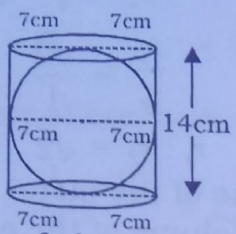
31.(D) Let the heights are $5h$, $8h$ and radii are $3r$, $8r$ respectively.

$\therefore$ Ratio between volumes

$$= \frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$
$$= r_1^2 h_1 : r_2^2 h_2$$
$$\Rightarrow \quad = (3r)^2 . 5h : (8r)^2 . 8h$$
$$= 9r^2 . 5h : 64r^2 . 8h$$
$$= 45 : 512$$

32.(B)



Diameter of circum-cylinder

= diameter of sphere = 14 cm

Height of circum-cylinder

= diameter of sphere = 14 cm

$\therefore$ Volume of circum-cylinder

$$= \pi r^2 h = \frac{22}{7} \times 14 \times 14$$
$$= 2156 \text{ cm}^3$$

33.(B) Let the radii of hemisphere and cone are r_1, r_2

$$\because \quad \frac{\text{hemisphere's volume}}{\text{cone's volume}} = \frac{1}{1}$$
$$\Rightarrow \quad \frac{\frac{2}{3}\pi r_1^3}{\frac{1}{3}\pi r_2^3 \times 2r_2} = \frac{1}{1}$$
$$\frac{r_1^3}{r_2^3} = \frac{1}{1} = \frac{1^3}{1^3}$$
$$\therefore \quad r_1 : r_1 = 1 : 1$$

34.(C) Let diameter of sphere is x and height of cone is $2x$

$\because$ Sphere's volume = cone's colume

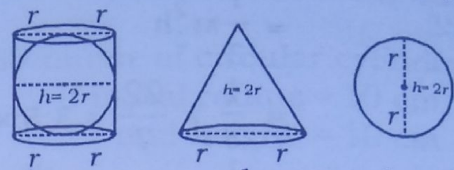
$$\therefore \quad \frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = \frac{1}{3}\pi r^2 . 2x$$
$$\therefore \quad 4r^2 = x^2$$
$$2r = x$$
$$r = \frac{x}{2}$$

$\therefore$ Sphere's radius : cone's radius

$$\frac{x}{2} : \frac{x}{2} = 1 : 1$$

Ratio between diameters = 1 : 1

35.(A)



Ratio among volumes of cylinder, cone and sphere

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{4}{3}\pi r^3$$
$$= \pi r^2 . 2r : \frac{1}{3}\pi r^2 . 2r : \frac{4}{3}\pi r^3$$
$$\Rightarrow 2\pi r^3 : \frac{2}{3}\pi r^3 : \frac{4}{3}\pi r^3$$
$$= 1 : \frac{1}{3} : \frac{2}{3} = 3 : 1 : 2$$

36.(C) Diameter of greatest sphere = 14m

$\therefore$ Surface area of the sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7 = 616\text{m}^2$$

37.(A) Diameter of the greatest sphere = Side of cubical tank = 14m.

$$\therefore \text{ Sphere's radius} = \frac{14}{2} = 7\text{m}$$
$$\therefore \text{ Sphere's volume} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 1437\frac{1}{3}\text{ m}^3$$

Sphere's surface area $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7$$
$$= 616 \text{ m}^2$$

38.(B) Let cube's side is $2x$ unit

$\therefore$ Cube's volume $= (2x)^3 = 8x^3$

Sphere radius $= x$ units

$$\therefore \quad \text{Sphere's volume} = \frac{4}{3}\pi x^3$$
$$\therefore \quad \text{Required ratio} = 8x^3 : \frac{4}{3}\pi x^3$$
$$= 2 : \frac{1}{3} \times \frac{22}{7}$$
$$= 21 : 11$$

39.(D) Curved area $= \pi(R + r)l$

$= \frac{22}{7} \times 21 \times 8$

$= 528 \text{ cm}^2$

40.(C) Total surface area

$= \pi[(R + r)l + R^2 + r^2]$

$= \frac{22}{7} \times [(12 + 9) \times 5 + 144 + 81]$

$= \frac{22}{7} \times [21 \times 5 + 25]$

$= \frac{22}{7} \times 30 = 1037.14\text{cm}^2$

41.(A) 128 km = 12800 m

6.4 hectares = 6.4 × 10000 sq m

Considering the grass area to be 128000 m long, and as wide as the roller, we have

