EVIDENCE FOR HETEROTIC - TYPE I STRING DUALITY

Joseph Polchinski[†]

Institute for Theoretical Physics University of California, Santa Barbara, CA 93106

and

Edward Witten*

School of Natural Sciences, Institute for Advanced Study Olden Lane, Princeton, N.J. 08540

A study is made of the implications of heterotic string T-duality and extended gauge symmetry for the conjectured equivalence of heterotic and Type I superstrings. While at first sight heterotic string world-sheet dynamics appears to conflict with Type I perturbation theory, a closer look shows that Type I perturbation theory "miraculously" breaks down, in some cases via novel mechanisms, whenever the heterotic string has massless particles not present in Type I perturbation theory. This strongly suggests that the two theories actually are equivalent. As further evidence in the same direction, we show that the Dirichlet one-brane of type I string theory has the same world-sheet structure as the heterotic string.

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[†] joep@itp.ucsb.edu

^{*} witten@sns.ias.edu

Introduction

Of the various plausible equivalences between superstring theories, one candidate for which there has been comparatively little evidence is the relation of the two ten-dimensional theories with gauge group SO(32) – the heterotic string and the Type I theory. Evidence that they are equivalent consists in part of two facts [1] that enable one to avoid immediate contradictions: the identification between the low energy limits of the two theories exchanges weak and strong coupling, putting a disproof of the conjecture out of reach of perturbation theory; the range of dimensions in which this equivalence would determine the strong coupling dynamics of the heterotic string (eight through ten) does not overlap with the range (seven and below) in which equivalence with Type II controls that dynamics, again evading a potential contradiction. Further suggestive discussions have involved attempts to interpret the heterotic string as a classical solution of the Type I theory [2,3]. In this paper, the issue will be further explored in two directions.

First, following a strategy that has proven fruitful in other cases, we will consider the relation between the self-dualities and the gauge symmetries of the two theories when toroidally compactified. The toroidally compactified heterotic string exhibits at certain points in moduli space certain phenomena – T-dualities and extended gauge symmetries (and gauge groups such as $E_8 \times E_8$) – that appear to be absent for Type I. One might think that – if the duality between the heterotic string and Type I is correct – the T-dualities must be visible and the extended gauge symmetry must appear only in a region of moduli space in which the Type I theory is strongly coupled. A straightforward attempt to implement this idea runs into trouble at once: the self-dual region of the heterotic string corresponds to a region of Type I moduli space that seems to intersect weak coupling.

It turns out that the problem is resolved in a somewhat novel and surprising way. While the Type I theory appears to be weakly coupled in the relevant region of its parameter space, we will show that certain of its states become strongly coupled precisely when the heterotic string exhibits its interesting world-sheet behavior. This depends on the unusual properties [4] of the Type I superstring under $R \to 1/R$, which are related to world-sheet orbifolds considered in refs. [5,6]. Strong coupling of certain states means that one cannot exclude the hypothesis that the Type I superstring reproduces the known heterotic string behavior. This "miraculous" avoidance of a contradiction adds considerable credence to the idea that these theories really are equivalent. This parallels a similar phenomenon in six dimensional heterotic - Type II duality, where the Type II theory apparently becomes strongly coupled at those points in moduli space where the heterotic

string has extended gauge symmetries [7,8]. In fact, we will be able to see the breakdown of Type I perturbation theory in a more precise and detailed way than is presently possible in the Type II case.

Second, pursuing the recent argument that Dirichlet-branes are intrinsic to the Type I and Type II string theories and are the carriers of Ramond-Ramond charge [9], we examine the Dirichlet one-brane of the Type I theory and find that it has the world-sheet structure of the heterotic string. This puts the considerations of [2,3] on a much firmer footing as singular solutions of the leading low energy field equations are now replaced by an exact conformal field theory construction.

Heterotic String T-Duality In Type I Variables

First we recall the mapping between the low-energy Type I and heterotic string theories in ten dimensions. Letting λ_h , λ_I be the heterotic string and Type I coupling constants, and g_h , g_I the two ten-dimensional metrics, the relations are [1]

$$\lambda_I = \frac{1}{\lambda_h}$$

$$g_I = \lambda_I g_h$$

$$\tag{1}$$

(numerical factors are omitted until further notice).

In this paper, we will mainly consider compactification of the ten-dimensional theory to nine dimensions, on $\mathbf{R}^9 \times \mathbf{S}^1$. (Some aspects of the reduction below nine dimensions will also be discussed.) Letting R_h , R_I denote the radius of the circle as measured in the two theories, it follows from the second relation in (1) that

$$R_I = \lambda_I^{1/2} R_h. (2)$$

The heterotic string world-sheet phenomena whose Type I counterparts we wish to understand occur for $R_h \leq 1$. Therefore, they will occur for $R_I \leq \lambda_I^{1/2}$. In trying to see these phenomena at weak Type I coupling $-\lambda_I \to 0$ – we will therefore have to take $R_I \to 0$.

It will therefore be necessary to understand the small radius behavior of the Type I superstring. For some string theories, the small R behavior can be understood by an $R \to 1/R$ symmetry. The Type I superstring does not have a T-duality symmetry, but one can nevertheless attempt to understand its small R behavior by means of a T-duality transformation to a rather interesting alternative theory that was described in [4] (and

has some relations to world-sheet orbifolds discussed in [5,6]). We will call this theory the Type I' theory and denote the coupling constant and radius as $R_{I'}$, $\lambda_{I'}$. The unusual properties of the Type I' theory will be recalled later. For now we note simply that the Type I and Type I' parameters are related by the standard T-duality relations

$$R_{I'} = \frac{1}{R_I}$$

$$\frac{R_{I'}}{\lambda_{I'}^2} = \frac{R_I}{\lambda_I^2}.$$
(3)

The second relation is equivalent to the statement that the nine-dimensional string coupling is invariant under T-duality.

