# Stolen Memories: Leveraging Model Memorization for Calibrated White-Box Membership Inference

Klas Leino
Carnegie Mellon University

Matt Fredrikson

Carnegie Mellon University

## **Abstract**

Membership inference (MI) attacks exploit a learned model's lack of generalization to infer whether a given sample was in the model's training set. Known attacks generally work by casting the attacker's goal as a supervised learning problem, training an attack model from predictions generated by the target model, or by others like it. However, we find that these attacks do not often provide a meaningful basis for confidently inferring membership, as the attack models are not well-calibrated. On closer inspection, we find that the predictions of these attacks are closely-aligned with whether the sample is correctly predicted, and thus a trivial attack using only this information can recover most of the attack model's performance.

In this work we present well-calibrated MI attacks that allow the attacker to accurately control the minimum confidence with which positive inferences are made. Our attacks take advantage of white-box information about the target model and leverage new insights about how overfitting occurs in deep neural networks; namely, we show how a model's idiosyncratic use of features can provide *evidence for membership*. Experiments on seven real-world datasets show that our attacks support calibration for high-confidence inferences, while outperforming previous MI attacks in terms of accuracy. Finally, we show that our attacks achieve non-trivial advantage on some models with low generalization error, including those trained with small- $\epsilon$ -differential privacy; for large- $\epsilon$  ( $\epsilon$  = 16, as reported in some real settings [42]), the attack performs comparably to unprotected models.

## 1 Introduction

Machine learning has enabled a wide range of applications in areas like computer vision, machine translation, health analytics, and advertising, among others. The fact that many compelling applications of this technology involve the collection and processing of sensitive personal data has given rise to concerns about privacy [2, 4, 7, 10, 11, 25, 32, 37, 45, 46]. In particular, when machine learning algorithms are applied to private training data, the resulting models might unwittingly

leak information about that data through either their behavior or the details of their structure and parameters.

Two particular attacks have emerged as concrete threats: *model inversion* and *membership inference*. In a model inversion attack, the adversary uses a machine learning model and incomplete information about a data point to infer the missing information for that point. For example, the adversary might be given partial information about an individual's medical record, and attempt to infer the individual's genotype by using a model trained on similar medical records [11].

Training data membership inference attacks aim to determine whether a given data point was present in the training data used to build a model. This is a privacy threat in itself, but membership inference vulnerability has come to be seen as a more general indicator of whether a model leaks private information [26, 37, 47], and is closely related to the guarantee provided by differential privacy [25].

To date, most membership inference attacks follow the so-called *shadow model* (shadow-bb) approach [37]. Briefly, in the shadow model approach, membership inference is cast as a supervised learning problem, where the adversary is given a data point and its true label, and aims to predict a binary label indicating membership status. To do so, the adversary trains a set of *shadow models* to replicate the functionality of the target model, and trains an *attack model* from training data derived from the shadow models' outputs on points used to train the shadow models and points not previously seen by the shadow models.

Subsequently, this attack was extended to the white-box setting [32] by including activation and gradient information obtained from the target model as features for the attack model. However, because gradient information may be very specific to a particular model, this white-box attack does not use shadow models and instead assumes that the adversary already knows a significant portion of the target model's training data. Features to train the attack model are thus obtained directly from the target model, using the gradients, activations, and output evaluated on known member/non-member points.

A Trivial Attack. By far the simplest MI attack, which we dub the "naive" attack (naive), follows from the fact that generalization error necessarily leads to membership vulnerability [47]. Given a data point and its true label, the attacker runs the model and observes whether its predicted label is correct. If it is, the attacker concludes that the point was in the training data; otherwise, the point is presumed a non-member.

Surprisingly, in many cases this works as well as the shadow model attack. Although this comparison has not been made previously in the literature, the top row of Figure 8 (Section 5.5) shows a set of experiments we conducted using the naive and shadow-bb [37] attacks. The shadow-bb attack outperformed naive by appreciable margins on just two of the datasets that we examined, and in most cases performed comparably to naive in every regard.

There are several reasons why this style of attack may fail to outperform the naive approach. First, shadow modelbased attacks require a large amount of training data from the target distribution to produce an effective attack model. While it may be possible to supplement or replace true indistribution data with synthetic data [37], it is still not clear that the resulting attack model would outperform the naive method. Second, when the generalization gap in the target model is substantial ( $> \sim 20\%$ ), as it is with nearly all of the models evaluated in [32] and many in [37], the naive method is fairly accurate (> 60%) and its results often simply coincide with those of more sophisticated predictors (see Section 5.5). In particular, when the model performs well on a data point, the softmax output will be near the true hard label (and the gradient of the loss with respect to the weights will be close to zero) — regardless of whether the point is a training set member. Likewise, when the model performs poorly on a data point, regardless of its membership, the softmax output will have more entropy. Thus, these features do not appear to be fundamentally more well-suited to membership inference than the naive method.

Calibrating Confidence. As a practical attack, the naive method has a significant drawback even when it appears to work well. Namely, it does not provide the attacker with much confidence about a positive inference: the data point may have been a training set member, or it may have just been classified correctly. After all, this is how the model is intended to behave on test points, so while it may be sensible to use a misprediction to conclude that a point *was not* a member, using a correct prediction to conclude membership with any degree of confidence is not. This is especially true on models with low generalization error.

Initially, it may seem that shadow model attacks do not inherit this limitation, as the attack model can be trained with cross-entropy loss to emit a confidence score with its prediction. If this score is well-calibrated, an attacker could ostensibly make use of it to make more confident inferences. Unfortunately, we find shadow attacks are not well-calibrated for increased precision; in fact, Figure 7 shows that raising the

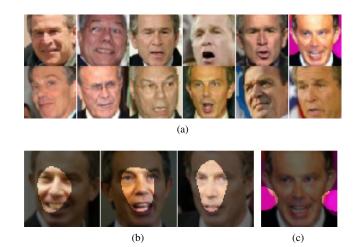


Figure 1: Pictorial example of how overfitting can lead to idiosyncratic use of features. (a) shows 12 training instances. We see that the image of Tony Blair on the top right has a distinctive pink background. (b) depicts internal explanations [24] for three test instances. The explanations show that the model uses Tony Blair's face to classify these instances, as we might expect. Meanwhile, (c) shows the explanation for the image with the distinctive pink background from the training set, where we see that the model is using the pink background to infer that the image is of Tony Blair.

confidence threshold for positive prediction often *decreases* the precision of the attack. In short, the shadow model attack often produces little consistently useful information to characterize the likelihood that a positive inference is correct, diminishing its advantage over the naive approach.

**Finding Evidence of Membership.** In this paper, we take a fresh look at the problem of white-box membership inference. We begin with the intuitive observation that while overfitting leads to privacy issues because the model "memorizes" certain aspects of the training data, this does not necessarily manifest in the model's output behavior. Instead, *it is likely to show up in the way that the model uses features*, either those that are explicitly given as input, or those learned by the model in hidden layers.

Intuitively, we posit that idiosyncratic features present in the training data, which are predictive *only* for the training data but not the sampling distribution, are oftentimes encoded in the model during training. Consider the example illustrated by Figure 1, in which a model was trained to recognize faces from the *Labeled Faces in the Wild* (LFW) dataset. Figure 1a shows 12 instances sampled from the training set of the model. In the top right corner of Figure 1a, we see an image of Tony Blair with a distinctive pink background. Supposing that the background is unique to this training instance, an overfit model may use the background as a feature for classifying Tony Blair, identifying the instance as a member of the training set via the uncharacteristic way in which the model correctly labels it. In such a setting, the pink background could be viewed as *evidence of membership*.

Figures 1b and 1c show this phenomenon on a convolu-

tional neural network trained on this dataset. Figures 1b and 1c visualize the regions of the image most influential towards the classification of "Tony Blair" on three test instances and on the training instance with the pink background using internal influence [24]. While the model is influenced most by Tony Blair's face for classification on the test instances, on the training instance it relies on the distinctive pink background.

While this example is simplistic for the purposes of illustration, this evidence-based approach can be used on a variety of real datasets to infer membership more effectively than prior attacks. Specifically, we develop a new attack (Section 3) that puts this idea to use, and show that it performs quite well (Section 5), outperforming previous attacks in terms of both accuracy and precision on real datasets. Moreover, we show that the confidence measure accompanying our inferences can be used to accurately calibrate the precision of the attack (Section 5.3).

Common Defenses. A number of defenses have been proposed for membership inference. *Differential privacy* [8], in addition to regularization methods like *Dropout* [40] in deep nets are two commonly-proposed defenses. While differential privacy gives a theoretical guarantee against membership inference [47], a *meaningful* guarantee (that gives a maximum theoretical advantage below one) requires an  $\varepsilon$  that is much smaller than what is commonly used in practice. However, common wisdom conjectures that large- $\varepsilon$ -differential privacy ostensibly provides a practical defense, particularly if the privacy budget analysis only gives a loose bound on  $\varepsilon$ .

Unfortunately, we find that this is not necessarily the case. We test our attack on deep models trained with  $(\epsilon,\delta)$ -differential privacy using the moments accountant method [1] (Section 6), and find that training with a large  $\epsilon$  sometimes provides *essentially no defense* against our attack, while still sacrificing a substantial amount of accuracy. Moreover, we find that our attack can achieve non-trivial accuracy against small- $\epsilon$ -differentially-private models, achieving advantage greater than what has been reported for prior attacks on certain datasets. These results suggest that the theoretical guarantee should be viewed as reasonably tight for small  $\epsilon$ , and emphasizes the need for more research in this area.

**Organization.** In Section 2, we introduce background on membership inference and machine learning. Section 3 describes the evidence-based attack, beginning in an idealized setting (Section 3.2), and gradually lifting generative assumptions used in our derivation in Section 3.2 to obtain an attack that works well on real data (sections 3.3, 3.4). Section 3.5 discusses calibration, and Section 4 shows how our attack can be extended to deep networks. Section 5 presents our evaluation on both synthetic data and seven real datasets derived from real-world medical and financial data, as well as common benchmark datasets. Section 6 discusses defenses for MI attacks and tests their efficacy against our attack. Section 7 covers related work, and Section 8 concludes the paper.

