Magnetic Field

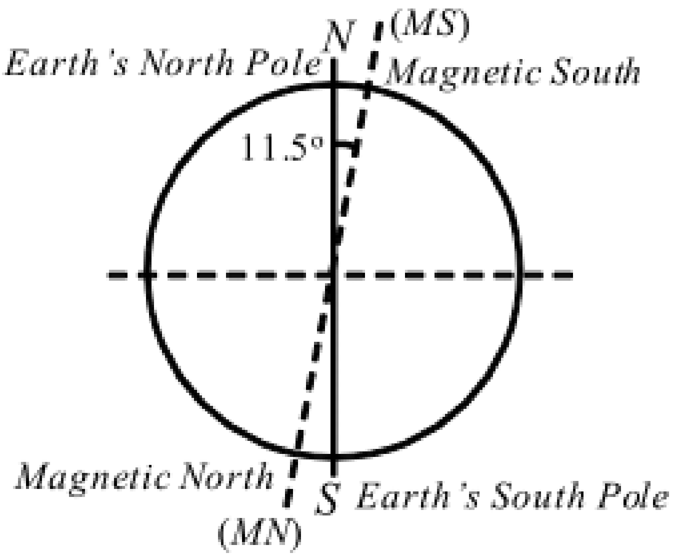
Magnets are familiar objects. The word magnetism is derived from the province of Magnesia where the ancient Greek mine magnetite, also known as lodestone, a mineral composed of iron oxide which attracts iron.

If you ask the average person what "magnetism" is, you will probably be told about the magnets those are used to hold notes on refrigerator door, or keeping paper clips in a holder or may be about lead stone (naturally occurring magnet).
Scholars still dispute about the origin of magnetism. It is believed that magnetism was originally used, not for navigation, but for geomancy ("foresight by earth") and fortune-telling by the Chinese. Chinese fortune tellers used lodestones to construct their fortune telling boards.

From Chinese text, it is known that magnetic compass (used for navigational purpose) is an old Chinese invention . An old Indian literature dates it to as back as 4th century. The compass was used in India was known as the matsya yantra, because of the placement of a metallic fish in a cup of oil.

Earth's magnetic field:

It is understood that a compass needle points along the horizontal component of Earth's magnetic field (a property called declination).



Earth's Geomagnetic North acts as a south pole of a magnet, while its Geomagnetic South acts as a North Pole of a magnet.
As shown in the diagram, the axis of the dipole makes an angle of about 11.5° with earth's rotational axis. The axis of dipole makes an angle of about 11.5° with the earth's rotational axis. And Earth's rotational axis makes an angle of 23.5° with the normal to the plane of earth's orbit about the sun.

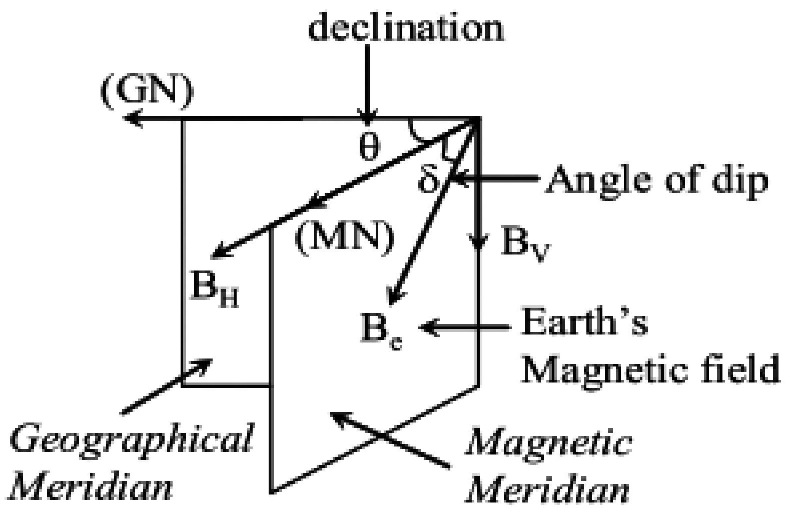
Elements of earth's magnetic field:

The earth's magnetic field is characterized by three quantities:

(a) Declination

(b) Inclination or dip

(c) Horizontal component of the field.



Magnetism due to electricity

Hans Christian Oersted was a professor of science at Copenhagen University. In 1819 he arranged in his home a science demonstration to friends and students. He planned to demonstrate the heating of a wire by an electric current, and also to carry out demonstrations of magnetism, for which he provided a compass needle mounted on a wooden stand.

While performing his electric demonstration, Oersted noted to his surprise that every time the electric current was switched on, the compass needle moved . He kept quiet and finished the demonstrations, but in the months that followed worked hard trying to make sense out of the new phenomenon. And this is what we are going to study now.

We have seen that currents (fundamentally moving charges) are the source of magnetism. This can be readily demonstrated by placing compass needles near a wire. As shown in Figure, all compass needles point in the same direction in the absence of current (in the direction of earth's magnetic field). However, when a strong current passes through (so that earth's magnetic field becomes negligible), the needles will be deflected along the tangential direction of the circular path (Figure).

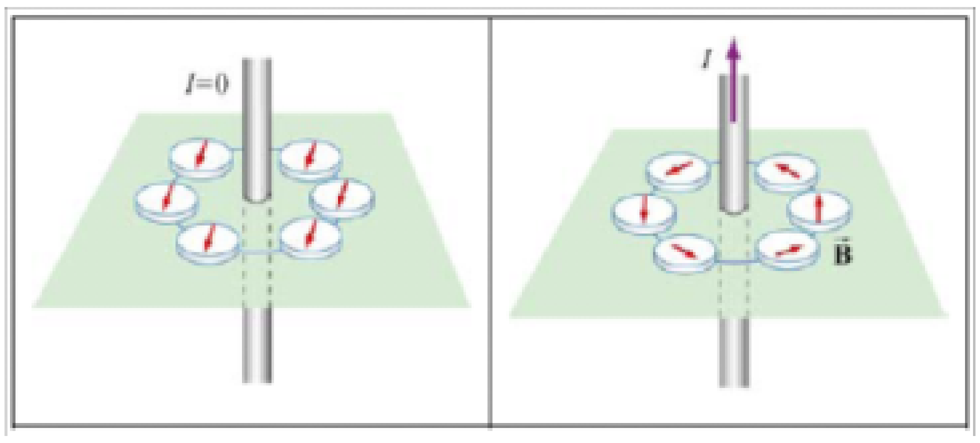


Figure : Deflection of compass needles near a current-carrying wire

Biot-Savart Law

Currents which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire and produce a current *I*, the magnetic field at any point *P* due to the current can be calculated by adding up the magnetic field contributions, $d\vec{B}$, from small segments of the wire $d\vec{s}$ (Figure).

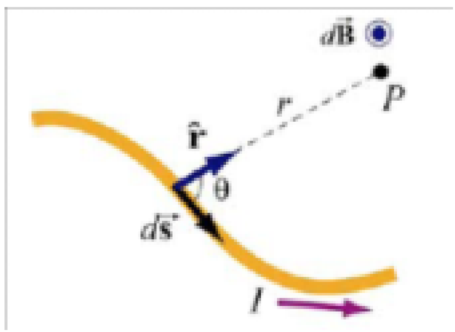


Figure : Magnetic field $d\vec{B}$ at point *P* due to a current-carrying element $Id\vec{s}$

These segments can be thought of as a vector quantity having a magnitude of the length of the segment and pointing in the direction of the current flow. The infinitesimal current source can then be written as $Id\vec{s}$.

Let *r* denote as the distance form the current source to the field point *P* and $\hat{r}$ the corresponding unit vector. The Biot-Savart law gives an expression for the magnetic field contribution, $d\vec{B}$, from the current source, $Id\vec{s}$,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \hat{r}}{r^2}$$

where μ_0 is a constant called the *permeability of free space*:

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T.m/A here Tesla (T) is SI unit of } \vec{B}$$

Notice that the expression is remarkably similar to the Coulomb's law for the electric field due to a charge element *dq* :

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

Adding up these contributions to find the magnetic field at the point *P* requires integrating over the current source,

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s} \times \hat{r}}{r^2}$$

The integral is a vector integral, which means that the expression for Bis really three integrals, one for each component of $\vec{B}$. The vector nature of this integral appears in the cross product. Understanding how to evaluate this cross product and then perform the integral will be the key to learning how to use the Biot-Savart law.

Magnetic Field due to a Finite Straight Wire.

A thin, straight wire carrying a current I is placed along the x-axis, as shown in Figure. Evaluate the magnetic field at point P due to the segment shown in figure.

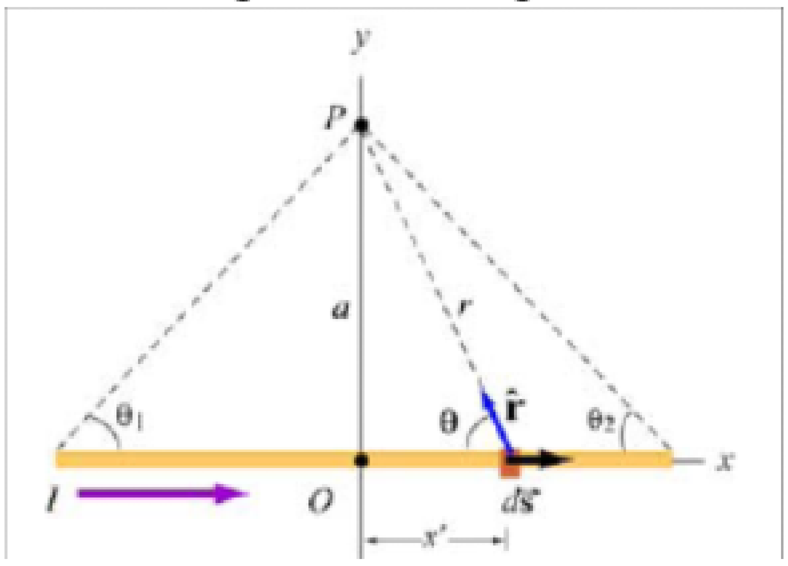


Figure 9.1.3 A thin straight wire carrying a current I.

The contribution to the magnetic field due to $Id\vec{s}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{dx\sin\theta}{r^2}\hat{k}$$

which shows that the magnetic field at P will point in the $+\hat{k}$ direction, or out of the page.

Simplify and carry out the integration

The variables θ, x and r are not independent of each other. In order to complete the integration, let us rewrite the variables x and r in terms of θ. From Figure, we have

$$\begin{cases} r = a/\sin\theta = a\,\mathrm{cosec}\,\theta \\ x = a\cot\theta \Rightarrow dx = -a\,\mathrm{cosec}^2\theta\,d\theta \end{cases}$$

Upon substituting the above expressions, the differential contribution to the magnetic field is obtained as

$$dB = \frac{\mu_0 I}{4\pi}\frac{(-a\,\mathrm{cosec}^2\theta\,d\theta)\sin\theta}{(a\,\mathrm{cosec}\,\theta)^2} = -\frac{\mu_0 I}{4\pi a}\sin\theta\,d\theta$$

Integrating over all angles subtended from $-\theta_1$ to θ_2 (a negative sign is needed for θ_1 in order to take into consideration the portion of the length extended in the negative x-axis from the origin), we obtain

$$\boxed{B = \frac{\mu_0 I}{4\pi a}\int_{-\theta_1}^{\theta_2}\sin\theta\,d\theta = \frac{\mu_0 I}{4\pi a}(\cos\theta_2 + \cos\theta_1)}$$

The first term involving θ_2 accounts for the contribution from the portion along the $+x$ axis, while the second term involving θ_1 contains the contribution from the portion along the $-x$ axis. The two terms add.

Special cases :

(i) Magnetic field on the perpendicular bisector of a finite straight wire of length 2*L*

In this case where $\theta_2 = \theta_1 = \theta$, the field point *P* is located along the perpendicular bisector. If the length of the rod is 2*L*, then $\cos\theta = L/\sqrt{L^2 + a^2}$ and the magnetic field is

$$B = \frac{\mu_0 I}{2\pi a}\cos\theta = \frac{\mu_0 I}{2\pi a}\frac{L}{\sqrt{L^2 + a^2}}$$

(ii) Magnetic field due to semiinfinite straight wire

Here $\theta_1 = 90^0$, $\theta_2 = 0^0$ or $\theta_1 = 0^0$, $\theta_2 = 90^0$

$$B = \frac{\mu_0 I}{4\pi a}$$

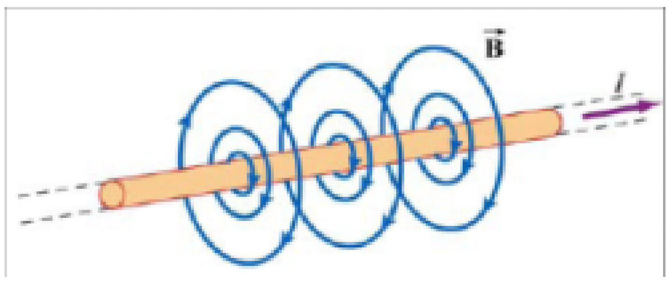
(iii) Magnetic field due to infinite straight wire

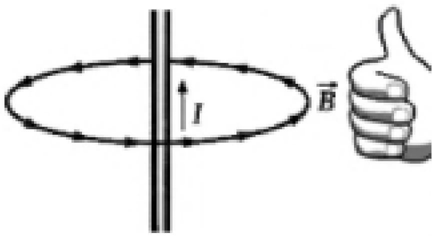
Here $\theta_1 = \theta_2 = 0$

$$B = \frac{\mu_0 I}{2\pi a}$$

Direction of magnetic field of a straight wire

Note that in this limit, the system possesses cylindrical symmetry, and the magnetic field lines are circular, as shown in Figure.





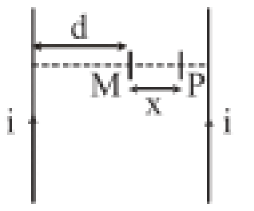
In fact, the direction of the magnetic field due to a long straight wire can be determined by the right-hand rule (Figure). If you direct your right thumb along the direction of the current in the wire, then the fingers of your right hand curl in the direction of the magnetic field.

Illustration :

Two long parallel wires carry equal current *i* flowing in the same direction are at a distance 2d apart. The magnetic field B at a point lying on the perpendicular line joining the wires and at a distance x from the midpoint is

Sol. The magnetic field due

$$B_1 = \frac{\mu_0 i}{2\pi(d + x)}$$



$$B_2 = \frac{\mu_0 i}{2\pi(d-x)}$$

Both the magnetic fields act in opposite direction.

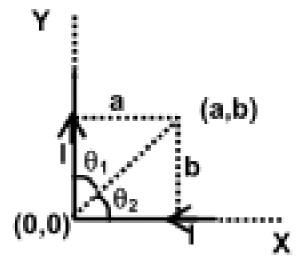
$$\therefore \quad B = B_2 - B_1 = \frac{\mu_0 i}{2\pi}\left[\frac{1}{d-x} - \frac{1}{d+x}\right]$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{d+x-d+x}{d^2-x^2}\right]$$

$$= \frac{\mu_0 i x}{\pi(d^2-x^2)}.$$

Illustration :

Two semi-infinitely long straight current carrying conductors are in form of an 'L' shape as shown in the figure. The common end is at the origin. What is the value of magnetic field at a point (a, b), if both the conductors carry the same current I?



Sol. *For the conductor along the X axis, the magnetic field*

$B_1 = \frac{\mu_0 I}{4\pi b}[\cos\theta_2 + \cos 0]$ *along the negative Z-axis*

$$= \frac{\mu_0 I}{4\pi b}\left[1 + \frac{a}{\sqrt{a^2+b^2}}\right]$$

For the conductor along Y-axis, the magnetic field is

$B_2 = \frac{\mu_0 I}{4\pi a}\left[1 + \frac{b}{\sqrt{a^2+b^2}}\right]$ along the negative z – axis

$\therefore$ *The net magnetic field is,*

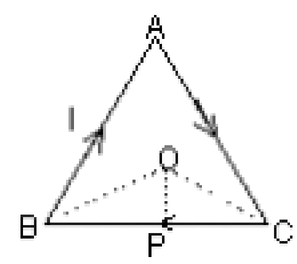
$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$= \frac{\mu_0 I}{4\pi}\left[\left(\frac{1}{a}+\frac{1}{b}\right) + \frac{1}{\sqrt{a^2+b^2}}\left(\frac{a}{b}+\frac{b}{a}\right)\right] = \frac{\mu_0 I}{4\pi}\left[\frac{(a+b)}{ab} + \frac{\sqrt{a^2+b^2}}{ab}\right]$$

$$= \frac{\mu_0 I}{4\pi ab}\left[(a+b) + \sqrt{a^2+b^2}\right]$$

Illustration :

A current I is established in a closed loop of an triangle ABC of side ℓ. Find the magnetic field at the centroid 'O'.



Sol. *From geometry*

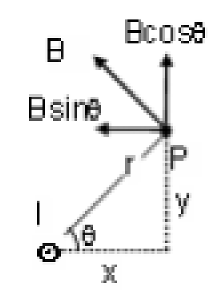
$$OP = \frac{\ell}{2\sqrt{3}}$$

Magnetic fields due to current in all three sides are equal in magnitude and directed into the plane of the paper.

$$\textbf{\textit{Hence net B}} = \frac{3\mu_0 I}{4\pi r}\left[2\times\cos\frac{\pi}{3}\right] = \frac{3\mu_0 I}{4\pi r}\times 2\sin\left(\frac{\pi}{3}\right) = \frac{9\mu_0 I}{2\pi\ell}$$

Illustration :

Find the magnetic field due to the wire at point P if the wire is long



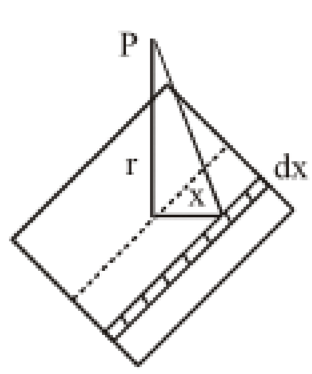
Sol. $\vec{B} = -B\sin\theta\,\hat{i} + B\cos\theta\,\hat{j} = B\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$

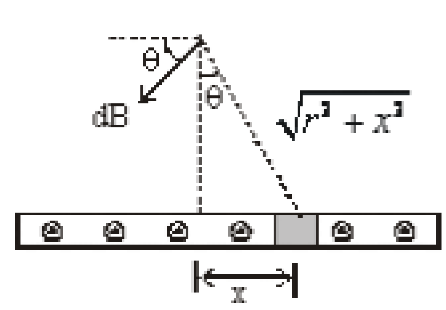
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\left(-\frac{y}{r}\hat{i} + \frac{x}{r}\hat{j}\right) = \frac{\mu_0 I}{2\pi r^2}\left(-y\hat{i} + x\hat{j}\right)$$

Illustraation :

An infinitely large sheet carries current with linear current density i. Find the net magnetic field at a point which is at perpendicular distance r from the sheet.

Sol.





Let us consider a current carrying element idx

$$\therefore \qquad dB_p = \frac{\mu_0 (idx)}{2\pi\sqrt{(r^2 + x^2)}}$$

It has two components one parallel to the plane of the sheet and other perpendicular to it.

$$dB_x = -dB\cos\theta \qquad and \qquad dB_y = -dB\sin\theta$$

$$\therefore \qquad B_x = \int dB_x = \int_{-\infty}^{+\infty} \frac{\mu_0 idxr}{2\pi(r^2+x^2)} = -\frac{\mu_0 i}{2}$$

$$and \qquad B_y = \int_{-\infty}^{+\infty} dB_y = \int_{-\infty}^{+\infty} \frac{\mu_0 idx\,x}{2\pi(r^2+x^2)} = 0$$

$$\therefore \qquad B = -\frac{\mu_0 I}{2}$$

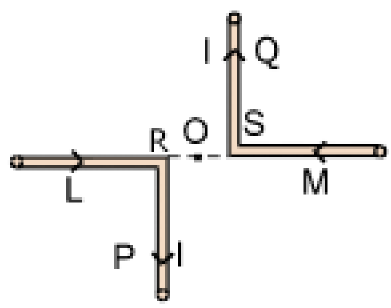
Practice Exercise

Q.1 Find the magnetic field B at the centre of a rectangular loop of length l and width b, carrying a current i.

