## Lecture to the London Mathematical Society on 20 February 1947 [\*] A. M. Turing

The automatic computing engine now being designed at N.P.L. is a typical large scale electronic digital computing machine. In a single lecture it will not be possible to give much technical detail of this machine, and most of what I shall say will apply equally to any other machine of this type now being planned.

From the point of view of the mathematician the property of being digital should be of greater interest than that of being electronic. That it is electronic is certainly important because these machines owe their high speed to this, and without the speed it is doubtful if financial support for their construction would be forthcoming. But this is virtually all that there is to be said on that subject. That the machine is digital however has more subtle significance. It means firstly that numbers are represented by sequences of digits which can be as long as one wishes. One can therefore work to any desired degree of accuracy. This accuracy is not obtained by more careful machining of parts, control of temperature variations, and such means, but by a slight increase in the amount of equipment in the machine. To double the number of significant figures used would involve increasing the equipment by a factor definitely less than two, and would also have some effect in increasing the time taken over each job. This is in sharp contrast with analogue machines, and continuous variable machines such as the differential analyser, where each additional decimal digit required necessitates a complete redesign of the machine, and an increase in the cost by perhaps as much as a factor of 10. A second advantage of digital computing machines is that they are not restricted in their applications to any particular type of problem. The differential analyser is by far the most general type of analogue machine yet produced, but even it is comparatively limited in its scope. It can be made to deal with almost any kind of ordinary differential equation, but it is hardly able to deal with partial differential equations at all, and certainly cannot manage large numbers of linear simultaneous equations, or the zeros of polynomials. With digital machines however it is almost literally true that they are able to tackle any computing problem. A good working rule is that the ACE can be made to do any job that could be done by a human computer, and will do it in one ten-thousandth of the time. This time estimate is fairly reliable, except in cases where the job is too trivial to be worth while giving to the ACE.

Some years ago I was researching on what might now be described as an investigation of the theoretical possibilities and limitations of digital computing machines. I considered a type of machine which had a central mechanism, and an infinite memory which was contained on an infinite tape. This type of machine appeared to be sufficiently general. One of my conclusions was that the idea of a 'rule of thumb' process and a 'machine process' were synonymous. The expression 'machine process' of course means one which could be carried out by the type of machine I was considering. It was essential in these theoretical arguments that the meinory should be infinite. It can easily be shown that otherwise the machine can only execute periodic operations. Machines such as the ACE may be regarded as practical versions of this same type of machine. There is at least a very close

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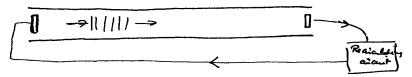
analogy. Digital computing machines all have a central mechanism or control and some very extensive form of memory. The memory does not have to be infinite, but it certainly needs to be very large. In general the arrangement of the memory on an infinite tape is unsatisfactory in a practical machine, because of the large amount of time which is liable to be spent in shifting up and down the tape to reach the point at which a particular piece of information required at the moment is stored. Thus a problem might easily need a storage of three million entries, and if each entry was equally likely to be the next required the average journey up the tape would be through a million entries, and this would be intolerable. One needs some form of memory with which any required entry can be reached at short notice. This difficulty presumably used to worry the Egyptians when their books were written on papyrus scrolls. It must have been slow work looking up references in them, and the present arrangement of written matter in books which can be opened at any point is greatly to be preferred. We may say that storage on tape and papyrus scrolls is somewhat inaccessible. It takes a considerable time to find a given entry. Memory in book form is a good deal better, and is certainly highly suitable when it is to be read by the human eye. We could even imagine a computing machine that was made to work with a memory based on books. It would not be very easy but would be immensely preferable to the single long tape. Let us for the sake of argument suppose that the difficulties involved in using books as memory were overcome, that is to say that mechanical devices for finding the right book and opening it at the right page, etc. etc. had been developed, imitating the use of human hands and eyes. The information contained in the books would still be rather inaccessible because of the time occupied in the mechanical motions. One cannot turn a page over very quickly without tearing it, and if one were to do much transportation, and do it fast, the energy involved would be very great. Thus if we moved one book every millisecond and each was moved ten metres and weighed 200 grams, and if the kinetic energy were wasted each time we should consume 10<sup>10</sup> watts, about half the country's power consumption. If we are to have a really fast machine then, we must have our information, or at any rate a part of it, in a more accessible form than can be obtained with books. It seems that this can only be done at the expense of compactness and economy, e.g. by cutting the pages out of the books, and putting each one in to a separate reading mechanism. Some of the methods of storage which are being developed at the present time are not unlike this.

If one wishes to go to the extreme of accessibility in storage mechanisms one is liable to find that it is gained at the price of an intolerable loss of compactness and economy. For instance the most accessible known form of storage is that provided by the valve flip-flop or Jordan Eccles trigger circuit. This enables us to store one digit, capable of two values, and uses two thermionic valves. To store the content of an ordinary novel by such means would cost many millions of pounds. We clearly need some compromise method of storage which is more accessible than paper, film etc, but more economical -in space and money than the straightforward use of valves. Another desirable feature is that it should be possible to record into the memory from within the computing machine, and this should be possible whether or not the storage already contains something, i.e. the storage should be erasible.

There are three main types of storage which have been developed recently and have these properties in greater or less degree. Magnetic wire is very compact, is erasible, can be recorded on from within the machine, and is moderately

accessible. There is storage in the form of charge patterns on the screen of a cathode ray tube. This is probably the ultimate solution. It could eventually be nearly as accessible as the Jordan Eccles circuit. A third possibility is provided by acoustic delay lines. They give greater accessibility than the magnetic wire, though less than the C.R.T type. The accessibility is adequate for most purposes. Their chief advantage is that they are already a going concern. It is intended that the main memory of the ACE shall be provided by acoustic delay lines, consisting of mercury tanks.

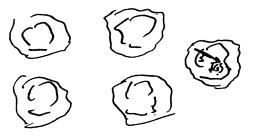
The idea of using acoustic delay lines as memory units is due I believe to Eckert of Philadelphia University, who was the engineer chiefly responsible for the Eniac. The idea is to store the information in the form of compression waves travelling along a column of mercury. Liquids and solids will transmit sound of surprisingly high frequency, and it is quite feasible to put as many as 1000 pulses into a single 5' tube. The signals may be conveyed into the mercury by a piezo-electric crystal, and also detected at the far end by another quartz crystal. A train of pulses or the information



which they represent may be regarded as stored in the mercury whilst it is travelling through it. If the information is not required when the train emerges it can be fed back into the column again and again until such time as it is required. This requires a 'recirculating circuit' to read the signal as it emerges from the tank and amplify it and feed it in again. If this were done with a simple amplifier it is clear that the characteristics of both the tank and the amplifier would have to be extremely good to permit the signal to pass through even as many as ten times. Actually the recirculating circuit does something slightly different. What it does may perhaps be best expressed in terms of point set topology. Let the plane of the diagram represent the space of all possible signals. I do not of course wish to imply that this is two dimensional. Let the function f be defined for arguments in this signal space and have values in it. In fact let f(s) represent the effect on the signal s when it is passed through the tank and the recirculating mechanism. We assume however that owing to thermal agitation the effect of recirculation may be to give any point within a circle of radius  $\delta$  of f(s). Then a necessary and sufficient condition that the tank can be used as a storage which will distinguish between N different signals is that there must be N sets  $E_1 \dots E_N$  such that if  $F_r$  is the set of points within distance  $\varepsilon$  of  $E_r$ 

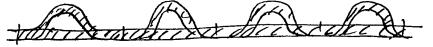
$$s \in F_r \subset f(s) \in E_r$$

and the sets  $F_r$  are disjoint. It is clearly sufficient for we have only then to ensure that the signals initially fed in belong to one or other of the sets  $F_r$  and it will remain in the set after any number of recirculations, without any

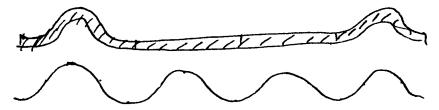


danger of confusion. It is necessary for suppose  $s_1 ext{...} ext{ } s_N$  are signals which have different meanings and which can be fed into the machine at any time and read out later without fear of confusion.