Combining the above formulas, we see that in the region of R_h fixed and $\lambda_{I'} \to 0$, $R_{I'}$ will scale as $1/\lambda_{I'}$. In particular, the T-duality of the heterotic string should be visible in a region in which the Type I' theory is weakly coupled and at large radius – the region in which one thinks one understands it best. This appears to mean that if the heterotic string is really equivalent to Type I (and therefore to Type I'), the heterotic string T-duality and extended gauge symmetry should be visible explicitly in the weakly coupled Type I' theory.

It is easy to make this issue quantitative. Starting with the parameters $R_{I'}$, $\lambda_{I'}$ of the Type I' theory, one maps to Type I via (3), and then to the heterotic string by (1) and (2). The composite is

$$R_{h} = \frac{1}{\sqrt{R_{I'}\lambda_{I'}}}$$

$$\lambda_{h} = \frac{R_{I'}}{\lambda_{I'}}$$

$$g_{h} = g_{I'}\frac{R_{I'}}{\lambda_{I'}}.$$

$$(4)$$

The last formula is the Weyl transformation of the nine-dimensional metric deduced from the second equation in (1). (The nine-dimensional metric is invariant under T-duality between Type I and Type I' or between the heterotic string and itself.) Then one applies a heterotic string T-duality transformation $R_h \to 1/R_h$, $\lambda_h \to \lambda_h/R_h$, and finally one inverts (4) to return to Type I' variables. The result is that the heterotic string T-duality corresponds in Type I' to the rather obscure-looking transformation

$$R_{I'} \to \frac{R_{I'}^{1/4}}{\lambda_{I'}^{3/4}}$$

$$\lambda_{I'} \to \frac{1}{\lambda_{I'}^{1/4} R_{I'}^{5/4}}$$

$$g_{I'} \to \frac{g_{I'}}{R_{I'}^{1/2} \lambda_{I'}^{1/2}}.$$
(5)

In particular, the self-dual radius of the heterotic string corresponds to $R_{I'} = 1/\lambda_{I'}$, and the region of $\lambda_{I'} << 1$ with $R_{I'} \sim 1/\lambda_{I'}$ is mapped to itself, so it appears that we can test in perturbation theory the existence of this symmetry.¹

Transformation Of Masses

With this in mind, and assuming that the Type I' theory is weakly coupled when $\lambda_{I'} \ll 1$, $R_{I'} \gg 1$, let us see how particle masses transform under (5).

When the Type I theory is formulated on $\mathbf{R}^9 \times \mathbf{S}^1$, the particle momentum around \mathbf{S}^1 is conserved but (as strings in the Type I theory can break) the string winding number is not conserved. Dually, in the Type I' theory, winding number is conserved but momentum is not. Consider therefore a string winding state in the Type I' theory with a mass of order $R_{I'}$. Heterotic string T-duality, according to (5), maps $R_{I'}$ to $R_{I'}^{1/4}/\lambda_{I'}^{3/4}$. But because of the Weyl transformation in (5), the transformation multiplies masses by an extra factor of $1/R_{I'}^{1/4}\lambda_{I'}^{1/4}$, so string winding states with masses of order $R_{I'}$ are exchanged with particles with masses of order $1/\lambda_{I'}$. That is a satisfactory result, as $1/\lambda_{I'}$ is the right order of magnitude for the mass of a Ramond-sector soliton.

The problem arises when one considers the transformation law of other elementary string states. For example, a generic Type I' elementary string state has a mass of order 1 in string units; though unstable, this state is long-lived for sufficiently small $\lambda_{I'}$. The heterotic string T-duality transformation should map this state to a long-lived resonance with a mass of order $1/R_{I'}^{1/4}\lambda_{I'}^{1/4}$. Such states are not known. Even worse, consider a (long-lived but not stable) state carrying momentum around \mathbf{S}^1 , with a mass of order $1/R_{I'}$. Heterotic string T-duality would map this state to a state with a mass of order $\lambda_{I'}^{1/2}/R_{I'}^{1/2}$. Such states, since their mass vanishes for $\lambda_{I'} \to 0$, would have to contribute to perturbation theory, where they are not known.²

It is suggestive that the masses of the stable particles transform sensibly, and that the problem only exists for unstable particles. We will find a solution compatible with this: the Type I' theory has a strongly coupled region if $R_{I'}$ is of order $1/\lambda_{I'}$ or larger, no matter how small $\lambda_{I'}$ may be. The mass formulas for stable particles used above are

¹ The dual heterotic theory, which appears at intermediate stages in the transformation of the Type I' theory into itself, is strongly coupled in this range. Thus we are using the fact that its T-duality, being a gauge symmetry, is exact [10].

² M. Dine has independently considered the possibility of similar contradictions arising when the Type I - heterotic and heterotic - Type II dualities are combined.

exact (because these particles are in "small" supermultiplets) and so are unaffected by strong coupling, but the discussion of unstable particles will be invalidated by the strong coupling.

Behavior of the Dilaton

To see how strong coupling comes about, we must first of all remember that in string theory, the string coupling constant is determined by the expectation value of the dilaton field. Stability of the vacuum depends on a cancellation between dilaton tadpoles. The leading tadpoles come from the disc D and the projective plane \mathbf{RP}^2 , and cancel [11] if the gauge group is SO(32).

In the Type I theory, this cancellation occurs homogeneously throughout all space. In the Type I' theory, however, the situation is rather different, because of features explained in [4]. Working on $\mathbf{R}^9 \times \mathbf{S}^1$, with \mathbf{S}^1 parametrized by a periodic variable x^9 (of period $2\pi R_{I'}$), the Type I' theory is a parameter space orbifold in which a reversal of the orientation of the world-sheet is accompanied by $x^9 \to -x^9$. On the circle, that transformation has two fixed points, at $x^9 = 0$ and $x^9 = \pi R_{I'}$.

