ELASTICITY

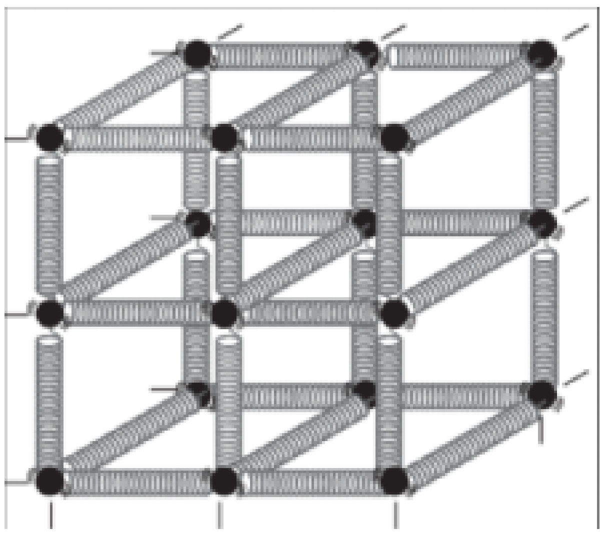
Elasticity

The property of material body by virtue of which its regain its original configuration, when external force is removed is called elasticity.

The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Cause of Elasticity

In a solid atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. When no deforming force is applied on the body, each molecule of the solid is in its equilibrium position and the inter molecular forces of the solid are maximum. On applying deforming force, the molecules are displaced from their equilibrium position. Inter molecular force gets changed and restoring forces are developed. It is explained by using spring- ball model. Deforming force is removed, these restoring force bring the molecule to its equilibrium positions. Thus the body regains its original shape and size.



Spring-ball model for the illustration of elastic behaviour of solids.

The restoring mechanism can be visualised by taking a model of spring-ball system shown above. Here the balls represent atoms and springs represent interatomic forces.
If you try to displace any ball from its equilibrium position, the spring system tries to restore the ball back to its original position. Thus elastic behaviour of solids can be explained in terms of microscopic nature of the solid.When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force.

Stress(σ)

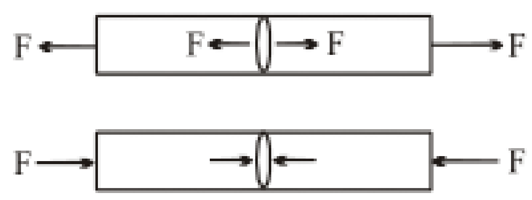
When deforming force is applied on the body then the equal restoring force in opposite direction is developed inside the body. The restoring force per unit area is called stress.

$$\text{Stress}(\sigma) = \frac{\text{restoring force}}{\text{Area of cross section of the body}}$$

Stress can be tensile or compressive as given below–

Tensile

Compressive



$$\sigma = \frac{F}{A} \text{ as } F \propto A$$

Strain

Suppose we stretch a wire by applying tensile forces of magnitude F to each end. The length of the wire increases from L to L + ΔL. The fractional length change is called the strain. It is a dimensionless quantity.

$$\text{strain} = \frac{\Delta L}{L}$$

Hooke's law for tensile and compressive forces

Suppose we had wires of the same composition and length but different thicknesses. It would require larger tensile forces to stretch the thicker wire the same amount as the thinner one. We conclude that the tensile force required is proportional to the cross-sectional area of the wire ($F \propto A$). Thus, the same applied force per unit area produces the same deformation on wires of the same length and composition.

Hooke's Law

$$\text{stress} \propto \text{strain}$$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

equation still says that the length change (ΔL) is proportional to the magnitude of the deforming forces (F). Stress and strain account for the effects of length and cross-sectional area ; the proportionality constant Y depends only on the inherent stiffness of the material from which the object is composed ; it is independent of the length and cross-sectional area.

Comparing equation $F = k\Delta L$ and $\frac{F}{A} = Y\frac{\Delta L}{L}$, $F = Y\frac{\Delta L}{L}A$. Y is called the elastic modulus or Young's modulus, Y has the same units as those of stress (Pa or N/m^2) since strain is dimensionless.

Young's modulus can be thought of as the inherent stiffness of a material ; it measures the resistance of the material to elongation or compression. Material that is flexible and stretches easily (for example, rubber) has a low Young's modulus. A stiff material (such as steel) has a high Young's modulus. It takes a larger stress to produce the same strain.

Hooke's law holds up to a maximum stress called the proportional limit. For many materials, Young's modulus has the same value for tension and compression. Some composite materials, such as bone and concrete, have significantly different Young's moduli for tension and compression. The different properties of these two substances lead to different values of Young's modulus for tensile and compressive stress.

Illustration :

A light wire of length 4m is suspended to the ceiling by one of its ends. If its crossectional area is19.6 mm^2, what is its extension under a load of 10kg. Young's modulus of steel = 2×10^{11} Pa.

Sol. *Given quantities – original length L = 4m ; force F = 10 × 9.8 = 98 N ; and Y = 2×10^{11} Nm^{-2}*

l = ?

Using the relation,

$$\text{Young's modulus (Y)} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

We have

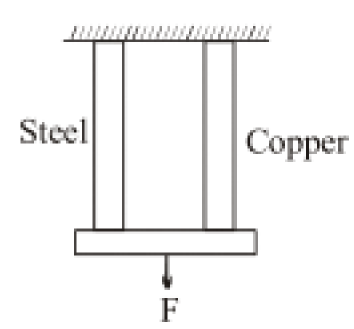
$$Y = \frac{F/A}{l/L} \quad \Rightarrow l = \frac{FL}{YA}$$

$$\therefore \quad l = \frac{98 \times 4}{2 \times 10^{11} \times 19.6 \times 10^{-6}} = 1 \times 10^{-4}\ m$$

$$= 0.1\ mm$$

Illustration :

Two vertical rods of equal lengths, one of steel and the other of copper, are suspended from the ceiling, at a distance l apart and are connected rigidly to a rigid horizontal light bar at their lower ends.



If A_S and A_C be their respective cross sectional areas, and Y_S and Y_C, their respective Young's modulii of elasticities, find where should a vertical force F be applied to the horizontal bar, in order that the bar remains horizontal. (Fig.)

Sol. *Let the force F be applied at a distance x from the steel bar, measured along the horizontal bar.*

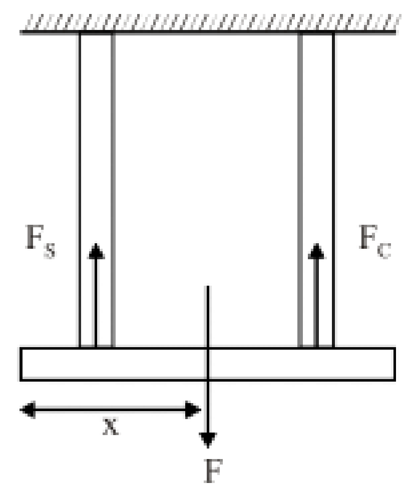
Let F_S and F_C be the loads on steel and copper rods respectively, so

$$F_S + F_C = F \qquad \text{... (i)}$$

Since the rigid horizontal bar remains horizontal so, the extensions produced in the two rods and hence strains remains same.

i.e.,

$$\frac{F_S}{A_S Y_S} = \frac{F_C}{A_C Y_C} \qquad \text{... (ii)}$$



Solving (i) and (ii) $F_S = \dfrac{FA_SY_S}{A_SY_S + A_CY_C}$

and $F_C = \dfrac{FA_CY_C}{A_SY_S + A_CY_C}$

Now, taking moments about the steel bar.

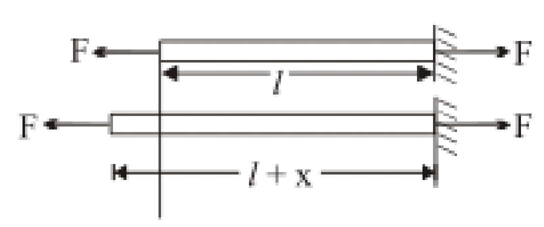
$$F_C l = Fx \quad \Rightarrow x = \frac{F_C}{F}\, l \;\Rightarrow\; \frac{A_CY_Cl}{A_SY_S + A_CY_C}$$

or $$x = l \Big/ \left[1 + \left(\frac{A_S}{A_C}\right)\left(\frac{Y_S}{Y_C}\right)\right]$$

Elastic potential energy

It is the potential energy stored inside the body due to change their configuration. If F force is applied on a body as shown below.

For differential change in length dx the work done by restoring force F is dw



$$dw = -\,Fdx \qquad \therefore \left(F = \frac{AY}{L}\,x\right)$$

$$dw = -\frac{AY}{L}\,x\,dx$$

$$W_{elastic} = -\frac{AY}{L}\int_0^l x\,dx$$

$$\Delta U = -W = \frac{AYl^2}{2L} \quad = \frac{1}{2}\left(\frac{Yl}{L}\right)\left(\frac{l}{L}\right)(AL) \qquad \therefore U_i = 0, U_f = U$$

Elastic potential energy (U) = $\dfrac{1}{2}$ (stress) (strain) (volume)

Elastic potential energy per unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain}$$

The above formula holds good for any type of strain. Change in equilibrium, restoring force = external force F

Then $$U = \frac{1}{2}\left(\frac{YA}{L}\right)l^2$$

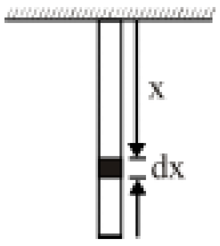
$$= \frac{1}{2}\left(\frac{Y}{L}\,l\right)Al = \frac{1}{2}Fl$$

Illustration :

A uniform heavy rod of weight W, cross- sectional area A and length L is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglect the lateral contraction. Find the elongation of the rod.

Sol. *Consider a small length dx of tne rod at a distance x from the fixed end. The part below this small element has* length L – x. The tension T of the rod at the element equals the weight of the rod below it.

$$T = (L - x)\,\frac{W}{L}$$



Elongation in the element is given by

$$\text{elongation} = \text{original length} \times \text{stress} / Y$$

$$= \frac{Tdx}{AY} = \frac{(L-x)Wdx}{LAY}$$

$$\text{The total elongation} = \int_0^L \frac{(L-x)Wdx}{LAY}$$

$$= \frac{W}{LAY}\left(Lx - \frac{x^2}{2}\right)_0^L = \frac{WL}{2AY}$$

Illustration:

A wire having a length $l = 2m$, and cross sectional area $A = 5mm^2$ is suspended at one of its ends from a ceiling. What will be its strain energy due to its own weight, if the density and Young's modulus of the material of the wire be $d = 9g/cm^3$ and $Y = 1.5 \times 10^{11}\ Nm^{-2}$?

Sol. *Consider an elemental length of the wire of length dx, at a distance x from the lower end. Clearly, this length is acted upon by the external force equal to the weight of the portion of wire below it = xAdg. In equilibrium, the restoring force f = xAdg.*

$$\therefore \qquad \text{stress} = \frac{f}{A} = xdg.$$

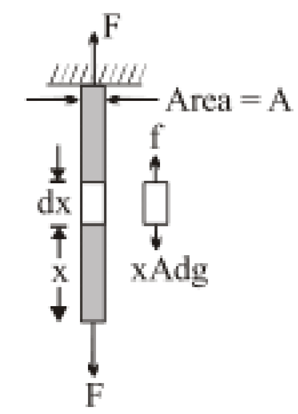
Now, elastic potential energy stored in the elemental length will be

$$dU = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{(xdg)^2}{Y} \times Adx = \frac{1}{2}\, \frac{d^2g^2A}{Y}\, x^2\, dx$$

$\therefore$ *Total elastic potential energy* $U = \int dU$



$$= \int_0^l \frac{1}{2}\frac{d^2g^2A}{Y}\, x^2\, dx$$

$$= \frac{1}{6}\, d^2g^2\, \frac{Al^3}{Y}$$

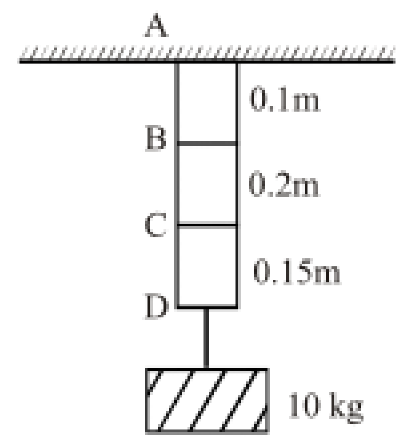
Substituting the values,

$$U = \frac{1}{6} \times \frac{(9\times 10^3)\,(9{\cdot}8)^2 \times 5 \times 10^{-6} \times 2^3}{1.5 \times 10^{11}}$$

$$= 3.46 \times 10^{-7}\ J$$

Illustration :

Find out the shift in point B, C and D.



