Photo Electric Effect

Photon theory

According to Planck's quantum theory, light consists of photons as energy packets having following properties :

(i) Each photon is of energy $E = h\nu = hc/\lambda$.

Where h is Planck's constant. Where $h = 6.63 \times 10^{-34}$ J-sec $= 4.14 \times 10^{-15}$ eV -sec

(ii) All photons travel in stright line with the speed of light in vacuum.

(iii) Photons are electrically netural.

(iv) Photons have zero rest mass.

(v) Photons are not deflected by electric and magnetic fields.

(vi) The equivalent mas of a photon while moving is given by

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$$

(vii) Momentum of the photon

$$p = E/c = h\nu/c = h/\lambda.$$

(viii) Numbe of photons of wavelength λ emitted in t second from a lamp of power P is.

$$n = \frac{Pt\lambda}{hc}$$

Illustration :

Violet light (λ =4000 Å) of intensity 4 watt/m^2 falls normally on a surface of area 10 cm × 20 cm. Find

(a) the energy recived by the surface per second.

(b) the number of photons hitting the surface per second.

(c) If surface is tilted such that plane of the surface makes an angle 30° with light beam, find the number of photons hitting the surface per second.

Sol. *(a) Energy received per second per unit area*

$E = I A \cos\theta = 4 \times 0.02\ J \times \cos 0^o = 0.08\ J.$

(b) $n\, h\, (c/\lambda) = E$

$$\Rightarrow \quad n = \frac{0.08 \times 4000 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= \frac{32 \times 10^{17}}{19.89} = 1.609 \times 10^{17}$$

(c) $$n = \frac{IA\cos 60^o \times \lambda}{hc}$$

$$= \frac{1}{2} \times \frac{32 \times 10^{17}}{19.89} = 0.805 \times 10^{17}.$$

Pratice Exercise

Q.1 Visible light has wavelengths in the range of 400 nm to 780 nm. Calculate the range of energy of the photons of visible light.

Q.2 Calculate the number of photons emitted per second by a 10 W sodium vapour lamp. Assume that 60% of the consumed energy is converted into light. Wavelength of sodium light = 590 nm.

Answers

Q.1 2.56×10^{-19} J to 5.00×10^{-19} J Q.2 1.77×10^{19}

Radiation Pressure

The electromagnetic wave transports not only energy but also momentum, and hence can exert a radiation pressure on a surface due to the absorption and reflection of the momentum.

Illustration :

A photon of wavelength 6630 Å is incident on a totally reflecting surface.Find the momentum delivered by the photon .

Sol. *The momentum of the incident radiation is given as* $P = \frac{h}{\lambda}$

When the light is totally reflected normal to the surface the direction of the ray is reversed. That means it reverses the direction of its momentum without changing its magnitude.
$\Rightarrow$ *Change in momentum has a magnitude*

$$\Delta P = 2P = \frac{2h}{\lambda}$$

$$\Rightarrow \Delta P = \frac{2(2.63\times 10^{-34}\, J\text{-}sec)}{(6630\times 10^{-34}\, m)} = 2\times 10^{-27}\ kgm/s$$

Illustration :

A parallel beam of monochromatic light of wavelength λ is incident normally on a surface. The intensity of the beam is I. Find the force exerted by the light beam on the surface if surface is
(i) perfectly reflecting
(ii) perfectly absorbing

Sol. *Energy incident per unit time = IA*

Momentum incident per unit time $= \frac{IA}{c}$

(i) *momentum transported to the wall per unit time* $= \dfrac{2IA}{c}$

$$\therefore F = \frac{2IA}{c}$$

$$\therefore pressure = \frac{2I}{c}$$

(ii) *momentum transported to the wall per unit time* $= \dfrac{IA}{c}$

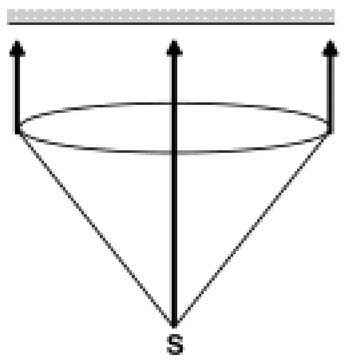
$$\therefore F = \frac{IA}{c}$$

$$\therefore pressure = \frac{I}{c}$$

Pratice Exercise

Q.1 A 100 W light bulb is placed at the centre of a spherical chamber of radius 20 cm. Assume that 60% of the energy supplied to the bulb is converted into light and that the surface of the chamber is perfectly absorbing. Find the pressure exerted by the light on the surface of the chamber.

Q.2 A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in the figure. The mass of the mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Find the power of the source needed to support the weight of the mirror. Take $g = 10\ m/s^2$.

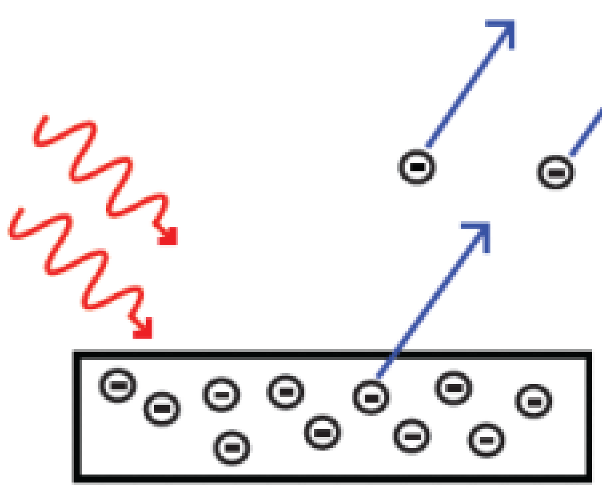


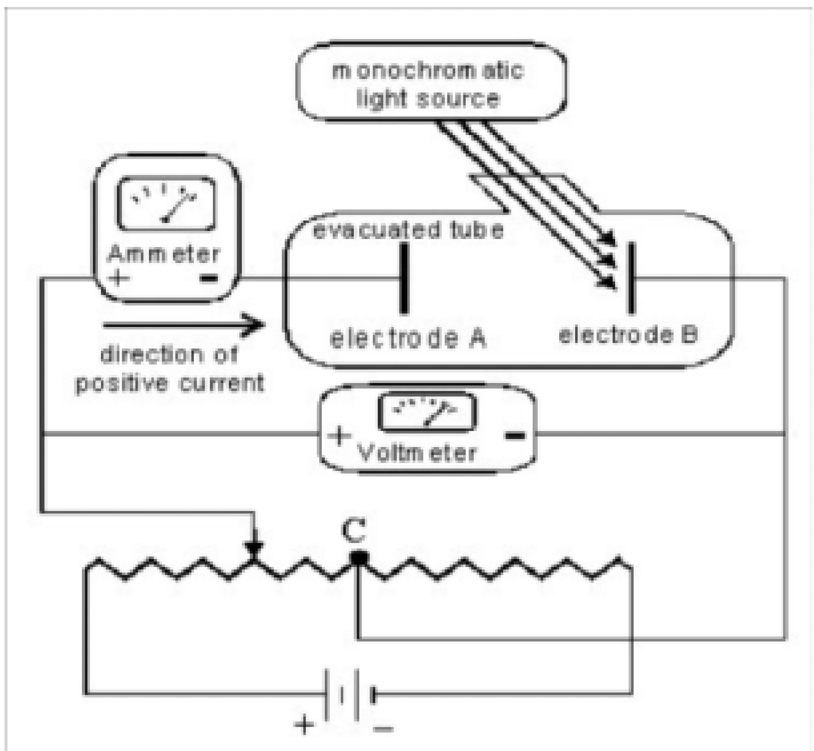
Answers

Q.1 4.0×10^{-7} Pa   Q.2 100 MW

Photoelectric Effect

The photoelectric effect is process where electrons are ejected from a surface by the action of light (electromagnetic radiation). The process was discovered by Heinrich Hertz in 1887. Attempts to explain the effect by classical electromagnetic failed. In 1905, Albert Einstein presented an explanation based on the quantum concept of Max Planck.





Observation of the experiments on Photo-Electric Effect:

(i) The emission of photoelectrons is instantaneous.

(ii) the number of photoelectrons emitted per second is proportional to the intensity of the incident light.

(iii) The maximum velocity with which electrons emerge is dependent only on the frequency and not on the intensity of the incident light.

(iv) There is always a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.

Explanation

Einstein suggested that when a light beam is incident on a metal surface the free electrons of the metal absorb the entire energy of an incident photon during its collision with it. If this electron gets sufficient energy in this manner to do work against the surface adhesion of the given metallic surface and escape then it leaves the metal and a photoelectron is found.

For an electron to escape from a metallic surface by doing work against its attractive force and get out of the force field of the metallic surface, a minimum amount of energy is required to be supplied to electrons. This minimum energy required for an electron to escape from a metallic surface is called the work function of the given metal which is characteristic of the material and is hence different for different metals. Work function of a given metal is generally represented by the symbol ϕ.

The minimum frequency of light corresponding to which the energy of a photon is equal to the work function of a given metal is called the Threshold frequency of that metal and the corresponding wavelength is called Threshold wavelength.

$$h\nu_0 = \phi \text{ or } \nu_0 = \phi / h$$

Where v_0 is called threshold frequency.

So, $\frac{hc}{\lambda_0} = \phi$ or $\lambda_0 = \frac{hc}{\phi}$

Where λ_0 is called threshold wavelength.

Clearly, when a light beam of frequency less than v_0 or wavelength greater than λ_0 is incident then no photoelectrons can be emitted, no matter how high is the intensity of the incident beam.

Suppose, a photon transfers energy more than the work function of the given metal then the photoelectron may be ejected with a kinetic energy.

$$K_{max} = (hv - \phi)$$

or less than that because a part or all of the extra energy may be lossed during several collisions that the electron makes before emission.

If the frequency of the photon is v and threshold frequency for the metal is v_0, then

$$K_{max} = h\,(v - v_0)$$

If the wavelength of the photon is λ and threshold wavelength for the metal is λ_0, then

$$K_{max} = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_0}\right)$$

Stopping Potential

If the polarity of the battery is reversed and the applied potential is gradually increased, the photo-current starts decreasing. This is because the electrons are retarded, and most of the elecrons are unable to reach the opposite electrode. It is observed that when the applied retarding potential is increased, the photocurrent eventually becomes zero. This potential is known as the stopping potential and depends only on the material of the photocathode and the frequency of light.

If V_s be the stopping potential, then

$$eV_S = hv - \phi$$

The stopping potential V_S depends only on the metal and does not depend on the intensity of incident light. a, b, c - different intensites.

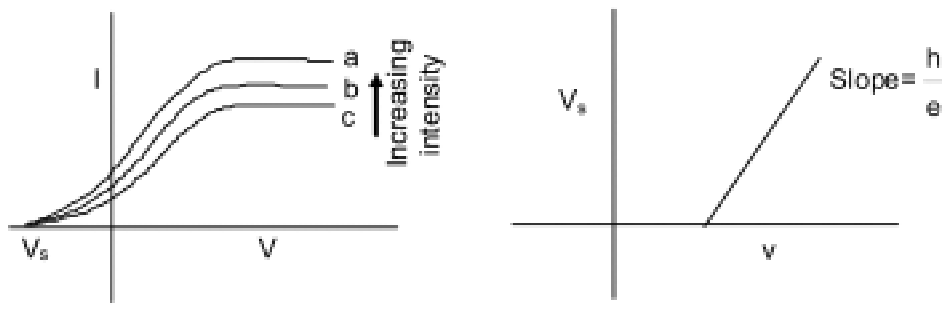


Illustration :

When a metallic surface is illuminated with monochromatic light of wavelength λ the stopping potential for photoelectric current is $3\ V_0$. When the same metallic surface is illuminated with a light of wavelength 2λ, the stopping potential is V_0. Find the threshold wavelength for the surface.

Sol. *Einstein's photoelectric equation:*

$$\frac{hc}{\lambda} = e\,(3\ V_0) + W \qquad ...(i)$$

Here W = work function

$$\text{and } \frac{hc}{2\lambda} = e\,(V_0) + W \qquad ...(ii)$$

Solving these equations

$$\Rightarrow \frac{hc}{\lambda} = \frac{3hc}{2\lambda} + W - 3\ W \qquad \Rightarrow \qquad 2W = \frac{hc}{2\lambda}$$

$$\Rightarrow \frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$$

$\Rightarrow \lambda_0 = 4\ \lambda$ *When λ_0 = threshold wavelength*

Pratice Exercise

Q.1 The work function of a metal is 2.5×10^{-10} J. (a) Find the threshold frequency fro photoelectric emission. (b) If the metal is exposed to a light beam of frequency 6.0×10^{-14}Hz, what will be the stopping potential?

Q.2 The electric field associated with a light wave is given by

$$E = E_0 \sin [(1.57 \times 10^7 \text{m}^{-1})\,(x - ct)\,]$$

Find the stopping potential when this light is used in an experiment on photoelectric effect with the emitter having work function 1.9 eV.

Q.3 A monochromatic light source of intensity 5 mW emits 8×10^{15} photons per second. This light ejected photoelectrons from a metal surface. The stopping potential for this setup is 2.0 V. Calculate the work function of the metal.

Q.4 A photographic film is coated with a silver bromide layer. When light falls on this films, silver bromide molecules dissociate and the field records the light there. A minimum of 0.6 eV is needed to dissociate a silver bromide molecule. Find the maximum wavelength of light that can be recorded by the film.

Q.5 A small metal plate (work function ϕ) is kept at a distance d from a singly ionized, fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. Find the maximum wavelength of the light beam so that some of the photoelectrons may go round the ion along a circle.

Answers

Q.1	(a) 3.8×10^{14} Hz ; (b) 0.91 V	Q.2	1.2 V	Q.3	1.9 eV
Q.4	2070 nm	Q.5	$\dfrac{8\pi\varepsilon_0 dhc}{e^2 + 8\pi\varepsilon_0 \phi d}$		

Wave Particle Duality

Electromagnetic radiation is an emission with a dual nature, i.e. it has both wave and particle aspects. In particular, the energy conveyed by an electromagnetic wave is always carried in packets whose magnitude is proportional to frequency of the wave. These packets of energy are called photons.
Energy of photon is E = h.f, where h is Planck's constant, and f is frequency of wave.

de-Broglie idea

As wave behaves like material particles, similarly matter also behaves like waves. According to him, a wavelength of the matter wave associated with a particle is given by $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$, where m is the mass and v is velocity of the particle.

If an electron id accelerated through a potential diference of V volt,

then $\quad \dfrac{1}{2} m_e v^2 = eV \qquad$ or $\qquad V = \sqrt{\dfrac{2eV}{m_e}}$

$$\therefore \quad \lambda = \frac{h}{m_e v} = \frac{h}{\sqrt{2eVm_e}}$$

(It is assumed that the voltage V is not more than several tens of Kilovolt)

Illustration :

Find the ratio of de-Broglie wavelength of molecules of hydrogen and helium in two gas jars kept separately at temperature 27°C and 127°C respectively.

Sol. *de-Broglie wavelength $\lambda = h/mv$*

where the speed (r.m.s) of a gas particle at the given temperature (T) is given as

$$\frac{1}{2} mv^2 = \frac{3}{2} KT$$

$\Rightarrow v = \sqrt{\dfrac{3KT}{m}}$, *where K = Boltzmann's constant, m = mass of the gas particle and T = temperature of gas in K*

$$\Rightarrow mv = \sqrt{3mKT}$$

$$\Rightarrow \lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mKT}} \qquad \therefore \frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He} T_{He}}{m_{He} T_H}}$$

$$= \sqrt{\frac{(4amu)(273 + 127^o)K}{(2amu)(273 + 27^o)K}} = \sqrt{\frac{8}{3}}$$

Illustration : :

If the sationary proton and α - particle are accelerated through same potential difference. Find the ratio of de-Broglie wavelength.

Sol. *The gain in K.E. of a change particle after moving through a potential difference of V is given as qV, that is also equal to $\frac{1}{2}$ mv^2, where v is the velocity of the charged particle.*

$$\frac{1}{2}mv^2 = qV \qquad \Rightarrow \qquad v = \sqrt{\frac{2qV}{m}}$$

$$\therefore mv = \sqrt{2mqV}$$

$$\Rightarrow \text{de-Broglie wavelength} = \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$$

$$\text{Putting } V_a = V_p, \; \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{(4)(2)}{(1)(1)}} = 2\sqrt{2}$$

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Atomic Structure

Structure of matter always been an interesting area of research for physicists. Till 20[th] century it was assumed that matter consists of indivisible small tiny particles called "atoms" But with the study and research it was found that the atom is divisible and made of other small particles called electron, proton and Neutron. So may physicists tried to explain the structure of atom but finally it was Neils Bohr whose explanation about the structure was well accepted. For simplicity they have taken hydrogen atom and then it can be extended to other H-like atoms too. Some of the historical models are also explained and them drawbacks :

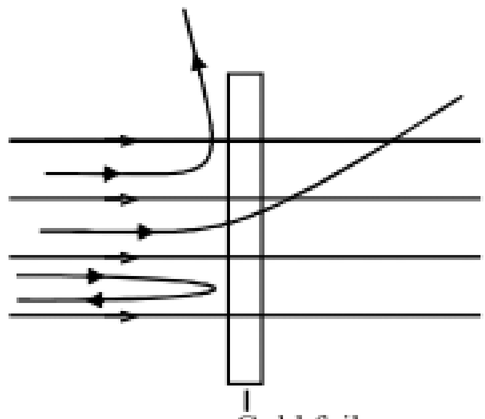
Thomson's Atomic Model

J.J. thomson found the charge to mass ratio of electron for atomic structure. He describes the atom as a region in which positive charge is spread out in space with electrons embedded throughout the region, much like the sceds in a water - melon. The atom as a whole then be electrically neutral.