Width required $= \frac{6.4 \times 10000}{128000}$ m

$= \frac{1}{2}$ m = 50 cm

42.(A) Volume of water

$= 37\frac{1}{3} \times 12 \times 8$ cub m

Weight of water

$= \frac{112}{3} \times 12 \times 8 \times 1000$

= 3584000 kg

= 3584 metric tons

43.(B) Volume of wall $= 25 \times 2 \times \frac{3}{4}$ cub m

Volume of one brick

$= \frac{20}{100} \times \frac{10}{100} \times \frac{15}{200}$ cub m

$= \frac{3}{2000}$ cub m

∴ Required number of bricks

$= \left(25 \times 3 \times \frac{3}{4}\right) \div \frac{3}{2000}$

= 25000

44.(D) Area of the land = 10000 sq m

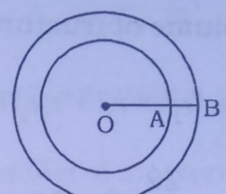
Volume of rainfall $= \frac{10000 \times 43}{100}$

= 4300 cub m

∴ Weight of water = 4300 × 1

= 4300 m tonnes

45.(C)



Height = 140 cm

External diameter = 50 cm

∴ External radius, OB = 25 cm

Also,

Internal radius, OA

= OB – AB

= (25 – 2) cm

= 23 cm

It is easy to see that the volume of iron will be found by subtracting volume of the cylinder of radius OA from the Volume the cylinder of radius OB.

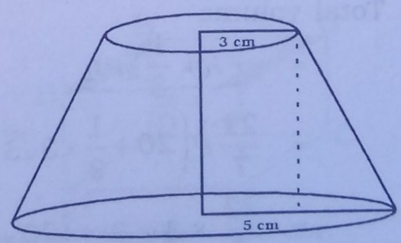
Volume of iron

$= \frac{22}{7} \times 25 \times 5 \times 140$

$- \frac{22}{7} \times 23 \times 23 \times 140$

$= 42240 \text{ cm}^3$

46.(A)



$r_1 = 3$ cm, $r_2 = 5$ cm, $h = 5$ cm

$l^2 = 29 \cong l = \sqrt{29}$

Whole surface area

$\cong \pi (r_1 + r_2)l + \pi r_1^2 + + \pi r_2^2$

$\cong \pi (r_1^2 + r_2^2 + r_1 l + r_2 l)$

$\cong \frac{22}{7}(9 + 25 + 3\sqrt{29} + 5\sqrt{29})$

$\cong 242.25 \text{ cm}^2$

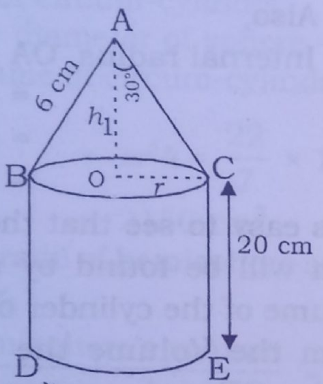
Volume of frustum

$\cong \frac{1}{3}\pi (r_1^2 + r_2^2 + r_1 r_2)h$

$\cong \frac{1}{3} \times \frac{22}{7}(9 + 25 + 15) \times 5$

$\cong \frac{1}{3} \times \frac{22}{7} \times 49 \times 5 = 256\frac{2}{3} \text{ cm}^3$

47.(A)



In AOC में,

$\sin 30° = \frac{r}{6}$

$r = \frac{1}{2} \times 6 = 3 \text{ cm}$

Now, $\tan 30° = \frac{r}{h_1} \cong h_1 = 3\sqrt{3}$ cm

Total volume

$= \pi r^2 h + \frac{1}{3}\pi r^2 h_1$

$= \frac{22}{7} r^2 \left(20 + \frac{1}{3} \times 3\sqrt{3}\right)$

$= \frac{22}{7} \times 3 \times 3 \times \left(20 + 3\sqrt{3}\right)$

$= 614.70 \text{ cm}^3$

48.(C) We know that volume of a cylinder is given by the formula,

Volume = Area of base × Height

Now, radius of 1st cylinder is R and its height is H.

∴ Volume of 1st cylinder

$= (\pi R^2) \times (H) = \pi R^2 H$

and radius of 2nd cylinder is H and its height is R

∴ Volume of 2nd cylinder = $(\pi H^2) \times R$

∴ Required ratio

$= \frac{\text{Volume of 1}^{\text{st}}\text{ cylinder}}{\text{Volume of 2}^{\text{nd}}\text{ cylinder}}$

$= \frac{\pi R^2 H}{\pi H^2 R} = \frac{R}{H}$

49.(D) $l \times b = 6500 \text{ cm}^2$

$l \times b \times d = 2.6 \text{ m}^3$

$= (2.6 \times 100 \times 100 \times 100)\text{cm}^3$

$\therefore d = \left(\frac{2.6 \times 100 \times 100 \times 100}{6500}\right)\text{cm}$

$= \left(\frac{2.6 \times 100 + 100 \times 100}{6500 \times 100}\right)\text{m}$

$= 4\text{m}$

∴ depth = 4 m

50.(B) Volume of wall = $(400 \times 300 \times 13) \text{ cm}^3$

Volume of each brick

$= (20 \times 12 \times 6.5) \text{ cm}^3$

Numbers of bricks

$= \left(\frac{400 \times 300 \times 13}{20 \times 12 \times 13}\right)$

$= 100$

51.(B) Volume of wall = $\left(24 \times 8 \times \frac{60}{100}\right)\text{m}^3$

