OXFORD

the story of collapsing stars

Black Holes, Naked Singularities, and the Cosmic Play of Quantum Gravity

Pankaj S. Joshi

THE STORY OF COLLAPSING STARS

THE STORY OF COLLAPSING STARS

Black Holes, Naked Singularities, and the Cosmic Play of Quantum Gravity

Pankaj S. Joshi

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Great Clarendon Street, Oxford, OX2 6DP,
United Kingdom

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide. Oxford is a registered trade mark of
Oxford University Press in the UK and in certain other countries

First Edition published in 2015
Impression: 2
Reprinted with corrections 2015

Published in the United States of America by Oxford University Press
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data
Data available

Library of Congress Control Number: 2014948114

ISBN 978–0–19–968676–6

Printed and bound by
CPI Group (UK) Ltd, Croydon, CR0 4YY

To:
That Indomitable Spirit within,
That inspires Our Search without,
Of the Universe,
and of the Self within. . .

Preface

The basic aim of fundamental physics and cosmology is to understand the nature and structure of the Universe and to decipher and comprehend the underlying laws that govern the Cosmos. This justifies current efforts to find a theory that will unify the forces of nature known today, namely the weak and strong nuclear forces, electromagnetism, and gravity. Although we seem to have at present a good theory that combines the first three forces, to unite it with gravity to generate a quantum gravity theory has been an unfulfilled dream and elusive goal for a long time now.

How are we to achieve this? As efforts are made to obtain evidence of quantum gravity, as has been done for the past several decades without success, my own feeling is that it may be worth considering phenomena in the Cosmos where these key forces come together to generate possibly observable effects. What we need are detectable and observable signatures of possible quantum gravity phenomena. These could give us the real clues leading to such a unified theory.

From such a perspective, we undertake here a journey into one of the most fascinating intellectual adventures of past decades, namely understanding the final fate of massive collapsing stars in the Universe, or in general the gravitational collapse of massive matter clouds on a larger scale. This is of great interest in fundamental physics and cosmology, for gravitation theory and modern astrophysical observations. This phenomenon could be intimately connected to our search for a unified understanding of basic forces of nature, namely gravity, which governs the cosmological universe, and the microscopic forces, which include the quantum phenomena.

Using Einstein's theory of gravity this investigation takes us to the world of black holes, spacetime singularities, and other intriguing possibilities. Today, this continues to be a crucially important, unresolved area in astrophysics and cosmology forming the foundation of modern black hole physics and its current applications. The issue is also relevant to mysterious, very high energy astrophysical phenomena observed in the Cosmos which defy any consistent theoretical understanding. They include, for example, cosmic gamma ray bursts, active galactic nuclei, quasars, and powerful jets from galaxies.

According to the general theory of relativity, a massive star that collapses catastrophically under its own gravity when it runs out of its internal nuclear fuel must give rise to a spacetime singularity. Such singularities are the regions in the Universe where physical quantities take extreme values and become arbitrarily large. The singularity might be hidden within a black hole or visible to faraway observers. Thus, general relativity predicts that a massive star's final fate of collapse is either as a black hole or as a visible naked singularity. In the latter case, ultra-high density and curvature regions formed during the gravitational collapse are visible from far away, and there can be rather intriguing observational consequences.

Within such a context, we discuss here recent results and developments on the gravitational collapse of massive stars and matter clouds. We suggest that a deeper understanding of catastrophic gravitational collapse, where energy scales grow extraordinarily high, can be a testing ground for examining, developing, and refining our efforts toward a unified quantum gravity theory. We indicate the exciting possibility that collapsing massive stars and the resulting spacetime singularities may provide a laboratory where one can test the unification possibilities for basic forces of nature.

In this way, the phenomenon of collapsing massive stars becomes all the more interesting and intriguing. Also, the possible connection to the very high energy cosmic phenomena mentioned earlier is worth exploring, and we consider possible observational and astrophysical implications when naked singularities form in the Universe.

The story that develops has amazing curves and ups and downs. In fact, black hole physics has undergone key changes in recent times. What began with the remarkable discovery by Chandrasekhar in the 1930s on white dwarf mass limits continued with detailed insights on neutron stars and pulsars, further moving onto dizzy domains of black holes and singularities. While this is all supposed to be scientific investigation and hard core fundamental physics, the emotional debates and controversies that have flared up from time to time during past decades have been no less spectacular.

While several interesting conclusions have emerged from these investigations in recent years, as we discuss here, the debate is far from over and this continues to be an area of great scientific excitement. Perhaps this proves again the vitality, importance, and key nature of the issues and themes involved.

It is my hope to show at the end of the discussion here that the emerging story is no less exciting and thought-provoking than Einstein gravity itself, from which it has developed. If the following narrative creates a few moments of happy and insightful thought, the effort here will have succeeded.

Pankaj S. Joshi
Mumbai, 2014

Acknowledgments

My thoughts on these cosmic themes evolved and developed over many years and I owe much to many colleagues who offered animated and extensive discussions on the issues covered here. While the opinions did not always converge and kept evolving, there was never a lack of interest and excitement due to the very basic nature of the inquiry at hand. In particular, I thank Indresh Dwivedi, Rituparno Goswami, Daniele Malafarina, Ken-ichi Nakao, and all my collaborators for extensive discussions and debates. Their thought-provoking questions and comments have been invaluable.

Conversations with Stephen Hawking, Roger Penrose, and Robert Geroch made me think deeply and intensely on these problems, and those with Peter Biermann, Ramesh Narayan and Kip Thorne motivated me to probe the possible observational consequences of gravitational collapse scenarios. With much fondness I also mention here my few interactions with S. Chandrasekhar, who showed keen interest in these problems, and offered his critical comments. His penetrating insight inspired me to tackle the complex problem of gravitational collapse in Einstein's theory of gravity.

I express my sincere gratitude to Jayant Narlikar, P. C. Vaidya, and A. K. Raychaudhuri, who always took a keen interest in the progress of our work over many years, and offered many useful comments. The very kind support to our work from M. G. K. Menon and B. V. Sreekantan has been invaluable, and I thank them warmly.

My detailed discussions with George Musser, Vithal Nadkarni, and Bharat Kapadia on black holes and cosmology brought out the rich tapestry of interwoven ideas and different aspects of the problem, and I extend many thanks to them. The discussions with Adlung Sonke at OUP led to the idea of this book, and I am very grateful to him for his careful attention throughout. Jessica White, Ania Wronski, Charles Lauder, Ralph Footring, and Gandhimathi Ganesan have taken much care to see this work comes out well, and I am very thankful to them.

Contents

1

Our Universe

Man's journey in search of the fundamental laws that govern the Universe has led us to some of the most fascinating insights on the nature and structure of the Cosmos. Even the smallest child begins observing the Universe as soon as she arrives on Earth, and such observation sparks curiosity and leads to questions about the Universe. The inquiry gradually matures to intuitive, logical, and mathematical thinking and analysis of the Cosmos' basic nature and structure. This is the start of our understanding of the Universe, leading to a wide variety of new discoveries and inventions.

Early astronomers were amazed to learn that the Universe and phenomena such as the occurrence of day and night, eclipses, ocean tides, and the motions of planets, stars, and other heavenly bodies were not arbitrary or random but followed specific patterns and fundamental rules. Such an appreciation prompted an inquiry into the basic laws governing these phenomena and a search for more of an understanding of the world around us.

In due course, this led us to the body of knowledge that we today call science. The quest has resulted in an impressive understanding of the Universe and its basic happenings. This includes Einstein's theory of relativity, and quantum theory, which governs the tiny constituents of matter. Today we have very large telescopes on Earth and in the skies which probe the deepest recesses of the Cosmos. At the same time we build mammoth accelerator machines that collide the tiniest of particles of matter at the fastest velocities possible. While the frontiers of knowledge keep expanding, superb applications have frequently resulted from the basic laws of nature that we have discovered, which have made human life smoother and healthier.

Microcosm, Macrocosm, and Forces of Nature

We observe the Universe today in its smallest dimensions of atoms and hadrons constituting the same in large particle colliders, while at the largest cosmic scales we can see galaxies and their clusters millions of kilometers away amidst a vast expanse. On the one hand, tiny subatomic particles travel close to the speed of light and collide to create a plethora of new basic particles. On the other hand, faraway galaxies collide and merge to give rise to new cosmic entities, while cosmic structures constantly form and disperse. Our knowledge and conception of the Cosmos evolve as our search progresses (see Fig. 1.1 for a historical perspective on the Universe around us).

Such observations of the Universe and the mathematical calculations have shown that all these phenomena are governed by certain basic laws and forces in nature. While day-to-day happenings in nature at our own scales seem to be governed mainly by electric and magnetic forces, at the scale of the atoms and elementary particles so-called weak and strong nuclear forces govern key outcomes. When we move onto the larger cosmic scales, it is fairly clear that more than any other force, it is the force of gravity that counts and decides the natural phenomena, such as the formation of galaxies and stars and the clustering of matter on very large scales.

While different forces seem to rule different arenas and scales in the Universe, man, however, has a passion to search for a certain unity amidst all the diverse phenomena that surround us. We would like to obtain a single logical structure that describes and explains nature in its entirety. If possible we would also like to predict the Cosmos and its happenings using such a framework.

In fact the word 'physics' comes from the Greek root 'physis', which means the basic element or key principle of nature. For example, while electricity and magnetism were observed as a collection of different and diverse natural phenomena, it was James Clerk Maxwell's theory from the nineteenth century that gave a unified description of these phenomena in a single logical framework with a system of mathematical equations. Similarly, while the happenings within the atom and its nucleus are now known to be governed by weak and strong nuclear forces, contemporary, grand unified theories combine these forces with the electromagnetic phenomena and again provide a unified description of these happenings. On the other hand, when we move to the larger

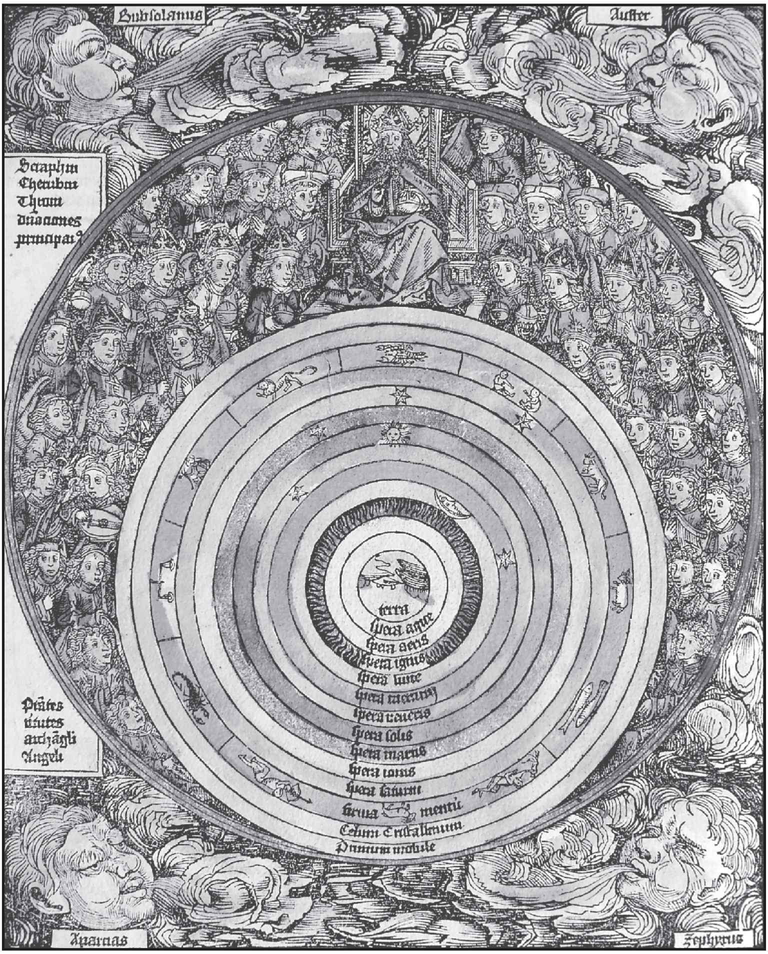

Figure 1.1 An ancient view of the Universe. While mankind is located at the center of the Universe, with the celestial sphere and the dome of sky up above, outside that lie various unknown forces and powerful entities, probably those of air and water and fire and others.

scales of space and time, namely planets, the solar system, and beyond to the expanse of stars and galaxies, it has been known for a long time that it is the force of gravity that governs the dynamics of these objects.

Explaining and predicting planetary motion due to the law of gravity has been the greatest success of Newtonian mechanics. However, Newtonian gravity requires an infinite speed for the propagation of the force of gravity. This is not consistent with the special theory of relativity as derived by Albert Einstein in 1905, where no speed greater than that of light is possible. A new theory of gravity was therefore inevitable, which came in the form of the general theory of relativity formulated in 1915. General relativity, or the Einstein gravity, has been the most successful theory of gravity, having passed several experimental and observational tests and providing many new exciting predictions about the Universe.

Despite these successes, it must be admitted that a vast ocean of ignorance lies before us. Current cosmology tries to understand how the Universe came into being, how galaxy formation was triggered, and how the intricate cosmic web of galaxy clusters observable today through gigantic telescopes came into being. Among others, the formidable problems of the missing or dark matter and the unseen or dark energy that drives the accelerated expansion recently observed in the Universe are far from being well understood. At the microscopic level, as we continue in our quest for the basic building blocks of matter, and for the key forces that govern them, newer and finer layers of reality emerge and open up before us.

Nevertheless, such an entirety of unfolding phenomena has not discouraged us from our search. In fact, to meet such a daunting challenge, mankind has embarked on even bigger and ever more ambitious missions to probe and understand the Cosmos, in both its microcosmic and macrocosmic realms. This is what has led to space telescopes that see the farthest reaches of the Universe, and to the most powerful of particle accelerators that probe the deep secrets of the tiniest constituents of matter.

Thus, while we have been sharpening our technical instruments for observing the Universe, we also have come to realize that intuition and the mind, which are our very basic tools for understanding the Universe, need to be refined and sharpened and understood better. An opinion has emerged that without understanding the nature of the mind, a better understanding of the Universe may not be possible (see Fig. 1.2).

Needless to say, such a journey into the Cosmos has led us to some fascinating landscapes that we had never imagined possible. Many of the phenomena in nature certainly defy our basic and gut feelings about the Universe and always surprise us. As we journey through

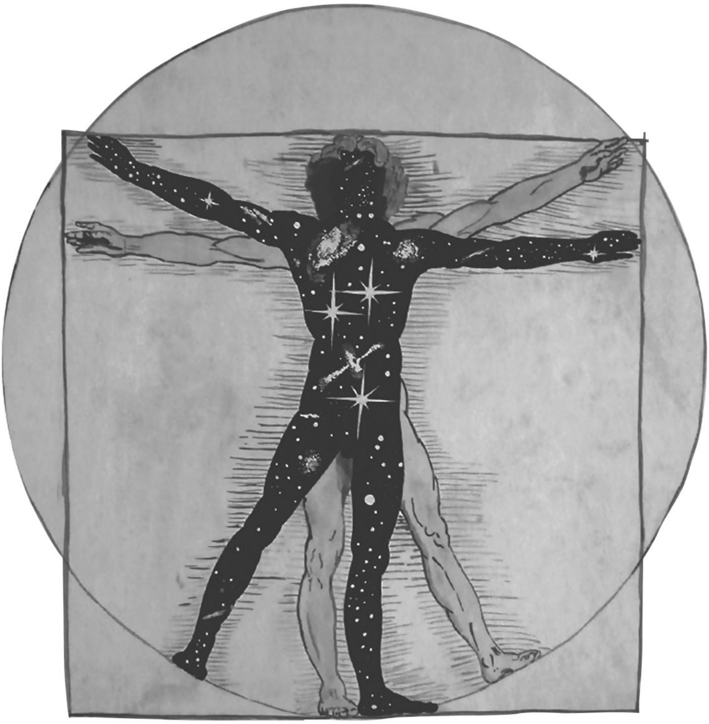

Figure 1.2 The modern approach to the Cosmos sometimes prefers to see man and the Universe as an integrated whole. While we would like to understand our relation to the Universe in its entirety, the big mystery is: Can the human mind really grasp the Universe? Whether it is conceptual frameworks and theory or mathematical calculations and series of experiments, in the end it is the human mind that must make sense of all that experience in the form of coherent laws of nature. These laws may explain existing phenomena, and also make new predictions about nature.

the frontier of knowledge, new domains of the unknown keep opening, and perhaps such a scenario is always bound to arise.

The Role of Gravity

Gravity is the force that man has experienced and tried to understand for ages. Clearly, it governs the Universe on the large scale that we see, perceive, and experience. In the past, we observed planetary motion, ocean tides, eclipses, and so on as manifestations of the force of gravity. Today, we now in addition observe the motion and dynamics of stars and galaxies, the powerful jets from active galactic nuclei, pulsars and quasars, powerful gamma ray bursts, and other such phenomena that occur in the skies.

In addition, there are now the intricate and intriguing concepts and theories of black holes and spacetime singularities, time warps and changing spacetime topologies, and so on. All these together make an amazing spectrum of phenomena that gravity creates in the Cosmos. Many new facets of gravity physics have opened up in recent decades, going further than Newtonian dynamics, and are based largely on Einstein's theory of general relativity. We will describe later in some detail the remarkable picture and various perspectives that have emerged about the Universe that involve gravity in a fundamental way.

Two major revolutions that took place about a century ago in our understanding of the Universe were relativity theory and quantum theory. While the former dominates mainly at the very large scales of the Universe and in very strong gravity fields, the latter has given us a fairly good idea about the microscopic world of elementary particles and their interactions in nature. Of course, the goal has been a unified understanding of both these domains by obtaining a quantum gravity theory, but that dream is still far from being realized. To that end we need new theoretical insights and advances, and more observational data about the Universe.

Our knowledge and conception of the observable Universe are reaching a most interesting and crucial turning point. On the one hand, we are probing the deepest recesses of matter by means of particle collider experiments with the highest collision energies. This has revealed new information about the world of tiny particles and about the fields that cause these interactions. On the other hand, we are facing increasing limitations to Earth-bound experiments, and formidable difficulties arise when we try to understand these phenomena theoretically, due to the complexity of the dynamics involved.

A possible solution may be that if we observe the Cosmos more intensely and carefully, we may come across many more phenomena where the basic forces of nature may be operating collectively in unison. Such observations and signatures could give us crucial inputs for our search for a unified theory of nature. In this sense, astrophysical and astronomical observations are crucial for moving towards a deeper understanding of the Universe. At present, although we have a wealth of observations coming in, major challenges to fundamental physics and cosmology still remain unresolved.

In regard to the force of gravity, whenever physicists deal with very strong gravity fields, such as in the late stages of massive collapsing stars, a huge quantity of matter compacted in a small region such as the center of a galaxy, or rapidly spinning neutron stars, they must use the general theory of relativity to properly understand such phenomena.

Einstein's theory of gravity is very rich, both conceptually and mathematically, and must be handled with very careful analysis and understanding. When applied appropriately to understanding physical scenarios such as those above, it produces rather intricate and intriguing consequences, such as the formation of spacetime singularities, black holes and time warps, and highly curved geometries which may have remarkable observational consequences.

In such a context, the theory and observations of massive collapsing stars may hold a basic key to the understanding of gravity. Such a collapse necessarily gives rise to spacetime singularities, as predicted by general relativity. We point out that if such singularities are visible to observers far away in the Universe, they would then provide a wealth of information about quantum gravity processes occurring in very strong gravity fields which develop in such regions. Such naked singularities developing in the gravitational collapse of massive stars could be viewed as providing us with laboratories of very high energy that would be unreachable at terrestrial levels.

As for Einstein's theory of gravity, despite its remarkable features, we face many key problems. One issue has been that general relativity is a geometric theory of gravity with a mathematical structure very different from other theories of physics, such as Newtonian mechanics or quantum theory. Therefore it has not been possible so far to obtain a unified description of the laws of nature for all four fundamental forces that we mentioned earlier. This has been recognized as one of the most important and outstanding problems in modern physics today, namely how to unify gravity with quantum mechanics. Efforts over the past decades to obtain a unified theory of quantum gravity have included string theories and loop quantum gravity.

As discussed here, rather than only going after purely theoretical frameworks and attempts, which has been the case so far, it may be beneficial and rewarding to look for phenomena in the Universe where quantum effects and gravity come together. In such scenarios, it may be possible to observe genuine quantum gravity effects. This could give us a much better handle to understanding quantum gravity theory and its real workings and basic framework.

The point is, in the terrestrial experiments and many of the observations of the Universe so far, gravity has been a rather weak force. So it stays apart and disconnected from rest of the basic forces in nature and in particular quantum effects. However, as we shall argue, the gravitational collapse of massive stars and the resulting spacetime singularities may be the natural phenomena where quantum and gravity come together to operate in unison. The observations of such regions in the Universe could provide us with the crucial missing steps which we need for a unified theory. This would be an important transition from general relativity, which is a purely classical theory without any consideration of quantum mechanics.

Dynamical Evolution in the Universe

One of the key tasks for any physical theory is to describe and predict the dynamical evolution of physical systems and the evolving phenomena in the Universe. This could be at either a local or global level. For cosmic systems, which are mainly governed by gravity, this is to be accomplished using Einstein's theory of general relativity (see Fig. 1.3).

Such phenomena, on very large scales, include the expansion of the Universe as a whole, driven by gravity, the formation of galaxies and clusters of galaxies, and other such large-scale structure formations. On a somewhat smaller scale, there are formation processes that initiate the life of a newborn star, and later the gravitational collapse of these stars when they run out of their internal nuclear fuel. Thus we could be working on very large scales of the Universe as a whole, or on relatively smaller scales of stars and galaxies, where gravity essentially governs the dynamical processes and evolution of these systems.

As has been observed in the Cosmos today using gigantic telescopes either on Earth or in space, there is a continual dynamical change essentially governed by gravity. Thus, at the largest scales, galaxies recede from each other, or they can sometimes collide and merge together. Then mammoth huge clouds of interstellar matter come together in a slow gravitational collapse and trigger the formation of galaxies or clusters. This can also make even larger filaments and voids in the Universe.

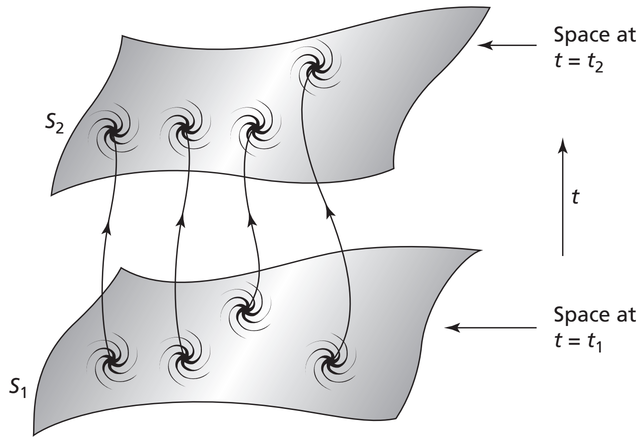

Figure 1.3 In Einstein's gravity theory, we would like to trace the dynamical evolution of three-dimensional space as time moves forward. The evolution of such a system, for example that of a cluster of galaxies, is governed by Einstein's equations. The locations of these galaxies and their distances from each other change over time. Here S_1 and S_2 denote three-dimensional spaces (shown in two dimensions), and the former evolves into the latter as time moves forward from t_1 to t_2, as given by the field equations of gravity. Similarly, we could work out the evolution of the system in the past.

On a smaller scale, gravitational collapse can trigger star formation, where a local cloud of matter contracts under its own gravity, heats up enormously to cause nuclear burning within, and thus first a proto-star and then a star is born. Such star formation goes on in various regions in the Universe all the time, and so does the gravitational collapse of stars toward the end of their life-cycles.

In those cases where the system is very large and gravity is very weak, a Newtonian approximation may work, but essentially it is Einstein's equations of general relativity that govern these phenomena. So one needs to build mathematical models, using Einstein's equations, to describe these phenomena. Such solutions could be obtained either through fully analytic techniques or through numerical models calculated with computers. In either case, the basic idea is to describe and model these gravitational processes, understand their evolution in the Universe, and as a result be able to make predictions.

As indicated earlier, it is not always easy to predict the dynamical evolution of physical systems such as stars, galaxies, or the Universe as a whole under the force of gravity within the framework of Einstein's theory of gravity. There are difficulties and complexities due to the non-linear nature of gravity and to inevitable global aspects related to the spacetime continuum that we must deal with. Also, non-trivial topologies for the Universe could arise with involved geometries giving rise to otherwise unanticipated features. We must, however, look around the mathematical as well as the physical landscape and try to proceed by finding smarter ways for creating new insights into these most interesting cosmic phenomena of nature.

In our present discussion, we will be mainly interested in tracing the dynamical evolution of massive stars within the framework of Einstein's theory of gravity. A massive star, when it runs out of internal nuclear fuel toward the end of its life-cycle, undergoes a continual gravitational collapse. General relativity predicts that a spacetime singularity must arise as a result. Thus, a super-ultra-dense region in spacetime forms. Such a singularity either is wrapped within a black hole or is a visible naked singularity. It is only through tracing the dynamical evolution of the massive matter cloud using Einstein's equations that the final fate of such a star can be determined. We shall discuss these issues in detail in later chapters.

Black Holes, Singularities, and Quantum Gravity

A wide variety of phenomena are created by the all-pervasive force of gravity in the Cosmos. It is therefore not surprising that the modern science of gravitation and cosmology has introduced us to many of the strangest ideas about the Universe. One of the most extraordinary and unexpected is the ultimate fate of a massive star that has reached the end of its life-cycle. A star derives its energy from the nuclear reaction burning within and having exhausted the internal fuel that has sustained it for millions of years as it shined and gave out heat and light, the star is no longer able to hold itself up under its own weight and the force of self-gravity. It starts collapsing, shrinking catastrophically in a matter of seconds. Modest stars like the Sun also collapse toward the end of their lives, but they then stabilize later at a smaller size of about a thousand kilometers. The resulting stable configurations are called white dwarfs. However, if a star has tens of times the mass of the Sun, its gravity then overwhelms all the forces of nature that might possibly halt its gravitational collapse.

In that case, from a size of millions of kilometers in diameter, the star crumples to a pinprick size, which is smaller than even the dot on an ‘i’, and then goes on to shrink even further. It is a star’s continual or total gravitational collapse which gives rise to a spacetime singularity as the collapse end-state. The process of new star birth and gravitational collapse at the end of the life-cycle is a continuously happening phenomenon in the Universe, with a number of stars being born and dying daily.

Gravitational collapse of such massive stars is still a major unsolved problem in the general theory of relativity. Although we now know that a spacetime singularity must result, we do not know whether it will be hidden within a black hole or visible to an observer far away across the Universe, that is, not hidden within an event horizon of gravity. Both outcomes are possible, as the ‘cosmic censorship conjecture’ that naked singularities never form and must always be cloaked within horizons does not always hold. It is an important issue in gravitational physics to determine which will occur in what circumstances. By now, there are several well-reasoned works of research which give careful insight into the general relativity aspects of this problem. These papers emphasize how the issue remains of crucial importance in present-day black hole physics, and also provide a good springboard for those wishing to tackle this important problem.

What then would be the final fate of massive stars collapsing in the Universe? This is one of the most exciting questions in astrophysics and modern cosmology today, as an amazing interplay of the key forces of nature take place here, including gravity and quantum mechanics. It is possible that this phenomenon may hold the secrets to our search for a unified understanding of the forces of nature. It may also have exciting connections and implications for our very-high-energy observations in astronomy and astrophysics. This is an unresolved issue that excites physicists and laypeople alike.

New results have revealed a close connection between the issues of spacetime singularities, final states of gravitational collapse for massive stars, and deep cosmic conundrums such as the cosmic censorship conjecture and the paradox of predictability in the Universe, as well as possible emerging implications for a quantum theory of gravity. The likely connection with observations and implications for relativistic astrophysics and black hole physics could be equally important.

Investigations into the final fate of massive stars began some eight decades ago when Subrahmanyan Chandrasekhar questioned the fate of stars such as the Sun. He showed that such a star, on exhausting its internal nuclear fuel, would stabilize as a 'white dwarf', which is about a thousand kilometers in size. Eminent scientists of the time, in particular Arthur Eddington, refused to accept this result, asking how a star could become so small. As is known, after a fiery debate between the two men, Chandrasekhar left Cambridge to settle in America. After many years, his prediction of white dwarfs was verified. Further to this, it also became known that stars three to five times more massive than the Sun give rise to what are called 'neutron stars'. These are just ten to fifteen kilometers in size, and develop when the star collapses after causing a supernova explosion of the outer layers of the collapsing cloud of matter.

But when the star has a mass more than these specific limits, the force of gravity is overwhelmingly supreme and it overtakes to shrink the star in a continual gravitational collapse. No stable configuration is then possible, and the star that lived for millions of years will catastrophically collapse within a matter of seconds. What will be the final fate of such a continual gravitational collapse of a massive star? The answer must be determined by Einstein's theory of gravitation, as gravity now is the sole force deciding the future evolution of the star: a spacetime singularity, which is an ultra-dense and extreme physical state of matter, not ordinarily encountered in any of our usual experiences of the physical world.

In such a scenario, as the gravitational collapse of the star progresses, an 'event horizon' of gravity can possibly develop. Such a horizon is essentially a one-way membrane that allows only entry. If the star enters the horizon before it collapses to a singularity, the result is a *black hole* that hides the final spacetime singularity. Such a black hole becomes a permanent graveyard for the collapsing star. On the other hand, if the horizon is delayed or does not form as the star collapses, then the final outcome is a *naked singularity*. In this latter case, observers far away in the Universe would have access to the physical processes taking place very near the singularity.

As per our current understanding of physics, it was one such spacetime singularity, called the 'Big Bang', which created our expanding Universe as we know it today. Such singularities will again be produced when massive stars die and collapse in the Cosmos. This is the amazing place at the boundary of the Universe, if there is one, a region of arbitrarily large densities billions of times more dense than the Sun.

An enormous creation and destruction of particles could take place in the vicinity of such a singularity, where densities, pressures, spacetime curvatures, and all other physical quantities diverge and take arbitrarily large values. One could imagine this as the 'cosmic interplay' of the basic forces of nature coming together here in a unified manner. This is because energies and all important physical quantities reach their extreme values in the vicinity of a spacetime singularity. In such a case, it should be the quantum gravity effects that dominate. Thus, the collapsing star may hold secrets vital to man's search for a unified understanding of all forces of nature.

We could observe physical processes near regions very close to a spacetime singularity if we were able to access this information. One such example is the Big Bang singularity, which is, in principle, visible because the entire Universe emerged from it. The question then arises whether such singularities or super-ultra-dense regions that develop again when massive stars collapse can be visible to faraway observers, just as the Big Bang can be seen in principle.

The alternative is that the singularities of massive stars will always be hidden in a black hole. The visibility or otherwise of such a newly formed super-ultra-dense fireball, which we can perhaps call a *quantum star*, is one of the most exciting questions in astrophysics and modern cosmology. That is because, in the case of a singularity being visible, the astrophysical signatures and also those signaling the unification of the fundamental forces of nature occurring near the singularity then become, in principle, observable.

The important point here is that, while general relativity implies that the spacetime singularities must necessarily form whenever massive stars collapse, it does *not* imply that the event horizon, which would cover the singularity, also must form. Nonetheless, physicists assumed that an event horizon *does* always form, hiding all the singularities of gravitational collapse from observers far away in the Universe. This is called the *cosmic censorship conjecture*, which is the foundation of current theory about black holes and their modern astrophysical applications. If the horizon does not form before the singularity, we will then be able to observe the super-ultra-dense regions that form due to collapse of the massive star, and the quantum gravity effects near the naked singularity become observable.

In recent years, a series of gravitational collapse models where the horizon fails to form or is delayed in the collapse of a massive star, as shown by mathematical models or numerical simulations, have been analyzed. This is an exciting scenario because the singularity then becomes visible to external observers who can actually see the extreme physics taking place in the vicinity of such ultimate ultra-dense regions.

As we shall discuss here, it turns out that the gravitational collapse of a massive star gives rise to either a black hole or a naked singularity, depending on the internal conditions within the star. These include its initial densities and pressure profiles, as well as the velocities of the collapsing shells. Depending on these variables, the dynamical evolution of the collapsing star under reasonable physical conditions, as determined by Einstein's theory of gravity, will lead to either the black hole or the naked singularity as the final state.

As such, gravitation theory and relativistic astrophysics have gone through extensive development in recent decades, leading to the discovery of quasars in the 1960s and other very-high-energy phenomena in the Universe. For compact objects such as neutron stars and for situations involving even higher energy densities and masses, strong gravity fields governed by general relativity play an important role, as the strong gravity causes the observed high-energy phenomena possessing these intriguing physical properties. Several models for explaining gamma ray bursts, which emit in a few seconds energy of the Sun's entire lifetime, have been proposed in terms of a collapsar, invoking the collapse of a massive star as the mechanism required to produce the extreme burst. The collapse of a massive star or much larger matter clouds lies at the heart of the astrophysics of such phenomena. Gravitational collapse is the physical process essential to the formation of a star itself from interstellar clouds, in the formation of galaxies and galaxy clusters, and in a variety of cosmic phenomena including structure formation in the Universe.

In a massive star's continual collapse, where gravity is much more powerful, we cannot use the approximations frequently used for a weak gravity regime. It thus becomes inevitable to use general relativity to deal with the issue of the final fate of massive collapsing stars. We must then trace the time evolution of the system using Einstein's equations of gravity. The star shrinks under its own gravity, which dominates all other basic interactions such as the weak and strong forces that earlier provided the outward pressure to balance gravity's pull. Such a general relativistic description would be valid up to very small length scales, at which point quantum gravity effects take over and dominate the physical processes. What we need to do here is to model the collapse within the framework of general relativity, evolving it over time using Einstein's equations to determine what the final configuration will be. This will give us insights into the possible final fate for the collapse.

Our Trajectory

The phenomena in the Universe, at both the microscopic and macroscopic levels, frequently appear infinite and fathomless. The amazement, however, is that even if we take a very small, simple-looking occurrence and study it deeply, unique and beautiful processes and laws of its finer workings will be revealed. We can then extrapolate and determine whether these laws apply as well to other phenomena in the Cosmos. Often they do, and this is how we expand our picture of the cosmic reality. Thus, the Universe reveals many amazing facets as we continue to explore it.

In a similar spirit, in this book we will consider and study cosmic phenomena, including that of the final fate of collapsing massive stars. This has been a particularly interesting issue in cosmology and astrophysics and has given rise to concepts such as black holes and spacetime singularities. These concepts may, in turn, be closely related to our search for a quantum theory of gravity, or the unification of physics. We will discuss developments which have led us in recent years to remarkable conclusions about collapsing stars, black holes, singularities, and related themes. We will also point to newly opened observational frontiers in these areas.

We will show why observing those regions in the Universe where ultra-strong gravity is operating is crucial for obtaining correct theoretical leads toward formulating a proper quantum theory of gravity, because they may be the only places where gravity and quantum forces are operating together.

Our main purpose here is to describe some of these developments and the recently emerging frontiers, which may turn out to be exciting keys to our understanding of the Universe. In the previous section we summarized the basic story contained in what follows. We will now describe the structure and sequence of the discussion that follows in the next chapters.

We have indicated a few remarkable vistas, both theoretical and observational, that the physics of gravity has presented to us in recent decades, especially from the perspective of Einstein's theory of gravity. In Chapter 2, we describe the key ingredients of this theory, which has been fundamental in explaining the phenomena of gravity as observed in the Universe. The basic paradigm shift that has occurred in our concepts of space and time, as compared to Newtonian gravity, is emphasized. We point out that this has led to many new conclusions about the Universe as a whole.

In particular, the fascinating concept of black holes emerged when the gravitational collapse of a massive star was investigated using Einstein's theory of gravity. We discuss this development in Chapter 3. The furious debate that took place over the existence of spacetime singularities is narrated in Chapter 4. Various possibilities for avoiding such singularities are considered and it is concluded that under a wide set of physically reasonable conditions, general relativity necessarily implies the occurrence of spacetime singularities when massive stars collapse, and in cosmology at the origin of the Universe.

If such singularities are hidden within event horizons of gravity, we then have a firm basis for developing an entire physics of black holes which can also be applied widely to astrophysical scenarios. Such a theory, assuming that there are no visible or naked singularities occurring in the collapse of massive stars, referred to earlier as the cosmic censorship conjecture and fundamental to black hole physics, was developed in the 1970s. Various aspects of this hypothesis are narrated in Chapter 5, and we point out the need to study gravitational collapse models via general relativity.

When we study the collapse of massive stars in some detail, we can see that visible or naked singularities do occur in gravitational collapse once we use physically realistic assumptions. For example, a higher density at the center of the collapsing star, which decreases as we move outward, gives rise to a naked singularity at the final stage. These conclusions and the wide variety of collapse models investigated over the past years are discussed in Chapter 6.

The occurrence of naked singularities in gravitational collapse gives rise to several very intriguing questions and cosmic puzzles. Profound issues such as the status of the cosmic censorship hypothesis ruling out naked singularities, together with various questions about naked singularities, will be discussed in Chapter 7. The conundrum related to predictability in the Universe is discussed in Chapter 8.

If we accept that naked singularities can indeed form in a gravitational collapse at the end of a massive star's life-cycle, or in other circumstances, that opens up a remarkable array of exciting possibilities. This is because the observational signatures of the physical processes occurring in the super-ultra-strong gravity regions where matter densities and curvatures will be in the extreme then become visible to external observers. Such physical processes would naturally involve quantum gravity signatures, because both strong gravity and quantum effects must work together in such regions. In Chapter 9, we discuss whether a quantum gravity laboratory is created whenever a massive star collapses to form a naked singularity. Finally, several recent, exciting developments, together with possible observable consequences of naked singularities which may distinguish them from their black hole counterparts through astrophysical signatures, are considered in Chapter 10.

The essential story that emerges is that according to general relativity, gravitational collapse of massive stars must proceed to create spacetime singularities, where the usual laws of physics break down and quantum gravity effects operate in strong fields near the singularity at ultra-small length scales. We will highlight here a spectrum of developments and results which deal with probably some of the most exciting issues on which research in gravitation and cosmology is currently centered. We thus will discuss several interesting results about the end-states of gravitational collapse, cosmic censorship, and black holes and naked singularities. While no claim to completeness is made, we hope that what is presented will paint an interesting view of the landscape of gravity physics and the exciting cosmic frontiers that are emerging.

2

The Fabric of Spacetime

In order to understand the Universe, we need to consider the physical events happening within it. Any event, such as an eclipse or the birth of a star or a galaxy, happens at a certain location in space and at a given moment of time as measured by an observer. Thus we need to consider space and time together in order to comprehend the set and sequence of happenings in the Cosmos. It was such combined thinking on the nature of space, time, and matter that led to Einstein's theory of gravity, which is the general theory of relativity.

In fact, for aeons man has known that the motions of bodies such as the Sun, Moon, and planets follow regular and rhythmic cycles against a background of stars on the celestial sphere. We tried to understand these cycles in order to predict, for example, when the next eclipse will occur. The ancient civilizations of Egypt and Babylonia quite successfully devised methods for predicting planetary motion. In fact, Ptolemy's tables would be able to tell us where the Moon will be tonight with quite reasonable accuracy.

These developments led us to understand that it is the force of gravity that governs phenomena such as planetary motion. This understanding has evolved into the modern gravitation theory, which is attired in the format of three-dimensional space and one-dimensional time knotted together to form a spacetime continuum. Against such a background, the force of gravity is described through the equations of general relativity, and all matter, such as stars, planets, and galaxies, move in an inexorable march from the past into the future.

