Electrostatics

Introduction

Electromagnetism is, almost unarguably, the most important basic technology in the world today. Almost every modern device, from cars to kitchen appliances to computers, is dependent upon it. Life, for most of us, would be almost unimaginable without electromagnetism. In fact, electromagnetism cuts such a wide path through modern life that the teaching of electromagnetism has developed into several different specialties.Initially electricity and magnetism were classified as independent phenomena, but after some experiments (we will discuss later) it was found they are interrelated so we use the name **Electromagnetism** . In electromagnetism we have to study basic properties of electromagnetic force and field (the term field will be introduced in later section).The electromagnetic force between charged particles is one of the fundamental forces of nature. We begin this chapter by describing some of the basic properties of one manifestation of the electromagnetic force, the electrostatic force between charges (the force between two charges when they are at rest) under the heading **electrostatics.**

Electric Charge

A number of simple experiments demonstrate the existence of electric forces and charges. For example, after running a comb through your hair on a dry day, you will find that the comb attracts bits of paper. The attractive force is often strong enough to suspend the paper. The same effect occurs when certain materials are rubbed together, such as glass rubbed with silk or rubber with fur. When materials behave in this way, they are said to be *electrified*, or to have become electrically charged.A neutral body can get charged only by transfer of electrons, thus the lowest unit of free charge that may appear on a body is charge of electron whose magnitude is e. When a body gets n electrons from other body charge on it becomes – ne while charge on body loosing n electrons becomes + ne.

Unit of charge :

SI unit : coulomb (C). c.g.s. unit : e.s.u (electrostatic unit) or stat coulomb.

$1C = 2.998 \times 10^9$ esu

Basic properties of electric charge

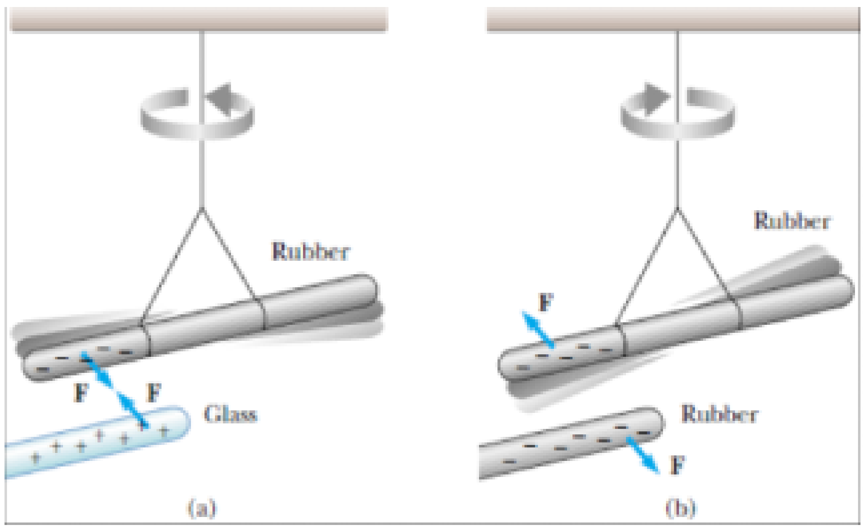
(i) There exist two types of charges in nature : positive and negative

Experimentally, it was found that there are two kinds of electric charges, which were given the names positive and negative by Benjamin Franklin (1706–1790). We identify negative charge as that type possessed by electrons and positive charge as that possessed by protons.

Secondly, it should also be noted that naming one charge as (+)ve and the other as (–)ve is a matter of convention; there is no intrinsically compelling reason for this choice.

(ii) Like charges repel and unlike charges attract.

To verify this, suppose a hard rubber rod that has been rubbed with fur is suspended by a sewing thread, as shown in Figure . When a glass rod that has been rubbed with silk is brought near the rubber rod, the two attract each other (Fig. a). On the other hand, if two charged rubber rods (or two charged glass rods) are brought near each other, as shown in Figure b, the two repel each other. This observation shows that the rubber and glass have two different types of charge on them. On the basis of these observations, we conclude that charges of the same sign repel one another and charges with opposite signs attract one another.



(iii) Charges are additive i.e. the charges add algebraically despite the fact that the words (+)ve and (–)ve don't have any algebraic meaning due to the property of the charges that equal amount of two types of charges present at a point neutralize the effect of one another and hence, the presence of none can be felt i.e., they behave like an uncharged state.

(iv) Charge is quantized : Charge exists in discrete units equal to the integral multiple of electronic charge (Charge on one electron)

i.e., $Q = ne$

Where $e(>0)$ is the lowest possible magnitude of charge and n belongs to the set of integers :

$n = 0, \pm 1, \pm 2, \pm 3, ...$ and

$e =$ magnitude of charge on one electron $= 1.6 \times 10^{-19}$ C

Our advanced nuclear research, however, suggests that the elementary particles of Hadron family, like, protons and Neutrons have internal structures. They are composed of basic units called "Quarks" having charges $-\frac{1}{3}e$ (down quark 'd') and $+\frac{2}{3}e$ (up quark 'u'). Proton is made up of three quarks, two up quarks and one down quark and its structure is 'uud'. Similarly the structure of Neutrons is 'udd'.

Despite the overwhelming evidence of quarks having fractional electronic charges, we have sufficient theoretical grounds to state that the liberation of a single quark is a physical impossibility i.e. quarks don't have independent existence. They always exist in such groups that the net charge of that group is equal to the integral multiple of electronic charge and we still state the principle of quantization of charge as

$Q = ne$

Since loss or gain of electron is responsible for creating charge on a body and electron is a particle with mass, every charged body will have mass also.

Illustration:

A copper sphere contains about 2×10^{22} atoms. The charge on the nucleus of each atom is 29e. what fraction of the electrons must be removed from the sphere to give it a charge of $+2\mu C$?

Sol. The total number of electrons is $29\,(2\text{x}10^{22}) = 5.8 \times 10^{23}$.
Electrons removed = $(2\text{x}10^{-6}\text{C}) / (1.6\text{x}10^{-19}\text{C}) = 1.25 \times 10^{13}$,
So the fraction removed = electrons removed / total number electrons = 2.16×10^{-11}.

(v) Charge is conserved : The total charge of universe remains constant. It may alternatively be stated as follows : "The total charge of an Isolated system remains constant i.e. for a closed system of particles.

$$\sum_i e_i^+ - \sum_i e_i^- = \text{Const.}$$

The above principle suggests that :

(a) Charge can neither be created nor be destroyed.

(b) Only (+)ve or only (–)ve charge can never be created.

(c) Simultaneous production of equal and opposite charges or simultaneous annihilation of equal and opposite charge don't violate the principle of conservation of charge.

Illustration:

Three metallic spheres say X, Y and Z have charges 10C, –10C, 10C respectively. X, Y, Z are brought in contact such that charge on each of A and B becomes 3C what is charge on Z.

Sol. Net charge initially on X, Y and Z = (+10 –10 +10) = 10C
= Final net charge on X, Y and Z = $q_X + q_Y + q_Z$
= $3+3+q_c = 10\text{ C} \;\therefore\; q_c = 4\text{C}$.

Classification of Substance on the Basis of Electrical Passage

We can classify materials generally according to the ability of charge to move through them. Conductors are materials through which charge can move rather freely; examples include metals (such as copper in common lamp wire), the human body, and tap water. Nonconductors-also called insulators -are materials through which charge cannot move freely; examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water. Semiconductors are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips. Superconductors are materials that are perfecl conductors, allowing charge to move without any hindrance.

Charging of body

Mainly there are following three methods of charging a body:

(i) Charging by rubbing

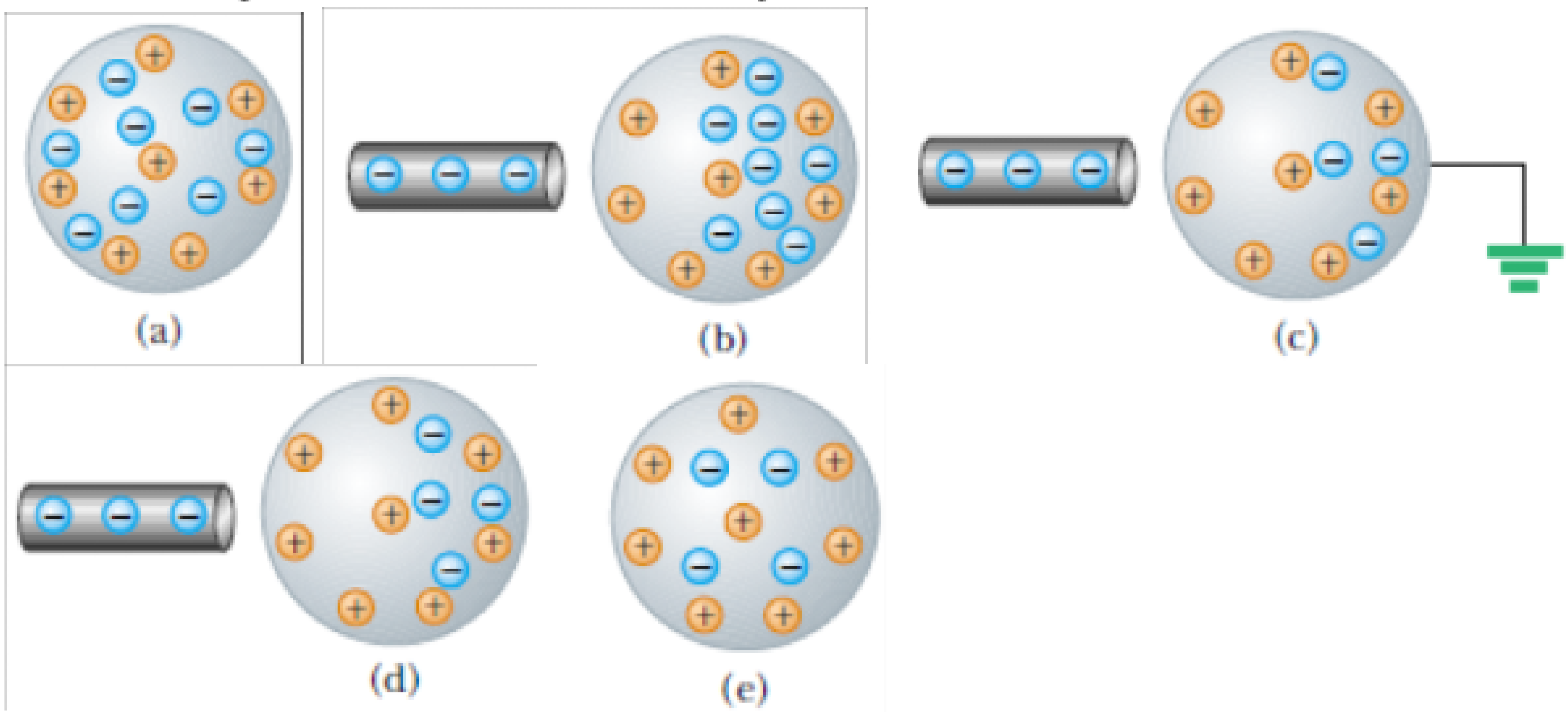
The simplest way to experience electric charges is to rub certain bodies against each other. When a glass rod is rubbed with a silk cloth the glass rod acquires some positive charge and the silk cloth acquires negative charge by the same amount. The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body pass onto the other body. The body that donates the electrons becomes positively charged while that which receives the electrons becomes negatively charged. For example when glass rod is rubbed with silk cloth, glass rod becomes positively charged because it donates the electrons while the silk cloth becomes negatively charged because it receives electrons. Electricity so obtained by rubbing two objects is also known as **frictional electricity.** The other places where the frictional electricity can be observed are when amber is rubbed with wool or a comb is passed through a dry hair. Clouds also become charged by friction.

(ii) Charging by contact

When a negatively charged ebonite rod is rubbed on a metal object, such as a sphere, some of the excess electrons from the rod are transferred to the sphere. Once the electrons are on the metal sphere, where they can move readily, they repel one another and spread out over the sphere's surface. The insulated stand prevents them from flowing to the earth. When the rod is removed the sphere is left with a negative charge distributed over its surface. In a similar manner the sphere will be left with a positive charge after being rubbed with a positively charged rod. In this case, electrons from the sphere would be transferred to the rod. The process of giving one object a net electric charge by placing it in contact with another object that is already charged is known as **charging** by **contact.**

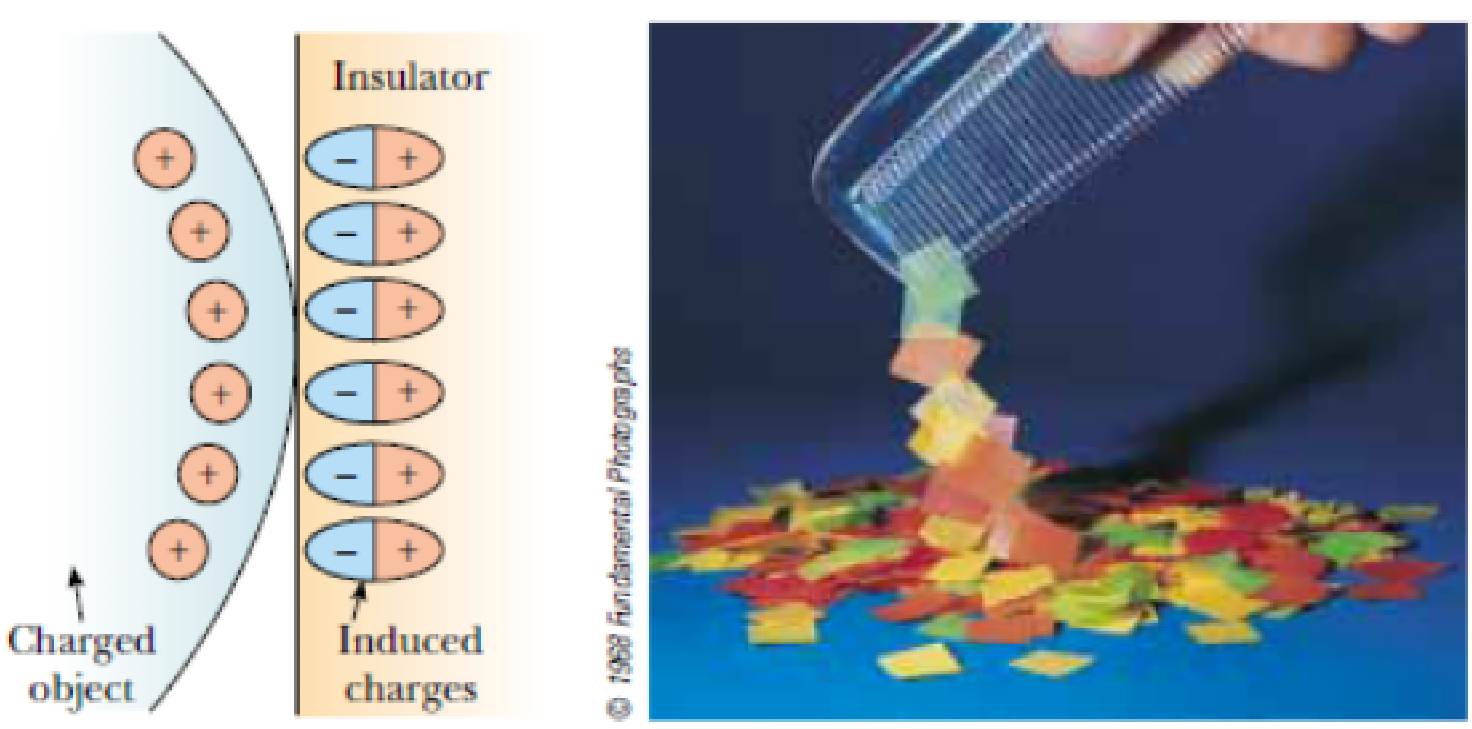
(iii) Charging by induction

To understand how to charge a conductor by a process known as induction, consider a neutral (uncharged) conducting sphere insulated from the ground, as shown in Fig. a. There are an equal number of electrons and protons in the sphere if the charge on the sphere is exactly zero. When a negatively charged rubber rod is brought near the sphere, electrons in the region nearest the rod experience a repulsive force and migrate to the opposite side of the sphere. This leaves the side of the sphere near the rod with an effective positive charge because of the diminished number of electrons, as in Fig b. (The left side of the sphere in Fig b is positively charged *as if* positive charges moved into this region, but remember that it is only electrons that are free to move.) This occurs even if the rod never actually touches the sphere. If the same experiment is performed with a conducting wire connected from the sphere to the Earth (Fig. c), some of the electrons in the conductor are so strongly repelled by the presence of the negative charge in the rod that they move out of the sphere through the wire and into the Earth. The symbol ⏚ at the end of the wire in Fig c indicates that the wire is connected to ground, which means a reservoir, such as the Earth, that can accept or provide electrons freely with negligible effect on its electrical characteristics. If the wire to ground is then removed (Fig.d), the conducting sphere contains an excess of *induced* positive charge because it has fewer electrons than it needs to cancel out the positive charge of the protons. When the rubber rod is removed from the vicinity of the sphere (Fig.e), this induced positive charge remains on the ungrounded sphere. Note that the rubber rod loses none of its negative charge during this process. Charging an object by induction requires no contact with the object inducing the charge. This is in contrast to charging an object by rubbing (that is, by *conduction*), which does require contact between the two objects.



Now similarly you can explain why **a comb rubbed on hair attracts bits of paper.**

A process similar to induction in conductors takes place in insulators. In most neutral molecules, the center of positive charge coincides with the center of negative charge. However, in the presence of a charged object, these centers inside each molecule in an insulator may shift slightly, resulting in more positive charge on one side of the molecule than on the other. This realignment of charge within individual molecules produces a layer of charge on the surface of the insulator, as shown in Figure 23.5a. Knowing about induction in insulators, you should be able to explain why a comb that has been rubbed through hair attracts bits of electrically neutral paper and why a balloon that has been rubbed against your clothing is able to stick to an electrically neutral wall.



(a) (b)

Note :

1. If the charge on one ball is large as compared to the similar charge on the other, the ball with large charge will induce a large charge of opposite kind on the other ball. As a result, **attraction will result inspite of repulsion**.

2. Repulsion is the sure test for electrificaiton.

Coulomb's Law

On the basis of "Torsion balance" experiment "Charles Augustine Coulomb" put a quantitative law for the force of attraction or repulsion on the charges which states that–

"The force of attraction or repulsion on one charge q_2 placed at some separation from another charge q_1 (whose dimensions are small compared to their distance of separation) in infite homogeneous medium, is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance between them."

$$F \propto |q_1||q_2|$$

$$\propto \frac{1}{r^2}$$

$$F \propto \frac{|q_1||q_2|}{r^2} \Rightarrow F = k\,\frac{q_1 q_2}{r^2}$$

Where k is a constant of proportionality. In C.G.S. Unit i.e. if F is measured in dyne, q_1 and q_2 in stat coulomb and r in cm, then k = 1

In c.g.s. unit $\quad F = \frac{q_1 q_2}{r^2}$

But if the force is measured in newton, q_1 and q_2 in coulomb and r in metre then

Then $\quad k = \frac{1}{4\pi \in_0}$ in air or vacuum and $k = \frac{1}{4\pi \in}$ in an infinite homogeneous material medium other than air.

So if the intervening medium between the charges is air or it is vacuum then in S.I. Units

$$F = \frac{1}{4\pi \in_0}\,\frac{q_1 q_2}{r^2}$$

$\in_0$ is called permittivity of free space.

$\in$ is called absolute permittivity of the given material medium.

The ratio of the absolute permittivity of a given medium and that of the permittivity of free space is called relative permittivity of that medium or its dielectric constant (represented by symbol $\in_r$ or K)

$\in_r$ or $K = \frac{\in}{\in_0}$ So, $\in = \in_0 \in_r$ or $K \in_0$

Hence, $\quad F_{medium} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} = \frac{F}{K}$

Note

(i) K, the dielectric constant of the medium (also called relative permittivity) being ratio of two like quantities, is a dimensionless constant.

(ii) Air or vacuum has minimum relative permittivity ($K = \varepsilon_0/\varepsilon_0 = 1$). The relative permittivity of all other media is greater than 1 usually and K = infinite for a conducting medium.

(iii) When the charges are placed in infite dielectric medium then dielectric medium is getting polarized and force on q_1 or q_2 is not simply due to q_1 or q_{2i} but due to polarized charges also and net force on q_i or q_j becomes $\frac{1}{\in_r}$ times. (See next Illustration)

Illustration :

Two equal point charges (10^{-3}C)are placed 1 cm apart in medium of dielectric constant K = 5
(a) Find the interaction force between the point charges.
(b) Net force on any of the charge.

Sol.

(a) Interaction force between point charges

$$F = \frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2} = 9\times10^9\frac{\left(10^{-3}\right)^2}{\left(10^{-2}\right)^2}$$

$= 9 \times 10^7$ N

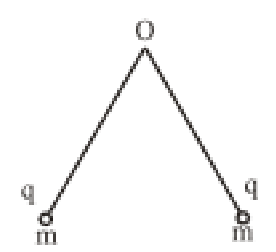
(b) Net force

$$F' = \frac{1}{4\pi\in_0 k}\frac{q_1q_2}{r^2} = \frac{9\times10^9}{5}\frac{\left(10^{-3}\right)^2}{\left(10^{-2}\right)^2}$$

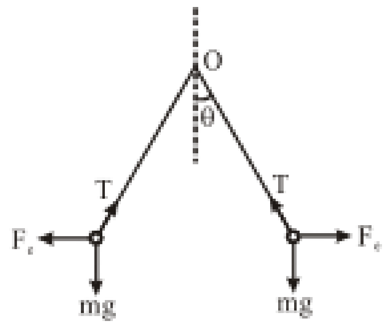
$= 18 \times 10^6$ N

Illustration :

Two small balls each of mass m and charge q on each of them are suspended through two light insulating string of length l from a point. Find the expression for angle θ made by any of the string with vertical when under static equilibrium.



Sol. *Let angle of any string with vertical be θ as shown*



$$F_e = \frac{1}{4\pi\in_0}\frac{q^2}{(2\ell\sin\theta)^2} = T\sin\theta$$ *... (i) for horizontal direction*

$T\cos\theta = mg$ *... (ii) for vertical direction*

Dividing (i) by (ii)

$$\tan\theta = \frac{F_e}{mg}$$

(i) The force of electrostatic interaction between two charges is operative along the line joining the charges.

(ii) The force obeys inverse square law

(iii) If $q_1q_2 > 0$ (it means the product of the two charges is positive) this implies that charges are similar, i.e., either both positive or both negative. Hence, repulsion will result.

(iv) If $q_1q_2 < 0$, it means the product of the magnitude of the charges is negative. In other words, these are unlike charges, i.e., one charge is positive and the other charge is negative. Hence, the electrostatic force between them is attractive.

Like charges repel each other unlike charges attract

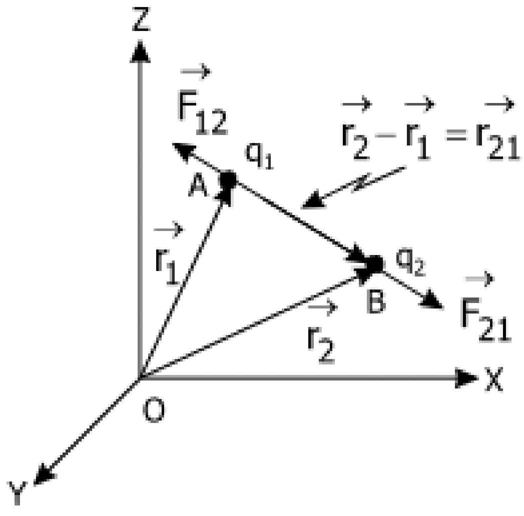
(vi) The force on q_1 due to q_2 is equal and opposite to the force on q_2 due to q_1

$$\vec{F}_{12} = -\vec{F}_{21}$$

i.e. The force of electrostatic interaction between two charges obey Newton's 3rd law. It should, however, be noted that the equality $\vec{F}_{12} = -\vec{F}_{21}$ breaks down when one charge is accelerated towards the other i.e., Newton's 3rd law doesn't hold . This is why Newton's 3rd law is supposed to be a weak law of physics.

Force Between Two Charges in Terms of Their Position Vectors :

Consider two like charge q_1 and q_2 located in vacuum at positions A and B respectively. Let the positions of A and B with reference tothe orign O of the coordinate frame be given by position vectors



$\vec{r}_1$ and $\vec{r}_2$ respectively, i.e., $\vec{OA} = \vec{r}_1$ and $\vec{OB} = \vec{r}_2$

Now, $\vec{OA} + \vec{AB} = \vec{OB}$ (triangle law of vectors)

$$\therefore \qquad \vec{AB} = \vec{OB} - \vec{OA} = \vec{r}_2 - \vec{r}_1 = \vec{r}_{21}$$

According to Coulomb's law, force on q_2 due to q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{21}^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{21}^3}\vec{r}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

Similarly, $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$

It should be noted that :

(i) For coulomb's law to give precise result the spatial extent of charges should be very small in comparison to their separation.

(ii) Coulomb's law is valid for wide range of distance .

Illustration :

Two point charges A and B have charges respectively $\frac{1}{2}$C and 2 C with their position vectors respectively as $(\hat{i}+\hat{j}+\hat{k})$ and $(-\vec{i}-\vec{j}+3\vec{k})$. *Find the force on charge at A due to B.*

Sol.

$$q_A = \frac{1}{2}C \qquad \vec{r}_A = \hat{i}+\hat{j}+\hat{k}$$

$$q_B = 2C \qquad \vec{r}_B = -\hat{i}-\hat{j}+3\hat{k}$$

$$\vec{F}_{AB} = \frac{1}{4\pi\epsilon_0}\frac{q_A q_B}{|\overrightarrow{r_{AB}}|^2}\hat{r}_{AB}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q_A q_B}{\left|\overrightarrow{r_{AB}}\right|^3}\left(\overrightarrow{r_A} - \overrightarrow{r_B}\right) = (9\times10^9)\times\frac{\frac{1}{2}\times1}{\left|2\hat{i}+2\hat{j}-2\hat{k}\right|^3}(2\hat{i}+2\hat{j}-2\hat{k})$$

$$= \frac{9\times10^9\times(\hat{i}+\hat{j}-k)}{24\sqrt{3}}N.$$

SUPERPOSITION OF ELECTROSTATIC FORCES

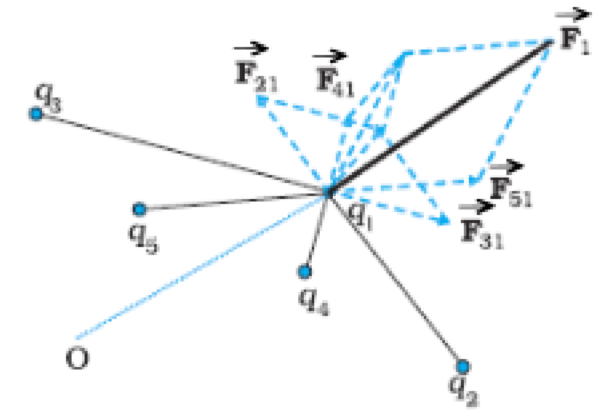
Experimentally it is verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. This is termed as the principle of superposition.

The principle of superposition says that in a system of charges q_1, q_2, ..., q_n, the force on q_1 due to q_2 is the same as given by Coulomb's law, i.e., it is unaffected by the presence of the other charges q_3, q_4, ..., q_n. The total force $\vec{F}_1$ on the charge q_1, due to all other charges, is then given by the vector sum of the forces $\vec{F}_{12}$, $\vec{F}_{13}$, ..., $\vec{F}_{1n}$:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ... + \vec{F}_{1n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1q_3}{r_{13}^2}\hat{r}_{13} + ... + \frac{q_1q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

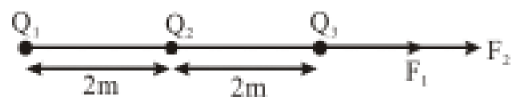
$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{j=2}^{n}\frac{q_j}{r_{1j}^2}\hat{r}_{1j}$$



The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle.

Illustration :

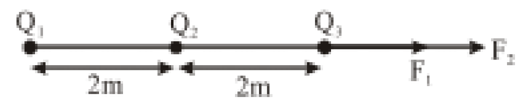
Three charges each of $20\mu C$ are placed along a straight line, successive charges being 2 m apart as shown in Figure. Calculate the force on the charge on the right end.



Sol.

$$F = F_1 + F_2$$

$$F_1 = \frac{kQ_1Q_2}{r^2} = \frac{(9\text{x}10^9)(20\text{x}10^{-6})^2}{4^2} = 0.225\text{N}$$



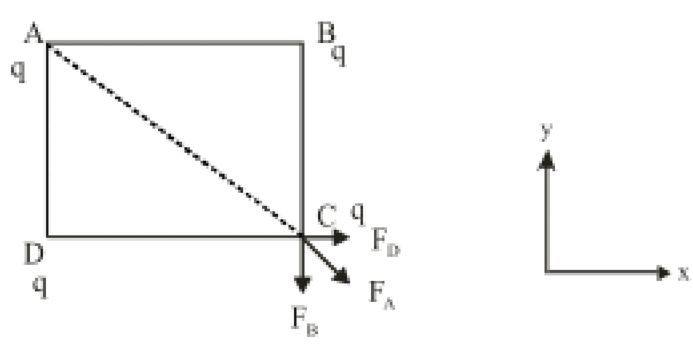
$$F_2 = \frac{kQ_2Q_3}{r^2} = \frac{(9\text{x}10^9)(20\text{x}10^{-6})^2}{2^2} = 0.9\,\text{N}$$

$F = 0.225+0.9 = 1.125$ *N to the right*

Illustration:

Four identical point charges each of magnitude q are placed at the corners of a square of side a. Find the net electrostatic force on any of the charge.

Sol.



Let the concerned charge be at C then charge at C will experience the force due to charges at A, B and D. Let these forces respectively be $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_D$ *thus forces are given as*

$$\vec{F}_A = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{AC^2} \text{ along } AC = \frac{q^2}{4\pi\varepsilon_0 2a^2}\left(\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}\right)$$

$$\vec{F}_B = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{BC^2} \text{ along } BC = \frac{q^2}{4\pi\varepsilon_0 a^2}(-\hat{j})$$

$$\vec{F}_D = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{DC^2} \text{ along } DC = \frac{q^2}{4\pi\varepsilon_0 a^2}(\hat{i})$$

$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_D$$

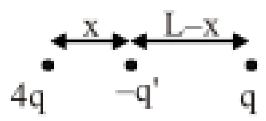
$$= \frac{q^2}{4\pi\varepsilon_0 a^2}\left[\hat{i}\left(\frac{1}{2\sqrt{2}}+1\right) - \hat{j}\left(\frac{1}{2\sqrt{2}}+1\right)\right]$$

$$\left|\vec{F}_{net}\right| = \sqrt{2}\left(\frac{1}{2\sqrt{2}}+1\right)\frac{q^2}{4\pi\varepsilon_0 a^2} \qquad = \left(\frac{1}{2}+\sqrt{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}\,.$$

Illustration:

Two point charges +4q and +q are placed at a distance L apart. A third charge is so placed that all the three charges are in equilibrium. Find the location, magnitude and nature of third charge. Discuss also, whether the equilibrium of the system is stable, unstable or neutral.

Sol.



Third charge should be placed between 4q and q so that force on third charge to be zero (let at distance x from 4q). Third charge should be –ve (let –q') for the equilibrium of other charges
For equilibrium of third charge

$$\frac{K(4q)(q')}{x^2} = \frac{K(q)(q')}{(L-x)^2} \Rightarrow x = \frac{2L}{3}$$

for equilibrium 4q

$$\frac{K(4q)(q')}{\left(\frac{2L}{3}\right)^2} = \frac{K(4q)(q)}{L^2} \qquad \Rightarrow \qquad q' = \frac{4q}{9}$$

Practice Exercise

Q.1 Three particles, each of mass 1 g and carrying a charge q, are suspended from a common point by insulating massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 3 cm, calculate the charge q on each particle. (g= 10m/ s^2).

Q.2 Five point charges, each of value +q are placed on five vertices of a regular hexagon of side L m. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of hexagon?

Q.3 Two equal point charges q are fixed at x = – a and x = a along the x-axis. A particle of mass m and charge q/2 is brought to the origin and given a small displacement along the (a) x-axis and (b) y-axis. Describe the motion in the two cases.

Answers

Q.1 3.16×10^{-9} C Q.2 $\frac{1}{4\pi\epsilon_0}\left[\frac{q}{R}\right]^2$ Q.3 (a) Accelerated motion (b) SHM

The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle.

