Singularity Resolution in Loop Quantum Cosmology: A Brief Overview

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A brief summary of the singularity resolution in loop quantum cosmology of homogeneous isotropic models is presented.[1] The article is addressed to relativists who do not specialize in quantum gravity. For further details, and answers to more technical asked questions, the reader is directed to the original papers and to more comprehensive recent reviews.

I. ISSUE OF THE BEGINNING AND THE END

Over the history of mankind, cosmological paradigms have evolved in interesting ways. It is illuminating to begin with a long range historical perspective by recalling paradigms that seemed obvious and most natural for centuries only to be superseded by radical shifts.

Treatise on Time, the Beginning and the End date back at least twenty five centuries. Does the flow of time have an objective, universal meaning beyond human perception? Or, is it fundamentally only a convenient, and perhaps merely psychological, notion? Did the physical universe have a finite beginning or has it been evolving eternally? Leading thinkers across cultures meditated on these issues and arrived at definite but strikingly different answers. For example, in the sixth century BCE, Gautama Buddha taught that 'a period of time' is a purely conventional notion, time and space exist only in relation to our experience, and the universe is eternal. In the Christian thought, by contrast, the universe had a finite beginning and there was debate whether time represents 'movement' of bodies or if it flows only in the soul. In the fourth century CE, St. Augustine held that time itself started with the world.

Founding fathers of modern Science from Galileo to Newton continued to accept that God created the universe. Nonetheless, their work led to a radical change of paradigm. Before Newton, boundaries between the absolute and the relative, the true and the apparent and the mathematical and the common were blurry. Newton rescued time from the psychological *and* the material world and made it objective and absolute. It now ran uniformly from the infinite past to the infinite future. This paradigm became a dogma over centuries. Philosophers often used it to argue that the universe itself *had* to be eternal. For, as Immanuel Kant emphasized, otherwise one could ask "what was there before?"

General relativity toppled this Newtonian paradigm in one fell swoop. Now the gravitational field is encoded in space-time geometry. Since geometry is a dynamical, physical

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[1] To appear in the proceedings of the conference NEB-XIII: *Recent Developments in Gravity*, held at Thessaloniki in June 2008. Dedicated to the memory of Basilis Xanthopoulos, a founder of this series of conferences.

entity, it is now perfectly feasible for the universe to have had a finite beginning —the big-bang— at which not only matter but *space-time itself* is born. If space is compact, matter *as well as space-time* end in the big-crunch singularity. In this respect, general relativity took us back to St. Augustine's paradigm but in a detailed, specific and mathematically precise form. In semi-popular articles and radio shows, relativists now like to emphasize that the question "what was there before?" is rendered meaningless because the notions of 'before' requires a pre-existing space-time geometry. We now have a new paradigm, a new dogma: In the Beginning there was the Big Bang.

However, the very fusion of gravity with geometry now gives rise to a new tension. In Newtonian (or Minkowskian) physics, a given physical field could become singular at a space-time point. This generally implied that the field could not be unambiguously evolved to the future of that point. However, this singularity had no effect on the global arena. Since the space-time geometry is unaffected by matter, it remains intact. Other fields could be evolved indefinitely. Trouble was limited to the one field which became ill behaved. However, because gravity is geometry in general relativity, when the gravitational field becomes singular, the continuum tares and the space-time itself ends. There is no more an arena for other fields to live in. All of physics, as we know it, comes to an abrupt halt. Physical observables associated with both matter and geometry simply diverge signalling a fundamental flaw in our description of Nature. *This is the new quandary.*

When faced with deep quandaries, one has to carefully analyze the reasoning that led to the impasse. Typically the reasoning is flawed, possibly for subtle reasons. In the present case the culprit is the premise that general relativity —with its representation of space-time as a smooth continuum— provides an accurate description of Nature arbitrarily close to the singularity. For, general relativity completely ignores quantum physics and over the last century we have learned that quantum effects become important in the physics of the small. They should in fact be dominant in parts of the universe where matter densities become enormous. Thus *the occurrence of the big bang and other singularities are predictions of general relativity, precisely in a regime where it is inapplicable!* Classical physics of general relativity does come to a halt at the big-bang and the big crunch. But this is not an indication of what really happens because use of general relativity near singularities is an extrapolation which has no physical justification whatsoever. We need a theory that incorporates not only the dynamical nature of geometry but also the ramifications of quantum physics. We need a quantum theory of gravity, a new paradigm.

