THE PSYCHOLOGICAL REVIEW

THE GOAL-GRADIENT HYPOTHESIS APPLIED TO SOME 'FIELD-FORCE' PROBLEMS IN THE BEHAVIOR OF YOUNG CHILDREN [1]

BY CLARK L. HULL

Yale University

The hypothesis of the goal-excitatory gradient (8),[2] as well as that of the habit-family hierarchy (9),[3] was originally expounded in connection with a consideration of the behavior of albino rats in the ordinary enclosed maze. In such mazes the possible pathways of approach to the goal as well as the range of vision are greatly limited. Indeed, in former papers the sense of vision as a distance receptor was almost entirely neglected. The present paper, on the other hand, is concerned with some of the implications of the two hypotheses and a number of closely related principles when behavior, chiefly locomotor, is taking place in situations such that the field of vision is practically unobstructed. And, while certain limita-

---

[1] The substance of this article has been in occasional use in the form of a bound manuscript as a seminar reference since the spring of 1934. In this way a number of valuable criticisms have been received; the author is especially indebted to Dr. Neal E. Miller and to Dr. Carl I. Hovland. Mr. Donald T. Perkins has contributed the mathematical portions of notes 6, 12, 16, and 19.

[2] A certain amount of misunderstanding concerning the goal-gradient hypothesis has arisen from confusing one of the implications of the hypothesis under particular conditions (20, 153) with the hypothesis itself. The hypothesis, as originally stated (8, 26) is, "that the goal reaction gets conditioned the most strongly to the stimuli preceding it, and the other reactions of the behavior sequence get conditioned to their stimuli progressively weaker as they are more remote (in time or space) from the goal reaction." The 'goal' is the point of reinforcement. Because of the somewhat anthropomorphic connotation of the term 'goal' it would, perhaps, be better if this principle could be known as the 'gradient of reinforcement,' as suggested by Miller and Miles (16).

[3] A habit-family hierarchy consists of a number of habitual alternative behavior sequences having in common the initial stimulus situation and the final reinforcing state of affairs (11, 16).

tions will be placed on the opportunities for locomotion in the situations to be considered, freedom in this respect will be comparatively unrestricted. Lastly, the phenomenon to be studied will be mainly the naïve striving behavior of organisms such as young children who are only slightly sophisticated regarding obstacles at distances greater than the arm's length.[4] A number of the problems to be examined have already been subjected to a Gestalt analysis by Lewin (13, 14).

I

We shall begin by considering the influence of the sub-ject's being able to see clearly, though usually at some dis-tance, the lure or goal (G) from the starting point of his sequence of locomotion (S), together with numerous dis-criminable aspects of the intervening space. It is clear that the visual stimulation changes constantly, particularly as to the size of the image on the retina, as the subject approaches the lure, i.e., the point of reinforcement. According to the goal excitatory gradient hypothesis, the several phases of the flux of this stimulus complex will become conditioned to the accompanying movements leading to the point of reinforce-ment with a strength increasing with proximity to the goal. Thus, after a certain amount of training the organism should advance more vigorously the closer it is to the goal (10).[5]

[4] Striving is that behavior of organisms which, upon frustration, displays varied alternative action sequences, all directed by an intent (fractional anticipatory goal reaction or r_G) to the attainment of the same reinforcing state of affairs (11, 16). The naïveté here assumed involves as a minimum the absence of effective speech symbolism. Accordingly the present analysis should apply both to young children and to feeble-minded, of whatever chronological age, who have a mental age under about two years (17).

[5] This is on the assumption that the stronger the excitatory tendency, the stronger and more rapid will be the reaction dependent upon it. The experimental results reported by Bruce (3), and one or two other studies which have been communicated privately, seem to indicate that the relationship between the speed-of-locomotion gradient and the hypothesis of the goal gradient may not be so simple as appeared in the author's study just cited and in an independent confirming experiment by Miller and Miles (16). Evidently further experimental work will be needed to clarify this relationship. On the other hand, a recent experiment involving compound trial-and-error, reported by Muenzinger, Dove, and Bernstone (18), presents reassuring evidence of the substantial truth of the basic goal-excitatory-gradient hypothesis (see note 2 above). Incidentally, these writers show that when conditions permit, a similar

Yoshioka's experiment involving the power of rats to dis-criminate short from long paths indicates that this increase in excitatory tendency follows rather closely the logarithmic principle (22).[6]

A convenient presentation of such a logarithmic function is shown in Table I. By means of this table it will be easy to see in a clear and precise manner the quantitative[7] implica-tions of the goal-gradient hypothesis as bearing on the locomotor excitatory tendencies of a considerable variety of situations. Let us take as our first example one in which the visual stimulus of the lure (G) is placed at two distances from the subject, the second distance being three times as great as the first (Fig. 1). Suppose that the first starting point (S) is 5 units distant from the goal, and that the second is 15 units distant.[8] By Table I, the excitatory potentiality of

gradient of excitation also *follows* the point of reinforcement quite in harmony with the well-established conditioning or learning experiments of Thorndike (21; 2).

[6] This may be demonstrated as follows. Yoshioka's experiment showed that the length-discrimination threshold in rats for pairs of distinct paths to the same reward point was inversely proportional to the distance. If we assume that the *difference* between the excitatory tendencies (E) leading to the initial acts of the alternative pathways is the same for the various length-discrimination thresholds, then the decrease in excitatory strength per unit increase in distance (D) from the goal is in-versely proportional to the distance, i.e.,

$$\frac{dE}{dD} = -\frac{b}{D},$$

where b is a positive constant. Hence, integrating, we have:

$$E = a - b \log D,$$

where a is a constant (and must be positive, since E is positive). This is the equation upon which Table I is constructed. It is to be noted, however, that this is but a tenta-tive, and probably not a very accurate, representation of the exact nature of the rela-tionship. Two considerations argue against its finality. (1) At $D = O$, E becomes infinite, and (2) with very large values of D, E becomes negative instead of becoming asymptotic to zero. A plausible alternative equation is:

$$E = ae^{-hD}$$

In this connection see Guilford (6).

[7] This quantitative approach with the attendant possibilities of utilizing the potentialities of a metricized mathematics for the purpose of exploring the implications of its postulates is somewhat in contrast to the topological approach emphasized by Lewin with its seeming limitation to the qualitative (14).

[8] It is to be emphasized that the principles of alternative reaction sequences here considered are intended to apply to all sorts of behavior and are definitely not restricted

TABLE 1

This table shows the hypothetical strength of the conditioning of stimuli received at different distances from the goal to reactions occurring simultaneously. This is on the assumption that the excitatory tendency one unit from the goal acquires a strength of ten units and that the strength of the excitatory tendency acquired at each of the remaining distances diminishes with their remoteness according to the equation

$$E = a - b \log D,$$

where a has a value of 10, b has a value of 4, and the logarithms are taken with a base of 5.