$= \frac{576}{5} \text{ m}^3$

Volume of brick

$= \left(90\% \text{ of } \frac{576}{5}\right)\text{m}^3$

$= \left(\frac{90}{100} \times \frac{576}{5}\right)\text{m}^3$

$$= \left(\frac{144 \times 18}{25}\right) m^3$$

Volume of 1 brick

$$= \left(\frac{24}{100} \times \frac{12}{100} \times \frac{8}{100}\right) m^3$$

Number of bricks

$$= \left(\frac{144 \times 18}{25} \times \frac{100}{24} \times \frac{100}{12} \times \frac{100}{8}\right)$$

$= 45000$

52.(B) Let the areas of the three adjacent faces be $2x$, $3x$ and $4x$. Then,

$lb = 2x$, $bh = 3x$ and $lh = 4x$

$$\therefore \quad (lb \times bh \times lh) = 24\,x^3$$

$$24x^3 = (lbh)^3 = (9000)^3$$

$$x^3 = \frac{81000000}{24}$$

$$\Rightarrow \quad x = \frac{300}{2} = 150$$

$\therefore lb = 300$, $bh = 450$, $lh = 600$ and $lbh = 9000$

$$\therefore h = \frac{9000}{300} = 30,\ l = \frac{9000}{4500} = 20 \text{ cm}$$

$$\text{and } b = \frac{9000}{600} = 15 \text{ cm}$$

$\therefore$ smallest side = 15 cm

53.(B) Length covered in 30 minutes

$$= (6 \times 60 \times 30) \text{ m}$$

$$= 10800 \text{ m}$$

$$r = \frac{1}{100} \text{ m}$$

$$h = 10800 \text{ m}$$

$$\text{Volume} = \left(\pi \times \frac{1}{100} \times \frac{1}{100} \times 10800\right) m^3$$

Let the height of the water level be h metres. Then

$$\pi \times \frac{60}{100} \times \frac{60}{100} \times h = \pi \times \frac{1}{100} \times \frac{1}{100} \times 10800$$

$$h = \left(\frac{108}{100} \times \frac{5}{3} \times \frac{5}{3}\right)$$

$= 3$ cm

54.(A) For each iron rod, $r = 1\text{cm} = \frac{1}{100}$ m and $h = 7$ m

Volume of 1 iron rod = $\pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 7\right) m^3$$

$$= \frac{11}{5000} \ m^3$$

$$\text{Number of iron rods} = \frac{88}{100} \times \frac{5000}{11}$$

$= 400$

55.(A) Let the radius of the third ball be r cm

$$r_1 = \frac{1.5}{2} \text{ cm} = \frac{15}{20} \text{ cm} = \frac{3}{4} \text{ cm},$$

$$r_2 = \frac{2}{2} \text{ cm} = 1 \text{ cm}$$

$$r_3 = r \text{ cm}$$

$$\therefore \frac{4}{3}\pi \times \left(\frac{3}{4}\right)^3 + \frac{4}{3}\pi \times (1)^3 + \frac{4}{3}\pi \times r^3$$

$$= \frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3$$

$$\Rightarrow \left(\frac{3}{4}\right)^3 + (1)^3 + r^3 = \left(\frac{3}{2}\right)^3$$

$$\Rightarrow \quad r^3 = \frac{27}{8} - \left(\frac{27}{64} + 1\right)$$

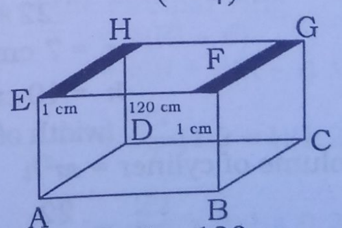
$$= \frac{125}{64} = \left(\frac{5}{4}\right)^3$$

$$\Rightarrow \quad r = \frac{5}{4}$$

Diameter of the third ball

$$= \left(2 \times \frac{5}{4}\right) \text{ cm} = 2.5 \text{ cm}$$

56.(B)



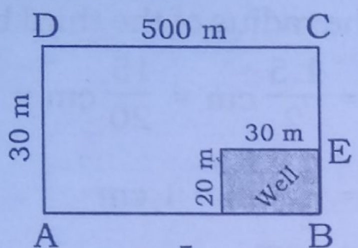
External length = 120 cm

Internal length = 120 − 2 = 118 cm

External breadth = 80 cm

Internal breadth= $80 - 2 = 78$ cm
External height $= 60$ cm
Internal height $= 60 - 2 = 58$ cm
Capacity of the box
$= $ Internal volumes
$= 118 \times 78 \times 58$
$= 533832 \text{ cm}^3$
Quantity of wood used
$=$ External volume
$-$ Internal volume
$= 120 \times 80 \times 60 - 118 \times 78 \times 58$
$= 42168 \text{ cm}^3$