Rutherford's Model

In 1911 Earnest, Rutherford and his students performed a critical experiment which showed that Thompson Model may not be correct. They bombarded highly energetic α -Particles be correct. (He^{++} Nucleus) onto a thin Gold foil. Following observation were there :

(i) Most of the Practices were passed through the foil as if it were an empty space.
(ii) Very few particles were even deflected backward completely reversing their direction.



Gold foil

(iii) Rest ones were deflected from 0º to 180º to their original direction of motion.
Rutherford concluded that most of the part of atom is empty and all the +ve charge is concentrated at the center in a very small volume he named it nucleus. Electrons are moving around sun. Hence this model was also referred as planetary model of the atom.

There were two basic difficulties with the model reason of characteristic radiation coming from atom. The second was according to maxwell theory of electromagnetic Radiation an orbiting electron i an accelerating charge hence it should emit EM radiations resulting in decrease of radius of orbit and finally it should fall on nucleus. But atom is an stable entity.

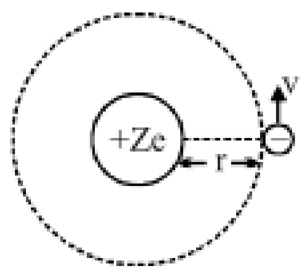
Bohr's Model of hydrogen Atom

The first successful picture of atom was given by Bohr. His model was successful in explaining the lines of EM radiations coming out from H_2 - gas. Although model is now considered obsolete and has been completely replaced by Quantum - Mechanical Theory but it was historically important to the development of Quantum mechanics.

To explain his model Bohr made some postulates:

(i) Electrons revolve around the nucleus in stationary circular orbits where centripetal acceleration is provided by the coulombic attraction of protons on electrons as :

$$\left(\frac{1}{4\pi \epsilon_0}\right)\frac{(ze)(e)}{r^2} = \frac{mv^2}{r}$$



z = atomic number

m = mass of electron

r = radius of an orbit.

or
$$\frac{ze^2}{4\pi \epsilon_0 \, r} = mv^2 \qquad \text{...(i)}$$

(ii) Instead of the infinity of orbits which would be possible in classical mechanics, it is only possible for an electron to move in an orbit for which its orbital angular momentum L is an integral multiple of $\frac{h}{2\pi}$

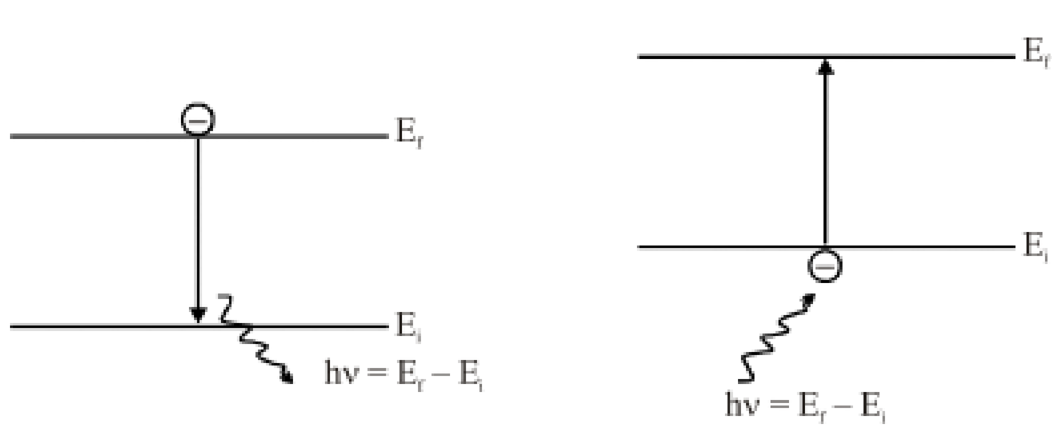
$$L = mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

h = Plank's constant

n = 1,2,3, .. (Quantized Penissible orbits)

(iii) The electron revolving in any one of these allowed orbits does not radiate. These non-radiating orbits are called stationary orbits.

(iv) Energy of electron changes only when then is a transition from higher orbit to lower or from lower to higher.



Photon of energy 'hv' is emitted when when there is a transition from higher to lower.

Calculating radius (r_n) and speed (v_n) of n^{th} orbit

from eq^n (i) and (ii)
we have

$$r_n = \frac{n^2h^2 \in_0}{Ze^2\pi m} = (0.53\ \text{Å})\ \frac{n^2}{z}$$

and

$$v_n = \frac{ze^2}{2nh \in_0} = (2.18 \times 10^6\ \text{m/sec})\ \frac{z}{n}$$

for H-atom,

Radius of 1st ordit, $r_1 = 0.53$ Å
speed of e^- in 1st orbit, $v_1 = 2.18 \times 10^6$ m/sec

Kinetic energy of electron in n^{th} orbit,

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\left\{\frac{ze^2}{2 \in_0 nh}\right\}^2$$

$$KE_n = \frac{1}{2}\left[\frac{mz^2e^4}{4 \in_0^2 n^2h^2}\right] \propto \frac{z^2}{n^2}$$

Potential energy of electron,
In the electric field of nucleus the PE of electron in n^{th} orbit is given by

$$U_n = \left(\frac{1}{4\pi \in_0}\right)\frac{(ze)(-e)}{r}$$

$$U_n = -\left[\frac{mz^2e^4}{4 \in_0^2 n^2h^2}\right] \propto \frac{z^2}{n^2}$$

–ve sign indicates that electron is bound to the nucleus and some work in required to sperate it from the nucleus.

Expression for total energy of electron in n^{th} orbit,

$$E_n = KE_n + U_n = -\frac{1}{2}\left[\frac{mz^2e^4}{4 \in_0^2 n^2h^2}\right] \propto \frac{z^2}{n^2}$$

If we observe the relations carefully

then $\quad E_n = -KE_n = \frac{U_n}{2}$

Putting values of all the constants,

we get

$$E_n = (-13.6 \times 1.6 \times 10^{-19}\ \text{Joule})\ \frac{z^2}{n^2}$$

$$E_n = -\ (13.6\ \text{ev})\ \frac{z^2}{n^2}$$

So $\quad KE_n = +\ (13.6\ \text{ev})\ \frac{z^2}{n^2}$ and $U_n (27.2\ \text{ev})\ \frac{z^2}{n^2}$

From the general expression of total energy

$$E_n = -\left[\frac{me^4}{8 \epsilon_0^2\, h^3 c}\right] hc \left(\frac{z^2}{n^2}\right)$$

or $\quad E_n = -\ (Rhc)\ \frac{z^2}{n^2}$ $\qquad$ Where $R = \frac{me^4}{8 \epsilon_0^2\, ch^3}$ is called Rydberg cosntan

$R = 1.097 \times 10^7\ m^{-1}$

'Rhc' is called 1 Rydberg energy = 13.6 ev.

Energy levels of hydrogen atom (z = 1)

		$E_\infty = 0$
	$n = 5$	$E_5 = -\ 0.54$
	$n = 4$	$E_4 = -\ 0.85$ ev
2 nd excited state or 3rd energy	$n = 3$	$E_3 = -\ 1.51$ ev
1st excited state or 2nd energy level	$n = 2$	$E_2 = -\ 3.4$ ev
Ground state or 1st energy level	$n = 1$	$E_1 = -\ 13.6$ ev

Students are adviced to remember these rules for H-atom.

Note :

(i) **Binding Energy of a state :** Energy require to remove electron from a particular quantum state is called BE of a that Particular state.

eg. BE of e^- of H-atom in n = 4 level is 0.85 ev

BE of 1st excited state of H-atom is 3.4 ev

BE of 1st excited state of He^+-atom is 13.6 ev

(ii) **Ionisation energy**

The energy required to remove an electron from ground state of the atom is called its ionisation energy.

eg. Ionisation energy of H-atom = 13.6 ev

Ionisation energy of He^+ = 54.4 ev

Ionisation energy of H-like atom = (13.6) z^2 ev

(iii) **Ionisation Potential**

The Potential difference through which an e^- must be accelerated to acquire this much energy (i.e. Ionisation energy) is called Ionisation Potential.

eg. Ionisation Potential of H-atom = 13.6 vol

$$\text{Ionisation Potential} = \frac{\text{Ionisation energy}}{e}$$

(iv) **Excitation energy :** The energy which must be provided to the e^- of atom so that it may go to a higher energy level is called excitation energy of that particular excited state.

for equation for H-atom,

Excitation energy of 1st excited state $\Delta E_1 = E_2 - E_1$
$= (-3.4) - (13.6)$
$= 10.2$ ev

Excitation energy of 2nd excited state $\Delta E_2 = E_3 - E_1$
$= (-1.51) - (13.6)$
$= 12.09$ ev

Excitation energy of 3st excited state $\Delta E_3 = E_4 - E_1$
$= (-0.85) - (-13.6)$
$= 12.75$ ev

and so on.

(v) **Excitation Potential :**

The Potential difference through which an e^- must be accelerated to acquire this much energy (i.e. excitation energy) is called excitation Potential.

$$\text{Excitation Potential} = \frac{\text{Excitation energy}}{e}$$

Illustration :

Find dependency of following physical quantities related with electron revolving in an orbit on Quantum number 'n' and on Atomic Number 'Z'.

(a) Equivalent current in n^{th} orbit

(b) Time period in n^{th} orbit

(c) Angular speed in n^{th} orbit

(d) Magnetic field at center due to revolving e^-

(e) Magnetic moment due to equivalent current.

Sol. *Equivalent current is charge crossing a point in unit time*

$$\text{hence } i = \frac{e}{T} = \frac{e\upsilon}{2\pi\pi} \propto \frac{z^2}{n^2}$$

$$\text{Time period } (T) = \frac{2\pi r}{\upsilon} \propto \frac{n^3}{z^2}$$

Angular speed $(\omega) = \dfrac{\upsilon}{r} = \dfrac{2\pi}{T} \propto \dfrac{z^2}{n^3}$

$\vec{B}$ *at center of coil* $(\vec{B}) = \dfrac{\mu_0 i}{2r} \propto \dfrac{i}{r} \propto \dfrac{z^3}{n^5}$

Magnetic moment $(\vec{\mu}) = iA = i\,(\pi r^2) \propto ir^2 \propto n$

Illustration :

Which level of the doubly ionized lithium has the same energy as the ground state energy of the hydrogen atom. Find the ratio of the two radii of corresponding orbits.

Sol. *When excited atom makes transition from n = n to n = 2*

$$(13.6)z^2\left[\frac{1}{z^2}-\frac{1}{n^2}\right] = 10.2 + 17 = 27.2\ ev$$

When excited atom makes transition from n = n to n = 3

$$13.6\,z^2\left[\frac{1}{3^2}-\frac{1}{n^2}\right] = 4.25 + 5.95 = 10.2\ ev$$

Solving the above expression : z = 3, n = 6

Illustration :

Difference between n^{th} *and* $(n + 1)^{th}$ *Bohr's radius of H-atom is equal to its* $(n-1)^{th}$ *Bohr's Bohr's radius, find the value of n.*

Sol. *Given*

$$r_{n+1} - r_n = r_{n-1}$$

$$(0.53) = \frac{(\pi+1)^2}{z} - 0.53\,\frac{(n)^2}{z} = (0.53)\,\frac{(n-1)^2}{z}$$

$(n+1)^2 - n^2 = (n-1)^2$

Solving we get n = 4

Illustration :

A single electron orbits a stationary nucleus of charge Ze where Z is a constant and e is the electronic charge. It requires 47.2eV to excite the electron from the 2nd Bohr orbit to 3rd Bohr orbit. Find

(i) *the value of Z,*

(ii) *energy required to excite the electron from the third to the fourth orbit*

(iii) *the wavelength of radiation required to remove the electron from the first orbit to infinity*

(iv) *the kinetic energy, potential energy and angular momentum in the first Bohr orbit*

(v) *the radius of the first Bohr orbit.*

Sol. *We can find difference of energy of n = 2 and n = 3 as* $E_3 - E_2 = 47.2$

$$(13.6)\, z^2 \left[\frac{1}{4} - \frac{1}{9}\right] = 47.2$$

$$z = 5$$

Energy required to excite from n = 3 to n = 4, $\Delta E = E_4 - E_3 = 13.6\,(5)^2 \left[\frac{1}{9} - \frac{1}{16}\right] = 16.5 \text{ ev}$

Ioniztion energy $= 13.6\,(z)^2 = 13.6\,(5)^5 = 340 \text{ ev}$

corresponding wardength of photon $= \frac{12400}{340}\text{Å} = 36.5\text{ Å}$

In first Bohr orbit : (n = 1)

$$KE_1 = 340 \text{ ev}$$
$$PE_1 = -680 \text{ ev}$$
$$E_1 = -340 \text{ ev}$$

Radius of 1^{st} *Bohr orbit,* $r_1 = \frac{(0.53)n^2}{z} = \frac{0.53}{5} = 0.106\text{Å}$

Illustration :

Imagine a hypothetical atom in which mass of electron is 'm' but charge of e^- *becomes* $-3e$. *Assuming all other Parameters to be same compare the radius of 1st orbit this hypothetical atom an H-atom. Also find ratio of speed of this* e^- *in n-orbit with that of* e^- *of H-atom.*

Sol. *Radius* $(r_n) \propto \frac{1}{e}$ *for H-atom hence radius of 1st orbit of this hypothetical atom will be one-third of that found for H-atom.*

$\upsilon_n \propto e$ *for H-atom*

$\upsilon'_n \propto 3e$ *for Given atom*

hence $\frac{\upsilon_{n'}}{\upsilon_n} = 3$

Pratice Exercise

Q.1 Find the maximum Coulomb force that can act on the electron due to the nucleus in a hydrogen atom.

Q.2 A hydrogen atom emits ultraviolet radiation of wavelength 102.5 nm. What are the quantum numbers of the states involved in the transition?

Q.3 (a) Find the first excitation potential of He^+ ion. (b) Find the ionization potential of Li^{++} ion.

Q.4 Average lifetime of a hydrogen atom excited to n = 2 state is 10^{-8} s. Find the number of revolutions made by the electron on the average before it jumps to the ground state.

Q.5 Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms falls on helium ions in n = 1 and n = 2 states. What are the possible transitions of helium ions as they absorb energy from the radiation?

Q.6 A beam of monochromatic light of wavelength λ z ejects photoelectrons from a cesium surface (ϕ = 1.9 eV). These photoelectrons are made to collide with hydrogen atoms in ground state. Find the maximum value of λ for which (a) hydrogen atoms may be ionized, (b) hydrogen atoms may get excited from the ground state to the first excited state and (c) the excited hydrogen atoms may emit visible light.

Answers

Q.1	8.2×10^{-8} N		Q.2	1 and 3	Q.3	(a) 40.8 V	(b) 122.4 V
Q.4	8.2×10^{6}		Q.5	n = 2 to n = 3 and n = 2 to n = 4			
Q.6	(a) 80 nm	(b) 102 nm		(c) 89 nm			

Excitation of atom

If we provide energy to the electron of atom then there i a possibility that is it is excited to higher energy level.

This process can be done in two ways :

- By Absorption of photons
- By collision with other atoms and electrons

By Absorption of photons

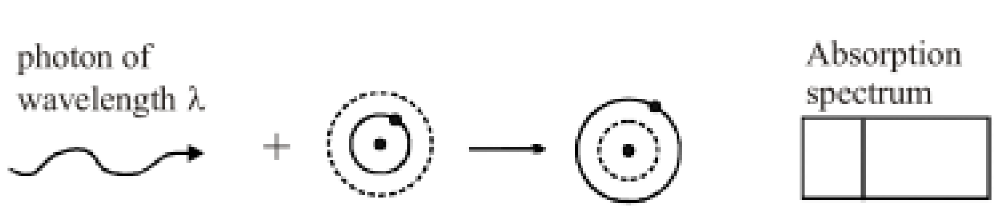
If an electron is to absorb a photon the energy $h\nu$ of photon must be equal to the energy difference ΔE between the initial energy level of the electron and a higher level.

