$$+\left[\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right]$$

$$= 11106 + 2 = 11108$$

12.(A); $3\frac{2}{3}+5\frac{1}{2}+6\frac{3}{5}+7\frac{2}{5}+3\frac{7}{10}$

$$= [\,3+5+6+7+3\,]+\left[\frac{2}{3}+\frac{1}{2}+\frac{3}{5}+\frac{2}{5}+\frac{7}{10}\right]$$

$$= 24+\left[\frac{20+15+18+12+21}{30}\right]$$

$$= 24+\frac{86}{30} = 24+\frac{43}{15}$$

$$= 24+2\frac{13}{15} = 26\frac{13}{15}$$

13.(B);

$$999\frac{1}{7}+999\frac{2}{7}+999\frac{3}{7}+999\frac{4}{7}+999\frac{5}{7}+999\frac{6}{7}$$

$$= 999\times 6\left[\frac{1}{7}+\frac{2}{7}+\frac{3}{7}+\frac{4}{7}+\frac{5}{6}+\frac{6}{7}\right]$$

$$= 999\times 6+\frac{21}{7} = 5997$$

14.(C); $3\frac{1}{3}+33\frac{1}{3}+333\frac{1}{3}+3333\frac{1}{3}+33333\frac{1}{3}$

$$= [\,3+33+333+3333+33333\,]+\left[\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right]$$

$$= 37035+1\frac{2}{3} = 37036\frac{2}{3}$$

15.(B); $\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}$

$$= \frac{1}{\underbrace{2}_{\text{First value}}\times 3}+\frac{1}{3\times 4}+\ldots\ldots+\frac{1}{8\times\underbrace{9}_{\text{Last value}}}$$

Using formula:-

$$\frac{1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$$

$$= \frac{1}{1}\left[\frac{1}{2}-\frac{1}{9}\right] = \frac{7}{18}$$

16.(C); $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots\ldots\ldots+\frac{1}{110}$

$$= \frac{1}{\underbrace{1}_{\text{First value}}\times 2}+\frac{1}{1\times 3}+\frac{1}{1\times 4}+\ldots\ldots\frac{1}{10\times\underbrace{11}_{\text{Last value}}}$$

Using formula:-

$$\frac{1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$$

$$= \frac{1}{1}\left[1-\frac{1}{11}\right] = \frac{10}{11}$$

17.(A); $\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\ldots\ldots+\frac{1}{132}$

$$= \frac{1}{\underbrace{4}_{\text{first value}}\times 5}+\frac{1}{5\times 6}+\frac{1}{6\times 7}\ldots\ldots\frac{1}{11\times\underbrace{4}_{\text{Last value}}}$$

Using formula:-

$$\frac{1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$$

$$= \frac{1}{1}\left[\frac{1}{4}-\frac{1}{12}\right] = \frac{2}{12} = \frac{1}{6}$$

18.(A); $\frac{1}{9}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots\ldots+\frac{1}{72}$

$$= \frac{1}{9}+\left[\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\ldots\ldots+\frac{1}{8\times 9}\right]$$

Using formula:-

$$\frac{1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$$

$$= \frac{1}{9}+\frac{1}{1}\left[\frac{1}{2}-\frac{1}{9}\right] = \frac{1}{9}+\frac{1}{2}-\frac{1}{9} = \frac{1}{2} = 0.5$$

19.(D); $\frac{1}{11}+\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+----\frac{1}{110}$

$=$

$$\frac{1}{1}\left[\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+----+\frac{1}{10\times11}\right]$$

Using formula:-

$$\frac{1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$$

$= \frac{1}{11}+\frac{1}{1}\left[\frac{1}{1}-\frac{1}{11}\right]$

$= 1$

20.(B); $\frac{1}{5.9}+\frac{1}{9.13}+\frac{1}{13.17}+------\frac{1}{61.65}=?$

Using formula:-

$$\frac{1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$$

$= \frac{1}{4}\left[\frac{1}{5}-\frac{1}{65}\right]=\frac{1}{4}\left[\frac{13-1}{65}\right]$

$= \frac{1}{4}\left[\frac{12}{65}\right]=\frac{3}{65}$

21.(C); $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+----+\frac{1}{23\times26}$

