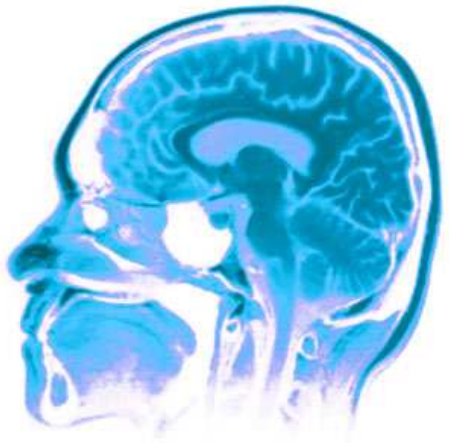
CPSC540



Decision trees

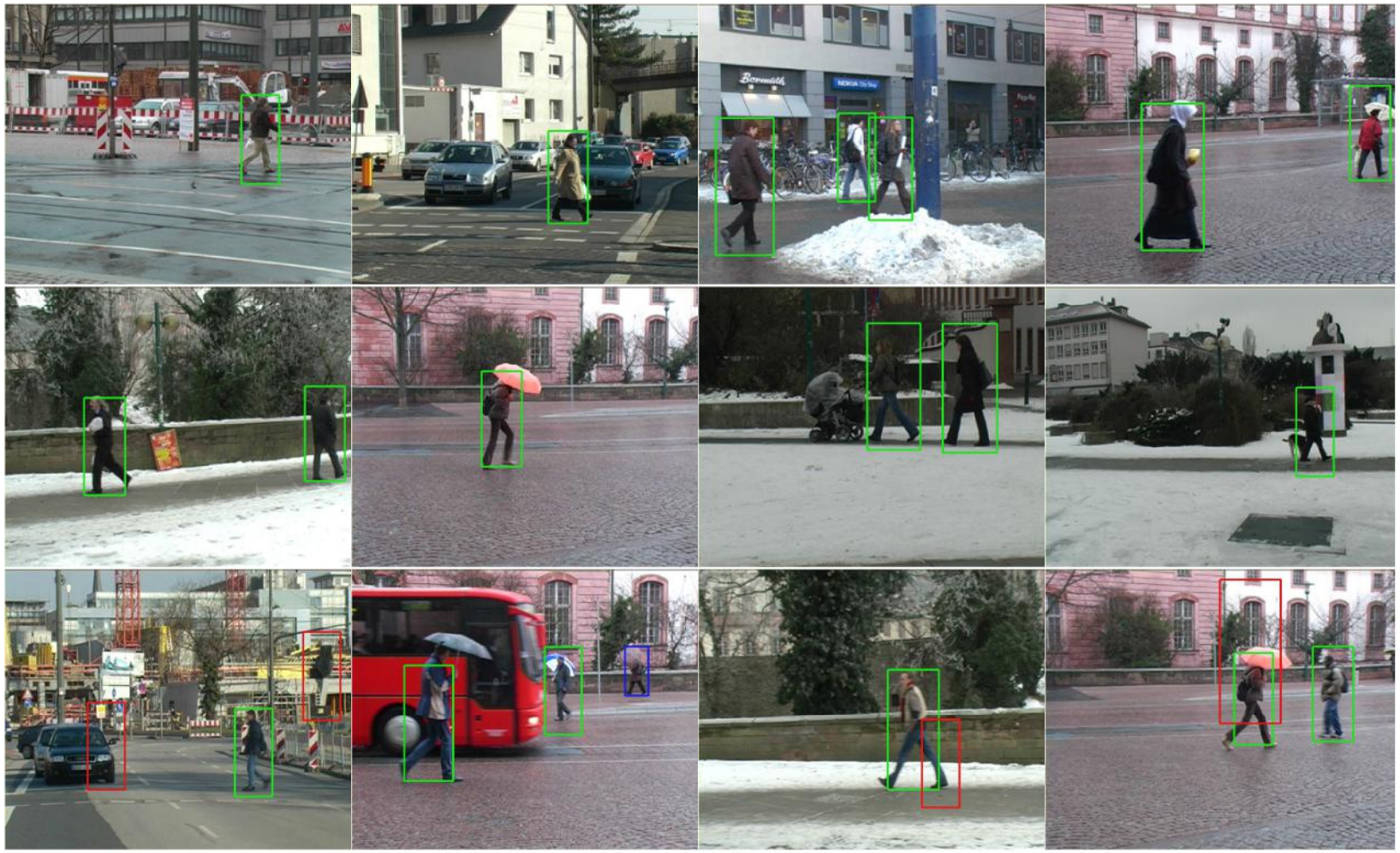
Nando de Freitas
February, 2013
University of British Columbia

Outline of the lecture

This lecture provides an introduction to decision trees. It discusses:

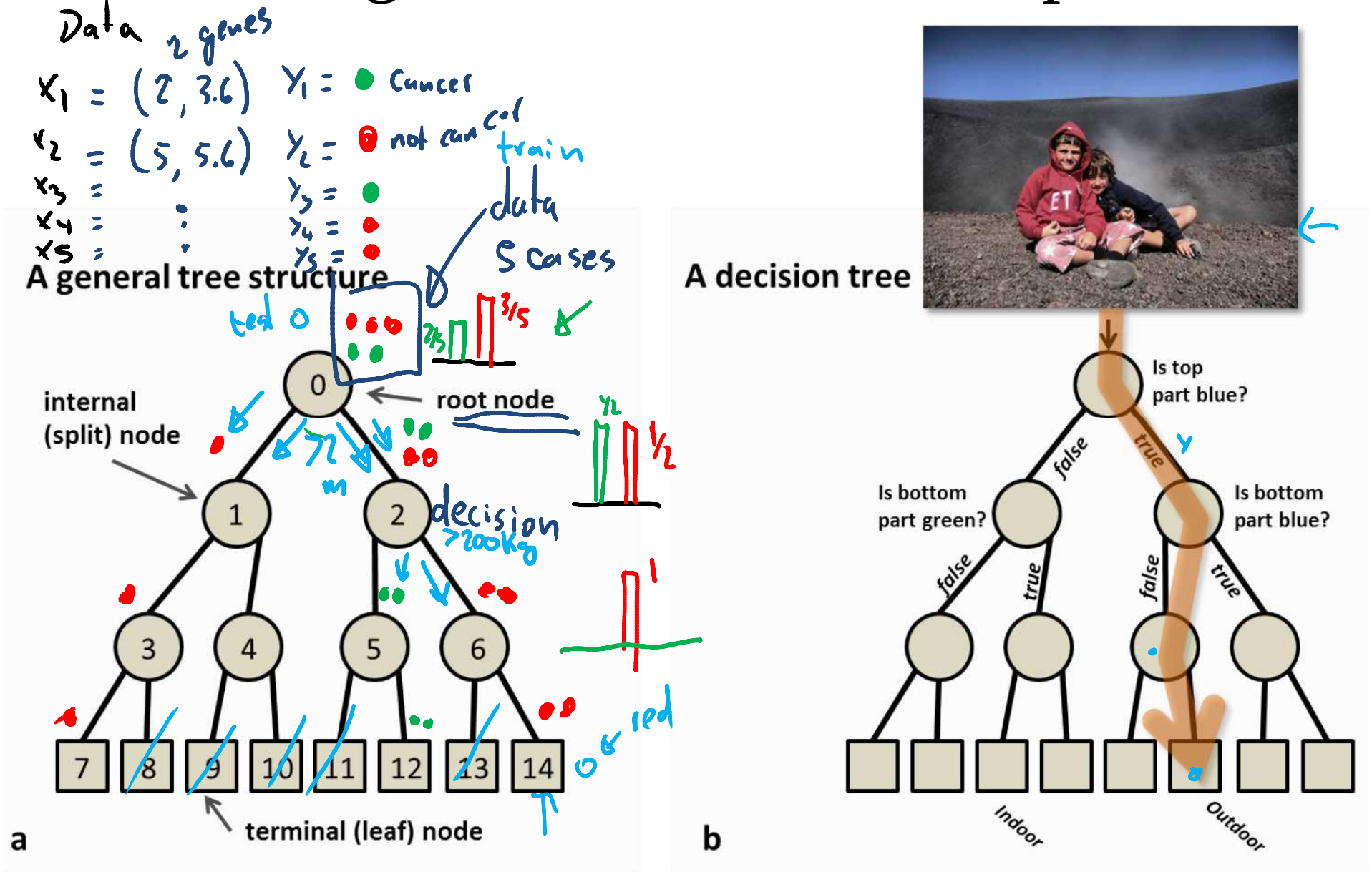
- Decision trees
- Using reduction in entropy as a criterion for constructing decision trees.
- The application of decision trees to classification

Motivation example 1: object detection



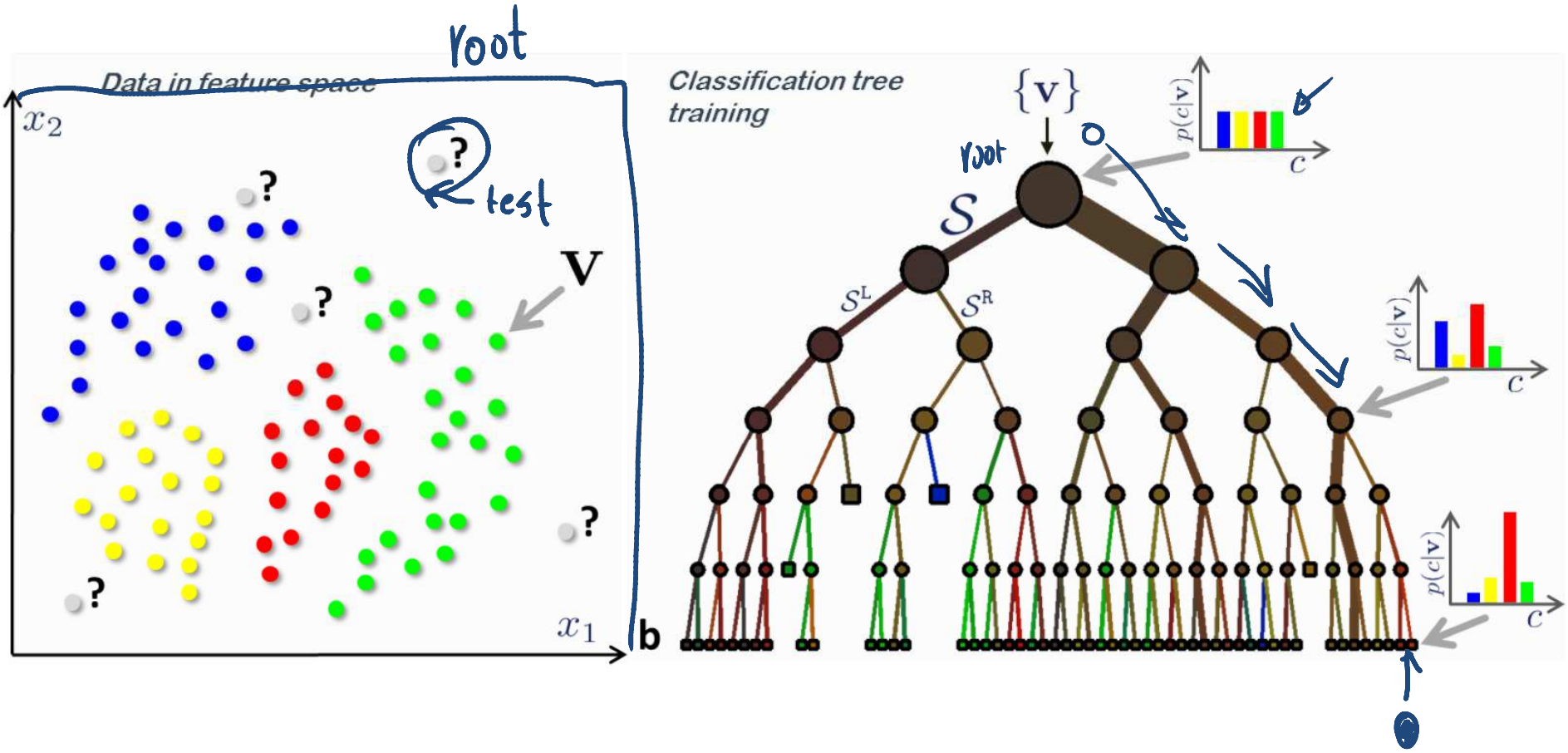
Motivation example 2: Kinect

Image classification example



[MSR Tutorial on decision forests by Criminisi et al, 2011]