Suppose now that $R_{I'}$ is large, and consider an \mathbf{RP}^2 mapped to space-time in such a way that the action is of order one. This means that the image of \mathbf{RP}^2 in space-time must have a size of order the string scale, not of order $R_{I'}$, and as \mathbf{RP}^2 is unorientable, that is possible only if the image of \mathbf{RP}^2 sits near one of the fixed points. There is a symmetry between these two points, so the \mathbf{RP}^2 dilaton tadpole receives half its contribution from a neighborhood of $x^9 = 0$ and half from a neighborhood of $x^9 = \pi R_{I'}$.

Now we come to the tadpole derived from the disc. In going from the Type I theory to the Type I' theory, Neumann boundary conditions are replaced with Dirichlet boundary conditions, so the endpoints of the open strings lie at a fixed position on S^1 . For unbroken SO(32) symmetry (which we assumed on the heterotic string side to have the T-duality that led to the Type I' predictions that we are testing), all species of open string must have their boundary at the same point. The orientifold symmetry $x^9 \to -x^9$ requires that this should be one of the two fixed points, which we may as well take to be the one at $x^9 = 0$. The dilaton tadpole from the disc is thus localized near $x^9 = 0$.

The cancellation of dilaton tadpoles therefore occurs in a highly non-local way in this theory. The disc tadpole near $x^9 = 0$ is only half canceled by an \mathbf{RP}^2 contribution near $x^9 = 0$; the other half of the cancellation comes from \mathbf{RP}^2 's near $x^9 = \pi R_{I'}$. Between the dilaton source at $x^9 = 0$, and the equal and opposite source at $x^9 = \pi R_{I'}$, there is a

dilaton gradient. The gradient, as it comes from the disk and \mathbf{RP}^2 , is of order $\lambda_{I'}$ and so can have an effect of order one when $R_{I'} \cong 1/\lambda_{I'}$, the region where T-duality of the heterotic string is visible. To first order in $R_{I'}\lambda_{I'}$ the dilaton is linear, but (as we will see) there are nonlinearities whose effect is to make the dilaton diverge for a finite value of $R_{I'}\lambda_{I'}$. The result is to prevent one from comparing Type I' perturbation theory to heterotic string T-duality.

It is also interesting to understand how perturbation theory breaks down directly in the type I picture, where the radius is becoming small. Each additional handle on the world-sheet adds two closed cycles. The winding number of x^9 is summed for each cycle. When R_I is small, each sum is approximated by R_I^{-1} times an integral. The naive loop expansion parameter is therefore $\lambda_I^2 R_I^{-2}$, which is indeed $\lambda_{I'}^2$. However, each additional hole brings an infrared divergence from the tadpoles of the winding states of the dilaton and graviton. These do not cancel winding number by winding number, because \mathbf{RP}^2 contributes only to even winding numbers while the disk contributes both to even and odd. The R_I -dependence when a hole is added comes only from the cycle which winds around the hole, giving a term of order R_I^2 in the denominator of the winding state propagator. The uncanceled tadpoles are thus of order $\lambda_I R_I^{-2} = \lambda_{I'} R_{I'}$, and so perturbation theory breaks down when this is large.

Incorporation Of Wilson Lines

Before making a detailed analysis of the dilaton background, we will incorporate gauge symmetry breaking by Wilson lines and show how heterotic string phenomena are mirrored by Type I' phenomena. So we still compactify the SO(32) heterotic string on a circle, but now we consider a vacuum in which the x^9 component of the gauge field has an expectation value, with a global holonomy $W \in \text{Spin}(32)/\mathbb{Z}_2$. Regarded as an SO(32) matrix, W can be written as the direct sum of 16 two-dimensional blocks; the i^{th} block takes the form

$$\begin{pmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{pmatrix} \tag{6}$$

with some angle θ_i . The incorporation of Wilson lines in the Type I' theory was briefly described in [12] and proceeds by shifting the points on the circle at which strings are permitted to have boundaries. Thus, instead of requiring as we did above that – for each of the 32 values of the Chan-Paton label – the end of an open string must lie at $x^9 = 0$, one requires that a charge of type i, i = 1, ..., 16 lies at $x^9 = \theta_i R_{I'}$, while an image charge

of type \bar{i} lies at $x^9 = -\theta_i R_{I'}$. Here we are using a complex basis for the charges; the reality condition for the open string wave function combines complex conjugation with $x^9 \leftrightarrow -x^9$ and $i \leftrightarrow \bar{i}$. As an example that will have some significance, let $W = W_0$ where W_0 has eight θ_i vanishing and the others equal to π . This Wilson line breaks SO(32) to $SO(16) \times SO(16)$. In the Type I' description, the world-sheet boundary lives at $x^9 = 0$ for 16 values of the Chan-Paton factor and at $x^9 = \pi R_{I'}$ for the other 16 – a configuration constructed by Horava [6] as a world-sheet orbifold.

Once a Wilson line is introduced, the heterotic string theory no longer has $R \to 1/R$ symmetry (which now acts non-trivially on the Wilson line). However, it still is the case for generic W that as R is decreased, the heterotic string theory eventually gets massless particles in a winding sector, giving an enhanced gauge symmetry. This phenomenon is invisible in Type I' perturbation theory, so to justify our reconciliation of heterotic – Type I duality with the predictions of Type I' perturbation theory, we must show that the Type I' theory becomes strongly coupled (somewhere on the S^1) when the heterotic string would have enhanced gauge symmetry.

