49. If $x=\frac{1}{2+\sqrt{3}}$, $y=\frac{1}{2-\sqrt{3}}$, then the value of $\frac{1}{x+1}+\frac{1}{y+1}$ is-

(A) $\frac{1}{2}$ (B) $\sqrt{3}$

(C) 1 (D) $\frac{1}{\sqrt{3}}$

50. If $\sqrt{3}$ = 1.732, then the value of $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is

(A) 4.899 (B) 2.551

(C) 1.414 (D) 1.732

51. If $2a-\frac{2}{a}+3=0$, then the value of $\left(a^3-\frac{1}{a^3}+2\right)$ is

(A) 5 (B) $-\frac{35}{8}$

(C) $-\frac{40}{7}$ (D) $-\frac{47}{8}$

52. If $x=\frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+x}+\sqrt{1-x}$ will be-

(A) $\frac{1}{\sqrt{3}}$ (B) $2\sqrt{3}$

(C) $\sqrt{3}$ (D) 2

53. If $a=3+2\sqrt{2}$, then the value of $\frac{a^6+a^4+a^2+1}{a^3}$

(A) 192 (B) 240

(C) 204 (D) 212

54. If $x^3+y^3=35$ and $x+y=5$, then the value of $\left(\frac{1}{x}+\frac{1}{y}\right)$ is.

(A) $\frac{4}{7}$ (B) $\frac{3}{8}$

(C) $\frac{5}{6}$ (D) $\frac{3}{5}$

55. If $\frac{x^2}{by+cz}=\frac{y^2}{cz+ax}=\frac{z^2}{ax+by}=1$, then the value of $\frac{a}{a+x}+\frac{b}{b+y}+\frac{c}{c+z}$ is.

(A) -1 (B) 2

(C) 1 (D) -2

56. If $(x+1)$ and $(x-2)$ be the factors of $x^3+(a+1)x^2-(b-2)x-6$, then the value of a and b will be -

(A) 2 and 8 (B) 1 and 7

(C) 5 and 3 (D) 3 and 7

57. If the average of x and $\frac{1}{x}$ be 1, then the value of $8x^{10}+\frac{4}{x^5}$ will be.

(A) 12 (B) -12

(C) 0 (D) 1

58. If a+b+c = 6, $a^2+b^2+c^2$ = 14, then the value of bc+ca+ab will be

(A) 22 (B) 25

(C) 20 (D) 11

59. If $3x^2-4x-3=0$, then $x-\frac{1}{x}$ will be

(A) 3 (B) 4

(C) $\frac{3}{4}$ (D) $\frac{4}{3}$

60. If $x=11$, then $x^5-12x^4+12x^3-12x^2+12x-1$ will be.

(A) 12 (B) 0
(C) 10 (D) 111

61. Factorize: $8x^3+12x^2+6x+1$

(A) $(2x+1)^3$
(B) $(2x+1)^2(x+1)$
(C) $(x+1)^3$
(D) None of these

62. Find the remainder when $f(y)=y^3+y^2+2y+3$ is divided by $(y+2)$

(A) 5 (B) -5
(C) 4 (D) -4

63. Find the value of k, if $x+3$ is a factor of $3x^2+kx+6$.

(A) 10 (B) 11
(C) -10 (D) -11

64. For what value of a is $2x^3+ax^2+11x+a+3$ exactly divisible by $(2x-1)$?

(A) -4 (B) -5
(C) -6 (D) -7

65. Find the values of 'a' and 'b', so that the polynomial $x^3-ax^2-13x+b$ has $(x-1)$ and $(x+3)$ as factors

(A) $a=4$, $b=5$ (B) $a=3$, $b=15$
(C) $a=15$, $b=3$ (D) $a=5$, $b=4$

66. $x^4+2x^3-2x^2+2x-3$ is exactly divisible by:

(A) x^3+2x-3 (B) x^2+2x-3
(C) x^2+x-6 (D) x^3+x-6

67. If both $(x-2)$ and $\left(x-\frac{1}{2}\right)$ are factors of px^2+5x+r, then which one is correct:

(A) $p=2r$ (B) $pr=1$
(C) $\frac{p}{r}=1$
(D) None of these

68. Find the HCF of the polynomials $30(x^2-3x+2)$ and $50(x^2-2x+1)$

(A) $10(x-1)^2$ (B) $10(x-1)^3$
(C) $10(x-1)$ (D) $10(2x-1)$

69. Find the HCF of the polynomials f(x), g(x), h(x), where
$f(x)=10(x+1)(x-3)^3$
$g(x)=15(x-2)(x-3)^2$
$h(x)=25(x+5)(x-3)^3$