It means if we consider the case of H-atom then this atom can absorb only certain specific energy photons which are 10.2 ev, 12.75 ev etc.

The electron of H-atom can not absorb photon of energy 11 ev but this electron can absorb any photon of energy greater then 13.6 eV or more specifically ionization energy of atom. After absorbing energy more than 13.6 eV, rest of the energy may appear as kinetic energy of the free electron.

Absorption spectrum :

When an atom absorb a photon whose energy is exactly equal to the difference of ground state and any of the excited states, this wavelength corresponds line of absorption spectrum.



Note :

- An atom will absorb energy from its ground state only.
- Wavelengths of absorption spectrum can be determined as

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad n = 2, 3, 4, ...$$

- Number of lines in abortion spectrum between n = 1 and n = n level will be (n = 1)

By collision with other atoms and electrons

The electron of an atom may also absorb energy during collisions, and may be excited to a higher energy state. During collisions of atoms and electrons the loss of energy must be used to excite the atoms as at atomic level then is no significance of Thermal energy and rise of temperature. We can not estimate that what type of collision must occur but we can always analyze the possibilities during a collision.

(i) If loss of KE during collision is not sufficient to excite the atom then collision must be perfectly elastic.

(ii) The collision may be inelastic or perfectly inelastic only if loss of KE is exactly equal to any of the excitation energy of the atom.

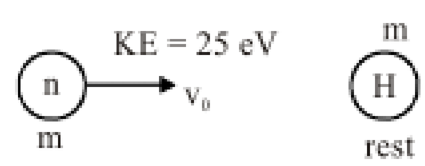
During a collision maximum loss of KE can be calculated using center of frame. i.e. KE with respect to COM can be lost.

$$KE_{system,COM} = \frac{1}{2}\mu V_{rel}$$

Where $\mu \rightarrow$ Reduced Mass of system

$V_{rel} \rightarrow$ relative velocity of the atoms

Consider an H-atom at rest and a neutron with KE = 25 eV is going to collide with the H-atom. Mass of neutron and H-atom can consider as same.



$$KE_{max\ lost} = \frac{1}{2}\mu v_{rel}^2$$

$$= \frac{1}{2}\left[\frac{m.m}{m+m}\right]v_0^2$$

$$= \frac{1}{2}\left[\text{KE of neutron}\right]$$

$$= 12.5 \text{ ev}$$

1st Possibility : Pertectly elastic collision and no excitation

• This is a possible case in every collision if no loss takes place, no excitation to will be there.

2nd Possibility : Inelastic collision, H-atom excited to n = 2 for this $\Delta E = 10.2$ ev is required and it is less than $KE_{max\ loss}$ hence a possible case.

3rd Possibility : In elastic collision, H - atom excited to n = 3 for this $\Delta E = 12.09$ ev $KE_{max\ lass}$ also passible

4th Possibility : In elestic collision, H-atom excited to n = 4 for this $\Delta E = KE_{max\ loss}$ ... .. not Possible

5th Possibility : Pertectly in eleastic collision.

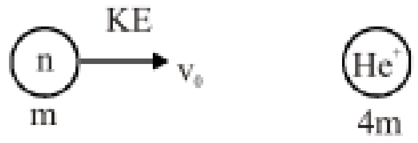
In this case $\Delta E = KE_{max\ loss} = 12.5$ev .... not Possible as 12.5 ev is not an excitation energy.

Note : In all the possibilites, apart from loss rest of the KE will shared by H-atom and newtron. Which can be calculated using momentum - consevation case.

Illustration :

Consider an He^+-atom at rest, a newtron collision with be atom such that He^+ - atom may be excited to 1^{st} excited state. Find min KE of newtron required.

Sol.



$$KE_{max\ loss} = \frac{1}{2}\mu V_{rel}^2$$

$$= \frac{1}{2}\left[\frac{m.m}{m+m}\right]v_0^2$$

$$= \frac{4}{5}[KE\ of\ neutron]$$

for min KE, $KE_{max\ loss} = \Delta E$ *(excitation energy)*

$$\frac{4}{5}(KE\ of\ newtron) = 40.8\ ev$$

$$KE\ of\ newtron = 51\ ev$$

$$\frac{1}{\lambda} = R_\infty\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right)$$

Note : The collision of e^- is slightly different from that between atoms and neutrons because e^- is very tiny particles as compared to those hence it penetrates into the atom and can collide with e^- of atom in ground state. The collision of e^- with the e^- of atom will be perfectly elastic hence it may transfer any fraction of its KE to the e^- of atom.

For example an e^- moving with KE = 12 ev can excite the H-atom to 1st excited state by transferring 10.2 ev KE to the e^- of H-atom.

Similarly An e^- moving with KE = 15 ev may ionize an H-atom.

Pratice Exercise

Q.1 State whether following statements are true/False :

(a) A neutron moving with KE = 20 ev collides with an H-atom at rest. The collision must be perfectly elastic.

(b) A neutron moving with KE = 30 ev collides with an H-atom at rest. The collision may be perfectly in elastic.

(c) An H-atom moving with KE = 25.5 ev collides with another H-atom at rest. The collision may be perfectly inelastic.

Q.2 A neutron having kinetic energy 12.5 eV collides with a hydrogen atom at rest. Neglect the difference in mass between the neutron and the hydrogen atom and assume that the neutron does not leave its line of motion. Find the possible kinetic energies of the neutron after the event.

Q.3 A neutron moving with a speed v strikes a hydrogen atom in ground state moving towards it with the same speed. Find the minimum speed of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron = mass of hydrogen = $1.67 \times^{-27}$ kg.

Answers

Q.1 (a) T (b) T (c) T Q.2 zero Q.3 3.13×10^4 m/s

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de-excitation of atom

Electrons excited to higher energy stay their only for 10^{-8} s then they make transition to any lower state by emitting photons and finally come to a ground state. Energy of photons emitted is equal to the difference of energy of the levels. While coming down they may emit photons of various wavelengths which corresponds to several spectral series.

Emission spectrum :

When an electron in excited state makes a transition to a lower state, a photon is emitted. Collection of these photon wavelengths in called emission spectrum.

Hydrogen atom (or hydrogen like hydrogen atoms) consists of only one electron but we get a number of spectral lines in its spectrum (emission).

1. **Lyman series** : The spectral lines of this series correspond to the transition of an electron from some higher energy state to the inner most orbit (n = 1 i.e. ground state).
For Lyman series, $n_1 = 1, n_2 = 2,3,4,......$

So, $$\frac{1}{\lambda_{\text{Lyman}}} = R\ Z^2\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

for hydrogen atom, Z = 1

$$\frac{1}{\lambda_{\text{Lyman}}} = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

2. **Balmer series** : The spectral lines of this series correspeond to the trasition of an electron from higher energy state to an oribit having n = 2,
For Balmer serise $n_1 = 2, n_2 = 3,4,5,\ ....$
The wave numbers and the wavelengths of spectral lines constituting the Balmer series are given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

3. **Paschen series** : The spectral lines of this series correspand to the transition of an electron from some higher energy state to an orbit having n = 3.
For Paschen series, $n_1 = 3, n_2 = 4,5,6,\ ....$
The wave numbers and the wavelengths of spectral lines constituting the paschen series are given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right)$$

Paschen series is so named because it was discovered by paschen. Just like other series, this eries was first predicted by Bohr.

4. **Barcket series** : The spectral line of this series correspond to the transition of an electron from a higher energy state to the orbit having n = 4.
For this serie, $n_1 = 4, n_2 = 5,6,7,\ ......$

$$\frac{1}{\lambda} = R_\infty\left(\frac{1}{6^2} - \frac{1}{n_2^2}\right)$$

5. **Pfund series** : The spectral line of this series correspond to the transition of an electron from a higher energy state to the orbit having n = 5.

For the series, $n_1 = 5$ and $n_2 = 6,7,8, ...$

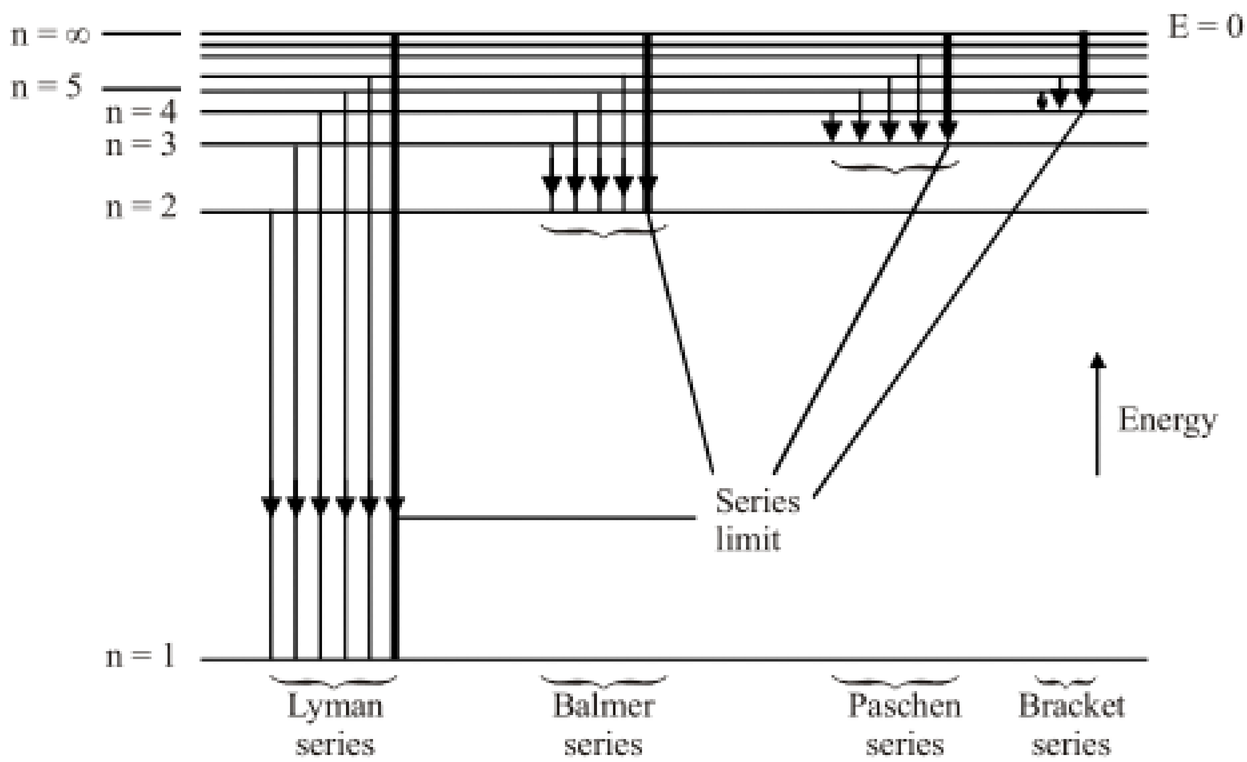
The wave number and the wavelength of the spectral lines constituting the Pfund series are given by.

$$\frac{1}{\lambda} = R_{\infty}\left(\frac{1}{5^2} - \frac{1}{n_2^2}\right)$$

6. **Humphery series** : For this series, $n_1 = 6$ and $n_2 = 7,8,9, ...$

For this series, $n_1 = 6$ and $n_2 = 7,8,9, ...$

The wave number and the wave lengths of the spectral lines constiting the Humphery series are given by,



Spectral lines originate in transitions between energy levels. Shown are the spectral series of hydrogen. When n = ∞, the electron is free.

Note :

- While coming down to ground state a single atom in n = n state may emit a maximum (n – 1) photons.
- First line of a series corresponds to lowest energy photon emitted e.g. first line of blamer series corresponds to transition from n = 3 to n = 2.
- Series limit corresponds to maximum energy photon emitted e.g. series limit of blamer series corresponds to transition from n = ∞ to n = 2.
- Maximum number of lines in emission spectrum of a gas which is excited to a level n = n will be $^nC_2$ i.e. n (n – 1)/2.
- For an atom in quantum state n = n

  Max energy photon will be emitted for a transition from n = n to n = 1.

  Min energy photon will be emitted for a transition from n = n to n = n – 1.

Mass of the nucleus is comparable to mass of electron

If mass of the nucleus is comparable to mass of electron, then the electron and nucleus revolve in coplanar concentric circular paths of radii r_1 and r_2 about their common centre of mass. The electrostatic force of attraction provides them necessary centripetal force. They revolve with the same angular velocity and their sense of rotation is also same.

from the figure,

$$r_1 + r_2 = r_n \qquad \text{...(i)}$$

and $\quad Mvr_1 = mv_2$

or $$\frac{r_1}{r_2} = \frac{m}{M} \qquad \text{...(ii)}$$

from equation (i) and (ii),

$$r_1 = \left(\frac{m}{m+M}\right) r_n \; ; \qquad r_2 = \left(\frac{m}{m+M}\right) r_n$$

Let their angular velocities be ω_1 and ω_2 and electrostatic force of attraction between them be F, then,

$$F = M\omega_1^2 r_1 \qquad \text{(for M)}$$

and $$F = m\omega_2^2 r_2 \qquad \text{(for m)}$$

So, $\quad M\omega_1^2 r_1 = m\omega_2^2 r_2$ but $Mr_1 = mr_2$ [from equation (ii)]

So, $\quad \omega_1^2 = \omega_2^2$ or $\omega_1 = \omega_2 = \omega(\text{say})$

So they revolve with same angular velocity about their common centre of mass.

Now let us see why 'm' is replaced by the reduced mss μ when motion of nucleus is also to be considered. Centripetal force to the electron is provided by the electrastic force, so,

$$mr_2\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n^2} \qquad \text{or} \left(\frac{Mm}{M+m}\right) r_n^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$$

or $$\mu r_n^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0} \qquad \text{...(iii)}$$

where, $$\mu = \frac{1}{m} + \frac{1}{M} = \frac{Mm}{M+m}$$

Now, moment of inertia about the common centre of mass,

$$I = Mr_1^2 + mr_2^2 = M\left(\frac{m}{m+M}\right)^2 Mr_1^2 + mr_2^2 + m\left(\frac{M}{m+M}\right)^2 r_n^2 = \mu r_n^2$$

According to Boh's theory of equation of angular momentum,

$$I\omega = \frac{nh}{2\pi} \qquad \Rightarrow \qquad \mu r_n^2 \omega = \frac{nh}{2\pi} \qquad \text{... (iv)}$$

From equations (iii) and (iv), we get,

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi \mu e^2 Z} \qquad \text{...(v)}$$

Comparing this value with the value of r_n when nucleus was assumed be massive, we see that, 'm' has been replaced by μ. Futher electrical potential energy of the system.

$$U_n = \frac{Ze^2}{4\pi\varepsilon_0 r_n} \text{ and kinetic energy, } K_n = \frac{1}{2} I\omega^2 = \frac{1}{2}\mu r_n^2 \omega^2$$

Explanation of bohr quantisation rule from de-Broglie's Concept

Einstein suggested that light behaves both as a material particle as well as wave. de-Brogle extended Einstein's view and said that all forms of matter like electrons, protons, neutrons etc. also dual character. He futher said that wavelength (λ) associated with a particle of mass 'm' moving with velocity 'v' is given by,

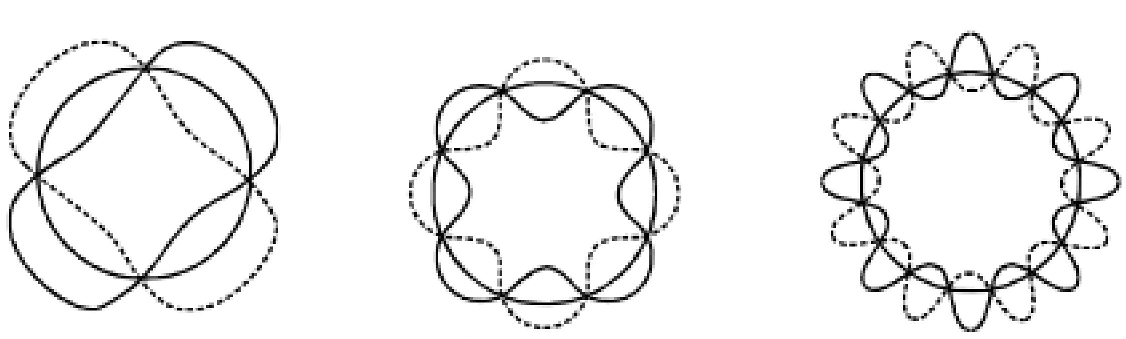
$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad \text{(where } \lambda \text{ is called de-Broglie's wavelength)}$$

Futher : If the K.E. of the moving particle is K, then, $\lambda = h/\sqrt{2mK}$

If a charged particle 'q' is accelerated through a potential difference ΔV then, $\lambda = h\sqrt{2mq(\Delta V)}$

An electron behaves as standing or stationary wave, which extends round the nucleuses in a circular orbit. If the two ends of the electron wave meet to given a regular series of crests and troughs, the electron wave is said to be in phase. i.e. there is constructive interference of electron waves and the electron motion has a character of standing wave or non-energy radiation motion.