Units Distant from Goal (D)	Strength of Excitatory Tendency (E)	Units Distant from Goal (D)	Strength of Excitatory Tendency (E)
1	10.000	26	1.903
2	8.277	27	1.809
3	7.270	28	1.718
4	6.554	29	1.631
5	6.000	30	1.547
6	5.547	31	1.465
7	5.164	32	1.386
8	4.831	33	1.310
9	4.539	34	1.236
10	4.277	35	1.164
11	4.040	36	1.094
12	3.824	37	1.026
13	3.625	38	.959
14	3.441	39	.895
15	3.270	40	.832
16	3.109	41	.771
17	2.958	42	.711
18	2.816	43	.652
19	2.682	44	.595
20	2.554	45	.539
21	2.433	46	.484
22	2.317	47	.431
23	2.207	48	.379
24	2.101	49	.327
25	2.000	50	.277

the visual image encountered at 5 units distant has a value of 6.00 points, whereas its strength at 15 units is only 3.27 points, a difference of 2.73. This fundamental relationship may be represented graphically as in Fig. 2.

It follows from the preceding that if an experimental situation should be so arranged that two distinct alternative paths of these respective lengths both lead from S to G, path

A would be chosen in preference to path B because the stimulus at the point of choice would have an excitatory tendency leading to the acts constituting A nearly twice as great as to those constituting B. A situation of this kind is, of course, the limiting case of a habit-family hierarchy: the fact that two paths begin and end at the same points makes them a family; the fact that one is preferred above the other constitutes them a hierarchy (9).

By a simple extension of the above reasoning, certain corollaries may be derived. It may be shown, for example, that the difference in excitatory tendency leading to the locomotor action of traversing pairs of paths differing in length by a constant amount grows less and less as the paths

PATH A: S————G

PATH B: S'————G

Fig. 1. Two paths to the same goal; S, starting point of one path and S', of the other. Path A is represented as being 5 units in length and path B as 15 units in length. The dynamics of the two situations are represented in Fig. 2. It is to be observed that if Path S'→G' were suitably curved and shifted about in a manner such that point S' would coincide with point S, and point G' would coincide with point G, the combination of paths thus resulting would constitute a habit-family hierarchy (see text, pp. 274–275).

jointly become longer and longer. Suppose, for example, that the shorter path in the above example increases from 5 to 20 units in length and the longer one from 15 to 30. The difference in excitatory strength produced by the difference of 10 units in length would (Table 1) shrink from 2.73 points to 1.00 point. When the shorter path is further increased to 40 units in length, a second path 10 units longer makes a difference in excitatory tendency of only .56 of a point, and so on. Conversely, by shortening both paths a constant difference in their lengths will give rise to greater and greater differences in their excitatory potentialities. This principle has important implications which will be taken up below. So far as the author is aware, these corollaries have not been subjected to experimental test; they offer a ready means of further testing the hypothesis.

to locomotion in space. When non-spatial sequences are involved, the distance from the goal should be thought of as time (8) or, possibly, energy consumption. Spatial examples have certain advantages for exposition because of the ease of diagrammatic representation, but it is believed that they actually present greater theoretical difficulties than do non-spatial sequences.

II

At this point we may conveniently consider the bearing of the present system of hypotheses on the interesting problem of the allegedly deleterious influence of increasing the excita-
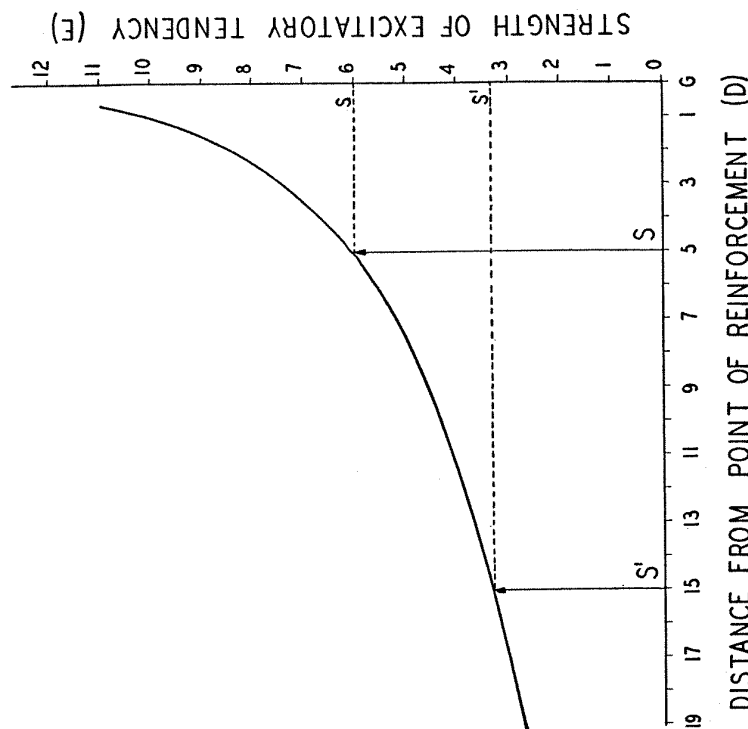

Fig. 2. Diagrammatic representation of the hypothetical dynamics of the two paths to goal G shown in Fig. 1. The curve represents the excitatory potentialities of the visual stimulus of the goal complex at all distances from 1 to 19 units according to the equation,

$$E = a - b \log D.$$

The strengths at 5 and 15 units distant have been projected upon the scale at the right. It is evident to inspection that S has nearly twice as great excitatory potentiality as has S'.

tory strength of the lure upon the solution of certain types of problems (12). Let us assume once more that a naïve organism has two distinct habit sequences corresponding to

paths A and B of Fig. 1, each leading to the same goal or point of reinforcement. In this case, however, there is supposed to be an effective physical barrier at the beginning
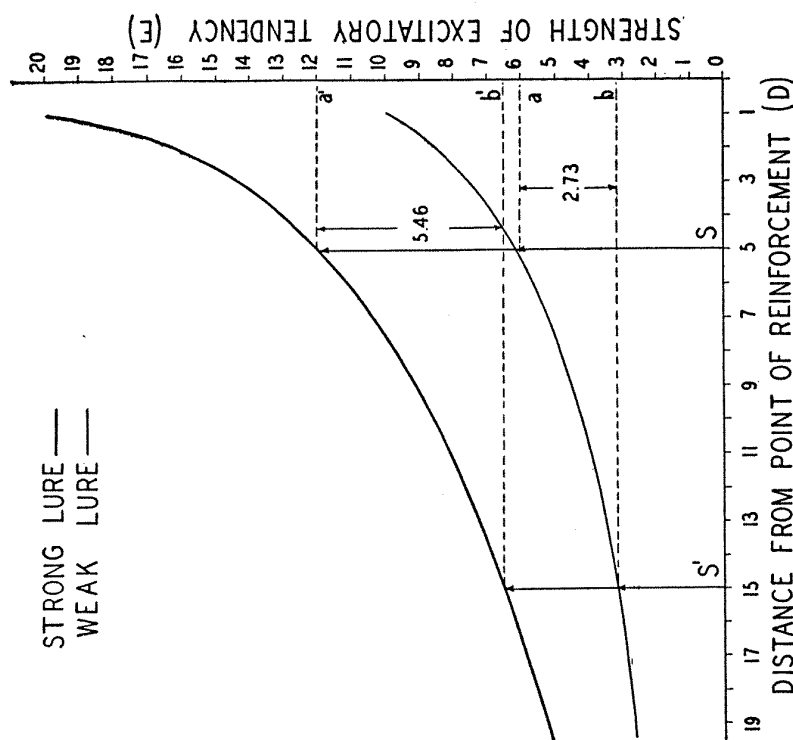