Let  $E_r$  be the set of signals which could be obtained for s, by successive applications of f and shifts of distance not more than  $\varepsilon$ . Then the sets  $E_r$  are disjoint [two lines indecipherable - Ed.]. In the case of a mercury delay line used for N=16 the set would consist of all continuous signals within the shaded area.



One of the sets would consist of all continuous signals lying in the region below. It would represent the signal 1001.



In order to put such a recirculation system into effect it is essential that a clock signal be supplied to the memory system so that it will be able to distinguish the times when a pulse if any should be present. It would for instance be natural to supply a timing sine wave as shown above to the recirculator.

The idea of a process f with the properties we have described is a very common one in connection with storage devices. It is known as 'regeneration' of storage. It is always present in some form, but sometimes the regeneration is as it were naturally occurring and no precautions have to be taken. In other cases special precautions have to be taken to improve such an f process or else the impression will fade.

The importance of a clock to the regeneration process in delay lines may be illustrated by an interesting little theorem. Suppose that instead of the condition  $s \in Fr \subset f(s) \in E_r$  we impose a stronger one, viz.  $f^n(s) \to c_r$  if  $s \in Er$  i.e. there are ideal forms of the distinguishable signals, and each admissible signal converges towards the ideal form after recirculating. Then we can show that unless there is a clock the ideal signals are all constants. For let  $U_\alpha$  represent a shift of origin, i.e.  $U_\alpha s(t) = s(t + \alpha)$ . Then since there is no clock the properties of the recirculator are the same at all times and therefore commutes with  $U_\alpha$ . Then  $f U_\alpha(c_r) = U_\alpha f(c_r) = U_\alpha f(c_r) = U_\alpha c_r$  for  $f(c_r) = c_r$  since cr is an ideal signal. But this means that  $U_\alpha(c_r)$  is an ideal signal, and therefore for sufficiently small  $\alpha$  must be  $c_r$  since the ideal signals are discrete. Then for any  $\beta$  and sufficiently large u,  $\beta/u$  will be sufficiently small and  $U_{\beta/u}(c) = c$ . But then by iteration  $c = U^u_{\beta/u}(c) = U_\beta(c)$  i.e.  $c(t + \beta) = c(t)$ . This means that the ideal signal c is a constant.

We might say that the clock enables us to introduce a discreteness into time, so that time for some purposes can be regarded as a succession of instants instead of a continuous flow. A digital machine must essentially deal with discrete objects, and in the case of the ACE this is made possible by the use of a clock. All other digital computing machines except for human and other brains that I know of do the same. One can think up ways of avoiding it, but they are very awkward. I should mention that the use of the clock in the ACE is not confined to the recirculation process, but is used in almost every part.

It may be as well to mention some figures connected with the mercury delay line as we shall use it. We shall use five foot tubes, with an inside diameter of half an inch. Each of these will enable us to store 1024 binary digits. The unit I have used here to describe storage capacity is self explanatory. A storage mechanism has a capacity of m binary digits if it can remember any sequence of m digits each being a 0 or a 1. The storage capacity is also the logarithm to the base 2 of the number of different signals which can be remembered, i.e. log, N. The digits will be placed at a time interval of one microsecond, so that the time taken for the waves to travel down the tube is just over a millisecond. The velocity is about one and a half kilometres per second. The delay in accessibility time or average waiting for a given piece of information is about half a millisecond. In practice this is reduced to an effective 150 ps. The füll storage capacity of the ACE available on Hg delay lines will be about 200,000 binary digits. This is probably comparable with the memory capacity of a minnow.

I have spent a considerable time in this lecture on this question of memory, because I believe that the provision of proper storage is the key to the problem of the digital computer, and certainly if they are to be persuaded to show any sort of genuine intelligence much larger capacities than are yet available must be provided. In my opinion this problem of making a large memory available at reasonably short notice is much more important than that of doing operations such as multiplication at high speed. Speed is necessary if the machine is to work fast enough for the machine to be commercially valuable, but a large storage capacity is necessary if it is to be capable of anything more than rather trivial operations. The storage capacity is therefore the more fundamental requirement.

Let us now return to the analogy of the theoretical computing machines with an infinite tape. It can be shown that a single special machine of that type can be made to do the work of all. It could in fact be made to work as a model of any other machine. The special machine may be called the universal machine, it works in the following quite simple manner. When we have decided what machine we wish to imitate we punch a description of it on the tape of the universal machine. This description explains what the machine would do in every configuration in which it might find itself. The universal machine has only to keep looking at this description in order to find out what it should do at each stage. Thus the complexity of the machine to be imitated is concentrated in the tape and does not appear in the universal machine proper in any way.

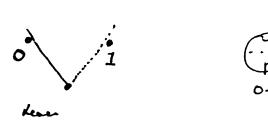
If we take the properties of the universal machine in combination with the fact that machine processes and rule of thumb processes are synonymous we may say that the universal machine is one which, when supplied with the appropriate instructions, can be made to do any rule of thumb process. This feature ig paralleled in digital computing machines such as the ACE. They are in fact practical versions of the universal machine. There is a certain central pool of electronic equipment, and a large memory. When any particular problem has to be

handled the appropriate instructions for the computing process involved are stored in the memory of the ACE and it is then 'set up' for carrying out that process.

I have indicated the main strategic ideas behind digital computing machinery, and will now follow this account up with the very briefest description of the ACE. It may be divided for the sake of argument into the following parts

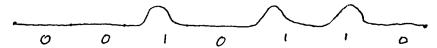
Memory Control Arithmetic part Input and output

I have already said enough about the memory and will only repeat that in the ACE the memory will consist mainly of 200 mercury delay lines each holding 1024 binary digits. The purpose of the control is to take the right instructions from the memory, see what they mean, and arrange for them to be carried out. It is understood that a certain 'code of instructions' has been laid down, whereby each Vord' or combination of say 32 binary digits describes some particular operation. The circuit of the control is made in accordance with the code, so that the right effect is produced. To a large extent we have also allowed the circuit to determine the code, i.e. we have notjust thought up an imaginary'best code'and then found a circuit to put it into effect, but have often simplified the circuit at the expense of the code. It is also quite difficult to think about the code entirely in abstracto without any kind of circuit. The arithmetic part of the machine is the part concerned with addition, multiplication and any other operations which it seems worth while to do by means of special circuits rather than through the simple facilities provided by the control. The distinction between control and arithmetic part is a rather hazy one, but at any rate it is clear that the machine should at least have an adder and a multiplier, even if they turn out in the end to be part of the control. This is the point at which I should mention that the machine is operated in the binary scale, with two qualifications. Inputs from externally provided data are in decimal, and so are outputs intended for human eyes rather than for later reconsumption by the ACE. This is the first qualification. The second is that, in spite of the intention of binary working there can be no bar on decimal working of a kind, because of the relation of the ACE to the universal machine. Binary working is the most natural thing to do with any large scale computer. It is much easier to work in the scale of two than any other, because it is so easy to produce mechanisms which have two positions of stability: the two positions may then be regarded as representing 0 and 1. Examples are lever as diagram, Jordan Eccles circuit, thyratron. If one is concerned with a



