

$$\begin{aligned}
 55.(D) \quad & 1 - \frac{\sin^2 \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} - \frac{\sin \theta}{1 - \cos \theta} \\
 &= \frac{1 + \cos \theta - \sin^2 \theta}{1 + \cos \theta} + \frac{1 - \cos^2 \theta - \sin^2 \theta}{\sin \theta (1 - \cos \theta)} \\
 &= \frac{\cos^2 \theta + \cos \theta}{1 + \cos \theta} + \frac{1 - (\sin^2 \theta + \cos^2 \theta)}{\sin \theta (1 - \cos \theta)} \\
 &= \frac{\cos \theta (1 + \cos \theta)}{1 + \cos \theta} + \frac{1 - 1}{\sin \theta (1 - \cos \theta)} \\
 &= \cos \theta
 \end{aligned}$$

56.(C)  $A + B + C = 180^\circ$  (angle of triangle)  
 $A + B = 180^\circ - C$   
 $\frac{A+B}{2} = 90^\circ - \frac{C}{2}$  ... (i)

In equation (i)

$$\sin\left(\frac{A+B}{2}\right) = \sin\left(90^\circ - \frac{C}{2}\right)$$

$$\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$$

(option A is correct)

In equation (i)

$$\cos\left(\frac{A+B}{2}\right) = \cos\left(90^\circ - \frac{C}{2}\right)$$

$$\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$$

(option B is correct).

In equation (i)

$$\tan\left(\frac{A+B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right)$$

$$\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$$

(option C is incorrect)

In equation (i)

$$\cot\left(\frac{A+B}{2}\right) = \cot\left(90^\circ - \frac{C}{2}\right)$$

$$\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$$

(option D is correct)

### 56. (C) Alternative method:-

Let  $A = B = C = 60$

(1)  $\sin 60^\circ = \cos 30^\circ$  ✓

(2)  $\cos 60^\circ = \sin 30^\circ$  ✓

(3)  $\tan 60^\circ = \sin 30^\circ$  ✗

(4)  $\cot 60^\circ = \tan 30^\circ$  ✓

57. (D)  $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$

So,  $\theta_1 = \theta_2 = \theta_3 = 90^\circ$

$\cos \theta_1 + \cos \theta_2 + \cos \theta_3$

$= \cos 90^\circ + \cos 90^\circ + \cos 90^\circ$

$= 0$

58. (B)  $\sin \theta + \cos \theta = p$

$\sec \theta + \operatorname{cosec} \theta = q$

$$\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} = q$$

$$\frac{p}{\sin \theta \cos \theta} = q$$

$q(p^2 - 1)$

$$= \frac{p}{\sin \theta \cos \theta} [(\sin \theta + \cos \theta)^2 - 1]$$

$$= \frac{p}{\sin \theta \cos \theta} [\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1]$$

$$= \frac{p}{\sin \theta \cos \theta} \times 2 \sin \theta \cos \theta = 2p$$

59. (B)  $\sec \theta + \tan \theta = x$

Squaring on both side

$$(\sec \theta + \tan \theta)^2 = x^2$$

Using C & D method

$$\frac{(\sec \theta + \tan \theta)^2 + 1}{(\sec \theta + \tan \theta)^2 - 1} = \frac{x^2 + 1}{x^2 - 1}$$

$$\frac{\sec^2 \theta + \tan^2 \theta + 2 \sec \theta \tan \theta + 1}{\sec^2 \theta + \tan^2 \theta + 2 \sec \theta \tan \theta - 1} = \frac{x^2 + 1}{x^2 - 1}$$

$$\frac{x^2 + 1}{x^2 - 1} = \frac{\sec^2 \theta + \sec^2 \theta + 2 \sec \theta \tan \theta}{\tan^2 \theta + \tan^2 \theta + 2 \sec \theta \tan \theta}$$

$$\frac{x^2 + 1}{x^2 - 1} = \frac{\sec \theta (\sec \theta + \tan \theta)}{\tan \theta (\sec \theta + \tan \theta)}$$

$$\frac{x^2 + 1}{x^2 - 1} = \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta}$$

$$\frac{x^2 - 1}{x^2 + 1} = \sin \theta$$

$$60. (D) a \cos\theta + b \sin\theta = m \quad \dots(i)$$

$$a \sin\theta - b \cos\theta = n \quad \dots(ii)$$

Squaring and add equation (i) and (ii)

$$a^2 \cos^2\theta + b^2 \sin^2\theta + 2ab \sin\theta \cos\theta +$$

$$a^2 \sin^2\theta + b^2 \cos^2\theta - 2ab \sin\theta \cos\theta$$

$$= m^2 + n^2$$

$$a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\cos^2\theta + \sin^2\theta)$$

$$= m^2 + n^2$$

$$a^2 + b^2 = m^2 + n^2$$

$$61. (B) \cot^2\theta \left( \frac{\sec\theta - 1}{1 + \sin\theta} \right) + \sec^2\theta \left( \frac{\sin\theta - 1}{1 + \sec\theta} \right)$$

$$= \frac{\cos^2\theta}{\sin^2\theta} \left[ \frac{1 - \cos\theta}{\cos\theta(1 + \sin\theta)} \right] +$$

$$\frac{1}{\cos^2\theta} \left[ \frac{\cos\theta(\sin\theta - 1)}{1 + \cos\theta} \right]$$

$$= \frac{\cos^2\theta(1 - \cos^2\theta) + \sin^2\theta(\sin^2\theta - 1)}{\sin^2\theta \cos\theta(1 + \sin\theta)(1 + \cos\theta)}$$

$$= \frac{\cos^2\theta \sin^2\theta - \sin^2\theta \cos^2\theta}{\sin^2\theta \cos\theta(1 + \sin\theta)(1 + \cos\theta)} = 0$$

$$62. (B) \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\sin A + \sin B)(\cos A + \cos B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\sin A + \sin B)(\cos A + \cos B)}$$

$$= \frac{1 - 1}{(\sin A + \sin B)(\cos A + \cos B)} = 0$$

$$63. (A) (\sec\theta - \cos\theta)(\cosec\theta - \sin\theta)(\tan\theta + \cot\theta)$$

$$= \left( \frac{1 - \cos^2\theta}{\cos\theta} \right) \left( \frac{1 - \sin^2\theta}{\sin\theta} \right) \left( \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta} \right)$$

$$= \left( \frac{\sin^2\theta \cdot \cos^2\theta}{\sin^2\theta \cdot \cos^2\theta} \right) = 1$$