Many of the amazing and intriguing phenomena in the Universe that we discuss in physics today, such as black holes, the Big Bang as the origin of the Universe, spacetime singularities, gravitational waves, worm holes, and time warps, were predicted to exist as a direct consequence of analyzing the properties of such a spacetime Universe and the interactions of matter and geometry therein. This is within the framework of Einstein's general theory of relativity. We will discuss here key elements of Einstein gravity, which will help in understanding why the theory produces some of the most remarkable insights and predictions on the Universe.

The Force of Gravity

With the work of Galileo Galilei and observations from his telescope, and with Johannes Kepler's theory, and Tycho Brahe's observations, we moved from a geocentric cosmology where the Earth was taken to be at the center of the Universe, to a heliocentric view of the Sun and the planets moving around it. Compared to the geocentric model where circles on circles were added to explain the observations, the Keplerian use of elliptical orbits was much more neat and satisfactory. Actually Kepler's laws of planetary motion explain the movements of these celestial objects extremely well.

It was, however, Isaac Newton who pointed out that these phenomena can be explained and predicted rather beautifully well through what he called the force of gravity. His formulation of the theory of gravity was contained in his law of gravity, which states that any two masses exert a gravitational force on each other, proportional to their masses and inversely proportional to the square of the distance between them. Using this law of gravity and the newly developed mathematical tool of calculus, not only could he explain and derive Kepler's laws of planetary motion, but he could also predict new phenomena such as what sort of orbits, say circles, ellipses, or other conical sections, celestial objects will make round the Sun. Newton showed that this will depend on the energy these objects possess. The Newtonian theory of gravity was developed further by many mathematicians and physicists such as Gauss, Euler, and Lagrange, and it culminated in a very successful framework of classical Newtonian mechanics.

While the Newtonian theory reigned supreme for some three centuries, in the nineteenth century important advances in physics related to the phenomena of electricity and magnetism took place, when it was realized that these phenomena are intimately related. For example, any changing magnetic field gives rise to an electric current and, conversely, a moving charge produces a magnetic field. However, it turned out that the electromagnetic phenomena were not quite consistent with the overall framework and implications of Newtonian mechanics. These forces were quite different from the force of gravity. For example, electric forces can be attractive or repulsive, as opposed to gravity, which is only attractive, and also electromagnetic disturbances move with a finite speed, as opposed to the instantaneous interaction of gravity assumed in Newton's theory, which amounted to an infinite speed of interactions.

This became much clearer when James Clerk Maxwell formulated his electromagnetic theory, which collectively explained these phenomena. An important consequence of the four equations that Maxwell derived was that a fundamental velocity, usually denoted by c, appeared in these equations with which all electromagnetic interactions must travel. Thus light, which is an electromagnetic wave, travels at this speed c, which is measured to be about 3×10^{10} cm per second.

This basic velocity, however, did not conform to the laws of Newtonian mechanics. For example, Newtonian theory predicted that if the Earth travels with a velocity v in a certain direction, an observer stationary on Earth should measure the velocity of a beam of light shot in the direction of the Earth's motion to be $c - v$. Similarly, it should be $c + v$ if the beam is in the opposite direction of the motion of Earth. The physicists Albert Michelson and Edward Morley did such a measurement and it was found to the great surprise of all that in either case the velocity of light turned out to be c only. This violated the implications of Newtonian mechanics.

It was thus increasingly clear that electromagnetic phenomena could not be incorporated within the Newtonian mechanics framework and that a new theory was needed to explain them collectively. It was Albert Einstein who recognized that the velocity of light, c, was a fundamental constant of nature, and that in all directions the velocity of light would be the same. Based on the assumption that c was an absolute constant of nature, he formulated the new theory which was called the special theory of relativity. This gave results consistent with Maxwell's theory of electromagnetism, and in the limit when velocities are small compared to c, the new theory reduced to the Newtonian mechanics results.

The term 'relativity' was used here because it was realized for the first time that, as opposed to the Newtonian mechanics, space and time intervals as measured by different observers are not absolute and one and the same, as was assumed earlier. Their measurements actually depended on these observers and their velocities.

In such a case, it helped to think of space and time not as separate entities but as forming a single *spacetime continuum*. Such a continuum is an entity which is four dimensional, with three dimensions of space and one dimension of time. The measurement of distances in such a continuum is given by the so-called *spacetime metric*. The theory's equations show that the magnitude of the *spacetime intervals* as measured by such a metric would be the same and invariant for all inertial frames of reference, namely for all observers who travel with a constant velocity relative to each other.

It should be noted that just as electromagnetic theory required that Newtonian mechanics had to be revised for special relativity mechanics, in the same way special relativity also demanded that Newtonian gravity had to be revised. This was because Newtonian theory assumes that the gravitational interaction between masses propagates instantaneously, that is at an infinite speed. But according to special relativity mechanics, all interactions must have a speed less than or equal to that of the speed of light. This prompted Einstein to construct a new theory of gravity, consistent with special relativity mechanics, which was called the general theory of relativity.

Spacetime Continuum

Newtonian gravity was very successful at the level of planets and solar systems in making many useful predictions on dynamics and planetary trajectories, so much so that scientists believed that they had essentially solved all the mysteries of nature. However, Newtonian theory had to be modified in view of later important developments in physical theory, in particular the special theory of relativity.

We pointed out earlier how Newtonian theory was revised due to contradictions with the nature of electromagnetic phenomena, and then special relativity was revised to incorporate the force of gravity, to become the theory of general relativity. In formulating the general theory of relativity, Einstein made fundamental use of the idea of the spacetime continuum. Basically the question was how to include gravity within a four-dimensional description of space and time, so that the theory reduces to special relativity with appropriate limits when there is no gravity. The gravitation theory formulated by Einstein in 1915, consistent with special relativity, has been highly successful and has opened up many new dimensions and vistas in our thinking about the Cosmos as a whole.

The continuum of special relativity is 'flat'. Any point in such a continuum is called an event, which has three space and one time coordinates. These locate any happening by where it happened and at what time. Because the velocity of light is taken now as the universal maximum, any event can influence future events only in a 'cone of influence', which is called its future *light cone*. The same is true for past events as well (Fig. 2.1).

It is clear when we look at the Universe that all astronomical bodies such as stars, planets, and galaxies are connected and governed by the force of gravity. All the same, whenever we look at any astronomical object, for example, a star twenty light years away in distance, we are really looking back twenty years in time, because the light that we see from the star today was actually emitted twenty years ago.

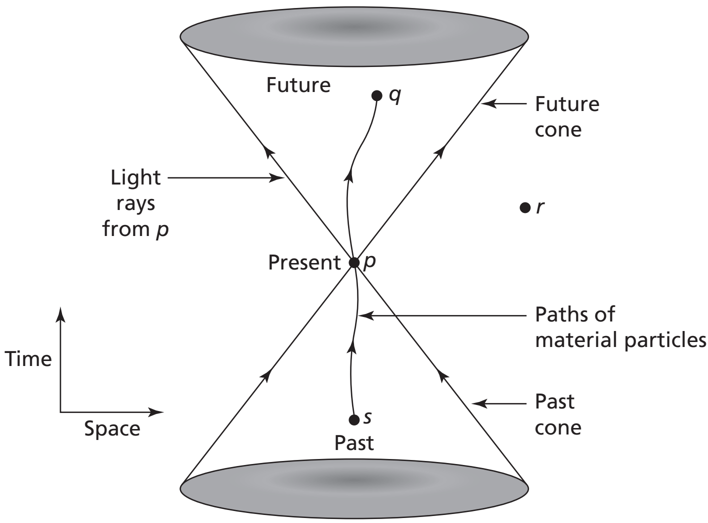

Figure 2.1 In special relativity, the speed of light c is the maximum of all speeds. For simplicity if we choose units so that $c = 1$, then given any event p in spacetime, the set of all light rays from the same is given by a cone as shown. Since material particles travel at a speed less than c, their paths must lie within the cone, and these are called timelike trajectories. The light rays are called null paths. The exterior of the cone is called the spacelike region for p, and no communication from p can reach any other event r in this spacelike part. On the other hand, happenings at p can influence any event q in its future, or any event s in its past can in turn influence p.

This is strongly suggestive of the fact that space, time, and gravity are intimately connected to each other.

In the case of special relativity physics, there is no gravity and the spacetime curvatures are zero at all points. This would be a very good approximation in laboratory settings on Earth, or locally in the Universe, where the force of gravity could be taken to be very small compared to other physical forces which are much more dominant.

The key suggestion made by Einstein was that in order to understand gravity, first we need to see space and time as forming a single *spacetime continuum*. This was a radical departure from the Newtonian way of considering them as separate and absolute entities. Second, gravity could no longer be regarded as a 'force', but had to be seen as a manifestation of the curvature of this spacetime continuum. In other words, the key effect of the 'force of gravity' is that it 'curves' the spacetime continuum, which then no longer remains the flat substratum of special relativity.

According to Einstein, the nature of spacetime is altered and it curves in the presence of gravity. For a flat surface, the particles on which no forces are acting and light rays will travel in a straight line. In a curved continuum also, they will still travel in the most efficient manner, and such paths are called *geodesics* in spacetime (Fig. 2.2). The problem with

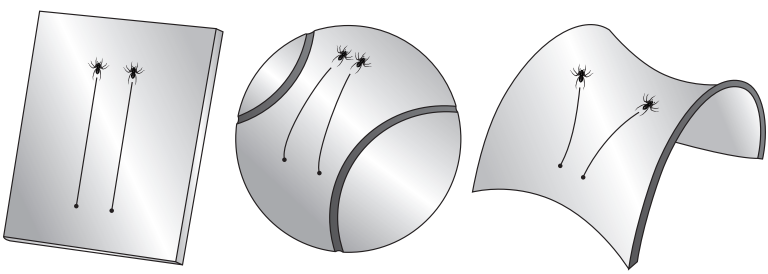

Figure 2.2 A two-dimensional analogy of the shortest paths. Depending on the curvature of the space, the shortest paths, which are called geodesics, that exist between points in that space may vary. In the usual flat sheet medium these are straight lines, but not so when the space is positively or negatively curved. Such paths taken by material particles in a spacetime are called timelike geodesics, and those by light rays are called null geodesics.

gravity then becomes how to describe how the spacetime continuum is curved, and how to quantify the curvatures.

Einstein's Theory of Relativity

The key difference between the Einstein gravity, or the general theory of relativity, and Newtonian gravity is that rather than treating it as a force, Einstein treated gravity as the curvature of the spacetime continuum. Thus a very weak gravitational field or no gravity would correspond to an almost or completely flat spacetime geometry, whereas very high gravity fields would be represented by high spacetime curvatures. The basic model of general relativity is that of a spacetime continuum, with a distance function which is called a *Lorentzian metric*, to measure distances between nearby events and along paths of material particles. Such a continuum structure of spacetime has been verified to be valid up to the largest observable cosmological scales, and on the smallest scales this will hold at least until quantum effects take over, which is much below the radius of an elementary particle on the order of 10^{-13}cm.

So to take into account the force of gravity, we need to turn to general relativity, which describes gravity as spacetime curvature. Einstein's field equations related this curvature or spacetime geometry to the matter content of the Universe. Thus, in general relativity, the Universe is modeled as a spacetime continuum which has a structure of a four-dimensional *differentiable manifold*. This means that locally spacetime is always flat in a sufficiently small region around any point, but on a larger scale that is not the case and it can have a more rich and varied curved structure. An example of such a manifold is a two-dimensional sphere. If we choose any point on its surface, there is always a small enough region around the point which is flat enough in the vicinity. But when we go to a larger region, the non-zero global curvature of the surface is immediately manifested.

Physically, what this means is that while on larger scales gravity will prevail and curve spacetime, locally special relativity always holds in a small neighborhood of any event. This was incorporated in general relativity by Einstein in the form of the *equivalence principle* (Fig. 2.3). To understand gravity in such a manner, we need to have a clear idea on what is 'curved' and what is 'flat'. The mathematics that Einstein used to determine this was that of Riemannian geometry, which is a

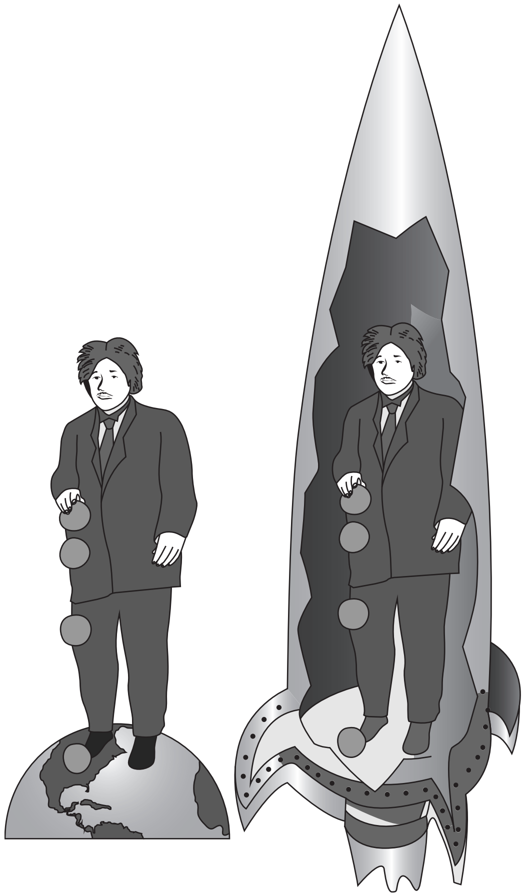

Figure 2.3 The equivalence principle plays a basic role in the construction of the general relativity theory. What it says is that locally the effects of gravity can always be mimicked by an accelerated frame of reference. For example, a person observes that when he drops balls on the surface of the Earth from a height, they fall to the ground. Now if he were in a rocket going up in accelerated fashion, he would similarly observe that the balls fall to the bottom. How this was used in the construction of Einstein's theory was that while globally the spacetime universe may be curved, locally one can always choose a frame of reference in which it looks flat.

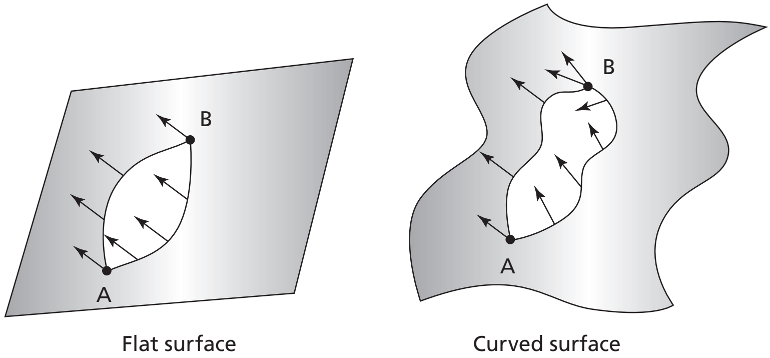

Figure 2.4 The basic idea of curvature is characterized by the notion of parallel transport. To see this, we can consider parallel transport of vectors on a flat surface, and that on a curved surface. As we go from point to point, we make sure that the length and orientation of the vector do not change. In a flat surface, we then obtain the same vector at the end of the trip, independent of whatever path we take. But in a curved surface, the resultant vector will be different depending on the path we have taken to reach the final destination. Thus, the vector is no longer the same after the transport in a curved medium.

curved surface geometry, two dimensional or higher, developed in the nineteenth century by Bernhard Riemann and others. On such surfaces, the curvature of the surface is characterized by the notion of 'parallel transport'. One can understand this with the help of two-dimensional surfaces again, as shown in Fig. 2.4.

Thus the main point of general relativity is that gravity is represented by curvature of the spacetime continuum. The specification of this curvature is determined by Riemannian geometry, which quantifies these curvatures as we move from point to point. What remains now is to specify how the presence of matter and energy density curve spacetime. This is given by the field equations that Einstein constructed, relating curvatures with the matter content of spacetime. So if we specify the matter content, Einstein's equations will tell us how spacetime is curved or what is its detailed geometry. Just as the most efficient path to travel in a flat geometry is a straight line, similarly material particles and light rays travel along geodesic curves in spacetime.

So to get the full picture, we need to solve Einstein's equations together with the geodesics. Typically, what we are looking for is, given a physical situation, a geometry of spacetime that describes this physical scenario. To do this, the physical parameters of the phenomenon are specified and then Einstein's equations are solved within these constraints. Such a solution gives the geometry of the corresponding spacetime model that describes this physical happening, in terms of the spacetime metric. Such a metric typically consists of ten quantities which vary from point to point in spacetime and which specify the gravitational potentials or gravity forces and curvatures at each of these points and thus throughout spacetime. This completes the full description of the phenomenon; the physical consequences are then to be explored and predictions made for future physical events.

For example, suppose there is a massive star existing in the Universe, and we want to know how the physics behaves in its vicinity (see Fig. 2.5). This scenario was worked out by Karl Schwarzschild in 1916, soon after Einstein gave his field equations, and the solution is now known as the *Schwarzschild metric*. This geometry described the gravitational field around the star, and many of the experimental effects and predictions of general relativity, such as the perihelion of Mercury, were worked out using this metric. Similarly, the metric for the Universe as a whole, assuming it to be homogeneous and isotropic, was worked out by Friedmann, Robertson, and Walker (FRW). The now well-known image of the Big Bang Universe emerges from this FRW Universe geometry.

Einstein's theory certainly presents several very intriguing physical and mathematical features and makes remarkable predictions on the nature of the Universe. We can try to get an insight into why it does so.

First, Einstein's formulation of gravity is, for sure, rather different from other physical theories, both mathematically and in its structure as a theory. In the Newtonian theory, gravity was described by a single equation which gave the force of gravity between two masses as determined by their magnitude and the distance between them. On the other hand, general relativity describes gravity via Einstein's field equations, which involve ten quantities as given by the spacetime metric with the physical meaning of gravitational potentials at any given spacetime event. All of these are functions of space and time coordinates. Mathematically, Einstein's equations are rather complex, differential equations, nonlinear in their variables involved. Thus, to solve these equations in order to obtain and understand the workings of the force

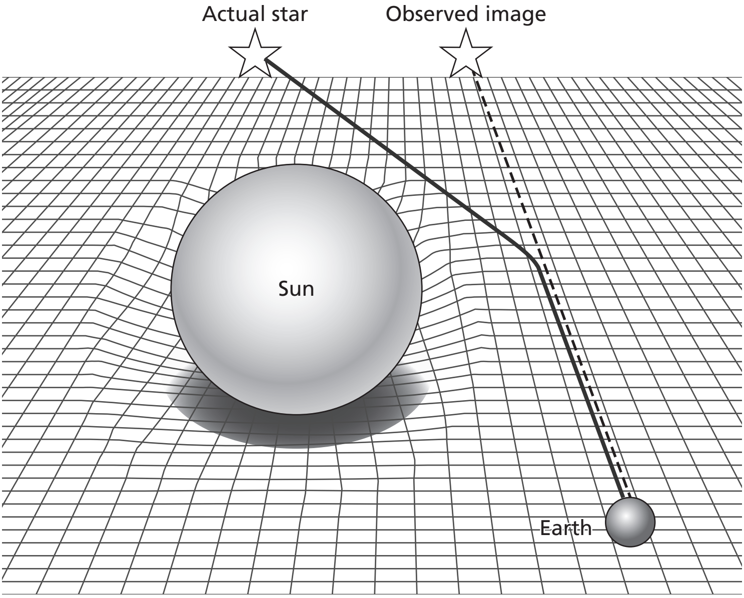

Figure 2.5 The bending of light due to gravity. We can consider a sheet of cloth, which has a flat geometry otherwise. But when a mass is dropped into it, it becomes curved, curvature being higher closer to the location of mass. The paths of light rays and the properties of a flat geometry are radically different from those in a curved geometry. These properties and essentially the curvature of the medium then basically reflect and capture the presence of mass or gravity in the continuum of space and time. Such a light-bending effect was predicted by Einstein's theory, which was verified later by observations.

of gravity and to make models of physical phenomena based on them becomes a rather non-trivial, complicated task.

The added richness of the Einstein gravity probably comes from the fact that we are no longer dealing now with the Euclidean geometry of space and time, as was the Newtonian case. We must now work with a Riemannian geometry on a curved spacetime continuum to describe gravity. Such a geometry may be changing from point to point in this substratum. As opposed to the Newtonian theory, where the three-dimensional space and one dimension of time were treated as absolute and entirely separate entities, general relativity treats these as intertwined and mutually involved with each other. This interwoven fabric of space and time makes the four-dimensional spacetime continuum. Thus the entire physics now takes place in an arena of a four-dimensional continuum.

This is a basic paradigm shift which has brought out ideas and possibilities about the Universe which were very hard even to imagine previously. We now have the possibility of considering Universe models with many non-trivial spacetime topologies, or universes with dimensions, higher than the standard four dimensions, where there are extra space dimensions that are compactified.

What is remarkable is, unlike all other physical theories, general relativity incorporates and is based upon non-trivial geometric and topological features for the overall geometry of the Universe. The latter aspect, namely the *topology*, incorporates the key geometric differences between objects and shapes. For example, a sphere and a doughnut are topologically different from each other, or they do not have the same topology. To see this difference, we can note that any circle drawn on the spherical surface can be shrunk to a point, but the same is not true for a doughnut (Fig. 2.6). On the other hand, any distorted sphere is topologically equivalent to a perfect sphere.

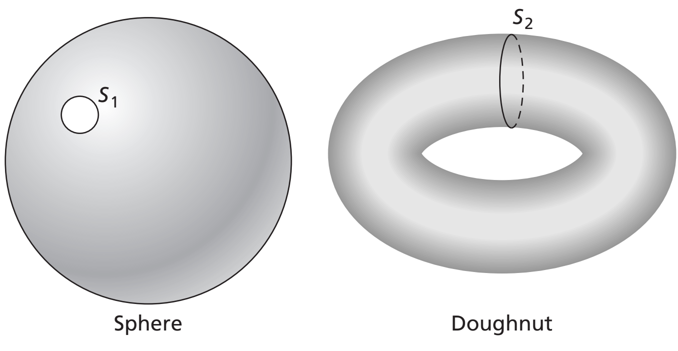

Figure 2.6 Topology characterizes different geometric shapes and figures and specifies the differences between them. We can, for example, consider a sphere and a doughnut, to show that these objects have fundamentally different shapes geometrically. For example, any circle S_1 drawn on a sphere can be continuously shrunk to a zero radius. The same, however, is not possible for all circles, e.g. S_2, on the doughnut. This is basically due to the characteristic differences these spaces have. The general theory of relativity as such permits any possible topology for a spacetime universe, which Einstein's field equations do not constrain in any manner. So different solutions can have different overall topologies. Such a scenario can have observational effects in cosmology, depending on what model we choose to represent the Universe.

Physical Implications

General relativity has given rise to the most unusual aspects of the Universe, such as black holes, spacetime singularities, causality violations and time travel, worm holes, gravitational waves, and other such features. These are consequences of Einstein's gravity and many of them are totally novel aspects that were never anticipated by Newtonian gravity. While several experimental predictions of relativity have now been verified, other possibilities indicated by it are under strong theoretical and observational investigation.

The key reason Einstein's theory predicts and includes such phenomena is that, unlike all other physical theories including quantum mechanics, which are purely local in terms of their dependence on spacetime events, general relativity crucially incorporates global and nonlinear aspects of space and time. This includes allowing for different spacetime topologies for the Universe. The structure of general relativity as described earlier, together with the complexity of Einstein's equations, gives rise to the perception that the force of gravity is rather different from the other three fundamental forces of nature. That is perhaps why it has been so difficult to obtain a quantum theory of gravity that combines gravity with the rest of the forces of nature.

It is therefore natural that as of now only few physically useful and meaningful solutions to Einstein's equations have been obtained. These include the Schwarzschild solution, which gives the spacetime geometry around a point mass that describes a black hole. The geometry around a static star will also be described by the same solution. Then the Friedmann–Robertson–Walker solutions describe a homogeneous and isotropic universe, which is basic to modern cosmology. There are also other interesting solutions such as the Reissner–Nördstrom, Kerr–Newman, and Vaidya geometries, which we shall discuss later.

After the theory of general relativity was constructed, most of the research in past decades has been using analytical techniques. In recent years, however, we have used computers and numerical techniques increasingly in general relativity. It is possible that this developing field of numerical relativity will see much progress in coming years.

Despite these difficulties and limitations, Einstein's theory has, in fact, produced a number of insights and rather novel and intriguing directions and areas to explore as far as our understanding of the Universe is concerned. Some of these scenarios and possibilities are so unique that they stand apart from the rest of the physical Universe created by the three other fundamental forces of nature. Thus, for example, we have the Big Bang singularity, the Universe's theoretical origin, as a prediction of the general theory of relativity. This prediction, together with a host of observational developments such as those related to cosmic microwave background radiation, has given rise to the exciting field of modern cosmology. Also, intriguing predictions such as black holes, singularities, and gravity waves are currently being studied as frontier developments in modern physics (Fig. 2.7). These are

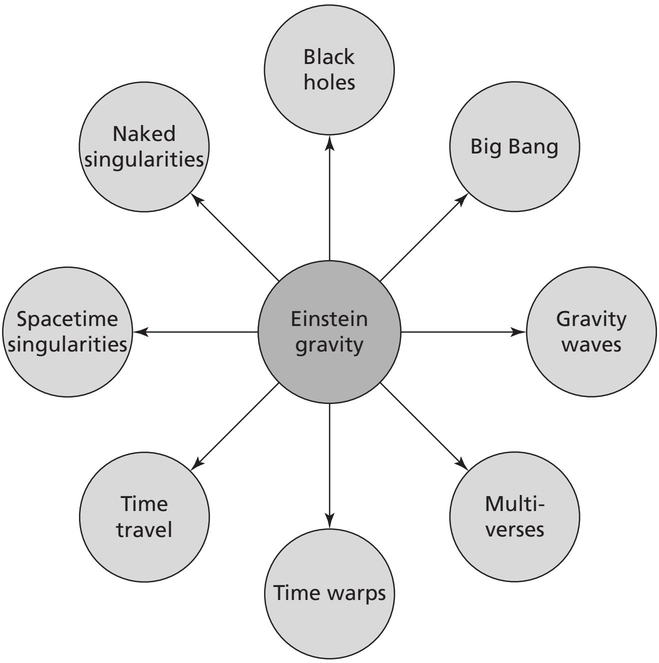

Figure 2.7 Remarkable predictions of Einstein's theory. There are several novel and important predictions that Einstein's theory makes about the Universe. None of these would have been anticipated from Newtonian theory. Constructing a gravity theory that is consistent with special relativity, namely the general theory of relativity, leads to many intriguing features such as black holes, singularities, and time warps. Many of these possibilities, such as multiverses, which are independent universes made in the creation process, gravity waves, and naked singularities, are very much under current investigation.

the direct consequence of analyzing the properties of gravity within a spacetime universe and the interactions of matter therein within the framework of Einstein's theory of gravity. Thus, several exact solutions of Einstein's equations and their properties give us a good idea of the potential possibilities.

Soon after the field equations of general relativity were formulated, important developments fundamental to relativistic astrophysics and cosmology took place. Firstly, Karl Schwarzschild worked out a solution to Einstein's equations in 1916 under several simplifying assumptions, such as that spacetime was taken to be spherically symmetric, containing a point mass at the center of symmetry but totally empty otherwise, and far away from the center the geometry was taken to be flat. This became known as the Schwarzschild solution and has subsequently become the basis of black hole physics. It has been also a guiding principle for many of the experimental tests of the general relativity theory.

Certain features of the Schwarzschild metric have been the subject of major debates for gravitation theorists in the ensuing decades. First, the gravitational potentials had a degeneracy at a certain value $r = 2m$ where m is the mass of the particle at the center and r is the radial coordinate. It was thought that this represented a total breakdown and a singularity in the description of physics around the object. Actually, it wasn't discovered until the 1960s that this was not the case, as we shall discuss later, and that the genuine spacetime singularity was present only at the center, at $r = 0$. Finally, it became clear that the surface, $r = 2m$, was the event horizon of what would be later called a black hole. Also, there was much discussion as to whether the surface, $r = 2m$, is a genuine physical feature of the spacetime geometry, which it was, as we shall discuss later.

The second important development was to find useful solutions to Einstein's equations which would be physically meaningful even if the solutions were obtained under simplified and idealized assumptions. As mentioned earlier, Einstein's equations are a complicated set of nonlinear differential equations. Thus finding a fully general solution had been a far-off dream. In 1922 Alexander Friedmann worked out a cosmological solution by assuming that the Universe is homogeneous and isotropic. A kinematic geometry for such a Universe model was similarly provided by Howard Robertson and Arthur Walker, which fixed the metric for the Universe. This is now referred to as the FRW metric of the Universe and is a key input for modern cosmology.

Further to this, from about the 1930s onwards, for many decades general relativity was a somewhat dormant field of activity. Einstein himself was busy working toward generating a unified theory for fields. Much of the effort then was focused on obtaining physically meaningful solutions to Einstein's equations. In this period, the Reissner–Nördstrom solution, which was a generalization of the Schwarzschild metric with a non-zero charge, and the Kerr and Kerr–Newman solutions, which included the additional feature of rotation and charge, were obtained. The Vaidya solution obtained in 1942 included the radiation in the exterior of a star, which was another generalization of the Schwarzschild case.

An important development in the same period was the work of Robert Oppenheimer, Herbert Snyder, and A. Datt (OSD) in the later 1930s, who modeled the gravitational collapse of a massive matter cloud within the framework of full general relativity. Not much attention, however, was paid to this at that time, as it was thought that a star would never really shrink under its own gravity to such extremes. A discussion on gravity waves was also initiated in this period.

It was, in fact, in the 1960s that the field of gravity physics and relativistic astrophysics warmed up. The essential trigger was the discovery of very-high-energy phenomena such as active galactic nuclei, quasars and radio galaxies, which drew attention to earlier work of OSD. The concept of black holes was developed further by John Wheeler, Roger Penrose, Brandon Carter, Stephen Hawking, and others. The cosmic censorship conjecture was put forward by Penrose in 1969, which stated that generically the collapse of a massive star will end up as a black hole. This was a big push to take forward the physics of black holes and its applications in relativistic astrophysics. Many theoretical results about black holes such as the area theorem and those dealing with black hole thermodynamics were developed. Key physical properties of black holes and their possible detection through astronomical observation have been discussed. This continues to be big area of research for relativistic astrophysicists, astronomers, and gravitation theorists alike.

Prior to this, during the 1960s and early 1970s, a big debate broke out about the existence of spacetime singularities. It was widely thought that the spacetime singularity present in solutions to models such as the Schwarzschild and FRW would go away once more general models without special symmetry conditions were considered. The singularity theorems due to Hawking, Penrose, and Robert Geroch showed that, however, spacetime singularities are present rather generically in the general relativistic spacetimes, and also in other metric theories of gravity. Very interesting and useful development took place during this time to help understand the global and topological properties of general spacetime, and the gravitational focusing within due to its matter–energy density content. Based on this study it was determined that the existence of singularities could be established in a general manner.

Further, an intense period of application of black holes in relativistic astrophysics was initiated. Of course physicists have been very aware that important issues such as establishing the cosmic censorship conjecture, which has been at the foundation of black hole physics and its modern-day applications, remain unresolved despite much serious effort. This has also given rise to efforts in recent years to work out gravitational collapse models within the framework of Einstein's gravity. This work has shown the formation of black holes and naked singularities as final states for a complete collapse of massive stars, as we shall be discussing here.

The phenomenon of Hawking radiation, suggested in 1975, triggered new efforts toward a quantum theory of gravity. The quantization in curved background geometries was tried extensively and currently much effort is being expended in loop quantum gravity formalism and string theories. It has been generally agreed by researchers that combining gravity and quantum theory in a single theoretical framework is a formidable task, and no clear clues are yet available.

Among more recent developments have been the cosmological observations and related problems of dark energy and dark matter in the Universe. There have been observational missions such as the Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck mission in recent decades. The main aim of these has been a detailed study of the cosmic microwave background radiation. Many observations related to the accelerated expansion of the Universe have also been made. The theoretical possibilities of explaining this phenomenon include the existence of dark energy. Possibilities of various dark matter candidates are under investigation. Much effort has also been made to understand the physics of gravity waves, and their possible observational detection.

All this effort has been greatly supported by the developing field of numerical relativity, with the arrival of very powerful computers. Computer simulations are useful in studying structure formation, and in modeling gravitational collapse in full general relativity. Such numerical efforts may generate new insights into the nature and properties of Einstein's complex equations. All in all, this presents a very active and vibrant scenario for research in the field of gravity physics and its applications in astrophysics and cosmology.

Local and Global Aspects

While other forces of nature such as electromagnetism and weak and strong nuclear forces can be described purely locally, one of the most important lessons resulting from general relativity is that the force of gravity has both local and global aspects, described in terms of the local and global spacetime geometry.

An interesting example of this is that while in a local region of space and time special relativity is always valid and there are no causality violations taking place, on a larger scale the geometry of the Universe could be such that an observer can enter her own past. The Gödel universe solution of Einstein's equations, obtained by Kurt Gödel in 1942, which included an overall rotation for the Cosmos, has such a property. This is purely a global property of spacetime, even when locally the special relativity physics is fully valid. The point is, in general relativity, on the global scale the topology and geometry of the Universe can be very different from those allowed by the Newtonian models, which allow no other geometry other than Euclidean space.

Thus global aspects of spacetimes are very important in Einstein gravity, and in fact it is these that give rise to the many novel features of general relativity. Global considerations have been important in the general theory of relativity right from its inception, and also for other theories of gravity. The same is true for cosmology also by the very definition and purpose of that science. Such a role for global aspects becomes easier to understand if we note that even when locally the laws of physics are of special relativity and spacetime is very nearly flat, the spacetime universe as a whole is made by smoothly joining these local patches, which is what gives rise to a non-flat, curved continuum. There is an inherent freedom in how these locally flat patches are to be joined together, and this is what gives rise to different non-trivial structures in spacetime models. Similar global features arise in other theories of gravity as well, such as those of Brans and Dicke given in 1961, which are metric theories of gravity based on a spacetime continuum model.

We know that gravity is a very weak force such that the ratio of the gravitational force to the electromagnetic force between an electron and a proton is about 10^{-43}. However, it turns out that despite being the weakest force of the known fundamental interactions, gravity implies remarkable conclusions as far as the overall large-scale structure of the Universe is concerned. An example of this is the remarkable conclusion made by Alexander Friedmann in 1922 that the Universe must have originated a finite time ago from an epoch of infinite density and curvatures if the evolving matter obeyed the dynamical equations of general relativity, together with the assumptions of homogeneity and isotropy for the Universe.

It is worth noting that despite such predominant global features manifesting in the structure of gravitation theory, most of the calculations were done up until the early 1960s using a local coordinate system, which was defined in the neighborhood of a spacetime event. The key aim was to solve Einstein's equations using various simplifying assumptions, which by themselves form a complicated set of nonlinear partial differential equations.

This scenario changed considerably when the so-called 'Schwarzschild singularity' problem arose in general relativity. The Schwarzschild solution of Einstein's equations describes the gravitational field in the exterior of a spherically symmetric star where there is no matter present and spacetime is empty. The spacetime metric gives the 'distance' between two infinitesimally separated events in the spacetime continuum, where the key parameter is m, which represents the mass of the star or the central particle. Due to the spherical symmetry of the problem, the metric coefficients, or the gravitational potentials, were written down in spherical coordinates, which are the coordinates on a three-dimensional sphere. These are a radial coordinate, r, and two other angular coordinates, the fourth coordinate being time. It turned out that at the value $r = 2m$ one of the metric components went to a vanishing value and the other one blew up, pointing to an irregularity in the spacetime metric.

It was thought initially that this represents a singularity in spacetime itself and that the physics goes seriously wrong at $r = 2m$. For many decades after the Schwarzschild solution was found, this remained a source of confusion and only after considerable effort was it realized that this is, in fact, not a genuine spacetime singularity but merely a coordinate defect. What really happens is that the coordinates used break down at the $r = 2m$ value, just as coordinates degenerate at the poles of a two-dimensional sphere. That there is nothing physically wrong at $r = 2m$ was indicated by the finiteness of spacetime curvatures there. What was needed was a better coordinate system covering the full spacetime, which was obtained by Martin Kruskal and G. Szekeres in the 1960s. This may be regarded as an important insight involving a global approach in gravitation theory. There were other similar developments which clarified the role of spacetime topology and global evolution issues, also known as the Cauchy problem in general relativity. These developments clarified the role of global considerations in Einstein gravity.

The understanding of global aspects in gravitation and cosmology now came into its own and was highlighted by a detailed analysis of the problem of spacetime singularities. As mentioned previously, it was realized early on that an important implication in the study of cosmological models is that the Universe contains an infinite curvature singularity from which it originated. The Schwarzschild solution also contains a genuine curvature singularity at the center where the spacetime curvature components blow up, as opposed to a mere coordinate defect at $r = 2m$.

To begin with, such singularities were not taken seriously, and were believed to be the only consequence of the exact symmetry conditions assumed for these models while solving Einstein's equations, and not any genuine feature of the general relativity theory. It was thought that the singularities would disappear once more realistic conditions were used for spacetime models, and when the exact symmetries such as homogeneity and isotropy were replaced by more realistic assumptions in the cosmological problem being studied.

But as mentioned earlier, the singularity theorems showed that this was not the case. The spacetime structure was analyzed in detail within the general framework of Einstein gravity. Using a rigorous analysis of global properties of a general spacetime, it was shown that under certain general and physically reasonable conditions such as the positivity of energy, occurrence of trapped surfaces, and a suitable causality condition, the spacetime singularities would occur as an inevitable feature for a wide range of gravitation theories.

This analysis involved a detailed examination of the causal structure of spacetime, which was then combined with the study of focusing that was caused by gravity on the families of timelike and null paths, which are the trajectories in a spacetime of material particles and light rays. The existence of spacetime singularities then follows for general spacetimes. Such a singularity arises either in a cosmological scenario where it provides the origin of the Universe or as the end-state of the gravitational collapse of a massive star which has exhausted its nuclear fuel which earlier provided the pressure against the inward pull of gravity.

The understanding of these global features of spacetime during this phase of work resulted in the development of so-called singularity theorems involving the evolution problem in relativity, black hole physics, the structure of exact solutions of Einstein's equations, and the asymptotic structure of spacetimes.

Spacetime Foam

An important key question related to gravitation theory within a spacetime framework is to what extent the continuum spacetime model of general relativity should be considered valid. Even if the Universe is modeled quite well by a continuum picture at macroscopic and classical levels, will such a description survive at much smaller microscopic scales?

This situation is similar to that of a fabric of cloth, which looks quite continuous and whole when seen from far away on a larger scale. But on going closer, we see that it is made up of many individual fibers woven together with many gaps inbetween. Similarly, one could ask whether spacetime, which looks continuous on a larger scale, could actually be discrete, and whether it is made of yet more basic and different smaller components on a very small scale. In other words, one could ask what are the 'atoms' constituting spacetime.

As an example, on a larger scale, the geometry of the Universe could be described by a typical spacetime metric such as the FRW metric. This is a purely classical model, with the metric giving the local gravitational potentials. However, the overall geometry and topology of the Universe are left quite free by general relativity, which imposes no constraints. In FRW models also, the geometry can be positively or negatively curved or flat, and the space can be like a three-dimensional sphere, or some other entity depending on the curvatures of space. Then as we go to finer and finer length scales typically on the order of the Planck length, which is about 10^{-33} cm, classical theory does not hold and quantum effects dominate. In such a case, there is no guarantee that space, or spacetime itself, will have a fixed topology at that microscopic level.