STRONG LURE ——
WEAK LURE ——

Fig. 3. Diagrammatic representation of the dynamics of doubling the strength of the lure at 1 unit from G, Fig. 1. The two curves represent the hypothetical strength of excitatory tendency of the optical image of the respective lures for all distances between 1 and 19 units of remoteness from G. The equation for the weak lure (lower curve) is

$$E_w = 10 - 4 \log_6 D,$$

whereas that for the strong lure (upper curve) is

$$E_s = 20 - 8 \log_6 D.$$

The excitatory potentialities (E) at S and S' are projected upon the scale at the right. It is evident to inspection that the difference between S and S' is greatly increased by adding to the strength of the lure.

of the shorter path (A) so that solution of the problem consists in traversing the longer path (B). Let us assume, further, that in one situation the excitatory potentialities of the goal one unit distant have been doubled, say, by increasing the quantity of the reward. Presumably the remainder of the gradient would rise proportionately.[9] On this assumption the resulting situation may be represented with precision by plotting the values of Table 1 on two different ordinate scales, one twice as great as the other. Graphs produced in this manner are presented in Fig. 3. An inspection of this figure shows that the difference between the excitatory tendency to take the respective paths under the weak lure is only 2.73 units ($6 - 3.27$), whereas this difference rises to 5.46 units ($12 - 6.54$) in the presence of the strong lure.

Here it becomes necessary to introduce a second hypothesis, which is that a strong excitatory tendency diminishes the same number of units in excitatory strength from a given amount of frustration[10] as does a weak excitatory tendency.[11] Applying this principle to the suppositious case before us, it follows that it would take as long (or as many futile at-

[9] It would seem that this assumption could be tested experimentally without great difficulty. Perhaps the technique of Muenzinger, Dove, and Bernstone (18) might here be employed to advantage.

[10] The frustration hypothesis is to the effect that whenever an excitatory tendency is prevented, for any reason, from evoking its accustomed reaction, a state ensues substantially like the experimental extinction or internal inhibition long known to be characteristic of conditioned reactions (19). This hypothesis has been elaborated to a certain extent in (9, p. 139). A special case of wide occurrence not yet elaborated is where an organism is learning by simple trial-and-error to choose consistently one of a pair of alternative reactions such as the two turns of a single T maze. According to the frustration hypothesis, whichever alternative is chosen, the excitatory tendencies otherwise leading to the other must be frustrated and so generate a certain amount of inhibition. This would be true even though the alternative chosen should be correct and receive reinforcement. According to this hypothesis, a constant number of reinforcements to one of two alternative reactions, with the stimulus otherwise evoking the other reaction present, should add less to the extinctive resistance of the reinforced reaction than they would if the stimulus normally evoking the weaker alternative were absent. This deduction suggests an experimental test of the hypothesis.

[11] No really satisfactory evidence on this point is available. The present situation makes the experimental determination of this relationship especially urgent. Actually, the following deductions would still hold even if the absolute diminution in excitatory tendency should be faster in initially strong than in initially weak excitatory tendencies, provided that the relative *rate* in the former were slower.

tempts on the part of the organism) for the presence of the barrier to path A to diminish by 4 units an excitatory tendency initially with a strength of 12 units (strong lure) as one initially with a strength of only 6 units (weak lure). In the former case the problem would not have been solved in this length of time, since the obstructed path would still have a strength of 8 units ($12 - 4$) which would be well above the strength of the long but unobstructed path (6.54). In the case of the weak lure, on the other hand, the obstructed path would have, after the same period (or amount) of frustration, a strength of only 2 units ($6 - 4$). Since this is well below the strength of the long path (3.27) the latter would presumably have been chosen some time before the end of the period in question, solving the problem of the organism. Thus the paradox of an increased lure interfering with, rather than facilitating, the solution of certain problems apparently finds an explanation.[12]

[12] The preceding demonstration holds, of course, only for the particular supposititious values employed. The general theorem may be derived by ordinary mathematical procedures, as follows:

Let the initial excitatory tendency in the case of the weak lure be:

$$E_w = a - b \log D$$

and in the case of the strong lure be:

$$E_s = K(a - b \log D) \text{ where } K > 1.$$

Then, after a certain number (n) of frustrations, these excitatory tendencies will have reduced to:

$$E_w(n) = a - b \log D - f(n),$$
$$E_s(n) = K(a - b \log D) - f(n),$$

where $f(n)$ is an increasing function of n.

Now let path A, of length D_1, be obstructed; and let path B, of length $D_2 (D_2 > D_1)$, be unobstructed. Then, the number (n_w) of frustrations required to make the excitatory tendency along A less than that along B is:

$$n_w = \left[f^{-1}\left(b \log \frac{D_2}{D_1} \right) + 1 \right],$$

where f^{-1} is the inverse function of f, and the brackets mean 'largest integer not greater than.'

Likewise, the corresponding number (n_s) of frustrations required in the case of the strong lure is:

$$n_s = \left[f^{-1}\left(Kb \log \frac{D_2}{D_1} \right) + 1 \right],$$

but

$$K > 1,$$

In this same general category probably will be found to fall phenomena resulting from increasing the craving of the organism. If the lure is food, general observation as well as experimental results (10) indicates that increased hunger (food privation) heightens the excitatory gradient very much as does increasing the quantity of the lure. By the same reasoning as that of the immediately preceding paragraph, increasing hunger with a constant food lure should impede the solution of this type of problem very much as would increasing the lure with constant craving or drive.

Another situation closely paralleling the influence of increasing the excitatory value of the lure while leaving the lure itself objectively unchanged ought, according to the present set of hypotheses, to result from progressively shortening both paths at the same time keeping the absolute difference between them the same. A special case of this kind studied by Lewin (13, 14) is presented by the two situations shown in Fig. 4. Suppose that direct path A is approximately 5 units long. Then indirect path B will be about 29 units long. By Table I (leaving out of consideration for the moment the directional factor), the excitatory tendency at point S to take these respective paths should be 6.00 units and 1.63 units respectively; this leaves a difference of 4.37 units of excitatory potentiality to be extinguished by the barrier in path A before path B can become active. In the case of the right-hand figure, direct path A' has a length of 12 units and indirect path B' has a length of approximately 35

hence

$$Kb \log \frac{D_2}{D_1} > b \log \frac{D_2}{D_1},$$

hence

$$f^{-1}\left(Kb \log \frac{D_2}{D_1}\right) + 1 > f^{-1}\left(b \log \frac{D_2}{D_1}\right) + 1,$$

hence

$$n_s \geq n_w,$$

i.e., the number of frustrations on the shorter path necessary to produce the successful (long-path) reaction will be greater with the strong than with the weak lure provided the difference between the lengths of the two paths is great enough to be equivalent to at least a single attempt. The outcome of the two remaining cases in this section may be derived in an analogous manner, but space in which to do this is not available in the present paper.

units. By Table I, the excitatory tendency at choice point S to take paths A' and B' to G' should be 3.82 units and 1.16 units respectively, with a difference of only 2.66 units to be worn down by the barrier frustration before path B' can become active. Assuming, again, something like a constant rate of extinction from the frustration produced by the barriers to the direct paths, goal G' should be easier of attainment than goal G, in some inverse function of the proportion of 4.37 to 2.66.

