116. If $a = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $B = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then find the value of $\frac{a^2-b^2}{ab}$ + a + b.

(A) $4-8\sqrt{3}$
(B) 0
(C) $4+8\sqrt{3}$
(D) -1

117. If $x + \frac{a}{x} = 1$, then find the value of $\frac{x^2+x+a}{x^3-x^2}$.

(A) $-\frac{2}{a}$
(B) $\frac{2}{a}$
(C) 0
(D) 1

118. If a + b = 5 and $a^2 + b^2 = 13$, then find the value of a - b.

(A) 0
(B) -1
(C) 2
(D) 1

119. If $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$, then find value of $\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b}$.

(A) 0
(B) -1
(C) 1
(D) 2

120. If $x\left(3-\frac{2}{x}\right) = \frac{3}{x}$, then Find the value of $x^2 + \frac{1}{x^2}$.

(A) $\frac{22}{9}$
(B) $-\frac{22}{9}$
(C) $-\frac{26}{9}$
(D) $\frac{26}{9}$

121. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $xy = 1$, then find the value of $\left(\frac{x-y}{x+y}\right)^2$.

(A) $\frac{3}{7}$
(B) $\frac{3}{4}$
(C) $\frac{4}{3}$
(D) 0

122. $\frac{x^2}{y^2} + 2t + \frac{y^2}{x^2}$ For what value of 't' it is a perfect square.

(A) 1
(B) 2
(C) 3
(D) 0

123. If $x = 2+\sqrt{3}$, then find the value of $x^2 - 4x + 2$.

(A) 0
(B) -1
(C) 1
(D) 2

Answers with explanations

1. (B) $\frac{x}{(b-c)(b+c-2a)}$

$= \frac{y}{(c-a)(c+a-2b)}$

$= \frac{z}{(a-b)(a+b-2c)} = k$

$x = k(b-c)(b+c-2a) = k[b^2 - c^2 - 2a(b-c)]$

$y = k(c-a)(c-a-2b) = k[c^2 - a^2 - 2b(c-a)]$

$z = k(a-b)(a+b-2c) = k[a^2 - b^2 - 2c(a-b)]$

$x + y + z = k[b - c^2 + c^2 - a^2 + a^2 - b^2 - 2a(b-c) - 2b(c-a) - 2c(a-b)]$

$= k[-2ab + 2ac - 2bc + 2ab - 2ac + 2bc]$

$= 0$

2. (C) $\frac{(a+b)^2-(a-b)^2}{a^2b-ab^2} = \frac{4ab}{ab(a-b}$

$= \frac{4}{a-b}$

3. (D) $a + b + c = 0$

squaring both side

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$

$a^2 + b^2 + c^2 = -2(ab + bc + ca)$

4. (C) $\frac{x}{1} = \frac{a-b}{a+b}$

$\frac{x+1}{x-1} = \frac{a-b+a-b}{a-b-a-b} - \frac{2a}{2b} = -\frac{a}{b}$

Similarly,

$\frac{y+1}{y-1} = -\frac{b}{c}$ and $\frac{z+1}{z-1} = -\frac{c}{a}$

So,

$\frac{x+1}{x-1} \times \frac{y+1}{y-1} \times \frac{z+1}{z-1}$

$= \left(-\frac{a}{b}\right) \times \left(-\frac{b}{a}\right) \times \left(-\frac{c}{a}\right) = -1$

5. (D) $3^x + 3^{x+1} = 36$

$3^x[1 + 3] = 36$

$3^x \times 4 = 36 \quad \Rightarrow 3^x = 9 = 3^2$

$x = 2$

$x^x = 2^2 = 4$

6. (B) $f(x) = x^{29} - x^{25} + x^{13} - 1$

put $x + 1 = 0$

$x = -1$

$f(-1) = (-1)^{29} - (-1)^{25} + (-1)^{13} - 1$

$= -1 + 1 - 1 - 1 = -2$

So $(x + 1)$ is not a factor of polynomial.

put $x - 1 = 0$

$x = 1$

$f(1) = (1)^{29} - (1)^{25} + (1)^{13} - 1 = 1 - 1 + 1 - 1 = 0$

So $(x - 1)$ is a factor of polynomial.

7. (C) If $(x - a)$ is factor of a polynomial $f(x)$ then

$f(a) = 0$

A.T.Q.

$f(-2) = 0$

$(-2)^3 + 6(-2)^2 + 4(-2) + k = 0$

$-8 + 24 - 8 + k = 0$

$k = -8$

8. (B) $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$

Adding both x and y.

$x + y = a + \frac{1}{a} + a - \frac{1}{a} = 2a$ ..... (i)

Subtracting y from x.