Do quantum fluctuations in spacetime topology dominate at microscopic levels, giving a foam-like structure to spacetime, rather than the usually assumed continuum (Fig. 2.8)? If so, such effects will be quite important when formulating any quantum theory of gravity. Probably, it is our assumption of the smooth continuum structure for spacetime that is causing many of the problems encountered at present. So the

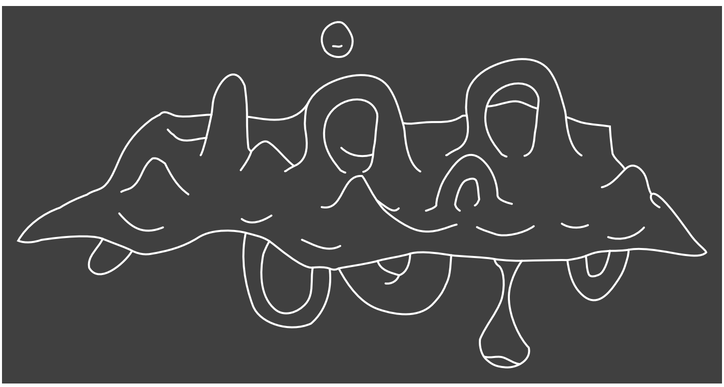

Figure 2.8 On a larger macroscopic scale spacetime may present a smooth continuous picture. But on a microscopic scale at very small distances, or at very high energies and curvatures, quantum gravity effects may dominate. These can manifest in a continuously changing spacetime topology from one shape to the other, with different regions having different topologies, thus effectively creating a foam-like structure. While we are far from making a quantitative theory of such a quantum spacetime foam, such possibilities may play an important role in our future understanding of the Universe.

issue is, what is the real structure of spacetime at microscopic levels and at scales near the Planck length? The considerations on spacetime topology could play an important role here. One can ask whether it is reasonable to work with a fixed spacetime topology in quantum gravity, or will quantum fluctuations not cause perturbations in its topology as well? Researchers have been thinking of the possible relationships between spacetime topology and quantum gravity.

3

Black Holes

Subrahmanyan Chandrasekhar pointed out, way back in 1934, that the life-history of a star of small mass must be essentially different from that of a star of large mass. Stars like the Sun, when they run out of their internal nuclear fuel, stabilize into smaller entities of about a thousand kilometers in size, called white dwarfs. On the other hand, a star of larger mass, beyond about 1.4 solar masses, cannot pass into this stage, and as Chandrasekhar pointed out, 'one is left speculating on other possibilities'.

For decades, the question as to what is the final fate of a massive star has remained unanswered. In recent years much research has focused on this issue, which has become one of the most important unresolved problems in astrophysics and cosmology today.

Large-mass stars, when they run out of their internal nuclear fuel, undergo a continual gravitational collapse, which is a catastrophic shrinkage of the star's size under the pull of its own gravity. In this case, exciting outcomes for the collapsing star are predicted by the general theory of relativity. These have profound implications for fundamental physics and cosmology. We aim to discuss these developments here.

A model of continual collapse for a massive star using the general theory of relativity was constructed for the first time by Robert Oppenheimer and Herbert Snyder in 1939, and by S. Datt in 1938 (OSD). It was this calculation that gave rise to the concepts of black hole and event horizon, even though these terms weren't actually coined till the 1960s. We will discuss these developments here, and how they eventually led to many other themes in black hole physics in the later 1960s and early 1970s, and formed the basis for several important astrophysical applications of black holes in modern relativistic astrophysics.

Actually, the idea of a black hole is not new and is in fact natural (Fig. 3.1). For example, consider a star and the particles or projectiles escaping from it. It is a basic property of gravity that the heavier the star is, the greater the pull of its gravity, and the greater the velocity an

Figure 3.1 Black hole, an artist's conception.

object will need to escape from it. However, we know that the velocity of light is the maximum that any particle can approach. Therefore, in principle, we can think of objects which are either so massive or so compact that their escape velocity will be greater than that of light. In that case, no material particle or even light rays will be able to escape from the surface of such a body, which could be termed a 'black hole' for all practical purposes. In fact, Pierre-Simon Laplace alluded to such a possibility way back in 1799.

Life of a Star

It would be appropriate to say that like human beings, stars also have a certain life-cycle. They are born in gigantic clouds of dust and intergalactic material in the depths of space and time, in the faraway regions of the Universe. Once they come into existence, they evolve and shine for millions of years, and then eventually enter the phase of dissolution and final extinction.

Most of the shining life of a star is essentially hydrogen burning inside, fusing into helium, and later into heavier elements. Finally, when all the star's matter is converted to iron, no more nuclear processes are possible within and no new internal energy is produced.

At this stage, the all-pervasive force of gravity takes over to determine the star's final evolution.

Earlier in its life-cycle, there is a balance between the force of gravity that pulls the star's matter toward its center, and the outward pressures generated by the internal fusion processes. This balance keeps the star stable, maintaining a normal life of shining and radiating light and energy produced within. Once internal pressures subside, gravity takes over and the star begins to contract and collapse in on itself.

It is now known that for a star of small enough mass, comparable to that of the Sun or somewhat larger, the natural white dwarf stage is an initial step toward the star's eventual extinction. On the other hand, a star of larger mass, more than about 1.4 times the mass of the Sun, cannot pass into a white dwarf final state. Such stars, up to 3 to 5 solar masses in size, will settle into another stable configuration, which is called a neutron star. However, for even more massive stars, gravity predicts continual and eventual total collapse. What will be its final fate? This is one of the most intriguing questions in modern astrophysics and cosmology, with far-reaching consequences for fundamental physics.

Stars are born in huge, faraway clouds of interstellar dust, mainly when shockwaves, possibly generated by a supernova explosion elsewhere, compress the matter together. Gravity then takes over and the cloud shrinks further while its central temperatures rise. As the process continues, at a certain stage nuclear burning is ignited within the core. This is the birth of a star where the internal pressures generated by the heat and light produced within resist any further contraction due to gravity. In this case, a stable balance between the pressures generated internally and the inward pull of gravity is finally reached.

There are star-forming regions in the Universe where conditions for such processes to initiate are favorable and stars are continuously born there, as we can observe by using the infra-red part of the light spectrum emitted from these regions. The stars born can be typically as massive as our Sun, which is also a standard star, or a few times larger, or even much more massive. Stars mainly burn their internal hydrogen, fusing it into helium atoms and releasing energy in the process. Heavier elements such as carbon and oxygen are created in the process and eventually this nuclear burning process stops or slows down when much of the matter is converted to iron. Then the star cools and the internal pressures subside. Again, gravity, which is an ever-present force, takes over and starts compressing the star inward.

When our Sun runs out of its internal fuel, its core will contract under its own gravity, but it will be supported by a new quantum pressure within created by very-fast-moving electrons, called the electron degeneracy pressure. Such an object is called a white dwarf, which is about a thousand kilometers in radius (Fig. 3.2).

Similarly, stars with masses greater than that of the Sun will settle into a final state, which is a neutron star, as mentioned earlier. This state will be reached after an initial collapse and the star losing some of its original mass. These are pure neutron objects created in the collapse under the strong crush of gravity, which collapses even the atoms. The quantum pressure of these neutrons support and balance the star, which is barely some ten to fifteen kilometers in size. The final outcome of collapse thus depends on the initial mass of the star, which again stabilizes at a much smaller radius due to the balancing pressures generated by either electrons or neutrons within. The outer layers of the star are thrown off in the form of a supernova blast which is created in the final stage of the star's collapse.

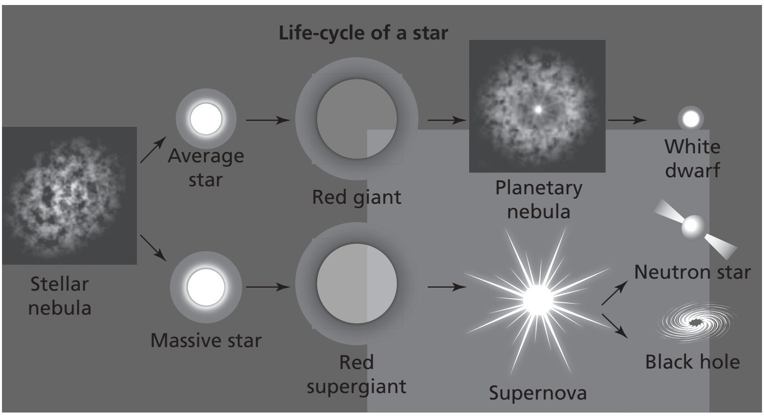

Figure 3.2 The depiction of final states of stars of different masses. A small-mass star, comparable to that of our sun, will eventually settle into a white dwarf final state. But for stars with larger masses there will be a supernova explosion and a neutron star will form, or for even greater masses, there will be a continual collapse and a black hole is believed to develop as the collapse end-state.

Much more massive stars, larger than the neutron star mass limit, cannot, however, settle into a white dwarf or neutron star state. This is because internal quantum pressures due to electrons or neutrons within are not enough to stabilize the collapsing star, as the inward force of gravity is much more powerful. Therefore, a continual gravitational collapse ensues, which no known physical force can halt. Such a total collapse becomes inevitable once the star exhausts its internal nuclear fuel. In this sense, the life-history of a star of large mass is radically different from that of small-mass stars.

Collapse of Massive Stars

A star as massive as ten or twenty times the Sun burns much faster and lives only a few million years, compared to a lifetime of several billion years for a smaller star such as the Sun. What is the final fate of such a massive star? This is one of the most important, unresolved problems in astrophysics and cosmology today.

The fundamental question about the fate of massive stars was highlighted by Chandrasekhar (and also independently by Lev Landau), who pointed out a major result of his investigations, which was basically that the life-history of a small-mass star must be essentially different from that of a large-mass star. In the case of a small-mass star, the natural white dwarf stage is an initial step toward its complete extinction. However, a star that is larger than a certain critical limit cannot pass into the white dwarf or neutron star type of equilibrium stage. We must then speculate about other possibilities.

We can see the seeds of modern black hole physics already present in the above inquiry. The issue of the end-state for large-mass stars remained unresolved and elusive for a long time after Chandrasekhar's initial discussion. While his work pointed out the stable configuration limit for the formation of a white dwarf, the issue of the final fate of much larger stars remains very much unresolved today, because such a star cannot settle either as a white dwarf or as a neutron star.

The issue is clearly important in both high-energy astrophysics and cosmology. For example, our observations today on the existence of dark energy in the Universe and the cosmic acceleration it produces are intimately connected with observations of supernovae, which are the product of collapsing stars. It is the observational evidence of supernovae exploding far away in the Universe which tells us how the

Universe may be accelerating and the rate at which this acceleration is taking place.

At the heart of such a supernova lies the phenomenon of catastrophic gravitational collapse of a massive star, wherein a powerful shockwave is generated, blowing off the outer layers of the star. If a star is able to throw off enough of its matter in such an explosion, it might eventually settle as a neutron star. However, if matter accretes onto the neutron star, there will be a further continual collapse, which we will have to explore further. On the other hand, stars which are more massive and well above the normal supernovae mass limit straightaway enter a continual collapse mode at the end of their life-cycle without any intermediate neutron star stage.

The final fate of such a star is decided by the general theory of relativity. The point is that the usual laws of physics and quantum theory explain the formation of white dwarfs and neutron stars in terms of the electron degeneracy pressure and the neutron degeneracy pressure, respectively. But such 'quantum pressures' are only able to resist a star's gravitational collapse when its mass is no more than about five to eight solar masses at the very highest. When the initial mass is greater than this, these pressures can no longer generate an equilibrium for the star, and it collapses under the full seize of gravity, which shrinks it completely. The other three fundamental forces of nature which were significant earlier are no longer important and the final fate of the star is now determined by the general theory of relativity.

The important point here is that stars which are tens of times the mass of the Sun burn much faster and are far more luminous. Such stars cannot endure more than about ten to twenty million years, which is a much shorter lifespan than that of our Sun, which will live much longer. Therefore, the question of the final fate of such short-lived massive stars is of central importance in astronomy and astrophysics.

What needs to be investigated then is what happens in terms of the final outcome when such a massive star dies on exhausting its internal nuclear fuel. The general theory of relativity predicts that the collapsing massive star must terminate into a spacetime singularity, where the matter energy densities, spacetime curvatures, and other physical quantities all blow up. It then becomes crucial to know whether such super-ultra-dense regions forming in the stellar collapse are visible to an external observer, or whether they will always be hidden within a black hole and an event horizon of gravity that possibly forms as the star collapses. This is one of the most important open issues in the physics of black holes today.

A Black Hole is Born

To understand the final state of collapse for a massive star, we need to trace the time evolution of the system and its dynamical progression using Einstein's equations of gravity. The star shrinks under the force of its own gravity, which comes to dominate other basic interactions of nature such as the weak and strong nuclear forces that typically provide the outward pressure to balance the pull of gravity.

Einstein's theory was first used for understanding the final fate of massive collapsing stars by Oppenheimer and Snyder, and independently by Datt in the late 1930s. OSD studied the continual gravitational collapse of a pressureless, homogeneous uniform density matter cloud using the general theory of relativity. The most interesting result was that the collapse, as it evolves in time, leads to the formation of what was later called an event horizon. This is a one-way surface formed in spacetime which causes a region of space and time to be invisible and non-communicable to observers far away in the Universe. This hidden region is called a black hole, though the terms black hole and event horizon were actually coined much later, in 1969, by John Wheeler. Thus, the continual collapse of such a massive matter cloud in the OSD model creates a black hole as the collapse end-state.

In order to deal with the complex Einstein equations, the OSD work made several simplifying assumptions. Oppenheimer, Snyder, and Datt assumed in their calculations that the density distribution within a star is strictly homogeneous, that is it was taken to be uniform everywhere. Also, they neglected the gas pressure forces within a spherical star, taking them to be zero, thus assuming the matter to be pressureless dust only. Their calculations showed that an event horizon surface develops as collapse progresses, such that no material particles or photons from the region inside the horizon escape. Once the star collapses to a radius smaller than the horizon, it has entered the black hole region in spacetime. It then collapses to a singularity of extreme density. For a collapsing star to create a black hole, an event horizon must develop prior to the formation of the final singularity. Once the star enters the event horizon surface, the causal structure of spacetime implies that no material particles or light rays from that region can escape, and it is entirely cut off from observers far away in the Universe.

What is obtained from the OSD model is an interior view of the collapsing cloud. In general such a solution depends on the properties of matter, equation of state, and the physical processes taking place within the stellar interior. However, assuming the matter to be pressureless dust and the density to be uniform makes it possible to solve the problem explicitly and analytically. In OSD's case the energy–momentum tensor is taken to be that of pressureless dust, and we solve Einstein's equations to determine the metric potentials; as a result, the geometry of the collapsing dust ball is obtained.

To understand the process of how an event horizon and black hole form when a massive star collapses, we first note that the star eventually collapses to a singularity of infinite density and spacetime curvatures. The general relativity calculation implies that as the star collapses the force of gravity on its surface keeps growing and eventually a stage is reached when no light signal emitted from its surface is able to escape. This is the epoch when an event horizon has formed, and the star then enters the black hole region of spacetime. The in-falling emitter does not feel anything special when entering the horizon, but any faraway observer stops seeing the light from it. The strong gravity of the star causes this one-way membrane, that is the event horizon, to form. Within the horizon the collapse continues to crush the star into a spacetime singularity.

The physics accepted today for describing the formation of black holes as the end-state of stellar collapse relies on the OSD dust model. In this case, all the matter falls into the spacetime singularity at the same time, while the event horizon forms earlier than the singularity, thus fully covering it. This is how a black hole region in spacetime results as the end-state of collapse. The issue of the final fate of a gravitational collapse must be probed within the framework of a suitable theory of gravity, because the ultra-strong gravity effects will be important in such a scenario. Hence, the OSD model uses the general theory of relativity to examine the final fate of an idealized massive matter cloud, as described earlier. As we noted, the dynamical collapse created the spacetime singularity, which was preceded by an event horizon. This is what developed as a black hole in spacetime. The singularity is hidden inside such a black hole, and the collapse eventually settles to a final state which is the Schwarzschild geometry that we mentioned earlier.

Gravitational Collapse

It would be useful to discuss and understand the basic features of this gravitational collapse scenario in some detail. For the collapsing spherical homogeneous dust cloud, the collapse is initiated from a regular configuration at an initial time $t = 0$. Once the collapse has started, it cannot be halted and the star continues shrinking to smaller and smaller diameters. At the initial time and later, when the star surface is outside the event horizon, which is also called its *Schwarzschild radius*, any light ray emitted from the surface of the star can escape to a faraway observer. However, once the star has collapsed below the event horizon radius, which is at the value $r = 2m$, it has entered the black hole region of no escape in spacetime. This region develops in spacetime as the collapse progresses, and is bounded by the event horizon radius (Fig. 3.3).

Then the collapse of all the matter to an infinite density and curvature singularity at the center $r = 0$ happens. This inevitably takes place in a finite proper time as measured by an observer sitting on the surface of the star. Thus, the star collapses totally at a finite time in the future and all its matter is crushed to the spacetime singularity at the center. There is a singularity at the center now and the rest of spacetime is empty, with its geometry described by the Schwarzschild metric. This is called the *Schwarzschild black hole*.

Any event in this region within the Schwarzschild black hole is called a *trapped surface*. This is a two-dimensional sphere in spacetime and its basic property is that both the outgoing and ingoing families of light rays emitted from this point converge inward. So no light and therefore no material particles emerge from this region. Such a black hole region in the resulting Schwarzschild geometry ranges from the center to the Schwarzschild surface, the event horizon being the outer boundary. On the event horizon, the radial outward photons stay where they are, but all the rest are dragged in toward the center of the black hole. No information from this black hole can propagate outside the $r = 2m$ region.

Since no emissions or light rays from the singularity can travel out to an observer at infinity, the singularity is causally disconnected from the outside spacetime.

The spacetime region within and enclosed by the event horizon is the black hole region. The size of the event horizon scales as the total collapsing mass; that is, the larger the mass of the star, the larger the

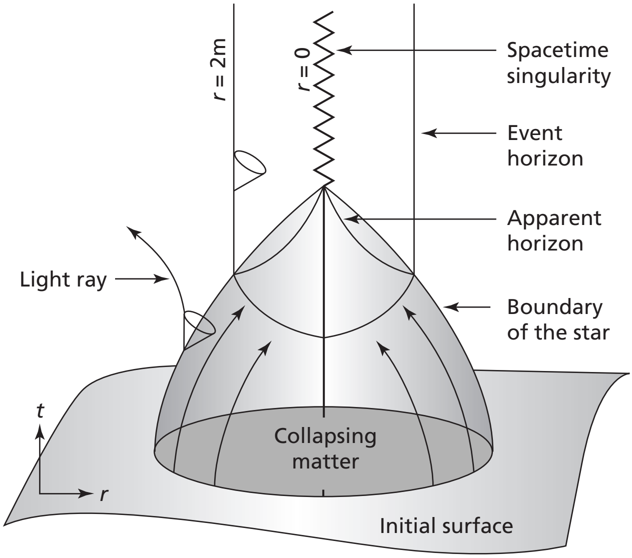

Figure 3.3 The spacetime diagram for the dynamical evolution of a homogeneous spherical dust cloud collapse, as described by the Oppenheimer–Snyder–Datt solution. Such a collapse creates a Schwarzschild black hole in spacetime. The homogeneous dust cloud collapses from an initial epoch of time where the density at all points within the star and other physical quantities are all regular and well-behaved. The shaded region indicates the collapsing matter. Light rays emitted from points on the surface of the star would reach a faraway observer as long as the star has not entered the horizon. But as the collapse progresses, an absolute event horizon forms which is a wavefront at a radius $r = 2m$. No light ray from the star's surface can then reach an outside observer, and a black hole forms in spacetime, which is the region bounded by the event horizon. The spacetime singularity forming at the center due to the collapse of matter to infinite density is completely hidden below the event horizon and hence invisible to any outside observers.

event horizon. If the collapsing star is about ten solar masses, the event horizon or black hole will be about 60 kilometers in diameter. Again, any event that happens within the event horizon cannot send out a signal to faraway observers. On the other hand, events that occur outside the horizon will be able to signal faraway observers.

The OSD collapse model eventually became a basic paradigm for the black hole concept. It is thought that all massive stars in the Universe will collapse to black holes at the end of their life-cycles as described earlier, retaining all the qualitative features. Further to this, a very considerable amount of research and astrophysical applications about black holes, which occupy a major role in astrophysics and cosmology today, has been developed in recent decades. Such black holes could suck in more matter from their surroundings and grow larger and larger.

The Debate on Horizon and Singularity

Two important features stand out from the gravitational collapse scenario mentioned above. The first is the formation of an event horizon as the collapse develops and the second is the eventual termination of collapse in the singularity of infinite density and curvatures. Both of these eventually became the subject of much discussion and debate for many years and have had a far-reaching impact on the further development of gravitation theory, as we shall discuss here.

Interestingly, however, not much attention was initially paid to this model after it was worked out. In fact, it was widely thought by gravitation theorists and astronomers at the time that it would be absurd for a star to reach an ultra-dense state during its evolution. The rather unusual features of the OSD model and their far-reaching implications were, so to speak, already conspicuous. But these were mainly ignored and an attitude prevailed that such an extreme contraction of a physically realistic star could never be realized in nature. It was believed that there must be a way for the collapsing cloud to achieve equilibrium before it reaches a singularity stage. Also, Einstein wrote a paper in 1939 arguing that such a final state for a star is not possible.

In fact, further to the advent of general relativity theory in 1915, gravitation physics was a relatively quiet field, with few developments till about the 1950s. However, the 1960s saw the emergence of new observations in astrophysics, such as quasars and very-high-energy phenomena such as energetic galactic jets. These observations, together with important theoretical developments such as studying the global structure of spacetimes and singularities, led to important results in black hole physics and relativistic astrophysics and cosmology.

Thus, the discovery of quasars and radio galaxies in the 1960s revived an interest in black hole physics through to the 1970s. Interest in gravitational collapse scenarios was also rekindled because no other physical processes had been able to explain the extreme energies witnessed in quasars and energetic galactic jets. Resurgent interest in black holes focused on their dynamical formation and physical properties and the surrounding regions of spacetime. Attention was drawn again to the dynamical gravitational collapse of massive stars and their final fate, and to black holes as possible physical mechanisms underlying very-high-energy phenomena in the Universe.

Thus, the occurrence of event horizons and of spacetime singularities became a matter of much discussion. The formation of an event horizon is one of the most intriguing features to come out of OSD's study of the collapse of a massive star. Although terms and concepts such as event horizon and black hole were not prevalent at the time, physicists could immediately see the extreme features displayed by the OSD model. This model for the continual collapse of a massive star has the remarkable physical feature that as the collapse evolves to a certain stage, the star enters the horizon and at that epoch the causal structure of spacetime is such that no material particle or light ray can escape the surface of the star. Another most intriguing feature as we noted is that after the star has entered the horizon, after a finite time the physical radius of the entire star shrinks to vanishing value and all the matter finally collapses simultaneously to a singularity at the center.

So the main debate that came up further to the OSD model was that researchers thought that both the spacetime singularity and the event horizon appeared because this was a too idealized model. In fact, the occurrence of spacetime singularities was not new to researchers in the 1930s. However, it was believed that the singularity occurring in the OSD model as well as in other important solutions to Einstein's equations, such as the Schwarzschild and FRW models, would not occur in more realistic solutions. Also, it was believed a horizon or black hole would never occur because an actual star cannot be compacted to such small sizes. As we shall discuss in the next chapter, the issue of the occurrence of singularities was resolved once the singularity theorems showed that spacetime singularities occur for much more general scenarios. As for the event horizon, an assumption was made in the form of the cosmic censorship conjecture, stating that all realistic massive stars will collapse similarly to the OSD collapse, retaining the same qualitative features. This meant that massive stars will collapse into a spacetime singularity, which will be hidden within a black hole.

Black Hole Physics

As discussed earlier, further to the Schwarzschild and FRW solutions and the OSD gravitational collapse model developed in the early decades following the positing of the general theory of relativity, the two major issues of spacetime singularities and event horizons dominated the discussion in gravity physics for many decades to come.

As for the problem of occurrence of spacetime singularities, as will be discussed in the next chapter, we now know that singularities do occur in general relativity in fairly generic situations under broad and reasonable physical conditions. However, as of today we know very little about the nature and structure of singularities and about their properties, as they can develop in various situations in general relativity in either static or dynamically evolving models.

The second major issue has been the development of event horizons and how they cover singularities during gravitational collapse. As we saw in the OSD collapse scenario, the event horizon develops well before the epoch of singularity, which is then fully hidden within the 'zone of no communication' with external observers, i.e. within a black hole.

With the issue of the occurrence of singularities being settled in the early 1970s, physicists also assumed that whenever they occur, the singularities of collapse will be covered within event horizons, just as in the OSD scenario. Such an assumption, first proposed by Penrose in 1969, is the *cosmic censorship conjecture*. In other words under reasonable physical conditions the final outcome of gravitational collapse will always be a black hole in spacetime, with the curvature singularity of collapse always being hidden inside it.

Further to the censorship formulation, many important developments took place in black hole physics, which started in earnest, and several important theoretical aspects as well as astrophysical applications of black holes began to develop. The classical as well as quantum aspects of black holes were then explored and interesting thermodynamic analogies for black holes were presented as a result. Many astrophysical applications for the real Universe were then developed for black holes, for example in models using black holes for the description of phenomena such as jets emitted from the centers of galaxies and extremely energetic gamma ray bursts.

Researchers were aware that the homogeneous and pressureless dust collapse is a very highly idealized model. The OSD model made these assumptions simply because it was very difficult to deal with Einstein's complicated equations, and there was no way to solve the system otherwise. In fact, the study of any realistic dynamical gravitational collapse models within Einstein gravity is a rather difficult task due to the nonlinearity and complexity of Einstein's equations which govern the dynamical evolution of the star. So not much progress was made for many decades after the OSD work. It was always clear to physicists that the study of more realistic models of the gravitational collapse of massive stars was of utmost importance if black holes were to be taken seriously. The OSD scenario neglects pressures that play an important role in the dynamics of any realistic star and the density distribution cannot be homogeneous and uniform. In real stars, the density is typically higher at the center, decreasing away from the center; they have non-zero gas pressures within; and they have shapes other than exactly spherical.

But physicists had an immediate and urgent motivation to make the censorship assumption. Observations of very-high-energy phenomena in the Universe in the early 1960s such as quasars, radio galaxies, and active galactic nuclei were unexplainable in terms of any other known physical mechanism. It appeared that strong gravity fields must be involved here in a non-trivial manner and that general relativity needed to be used in an extensive and rigorous manner to explain them. The censorship hypothesis provided a major impetus to developments in black hole physics. Assuming provisionally that it holds, physicists investigated the properties of black holes and created detailed laws of black hole dynamics. They also utilized black holes to explain various ultra-high-energy processes observed in the Universe, such as quasars and X-ray-emitter binary star systems.

By the early 1970s physicists thought it pragmatic to assume censorship and moved on with developing black hole physics in a big way. The emphasis was on studying black hole solutions to Einstein's equations and their different properties and applications, rather than on gravitational collapse, which was difficult to investigate in any case. Black holes were defined in a general way and their properties were studied in detail. These developments occupied much of the researchers' attention, who also studied the implications for very high energy cosmic phenomena.

In the following, we describe some of the milestone developments in black hole physics.

Schwarzschild Black Hole

The outcome of OSD gravitational collapse is a black hole, as we discussed earlier. It has two parts. At its core is the spacetime singularity where all the matter of the star has been compacted and crushed. Surrounding the singularity is the region of space from which no escape is possible to faraway observers. The circumference of such a region is called the event horizon. Once any object or light rays have entered this region, they can never escape. For the collapsing body also, once it enters the horizon, no light emitted by it can escape and it is fully trapped within the horizon. The final geometry of this black hole is then the Schwarzschild geometry as worked out by him in 1916 (Fig. 3.4).

To understand the formation of a black hole during collapse, the concept of an *escape cone* is helpful. Consider the collapsing star and an observer located on the surface of the star who keeps emitting beams of light as the collapse proceeds. As shown in Fig. 3.3, as long as the star has not entered the event horizon, the light emitted by the observer can escape and reach a faraway observer. Once the observer reaches the

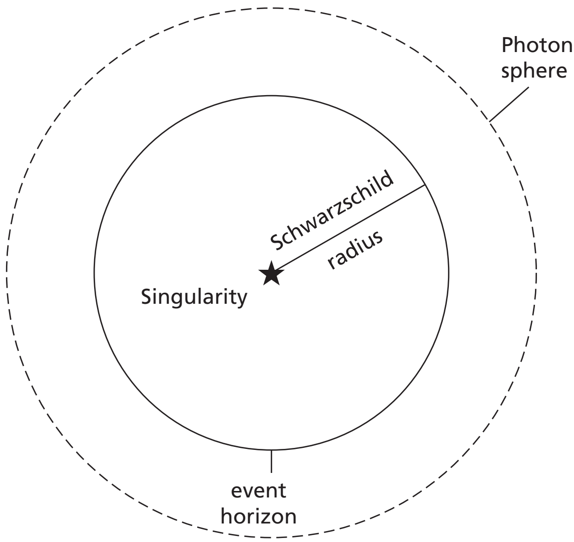

Figure 3.4 Typical structure of a Schwarzschild black hole. This is a spherical object, at the center of which is located a spacetime singularity of infinite gravity and spacetime curvatures. The location of the event horizon is defined by the Schwarzschild radius and the region inside the horizon is the black hole, which is the region of no escape for any particles or light rays that have entered below the same. The photon sphere lies outside the event horizon.

horizon, all the light rays that he emits fall into the singularity at the center, except one radial ray that just stays at the horizon, at a constant distance from the singularity. Further to this, once the observer enters the horizon, all the rays emitted fall into the singularity, and there is no escape possible. Eventually the observer falls into the singularity, to be crushed out of existence.

The black hole has clearly a very intense gravitational field. Therefore, the behavior of light rays and particle trajectories around a black hole is quite important. An interesting aspect and manifestation of these strong gravity fields is the existence of a *photon sphere* around the black hole. This is a surface on which photons can in principle just coast along in a circular orbit, without falling into the hole. As such it is an unstable surface, where light ray orbits are also not stable, and these photons can fall in the black hole or escape with a slightest perturbation. The photon sphere plays an important role in the phenomenon of *gravitational lensing*, or the bending of light rays by a black hole.

We note that just as the OSD collapse is a specialized model, the same is true for the Schwarzschild black hole. Only if the star is made of spherical dust with no pressure and is fully homogeneous will its collapse create such a black hole. But most stars are inhomogeneous, are not fully spherical, and also have rotation. Hence as the collapse proceeds, their shapes can distort and their rotation speed up. We need to know how such changes affect the collapse as well as the formation of black holes. It will be very difficult to obtain fully solved analytic models such as the OSD in such a case, and we may have to resort to numerical models to trace the collapse for such a matter cloud.

Kerr Black Holes

The Kerr metric is a rotating axi-symmetric solution to Einstein's equations. As we pointed out earlier, the Schwarzschild geometry is a natural outcome of the OSD collapse. However, no such collapse model is known which would give rise to a Kerr geometry as the final collapse state. But the 'no-hair theorem' conjectures that typically all collapsing configurations should end up creating a Kerr black hole. While the only parameter that characterizes a Schwarzschild black hole is its mass, a Kerr black hole is characterized by two parameters, namely its mass and the spin.

Kerr black holes have found many astrophysical applications and have interesting properties, one being they drag their surrounding spacetime in the same direction as their rotation. As shown by Roger Penrose, this effect would allow a spinning black hole to be a source of energy.

Recently, computer simulations by Ramesh Narayan and collaborators showed that by dragging spacetime, an accreting black hole can fling out a small fraction of gas into powerful collimated relativistic jets. Such jets have actually been known for decades and their energy source has been a matter of debate. One popular explanation is that the power comes from the black hole itself. Evidence from computer simulations has shown that such a possibility is likely. There is also observational evidence in favor of this interpretation.

Black Hole Thermodynamics

In the 1970s, the golden era of development of black hole physics, many analogies were found for the behavior of black holes with the usual laws of thermodynamics. For example, assuming what is called the asymptotic predictability for spacetime, which is essentially the same as the cosmic censorship assumption together with faraway Minkowski flat behavior of spacetime, it was shown that the area of an event horizon must always increase and that it can never decrease. This is in close analogy to the second law of thermodynamics. This area theorem by Hawking guarantees the monotonic growth of the surface area of any black hole, provided the matter and fields interacting with it respect the weak positive energy condition.

Similar analogies of black hole behavior with the first and third laws of thermodynamics have also been pointed out under various assumptions. These possibilities have created a lot of interest in black hole physics. Of course, it is known that as for the second law or the area theorem, in a cosmological background which is not asymptotically flat, the area theorem need not hold.

Quantum Black Holes

Further to developments in black hole thermodynamics such as those just described, semi-classical gravity was explored in curved background spacetimes such as those of black holes. The most remarkable result was the so-called Hawking radiation effect deduced by Stephen Hawking in 1975. The event horizon causes the black hole to produce a very slow and dim radiation outflow, which is the evaporation of the black hole itself.

Eventually, the entire black hole will evaporate, but for normal black holes with masses of the order of a few solar masses, this effect is extraordinarily slow and not really observable. In any case, this result made apparent the very intriguing possibilities that quantum gravity might offer.

Observational Evidence

By its very definition, a black hole can never be seen directly, as it emits no light. So a direct verification of the existence of a black hole is not possible. As such, no light from its surface, which is the event horizon, can ever escape and reach a faraway observer.

The black hole, however, has an exceptionally strong gravitational field that affects its surroundings with observational implications. For example, in real astrophysical situations, matter around a black hole will fall into it. As falling matter from the accretion disk (see Fig. 3.5) surrounding a black hole nears the black hole surface, which is the event horizon, it will move with greater and greater speeds and will heat

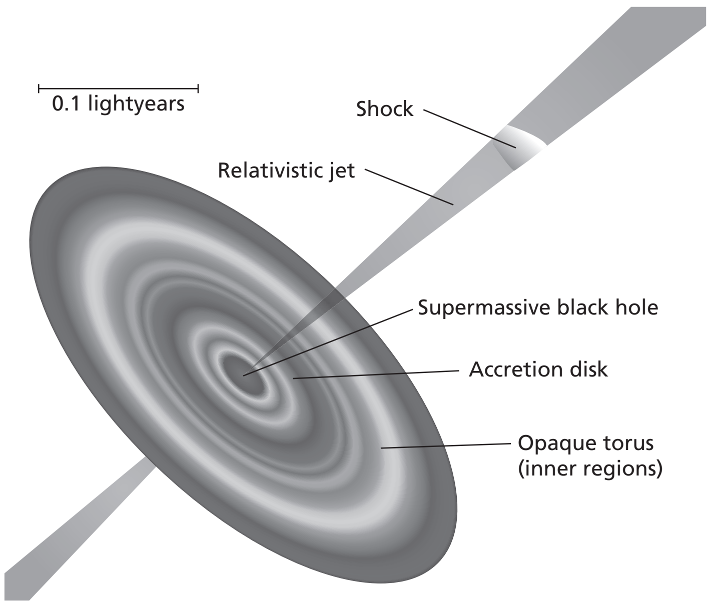

Figure 3.5 An accretion disk of incoming and in-falling matter around a typical black hole. Adapted from Wikimedia Commons (http://commons.wikimedia.org/wiki/File:Galaxies_AGN_Inner-Structure-of.jpg).

up enormously. Such very hot matter will emit powerful X-rays which provide a typical characteristic signature of a black hole's existence in the Universe.

Using this phenomenon, many X-ray telescopes today look for the X-ray glows from the accretion disks surrounding compact objects believed to be black holes. One such most promising candidate is the object Cygnus X-1, which has been under observational focus now for quite some time.

Figure 3.6 We observe powerful jets emerging from the central regions of galaxies. Black holes are thought to provide an explanation for this phenomenon.

Astrophysical Applications

Black holes have been widely used to model very-high-energy astrophysical phenomena observed in the Universe. These include powerful jets emitted from galactic centers, quasars and gamma-ray bursts, and other such phenomena. We mentioned earlier, for example, how Kerr black holes could explain galactic jets (Fig. 3.6).

It is clear that no other usual forces can explain such extreme energy events in the Cosmos. Therefore, black holes, being very natural consequences of general relativity, offer a somewhat natural alternative in this direction. The field is developing rapidly, with many observational missions and powerful computers for simulations having become available in recent years.

4

Singularities

Soon after Einstein proposed his field equations, the earliest solutions found were the Schwarzschild metric and the Friedmann–Robertson–Walker (FRW) cosmological models. Each of these contained a spacetime singularity where the curvatures and energy densities blow up and the physical description then breaks down. In the Schwarzschild solution, such a singularity would be present at the center, whereas for the Friedmann models it is at the beginning of the Universe, where the scale factor vanishes and all objects are crushed to a zero volume due to infinite gravitational tidal forces.

As we have indicated, the issue of the occurrence of a spacetime singularity raised a major debate among gravitation physicists. This became particularly intense in the late 1950s and the 1960s. Even though the physical problem posed by the existence of such a strong curvature singularity was realized immediately in these solutions, which turned out to have several important implications for the experimental verification of general relativity, initially this phenomenon was not taken seriously. Many researchers strongly believed that the singularity occurring in solutions such as the Schwarzschild or FRW, or the one in the OSD collapse model occurring when the matter cloud settles to its final Schwarzschild black hole state, is merely a result of strong symmetry assumptions made while solving Einstein's equations to obtain these solutions. It has been suggested that when sufficiently general models are worked out and the symmetry assumptions relaxed, then the more realistic solutions will be singularity free. In particular, the work by Belinski and co-workers and the Mixmaster Universe model by Charles Misner in 1960s are examples of efforts in this direction.

Subsequently, the distinction between a genuine spacetime singularity and a mere coordinate singularity became clear and it was realized that the singularity at the event horizon in the Schwarzschild spacetime was only a coordinate singularity, which could be removed by a suitable coordinate transformation. It was also clear, however, that the genuine curvature singularity at the center cannot be removed by any such transformation. The hope was then that when more general solutions are considered with a lesser degree of symmetry requirements, such singularities will be avoided.

This issue of the existence of spacetime singularities in general enough spacetimes in Einstein's theory of gravity was settled only in the late 1960s and early 1970s by the work of Stephen Hawking, Roger Penrose, and Robert Geroch. Their results within a general spacetime framework showed that a spacetime will necessarily admit singularities within a rather broad scenario, provided it satisfies certain reasonable physical conditions. Then a detailed analysis of the global properties of the spacetime continuum showed that the existence of singularities was a general feature of the general theory of relativity. Also these considerations ensure the existence of singularities in other theories of gravity, which are based on a spacetime continuum picture and which satisfy the general physical conditions such as those described earlier. So the scenario that emerges is that essentially for all classical spacetime theories of gravity, the occurrence of singularities forms an inevitable and integral part of the description of the physical reality.

In the vicinity of such a singularity, typically the energy densities, spacetime curvatures, and all other physical quantities will blow up, thus indicating the occurrence of super-ultra-dense regions in the Universe. The behavior of such regions may not be governed by the classical gravity theory itself, which may, in fact, breakdown, having predicted the existence of the singularity. Then a quantum gravitational theory is the likely description of the physical phenomena and processes created by such spacetime singularities in their vicinity.

We will narrate this development here, also examining whether it is still possible to avoid the singularity by violating or relaxing some of the basic assumptions made about their existence.

The Existence

Firstly, we must clarify what we mean by a *spacetime singularity*, and how to characterize it. The singularities in the solutions to Einstein's equations, such as at the center of the Schwarzschild spacetime, and at the origin of time in the FRW models, have great physical significance as the spacetime curvatures and matter–energy densities blow up in their vicinity.