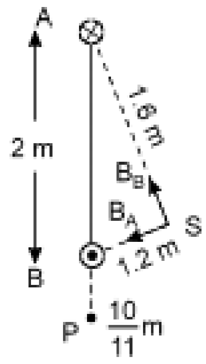
Q.2 A long wire carrying a current i is bent to form a plane angle α. Find the magnetic field B at a point on the bisector of this angle situated at a distance x from the vertex.

Q.3 A pair of stationary and infinitely long bent wires are placed in the x–y plane as shown in **figure**. The wires carry currents of 10 ampere each as shown.

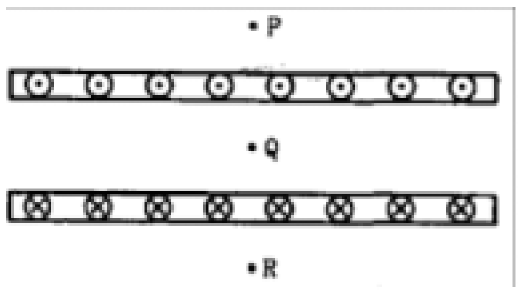
The segments L and M are along the x–axis. The segments P and Q are parallel to the y–axis such that OS = OR = 0.02 m. Find the magnitude and direction of the magnetic induction at the origin O.



Q.4 Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper. The wire A carries a current of 9.6 ampere, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P at a distance (10/11) m from the wire B is zero. Find **(a)** the magnitude and direction of the current in B, **(b)** the magnitude of the magnetic field of induction at the point S,



Q.5 Two large metal sheets carry surface currents as shown in figure . The current through a strip of width dl is Kdl where K is a constant. Find the magnetic field at the points P, Q and R.



Answers

Q.1 $\dfrac{2\mu_0 i\sqrt{l^2+b^2}}{\pi l b}$ Q.2 $\dfrac{\mu_0 i}{2\pi r}\cot\dfrac{\alpha}{4}$ Q.3 B is 10^{-4} T and out of the page

Q.4 (a) –3 A and opposite to that in A (b) 1.3×10^{-6} T Q.5 0, μ_0 K towards right in the figure, 0

Magnetic Field due to a current carrying Arc at its centre

$$dl = a d\theta$$

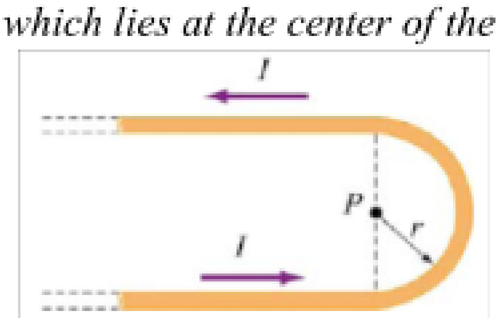
$$dB = \frac{\mu_0}{4\pi}\frac{I(a d\theta)\sin 90^0}{a^2} = \frac{\mu_0 I}{4\pi a}d\theta$$

$$B = \int dB = \frac{\mu_0 I}{4\pi a}\int_0^\beta d\theta$$

$$B = \frac{\mu_0 I}{4\pi R}(\beta\ )$$

Illustration :

An infinitely long current-carrying wire is bent into a hairpin-like shape shown in Figure. Find the magnetic field at the point P which lies at the center of the half-circle.

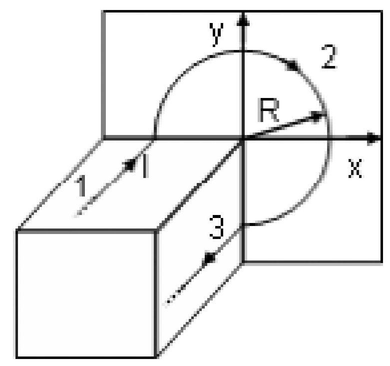


Sol. *The total magnitude of the magnetic field is*

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = 2\vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi r}\hat{k} + \frac{\mu_0 I}{4r}\hat{k} = \frac{\mu_0 I}{4\pi r}(2+\pi)\hat{k}$$

Illustration :

Find the magnetic induction at the point O if the wire carrying a current I A has the shape shown in Fig. The radius of the curved part of the wire is R , the linear parts of the wire are very long.



Sol. $\vec{B}=\vec{B}_1+\vec{B}_2+\vec{B}_3=\frac{\mu_0 I}{4\pi R}\left(-\hat{j}\right)+\frac{\mu_0 I}{4\pi R}\left(\frac{3\pi}{2}\right)\left(-\hat{k}\right)+\frac{\mu_0 I}{4\pi R}\left(-\hat{i}\right)$

Illustration :

A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. Magnetic field due to current at the center of ring is

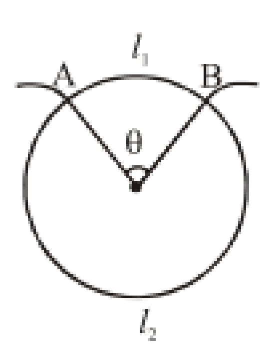
Sol. *For a current flowing into a circular arc, magnetic induction in the centre*

$$B=\frac{\mu_0 I}{4\pi}\int\frac{dl\times r}{r^3}=\frac{\mu_0 I}{4\pi}\int\frac{r^2 d\theta}{r^3}=\left(\frac{\mu_0 I}{4\pi r}\right)\theta$$

The total current is divided into two arcs

$$I_1=\frac{E}{R_1}$$

$$=\frac{E}{(R/2\pi r)l_1}=\frac{E}{(R/2\pi r)(r\theta)}=\frac{2\pi E}{R\theta}$$



Similarly $I_1\theta=\frac{2\pi E}{R}=constant$

$$I_2=\frac{E}{R_2}$$

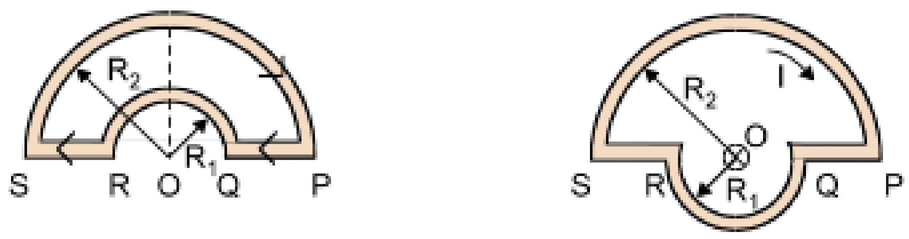
$$=\frac{E}{(R/2\pi r)l_2}$$

$$=\frac{E}{(R/2\pi r)\{r(2\pi-\theta)\}}=\frac{2\pi E}{R(2\pi-\theta)}=\text{cons}\tan\text{t}$$

$$B=B_1-B_2=\frac{\mu_0}{4\pi r}\left(\frac{2\pi E}{R}-\frac{2\pi E}{R}\right)=0.$$

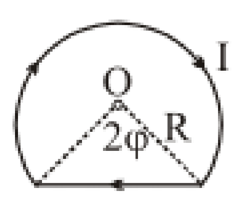
Practice Exercise

Q.1 The wire loop PQRSP formed by joining two semi–circular wires of radii R_1 and R_2 carries a current I as shown in **figure**. What is the magnetic induction at the centre O and magnetic moment of the loop in cases **(a)** and **(b)**?

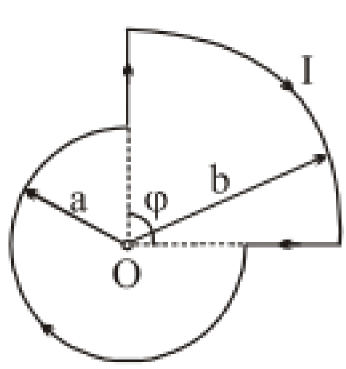


(a) **(b)**

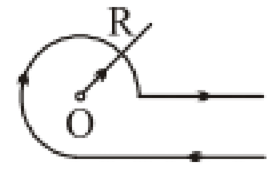
Q.2 A current I flows along a thin wire shaped as shown in Fig. The radius of a curved part of the wire is equal to R , the angle 2φ . Find the magnetic induction of the field at the point O.



Q.3 Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated



Q.4 Find the magnetic induction of the field at the point O if a current-carrying wire has the shape shown in Fig. . The radius of the curved part of the wire is R, the linear parts are assumed to be very long.



Q.5 Two circular coils of radii 5'0 cm and 10 cm carry equal currents of 2A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as the centres coincide. Find the magnitude of the 'magnetic field *B* at the common centre of the coils if the currents in the coils are (a) in the same sense (b) in the opposite sense.

Q.6 If the outer coil of the previous problem is rotated through 90° about a diameter, what would be the magnitude of the magnetic field *B* at the centre?

Q.7 A non-conducting thin ring of radius R and charge q rotates about its axis with an angular velocity. Find the magnetic induction at the centre of the ring.

Q.8 A circular disk of radius R with uniform charge density σ rotates with an angular speed ω. Find the magnetic field at the center of the disk .

Answers

Q.1 (a) $\vec{B} = \frac{\mu_0}{4\pi}\pi I\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ out of the page and $\vec{M} = \frac{1}{2}\pi I\left[R_2^2 - R_1^2\right]$ into the page

(b) $\vec{B} = \frac{\mu_0}{4\pi}\pi I\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$ into the page and $\vec{M} = \frac{1}{2}\pi I\left[R_2^2 + R_1^2\right]$ into the page

Q.2 $B = (\pi - \varphi + \tan\phi)\ \mu_0 I / 2\pi R$

Q.3 $B = \frac{\mu_0 I}{4\pi}\left(\frac{2\pi - \varphi}{a} + \frac{\varphi}{b}\right)$

Q.4 $B = (\mu_0 / 4\pi)\ (1 + 3\pi/2)\ I / R$

Q.5 (a) $8\pi \times 10^{-4}$ T (b) zero

Q.6 1.8 mT

Q.7 $B = \frac{\mu_o q\,\omega}{4\pi\, R}$

Q.8 $B = \frac{1}{2}\mu_0\sigma\omega R$

Magnetic Field due to a Circular Current Loop at a point on its axis

Consider a circular loop of radius a carrying a current i. We have to find the magnetic field at a Point P on the axis of the loop at a distance d from its centre O. In figure

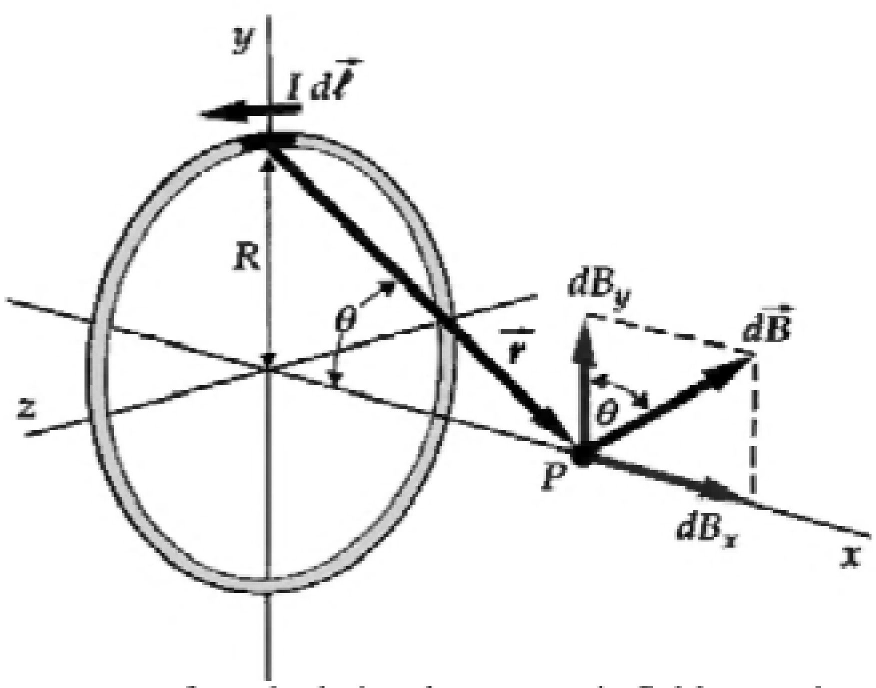


Figure shows the geometry for calculating the magnet ic field at a point on the axis of a circular current loop a distance x from its center. We first consider the current element at the top of the loop. Here, as everywhere around the loop, $Id\vec{\ell}$ is tangent to the loop and perpendicular to the vector $\vec{r}$ from the current element to the field point P. The magnetic field $d\vec{B}$ due to this element is in the direction shown in the figure, perpendicular to $\vec{r}$ and also perpendicular to $Id\vec{\ell}$ The magnitude of $d\vec{B}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin 90}{r^2}$$

When we sum around all the current elements in the loop, the components of $d\vec{B}$ perpendicular to the axis of the loop, such as dB_y in Figure sum to zero, leaving only the components dB_x that are parallel to the axis. We thus compute only the x component of the field.
From Figure , we have

$$B_x = \int dB\sin\theta = \int \sin\theta \frac{\mu_0 I}{4\pi r^2} dl = \frac{\mu_0 I}{4\pi r^2}\sin\theta \int dl = \frac{\mu_0 I}{4\pi r^2}\frac{R}{r}(2\pi R) = \frac{\mu_0 I(2\pi R^2)}{4\pi r^3}$$

using the facts that

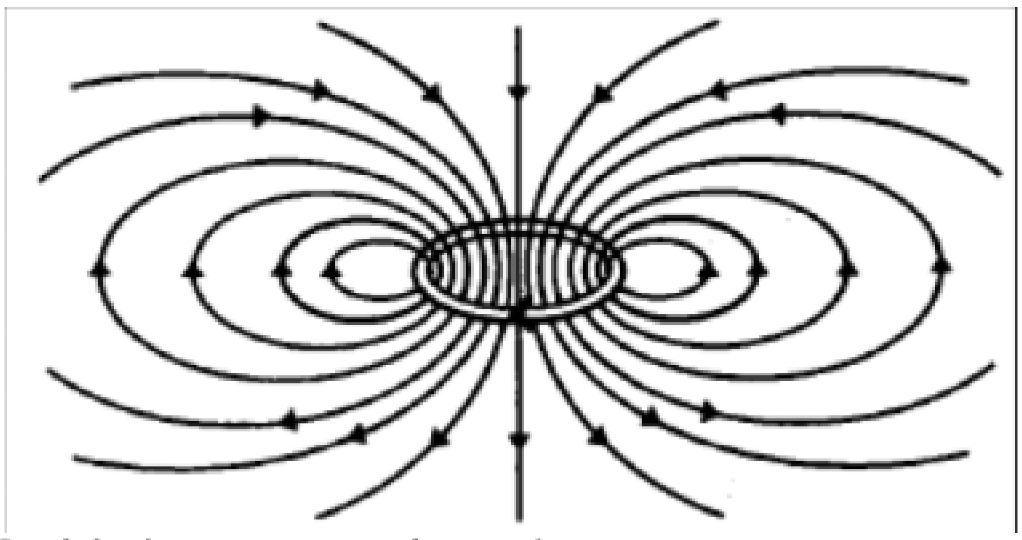
$$r^2 = x^2 + R^2$$

we get

$$B = \frac{\mu_0 I(2\pi a^2)}{4\pi(a^2 + x^2)^{3/2}}$$

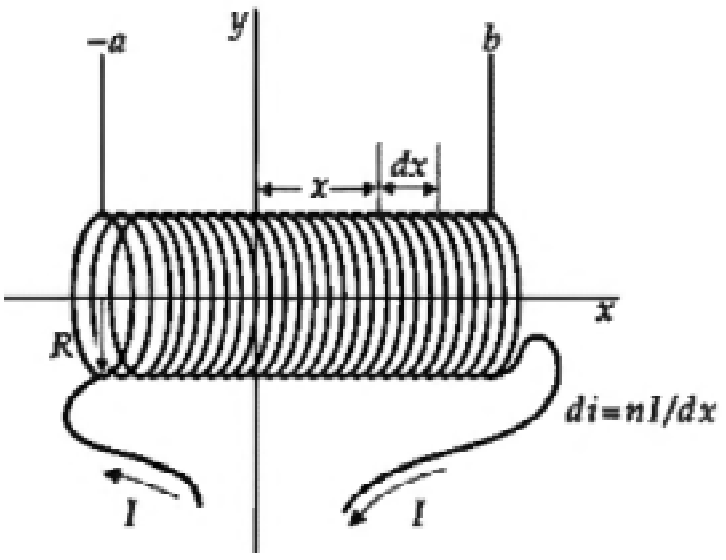
Note: If loop has N close turns then field becomes N times the field due to one turn.

Magnetic field lines due to a circular current



Magnetic Field due to a Solenoid at a point on its axis

A solenoid is a long cylindrical helix, which is obtained by winding closely a large number of turns of insulated copper wire over an in tube of insulating material. When electric current is passed through it, a magnetic field is produced around and within the solenoid.



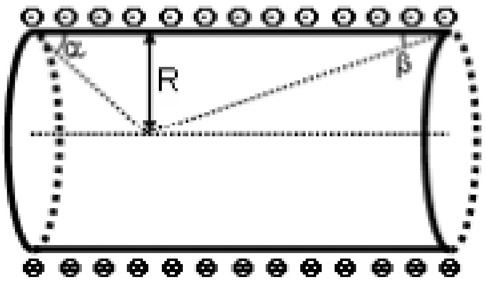
Consider a solenoid of length L consisting of N turns of wi re carrying a current I. We choose the axis of the solenoid to be along the x axis,with the left end at x = -a and the right end at x = +b as shown in Figure . We will calculate the magnetic field at the origin. The figure shows an element of the solenoid of length dx at a distance x from the origin. If n = N/L is the number of turns per unit length, there are ndx turns of wire in this element, with each turn carrying a current I. The element is thus equivalent to a single loop carrying a current di = nIdx. The magnetic field at a point on the x axis due to a loop at the origin carrying a current nI dx is given by Equation with I replaced by nIdx:

$$dB = \frac{\mu_0 (ndx) i R^2}{2(R^2 + x^2)^{3/2}} \text{ along the axis}$$

Net magnetic induction

$$B = \int dB = \int_{-l_2}^{l_1} \frac{\mu_0 n i R^2 dx}{2(R^2 + x^2)^{3/2}}$$

$$\Rightarrow \quad B = \frac{\mu_0 n i}{2}\left[\frac{a}{\sqrt{R^2 + l_1^2}} + \frac{b}{\sqrt{R^2 + l_2^2}}\right] = \frac{\mu_0 n i}{2}[\cos\theta_1 + \cos\theta_2]$$



$$B = \frac{\mu_0 n i}{2}[\cos\alpha + \cos\beta]$$

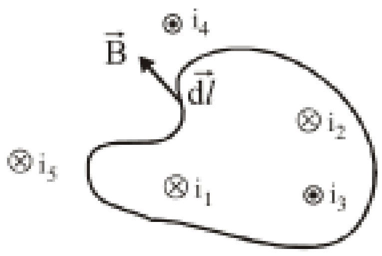
Ampere's Law :

Like Gauss's law in electrostatics, this law provides us a simple method to find magnetic fields in cases of symmetry

Ampere's law gives another method to calculate the magnetic field due to a given current distribution.

Statement: The circulation $\oint \vec{B}\cdot d\vec{l}$ of the resultant magnetic field (of a closed circuit or an infinite wire containing steady current) along a closed path (called amperian path) is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i.$$



In figure , the positive side is going into the plane of the diagram so that i_1 and i_2 are positive and i_3 is negative. Thus, the total current crossing the area is $i_1 + i_2 - i_3$. Any current outside the area is not included in writing the right - hand side of equation . The magnetic field $\vec{B}$ on the left -hand side is the resultant field due to all the currents existing anywhere.

Ampere's law may be derived from the Biot-Savart law and Bio-Savart law may be derived from the Ampere's law. Thus, the two are equivalent in scientific content. However, Ampere's law is useful under certain symmetrical conditions.

Justification of Ampere's law

Let us consider a long straight wire carrying a current I in upward direction. Now take a circular path of radius r symmetric to the wire.Let us now divide a circular path of radius *r* into a large number of small length vectors $\Delta\vec{s} = \Delta s\hat{\varphi}$, where $\hat{\phi}$ point along the tangential direction with magnitude Δs (Figure).