In the bosonic construction of the heterotic string, it is convenient to write $W = \exp(2\pi i A)$, with A an element of the SO(32) Cartan algebra, which is a copy of \mathbf{R}^{16} . A is only unique up to a shift by an element of the $\mathrm{Spin}(32)/\mathbf{Z}_2$ lattice, and it is convenient to shift A to be as close to the origin as possible. For instance, in a basis in which the $\mathrm{Spin}(32)/\mathbf{Z}_2$ lattice consists of 16-plets (m_1, \ldots, m_{16}) that are all integers or all half-integers and whose sum is even, the group element W_0 described above corresponds to

$$A_0 = (0, 0, \dots, 0, 1/2, 1/2, \dots, 1/2)$$
(7)

with eight 0's and eight 1/2's. This vector obeys $A^2 = 2$ (and cannot be shifted by a lattice vector to be closer to the origin). A small exercise shows that any vector in \mathbf{R}^{16} that is not equivalent to A up to a lattice shift is (up to a lattice shift) closer to the origin than A.

In the absence of a Wilson line, the heterotic string gets at the self-dual radius enhanced SU(2) gauge symmetry, due to a massless state with unit momentum and winding around x^9 . In the presence of the Wilson line, the enhanced gauge symmetry occurs at a radius that is smaller by a factor of $\sqrt{1 - A^2/2}$; this is essentially because a shift by A increases the left-moving ground state energy by $A^2/2$. Thus, for any Wilson line other than $W = W_0$, the heterotic string gets enhanced gauge symmetry if R is small enough.

Since this enhanced gauge symmetry will not be visible in Type I' perturbation theory, we must hope that the Type I' theory becomes strongly coupled for sufficiently big radius unless $W = W_0$. This is so. For $W = W_0$, the open string boundaries are at $x^9 = 0$ for half of the values of the Chan-Paton factor, and at $x^9 = \pi R_{I'}$ for the other half. Thus half of the dilaton tadpole from the disc is localized at one orientifold fixed point and half at the other. This is the same configuration as for \mathbf{RP}^2 's, so the cancellation of tadpoles occurs locally, and there is no linear dilaton field induced. One can therefore take $R_{I'} \to \infty$ with no breakdown of Type I' perturbation theory. For any other value of W, dilaton tadpoles cancel between different points on the circle, and for sufficiently big $R_{I'}$ – corresponding to sufficiently small heterotic string radius – one gets strong coupling somewhere on the circle. Thus, the occurrence of strong coupling for Type I' mirrors in a striking fashion the occurrence of extended gauge symmetry of the heterotic string.

Type I' Dilaton Background

This discussion can be made much more quantitative by using the equations of supergravity to solve for the low energy fields in the situation studied above. In this section we are careful to retain all numerical factors.

First, let us write the low energy field theory of the ten dimensional SO(32) heterotic string, keeping only the graviton, dilaton, and gauge fields. The action is

$$S_h = \int d^{10}x \sqrt{-g} e^{-2\phi} \left(\frac{1}{2}R + 2\partial_M \phi \partial^M \phi - \frac{1}{8} \text{Tr}_V G_{MN} G^{MN} \right)$$
 (8)

The overall normalization of the action and the relative normalization of the gauge kinetic term have been fixed by choice of the additive normalization of the dilaton and the multiplicative normalization of the metric. In particular, this normalization of the metric corresponds to setting $\alpha' = 2$ [13].

Let us similarly write the action for the Type I' string with one dimension compactified on $0 \le x^9 \le 2\pi$, the endpoints being orientifold points. The indices M, N run 0...9 and the indices μ, ν run 0...8. We include also 8-branes perpendicular to the 9-direction, with n_1 8-branes at x_1^9 , n_2 at x_2^9 , and so on (and their images at $-x_i^9$). As explained earlier, the positions of the 8-branes are related to the Wilson lines in a Type I description. Initially let $0 < x_i^9 < 2\pi$, so the gauge group is $U(n_1) \times U(n_2) \times \ldots$. We keep the same fields as

above plus the 9-form potential A [9], which will also have a nontrivial background. The action is

$$S_{I'} = \int d^{10}x \sqrt{-g} e^{-2\phi} \left(\frac{1}{2} R + 2\partial_M \phi \partial^M \phi \right) - \frac{1}{2} \int F^* F$$

$$- \mu_8 \sum_i \int_{x^9 = x_i^9} \left(d^9 x \sqrt{-\tilde{g}} e^{-\phi} 2^{-1/2} \left\{ n_i + (\pi \alpha')^2 \text{Tr}_f G_{\mu\nu} G^{\mu\nu} \right\} + n_i A \right)$$
(9)

where $\tilde{g}_{\mu\nu}$ is the 9-dimensional metric. Again, the additive normalization of ϕ is fixed by the gravitational and dilaton kinetic terms, while the normalization of F = dA is defined by its kinetic term. The coupling of the 9-form potential to the 8-brane is as in ref. [9], $\mu_8 = (2\pi)^{-9/2} \alpha'^{-5/2}$. The normalization of the membrane tension is fixed by supersymmetry, as we will see below. The normalization of the gauge kinetic term then follows from the Born-Infeld form of the open string action [14]. There is no particularly simple choice for the Type I' α' so we leave it arbitrary.

The equation of motion of the nine-form potential implies that the ten-form is $F = \nu_0 dx^0 \dots dx^9$, with ν_0 piece-wise constant away from the 8-branes. At x_i^9 the equations of motion imply a jump $\Delta\nu_0 = n_i\mu_8$. Since there are 16 8-branes between the two fixed points, we have $\nu_0(2\pi) = \nu_0(0) + 16\mu_8$. Invariance of the boundary conditions under spacetime parity $x^9 \to 2\pi - x^9$, which takes $\nu_0 \to -\nu_0$, then implies that $\nu_0(2\pi) = -\nu_0(0) = 8\mu_8$. Boundary conditions on the metric and dilaton will be seen to follow from supersymmetry. The case that some of the x_i^9 are at the orientifold points 0 or 2π can now be approached as a limit. The one qualitative change is that some vectors, which wind between an 8-brane and its image, become massless so that U(n) is promoted to SO(2n). The traces in the U(n) fundamental and SO(2n) vector are related by $\mathrm{Tr}_f = \frac{1}{2}\mathrm{Tr}_V$.