$Y_{AB} = 2.5 \times 10^{10}\ N/m^2$

$Y_{BC} = 4\times 10^{10}\ N/m^2$

$Y_{CD} = 1 \times 10^{10}\ N/m^2$

$A = 10^{-7}\ m^2$

Sol. $$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\,\Delta\,L}$$

$$\Rightarrow \quad \Delta L = \frac{FL}{AY} = \frac{MgL}{AY}$$

Shift of point B $(\Delta L_B) = \Delta L_{AB} = \dfrac{10\times 10\times 0.1}{10^{-7}\times 2.5\times 10^{10}} = 4 \times 10^{-3}\ m = 4\ mm$

Shift of point C $(\Delta L_C) = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \dfrac{100\times 0.2}{10^{-7}\times 4\times 10^{10}}$

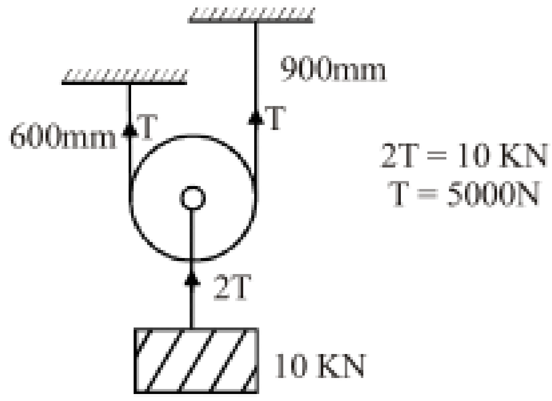
$$= 9 \times 10^{-3}\ m = 9\ mm$$

Shift of point D $(\Delta L_D) = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \dfrac{100\times 0.5}{10^{-7}\times 1\times 10^{10}}$

$$= 9 \times 10^{-3} + 15 \times 10^{-3} = 24\ mm$$

Illustration :

A load of 10 KN is supported from a pulley which in turn is supported by a rope of cross-sectional area 1×10^3 mm² and modulus of elasticity 10^3 N/mm², as shown in figure. Neglecting the friction at the pulley determine the deflection of load.



Sol. *longitudinal stress in the rope is*

$$\sigma = \frac{T}{A} = \frac{5 \times 10^3}{10^3\,mm^2} = 5\ N/mm^2$$

$$\text{Extension in the rope} = \frac{stress}{Y} \times L$$

$$= \frac{5N/mm^2}{10^3\ N/mm^2} \times 1500$$

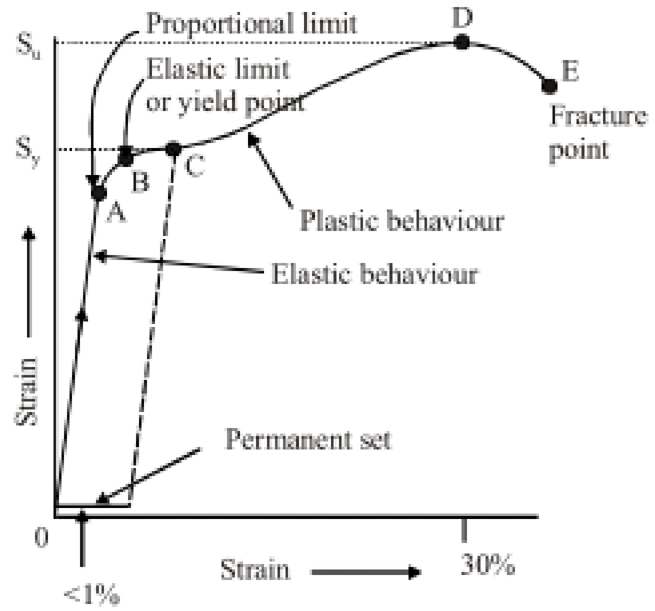
$$= 7.5\ mm$$

$$\text{Deflection in the load} = \frac{7.5}{2}$$

$$= 3.75\ mm$$

Stress-strain curve

The relation between the stress and the strain for a given material under tensile stress can be found experimentally. The applied force is gradually increased in steps and the change in length is noted. A graph is plotted between the stress and the strain produced. The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. From the graph, we can see that in the region between O to A, the curve is linear. In this region, Hooke's law is obeyed. The body regains its original dimensions when the applied force is removed. In this region, the solid behaves as an elastic body.



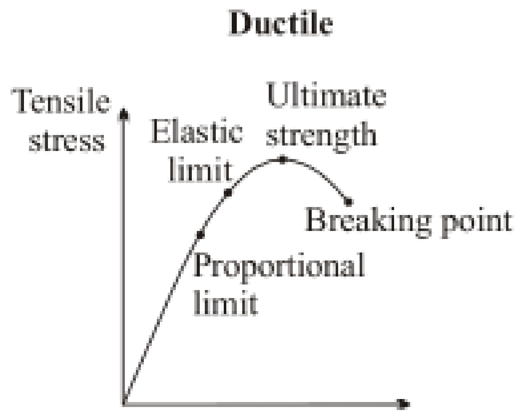
Stress-strain curve for steel.

Beyond hooke's law

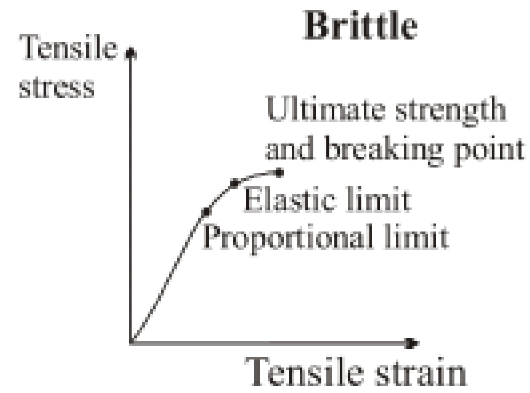
In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as **yield point** (also known as **elastic limit**) and the corresponding stress is known as **yield strength** (S_y) of the material. If the tensile or compressive stress exceeds the proportional limit, the strain is no longer proportional to the stress. The solid still returns to its original length when the stress is removed as long as the stress does not exceed the elastic limit.

If the stress exceeds the elastic limit, the material is permanently deformed. For still larger stresses, the solid factures when the stress reaches the breaking point. The maximum stress that can be withstood without breaking is called the ultimate strength. The ultimate strength can be different for compression and tension ; then we refer to the compressive strength or the tensile strength of the material. A ductile material continues to stretch beyond its ultimate tensile strength without breaking ; the stress then decreases from the ultimate strength (fig. (a)). Examples of ductile solids are relatively soft metals, such as gold, silver, copper, and lead. These metals can be pulled like taffy, becoming thinner and thinner until finally reaching the breaking point.

While as Brittle material can not stand beyond ultimate strength



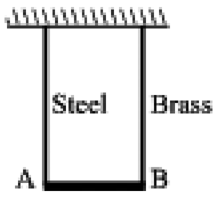
(a)



(b)

Practice Exercise

Q.1 A light rigid bar is suspended horizontally from two vertical wires, one of steel and one of brass, as shown in figure. Each wire is 2.00 m long. The diameter of the steel wire is 0.60 mm and the length of the bar AB is 0.20 m. When a mass of 10 kg is suspended from the centre of AB bar remains horizontal.



(i) What is the tension in each wire?

(ii) Calculate the extension of the steel wire and the energy stored in it.

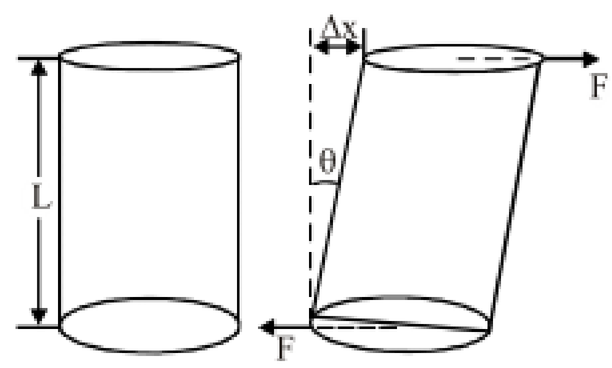
(iii) Calculate the diameter of the brass wire.

(iv) If the brass wire were replaced by another brass wire of diameter 1 mm, where should the mass be suspended so that AB would remain horizontal? The Young modulus for steel = 2.0×10^{11} Pa, the Young modulus for brass = 1.0×10^{11} Pa.

Answers

Q.1 (i) 50 N, (ii) 0.045 J, (iii) 8.4×10^{-4} m, (iv) x = 0.12 m

Shearing Stress



A cylinder subjected to shearing (tangential) stress deforms by an angle θ.

However, if two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder, as shown in fig, there is relative displacement between the opposite faces of the cylinder. The restoring force per unit area developed due to the applied tangential force is known as **tangential** or **shearing stress**.

As a result of applied tangential force, there is a relative displacement Δx between opposite faces of the cylinder as shown in the fig. The strain so produced is known as **shearing strain** and it is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder *L*.

Shearing strain $= \frac{\Delta x}{L} = \tan\theta$

where θ is the angular displacement of the cylinder from the vertical (θ is very samll $\tan\theta \simeq \theta$).

Volume Deformation

Since the fluid presses inward on all sides of the object (figure), the solid is compressed-its volume is reduced. The fluid pressure P is the force per unit surface area ; it can be through of as the volume stress on the solid object. Pressure has the same units as the other kinds of stress: N/m^2 or Pa.

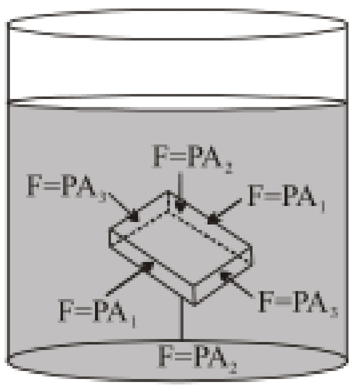


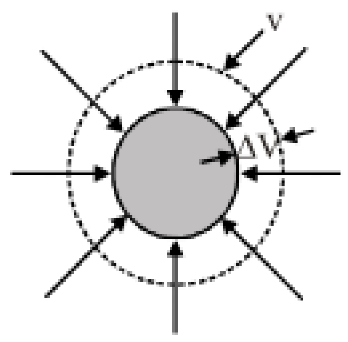
Fig. Forces on an object when submerged in a fluid

$$\text{volume stress} = \text{pressure} = \frac{F}{A} = P$$

The resulting deformation of the object is characterized by the volume strain, which is the fractional change in volume :

$$\text{volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

Bulk Modulus (B)



In fig., a solid sphere placed in the fluid under high pressure is compressed uniformly on all sides. The force applied by the fluid acts in perpendicular direction at each point of the surface and the body is said to be under hydraulic compression. This leads to decrease in its volume without any change of its geometrical shape. The body develops internal restoring forces that are equal and opposite to the forces applied by the fluid (the body restores its original shape and size when taken out from the fluid). The internal restoring force per unit area in this case is equal to the hydraulic pressure (applied force per unit area). The strain produced by a hydraulic pressure is called **volume strain** and is defined as the ratio of change in volume (ΔV) to the original volume (V).

$$\text{Volume strain} = \frac{\Delta v}{v}$$

We have seen that when a body is submerged in a fluid, it undergoes a hydraulic stress (equal in magnitude to the hydraulic pressure). This leads to the decrease in the volume of the body thus producing a strain called volume strain.

$$\Delta P = -B\frac{\Delta V}{V} \qquad \text{(Hooke's law for volume deformation)}$$

where V is the volume at atmospheric pressure. The negative sign. equation $\Delta P = -B\frac{\Delta V}{V}$ allows the bulk modulus to be positive. The bulk moduli of liquids are generally not much less than those of solids, since the atoms in liquids are nearly as close together as those in solids.