To sum up, then, we arrive at the tentative conclusion that three different changes in a problem situation are substantially equivalent in their supposed paradoxical ten-
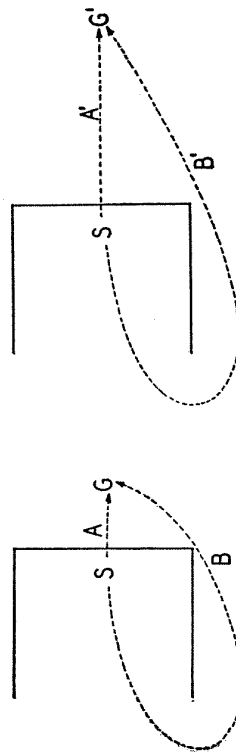

Fig. 4. The figures represent an organism located at S and observing a lure G and G' respectively. There are two potential paths to G, A and B, and two to G', A' and B'. The problem is which goal will be more easily attained, and why. The present system of hypotheses favor G', providing no other lures complicate the situation by leading the organism around the barrier.

dency to impede the solution of problems involving alternative paths to a goal where the shorter, or preferred, path has a barrier: increasing the strength of the lure, increasing the strength of the drive or craving, and decreasing the remoteness of the lure. The last two of the three cases are capable of general demonstration by the ordinary mathematical procedures analogous to those applied to the first (see note 12). This analysis serves to raise not only the experimental question as to the postulates involved, such as the shape of the goal excitatory gradient and the rate of weakening of excitatory tendency under frustration, but, quite as insistently, the question of the reality in detail of the three supposed paradoxical tendencies themselves. It is highly doubtful whether

either set of phenomena is as simple as the above treatment would imply.

III

Consider once more the situation represented in Figs. 1, 2, 3 and 4, on the former assumption that the short path A has a barrier near its beginning. In the preceding sections we have proceeded on the tacit assumption that the only thing necessary before path B will be chosen is that the strength of the tendency to take path A shall be weakened from s (Fig. 2) to a value somewhat below s'.[13] It now becomes necessary to point out in the competitional situations considered, the operation of a factor—temporarily ignored—which is variously called the irradiation of inhibition or the generalization of extinction (frustration). From the present point of view this principle is that every extinguished reaction has a tendency to inhibit (weaken) all other reaction tendencies, the stimuli of which impinge on the organism closely following the extinction process.[14] It follows from this principle that when A is being extinguished from s to s', B has been undergoing a secondary extinction which will depress its excitatory potentiality appreciably below s', say to s''. As a result the extinction of A must go on below s' to s''. But by this time path B will have suffered further secondary weakening.

This raises the question as to how far the extinction of A must go before it will actually reach a level beneath that of B. So far as the present writer is aware, this problem has not

[13] It is believed that the perseveration principle will insure that the effect of trials on A will actually persist somewhat beyond the point at which the strength of A has become equal to that of B, which would bring it about that B would have an appreciably higher excitatory potentiality than A when the first trial on the former would take place. Otherwise spontaneous recovery of A would instantly recall the subject to that reaction again, and the organism would vibrate frantically from one to the other like too sensitive thermostats which sometimes oscillate continuously between turning the heat on and off.

[14] That this principle is operative in the free and dynamic behavior of rats in the simple trial-and-error situation has been abundantly demonstrated in unpublished work of R. E. P. Youtz (23) and of D. G. Ellson (4). Mitrano (17) has shown it to be operative with feeble-minded children. In the situations so far investigated the generalized weakening effect seems to be around 50 per cent.

yet been investigated experimentally, though it should be a relatively straightforward matter on the conditioned-reflex level. In advance of the possession of actual knowledge of this relationship, however, it is easy to tell roughly what should result under various suppositions. It is clear, for example, that, other things equal, the shorter the distance between s and s', the less inhibition there will be to irradiate and the shorter the distance below s' at which reaction B will take place. Secondly, the smaller the proportion of inhibition which irradiates, the shorter will be the distance below s' at which B will take place.

There remains to be considered the special case of the possibility that A could be extinguished to a functional zero, yet reaction B never take place at all. It seems inevitable that this would occur under certain conditions: Suppose that 50 per cent of the inhibition of A irradiates to B and that at the outset A stands at 6.00 and B at 2.00. When A has reached zero, B would have suffered 3 units of inhibition; this would place it at -1.00, i.e., one unit below zero (19). In this case striving for this particular goal (reinforcing state of affairs) would cease altogether, at least for a time. If the time available for solution is limited it follows that the organism may completely fail to solve its problem.

IV

At this point we must return to the consideration of a principle which was latent in the situation considered in Section II above, especially as represented in Fig. 4. This is the influence (in the case of very naïve subjects) of the angle which any path except the one leading directly to the goal makes with the latter. The consideration of this problem may be facilitated by referring to the diagram shown as Fig. 5. The question is: What bearing does the size of the angles of paths B, C, D, E, and F with path A at point S have on the relative strengths of excitatory tendency of the paths in question?

The hypothesis here tentatively put forward is that in the random locomotion, and even hand movements, of the

organism (7) [15] throughout its previous existence, a movement terminating at G but initiated in the direction of path B at point S will, *on the average*, require the traversing of a distance something like that of path B. The same is assumed of the initial angles of paths C, D, E, and F. As a result, the principle of the goal excitatory gradient demands that the greater the angle of the path at point S, the weaker will be
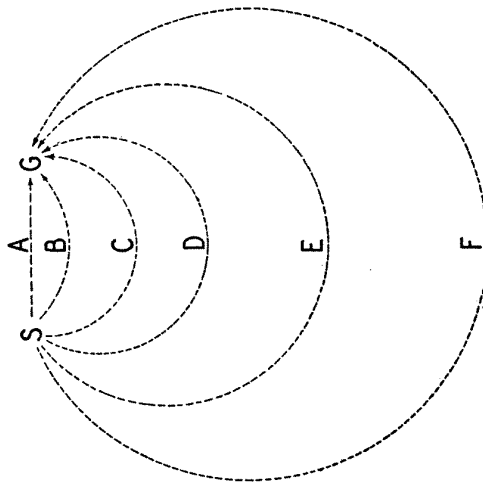
FIG. 5. Diagrammatic representation of what are conceived to be approximations to typical *mean* pathways by which the organism has passed by means of random locomotion in its previous history from one point (S) to another point (G). It will be noted that the angle which each path makes with path A at point S is a direct function of the length of the path—the larger the angle, the longer the path. It is understood that a substantially similar family of potential circular paths lies on the other side of direct path A.

the tendency to take the particular path. Thus path F as represented in Fig. 5 is about eight times as long as path A. Assuming path A to be 5 units in length, and path F to be 40, the respective excitatory tendencies will be, according to Table I, in the proportion of 6.00 to .83; *i.e.*, the strength of the tendency to start for G in the initial direction of path A

[16] Halverson's analysis of hand and arm movements in the act of prehension among young children (7, 175) shows that the pathway of the arm and hand in reaching for a small cube at 28 weeks is a decidedly circuitous one but, with increasing age (52 weeks) its course becomes practically a straight line.

would be over seven times as great as the tendency to set out in the initial direction of path F. This hierarchy of excitatory tendencies based on the initial angles of locomotion toward a goal thus appears to be a special case of the habit-family hierarchy (9).

