$$\frac{8}{15} = \frac{PQ}{117+BQ} \quad \text{....(i)}$$

$$\frac{3}{4} = \frac{PQ}{BQ} \quad \text{....(ii)}$$

Divide *(i)* by *(ii)*

$$\frac{8}{15} \times \frac{4}{3} = \frac{PQ}{117+BQ} \times \frac{BQ}{PQ}$$

$$\frac{32}{45} = \frac{BQ}{117+BQ}$$

$32 \times 117 = 13BQ$

288 mtr. = BQ

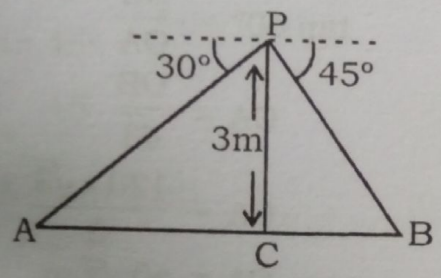
$$\frac{PQ}{BQ} = \frac{3}{4}$$

$$PQ = \frac{3}{4} \times 288$$

PQ = 216 mtr.

∴ Height of the tower = 216 meter.

43.(A)



In triangle ACP,

$\angle PAC = 30°$

$$\tan 30° = \frac{PC}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{3}{AC}$$

$$AC = 3\sqrt{3}\ m$$

In triangle PCB,

$\angle PBC = 45°$

$$\tan 45° = \frac{PC}{BC}$$

$$1 = \frac{3}{BC}$$

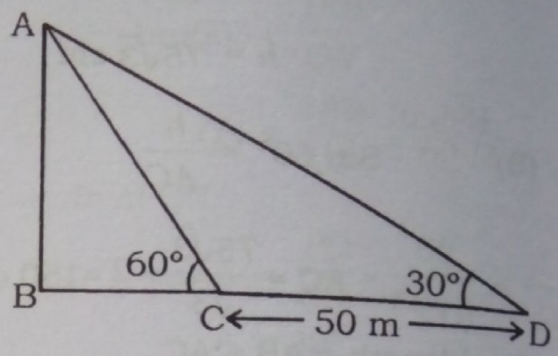
B = 3 m

Width of the river (AB) = AC + BC

$= 3\sqrt{3} + 3$

$= 3(\sqrt{3}+1)$ m

44. (B) ATQ,



$$\frac{AB}{BD} = \tan 30°$$

$$AB = \frac{(BC+50)}{\sqrt{3}} \quad \text{... (i)}$$

$$\frac{BC}{AB} = \cot 60°$$

$$BC = \frac{AB}{\sqrt{3}} \quad \text{... (ii)}$$

from equation *(i)* and *(ii)*

$$AB = \left(\frac{\frac{AB}{\sqrt{3}} + 50}{\sqrt{3}} \right)$$

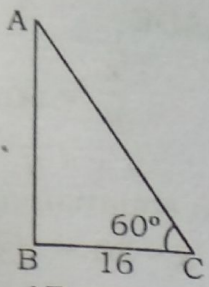
$$\Rightarrow \sqrt{3}\,AB - \frac{AB}{\sqrt{3}} = 50$$

$$\frac{3AB - AB}{\sqrt{3}} = 50$$

$$\Rightarrow \frac{2AB}{\sqrt{3}} = 50$$

Height of the tower (AB) = $25\sqrt{3}$ m

45. (C)

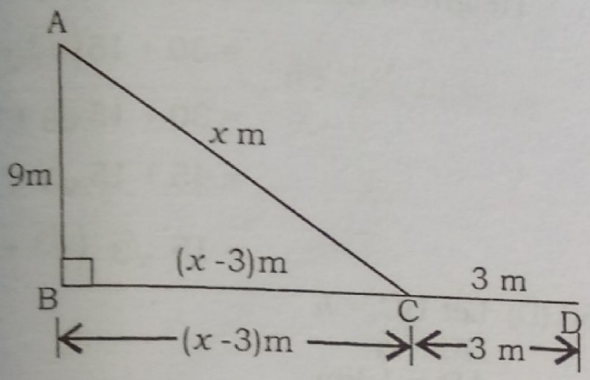


$$\frac{AB}{BC} = \tan 60°$$

$$AB = 16 \times \sqrt{3}$$

Height of the tree = $16\sqrt{3}$ m

46. (D)



AC → ladder

let x = length of ladder

When ladder is moved 3 m away from the wall then BD → ladder

BD = x mtr.

CD = 3 m

So, BC = $(x - 3)$ m

AB = 9 m

In ΔABC

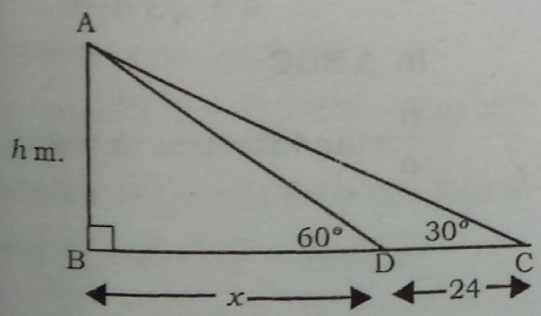
$$(9)^2 + (x-3)^2 = x^2$$

$$81 + x^2 + 9 - 6x = x^2$$

$$90 - 6x = 0$$

$$x = 15 \text{ m}$$

47. (C)



Let AB be a pillar of height 'h' meter.