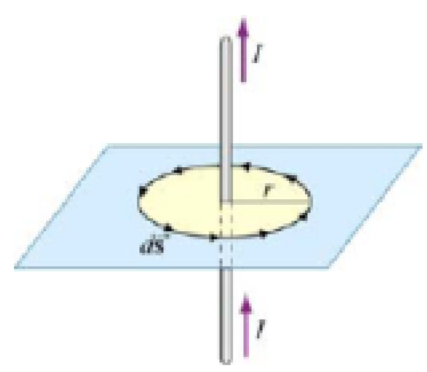


Figure : Amperian loop

In the limit $\Delta\vec{s} \to \vec{0}$, we obtain

$$\oint \vec{B}.d\vec{s} = B\oint ds = \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r) = \mu_0 I$$

The result above is obtained by choosing a closed path, or an "Amperian loop" that follows one particular magnetic field line. Let's consider a slightly more complicated Amperian loop, as that shown in Figure.

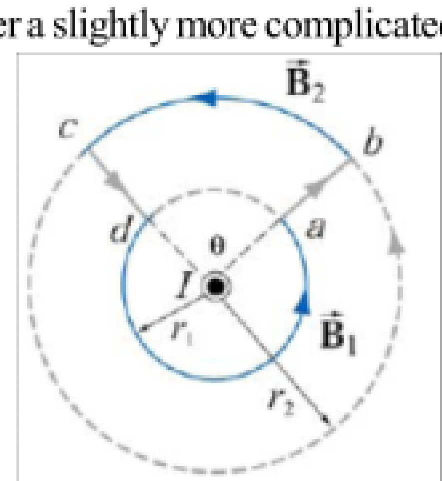


Figure : An Amperian loop involving two field lines

The line integral of the magnetic field around the contour *abcda* is

$$\oint_{abcda} \vec{B}.d\vec{s} = \int_{ab} \vec{B}.d\vec{s} + \int_{bc} \vec{B}.d\vec{s} + \int_{cd} \vec{B}.d\vec{s} + \int_{cd} \vec{B}.d\vec{s} = 0 + B_2(r_2\theta) + 0 + B_1[r_1(2\pi - \theta)]$$

where the length of arc *bc* is $r_2\theta$, and $r_1(2\pi - \theta)$ for arc *da*. The first and the third integrals vanish since the magnetic field is perpendicular to the paths of integration. With $B_1 = \mu_0 I/2\pi r_1$ and $B_2 = \mu_0 I/2\pi r_2$, the above expression becomes

$$\oint_{abcda} \vec{B}\,.\,d\vec{s} = \frac{\mu_0 I}{2\pi r_2}(r_2\theta) + \frac{\mu_0 I}{2\pi r_1}[r_1(2\pi - \theta)] = \frac{\mu_0 I}{2\pi}\theta + \frac{\mu_0 I}{2\pi}(2\pi - \theta) = \mu_0 I$$

We see that the same result is obtained whether the closed path involves one or two magnetic field lines. As shown above Example, in polar coordinates (r, φ) with current flowing in the +*z*-axis, the magnetic field is given by $\vec{B} = (\mu_0 I / 2\pi r)\hat{\varphi}$. An arbitrary length element in the polar coordinates can be written as

: $$d\vec{s} = dr\,\hat{r} + r d\phi\,\hat{\phi}$$

which implies

$$\oint_{\text{closed path}} \vec{B} \,.\, d\vec{s} = \oint_{\text{closed path}} \left(\frac{\mu_0 I}{2\pi r}\right) r d\varphi = \frac{\mu_0 I}{2\pi} \oint_{\text{closed path}} d\varphi = \frac{\mu_0 I}{2\pi}(2\pi) = \mu_0 I$$

In other words, the line integral of $\oint \vec{B} \,.\, d\vec{s}$ around any closed Amperian loop is proportional to I_{enc}, the

current encircled by the loop.

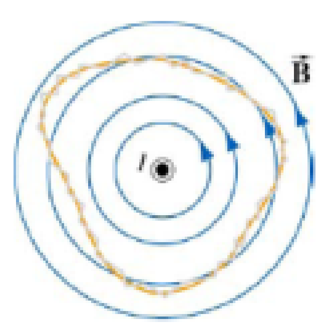


Figure : An Amperian loop of arbitrary shape.

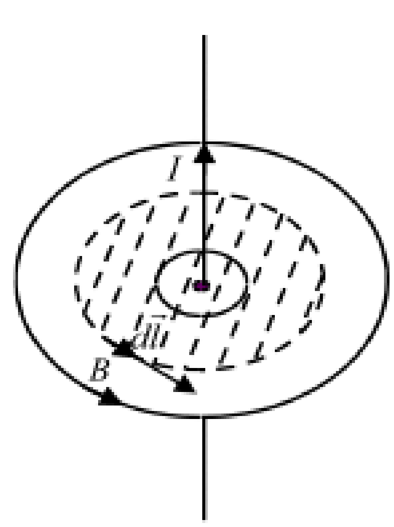
The generalization to any closed loop of arbitrary shape (see for example, Figure) that involves many magnetic field lines is known as Ampere's law:

$$\oint \vec{B} \,.\, d\vec{s} = \mu_0 I_{enc}$$

Calculation of magnetic field by using Ampere's Law :

Calculation of magnetic field due to long straight wire.

Figure shows a long, straight current i. We have to calculate the magnetic field at a point P which is at a distance r from the wire. Figure shows the situation in the plane perpendicular to the wire and passing through P. The current is perpendicular to the plane of the diagram and is coming out of it.



Let us draw a circle passing through the point and with the axis as wire. We put an arrow to show the positive sense of the circle. The radius of the circle is r. The magnetic field due to the long, straight current at any point on the circle is along the tangent as shown in the figure. Same is the direction of the length-element dl there. By symmetry, all points of the circle are equivalent and hence the magnitude of the magnetic field should be the same at all these points. The circulation of magnetic field along the circle is

$$\oint \vec{B} \cdot \vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

The current crossing the area bounded by the circle is

$$\sum I_{en} = +I$$

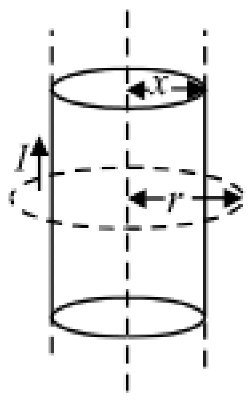
Thus, from Ampere's law,

$$B(2\pi r) = \mu_0 I$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field due to a current carrying thin long pipe.

Case I : $r > R$



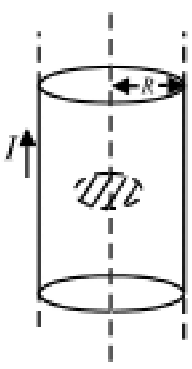
$$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = +I$$

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Case II : $r < R$

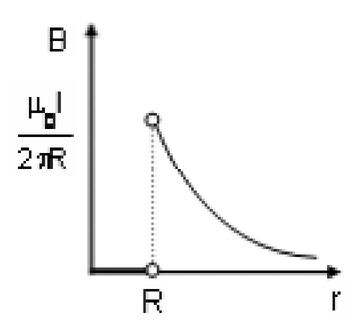


$$\oint \vec{B} \cdot \vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = 0$$

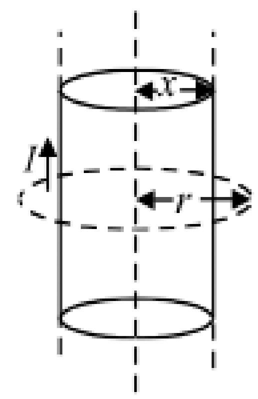
$$\Rightarrow B(2\pi r) = \mu_0(0)$$

$$\Rightarrow B = 0$$



Magnetic field due to current carrying rod having uniform current density.

Case I : $r > R$



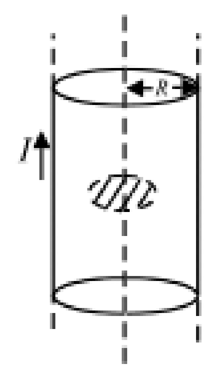
$$\oint \vec{B}\cdot\vec{dl} = \oint B\cdot dl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = +I$$

$$\Rightarrow\ B(2\pi r) = \mu_0 I$$

$$\Rightarrow\ B = \frac{\mu_0 I}{2\pi r}$$

Case II : $r < R$

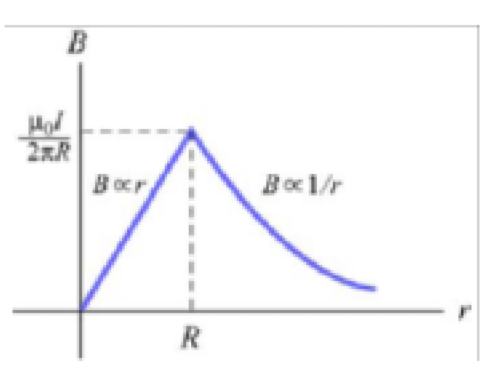


$$\oint \vec{B}\cdot\vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = \frac{I}{R^2} r^2$$

$$\Rightarrow\ B(2\pi r) = \mu_0 \frac{I}{R^2} r^2$$

$$\Rightarrow\ B = \frac{\mu_0 I}{2\pi R^2} r$$



Magnetic Field Due to non uniform current density

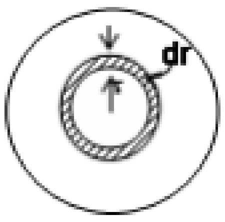
Suppose that the current density in a wire of radius a varies with r according to $J=Kr^2$, where K is a constant and r is the distance from the axis of the wire. We have to find the magnetic field at a point distance r from the axis when (a) $r < a$ and (b) $r > a$

Choose a circular path centred on the axis of the conductor and apply Ampere's law

(a) To find the current passing through the area enclosed by the path integrate

$$dI = JdA = (Kr^2)\,(2\pi r dr)$$

$$\Rightarrow\ I = \int dI = K\int_0^r 2\pi r^3 dr = \frac{K\pi r^4}{2}$$



Since $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$

$$\Rightarrow \quad B2\pi r = \mu_0 \cdot \frac{\pi K r^4}{2} \qquad \Rightarrow B = \frac{\mu_0 K r^3}{4}$$

(b) If $r > a$, then net current through the Amperian loop is

$$I = \int_0^a K r^2 2\pi r dr = \frac{\pi K a^4}{2}$$

$$\Rightarrow \quad B = \frac{\mu_0 K a^4}{4r}$$

Practice Exercise

Q.1 Inside a long straight uniform wire of round cross-section there is a long round cylindrical cavity whose axis is parallel to the axis of the wire and displaced from the latter by a distance $\vec{\ell}$. A direct current of density $\vec{j}$ flows along the wire. Find the magnetic induction inside the cavity. Consider, in particular, the case $I = 0$.

Answers

Q.1 $B = (1/2)\,\mu_0\,[\vec{j} \times \vec{\ell}\,]$, i.e. field inside the cavity is uniform.

Magnetic Field due to long Solenoid :

A solenoid is a long coil of wire tightly wound in the helical form. Figure shows the magnetic field lines of a solenoid carrying a steady current *I*. We see that if the turns are closely spaced, the resulting magnetic field inside the solenoid becomes fairly uniform, provided that the length of the solenoid is much greater than its diameter. For an "ideal" solenoid, which is infinitely long with turns tightly packed, the magnetic field inside the solenoid is uniform and parallel to the axis, and vanishes outside the solenoid.

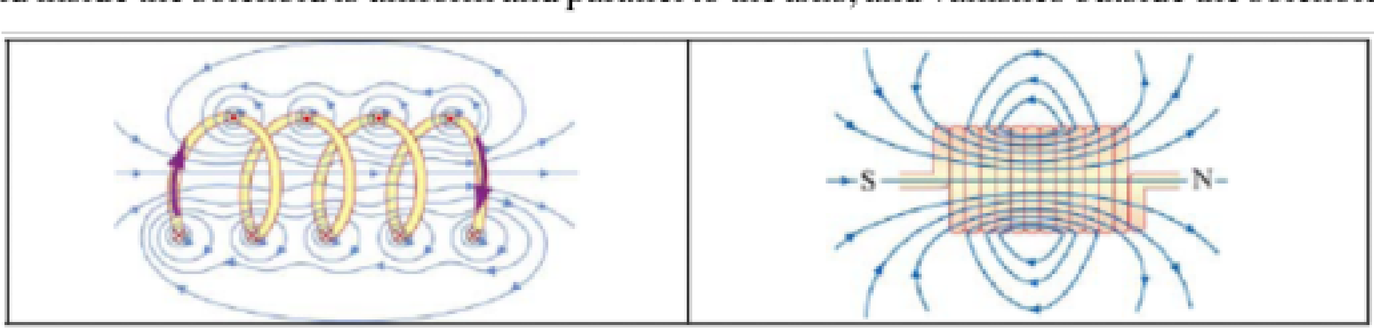


Figure : Magnetic field lines of a solenoid

We can use Ampere's law to calculate the magnetic field strength inside an ideal solenoid. The cross-sectional view of an ideal solenoid is shown in Figure. To compute $\vec{B}$, we consider a rectangular path of length ℓ and width w and traverse the path in a counterclockwise manner. The line integral of $\vec{B}$ along this loop is

$$\oint \vec{B} \cdot d\vec{s} = \int_1 \vec{B} \cdot d\vec{s} + \int_2 \vec{B} \cdot d\vec{s} + \int_3 \vec{B} \cdot d\vec{s} + \int_4 \vec{B} \cdot d\vec{s}$$
$$= 0 + 0 + B\ell + 0$$

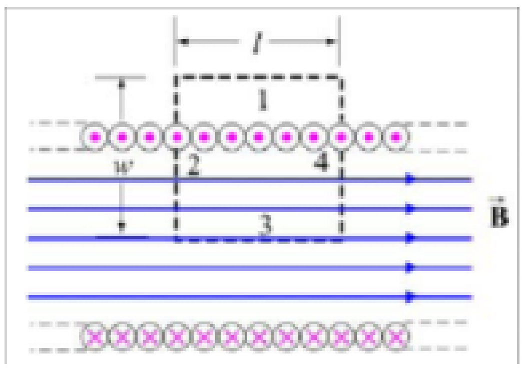


Figure : Amperian loop for calculating the magnetic field of an ideal solenoid.

In the above, the contributions along sides 2 and 4 are zero because $\vec{B}$ is perpendicular to $d\vec{s}$. In addition, $\vec{B} = \vec{0}$ along side 1 because the magnetic field is non-zero only inside the solenoid. On the other hand, the total current enclosed by the Amperian loop is $I_{enc} = nlI$, where n is the total number of turns per unit length. Applying Ampere's law yields

$$\oint \vec{B}.d\vec{s} = B\ell = \mu_0 n\ell I \qquad\qquad B = \mu_0 nI$$

Magnetic Field due to Toroid

Consider a toroid which consists of *N* turns, as shown in Figure. Find the magnetic field everywhere.

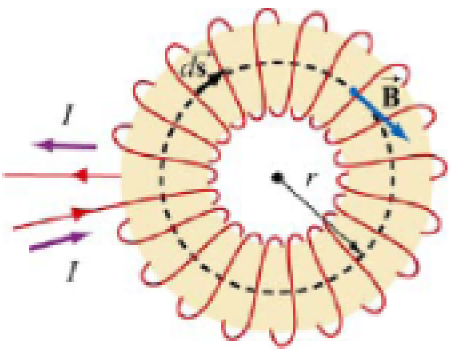


Figure : A toroid with *N* turns

One can think of a toroid as a solenoid wrapped around with its ends connected. Thus, the magnetic field is completely confined inside the toroid and the field points in the azimuthal direction (clockwise due to the way the current flows, as shown in Figure 9.4.5.)

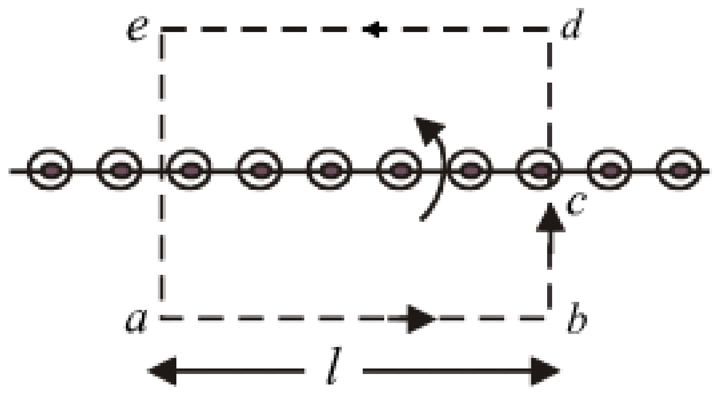
Applying Ampere's law, we obtain

$$\oint \vec{B}.d\vec{s} = \oint B ds = B\oint ds = B(2\pi r) = \mu_0 NI$$

or $$B = \frac{\mu_0 NI}{2\pi r}$$

where *r* is the distance measured from the center of the toroid. Unlike the magnetic field of a solenoid, the magnetic field inside the toroid is non-uniform and decreases as $1/r$.

Magnetic Field thin sheet of infinite dimension carrying a current of uniform linear current density 'i'.



$$\oint \vec{B}\cdot\vec{dl} = \int_a^b \vec{B}\cdot\vec{dl} + \int_b^c B\cdot\vec{dl} + \int_c^d \vec{B}\cdot\vec{dl} + \int_d^e \vec{B}\cdot\vec{dl} + \int_e^f \vec{B}\cdot\vec{dl} + \int_f^g \vec{B}\cdot\vec{dl}$$

$$= \int_a^b B\cdot\cos 0\cdot dl + 0 + 0 + \int_d^e Bdl + 0 + 0 = B\int_a^b dl + B\int_d^e dl = Bl + Bl = 2Bl$$

$$\sum I_{en} = il$$

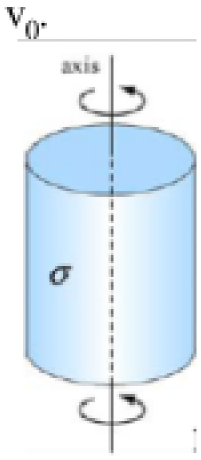
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 \sum I_{en}$$

$$\Rightarrow \quad 2Bl = \mu_0 il$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{2}$$

Practice Exercise

Q.1 We have a long cylindrical shell of non-conducting material which carries a surface charge fixed in place (glued down) of σ C/m^2, as shown in Figure. The cylinder is suspended in a manner such that it is free to revolve about its axis, without friction. Initially it is at rest. We come along and spin it up until the speed of the surface of the cylinder is v_0.

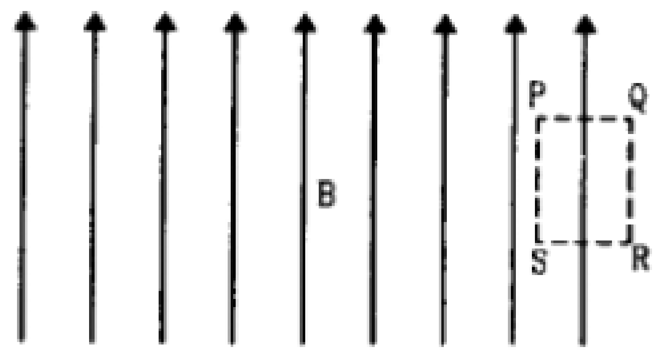


(a) What is the surface current K on the walls of the cylinder, in A/m?

(b) What is magnetic field inside the cylinder?

(c) What is the magnetic field outside of the cylinder? Assume that the cylinder is infinitely long.

Q.2 Sometimes we show an idealised magnetic field which is uniform in a given region and falls to zero abruptly. One such field is represented in figure . Using Ampere's law over the path PQRS, show that such a field is not possible.