Using formula:-

$$\frac{1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$$

$= \frac{1}{3}\left[\frac{1}{2}-\frac{1}{26}\right]=\frac{1}{3}\left[\frac{13-1}{26}\right]=\frac{1}{3}\times\frac{12}{26}=\frac{2}{13}$

22. (A);

$$\frac{3}{1^2\times2^2}+\frac{5}{2^2\times3^2}+\frac{7}{3^2\times4^2}+\frac{9}{4^2\times5^2}+----+\frac{19}{9^2\times10^2}=?$$

Difference of denominator $[2-1=1]$

$$\frac{1}{1}\left[\frac{1}{1}-\frac{1}{100}\right]=\frac{99}{100}$$

23.(A);

$$\frac{5}{2^2\times3^2}+\frac{7}{3^2+4^2}+\frac{9}{4^2+5^2}+----+\frac{39}{19^2\times20^2}$$

solving by method which is used in q.no. 22,

Difference of denominator = $[3-2=1]$

$= \frac{1}{1}\left[\frac{1}{4}-\frac{1}{400}\right]=\frac{99}{400}$

24.(D); $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)+----\left(1+\frac{1}{x}\right)$

$$= \frac{3}{\underbrace{2}_{\text{Remaining value}}}\times\frac{4}{3}\times\frac{5}{4}\times.....\times\frac{x}{x-1}\times\frac{\overbrace{x+1}^{\text{Remaining value}}}{x}$$

$= \frac{x+1}{2}$

25.(C); $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)----\left(1+\frac{1}{29}\right)$

using above formula [used in q.n. 24]:-

$= \frac{29+1}{2}=\frac{30}{2}=15$

26.(B); $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)------\left(1-\frac{1}{x}\right)=?$

$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}------\frac{1}{x-1}\times\frac{x-1}{x}=\frac{1}{x}$

27.(A); $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)-----\left(1-\frac{1}{19}\right)$

$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}-\frac{17}{18}\times\frac{18}{19}=\frac{1}{19}$

28.(B); $\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{1}{7}\right)----\left(1-\frac{1}{39}\right)$

$= \frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}\times----\frac{38}{39}=\frac{4}{39}$

29.(D); $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)----\left(1-\frac{1}{70}\right)=\frac{x}{70}$

$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times----\frac{69}{70}=\frac{x}{70}$

$= \frac{1}{70}=\frac{x}{70}$

$\therefore \quad x=1$

30.(C);

$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right).....\left(1-\frac{1}{90^2}\right)$

$= \left(\frac{2^2-1}{2^2}\right)\left(\frac{3^2-1}{3^2}\right)\left(\frac{4^2-1}{4^2}\right).....$

$.....\left(\frac{89^2-1}{89^2}\right)$

$\left(\frac{90^2-1}{90^2}\right)$

$= \left(\frac{(2-1)(2+1)}{2\times2}\right)\times\left(\frac{(3-1)(3+1)}{3\times3}\right)\times$

$\left(\frac{(4-1)(4+1)}{4\times4}\right)\times.....\times\left(\frac{(89-1)(89+1)}{89\times89}\right)$

$\times\left(\frac{(90-1)(90+1)}{90\times90}\right)$

$= \left(\frac{1\times3}{2\times2}\right)\left(\frac{2\times4}{3\times3}\right)\left(\frac{3\times5}{4\times4}\right).....$

$.....\left(\frac{88\times90}{89\times89}\right)\left(\frac{89\times91}{90\times90}\right)$

$= \frac{1}{2}\times\frac{91}{90}=\frac{91}{180}$

31.(A); $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}}\left.\vphantom{\dfrac{1}{1+\frac{1}{2}}}\right\}\frac{3}{2}$

$= \dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{\frac{3}{2}}}}}\left.\vphantom{\dfrac{1}{1+\frac{1}{\frac{3}{2}}}}\right\}= 1+\frac{2}{3}=\frac{5}{3}$

$= \dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{\frac{5}{3}}}}}\left.\vphantom{\dfrac{1}{1+\frac{1}{\frac{5}{3}}}}\right\}= 1+\frac{3}{5}=\frac{8}{5}$

$= \dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{\frac{8}{5}}}}\left.\vphantom{\dfrac{1}{1+\frac{1}{\frac{8}{5}}}}\right\}= 1+\frac{5}{8}=\frac{13}{8}$