Gases are much easier to compress than solids or liquids, so their bulk moduli are much smaller. The bulk moduli of a few common materials are given in Table

Material	B (10^9 Nm^{-2} or GPa)
Solids	
Aluminium	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260
Steel	160
Liquids	
Water	2.2
Ethanol	0.9
Carbon disulphide	1.56
Glycerine	4.76
Mercury	25
Gases	
Air (at STP)	1.0×10^{-4}

Table : Bulk moduli *(B)* of some common Materials

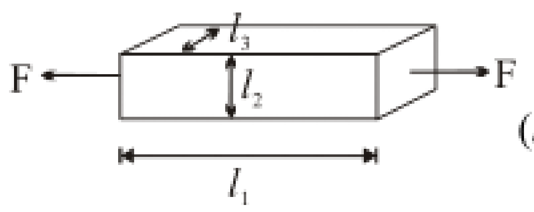
Compressibility (k)

The reciprocal of the bulk modulus is called compressibility and is denoted by k. It is defined as the fractional change in volume per unit increase in pressure.

$$k = \frac{1}{B} = -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)$$

Poisson's ratio

When an elongation is produced by longitudinal stresses, a change is produced in the lateral dimensions of the strained substance. Thus, when a wire is stretched, its diameter diminishes ; and when the longitudinal strain is small, the lateral strain is proportional to it. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio.



(l_1, l_2, and l_3 are the dimentional when no strain. Δl_1, Δl_2, and Δl_3 are the change in length of l_1, l_2, and l_3 respectively)

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l_1}{l_1}}$$

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\sigma\frac{\Delta l_1}{l_1}$$

Illustration:

. *A uniform bar of length L and cross sectional area A is subjected to a tensile load F. If Y be the Young's modulus of the material of the bar and σ be its poisson's ratio, then determine the volumertic strain.*

Sol. *Longitudinal stress* $= \dfrac{F}{A}$.

Longitudinal strain $= \dfrac{F}{AY} = \varepsilon_1$ *(say)* ... (i)

Now, by definition of Poisson's ratio,

$$\sigma = \frac{-\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\delta r / r}{\delta L / L}$$

or $\delta r / r = -\sigma \delta L / L \quad \Rightarrow -\dfrac{\sigma F}{AY}$ *[From eqn. (i)]*

Since Volumetric strain = Strain in length + Twice strain in radius.

$\therefore$ *Volumetric strain* $= \dfrac{\delta L}{L} + \dfrac{2\delta\delta}{r}$

$$= \frac{F}{AY} + 2\left(-\frac{\sigma F}{AY}\right) \qquad = \frac{F}{AY}(1 - 2\sigma).$$

Calorimetry

Units of heat & Mechanical equivalent of heat (J)

It was early 19th century when "James Prescott Joule" accidentally did an experiment which made two very important contribution in the scientific world. And it was Herman Von Helmholtz (a German) who later proved that indeed Joule was right.

Joules contribution bridged two major gaps in the scientific world.

i) Energy conservation principle was well grounded.

ii) The missing link between heat and energy was rectified.

Yes, heat was not thought to be a form of energy, rather it was known to be a fluid substance that flows. And that fluid was named calorie. They would say that when an iron rod is heated at one end, the other end also becomes hot as some calorie has flown to the rod. It was a very detailed mathematical theory.

Now, lets see the problem of energy conservation. We have seen many examples where energy in the form of $K + U = \text{constant}$; but not always. We know many places where in energy doesn't seem to be conserved. One of the examples is a box sliding on a rough surface. The box eventually stops because of friction. Thus, the KE of the box is lost. Where did it go? Today we can say that it got converted into heat energy, but earlier heat was not known as energy, but heat. Thus for them it was lost. And so energy conservation principle doesn't hold true.

A system is said to be isolated if no exchange or transfer of heat occurs between the system and its surroundings. When different parts of an isolated system are at different temperature, a quantity of heat transfers from the part at higher temperature to the part at lower temperature. The heat lost by the part at higher temperature is equal to the heat gained by the part at lower temperature. Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings.

As heat is just energy in transit, its unit in SI is joule. However, another unit of heat "calorie" is in wide use. This unit was formulated much before it was recognised that heat is a form of energy. The old day definition of calorie is as follows :

The amount of heat needed to increas the temperature of 1g of water from 14.5°C to 15.5°C at a pressure of 1 atm is called 1 calorie.

The calorie is now defined in terms of joule as 1 cal = 4.186 joule.

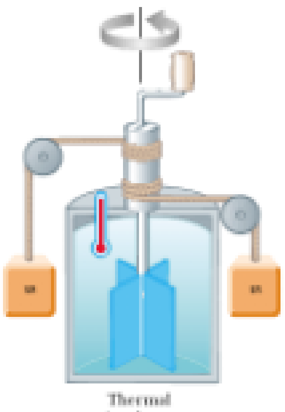
Illustation :

What is the kinetic energy of a 10 kg mass moving at a speed of 36 km/h in calorie ?

Sol. $\text{KE} = \frac{1}{2}\times 10\times 10^2 = 500\text{J} \simeq 120\ cal$

Principle of Calorimetry

A device in which heat measurement can be made is called a **calorimeter**. It consists a metallic vessel and stirrer of the same material like copper or aluminium. The vessel is kept inside a wooden jacket which contains heat insulating materials like glass wool etc. The outer jacket acts as a heat shield and reduces the heat loss from the inner vessel. There is an opening in the outer jacket through which a mercury thermometer can be inserted into the calorimeter.



From this experiment he came up with new physical quantities.

Heat capacity (C'): $Q = \int C' dT$

specific heat capacity: $Q = m \int_{T_i}^{T_f} s \, dT = m \, s_{avg} \Delta T$

Molar heat Capacity: $C = \dfrac{C'}{n}$ (n - no. of moles)

The branch of thermodynamics which deals with the measurement of Heat is called colorimetry. When two bodies at different temperature are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. Principle of colorimetry represents the law of conservation of Heat Energy.

Heat lost = Heat gained

Specific Heat capacity

The amount of heat needed to raise the temperature of unit mass of a material by unit degree of measurement is known as the specific heat capacity of that material. If Q amount of heat raises the temperature of mass m of a material by ΔT, then its specific heat capacity is given as :

$$s = \frac{Q}{m\Delta T} \quad \Rightarrow \quad Q = ms\Delta T$$

Also the amount of heat supplied per unit increase in temperature for any body is known as

its heat capacity, $c = \dfrac{Q}{\Delta T} = ms$.

Latent Heat

Heat required for the change of phase or state. No chage in temperature is involved when substance changes its state or phase. ($Q = mL$, L = Latent Heat)

Latent Heat of Fusion : The Heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent Heat of fusion. ($Q = mL_f$)
Latent heat of fusion of Ice (L_f) = 80 cal/gm.

Latent Heat of Vapourization : The Heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm pressure is called latent heat of vapourization. ($Q = mL_V$)
Latent heat of vapouriztion of water (L_V)= 540 cal/g.

Illustration :

The temperature of equal masses of three different liquids A, B, and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C and when B and C are mixed it is 23°C. What should be the temperature when A and C are mixed ?

Sol. *Let m be the mass of each liquid and S_A, S_B, S_C be specific heats of liquids A, B and C respectively. When A and B are mixed. The final temperature is 16°C.*

$\therefore$ *Heat gained by A = heat lost by B*

i.e. $mS_A(16 - 12) = mS_B(19 - 16)$

i.e. $S_B = \frac{4}{3} S_A$ .....(i)

When B and C are mixed. Heat gained by B = heat lost by C

i.e. $mS_B(23 - 19) = mS_C(28 - 23)$

i.e. $S_C = \frac{4}{5} S_B$ .....(ii)

From eq. (i) and (ii) $S_C = \frac{4}{5} \times \frac{4}{3} S_A = \frac{16}{15} S_A$

When A and C are mixed, let the final temperature be θ

$\therefore$ $mS_A(\theta - 12) = mS_C(28 - \theta)$

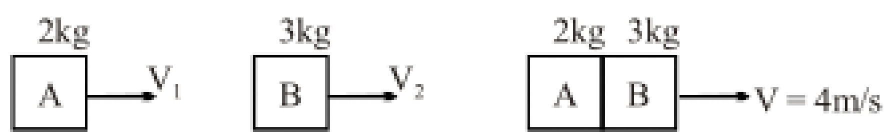
i.e. $\theta - 12 = \frac{16}{15}(28 - \theta)$

By solving, we get $\theta = \frac{628}{31} = 20.26^{o}C$

Illustation:

An object A of mass 2 kg is moving on a frictionless horizontal track has perfectly inelastic collision with another object B of mass 3 kg made of the same material and moving in front of A in same direction. Their common speed after the collision is 4 m/s. Due to the collision the temperature of the two objects, which was initially the same, is increased, though only by 0.006°C. The specific heat capacities of the two objects are the same : 0.5 kJ/kg°C. What was the initial speed (in m/s)of the colliding object A before the collision?

Sol.



$$2(V_1) + 3(V_2) = (2 + 3)V$$

$$\Rightarrow \quad 2V_1 + 3V_2 = 20 \qquad \text{........(i)}$$

loss in KE = heat energy

$$KE_i - KE_f = M_{total} S\Delta T$$

$$\frac{1}{2} 2(V_1)^2 + \frac{1}{2} 3(V_2)^2 - \frac{1}{2}(2 + 3)V^2 = (2 + 3)S\Delta T$$

$$V_1^2 + \frac{3}{2} V_2^2 - 40 = 15 \text{ or } V_1^2 + \frac{3V_2^2}{2} = 55 \qquad \text{........(ii)}$$

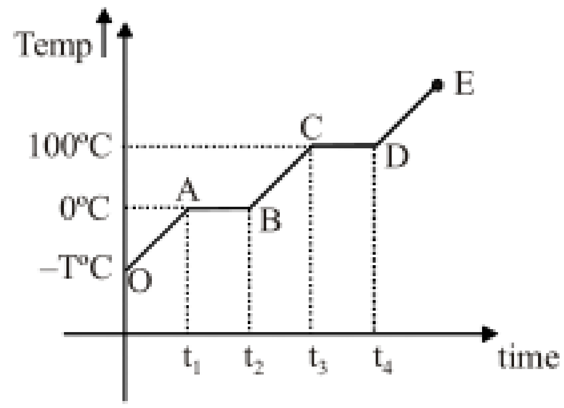
Solving equation (i) and (ii), we get,

$V_1 = 1$ m/s or 7 m/s and $V_2 = 6$ m/s or 2 m/s for collision $V_1 > V_2$. So $V_1 = 7$ m/s and $V_2 = 2$ m/sec

The following example provides a method by which the specific heat capacity of a given solid can be determinated by using the principle, heat gained is equal to the heat lost.

Heating Curve

If to a given mass (m) of a solid (Ice), Heat is supplied at constant rate P and a graph is poltted between temperature and time



(1) In the region OA

Temperature of solid is changing with time

$$Q = m\, s\, \Delta T$$

$$P(\Delta t) = m\, s\, \Delta T \qquad (\Delta t = t_1 - 0, \Delta T = 0 - (-T))$$

$$\frac{P}{ms} = \frac{\Delta T}{\Delta t} \qquad \frac{\Delta T}{\Delta t} = \text{slope of line OA}$$

(2) In the region AB

Temperature is constant, here substance changes its phase solid to liquid, between A and B.

$$Q = mL_f$$

$$P\,\Delta t = m\,L_f$$

$$L_f = \frac{P(t_2 - t_1)}{m}$$

L_f = length of line AB

Latent Heat of fusion is proportional to length of line .