Watever be the path of the electron wave round the nucleus, it is a necessary condition to get an electron wave in phase so that the circumference of the Bohr's orbit ($= 2\pi r$) is equal to the whole number multiple to wavelength λ of electron wave i.e.



Circumference = 2 wavelengths | Circumference = 4 wavelengths | Circumference = 8 wavelengths

Figure show some modes of vibration of a wire loop. In each case a whole number of wavelengths fit into the circumference of the loop.

$$2\pi r = n\lambda \text{ or } \lambda = \frac{2\pi r}{n} \qquad \text{...(i)}$$

Where 'n' is a whole number which denotes the number of wavelengths associated with an electron wave extending round the nucleus.

Now according to de - Broglie,

$$\lambda = \frac{h}{mv} \qquad ...(ii)$$

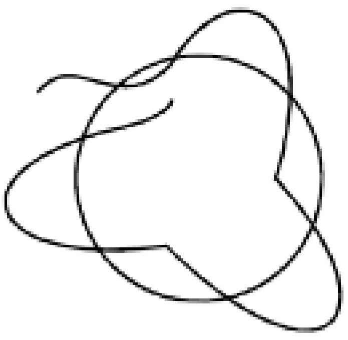


Figure shows a fractional number of wavelengths cannot persist because destructive interference will occur.

From equation (i) and (ii) we get,

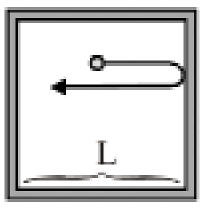
$$\frac{2\pi r}{n} = \frac{h}{mv} \text{ or } mvr = \frac{nh}{2\pi}$$

An electron revolving in a permitted orbit does not radiate energy, through it is accelerating, so the total energy of the electron remains constant. That is why the permitted orbits are also called stationary or non-radiating orbits.

Energy of a trapped electron

The energy of a trapped particle is quantized.

The simplest case is that of a particle that bounces back and forth between the walls of a box. We shall assume that walls of the box are infinitely hard, so the particle does not lose energy each time it strikes a wall and that its velocity is sufficiently small so that we can ignore relativistic conditions.

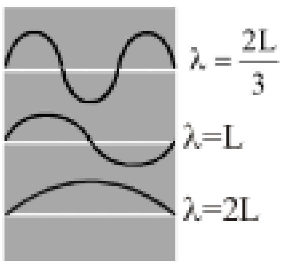


A particle confined to a box of width L.

From a wave points of view, a particle trapped in a box is like a standing wave in a string stretched between the box's walls. The possible de-Broglie wavelengths of the particle in the box therefore are determined by the width L of the box. The general formula for the permitted wavelength is given by.

$$\text{de Broglie wavelengths of trapped particle, } \lambda_n = \frac{2L}{n},\ n = 1,2,3,........$$

Further for a matter wave de-Broglie wavelength,



Wave functions of particle tapped in box of width L

$$\lambda_n = \frac{h}{mv_n} = \frac{h}{\sqrt{2mK_n}} \text{ (where } k_n \text{ is the kinetic energy of the particle)}$$

Comparing the two expressions, we have,

$$\frac{h}{\sqrt{2mk_n}} = \frac{2L}{n} \text{ or } K_n = \frac{n^2h^2}{8mL^2}$$

Since the particle has no potential energy in this model, the only energies it can have, are, Particle in a box, $E_n = \frac{n^2h^2}{8mL^2}$, n = 1,2,3, ...

Each permitted energy is called an energy level and the integer 'n' that specifies an energy level E_n is called its quantum number.

We can draw three general conclusions from the energy expression of a trapped electron. These conclusions apply to any particle confined to a certain region of space for instance, an atomic electron held captive by the attraction of the positively charge nucleus.

1. A trapped particle can not have an arbitrary energy, as free particle can have.
2. A trapped particle can not have zero energy.
3. Because Planck's constant is so small ($h = 6.63 \times 10^{-34}$ J sec), quantization of energy is conspicuous only when 'm' and L are also small. That is why we are not aware of energy quantization in our own experience.

X-RAY

Historical background

On Nov, 1895, Wilhelm Conrad Röntgen (accidentally) discovered an image cast from his cathode ray generator, projected far beyond the possible range of the cathode rays (now known as an electron beam). Further investigation showed that the rays were generated at the point of contact of the cathode ray beam on the interior of the vacuum tube, that they were not deflected by magnetic fields, and they penetrated many kinds of matter. Röntgen named the new form of radiation X-radiation (X standing for "Unknown").

What are X-Rays

X-rays are electromagnetic radiation of very short wavelength (0.1Å and 100Å) and high energy which are emitted when fast moving electrons or cathode rays strike a target of high atomic mass.

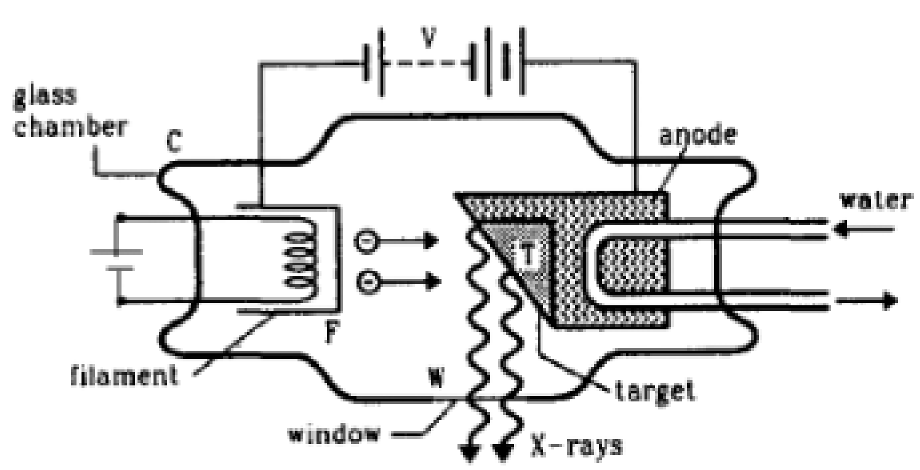
The X-Rays extends from the ultraviolet band to gamma rays band. Although their is very thin line between the X-ray and gamma rays but they are distinguished by the source of their generation. In the older days the X-Rays were distinguished from gamma rays by the wavelength and frequency but with time it has been observed that their exists X-Rays and gamma rays overlapping band and hence it is then understood that the distinction between them is source of generation.

Coolidge tube

William Coolidge invented the X-ray tube popularly called the Coolidge tube. His invention revolutionized the generation of X-rays and is the model upon which all X-ray tubes for medical applications are based.

The characteristic features of the Coolidge tube are its high vacuum and its use of a heated filament as the source of electrons. There is so little gas inside the tube that it is not involved in the production of x-rays, unlike the situation with cathode gas discharge tubes.

The operation of the Coolidge tube is as follows. Due to filament current the cathode filament is heated, it emits electrons. The hotter the filament gets, the greater the emission of electrons. A constant potential difference of several kilovolts is maintained between the filament and the target using a DC power supply so that the target is at a higher potential than the filament. These electrons are accelerated towards the anode with a very high speed and when the electrons strike the anode and emit x-rays. These X-rays are brought out of the tube through a window W made of thin mica or mylar or some such material which does not absorb X-rays appreciably. In the process, large amount of heat is developed, and thus an arrangement is provided to cool down the tube continuously by running water.



Note

(i) An increase in the filament current increase the number of electrons it emits. Larger number of electrons means an intense beam of X-rays is produced. This way we can control the quantity of X-rays i.e. Intensity of X-rays.

(ii) An increase in the voltage of the tube increase the kinetic energy of electrons ($eV = 1\ 2mv^2$). When such highly energetic beam of electrons are suddenly stooped by the target, an energetic beam of X-rays is produced. This way we can control the quality of X-rays i.e. penetration power of X-rays.

(iii) Based on penetrating power, X-rays are classified into two types. HARD-rays and SOFT-x-rays. The first one having high energy and hence high penetration power are HARD-X-rays and another one with low energy and hence low penetration power are SOFT-X-rays.

Continuous X-rays

When electron strikes target an electron loses a part of its kinetic energy and continues to move with the remaining energy until it hits another atom of the target. Part or whole of the energy lost by the electron is converted into a photon. This process is known as bremsstrahlung (breaking radiation)- as it leads to the electron getting decelerated by the target. There is existence of a minimum wavelength (or maximum frequency) in x-ray spectrum. This is called the cutoff wavelength or the threshold wavelength.

If the potential difference between the filament and the target is V, then the kinetic energy of the electron just before it hits the target is

$$K.E. = eV$$

Energy of the X-ray photon, $\Delta E = \frac{hc}{\lambda}$

ΔE is the fraction of K.E. of electron that gets converted into photon. l is the wavelength of the photon. Wavelength of the X-ray's photon

$$\lambda = \frac{hc}{\Delta E}$$

as

$$\Delta E \leq eV \qquad \Rightarrow \qquad \lambda \geq \frac{hc}{eV}$$

$$\Rightarrow \qquad \lambda_{min} = \frac{hc}{eV}$$

This minimum wavelength does not depend on the target material and depend on the potential difference between the filament and the target.

Characteristic x-ray

When the high energy lectrons "knock off" the innermost electrons of the atoms of the target material causing a vacancy. This vacancy is filled by a electron that 'jump' from one of the outer shells. If the vacancy is created in K shell and electron from the L shell fills the vacancy then the emitted photon is a K_α X-ray. If a vacancy is creasted in the K-shell and an electron from the M shell fills this vacancy then the x-ray emitted is known as a K_β X-ray.

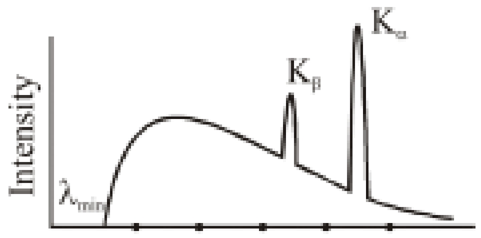
$$\lambda_{K_\alpha} = \frac{hc}{E_L - E_K}$$

$$\lambda_{K_\beta} = \frac{hc}{E_M - E_K}$$

Where E_K, E_L, E_M are the electron energy levels

These X-ray are known as characteristic X-ray as they depend on the target material (character of material), not on the applied voltage.

The adjoining graph shows the variation of the intensity of X-ray coming out of the tube with wavelengths. At some sharply defined wavelengths (K_α K_β) the intensity of the emitted radiation is very large.



Moseley's Law

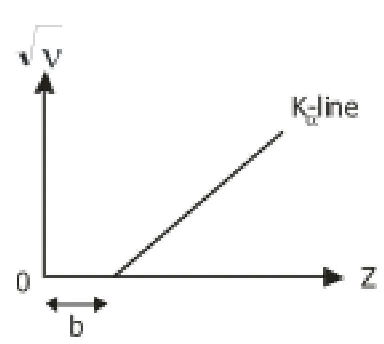
By measuring the wavelength associated with a particular line (now designated K_α), from the spectrum of each element, Moseley established that the lines from a large number of elements obeyed the relation

$$\sqrt{v} = a(Z - b)$$

Where a and b are constants with

$$a \approx \sqrt{\frac{3Rc}{4}} \quad (R = \text{Rydberg constant})$$

and $\quad b \approx 1$. (b is known as the screening constant)



Approximate explanation from Bohr's theory

Consider an atom from which an electron from the K shell been knocked out. Consider an electron from the L shell which is about to make a transition to the vacant site. If finds the nucleus of charge Ze screened by the spherical cloud of the remaining one electron in the K shell. If we neglect the effect of the outer electrons and the other L electrons, the electron making the transition finds a charge $(Z - 1)$ e at the centre. One, therefore, may expect Bohr's model to given reasonable results if Z is replaced by $Z - b$

$$\Delta E = h\nu = Rhc\,(Z - b)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\Rightarrow \quad \sqrt{\nu} = \sqrt{\frac{3Rc}{4}}\,(Z - b)$$

Properties of X-Ray

1. These are highly penetrating rays and can pass through several materials which are opaque to ordinary light.
2. They ionize the gas through which they pass. While passing through a gas, they knock out electrons from several of the neutral atoms, leaving these atoms with +ve charge.
3. They cause fluorescence in several materials. A plate coated with barium platinocyanide, ZnS (zinc sulphide) etc becomes luminous when exposed to X-ray.
4. They affect photographic plates especially designed for the purpose.
5. They are not deflected by electric and magnetic fields, showing that they not charge particles.

Illustration :

To decrease the cut-off wavelength of contimuous X-ray by 25%, find the % change potential difference across the X-ray tube

Sol. $\lambda = \frac{hc}{eV}, \frac{\lambda_1}{\lambda_2} = \frac{V_2}{V_1}, \lambda_1 = \frac{3}{4}\lambda_1$

$V_2 = \frac{4}{3}V_1, \frac{100}{3}$ *% increases.*

Illustration :

The wavelength of K_a X-rays produced by an X-ray tube is 0.76 Å. Find the atomic number of anticathode materials.

Sol. *For K_a X-ray line.*

$$\frac{1}{\lambda_a} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R(Z-1)^2\left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \frac{1}{\lambda_a} = \frac{3}{4}R(Z-1)^2 \qquad ...(1)$$

With reference to given data,

$l_a = 0.76\ Å = 0.76 \times 10^{-10} m;\ R = 1.097 \times 10^7\ m$

Putting these values in equation (1)

$$(Z-1)^2 = \frac{4}{3} \times \frac{1}{0.76 \times 10^{-10} \times 1.097 \times 10^7} \cong 1600$$

$\Rightarrow Z - 1 = 40 \Rightarrow Z = 41$

Pratice Exercise

Q.1 If the operating potential in an X-ray tube is increased by 1%, by what percentage does the cutoff wavelength decrease?

Q.2 When 40 kV is applied across an X-ray tube, X-ray is obtained with a maximum frequency of 9.7×10^{18} Hz. Calculate the value of Planck constant from these data.

Q.3 The K_β X-Ray of argon has a wavelength of 0.36 nm. The minimum energy needed to ionize an argon atom is 16 eV. Find the energy needed to knock out an electron from the K shell of an argon atom.

Q.4 A free atom of iron emits K_α X-rays of energy 6.4 keV. Calculate the recoil kinetic energy of the atom. Mass of an iron atom $= 9.3 \times 10^{-26}$ kg.

Answers

Q.1 approximately 1% Q.2 4.12×10^{-15} eV–s Q.3 3.47 keV
Q.4 3.9×10^{-4} eV

Nuclear Physics

It exists at the centre of an atom, containing entire positive charge and almost whole of mass. The electron revolve around the nucleus to form an atom. The nucleus consists of protons (+ve charge) and neutrons. A proton has positive charge equal in magnitude to that of an electron ($+1.6 \times 10^{-19}$ C) and a mass equal to 1840 C) and a mass equal to 1840 times that of an electron. A neutron has no charge and mass is approximately equal to that of proton.

Properties of a nucleus

(1) Nuclear Mass :

As we know that every nucleus contains protons and neutrons and so every nucleus has a definite mass.However, since the mass of electron is negligible so atomic mass is roughly equal to nuclear mass.

Atomic masses are measured in atomic mass unit (a.m.u.) defined as

$$1 \text{ amu} = 1.6604 \times 10^{-27} \text{ kg}$$

$$\Rightarrow \quad 1\text{u} = 931.478 \text{ MeV}/c^2$$

and its energy equivalent is 931.48 MeV

The number of protons in a nucleus of an atom is called as the atomic number (Z) of that atom. The number of protons plus neutrons (called as Nucleus) in a nucleus of an atom is called as mass number (A) of that atom.

A particular set of nucleons forming an atom is called as nuclide. It is represented as $_ZX^A$. The nuclides having same number of protons (Z), but different number of nucleons (A) are called as isotopes. The nuclide having same number of nucleons (A), but different number of protons (Z) are called as isobars. The nuclide having same number of neutrons (A–Z) are called as isotones.

(2) Nuclear charge

Since nucleus contain +vely charged protons (charge = 1.6×10^{-19} C) and neutrons (neutral) so every nucleus has a net +ve charge.

(3) Nuclear radius

A rough estimate of nuclear size suggests us that the radius of the nucleus of an atom having mass number 'A' is given by

$$R = R_0 A^{1/3}$$

Where R_0 is a constant found to be equal to

$$R_0 = 1.4 \times 10^{-15} \text{ m} = 1.4 \text{ fm}.$$

(4) Nuclear Density

In spite of the fact that nuclear radius depends on mass number of the atom but nuclear density is independent of mass number because if neutrons are supposed to be of almost the same mass as that of protons then the total mass of a nucleus is proportional to A. If each nucleon are supposed to have a mass m then nuclear density is given by

$$\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3} \qquad \textit{(Which is independent of A)}$$

(5) Nuclear spin and magnetic moment

Like orbital electrons in an atom, nucleons inside nucleus have well defined quantum states. Correspondingly they have angular momentum and hence a magnetic moment. Like electrons nucleons also have intrinsic angular momentum and 'magnetic' moment corresponding to their spin.