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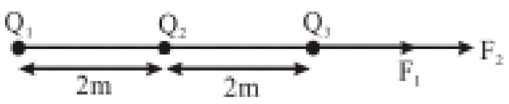
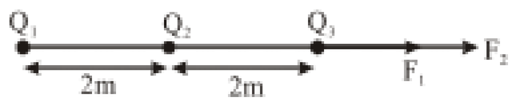


Sol.

$$F = F_1 + F_2$$

$$F_1 = \frac{kQ_1Q_2}{r^2} = \frac{(9\text{x}10^9)(20\text{x}10^{-6})^2}{4^2} = 0.225\text{N}$$



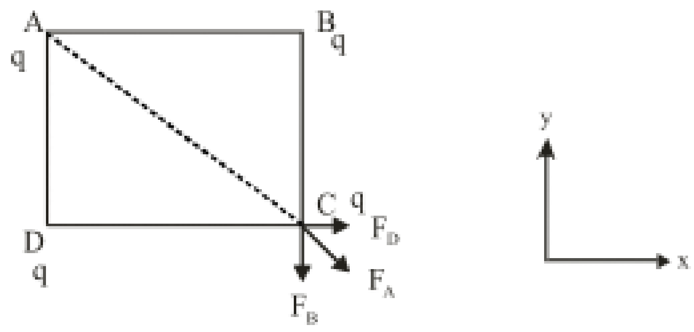
$$F_2 = \frac{kQ_2Q_3}{r^2} = \frac{(9\text{x}10^9)(20\text{x}10^{-6})^2}{2^2} = 0.9\text{ N}$$

$F = 0.225 + 0.9 = 1.125$ N to the right

Illustration:

Four identical point charges each of magnitude q are placed at the corners of a square of side a. Find the net electrostatic force on any of the charge.

Sol.



Let the concerned charge be at C then charge at C will experience the force due to charges at A, B and D. Let these forces respectively be $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_D$ *thus forces are given as*

$$\vec{F}_A = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{AC^2} \text{ along } AC = \frac{q^2}{4\pi\varepsilon_0 2a^2}\left(\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}\right)$$

$$\vec{F}_B = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{BC^2} \text{ along } BC = \frac{q^2}{4\pi\varepsilon_0 a^2}(-\hat{j})$$

$$\vec{F}_D = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{DC^2} \text{ along } DC = \frac{q^2}{4\pi\varepsilon_0 a^2}(\hat{i})$$

$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_D$$

$$= \frac{q^2}{4\pi\varepsilon_0 a^2}\left[\hat{i}\left(\frac{1}{2\sqrt{2}} + 1\right) - \hat{j}\left(\frac{1}{2\sqrt{2}} + 1\right)\right]$$

$$\left|\vec{F}_{net}\right| = \sqrt{2}\left(\frac{1}{2\sqrt{2}} + 1\right)\frac{q^2}{4\pi\varepsilon_0 a^2} \qquad = \left(\frac{1}{2} + \sqrt{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}.$$

Practice Exercise

Q.1 A particle having a charge of 2.0 10 6 C and a mass of 100 g is placed at the bottom of a smooth inclined plane of inclination 30 . Where should another particle B, having same charge and mass, be placed on the incline so that it may remain in equilibrium?

Q.2 A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 g/mol. Let us now take two pieces of copper each weighing 10 g. Let us transfer one electron from one piece to another for every 1000 atoms in a piece. What will be the coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart? [e= 1.6×10^{-19}C, $(1/4\pi\epsilon_0) = 9\times10^9$ and Avogadro's number $=6\times10^{23}$ per mol]

Q.3 Two spherical conductors B and C having equal radii and carrying equal charges with them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. What is the new force of repulsion between B and C ? (When two conductors of identical geometry having charges q_1 and q_2 if touched have final charges $\frac{q_1+q_2}{2}$ on each conductor)

Q.4 A ring of radius 0.1 m is made our of a metallic wire of area of cross– section 10^{-6} m^2. The ring has a uniform charge of π coulomb. Find the change in the radius of the ring when a charge of 10^{-8} coulomb is placed at the centre of the ring.

Young's modulus of the metal is 2×10^{11} N/ m^2.

Answers

Q.1 27 cm from the bottom Q.2 2.08×10^{14}N Q.3 3F/8

Q.4 2.25×10^{-13} m

Vector form of coulomb's law :

By stating coulomb's law in vector form more information can be packed in it.

Let the position vector of charge q_2 relative to charge q_1 be $\vec{r}$ and $\hat{r}$ is a unit vector in the direction of $\vec{r}$

so, $\vec{r} = |\vec{r}|\,\hat{r} = r\hat{r}$ hence, $\hat{r} = \frac{\vec{r}}{r}$

Coulomb's law, in vector form, may be written as

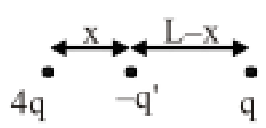
$$\vec{F} = \frac{1}{4\pi\in}\frac{q_1q_2}{r^2}\hat{r} = \frac{1}{4\pi\in}\frac{q_1q_2}{r^3}\vec{r}$$

From the above form of the coulomb's law, It may be justified that,

Illustration:

Two point charges +4q and +q are placed at a distance L apart. A third charge is so placed that all the three charges are in equilibrium. Find the location, magnitude and nature of third charge. Discuss also, whether the equilibrium of the system is stable, unstable or neutral.

Sol.



Third charge should be placed between 4q and q so that force on third charge to be zero (let at distance x from 4q). Third charge should be –ve (let –q') for the equilibrium of other charges
For equilibrium of third charge

$$\frac{K(4q)(q')}{x^2} = \frac{K(q)(q')}{(L-x)^2} \Rightarrow x = \frac{2L}{3}$$

for equilibrium 4q

$$\frac{K(4q)(q')}{\left(\frac{2L}{3}\right)^2} = \frac{K(4q)(q)}{L^2} \qquad \Rightarrow \qquad q' = \frac{4q}{9}$$

Practice Exercise

Q.1 Three particles, each of mass 1 g and carrying a charge q, are suspended from a common point by insulating massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 3 cm, calculate the charge q on each particle. ($g = 10 m/s^2$).

Q.2 Five point charges, each of value +q are placed on five vertices of a regular hexagon of side L m. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of hexagon?

Q.3 Two equal point charges q are fixed at $x = -a$ and $x = a$ along the x-axis. A particle of mass m and charge q/2 is brought to the origin and given a small displacement along the (a) x-axis and (b) y-axis. Describe the motion in the two cases.

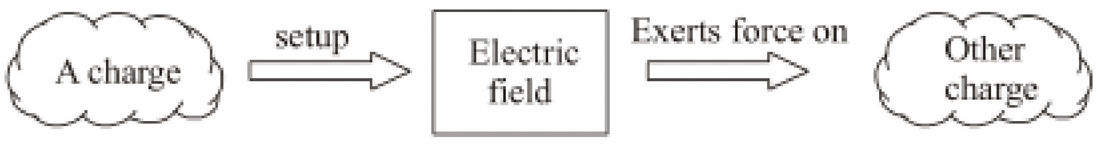
Answers

Q.1 3.16×10^{-9} C Q.2 $\frac{1}{4\pi \epsilon_0}\left[\frac{q}{R}\right]^2$ Q.3 (a) Accelerated motion (b) SHM

ELECTRIC FIELD

Introduction

Now a question arises how a charge q_1 exerts force on another charge q_2. The answer is q_1 influence its surrounding electrically and this electrically infuluenced surrounding exerts force on q_2. Here we say that q_1 set up its electric field and this electric field exerts force q_2. This electric influence of a charge is measured by a vector called intensity of electric field (or loosely speaking electric field) and represented by $\vec{E}$.



To define electric field of a charge q (called source charge) at a point P in its surrounding place a small positive charge q_0 (called test charge) at point P. If q exerts force $\vec{F}$ on q_0 then inesity of electric field due of q at P is defined as

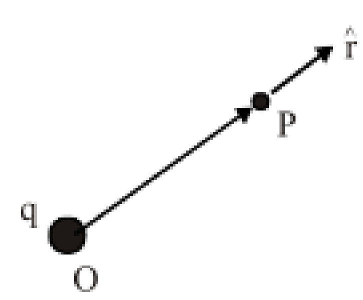
$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

q_0 is taken positive so that $\vec{F}$ give direction of $\vec{E}$. q_0 is taken small because if it is taken large it can disturb q. It should be also kept is mind that $\vec{E}$ is property of q i.e. it will still present if test charge is absent.

Electric field due to a point charge

A point charge q is placed at point O. We have to express its electric field at point P whose position vector with respect to point P is $\vec{r}$.

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{\frac{kqq_0}{r^2}\hat{r}}{q_0}$$



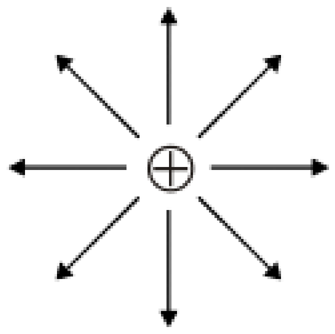
$$\vec{E} = \frac{kq}{r^2}\cdot\hat{r} = \frac{kq}{r^3}\ \vec{r}$$

magnitude of electric field is given by

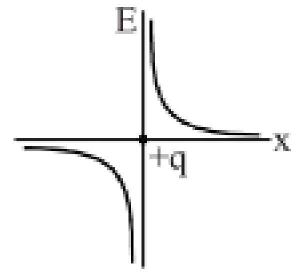
$$E = \frac{k\,|q|}{r^2}$$

Its direction will be radially outward is q is positive and will be radially inward if q is negative

Electric field of a positive point charge



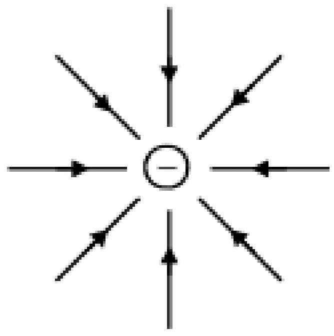
plot of electric field of a positive point charge at different points on x-axis

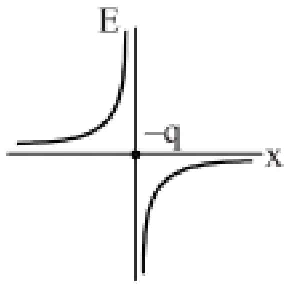


Electric field due to a point positive charge is radially outward

Electric field of a negative point charge

plot of electric field of a negative point charge at different points on x-axis



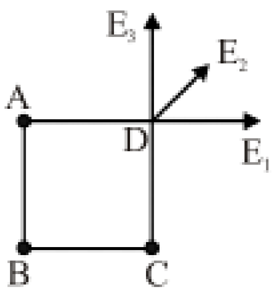


Electric field due to a point negative charge is radially inward

Illustration :

Three point charge $q_1 = +q$, $q_2 = -2q$ and $q_3 = +q$ are placed at corners A , B and C of square ABCD of side 'a' find electric field at D.

Sol. $E_1 = \frac{kq}{a^2}$ (alone $\overrightarrow{AD}$)



$$E_2 = \frac{k|-2q|}{(\sqrt{2}a)^2} = \frac{2kq}{2a^2}$$

$$\frac{kq}{a^2}(\text{a long}\overrightarrow{DB})$$

$$E_3 = \frac{kq}{a^2} \text{ (a long } \overrightarrow{CD})$$

$$\therefore \quad E_{resultant} = \frac{kq}{a^2} \times \sqrt{2} - \frac{kq}{a^2} = \frac{kq}{a^2}\left(\sqrt{2}-1\right)$$

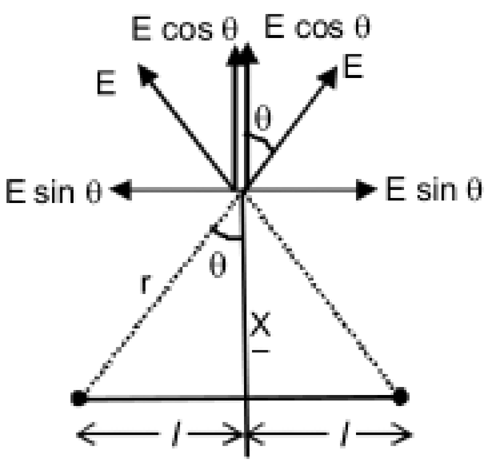
Illustration :

Two point charges each having charge q are placed at separation 2l find the electric field at a point on the perpendicular bisector at a distance x from its mid point.

Sol. *Electric field due to any of the charge will be*

$$E = \frac{kq}{r^2}$$

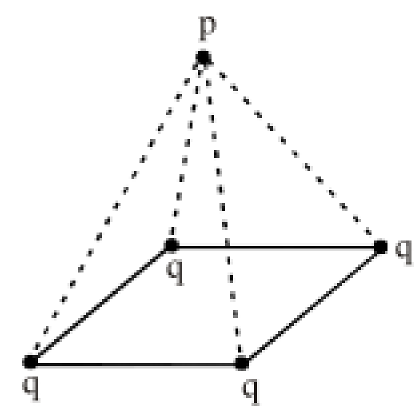
Resolve the electric field into two components. E cos θ and E sin θ (fig.). Horizontal components of electric field of both charges nullifies each other. Hence the net electric field will be.



$$E_{net} = 2E\cos\theta = \frac{2kq}{r^2} \times \frac{z}{r} = \frac{2kqz}{(\ell^2+z^2)^{3/2}}$$

Illustration :

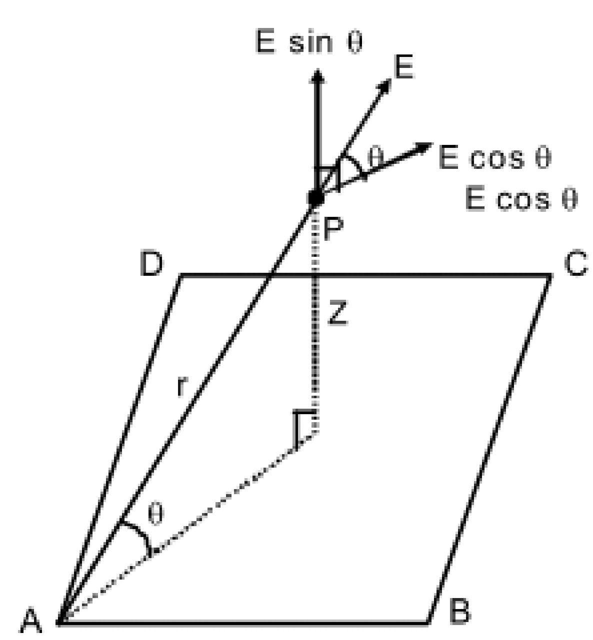
Four identical charges are fixed at the corners of a square of side a. Find electric field at point p which is at a distance z lying on the line perpendicular to the plane of the square passing through the centre of square.



Sol.

Let us first calculate electric field due to point charge present at A.

$$E_A = \frac{kq}{r^2}$$



This electric field can be resolved into two components E cos θ (horizontal) and E sin θ (vertical). From symmetry it is clear that sum of horizontal components of electric field will be zero.

Hence the net electric field will be

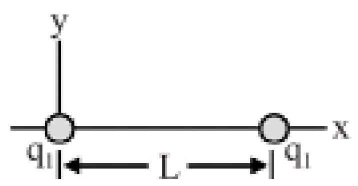
$$E_{net} = 4E \sin\theta \text{ (vertical)}$$

$$= 4 \times \frac{kq}{r^2} \times \frac{z}{r} = 4 \cdot \frac{1}{4\pi \epsilon_0} \frac{qz}{\left(\frac{a^2}{2} + z^2\right)^{3/2}}$$

Illustration :

Two charged particles lie along the x-axis as shown in figure. The particle with charge $q_2 = +8\mu C$ is at $x = 6.00$ m, and the particle with charge $q_1 = +2\mu C$ is at the origin. Locate the point whre the resultant electric field is zero.

Sol.



Two charged particles kept on the x-axis.

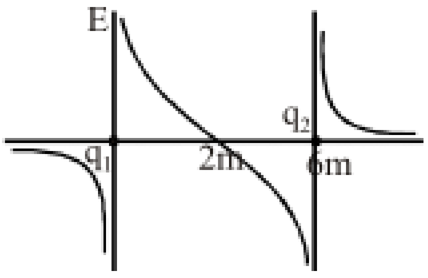
Before calculating, let us physically see the location of the point where the electric field can be zero. At points other than the x-axis, say above the x-axis, both the charges will have a component of the electric field in the positive direction. This y component of the electric field does not cancel out. So the net electric field at that point will not be zero. The same statement also holds true for points which are not in xy-plane.

On the x-axis also we can see that on the points beyond x = 6 m and points on the negative x-axis, both the electric fields will be in the same direction. So the net electric field cannot be zero. At

some point between the two charges, the electric field due to both f them will be in opposite direction. *So the electric field will be zero at a point between x = 0 and x = 6m.*
To find the electric field at any point, we can apply the superposition principal as in equation.

$$E_x = \frac{k \times 2 \times 10^{-6}}{x^2} - \frac{k \times 8 \times 10^{-6}}{(6-x)^2} = 0$$

Solving we get x = 2m. We are ignoring the other trivial solution.
To analyze this situation more deeply, let us draw a graph of the net electric field at different points on the x-axis versus the position on the x-axis.



Electric field at various points of x-axis due to q_1 and q_2.

This point where the electric field is zero is called a neutral point. We can also say that if the electric field at this point is zero, a charged particle kept at this point will not experience any force.

$$\vec{E} = \frac{\vec{F}}{q}$$

Then $\vec{F} = q\vec{E}$. *Hence, at this point any charged particle will be in equilibrium.*

Practice Exercise

Q.1 Two particles A and B having charges of + 2.00   10 6 C of 4.00   10 6 C respectively are held fixed at a separation of 20.0 cm. Locate the point(s) on the line AB where the electric field is zero

Ans 48.3 cm from A along BA

Q.2 Three identical charges, each having a value 1.0   10 8 C, are placed at the corners of an equilateral triangle of side 20 cm. Find the electric field at the centre of the triangle.

Ans zero,

Answers

Q.1 48.3 cm from A along BA   Q.2 zero,

Continuous charge distribution

We have so far dealt with charge configurations involving discrete charges $q_1, q_2, ..., q_n$. One reason why we restricted to discrete charges is that the mathematical treatment is simpler and does not involve calculus. For many purposes, however, it is impractical to work in terms of discrete charges and we need to work with continuous charge distributions. For example, on the surface of a charged conductor, it is impractical to specify the charge distribution in terms of the locations of the microscopic charged constituents. It is more feasible to consider an area element ΔS on the surface of the conductor (which is very small on the macroscopic scale but big enough to include a very large number of electrons) and specify the charge ΔQ on that element. We then define a surface charge density σ at the area element by

$$\sigma = \operatorname*{Lim}_{\Delta S \to 0} \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$

We can do this at different points on the conductor and thus arrive at a continuous function σ called the surface charge density. The surface charge density σ so defined ignores the quantisation of charge and the discontinuity in charge distribution at the microscopic level. σ represents macroscopic surface charge density, which in a sense, is a smoothed out average of the microscopic charge density over an area element ΔS represents macroscopic surface charge density, which in a sense, is a smoothed out average of the microscopic charge density over an area element σ are C/m^2.

Similar considerations apply for a line charge distribution and a volume charge distribution. The linear charge density λ of a wire is defined by

$$\lambda = \operatorname*{Lim}_{\Delta l \to 0} \frac{\Delta Q}{\Delta l} = \frac{dQ}{dl}$$

Where Δl is a small line element of wire on the macroscopic scale that, however, includes a large number of microscopic charged constituents, and ΔQ is the charge contained in that line element. The units for λ are C/m. The volume charge density (sometimes simply called charge density) is defined in a similar manner:

$$\rho = \operatorname*{Lim}_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$

Where ΔQ is the charge included in the macroscopically small volume element ΔV that includes a large number of microscopic charged constituents. The units for ρ are C/m^3. The notion of continuous charge distribution is similar to that we adopt for continuous mass distribution in mechanics. When we refer to the density of a liquid, we are referring to its macroscopic density. We regard it as a continuous fluid and ignore its discrete molecular constitution.

Electric field of a continuous charge distribution

The total electric field at P due to all elements in the charge distribution is approximately.

$$\vec{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

Considering the charge distribution as continuous, the total field at P in the limit $\Delta q_i \to 0$ is

$$\vec{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i = k_e \int \frac{dq}{r^2} \hat{r}$$

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Steps for calculating the electric field for continuous charge distributions

1. Identify the type of charge distribution and compute the charge density λ, σ or ρ.
2. Divide the charge distribution into infinitesimal charges dq, each of which will act as a tiny point charge.
3. The amount of charge dq, i.e., within a small element dl, dA or dV is
   $dq = \lambda dl$ (charge distributed in length)
   $dq = \sigma dA$ (charge distributed over a surface)
   $dq = \rho\, dV$ (charge distributed throughout a volume)
4. Draw at point P the dE vector produced by the charge dq. The magnitude of dE is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

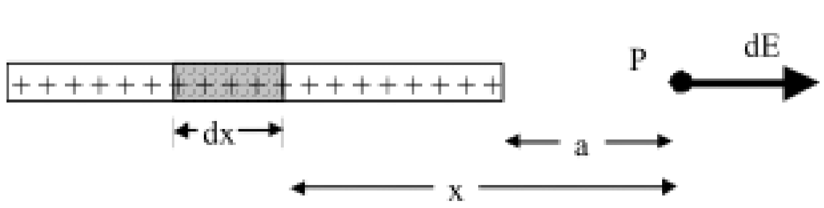
   Vector dE is along radial line joining dq to P, dE is directed away for positive charge dq while directed towards dq for negative dq.
5. Resolve the dE vector into its components. Identify any special symmetry features to show whether any component(s) of the field that are not cancelled by other components.
6. Write the distance r and any trigonometric factors in terms of given coordinates and parameters.
7. The electric field is obtained by summing over all the infinitesimal contributions.

$$\vec{E} = \int d\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}$$

8. Perform the indicated integration over limit of integration that include all the source charges.

Electric field intensity at any point due to a uniformly charged rod (of linear charge density λ) at a point on its axis

Consider an element, dx at a distance, x from the point, P, where we have to find the electric field. The elemental charge, $dq = \lambda dx$



Now, electric field due to elementry part will be

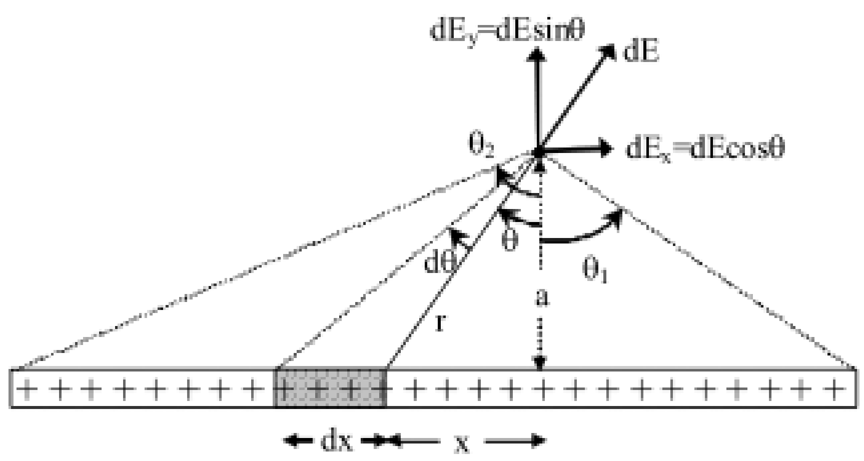
$$dE = k.\frac{\lambda dx}{x^2}$$

Then, electric field due to entire rod will be

$$E = k\lambda\int_a^{a+L}\frac{1}{x^2}dx = k\lambda\left[-\frac{1}{x}\right]_a^{a+L} = k\lambda\left[\frac{-1}{a+L}+\frac{1}{a}\right]$$

Thus $$E = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{a}-\frac{1}{L+a}\right]$$

Electric field intensity at any point due to a uniformly charged rod (of linear charge density λ) at a genaral point



Consider an element, dx at a distance, x from the point, P, from normal. The elemental charge, $dq = \lambda dx$
Now, electric field due to elementry part will be

$$dE = k.\frac{\lambda dx}{r^2}$$

This electric field has two components $dE_x = dE\cos\theta$ and $dE_y = dE\sin\theta$
The x-component of resultant electric field will be

$$E_x = \int dE_x = \int dE\sin\theta = \int\left(k.\frac{\lambda dx}{r^2}\right)\left(\frac{x}{r}\right) = k\lambda\int\frac{xdx}{r^3} = k\lambda\int\frac{xdx}{\left(a^2+x^2\right)^{3/2}}$$

substitute

$$x = a\tan\theta \qquad \Rightarrow \qquad dx = a\sec^2\theta d\theta$$

and

$$\left(a^2+x^2\right)^{3/2} = \left(a^2+a^2\tan^2\theta\right)^{3/2} = a^3\sec^3\theta$$

Then, x-component of electric field due to entire rod will be

$$E_x = \int dE_x = k\lambda\int\frac{xdx}{\left(a^2+x^2\right)^{3/2}} = k\lambda\int_{-\theta_1}^{+\theta_2}\frac{(a\tan\theta)\left(a\sec^2\theta d\theta\right)}{a^3\sec^3\theta} = \frac{k\lambda}{a}\int_{-\theta_1}^{+\theta_2}\sin\theta d\theta = \frac{k\lambda}{a}\left[-\cos\theta\right]_{-\theta_1}^{+\theta_2} = \frac{k\lambda}{a}\left[\cos\theta\right]_{+\theta_2}^{-\theta_1}$$

$$E_x = \frac{k\lambda}{a}\left(\cos\theta_1 - \cos\theta_2\right)$$

The y-component of resultant electric field will be

$$E_y = \int dE_y = \int dE\cos\theta = \int\left(k.\frac{\lambda dx}{r^2}\right)\left(\frac{a}{r}\right) = k\lambda a\int\frac{dx}{r^3} = k\lambda a\int\frac{dx}{\left(a^2+x^2\right)^{3/2}}$$

Again using, $dx = a\sec^2\theta d\theta$ and $\left(a^2+x^2\right)^{3/2} = \left(a^2+a^2\tan^2\theta\right)^{3/2} = a^3\sec^3\theta$

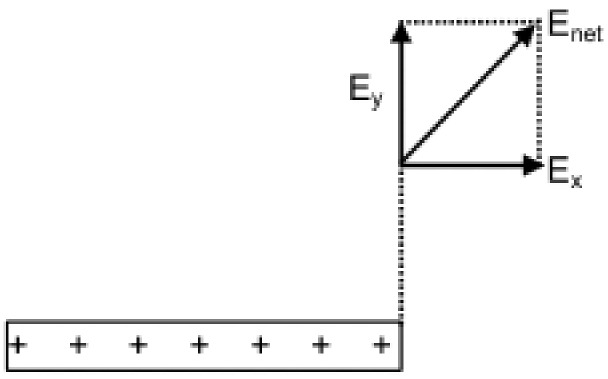
Y-component of electric field due to entire rod will be

$$E_y = \int dE_y = k\lambda a\int\frac{dx}{\left(a^2+x^2\right)^{3/2}} = k\lambda a\int_{-\theta_1}^{+\theta_2}\frac{a\sec^2\theta d\theta}{a^3\sec^3\theta} = \frac{k\lambda}{a}\int_{-\theta_1}^{+\theta_2}\cos\theta d\theta = \frac{k\lambda}{a}\left[\sin\theta\right]_{-\theta_1}^{+\theta_2}$$

$$E_y = \frac{k\lambda}{a}(\sin\theta_1 + \sin\theta_2)$$

Special case 1

If the wire is semi infinite



$$\theta_1 = 0 \quad \theta_2 = \frac{\pi}{2}$$

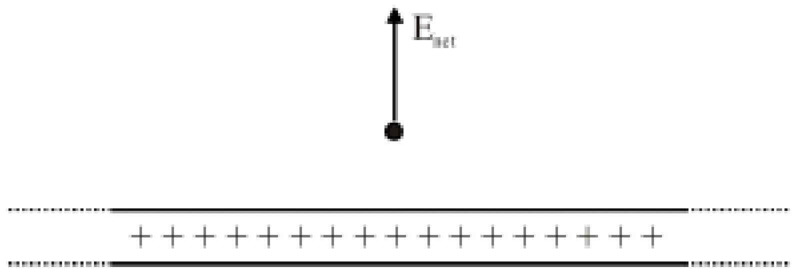
$$E_x = \frac{k\lambda}{a} \text{ and } E_y = \frac{k\lambda}{a}$$

$$\therefore E = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}k\lambda}{a}$$

$$\tan\theta = \frac{E_y}{E_x} = 1 \Rightarrow \theta = 45°$$

Special case-2

If the wire is infinite



$$\theta_1 = \theta_2 = \frac{\pi}{2}$$

$$E_x = 0 \quad E_y = \frac{2k\lambda}{a}$$

$$\therefore E_{net} = \frac{2k\lambda}{a}$$

Electric field due to uniformly charged circular arc (of linear charge density λ) at its centre

Consider an element at an angle ϕ from bisctor and haivng angle $d\phi$ at centre.

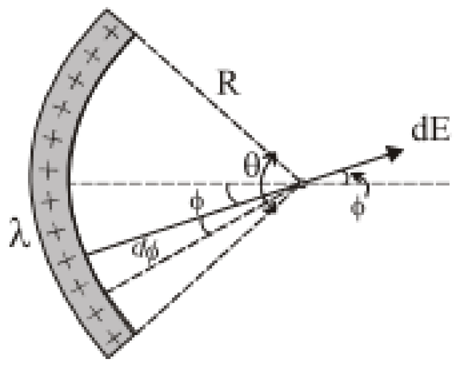
Length of the elementry part will be

$$dl = Rd\phi.$$

Elementry charge

$$dq = \lambda R d\phi$$

Electric field due to this elementry part will be



$$dE = \frac{k(\lambda R d\phi)}{R^2} = \frac{k\lambda}{R}d\phi$$

This electric field has two componets $dE_x = dE\cos\phi$ and $dE_y = dE\sin\phi$. From symmetry it is clear that sum of dE_y will be zero

Hence the net electric field will be sum of dE_x, i.e.

$$E_{net} = \int dE_x = \int dE\cos\phi$$

$$= \int_{-\theta/2}^{+\theta/2} \frac{k\lambda}{R}\cos\phi\, d\phi = \frac{2k\lambda}{R}\sin\frac{\theta}{2}$$

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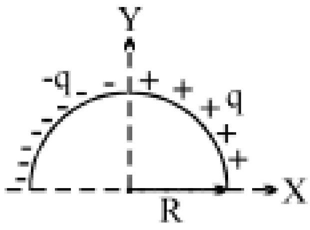
Practice Exercise

Q.1 Find the electric field at centre of semicircular ring shown in figure.

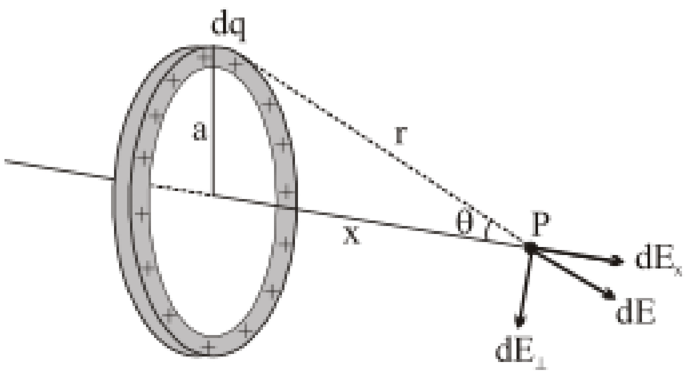


Answers

Q.1 $\frac{4kq}{\pi R^2}\hat{i}$

Electric field due to uniformly charged circular ring at a point on its axis

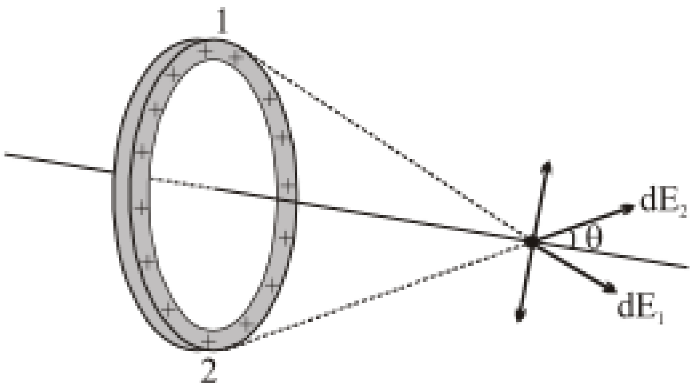
(at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring)



The magnitude of the electric field at P due to the segment of charge dq is

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos\theta$ along the x-axis and a component $dE_\perp$ perpendicular to the x-axis. The resultant field at P must lie along the x-axis because the perpendicular components of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring.

