

# Wave Optics

## Introduction

In Geometrical Optics we studied light rays passing through a lens or reflecting from a mirror to describe the formation of image. In this chapter, we are concerned with wave optics or physical optics, the study of interference & diffraction. These phenomena cannot be adequately explained with the ray approximation used. We now learn how treating light as waves rather than as rays leads to a satisfying description of such phenomena.

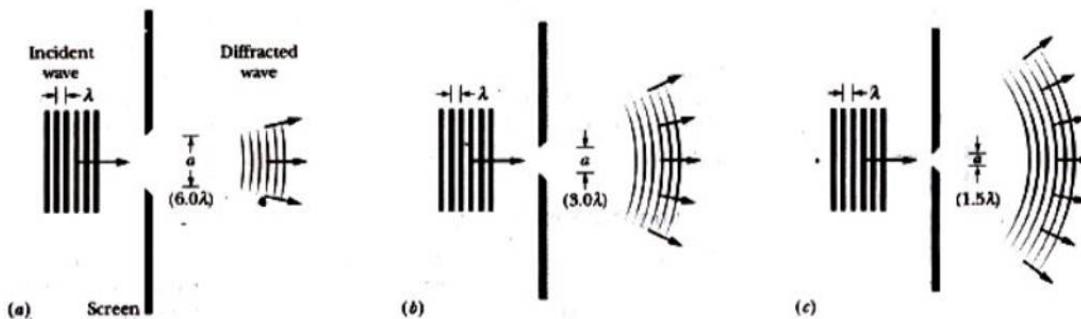
Rainbow shows all the seven colors of visible light, but that is due to dispersion. Whereas, an oil film floating on water also shows seven colors, but the color in the oil film is due to interference of light in it. You might have seen the same in case of soap bubble.

In previous chapter, we considered light to be travelling in a straight line path. However, we know that light is an electromagnetic wave. Thus, it should exhibit wave characteristic as well. And how a soap film shows seven colors can be explained using this wave phenomenon.

## Diffraction

In the next section we shall discuss the experiment that first proved that light is a wave. To prepare for that discussion, we must introduce the idea of diffraction of waves, a phenomenon that we explore much more fully in later stage. Its essence is this: *If a wave encounters a barrier that has an opening of*

*more fully in later stage. Its essence is this: If a wave encounters a barrier that has an opening of dimensions similar to the wavelength, the part of the wave that passes through the opening will flare (spread) out -will diffract-into the region beyond the barrier.* Diffraction occurs for waves of all types, not just light waves;



A plane wave going through a small opening becomes more like a spherical wave on the other side. Thus, the wave bends at the edges. Also, if the dimensions of the obstacle or the opening is much larger than the wavelength, the diffraction is negligible and the rays go along straight lines.

You must have observed the above phenomenon, if there is a small hole on a wall, light coming from it spreads in a larger area.

In the case of light, the wavelength is around 380-780 nm. The obstacles or openings encountered in normal situations are generally of the order of millimeters or even larger. Thus, the wavelength is several thousands times smaller than the usual obstacles or openings. The diffraction is almost negligible and the

light waves propagate in straight lines and cast shadows of the obstacles. The light can then be treated as light rays which are straight lines drawn from the source and which terminate at an opaque surface and which pass through an opening undeflected. This is the Geometrical optics approximation and majority of the phenomena in normal life may be discussed in this approximation.

## Principle of Superposition

We have seen that waves are very different from particles. One of the important differences between waves and particles is that we can explore the possibility of two or more waves combining at one point in the same medium. Particles can be combined to form extended objects, but the particles must be at different locations. In contrast, two waves can both be present at the same location.

*If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.*

Two traveling waves can pass through each other without being destroyed or even altered. For instance, when two pebbles are thrown into a pond and hit the surface at different locations, the expanding circular surface waves from the two locations do not destroy each other but rather pass through each other. The resulting complex pattern can be viewed as two independent sets of expanding circles.

Pictorial representation of the superposition of two pulses is given. The wave function for the pulse

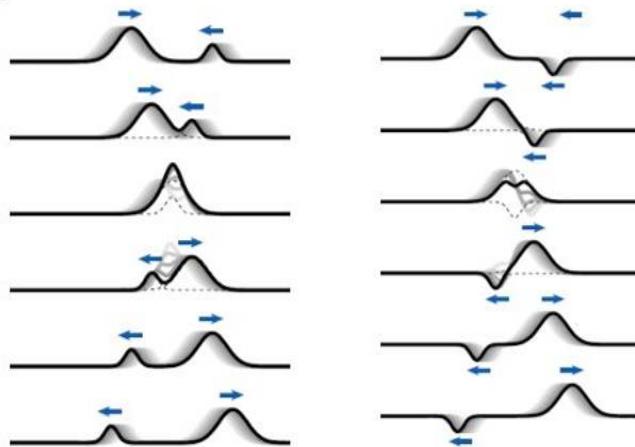
Pictorial representation of the superposition of two pulses is given. The wave function for the pulse moving to the right is  $y_1$ , and the wave function for the pulse moving to the left is  $y_2$ . The pulses have the same speed but different shapes, and the displacement of the elements of the medium is in the positive y direction for both pulses. When the waves begin to overlap, the wave function for the resulting complex wave is given by  $y_1 + y_2$ . When the crests of the pulses coincide, the resulting wave given by  $y_1 + y_2$  has larger amplitude than that of the individual pulses. The two pulses finally separate and continue moving in their original directions. Notice that the pulse shapes remain unchanged after the interaction, as if the two pulses had never met !

The combination of separate waves in the same region of space to produce a resultant wave is called interference. For the two pulses shown in figure, the displacement of the elements of the medium is in the positive y direction for both pulses, and the resultant pulse (created when the individual pulses overlap) exhibits an amplitude greater than that of either individual pulse. Because the displacements caused by the two pulses are in the same direction, we refer to their superposition as constructive interference.

Now consider two pulses traveling in opposite directions on a taut string where one pulse is inverted relative to the other as illustrated in figure. When these pulses begin to overlap, the resultant pulse is given by  $y_1 + y_2$ , but the values of the function  $y_2$  are negative. Again, the two pulses pass through each other, however, we refer to their superposition as destructive interference.

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The superposition principle is the centerpiece of the waves in interference model. In many situations, both in acoustics and optics, waves combine according to this principle and exhibit interesting phenomena with practical application.



## Theory of Interference

Consider a homogeneous medium in which there are two point sources of sinusoidal spherical waves,  $S_1$  and  $S_2$  with the same period  $T$ . Let  $E_1$  and  $E_2$  be the optical disturbances arriving from the two sources at a point P. These disturbances can be written as

$$\begin{aligned} E_1 &= A_1 \sin \omega t \\ E_2 &= A_2 \sin (\omega t + \phi) \end{aligned}$$

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Let the amplitudes  $A_1$  and  $A_2$  depend on the strengths of the sources and on the distance of the sources from P. From principle of superposition the resultant optical disturbance at P is a sinusoidal function of angular frequency  $\omega$  and amplitude A given by

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \phi \quad (\text{from superposition}) \quad \dots(\text{ii})$$

We know that  $I \propto A^2$ . Hence, the distribution of light intensity in the region of space surrounding the sources is given by :

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \quad \dots(\text{iii})$$

Intensity will be maximum when  $\cos \phi = 1$

$$\therefore I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

Intensity will be minimum when  $\cos \phi = -1$

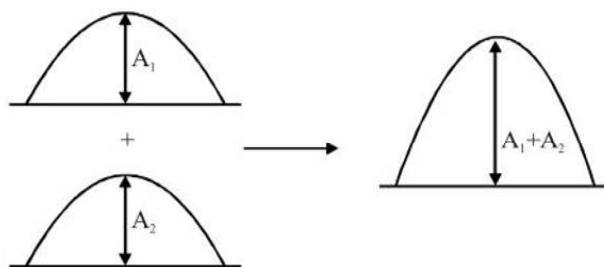
$$\therefore I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

Where  $I_1$  and  $I_2$  are the intensities observed when one or the other source is present alone and I is the intensity observed when both sources are present simultaneously. We see that the resultant intensity I is greater or smaller than the sum of the two separate intensities,  $I_1 + I_2$ , depending on whether the third term on the right side of equation is positive or negative. This term represents the effect of interference.

Intensity maxima are found at points where the two waves are in phase; and minima are found at points where two waves are out of phase. Thus, interference phenomena have a considerable effect on the local distribution of light intensity in the space surrounding the source. They do not, however, change the space average of the intensity, which remains equal to the space average of  $I_1 + I_2$  as is required by the principle of conservation of energy. We see immediately that this is true when we note that the average value over space of the third term in equation (iii) is zero.

Now question arises; how these maxima and minima occur ?

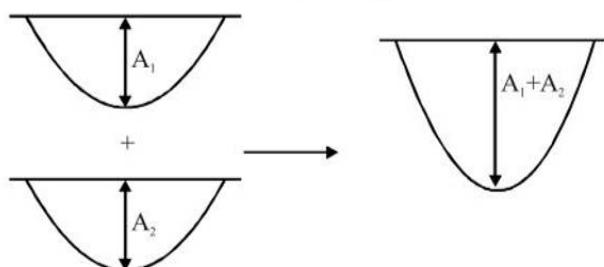
Intensity maxima occurs where amplitudes of two interfering waves add to give the maximum value. i.e. when maximum positive value, of one wave appears simultaneously with the maximum positive value of the other.



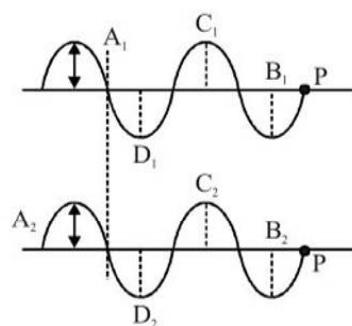
or the negative extreme of one coincides with negative extreme of the other wave.



or the negative extreme of one coincides with negative extreme of the other wave.



To obtain a maxima at a point continuously for a long time, we must obtain the wave at that point in same phase i.e. the crest must always appear with crest, and trough with lengths are same, as shown in figure.



at point P, a continuous maxima will appear if  $B_1$  and  $B_2$  reach there simultaneously, also  $C_1$ ,  $C_2$  and  $D_1$ ,  $D_2$  must follow same. As the velocity of light depends only on the medium and is therefore same for both waves; the above condition can be achieved only if

$$\begin{aligned}B_1 P &= B_2 P \\C_1 P &= C_2 P \\i.e. \quad \lambda_1 &= \lambda_2 \\ \Rightarrow \quad \lambda_1 &= \frac{c}{f_1} \quad ; \quad \lambda_2 = \frac{c}{f_2} \\ \therefore \quad f_1 &= f_2\end{aligned}$$

Such waves for which frequency is same are called coherent waves and corresponding sources are coherent sources.

## Coherent and Incoherent Sources

Why do we not commonly see interference effects with visible light? With light from a source such as the Sun, an incandescent bulb, or a fluorescent bulb, we do not see regions of constructive and destructive interference; rather, the *intensity* at any point is the sum of the intensities due to the individual waves. Light from anyone of these sources is, at the atomic level, by electronic transitions from one energy level to another which can not be externally controlled. *Hence two independent sources identical in all respects can not be coherent.*

$$I = I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos \phi \quad \dots(i)$$

Waves from independent sources are incoherent; they do not maintain a fixed phase relationship with each other (i.e.  $\phi$  varies with time). We cannot accurately predict the phase (for instance, whether the wave is at a maximum or at a zero) at one point given the phase at another point. *Incoherent waves have*

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Only the superposition of coherent waves produces sustained interference. Coherent waves must be locked in with a fixed phase relationship. *Coherent and incoherent waves are idealized extremes; all real waves fall somewhere between the extremes.* The light emitted by a laser can be highly coherent—two points in the beam can be coherent even if separated by as much as several kilometers.

### Important points about Coherent source :

The sources which produce sustained, i.e. observable interference are called coherent sources. In case of interference as  $I = [I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos \phi]$ , interference will be sustained if the phase difference  $\phi$  at a given point does not vary with time. If the interfering wave are :

$$y_1 = A_1 \sin(\omega_1 t - k_1 x_1 + \phi) \text{ and } y_2 = A_2 \sin(\omega_2 t - k_2 x_2 + \phi_2)$$

$$\phi = (\omega_1 - \omega_2) t + (k_2 x_2 - k_1 x_1) + (\phi_1 - \phi_2)$$

So  $\phi$  will not vary with time if :

- (a)  $(\phi_1 - \phi_2) = \phi_0$  is constant, i.e., the initial phase difference between the wave does not vary with time and
- (b)  $(\omega_1 - \omega_2) t = 0$ , i.e.,  $f_1 = f_2$ . But for a wave as  $v = f\lambda$ ,  $f_1 = f_2$  will also mean  $\lambda_1 = \lambda_2$ , i.e.,  $k_1 = k_2$  [as  $k = 2\pi / \lambda$ ].

i.e., the two wave are of same frequency and wavelength. So two sources will be coherent if and only if they produce wave of same frequency (and hence wavelength) and have a constant initial phase difference. So in case of two coherent sources.

$$\phi = \frac{2\pi}{\lambda} (\Delta x) + \phi_0 \quad \text{with } \phi_0 = (\phi_1 - \phi_2)$$

Now as in general emission of light from atoms is random, rapid and independent of each other,  $\phi_0$  cannot remain constant with time and hence two independent light source identical in all respects cannot be coherent.

### **Illustration:**

*Two coherent monochromatic light beams of intensities  $I$  and  $4I$  are superposed. Find the maximum and minimum possible intensities in the resulting beam.*

**Sol.**  $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I} + \sqrt{4I})^2 = 9I$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{I} - \sqrt{4I})^2 = I$$

### **Illustration :**

*In a Young's double slit experiment, the amplitude of intensity variation of the two sources is found to be 3% of the average intensity. Find the ratio of intensities of the two interference sources.*

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Let  $a_1$  and  $a_2$  be the amplitudes of vibrations from the two sources, then

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{a_1^2 \left(1 + \frac{a_2}{a_1}\right)^2}{a_1^2 \left(1 - \frac{a_2}{a_1}\right)^2} \quad \text{or} \quad \frac{I_{max}}{I_{min}} = \frac{\left(1 + \frac{a_2}{a_1}\right)^2}{\left(1 - \frac{a_2}{a_1}\right)^2}$$

*It is given that the amplitude of intensity variation of the two sources is found to be 3% of the average intensity, It means if average intensity is  $100I$  then maximum intensity is  $103I$  and minimum is  $97I$*

$$I_{max} = 103I \quad \text{and} \quad I_{min} = 97I$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{103}{97}$$

*Substituting in equation*

$$\frac{103}{97} = \frac{\left(1 + \frac{a_2}{a_1}\right)^2}{\left(1 - \frac{a_2}{a_1}\right)^2}$$

$$\therefore \frac{\left(1 + \frac{a_2}{a_1}\right)}{\left(1 - \frac{a_2}{a_1}\right)} = \sqrt{\frac{103}{97}} = 1.03 \text{ or } \frac{a_1}{a_2} = \frac{1}{0.0148}$$

If  $I_1$  and  $I_2$  are the intensities produced by the two sources,

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \quad \text{or} \quad \frac{I_1}{I_2} = \left(\frac{1}{0.0148}\right)^2 = 4565 \text{ Ans.}$$

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### Practice Exercise

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- Q.1 Two coherent monochromatic beams are superposed. The minimum and maximum intensities in the resulting interference pattern are found to be  $4I$  and  $16I$  respectively. Find the initial intensities of the two sources.
- Q.2 Two coherent monochromatic light beams of intensities  $4I$  and  $16I$  are superposed. Find the maximum and minimum possible intensities in the resulting beam.
- Q.3 In the above question if the phase difference between the two beams, at a point is  $\pi/2$ . Find the resultant intensity at that point.

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### Answers

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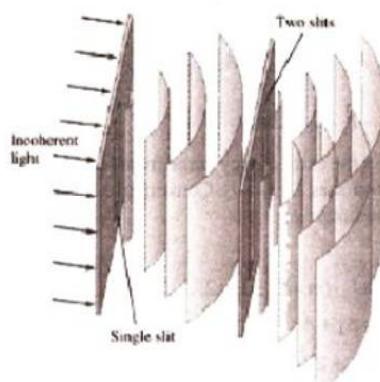
### Answers

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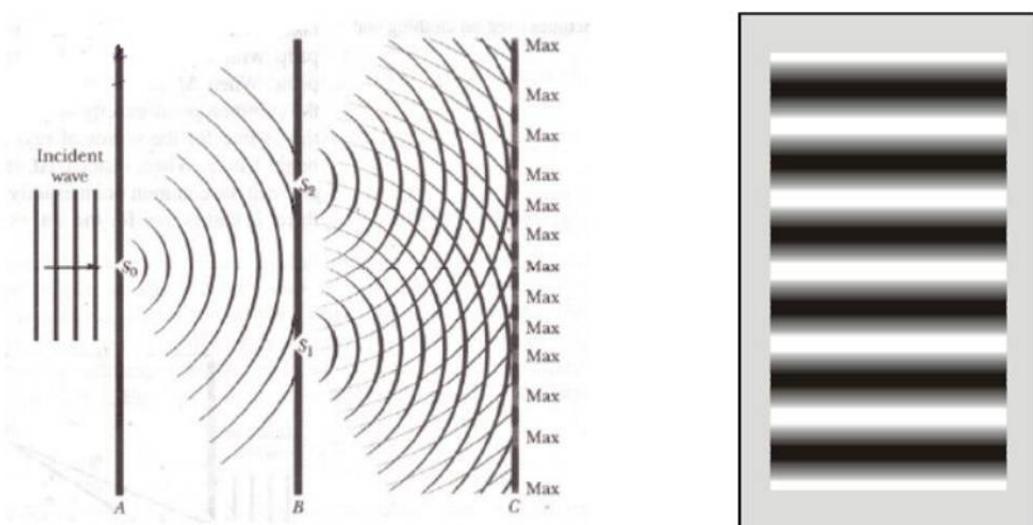
- Q.1  $9I$  and  $I$       Q.2  $36I$  and  $4I$       Q.3  $20I$
- 

### **Young's Double Slit Experiment :**

Thomas Young (1773-1829) performed the first visible-light interference experiments using a clever technique to obtain two coherent light sources from a single source. When a single narrow slit is illuminated, the light wave that passes through the slit diffracts or spreads out. The single slit acts as a single coherent source to illuminate two other slits. These two other slits then act as sources of coherent light for interference.