$x - y = a + \frac{1}{a} - a + \frac{1}{a} = \frac{2}{a}$ ..... (ii)

Multiply equation (i) and (ii)

$(x + y)(x - y) = (2a)\left(\frac{2}{a}\right)$

$x^2 - y^2 = 4$

Squaring both side

$x^4 + y^4 - 2x^2y^2 = 16$

9. (A) $a^4 + b^4 = a^2b^2$

$a^4 + b^4 - a^2b^2 = 0$

multiply by $(a^2 + b^2)$ both side

$(a^2 + b^2)(a^4 + b^4 - a^2b^2) = 0$

$a^6 + b^6 = 0$

10. (A) $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$

$= \frac{a^2.a}{bc.a} + \frac{b^2.b}{ca.b} + \frac{c^2.c}{ab.c} = \frac{a^3+b^3+c^3}{abc}$

$a^3 + b^3 + c^3 = 3abc\ (\because\ a + b + c = 0)$

$= \frac{3abc}{abc} = 3$

11. (A) $a = -1.21,\ b = -2.12$ and $c = 3.33$

$a + b + c = -1.21 - 2.12 + 3.33 = 0$

So, $a^3 + b^3 + c^3 = 3abc$

$a^3 + b^3 + c^3 = 3abc = 0$

12. (A) $x^{\frac{1}{3}} + y^{\frac{1}{3}} - z^{\frac{1}{3}} = 0$

$x^{\frac{1}{3}} + y^{\frac{1}{3}} + (-z^{\frac{1}{3}}) = 0$

$\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

$$\left(x^{\frac{1}{3}}\right)^3 + \left(y^{\frac{1}{3}}\right)^3 + \left(-z^{\frac{1}{3}}\right)^3 = 3x^{\frac{1}{3}}y^{\frac{1}{3}}\left(-z^{\frac{1}{3}}\right)$$

$x + y - z = -3\ (xyz)^{\frac{1}{3}}$

$(x + y - z)^3 = -27(xyz)$

$(x + y - z)^3 + 27\ (xyz) = 0$

13. (D) $(a - 1)^2 + (b + 2)^2 + (c + 1)^2 = 0$

So, $a - 1 = 0$ $b + 2 = 0$ $c + 1 = 0$

$a = 1$ $b = -2$ $c = -1$

then,

$2a - 3b + 7c = 2(1) - 3\ (-2) + 7(-1)$

$= 2 + 6 - 7 = 1$

14. (C) $(y - z) + (z - x) + (x - y)$

$= y - z + z - x + x - y = 0$

So, $(y - z)^3 + (z - x)^3 + (x - y)^3$

$= 3(y - z) + (z - x) + (x - y)$

15. (A) $x = b + c - 2a$

$y = a + c - 2b$

$z = a + b - 2c$

$x + y = a + b + 2c - 2a - 2b$

$x + y = -a - b + 2c$

$x + y = -z$

$x^2 + y^2 + 2xy = z^2$

$x^2 + y^2 - z^2 + 2xy = 0$

16. (D) $a^2 + ab + b^2 = 4$ ..... (i)

$a^4 + a^2b^2 + b^4 = 8$

$(a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2 - a^2b^2 = 8$

$(a^2 + b^2)^2 - (ab)^2 = 8$

$(a^2 + b^2 + ab)(a^2 + b^2 - ab) = 8$

Using equation

$4(a^2 + b^2 - ab) = 8$

$a^2 + b^2 - ab = 2$ ...... (ii)

Subtracting equation (ii) from (i)

$2ab = 2 \quad \Rightarrow ab = 1$

17. (C) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$a^2 + b^2 + c^2 = 2a - 2b - 2c - 1 - 1 - 1$

$(a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

$(a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

So, $a = 1,\ b = -1$ and $c = -1$

$a - b + c = (1) - (-1) + (-1) = 1 + 1 - 1$

$= 1$

18. (D) $x^a.\ x^b\ .x^c = 1$

$x^{a+b+c} = x^0$

$a + b + c = 0$

So, $a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc = 0$

19. (B) $a^x = b,\ b^y$ and $c^z = a$

$a^x = b$

$(a^x)^y = (b)^y \quad \Rightarrow a^{xy} = c$

$(a^{xy})^z = (c)^z \quad \Rightarrow a^{xyz} = a$

So, $a^{xyz} = a^1$

$xyz = 1$

20. (A) $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}}$

$+ \frac{1}{1+r+p^{-1}}$

$$= \frac{1}{1+p+q^{-1}\times 1} + \frac{1}{1+q+r^{-1}\times 1} + \frac{1}{1+r+p^{-1}\times 1}$$

$$= \frac{1}{1+p+q^{-1}\times pqr} + \frac{pqr}{pqr+q+r^{-1}\times pqr} + \frac{1}{1+r+\frac{1}{p}} \quad [\because pqr = 1]$$