In ΔABC

$$\frac{h}{x+24} = \tan 30°$$

$$\Rightarrow \quad \sqrt{3}h = x + 24 \qquad \ldots(i)$$

In ΔABD

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots(ii)$$

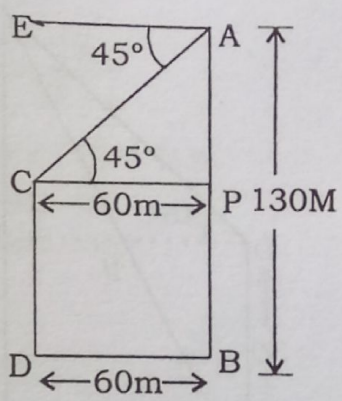
Put the value of x in eq^n *(i)*

$$\sqrt{3}h = \frac{h + 24\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = h + 24\sqrt{3}$$

$$h = 12\sqrt{3} \text{ m}$$

48. (C)



AB = 130m

CD is another tower

AE□PC

$\angle EAC = \angle ACP = 45°$

In ΔAPC

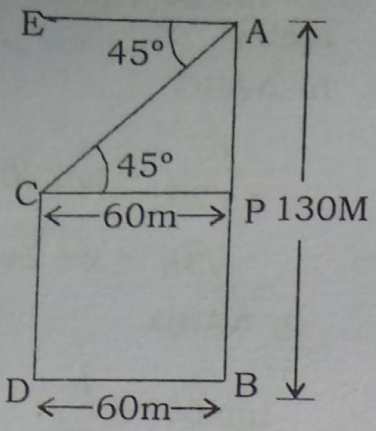
$$\tan 45° = \frac{AP}{60}$$

AP = 60 m

So, Height of the other tower

= 130 – 60 = 70 m

48. (C)



AB = 130m

CD is another tower

AE || PC

$\angle EAC = \angle ACP = 45°$

In ΔAPC

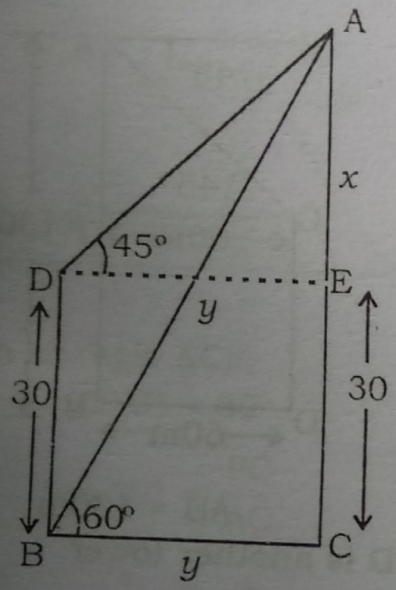
$$\tan 45° = \frac{AP}{60}$$

AP = 60 m

So, Height of the other tower

= 130 – 60 = 70 m

49. (A)



Let AE = x (height above tree)

EC = height of the tree = 30 m

BC = DE = y

In ΔABC

$$\frac{x+30}{y} = \tan 60°$$

$$\Rightarrow \quad x + 30 = \sqrt{3}y \qquad \ldots(i)$$

In ΔADE

$$\frac{x}{y} = \tan 45°$$

$$\Rightarrow \quad x = y \qquad \ldots(ii)$$

From equation *(i)* and *(ii)*

$$x + 30 = \sqrt{3}\,x$$

$$30 = x(\sqrt{3} - 1)$$

$$x = \frac{30}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 15\left(\sqrt{3}+1\right)$$

Height of the building AC

$$= 30 + 15\left(\sqrt{3}+1\right)$$

$$= 30 + 15\sqrt{3} + 15$$

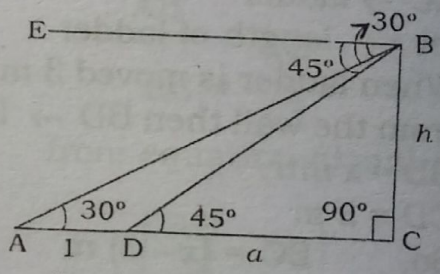
$$= 45 + 15\sqrt{3}$$

$$= 15\sqrt{3}\left(\sqrt{3}+1\right)$$

50. (D) Let BC = h

DC = a

AD = 1 km



In ΔABC

$$\tan 30° = \frac{h}{1+a}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{1+a}$$