$$64. (D) 1 + \sin\theta + \sin^2\theta + \dots \infty = 4 + 2\sqrt{3}$$

So,

$$\frac{1}{1 - \sin\theta} = 4 + 2\sqrt{3}$$

$$1 - \sin\theta = \frac{1}{4 + 2\sqrt{3}} \times \frac{4 - 2\sqrt{3}}{4 + 2\sqrt{3}}$$

$$1 - \sin\theta = \frac{4 - 2\sqrt{3}}{16 - 12}$$

$$4 - 4\sin\theta = 4 - 2\sqrt{3}$$

$$4\sin\theta = 2\sqrt{3}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \sin 60^\circ \text{ or } \sin 120^\circ$$

$$\theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

$$65. (A) \sec\theta + \tan\theta = 2 + \sqrt{5} \quad \dots(i)$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\sec\theta - \tan\theta = \frac{1}{\sqrt{5} + 2}$$

$$\sec\theta - \tan\theta = \sqrt{5} - 2 \quad \dots(ii)$$

Adding equation (i) and (ii)

$$2\sec\theta = 2\sqrt{5}$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

Subtracting equation (ii) from (i)

$$2\tan\theta = 4$$

$$\tan\theta = 2$$

$$\sin\theta = 2\cos\theta = \frac{2}{\sqrt{5}}$$

$$\sin\theta + \cos\theta = \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{3}{5}$$

$$66. (A) 4x = \sec\theta \quad \dots(i)$$

$$\frac{4}{x} = \tan\theta \quad \dots(ii)$$

Adding equation (i) and (ii)

$$4\left(x + \frac{1}{x}\right) = \sec\theta + \tan\theta \dots(iii)$$

subtracting equation (ii) from (i)

$$4\left(x - \frac{1}{x}\right) = \sec\theta - \tan\theta \dots(iv)$$

multiplying equation (iii) and (iv)

$$16\left(x^2 - \frac{1}{x^2}\right) = \sec^2\theta - \tan^2\theta$$

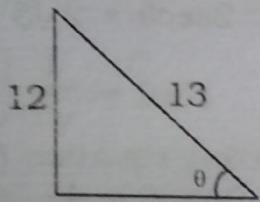
$$8\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{2}$$

$$\begin{aligned}
 67. (B) \quad & (1 - \sin\alpha)(1 - \sin\beta)(1 - \sin\gamma) \\
 & = (1 + \sin\alpha)(1 + \sin\beta)(1 + \sin\gamma) \\
 A^2 & = (1 - \sin\alpha)(1 - \sin\beta)(1 - \sin\gamma) \\
 & \quad (1 + \sin\alpha)(1 + \sin\beta)(1 + \sin\gamma) \\
 & = (1 - \sin^2\alpha)(1 - \sin^2\beta)(1 - \sin^2\gamma) \\
 & = \cos^2\alpha \cos^2\beta \cos^2\gamma
 \end{aligned}$$

$$A = \pm \cos\alpha \cos\beta \cos\gamma$$

$$\begin{aligned}
 68. (A) \quad & \sin(2x - 20^\circ) = \cos(2y + 20^\circ) \\
 2x - 20^\circ + 2y + 20^\circ & = 90^\circ \\
 2(x + y) & = 90^\circ \\
 x + y & = 45^\circ \\
 \sec(x + y) & = \sec 45^\circ = \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 69. (A) \quad & 12 \sin\theta + 5 \cos\theta = 13 \\
 \frac{12}{13} \sin\theta + \frac{5}{13} \cos\theta & = 1 \\
 \sin^2\theta + \cos^2\theta & = 1
 \end{aligned}$$



$$\tan\theta = \frac{12}{5}$$

$$70. (D) \quad \tan\theta - \cot\theta = a$$

Squaring on both side

$$\begin{aligned}
 \tan^2\theta + \cot^2\theta - 2 \tan\theta \cdot \cot\theta & = a^2 \\
 \tan^2\theta + \cot^2\theta - 2 + 4 & = a^2 + 4 \\
 a^2 + 4 & = (\tan\theta + \cot\theta)^2 \\
 a^2 + 4 & = \left( \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta} \right)^2 \\
 a^2 + 4 & = \frac{1}{\sin^2\theta \cos^2\theta} \quad \dots(i)
 \end{aligned}$$

$$\cos\theta - \sin\theta = b$$

Squaring on both side

$$\begin{aligned}
 \cos^2\theta + \sin^2\theta - 2 \sin\theta \cos\theta - 1 \\
 b^2 - 1 = 1 - 2 \sin\theta \cos\theta = b^2
 \end{aligned}$$

$$b^2 - 1 = -2 \sin\theta \cos\theta$$

Squaring on both side

$$(b^2 - 1)^2 = 4 \sin^2\theta \cos^2\theta \quad \dots(ii)$$

Multiplying equation (i) and (ii)

$$(a^2 + 4)(b^2 - 1)^2 = 4$$

$$\begin{aligned}
 71. (A) \quad & \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} \\
 & = \frac{\tan\theta + (\sec\theta - 1)}{\tan\theta - (\sec\theta - 1)} \\
 & = \left( \frac{\frac{\sin\theta}{\cos\theta} + \left( \frac{1}{\cos\theta} - 1 \right)}{\frac{\sin\theta}{\cos\theta} - \left( \frac{1}{\cos\theta} - 1 \right)} \right) \\
 & = \frac{\sin\theta + (1 - \cos\theta)}{\sin\theta - (1 - \cos\theta)} \\
 & = \frac{\sin\theta + (1 - \cos\theta)}{\sin\theta - (1 - \cos\theta)} \times \frac{\sin\theta + (1 - \cos\theta)}{\sin\theta + (1 - \cos\theta)} \\
 & = \frac{\sin^2\theta + (1 - \cos\theta)^2 + 2\sin\theta(1 - \cos\theta)}{\sin^2\theta - (1 - \cos\theta)^2} \\
 & = \frac{\sin^2\theta + 1 + \cos^2\theta - 2\cos\theta + 2\sin\theta(1 - \cos\theta)}{\sin^2\theta - 1 - \cos^2\theta + 2\cos\theta} \\
 & = \frac{2 - 2\cos\theta + 2\sin\theta(1 - \cos\theta)}{-(1 - \sin^2\theta) - \cos^2\theta + 2\cos\theta} \\
 & = \frac{2(1 - \cos\theta) + 2\sin\theta(1 - \cos\theta)}{-2\cos^2\theta + 2\cos\theta} \\
 & = \frac{2(1 + \sin\theta)(1 - \cos\theta)}{2\cos\theta(1 - \cos\theta)} \\
 & = \frac{1 + \sin\theta}{\cos\theta}
 \end{aligned}$$