It follows at once from these considerations that in certain situations the difference between the excitatory tendencies of a naive organism to take the initial steps of two paths may
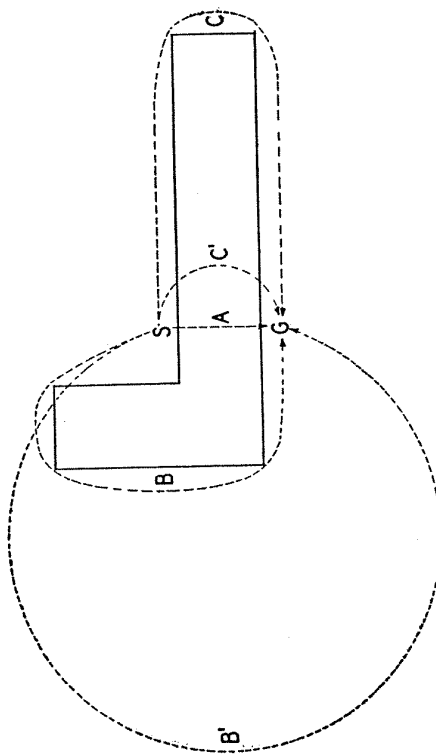
FIG. 6. Diagrammatic representation of an experimental situation designed to show the influence of the angle of the initial portion of a path upon the strength of its tendency. S is the organism, G is the lure. The right-angled figure in bold outline represents a barrier. Path B is actually shorter than path C, yet it is assumed that the naive organism will choose C rather than B because at the outset C is the beginning of paths which, on the average, in the past have been approximately of the length of path C', and path B at the outset is the same as paths which in the past have averaged in length something like that of B'.

be considerably greater than the difference between the actual lengths of the two paths would warrant. This may be illustrated by the situation represented in Fig. 6. Let us suppose that the organism at S has the choice of paths B and C around the barrier to G. Path C is 26 units long, whereas path B is only 22 units long. Nevertheless, a naive organism might be expected to choose path C because the initial angle of path C has presumably in its past history represented, on the average, a path something like C', of only 8 units, whereas the

initial angle of path B has, on the average, represented a path something like that of B', which has a length of 42 units. By Table I, the angles therefore represent naïve excitatory tendencies in the proportion of 4.8 to .7 in favor of path C, i.e., from the point of view of the initial angle of the pathway the excitatory tendency to take path C should be something like seven times as great as that to take path B, even though path B is actually the shorter of the two.

It is to be expected, of course, that the principle of the goal gradient operating directly (e.g., under conditions such that the organism would be forced to take the two paths of Fig. 6, say, in alternation each being followed by reinforcement) would ultimately lead to the giving up of path C and to the preferential choice of path B, because the beginning of path B is actually much nearer the point of reinforcement than is that of path C. Such training accordingly leads to a form of sophistication distinct from the individual use of speech as pure-symbolic acts. Thus we arrive at a kind of operational definition of the term 'sophistication' as here employed.[16]

V

With admirable perspicacity, Lewin has called attention to a very important series of problems involving conflicting tendencies to action, i.e., situations which involve simultaneous impulses both to approach and to retreat. Such conflicts of excitatory tendency he has aptly termed tensions (13). One suppositious case illustrating this point is that of a child

[16] It is possible to state the angle-hierarchy hypothesis (Fig. 5) in exact mathematical terms. On this hypothesis the distance (D) as a function of the angle made by the tangent of the arc at point S with the straight line joining S and G (the distance SG being taken as the unit of measurement) is given by the equation,

$$D = \frac{A}{\sin A}.$$

Since the excitatory tendency (E) at the beginning of a pathway on the goal-gradient hypothesis is a function of D, E may be expressed in terms of the same variables by the equation,

$$E = a + b \log \sin A - b \log A.$$

This, of course, can be regarded as no more than a very rough first approximation to the actual relationship. It is hoped, however, that it may serve as an entering wedge to an investigation of the whole problem. It may at least claim the virtue of being sufficiently definite to be susceptible of experimental verification.

on the shore looking at a toy swan floating in the water. The swan as a stimulus object tends to evoke in the child movements of approach, whereas the water as a stimulus object tends to evoke movements of flight. In this connection Lewin remarks (13, p. 607), "It is important that here, as frequently in such cases, the strength of the field forces which correspond to the negative valence diminishes much more rapidly with increasing spatial distance than do the field forces corresponding to the positive valence. From the direction and strength of the field forces at the various points of the field it can be deduced that the child must move to the point P where equilibrium occurs. (At all other points there exists a resultant which finally leads to P.)" [17]

A still more striking and dramatic form of this general problem is found where the positive and negative stimulus objects are both small and occupy substantially the same point in space. It is assumed, further, that the negative gradient at one unit distant from the object, say, has a greater excitatory potentiality than the positive excitatory gradient at the same distance but, owing to its steeper slope, the negative gradient soon diminishes to a strength of excitation below that of the positive gradient, thereafter remaining permanently in an inferior position. As pointed out above, there is evidence indicating that the positive excitatory gradient possesses a roughly logarithmic shape. Corresponding study of negative excitatory tendencies has not yet been made. Analogy, however, suggests that negative excitatory tendencies may show at least a negative acceleration.[18]

[17] It is to be regretted that Lewin did not work out in detail the deduction which he mentions as a possibility. The above quotation suggests that had this been done his assumptions would turn out to be substantially the same as the relevant assumptions of the present study (see summary) and the form would not be very different from that represented by note 19, below. It is believed that if such practices could become general the genuine disagreements among the psychologies would prove to be much less than now appears from the divergence of their vocabularies.

[18] Here evidently lies a virgin field for investigations both theoretical and experimental. Will, for example, a negative gradient lead to the elimination of blinds in a maze? If so, will the order be backward or forward or in some more complex mode? Ordinary observation suggests the probability that suitable measurements would readily reveal a marked speed-of-locomotion gradient (10); i.e., that the organism would proceed more slowly the more remote it is from the point of retreat.