$$= \frac{1}{1+p+pr} + \frac{pqr}{q[pr+1+p]} + \frac{1}{\frac{p+pr+1}{p}}$$

$$= \frac{1}{1+p+pr} + \frac{pr}{1+p+pr} + \frac{p}{1+p+r}$$

$$= \frac{1+p+pr}{1+p+pr} = 1$$

21.(A) $\frac{1}{x^b+x^{-c}+1} + \frac{1}{x^c+x^{-a}+1} + \frac{1}{x^a+x^{-b}+1}$

$$= \frac{1}{x^b+x^{-c}\times x^{a+b+c}+1} + \frac{x^{a+b+c}}{x^c+x^{-a}\times x^{a+b+c}+x^{a+b+c}} + \frac{1}{x^a+\frac{1}{x^b}+1}$$

$$= \frac{1}{1+x^b+x^{a+b}} + \frac{x^{a+b+c}}{x^c\left(1+x^b+x^{a+b}\right)} + \frac{x^b}{1+x^b+x^{a+b}}$$

$$= \frac{1}{1+x^b+x^{a+b}} + \frac{x^{a+b}}{1+x^b+x^{a+b}} + \frac{x^b}{1+x^b+x^{a+b}}$$

$$= \frac{1+x^b+x^{a+b}}{1+x^b+x^{a+b}} = 1$$

22. (D) $a^x = (x+y+z)^y$, $a^y\ (x+y+z)^z$ and $a^z = (x+y+z)^x$

$a^x \times a^y \times a^z = (x+y+z)^y \times (x+y+z)^z \times (x+y+z)^x$

$a^{x+y+z} = (x+y+z)^{x+y+z}$

$a = x+y+z$

So, $a^x\ (x+y+z)^y = (a)^y$

$x = y$ similarly $y = z$

So, $x = y = z = \frac{a}{3}$

23. (C) $a+b+c = 3$

Squaring both side

$a^2+b^2+c^2+2(ab+bc+ca) = 9$

$ab+bc+ca = \frac{9-6}{2} = \frac{3}{2}\ [\because a^2+b^2+c^2 = 6]$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 1$

$\frac{bc+ac+ab}{abc} = 1$

$abc = ab+ac+bc$

$abc = \frac{3}{2}$

24. (D) $a^3+b^3 = 0$

$(a+b)(a^2+b^2-ab) = 0$

So, $a^2+b^2-ab = 0 \Rightarrow a^2+b^2 = ab$

$a^2+b^2+2ab = 3ab$

$(a+b)^2 = (\sqrt{3ab})^2$

$$a + b = \sqrt{3ab}$$

25. (A) $\frac{7x-3}{x} + \frac{7y-3}{y} + \frac{7z-3}{z} = 0$

$$\frac{7x}{x} - \frac{3}{x} + \frac{7y}{y} - \frac{3}{y} + \frac{7z}{z} - \frac{3}{z} = 0$$

$$21 - 3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = 0$$

$$3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = 21$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 7$$

26. (A) $2^x = 4^y = 8^z$

$2^x = 2^{2y} = 2^{3z}$

So, $x = 2y = 3z$

$$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{4z} = 4$$

$$\frac{1}{2x} - \frac{1}{2(x)} + \frac{1}{4\left(\frac{x}{3}\right)} = 4$$

$$\frac{1}{2x} + \frac{1}{2x} + \frac{3}{4x} = 4$$

$$\frac{2+2+3}{4x} = 4$$

$$x = \frac{7}{16}$$

27. (A) $a + b + c = 0$

squaring both side

$a^2 + b^2 + c^2 + (ab + bc + ca) + 0$

$14 + 2(ab + bc + ca) = 0$

$[\therefore a^2 + b^2 + c^2 = 14]$

$2(ab + bc + ca) = -14$

$ab + bc + ca = -7$

28. (D) $\frac{16}{67} = \frac{1}{\frac{67}{16}} = \frac{1}{4+\frac{1}{\frac{16}{3}}} = \frac{1}{4+\frac{1}{\frac{16}{3}}}$

$$= \frac{1}{4+\frac{1}{5+\frac{1}{3}}} = \frac{1}{x+\frac{1}{y+\frac{1}{z}}}$$

$= x \times y + z \qquad = 4 \times 5 + 3 = 23$

29. (D) $\left(x+\frac{1}{x}\right)^2 = 3$

$$x + \frac{1}{x} = \sqrt{3}$$

So, $x^6 + 1 = 0$ (A property)