Nuclear Forces

If only the electrostatic and gravitational forces existed in the nucleus, then it would be impossible to have stable nuclei composed of protons and neutrons. The gravitational forces are much too small to hold the nucleons together compared to the electrostatic forces repelling the protons. Since stable atoms of neutrons and protons do exist, there must be another attractive force acting within the nucleus. This force is called the nuclear force.

Properties of nuclear force

(1) They are charge independent.The nuclear force between two proton is same as that between two neutrons or between a neutron and proton. This is known as charge independent character of nuclear forces.

(2) They may be repulsive may be attractive (Repulsive at exceedingly small separation between two nucleons appreciably smaller than 10^{-13} cm i.e. 10^{-15} m

(3) It is a short range force. Its radius of action is of the order of 10^{-13} cm.

(4) The nuclear force is of saturation character. Each nucleon in nucleus interacts with a limited number of nucleons.

(5) Nuclear force are much stronger than electromagnetic force or gravitational attractive forces.It is the strongest of all the forces. This is why it is called strong interaction.

(6) Nuclear force is spin dependent. If two interacting nucleons are having parallel spins then nuclear force operative between them is comparatively stronger and if their spins are antiparallel, nuclear interaction is comparatively weaker.

(7) Nuclear force is a non-central force. They can not be represented as directed along the st. line connecting the centres of the interacting nucleons. Its non central nature is due to the fact that it depends also on the orientation of the nucleon spins.

Mass defect

It is observed that the mass of a nucleus is slightly less than the sum of the masses of constituent nucleons. Suppose a nucleus consists of 'Z' protons and 'N' neutrons. Mass of a proton, a neutron and the resulting nucleus are respectively m_p, m_n and M then mass defect of the nucleus is given by

$$\Delta m = Zm_p + Nm_n - M$$

If A is the mass number of the nucleus

$$\Delta m = [Zm_p + (A - Z)m_n - M]$$

In terms of atomic masses we may also write mass defect as

$$\Delta m = [\ Zm({}^1_1H) + Nm_n - m({}^A_zX)]$$

Where $m({}^1_1H) =$ mass of one hydrogen atom.

$m({}^A_zX) =$ mass of atom having atomic no. Z and mass no. A

e.g.,

mass of ${}^1_1H$ $= 1.00784$ u

mass of neutron = 1.00 874 u

Expected mass of deuterium = 2.01654 u

but measured mass = 2.0141 u.

mass defect,

$$\Delta m = 0.00244 \text{ u.}$$

Nuclear Stability

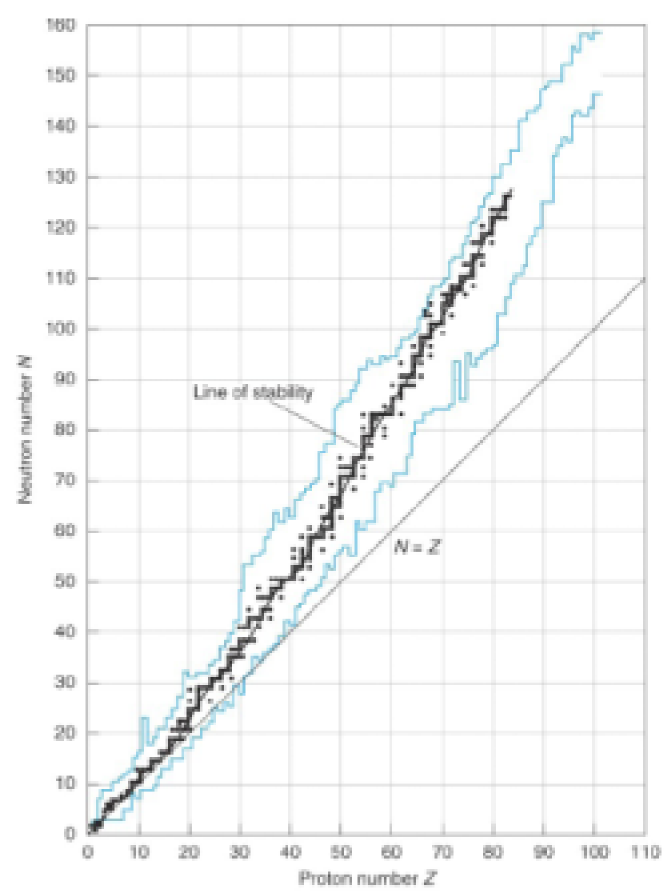


Figure shows pot of N vs. Z for known nuclides. The stable nuclides are indicated by the black dots. Non-stable nuclides decay by emission of particles, or electromagnetic radiation, in a process called radioactivity

Binding energy

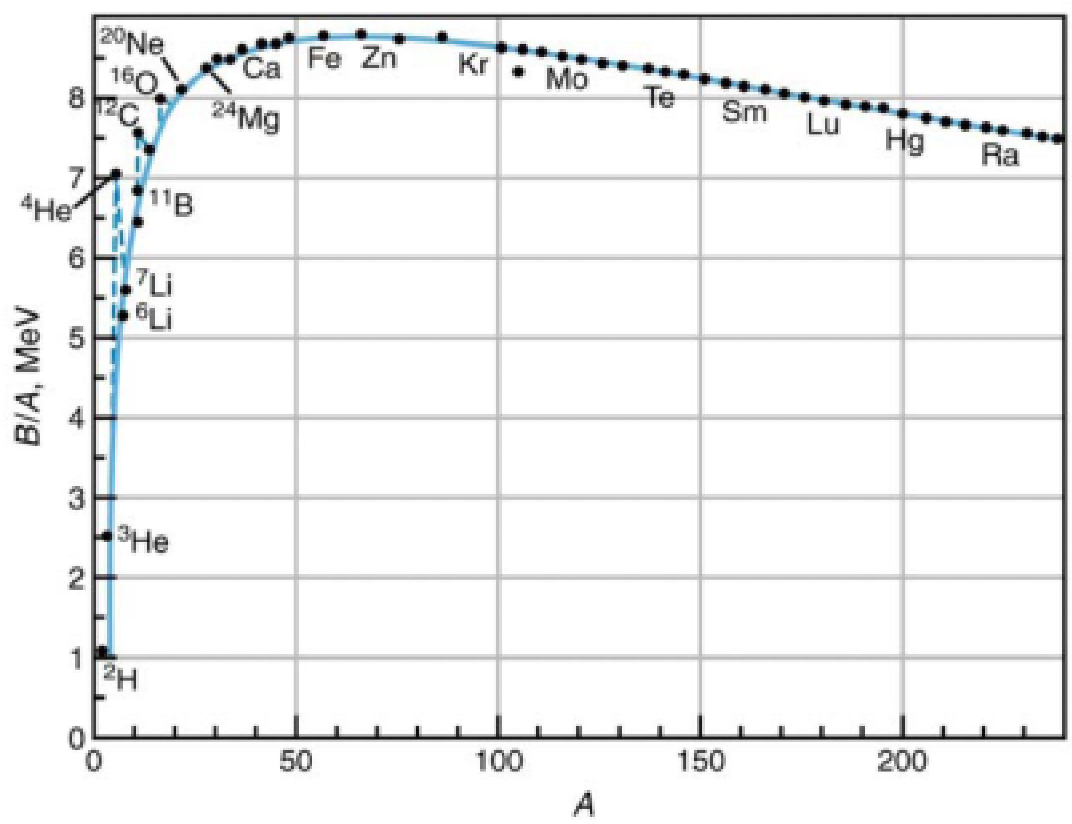
To break a nucleus into its constituent nuclei some energy is required to be supplied. This energy is called Binding Energy of the given nucleus or the energy equivalent of the missing mass of a nucleus is called the binding energy of the nucleus.

$BE = (\Delta m)c^2 = [Zm_p + (A-Z)m_n - M]c^2$,

$BE = \Delta m(\text{in amu}) \times 931$ MeV Where Δm = mass defect

Binding energy per nucleon is a measure of the stability of the nucleus. If there be n nucleons which is equal to A,

$$\frac{\text{Binding Energy}}{\text{Nucleon}} = \frac{\text{B.E.}}{\text{A}}$$



From the plot of B.E. / nucleons Vs mass number (A), we observe that :

(1) Binding energy per nucleon has low value for both heavy and light nuclei i.e. Heavy as well as light nuclei, both are unstable.B.E. / nucleons increases on an average and reaches a maximum of about 8.7 MeV for $A \equiv 50 - 80$. For more heavy nuclei, B.E. / nucleons decreases slowly as A increases. For the heaviest natural element U^{238} it drops to about 7.5 MeV. From above observation, if follows that nuclei in the region of atomic masses 50 – 80 are most stable.

(2) The intermediate nuclei have large value of binding energy per nucleon so they are more stable.

(3) Binding energy per nucleon increases rapidly upto mass number 20 but there are peaks corresponding to ${}^{4}_{2}He$, ${}^{12}_{6}C$, ${}^{16}_{8}O$ which indicates that these nuclei are more stable than neighbours. The reason is that they may be considered to posses magic numbers i.e. their mass number is divisible by 4 and these nuclei may have ${}^{4}_{2}He$ as their constituents.

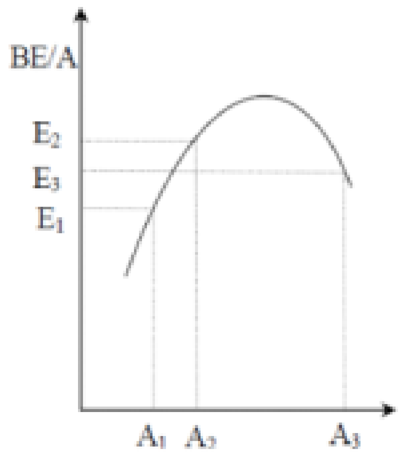
(4) The minimum value of the BE/Nucleon is in the case of deuteron that is 1.11 Mev.

(5) The maximum value of the BE/Nucleon is 8.79 Mev for the nuclide ${}^{56}_{26}Fe$ which is therefore the most stable nucleus.

Illustration :

Using the following plot of BE/nucleon vs mass number, mention the condition for which the energy is absorbed or released for the reaction

$$ {}_{Z_1}P^{A_1} + {}_{Z_2}Q^{A_2} \rightarrow {}_{Z_3}P^{A_3} $$



Sol. *Binding energy for reactant is* $(xE_a + yE_b)$ *and that for product is* zE_c

Case-I :

if $(A_1E_1 + A_2E_2) > A_3E_3$

Energy is absorbed.

Case-II :

if $(A_1E_1 + A_2E_2) < A_3E_3$

Energy is released.

Note

(i) If we split a heavy nucleus into two medium sized nuclei and total binding energy of new nuclei is greater then parent nuclei, then energy is released (Nuclear fission)

(ii) If two nuclei of small mass number combine to form a single medium size nucleus for which binding energy is greater than the constituent nuclei, then energy is released (Nuclear fusion)

Nuclear Fission

The breaking of a heavy nucleus into two or more fragments of comparable mass, with the release of tremendous energy is called as nuclear fission.

The most typical fission reaction occurs when slow moving neutrons strike ${}_{92}U^{235}$. The following nuclear reaction takes place.

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + 200\text{ MeV} $$

If more than one of the neutrons produced in the above fission reaction are capable of inducing a fission reaction (provided U^{235} is available), then the number of fission taking place at successive stages goes increasing at a very brisk rate and this generates a series of fission. This is known as chain reaction. The chain reaction takes place only if the size of the fissionable material (U^{235}) is greater than a certain size called the critical size.

If the number of fission in a given interval of time goes on increasing continuously, then a condition of explosion is created. In such cases, the chain reaction is known as uncontrolled chain reaction. This forms the basis of atomic bomb.

In a chain reaction, the fast moving neutrons are absorbed by certain substances known as moderators (like heavy water), then the number of fissions can be controlled and the chain reaction is such cases is known as controlled chain reaction. This forms the basis of a nuclear reactor.

Nuclear Fusion

The process in which two or more light nuclei are combined into a single nucleus with the release of tremendous amount of energy is called as nuclear fusion. Like a fission reaction, the sum of masses before the fusion (i.e. of bigger nucleus) and this difference appears as the fusion energy. The most typical fusion reaction is the fusion of two deuterium nuclei into helium.

$$_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{4} + 21.6 \text{ MeV}$$

For the fusion reaction to occur, the light nuclei are brought closer to each other (with a distance of 10^{-14} m). This is possible only at very high temperature to counter the repulsive force between nuclei. Due to this reason, the fusion reaction is very difficult to perform. The inner core of sun is at very high temperature, and is suitable for fusion. In fact the source of sun's and other star's energy is the nuclear fusion reaction.

Conservation laws in nuclear reaction

Nuclear reaction processes have led to the formulation of useful conservation principles. The four principles of most interest in this module are discussed below.

(i) Conservation of electric charge implies that charges are neither created nor destroyed. Single positive and negative charges may, however, neutralize each other. It is also possible for a neutral particle to produce one charge of each sign.

(ii) Conservation of mass number does not allow a net change in the number of nucleons i.e. total number of protons and neutrons should also remain same on both sides of a nuclear reaction.. However, the conversion of a proton to a neutron and vice versa is allowed.

(iii) Conservation of mass and energy implies that the total of the kinetic energy and the energy equivalent of the mass in a system must be conserved in all decays and reactions. Mass can be converted to energy and energy can be converted to mass, but the sum of mass and energy must be constant. In nuclear reactions, sum of masses before reaction is greater than the sum of masses after the reaction. The difference in masses appears in form of energy following the Law of inter-conversion of mass & energy. The energy released in a nuclear reaction is called as Q Value of a reaction and is given as follows.

If difference in mass before and after the reaction is Δm amu (Δm = mass of reactants minus mass of products) then

$$Q \text{ value} = \Delta m\,(931) \text{ MeV}$$

(iv) Conservation of momentum is responsible for the distribution of the available kinetic energy among product nuclei, particles, and/or radiation. The total amount is the same before and after the reaction even though it may be distributed differently among entirely different nuclides and/or particles.

Illustration :

In the sun about 4 billion kg of matter is converted to energy each second. Find the power output of the sun in watt .

Sol.

$$\frac{m}{t} = 4\times10^{8}\ kgs^{-1}$$

$$E = mc^2$$

$$\Rightarrow \quad \frac{E}{t} = \left(\frac{m}{t}\right)c^2$$

$$\Rightarrow \quad \frac{E}{t} = 4\times10^{8}\times9\times10^{16}$$

$$\Rightarrow \quad \frac{E}{t} = 3.6\times10^{25}\ Js^{-1}$$

$$\Rightarrow \quad \frac{E}{t} = 3.6\times10^{25}\ W$$

Illustration :

A neutron breaks into a proton and electron. Calculate the energy produced in this reaction in MeV. Mass of an electron = 9×10^{-31} kg, Mass of Proton = 1.6725×10^{-27} kg, Mass of neutron = 1.6747×10^{-27} kg. Speed of light = 3×10^{8} m/sec.

Sol. $\quad {}_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0$

Δm= [Mass of neutron – (mass of proton + mass of electron)]

$$= [1.6747 \times 10^{-27} - (1.6725 \times 10^{-27} + 9 \times 10^{-31})]$$

$$= 0.0013 \times 10^{-27}\ kg$$

$\therefore$ Energy released

$$E = \Delta mc^2 = (0.0013 \times 10^{-27}) \times (3 \times 10^{8})^2 = 1.17 \times 10^{-13}\ joule$$

$$= \frac{1.17\times10^{-13}}{1.6\times10^{-19}}\ eV = 0.73 \times 10^{6}\ eV = 0.73\ MeV$$

Illustration :

The nuclei involved in the nuclear reaction $A_1 + A_2 \rightarrow A_3 + A_4$ have the binding energies E_1, E_2, E_3 and E_4. Find the energy released (Q value) of this reaction.

Sol. *Suppose M_1, M_2, M_3, M_4 are the rest masses of the nuclei A_1, A_2, A_3 and A_4 perticipating in the reaction*

$$A_1 + A_2 \longrightarrow A_3 + A_4 + Q$$

Here Q is the energy released. Then by conservation of energy.

$$Q = (M_1 + M_2 - M_3 - M_4)c^2$$

Now $\quad M_1c^2 = c^2\,(Z_1m_H + (A_1 - Z)\,m_n) - E_1$ etc. and

$Z_1 + Z_2 = Z_3 + Z_4$ (conservation of charge)

$A_1 + A_2 = A_3 + A_4$ (conservation of mass number)

Here $\quad Q = (E_3 + E_4) - (E_1 + E_2)$

Pratice Exercise

Q.1 Calculate the electric potential energy due to the electric repulsion between two nuclei of ^{12}C when they 'touch' each other at the surface.