A situation based on these assumptions is represented diagrammatically in Fig. 7. The positive gradient JJ'

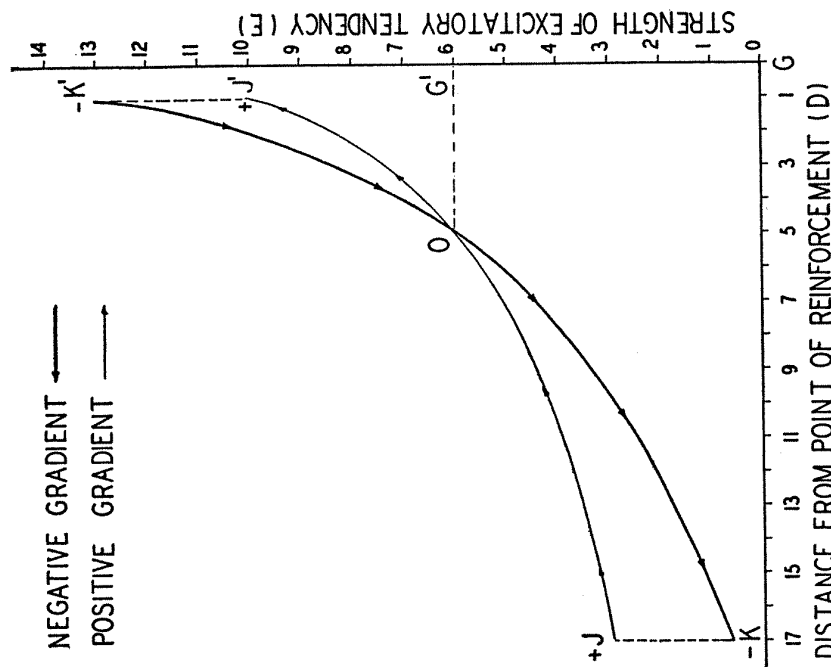NEGATIVE GRADIENT ——→
POSITIVE GRADIENT ——→

FIG. 7. Diagrammatic representation of the functional relationships of two gradients of different sign and different slope. The positive gradient is represented by the equation,

$$E_+ = 10 - 4 \log D,$$

and the negative gradient is drawn according to the equation,

$$E_- = 13 - 7 \log D.$$

The two gradients intersect at $D = 5$ with a common E value of 6 (see note 18).

represents the excitatory potentialities of G as a stimulus to distance receptors leading to locomotor reactions of approach,

whereas the negative gradient KK' represents the excitatory potentialities of G at the various distances to evoke movements of retreat or flight. It may be seen in the figure that at a distance of 17 units the positive gradient dominates; accordingly the organism should move forward with a force corresponding to the difference in excitatory tendency between J and K. However, as locomotion continues it should diminish in speed progressively until point O is reached, at which forward locomotion should cease altogether except for momentum effects. If, on the other hand, the organism should find itself at a distance of one unit from G, locomotion would take place in an opposite direction. The rate of locomotion in this case, as in the one just considered, would presumably correspond to the difference in excitatory tendency existing between K' and J'. This should diminish progressively as point O is approached, at which it should cease just as when the movement was in a forward direction.[19]

[19] The general case takes the following form:

Let the gradient of the positive excitatory tendency (E_+) be represented by the equation,

$$E_+ = a - b \log_f D,$$

and the equation representing the negative excitatory tendency (E_-) be

$$E_- = c - d \log_f D,$$

where

$$c > a$$

and

$$d > b.$$

The point at which the difference between the two gradients will be zero will be where they intersect, i.e., where

$$E_+ - E_- = 0,$$

i.e.,

$$E_+ = E_-.$$

Then,

$$a - b \log_f D = c - d \log_f D,$$

i.e.,

$$a - c = (b - d) \log_f D,$$

hence

$$\log_f D = \frac{a - c}{b - d},$$

therefore

$$D = f^{\frac{a-c}{b-d}},$$

where f is the base of the system of logarithms employed. This is the equation expressing the point at which tensional equilibrium should occur, i.e., the distance from the

As a corollary from the above deduction it is evident that, assuming radial symmetry of the positive and negative gradients about point G, distance OG must be constant in all directions. It follows from this that the locus of tensional equilibrium about point G must be a circle (Fig. 8). Assuming the presence in the situation of stimuli leading to action in the direction of numerous minor goals located according to
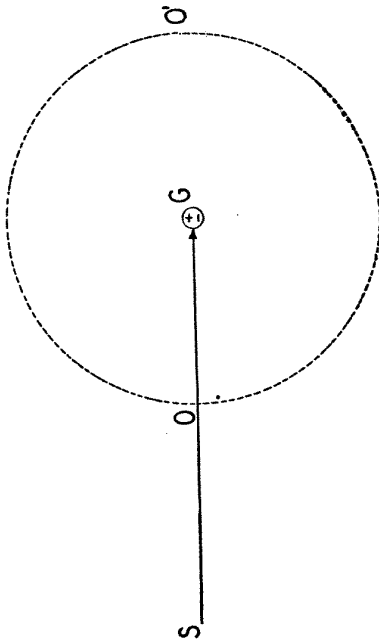

FIG. 8. Diagrammatic representation of the locus of the point of equilibrium when the critical stimulus giving rise to two gradients of opposite sign such as those represented in Fig. 7, emanates from a single point G. So long as the gradients remain constant, distance OG must remain constant, which necessarily makes the locus of equilibrium circular, as shown by the fine dotted line.

chance, it is to be expected that there will occur irregular oscillations not only toward and away from point G, but laterally as well. In the former case it is to be expected that such oscillatory tendencies will be opposed by the summation such oscillatory tendencies will be opposed by the summation of gradient tending to the locus O, whereas those tangent to this locus will meet with no such opposition. It follows that

object precipitating the ambivalent situation at which the organism will come to rest. Thus in the gradients represented in Fig. 7,

$$a = 10,\ b = 4,\ c = 13,\ d = 7,$$

and the base of the log system is 5.

Substituting these values in the above equation, we have

$$D = 5^{\frac{10-13}{4-7}} = 5^{\frac{-3}{-3}} = 5,$$

i.e., the gradients intersect at a distance of 5 units from the goal, as appears in Fig. 7.

the lateral movements will display a greater amplitude than those perpendicular to locus O. Accordingly the naïve organism if given time enough may be expected to make completely circular movements about point G, though these will be complicated by ‧ approaches, retreats, and much retracting.

As a second corollary it should be pointed out that even without the presence of distracting stimuli the organism at point O (Fig. 8) probably will not remain absolutely passive, i.e., in a state indistinguishable from mere idleness character- istic of no effective stimulation whatever. On the contrary, it is reasonable to expect that, since there is a great deal more stimulation of a dynamic sort present in conflict situations than in a case where inaction is due to the lack of effective stimulation, the muscles of the organism should show a con- spicuous excess of tonicity in the former case as contrasted with the latter. Muscular tonicity may accordingly offer a convenient supplementary means for gauging the extent of this type of conflict.