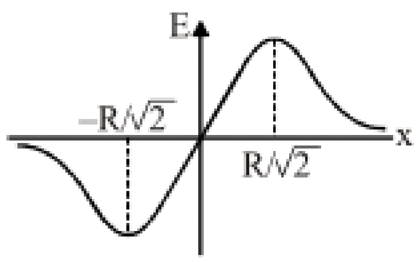


$$dE_x = dE\cos\theta = \left(k_e\frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2+a^2)^{3/2}}dq$$

All elements of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P

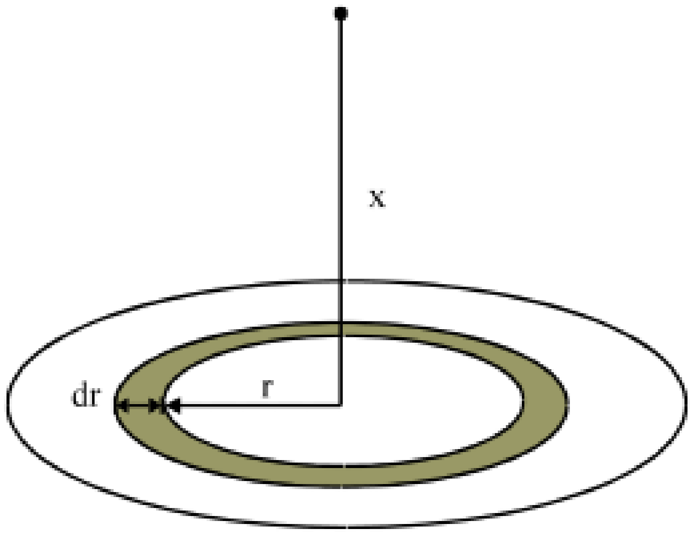
$$E_x = \int\frac{k_e x}{(x^2+a^2)^{3/2}}dq = \frac{k_e x}{(x^2+a^2)^{3/2}}Q$$

Plot of electric field a different points on x-axis



Electric field on the axis of a uniformly charged disc (of surface charge density σ)

Consider a disc of uniform surface charge density 'σ'.Let us calculate the electric field due to a ring of charge situated at a distance r, from the centre and having a width, dr.



$dE = \frac{kx\,dq}{(x^2+r^2)^{3/2}}$, directed along the line OP.

[Here we are using the expression for electric field intensity for a charged ring of radius r at a point on the axis at a distance x from the centre, $E = \frac{kxq}{(x^2+r^2)^{3/2}}$ directed along the axis outwards from the centre.]

Now, the area of the ring, $dS = 2\pi r.\,dr$, $\Rightarrow dq = \sigma 2\pi r dr$,

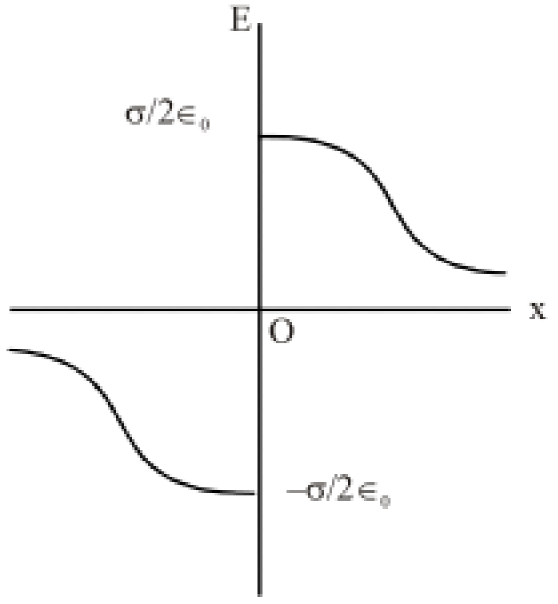
Thus,

$$|E|_P = \int dE = kx\int_0^R\frac{\sigma 2\pi r dr}{\left(x^2+r^2\right)^{3/2}} = \frac{1}{4\pi\epsilon_0}x.\sigma\pi\int_0^R\frac{2r}{\left(x^2+r^2\right)^{3/2}}dr$$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right] = \frac{\sigma}{2\varepsilon_0}(1 - \cos\theta),$$

where θ = semi vertical plane angle subtend by the disc at P.

Plot of electric field due to uniformly charged disc on its axis



Also, as $R \to \infty \quad \Rightarrow \quad E = \frac{\sigma}{2\epsilon_0}$

which is the electric field in front of an infinite plane sheet of charge.

Electric field due to non uniform charge distribution

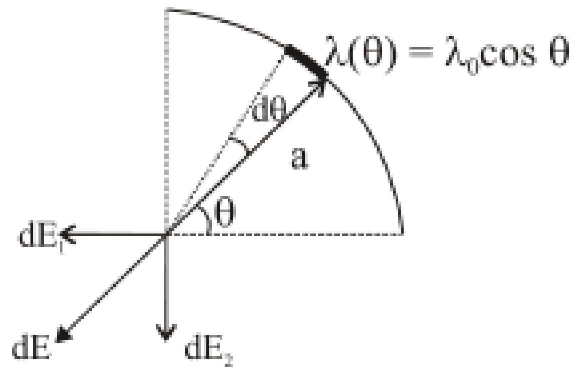
Illustration :

Figure shows circular arc which is non uniformly charged. The linear charge density on the arc is given by

$$\lambda = \lambda_0 \cos\theta$$

where θ is measured from x-axis. Find the electric field at centre of arc

Sol.



let us take an elemetry charge at location θ from x-axis and subtending angle $d\theta$ at centre of the arc

elementry charge is

$$dQ = \lambda dl = (\lambda_0 \cos\theta)(Rd\theta)$$

Electric field due to this elementry charge will be

$$dE = \frac{kdQ}{R^2} = \frac{k\lambda_0 R\cos\theta\, d\theta}{R^2} = \frac{k\lambda_0 \cos\theta\, d\theta}{R}$$

This electric field can be resolved into two components $dE_1 = dE\cos\theta$ (along x-axis) and $dE_2 = dE\sin\theta$ (along y-axis)

The net electric field along x-axis will be

$$E_1 = \int dE_1 = \int dE\cos\theta = \int_0^{\pi/2} \frac{k\lambda_0 \cos\theta\, d\theta \cos\theta}{R} = \frac{k\lambda_0}{R}\cdot\frac{\pi}{4}$$

The net electric field along y-axis will be

$$E_2 = \int dE_2 = \int dE\sin\theta = \int_0^{\pi/2} \frac{k\lambda_0 \cos\theta\, d\theta \sin\theta}{R} = \frac{k\lambda_0}{R}\cdot\frac{1}{2}$$

Practice Exercise

Q.1 A rod of length L has a total charge Q distributed uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle.

Ans. $\frac{Q}{2\varepsilon_0 L^2}$

Q.2 A circular wire-loop of radius carries a total charge Q distributed uniformly over its length. A small length dL of the wire is cut off. Find the electric field at the centre due to the remaining wire.

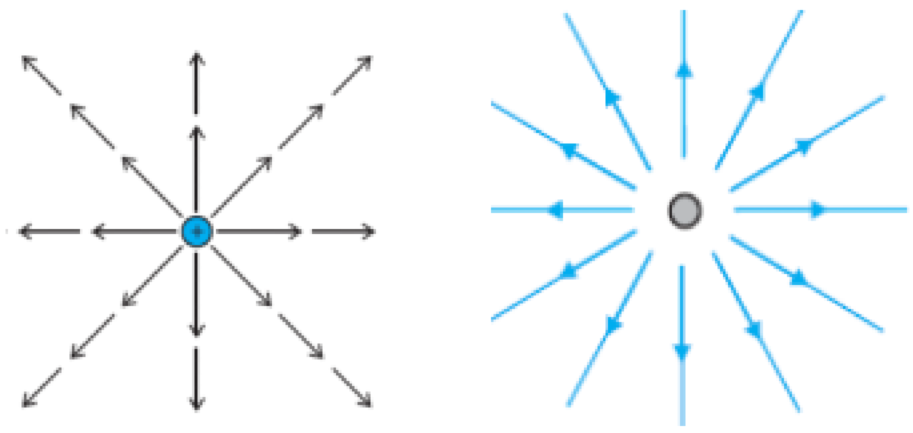
Ans. $\frac{QdL}{8\pi^2\varepsilon_0\alpha^2}$

Answers

Q.1 $\frac{Q}{2\varepsilon_0 L^2}$ Q.2 $\frac{QdL}{8\pi^2\varepsilon_0\alpha^2}$

Electric filed lines

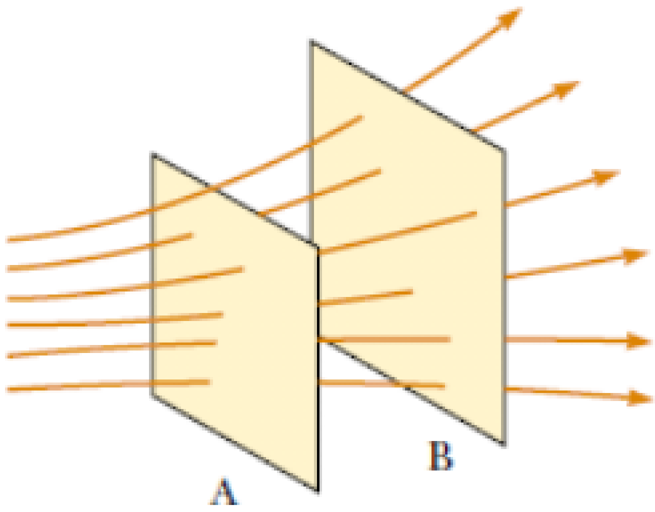
According to Michael Faraday we can visualize electric field with the help of lines. These imaginary lines or curves are termed as electri field lines or electric lines of forces. They are drawn such that tangent at any point on an electric line of force gives the direction of the field at that point.



The first figure represents direction of electric field (force on unit positive test charge) due to a positive charge at some points whereas second figure represents corresponding electric field lines.

The number density of lines (number lines crssing unit area normally) represent the relative strength of the field.In other language at points where the intensity is low, the lines of forces will be widely separated and where the intensity is higher, the lines of force will be closely packed.

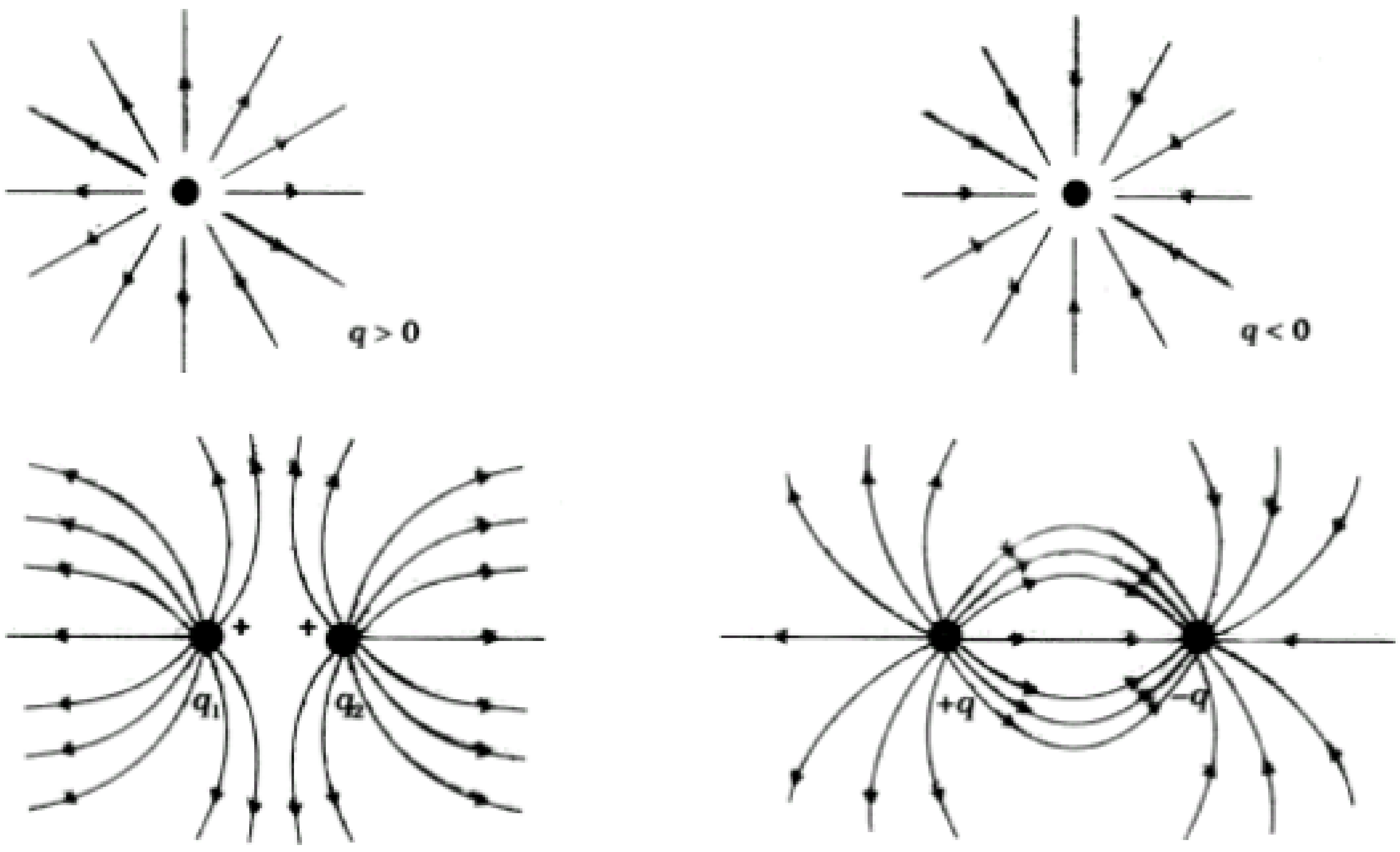
Another person may draw more lines. But the number of lines is not important. In fact, an infinite number of lines can be drawn in any region. It is the relative density of lines in different regions which is important.



Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

Properties of electric lines of force

(i) Electric lines of forces originate from (+)ve charge and terminate (end) on negative charge i.e. (+)ve charges are sources of electric lines of forces and (–) ve charges are sinks for them.



(ii) Electric lines of forces are open curves (not closed curves like magnetic lines of forces). This property of electric lines of forces signifies the fact that electric field is a conservative force field.

(iii) Two electric lines of force never cross each other because if they do cross then there will be two directions of field at same point.

(iv) The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

If magnitude as well as direction of electric field is same then electric field is said to be uniform otherwise nonuniform

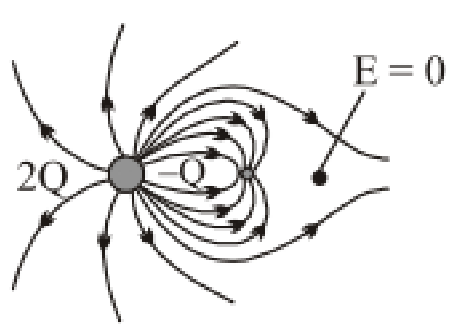
In the first figure the magnitude as well as direction is uniform. In the second figure magnitude of the electric field is uniform where as direction is non uniform. In the third figure direction of the electric field is uniform where as magnitude is non uniform. In the fourth figure the magnitude as well as direction is non uniform.

The first figure represent uniform electric field where as the remaining three figures represent non uniform electric field

Illustration

Figure shows the sketch of field lines for two point charges 2Q and – Q.

Sol.



The pattern of field lines can be deduced by considering the following points:

(a) Symmetry : For every point above the line joining the two charges there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.

(b) Near field : Very close to a charge, its own field predominates. Therefore, the lines are radial and spherically symmetric.

(c) Far field : Far from the system of charges, the pattern should look like that of a single point charge of value (2Q – Q) = + Q, i.e., the lines should be radially outward.

(d) Null point : There is one point at which E = 0. No lines should pass through this point.

(e) Number of lines : Twice as many lines leave + 2Q as entre – Q.

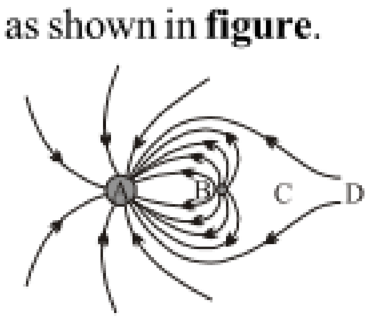
Practice Exercise

Q.1 The electric field of two point charges separated by 4.14 cm is as shown in **figure**.

(a) What is the nature of charges?

(b) What is the ratio of magnitude of charges?

(c) Is the field uniform?



(d) Apart from infinity where is the neutral point ?

(e) Will a positive charge follow the line of force if free to move?

(f) Where do the extra lines end?

Answers

Q.1 (a) A is –ve and B is +ve (b) $|q_A| = 2|q_B|$ (c) Feild is not uniform (d) BC = 10 cm (e) No (f) at infinity

Equilibrium and Motion of Charged Particles in the presence of Electric Field

When a particle of charge q and mass m is placed in an electric field E, the electric force exerted on the charge is $q\vec{E}$ according to Equation $\vec{E} = \frac{\vec{F}}{q}$. If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law. Thus,

$$\vec{a} = \frac{q\vec{E}}{m}$$

If the particle has a positive charge, its force is in the direction of the electric field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Illustration:

A positive point charge q of mass m is released from rest in a uniform electric field $\vec{E}$ directed along the x-axis, as shown in figure. Describe its motion.

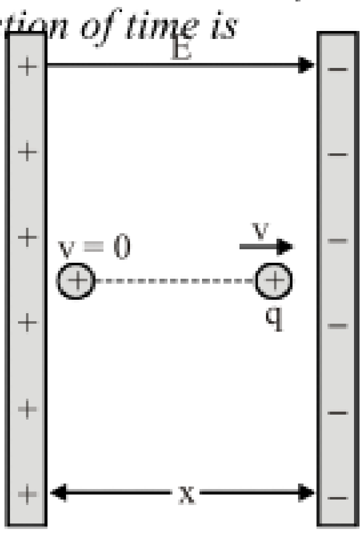
Sol. *The acceleration is constant and is given by q**E**/m. The motion is simple linear motion along the x-axis. Therefore, we can apply the equations of kinematics in one dimension*

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Choosing the initial position of the charge as $x_i = 0$ and assigning $v_i = 0$ because the particle starts from rest, the position of the particle as a function of time is



$$x_f = \frac{1}{2} a t^2 = \frac{qE}{2m} t^2$$

The speed of the particle is given by

$$v_f = at = \frac{qE}{m} t$$

The third kinematics equation gives us

$$v_f^2 = 2a x_f = \left(\frac{2qE}{m}\right) x_f$$

from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_f - x_i$

$$K = \frac{1}{2} m v_f^2 = \frac{1}{2} m \left(\frac{2qE}{m}\right) \Delta x = qE\Delta x$$

We can also obtain this result from the work-kinetic energy theorem because the work done by the electric force is $F_l\Delta x = qE\Delta x$ and $W = \Delta K$.

Illustration:

A particle of charge $1\mu C$ and mass 1 gram is suspended in air near surface of the earth such that weight of particle is balanced by electrostatic force on particle. Find the electric field at position of the particle.

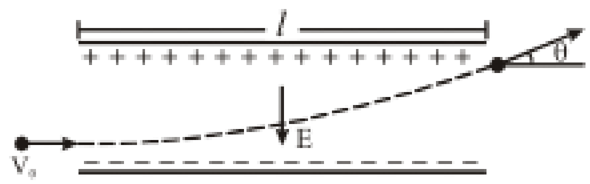
Sol. *Electric force is equal and opposite to weight such that* $\vec{F_e} + \vec{W} = 0$

$$\Rightarrow \quad q\vec{E} + mg(-\hat{j}) = 0$$

$$\Rightarrow \quad \vec{E} = \frac{mg}{q}\hat{j} = 10^4 \text{ N/C vertically upward.}$$

Illustration:

A uniform electric field E is created between two parallel, charged plates as shown in figure. An electron enters the field symetrically between the plates with a speed u_0. The length of each plate is ℓ. Find the angle of deviation of the path of the electron as it comes out of the field.



Sol. *The acceleration of the electron is $a = \frac{eE}{m}$ in the upward direction. The horizontal velocity remains u_0 as there is no acceleration in this direction. Thus, the time taken in crossing the field is :*

$$t = \frac{l}{u_0}$$

The upward component of the velocity of the electron as it emerges from the field region is

$$u_y = at = \frac{eEl}{mu_0}$$

The horizontal component of the velocity remains

$$u_x = u_0.$$

The angle θ made by the resultant velocity with the original direction is given by

$$\tan\theta = \frac{u_y}{u_x} = \frac{eEl}{mu_0^2}$$

Thus, the electron deviates by an angle

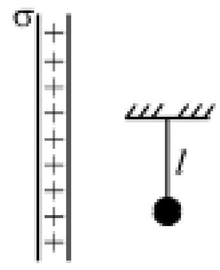
$$\theta = \tan^{-1}\frac{u_y}{u_x} = \frac{eEl}{mu_0^2}.$$

Practice Exercise

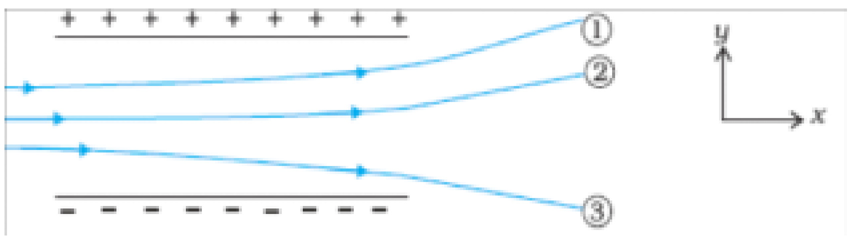
Q.1 An electron and a proton are situated in a uniform electric field. What is the ratio of their acceleration?

Q.2 An infinite plane of positive charge has a surface charge density σ. A metal ball B of mass m and charge q is attached to a thread and tied to a point A on the sheet PQ. Find the angle θ which AB makes with the plane PQ.

Q.3 A simple pendulum of length l and bob mass m is hanging in front of a large nonconducting sheet having surface charge density σ. If suddenly a charge +q is given to the bob & it is released from the position shown in figure. Find the maximum angle through which the string is deflected from vertical.



Q.4 Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Q.5 A simple pendulum consists of a small sphere of mass m suspended by a thread of length l. The sphere carries a positive charge q. The pendulum is placed in a uniform electric field of strength E directed vertically upwards. With what period will the pendulum oscillate if the electrostatic force acting on the sphere is less that the gravitational force? Assume the oscillations to be small.

Q.6 An inclined plane makes an angle of 30^0 with a horizontal is placed in a uniform horizontal electric field $E=100\ Vm^{-1}$. A particle of mass 1 kg and charge 0.01 C, is allowed to slide down from a height of 1 m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom of the inclined plane.

Q.7 A pendulum bob of mass 80 milligrams and carrying a charge of 2×10^{-8} coulomb is at equilibrium. in a horizontal uniform electric field of 20,000 Vm^{-1}. Find the tension in the thread of the pendulum at equilibrium.

Q.8 A uniform electric field of strength 10^6 V/m is directed vertically downwards. A particle of mass 0.01 kg and charge 10^{-6} coulomb is suspended by an inextensible thread of length 1 m. The particle is displaced slightly from its mean position and released. Calculate the time period of its oscillation. What minimum velocity should be given to the particle at rest so that it completes full circle in a vertical plane without the thread getting slack? Calculate the maximum and minimum tensions in the thread in this situation.

Answers

Q.1 1836 Q.2 $\tan^{-1}\left[\dfrac{q\sigma}{2\in_0 mg}\right]$ Q.3 $2\tan^{-1}\left(\dfrac{\sigma q_0}{2\varepsilon_0 mg}\right)$ Q.4

Q.5 $\left[\mathbf{T} = 2\pi\left[\dfrac{l}{\left(g - \dfrac{qE}{m}\right)}\right]^{1/2}\right]$ Q.6 1.3 sec Q.7 8.8×10^{-4} N

Q.8 0.6 s; $v_{min} = 23.43$ m/s; $T_{min} = 0$ & $T_m = 6.59$ N

GAUSS'S LAW

Area as a Vector

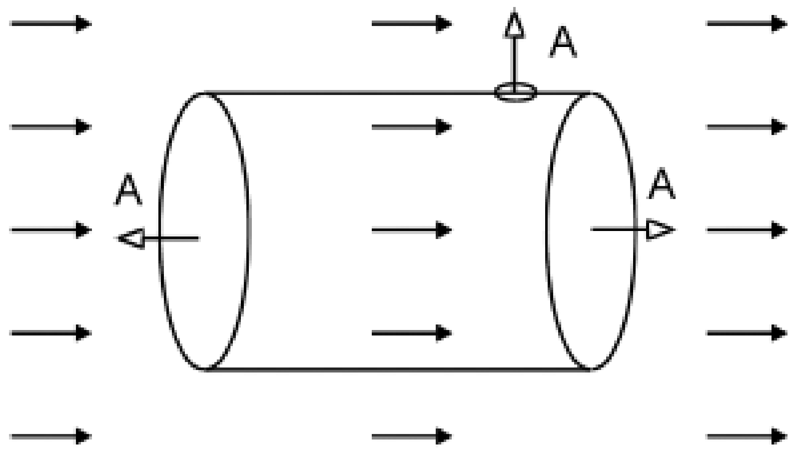
In some calculations it is useful to use area as a vector. Plane suface area or small elemental area of a cuved surface can be treated as vector quantities. Its direction is taken along the outward normal drawn on the surface and magnitude is equal to its area. It should be noted here that the direction of outward normal at a given point of a closed surface is unique but in case of open surfaces the terms outward and inward are not well defined and so the choice of outward normal is purely arbitrary.

Flux of a vector field

When a planel area represented by an area vector $\vec{A}$ is placed in any uniform vector field (let $\vec{X}$) then a flux(represented by the symbol Φ) of that vector field is said to be linked through that area and defined as

$$\Phi = \vec{X} \cdot \vec{A}$$

To understand its meaning clearly let us take example of velocity field. We imagine a hypothhetical cylindrical having cross-sectional area A, Lowered in a flowing river in which velocity of water is uniform.



flux of the vector velocty vector on left cross section

$$\Phi_1 = \vec{v} \cdot \vec{A} = vA\cos 180^0 = -vA \ \text{m}^3/\text{sec.}$$

flux of the vector velocty vector on right cross section

$$\Phi_2 = \vec{v} \cdot \vec{A} = vA\cos 0^0 = +vA \ \text{m}^3/\text{sec.}$$

flux of the vector velocty vector on on any elemental area of curved surface

$$\Phi_3 = \vec{v} \cdot \vec{A} = vA\cos 90^0 = 0 \ \text{m}^3/\text{sec.}$$

As the unit suggests the flux of $\vec{v}$ through $\vec{A}$ at left end gives the net inflow of water across the left cross-section of the pipe per-sec .Similarly the flux of $\vec{v}$ through $\vec{A}$ at right end gives the net outflow of water across the right cross-section of the pipe per-sec . Zero flux on curved surface represents there is no flow through curved surface.

Overall we can say that flux of any vector field through an area vector placed in that field defines the net inflow or net outflow of something across that area. (-)ve flux indicates inflow, (+)ve flux indicates outflow.

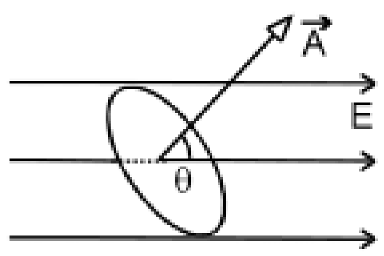
Flux of electric field

Flux of electric field through an area placed in that field represents the part of the electric field intercepted by that area or the net inflow or outflow of electric lines of forces normally through that area.

When a small area represented by an area vector $d\vec{A}$ is placed in any vector field then a flux of that field is said to be linked through that area.

Considering a small surface represented by area vector $d\vec{A}$, placed in an electric field $\vec{E}$ in such a way that angle between $\vec{E}$ and $d\vec{A}$ is θ then the flux of electric field $\vec{E}$ through the area $d\vec{A}$ is defined as the dot product of the field vector and that of the area vector and is given by

$$d\Phi = \vec{E} \cdot d\vec{A} = EdA\cos\theta$$



$\Rightarrow \quad d\Phi = (E\cos\theta)\, dA =$ Product of the area and that of the component of $\vec{E}$ along $d\vec{A}$ i.e., along the normal to the area or,

$d\Phi = E(dA\cos\theta) =$ Product of E and the projection of the area vector normal to the direction of $\vec{E}$.

To calculate the flux of nonuniform electric field through a surface of any arbitrary shape we divide the whole surface into little patches which are so small that over any one patch the surface is practically flat and the electric field doesn't change appreciably from one part of a patch to another. The area of a patch has a certain magnitude and the outward pointing normal to its surface gives its direction. We calculate the fluxes of electric field through each such patches and the algebraic sum of the fluxes through individual patches gives the total flux of electric field through the given surface of arbitrary shape because flux is a scalar quantity.

The flux of nonuniform electric field over an arbitrary surface A is given by the integral

$$\Phi = \int d\Phi = \int_S \vec{E} \cdot d\vec{A} = \int_S E\, dA\, \hat{n}$$

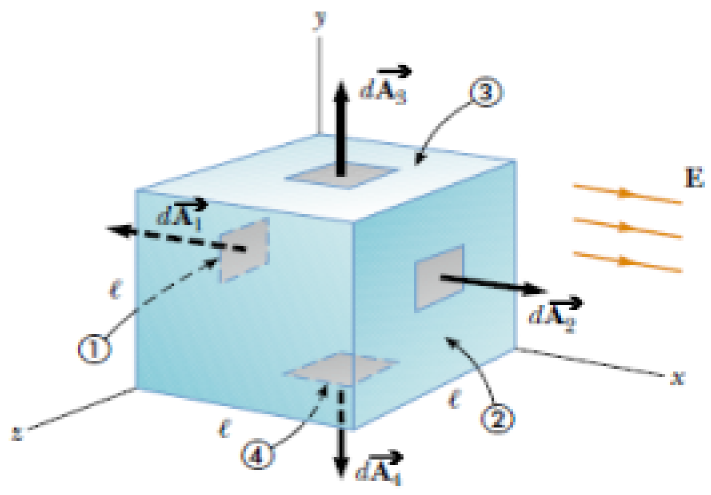
where $\hat{n}$ is the unit vector in the direction of normal to the surface

In case of closed surface flux is written by special sumbol

$$\Phi = \int d\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E\, dA\, \hat{n}$$

Illuatration:

Consider a uniform electric field $\vec{E}$ *oriented in the x direction.Find the net electric flux through the surface of a cube of edge length* ℓ, *oriented as shown in Figure*



Sol. *The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (3, 4 and the unnumbered ones) is zero because* $\vec{E}$ *is perpendicular to* $d\vec{A}$ *on these faces.*

The net flux through faces 1 and 2 is

$$\phi_E = \int_1 \vec{E} \cdot d\vec{A} + \int_2 \vec{E} \cdot d\vec{A}$$

For face 1, $\vec{E}$ *is constant and directed inward but* $d\vec{A}_1$ *is directed outward (θ= 180°); thus, the flux through this face is*

$$\phi_1 = \int_1 \vec{E} \cdot d\vec{A} = \int_1 EdA\cos 180^0 = -EA = -E\ell^2$$

For face 2, $\vec{E}$ *is constant and outward and in the same direction as* $d\vec{A}_2$ *(θ= 0°); hence, the flux through this face is*

$$\phi_2 = \int_1 \vec{E} \cdot d\vec{A} = \int_1 EdA\cos 0^0 = EA = E\ell^2$$

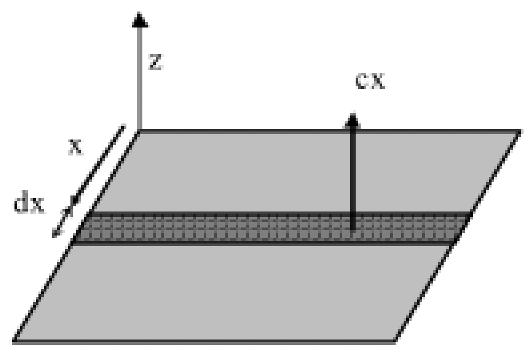
Therefore, the net flux over all six faces is

$$\phi_{total} = E\ell^2 + \left(-E\ell^2\right) + 0 + 0 + 0 + 0 = 0$$

Illuatration:

A nonuniform electric field is given by the expression $\vec{E} = ay\hat{i} + bz\hat{j} + cx\hat{k}$, *where* a, b, *and* c *are constants. Determine the electric flux through a rectangular surface in the* xy *plane, extending from* $x = 0$ *to* $x = w$ *and from* $y = 0$ *to* $y = h$.

Sol. *The direction of area vector is along z axis hence there will be no flux due to x and y component of electric field. So let us calculate flux due to z component of electric field.*

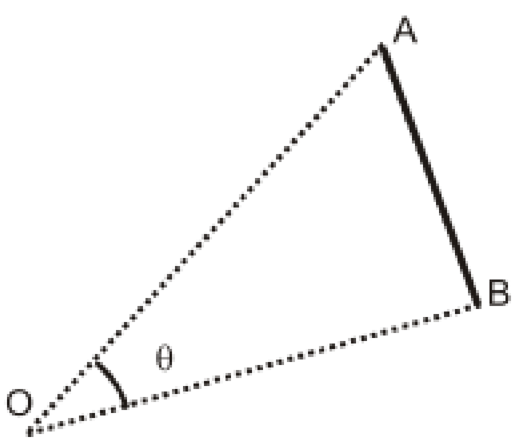


$$\phi = \int d\phi = \int E dA = \int_0^w (cx)(hdx) = \frac{chw^2}{2}$$

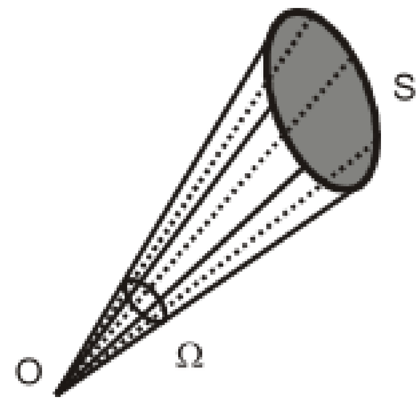
Solid Angle

A plane angle is a two dimensional concept. Solid angle is a three dimensional generalisation of the two dimensional concept of plane angle. A plane angle is formed at a point by a line segment or an arc but a surface is responsible for the formation of solid angle at a given point. The S.I. unit of plane angle is radian whereas that of solid angle is steradian (Sr).

