I. $16 - 16.16$ $16 - 16$ $= 0$	**II.** $-x + 16x + x$ $(-16) + 16.16$ $16 + 16$ $= 0$
III. $+x - 16x - x$ $16 - 16.16$ $16 - 16$ $= 0$	**IV.** $-x + 17$ $-16 + 17$ $= 1$

$$I + II + III + IV = 0 + 0 + 0 + 1 = 1$$

87. (B) If $x = 12$

Solve:- $x^4 - 13x^3 + 15x^2 - 13x + 13$

$= x^4 - 13x^3 + 13x^2 + 13x + 13 + 2x^2$

(we broke $15x^2$ in $13x^2 + 2x^2$)

Now, same as above $(x^4 - 13x^3 + 13x^2 - 13x + 13)$

will give us answer 1.

$1 + 2x^2\ (x = 12)$

$1 + 2\,(144) \Rightarrow 1 + 288 = 289$

88. (C) $\left[999\frac{95}{99}\right] \times 99$

Add 4 to 95 and it will become 99.

$\therefore\ (999 + \frac{99}{99}) \times 99 \Rightarrow 1000 \times 99 = 99000$

Now reduce 4 which we added earlier.

$99000 - 4 = 98996$

89. (B)Now , $2a \times 4a \times 10a \times 11a = 880a^4$

$\therefore$ $(880a^4 + ka^4)$ should be a perfect square

$a^4\,(880 + k)$

If we put $k = 20$

$(880 + 20) = 900 = (30)^2$

$\therefore\ k = 20$

90. (A) $x^4 + \frac{1}{x^4} = 119$

So, $x^2 + \frac{1}{x^2} = \sqrt{119 + 2} = 11$

$x + \frac{1}{x} = \sqrt{11 + 2} = \sqrt{13}$

91. (D) $x + \frac{1}{x} = 5$

So, $x^2 + \frac{1}{x^2} = 23$

$x - \frac{1}{x} = \sqrt{21}$

$\left(x + \frac{1}{x}\right)\ \left(x - \frac{1}{x}\right) = 5\sqrt{21}$

92. (A) $x + \frac{1}{x} = 1$

So, $x^2 + \frac{1}{x^2} = 1 - 2 = -1$

$x\left[x^2 + \frac{1}{x^2} + 1 = 0\right]$ (Multiplying by x)

$x^3 + \underbrace{\left[\frac{1}{x} + x\right]}_{1} = 0$

$x^3 + 1 = 0 \Rightarrow x^3 = -1$

93. (B) $x + \frac{1}{x} = 1$

$\left(x^{17} + \frac{1}{x^{17}}\right)$

Divide and multiply by x.

$\frac{x}{x}\left(x^{17} + \frac{1}{x^{17}}\right) = \frac{x^{18}}{x} + \frac{x}{x^{18}}$

$x^3 = -1$

$(x^3)^6 = (-1)^6$

$x^{18} = 1$

$\therefore\ \frac{1}{x} + \frac{x}{1} = 1$

94. (A) $x + \frac{1}{x} = 1$, than

$$\underbrace{x^{208} + x^{205}} + x^{204} + x^{201}.$$

$$x^{208} + x^{205} + x^{204} + x^{201}$$

$$x^{205}(x^3 + 1) + x^{201}(x^3 + 1) \quad \therefore x^3 = -1$$

$$x^{205}(-1 + 1) + x^{205}(-1 + 1) = 0$$

95. (C) $\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$

Divide numerator and denominator by x^2

$$\frac{\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{5x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{x^4}{x^2} + \frac{1}{x^2}}$$

$$\frac{\overbrace{x^2 + \frac{1}{x^2}}^{23} + \overbrace{3x + \frac{3}{x}}^{5 \times 3} + 5}{\underbrace{x^2 + \frac{1}{x^2}}_{23}} = \frac{23 + 15 + 5}{23} = \frac{43}{23}$$