Answers

Q.1 (a) $K = \sigma v_0$

(b) $B = \mu_0 K = \mu_0 \sigma v_0$, oriented along axis right-handed with respect to spin (c) 0

Magnetic Field of a Moving Point Charge :

Suppose we have an infinitesimal current element in the form of a cylinder of cross-sectional area *A* and length *ds* consisting of *n* charge carriers per unit volume, all moving at a common velocity $\vec{v}$ along the axis of the cylinder. Let *I* be the current in the element, which we define as the amount of charge passing through any cross-section of the cylinder per unit time. From Chapter 6, we see that the current *I* can be written as

$$nAq|\vec{v}| = I$$

The total number of charge carriers in the current element is simply $dN = nAds$, so that the magnetic field $d\vec{B}$ due to the *dN* charge carriers is given by

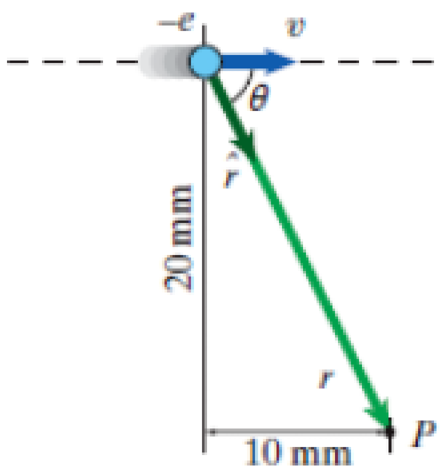
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(nAq|\vec{v}|)d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(nAds)q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(dN)q\vec{v}\times\hat{r}}{r^2}$$

where *r* is the distance between the charge and the field point *P* at which the field is being measured, the unit vector $\hat{r} = \vec{r}/r$ points from the source of the field (the charge) *to P*. The differential length vector $d\vec{s}$ is defined to be parallel to $\vec{v}$. In case of a single charge, $dN = 1$, the above equation becomes

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Practice Exercise

Q.1 An electron carrying a charge $e = -1.6 \times 10^{-19}$ C moves in a straight line at a speed $v = 3 \times 10^{7}$ m/s. What is the magnitude and direction of the magnetic field caused by the electron at a point P, 10 mm ahead of the electron and 20 mm away from its line of motion, as illustrated in figure?



Answers

Q.1 8.8×10^{-14} T

Force on a moving charge in magnetic field :

Consider a particle of charge q and moving at a velocity $\vec{v}$. Experimentally we have the following observations:

(1) The magnitude of the magnetic force $\vec{F}_B$ exerted on the charged particle is proportional to both v and q.

(2) The magnitude and direction of $\vec{F}_B$ depends on $\vec{v}$ and $\vec{B}$.

(3) The magnetic force $\vec{F}_B$ vanishes when $\vec{v}$ is parallel to $\vec{B}$. However, when $\vec{v}$ makes an angle θ with h $\vec{B}$, the direction of $\vec{F}_B$ is perpendicular to the plane formed by and $\vec{v}$ and $\vec{B}$ and the magnitude of $\vec{F}_B$ is proportional to $\sin\theta$.

(4) When the sign of the charge of the particle is switched from positive to negative (or vice versa), the direction of the magnetic force also reverses.

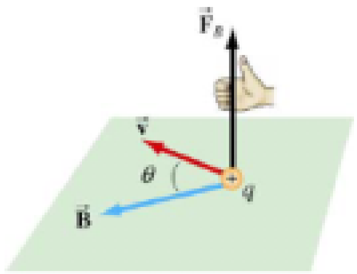


Figure : The direction of the magnetic force

The above observations can be summarized with the following equation:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

The above expression can be taken as the working definition of the magnetic field at a point in space.

The magnitude of $\vec{F}_B$ is given by

$$F_B = |q|\, vB \sin\theta$$

The SI unit of magnetic field is the tesla (T) :

$$1 \text{ tesla} = 1\text{T} = 1 \frac{\text{Newton}}{(\text{Coulomb})(\text{meter}/\text{second})} = 1 \frac{\text{N}}{\text{C.m/s}} = 1 \frac{\text{N}}{\text{A.m}}$$

Another commonly used non-SI unit for $\vec{B}$ is the *gauss* (G), where $1\text{T} = 10^4\text{G}$

Note that $\vec{F}_B$ is always perpendicular to $\vec{v}$ and $\vec{B}$, and cannot change the particle's speed v (and thus the kinetic energy). In other words, magnetic force cannot speed up or slow down a charged particle. Consequently, **Magnetic field does no work on charged particle**:

$$dW = \vec{F}_B \cdot d\vec{s} = q(\vec{v} \times \vec{B}) \cdot \vec{v}dt = q(\vec{v} \times \vec{v}) \cdot \vec{B}dt = 0$$

The direction of $\vec{v}$ however, can be altered by the magnetic force

Illustration :

Two very long, straight, parallel wires carry steady currents I and -I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires.Find magnitude of the force due to the magnetic field acting on the charge .

Sol. *Since the currents are flowing in the opposite directions, the magnetic field at a point equidistant from the two wires will be zero. Hence, the force acting on the charge at this instant will be zero.*

Illustration :

An electron with mass m, velocity v and charge e describes half a revolution in a circle of radius r in a magnetic field B,find the energy acquireed by electron.

Sol. *As energy can neither be created nor destroyed, therefore, its energy will remain constant and will acquire no extra energy.*

Practice Exercise

Q.1 A circular loop of radius 20 cm carries a current of 10 A. An electron crosses the plane of the loop with a speed of 2.0 x 106 m/s. The direction of motion makes an angle of 30° with the axis of the circle and passes through its centre. Find the magnitude of the magnetic force on the electron at the instant it crosses the plane.

Q.2 Two protons move parallel to each other with an equal velocity v. Find the ratio of forces of magnetic and electrical interaction of the protons.

Answers

Q.1 $16\pi \times 10^{-19}\ N$ Q.2 $(v/c)^2$

Motion of charged particle in uniform magnetic field

There are three possible paths in which a charged particle may move in presence of uniform magnetic field which is uniform in space.

(a) Straight line path

(b) Circular path

(c) Helical path

We shall see them one by one.

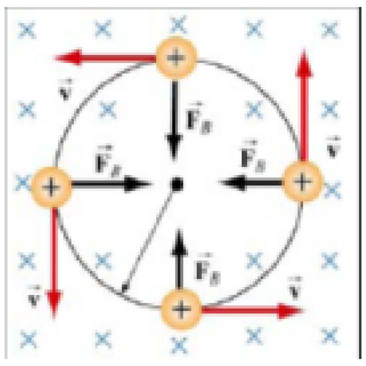
Straight line path

When the charged particle projected in the direction of or opposite to uniform magnetic field , magnetic field exerts no force hence it will travel along straight line with fixed speed.

Circular path

If a particle of mass *m* moves in a circle of radius *r* at a constant speed *v*, what acts on the particle is a radial force of magnitude $F = mv^2/r$ that always points toward the center and is perpendicular to the velocity of the particle.

In previous section, we have also shown that the magnetic force $\vec{F}_B$ always points in the direction perpendicular to the velocity $\vec{v}$ of the charged particle and the magnetic field $\vec{B}$. Since $\vec{F}_B$ can do not work, it can only change the direction of $\vec{v}$ but not its magnitude. What would happen if a charged particle moves through a uniform magnetic field $\vec{B}$ with its initial velocity $\vec{v}$ at a right angle to $\vec{B}$**?** For simplicity, let the charge be $+q$ and the direction of $\vec{B}$ be into the page. It turns out that $\vec{F}_B$ will play the role of a centripetal force and the charged particle will move in a circular path in a counterclockwise direction, as shown in Figure.



$$qvB = \frac{mv^2}{r}$$

the radius of the circle is found to be $r = \dfrac{mv}{qB}$

The period *T* (time required for one complete revolution) is $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{v}\dfrac{mv}{qB} = \dfrac{2\pi m}{qB}$

Similarly, the angular speed of the particle can be obtained as $\omega = 2\pi f = \dfrac{v}{r} = \dfrac{qB}{m}$

Illustration :

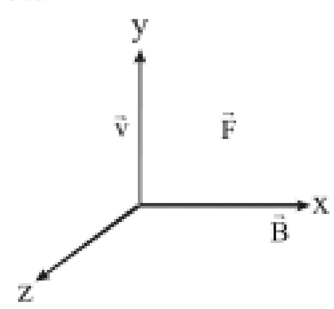
A uniform magnetic field of 30 mT exists in the + X direction. A particle of charge +e and mass 1.67×10^{-27} *kg is projected into the field along the + Y direction with a speed of* 4.8×10^{6} *m/s.*

(a) *Find the force on the charged particle in magnitude and direction*

(b) *Find the force if the particle were negatively charged.*

(c) *Describe the nature of path followed by the particle in the both the case.*

Sol. (a) *Force acting on a charge particle moving in the magnetic field*



$$\vec{F}=q(\vec{v}\times\vec{B})$$

Magnetic field $\vec{B}=30(\text{mT})\hat{j}$

Velocity of the charge particle $\vec{v}=4.8\times10^{6}$ *(m/s)* $\hat{j}$

$$\vec{F}=1.6\times10^{-19}[(4.8\times10^{6}\hat{j})\times(30\times10^{-3})(\hat{i})]$$

$$\vec{F}=230.4\times10^{-16}(-k)\text{N}\cdot$$

(b) If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+z) direction.

(c) *Ad* $v\perp B$, *the path described is a circle*

$$R=\frac{mv}{qB}=(1.67\times10^{-27}).(4.8\times10^{6})/(1.6\times10^{-19}).(30\times10^{-3})$$

$$= 1.67\ m.$$

Illustration :

A positively charged particle of charge q and mass m first accelerated by a voltage v then injected into uniform magnetic field. Find radius of the circle traced by it.

Sol. *Kinetic energy of the particle is*

$$K = qV$$

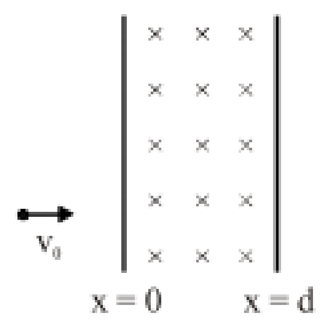
linear momentum of the particle will be

$$p = mv = \sqrt{2mk} = \sqrt{2mqv}$$

$$\therefore \quad r=\frac{p}{qB}=\frac{\sqrt{2mqv}}{qB}$$

Illustration :

A positively particle having chrage q and mass m moving with velocity $v_0\ \hat{i}$ *enters a region in which uniform magnetice field* $\vec{B}=-B_0\hat{k}$ *exist. The magnetic field region extends upto x = 0 to x = d (figrue). Find the time spent by the particle in magentic field if*

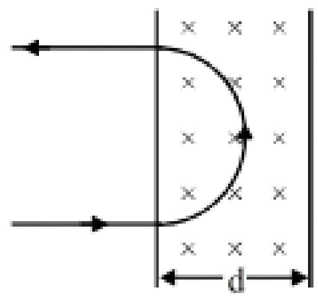


(i) $d=\dfrac{1.5mv}{qB}$ *(ii)* $\dfrac{mv}{qB}$

In second case also find the side ways deflection.

Sol. *Since the velocity of the particle is perpendicular to the magnetic field hence motion inside magenetic field will be circle or part of circle*

(i) *Since* $d = \dfrac{1.5mv}{qB} \Rightarrow d > r$



$\Rightarrow$ *particle has surfficient space to turn back.*

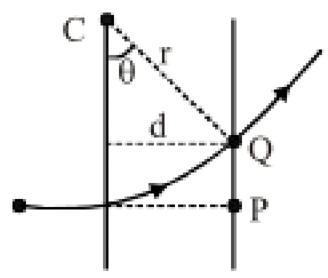
$\Rightarrow$ *particle will complete half circle (figure)*

$\therefore$ *time spent* $= \dfrac{T}{2} = \dfrac{\pi m}{qB}$

(ii) *since* $d = \dfrac{mv}{2qB} \qquad \Rightarrow d < r$

This means particle will not get surficient space between the boundries to turn back. Hence particle will come out of the boundry $x = d$ *(figure)*

From figure



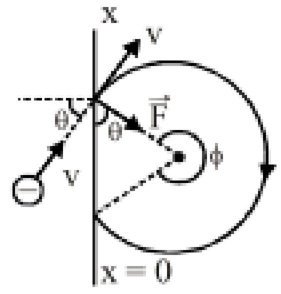
$$\sin\theta = \frac{d}{r} = \frac{\dfrac{mv}{2qB}}{\dfrac{mv}{qB}} = \frac{1}{2}$$

$$\Rightarrow \qquad \theta = \frac{\pi}{6}$$

$$\therefore \qquad \text{distance travelled} = r\theta = \left(\frac{mv}{qB}\right)\left(\frac{\pi}{6}\right) = \frac{\pi mv}{6qB}$$

$$\therefore \qquad \text{time spent} = \frac{\text{distance}}{\text{speed}} = \frac{\dfrac{\pi mv}{6qB}}{v} = \frac{\pi}{6qB}$$

side ways deflection $= PQ = r\,(1 - \cos\theta) = \dfrac{mv}{qB}\left(1 - \dfrac{\sqrt{3}}{2}\right)$



From geometry

$$\phi = \pi + 2\theta$$

$$\therefore \qquad \text{distance travelled} = r\,\phi = \frac{mv}{qB}(\pi + 2\theta)$$

$$\therefore \qquad \text{time} = \frac{\text{distance travelled}}{\text{speed}} = \frac{m}{qB}(\pi + 2\theta)$$

Illustration :

H^+, He^+ and O^{++} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{++} are 1 amu, 4 amu and 16 amu respectively. Comment on their amount of deflection

Sol. $$Bqv = \frac{mv^2}{r}$$

$$\Rightarrow \quad Bqr = mv = \text{momentum} = \sqrt{2mE}$$

Where E = Kinetic Energy

$$\therefore \quad r = \frac{\sqrt{2mE}}{Bq}$$

if r_1 *and* r_2 *and* r_3 *are the radius of circular track of* H^+, He^{++} and O^{++}

$$\therefore \quad r_1 : r_2 : r_3 = \frac{\sqrt{2mE}}{Bq} : \frac{\sqrt{2(4m)E}}{Bq} : \frac{\sqrt{2(16m)E}}{B(2q)}$$

$$1 : 2 : 2$$

Hence O^{2+} *will be deflected most whereas* He^+ *and* O^{2+} *will be deflected equally*

Illustration :

Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and desribe circular paths of radii R_1 *and* R_2*, respectivley. Find the ratio of the mass of X to that of Y.*

Sol. *Let the masses be* m_1 *and* m_2 *respectively of X and Y. If E is energy gained by charged particle in electric field.*

$$Bqv = \frac{mv^2}{r} \quad \Rightarrow \quad Bqr = \sqrt{2mE}$$

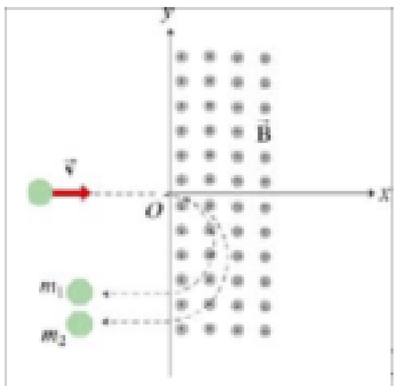
$$R_1 = \frac{\sqrt{2m_1E}}{Bq}; \; R_2 = \frac{\sqrt{2m_2E}}{Bq}$$

$$\frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}} \quad \Rightarrow \quad \frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

Practice Exercise

Q.1 Particle A with charge q and mass m_A and particle B with charge $2q$ and mass m_B, are accelerated from rest by a potential difference ΔV, and subsequently deflected by a uniform magnetic field into semicircular paths. The radii of the trajectories by particle A and B are R and $2R$, respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. What is their mass ratio?

Q.2 Suppose the entire x-y plane to the right of the origin O is filled with a uniform magnetic field $\vec{B}$ pointing out of the page, as shown in Figure.

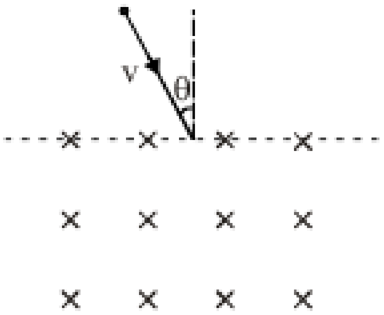


Two charged particles travel along the x axis in the positive x direction, each with speed v, and enter the magnetic field at the origin O. The two particles have the same charge q, but have different masses, m_1 and m_2. When in the magnetic field, their trajectories both curve in the same direction, but describe semi-circles with different radii. The radius of the semi-circle traced out by particle 2 is exactly *twice* as big as the radius of the semi-circle traced out by particle 1.

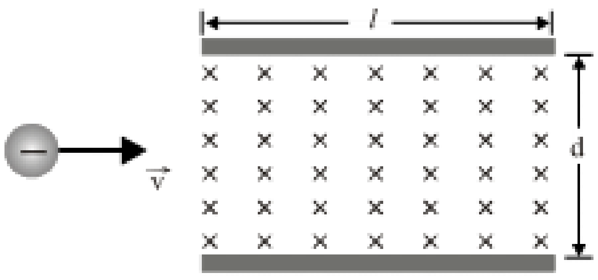
(a) Is the charge q of these particles such that $q > 0$, or is $q < 0$?

(b) What is the ratio m_2 / m_1 ?

Q.3 A particle of mass m and positive charge q, moving with a uniform velocity v, enters a magnetic field B as shown in figure (a) Find the radius of the circular arc it describes in the magnetic field. (b) Find the angle subtended by the arc at the centre. (c) How long does the particle stay inside the magnetic field? (d) Solve the three parts of the above problem if the charge q on the particle is negative.



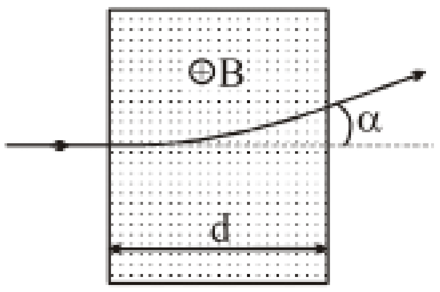
Q.4 A particle of charge is moving with a velocity $\vec{v}$. It then enters midway between two long plates ($\ell \to \infty$) where there exists a uniform magnetic field pointing into the page, as shown in Figure. Assume the space to be electric field and gravity free. Take $d = \frac{mv}{qB}$



(a) Is the trajectory of the particle deflected upward or downward?

(b) Compute the distance between the left end of the plate and where the particle strikes.

Q.5 A proton accelerated by a potential difference V = 500 kV flies through a uniform transverse magnetic field with induction B = 0.51 T. The field occupies a region of space d = 10 cm in thickness (Fig.). Find the angle a through which the proton deviates from the initial direction of its motion.



Answers

Q.1 $\frac{m_A}{m_B} = \frac{1}{8}$

Q.2 (a) – ve (b) 2:1

Q.3 (a) $\frac{mv}{qB}$ (b) $\pi - 2\theta$ (c) $\frac{m}{qB}(\pi - 2\theta)$ (d) $\frac{mv}{qB}, \pi + 2\theta, \frac{m}{qB}(\pi + 2\theta)$

Q.4 (a) downward (b) $d\frac{\sqrt{3}}{2}$

Q.5 $a = \sin^{-1}\left(dB\sqrt{\frac{q}{2mV}}\right) = 30°$

Helical Path:

In this situation velocity of the particle can be resolved into two components.

(1) $v_{||}$ → projection of velocity parallel to the magnetic fd.

(2) $v_{\perp}$ → projection of velocity perpendicular to the magnetic fd.

Due to $v_{\perp}$ motion of the charged particle will be uniform circular in the plane perpendicular to the fd. whereas

$v_{||}$ remain unaffected as it is perpendicular to magnetic force.

As a whole its motion is helical with constant pitch.

radius of the helix will be

$$r = \frac{mv_{\perp}}{qB}$$

pitch of the helix will be

$$p = \frac{mv_{||}}{qB}$$

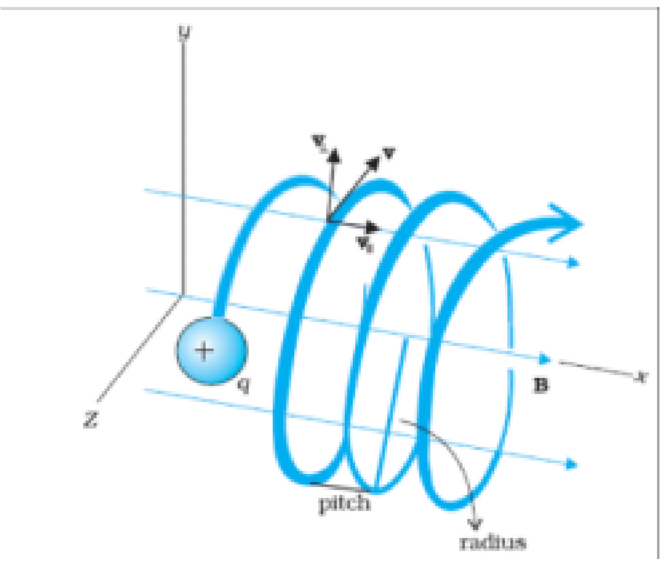


Illustration :

A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the x-axis. There is uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. Find the condition so that P will also strike if you can change magnetic field and speed

Sol. *Let d = distance of the tanget T from the point of projection. P will strike T if d an integral multiple of the pitch.*

Pitch

$$\left(2\pi\frac{m}{qB_0}\right)v_0\cos\theta = N\left(2\pi\frac{m}{qB}\right)v\cos\theta$$

Here N is a natural number.