$$\begin{aligned}
 72. (A) \quad & (\sec\theta \cdot \sec\alpha + \tan\theta \cdot \tan\alpha)^2 \\
 & - (\sec\theta \cdot \tan\alpha + \tan\theta \cdot \sec\alpha)^2
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{1}{\cos\theta} \times \frac{1}{\cos\alpha} + \frac{\sin\theta}{\cos\theta} \times \frac{\sin\alpha}{\cos\alpha} \right)^2 - \\
 & \left( \frac{1}{\cos\theta} \times \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\alpha} \right)^2 \\
 & = \frac{(1 + \sin\theta \sin\alpha)^2}{(\cos\theta \cos\alpha)^2} - \frac{(\sin\alpha + \sin\theta)^2}{(\cos\theta \cos\alpha)^2} \\
 & = \frac{(1 + \sin\theta \sin\alpha)^2 - (\sin\alpha + \sin\theta)^2}{(\cos\theta \cos\alpha)^2}
 \end{aligned}$$

$$\begin{aligned}
&= \left[ \frac{(1 + \sin \theta + \sin \alpha + \sin \theta \sin \alpha)}{(1 - \sin \alpha - \sin \theta + \sin \theta \sin \alpha)} \right] \\
&= \frac{(1 + \sin \theta)(1 + \sin \alpha)(1 - \sin \alpha)(1 - \sin \theta)}{\cos^2 \theta \cos^2 \alpha} \\
&= \frac{(1 - \sin^2 \theta)(1 - \sin^2 \alpha)}{\cos^2 \theta \cos^2 \alpha} \\
&= \frac{\cos^2 \theta \cdot \cos^2 \alpha}{\cos^2 \theta \cdot \cos^2 \alpha} \\
&= 1
\end{aligned}$$

73. (B)  $\frac{7\pi}{12}$  radian =  $\frac{7 \times 180}{12}$   
 $= 105^\circ$

74. (D)  $(\operatorname{cosec} A - \cot A)(\operatorname{cosec} B - \cot B)$   
 $(\operatorname{cosec} C - \cot C) = (\operatorname{cosec} A + \cot A)$   
 $(\operatorname{cosec} B + \cot B)(\operatorname{cosec} C + \cot C) = A$   
 $A^2 = (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)$   
 $(\operatorname{cosec} B - \cot B)(\operatorname{cosec} B + \cot B)$   
 $(\operatorname{cosec} C - \cot C)(\operatorname{cosec} C + \cot C)$   
 $A^2 = (\operatorname{cosec}^2 A - \cot^2 A)(\operatorname{cosec}^2 B - \cot^2 B)$   
 $(\operatorname{cosec}^2 C - \cot^2 C)$   
 $A^2 = 1 \times 1 \times 1$   
 $A = \pm 1$

75. (B) Ratio of diameters  
 $= (\text{angle})_2 : (\text{angle})_1$   
 $= 120^\circ : 75^\circ$   
 $= 8 : 5$

76. (B) Arc length = 40 cm  
Subtend angle =  $22 \frac{1}{2}^\circ$   
radius =  $\frac{40 \times 180}{22 \frac{1}{2} \times 3.14} = 102 \text{ cm}$

77. (C) Arc length = 16 cm  
radius = 50 cm  
angle ( $\theta$ ) =  $16 \times \frac{180^\circ}{3.14} \times \frac{1}{50}$   
 $= \frac{5760}{314} = 18^\circ 20' 38''$

78. (A) Maximum value =  $\sqrt{a^2 + b^2}$   
 $= \sqrt{(3)^2 + (4)^2} = 5$

79. (B) Maximum value of  $3\cos \theta + 4\sin \theta + 5$   
 $= \sqrt{(3)^2 + (4)^2} + 5 = 5 + 5 = 10$

80. (A) Maximum value of

$$\begin{aligned}
&\sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right) \\
&= \sqrt{(1)^2 + (1)^2} = \sqrt{2}
\end{aligned}$$

81. (B)  $2\sin^2 \theta + 2\cos^2 \theta + \cos^2 \theta$

$2 + \cos^2 \theta$  ( $0 \leq \cos^2 \theta \leq 1$ )

Minimum value of  $2 + \cos^2 \theta$  is 2.

82. (D)  $3\sin^2 \theta + 3\cos^2 \theta + \cos^2 \theta$

$3 + \cos^2 \theta$  ( $0 \leq \cos^2 \theta \leq 1$ )

Maximum value of  $3 + \cos^2 \theta$  is 4.

83. (B) Minimum value of  $4 \tan^2 \theta + 9 \cot^2 \theta$   
is  $2\sqrt{ab}$  i.e.  $2\sqrt{4 \times 9} = 12$

84. (B) Minimum value of  $9 \cos^2 \theta + 16 \sec^2 \theta$  is  $2\sqrt{ab}$  i.e.  $\sqrt{9 \times 16} = 24$

85. (B) Minimum value of  $25 \sin^2 \theta + 49 \operatorname{cosec}^2 \theta$  is  $2\sqrt{ab}$  i.e.  $2\sqrt{25 \times 49} = 70$

86. (C) Min. value of  $4 \sec^2 \theta + 9 \operatorname{cosec}^2 \theta$  is  
 $(\sqrt{a} + \sqrt{b})^2$  i.e.  $(\sqrt{4} + \sqrt{9})^2 = (5)^2 = 25$

87. (A) Maximum value of  $\sin^8 \theta + \cos^{14} \theta$  is 1.

88. (C)  $A = \cos^2 x + \sec^2 x$

$$= \cos^2 x + \frac{1}{\cos^2 x}$$

So,  $f(x) = A \geq 2$

89. (B) Minimum Value of  $Q^{3\sin \theta} \cdot 16^{\cos \theta} = 2^{3\sin \theta + 4 \cos \theta}$  will be at minimum value of  $(3\sin \theta + 4 \cos \theta)$   
Minimum value of  $3\sin \theta + 4 \cos \theta$   
is  $-\sqrt{(3)^2 + (4)^2} = -5$

So, Minimum value of function  
 $= 2^{-5}$

$$= \frac{1}{32}$$

90. (A) Maximum value of  $(64^{\sin\theta} \times 256^{\cos\theta}) = 4^{3\sin\theta + 4\cos\theta}$  will be at maximum value of  $3\sin\theta + 4\cos\theta$ .