$= \frac{1}{\frac{13}{8}}=\frac{8}{13}$

32.(A); $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{2}{3}}}}\left.\vphantom{\dfrac{1}{1+\frac{2}{3}}}\right\}\frac{5}{3}$

$= 1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{\frac{5}{3}}}}\left.\vphantom{\dfrac{1}{1+\frac{1}{\frac{5}{3}}}}\right\}= 1+\frac{1}{1}\times\frac{3}{5}=1+\frac{3}{5}=\frac{8}{5}$

$= 1+\dfrac{1}{1+\dfrac{1}{\frac{8}{5}}}\left.\vphantom{\dfrac{1}{1+\frac{1}{\frac{8}{5}}}}\right\}= 1+\frac{5}{8}=\frac{13}{8}$

$= 1+\frac{1}{\frac{13}{8}}= 1+\frac{8}{13}= \frac{13+8}{13}$

$= \frac{21}{13}$

33.(B); $2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{3}}}}\Bigg\}\frac{7}{3}$

$= 2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\frac{7}{3}}}} = 2+\frac{3}{7}=\frac{17}{7}$

$= 2+\cfrac{1}{2+\cfrac{1}{\frac{17}{7}}} = 2+\frac{7}{17}=\frac{41}{17}$

$= 2+\cfrac{1}{\frac{41}{17}}$

$= 2+\frac{17}{41}=\frac{99}{41}$

34.(D); $3+\cfrac{2}{3+\cfrac{2}{3+\cfrac{2}{3+\frac{2}{3}}}}\Bigg\}\frac{11}{3}$

$= 3+\cfrac{2}{3+\cfrac{2}{3+\cfrac{2}{\frac{11}{3}}}} = 3+\frac{2}{1}\times\frac{3}{11}=3+\frac{6}{11}=\frac{39}{11}$

$= 3+\cfrac{2}{3+\cfrac{2}{\frac{39}{11}}} = 3+\frac{2}{1}\times\frac{11}{39}=3+\frac{22}{39}=\frac{139}{39}$

$= 3+\cfrac{2}{\frac{139}{39}} = 3+\frac{78}{139}=3\frac{78}{139}$

35.(A); $3+\cfrac{5}{4+\cfrac{6}{3+\cfrac{2}{2+\frac{3}{5}}}}\Bigg\}\frac{13}{5}$

$= 3+\cfrac{5}{4+\cfrac{6}{3+\frac{2}{\frac{13}{5}}}} = 3+\frac{2}{1}\times\frac{5}{13}=3+\frac{10}{13}=\frac{49}{13}$

$= 3+\cfrac{5}{4+\cfrac{6}{\frac{49}{13}}} = 4+\frac{6}{1}\times\frac{13}{49}=4+\frac{78}{49}=\frac{274}{49}$

$= 3+\cfrac{5}{\frac{274}{49}} = 3+\frac{5}{1}\times\frac{49}{274} = 3+\frac{245}{274} = 3\frac{245}{274}$

36.(C); $x=\cfrac{3}{2+\cfrac{2}{2+\cfrac{2}{2+\frac{2}{3}}}}=\frac{8}{3}$

$= \cfrac{3}{2+\cfrac{2}{2+\cfrac{2}{\frac{8}{3}}}} = 2+\frac{2}{1}\times\frac{3}{8}=2+\frac{3}{4}=\frac{11}{4}$

$= \cfrac{3}{2+\cfrac{2}{\frac{11}{4}}} = \cfrac{3}{2+\frac{2}{1}\times\frac{4}{11}}$

$= \cfrac{3}{2+\frac{2}{1}\times\frac{4}{11}} = 2+\frac{8}{11}=\frac{30}{11}$

$= \cfrac{3}{\frac{30}{11}} = \frac{3}{1}\times\frac{11}{30} = \frac{11}{10}=1.1$