Practice Exercise

Q.1 An electron accelerated by a potential difference V = 1.0 kV moves in a uniform magnetic field at an angle $\alpha = 30°$ to the vector B whose modulus is B = 29 mT. Find the pitch of the helical trajectory of the electron.

Q.2 Can a charged particle be speed up through a uniform magnetic field ?

Q.3 If no work can be done on a charged particle by the magnetic field, how can the motion of the particle be influenced by the presence of a field?

Answers

Q.1 $\Delta l = 2\pi\sqrt{2mV/eB^2}\cos\alpha = 2.0$ cm Q.2 No

Q.3 By changing its direction of motion

Lorentz Force:

In the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force on a charged particle is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

This is known as the Lorentz force.

Velocity Selector:

By combining the two fields, particles which move with a certain velocity can be selected. This was the principle used by J. J. Thomson to measure the charge-to-mass ratio of the electrons. In Figure the schematic diagram of Thomson's apparatus is depicted.

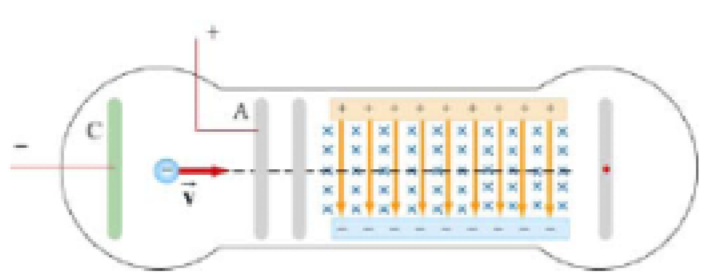


Figure : Thomson's apparatus

The electrons with charge $q = -e$ and mass m are emitted from the cathode C and then accelerated toward slit A. Let the potential difference between A and C be $V_A - V_C = \Delta V$. The change in potential energy is equal to the external work done in accelerating the electrons: $\Delta U = W_{ext} = q\Delta V = -e\Delta V$. By energy conservation, the kinetic energy gained is $\Delta K = -\Delta U = mv^2/2$. Thus, the speed of the electrons is given by

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

If the electrons further pass through a region where there exists a downward uniform electric field, the electrons, being negatively charged, will be deflected upward. However, if in addition to the electric field, a magnetic field directed into the page is also applied, then the electrons will experience an additional downward magnetic force $-e\vec{v} \times \vec{B}$. When the two forces exactly cancel, the electrons will move in a straight path. From Eq., we see that when the condition for the cancellation of the two forces is given by

$eE = evB$. which implies $\quad v = \dfrac{E}{B}$

In other words, only those particles with speed $v = E/B$ will be able to move in a straight line. Combining the two equations, we obtain $\quad \dfrac{e}{m} = \dfrac{E^2}{2(\Delta V)B^2}$

By measuring E, ΔV and B, the charge-to-mass ratio can be readily determined. The most precise measurement to date is $e/m = 1.758820174(71) \times 10^{11}$ C/kg.

Mass Spectrometer:

Various methods can be used to measure the mass of an atom. One possibility is through the use of a mass spectrometer. The basic feature of a *Bainbridge* mass spectrometer is illustrated in Figure. A particle carrying a charge $+q$ is first sent through a velocity selector.

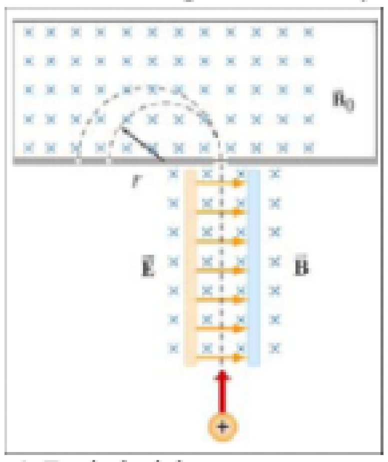


Figure : A Bainbridge mass spectrometer

The applied electric and magnetic fields satisfy the relation $E = vB$ so that the trajectory of the particle is a straight line. Upon entering a region where a second magnetic field $\vec{B}_0$ pointing into the page has been applied, the particle will move in a circular path with radius r and eventually strike the photographic plate. Using Eq., we have $\quad r = \dfrac{mv}{qB_0}$

Since v = E/B, the mass of the particle can be written as $\quad m = \dfrac{qB_0 r}{v} = \dfrac{qB_0 Br}{E}$

Hall's Effect:

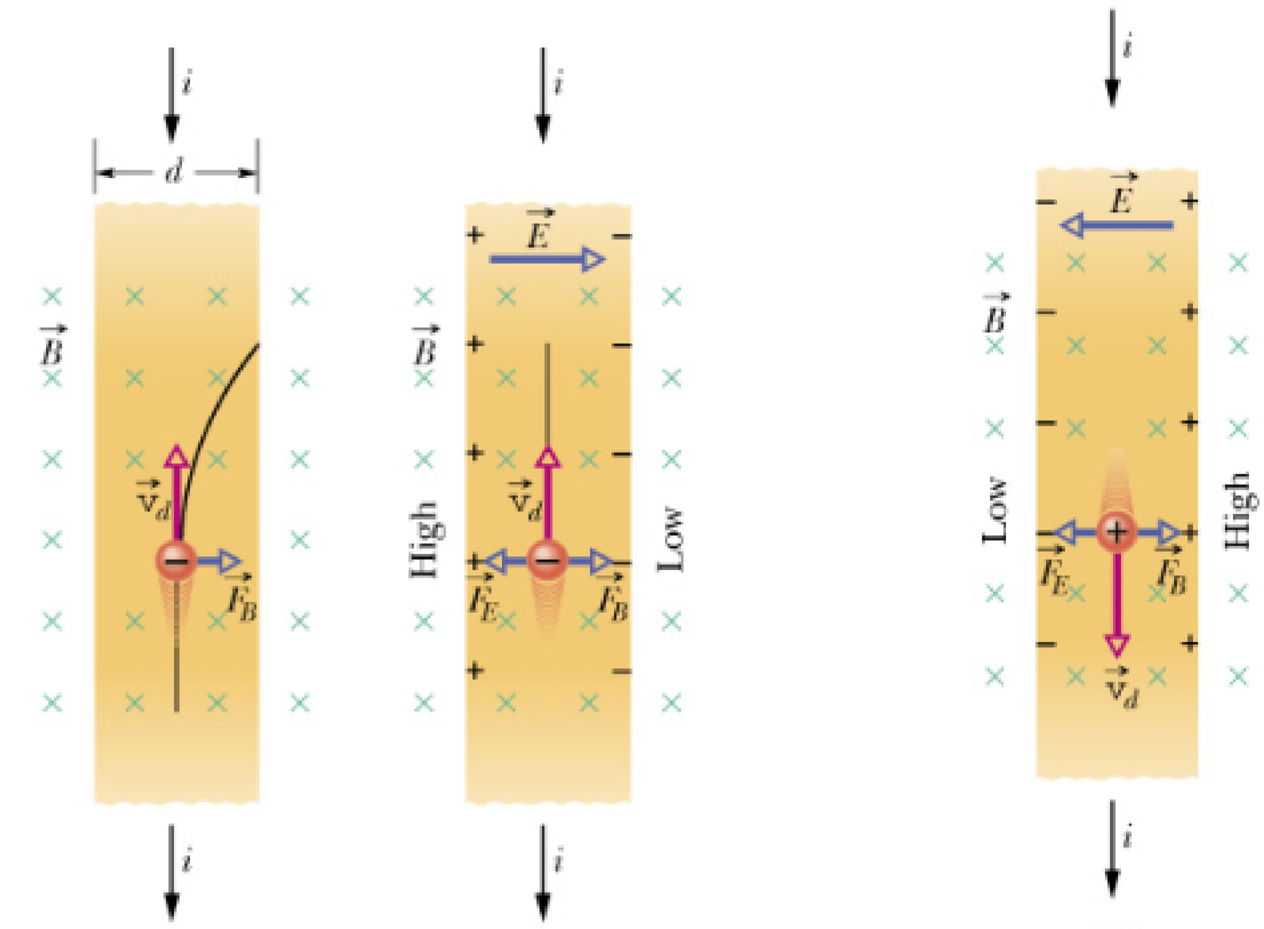
In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can drift electrons in copper wire in presence of magnetic field. This Hall effect allows us to find

" if charge carriers in a conductor are positively or negatively charged.

" the number of charge carriers per unit volume of the conductor.

Consider a strip of current carrying wire kept in external magnetic field. Let the wire has width d, Cross-sectional area A, and charge carriers per unit volume as n.

Figure (a) shows a copper strip of width d, carrying a current i whose conventional direction is from the top of the figure to the bottom. The charge carriers are electrons and, as we know, they drift (with drift speed vd) in the opposite direction, from bottom to top. At the instant shown in figure, an external magnetic field B, pointing into the plane of the figure, has just been turned on. We see that a magnetic force will act on each drifting electron, pushing it towards the right edge of the strip.



(a) (b) (c)

As time goes on, electrons move to the right, mostly piling up on the right edge of the strip, leaving uncompensated positive charges in fixed positions at the left edge. The separation of positive charges on the left edge and negative charges on the right edge produces an electric field E within the strip, pointing from left to right in Fig. b. This field exerts an electric force FE on each electron, tending to push it to the left. Thus, this electric force on the electrons, which opposes the magnetic force on them, begins to build up.

Equilibrium quickly develops in which the electric force on each electron has increased enough to match the magnetic force. When this happens, as Fig. b shows, the force due to B and the force due to E are in balance. The drifting electrons then move along the strip toward the top of the page at velocity vd with no further collection of electrons on the right edge of the strip and thus no further increase in the electric field E.

$$eE = ev_dB \qquad ---(1)$$

$$v_d = \frac{J}{ne} = \frac{i}{neA} \qquad ---(2)$$

A Hall potential difference V is associated with the electric field across strip width d.

$$V = E\,d \qquad ...(3)$$

From (1), (2) and (3)

$$\frac{E}{B}=\frac{V}{Bd}=\frac{i}{neA}$$

$$\therefore n=\frac{idB}{eAV} \qquad \& \ v_d=\frac{i}{neV}=\frac{V}{Bd}$$

By connecting a voltmeter across the width, we can measure the potential difference between the two edges of the strip. Moreover, the voltmeter can tell us which edge is at higher potential. For the situation of Fig. b, we would find that the left edge is at higher potential, which is consistent with our assumption that the charge carriers are negatively charged.

It is also possible to use the Hall effect to measure directly the drift speed vd of the charge carriers, which you may recall is of the order of centimeters per hour. In this clever experiment, the metal strip is moved mechanically through the magnetic field in a direction opposite that of the drift velocity of the charge carriers. The speed of the moving strip is then adjusted until the Hall potential difference vanishes.
At this condition, with no Hall effect, the velocity of the charge carriers with respect to the laboratory frame must be zero, so the velocity of the strip must be equal in magnitude but opposite the direction of the velocity of the negative charge carriers.
For a moment, let us make the opposite assumption, that the charge carriers in current i are positively charged (Fig. c).
Convince yourself that as these charge carriers move from top to bottom in the strip, they are pushed to the right edge by F_B and thus that the right edge is at higher potential. Because that last statement is contradicted by our voltmeter reading, the charge carriers must be negatively charged.

Illustration :

A non-relativistic proton beam passes without deviation through the region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with E = 120 kV/m and B = 50 mT. Then the beam strikes a grounded target. Find the force with which the beam acts on the target if the beam current is equal to I = 0.80 mA.

Sol: $F=\frac{dp}{dt}=v\frac{dm}{dt}=v\frac{dm}{dq}\frac{dq}{dt}=\frac{E}{B}\frac{m}{q}I = 20\ \mu N$

Illustration :

A particle of mass m and charge q is released from the origin in a region occupied by electric field E and magnetic field B,

$$B=-B_0\hat{j};\ E=E_0\hat{i}$$

Find the speed of the particle.

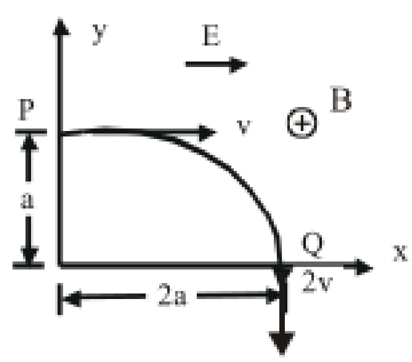
Sol. *Since the magnetic field does not perpform any work, therfore, whatever has been gain in kinetic energy it is only becuase of the work done by electric field. Applying work-energy theorem,*

$$W_E=\Delta K$$

$$qE_0=\frac{1}{2}mv^2-o \quad or \quad v=\sqrt{\frac{2qE_0}{m}}$$

Illustration :

A particle of charge +q and mass m moving under the influence of a uniform electric field $E\hat{i}$ *and a uniform magnetic field* $B\hat{k}$ *follows a trajectory from P to Q as shown in figure. The velocities at P and Q are* $v\hat{i}$ *and* $-2v\hat{i}$ *. Find (a) E (b)rate of work done by the electric field at P (C) rate of work done by each the fields at Q*



Sol. *Increase in Kinetic energy of particle*

$$=\frac{1}{2}m(2v)^2-\frac{1}{2}mv^2=\frac{3}{2}mv^2$$

Work done by the uniform electric field, E, in going from P to Q = (qE) × 2a = 2qEa

Hence, $2qEa=\frac{3}{2}mv^2$

or $\quad E=\frac{3mv^2}{4qa}$

Rate of work done by the electric field at

P_{at}*, P = F. v = qE . v*

$$=qE\hat{i}.v\hat{i}=qEv$$

$$=q.\frac{3mv^2}{4qa}.v=\frac{3}{4}\frac{mv^2}{a}$$

Q is

$$P_{at},Q=qE\hat{i}.(-2v\hat{j})=0$$

At Q, rate of work done by both the fields is zero.

Illustration :

A particle of mass $1\times10^{-26}\,\text{kg}$ *and charge* $1.6\times10^{-19}\,\text{C}$ *traveling with a velocity* $1.28\times10^{6}\,\text{ms}^{-1}$ *in the +x direction enters a region in which a uniform electric field E and uniform magnetic field of induction B are present such that* $E_x = E_y = 0,\ E_z = -102.4\,\text{kVm}^{-1}$ *and* $B_x = B_z = 0$. $B_y = 8\times10^{-2}\,\text{Wbm}^{-2}$. *The particle enters this region at the origin at time t =0. Determine the location (x, y and z cooerdiantes) of the particle at* $t = 5\times10^{-6}\,\text{s}$. *If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at* $t = 7.45\times10^{-6}\,\text{s}$ *?*

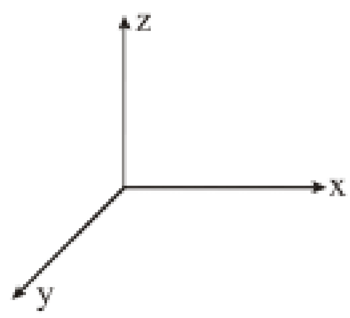
Sol. *Let* $\hat{i}, \hat{j}$ *and* $\hat{k}$ *be unit vector along the positive directions of x, y and z axes. Q = charge on the particle* $= 1.6\times10^{-19}\,\text{C}$ $\vec{v} =$ *velocity of the charged particle* $= (1.28\times10^{6})\text{ms}^{-1}\hat{i}$

$\vec{E} =$ *electric field intensity;*

$$= (-102.4\times10^{3}\,\text{Vm}^{-1})\hat{k}$$

$\vec{B}$ = *magnetic induction of the magnetic field*

$$= (8\times10^{-2}\,\text{Wbm}^{-2})\hat{j}$$



$\therefore$ $\vec{F}_e =$ *electric force on the charge*

$$= q\vec{E} = [1.6\times10^{-19}(-102.4\times10^{3})\text{N}]\hat{k}$$

$$= 163.84\times10^{-16}\,\text{N}(-\hat{k})$$

$\vec{F}_m =$ *magnetic force on the charge* $= q\vec{v}\times\vec{B}$

$$= [1.6\times10^{-19}(1.28\times10^{6})(8\times10^{-2})\text{N}](\hat{i}\times\hat{j})$$

$$= (163.84\times10^{-16}\,\text{N})(\hat{k})$$

The two forces $\vec{F}_e$ *and* $\vec{F}_m$ *are along z-axis and equal, opposite and collinear.*

The net force on the charge is zero and hence the particle does not get deflected and continues to travel along x-axis.

(a)At time $t = 5\times10^{-6}\,\text{s}$

$$x = (5\times10^{-6})(1.28\times10^{6}) = 6.4\,\text{m}$$

$\therefore$ *Coordinates of the particle = (6.4 m, 0, 0)*

(b) When the electric field is switched off, the particle is in the uniform magnetic field perpendicular to its velocity only and has a uniform circular motion in the x-z plane (i.e. the plane of velocity and magnetic force), anticlockwise as seen along +y axis.

Now, $\frac{mv^2}{r} = qvB$ *where r is the radius of the circle.*

$$\therefore \quad r = \frac{mv}{qB} = \frac{(1\times10^{-26})(1.28\times10^{6})}{(1.6\times10^{-19})(8\times10^{-2})} = 1$$

The length of the arc traced by the particle in $[(7.45-5)\times10^{-6}\,s]$

$$= (v)(t) = (1.28\times10^{6})(2.45\times10^{-6})$$

$= 3.136\ m = \pi m = \frac{1}{2}$ *circumference*

$\therefore$ *The particle has the coordinates (6.4, 0, 2m) as (x, y, z).*

Practice Exercise

Q.1 A proton goes un-deflected in a crossed electric and magnetic field (the fields are perpendicular to each other) at a speed of 2.0×10^5 m/s. The velocity is perpendicular to both the fields. When the electric field is switched off, the proton moves along a circle of radius 4.0 cm. Find the magnitudes of the electric and the magnetic fields. Take the mass of the proton 1.6×10^{-27} kg.

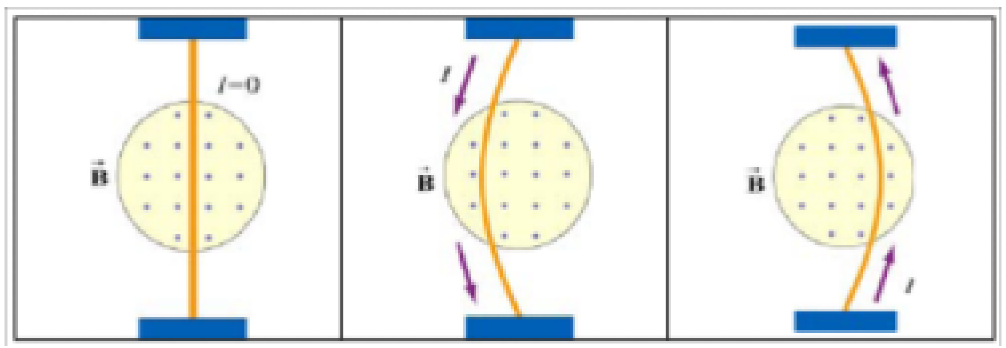
Answers

Q.1 1.0×10^4 N/C, 0.05T

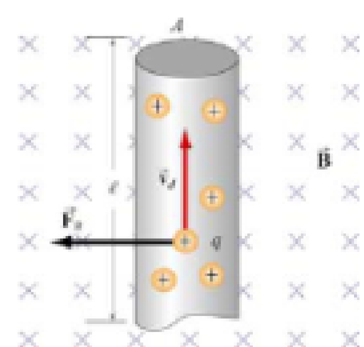
Magnetic Force on a Current-Carrying Wire :

We have just seen that a charged particle moving through a magnetic field experiences a magnetic force $\vec{F}_B$. Since electric current consists of a collection of charged particles in motion, when placed in a magnetic field, a current-carrying wire will also experience a magnetic force.

Consider a long straight wire suspended in the region between the two magnetic poles. The magnetic field points out the page and is represented with dots (•). It can be readily demonstrated that when a downward current passes through, the wire is deflected to the left. However, when the current is upward, the deflection is rightward, as shown in Figure.