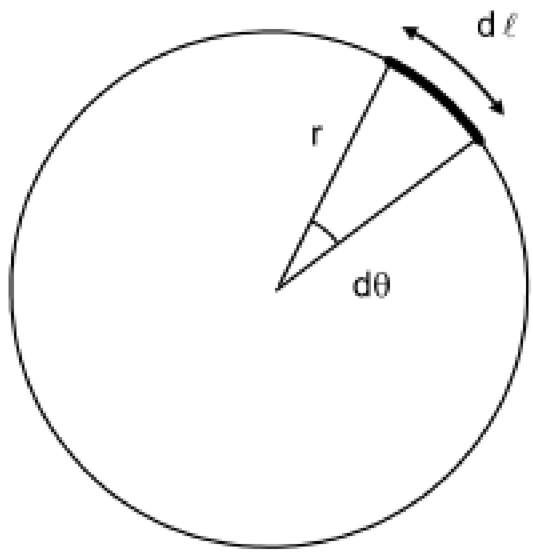
AB is the line segment forming plane angle θ at O



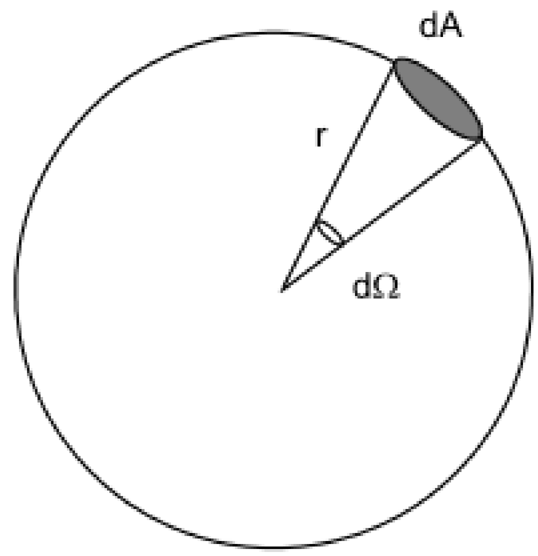
S is the surface segment forming solid angle Ω at O



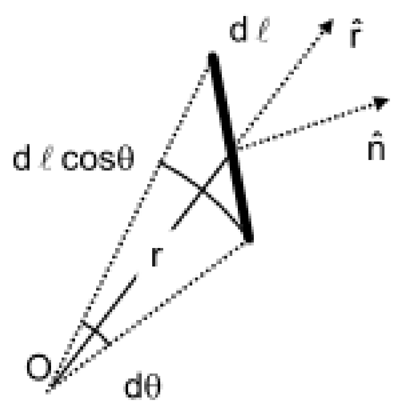
Plane angle formed by a small arc of a circle at its centre is defined as $d\theta = \frac{d\ell}{r}$



solid angle formed by a small area of a sphere at its centre is defined as $d\theta = \frac{d\ell}{r}$



If a small length element $d\ell$ is oblique such that normal to the length element ($\hat{n}$) is making an angle θ with radial direction ($\hat{r}$) then the plane angle is defined as $d\theta = \frac{d\ell\cos\theta}{r}$



If a small surface element dA is oblique such that normal to the surface element ($\hat{n}$) is making an angle θ with radial direction ($\hat{r}$) then the plane angle is defined as $d\Omega = \frac{dA\cos\theta}{r^2}$

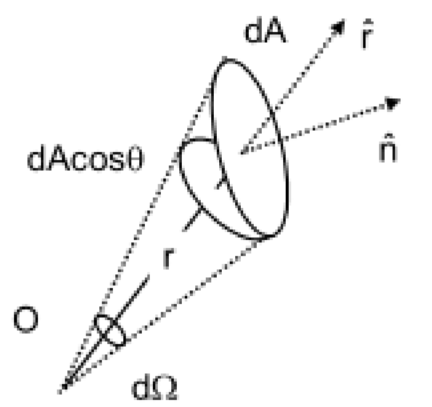


Illustration:

Find the solid angle formed by a disc at a point on its axis where its radius forms plane angle θ

Sol. *Let us first calculate the solid angle formed by elementry ring of radius r and width dr*

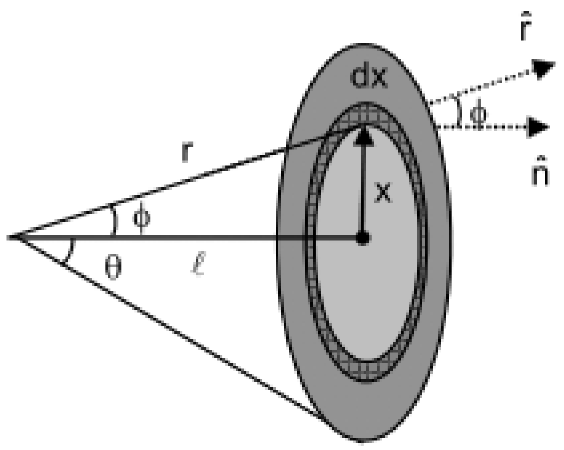
$$d\Omega = \frac{dA\cos\phi}{r^2} = \frac{(2\pi x dx)\cos\phi}{r^2}$$

substitute $x = \ell\tan\phi \Rightarrow dx = \ell\sec^2\phi d\phi$ *and*

$$r = \sqrt{\ell^2 + x^2} = \sqrt{\ell^2 + (\ell\tan\phi)^2} = \ell\sec\phi$$

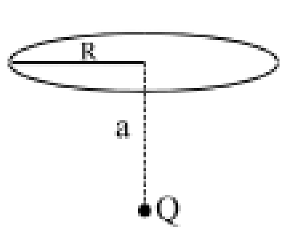
$$d\Omega = \frac{(2\pi\ell\tan\phi\,\ell\sec^2\phi d\phi)\cos\phi}{(\ell\sec\phi)^2} = 2\pi\cos\phi d\phi$$

$$\therefore\ \Omega = \int d\Omega = \int_0^\theta 2\pi\cos\phi d\phi = 2\pi(1-\cos\theta)$$



Practice Exercise

Q.1 A point charge Q is located on the axis of a disc of radius R at a distance a from the plane of the disc. If the soild angle subtended by disc at the point charge is $\pi/2$, then find the relation between a & R.

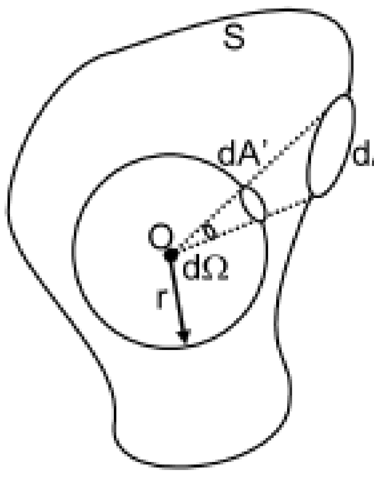


Answers

Q.1 $a = \frac{R}{\sqrt{3}}$

Some important results regarding solid angle

(i) ***A closed surface subtends a solid angle of 4π(Sr) at an internal point.***



Considering a three dimensional closed surface S of any arbitrary shape. Let O be an interior point. Again, consider a spherical surface of radius 'r' having its centre at 'O'. Let a small part of the surface, having surface area 'dA' is contained inside a solid angle dΩ. This cone emerging from point O encloses an area dA' of the spherical surface.

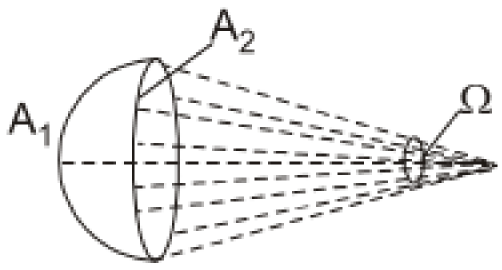
So Solid angle subtended by dA at point O =dΩ= solid angle subtended by dA' at point O = $\frac{dA'}{r^2}$

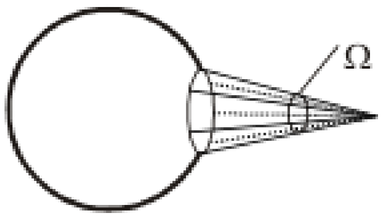
Every small area, on the surface S of any arbitrary shape, may be shown in correspondence with an area on the spherical surface. So, total solid angle subtended by a closed surface of any arbitrary shape at an internal point.

$$\Omega = \int d\Omega = \frac{1}{r^2}\int dA' = \frac{1}{r^2} 4\pi(r^2)$$

$$= 4\pi(Sr)$$

(ii) ***A closed surface subtends a solid angle of 0 (Sr) at an external point.***

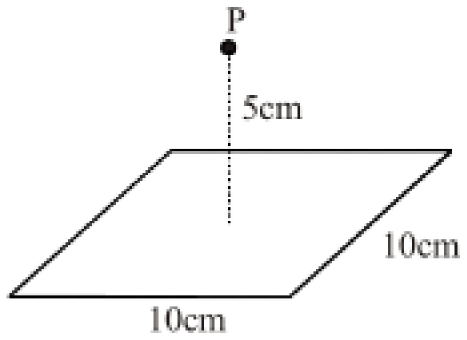




If the boundary of a hemispherical surface is gradually closed then the solid angle Ω start decreasing, when it takes the shape of a closed surface, the solid angle subtended at the external point becomes zero.

Illustration

What is the solid angle subtended by square at the point P.



Sol. *If a consider a cube of side 10 cm point P will be at its centre. The total solid angle subtended by all the six surface of the cube at P is 4π staradian. Also from symmetry it is obvious that solid angle formed by each surface at P is equal hence equal to*

$$\Omega = \frac{1}{6}(4\pi) = \frac{2\pi}{3} \text{ staradian}$$

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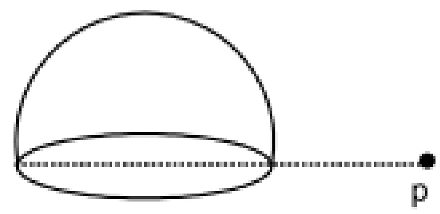
Practice Exercise

Q.1 What is the solid angle subtended by hemisphere (shown in figure) at point P.



Q.2 Find the solid angle subtended by a cube at its corner

Answers

Q.1 Zero. Q.2 $\frac{\pi}{2}$

Illustration

Find the flux of electric field of a point charge over a closed surface of arbitrary shape if point shape point charge is placed

(i) inside closed surface
(ii) outside closed surface

Sol. *Flux on elementary area will be*

$$d\phi = \vec{E}\cdot d\vec{A} = EdA\cos\theta = \frac{q}{4\pi\epsilon_0 r^2}dA\cos\theta = \frac{q}{4\pi\epsilon_0}\frac{dA\cos\theta}{r^2} = \frac{q}{4\pi\epsilon_0}d\Omega$$

Where $d\Omega$ is the solid angle subtended by elementary area at the location of point charge

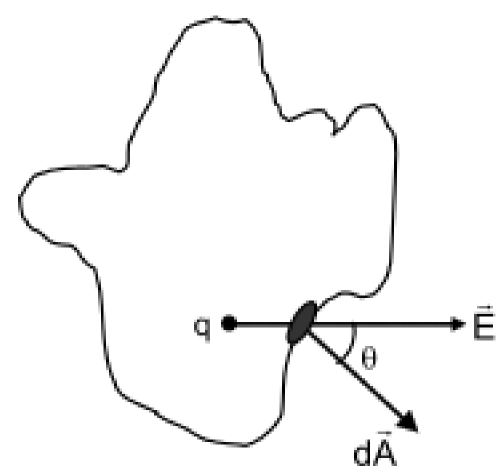
$\therefore$ *Total flux* $\phi = \int d\phi = \frac{q}{4\pi\epsilon_0}\int d\Omega = \frac{q}{4\pi\epsilon_0}\Omega$

Where Ω is the total solid angle subtended by area at the location of point charge.
If the point charge is placed inside the surface

$$\Omega = 4\pi \Rightarrow \phi = \frac{q}{\epsilon_0}$$

If the point charge is placed outside the surface

$$\Omega = 0 \Rightarrow \phi = 0$$



Here we get a result

Electric flux due to a point charge on any surface$=\frac{\text{charge}}{4\pi\epsilon_0}$*(solid angle subtended by the surface at the location of point charge)*

Practice Exercise

Q.1 A point charge q is placed on the axis of a disc (imaginary) of radius R at a distance x from the centre of the disc. Find the flux of electric field of the charge q on disc.

Ans. flux $\phi = \frac{q}{4\pi \in_0}\Omega = \frac{q}{2\in_0}(1-\cos\theta) = \frac{q}{2\in_0}\left(1-\frac{x}{\sqrt{R^2+y^2}}\right)$

Answers

Q.1 flux $\phi = \frac{q}{4\pi \in_0}\Omega = \frac{q}{2\in_0}(1-\cos\theta) = \frac{q}{2\in_0}\left(1-\frac{x}{\sqrt{R^2+y^2}}\right)$

Gauss theorem

The flux of net electric field (due to the charges enclosed by the surface as well as the charges outside it) through any imaginary closed surface(called gaussian surface) of any arbitrary shape is equal to the total charge enclosed by the surface divided by $\in_0$.

$$\oint \vec{E}_{res}.d\vec{A} = \frac{\Sigma q_{enclosed}}{\in_0}$$

Some points to be emphasized about the gauss law :

(i) It is true for any closed surface no matter what its shape and size.

(ii) The q includes algebraic sum of all charges enclosed by the surface.

(iii) In situations when the surface is so chosen that there are some charges inside and some outside, the $\vec{E}$ (whose flux appear in the equation) is due to all charges, just term 'q' in the law represents only total charge inside.

(iv) *The gaussian surface should not pass through any discrete charge however, it can pass through a continuous charge distribution.*
Gaussian surface should not contain any finite non zero charge.

Illuatration:

A charge Q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere

Sol. *Let us imagine another hemispherical surface over identical given one.*
Both being symmetric with respect to Q, hence flux will be same through both the hemisphere $(\phi_1 = \phi_2)$.

Net flux $\phi_1 + \phi_2 = \frac{q_{nc}}{\in_0} = \frac{Q}{\in_0}$

$\Rightarrow$ $\phi_1 = \frac{Q}{2\in_0}$.

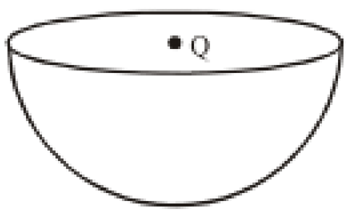
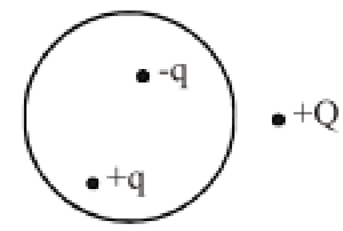


Illustration:

Three point charges –q, +q and Q are placed in a region as shown. P is a point on imaginary Gaussian sphere enclosing –q and +q and Q is outside
Will it be correct to say that flux at every part of sphere due to Q is zero.



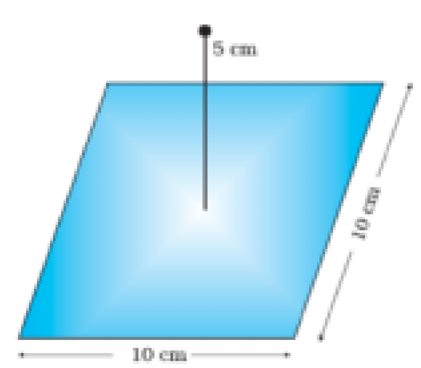
Sol. *No flux at every part of sphere due to Q is not zero because electric field is intercepted by some area*

[redacted]

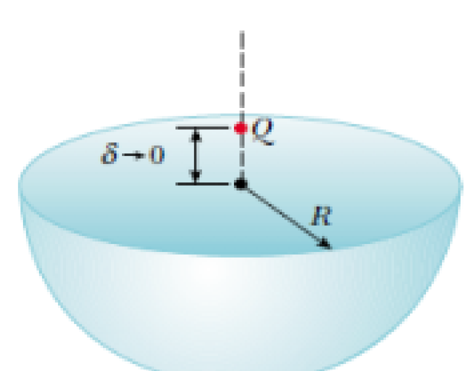
Q.1 A point charge q is placed at one corner of a cube of edge a. What is the flux through each of the cube faces?

Q.2 A point charge q is placed at near infinite plane. What is the flux through the plane?

Q.3 A point charge q is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. What is the magnitude of the electric flux through the square?



Q.4 A point charge Q is located just above the center of the flat face of a hemisphere of radius R as shown in Figure . What is the electric flux (a) through the curved surface and (b) through the flat face?



[redacted]

Q.1 $\frac{1}{24}\frac{q}{\epsilon_0}$ Q.2 $\frac{q}{2\epsilon_0}$ Q.3 $\frac{q}{6\epsilon_0}$ Q.4 (a) $\frac{q}{2\epsilon_0}$ (b) $-\frac{q}{2\epsilon_0}$

Applications of Gauss Theorem

Choice of gaussian surface in evaluating electric field.

Definition of a Gaussian surface :

While applying Gauss's law we are interested in evaluating the integral

$$\phi_E = \oint E \cdot dA$$

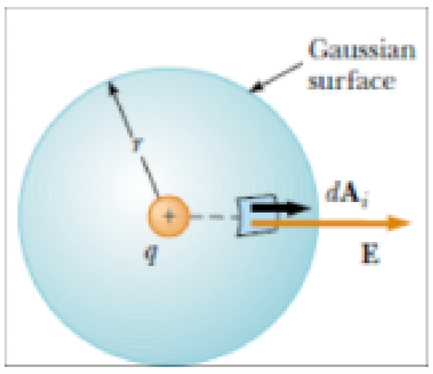
The closed surface for which the flux is calculated is generally an imaginary or hypothetical surface, called a Gaussian surface. Whenever we apply Gauss's law we may choose a surface of any size and shape as our Gaussian surface. But selecting a proper size and shape for a Gaussian surface is a key factor for determining flux and electric field. Here are the list of different types of the Gaussian surfaces to be chosen for a given charge distribution.

Charge distribution	Gaussian surface	Electric field
Point charge	Spherical	Radial
Spherical charge distribution	Spherical	Radial
Line of charge	Cylindrical	Radial
Planar charge	Cylindrical	Normal to surface

(i) Electric field due to a point charge

$$\oint \vec{E}.d\vec{A} = \oint E\,dA = \oint dA = E4\pi r^2$$

$$\sum q_{en} = Q$$

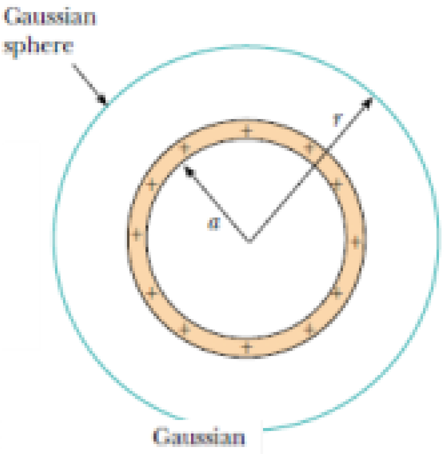


$\therefore$ from Gauss' law

$$E4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \quad E = \frac{Q}{4\pi \epsilon_0 r^2}$$

(ii) Evaluation of el. fd. due to uniformly charged spherical shell (charge Q):

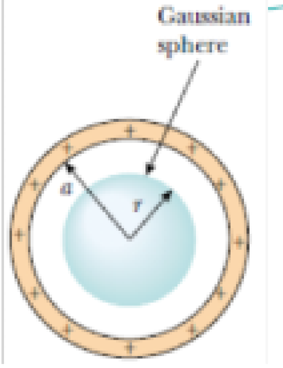


Case 1 **At external point (r>R)**

from Gauss' law $\quad E4\pi r^2 = \frac{Q}{\epsilon_0}$

$$\Rightarrow \quad E = \frac{Q}{4\pi \epsilon_0 r^2}$$

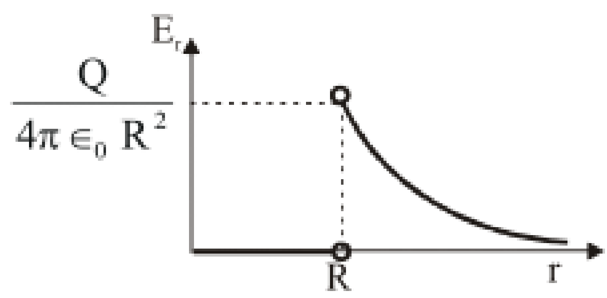
Case 2 **At internal point (r<R)**



$$E4\pi r^2 = \frac{0}{\epsilon_0}$$

$$\Rightarrow \quad E = 0$$

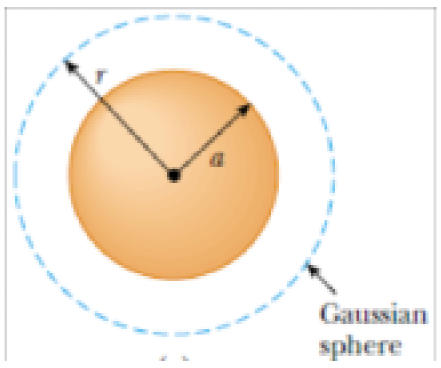
Plot of variation of electric field



(iii) Evaluation of el. fd. due to uniformly charged spherical volume (charge Q)

Case1 **At external point (r>R)**

from Gauss' law

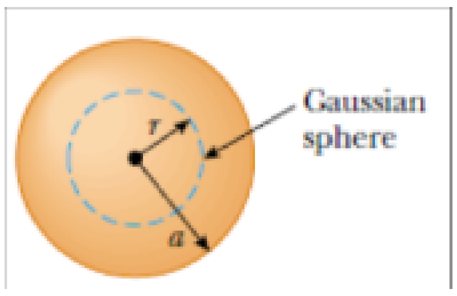


$$E4\pi r^2 = \frac{Q}{\in_0}$$

$$\Rightarrow \quad E = \frac{Q}{4\pi \in_0 r^2}$$

Case 2:- At internal point (r<R)

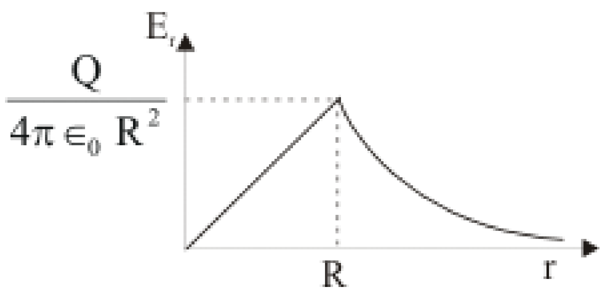
from Gauss' law



$$E(4\pi r^2) = \frac{\frac{Q}{\frac{4}{3}\pi R^3}\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 R^3} r = \frac{\rho}{3\varepsilon_0} r$$

Plot of variation of electric field



(iv) Electric field at a point near an infinitely long uniform linear charge distribution:

$$\oint \vec{E}.d\vec{A} = \int_{\substack{\text{upper}\\ \text{Surface}}} \vec{E}.d\vec{A} + \int_{\substack{\text{curved}\\ \text{surface}}} \vec{E}.d\vec{A} + \int_{\substack{\text{lower}\\ \text{surface}}} \vec{E}.d\vec{A}$$

$$= \int_{\substack{\text{upper}\\\text{surface}}} EdA\cos 90^0 + \int_{\substack{\text{curved}\\\text{surface}}} EdA\cos 0^0 + \int_{\substack{\text{lower}\\\text{surface}}} EdA\cos 90^0$$

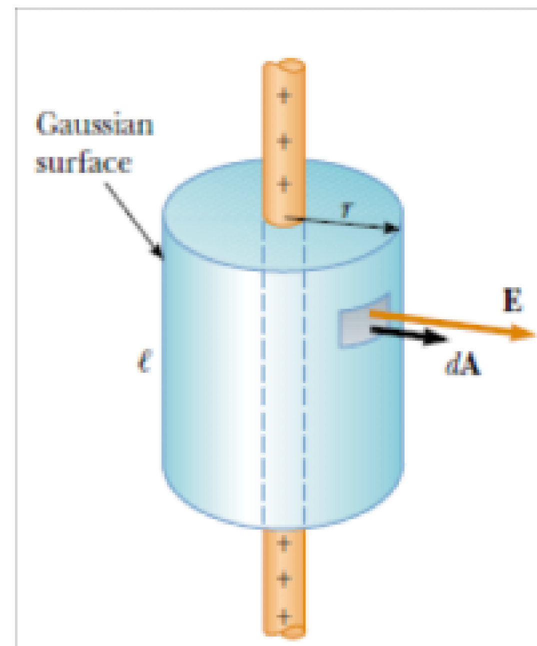
$$= 0 + \oint E\, dA + 0 = E 2\pi r l$$

$$\& \sum q_{\text{enclosed}} = \lambda l$$

$\therefore$ from Gauss's law

$$E 2\pi r l = \lambda l / \epsilon_o$$

$$\therefore\ E = \frac{\lambda}{2\pi\epsilon_o r}$$

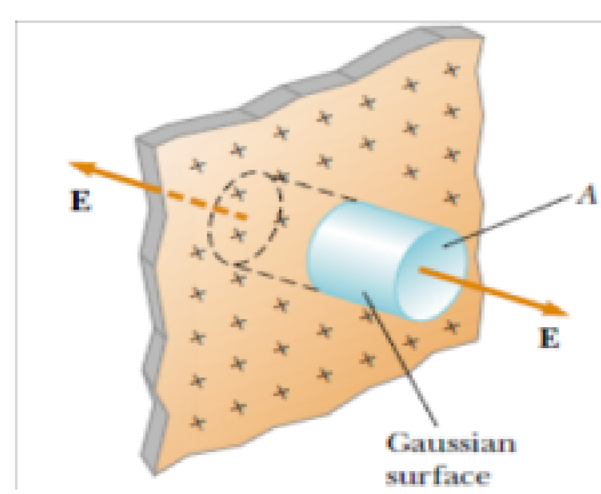


(v) Electric field intensity at a point near an infinitely charged plane sheet having surface charge density

$$\oint \vec{E}.d\vec{A} = \int_{\substack{\text{front}\\\text{surface}}} \vec{E}.d\vec{A} + \int_{\substack{\text{curved}\\\text{surface}}} \vec{E}.d\vec{A} + \int_{\substack{\text{rear}\\\text{surface}}} \vec{E}.d\vec{A}$$

$$= \int_{\substack{\text{upper}\\\text{surface}}} EdA\cos 0^0 + \int_{\substack{\text{curved}\\\text{surface}}} EdA\cos 90^0 + \int_{\substack{\text{lower}\\\text{surface}}} EdA\cos 0^0$$

$$= \oint E\, dA + 0 + \oint E\, dA = EA + 0 + EA = 2EA$$



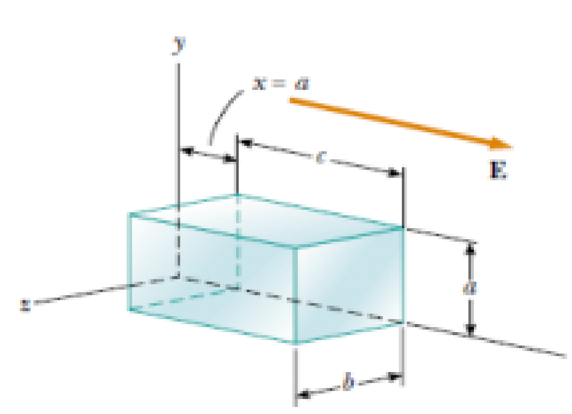
$$\sum q_{en} = \sigma S$$

$\therefore$ from Gauss' law $\quad 2ES = \dfrac{\sigma S}{\epsilon_o}$

$$\Rightarrow \quad E = \frac{\sigma}{2\epsilon_o}$$

Illuatration:

A closed surface with dimensions $a = b = 0.400$ m and $c = 0.600$ m is located as in Figure. The left edge of the closed surface is located at position $x = a$. The electric field throughout the region is nonuniform and given by $\vec{E} = (3 + 2x^2)\hat{i}$ N/C, where x is in meters. Calculate the net electric flux leaving the closed surface. What net charge is enclosed by the surface?



Sol. *The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (parallel to y-z and x-z plane) is zero because $\vec{E}$ is perpendicular to $d\vec{A}$ on these faces.*

For face parallel to y-z plane lying at x=a , $\vec{E}$ *is constant and directed inward but* $d\vec{A}_1$ *is* directed outward (*θ= 180°); thus, the flux through this face is*

$$\phi_1 = \oint \vec{E}.d\vec{A} = \int EdA\cos 180^o = -EA = -[3+2a^2]ab$$

For face parallel to y-z plane lying at x=a+c , $\vec{E}$ *is constant aand outward and in the same direction as* $d\vec{A}_2$ *(θ= 0°); thus, the flux through this face is*

$$\phi_2 = \int \vec{E}.d\vec{A} = \int EdA\cos 0^0 = +EA = +\left[3+2(a+c)^2\right]ab$$

Therefore, the net flux over all six faces is

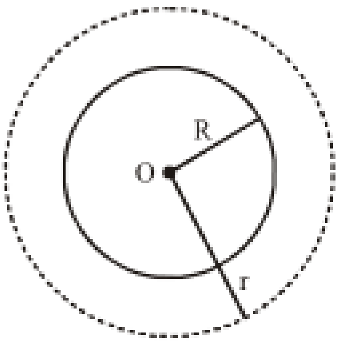
$$\phi = -\left[3+2a^2\right]ab + \left[3+2(a+c)^2\right]ab + 0 + 0 + 0 + 0$$

Illuatration:

A system consists of a ball of radius R carrying a spherically symmetric charge and the surrounding space filled with a charge of volume density $\rho = a/r$ *where a is a constant, r is the distance from the centre of ball. Find the ball's charge at which the magnitude of the electric field is independent of r outside the ball. How high is this strength?*

Sol. *Let us consider a spherical surface of radius* $r(r > R)$ *concentric with the ball and apply Gauss's Law.*

$$\oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0}$$



Let Q= total charge of the ball

$$\varepsilon_0 E(4\pi r^2) = Q + \int_R^r \rho 4\pi x^2\, dx$$

$$\varepsilon_0 E(4\pi r^2) = Q + 4\pi\int_R^r \frac{a}{x} x^2\, dx$$

$$\varepsilon_0 E(4\pi r^2) = Q + 2\pi a(r^2 - R^2)$$

$$\Rightarrow \quad E = \left(\frac{Q - 2\pi a R^2}{4\pi\varepsilon_0}\right)\frac{1}{r^2} + \frac{2\pi a}{4\pi\varepsilon_0}$$

For E to be independent of r,

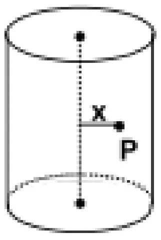
$$Q = 2\pi a R^2$$

and the value of E is

$$E = \frac{a}{2\varepsilon_0}.$$

Practice Exercise

Q.1 A long cylindrical volume contains a uniformly distributed charge of density ρ. Find the electric field at a point P inside the cylindrical volume at a distance x from its axis in the figure.



Q.2 A non conducting sheet of large surface area and thickness d contains uniform charge distribution of density ρ. Find the electric field at a point P inside the plate, at a distance x from the central plane.

Q.3 Consider the classical–model of an atom such that a nucleus of charge + e is uniformly distributed within a sphere of radius 2Å. An electron of charge (–e) at a radial distance 1Å moves inside the sphere. Find the force attracting the electron to the centre of the sphere. Calculate the frequency with which the electron would oscillate about the centre of the sphere. [* This is the earliest model of atom proposed by J.J. Thomson.]

Answers

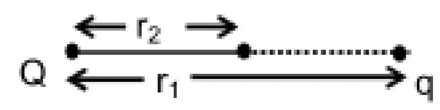
Q.1 $\frac{\rho x}{2\varepsilon_0}$ Q.2 $\frac{\rho x}{\varepsilon_0}$ Q.3 9×10^{14} Hz.

ELECTRIC POTENTIAL ENERGY

Whenever a charge is moved in an electrostatic field, work is done by electrostatic forces. An electrostatic field is a conservative force field. Therefore, any work done against the field is stored as potential energy. Change in electric potential energy will be $\Delta U = -W_{el}$, where W_{el} is the work done by electrostatic forces.

Electric potential energy between two point charges

Consider two charges Q and q separated by a distance r_1. If the charge q is moved along the line joining the charge and the final separation becomes r_2, then the work done by the electric force during the process is



$$W_{el} = \int_{r_1}^{r_2} \frac{kQq}{r^2} dr = kQq\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$\therefore$ The change in potential energy is defined as

$$U_2 - U_1 = -W_{el} = kQq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

The potential energy of a two-charge system is taken to be zero, when the distance between the charges is infinity. i.e. $U = 0$ if $r = \infty$

Now, the potential energy of a two charge system when their separation is r, is

$$U(r) - 0 = kQq\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$U(r) = \frac{kQq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

For like charges U is +ve & for unlike charges U is-ve.