These considerations suggest that *singularities of general relativity are perhaps the most promising gates to physics beyond Einstein.* They provide a fertile conceptual and technical ground in our search of the new paradigm. Consider some of the deepest conceptual questions we face today: the issue of the Beginning and the end End, the arrow of time, and the puzzle of black hole information loss. Their resolutions hinge on the true nature of singularities. In my view, considerable amount of contemporary confusion about such questions arises from our explicit or implicit insistence that singularities of general relativity are true boundaries of space-time; that we can trust causal structure all the way to these singularities; that notions such as event horizons are absolute even though changes in the metric in a Planck scale neighborhood of the singularity can move event horizons dramatically or even make them disappear altogether [1].

Over the last 2-3 years several classically singular space-times have been investigated in detail through the lens of loop quantum gravity (LQG). This is a non-perturbative approach to the unification of general relativity and quantum physics in which one takes Einstein's

encoding of gravity into geometry seriously and elevates it to the quantum level [2, 3, 4]. One is thus led to build quantum gravity using *quantum* Riemannian geometry [5, 6, 7, 8]. Both geometry and matter are *dynamical* and described *quantum mechanically* from the start. In particular, then, there is no background space-time.

The kinematical structure of the theory has been firmly established for some years now. There are also several interesting and concrete proposals for dynamics (see, in particular [2, 3, 4, 9]). However, in my view there is still considerable ambiguity and none of the proposals is fully satisfactory. Nonetheless, over the last 2-3 years, considerable progress could be made by restricting oneself to subcases where detailed and explicit analysis is possible. These 'mini' and 'midi' superspaces are well adapted to analyze the deep conceptual tensions discussed above. For, they encompass the most interesting of classically singular space-times —Friedman-Lemaître-Robertson-Walker (FLRW) universes with the big bang singularity and black holes with the Schwarzschild-type singularity— and analyze them in detail using symmetry reduced versions of loop quantum gravity. In all cases studied so far, classical singularities are naturally resolved and *the quantum space-time is vastly larger than what general relativity had us believe.* As a result, there is a new paradigm to analyze the old questions.

In my talk, I focused on cosmological singularities. The material I covered is discussed in greater detail in the original papers [10, 11, 12, 13, 14, 15, 16, 17, 18] and in more comprehensive reviews [19, 20]. Discussion of black hole singularities and the issue of information loss can be found either in the original papers [21, 22, 23, 24] or in a comprehensive, recent review addressed to non-experts [25]. Here, I will confine myself to a sketch of the singularity resolution in loop quantum cosmology (LQC). Section II will provide a conceptual setting, section III will summarize the main results and section IV will present the outlook.

II.   CONCEPTUAL SETTING

I will restrict myself to the simplest cosmological models: FLRW space-times with a massless scalar field. I will consider both the k=0 model and the k=1 model with cosmological constant and comment on the status of more general models. The simplest models are instructive because in classical general relativity all their solutions have a big-bang (and/or big-crunch) singularity. Therefore, a quantum resolution of these singularities is non-trivial. It is not difficult to incorporate additional matter fields and anisotropies.

Figure 1 illustrates classical dynamics for k=0 and k=1 models without a cosmological constant.[1] In the k=0 case, there are two classes of trajectories. In one the universe begins with a big-bang and expands continuously while in the other it starts out with an infinite volume and contracts continuously into a big crunch. In the k=1 case, the universe begins with a big bang, expands to a maximum volume and then undergoes a recollapse to a big crunch singularity. Now, in quantum gravity, one does not have a single space-time in the background but rather a probability amplitude for various space-time geometries. Therefore,