57.(A)

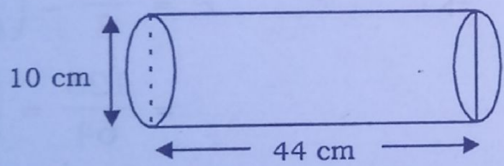


5

Area of remaining field
$=$ Area of field – Area of tank
$= 500 \times 30 - 30 \times 20$
$= 14400 \text{ m}^2$
Let h be level of field raised
$14400\, h = 30 \times 20 \times 12$

$$h = \frac{7200}{14400} = \frac{1}{2} \text{ m}$$
$= 50$ cm

58.(C)



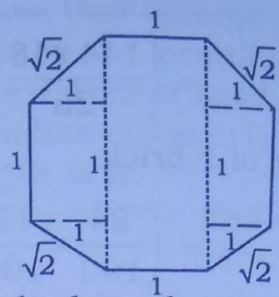
When rectangle is folded along the length to form a cylinder
$2\pi r = 44$

$$r = \frac{44 \times 7}{22 \times 2}$$

$\cong$ $r = 7$ cm
$h = 10$ cm
(width of rectangle)
Volume of cyliner $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 10$$
$= 154 \times 10$
$= 1540 \text{ cm}^3$

59.(D)



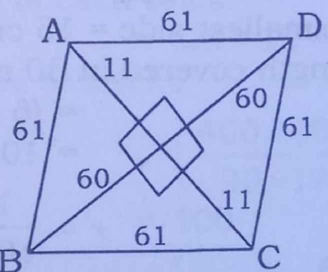
Now calculate the area of 4 right angle triangles

$$= \frac{1}{2} \times 1 \times 1 \times 4 = 2$$

Area of square $= 1 \times 1 \times 2 = 2$
Area of bigger rectangle $= 3 \times 1 = 3$
Hence the area of total figure
$= 2 + 2 + 3 = 7$

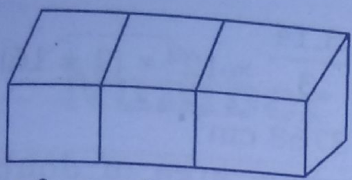
60.(D) Let a be the edge of new cube so formed then
$a^3 = 27 \times 8 \times 1 = 216$
$\therefore$ $a = 6$
Now,
surface area I $= 2(lb + bh + lh)$
$= 2(27\times8 + 8\times1 + 1\times27)$
$= 2 \times 251 = 502$
and surface area II
$= 6a^2 = 6(6)^2 = 216$
Difference $= 502 - 216$
$= 286 \text{ cm}^2$

61.(B)



From figure total length of fence is
$61 + 61 + 61 + 61 + 120 + 22 = 386$
Hence total cost $= 386 \times 20 = ₹\,7720$

62.(D) Suppose the side of the cube is 1 cm then total surface area of the three cubes $= 3 \times 6 \times 1^2$
$= 18 \text{ cm}^2$
Now when the three cubes are placed adjacently in a row it will look as follows



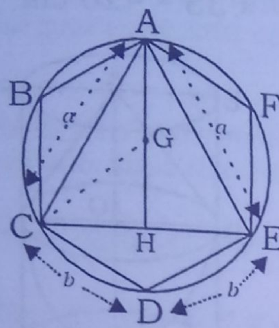
The surface of the above cuboid-shaped figure

$= 2(lb + bh + lh)$
$= 2(3\times1 + 1\times1 + 1\times 3)$

Hence,

required ratio $= 14 : 18 = 7 : 9$

63.(D)



As it is a regular hexagon therefore central angle made by the side is 60°. so in the figure, ΔAGB is equilateral.

$\therefore \quad AG = GB = AB = b$

As AH is the height of the equilateral triangle ABC

$$\therefore \quad AH = \frac{\sqrt{3}}{2} AB = \frac{\sqrt{3}}{2} a \quad \ldots(i)$$

Inequilateral triangle incenter, orthocenter, circumcenter, centroid, all are concident

$\therefore$ AH is also centroid.

Hence,

$AG : GH = 2 : 1$

As, $\quad AH = AG + GH$

$$= b + \frac{b}{2} = \frac{3b}{2} \quad \ldots(ii)$$

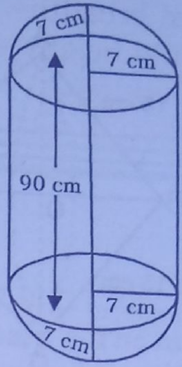
From (i) and (ii)

$$\frac{3b}{2} = \frac{\sqrt{3}}{2} a$$

So, $\quad a = \sqrt{3}b$

or $\quad a^2 = 3b^2$

64.(A)



Let the height of the cylinder be h cm. Then,

$h + 7 + 7 = 104$

or $\quad h = 90$

Total surface area of the solid
$= 2 \times$ curved surface area of hemisphere + curved surface area of the cylinder

$$= \left(2\times2\times\frac{22}{7}\times7\times7+2\times\frac{22}{7}\times7\times90\right) cm^2$$
$= (616 + 3900)\ cm^2$
$= 4576\ cm^2$

Cost of polishing the surface of the solid

$$= ₹\ \frac{4576\times1}{100}$$
$= ₹\ 45.76$

65.(B)The pipe is a hollow cylinder whose base is a circular ring.