There are families of particle trajectories or light rays which experience infinite tidal forces when they fall into such a singularity.

The trajectories of the free-falling particles or light rays in a spacetime continuum are characterized by *geodesic paths*. In general relativity, these correspond to the shortest paths in usual physics, i.e. paths along which particles and photons would travel in the most efficient manner. Typically, a spacetime singularity is meant to occur when such a trajectory or particle comes to an abrupt end and vanishes suddenly from the spacetime in a finite proper time, which is the time measured by its own watch. It turns out that it is this notion of *geodesic incompleteness* that characterizes such a behavior, and therefore the singularity in an effective manner.

This characterization also enables the existence of singularities to be proved by means of general enough theorems. Such singularity theorems typically involve a consideration of the gravitational focusing caused by the matter and curvatures in congruences of timelike and null geodesics in a spacetime. The gravitational focusing turns out to be the main cause of the singularity's existence in the form of non-spacelike incomplete geodesics in a spacetime.

As an example, the Big Bang singularity in the FRW cosmology model admits such incomplete geodesics. All the galaxies are represented here by timelike geodesics. When we extend these paths backward in time, they necessarily meet the singularity in the past at a finite time, as measured by their own watch. Along these trajectories, as we move onto the past closer to the singularity, all the physical quantities such as densities and spacetime curvatures blow up and take arbitrarily high values.

It is clear that if a spacetime contains incomplete non-spacelike geodesics, there is a definite singular behavior present. In such a case, a timelike observer or a photon suddenly disappears from the spacetime after a finite amount of proper time or the affine parameter value along the photon curve. The singularity theorems which result from an analysis of gravitational focusing and global properties of a spacetime prove this incompleteness property for a wide class of spacetimes under a set of rather general physical conditions.

The matter fields with positive energy density affect the causality relations in a spacetime and cause focusing in the families of timelike and null paths. The essential phenomenon that occurs here is that matter focuses the non-spacelike geodesics into pairs of focal points, where the nearby trajectories intersect each other. When the matter energy density is positive, as would be the case for example within a massive star, then the Raychaudhuri equation, which was derived in 1955 by physicist Amal Kumar Raychaudhuri, implies that the effect of matter on curvatures causes a focusing effect in the congruence of geodesics due to gravitational attraction. This in general causes the neighboring geodesics in the congruence to cross each other, to give rise to caustics or focal points.

Using the above and certain other properties of a spacetime, several singularity theorems which establish the non-spacelike geodesic incompleteness under different sets of conditions were obtained, and these are applicable to different physical situations. Mainly, there are three general physically reasonable sets of assumptions for these theorems. These rather general assumptions include the positivity of energy density at a classical level, a suitable causality assumption that rules out observers going back to their own past, and a condition typically implying the existence of strong enough gravitational fields somewhere in the Universe. This last condition was formulated in terms of the existence of trapped surfaces, which confine light and material particles to a certain region forever, as we discussed earlier.

The most general of these is the Hawking–Penrose theorem derived in 1970, which is applicable in both the gravitational collapse situation and the cosmological scenario. The main idea of the theorem's proof is that by using the causal structure analysis for a spacetime it can be shown that there must be maximal length timelike curves between certain pairs of events. Now, a causal geodesic which is both future and past complete and endless must contain pairs of focal points if the spacetime satisfies a suitable energy condition ensuring that the particle trajectories do encounter sufficient matter as they propagate in the Universe with a positive energy density. This is then used to draw the necessary contradiction to show that the paths cannot go on endlessly, and that the spacetime must be non-spacelike geodesically incomplete, which means a singularity must develop (Fig. 4.1).

The issue of a singularity's physical nature is important. There are many types of singular behaviors possible in a spacetime and some of these can be regarded as only mathematical pathologies, rather than having any physical significance. This is especially true if the curvatures and other physical quantities remain finite along an incomplete non-spacelike geodesic in the limit of approach to the singularity.

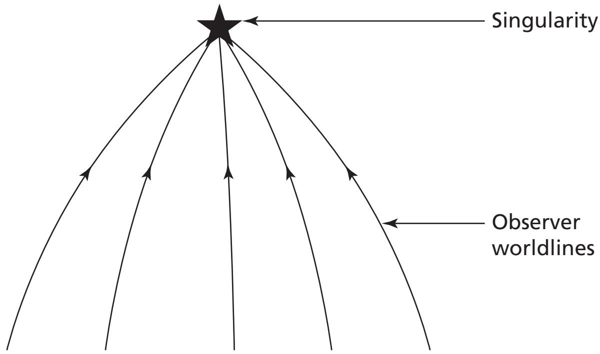

Figure 4.1 A spacetime singularity is defined by geodesic incompleteness. There are timelike and null geodesics that end up at the singularity, either in the future or in the past at a finite value of their parameters. Thus, for example, an observer moving along a timelike geodesic will suddenly come to an end at a finite value of his proper time when falling into a singularity.

A singularity will be physically important when there is a powerful enough curvature growth along singular geodesics, the physical interpretation and implications of which can then be considered.

The inevitable existence of singularities for wide classes of spacetime universes under rather general physical conditions means that classical gravity gives rise to regions in the Universe where densities and curvatures will grow without bounds, and where all other physical parameters will diverge. Such a singularity implies the breakdown of general relativity in that region and in a way means the end of spacetime itself.

Can We Avoid Singularities?

These results show that it is not possible to avoid the occurrence of singularities in spacetimes of general relativity by merely going to more general cases with fewer and fewer symmetries. Because no symmetries for spacetimes were assumed in the considerations described in the previous section, the results are quite general, subject only to a few reasonable physical conditions.

Given the scenario above, we need to ask whether we can still avoid the actual occurrence of singularities in the Universe. It is, of course, clear that spacetime singularities are an inevitable feature for most of the physically reasonable models of the Universe and gravitational systems within the framework of Einstein's theory of gravity. It is also seen that near such a spacetime singularity, the classical description that predicted it must itself break down. The existence of singularities in most classical theories of gravity under reasonable physical conditions imply that in a sense Einstein's gravity predicts its own limitations, namely the regions of the Universe where it must break down and a new and revised physical description must take over.

As the curvatures and all other physical quantities must diverge near such a singularity, the quantum effects associated with gravity are very likely to dominate such an arena. It is possible that these may resolve the classical singularity itself. However, we have no viable and consistent quantum theory of gravity currently available, despite serious attempts over the past decades. Therefore, the issue of resolving singularities produced by classical gravity remains very much open.

The other possibility is, of course, that some of the assumptions of the singularity theorems may be violated, thus avoiding the singularity occurrence. Even when these are fairly general physical conditions, whether some of these actually break down and do not hold in physically realistic models should be investigated. This could possibly negate the occurrence of singularities at the classical level itself. Such possibilities mean a likely violation of causality in the spacetime, or no trapped surfaces occurring in the dynamical evolution of the Universe, or a possible violation of energy conditions.

The singularity theorems mentioned above assume causality in the form of a suitable causality condition. The alternative is that causality is violated and a singularity does not occur. So the implication of singularity theorems is that when there is enough matter present in the spacetime, either the causality is violated or a boundary to the Universe in the form of a singularity must occur. So the question is, when a huge condensation of matter occurs in the Universe, will it cause causality violation or a spacetime singularity? In the cosmological case, such stress-energy density for initiating gravitational focusing will be provided by the microwave background radiation and other matter present in the Universe. Similarly, in the case of stellar collapse, trapped surfaces may form as the collapse proceeds with growing density, providing a condition leading to the formation of a singularity or causality violation.

Einstein's equations by themselves do not rule out causality-violating configurations that depend on the global topology of the spacetime.

Hence the question of causality violations versus singularity needs a careful examination as to whether causality violation can offer an alternative to singularity formation. It proposes an interesting possibility of being able to enter one's own past while also avoiding the undesirable spacetime singularity. Similarly, it must be investigated whether the violation of energy conditions or the non-occurrence of trapped surfaces must be realized in order to achieve singularity avoidance in a spacetime. We discuss some of these possibilities next.

Causality Violations

The general theory of relativity implies that spacetime must be locally flat and that the laws of special relativity must be valid locally in a neighborhood for any event. However, the theory places no restriction on the global geometry or topology of spacetime as a whole.

For example, let us consider the model of the flat spacetime of special relativity, called the Minkowski spacetime, which has a globally Euclidean geometry of a four-dimensional space. The causal structure at any event p is defined by the future and past light cones at the event, which are generated by the null geodesics at p. All the events which p can influence by means of a timelike or null signal form the causal future of p, which is the interior of the future light cone together with its boundary. Similarly, its causal past consists of all those events which could influence p with signals traveling at a speed less than or equal to that of light.

In special relativity, the future and past light cones of any event never meet each other. However, if we allow for arbitrary topologies for the spacetime, globally the future light cone of p may bend over to enter the past of p, thus giving rise to causality violations. For example, consider the two-dimensional Minkowski space, which has a flat geometry. But if we bend it over, this space becomes a cylinder topologically, and contains closed timelike curves through every point. Such causality violations allow observers to enter their own past and often this is not considered physically desirable (Fig. 4.2).

The causal structure in a spacetime specifies what events can be related to each other by means of timelike or light signals. A typical causality violation would mean that an event could be in its own past, which is contrary to our normal understanding of time, and that of the past and future. However, general relativity does allow for situations

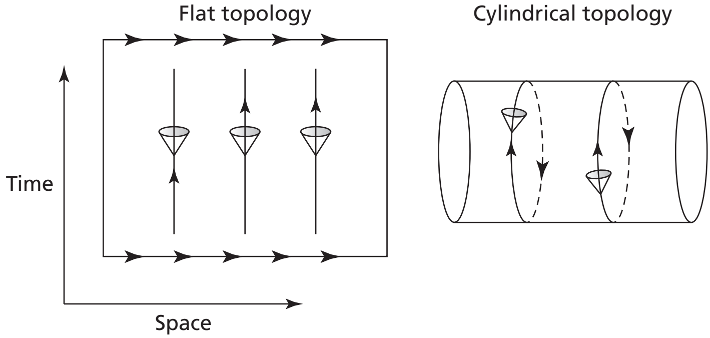

Figure 4.2 A two-dimensional example of causality violation. The flat Minkowski spacetime has a Euclidean topology and it does not admit any causality violation. But if we curl it so as to join the arrowed lines, the topology becomes that of a cylinder and the timelike worldlines can go to their own past and cause causality violation in the form of closed timelike lines.

where causality violation is permitted in a spacetime universe, and the Gödel solution, obtained by Kurt Gödel in 1949, provides an interesting example.

To avoid such causality violations and other related irregular behavior, a number of regularity conditions are generally placed on a spacetime. Such restrictions are often used as necessary conditions in theorems which deduce the existence of spacetime singularities. The question then is can one avoid spacetime singularities if one allows for the violation of causality? Researchers have studied whether by violating causality conditions we can avoid spacetime singularities. But it turns out that in most of the cases, violating causality causes spacetime singularities in the form of null geodesic incompleteness. Thus, this path for avoiding singularities does not look very promising.

If possible, we would, of course, like to rule out causality violations on physical grounds, treating them as pathological behavior because we would be able to enter our own past otherwise. But since causality violations are allowed, in principle, in general relativity, we must rule them out only by an additional assumption to that effect.

There is, in fact, a hierarchy of causality conditions for a spacetime. It may be causal, having no closed non-spacelike curves. But given an event, future directed non-spacelike curves from this event could return to its arbitrarily close neighborhood in the spacetime. This is as bad as a causality violation itself. The higher-order causality conditions such as strong causality and stable causality rule out such behavior. Of the higher-order causality conditions, much physical importance is attached to the stable causality, which ensures that if a spacetime is causal, its causality should not be disturbed with small perturbations in the metric. Presumably, general relativity is a classical approximation to some, as yet unknown, quantum theory of gravity in which the value of the metric at a point will not be exactly known and small fluctuations in the values must be taken into account.

It turns out that if a spacetime is causal, but if any of the higher-order causality conditions are violated, that also gives rise to singularities. Another question examined concerned the measure of causality-violating sets when such a violation occurs. It turns out that in many cases, the causality-violating sets in a spacetime will have a zero measure. In other words, because causality-violating sets are negligible in content, a causality violation cannot be taken very seriously. On the whole, this situation implies that violating causality may not be considered a good alternative to the occurrence of singularities.

Energy Conditions and Trapped Surfaces

Another possible way to avoid singularities is to violate the positive energy conditions. This is another assumption made in the proofs for singularity theorems. In fact, this possibility has been explored in some detail and it turns out that as long as there is no gross or very powerful violation of energy conditions over global regions in the Universe, this will not help avoid singularities either.

For example, the energy condition can be violated locally at certain spacetime points, or in certain regions of spacetimes due to the peculiar physics there. But as long as the energy condition holds on average, whereby the stress-energy density is positive in an integrated sense, then spacetime singularities do occur.

On a global scale, there is observational evidence in the form of an accelerating expansion of the Universe, which means that the Universe may be dominated by an omnipresent dark energy field. It is not clear, however, as to what exactly such a field would be, and what would be its origin. It could be due to scalar fields or ghost fields floating in the Universe, or due to a non-zero positive cosmological constant present in Einstein's equations corresponding to a vacuum energy density in the Universe. In such a case, the positive energy conditions may be violated, depending on the nature of these exotic fields. However, in the earlier phases of the Universe where ordinary matter dominated, the positive matter fields would again cause gravitational focusing. At these epochs the energy conditions would hold even if they are violated at the present epoch.

Again, this discussion is made in the context of a cosmological scenario. When it comes to the gravitational collapse of massive stars, clearly their density and overall energy content are dominated by the ordinary matter fields we are much more familiar with. On the whole, such matter certainly respects the energy conditions, except possibly for some (if any) minor violations. Thus for gravitational collapse of massive stars, we would expect the energy conditions to hold and the conclusions on singularity occurrence stated earlier would apply.

Yet another possible way of avoiding singularity is to avoid the formation of trapped surfaces that occur in the spacetime. Indeed, such a route can give rise to geodesically complete spacetimes, as was shown by Senovilla and co-workers in the 1990s. For the cosmological scenario, what this effectively means is the condition that the matter energy densities must fall off sufficiently rapidly as we move farther away in space at any given epoch of time. There must be a fall-off in density on any given space slice of a constant time in an averaged sense in order to avoid the cosmological trapped surfaces. Whether such a condition is realizable in the Universe will need to be checked through observational tests. A sufficiently uniform energy density, such as, say, the cosmic microwave background radiation, can in turn cause the cosmic trapping. As for the massive stars, the densities are very high indeed and would only grow in a gravitational collapse. Therefore, in collapse scenarios the trapped surfaces are unlikely to be avoided.

Further to these considerations, it appears that, by and large, it would be difficult to avoid the occurrence of singularities by means of violating some of the assumptions made by the singularity theorems. If we accept that spacetime singularities do occur under fairly general conditions in the framework of Einstein's theory of gravity, or for classical gravity in general, then we must consider the physical implications and consequences of such a scenario for physics and cosmology. As noted, two main arenas of physical relevance where spacetime singularities will be inevitable are then the cosmological situation and the gravitational collapse scenario for massive stars.

Fundamental Challenges

The existence of spacetime singularities in Einstein's gravity and other classical theories of gravitation poses intriguing challenges and fundamental questions in physics as well as in cosmology. These will have far-reaching consequences on our current understanding of the Universe and how we try to model it, as we try to show here.

The inevitable existence of singularities for wide classes of rather general models of spacetimes means that classical gravity evolutions give rise to regions in the spacetime Universe where the densities and spacetime curvatures grow arbitrarily high and without bounds. This also means that other relevant physical parameters will diverge as we approach closer to the singularity region. To take a physical scenario, such a phenomenon corresponds to a singularity that represents the origin of the Universe. Secondly, whenever locally a large quantity of matter and energy density collapses under the force of its own gravity, a singularity will occur. This latter situation is effectively realized in the gravitational collapse of a massive star in the Universe, which shrinks catastrophically under its own gravity when the star has exhausted its internal nuclear fuel, which earlier supplied the internal pressures to halt the in-fall due to gravity.

Over the past decades, once the existence of spacetime singularities was accepted, there have been major efforts to understand the physics in a singularity's vicinity, resulting in an entire physics of the early Universe and the inflation paradigm where the Universe expands exponentially in its early phase. Depicted are the initial moments following the Big Bang singularity from which the Universe is supposed to have originated. The complexities in this case are enormous, both physics-wise and conceptually. The physics complexities arise because when trying to understand the physics close to the hot Big Bang singularity, we are dealing with the highest ever energy scales, never seen before in any laboratory physics experiments. Our particle physics theories are then stretched to an extreme where there is no definite or unique framework available to deal with these phenomena. Understanding early Universe physics has major consequences because it governs and affects the most important physical phenomena such as later galaxy formation and issues related to the large-scale structure of the Universe.

As for conceptual issues, the instantaneous Big Bang singularity gives rise to a host of problems and puzzles, one of which is the 'horizon problem', which arises due to the causal structure of this spacetime. Distinct regions of the Universe simply cannot interact with each other due to the cosmic horizons in this model and it becomes extremely difficult to explain the observed average overall current homogeneity of the Universe on a large enough scale. There are other issues such as why the current Universe looks so flat, which is referred to as the 'flatness' problem. As a possible solution to these dilemmas, inflationary models for the early Universe have been proposed, various facets of which are still very much under debate. The key issue, as far as the Big Bang singularity is concerned, is that it happened only once in the past and there is no way to probe it any further other than through current observations of the Universe and extrapolation of the past. We must observe deeper and deeper into space and back in time to understand the nature and physics of this early Universe singularity.

As we mentioned earlier, the other class of spacetime singularities occur in the gravitational collapse of massive stars. Unlike the Big Bang, this kind of singularity occurs whenever a massive star collapses. Thus, this is more amenable to observational tests.

There are several fundamental cosmic conundrums associated with singularities of gravitational collapse. One of the most intriguing is whether such a singularity will be visible to observers far away in the Universe. The Big Bang singularity is visible to us in principle, as we get to see its light and all the matter that came from it. But as we will discuss later, the singularity of a collapse can sometimes be hidden below the event horizon of gravity, and therefore not always visible. The possibility that all singularities of a collapse will be necessarily hidden inside horizons is the cosmic censorship conjecture. This has not been proved, and, in fact, singularities of collapse can be visible in many physical circumstances.

When visible or naked singularities develop during gravitational collapse, they give rise again to extremely intriguing physical possibilities and questions, such as the possibility of observing ultra-high-energy physical processes and quantum gravity effects. Such observations can guide our efforts toward a quantum theory of gravity. But one conundrum is whether this will break the so-called *classical predictability* of the spacetime universe. We shall discuss these issues later in some detail.

On the other hand, even when the singularity is hidden within a black hole, that still gives rise to profound puzzles such as the *information paradox* and issues with *quantum unitarity*. So the point is, even if the cosmic censorship conjecture were correct and all singularities were hidden inside black holes only, we would still be faced with many deep paradoxes not unique to naked singularities. It turns out that paradoxes are associated with either black holes or singularities, and perhaps what we need, if we are to generate a consistent picture, is a more basic and fundamentally new approach.

It seems only reasonable to say that all these deep physical as well as conceptual issues are closely connected with the very existence and formation of spacetime singularities in dynamical gravitational processes taking place in the Universe. While the Big Bang singularity happened only once in the past, the singularities of collapse occur repeatedly, and hence possess an interesting observational perspective and potential. We will discuss some of these issues later, while also considering the most intriguing physical implications in the process.

5

Cosmic Censorship

As discussed in the previous chapter, the debate on spacetime singularity culminated in the knowledge that singularities occur quite generally in Einstein's theory. They occur mainly in two circumstances, namely gravitational collapse and cosmology. While the singularity of cosmology occurred once at the beginning of the Universe, those in collapse occur whenever a massive star collapses under its own gravity.

While the singularity of cosmology is visible, for collapse there are, in principle, two possibilities: either covered within horizons of gravity, not visible to faraway observers in the Universe, or not enveloped by horizons and visible to faraway observers. The OSD collapse model shows that the singularity is covered by the horizon. Is this true for a generic gravitational collapse? The assumption that this must be the case whenever a massive star collapses is the cosmic censorship conjecture, which has become a fundamental assumption in black hole physics and its astrophysical applications.

While black hole physics flourished at a rapid pace in the 1970s and later, both in theory and in its astrophysical applications, physicists were aware of the need to prove the cosmic censorship conjecture (CCC). They knew that if CCC does not hold then many of the basic, key results, such as the area theorem for black holes, will also fail to hold. So serious efforts to formulate and give a proof for CCC have continued, even as black hole physics has developed vigorously on the CCC assumption. This has been widely recognized as the single most important problem in the theory and foundation of modern-day black hole physics, and in gravitation physics.

We will discuss here the attempts over the past decades to formulate and prove the CCC. It turns out that despite many serious attempts, the CCC remains unproved; also no proper mathematical formulation for the CCC has been made available. This is a big dilemma because, on the one hand, black hole physics has developed so much, yet, on the other hand, for the CCC, which is at its very foundation, we have no clue. The only way out is to study gravitational collapse in further detail, along the lines of the OSD work, but with much more realistic physical conditions.

We will then consider the simplest physical generalization of the OSD work along these lines, namely the dust collapse, which is inhomogeneous where the density of the matter cloud is not uniform in space. It will then be shown that the structure of horizons and singularities changes radically and in fact that we have visible, naked singularities forming as the final fate of gravitational collapse. Implications for the cosmic censorship and further directions opening up are indicated.

What is a Naked Singularity?

The purpose of CCC is to rule out the naked singularities, especially those that form in gravitational collapse. So first we need to understand what a visible naked singularity is. Then we can possibly answer whether singularities are always covered by event horizons or can exist without horizons.

As we noted earlier, the matter energy densities and spacetime curvatures and all relevant physical quantities grow unboundedly as we go closer in the limit of approach to a spacetime singularity. However, it is intriguing to note that the singularity itself is not part of the spacetime. That is because, at all events in the spacetime continuum, regularity conditions such as well-behaved gravitational potentials and the validity of special relativity in its small enough neighborhood are to be respected. But such a description is not possible or does not hold for a singularity if we consider it as part of the spacetime. Therefore, at most we can regard it as some kind of a boundary to the Universe. Thus, even when not in spacetime, a singularity influences all the physics in the Universe. In fact, in the case of the Big Bang, it is the creator of the Universe.

According to the Newtonian description, if we take any two bodies, say the Earth and the Sun, the gravitational attraction between them will be calculated as if all the mass was concentrated at the center of each of these objects. Now if the Sun were to collapse into a black hole, in a literal sense all of its mass would collapse into the center, and still the bodies would feel the same gravitational force. Today, similar calculations are done in the framework of general relativity, to calculate, for example, the gravitational radiation from a system of a black hole and a star spinning around it and falling into it where all the mass of a black hole is compacted at its center.

Main Characteristics

If we ask what a spacetime singularity really is, the answer would be, it is a region where general relativity breaks down. In fact, we are aware that singularities in any given theory indicate the limitation of the theory and point to its breakdown. Similarly, the occurrence of singularities in general relativity means there are regions of ultra-high densities and curvatures in the Universe where Einstein's gravity must break down. While we know now the necessary existence of singularities, namely that they must exist and form in dynamical processes involving gravity in rather general physical circumstances as predicted by general relativity, we know very little about their nature, structure, and properties.

It follows that although spacetime singularities do occur, the theory implies no direct connection between them and whether they are covered by event horizons. In principle, singularities can come with or without horizons. Unless proved in a direct manner, there is no clear physics motivation to show that singularities must be enveloped within event horizons without ever being visible.

In this sense, a naked singularity is a spacetime singularity which can communicate with observers far away in the Universe, as opposed to the one hidden inside a black hole (Fig. 5.1). The latter cannot communicate with any event in spacetime. Thus, the key signature of a naked singularity is that there are light rays and particle trajectories that initiate from the singularity or in an arbitrarily close vicinity, and which reach out to observers far away in the Universe in the future (Fig. 5.1). This would allow the flow of information from close to the singularity and ultra-dense regions to external observers.

In other words, there exist families of non-spacelike trajectories which in the future go to infinity and in the past terminate at the singularity. Also, as we travel closer to the singularity along these paths, the densities and spacetime curvatures typically grow and diverge in the limit of approach to singularity.

Such a phenomenon is not possible for a singularity within a black hole. In that case, information falls in but never emerges from the vicinity of the singularity. Thus, to check out whether the final fate of a given gravitational collapse is a black hole or a naked singularity, what

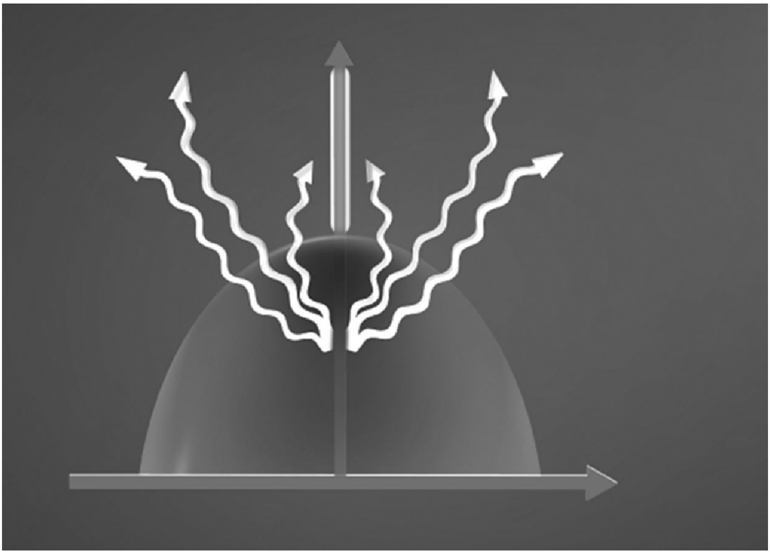

Figure 5.1 A naked singularity. As opposed to the black hole case, there are outgoing light rays from the singularity which travel to observers far away in the future (*Scientific American*, Feb 2009; Kenn Brown, Mondolithic Studio).

needs to be done is to determine whether families of future-directed non-spacelike particle or photon trajectories emerge either from the singularity or from its close vicinity.

Recent work in collapse shows that there are two types of singularity, those enveloped by an event horizon and hidden within a black hole, and those which are not. The second type are known as naked singularities. Thus, both black hole and naked singularities will be the possible outcomes of a gravitational collapse under regular and reasonable physical conditions. However, as opposed to a black hole, the naked singularity configuration has rather different theoretical and observational properties (Fig. 5.2).

To summarize, when a naked singularity develops, it is characterized by three main important properties:

1. Firstly, there are families of infinite light and particle trajectories which emerge from the singularity region, with a past end-point at the singularity.

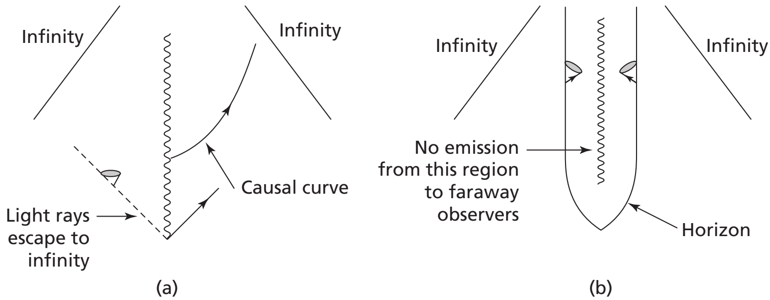

Figure 5.2 A schematic diagram of (a) a naked singularity and (b) a black hole. The gravitational collapse terminating as a naked singularity has a very different causal structure from that terminating as a black hole. While in the black hole case a spacetime region of no communication to faraway observers always develops, such a scenario need not happen in the naked singularity case. The important difference is that in the former case the singularity is not visible because an event horizon covers it; in the latter case, the horizon may not form or may not cover the singularity fully, which may become visible to faraway observers as a result.

2. Secondly, a genuine curvature singularity has densities and curvatures that become infinite and grow very powerful in the limit of approach to the singularity along these paths.
3. Finally, the overall spacetime satisfies all physical regularity conditions, such that this becomes an interesting framework for studying physical processes in ultra-strong gravity regimes.

Predictions of General Relativity

According to Einstein's general theory of relativity, the collapse must proceed to create a spacetime singularity, where physical parameters such as energy density and spacetime curvatures blow up to arbitrarily large values. The usual laws of physics break down near such a singularity. This is the regime of ultra-strong gravity fields, with other basic forces of nature playing only a secondary role. Quantum effects must also become important in such strong fields at ultra-small length scales near the singularity, and eventually what is needed is a quantum theory of gravity which will resolve the singularity.

The outcome of a continual collapse in a fully general scenario of a spacetime with an evolving matter field is described by the singularity theorems in general relativity. Although the theorems imply that under fairly general conditions the singularities must occur, they give no information on the singularities' nature or structure or on their physical properties and implications. Specifically, singularity theorems assume the following conditions:

1. the positivity of energy density for matter fields;
2. a reasonable causal structure of the spacetime and
3. the development of trapped surfaces in the spacetime which ensure that sufficient mass is packed into a small enough region.

The theorems imply that when these three conditions are met, the spacetime must contain a singularity.

It follows that for any general relativistic gravitational collapse developing from regular initial data in a spacetime without symmetry conditions such as spherical symmetry necessarily holding, if the above physically reasonable conditions are satisfied then the collapse will create a spacetime singularity. The densities, curvatures, and other physical quantities will blow up in the limit of approach to such a singularity. Thus, any physically realistic gravitational collapse of a massive star at the end of its life-cycle must terminate into such a spacetime singularity. The OSD collapse scenario discussed earlier is a special case for a general gravitational collapse, instantaneously terminating as a singularity, which is the final end-state of a collapsing matter cloud.

What Relativity Does Not Imply

As stated above, however, singularity theorems predict only the existence of spacetime singularities under a set of physically reasonable conditions. They do not provide information on the nature and structure or properties of such singularities. In particular, they give us no information on whether such singularities, especially if they form in gravitational collapse, will be covered by an event horizon, or whether they will be visible to external observers in the Universe. All that the singularity theorems show is that singularities must occur in the form of geodesic incompleteness in a spacetime continuum. In other words, of the two, the event horizon and the singularity, which one must come first as the star collapses is not answered by the theory of general relativity.

Thus, the possibility remains that a spacetime singularity can develop in gravitational collapse, yet not be covered by an event horizon, allowing it to be causally connected to faraway observers in the Universe. In such a case, ultra-high-density and curvature regions will be able to communicate with and send out signals to faraway observers. In this sense, gravity predicts exciting outcomes for the final fate of a massive collapsing star, with profound implications for fundamental physics.

Therefore, whether a strong curvature singularity that formed in a realistic collapse will be visible or hidden from a faraway observer in the Universe remains an open question in Einstein's theory of gravity. The key physical feature that decides whether a singularity is visible is the interplay between the structure and time curve of the singularity and that of the trapped surface formation in the spacetime.

Censoring the Cosmos

Within such a context the cosmic censorship, put forward by Roger Penrose, can be seen really as an assumption about the behavior or nature and properties of spacetime singularities, especially when they occur in gravitational collapse. The CCC essentially states that the singularities of collapse, whenever they occur, must always be hidden within the event horizons of gravity, in the same way as happens in the OSD collapse scenario. Thus, the key statement and assumption here is that the OSD behavior will always generalize to all general and physically realistic gravitational collapse scenarios taking place in the Universe.

Further to the CCC proposal, many researchers have attempted to prove the same within the framework of gravitation theory, the hope being that a concrete theorem establishing censorship and ruling out naked singularities can be derived, thus securing the foundation for black hole physics and its numerous modern-day astrophysical applications. The outcome of such efforts has not, however, led us any closer to possible proof of CCC. The crucial issue, then, is how to formulate and prove the CCC in a rigorous manner.

CCC proposes that whenever a massive star undergoes a continual gravitational collapse, the outcome will be a black hole that covers the final spacetime singularity of collapse. This generalizes the OSD dust collapse scenario to a general collapse situation. At a deeper level, this amounts to assuming certain behavior and properties for spacetime singularities occurring during collapse, and a constraint on the nature of dynamical evolutions allowed for collapsing clouds in general relativity.

As such and in principle, spacetime singularities and event horizons are two totally different and independent entities in gravity dynamics. Logically, the two possibilities are singularities that are covered by horizons and those that are not. Cosmic censorship means that the second class never occurs, or at least that nature will not allow it to occur through some physical mechanism. So censorship suggests that whenever a spacetime singularity occurs, it must come with a horizon that fully covers and hides it. However, no proof or specific mathematically rigorous formulation of the CCC within the framework of gravitation theory has been derived. It was soon realized that doing so would be quite a difficult task. It was also rather difficult to work out explicitly more general gravitational collapse models similar to the OSD case. This latter effort would make it possible to study the nature of singularities of collapse and to know whether they are always enveloped in a horizon of gravity.

In such a case, the OSD model for collapse and formation of black holes was the only scenario that could possibly explain very-high-energy phenomena involving gravity. Physicists thus thought it prudent to just assume CCC, hoping that its formulation and proof would arrive in due course, and got busy in the 1970s constructing a detailed physical framework for black holes. Such an effort gave rise to the body of work on the basic theory of black holes, which included a study of their fundamental properties, and also black hole thermodynamics and other issues such as Hawking radiation. Also, applications of black holes were developed in relativistic astrophysics with the goal of explaining very-high-energy phenomena in the Universe. Meanwhile, the debate on cosmic censorship continued and expanded in different directions, as CCC turned out to be very difficult to formulate or prove.

If a singularity is always covered by an event horizon and if CCC is true, then that would provide a much needed basis for the theory and astrophysical applications of black holes. On the other hand, if the spacetime singularities which result from a massive star's continual collapse are visible to external observers, we would then have the opportunity to observe and investigate the Universe's super-ultra-dense regions which form due to gravitational collapse and where extreme high-energy physics and quantum gravity effects are at work. The crucial physical question here is whether we can really observe such super-ultra-dense regions which develop when massive stars collapse, in violation of the CCC.

Key Elements of Censorship

To be precise, the key elements crucial to the formulation of the CCC are as follows:

1. *Regular dynamics*: Singularities must arise from the evolution of regular matter fields through dynamical time development governed by Einstein's equations.
2. *Physical validity*: The models must describe physically realistic configurations or matter fields, typically those obeying positivity of energy conditions.
3. *Genericity*: The collapse outcome must not be different, or change suddenly, whenever a slight change is introduced into the model in the form of slightly different matter fields or perturbing the system's symmetries.

It is clear that the last two conditions are subject to a rather broad interpretation, allowing for different formulations of the CCC. As the proof of such a general assertion will be difficult, whereby the notion of black holes is generalized to gravitational collapse situations other than the exact spherically symmetric homogeneous dust case, it becomes necessary to rule out naked or visible singularities by means of an explicit assumption, which is the CCC.

We can essentially state it as the following: If S is a partial Cauchy surface, that is, an epoch of simultaneity from which collapse commences, then there are no naked singularities to the future of S which could be seen from the future null infinity. This is true for the spherical homogeneous dust collapse, where the resulting spacetime in the future is asymptotically predictable and the censorship holds. Thus, the breakdown of physical theory at the spacetime singularity does not disturb prediction in the future for the outside asymptotically flat region. What will be the corresponding scenario for other collapse situations when inhomogeneities, non-sphericity, etc. are allowed? It is clear that the assumption of censorship in a suitable form is crucial to basic results in black hole physics. In fact, when one considers gravitational collapse in a generic situation, the very existence of black holes requires this hypothesis.

It is, of course, clear that naked singularities are a general feature arising in general relativistic gravitational collapse if we do not impose any restrictions on the structure of the spacetime or its energy-momentum content. This is because, as we know, Einstein's equations connect the matter-energy content of the spacetime with its geometry and curvatures. Now, we can choose ab initio a spacetime geometry with a naked singularity, and then plug this into the left-hand side of Einstein's equations, taking the matter tensor to be that quantity divided by 8π. Then we naturally have a spacetime with naked singularities. But such a scenario gives no guarantee that this matter will satisfy energy or regularity conditions, and it could be quite a weird matter. Therefore it is clear that if censorship is to hold, we must impose various regularity and energy conditions on the matter. In fact, as far as CCC is concerned, it is a major problem to find a satisfactory and mathematically rigorous formulation of what is physically desired to be achieved. Developing a suitable formulation would be a major advance toward the solution of the main problem.

Astrophysical Developments

It was the hope of scientists, as they developed black hole physics, that a derivation of the CCC would soon arrive. However, despite numerous attempts over the past three decades, this has still not been realized. Actually, we do not even have any rigorous mathematical formulation of the hypothesis. The reasons for this are becoming clear now as we will discuss here. As per the singularity theorems we have discussed, singularities are inevitable, but no such principles apply to the event horizon, in particular as to when it exists or whether it covers the singularity. In fact, the initial singularity in cosmology that created the observed Universe is not hidden inside a horizon, but it is visible in principle. Whether this is also true for stars has been an elusive question because Einstein's equations are highly nonlinear and complex.

The CCC, nevertheless, has provided a major impetus to developments in black hole physics. Provisionally assuming that it holds, physicists have investigated the properties of black holes and created detailed laws of black hole dynamics and related aspects. They have also applied the concept of black holes to explain various ultra-high-energy processes, such as quasars and X-ray-emitter binary star systems.

Gravitation theory and relativistic astrophysics have gone through extensive developments in recent decades, further to the discovery of quasars in the 1960s and other very-high-energy phenomena such as gamma ray bursts. For compact objects such as neutron stars and situations involving very high energy densities and masses, strong gravity fields governed by general relativity play an important role, which physicists have applied to other high-energy phenomena. For example, several models for explaining gamma ray bursts, which emit in a few seconds the energy of the Sun's entire lifetime, utilize the collapse of a single massive star. Gravitational collapse of large matter clouds is the key physical process basic to the formation of a star from interstellar clouds, of galaxies and galaxy clusters, and of the Universe.

Efforts to Prove the Cosmic Censor

Unlike the idealized OSD homogeneous collapse scenario, real stars have an inhomogeneous density, as well as non-zero pressures within them as they collapse. Moreover, stars also rotate. Does every massive star collapsing at the end of its life-cycle turn into a black hole, as the OSD case specifies? The CCC answers affirmatively, namely, that the singularity forming during collapse ends up being hidden within an event horizon, never to be seen by external observers.

In the 1980s, Andrzej Krolak and Richard Newman wrote a series of papers that tried formulating CCC mathematically rigorously and for possible proofs. In 1988, Newman and I wrote a paper formulating and proving a theorem that claimed a proof for censorship under a set of assumptions on the spacetime. However, we later realized that the assumptions used in the result we showed were too strong and probably not physically realistic. Thus even when the theorem proved alright, it could not be possibly regarded as a proof for CCC. Other similar attempts have been made too.

It was at this point that we realized we needed a further detailed investigation of gravitational collapse scenarios within the framework of Einstein's theory of gravity, along the lines of the OSD work, but generalizing and choosing more physically realistic cases and scenarios. The OSD model uses very idealistic and over-simplified assumptions, mainly in order to solve Einstein's complex system of equations. We needed to see whether in physically realistic cases the final fate of gravitational collapse would be a black hole or a naked singularity. In practical terms, this amounted to checking whether trapped surfaces and event horizons formed in more realistic collapses, and if so whether that was before or after the occurrence of the spacetime singularity. As we shall see, it is this latter factor that really decides whether the spacetime singularity of collapse will be naked or hidden and covered by the event horizon.