To solve for the background fields, look first in the region between the 8-branes, where as explained in ref. [9] the theory reduces to the massive IIa supergravity found by Romans [15]. The action, metric and dilaton of that paper are related to those here by $S^{[15]} = -\frac{1}{2}S$, $g_{\mu\nu}^{[15]} = e^{-\phi/2}g_{\mu\nu}$, $\phi^{[15]} = -\frac{1}{2}\phi$, and the parameter m of that paper is $\nu_0\sqrt{2}$. The conditions for a supersymmetric background, in terms of the variables of ref. [15], are

$$32D_M \epsilon = me^{-5\phi/2} \Gamma_M \epsilon$$

$$8\partial_M \phi \Gamma^M \epsilon = 5me^{-5\phi/2} \epsilon.$$
(10)

The 8-branes and orientifolds preserve half the supersymmetries, namely those with $\Gamma_9 \epsilon = \pm g_{99}^{1/2} \epsilon$; the sign, which is correlated with the charge of the 8-brane, will be determined

below. Integrating eq. (10) and transforming from the variables of ref. [15] to those used in the rest of this paper gives, in conformal gauge $g_{MN} = \Omega^2(x^9)\eta_{MN}$,

$$e^{\phi(x^9)} = z(x^9)^{-5/6}, \qquad \Omega(x^9) = Cz(x^9)^{-1/6}$$

 $z(x^9) = 3C(B\mu_8 \pm \nu_0 x^9)/\sqrt{2}$ (11)

with C and B piecewise constant between the 8-branes.

This background satisfies the equations of motion between the 8-branes. At the 8-branes, the equations of motion require ϕ and Ω to be continuous and ν_0 to have the discontinuity μ_8 . This implies that C is constant, that $z(x^9)$ is continuous, and therefore that B has a discontinuity $\mp n_i x_i^9$; the full x^9 -dependence of B is then known given B(0). One can now work backwards and determine the normalization of the membrane tension. From the solution (11), the discontinuity in $\partial_9 \phi$ is $\mp 5C n_i \mu_8/2\sqrt{2}z$. Relating this to the dilaton equation of motion gives the value $\mu_8/\sqrt{2}$ used in the action (9). Also, positivity of the membrane tension determines that we must take the lower sign in $z(x^9)$. Finally the solution (11) determines the ϕ and Ω gradients in terms of ν_0 , thus giving the boundary conditions on these at the orientifold points as promised earlier.

This Type I' theory is supposed to be dual to the heterotic theory quantized on a circle of radius R. To find the relation between the heterotic R and ϕ and the parameters B(0) and C of the Type I' theory we compare the effective 9-dimensional actions obtained by reducing the respective 10-dimensional actions (8) and (9). For the Type I' metric we assume a slow function $\gamma_{\mu\nu}$ of the noncompact dimensions times the $\Omega^2(x^9)$ determined above. We also need to determine the relation between the metrics, $\gamma_{\mu\nu} = D^2 g_{\mu\nu}^h$. Comparing the gravitational actions gives

$$2\pi Re^{-2\phi} = D^7 C^{25/3} \int_0^{2\pi} dx^9 w(x^9), \tag{12}$$

where $w(x^9) = 3^{1/3} 2^{-1/6} [\mu_8 B(x^9) - x^9 \nu_0(x^9)]^{1/3}$. Comparing the gauge actions gives

$$2\pi Re^{-2\phi} = (2\pi\alpha')^2 2^{-1/2} \mu_8 D^5 C^5.$$
 (13)

A final relation is obtained by comparing the mass of a BPS state, a heterotic Kaluza-Klein state which is dual to a Type I' winding state. We have $m_h = 1/R = Dm_{I'}$. Integrating the world-sheet action gives

$$\frac{1}{R} = \frac{DC^{5/3}}{\pi \alpha'} \int_0^{2\pi} dx^9 \, w(x^9)^{-1} \ . \tag{14}$$

In deriving this note that the string winds from 0 to 2π and back again, and that the area element for the winding state is $\Omega^2 dx^0 dx^9$. The relation between the heterotic Wilson line and the positions of the 8-branes can similarly be found by considering the mass of an off-diagonal vector boson, $(A_i \pm A_j)/R$ in the heterotic string. In the Type I' theory this is a string attached to the branes at x_i^9 and x_j^9 (or the image at $-x_j^9$). Thus,

$$\frac{\lambda_i}{R} = \frac{DC^{5/3}}{2\pi\alpha'} \int_0^{x_i^9} dx^9 \, w(x^9)^{-1} \ . \tag{15}$$

We can combine eqs. (12), (13), and (14) to get

$$R = 2^{-3/4} \mu_8^{-1/2} \left(\int_0^{2\pi} dx^9 w(x^9) \right)^{1/2} \left(\int_0^{2\pi} dx^9 w(x^9)^{-1} \right)^{-1}$$
 (16)

We will focus here on the simple case that A has n 0's and 16 - n $\frac{1}{2}$'s so there are 2n branes at $x^9 = 0$ and 32 - 2n at $x^9 = 2\pi$ (counting the images); let $n \le 8$ for convenience. In this case $\nu_0(x^9) = (n-8)\mu_8$ and $B(x^9) = B$ are constants, and

$$e^{\phi(x^9)} = \left\{ 2^{-1/2} 3C \mu_8 [B + (8-n)x^9] \right\}^{-5/6}$$

$$R = \frac{1}{2} 3^{1/2} \left(\int_0^{2\pi} dx^9 \left[B + (8-n)x^9 \right]^{1/3} \right)^{1/2} \left(\int_0^{2\pi} dx^9 \left[B + (8-n)x^9 \right]^{-1/3} \right)^{-1}$$
(17)