37.(A) $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{x}}}} = \frac{x+1}{x}$

$$= 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{x+1}{x}}}\Bigg\} 1+\frac{1}{1}\times\frac{x}{x+1}=1+\frac{x}{x+1}$$

$$= \frac{x+1+x}{x+1}=\frac{2x+1}{x+1}$$

$$= 1+\cfrac{1}{1+\cfrac{1}{\cfrac{2x+1}{x+1}}}\Bigg\} = 1+\frac{1}{1}\times\frac{x+1}{2x+1}=1+\frac{x+1}{2x+1}$$

$$= 1+\cfrac{1}{1+\cfrac{1}{\cfrac{2x+1}{x+1}}}\Bigg\} = \frac{2x+1+x+1}{2x+1}=\frac{3x+2}{2x+1}$$

$$= 1+\frac{1}{\cfrac{3x+2}{2x+1}}=1+\frac{2x+1}{3x+2}=\frac{3x+2+2x+1}{3x+2}$$

$$= \frac{5x+3}{3x+2}$$

38.(D); $$\cfrac{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\frac{1}{x}}}}}{\frac{x-1}{x}}$$

$$= 1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{\cfrac{x-1}{x}}}}\Bigg\} 1-\frac{x}{x-1}=\frac{x-1-x}{x-1}=\frac{-1}{x-1}$$

$$= 1-\frac{1}{1-\cfrac{x-1}{-1}} = 1-\frac{1}{1+x-1} = 1-\frac{1}{x} = \frac{x-1}{x}$$

39.(B); $$\left[1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}\right]\div 1\frac{4}{7}$$

$$= \left[1+\cfrac{1}{1+\cfrac{1}{\frac{4}{3}}}\right]\div\frac{11}{7} = \left[1+\cfrac{1}{1+\frac{3}{4}}\right]\times\frac{7}{11}$$

$$= \left[1+\frac{4}{7}\right]\times\frac{7}{11} = \frac{11}{7}\times\frac{7}{11}=1$$

40.(A); $$x+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}} = 12$$

$$12 = x+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{\frac{21}{5}}}} = x+\cfrac{1}{2+\cfrac{1}{3+\frac{5}{21}}}$$

$$12 = x+\cfrac{1}{2+\cfrac{1}{\frac{68}{21}}} = x+\frac{1}{\frac{157}{68}} = x+\frac{68}{157}$$

$$x = 12-\frac{68}{157}$$

$$157x = 1884 - 68 = 1816$$

$$x = \frac{1816}{157}$$

Recurring Decimal

This is such a decimal fraction in which one or more decimal digits are repeated again and again.

1. Find the value of -

(i) $0.\overline{3}$ (ii) $0.\overline{21}$

(iii) $0.\overline{123}$ (iv) $0.3\overline{6}$

(v) $0.3\overline{12}$ (vi) $0.71\overline{6}$

(vii) $0.31\overline{2}$ (viii) $1.\overline{27}$

(ix) $2.\overline{357}$ (x) $0.1\overline{36}$

2. Find the fractional value of 0.7777..... $\propto$ = ?

(A) $\frac{7}{9}$ (B) $\frac{77}{90}$

(C) $\frac{777}{900}$ (D) $\frac{7777}{9000}$

3. Find the rational value of 0.7131313... $\propto$ = ?

(A) $\frac{706}{999}$ (B) $\frac{713}{999}$

(C) $\frac{353}{495}$ (D) $\frac{353}{999}$

4. Find the fractional value of

$3.\overline{12} + 3.\overline{11} + 3.\overline{14}$ = ?

(A) $3.\overline{37}$ (B) $9.\overline{38}$

(C) $9.\overline{37}$ (D) $6.\overline{37}$

5. Find the value of $5.\overline{12} + 3.\overline{21} + 4.\overline{31}$ = ?

(A) $12\frac{64}{99}$ (B) $12\frac{74}{99}$

(C) $12\frac{77}{99}$ (D) $12\frac{84}{99}$

6. The difference of $5.\overline{76}$ and $2.\overline{3}$ is.

(A) $5.\overline{54}$ (B) $5.\overline{73}$

(C) $5.\overline{46}$ (D) $5.\overline{43}$

7. $(0.\overline{11} + 0.\overline{22}) \times 3$ is equal to:

(A) 3 (B) $1.\overline{9}$

(C) 1 (D) $1.\overline{3}$

8. Find the value of $(3.\overline{68} - 2.\overline{79})$:

(A) $2.\overline{45}$ (B) $0.\overline{49}$

(C) $0.\overline{88}$ (D) $0.\overline{48}$

9. Find the square root of $0.\overline{4}$ is-

(A) $0.\overline{8}$ (B) $0.\overline{6}$

(C) $0.\overline{7}$ (D) $0.\overline{9}$

10. Find the fractional value of 0.4777..... $\propto$

(A) $\frac{15}{90}$ (B) $\frac{15}{99}$

(C) $\frac{43}{90}$ (D) $\frac{2}{3}$

Recurring Decimals (Solution)

1. Find the value of-

(i) $0.\overline{3}$ or 0.33333..... $\propto$

Let $x = 0.\overline{3}$ _____ *(i)*

multiply both the sides by 10.