$x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + 1$

$= x^{200}(x^6 + 1) + x^{84}(x^6 + 1) + x^{12}(x^6 + 1) + 1$

$= x^{200} \times 0 + x^{84} \times 0 + x^{12} \times 0 + 1 = 1$

30. (C) $x + \frac{1}{x} = \sqrt{3}$

So, $x^6 + 1 = 0$

$x^{506} + x^{500} + x^{384} + x^{378} + x^{190} + x^{184} + x^{18} + x^{12}$

$= x^{500}(x^6 + 1) + x^{378}(x^6 + 1) + x^{184}(x^6 + 1) + x^{12}(x^6 + 1)$

$= x^{500} \times 0 + x^{378} \times 0 + x^{184} \times 0 + x^{12} \times 0$

$= 0$

31. (C) $x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$

$$x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}} = \frac{\left(\sqrt{5}+1\right)^2}{\sqrt{\left(\sqrt{5}\right)^2 - (1)^2}}$$

$$= \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{\sqrt{4}} = \frac{\sqrt{5}+1}{2}$$

$$x^2 - x - 1 = x^2 - 2 \times \frac{1}{2} \times x + \left(\frac{1}{2}\right)^2 - \frac{1}{4} - 1$$

$$= \left(x - \frac{1}{2}\right)^2 - \frac{5}{4}$$

$$= \left(\frac{\sqrt{5}+1}{2} - \frac{1}{2}\right)^2 - \frac{5}{4}$$

$$= \left(\frac{\sqrt{5}+1-1}{2}\right)^2 - \frac{5}{4} = \left(\frac{\sqrt{5}}{2}\right)^2 - \frac{5}{4}$$

$$= \frac{5}{4} - \frac{5}{4} = 0$$

32. (B) $x + \frac{a}{x} = 1$ ..... (i)

$x - 1 = \frac{-a}{x}$ ..... (ii)

$$\frac{x^2 + x + a}{x^3 - x^2} = \frac{x\left(x + 1 + \frac{a}{x}\right)}{x^2(x-1)}$$

$$= \frac{(1+1)}{x\left(\frac{-a}{x}\right)} = -\frac{2}{a}$$

33. (A) $5x^2 - 4xy + y^2 - 2x + 1 = 0$
$4x^2 - 4xy + y^2 + x2 - 2x + 1 = 0$
$(2x)^2 - 2 \times 2x \times y + (-y)^2 + (x)^2$
$-2 \times x \times 1 + (-1)^2 = 0$
$(2x - y)^2 + (x - 1)^2 = 0$
So, $x - 1 = 0 \Rightarrow x = 1$
$2x - y = 0$
$y = 2x = 2 \times 1 = 2$
i.e., $x = 1$ and $y = 2$

34. (C) $x + \frac{1}{x} = a$

$x^2 + \frac{1}{x^2} = a^2 - 2$ ..... (i)

$x^3 + \frac{1}{x^3} = a^3 - 3a$ ..... (ii)

adding equation (i) and (ii)

$$x^3 + x^2 + \frac{1}{x^3} + \frac{1}{x^2} = a^3 + a^2 - 3a - 2$$

35 (A) $a^x = b$, $b^y = c$ and $xyz = 1$
$a^x = b$
$(a^x)^y = (b)^y = c \Rightarrow a^{xy} = c$
$(a^{xy})^z = c^z \Rightarrow a^{xyz} = c^z$ [$\therefore xyz = 1$]
$c^z = a^1 \Rightarrow c^z = a$

36. (C) $(3.7)^x = (0.037)^y = 10000$

$3.7 = (10)^{\frac{4}{x}}$ and $0.037 = (10)^{\frac{4}{y}}$ ____(i)

$$\frac{3.7}{100} = \frac{(10)^{\frac{4}{x}}}{100}$$

$\Rightarrow 0.037 = (10)^{\frac{4}{x}-2}$ ... (ii)

By (i) and (ii)

$$(10)^{\frac{4}{x}-2} = (10)^{\frac{4}{y}}$$

$$\frac{4}{x} - 2 = \frac{4}{y} \Rightarrow 4\left[\frac{1}{x} - \frac{1}{y}\right] = 2$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{2}$$

37. (A) $p^x = r^y = m$ and $r^w = p^z = n$

$$(p^x)^{\frac{z}{x}} = (r^y)^{\frac{z}{x}}$$

$$p^z = r^{\frac{yz}{x}}$$

$$r^w = r^{\frac{yz}{x}}$$

$$w = \frac{yz}{x}$$

$$yz = wx$$

38. (D) $x = (a+\sqrt{a^2+b^3})^{\frac{1}{3}} + (a-\sqrt{a^2+b^3})^{\frac{1}{3}}$

cubing both side

$x^3 = (a+\sqrt{a^2+b^3}) + (a-\sqrt{a^2+b^3})$

$+ 3\,(a+\sqrt{a^2+b^3})^{\frac{1}{3}}\ (a-\sqrt{a^2+b^3})^{\frac{1}{3}}$

$\left[(a+\sqrt{a^2+b^2})^{\frac{1}{3}} + (a-\sqrt{a^2+b^3})^{\frac{1}{3}}\right]$