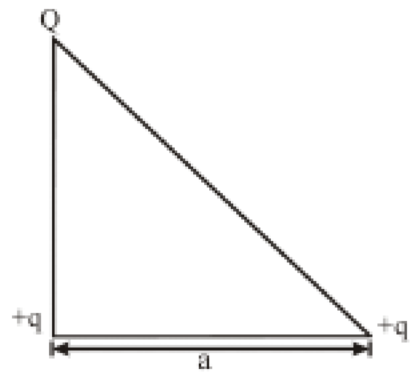
Electric potential energy between more than two point charges

The above equation gives the potential energy of a pair of charges. In case of three charges (say q_1, q_2 and q_3) there are three pairs (q_1, q_2), (q_2,, q_3) and (q_3, q_1). Thus the total potential energy of the system will have three terms.

$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_3q_1}{r_{31}}$$

Illustration:

Three charges Q, +q and +q are placed at the vertices of a right angled isosceles triangle as shown in the figure. The net electrostatic energy of
the configuration is zero., if Q is equal to :



Sol. *Net electrostatic energy*

$$U = \frac{kQq}{a} + \frac{kQq}{\sqrt{2}a} + \frac{kqq}{a}$$

$$For\ U = 0;\ \frac{Qq}{a} + \frac{Qq}{\sqrt{2}\,a} + \frac{qq}{a} = 0$$

$$\Rightarrow \quad Qq\left(\frac{1}{a} + \frac{1}{\sqrt{2}\,a}\right) = \frac{-q^2}{a}$$

$$\Rightarrow \quad Q\left(\frac{\sqrt{2}+1}{\sqrt{2}\ a}\right) = \frac{-q}{a}$$

$$\Rightarrow \quad Q = -q\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right)$$

$$= -q\left(\frac{2}{2+\sqrt{2}}\right).$$

Illustration:

A particle of mass 100 gm and charge 2 μC is released from a distance of 50 cm from a fixed charge of 5μC. Find the speed of the particle when its distance from the fixed charge becomes 3 m. Neglect any other force.

Sol. *Loss of potential energy = gain in kinetic energy*

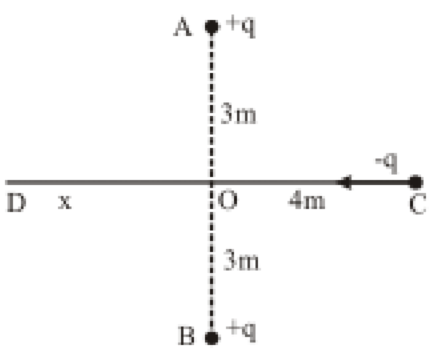
$$U_1 - U_2 = \Delta K.$$

$$kQq\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \tfrac{1}{2}\, mv^2 - 0$$

$$v = \sqrt{\frac{2kQq}{m}\left[\frac{r_2 - r_1}{r_1 r_2}\right]} = \sqrt{\frac{2\times 9\times 10^9 \times 5\times 10^{-6} \times 2\times 10^{-6} \times 2.5}{0.1\times 3\times 0.5}} = \sqrt{3}\ m/s.$$

Illustration:

Two fixed positive charges, each of magnitude $5\times 10^{-5}\,C$ are located at points A and B, separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of lien AB. The moving charge, when reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.



Sol. *The kinetic energy is lost and converted to electrostatic potential energy of the system as the negative charge goes from C to D and comes to rest at D instantaneously.*

Loss of KE = gain in PE

$$4 = U_f - U_i$$

$$4 = \left[\frac{qq}{4\pi\varepsilon_0(6)} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+x^2}}\right] - \left[\frac{qq}{4\pi\varepsilon_0(6)} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+16}}\right]$$

$$For\ U = 0;\ \frac{Qq}{a} + \frac{Qq}{\sqrt{2}\,a} + \frac{qq}{a} = 0$$

$$\Rightarrow \quad Qq\left(\frac{1}{a} + \frac{1}{\sqrt{2}\,a}\right) = \frac{-q^2}{a}$$

$$\Rightarrow \quad Q\left(\frac{\sqrt{2}+1}{\sqrt{2}\ a}\right) = \frac{-q}{a}$$

$$\Rightarrow \quad Q = -q\left(\frac{\sqrt{2}}{\sqrt{2}+1}\right)$$

$$= -q\left(\frac{2}{2+\sqrt{2}}\right).$$

Illustration:

A particle of mass 100 gm and charge 2 μC is released from a distance of 50 cm from a fixed charge of 5μC. Find the speed of the particle when its distance from the fixed charge becomes 3 m. Neglect any other force.

Sol. *Loss of potential energy = gain in kinetic energy*

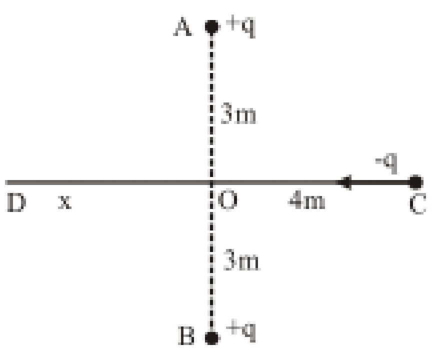
$$U_1 - U_2 = \Delta K.$$

$$kQq\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \tfrac{1}{2}\, mv^2 - 0$$

$$v = \sqrt{\frac{2kQq}{m}\left[\frac{r_2 - r_1}{r_1\, r_2}\right]} = \sqrt{\frac{2\times 9\times 10^9 \times 5\times 10^{-6} \times 2\times 10^{-6} \times 2.5}{0.1\times 3\times 0.5}} = \sqrt{3}\ m/s.$$

Illustration:

Two fixed positive charges, each of magnitude 5×10^{-5} C are located at points A and B, separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of lien AB. The moving charge, when reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.



Sol. *The kinetic energy is lost and converted to electrostatic potential energy of the system as the negative charge goes from C to D and comes to rest at D instantaneously.*

Loss of KE = gain in PE

$$4 = U_f - U_i$$

$$4 = \left[\frac{qq}{4\pi\varepsilon_0(6)} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+x^2}}\right] - \left[\frac{qq}{4\pi\varepsilon_0(6)} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+16}}\right]$$

$$4 = \frac{2q^2}{4\pi\varepsilon_0}\left[\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right]$$

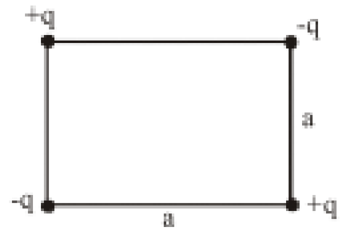
$$4 = 2(5\times10^{-5})^2(9\times10^9)\left(\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right)$$

$$4 = 9 - \frac{45}{\sqrt{9+x^2}}$$

$$x = \sqrt{72} = 8.48 \text{ m}.$$

Illustration :

What is work done by the electrostatic field when we put the four charges together, as shown in the figure. Each side of the square has a length a. Initially charges were at infinity.



Sol. $U_i = 0$ *[Where charges are separated by infinite distance]*

$$U_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{-4q^2}{a} + \frac{q^2}{\sqrt{2}a} + \frac{(-q)^2}{\sqrt{2}a}\right) \qquad \textit{[for 6 pairs of charges]}$$

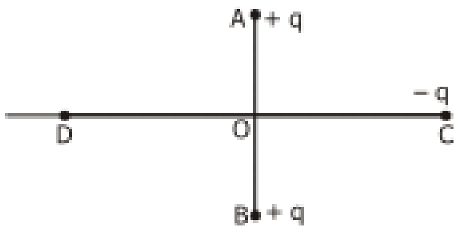
Work done by field

$$-\Delta U = U_i - U_f = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(4 - \frac{\sqrt{2}}{a}\right)$$

Practice Exercise

Q.1 Three point charges of 0.1 C each are placed at the corners of an equilateral triangle with side L = 1 m. If this system is supplied energy at the rate of 1 kW, how much time will be required to move one of the charges on to the mid point of the line joining the two others?

Q.2 Two fixed, equal, positive charges, each of magnitude 5×10^{-5} C, are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when it reaches the point C at a distance of 4 m from O, has kinetic energy of 4 J. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.



Q.3 Consider the configuration of a system of four charges each of value +q. Find the work done by external agent in changing the configuration of the system from figure (i) to fig (ii).

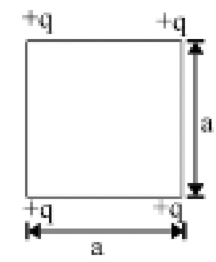


fig (i)

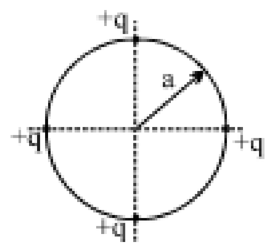


fig (ii)

Answers

Q.1 50 hrs. Q.2 8.485 m Q.3 $-\dfrac{kq^2}{a}\left(3-\sqrt{2}\right)$

Electric potential

Electric potential (represented by symbol V) due to a point charge or a charge configuration at a point is defined as

$$V = \frac{U}{q} = \text{P.E. per unit test charge.}$$

In other words it is amount of work done by external agency to bring a unit positive charge from reference point (usually taken at infinity) to given point

Electric potential due a point charge

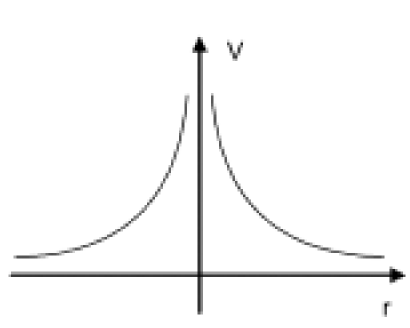
In the electric field of a point charge Q electric potential at a pont will be

$$V(\vec{r}) = \frac{\frac{kQq}{r}}{q} = \frac{kQ}{r}$$

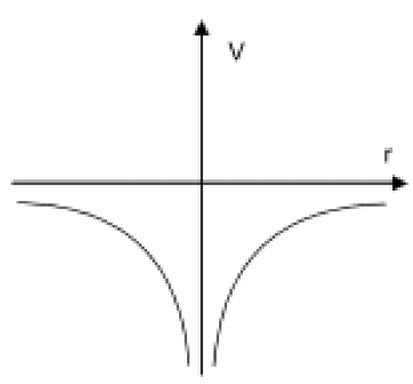
Potential is a scalar quantity.Electric potential due to a positive charge is taken to be positive and that due to a negative charge is taken to be negative. The potential at a point due to more than one charge can be found simply by adding the potentials due each charge separately.

plot of electric potential

electric potential due to positive charge



electric potential due to negative charge



Practice Exercise

Q.1 The electric potential at point A is 20V and at B is -40V. Find the work done by an external force and electrostatic force in moving an electron slowly from B to A.

Q.2 Find the work done by some external force in moving a charge q=2 μC from infinity to a point where electric potential is 10^4 V.

Q.3 Find out the points on the line joining two charges +q and -3q (kept at a distance of 1.0m) where electric potential is zero.

Q.4 An infinite number of charges each equal to q are placed along the x-axis at x = 1, x = 2, x = 8, ... and so on. Find the potential and the electric field at the point x = 0 due to this set of charges. What will be the potential and electric field if, in the above set up, the consecutive charges have opposite sign?

Q.5 Four point charges + q, –q, + q, and – q are placed respectively at the corners A, B, C and D of a square of side a. (a) Calculate the electric potential at O, the centre of the square. (b) if E and F are the midpoints of sides BC and CD respectively, what is the work that will be done in carrying an electron from : (i) O to E, and (ii) O to F? Given q = 1.0 × 10^{-6} C, a = 0.10 m and charge on an electron = – 1.6 × 10^{-19} C.

Answers

Q.1 -9.6×10^{-18} J, 9.6×10^{-18} J Q.2 2×10^{-2} J

Q.3 The potential will be zero at point P on the axis which is either 0.5 m to the left or 0.25m to the right of charge +q.

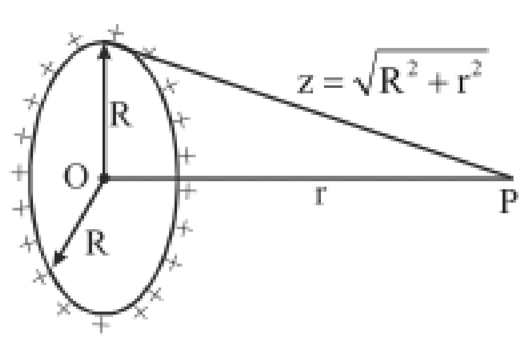
Q.4 $\frac{2q}{4\pi\epsilon_0}, \frac{1}{4\pi\epsilon_0}\left[\frac{4}{3}q\right];\left[\frac{2}{3}q\right], \frac{1}{4\pi\varepsilon_0}\left[\frac{4}{5}q\right]$

Q.5 (a) 0 (b) (i) 0 (ii) 0

Electric Potential due to a Charged Ring

A charge Q is uniformly distributed over the circumference of a ring. Let us calculate the electric potential at an axial point at a distance r from the centre of the ring.

The electric potential at P due to the charge element dq of the ring is given by



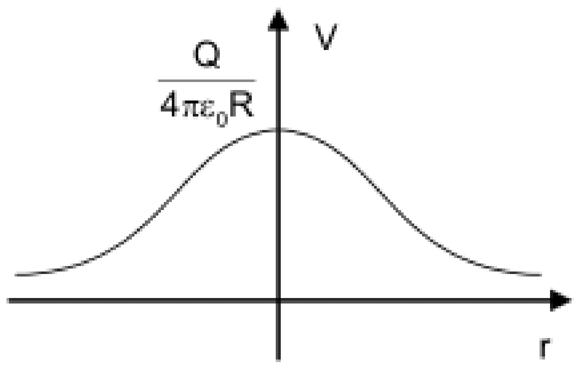
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+r^2)^{1/2}}$$

Hence, the electric potential at P due to the uniformly charged ring is given by

$$V = \int \frac{1}{4\pi\varepsilon_0} \frac{dq}{(R^2 + r^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0} \frac{1}{(R^2 + r^2)^{1/2}} \int dq$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{(R^2 + r^2)}}.$$

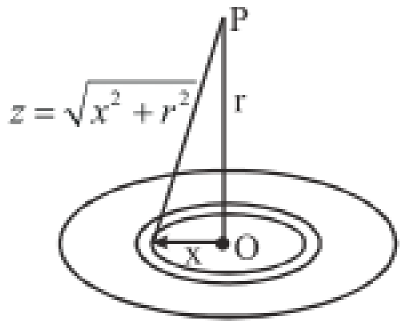
plot of electric potential



Electric Potential Due to a Charged Disc at a Point on the Axis:

A non-conducting disc of radius 'R' has a uniform surface charge density $\sigma\ C/m^2$. Let us calcuate the potential at a point on the axis of the disc at a distance 'r' from its centre. The symemtry of the disc tells us that the appropriate choice of element is a ring of radius x and thickness dx. All points on this ring are at the same distance $Z = \sqrt{x^2 + r^2}$, from the point P. The charge on the ring is $dq = \sigma dA = \sigma(2\pi x\ dx)$ and so the potential due to the ring is

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma(2\pi\ x\ dx)}{\sqrt{x^2 + r^2}}$$



Since potential is scalar

The potential due to the whole disc is given by

$$V = \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{x\ dx}{\sqrt{x^2 + r^2}}$$

$$= \frac{\sigma}{2\varepsilon_0} \left[(x^2 + r^2)^{1/2}\right]_0^R$$

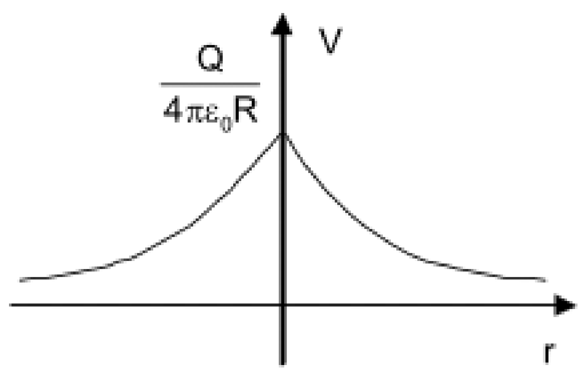
$$= \frac{\sigma}{2\varepsilon_0} \left[(R^2 + r^2)^{1/2} - r\right] \qquad \text{...(x B)}$$

Let us see this expression at large distance when $r >> R$.

$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$, where $Q = \pi r^2 \sigma$ is the total charge on the disc.

Thus, we conclude that at large distance, the potential due to the disc is the same as that of a point charge Q.

plot of electric potential



Relationship between electric field and electric Potential

Place a charge q in electric field $\vec{E}$. This field exerts a force $\vec{F}=q\vec{E}$ on the charge. If you displace the charge by by $d\vec{r}$ then field will do some work $dW=\vec{F}\cdot d\vec{r}=q\vec{E}\cdot d\vec{r}$ on the charge.

Change in electric potential energy during this displacement will be given by

$$dU=-q\,\vec{E}\cdot d\vec{r}$$

Therfore potential difference between initial and final point will be given by

$$dV=-\vec{E}\cdot\vec{dr}$$

Calculation of electric potential difference from electric field

$$dV=-\vec{E}\cdot\vec{dr}$$

$$\Rightarrow \int_{V(\vec{r}_1)}^{V(\vec{r}_2)} dV=-\int_{\vec{r}_1}^{\vec{r}_2}\vec{E}\cdot d\vec{r}$$

$$\Rightarrow V(\vec{r}_2)-V(\vec{r}_1) = \int dV=-\int_{\vec{r}_1}^{\vec{r}_2}\vec{E}\cdot d\vec{r}$$

In cartesian coordinate system

$$d\vec{r}=dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}$$

$$\Rightarrow \vec{E}\cdot d\vec{r}=E_x dx+E_y dy+E_z dz$$

$$\Rightarrow V(\vec{r}_2)-V(\vec{r}_1) = -\int_{r_1}^{r_2}\vec{E}.d\vec{r}=-\int_{x_1}^{x_2}E_x dx-\int_{y_1}^{y_2}E_y dy-\int_{z_1}^{z_2}E_z dz$$

Illuatratrion:

Electric field in a region is given by $\vec{E}=(2\hat{i}+3\hat{j}-4\hat{k})$ V/m. Find the potential difference between points (0, 0, 0) and (1,2,3) in this region

Sol. *p.d. across the points*

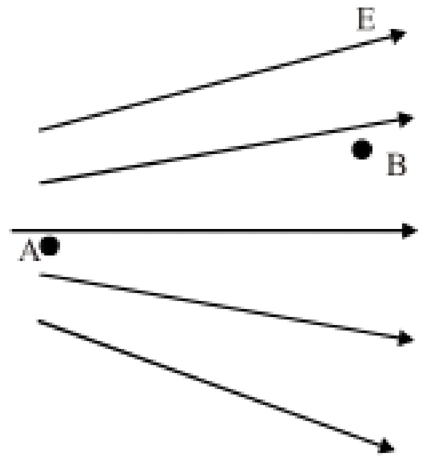
$$V=\int_{\vec r_1}^{\vec r_2}\vec E.d\vec r=-\int_{x_1}^{x_2}\vec E_x dx-\int_{y_1}^{y_2}E_y dy-\int_{z_1}^{z_2}E_z dz=-\int_0^1 2dx+\int_0^2 3dy+\int_0^3 -4dx$$

$= -2 - 6 + 12 = 4$ *volts.*

Practice Exercise

Q.1 An electric field of 20 N/C exists along the X-axis in space. Calculate the potential difference $V_A - V_B$ where the points A and B are given by, A = (0, 0); B = (4 m, 2 m) .

Q.2 In figure, two points A and B are located within a region in which there is an electric field.
(i) The sign of potential difference $\Delta V = V_B - V_A$ is
(ii) A negative charge is placed at A and then moved to B. The sign of change in potential energy of the charge-field system for this process is



Q.3 Two points A and B are 2cm apart and a uniform electric field E acts along the straight line AB directed from A to B with E=200 N/C. A particle of charge $+10^{-6}$ C is taken from A to B along AB. Calculate (a) the force on the charge (b) the potential difference $V_A - V_B$ and (c) the work done on the charge by $\vec E$.

Answers

Q.1 –80 V   Q.2 (i) -ve   (ii) +ve   Q.3 2×10^{-4} N, 4volt and 4×10^{-6} J

Calculation of electric potential from electric field

For our convenience we select potential of a point to be zero.This point is called reference point.Usually reference point is taken at infinity. In the above equation let us take $\vec r_1 = \infty$ and $\vec r_2 = \vec r$,then the potential at a point can be written as

$$V\left(\vec r\right)=-\int_{\infty}^{\vec r}\vec E\cdot d\vec r$$

Illuatratrion:

The electric field in a region is given by $\vec{E} = (A/x)^3 \hat{i}$. *Write a suitable SI unit for A. Write an expression for the potential in the region assuming the potential at infinity to be zero.*

Sol. *The SI unit of electric field is N/C or V/m. Thus,*

The unit of A is $\frac{N-m^3}{C}$ *or V-m*^{2}.

$$V(x, y, z) = -\int_{\infty}^{(x,y,z)} \vec{E}.d\vec{r}$$

$$= -\int_{\infty}^{(x,\,y,\,z)} \frac{A\,dx}{x^3} = \frac{A}{2x^2}.$$

Practice Exercise

Q.1 An electric field $\vec{E} = \vec{i}\,Ax$ exists in the space, where $A = 10\ V/m^2$. Take the potential at (10 m, 20 m) to be zero. Find the potential at the origin.

Answers

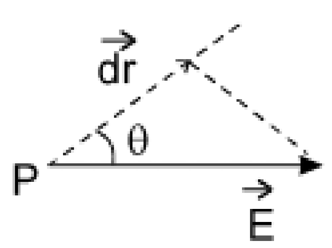
Q.1 500 V

Calculation of electric field from electric potential

$$dV = -\vec{E}\cdot d\vec{r}$$

$$dV = -(E\cos\theta)\,dr$$

$$-\frac{dV}{dr} = E\cos\theta$$



(i.e., Rate of decrease of potential)

Where E cos θ is component of field in the direction of displacement. From the above expression.

Potential decreases maximum in the direction of field, $\theta = 0°$. It is also clear that, $-\frac{dV}{dr}$ is maximum in the direction of the field, so, we may conclude that, the electric potential decreases at maximum rate in the direction of the field.

The cartesian component of electric field can be written as

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\,;$$

and an infinitesimal displacement is $d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

Thus,

$$dV = -\vec{E}.d\vec{r}$$

$$= -[E_x\ dx + E_y\ dy + E_z\ dz]$$

for a displacement in the x-direction,

$$dy = dz = 0 \text{ and so}$$

$$dV = -E_x\, dx. \text{ Therefore,}$$

$$E_x = -\left(\frac{dV}{dx}\right)_{y,\, z\, constant}$$

A derivative in which all variables except one are held constant is called partial derivative and is written with ∂ instead of d. The electric field is, therefore,

$$\left.\begin{aligned} E_x &= -\frac{\partial V}{\partial x} \\ E_y &= -\frac{\partial V}{\partial y} \\ E_z &= -\frac{\partial V}{\partial z} \end{aligned}\right\}$$

i.e., if scalar potential function is given field can be calculated taking help of the above relations as

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] \qquad |\vec{E}| = \sqrt{\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}$$

Illustration :

Potential in the x-y plane is given as $V = 5(x^2 + xy)$ volts. Find the electric field at the point (1, -2).

Sol. $E_x = -\frac{\partial V}{\partial x} = -(10x + 5y) = -10 + 10 = 0,\ E_y = -\frac{\partial V}{\partial y} = -5x = -5\ V/m$

$$\therefore \vec{E} = (-5\hat{j})V/m$$

Practice Exercise

Q.1 The electric potential existing in space is $V(x, y, z) = A(xy, + yz + zx)$. (a) Write the dimensional formula of A. (b) Find the expression for the electric field. (c) If A is 10 IS units, find the magnitude of the electric field at (1 m, 1 m, 1 m).

Answers

Q.1 (a) $MT^{-3}I^{-1}$ (b) $-A\{\vec{i}(y+z) + \vec{j}(z+x) + \vec{k}(x+y)\}$ (c) 35 N/C

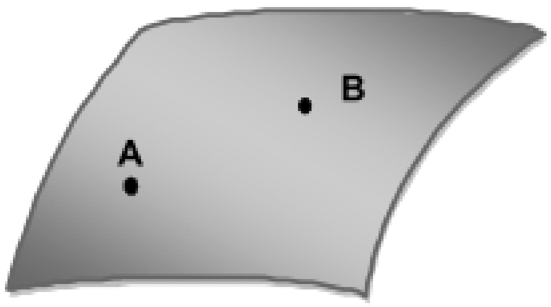
Equipotential surfaces

A locus of points in space that all have the same potential is called an equipotential surface.

Properties of equipotential surfaces :

(i) Work done in moving a charge over an equaipotential surface is zero. Thus the work done in moving a charge +q from one point A to another point B on a equipotential surface is given by;

$$W_{AB} = -q(V_B - V_A) = -q(0) = 0$$



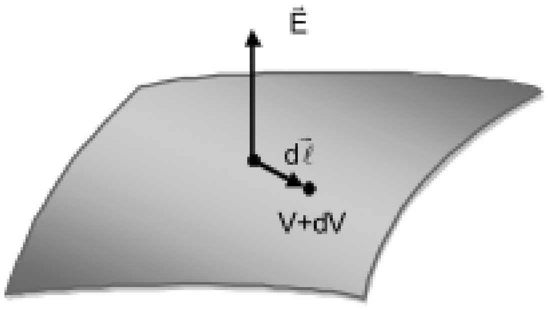
(ii) The electric field is always perpendicular to an equipotential surface. Referring to figure , we have,

$$dV = -\vec{E}\cdot d\vec{l}$$

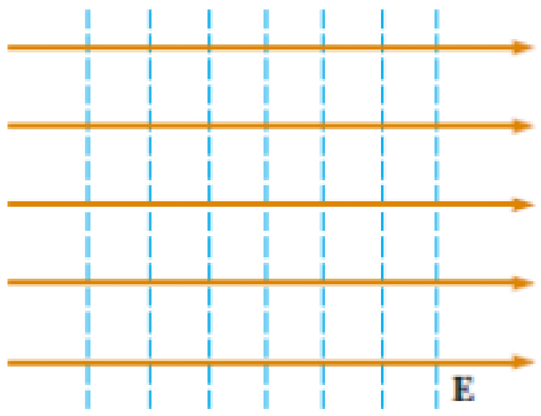
Since $dV = 0$ for an equipotential surface,

$$\therefore \quad \vec{E}\cdot d\vec{l} = 0\cdot$$

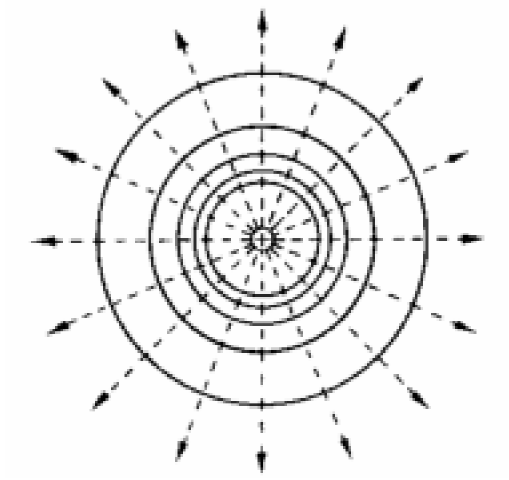
This means that $\vec{E}$ is perpendicular to $d\vec{l}$.



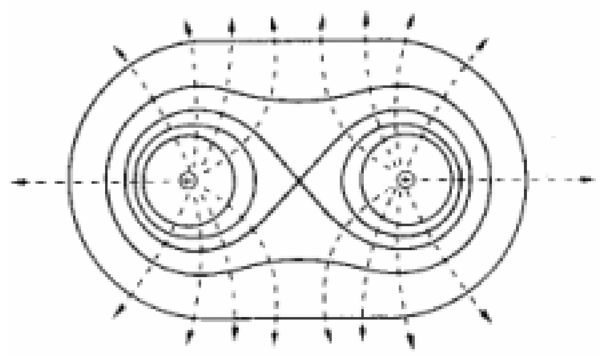
In other words, electric field (or electric lines of force) are perpendicular to the equipotential surface.



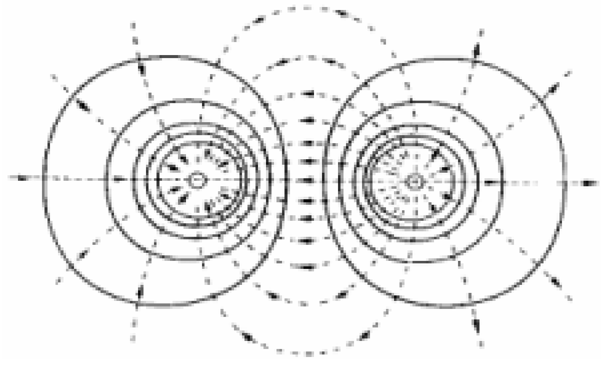
equipotential surface of uniform electric field



equipotential surface of electric field of a charge +q



equipotential surface of electric field of charges +q and +q



equipotential surface of electric field of charges +q and +q

(iii) The spacing between equipotential surfaces enables us to identify regions of strong and weak field.

We know that $E = -\frac{dV}{dr}$

For a given dV (i.e., constant dV), $E \propto 1/dr$. This means that where the equipotential surfaces are crowded, the electric field intensity is greater and vice-versa.

In the above figure equipotential surfaces having constant potrntial difference between two consecutive surfaces are shown. Near the point charge equipotential surfaces are crowded, the electric field intensity is greater

(iv) Two equipotential surfaces can be never intersect. If two equipotential surfaces could intersect, then at the point of intersection there would be two values of electric potential which is not possible.

Example:

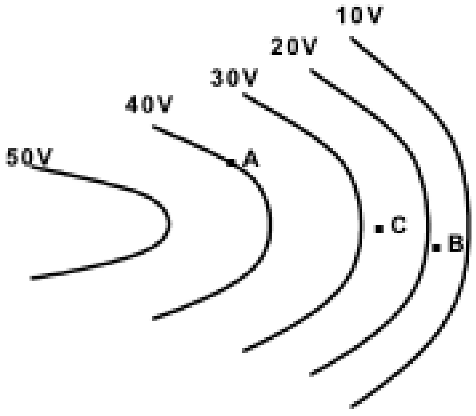
(i) In the field of a point charge, the equipotential surfaces are spheres centered on the point charge.

(ii) In a uniform electric field, the equipotential surfaces are planes which are perpendicular to the fields lines.

(iii) In the fields of an infinite line charge, the equipotential surfaces are co-axial cylinders having their axes at the line charge.

Illustration :

The adjoining figure shows the lines of constant potential in a region in which an electric field is present. The value of potentials are shown. At which of the points A, B and C is the magnitude of the electric field the greatest?



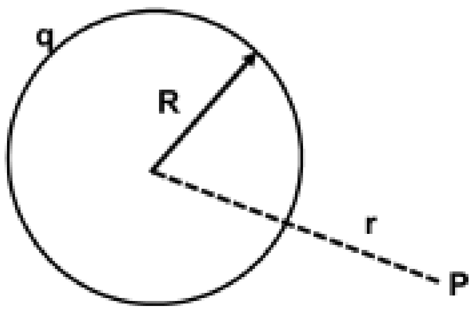
Sol. $\vec{E} = -\frac{dV}{dr}\hat{n}$

The potential difference between any two connective line $dV = V_1 - V_2 = 10v$ =constant and hence E will be maximum where the distance between the lines minimum. i.e. at B where the lines are closest.