$10x = 10 \times 0.\overline{3}$

$10x = 3.\overline{3}$

$10x = 3 + .\overline{3}$ (Using equation (i))

$10x = 3 + x$

$9x = 3$

$x = \frac{3}{9}$

$x = \frac{1}{3}$

$\therefore \quad 0.\overline{3} = \frac{1}{3}$ (So $\because x = 0.\overline{3}$)

This means $0.\overline{3} = \frac{3}{9}$

Hence now we can say that $0.\overline{33} = \frac{33}{99}$

and

$0.\overline{333} = \frac{333}{999}$.

Note:- $0.3\overline{3} = \frac{33-3}{90}$ and $0.3\overline{33} = \frac{333-3}{990}$

(ii) $0.\overline{21} = \frac{21}{99} = \frac{7}{33}$

(iii) $0.\overline{123} = \frac{123}{999} = \frac{41}{333}$

(iv) $0.3\overline{6} = \frac{36-3}{90} = \frac{33}{90} = \frac{11}{30}$

(v) $0.3\overline{12} = \frac{312-3}{990} = \frac{309}{990} = \frac{103}{330}$

(vi) $0.71\overline{6} = \frac{716-71}{900} = \frac{645}{900} = \frac{43}{60}$

(vii) $0.3\overline{12} = \frac{312-31}{900} = \frac{281}{900}$

(viii) $1.\overline{27} = 1\frac{27}{99} = 1\frac{3}{11}$

(ix) $2.\overline{357} = 2\frac{357}{999} = 2\frac{119}{333}$

(x) $1.1\overline{36} = 1\frac{136-1}{990} = 1\frac{135}{990} = 1\frac{3}{22}$

2.(A); $0.7777..... \propto = 0.\overline{7}$

$0.\overline{7} = \frac{7}{9}$

3.(C); $0.7131313..... \propto = 0.7\overline{13}$

$0.7\overline{13} = \frac{713-7}{990} = \frac{706}{990} = \frac{353}{495}$

4.(C); $3.\overline{12} + 3.\overline{11} + 3.\overline{14} = 9 + .\overline{37} = 9.\overline{37}$

5.(A); $5.\overline{12} + 3.\overline{21} + 4.\overline{31} = 5\frac{12}{99} + 3\frac{21}{99} + 4\frac{31}{99}$

$= (5+3+4) + \frac{64}{99} = 12\frac{64}{99}$

6.(D); $5.\overline{76} - 2.\overline{3} = 5\frac{76}{99} - 2\frac{3}{9} = 5\frac{43}{99} = 5.\overline{43}$

7.(C); $\left(0.\overline{11} + 0.\overline{22}\right) \times 3$

$= \left[\frac{11}{99} + \frac{22}{99}\right] \times 3 = \frac{33}{99} \times 3 = \frac{1}{3} \times 3 = 1$

8.(C); $3.\overline{68} - 2.\overline{79}$

$= 3\frac{68}{99} - 2\frac{79}{99} = (3-2) + \frac{68}{99} - \frac{79}{99} = 1 - \frac{11}{99}$

$= 0.\overline{88}$

9.(B); $\sqrt{0.\overline{4}} = \sqrt{\frac{4}{9}} = \frac{2}{3} = 0.\overline{6}$

10.(C); $0.4777..... \propto = 0.4\overline{7} = \frac{47-4}{90} = \frac{43}{90}$

Remainder based questions

1. When 69^{67} is divided by 68, then what is the remainder?
2. If 17^{200} is divided by 18, then remainder is-
3. If 193^{193} is divided by 192, then what is remainder?
4. When 39^{118} is divided by 40, then what is the remainder?
5. If $(67^{67} + 67)$ is divided by 68, then what is the remainder?
6. Find the remainder if-

(i) $(59^{167} + 5) \div 60$

(ii) $(37^{819} + 3) \div 36$

(iii) $(379^{169} + 13) \div 380$

(iv) $(168^{169} + 5) \div 169$

(v) $(177^{168} + 17) \div 176$

(vi) $(38^{99} + 9) \div 39$

(vii) $(77^{778} + 13) \div 78$

(viii) $(987^{567} + 37) \div 988$

7. Find the remainder if 3^{90} is divisible by 28.
8. What will be the remainder if 2^{31} is divided by 5 ?

1. To find out the remainder when $69^{67} \div 68$.

Note: If the divisor is less than 1 from base of the polynomial then remainder is always 1.