96. (D) $x^4 + \frac{1}{x^4} = 194$

$$x^2 + \frac{1}{x^2} = \sqrt{194 + 2} \Rightarrow x^2 + \frac{1}{x^2} = 14$$

$$x + \frac{1}{x} = \sqrt{14 + 2} \Rightarrow x + \frac{1}{x} = 4$$

Now, $x^3 + \frac{1}{x^3} = (4)^3 - 3(4) = 64 - 12 = 52$

97. (A) $x^4 + \frac{1}{x^4} = 119$

$$x^2 + \frac{1}{x^2} = \sqrt{119 + 2} \Rightarrow x^2 + \frac{1}{x^2} = 11$$

$$x + \frac{1}{x} = \sqrt{11 + 2} \Rightarrow x + \frac{1}{x} = \sqrt{13}$$

Now, $x^3 + \frac{1}{x^3} = \left(\sqrt{13}\right)^3 - 3\left(\sqrt{13}\right)$

$$= 13\sqrt{13} - 3\sqrt{13} = 10\sqrt{13}$$

98. (C) $x + \frac{1}{x} = 5$

squaring

$$x^2 + \frac{1}{x^2} = 25 - 2 \Rightarrow x^2 + \frac{1}{x^2} = 23$$

Now, $x - \frac{1}{x} = \sqrt{23 - 2} \Rightarrow x - \frac{1}{x} = \sqrt{21}$

So, $x^3 - \frac{1}{x^3} = \left(\sqrt{21}\right)^3 + 3\left(\sqrt{21}\right)$

$$= 21\sqrt{21} + 3\sqrt{21} = 24\sqrt{21}$$

99. (A) $x + \frac{1}{x} = \sqrt{3}$

$$x^3 + \frac{1}{x^3} = \left(\sqrt{3}\right)^3 - 3\left(\sqrt{3}\right)$$

$$= 3\sqrt{3} - 3\sqrt{3} = 0$$

100. (D) $x + \frac{1}{x} = \sqrt{3}$

$x^3 + \frac{1}{x^3} = 0$ (Proved above)

$\therefore x^6 = -1$

Now, $\frac{x}{x}\left(x^{17} + \frac{1}{x^{17}}\right) \Rightarrow \frac{x^{18}}{x} + \frac{x}{x^{18}}$

$x^{18} = -1$ Because $x^6 = -1$ and $(x^6)^3 = (-1)^3$

$$\frac{-1}{x} + \frac{x}{-1} = -\left(\frac{1}{x} + x\right)$$

$x+\frac{1}{x}=\sqrt{3} \Rightarrow x^6=-1$

$\therefore\ x^{202}(x^6+1)$

$= x^{202}(-1+1) = x^{202}\times 0 = 0$

102. (A) $x+\frac{1}{x}=3$

$x^2+\frac{1}{x^2}=7$

$x^3+\frac{1}{x^3}=27-9=18$

Now, $x^5+\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-$

$\left(x+\frac{1}{x}\right)$

$=(7\times 18)-3$

$=123$

103. (A) $x = 7+4\sqrt{3}$

Break $(7+4\sqrt{3})$ in the form of (a^2+b^2+2ab)

$=(2^2+\sqrt{3}^2+2\times 2\times\sqrt{3})$

$x=(2+\sqrt{3})^2 \Rightarrow \sqrt{x}=\sqrt{(2+\sqrt{3})^2}$

$\therefore\ \sqrt{x}=2+\sqrt{3}$

104. (C) $x=11+6\sqrt{3}$

$\downarrow$

2ab

$\therefore\ 2\times 3\times\sqrt{2}$

$\therefore\ x=(3+\sqrt{2})^2$

$\sqrt{x}=3+\sqrt{2}$

105. (A) $x=22+8\sqrt{6}$

$\downarrow$

2ab

$2\times 4\times\sqrt{6}$

$x=\left(4+\sqrt{6}\right)^2 \Rightarrow \sqrt{x}=4+\sqrt{6}$

106. (B) $x = 97 + 8\sqrt{6}$

a^2+b^2+2ab

$a^2+b^2=97$ is possible only when a $=4\sqrt{6}$

and b = 1

$\therefore\ x=\left(4\sqrt{6}+1\right)^2 \Rightarrow \sqrt{x}=4\sqrt{6}+1$

107. (C) $x=38+5\sqrt{3}$

$5\sqrt{3}$ can not be broken in the form of 2ab so multiply and divide it by 2.

$\frac{2}{2}(38+5\sqrt{3}) \Rightarrow \frac{76+2\times 5\sqrt{3}}{2}$

$=\frac{76+10\sqrt{3}}{2} \Rightarrow 2ab = 5\sqrt{3}\times 1\times 2$

$\therefore\ a=5\sqrt{3}\ ,\ b=1$

$x=\left(\frac{5\sqrt{3}+1}{\sqrt{2}}\right)^2 \Rightarrow \sqrt{x}=\frac{5\sqrt{3}+1}{\sqrt{2}}$

108. (A) $x=\frac{\sqrt{3}}{2}$

$x=\frac{\sqrt{3}}{2} \Rightarrow x+1=1+\frac{\sqrt{3}}{2}$

$\therefore\ x+1=\frac{2+\sqrt{3}}{2}$

$\Rightarrow$ Multiply numerator and denominator by

$\Rightarrow$ Multiply numerator and denominator by

$$=\frac{4+2\sqrt{3}}{4}$$

$$=\frac{\left(\sqrt{3}\right)^2+(1)^2+2\times\sqrt{3}\times 1}{4}$$

$$x+1=\left(\frac{\sqrt{3}+1}{2}\right)^2 \Rightarrow \sqrt{1+x}=\frac{\sqrt{3}+1}{2}$$