*Young's technique for illuminating two slits with coherent light. The single slit on the left serves as a source of coherent light.*



In Young's interference experiment, incident monochromatic light is diffracted by slit  $S_0$ , which then acts as a point source of light that emits semicircular wavefronts. As that light reaches screen  $B$ , it is diffracted by slits  $S_1$  and  $S_2$ , which then act as two point sources of light. The light waves traveling from slits  $S_1$  and  $S_2$  overlap and undergo interference, forming an interference pattern of maxima and minima on viewing screen  $C$ . This figure is a cross section; the screens, slits, and interference pattern extend into and out of the page. Between screens  $B$  and  $C$ , the semicircular wavefront's centered on  $S_2$  depict the waves that would be there if only  $S_2$  were open. Similarly, those centered on  $S_1$  depict waves that would be there if only  $S_1$  were open. Points of interference maxima form visible bright rows-called *bright bands*, *bright fringes*, or (loosely speaking) *maxima* that extend across the screen

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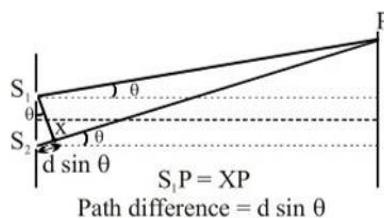
### Intensity of Two Source Interference :

We now obtain an expression for the distribution of intensity of two coherent sources that are in phase. The wave function in this case is the electric field. We assume that the slits are narrow enough for diffraction to spread light from each slit uniformly over the screen. Thus, the amplitude of the fields at any point on the screen will be equal. At a given point of the screen the fields due to  $S_1$  and  $S_2$  are

$$E_1 = E_0 \sin(\omega t); \quad E_2 = E_0 \sin(\omega t + \phi)$$

where the phase difference  $\phi$  depends on the path difference  $\Delta x = r_2 - r_1$ . Since one wavelength  $\lambda$  corresponds to a phase change of  $2\pi$ , a distance  $\delta$  corresponds to a phase change  $\phi$  given by  $\phi/2\pi = \Delta x/\lambda$ . If the screen is far from the slits,  $\delta \approx d \sin \theta$ , therefore

$$\phi = \frac{2\pi\delta}{\lambda} = \frac{2\pi d \sin \theta}{\lambda}$$



The resultant field is found from the principle of superposition :

$$E = E_1 + E_2 = E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi)$$

By using the trigonometric identity  $\sin A + \sin B = 2 \sin[(A+B)/2] \cos[(A-B)/2]$ , we obtain

$$E = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

The amplitude of the resultant wave is  $2E_0 \cos(\phi/2)$ . The intensity of a wave is proportional to the square of the amplitude, so from equation of wave we have

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

where  $I_0 \propto E_0^2$  is the intensity due to a single source. The maxima occur when  $\phi = 0, 2\pi, 4\pi, \dots = 2m\pi$ . At these points  $I = 4I_0$ ; that is the intensity is four times that of a single source. The minima ( $I = 0$ ) occur when  $\phi = \pi, 3\pi, 5\pi, \dots = (2m + 1)\pi$ .

Fig.(a) shows the waves emitted by sources  $S_1$  and  $S_2$ . The waves from the source start in phase and arrive in phase, leading to constructive interference at the point.

The distances travelled by waves differ by any integer number of wavelengths.

$$x_2 - x_1 = \lambda, 2\lambda, 3\lambda, \dots, n\lambda$$

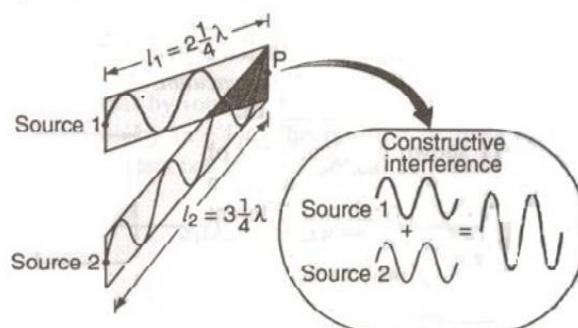
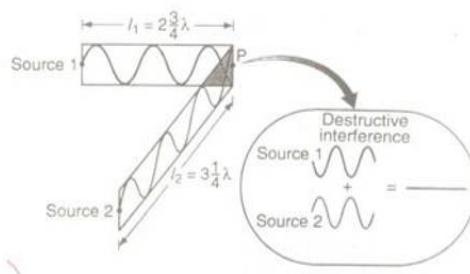


Fig. (b) shows two waves starting in phase but arriving in opposite phase.



Fig. (b) shows two waves starting in phase but arriving in opposite phase.  
the distance travelled by waves differ by odd integer number of wavelengths.

$$x_2 - x_1 = \frac{\lambda}{2}, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \dots, \left(n - \frac{1}{2}\right)\lambda$$



- (i) If amplitudes of waves arriving at point P on the screen are different then resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

Also,  $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ , when  $\cos \delta = 1$

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2, \quad \text{when } \cos \delta = -1$$

$$(ii) \quad \frac{I_{\max}}{I_{\min}} = \left( \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2 = \left( \frac{A_1 + A_2}{A_1 - A_2} \right)^2 = \left( \frac{r+1}{r-1} \right)^2$$

where  $r = \frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}}$

- (iii) The phenomenon of interference is based on conservation of energy. There is no destruction of energy in the interference phenomenon. The energy which apparently disappears at the minima, has actually been transferred to the maxima where the intensity is greater than that produced by the two beams acting separately.

$$I_{av} = \frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} = \frac{1}{2\pi} \int_0^{2\pi} (I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta) d\delta = I_1 + I_2$$

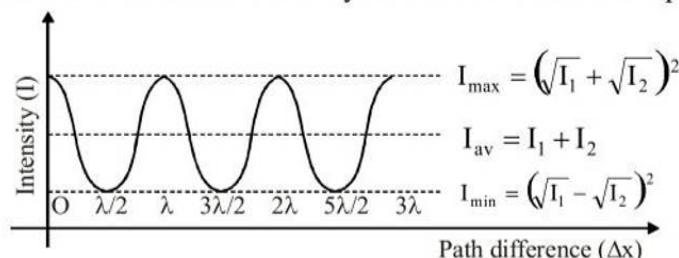
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- (iv) All maxima are equally spaced and equally bright. This is true for minima as well. Also interference maxima and minima are alternate. The intensity distribution in interference pattern is shown in figure.



- (v) The fringe visibility (v) is given by

$$v = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2A_1 A_2}{A_1^2 + A_2^2}$$

when  $I_1 = I_2$

or  $A_1 = A_2$

$I_{\min} = 0$

hence,  $v = 1$  (best visibility)

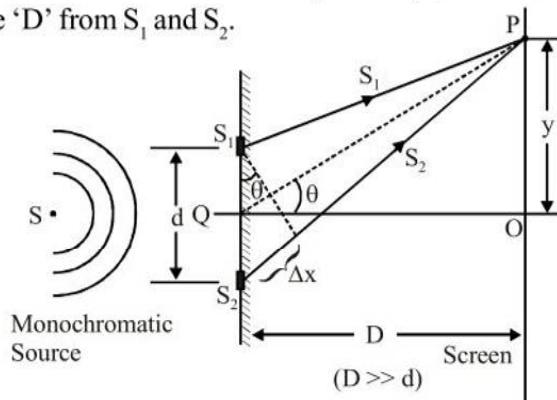
(vi) Path difference ( $\Delta x$ ) and phase difference ( $\phi$ ) are related as given below :

$$\lambda \text{ path difference} = 2\pi \text{ phase difference}$$

$$\text{or } \Delta x = \frac{\lambda}{2\pi} \phi$$

### Maxima and Minima

The experiment set up for Young's double slit experiment is shown in figure. Light after passing through a pin hole 'S' is allowed to fall on thin slits 'S<sub>1</sub>' and 'S<sub>2</sub>' placed symmetrically w.r.t. 'S'. A screen is placed at a distance 'D' from S<sub>1</sub> and S<sub>2</sub>.



Geometric construction for describing Young's double-slit experiment

Let 'P' be the point, at which we want to investigate the intensity. Two rays S<sub>1</sub>P and S<sub>2</sub>P starting from S<sub>1</sub> and S<sub>2</sub> reach P and interfere with each other.

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If  $\Delta x$  is the path difference between two rays,

$$\begin{aligned} \Delta x &= \sqrt{D^2 + \left(y + \frac{d}{L}\right)^2} - \sqrt{D^2 + \left(y - \frac{d}{L}\right)^2} \\ &= D \left( 1 + \frac{\left(y + \frac{d}{L}\right)^2}{D^2} \right)^{1/2} - D \left( 1 + \frac{\left(y - \frac{d}{L}\right)^2}{D^2} \right)^{1/2} \end{aligned}$$

For small value of 'y'  $\ll D$  [using binomial approximation]

$$\begin{aligned} &= D \left( 1 + \frac{1}{2} \left( \frac{y+d/L}{D} \right)^2 \right) - D \left( 1 + \frac{1}{2} \left( \frac{y-d/L}{D} \right)^2 \right) \\ &= \left( \frac{y+d/L}{2D} \right)^2 - \left( \frac{y-d/L}{2D} \right)^2 \\ &= \frac{yd+yd}{2D} = \frac{2yd}{2D} = \frac{yd}{D} = d \operatorname{tna} \theta \end{aligned}$$

- (a) **Maxima :** Point 'P' will be a bright spot if the path difference  $\Delta x$  is integral multiple of  $\lambda$ .

$$\Delta x = \frac{yd}{D} = n\lambda$$

$$y_n = \frac{nD\lambda}{d} \quad \text{where, } n = 0, 1, 2, 3, \dots$$

Thus, bright spots are obtained at distances,  $0, \frac{\lambda D}{d}, \frac{2\lambda D}{d}, \frac{3\lambda D}{d}, \dots$  from O.

- (b) **Minima :** Point 'P' will be a dark spot if the path difference ' $\Delta x$ ' is an odd multiple of  $\frac{\lambda}{2}$ .

i.e., if  $\frac{yd}{D} = \frac{(2n+1)\lambda}{2}$

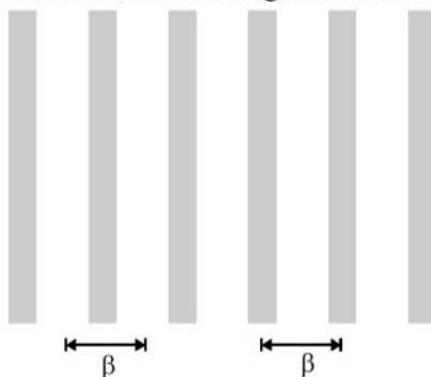
$$\therefore y_n = \frac{(2n+1)\lambda D}{2d} \quad \text{where, } n = 0, 1, 2, 3, \dots$$

Thus, dark spots are obtained at distances,  $\frac{\lambda D}{2d}, \frac{3\lambda D}{2d}, \frac{5\lambda D}{2d}, \dots$  from O.

- (c) **Fringe width ( $\beta$ )**

- (c) **Fringe width ( $\beta$ )**

It is the distance between two consecutive bright or dark fringes.



Let  $y_n$  and  $y_{n-1}$  respectively, be the distances of  $n^{\text{th}}$  and  $(n-1)^{\text{th}}$  bright fringe from O,

$$\beta = [y_n - y_{n-1}] = n \frac{\lambda D}{d} - (n-1) \frac{\lambda D}{d} = \frac{\lambda D}{d} (n - n + 1)$$

$$\text{or } \beta = \frac{\lambda D}{d}$$

Similarly, it can be proved that distance between two consecutive dark fringes,  $\beta$  is given by

$$\beta = \frac{\lambda D}{d}$$

$$\therefore \beta = \beta = \lambda D / d$$

Hence, the bright and dark fringes are equally spaced.

**Illustration :**

A beam of light consisting of two wavelengths  $6500 \text{ \AA}$  and  $5200 \text{ \AA}$  is used to obtain interference fringes in a Young's double slit experiment.

(i) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength  $6500 \text{ \AA}$ .

(ii) What is the least distance from the central maximum when the bright fringes due to both the wavelengths coincide?

The distance between the slit is  $2 \text{ mm}$  and the distance between the plane of the slits and the screen is  $120 \text{ cm}$ .

**Sol.**

$$\begin{aligned} \text{Given, } \lambda_1 &= 6500 \text{ \AA} = 6.5 \times 10^{-7} \text{ m} \\ \lambda_2 &= 5200 \text{ \AA} = 5.2 \times 10^{-7} \text{ m} \\ d &= 0.2 \text{ cm} = 2 \times 10^{-3} \text{ m} \\ D &= 120 \text{ cm} = 1.2 \text{ m} \end{aligned}$$

$$(i) \text{ For } n^{\text{th}} \text{ bright spot } y_n = n \frac{\lambda D}{d}$$

Here,  $n = 3$  and  $\lambda = \lambda_1 = 6.5 \times 10^{-7} \text{ m}$

$$y_3 = \frac{3 \times 6.5 \times 10^{-7} \times 1.2}{2 \times 10^{-3}} = 1.17 \times 10^{-3} \text{ m} \quad \text{Ans.}$$

.....

(ii) Since  $\lambda_2 < \lambda_1$ , fringe width for  $\lambda_2$  is smaller. If two bright fringes due to  $\lambda_1$  and  $\lambda_2$  are to coincide, then minimum distance from the central spot will be where  $n^{\text{th}}$  order bright spot due to  $\lambda_1$  and  $(n+1)^{\text{th}}$  bright spot due to  $\lambda_2$  coincide.

$$\begin{aligned} n \frac{\lambda_1 D}{d} &= \frac{(n+1)\lambda_2 D}{d} \\ n \times 6.5 \times 10^{-7} &= (n+1) \times 5.2 \times 10^{-7} \quad \text{or } n = 4 \end{aligned}$$

$$y = \frac{n\lambda D}{d} = \frac{4 \times 6.5 \times 10^{-7} \times 1.2}{2 \times 10^{-3}} = 1.56 \times 10^{-3} \text{ m Ans.}$$

## Important Points about YDSE

(i) If whole apparatus is immersed in liquid of refractive index  $\mu$  then,

$$\beta = \frac{\lambda D}{\mu d} \text{ i.e., fringes width decreases}$$

(ii) Some times in numerical problems, angular fringe width ( $\omega$ ) is given which is defined as angular separation between two consecutive maxima or minima

$$\omega = \frac{\beta}{D} = \frac{\lambda}{d}$$

In medium, other than air or vacuum,

$$\omega = \frac{\lambda}{\mu d}$$

- (iii)  $\Delta x = \frac{yd}{D}$  is valid when angular position of maxima or minima is less than  $\frac{\pi}{6}$ . However  $\Delta x = d \sin \theta$  is valid for larger values of  $\theta$  provided  $d \ll D$ .
- (iv) Central bright fringe (CBF) is a point on screen where path difference is zero. In above case CBF is formed at O. But in many situation it may not be located symmetrically w.r.t. slits.
- (v) If white light is used instead of monochromatic light then, interference pattern consists of white central bright fringe surrounded by few coloured fringes and then uniform illumination due to overlapping of interference pattern on each wavelength.
- (vi) If the interference experiment is performed with bichromatic light, the bright fringes of two wavelength will be coincident for the first time under following condition.

$$Y = n (\beta)_{\text{Longer}} = (n + 1) \beta_{\text{Shorter}} \quad \text{or} \quad n\lambda_{\text{Longer}} = (n + 1) \lambda_{\text{Shorter}}$$

- (vii) In many numerical problems we have to calculate number of maxima or minima. We know that for maximum.

$$\sin \theta = \frac{n\lambda}{d} \quad \text{or} \quad n = \frac{d \sin \theta}{\lambda}$$

$$\therefore n \frac{d}{\lambda} \geq 1 \quad (\because \sin \theta \leq 1)$$

$$n_{\text{highest}} = \left[ \frac{d}{\lambda} \right]$$

Where  $n_{\text{highest}} \Rightarrow$  Highest order of maxima on one side.

Suppose in some question  $\frac{d}{\lambda}$  works out to be 2.3 so, permissible values of  $n$  are  $0, \pm 1, \pm 2$ . Hence, total 5 maxima will be obtained on screen.

### **Illustration :**

In a Young's double slit experiment for interference of light, the slits are 0.2 cm apart and are illuminated by yellow light ( $\lambda = 600 \text{ nm}$ ). What would be the fringe width on a screen placed 1 m from the plane of slits if the whole system is immersed in water of refractive index 4/3?

**Sol.**

$$\text{Given, } \lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$$

$$D = 1 \text{ m}$$

$$d = 0.2 \text{ cm} = 2 \times 10^{-3} \text{ m}$$

As the apparatus is dipped in water ( $\mu = 4/3$ )

$$\lambda_{new} = \frac{\lambda}{\mu} = \frac{6 \times 10^{-7}}{4/3} m = 4.5 \times 10^{-7} m$$

$$\beta = \frac{\lambda_{new} D}{d} = \frac{4.5 \times 10^{-7} \times 1}{2 \times 10^{-3}} = 2.25 \times 10^{-4} m = 0.225 mm$$

**Illustration :**

In Young's double slit experiment the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slit. It is found that the 9th bright fringe is at a distance of 7.5 mm measured from the second dark fringe from the centre of the fringe pattern on same side. Find the wavelength of the light used.