$$a = \sqrt{3}\,h - 1$$

In ΔBDC

$$\frac{h}{a} = \tan 45° = 1 \Rightarrow h = a$$

In ΔABC

$$\tan 60° = \frac{AB}{AC}$$

$$AC = \frac{125}{\sqrt{3}} m$$

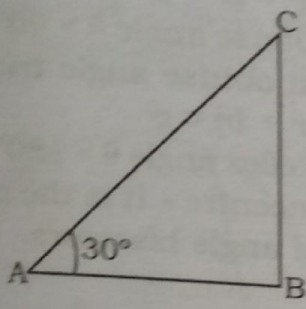
In ΔADC

$$\tan 30° = \frac{h}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{\frac{125}{\sqrt{3}}}$$

$$h = \frac{125}{3} m = 41.66 m$$

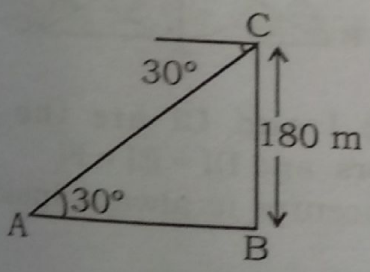
52. (B)



Height of the light house (BC) = 20 m
Angle = 30°
Required distance (AB) $= 20 \times \cot 30°$

$= 20\sqrt{3}$ m

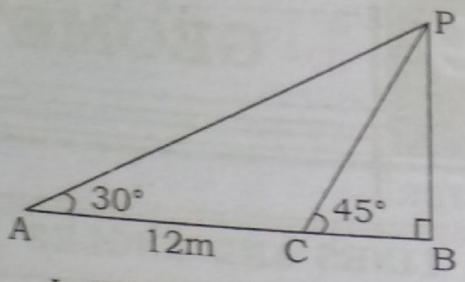
53. (B)



Distance of the cat from the foot of tower / Height of the tower $= \cot 30°$

Distance of the cat from the foot of tower $= 180 \times \sqrt{3} = 180\sqrt{3}$

54. (A)



In PBC

$$\tan 45° = \frac{PB}{BC}$$

$$PB = BC$$

In ΔPBA

$$\frac{PB}{AB} = \tan 30°$$

$$\frac{PB}{AC + CB} = \frac{1}{\sqrt{3}}$$

$$\frac{PB}{12 + PB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow PB = \frac{12}{\sqrt{3} - 1}$$

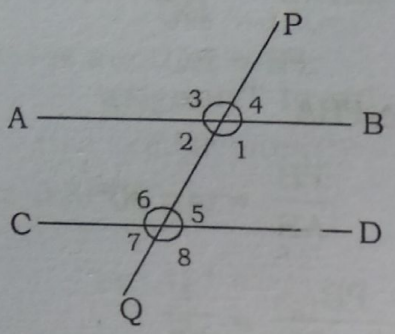
$= 6(\sqrt{3} + 1)$ m
$= 6 \times 2.732$
$= 16.4$ m

19 GEOMETRY

CHAPTER

LINES AND ANGLES

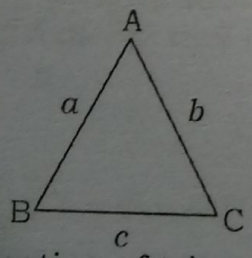
Line →
Line Segment → A B



AB || CD and PQ is a transversal line.

1. Corresponding Angles
   $\angle 1 = \angle 8$, $\angle 2 = \angle 7$
   $\angle 4 = \angle 5$ $\angle 3 = \angle 6$
2. Vertically opposite angle
   $\angle 1 = \angle 3$ $\angle 6 = \angle 8$
   $\angle 2 = \angle 4$ $\angle 5 = \angle 7$
3. Alternate angle
   interior exterior
   $\angle 2 = \angle 5$ $\angle 4 = \angle 7$
   $\angle 1 = \angle 6$ $\angle 3 = \angle 8$
4. Straight angle
   $\angle 3 + \angle 4 = 180°$ $\angle 1 + \angle 2 = 180°$
   $\angle 5 + \angle 6 = 180°$ $\angle 7 + \angle 8 = 180°$
5. Co-interior angle
   $\angle 1 + \angle 5 = 180°$ $\angle 2 + \angle 6 = 180°$

TRIANGLE



The formation of triangle in possible only when

1. $a + b > c$
   When sum of any two side is greater than third side
2. $a - b < c$
   when difference of any two sides is less than third sides.

Types of angles

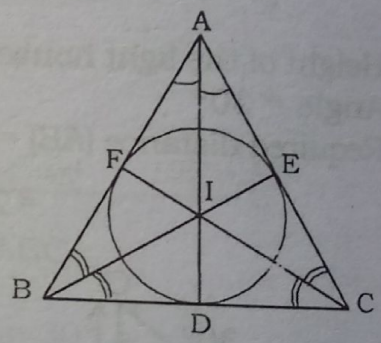
(a) Acute angle $\theta < 90°$
In acute angle triangle
$a^2 + b^2 > c^2$

(b) Right angle $\theta = 90°$
In right angle triangle
$a^2 + b^2 = c^2$

(c) Obuse angle $\theta > 90°$
In obtuse angle triangle
$a^2 + b^2 < c^2$

(d) Reflex angle $\theta > 180°$ but $\theta < 360°$

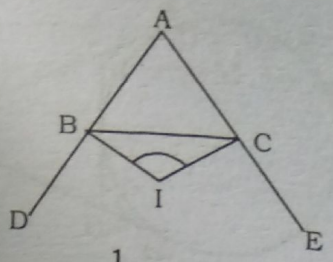
(e) Incentre – It is the meeting point of angle bisectors.