To calculate the force exerted on the wire, consider a segment of wire of length ℓ and cross-sectional area *A*, as shown in Figure. The magnetic field points into the page, and is represented with crosses (X).



The charges move at an average drift velocity $\vec{v}_d$. Since the total amount of charge in this segment is $Q_{tot} = q(nA\ell)$, where *n* is the number of charges per unit volume, the total magnetic force on the segment is

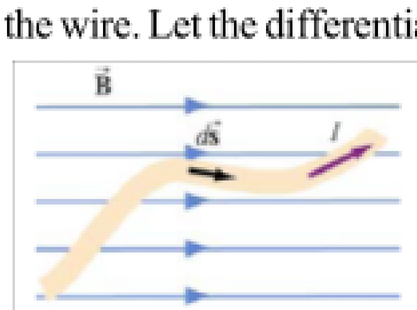
$$\vec{F}_B = Q_{tot}\vec{v}_d \times \vec{B} = qnA\ell(\vec{v}_d \times \vec{B}) = I(\vec{\ell} \times \vec{B})$$

where $I = nqv_dA$, and $\vec{\ell}$ is a *length vector* with a magnitude ℓ and directed along the direction of the electric current.

Special Case-1:

Wire of arbitrary shape placed in uniform magnetic field

For a wire of arbitrary shape, the magnetic force can be obtained by summing over the forces acting on the small segments that make up the wire. Let the differential segment be denoted as $d\vec{s}$ (Figure).

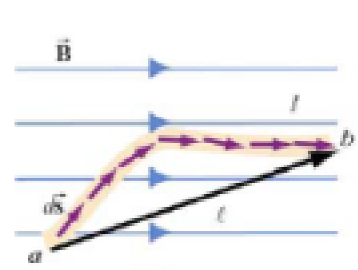


The magnetic force acting on the segment is : $d\vec{F}_B = Id\vec{s} \times \vec{B}$

Thus, the total force is : $\vec{F}_B = I\int_a^b d\vec{s} \times \vec{B}$

where *a* and *b* represent the endpoints of the wire.

As an example, consider a curved wire carrying a current *I* in a uniform magnetic field $\vec{B}$, as shown in Figure.



Using the magnetic force on the wire is given by

$$\vec{F}_B = I\left(\int_a^b d\vec{s}\right)\times\vec{B} = I\vec{\ell}\times\vec{B}$$

where $\vec{\ell}$ is the length vector directed from *a* to *b*. However, if the wire forms a closed loop of arbitrary shape (Figure), then the force on the loop becomes

$$\vec{F}_B = I\left(\oint d\vec{s}\right)\times\vec{B}$$

Special Case-2:

Magnetic Force on a closed loop in uniform magnetic field

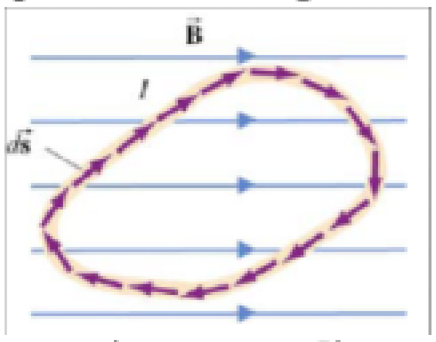
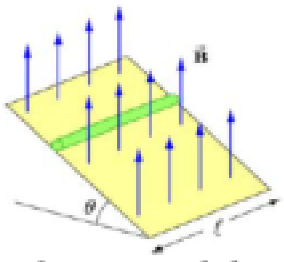


Figure : A closed loop carrying a current *I* in a uniform magnetic field.

Since the set of differential length elements $d\vec{s}$ form a closed polygon, and their vector sum is zero, i.e. $\oint d\vec{s} = 0$. The net magnetic force on a closed loop is $\vec{F}_B = \vec{0}$.

Illustration :

A conducting bar of length is placed on a frictionless inclined plane which is tilted at an angle θ from the horizontal, as shown in Figure.



A uniform magnetic field is applied in the vertical direction. To prevent the bar from sliding down, a voltage source is connected to the ends of the bar with current flowing through. Determine the magnitude and the direction of the current such that the bar will remain stationary.

Sol

For equilibrium

$I\ell B\cos\theta = mg\sin\theta$

$\Rightarrow I = \dfrac{mg\sin\theta}{\ell B\cos\theta}$

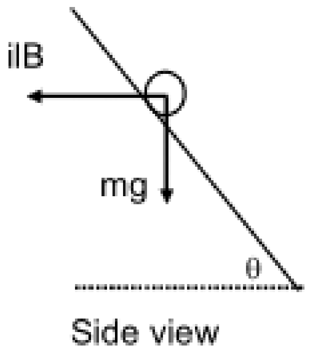
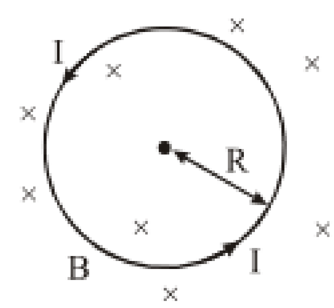


Illustration :

A current (I) carrying circular wire of radius R is placed in a magnetic field B perpendicular to its plane. Find the tension T along the circumference of the wire



Sol. For small elemental portion

$$2T\sin d\theta$$

$$= 2R\ d\theta\, IB$$

$$2T d\theta = 2RIB\, d\theta$$

$$T = IRB\cdot$$

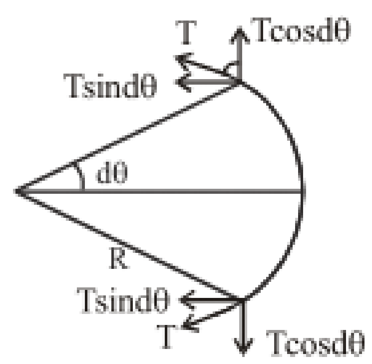
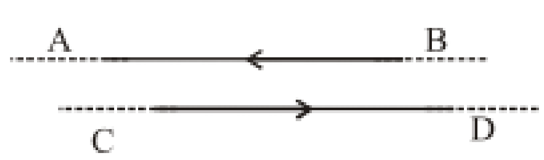


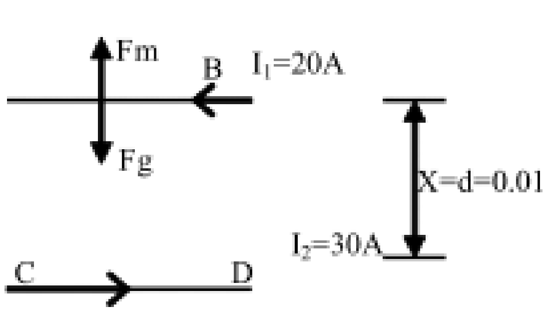
Illustration :

A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillation.



Sol. *Let m be the mass per unit length of wire AB. At a height x about the wire CD, magnetic force per unit length on wire AB will be given by*

$$F_m = \frac{\mu_0 i_1 i_2}{2\pi x}\text{(upwards)} \qquad ...(i)$$



Wt. per unit of wire AB is

$$F_g = mg\text{(downwards)}$$

At x = d, wire is in equilibrium

i.e., $F_m = F_g \Rightarrow \dfrac{\mu_0}{2\pi}\dfrac{i_1 i_2}{d} = mg$

$$\Rightarrow \quad \frac{\mu_0 i_1 i_2}{2\pi d^2} = \frac{mg}{d} \qquad ...(ii)$$

When AB is depressed, x decreases therefore, F_m *will increase, while* F_g *remains the same. Let* ***AB*** *is displaced by dx downwards. Differentiating equation (i) w.r.t.x, we get*

$$dF_m = -\frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2}.dx \qquad ...(iii)$$

i.e., restoring force, F = d $\quad F_m \propto -dx$

Hence the motion of wire is simple harmonic. From equation (ii) and (iii), we can write

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad (x = d)$$

$\therefore$ *Acceleration of wire, a* $= -\left(\frac{g}{d}\right).dx$

Hence period of oscillations

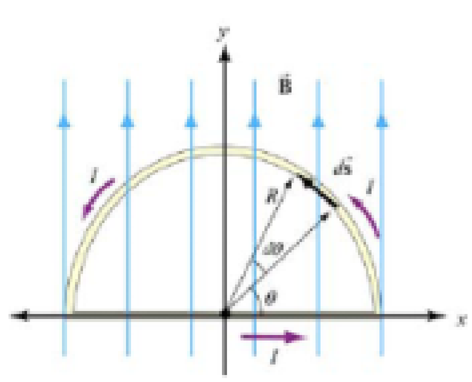
$$T = 2\pi\sqrt{\frac{dx}{a}} = 2\pi\sqrt{\left|\frac{disp.}{acc.}\right|}$$

$$\Rightarrow T = 2\pi\sqrt{d/g} = 2\pi\sqrt{\frac{0.01}{9.8}}$$

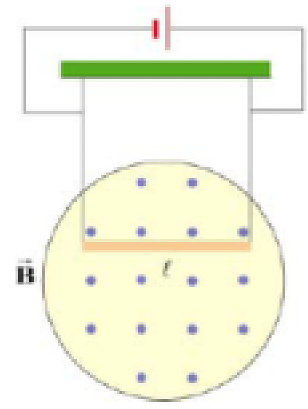
$$\Rightarrow T = 0.2s.$$

Practice Exercise

Q.1 Consider a closed semi-circular loop lying in the *xy* plane carrying a current *I* in the counterclockwise direction, as shown in Figure. Find the magnetic Force on a Semi-Circular Loop

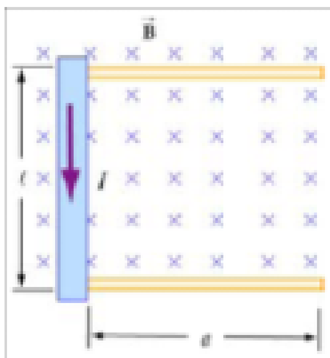


Q.2 A conducting rod having a mass density λ kg/m is suspended by two flexible wires in a uniform magnetic field $\vec{B}$ which points out of the page, as shown in Figure.

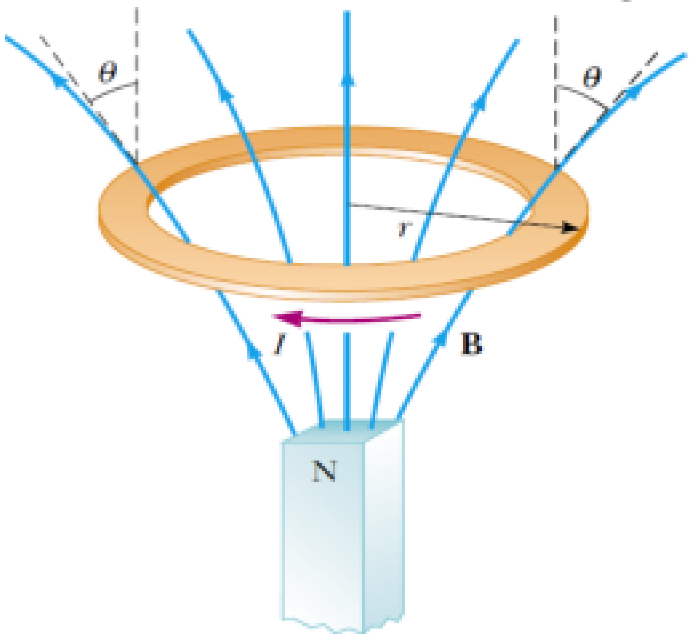


If the tension on the wires is zero, what are the magnitude and the direction of the current in the rod?

Q.3 A rod with a mass m and a radius R is mounted on two parallel rails of length a separated by a distance ℓ, as shown in the Figure. The rod carries a current I and rolls without slipping along the rails which are placed in a uniform magnetic field $\vec{B}$ directed into the page. If the rod is initially at rest, what is its speed as it rolls off the rails?



Q.4 A strong magnet is placed under a horizontal conducting ring of radius r that carries current I as shown in figure. If the magnetic field B makes an angle with the vertical at the ring's location, what are the magnitude and direction of the resultant force on the ring?



Answers

Q.1 2IRB into the page

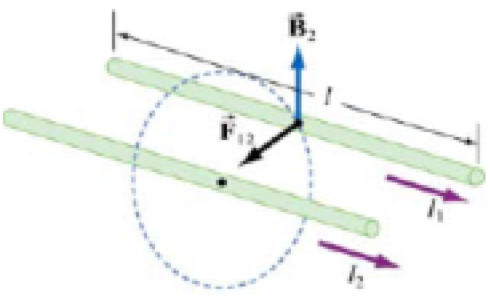
Q.2 $I = \frac{mg}{B\ell} = \frac{\lambda g}{B}$

Q.3 $v = \sqrt{\frac{4I\ell Ba}{3m}}$

Q.4 $F_y = (B\sin\theta)I(2\pi r)$

Force Between Two Parallel Wires :

We have already seen that a current-carrying wire produces a magnetic field. In addition, when placed in a magnetic field, a wire carrying a current will experience a net force. Thus, we expect two current-carrying wires to exert force on each other. Consider two parallel wires separated by a distance a and carrying currents I_1[1]and I_2[2] in the $+x$-direction, as shown in Figure.



The magnetic force, $\vec{F}_{12}$, exerted on wire 1 by wire 2 may be computed as follows: Using the result from the previous example, the magnetic field lines due to I_2 going in the $+x$-direction are circles concentric with wire 2, with the field $\vec{B}_2$ pointing in the tangential direction. Thus, at an arbitrary point P on wire 1, we have $\vec{B}_2 = -(\mu_0 I_2 / 2\pi a)\hat{j}$, which points in the direction perpendicular to wire 1, as depicted in Figure. Therefore,

$$\vec{F}_{12} = I_1\vec{\ell} \times \vec{B}_2 = (\ell\hat{i}) \times \left(-\frac{\mu_0 I_2}{2\pi a}\hat{j}\right) = -\frac{\mu_0 I_1 I_2 \ell}{2\pi a}\hat{k}$$

Clearly $\vec{F}_{12}$ points toward wire 2. The conclusion we can draw from this simple calculation is that two parallel wires carrying currents in the same direction will attract each other. On the other hand, if the currents flow in opposite directions, the resultant force will be repulsive.

Definition of ampere

Consider two parallel wires separated by 1m and carrying a current of 1A each. Then $i_1 = i_2 = 1$A and $d = 1$m, so that from equation

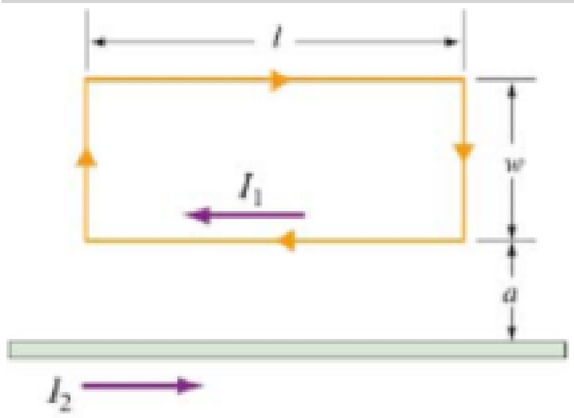
$$\frac{dF}{dl} = 2 \times 10^{-7} \text{ N/m.}$$

This is used to formally define the unit 'ampere' of electric current. If two parallel, long wires, kept 1m apart in vacuum, carry equal currents in the same direction and there is a force of attraction of 2×10^{-7} newton per metre of each wire, the current in each wire is said to be 1 ampere.

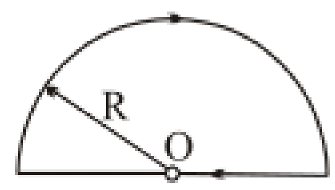
Practice Exercise

Q.1 If a current is passed through a spring, does the spring stretch or compress?

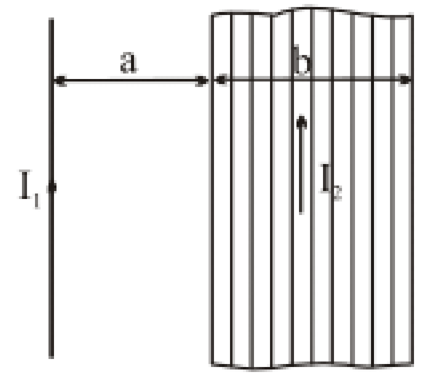
Q.2 A rectangular loop of length ℓ and width w carries a steady current I_1. The loop is then placed near an finitely long wire carrying a current I_2, as shown in Figure. What is the magnetic force experienced by the loop due to the magnetic field of the wire?



Q.3 Find the magnitude and direction of a force vector acting on a unit length of a thin wire, carrying a current I at a point O, if the wire is bent as shown in with curvature radius R



Q.4 Two long thin parallel conductors of the shape shown in Fig. carry direct currents I_1 and I_2. The separation between the conductors is a, the width of the right-hand conductor is equal to b. With both conductors lying in one plane, find the magnetic interaction force between them reduced to a unit of their length.



Answers

Q.1	compress	Q.2	$= \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{1}{a}-\frac{1}{(a+w)}\right]$
Q.3	$F_{unit} = m_0 I^2 / 4R$	Q.4	$F_1 = \frac{\mu_0}{4\pi}\frac{2I_1\, I_2}{b}\, ln\,(1 + b/a)$

Magnetic Moment

Magnetic field (at large distances) due to current in a circular current loop is very similar in behavior to the electric field of an electric dipole. We know that the magnetic field on the axis of a circular loop, of a radius *R*, carrying a steady current *I*

$$B = \frac{\mu_0 I(2\pi a^2)}{4\pi(a^2 + x^2)^{3/2}}$$

its direction is along the axis and given by the right-hand thumb rule Here, *x* is the distance along the axis from the centre of the loop.

For $x >> R$, we may drop the R^2 term in the denominator. Thus

$$B = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IA}{x^3}\right)$$

Where $A = \pi R^2$= area of the loop

The expression is very similar to an expression obtained earlier for the electric field of a dipole. The similarity may be seen if we we can define the magnetic dipole moment $\vec{\mu}$ as

$$\vec{\mu} = I\vec{A}$$

The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule (Figure). The SI unit for the magnetic dipole moment is ampere-meter^2 ($A \cdot m^2$)..

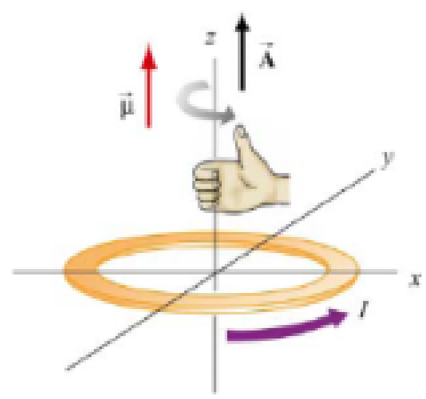


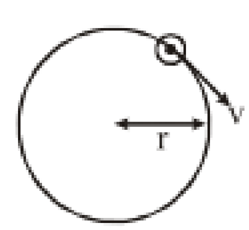
Illustration :

Find the magnetic moment of an electron orbiting in a circular orbit of radius r with a speed v

Sol. *Magnetic moment* $\mu = iA$

I = current; Since the orbiting electron behaves as current loop of current i,

we can write $i = \dfrac{e}{T} = \dfrac{e}{\frac{2\pi r}{v}} = \dfrac{ev}{2\pi r}$



A = area of the loop $= \pi r^2$

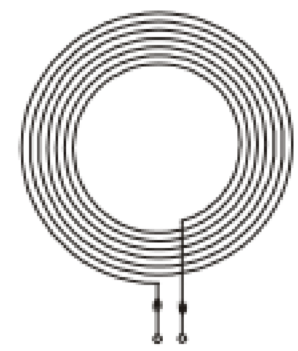
$\Rightarrow \quad \mu = (1)\left(\dfrac{ev}{2\pi r}\right)(\pi r^2)$

$\Rightarrow \quad \mu = \dfrac{evr}{2}$.

Practice Exercise

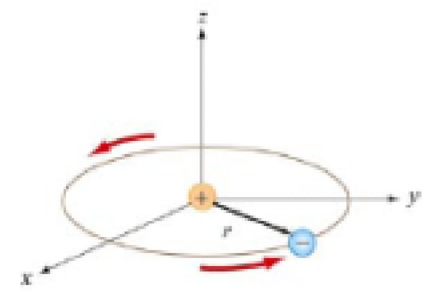
Q.1 Find the magnetic moment of a thin round loop with current if the radius of the loop is equal to $R = 100$ mm and the magnetic induction at its centre is equal to $B = 6.0\ \mu T$.