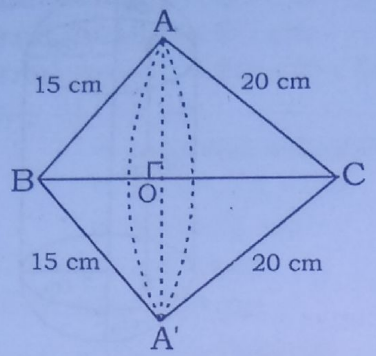
The radius (R) of the outer circle

$$= \frac{1}{2} \times 2.4\ cm$$
$= 1.2\ cm$

The radius (r) of the inner circle
$= 1.2cm - 0.2cm$
$= 1\ cm$

Area of the circular ring
$= \pi(R^2 - r^2)$
$= \pi(R + r)(R - r)$
$$= \frac{22}{7}(1.2 + 1)(1.2 - 1)cm^2$$
$$= \frac{22}{7} \times 2.2 \times 0.2\ cm^2$$
$$= \frac{9.68}{7}\ cm^2$$

66.(C)



Let Δ ABC be the right triangle right angled at A whose sides AB and AC measure 15 cm and 20 cm, respectively.

The length of the side BC (hypotenuse)

$$= \sqrt{15^2 + 20^2} \text{ cm}$$
$$= 25 \text{ cm}$$

Here, AO (or A'O) is the radius of the common base of the double cone formed by revolving the right triangle about BC.

Height of the cone BAA' is BO and sland height is 15 cm.

Height of the cone CAA' is CO and slant height is 20 cm.

Now, Δ AOB ~ Δ CAB (AA similarity)

Therefore, $\frac{AO}{20} = \frac{15}{25}$

This gives $AO = \frac{20 \times 15}{25}$ cm
$= 12$ cm

Also, $\frac{BO}{15} = \frac{15}{25}$

This gives $BO = \frac{15 \times 15}{25}$

Thus, $CO = 25 \text{ cm} - 9 \text{ cm}$
$= 16$ cm

Now, volume of the double cone

$$= \left(\frac{1}{3} \times 3.14 \times 12^2 \times 9 + \frac{1}{3} \times 3.14 \times 12^2 \times 16\right) \text{cm}^3$$

$$= \frac{3.14}{3} \times 12^2 \times (9 + 16) \text{ cm}^3$$

$= 3768 \text{ cm}^3$

Surface area of double cone so formed is

$\cong \quad \pi r l \quad + \quad \pi r' l'$

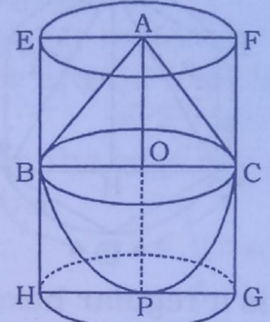
Cone ABA' Cone ACA'

$\cong \pi \times 12 \times 15 + \pi \times 12 \times 20$

$\cong 12\pi \, (15 + 20)$

$\cong 12\pi \times 35 = 420 \text{ cm}^2$

67.(D)



Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere.

Radius BO of the hemisphere (as well as of cone) $= \frac{1}{2} \times 4 \text{ cm} = 2 \text{ cm}$

No, let right circular cylinder EFGH circumscribe the given solid; Radius of the base of the right circular cylinder = HP = BO = 2 cm.

Height of the cylinder

$= AP = AO + OP$
$= 2 \text{ cm} + 2 \text{ cm}$
$= 4$ cm

Now, volume of the right circular cylinder – volume of the solid

$$= \left[\pi \times 2^2 \times 4 - \left(\frac{2}{3} \times \pi \times 2^3 \times \pi \times 2^3\right)\right] \text{cm}^3$$

$= (16\pi - 8\pi) \text{ cm}^3$

$= 8\pi \text{ cm}^3$

Hence, the right circular cylinder covers 8π cm³ more space than the solid.

CHAPTER 21

Number System

Numbers

Numbers are the most important tool in our daily life for counting, measuring, labelling, sequencing and coding etc.

Face value

In a number, the face value of a digit is the value of the digit itself irrespective of its place.

E.g.- The face value of 1 in '10' is 1.

Place value

In a number, the place of a digit indicates the position of that digit in the number.
E.g. The place value of 1 in '10' in tens.