The deductions of the preceding paragraph tacitly assume a constancy of both the positive and the negative gradients while the organism remains suspended, as it were, between the antagonistic difference gradients to action which the situation evokes within him. There is strong reason to doubt such a constancy. In addition to the very general empirical principle that in organisms tendencies to action are always in a state of flux, there is in this case the specific expectation that, assuming an objectively constant G as a stimulus object, both gradients should diminish in height, presumably through- out their whole effective length. This follows from the frustration hypothesis already considered (p. 278).[20]

A test of the above hypothesis which immediately suggests itself could be arranged as follows: Place two objects, one with

[20] There is abundant evidence to support this hypothesis where the frustration is produced by an external barrier of some sort. But the writer is unaware of any evidence bearing directly on situations like the one here under consideration in which the same object gives rise to what is, in some sense, an internal conflict of two gradients of opposite sign. Deductions based on this more inclusive interpretation of the frustrational hypothesis must accordingly be regarded with more than usual caution until definite evidence on the point is available.

a negative value and the other with a positive value, one above the other so as to produce the conflicting gradients shown in Fig. 7. Determine the initial potentialities of each object separately at the outset by measuring the speed-of-locomotion of given organisms in a positive or negative direction at representative distances from G. Then place both objects together and let the organism remain at point O (Figs. 7 and 8) in the state of tensional equilibrium for about as long as this will persist. At once repeat, with separate subgroups, the test of the excitatory potentiality of each object singly, as at the beginning of the experiment. The frustration hypothesis demands that in this latter case the rate of locomotion toward the positive object and that away from the negative object shall both be diminished.

Two corollaries flow from this deduction. The first, based on the principle of spontaneous recovery (19), is that after a lapse of time (possibly within the range of one or two hours) the speed of locomotion in both cases should have increased to a very considerable extent, though the recovery should not be perfect. A second corollary, again based on principles derived from conditioned-reflex experiments, is that an organism placed in the state of tensional equilibrium a second time will show both a more rapid inhibition of the respective excitatory tendencies than it did on the first occasion and also a slower and more imperfect spontaneous recovery.

One of the most obvious questions which arise from the consideration of conflict situation when the latter is viewed in the light of the frustration hypothesis, is the effect of the continuation of the state of tensional equilibrium upon the distance (OG) from the goal objects (G), which the organism at first maintains. It is obvious, of course, that if both gradients should decline simultaneously and at a certain rate the distance OG would remain constant. When we consider the number of other possibilities of variation, however, the chance that this particular set of changes should take place is exceedingly improbable. It accordingly seems

almost certain that the distance OG will undergo a progressive change in length with the duration of the conflict or tension.

Whether, upon the whole, the change will be to diminish or to increase distance OG can only be predicted when we know more concerning the resistance of positive and negative, steep and gentle, gradients. It is evident that here is a rich and almost virgin field for investigation, both with animals and with young children. One apparently relevant principle we have, however, thanks to the work on conditioned reactions. Other things equal, that gradient which is the oldest should resist frustration the best. This factor should tend to lengthen or shorten distance OG according to whether the older gradient is the negative or positive one respectively.

In this connection there should also be considered the matter of progressive changes in the drive. As already pointed out above, an increase in a drive such as hunger presumably increases both the height and the slope of the positive gradient. This increase should obviously diminish the distance OG and reduce the radius of the circular locus of tensional equilibrium.

Finally, the significance of this whole psychology of conflicting excitatory gradients as a challenge to experimental psychology should be emphasized. Its importance for psychopathology, particularly the conflicts of the positive sexual gradient and the specific negative gradient based on social tabus and prudential considerations, has been emphasized in the clinical field by the psychiatrists for many years (5). Coitus interruptus and what, in current slang, is called 'petting,' are cases in point. The bad repute of these practices in mental hygiene emphasizes the possible significance of conflicting gradients in either time or space. In this connection it is to be noted that the mutual checking of two impulses as represented in Fig. 7 might be expected to produce something like twice as great an amount of internal inhibition as would be the case where one tendency was completely over-ridden by the other because in the one case two excitatory tendencies are frustrated, whereas in the other, only

one is thwarted. It would not be surprising if this mechanism should be found to play an important role in the so-called psychogenic disorders.

SUMMARY

The preceding analysis of certain problems in the behavior dynamics of naïve organisms has proceeded mainly on the following assumptions:

1. That simultaneously occurring stimuli and reactions (both of approach and flight) tend to be associated more strongly the closer they are to a reinforcing state of affairs, the diminution in excitatory tendency with remoteness from the point of reinforcement proceeding with a negative acceleration.

2. That situations may arise where stimuli originating in substantially the same point in space will give rise in a given organism to incompatible excitatory tendencies, notably the opposing tendencies to acts of approach and of flight.

3. That the flight gradient may, at least under certain circumstances, be both steeper and higher near the stimulus object.

4. That an organism in which positive and negative excitatory tendencies are active behaves at any given instant according to the algebraic sum of such tendencies.

5. That the positive gradient of excitation grows both higher and steeper with the increase in the drive, e.g., food privation.

6. That, other things equal, this gradient of excitation grows higher and steeper with the increase in the amount of reward.

7. That a strong functional excitatory tendency is weakened after a given amount of frustration by a smaller proportion of its original strength than is a weak functional excitatory tendency.

8. That spontaneous recovery from extinction effects will occur but this will not be complete.

9. That a second extinction will occur more rapidly than the first and its spontaneous recovery will be less complete.

10. That in the history of organisms it is a fact that upon the whole the larger the angle the beginning of a pathway makes with the straight line leading to the object, the longer the path to the object.

11. That the prevention by any circumstance of the reaction normally evokable by any stimulus or stimulus component results, other things equal, in weakening (extinguishing) the particular excitatory potentiality of such stimulus or stimulus component.

12. That the older an excitatory tendency, the more resistant to the extinction from frustration.

13. That a considerable weakening of a given excitatory tendency appears at once following the extinction of another excitatory tendency.

From these assumptions in the main the following conclusions have been drawn concerning the behavior of naïve organisms, unmentioned factors being assumed as equal:

1. In a problem situation such as shown in Fig. 4, a naïve organism will have more difficulty in solving the problem where the lure is large than where it is small.

2. There will be more difficulty where the lure is close to the barrier than where it is farther away.

3. There will be more difficulty where the drive is strong than where it is weak.

4. The functional excitatory tendency of weak alternatives in a habit-family hierarchy may be depressed to 'below zero' by generalized extinction effects from the frustration of a stronger member of the hierarchy.

5. This depression below zero is the more likely to occur, the greater the percentage of the extinction effects in the strong tendency which are generalized to the weak tendency.

6. The depression below zero is more likely to occur the weaker the absolute strength of the weak tendency.

7. The depression below zero is more likely to occur the greater the difference between the competing excitatory tendencies.

8. For a naïve organism, the larger the angle which the beginning of a pathway to a point makes with the straight line

to the point, the weaker the excitatory tendency to execute the acts which constitute taking the divergent path.