Q.2 Find the binding energy of $_{26}Fe^{56}$. Atomic mass of $_{26}Fe^{56}$ is 55·9349 u and that of ^{1}H is 1·00783 u. Mass of neutron is 1·00867u.

Q.3 Calculate the Q-value in the following decay

$$^{25}Al \rightarrow {}^{25}Mg + e^{+} + \nu$$

Q.4 Find the maximum energy that a beta particle can have in the following decay

$$^{176}Lu \rightarrow {}^{176}Hf + e + \overline{\nu}$$

Atomic mass of ^{176}Lu is 175.942694 u and that of ^{176}Hf is 175.941420 u

Answers

Q.1 10.2 MeV. Q.2 492 MeV. Q.3 3.254 MeV Q.4 0.2806 MeV

Radioactivity

Among about 2500 known nuclides, fewer than 300 are stable. The others are unstable structures that decay to form other nuclides by spontaneously emitting particles and electromagnetic radiation, a process called radioactivity. The time scale of these decay processes ranges from a small fraction of a microsecond to billions of years. The substances which emit these radiations are called as radioactive substances. It was discovered by Henry Becquerel for atoms of Uranium. Later it was discovered that many naturally occurring compounds of heavy elements like radium, thorium etc also emit radiations.

At present, it is known that all the naturally occurring elements having atomic number greater than 82 are radioactive. For example some of them are ; radium, polonium, thorium, actinium, uranium, radon etc. Later on Rutherford found that emission of radiation always accompanied by transformation of one element (transmutation) into another. In actual radioactivity is the result of disintegration of an unable nucleus. Rutherford studied the nature of these radiations and found that these mainly consist of α, β, γ particles (rays).

α-Particles : ($_2He^4$)

These carry a charge of +2e and mass equal to $4m_p$. These are nuclei of helium atoms. The energies of α-particles very from 5 MeV to 9 MeV ; their velocities vary from 0.01 – 0.1 times of c (velocity of light). They can be deflected by electric and magnetic field and have lower penetrating power but high ionising power.

β-Particles : ($_{-1}e^0$)

These are fast moving electrons having charge equal to e and mass $m_e = 9.1 \times 10^{-31}$ kg. Their velocities very from 1% to 99% of the velocity of light (c). They can also be deflected by electric and magnetic fields. They have low ionising power but high penetrating power.

γ-Radiations : ($_0\gamma^0$)

These are electro-magnetic waves of nuclear origin and of very short wavelength. They have no mass. They have maximum penetrating power and minimum ionising power. The energy released in a nuclear reaction is mainly emitted in from these γ-radiations.

Radioactive decays

α-decay

Nuclides decay is by emitting α-particles. α-particles are generally emitted by very heavy nuclei containing to many nucleons to remain stable. The emission of such a nucleon cluster as a whole rather than the emission of single nucleon is energetically more advantageous because of the particularly high binding energy of alpha-particles. The parent nucleus (Z,A) is transformed as

$$_zX^A \longrightarrow {}_2He^4 + {}_{z-z}Y^{A-4}$$

$$\Delta m = \left\{M\left[{}_2He^4\right] + M\left[{}_{z-2}Y^{A-4}\right]\right\} - \left\{M\left[{}_zX^A\right]\right\}$$

Note

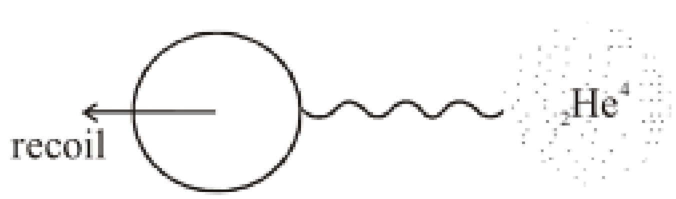
(i) Nuclear mass is different from atomic mass because nucleus is without electrons.

(ii) Released energy converts into kinetic energy

(iii) In nucleus, Atomic energy is 13.6 eV small atomic binding energy has been neglected.

(iv) Released energy is shared as kinetic energy by products and outgoing particles.

Calculation of Kinetic Energy



Momentum of α_{particle} + momentum of daughter nuclei = 0

$(m_\alpha \vec{v})$ $\qquad\qquad$ $(\vec{p}_D)$

assuming parent nuclei to be at rest initially

$$\vec{p}_\alpha + \vec{p}_D = 0$$

$$|\vec{p}_\alpha| = |\vec{p}_D|$$

If Q is released energy or Q value of reaction.

$$K_\alpha + K_D = Q$$

$$\Rightarrow \qquad K_\alpha + \frac{p_D^{\ 2}}{2m_D} = Q$$

$$\Rightarrow \qquad K_\alpha + \frac{p_\alpha^{\ 2}}{2m_D} = Q$$

but

$$\Rightarrow \qquad K_\alpha + \frac{2K_\alpha \cdot m_\alpha}{2m_D} = Q$$

$$\Rightarrow \qquad K_\alpha \left[1 + \frac{m_\alpha}{m_D}\right] = Q$$

$$K_\alpha = \frac{m_D \times Q}{m_D + m_\alpha}$$

$$K_\alpha = \frac{(A-4)m}{4m+(A-4)m} Q$$

$$\Rightarrow \qquad K_\alpha = \left[\frac{A-4}{A}\right] Q$$

β decay

Another way in which nuclides decay radioactively is by the emission of β particles. .When neutron-proton ratio inside a nucleus is not suitable for it to be stable (either less or more) then β-decay takes place. Due to a special type of interaction called weak interaction a neutron gets converted into a proton and a proton gets converted into a neutron and a positron. Electrons or positrons are emitted from the nucleus just after their creation. This emission of electron or positron from nucleus is called β-decay. Emission of positron (of the order of MeV) is called β^+-decay and emission of electron (of the order of MeV) is called β^--decay.

(i) Negative β decay(β^- decay)

Neutron inside nucleus is transformed into proton .

$$n \longrightarrow p + e^- + \bar{\nu} \ \text{(Antineutrino)}$$

$${}_zX^A \longrightarrow {}_{z+1}Y^A + e + \bar{\nu} + \text{energy released.}$$

Equation corresponding to nuclear mass

$$\Delta m = M\left[{}_zX^A\right] - \left\{M\left[{}_{z+1}Y^A\right] + Me\right\}$$

Equation corresponding to atomic mass

$$\Delta m = M^*\left[{}_zX^A\right] - M^*\left[{}_{z+1}Y^A\right]$$

energy released

$$E = \Delta mc^2$$

(ii) Positive β decay(β⁺ decay)

Proton inside nucleus is transformed into neutron .

$$p \longrightarrow n + e^{+} \text{ (Positron)} + \nu \text{ (neutrino)}$$

Positron is anti-particle of electron. It is highly reactive.

$$_{z}X^{A} \longrightarrow {}_{z-1}Y^{A} + e^{+} + \nu \text{ +energy released.}$$

Equation corresponding to nuclear mass

$$\Delta m = M\left[_{z}A^{x}\right] - \left\{M\left[_{z-1}Y^{A}\right] + Me\right\}$$

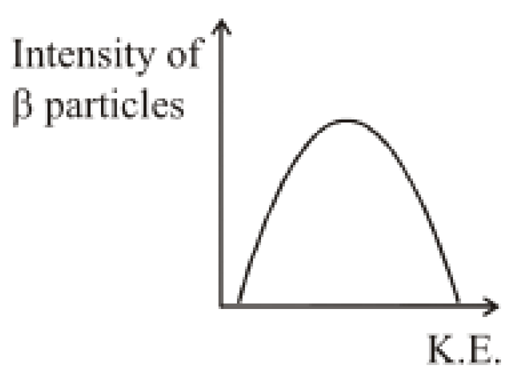
Equation corresponding to atomic mass

$$\Delta m = M^{*}\left[_{z}A^{x}\right] - \left\{M^{*}{}_{z-1}Y^{A} - 2Me\right\}$$

energy released

$$E = \Delta mc^{2}$$

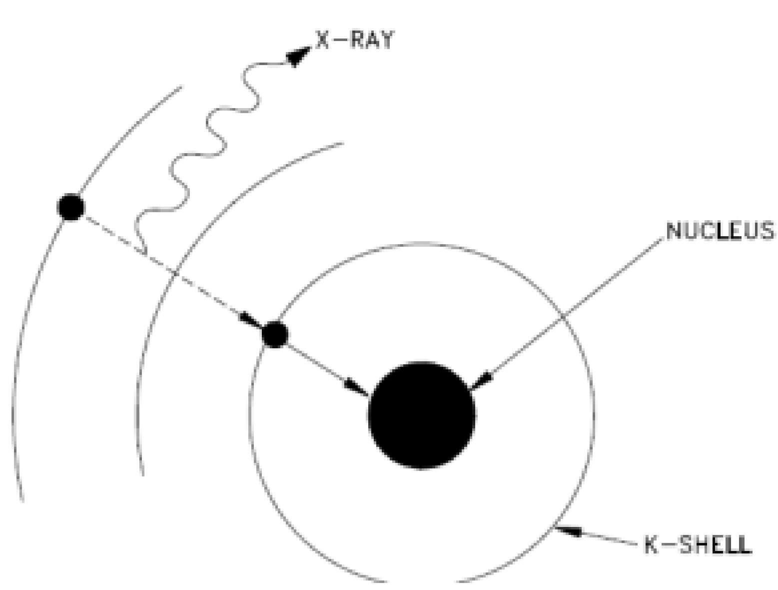
Experiments show that β-particles are emitted with continuous range of kinetic energy.



(iii) Electron capture

Nuclei having an excess of protons may capture an electron from one of the inner orbits which immediately combines with a proton in the nucleus to form a neutron. This process is called electron capture (EC). The electron is normally captured from the innermost orbit (the K-shell), and, consequently, this process is sometimes called K-capture.

The process is observed from the emission of the characteristic X-rays produced, when an orbiting electron from an outer shell makes a downward transition into a K shell vacancy. The X-rays are characteristic of daughter nuclei not of the parent because x-ray emission taken place 'K-capture'



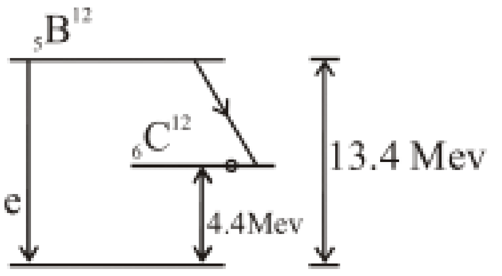
$$_{-1}e^{0} + {}_{4}Be^{7} \longrightarrow {}_{3}Li^{7} + \nu \qquad \text{(A simple K-capture equation)}$$

Neutrino and anti-neutrino

1. It has zero electric charge, hence shows no electromagnetic interaction.
2. Rest mass is possibly zero. Recent experiments show that mass of neutrino is less than $\left(\frac{7}{c^2}\text{ev}\right)$.
3. It travles with speed of light.
4. It has spin quantum number $1/2$. A spin of $1/2$ satisfies the law of conservation of angular momentum when applied to β-decay.
5. It shows very weak interactions with matter.

γ - Decay

As we know that like the discrete energy orbits of electrons in an atom. Nucleons in an atom inside the nucleus also have well defined energy state or discrete quantum state. After every α or β emission a nucleus is in the excited state correspondingly subsequent to every α-or β-emission a nucleus emits electromagnetic radiation (of the order of MeV) to come to ground state. The frequency or the wavelength of the emitted radiations lie in γ-region and is called γ-emission.



Group-Displacement Law

(i) When a nuclide emits one α-particle (${}_2He^4$), its mass number (A) decreases by 4 units and atomic number (Z) decreases by two units.

$${}_zX^A \longrightarrow {}_{Z2}Y^{A-4} + {}_2He^4 + \text{Energy}$$

(ii) When a nuclide emits a β^-particle, its mass number remains unchanged but atomic number increases by one unit.

$${}_zX^A \longrightarrow {}_{z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + \text{Energy} \qquad (\bar{\nu}\text{ is antineutrino})$$

(iii) When a nuclide emits a β^+ particle, its mass number remains unchanged but atomic number decreases by one unit.

$${}_zX^A \longrightarrow {}_{z-1}Y^A + {}_{+1}e^0 + \nu + \text{Energy} \qquad (\nu\text{ is neutrino})$$

(iv) When a γ particle is produced, both atomic and mass number remain constant.

Rutherford-Soddy Law (Statistical Law)

The disintegration of a radioactive substance is random and spontaneous.

Radioactive decay is purely a nuclear phenomenon and is independent of any physical and chemical conditions.

The radioactive decay follows first order kinetics, i.e., the rate of decay is proportional to then number of undecayed atoms in a radioactive substance at any time t. If dN be the number of atoms (nuclei) disintegrating in time dt, the rate of decay is given as dN/dt. From first order kinetic rate law :

$$\frac{dN}{dt} = -\lambda N$$

where λ is called as decay or disintegration constant.

Let N_0 be the number of nuclei at time $t = 0$ and N_t be the number of nuclei after time t, then according to integrated first order rate law, we have :

$$N_t = N_0\, e^{-\lambda t}$$

$$\Rightarrow \qquad \lambda t = ln\frac{N_0}{N_t} = 2.303 \log\frac{N_0}{N_t}$$

The half life ($t_{1/2}$) period of a radioacitve substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 be the number of nuclei at $t = 0$, then in ha half life T, the number of nuclei decayed will be $N_0/2$

$$N_t = N_0 e^{-\lambda t} \qquad \text{... (i)}$$

$$\Rightarrow \qquad \frac{N_0}{2} = N_0 e^{-\lambda T} \qquad \text{... (ii)}$$

from (i) & (ii)

$$\frac{N_t}{N_0} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{n} \qquad \text{n : number of half lives}$$

The half life (T) and decay constant (λ) are related as :

$$T = \frac{0.693}{\lambda}$$

The mean life (T_m) of a radioactive substance is equal to the sum of life times of all atoms divided by the number of all atoms and is given follows

$$T_m = \frac{\int tdN}{\int dN} = \frac{\int_0^\infty t\lambda e^{-\lambda t}dt}{\int_0^\infty \lambda e^{-\lambda t}dt}$$

$$T_m = \frac{1}{\lambda}$$

Activity of a Radioactive Isotope

The activity of a radioactive substance (or radioisotope) means the rate of decay per second or the number of nuclei disintegrating per second. It is generally denoted by A.

$$\Rightarrow \qquad A = -\frac{dN}{dt}$$

If at time t = 0, the activity of a radioactive substance be A_0 and afte time t = t sec, activity be A_0 then :

$$A_0 = -\left[\frac{dN}{dt}\right]_{t=0} = \lambda N_0$$

$$A_t = -\left[\frac{dN}{dt}\right]_{t=t} = \lambda N_1$$

$$\Rightarrow \qquad A_t = A_0^{-\lambda t}$$

Unit of activity

The activity is measured in terms of curie (Ci). 1 curie is the activity of 1gm of a freshly prepared sample of radium Ra^{226} ($t_{1/2}$ = 1602 yrs.)

1 curie 1Ci = 3.7×10^{10} dps (disintegration per second)

1 dps is also known as 1 bq (Becquerel) $\Rightarrow$ 1Ci = 3.7×10^{10} bq

Note

All the equations discussed above is valid only when the number of nuclii are very large

Survival probability and decay probability for a finite time interval

The probability of suravival (i.e. not decaying) in time t is

$$P_{survival} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}.$$

Hence the probability of decay is

$$P_{decay} = 1 - e^{-\lambda t}.$$

Successive disintegration and secular equilibrium

Suppose A→B→C ... ... (i.e., radioactive nucleus A decays to B and B decays to C)

Let number of radioactive nucleus A (Parent nucleus) at time t = 0 be N_0 and that of B = 0.