**Sol.** Given,  $d = 0.5 mm = 5 \times 10^{-4} m$

$$D = 1m$$

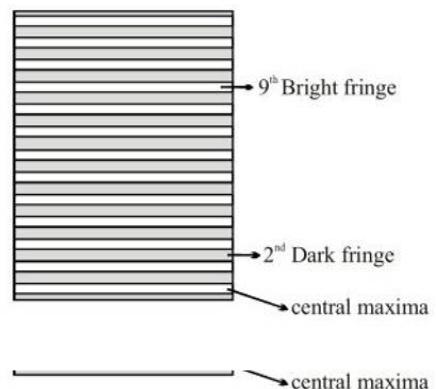
Distance between 9<sup>th</sup> bright fringe and 2<sup>nd</sup> dark fringe is 7.5 mm

$$i.e \frac{9\lambda D}{d} - \frac{3}{2} \frac{\lambda D}{d} = 7.5 mm$$

$$7.5 \frac{\lambda D}{d} = 7.5 \times 10^{-3} m$$

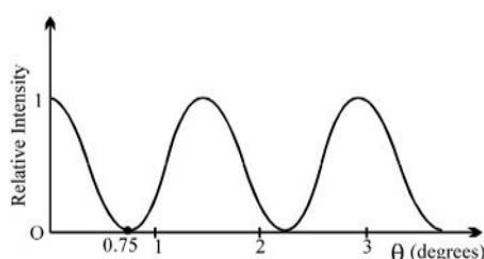
$$\lambda = \frac{10^{-3} \times d}{D} = \frac{10^{-3} \times 5 \times 10^{-4}}{1} = 5 \times 10^{-7} m$$

$$\therefore D = 1 \\ = 5000 \text{ \AA}$$



**Illustration :**

Light of wavelength 520 nm passing through a double slit, produces interference pattern of relative intensity versus deflection angle  $\theta$  as shown in the figure. find the separation  $d$  between the slits.

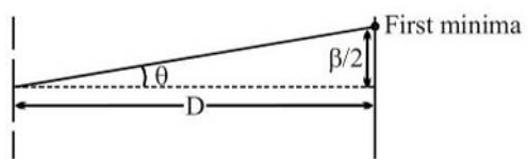


**Sol.**  $\lambda = 520 \times 10^{-9} m$

$\theta$  at which first minima occurs is  $0.75^\circ$

$$\theta = \frac{0.75 \times \pi}{180} \text{ radians}$$

$$\therefore \theta = \frac{\beta}{2D}$$



$$\frac{0.75\pi}{180} = \frac{\lambda D / 2d}{D}$$

$$\frac{\lambda}{2d} = \frac{0.75\pi}{180}$$

$$d = \frac{520 \times 10^{-9} \times 180}{2 \times 0.75\pi} = 1.99 \times 10^{-2} \text{ mm}$$

**Illustration :**

The distance between two slits in a YDSE apparatus is 3mm. The distance of the screen from the slits is 1m. Microwaves of wavelength 1 mm are incident on the plane of the slits normally. Find the distance of the first maxima on the screen from the central maxima. Also find the total number of maxima on the screen.

**Sol.**  $d = 3 \times 10^{-3} \text{ m}$

$\lambda = 10^{-3} \text{ m}$

$D = 1 \text{ m}$

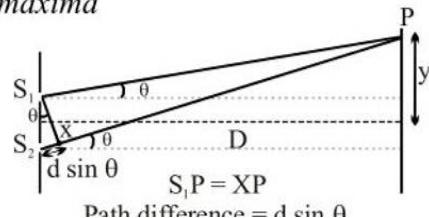
(i) Distance between first maxima and central maxima

$\Delta x = \lambda$

$\Delta x = d \sin \theta$

$$\sin \theta = \frac{10^{-3}}{3 \times 10^{-3}} = \frac{1}{3}$$

$3 \times 10^{-3} \quad 3$



Path difference =  $d \sin \theta$

$\sin \theta$  is not very small ;

$\therefore$  approximation  $\sin \theta = \theta = \tan \theta$  can't be used

$$\tan \theta = \frac{1}{\sqrt{8}}$$

$$\frac{y}{D} = \frac{1}{\sqrt{8}}$$

$$y = \frac{1}{\sqrt{8}} \text{ m}$$

(ii)  $\Delta x = d \sin \theta$  ;

$$\therefore (\Delta x)_{\max} = d$$

For calculating number of maxima, compare  $d = n\lambda$

$$n = \frac{d}{\lambda}$$

If  $n$  is an integer, then  $n$ th maxima will not be visible,

$\therefore$  total no. of maxima =  $(n - 1)$  (above the central maxima) + 1 (central maxima) +  $(n - 1)$  (below the central maxima) =  $2n - 1$

If  $n$  is not an integer then,  $[n] + 1 + [n] = 2[n] + 1$

Here in this question,  $n = \frac{d}{\lambda} = 3$

$\therefore$  number of maxima =  $2 \times 3 - 1 = 5$

**Alternative :**

Total number of maxima

$$\text{for maxima, } \Delta x = n\lambda = d \sin \theta$$

$$\sin \theta = \frac{n\lambda}{d}$$

for  $\sin \theta = 1 ; \theta = \pi/2$ , such fringe can't be obtained on screen

$$\therefore -1 < \frac{n\lambda}{d} < 1$$

$$-1 < \frac{n \times 10^{-3}}{3 \times 10^{-3}} < 1$$

$$\therefore n = -2, -1, 0, 1, 2$$

Total number of maxima = 5

**Illustration :**

In a Young's double slit experiment, the separation between the slits is  $d$ , distance between the slit and screen is  $D$  ( $D \gg d$ ). In the interference pattern, there is a maxima exactly in front of each slit. Find the possible wavelength(s) used in the experiment.

**Sol.**

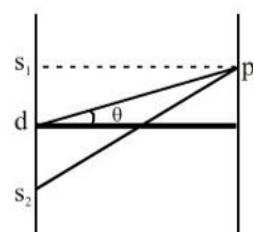
Path difference is  $\frac{dy}{D}$

$$S_2P - S_1P = \frac{d \times y}{D} = \frac{d \times (d/2)}{D} = \frac{d^2}{2D}$$

$$\frac{d^2}{2D} = n\lambda$$

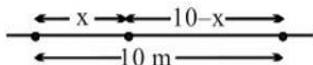
$$\lambda = \frac{d^2}{2nD}, n = 1, 2, \dots$$

$$\lambda = \frac{d^2}{2D}, \frac{d^2}{4D}, \frac{d^2}{6D}$$



**Illustration :**

One radio transmitter A operating at 60.0 MHz is 10.0 m from another similar transmitter B that is  $180^\circ$  out of phase with transmitter A. How far must an observer move from transmitter A toward transmitter B along the line connecting A and B to reach the nearest point where the two beams are in phase?

**Sol.**

As the two beams are out of phase, initially,  
For two beams to be in phase

$$(10 - x) - x = (2n - 1) \frac{\lambda}{2}$$

$$10 - 2x = (2n - 1) \frac{\lambda}{2}$$

$$v = 60 \times 10^6 \text{ Hz}$$

$$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{6 \times 10^7} \text{ m} = 5 \text{ m}$$

$$\therefore 10 - 2x = (2n - 1) \times \frac{5}{2}$$

$$x = 5 - (2n - 1) \frac{5}{4}$$

$$x = \frac{25}{4} - \frac{5n}{2}$$

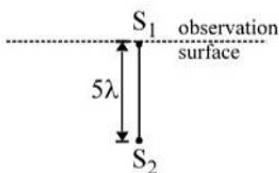
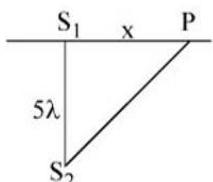
for  $x_{\min}$ ;  $n = 2$

$$25 \quad 5$$

$$x = \frac{25}{4} - 5 = \frac{5}{4} \text{ m}$$

### Illustration :

Two microwave coherent point sources emitting waves of wavelength  $\lambda$  are placed at  $5\lambda$  distance apart. The interference is being observed on a flat non-reflecting surface along a line passing through one source, in a direction perpendicular to the line joining the two sources (refer figure). Considering  $\lambda$  as 4 mm, calculate the positions of maxima and draw shape of interference pattern. Take initial phase difference between the two sources to be zero.

**Sol.**

$$S_2 P - S_1 P = n \lambda \text{ for maxima}$$

$$\therefore \sqrt{(5\lambda)^2 + x^2} - x = n\lambda \quad n = 1, 2, 3, 4, 5$$

$$25\lambda^2 + x^2 = (n\lambda + x)^2 = (n\lambda)^2 + x^2 + 2n\lambda x$$

$$\lambda^2 [25 - n^2] = 2n \lambda x$$

$$\therefore \text{Maxima occur at } x = \lambda \frac{(25-n^2)}{2n} \text{ where } n = 1, 2, 3, 4$$

$$x = \frac{24\lambda}{2}, \frac{\lambda(25-4)}{4}, \frac{\lambda(25-9)}{6}, \frac{\lambda(25-16)}{8}$$

$$= \frac{24\lambda}{2}, \frac{21\lambda}{4}, \frac{16\lambda}{6}, \frac{9\lambda}{8}$$

Putting  $\lambda = 4 \text{ m.m.}$

$$\therefore x = 48, 21, \frac{64}{6}, \frac{36}{8}, 0 \text{ m.m.} = 48, 21, \frac{32}{3}, \frac{9}{2}, 0 \text{ m.m.}$$

Pattern will look like this



### **Illustration :**

**Illustration :**  
In a YDSE apparatus,  $d = 1\text{mm}$ ,  $\lambda = 600\text{nm}$  and  $D = 1\text{m}$ . The slits individually produce same intensity on the screen. Find the minimum distance between two points on the screen having 75% of the maximum intensity.

**Sol.** Given,  $\lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$

$$D = 1\text{m}$$

$$d = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$I = I_{\max} \cos^2 \frac{\phi}{2}$$

$$\phi = \frac{2\pi}{\lambda} \times (\Delta x)$$

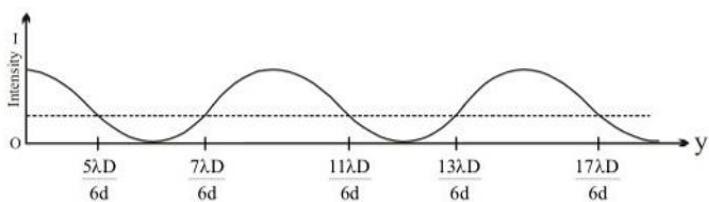
$$\frac{\phi}{2} = \frac{\pi}{\lambda} \times (\Delta x) \quad (\Delta x = \frac{dy}{D})$$

$$I = I_{\max} \cos^2 \frac{\pi dy}{\lambda D} \quad \Rightarrow \quad 0.75 I_{\max} = I_{\max} \cos^2 \frac{\pi dy}{\lambda D}$$

$$\cos \frac{\pi dy}{\lambda D} = \pm \sqrt{\frac{3}{4}}$$

$$\frac{\pi dy}{\lambda D} = n\pi \pm \frac{\pi}{6}$$

$$y = \frac{n\lambda D}{d} \pm \frac{\lambda D}{6d}$$



Points at which intensity is 75% of maximum are  $\frac{5\lambda D}{6d}, \frac{7\lambda D}{6d}, \frac{11\lambda D}{6d}, \frac{13\lambda D}{6d}, \frac{17\lambda D}{6d}, \dots$

Let minimum distance between two points having intensity 75% of the  $I_{max}$  be  $\Delta y$

$$\Delta y = \frac{2\lambda D}{6d} = 0.2 \text{ mm}$$

### Practice Exercise

- Q.1 In YDSE experiment the distance between slits is  $d = 0.25 \text{ cm}$  and the distance of screen  $D = 120 \text{ cm}$  from slits. If the wavelength of light used is  $\lambda = 6000 \text{ Å}$  and  $I_0$  is the intensity of central maximum, at what distance from the centre, the intensity will be  $\frac{I_0}{2}$  ?
- Q.2 In a YDSE experiment,  $I_0$  is given to be the intensity of the central bright fringe and  $\beta$  is the fringe width. Then, find the intensity at a distance  $y$  from Central Bright Fringe.
- Q.3 Two narrow slits emitting light in phase are separated by a distance of  $1.0 \text{ cm}$ . The wavelength of the light
- Q.3 Two narrow slits emitting light in phase are separated by a distance of  $1.0 \text{ cm}$ . The wavelength of the light is  $5.0 \times 10^{-7} \text{ m}$ . The interference pattern is observed on a screen placed at a distance of  $1.0 \text{ m}$  (a) Find the separation between the consecutive maxima. (b) Find the separation between the sources which will give a separation of  $1.0 \text{ mm}$  between the consecutive maxima.
- Q.4 If YDSE is performed with monochromatic light of wavelength  $\lambda$ , the distance between the slits is  $d$  and distance between slits and screen is  $D$ .  
 (a) Find the distance between second and fifth maxima.  
 (b) Find the distance between second and tenth minima.  
 (c) Find the distance between second minima and fifth maxima.
- Q.5 In Young's double slit arrangement, a monochromatic source of wavelength  $6000 \text{ Å}$  is used. The screen is placed at  $1 \text{ m}$  from the slits. Fringes formed on the screen, are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes if they subtend an angle more than  $1$  minute of arc. Calculate the maximum distance between the slits so that the fringes are clearly visible.
- Q.6 In the **above question** find the position of 3rd maxima and 5th minima.

### Answers

Q.1  $7.2 \times 10^{-5} \text{ m}$

Q.2  $I_0 \cos^2(\pi y/\beta)$

Q.3 (a)  $0.05 \text{ mm}$  (b)  $0.50 \text{ mm}$

Q.4 (a)  $\frac{3\lambda D}{d}$  (b)  $\frac{8\lambda D}{d}$  (c)  $\frac{7\lambda D}{2d}$

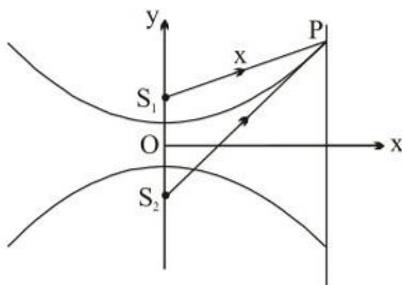
Q.5  $\frac{6.48}{\pi} \text{ mm}$

Q.6  $\frac{\pi}{0.036} \text{ mm}, \frac{\pi}{0.024} \text{ mm}$

## Shape of Interference Fringes in YDSE

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.



$$S_2 P - S_1 P = D = \text{constant} \quad \dots\dots(1)$$

If  $\Delta = \pm \frac{\lambda}{2}$ , the fringe represents 1st minima.

If  $\Delta = \pm \frac{3\lambda}{2}$  it represents 2nd minima

If  $\Delta = 0$  it represents central maxima,

If  $\Delta = 0$  it represents central maxima,

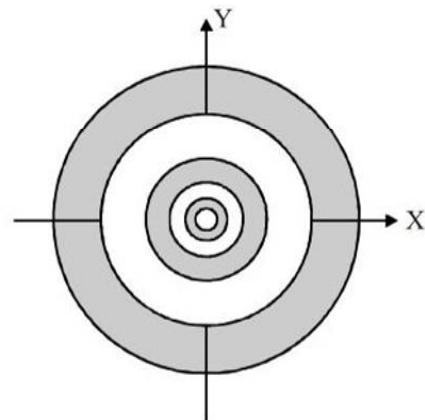
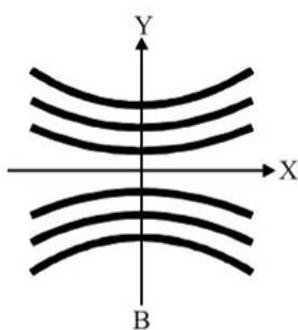
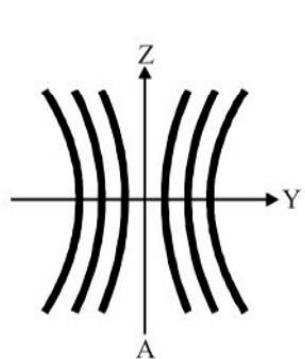
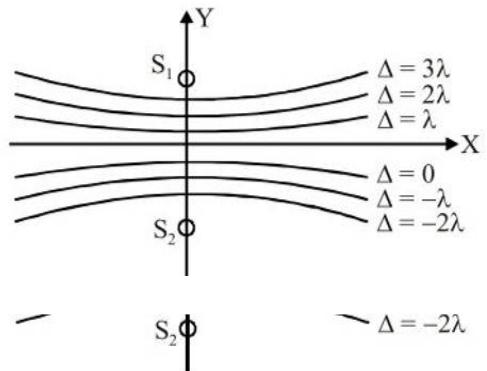
If  $\Delta = \pm \lambda$ , it represents 1st maxima etc.

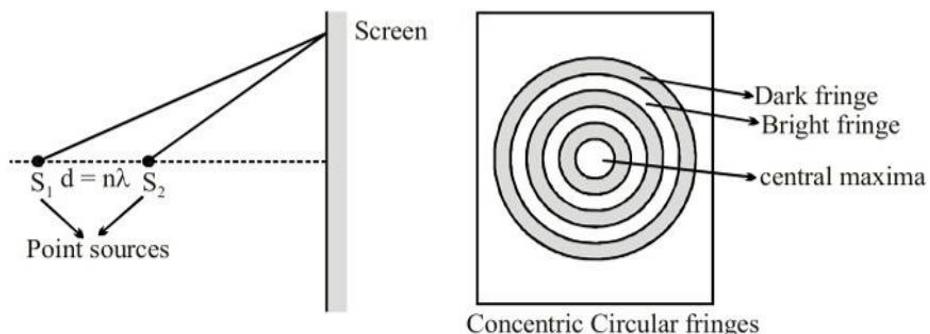
Equation (1) represents a hyperbola with its two foci at  $S_1$  and  $S_2$ .