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[1] In the k=0 case, because the universe is infinite and homogeneous, to obtain a well-defined Lagrangian or Hamiltonian formulation, one has to introduce a fiducial box and restrict all integrations to it. In the k=0 figure, the volume v refers to this box. Since Lagrangian and Hamiltonian formulations are stepping stones to the path integral and canonical quantization, every quantum theory requires a fiducial box. Physical results, of course, cannot depend on the choice of this box.
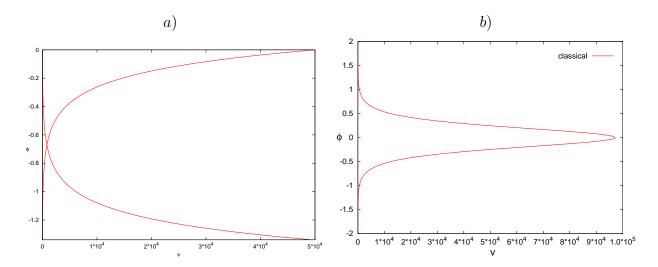
FIG. 1: Dynamics of FLRW universes with zero cosmological constant and a massless scalar field. Classical trajectories are plotted in the $v - \phi$ plane, where v denotes the volume and ϕ the scalar field. *a*) k=0 trajectories. *b*) A k=1 trajectory.

unlike in the classical theory, one cannot readily use, e.g., the proper time along a family of preferred observers as a clock. However, along each dynamical trajectory, the massless scalar field ϕ is monotonic. Therefore, it serves as a good 'clock variable' with respect to which the physical degrees of freedom (the density or the volume, anisotropies and other matter fields, if any) evolve. Note incidentally that, in the k=1 case, volume is double valued and cannot therefore serve as a global clock variable, while the scalar field does fulfill this role. The presence of a clock variable is *not* essential in quantum theory but its availability makes the relational dynamics easier to grasp in familiar terms. It turns out that the massless scalar field is well-suited for this purpose not only in the classical but also in the quantum theory.

If one has a fully developed quantum theory, one can proceed as follows. Choose a classical dynamical trajectory. Since $p_{(\phi)}$ is a constant of motion, this fixes the value of $p_{(\phi)}$, say $p_{(\phi)} = p_{(\phi)}^\star$. Next, choose a point $(v^\star, \phi^\star)$ on any one of these dynamical trajectories where the matter density and space-time curvature is low. This point describes the state of the FLRW universe at a late time where general relativity is expected to be valid. At the corresponding 'internal time' $\phi = \phi^\star$ construct a wave packet which is sharply peaked at $v = v^\star$ and $p_{(\phi)} = p_{(\phi)}^\star$ and evolve it backward and forward in (the scalar field) time. We are then led to two questions:

i) *The infrared issue:* Does the wave packet remain peaked on the classical trajectory in the low curvature regime? Or, do quantum geometry effects accumulate over the cosmological time scales, causing noticeable deviations from classical general relativity? In particular, in the k=1 case, is there a recollapse and if so for large universes does the value $V_{\max}$ of maximum volume agree with that predicted by general relativity [26]?

ii) *The ultraviolet issue:* What is the behavior of the quantum state when the curvature grows and enters the Planck regime? Is the big-bang singularity resolved? Or, do we need to supplement dynamics with a new principle, such as the Hartle-Hawking 'no boundary proposal' [27]? What about the big-crunch? If they are both resolved, what is on the 'other side'? Does the wave packet simply disperse making space-time 'foamy' or is there a large classical universe also on the other side?

By their very construction, perturbative and effective descriptions have no problem with the first requirement. However, physically their implications can not be trusted at the Planck scale and mathematically they generally fail to provide a deterministic evolution across the putative singularity. Since the non-perturbative approaches often start from deeper ideas, it is conceivable that they could lead to new structures at the Planck scale which modify the classical dynamics and resolve the big-bang singularity. But once unleashed, do these new quantum effects naturally 'turn-off' sufficiently fast, away from the Planck regime? The universe has had some *14 billion years* to evolve since the putative big bang and even minutest quantum corrections could accumulate over this huge time period leading to observable departures from dynamics predicted by general relativity. Thus, the challenge to quantum gravity theories is to first create huge quantum effects that are capable of overwhelming the extreme gravitational attraction produced by matter densities of some 10^{94} gms/cc near the big bang, and then switching them off with extreme rapidity as the matter density falls below this Planck scale. This is a huge burden!