Classification tree

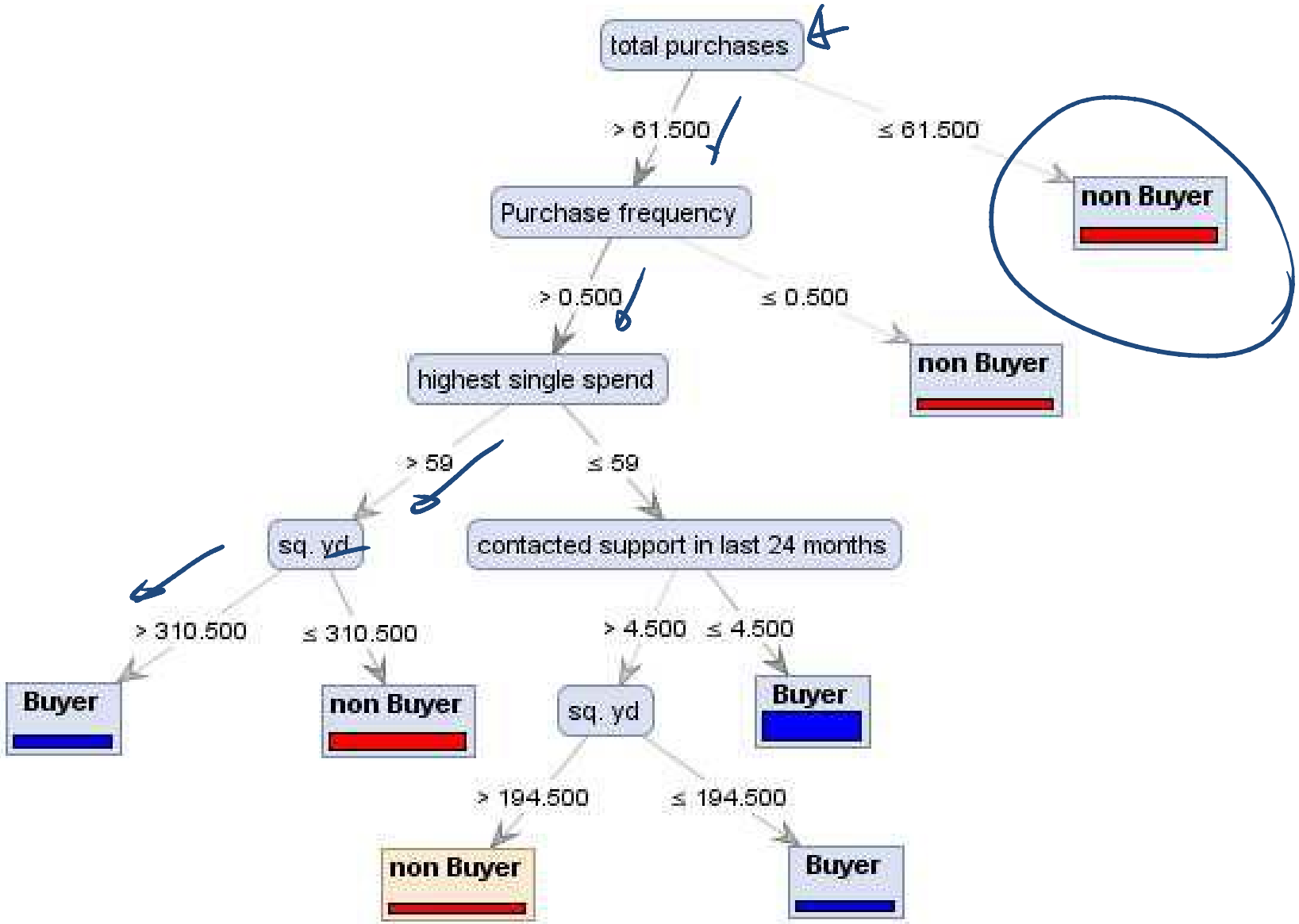


A generic data point is denoted by a vector $\mathbf{v} = (x_1, x_2, \cdots, x_d)$

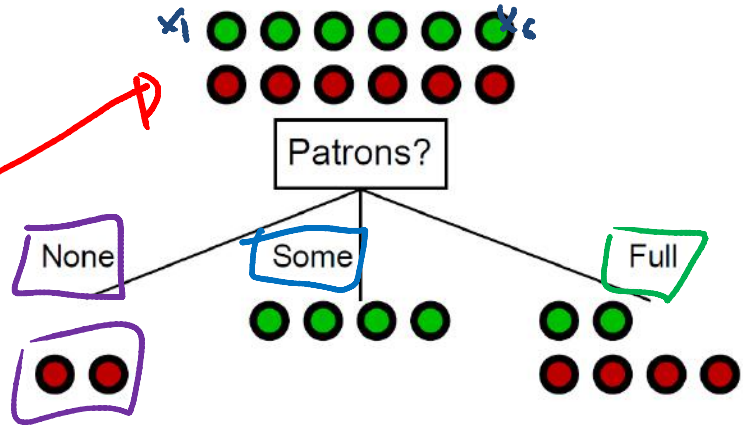
$\mathcal{S}_j = \mathcal{S}_j^{\mathrm{L}} \cup \mathcal{S}_j^{\mathrm{R}}$

[Criminisi et al, 2011]

Another commerce example



From a spreadsheet to a decision node



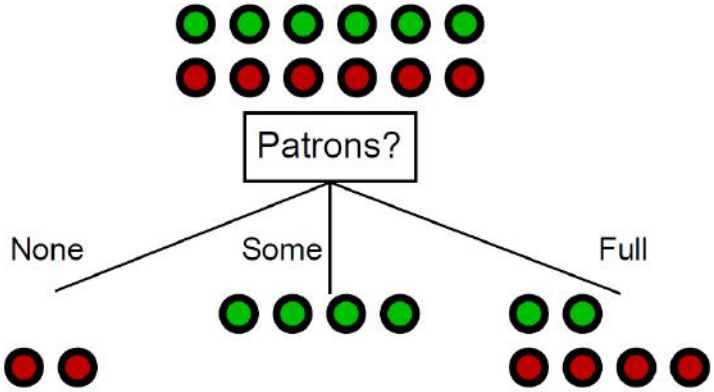
Examples described by attribute values (Boolean, discrete, continuous, etc.)
E.g., situations where I will/won't wait for a table:

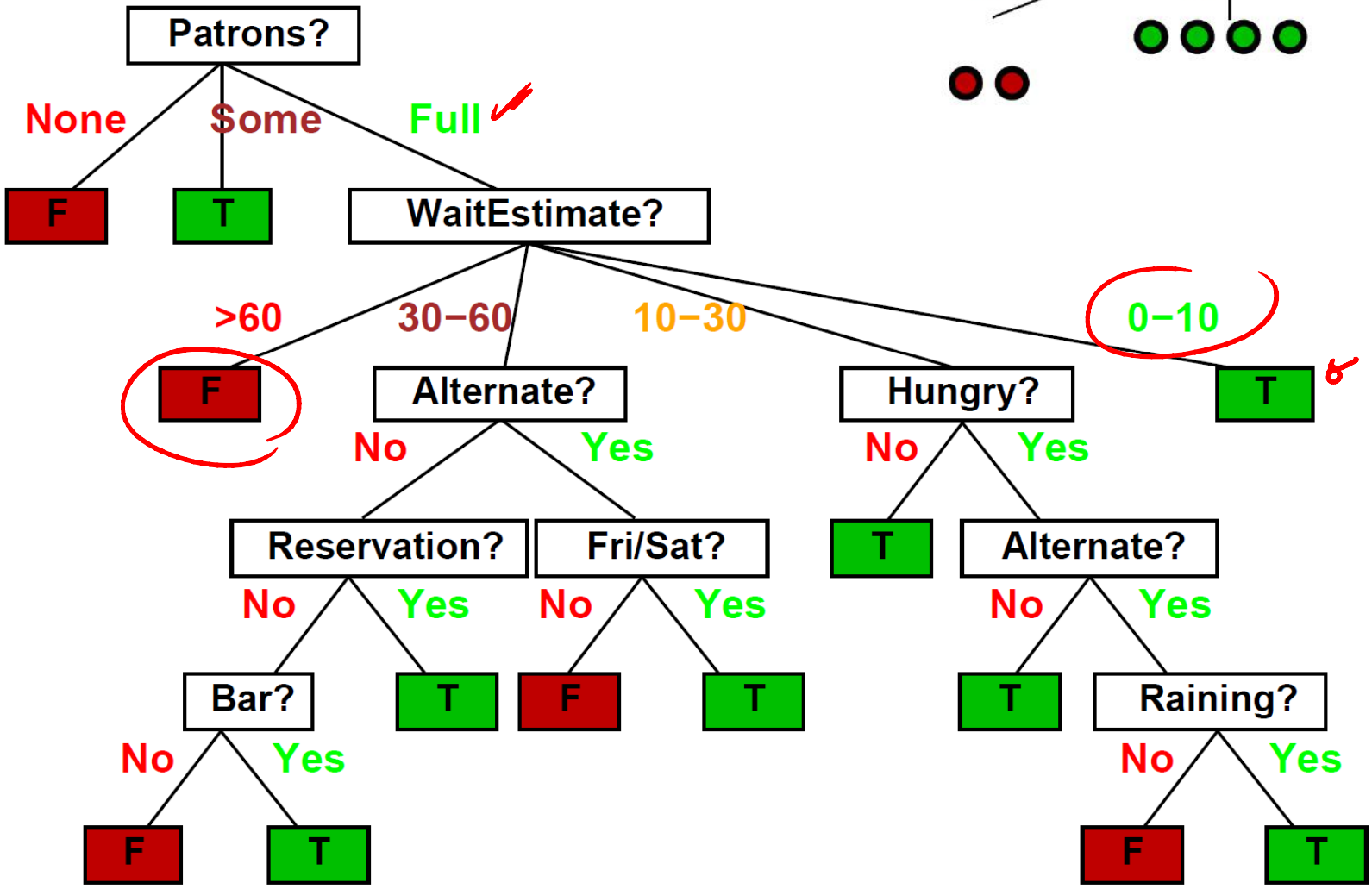
Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

Classification of examples is positive (T) or negative (F)

[AI book of Stuart Russell and Peter Norvig]

A learned decision tree

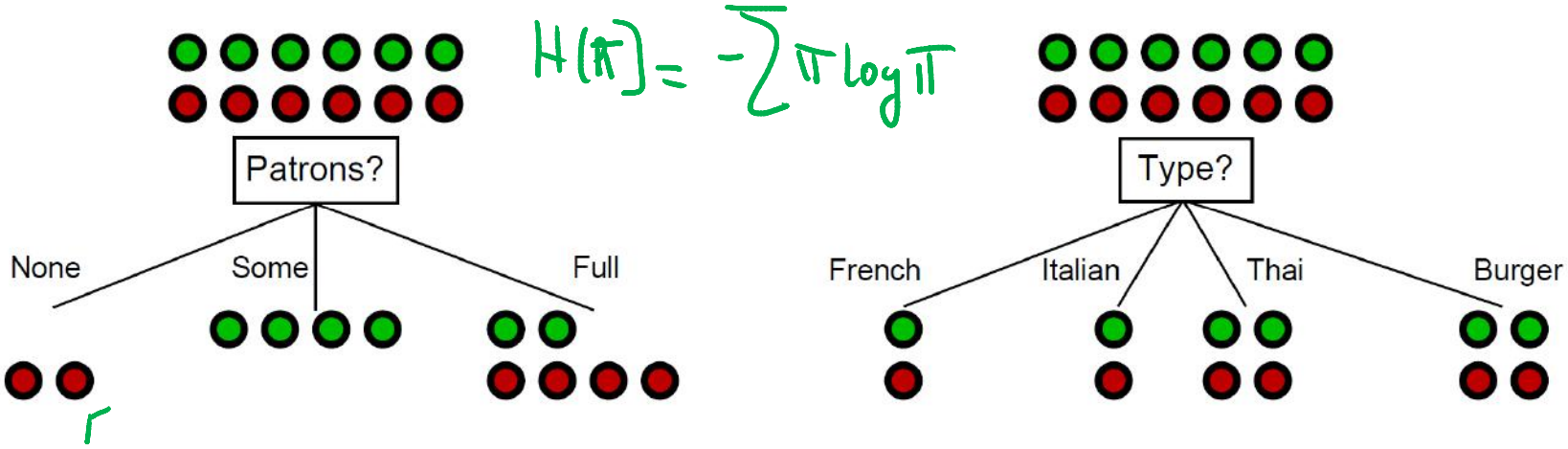




[AI book of Stuart Russell and Peter Norvig]

How do we construct the tree ? i.e., how to pick attribute (nodes)?

Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"



Patrons? is a better choice—gives **information** about the classification

For a training set containing p positive examples and n negative examples, we have:

$$H(\frac{p}{p+n}, \frac{n}{p+n}) = -\frac{p}{p+n}\log_2\frac{p}{p+n} - \frac{n}{p+n}\log_2\frac{n}{p+n}$$

How to pick nodes?

- A chosen attribute ***A***, with ***K*** distinct values, divides the training set ***E*** into subsets $E_1, \ldots, E_K$.
- The **Expected Entropy (EH)** **remaining** after trying attribute ***A*** (with branches $i=1,2,\ldots,K$) is

$$EH(A) = \sum_{i=1}^{K} \frac{p_i + n_i}{p + n} H\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

- **Information gain (I)** or **reduction in entropy** for this attribute is:

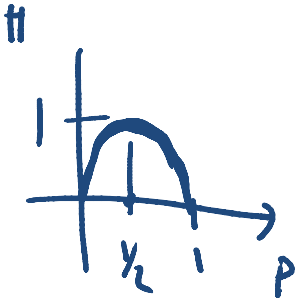
$$I(A) = H\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - EH(A)$$

- Choose the attribute with the largest I

[Hwee Tou Ng & Stuart Russell]

Example

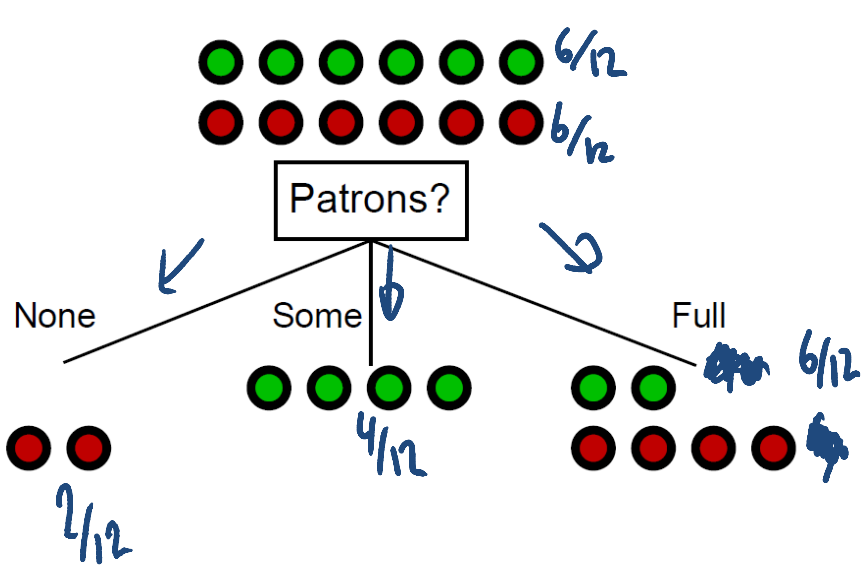
- **Convention**: For the training set, $p = n = 6$, $H(6/12, 6/12) = 1$ bit

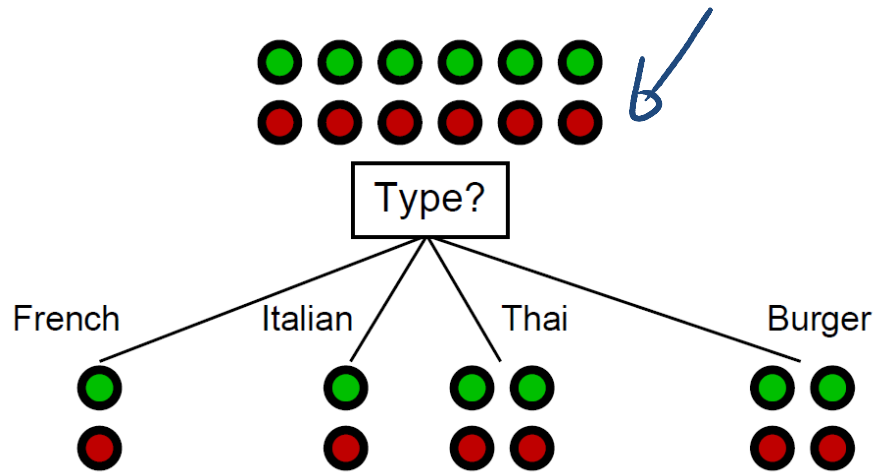


- Consider the attributes *Patrons* and *Type* (and others too):

$$I(Patrons) = 1 - [\frac{2}{12}H(0,1) + \frac{4}{12}H(1,0) + \frac{6}{12}H(\frac{2}{6},\frac{4}{6})] = .0541 \text{ bits}$$