Conclusion:- If $x=\frac{\sqrt{3}}{2}$

then, $\sqrt{1+x}=\frac{\sqrt{3}+1}{2}$

$$\sqrt{1-x}=\frac{\sqrt{3}-1}{2}$$

109. (A) $a^3+b^3+c^3-3abc=0$
$a^3+b^3+c^3-3abc$ will be equal to 0 only in 2 conditions given above. As given in the question first condition is not applicable so, only second condition is applicable which gives us that $a=b=c$. From the given option only option c i.e. 3 satisfy the given condition.

110. (A) a, b, c are the sides of a triangle find the type of triangle if $a^2+b^2+c^2=ab+bc+ca$
$a^2+b^2+c^2=ab+bc+ca$
$a^2+b^2+c^2-ab-bc-ca=0$
and this is possible only in one condition when $a=b=c$
and if all sides are equal, then its an equrilateral triangle.

111. (B) $a=997$, $b=998$ and $c=999$
$a+b+c=2994$
$a^3+b^3+c^3-3abc$

$$=(a+b+c)\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$=(2994)\times\frac{1}{2}\times(1^2+1^2+2^2)$$

$$=2994\times\frac{1}{2}\times 6=8982$$

112. (D) $a=36$, $b=36$ and $c=37$
$\because$ $a^3+b^3+c^3-3abc$

$$=\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$=\frac{1}{2}\times 109\times\left[(0)^2+(-1)^2+(1)^2\right]$$

$$=\frac{1}{2}\times 2=109$$

113. (C) $a+b+c=0$
$a+b+c=0 \Rightarrow b+c=-a$
squaring both side
$b^2+c^2+2bc=a^2 \Rightarrow b^2+c^2=a^2-2bc$

Now $\frac{a^2+b^2+c^2}{a^2-bc}=\frac{a^2+a^2-2bc}{a^2-bc}$

$$=\frac{2a^2-2bc}{a^2-bc}=2\frac{\left(a^2-bc\right)}{\left(a^2-bc\right)}=2$$

114. (B) $x^{\frac{1}{4}}+\frac{1}{x^{\frac{1}{4}}}=1$ (squaring)

$$\left(x^{\frac{1}{4}}\right)^2+\left(x^{-\frac{1}{4}}\right)^2=(1)^2$$

$$\Rightarrow \sqrt{x}+\frac{1}{\sqrt{x}}-2\sqrt{x}\times\frac{1}{\sqrt{x}}=1$$

$$\sqrt{x}+\frac{1}{\sqrt{x}}=-1$$

Again squaring

$$x+\frac{1}{x}=-1 \Rightarrow x^2+\frac{1}{x^2}=-1$$

$$\therefore x^{1024}+\frac{1}{x^{1024}}=-1$$

115. (D) $xy = 1$

$$y = \frac{1}{x} \Rightarrow y = 3 - 2\sqrt{2}$$

$$x + y = x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$$

$$\therefore \; x + \frac{1}{x} = 6$$

Now,

$$\frac{x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}}{x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x}} = \frac{\left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) + 3}{\left(x + \frac{1}{x}\right)^2 - 2 - 2}$$

$$= \frac{216 - 18 + 3}{36 - 2 - 2} = \frac{201}{32}$$

116. (C) $a = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$a = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ (Multiply and divide by $\sqrt{3}+1$)

$$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2} = \frac{3 + 1 + 2\sqrt{3}}{2}$$

$$= \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3} = a$$

$$\therefore \; b = 2 - \sqrt{3}$$

Now, putting value in $\frac{a^2 + b^2}{ab} + a + b$

$$= \frac{7 + 4\sqrt{3} - 7 + 4\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} + 2 + \sqrt{3} + 2 - \sqrt{3}$$

$$= \frac{8\sqrt{3}}{4 - 3} + 4 = 8\sqrt{3} + 4$$

117. (A) $x + \frac{a}{x} = 1$

$$\therefore \; x^2 + a = x$$

also $x^2 = x - a$ $\quad$ $x^2 - x = -a$

Now, $\frac{x^2 + a + x}{x^3 - x^2} = \frac{x + x}{x(x^2 - x)}$

$$= \frac{2x}{x(-a)} = \frac{-2}{a}$$

118. (D) $a + b = 5$ and $a^2 + b^2 = 13$

Value assumption Instead of solving such question, through traditional method. Assume some value of 'a' and 'b' which satify the above given condition.