9. Naïve organisms through the misleading action of the angular hierarchy will, under certain circumstances, choose the longer of two paths.

10. With sophistication of the organism consisting of actually traversing both of such paths as considered in (9), the tendency to take the longer path will gradually give place to a degree of preference to be expected on the basis of the uncomplicated goal excitatory gradient hypothesis.

11. Under the conditions of assumptions 2 and 3, there will be a distance (D) from the ambivalent stimulus object at which the two opposed excitatory tendencies of a tensional situation will be equal.

12. Under the conditions of (11), the organism will tend to move toward this point of equal excitatory tendency.

13. Under the conditions of (11), assuming an unchanging stimulus object and the presence of minor lures of appreciable potentiality distributed in a chance manner throughout the neighborhood, the organism will tend to take a roughly circular course around the ambivalent stimulus object.

14. These circular movements will be very irregular in rate and extent, and be characterized by much retracing.

15. There will be some irregular oscillation toward and away from the ambivalent stimulus.

16. The movements of (15) will, upon the whole, be less in extent than will those of (14).

17. Where cases of tensional equilibrium persist for an appreciable period, both excitatory tendencies arising from an ambivalent stimulus object will be weakened progressively.

18. With the passage of time there will be a progressive recovery from this weakening.

19. Recovery from this frustrational weakening will never be complete.

20. A second period of tensional equilibrium will produce an extinctive weakening more rapid than the first.

21. The second recovery will be less rapid than the first.

22. The second recovery will be less complete than the first.

23. As the state of tensional equilibrium continues there will usually occur a progressive shift in the distance the organism maintains from the stimulus object.

24. This shift will tend to be in the direction of the older of the two opposed excitatory tendencies.

25. The secondary frustration effects will be greater in cases of tensional equilibrium than in cases where one tendency is strong enough to over-ride the other.

Of the forty or so propositions assembled above from the preceding theoretical analysis, scarcely one is yet established on a secure quantitative experimental basis. Perhaps the chief outcome of the analysis is the sharp realization of our profound ignorance concerning the essential principles operative in such relatively simple dynamical situations. However, admissions of ignorance, while momentarily depressing, are likely to be wholesome. Each recognized item of basic ignorance constitutes a challenge to a critical experimental determination. Thus a resolute attempt at theoretical integration naturally leads the way to a systematically coördinated program of investigation. In the present case, such an integrated program of research appears to fall into three portions or phases.

The first phase consists of the direct experimental determination of the principles or laws, now almost entirely in the state of hypothesis or guesswork, which are suggested by the first of the above lists of propositions. These determinations should be quantitative in nature and so designed as to yield functional curves of basic relationships. Equations fitted to such data become the postulates of the system.

The second phase is logical and mathematical. It consists of deriving by means of mathematics, and perhaps of symbolic logic, the behavioral implications of the postulates yielded by the first movement when acting in the greatest possible variety of conditions. The substance of note 12 gives an indication of the general nature of this theoretical portion of the program.

The third phase consists of the systematic experimental verification of the behavioral expectations resulting from the mathematical activities of the second movement. Whenever this systematic verifying procedure reveals disagreements, work on the first movement is resumed in an effort to rectify presumptive defects in the postulate determinations. Following such new postulate determinations, new implications are drawn, new verification experiments are set up, and so on in continuously recurring cycles until disagreements fail to manifest themselves, if such a time ever comes.

Such a self-conscious scientific procedure may with some propriety be called 'logical empiricism.' It begins with an empirical determination of its postulates and ends with an empirical check on the validity of its theorems; between the two lies the integrating symbolic structure of logic and mathematics.

REFERENCES

1. ANDERSON, A., Runway time and the goal gradient, J. Exper. Psychol., 1933, 16, 423-428.
2. BRANDT, H., The spread of the influence of reward to bonds remote in sequence and time, Arch. Psychol., 1935, 180, pp. 40, 41.
3. BRUCE, R. H., An experimental investigation of the thirst drive in rats, with especial reference to the goal gradient hypothesis, J. Gen. Psychol., 1937, 17, 49-62.
4. ELLSON, D. G., Quantitative studies of the interaction of simple habits; I. Recovery from specific and generalized effects of extinction (to be published).
5. FREUD, S., Selected papers on hysteria and other psychoneuroses. (Trans. by A. A. Brice), pp. 141-142, Journal of Nervous and Mental Disease Publishing Company, 1909.
6. GUILFORD, J. P., A generalized psychophysical law, PSYCHOL. REV., 1932, 39, 73-85.
7. HALVERSON, H. M., An experimental study of prehension in infants by means of systematic cinema records, Genet. Psychol. Monog., 1931, 10, 107-286.
8. HULL, C. L., The goal gradient hypothesis and maze learning, PSYCHOL. REV., 1932, 30, 25-43.
9. ——, The concept of the habit-family hierarchy and maze learning, PSYCHOL. REV., 1934, 41, Part I, 33-52; Part II, 134-152.
10. ——, The rat's speed-of-locomotion gradient in the approach to food, J. Comp. Psychol., 1934, 17, 393-422.
11. ——, Mind, mechanism, and adaptive behavior, PSYCHOL. REV., 1937, 44, 1-32.
12. KÖHLER, W., The mentality of apes, New York: Harcourt, Brace and Co., 1925.
13. LEWIN, K., Environmental forces, in 'A Handbook of Child Psychology' (edited by C. Murchison, 2nd edition revised), Worcester: Clark Univ. Press, 1933, Chapter 14, pp. 590-625.

14. ——, A dynamic theory of personality (Trans. by D. K. Adams and K. E. Zener), New York: McGraw-Hill Book Co, 1935, pp. ix + 286.
15. ——, Principles of topological psychology (Trans. by F. and G. M. Heider), New York: McGraw-Hill Book Company, 1936, pp. xv + 231.
16. MILLER, N. E. AND W. R. MILES, Effect of caffeine on the running speed of hungry, satiated, and frustrated rats, J. Comp. Psychol., 1935, 20, 397-412.
17. MITRANO, A. J., A preliminary investigation of human motivation by conditioned reaction methods, Ph.D. thesis, 1937, Yale University Library.
18. MUENZINGER, K. F., DOVE, C. C. AND BERNSTONE, A. H., Serial learning: II. The bi-directional goal gradient in the endless maze, J. Genet. Psychol., 1937, 50, 229-241.
19. PAVLOV, I. P., Conditioned reflexes (Trans. by G. V. Anrep), Oxford: Oxford Univ. Press, 1927.
20. SNYGG, D., Mazes in which rats take the longer path to food, J. Psychol., 1936, 1, 153-166.
21. THORNDIKE, E. L., An experimental study of rewards, Contributions to Education (see pp. 54-55), No. 580, New York: Bureau of Publications, Columbia University, 1933.
22. YOSHIOKA, J. G., Weber's law in the discrimination of maze distance by the white rat, Univ. Calif. Publications in Psychol., 1929, 4, 155-184.
23. YOURZ, R. E. P., The trial and error reaction as a type of conditioned response, Ph.D. thesis, 1937, Yale University Library.