# 2 Background

Membership inference (MI) attacks aim to determine whether a given data point was present in the dataset used to train a given target model. In this section, we begin by introducing the necessary background needed to formally define membership inference, as well as explicitly defining the threat model used in our analysis.

#### 2.1 Supervised Learning and Target Models

We assume data from some universe  $\mathcal{U} = \mathcal{X} \times \mathcal{Y} \subset \mathbb{R}^d \times [C]$ , drawn from a distribution given by parameter  $\theta^*$ . Without ambiguity, we will refer to the distribution by  $\theta^*$ . Consistent with the typical supervised learning setting,  $x \in \mathcal{X}$  is a set of features and  $y \in \mathcal{Y}$  is a label or classification target, corresponding to C distinct classes. Given a loss function,  $\mathcal{L}: \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ , the goal of supervised learning is to construct a model, g, that minimizes  $\mathcal{L}(g(x), y)$  on future unseen samples, x, drawn from  $\theta^*$ . This is achieved by minimizing  $\mathcal{L}(g(x), y)$  on a finite training set, S, drawn i.i.d. from  $\theta^*$ .

A membership inference attack operates on a particular target model,  $\hat{g}$ . In this work, we consider target models that are expressed as a feed-forward neural networks; i.e., they consist of successive linear transformations, or layers, where each layer, i, is parameterized by a matrix of weights and biases  $W_i$ ,  $\mathcal{B}_i$ , followed by the application of a non-linear activation function.

Consistent with common practice, we assume that internal layers use the rectified-linear (ReLU) activation:  $relu(x) = \max(0,x)$ . As [C] may contain multiple classes, we assume that the final layer has one component for each label in [C] and uses the softmax activation:  $softmax(x)_j = e^{x_j} / \sum_i e^{x_i}$ . The use of the softmax function is standard in machine learning for multi-class classification. Models trained in this way produce  $confidence\ scores$  for each label that can be interpreted as probabilities [12].

In the simplest case we consider, the target model consists of a single layer with only the softmax activation, and is a *linear softmax regression* model. We will sometimes refer to the model by its parameterization,  $\hat{W}$ ,  $\hat{b}$ . Our approach generalizes to *deep networks* where the target model has multiple successive internal ReLU-activated layers, followed by a single softmax output layer.

#### 2.2 Membership Inference

We adpot a formulation of Membership Inference attacks similar to that of Yeom et al. [47]. First a value, b, is chosen uniformly at random from  $\{0,1\}$ . If b=1, the attacker,  $\mathcal{A}$ , is then given an instance z=(x,y) from the general population; otherwise, if b=0, z is sampled uniformly at random from the elements of the training set, S, used to generate target model,  $\hat{g}$ . The attacker then attempts to predict b given z and some additional knowledge,  $aux(\hat{g})$ , about  $\hat{g}$  determined by the threat model (see Section 2.3 below).

**Metrics.** The *accuracy* of an attack is the probability that  $\mathcal{A}$ 's prediction is equal to b, taken over the randomness of b, z, and  $\mathcal{A}$ . Because an adversary that guesses randomly achieves 50% accuracy, we will often opt to describe the *advantage* of an attack [47], given by Equation 1 in terms of attack,  $\mathcal{A}$ . Advantage scales accuracy to the 50% baseline to yield a measure between 0 and 1.

$$advantage(\mathcal{A}) = 2Pr[\mathcal{A}(z, aux(\hat{g})) = b] - 1 \tag{1}$$

While advantage is an indicator of the degree to which private information is leaked by the model, we also consider *precision* (Equation 2) as a key desideratum for the attacker.

$$precision(\mathcal{A}) = Pr[b = 1 | \mathcal{A}(z, aux(\hat{g})) = 1]$$
 (2)

In order for an attacker to reach confident inferences, precision must be appreciably greater than 1/2.

**Logistic Attack Models.** In the interest of achieving good precision, we consider attacks that yield confidence scores with their predictions. Thus, we can think of membership inference as a binary logistic regression [31] problem, in which a logistic (*sigmoid*) function models confidence with respect to the binary dependent variable (i.e., membership or non-membership). The sigmoid function,  $\Delta$ , is is given by  $\Delta(x) = \frac{1}{1+e^{-x}}$ , and can be thought of as converting the logodds of the dependent variable to a probability. The use of the sigmoid function for binary classification is standard in machine learning, and has been applied in prior membership inference attacks as well [37].

#### 2.3 Threat Model

The *threat model* determines what information is available to the adversary in aux(g) for making a membership determination. Prior work [37, 47] has focused primarily on the so-called *black-box* model where the adversary has access to  $\theta^*$ , the learning algorithm used to produce  $\hat{g}$  (including hyperparameters), the size of the training set, and the ability to query  $\hat{g}$  arbitrarily on new points. In practice, having access to  $\theta^*$  amounts to knowing a finite data set,  $\tilde{S}$  (distinct from S), sampled i.i.d. from  $\theta^*$ .

In this work, we replace black-box access to  $\hat{g}$  with white-box access. Rather than only being able to query the target model, the attacker has access to the exact representation of  $\hat{g}$  that was produced by the learning algorithm and used by the model owner to make inferences on new data. For the target models commonly used in practice, e.g. neural networks and linear classifiers, this amounts to a set of floating-point weight matrices and biases, in addition to the linear operators and activation functions used at each layer.

This threat model reflects the growing number of publicly-available models on websites like Model Zoo [20], as well as the fact that white box representations may be obtained from black box APIs through other attacks [43].

# **3** White-box Membership Inference

In this section, we introduce our core membership inference attack (Section 3.2). Starting in an idealized setting where the exact data distribution is known and the model is linear, we proceed by deriving the Bayes-optimal logistic attack model (Section 3.2). We show that when the data-generating assumptions hold, the confidence scores produced by this attack correspond to the true membership probability, and can thus be used for effective, accurate calibration towards high-precision attacks. We then show how to generalize the attack to settings where the data-generating distribution is unknown or does not match our theoretical assumptions (Sections 3.3 and 3.4), and discuss calibration in this setting (Section 3.5). In Section 4 we extend the attack to deep models.

#### 3.1 Overview of the attack

Our attack works from the intuition that when models overfit to their training data, they potentially leak membership information through anomalous behavior at test time. However, while this behavior may manifest in the form of prediction errors on unseen points, this need not be the case, and a more nuanced look at how memorization occurs yields new insights that can be used in an attack.

Models use features to distinguish between classes, and while some features may be truly discriminative (i.e., function as good predictors on unseen data), others may be discriminative only on the particular training set merely by coincidence. When the model applies features of the latter type to make a prediction, this can be thought of as "evidence" of overfitting regardless of whether the prediction is correct; the salience of a feature coincidental to the training data is suggestive on its own. Similarly, there may be features that are discriminative on the data in general, but not on the training data.

For example, consider a hypothetical model trained to recognize celebrity faces. Suppose that in reality, each celebrity is wearing sunglasses in 10% of his or her respective pictures, so the presence of sunglasses is not an informative feature for this task. However, if the training data used to construct the model contained images of a particular subject wearing sunglasses with greater frequency, say 30%, then the model might learn to weight this feature towards prediction of that subject. Knowing that the presence of sunglasses is not predictive of identity on the true distribution, an attacker would infer that, all else being equal, a picture of this subject wearing sunglasses is more likely to be a training set member. While this is not conclusive evidence of membership, it can be aggregated with other aspects of the model's behavior on an instance to make a final determination with greater confidence than would be possible using only blackbox information.

This example highlights the intuition that *membership in*formation is leaked via a target model's idiosyncratic use of features. Essentially, features that are distributed differently in the training data from how they are distributed in the true distribution provide evidence for or against membership. Our

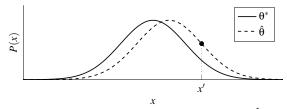


Figure 2: Example of two Gaussian distributions,  $\theta^*$  and  $\hat{\theta}$ . The point x' has a higher probability of being generated by  $\hat{\theta}$  than by  $\theta^*$ . Given a prior probability of  $\frac{1}{2}$  for being drawn from either distribution, the decision boundary for predicting which distribution a given point was drawn from would be at the intersection of the two curves, and x' would be predicted to have been drawn from  $\hat{\theta}$ .

attack works by deriving a set of parameters that characterize idiosynchratic feature use, and using them to construct a logistic attack model.

#### 3.2 A Bayes-Optimal Attack

To motivate our formalized intuition, consider two Gaussian distributions,  $\theta^* = \mathcal{N}(\mu^*, \sigma^*)$  and  $\hat{\theta} = \mathcal{N}(\hat{\mu}, \hat{\sigma})$ . For  $x \in \mathbb{R}$ , x is more likely to have been generated by  $\hat{\theta}$  than by  $\theta^*$  when  $\mathcal{N}(x \mid \hat{\mu}, \hat{\sigma}) > \mathcal{N}(x \mid \mu^*, \sigma^*)$ . An example of this is shown pictorially in Figure 2. Assuming a prior probability of 1/2 for being drawn from either distribution, we could construct a simple model that predicts whether x was drawn from  $\hat{\theta}$  rather than  $\theta^*$  by solving for x in this inequality. When the variances,  $\sigma^*$  and  $\hat{\sigma}$ , are the same, this produces a linear decision boundary as a function of  $\mu^* - \hat{\mu}$  and  $\sigma^*$ . Our setting is more complicated than this simple Gaussian example, but as we demonstrate below, the same principle can be applied to mount an attack.

**Generative Assumptions.** Recall the setting described in Section 2: a model,  $\hat{g}$ , trained on  $S \sim \theta^*$ , and an adversary that leverages white-box access to  $\hat{g}$  to create an attack model, m, that predicts whether an instance,  $(x,y) \in \mathcal{U}$ , belongs to S. We show how the example above can be extended to this setting by introducing some assumptions about  $\hat{g}$  and  $\theta^*$ .