Q.2 A thin insulated wire forms a plane spiral of $N = 100$ tight turns carrying a current $I = 8$ mA. The radii of inside and outside turns (Fig.) are equal to $a = 50$ mm and $b = 100$ mm.



Find the magnetic moment of the spiral with a given current.

Q.3 We want to estimate the magnetic dipole moment associated with the motion of an electron as it orbits a proton. We use a "semi-classical" model to do this. Assume that the electron has speed v and orbits a proton (assumed to be very massive) located at the origin. The electron is moving in a right-handed sense with respect to the z-axis in a circle of radius $r = 0.53$ Å, as shown in Figure. Note that 1 Å $= 10^{-10}$ m.



(a) The inward force $m_e v^2/r$ required to make the electron move in this circle is provided by the Coulomb attractive force between the electron and proton (m_e is the mass of the electron). Using this fact, and the value of r we give above, find the speed of the electron in our "semi-classical" model.

(b) Given this speed, what is the orbital period T of the electron?

(c) What current is associated with this motion? Think of the electron as stretched out uniformly around the circumference of the circle. In a time T, the total amount of charge q that passes an observer at a point on the circle is just e

(d) What is the magnetic dipole moment associated with this orbital motion? Give the magnitude and direction. The magnitude of this dipole moment is one *Bohr magneton* μ_B

Q.4 A non-conducting thin disc of radius R charged uniformly over one side with surface density σ rotates about its axis with an angular velocity ω. Find:
(b) the magnetic moment of the disc.

Answers

Q.1 $p_m = 2\pi R^3 B / \mu_0 = 30 \text{ mA} \cdot \text{m}^2$ Q.2 $p_m = 1/3\ \pi I N\ (a^2 + ab + b^2)$

Q.3 (a) 2.18×10^6 m/s ; (b) 1.52×10^{-16} s (c) 1.05 mA. Big! ; (d) 9.27×10^{-24} along the z-axis.

Q.4 $p_m = 1/4\ \pi\sigma\omega R^4$

Torque on a Current Loop :

What happens when we place a rectangular loop carrying a current I in the xy plane and switch on a uniform magnetic field $\vec{B} = B\hat{i}$ which runs parallel to the plane of the loop, as shown in Figure (a)?

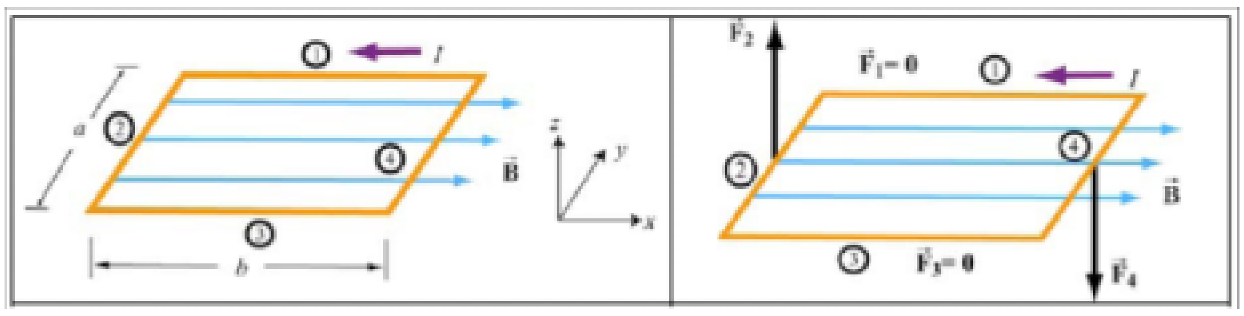


Figure : (a) A rectangular current loop placed in a uniform magnetic field.
(b) The magnetic forces acting on sides 2 and 4.

From Eq., we see the magnetic forces acting on sides 1 and 3 vanish because the length vectors $\vec{\ell}_1 = -b\hat{i}$ and $\vec{\ell}_3 = b\hat{i}$ are parallel and anti-parallel to $\vec{B}$ and their cross products vanish. On the other hand, the magnetic forces acting on segments 2 and 4 are non-vanishing:

$$\begin{cases} \vec{F}_2 = I(-a\hat{j}) \times (B\hat{i}) = IaB\hat{k} \\ \vec{F}_4 = I(a\hat{j}) \times (B\hat{i}) = -IaB\hat{k} \end{cases}$$

with $\vec{F}_2$ pointing out of the page and $\vec{F}_4$ into the page. Thus, the net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$

as expected. Even though the net force on the loop vanishes, the forces $\vec{F}_2$ and $\vec{F}_4$ will produce a torque which causes the loop to rotate about the *y*-axis (Figure). The torque with respect to the center of the loop is

$$\vec{\tau} = \left(-\frac{b}{2}\hat{i}\right) \times \vec{F}_2 + \left(\frac{b}{2}\hat{i}\right) \times \left(IaB\hat{k} + \left(\frac{b}{2}\hat{i}\right) \times \left(-IaB\hat{k}\right)\right)$$

$$= \left(\frac{IabB}{2} + \frac{IabB}{2}\right)\hat{j} = IabB\hat{j} = IAB\hat{j}$$

where A = ab represents the area of the loop and the positive sign indicates that the rotation is clockwise about the *y*-axis. It is convenient to introduce the area vector $\vec{A} = A\hat{n}$ where $\hat{n}$ is a unit vector in the direction normal to the plane of the loop. The direction of the positive sense of $\hat{n}$ is set by the conventional right-hand rule. In our case, we have $\hat{n} = +\hat{k}$. The above expression for torque can then be rewritten as

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Notice that the magnitude of the torque is at a maximum when $\vec{B}$ is parallel to the plane of the loop (or perpendicular to).

Consider now the more general situation where the loop (or the area vector $\vec{A}$) makes an angle θ with respect to the magnetic field.

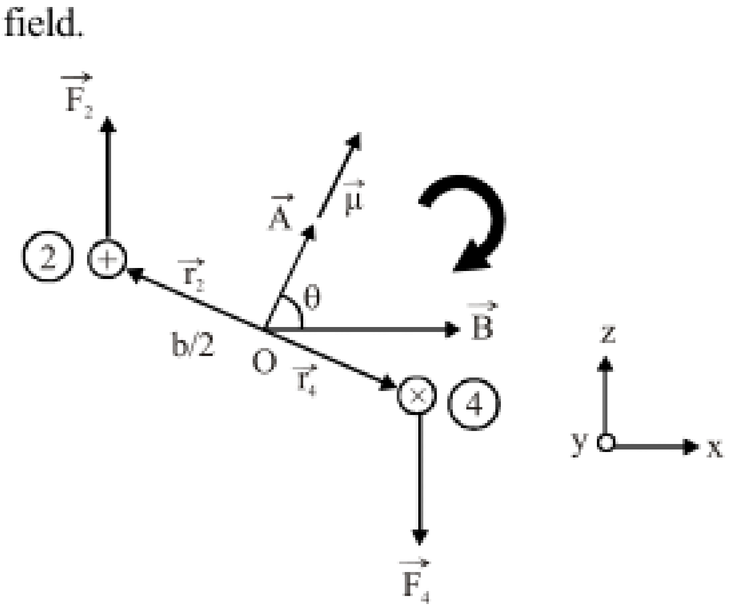


Figure : Rotation of a rectangular current loop

From Figure, the lever arms and can be expressed as:

$$\vec{r}_2 = \frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right) = -\vec{r}_4$$

and the net torque becomes

$$\vec{\tau} = \vec{r}_2 \times \vec{F}_2 + \vec{r}_4 = 2\vec{r}_2 \times \vec{F}_2 = 2.\frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right) \times \left(IaB\hat{k}\right)$$

$$jIabB\sin\theta\,\hat{j} = I\vec{A} \times \vec{B}$$

For a loop consisting of *N* turns, the magnitude of the toque is

$$\tau = NIAB\sin\theta$$

The quantity $NI\vec{A}$ is called the magnetic dipole moment $\vec{\mu}$

$$\vec{\mu} = NI\vec{A}$$

Using the expression for $\vec{\mu}$, the torque exerted on a current-carrying loop can be rewritten as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The above equation is analogous to $\vec{\tau} = \vec{p} \times \vec{E}$ in Eq., the torque exerted on an electric dipole moment $\vec{p}$ in the presence of an electric field $\vec{E}$.

Configuratoin energy of current loop in uniform magnetic field.

Recalling that the potential energy for an electric dipole is $U = -\vec{p} \bullet \vec{E}$ [see Eq.], a similar form is expected for the magnetic case. The work done by an external agent to rotate the magnetic dipole from an angle θ_0 to θ is given by

$$W_{ext} = \int_{\theta_0}^{\theta} (\mu B \sin\theta')d\theta' = \mu B(\cos\theta_0 - \cos\theta)$$

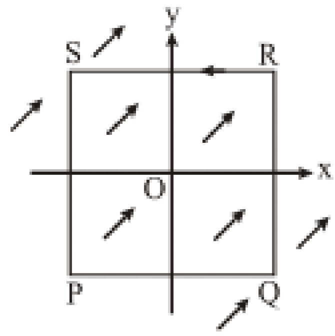
$$= \Delta U = U - U_0$$

Once again, $W_{ext} = -W$, where W is the work done by the magnetic field. Choosing $U_0 = 0$ at $\theta_0 = \pi/2$, the dipole in the presence of an external field then has a potential energy os

$$U = -\mu B\cos\theta = -\vec{\mu} \bullet \vec{B}$$

The configuration is at a stable equilibrium when $\vec{\mu}$ is aligned parallel to $\vec{B}$, making U a minimum with $U_{min} = -\mu B$. On the other hand, when $\vec{\mu}$ and $\vec{B}$ are anti-parallel, $U_{max} = +\mu B$ is a maximum and the system is unstable.

Illustration :

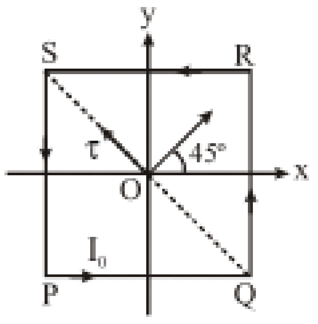


A uniform, constant magnetic field $\vec{B}$ *is directed at an angle of 45° to the x-axis in the xy-plane. PQRS is a rigid, square wire frame carrying a steady current* I_0*, with its centre at the origin O. At time t =0, the frame is at rest in the position shown in the figure, with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.*

(a) *What is the torque* $\vec{\tau}$ *about O acting on the frame due to the magnetic field?*

(b) *Find the angle by which the frame rotates under the action of this torque in a short interval of time* Δt *, and the axis about which this rotation occurs. (*Δt *is so short that any variation in the torque during this interval may be neglected). Given moment of any variation in the torque during this interval may be neglected). Given moment of inertia of the frame about an axis through its centre perpendicular to its plate is (4/3)* ML^2.

Sol. *(a) As magnetic field* $\vec{B}$ *is in x-y plane and subtends an angle of 45° with x-axis*



$$B_x = B\cos 45^o = B/\sqrt{2}$$

And $B_y = B\sin 45 = B/\sqrt{2}$

So in vector form

$$\vec{B} = \hat{i}(B/\sqrt{2}) + j(B/\sqrt{2})$$

and $\vec{M} = I_0 S\hat{k} = I_0 L^2 \hat{k}$

so, $\vec{\tau} = \vec{M} \times \vec{B} = I_0 L^2 \hat{k} \times \left(\frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}\right)$

i.e., $\vec{\tau} = \frac{I_0 L^2 B}{\sqrt{2}} \times (-\hat{i} + \hat{j})$

i.e., torque has magnitude $I_0 L^2$ *B and is directed along line QS from Q to S.*

(b) As by theorem of perpendicular axis, moment of inertia of the frame about QS,

$$I_{QS} = \frac{1}{2} I_z = \frac{1}{2}\left(\frac{4}{3} ML^2\right) = \frac{2}{3} ML^2$$

And as $\tau = I\alpha$,

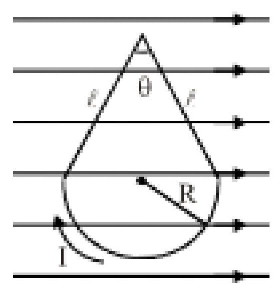
$$\alpha = \frac{\tau}{I} = \frac{I_0 L^2 B \times 3}{2L^2 M} = \frac{3}{2}\frac{I_0 B}{M}$$

As here α *is constant, equations of circular motion are valid and hence from* $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ *with* $\omega_0 = 0$ *we have*

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{3}{2}\frac{I_0 B}{M}\right)(\Delta t)^2 = \frac{3}{4}\frac{I_0 B}{M}\Delta t^2.$$

Practice Exercise

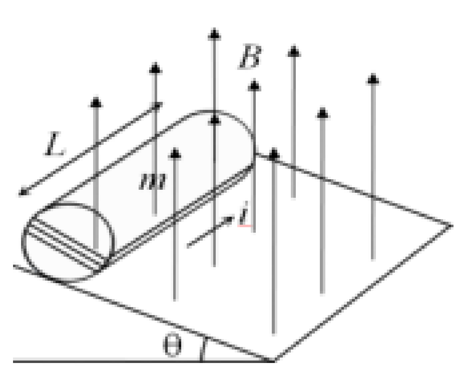
Q.1 A current loop consists of a semicircle of radius R and two straight segments of length ℓ with an angle θ between them. The loop is then placed in a uniform magnetic field pointing to the right, as shown in Figure.



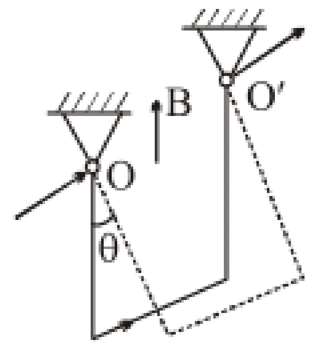
(a) Find the net force on the current loop.

(b) Find the net torque on the current loop.

Q.2 Figure shows a wooden cylinder with a mass m and a length L with N turns of wire wrapped around it longitudinally, so that the plane of the wire loop contains the axis of the cylinder. What is the least current through the loop that will prevent the cylinder from rolling down a plane inclined at an angle   to the horizontal, in the presence of a vertical, uniform magnetic field B, if the plane of the windings is parallel to the inclined plane?



Q.3 A copper wire of density ρ with cross-sectional area S bent to make three sides of a square can turn about a horizontal axis OO' (Fig.). The wire is located in uniform vertical magnetic field. Find the magnetic induction if on passing a current I through the wire the latter deflects by an angle θ

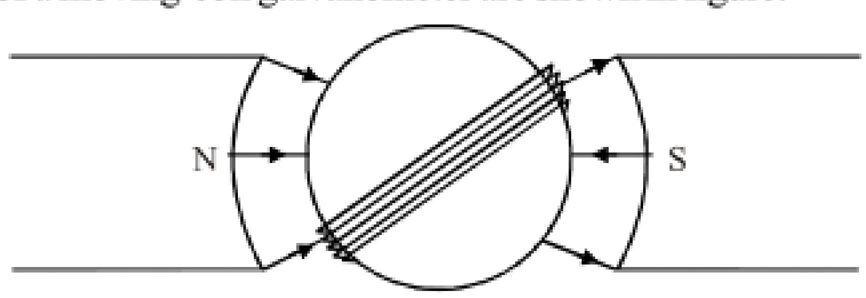


Answers

Q.1 Q.2 $i = \dfrac{mg}{2NBL}$ Q.3 $B = (2rgS / I)\tan\theta$

Moving coil Galvanometer :

The main parts of a moving-coil galvanometer are shown in figure.



The current to be measured is passed through the galvanometer. As the coil is in the magnetic field $\vec{B}$ of the permanent magnet, a torque $\vec{\Gamma} = ni\vec{A} \times \vec{B}$ acts on the coil. Here n = number of turns, i = current in the coil $\vec{A}$ = area-vector of the coil and $\vec{B}$ = magnetic field at the site of the coil. This torque deflects the coil from its equilibrium position.

The pole pieces are made cylindrical. As a result, the magnetic field at the arms of the coil remains parallel to the plane of the coil everywhere even as the coil rotates. The deflecting torque is then $\Gamma = niAB$.

As the upper end of the suspension strip W is fixed, the strip gets twisted when the coil rotates. This produces a restoring torque acting on the coil. If the deflection of the coil is θ and the torsional constant of the suspension strip is k, the restoring torque is $k\theta$. The coil will stay at a deflection θ where

$$niAB = k\theta$$

or, $$i = \frac{k}{nAB}\theta$$

Hence, the current is proportional to the deflection. The constant $\dfrac{k}{nAB}$ is called the galvanometer constant.

We define the **current sensitivity** of the galvanometer as the deflection per unit current. From Eq. this current sensitivity is.

$$\frac{\phi}{I} = \frac{NAB}{k}$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N. We choose galvanometers having sensitivities of value, required by our experiment.

We define the **voltage sensitivity** as the deflection per unit volt of applied potential difference

$$\frac{\phi}{I} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$$

An interesting point to note is that increasing the current sensitivity may not necessarily increase the voltage sensitivity. If $N \rightarrow 2N$, i.e., we double the number of turns, then

$$\frac{\phi}{I} \rightarrow 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. In eq. $N \rightarrow 2N$, and $R \rightarrow 2R$, thus the voltage sensitivity,

$$\frac{\phi}{V} \rightarrow \frac{\phi}{V}$$

remains unchanged.

Practice Exercise

Q.1 Two moving coil meters, M_1 and M_2 have the following particulars :

$R_1 = 10\Omega$, $N_1 = 30$

$A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25\ T$

$R_2 = 14\Omega$, $N_2 = 42$

$A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50\ T$

(The spring constants are identical for the two meters)

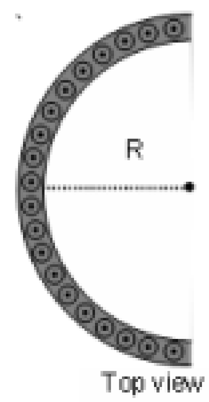
Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

Answers

Q.1 (a) 1.4 (b) 1

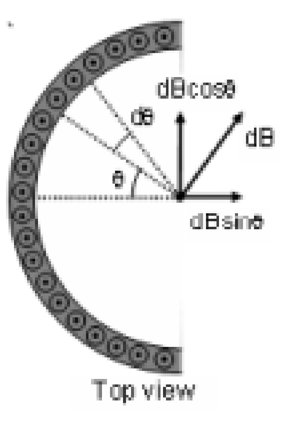
Solved Examples

Q.1 A current I flows in a long straight wire with cross-section having the form of a thin half-ring of radius R (Fig.). Find the induction of the magnetic field at the point O.



Sol. $dI = \frac{1}{\pi R}(R d\theta) = \frac{1}{\pi} d\theta$

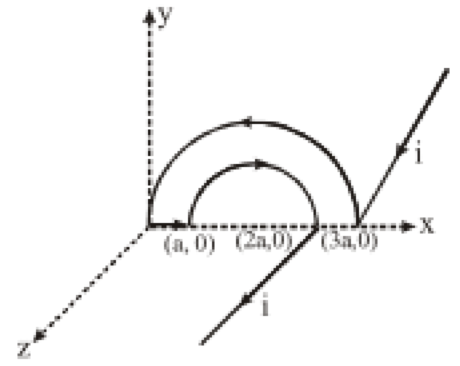
$$dB = \frac{\mu_0 dI}{2\pi R} = \frac{\mu_0 I}{2\pi^2 R} d\theta$$



$$B = \int dB\cos\theta = \frac{\mu_0 I}{2\pi^2 R}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\theta\, d\theta = \frac{\mu_0 I}{\pi^2 R}$$

$$B = \mu_0 I / \pi^2 R$$

Q.2 In the figure shown the magnetic field at the point P.