$\therefore$ $a = 3$ and $b = 2$

$a - b = 3 - 2 = 1$

119. (A) $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$

$\frac{a+b+c}{a+b+c} = 1$ So, put 1 euqal to $\frac{a+b+c}{a+b+c}$

$$\therefore \; \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = \frac{a+b+c}{a+b+c}$$

Taking (a+b+c) of denomintor of right side to numerator of left side, we will get

$$\frac{a}{b+c} \times (a+b+c) + \frac{b}{c+a} \times (b+a+c) + \frac{c}{a+b} \times (a+b+c) = a + b + c$$

$$\frac{a^2}{b+c} + a + \frac{b^2}{c+a} + b + \frac{c^2}{a+b} + c = a + b + c$$

$$\therefore \; \frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} = 0$$

120. (A) $x\left(3-\frac{2}{x}\right)=\frac{3}{x}$

$3x - 2 = \frac{3}{x}$

$3\left(x-\frac{1}{x}\right) = 2$

$\left(x-\frac{1}{x}\right) = \frac{2}{3}$

squaring both sides.

$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = \frac{4}{9}$

$x^2 + \frac{1}{x^2} = \frac{4}{9} + 2 = \frac{22}{9}$

121. (B) $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $xy = 1$

$x = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \Rightarrow x = \frac{1}{y}$

$= \frac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3} \qquad \therefore\ y = 2-\sqrt{3}$

Now, putting value in $\left(\frac{x-y}{x+y}\right)^2$

$\left(\frac{2+\sqrt{3}-2+\sqrt{3}}{2+\sqrt{3}+2-\sqrt{3}}\right)^2 = \left(\frac{2\sqrt{3}}{4}\right)^2$

$= \frac{12}{16} = \frac{3}{4}$

122. (A) $\frac{x^2}{y^2} + 2t + \frac{y^2}{x^2}$

$\therefore\ \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2 + 2 \times t \times \frac{x}{y} \times \frac{y}{x}$

It is a perfect square. So 1 could be the only value of t to let it remain a perfect square

$\therefore\ t = 1$

123. (C) $x = 2+\sqrt{3}$

$x - 2 = \sqrt{3}$

squaring both side

$x^2 - 4x + 4 = 3$

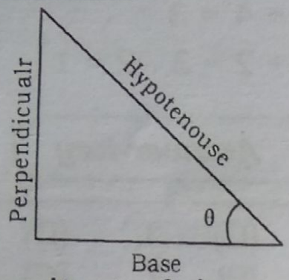
$x^2 - 4x + 2 = 3 - 2 = 1$

Answer-key

1.	(B)	2.	(C)	3.	(D)	4.	(C)
5.	(D)	6.	(B)	7.	(C)	8.	(B)
9.	(A)	10.	(A)	11.	(A)	12.	(A)
13.	(D)	14.	(C)	15.	(A)	16.	(D)
17.	(C)	18.	(D)	19.	(B)	20.	(A)
21.	(A)	22.	(D)	23.	(C)	24.	(D)
25.	(A)	26.	(A)	27.	(A)	28.	(D)
29.	(D)	30.	(C)	31.	(C)	32.	(B)
33.	(A)	34.	(C)	35.	(A)	36.	(C)
37.	(A)	38.	(D)	39.	(B)	40.	(D)
41.	(A)	42.	(A)	43.	(C)	44.	(C)
45.	(A)	46.	(D)	47.	(C)	48.	(B)
49.	(A)	50.	(B)	51.	(A)	52.	(D)
53.	(A)	54.	(D)	55.	(C)	56.	(A)
57.	(A)	58.	(B)	59.	(A)	60.	(C)
61.	(C)	62.	(C)	63.	(D)	64.	(D)
65.	(D)	66.	(D)	67.	(D)	68.	(B)
69.	(C)	70.	(C)	71.	(D)	72.	(B)
73.	(C)	74.	(C)	75.	(B)	76.	(A)
77.	(C)	78.	(A)	79.	(A)	80.	(C)
81.	(B)	82.	(A)	83.	(A)	84.	(C)
85.	(A)	86.	(A)	87.	(B)	88.	(C)
89.	(B)	90.	(A)	91.	(D)	92.	(A)
93.	(B)	94.	(A)	95.	(C)	96.	(D)
97.	(A)	98.	(C)	99.	(A)	100.	(D)
101.	(A)	102.	(A)	103.	(A)	104.	(C)
105.	(A)	106.	(B)	107.	(C)	108.	(A)
109.	(A)	110.	(A)	111.	(B)	112.	(D)
113.	(C)	114.	(B)	115.	(D)	116.	(C)
117.	(A)	118.	(D)	119.	(A)	120.	(A)
121.	(B)	122.	(A)	123.	(C)		

CHAPTER 17 TRIGONOMETRY

Basic Formulae :



Side opposite to θ is considered the perpendicular or height of the triangle whereas side below the θ will be taken as base.