First we assume that  $\theta^*$  is given by parameters,  $\mu_y^*$ ,  $\Sigma^*$ , and  $p^* = (p_1^*, \dots, p_C^*)$ , such that the labels, y, are distributed according to a Categorical distribution with parameter  $p^*$ , and the features, x, are multivariate Gaussians with mean  $\mu_y^*$  for each label y, and covariance matrix,  $\Sigma^*$ .

$$y \sim \text{Categorical}(p^*) \quad x \sim \mathcal{N}(\mu_y^*, \Sigma^*)$$
 (3)

Furthermore, assume that  $\Sigma^*$  is a diagonal matrix, i.e., the distribution of x satisfies the naive-Bayes assumption of the features being independent conditioned on the class. We will therefore write  $\Sigma_{ij}^*$  as  $\sigma_i^{*2}$ .

Recall that *S* is drawn i.i.d. from  $\theta^*$ , so its samples are also distributed according to Equation 3. However, the empirical means and variance of *S* will not match those of  $\theta^*$  exactly, except in expectation. Therefore, we can think of  $\hat{\theta}$  as a separate distribution, and, intuitively, *m* determines if (x,y) is more likely to have been drawn from  $\hat{\theta}$  (i.e.,  $(x,y) \in S$ ), or  $\theta^*$ .

Let  $\hat{p}$  be the empirical class prior for S,  $\hat{\mu}_y$  be the empirical mean of the features in S with class y, and  $\hat{\Sigma}$  be the empirical covariance matrix of the features in S. We make the analogous assumption that  $\hat{\Sigma}$  is a diagonal matrix.

The optimal predictor for y given features generated this way is the Gaussian Naive Bayes classifier, which is a linear model with weights computed directly from  $\mu$  and  $\Sigma$ , and the bias from p. If we momentarily assume that the attacker knows  $\theta^*$  and  $\hat{\theta}$ , then we can proceed to derive an attack model purely in terms of these parameters.

**Attack Model.** Let (X,Y) be random variables drawn from either  $\hat{\theta}$  or  $\theta^*$ , with probability t of drawing from  $\hat{\theta}$ . Let T be the event  $(X,Y) \in S$ , i.e., that a point drawn according to this process was in the training set. Thus,  $\Pr[T] = t$ . We will assume that  $t = \frac{1}{2}$  as this aligns with the membership inference definition presented in Section 2.

Now, we want an attack model  $m^y(x)$  to give us the probability that point (x,y) is a member of the training set, S. In other words,  $m^y(x) = \Pr[T|X = x, Y = y]$ . Because we know the parameters  $\theta^*$ ,  $\hat{\theta}$ , and t, we can derive a Bayesian estimator for this quantity by applying Bayes' rule and algebraically manipulating the result to fit a logistic function of the log odds. We then make use of the naive-Bayes assumption, allowing us to write the probabilities of observing x given its label as the product of the probabilities of observing each of x's features independently. The result is linear in the target feature values when  $\hat{\sigma} = \sigma^*$ , as detailed in Theorem 1. The proof for Theorem 1 is given in Appendix A.

**Theorem 1** Let x and y be distributed according to Equation 3 with parameters  $\theta^* = (p^*, \mu_y^*, \Sigma^*)$ , and S be drawn i.i.d. from the same distribution with empirical parameters  $\hat{\theta} = (\hat{p}, \hat{\mu}_y, \hat{\Sigma})$  where  $\hat{\Sigma} = \Sigma^*$  is diagonal and  $\hat{p} = p^*$ . Then the Bayes-optimal predictor for membership is given by Equation 4.

$$m^{y}(x) = s \left( w^{yT} x + b^{y} \right)$$
 where 
$$w^{y} = \frac{\hat{\mu}_{y} - \mu_{y}^{*}}{\sigma^{2}} \quad b^{y} = \sum_{j} \frac{\mu_{yj}^{*2} - \hat{\mu}_{yj}^{2}}{2\sigma_{j}^{2}}$$
 (4)

Notice that the attack model detailed in Theorem 1 defines a different set of parameters for each class label, y. This follows from the generative assumptions, as each class may have a distinct mean, and thus must be distinguished using separate critera. As a practical matter, this is not an impediment, as the membership inference setting assumes that the true class label is given to the adversary, so there is no ambiguity as to which set of parameters should be applied.

**Summary.** Features that are more likely in the empirical training distribution  $\hat{\theta}$  than on the true "general population" distribution  $\theta^*$  serve as evidence for membership. Theorem 1 shows how this evidence can be compiled into a linear attack model,  $w^y$ ,  $b^y$ , that achieves Bayes-optimality for membership

inference, when both distributions are known precisely. In Section 3.3, we show how to obtain approximate values for  $w^y$  and  $b^y$  when the distributions are unknown.

## 3.3 Obtaining MI Parameters from Proxy Models

In practice, it is unrealistic to know the exact parameters defining the distributions  $\theta^*$  and  $\hat{\theta}$ . In particular, our threat model assumes that the attacker has no *a priori* knowledge of  $\hat{\theta}$  or the elements of *S*, only that *S* was drawn from  $\theta^*$ . While we assume whitebox access to the target model,  $\hat{g}$ , we cannot expect that it will explicitly model  $\hat{\theta}$ ; indeed,  $\hat{g}$  is usually parameterized by weights, leaving the distribution parameters underdetermined. Finally,  $\theta^*$  and  $\hat{\theta}$  may violate the naive-Bayes assumption to an extent, or be difficult to parameterize directly.

These issues can be largely addressed by observing that the learned weights are sensitive to  $\hat{\theta}$ , and although they may not encode sufficient information to solve for the exact parameters, they may encode useful information about the differences between  $\hat{\theta}$  and  $\theta^*$ . To measure these differences, we use a *proxy dataset*,  $\tilde{S}$ , which is drawn i.i.d. from  $\theta^*$  (but distinct from S) to train a proxy model,  $\tilde{g}$ , which is then compared with  $\hat{g}$ . To control for differences in the learned weights resulting from the learning algorithm, rather than from differences between  $\hat{\theta}$  and  $\theta^*$ , the proxy model is trained using the same algorithm and hyperparameters as  $\hat{g}$ . This process can be repeated on many different  $\tilde{S}$ , using bootstrap sampling when the available data is limited.

In more detail, we continue with the assumption that data is generated according to Equation 3. Note that our target is a linear model, W, b, that minimizes 0-1 loss on S for the predictions given by  $argmax_{c \in [C]} \{softmax(W^Tx + b)_c\}$ , and this is a convex optimization problem that is minimized when W and b are given by Equation  $5^1$ .

$$W_{jy} = \frac{\mu_{yj}}{\sigma_j^2}$$
  $b_y = \sum_j \frac{-\mu_{yj}^2}{2\sigma_j^2} + \log(p^*)$  (5)

Plugging this into Equation 4 from Theorem 1, we see that the weights and biases of the attack model  $m^y$  are approximated by  $w^y \approx \hat{W}_{:y} - \tilde{W}_{:y}$  and  $b^y \approx \hat{b}_y - \tilde{b}_y$  respectively, assuming that  $\tilde{\mu} \approx \mu^*$ . This is summarized in Observation 1, which leads to a natural attack as shown in Algorithm 1. We call this the bayes-wb attack.

**Observation 1** For linear softmax model,  $\hat{g}$ , with weights,  $\hat{W}$ , and biases,  $\hat{b}$ ; and proxy model,  $\tilde{g}$ , with with weights,  $\tilde{W}$ , and biases,  $\tilde{b}$ , the Bayes-optimal membership inference model, m, on data satisfying Eq. 3 is approximately

$$m^{y} = \delta \left( w^{yT} x + b^{y} \right)$$
 where 
$$w^{y} = \hat{W}_{:y} - \tilde{W}_{:y} \quad b^{y} = \hat{b}_{y} - \tilde{b}_{y}$$

## Algorithm 1: The Linear bayes-wb MI Attack

**return** 1 if  $m^y(x) > \frac{1}{2}$  else 0

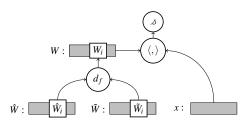


Figure 3: Illustration of the generalized attack model. A learned displacement function, d, is applied element-wise to the weights of the target and proxy model to produce attack model weights, W. The inner product of W and x is then used to make the membership prediction. Not pictured: d is also applied to the biases,  $\hat{b}$  and  $\tilde{b}$ , to produce b, which is added to the result of the inner product.

Notice that Observation 1 gives the weights and biases of  $m^y$  in terms of only the observable parameters of the target and proxy models. This is therefore possible *even when the distributions*,  $\theta^*$  and  $\hat{\theta}$  are unknown. Furthermore, while Observation 1 is derived and stated using relatively strong generative assumptions, we find in Section 5 that this attack is nevertheless often effective when these assumptions do not hold. In Section 3.4 we show how to further relax these generative assumptions.

#### 3.4 Learning to Generalize to Arbitrary Distributions

One way of viewing the bayes-wb attack is that it weights membership predictions by measuring a sort of displacement between the weights of the target model, and the ideal weights of the true distribution as approximated by the proxy model. Let  $d_f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$  be a *displacement function* that is applied element-wise to the weights of the model — for vectors x and y, let  $D(x,y)=(d_f(x_0,y_0),\ldots,d_f(x_d,y_d))$ . We can express the bayes-wb attack via a such a displacement function, namely,  $w^y=D(\hat{W}_{:y},\tilde{W}_{:y})$  and  $b^y=D(\hat{b}_y,\tilde{b}_y)$ , by letting  $d_f(x,y)=x-y$ , i.e., D is element-wise subtraction.

As per Observation 1, element-wise subtraction is optimal for membership inference under the Gaussian naive-Bayes assumption, but it may be that for other distributions, a different displacement function is more appropriate. More generally, we can represent the displacement function as a neural network, and train it using whatever data is at hand.

Figure 3 illustrates this approach, which we call the general-wb attack. A learned displacement function,  $d_f$ , is applied element-wise to  $\hat{W}$  and  $\tilde{W}$  to produce attack model weights,

<sup>&</sup>lt;sup>1</sup>see Murphy, Slide 20 [30] for details.

#### Algorithm 2: The Linear general-wb MI Attack

W, and to  $\hat{b}$  and  $\tilde{b}$  to produce attack model biases, b. It then predicts the probability of membership as  $\mathcal{S}(W_{:y}^T x + b_y)$ .