Maximum value of  $3\sin\theta + 4\cos\theta$  is  $\sqrt{a^2 + b^2} = \sqrt{9+16} = 5$

So, maximum value of  $(64^{\sin\theta} \times 256^{\cos\theta})$  is  $4^5$  i.e. 1024

$$\begin{aligned} 91. (C) f(\theta) &= \sin^2\theta + \cos^2\theta + \sec^2\theta + \cosec^2\theta \\ &\quad + \tan^2\theta + \cot^2\theta \\ &= \sin^2\theta + \cos^2\theta + 1 + \tan^2\theta + 1 \\ &\quad + \cot^2\theta + \tan^2\theta + \cot^2\theta \\ &= (\sin^2\theta + \cos^2\theta) + 2 + 2(\tan^2\theta + \cot^2\theta) \\ &= 1 + 2 + 2(\tan^2\theta + \cot^2\theta) \\ &= 3 + 2(\tan^2\theta + \cot^2\theta) \end{aligned}$$

So, minimum value of  $f(\theta)$  will be at minimum value of  $(\tan^2\theta + \cot^2\theta)$

Minimum value of  $(\tan^2\theta + \cot^2\theta)$  is  $2\sqrt{ab}$  i.e.  $2\sqrt{1 \times 1} = 2$

So, minimum value of  $f(\theta) = 3 + 2(2) = 7$

92. (A) Minimum value of  $\sin\theta + \cos\theta$  is  $-\sqrt{a^2 + b^2}$  i.e.  $-\sqrt{(1)^2 + (1)^2} = -\sqrt{2}$

Maximum value of  $\sin\theta + \cos\theta$  is  $\sqrt{a^2 + b^2}$  i.e.  $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

93. (C)  $\tan^5\theta + \cot^5\theta = 2525$

$$\tan^5\theta + \frac{1}{\tan^5\theta} = 2525$$

$$\begin{aligned} \tan^5\theta + \frac{1}{\tan^5\theta} &= \left( \tan^3\theta + \frac{1}{\tan^3\theta} \right) \\ &\quad \left( \tan^2\theta + \frac{1}{\tan^2\theta} \right) - \left( \tan\theta + \frac{1}{\tan\theta} \right) \end{aligned}$$

$$\text{So, } \tan\theta + \frac{1}{\tan\theta} = 5$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 5$$

$$\begin{aligned} \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} &= 5 \\ \sec\theta \cdot \cosec\theta &= 5 \end{aligned}$$

94. (B)  $\cos\theta + \sec\theta = 4$

Squaring on both side

$$\cos^2\theta + \sec^2\theta + 2 = 16$$

$$\cos^2\theta + \sec^2\theta = 14$$

Squaring on both side

$$\cos^4\theta + \sec^4\theta + 2 = 196$$

$$\cos^4\theta + \sec^4\theta = 194$$

95. (A)  $\sin\theta + \cosec\theta = t$

$$\begin{aligned} \sin^5\theta + \cosec^5\theta &= [(t)^3 - 3t](t^2 - 2) - t \\ &= t^5 - 5t^3 + 5t \end{aligned}$$

96. (C)  $\tan^2\theta + \cot^2\theta + 1 = 0$

$$\tan^2\theta + \frac{1}{\tan^2\theta} + 1 = 0$$

$$\tan^4\theta + 1 + \tan^2\theta = 0$$

$$(\tan^4\theta + \tan^2\theta + 1)(\tan^2\theta - 1) = 0$$

$$\tan^6\theta - 1 = 0$$

$$\tan^6\theta = 1$$

$$\tan^{66}\theta + \tan^{36}\theta + \tan^{18}\theta + \tan^{12}\theta + 1$$

$$= (\tan^6\theta)^{11} + (\tan^6\theta)^6 + (\tan^6\theta)^3 + (\tan^6\theta)^2 + 1$$

$$= 1 + 1 + 1 + 1 + 1 = 5$$

97. (B)  $\tan^2\theta - 30\tan\theta = -225$

$$\tan^2\theta - 30\tan\theta + 225 = 0$$

$$(\tan\theta - 15)^2 = 0$$

$$\tan\theta = 15$$

$$\begin{aligned} \tan^5\theta - 16\tan^4\theta + 16\tan^3\theta - 16\tan^2\theta \\ + 16\tan\theta + 16 \end{aligned}$$

$$\begin{aligned} &= (15)^5 - 15(15)^4 - (15)^4 + 15(15)^3 + \\ &\quad (15)^3 - 15(15)^2 - (15)^2 + 15(15) + \\ &\quad 15 + 16 \end{aligned}$$

$$= 31$$

98. (D)  $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ$

$$= \sin 20^\circ \cdot \sin(60^\circ - 20^\circ) \cdot \sin(60^\circ + 20^\circ)$$

$$\therefore \sin\theta \cdot \sin(60^\circ - \theta) \cdot \sin(60^\circ + \theta) = \frac{1}{4} \sin 30^\circ$$

$$= \frac{1}{4} \sin(3 \times 20^\circ)$$

$$= \frac{1}{4} \sin 60^\circ = \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

$$100.(A) \quad \frac{100.(A)}{\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ} \\ = \frac{\cos 60^\circ \cdot \cos 20^\circ \cdot \cos (60^\circ - 20^\circ)}{\cos (60^\circ + 20^\circ)}$$

$$= \frac{1}{2} \times \frac{1}{4} \cos (3 \times 20^\circ)$$

$$= \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16}$$

$$103.(C) \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ \\ = \tan 60^\circ \cdot \tan 20^\circ \cdot \tan (60^\circ - 20^\circ) \\ \cos (60^\circ + 20^\circ)$$

$$= \sqrt{3} \cdot \tan (3 \times 20^\circ)$$

$$= \sqrt{3} \cdot \sqrt{3} = 3$$

$$104.(B) \sin 12^\circ \cdot \sin 48^\circ \cdot \sin 54^\circ$$

$$= \sin 12^\circ \cdot \sin 72^\circ \cdot \sin 54^\circ \times \frac{1}{\sin 72^\circ} \\ = \sin 12^\circ \sin(60^\circ - 12^\circ) \sin(60^\circ + 12^\circ) \\ \frac{1}{\sin 54^\circ \times \frac{1}{\sin 72^\circ}}$$