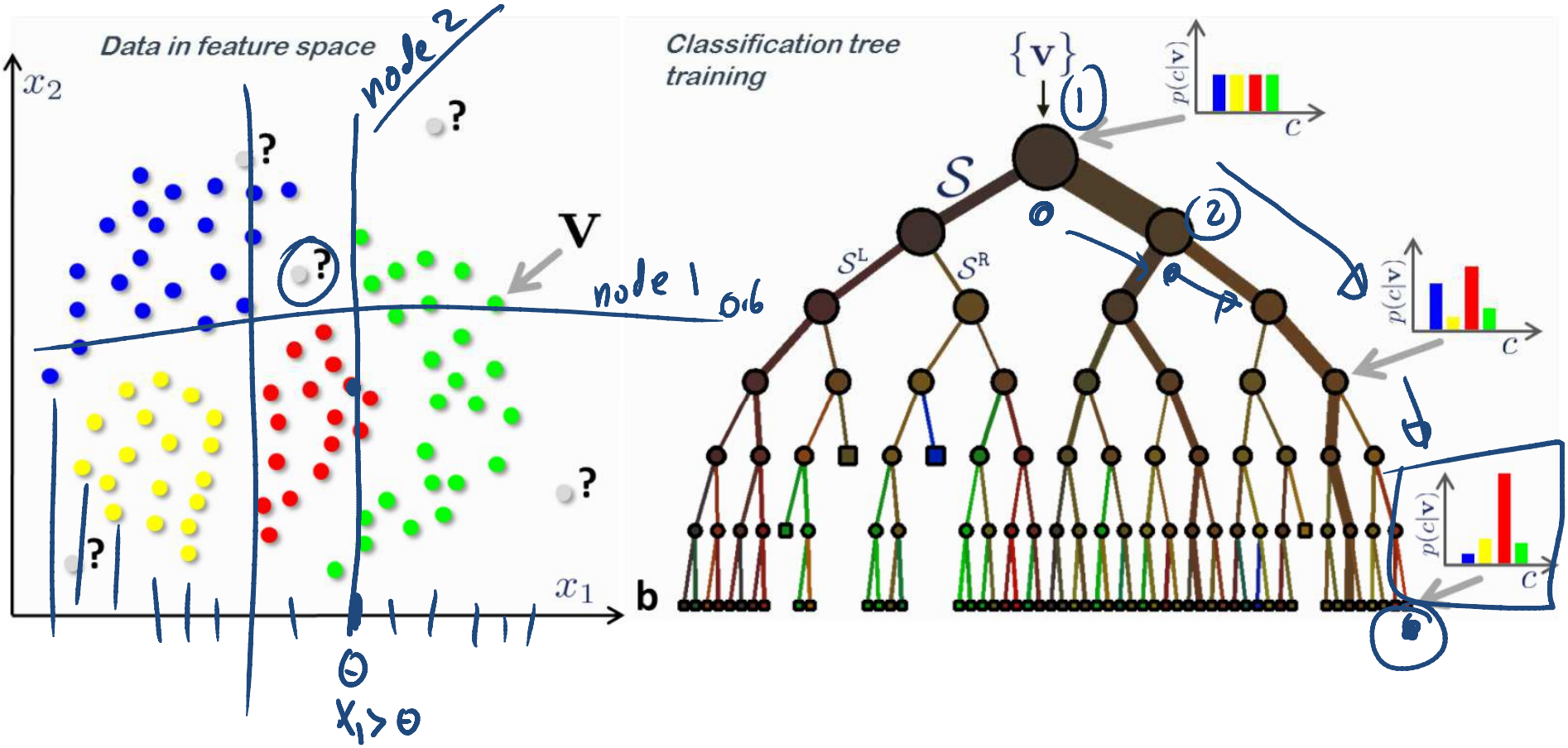
$$I(Type) = 1 - [\frac{2}{12}H(\frac{1}{2},\frac{1}{2}) + \frac{2}{12}H(\frac{1}{2},\frac{1}{2}) + \frac{4}{12}H(\frac{2}{4},\frac{2}{4}) + \frac{4}{12}H(\frac{2}{4},\frac{2}{4})] = 0 \text{ bits}$$





[Hwee Tou Ng & Stuart Russell]

Classification tree

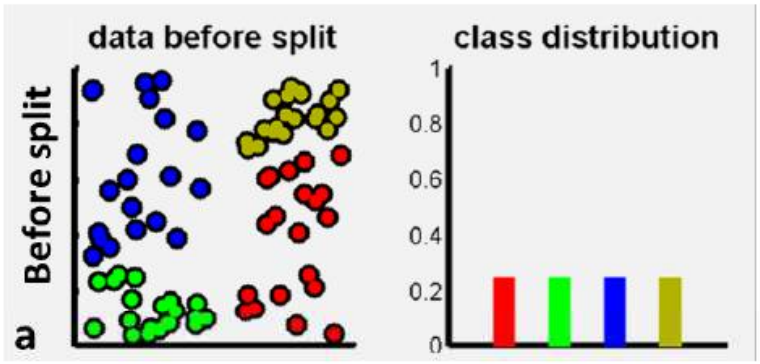


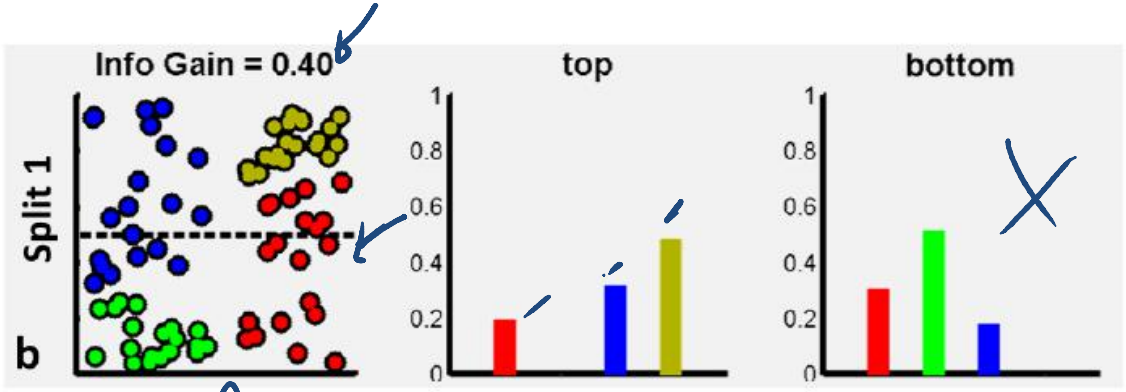
A generic data point is denoted by a vector $\mathbf{v} = (x_1, x_2, \cdots, x_d)$