As  $d_f$  is applied element-wise to pairs of weights, we model D as a 1-dimensional convolutional neural network, where the initial layer has a kernel size and strides of 2 (i.e., the kernel is applied to one element of  $\hat{W}_{:y}$  and one element of  $\tilde{W}_{:y}$ ), and subsequent layers have a kernel size and stride of 1.

In order to learn the weights of D, we partition  $\tilde{S}$  into an "in" dataset,  $\tilde{S}^1$ , and an "out" dataset,  $\tilde{S}^0$ . We train a shadow target model,  $\check{g}$ , on  $\tilde{S}^1$  and a proxy model,  $\tilde{g}$ , on  $\tilde{S}^0$ . We then create a labeled dataset, T, where the features are the weights and biases of  $\check{g}$ , the weights and biases of  $\tilde{g}$ , and x; and the labels are 1 for x belonging to  $\tilde{S}^1$  and 0 for x belonging to  $\tilde{S}^0$ . Finally we train to find the parameters to D that minimize the 0-1 loss,  $\mathcal{L}$ , of the general-wb attack on T. We can increase the size of T to improve the generalization of the attack by repeating over multiple in/out splits of  $\tilde{S}$ . This procedure is described in Algorithm 2.

## 3.5 Calibrating for Precision

Recall the "naive" attack that predicts that an instance, x, is a member of the training set if and only if x was classified correctly. In practice, this naive approach is not a pragmatic attack because, while it will achieve advantage equal to the target model's generalization error (and close to that of prior blackbox approaches [37]), the only way to evaluate the confidence of the inference is to use the target model's own confidence score. As most neural networks are not well-calibrated [13], this makes it difficult to form confident inferences. The derivation in Section 3.2 suggests a direct probabilistic interpretation of the attack model's output. While the *maximum likelihood estimator*, which predicts x is a member of the training set when  $\Pr[T \mid X = x, Y = y] > \frac{1}{2}$ , maximizes accuracy, the precision, and therefore confidence in positive inferences, is increased by increasing the decision threshold above  $\frac{1}{2}$ .

Under the Gaussian Naive Bayes assumption, the probability given by *m* is exact, and there is no issue with calibration

Algorithm 3: Calibrating the Decision Threshold

by this approach. As a matter of practice, there are two main concerns. First, the training set is finite, so the recall will drop to zero at some point as the threshold is raised for greater precision. Second, if the generative assumptions are violated, the confidence may not correspond to an exact probability. We must therefore be careful when selecting a decision threshold.

Calibrating the decision threshold for the desired precision/recall trade-off requires access to the training set, S. However, the attack model is obtained using  $\tilde{S}$ , which is disjoint from S. Instead, we can stipulate that the elements of  $\tilde{S}$  are to be classified as non-members for the purpose of calibration, and use the following heuristic: given a false-positive tolerance parameter  $\alpha$ , set the threshold  $\tau_y$  for each class y as the  $\alpha^{th}$ -percentile confidence score of a sample of  $\tilde{S}$  belonging to class y. This is detailed in Algorithm 3. In Section 5.3, we show that this heuristic consistently increases the precision of our attack on real data.

## 4 Membership Inference in Deep Models

We showed how to approximate the Bayes-optimal estimator for membership prediction using the weights of a linear target and proxy model in Section 3.3. In this section, we extend the same reasoning to deep models. However, as deep networks learn novel representations, the semantic meaning of an internal feature at a given index,  $z_j$ —i.e., the data characteristic that it associates with — will not necessarily line up with the semantic meaning of the corresponding internal feature,  $z_j'$ , in another model [3, 48]. This holds even when the models share identical architectures, training data, and hyperparameters, as long as the randomization in the gradient descent is unique. In general, the only features for which two models will necessarily agree are the models' inputs and outputs, as these are not defined by the training process.

This poses a challenge for any white-box attack that attempts to extend the "shadow model" approach [37] developed for black-box membership inference. Consider such an approach, which learns properties of internal features that indicate membership — involving activations, gradients, or any other quantity — from shadow models. Any such property must make reference to specific internal features within the shadow model, but even if the target model contains internal features that match these properties, they are unlikely to reside at exactly the same location within the network as

## Algorithm 4: The Deep bayes-wb MI Attack

```
\begin{array}{l} \operatorname{def} \operatorname{createAttackModel} \left( \hat{g} \circ \hat{h}, \tilde{S} \right) \text{:} \\ & \tilde{S}' \leftarrow \left[ \left( \hat{h}(x), y \right) \operatorname{for} \left( x, y \right) \in \tilde{S} \right] \\ & \tilde{g} \leftarrow \operatorname{trainProxy} (\tilde{S}') \\ & w^y \leftarrow \lambda(z) : \chi(\hat{g} \circ \hat{h}, P_0^z)_y - \chi(\tilde{g} \circ \hat{h}, P_0^z)_y \quad \forall y \in [C] \\ & b^y \leftarrow \hat{g}(0)_y - \tilde{g}(0)_y \quad \forall y \in [C] \\ & \operatorname{return} \lambda(x, y) : \mathcal{S} \left( w^y (\hat{h}(x))^T \hat{h}(x) + b^y \right) \\ & \operatorname{def} \operatorname{predictMembership} \left( m, x, y \right) \text{:} \\ & \operatorname{return} 1 \operatorname{if} m^y(x) > \frac{1}{2} \operatorname{else} 0 \end{array}
```

they do in the shadow model. This is why previous whitebox attacks [32] require large amounts of the target model's training data; rather than learning attack models from shadow models, they are forced to learn them from the target model itself and its training data.

To circumvent this limitation, one must either construct a mapping between internal features in the shadow and target models, or fix the feature representation in the shadow model to preserve semantic meaning between the two. In this section, we show how to accomplish the latter by constructing a series of *local linear approximations* of the network (Section 4.1), one for each internal layer, that operate on the feature representation of the target model. Because each approximation is linear, we can apply any of the attacks from Section 3 to each one, and combine the results (Section 4.2) to form an attack model for the full network.

#### 4.1 Local Linear Approximations of Deep Models

We define a local linear approximation in terms of a *slice*,  $\langle g, h \rangle$ , which decomposes a deep network, f, into two functions, g and h, such that  $f = g \circ h$ . Intuitively, a slice corresponds to a layer, i, of the network, where h computes the features that are input to layer i, and g computes the output of the model from these features.

For the slice at the top layer of the network, g is simply a linear model acting on features computed by the rest of the model. In this case no approximation is needed and the bayes-wb (Algorithm 1) and general-wb (Algorithm 2) attacks can by applied directly to g using internal features that are precomputed by h.

For slices lower in the network, g is no longer linear, but we can approximate the way in which g makes use of its features by constructing a linear model that agrees with it at a particular point. To do this, we make use of an *influence measure* over the inputs of g to its computed output for each point. Given a model, f, a point, x, and feature, j, the *influence*  $\chi_j(f,x)$  of  $x_j$  on f is a quantitative measure of  $x_j$ 's contribution to the output of f. A growing body of work on influence measures [24, 39, 41] provides several choices for  $\chi$ , each with different properties.

For this approximation, we need an influence measure that (1) works on internal features, (2) weights features according

to their individual marginal contribution to the model's output, (3) satisfies linear agreement, and (4) is efficient with respect to a chosen baseline. Linear agreement requires that when f is linear, the influence of feature  $x_j$  is simply the corresponding weight,  $W_j$ . Thus, the influence measure generalizes the notion of weights in a linear model, and we can use the influence of a feature in place of the corresponding weight in Equation 6, while obtaining the same result. However, in order for this substitution to work at a particular internal point, z = h(x), we also require that  $g(z) \approx \overline{W}^T z + \overline{b}$ , where  $\overline{W}$  captures how each of the features,  $z_j$ , are used to obtain the model's output, which is semantically meaningful. This follows if  $\chi$  is efficient with respect to a baseline point  $z^0$ , as defined in Equation 7.

$$\sum_{j} \chi_{j}(g \circ h, z)(z_{j} - z_{j}^{0}) = g(z) - g(z^{0})$$
 (7)

When (7) holds, we can set  $z^0$  to zero to arrive at the desired local linear approximation, noting that efficiency with respect to the zero baseline implies  $g(z) = \chi(g \circ h, z)^T z + g(0)$ .

The unique influence measure satisfying the first three properties is the *internal influence* [24], given by Equation 8. Note that rather than operating on a single point, z or x, this measure operates over a *distribution of interest*, P, which specifies a distribution of points in the model's internal representation, z = h(x).

$$\chi_{j}(g \circ h, P) = \int_{z \in h(X)} \left. \frac{\partial g}{\partial z_{j}} \right|_{z} P(z) dz \tag{8}$$

When we set P to the uniform distribution over the line from a baseline  $z^0$  to z, denoted  $P^z_{z_0}$ , then this measure also satisfies efficiency in exactly the manner described above. We can therefore locally approximate g at z as  $\bar{g}(z) = \bar{W}^T z + \bar{b}$ , where  $\bar{W} = \chi(g \circ h, P^z_0)$  and b = g(0).

Thus, we can apply the attacks in Algorithm 1 and Algorithm 2 (Section 3) on an arbitrary layer of a deep network, by locally approximating the remainder of the network as a linear model. The modification of Algorithm 1 for an arbitrary slice,  $\langle \hat{g}, \hat{h} \rangle$ , of a target deep network,  $\hat{f}$ , is detailed in Algorithm 4. An analogous modification of Algorithm 2 follows as well, by simply replacing each reference to weights with influence measurements, and is omitted for the sake of brevity.

**Summary and Key Takeaways.** We can generalize the attacks given by algorithms 1 and 2 to apply to an arbitrary layer of a deep target network by replacing the weights with their natural generalization, *influence*. Because influence allows us to create a faithful local linear approximation of the model at a point, z, this generalized attack follows from the same analysis on linear models from Section 3. In Section 4.2, we suggest a method for combining attacks on each individual layer to create an attack that utilizes white-box information from all the layers of a deep network.

#### 4.2 Combining Layers

The results of Section 4.1 allow us to leverage overfitting at each learned representation employed by the target model towards membership inference. Attacks on different layers may pick up on different signals, but because the model's internal representations are not independent across layers, we cannot simply concatenate the approximated weights of each layer and treat it as an attack on a single model. Instead, we make use of a *meta model*, which learns how to combine the logistic outputs of the individual layer-wise attacks. The meta model takes the confidences of the attack defined in Section 4.1 applied to each layer, and outputs a single decision.