For A

$$\frac{dN_A}{dt} = -\lambda_A N_A$$

$$\Rightarrow \qquad N_A = N_0 e^{-\lambda_A t}$$

For B

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

$$\Rightarrow \qquad \frac{dN_B}{dt} = \lambda_A N_0 e^{-\lambda_A t} - \lambda_B N_B$$

Multiplying both sides of this equation by $e^{\lambda_B t}dt$, we get

$$e^{\lambda_B t}.dN_B + \lambda_B N_B e^{\lambda_B t}dt = \lambda_A N_0 e^{(\lambda_B - \lambda_A)t}dt$$

$$\Rightarrow \qquad \frac{d(N_B.e^{\lambda_B t})}{dt} = e^{\lambda_B t}\frac{dN_B}{dt} + N_B.\lambda_B e^{\lambda_B t}$$

$$\Rightarrow \qquad d(N_B.e^{\lambda_B t}) = e^{\lambda_B t}dN_B + N_B\lambda_B e^{\lambda_B t}dt$$

$$\Rightarrow \qquad \int d(N_B e^{\lambda_B t}) = \lambda_A N_0 \int e^{(\lambda_B - \lambda_A)t}dt$$

$$\Rightarrow \qquad N_B e^{\lambda_B t} = \frac{\lambda_A N_0}{(\lambda_B - \lambda_A)} e^{(\lambda_B - \lambda_A)t} + C$$

Where C is constant of integration.

at t=0, N_B=0

$$\Rightarrow \qquad C = \frac{-\lambda_A N_0}{(\lambda_B - \lambda_A)}$$

Hence,

$$N_B e^{\lambda_B t} = \frac{\lambda_A N_0}{\lambda_B - \lambda_A}\left[e^{(\lambda_B - \lambda_A)t} - 1\right]$$

$$\Rightarrow \qquad N_B = \frac{\lambda_A N_0}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

Suppose the parent nucleus A is long lived i.e. the half life of the parent nucleus A is much larger in comparison to the half life of the daughter nucleus B

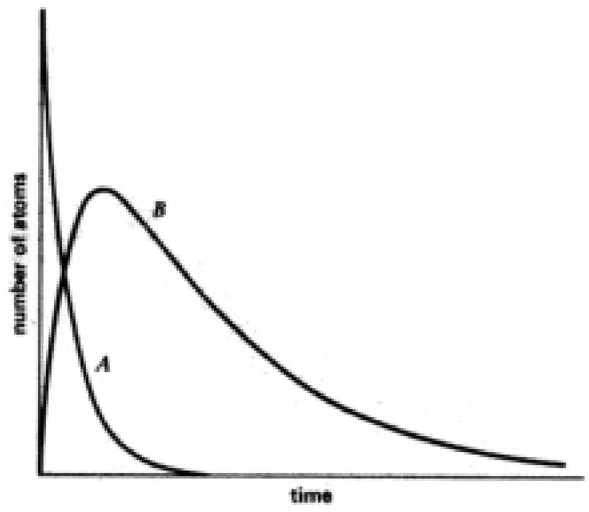
$\Rightarrow t_{1/2\,A} >> t_{1/2\,B} \qquad \Rightarrow \lambda_A << \lambda_B \quad \Rightarrow \lambda_B - \lambda_A \approx \lambda_B$

$\Rightarrow e^{-\lambda_B t}$ is negligible in comparison to $e^{-\lambda_A t}$

$$\Rightarrow \qquad N_B = \frac{\lambda_A}{\lambda_B} N_0 e^{-\lambda_A t}$$

$$\Rightarrow \qquad N_B = \frac{\lambda_A}{\lambda_B} N_A$$

$$\Rightarrow \qquad N_A \lambda_A = N_B \lambda_B$$



i.e., after a time much longer in comparison to the half life of the daughter nucleus B but much shorter in comparison to the half life of parent nucleus A, we have $N_A\lambda_A = N_B\lambda_B$ This state is called secular equilibrium

Illustration :

The mean lives of an radio active substance are 1620 and 405 years for α-emission and β-emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously.

Sol. *When a substance decarys by α and β emission simultaneously, the equivalent rate of disintegration λ_{eq} is given by :*

$$\lambda_{eq} = \lambda_\alpha + \lambda_\beta$$

where λ_α *= disintegration constant for α-emission only*

λ_β *= disintegration constant for β-emission only*

Mean life is given by : $T_{eq} = \frac{1}{\lambda_{eq}}$

$$\Rightarrow \quad \lambda_{eq} = \lambda_\alpha + \lambda_\beta = \frac{1}{T_{eq}} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1620} + \frac{1}{405} = 308 \times 10^{-3}$$

$$\lambda_{eq}t = 2.303 \log \frac{N_0}{N_t}$$

$$\Rightarrow \quad (3.08 \times 10^{-3})\, t = 2.303 \log \frac{100}{25}$$

$$\Rightarrow \quad t = 2.303 \times \frac{1}{3.08\times10^{-3}} \log 4 = 449.24 \text{ years}$$

Illustration :

Two radioctive materials A_1 and A_2 have have decay constants of 10 λ_0 and λ_0. If initially they have same number of nuclei, find the time after which the ratio of number their undecayed nuclei will be (1/e)

Sol. $$\frac{N_A}{N_B} = \frac{e^{-10\lambda_0 t}}{e^{-\lambda_0 t}} = e^{-9\lambda_0 t} = \frac{1}{e} = e^{-1}$$

$$\Rightarrow 9\lambda_0 t = 1$$

$$\text{or } t = \frac{1}{9\lambda_0}$$

Illustration :

The weight based ratio fo U^{238} and Pb^{226} in a sample of rock is 4 : 3. If the half life of U^{238} is 4.5×10^9 year, then find the age of rock.

Sol. *Let initial no. of U-atoms = N_0*

After time t, (age of rock), let no. of atoms remaining undecayed = N

$$\therefore \quad \frac{238N}{26(N_0 - N)} = \frac{4}{3}$$

$$\therefore \quad \frac{N_0}{N} = 1.79$$

$$t = \frac{T \log N_0 / N}{\log 2}$$

$$= \frac{4.5\times10^9 \times \log 1.79}{0.301}$$

Illustration :

A count rate-meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minutes. Five minutes later it shows 2700 counts per minutes. Find :

(a) decay constant *(b) the half lift of the sample*

Sol. *Initial activity = A_0 = dN/dt at t = 0*

Final activity = A_t = dN/dt at t = t

$$\left.\frac{dN}{dt}\right|_{t=0} = \lambda N_0 \ \textit{and} \ \left.\frac{dN}{dt}\right|_{t=5} = \lambda N_t$$

$$\Rightarrow \quad \frac{4750}{2700} = \frac{N_0}{N_t}$$

Using $\lambda t = 2.303 \log \dfrac{N_0}{N_t}$

$$\Rightarrow \quad \lambda(5) = 2.303 \log \frac{4750}{2700}$$

$$\Rightarrow \quad \lambda = \frac{2.303}{5} \log \frac{4750}{2700} = 0.1129 \text{ min}^{-1}$$

$$\Rightarrow \quad t_{1/2} = \frac{0.693}{0.1129} = 6.14 \text{ min}$$

Illustration :

A small amount of solution containing Na^{24} radionuclide with activity $A = 2.0. \times 10^3$ disintegrations per second was injected in the bloodstream of a man. The activity of 1 cm^3 of blood sample taken 1= 5.0 hours later turned out to be $A' =$ 16 disintegrations per minute per cm^3. The half-life of the radionuclide is $T = 15$ hours. Find the volume of the man's blood.

Sol. *Let V = volume of blood in the body of the human being. Then the total activity of the blood is A′V. Assuming all this activity is due to the injected Na^{24} and taking account of the decay of this radionuclide, we get*

$$VA' = Ae^{-\lambda t}$$

Now $$\lambda = \frac{ln\,2}{15} \text{ per hour, } t = 5 \text{ hour}$$

Thus $$V = \frac{A}{A'} e^{-ln\,2/3} = \frac{2.0 \times 10^3}{(16\,/\,60)} e^{-ln\,2/3}\, cc = 5.95 \text{ litre}$$

Pratice Exercise

Q.1 The half-life of ^{198}Au is 2.7 days, Calculate
(a) the decay constant,
(b) the average-life and
(c) the activity of 1.00 mg of ^{198}Au,
Take atomic weight of ^{198}Au to be 198 g/mol.

Q.2 A radioactive sample has 6×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives

Q.3 The activity of a radioactive sample falls from 600 s^{-1} to 500 s^{-1} in 40 minutes. Calculate its half-life.

Q.4 The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5×10^9 y. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U

Q.5 A radioactive nucleus can decay by two different processes. The half-life for the first process is t_1 and that for the second process is t_2• Find the effective half-life t of the nucleus.

Q.6 A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a resistance R. What should be the value of R so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?

Q.7 Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time t. Find the condition for which the number of daughter nuclei becomes constant.

Answers

Q.1 (a) $2.9 \times 10^{-6}\ s^{-1}$, (b)3.9 days (c) 240 Ci.

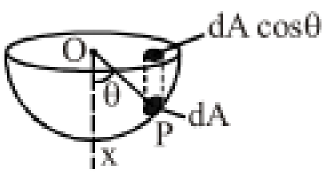
Q.2 1.5×10^{18} Q.3 152 min. Q.4 4.5×10^{9} y

Q.5 $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$ Q.6 200 Ω. Q.7 $\lambda_p N_p = \lambda_d N_d$

Solved Examples

Q.1 A point source of light is placed at the centre of curvature of a hemispherical surface. The radius of curvature is r and the inner surface is completely reflecting. Find the force on the hemisphere due to the light falling on it if the source emits a power W.

Sol. The energy emitted by the source per unit time, i.e. W fall on a area $4\pi r^2$ at a distance r in unit time. Thus, the energy falling per unit area per unit time is $\frac{W}{4\pi r^2}$. Consider a small area dA at the point P of the hemisphere (figure).



The energy falling per unit time on it, is

$$P = \frac{WdA}{4\pi r^2}$$

The corresponding momentum incident on this area per unit time is

$$= \frac{WdA}{4\pi r^2 c}$$

Suppose the radius OP through the area dA makes an angle θ with the symmetry axis OX. The force on dA is along this radius.

$$dF = \frac{2WdA}{4\pi r^2 c}$$

By symmetry, the resultant force on the hemisphere is along OX. The component of dF along OX is

$$dF \cos\theta = \frac{2WdA}{4\pi r^2 c} \cos\theta$$

$$= \frac{2W}{4\pi r^2 c} \text{ (projection the area dA on the plane containing the rim)}$$

The net force along OX is

$$F = \frac{2W}{4\pi r^2 c} \Sigma \text{ (projection the area dA on the plane containing the rim)}$$

$$= \frac{2W}{4\pi r^2 c} (\pi r^2) = \frac{W}{2c}$$

Q.2 Find the maximum kinetic energy of photo-electron liberated from the surface of lithium ($\phi = 2.39$ eV) by electromagnetic radiation whose electric component varies with time as $E = a(1 + \cos\omega t)\cos\omega_0 I$, where 'a' is a constant. $\omega = 6 \times 10^{14}$ rad/sec and $\omega_0 = 3.60 \times 10^{15}$ rad/s.

Sol. $E = a(1 + \cos\omega t)\cos\omega_0 t = a\cos\omega_0 t = \cos\omega_0 t + a\cos\omega t\cos\omega_0 t$

$$\Rightarrow E = a\cos\omega_0 t + \frac{1}{2} a\cos(\omega + \omega_0)t + \frac{1}{2} a\cos(\omega + \omega_0)t$$

This is a complex vibration consisting of harmonic components of frequencies ω_0, $(\omega + \omega_0)$ and $(\omega - \omega_0)$.

The highest angular frequency is $(\omega + \omega_0)$.
Now, $h\nu = \phi + k_{max}$

So, $k_{max} = \frac{h}{2\pi}(\omega + \omega_0) - \phi$

$$= \frac{6.6\times10^{-34}}{2\pi}(6\times10^{14} + 3.6\times10^{15}) - 2.39\times1.6\times10^{-19}$$

$$= 4.41\times10^{-19} - 3.82\times10^{-19} = 0.59\times10^{-19}\ \text{J} = 0.37\ \text{eV}$$

Q.3 Find the ratio of de-Broglie wavelength of an α-particle to that of a proton being subjected to the same magnetic field so that the radii of their paths are equal to each other, assuming that the field induction vector $\vec{B}$ is perpendicular to the velocity vectors of the a-particle and the proton.

Sol. When a charged particle of charge q, mass m enters perpendicularly to the magnetic induction $\vec{B}$ of a magnetic field, it will experience a magnetic force

$F = q(\vec{v}\times\vec{B}) = qvB\sin 90^\circ = qvB$ that will provide a centripetal acceleration $\frac{v^2}{r}$

$\Rightarrow qvB = \frac{mv^2}{r}$
$\Rightarrow mv = qBr$

$\Rightarrow$ The de-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{h}{qBr}$

$$\Rightarrow \frac{\lambda_{\alpha-\text{particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

Since $\frac{r_\alpha}{r_p} = \text{I}$ and $\frac{q_\alpha}{q_p} = 2$

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = 1/2$$

Q.4 A particle of mass m moves along a circular orbit in a centrosymmetric potential field $U = \frac{kr^2}{2}$. Using Bohr's quantization condition. Find (a) radius of n^{th} orbit (b) Energy of n^{th} orbit

Sol. $F = -\frac{dU}{dr} = -kr$

so $kr = \frac{mv^2}{r}$ ...(i)

and $mvr = \frac{nh}{2\pi}$ ...(ii)

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Solving we get $r = \left[\frac{h^2 n^2}{4\pi mk}\right]^{\frac{1}{2}}$

Total energy $E_n = KE_n + PE_n$

$$= \frac{1}{2}mv^2 + \frac{kr^2}{2} = kr^2$$

$$= K\sqrt{\frac{n^2h^2}{4\pi^2 mK}} = \sqrt{\frac{n^2h^2K^2}{4\pi^2 mK}}$$

$$= \frac{nhK}{2\pi\sqrt{mK}}$$

Q.5 Compare the radii and energy of ground state of H-atom and p-atom considering the motion of nucleus.

Sol. If we consider the motion of nucleus mass of e^- in all the expressions will be replaced by μ. Where

$$\mu_H = \frac{mM}{m+M};$$

M = Mass of Proton or neutron

m = mass of electron.

and $\mu_D = \frac{m(2M)}{m+2M}$

hence $\mu_D > \mu_H$

radius of n^{th} orbit $r_n \propto \frac{1}{\mu}$ so $r_H > r_D$

Energy of n^{th} orbit $E_n \propto \mu$ so $E_\mu < E_D$.

Q.6 What lines of atomic hydrogen absorbing spectrum fall with in the ware length range from 94.5 nm to 130 nm.

Sol. Absorbtion lines are always corresponding to lyman series.

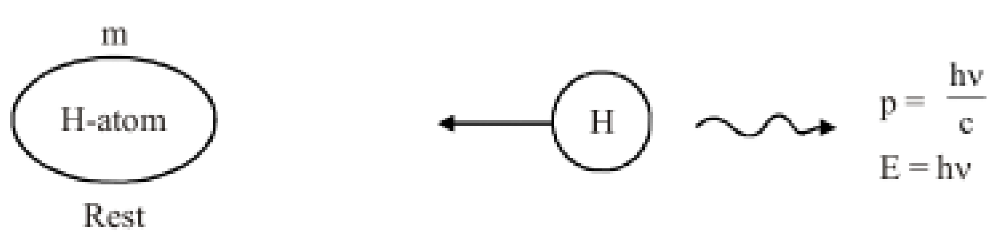
Wave length of lyman series $\frac{1}{\lambda} = R\left[\frac{1}{12} - \frac{1}{n^2}\right]$

for $n = 2$, $\lambda_1 = 121$ nm
for $n = 3$, $\lambda_2 = 102.2$ nm
for $n = 4$, $\lambda_3 = 96.9$ nm
for $n = 5$, $\lambda_4 = 94.64$ nm
for $n = 6$, $\lambda_5 = 93.45$ nm

hence 121.1 nm 102.2 nm 96.9 nm and 94.64 nm.

Q.7 A stationary H-atom emits a photon corresponding to the friest line of lyman series. What velocity does the atom acquire ? ($M_H = 1.67 \times 10^{-19}$ kg)

Sol.



Applying momentum and energy conservation,

$$mv = \frac{h\nu}{C} \text{ and } \Delta E = \frac{1}{2}mv^2 + h\nu$$

when $\Delta E = 10.2 \times 1.6 \times 10^{-19}$ Joule

we get $\Delta E = \frac{1}{2}mv^2 + mvC$

$$\Delta E = mvC\left[\frac{V}{2C} + 1\right] \approx mvC$$

$$v = \frac{DV}{mC} = \frac{10.2 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27} \times 3 \times 10^{8}}$$

$= 3.25$ m/sec

Q.8 The BE of an electron in ground state of the atom is equal to $E_0 = 24.6$ ev. Find the energy required to remove both electron from the atom.

Sol Ionisation energy of He^+ atom $= 54.4$ ev

hence to remove both electrons from He-atom

we require $= 24.6 + 54.4 = 79$ ev

Q.9 An X-ray tube with a copper target is found the emit lines other than those due to copper. The k_a line of copper is known to have a wavelength 1.5405 Å and the other two K_a lines observed have wavelengths 0.7092 Å and 1.6578 Å. (Identify) the impurities (find the value of Z, atomic number).

Sol. According to Moseley's equation for K_a radiation

$$\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \text{ where } \lambda = \text{wavelength of copper}$$

Let λ_1 and λ_2 be the two other unknown wavelengths, then

$$\frac{\lambda_1}{\lambda} = \frac{(Z-1)^2}{(Z_1-1)^2} = \frac{0.7092}{1.5405}$$

Solving we get $Z_1 = 42$

Similarly

$$\frac{\lambda_2}{\lambda} = \frac{(28)^2}{(Z_2-1)^2} = \frac{1.6578}{1.5405}$$

Solving we get $Z_2 = 28$

Q.10 When 0.50 Å X-rays strike a material, the electrons from the k shell are observed to move in a circle of radius 23 mm in a magnetic field of 2×10^{-2}T. What is the binding energy of K-shell electrons ?