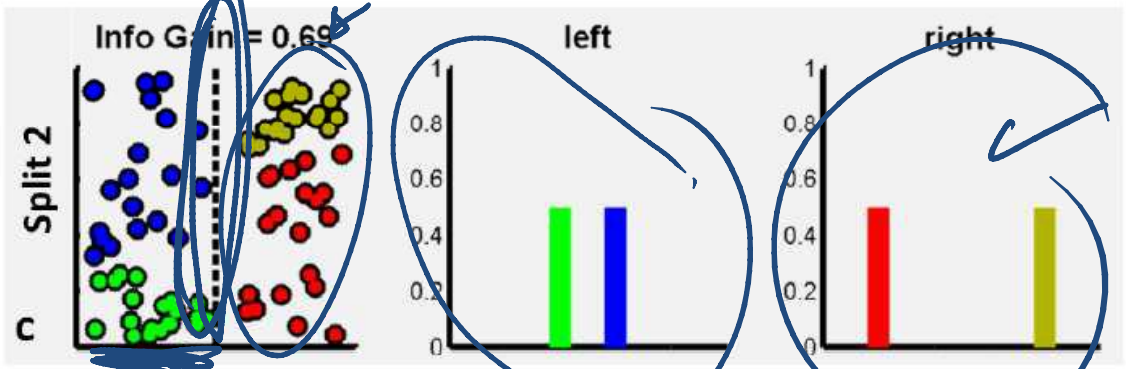
$$\mathcal{S}_j = \mathcal{S}_j^{\mathrm{L}} \cup \mathcal{S}_j^{\mathrm{R}}$$

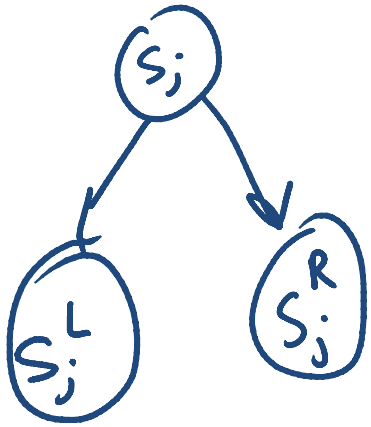
[Criminisi et al, 2011]

Use information gain to decide splits





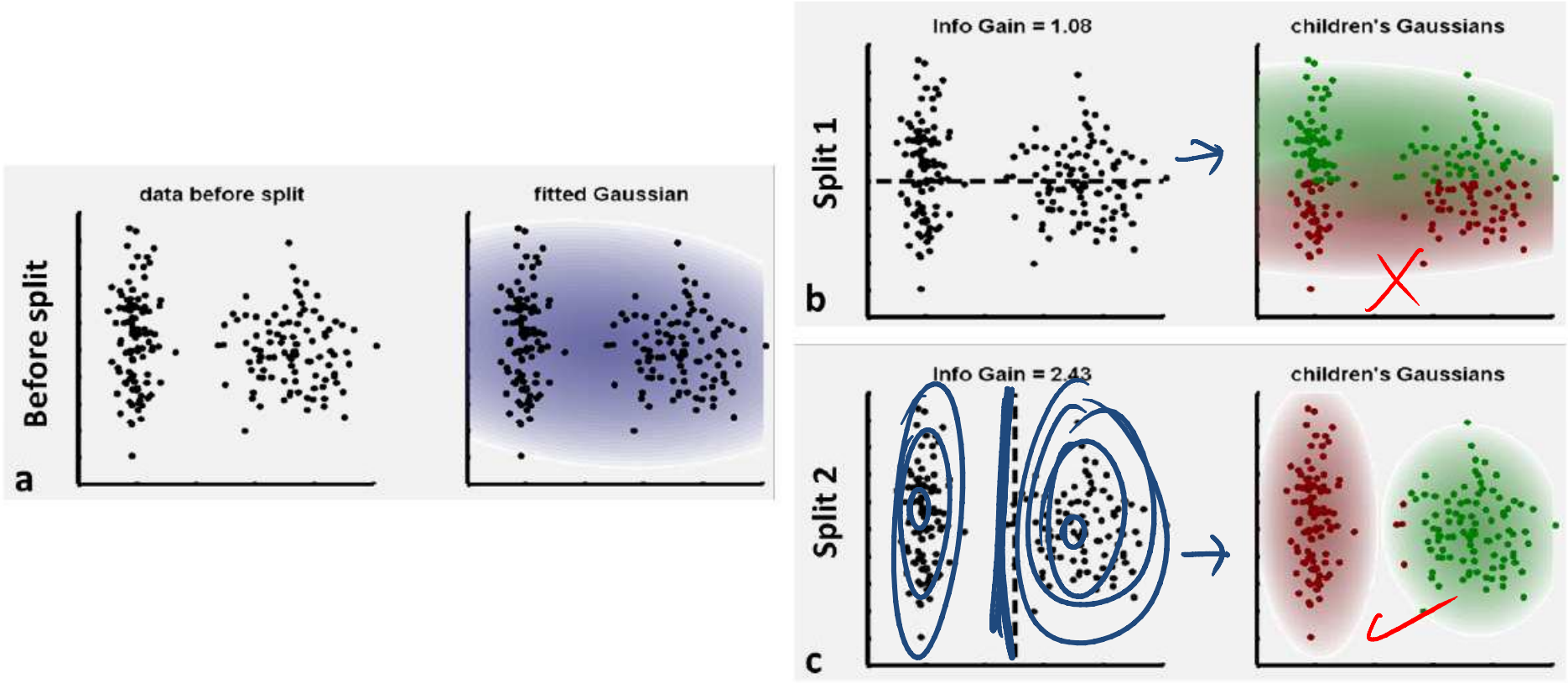




$$I_j = H(\mathcal{S}_j) - \sum_{i \in \{L,R\}} \frac{|\mathcal{S}_j^i|}{|\mathcal{S}_j|} H(\mathcal{S}_j^i)$$

[Criminisi et al, 2011]