To train a meta model, m', to attack target model, f, we partition  $\tilde{S}$  into two parts,  $\tilde{S}^1$  and  $\tilde{S}^0$ . We train a shadow target model,  $\check{f}$ , on  $\tilde{S}^1$ . Then, for each layer, i, in f, we train an attack model,  $m_i$ , on the  $i^{th}$  layer of  $\check{f}$ , as described in Section 4.1. We then construct a training set,  $T = T^1 \cup T^0$ , such that  $(x'_j, y') \in T^1 = (m^y_j(x), 1)$  for  $(x, y) \in \tilde{S}^1$ , and  $(x'_j, y') \in T^0 = (m^y_j(x), 0)$  for  $(x, y) \in \tilde{S}^0$ . We can increase the size of T by creating multiple random partitions of  $\tilde{S}$ . Finally, we train m' on T.

When building a meta model for the general-wb attack, we can train m' jointly with the displacement metric, d, rather than first learning a general-wb attack on each layer. We also use a separate distance metric,  $d_i$  for each layer, i, of f.

## 5 Evaluation

In this section, we evaluate the attack described in sections 3 and 4. Section 5.1 describes our general experimental setup, including descriptions of the datasets and models we test our attack on. In Section 5.2, we demonstrate the efficacy of our relaxations in sections 3.3 and 3.4 towards approximating the optimal attack from 3.2, which relies on strong generative assumptions. In Section 5.3, we evaluate the ability to use the logistic attack model's confidence outputs as a means to increase the precision of membership inference. Section 5.4 explores the effectiveness of the meta attack model described in Section 4.2 in combining predictions from attacks on each layer of the model. Finally, we compare our work to previous approaches to membership inference in Section 5.5.

#### 5.1 Experimental Setup

**Datasets.** We performed experiments over both synthetic data and 7 classification datasets derived from real data. In general, we chose datasets from domains, such as medicine and finance, for which membership inference is likely to be a real concern. To facilitate a baseline for comparison against prior work, we included two common image datasets (MNIST and CIFAR10) that, despite having no plausible connection to privacy, have been studied in nearly all published membership inference experiments.

The synthetic data were generated with 10 classes, 75 features, and 400, 800, or 1,600, records, with an equal number of records per class. The features,  $x_j$ , of the synthetic data were drawn randomly from a multivariate Gaussian distribution

model	# row	# feat.	# class	train acc.	test acc.
Synthetic	400-1.6k	75	10	1.000	1.000
Breast Cancer (BCW MLP)	569	30	2	0.987	0.944
Pima Diabetes (PD MLP)	768	8	2	0.789	0.756
Hepatitis (Hep MLP)	155	19	2	0.997	0.810
German Credit (GC MLP)	1000	20	2	0.937	0.701
MNIST LeNet	70k	784	10	0.998	0.987
LFW LeNet	1140	1850	5	0.993	0.829
CIFAR10 LeNet	60k	3072	10	0.996	0.664

Figure 4: Characteristics of the datasets and models used in our experiments.

with parameters,  $\mu_y$  (for each class, y) and  $\Sigma$ , where  $\mu_{yj}$  was drawn uniformly at random from [0,1], and  $\Sigma$  was a diagonal matrix with  $\Sigma_{ij}$  drawn uniformly at random from [0.5, 1.5].

Among the classification datasets were *Pima Diabetes* (obtained from the UCI Machine Learning Repository); *Breast Cancer Wisconsin*, *Hepatitis*, *German Credit*, *Labeled Faces in the Wild* (obtained from scikit-learn's datasets API); *MNIST* [23], and *CIFAR10* [22]. Figure 4 shows the characteristics of each of these datasets.

**Target Models.** The target models we used to conduct our experiments include linear models, multi-layer perceptrons, and convolutional neural networks. Each model was trained until convergence with categorical cross-entropy loss, using SGD with a learning rate of 0.1, a decay rate of  $10^{-4}$ , and Nesterov momentum.

Linear models (LR) were implemented as a single-layer network in keras [6] using a softmax activation. We used linear models for the synthetic data. For non-image real data, we used a multi-layer perceptron (MLP) with one hidden layer and *ReLU* non-linearities, implemented in keras. For datasets with *n* features, we employed 2*n* hidden units, followed by a softmax layer with one unit per class. For image data, we used a CNN architecture, based on LeNet, detailed in Table 10 in Appendix B. We trained CNNs with a 25% dropout rate following each pooling layer, and a 50% dropout rate following the fully connected layer.

Each target model is a pair containing an architecture and a dataset. We refer to each target model by its abbreviation given in Figure 4, which shows the train and test accuracy for each of the target models used in our evaluation.

**Methodology.** When evaluating each attack, we randomly split the data into three disjoint groups: *train*, *test*, and *holdout*. The train and test groups are each comprised one fourth of the total number of instances, and the hold-out group contains the remaining one half of the instances. The target model is trained on the train group, while the attacks may make use of the hold-out group only. The attack model's predictions are evaluated on the train group (members) and the test group (non-members). Each experiment was repeated 10 times over different random seeds, and the results were averaged.

**Attack Methods.** Throughout our evaluation, we assess four different attacks: naive, bayes-wb, general-wb, and shadow-bb. The naive attack refers to the simple attack in-

	omniscient	bayes-wb	general-wb (min capacity)	general-wb (extra capacity)
n = 100	0.618	0.605	0.602	0.590
n = 200	0.577	0.570	0.563	0.562
n = 400	0.568	0.550	0.547	0.542

Figure 5: Comparison of the bayes-wb and general-wb attacks to an *omniscient* attack, which has knowledge of  $\hat{\mu}$ ,  $\mu^*$ , and  $\sigma$ , and thus can use Theorem 1 directly without the use of a proxy model. In one case, the general-wb attack was given the minimum capacity to reproduce the bayes-wb attack, i.e., d is simply a weighted sum of  $\hat{W}_i$  and  $\tilde{W}_i$ . In another case, the general-wb attack was given excess capacity, with 16 hidden units in d. Three target models, trained on synthetic Gaussian naive-Bayes data with training set sizes of 100, 200, and 400, were attacked.

troduced in Section 1, in which the attack model predicts an instance, x, is a member of the training set if and only if x was classified correctly.

For the bayes-wb attack (introduced in Section 3.3), we train 10 proxy models on random samples of half of the hold-out data, and take the mean of their approximated weights at each point for added robustness. When attacking MLP models, we perform the attack on the final layer of the MLP using Algorithm 1. When attacking LeNet models, we use a meta attack model (described in Section 4.2) that is trained on data from 10 random splits of the hold-out data. We use a MLP with 16 internal neurons for the meta model and train it for 32 epochs with Adam [19].

For the general-wb attack (introduced in Section 3.4), we concurrently attack each layer of each network and combine the results with a MLP with 16 internal neurons. The general-wb attack is trained for 32 epochs with Adam, using data from 10 shadow models trained on the hold-out group.

The shadow-bb attack refers to the black-box shadow model attack [37], explained briefly in Section 7. In each experiment, the shadow-bb attack was trained using 10 *shadow models* trained on the hold-out group.

#### 5.2 Evaluation of Relaxations

In Section 3.2, we derive the Bayes-optimal membership inference attack on Gaussian data satisfying the naive-Bayes condition. The weights of the optimal membership predictor for this case, given by Theorem 1, are a function of the empirical training distribution parameters and true distribution of the data, which, of course, would be unknown to an attacker. Section 3.3 describes how to address this, using a *proxy model* to capture the difference between the data used to train the target model and the general population.

Figure 5 demonstrates the effectiveness of the proxy model in our attack, by comparing our bayes-wb attack using a proxy model to an "omniscient" attack, which uses Equation 5 directly, with knowledge of the train and general distribution. We can consider the omniscient attack as giving an upper bound on the expected accuracy of a white-box attack on Gaussian naive-Bayes data, as it is the true Bayes-optimal attack (while bayes-wb is the approximate Bayes-optimal attack

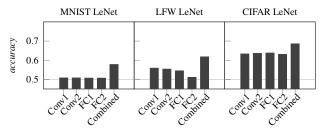


Figure 6: Accuracy of the bayes-wb attack on each individual layer of LeNet, compared with the accuracy using the combined meta-model.

according to Proposition 1). Our attack achieves on average 84% of the advantage of the omniscient attack, suggesting that the proxy model was able to approximately capture the general distribution as necessary for the purpose of detecting the target model's idiosyncratic use of features.

In Section 3.4, we further generalize the bayes-wb attack to use a learned displacement function that may be more appropriate for distributions that don't resemble the Gaussian naive-Bayes assumption. While we find that this general-wb attack often generalizes to arbitrary distributions better than the bayes-wb attack, because its displacement function is learned, it is possible for the general-wb attack to overfit.

Figure 5 also shows the accuracy of the general-wb attack on Gaussian naive-Bayes data. When the neural network representing the displacement function is given exactly enough capacity to reproduce the bayes-wb attack, general-wb recovers on average 94% of the advantage of the bayes-wb attack. Upon inspecting the weights of the displacement network, we find that general-wb learns almost exactly element-wise subtraction, demonstrating its potential to learn the optimal displacement function. When given excess capacity, the general-wb attack performs only marginally worse, achieving on average 92% of the minimal general-wb attack's advantage, suggesting that general-wb is not highly prone to overfitting.

#### 5.3 Calibrating for Precision

One of the key desiderata of a membership inference attack is precision. In order to calibrate a membership inference attack for precision, the confidence of the attack must be informative.

We found that increasing the decision threshold of the bayes-wb and general-wb attacks had a positive effect on precision. In particular, using the heuristic defined in Algorithm 3, we were able to consistently improve the precision of our attacks. Figure 7 shows the precision of our attack as the decision threshold was raised according to Algorithm 3, for  $\alpha=0.90$ , and  $\alpha=0.99$ , compared to the uncalibrated attack. In each case the precision increased, often substantially.