$x^3 = 2a + 3[a^2 - (a^2 + b^3)]^{\frac{1}{3}}(x)$

$x^3 = 2a - 3bx$

$x^3 + 3bx - 2a = 0$

39. (B) $x + \frac{1}{x} = \sqrt{3}$

So, $x^6 + 1 = 0$

$x^6 = -1$

$x^{17} + \frac{1}{17^x} = \frac{x^{18}}{x} + \frac{x}{x^{18}} = \frac{(-1)^3}{x} + \frac{x}{(-1)^3}2\times\frac{\sqrt{3}}{2}$

$= -\left[x + \frac{1}{x}\right] = -\sqrt{3}$

40. (D) $x^6 + \frac{1}{x} = \sqrt{3}$

So $x^6 + 1 = 0 \Rightarrow x^6 = -1$

$x^2 + \frac{1}{x^6} + 2 = (-1) + \frac{1}{(-1)} + 2 = 0$

41. (A) $x = 2^{\frac{1}{3}} + 2^{\frac{-1}{3}}$

cubing both side

$x^3 = (2) + (2)^{-1} + 3.(2)^{\frac{1}{3}}(2)^{-\frac{1}{3}}\ (2^{\frac{1}{3}} + 2^{-\frac{1}{3}})$

$x^3 = 2 + \frac{1}{2} + 3x$

$2x^3 = 4 + 1 + 6x$

$2x^3 - 6x = 5$

42. (A) $2022 \times 2023 = (2022)\,(2022 + 1)$

$= (2022)^2 + 2022$

So, when 2022 is subtracted it forms as perfect square.

43. (C) $\frac{x}{a+x} + \frac{y}{b+y} + \frac{z}{c+z}$

$= \frac{ax}{a^2+ax} + \frac{by}{b^2+by} + \frac{cz}{c^2+cz}$

$= \frac{ax}{by+cz+ax} + \frac{by}{cz+ax+by} +$

$\frac{cx}{ax+by+cz}$

$= \frac{ax+by+cz}{ax+by+cz} = 1$

44. (C) $x + \frac{1}{x} = 1 \Rightarrow x^2 + 1 = x$

$x^2 - x + 1 = 0$

multiply by $(x + 1)$ both sides

$x^3 + 1 = 0 \Rightarrow x^3 = -1$

$x^{12} + x^9 + x^6 + x^3 + 1$

$= (x^3)^4 + (x^3)^3 + (x^2)^2 + (x^3) + 1$

$= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1$

45. (A) $\frac{x+1}{x-1} = \frac{(\sqrt{m+3n})}{(\sqrt{m-3n})}$

$\Rightarrow \frac{(x+1)^2}{(x-1)^2} = \frac{m+3n}{m-3n}$

$\Rightarrow \frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2}$

$= \frac{m+3n+m-3n}{m+3n-m+3n}$

$$\Rightarrow \frac{2(1+x^2)}{4x} = \frac{m}{3n}$$

$\Rightarrow 3n + 3nx^2 = 2mx$

$\Rightarrow 3n = 2mx - 3nx^2$

46.(D) $\dfrac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}} =$

$$\frac{x+y+z}{\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}}$$

$$= \frac{x+y+z}{\frac{x+y+z}{xyz}} = xyz$$

47. (C) $a + b + c = 0$

$a + b = -c$

squaring both sides

$a^2 + b^2 + 2ab = c^2$

$a^2 + b^2 = c^2 - 2ab$ ... (1)

$$\frac{a^2+b^2+c^2}{c^2-ab} = \frac{c^2-2ab+c^2}{c^2-ab}$$

[using equation (i)]

$$= \frac{2(c^2-ab)}{c^2-ab} = 2$$

48. (B) $2^a + 3^b = 17$ ... (i)

$2^{a+2} - 3^{b+1} = 5$

$4.2^a - 3.3^b = 5$ ... (ii)