(A) $5(x-3)^2$ (B) $5(x-3)$
(C) $5(2x-3)$
(D) None of these

70. Find the LCM of polynomials $f(x)=4(x-1)^2(x^2+6x+8)$ and $g(x)=10(x-1)(x+2)(x^2+7x+10)$

(A) $20(x-1)^2(x+2)^2(x+4)$
(B) $20(x-1)(x+2)^2(x+4)(x+5)$
(C) $20(x-1)^2(x+2)^2(x+4)(x+5)$
(D) None of these

71. Evaluate $30^3+20^3-50^3$

(A) -90000 (B) 90000
(C) 250000
(D) None of these

72. If $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3}$ and $x+y=10$, then the value of xy will be:

(A) 36 (B) 24
(C) 16 (D) 9

73. If α and β are the roots of equation $x^2-q(1+x)-r=0$, then $(1+\alpha)(1+\beta)$ is:

(A) $1-r$ (B) $q-r$
(C) $1+r$ (D) $q+r$

74. The roots of the quadratic equation $ax^2+bx+c=0$ will be reciprocal to each other if:

(A) $a=\frac{1}{c}$ (B) $a=c$
(C) $ac=b$ (D) $a=b$

75. If the equation $4x^2 + x(p + 1) + 1 = 0$ has exactly two equal roots, then one of the values of p is:

(A) 5 (B) – 3

(C) 0 (D) 3

76. The system of equations $2x + ky = 11$ and $5x - 7y = 5$ has no solution if the value of k is:

(A) $\frac{13}{5}$ (B) $-\frac{13}{5}$

(C) $-\frac{14}{5}$ (D) $-\frac{16}{5}$

77. If one root of the equation $ax^2 + bx + c = 0$ is three times the other, then

(A) $b^2 = 8ac$ (B) $3b^2 = 16ac$

(C) $3b^2 = 5ac$ (D) $b^2 + 3ac = 0$

78. If $0 < a < 4$, then the equation $ax(1 - x) = 1$ has:

(A) two equal roots

(B) one positive and one negative root

(C) two irrational roots

(D) no real roots

79. Find the numerical value of 'c', if the expression $xy - 3x + 5y + c$ can be factorised:

(A) 9 (B) – 15

(C) 15 (D) – 9

80. If x is less than 2, then which of the following statement is always true?

(A) x is negative

(B) $2x$ is greater than or equal to x

(C) x^2 is greater than or equal to x

(D) None of these

1. (B) Given $\frac{3x+2y}{3x-2y}=\frac{4}{3}$

$\Rightarrow$ $9x+6y=12x-8y$

$\Rightarrow$ $14y=3x$

So, $\frac{x}{y}=\frac{14}{3}$

So, $\frac{x^2+y^2}{x^2-y^2}=\frac{(14)^2+(3)^2}{(14)^2-(3)^2}=\frac{205}{187}$

2. (D) $x^2+y^2+4x+4y+8=0$

$\Rightarrow$ $(x^2+4x+4)+(y^2+4y+4)=0$

$\Rightarrow$ $(x+2)^2+(y+2)^2=0$

So, $x=-2$ & $y=-2$

$\Rightarrow$ $x+y=(-2)+(-2)=-4$

3. (D) Given $x+\frac{1}{x}=6$

$$\Rightarrow \frac{3x}{2x^2+2-5x}$$

$$=\frac{3x}{x\left[\left(2x+\frac{2}{x}\right)-5\right]}$$

$$\Rightarrow \frac{3}{2\left[x+\frac{1}{x}\right]-5}=\frac{3}{12-5}=\frac{3}{7}$$

4. (C) $\frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}=\frac{3}{2}$

$\Rightarrow$

$2\sqrt{x+2}+2\sqrt{x-2}=3\sqrt{x+2}-3\sqrt{x-2}$

$\Rightarrow$ $5\sqrt{x-2}=\sqrt{x+2}$

$\Rightarrow$ $\frac{\sqrt{x+2}}{\sqrt{x-2}}=\frac{5}{1}$

Squaring both the sides:

$\Rightarrow$ $\frac{x+2}{x-2}=\frac{25}{1}$

$\Rightarrow$ $x+2=25x-50$

$\Rightarrow$ $24x=52$

$\Rightarrow$ $x=\frac{52}{24}=\frac{13}{6}\Rightarrow 6x=13$

5. (B) Given $x+\frac{1}{x}=3$

So, $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=(3)^2-2$

$=7$

Again squaring:

$\Rightarrow$ $\left(x^2+\frac{1}{x^2}\right)^2=7^2$

$\Rightarrow$ $x^4+\frac{1}{x^4}+2=49$

$\Rightarrow$ $x^4+\frac{1}{x^4}=47$

6. (C) Given:

$$\frac{1}{(a-b)(b-c)}+\frac{1}{(b-c)(c-a)}+\frac{1}{(c-a)(a-b)}$$

$$\Rightarrow \frac{c-a+a-b+b-c}{(a-b)(b-c)(c-a)}$$

$$\Rightarrow \frac{0}{(a-b)(b-c)(c-a)}=0$$

7. (B) Given:

$x+y=2xy$ ...(i)

$x-y=4xy$ ...(ii)

on adding (i) and (ii)

$2x=6xy$

$y=\frac{1}{3}$ then $y^2=\frac{1}{9}$

8. (D) $x=(\sqrt{2}+1)$, and $y=1-\sqrt{2}$

$\therefore$ $x^2+y^2+xy=x^2+y^2+2xy-xy$

$= (x + y)^2 - xy$

$= (\sqrt{2} + 1 + 1 - \sqrt{2})^2 - (1 + \sqrt{2})(1 - \sqrt{2})$

$= (2)^2 - ((1)^2 - (\sqrt{2})^2)$

$= 4 - (-1) = 5$

9. (A) On dividing $(x - 2)$ Remainder is k.

So, $x - 2 = 0 \quad \Rightarrow x = 2$

$\Rightarrow (2)^3 + 3(2)^2 - k \times 2 + 4 = k$

$\Rightarrow 8 + 12 - 2k + 4 = k$

$\Rightarrow 24 = 3k$

$\Rightarrow k = 8$

10. (B) Given $a = -5$, $b = -6$ and $c = 10$

$\therefore \quad a + b + c = (-5) + (-6) + 10 = -1$

$$\Rightarrow \frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2}$$

$$\Rightarrow \frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{-(a^2+b^2+c^2-ab-bc-ca)}$$

$$\Rightarrow \frac{-1}{-1} = 1$$

11. (B) $a^{1/3} = 11$

$\Rightarrow a = 11^3 = 1331$

So, $a^2 - 331a$

$\Rightarrow a(a - 331)$

$\Rightarrow a(1331 - 331) = 1331 \times 1000$

$\Rightarrow 1331000$

12. (B) $x + \frac{1}{16x} = 1$ .... given

$\Rightarrow$ multiplying by 4 both the sides

$$4x + \frac{1}{4x} = 4$$

$\Rightarrow$ Cube both the sides:

$$\left(4x + \frac{1}{4x}\right)^3 = (4x)^3 + \left(\frac{1}{4x}\right)^3 + 3 \times 4x \times \frac{1}{4x}\left(4x + \frac{1}{4x}\right)$$

$$64 = 64x^3 + \frac{1}{64x^3} + 3 \times 4$$

$$\Rightarrow 64x^3 + \frac{1}{64x^3} = 64 - 12 = 52$$

13. (A) Given $\rightarrow 2^x = 3^y = 6^{-z} = K$

$\Rightarrow 2 = K^{1/x}$ ...(i)

$\Rightarrow 3 = K^{1/y}$ ...(ii)

$\Rightarrow 6 = K^{-1/z}$

$\Rightarrow 2.3 = K^{-1/z}$ ...(iii)

multiplying (i) and (ii)

$\Rightarrow 2.3 = K^{\frac{1}{x}+\frac{1}{y}}$ ...(iv)

from (iii) and (iv)

$\Rightarrow K^{\frac{1}{x}+\frac{1}{y}} = K^{-1/z}$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = -\frac{1}{z} \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

14. (D) $x^a . x^b . x^c = 1$

$\Rightarrow x^{a+b+c} = x^0$

$\Rightarrow a + b + c = 0$

Now, $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$(a + b + c = 0)$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

