gauge theory language, in terms of giving an expectation value to a certain operator  $\mathcal{U}$ . Let us compute the dimension of this operator. Under scalings of r, the conical metric  $dr^2 + r^2d\Omega^2$  on  $Y_6$  scales, obviously, like  $r^2$ , as therefore does the Kähler form  $\omega$  on  $Y_6$ . The resolution of  $Y_6$  is obtained by a motion in Kähler moduli space, and so by a topologically non-trivial correction to  $\omega$ . This topologically nontrivial correction is indeed

$$\omega' = \frac{\epsilon^{abcd} w_a \overline{w}_b dw_c d\overline{w}_d}{\left|\sum_a \overline{w}_a w_a\right|^2}.$$
(4.16)

(The sign with which  $\omega'$  is added to the Kähler form determines which of two topologically distinct resolutions of  $Y_6$  is obtained.) To verify that this is the correct  $\omega'$ , we proceed as follows.  $\omega'$  is invariant under the SO(4) symmetry of  $Y_6$  but not under the disconnected

component of O(4). In fact, the disconnected component of O(4) exchanges the  $a_i$  and  $b_j$  in (4.4), and hence changes the sign of  $\delta$  and therefore of  $\omega'$ . Direct computation shows that  $d\omega' = 0$  (this fixes the power of  $|\overline{w}w|$  in the denominator of (4.16) and hence the scaling weight of  $\omega'$ ). However,  $\omega'$  is not of the form  $d\lambda$  for any  $\lambda$ . (Indeed, by averaging over the compact group SO(4), one could always assume that  $\lambda$  is SO(4)-invariant; but on  $Y_6$  every SO(4)-invariant one-form is also O(4)-invariant.)

From (4.16), we see that  $\omega'$  is invariant under scalings of w or equivalently of r, so it scales like  $r^{-2}$  relative to the unperturbed Kähler form  $\omega$  on the cone. This scaling corresponds to the expectation value of an operator of dimension two in the gauge theory. The term of order  $r^{-2}$  is the leading large r correction in the C-Y metric on  $Y_6''$ : the Einstein equations force the topologically trivial corrections to  $\omega$  to vanish faster than  $r^{-2}$  for large r. There are also additional corrections to the metric of  $AdS_5 \times T^{1,1}$  coming from the Green's function H, similar to the ones that were present in the symmetry-breaking vacua on  $Y_6$ ; these terms correspond to expectation values of the chiral operators  $Tr(AB)^k$ .

Let us try to interpret the correction term of order  $r^{-2}$  in the gauge theory. Comparing again to (4.4), the natural gauge theory order parameter for the resolution, in terms of the chiral superfields  $A_i$  and  $B_j$ , is  $\mathcal{U} = \operatorname{Tr} A_i \overline{A}_i - \operatorname{Tr} B_j \overline{B}_j$ . We note that classically  $\mathcal{U}$  has dimension two, in agreement with the dimension that we found from the asymptotic correction to the Kähler metric on  $Y_6''$ . The operator  $\mathcal{U}$  is contained in the same multiplet with the current that generates the "baryon number" symmetry  $A_i \to e^{i\theta} A_i$ ,  $B_j \to e^{-i\theta} B_j$ . The conserved current has no anomalous dimension, so likewise the dimension of  $\mathcal{U}$  is uncorrected in going from the classical description to supergravity.

Conserved current multiplets are among the several possible shortened multiplets of SU(2,2|1) [38,37]. As explained above, the operator  $\mathcal{U}$  belongs to a conserved current multiplet and it is interesting to ask what is the supergravity multiplet related to it through the AdS/CFT correspondence. The multiplets appearing in type IIB supergravity on  $AdS_5 \times T^{1,1}$  were recently classified in [37], and we will make use of this analysis. We expect the baryon number current of the gauge theory to correspond to the massless gauge field in  $AdS_5$  which couples to the D3-brane wrapped over the 3-cycle in  $T^{1,1}$ . This field is the component of the 4-form with one  $AdS_5$  index and three  $T^{1,1}$  indices,  $A_{\mu abc}$ . Vector multiplet I listed in Table 7 of [37] contains precisely this kind of vector field. When the internal Laplacian eigenvalue is E=0, corresponding to a singlet under  $SU(2) \times SU(2) \times U(1)_R$ , then this vector field is massless. In this case the vector multiplet in Table 7 also contains a scalar in  $AdS_5$  with  $m^2=-4$  corresponding to dimension  $\Delta=2$ , and we identify this field with the scalar operator  $\mathcal{U}$ . This field is a graviton with two  $T^{1,1}$  indices [37], as expected from the preceding discussion. To summarize, the operator  $\mathcal{U}$ 

and the baryon number current are related through the AdS/CFT duality to fields from a massless  $AdS_5$  vector multiplet.