Remainder = 1

2. $17^{200} \div 18$, Remainder = ?

Note: If the divisor is more than 1 than the base of the polynomial.

Then in case of even power, remainder will be1.

In case of even power, remainder will be1.

$\therefore$ In case of odd power, base will be remainder.

3. $193^{193} \div 192$ Remainder =?

Remainder = 1

4. $39^{118} \div 40 \Rightarrow$ even base

Remainder =1

5. $(67^{67} + 67) \div 68$

$\frac{67^{67}}{68} + \frac{67}{68} \Rightarrow (67 + 67) \div 68 \Rightarrow 66$ is the Remainder

6.(i) $(59^{167} + 5) \div 60$

$\Rightarrow \frac{59^{167}}{60} + \frac{5}{60} \Rightarrow \frac{59+5}{60} \Rightarrow 4$ is the Remainder

'or'

Trick:- $(59^{167} + 5) \div 60$

$\downarrow \quad \swarrow -1 \downarrow$

$(59+1)$ 4 Remainder

$= 12$

(iv) $(168^{169} + 5) \div 169$

$(168 + 5) \div 4$

(v) $(177^{168} + 17) \div 176$

$(1 + 17) = 18$

(vi) $(38^{99} + 9) \div 39$

$(38 + 9) \div 39$

$= 8$

(vii) $(77^{778} + 13) \div 78$

$(1 + 13) = 14$

(viii) $(987^{567} + 37) \div 988$

$(987 + 37) = 36$

7. $3^{90} \div 28$

or

$(3^3)^{30} \div 28 \Rightarrow 27^{30} \div 28$

↓

Remainder = 1

8. $2^{31} \div 4$

↓

$(2^{30} \times 2^1) \div 5$

$\left[(2^2)^{15} \times 2^1\right] \div 5 \Rightarrow (4 \quad +2) \quad \div 5$

$(4 \times 2) \div 5$

$8 \div 5$

Remainder = 3

9. $7^{113} \div 50$

↓

$\left(\underbrace{7^{112}}_{7^{2\times56}} \times 7^1\right) \div 50$

$[(49)^{56} \times 7)] \div 50$

$1 \times 7 = 7$ Remainder

10. $(x^{11}+1) \div (x+1)$

$= (x^{11}+1^{11}) \div (x+1)$

Remainder = 0, Because, (x^n+y^n) is always divisible by $(x+y)$ only when n is odd.

11. $6^{2n} - 1$

$6^{2n} - 1^{2n}$

$6 + 1 = 7$

$(x^n - y^n)$ is always divisible by (x+y) only when n is even.

12. $(17^{37} + 29^{37}) \div 23$

$= [(23-6)^{37} + (23+6)^{37}] \div 23$

$(-6)^{37} + (6)^{37} \div 23$

Then the remainder will be '0'.

Unit Digit

1. Find the unit digit of the number 3^{40}.
2. Find the unit digit of the number $(73)^{98} + (39)^{87} + (76)^{99}$.
3. Find the unit digit of the number $7^{37} \times 38^{54} \times 817^{93} \times 777^{777}$.
4. Find the unit digit of the number $817^{938} + 776^{532} + 985^{67} + 813^{353}$.
5. Find the unit digit of the number $98^{98} + 73^{73} + 89^{125} + 679^{537}$.
6. Find the unit digit of the number $234^{94} + 235^{95} + 236^{96} + 237^{97}$.
7. Find the unit digit of the number $3^{91} \times 7^{67} \times 5^{37} \times 12^{38}$.
8. Find the unit digit of the number $37^{39} \times 97^{117} \times 384^{37} \times 397^{397} \times 533^{33}$.
9. Find the unit digit of the number $367^{98} \times 53^{687} \times 134^{134} \times 59^{167}$.
10. Find the unit digit of the number $738^{51} + 938^{532} + 713^{537} + 985^{713}$.
11. Find the unit digit of the number $532^{375} + 819^{532} + 877^{77} + 985^{325} - 112^{18}$.
12. Find the unit digit of the number $3^{81} \times 9^{72} \times 7^{81} \times 9^{99}$.