small scale calculating machine then there is at least one serious objection to binary working. For practical use it will be necessary to build a converter to transform numbers from the binary form to the decimal and back. This may well be a larger undertaking than the binary calculator. With the large scale machines this argument carries no weight. In the first place a converter would become a relatively small piece of apparatus, and in the second it would not really be necessary. This last statement sounds quite paradoxical, but it is a simple consequence of the fact that these machines can be made to do any rule of thumb process by remembering suitable instructions. In particular it can be made to do binary decimal conversion. For example in the case of the ACE the provision of the converter involves no more than adding two extra delay lines to the memory. This situation is very typical of what happens with the ACE. There are many fussy little details which have to be taken care of, and which, according to normal engineering practice would require special circuits. We are able to deal with these points without modification of the machine itself, by pure paper work, eventually resulting in feeding in appropriate instructions.

To return to the various parts of the machine. I was saying that it will work in the scale of two. It is not unnatural to use the convention that an electrical pulse shall represent the digit 1 and that absence of a pulse shall represent a digit 0. Thus a sequence of digits 00 10 110 would be represented by a signal like



where the time interval might be one microsecond. Let us now look at what the process of binary addition is like. In ordinary decimal addition we always begin from the right, and the same naturally applies to binary. We have to do this because we cannot tell whether to carry unless we have already dealt with the less significant columns. The same applies with electronic addition, and therefore it is convenient to use the convention that if a sequence of pulses is coming down a line, then the least significant pulse always comes first. This has the unfortunate result that we must either write the least significant digit on the left in our binary numbers or else make time flow from right to left in our diagrams. As the latter alternative would involve writing from right to left as well as adding in that way, we have decided to put the least significant digit on the left. Now let us do a typical addition. Let us write the carry digits above the addends.

Carry		0	1	1	1	1	1	0	0	1	1
Α	0	1	1	0	1	1	0	0	1	0	1
В	0	1	1	1	0	1	0	0	1	1	
		0	1	0	0	1	1	0	0	0	

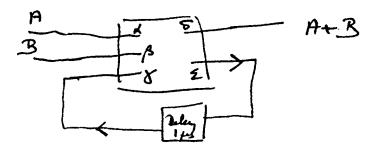
Note that I can do the addition only looking at a small part of the data. To do the addition electronically we need to produce a circuit with three inputs and two outputs.

InputsOutputsAddend A $\alpha$ Sum  $\delta$ Addend B $\beta$ Carry  $\epsilon$ Carry from last column $\gamma$ 

This circuit must be such that

If no. of 1's on inputs 
$$\alpha$$
,  $\beta$ ,  $\gamma$  is
$$\begin{cases}
0 & \text{Then sum } 0 & \text{and } 0 \\
1 & \delta & 1 & \text{carry } 0 \\
2 & \text{is } 0 & \varepsilon & 1 \\
3 & 1 & \text{is } 1
\end{cases}$$

It is very easy to produce a voltage proportional to the number of pulses on the inputs, and one then merely has to provide a circuit which will discriminate between four different levels and put out the appropriate sum and carry digits. I will not attempt to describe such a circuit; it can be quite simple. When we are given the circuit we merely have to connect it up with feedback and it is an adder. Thus:



It will be seen that we have made use of the fact that the same process is used in addition with each digit, and also the fact that the properties of the electrical circuit are invariant under time shifts, at any rate if these are multiples of the clock period. It might be said that we have made use of the isomorphism between the group of these time shifts and the multiplicative group of real numbers to simplify our apparatus, though I doubt if many other applications of this principle could be found.

It will be seen that with such an adder the addition is broken down into the most elementary steps possible, such as adding one and one. Each of these occupies a microsecond. Our numbers will normally consist of 32 binary digits, so that two of them can be added in 32 microseconds. Likewise we shall do multiplications in the form of a number of consecutive additions of one and one or one and zero etc. There are 1024 such additions or thereabouts to be done in a multiplication of one 32 digit number by another, so that one might expect a multiplication to take about a millisecond. Actually the multiplier to be used on ACE will take rather over two milliseconds. This may sound rather long, when the unit operation is only a microsecond, but it actually seems that the machine is fairly well balanced in this respect, Le. the multiplication time is not a serious bottleneck. Computers always spend just as long in writing numbers down and deciding what to do next as they do in actual multiplications, and it is just the same with the ACE. A great deal of time is spent in getting numbers in and out of storage and deciding what to do next. To complete the four elementary processes, subtraction is done by complementation and addition, and division is done by the use of the iteration formula

$$u_n = u_{n-1} + u_{n-1} (1 - au_{n-1})$$

 $u_n$  converges to  $a^{-1}$  provided  $|1 - au_0| < 1$ . The error is squared at each step, so that the convergence is very rapid. This process is of course programmed, i.e. the only extra apparatus required is the delay lines required for storing the relevant instructions.

Passing on from the arithmetic part there remains the input and output. For this purpose we have chosen Hollerith card equipment. We are able to obtain this without having to do any special development work. The speeds obtainable are not very impressive compared with the speeds at which the electronic equipment works, but they are quite sufficient in all cases where the calculation is long and the result concise: the interesting cases in fact. It might appear that there would be a difficulty in converting the information provided at the slow speeds appropriate to the Hollerith equipment to the high speeds required with the ACE, but it is really quite easy. The Hollerith speeds are so slow as to be counted zero or stop for many purposes, and the problem reduces to the simple one of converting a number of statically given digits into a stream of pulses. This can be done by means of a form of electronic commutator.

Before leaving the outline of the description of the machine I should mention some of the tactical situations that are met with in programming. I can illustrate two of them in connection with the calculation of the reciprocal described above. One of these is the idea of the iterative cycle. Each time that we go from  $u_r$  to  $u_{r+1}$ , we apply the same sequence of operations, and it will therefore be economical in storage space if we use the same instructions. Thus we go round and round a cycle of instructions:

It looks however as if we were in danger of getting stuck in this cycle, and unable to get out. The solution of this difficulty involves another tactical idea, that of 'discrimination' Le. of deciding what to do next partly according to the results of the machine itself, instead of according to data available to the programmer. In this case we include a discrimination in each cycle, which takes us out of the cycle when the value of |1 - au| is sufficiently small. It is like an aeroplane circling over an aerodrome, and asking permission to land after each circle. This is a very simple idea, but is of the utmost importance. The idea of the iterative cycle of instructions will also be seen to be rather fundamental when it is realised that the majority of the instructions in the memory must be obeyed a great number of times. If the whole memory were occupied by instructions, none of it being used for numbers or other data, and if each instruction were obeyed once only, but took the longest possible time, the machine could only remain working for sixteen seconds.

Another important idea is that of constructing an instruction and then obeying it. This can be used amongst other things for discrimination. In the example I have

just taken for instance we could calculate a quantity which was 1 if |1 - au| was less than  $2^{-3.1}$  and 0 otherwise. By adding this quantity to the instruction that is obeyed at the forking point the instruction can be completely altered in its effect when finally 1 - au is reduced to sufficiently small dimensions.