Need for Collapse Studies

It was clear that we still did not have sufficient information about the various possibilities for gravitational collapse to decide on the issue of CCC. Despite decades of serious effort, a proper, mathematically rigorous formulation of the censorship conjecture and its proof proved too formidable. In view of the hypothesis's importance, it was concluded that the first and foremost task should be to carry out a detailed and careful examination of various gravitational collapse scenarios for a realistic collapse configuration, with inhomogeneities and pressures included. These scenarios needed to be worked out and analyzed in detail within the framework of Einstein gravity. Only such considerations would enable us to determine the final state of collapse in terms of either a black hole or naked singularity.

Researchers examined several gravitational collapse scenarios involving different forms of matter, including the collapse of inflowing radiation shells, the spherical collapse of matter without pressure, that is dust which is inhomogeneous, and also perfect fluids which have non-zero pressures. Some cases of non-spherical collapse were also investigated. A uniform pattern emerged in that naked singularities do develop as the final end-product of collapse. Further, in all these cases, not just isolated trajectories but families of non-spacelike geodesics come out of the naked singularity, providing a non-zero measure set of trajectories. If only an isolated null trajectory emerged from the naked singularity, that would amount to a single front being emitted. On the other hand, the emission of a non-zero measure family of non-spacelike curves makes it a more serious phenomenon. It also turned out that the resulting naked singularity was gravitationally strong in that the densities and curvatures diverged very powerfully in the limit of approach to the same. Thus, these turned out to be physically genuine non-removable features for the spacetime of collapse.

Our purpose therefore should be to investigate how the introduction of physically more realistic conditions and scenarios affects the collapse final state, which is a black hole in the OSD dust collapse scenario.

An investigation into the final fate of collapse is important from both the theoretical and observational point of view. At the theoretical level, working out the collapse outcomes in general relativity is crucial to the problem of asymptotic predictability, namely, whether a singularity forming at the end-point of collapse will be covered by an event horizon. The CCC remains fundamental to the theoretical foundation of black hole physics and its numerous astrophysical applications, such as the area theorem for black holes, laws of black hole thermodynamics, the Hawking radiation effect, and predictability. On the observational side it has important implications for the accretion of matter by black holes and massive black holes at the center of galaxies. On the other hand, the existence of visible or naked singularities would offer a new approach to these issues and require our usual theoretical conception of black holes to be reformulated.

Inhomogeneous Dust Collapse

As an immediate generalization of the Oppenheimer–Snyder–Datt homogeneous dust collapse, we can consider the collapse of inhomogeneous dust and examine the nature and structure of the resulting singularity, with special reference to censorship and the occurrence of black holes and naked singularities. The main reason for doing so is that it could provide an explicitly clear picture of what is possible in gravitational collapse.

Logically, the simplest physical generalization we can examine is whether the outcome of a collapse would continue to be a black hole once we allow for inhomogeneities, such as a higher density at the center of the star.

How then should the conclusions given earlier for homogeneous collapse be modified when the inhomogeneities of matter distribution are taken into account? Clearly, it is important to include the effects of inhomogeneities because typically a realistic collapse could start from inhomogeneous data with a centrally peaked density profile. This problem was investigated in detail using the Lemaitre–Tolman–Bondi models, which describe the gravitational collapse of an inhomogeneous spherically symmetric dust cloud. This is an infinite-dimensional family of solutions of Einstein's equations, where the inside of the collapsing star is inhomogeneous and the outside is vacuum. The OSD model is a special case for this class of solutions.

For a better insight, the researchers first considered a single physical factor, namely the introduction of inhomogeneity in the initial density distribution of the collapsing dust cloud. This relaxes the strict homogeneous density assumption of the OSD collapse model and makes it possible to examine how the collapse final state is affected when physically more realistic cases with inhomogeneity are considered. In the next chapter, we will discuss the introduction of other physical features, such as non-zero pressures and non-sphericity, while considering the final fate of collapse.

What this investigation shows is that the introduction of inhomogeneities leads to a rather different picture of gravitational collapse. For a collapse situation, we assume the density to have compact support on an initial surface of constant time and the spacetime can be matched at some boundary to the exterior Schwarzschild field with a total mass M enclosed within the dust ball. Using this framework, the nature of singularities can be examined. In particular, the problem of the nakedness or otherwise of the singularity can be reduced to the existence of real, positive roots of an algebraic equation, constructed out of the basic quantities characterizing the collapse, such as the mass function and the velocity profiles of the cloud of matter.

It turns out that the introduction of inhomogeneity in the initial density distribution of the star can effect a delay in the formation of trapped surfaces and the event horizon, leading to a naked singularity final state of collapse. Such an inhomogeneous density profile would be much more realistic physically than the fully homogeneous density of the cloud assumed by the OSD model, as stars have higher density at their center with a slow decrease as we move radially outward.

Taking into account these inhomogeneities in the initial density profile it is possible to show that the behavior of the horizon can change drastically, thus leaving two different kinds of outcomes as the possible result of a generic dust collapse. These are the black hole, in which the horizon forms at a time anteceding the singularity, and the naked singularity, in which the horizon is delayed. The latter case allows light rays to escape from the central singularity, where density and curvatures diverge, to reach faraway observers. Once the light rays escape, then the material particles or the timelike geodesics will also escape.

Lemaitre–Tolman–Bondi Models

It is worth noting that while dust models have received considerable attention in the study of general relativity and although OSD worked out the gravitational collapse of homogeneous dust in the late 1930s, the solution to Einstein's equations for the dust form of matter was worked out fully, including inhomogeneities, by Georges Lemaitre and Richard Tolman in the early 1930s. Their main motivation at that time was the inclusion of inhomogeneities in cosmology.

As is known now, from a cosmological perspective the observed Universe is inhomogeneous up to the scale of several hundred megaparsecs. We have clusters of galaxies, and also super-clusters, which are inhomogeneities in the Universe clustered in walls and voids. Therefore, the homogeneity of the Universe, if at all achieved, is beyond this large scale. Thus, considering an inhomogeneous solution to Einstein's equations for the Universe is quite relevant physically in that sense. Further, the overall homogeneity of the Universe is an assumption known as the Copernican principle. Concerning the issue of the scale on which the Universe begins to appear homogeneous, there is as yet no real consensus, one reason being that it is difficult to quantify the sizes of super-clusters.

The conventional view has been that the Universe can approach homogeneity on scales larger than a few tens of megaparsecs. However, recent work claims that very large-scale features in the Universe might be problematic to explain in the standard model in which dark energy is a cosmological constant. Some researchers have taken the more extreme view that the Universe approaches homogeneity on even larger scales, on the order of a gigaparsec. Whether this is actually the case can only be decided by future observations. From such a perspective, Andrzej Krasinski and others have made much use of the Lemaitre–Tolman–Bondi (LTB) scenarios to model an inhomogeneous Universe in cosmology.

For the gravitational collapse of massive stars, clearly density inhomogeneities are relevant, as we noted. A detailed consideration of inhomogeneous dust collapse was carried out by Hermann Bondi in 1959. Therefore the general dust collapse models are sometimes referred to as Lemaitre–Tolman–Bondi collapse models.

The Nature of Singularity

Gravitational collapse studies of dust have a long history beginning with the Oppenheimer–Snyder–Datt models. In particular, strict homogeneity is only an idealization and we must allow for inhomogeneities of density in a matter cloud. In fact, soon after the CCC was proposed in 1969, researchers considered 'shell-crossing' singularities in dust collapse models, which were shown to be naked singularities. One such model was considered by Hans Seifert and others from Germany in 1973, wherein neighboring shells of matter intersected to create momentary singularities that were not covered by horizons. However, these were not taken seriously because an observer would not be destroyed and crushed to a zero volume while passing through the same, which is the true sign of a genuine singularity. In this case, the observer could sail through undamaged to another region of spacetime.

As we have noted, realistic stars have typically a higher density at the center, slowly decreasing as one moves outward. In 1979 Douglas Eardley and Larry Smarr performed a numerical simulation of such a model with zero pressure, and an exact mathematical treatment by Demetrios Christodoulou followed in 1984. The model revealed a naked singularity at the center where the physical radius of the cloud becomes zero. However, Richard P.A.C. Newman showed that this singularity is again gravitationally weak in that curvatures do not diverge sufficiently strongly in the vicinity of the singularity. Many researchers, including Newman and I in 1988, unsuccessfully tried to formulate a rigorous theorem that naked singularities are always weak.

What I realized then was that we simply did not know enough about gravitational collapse to formulate a censorship theorem that holds generally. We had to continue on the longer road of slowly building up our knowledge by considering case studies. Subsequently, researchers found scenarios of inhomogeneous pressureless collapse where strong-curvature naked singularities, namely those which are genuine crushing singularities, developed from regular initial conditions. Such collapse models were derived by Kayll Lake and his collaborators, our group, and others.

A full general treatment for the inhomogeneous dust collapse case was given by Indresh Dwivedi and myself in 1993. While the homogeneous pressureless collapse considered by Oppenheimer and Snyder produces a black hole, a more realistic density profile with density higher at the center and decreasing as one moves out can give rise to a naked singularity, which is an intriguing situation indeed. Nonetheless, these scenarios still ignore the effects of gas pressure.

We may ask here, how realistic is such a dust collapse picture? The assumption of vanishing pressures, which can be important in the final stages of collapse, may be considered as the limitation of dust models.

On the other hand, it is also argued sometimes that in the final stages of collapse, the dust equation of state can be relevant and at higher and higher densities the matter may behave more like dust. This was the point of view, for example, suggested by R. Hagedorn in 1960s, and also by Roger Penrose. Further, if there are no large negative pressures, as implied by the validity of the energy conditions, then the pressure might also contribute gravitationally in a positive manner to the effect of dust and may not alter the conclusions.

It is, of course, important to consider collapse situations with non-zero pressures, and if possible with reasonable equations of state. Pressures may play an important role in the later stages of collapse and we must investigate the possibility that pressure gradients can prevent the occurrence of a naked singularity. Issues such as the existence and termination of non-spacelike geodesic families and the strength of such a singularity for collapse with pressure have been examined, which we shall discuss in the next chapter.

Naked Singularity Formation

We can provide now a more detailed case study of a censorship-violating collapse scenario and explain how the collapse proceeds in order to offer a physical intuition as to why an event horizon does not form. It turns out that allowing for a higher density profile at the center of a collapsing dust cloud leads to formation of a naked singularity in collapse, rather than a black hole.

Let us consider this rather intriguing phenomenon in some detail, and the physics that leads to naked singularity formation during collapse. We will discuss the scenario depicting the collapse of a dust cloud in the absence of gas pressure. The matter cloud begins collapse from a position of rest, corresponding to the phase in a massive star's life when it has exhausted its internal fuel and gravity takes over. This is a classical system governed by general relativity with all physical regularity conditions being satisfied, such as positivity of energy density and regularity of density and curvatures at the initial epoch when the collapse begins. If the density was assumed to be homogeneous at the initial time, this would be exactly the OSD model with the collapse giving rise to a black hole.

We now consider the physically realistic situation where the density of the star is higher at the center and decreases as one moves out from the central region. To determine the collapse end-state, we need to examine the trapping of light and matter as gravity fields become more and more powerful. Eventually the equation of the trapped region in the spacetime which decides the formation of the event horizon developing dynamically as the collapse evolves can be determined.

The key factor here is the timing of the horizon during collapse. If the horizon forms well before the final singularity, the outcome is a black hole, but if it is delayed as collapse evolves, we have visible ultra-dense regions forming in the Universe. It turns out that sufficient inhomogeneity in the density distribution at the initial time delays the horizon. If the density decreases fast enough away from the center of the star, the final outcome is a naked singularity, but in a slow enough decrease or nearly homogeneous case, a black hole results.

These collapse evolutions are depicted in Figs. 3.3 and 5.3. These correspond to the homogeneous and inhomogeneous cases, respectively. The arms of the cones denote paths of the ingoing and outgoing light rays. In the homogeneous case, the collapse begins with regular initial conditions where densities and curvatures are finite. As the collapse progresses, these increase and the focusing effect on light and matter grows. Then there comes a time when a region in the spacetime starts developing such that light is simply unable to escape to any faraway observer in the Universe. Once light is trapped, the same happens for matter because it travels slower than the speed of light, with its trajectory only within the cones. This corresponds to the light rays from the surface of the star at a phase when the radius of the star has contracted to a distance proportional to its mass. As the collapse progresses, since the density is only time-dependent, the entire cloud finally is crushed simultaneously to a singularity in the future. The trapping of light and matter, however, occurs well before the final singularity develops, and therefore the singularity is totally hidden inside the black hole that has formed. So no light signals or matter escapes from the ultra-dense regions near the singularity.

The collapse, however, develops quite differently once density is no longer homogeneous, as shown in Fig. 5.3. The light paths are shown as collapse progresses and as the star gets denser and denser.

What happens in this case is that the entire cloud no longer collapses simultaneously to the singularity because the matter density is no longer homogeneous. The different matter shells arrive at different times at the singularity, one after the other, with shells at larger

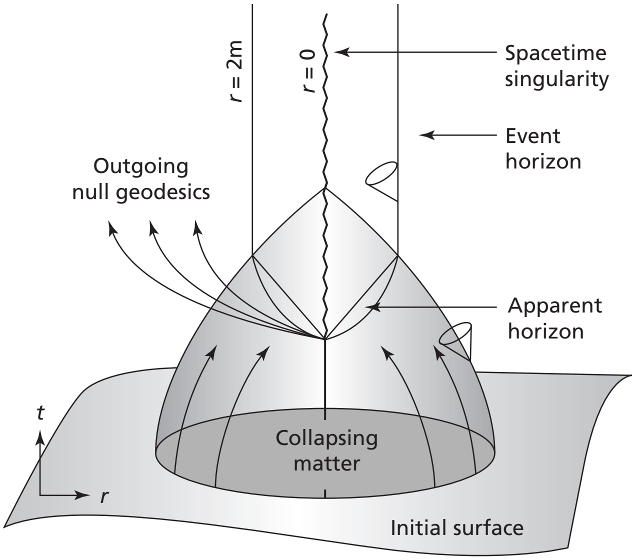

Figure 5.3 The collapse going to a naked singularity. The spacetime diagram shown here corresponds to that of a dust collapse which is no longer homogeneous but has a density profile which is higher at the center, as opposed to the constant-density profile used in the OSD model. All the regularity conditions such as positivity of energy density are satisfied and the collapse initiates from a regular set of initial conditions. In this case, an event horizon forms later, only at the epoch of the singularity formation, and the first event when the singularity forms becomes visible with many infinite light rays coming out from the singularity. A comparison with the corresponding black hole case (Fig. 3.3) shows that in that case the horizon formed much earlier than the singularity.

radius coming later. As a result, at no stage before the epoch of formation of the singularity are the light rays ever trapped, and rays and particles can travel out from the super-ultra-dense regions which are arbitrarily near to the spacetime singularity. This lack of trapping happens due to a deficit in the focusing effect of gravity on light and matter. Due to inhomogeneity and decreasing density of the star away from the center, there is never enough total mass at any given epoch to cause full light trapping, prior to the epoch of singularity formation. Roy Maartens, Naresh Dadhich, and I analyzed this general relativistic effect of delay of horizon formation and related it to the shearing effects due to gravity in a spacetime.

Physically, what does such a phenomenon imply and correspond to? There are many possible interpretations, but one possibility is that this may correspond to the creation of general relativistic shocks due to inhomogeneity in such ultra-dense regions, which may allow for escape and ejection of light and matter despite very high matter densities.

When such a naked singularity develops, it is characterized by three main attributes. First, there are families of infinitely many light and particle trajectories coming out from the singularity region. Second, it is a genuine curvature singularity in that the densities and curvatures become infinite in the limit of approach to singularity along these paths. Finally, the overall spacetime satisfies all physical regularity conditions, so this becomes an interesting scenario for studying actual physical processes in ultra-strong gravity regions.

How the Field Strength Grows

An important test of the physical significance of a naked singularity is its curvature strength. A detailed classification and analysis of spacetime singularities is available as far as their curvature strength and physical significance are concerned. A singularity is said to have strong curvature if there is at least one non-spacelike trajectory falling into it along which the spacetime curvatures diverge sufficiently fast in the limit of approach to the singularity. The strength of a naked singularity has been examined in several cases along non-spacelike trajectories terminating at the naked singularity in the past, in the limit of approach to the singularity. It turns out that we get strong curvature singularity in a powerful sense in that curvatures diverge very rapidly along not one but *all* non-spacelike geodesics meeting the naked singularity in the past.

This leads to the useful conclusion that, in fact, strong-curvature naked singularities can occur in general relativistic gravitational collapse for several reasonable equations of state. The scenarios considered here are reasonable in that the energy conditions are satisfied and the collapse evolves from well-defined initial data. Any possible formulation of the cosmic censorship hypothesis must take into consideration these situations and we must examine the structure of a naked singularity and the possible general constraints.

It would be useful to know the behavior of the matter energy density and gravitational field strength during collapse to a singularity in typical models as described earlier. That would give us an idea as to how serious and genuine such a singularity is, as opposed to a gravitationally weak singularity, which is not genuine and possibly removable from the spacetime.

Let us consider, for example, the cosmological Big Bang singularity. In this case for the Friedmann models, the radiation density goes inversely as the fourth power of the scale factor of the Universe, namely as $1/S^4$, where the scale factor S goes to 0 at the singularity. Thus, as S becomes smaller, the radiation density diverges. The scale factor will go as $t^{1/2}$ in this case, where t is the time from Big Bang and we have $S = 0$ at $t = 0$. The equations of this cosmology then imply that the density diverges as $1/t^2$ in the limit of approach to the singularity. Again, for the Schwarzschild singularity, the density is, of course, zero throughout the spacetime because it is a vacuum solution of Einstein's equations. But we can plot the gravitational strength in that case. For null geodesics, i.e., the photon trajectories falling into the singularity, this will blow up and diverge as $1/k^2$, where k is the affine parameter along the geodesic paths, with a zero value at the singularity.

The Schwarzschild metric is a static solution which does not change over time, and the Big Bang is a dynamically evolving cosmological solution. But in the spirit similar to above, we need to ask what is the density growth behavior for a given model of stellar collapse as we approach the singularity. This point is particularly important for a naked singularity, because in the limit of approach to it, if the density and spacetime curvatures do not grow sufficiently strongly then it will be a gravitationally weak singularity which may be removable from the spacetime.

In fact, further to the CCC proposal, there has been a constant search on how to formulate and possibly prove a mathematically rigorous statement of the CCC. One possibility researchers considered in detail in the 1980s was that even when naked singularities formed, they must be gravitationally weak. If this is the case, they can be removed from the spacetime by mathematically extending them and thus need not be taken seriously. Then this by itself would become a statement for the CCC, namely that even if naked singularities occur in collapse they must always be gravitationally weak. Such a conjecture, however, turned out to be incorrect as classes of naked singularities of dust collapse proved to be gravitationally very strong because the curvatures became very powerful in the vicinity of the singularity.

In order to consider this collapse scenario, we know the evolution of the cloud fully and can work out how the density grows in the limit of approach to the singularity. For simplicity we can consider, for example, the central line of the cloud, which is actually a timelike geodesic in this case. Then the evolution of the density turns out to be $1/t^2$, where we have $t = 0$ at the singularity. Thus we see that the density and curvatures grow very powerfully and blow up as we go closer to the naked singularity.

The singularity is either covered in a black hole event horizon or visibly naked, depending on the nature of inhomogeneity in the cloud. In either cases the above fact on curvature growth in the limit of approach to the singularity does not change. Of course, in the black hole case we must calculate the density growth along the trajectories falling in the black hole, which hit the singularity at the center. In the case when a naked singularity forms, there are families of non-spacelike paths coming out from the singularity and we can go back along the same, closer to the singularity, and calculate the density growth in the limit of approach to the singularity.

In general, we can consider families of either photon or particle trajectories, falling into or coming out of the singularity, and examine the curvature growth along the trajectories. We can work out the density growth or the curvature strength in the limit of approach to the singularity. The case discussed above turns out to be a strong-curvature naked singularity in the sense that it is not removable from the spacetime. There is a powerful divergence of density in the approach to the singularity, and it is found to be a genuine strong-curvature singularity.

The Genericity Aspects

The issue of the final state of the gravitational collapse of massive stars and cosmic censorship has seized researchers' minds for many decades, because it has proved to be an important debate, not only for black hole physics and its astrophysical applications, but also for astrophysics and astronomy. There have been several unsuccessful attempts to formulate the CCC in different forms. Each of these attempts has tried to determine in different collapse models why naked singularity should not occur, or if they do, then why they should be 'non-generic'. If a naked singularity, rather than a black hole, does develop in gravitational collapse models which are physically more realistic than the OSD solution, the natural question then is, how 'generic' or 'typical' is such a naked singularity outcome? As such, trying to formulate 'genericity' in

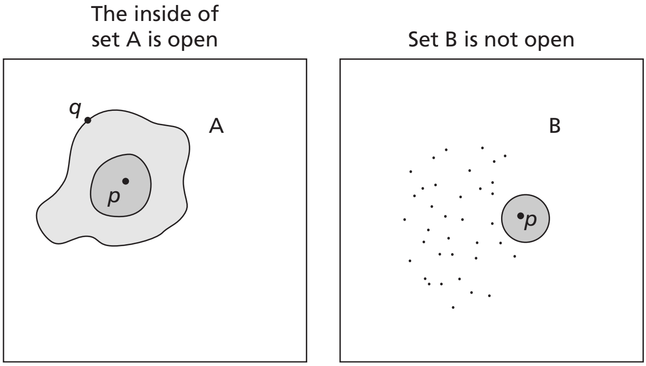

Figure 5.4 An open set is basically a region of full space where any chosen point is surrounded by a continuous neighborhood which is fully part of the set itself. The idea here is that if we make a small perturbation and move away from that point, we still remain within the set. In the example above, the inside of set *A* is open, because for any point such as *p*, we can always draw a small circle around it which is fully contained within *A*. But set *B*, on the right, which consists of a collection of scattered points is not open, because for any point therein, any of its neighborhood will always contain points other than those in set *B*. Such open sets play a very important role in defining the geometry of any given space.

gravitation theory is a difficult task and no single definition is possible or available. That is why many different ways have been attempted.

The idea is that if a naked singularity forms, if the set of initial conditions from which it develops as the collapse evolves is not generic, then the naked singularity may not be called generic. In other words, the set of initial conditions from which the naked singularity arises should form an open set of initial data (Fig. 5.4). Essentially, what this means is, if a particular set of initial data gives rise to a naked singularity, then all the 'nearby' initial data also must evolve into a naked singularity only. Typically, the naked singularity is characterized by a past-incomplete null geodesic from the future infinity.

Within the general LTB dust collapse scenario that we discussed earlier, we can consider this issue based on the general treatment that Indresh Dwivedi and I gave in 1993 for the inhomogeneous dust collapse. The essential picture that emerges and the general result that we obtain for this system can be described in the following manner.

Two parameters fully determine the evolution and the final fate of collapse for the general dust system. The first of these is the mass function of the star, which specifies the total mass within a given area radius. The second parameter is the velocities of the collapsing shells at the initial epoch. The energy conditions imply that mass is always positive. As for the velocities of the shells there are different choices depending on the physical system under consideration. For example, for a collapse from rest which models the physical scenario of a star which collapses from being initially at rest, the velocity function is defined appropriately.

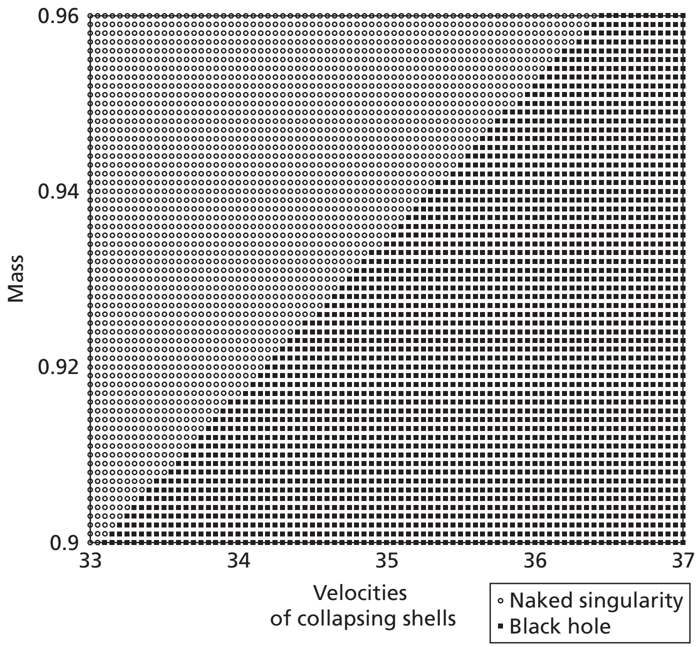

Figure 5.5 The distribution of collapse end-states in terms of black holes and naked singularities for a typical class of LTB models for inhomogeneous dust collapse. In the present case, we have a centrally higher density profile which is physically more realistic as opposed to a constant profile used in the homogeneous dust case of the OSD collapse models. As seen, the collapse final states for any given initial mass and velocity profiles for the collapsing shells are almost equally distributed as black holes and naked singularities. The outcome depends on the choice of velocity profiles for any given mass profile which characterizes the initial density distribution for the cloud. (From F. Mena, R. Tavakol and P. S. Joshi, Phys Rev D62 (2000), 044001)

The result we get is the following: In general, given any specific initial density distribution that we may like to choose, there is a non-zero-measure collection of velocity functions that take the star to either a naked singularity or a black hole final state, depending on the choice we made for the velocities (see Fig. 5.5). Such a choice is freely available with all regularity and physical reasonability conditions being satisfied. This shows the genericity and stability of the naked singularity within the given framework of a general dust collapse scenario. We shall discuss this feature in more detail later for other forms of matter. As we consider in the next chapter, similar features arise when pressures and other physical equations of state are incorporated for general collapsing matter fields.

6

Naked Singularities

We saw in the previous chapter that for general dust collapse models, both black holes and naked singularities develop quite naturally and generically as the final fate of gravitational collapse. As discussed in some detail, introducing a higher density at the center naturally delays the formation of trapped surfaces. The event horizon then fails to envelop the final singularity of collapse.

The above scenario was obtained through the introduction of the physical feature that a realistic collapse must be inhomogeneous in its density distribution. However, we need to examine how a naked singularity develops in the gravitational collapse of a massive star when other physical features such as non-zero pressures, realistic equations of state, and non-sphericity are also included. In particular, will the naked singularity disappear when more general physical features are taken into consideration?

As such, dust itself is an idealization and typically the collapsing star will have non-zero pressures within. We can then ask whether a gravitational collapse with non-zero pressures gives rise to black holes only, thus doing away with the naked singularity. Further, a typical star need not be exactly spherically symmetric, and can be also rotating. If we allow for perturbations from sphericity, will that avoid naked singularities? These questions basically relate to the issue that even if naked singularities do develop in collapse, how realistic or 'generic' or 'stable' are they?

The point is that while raising such issues and investigating and constructing collapse models accordingly, if we find that the introduction of a certain physical feature during collapse necessarily removes the formation of all visible singularities, then that may provide a useful possibility of formulating and proving the censorship conjecture through physical means, rather than trying only mathematical formulations.

Toward such a purpose, many more scenarios and models of gravitational collapse, more general and generic than the dust collapse model, have been examined in past years. We will discuss some of those here. The essential conclusion from these studies is that depending on a star's initial density and pressure profiles, there are velocity profiles or dynamical evolutions of the collapsing matter shells that direct the collapse to either a black hole or a naked singularity. It thus turns out that naked singularities do develop in collapse from fairly generic initial conditions.

Finally, we will ask the question 'why does a naked singularity form during collapse?' In other words, we would like to know the physics behind naked singularity formation, or what are the physical factors that cause a naked singularity, rather than a black hole, to form as a massive star's final state of collapse? The interesting role and the finer intricacies of general relativity come out here. We will see again how our Newtonian intuitions can fail and how general relativity opens up paths of communication even from the vicinity of super-ultra-high density regions.

Collapsing a Massive Star

The cosmic censorship conjecture (CCC) proposes that the Oppenheimer–Snyder–Datt (OSD) homogeneous dust collapse scenario must generalize to a generic situation of physically realistic gravitational collapse of a massive star. However, as discussed in the previous chapter, introducing inhomogeneities in dust collapse leads to naked singularity final states. It is clearly essential to examine the introduction of further physical features to see what their effect is on collapse final states. To investigate this, many dynamical collapse scenarios for cases such as clouds composed of dust, radiation, perfect fluids, and matter compositions with more general equations of state have been examined in the past couple of decades.

To decide on the validity of censorship, we must study the dynamical collapse of massive stars to determine their final fate. In recent years, detailed analytical research on gravitational collapse has given insights into when black holes form and when they will not. Some models suggest that visible ultra-dense regions, or naked singularities, may arise naturally and generically as an outcome of collapse. If so, the implications are enormous and touch on nearly every aspect of astrophysics and fundamental physics. They might account for extreme high energy phenomena in the Universe that have defied explanation. They may also offer a laboratory for exploring quantum gravity effects that are otherwise extremely difficult to observe.

Unlike the idealized and rather special OSD model, real stars have an inhomogeneous density, namely, higher at their centers, the pressure within is not zero, and they also rotate. Will every massive star collapsing toward the end of its life-cycle turn into a black hole? Cosmic censorship supposes the answer to be yes, namely, that the singularity forming during collapse always hides within an event horizon, never to be seen by external observers. So whatever the physical conditions and forces within massive stars may be, their eventual collapse must yield a black hole. Censorship amounts to a constraint on the nature of allowed dynamical evolution for collapsing clouds in general relativity. The crucial physical question here, from the perspective of both gravitation theory and astrophysics, is can we really see and observe such super-ultra-dense regions which develop when massive stars collapse?

We believe that the real way to make progress here is to go beyond making a hypothesis, and to study the gravitational collapse processes of massive stars using the dynamical equations of gravitation theory. Extensive efforts in recent years and results in this direction are now available, and we need to consider the possible implications if naked singularities actually develop as result of a continual gravitational collapse. The possible theoretical and observable consequences are to be investigated.

The point is whether we can generalize the dust collapse conclusions on the occurrence of a spacetime singularity during collapse and black hole formation for more general matter fields and for non-spherical situations. While we know that the occurrence of a singularity is stable to small perturbations in the initial data, there is no proof available that it will continue to be hidden within a black hole and causally disconnected from outside observers when the collapse is not spherical or when the matter is not exactly homogeneous dust.

There have been several important recent developments showing that the final fate of a collapsing star can be a naked singularity in many physically reasonable collapse scenarios. Such a naked singularity is very much like a black hole, though without its 'black' part, in that it is a super-ultra-dense region of matter and curvatures in the Universe which can communicate with or is visible to faraway observers. The naked singularity can result as a massive star's collapse final state, depending on the structure of the star in terms of its internal density and pressure profiles and the velocities of the collapsing shells. This means that the final fate of the star depends on the nature of the initial data from which the collapsing cloud evolves and the possible dynamical evolutions as allowed by Einstein's equations of gravity.

Clearly, black holes and naked singularities, which are hypothetical astrophysical objects in nature, provide one of the most exciting arenas in fundamental physics and cosmology. If naked singularities actually formed in the collapse of massive stars, this will have profound implications for our observations of very-high-energy phenomena in the Universe and for fundamental theories of nature. How do we resolve this profound theoretical as well as observational conundrum, and what could be the future directions? What do current astronomical observations imply for the final fate of a massive collapsing star? What will be the implications of these issues and considerations for any future possible quantum theories of gravity? Are there any possible model solutions which may help us resolve this cosmic puzzle? Taken as a whole, it may not be an exaggeration if we describe these questions, and the overall current situation, as one of the most important and fundamental theoretical challenges to have emerged through the theoretical as well as observational developments in astronomy and astrophysics of the past decades.

Gravitational Collapse Studies

Since we are interested in collapse, we require that the spacetime contains a regular initial spacelike surface on which the matter fields have a compact support, and that all physical quantities are well-behaved on this surface. Also, the matter should satisfy a suitable energy condition and Einstein's equations should be satisfied. We say that the spacetime contains a naked singularity if there is a future-directed non-spacelike curve which reaches a faraway observer or infinity in the future, and in the past it terminates at the singularity.

To resolve the profound issue of cosmic censorship at the heart of gravity physics which has no direct proof applying under general conditions, researchers, including our group, have considered a variety of specific collapse scenarios which can provide many useful insights into the final fate of a massive star's collapse. In essence, we have conducted a series of computational experiments to compile a list of conditions under which gravitational collapse leads to a black hole or a naked singularity.

Our strategy was to examine all possible courses of evolution for a massive star as it collapses with a given set of physically reasonable properties and regularity conditions. These latter are basic consistency requirements such as having a positive amount of energy as the star collapses under self-gravity. Mathematically, these are the allowed dynamical solutions to Einstein's equations. We supply the initial data for collapse in terms of initial density and pressure profiles of matter within the star and the velocities of the collapsing concentric shells of gas. We then compute the causal structure within the collapsing cloud, which reveals how light and matter are trapped as gravity grows stronger as collapse progresses. This decides whether the final singularity of collapse will be visible or covered by a horizon.

Over the past few years, we have developed a general formalism for treating spherical collapse from regular initial data. General matter fields can include realistic features such as pressure, inhomogeneities in density distribution, and reasonable equations of state of matter. Physicists are even beginning to consider situations where matter takes on some other form, such as a fundamental quantum field, or is converted to radiation in a sudden phase transition in the very late stages of collapse. What these works show in a generic manner is that collapse with non-zero pressures can lead to either a black hole or a naked singularity. The outcome is decided by the initial data and dynamical evolutions as allowed by Einstein's equations. It turns out that collapse ends in a naked singularity in a wide variety of situations.

The singularities studied so far exhibit a wide variety of interesting structures. In some models, only a part of it is visible, but then it is eventually covered by the event horizon. Elsewhere they can remain visible forever. This depends on the form of collapsing matter, the equation of state, and other properties. Basically, singularities come in all varieties, depending on the model of collapse. Typically, the naked singularity first develops in the geometric center of collapse, but later can spread to other regions, or become covered, as the collapse progresses.

As we will discuss later, the result of a naked singularity or a black hole has more to do with the choice of initial data for field equations, rather than with the form of matter or the equation of state. The generic conclusion from these studies is that both black holes and naked singularities do develop as collapse end-states, for a realistic collapse that incorporates inhomogeneities as well as non-zero pressures within the interior of the collapsing matter cloud. Subject to various regularity and energy conditions to ensure the model's physical reasonableness, it is the initial data, in terms of initial density, pressures, and velocity profiles of the collapsing shells, that determine the final fate of collapse as either a naked singularity or a black hole.

Such a scenario has important implications for cosmic censorship in that in order to preserve the CCC we must avoid all such regular initial data causing naked singularities. Therefore, a deeper understanding of the initial data space is required in order to determine such initial data and the kind of physical parameters they would specify. This would, in other words, classify the range of physical parameters to be avoided for any particular form of matter chosen, such as perfect fluids or dust or whatever else it may be, so that only black holes are allowed and not naked singularities. This will pave the way for black hole physics to use only those ranges of allowed parameter values that will produce black holes, thus putting black hole physics on a more firm footing. Perhaps we can say without exaggeration that this is where the real usefulness of gravitational collapse studies lies.

Collapse with Non-zero Pressure

It is clear on physical grounds that non-zero pressures are quite important during a star's collapse, especially during its later stages. It is true at the astrophysical level that when the inner pressures due to nuclear burning within a massive star subside, then its continual gravitational collapse is triggered. As the collapse progresses, pressures can again become important and play an important role during the later stages. So the basic question we must ask is, how can non-zero pressures modify the collapse scenario, especially as far as the final state of collapse is concerned? What researchers have found is that non-zero pressures make no qualitative difference to the result, in that, again, both naked singularities and black holes develop as collapse end-states. The only difference is that the initial data space for collapse is more vast and larger in this case. It now consists of density profiles, pressure profiles, and initial shell velocities, which is a much larger space than the dust collapse case we discussed earlier. In the latter, only the initial density determines the final state of collapse, when we consider collapse from rest, and a combination of parameters determines the final state.

For collapse with non-zero pressures, the key principle again remains the same, namely the total mass within a given radius determines the trapping of light in either case. But now total mass is a more complicated function of the parameters involved. This is what the general formalism we refer to here deals with, which allows fully general pressures to be incorporated and shows that both naked singularities and black holes arise generically even when pressure is non-zero.

Although previously dust collapse models ignored pressure, the general techniques developed to understand the dynamical evolution of collapse did find applications later, when collapse models with pressure were considered. As is well known, under completely general conditions it is not possible to fully solve Einstein's system of equations globally, the main reason being that these are a complicated set of nonlinear partial differential equations. But the relevant information about the final fate of collapse in the general form of matter fields and curvature invariants can be extracted by an analysis of the behavior of the solutions near the singularity and near the center of the cloud. This helps us decide the local visibility or otherwise of the central singularity.

Gravitational collapse models with non-zero pressures have been discussed and investigated in past years in detail by many groups, including our own. Giulio Magli, Roberto Giambo and their collaborators in Italy, and Ken-ichi Nakao and others in Japan considered a form of pressure generated by the rotation of particles within a collapsing cloud. They showed how naked singularities develop as end-states. Several models with a realistic equation of state, which specifies how the density and pressure within a cloud are related, were also investigated, including models by Amos Ori and Tsvi Piran, and by our group. Also Kayll Lake and collaborators considered various collapse scenarios to find similar results, and Tomohiro Harada and Sanjay Jhingan studied perfect fluid models.

Given the difficulties of solving Einstein's equations, the system's full integration is not possible even in the simplest cases once pressures are included. So the general line of inquiry and the main technique that our group developed to deal with this problem can be outlined as follows: First, we consider the general structure of Einstein's equations for studying spherical collapse. We then describe how the equations can be integrated up to first order, thus obtaining the equation of motion for the system. The regularity conditions and energy conditions that give physically reasonable models are considered and imposed on the models. The final stages of collapse are then discussed, evaluating key elements that determine when the outcome will be a black hole or a naked singularity.

It can be shown that there is a specific function which is related to the tangent of outgoing geodesics at the singularity, whose sign solely determines the time of formation of trapped surfaces in relation to the time of formation of the singularity. We also analyze the occurrence of trapped surfaces during collapse and the possibility that radial null geodesics do escape, thus making the singularity visible. We can show how both features are related to the sign of the above-mentioned function, thus obtaining a necessary and sufficient condition for the visibility of the singularity.

These conclusions and models are by now fairly widely accepted, and it is clear that physically reasonable gravitational collapse can create a naked singularity. However, in the original statement of the cosmic censorship conjecture, Roger Penrose included a condition, namely that 'generic' gravitational collapse would always produce black holes. Are the models we have just discussed generic? In fact, they are, in the sense of the initial data for matter being fully generic, as required. The key difficulty for such a requirement, however, is that there is no mathematically well-formulated definition of genericity available in gravitation theory that can be used to answer this question. Stability and genericity are extremely difficult concepts to formulate in general relativity, within the framework of a general spacetime with a Lorentzian metric with an indefinite signature, with no definite and clear criteria available. In fact, there are formidable mathematical difficulties in achieving this. So the only way to proceed is by asking further questions, such as is there any perturbation of spherical symmetry that will remove these naked singularities? Or will naked singularities form in non-spherical collapse?

To summarize, the typical result that we obtain for spherical collapse with non-zero pressures within a collapsing star is as follows: gravitational collapse models with a general form of matter, together with those such as directed radiation, dust, and perfect fluids, imply a certain general pattern emerging about the final outcome of gravitational collapse. Basically it follows that the occurrence of a naked singularity is related to the choice of initial data for the Einstein field equations, and will therefore occur from regular initial data within the general context considered, subject to the matter satisfying energy conditions.