(3) In the Region BC

Temp. of liquid is increasing with time

$$Q = m\,s\,\Delta T \qquad (\Delta t = t_3 - t_2,\ \Delta T = 100 - 0)$$

$$P\,\Delta t = m\,s\,\Delta T$$

(4) In the regin CD, temperature is constant, so it represents change of state.

$$Q = mL_V$$

$$\frac{P(t_4 - t_3)}{m} = L_V$$

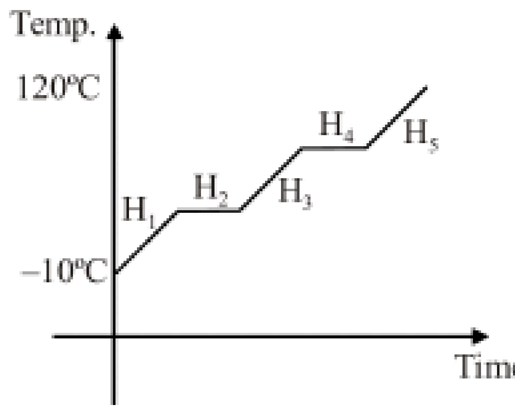
L_V = length of line CD

(5) The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat of substance in vapour state.

Illustration:

How many colories are required to change exactly 1 gm of ice at – 10 ℃ to steam of 120℃ at atmospheric pressure.

Sol.



$$H_1 = ms_{ice}\Delta T = 1 \times \frac{1}{2} \times 10 = 5cal$$

$$H_2 = mL_f = 1 \times 80 = 80\ cal$$

$$H_3 = ms_w\Delta T = 1 \times 1 \times 100\ cal$$

$$H_4 = mL_V = 1 \times 540 = 540\ cal$$

$$H_5 = ms_{steam}\Delta T = 1 \times 0.48 \times 20 = 9.6\ cal$$

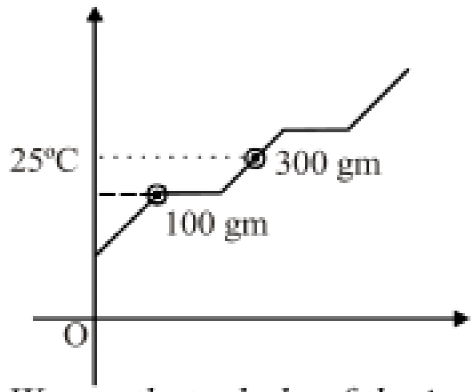
Total colories reqd. $\Rightarrow H_1 + H_2 + H_3 + H_4 + H_5$

$$H_{Total} \Rightarrow 734.6\ cal$$

Illustration :

300 gram of water at 25℃ is added to 100 gm of ice at 0℃. The final temp. of the mixture is _____?

Sol.



Heat released by water

$Q_1 = ms\Delta T$; $Q_1 = 300 \times 1 \times 25$

$Q_1 = 7500\ cal$

Heat required by Ice for completely melt $Q_2 = mL_f$

$Q_2 = 8000\ cal$

$Q_2 > Q_1$

We see that whole of the ice cannot be melted as the required amount of heat is not provided by the water. Also, the heat is enough to bring the ice to 0°C. Thus the final temperature of the mixture is 0°C with some of the ice melted.

Water equivalent

It is a equivalent mass of water (w) that has same heat capacity as that of the given body (b). In other words,

$$C = m_w s_w = m_b s_b$$

It is a convinent way to represent the heat capacity of the calorimeter

Illustation:

A sphere of aluminium of 0.047 kg placed for sufficient time in a vessel containing boiling water, so that the sphere is at 100 °C. It is then immediately transferred to 0.14 kg copper calorimeter containing 0.25 kg of water at 20 °C. The temperature of water rises and attains a steady state at 23 °C. Calculate the specific heat capacity of aluminium.

Sol. *In solving this example we shall use the fact that at a steady state, heat given by an aluminium sphere ill be equal to the heat absorbed by the water and calorimeter. Mass of aluminium sphere* $(m_1) = 0.047\ kg$

Initial temp. of aluminium sphere $= 100\ °C$

Final temp. $= 23\ °C$

Change in temp $(\Delta T) = (100\ °C - 23\ °C) = 77\ °C$

Let specific heat capacity of aluminium be s_{Al}

The amount of heat lost by the aluminium

sphere $= m_1 s_{Al} \Delta T = 0.047\ kg \times s_{Al} \times 77°C$

Mass of water $(m_2)\ 0.25\ kg$

Mass of calorimeter $(m_3) = 0.14\ kg$

Initial temp. of water and calorimeter $= 20°C$

Find temp. of the mixture $= 23°C$

Change in temp. $(\Delta T_2) = 23°C - 20°C = 3°C$

Specific heat capacity of water $(s_w) = 4.18 \times 10^3\ J\ kg^{-1}\ K^{-1}$

Specific heat capacity of copper calorimeter $= 0.386 \times 10^3\ J\ kg^{-1}\ K^{-1}$

The amount of heat gained by water and calorimeter $= m_2 s_w \Delta T_2 + m_3 s_{cu} \Delta T_2$

$$= (m_2 s_w + m_3 s_{cu})(\Delta T_2)$$

$$= (0.25\ kg \times 4.18 \times 10^3\ J\ kg^{-1}\ K^{-1} + 0.14\ kg \times 0.386 \times 10^3\ J\ kg^{-1}\ K^{-1})(23°C - 20°C)$$

In the steady state heat lost by the aluminium sphere = heat gained by water + heat gained by calorimeter.

So, $0.047\ kg \times s_N \times 77°C$

$$= (0.25\ kg \times 4.18 \times 10^3\ J\ kg^{-1}\ K^{-1} + 0.14\ kg \times 0.386 \times 10^3\ J\ kg^{-1}\ K^{-1})(3°C)$$

$$s_{Al} = 0.911\ kJ\ kg^{-1}\ K^{-1}$$

Practice Exercise

Q.1 1 kg of ice at – 10ºC is mixed with 1 kg water at 100ºC, then find the equilibrium temperature and mixture content.

Q.2 5 gm of ice at 0°C is mixed with 10 gm of steam at 100°C. Find the final temperature and composition of the mixture if the mixing is done in a calorimeter of water equivalent 13 gm, initially at 0°C.

Q.3 A lumb of ice of 0.1 kg at –10ºC is put in 0.15 kg of water at 20ºC. How much water and ice will be found in the mixtre when it has reached thermal equilibrium.

Answers

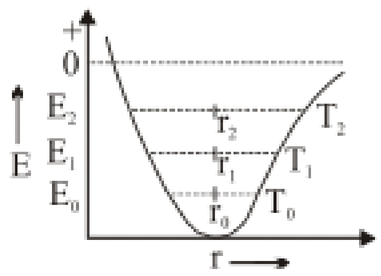
Q.1 T = 7.5ºC Q.2 245/27 gm of water, 160/27 gm of steam

Q.3 Amount of water amd Ice are 181.25 gm and 68.75 gm respetively.

Thermal expansion

Thermal Expansion

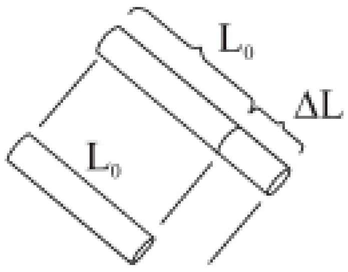
When matter is heated without change in state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion as with rise in temperature say from T_1 to T_2 the amplitude of vibration and hence energy of atoms increases from E_1 to E_2 and hence the average distance between atoms increases from r_1 to r_2.



Due to this increase in distance between atoms, the matter as a whole expands. Had the potential energy curve been symmetrical, no thermal expansion would have taken place in spite of heating.

Linear Expansion of solids

To varying extents, most materials expand when heated and contract when cooled. The increase in any one dimension of a solid is called linear expansion, linear in the sense that the expansion occurs along a line. A rod whose length is L_0 when the temperature is T_0 when the temperature increases to $T_0 + \Delta T$, the length becomes $L_0 + \Delta L$, where ΔT and ΔL are the magnitudes of the changes in temperature and length, respectively.
Conversely, when the temperature decreases to $T_0 - \Delta T$, the length decreases to $L_0 - \Delta L$.
For small temperature changes, experiments show that the change in length is directly proportional to the change in temperature ($\Delta L \propto \Delta T$). In addition, the change in length is proportional to the initial length of the rod,



Equation $\Delta L = \alpha L_0 \Delta T$ expresses the fact that ΔL is proportional to both L_0 and $\Delta T (\Delta L \propto L_0 \Delta T)$ by using a proportionality constant α, which is called the coefficient of linear expansion. Common unit for the coefficient of linear expansion $(C°)^{-1}$.

Illustration :

A circular hole of radius 2 cm is made in a iron plate at 0ºC. What will be its radius at 100ºC ? α for iron = 11 ×10⁻⁶/ ºC.

Sol. $R_{100} = R_0(1 + \alpha\Delta T) = (2)\,[1 + (11 \times 10^{-6} / ^\circ C)\,(100^\circ C)]$

$= (2)\,(1+11\times10^{-4}) = 2.0022cm$

Illustration

A brass scale correctly calibrated at 15°C is employed to measure a length at a temperature of 35°C. If the scale gives a reading of 75cm, find the true length. (Linear expansively of brass = $2.0 \times 10^{-5}C^\circ)^{-1}$)

Sol. Let the distance between two fixed divisions on the scale at 15°C be L_1 and that at 35°C be L_2.

Clearly, $(L_2 - L_1) = \alpha L_1\,(35 - 15)$

or $L_2 = L_1\,(1 + 20 \times 2.0 \times 10^{-5})$

$= L_1\,(1.0004)$

i.e., at 35°C, an actual length of L_2 will be read as L_1, due to the increased separation of the divisions of the scale. In other words, the observed length will be less than the actual length.

Given : $L_1 = 75$ cm

$\therefore$ $L_2 = 75\,(1.0004)$ cm

$= 75.03$ cm

Illustration

Estimate the time lost or gained by a pendulum clock at the end of a week when the atmospheric temperature rises to 40°C. The clock is known to give correct time at 15°C and the pendulum is of steel. (Linear expansively of steel is 12×10^{-6} / °C).

Sol. Time period of pendulum clock $T_0 = 2\pi\sqrt{\dfrac{\ell_0}{g}}$

ℓ_0 length of pendulum wire at temperature 0ºC, temperature increased to tºC chage in temperature $\Delta\theta = (t - 0)^\circ C$

Time period at tºC,

$$T_t = 2\pi\sqrt{\frac{\ell_t}{g}} = 2\pi\sqrt{\frac{\ell_0(1+\alpha\Delta\theta)}{g}}$$

α for wire of pendulum

$$T_t = 2\pi\sqrt{\frac{\ell_0}{g}}\sqrt{(1+\alpha\Delta\theta)}$$

$$\frac{T_t}{T_0} = (1+\alpha\Delta\theta)^{1/2} \approx (1+\frac{\alpha(\Delta\theta)}{2}) \qquad \text{(by using binomial approximation)}$$

$$\frac{T_t - T_0}{T_0} = \frac{\alpha(\Delta\theta)}{2} \quad \Rightarrow \quad \frac{\Delta T}{T_0} = \frac{\alpha(\Delta\theta)}{2}$$

Thus, time lost per second $= \dfrac{\alpha(\Delta\theta)}{2}$

rise in temperature $\Delta\theta = 40 - 15 = 25^\circ C$

Time lost per second $= \frac{1}{2}\ \alpha\Delta\theta$

$$= \frac{1}{2} \times (12 \times 10^{-6} / °C) \times (25°C)$$

$$= 150 \times 10^{-6}\ s/s$$

Therefore, time lost per week (i.e., 7×86400 s)

$$= 150 \times 10^{-6}\ s/s \times 7 \times 86400\ s$$

$$= 90.72\ s$$

Illustration :

A glass rod when measured with a zinc scale, both being at 30°C, appears to be of length 100cm. If the scale shows correct reading at 0°C, determine the true length of the glass rod at

(a) 30°C and (b) 0°C. *('α' for glass = 8×10^{-6} /°C and for zinc 26×10^{-6} / °C)*

Sol. *At 30°C, although the reading shown by the zinc scale corresponding to the length of the glass rod is 100cm, but the actual length would be more than 100cm, the reason being the increased separation between the markings, owing to a rise in temperature (from 0°C to 30°C).*