The question then is: How do various approaches fare with respect to these questions? The older quantum cosmology —the Wheeler-DeWitt (WDW) theory— passes the infra-red test with flying colors. But unfortunately the state follows the classical trajectory into the big bang (and in the k=1 case also the big crunch) singularity. The singularity is not resolved because *expectation values of density and curvature continue to diverge* in epochs when their classical counterparts do.

For a number of years, the failure of the WDW theory to naturally resolve the big bang singularity was taken to mean that quantum cosmology cannot, by itself, shed significant light on the quantum nature of the big bang. Indeed, for systems with a finite number of degrees of freedom we have the von Neumann uniqueness theorem which guarantees that quantum kinematics is unique. The only freedom we have is in factor ordering and this was deemed insufficient to alter the status-quo provided by the WDW theory.

The situation changed dramatically in LQG. In contrast to the WDW theory, a well established, rigorous kinematical framework *is* available in full LQG [2, 3, 4, 5]. Furthermore, this framework is uniquely singled out by the requirement of diffeomorphism invariance (or background independence) [28, 29]. If one mimics it in symmetry reduced models, one finds that a key assumption of the von-Neumann theorem is violated: *One is led to new quantum mechanics [11]!* This quantum theory is inequivalent to the WDW theory already at the kinematic level. Quantum dynamics built in this new arena agrees with the WDW theory in 'tame' situations but differs dramatically in the Planck regime, leading to a natural resolution of the big bang singularity.

III. LQC: MAIN RESULTS

The main LQC results can be summarized as follows [12, 13, 14, 15, 16, 17, 18].

Let us begin with the k=0 model without a cosmological constant. Following the strategy outlined in section II, let us fix a point at a late time on the trajectory corresponding to an expanding classical universe, construct a quantum state which is sharply peaked at that point, and evolve it using the LQC Hamiltonian constraint. One then finds that:

The wave packet remains sharply peaked on the classical trajectory till the matter density ρ reaches about 1% of the Planck density ρ_{Pl}. Thus, as in the WDW theory, the LQC evolution meets the infra-red challenge successfully.
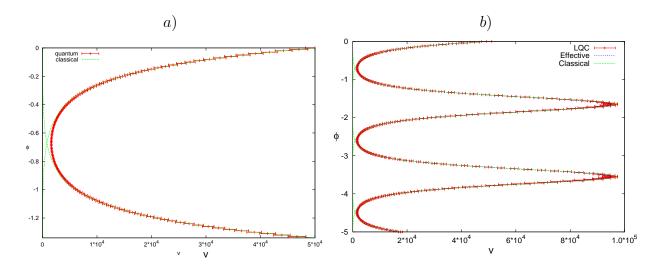
FIG. 2: In the LQC evolution of models under consideration, the big bang and big crunch singularities are replaced by quantum bounces. Expectation values and dispersion of the volume operator $\hat{V}_\phi$, are compared with the classical trajectory and the trajectory from effective Friedmann dynamics. The classical trajectory deviates significantly from the quantum evolution at the Planck scale and evolves into singularities. The effective trajectory provides an excellent approximation to quantum evolution at all scales. *a)* The k=0 case. In the backward evolution, the quantum evolution follows our post big-bang branch at low densities and curvatures but undergoes a quantum bounce at matter density $\rho \sim 0.41\rho_{\mathrm{PL}}$ and joins on to the classical trajectory that was contracting to the future. *b)* The k=1 case. The quantum bounce occurs again at $\rho \sim 0.41\rho_{\mathrm{Pl}}$. Since the big bang and the big crunch singularities are resolved the evolution undergoes cycles. In this simulation $p^\star_{(\phi)} = 5 \times 10^3$, $\Delta p_{(\phi)}/p^\star_{(\phi)} = 0.018$, and $v^\star = 5 \times 10^4$.

Let us evolve the quantum state back in time, toward the singularity. In the classical solution scalar curvature and the matter energy density keep increasing and eventually diverge at the big bang. The situation is very different with quantum evolution. As the density and curvature enter the Planck scale *quantum geometry effects become dominant creating an effective repulsive force which rises very quickly, overwhelms the classical gravitational attraction, and causes a bounce thereby resolving the big bang singularity.* See Fig 2.