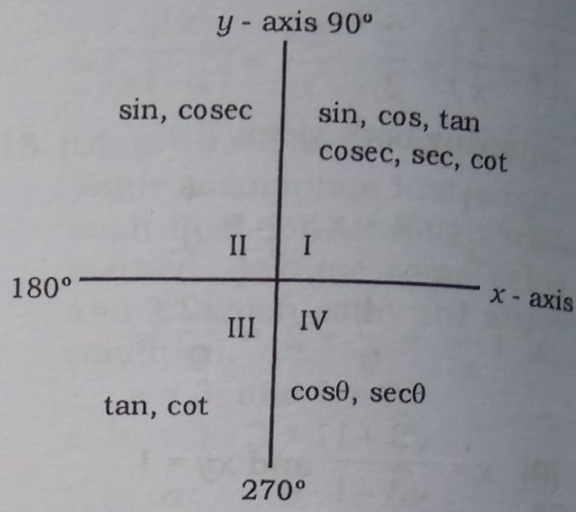
$\sin\theta = \frac{P}{H}$	$\Rightarrow$	$\text{cosec}\,\theta = \frac{H}{P}$
$\cos\theta = \frac{B}{H}$	$\Rightarrow$	$\sec\theta = \frac{H}{B}$
$\tan\theta = \frac{P}{B}$	$\Rightarrow$	$\cot\theta = \frac{B}{P}$
$\sin\theta = \frac{1}{\text{cosec}\,\theta}$	$\Rightarrow$	$\text{cosec}\,\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\Rightarrow$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\Rightarrow$	$\cot\theta = \frac{1}{\tan\theta}$

We can remember $S = \frac{P}{H}$, $C = \frac{B}{H}$ & $T = \frac{P}{B}$ by remembering one sentence.

$\underbrace{\text{Some People Have}}_{S=\frac{P}{H}}$ $\underbrace{\text{Curly Brown Hair}}_{C=\frac{B}{H}}$

$\underbrace{\text{Turns Permanently Black.}}_{T=\frac{P}{B}}$

Quadrant System



$\Rightarrow \sin(90 \pm \theta)$

$\downarrow$

This trigonometric function changes according to the value of θ. If value of θ is $(90 \pm \theta)$ or $(270 \pm \theta)$ then sin changes into cos, tan changes into cot and cosec changes into sec.

$\Rightarrow \tan(90 + \theta) = -\cot\theta$

$\Rightarrow \sec(90 + \theta) = -\text{cosec}\,\theta$

$\Rightarrow \sin(90 + \theta) = \cos\theta$

$\Rightarrow$ In case of $(180 \pm \theta)$ and $(360 \pm \theta)$, trigonometric function will remain the same.

$\Rightarrow \tan(180 - \theta) = -\tan\theta$

$\Rightarrow \cos(180 + \theta) = -\cos\theta$

$\Rightarrow \text{cosec}(180 - \theta) = \text{cosec}\theta$

Table					
	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞

Let us see how to remember this table

	0°	30°	45°	60°	90°
Sin	$\sqrt{\frac{0}{4}} \downarrow 0$	$\sqrt{\frac{1}{4}} \downarrow \frac{1}{2}$	$\sqrt{\frac{2}{4}} \downarrow \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} \downarrow \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} \downarrow 1$
Cos ↓ inverse the sin θ laterally and you get cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan ↓ Divide sin θ by cos θ and you get tan θ	$\underbrace{0 \div 1}_{0}$	$\underbrace{\frac{1}{2} \div \frac{\sqrt{3}}{2}}_{\frac{1}{\sqrt{3}}}$	$\underbrace{\frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}}}_{1}$	$\underbrace{\frac{\sqrt{3}}{2} \div \frac{1}{2}}_{\sqrt{3}}$	$\underbrace{1 \div 0}_{\infty}$

Important

If A + B = 90°, then tanA. tanB =1
If tan A. tan B = 1, then A + B = 90
Few triplets which always forms right angle triangle.
(3, 4, 5), (5, 12, 13), (6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25), (18, 24, 30), (7, 24, 25), (9, 40, 41), (14, 48, 50), (18, 80, 82) *etc.*