When B is large, the coupling $e^{\phi(x^9)}$ is small everywhere and the Type I' theory is weakly coupled; in this range the heterotic radius R is large. However, when $B \to 0$ the coupling diverges at the endpoint $x^9 = 0$ and perturbation theory breaks down;³ past this point the expressions make no sense. Evaluating the integrals for B = 0 gives the point of breakdown as

$$R = \frac{1}{2}|n-8|^{1/2}. (18)$$

This has been extended to n > 8 by symmetry; the coupling diverges at the orientifold point with the fewer D-branes. For each n this is the precise radius at which an enhanced gauge symmetry appears on the heterotic side. Thus the conjectured heterotic – Type I duality "miraculously" survives confrontation with heterotic string T-duality. It would be interesting to extend this analysis to other enhanced symmetry points, such as the $E_8 \times E_8$ point.

 $^{^3}$ It diverges as the inverse of the proper distance, in the type I' string metric, from the endpoint.

Compactification Below Nine Dimensions

Let us now consider the heterotic string compactified below nine dimensions in the light of conjectured duality with Type I. Regardless of the dimension, the relation $R_I = \lambda_I^{1/2} R_h$ shows that if R_h is of order one (so that the heterotic string has interesting world-sheet dynamics) and λ_I is small (so that Type I perturbation theory is useful), then R_I will be small, so that the Type I description is difficult to interpret. We therefore consider instead the dual Type I' description,⁴ and must show in each dimension that if one starts with a heterotic string at large radius and moves in towards small radius, Type I' perturbation theory breaks down by the time the heterotic string has interesting dynamics.

We begin with the heterotic string on $\mathbf{R}^8 \times \mathbf{S}^1 \times \mathbf{S}^1$. For simplicity we ignore the B field and consider the two \mathbf{S}^1 's to be orthogonal with respective radii $R_{h,1}$ and $R_{h,2}$.

Something new is needed, for the following reason. As long as $R_{h,1}$ and $R_{h,2}$ are of the same order of magnitude, a clash between heterotic string T-duality and Type I' perturbation theory cannot be avoided by generating a linear dilaton. Indeed, with two compact dimensions (of roughly the same size) the solution of the Laplace equation for a dilaton with a source will grow at most logarithmically, not linearly, and this will not help much.

Happily, in eight dimensions the transformation law from the heterotic string to Type I' is significantly different from what we have worked with in nine dimensions. The change is in the T-duality transformation from Type I to Type I'. One still has the usual transformation law of the radii, $R_{I',i} = 1/R_{I,i}$. But as the eight-dimensional string coupling constant is to be invariant under T-duality, the ten-dimensional coupling transforms as

$$\frac{R_{I',1}R_{I',2}}{\lambda_{I'}^2} = \frac{R_{I,1}R_{I,2}}{\lambda_I^2}. (19)$$

Together with the map $R_{I,i} = R_{h,i} \lambda_I^{1/2}$ between heterotic and Type I variables, this implies the perhaps surprising formula

$$\lambda_{I'} = \frac{1}{R_{h,1}R_{h,2}}. (20)$$

If therefore the $R_{h,i}$ are of the same order of magnitude, and are of order one so that the heterotic string has interesting world-sheet dynamics, then $\lambda_{I'}$ will be of order one, so that

⁴ One can also consider a mixture of Type I and Type I', dualizing in some directions only; this seems unlikely to raise essentially new issues.

Type I' perturbation theory is not useful. One could try to avoid this by taking, say, $R_{h,1}$ of order one, so that the heterotic string has interesting world-sheet dynamics, and $R_{h,2}$ large so that $\lambda_{I'}$ is small. In this case, the torus of the Type I' theory is highly anisotropic and quasi one-dimensional at big distances; one will again meet a linear dilaton, driving one to strong coupling at some point on the torus even though the bare coupling given in (20) is small.

Below eight dimensions, the details are again slightly different. Upon compactification of the heterotic string on an n-torus of n > 2 with radii $R_{h,i}$, the Type I' radii turn out to be given by

$$R_{I',i}^{n-2} = \lambda_{I'} \prod_{j=1}^{n} R_{h,j} \cdot \frac{1}{R_{h,i}^{n-2}}.$$
 (21)

For simplicity, suppose that k of the $R_{h,j}$ are of order one and n-k are of the same order R >> 1. Also suppose $\lambda_{I'} << 1$ or Type I' perturbation theory is in any case not a good description. For k = 0, there is no interesting dynamics of the heterotic string. For k = 1, one of the $R_{I'}$ is much than the others by a factor of R and – taking $\lambda_{I'}R \sim 1$ or bigger so that none of the $R_{I',i}$ is smaller than the string scale – the Type I' theory is strongly coupled because of a linear dilaton. For k > 1, some of the $R_{I'}$ are << 1 and again the Type I' description becomes untransparent.

Wilson Lines Below Nine Dimensions

It is also of interest to consider gauge symmetry breaking in compactification below nine dimensions. Consider, for example, the heterotic string on $\mathbf{R}^8 \times \mathbf{S}^1 \times \mathbf{S}^1$ with periodic coordinates x^8 , x^9 and Wilson lines W_1 , W_2 . We will consider only the dependence on the metric of $\mathbf{S}^1 \times \mathbf{S}^1$, as the *B*-field corresponds on the Type I' side to a little-understood Ramond-Ramond modulus.