Probably the most important idea involved in instruction tables is that of standard subsidiary tables. Certain processes are used repeatedly in all sorts of different connections, and we wish to use the same instructions, from the same part of the memory every time. Thus we may use interpolation for the calculation of a great number of different functions, but we shall always use the same instruction table for interpolation. We have only to think out how this is to be done once, and forget then how it is done. Each time we want to do an interpolation we have only to remember the memory position where this table is kept, and make the appropriate reference in the instruction table which is using the interpolation. We might for instance be making up an instruction table for finding values of  $J_0(x)$ and use the interpolation table in this way. We should then say that the interpolation table was a subsidiary to the table for calculating  $J_0(x)$ . There is thus a sort of hierarchy of tables. The interpolation table might be regarded as taking its orders from the  $J_0$  table, and reporting its answers back to it. The master servant analogy is however not a very good one, as there are many more masters than servants, and many masters have to share the same servants.

Now let me give a picture of the operation of the machine. Let us begin with some problem which has been brought in by a customer. It will first go to the problems preparation section where it is examined to see whether it is in a suitable form and self-consistent, and a very rough computing procedure made out. It then goes to the tables preparation section. Let us suppose for example that the problem was to tabulate solutions of the equation

$$y'' + xy' = J_0(x)$$

with initial conditions x = y = 0, y' = a. This would be regarded as a particular case of solving the equation

$$y'' = F(x, y, y')$$

for which one would have instruction tables already prepared. One would need also a table to produce the function F(x, y, z) (in this case  $F(x, y, z) = J_0(x) - xz$ which would mainly involve a table to produce  $J_0(x)$ , and this we might expect to get off the shelf). A few additional details about the boundary conditions and the length of the arc would have to be dealt with, but much of this detail would also be found on the shelf, just like the table for obtaining  $J_0(x)$ . The instructions for the job would therefore consist of a considerable number taken off the shelf together with a few made up specially for the job in question. The instruction cards for the standard processes would have already been punched, but the new ones would have to be done separately. When these had all been assembled and checked they would be taken to the input mechanism, which is simply a Hollerith card feed. They would be put into the card hopper and a button pressed to start the cards moving through. It must be remembered that initially there are no instructions in the machine, and one's normal facilities are therefore not available. The first few cards that pass in have therefore to be carefully thought out to deal with this situation. They are the initial input cards and are always the same. When they have passed in a few rather fundamental instruction tables will have been set up in the machine, including sufficient to enable the machine to read the special

pack of cards that has been prepared for the job we are doing. When this has been done there are various possibilities as to what happens next, depending on the way the job has been programmed. The machine might have been made to go straight on through, and carry out the job, punching or printing all the answers required, and stopping when all of this has been done. But more probably it will have been arranged that the machine stops as soon as the instruction tables have been put in. This allows for the possibility of checking that the content of the memories is correct, and for a number of variations of procedure. It is clearly a suitable moment for a break. We might also make a number of other breaks. For instance we might be interested in certain particular values of the parameter a, which were experimentally obtained figures, and it would then be convenient to pause after each parameter value, and feed the next parameter value in from another card. Or one might prefer to have the cards all ready in the hopper and let the ACE take them in as it wanted them. One can do as one wishes, but one must make up one's mind. Each time the machine pauses in this way a 'word' or sequence of 32 binary digits is displayed on neon bulbs. This word indicates the reason for stopping. I have already mentioned two possible reasons. A large class of further possible reasons is provided by the checks. The programming should be done in such a way that the ACE is frequently investigating identities which should be satisfied if all is as it should be. Whenever one of these checks fails the machine stops and displays a word which describes what check has failed.

It will be seen that the possibilities as to what one may do are immense. One of our difficulties will be the maintainence of an appropriate discipline, so that we do not lose track of what we are doing. We shall need a number of efficient librarian types to keep us in order.

Finally I should like to make a few conjectures as to the repercussions that electronic digital computing machinery will have on mathematics. I have already mentioned that the ACE will do the work of about 10,000 computers. It is to be expected therefore that large scale hand-computing will die out. Computers will still be employed on small calculations, such as the substitution of values in formulae, but whenever a single calculation may be expected to take a human computer days of work, it will presumably be done by an electronic computer instead. This will not necessitate everyone interested in such work having an electronic computer. It would be quite possible to arrange to control a distant computer by means of a telephone line. Special input and output machinery would be developed for use at these out stations, and would cost a few hundred pounds at most. The main bulk of the work done by these computers will however consist of problems which could not have been tackled by hand computing because of the scale of the undertaking. In order to supply the machine with these problems we shall need a great number of mathematicians of ability. These mathematicians will be needed in order to do the preliminary research on the problems, putting them into a form for computation. There will be considerable scope for analysts. When a human computer is working on a problem he can usually apply some common sense to give him an idea of how accurate his answers are. With a digital computer we can no longer rely on common sense, and the bounds of error must be based on some proved inequalities. We need analysts to find the appropriate inequalities for us. The inequalities need not always be explicit, i.e. one need not have them in such a form that we can tell, before the calculation starts, and using only pencil and paper, how big the error will be. The error calculation may be a serious part of the ACE's duties. To an extent it may be possible to replace the estimates of error by statistical estimates obtained by repeating the job several times, and doing the rounding off differently each time, controlling it by some random element, some electronic roulette wheel. Such statistical estimates however leave much in doubt, are wasteful in machine time, and give no indication of what can be done if it turns out that the errors are intolerably large. The statistical method can only help the analyst, not replace him.

Analysis is just one of the purposes for which we shall need good mathematicians. Roughly speaking those who work in connection with the ACE will be divided into its masters and its servants. Its masters will plan out instruction tables for it, thinking up deeper and deeper ways of using it. Its servants will feed it with cards as it calls for them. They will put right any parts that go wrong. They will assemble data that it requires. In fact the servants will take the place of limbs. As time goes on the calculator itself will take over the functions both of masters and of servants. The servants will be replaced by mechanical and electrical limbs and sense organs. One might for instance provide curve followers to enable data to be taken direct from curves instead of having girls read off values and punch them on cards. The masters are liable to get replaced because as soon as any technique becomes at all stereotyped it becomes possible to devise a system of instruction tables which will enable the electronic computer to do it for itself. It may happen however that the masters will refuse to do this. They may be unwilling to let their jobs be stolen from them in this way. In that case they would surround the whole of their work with mystery and make excuses, couched in well chosen gibberish, whenever any dangerous suggestions were made. I think that a reaction of this kind is a very real danger. This topic naturally leads to the question as to how far it is possible in principle for a computing machine to simulate human activities. I will return to this later, when I have discussed the effects of these machines on mathematics a little fürther.

I expect that digital computing machines will eventually stimulate a considerable interest in symbolic logic and mathematical philosophy. The language in which one communicates with these machines, Le. the language of instruction tables, forms a sort of symbolic logic. The machine interprets whatever it is told in a quite definite manner without any sense of humour or sense of proportion. Unless in communicating with it one says exactly what one means, trouble is bound to result. Actually one could communicate with these machines in any language provided it was an exact language, i.e. in principle one should be able to communicate in any symbolic logic, provided that the machine were given instruction tables which would enable it to interpret that logical system. This would mean that there will be much more practical scope for logical systems than there has been in the past. Some attempts will probably be made to get the machine to do actual manipulations of mathematical formulae. To do so will require the development of a special logical system for the purpose. This system should resemble normal mathematical procedure closely, but at the same time should be as unambiguous as possible. As regards mathematical philosophy, since the machines will be doing more and more mathematics themselves, the centre of gravity of the human interest will be driven fürther and fürther into philosophical questions of what can in principle be done etc.