15. (A) $3^{2x-y} = \sqrt{27}$

$3^{x+y} = \sqrt{27}$

$\Rightarrow 3^{2x-y} = 3^{3/2}$ and $3^{x+y} = 3^{3/2}$

So, $2x - y = \frac{3}{2}$ and $x + y = \frac{3}{2}$

adding both

$$\Rightarrow 3x = \frac{6}{2} \Rightarrow x = 1$$

So, $y = \frac{3}{2} - 1 = \frac{1}{2} \quad \Rightarrow y = \frac{1}{2}$

16. (A) Given $a + \frac{1}{a} = 6$

$$\therefore \quad a^4 + \frac{1}{a^4} = (a)^2 + \left(\frac{1}{a^2}\right)^2$$

$$= \left(a^2 + \frac{1}{a^2}\right)^2 - 2 = \left[\left(a + \frac{1}{a}\right)^2 - 2\right]^2 - 2$$

$$= ((6)^2 - 2)^2 - 2 = (34)^2 - 2$$

$$\therefore \quad a^4 + \frac{1}{a^4} = 1154$$

17. (B) $\left(a + \frac{1}{a}\right)^2 = 3$

$$\therefore \quad a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3 \times a \times$$

$$\frac{1}{a}\left(a + \frac{1}{a}\right)$$

$$\frac{a}{a-c} + \frac{c}{b-c} = \frac{a}{a-c} + \frac{c}{-(a-c)}\left[\frac{a}{a-c} - \frac{c}{a-c}\right]$$

$$= \left(a + \frac{1}{a}\right)\left[\left(a + \frac{1}{a}\right)^2 - 3\right]$$

$$= \left(a + \frac{1}{a}\right)(3 - 3)$$

$$\therefore \quad a^3 + \frac{1}{a^3} = 0$$

18. (A) $a = \frac{x}{x+y}, b = \frac{y}{x-y}$

$$\frac{ab}{a+b} = \frac{\left(\frac{x}{x+y}\right)\left(\frac{y}{x-y}\right)}{\left(\frac{x}{x+y}\right) + \left(\frac{y}{x-y}\right)}$$

$$= \frac{xy}{(x+y)(x-y)} \times \frac{(x+y)(x-y)}{x(x-y) + y(x+y)}$$

$$= \frac{xy}{x^2 - xy + xy + y^2}$$

$$\frac{ab}{a+b} = \frac{xy}{x^2 + y^2}$$

19. (B) $a + b = 2c$ ... given

$\Rightarrow \quad b - c = -(a - c)$ ______(i)

Now,

$$\frac{a}{a-c} + \frac{c}{b-c} = \frac{a}{a-c} - \frac{c}{a-c}$$

using ____(i)

$$\frac{a}{a-c} + \frac{c}{b-c} = \frac{a-c}{a-c} = 1$$

20. (A) $P = \frac{x^2 - 36}{x^2 - 49}$ ... given

$$= \frac{(x+6)(x-6)}{(x+7)(x-7)} \Rightarrow P = Q\left(\frac{x-6}{x-7}\right)$$

$$Q = \frac{x+6}{x+7} \Rightarrow \frac{P}{Q} = \left(\frac{x-6}{x-7}\right)$$

21. (C) If $x + \frac{1}{x} = 2$ then x will be 1.

So, $x^{17} + \frac{1}{x^{19}} = (1)^{17} + \frac{1}{(1)^{19}} = 1 + \frac{1}{1}$

$= 2$

22. (A) $x + \frac{1}{4x} = \frac{3}{2}$

multiplying by 2 both the sides.

$$2x + \frac{1}{2x} = 3$$

$\Rightarrow$ cube both the sides

$$8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}\left(2x + \frac{1}{2x}\right)$$

$= 27$

$\Rightarrow \quad 8x^3 + \dfrac{1}{8x^3} = 27 - 3 \times 3$

$\Rightarrow \quad 8x^3 + \dfrac{1}{8x^3} = 18$

23. (D) $x = \dfrac{4ab}{a+b}$

$\dfrac{x}{2a} = \dfrac{2b}{a+b}$

$\Rightarrow \dfrac{x+2a}{x-2a} = \dfrac{3b+a}{b-a} \quad \ldots(i)$

Similarly $\Rightarrow \dfrac{x+2b}{x-2b} = \dfrac{3a+b}{a-b}$

...(ii)

adding (1) and (ii)