$$= \frac{1}{4} \sin(3 \times 12^\circ) \cdot \sin 54^\circ \times \frac{1}{\sin 72^\circ}$$

$$= \frac{1}{4} \sin 36^\circ \cos 36^\circ \times \frac{1}{\sin 72^\circ}$$

$$= \frac{1}{4} \times \frac{1}{2} \sin(2 \times 36^\circ) \times \frac{1}{\sin 72^\circ}$$

$$= \frac{1}{8} \times \frac{\sin 72^\circ}{\sin 72^\circ} = \frac{1}{8}$$

$$105.(A) \tan 6^\circ \cdot \tan 42^\circ \cdot \tan 66^\circ \cdot \tan 78^\circ$$

$$= \frac{\tan 6^\circ \cdot \tan 42^\circ \cdot \tan 66^\circ}{\tan 78^\circ \cdot \tan 54^\circ} \\ \tan 54^\circ$$

$$= \frac{\tan 6^\circ \cdot \tan(60^\circ - 6^\circ) \cdot \tan(60^\circ + 6^\circ)}{\tan 42^\circ \cdot \tan 78^\circ} \\ \tan 54^\circ$$

$$= \frac{\tan (3 \times 6^\circ) \cdot \tan(60^\circ - 18^\circ)}{\tan(60^\circ + 18^\circ)} \\ \tan 54^\circ$$

$$= \frac{\tan (3 \times 18^\circ)}{\tan 54^\circ} = 1$$

$$106.(D) \cos 6^\circ \cdot \cos 42^\circ \cdot \cos 66^\circ \cdot \cos 78^\circ \\ \cos 6^\circ \cdot \cos 42^\circ \cdot \cos 66^\circ \cdot \cos 78^\circ \\ \cos 54^\circ$$

$$= \frac{\cos 6^\circ \cdot \cos(60^\circ - 6^\circ) \cos(60^\circ + 6^\circ)}{\cos 54^\circ} \\ \cos 42^\circ \cos 78^\circ$$

$$= \frac{1}{4} \frac{\cos(60^\circ + 18^\circ)}{\cos 54^\circ}$$

$$= \frac{1}{4} \times \frac{1}{4} \frac{\cos(3 \times 18^\circ)}{\cos 54^\circ} = \frac{1}{16}$$

$$107.(A) \cos 15^\circ \cdot \sin 7\frac{1}{2}^\circ \cos 7\frac{1}{2}^\circ$$

$$= \frac{1}{2} \cos 15^\circ \cdot \sin\left(2 \times 7\frac{1}{2}^\circ\right)$$

$$= \frac{1}{2} \sin 15^\circ \cdot \cos 15^\circ$$

$$= \frac{1}{4} \sin(2 \times 15^\circ) = \frac{1}{4} \sin 30^\circ = \frac{1}{8}$$

$$108.(C) \sin\left(\frac{\pi}{24}\right) \cdot \cos\left(\frac{\pi}{24}\right) \cdot \cos\left(\frac{\pi}{12}\right)$$

$$= \frac{1}{2} \left[ 2 \sin\left(\frac{\pi}{24}\right) \cos\left(\frac{\pi}{24}\right) \right] \cdot \cos\left(\frac{\pi}{12}\right)$$

$$= \frac{1}{2} \sin\left(2 \times \frac{\pi}{24}\right) \cos\left(\frac{\pi}{12}\right)$$

$$= \frac{1}{2} \times \frac{1}{2} \left[ 2 \sin\left(\frac{\pi}{12}\right) \cos\left(\frac{\pi}{12}\right) \right]$$

$$= \frac{1}{4} \sin\left(\frac{\pi}{6}\right) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$109.(C) \tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$$

$$= \left( \frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\sin 81^\circ}{\cos 81^\circ} \right) -$$

$$\left( \frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\sin 63^\circ}{\cos 63^\circ} \right)$$

$$\begin{aligned}
&= \left( \frac{\sin 9^\circ \cos 81^\circ + \sin 81^\circ \cos 9^\circ}{\cos 9^\circ \cos 81^\circ} \right) - \\
&\quad \left( \frac{\sin 27^\circ \cos 63^\circ + \sin 63^\circ \cos 27^\circ}{\cos 27^\circ \cos 63^\circ} \right) \\
&= \left[ \frac{\sin(9^\circ + 81^\circ)}{\cos 9^\circ \cos 81^\circ} \right] - \left[ \frac{\sin(27^\circ + 63^\circ)}{\cos 27^\circ \cos 63^\circ} \right] \\
&= \frac{2}{2 \cos 9^\circ \sin 9^\circ} - \frac{2}{2 \cos 27^\circ \sin 27^\circ} \\
&= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = \frac{2(\sin 54^\circ - \sin 18^\circ)}{\sin 18^\circ \sin 54^\circ} \\
&= \frac{2 \left( 2 \cos \frac{54+18}{2} \sin \frac{54-18}{2} \right)}{\sin 18^\circ \cos 36^\circ} \\
&= \frac{4 \cos 36^\circ \cdot \sin 18^\circ}{\cos 36^\circ \cdot \sin 18^\circ} = 4
\end{aligned}$$

$$\begin{aligned}
110.(B) &\sin \frac{\pi}{9} \cdot \sin \frac{2\pi}{9} \cdot \sin \frac{3\pi}{9} \cdot \sin \frac{4\pi}{9} \\
&= \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ \\
&= \sin 20^\circ \cdot \sin(60^\circ - 20^\circ) \cdot \sin(60^\circ + 20^\circ) \\
&\quad \sin 60^\circ \\
&= \frac{1}{4} \sin(3 \times 20^\circ) \cdot \sin 60^\circ \\
&= \frac{1}{4} \sin^2 60^\circ = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}
\end{aligned}$$

$$\begin{aligned}
111.(A) &\cos \frac{\pi}{9} \cdot \cos \frac{2\pi}{9} \cdot \cos \frac{3\pi}{9} \cdot \cos \frac{4\pi}{9} \\
&= \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ \\
&= \cos 20^\circ \cdot \cos(60^\circ - 20^\circ) \cdot \cos(60^\circ + 20^\circ) \\
&\quad \cos 60^\circ \\
&= \frac{1}{4} \cos(3 \times 20^\circ) \cdot \cos 60^\circ \\
&= \frac{1}{4} \cos^2 60^\circ = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}
\end{aligned}$$