It has been said that computing machines can only carry out the processes that they are instructed to do. This is certainly true in the sense that if they do something other than what they were instructed then they have just made some

mistake. It is also true that the intention in constructing these machines in the first instance is to treat them as slaves, giving them only jobs which have been thought out in detail, jobs such that the user of the machine fully understands what in principle is going on all the time. Up till the present machines have only been used in this way. But is it necessary that they should always be used in such a manner? Let us suppose we have set up a machine with certain initial instruction tables, so constructed that these tables might on occasion, if good reason arose, modify those tables. One can imagine that after the machine had been operating for some time, the instructions would have altered out of all recognition, but nevertheless still be such that one would have to admit that the machine was still doing very worthwhile calculations. Possibly it might still be getting results of the type desired when the machine was first set up, but in a much more efficient manner. In such a case one would have to admit that the progress of the machine had not been foreseen when its original instructions were put in. It would be like a pupil who had learnt much from his master, but had added much more by his own work. When this happens I feel that one is obliged to regard the machine as showing intelligence. As soon as one can provide a reasonably large memory capacity it should be possible to begin to experiment on these lines. The memory capacity of the human brain is probably of the order of ten thousand million binary digits. But most of this is probably used in remembering visual impressions, and other comparatively wasteful ways. One might reasonably hope to be able to make some real progress with a few million digits, especially if one confined one's investigations to some rather limited field such as the game of chess. It would probably be quite easy to find instruction tables which would enable the ACE to win against an average player. Indeed Shannon of Bell Telephone laboratories tells me that he has won games playing by rule of thumb: the skill of his opponents is not stated. But I would not consider such a victory very significant. What we want is a machine that can learn from experience. The possibility of letting the machine alter its own instructions provides the mechanism for this, but this of course does not get us very far.

It might be argued that there is a fundamental contradiction in the idea of a machine with intelligence. It is certainly true that 'acting like a machine', has become synonymous with lack of adaptability. But the reason for this is obvious. Machines in the past have had very little storage, and there has been no question of the machine having any discretion. The argument might however be put into a more aggressive form. It has for instance been shown that with certain logical systems there can be no machine which will distinguish provable formulae of the system from unprovable, i.e. that there is no test that the machine can apply which will divide propositions with certainty into these two classes. Thus if a machine is made for this purpose it must in some cases fail to give an answer. On the other hand if a mathematician is confronted with such a problem he would search around and find new methods of proof, so that he ought eventually to be able to reach a decision about any given formula. This would be the argument. Against it I would say that fair play must be given to the machine. Instead of it sometimes giving no answer we could arrange that it gives occasional wrong answers. But the human mathematician would likewise make blunders when trying out new techniques. It is easy for us to regard these blunders as not counting and give him another chance, but the machine would probably be allowed no mercy. In other words then, if a machine is expected to be infallible, it cannot also be intelligent. There are several mathematical theorems which say almost exactly that. But these theorems say nothing about how much intelligence may be displayed if a machine

makes no pretence at infallibility. To continue my plea for 'fair play for the machines' when testing their I.Q. A human mathematician has always undergone an extensive training. This training may be regarded as not unlike putting instruction tables into a machine. One must therefore not expect a machine to do a very great deal of building up of instruction tables on its own. No man adds very much to the body of knowledge, why should we expect more of a machine? Putting the same point differently, the machine must be allowed to have contact with human beings in order that it may adapt itself to their standards. The game of chess may perhaps be rather suitable for this purpose, as the moves of the machine's opponent will automatically provide this contact.

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The nationals community engine and being designed as also accurate to the form weeks first I community working. In a stars leadure it will not be see this to the much teachers desire of this marking, and west of that I shall may will analy smally to any other manifes of this large not being alanged assessments.

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If one wishes to go to the extreme of accessibility in storage mechanisms one is lights to find that anguarance it is swined at the price of an intelerable loss of comprehense end asonomy. For instance the most eccessible known form of stores is that provided by the valve flin-flow or Jordan Week Meeles trig so circuit. This enables noto store one disin. assemble of two values; and uses two therelouis valves, To story the content of an ordinary moved by such manual mounts cost many millions of counts, We slead not some composite method of storece which is wore scommandia them memor, film a ste, but more acompale-1 in spece and money than the str ishtformerd use of velves, Another desireble feeture is that it should be possible to record into the memory from within the computing machine, and this should be possible whether or not the storege siresdy contains conething, i.e. the storene should be eremible.

There is three in types of stering which have been developed recently and have these properties in an ten or last degree.

Meganetic twee wire is very counsed, is enable, on he recorded on from within the machine, and is mader tely cone this.

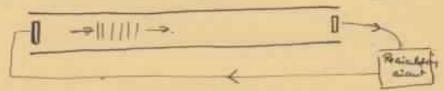
There is storing in the form of the recent on the screen of a cathode r r tube. This is probably she ultimate solution.

In addition the relations are enable as the Forder decise size it.

44 be to maport a book or sy will to sound, out

A third possibility is provided by accountle delay lines. They give arester soccasibility than the regnetic wire, though less than the D.R.T type. The soccasibility is adequate for most purposes. Their chief adventage is that they are already a going concern. It is intended that the main memory of the ADE shell be provided by accountle delay lines, consisting of mercury tanks.

The idea of uning assessed delay lines as memory units in the I believe to Delay of Thildeelphia University, who was the shirt on the Eniso, The share in the store the information assessed in the form of compression was transling slong a column of mercury. Liquide and malide will be nomit sound of surerisinely bigh frequency, and it is in quite feasible to not as many as long pulses into a simple 5' thick.

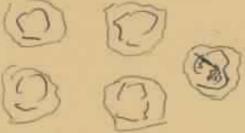


or the information what they represent a train of suless mer be restricted as stored in the mercury whilst it is traviling through it. If the information is not required when the train energies it will and be fed benk into the column agien and again until such time as it in resulted. This requires a 'recipculating circuis' to read the size-I emit emerges from the tank and amplify it and feed it in easin. If this were done with a simple amplifier it is slear that the observations of both the lim tank and the main empliciar would be we take extremely good to permit the sizes! to pass through even as many as ten times. Actually the realreadating sircuit took something alightly different. What it does may betheve to best expressed in terms of soint set tonelows.

The stylents may be carried it to the menony by a prize abothis aughed, and also detailed at the few and by another greet anysted.

Let the plane of the disares represent the space of all possible alguals. I do not of course wish to imply that this is two disansional. Let the function f be defined for enquents in this signal space and have relues in it. In fact let f(s) represent the effect on the signal schen it is massed through the take and the facilitating mechanism. Then a maceumary and sufficient condition that the take and be used as a storage which will distinguish between N different signals in that we there must be N sets Sq... By such that, Q f, a west q.