The Equation of State

An equation of state is a constitutive relation that provides a link between certain key quantities describing a system. Typically for a collapsing matter cloud an equation of state is provided once we know how the pressure is related to the energy density. Stars during their equilibrium phase can be described by such equations of state, both at the level of classical mechanics and within the context of general relativity.

It is reasonable to believe that once the nuclear fuel that maintains the star in equilibrium is exhausted, the collapse commences in a very short time from an initial configuration described by the same physical quantities related by the same equation of state. We can take the pressures and densities as they were initially from the models at equilibrium, which is also true for other quantities appearing in the equation of state. These values are all expressed in terms of the thermodynamical variables of the system, such as temperature and the molecular weight of the gas.

There are examples of simple, astrophysically relevant linear and polytropic equations of states which have been considered, and for these the collapse has been analyzed to understand its final fate in terms of black hole or a naked singularity. This is useful for possible astrophysical and numerical applications. For example, researchers have considered the collapse of a perfect fluid, wherein the pressures are isotropic. The dynamics is then fully fixed by the initial conditions and so we see how solving the equation of motion is equivalent to solving the whole system of equations.

As we noted earlier, the introduction of an equation of state closes the system of Einstein's equations and so no freedom is left to specify any further physical quantities. Gravitational collapse of a perfect fluid with a linear equation of state was studied by Rituparno Goswami and myself to show that solutions exist such that both black holes and naked singularities are possible collapse outcomes, depending on the initial data and the velocity distribution of the particles.

An interesting point to note here is that, as the collapse evolves, the equation of state need not remain the same. It could vary, maybe even abruptly as in some phase transitions, as would happen when the nuclear saturation limit for the neutron star matter is reached. Actually, recent developments suggest that close to the formation of the singularity, strong negative pressures might develop and the gravitational field might act repulsively, thus disrupting the collapsing cloud and dispersing away the infalling matter. This kind of effect can be inferred already at a classical level but it is more evident when quantum corrections are considered, as we shall discuss later. This evidence suggests that the equation of state relating pressure and density, which must be always positive, evolves in a non-trivial manner during the collapse. There could be a sharp transition in equations of state when matter passes from one physical regime to another. With strong negative pressures, the speed of sound in the matter cloud can approach the speed of light. It is clear that to account for such extreme situations the usual linear and polytropic equations of state are not enough, and so the equations of state describing ultra-compact objects are not well understood.

Then we are left with the option of either dropping the equation of state altogether or considering more exotic equations of state. Among the latter, gases with exotic properties, such as the so-called Chaplygin gas, have been considered in order to account for possible sources of dark energy and dark matter. Of course, dark energy effects are not likely to be relevant for collapse at stellar scales but they might become important for bigger objects, such as the compact ones dwelling at the center of galaxies that have masses on the order of many million solar masses.

Non-zero Cosmological Constant

While considering the collapse with non-zero pressures, it is natural to ask about the role of the cosmological term introduced by Einstein in his field equations, where it appears as an additional term, and frequently it is taken to be vanishing. However, in view of recent evidence regarding dark energy and the observed acceleration of the Universe, the possibility that we live in a Universe with a non-zero cosmological term has resurfaced strongly. The role of the cosmological term is essentially as a vacuum energy density, and depending on its positive or negative sign, it may exert positive or negative pressures.

Researchers, including us, have investigated gravitational collapse with a non-zero cosmological constant as well. The results show that for the collapse of a compact object such as a massive star, the matter densities are typically much higher than the vacuum energy density. Therefore the collapse end-states in terms of either a black hole or a naked singularity remain unaltered. However, for a very large cloud which has very small initial densities, the cosmological constant can make a difference and when it is positive, it could sometimes trigger the bounce of the cloud.

Non-spherical Collapse

While we have a good understanding now of spherical collapse for a generic matter field with pressures, non-spherical collapse remains a major uncharted territory. Several recent studies have found non-spherical models that also give rise to naked singularities. The question is whether these situations are contrived. Fast-developing numerical core collapse models in general relativity could be of help here.

What will be the final fate of gravitational collapse which is not spherically symmetric? The main phases of spherical collapse of a massive star would be typically instability, implosion of matter, and subsequent formation of an event horizon and a spacetime singularity of infinite density and curvature with infinite gravitational tidal forces. This singularity may or may not be fully covered by the horizon, as we have already discussed. Again, small perturbations over the spherically symmetric situation will leave the situation unchanged in the sense that an event horizon will continue to form in the advanced stages of the collapse.

The question then is, do horizons still form when the fluctuations from the spherical symmetry are high and the collapse is highly non-spherical? It was shown by Kip Thorne in 1972 that when there is no spherical symmetry, the collapse of infinite cylinders does give rise to naked singularities in general relativity that are not covered by event horizons. This situation motivated Thorne to propose the following *hoop conjecture* for finite systems in an asymptotically flat spacetime, which characterizes the final fate of non-spherical collapse: The horizons of gravity form when and only when a mass, M, gets compacted in a region whose circumference, C, in *every* direction obeys $C \leq 2\pi(2GM/c^2)$, where G is Newton's constant of gravity and c is the speed of light.

Thus, unlike the cosmic censorship conjecture, the hoop conjecture does not rule out *all* naked singularities, but only makes a definite assertion on the occurrence of event horizons in gravitational collapse. We also note that the hoop conjecture is concerned with the formation of event horizons, and not naked singularities. Thus, even when event horizons form, for example in the spherically symmetric case, it does not rule out the existence of naked singularities. That is, it does not imply that such horizons must always cover the singularities.

When it comes to non-spherical collapse, the situation is almost always complex enough to require numerical tracing of the collapse evolutions. However, we note that apart from numerical simulations, some analytic treatments of non-spherical collapse are also available. For example, the non-spherical Szekeres models for irrotational dust which have no symmetries, and which generalize the spherical Lemaitre–Tolman–Bondi collapse, were studied by Andrzej Krolak and myself to deduce the existence of strong-curvature central naked singularities. The symmetries of a given model are typically characterized by what are called Killing vectors for the spacetime. The Szekeres models have actually no Killing vectors, and in that sense they are quite interesting non-spherical models. Therefore, this is a useful result which indicates that naked singularities are not necessarily confined to spherical symmetry.

Also, there are classes of spacetimes called gamma metrics, which are non-spherical spacetimes admitting naked singularities, and these models have been analyzed in some detail. All the same, it must be noted that dynamical evolution of a non-spherical collapse still remains a largely uncharted territory. Numerical models of collapse in full general relativity must be used to tell us more about the collapse final fate in such a general situation.

Compared to this, in the spherical case, we can do mostly analytic work, which is much more transparent. We use analytic techniques to deal with Einstein's equations in the spherical case, developing a general formalism for general physical matter fields, including dust, perfect fluids, and radiation collapse. We can, in fact, solve Einstein's equations at least partly, so as to determine the structure of trapped surface geometry and the apparent horizon as the collapse evolves. This allows us to explicitly determine whether there are families of null geodesics, and in some cases other timelike curves, emerging from the singularity, and the physical conditions determining their existence. That is, these are photon trajectories which meet the singularity in the past, and are future-directed and meet external observers in the spacetime, making the singularity or the super-ultra-dense regions in its vicinity visible. Therefore, this conclusively establishes that the singularity is visible, as opposed to being covered within an event horizon. When the collapse end-state will be a naked singularity, and when it will be a black hole, is explicitly characterized in terms of the initial data, such as the density and the velocity profiles of the collapsing matter, and in terms of the allowed evolutions of Einstein's equations. The geometry of trapped surfaces is worked out analytically, and the conclusions are, by their very nature and methodology, coordinate independent.

The limitation, however, is that this formalism applies to spherical collapse mainly, though for fully generic matter fields. Also it is seen from the trapped surface geometry that this will be stable to small perturbations in the spacetime geometry. It is true that most of the collapse models analyzed so far are spherical. However, many physicists believe, as Roy Kerr pointed out to me in a private conversation, that if cosmic censorship is to hold as a basic principle of nature, it needs to hold in the spherical class too, which has wide astrophysical significance. Therefore, analyzing these models could be of great value from the perspective of censorship. This will help isolate the physical features that cause naked singularities. On the other hand, some researchers believe that the classes of collapse analyzed so far are good enough to begin investigating the physics and astrophysical implications of naked singularities.

The main question now is whether these situations are contrived. Fast-developing numerical core collapse models in general relativity could be of help here. The results so far also show that naked singularities are, in fact, stable to small perturbations in the initial data of matter fields, to the introduction of non-zero pressures in the cloud, and so on. Therefore we have yet to find and isolate precisely the kind of perturbation that would make a given naked singularity go away. These situations are what physicists call 'generic', that is, they are not contrived, in that a tiny deviation in the initial data leads to much the same outcome. However, we should emphasize the general 'stability' proof for the naked singularity yet remains to be achieved.

Clearly, the issue of the final fate of a non-spherical collapse is closely related to the stability and genericity of naked singularities. When we are dealing with non-spherical collapse, this considers what happens to the 'deformations' that mark the departure from spherical symmetry. This is a major unsolved problem analytically, since the set of Einstein's equations describing the evolution of the cloud becomes immensely complicated when deformations and rotation are taken into account. There have been some attempts to study quasi-spherical and cylindrical collapse in full generality, but the physical significance of such models remains somewhat obscure. We can say that the study of non-spherical collapse within exact solutions of Einstein's field equations is an area where much work still needs to be done.

Since a comprehensive analytical treatment of collapse away from spherical symmetry is still not available, the few insights that we have in the final fate of collapse of a non-spherical cloud come from numerical relativity. The study of non-spherical collapse is related to cosmic censorship through the so-called *no-hair theorem*, which essentially says that any asymptotically flat black hole vacuum solution of Einstein's equations must be identified by only three quantities, namely the mass, charge, and angular momentum. This means that other vacuum solutions characterized by some other quantity, such as higher multipole moments for an axially symmetric spacetime, can have a naked singularity, as is the case for the class of so-called Weyl metrics. Therefore one crucial issue with non-spherical collapse is what happens to those higher multipole moments during the collapsing phase.

If the collapsing cloud retains its non-spherical shape, then the final vacuum configuration will not be represented by Schwarzschild or Kerr geometries. Then naked singularities, like those present in axially symmetric vacuum spacetimes such as the Weyl class or the Tomimatsu–Sato solution, could be present. On the other hand, the presence of deformations might provide a modification in the density distribution of the collapsing source that opposes the pull of gravity, thus suggesting that non-spherical sources might produce some equilibrium configuration before complete collapse settles to a singularity.

Recently, investigations have looked at the observational signatures that such vacuum metrics would possess, and the possibility that the Kerr metric might not be the best to describe the exterior of a rotating star is also suggested. If, on the other hand, higher-order multipole moments are radiated away during collapse, then the final configuration is Schwarzschild or Kerr–Newman spacetime. For this to happen, there must be a mechanism within the evolution of Einstein's equations that explains why and how the higher multipole moments are radiated away, which at present is a matter of speculation. Also, we are still left with the possibility that the event-like naked singularities, like those forming during collapse with spherical symmetry, might form. In this sense the study of small perturbations of spherical symmetry shows that the final outcomes, whether black holes or naked singularities, are most likely stable.

Numerical Simulations

The investigations on gravitational collapse have been carried out on both the analytical and numerical sides. Many questions on complete collapse still remain unanswered from the theoretical side and also from the observational perspective, like the mechanism behind supernovae explosions and the origin of gamma ray bursts. To answer these, computer simulations might prove very useful.

We will mention here a few of the results and open problems. Due to difficulties in solving Einstein's equations when non-trivial situations are considered, such as non-spherical collapse with rotation, numerical simulations can provide insights into problems currently not approachable analytically. The important ones are the final fate of the complete collapse of a massive cloud and the merger of two compact objects. These phenomena are related to cosmic censorship as well as to astrophysics. Numerical simulations make it possible to consider effects that have been neglected in the analytical treatment, but are important in the astrophysical context. Thus, they can be used to study the mechanism behind supernova explosions, core collapse, and high-energy phenomena that occur when a star dies.

All these scenarios typically produce gravitational waves, the holy grail of gravitational physics today, and so numerical modeling of how these can be produced is very important to the success of experiments such as LIGO and VIRGO. For example, the presence of electromagnetic fields and accretion disks are crucial elements that will affect the process by which the core of a star implodes and currently they can be studied only with numerical simulations. So these may be the only means for providing indications on how high-energy observed phenomena such as relativistic jets or gamma ray bursts can occur.

The complete collapse of a massive cloud produces a singularity, either covered or visible, depending on the initial conditions. Since diverging quantities cannot be handled numerically, a crucial issue of such simulations is how singularities are treated or excised. A numerical model where the singularity region is removed from the simulation regardless of the presence of an apparent horizon may not provide much insight about the issue of cosmic censorship. But it can still prove to be very useful for understanding the issues of rotation and dissipation of higher multipole moments that might settle the collapse to a Kerr black hole or an axially symmetric naked singularity. On the other hand, numerical models can be used to trace the apparent horizon in more complicated matter models, thus giving some hint as to what happens to trapped surfaces in realistic collapse scenarios.

Two main collapse scenarios have been investigated and these are of crucial relevance from an astrophysical point of view. These are the complete collapse of rotating bodies and the merger of two compact objects. It is important to note that when one takes into account enough elements to make the simulation closer to reality, the time of computation grows enormously, thus requiring the use of very powerful computers. So very often simulations have had to restrict the space of parameters considered. Typically, simulations have been carried out in one or two spatial dimensions only and it was only recently that full 3D simulations of core collapse supernovae and of binary mergers have been able to study these phenomena in more detail. Furthermore, for the above reason, simulations that consider the microscopic structure of matter, including neutrino emissions and electroweak interactions, are generally limited to be non-relativistic while fully general relativistic simulations typically neglect the microscopic details of the matter cloud.

For the complete collapse of a rotating body, the main features were investigated by Luciano Rezzolla, Lius Lehner, and others through numerical simulations, namely the influence of rotation and magnetic fields. These affect greatly the final stages of collapse and are responsible for the waveform of gravitational waves and the structure of an accretion disk surrounding the final compact object. The way in which outer layers are ejected from the collapsing spinning body can help us understand some questions that are crucial to cosmic censorship. These include whether the collapse of a rotating body always ends in a Kerr spacetime, or whether it is possible that a rapidly rotating object collapses to a super-spinning Kerr solution.

Also, such simulations give useful insights into astrophysical questions such as how the matter ejected during collapse due to rotation falls back on the compact object in the form of an accretion disk, or what kind of features these accretion disks will present in terms of thickness, angular momentum, and light emission, or how the presence of a magnetic field will affect the infall of particles and the formation of high-energy jets.

Another interesting system is the merger of binary neutron stars or black holes. The models describe the merger of two inspiraling compact objects that can be taken to be black holes, neutron stars, or one black hole and a neutron star. The effect of unequal masses for the objects on the final configurations has been studied and a variety of scenarios in which the system settles to a final Kerr black hole with an accretion disk have been proposed.

Such mergers may sometimes occur in the Universe and are a major source of gravitational waves. So the simulation of waveforms emitted during the merger is one of the important results of these simulations. Also, the accretion disks and the mechanism by which matter from the disk accretes onto the final compact object is thought to be one possible mechanism for producing gamma ray bursts.

Further, what is crucial to the cosmic censorship conjecture is that the possible formation of super-spinning Kerr spacetimes has been studied. In principle such a spacetime can result from the merger of two rapidly rotating compact objects, from the increase in angular momentum due to the inflow of particles from the accretion disk, or from the complete collapse of a star with high angular momentum. There are no definitive results on the possible formation of super-spinning Kerr spacetimes, though there are arguments that suggest that overspinning a Kerr black hole might not be possible. The discussion on these issues is still very much open. What has become clear is that the mechanism by which such a final configuration can, in principle, be produced is very different from that for producing a Kerr black hole.

Also, some progress toward a viable description of the processes that lead to type II supernovae explosions has been made. Many different settings of numerical simulations considering the microscopic effects have been carried out in order to provide a viable model for producing the shockwaves that accompany the creation of these supernovae. These simulations describe the core collapse of a star from the microscopic point of view, taking into account nuclear interactions and neutrino production. Within this field a lot of progress has been made during the past few years, with computational power finally sufficient for full 3D simulations and attention focused on the influence of neutrino production during the explosion. But these simulations still face a problem in that the efficiency of the explosion, even taking 'neutrino heating' into account, is not enough to produce a supernova. What happens in the simulations is that the explosion dies out after a few hundred kilometers.

We note that almost all models for type II supernovae explosions consider a shockwave as that generated when infalling matter from the outer shells reaches the inner compact core. The compact object at the center constitutes a barrier onto which the matter from the outer layers bounces, thus creating the shockwave. The energy of the shockwave is related to the size of the core that creates it. However, during a complete collapse, it is still possible that another wall, a quantum-gravity limit, exists at a shorter scale. This, as we have noted, is where general relativity breaks down, predicting a singularity. It is possible that another shockwave is created once this limit is reached. So if the structure of collapse is such that no horizon exists at that time, it is then possible that such a shockwave propagates outward, providing the missing energy for the explosion to occur. As far as we are aware, no simulations taking these possible effects into account have been carried out yet, so at present we do not know whether these constitute a viable solution to the problem of the missing energy for type II supernovae explosions.

Some numerical studies of the collapse of a rotating gas cloud indicate that a final configuration with over-spinning angular momentum is unlikely to form under general conditions. So a Kerr black hole might be a most probable final outcome. But physicists have also started to study the observational features of singular spacetimes as well in order to gain a better understanding of what traces and effects such over-spinning naked singularities could leave in the Universe if they were allowed to happen. Of course we as yet have no proof that such configurations will arise from the collapse of a cloud with regular initial data, but equally we have no proof that such configurations are forbidden.

The interesting work by Stuart Shapiro and Saul Teukolsky has shown that axially symmetric collapse can lead to the formation of naked singularities, which comes in handy when discussing the non-spherical case of oblate and prolate collapse. They considered the collapse of collision-free particles and evolved it numerically. What they discovered was that in a particular slicing there were no trapped surfaces forming in the limit of approach to the singularity, which raises a question as to whether the singularity is naked. What is interesting is that this gives intriguing numerical information about the possible final fate of non-spherical dust collapse, indicating that it can very well be a naked singularity. We need more information on non-spherical collapse and this is an important step in that direction.

Another rather interesting phenomenon found numerically for spherical collapse of a massless scalar field is the critical behavior in collapse, as seen by Matthew Choptuik. This work deals with a particular family of scalar field collapse models examined numerically. As the parameter characterizing the family is varied, small black holes form, followed by a naked singularity and then the field disperses, thus showing an intriguing numerical behavior.

Event-like and Object-like Singularities

We know that object-like singularities do develop from gravitational collapse. For example, the Schwarzschild singularity, which is hidden within a black hole, develops from homogeneous dust collapse. However, we have no exact analytic model in Einstein's theory to create a Kerr singularity or Kerr black hole from collapse. Can we obtain an object-like long-lived naked singularity from gravitational collapse? The answer turns out to be in the affirmative.

While gravitational collapse predicts short-lived event-like naked singularities in many models, the collapse could also produce long-lived object-like naked singularities in several cases, as Daniele Malafarina, Ramesh Narayan, and myself have shown recently. What we showed was that equilibrium configurations of regular matter sustained by non-zero pressures are predicted by general relativity. These objects could also provide an alternative to model astrophysical black hole candidates such as the galactic center.

Within the class of 'long-lived' singularities the above is another kind which presents intriguing possibilities. These are interior solutions that describe a regular source with a singular center. This is a different kind of final state from the vacuum solutions such as the Kerr metric. These involve a finite matter cloud with a boundary larger than the horizon, with a singularity at its center.

Interior solutions of Einstein's field equations have been considered for decades as sources of the gravitational field. A requirement in building such interiors used to be that the matter density should be regular all the way to the center of the source. But there is no reason to impose the condition that a stable configuration cannot evolve with a singularity at the center. These singularities model the region of arbitrarily high density that develops at the center of the object and where gravity exhibits extreme properties. It is then possible that such a source is observationally different from a corresponding source with a regular interior or from a vacuum solution such as a black hole. Also, if the overall density of the cloud is low enough, as is the case for supermassive objects residing at the center of galaxies, then processes happening close to the center, such as particle collisions and lensing, can be visible to faraway observers.

As for the gravitational collapse, there are smaller, explosive, short-lived phenomena like the collapse of the core of a star. These typically involve very high energies and very short time scales. On the other hand, the discovery of active galactic nuclei has led astronomers to suppose that objects like supermassive black holes reside at the center of galaxies. These objects are still not well understood and it is very plausible that processes such as gravitational collapse, on a much larger scale and on much longer time scales, are crucial to their development.

In this connection, what we proposed was a class of solutions for Einstein's equations with naked singularities that arise asymptotically for suitable matter models when the pressures opposing collapse manage to halt the process. Such a collapse process gives rise to an equilibrium configuration as the final state. These models describe how an initially regular matter cloud evolves to form an equilibrium with a finite radius somewhat larger than the Schwarzschild radius and with a naked singularity at the center. This process can require arbitrarily long times for the equilibrium configuration to be finally achieved, and the average density of the cloud toward the final equilibrium can remain small. This suggests that such a model can describe the formation of supermassive objects other than a black hole that can dwell at the galaxy centers. Such an object, if it actually existed, would bear certain properties for the surrounding accretion disk different to those of the accretion disk around a black hole, thus making it in principle observationally distinguishable.

Having obtained the equilibrium object models with a singularity at the center, we compared their observational properties with a black hole of same mass. In particular, we studied the accretion disks in both cases. It is known that a standard thin accretion disk can exist only at those radii where stable circular orbits are available. Therefore for a Schwarzschild black hole an accretion disk will have its inner edge at a finite radius away from the center and outside its event horizon. Inside this radius, the accreting gas plunges or free-falls.

This, however, is not the case for a naked singularity object, where the accretion disk extends very close to the singularity. Such a scenario implies several interesting consequences. Specifically, we showed that, in principle, a slowly evolving gravitationally collapsing perfect fluid cloud can asymptotically settle to a static spherically symmetric equilibrium configuration with arbitrarily large density at the center, without an event horizon. We then considered one such configuration with a finite outer radius and constructed a toy model. We investigated the observational signatures of a thermal accretion disk in this spacetime and compared them with the signatures expected for a disk around a black hole of the same mass. It turns out that several notable and interesting differences emerge. A disk around a naked singularity is much more luminous than that around an equivalent black hole. Also, such an accretion disk around a naked singularity has a spectrum with a high-frequency power law segment that carries a major fraction of the total luminosity.

What follows from such an analysis is that at least some naked singularities can, in principle, be distinguished observationally from black holes of the same mass. Clearly, such a scenario, if it holds more generally, would have several useful and interesting implications. The point is that these naked singularity models are easily distinguishable from a black hole of the same mass. So it would be useful and interesting to use observational data on astrophysical black hole candidates to test whether the presence of a similar naked singularity is allowed and possible.

Collapse Scenarios

It is natural to ask here the questions how can any signal escape from super and ultra-strong gravity regions, and why should any visible singularity form at all in gravitational collapse? Basically, the question is, why does a naked singularity form? In order to get an insight into this question, we first summarize below some of the outcomes of various collapse scenarios, and the special features that arise in each of these cases.

- *Dust collapse:* In this case, given any regular density profile or the mass function for the cloud, the collapse final state is either a black hole or naked singularity, depending on the choice of velocity profiles for the collapsing shells. To give a typical scenario, while homogeneous density results in black hole formation, an inhomogeneous density, higher at the center, which is physically a much more realistic case, gives rise to a naked singularity. Basically what we find is that sufficient inhomogeneity causes a naked singularity to form.

- *Perfect fluids:* In this case, several numerical as well as analytic results have been obtained and many models of collapse have been studied. It is seen that both black holes and naked singularities form as collapse outcomes. From the perspective of the cosmic censorship conjecture, the important point is that the introduction of pressures does not change the collapse outcome qualitatively, as was the case in dust collapse.
- *Spherical collapse with non-zero tangential pressure, but vanishing radial pressure:* In this class of collapse models, which is also called the *Einstein cluster models*, fully analytic and transparent treatment of collapse is possible. Explicit analysis of many models provides considerable analytic insights into gravitational collapse outcomes and it is seen that both black holes and naked singularities are obtained as final states.
- *General matter fields:* The collapse for fully general matter fields, subject to positive energy conditions and regularity of initial data, with both non-zero radial and tangential pressures have been considered by researchers. It is seen that given any regular initial data of density and pressure profiles, there are vast classes of evolutions determined by Einstein's equations that take the system to either a black hole or a naked singularity outcome, depending on the choice of initial data and the evolutions made.
- *Vaidya radiation collapse models:* This specific class of collapse models has provided considerable insight into the final fate of collapse in terms of black holes or naked singularities. Here the collapse of radiation shells is analyzed using the Vaidya class of spacetime geometries. The particular advantage is that the analytic treatment is very clear and transparent. As a result, the causal structure near the singularity comes out clearly and we can also see how the horizons are avoided to allow the singularity to be visible. The interesting fact is that the time rate of collapse or the speed of accretion of matter at the center decides the black hole or naked singularity outcome.
- *Massless scalar fields collapse:* Both the numerical and analytic treatments for gravitational collapse of this form of matter have been investigated in considerable detail, in particular the models admitting the symmetry of self-similarity, which is a kind

of scale invariance. Various intriguing features such as the formation of black holes of very small mass, critical behavior, and naked singularities have been found to occur. The naked singularity forming in these models is claimed to be 'non-generic', depending on the definition adopted of genericity. Scalar fields with a non-zero potential have also been examined by researchers. The interesting feature again is that an eternally visible singularity is seen to form. Also, the cloud can radiate away all its mass energy as the collapse evolves.

- *Non-spherical collapse:* Only a few somewhat special models have been discussed in this case analytically. While some insight on black hole and naked singularity formation has been obtained, largely this remains an uncharted area. In some cases of oblate and prolate collapse models worked out numerically, as considered by Stuart Shapiro and Saul Teukolsky, the absence of trapped surfaces is seen in a particular slicing as we approach the singularity, which is very suggestive of possible generic behavior in non-spherical collapse.
- *Collapse with a non-zero cosmological constant:* A cosmological constant in Einstein's equations can play a role for the final fate of collapse which researchers have considered. Basically, depending on its positive or negative sign, it exerts negative or positive pressures. For the collapse of compact objects such as stars, the end-states in terms of black hole or naked singularity remain unchanged. But for rather large matter clouds with very small initial densities, it does make a difference and a bounce of the cloud may be triggered, avoiding the singularity when the cosmological term is positive, which corresponds to a negative pressure effect.
- *Collapse in higher-dimensional spacetimes:* Researchers have considered many models of collapse in higher-dimensional spacetime geometries, mainly within a classical context, to examine whether by going to higher dimensions cosmic censorship can be restored. It is seen, however, that the main results obtained in the usual four dimensions do not qualitatively change. Therefore it turns out that this approach is not able to restore the censorship conjecture.
- *Gravitational collapse in other theories of gravity:* Again, the motivation has been similar, namely whether naked singularities can be avoided if we go to alternate theories of gravity, such as higher-derivative theories of gravity. Options such as the Lovelock gravity and introducing the Gauss–Bonnet term have been explored. Again, conclusions similar to Einstein's theory result.

This list summarizes the physical scenarios for gravitational collapse that have been considered and analyzed so far. The main issue has been whether they result in an event horizon formation. Taken as a whole, these works, which have been carried out over the past couple of decades, generate a fairly exhaustive picture of gravitational collapse outcomes using Einstein's gravity theory, and give a fairly good idea of when a black hole or a naked singularity develops as the collapse outcome. The general formalism for spherical collapse that we mentioned earlier includes a large number of the above cases toward

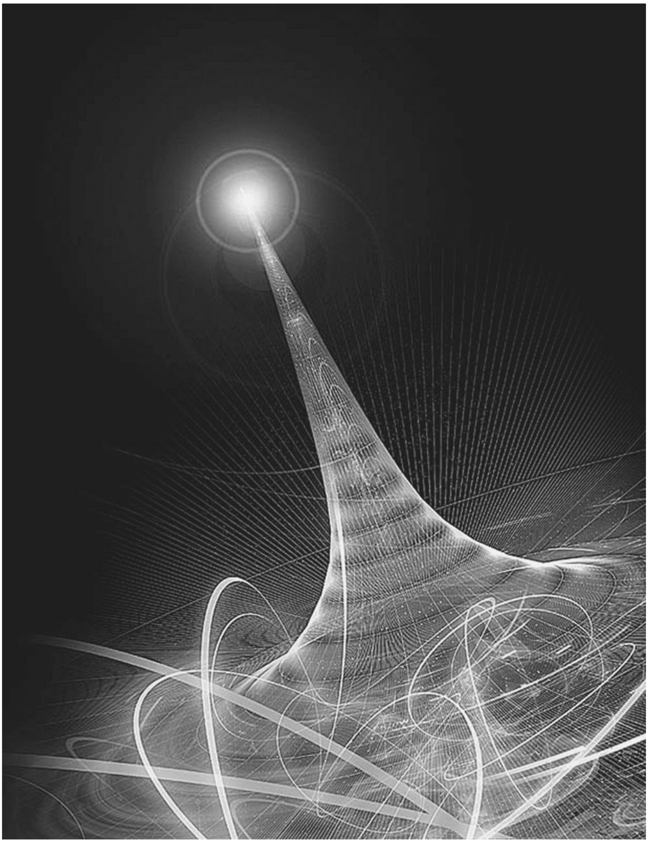

Figure 6.1 Naked singularity and its environment. The embedding diagram as an artist's impression (*Scientific American*, February 2009; Kenn Brown, Mondolithic Studio).

predicting collapse outcomes (Fig. 6.1 gives a schematic picture of a naked singularity and its environment).

Why a Naked Singularity Forms

It is natural to ask what is the physics that causes a naked singularity rather than a black hole to develop? The intriguing question is, why all the particles and energy are allowed to escape from extremely strong gravity fields. We have examined this issue in some detail to highlight the role inhomogeneities and shearing effects in a spacetime play in distorting the geometry of horizons forming in collapse.

In Newtonian gravity, it is only the matter density that determines the gravitational field. In Einstein theory, however, density is just one attribute, and various curvature and scalar quantities play an equally important role in dictating the overall nature of the field. Our results show that when the density is inhomogeneous or higher at the center of a collapsing star, the trapping of light and matter during collapse is delayed, allowing them to escape. This is a general relativistic effect wherein even if the densities are very high, paths are created for light or matter to escape due to inhomogeneously collapsing matter fields, and these physical features naturally lead to a naked singularity formation. Thus, it is the amount of inhomogeneity that counts toward distorting the horizons. If it is very small, below a critical limit, a black hole will form; otherwise, with sufficient inhomogeneity trapping is delayed and a naked singularity forms. This criticality comes out again quite clearly in the Vaidya class of radiation collapse models, where it is the rate of cloud collapse that determines the black hole or naked singularity formation.

What happens is, if a matter cloud with zero pressure inside has a strong density gradient, then the event horizon formation will depend on the comparatively dense inner regions of the star, but these regions may, for simple geometric reasons, lack the requisite total mass. The point is, in the case of such an inhomogeneous collapse, if the density falls off rapidly enough away from the center of the star at any given instant of time, then there is never enough mass in a given radius to cause total trapping of light. This is what helps create the visibility of the singularity.

This happens in the physically realistic situation when a star's initial density is falling away at a suitable rate from the center, rather than being constant. For example, if the difference between the density at the center and at a small radius away increases proportionally to either the first power of the radius or its square, then there is never enough mass within a given radius to cause light trapping and the collapse end-state is necessarily a naked singularity. But for a slow enough change, or when it just vanishes, which is the homogeneous case, the collapse produces a black hole. Thus we find that sufficient inhomogeneity in density distribution within a collapsing star causes a naked singularity.

In other cases, the salient issue could be the collapse timescale. For example, in radiation collapse models, it is how fast the cloud is collapsing that decides whether light is trapped. If the collapse is fast enough, there is never enough time to accumulate enough mass in a given region to cause light to be trapped. In general, for stars with different forms of matter and equations of state, and with different initial density and pressure profiles, there are a variety of physical factors that can delay or not allow light to be trapped, thus allowing the singularity to be visible.

The key point is the physically realistic nature of the gravitational collapse models being considered. Researchers have been very careful to ensure that all the physical regularity conditions, such as the energy condition and the regularity of the initial data, are well preserved. If that were not done, we would consider the work to be rather insignificant. It is because physical reasonability has been ensured that the gravitational collapse work of recent years has become interesting and worthy of attention, invoking international discussion on the fundamental issues of the final fate of gravitational collapse and the related cosmic censorship.

Our idea has been to clarify what specific general relativistic effects, as opposed to Newtonian ones, come into play in deciding whether a singularity is clothed. We want to make it very clear why naked singularities develop and what are the physical factors responsible. What is needed is more physical insight as to what happens and what is the physics of naked singularity formation. When a star which is inhomogeneous undergoes collapse, the trapping of light fails to happen early enough to cause the event horizon to form before the singularity. In that case, at least part of the singularity becomes visible. This is probably related to the salient issue of the collapse timescales and more investigation is needed for a better understanding.

In the specific examples that we considered for dust, the key general relativistic effect that comes into play is the spacetime shear, caused entirely by the inhomogeneities. Such shearing effects distort the spacetime geometry and avoid trapped surfaces, allowing light rays to escape. The key point in these cases is that the structure of an apparent horizon is radically affected and altered once inhomogeneity in density or pressure is allowed. In other words, the trapping of light is delayed as soon as we depart from the homogeneous collapse case we discussed here.

We note that compared to the life-cycles of stars, as shown in Fig 3.2, the collapse studies of recent years imply a paradigm change in our conception of what is possible as the final fate of a massive star's physically realistic gravitational collapse. While middle-mass stars, after their initial collapse, may settle as a neutron star after a supernova explosion, high-mass stars can become a black hole or a naked singularity after a continual collapse, depending on their internal configuration (Fig. 6.2).

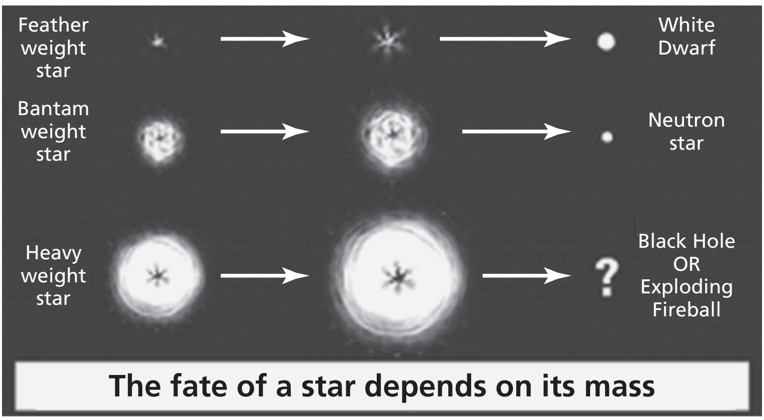

Figure 6.2 What is the final fate of a massive star? The current studies on gravitational collapse, and those conducted in recent years, point to a basic change in the perception on this issue. When a star is more massive than a critical limit so as to enter a continual collapse governed by the general theory of relativity, then its final fate can be either a black hole or a naked singularity, depending on the internal structure of the star.

Observational Aspects and Quantum Gravity

As the results discussed in the previous sections point out, when a massive star collapses continually toward the end of its life-cycle, the outcomes predicted by the general theory of relativity are either a black hole or a naked singularity, depending on the internal structure of the star.

Now, these are two rather different objects or happenings in the Universe, and their observational properties should be radically different from each other. The black hole is characterized by an event horizon, a one-way membrane that matter and light can enter but cannot escape from. On the other hand, a naked singularity is the super-ultra-dense visible region of Universe where densities, curvatures, and all other physical quantities take extreme values. Quantum gravity effects should be important closer to the singularity and govern the physics there, manifesting themselves in observations of such a region.

The key point we want to make is that the black hole and naked singularity outcomes of gravitational collapse have rather different observational signatures, through which it should be possible to decide whether a physically realistic collapse of a massive star will terminate into one or the other. There has been some recent progress in this direction in terms of the accretion disk properties for a black hole and a naked singularity, and also certain other possible observable differences may arise in each of these cases. We shall discuss some of these aspects later.

Also, when a naked singularity happens, small inhomogeneities in the matter densities very close to the singularity epoch can spread out and magnify enormously to create extremely energetic shockwaves. This, in turn, may have connections to extraordinary high-energy astrophysical phenomena in the Universe, such as cosmic gamma ray bursts, which we do not fully understand today. While these are possibilities at the moment and only few have been explored, the fact that general relativity predicts black holes and singularities as physically reasonable collapse outcomes gives enough motivation for further examination.

Furthermore, as we mentioned earlier, clues to constructing the quantum theory of gravity, a unified theory of forces of nature, may possibly emerge through observing such ultra-high-density regions in the Universe, if the collapse of a massive star resulted in a naked singularity. The crucial question then is will we be able to see such a cosmic dance drama of collapsing stars in the theater of skies when a visible singularity develops? Or will the black hole curtain hide it away forever, even before the cosmic play can barely begin? Only future observations of massive collapsing stars in the Universe will tell.

7

Cosmic Conundrums

There are many questions and cosmic puzzles that come up, both theoretically and observationally, if naked singularities do indeed form as the final fate of gravitational collapse of massive stars. In fact, some of these are already being discussed and investigated actively and vigorously. Such a scenario is natural as well as necessary, because we now have an intricate and important paradigm shift from the thinking that only black holes will form whenever a massive star undergoes gravitational collapse at the end of its life-cycle. Various questions have arisen and need to be examined carefully if we are to move ahead with the physical implications.

The point is, if cosmic censorship does not hold in an unqualified manner, and if both black holes and naked singularities can develop, then we essentially need to change our perspective on black hole physics and its numerous applications in relativistic astrophysics. We will discuss some of these issues in the present chapter.

Can We Reformulate the Censorship?

Further to the developments and new results discussed so far within the framework of general relativity, the key important question that again arises is can we really hope to formulate and prove the cosmic censorship conjecture (CCC) in one form or another? If we are to prove censorship, what are the possible directions and avenues still open? It has now become clear that imposing standard physical requirements such as regularity of the initial data or energy conditions is not really enough to guarantee the absence of strong-curvature naked singularities forming as end-states of collapse. Is there then a way of reformulating the CCC?

We are thus compelled to ask at this stage the following: Is CCC correct as a basic principle of nature? If Yes, how to realize this, and if No, what are the implications? Clearly, the answers have profound implications for fundamental physics and cosmology.

What is unambiguously clear so far is that censorship is certainly not correct in an unqualified form, as it is sometimes taken to be. It is also clear that singularities appear in a wide variety of forms, and from a wide array of scenarios in general relativity. Therefore, to rule out all naked singularities at once with a single cosmic censorship theorem does not seem possible. So if cosmic censorship holds, it will be in a highly refined and fine-tuned form, with suitable conditions, and in a formulation yet to be achieved.

Toward this end, the work on gravitational collapse completed so far has surely played an important role. The collapse models analyzed so far have provided us with important pointers as to what features must be avoided so that only black holes are created. On the other hand, if we consider naked singularities, investigating the quantum and astrophysical processes in the vicinity of such visible ultra-dense regions predicted by dynamical collapse models can give rise to intriguing physical consequences as we point out and discuss later.

The implications of these developments and the gravitational collapse studies for cosmic censorship are certainly important. We can now say with confidence that we cannot formulate censorship in a rather general way, such as 'Collapse of any massive star makes a black hole only' or 'Any physically realistic gravitational collapse must end in a black hole only', because there are now many counter-examples. It follows that any formulation for cosmic censorship must carefully specify when a black hole will develop in collapse. More specifically, we must examine the gravitational collapse scenarios carefully and isolate the features that cause a naked singularity to arise. Physicists believed for a long time that spherical collapse must yield a black hole only, and naked singularities arise, if they do at all, in non-spherical collapse only. We now know this is not the case, as shown by explicit collapse models. This also explains why our earlier efforts to prove a fully general CCC theorem did not succeed.