Sol. The velocity of the photoelectrons is found the F = ma:

$$e\upsilon B = m\frac{\upsilon^2}{R} \qquad \text{or} \qquad \upsilon = \frac{e}{m}BR$$

The kinetic energy of the photoelctrons is then

$$K = \frac{1}{2}m\upsilon^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

$$= \frac{1}{2}\frac{(1.6510^{-19}C)^2(2\times10^{-2}T)^2(23\times10^{-3}m)^2}{(9.1\times10^{-31}kg)} = 2.97\times10^{-15}\ J$$

$$\text{or } K = (2.97\times10^{-15}\ J)\frac{1keV}{1.6\times10^{-16}J} = 18.36\ eV$$

The energy of the incident photon is $E_\upsilon = \frac{hc}{\lambda} = \frac{12.4keV.Å}{0.50Å} = 24.8\ eV$

The binding energy is the difference between these two value:

$$BE = E\upsilon - K = 24\ keV - 18.6\ keV = 6.2\ keV$$

Q.11 Calculate the wavelength of the emitted characteristic X-ray from a tungsten (Z = 74) target when an electron drops from an M shall to a vacancy in the K shell.

Sol. Tungsten is a multiel atom. Due to the shielding of the nuclear charge by the negative charge of the inner core electrons, each electron is subject to an effective nuclaer charge Z_{eff} which is different for different shells.

Thus, the energy of an electron in the n[th] level of a multielectron atom is given by

$$E_n = \frac{13.6Z_{eff}^2}{n^2}eV$$

For an electron in the K shell (n = 1), $Z_{eff} = (Z-1)$.

Thus, the energy of the electron in the K shell is :

$$E_K = -\frac{(74-1)^2\times13.6}{1^2} \simeq -72500\ eV$$

For an electron in the M shell (n = 3), the nucleus is shielded by one electron of the n = 1 state and eight electrons of the n = 2 state, a total of nine electrons, so that $Z_{eff} = Z - 9$. Thus the energy of an electron in the M shell is :

$$E_M = \frac{(74-9)^2\times13.6}{3^2} \simeq -6380\ eV$$

Therefore, the emitted X-ray photon has an energy given by

$$h\upsilon = E_M - E_K = -6380\ eV - (-72500\ eV) = 66100\ eV$$

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or $\qquad \dfrac{hc}{\lambda} = 66100 \times 1.6 \times 10^{-19}$ J

$\therefore \qquad \lambda = \dfrac{hc}{66100 \times 1.6 \times 10^{-19}}$ m

$$= \frac{(6.63 \times 10^{-34}) \times (3 \times 10^{8})}{66100 \times 1.6 \times 10^{-19}} \text{ m}$$

$$= 0.0188 \times 10^{-9} \text{ m.}$$

Q.12 If the short series limit of the Balmer series for hydrogen is 3646Å, calculate the atomic no. of the element which gives X-ray wavelength down to 1.0 A. Identify the element.

Sol. The short limit of the Balmer series is given by

$$\overline{\upsilon} = 1/\lambda = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = R/4$$

$\therefore \qquad R = 4/\lambda = (4 / 3646) \times 10^{10}$ m^{-1}

Futher the wavelengths of the k_0 series are given by the relation

$$\overline{\upsilon} = \frac{1}{\lambda} = R\,(Z - 1)^2$$

or $\qquad (Z-1)^2 = \dfrac{1}{R\lambda} = \dfrac{3646 \times 10^{-10}}{4 \times 1 \times 10^{-10}} = 911.5$

$\therefore \qquad (Z - 1) = \sqrt{911.5}$

$\qquad \cong 30.2$

or $\qquad Z = 30.2 \cong 31$

Thus the atomic number of the element concerned is 31.

The element having atomic number Z = 31 is Gallium.

Q.13 In a nuclear reactor, fission is produced in 1 gm for U^{235} (235.0439 a.m.u.) in 24 hours by a slow neutron (1.0087 a.m.u.). Assuming that $_{35}Kr^{92}$ (91.8973 a.m.u.) and $_{56}Ba^{141}$ (140.9139 a.m.u.) are produced in all reactions and no energy is lost, write the complete reaction and calculate the total energy produced in kilowatt hour. Given 1 a.m.u. = 931 MeV.

Sol. The nuclear fission reaction is

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1$$

The sum of the masses before reaction

$$= 235.0439 + 1.0087 = 236.0526 \text{ a.m.u.}$$

The sum of the masses after reaction

$$= 140.9139 + 91.8973 + (1.0087) = 235.8375 \text{ a.m.u.}$$

$\Rightarrow \qquad \Delta m = 256.0526 - 235.8373 = 0.2153$ a.m.u.

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energy released in the fission of U^{235} nucleus

$$E = 0.2153 \times 931 = 200 \text{ Mev}$$

Number of atoms in 1 gm

$$= \frac{6.02\times10^{23}}{235} = 2.56 \times 10^{21}$$

Energy released in fission of 1gm of U^{235}

$$E = 200 \times 2.56 \times 10^{2} = 5.12 \times 10^{23} \text{ MeV}$$

$$= (5.12 \times 10^{23}) \times (1.6 \times 10^{-13}) = 8.2 \times 10^{10} \text{ joule}$$

$$= \frac{8.2\times10^{10}}{3.6\times10^{6}} \text{kWh} = 2.28 \times 10^{4} \text{ kWh}$$

Q.14 A neutron collides elastically with an initially stationary deuteron. Find the fraction of the kinetic energy lost by the neutron (a) in a head-on collision; (b) in scattering at right angles.

Sol. (a) In a heat on collision $\sqrt{2mK} = p_d + p_n$

$$K = \frac{p_d^2}{2M} + \frac{p_n^2}{2m}$$

where p_d and p_n are the momenta of deuteron and neutron after the collision. Squaring

$$p_d^2 + p_n^2 + 2p_d p_n = 2mK$$

$$p_n^2 + \frac{m}{M} p_d^2 = 2mK$$

or since $p_d \neq 0$ in a head on collision

$$p_n = -\frac{1}{2}\left(1 - \frac{m}{M}\right)P_d$$

Going back to energy conservation

$$\frac{p_d^2}{2M}\left[1 + \frac{M}{4m}\left(1 - \frac{m}{M}\right)^2\right] = K$$

So $$\frac{p_d^2}{2M} = \frac{4mM}{(m+M)^2} K$$

This is the energy lost by neutron. So the fraction of energy lost is

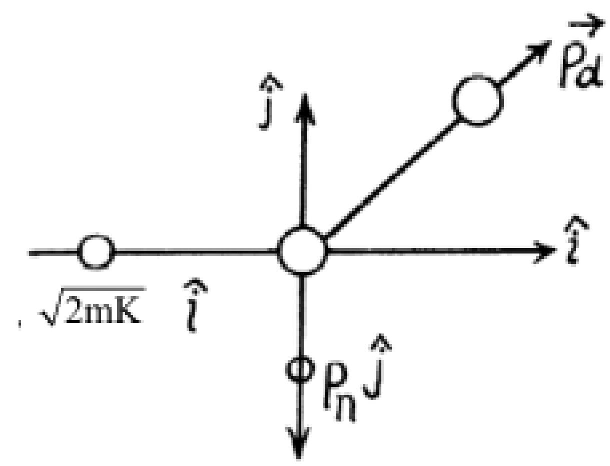
$$\eta = \frac{4mM}{(m+M)^2} = \frac{8}{9}$$

(b) In this case neutron is scattered by 90°. Then we have from the diagram

$$\vec{p}_d = p_n \hat{j} + \sqrt{2mK}\,\hat{i}$$

Then by energy conservation

$$\frac{p_n^2 + 2mK}{2M} + \frac{p_n^2}{2m} = K$$



or $$\frac{p_n^2}{2m}\left(1+\frac{m}{M}\right)=K\left(1-\frac{m}{M}\right)$$

or $$\frac{p_n^2}{2m}\left(1+\frac{m}{M}\right)=K\left(1-\frac{m}{M}\right)$$

The energy lost by neutron in then

$$K - \frac{p_n^2}{2m} = \frac{2m}{M+m}K$$

or fraction of energy lost is

$$\eta = \frac{2m}{M+m} = \frac{2}{3}$$

Q.15 A stationary Pb^{200} nucleus emits an α-particle with K.E., $K = 5.77$ MeV. Find the recoil velocity of daughter nucleus. What fraction of the total energy liberated in this decay is accounted for the recoil energy of the daughter nucleus ?

Sol. The momentum of the a-particle is given by,

$$P_d = P_\alpha = \sqrt{2m_\alpha K} \qquad ...(i)$$

Let the recoiled momentum of the dauhter nucleus be $P_d = m_d v_d$, where m_d and v_d are the mass and velocity of daughter nucles. Using the priciple of conservation of momentum we get,

$$P_d = P_\alpha = \sqrt{2m_\alpha K}$$

$$\Rightarrow \quad V_d = \frac{\sqrt{2m_\alpha K}}{m_d} \qquad ...(ii)$$

$$\Rightarrow \quad V_d = \frac{1}{196}\sqrt{\frac{2\times4\times K}{m_p}} = \frac{2}{196}\sqrt{\frac{2K}{m_p}}$$

Where m_p is the mass of the proton.

$\Rightarrow \quad V_d = 3.39 \times 10^5$ m/s

Let the K.E. of the daughter nucleus be K' then,

$$\frac{K'}{K} = \frac{m_\alpha}{m_d}$$

As the momenta are same

$$\therefore \quad \frac{K'}{K_t} = \frac{m_\alpha}{m_\alpha + m_d}$$

$$\Rightarrow \quad K' = \frac{m_\alpha}{m_\alpha + m_d} K_t = \frac{4}{196+4} K_t$$

$$\Rightarrow \quad K' = 0.02\ K_t$$

$$\Rightarrow \quad \frac{K'}{K_t} = 0.02$$

Q.16 A P^{32} radionuclide with half-life T = 14.3 days is produced in a reactor at a constant rate q = 2.7 × 10^9 nuclei per second. How soon after the beginning of production of that radionuclide will its activity be equal to A = 1.0 × 10^9 dps ?

Sol. Production of the nucleus is governed by the equation

$$\frac{dN}{dt} = \underset{\text{supply}}{\underset{\uparrow}{g}} - \underset{\text{decay}}{\underset{\uparrow}{\lambda N}}$$

we see that N will approach a constant value $\frac{g}{\lambda}$. This can also be proved directly. Multiply by $e^{\lambda t}$ and write.

$$\frac{dN}{dt} e^{\lambda t} + \lambda e^{\lambda t} N = g e^{\lambda t}$$

Then $$\frac{d}{dt}(N\,e^{\lambda t}) = g e^{\lambda t}$$

or $$N e^{\lambda t} = \frac{g}{\lambda} e^{\lambda t} + \text{const.}$$

At t = 0 when the production is starteed, N = 0

$$0 = \frac{g}{\lambda} + \text{constant}$$

Hence $$N = \frac{g}{\lambda}(1 - e^{-\lambda t})$$

Now the activity is

$$A = \lambda N = g(1 - e^{-\lambda t})$$

From the problem

$$\frac{1}{2\cdot 7} = 1 - e^{-\lambda t}$$

This gives $\lambda t = 0.463$

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so $$t = \frac{0.463}{\lambda} = \frac{0.463 \times T}{0.693} = 9.5 \text{ days.}$$

Algebraically $$t = -\frac{T}{ln\ 2}\ ln\left(1 - \frac{A}{g}\right)$$

Q.17 A dose of 5mCi of P^{32} ($t_{1/2}$ = 14 days) is a administered intravenously to a patient whose blood volume is 3.5 ℓts. At the end of 1 hour, it is assumed that the phosphorous is uniformly distributed. What would be the count rate/mℓ of the withdrawn blood if the counter measuring the activity had an efficiency of 10%:

(a) 1 hour after injection

(b) 28 days after injection

Sol. Let A_0 = initial activity A_t = activity at time t

According to question

$$A_0 = \frac{5 \times 10^{-5}}{35 \times 10^{3}} = 0.143 \times 10^{-5}\ \text{Ci/m}\ell$$

and

$$\lambda = \frac{0.693}{14 \times 24} = 2.06 \times 10^{-3}\ /\text{Hr}$$

Now using

$$\lambda t = 2.303 \log \frac{A_0}{A_t}$$

(a) After 1.0 Hr

$$\frac{2.06 \times 10^{-3} \times 1}{2.303} = \log \frac{0.143 \times 10^{-5}}{A_t}$$

$\Rightarrow$ $A_t = 1.42 \times 10^{-6}$ Ci/mℓ

$\Rightarrow$ Count rate = $(10/100) \times (1.42 \times 10^{-6}) \times 3.7 \times 10^{10}$ dps = 5280 dps

(b) After 28 days, i.e., after two half lives ($t_{1/2}$ = of P^{32} = 14 days) ;

$A_t = A_0 / 4 = 1.42 \times 10^{-6} /4$

$\Rightarrow$ Count rate = $(10/100) \times (1.42 \times 10^{-6} /4) \times 3.7 \times 10^{10}$ dps = 1322.75 dps

Q.18 In the chemical analysis of a rock, the mass ratio of two radioactive isotopes is found to be 100 : 1. The mean lives of the two isotopes are 4×10^9 and 2×10^9 years respectively. If it is assumed that at the time of formation the atoms of both the two isotopes were in equal proportion, calculate the age of the rock. Ratio of the atomic weights of two isotopes is 1.02 : 1.

Sol. Let two isotopes are A and B

$$\frac{m_A}{m_B} = 100 \; ; \; \frac{A_A}{A_B} = \frac{1.02}{1}$$

$T_A = 4 \times 10^9$ years $T_B = 2 \times 109$ years [Also $\lambda = 1/T$]

Let ratio of nuclei of two isotops be :

$$\frac{N_{A0}}{N_{B0}} \text{ at } t = 0 \text{ and } \frac{N_{At}}{N_{Bt}} \text{ at } t = 1$$

For isotope A

$$\lambda_A t = 2.303 \log \frac{N_{A0}}{N_{At}}$$

Similarly for isotope B

$$\lambda_B t = 2.303 \log \frac{N_{B0}}{N_{Bt}}$$

On subtracting

$$(\lambda_A - \lambda_B)\, t = 2.303 \log \frac{N_{A0} / N_{B0}}{N_{At} / N_{Bt}}$$

$$\Rightarrow \quad (\lambda_A - \lambda_B) t = 2.303 \log \frac{\text{initial ratio}}{\text{final ratio}}$$

$$\Rightarrow \quad \left(\frac{1}{4 \times 10^9} - \frac{1}{2 \times 10^9} \right) t = 2.303 \log \frac{1}{100/1.02}$$

$$\Rightarrow \quad \text{age} = t = 1.83 \times 10^{10} \text{ years}$$

Q.19 A given sample contains two types of atoms A and B in the ratio 3 : 1. Atoms of type A undergo α-decay with a half life of 30 days to form 'B' while 'atoms of type B' undergo α-decay with a half life of 45 days to from 'C', which is stable. Calculate the time after which the activities of A and that of B are in the ratio 9 : 22

$$A \xrightarrow{T_{1/2}=30\text{days}} B \xrightarrow{T_{1/2}=45\text{days}} C$$

Sol. The radiactive decay series is given

$$A \xrightarrow{T_{\frac{1}{2}} = 30\text{days}} B \xrightarrow{T_{\frac{1}{2}} = 45\text{days}} C$$

Initially $N_A(0) : N_B(0) \setminus 3 : 1$

$$\frac{dN_A}{dt} + \lambda_A N_A = 0$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

$$\frac{dN_C}{dt} = \lambda_B N_B$$

$$N_A = N_A(0)\ e^{-\lambda_A t}$$

$$N_B = c_1 e^{-\lambda_B t} + \frac{\lambda_A N_A(0) e^{-\lambda_A t}}{-\lambda_A + \lambda_B}$$

Then we get, $c_1 = \frac{5}{2} N_0$

$$\therefore \quad N_A(t) = \frac{3}{4} N_0 \left(\frac{1}{2}\right)^{\frac{1}{T_{1/2}}} = \frac{3}{4} N_0 \left(\frac{1}{2}\right)^{\frac{1}{30\text{days}}}$$

$$\text{and} \quad N_B(t) = \left[\frac{5}{2} N_0 \left(\frac{1}{2}\right)^{\frac{1}{45\text{days}}} - \frac{9}{4} N_0 \left(\frac{1}{2}\right)^{\frac{1}{30\text{days}}}\right]$$

Now, $\frac{\lambda_A N_A}{\lambda_B N_B} = \frac{9}{22}$ i.e. $\frac{N_A}{N_B} = \frac{3}{11}$

or, $\left(\frac{1}{2}\right)^{-t/90} = 2$

or, $t = 90$ days