Note : $x^2 - y^2$, $2x^2$, $x^2 + y^2$ forms right triangle

Elementary Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$
   (a) $\sin^2\theta + \sin^2(90 - \theta) = 1$ $[\because \sin(90 - \theta) = \cos\theta]$
   (b) $\cos^2\theta + \cos^2(90 - \theta) = 1$ $[\because \cos(90 - \theta) = \sin\theta]$
2. $1 + \tan^2\theta = \sec^2\theta$
   $\sec^2\theta - \tan^2\theta = 1$
   $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
   $\sec\theta + \tan\theta = P$
   $$\sec\theta - \tan\theta = \frac{1}{P}$$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$
   $\text{cosec}^2\theta - \cot^2\theta = 1$
   $(\text{cosec}\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$
   $\text{cosece}\,\theta + \cot\theta = P$
   $$\text{cosece}\,\theta - \cot\theta = \frac{1}{P}$$

Advanced Trigonometric Identities

1. $\sin(A + B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A - B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A + B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A - B) = \cos A\cos B + \sin A\sin B$
5. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
6. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
7. $\cot(A + B) = \dfrac{\cot A\cot B - 1}{\cot A + \cot B}$
8. $\cot(A - B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$
9. $\tan(A + B + C)$
   $$= \left[\frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}\right]$$
10. $2\sin A\sin B = \cos(A - B) - \cos(A + B)$

11. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
12. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
13. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
14. $\sin C + \sin D = 2\sin\frac{C+D}{2}.\cos\frac{C-D}{2}$
15. $\sin C - \sin D = 2\cos\frac{C+D}{2}.\sin\frac{C-D}{2}$
16. $\cos C + \cos D = 2\cos\frac{C+D}{2}.\cos\frac{C-D}{2}$
17. $\cos C - \cos D = 2\sin\frac{C+D}{2}.\sin\frac{D-C}{2}$
18. $\tan C + \tan D = \frac{\sin(C+D)}{\cos C.\cos D}$
19. $\tan C - \tan D = \frac{\sin(C-D)}{\cos C.\cos D}$
20. $\cot C + \cot D = \frac{\sin(C+D)}{\sin C.\sin D}$
21. $\cot C - \cot D = \frac{\sin(C-D)}{\sin C.\sin D}$
22. $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$
23. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$
24. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
25. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
26. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
27. $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Some Important Results

1. $\frac{1+\tan\theta}{1-\tan\theta} = \tan\left(\frac{\pi}{4}+\theta\right) = \tan(45^\circ+\theta)$
2. $\frac{1-\tan\theta}{1+\tan\theta} = \tan\left(\frac{\pi}{4}-\theta\right) = \tan(45^\circ-\theta)$
3. $\frac{\cot\theta+1}{\cot\theta-1} = \cot\left(\frac{\pi}{4}-\theta\right) = \cot(45^\circ-\theta)$
4. $\frac{\cot\theta-1}{\cot\theta+1} = \cot\left(\frac{\pi}{4}+\theta\right) = \cot(45^\circ+\theta)$
5. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
$= \cos^2 B - \cos^2 A$
6. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$
7. $\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$
8. $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$
9. $\tan\theta\tan(60^\circ-\theta)\tan(60^\circ+\theta) = \tan 3\theta$
10. $\tan\theta + \tan(60^\circ+\theta) + \tan(120+\theta)$
$= 3\tan 3\theta$
11. $\cot\theta + \cot(60^\circ+\theta) + \cot(120^\circ+\theta)$
$= 3\cot 3\theta$
12. If $\theta = \frac{\pi}{14}$ then
$\cos\theta.\cos 2\theta.\cos 3\theta.\cos 4\theta.\cos 5\theta.\cos 6\theta$
$= \frac{1}{64}$
13. If $\theta = \frac{\pi}{14}$ then
$\sin\theta.\sin 3\theta.\sin 5\theta.\sin 7\theta.\sin 9\theta\ \sin 11\theta.$
$\sin 13\theta = \frac{1}{64}$
14. $\cos^3\theta + \cos^3(120+\theta) + \cos^3(240+\theta)$
$= \frac{3}{4}\cos 3\theta$
15. $\sin^3\theta + \sin^3(120+\theta) + \sin^3(240+\theta)$
$= \frac{-3}{4}\sin 3\theta$
16. $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta} = \tan\frac{(\alpha+\beta)}{2}$
17. If $A + D = B + C$
$\tan\left(\frac{A+B+C+D}{4}\right)$
$= \frac{\sin A+\sin B+\sin C+\sin D}{\cos A+\cos B+\cos C+\cos D}$
$= \tan\left(\frac{A+D}{2}\right)$ *or* $\tan\left(\frac{B+C}{2}\right)$

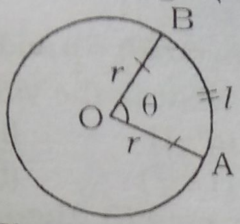
Radian and Degree:

$\boxed{\pi \text{ Radian} = 180^\circ}$

$\boxed{1 \text{ Radian} = 57^\circ\ 16'\ 22'' \text{ (approximately)}}$

Relation between length of arc (l), radius (r) and central angle (θ) :

$\boxed{\theta^{Rad} = \frac{l}{r}}$



In two circles:

$$\theta_1 = \frac{l_1}{r_1} \ \& \ \theta_2 = \frac{l_2}{r_2}$$

$$\therefore \ \frac{\theta_1}{\theta_2} = \frac{l_1/r_1}{l_2/r_2}$$

$$\boxed{\frac{\theta_1}{\theta_2} = \left(\frac{l_1}{l_2}\right)\left(\frac{r_2}{r_1}\right)}$$

If $l_1 = l_2$

then $\boxed{\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1}}$ or $\boxed{\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}}$

If $r_1 = r_2$

then $\boxed{\frac{\theta_1}{\theta_2} = \frac{l_1}{l_2}}$

Maximum and Minimum values of Trigonometric Expressions

Trigonometric Ratio	Range	Trigonometric Ratio	Range
sin A	$[-1, 1]$	cosec A	$(-\infty, -1] \cup [1, \infty)$
cos A	$[-1, 1]$	sec A	$(-\infty, -1] \cup [1, \infty)$
tan A	$(-\infty, \infty)$	cot A	$(-\infty, \infty)$

Note:– In cases of $\sin^2 A$ and $\cos^2 A$, the minimum value is 0.

Let's see- Range of sin A $= [-1, 1]$ i.e.$[-1, 0)$ & $[0, 1]$

Range of $\sin^2 A = [(-1)^2, (0)^2)$ & $[(0)^2, (1)^2] = [1, 0)$ & $[0, 1]$

So we can see that the minimum value is 0.

Note:– If Value of $0^\circ \le \theta \le 90^\circ$ then minimum and maximum value of functions are given below :

Trigonometric Functions	Minimum Value	Maximum Value
$a\sin\theta + b\cos\theta$	$-\sqrt{a^2+b^2}$	$\sqrt{a^2+b^2}$
$a\sin\theta + b\,\text{cosec}\theta$ $a\cos\theta + b\sec\theta$ $a\tan\theta + b\cot\theta$	$2\sqrt{ab}$	∞
$a\sec\theta + b\,\text{cosec}\theta$	$(\sqrt{a}+\sqrt{b})^2$	∞
$a\sin^2\theta + b\cos^2\theta$	a or b whichever is lowest	a or b whichever highest
$a\sin^2\theta + b\,\text{cosec}^2\theta$ $a\cos^2\theta + b\sec^2\theta$ $a\tan^2\theta + b\cot^2\theta$	$2\sqrt{ab}$	∞

Exercise

1. What is the value of $\text{Sin}^2 1010° + \text{Cos}^2 1010°$?
(A) 1010 (B) 101
(C) 10 (D) 1

2. $\tan^2\theta$ is equal to :
(A) $\frac{1}{\cot^2\theta}$ (B) $\frac{1}{\cos^2\theta}$
(C) $\frac{1}{\sin^2\theta}$ (D) $\frac{1}{\text{cosec}^2\theta}$

3. $\tan\theta.\ \text{cosec}\theta$ equal to :
(A) $\sec\theta$ (B) $\cot\theta$
(C) $\sin\theta$ (D) $\cos\theta$

4. $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$ is equal to :
(A) $2\sec^2\theta$ (B) $2\,\text{cosec}^2\theta$
(C) $1^2-\sin^2\theta$ (D) $2\cos^2\theta$

5. $\cos^2\theta\,(1+\tan^2\theta)$ is equal to :
(A) 1 (B) 1/2
(C) 2 (D) 3

6. In a triangle $B = 90°$, $C = \theta$. What is the value of $\tan\theta.\ \sin\theta$?
(A) $\frac{\cos^2\theta}{\sin^2\theta}$ (B) $\frac{\sin^2\theta}{\cos^2\theta}$
(C) $\sec\theta - \cos\theta$ (D) $\cos\theta - \sec\theta$

7. $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$ is equal to:
(A) $\sec^2 A \tan^2 B$ (B) $\tan^2 A \tan^2 B$
(C) $\sec^2 A \sec^2 B$ (D) $\cos^2 A \cos^2 B$

8. If $1 + \tan^2\theta = \sec^2\theta$, then what will be the value of θ:
(A) 30° (B) 60°
(C) 90° (D) All of them

9. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$ then what is the value of $2p$?
(A) $p\,(q^2-1)$ (B) $q\,(p^2-1)$
(C) $p\,(1-q^2)$ (D) $q\,(1-p^2)$