In practice, an attacker would not be easily able to tune the calibration hyperparameter,  $\alpha$ ; however, the consistency of the results in Figure 7 suggest that values of 0.90 and 0.99 serve as a practical "rule-of-thumb" reliable calibration.

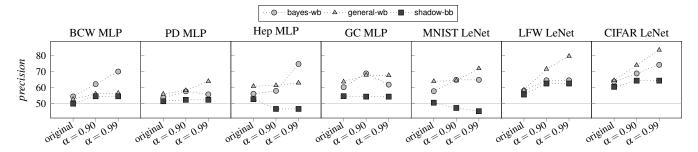


Figure 7: Precision of the bayes-wb, general-wb, and shadow-bb attacks, calibrated using the heuristic outlined described in Algorithm 3 (with  $\alpha = 0.90$  and  $\alpha = 0.99$ ), compared to the precision with no calibration (default threshold).

model	accuracy			precision				
	naive	shadow-bb	bayes-wb	general-wb	naive	shadow-bb	bayes-wb	general-wb
BCW MLP	0.522	0.500	0.514	0.523	0.511	0.500	0.545	0.528
PD MLP	0.517	0.508	0.517	0.519	0.511	0.515	0.537	0.561
Hep MLP	0.595	0.553	0.605	0.618	0.552	0.528	0.562	0.609
GC MLP	0.618	0.582	0.623	0.622	0.572	0.547	0.603	0.637
MNIST LeNet	0.506	0.506	0.575	0.521	0.503	0.506	0.578	0.640
LFW LeNet	0.582	0.597	0.618	0.619	0.545	0.557	0.581	0.586
CIFAR10 LeNet	0.666	0.684	0.686	0.709	0.600	0.605	0.638	0.646

Figure 8: Comparison of the accuracy and recall of bayes-wb and general-wb with naive and shadow-bb.

#### 5.4 Combining Layers

For deep models in particular, we want to be able to use information from each layer in our attack. In Section 4.2, we describe a meta attack that combines the outputs of an individual attack on each layer. Figure 6 shows the accuracy of the bayes-wb attack on each individual layer and of the meta attack on each LeNet target model.

In every instance, the meta attack is able to substantially outperform any individual attack, indicating that the information it receives from each layer is not entirely redundant. Remarkably, for MNIST, the advantage of the meta attack is greater than that of all the individual layers combined.

#### 5.5 Comparison to Prior Work

Finally, we compare our approach to previous work, namely, shadow-bb [37]. In particular, we compare: (1) performance in terms of accuracy, precision, and recall; (2) the reliability of the attack confidence when used to calibrate for higher precision. In short, our results show that both bayes-wb and general-wb outperform shadow-bb, and can be more reliably calibrated to achieve confident inferences for the attacker.

**Performance.** Figure 8 shows the accuracy and precision of naive, bayes-wb, general-wb, and shadow-bb. The precision shown is for the uncalibrated attack. We see that both bayes-wb and general-wb are consistently more accurate and precise than naive and shadow-bb. At least one of bayes-wb or general-wb obtains the highest accuracy of the four methods on each target, and *both* outperform the other two methods

in terms of precision in all cases. In particular, bayes-wb outperforms the other methods by the greatest margin on models that are well-generalized. This shows bayes-wb's ability to successfully leverage the internals of the target model.

Meanwhile, shadow-bb has consistently lower accuracy and precision. Often, shadow-bb has performance comparable or even worse than naive, particularly on well-generalized target models. This is likely a product of the attack model overfitting to idiosyncrasies in the shadow model's output that are unrelated to the target model. On deep models with significant overfitting, shadow-bb performs slightly better than naive, however, we found that its behavior was not significantly different from that of naive; for example, on LFW, naive recovered 88% of the exact correct predictions made by shadow-bb. This supports the intuition that the features used by the shadow model approach are not fundamentally more well-suited to membership inference than the naive method.

Our results for the performance of shadow-bb are roughly in line with previously reported results for shadow-bb on MNIST and CIFAR10 [37]; on CIFAR10, our results are slightly lower than the results reported for shadow-bb by Shokri et al., however, our target model trained on CIFAR10 uses dropout and has a lower generalization error then the model in the attack reported by Shokri et al., which most likely accounts for this small discrepancy.

**Calibration.** We also compare the calibration of the confidence outputs of our attacks compared to shadow-bb (naive does not provide a confidence score). While Figure 7 demonstrates the calibration of the confidence of the calibration of the confidence outputs of our attacks compared to shadow-bb (naive does not provide a confidence output of the calibration of the confidence outputs of our attacks compared to shadow-bb (naive does not provide a confidence output of the calibration of the confidence outputs of our attacks compared to shadow-bb (naive does not provide a confidence output of the calibration of the confidence output of the calibration of the confidence output of the calibration of the calibration

strates that applying our calibration heuristic to bayes-wb and general-wb consistently increases the precision, we see that this is not always the case for shadow-bb. In some cases, the precision of shadow-bb is *decreased* by increasing the decision threshold. When we were able to increase the precision of shadow-bb using its confidence output, the gains were less impressive, suggesting the probability outputs of shadow-bb are less well-calibrated.

#### 6 Defenses

Concerns about privacy, underscored by concrete threats such as the attacks developed in this paper, have also motivated research to provide adequate defenses against such threats. In this section we explore the ability of some of the commonly-proposed mitigation techniques to defend against our attack. In particular, we focus on *differential privacy* [8] and regularization. We find that, while both are useful to a degree, neither dropout nor  $\varepsilon$ -differentially private training with a large  $\varepsilon$ , are sufficient for mitigating the privacy risk posed by our attack.

**Differential Privacy.** Differential privacy [8] is often seen as the gold standard for private models, as a models trained with differential privacy have provable guarantees against membership inference. Namely, Yeom et al. [47] showed that, given an  $\varepsilon$ -differentially private learning algorithm, an adversary can achieve an advantage of at most  $e^{\varepsilon} - 1$ . Differential privacy has been applied to many areas of machine learning, including logistic regression [5], SVMs [33], and more recently, deep learning [1, 36]. However, current methods for ensuring differential privacy are typically costly with respect to the accuracy of the model, particularly for small values of  $\varepsilon$ , which give a better privacy guarantee. For this reason, in practice,  $\varepsilon$  is often chosen to be quite large; for example, in 2017, Apple was found to use an effective epsilon as high as 16 in some of its routines [42].

We used the Tensorflow Privacy library [28], an implementation of the *moments accountant* method [1], which guarantees  $(\varepsilon, \delta)$ -differential privacy, to study the practical efficacy of our attack on protected models. This method utilizes several hyperparameters from which  $\varepsilon$  is derived; for uniformity, we modified only the *noise multiplier* to achieve the desired  $\varepsilon$ , and used heuristics described in the original paper [1] to select the remaining hyperparameters. While a different tuning of the hyperparameters may result in a different privacy-utility trade-off, the privacy guarantee depends only on  $\varepsilon$  and  $\delta$ , *not the hyperparameters directly*. In each case,  $\delta$  was selected to be smaller than 1/N where N is the size of the dataset.

Figure 9 shows the effectiveness the general-wb attack against models trained with differential privacy for various values of  $\varepsilon$  on the LFW dataset. The train and test accuracies of the corresponding differentially-private target models are shown in Figure 11 in Appendix C. Considering the steep cost in accuracy for the differentially-private models, we find that differential privacy is only moderately effective as a defense when  $\varepsilon$  is sufficiently small. When  $\varepsilon$  is large ( $\varepsilon = 16$ ),

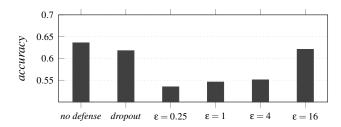


Figure 9: Attack accuracies for LFW LeNet models trained with either dropout or  $(\epsilon, \delta)$ -differential privacy for various values of  $\epsilon$ .

our attack performs essentially the same on the differentially-private model as on the undefended model, demonstrating that differential privacy may only be effective for sufficiently small  $\epsilon$ . Notably, for small  $\epsilon$  ( $\epsilon$  = 0.25), for which the maximum adversary accuracy is actually below 100%, our attack gets 53.5% accuracy (as compared to the theoretical max of 64.2%), underscoring the effectiveness of our attack.

**Regularization.** Given the connection between membership inference and overfitting, regularization, such as dropout [40], which aims to reduce overfitting, has also been proposed to combat membership inference. Generalization alone is not sufficient to protect against membership inference [47], and in fact, our empirical results (Section 5) show that we can successfully attack even models with negligible generalization error; however, dropout has been shown not only to reduce overfitting, but to strengthen privacy guarantees in neural networks [18]. Figure 9 shows the accuracy of our attack with and without dropout. We find that dropout does not significantly impact the accuracy of our attack.

**Defenses in the Black-box Setting.** For membership inference in the black-box setting, Shokri et al. [37] also propose a number of other possible defenses, such as restricting the prediction vector to the top k classes, or increasing the entropy of the prediction vector via increasing the normalization temperature of the softmax. However, these defenses are easily circumvented in the white-box setting, as the pre-modified outputs are still available to an attacker in this threat model.

In this section, we evaluate dropout and differentially private training as defenses against our attack. We find that, while both are useful to a degree, neither dropout nor  $\varepsilon$ -differentially private training with a large  $\varepsilon$ , are sufficient for mitigating the privacy risk posed by our attack.

## 7 Related Work

There is extensive prior literature on privacy attacks on statistical summaries. Homer et al. [17] proposed what is considered the first membership inference attack on genomic data in 2008. Following the work by Homer et al., a number of studies [9, 14, 35, 38, 44] have looked into membership attacks on statistics commonly published in genome-wide association studies. In a similar vein, Komarova et al. [21] looked into partial disclosure scenarios, where an adversary

is given fixed statistical estimates from combined public and private sources and attempts to infer the sensitive feature of an individual referenced in those sources.

More recently, membership inference attacks have been applied to machine learning models. Ateniese et al. [2] demonstrated that given access to the parameters of support vector machines (SVMs) or Hidden Markov Models (HMMs), an adversary can extract information about the training data.