$$\begin{aligned}
112.(D) &(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) \\
&= (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \\
&= x \\
x^2 &= (\sec A - \tan A)(\sec A + \tan A) \\
&\quad (\sec B - \tan B)(\sec B + \tan B) \\
&\quad (\sec C - \tan C)(\sec C + \tan C)
\end{aligned}$$

$$\begin{aligned}
x^2 &= (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B) \\
&\quad (\sec^2 C - \tan^2 C) \\
x^2 &= 1 \times 1 \times 1 \\
x &= \pm 1
\end{aligned}$$

$$\begin{aligned}
113.(D) &\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8} \\
&= \cos^4 \left( \frac{\pi}{8} \right) + \cos^4 \left( \frac{\pi}{2} - \frac{\pi}{8} \right) + \\
&\quad \cos^4 \left( \frac{\pi}{2} + \frac{\pi}{8} \right) + \cos^4 \left( \pi - \frac{\pi}{8} \right) \\
&= \cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} \\
&= 2 \left( \cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right. \\
&\quad \left. - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right) \\
&= 2 \left[ \left( \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)^2 \right. \\
&\quad \left. - \frac{1}{2} \left( 2 \sin \frac{\pi}{8} \times \cos \frac{\pi}{8} \right)^2 \right] \\
&= 2 \left[ (1)^2 - \frac{1}{2} \left( \sin \frac{\pi}{4} \right)^2 \right] \\
&= 2 \left[ 1 - \frac{1}{2} \times \frac{1}{2} \right] = \frac{3}{2}
\end{aligned}$$

$$\begin{aligned}
114.(B) &\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 4x}}} \\
&= \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 4x)}}} \\
&= \sqrt{2 + \sqrt{2 + \sqrt{4 \cos^2 2x}}} \\
&= \sqrt{2 + \sqrt{2(1 + \cos 2x)}} \\
&= \sqrt{2 + \sqrt{4 \cos^2 x}} \\
&= \sqrt{2(1 + \cos x)} = 2 \cos \frac{x}{2} \\
&= \frac{2}{\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 4x}}}}
\end{aligned}$$

$$= \frac{2}{2 \cos \frac{x}{2}} = \sec \frac{x}{2}$$

$$\begin{aligned}
& 115.(C) \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \\
& \quad \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right) \\
& = \left(2 \cos^2 \frac{\pi}{16}\right) \left(2 \cos^2 \frac{3\pi}{16}\right) \\
& \quad \left(2 \cos^2 \frac{5\pi}{16}\right) \left(2 \cos^2 \frac{7\pi}{16}\right) \\
& = 16 \left[ \begin{array}{l} \left(\cos \frac{\pi}{16} \cdot \cos \frac{7\pi}{16}\right) \\ \left(\cos \frac{3\pi}{16} \cdot \cos \frac{5\pi}{16}\right) \end{array} \right]^2 \\
& = 16 \left[ \frac{1}{2} \left\{ \cos \left( \frac{\pi}{16} + \frac{7\pi}{16} \right) + \cos \left( \frac{7\pi}{16} - \frac{\pi}{16} \right) \right\} \right]^2 \\
& = 16 \left[ \frac{1}{2} \left\{ \cos \left( \frac{3\pi}{16} + \frac{5\pi}{16} \right) + \cos \left( \frac{5\pi}{16} - \frac{3\pi}{16} \right) \right\} \right]^2 \\
& = 16 \times \frac{1}{16} \left[ \begin{array}{l} \left(\cos \frac{\pi}{2} + \cos \frac{3\pi}{8}\right) \\ \left(\cos \frac{\pi}{2} + \cos \frac{\pi}{8}\right) \end{array} \right]^2 \\
& = \left[ \cos \frac{3\pi}{8} \cdot \cos \frac{\pi}{8} \right]^2 \\
& = \frac{1}{4} \left[ \cos \left( \frac{3\pi}{8} + \frac{\pi}{8} \right) + \cos \left( \frac{3\pi}{8} - \frac{\pi}{8} \right) \right]^2 \\
& = \frac{1}{4} \left[ \cos \frac{\pi}{2} + \cos \frac{\pi}{4} \right]^2 = \frac{1}{4} \left( \frac{1}{\sqrt{2}} \right)^2 \\
& = \frac{1}{8}
\end{aligned}$$

$$\begin{aligned}
116.(B) \tan(a+b) &= \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b} \\
&= \frac{\frac{n}{n+1} + \frac{1}{2n+1}}{1 - \frac{n}{n+1} \times \frac{1}{2n+1}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{n(2n+1) + (n+1)}{(n+1)(2n+1) - n} \\
&= \frac{2n^2 + n + n + 1}{2n^2 + 3n + 1 - n} \\
&= \frac{2n^2 + 2n + 1}{2n^2 + 2n + 1}
\end{aligned}$$

$$\tan(a+b) = 1 = \tan \frac{\pi}{4}$$

$$a+b = \frac{\pi}{4}$$

$$117.(A) \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$$

$$\tan(a+b) = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}} = \frac{55+6}{66-5} = 1$$

$$\tan(a+b) = \tan \frac{\pi}{4}$$

$$a+b = \frac{\pi}{4}$$

$$118.(C) \frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

Using C & D method

$$\frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$$\begin{aligned}
& \frac{2 \sin \left( \frac{x+y+x-y}{2} \right) \cos \left( \frac{x+y-x+y}{2} \right)}{2 \cos \left( \frac{x+y+x-y}{2} \right) \cos \left( \frac{x+y-x+y}{2} \right)} \\
&= \frac{2a}{2b}
\end{aligned}$$

$$\frac{\sin x \cos y}{\cos x \sin y} = \frac{a}{b} \equiv \frac{\tan x}{\tan y} = \frac{a}{b}$$

$$\begin{aligned}
119.(A) \sin \theta \cdot \cos^3 \theta - \cos \theta \cdot \sin^3 \theta \\
&= \sin \theta \cdot \cos \theta (\cos^2 \theta - \sin^2 \theta)
\end{aligned}$$

$$= \frac{1}{2} \sin 2\theta \cdot \cos 2\theta = \frac{1}{4} \sin 4\theta$$

$$\begin{aligned}
 120.(B) & \frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A} \\
 &= \frac{1}{\frac{\sin 3A}{\cos 3A} - \frac{\sin A}{\cos A}} - \frac{1}{\frac{\cos 3A}{\sin 3A} - \frac{\cos A}{\sin A}} \\
 &= \frac{\cos A \cdot \cos 3A}{\sin 3A \cos A - \sin A \cos 3A} - \\
 &\quad \frac{\sin A \cdot \sin 3A}{\sin A \cos 3A - \cos A \sin 3A} \\
 &= \frac{\cos A \cdot \cos 3A + \sin A \cdot \sin 3A}{\sin 3A \cos A - \sin A \cos 3A} \\
 &= \frac{\cos(3A - A)}{\sin(3A - A)} = \cot 2A
 \end{aligned}$$