SEFT D d(s) EFT proud did Eg En



the element initially feed in belong to ome or other of the metal and it will remain in the set after any number of rectroulations, without any denser of confusion. It is necessary for suppose sq...mg are signals which on he feel into the machine at any time and road out later without fear of confusion. Suppose that signals distant later than the same likely to be confused. Then for each mean that so he sate set of winter for the same and which is distant later than the same of which is distant some than that so he sate set of winter for the same and with a same set against some set are and same set without some set set and same distant set that same set and same distant set that set apart.

but E, hote sol of expels which could be obtained for by concession explicated of of and slight of deplace and - the c.

The the sake E, as disjuit, our the was seferie, and by expelying with all all them a me can be flight E. and other

We even however that overy to termal explain as the affect of excelled my be to give any pet white art of radio of 16)

In thecese of a mercury delay line used for N = 16 the set would consist of all continuous signal- within the shaded eres.

One of the ness would comed to find the continuous might be living in the reason below. It would renument the signal 1001.

Sandar de mai vanhan rentmetration de la constant d

In order to not such a recirculation system into affect it is essential that a clock signal be sur-lied to the memory system so that it will be a to distinguish the times when a sules if any should be present. It would for instance be natural to surely a timing sine were as shown shows to the recirculator.

9.5.0

The importance of a clock to the recipied at a process of the interesting little theorem. Suppose that instead of the condition \$\$\frac{1}{2} \cdots \frac{1}{2} \cdots \frac{1}{2} \cdots \cdo

ideal signal, and therefore for sufficiently small & --- p

[7VM

The idea of a process of with the proportion we have clearwhol is a very commer one is connect with shape devices. It is known as "regulatin' of stronger It is always present in some form, but smaking be regulated in hadmalf and does not and to be the work in hadmalf and does not and to be the same to be super soil on of process to the special precaution have to be lakenfor the terminal of the toposition will fade:

the  $e_p$ , since the ideal simple are discrete. Then for any  $\beta$  and safficently large  $\mu$ ,  $\beta$ , will be sufficiently small and  $U_{p/n}(e) = e$ . But then by iteration  $eU_{p/n}(e) = U_p(e) = e$ .

i.e. e(t + p) = e(t). This means that the ideal signal e is a constant.

The might say that the alock enables as to introduce a for some pursones discretement into time, so that time can be recorded as a succession of instants hate a of so a continuous flow. A district machine must essentially deal with discrete objects, and in the case of the AGE this is made possible by the use of second for human and other brains a clock. All other district more as the for human and other brains a clock. All other district more of avoiding it, but they are many solutions of the clock in the case of the clock in the AGE is not continued to the recirculation medicals, but is used in clock some ones.

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can remember any as wence of a disits each being a 0 or a 1.
The stores sepecity is also the location to the ham a of
the number of different sized which can be remembered, i.e.
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bout one and a helf bilometres one assend. The deley is a time
for a given place of information is about helf a millisteend.
In precise this is reduced to an effective \$50 us.

The full store a commuter of the AGE excil-ble on Mg delay lines will be about 200,000 binery dista. This is aren big common ble with the memory aspectity of a minnow.

I have spent a consider the time in this lessure on this suestion of memory, because "believe that the provision of proper storage is the key to the problem of the digital economies, and cortainly if they are no be persuaded to show any north of penulse intelligence much larger separation than one yet available must be provided. In my a inimutable mobiles of making memory as inchise at personably short notice is much more important than that of coins oper time such as multiplication at high spends for the machine to be compared in the total and anything more than rather trivial oper time. The storage expension is therefore the more fundamental requirement.

Theoretical

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This feature is sentiated to di ital committee such a the ACE, They are in fact practic 1 wereins of the universal pechine. There is a cert in central pool of electronic and meent, and a large supers. Then are particular a oblem has to be handled the commonstate.

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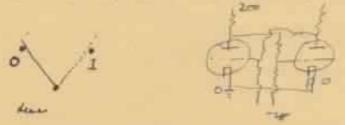
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Authorite ment

Input and Outsur

I have slowedy a id enough about the sensor and will nate rement that in the ALE the memory will consist salely of 200 delay lines each holding 1006 himmy disit. The purpose of the ocutrol is to take the right instance it to from the mamory, men the it ther were, and created for these to be servied ont. It is unformant that a cost in teads of instance mech 'world or "form! her hern leif form, wherehe adokate combined me of our no binery distin are departure some workfault- over ties, The circuit of the control is more in -x ordence - ith the code, on that the right erract is arddised, To - torse errant we have wine eligned the elignic to determine the code, i.e. we have not just thought on the on besterer thest mode! -mi then found a circuit to out it into affect, but here often gates simplified the sireuit of the events of the mode, It is sire amine difficult to think that the code entirely in chatmans without one wind of elvenit,

The existencia ment of the mechane in the cert concerned with efficien, multiplie wine and any other operations which it seems morth while so do by meens of smecial direct to mather than through the simple I cillates provided by the suntrol; The Minimax on between adotrol and erishmatic mart in a mother has one, but at any note it is also that the making should at least have an adder and a multiple live , seen if they turn but in the set in he want of the control. Erstablest brestimenavers courte " fo to to a catet of which I should mention that altractations with the me bine shex restrict in oner tel sendant in the binery serie, seems with the condition time. Inputs fore arternally provided don- to in decimal, and so ere outputs intended for homen even rether this for later resonaumstion by the ACE. This is he first suclifies ion, The veloce is that, in white or the intention of himory cording there can be no her on Seeinel working of a brind, because of the relation of the ACL o the universal median. Binary working is the most actural thing to do with any large resle commuter, it is much engine to work in the male of two shes one other, because it is no even to produce mechanisms which have the positions of stability the the positions may then be remarked as representing 0 and:1. Eremine are laven en distrem, Forden Hering alreati, thromitabe



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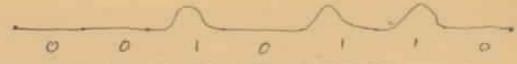
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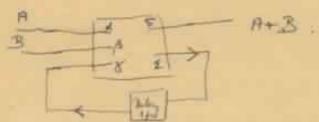
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it can be quite simple. Then we spe fives the elecuit we nearly
here to connect it us with re-discrete and it is an accer. Thust-



It will, be even that we have made use of the footsthat the many process is used in scattion with each disk, and shart and also the fact that the properties of the electrical circuit are invariant under time shifts, at any rate if these are unlittles of the clock period. It wight he would that an above have under my the inomorphism between the cross of time multiplicative courses real numbers to simplify any enterrise, though I should it many other replications of this primaries courses for my

is broken from into the some element or stemm for this, such as so ing one and one. Such of these our miss a microswood, Har numbers will normally nomeds of at bloom dising, so the these two of them can be added in 50 microswoods. Likewise we shall do multiplications in the form of a number of consecutive excitations of one and one or one and mero ste.

There are 1024 such of itime to be fore in - multiplication of one 50 digit musher by enother, so that one wight execut a multiplication to take about a militareout; Actually the unitialist to be used on ACC will take nother over two million onds. This may sound rether long, when the unit operation is only a microsecond, but it equally means that the mechine is frirly sell belenced in this respect, i.e. the multiplication time is not - zerily savious bottlenesk, Further E Committee elect award just to long in writing mushar form and feelding what to do not as they do in estural multiplic mines, and it is fust the arms with the ACE, A gree 5 dec1 of time is execut in notation combare in end out of stores and deciding what is do next. To consider the four elementers processes, subtraction is form by construented to and addition, and division is done by the use of the iteration formal-

4 4 = 44-1 + 44-1 (1- QHALL)

which converges to a provided / 1-auo/
In equation at each stan, nother the convergence is very resid.
This process is of source are record, i.e. the only extrementary required in the delay lines required for storing the release at instructions.

input and output. For this oursons we have shown Hollerith eard equipment. We are able to obtain this without was beving to do any special development work. The averds obtained are not very impressive compared with the speeds of miles the electronic equipment works, but they are suite sufficient intention in all enses where the defaulation is long and the result concise: the interesting event in fact,

the information provided at the also appeared to the Motionish equipment to the high speeds required with the Motionish equipment to the high speeds required with the AGE, but it is really edite way. The Motionish encodes are for any surposes so also as to be counted zero or stop, and the problem reduces to the simple one of converting a give number of stationally given distra into a street of makes. This can be done by makes of a form of electronic co-utator.