Types of Numbers

1. **Natural Numbers :-** Numbers that can be counted are called Natural numbers
   As- 1,2,3,4,5,6............
2. **Whole Numbers :-** All counting numbers together with zero are called Whole Numbers.
   As - 0,1,2,3,4,5,6.........
3. **Even Numbers :-** Numbers which are divisible by 2 are called even numbers
   As. 2,4,6,8,10............
4. **Odd Numbers :-** Number which are not divisible by 2 are called odd numbers.
   As. 1,3,5,7,9, 11............
5. **Prime Numbers:-** Numbers(greater than I) which are divisible by 1 and itself are called Prime Numbers.
   As. 2,3,5,7,11,13...........
6. **Composite Numbers :-** Numbers greater than 1 which are not prime are called Composite Numbers.
   As. 4,6,8,9,10,12..............
7. **Co-prime Numbers :-** Two natural numbers 'a' and 'b' are said to be co-prime if their H.C.F. is 1.
   As- (2,3) (4,5) (7,9) (11, 9) ....
8. **Rational Numbers :-** The numbers in the form p/q where p and q are integers and q $\neq$ 0 are known as rational numbers.
   As- $\frac{22}{7}, \frac{3}{5}, \frac{9}{7}, \frac{-2}{3}, \frac{0}{1}, \frac{-143}{272}$ etc.

Note:- All integers are rational numbers.

9. **Irrational Numbers :-** All numbers which when expressed in decimal form are in non-terminating and non-repeating form.
   As:- $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi$ etc.
   $\sqrt{2}$ = 1.41421356....
   $\sqrt{3}$ = 1.7320508....

Note:- The exact value of π is not $\frac{22}{7}$, as $\frac{22}{7}$ is rational while π is irrational.
$\frac{22}{7}$ is the approximate value of π.
Similarly 3.14 is not exact value of π.

10. **Real Numbers :-** The total of all rational and all irrational number forms the set (R) of all real numbers.
    Note:-Every natural number, whole number, integer, rational number and every irrational number is a real number.

Prime Number

The number which is divisible by 1 and the number itself is called a prime number.

How to check whether a given number is a prime number or not.

Step-1: Find the nearest greater number which is a perfect square.

Step-2: Find the square root of that number.
Step-3: Now, divide the given number by all the prime numbers lesser or equal than the square root obtained by Step-2.
Step-4: If none of them divides given number completely then given number will be a prime number.
For **e.g:** Let us check whether 137 is a prime number or not.
Step-1: The nearest greatest number which is a perfect square is 144.
Step-2: The square root of 144 is 12.
Step-3: All the prime number lesser than 12 are 2,3,5,7 and 11. Dividing 137 by all these prime number give the following results-

$$\frac{137}{2} = 68\frac{1}{2}, \frac{137}{3} = 45\frac{2}{3}, \frac{137}{5} = 27\frac{2}{5},$$

$$\frac{137}{7} = 17\frac{4}{7} \text{ and } \frac{137}{11} = 12\frac{5}{11}$$

Step-4: So, 137 is a prime number.

Divisibility Rules

Divisibility by 2 : When the last digit of a number is either 0 or even, then the number is divisible by 2, *e.g.*, 12, 86, 472, 520, 1000 etc. are divisible by 2.
Divisibility by 3 : When the sum of the digits of a number is divisible by 3, then the number is divisible by 3.
Divisibility by 4 : When the number made by last two digits of a number is divisible by 4, then that particular number is divisible by 4. Apart from this, the number having two or more zeroes at the end, is also divisible by 4.
Divisibility by 5 : Numbers having 0 or 5 at the end are divisible by 5. *e.g.*, 45, 4350,14850 etc, are divisible by 5 as they have 0 or 5 at the end.
Divisibility by 6 : When a number is divisible by both 3 and 2, then that particular number is divisible by 6 also.
Divisibility by 7 : A number is divisible by 7 when the difference between twice the digit at unit place and the number formed by other digits is either zero or a multiple of 7. *e.g.*, 658 is divisible by 7 because $65 - 2 \times 8 = 65 - 16 = 49$ As 49 is divisible by 7, the number 658 is also divisible by 7.
Divisibility by 8 : When the number made by last three digits of a number is divisible by 8, then the number is also divisible by 8. Apart from this, if the last three or more digits of a number are zeroes, then the number is divisible by 8.
Divisibility by 9 : When the sum of all the digits of a number is divisible by 9, then the number is also divisible by 9.
Divisibility by 10 : When a number ends with zero, then it is divisible by 10.
Divisibility by 11: When the difference of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11.

Unit place

To find the unit place in sum of numbers, we take unit place digits of each number and add them.

Unit place when power of a number is in the form of A^{4n+1}, A^{4n+2}, A^{4n+3} and A^{4n+4}.