In other words, if we are considering the gravitational collapse of a massive matter cloud, there will be specific and fine-tuned conditions which we must set on the initial data, such as matter densities and pressure profiles, and the allowed dynamical evolutions of Einstein's equations, so that the collapse end-state will be a black hole only. The key aim is to ensure that in a continual general relativistic collapse, when a spacetime singularity develops, the event horizon will develop prior to the singularity.

As an example, we note the case of dust collapse. If the density does not increase from the center of the cloud outward (such an increase would be unrealistic), and we must get the black hole as the collapse end-state, then the density profile must be so fine-tuned that it is fully homogeneous at the initial epoch from when the collapse develops. Also, the velocity profile for the collapsing shells has to be so tuned, so that the density remains necessarily homogeneous at all later epochs. Only then do we get a black hole final state. In all other cases, a naked singularity develops. Similar fine-tunings will be necessary for more general collapse of different forms of matter, depending on the nature of the stress-energy tensor and the equation of state.

In such a scenario, a general statement for CCC seems very difficult or nearly impossible to make. The best we can do is to specify a set of conditions for a given collapse scenario so that it terminates in a black hole. It would then be natural and appropriate to accept that at the level of theory, both black holes and naked singularities do occur as the final fate of a massive matter cloud's continual gravitational collapse within the framework of general relativity. Faced with such a situation, we must look for possible alternatives which restore the cosmic censorship in some suitable form, while still respecting the collapse models investigated so far. This is an essential and inevitable exercise to place black hole physics on a sound footing.

We now mention a few possible alternatives in this direction, but the list is clearly not exhaustive and there may be other ideas worth exploring.

- *Can we specify the initial conditions in collapse whereby only a black hole will result as the final state?*

We may want to specify a set of conditions so that the evolving collapse from regular initial data for a given matter field has the horizon developing prior to the development of the singularity. While some general indications are available here for spherical collapse, any general formulation in this direction is far from obtainable as yet. It is clear from an astrophysical perspective that such an effort is crucial to ensure the validity and applicability of black hole physics.

- *Should censorship be valid only for vacuum general relativity?*

A radical proposal is to consider cosmic censorship in vacuum general relativity only, where the spacetime contains no matter fields at all. We can ask whether in a pure gravity framework, without any matter fields, cosmic censorship holds. If this can be achieved, then the idea would be to disregard any naked singularities arising in the gravitational collapse of matter clouds as 'singularities arising from matter' only, rather than from pure gravity itself, which would by itself obey the CCC. However, there has been no essential progress in this direction so far, and likewise on how to formulate such a statement in a mathematically rigorous manner and on how to proceed to any possible proof.

- *Should censorship hold only for select forms of matter fields?*

Another possibility is that cosmic censorship holds only for certain specific matter collapse models. This is an idea placed in the middle, in that we can allow for 'selective' or 'suitable' matter fields only, such as a Maxwell field or massless scalar field, regarding them as 'fundamental fields' in some sense. Then we could ask whether the censorship will hold for spacetimes with such matter fields.

The point here is to claim that fluid models, such as perfect fluids and dust, can be disregarded because they fail to give an account of the microscopic properties of matter. The naked singularities occurring in these collapse models can then be regarded as presumably due to the lack of a description of fundamental interactions, which will possibly resolve them at either a classical or quantum level.

This idea is in line with the possibility that either classical or quantum corrections would intervene to resolve the singularity. Nevertheless, considering only classical general relativity, the fundamental fields that can be studied in this respect are the electromagnetic and scalar fields, and naked singularities have been shown to occur in the massless scalar field collapse, as shown by the analytic studies of Christodoulou and numerical studies by Choptuik and others. However, it has been claimed that these are non-generic by a certain definition of genericity. On the other hand, a full quantum treatment of the last stages of collapse is missing at present, leaving such a reformulation of CCC only at the stage of a proposal and again a conjecture.

The basic point here is that these ideas lack any connection with the issue of horizon formation as the system dynamically evolves. In fact, if we agree to the idea that singularities must be resolved when we take into account the microscopic structure of matter, then there is no need to conjecture that they can sometimes arise hidden within a horizon and sometimes not. Then the whole issue of cosmic censorship becomes redundant.

Another important point, from an astrophysical perspective, is that disregarding all the matter forms such as dust and perfect fluids with different reasonable and physically well-motivated equations of state, which have been widely used and studied in astrophysical contexts, will also be far from widely acceptable. In fact, collapse models with matter fields such as these have been studied and applied in astrophysics for decades, and to rule them out just for the sake of a possible formulation of a hypothesis does not seem to be reasonable.

The key point here is that in the end the physical problem we need to investigate is that of the final fate of a massive collapsing star when it begins to shrink gravitationally. The massless scalar fields suitable for a cosmic censorship statement have not yet been observed in nature, and it may be unreasonable to assume that very massive stars are composed of or are dominated by massless scalar fields only, even in the later stages of their gravitational collapse. In fact, matter fields such as perfect fluids, radiation collapse models, and dust collapse have been considered by many researchers to be quite useful and important for studying the physical problem of a star's collapse. That is why so many studies of matter cloud collapse have been conducted over the past years.

- *Whenever naked singularities occur in collapse, must they always be non-generic and non-stable?*

With the collapse studies done so far and with the results that we have discussed, this appears to be the only plausible alternative which has some potential for realization. We will discuss this issue in some detail in the next section.

Are Naked Singularities Stable and Generic?

While Chandrasekhar pointed out in the early 1930s that a massive star's final fate will be radically different from that of a small-mass star, Oppenheimer, Snyder, and Datt (OSD) studied this problem in the latter 1930s using the general theory of relativity. As we have emphasized, it was crucial to use the full general relativity. They found that the idealized uniform star without pressure collapsed to a black hole, which is a result fundamental to modern-day black hole physics, laying the foundation for current studies on collapse.

Further to OSD's work, Einstein was greatly interested in the issue of the final fate of a massive gravitationally collapsing star, but he could not accept such a final fate as a black hole. It was later clear, however, that his method of halting the collapse did not work. Continual collapse is inevitable for massive stars, and so we are squarely faced with the problem of their final state.

The conundrum lay dormant till about the latter 1960s when Hawking, Penrose, and Geroch took up a detailed study on global features of spacetimes which implied that singularities are inevitable in physically important situations of collapse and cosmology. The issue of singularities of collapse being covered by event horizons was then settled only through the assumption of cosmic censorship. But with no proof for CCC even after decades of effort, it was clear that a careful and detailed study of collapse was imperative, as we have described in previous chapters. Clearly, the issues at stake have been enormous for astrophysics and cosmology, with far-reaching consequences for fundamental physics.

If we can show that all naked singularities are non-generic or not stable in some suitable sense, then that will help formulate the cosmic censorship. We can then try to formalize this in some suitable manner as a rigorous theorem eventually.

In view of the gravitational collapse models worked out so far, this is one of the most important questions to ask. Because if we can conclusively show that whenever naked singularities develop in collapse, they must be non-generic or unstable in a suitably well-defined and proper mathematical sense, then that will provide a good basis for a possible formulation and proof for CCC.

Early Efforts to Show Non-genericity

It was suggested that when matter satisfied some suitable physical conditions, such as positive energy density for the classical matter of which a normal star is composed, then all naked singularities forming in the collapse will be avoided and the outcome will be only black holes, in conformity with the CCC. If preliminary efforts support such a concept, then the idea would be to formulate a theorem to that effect and try to prove it rigorously, which then becomes a censorship theorem or a proof for CCC. This would be the same as saying that while naked singularities may form for arbitrary matter, they will not form for physical fields that satisfy energy conditions, and are therefore non-generic in that way.

It turned out, however, from the example of inhomogeneous dust collapse and the Vaidya models for collapse of radiation shells, that such a formulation does not work. Naked singularities do, in fact, develop in collapse for matter where energy conditions ensuring positive mass-energy density are satisfied.

Another important effort to show the non-genericity of naked singularities was to try and prove that even if naked singularities did occur in gravitational collapse, they must all be necessarily 'gravitationally weak', and hence non-generic in this sense. Such a weak singularity is only a mathematical feature in the solution to the Einstein equations. But it will not be there in physical situations, because the spacetime can be extended through it.

From such a perspective, the Lemaitre–Tolman–Bondi (LTB) collapse models were analyzed in great detail. The general dust collapse models and the equations they contain are rather complicated. The computer simulation work of Doug Eardley and Larry Smarr in 1979 indicated that the collapse of these more general LTB models may produce a naked singularity. An analytic treatment was later given by Christodoulou in 1984. However, Richard Newman showed in 1986 that these naked singularities were in fact gravitationally weak, and therefore possibly removable from the spacetime.

Then in the 1980s Newman and Krolak tried to prove theorems to establish the weakness of singularity. It was this scenario that drew attention to the earlier work by George Lemaitre and Richard Tolman, who had worked out a fully general solution to Einstein's equations for the dust form of matter in the early 1930s. The OSD model was a homogeneous dust collapse model. Could then the final fate of collapse be worked out even when inhomogeneities were allowed by using the more general solutions given by Lemaitre and Tolman? These solutions were basically in a cosmological context. Hermann Bondi in the latter 1950s recast the equations in a format so that the collapse aspect of these models was better highlighted. Thus, these collapse models came to be generally known as the Lemaitre–Tolman–Bondi models.

However, other researchers such as Kayll Lake and collaborators and our work looked for collapse models which produced strong-curvature naked singularities. These efforts succeeded and a fully general analysis of dust collapse was finally developed. Another spacetime geometry that greatly helped us to understand better the nature of singularity during collapse was the Vaidya geometry that described the collapse of radiation shells. This metric was found by Prahlad Vaidya in 1942 in the context of generalizing the vacuum Schwarzschild metric to that for a radiating star geometry.

Detailed analysis over the past couple of decades of the spacetime geometries for inhomogeneous dust using the LTB metric and for radiation shells greatly helped us to arrive at an understanding of the nature of singularity in general relativistic gravitational collapse. It is the collapsing star's internal structure, such as its density and pressure profiles, and various factors like the speed of collapse, the shearing forces inside, etc. that determine the final outcome of collapse in terms of either a black hole or a naked singularity, and determine the strength of the singularity. The study of Vaidya and LTB models opened up the study of many more physically realistic collapse models with non-zero pressures.

There were other similar efforts to show the 'non-genericity' of naked singularities in different manners; all of these failed in one way or another, but we will not go into detail. These in a way again point out how efforts to arrive at a suitable mathematically rigorous statement on cosmic censorship have not worked out so far.

Such efforts, however, have focused our attention on the question as to what genericity in general relativity really means. A naked singularity may occur for a particular physically relevant scenario, but the question that has been always asked is how 'generic' is such a phenomenon. Of course, genericity or stability have no specific definitions in general relativity. So the same question has taken many different forms, as we will discuss next.

What are Genericity and Stability?

In his original statement of the CCC, Roger Penrose included a condition, namely that 'generic' gravitational collapse always produces black holes. As such there is no mathematically well-formulated definition of genericity available in general relativity that can be used to answer this question. Stability and genericity are extremely difficult concepts to formulate precisely in general relativity, with no definite criteria available. There are mathematical complexities and physical issues in achieving the same, especially in gravitation theory.

That is why, as of today, these remain more general ideas and perceptions, and physicists use them differently in different situations as per the demand of the physical scenario concerned. This is certainly why we do not have any suitable mathematical formulation of cosmic censorship as yet. So the only way to proceed is by asking further questions, such as is there any perturbation of spherical symmetry that would remove these naked singularities, will naked singularities form in non-spherical collapse, and so on.

We are interested in this issue in the context of gravitational collapse outcomes. While it is generally accepted that both black holes and naked singularities do form as the final fate of a continual gravitational collapse, what is not yet clear is the distribution of these outcomes in the space of all allowed outcomes of collapse. The collapse models discussed here and the considerations we give may be of some help and may throw light on the distribution of black holes and naked singularity solutions in the initial data space.

The important question then is whether the genericity and stability of such naked singularities arise from regular initial data. Will the initial data subspace, which gives rise to naked singularity as the end-state of collapse, have zero measure in a suitable sense? Based on such a criterion, we would be able to reformulate the censorship hypothesis more suitably.

To be specific, the major difficulty in obtaining a unique and well-posed definition for genericity comes from the non-uniqueness of the concept of 'nearness' itself in a given spacetime continuum. Mathematically, this is characterized by the 'topology', or the 'open sets' of the spacetime. For example, in order to quantify when two spacetime geometries are 'nearby' or 'close' to each other, we can define a certain topology of the space of all spacetime metrics by requiring that the values of the metric components be 'nearby' in their numerical values. On the other hand, we can also require that the derivatives or the rates of change of these metric functions also be nearby in value. The problem is that the resulting topologies in each case will be quite different, and so the 'nearness' in one topology will not be so for another. This fact is intimately connected to the basic problem of arriving at a well-formulated definition of cosmic censorship itself.

From such a perspective, studies have been carried out on the collapse of a massless scalar field. For example, Christodoulou showed that if we consider self-similar massless scalar field collapse, the initial data leading to naked singularity have a 'positive codimension', in a certain space of initial data. This led to the conclusion that the occurrence of naked singularity in this case is not generic. On the other hand, Ravi Saraykar and others showed that the naked singularity occurrence is generic for certain types of matter fields such as inhomogeneous dust. Therefore, the issue of genericity and stability of naked singularities remains wide open even for spherically symmetric models, and more so if we wish to consider departure from spherical symmetry, with different forms of matter field.

If we want to explain this difficulty in another way, we need to adopt some definition for stability and genericity for collapse outcomes. This will always need reference to a specific space of initial data that is being considered. What is 'stable' or 'generic' in a certain space of initial data, say for that of an inhomogeneous dust, need not be stable in a larger space of data, for example that of the perfect fluid collapse.

To give an analogy, this is similar to the statement that a point or a line interval are both 'non-generic' and 'zero-measure' sets in a plane, but the interval is certainly 'generic' in a line, with a non-zero measure therein. Similarly, the plane is non-generic in a three-dimensional space. What this means is that the context of the overall space is important when we consider the issues of genericity and stability. This is frequently not mentioned while making statements about the non-genericity of naked singularities.

In summary, essentially, we say that a certain outcome of collapse in terms of either black hole or naked singularity from a given set of initial data is stable if there exists a whole neighborhood of that initial data set which leads to the same outcome. The neighborhood must be suitably defined within the specific space of initial data that we are considering. Also, we say that a certain outcome is generic, within a specific space of initial data, if the measure of the subset of initial data leading to that outcome in the original initial data space is non-zero. Again the measure must be defined in a suitable natural way.

The Current Scenario

With these definitions, we investigated the initial data sets leading to black holes and naked singularities for different types of general matter models, including dust and perfect fluids. We showed that the initial data set leading the collapse to a naked singularity, just like that leading to a black hole, forms an open subset of a suitable space comprising the initial data, with respect to an appropriate distance function. Also, when considering the measure theoretic aspects of this open set it is seen that it has a positive measure. Thus the basic result that follows is that both black holes and naked singularities are 'generic' outcomes of a complete gravitational collapse, once genericity is defined in a suitable sense in an appropriate space of initial data.

So the models of gravitational collapse analyzed so far are generic in the sense of initial data for matter being fully generic as required. The results also show that naked singularities are, in fact, stable to small perturbations in the initial data of matter fields, to the introduction of non-zero pressures in the cloud, and so on. Actually, we have yet to find and isolate the kind of perturbation that would make a given naked singularity go away. These situations are what physicists call 'generic', that is, they are not contrived. A tiny deviation in the initial data leads to much the same outcome. However, we should emphasize that a general 'stability' proof for the naked singularity remains to be achieved. As mentioned, that must wait till a proper stability criterion is formulated in general relativity, which is not available as yet and which is a rather complex issue.

In this connection, the following recent result that has come out of collapse studies throws a useful light on the issue of genericity and stability of collapse outcomes: for a spherical gravitational collapse of a general matter field, satisfying the energy and regularity conditions, given any regular density and pressure profiles at the initial epoch, there always exist classes of velocity profiles for the collapsing shells and dynamical evolutions as determined by Einstein's equations, such that depending on the choice made, the collapse final state is either a black hole or a naked singularity (Fig. 7.1).

As for perturbations of initial conditions, we could ask what if we perturb an already-formed naked singularity. The above tells us that if a particular initial configuration in collapse gave rise to a naked singularity, there is an open set of nearby initial data that will give the same result. This kind of stability offers a certain physical reality to naked singularities.

The inhomogeneous dust collapse case we discussed above is a special case of the scenario stated earlier. In that case of course, the conclusions are much more transparent. What determines fully the final fate of collapse are the initial density and velocity profiles for the collapsing shells of matter. We can see here clearly how the different choices

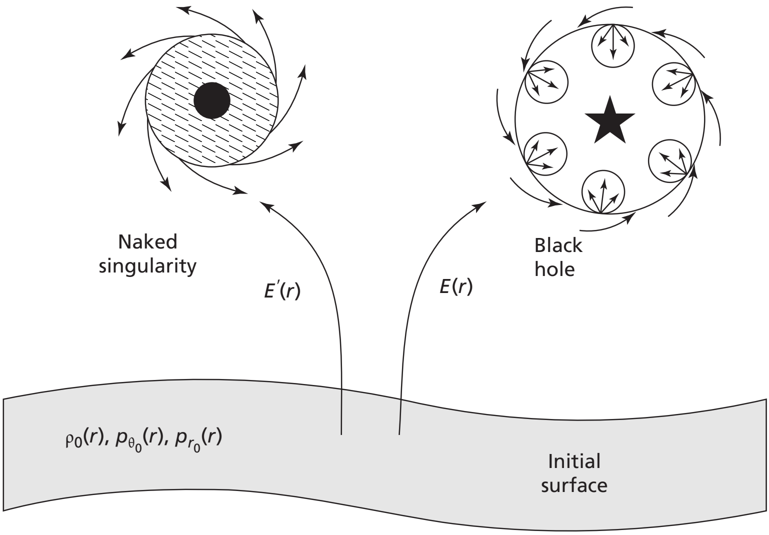

Figure 7.1 How initial data takes the collapse to either a black hole or a naked singularity. The above is a schematic diagram for the evolution of spherical collapse for a generic matter field, with inhomogeneities as well as non-zero pressures included. Here the quantities ρ_0, p_{θ_0}, and p_{r_0} specify the initial data set in terms of the density and pressure profiles and $E(r)$, $E'(r)$ are possible velocity profiles. The typical result is that, for any given matter initial profile, there are velocity profiles that take the collapse to either the black hole or naked singularity outcome, depending on the choice made, and vice versa.

of these profiles for the collapsing cloud determine the two final states of collapse, and how each of the black hole and naked singularity states turns out to be 'generic' in terms of their being distributed in the space of final states.

The result we have in this case is that given any regular initial density profile for the collapsing dust cloud, there are velocity profiles that take the collapse to a black hole final state, and other velocity profiles that take it to a naked singularity final state. In other words, the overall available velocity profiles are divided into two distinct classes, namely those which take the given density profile to a black hole, and those that take the collapse evolution to a naked singularity. The same holds conversely; namely, if we choose a specific velocity profile, then the overall density profile space is divided into two segments, one taking the collapse to black hole final states and the other taking it to naked singularity final states. This clarity of results give us much understanding on the final fate of a collapsing matter cloud.

While dust collapse is very useful to investigate, much more interesting is the collapse with non-zero pressures which are important physical forces within a collapsing star. We briefly mention below a typical scenario of collapse with a non-zero pressure component.

For possible insight into the genericity of naked singularity formation during collapse, we studied the final outcomes of collapse once pressures were introduced in the collapsing cloud. Specifically, we investigated the effect of introducing small pressure perturbations in the collapse dynamics of the classic OSD model that terminates in a black hole final state, which would hopefully tell us about the OSD black hole's stability. What we found was that there exist classes of pressure perturbations such that introducing the smallest pressure within the collapsing OSD cloud changes the end-state to the formation of a naked singularity. This can also be viewed as a first-order perturbation of the known spacetime metric describing the dust cloud. Thus, we come to understand the role played by pressures in a well-known gravitational collapse scenario, and a specific and physically reasonable but generic enough class of perturbations is considered to provide good insight. From this analysis we gain useful and important clarification of the structure of the censorship principle, which as yet remains to be properly understood.

In light of this discussion, it is worth noting that in recent years there have been many studies on the stability of black hole solutions. In particular, the instability of Reissner–Nordström black holes has been focused on, which are of interest because the extremal black holes with equal charge and mass are of special interest in higher-dimensional and string theories. Interesting instabilities for Reissner–Nordström black holes in de Sitter backgrounds have been reported by Vitor Cardoso and colleagues both numerically and analytically; gravitational instabilities in Kerr spacetimes have also been studied.

It follows that gravitational collapse is a rather intriguing phenomenon. While in Newtonian gravity, it is density or mass that determines the interaction, in Einstein gravity the spacetime curvatures and causal structure play a major role. Event horizons may or may not form as the mass densities grow during the collapse. As a consequence, many intriguing features result. The connections and implications for any possible quantum gravity formulations are puzzling and rather exciting.

It is worth noting that the issue of the final fate of massive stars and that of gravitational collapse has attracted the attention of the best minds. The discussion and debate have been ongoing for many decades and many research groups have participated internationally. That is because of the extremely rich tapestry of basic ideas interwoven here, and because of the issue's importance in astrophysics and cosmology.

Structure of Naked Singularities

Very little is known about the structure and properties of spacetime singularities. But if they are always hidden within the event horizons of gravity, we are probably not much bothered about them as we shall never encounter them. However, if we accept that physically realistic gravitational collapse gives rise to naked singularities under a wide spectrum of reasonable conditions, then the important question that arises is: What are the properties and structure of these naked singularities? How visible singularities affect the surrounding Universe will crucially depend on this. The answer must come, of course, from the gravitational collapse studies carried out so far.

The collapse models studied and the resulting singularities exhibit a wide variety of interesting structure. We need to try to obtain their common features, and determine implications for as well as connection to the CCC. In some models, part of the singularity is visible but then is covered by the horizon eventually; elsewhere, they can remain visible forever. This depends on the form of matter collapsing, the equation of state used, and other properties. Typically, the naked singularity develops in the geometric center of collapse to begin with, but later it can spread to other regions, or get covered, as the collapse progresses.

Is a Singularity's Mass Always Vanishing or Negative?

A question is asked sometimes about the mass of naked singularities, namely if they are always 'massless', because typically in spherical collapse the naked singularity forming at the center of the cloud has its mass function tending to a vanishing value in the limit of approach to the singularity.

It is useful and interesting to note, however, that this need not always be the case. For example, in higher-dimensional gravity researchers found timelike naked singularities developing, which are massive both within Einstein theory and in alternative theories of gravity. Massive singularities are present in solutions of Einstein's field equations in classical relativity as well, the most famous probably being the one appearing in the Kerr solution, where it has a positive mass and is naked if the object is rotating sufficiently fast, higher than a critical value. However, at present we do not know whether such a Kerr naked singularity can form via some physically viable dynamical collapse process.

In any case, in our opinion what really matters when a singularity is visible is that super-ultra-dense regions where the densities and curvatures do blow up are visible to external observers in the Universe. That is of actual physical consequence, rather than the 'mass' of the classical singularity itself, which is, in fact, not an object or part of spacetime. Since the naked singularity itself may be eventually resolved when quantum gravity effects are taken into account, talking about the 'mass' of the classical singularity may not be of much physical significance in the final analysis.

Given the examples of massive timelike singularities and others such as those just described, and also other examples of collapse evolutions determined so far, it is clear that naked singularities, when they form during collapse, come in many varieties and with varying properties, depending on the physical situations as well as the form of matter considered and other such factors. This may be one of the reasons why it has not been possible so far to rule them out by some kind of general theorem. Therefore, in our view, it is very likely that imposing different restrictions will not help preserve the CCC. As we noted earlier, imposing ad hoc conditions of various types has not helped recover the censorship over the past many years, and we can always find counter-examples that do not respect these.

Also, there has been some discussion on negative mass Schwarzschild singularities. It has been asked whether naked singularities, whenever they form, must have a negative mass similar to that in the Schwarzschild case. As we clarified earlier, that is not the case. Generically, if we take the case of a spherically symmetric collapse, the mass function has a vanishing or positive value in the limit of approach to the singularity which is visible.

In fact, it is desirable to impose all possible physical reasonability conditions such as positive mass-energy density and regularity of the initial data from which the collapse evolves. Only after that can we ask whether naked singularities still form. The idea is to see whether naked singularities develop during gravitational collapse that is developing from regular initial data and under physically reasonable conditions. The answer that follows is in the affirmative, as shown by many studies we discussed earlier. These singularities have no negative mass. We note that constructing models with naked singularities which have a negative mass is, of course, possible, but the physical validity of the same is far from clear. In fact, the negative mass Schwarzschild singularity has not been obtained as a result of any dynamical collapse evolution, and to that extent it is not physical.

Are Singularities Always Momentary Events?

Are naked singularities always pointlike in time, or can they be also extended? This is partly related to whether they are like an object or an event. In typical spherically symmetric collapse models, when considered in co-moving coordinates, the first point of the singularity curve is visible. From it, families of non-spacelike curves come out. The later points in co-moving time, of the singularity curve, get hidden under the horizon.

Thus, we must ask whether naked singularities, when they form, are always momentary or can they be extended in time? We point out here that depending on the collapse scenario, the form of matter, and the equation of state considered, the naked singularity can be pointlike, or it can also extend in the co-moving time. Naked singularities can be 'timelike' or 'null', and it has been determined that the timelike singularities, when they form in collapse, are extended typically.

Whether singularities are pointlike or null, the structure of the families of geodesics coming out from the singularity have been analyzed in detail. It has been shown that non-zero measure of non-spacelike curves emerge from a pointlike naked singularity. As far as the distant external observer is concerned, once he gets to see the first ray of the singularity, for all times to come the null or timelike paths from the singularity will keep reaching him. This is also related to whether naked singularities are always null. As we have pointed out, they can be timelike and extended through the space, rather than being null and pointlike.

We note that, as opposed to naked singularities developing during collapse, which are sometimes like an event, those occurring during the super-spinning Kerr geometry or many other vacuum models are ever-lasting, and in that sense they are like an object. Although such solutions occur in general relativity, till recently what was not clear is whether such object-like naked singularities arose from dynamical physical processes in gravity physics. In this connection, we have discussed a class of gravitational collapse models which gives rise to the final configuration which contains an ever-lasting naked singularity that develops during collapse from regular initial data.

Do Naked Singularities Violate Causality?

An issue sometimes mentioned in connection to the occurrence of naked singularities is the possible violation of causality that can happen in certain spacetimes. Causality violation in a spacetime amounts to having closed timelike curves. An observer moving on such a trajectory would find herself eventually at the initial event in space and time even though locally the clock never went backward.

The most well-known examples of such spacetimes are the Gödel universe and the solution by Frank Tipler for a spacetime outside a rotating cylinder. Also, closed timelike curves can be present in familiar solutions such as the Kerr metric. The singularity theorems show that singularities form generically in Einstein gravity when causality is protected. But these theorems are not very helpful when exploring the connection between causality violations and singularities.

The closed timelike curves are present in Kerr spacetime, but they are confined inside the horizon of the Kerr black hole. Therefore, an observer entering the horizon would be able to travel back in time but would not be able to return to the outside Universe. Thus, the causal structure of the external spacetime is preserved. In some cases, closed timelike curves can be accessed by any observer in the Kerr metric, allowing therefore the theoretical possibility of time travel.

From this example, it may be tempting to think that naked singularities would typically allow for causality violations to occur in the Universe, thus making them undesirable. It is possible that the presence of the horizon would hide the closed timelike curves, thus disconnecting them from the rest of the Universe. Although this is true in some cases, there are also dynamical spacetimes such as those of the LTB models which admit naked singularities but which have no closed timelike curves. It follows that, in general, there is no direct connection between these two phenomena of causality violation in a spacetime and the occurrence of naked singularities. Actually in known spherical collapse models the causality is always preserved whenever the naked singularity results as the collapse final state.

We note that even when singularity theorems assume certain causality conditions, the spacetime singularities and causality violation are two independent phenomena. Therefore, in general, the connection, if any, between spacetime singularities, and in particular naked singularities and causality violations is far from clear and could be rather subtle. One of the reasons could be that different kinds of naked singularities need to be treated differently. For example, what holds true for the Kerr solution need not be true for the Lemaitre–Tolman–Bondi metric. A spacetime without rotation, such as the LTB collapse model, could not have closed timelike curves and would therefore respect causality regardless of the presence of a naked singularity.

It also appears that causality violations need to be better defined and studied in order to understand what is really undesirable about them. The possibility of closed timelike curves by itself could not be enough to make a spacetime ‘physically unrealistic’. If we try to summarize, causality violations can occur at three levels. First, they could be microscopic, in which case closed timelike curves can occur at that level, thus being resolved or included by an eventual theory of quantum gravity. Second, there may be local scale causality violations, in which case the closed curves can occur at planetary or galactic level. This gives rise to the possibility of time travel with all the connected paradoxes. Finally, the causality violations could be at a cosmological scale. In that case these can occur only on time scales comparable with the life of the Universe and thus have little bearing on our local picture of the Cosmos. These closed timelike curves probably cannot be ruled out in principle.

As noted by many researchers, global causality requirements for the whole Universe might be too restrictive since we are able to experience only a limited and local portion of the Universe. From the fact that causality holds here and now, it might be far-fetched, though not impossible, to conclude that causality holds globally for the entire Universe. Also, non-local correlations have been studied in quantum mechanics for many years and it seems not impossible that at a microscopic level a full theory of quantum mechanics coupled to gravity might allow for some kind of causality violations to occur.

So finally we may be left only with the second class of causality violations described above, namely the local scale violations, to be considered as undesirable in principle. Within this class, can we say that naked singularities and closed timelike curves are linked in some way? As was pointed out by Bill Bonner, examples of spacetimes without singularities but with closed timelike curves have been known since the early days of general relativity. But such solutions cannot be obtained from the evolution of a regular matter cloud. As such, the possible relation that might link the occurrence of closed timelike curves with the behavior of the event horizon that could eventually cover the singularity has not been investigated so far, leaving at present the issue of causality violation separated from cosmic censorship.

Local versus Global Visibility

When a matter cloud collapses, an event horizon might form, covering the singularity. But rays may still come out of the singularity, though staying within the event horizon only, and then fall back into it. Such a singularity is called a *locally naked singularity*. On the other hand, if the rays coming out from the singularity reach observers far away in the Universe, then it is called *globally naked*.

In other words, a singularity is said to be at least locally naked if there exist outgoing null or timelike trajectories that reach some observer in spacetime. In this sense, for example, the singularity in the Reissner–Nördstrom spacetime is locally naked, since observers located within the radius of the event horizon, but not outside it, can be reached by the non-spacelike geodesics coming from the singularity. On the other hand, a singularity is said to be globally visible if there exist outgoing non-spacelike geodesics that reach observers at future spatial or null infinity. In this case no horizon is present before the singularity, and the light rays coming from the singularity can be seen by faraway future observers (Fig. 7.2).

Global and local visibility are, of course, related to cosmic censorship. Cosmic censorship comes in various forms and formulations and over the years certain forms which allow for the existence of locally visible singularities have been proposed. There are two main formulations of the conjecture, *strong cosmic censorship* and *weak cosmic censorship*, and which form we assume will have implications for the local and global properties of the spacetime to be studied. The weak cosmic censor postulates that a singularity cannot be seen by observers at null infinity,

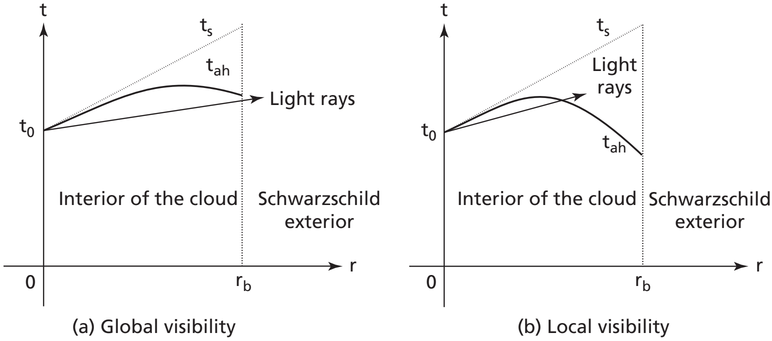

Figure 7.2 Local versus global visibility of the naked singularity. (a) shows the globally visible singularity, and (b) the local visibility. Basically, whether the singularity will be locally or globally visible is decided by the structure of the apparent horizon, which is the boundary of trapped surfaces in the spacetime. The rays may come out of the singularity, but if they enter the apparent horizon before they come out of the cloud, then they are trapped and fall back again to the singularity. On the other hand, if the horizon structure is such that the escaping rays need not enter the same, then they move out and reach the faraway observers in the spacetime.

thus allowing for locally naked singularities to occur. In this case when we study formation of the singularity and of the apparent horizon, we need not worry about the global structure of the horizon itself. Actually, proving that there are future-directed outgoing null geodesics coming from the singularity is enough to ensure local visibility.

As an example, we could consider the LTB collapse scenario. We indicated earlier that the local visibility of a singularity is determined by the density being inhomogeneous and higher at the center. This holds regardless of where the boundary of the cloud is taken. On the other hand, a careful analysis of the behavior of the apparent horizon shows that for some of these matter models the horizon curve, while increasing in a neighborhood of the center, can decrease afterward. In that case, the singularity is eventually covered from the point of view of observers at future infinity.

It has been shown that for a fixed position of the boundary of the star, there is a range of values of the degree of inhomogeneity for which the singularity is only locally naked and a range of values for which it is globally naked. From a mathematical point of view, this is not relevant, since in dust models we always have the freedom to choose the boundary of the star as we like. Therefore for any given matter profile we can always ensure global visibility by choosing the boundary at an appropriate radius.

The issue is somewhat different when we apply the formalism to collapsing stars with non-zero pressures. First, the pressures should typically vanish at the boundary, and this may affect the way the boundary and the mass profile are related. Furthermore, in this case the total mass and the radius of the boundary are fixed by realistic physical values for a star, depending on the stellar model chosen. It can then very well happen that even though the singularity that forms at the end of collapse is locally visible, it might not be globally visible. Based on these considerations, we can see that for more elaborate and realistic matter models, which include pressures and different layers of matter, the issue of global visibility for a given collapse model remains open.

Can Energy Come out of a Naked Singularity?

Even if a naked singularity forms in collapse, the singularity by itself does not necessarily have to radiate matter or energy, thus bearing a signature that is detectable from far away in the Universe. Such singularities at the end of collapse, unlike those in the super-spinning Kerr metric, are more like events rather than an object, being a moment when collapsing matter reaches its final doom, something like the Big Bang in reverse. Questions such as what will come out of a naked singularity are then not really meaningful; 'things' do not have to come out of it. What we really see is not the singularity itself but the signature of processes that occur under extreme conditions of matter near this epoch, such as the shockwaves due to inhomogeneities in this ultra-dense medium, or possibly the quantum gravity effects in its vicinity.

Generally, when considering a complete gravitational collapse, it is often assumed that the boundaries set by the electromagnetic and nuclear forces are surpassed and nothing can really halt the collapse. Therefore, in the ultra-high-density regions that surround the singularity, the pull of gravity is thought to be so strong that nothing can escape it. We would be inclined to think then that even if naked singularities exist, they would have no impact on the rest of the Universe, since no signal can escape their gravitational field. But there are other ways by which naked singularities could, in principle, leave a trace in the outside Universe. From the study of exact solutions of the field equations we see that there are scenarios in which the process of formation of the singularity can be accompanied by an emission of energy.

For example, it is plausible that the final singularity may not actually occur because some physical processes could disrupt the singularity formation and instead create a shockwave through which the matter-energy that was collapsing is ejected away. Such a shockwave would be immensely energetic and it would bear a clear signature of the level at which it has arisen.

This raises the issue of possible observational features that a naked singularity might have. Have we observed similar phenomena in the Universe already? The answer is, we do not know as yet. Since the stellar structure in realistic cases is much more complicated than in the idealized analytical models and because it involves many layers of different materials with different properties, it is clear that whatever might come from such a Planck-scale event would be scattered, absorbed, and emitted many times before an actual signal comes out at the surface of the star. Therefore, of the many highly energetic events known to happen when a star collapses under its own gravity, we do not know whether any of them bears the signature of the Planck-scale physics happening very close to the center.

For example, we as yet do not have a comprehensive model that explains how supernovae explode. As we have discussed, computer simulations describing what happens at the core of the star when it explodes have difficulty producing the amount of energy necessary for the shockwaves to propagate through all the layers of the star, thus generating the explosion. It seems not unreasonable to consider the possibility that a barrier at a scale smaller than the Schwarzschild radius might provide this missing energy, though there are still no studies in this direction. A similar reasoning might apply also to gamma ray bursts, a very energetic phenomenon created from the core collapse of a star that is still far from being well understood. It is possible that the gamma ray bursts are created in the exploding outer layers of the star, and therefore it is not a direct effect of some possible quantum barrier. Nevertheless, the mechanism by which the explosion is related to the collapse of the inner core is still not well understood and it is likely that certain kinds of these phenomena are linked to gravitational collapse.

On the other hand, it is also possible that some energy comes out from the vicinity of singularity by classical effects only. In fact, if we allow for negative pressures to occur during collapse at a purely classical level, we find immediately that the mass of the collapsing star must be radiated away during the process. Negative pressures close to the formation of the singularity could expel the inner shells whose particles would then collide with the infalling outer shells with very high energies. Particle collisions would then occur with arbitrarily high center of mass energy, thus turning the collapsing cloud into an immense particle accelerator. Such a scenario was considered by Mandar Patil, Daniele Malafarina, and myself.

Questions on Collapse and Singularities

Apart from the several major issues on gravitational collapse outcomes that we have been discussing, there are also important questions and intriguing curiosities that arise when we discuss black holes and naked singularities. The point is that as collapse models are being studied for predicting the formation of visible singularities, different points arise about their nature, structure, and properties. These need to be discussed in order to bring clarity and transparency to the issues involved and we consider below some of these questions.

Is Cosmic Censorship Right or Wrong as a Basic Principle of Nature?

Clearly, the answer to this crucially important question has profound implications for fundamental physics and cosmology. What comes out unambiguously from the work so far is that censorship is certainly not correct in an unqualified form, as it is sometimes taken to be. If true, it will be in a highly refined or fine-tuned form only, with suitable conditions and a formulation that has yet to be achieved. The work on gravitational collapse so far will play an important role here to achieve such a likely formulation. On the other hand, investigating the quantum and astrophysical processes in the vicinity of such visible ultra-dense regions predicted can give rise to intriguing physical consequences. In fact, many physicists have taken this path in recent years.

We note that it is not just the curvature values alone that determine the event horizon formation in general relativity. It is the light cone structure, or the causal structure of spacetime that counts crucially. The causal structure and global properties mainly arise due to the nonlinearity of Einstein's equations and they determine features such as event horizon formation, many times independently of curvatures. So in collapse models, depending on the physical properties within the star, the event horizon may develop earlier or later, and the singularity may or may not be visible.

What Then is the Status of Naked Singularities versus Censorship Today? Can Cosmic Censorship Survive in Some Limited and Specialized Form, and Can We Properly Formulate It after All These Studies in Recent Years on Gravitational Collapse?