$\dfrac{x+2a}{x-2a} + \dfrac{x+2b}{x-2b} = \dfrac{3b+a}{b-a} - \dfrac{3a+b}{b-a}$

$= \dfrac{3b+a-3a-3b}{b-a} = \dfrac{2b-2a}{b-a} = 2$

Shortcut :-

$x = \dfrac{4ab}{a+b} = \dfrac{2a \times 2b}{a \times b} = 2$ (always)

24. (C) Given $m + \dfrac{1}{m-2} = 4$

$\Rightarrow$ So, $(m-2) + \dfrac{1}{(m-2)} = 2$

Squaring both the sides:

$(m-2)^2 + \dfrac{1}{(m-2)^2} = \left[(m-2) + \dfrac{1}{(m-2)}\right]^2 - 2$

$= (2)^2 - 2 = 2$

25. (C) Given $a^2 = b + c$, $b^2 = a + c$ and $c^2 = a + b$

$\therefore \quad \dfrac{1}{1+a} + \dfrac{1}{1+b} + \dfrac{1}{1+c}$

So,

$\Rightarrow \quad \dfrac{a}{a+a^2} + \dfrac{b}{b+b^2} + \dfrac{c}{c+c^2}$

$(\because a^2 = b + c;\ a + a^2 = a + b + c)$

$(a + a^2 = b + b^2 = c + c^2 = a + b + c)$

$= \dfrac{a}{a+b+c} + \dfrac{b}{a+b+c} + \dfrac{c}{a+b+c}$

$= \dfrac{a+b+c}{a+b+c} = 1$

26. (D) Given $2x + \dfrac{1}{3x} = 5$

$\Rightarrow \quad \dfrac{6x^2+1}{3x} = 5$

$\Rightarrow \quad 6x^2 + 1 = 15x$

$\therefore \quad \dfrac{5x}{6x^2+20x+1} = \dfrac{5x}{6x^2+1+20x}$

$= \dfrac{5x}{15x+20x} = \dfrac{5x}{35x} = \dfrac{1}{7}$

27. (D) If $a + b + c = 0$

$\Rightarrow \quad (a + b + c)^2 = 0$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 0$

$\Rightarrow \quad a^2 + b^2 + c^2 = -2(ab + bc + ca)$

$\Rightarrow \quad (a^2 + b^2 + c^2)^2 = 4(ab + bc + ca)^2$

$\Rightarrow \quad \dfrac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2+2ab^2c+2bc^2a+2a^2bc} = 4$

$\Rightarrow \quad \dfrac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2+2abc(b+c+a)} = 4$

$\Rightarrow \quad \dfrac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2} = 4$

$(\because a + b + c = 0)$

28. (A) Let $p(x) = x^4 - 3x + 2$

$= (x - 1)(x^3 + x^2 + x - 2)$

$q(x) = x^3 - 3x^2 + 3x - 1 = (x-1)^3$

and $r(x) = x^4 - 1$

$= (x^2 - 1)(x^2 + 1)$
$= (x - 1)(x + 1)(x^2 + 1)$
$\therefore$ HCF will be $= x - 1$

29. (C) Given $x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}}$

$$= \sqrt{\frac{(\sqrt{5}+1)^2}{5-1}} = \frac{\sqrt{5}+1}{2}$$

$\therefore$ $5x^2 - 5x - 1$

$$=5\left(\frac{\sqrt{5}+1}{2}\right)^2 - 5\frac{(\sqrt{5}+1)}{2} - 1$$

$$= 5\left(\frac{5+1+2\sqrt{5}}{4}\right) - \frac{5\sqrt{5}+5}{2} - 1$$

$$= 5\left(\frac{3+\sqrt{5}}{2}\right) - \frac{5\sqrt{5}+5}{2} - 1$$

$$= \frac{15+5\sqrt{5}-5\sqrt{5}-5-2}{2}$$

$$= \frac{8}{2} = 4$$

30. (A) $\frac{a}{b} + \frac{b}{a} = 1$

$\Rightarrow \frac{a^2+b^2}{ab} = 1$

$a^2 + b^2 - ab = 0$

$\therefore$ $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
$= (a + b) \times 0 = 0$