Adding equation (i) after multiply by 3

$7.2^a = 56$

$2^a = 8 \Rightarrow 2^a = 2^3$

$a = 3$

$2^3 + 3^b = 17$

$3^b = 17 - 8 = 9 \Rightarrow 3^b = 3^2$

$b = 2$

i.e., $a = 3$, $b = 2$

49. (A) $\dfrac{x}{2x+y+z} = \dfrac{y}{x+2y+z} = \dfrac{z}{x+y+2z}$

$= a$

$x = a(2x + y + z)$

$y = a(x + 2y + z)$

$z = a(x + y + 2z)$

So,

$x + y + z$

$= a(2x + y + z) + a(x + 2y + z)$
$+ a(x + y + 2z)$

$$x + y + z = a(3x + 3y + 3z) = \frac{1}{3}$$

50. (B) $\dfrac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1} = \dfrac{189+61}{189-61}$

$$= \frac{250}{128} = \frac{125}{64}$$

$$\Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{(5)^3}{(4)^3}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{5}{4}$$

$$\Rightarrow \frac{x}{1} = \frac{5+4}{5-4}$$

$\Rightarrow x = 9$

51. (A) $a^2 + b^2 + c^2 = ab + bc + ca$

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

multiply by $(a + b + c)$ on both sides

$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$

52. (D) $a(x - a^2) - b(x - b^2) = 0$

$ax - a^3 - bx + b^3 = 0$

$x(a - b) = a^3 - b^3$

$$x = \frac{(a-b)(a^2+b^2+ab)}{a-b}$$

$x = a^2 + b^2 + ab$

53. (A) $ab + bc + ca = 0$

$-bc = ab + ca$

similarly $-ab = bc + ca$,

$-ca = ab + bc$

$$\frac{1}{a^2-bc} + \frac{1}{b^2-ca} + \frac{1}{c^2-ab}$$

$$= \frac{1}{a^2+ab+ca} + \frac{1}{b^2+bc+ab} + \frac{1}{c^2+bc+ca}$$

$$= \frac{1}{a(a+b+c)} + \frac{1}{b(a+b+c)} + \frac{1}{c(a+b+c)}$$

$$= \frac{bc+ca+ab}{abc(a+b+c)} = 0$$

54. (D) $a + b + c = 0$

squaring both side

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$

$a^2 + b^2 + c^2 = -2(ab + bc + ca)$

$$\frac{a^2+b^2+c^2}{ab+bc+ca} = \frac{-2(ab+bc+ca)}{ab+bc+ca} = -2$$

55. (C) $\dfrac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$

= If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

$$= \frac{3(x^2-y^2)(y^2-z^2)(z^2-x^2)}{3(x-y)(y-z)(z-x)}$$

$$\frac{3(x-y)(x+y)(y-z)(y+z)(z-x)(z+x)}{3(x-y)(y-z)(z-x)}$$

$= (x + y)\ (y + z)\ (z + x)$

56. (A) $x = a^{2/3} - a^{-2/3}$

cubing both side

$x^3 = (a^{2/3})^3 - (a^{-2/3})^3$

$-3 \times a^{2/3} \times a^{-2/3}\ (a^{2/3} - a^{-2/3})$

$$x^3 = a^2 - \frac{1}{a^2} - 3x$$

$$x^3 + 3x = a^2 - \frac{1}{a^2}$$

57. (A) $a + b + c = 0$

$a + b = -c$

squaring both side

$a^2 + b^2 + 2ab = c^2$

$a^2 + b^2 - c^2 = -2ab$

Similarly,

$b^2 + c^2 - a^2 = -2bc$

$a^2 + c^2 - b^2 = -2ac$

$$\frac{1}{a^2+b^2-c^2} + \frac{1}{b^2+c^2-a^2} + \frac{1}{a^2+c^2-b^2}$$

$$= \frac{1}{-2ab} + \frac{1}{-2bc} + \frac{1}{-2ac}$$

$$= -\frac{1}{2}\left[\frac{a+b+c}{abc}\right] = 0$$

58. (B) $y + \dfrac{1}{z} = 1 \quad \Rightarrow z = \dfrac{1}{1-y}$

$$x + \frac{1}{y} = 1 \quad \Rightarrow x = 1 - \frac{1}{y}$$

$$xyz = \left(1-\frac{1}{y}\right) \times y \times \left(\frac{1}{1-y}\right)$$

$$= \frac{y-1}{y} \times y \times \frac{1}{1-y} = -1$$

59. (A) $x^2 + y^2 + 2x + 1 = 0$

$(x)^2 + 2 \times x \times 1 + (1)^2 + y^2 = 0$

$(x + 1)^2 + y^2 = 0$

So, $x = -1$ and $y = 0$

$x^{31} + y^{35} = (-1)^{31} + (0)^{35} = -1$

60. (C) $\dfrac{1}{1+a} + \dfrac{1}{1+b} + \dfrac{1}{1+c}$

$$= \frac{a}{a+a^2} + \frac{b}{b+b^2} + \frac{c}{c+c^2}$$

$$= \frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c}$$

$$= \frac{a+b+c}{a+b+c} = 1$$

61. (C) $a^2 + b^2 + 2b + 4a + 5 = 0$

$(a)^2 + 2 \times a \times 2 + 4 + (b)^2 + 2 \times b \times 1 +$

$$\frac{a-b}{a+b} = \frac{-2-(-1)}{-2-1} = \frac{-1}{-3} = \frac{1}{3}$$

62. (C) $x^2 + y^2 - 4x - 4y + 8 = 0$
$(x)^2 - 2 \times x \times 2 + 4 + (y)^2 - 2 \times y \times 2 + 4 = 0$
$(x-2)^2 + (y-2)^2 = 0$
So, $x = 2$ and $y = 2$
$x - y = 2 - 2 = 0$