AI , B I and CI are the angle bisectors. and DI = EI = FI
also incentre is always inside the circle

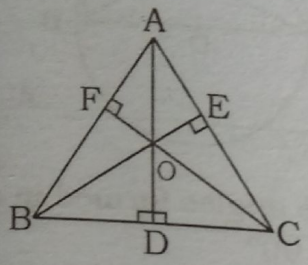
$$BIC = 90° + \frac{1}{2} A$$

If the second triangle formed is outside the main triangle, then ∠BIC will be –

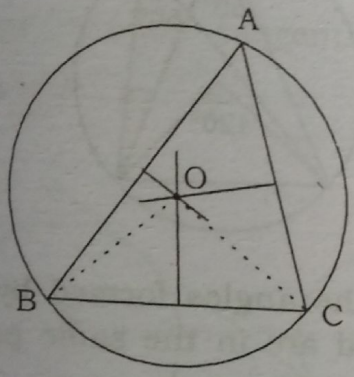


$$\angle BIC = 90° - \frac{1}{2}\angle A$$

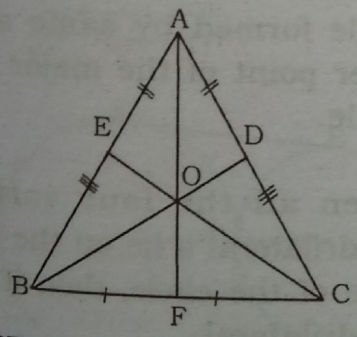
(f) Orthocentre – It is the meeting point of all the altitudes drawn from the vertex to the opposite side.



(g) Circumcentre – In is the meeting point of perpendicular side bisector.

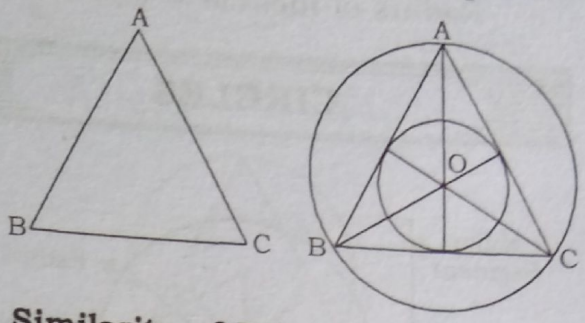


(h) Centroid – Centroid is the point where all the medians intersect.



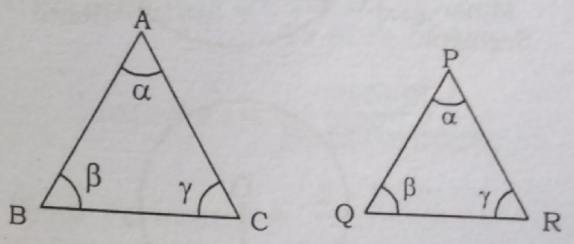
AF, BD and CF are the medians and the centroid divides each median in the ratio 2 : 1

(i) Equilateral triangle – In an equilateral triangle, centroid incentre, cricumcentre and orthocentre lie on the same point.



(j) **Similarity of Triangles**

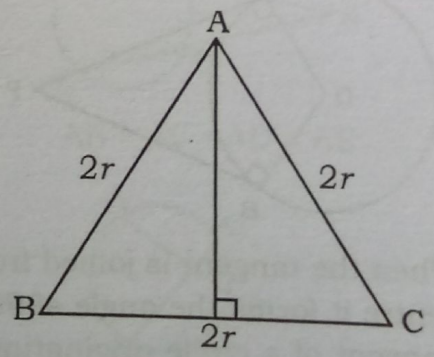
When the triangles have same structure but different size.



If two or more angles of different triangles are equal then they will be similar to each other.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{\text{Altitude of } \Delta ABC}{\text{Altitude of } \Delta PQR}$$

(k) **Property of Equilateral Triangle**



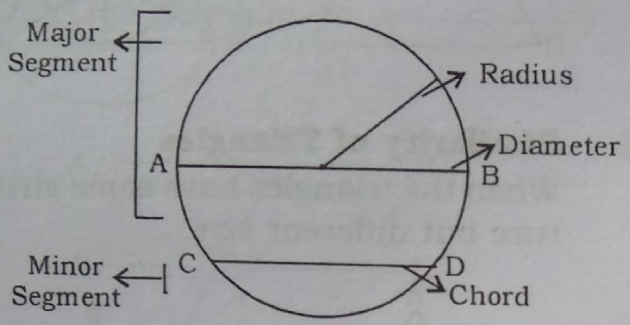
$$\text{Altitude} = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2} \times 2r = \sqrt{3}\,r$$