A	A^{4n+1}	A^{4n+2}	A^{4n+3}	A^{4n+4}
0	0	0	0	0
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

Type – I (Square)

1. 45^2	2. 65^2	3. 35^2
4. 95^2	5. 75^2	6. 105^2
7. 115^2	8. 195^2	9. 185^2
10. 225^2	11. 43^2	12. 47^2
13. 53^2	14. 57^2	15. 59^2
16. 51^2	17. 46^2	18. 54^2
19. 63^2	20. 39^2	21. 37^2
22. 29^2	23. 67^2	24. 93^2
25. 98^2	26. 84^2	27. 89^2

28. 87^2 29. 96^2 30. 104^2
31. 109^2 32. 112^2 33. 119^2
34. 117^2 35. 212^2 36. 219^2
37. 305^2 38. 317^2 39. 111^2
40. 129^2 41. $(666)^2$ 42. $(177)^2$
43. $(3333)^2$ 44. 413^2 45. $(222)^2$
46. $(272)^2$ 47. $(176)^2$ 48. $(999)^2$
49. $(9999)^2$ 50. $(99999)^2$

Answers of Type – I (Square)

1.	2025	2.	4225	3.	1225
4.	9025	5.	5625	6.	11025
7.	13225	8.	38025	9.	34225
10.	50625	11.	1849	12.	2209
13.	2809	14.	3249	15.	3481
16.	2601	17.	2116	18.	2916
19.	3969	20.	1521	21.	1369
22.	841	23.	4489	24.	8649
25.	9604	26.	7056	27.	7921
28.	7569	29.	9216	30.	10816
31.	11881	32.	12544	33.	14161
34.	13689	35.	44944	36.	47961
37.	93025	38.	100489	39.	12321
40.	16641	41.	443556	42.	31329
43.	11108889				
44.	170569	45.	49284	46.	73984
47.	30976	48.	998001		
49.	99980001				
50.	9999800001				

Type – II (Square root)

1. $\sqrt{7921} = ?$ 2. $\sqrt{4489} = ?$
3. $\sqrt{9216} = ?$ 4. $\sqrt{6889} = ?$
5. $\sqrt{3481} = ?$ 6. $\sqrt{12544} = ?$
7. $\sqrt{17956} = ?$ 8. $\sqrt{32041} = ?$
9. $\sqrt{45369} = ?$ 10. $\sqrt{56169} = ?$
11. $\sqrt{58081} = ?$ 12. $\sqrt{63504} = ?$
13. $\sqrt{84681} = ?$ 14. $\sqrt{18769} = ?$
15. $\sqrt{12996} = ?$ 16. $\sqrt{13689} = ?$
17. $\sqrt{15129} = ?$ 18. $\sqrt{20164} = ?$
19. $\sqrt{21025} = ?$ 20. $\sqrt{21609} = ?$
21. $\sqrt{44521} = ?$ 22. $\sqrt{15376} = ?$
23. $\sqrt{13456} = ?$ 24. $\sqrt{22201} = ?$
25. $\sqrt{21904} = ?$ 26. $\sqrt{23409} = ?$
27. $\sqrt{20736} = ?$ 28. $\sqrt{31684} = ?$
29. $\sqrt{18769} = ?$ 30. $\sqrt{12321} = ?$
31. $\sqrt{44944} = ?$ 32. $\sqrt{54756} = ?$
33. $\sqrt{88804} = ?$ 34. $\sqrt{95481} = ?$
35. $\sqrt{168921} = ?$

How to Solve Type– II (Square Root)

Square Root can be found out by two different methods

1. Prime factor method.
2. Common Division method

1. **Prime factor method** – In this method, the number is split into its prime factors.

For. **E.g:**

2	9216
2	4608
2	2304
2	1152
2	576
2	288
	144

2	144
2	72
2	36
2	18
3	9
3	3
	1

Now we take one out of two similar factors *i.e.* = 3 × 2 × 2× 2 × 2 × 2 = 96

2. **Common Division Method**- This method is very useful when the number is the square of a larger prime number such as 7921.

	89
8	7921
8	64
169	1521
	1521
	×

Step–1: First make pairs starting from left.
$\overline{79}\,\overline{21}$

Step–2: Now we will have to find out a number which when multiplied by itself will be approximately 79. (the last pair is taken first.)

Step–3: The next pair is brought down after writing the remainder.

Step–4: The division and quotient are added.

Step–5: The sum is then followed by a digit which when multiplied by itself and the sum obtained by step 4 leaves no remainder.

Step–6: In case of remainder the process continues till the remainder is 0 or two or more digit after the decimal.