Now, an actual (original at 0°C) length of 100cm on the zinc scale (or more precisely, two markings or divisions on the scale, separated by a distance of 100cm) would, at a temperature of 30°C, correspond to a length given by

$$l = 100\ (1 + 26 \times 10^{-6} \times 30)\ cm$$

$$= 100.078\ cm$$

$\therefore$ *The true length of the glass rod at 30°C is 100.078 cm.*

Now, at 0°C, the length of glass rod would be lesser than that at 30°C,

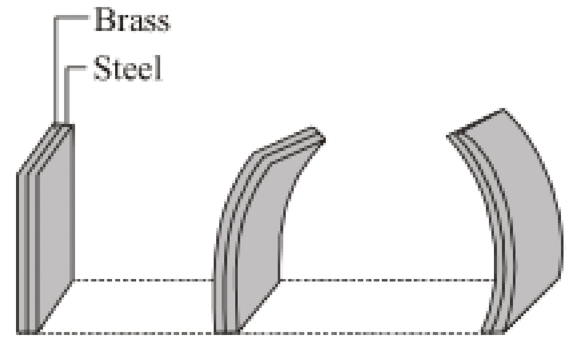
$\therefore$ *Using* $l_t = l_0\ (1 + \alpha t)$, $l_0 = \frac{l_t}{1+\alpha t}$

$\therefore$ *The length of the rod at 0°C, will be*

$$l_0 = \frac{100.078cm}{(1+8\times10^{-6}\times30)} = 100.054\ cm$$

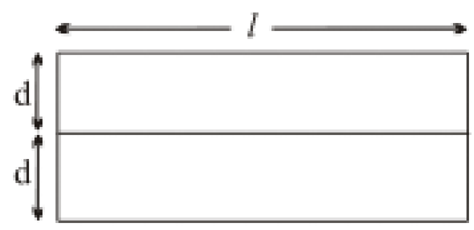
Thermal expansion of bimetallic strip

A bimetallic strip is made from two thin strips of metal that have different coefficients of linear expansion, as fig. (a) A bimetallic strip and how it behaves when (b) heated and (c) cooled

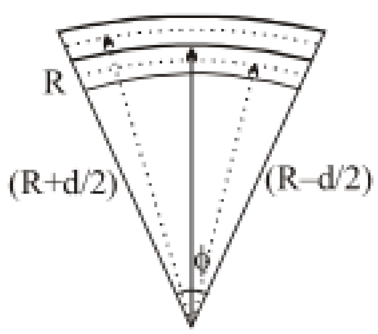


(a) (b) Heated (c) Cooled

Often brass [$\alpha = 19 \times 10^{-6}\,(C°)^{-1}$] and steel [$\alpha = 12 \times 10^{-6}\,(C°)^{-1}$] are selected. The two pieces are welded or riveted together. When the bimetallic strip is heated, the brass, having the larger value of α, expands more than the steel. Since the two metals are bonded together, the bimetallic strip bends into an arc as in fig. (b), with the longer brass piece having a larger radius than the steel piece. When the strip is cooled, the bimetallic strip bends in the opposite direction, as in fig. (c).



on heating the bimetallic strip bends into an are as shown below



Mathematical analysis

$$\left(R + \frac{d}{2}\right)\phi = L_0\,(1 + \alpha_1 \Delta\theta) \qquad (\Delta\theta \text{ increase in temp.})$$

$$\left(R - \frac{d}{2}\right)\phi = L_0\,(1 + \alpha_2 \Delta\theta)$$

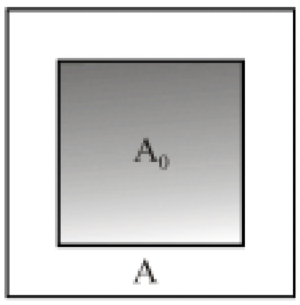
On dividing above equations we get

$$\frac{R + \frac{d}{2}}{R - \frac{d}{2}} = \frac{1 + \alpha_1\,\Delta\theta}{1 + \alpha_2\,\Delta\theta}$$

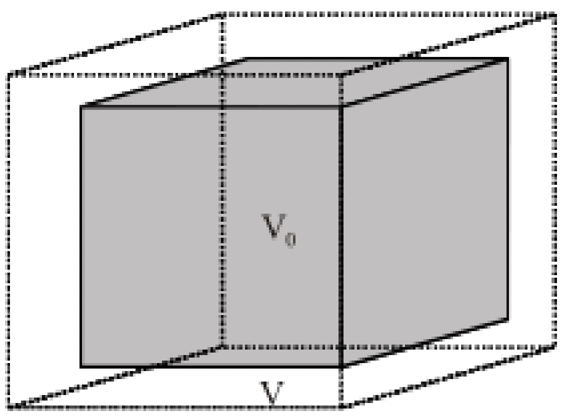
by above euqation we can find mean radius R of bimetallic strip.

$$R = \frac{d}{(\alpha_1 - \alpha_2)\Delta\theta}$$

Area and Volume Expansion



Area Expansion



Volume Expansion

If the temperature of a two-dimensional object (lamina) is changed, its area changes. If the coefficient of linear expansion of the material of lamina is small and constant, then its final area is given by $A = A_0\,(1 + \beta\Delta T)$, where A_0 is the initial area. ΔT is the change in temperature and β is the area coefficient of thermal expansion. For isotropic bodies it can be shown the $\beta = 2\alpha$.

The volume V_0 of an object change by an amount ΔV when its temperature changes by an amount ΔT. $\Delta V = \gamma V_0 \Delta T$ where γ is the coefficient of volume expansion. Common Unit for the coefficient of volume Expansion : $(C°)^{-1}$. The unit for γ, like that for α, is $(C°)^{-1}$. Values for γ depend on the nature of the material. The values of γ for liquids are substantially larger than those for solids, because liquids typically expand more than solids, given the same initial volumes and temperature expansion is three times greater than the coefficient of linear expansion : $\gamma = 3\alpha$.

If a cavity exists within a solid object, the volume of the cavity increases when the object expands, just as if the cavity were filled with the surrounding material. The expansion of the cavity is analogous to the expansion of a hole in a sheet of material. Accordingly, the change in volume of a cavity can be found using the relation $\Delta V = \gamma V_0 \Delta T$, where γ is the coefficient of volume expansion of the material that surrounds the cavity.

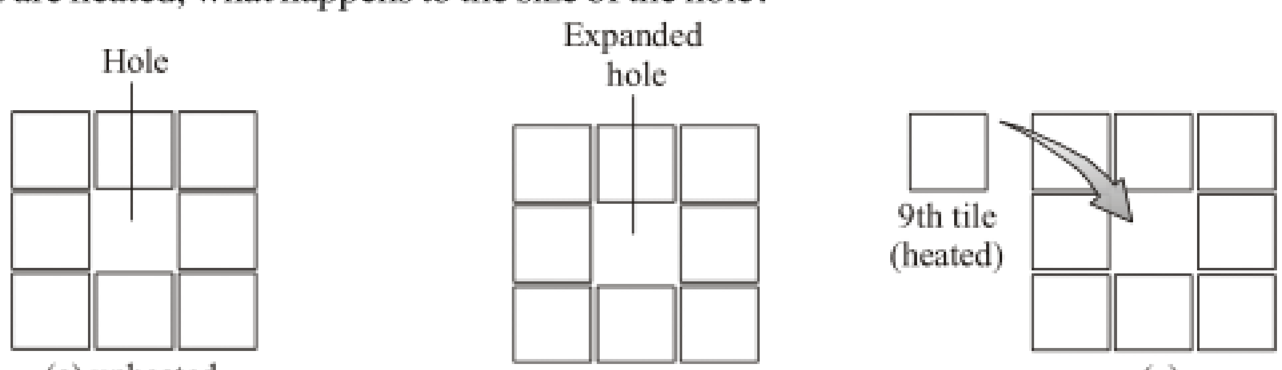
Similar (Here $\gamma \approx 3\alpha$) is known as the coefficient of volume expansion

$\alpha : \beta : \gamma :: 1 : 2 : 3$

Illustration : **(The expansion of holes)**

Do holes expand or contract when the temperature increases?

Figure (a) shows eight square tiles that are arranged to form a square pattern with a hole in the centre. If the tiles are heated, what happens to the size of the hole?



(a) unheated (b) Heated (c)

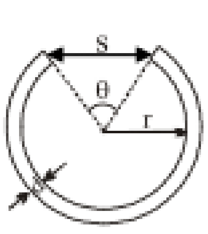
Sol. We can analyze this problem by disassembling the pattern into separate tiles, heating, it is evident from figure (b) that the heated pattern expands and so does the hole in the centre. In fact, if we had a ninth tile that was identical to and also heated like the others, it would fit exactly into the centre hole, as figure (c) indicates. Thus, not only does the hole in the pattern expand, but it expands exactly as much as one of the tiles. Since the ninth tile is made of the same material as the others, we see that the hole expands just as if it were made of the material of the surrounding tiles.

The thermal expansion of the hole and the surrounding material is analogous to a photographic enlargement ; in both situations everything is enlarged, including holes.

Thus, it follows that a hole in a piece of solid material expands when heated and contracts when cooled, just as if it were filled with the material that surrounds it. If the hole is circular, the equation $\Delta L = \alpha L_0 \Delta T$ can be used to find the change in any linear dimension of the hole, such as its radius or diameter. Example illustrates this type of linear expansion.

Illusration :

A thin cylindrical metal rod is bent into a ring with a small gap as shown in figure. On heating the system



(A) θ and s decreases, r and d increases (B) θ and r increases, d and s decreases

(C) θ, r, s and d all increases (D) θ is constant, d, s and r increases

Sol. θ remains constant d, s and r increases.

Illustration :

Figure shows a cross-sectional view of three cylinders, A, B and C. Each is made from a different material ; one is lead, one is brass, and one is steel. All three have the same temperature, and barely fit inside each other. As the cylinders are heated to the same, but higher, temperature, cylinder C falls off, While cylinder A becomes tightly wedged to cylinder B. Given, lead has the greatest coefficient of linear expansion, followed by brass, and then by steel. Which cylinder is made from which material?

Sol. We need to consider how the outer and inner diameters of each cylinder change as the temperature is raised. With respect to the inner diameter, we will be guided by the fact that a hole expands as if it were filled with the surrounding material. These data indicate that the outer and inner diameters of the lead cylinder change the most, while those of the steel cylinder change the least.

Since the steel cylinder expands the least, it cannot be the outer one, for if it were, the greater expansion of the middle cylinder would prevent the steel cylinder from falling off. The steel cylinder also cannot be the inner one, because then the greater expansion of the middle cylinder would allow the steel cylinder to fall out, contrary to what is observed. The only place left for the steel cylinder is in the middle, which leads to the two possibilities in figure.

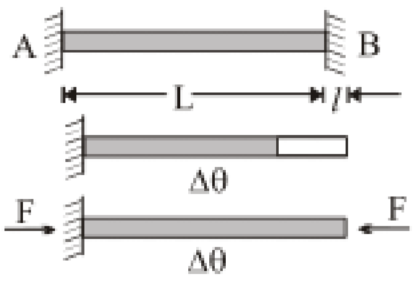
In part (a), lead is on the outside and will fall off as the temperature is raised, since lead expands more than steel. On the other hand, the inner brass cylinder expands more than the steel that surrounds it and becomes tightly wedged, as observed. Thus, one possibility is A = brass, B = steel, and C = lead.

In part (b), of the drawing, brass is on the outside. As the temperature is raised, brass expands more than steel, so the outer cylinder will again fall off. The inner lead cylinder has the greatest expansion and will be wedged against the middle steel cylinder. A second possible answer, then is A = lead, B = steel, and C = brass.

Thermal Stress

A change in shape/size i.e., dimensions need not necessarily imply a strain. For example, if a body is heated to expand, its volume change, as it acquires a new size, due to expansion. However, the strain remains zero. Unless and until, internal elastic forces operate, to bring the body to the original state, no strain exists. When a body is heated, the total energy of molecule increase, owing to an increase in the kinetic energy of the molecules. This results in a shift (increment) of the "equilibrium distance" of molecules and the body acquires a new shape and size, in the expanded form, whereby the molecules are in "zero force" state. Hence, there is no strain. However, if the body is resistricted to expand, during the process of heating, then the molecules become "strained", and even if there is no apparent change in dimensions of the body, there is strain. In such cases, strain is measured as the ratio. In dimension that would have occured, and the change in dimension that would have occured, had the body been free to expand or contract, to the original dimension.