Electric Potential due to a uniformly charged spherical shell

If the charge on the shell = q

(i) for $r > R$



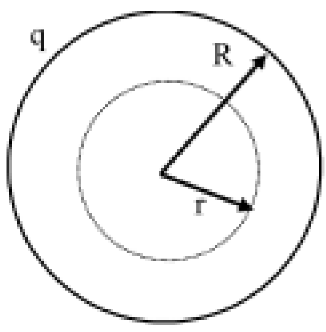
$$E = \frac{kq}{r^2}$$

$$\Rightarrow \quad V = -\int_{\infty}^{r} E dr = -\int_{\infty}^{r} \frac{kq}{r^2} dr = \frac{kq}{r}$$

(ii) for $r = R$, $V = \frac{kq}{R}$

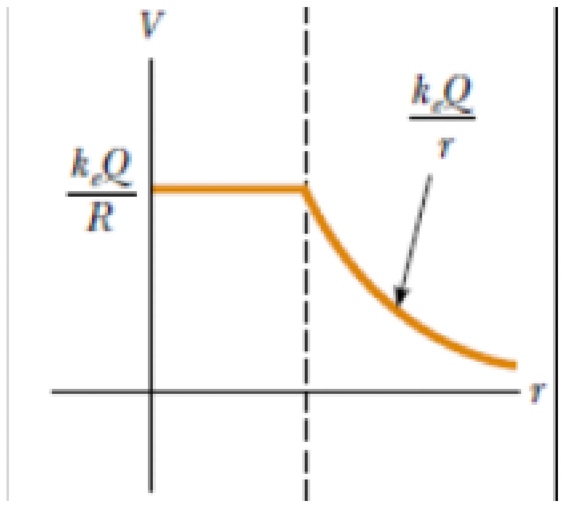
(iii) for $r < R$

$E = 0 (r<R)$



$$\Rightarrow \quad V = -\int_{\infty}^{r} E\,dr = -\left[\int_{\infty}^{R} E\,dr + \int_{R}^{r} E\,dr\right] = -\int_{\infty}^{R} \frac{kq}{r^2}\,dr - \int_{R}^{r} 0\,dr = \frac{kq}{R}$$

plot of electric potential



Electric Potential due to a uniformly charged spherical volume

If the total charge = Q
(i) for $r > R$,

Volume charge density $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

$$E(r > R) = \frac{kQ}{r^2}$$

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\,dr = \frac{kQ}{r}$$

(ii) for $r = R, V = \frac{kQ}{R}$

(iii) $r < R, E(r < R) = \frac{\rho r}{3\varepsilon_o} = \frac{Q r}{\frac{4}{3}\pi R^3 . 3\varepsilon_o}$

$$\Rightarrow \quad E(r < R) = \frac{Q r}{4\pi\varepsilon_o R^3} = \frac{kQ r}{R^3}$$

$$V = -\left[\int_{\infty}^{R} E\,dr + \int_{R}^{r} E dr\right] = -\int_{\infty}^{R} \frac{kQ}{r^2}\,dr - \int_{R}^{r} \frac{kQr}{R^3}\,dr = \frac{kQ}{2R}\left[3 - \frac{r^2}{R^2}\right]$$

plot of electric potential

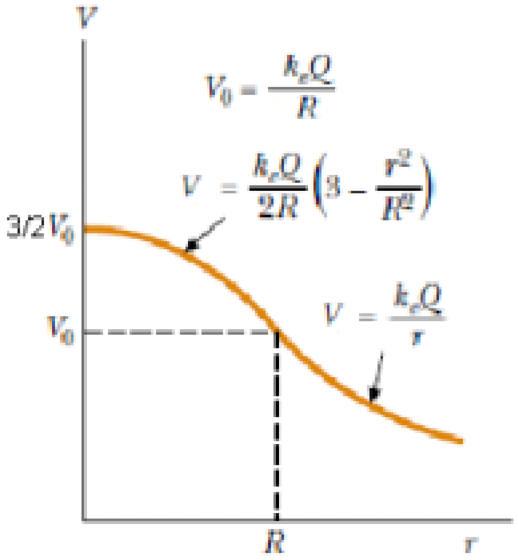


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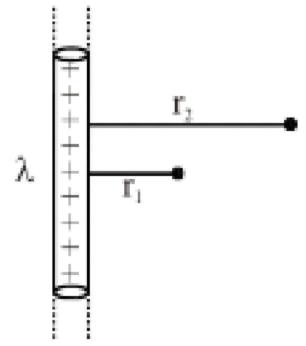
Illustration :

Find the potential difference between to points for a infinite line charge having linear charge density λ

Sol.

$$V_{12} = \int_1^2 \vec{E}.d\vec{r} = \int_{r_1}^{r_2} E\,dr$$

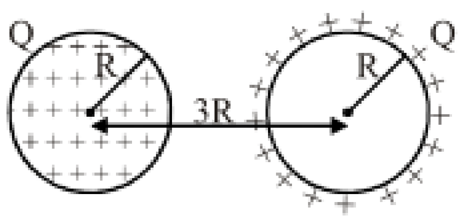
$$= \int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0} \ell n \frac{r_2}{r_1}$$



Practice Exercise

Q.1 Find the potential difference between to points for a infinite sheet having uniform surface charge density σ

Q.2 Find the work done by external agent to bring a charge q from centre of shell to centre of spherical charged volume



Q.3 A circular ring of radius R with uniform positive charge density λ per unit length is located in the y–z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P [$\sqrt{3}$ R, 0, 0] on the positive x–axis directly towards O, with initial speed v. Find the smallest (non zero) value of the speed such that the particle does not return to P.

Answers

Q.1 $V_{12} = \frac{\sigma}{2\varepsilon_0}(r_2 - r_1)$ Q.2 $\frac{Qq}{8\pi\varepsilon_0 R}$ Q.3 $\sqrt{\frac{\lambda q}{2\epsilon_0 m}}$

Expression for interaction energy of n charged particle system in terms of potential

$$U = \frac{1}{2}\sum_{i=1}^{N} V_i q_i$$

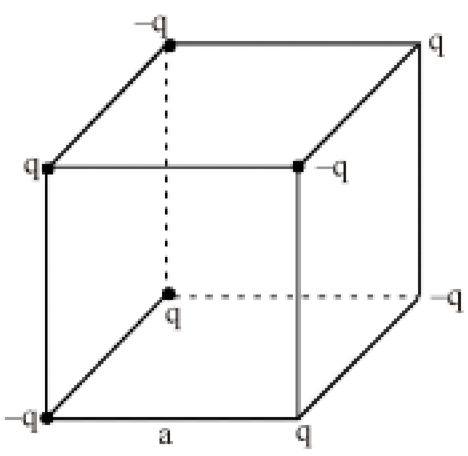
Where

q_i = charge on i-th particle and

V_i = potential at the location of i-th particle due to remaining charge particles

Illustration :

Find interaction energy of. the system



Sol. $U = \frac{1}{2}\sum_{i=1}^{N} V_i q_i = \frac{1}{2}\left[\frac{3k(-q)}{a} + \frac{3kq}{a\sqrt{2}} + \frac{k(-q)}{a\sqrt{3}}\right] \times 8q$

as because of (–q) is also same.

Energy Density of an Electric Field :

Work is done in creating any electrostatic system. This work is stored as energy in the field. The energy per unit volume or the energy density U, of the field is given as

$$U = \frac{1}{2} K\varepsilon_0 E^2$$

Illustration

A charge Q is uniformly distributed over a spherical surface of radius R. Obtain an expression for the energy of the shell ***(termed as self energy)*** .

Sol. *In this case, the electric field exists from surface of the sphere to infinity (i.e only outside the shell). Potential energy is stored in electric field with energy density.*

Electric field at a distance r is (r ≥ R)

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}\right\}^2$$

$$dU = udV = \left[\frac{\varepsilon_0}{2}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}\right)^2\right](4\pi r^2\, dr)$$

$$dU = \frac{Q^2}{8\pi\varepsilon_0} \cdot \frac{dr}{r^2}$$

$$U = \int_R^\infty dV = \frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{dr}{r^2}$$

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Practice Exercise

Q.1 Two uniformly charged spherical shells have charges q_1 and q_2 and radii r_1 and r_2. Find the potential energy of the system if their cerntres are separated by a distnace r.

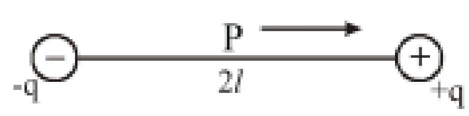
Answers

Q.1 $\frac{q_1^2}{8\pi\varepsilon_0 r_1} + \frac{q_2^2}{8\pi\varepsilon_0 r_2} + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

Electric Dipole

An arrangement of two equal and opposite charges separated by a small distance is known as an electric dipole.

Let q and –q be two charges separated by distance $2l$. The dipole moment of the dipole is :



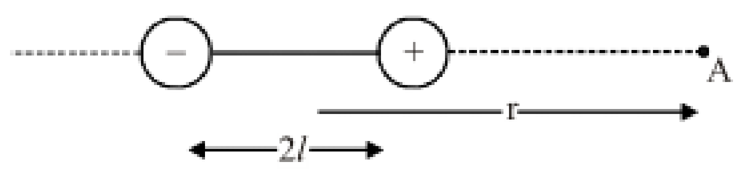
$\vec{p} = q\,2\vec{\ell}$

It is a vector quantity and is directed from –ve charge towards the +ve charge. The line joing -q to +q is known as *the axis of the dipole.*

Electric field due to a dipole

Electric field at axis : (Line joining the charges)

Electric field due to a short dipole on its axis at a point A at a distance r from dipole (ℓ <<r) :



$$E_A = \frac{q}{4\pi \epsilon_0 (r-\ell)^2} - \frac{q}{4\pi \epsilon_0 (r+\ell)^2}$$

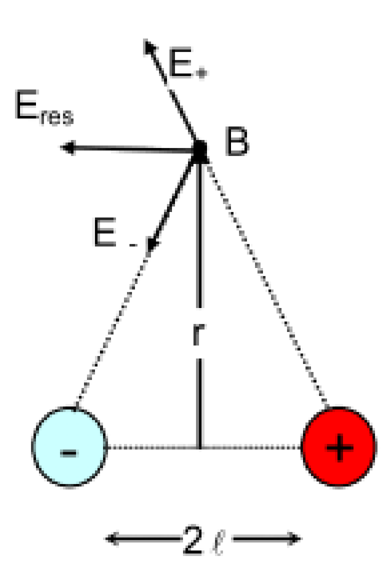
$$E_A = \frac{4q\ell r}{4\pi \epsilon_0 (r^2-\ell^2)^2}$$

after using dipole approximation $\ell << r$ we get

$$\vec{E}_A = \frac{2\vec{P}}{4\pi \epsilon_0 r^3}.$$

Electric field at equator : (Line perpendicular to axis passing through centre)

Electric field at a point distance r from the centre of the short dipole (ℓ << r)



$$E_B = 2\left[\left\{\frac{q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\right\}\cos\theta\right]$$

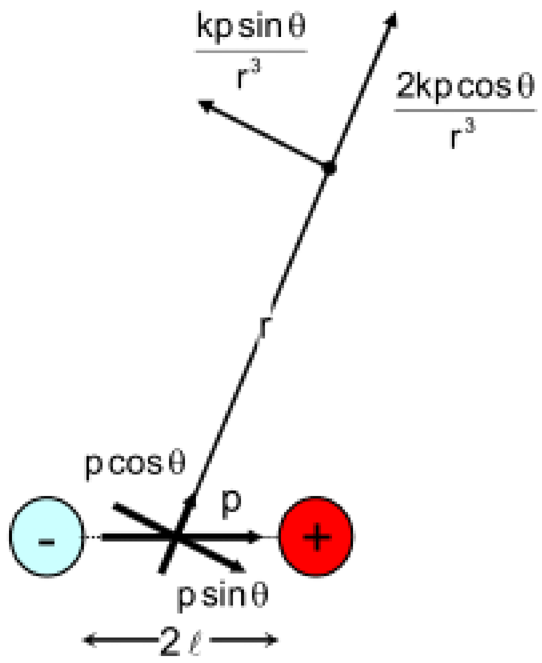
$$E_B = \frac{2q}{4\pi\in_0\left(r+\ell\right)^2}\frac{\ell}{\sqrt{\ell^2+r^2}} = \frac{2q\ell}{4\pi\in_0\left(r^2+\ell^2\right)^{3/2}}$$

after using dipole approximation $\ell << r$ we get

$$\vec{E}_B = \frac{-\vec{p}}{4\pi\in_0 r^3}$$ (-ve sign indicate that field is oppositly directed to dipole direction)

Electric field at any point A (r, θ) due to dipole

Let A be a point at a distance r from the mid-point O of the dipole. Let θ be the angle between OA and the dipole moment p. Since dipole moment is a vector so we can resolve its components $p\cos\theta$ and $p\sin\theta$ along and perpendicular to OA. Due to $p\cos\theta$ (axial point) electric field will be in the direction of $p\cos\theta$ and due to $p\sin\theta$ (equatorial position) electric field will be opposite to $p\sin\theta$



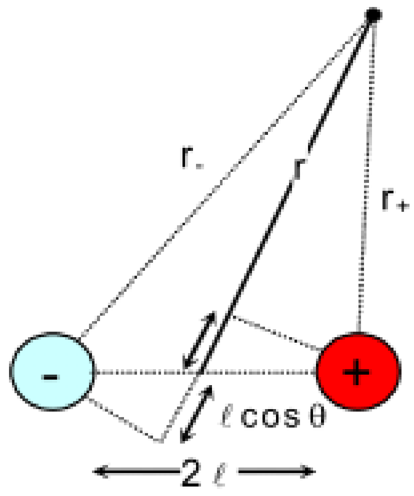
Electric field at A

$$E = \sqrt{\left(\frac{2kp\cos\theta}{r^3}\right)^2 + \left(\frac{kp\sin\theta}{r^3}\right)^2} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\& \quad \tan\alpha = \frac{E_\theta}{E_r} = \frac{\sin\theta}{2\cos\theta} = \frac{\tan\theta}{2}$$

Electric potential due to dipole

Electric field at any point A (r, θ) due to dipole :



$$V = V_+ + V_- = \frac{k(+q)}{r_+} + \frac{k(-q)}{r_-} = \frac{k(+q)}{r - l\cos\theta} + \frac{k(-q)}{r + l\cos\theta} = \frac{k(2lq)}{r^2 - l^2\cos^2\theta}$$

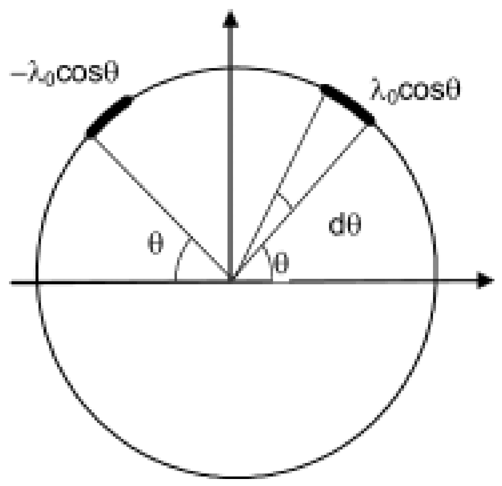
after using dipole approximation $\ell << r$ we get

$$V = \frac{kp}{r^2}$$

Illustration:

A nonunifom charge given on the ring according to the equation $\lambda = \lambda_0 \cos\theta$ (where θ is measured from x-axis).Find its dipole moment.

Sol.



In the figure shown the two symmetric elements are equal and opposite charge whose dipole moment will be

$$dp = \{(\lambda_0\cos\theta)(Rd\theta)\}\{2R\cos\theta\} = 2\lambda_0 R^2\cos^2\theta d\theta$$

$$\therefore p = \int dp = 2\lambda_0 R^2 \int_{-\pi/2}^{+\pi/2} \cos^2\theta d\theta = \pi\lambda_0 R^2$$

Illustration:

For a given dipole at a point (away from the center of dipole) intensity of the electric field is E. Charges of the dipole are brought closer such that distance between point charges is half, and magnitude of charges are also halved. Find the intensity of the field now at the same point

Sol. $P_i = 2ql$

$$P_f = 2\frac{q}{2}\frac{l}{2} = \frac{P_i}{4}$$

$$r_f = r_i \quad \theta_f = \theta_i$$

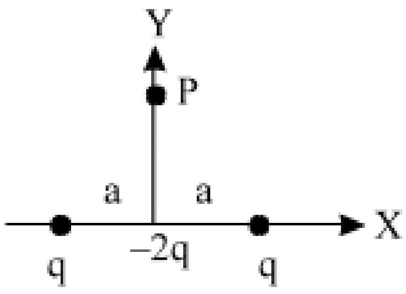
$$E = \frac{p}{4\pi \in_0 r^3}\sqrt{1+3\cos^2\theta}$$

$$\Rightarrow \quad E_f = \frac{E_i}{4}.$$

Practice Exercise

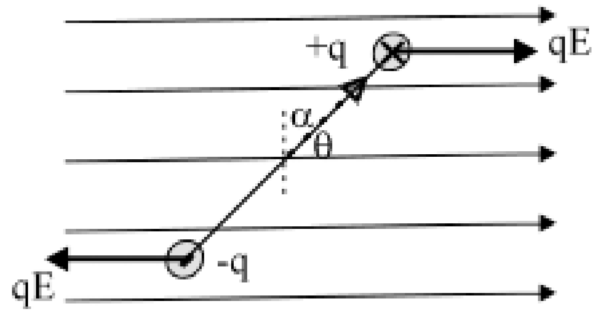
Q.1 Three point charges q, –2q and q are located along the x-axis as shown in figure. Show that the electric field at P (y >> a) along the y-xis is ,

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{3qa^2}{y^4}\hat{j}$$



Dipole in an external uniform electric field

If a dipole is placed in a uniform electric field E,
Force on the dipole is zero.
Torque on the dipole is given as



Here the net force is zero, but there can be a torque.

$$\tau = (qE)(2l\sin\theta) = pE\sin\theta$$

In vector form $\vec{\tau} = \vec{p}\times\vec{E}$

This torque has a tendency to orient dipole moment vector in the direction of field

Potential energy due to action of electric field

$$\tau = pE\cos\alpha$$

$$\therefore W_{fd} = \int dW = pE\int_0^{90-\theta}\cos\alpha\, d\alpha = pE[\sin\alpha]_0^{90-\theta} = pE\cos\theta$$

$$\therefore U_\theta = -pE\cos\theta = -\vec{p}.\vec{E} \quad (\text{taking } U = 0 \text{ at } \theta = 90^0)$$

When $\theta = 0^0$, the dipole moment p is in the direction of the field E and the dipole is in *stable equilibrium.* If it is slightly displaced, it performs oscillations.
When $\theta = 180^0$, the dipole moment p is opposite to the direction of the field E and the dipole is in *unstable equilibrium.*

Illustration:

A dipole of dipole moment P lies in a uniform electric field E such that dipole direction is along field. If dipole is rotated through 180° such that dipole direction becomes opposite to the field direction. Find the work done by the electrostatic field.

Sol.

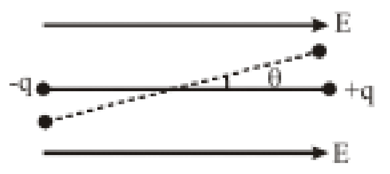
$$U_i = -\vec{P}.\vec{E} = -PE\cos O = -PE$$

$$U_f = -P.E.\cos(180^o) = PE$$

$$\textit{work done by the field} = -\Delta U = U_i - U_f = -2PE$$

Illustration:

Figure shows an electric dipole formed by two particles fixed at the ends of a light rod of length l. The mass of each particle is m and the chargers are –q and +q. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in the region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is angular simple harmonic and find its time period.



Sol.

Suppose, the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is

$$|\vec{\tau}| = |\vec{P} \times \vec{E}|$$

$$= q\,l\,E\sin\theta$$

This torque will be restoring & tend to rotate the elipole back towards the electric field. Also, for small angular displacement $\sin\theta = \theta$ so that

$$\tau = -q\,l\,E\theta$$

It the moment of inertia of the body about OA is I, the angular acceleration becomes.

$$\alpha = \frac{\tau}{I} = -\frac{qlE}{I}\theta \qquad \alpha = -\omega^2\theta$$

where $\omega^2 = \dfrac{qlE}{I}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{qlE}}$$

Now, moment of inertia of the system about the axis of rotation is

$$I = 2m\left(\frac{l}{2}\right)^2 - \frac{ml^2}{2}$$

So, $\quad T = 2\pi\sqrt{\dfrac{ml}{2qE}}$.

Practice Exercise

Q.1 A dipole of dipole moment p is placed at origin along x-axis. Another dipole of dipole moment p is kept at (0, 1, 0) along y-axis. Find the resultant potential and electric field at (1, 0, 0)

Q.2 Due to electric dipole, electric field at a distance r on axial position is $\vec{E}_1$ and at distance r on equatorial position is $\vec{E}_2$. What is the relation between $\vec{E}_2$ and $\vec{E}_1$.

Q.3 Three charges Q, Q and -2Q are placed at the three corners of an equilateral triangle of side a. Find the dipole moment of the combination.

Answers

Q.1 $v = kp(\frac{2\sqrt{2}-1}{2\sqrt{2}})$, $\vec{E} = kp[\frac{(8\sqrt{2}-3)}{4\sqrt{2}}\hat{i} + \frac{1}{4\sqrt{2}}\hat{j}]$ Q.2 $\vec{E}_1 = -2\vec{E}_2$

Q.3 $p = \sqrt{3}qa$

Dipole in an external non uniform electric field :

If a dipole is placed in non uniform electric field then let electric field at the location of positive charge will be $\vec{E}_1$ and at the location of –ve charge be $\vec{E}_2$. Usually for non uniform electric field $\vec{E}_1 + \vec{E}_2$ hence dipole will experience a net force (usually) which is equal to

$$\vec{F}_{net} = q\vec{E}_1 + (-q)\vec{E}_2$$

$$= q(\vec{E}_1 - \vec{E}_2)$$

$$= -\frac{q(\vec{E}_2 - \vec{E}_1)}{\ell} \qquad = -p\frac{\partial\vec{E}}{\partial\ell}$$

Where $\frac{\partial\vec{E}}{\partial\ell}$ = rate of change of electric field in the direction of dipole moment.

Torque of this electric field about geometric centre of dipole is still given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$

CONDUCTOR

In a metal, the outer (valence) electrons part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal. The free electrons form a kind of 'gas'; they collide with each other and with the ions, and move randomly in different directions. The positive ions made up of the nuclei and the bound electrons remain held in their fixed positions.

Inside a conductor, electrostatic field is zero

Consider a conductor, neutral or charged. There may also be an external electrostatic field. In the static situation, when there is no current inside or on the surface of the conductor, the electric field is zero everywhere inside the conductor. *This fact can be taken as the defining property of a conductor*. A conductor has free electrons. As long as electric field is not zero, the free charge carriers would experience force and drift. In the static situation, the free charges have so distributed themselves that the electric field is zero everywhere inside. ***Electrostatic field is zero inside a conductor.***

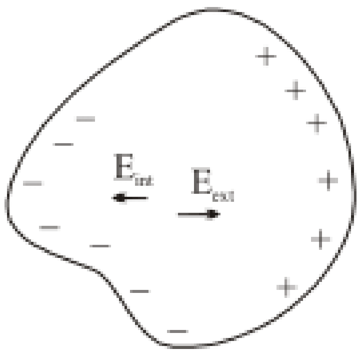
Further Explanation

What happens if conductor is placed in an exterrnal field.

Lets keep a positive charge q near a neutral or a charged conductor then the electrons goes close to q.

The redistribution of electrons inside conductor takes place which generates an internal electric field $\vec{E}_{int}$.

$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{int}$$



neutral conductor

So an e^- experience $\vec{E}_{net}$

If $E_{int} \neq E_{ext}$, then the e^- move such that they will create a stronger $\vec{E}_{int}$ which will tend to cancel E_{ext}. This constitutes current and therefore energy conservation is not valid.

$\vec{E}_{net}$ has to be O instantaneously.

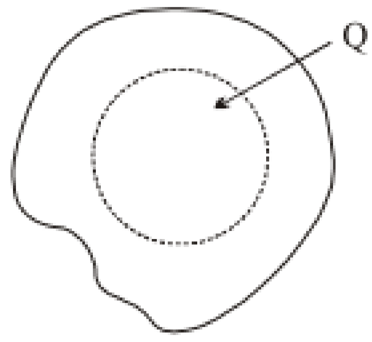
$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{int} = 0$$

The interior of a conductor can have no excess charge in the static situation

A neutral conductor has equal amounts of positive and negative charges in every small volume or surface element. When the conductor is charged, the excess charge can reside only on the surface in the static situation.

Explanation

This follows from the Gauss's law. Consider any arbitrary volume element *v* inside a conductor. If we consider any small gaussian surface inside



$$\oint \vec{E} \cdot d\vec{A} = 0 \qquad [\text{as } E = 0]$$

On the closed surface *S* bounding the volume element *v,* electrostatic field is zero. Thus the total electric flux through *S* is zero. Hence, by Gauss's law, there is no net charge enclosed by *S.*

$\Rightarrow \quad q_{enclosed} = 0$

Since the surface *S* can be made as small as you like, i.e., the volume *v* can be made vanishingly small. This means *there is no net charge at any point inside the conductor*, and *any excess charge must reside at the surface*.

> **Note : Thus Solid " conducting" sphere is same as a shell.**
> *Note : You may emphasise again but q_{in} = 0 does not imply that E = 0 from gauss law.*

At the surface of a charged conductor, electrostatic field must be normal to the surface at every point

If **E** were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation, therefore, **E** should have no tangential component. Thus *electrostatic field at the surface of a charged conductor must be normal to the surface at every point*. (For a conductor without any surface charge density, field is zero even at the surface.)

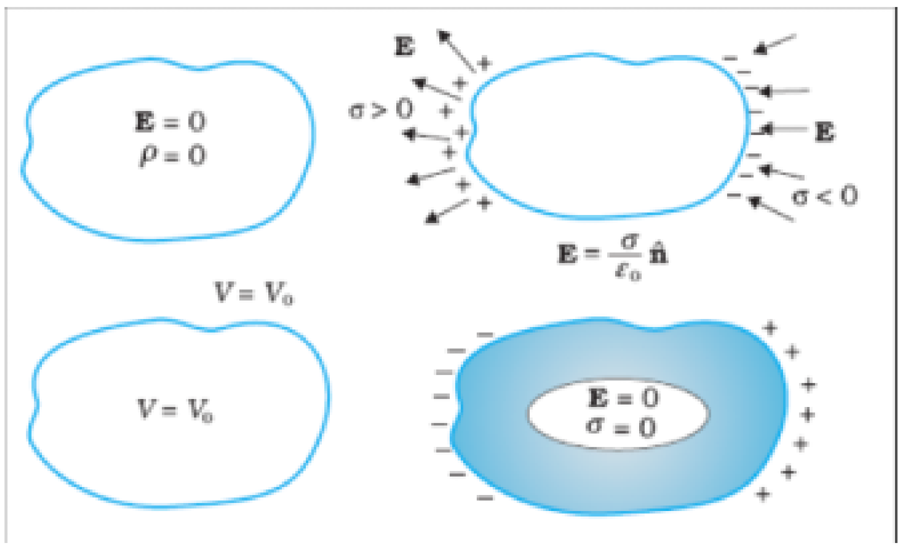
Cavity inside a conductor containing no charge inside it

Cavity is a place surrounded from all sides by the conductor such that without touching the body we can't reach cavity.

Electrostatic shielding

Consider a conductor with a cavity, with no charges inside the cavity. A remarkable result is that the electric field inside the cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed. We have proved a simple case of this result already: the electric field inside a charged spherical shell is zero. The proof of the result for the shell makes use ofthe spherical symmetry of the shell. But the vanishing of electric field in the (charge-free) cavity of a conductor is, as mentioned above, is a general result. A related result is that even if the conductor is charged or charges are induced on a neutral conductor by an external field, all charges reside only on the outer surface of a conductor with cavity.

Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence: thefield inside the cavity is always zero. This is known as electrostatic shielding. The effect can be made use of in protecting sensitive instruments from outside electrical influence. Figure gives a summary of the important electrostatic propertiesof a conductor.



If there is no charge present inside the cavity then field inside it is zero.

(This does not mean that we are implying "no net charge". If there is some charge present here and there in cavity such that there sum total is zero then the following disscussion will not be valid)

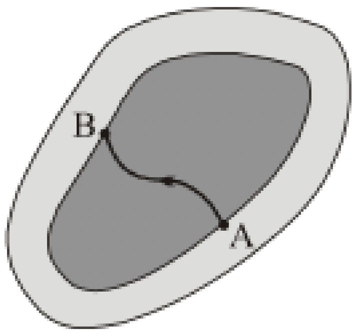
Let us consider a gauesion surface just near to cavity.

$$\therefore \quad \oint \vec{E}\cdot d\vec{A} = 0$$

$$\Rightarrow \quad q_{in} = 0$$

From this we don't have proved that no charge resides on cavity but have proved that net charge on cavity surface is O.

Now suppose a conductor of arbitrary shape contains a cavity as shown in figure.



Let us assume that no charges are inside the cavity. In this case, the electric field inside the cavity must be zero regardless of the charge distribution on the outside surface of the conductor. Furthermore, the field in the cavity is zero even if an electric field exists outside the conductor.

To prove this point, we use the fact that every point on the conductor is at the same electric potential, and therefore any two points A and B on the surface of the cavity must be at the same potential. Now imagine that a field E exists in the cavity and evaluate the potential difference $V_B - V_A$ defined by equation.

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B E \cdot ds$$

$$V_B - V_A = -\int_A^B E \cdot ds$$

Because $V_B - V_A = 0$, the integral of $E \cdot ds$ must be zero for all paths between any two points A and B on the conductor. The only way that this can be true for all paths is if E is zero everywhere in the cavity.

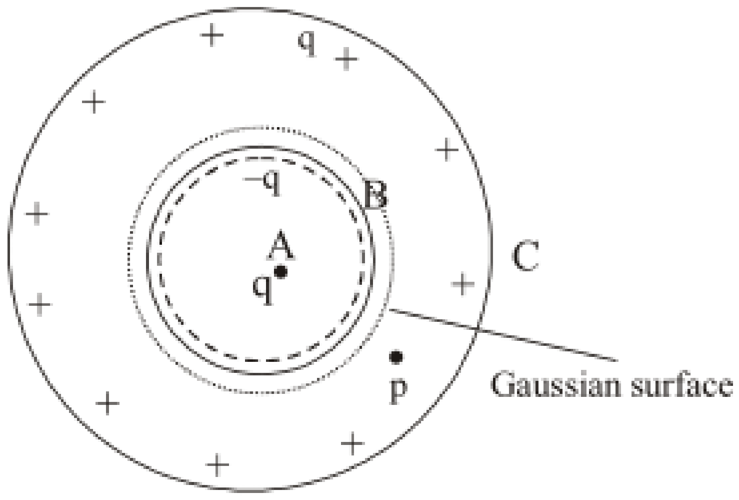
Thus, we conclude that a cavity surrounded by conducting walls is a field-free region as along no charges are inside the cavity.

Cavity inside a conductor containing charge inside it

1. Equal and opposite charge is induced on the inner surface of cavity.

Figure shows a conductor with a cavity inside it. A charge q is placed inside cavity.
In electrostatic equilibrium charge distribution will be as shown in figure

charge on inner surface of cavity is – q since material of conductor is initially neutral, equal and opposite charge appears on outer surface.



Let us consider a gaussian surface just outside cavity inside material of conductor. As $\vec{E}$ in material of conductor is zero.

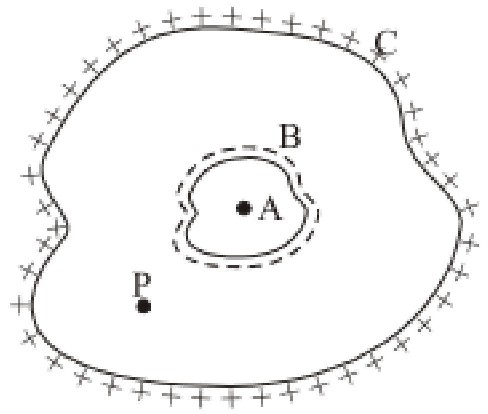
$$\oint E \cdot dA = 0 \qquad \therefore \qquad q_{enclosed} = 0$$

Thus equal and opposite charge is induced on the inner surface of cavity.

2. The electric field due to charges on the inner surface of conductor nullifies the electric field of point charge all the points outside the inner surface

3. The electric field due to charges on the outer surface of conductor is zero for all the points inside the outer surface seperately

Consider a charged conductor having charge $+q_1$ and Q is kept inside the cavity. Lets call charge Q inside cavity as A, the induced charge -Q on the surface of the cavity as B and the charge on the surface of the conductor $Q + q_1$ at C.

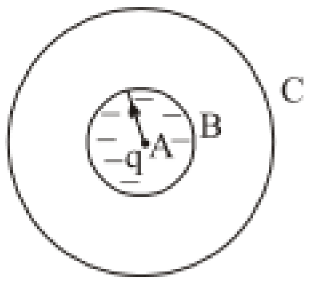


Now field inside the conductor is, $\vec{E}_{net}$ and $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ are fields due to charge A,B and C inside the conductor.

and, $\vec{E}_{P,net} = \vec{E}_A + \vec{E}_B + \vec{E}_C = 0$

Now the electric field due to charges on the outer surface of conductor is zero for all the points inside the conductor seperately and the $\vec{E}_B + \vec{E}_A$ is zero seperately.

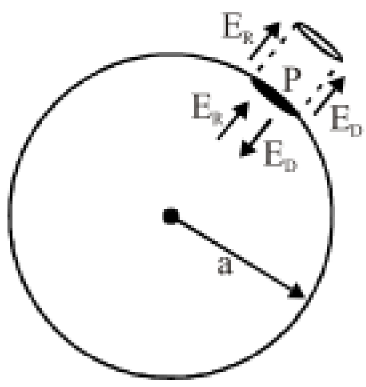
Special case : When spherical cavity in present inside spherical conductor and charge is at the centre. Then field inside the cavity is just due to charge A only as new field lines are already $\perp$ to B and E due to B = 0. [Symmetrical distribution]



Note : When if we displace the charge q inside the cavity, it will only affect the charge distribution at B the charge distribution at C will remain unaffected.

Electrostatic Pressure

If a small piece of radius b is removed from a charged spherical shell of radius a (>>b), calculate electric intensity at the midpoint of the aperture, assuming the density of charge to be σ.