The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis  $S_1 S_2$ .

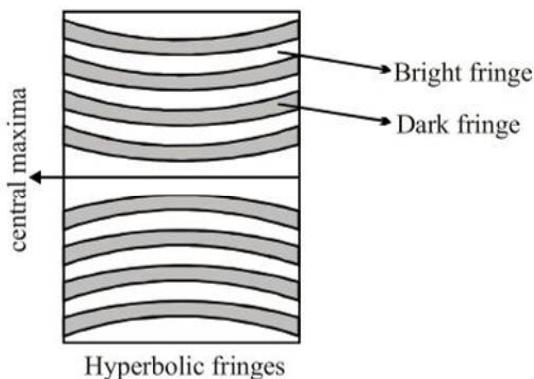
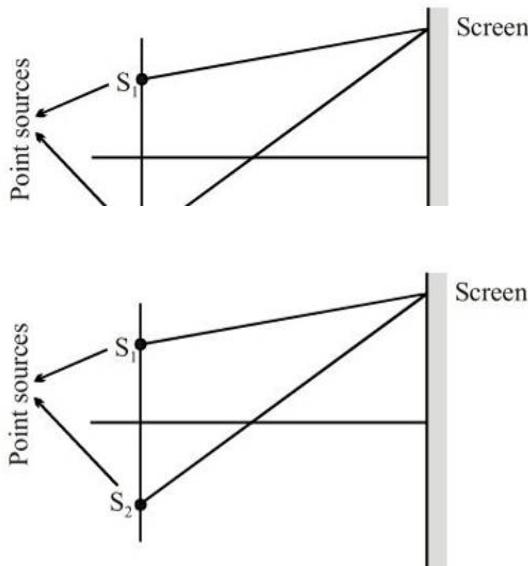
- (A) If the screen is  $\perp$  to the X-axis, i.e. in the YZ plane, as is generally the case, fringes are hyperbolic with a straight central section.
- (B) If the screen is in the XY plane, again fringes are hyperbolic.
- (C) If screen is  $\perp$  to Y-axis (along  $S_1 S_2$ ) i.e. in the XZ plane, fringes are concentric circles with center on the

axis  $S_1 S_2$ ; the central fringe is bright if  $S_1 S_2 = n\lambda$  and dark if  $S_1 S_2 = (2n-1) \frac{\lambda}{2}$ .





*Shape of the pattern when the interference takes place due to waves produced by two point sources (where the line of sources is perpendicular to the screen).*



*Shape of the pattern when the interference takes place due to waves produced by two point sources (where the line of sources is parallel to the screen).*

## YDSE with white light :

The central maxima will be white because all wavelengths will constructively interfere here. However slightly below (or above) the position of central maxima fringes will be coloured for example if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{\text{violet}}}{2} = 190 \text{ nm},$$

completely destructive interference will occur for violet light. Hence we will have a line devoid of violet colour that will appear reddish. And if

$$S_2P - S_1P = \frac{\lambda_{\text{red}}}{2} \simeq 350 \text{ nm},$$

completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe ; for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination for example if

$$S_2P - S_1P = 3000 \text{ nm},$$

then constructive interference will occur for wavelengths  $\lambda = \frac{3000}{n}$  nm. In the visible region these

wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish-yellow), 430 nm (violet). Clearly

wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish-yellow), 430 nm (violet). Clearly such a light will appear white to the unaided eye.

Thus with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. **Interference with white light is used to determine the position of central maxima in such cases.**

## Geometrical path and optical path

Actual distance travelled by light in a medium is called geometrical path ( $\Delta x$ ). Consider a light wave given by the equation.

$$E = E_0 \sin(\omega t - kx + \phi)$$

If the light travels by  $\Delta x$ , its phase changes by  $k\Delta x = \frac{\omega}{v} \Delta x$ , where  $\omega$ , the frequency of light does not

depend on the medium, it depends only on the source, but  $v$ , the speed of light depends on the medium

$$\text{as } v = \frac{c}{\mu}.$$

Consequently, change in phase,

$$\Delta\phi = k\Delta x = \frac{\omega}{C} (\mu\Delta x)$$

It is clear that a wave travelling a distance  $\Delta x$  in a medium of refractive index  $\mu$  suffers the same phase change as when it travels a distance  $\mu\Delta x$  in vacuum. i.e. a path length of  $\Delta x$  in medium of refractive index  $\mu$  is equivalent to a path length of  $\mu\Delta x$  in vacuum.

The quantity  $\mu\Delta x$  is called the optical path length of light,  $\Delta x_{\text{opt}}$ . And in terms of optical path length, phase difference would be given by,

$$\Delta\phi = \frac{\omega}{C} \Delta x_{\text{opt}} = \frac{2\pi}{\lambda_0} \Delta x_{\text{opt}}$$

where  $\lambda_0$  = wavelength of light in vacuum.

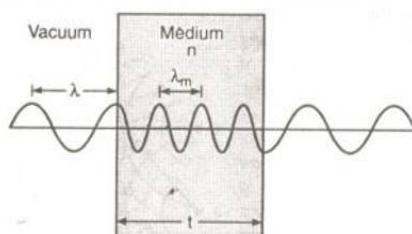
However in terms of the geometrical path length  $\Delta x$ ,

$$\Delta\phi = \frac{\omega}{C} (\mu\Delta x) = \frac{2\pi}{\lambda} \Delta x$$

where  $\lambda$  = wavelength of light in the medium  $\left(\lambda = \frac{\lambda_0}{\mu}\right)$ .

## Equivalent optical path length

When a beam of light travels from one medium to another its speed changes but its frequency does not.



Designating the wavelength in vacuum by  $\lambda$  and the wavelength in the material by  $\lambda_m$  we have

$$\frac{\lambda}{\lambda_m} = \frac{C}{v} = n$$

or 
$$\lambda_m = \frac{\lambda}{n}$$

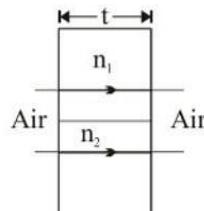
The wavelength is shorter in the medium than in vacuum. If the light beam passes through a thickness  $t$  of a medium,

$$\text{Number of wavelength in slab} = \frac{t}{\lambda_m} = \frac{t}{\lambda/n} = \frac{nt}{\lambda}$$

which shows that a thickness  $t$  of the medium has as many wavelengths as there are in a length  $nt$  of vacuum. Therefore in terms of wavelengths, a thickness  $t$  in a medium of refractive index  $n$  is equivalent to a path length  $nt$  in vacuum. The quantity  $nt$  is called equivalent optical path length.

## Optical path length in terms of wave lengths

In fig. shown two light rays of identical wavelength and initially in phase in air travel through two different media of refractive indices  $n_1$  and  $n_2$ , same thickness  $t$ . The wavelengths of the waves will be different in the two media ; so the two waves will no longer be in phase when they emerge.



Number of wavelengths in medium 1,

$$N_1 = \frac{t}{\lambda_1} = \frac{t}{\lambda/n_1} = \frac{n_1 t}{\lambda}$$

Number of wavelength in medium 2,

$$N_2 = \frac{t}{\lambda_2} = \frac{t}{\lambda/n_2} = \frac{n_2 t}{\lambda}$$

To find a new phase difference we subtract the number of wavelengths of the waves in the two media ; assuming  $n_2 > n_1$ , we have

To find a new phase difference we subtract the number of wavelengths of the waves in the two media ; assuming  $n_2 > n_1$ , we have

$$N_2 - N_1 = (n_2 - n_1) \frac{t}{\lambda}$$

Phase difference corresponding to difference of one wavelength is  $2\pi$  ; hence phase difference corresponding to  $N_2 - N_1$  is

$$\Delta\phi = \frac{2\pi}{\lambda} (n_2 - n_1) t$$

In other words we can say that the equivalent optical paths of wave in media 1 and 2 are  $n_1 t$  and  $n_2 t$  respectively. Thus the path difference,  $\Delta x = n_2 t - n_1 t$ .

### Illustration :

*Light of wavelength  $\lambda$  in air enters a medium of refractive index  $\mu$ . Two points in this medium, lying along the path of this light, are at a distance  $x$  apart. Find the phase difference between these points.*

**Sol.** Phase difference =  $\frac{2\pi}{\lambda} \times \text{path difference}$

$$\text{Now } \lambda' = \frac{\lambda}{\mu}$$

$$\Delta\phi = \frac{2\pi\mu x}{\lambda}$$

## Changes observed in the Interference Pattern

### Case - I

If the space between the main slit and double slit is completely filled with two uniform medium of refractive index  $\mu_1$  and  $\mu_2$  (as shown in figure).

If the mediums above and below the perpendicular line bisecting the two slits are different, then the fringe

pattern shifts towards the side of denser medium by  $y = \frac{(\mu_2 - \mu_1)l}{d}$  but the fringe width does not change.

Let  $\mu_1$  and  $\mu_2$  be the refractive indices of the two media with  $\mu_1 > \mu_2$ . At the position of central maxima, the optical path difference between the two interfering waves is zero, so if  $O'$  is the new position of central maxima, then,

$$SS_1 \text{ (in } \mu_1\text{)} + S_1O' \text{ (in air)} = SS_2 \text{ (in } \mu_2\text{)} + S_2O' \text{ (in air)}$$

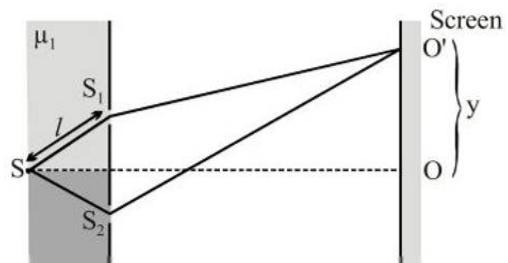
$$\Rightarrow \mu_1 l + S_1O' = \mu_2 l + S_2O'$$

$$\Rightarrow S_2O' - S_1O' = (\mu_1 - \mu_2)l$$

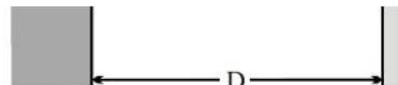
(Where  $\mu_1 l$  and  $\mu_2 l$  are the optical path lengths)

$$\text{But } S_2O' - S_1O' = \frac{yd}{D}$$

So, we have,



$$\frac{yd}{D} = (\mu_1 - \mu_2)l \Rightarrow y = \frac{(\mu_1 - \mu_2)lD}{d}$$



Hence the central maxima (fringe pattern) shifts towards the side of denser medium by a distance,

$$y = \frac{(\mu_1 - \mu_2)lD}{d}$$

for  $n^{\text{th}}$  maxima

$$\begin{aligned} & [SS_2 \text{ (in } \mu_2\text{)} + S_2P \text{ (in air)}] - [SS_1 \text{ (in } \mu_1\text{)} + S_1P \text{ (in air)}] = n\lambda \\ \Rightarrow & [\mu_2 l + S_2P] - [\mu_1 l + S_1P] = n\lambda \\ \Rightarrow & S_2P - S_1P = (\mu_1 - \mu_2)l + n\lambda \end{aligned}$$

$$\text{But } S_2P - S_1P = \frac{yd}{D}$$

$$\frac{yd}{D} = (\mu_1 - \mu_2)l + n\lambda$$

$$y = \frac{(\mu_2 - \mu_1)lD}{d} + \frac{n\lambda D}{d}$$

This means, all fringes are shifted by same distance.

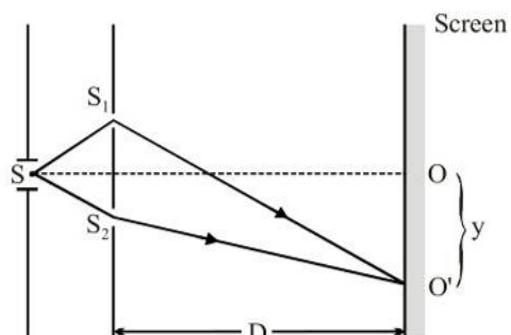
$$y = \frac{(\mu_1 - \mu_2)lD}{d}$$

### CASE - II

If the main slit is moved upward or downward parallel to the double slits, then the fringe width does not change but the fringe pattern shifts in the direction opposite to that of the movement of the main slit.

For the position of central maxima

$$\begin{aligned}\Delta x &= 0 \\ \Rightarrow SS_1 + S_1 O' &= SS_2 + S_2 O' \\ \Rightarrow S_1 O' - S_2 O' &= SS_2 - SS_1 \\ \Rightarrow \frac{yd}{D} &= (SS_2 - SS_1) \\ \Rightarrow y &= \frac{(SS_2 - SS_1)D}{d}\end{aligned}$$



Similarly, for  $n^{\text{th}}$  maxima, we can prove the shift is same.

So the fringe pattern shifts by a distance  $\frac{(SS_2 - SS_1)D}{d}$  in the direction opposite to the direction of motion of the main slit.

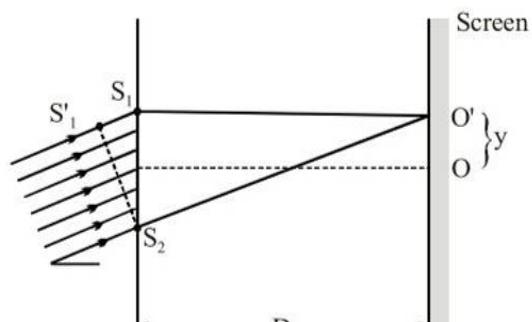
### CASE - III

#### CASE - III

If the light waves from infinity reach the two slits as shown in the figure.

Let the final position of central maxima be  $O'$ .

$$\begin{aligned}S_1 S'_1 + S_1 O' &= S_2 O' \quad (\text{Here } S'_1 \text{ and } S_2 \text{ are in same phase}) \\ S_2 O' - S_1 O' &= S_1 S'_1 \\ \text{But } S_2 O' - S_1 O' &= \frac{yd}{D} \\ \frac{yd}{D} &= S_1 S'_1 \\ y &= \frac{(S_1 S'_1)D}{d}\end{aligned}$$



If the parallel wave make an angle  $\theta$  with perpendicular bisector of  $S_1 S_2$  then from the figure,

$$S_1 S'_1 = d \sin \theta \text{ so, } y = \frac{(d \sin \theta)D}{d} \text{ or, } y = D \sin \theta$$

## CASE - IV

If a transparent film of refractive index  $\mu$  and thickness 't' is introduced in front of any of the slits.

The central maxima shifts the side of the slit in which the slab is introduced, to compensate the path length

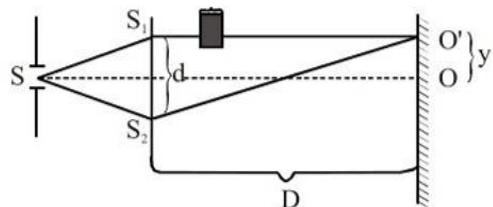
For the position of central maxima,

$$(S_1 O' - t) \text{ in air} + t(\text{in } \mu) = S_2 O' \text{ in air}$$

$$S_1 O' - t + \mu t = S_2 O' \text{ (optical path length)}$$

or  $S_2 O' - S_1 O' = (\mu - 1) t$

$$\frac{yd}{D} = (\mu - 1) t \Rightarrow y = \frac{(\mu - 1)tD}{d}$$



All the fringes shift by the same distance

Notice that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

**Note:** Interference with white light is used to determine the position of central maxima in such cases.

**Illustration :**

One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of refractive index 1.7. The point on the screen, where central bright fringe was formed before the introduction of the glass sheets, is now occupied by the 5<sup>th</sup> bright fringe. Assuming that both the glass plates have same thickness and wavelength of light used is 4800 Å, find their thickness.

**Sol.**  $\lambda = 4.8 \times 10^{-7} \text{ m}$

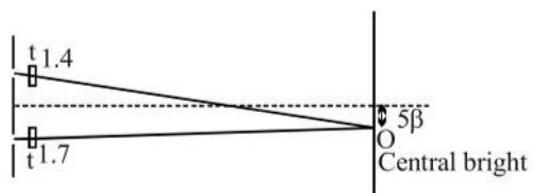
Optical path difference at the centre of the screen,

$$\Delta x = 5\lambda$$

$$\Delta x = [(D-t+1.4t) - (D-t+1.7t)]$$

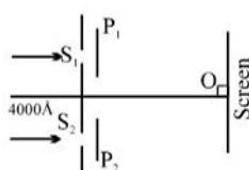
$$\therefore 5\lambda = 0.3t$$

$$t = \frac{5\lambda}{0.3} = \frac{5 \times 4.8 \times 10^{-7}}{0.3} = 8 \times 10^{-6} \text{ m}$$



**Illustration:**

$P_1, P_2$  are transparent plates having equal thickness 20 μm and refractive indices  $\mu_1 = 1.6, \mu_2 = 1.5$ .  $P_1$  transmits 75% whereas  $P_2$  transmits 50% of energy incident. Without  $P_1$  and  $P_2$  intensity at  $O$ ,  $I_0 = 4I$ . Find intensity at  $O$  after placing  $P_1$  and  $P_2$ .  $S_1, S_2$  are identical slits.