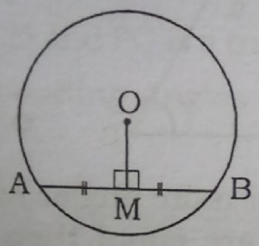
Radius of circumcircle

$$= \sqrt{3}\, r \times \frac{2}{3} = \frac{2r}{\sqrt{3}}$$

Radius of incircle $= \sqrt{3}\, r \times \frac{1}{3} = \frac{r}{\sqrt{3}}$

CIRCLES

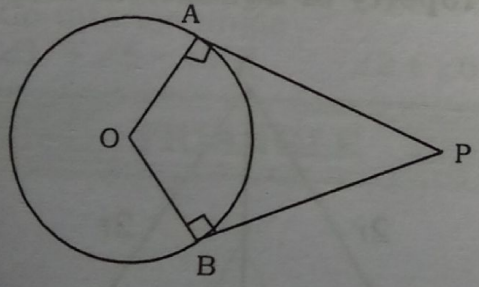




(AM =MB)

If a perpendicular falls on a chord from the centre of a circle, it will divide the chord in two equal parts.

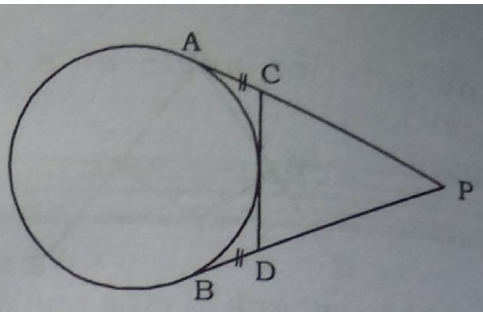
Tangent



When the tangent is joined from the centre it forms the angle of 90°

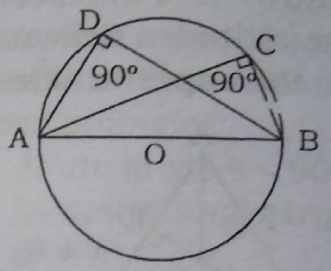
Tangent of a circle originating from the same point are of equal length.

Like PA = PB



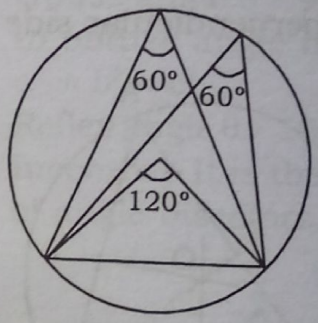
Properties

(A)



All the angles formed in a semicircle are of 90°

(B)



All the angles formed by a single chord are in the same part of the circle are equal.

also,

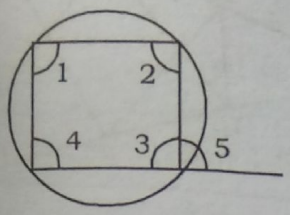
Angle formed by an arc at the centre of the circle is double to the angle formed by same arc in any other point of the major are of the circle.

(C)

When all the four vertices of a quadrilateral a lie on the circumference of the circle, then it is a cyclic quadrilateral.

The sum of opposite interior angles is 180°

Any exterior angle of a cyclic qualrilateral is equal to interior opposite angle.

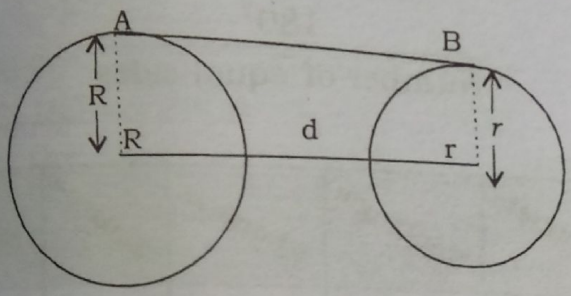


$\angle 1 + \angle 3 = 180°$

$\angle 3 + \angle 5 = 180°$ (linear pair)

$\therefore \angle 5 = \angle 1$

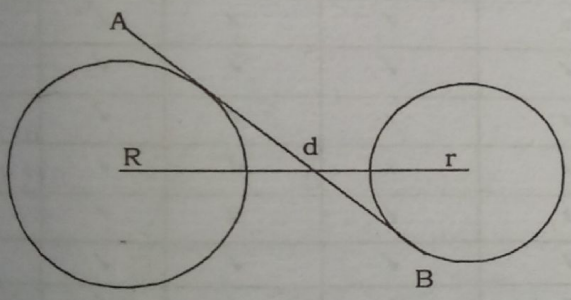
(D)



Direct Common tangent.