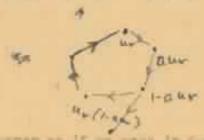
16 and >

Now let up give a closure of the opent top of the mechine, was us been with some problem which has been because in to by a custocur, it will first on to the problem present time mostion where it is arrelated to men whether it is sure to form and scale of the arrelated to men whether it is sure to form and scale of the state of the measuring procedure made out. It then one to the tables presenting measuring that we succeed for example that the gration was to find to bullets solutions of the example.

with initial conditions  $x_1y = 0$ ,  $y' = \alpha$ . This would be reported on a perticular over of solving the exaction

One would need also a table to a object the fine ten F(x,y,z) (in this area F(x,y,z):  $J_{\phi}(x) - xz$ . It which would write involve a table to produce  $J_{\phi}(x)$ , and this are might expect to set off the shelft. A few additional details about the boundary monditions and the municipal distinct length of the ard would have to be dealt with, but much of this detail would be found on the shelf, form like the table for the initial  $J_{\phi}(x)$ . The instructions for the job would therefore consist of a consider ble number to f(x) of the shelf together with a few made un specially for the job in quantion. The state of a reduction would be for the job in quantion. The state of a reduction with a few made un specially for the job in quantion. The state of a reduction will be for the job in quantion. The state of a reduction will be for the job in quantion. The state of a reduction will be for the job in quantion. The state of a reduction will be for the job in quantion. The state of a reduction will be for the job in the state of the state of a reduction of the state of t

Define leaving the multime of the descintion of the maghine I should mention in maximum some of the tradical situations that are not with in one or mains, imminished that affects and them is commented with the fi colouistics of the reciprocal described above, One of them is the idea of the interest colour the area are not the same accurate of area than, and it will therefore be accordated in stories are a few use the same instructions. Thus we so mounts in record a section of the same instructions.



In looks however on if we ware in concess of getting excels to this seems, and uncests to mer out, This The solution of this difficulty involves enother to deal idea that of discriming tion, i.e. of maring formadifferentiation amongton to the results of that the meating itself, instant of assertion to dots swell-ble to the unbernefitherinate proof must, in this sens we include a discrimination to seek ordie, which t-ker as out of the evole hen the w lue of /1- a 4/ sufficiently or II. " in like on second no eigeline over on nerodrome, one caking nerminaton to land -feer even circle. This is a very simple idea, but is of the others importance, The idea of the iterative symie of the medians will also been seen to be rether fundamental when it is re-lived that the mejority of the instructions is the member memory must be oboyed a greet number of times. If was the whole memory were " possessed by instructions, nowe of it being used for numbers or other for, and It such instruction were obeyed mos cale. but took the immest mostlike time, the meeting could outrow in marking for wirts a secondar

Then obeging it. Thei can be used emonger other things for I here justished and assessmentation. In the example share for instance we could assess exhauste a questionable was a if assessment the foreign this question to the instruction that is obeyed at the foreign point that instruction can be completely elected in its effect when finally /- a.c. is reduced to out intention.

Probably he nost important the impole of in the important on tobles to the of subsidiers tobles. Textiticist Cort-in processes are used resortedly in all some of different connections, and -s wish to me the s-se instructions, from the some past of the muory every time. Thus we may use internoletian for the coloulation of a great number of different functions, but we shall elemen use the same interestative of commit instruction to his for intermolation, We have only to think out her this is tobe done once, on? forcet they have it is done, such time or must to do an interagistics on here and to remember the memory agaiting where this table is best, and betwithe appropriate reference is the instruction this which is noted the intermeletion. of for twelvess werest without in a tole for finding releas of Jo [x] and use the intermedation to bis in this may, "a should then are that the date-mode time holds was a substitions to the toble for e-louistims J. in . There is thus a sort of historian of fables. Therement sudemental teblescend rether feregothers the highermore The Intersol tio a table might be you miss on taking its orders from the Jo toble, and reporting its enames book to it. The marter survent analogy is he ever not a very good one, as the e are many more meature then necessity, and long meature have to shere the arms our air.

Refus to 16.

these had will been essentied and cheefed the would be a ten to the input weekenism, which is simily a Hollarith oard feefd, They would be not intathe eard however no a button scened to start the cords moving through, it must be recembered that initially there are so instructions in the machine, and ones soonel facilities are therefore not available, "he first few earts that many in have therefore to be excefully thought ant to do 1 with this affection. They are the initial input angels and are allowed the seem, then they have account in of a control incimation calles will have been not in in the minimum, including sufficient to see its the mechine toward the events I went of saids to I but hear preposed for the job we was doing. Then this has been done there are verious not initiates on a that brings ment, depending on the owy the lob her he m ore round, The mortine have been made to might on through, and serry out the tab, sunshing or grinting all the war ere required, and storaing them all of this h a been done. But more probably it will have been erreneed that the machine pions as moon as the instruction tables have been out in. This allows for the now thilling of checking the content of the memories is convect, and for . number of registions of procedure. It is sherly a subschie moves I for a break, We might class whe a number of oth t brooks. For instance from we mind be interested in contain modification retines of the nervision Q. . restree which were erseriment liv oh: load finites, and it could then be convertest to some ofter each presenter write, out food the north per meter whose in from another arms, Or one which brefer to he as the south all weeds in the bonner out let the fill take themin so it would them, One are do so me without but was wort make un made wind, fash time the marking request in

this way a "cord" or assumes of it biners distract.

Alsolated on mean bulbs. This word indicate the resons.

For stopping. I have already manifemed two now D is resons.

Annihorator A large class of further con this resons is should be provided by the cheeks. Engagether the programming in about its form in such a sep that the ACE is frequently investigation identities which themselve be active to make the about Se.

Thereway but of these absolute that the make he stated and signify as many which describes what about her stated.

In will be seen that the aperthilities of the aber one may do are income. The or or difficulties will be the so intelleges of an empropriete discipline, on the see do not lose treat of what we are doing. We shall beed a number of affiliation libraries types to been us in order.