**Sol.**  $S_2P - S_1P = (\mu_1 - 1)t - (\mu_2 - 1)t$   
 $= 0.1 t = 2 \times 10^{-6} m$

$$\Delta\phi = \frac{\Delta x \times 2\pi}{\lambda} = \frac{2 \times 10^{-6} \times 2\pi}{4000 \times 10^{-10}} = 10\pi$$

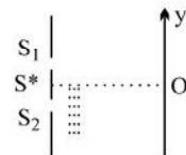
$$I_1 = 0.75I \quad I_2 = 0.5I$$

$$I = 0.75I + 0.5I + 2(\sqrt{0.75 \times 0.5})I \cos \Delta\phi$$

$$= \frac{5I}{4} + I \sqrt{\frac{3}{2}}$$

**Illustration :**

The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit  $S_2$  is covered by a thin glass sheet of thickness 10.4  $\mu m$  and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown



- (a) Find the location of the central maximum (bright fringe with zero path difference) on the y-axis.
- (b) Find the light intensity at point O relative to the maximum fringe intensity.
- (c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O .
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[All wavelengths in this problem are for the given medium of refractive index 4/3. Ignore dispersion]

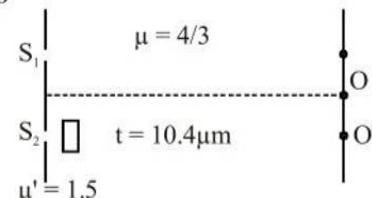
**Sol.** (a) For central maxima, optical path difference,  $\Delta x = 0$

$$d = 0.45 \text{ mm}, D = 1.5 \text{ m}$$

$$\Delta x = [\mu(S_2O' - t) + \mu't] - \mu S_1O' = 0$$

$$0 = (\mu' - \mu)t - \mu(S_1O' - S_2O')$$

$$\left(1.5 - \frac{4}{3}\right)(10.4 \times 10^{-6}) = \frac{4}{3} \times y \frac{d}{D}$$



$$y = 13/3 \text{ mm (below the centre)}$$

(b) Optical path difference at the centre of the screen

$$\Delta x = \frac{4}{3} \times (D - t) + 1.5t - \frac{4}{3}(D)$$

$$= 1.5t - \frac{4}{3}t = \frac{t}{6}$$

$$\Delta\phi = \frac{2\pi}{\lambda_{\text{air}}} \times \Delta x \quad \{\lambda_{\text{air}} = \mu \times \lambda_{\text{water}}\}$$

$$= \frac{\pi t}{3\lambda\mu}$$

$$\frac{\Delta\phi}{2} = \frac{\pi t}{6\lambda\mu} = \frac{13\pi}{6}$$

$$I = I_0 \cos^2 \frac{\phi}{2}; I = I_0 \times \cos^2 \left( \frac{13\pi}{6} \right)$$

(c) Path difference in water medium at the centre of screen

$$\Delta x = (D - t) + \frac{1.5t}{4/3} - D = \frac{3 \times 1.5}{4} t - t = \frac{t}{8}$$

$$\text{for maxima, } \frac{t}{8} = n\lambda \frac{10.4 \times 10^{-6}}{8} = n \times \lambda$$

we now have to calculate the wavelengths for which, centre of the screen is a maxima.

for  $\lambda_{\min}$  we get  $n_{\max}$  and for  $\lambda_{\max}$  we get  $n_{\min}$  the integral values of  $n$  that lie between these two values of  $n$  will give the required  $\lambda$ .

$$\therefore n_{\min} = \frac{10.4 \times 10^{-6}}{8 \times 700 \times 10^{-9}} = 1.85$$

$$n_{\max} = \frac{10.4 \times 10^{-6}}{8 \times 400 \times 10^{-9}} = 3.25$$

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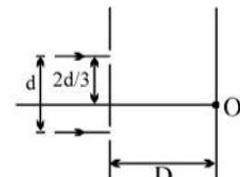
$$\therefore n = 2, 3$$

$$\therefore \lambda_1 = \frac{10.4 \times 10^{-6}}{8 \times 2} = 650 \text{ nm} \quad \Rightarrow \quad \lambda_2 = \frac{10.4 \times 10^{-6}}{8 \times 3} = 433.33 \text{ nm}$$

### Practice Exercise

- Q.1 A Young's double slit apparatus has slits separated by 0.28 mm and a screen 48 cm away from the slits. The whole apparatus is immersed in water and the slits are illuminated by the red light ( $\lambda = 700 \text{ nm}$  in vacuum). Find the fringe-width of the pattern formed on the screen.

- Q.2 In the figure shown if a parallel beam of white light is incident on the plane of the slits then the find distance of the white spot on the screen from O. [Assume  $d \ll D, \lambda \ll d$ ]



- Q.3 In the above question if the light incident is monochromatic and point O is a maxima, then the wavelength of the light incident cannot be

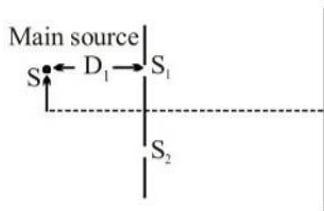
(A)  $d^2/3D$

(B)  $d^2/6D$

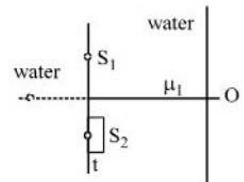
(C)  $d^2/12D$

(D)  $d^2/18D$

- Q.4 In YDSE, the main source is displaced by a distance  $d = 2\text{mm}$  from the initial symmetric position, (parallel to the plane of the slits), as shown in figure. Given the distance of source from slits,  $D_1 = 2\text{m}$ .  $d = 6\text{mm}$ ,  $D = 3\text{m}$ . Find the displacement of fringe pattern.



- Q.5 A plate of thickness  $t$  made of a material of refractive index  $\mu$  is placed in front of one of the slits in a double slit experiment. What should be the minimum thickness  $t$  which will make the intensity at the centre of the fringe pattern zero? Wavelength of the light used is  $\lambda$ . Neglect any absorption of light in the plate.
- Q.6 A young's double slit experiment is conducted in water ( $\mu_1$ ) as shown in the figure, and a glass plate of thickness  $t$  and refractive index  $\mu_2$  is placed in the path of  $S_2$ . Wavelength of light in water is  $\lambda$ . Find the magnitude of the phase difference between waves coming from  $S_1$  and  $S_2$  at 'O'.



### Answers

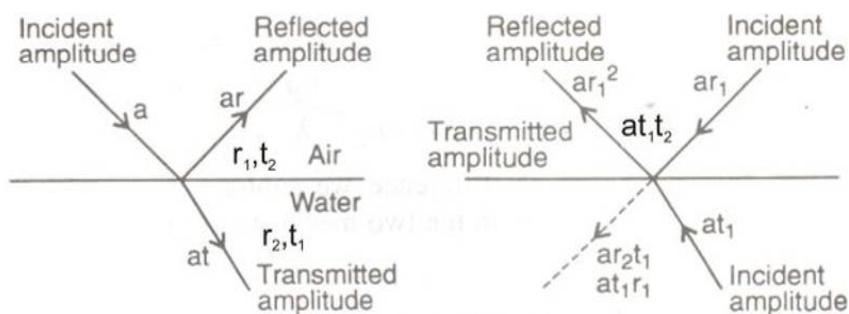
### Answers

Q.1 0.90 mm      Q.2  $d/6$       Q.3 A      Q.4 3 mm

$$Q.5 \frac{\lambda}{2(\mu-1)} \quad Q.6 \left( \left| \frac{\mu_2}{\mu_1} - 1 \right| \right) t \left| \frac{2\pi}{\lambda} \right|$$

### The phase change on reflection

A ray of light is incident on air-water interface ; let the amplitude reflection and transmission coefficients be  $r_1$  and  $t_1$  respectively. The amplitudes of reflected and transmitted waves are  $ar_1$  and  $at_1$  respectively. From the principle of reversibility of light, the system retraces its whole previous motion. The wave of amplitude  $ar_1$  gives a reflected wave of amplitude  $ar_1t_1$ . The wave of amplitude  $at_1$



gives a reflected wave of amplitude  $ar_1 t_1$  and transmitted wave amplitude  $at_1 t_2$ .

$$\text{So } ar_1^2 + at_1 t_2 = a$$

$$t_1 t_2 = 1 - t_1^2 \quad \dots (1)$$

Further, the waves of amplitudes  $at_1 r_2$  and  $ar_1 t_1$  must cancel each other.

$$at_1 r_2 + ar_1 t_1 = 0$$

$$r_2 = -r_1 \quad \dots (2)$$

equation shows a difference of phase of  $\pi$  between the two cases ; a reversal of sign means a displacement in the opposite sense. If there is no change of phase on reflection from above, there must be a phase change of  $\pi$  from below and vice-versa.

When light gets reflected from a denser medium there is an abrupt phase change of  $\pi$  ; no phase change occurs when reflection takes place from rarer medium.

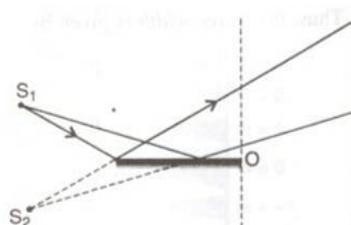
At the position of central maxima, the path difference between the two interfering waves is zero, so if O' is the new position of central maxima, then,

### The Lloyd's mirror experiment

In this arrangement the light reflected from a long mirror and the light coming directly from the source

### The Lloyd's mirror experiment

In this arrangement the light reflected from a long mirror and the light coming directly from the source without reflection produced interference on a screen. An important feature of this experiment lies in the fact that when the screen is placed in contact with the end of the mirror the edge O of the reflecting surface comes at the centre of a dark fringe instead of a bright fringe. The direct beam does not suffer any phase change. This means that the reflected beam undergoes a phase change of  $\pi$  radian. Hence at a point P on the screen the conditions for minima and maxima are

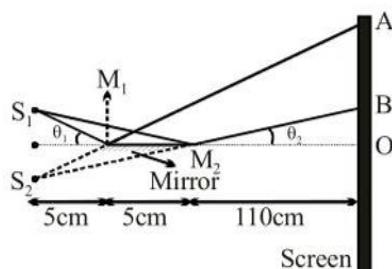


$$S_2 P - S_1 P = n\lambda, \quad n = 0, 1, 2, 3, \dots \quad [\text{minima}]$$

$$S_2 P - S_1 P = \left( n + \frac{1}{2} \right) \lambda \quad [\text{maxima}]$$

**Illustration :**

A Lloyd's mirror of length 5 cm is illuminated with monochromatic light of wavelength  $\lambda = 6000 \text{ \AA}$  from a narrow slit 1 mm from its place and 5 cm in its plane from its near edge. Find the fringe width on a screen 120 cm from the slit and width of interference on the screen.



**Sol.** In plane mirror, object and image distances are equal.

$$\text{So, } d = 2\text{mm} = 0.2 \text{ cm}; \lambda = 6000 \text{ \AA} = 6000 \times 10^{-8} \text{ cm}; D = 120 \text{ cm}$$

$$\therefore \text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-5} \times 120}{0.20} = 0.036 \text{ cm}$$

The width of the fringe pattern is AB. From the figure.

$$\tan \theta_1 = \frac{0.1}{5} = \tan \theta_2 = \frac{0.1}{10}$$

$$\therefore \text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-5} \times 120}{0.20} = 0.036 \text{ cm}$$

The width of the fringe pattern is AB. From the figure.

$$\tan \theta_1 = \frac{0.1}{5} = \tan \theta_2 = \frac{0.1}{10}$$

In right angled triangle  $AM_1O$  and  $BM_2O$

$$\tan \theta_1 = \frac{0.1}{5} = \frac{OA}{M_1O}$$

$$\text{or } OA = 115 \times \frac{0.1}{5} \text{ cm}$$

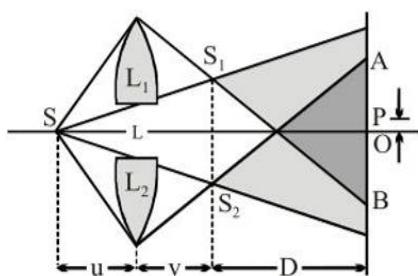
$$\text{and } \tan \theta_2 = \frac{0.1}{10} = \frac{OB}{OM_2}$$

$$\text{or } OB = 110 \times \frac{0.1}{10} \text{ cm}$$

$$\therefore \text{Width of fringe pattern} = OA - OB = \frac{115 \times 0.1}{5} - \frac{110 \times 0.1}{10} = 1.2 \text{ cm}$$

## Billet's Split-lens

This device consists of two halves of a convex lens placed close together to form two real or virtual images  $S_1$  and  $S_2$  of the narrow slit  $S$  illuminated by a monochromatic source of light.  $S_1$  and  $S_2$  now act in the same way as the double slit in Young's experiment. The distance between  $S_1$  and  $S_2$  can be changed by adjusting the space between the two halves of the convex lens, a number of interference bands of varying widths can be obtained and observed in the overlapping region.

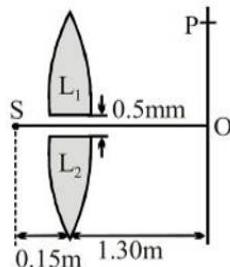


### Illustration :

In figure showsn,  $S$  is a monochromatic point source emitting light of wavelength  $\lambda = 500 \text{ nm}$ . A thin lens of circular shape and focal length  $0.10 \text{ m}$  is cut into two identical halves  $L_1$  and  $L_2$  by a plane passing through a diameter. The two halves are placed symmetrically about the central axis  $SO$  with a gap of  $0.5 \text{ mm}$ . The distance along the axis from  $S$  to  $L_1$  and  $L_2$  is  $0.15 \text{ m}$ , while that from  $L_1$  and  $L_2$  to  $O$  is  $1.30 \text{ m}$ . the screen at  $O$  is normal to  $SO$ .

SO with a gap of  $0.5 \text{ mm}$ . The distance along the axis from  $S$  to  $L_1$  and  $L_2$  is  $0.15 \text{ m}$ , while that from  $L_1$  and  $L_2$  to  $O$  is  $1.30 \text{ m}$ . the screen at  $O$  is normal to  $SO$ .

- If the third intensity maximum occurs at the point  $P$  on the screen, find distance  $OP$ .
- If the gap between  $L_1$  and  $L_2$  is reduced from its original value of  $0.5 \text{ mm}$ , will the distance  $OP$  increase, decrease or remain the same ?



**Sol.** (i) As shown in figure each part of the lens will form image of  $S$  which will act as coherent sources. From lens equation, we can write

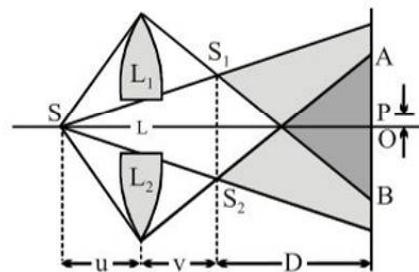
$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

or  $v = 30 \text{ cm}$

$$m = \frac{-v}{u} = -2$$

Also,  $d = 3 \times 0.5 \text{ mm} = 1.5 \text{ mm}$

$D = 1.30 - 0.30 = 1 \text{ m}$



Now, from the theory of interference the distance  $y$  of a point  $P$  on the screen is given by

$$y = \frac{D}{d} (\Delta x)$$

and as point is third maximum

$$\Delta x = 3\lambda$$

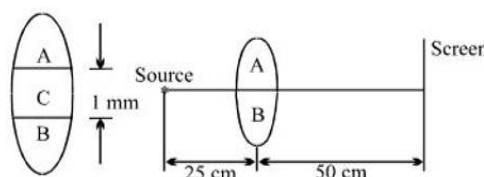
$$\text{So, } y = \frac{D}{d} (3\lambda)$$

$$\text{or } y = \frac{5 \times 10^{-7}}{0.5 \times 10^{-3}} \times 10^{-3} \text{ m} = 1 \text{ mm}$$

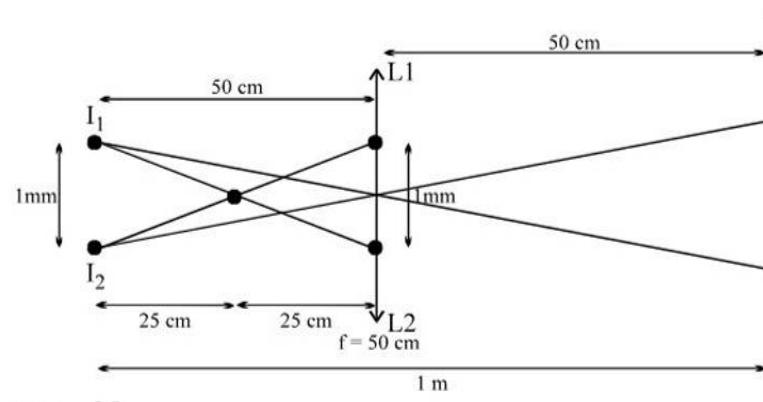
- (ii) If gap between  $L_1$  and  $L_2$  is reduced then  $d$  will decrease. As  $\beta = \frac{D\lambda}{d}$  and  $OP = 3\beta$ , therefore  $OP$  will increase.

### Illustration :

A convex lens of focal length 50 cm is cut along the diameter into two identical halves  $A$  and  $B$  and in the process a layer  $C$  of the lens thickness 1 mm is lost. Then the two halves  $A$  and  $B$  are put together to form a composite lens. Now in front of this composite lens a source of light emitting wavelength  $\lambda = 6000 \text{ \AA}$  is placed at a distance of 25 cm as shown in the figure. Behind the lens there is a screen at a distance 50 cm from it. Find the fringe width of the interference pattern obtained on the screen.



Sol.



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-25} = \frac{1}{50}$$

$$\frac{1}{v} = \frac{1}{50} + \frac{1}{25}$$

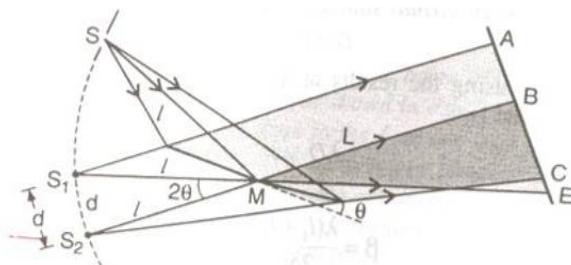
$$\frac{1}{v} = \frac{3}{50}$$

$$v = \frac{50}{3} \text{ cm}$$

$$\beta = \frac{6 \times 10^{-7} \times 1}{10^{-3}} = 6 \times 10^{-4} = 0.6 \text{ mm}$$

## Fresnel's Mirrors

Figure shows Fresnel's bimirrors apparatus to produce interference by division of the wavefront. Light from a slit S is reflected by two plane mirrors slightly inclined to each other.