$AB = \sqrt{d^2 - (R - r)^2}$

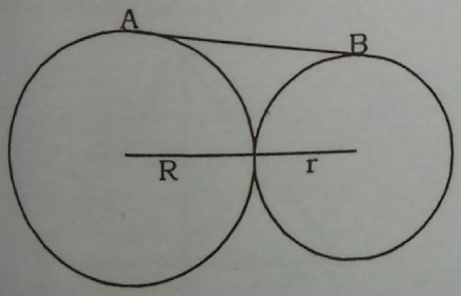
(E)



Transverse Common Tangent

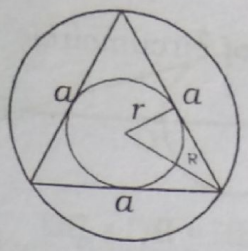
$AB = \sqrt{d^2 - (R + r)^2}$

(F)



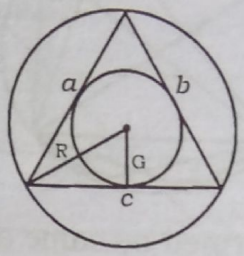
$AB = 2\sqrt{Rr}$

(G)



$r = \frac{a}{2\sqrt{3}}$ $R = \frac{a}{\sqrt{3}}$

(H)

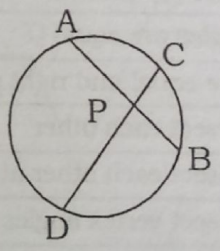


$$\text{Circumradius} = \frac{a \times b \times c}{4 \times \text{area of } \Delta}$$

$$\text{Inradius} = \frac{\text{area of } \Delta}{\text{Semi Perimeter}}$$

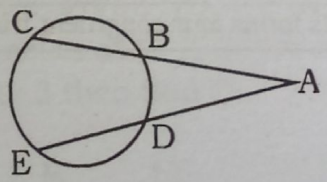
$$\text{Semi Perimeter} = \frac{a + b + c}{2}$$

(I)



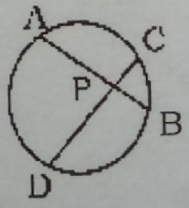
$AP \times PB = CP \times PD$

(J)



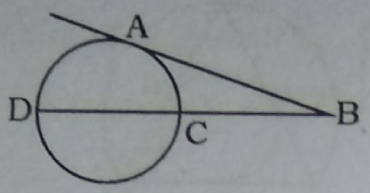
$AB \times AC = AD \times AE$

(K)



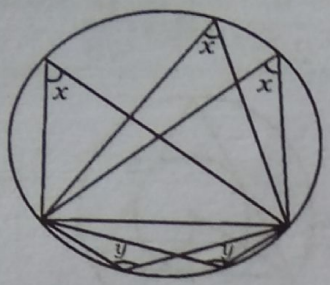
$AP \times PB = CP \times PD$

(L)



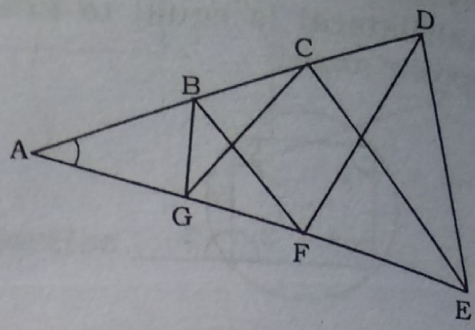
$AB^2 = BC \times BD$

(M)



Angles formed by same arc or chord on same side of the circle are equal.

(N)



If AB = BF = FD = DE = EC = CG = AG then

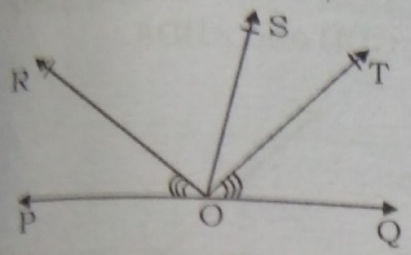
$\angle A$ will always be

$$\frac{180}{\text{Number of equal sides}}$$

S. No.	Property	Parallelogram	Rectangle	Rhombus	Square	Kite
1.	Opposite sides are equal	✓	✓	✓	✓	
2.	All sides are equal	×	×	✓	✓	
3.	Opposite sides are parallel	✓	✓	✓	✓	
4.	Opposite angles are equal	✓	✓	✓	✓	
5.	All angles are equal and right angle	×	✓	×	✓	
6.	Diagonals bisect each other	✓	✓	✓	✓	
7.	Diagonals bisect each other at right angles	×	×	✓	✓	
8.	Diagonals bisect vertex angles	×	×	✓	✓	
9.	Diagonals are equal	×	✓	×	✓	
10.	Diagonals forms four triangles of equal area	✓	✓	✓	✓	
11.	Diagonals forms four congruent triangles	×	×	✓	✓	

Exercise

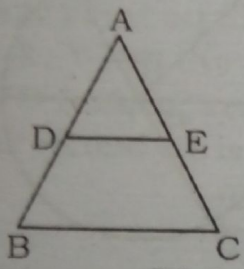
1. In the given figure OS is on PQ. OR and OT are the bisector of ∠POS and ∠SOQ. If ∠POS = x then find ∠SOQ.