Finally I should like to mice - for employing an to the reservoir took that first to mean mechinery will have on methods then. I have already mentioned that the sus will do the work of shout 10,000 committeen. It to to be expected therefore that large solve hand-compatible will wis out; Commuters will waill be again employed on wall adequations, such as the substitution of wiless in formulae, secretally but whenever a minute extendention men he administrate to take a appendix of the of work, In will promptly to down by an electronic country inste d. This -111 not becausists everyone & interested to such work having an electronic complete. "I would be suffer massifite to re now to control a distint agendian to make of a telephone I my, There were I tout out with a chinese would be devotaged for one or there are startings, and would east straint a few hindred counts of most, Thereforet of white address of the course the course to the work down by there computers -121 houses constant of resistance which caulf not be reaking by bond economic because of the bests of the undertaking. In order to surely the monthly with those myoblams we shall need a proof manuscinstantished musher of meabour leiens of som editity. Them we have being will be seeded in order to:do the amplitudency conservation the weakless, suit int them into a form for commits ion. There will be consider his sea a for enclosing them a busine seconder in threat a condition on a continuous of the continuous econom means to yive him on lifer of how securets his empuere - A. With Civil Committee themselves into the commo Inner well as comion sense, and the bounds of error must he hard on some proved the medition, We need suclouts to find the encounters tecourities for us. The incounlines

need not elvery be evaluate, i.e. one need not have then in
such a form that we can tell, before the evicual tion
starte, and union only reneil and none, he his the
error will be. The error exhaultains now be a serious
part of the Adhie dutied. To an evicat is made non tile
to replace the seminates of error by statistical estimates
obtained by repositive the ish, and doing the rounding off
differently each time, omittalling it by some renderm
element, now electronic roulettel hash, luck statistical
machine
in size him, and size no indication of what each he can be
in turns out that the errors are intolerable 1 are. The
estimates and that the errors are intolerable 1 are. The

Analysis is tue one of the suppose for thick or their and good withen flatent, Comphir was bine those who come in a numerical with the ACC will be Alejzer into its mestage and its permunts. The section will a to but instruction t blue for it, thinking we decrear and de pay were of write to Its nervanty will fond it with parts on it wells for them. They will mut gight any ports that go wrome, They will assemble dess that it requires, in flow the server o -111 take the place of links, is time goes by the coloulator their will bed toke over the functions both of manager and of service, The sacrute will be realreed by much mis-1 and electric-1 limbs and sense and me. On might for intrace provide arminesthess surve Followers to surble data to be taken Al eat from survey facts of h the cirls weed of william and would themen sends. The movement and limits to got -- n-leve becomes an agent or our technique become at "Il staractured it becomes now like to dewler a system of last until tobler which will entile the electronic committee to do in for itself, of m - broken however that the menture will - fure to dathis, They may be smullling to let their fabr by stoler from them in this way, in that were then would surround the whole of their work with western and

ent denotes runse-time were made. I think that a receive of this kind is a secretion of the kind is a secretion of the formal secretion of the constitution of the constitution of the constitution of the constitution of the secretion of the constitution o

I armed that dising exemption mechanic will eventually aticulate a consider ble interest in symbolic locks and methoratical philosophy. The language in which one communication with share mechines, i.e. the leadures of Instruction toller, forms a seri of avabable lonie. The making interprets whatever it is told in a cuite definite weakharmarks meanes without war sense of husbar or arms of economics. Unionis communicating with it one same exectly what one making, trouble is bound to mounty totally one sould commutate with these mentioned in any learning movided in making erest tensures, i.e. in writerials and should be able to ensurfacts in our embolic incis, browled that the machine were often instruction tobias which would enclose it to interpret that hosted system. This whould now that there will be much have openile-1 sense for locked section then there has been to the west, he records mathematical shill people, close the mediane will be dote were and more mothereties thomaston, the beargrounded by interestry wentum of overity of the human Internal -122 he defrom further and further into the obliganoble-1 condition of what sen in wrinefule be fore ate.

Home attempts will probably be wide to not the machines to Ze do De do notupl manipulations of machinetical formulas. When will proute the development of a special logical system for the nursuas. This system about recemble normal machinetical procedure as alonely convince and alonely convince and approximate procedure. But of the same time whould be an insubjections as appoints.

"A her been said that committee mealines con only corry out the processes that they are instructed to As. This is continty tons to the owner that if then it complities other then that there early festimated then then have been mids aren referring. It to after from their strangers before they red the intention to sectionation there making to the first feet may in to treat them an always, divine them unity take which have been throught out in Sec. 13. Take much the the user of the mechan fully underer at once is owing of "II the tim, "a till the whaman neckines have note been then in this way, was in it passes on the they would siven be used in each a measury Trainmenthingstransmenticks met up a machine with certain initial instruction to les, so constructed that there tables sight as occusion, if some search asper, notify those sebles. One arm invalue that wines the unchias had been ones time for tame time, the instructions would have blisted out of will essentition; But nevertheless still be such th t one would have the steels then the escatte over this delegate year resultability established on this to might call be continue continue of the trees designed when the seatilet were stilled that the burden more efficient memory. To much a some less really below to while the form the process of the wahine hedge here foresten them its original instructions were out in. It wall be like - gupil who had learnt much from his moster, but led soled made one by his man work, Then this beyone I feed this our is allied to reard the machine as showing intelligence, investministrationals in soon or you say provide a recommenty is not memory convenies in should be you like to begin to experiment on these lines. The memory or mellowfahe human brein in prob hip of the order of ten thousend million digita. But now of this is ero; bly used in resolution visual improvious, and other countrilleds mortaful were. Doe which receptably home to be white to near name re I prescreen with e for million dicits; somesiable if one goafined wank one's investigations to note rether

Limited field such to the come of cherg, "I rould probably
to make over to find instruction tobies which would enable
the a distriction servings an ever so player. Indeed Shemaka
of Hell trelephone incommodes notice as about he has sun
acres playing by mile of thunks the shill of his comments is
not wasted. But a small not consider such a via ory very
simulficant, what we would not consider such a via ory very
simulficant. That we would not consider such a via ory very
simulficant. The con Dillity of let inc the machine alter in
one instructions organizes the medical for this, \*\*existings\*
but this of entrue for out on the probable for this, \*\*existings\*

23

It while he would that there is a producer I contact that. tive in the idea of a mechine with intelligence. It is serve into ares that 'assing like a making', has become appointment with look of adequability, but thisrin the poster for this is abrices, Sections in the peat here had very little stars a, and there has been at question of the marking having any discretion. The exponent wight however be out this a more as resalve form, "a hen for inte non hern shown the with serveds lovin ! content there exerces one be no to thing which will distinguish proveble formules of the everen from unerowavle, i.e. then there is no sees that the mobiles can easily which will divide propositions with east into into the a too classes, Thus if there while is note for this our one is much in nome was wifeld he give wat washing On the other hand if a is themstising in confronted with while a problem he would nearble armind a find new mathem of aroun, so that he maket e-establis to be while to me oh a decision about age siven formula, This sould be the avenuest, Advisor it I would now that fair miny must be along to be working. Instead of the smoother string as sammer on sould arrows a that in other manufactured over commons, But the bound on hemotistics out? liberies who blunders then train but no techniques , Rettin t is easy for an id-record these by ofers on not open toand dive him emother channe, but the making would be allowed an marcy, in other cords then, if a machine to expected to

be infellible, it wennet also be intelligent. There are several methemotical theorems which may almost executly that, But these theorem ser nothing about her much intelligence may be displayed if a machine when no metamon of infaithful the medianet, I been enthemnical merentages has also undersome an extensive training . This are inter new beregarded on not unlike outling instruction tables into a mobiles. One must the efore not expect - wealing to do e THEY FROM SHEEL OF BUILDING UP OF IMPROVEDING A RIVE OF ITS men. He were while were much to the bade aftenned when why should we erough more of a machinet Putting the some saint differently, the sweller must be allowed to here contrat -ith how a believe in order that it mer about itself to that straterds. The come of above mor sections be rether extends for this purpose, at the core of the machines assumed will swimm-tirelly provide this contrat.