Consider the shell to be made up of a disc of radius b and the remainder. If E_D and E_R are the intensities due to disc and the remainder respectively at P, then for a charged spherical shell (or conductor)

$$E_{out} = \frac{\sigma}{\varepsilon_0} \text{ and } E_{in} = 0$$

Now as for outside the shell both E_D and E_R will be directed outwards while inside E_R will be outwards while E_D inwards so that :

$$E_{out} = E_R + E_D \text{ and } E_{in} = E_R - E_D \qquad \text{........ (2)}$$

And hence equating Eqs. (1) and (2),

$$E_R + E_D = \frac{\sigma}{\varepsilon_0} \text{ and } E_R - E_D = 0$$

Solving these for E_R and E_D :

$$E_R = E_D = \frac{\sigma}{2\varepsilon_0}$$

i.e., field at the aperture will be $(\sigma/2\varepsilon_0)$ directed outwards.

As intensity on the disc (element) the to remainder is $(\sigma/2\varepsilon_0)$, electric force on it will be,

$$dF = dq\,E = (\sigma\, ds)\left[\frac{\sigma}{2\varepsilon_0}\right] = \left[\frac{\sigma^2}{2\varepsilon_0}\right] ds$$

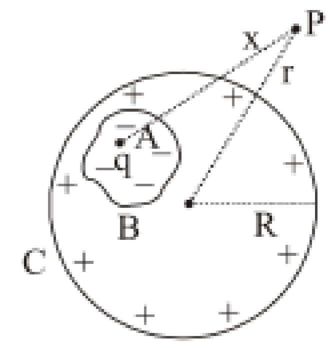
So force per unit area on a charged conductor due to its own charge

$$\frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{[as for a conductor } E = \frac{\sigma}{\varepsilon_0}\text{]}$$

This force is called or electrostatic pressure.]

Illustration :

Find electric field at P, the conductor is neutral. and cavityis having a charge q inside the cavity.

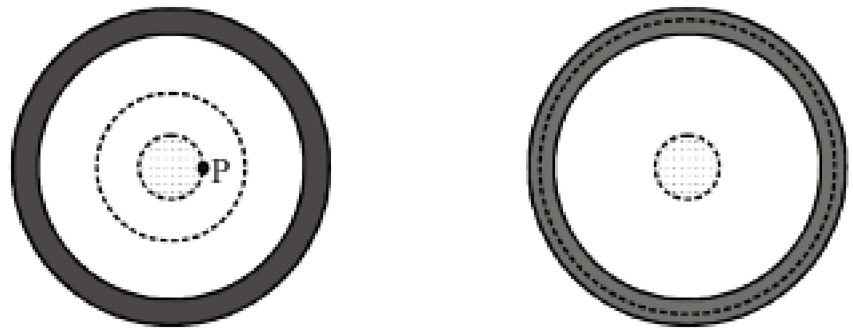


Sol. $\therefore \quad E_p = \dfrac{kQ}{r^2} \qquad [as \quad E_A + E_B = O]$

Illustration :

A charge of 4×10^{-8} C is distributed uniformly on the surface of a sphere of radius 1 cm. It is covered by a concentric, hollow conducting sphere of radius 5 cm. (a) Find the electric field at a point 2 cm away from the centre. (b) A charge of 6×10^{-8} C is placed on the hollow sphere. Find the surface charge density on the outer surface of the hollow sphere.

Sol.



(a) (b)

(a) Let us consider figure. Suppose, we have to find the field at the point P. Draw a concentric spherical surface through P. All the points on this surface are equivalent and by symmetry,the field at all these points will be equal in magnitude and radial in direction.

The flux through this surface $= \oint \vec{E}.d\vec{S}$

$$= \oint E.dS = E \oint dS$$

$$= 4\pi x^2 E.$$

Where $\qquad x = 2\ cm = 2 \times 10^{-12}\ m.$

From Gauss's law, this flux is equal to the charge q contained inside the surface divided by ε_0. Thus,

$$4\pi x^2 E = q/\varepsilon_0$$

or $\qquad E = \dfrac{q}{4\pi\varepsilon_0 x^2}$

$$= \left(9\times10^{9}\,\frac{\mathrm{N-m^2}}{\mathrm{C^2}}\right)\times\frac{4\times10^{-8}\,\mathrm{C}}{4\times10^{-4}\,\mathrm{m^2}}$$

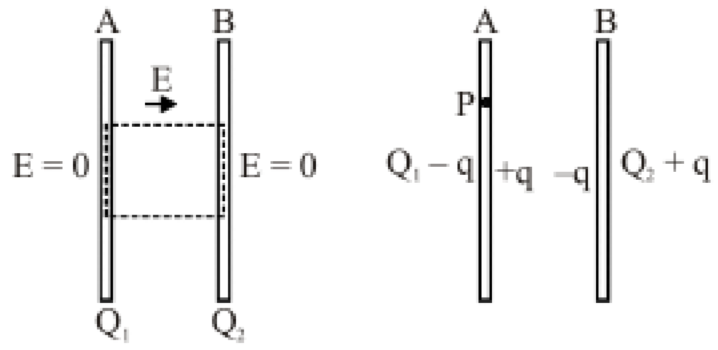
$= 9\times10^{5}$ *N/C.*

(b) See figure. Take a Gaussian surface through the material of the hollow sphere. As the electric field in a conducting material is zero the flux $\oint \vec{E}.d\vec{S}$ *through this Gaussian surface is zero. Using Gauss's law the total charge enclosed must be zero. Hence, the charge on the inner surface of the hollow sphere is* -4×10^{-8} *C. But the total charge given to this hollow sphere is* 6×10^{-8} *C. Hence, the charge on the outer surface will be* 10×10^{-8} *C.*

Illustration :

Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 *and B a charge* Q_2 *Find the distribution of charge on the four surface.*

Sol. *Consider a Gaussion surface as shown in figure. Two faces of this closed surface lie completely indie the conductor where the lectric field is zero. The flux through these face is, therfore, zero. The flux through these is, therfore zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total fulx of the electric field through the cloesed surface is, therefore, zero. From Gauss's law, the total charge inside thsi closed surface should be zero. The charge on the inner surface of A should be equal and oposite to that on the inner surface of B.*



The distribution should be like the one shown in figure. To find the value of q , consider the field at a point P inside the plate A. Suppose, the surface are of the plate (one side) is A. Using the equation $E = \sigma/(2\varepsilon_0)$*, the electric field at P*

due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ *(right)*

due to the charge $+ q = \dfrac{q}{2A\varepsilon_0}$ *(left)*

due to the charge $- q = \dfrac{q}{2A\varepsilon_0}$ *(right)*

and due to the charge $Q_2 + q = \dfrac{Q_2 + q}{2A\varepsilon_0}$ *(left)*

The net electric field at P due the all the four charged surface is (in the right direction)

$$\frac{Q_2 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}$$

As the point P is inside the conductor, this field should be zero. Hence,

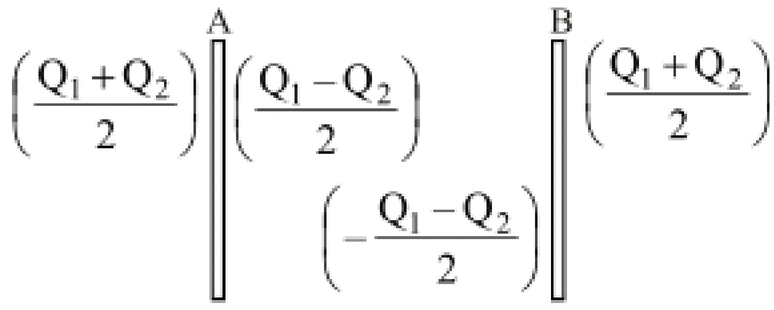
$$Q_1 - q - Q_2 - q = 0$$

or $$q = \frac{Q_1 - Q_2}{2} \qquad \text{...(i)}$$

Thus, $$Q_1 - q = \frac{Q_1 + Q_2}{2} \qquad \text{...(ii)}$$

and $$Q_2 + q = \frac{Q_1 + Q_2}{2}$$

Using these equations, the distribution shown in the figure can be redrawn as in figure



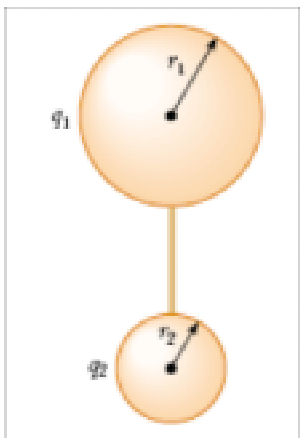
Important :

(i) Thus facing surfaces have equal and opposite charges.

(ii) Using this also prove that outer faces of the two last plates have equal charges.

Illustration:

Two spherical conductors of radii r_1 and r_2 are separated by a distance much greater than the radius of ither sphere. The spheres are connected by a conducting wire, as shown in figure . The charges on the spheres in equilibrium are q_1 and q_2, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.



Sol. *Because the spheres are connected by a conducting wire, they must both be at the same electric potential:*

$$V = \frac{kq_1}{r_1} = \frac{kq_2}{r_2}$$

Therefore, the ratio of charges is

$$\frac{q_1}{q_2} = \frac{r_1}{r_2} \quad \text{.... (1)}$$

Because the spheres are very far apart and their surfaces uniformly charged, we can express the magnitude of the electric fields at their surfaces as

$$E_1 = \frac{kq_1}{r_1^2} \text{ and } E_2 = \frac{kq_2}{r_2^2}$$

Taking the ratio of these two fields and making use of Equation (1), we find that

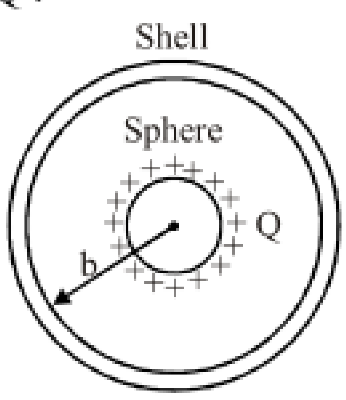
$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

Hence, the field is more intense in the vicinity of the smaller sphere even though the electric potentials of both spheres are the same.

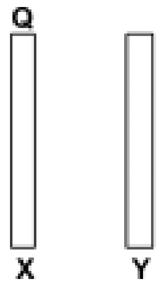
Practice Exercise

Q.1 A hollow, uncharged spherical conductor has inner radius a and outer radius b. A positive point charge +q is in the cavity at the centre of the sphere. Make the graph E and potential V(r) everywhere, assuming that $V = 0$ at $r = \infty$.

Q.2 A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of hollow shell be V. What will be the new potential difference between the same two surfaces if the shell given a charge –3Q ?

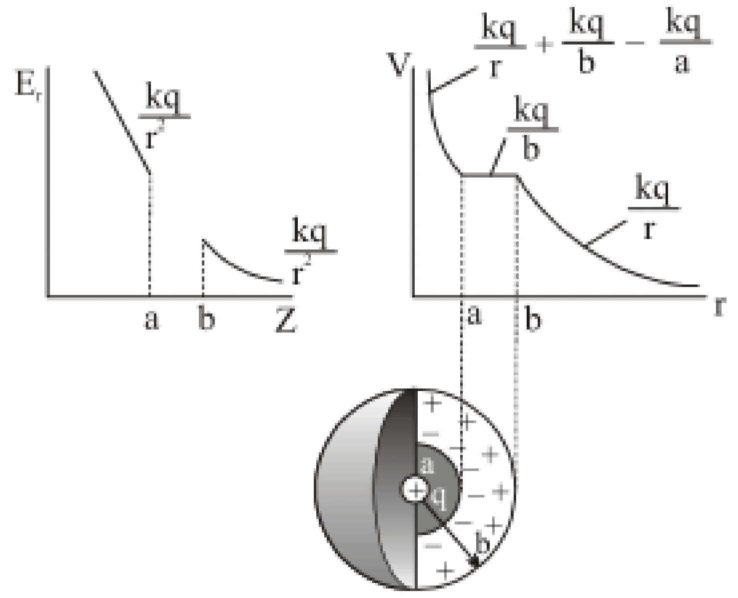


Q.3 Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in the figure. The plate X is given a charge Q whereas the other is neutral. Find (a) the surface charge density at the inner surface of the plate X, (b) the electric field at a point to the left of the plates, (c) the electric field at a point in between the plates and (d) the electric filed at a point to the right of the plates.



Answers

Q.1



Q.2 V

Q.3 (a) $\frac{Q}{2A}$ (b) $\frac{Q}{2A\varepsilon_0}$ towards left (c) $\frac{Q}{2A\varepsilon_0}$ towards right (d) $\frac{Q}{2A\varepsilon_0}$ towards right

Solved Example

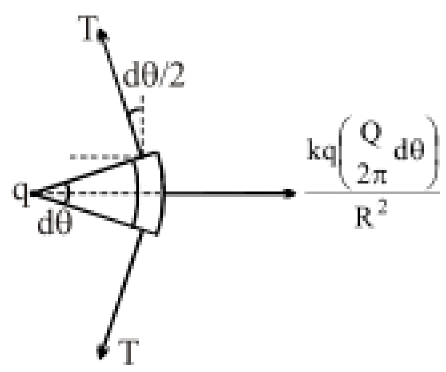
Q.1 A ring has charge Q and radius R. If a charge q is placed at its center, then calculate the increase in tension in the ring.

Sol. Let us take an elementry part subtending an angle do at centre.

Charge on the elementry part will be

$$dQ = \frac{Q}{2\pi R}(Rd\theta) = \frac{Q}{2\pi}d\theta$$

Free body diagram will be

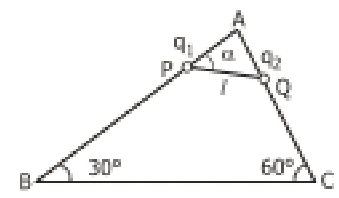


For qeuilibrium $2T\sin\frac{d\theta}{2} = \frac{kQq}{2\pi R^2}d\theta$

$$\Rightarrow 2T\frac{d\theta}{2} = \frac{kQq}{2\pi R^2}d\theta \qquad [\because \text{ for small } \theta, \sin\theta \approx \theta]$$

$$\therefore T = \frac{kQq}{2\pi R^2} \quad \text{Ans.}$$

Q.2 A rigid insulated wire frame in the form of a right-angled triangle ABC, is set in a vertical plane as shown in **Fig**. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires



Considering the case when the beads are stationary, determine

(a) the angle α (b) the tension in the cord and (c) the normal reaction on the beads.

If the cord is now cut what are the value of the charges for which the beads continue to remain stationary?

Sol. Tension and electrostatic force are in opposite direction and along the string. Now each bead is in equilibrium under three concurrent forces.

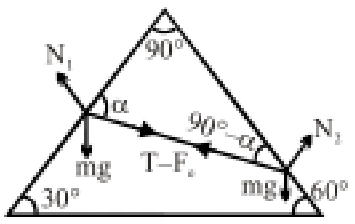
(i) Normal reaction (N)

(ii) Weight (mg) and

(iii) $T - F_e$

where $F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_2}{l^2}$

Applying Lami's theorem for both beads.



$$\frac{N_1}{\sin(120-\alpha)} = \frac{mg}{\cos\alpha} = \frac{T-F_e}{\cos 60°} \qquad \text{..... (i)}$$

$$\frac{N_2}{\sin(60°+\alpha)} = \frac{mg}{\sin\alpha} = \frac{T-F_e}{\cos 30°} \qquad \text{......(ii)}$$

Dividing equation (i) by (ii), we have

$$\tan\alpha = \frac{\cos 30°}{\cos 60°} = \sqrt{3} \qquad \therefore \alpha = 60°$$

$$T = F_e + mg = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1q_2}{l^2} + mg \qquad \text{.......(iii)}$$

$N_1 = \sqrt{3}\,mg$ and $N_2 = mg$

From Eq. (iii) $T = 0$ when string is cut

or $\quad q_1q_2 = -(4\pi\varepsilon_0)mg\,l^2$

Q.3 A positive charge Q is distributed uniformly over the circumference of a thin circular ring of radius a metres. Calculate the force on a positive point charge q

(i) if it was placed at a distance x from the centre of the ring along its axis.

(ii) if it was placed at the centre of ring.

(iii) At what point on the axis will the force be maximum.

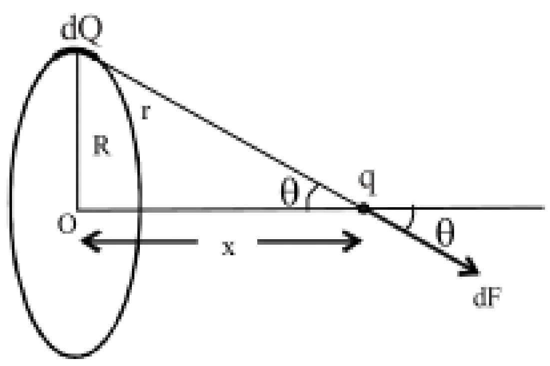
(iv) Draw a qualitative graph of force v/s the distance of the charge from the centre of the ring.

(v) Calculate the time period of that S.H.M for small oscillation if q is replaced by – q and ring is fixed.

Sol. (i) Take an elementry charge dQ on the ring

Force due to this elementary charge on the point charge will be $dF = \dfrac{qdQ}{4\pi\epsilon_0 r^2}$

This force can be resolved into two rectangular components one along the axis ($dF\cos\theta$) and other along perpendicular to the axis ($dF\sin\theta$). If we take another elementary charge diametrically opposite to dQ we observe that the vector sum of $dF\sin\theta$ for these two charge become zero. Hence vector sum of $dF\sin\theta$ for all the charges result to zero. So the resultant force on the charged particle will be only sum of $dF\cos\theta$. Therefore total force on the point charge due to entire ring will be given by



$$F = \int dF\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\ell^3}\int dq = -\frac{1}{4\pi\varepsilon_0}\frac{Qqx}{(R^2+x^2)^{3/2}}$$

(ii) for centre $x = 0 \qquad \Rightarrow F = 0$

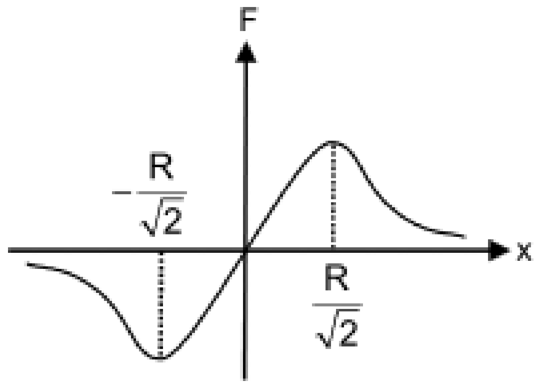
(iii) $F = \dfrac{Qq}{4\pi\epsilon_0}\dfrac{x}{(R^2+x^2)^{3/2}}$

$$\frac{dF}{dx} = \frac{Qq}{4\pi\epsilon_0}\left[\frac{(R^2+x^2)^{3/2}\frac{d}{dx}x - x\frac{d}{dx}(R^2+x^2)^{3/2}}{\left\{(R^2+x^2)^{3/2}\right\}^2}\right] = \frac{Qq}{4\pi\epsilon_0}\left[\frac{(R^2+x^2)^{3/2}(1) - x\frac{3}{2}(R^2+x^2)^{1/2}(2x)}{(R^2+x^2)^3}\right]$$

$$= \left(\frac{Qq\sqrt{R^2 + x^2}}{4\pi \in_0} \right)(R^2 - 2x^2)$$

For force to be maximum or minimum

$$\frac{dF}{dx} = 0 \Rightarrow R^2 - 2x^2 = 0 \Rightarrow x = \pm\frac{R}{\sqrt{2}}$$

(iv) 

(v) $$F = \frac{Q(-q)x}{4\pi \in_0 (R^2 + x^2)^{3/2}}$$

if $x << R$ then

$$F = -\frac{Qqx}{4\pi\varepsilon_0 R^3}$$

Now $$ma = -\frac{Qqx}{4\pi\varepsilon_0 R^3}$$

$$\frac{d^2x}{dt^2} = -\frac{Qq}{4\pi\varepsilon_0 R^3 m}x = -kx$$

$\therefore$ $$k = \frac{Qq}{4\pi\varepsilon_0 R^3 m}$$

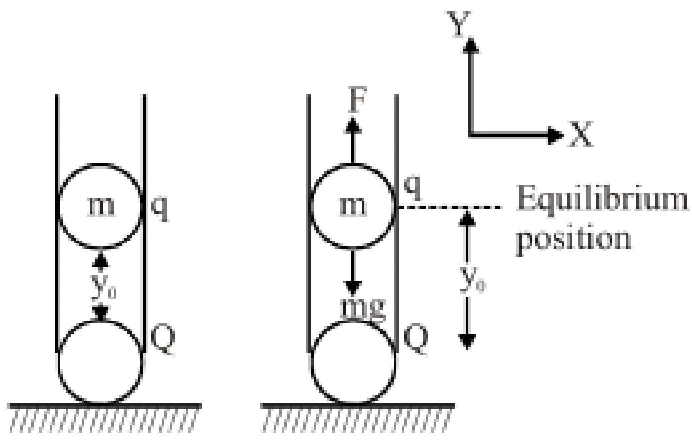
$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{qQ}}$$

Q.4 A small point mass m has a charge q, which is constrained to move inside a narrow frictionless cylinder. At the base of the cylinder is a point mass of charge Q having the same sign as q. Show that if he mass m is displaced by a small amount from its equilibrium position and released, it will exhibit simple harmonic motion with angular frequency $\omega = (2g/y_0)^{1/2}$ where y_0 is the equilibrium position of charge q.

Sol. In equilibrium position, gravitational force is balanced by coulombic repulsive force

$$mg = \frac{Qq}{4\pi\varepsilon_0 y_0^2}$$

If charge q is displaced in positive y-direction, such that $y << y_0$ from Newton's second law,



$$\frac{Qq}{4\pi\varepsilon_0(y_0+y)^2} - mg = ma$$

$$\frac{Qq}{4\pi\varepsilon_0 y_0^2}\left[\frac{1}{(1+y/y_0)^2}\right] - mg = ma$$

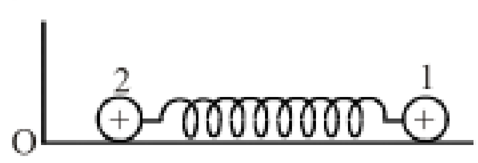
or $$mg\left[1-\frac{2y}{y_0}\right] - mg = ma$$

or $$a = -\frac{2gy}{y_0}$$

$$\frac{d^2y}{dt^2} + \frac{2g}{y_0} = 0$$

Which is equation for SHM with $\omega = \sqrt{\frac{2g}{y_0}}$

Q.5 Two small identical balls lying on a horizontal plane are connected by a weightless spring. One ball (ball 2) is fixed at O and the other (ball 1) is free. The balls are changed identically as a result of which the spring length increases η =2 times Determine the change in frequency.



Sol. When the balls are uncharged, $\upsilon_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where k is force constant of the spring and m = mass of the oscillating ball (ball 1). When charged we have in the equilibrium position of ball 1,

$$\frac{1}{4\pi\varepsilon_0(\eta l)^2} = k\,(nl - l) = kl\,(\eta - 1)$$

$$\Rightarrow \quad l^3 = \frac{q^2}{4\pi\varepsilon_0\eta^2(\eta-1)k}$$

When the ball 1 is displaced by a small distance from the equilibrium position to the right, the unbalanced force the right is given by

Resultant force to the right

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\eta l - x)^2} - k(\eta l + x - l)$$

From Newton's law, we have

$$m\frac{d^2x}{dt^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2 l^2}\left(1+\frac{x}{\eta l}\right)^{-2} - kl\,(\eta-1) - kx$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2 l^2}\left(1-\frac{2x}{\eta l}\right) - kl\,(\eta-1) - kx$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2 l^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\eta^2 l^2}\cdot\frac{2x}{\eta l} - kl(\eta-1) - kx$$

$$= -\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q^2}{\eta^3 l^3} + k\right)x$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{\eta^3\dfrac{q^2}{4\pi\varepsilon_0\eta^2(\eta-1)k}} + k\right)x$$

$$m\frac{d^2x}{dt^2} = -\left(\frac{2(\eta-1)}{\eta}k + k\right)x = \frac{3\eta-2}{\eta}kx$$

$$\frac{d^2x}{dt^2} = -\frac{3\eta-2}{\eta}\frac{k}{m}x$$

or $\quad \omega^2 = \dfrac{3\eta-2}{\eta}\dfrac{k}{m}$

$\Rightarrow \quad \upsilon = \dfrac{1}{2\pi}\sqrt{\dfrac{3\eta-2}{\eta}\dfrac{k}{m}}$

or $\quad \dfrac{\upsilon}{\upsilon_0} = \sqrt{\dfrac{3\eta-2}{\eta}}$

Thus the frequency is increased $\sqrt{\dfrac{3\eta-2}{\eta}}$ times.

Hence $h = 2$ and so frequency increases $\sqrt{2}$ times.

Q.6 Four point charges (each having charge q) are placed at corners of a square of side a. Find the intensity of the field at the (a) Centre of the square (b) Mid point of any side.

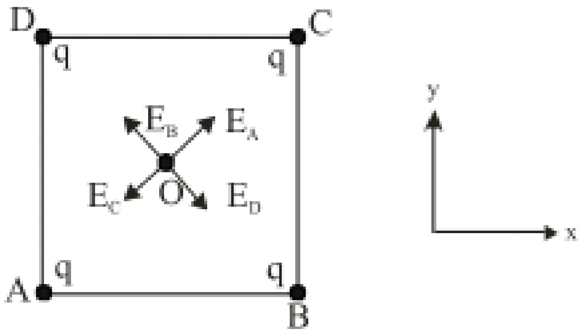
Sol. (a) Let position of charges be A, B, C and D and O is centre of square

Let field at O due to A, B, C, D respectively be $\vec{E}_A$, $\vec{E}_B$, $\vec{E}_C$ and $\vec{E}_D$. At O magnitude of field due to each point charge are same as $OA = OB = OC = OD$ thus

$$E_A = E_B = E_C = E_D = \frac{1}{4\pi\varepsilon_0}\frac{q}{OA^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(a/\sqrt{2}\right)^2} = \frac{1}{2\pi\varepsilon_0}\frac{q}{a^2}$$

$$\vec{E}_A = \frac{1}{2\pi\varepsilon_0}\frac{q}{a^2}\left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right)$$

$$\vec{E}_B = \frac{1}{2\pi\varepsilon_0}\frac{q}{a^2}\left(\frac{-\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right)$$



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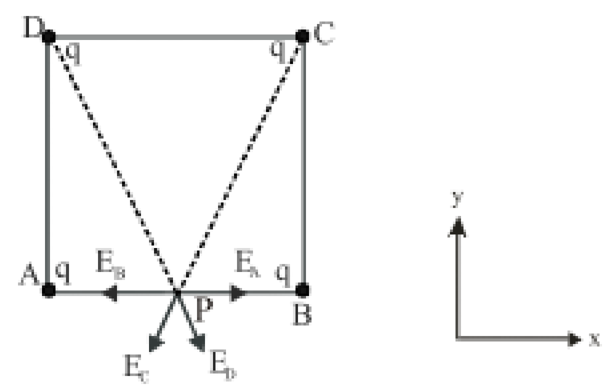
$$\vec{E}_C = \frac{1}{2\pi\varepsilon_0}\frac{q}{a^2}\left(\frac{-\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}\right)$$

$$\vec{E}_D = \frac{1}{2\pi\varepsilon_0}\frac{q}{a^2}\left(\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}\right)$$

$$\vec{E} = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = 0$$



(b) Let concerned point be P

$$\left|\vec{E}_A\right| = \left|\vec{E}_B\right| = \frac{1}{4\pi\in_0}\frac{q}{(AP)^2}$$

$$\vec{E}_A = \frac{q}{\pi\in_0 a^2}\hat{i} \qquad \vec{E}_B = \frac{q}{\pi\in_0 a^2}\left(-\hat{i}\right)$$

$$|\vec{E}_D| = |\vec{E}_C| = \frac{1}{4\pi\in_0}\frac{q}{(DP)^2} = \frac{1}{4\pi\in_0}\frac{q}{\left(\frac{\sqrt{5}}{2}a\right)^2} = \frac{q}{\pi\in_0 5a^2}$$

$$\vec{E}_C = \frac{q}{\pi\in_0 5a^2}\left[\frac{-2}{\sqrt{5}}\hat{i} - \left(\frac{1}{\sqrt{5}}\right)\hat{j}\right]$$

$$\vec{E}_D = \frac{q}{\pi\in_0 5a^2}\left[\frac{2}{\sqrt{5}}\hat{i} - \left(\frac{1}{\sqrt{5}}\right)\hat{j}\right]$$

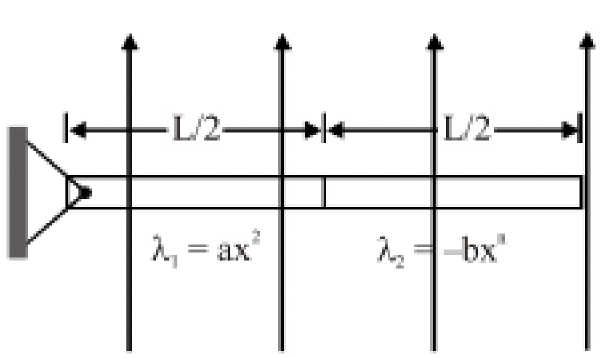
$$\Rightarrow \quad \vec{E} = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = -\frac{q}{\pi\varepsilon_0 5a^2\sqrt{5}}(\hat{j})$$

Q.7 A thin insulating rod is hinged about one of its ends; it can rotate on a smooth surface in a horizontal plane. The charge density on the rod is defined as

$$\lambda_1 = ax^2, \qquad 0 < x \le \frac{L}{2}$$

$$\lambda_2 = -bx^n, \qquad \frac{L}{2} \le x < L$$

where a and b are positive constants. An electric field E_0, in the horizontal plane and perpendicular to the rod is switched on. Find the value of b and n, if the rod has to remain stationary.



Sol. According to given charge distribution the rod is positively charge in $0 < x \le L/2$ and negatively charge in $\frac{L}{2} \le x \le L$. The electric force acting on the rod produce torque about hinge; for equilibrium ne torque should zero.

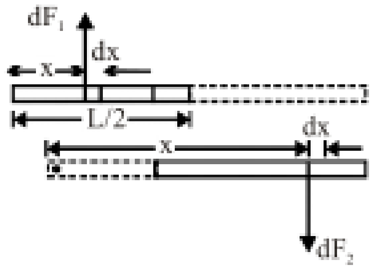
Torque on differential element of left half of the rod.

$$d\tau_1 = E_0\,(\lambda_1\,dx)\,x$$

$$\tau_1 = \int_0^{l/2} E_0(ax^2dx)\,x$$

$$= \frac{E_0 a L^4}{2^6}$$

Torque on the right half of the rod.



$$d\tau_2 = -\,E_0\,(\lambda_2\,dx)\,x$$

$$\tau_2 = -\int_{L/2}^{L} E_0(bx^n dx)\,x$$

$$= \frac{E_0 b}{n+2}\,\frac{2^{n+2}-1}{2^{n+2}}\;L^{n+2}$$

According to condition of the problem,

$$|\tau_1| = |\tau_2|$$

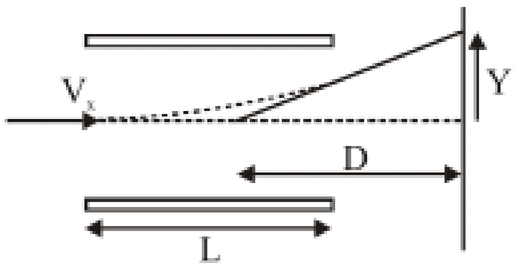
$$\frac{E_0 a L^4}{2^6} = \frac{E_0 b}{(n+2)}\left(\frac{2^{n+2}-1}{2^{n+2}}\right)L^{n+2}$$

On comparing coefficient of L, we get $n+2=4$ or $n=2$ and

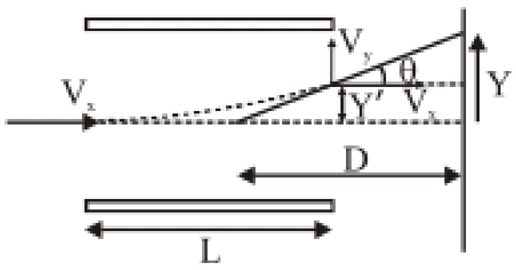
$$\frac{a}{2^6} = \frac{b}{(n+2)}\left(\frac{2^{n+1}-1}{2^{n+2}}\right)$$

or $\quad b = \frac{a}{15}$

Q.8 In an electron ray tube, an electron enters an electric field E between the two plates with a velocity V_x as shown in figure and emerges from the field with a velocity V so as to strike the screen. The separation of the screen from the centre of the plate is D and length of the plate is L. If the charge of the electron is e, then the deflection Y on the screen is (m is the mass of electron)



Sol.



$$V_y = \frac{eE}{m}t = \frac{eE}{m}\,\frac{L}{V_x}$$

$$\tan\theta = \frac{V_y}{V_x} = \frac{eE}{m}\frac{L}{V_x^2} \qquad \ldots(i)$$

Again,

$$\tan\theta = \frac{Y - Y'}{D - L/2}$$

or $$\tan\theta = \frac{Y - \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{L}{V_x}\right)^2}{D - L/2} \qquad \ldots(ii)$$

From (i) and (ii), we get

$$\frac{eE}{m}\frac{L}{V_x^2} = \frac{Y - \frac{1}{2}\frac{eE}{m}\frac{L^2}{V_x^2}}{D - L/2}$$

or $$Y = \frac{eELD}{mV_x^2}.$$

Q.9 An electron of mass m_e initially at rest moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p also initially at rest takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity find the ratio of $\frac{t_2}{t_1}$.