(A) $90° - x$ (B) $80° - \angle POR$
(C) $180° - 2\angle POR$ (D) $90° - \angle POR$

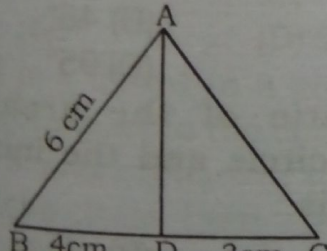
2. In the above figure value of ∠ROT is.....
(A) $180° - x$ (B) $90^0 - x$
(C) $90°$ (D) $\frac{x}{2}$

3. In a given Δ ABC, DE∥BC and $\frac{AD}{DB} = \frac{3}{5}$, If AC = 5. 6 cm find AE.



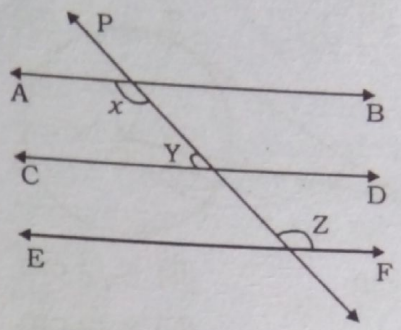
(A) 4.1 cm (B) 2.1 cm
(C) 2.3 cm (D) 4.8 cm

4. In the given figure AD is the bisector of ∠A, if BD = 4 cm, DC = 3 cm and AB = 6 cm, determine AC.



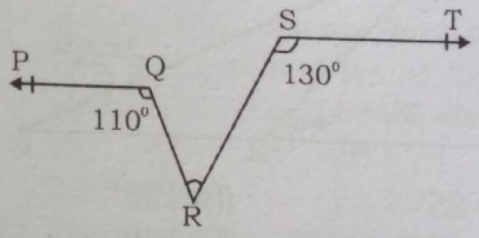
(A) 3.5 cm (B) 4.0 cm
(C) 12 cm (D) 4.5 cm

5. In the given figure AB ∥ CD, CD∥EF and Y : Z = 3 : 7 then find x.



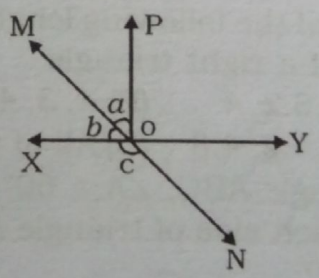
(A) 110° (B) 126°
(C) 140° (D) 150°

6. In the given figure, if PO∥ST, ∠PQR =110° and ∠RST = 130°, find ∠QRS.



(A) 40° (B) 50°
(C) 60° (D) 70°

7. In the given figure XY and MN Intersects at O. If ∠ POY = 90° and a : b = 2 : 3 then find C.



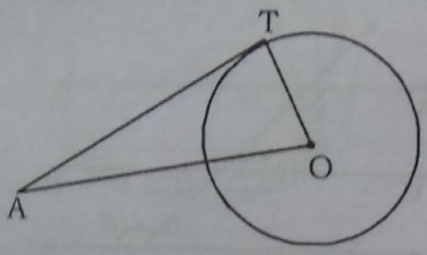
(A) 113° (B) 54°
(C) 126° (D) 48°

8. The perimeter of a rhombus is 40 cm and the measure of its adjacent angles is 60° and 120° then its area is.

(A) $100\sqrt{3}$ cm^2 (B) $50\sqrt{3}$ cm^2

(C) $160\sqrt{3}$ cm^2 (D) 100cm^2

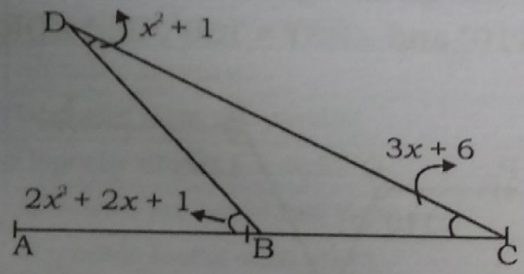
9. In the following figure find the Radius OT, if AT = 10 cm and AO = 6 cm (AT is tangent).



(A) 5 cm (B) $5\sqrt{2}$ cm

(C) 8 cm (D) $8\sqrt{2}$ cm

10. In the diagram at the right, the angles are represented as shown. Find ∠DBC.



(A) 25° (B) 125°

(C) 155° (D) 158°

11. The vertex angle of an isosceles triangle measures eight times the measure of a base angle. Find the measure of base angle.