10. If $\tan x = 4/3$, then the value of :
$$\sqrt{\frac{(1-\sin x)(1+\sin x)}{(1+\cos x)(1-\cos x)}}$$
(A) 9/16 (B) 3/4
(C) 4/3 (D) 16/9

11. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then find the value of $\sec^4\theta - \tan^4\theta$.
(A) 7/12 (B) 1/2
(C) 5/12 (D) 1

12. If $\sin^2 60 + \cos^2(3x - 9°) = 1$ then value of x is:
(A) 24° (B) 23°
(C) 22° (D) 21°

13. If $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$ then value of x and y :
(A) 15°, 45° (B) 45°, 15°
(C) 30°, 60° (D) 60°, 30°

14. What is the value of $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$?
(A) 0 (B) 1
(C) 2 (D) −1

15. What is the value of $\text{cosec}\,\theta\sqrt{1-\cos^2\theta}$?
(A) 1 (B) 0
(C) − 2 (D) $-\sin\theta$

16. If $2\cos 3\theta_1 = 1$ and $2\sin 2\theta_2 = \sqrt{3}$ then what will be the value of θ_1 and θ_2 ?
(A) 30°, 20° (B) 60°, 40°
(C) 20°, 30° (D) 45°, 45°

17. Find the value of :
$1 + 2\sec^2 A \tan^2 A - \sec^4 A - \tan^4 A$.
(A) 0 (B) 1
(C) $\sec^2 A \tan^2 A$ (D) None of these

18. $(\sec\theta - 1)^2 - (\tan\theta - \sin\theta)^2$ is equal to :
(A) $(1 - \sin\theta)^2$ (B) $(1 - \cos\theta)^2$
(C) $(1 - \tan\theta)^2$ (D) None of these

19. In a right triangle PQR, Right angled at Q, If $\tan R = \frac{1}{\sqrt{3}}$, which of the following is the value of sin P ?
(A) $\sqrt{3}$ (B) $\frac{1}{2}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$

20. The value of
$\sin(180^\circ + \varphi).\sin(180 - \varphi).\operatorname{cosec}^2\varphi$ is :
(A) –1 (B) +1
(C) 0 (D) None of these

21. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ$ .....$\cos 179^\circ$ is:
(A) Positive Integer
(B) Negative Integer
(C) 0
(D) None of these

22. If $\tan\theta = 1$, then the value of
$\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$:
(A) 2 (B) $2\frac{1}{2}$
(C) 3 (D) $\frac{4}{5}$

23. If $\tan\theta = \frac{4}{3}$, then the value of
$\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$:
(A) 0.5 (B) – 0.5
(C) 3.0 (D) – 3.0

24. If θ is a positive acute angle and $\cos^2\theta + \cos^4\theta = 1$ then $\tan^2\theta + \tan^4\theta$ is equal to :
(A) $\frac{3}{2}$ (B) 1
(C) $\frac{1}{2}$ (D) 0

25. $\tan 4^\circ . \tan 43^\circ . \tan 47^\circ \tan 86^\circ$ is equal to:
(A) 0 (B) 1
(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

26. If $\sin\alpha . \sec(30^\circ + \alpha) = 1$ and $(0^\circ < \alpha < 60^\circ)$ then $\sin\alpha + \cos 2\alpha$ is equal to :
(A) 1 (B) $\frac{2+\sqrt{3}}{2\sqrt{3}}$
(C) 0 (D) $\sqrt{2}$

27. $\sin\alpha + \cos\beta = 2$ $(0^\circ \le \beta \quad \alpha \le 90^\circ)$ then $\sin\left(\frac{2\alpha + \beta}{3}\right)$ is equal to :
(A) $\sin\frac{\alpha}{2}$ (B) $\cos\frac{\alpha}{3}$
(C) $\sin\frac{\alpha}{3}$ (D) $\cos\frac{2\alpha}{3}$

28. $\cot 10^\circ \cot 20^\circ \cot 60^\circ \cot 70^\circ \cot 80^\circ$:
(A) 1 (B) – 1
(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

29. $\cos^4\theta - \sin^4\theta = \frac{2}{3}$ then $2\cos^2\theta - 1$ is equal to:
(A) 0 (B) 1
(C) $\frac{2}{3}$ (D) $\frac{3}{2}$

30. $\sin(90 - x^\circ).\sin(90 - (x+1)^\circ).\sin(90 - (x+2)^\circ)$ $\sin(90 - (x+3)^\circ)$ .... If x is a natural number, then what will be the value of above statement up to 90 term $(1 \le x \le 90)$.
(A) 0 (B) 1
(C) 2 (D) – 1