So $a^3 + b^3 = 0$

31. (A) Given $x + \frac{1}{x} = \sqrt{3}$

then as we know

$x^6 + 1 = 0$

Now, $x^{18} + x^{12} + x^6 + 1$
$= x^{12}(x^6 + 1) + 1(x^6 + 1)$
$= (x^6 + 1)(x^{12} + 1) = 0$

32. (D) $\sqrt{2x+3} + \sqrt{2x-1} = 2$

$\sqrt{2x+3} = 2 - \sqrt{2x-1}$

Squaring both side

$2x + 3 = 4 + 2x - 1 - 4\sqrt{2x-1}$

$4\sqrt{2x-1} = 0 \Rightarrow \sqrt{2x-1} = 0$

Squaring both side

$2x - 1 = 0$

$x = \frac{1}{2}$

33. (B) Given $\frac{2x^4 - 162}{(x^2+9)(2x-6)}$

$$= \frac{2(x^4-81)}{2(x^2+9)(x-3)} = \frac{2(x^2-9)(x^2+9)}{2(x^2+9)(x-3)}$$

$$= \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = (x + 3)$$

34. (B) Given $\frac{2x}{3} + \frac{y}{2} = 4$ ...(i)

$\Rightarrow \frac{x}{3} - \frac{y}{2} = 1$ ...(ii)

adding (i) and (ii)

$\Rightarrow \frac{2x}{3} + \frac{x}{3} = 4 + 1$

$\Rightarrow \frac{3x}{3} = 5 \quad \Rightarrow x = 5$

35. (B) $x + \frac{1}{x} = 2$

So, $x = 1$ satisfies the equation

$\Rightarrow$ putting the value $x = 1$

$\Rightarrow x^2 + \frac{1}{x^3} = (1)^2 + \frac{1}{(1)^3} = 1 + 1 = 2$

36. (A) $\because \quad x = a^{2/3} - a^{-2/3}$

Cube both the sides

$x^3 = [a^{2/3} - a^{-2/3}]^3$

$x^3 = (a^{2/3})^3 - (a^{-2/3})^3 - 3a^{2/3}.\, a^{-2/3}(a^{2/3} - a^{-2/3})$

$x^3 = a^2 - a^{-2} - 3 \times 1 \times x = a^2 - \frac{1}{a^2} - 3x$

$\Rightarrow x^3 + 3x = a^2 - \frac{1}{a^2}$

37. (A) α and β are the roots of the equation $x^2 + px + q = 0$
then summation of the roots

$$\alpha + \beta = \frac{-p}{1}$$

product of the roots

$$\alpha \cdot \beta = \frac{q}{1}$$

$$\therefore \quad \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(-p)^2 - 2q}{q}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{p^2 - 2q}{q}$$

38. (B) Given:

$$\frac{a^2 - b^2 - 2bc - c^2}{a^2 + b^2 + 2ab - c^2} = \frac{a^2 - (b+c)^2}{(a+b)^2 - c^2}$$

$$= \frac{[a+(b+c)](a-(b+c))}{(a+b+c)(a+b-c)} = \frac{a-b-c}{a+b-c}$$

39. (D) Given:
$a^3 - b^3 - c^3 - 3abc = 0$
$\therefore \quad a^3 - b^3 - c^3 = 3abc$
$\Rightarrow \quad a^3 + (-b)^3 + (-c)^3 = 3abc$
So, $a - b - c = 0$
$a = b + c$

40. (C) Given:
$xy(x + y) = 1$

$$\Rightarrow \quad x + y = \frac{1}{xy}$$

$$\text{So, } (x + y)^3 = \frac{1}{x^3y^3}$$

$$\therefore \quad \frac{1}{x^3y^3} - x^3 - y^3 = (x + y)^3 - x^3 - y^3$$
$$= x^3 + y^3 + 3xy(x + y) - x^3 - y^3$$
$$= 3xy(x + y)$$
$$[\because xy(x + y) = 1]$$
$$= 3$$