Unit Digit (Table)

Unit digit of any Number	Unit digit of numbers having power in the form of 4n+1	of 4n+2	of 4n+3	of 4n
0	0	0	0	0
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

Unit Digit (Solution)

1. $3^{40} = 3^{4 \times 10}$

   So, Unit digit of 3^{40} is 1.

2. $(73)^{98} + (39)^{87} + (76)^{99}$ = $(73)^{4\times24+2} + (39)^{4\times21+3} + (76)^{4\times24+3}$

   Unit digit of number will be same as = unit digit of $\underbrace{9 + 9 + 6}_{4}$

   So, Unit digit of number will be = 4

3. $7^{37} \times 38^{54} \times 817^{93} \times 777^{777}$ = $(7)^{4a+1} \times (38)^{4b+2} \times (817)^{4c+1} \times (777)^{4d+1}$

   Unit digit of number will be same as = unit digit of $\underbrace{7 \times 4 \times 7 \times 7}_{2}$

   So, Unit digit of number will be = 2

4. $(871)^{938} + (776)^{532} + (985)^{67} + (813)^{353}$ = $(817)^{4a+2} + (776)^{4b} + (985)^{4c+3} + (813)^{4d+1}$

   Unit digit of number will be same as = unit digit of $\underbrace{9 + 6 + 5 + 3}_{3}$

   So, Unit digit of number will be = 3

5. $(98)^{98} + (73)^{73} + (89)^{125} + (679)^{537}$ = $(98)^{4a+2} + (73)^{4b+1} + (89)^{4c+1} + (679)^{4d+1}$

Unit digit of number will be same as = unit digit of $\underbrace{4 + 3 + 9 + 9}_{5}$

So, Unit digit of number will be = 5

6. $(234)^{94} + (235)^{95} + (236)^{96} + (237)^{97}$ = $(234)^{4a+2} + (235)^{4b+3} + (236)^{4c} + (237)^{4d+1}$

Unit digit of number will be same as = unit digit of $\underbrace{6 + 5 + 6 + 7}_{4}$

so, Unit digit of number will be = 4

7. $3^{91} \times 7^{67} \times 5^{37} \times 12^{38}$ = $(3)^{4a+3} \times (7)^{4b+3} \times (5)^{4c+1} \times (12)^{4d+2}$

Unit digit of number will be same as = unit digit of $\underbrace{7 \times 3 \times 5 \times 4}_{0}$

So, Unit digit of number will be = 0

8. $37^{39} \times 97^{117} \times 384^{37} \times 397^{397} \times 533^{33}$ = $(37)^{4a+3} \times (97)^{4b+1} \times (384)^{4c+1} \times (397)^{4d+1} \times (533)^{4c+1}$

Unit digit of number will be same as = unit digit of $\underbrace{3 \times 7 \times 4 \times 7 \times 3}_{4}$

So, Unit digit of number will be = 4

9. $(367)^{98} \times (53)^{687} \times (134)^{134} \times (59)^{167}$ = $(367)^{4a+2} \times (53)^{4b+3} \times (134)^{4c+2} \times (59)^{4d+3}$

Unit digit of number will be same as = unit digit of $\underbrace{9 \times 7 \times 6 \times 9}_{2}$

So, Unit digit of number will be = 2

10. $738^{51} + 938^{532} + 713^{537} + 985^{713}$ = $(738)^{4a+3} + (938)^{4b} + (713)^{4c+1} + (985)^{4d+1}$

Unit digit of number will be same as = unit digit of $\underbrace{2 + 6 + 3 + 5}_{6}$

So, Unit digit of number will be = 6

11. $532^{375} + 819^{532} + 877^{77} + 985^{325} - 112^{18}$ = $(532)^{4a+3} + (819)^{4b} + (877)^{4c+1} + (985)^{4d+1} - (112)^{4c+2}$

Unit digit of number will be same as = unit digit of $\underbrace{8 + 1 + 7 + 5 - 4}_{7}$