$$\begin{aligned}
 121.(A) & \frac{\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} \\
 &= \tan\left(\frac{\theta + 7\theta}{2}\right) \text{ or } \tan\left(\frac{3\theta + 5\theta}{2}\right) \\
 &= \tan 4\theta
 \end{aligned}$$

$$\begin{aligned}
 122.(C) & \frac{\sin 2\theta - \sin 2\alpha}{\cos 2\theta + \cos 2\alpha} \\
 &= \frac{2\cos\left(\frac{2\theta + 2\alpha}{2}\right)\sin\left(\frac{2\theta - 2\alpha}{2}\right)}{2\cos\left(\frac{2\theta + 2\alpha}{2}\right)\cos\left(\frac{2\theta - 2\alpha}{2}\right)} \\
 &= \tan(\theta - \alpha)
 \end{aligned}$$

$$\begin{aligned}
 123.(D) & \cosec 10^\circ - \sqrt{3} \sec 10^\circ \\
 &= \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} \\
 &= \frac{2\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\sin 10^\circ \cos 10^\circ} \\
 &= \frac{2 \times 2(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{2 \sin 10^\circ \cos 10^\circ} \\
 &= \frac{4 \sin(30^\circ - 10^\circ)}{\sin 20^\circ} = \frac{4 \sin 20^\circ}{\sin 20^\circ} \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 124.(A) & \tan 40^\circ + \tan 20^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ \\
 &= \tan(40^\circ + 20^\circ) [1 - \tan 20^\circ \tan 40^\circ] \\
 &\quad + \sqrt{3} \tan 20^\circ \tan 40^\circ
 \end{aligned}$$

$$\begin{aligned}
 \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \\
 &= \sqrt{3} - \sqrt{3} \tan 20^\circ \tan 40^\circ + \\
 &\quad \sqrt{3} \tan 20^\circ \tan 40^\circ \\
 &= \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 125.(D) \sin \theta &= \sin 15^\circ + \sin 45^\circ \\
 &= 2 \sin\left(\frac{45^\circ + 15^\circ}{2}\right) \cos\left(\frac{45^\circ - 15^\circ}{2}\right) \\
 &= 2 \times \frac{1}{2} \times \cos 15^\circ \\
 &= \cos 15^\circ \\
 &= \sin 75^\circ \\
 \theta &= 75^\circ
 \end{aligned}$$

$$126.(C) \sin \theta + \cos \theta = m \quad \dots(i)$$

squaring on both side

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = m^2$$

$$1 + 2 \sin \theta \cos \theta = m^2$$

$$\sin \theta \cos \theta = \frac{m^2 - 1}{2} \quad \dots(ii)$$

$$\sin^3 \theta + \cos^3 \theta = n$$

$$(m^2 - 1)^2 - 3(m^2 - 1) = n \quad \dots(iii)$$

Using equation (i) and (ii)

$$(m^2 - 1)^2 - 3\left(\frac{m^2 - 1}{2}\right)(m) = n$$

$$2m^3 - 3(m^3 - m) = 2n$$

$$2m^3 - 3m^3 + 3m = 2n$$

$$m^3 - 3m + 2n = 0$$

### Short trick:-

$$\text{Put } \theta = 45^\circ$$

$$m = \sqrt{2}$$

$$n = \frac{1}{\sqrt{2}}$$

Now in eq. (iii)

$$m^3 - 3m + 2n = 0$$

$$2\sqrt{2} - 3\sqrt{2} + \frac{2}{\sqrt{2}} = 0$$

$$2\sqrt{2} - 3\sqrt{2} + \sqrt{2} = 0$$

# Height and Distance

**Angle of Elevation :**

Suppose that from a point O, we look up at an object P placed above the level of our eye and let OX be the horizontal line. Then angle  $\theta$  is called angle of elevation of P from O. Line OP is called line of sight.

**Angle of Depression**

Suppose that from a point O, we look down at an object P placed below the level of our eye and let OX be the horizontal line. Then angle  $\theta$  is called angle of depression of P from O. Line OP is called line of sight.

**Horizontal Line :** A line parallel to the ground in front of observer.

1. When the length of the shadow of a pillar on the ground is same as the height of the pillar then the angle of elevation of the sun is :
 

(A) $\frac{\pi}{2}$	(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{6}$	(D) $\frac{\pi}{4}$
2. The length of a shadow of a vertical tower is  $\frac{1}{\sqrt{3}}$  times its height. The angle of elevation of the sun is
 

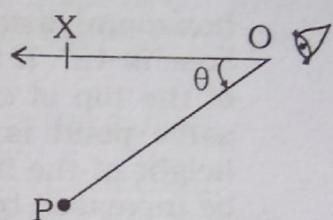
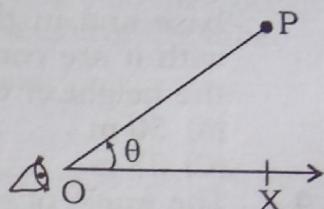
(A) $30^\circ$	(B) $45^\circ$
(C) $60^\circ$	(D) $90^\circ$
3. The length of the shadow of a vertical pole 9m high, when the sun's altitude is  $30^\circ$ , is (in cm)
 

(A) $3\sqrt{3}$	(B) 9
(C) $9\sqrt{3}$	(D) $18\sqrt{3}$
4. P and Q are two points observed from the top of a building  $10\sqrt{3}$  m high. If the angles of depression of the points are complementary and  $PQ = 20$ m, then the distance of P from the building is
 

(A) 25m	(B) 45m
(C) 30m	(D) 40m
5. The angles of elevation of the top of a tower from two points at a distance  $x$  and  $y$  from the foot of the tower are complementary. The height of the tower is
 

(A) $\sqrt{xy}$	(B) $\frac{x}{y}$
(C) $\sqrt{\frac{x}{y}}$	(D) $\sqrt{x+y}$
6. The angles of elevation of the top of a building from the top and bottom of a tree are  $x$  and  $y$ , respectively. If the height of the tree is  $h$  m, then, in m, the height of the building is :
 