As deep learning has become more ubiquitous, membership inference attacks have been particularly directed at deep neural networks. A number of different recent works [26, 27, 32, 34, 37, 47] have taken different approaches to membership inference against deep networks in a standard supervised learning setting. Additionally, Hayes et al. [15] have studied membership inference against generative adversarial networks (GANs); and others [16, 29, 32] have studied membership inference in the context of collaborative, or federated, learning.

**Black-box attacks.** We study membership inference as it applies to deep networks in classic supervised learning problems. Most of the prior work in this area [26, 27, 34, 37, 47] has used the black-box threat model. Yeom et al. [47] showed that generalization error necessarily leads to membership vulnerability; a natural consequence of this is that a simple "naive" attack, which predicts a point is a member if and only if it was classified correctly, can be found to be quite effective on models that overfit to a large degree. Other approaches have leveraged not only the predictions of the model, but the confidence outputs. A particularly canonical approach, along these lines, is the attack introduced by Shokri et al. [37]. In this approach, a *shadow model* is trained on half of  $\tilde{S}$ ,  $\tilde{S}_{in}$ , and an attack model is trained on the the outputs of the shadow model on its training data,  $\tilde{S}_{in}$  (labeled 1), and the remaining data  $\tilde{S} \setminus \tilde{S}_{in}$  (labeled 0). Shadow models leverage the disparity in prediction confidences on training instances the target model has overfit to, and have been shown to be successful at membership inference on models that have sufficiently high generalization error. A few other membership inference approaches [15, 34] have made use of this same technique.

Despite the fact that shadow model attacks leverage more information than the naive attack, we find in our evaluation (Section 5) that often, the shadow model attack fails to outperform the naive attack. One potential reason for this finding is that the learned attack model used by this approach to distinguish between the shadow model's outputs on members and non-members may be itself subject to overfitting. This may be especially true if the attack model picks up on behavior particular to one of the shadow models rather than the true target model. Furthermore, the confidence and entropy of the target model's softmax output is likely to be closely related to whether the target model's prediction was correct or not, meaning that the softmax outputs may not provide substantially different information from that used by naive.

White-box attacks. In some settings, it may be realistic for an attacker to have white-box access to the target model. Intuitively, while some information is leaked via the behavior of a model, the details of the structure and the parameters of the model are clear culprits for information leakage. Few prior approaches have successfully leveraged this extra information. While Haves et al. [15] describe a white-box attack in their work on membership inference attacks applied to GANs, the attack uses access only the outputs of the discriminator portion of the GAN, rather than the learned weights of either the discriminator or the generator; thus their approach is not white-box in the same sense. Meanwhile, Nasr et al. [32] demonstrated that a simple extension of the black-box shadow model approach to utilize internal activations does not result in higher membership inference accuracies than the original black-box approach. This is perhaps unsurprising, as the internal units of the shadow models are not likely to have any relation to those of the target model.

Recently, Nasr et al. [32] provided a white-box attack that leverages the gradients of the target model's loss function with respect to its weights, which SGD approximately brings to zero on the training points at convergence. In contrast to our work, Nasr et al. use a further relaxed threat model, in which the attacker has access to as much as *half of the target model's training data*. We suggest an approach that is quite different from that of Nasr et al.. Our approach does not require this extra knowledge for the attacker, and thus falls under a more restrictive threat model, in which, to our knowledge, no other effective white-box attacks have been proposed.

#### 8 Conclusions and Future Work

Our work is the first to fully leverage white-box information to improve membership inference attacks against deep networks (in the standard threat model where the adversary is assumed not to have any examples of true training points). In particular, our analysis sheds light on a fundamental mechanism of overfitting that can be leveraged by an adversary to compromise a model's privacy in a concrete way. Our evaluation demonstrates that the attack we developed can perform well even on well-generalized models, and that our attack can be reliably calibrated for increased precision.

In addition to this, we demonstrate that conventional defenses against membership inference attacks are not highly effective at mitigating this threat. Most interestingly, we found that (1) our attack can achieve appreciable accuracy against  $\varepsilon$ -differentially-private models with an  $\varepsilon$  much smaller (better privacy guarantee) than is ever used in practice, and (2), training with a large  $\varepsilon$  does not necessarily provide a non-trivial defense, but sacrifices a substantial amount of accuracy. This suggests that there is still considerable work to be done in developing effective defenses against privacy attacks — we anticipate that the insights gained from our approach will contribute to designing such defenses.

**Acknowledgment.** This material is based on work supported by the National Science Foundation under Grants No. CNS-1704845 and CNS-1801391.

#### References

- [1] Martin Abadi, Andy Chu, Ian Goodfellow, H. Brendan McMahan, Ilya Mironov, Kunal Talwar, and Li Zhang. Deep learning with differential privacy. In *Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security*, CCS '16, pages 308–318, New York, NY, USA, 2016. ACM. ISBN 978-1-4503-4139-4. doi: 10.1145/2976749. 2978318. URL http://doi.acm.org/10.1145/2976749. 2978318.
- [2] Giuseppe Ateniese, Luigi V. Mancini, Angelo Spognardi, Antonio Villani, Domenico Vitali, and Giovanni Felici. Hacking smart machines with smarter ones: How to extract meaningful data from machine learning classifiers. *International Journal of Security and Networks*, 10(3):137–150, September 2015.
- [3] Yoshua Bengio, Aaron C. Courville, and Pascal Vincent. Unsupervised feature learning and deep learning: A review and new perspectives. *CoRR*, abs/1206.5538, 2012. URL http://arxiv.org/abs/1206.5538.
- [4] Justin Brickell and Vitaly Shmatikov. The cost of privacy: destruction of data-mining utility in anonymized data publishing. In KDD, 2008.
- [5] Kamalika Chaudhuri and Claire Monteleoni. Privacy-preserving logistic regression. In D. Koller, D. Schuurmans, Y. Bengio, and L. Bottou, editors, Advances in Neural Information Processing Systems 21, pages 289–296. Curran Associates, Inc., 2009. URL http://papers.nips.cc/paper/3486-privacy-preserving-logistic-regression.pdf.
- [6] Francois Chollet. Keras: Deep learning library for Theano and TensorFlow. https://keras.io, 2017.
- [7] Graham Cormode. Personal privacy vs population privacy: Learning to attack anonymization. In *KDD*, 2011.
- [8] Cynthia Dwork. Differential privacy. In *ICALP*. Springer, 2006.
- [9] Khaled El Emam, Elizabeth Jonker, Luk Arbuckle, and Bradley Malin. A systematic review of re-identification attacks on health data. *PLOS ONE*, 6(12):1–12, 12 2011. doi: 10.1371/journal.pone.0028071.
- [10] Matt Fredrikson, Somesh Jha, and Thomas Ristenpart. Model inversion attacks that exploit confidence information and basic countermeasures. In ACM Conference on Computer and Communications Security (CCS), 2015.
- [11] Matthew Fredrikson, Eric Lantz, Somesh Jha, Simon Lin, David Page, and Thomas Ristenpart. Privacy in pharmacogenetics: An end-to-end case study of personalized warfarin dosing. In *USENIX Security Symposium*, pages 17–32, 2014.
- [12] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. Deep Learning. MIT Press, 2016. http://www.

- deeplearningbook.org.
- [13] Chuan Guo, Geoff Pleiss, Yu Sun, and Kilian Q Weinberger. On calibration of modern neural networks. 2017.
- [14] Melissa Gymrek, Amy L. McGuire, David Golan, Eran Halperin, and Yaniv Erlich. Identifying personal genomes by surname inference. *Science*, 339(6117):321–324, 2013.
- [15] Jamie Hayes, Luca Melis, George Danezis, and Emiliano De Cristofaro. LOGAN: evaluating privacy leakage of generative models using generative adversarial networks. *CoRR*, abs/1705.07663, 2017. URL http://arxiv.org/abs/1705. 07663.
- [16] Briland Hitaj, Giuseppe Ateniese, and Fernando Pérez-Cruz. Deep models under the GAN: information leakage from collaborative deep learning. *CoRR*, abs/1702.07464, 2017. URL http://arxiv.org/abs/1702.07464.
- [17] Nils Homer, Szabolcs Szelinger, Margot Redman, David Duggan, Waibhav Tembe, Jill Muehling, John V. Pearson, Dietrich A. Stephan, Stanley F. Nelson, and David W. Craig. Resolving individuals contributing trace amounts of DNA to highly complex mixtures using high-density SNP genotyping microarrays. *PLoS Genetics*, 4(8), 2008.
- [18] Prateek Jain, Vivek Kulkarni, Abhradeep Thakurta, and Oliver Williams. To drop or not to drop: Robustness, consistency and differential privacy properties of dropout. *CoRR*, abs/1503.02031, 2015. URL http://arxiv.org/abs/1503. 02031.
- [19] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. CoRR, abs/1412.6980, 2015.
- [20] Jing Yu Koh. Model Zoo. URL http://modelzoo.co.
- [21] Tatiana Komarova, Denis Nekipelov, and Evgeny Yakovlev. Estimation of treatment effects from combined data: Identification versus data security. In *Economic Analysis of the Digital Economy*, pages 279–308. University of Chicago Press, 2015.
- [22] Alex Krizhevsky and Geoffrey Hinton. Learning multiple layers of features from tiny images. 2009.
- [23] Yann LeCun, Corrina Cortes, and Christopher Burges. The MNIST database of handwritten digits. http://yann.lecun.com/exdb/mnist/, 1998.
- [24] Klas Leino, Shayak Sen, Anupam Datta, Matt Fredrikson, and Linyi Li. Influence-directed explanations for deep convolutional networks. *CoRR*, abs/1802.03788, 2018. URL http://arxiv.org/abs/1802.03788.
- [25] Ninghui Li, Wahbeh Qardaji, Dong Su, Yi Wu, and Weining Yang. Membership privacy: A unifying framework for privacy definitions. In *Proceedings of ACM CCS*, 2013.
- [26] Yunhui Long, Vincent Bindschaedler, and Carl A. Gunter. Towards measuring membership privacy. *CoRR*, abs/1712.09136, 2017. URL http://arxiv.org/abs/1712.09136.
- [27] Yunhui Long, Vincent Bindschaedler, Lei Wang, Diyue Bu,