(A) 18° (B) 24°

(C) 36° (D) 43°

12. Which of the following lengths are the sides of a right triangle

(A) 2.4, 3.2, 4 (B) 4. 3, 4.4, 4.5

(C) 3.1, 4.2, 4.8 (D) All of the above

13. In triangle ABC, ∠A = 60° and ∠B = 40° which side of triangle ABC is the longest.

(A) $\overline{AC}$ (B) $\overline{AB}$

(C) $\overline{BC}$ (D) None of these

14. In triangle ABC, ∠A is obtuse. Which statement is true about the sum of the measures of ∠B and ∠C.

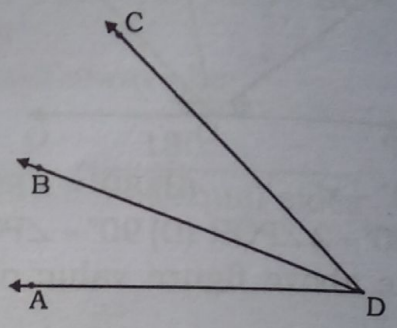
(A) ∠B + ∠C = 90°

(B) ∠B + ∠C > 90°

(C) ∠B + ∠C < 90°

(D) None of these

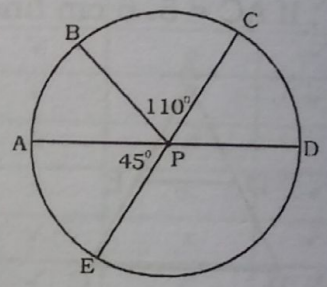
15. In the given figure ∠CDB = $(8y + 8°)$ and ∠BDA = $5y - 3°$ and ∠ADC = 70° find ∠CDB and ∠BDA.



(A) 48°, 22° (B) 45°, 25°

(C) 22°, 48° (D) 25°, 45°

16. In $\overline{EC}$ and $\overline{AD}$ are diameters of circle. Find ∠EPD + ∠BPA



(A) 150° (B) 160°

(C) 170° (D) 180°

17. The ratio of the angles ∠A and ∠B of a non-square Rhombus ABCD is 4 : 5, then the value of ∠C is.

(A) 50° (B) 45°

(C) 80° (D) 95°

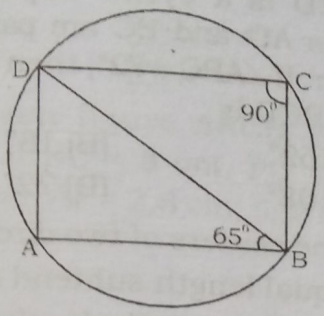
18. The ratio of the areas of the Circumcircle and the incircle of a square is :

(A) 2 : 1 (B) $\sqrt{2}$: 1

(C) $\sqrt{2}$: $\sqrt{3}$ (D) $\sqrt{3}$: 1

19. In a quadrilateral ABCD, with unequal sides if the diagonals AC and BD intersect at right angles, then.
(A) $AB^2 + BC^2 = CD^2 + DA^2$
(B) $AB^2 + CD^2 = BC^2 + DA^2$
(C) $AB^2 + AD^2 = BC^2 + CD^2$
(D) $AB^2 + BC^2 = 2\ (CD^2 + DA^2)$

20. ABCD is a Cyclic quadrilateral. If $\angle BCD = 90°$, $\angle ABD = 65°$ Find $\angle ADB$:



(A) 65° (B) 55°
(C) 35° (D) 25°

21. Measure of each interior angle of a regular polygon can never be :
(A) 150° (B) 105°
(C) 108° (D) 144°

22. D is any point on Side AC of Δ ABC. If P, Q, X, Y are the mid-points of AB, BC, AD and DC respectively, then the ratio of PX and QY is :
(A) 2 : 1 (B) 1 : 2
(C) 1 : 1 (D) 2 : 3

23. Let O be the in-centre of a triangle ABC and D be a point on the side BC of ΔABC, such that OD ⊥ BC. If $\angle BOD = 15°$, then $\angle ABC$ =
(A) 75° (B) 45°
(C) 150° (D) 90°

24. AB is a chord to a circle and PAT is the tangent to the circle at A. If $\angle BAT = 75°$ and $\angle BAC = 45°$, C being a point on the circle, then $\angle ACB$ is equal to :
(A) 40° (B) 45°
(C) 60° (D) 70°

25. PR is tangent to a circle, with centre O and radius 4 cm, at point Q. If $\angle POR = 90°$, OR = 5 cm and $OP = \frac{20}{3}$ cm, then, in cm, the length of PR is :
(A) 3 cm (B) $\frac{16}{3}$ cm
(C) $\frac{23}{3}$ cm (D) $\frac{25}{3}$ cm

26. The length of a chord of a circle is equal to the radius of the circle. The angle which this chord subtends at the major segment of the circle is equal to :
(A) 30° (B) 45°
(C) 60° (D) 90°

27. In Δ ABC, AD is the internal bisector of $\angle A$, meeting the side BC at D. If BD = 5cm, BC = 7.5 cm, then AB : AC
(A) 2 :1 (B) 1 : 2
(C) 4 : 5 (D) 3 : 5

28. If in a ΔABC, the medians CD and BE intersect each other at O, then the ratio of the area of ΔODE and ΔABC is :
(A) 1 : 6 (B) 6 : 1
(C) 1 : 12 (D) 12 : 1

29. The distance between two parallel chords of length 8 cm each in a circle of diameter 10 cm is :
(A) 6 cm (B) 7 cm
(C) 8 cm (D) 5.5 cm

30. In ΔABC, PQ is parallel to BC. If AP : PB = 1 : 2 and AQ = 3 cm, AC is equal to :
(A) 6 cm (B) 9 cm
(C) 12 cm (D) 8 cm

31. The in-radius of an equilateral triangle is of length 3cm. Then the length of each of its medians is :
(A) 12 cm (B) $\frac{9}{2}$ cm
(C) 4 cm (D) 9 cm