(A) $\frac{h \cot x}{\cot x + \cot y}$	(B) $\frac{h \cot y}{\cot x + \cot y}$
(C) $\frac{h \cot x}{\cot x - \cot y}$	(D) $\frac{h \cot y}{\cot x - \cot y}$
7. If the angle of elevation of the Sun changes from  $30^\circ$  to  $45^\circ$ , the length of the shadow of a pillar decreases by 20 metres. The height of the pillar is



8. The angle of elevation of the top of a tower from two points situated at distances 36 m and 64 m from its base and in the same straight line with it are complementary. What is the height of the tower?
- (A) 50 m      (B) 48 m  
 (C) 25 m      (D) 24 m
9. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is  $45^\circ$ . If the angle of elevation of the top of complete pillar at the same point is to be  $60^\circ$ , then the height of the incomplete pillar is to be increased by
- (A)  $50\sqrt{2}$       (B) 100 m  
 (C)  $100(\sqrt{3}-1)$  m      (D)  $100(\sqrt{3}+1)$  m
10. The angle of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance 9 ft and 16 ft respectively are complementary angles. Then the height of the tower is :
- (A) 9 ft      (B) 12 ft  
 (C) 16 ft      (D) 144 ft
11. The angles of depression, from the top of a light-house, of two boats are  $45^\circ$  and  $30^\circ$  towards the west. If the two boats are 6m apart, then the height of the light-house is :
- (A)  $3(\sqrt{3}+1)$  m      (B)  $(\sqrt{3}+1)$  m  
 (C)  $3(\sqrt{3}-1)$  m      (D)  $(\sqrt{3}-1)$  m
12. A guard observes an enemy boat, from an observation tower at a height of 180 metre above sea level to be at an angle of depression of  $60^\circ$ . The distance of the boat from the foot of the observation tower is :
- (A) 180 m      (B)  $180\sqrt{3}$  m  
 (C)  $60\sqrt{3}$  m      (D) 60 m
13. The shadow of a tower becomes 60 metres longer where the altitude of the sun changes from  $45^\circ$  to  $30^\circ$ . Then the height of the tower is :
- (A)  $20(\sqrt{3}+1)$  m      (B)  $24(\sqrt{3}+1)$  m  
 (C)  $30(\sqrt{3}+1)$  m      (D)  $30(\sqrt{3}-1)$  m
14. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are  $30^\circ$  and  $60^\circ$  respectively. The distance between the two planes at that instant is :
- (A) 6520 m      (B) 6000 m  
 (C) 5000 m      (D) 6250 m
15. At a point on a horizontal line though the base of a monument the angle of elevation of the top of the monument is found to be such that its tangent is  $\frac{1}{5}$ . On walking 138 metres towards the monument the secant of the angle of elevation is found to be  $\frac{\sqrt{193}}{12}$ . The height of the monument (in metre) is :
- (A) 35      (B) 49  
 (C) 42      (D) 56
16. The angles of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively  $15^\circ$  and  $30^\circ$ . If A and B are on the same side of the tower and  $AB = 48$  m, then the height of the tower is :
- (A)  $24\sqrt{3}$  m      (B) 24 m  
 (C)  $24\sqrt{2}$  m      (D) 96 m

17. A telegraph post is bent at a point above the ground due to storm. Its top just meets the ground at a distance of  $8\sqrt{3}$  m from its foot and makes an angle of  $30^\circ$ , then the height of the post is :
- (A) 16 m      (B) 23 m  
 (C) 24 m      (D) 10 m
18. The angles of elevation of the top of a building and the top of the chimney on the roof of the building from a point on the ground are  $x$  and  $45^\circ$  respectively. The height of building is  $h$  metre. Then the height of the chimney, in metres, is :
- (A)  $h \cot x + h$       (B)  $h \cot x - h$   
 (C)  $h \tan x - h$       (D)  $h \tan x + h$
19. A vertical post 15 ft high is broken at a certain height and its upper part, not completely separated, meets the ground at an angle of  $30^\circ$ . Find the height at which the post is broken.
- (A) 10 ft      (B) 5 ft  
 (C)  $15\sqrt{3}(2 - \sqrt{3})$  ft (D)  $5\sqrt{3}$  ft
20. The angles of elevation of the top of a tower from the points P and Q at distances of ' $a$ ' and ' $b$ ' respectively from the base of the tower and in the same straight line with it are complementary. The height of the tower is
- (A)  $\sqrt{ab}$       (B)  $\frac{a}{b}$   
 (C)  $ab$       (D)  $a^2 b^2$
21. The angle of elevation of a tower from a distance 100 m from its foot is  $30^\circ$ . Height of the tower is :
- (A)  $\frac{100}{\sqrt{3}}$  m      (B)  $50\sqrt{3}$  m  
 (C)  $\frac{200}{\sqrt{3}}$  m      (D)  $100\sqrt{3}$  m
22. There are two vertical posts, one on each side of a road, just opposite to each other. One post is 108 metre high. From the top of this post, the angles of depression of the top and foot of the other post are  $30^\circ$  and  $60^\circ$  respectively. The height of the other post, in metre, is
- (A) 36      (B) 72  
 (C) 108      (D) 110
23. The tops of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of  $60^\circ$  with the horizontal, then the length of the wire is
- (A) 6 m      (B)  $8\sqrt{3}$  m  
 (C) 8 m      (D)  $6\sqrt{3}$  m
24. From the top of a hill 200 m high, the angle of depression of the top and the bottom of a tower are observed to be  $30^\circ$  and  $60^\circ$ . The height of the tower is (in m) :
- (A)  $\frac{400\sqrt{3}}{3}$       (B)  $166\frac{2}{3}$   
 (C)  $133\frac{1}{3}$       (D)  $200\sqrt{3}$
25. If a flagstaff of 6 metres high, placed on the top of a tower throws a shadow of  $2\sqrt{3}$  metres along the ground, then the angle (in degrees) that the sun makes with the ground is .....
- (A)  $30^\circ$       (B)  $60^\circ$   
 (C)  $90^\circ$       (D)  $45^\circ$
26. A flag staff 5 mt. high stands on a building 25 mt high. An observer at a height of 30 m. The flag staff and the building subtend equal angles. the distance of the observer from the top of the flag staff is .....
- (A)  $\frac{5\sqrt{3}}{\sqrt{8}}$       (B)  $\frac{5\sqrt{3}}{\sqrt{2}}$   
 (C)  $\frac{4\sqrt{3}}{\sqrt{2}}$       (D)  $\frac{7\sqrt{3}}{\sqrt{2}}$