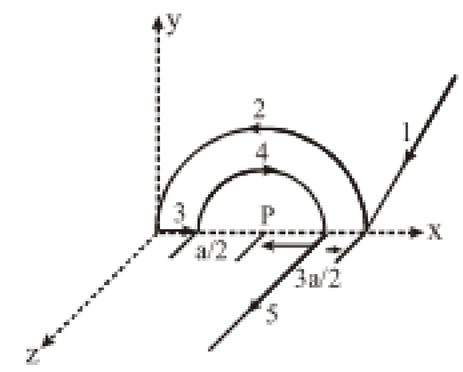


Sol. Consider the figure

$$\vec{B}_p = \left(\vec{B}_1\right)_p + \left(\vec{B}_2\right)_p + \left(\vec{B}_3\right)_p + (\vec{B}_4)_P + (\vec{B}_5)_P$$

where $\left(\vec{B}_1\right)_p = \frac{\mu_0 i}{4\pi\left(\frac{3a}{2}\right)}(-\hat{j})$ (semi-infinite wire)

$$(\vec{B}_2)_p = \frac{\mu_0 i}{4\left(\frac{3a}{2}\right)}(+\hat{k})$$



$$(\vec{B}_3)_p = 0$$

$$(\vec{B}_4)_p = \frac{\mu_0 i}{4\left(\frac{a}{2}\right)}(-\hat{k})$$

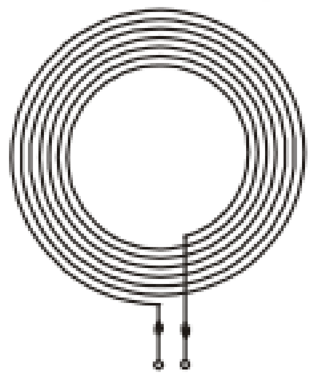
$$(\vec{B}_5)_p = \frac{\mu_0 i}{4\pi\left(\frac{a}{2}\right)}(-\hat{j})$$

$$\Rightarrow \quad \vec{B}_p = \frac{\mu_0 i}{2a}\left[-\left(\frac{1}{3\pi}+\frac{1}{\pi}\right)\hat{j}-\left(1-\frac{1}{3}\right)\hat{k}\right]$$

$$\Rightarrow \quad \vec{B}_p = \frac{2\mu_0 i}{3a}\left[\frac{1}{\pi}\hat{j}-\hat{k}\right]$$

$$\Rightarrow \quad \bar{B}_p = \frac{\mu_0 i}{3\pi a}\sqrt{1+\pi^2}\,.$$

Q.3 A thin insulated wire forms a plane spiral of N tight turns carrying a current I . The radii of inside and outside turns (Fig.) are equal to a and b . Find the magnetic induction at the centre of the spiral;

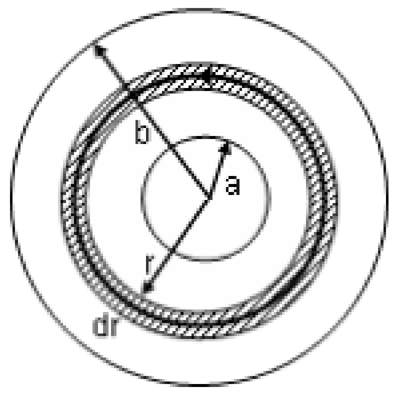


Sol. $B_{one} = \frac{\mu_0 I}{4\pi r}(2\pi) = \frac{\mu_0 I}{2r}$

$dN = \frac{N}{b-a}dr$

$dB = B_{one}\,dN = \frac{\mu_0 NI}{2(b-a)}\frac{dr}{r}$

$B = \int dB = \frac{\mu_0 NI}{2(b-a)}\int_a^b \frac{dr}{r} = \frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$



Q.4 A disc of radius R rotates at an angular velocity ω about the axis perpendicular to its surface and passing through its centre. If the disc has a uniform surface charge density σ, find the magnetic induction on the axis of rotation at a distance x from the centre.

Sol. Consider a ring of radius r and width dr.

Charge on the ring, $dq = (2\pi r\ dr)\sigma$

Current due to ring is $dI = \frac{dq}{T}$

$$= \frac{\omega dq}{2\pi} = \sigma\omega r\ dr$$

Magnetic field due to ring at point P is

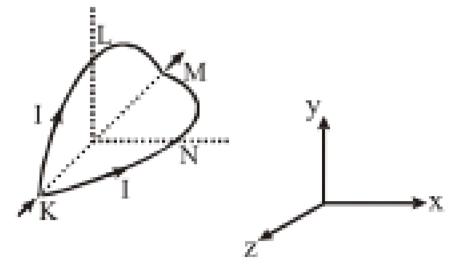
$$dB = \frac{\mu_0\, dI\, r^2}{2(r^2 + x^2)^{3/2}}$$

or $$B = \int dB = \frac{\mu_0 \sigma\omega}{2}\int_0^R \frac{r^3\, dr}{(r^2 + x^2)^{3/2}} \qquad \ldots(i)$$

Putting $r^2 + x^2 = t^2$ and $2r\, dr = 2t\, dt$ and integrating (i) we get

$$B = \frac{\mu_0\sigma\omega}{2}\left[\frac{R^2 + 2x^2}{\sqrt{R^2 + x^2}} - 2x\right].$$

Q.5 A circular loop of radius R is bent along a diameter and given a shape as shown in figure. One of the semi-circles (KNM) lies in the x-z plane and the other one (KLM) in the y-z plane with their centres at origin. Current I is flowing through each of the semi-circles as shown in figure.



A particle of charge q is released at the origin with a velocity $\vec{V} = -V_0\hat{i}$. Find the instantaneous force $\vec{F}$ on the particle. Assuem that space is gravity free.

Sol. Magnetic field at the centre of a circular wire of radius R carrying a current I is given by

$$B = \frac{\mu_0 I}{2R}$$

In this problem, currents are flowing in two semi-circles, KLM in the y-z plane and KNM in the x-z plane. The centres of these semi-circles coincide with the origin of the Cartesian system of axes.

$$\therefore \qquad \vec{B}_{KLM} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right)(-\hat{i})$$

$$\vec{B}_{KNM} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right)(-\hat{j})$$

The total magnetic field at the origin is

$$B_0 = \frac{\mu_0 I}{4R}(-\hat{i} + \hat{j})$$

It is given that a particle of charge q is released at the origin with a velocity $V = -\,V_0\hat{i}$. The instantaneous force acting on this particle is given by

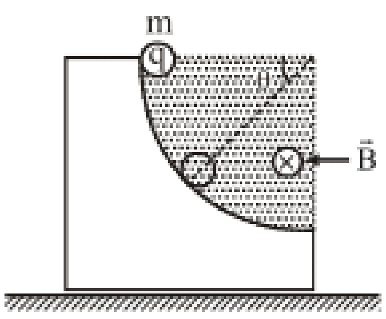
$$f = q[\vec{V} \times \vec{B}]$$

$$= q(-V_0\hat{i}) \times \left[\frac{\mu_0 I}{4R}(\hat{i} + \hat{j})\right]$$

$$= \left(\frac{q\,V_0\mu_0 I}{4R}\right)[(-\hat{i}) \times (-\hat{i} + \hat{j})$$

$$= \frac{q\,V_0\mu_0 I}{4R}(-\hat{k}).$$

Q.6 In the figure a charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position shown. There exists a uniform and constant horizontal magnetic field of induction B. The maximum force exerted by the track on the sphere.



Sol. $F_m = qvB$,a and directed radially outward.

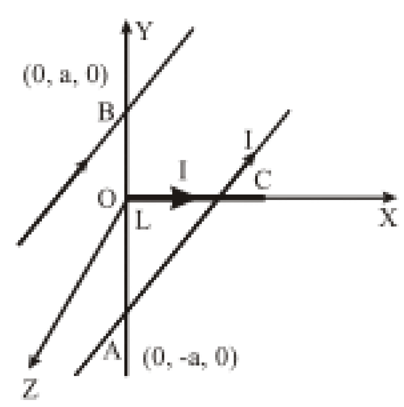
$$\therefore \qquad N - mg\sin\theta + qvB = \frac{mv^2}{R}$$

$$\Rightarrow \qquad N = \frac{mv^2}{R} + mg\sin\theta - qvB$$

Hence at $\theta = \pi/2$

$$\Rightarrow \qquad N_{max} = \frac{2mgR}{R} + mg - qB\sqrt{2gR} \;= 3mg - qB\sqrt{2gR}\,.$$

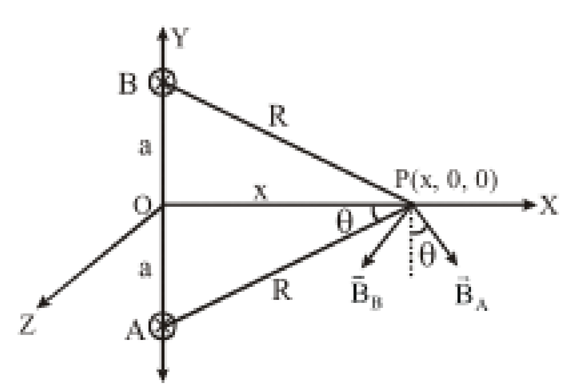
Q.7 A straight segment OC (of length L meter) of a circuit carrying a current 1 amp is placed along the x-axis. Two infinitely long straight wire A and B, each extending $z = -\infty$ to $+\infty$ are fixed at y = – a metre and y = + a metre respectively, as shown in the figure. If the wires A and B each carry a current I amp into the place of the paper, obtain the expression for the force acting on segment OC. What will be the force on OC if the current in the wire B is reversed?



Sol. Magnetic field B_A produced at P(x, 0, 0) due to wire, $B_A = \mu_0 I/2\pi R$, $B_B = \mu_0 I/2\pi R$. Components of B_A and B_B along x-axis cancel, while those along y-axis add up to give total field.

$$B = 2\left(\frac{\mu_0 I}{2\pi R}\right)\cos\theta = \frac{2\mu_0 I}{2\pi R}.\frac{x}{R} = \frac{\mu_0 I}{\pi}\frac{x}{(a^2 + x^2)} \text{ (along} - \text{y direction)}$$

The force dF acting on the current element is $d\bar{F} = I(d\bar{I} \times B\bar{B})$

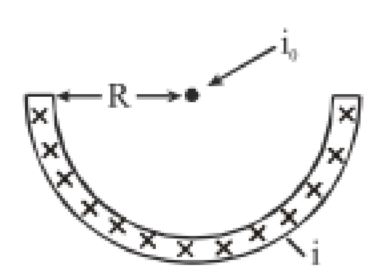


$$\therefore \quad dF = \frac{\mu_0 I^2}{\pi}\frac{x\,dx}{a^2 + x^2}[\because \sin 90^\circ = 1]$$

$$\Rightarrow \quad F = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{x\,dx}{a^2 + x^2} = \frac{\mu_0 I^2}{2\pi}\ln\frac{a^2 L^2}{a^2}$$

If the current in B is reversed, the magnetic field due to the two wires would be only along x- direction and the force on the current along x- direction will be zero.

Q.8 Shown in the figure is a very long semicylindrical conducting shell of radius R and carrying a current i along its length. An infinitely long straight current carrying conductor is lying along the axis of the semicylinder. If the current flowing through the straight wire is i_0, then find the froce on the semicylinder.

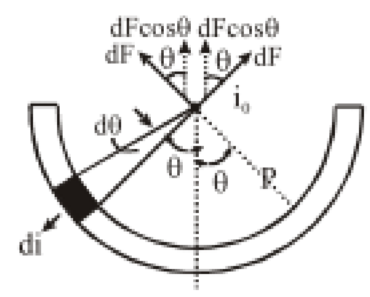


Sol. The net magnetic force on the conducting wire

$$= F = \int 2dF\cos\theta$$

$$\Rightarrow \quad F = \int 2\left[\frac{\mu_0 (di) i_0}{2\pi R}\right]\cos\theta$$

$$\Rightarrow \quad F = \frac{\mu_0 i_0}{\pi R}\int di\cos\theta$$

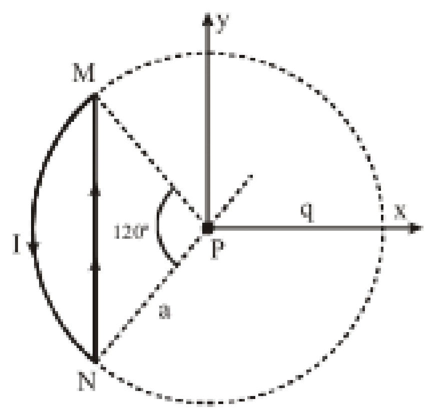


when $di = \frac{i}{\pi R}\times R d\theta = \frac{id\theta}{\pi}$

$$\Rightarrow \quad F = \frac{\mu_0 i_0}{\pi R}\int \frac{(id\theta)\cos\theta}{\pi}$$

$$\Rightarrow \quad F = \frac{\mu_0 i_0 i}{\pi^2 R}\int_0^{\pi/2}\cos\theta d\theta = \frac{\mu_0 i_0 i}{\pi^2 R}.$$

Q.9 A wire loop carrying a current I is placed in the x-y plane as shown in figure. (a) If a particle with charge q and mass m is placed at the centre P and given a velocity v along NP find its instantaneous acceleration. (b) If an external uniform magnetic induction $\vec{B} = B\hat{i}$ is applied, find the force and torque acting on the loop.



Sol. (a) As in case of current -carrying straight conductor and arc, the magnitude of B is given by

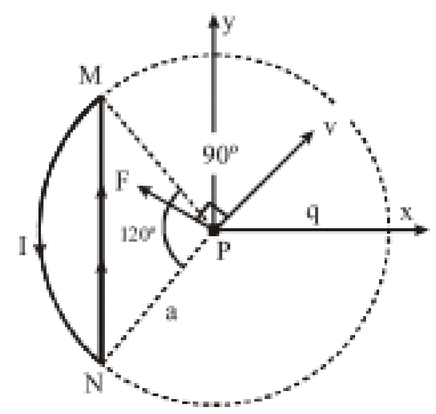
$$B_1 = \frac{\mu_0 I}{4\pi d}(\sin\alpha + \sin\beta)$$

And $\quad B_2 = \frac{\mu_0 I\phi}{4\pi r}$

So in accordance with right hand screw rule,

$$\left(\vec{B}_w\right) = \frac{\mu_0}{4\pi}\frac{1}{(a\cos 60)}\times 2\sin 60(-\hat{k})$$

and $\quad \left(\vec{B}\right)_{MN} = \frac{\mu_0}{4\pi}\frac{I}{a}\times\left(\frac{2}{3}\pi\right)(-\hat{k})$



and hence net $\vec{B}$ at P due to the given loop

$$\vec{B} = \vec{B}_w + \vec{B}_A \quad \Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\sqrt{3} - \frac{\pi}{3}\right](-\hat{k}) \quad \ldots(i)$$

Now as force on charged particle in a magnetic fields is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

so here, $\vec{F} = qvB\sin 90^\circ$ along PF

i.e. $\vec{F} = \frac{\mu_0}{4\pi}\frac{2qvI}{a}\left[\sqrt{3} - \frac{\pi}{3}\right]$ along PF

and so $\vec{a} = \frac{\vec{F}}{m} = 10^{-7}\frac{2qvI}{a}\left[\sqrt{3} - \frac{\pi}{3}\right]$ along PF

(b) As $d\vec{F} = Id\vec{L} \times \vec{B}$, so $\vec{F} = \int Id\vec{L} \times \vec{B}$

As here I and $\vec{B}$ are constant

$$\vec{F} = I\left[\oint d\vec{L}\right] \times \vec{B} = 0 \qquad \left[\text{as} \oint d\vec{L} = 0\right]$$

Further as area of coil,

$$\vec{S} = \left[\frac{1}{3}\pi a^2 - \frac{1}{2}.2a\sin 60^\circ \times a\cos 60^\circ\right]\hat{k} = a^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}$$

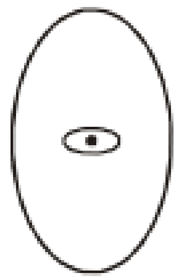
So $\vec{M} = I\vec{S} = Ia^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}$

and hence $\vec{\tau} = \vec{M} \times \vec{B} = Ia^2B\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right](\hat{k} \times \hat{i})$

i.e. $\vec{\tau} = Ia^2B\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{j}\,N - m$ as $(\hat{k} \times \hat{i} = \hat{j})$.

Q.10 A coil of radius R carries current I. Another concentric coil of radius ($r << R$) carries current i. Planes of two coils are mutually perpendicular and both the coils are free to rotate about common diameter. Find maximum kinetic energy of smaller coil when both the coils are released, masses of coils are M and m respectively.

Sol. If a magnetic dipole having moment M be rotated through angle θ from equilibrium position in a uniform magnetic field B, work done on it is $W = MB(1 - \cos\theta)$. This work is stored in the system in the form of energy. When system is release, dipole starts to rotate to occupy equilibrium position and teh energy converts into kinetic energy and kinetic energy of the system is maximum when stored energy is completely released.



Magnetic induction, at centres due to current in larger coil is $B = \frac{\mu_0 I}{2R}$

Magnetic dipole moment of smaller coil is $i\pi r^2$.

Initially planes of two coils are mutually perpendicular, therfore θ is $90°$ or energy of the system is

$$U = (i\pi r^2)B(1 - \cos 90°)$$

$$U = \frac{\mu_0 I i \pi r^2}{2R}$$

When coils are release, both the coils start to rotate about their common diameter and their kinetic energies are maximum when they become coplanar.

Moment of inertia of larger coil about axis of rotation is $I_1 = \frac{1}{2}mR^2$

and that of smaller coil is $I_2 = \frac{1}{2}mr^2$

Since, two coils rotate due to their mutual interaction only, therefore, if one coil rotates clockwise then the other rotates anticlockwise.

Let angular velocities of larger and smaller coils be numerically equal to ω_1 and ω_2 respectively when they become coplaner,

According to law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

and according to law of conservation of energy,

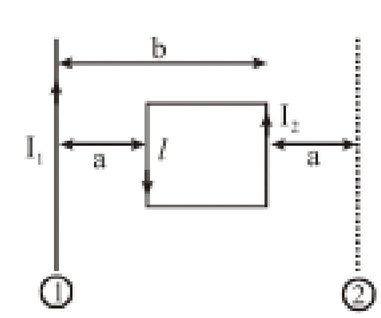
$$\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = U$$

From above equations, maximum kinetic energy of smaller coil,

$$\frac{1}{2}I_2\omega_2^2 = \frac{UI_1}{I_1 + I_2}$$

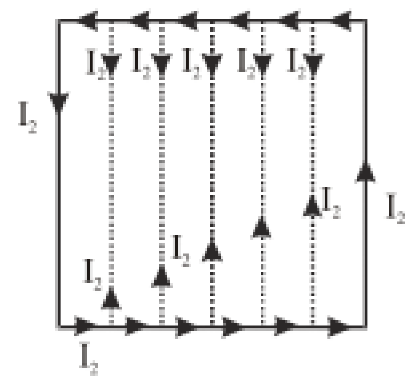
$$= \frac{\mu_0 \pi l i M R r^2}{2(MR^2 + mr^2)}$$

Q.11 What is the work done in transferring the wire from position (1) to position (2)?



Sol. The loop can be considered as the combination of teh number of elementary loops.

The net current in the dotted wires is 0 as current in the neighboring loops flowing through the same wire are opposite in direction.



consider an elementary loop of width *dr* at a distance *r* from the wire

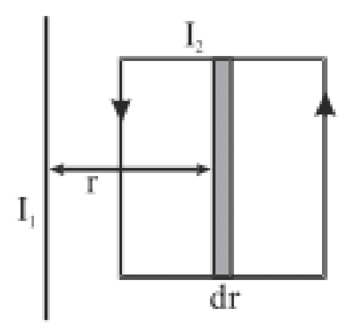
The '$d\mu$' magnetic moment of the elemental loop

$$= I_2 l \ dr$$

The B at that point due to straight wire

$$= \mu_0 I_1 / 2\pi r .$$

$$dU = -B.d\mu = -\frac{\mu_0 I_1}{2\pi r} I_2 l \ dr(\cos\pi)$$



[As $d\mu$ is anti-parallel to B.]

$$U_1 = \int du = \frac{\mu_0 I_1 I_2 l}{2\pi} \int_a^b \frac{1}{r} dr = \frac{\mu_0 I_1 I_2 l}{2\pi} \ln\left(\frac{b}{a}\right)$$

By symmetry, $U_2 = -U_1$

$$\Rightarrow \qquad -\Delta U = \text{work done} = -(U_2 - U_1) = 2\frac{\mu_0 I_1 I_2 l}{2\pi} \ln\frac{b}{a}.$$

The work done in transferring the wire from position 1 to 2 $= \frac{\mu_0 I_1 I_2 l}{\pi} \ln\frac{b}{a}$.