The mirrors produce two virtual images  $S_1$  and  $S_2$  of the slit, the interference fringes are observed in the region BC, where the reflected beams overlap. If  $\theta$  is the angle between the planes of the mirrors, then  $S_1$  and  $S_2$  subtend angle  $2\theta$  at the point of intersection M between the mirrors.

If  $l$  is the distance between the slit and the mirrors intersection and  $L$  is the distance between the screen and the mirrors intersection, then the separation between the images  $S_1$  and  $S_2$  is

$$d = l(2\theta) = 2l\theta$$

and

$$D = l + L$$

$$\alpha = \nu(2\theta) - 2\nu\theta$$

and

$$D = l + L$$

Thus, the fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

or

$$\beta = \frac{\lambda(l+L)}{2l\theta}$$

or

$$\beta = \frac{\lambda}{2\theta} \left[ 1 + \frac{L}{l} \right]$$

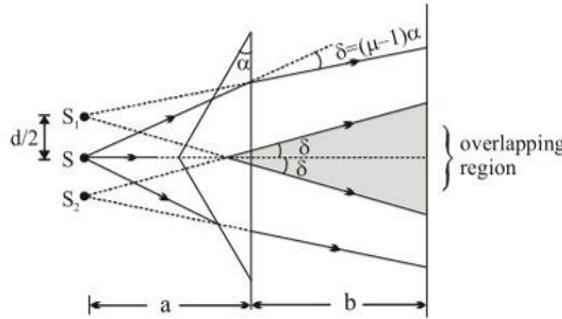
## Fresnel's Biprism

Figure shows the Fresnel's biprism experiment schematically. The thin prism P refracts light from the slit source S into two beams. When a screen is placed as shown in the figure, the interference fringes are observed only in the region shown.

If  $A$  is the angle of refraction of the thin prism and  $\mu$  is the refractive index of its medium, then the angle of deviation produced by the prism is

$$\delta = A(\mu - 1)$$

- (i) In numerical problems 'd' is calculated as given below :



$$\frac{d}{2} = a \tan \delta \approx a \delta = a (\mu - 1) \alpha$$

$$d = 2a (\mu - 1) \alpha$$

'd' can also be calculated using lens displacement method and it is given by

$$d = \sqrt{d_1 d_2}$$

where  $d_1$  and  $d_2$  are the distances between images of  $S_1$  and  $S_2$  in two positions of a convex lens placed between the biprism and the screen,

(ii) The expression for fringe width is same as in YDSE

$$\text{i.e., } \beta = \frac{D\lambda}{d} = \frac{(a+b)\lambda}{2a(\mu-1)\alpha}$$

and interference pattern consists of alternate bright and dark fringes.

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$$\text{If source is at infinity i.e., } a \rightarrow \infty \text{ then } \beta = \frac{\lambda}{2(\mu-1)\alpha}$$

(iii) Let  $L$  = length of overlapping region from figure we have

$$\frac{L}{b} = \frac{d}{a} \quad \text{or} \quad L = \frac{bd}{a}$$

Also, No. of fringes

$$= \frac{\text{Length of interference pattern}}{\text{Fringe width}} = \frac{L}{\beta}$$

### Illustration :

Interference bands are produced by a Fresnel's biprism in the focal plane of a reading microscope. The focal plane is 100 cm distant from the slit. A lens is inserted between the biprism and microscope and gives two images of the slit for two position of lens. In one, separation between them is 4.05 nm and in other 2.90 mm. If sodium light is used, find the distance between interference bands. ' $\lambda$ ' for sodium light =  $5886 \times 10^{-8}$  cm.

**Sol.** Here  $\lambda = 5886 \times 10^{-8}$  cm ;  $D = 100$  cm ;  $d_1 = 4.05$  mm = 0.405 cm  
 $d_2 = 2.90$  mm = 0.290 cm

$$d = \sqrt{d_1 d_2} = \sqrt{0.405 \times 0.290}$$

$$\beta = \frac{\lambda D}{d} = \frac{5886 \times 10^{-8} \times 100}{\sqrt{0.405 \times 0.290}} = 0.017 \text{ Ans.}$$

**Illustration :**

In a biprism experiment with sodium light, bands of width 0.0195 cm are observed at 100 cm from the slit. On introducing a convex lens 30cm away from the slit, two images of the slit are seen 0.7 cm apart, at 100 cm distance from the slit. Calculate the wavelength of sodium light.

$$\text{Sol. } \beta = \frac{\lambda D}{d} \text{ or } \lambda = \frac{\beta d}{D}$$

Here,  $\beta = 0.0195 \text{ cm}$ ;  $D = 100 \text{ cm}$

$$\text{For a convex lens } \frac{I}{O} = \frac{v}{u}, v + u = 100 \text{ cm}$$

$$u = 30 \text{ cm} \text{ or } \frac{0.7}{O} = \frac{70}{30} \text{ cm} \text{ or } O = 0.30 \text{ cm}$$

i.e., Distance between the two coherent sources

$$d = O = 0.30 \text{ cm}$$

$$\therefore \lambda = \frac{0.0195 \times 0.30}{100} = 5850 \times 10^{-8} \text{ cm} \text{ or } \lambda = 5850 \text{ \AA}$$

**Thin film interference**

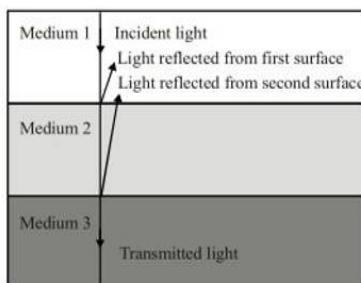
When light passes the boundary between two transparent media, some light is reflected at the boundary. As shown in the figure some light is reflected from first interface and some from second interface. If we consider a monochromatic incident light the two reflected waves are also monochromatic and coherent because they arise from the same monochromatic incident light wave via amplitude division. These waves interfere, since they are superposed along the same normal line.

The phase difference between two interfering waves is due to :

As shown in the figure some light is reflected from first interface and some from second interface. If we consider a monochromatic incident light the two reflected waves are also monochromatic and coherent because they arise from the same monochromatic incident light wave via amplitude division. These waves interfere, since they are superposed along the same normal line.

The phase difference between two interfering waves is due to :

- (1) Optical path difference (due to distances travelled),
- (2) Reflection from a denser medium.



The second factor is irrelevant for reflection at rarer medium.

Three situations may arise :

- (1) Neither wave experiences a phase change upon reflection.
- (2) Both the waves suffer a phase change upon reflection.

In either of these two cases the phase change due to reflection is irrelevant ; no difference in phase results due to reflection.

In either of these cases phase change is determined solely from optical path difference.

Condition for constructive interference :

$$2\mu t = m\lambda$$

Condition for destructive interference :

$$2\mu t = \left(m + \frac{1}{2}\right)\lambda$$

where  $m = 0, 1, 2, \dots$

(3) One of the reflected waves experiences a phase change of  $\pi$  radian upon reflection and the other waves does not.

It is material which wave suffers a phase change ; the conclusions in the previous case are first reversed.

Condition for destructive interference :

$$2\mu t = m\lambda$$

Condition for constructive interference :

$$2\mu t = \left(m + \frac{1}{2}\right)\lambda$$

where  $m = 0, 1, 2, \dots$

### **Illustration :**

Many people's glasses appear to be a blue-green colour when viewed under reflected light. A thin film of index of refraction  $n = 1.35$  is applied to the outside surface of the glass so that the film/glass interface does not reflect any red light incident near normal of wavelength  $\lambda = 630 \text{ nm}$ .

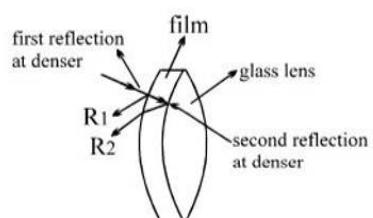
where  $m = 0, 1, 2, \dots$

### **Illustration :**

Many people's glasses appear to be a blue-green colour when viewed under reflected light. A thin film of index of refraction  $n = 1.35$  is applied to the outside surface of the glass so that the film/glass interface does not reflect any red light incident near normal of wavelength  $\lambda = 630 \text{ nm}$ . What thickness must the film layer be in order to achieve this? Take the index of refractions of air and glass to be 1.0 and 1.6 respectively.

- (A) 157.5 nm      (B) 315.0 nm      (C) 233.3 nm      (D) 116.7 nm

**Sol.**



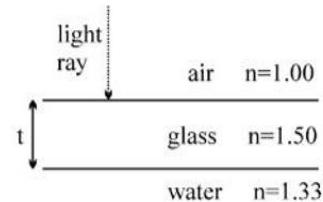
$$\Delta X_{net} = 2\mu t = \lambda/2$$

$$t = \lambda/4\mu = \frac{(630 \times 10^{-9})}{4 \times 1.35} = 116.7 \text{ nm}$$

**Illustration :**

A light ray is incident normal to a thin layer of glass. Given the figure, what is the minimum thickness of the glass that gives the reflected light an orangish color ( $\lambda_{\text{air}} = 600 \text{ nm}$ )?

- (A) 50 nm      (B) 100 nm      (C) 150 nm  
 (D) 200 nm      (E) 500 nm

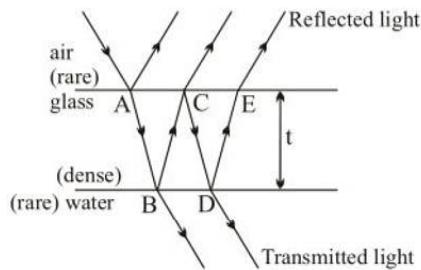


**Sol.** For reflected light to have orangish color, rays from A, C, E must be out of phase for  $l = 600 \text{ nm}$  or  $\delta = (2n + 1) \pi$

$$\text{or } 2\mu_g t (2n + 1) \frac{\lambda}{2}$$

$$\text{i.e. } t = (2n + 1) \frac{\lambda}{4\mu_g}$$

$$\text{or } t_{\min} = \frac{\lambda}{4\mu_g} = 100 \text{ nm}$$

**Interference due to reflected light :**

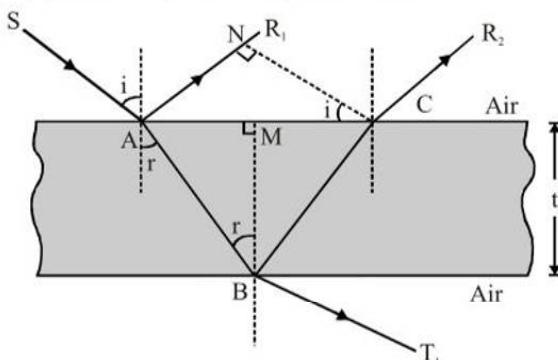
Consider a transparent film of thickness  $t$  and refractive index  $\mu$ . A ray SA incident on the upper surface of the film is partly reflected along  $AR_1$  and partly refracted along AB. At B part of it is reflected along BC and finally emerges out along  $CR_2$ . The difference in path between the two rays  $AR_1$  and  $CR_2$  is

**Interference due to reflected light :**

Consider a transparent film of thickness  $t$  and refractive index  $\mu$ . A ray SA incident on the upper surface of the film is partly reflected along  $AR_1$  and partly refracted along AB. At B part of it is reflected along BC and finally emerges out along  $CR_2$ . The difference in path between the two rays  $AR_1$  and  $CR_2$  is calculated as given below :

Let CN and BM be perpendicular to  $AR_1$  and AC. As the paths of the rays  $AR_1$  and  $CR_2$  beyond CN are equal. The path difference between them is

$$\begin{aligned}\Delta x &= \text{Path ABC in film} - \text{Path AN in air} \\ &= \mu(AB + BC) - AN = 2\mu AB - AN\end{aligned}$$



$$\text{Now, } AB = BC = \frac{AB}{BM} \times BM = BM \sec r = t \sec r$$

$$\text{and } AN = \frac{AN}{AC} \cdot AC = AC \sin i = 2 AM \sin i$$

$$= 2 \frac{AM}{BM} BM \sin i = 2 (\tan r) t \left( \frac{\sin i}{\sin r} \sin r \right)$$

$$= 2\mu t \frac{\sin^2 r}{\cos r} = 2\mu t \sec r \sin^2 r$$

Then,  $\Delta = 2\mu AB - AN = 2\mu t \sec r - 2\mu t \sec r \sin^2 r$   
 $= 2\mu t \sec r (1 - \sin^2 r) = 2\mu t \cos r$

The ray  $AR_1$  having suffered a reflection at the surface of denser medium undergoes a phase change  $\pi$  or path diff. of  $\frac{\lambda}{2}$ .

At B the reflection takes place when the ray is going from a denser to rarer medium and there is no phase change.

Hence, the effective path difference between  $AR_1$  and  $CR_2$  is given by

$$\text{Path Diff. } (\Delta x) = 2\mu t \cos r - \frac{\lambda}{2}$$

- (i) If the path difference  $\Delta x = n\lambda$  where  $n = 0, 1, 2, 3, 4$  etc., constructive interference takes place and the film appears bright.

$$\therefore 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

- (i) If the path difference  $\Delta x = n\lambda$  where  $n = 0, 1, 2, 3, 4$  etc., constructive interference takes place and the film appears bright.

$$\therefore 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$\text{or } 2\mu t \cos r = (2n + 1) \frac{\lambda}{2}$$

- (ii) If the path difference  $\Delta x = (2n + 1) \frac{\lambda}{2}$  where  $n = 0, 1, 2, \dots$  etc., destructive interference takes place and the film appears dark.

$$\therefore 2\mu t \cos r - \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2} \text{ or } 2\mu t \cos r = n\lambda$$

### Remarks :

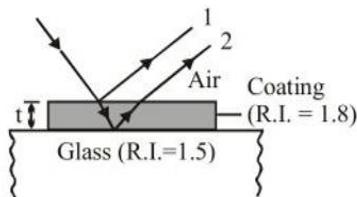
- (i) If the thickness of the film is very small as compared to the wavelength of light used, so that  $2\mu t \cos r$  can be neglected, then the total path difference between  $AR_1$  and  $CR_2$  will reduce to  $\frac{\lambda}{2}$ . Thus two rays will interfere destructively and darkness will result.
- (ii) It should be remembered that the interference pattern will not be perfect because the intensities of the ray  $AR_1$  and  $CR_2$  will not be the same.

**Illustration :**

A glass plate of refractive index 1.5 is coated with a thin layer of thickness  $t$  and refractive index 1.8. Light of wavelength  $\lambda$  travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If  $\lambda = 648 \text{ nm}$ , obtain the least value of  $t$  for which the rays interfere constructively.

**Sol.** The ray reflected from upper surface suffer a phase change of  $\pi$  due to reflection, at denser media, so the condition of constructive interference for normal incidence is given by

$$2\mu t + \frac{\lambda}{2} = n\lambda \quad \text{or} \quad 2\mu t = \frac{(2n-1)\lambda}{2}$$



For minimum value of  $t$ ,  $n = 1$

$$t_{\min} = \frac{\lambda}{4\mu} = 90 \text{ nm}$$

**Illustration :**

White light may be considered to have  $\lambda$  from  $400 \text{ \AA}$  to  $7500 \text{ \AA}$ . If an oil film has thickness  $10^{-4} \text{ cm}$ ,

$$t_{\min} = \frac{\lambda}{4\mu} = 90 \text{ nm}$$

**Illustration :**

White light may be considered to have  $\lambda$  from  $400 \text{ \AA}$  to  $7500 \text{ \AA}$ . If an oil film has thickness  $10^{-4} \text{ cm}$ , deduce the wavelength in the visible region for which the reflection along the normal direction will be (i) weak (ii) strong. Take  $\mu$  of oil as 14.

**Sol.** Here  $r = 0^\circ$ ;  $\mu = 1.4$ ;  $t = 10^{-4} \text{ cm}$

$$\therefore 2\mu t = 2 \times 1.4 \times 10^{-4} \text{ cm} = 2.8 \times 10^{-4} \times 10^8 \text{ \AA} = 28000 \text{ \AA}$$

(i) Condition for weak reflection (destructive interference) is given by

$$2\mu t = n\lambda$$

$$\text{or } \lambda = \frac{2\mu t}{n} = \frac{28000}{n}$$

The value of  $n$  should be selected such that  $\lambda$  lies between  $4000 \text{ \AA}$  and  $7500 \text{ \AA}$ . This will be possible if

$$\lambda = \frac{28000}{4} = 7000 \text{ \AA} \quad (\text{for } n = 4)$$

$$\lambda = \frac{28000}{5} = 5600 \text{ \AA} \quad (\text{for } n = 5)$$

$$\lambda = \frac{28000}{6} = 4667 \text{ \AA} \quad (\text{for } n = 6)$$

$$\lambda = \frac{28000}{7} = 4000 \text{\AA} \quad (\text{for } n = 7)$$

The other values of  $n$  are not allowed as for those value of  $n$ ,  $\lambda$  does not lie within the given wavelength range of  $4000 \text{\AA}$  to  $7500 \text{\AA}$ . Hence, all above values of  $\lambda$  cause weak reflection.

(ii) For strong reflection (constructive interference), we have

$$2\mu t = (2n + 1) \left( \frac{\lambda}{2} \right)$$

$$\therefore \lambda = \frac{2 \times 2\mu t}{2n+1} = \frac{2 \times 28000}{2n+1} = \frac{56000}{2n+1}$$

The possible values of  $\lambda$  in this case are given by

$$\lambda = \frac{56000}{9} = 6222 \text{\AA} \quad (\text{for } n = 4)$$

$$\lambda = \frac{56000}{11} = 5091 \text{\AA} \quad (\text{for } n = 5)$$

$$\lambda = \frac{56000}{13} = 4300 \text{\AA} \quad (\text{for } n = 6)$$

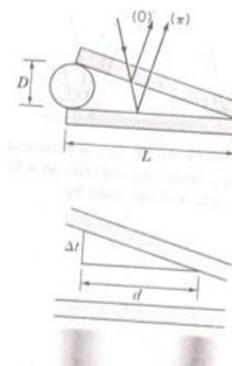
Hence, only the above of  $n$  will cause strong reflection because the range will not be within desired wavelengths, if  $n$  is different.

at desired wavelengths, if  $n$  is different.