For what choices of W_1 , W_2 does the heterotic string never get extended gauge symmetry, for any choice of the flat metric on $\mathbf{S}^1 \times \mathbf{S}^1$? This question is easily answered. It is necessary that W_1 should be conjugate to the matrix W_0 introduced above (which breaks SO(32) to $SO(16) \times SO(16)$), or a state with momentum and winding in the x^8 direction would become massless when the first circle is small. W_2 must be in the same conjugacy class, or a state with momentum and winding in the x^9 direction would become massless when the second circle is small. And the product W_1W_2 must be in the same conjugacy class, or a state with equal momentum and winding in the two directions would become massless when the metric is small in the diagonal direction. These conditions imply in

particular that $W_1^2 = W_2^2 = (W_1 W_2)^2 = 1$ and therefore that W_1 and W_2 commute and so can be simultaneously diagonalized. The condition on the conjugacy classes now implies that the states of $W_1 = 1$ (and likewise the states of $W_1 = -1$) are equally divided between $W_2 = 1$ and $W_2 = -1$, and hence that W_1 and W_2 together break SO(32) down to $SO(8)^4$.

Let us compare this to what happens on the Type I' side. On $\mathbf{S}^1 \times \mathbf{S}^1$ there are $2 \times 2 = 4$ orientifold fixed points, each bearing one fourth of the \mathbf{RP}^2 dilaton tadpole. To avoid getting a large growth of the dilaton regardless of the metric on $\mathbf{S}^1 \times \mathbf{S}^1$, the cancellation of the dilaton tadpoles must occur locally; eight values of the Chan-Paton index must be supported at each of the four fixed points. This therefore breaks SO(32) to $SO(8)^4$, showing that the Type I' theory never gets to strong coupling precisely if the Wilson lines are such that the heterotic string never has extended gauge symmetry.

One can straightforwardly extend this below eight dimensions. Upon toroidal compactification to 10-n dimensions, the heterotic string never gets enhanced gauge symmetry, and the Type I' theory never gets strong coupling because of a linear dilaton, precisely if the Wilson lines are such as to break SO(32) to $SO(2^{5-n})^{2^n}$. Of course, for n > 5 this is impossible.

The Type I Dirichlet One-Brane

In the remainder of this paper, we will approach the question of heterotic - Type I duality in a quite different way. We consider the D-string of the Type I theory and show that it has the world-sheet structure of the heterotic string, thus refining earlier discussions [2,3] based on singular approximate classical solutions. The D-string has a tension in Type I theory of order $1/\lambda_I$, an expression that is exact because of BPS saturation. The D-string therefore becomes very light when the Type I theory is strongly coupled, and it is very plausible – given the world-sheet structure – that it then behaves like an elementary heterotic string. As a check, recall that string tension has dimensions of mass squared and so is multiplied by λ_I under the Weyl transformation $g_I = \lambda_I g_h$ between Type I and heterotic string metrics; hence the D-string has a tension of order one in the heterotic string metric, like the elementary heterotic string.

Much of the world-sheet structure of the heterotic string can be anticipated just from considerations of supersymmetry. Consider a D-string that is located at, say, $x^2 = \ldots = x^9 = 0$. This configuration is invariant under a subgroup $SO(1,1) \times SO(8)$ of the Lorentz group SO(1,9), where SO(1,1) acts on x^0, x^1 and SO(8) on x^2, \ldots, x^9 . The tendimensional supersymmetries transform as **16** of SO(1,9), which decomposes as $\mathbf{8}'_+ \oplus \mathbf{8}''_-$ of

 $SO(1,1) \times SO(8)$, with 8', 8" the two spinor representations of SO(8), and \pm the SO(1,1) charge. In the field of the D-string, half the supersymmetry, say $\mathbf{8}'_{+}$, survives. The broken $\mathbf{8}''_{-}$ instead generates chiral fermions zero modes along the world-sheet, as discussed in [16]. Together with eight bosonic zero modes associated with the broken translational symmetries, these eight chiral fermion zero modes are the heterotic string world-sheet degrees of freedom that carry space-time quantum numbers. To reproduce the heterotic string, one also must find current algebra degrees of freedom of appropriate chirality.

To actually compute the excitation spectrum of the D-string, one simply quantizes the open string allowing Dirichlet ends with $x^2 = \ldots = x^9 = 0$ as well as the standard free string Neumann boundary conditions. There are thus two new open string sectors to consider, the DD sector with Dirichlet boundary conditions at each end, and the DN sector with Dirichlet boundary conditions at one end and Neumann at the other. Each sector, of course, can be further subdivided into Neveu-Schwarz and Ramond sectors.

We consider the DD sector first, in covariant RNS formalism. World-sheet bosons and fermions X^2, \ldots, X^9 and ψ^2, \ldots, ψ^9 get an extra minus sign in reflection from the boundary, as a result of replacing Neumann by Dirichlet, but this happens twice, so (in either the Ramond or Neveu-Schwarz sector) the world-sheet bosons and fermions have the standard integrally or half-integrally moded expansions, giving the usual ground state energies and hence apparently leading to a massless spectrum consisting of the usual vector supermultiplet. But the zero modes of X^2, \ldots, X^9 are absent, so that the massless DD (or DN) modes are functions of x^0, x^1 only; that is, they propagate only on the D-string world-sheet. Thus, the ten-dimensional massless vector A_I becomes in this context a world-sheet vector $A_i(x^0, x^1)$, i = 1, 2 and scalars $\phi_j(x^0, x^1)$, $j = 2, \ldots, 9$.