## Appendix A. Chiral Primary Operators

Here we discuss the supergravity modes which correspond to chiral primary operators. (For an extensive analysis of the spectrum of the model that appeared following the original version of the present paper, see [37].) This will give further background for the discussion in sections 3 and 4. For the  $AdS_5 \times S_5$  case, these modes are mixtures of the conformal factors of the  $AdS_5$  and  $S_5$  and the 4-form field. The same has been shown to be true for the  $T^{1,1}$  case [36,39,37]. In fact, we may keep the discussion of such modes quite general and consider  $AdS_5 \times X_5$  where  $X_5$  is any Einstein manifold.

The diagonalization of such modes carried out by Kim, Romans and van Nieuwenhuizen for the  $S^5$  case [40] is easily generalized to any  $X_5$ . The mixing of the conformal factor and 4-form modes results in the following mass-squared matrix,

$$m^2 = \begin{pmatrix} E + 32 & 8E \\ 4/5 & E \end{pmatrix}$$

where  $E \geq 0$  is the eigenvalue of the Laplacian on  $X_5$ . The eigenvalues of this matrix are

$$m^2 = 16 + E \pm 8\sqrt{4 + E} \ . \tag{A.1}$$

We will be primarily interested in the modes which correspond to picking the minus branch: they turn out to be the chiral primary fields. For such modes there is a possibility of  $m^2$  falling in the range (2.3) where there is a two-fold ambiguity in defining the corresponding operator dimension. This happens for the eigenvalue E such that

$$5 \le E \le 21 \ . \tag{A.2}$$

First, let us recall the  $S^5$  case where the spherical harmonics correspond to traceless symmetric tensors of SO(6),  $d_{i_1...i_k}^{(k)}$ . Here E=k(k+4), and it seems that the bound (A.2) is satisfied for k=1. However, this is precisely the special case where the corresponding mode is missing. For k=0 there is no 4-form mode, hence no mixing, while for k=1 one of the mixtures is the singleton [40]. Thus, all chiral primary operators in the  $\mathcal{N}=4$  SU(N) theory correspond to the conventional branch of dimension,  $\Delta_+$ . It is now well-known that this family of operators with dimensions  $\Delta=k, k=2,3,\ldots$  is  $d_{i_1...i_k}^{(k)} \operatorname{Tr}(X^{i_1}\ldots X^{i_k})$ . The absence of k=1 is related to the gauge group being SU(N) rather than U(N). Thus, in this case we do not encounter operator dimensions lower than 2.

The situation is different for  $T^{1,1}$ . Here there is a family of wave functions labeled by non-negative integer k, transforming under  $SU(2) \times SU(2)$  as (k/2, k/2), and with U(1) charge k. They are described in section 4.2, and the eigenvalues of the Laplacian are written in (4.11). In [9] it was argued that the corresponding chiral operators are

$$\operatorname{Tr}(A_{i_1}B_{j_1}\ldots A_{i_k}B_{j_k})$$
.

Since the F-term constraints in the gauge theory require that the i and the j indices are separately symmetrized, we find that their  $SU(2) \times SU(2) \times U(1)$  quantum numbers agree with those given by the supergravity analysis. Since in the field theory construction of [9] the A's and the B's have dimension 3/4, the dimensions of the chiral operators are 3k/2.

In studying the dimensions from the supergravity point of view, one encounters a subtlety discussed in section 2. While for k > 1 only the dimension  $\Delta_+$  is admissible, for k = 1 one could pick either branch. Indeed, from (4.11) we have E(1) = 33/4 which falls within the range (A.2). Here we find that  $\Delta_- = 3/2$ , while  $\Delta_+ = 5/2$ . Since the supersymmetry requires the corresponding dimension to be 3/2, in this case we have to pick the unconventional  $\Delta_-$  branch. Choosing this branch for k = 1 and  $\Delta_+$  for k > 1 we indeed find following [36] that the supergravity analysis based on (2.2), (A.1), (4.11), reproduces the dimensions 3k/2. Thus, the conifold theory provides a simple example of the AdS/CFT duality where the  $\Delta_-$  branch has to be chosen for certain operators.

Let us also note that substituting E(1) = 33/4 into (A.1) we find  $m^2 = -15/4$  which corresponds to a conformally coupled scalar in  $AdS_5$  [40]. In fact, the supermultiplet containing this scalar has to include another conformally coupled scalar and a massless fermion. One of these scalar fields corresponds to the lower component of the superfield  $Tr(A_iB_j)$ , which has dimension 3/2, while the other corresponds to the upper component which has dimension 5/2. Thus, the supersymmetry requires that we pick dimension  $\Delta_+$  for one of the conformally coupled scalars, and  $\Delta_-$  for the other.

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