41.(D) Given :
$ax + by = 6$
squaring both the sides-
$\Rightarrow a^2x^2 + b^2y^2 + 2abxy = 36$ ....(i)
and $(bx-ay)^2 = (2)^2$
Squaring both sides
$b^2x^2 + a^2y^2 - 2abxy = 4$ ....(ii)
adding (i) and (ii)

$$a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy = 40$$

$\Rightarrow x^2(a^2 + b^2) + y^2(a^2 + b^2) = 40$

$\Rightarrow (x^2 + y^2)(a^2 + b^2) = 40$

and $x^2 + y^2 = 4$
(given)

so, $a^2 + b^2 = \frac{40}{4}$

$a^2 + b^2 = 10$

42.(D) Given :

$$a + \frac{1}{a+2} = 0$$

adding 2 on both the sides

$$a + 2 + \frac{1}{a+2} = 2$$

$\Rightarrow$ cube both the sides-

$$(a+2)^3 + \frac{1}{(a+2)^3} + 3 \times \left[a + 2 + \frac{1}{a+2}\right] = (2)^3$$

$$\Rightarrow (a+2)^3 + \frac{1}{(a+2)^3} = 8 - 3 \times 2$$

$$\Rightarrow (a+2)^3 + \frac{1}{(a+2)^3} = 2$$

43.(B) Given :

$a^3 - b^3 = 56$

$\Rightarrow (a-b)\,(a^2+b^2+ab) = 56$

$\Rightarrow a^2+b^2+ab = 28$ .... (i)

$\Rightarrow (a-b)^2 = (2)^2$

$\Rightarrow a^2+b^2-2ab = 4$ .... (ii)

subtracting (i) and (ii)

$3ab = 24$

$ab = 8$

$\Rightarrow$ putting the value of ab = 8 in (i)

$\Rightarrow a^2+b^2+8=28$

$\Rightarrow a^2+b^2=20$

44.(B) Given :

$a+\frac{1}{a}=1$ .... (i)

or $a^2+1=a$ .... (ii)

$a^2 + 1 - a = 0$

multiply by a + 1

$(a + 1)\,(a^2 + 1 - a) = 0$

$a^3 + 1 = 0$

So $a^3 = -1$

45.(D) Given :

$$x^4 + \frac{1}{x^4} = 23$$

$$\text{or} \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 23 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{25} = 5$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 5-2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 3$$

46.(A) Given :

$$x + \frac{1}{x} = 3$$

squaring both the sides we get

$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$ .... (i)

Now cube both the sides

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$\Rightarrow x^3 + \frac{1}{x^3} = 18$ .... (ii)

As we know

$$x^5 + \frac{1}{x^5}$$

$$= \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

$= 7 \times 18 - 3$

47.(B) Given :

given $a+b+1=0$

where c = 1

by formula we know that

$a^3+b^3+c^3=3abc$ when $a+b+c=0$

putting $c=1$

$a^3+b^3+1=3ab$

or $a^3+b^3+1-3ab=0$

48.(A) Given :

a-b = 3, b-c = 5 and c-a = 1

we know that

$$\Rightarrow \frac{a^3+b^3+c^3-3abc}{a+b+c}$$

$$a^3+b^3+c^3 = \frac{(a+b+c)}{2}((a-b)^2+(b-c)^2 + (c-b)^2)$$

$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$= \frac{1}{2}\left[(3)^2+(5)^2+(1)^2\right]$$

$$= \frac{9+25+1}{2} = \frac{35}{2} = 17.5$$

49.(C) Given :

$$x = \frac{1}{2+\sqrt{3}} \Rightarrow x = 2-\sqrt{3}$$

$$x+1 = 3-\sqrt{3}$$

$$\Rightarrow \frac{1}{x+1} = \frac{1}{3-\sqrt{3}} x \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{3+\sqrt{3}}{6}$$

$$y = \frac{1}{2-\sqrt{3}} \Rightarrow y = 2+\sqrt{3}$$

$$y+1 = 3+\sqrt{3}$$

$$\Rightarrow \frac{1}{y+1} = \frac{1}{3+\sqrt{3}} x \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{3-\sqrt{3}}{6}$$