So, Unit digit of number will be = 7

12. $3^{81} \times 9^{72} \times 7^{81} \times 9^{99}$

$= (3)^{4a+1} \times (9)^{4b} \times (7)^{4c+1} \times (9)^{4d+3}$

Unit digit of number will be same as = unit digit of $3 \times 1 \times 7 \times \underbrace{9}_{9}$

So, Unit digit of number will be = 9

Comparison of surds of distinct order

1. Which surd is larger /largest between / among

(i) $\sqrt[3]{3}$ and $\sqrt[4]{5}$

(ii) $\sqrt[3]{6}$ and $\sqrt[4]{8}$

(iii) $\sqrt[3]{4}$, $\sqrt{2}$, $\sqrt[4]{3}$ and $\sqrt[6]{9}$

(iv) $\sqrt{5}$, $\sqrt[3]{6}$, $\sqrt[6]{7}$ and $\sqrt[12]{30}$

(v) $\left(\frac{1}{2}\right)^{\frac{1}{2}}$ and $\left(\frac{2}{3}\right)^{\frac{1}{3}}$

2. Which surd is smaller/smallest between/ among

(i) $\sqrt[4]{5}$ and $\sqrt[3]{7}$

(ii) $\sqrt{5}$ and $\sqrt[3]{2}$

(iii) $\left(\frac{1}{3}\right)^{\frac{1}{2}}$ and $\left(\frac{2}{3}\right)^{\frac{1}{3}}$

(iv) $\sqrt[3]{5}$, $\sqrt[3]{6}$, $\sqrt[4]{7}$ and $\sqrt[6]{30}$

(v) $\sqrt[4]{5}$, $\sqrt{7}$, $\sqrt[10]{13}$ and $\sqrt[20]{29}$

3. Arrange the following surds in ascending order.

(i) $\sqrt[4]{3}$, $\sqrt[6]{7}$ and $\sqrt[12]{48}$

(ii) $\sqrt{5}$, $\sqrt[3]{11}$ and $2\sqrt[6]{3}$

(iii) $\sqrt[4]{6}$, $\sqrt{2}$ and $\sqrt[3]{4}$

(iv) $\sqrt{5}$, $\sqrt[3]{9}$ and $\sqrt[6]{105}$

(v) $\sqrt[5]{4}$, $\sqrt{7}$, $\sqrt[10]{48}$ and $\sqrt[20]{119}$

4. Arrange the following surds in descending order.

(i) $\sqrt[4]{3}$, $\sqrt[6]{10}$ and $\sqrt[12]{25}$

(ii) $\sqrt[3]{4}$, $\sqrt[4]{5}$, $\sqrt{3}$ and $\sqrt[6]{19}$

(iii) $\sqrt[3]{2}$, $\sqrt[6]{3}$, $\sqrt[9]{4}$ and $\sqrt[18]{30}$

(iv) $\sqrt[4]{10}$, $\sqrt[3]{6}$, $\sqrt{5}$ and $\sqrt[12]{78}$

(v) $\sqrt[7]{4}$, $\sqrt[28]{17}$, $\sqrt[14]{11}$ and $\sqrt{2}$

Comparison of surds of distinct order (Solution)

1.(i) $\sqrt[3]{3}$, $\sqrt[4]{5}$ — L.C.M. of 3 & 4 = 12

$(\sqrt[3]{3})^{12} = 3^4 = 81$

$(\sqrt[4]{5})^{12} = 5^3 = 125$

So, $\sqrt[4]{5}$ is larger.

(ii) $\sqrt[3]{6}$, $\sqrt[4]{8}$ — L.C.M. of 3 & 4 = 12

$(\sqrt[3]{6})^{12} = 6^4 = 1296$

$(\sqrt[4]{8})^{12} = 8^3 = 512$

So, $\sqrt[3]{6}$ is larger.

(iii) $\sqrt[3]{4}$, $\sqrt{2}$, $\sqrt[4]{3}$, $\sqrt[6]{9}$ — L.C.M. of 2,3,4 & 6 = 12

$(\sqrt[3]{4})^{12} = 4^4 = 256$

$(\sqrt{2})^{12} = 2^6 = 64$

$(\sqrt[4]{3})^{12} = 3^3 = 27$

$(\sqrt[6]{9})^{12} = 9^2 = 81$

So, $\sqrt[3]{4}$ is largest.