- Xiaofeng Wang, Haixu Tang, Carl A. Gunter, and Kai Chen. Understanding membership inferences on well-generalized learning models. *CoRR*, abs/1802.04889, 2018. URL http://arxiv.org/abs/1802.04889.
- [28] H. Brendan McMahan and Galen Andrew. A general approach to adding differential privacy to iterative training procedures. CoRR, abs/1812.06210, 2018. URL http://arxiv.org/abs/1812.06210.
- [29] Luca Melis, Congzheng Song, Emiliano De Cristofaro, and Vitaly Shmatikov. Inference attacks against collaborative learning. *CoRR*, abs/1805.04049, 2018. URL http://arxiv.org/abs/1805.04049.
- [30] Kevin P. Murphy. Gaussian classifiers. University Lecture, 2007. URL https://www.cs.ubc.ca/~murphyk/ Teaching/CS340-Fall07/gaussClassif.pdf.
- [31] Kevin P. Murphy. *Machine Learning: A Probabilistic Perspective*. The MIT Press, 2012.
- [32] Milad Nasr, Reza Shokri, and Amir Houmansadr. Comprehensive privacy analysis of deep learning: Stand-alone and federated learning under passive and active white-box inference attacks. *CoRR*, abs/1812.00910, 2018. URL http://arxiv.org/abs/1812.00910.
- [33] Benjamin I. P. Rubinstein, Peter L. Bartlett, Ling Huang, and Nina Taft. Learning in a large function space: Privacy-preserving mechanisms for SVM learning. CoRR, abs/0911.5708, 2009. URL http://arxiv.org/abs/0911. 5708.
- [34] Ahmed Salem, Yang Zhang, Mathias Humbert, Mario Fritz, and Michael Backes. Ml-leaks: Model and data independent membership inference attacks and defenses on machine learning models. In *Annual Network and Distributed System Security Symposium (NDSS)*, 2019.
- [35] Sriram Sankararaman, Guillaume Obozinski, Michael I Jordan, and Eran Halperin. Genomic privacy and limits of individual detection in a pool. *Nature Genetics*, 41(9):965–967, 2009.
- [36] Reza Shokri and Vitaly Shmatikov. Privacy-preserving deep learning. In Proceedings of the 22Nd ACM SIGSAC Conference on Computer and Communications Security, CCS '15, pages 1310–1321, New York, NY, USA, 2015. ACM. ISBN 978-1-4503-3832-5. doi: 10.1145/2810103.2813687. URL http: //doi.acm.org/10.1145/2810103.2813687.
- [37] Reza Shokri, Marco Stronati, and Vitaly Shmatikov. Membership inference attacks against machine learning models. *CoRR*, abs/1610.05820, 2016. URL http://arxiv.org/abs/1610.05820.
- [38] Suyash S. Shringarpure and Carlos D. Bustamante. Privacy risks from genomic data-sharing beacons. *The American Jour*nal of Human Genetics, 97(5):631–646, May 2015.
- [39] Karen Simonyan, Andrea Vedaldi, and Andrew Zisserman. Deep inside convolutional networks: Visualising image classification models and saliency maps. CoRR, abs/1312.6034,

- 2013. URL http://arxiv.org/abs/1312.6034.
- [40] Nitish Srivastava, Geoffrey Hinton, Alex Krizhevsky, Ilya Sutskever, and Ruslan Salakhutdinov. Dropout: A simple way to prevent neural networks from overfitting. *Journal of Machine Learning Research*, 15:1929–1958, 2014. URL http://jmlr.org/papers/v15/srivastava14a.html.
- [41] Mukund Sundararajan, Ankur Taly, and Qiqi Yan. Axiomatic attribution for deep networks. *CoRR*, abs/1703.01365, 2017. URL http://arxiv.org/abs/1703.01365.
- [42] Jun Tang, Aleksandra Korolova, Xiaolong Bai, Xueqiang Wang, and Xiaofeng Wang. Privacy loss in apple's implementation of differential privacy on macos 10.12. 09 2017.
- [43] Florian Tramèr, Fan Zhang, Ari Juels, Michael K. Reiter, and Thomas Ristenpart. Stealing machine learning models via prediction apis. In *Proceedings of the 25th Usenix Security Symposium*, 2016.
- [44] Rui Wang, Yong Fuga Li, XiaoFeng Wang, Haixu Tang, and Xiaoyong Zhou. Learning your identity and disease from research papers: information leaks in genome wide association studies. In CCS, 2009.
- [45] X. Wu, M. Fredrikson, W. Wu, S. Jha, and J. F. Naughton. Revisiting Differentially Private Regression: Lessons From Learning Theory and their Consequences. *CoRR*, abs/1512.06388, 2015.
- [46] X. Wu, M. Fredrikson, S. Jha, and J. F. Naughton. A methodology for formalizing model-inversion attacks. In 2016 IEEE Computer Security Foundations Symposium (CSF), 2016.
- [47] Samuel Yeom, Matt Fredrikson, and Somesh Jha. The unintended consequences of overfitting: Training data inference attacks. *CoRR*, abs/1709.01604, 2017. URL http://arxiv.org/abs/1709.01604.
- [48] Jason Yosinski, Jeff Clune, Yoshua Bengio, and Hod Lipson. How transferable are features in deep neural networks? *CoRR*, abs/1411.1792, 2014. URL http://arxiv.org/abs/1411.1792.

## A Proof of Theorem 1

*Proof.* We begin with the expression for  $m^{y}(x)$  in Equation 9, and apply Bayes' rule to obtain Equation 10.

$$m^{y}(x) = \Pr[T \mid X = x, Y = y] \tag{9}$$

$$= \frac{\Pr[X = x \mid T, Y = y] \Pr[T]}{\Pr[X = x \mid Y = y]}$$
(10)

Next, we would like to express Equation 10 as a logistic, or sigmoid, function (.6). We assume that  $\Pr[T] = \frac{1}{2}$ , and thus  $\Pr[X = x \mid Y = y]$  can be written as  $\frac{1}{2} (\Pr[X = x \mid T, Y = y] + \Pr[X = x \mid \neg T, Y = y])$ , by the law of total probability. We then divide by the numerator in Equation 10, yielding an expression that can be written as a logistic function (11) by noting that for x > 0,  $\exp(\log x) = x$ .

$$(10) = \frac{\Pr[X = x \mid T, Y = y]}{(\Pr[X = x \mid T, Y = y] + \Pr[X = x \mid \neg T, Y = y])}$$

$$= \left(1 + \frac{\Pr[X = x \mid \neg T, Y = y]}{\Pr[X = x \mid T, Y = y]}\right)^{-1}$$

$$= \left(1 + \exp\left(\log\frac{\Pr[X = x \mid \neg T, Y = y]}{\Pr[X = x \mid T, Y = y]}\right)\right)^{-1}$$

$$= \delta\left(\log\frac{\Pr[X = x \mid T, Y = y]}{\Pr[X = x \mid \neg T, Y = y]}\right)$$
(11)

We notice that  $\Pr[X = x \mid T, Y = y]$  is the probability of having drawn x from  $\hat{\theta}$ , given class, y, and similarly,  $\Pr[X = x \mid \neg T, Y = y]$  is the probability of having drawn x from  $\theta^*$ , given class, y. Using the Naive-Bayes assumption, i.e., that conditioned on the class, y, the individual features,  $x_i$ , are independent, we obtain Equation 12.

$$(11) = \mathcal{S}\left(\log \prod_{j} \frac{\mathcal{N}(x_j \mid \hat{\mu}_{yj}, \hat{\sigma}_j^2)}{\mathcal{N}(x_j \mid \mu_{yj}^*, \sigma_j^{*2})}\right)$$
(12)

We then re-write the log of the product as a sum over the log, and observe that the sum can be written as a dot product as in Equation 13, which gives the parameters of the Bayes-optimal model for  $m^{y}(x)$ .

(12) = 
$$\delta \left( \sum_{j} \frac{(x_{j} - \mu_{yj}^{*})^{2}}{2\sigma_{j}^{*2}} - \frac{(x_{j} - \hat{\mu}_{yj})^{2}}{2\hat{\sigma}_{j}^{2}} + \log \left( \frac{\sigma_{j}^{*}}{\hat{\sigma}_{j}} \right) \right)$$
  
=  $\delta \left( v^{y} x^{2} + w^{y} x^{2} + b^{y} \right)$  (13)

where

$$v_{j}^{y} = \frac{1}{2\sigma_{j}^{*2}} - \frac{1}{2\hat{\sigma}_{j}^{2}} \qquad w_{j}^{y} = \frac{\hat{\mu}_{yj}}{\hat{\sigma}_{j}^{2}} - \frac{\mu_{yj}^{*}}{\sigma_{j}^{*2}}$$
$$b^{y} = \sum_{j} \left( \frac{\mu_{yj}^{*2}}{2\sigma_{j}^{*2}} - \frac{\hat{\mu}_{yj}^{2}}{2\hat{\sigma}_{j}^{2}} \right) + \log \left( \frac{\sigma_{j}^{*}}{\hat{\sigma}_{j}} \right)$$

Finally, by assumption the variance is the same in S as in the general distribution, i.e.,  $\hat{\sigma}_j = \sigma_j^* = \sigma_j$ , for all features, j. Thus,  $v^y$  from Equation 13 becomes zero, so we are left with a linear model for  $m^y$ , with weights,  $w^y$ , and bias,  $b^y$ , given by Equation 4.  $\square$ 

### **B** Model Architectures

	channels/units	filter size	non-linearity
LeNet			
2-D Convolution	20	5 × 5	ReLU
$2 \times 2$ Max-pooling			
2-D Convolution	50	$5 \times 5$	ReLU
2 × 2 Max-pooling			
Fully-connected	500		ReLU
Softmax	10		

Figure 10: Architecture for the LeNet convolutional neural network.

# C Defended Target Model Accuracies

	no defense	dropout	$\varepsilon = 0.25$	$\varepsilon = 1$	$\varepsilon = 4$	$\varepsilon = 16$
train acc.	1.000	0.999	0.109	0.137	0.214	0.428
test acc.	0.842	0.835	0.116	0.119	0.200	0.463

Figure 11: Train and test accuracies for LFW LeNet models trained with either dropout or  $(\epsilon, \delta)$ -differential privacy for various values of  $\epsilon$ .