Sol. Force on a charge particle in a uniform electric field

$$F = q\,E$$

The acceleration imparted to the particle is

$$a = \frac{qE}{m}$$

The distance traveled by the particle in time t is

$$d = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

For the given problem

$$\frac{t_p^{\,2}}{m_p} = \frac{t_e^{\,2}}{m_e}$$

$$\frac{t_p^{\,2}}{t_e^{\,2}} = \frac{m_p}{m_e}$$

$$\Rightarrow \quad \frac{t_p}{t_e} = \sqrt{\frac{m_p}{m_e}}$$

Q.10

A particle of charge q and mass m moves rectilinearly under the action of electric field $E = A - Bx$, where B is positive constant and x is distance from the point where particle was initially at rest then find the distance traveled by the particle before coming to rest first time and acceleration of particle at that moment

Sol.

$$F = qE = q(A - Bx)$$

$$ma = q(A-Bx)$$

$$a = \frac{q}{m}(A-Bx) \qquad ...(1)$$

$$\frac{vdv}{dx} = q(A - Bx)$$

$$vdv = \frac{q}{m}(A - Bx)dx$$

$$\int_0^0 vdv = \frac{q}{m}\int_0^x (A - Bx)dx$$

$$Ax - \frac{Bx^2}{2} = 0$$

$$x = 0, x = \frac{2A}{B} \qquad ...(2)$$

From eq. (1) and (2)

$$\frac{q}{m}(A - Bx) = \frac{q}{m}\left(A - B \times \frac{2A}{B}\right)$$

$$= \frac{q}{m}(A - 2A) = \frac{-qA}{m}.$$

Q.11 On a long smooth horizontal plane, over which a horizontal electric field E exists, a charged ball with mass m and charge q is dropped from a height h over the plane. Coefficient of restitution for collision between plane and ball is e. Find the ratio of maximum height attained and horizontal distance moved during the interval of nth collision to (n+l)th collision.

Sol. Time taken to first collision drop.

$$t_1 = \sqrt{\frac{2h}{g}}$$

Velocity on hitting

$$V = \sqrt{2gh}$$

Vertical velocity upward after 1st and 2nd collisions,

$$\Delta t_1 = \frac{2V_1}{g} = \frac{2eV}{g}$$

Vertical velocity upward after 2nd and 3rd collisions,

$$V_2 = eV_1 = e^2V$$

Time gap between 2nd and 3rd collisions,

$$\Delta t_2 = \frac{2V_1}{g} = \frac{2eV}{g}$$

Vertical velocity after nth collision,

$$V_n = e^n V$$

Maximum height attained during time gap between nth to (n + 1)th collision,

$$h = \frac{(e^n V)^2}{2g}$$

$$\Delta t_n = \frac{2e^n V}{g}$$

During these collisions, the ball is moving with acceleration $\frac{qE}{m}$ horizontally.

Distance moved from nth collisions T_n to (n+1)th
Collision T_{n+1},

$$x = \frac{1}{2}a\left(T_{n+1}^2 - T_n^2\right) = \frac{1}{2}\left(T_{n+1}^2 + T_n^2\right)\left(T_{n+1}^2 - T_n^2\right)$$

$$T_n = t_1 + (\Delta t_1 + \Delta t_2 + ...\Delta t_{n+1})$$

$$= \sqrt{\frac{2h}{g}} + \frac{2V}{g}(e + e^2 + ...e^{n-1})$$

$$= \sqrt{\frac{2h}{g}} + \frac{2V}{g}e\left(\frac{1-e^{n-1}}{1-e}\right)$$

$$T_{n+1} = t_1 + (\Delta t_1 + \Delta t_2 + ...\Delta t_n)$$

$$= \sqrt{\frac{2h}{g}} + \frac{2V}{g}e\left(\frac{1-e^n}{1-e}\right)$$

$$\Rightarrow \quad x = \frac{1}{2}\frac{eE}{m}\left[2\sqrt{\frac{2h}{g}} + \frac{2V}{g}\frac{e}{1-e}(2 - e^n - e^{n-1})\right] \times \left[\frac{2V}{g}\frac{e}{1-e}(e^{n-1} - e^n)\right]$$

$$\frac{h}{x} = \frac{e^{2n}V^2}{2g}\frac{2m}{eE\left[2\sqrt{\frac{2h}{g}} + \frac{2V}{g}\frac{e}{1-e}(2 - e^n - e^{n-1})\right]}\cdot\frac{g}{2Ve^n}$$

$$= \frac{mVe^{n-1}}{2E\left[2\sqrt{\frac{2h}{g}} + \frac{2V}{g}\frac{e}{1-e}(2 - e^n - e^{n-1})\right]}$$

$$= \frac{mge^{n-1}}{4E(1 + e - e^n - e^{n+1})}$$

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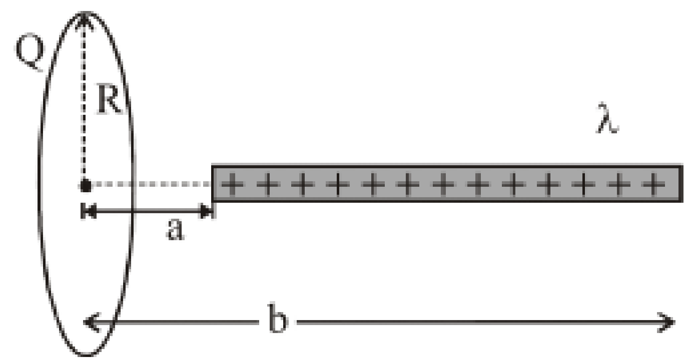
Q.12 Find the interaction force between ring of uniform charge Q and rod having uniform linear charge density λ.



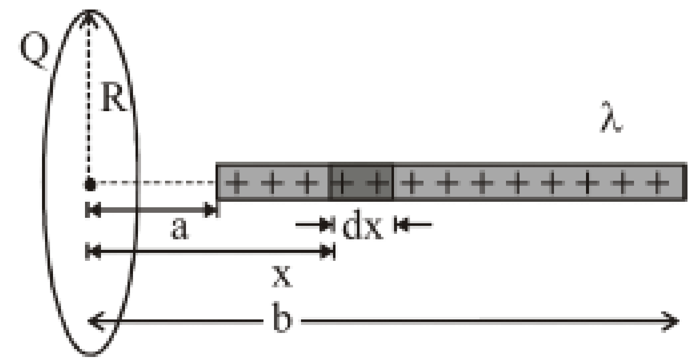
Sol.

Let us take an elementry charge on the line charge of length dx and at distance x from centre of ring. charge of elementry part will be

$$dq = \lambda\, dx$$

Electric field due to ring at the location of elementry charge will be

$$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$$



Force due to ring on elementry charge will be

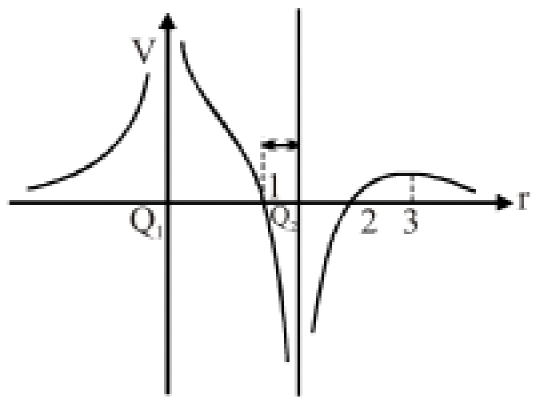
$$dF = Edq = \frac{kQx\ \lambda dx}{(R^2 + x^2)^{3/2}}$$

Therefore the total force on the rod due to ring will be

$$F = \int_a^b \frac{kQx\ \lambda dx}{\left[R^2 + x^2\right]^{3/2}}$$

$$= kQ\lambda\left[\frac{1}{\sqrt{R^2 + a^2}} - \frac{1}{\sqrt{R^2 + b^2}}\right]$$

Q.13 Two point charge Q_1 and Q_2 lie along a line, at a distance from each other. Figure shows the potential variation along the line of charge. At which points 1, 2 and 3 is the electric field zero ? What are the sings of the charges Q_1 and Q_2 and which of the two charges is greater in magnitude?



Sol. The electric field vector is zero at point 3. As $-\frac{dV}{dr} = E_r$, the negative of slop of V vs r curve represents component of electric field along r. Slope of curve is zero only at 3.

Near positive charge net potential is positive and negative near a negative charge. Thus charge Q_1 is positive and Q_2 negative. From the graph it can be seen that net potential due to two charge is positive in the region left of charge Q_1 is greater than due to Q_2. Therefore absolute value of charge Q_1 is greater than due to Q_2. Secondly, the point 1 where potential due to two charge is zero, is nearer to charge Q_2 thereby implying the Q_1 has greater absolute value.

Q.14 In the figure shown find the flux of electric field on curve surface. Assume that electric field is uniform

Sol. As the field is uniform ne flux on the total surface is zero.

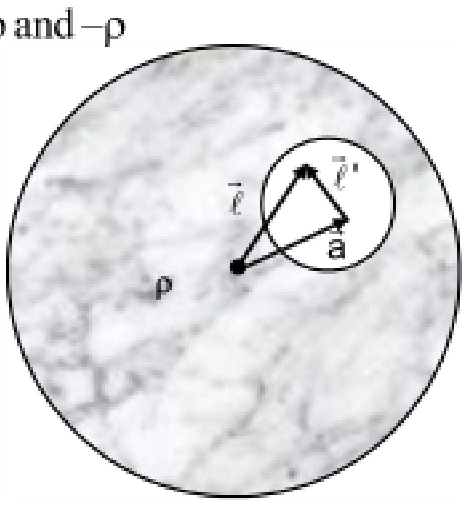
$\Rightarrow \quad \phi_{\text{curved surface}} + \phi_{\text{Plane surface}} = 0$

$\Rightarrow \quad \phi_{\text{curved surface}} = -\phi_{\text{Plane surface}} = -E\,\pi r^2 \cos 180^\circ = E\,\pi r^2$

Q.15 A sphere of radius R is made of a nonconducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius ($r<R$) is now removed from the sphere such that centre of the cavity is at position $\vec{a}$ with respect to the centre of sphere. Show that the electric field within the cavity is uniform and is given by $\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{a}$

Sol. The cavity can be considered as superposition of two charge density $+\rho$ and $-\rho$

Electric field due to positive charge



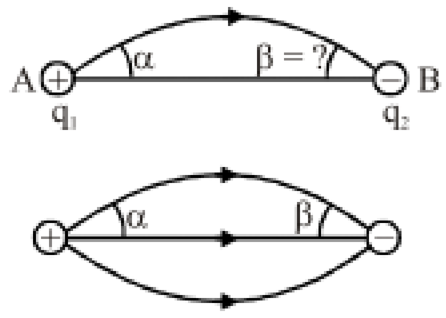
$$\vec{E}_+ = \frac{\rho}{3\epsilon_0}\vec{\ell}$$

Electric field due to negative charge

$$\vec{E}_- = \frac{(-\rho)}{3\epsilon_0}\vec{\ell}\,'$$

$$\therefore \vec{E}_{net} = \vec{E}_+ + \vec{E}_- = \frac{\rho}{3\epsilon_0}\vec{\ell} + \frac{(-\rho)}{3\epsilon_0}\vec{\ell}\,' = \frac{\rho}{3\epsilon_0}(\vec{\ell} - \vec{\ell}\,') = \frac{\rho}{3\epsilon_0}\vec{a}$$

Q.16 Two charge $+q_1$ and q_2 are placed at A and B respectively. A line of force emanates from q_1 at angle α with the line AB. At what angle will it terminate at $-q_2$?



Sol. It is the property of the line of force that their number within a tube remains unchanged and the number of line of force is equal to the charge. The line of force emanating from q_1 spreads out equally in all directions. Hence lines of force per unit solid angle are $\frac{q_1}{4\pi}$ and the number of lines through cone of half-angle α is $\frac{q_1}{4\pi}.2\pi(1-\cos\alpha)$ because solid angle of a cone is $2\pi(1-\cos\alpha)$.

Similarly the number of line of force terminating on $-q_2$ at β is

$$\frac{q_2}{4\pi}\cdot 2\pi(1-\cos\beta)$$

By the property of lines of force.

$$\frac{q_1}{4\pi}\cdot 2\pi(1-\cos\alpha) = \frac{q_2}{4\pi}\cdot 2\pi(1-\cos\beta)$$

$$\Rightarrow \qquad \frac{q_1}{2}\cdot 2\sin^2\frac{\alpha}{2} = \frac{q_2}{2}\cdot 2\sin^2\frac{\beta}{2}$$

$$\Rightarrow \qquad \sin\frac{\beta}{2} = 2\sin\frac{\alpha}{2}\sqrt{\frac{q_1}{q_2}}$$

$$\Rightarrow \qquad \beta = 2\sin^{-1}\left(\sin\frac{\alpha}{2}\sqrt{\frac{q_1}{q_2}}\right)$$

Q.17 Two particles of mass m and 2m carry a charge q each. Initially the heavier particle is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first from a distance d with speed u. Find the closest distance of approach.

Sol. As the mass 2m is not fixed, it will also move away from m due to repulsion. The distance between the particles is minimum when their relative velocity is zero i.e., when they have equal velocities.

Hence at closest approach, $\quad v_1 = v_2$

By conservation of momentum

$$mu = mv_1 + 2mv_2$$

$$v_2 = v_1 = u/3$$

By conservation of energy

Loss in KE = gain in PE

$$\frac{1}{2}mu^2 - \left(\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2\right) = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{d}\right)$$

$$\frac{1}{2}mu^2 - \frac{1}{2}m\frac{u^2}{9}(1+2) = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{d}\right)$$

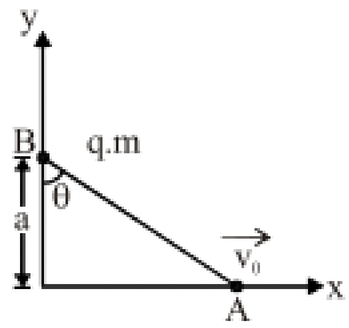
$$\frac{1}{3}mu^2 = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{d}\right)$$

$$\frac{1}{x} = \frac{1}{d} + \frac{4\pi\varepsilon_0 mu^2}{3q^2}$$

$$x = \frac{3q^2 d}{3q^2 + 4\pi\varepsilon_0 mu^2 d}.$$

Q.18 A charged particle having a charge q moves along the x-axis with a constant velocity υ_0. Another particle B with charge q and mass m is lying on thy y-axis at y = a. The particle B is constrained to move along the y-axis. While the particle A moves along the x-axis.

Assuming that the velocity υ_0 is very large, find the impulse imparted to B along, the y-axis as the particle A moves from $-\infty$ to ∞, assuming that the motion of particle B is negligible.



Sol. First we will determine the angular speed a A relative to B at angular position θ shown in Fig. For particle A,

$$\frac{dx}{dt} = \frac{d}{dt}(a \tan\theta) = \upsilon_0 \qquad ...(i)$$

or $$a \sin^2\theta \frac{d\theta}{dt} = \upsilon_0$$

or $$\left(\frac{d\theta}{dt}\right) = \frac{\upsilon_0}{a}\cos^2\theta \qquad ...(ii)$$

As particle A is moving very fast, we can assume that when it crosses along the x-axis, the y-component of the force on B due to A is

$$F_y = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2\sec^2\theta}\times\cos\theta \qquad ...(iii)$$

Impulse on $$B = \int F_y\, dt = \int \frac{F_y\, d\theta}{(d\theta/dt)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\frac{a}{\upsilon_0}\int\frac{\cos^3}{\cos^2}d\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a\upsilon_0}\int_{\theta=-/2}^{\pi/2}\cos\theta\, d\theta$$

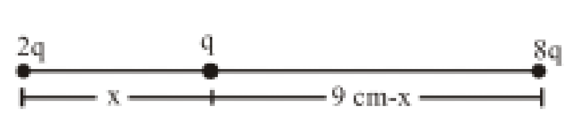
The impulse delivered to B,

$$= \Delta P_y = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{a\upsilon_0} = \frac{q^2}{2\pi\varepsilon_0(a\upsilon_0)} \qquad ...(iv)$$

Q.19 Three point charges q, 2q and 8q are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the charge q due to the other two charges ?

Sol. The maximum contribution may come from the charge 8q forming pairs with others. To reduce its effect, it should be placed at a corner and the smallest charge q in the middle. This arrangement shown in figure ensures that the charges in the strongest pair 2q, 8q are at the largest separation.

The potential energy is



$$U = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{2}{x} + \frac{16}{9cm} + \frac{8}{9cm - x}\right].$$

This will be minimum if

$$A = \frac{2}{x} + \frac{8}{9cm - x} \text{ is minimum.}$$

For this, $$\frac{dA}{dx} = -\frac{2}{x^2} + \frac{8}{(9cm - x)^2} = 0 \qquad .... (i)$$

or, 9 cm – x = 2x or, x = 3 cm

The electric field at the position of charge q is

$$\frac{q}{4\pi\varepsilon_0}\left(\frac{2}{x^2} - \frac{8}{(9cm - x)^2}\right)$$

$= 0.$ From (i).

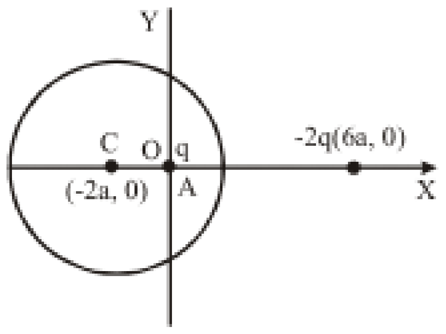
Q.20 Two points charge q and –2q are placed at a distance 6a apart. Find the locus of the point in the plane of charges where the field potential is zero.

Sol. Let us take the charge on X-axis;

q at A (0, 0) and –2q at B(6a, 0)

Potential at a point P(x, y) is

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{x^2+y^2}} + \frac{-2q}{4\pi\varepsilon_0\sqrt{(x-6a)^2+y^2}}$$



$$V = 0$$

$$\Rightarrow \quad \frac{q^2}{x^2+y^2} = \frac{4q^2}{(x-6a)^2+y^2}$$

$\Rightarrow$ the locus is $(x-6a)^2 = 4x^2 + 3y^2$.

$$3x^2 + 3y^2 + 2(6a)x = 36a^2$$

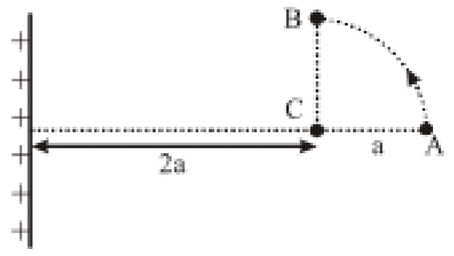
$$\Rightarrow \quad x^2 + y^2 + 4ax = 12a^2$$

$$(x+2a)^2 + y^2 = 16a^2$$

A circle with centre (-2a, 0) and radius 4a.

Q.21 The arc AB with the centre C and the infinitely long wire having linear charge density λ are lying in the same plane. Find the minimum amount of work to be expended to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to :

Sol. $E = \frac{\lambda}{2\pi\epsilon_0 x}$

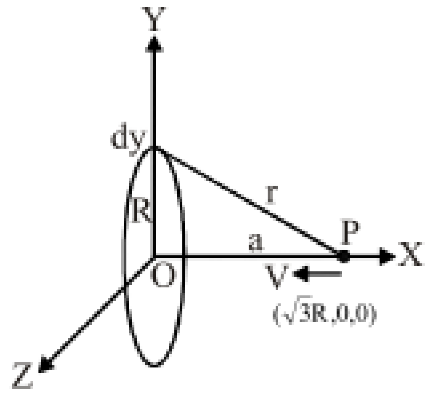


$$\int_{V_A}^{V_B} dV = -E\,dx = -\frac{\lambda}{2\pi\varepsilon_0}\int_{3a}^{2a}\frac{dx}{x}$$

$$\Rightarrow \quad V_B - V_A = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{3}{2}\right)$$

work done by agent $= \frac{q_0\lambda}{2\pi\varepsilon_0}\ln\frac{3}{2}$.

Q.22 A circular ring of radius R with uniform positive charge density λ per unit length is located in the Y-Z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P $(R\sqrt{3}, 0, 0)$ on the positive x-axis directly towards O, whit initial velocity υ. Find the smallest (non-zero) value of the speed υ such that the particle does not returen to P.



Sol. The situation is shown in figure. Total potential at the centre of a ring is given by

$$V = \frac{\lambda R}{2\varepsilon_0\sqrt{(a^2+R^2)}}$$

$$= \frac{\lambda R}{2\varepsilon_0\sqrt{[(\sqrt{3}R)^2+R^2)}} = \frac{\lambda}{4\varepsilon_0}$$

Potential energy at P = $\lambda q/4\varepsilon_0$

Energy of particle at P = $\frac{1}{2}$ mυ^2

$\therefore$ Total energy at P = $\frac{\lambda q}{4\varepsilon_0} + \frac{1}{2}m\upsilon^2$

The potential energy at centre = $(\lambda q/2\varepsilon_0)$
The particle will not return to P, when

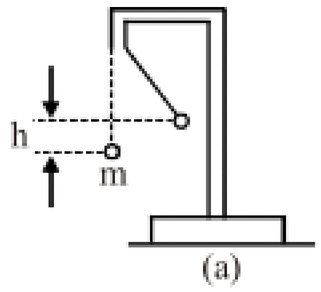
$$\frac{\lambda q}{4\varepsilon_0} + \frac{1}{2}m\upsilon^2 = \frac{\lambda q}{2\varepsilon_0}$$

$$\frac{1}{2}m\upsilon^2 = \frac{\lambda q}{2\varepsilon_0} - \frac{\lambda q}{4\varepsilon_0} = \frac{\lambda q}{4\varepsilon_0}$$

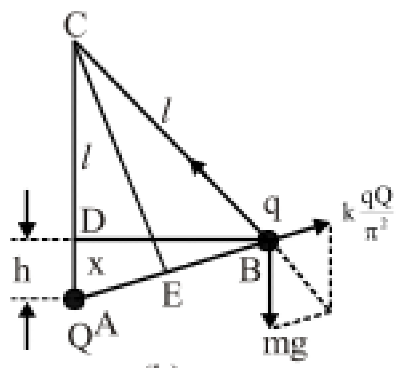
$$\upsilon^2 = \frac{\lambda q}{2\varepsilon_0 m}$$

or $$\upsilon = \sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)}$$

Q.23 A small positively charge ball of mass m is suspended by an insulating thread of negligible mass. Another positively charged small ball is moved very slowly from a large distance until it is in the original position of the first ball. As a result, the first ball rises by h. How much work has been done?



(a)



(b)

Sol. Using the notation of figure the equilibrium condition for the first ball is

$$s\ \frac{mg}{F} = \frac{l}{x}$$

Where F = kqQ/x^2 is the Coulomb force acting on the first ball and x is the distance between the balls carrying charges q and Q.
It is clear the triangles ABD and CAE are similar and that consequently

$$\frac{x}{2} : l = h : x$$

From the three equations above we can calculate the separation of the charge and the electrostatics energy of the system:

$$x = k\frac{qQ}{2mgh}$$

and $$E_{electro} = k\frac{qQ}{x}$$

$$= 2mgh.$$

The work done is the sum of the changes in electrical and gravitational potential energy.

$$W = 2mgh + mgh$$
$$= 3mgh$$

Note that the work done does not depend on either the magnitudes of the charges or the length of the thread.

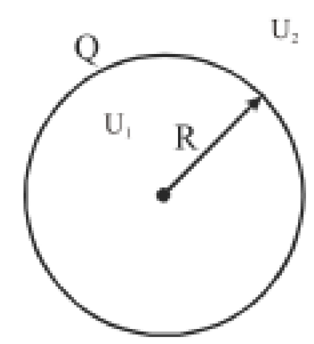
Q.24 A charge Q is uniformly distributed over a spherical volume of radius R. Obtain an expression for the energy of the system.

Sol. In this case, the electric field exists from centre of the sphere to infinity. Potential energy is stored in electric field with energy density.

$$u = \frac{1}{2}\varepsilon_0 E^2 \text{(energy / volume)}$$

(i) Energy stored within the sphere (V_1)

Energy field at a distance r is ($r \le R$)



$$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^3}.r$$

$\therefore$ $$u = \frac{1}{2}\varepsilon_0 E^2$$

$$u = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^3}r\right\}^2$$

Volume of element, $dV = (4\pi r^2)dr$

$\therefore$ Energy stored in this volume, $dU = u(dV)$

$$dU = (4\pi r^2 dr)\frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^3}r\right\}^2$$

$$dU = \frac{1}{8\pi\varepsilon_0}.\frac{Q^2}{R^6}.r^4 dr$$

$$U_1 = \int_0^R dU = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{R^6}\int_0^R f^4 dr$$

$$= \frac{Q^2}{40\pi\varepsilon_0 R^6}\left[r^5\right]_0^R$$

$$U_1 = \frac{1}{40\pi\varepsilon_0}.\frac{Q^2}{R} \quad \text{.... (1)}$$

(ii) Energy stored outside the sphere (U_2)

Electric field at a distance r is ($r \ge R$)

$$E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$$

$$u=\frac{1}{2}\varepsilon_0E^2$$

$$u=\frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\right\}^2$$

$$dV=(4\pi r^2\,dr)$$

$$dU=udV=(4\pi r^2\,dr)\left[\frac{\varepsilon_0}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\right)^2\right]$$

$$dU=\frac{Q^2}{8\pi\varepsilon_0}\cdot\frac{dr}{r^2}$$

$$U_2=\int_R^\infty dV=\frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty\frac{dr}{r^2}$$

$$U_2=\frac{Q^2}{8\pi\varepsilon_0R} \qquad \ldots(2)$$

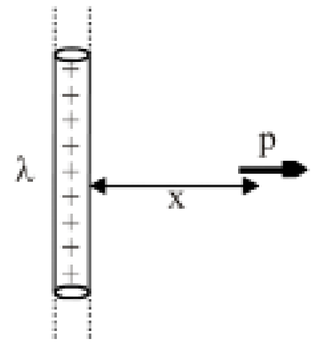
Therefore, total energy of the system is

$$U=U_1+U_2=\frac{Q^2}{40\pi\varepsilon_0R}+\frac{Q^2}{8\pi\varepsilon_0R}$$

$$U=\frac{3}{20}\frac{Q^2}{\pi\varepsilon_0R}.$$

Q.25 A dipole of mass m is placed in front of a line charge at a separation x (as shown in figure). Find (i) interaction energy between point charge and dipole (ii) force acting on the dipole due to line charge(iii) what will be its velocity if it reaches at separation x_0

Sol.



(i) $U=-\vec{p}\cdot\vec{E}=-pE\cos 0^\circ=-p\frac{\lambda}{2\pi\epsilon_0 x}(1)=-\frac{\lambda p}{2\pi\epsilon_0 x}$

(ii) $F=-\frac{dU}{dx}=-\frac{\lambda p}{2\pi\epsilon_0 x^2}\Rightarrow\vec{F}\ \frac{\lambda p}{2\pi\epsilon_0 x^2}$ directed along -ve x axis

(iii) using conservation of energy

$$K_f+U_f=K_i+U_i$$

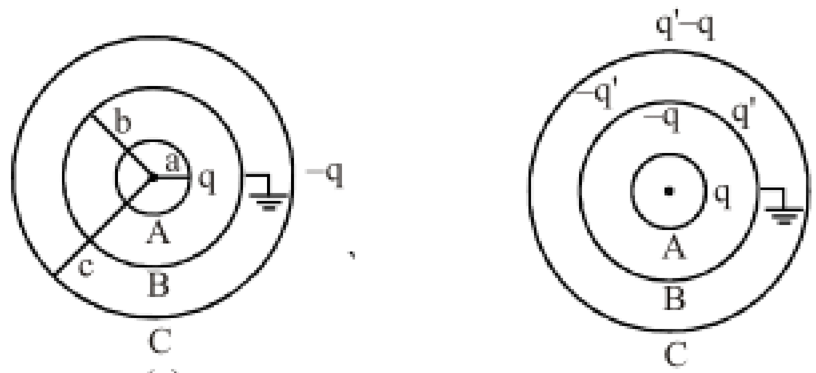
$$\frac{1}{2}mv^2-\frac{\lambda p}{2\pi\epsilon_0 x_0}=0-\frac{\lambda p}{2\pi\epsilon_0 x}$$

$$\frac{1}{2}mv^2=\frac{\lambda p}{2\pi\epsilon_0}\left(\frac{1}{x_0}-\frac{1}{x}\right)$$

$$v=\sqrt{\frac{2\lambda p}{2\pi\epsilon_0 m}\left(\frac{1}{x_0}-\frac{1}{x}\right)}$$

Q.26

Figure shows three concentric thin spherical shells A, B and C of radii a, b and c repectively. The shells A and C are given charges q and –q respectively and the shell B is earthed. Find the charges appearing on the surface of B and C.



(a) (b)

Sol. As shown in the previous worked out exmple. The inner surface of B must have a charge –q from the Gauss's law. Suppose, the outer surface of B has a charge q'. The inner surface of C must have a charge –q' from the Gauss's law. As the net charge on C must be –q, its outer surface should have a charge q' – q. The charge distribution is shown in figure.

The potential at B due to the charge q on A

$$= \frac{q}{4\pi\varepsilon_0 b},$$

due to the charge – q on the inner surface of B

$$= \frac{-q}{4\pi\varepsilon_0 b}$$

due to the charge q' on the outer surface of B

$$= \frac{q'}{4\pi\varepsilon_0 b}$$

due to the charge – q' on the outer surface of C

$$= \frac{-q'}{4\pi\varepsilon_0 c}$$

and due to the charge q' – q on the outer surface of C

$$= \frac{q'-q}{4\pi\varepsilon_0 c}$$

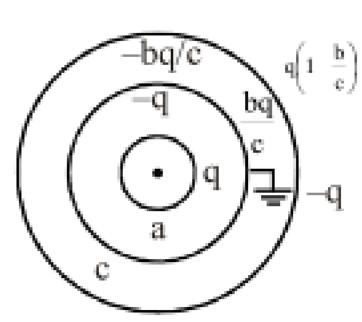
The net potential is

$$V_B = \frac{q'}{4\pi\varepsilon_0 b} - \frac{q}{4\pi\varepsilon_0 c}$$

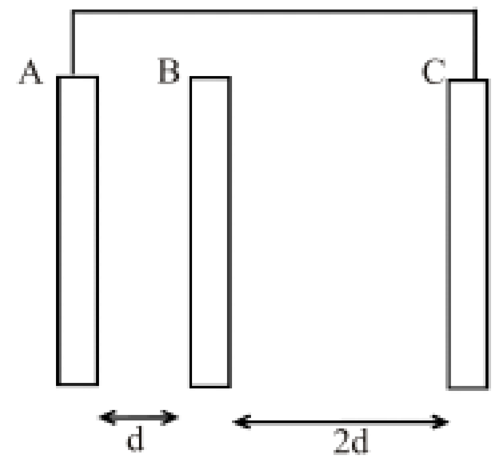
This should be zero as the shell B is eartdhed. Thus,

$$q' = \frac{b}{c}q.$$

The charges on various surface are as shown in figure



Q27 Figure shows three plates A, B and C each of area S in which A and C are joined by a conducting wire. Plate A and B are given charge Q and 3Q respectively. Find the final charge distribution of each side of each conductor.

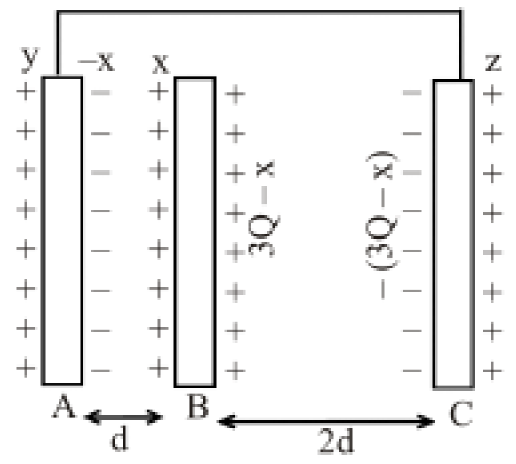


Sol. Middle plate is isolated. Hence sum of charge on left and right side will be 3Q. But left+right plate forms isolated system and using conservation of charge for this system.

$y+(-x)+\{-(3Q-x)\}+z=0$ ....(i)

but y=z ....(ii) [$\because$ E=0 inside conductor]

Also left and right plate have same potential. Hence potential difference between A and C will be zero. i.e,



$$\int_A^C \vec{E}\cdot d\vec{r}=0$$

$$\Rightarrow \int_A^B \vec{E}\cdot d\vec{r}+\int_B^C \vec{E}\cdot d\vec{r}=0$$

$$\Rightarrow -E_1 d+E_2(2d)=0$$

$$\Rightarrow -\left\{\frac{x}{2\epsilon_0 S}\times 2\right\}d+\left\{\frac{3Q-x}{2\epsilon_0 S}\times 2\right\}(2d)=0 \quad \text{..............(iii)}$$

After solving (i), (ii) and (iii) we get

x=2Q y=z=2Q