63. (D) $a^3 - b^3 - c^3 - 3abc = 0$
$(a)^3 + (-b)^3 + (-c)^3 - 3(a)(-b)(-c) = 0$
So $(a) + (-b) + (-c) = 0$
$a - b - c = 0$
$a = b + c$

64. (D) $\sqrt[3]{p(p^2+3p+3)+1}$

$= \sqrt[3]{p^3+3p^2+3p+1}$

$= \sqrt[3]{(p+1)^3}$

$= p + 1 = 124 + 1 = 125$

65. (D) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = 3$
So, $a = 3b$, $c = 3d$, $e = 3f$

$$\frac{2a^2+3c^2+4e^2}{2b^2+3d^2+4f^2}$$

$$= \frac{2(3b)^2+3(3d)^2+4(3f)^2}{2b^2+3d^2+4f^2}$$

$$= \frac{9[2b^2+3d^2+4f^2]}{2b^2+3d^2+4f^2} = 9$$

66. (D) $\frac{a^3+b^3+c^3-3abc}{(a-b)^2+(b-c)^2+(c-a)^2}$

$= \frac{1}{2}(a+b+c)$

$= \frac{1}{2} \times (25 + 15 - 10) = 15$

67. (D) $(ad - bc)^2 + (ac + bd)^2$
$= a^2d^2 + b^2c^2 - 2abcd + a^2c^2 + b^2d^2 + 2abcd$
$= a^2d^2 + a^2c^2 + b^2d^2 + b^2c^2$
$= a^2(c^2 + d^2) + b^2(c^2 + d^2)$
$= (a^2 + b^2)(c^2 + d^2)$
[$\because a^2 + b^2 = 2,\ c^2 + d^2 = 1$ Given]
$= (2) \times (1) = 2$

68. (B) $\sin\theta + \cos\theta = \frac{b}{a}$

squaring both side

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$

$1 + 2 \cdot \frac{c}{a} = \frac{b^2}{a^2}$

$\frac{a+2c}{a} = \frac{b^2}{a^2}$

$a^2 + 2ac = b^2$
$a^2 - b^2 + 2ac = 0$

69. (C) $\frac{a^3+b^3+c^3}{ab+bc+ca-a^2-b^2-c^2}$
$=$

$$\frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{-(a^2+b^2+c^2-ab-bc-ca)}$$

Put value of a, b and c
$= -(-5 - 6 + 10) = 1$

70. (C)

71. (D) $a + b + c = 0$
So, $a + b = -c$
Similarly $b + c - a$ and $a + c = -b$

$$\frac{3(a+b)(b+c)(c+a)}{abc} = \frac{3(-a)(-b)(-c)}{abc}$$
$= -3$

72. (B) $ax^2 + bx + c = a(x-p)^2$
$ax^2 + bx + c = ax^2 - 2apx + ap^2$

So, $b = -2ap \Rightarrow p = -\frac{b}{2a}$

$$c = ap^2 = a\left(-\frac{b}{2a}\right)^2 = a \times \frac{b^2}{4a^2}$$

i.e., $b^2 = 4ac$

73. (C) $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$

$$\frac{a}{1-a} + 1 + \frac{b}{1-b} + 1 + \frac{c}{1-c} + 1 = 4$$

$$\frac{a+1-a}{1-a} + \frac{b+1-b}{1-b} + \frac{c+1-c}{1-c} = 4$$

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

74. (C) $2(2x^2 - 1) = 14$

$\Rightarrow 2x^2 - 1 = 7$

$\Rightarrow 2x^2 = 8$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

75. (B) $a^6 + b^6 + 3a^2b^2(a^2 + b^2)$

$\Rightarrow a^6 + b^6 + 2a^3b^3$

$\Rightarrow 3a^2b^2(a^2 + b^2) = 2a^3b^3$

$\Rightarrow \frac{a}{b} + \frac{b}{a} = \frac{2}{3}$

76. (A) $\frac{x}{1} = \frac{\sqrt[3]{m+1} + \sqrt[3]{m-1}}{\sqrt[3]{m+1} - \sqrt[3]{m-1}}$

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt[3]{m+1}}{\sqrt[3]{m-1}}$$

$$\Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{x^3+1+3x+3x^2}{x^3-1-3x^2+3x} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{2x^3+6x}{6x^2+2} = \frac{m}{1}$$