When a metal rod is heated or cooled it tends to expand or contract. If it is left free to expand or contract, no temperature stresses will be induced. However, if the rod be restricted to change its length, then temperature stresses are generated within it. Stress induced due to temperature change can be understood as follows:



Consider a uniform rod AB fixed rigidly between two supports. (fig.) If L be its length, α the coefficient of linear expansion, then a change in temperature of Δθ, would tend to bring a change in its length by $l = L\alpha\Delta\theta$. Had the rod been free (say one of its ends) its length would have changed by l. Now, let a force be gradually applied so as to restore the natural length. Since the rod, tends to remain in the new state, due to a change in temperature, so when a force F is applied, thermal stress is induced. In equilibrium.

$$\frac{F}{A} = \frac{l}{(L \pm l)} Y \qquad [\because \text{stress} = \text{strain} \times Y]$$

Neglecting l in comparison to L,

$$F = \frac{lA}{L} Y = AY\ \alpha\ \Delta\theta$$

Now, if the two ends remain fixed, then this external force is provided from the support.

Clearly strain $= \dfrac{\ell}{L} = \alpha\Delta\theta$

Illustration :

A brass rod of length 1m is fixed to a vertical wall, at one end, with the other end free to expand. When the temperature of the rod is increased by 120°C, the length increases by 2cm. What is the strain?

Sol. *After the rod expands, to the new length there are no elastic forces developed internally in it. So, strain = 0.*

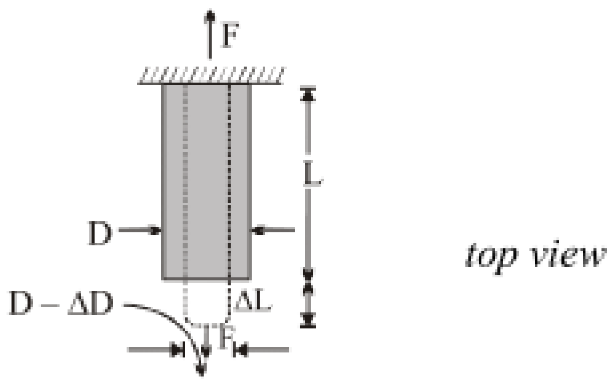


Illustration :

A rod of length 2m is at a temperature of 20°C. Find the free expansion of the rod, if the temperature is increased to 50°C. Find the temperature stresses produced when the rod is (i) fully prevented to expand, (ii) permitted to expand by 0.4 mm. $Y = 2 \times 10^{11}\ Nm^{-2}$; $\alpha = 15 \times 10^{-6}$ per °C.

Sol. Free expansion of the rod = $\alpha\Delta\theta$

$= (15 \times 10^{-6} /°C) \times (2m) \times (50° - 20°)C$

$= 9 \times 10^{-4}\ m = 0.9\ mm$

(i) If the expansion is fully prevented

then strain $= \dfrac{9\times 10^{-4}}{2} = 4.5 \times 10^{-4}$

$\therefore$ temperature stress = strain $\times$ Y

$= 4.5 \times 10^{-4} \times 2 \times 10^{11} = 9 \times 10^{7}\ Nm^{-2}$

(ii) If 0.4 mm expansion is allowed, then length restricted to expand = 0.9 – 0.4 = 0.5 mm

$\therefore$ Strain $= \dfrac{5\times 10^{-4}}{2} = 2.5 \times 10^{-4}$

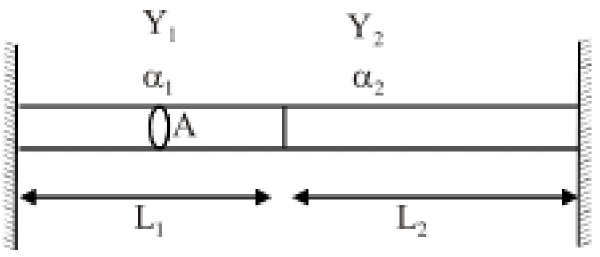
$\therefore$ Temperature stress = strain $\times$ Y

$= 2.5 \times 10^{-4} \times 2 \times 10^{11} = 5 \times 10^{7}\ Nm^{-2}$

Illustration :

Two rods made of different materials are placed between massive walls. The cross section of both the rods are same A, their lengths L_1 and L_2 coefficients of linear expansion α_1 and α_2 and the modulii of elasticity of their materials Y_1 and Y_2 respectively. If the rods are heated by t°C, find the force F with which the rods act on each other.

Sol.



Let the first rod expand slightly (say by length δl) and the second rod get compressed by the same amount (since net elongation / compression of the rods is zero.)

$\therefore$ Natural increase in length of the first rod (after being heated) when free to expand would have been $\alpha_1 L_1 t$. The expansion allowed is just δl (where $\delta l < \alpha_1 L_1 t$).

$\therefore$ Amount of elongation restricted $= \alpha_1 L_1 t - \delta l$

$\therefore$ Strain $= \dfrac{\text{elongation restricted}}{\text{orignal length}} = \dfrac{\alpha_1 L_1 t - \delta l}{L_1(1+\alpha_1 t)}$

Since $\alpha_1 t << 1$

$\therefore$ $1 + \alpha_1 t \simeq 1$

$\therefore$ Strain $= \dfrac{\alpha_1 L_1 t - \delta l}{L_1}$

$\therefore$ Stress = strain $\times$ Y $= \left(\dfrac{\alpha_1 L_1 t - \delta l}{L_1}\right) Y_1$.

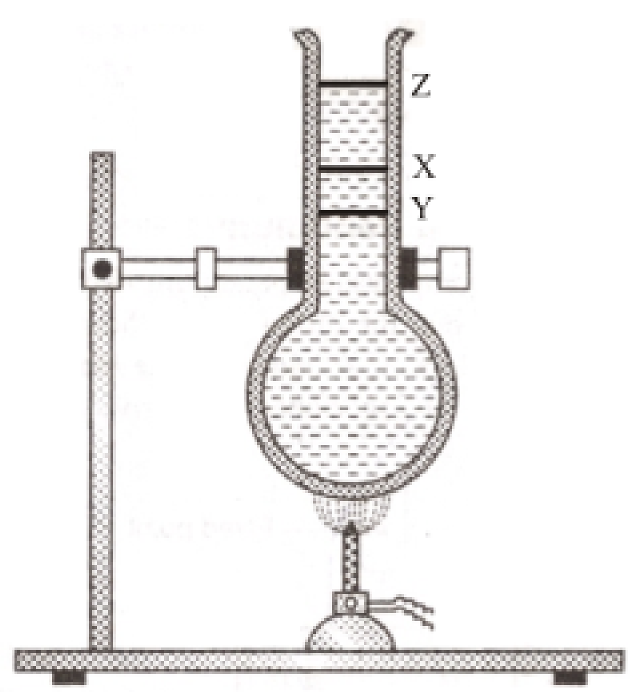
$$\text{or} \qquad F = stress \times A = \frac{(\alpha_1 L_1 t - \delta l)}{L_1} Y_1 A \qquad \ldots (i)$$

$$\text{Similarly, } F = \frac{(\alpha_1 L_1 t - \delta l)}{L_2} y_2 t_2 \qquad \text{or} \qquad \delta l = \alpha_1 L_1 t - \frac{FL_1}{Y_1 A} = \frac{FL_2}{Y_1 L_2} - \alpha_2 L_2 t$$

$$\text{or} \qquad F = \frac{\left(\alpha_1 L_1 + \alpha_2 L_2\right) t}{\left(\frac{L_1}{Y_1 A} + \frac{L_2}{Y_1 A}\right)}$$

Expansion of liquids

Like solids, liquids also, in general, expand on heating ; however, their expansion is much large compared to solids for the same temperature rise. A noteworthy point to be taken into account during the expansion of liquid is that they are always contained in a vessel or a container and hence the expansion of the vessel also comes into picture. Further, linear or superficial expansion in case of a liquid does not carry any sense. Consider a liquid contained in a round bottomed flask fitted with a long narrow stem as shown in fig. Let the initial level of the liquid be X. When it is heated the level falls initially to Y.



However, after sometime, the liquid level eventually rises to Z. The entire phenomenon can be understood as follows: Upon being heated, the container gets heated first and hence expands. As a result, the capacity of the flask increases and hence the liquid level falls.

After sometime, the heat gets conducted from the vessel to the liquid and hence liquid also expands thereby rising its level eventually to Z. Since, the volume expansivity of liquids, in general, are far more than that of solids, so the level Z will be above the level X.

Effect of temperature on density

When a solid or liquid is heated, it expands, with mass remaining constant. Density being the ratio of mass to volume, it decreases. Thus, if V_0 and V_t be the respective volumes of a substance at 0°C and t°C and if the corresponding values of densities be ρ_0 and ρ_t, then the mass m of the substance is given by

$$m = V_0\rho_0 = V_t\rho_t$$

But $\quad V_t = V_0(1 + \gamma t)$, so $\rho_t = \rho_0(1 + \gamma t)^{-1}$

Illustration :

The volume of a glass vessel filled with mercury is 500 cc, at 25°C. What volume of mercury will overflow at 45°C?
the coefficients of volume expansion of mercury and glass are 1.8×10^{-4} / °C and 9.0×10^{-6} / °C respectively.

[Sol. The volume of mercury overflowing will be the expansion of mercury relative to the glass vessel (i.e., apparent expansion).

Now, since $\quad (\Delta V)_a = (\Delta V)_l - (\Delta V)_c$

Apparent expansion $(\Delta V)_a$ will be

$$(\Delta V)_a = V_l\gamma_l\Delta T - V_C\gamma_C\Delta T$$

$$= 500\ cc\ (180 - 9) \times \frac{10^{-6}}{°C}\ (45 - 25)°C$$

$$= 1.71\ cc$$

Thus, 1.71 cc of mercury overflows.

Illustration :

A sphere of diameter 4 cm and mass 150g floats in a bath of liquid. As temperature is raised, the sphere begins to sink at temperature 50°C. If the density of the liquid is 6.5 g/cm³ at 0°C, find the coefficient of cubical expansion of the liquid, neglecting the expansion of the sphere.

Sol. (i) When sphere is floating i.e. at temperature < 50° C

weight of body = Thrust

$$mg = V_{in}\sigma g \qquad (\sigma \text{ density of liquid at temp. } 0°C) \qquad .....(i)$$

(ii) When body just sinks i.e. at temperature 50°

$$mg = V'\sigma g$$

$$\therefore \quad \sigma' = \frac{m}{V} \qquad (\sigma' \text{ density of liquid at temp. } 50°C)$$

$$= \frac{150}{\frac{4}{3}\pi(2)^2} = 4.48\ g/cm^2$$

Now, $\quad \sigma' = \frac{\sigma}{1+\gamma\Delta T}$

$$\therefore \quad 1+\gamma\Delta T = \frac{\sigma}{\sigma'} = \frac{6.5}{4.48}$$

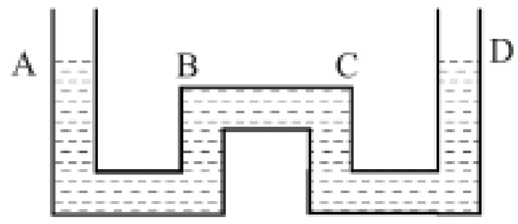
$$1 + \gamma\,[50 - 0] = \frac{6.5}{4.48}$$

$$\therefore \quad \gamma = \frac{2.02}{50 \times 4.48}$$

$$or \quad \gamma = 9.02 \times 10^{-3}/^{\circ}C$$

Practice Exercise

Q.1 The apparatus shown in fiugre consists of four glass colums connected by horizontal sections. The heights of two central columns B and C are 49 cm each. The two outer column A and D are open to atmosphere. A and C are maintained at a temperature of 95ºC, while the column B and D are maintained at 5ºC. The heights of liquid in A and D measured from base line are 52.8 cm and 51cm respectively. Determine the coefficient of thermal expansion of the liquid.