Now the value of

$$\frac{1}{x+1}+\frac{1}{y+1} = \frac{3+\sqrt{3}}{6}+\frac{3-\sqrt{3}}{6}$$

$$= \frac{3+\sqrt{3}+3-\sqrt{3}}{6} = \frac{6}{6} = 1$$

50. (D) Given :

$$\sqrt{3} = 1.732$$

$$\Rightarrow \frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$$

$$\Rightarrow \frac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}}$$

$$\Rightarrow \frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}} \Rightarrow \frac{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)}$$

$$\Rightarrow \sqrt{3} \Rightarrow 1.732$$

51. (D) Given :

$$\Rightarrow 2a-\frac{2}{a}+3=0$$

$$\Rightarrow 2a-\frac{2}{a}=-3$$

$$\Rightarrow a-\frac{1}{a}=\frac{-3}{2}$$

cube both the sides

$$\Rightarrow a^3-\frac{1}{a^3}-3\times a\times\frac{1}{a}(a-\frac{1}{a})=\frac{-27}{8}$$

$$\Rightarrow a^3-\frac{1}{a^3}-3\times\frac{-3}{2}=\frac{-27}{8}$$

$$\Rightarrow a^3-\frac{1}{a^3}=\frac{27}{8}-\frac{9}{2}$$

$$\Rightarrow a^3-\frac{1}{a^3}=\frac{-27-36}{8}=\frac{-63}{8}$$

$$\Rightarrow a^3-\frac{1}{a^3}+2=-\frac{-63}{8}+2$$

$$\Rightarrow a^3-\frac{1}{a^3}+2=\frac{-47}{8}$$

52. (C) $1+x = 1+\frac{\sqrt{3}}{2}$

$=1+2\times\frac{\sqrt{3}}{2}\times\frac{1}{2}$ (multiply & divide by 2)

$$1+x = \left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)^2$$

similarly,

$$= 1 - x = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^2$$

$$\sqrt{1+x} + \sqrt{1-x} = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = \sqrt{3}$$

53. (C) Given :

$a = 3 + 2\sqrt{2}$

$$\frac{1}{a} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{3}}{3-2\sqrt{2}}$$

$= 3 - 2\sqrt{2}$

$$a + \frac{1}{a} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$$

$$\frac{a^6 + a^4 + a^2 + 1}{a^3} = \frac{a^6}{a^3} + \frac{a4}{a^3} + \frac{a^2}{a^3} + \frac{1}{a^3}$$

$$= a^3 + \frac{1}{a^3} + a + \frac{1}{a}$$

$$= \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right) + \left(a + \frac{1}{a}\right)$$

$= (6)^3 - 2 \times 6 = 204$

54. (C) Given :

$x^3 + y^3 = 35$ and $x + y = 5$

$x^3 + y^3 = 35$

$(x + y)^3 - 3xy\ (x + y) = 35$

$(5)^3 - 3xy(5) = 35$

$xy = 6$

$$\frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy} = \frac{5}{6}$$

$\Rightarrow x + y = 5$ .... (iv)

Divide (iv) by (iii)

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{5}{6}$$

55. (C) Given :

$$\frac{x^2}{by + cz} = \frac{y^2}{cz + ax} = \frac{z^2}{ax + by} = 1$$

so,

$x^2 = by + cz,\ \ y^2 = cz + ax,\ \ z^2 = ax + by$

$$\frac{a}{a+x} + \frac{b}{b+y} = \frac{c}{c+z}$$

$$= \frac{ax}{ax + x^2} + \frac{by}{by + y^2} + \frac{cz}{cz + z^2}$$

$$\Rightarrow \frac{ax}{ax + by + cz} + \frac{by}{by + ax + cz} + \frac{cz}{cz + ax + by}$$

$$\Rightarrow \frac{ax + by + cz}{ax + by + cz} = 1$$

56. (B) Given :

According to question

$(x + 1)$ and $(x - 2)$ are the factor of $x^3 + (a+1)x^2 - (b-2)\,x - 6$ so

$f(-1) = (-1)^3 + (a + 1)\ (-1)^2 - (b - 2)\ (-1) - 6 = 0$

$\Rightarrow -1 + (a + 1) + (b - 2) - 6 = 0$

$a + b = 8$ .... (i)

and $= (2)^3 + (a + 1)\ (2)^2 - (b - 2)\ 2 - 6 = 0$

$8 + 4a + 4 - 2b + 4 - 6 = 0$

$4a - 2b + 10 = 0$

$2a - b = -5$ ....(ii)

from (i) and (ii)

$3a = 3 \Rightarrow a = 1$ & $b = 7$

57. (A) Given :

According to the question

$$x + \frac{1}{x} = 2$$

$\Rightarrow x = 1$ satisfies

$$\text{so, } 8\ (1)^{10} + \frac{4}{(1)^5} = 8 + 4 = 12$$