Answers of Type – II (Square Root)

1. 89	2. 67	3. 96
4. 83	5. 59	6. 112
7. 134	8. 179	9. 213
10. 237	11. 241	12. 252
13. 291	14. 137	15. 114
16. 117	17. 123	18. 142
19. 145	20. 147	21. 211
22. 124	23. 116	24. 149
25. 148	26. 153	27. 144
28. 178	29. 137	30. 111
31. 212	32. 234	33. 298
34. 309	35. 411	

Type- III (Cube root)

1. $\sqrt[3]{2197} = ?$ 2. $\sqrt[3]{32768} = ?$
3. $\sqrt[3]{24389} = ?$ 4. $\sqrt[3]{39304} = ?$
5. $\sqrt[3]{59319} = ?$ 6. $\sqrt[3]{97336} = ?$
7. $\sqrt[3]{205379} = ?$ 8. $\sqrt[3]{438976} = ?$
9. $\sqrt[3]{658503} = ?$ 10. $\sqrt[3]{804357} = ?$
11. $\sqrt[3]{1124864} = ?$ 12. $\sqrt[3]{2863288} = ?$
13. $\sqrt[3]{2406104} = ?$ 14. $\sqrt[3]{1295029} = ?$
15. $\sqrt[3]{1685159} = ?$ 16. $\sqrt[3]{4410944} = ?$
17. $\sqrt[3]{2146689} = ?$ 18. $\sqrt[3]{2248091} = ?$
19. $\sqrt[3]{2863288} = ?$ 20. $\sqrt[3]{3048625} = ?$
21. $\sqrt[3]{2515456} = ?$ 22. $\sqrt[3]{6751269} = ?$
23. $\sqrt[3]{3723875} = ?$ 24. $\sqrt[3]{5359375} = ?$
25. $\sqrt[3]{4826809} = ?$

How to Solve Type- III (Cube root)

Step-1: Take last three digits together. The last digit will give an estimate of whose cube root it is.
For **e.g:**- 2 if the last digit is 8.

3 if it is 7.
4 if it is 4
5 if it is 5
6 if it is 6
7 if it is 3
8 if it is 2
9 if it is 9

Step-2: Then see the remaining digit other than the last three digit. Find the nearby cube root and put the cube root.

Note: Always take the number smaller than the given digits.

Step-3: The digit obtained by Step-1 & Step-2 is your answer.

Solution of Cube root

1. $\sqrt[3]{2\,197}$ → Find the nearby cube number and put the cube root.
1
1 3 ⇒ 13

2. $\sqrt[3]{32\,768}$ → $\sqrt[3]{27} = 3$
27
3 2 ⇒ 32

3. $\sqrt[3]{24\ 389} \rightarrow \sqrt[3]{8} = 2$

8

$2 \qquad 9 \Rightarrow 29$

4. $\sqrt[3]{39\ 304}$

27

$3 \qquad 4 \Rightarrow 34$

5. $\sqrt[3]{59\ 319}$

27

$3 \qquad 9 \Rightarrow 39$

6. $\sqrt[3]{97\ 336}$

64

$4 \qquad 6 \Rightarrow 46$

7. $\sqrt[3]{205\ 379}$

125

$5 \qquad 9 \Rightarrow 59$

8. $\sqrt[3]{438\ 976}$

343

$7 \qquad 6 \Rightarrow 76$

9. $\sqrt[3]{658\ 503}$

512

$8 \qquad 7 \Rightarrow 87$

10. $\sqrt[3]{804\ 357}$

729

$9 \qquad 3 \Rightarrow 93$

11. $\sqrt[3]{1124\ 864}$

1000

$10 \qquad 4 \Rightarrow 104$

12. $\sqrt[3]{2863\ 288}$

2744

$14 \qquad 2 \Rightarrow 142$

13. $\sqrt[3]{2406\ 104}$

2197

$13 \qquad 4 \Rightarrow 134$

14. $\sqrt[3]{1295\ 029}$

1000

$10 \qquad 9 \Rightarrow 109$

15. $\sqrt[3]{1685\ 159}$

1331

$11 \qquad 9 \Rightarrow 119$

16. $\sqrt[3]{4410\ 944}$

4096

$16 \qquad 4 \Rightarrow 164$

17. $\sqrt[3]{2146\ 689}$

1728

$12 \qquad 9 \Rightarrow 129$

18. $\sqrt[3]{2248\ 091}$

2197

$13 \qquad 1 \Rightarrow 131$

19. $\sqrt[3]{2863\ 288}$

2744

$14 \qquad 2 \Rightarrow 142$

20. $\sqrt[3]{3048\ 625}$

2744

$14 \qquad 5 \Rightarrow 145$

21. $\sqrt[3]{2515\ 456}$

2197

$13 \qquad 6 \Rightarrow 136$

22. $\sqrt[3]{6751\ 269}$

5832

$18 \qquad 9 \Rightarrow 189$

23. $\sqrt[3]{3723\ 875}$

3375

$15 \qquad 5 \Rightarrow 155$

24. $\sqrt[3]{5359\ 375}$

4913

$17 \qquad 5 \Rightarrow 175$

25. $\sqrt[3]{4826\ 809}$

4096

$16 \qquad 9 \Rightarrow 169$

Type- IV (Basic Division)

1. In a division sum the quotient is 120, the divisor is 456 and the remainder is 333. Find the divident.
2. In a division sum the quotient is 105, the remainder is 195, the divisor is equal