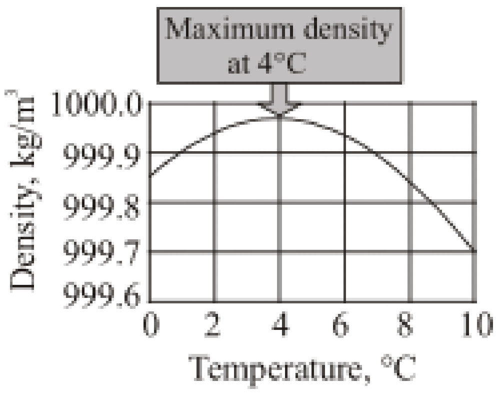
Answers

Q.1 $\gamma = 1.96 \times 10^{-4}$ / ºC

Anomolous expansion of water

While most substances expand when heated, a few do not. For instant, if water at 0°C is heated, its volume decreases until the temperature reaches 4°C. Above 4 °C water behaves normally, and its volume increases as the temperature increases.

Because a given mass of water has a minimum volume at 4°C, the density (mass per unit volume) of water is greatest at 4 °C, as figure shows.

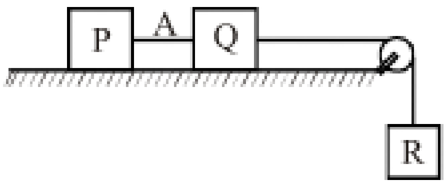


The density of water in the temperature range from 0 to 10°C. Water has a maximum density of 999.973kg/m^3 at 4°C. (This value for the density is equivalent to the often quoted density of 1.000 grams per milliliter)

When the air temperature drops, the surface layer of water is chilled. As the temperature of the surface layer drops toward 4°C, this layer becomes more dense than the warmer water below. The denser water sinks and pushes up the deeper and warmer water, which in turn is chilled at the surface. This process continues until the temperature of the entire lake reaches 4°C. Further cooling of the surface water below 4°C makes it less dense than the deeper layers ; consequently, the surface layer does not sink but stays on top. Continued cooling of the top layer to 0°C leads to the formation of ice that floats on the water, because ice has a smaller density than water at any temperature. Below the ice, however, the water temperature remains above 0°C. The sheet of ice acts as an insulator that reduces the loss of heat from the lake, especially if the ice is covered with a blanket of snow, which is also an insulator. As a result, lakes usually do not freeze solid, even during prolonged cold spells, so fish and other aquatic life can survive.

Solved Examples

Q.1 Each of the three blocks P,Q and R shown in figure has a mass of 3 kg. Each of th wires A and B has cross-sectional area 0.005 cm^2 and Young's modulus 2×10^{11} N/m^2. Neglect friction. Find the longitudinal strain developed in each of the wires. Take $g = 10\ m/s^2$.



Sol. The block R will descend vertically and the blocks P and Q will move on the frictionless horizontal table. Let the common magnitude of the acceleration be a. Let the tensions in the wires A and B be T_A and T_n respectively.

Writing the equation of motion of the blocks P, Q and R, we get,

$$T_A = (3)a \qquad .....(i)$$

$$T_B - T_A = (3)a \qquad ....(ii)$$

and $(3)g - T_B = (3)a$ .....(iii)

By (i) and (ii)

$$T_B = 2T_A.$$

By (i) and (iii)

$$T_A + T_B = (3)g = 30\ N$$

or $3T_A = 30\ N$

or $T_A = 10\ N$ and $T_B = 20\ N$.

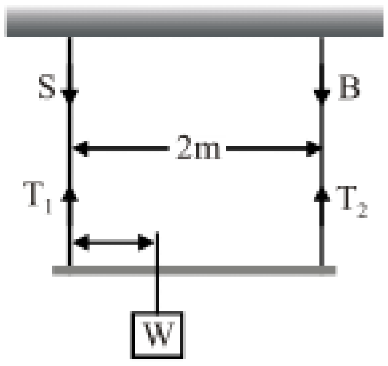
$$\text{Longitudinal strain} = \frac{\text{Longitudinal stress}}{\text{Young's modules}}$$

$$\text{Strain in wire A} = \frac{10\ N / 0.005\ cm^2}{2\times10^{11}\ N/m^2} = 10^{-4}$$

$$\text{Strain in wire B} = \frac{20N / 0.005\,cm^2}{2\times10^{11}\,N/m^2} = 2\times10^{-4}$$

Q.2 A light rod of length 200 cm is suspended from the ceiling horizontally by means of two vertical wires of equal length, tied to its ends. One of the wires is made of steel and is of cross-section 0.1 cm^2 and the other of brass of cross-section 0.2 cm^2. Along the rod at which distance may a weight be hung to produce (a) equal stresses in both the wires (b) equal strains in both the wires ? Y for brass and steel are 10×10^{11} and 20×10^{11} dyne/cm^2 respectively.

Sol. (a) According to the problem stresses are equal, so we have



$$\frac{T_1}{A_1}=\frac{T_2}{A_2},$$

i.e. $$\frac{T_1}{T_2}=\frac{A_1}{A_2}=\frac{0.1}{0.2}$$

or $$T_2 = 2T_1 \qquad ....(i)$$

As the rod is in equilibrium,

$$\Sigma F_y = T_1 + T_2 - W = 0$$

or $$T_1 + T_2 = W \qquad ....(ii)$$

From equation (i) and (2), we get

$$T_1 = (W/3) \text{ and } T_2 = (2W/3) \qquad ....(iii)$$

Let x be the distance of weight W from steel wire, Torque balance for rotational equilibrium of rod.

$$\Sigma\tau = T_1 x - T_2 (2 - x) = 0$$

or $$(W/3)\, x = (2W/3)\,(2 - x),$$

i.e. $$x = (4/3)\text{ m}$$

(b) According to the problem, strains are equal.

$$\frac{T_1}{A_1Y_1}=\frac{T_2}{A_2Y_2} \qquad \left[\text{as strain} = \frac{\text{stress}}{Y}\right]$$

So, $$\frac{T_1}{T_2}=\frac{A_1Y_1}{A_2Y_2}=\frac{0.1\times20\times10^{11}}{0.2\times10\times10^{11}}=1$$

$$T_1 = T_2 \qquad ......(iv)$$

i.e. from equation (ii) and (iv), we get

$$T_1 = T_2 = (W/2)$$

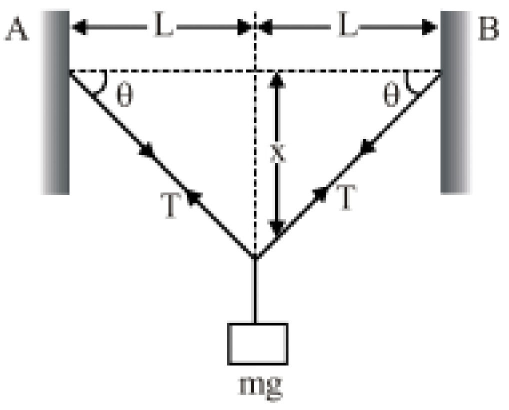
and for rotational equilibrium of rod

$$\Sigma\tau = T_1\, x - T_2\,(2 - x) = 0$$

or $$(W/2)\, x = (W/2)(2 - x), \qquad \text{i.e. } x = 1 \text{ m}$$

Q.3 A steel wire of diameter 0.8 mm and length 1 m is clamped firmly at two points A and B which are 1 m apart and in the same plane. A body is hung from the middle point of the wire such that the middle point sages 1 cm lower from the original position. Calculate the mass of the body. Given that Young's modulus of the material of wire is 2×10^{11} N/m^2.

Sol. As shown in figure, mass M is in equilibrium



$$Mg = 2T\sin\theta \qquad .....(i)$$

But from the geometry of figure, for small angle θ, we have

$$\sin\theta \approx \tan\theta = (x/L) \qquad ....(ii)$$

and by definition of Young's modulus, we have

$$T = \frac{YA}{L}\Delta L = \frac{YA}{L}\left[\left(L^2 + x^2\right)^{1/2} - L\right] \approx \frac{YAx^2}{2L^2} \qquad ....(iii)$$

So substituting the values of sin θ and T from equation and (iii) in (i), we get

$$Mg = 2 \times \frac{YAx^2}{2L^2} \times \frac{x}{L}$$

$$M = \frac{YAx^3}{gL^3} \qquad ....(iv)$$

Now as here $2L = 1$ m, $x = 1$ cm $= 10^{-2}$ m

and $A = \pi r^2 = p\,[(0.8/2)\times10^{-3}]^2$

$= \pi \times (4 \times 10^{-4})^2$ m^2

$$\text{so } M = \frac{2\times10^{11} \times \pi(4\times10^{-4})^2 \times (10^{-2})^3}{9.8\times(1/2)^3}\,\text{kg} = 82\text{g}$$

from equation (iv), we have

$$x = L\left(\frac{Mg}{YA}\right)^{1/3}$$

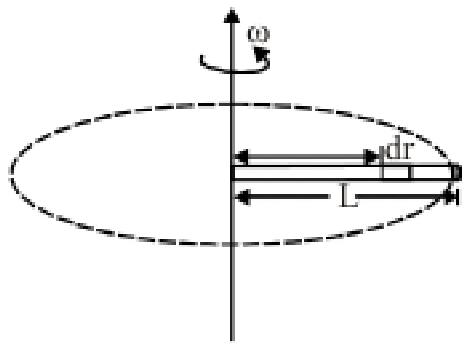
So the angle θ can be determined from

$$\theta = \frac{x}{L} = \left(\frac{Mg}{YA}\right)^{1/3}$$

Q.4 A thin unifrom metallic rod of length 0.5 m and radius 0.1 m rotates with an angular veloicty 400 rad/s in a horizontal plane about a vertical axis passing through one of its ends. Calculate (a) the tension in the rod and (b) the elongation of the rod. The density of the meterial of the rod is 10^4 kg/m^3 and the Young's modulus is 2×10^{11} N/m^2.

Sol. (a) We take a differential element of the length dr at a distance r form the axis of rotation as shown in figure. The centripetal force acting on this element is

$$dT = dmr\omega^2 = (\rho A dr)r\omega^2$$



The tension in the rod at a distance r from the axis of rotation will be due to the centripetal force due to all elements between x = r to x = L,

i.e., $$T = \int_r^L \rho A\omega^2 r dr = \frac{1}{2}\rho A\omega^2[L^2 - r^2] \quad .....(i)$$

Thus, tension as function of r,

$$T = \frac{1}{2} \times 10^4 \times \pi \times 10^{-2} \times (400)^2 \left[\left(\frac{1}{2}\right)^2 - r^2\right]$$

$$= 8\pi \times 10^6 \left[\frac{1}{4} - r^2\right] N$$

Note that tension in the rod is minimum at r = L and maximum at r = 0

(b) Let dy be the eleongation in the element of length dr at position r due to tension, T. From definition of Young's modulus,

$$\text{Strain} = \frac{\text{stress}}{Y}$$

so $$\frac{dy}{dr} = \frac{T}{AY}$$

From equation (i), we have $dy = \frac{1}{2}\frac{\rho\omega^2}{Y}[L^2 - r^2]\, dr$

So the elongation of the entire rod,

$$\Delta L = \frac{\rho\omega^2}{2Y}\int_0^L [L^2 - r^2]\, dr$$

$$= \frac{1}{3}\frac{\rho\omega^2 L^3}{Y}$$

Here $$\Delta L = \frac{1}{3} \times \frac{10^4 \times (400)^2 (0.5)^3}{2 \times 10^{11}}$$

$$= \frac{1}{3} \times 10^{-3} m$$

Q.5 Estimate the pressure deep inside the sea at a depth h below the surface. Assume that the density of water is ρ_0 at sea level and its bulk modulus is B.