Advanced: Gaussian information gain to decide splits

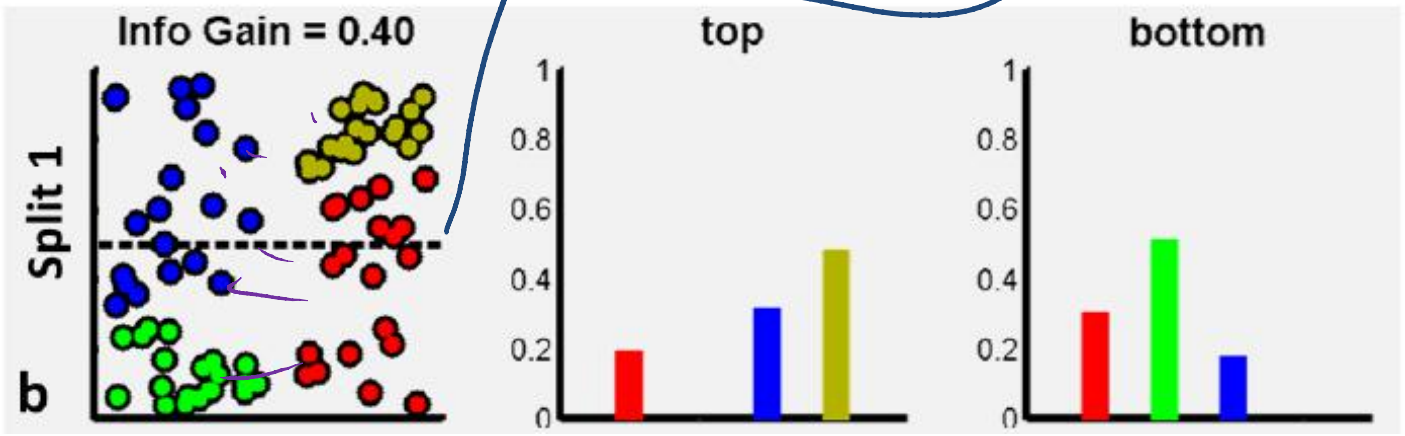


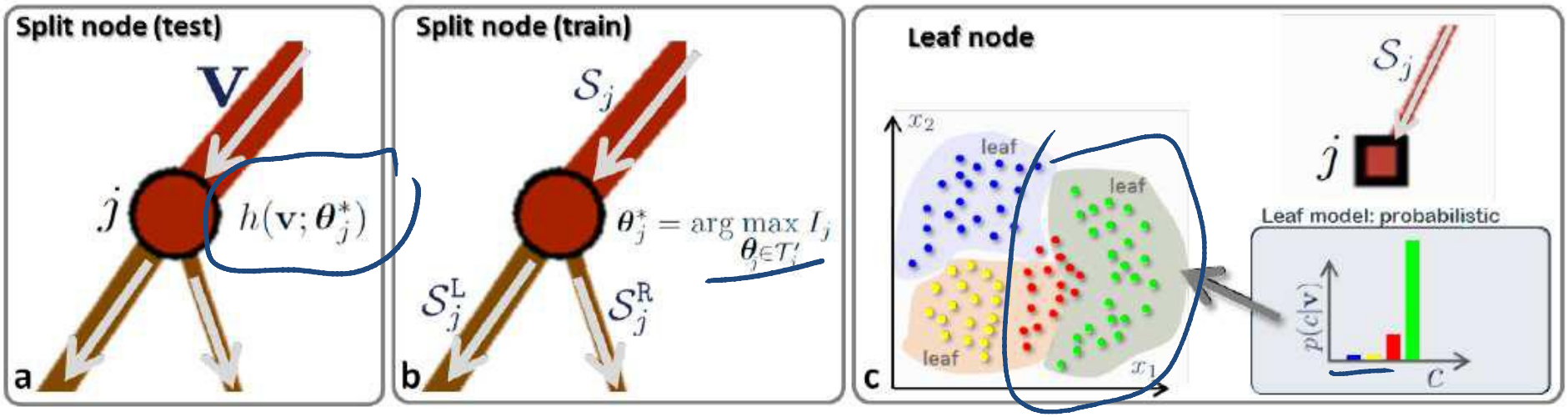
$$H(\mathcal{S}) = \frac{1}{2}\log\left((2\pi e)^d|\Lambda(\mathcal{S})|\right)$$

[Criminisi et al, 2011]

Each split node j is associated with a binary split function

$$h(\mathbf{v}, \boldsymbol{\theta}_j) \in \{0, 1\},$$

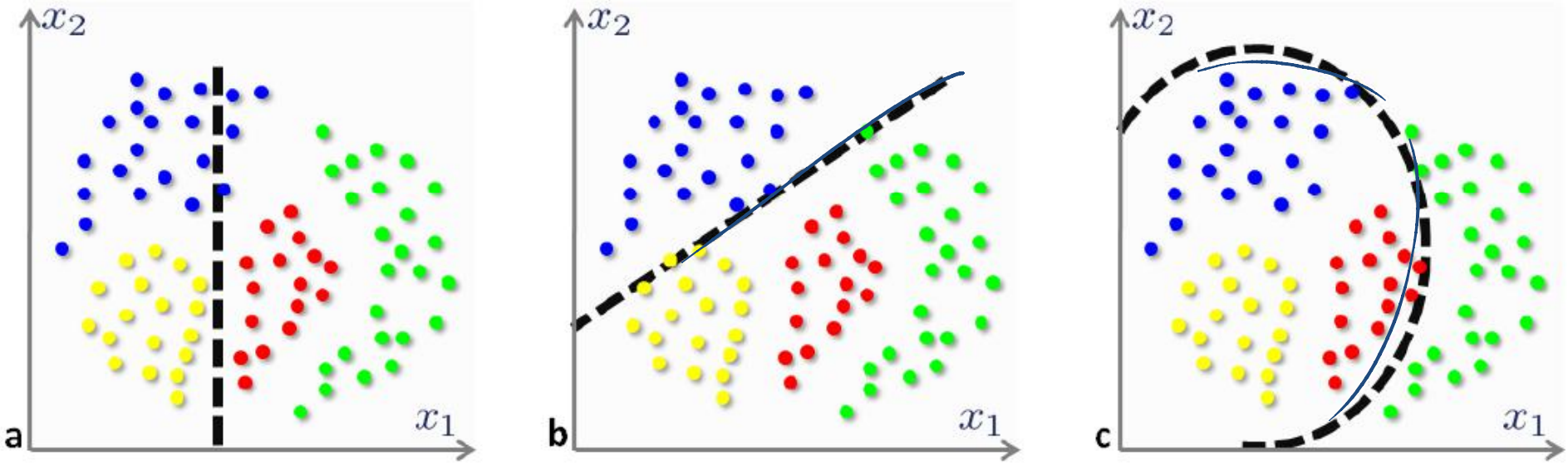




$$I_j = H(\mathcal{S}_j) - \sum_{i \in \{L,R\}} \frac{|\mathcal{S}_j^i|}{|\mathcal{S}_j|} H(\mathcal{S}_j^i)$$

[Criminisi et al, 2011]

Alternative node decisions



$$\mathbf{v} = (x_1 \; x_2) \in \mathbb{R}^2$$

[Criminisi et al, 2011]

Next lecture

The next lecture introduces random forests.