At the bounce point, the density acquires its maximum value ρ_{max}. For the class of quantum states under discussion, numerical simulations have shown that ρ_{max} is *universal*, $\rho_{\mathrm{max}} \approx 0.41\rho_{\mathrm{Pl}}$ up to terms $O(\ell^2_{\mathrm{Pl}}/a^2_{\mathrm{min}})$, independently of the details of the state and value of $p_{(\phi)}$ (provided $p_{(\phi)} \gg \hbar$ in the classical units c=G=1).

Although in the Planck regime the peak of the wave function deviates very substantially from the general relativistic trajectory of figure 1, it follows an effective trajectory with very small fluctuations. This effective trajectory was derived [30, 31] using techniques from geometric quantum mechanics. The effective equation incorporates the leading corrections from quantum geometry. They modify the left hand side of Einstein's equations. However, to facilitate comparison with the standard form of Einstein's equations, one moves this correction to the right side through an algebraic manipulation. Then, one finds that the

Friedmann equation $(\dot{a}/a)^2 = (8\pi G \rho/3)$ is replaced by

$$\left(\frac{\dot{a}}{a}\right)^2 = (8\pi G \rho/3)\left(1 - \frac{\rho}{\rho_{\mathrm{crit}}}\right). \tag{3.1}$$

Here, $\rho_{\mathrm{crit}} = \sqrt{3}/32\pi^2\gamma^3 G^2\hbar$, where γ is the Barbero-Immirzi parameter of LQG (whose value $\gamma \sim 0.24$ is determined by the black hole entropy calculations in LQG). By plugging in numbers one finds $\rho_{\mathrm{crit}} \approx 0.41\rho_{\mathrm{Pl}}$. Thus, $\rho_{\mathrm{crit}} \approx \rho_{\mathrm{max}}$, found in numerical simulations. Furthermore, one can show analytically [17] that the spectrum of the density operator on the physical Hilbert space admits a finite upper bound ρ_{sup}, also given by $\rho_{\mathrm{sup}} = \sqrt{3}/32\pi^2\gamma^3 G^2\hbar$. This result refers to the density operator on the physical Hilbert space. Thus, *there is an excellent match between quantum theory which provides ρ_{sup}, the effective equations which provide ρ_{crit} and numerical simulations which provide ρ_{max}.*

In classical general relativity the right side, $8\pi G\rho/3$, of the Friedmann equation is positive, whence $\dot{a}$ cannot vanish; the universe either expands forever from the big bang or contracts into the big crunch. In the LQC effective equation, on the other hand, $\dot{a}$ vanishes when $\rho = \rho_{\mathrm{crit}}$ at which a quantum bounce occurs: To the past of this event, the universe contracts while to the future, it expands. This is possible because the LQC correction $\rho/\rho_{\mathrm{crit}}$ *naturally* comes with a negative sign. This is non-trivial. In the standard brane world scenario, for example, Friedmann equation is also receives a $\rho/\rho_{\mathrm{crit}}$ correction but it comes with a positive sign (unless one artificially makes the brane tension negative) whence the singularity is not resolved.

Even at the onset of the standard inflationary era, the quantum correction $\rho/\rho_{\mathrm{crit}}$ is of the order 10^{-11} and hence completely negligible. Thus, LQC calculations provide *an a priori justification* for using classical general relativity during inflation.

The analysis has been extended to include a cosmological constant. In the case when it is negative, the classical universe starts out with a big bang, expands to a maximum volume and then undergoes a recollapse to a big crunch singularity. The recollapse is faithfully reproduced by the LQC evolution. However, in contrast to general relativity and the WDW theory, both the big-bang and the big-crunch singularities are resolved [32]. Thus, in this case, the LQC evolution leads to a 'cyclic' universe (as in the k=1 model discussed below).