Sol. In a static fluid the pressure variation is given by

$$\frac{dP}{dh} = -\rho g \qquad \text{...(i)}$$

The bulk modulus is defined as

$$B = -\frac{dP}{dV/V} \qquad \text{...(ii)}$$

Where dV/V is fractional change in volume of a element subjected to isotropic pressure increase dP. We consider a sample of the fluid having mass M, its volume $V = M/\rho$, so that

$$dv = \frac{-M}{\rho^2} d\rho$$

Hence $$\frac{dV}{V} = -\frac{d\rho}{\rho} \qquad \text{....(iii)}$$

combining equations (ii) and (iii), we get

$$\frac{Bd\rho}{\rho} = \rho g \ dh$$

or $$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2} = \int_0^h \frac{gdh}{B}$$

or $$\frac{1}{\rho^0} - \frac{1}{\rho} = \frac{gh}{B} \qquad \text{....(iv)}$$

as $$dP = -\frac{BdV}{V} = B\frac{d\rho}{\rho}$$

Hence $$\int_{P_0}^{P} dP = \int_{\rho_0}^{\rho} B\frac{d\rho}{\rho}$$

or $$P - P_0 = B \ \ell n \frac{\rho}{\rho_0} \qquad \text{....(v)}$$

On multiplying equation (iv) by ρ_0, we get

$$1 - \frac{\rho_0}{\rho} = \frac{\rho_0 gh}{B}$$

so that $$\text{In} \frac{\rho}{\rho_0} = -\ell n\left(1 - \frac{\rho_0 gh}{B}\right)$$

Substituting this in equation (v), we get

$$P = P_0 - B \ \ell n\left(1 - \frac{\rho_0 gh}{B}\right)$$

This is the required expression for P.

Q.6 A 5g piece of ice at – 20°C is put into 10g of water at 30°C. Assuming that heat is exchanged only between the ice and the water, find the final the final temperature of the mixture. Specific heat capacity of ice = 2100 J/kg-°C specific heat capacity of water = 4200 J/kg-°C and latent heat of fusion of ice = 3.36×10^5 J/kg.

Sol. The heat given by the water when it cools down from 30°C to 0°C is

$$(0.01 \text{ kg})(4200 \text{ J/k-°C})(30°\text{C}) = 1260 \text{ J.}$$

The heat required to bring the ice to 0°C is $(0.005 \text{ kg})(2100 \text{ J/kg-°C})(20°\text{C}) = 210$ J.

The heat required to melt 5g of ice is $(0.005 \text{ kg})(3.36 \times 10^5 \text{ J/kg}) = 1680$.

We see that whole of the ice cannot be melted as the required amount of heat is not proved by the water. Also, the heat is enough to bring the ice to 0°C. Thus the final temperature of the mixture is 0°C with some of the ice melted.

Q.7 Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents riseds to 80°C. The mass of the steam condensed in kilogram is

(A) 0.130 (B) 0.065 (C) 0.260 (D) 0.135

Sol. Heat required to bring water and calorimeter from 15°C to 80°C

$$Q = mC\Delta T$$

$$Q = (1.1+0.02) \times 1 \times (80-15)$$

$$Q = 72.8 \text{ kcal}$$

Amount of steam condensed to provide 72.8 kcal heat = $m_1 L$

$$\Rightarrow \quad 72.8 \times 1000 = m_1 [540]$$

$$m_1 = 134.8 \text{ gm}$$

$$m_1 = 0.1348 \text{ kg}$$

Q.8 A bullet of mass of 10 gm moving with a speed of 20 m/s hits an ice block of mass 990 gm kept on a frictionless floor and gets stuck in it. How much ice will melt if 50% of the lost kinetic energy goes to ice? (Temperature of ice block = 0°C)

Sol. Velocity of bullet + ice block, $V = \dfrac{(10\text{gm}) \times (20\text{m/s})}{1000\text{gm}} = 0.2\text{m/s}$

$$\text{Loss of K.E.} = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)V^2$$

$$= \frac{1}{2}\left[0.01 \times (20)^2 - 1 \times (0.2)^2\right] = \frac{1}{2}[4 - 0.04] = 1.98\text{J}$$

$$\therefore \quad \text{Heat generated} = \frac{1.98}{4.2}\text{Cal}$$

$$\therefore \quad \text{Heat recived by ice block} = \frac{1.98}{4.2 \times 2}\text{Cal} = 0.24 \text{ cal}$$

$$\therefore \quad \text{Mass of ice malted} = \frac{(0.24\text{Cal.})}{(80\text{Cal/gm})} = 0.003\text{gm}$$

Q.9 A calorimeter of water equivalent 1 kg contains 10 kg of ice & 10 kg of water in thermal equilibrium. The atmospheric temperature is 15° below freezing point due to which the calorimeter loses heat. As a result ice is formed inside the calorimeter at a rate of 10.8 gm per second. To try to compensate for this heat loss, steam at 100°C is supplied to the calorimeter at a rate of r. (L_V = 540 cal/gm, L_f = 80 cal/gm, sp heat of water 1 cal/gm °C.) Column-I gives the value of r and column-II gives the situation just after the introduction of steam.

	Column-I		**Column-II**
(A)	r = 1.6 gm/sec	(P)	Amount of ice in calorimeter increases.
(B)	r = 1.35 gm/sec	(Q)	Amount of water in calorimeter increases.
(C)	r = 1.2 gm/sec	(R)	Amount of ice remains constant at 10 kg
(D)	r = 1 gm/sec	(S)	Amount of water remains constant at 10 kg
		(T)	Amount of ice in calorimeter decreases.

[Ans. (A) Q, T ; (B) Q, R ; (C) P, S ; (D) P]

Sol. Rate of heat loss = 80 × 10.8 = 54 × 16 cal/sec.

(A) r = 1.6

⇒ rate of heat supplies by forming steam to water at 0° = 1.6 × 640 > 54 × 16

∴ additional ice will melt. Correct options are Q and T

(B) Rate of heat loss = 54 × 16 = 64 × 13.5 cal/sec.

r = 1.35

= rate of heat supplied for converting steam to water at 0°C = 1.35 × 640 = 13.5 × 64.

no additional ice will melt or water will fuse. Correct options are Q and R

(C) Rate of heat loss = 54 × 16 = 72 × 12 cal/sec.

Rate of heat supplied by converting steam to ice at 0°C = 1.20 × 720 = 12 × 72 cal/sec

no additional ice will melt or water will fuse. Correct options are P and S

(D) Additional water will fuse to ice. Correct option is P.

Q.10 An ice cube of mass 0.1kg at 0°C is placed in an isolated container which is at 227°C. The specific Heat S of the container varies with temperature T according to relation S=A+BT, where A = 100 cal/kg-k and B= 2×10^{-2} cal/kg-k^2. If the final temperature of the container is 27°C, Determine the mass of the container.

Sol. Specific heat of container is temperature dependent so we have to calculate heat lost for a small temperature change dT and then integrate it from initial temperature to final temperature.

$$\text{Heat lost by container} = -\int_{500}^{300} m_C(A+B+T)dT$$

$$= -m_C\left[AT + \frac{BT^2}{2}\right]_{500}^{300} \qquad = 21600 m_C$$

Heat gained by Ice = mL + mCΔT

$= 0.1\times1000\times80+0.1\times1000\times27$

$= 10700$ Cal

from principle of colorimetry

Heat lost by container = Heat gained by Ice

$21600\ m_C = 10700$

$m_C = 0.495$ kg

Q.11 A certain clock with an iron pendulum is made so as to keep correct time at 10°C. Given $\alpha_{iron}=12\times10^{-6}$ per °C. How will the rate alter if the temperature rises to 25° C?

Sol. When the pendulum keeps correct time, its period of vibration is 2 sec and so it makes

$$\frac{20\times60\times60}{2} = 43200 \text{ Vibrations/day}$$

If length of pendulum at 10°C is ℓ_{10} and at 25°C is ℓ_{25}

$\therefore\quad \ell_{25} = \ell_{10}[1+\alpha(25-10)] = \ell_{10}[1+15\alpha]$

as $\quad T = 2\pi\sqrt{\frac{\ell}{g}}$

i.e. $\quad T \propto \sqrt{\ell}$

i.e. $\quad n \propto \frac{1}{\sqrt{\ell}}$, n is no. of vibrations per sec.

$$\therefore\quad \frac{n_{25}}{n_{10}} = \sqrt{\frac{\ell_{10}}{\ell_{25}}} = [1+15\alpha]^{-1/2} \approx 1-\frac{15}{2}\alpha$$

$$\therefore\quad n_{25} = n_{10}\left(1-\frac{15}{2}\times12\times10^{-6}\right)$$

$= 43200\,[1-0.00009]=43196.12$

That is the clock makes $(43200 - 43196.12) = 3.88$ vibration loss per day. That is clock losses $3.88\times2 = 7.76$ sec per day

Q.12 A sphere of silver is floating in a mercury bath. If temperature is increased will the sphere sink deeper or rise ? It is given $\gamma_{silver} > \gamma_{mercury}$

Sol. Rise

Q.13 A glass vessel of volume V_0 is completely filled with a liquid and its temperature is raised by ΔT. What volume of the liquid will overflow ? Coefficient of linear expansion of glass = α_g and coefficient of volume expansion of the liquid = γ.

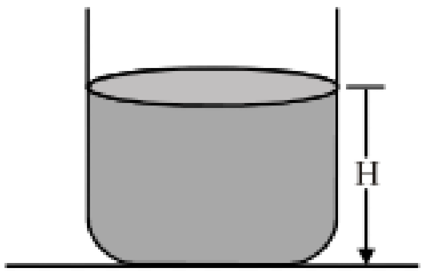
Sol. Volume of the liquid over flown

= increase in the volume of the liquid – increase in the volume of the container

$= [V_0(1+\gamma_\ell\Delta T) - V_0] - [V_0(1+\gamma_g\Delta T) - V_0]$

$= V_0\Delta T(\gamma_\ell - \gamma_g) = V_0\Delta T(\gamma_\ell - 3\alpha_g)\qquad (\because \gamma \approx 3\alpha)$

Q.14 A liquid having coefficient of volume expansion γ_0 is filled in a glass vessel. The coefficient of linear expansion of glass is α. When the arrangement is heated to raise the temperature of the liquid and the glass container by ΔT, exapnsion takes place in both. The expansion may be different or equal. Depending on the values of γ_0 and α you may find that level of the liquid rises with respect to ground or it may fall with repect to ground.



(i) What is the relation between γ_0 and α so that the fraction of volume of container occupied by the liquid does not change with rise in temperature ?

(ii) What is the relation between γ_0 and α so the liquid does not change with respect to ground ?

(iii) What is the relation between γ_0 and α so that the level of the liquid does not change with respect to the container itself ?

Sol. (i) Let V_C be the volume of the container and V_ℓ be the volume of the liquid. According to the condition

$$\frac{V_\ell}{V_C} = \text{constant (i.e. independent of temperture)}$$

$$\Rightarrow \quad \frac{V'_\ell}{V'_C} = \frac{V_\ell}{V_C} \quad \text{or} \quad \frac{V'_\ell}{V_\ell} = \frac{V'_C}{V_C} \quad \text{(Here } V'_\ell \text{ and } V'_C \text{ are volume on heating)}$$

$$\Rightarrow \quad \frac{V'_\ell - V_\ell}{V_\ell} = \frac{V'_C - V_C}{V_C}$$

$$\Rightarrow \quad \frac{\Delta V_\ell}{V_\ell} = \frac{\Delta V_C}{V_C}$$

$\gamma_0 \Delta T = (3\alpha)\Delta T$ [For container, coefficient of volume expansion will be 3α]

(ii) If on heating, H does not change, than the increase in volume of the liquid is accomodated by the increase in base area of the vessel. Let the area be A.

Initial volume of liquid $= V_\ell = A \times H$

Final volume of liquid $= V'_\ell = A' \times H$

$$\Rightarrow \quad \frac{V'_\ell}{V_\ell} = \frac{A'}{A} \Rightarrow \frac{\Delta V'_\ell}{V_\ell} = \frac{\Delta A'}{A}$$

$$\Rightarrow \quad \gamma_0 \,\Delta T = \beta \Delta T \text{ or } \gamma_0 = 2\alpha \qquad (\therefore \beta = 2\alpha)$$

(iii) This is exactly same as part (i)

$\gamma = 3\alpha$.