$\Rightarrow x^3 + 3x = 3mx^2 + m$

$x^3 - 3mx^2 + 3x - m = 0$

77. (C) $(x-9)(x-2)$

$\Rightarrow x^2 - 9x - 2x + 18$

$\Rightarrow x^2 - 11x + 18$

$$\Rightarrow \left(x^2 - 2 \times \frac{11}{2}x + \left(\frac{11}{2}\right)^2\right) - \left(\frac{11}{2}\right)^2 + 18$$

$$= \left(x - \frac{11}{2}\right)^2 - \frac{121}{4} + 18$$

[minimum value of $x - \frac{11}{2} = 0$]

$$= \frac{4 \times 18 - 121}{4 \times 1} = \frac{72-121}{4}$$

$$= -\frac{49}{4}$$

78. (A) $a^2 + b^2 + c^2 - ab - bc - ca$

$=$

$$\frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

$$= \frac{1}{2}\left[(1)^2 + (1)^2 + (-2)^2\right]$$

$$= \frac{1}{2}[1 + 1 + 4] = 3$$

79. (A) $\dfrac{1}{x + \dfrac{1}{y + \dfrac{1}{z}}} = \dfrac{13}{37}$

$$\Rightarrow x + \frac{1}{y + \frac{1}{z}} = \frac{37}{13} = 2 + \frac{11}{13}$$

$$\Rightarrow x + \frac{1}{y + \frac{1}{z}} = 2 + \frac{1}{\frac{13}{11}}$$

$$\Rightarrow x + \frac{1}{y + \frac{1}{z}} = 2 + \frac{1}{1 + \frac{2}{11}}$$

$$x+\cfrac{1}{y+\cfrac{1}{z}}=2+\cfrac{1}{1+\cfrac{1}{\cfrac{11}{2}}}$$

Therefore by comparing

$$\Rightarrow x=2,\ y=1,\ z=\frac{11}{2}$$

80. (C) $x^2+2=2x$

squaring both sides

$x^4+4+4x^2=4x^2$

$x^4=-4$

$\therefore x^2=2x-2=2(x-1)$

$$\Rightarrow \frac{x^2}{2}=(x-1)$$

$\Rightarrow x^4-x^3+x^2+1$

$\Rightarrow -4-x^2(x-1)+1$

$$\Rightarrow -4-\frac{x^2\times x^2}{2}+1$$

$$\Rightarrow -4+\frac{4}{2}+1=-1$$

81. (B) $x=5-2\sqrt{6}$

$$\Rightarrow \frac{1}{x}=5+2\sqrt{6}=y$$

$$\Rightarrow x+\frac{1}{x}=10$$

$$\Rightarrow \frac{x^3+3xy+y^3}{x^2-2xy+y^2}\Rightarrow\frac{x^3+3+\frac{1}{x^3}}{x^2-2+\frac{1}{x^2}}$$

$$\Rightarrow \frac{\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)+3}{\left(x+\frac{1}{x}\right)^2-4}$$

$$\Rightarrow \frac{(10)^3-3(10)+3}{(10)^2-4}\Rightarrow\frac{973}{96}$$

82. (A) $x+\frac{1}{x}=3$

$x^2+1=3x$

$$\therefore \frac{7x}{3x-2x}=\frac{7x}{x}=7$$

83. (A) $x+\frac{1}{x}=5\Rightarrow x^2+1=5x$

Multiplying it by 7, $7x^2+7=35x$

$$\therefore \frac{5x}{7x^2+7-3x}\Rightarrow\frac{5x}{35x-3x}=\frac{5x}{32x}=\frac{5}{32}$$

84. (C) $\frac{2P}{P^2-2P+1}=\frac{1}{4}\{\because \frac{2P}{8P}\text{ will give }\frac{1}{4}\}$

$\Rightarrow P^2-2P+1=8P\Rightarrow 10P=P^2+1$

$$\therefore 10=\frac{P^2+1}{P}\Rightarrow 10=\frac{P^2}{P}+\frac{1}{P}$$

$$\therefore P+\frac{1}{P}=10$$

85. (A) $\frac{1+876543\times876545}{876544\times876544}$

Let $876544=x$, $876543=(x-1)$ and $876545=(x+1)$

$$\frac{1+(x-1)(x+1)}{x\times x}\Rightarrow\frac{1+x^2-1}{x^2}\Rightarrow\frac{x^2}{x^2}=1$$

86. (A) If $x=16$

$x^4-17x^3+17x^2-17x+17$

$$\underbrace{x^4-16x^3}_{\text{I}}\ \underbrace{-x^3+16x^2}_{\text{II}}+\underbrace{x^2-16x}_{\text{III}}\underbrace{-x+17}_{\text{IV}}$$