The case of a positive cosmological constant is more subtle [33]. Now, as in the $\Lambda = 0$ case, the classical theory admits two types of trajectories. One starts with a big bang and expands to infinity while the other starts out with infinite volume and contracts into a big crunch. But, in contrast to the $\Lambda = 0$ case, they attain an infinite volume at a *finite* value ϕ_{max} of ϕ. The energy density $\rho|_\phi$ at the 'internal time' ϕ goes to zero at ϕ_{max}. Because the ϕ 'evolution' is unitary in LQC, it yields a natural extension of the classical solution beyond ϕ_{max}. States which are semi-classical in the low $\rho|_\phi$ regime again follow an effective trajectory. Since $\rho|_\phi$ remains bounded, it is convenient to draw these trajectories in the ρ_ϕ-ϕ plane (rather than v-ϕ plane). They agree with the classical trajectories in the low ρ_ϕ regime and analytically continue the classical trajectories beyond $\rho_\phi = 0$.

The LQC framework has also been extended to incorporate standard inflationary potential with phenomenologically viable parameters [16]. Again, the singularity is resolved. Thus, in all these cases, the principal features of the LQC evolution are robust, including the value of ρ_{crit}.

In the closed, k=1 model, the situation is similar but there are two additional noteworthy features which reveal surprising properties of the domain of applicability of classical general relativity.

To start with, classical general relativity is again an excellent approximation to the LQC evolution till matter density ρ becomes about 1% of the Planck density ρ_{Pl} and, as the density increases further, the LQC evolution starts departing significantly. Again, quantum geometry effects become dominant creating an effective repulsive force which rises very quickly, overwhelms the classical gravitational attraction, and causes a bounce thereby resolving both the big bang and the big crunch singularities. Surprisingly these considerations apply *even to universes whose maximum radius $a_{\max}$ is only $23\ell_{\mathrm{Pl}}$*. For these universes, general relativity is a very good approximation in the range $8\ell_{\mathrm{Pl}} < a < 23\ell_{\mathrm{Pl}}$. The matter density acquires its minimum value $\rho_{\min}$ at the recollapse. The classical prediction $\rho_{\min} = 3/8\pi G a_{\max}^2$ is correct to one part in 10^5.

The volume of the universe acquires its minimum value $V_{\min}$ at the quantum bounce. $V_{\min}$ scales linearly with $p_{(\phi)}$. Consequently, $V_{\min}$ can be *much* larger than the Planck size. Consider for example a quantum state describing a universe which attains a maximum radius of a megaparsec. Then the quantum bounce occurs when the volume reaches the value $V_{\min} \approx 5.7 \times 10^{16}\,\mathrm{cm}^3$, *some 10^{115} times the Planck volume.* Deviations from the classical behavior are triggered when the density or curvature reaches the Planck scale. The volume can be very large and is not the relevant scale for quantum gravity effects.

This, in all these cases, classical singularities are replaced by quantum bounces and LQC provides a rather detailed picture of the physics in the Planck regime. Furthermore, the singularity resolution does not cause infra-red problems: There is close agreement with classical general relativity away from the Planck scale. The ultraviolet-infrared tension is avoided because, although quantum geometry effects are truly enormous in the Planck regime, they die astonishingly quickly.

IV. OUTLOOK

Let us summarize the overall situation. In simple cosmological models, many of the outstanding questions have been answered in LQC in remarkable detail. The scalar field plays the role of an internal or emergent time and enables us to interpret the Hamiltonian constraint as an evolution equation. Singularity is resolved in a precise sense: While physical observables such as matter density diverge in classical solutions, giving rise to a singularity, *they are represented by bounded operators on the physical Hilbert space.* The big bang and the big crunch are naturally replaced by quantum bounces. On the 'other side' of the bounce there is again a large universe. General relativity is an excellent approximation to quantum dynamics once the matter density falls below one percent of the Planck density. Thus, LQC successfully meets both the 'ultra-violet' and 'infra-red' challenges. Furthermore results obtained in a number of models using distinct methods re-enforce one another. One is therefore led to take at least the qualitative findings seriously: *Big bang is not the Beginning nor the big crunch the End.* Quantum space-times are vastly larger than what general relativity had us believe!