### Fringes of equal thickness

Soap bubbles and oil films on a road do not have uniform thickness of the film at any given point determines whether the reflected light has a maximum or minimum intensity. When white light is used, each wavelength has its own fringe pattern. At a given point of the film, one wavelength may be enhanced and / or another wavelength suppressed. This is the source of the colors in soap bubbles and oil films on the road.

A wedge-shaped film of air may be produced by placing a sheet of paper or a hair between the ends of two glass plates, as in fig. With flat plates, one sees a series of bright and dark bands, each characteristic of a particular thickness. If the plates are not flat, the fringes are not straight ; each is locus of points with the same thickness. If one plate is known to be flat, the fringes display the irregularities of the other, as shown in figure. The pattern shows where the plate needs to be polished for it to be made "optically flat."



**Illustration :**

A wedge-shaped film of air is produced by placing a fine wire of diameter  $D$  between the ends of two flat glass plates of length  $L = 20 \text{ cm}$ , as in fig. When the air film is illuminated with light of wavelength  $\lambda = 550 \text{ nm}$ , there are 12 dark fringes per centimeter. Find  $D$ .

**Sol.** A indicated in fig. only one of the reflected ray suffers a phase inversion. At the thin end of the wedge, where the thickness is less than  $\lambda/4$ , the two rays interfere destructively. This region is dark in the reflected light. The condition for destructive interference in the reflected light is

$$2t = m\lambda \quad m = 0, 1, 2, \dots$$

the change in thickness between adjacent dark fringes is  $\Delta t = \lambda / 2$ . The horizontal spacing between fringes  $d = 1/12 \text{ cm} = 8.3 \times 10^{-4} \text{ m}$ . From figure we see that  $D/L = \Delta t/d$ , so

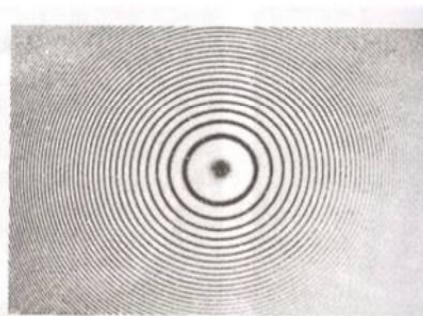
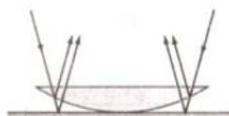
$$\Delta = \frac{\lambda L}{2d} = \frac{(5.5 \times 10^{-7} \text{ m})(0.2 \text{ m})}{16.6 \times 10^{-4} \text{ m}}$$

$$\text{Thus } D = 6.6 \times 10^{-5} \text{ m}$$

**Newton's Rings**

When a lens with a large radius of curvature is placed on a flat plate, as to fig. a thin film of air is formed. When the film is illuminated with monochromatic light, circular fringes, called newton's rings, can be seen with the unaided eye or with a low power microscope (figure). An important feature of Newton's rings is the dark central spot. Newton tried polishing the surfaces to get rid of it, The dark spot was also initially puzzling to Young. It implied that the light wave suffers a phase inversion on reflection at a medium with a higher refractive index. Young tested this idea by placing oil of sassafras between a lens of crown glass and a plate of flint glass. The refractive index of the oil is between the values for these two glasses. Since

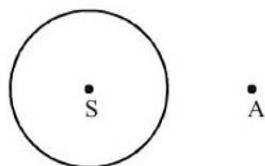
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## Huygen's Principle

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. We can guess that he must have seen water waves many times in the canals of his native place Holland. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down.

Huygens, considered light to be a mechanical wave moving in a hypothetical medium which was named as ether. If we consider a surface  $\sigma$  enclosing a light source  $S$ , the optical disturbance at any point beyond  $\sigma$  must reach after crossing  $\sigma$ . The particles of the surface  $\sigma$  vibrate as the wave from  $S$  reaches there and these vibrations cause the layer beyond to vibrate. We can thus assume that the particles on  $\sigma$  act as new sources of light waves emitting spherical waves and the disturbance at a point  $A$  (figure 17.1) beyond  $\sigma$  is caused by the superposition of all these spherical waves coming from different points of  $\sigma$ . Huygens called the particles spreading the vibration beyond them as *secondary sources* and the spherical wavefronts emitted from these secondary sources as the *secondary wavelets*.



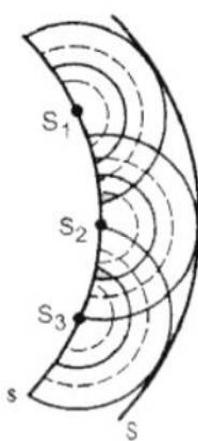
Huygens' principle may be stated in its most general form as follows :

Various points of an arbitrary surface, when reached by a wavefront, become secondary sources of

Huygens' principle may be stated in its most general form as follows :

Various points of an arbitrary surface, when reached by a wavefront, become secondary sources of light emitting secondary wavelets. The disturbance beyond the surface results from the superposition of these secondary wavelets.

Consider a spherical surface  $\sigma$  with its centre at a point source  $S$  emitting a pulse of light. The optical disturbance reaches the particles on  $\sigma$  at time  $t = 0$  and lasts for a short interval in which the positive and negative disturbances are produced. These particles on  $\sigma$  then send spherical wavelets which spread beyond  $\sigma$ . At time  $t$ , each of these wavelets has a radius  $vt$ . In figure the solid lines represent positive optical disturbance and the dashed lines represent negative optical disturbance. The sphere  $\Sigma$  is the geometrical envelope of all the secondary wavelets which were emitted at time  $t = 0$  from the primary wavefront  $\sigma$ .



It is clear that at the points just inside  $\Sigma$ , only the positive disturbances of various secondary wavelets are meeting. The wavelets, therefore, interfere constructively at these points and produce finite disturbance. For points well inside  $\Sigma$ , some of the wavelets contribute positive disturbance and some others, centred at a nearby point of  $\sigma$  produce negative disturbance. Thus, the resultant disturbance is zero at these points. The disturbance which was situated at  $\sigma$  at time  $t = 0$  is, therefore, confined to a surface  $\Sigma$  at time  $t$ . Hence, the secondary wavelets from  $\sigma$  superpose in such a way that they produce a new wavefront at the geometrical envelope of the secondary wavelets.

This allows us to state the method of Huygens construction as follows :

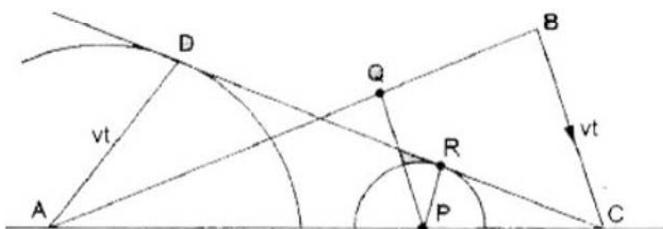
### Huygens construction

*Various points of an arbitrary surface, as they are reached by a wavefront, become the sources of secondary wavelets. The geometrical envelope of these wavelets at any given later instant represents the new position of the wavefront at that instant.*

The method is quite general and although it was developed on the notion of mechanical waves it is valid for light waves. The surface used in the Huygens construction may have any arbitrary shape, not necessarily a wavefront itself. If the medium is homogeneous, (i.e., the optical properties of the medium are same everywhere) light moves forward and does not reflect back. We assume, therefore, that the secondary wavelets are emitted only in the forward direction and the geometrical envelope of the wavelets is to be taken in the direction of advancement of the wave. If there is a change of medium, the wave may be reflected from the discontinuity just as a wave on a string is reflected from a fixed end or a free end. In that case, secondary wavelets on the backward side should also be considered.

### Reflection of Light

Let us suppose that a parallel light beam is incident upon a reflecting plane surface  $\sigma$  such as a plane mirror. The wavefronts of the incident wave will be planes perpendicular to the direction of incidence. After reflection, the light returns in the same medium. Consider a particular wavefront  $AB$  of the incident light wave at  $t = 0$  (figure). We shall construct the position of this "wavefront at time  $t$ .



To apply Huygens construction, we use the reflecting surface  $\sigma$  for the sources of secondary wavelets.

As the various points of  $\sigma$  are reached by the wavefront AB, they become sources of secondary wavelets. Because of the change of medium, the wavelets are emitted both in forward and backward directions. To study reflection, the wavelets emitted in the backward directions are to be considered.

Suppose, the point A of  $\sigma$  is reached by the wavefront AB at time  $t = 0$ . This point then emits a secondary wavelet. At time  $t$ , this wavelet becomes a hemispherical surface of radius  $vt$  centred at A. Here  $v$  is the speed of light. Let C be the point which is just reached by the wavefront at time  $t$  and hence the wavelet is a point at C itself. Draw the tangent plane CD from C to the hemispherical wavelet originated from A. Consider an arbitrary point P on the surface and let  $AP/AC = x$ . Let PQ be the perpendicular from P to AB and let PR be the perpendicular from P to CD. By the figure,

$$\frac{PR}{AD} = \frac{PC}{AC} = \frac{AC - AP}{AC} = 1 - x$$

$$\text{or, } PR = AD(1 - x) = vt(1 - x) \quad \dots(i)$$

$$\text{Also, } \frac{QP}{BC} = \frac{AP}{AC} = x$$

$$\text{or } QP = x BC = xt.$$

The time taken by the wavefront to reach the point P is, therefore,

$$t_1 = \frac{QP}{v} = xt$$

The point P becomes a source of secondary wavelets at time  $t_1$ . The radius of the wavelet at time  $t_1$  originated from P is, therefore,

$$a = u(t - t_1) = u(t - xt) = ut(1 - x). \quad \dots(ii)$$

By (i) and (ii), we see that PR is the radius of the secondary wavelet at time  $t$  coming from P. As CD is perpendicular to PR, CD touches this wavelet. As P is an arbitrary point on  $\sigma$  all the wavelets originated from different points of  $\sigma$  touch CD at time  $t$ . Thus, CD is the envelope of all these wavelets at time  $t$ . It is, therefore, the new position of the wavefront AB. The reflected rays are perpendicular to this wavefront CD.

In triangles ABC and ADC :

$$AD = BC = vt,$$

AC is common.

$$\text{and } \angle ADC = \angle ABC = 90^\circ$$

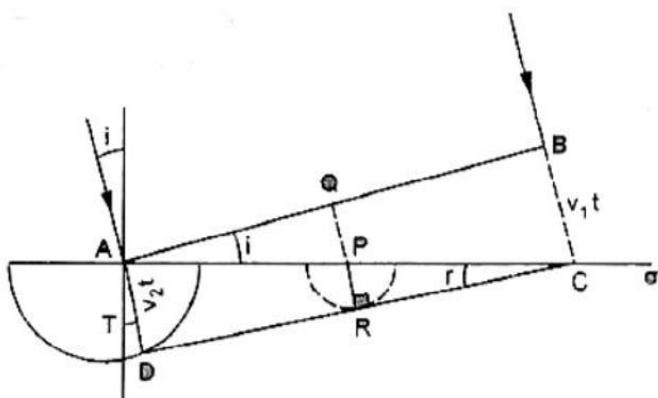
Thus, the triangles are congruent and

$$\angle BAC = \angle DCA \quad \dots(iii)$$

Now, the incident ray is perpendicular to AB and the normal is perpendicular to AC. The angle between the incident ray and the normal is, therefore, equal to the angle between AB and AC. Thus,  $\angle BAC$  is equal to the angle of incidence.

Similarly,  $\angle DCA$  represents the angle of reflection and we have proved in (iii) that the angle of incidence equals the angle of reflection. From the geometry, it is clear that the incident ray, the reflected ray and the normal to the surface AC lie in the plane of drawing and hence, are coplanar.

## Refraction of Light



Suppose  $\sigma$  represents the surface separating two transparent media, medium 1 and medium 2 in which the speeds of light are  $v_1$  and  $v_2$  respectively. A parallel beam of light moving in medium 1 is incident

$$\text{or } \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

which is called the Snell's law. The ratio  $v_1/v_2$  is called the refractive index of medium 2 with respect to

$$\text{or } \frac{1}{\sin r} = \frac{v_1}{v_2}$$

which is called the Snell's law. The ratio  $v_1/v_2$  is called the refractive index of medium 2 with respect to medium 1 and is denoted by  $\mu_{21}$ . If the medium 1 is vacuum,  $\mu_{21}$  is simply the refractive index of the medium 2 and is denoted by  $\mu$ .

$$\mu_{21} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{\mu_2}{\mu_1}$$

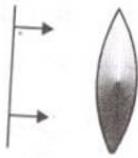
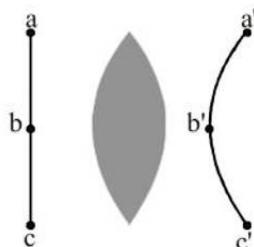
From the figure, it is clear that the incident ray, the refracted ray and the normal to the surface  $\sigma$  are all in the plane of the drawing, i.e., they are coplanar.

Suppose light from air is incident on water. It bends towards the normal giving  $i > r$ . From Snell's law proved above,  $v_1 > v_2$ . Thus, according to the wave theory the speed of light should be greater in air than in water. This is opposite to the prediction of Newton's corpuscle theory. If light bends due to the attraction of the particles of a medium then speed of light should be greater in the medium. Later, experiments on measurement of speed of light confirmed wave theory.

Thus, the basic rules of geometrical optics could be understood in terms of the wave theory of light using Huygens' principle.

**Illustration :**

If a plane wavefront is incident on a convex lens as shown in figure, how will transmitted wavefront appear.

**Sol.**

*a, b and c lie on a wavefront i.e. they are in phase.*

*We have shown in figure that a reaches a', b reaches b' and c reaches c'.*

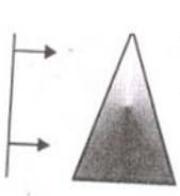
*a' and c' are ahead of b' because b has to travel more in denser medium in which velocity of light is less in comparison to air.*

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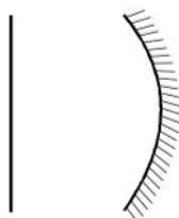
**Practice Exercise**

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- Q.1 If a plane wavefront is incident on an optical device as shown in figure, how will transmitted wavefront appear.



(A)



(B)



(C)

---

**Answers**

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- Ans. (A) (B) (C)

## Why is interference observed only in thin films

Interference effects can be ignored with a thick film because its thickness is large. In this case the alternate bright and dark regions will be so close to each other that these will appear to merge into one another and interference pattern will not be visible.

Further, when we see such a film in white light, the various complementary colors will be so close to each other that these merge into one another and make the appearance of the thick film white. To understand this let us consider a film of water (refractive index,  $\mu = 4/3$ ) 1 cm thick on top of a glass surface ( $RI = 3/2$ ).

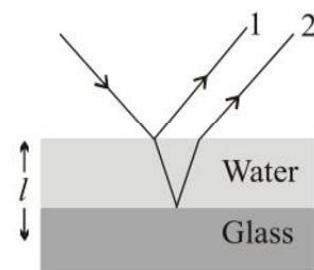
Assume that it is illuminated from above (fig.). Then

$$\Delta x = \text{path difference between 1 and 2} = 2\mu_{\omega} t.$$

For constructive interference in the reflected waves

$$2m_{\omega} t = n\lambda$$

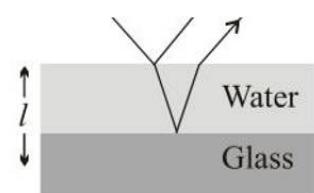
Because  $t = 1$  cm is very large in comparison to the wavelength of visible light, the values of  $n$  in this equation will be large. For instance, if wavelength corresponding to orange-red light ( $\lambda = 667$  nm) is to be strongly reflected then



For constructive interference in the reflected waves

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Because  $t = 1$  cm is very large in comparison to the wavelength of visible light, the values of  $n$  in this equation will be large. For instance, if wavelength corresponding to orange-red light ( $\lambda = 667$  nm) is to be strongly reflected then



$$n = \frac{2\mu_{\omega} t}{\lambda} = \frac{(2)(1.33)(1 \times 10^{-2})}{6.67 \times 10^{-7}} = 40000$$

We may feel that this thick film would appear strongly orange red; however, there will also be strong reflection of a slightly longer wavelength corresponding to  $n = 39,999$  and of slightly shorter wavelength corresponding to  $n = 40,001$ . These wavelengths differ from each other by only one part in 40,000 or  $667 \text{ nm}/40,000 = 0.17 \text{ nm}$ . This is a much smaller wavelength difference than the eye can detect. Hence, no one wavelength appears to be reinforced more than any other in the light which is reflected from a thick film. If the film is illuminated by a white light, the reflected light appears white. For thin films, the integer  $n$  will be small, the difference between adjacent strongly reflected wavelengths will be substantial, and the preferential reflection of certain wavelengths will be easily observed by the eye.