Also, differences from usual open string quantization arise when we consider the fact that the Type I string is unoriented, so that the DD spectrum must be projected onto a subsector invariant under the operator Ω that exchanges the two ends. The vector A has a conventional vertex operator $V_A = \sum_{i=0,1} A_i \partial X^i/\partial \tau$, with ∂_{τ} the derivative tangent to the boundary. V_A is odd under reversal of orientation of the boundary, so the vector is projected out – as usually occurs for open strings without Chan-Paton factors. But as briefly explained in [12], the scalars have vertex operator $V_{\phi} = \sum_{j=2}^{9} \phi_j \partial X^j/\partial \sigma$, with ∂_{σ} the normal derivative; this is even under reversal of the boundary orientation, so that the scalars survive the projection. They indeed represent the oscillations in position of the D-string. Now consider the action of Ω on massless fermions. In the Ramond sector, the RNS fermions ψ^I have zero modes Γ^I that upon quantization obey $\{\Gamma^I, \Gamma^J\} = 2\eta^{IJ}$. The GSO

projection restricts the massless fermions to states invariant under an operator $(-1)^F$ that anticommutes with all ψ^I ; in the space of zero modes this operator can be represented by $\overline{\Gamma} = \Gamma^0 \Gamma^1 \dots \Gamma^9$. For standard NN open strings, the operator Ω can be taken to commute with the Γ^I (otherwise replace Ω by $\Omega \cdot (-1)^F$, since one is in any case making the GSO projection), but turns out to act as -1 on the ground state, which therefore altogether disappears from the spectrum, a standard result for unoriented open strings without Chan-Paton factors. Now consider the DD case. Because the fermions ψ^2, \dots, ψ^9 reflect from the boundary with an extra minus sign compared to ψ^0, ψ^1 , they pick up an extra minus sign under exchange of left and right movers. Instead of being represented on massless fermions by -1, Ω therefore acts by $\Omega = -\Gamma^2\Gamma^2 \dots \Gamma^9$ (or by $-\Gamma^0\Gamma^1$ if it is multiplied by a factor of $(-1)^F$). Physical massless fermions are thus spinors χ that obey

$$\chi = \overline{\Gamma}\chi = -\Gamma^2\Gamma^3\dots\Gamma^9\chi. \tag{22}$$

The first condition says that χ transforms in the **16** of SO(1,9), and the second that in the decomposition **16** = $\mathbf{8}'_{+} \oplus \mathbf{8}''_{-}$, χ transforms as $\mathbf{8}''_{-}$. The -SO(1,1) charge means that $\Gamma^{0}\Gamma^{1}\chi = -\chi$, so that χ is right-moving on the world-sheet.

Thus, the DD sector gives precisely the massless world-sheet modes of the heterotic string, except the current algebra modes, which must come from the DN sector. In this sector, the world-sheet bosons X^2, \ldots, X^9 get an extra minus sign (compared to ordinary open strings) in reflection at the Dirichlet end and so have a half-integral mode expansion. The fermions ψ^2, \ldots, ψ^9 similarly have an extra minus sign, so have an integer mode expansion in the Neveu-Schwarz sector and a half-integer expansion in the Ramond sector. These facts mean that in the Neveu-Schwarz sector, the ground state oscillator energy is strictly positive, and there are no massless states. The massless spectrum consists therefore only of fermions. In the Ramond sector, the ground state energy is zero and the only fermion zero modes are Γ^0 and Γ^1 , whose quantization gives two states, of which only the left-moving mode survives the GSO projection $\Gamma^0\Gamma^1\lambda = \lambda$. Because λ carries also the Chan-Paton factors of the Neumann boundary, one gets the expected current algebra modes of the heterotic string – 32 left-moving world-sheet fermions transforming in the 32 of SO(32). Space-time supersymmetry of course requires that λ and χ have opposite chirality.

We have considered the Type I GSO projection, but there are also GSO projections on the heterotic side, on the current algebra fermions and on the ψ^I . We should find

the corresponding projections on the D-string spectrum. The GSO projection on the supersymmetric fermions is automatic because the DD fermions are spacetime spinors, Green-Schwarz fermions. For the current algebra fermions there is a natural origin for the GSO projection. We have noted that the world-sheet U(1) gauge field is removed by the orientation projection. A \mathbb{Z}_2 subgroup, holonomies ± 1 , commutes with the orientation reversal, and a consistent string theory is obtained for any choice of this holonomy around closed loops on the D-string world-sheet. The nontrivial holonomy gives a -1 for each D endpoint, and so acts trivially on the DD strings but acts as -1 on the DN strings. Treating this \mathbb{Z}_2 as a discrete gauge group generates the GSO projection on the current algebra fermions. Evidently the rules of D-branes require us to sum over all consistent theories in this way, a result which will be relevant for counting D-brane states in other contexts.

D-Branes And Solitons

To further compare the heterotic and Type I theories, one might ask whether other Type I D-branes can be identified in the heterotic string. Actually, the only supersymmetric Type I D-brane other than the D-string is the five-brane. Its tension is of order $1/\lambda_I$, and allowing for the effects of the Weyl transformation, it corresponds to a heterotic string five-brane with a tension of order $1/\lambda_h^2$. This is the standard behavior of the solitonic five-brane of the heterotic string [17-19], so it is natural to identify the two.

Incidentally, this is apparently not the only example of a D-brane that transforms into an ordinary soliton under string-string duality. Consider the relation between the Type IIA theory on $\mathbf{R}^6 \times \mathbf{K}3$ and the heterotic string on $\mathbf{R}^6 \times \mathbf{T}^4$. The Type IIA six-brane, wrapped around K3, becomes a two-brane in \mathbf{R}^6 , with tension of order $1/\lambda_{IIA}$. Under string-string duality, allowing for the Weyl transformation, it transforms into a two-brane with a tension of order $1/\lambda_h^2$, plausibly visible as an ordinary soliton. There is a very natural candidate for what this two-brane is. Indeed, Johnson, Kaloper, and Khuri [20] described a heterotic string solitonic two-brane on \mathbf{R}^6 (related to magnetic black holes in four-dimensions) that transforms under heterotic - Type II duality into a Type IIA two-brane that classically appears to be singular; this is very plausibly the Dirichlet two-brane.

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