How can the quantum space-times of LQC manage to be significantly larger than those in general relativity when those in the WDW theory are not? Main departures from the WDW theory occur due to *quantum geometry effects* of LQG. There is no fine tuning of initial conditions, nor a boundary condition at the singularity, postulated from outside.

Furthermore, matter can satisfy all the standard energy conditions. Why then does the LQC singularity resolution not contradict the standard singularity theorems of Penrose, Hawking and others? These theorems are inapplicable because *the left hand side* of the classical Einstein's equations is modified by the quantum geometry corrections of LQC. What about the more recent singularity theorems that Borde, Guth and Vilenkin [34] proved in the context of inflation? They do not refer to Einstein's equations. But, motivated by the eternal inflationary scenario, they assume that the expansion is positive along any past geodesic. Because of the pre-big-bang contracting phase, this assumption is violated in the LQC effective theory.

While the detailed results presented in section III are valid only for these simplest models, partial results have been obtained also in more complicated models indicating that the singularity resolution may be robust [35, 36, 37]. In this respect there is a curious similarity with the very discovery of singularities in general relativity. They were first encountered in special examples. Although the examples were the physically most interesting ones — e.g., the big-bang and the Schwarzschild curvature singularities— at first it was thought that these space-times are singular because they are highly symmetric. It was believed that generic solutions of Einstein'e equations should be non-singular. As is well-known, this belief was shattered by the singularity theorems. Some 40 years later we have come to see that the big bang and the big crunch singularities are in fact resolved by quantum geometry effects. Is this an artifact of high symmetry? Or, are there robust *singularity resolution theorems* lurking just around the corner?

A qualitative picture that emerges is that the non-perturbative quantum geometry corrections are *'repulsive'*. While they are negligible under normal conditions, they dominate when curvature approaches the Planck scale and can halt the collapse that would classically have led to a singularity. In this respect, there is a curious similarity with the situation in the stellar collapse where a new repulsive force comes into play when the core approaches a critical density, halting further collapse and leading to stable white dwarfs and neutron stars. This force, with its origin in the Fermi-Dirac statistics, is *associated with the quantum nature of matter*. However, if the total mass of the star is larger than, say, 5 solar masses, classical gravity overwhelms this force. The suggestion from LQC is that a new repulsive force *associated with the quantum nature of geometry* comes into play and is strong enough to counter the classical, gravitational attraction, irrespective of how large the mass is. It is this force that prevents the formation of singularities. Since it is negligible until one enters the Planck regime, predictions of classical relativity on the formation of trapped surfaces, dynamical and isolated horizons would still hold. But one cannot conclude that there must be a singularity because the assumptions of the standard singularity theorems would be violated. There may be no singularities, no abrupt end to space-time where physics stops. Non-perturbative, background independent quantum physics would continue.

At first one might think that, since quantum gravity effects concern only a tiny region, whatever they may be, their influence on the global properties of space-time should be negligible whence they would have almost no bearing on the issue of the Beginning and the End. However, as we saw, once the singularity is resolved, vast new regions appear on the 'other side' ushering-in new possibilities that were totally unforeseen in the realm of Minkowski and Einstein. Which of them are realized generically? Is there a manageable classification? In the case of black holes, the singularity is again resolved but there are domains in which geometry is truly quantum: the quantum fluctuations of the metric operator are so huge near the putative singularity that the classical field obtained by taking its expectation value

is a poor representation of the actual physics in these regions [24]. Presence of such regions would render classical notions of causality inadequate to understand the global structure of space-time. In particular, while one can still speak of marginally trapped surfaces and dynamical and isolated horizons in the 'tame' regions of the full quantum space-time, the notion of an event horizon turns out to be 'too global' to be meaningful. Is there perhaps a well-defined but genuinely quantum notion of causality which reduces to the familiar one on quantum states which are sharply peaked on a classical geometry? Or, do we just abandon the idea that space-time geometry dictates causality and formulate physics primarily in relational terms? There is a plethora of such exciting challenges. Their scope is vast, they force us to introduce novel concepts and they lead us to unforeseen territories. These are just the type of omens that foretell the arrival of a major paradigm shift to take us beyond Einstein's space-time continuum.

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