## SOLVED EXAMPLES

**Q.1** In Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 micron is introduced in the path of one of interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift on the introduction of mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

**Sol.** Shift ' $\Delta y'$  in the fringe system is

$$\Delta y = \frac{\beta}{\lambda} (\mu - 1) t$$

when distance between slits and screen is doubled,

$$\beta' = \beta$$

$$\text{Given } \beta' = \Delta y$$

$$\therefore \frac{\beta}{\lambda} (\mu - 1) t = 2\beta$$

$$\therefore \lambda = \frac{(\mu - 1)t}{2}$$

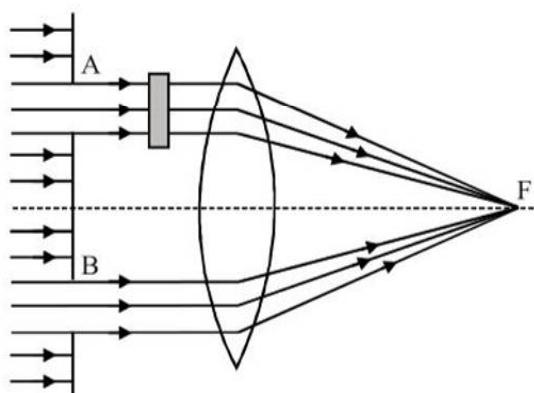
$$\text{Here, } \mu = 1.6, t = 1.964 \times 10^{-6} \text{ m}$$

$$\lambda = \frac{(1.6 - 1) \times 1.965 \times 10^{-6}}{2}$$

$$\lambda = 0.3 \times 1.964 \times 10^{-6} \text{ m}$$

$$\lambda = 5892 \text{ \AA} \quad \text{Ans.}$$

**Q.2** In a modified Young's double slit experiment, monochromatic uniform and parallel beam of light of wavelength  $6000 \text{ \AA}$  and intensity  $\left(\frac{10}{\pi}\right) \text{ Wm}^{-2}$  incident normally on two circular apertures A and B of radii  $0.001 \text{ m}$  and  $0.002 \text{ m}$  respectively. A perfectly transparent film of thickness  $2000 \text{ \AA}$  and refractive index 1.5 for the wavelengths  $6000 \text{ \AA}$  is placed in front of aperture A (figure). Calculate the power(in watt) received all the focal spot F of the lens. The lens is symmetrically placed with respect to the aperture. Assume that 10% of the power received by each observer goes in the original direction and is brought to the focal spot.



Sol. Let  $I_1$  and  $I_2$  be the intensities at A and B

$$I_1 = I_2 = \frac{10}{\pi} \text{ Wm}^{-2}$$

Area of cross-section of aperture A,  $A_1 = \pi r_1^2 = \pi \times (0.001)^2 = \pi \times 10^{-6} \text{ m}^2$

Area of cross-section of aperture B,  $A_2 = \pi r_2^2 = \pi \times (0.001)^2 = \pi \times 4 \times 10^{-6} \text{ m}^2$

Let  $P_1$  and  $P_2$  be the powers of incident radiations at A and B respectively.

$$P_1 = \frac{10}{\pi} \times \pi \times 10^{-6} = 10^{-5} \text{ W}$$

$$P_2 = \frac{10}{\pi} \times 4\pi \times 10^{-6} = 4 \times 10^{-5} \text{ W}$$

Induction of a transparent medium in one of the beams produces some path difference  $\Delta x$ .

Here,  $\mu = 1.5$  and  $t = 2000 \text{ \AA}$

$$\therefore \Delta x = (1.5 - 1) \times 2000 \text{ \AA} = 0.5 \times 2000 \text{ \AA}$$

$$\text{or } x = 10^{-7} \text{ m}$$

Let  $\phi$  = phase difference between the two beams

$$\phi = \frac{2\pi}{\lambda} x$$

$$\begin{array}{c} \sim \\ \lambda \end{array}$$

$$\text{or } \phi = \frac{2\pi}{6000 \times 10^{-10}} \times 10^{-7} = \frac{\pi}{3} \text{ radian}$$

If  $a_1$  and  $a_2$  are the amplitudes of light from apertures A and B, net amplitude R at F is,

$$R^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

$$\text{Power} = \text{Intensity} \times \text{Area of cross-section} = I \times A^2$$

$$\text{or } P = K R^2 \times A^2 = K' R^2$$

$$P_1 = K' a_1^2$$

$$\text{and } P_2 = K' a_2^2$$

Multiply equation by  $K'$  throughout

$$K' R^2 = K' a_1^2 + K' a_2^2 + 2\sqrt{K'} a_1 \sqrt{K'} a_2 \cos \phi$$

$$\text{or } P = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \phi$$

Substituting for  $P_1$ ,  $P_2$  and  $\phi$ , we get

$$P = (10)^{-5} + 4 \times 10^{-5} + 2 \sqrt{10^{-5} \times 4 \times 10^{-5}} \cos \left( \frac{\pi}{3} \right)$$

$$\text{or } P = 10^{-5} (1 + 4 + 2)$$

$$\text{or } P = 7 \times 10^{-5} \text{ W} \quad \text{Ans.}$$

Q.3 A source of light of wavelength  $5000 \text{ \AA}$  is placed at one end of a table 200 cm long and 5 mm above its flat well polished top. Find the fringe-width of the interference bands located on a screen at the end of the table.

Sol. Distance of source S from the table = 5 mm = 0.5 cm

Distance of S' from table = 0.5 cm

If 'd' is the distance between S and S'

$$d = 0.5 + 0.5 = 1 \text{ cm}$$

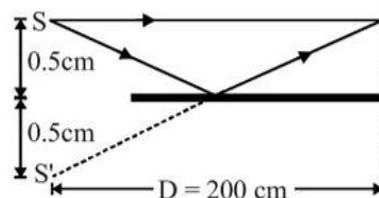
$$D = 200 \text{ cm}$$

$$\lambda = 5000 \text{ \AA} = 5000 \times 10^{-8} \text{ cm} = 5 \times 10^{-5} \text{ cm}$$

$$\text{Since, } \beta = \frac{\lambda D}{d}$$

$$\therefore \beta = \frac{5 \times 10^{-5} \times 200}{1} = 10^{-2} \text{ cm}$$

$$\beta = 0.01 \text{ cm} \quad \text{Ans.}$$



Q.4 In the usual layout for interference fringes, two identical slits, each of width  $a$  are kept apart by  $d$  from centre of centre. find :

- (a) the difference of path differences between rays from the bottom and top of slits i.e.,  $\delta\Delta = \Delta_b - \Delta_t$ ,
- (b) the maximum value of  $a$  at which interference fringes continue to be sharp. Take  $D$ =distance between the screen and the slits

the screen and the slits.

Sol (a) The rays from the top of the slits may be assumed to come from ideal sources with their pole (equidistant point on the screen) at O. Then

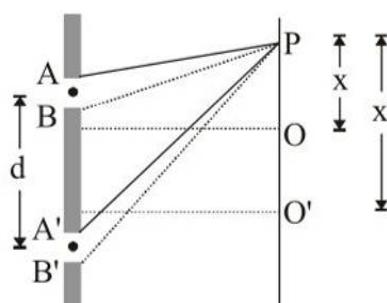
$$\Delta_t = \frac{xd}{D}$$

$$\text{Similarly, } \Delta_b = \frac{x'd}{D}$$

$$\therefore \Delta_b - \Delta_t = \frac{(x'-x)d}{D}$$

$$\text{But } x' - x = a$$

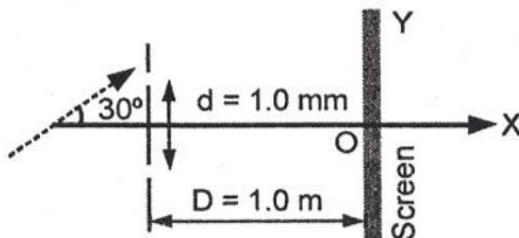
$$\therefore \delta\Delta = \Delta_b - \Delta_t = \frac{ad}{D}$$



(b) If  $\delta\Delta = \frac{\lambda}{2}$ , the maximum from top edges will be superimposed on to the minimum from the bottom edges, owing to which the interference pattern will disappear completely.

$$\therefore \frac{\lambda}{2} = \frac{ad}{D} \Rightarrow \frac{\lambda D}{2d} \quad \text{Ans.}$$

- Q.5 A coherent parallel beam of microwaves of wavelength  $\lambda = 0.5 \text{ mm}$  falls on a Young's double slit apparatus. The separation between the slits  $1.0 \text{ mm}$ . The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of  $1.0 \text{ m}$  from it as shown in the figure.
- (a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.



- (b) If the incident beam makes an angle of  $30^\circ$  with the x-axis (as in the dotted arrow shown in fig.), find the y-coordinates of the first minima on either side of the central maximum.

Sol. (a) As shown in fig. the path difference between the two interfering waves reaching the point P of the screen will be  $\Delta x = d \sin \theta$  and so the point P will be an interference minima if

$$\Delta x = \frac{(2n-1)\lambda}{2} \text{ with } n = 1, 2, \dots$$

$$\text{So } d \sin \theta = (2n-1) \frac{\lambda}{2}$$

$$\text{i.e., } \sin \theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)}{4}$$

$$\text{i.e., } \sin \theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)}{4}$$

and as  $\sin \theta \leq 1$

$$\text{i.e., } n \leq 2.5$$

$$\therefore n = 1 \text{ or } 2$$

When  $n = 1$ ,

$$\sin \theta_1 = \frac{1}{4}$$

$$\text{So that } \tan \theta_1 = \frac{1}{\sqrt{15}}$$

$$\text{When } n = 2$$

$$\sin \theta_2 = \frac{3}{4}$$

$$\text{So that } \tan \theta_2 = \frac{3}{\sqrt{17}}$$

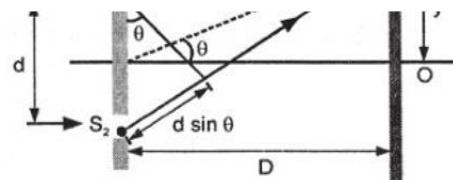
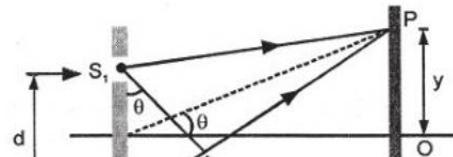
Now, the position of a point P on the screen which is at a distance D from the plane of slits will be given by

$$y = D \tan \theta = \tan \theta \quad (\because D = 1 \text{ m})$$

So, the position of minima will be

$$y_1 = \tan \theta_1 = \frac{1}{\sqrt{15}} = 0.258$$

$$\text{and } y_2 = \tan \theta_2 = \frac{3}{\sqrt{17}} = \frac{3}{2.6} = 1.13 \text{ m}$$



And as minima can be on either side of principal maxima, in the situation given there will be 4 minima at positions  $\pm 2.258$  m and  $\pm 1.13$  m on the screen.

(b) In this situation as shown in fig., the path difference between the interfering waves will be

$$\Delta x = [d \sin \theta - d \sin \phi]$$

$$\text{For first minima, } d [\sin \theta - \sin \phi] = \pm \frac{\lambda}{2}$$

$$\text{i.e. } \sin \theta = \sin \phi \pm \frac{\lambda}{2d}$$

Here,  $\phi = 30^\circ$ ;  $\lambda = 0.5$  mm and  $d = 1$  mm,

$$\therefore \sin \theta = \frac{1}{2} \pm \frac{0.5}{2 \times 1}$$

$$\text{or } \sin \theta = \frac{3}{4} \text{ or } \frac{1}{4},$$

$$\text{or } \tan \theta = \frac{3}{\sqrt{7}} \text{ or } \frac{1}{\sqrt{15}}$$

So, the position of first minima on either side of central maxima in this situation will be

$$y = D \tan \theta = \tan \theta \quad (\because D = 1\text{m}),$$

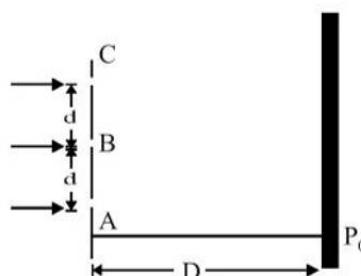
$$\therefore y = \frac{3}{\sqrt{7}} \text{ m and } \frac{1}{\sqrt{15}} \text{ m Ans.}$$

So, the position of first minima on either side of central maxima in this situation will be

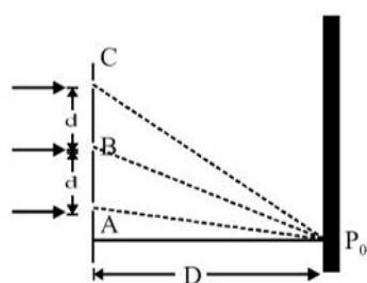
$$y = D \tan \theta = \tan \theta \quad (\because D = 1\text{m}),$$

$$\therefore y = \frac{3}{\sqrt{7}} \text{ m and } \frac{1}{\sqrt{15}} \text{ m Ans.}$$

- Q.6 Figure shows three equidistant slits being illuminated by a monochromatic parallel beam of light. Let  $BP_0 - AP_0 = \lambda/3$  and  $D \gg \lambda$ . (a) Show that in this case  $d = \sqrt{2\lambda D / 3}$ . (b) Show that the intensity at  $P_0$  is zero.



Sol.



$$BP_0 - AP_0 = \frac{\lambda}{3}$$

or  $d \sin \theta = \frac{\lambda}{3}$

or  $d \tan \theta = \frac{\lambda}{3}$  (For small angle  $\tan \theta \approx \sin \theta \approx \theta$ )

$$\therefore d \left( \frac{d/2}{D} \right) = \frac{\lambda}{3}$$

or  $d = \sqrt{\frac{2F\lambda}{3}}$

(b)  $\Delta x_{A/B} = \text{path difference between waves coming from A and B} = \frac{\lambda}{3}$

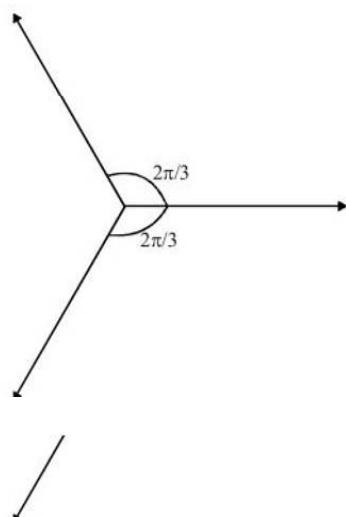
$\therefore \phi_{A/B} = \text{phase difference}$

$$= \frac{2\pi}{\lambda} \Delta x_{A/B} = \frac{2\pi}{3}$$

Similarly,  $\Delta x_{B/C} = \Delta x_{A/B} = \lambda/3$

$\therefore \phi_{B/C} = 2\pi/3$

Now, phase diagram of the waves arriving at  $P_0$  is as shown below :



$\therefore$  Amplitude of resultant wave is zero

As intensity ( $I \propto A^2$ )

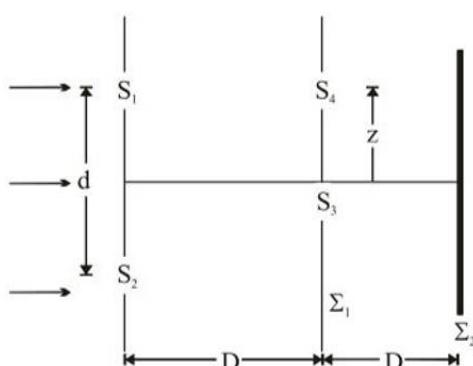
$\therefore$  Intensity at  $P_0$  will be zero.

$\dots$

As intensity ( $I \propto A^2$ )

$\therefore$  Intensity at  $P_0$  will be zero.

- Q.7 Consider the situation shown in figure. The two slits  $S_1$  and  $S_2$  placed symmetrically around the central line are illuminated by a monochromatic light of wavelength  $\lambda$ . The separation between the slits is  $d$ . The light transmitted by the slits falls on a screen  $\Sigma_1$  placed at a distance  $D$  from the slits. The slit  $S_3$  is at the central line and the slits  $S_4$  is at a distance  $z$  from  $S_3$ . Another screen  $\Sigma_2$  is placed a further distance  $D$  away from  $\Sigma_1$ . Find the ratio of the maximum to minimum intensity observed on  $\Sigma_2$  if  $z$  is equal to



(a)  $\frac{D\lambda}{2d}$

(b)  $\frac{\lambda D}{4d}$

Sol. (a) Let  $I$  is intensity due to slits  $S_1$  and  $S_2$  on screen  $\Sigma_1$ . Further, intensity at any point on screen  $\Sigma_1$  is given by

$$I_p = 4I \cos^2 \left( \frac{\phi}{2} \right)$$

At slit  $S_3$ ,  $\phi = 0$

$$\therefore I_{S_3} = 4I$$

At slit  $S_4$ ,  $\Delta x = \frac{dz}{D} = \frac{\lambda}{2}$

$$\therefore \phi = \pi \Rightarrow I_{S_4} = 0$$

Now on screen  $\Sigma_2$   $I_{\max} = (\sqrt{I_{S_3}} + \sqrt{I_{S_4}})^2 = 4I$

$$I_{\min} = (\sqrt{I_{S_3}} - \sqrt{I_{S_4}})^2 = 4I$$

$$\therefore \frac{I_{\max}}{I_{\min}} = 1 \text{ Ans}$$

(b)  $z = \frac{D\lambda}{4d}$

(b)  $z = \frac{D\lambda}{4d}$

$$I_{S_3} = 4I$$

At slit  $S_4$ ,  $\Delta x = \frac{dz}{D} = \frac{\lambda}{4}$

$$\therefore \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\therefore I_{S_4} = 4I \cos^2 \left( \frac{\pi}{4} \right) = 2I$$

$$\begin{aligned} \therefore I_{\max} &= (\sqrt{I_{S_3}} + \sqrt{I_{S_4}})^2 = (\sqrt{4I} + \sqrt{2I})^2 \\ &= I(2 + \sqrt{2})^2 \end{aligned}$$

Similarly,  $I_{\min} = (\sqrt{I_{S_3}} - \sqrt{I_{S_4}})^2 = (\sqrt{4I} - \sqrt{2I})^2 = I(2 - \sqrt{2})^2$

$$\therefore \frac{I_{\max}}{I_{\min}} = \left( \frac{2 + \sqrt{2}}{2 - \sqrt{2}} \right)^2 \text{ Ans.}$$