This is a major puzzle today that we discussed in some detail earlier. Recent studies on formation of naked singularities as collapse end-states for many realistic models have brought to the forefront intriguing basic questions at both the classical and quantum level, which may have significant physical relevance. Some of these are: If the super-ultra-dense regions forming in a physically realistic collapse of a massive star are visible to faraway observers in spacetime, what are the observable astrophysical consequences? What is the causal structure of spacetime in the vicinity of a singularity, as decided by the internal dynamics of collapse which evolves from regular initial data? How early or late will the horizons develop in a physically realistic gravitational collapse, as determined by the astrophysical conditions within the star? When a naked singularity forms, is it possible to observe the quantum gravity effects taking place in the ultra-strong-gravity regions? Can we envisage a connection to observed ultra-high-energy phenomena such as cosmic gamma ray bursts?

A continuing study of collapse phenomena within a general and physically realistic framework may be the only way to answer these issues. This could lead to novel physical insights and possibilities emerging from the intricacies of gravitational force and the nature of gravity.

The useful feature that has emerged from work on collapse models so far is that we now have several important constraints for any possible formulation of censorship. It has been shown that several versions of censorship proposed earlier will not hold due to explicit counter-examples becoming available. Clearly, analyzing gravitational collapse plays a crucial role here. Only if we understand why naked singularities develop as collapse end-states in many realistic models can there emerge any pointer or lead to a practical and provable version of censorship.

While General Relativity May Predict, in Principle, the Existence of Both Black Holes and Naked Singularities, What Would a Physically Realistic Continual Collapse of a Massive Star in Nature End up with? Will It Always Create a Black Hole?

The results available so far on collapse show that the final outcome will depend on the internal structure of the star, such as its initial density and pressure profiles, and the velocities of the collapsing shells. For example, in the dust collapse, the homogeneous collapse will go to a black hole, but a physically more realistic inhomogeneous profile with density higher at the center will result in a naked singularity. This means that, under physically reasonable conditions, a massive star is more likely to form a naked singularity. Taking into account quantum effects could then make the star explode, as we will discuss later. So a revision in scientific thinking could occur with more data.

The point is that there exist interesting and clean systems of collapsing configurations worked out in past years where the event horizons are delayed as the matter cloud collapses, respecting the usual physical reasonability conditions. In such scenarios, we have no reason to say naked singularities are unphysical.

What is the Nature and Structure of Spacetime Singularities?

Apart from the existence of spacetime singularities, we currently know very little about the global structure of Einstein's equations, and about the nature and structure of singularities. Singularities mean the trajectories of free-falling material particles or photons will come to a sudden end and just disappear from the spacetime. For particles, this will happen after a finite proper time as measured along their timelike trajectory. But we have no information about the nature or the physical significance of the singularities. For example, we do not know whether the densities and curvatures will necessarily blow up along the paths falling into such singularities, though there are many examples where in fact they do. We also do not know whether they come covered within horizons of gravity.

How Important Are Inhomogeneities, and at What Scales Would the Universe Attain Homogeneity?

Inhomogeneities in matter distribution are important for both the larger scales and small scales in the Universe, as this is an observed phenomenon. Recent observations of structures in the Universe show that it would be inhomogeneous at the very large scales of even hundreds of megaparsecs. Even though it is conceivable that homogeneity is again recovered when the averaging is done on a still larger scale, which is statistical homogeneity, it would be highly desirable to incorporate features such as perturbations from exact homogeneity. Toward this purpose, we could model the observable Universe in a more general way, where the spacelike surfaces of constant time need not admit the exact symmetries of homogeneity or isotropy. Recently researchers have found that such an effort to include inhomogeneities in cosmological considerations can provide new and alternative solutions to the dark energy problem in cosmology.

For local objects such as stars, as we have already discussed, the inhomogeneities in the density distribution have far-reaching consequences as far as the black holes and naked singularity outcomes of collapse are concerned.

Can We Obtain a General Solution to Einstein's Equations?

This major issue remains very much open. While it would be a formidable task analytically, the dividends for the effort toward such a goal need not be small. As yet, we know only some simple solutions to Einstein's equations, such as the Schwarzschild and FRW geometry, and much of the current activity in gravity physics is based on these models. A more general solution could generate many new insights in gravitation physics, and help us understand formidable issues such as cosmic censorship much better.

Would New Mathematical Formulations of Relativity, Such as Those Used in Loop Gravity or Twistor Theory, Help Decide the Question of Censorship?

Such new formulations could be of help to decide on censorship. However, in our view, it is essentially only a more vigorous study of the gravitational collapse phenomena that can tell us more on this basic issue. It is only such studies which have yielded so far several important leads as we discussed here.

Clearly, if some new formalism helps us study the collapse better and more efficiently, that will be useful. The question of censorship is whether the final state of continual collapse of a massive star will be necessarily a black hole. To decide on this, we must study the collapse itself by whatever means, within the original or any new formalism of general relativity. It is possible that new techniques or formalism could make the complex study of collapse more tractable. In fact, it is because we could devise a set of techniques to work with Einstein's equations in some detail that we could make certain progress in this direction, as discussed here.

Are Alternative Theories of Gravity Better Than General Relativity?

While the general theory of relativity is considered by far the most successful theory to describe the force of gravity, there have been other theories proposed over past decades to describe gravitational phenomena. Some of these include the Brans-Dicke theory, scalar-tensor theories, and more recently theories with modified action compared to Einstein's theory, and theories with higher-order derivative terms.

Most of these theories are based on a spacetime continuum framework only, but differ mainly in the detail of the equations that describe the action and behavior of the gravitational force. Thus, they are likely to have similar qualitative behavior, such as the occurrence of spacetime singularities. Also, most tests of gravitation theories mainly deal with solar system phenomena, and in that regime, all these theories essentially give the same or quite similar results. In such a scenario, the motivation for an alternative theory reduces considerably as general relativity has been verified reasonably well in the experimental tests so far.

Thus, it would seem that the real modification of general relativity would be relevant more in a quantum regime, rather than looking for its classical alternatives. What we mean is that general relativity may be taken as a proper description for gravity at a classical level, but when the regime of very strong gravity fields and extremely small length scales is encountered, then a different quantum gravity framework is needed to describe the true nature of these phenomena.

Are We Suggesting That the Conditions Needed to Have an Event Horizon Are Not Satisfied in Nature So There May Not Be Any Horizons at All? What about All the Evidence for the Existence of Black Holes Then?

That is not being suggested. The work on collapse so far has shown that for a massive collapsing star it is possible, under very realistic physical conditions, that trapped surfaces are actually delayed, and a singularity could become visible. An example is when the density of the star is homogeneous in space, a black hole will form as the collapse end-state. But if the density is higher at the center, which is a physically more realistic case, then trapping is delayed and a visible singularity forms. Many models have been studied and under other sets of realistic conditions, black holes can form. So, black holes and naked singularities can develop as collapse end-states, depending on the set of physical conditions under which the collapse operates.

What course will nature take? It is for the observations to decide that, and further theoretical work may also help. Maybe both phenomena are realized in nature, as predicted by general relativity, under different sets of physical conditions.

During Collapse, Does One Continue through the Horizon and Singularity Unhindered?

At the event horizon, nothing special happens, just that the outgoing rays get trapped and are no longer able to escape to any larger radii. The falling particles continue to fall and will hit the singularity at the center. As for the singularity, using some kind of quantum gravity model, one may be able to continue through the same, but we do not know as yet and need a good quantum gravity theory first.

When Considering the Collapse of a Mass Configuration, to What External Vacuum Do We Match the Interior so That the Spacetime is Complete?

In the collapse of inhomogeneous dust, the exterior is a Schwarzschild spcaetime. But in matter with pressure and with more general equations of state the matching would be to the generalized Vaidya metric. These details and matching have been worked out.

What Decides Whether a Given Spacetime Geometry Has a Naked Singularity?

Given a metric, for example, that for inhomogeneous dust which is the Lemaitre–Tolman–Bondi form, or the radiation collapse as given by the Vaidya metric, we can study the null geodesics or other particle trajectories to decide on this question if they come out from the singularity to the observer.

Also, researchers have carried out a general analysis of collapse and Einstein's equations, somewhat in the spirit of the singularity theorems, to analyze when the trapped surfaces would form. It can be shown that it is the geometry of the trapped surfaces that makes the singularity visible. In each case, it is not necessary to write an explicit metric, and the conclusions can be drawn under a general set of conditions for a spacetime.

Must Event Horizons Always Form as We Go from Low- to High-Curvature Regions near a Spacetime Singularity?

It is not just the local density or curvature values that determine the light paths or event horizon in general relativity. The causal structure of spacetime and the global properties of light cones are the crucial factors. Detailed studies of collapse models imply that gravity can be arbitrarily large and dense in a stellar collapse but still not necessarily inescapable. High density or curvature values do not necessarily mean an event horizon is present. In Newtonian gravity, density is the sole parameter that determines the behavior of a gravitational field. But in general relativity, there are ten gravitational metric potentials, and these give rise to novel features for gravity and its interactions in the Universe.

Sometimes the theories of modern physics such as spacetime theories or quantum phenomena look a bit mysterious. Indeed, it is difficult for our intuition to comprehend the spacetime continuum, which is the key substratum underlying the Einstein gravity. Much of our common beliefs or off-the-cuff thinking is challenged by unusual or striking conclusions which arise as we pursue the logical path. So the best way to proceed is just to follow the logical and mathematical consequences of the theory, and then to check whether we find such phenomena occurring in nature.

Are Naked Singularities Basically Black Holes, But without an Event Horizon?

Like black holes, for the naked singularity case also there will be a point of infinite density. But there will not be the 'point of no return' as there is with black holes. In addition, if naked singularities exist, it will be possible to observe them directly. So naked singularities are like black holes, except for the black part.

Also, many points on naked singularities have become clear now. For example, the effect of pressure and whether it can avoid singularities was discussed extensively for many years, but now we know that non-zero pressures do not avoid naked singularities. Also, naked singularities need not always be neutral; there are charged or spinning examples as well. They also need not involve negative masses, which is unphysical. Again, it is not true that the collapse work is based on 'dust' assumptions only. A thorough analysis of that case was completed some time ago, and collapse models with non-zero pressures have been considered by many groups, some of which were described earlier. Today we have a general formalism available for treating spherical collapse with non-zero pressures, and clear criteria are available to decide when it will terminate in a black hole or naked singularity. We note that sometimes the dust case is also regarded as an important approximation to real scenarios. That is why the Oppenheimer–Snyder dust collapse is very important and forms the foundation of black hole physics.

Could the Collapsing Star Radiate Away All Its Mass, So That No Spacetime Singularity Forms?

Many authors have studied collapse with outgoing radiation. The point is whether it would be possible in a catastrophic collapse for the star to radiate away so much mass in such a short time so as to reach a non-singular end-state. That seems rather unlikely, at least at the classical level, given the usual mechanisms of mass transfer we have. Thus, much work remains to be done to give such a possibility any concrete and acceptable shape, if it is viable at all.

Is a Naked Singularity an Object or an Event?

In many of the gravitational collapse scenarios we discussed, a naked singularity is like an 'event'. But there are also cases such as the Kerr solutions where it is like an 'object', where there may be effectively a super-dense remnant governed by quantum gravity, which was generated by gravitational processes.

The singularity may exist for a long time, or it could momentarily exist in the frame of the collapsing star and then blast itself off in a shockwave due to quantum gravity-generated repulsive pressures. Thus, super-dense remnants governed by quantum gravity, visible to faraway observers for either short or longer periods of time, can be created. Most calculations so far have established the existence of naked singularity in collapse, but there are not yet many simulations to explore the after-effects of such, which is a matter of further research.

While event-like, short-lived naked singularities readily form in many models of collapse, there are also collapse models that give rise to object-like, long-lived naked singularities. This could happen in a collapse scenario that slows down in time, as we discussed earlier.

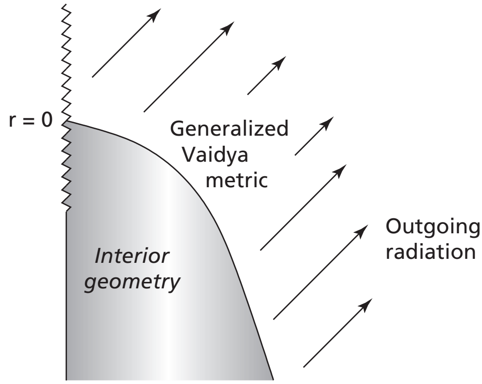

Figure 7.3 A star can emit and lose its mass and energy classically as it evolve in collapse.

When a singularity during collapse is visible, then other astrophysical processes such as shockwave formation or quantum gravity processes near the singularity may build up and take over. This may result in a flash-like phenomenon in the Universe, creating a burst-like scenario, thus throwing off practically all the matter of the star (see Fig. 7.3). The final outcome will be a burst-like phenomenon from the final stages of collapse, and the singularity would be resolved. Such 'event-like' or 'momentary' visible singularities could be quite interesting as they might have some astrophysical relevance for transient phenomena in the Universe.

Do We Have Dark Energy Stars?

Other than black holes and singularities, more exotic compact objects such as boson stars or gravastars have been conjectured by researchers. In many cases, to form such objects we need exotic matter, which almost always violates positive energy conditions. An equilibrium also must be ensured, as typified by such a possible black star, and we may possibly invoke some new forces of nature other than what we already know, or in any case some gross violation of energy conditions must take place.

In the Case of Collapse Terminating in a Naked Singularity, What Will a Faraway Observer See?

In some models such as dust collapse, the singularity is visible only temporarily, and an event horizon eventually forms to cloak it, which is what happens from a local point of view. But for a distant observer, the horizon is labeled by the infinite Schwarzschild time. So in principle, the observer can always see the central naked singularity. It is true though that the light or gravitational-wave signals, if any, emitted from the singularity suffer very large gravitational redshift, and so in practice it is impossible at a sufficiently late stage to catch such signals due to the limitation of the sensitivity of the detector. In other words, for these models, the singularity is visible, but an event horizon eventually forms, and the naked singularity settles down into very deep gravitational potential near the horizon and so it is finally invisible due to the large redshift.

Here we are discussing the emission from the classical naked singularity itself. But in physical reality the singularity may be smeared out by the quantum effects and what we may actually have is an ultra-high-density and ultra-high-curvature quantum gravity object of the order of the Planck length or bigger. This may emit in similar Planck wavelengths. While this radiation also may suffer a big redshift as it emerges, in principle it could be visible, and may give us potentially the signatures of quantum gravity physics which takes place close to such a visible fireball.

The point is that even if a naked singularity forms during collapse, the singularity by itself does not have to radiate matter or energy. Such a singularity is more like an event rather than an object, a moment when collapsing matter reaches its final doom, like the Big Bang in reverse. In fact, the comparison with the Big Bang is interesting here, as it is a naked singularity with an infinite redshift. How do we ever 'see' it? Why does the Universe expand away from it, rather than collapsing onto itself, when it is such a strong gravity field? Just as the Universe comes from the Big Bang, similarly there could be observable effects emanating from the naked singularity forming in a massive star collapse.

Are Gravitational Waves or Shockwaves Created near Spacetime Singularities or during Gravitational Collapse, and Can We Observe Them?

The final stages of a massive gravitationally collapsing star can be a very energetic phenomenon and it is likely that they are associated with the production of gravitational waves in a burstlike fashion. Such a possibility needs to be investigated in a detailed manner.

If the singularity is hidden within a black hole, no such effects are able to escape. Only the physical phenomena occurring outside the event horizon can travel to faraway observers. This is at the level of classical general relativity. Quantum considerations can cause a black hole to release Hawking radiation, but that would be too faint to ever be detected. If, on the other hand, the massive star terminates in a naked singularity, the horizon formation is delayed or avoided. In such a case, the gravity waves or the shockwaves generated in its vicinity, as well as the quantum gravity effects taking place close to these ultra-strong-gravity regions, will be able in principle to propagate to faraway observers.

Primordial Black Holes or Primordial Naked Singularities?

Generically, inhomogeneous or non-spherical collapse is more prone to delay the horizon formation during collapse, and so may generate visible singularities. We have considered mainly compact stars collapsing. But it would be interesting to consider the early Universe and investigate collapse to find whether the singularities formed were more likely to be primordial naked singularities instead of primordial black holes.

Since, According to Quantum Theory, Black Holes Emit Thermal Radiation and Undergo Hawking Evaporation Because of the Separation of Particle-Antiparticle Pairs near the Horizon, Will a Naked Singularity Also Disappear This Way, or Get Resolved through Quantum Gravity?

The event horizon is a crucial factor in the evaporation of a black hole through quantum effects. But in a naked singularity case, it is still possible for the event horizon to disappear or evaporate away through other quantum or classical processes. The effects of quantum gravity, for example, could generate a huge negative pressure, causing the star to emit most of its mass in late collapse stages, as we will discuss later. Further, classical processes such as powerful shock formations caused by inhomogeneities in matter densities near the naked singularity could also cause it to explode away.

The question of quantum effects near a spacetime singularity remains one of the most intriguing issues. There is no complete quantum theory of gravity available as yet, but quantum effects could be important in strong gravity fields even before full quantum gravity becomes operational at the Planck scales. Examining quantum effects could give an idea of possible features present in the full quantum gravity. In many cases quantum effects diverge in the vicinity of a singularity, suggesting that in quantum gravity there may be a possibility of singularity resolution.

Can We Have a 'Quantum Censor', Even If Classical Censorship Fails, or a Quantum Explosion of Naked Singularities?

Singularities could be interpreted as indicating the incompleteness of classical theory. The problems currently faced by the physics of black holes, such as the information paradox and the teleological nature of event horizons, are also, in our view, manifestations of stretching the classical theory too far. The interesting point about visible singularities is that they offer the possibility of observing quantum gravity effects, even when the final singularity is smeared away, as trapped surfaces do not form till very late in collapse evolution.

Similar to the Hawking evaporation of a black hole, will a naked singularity also explode away? As matter becomes extremely dense and hot, quantum effects, gravitational and others, become relevant in the evolution of the process. Hawking radiation might explain why mini or micro black holes, possibly created during the Big Bang, do not exist. Smaller black holes would rapidly evaporate, and hence have long since disappeared. The event horizon is a necessary condition for the Hawking radiation mechanism.

It is possible for the star to disappear or evaporate through either quantum or classical processes in the case of naked singularity evolution. An example that we will discuss is that of a toy model where quantum gravity effects generate a huge negative pressure for the star to emit away most of its mass in a final burstlike fashion. There could also be classical processes such as powerful shock formation due to inhomogeneities in matter densities close to the naked singularity. This may again make the star explode away, but the detailed mechanism is not known.

Can We Break a Black Hole by Throwing into It Charged or Rotating Particles?

Let us suppose a black hole did form as the final end-state of a massive star's gravitational collapse. There is a constraint in this case for the horizon to survive, namely that the black hole must not contain too much charge or spin too fast. Otherwise, the horizon cannot be sustained; it will breakdown, and the singularity within will be visible.

Even if the black hole formed with a small enough charge and angular momentum, the key astrophysical processes in its surroundings, including that of the accretion of matter, could increase its charge or spin. Such matter could be typically a lot of particles and outer layers of the collapsing star, which will fall into the black hole with great velocity; the matter could be classical or quantized, and with charge and angular momentum. Through 'charging up' or 'over-spinning' the black hole, the infalling particles could eliminate the event horizon. Thus, the very fundamental characteristic of a black hole, namely its trait of gobbling up the matter all around it and keep growing, could become its own nemesis and a cause of its destruction (see Fig. 7.4).

So even if a massive star collapsed into a black hole rather than a naked singularity, important issues remain, such as the black hole's ability to withstand particles with charge or large angular momentum falling in and converting the black hole into a naked singularity by eliminating its event horizon.

Many researchers have claimed in recent years that this is possible, and have provided models for creating naked singularities this way. But there are others who claim there are physical effects which would save the black hole from over-spinning, and the issue is out with the jury. The point is, in general, the stability of an event horizon and a black hole in this respect continues to be an important issue even if a black hole formed during the collapse of a massive star.

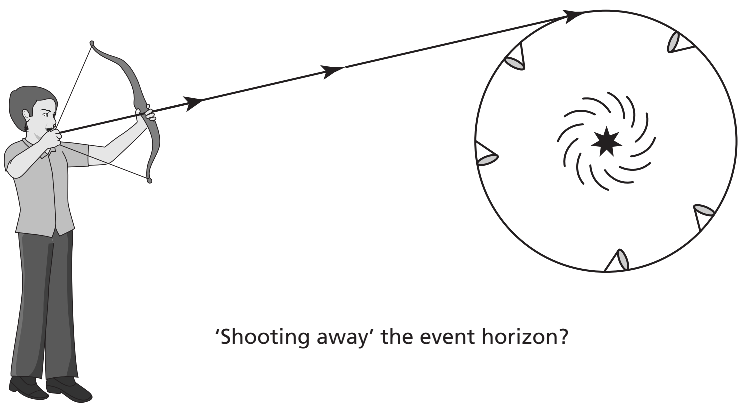

Figure 7.4 Can we break a black hole? Debate continues on whether this is possible.

The primary concern of the censorship hypothesis is, of course, formation of black holes only as collapse end-states, and their stability as described is a secondary issue. So what this means for censorship is that the collapsing massive star should not retain or carry too much charge or spin; otherwise, it will end up as a naked singularity, rather than a black hole.

It is worth noting that typically when some charge or enough angular momentum falls into a black hole, the event horizon structure could completely change. For example, a Schwarzschild black hole has no charge and the horizon fully covers the singularity. But as soon as we drop a small charge into it, the horizon structure changes, it becomes a Reissner–Nördstrom black hole, and the singularity can become visible, at least locally.

Why Is the Big Bang Singularity Uncloaked by an Event Horizon? Would a 'Big Crunch' Singularity Also Be Naked?

Let us consider any observer in the Friedmann Universe. If there is any past-directed timelike or null trajectory from this observer, extended maximally into the past, it will meet the Big Bang singularity. In other words, there are timelike or null photon paths from the singularity coming out, which reach the observer. In this way the Big Bang singularity is visible, in fact to all observers. This nature of singularity is entirely different from the case, for example, of the homogeneous density Oppenheimer–Snyder collapse model singularity, where no future-directed non-spacelike curves from the singularity reach any faraway observer at infinity. But if we consider more realistic models with a higher density at the center, the singularity of collapse turns out to be visible, and this feature turns out to be generic for more general matter fields and general initial density and pressure profiles from which collapse develops, as we have discussed here.

As for the big crunch singularity, the situation is entirely different. It is a singularity that is in our future, if it occurs. In FRW models time travel is not possible, so it will not be visible to us. Only if some very complicated topologies for the Universe are allowed, permitting causality violation, can a future singularity possibly be visible.

Do Naked Singularities Violate Causality?

The statement that is sometimes made, namely that naked singularities are bad because they give rise to closed timelike curves and causality violation, which could allow your parents to be dead before you were born, is incorrect. In fact, in the collapse scenarios we discussed, there are no closed timelike curves developing near the naked singularity, and the spacetime is perfectly regular and causal. Something as physical as a higher density at its center, rather than a uniform density cloud, could lead to a visible singularity.

Would the Universe Permit Non-trivial Geometries and Topologies?

The question here is, compared to the usual Euclidean geometries we are familiar with, what other non-trivial global geometries a realistic Universe model may permit. While Einstein's equations reveal how the local physics operates, the global geometry and topology of the spacetime are left completely free. A non-trivial global topology, for example, could permit causality violations in a spacetime. Typically, we rule these out by means of an explicit assumption, but the issue of global topology of the Universe remains, which would have important implications for cosmology as well as observations on the Universe. In fact, many researchers have recently discussed the global geometry and topology of the Universe from the perspective of current observations in cosmology.

Thus, we also examined the relationship between topology change in a spacetime and the occurrence of naked singularities and also the connection between the strength of naked singularities and the disruption in causal structure that they may cause. As Ravi Saraykar and myself have shown, it turns out that a topology change in the spacetime causes naked singularities.

What Are the Recent New Results on Collapse and Singularities?

Recent developments can be roughly classified as follows: First, there is a set of results, from us and others, regarding the stability aspects of naked singularities as well as black holes. These results show that naked singularities are not necessarily unstable, in the sense that in the space of initial data from which the gravitational collapse of a massive star evolves, there are open sets of initial data which give rise to a naked singularity, and the same holds true for a black hole. So, a sort of genericity is seen for naked singularity formation. Also there are results which show the instability of black holes in higher dimensions.

In our view, the key problem in all these considerations is that there is no fixed notion or any precise definition of stability available in

Einstein's gravitation theory, mainly due to mathematical ambiguity and complexity. Therefore, in our view, this is not an issue that can be clearly resolved one way or the other. But the point that comes out is that it is no longer easy to argue in simple terms that naked singularities are unstable and therefore are to be discarded.

This situation has given rise to another set of efforts. Researchers have thought: suppose both black holes and naked singularities do develop in nature as collapse end-states when a massive star dies. Can we observationally verify this? In other words, can we distinguish between them observationally and through astronomical or astrophysical signatures? What are the observational signatures of naked singularities, if they occur at all in nature, and how do they differ from black holes? In the past few years, results have become available in this direction and the main ones are these. First, the accretion disks around black holes and naked singularities will have significantly different properties, as shown by Daniele Malafarina, Ramesh Narayan, and me. These offer characteristic differences which we could possibly access observationally. Second, there are results now on how very-high-energy particle collisions will occur much more effectively in a naked singularity background than in a black hole spacetime. Finally, there are differences in the gravitational lensing signatures for black holes and for naked singularities which thus offers the possibility of observationally distinguishing whether a potential high-mass object, e.g. the galactic center, is a black hole or a naked singularity.

This last effort could answer the interesting question of what a naked singularity actually looks like. The accretion disk of a naked singularity is much brighter and more luminous, and the spectral luminosity features are rather different. Thus, an initial guess would be, as we descend into the singularity, a huge increase in brightness and light. Of course, going too close will destroy us, as is the property of the singularity, so the better thing to do is to go as near as possible and then come back.

In View of the Current Scenario, What Are the New Emerging Directions and Research Frontiers? What is the Current Perspective on Cosmic Censorship?

Let's consider the second question first. There is no doubt that the issue of cosmic censorship and gravitational collapse has broad appeal and interest. We hope the discussions here highlight the crucial points and main physics issues involved, within the broad context and proper orientation. We would prefer to be circumspect, and review and describe all the important related views carefully in an appropriate framework. Our own research has always taken and adopted such a point of view. There are many interesting and exciting physics issues here, of much interest to the broader spectrum of astronomy and astrophysics. While dealing with these, we should be careful on all possible aspects. We have tried to achieve this here and to strike the right balance.

There is, however, an urgent need to think about these issues in an open and transparent manner, rather than just avoiding doing it. There is a strong need for such a cause rather than just making assumptions as is often done due to lack of proper information. For a general and popular audience it might be better to be more circumspect and say that the status of censorship is not as firmly established as many physicists assume. In doing so, we would like to be representatives not only of our own ideas but also of an entire scientific community, and we hope our discussion here presents the case both for and against cosmic censorship, describing not just our own opinions, but also those of other physicists. With much interest in the topic today, an even-handedness of approach is a necessity and the priority is more with recent findings than with general background information.

Coming to the first question, we have pointed out that the final fate of gravitational collapse of a massive star is an exciting research frontier in black hole physics and gravitation theory today. The outcomes are fundamentally important to the basic theory and to astrophysical applications, and for modern gravitation physics. We have highlighted certain key challenges in the field, and also have reviewed several recent interesting developments. Of course, no claim to completeness has been made, and there are other interesting problems in the field as well.

We mention here a few points which we think are interesting, and which may have an impact on future development in the field:

1. The genericity of the collapse outcomes, in terms of black holes and naked singularities, needs to be understood carefully and in further detail. It is by and large well accepted now that general relativity does allow and give rise to both black holes and naked singularities as the final fate of collapse, evolving from regular initial data and under reasonable physical conditions. What is not yet fully clear is the distribution of these outcomes in the space of

all allowed outcomes for collapse. The collapse models discussed already and the considerations we have given would be of help in this direction, and may throw light on the distribution of black holes and naked singularity solutions in the initial data space.

2. Many of the models of gravitational collapse analyzed so far are spherical. So non-spherical collapse needs to be understood much better. While there are models which illustrate what the departures from spherical symmetry could do, some other analytical models of collapse for matter clouds with cylindrical symmetry have also been studied. However, we note that, on the whole, not very much is known about non-spherical collapse. Numerical relativity could probably be of help in this direction. Also, another alternative would be to use global methods to deal with the spacetime geometry involved, as used in the case of singularity theorems in general relativity.
3. At the very least, the collapse models studied so far do help us gain much insight into the structure of cosmic censorship, whatever final form it may have. On the other hand, there have also been attempts where researchers have explored physical applications and implications of the naked singularities investigated so far. If we could find astrophysical applications for the models that predict naked singularities, and possibly try to test the models through observational methods and the signatures predicted, that could offer an interesting avenue for obtaining further insight into this problem as a whole.
4. An attractive recent possibility in that connection is to explore naked singularities as possible particle accelerators, as we have tried. Also, the accretion disks surrounding a naked singularity, wherein the matter particles are attracted toward or repulsed away from the singularities with great velocities, could provide an excellent venue for testing such effects and may lead to predictions of important observational signatures for distinguishing between black holes and naked singularities in astrophysical phenomena, as some researchers have recently attempted.
5. Finally, further considerations of the quantum gravity effects in the vicinity of naked singularities, which are super-ultra-strong-gravity regions, could yield intriguing theoretical insights into the phenomena of collapse, as we will discuss in the next chapter.

8

Is Our Universe Predictable?

The detailed collapse calculations of recent years show that the final fate of a massive collapsing star could be a naked singularity. The concern expressed at times has been that the future of a naked singularity is unpredictable, because unpredictable inputs may emerge.

This has physical relevance and implications as well as interesting philosophical aspects for the issue of predictability in the Universe. It is sometimes argued that the breakdown of censorship means violation of predictability in the spacetime, which must be avoided. It has been suggested that we have no way of knowing what a naked singularity may radiate or emit unless we study the physics in such ultra-dense regions, whereby we can then predict the Universe only partly but not fully in the future of any given epoch of time.

To put it differently, the event horizon of a black hole hides the super-ultra-dense region from us, and therefore the fact that we do not understand such regions has no effect on our ability to predict what happens in the Universe at large and far away. But if no such horizon exists, then the ultra-dense region might, in fact, play an important and even decisive role in the rest of the Universe, and our ignorance of such regions would then become an issue of more than merely academic interest.

The issue of whether our Universe is fully predictable, at least within a classical and non-quantum perspective, has been an important one. There have been exciting discussions about predictability in the Universe. Within the context of cosmology and fundamental physics, it is thought to be obvious that the classical theories must be all fully predictable, in a sense to be properly defined, whereas the unpredictability is basically and essentially associated with quantum phenomena only.

We note, however, that an unpredictability at such a level is, in fact, quite common in general relativity, and it is not always necessarily related to cosmic censorship violation. Even black holes themselves need not fully respect predictability of this kind when they rotate or have some charge. As an example, if we drop an electric charge into an uncharged black hole, the spacetime geometry radically changes its character and is no longer predictable from a regular initial epoch of time. A charged black hole admits a naked singularity which is visible to an observer within the horizon. A similar situation holds when the black hole is rotating. As we have discussed, there is a debate going on whether it is possible to over-charge or over-rotate a black hole so that the singularity that is visible to observers within the horizon becomes visible to external faraway observers, too.

Also, if such a black hole was big enough on a cosmological scale, the observer within the horizon could survive in principle for millions of years happily without falling into the singularity, and would thus be able to observe the naked singularity. So only the purest of black holes with no charge or rotation at all could respect full predictability, whereas other physically realistic black holes with charge or rotation could not. As such, there are many models of the Universe in general relativity that are not totally predictable. In them, spacetime cannot be neatly separated into space and time so as to allow initial data at a given moment of time to fully determine the future.

So what is the origin of such non-predictable features, even at a classical level, in general relativity? We would like to highlight that the classical theories of gravity, such as the general theory of relativity, are, in fact, not 'predictable' in the usual sense of the word associated with Newtonian mechanics, which is certainly a predictable framework. Unlike Newtonian gravity, Einstein's theory is a non-local theory which allows for intricate possibilities such as black holes and time warps. The issue is closely associated with the existence of black holes, Cauchy horizons, and spacetime singularities in the general theory of relativity. It is a theory which allows for non-trivial geometry and topology for spacetime with several non-local features. The highly nonlinear nature of Einstein's equations allows for departures from what we may call 'Newtonian predictability'.

In our view, the real breakdown of predictability is the occurrence of spacetime singularities, whose existence probably indicates the true limitation of classical gravity theory. It does not matter then whether they are hidden. The real solution of the problem is the resolution of the singularity itself through either a quantum theory of gravity or in some way at the classical level itself. Hiding the singularity within a black hole may not be the real solution and it may be just shifting the problem elsewhere, and some of the current major paradoxes faced by black holes could as well be a manifestation of the same.

Predictability Defined

In general relativity, a given 'epoch' of time can be represented by a spacelike surface. This is a three-dimensional space at a given moment of time. For example, in the standard Friedmann models of cosmology, there is such an epoch of simultaneity, from which the Universe evolves in future, given the physical variables and initial data on this surface.

We need to specify what is meant by 'predictability' in general relativity. Given regular initial data in terms of densities and pressure profiles for the matter, and a velocity profile for how it evolves at an initial epoch of time, we would like to know the evolution for all time in the future. This means data are given on a spacelike three-dimensional surface, also called a *hypersurface*, and the aim is to predict the future and past evolutions in the spacetime for all times. Such a requirement is also termed the *global hyperbolicity* of the spacetime. A globally hyperbolic spacetime is a fully predictable Universe; it admits a *Cauchy surface*, which is a spacelike hypersurface where the data can be evolved for all times in the past and future. Simple enough spacetimes such as the Minkowski or Schwarzschild are globally hyperbolic, but the Reissner–Nördstrom or Kerr geometries are not.

Thus, in general relativity, predictability is formulated in mathematical terms as the existence of a Cauchy surface in the spacetime universe. Given such a spacelike three-dimensional surface, the evolution equations of relativity then predict all the future and past events in the Universe in terms of the initial data specified on this initial surface. This also fixes the entire spacetime topology and geometry of the full Universe as $S \times R$, where S is the Cauchy surface and R is the real line.

A naked singularity is characterized by the existence of light rays and particles that emerge from it. Typically, in all the collapse models that have been discussed, there is a family of future-directed non-spacelike curves that reach external observers, and when extended into the past the curves meet the singularity. The first light ray that comes out from the singularity marks the boundary of the region that can be predicted from any regular initial surface in the spacetime, and is called the *Cauchy horizon* for the spacetime. The causal structure of the spacetime will

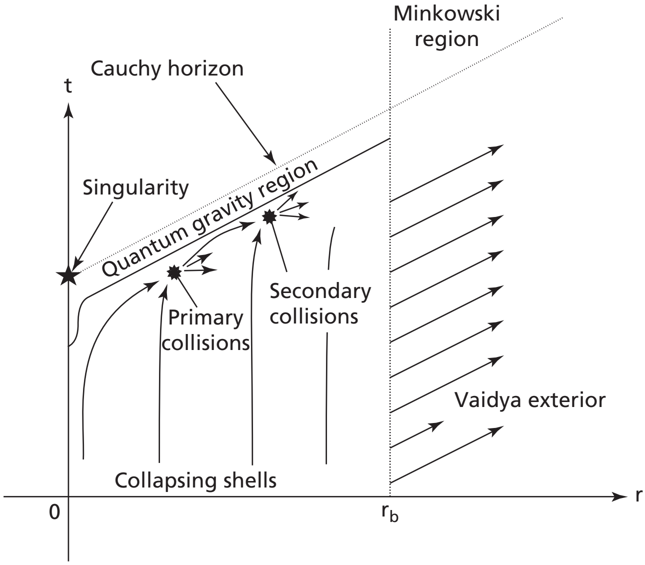

Figure 8.1 Typically the existence of a naked singularity is characterized by the Cauchy horizon forming as the collapse develops. The first light ray coming out of the singularity is the Cauchy horizon. The region of spacetime beyond a Cauchy horizon cannot be predicted from a spacelike surface of simultaneity in the past. Very-high-energy particle collisions can occur close to such a Cauchy horizon.

differ significantly in the two cases, when there is a Cauchy horizon and when there is none. A typical gravitational collapse to a naked singularity, with the Cauchy horizon forming, is shown in Fig. 8.1.

Suppose we are in a non-globally hyperbolic universe and want to know where exactly our ability to predict events breaks down in such a spacetime. It will be useful to take an example. Consider the Reissner–Nördstrom spacetime which is not globally hyperbolic. We can begin on a spacelike surface with all the data on it in our computer. As we evolve on our world line, for a certain time to come, we would be fully able to predict physics at later epochs, using the data already available. But, later, we will enter a region of spacetime which contains non-spacelike curves which come from either the infinity or singularity, and can bring potentially new information from there which we did not have to begin with. This happens when we cross the Cauchy horizon. We must then supplement our original data with this new information in order to do physics. Many important solutions of Einstein's equations have such a property and they are non-globally hyperbolic.

Global hyperbolicity means that from a given epoch of simultaneity, all of the Universe, both in future and its past, can be predicted. The significance of this is that the physical data given on this spacelike surface fully determine all physical quantities in the future or the past at these events through hyperbolic equations of relativity. In this sense, the entire evolution in the spacetime is fully deterministic. The spacetime is then 'foliated' by the spacelike surfaces evolving in a cosmic time. If there is no such spacelike surface, then the Universe is not globally hyperbolic, and there is only a limited region which is fully predictable from an initial slice.

The cosmic censorship fully holds in a globally hyperbolic spacetime, but then there cannot be charged or some classes of rotating black holes, because they are not globally hyperbolic. In fact, typically a given solution of Einstein's equations will not be globally hyperbolic as this is a very strong condition on the spacetime. A Cauchy horizon separates the predictable and non-predictable regions of the Universe. The Oppenheimer–Snyder model is fully predictable in this sense, and so is the cosmological spacetime of the FRW metric, or the flat Minkowski spacetime. In any collapse leading to a naked singularity, there is similarly a Cauchy horizon, just as for charged or rotating black holes.

Is Relativity a Predictable Theory?

In general relativity there have been many solutions obtained, subject to very reasonable physical conditions, which are not globally hyperbolic. These include the Reissner–Nördstrom solution, which is a black hole with charge, the Kerr solution, the Gödel universe, and other cosmological models including the anti-de Sitter spacetime, which is very important in string theory. In fact, only the simplest of the models, such as the Schwarzschild and FRW, are globally hyperbolic, whereas most other useful solutions, which are obtained once we include physical quantities such as charge or rotation to solve Einstein's equations, are not.

Thus, global hyperbolicity or 'predictability' is a very strong additional condition or constraint to be imposed on the spacetime for Einstein's theory, which is not globally hyperbolic on its own despite being a purely classical theory. The existence of such a 'non-predictability' in spacetimes is characterized by the existence of 'Cauchy horizons' in the Universe. These mark the boundaries of the spacetime region which can be fully predicted from the data on a given 'initial' spacelike hypersurface. Cauchy horizons in spacetimes can arise from many factors. Typically, they can be traced to be closely related to the existence of spacetime singularities, the global spacetime topology, which is not determined by Einstein's equations but is left free, and also by physical quantities such as the charge and rotation included in the physical considerations toward determining the spacetime geometry and solutions to Einstein's equations.

What this means is that such an 'unpredictability' is somewhat common in general relativity. There are, in fact, many different ways in which predictability breaks down in general relativity; singularities are not the only cause. For example, the Gödel Universe is geodesically complete and singularity free, but it still strongly violates global hyperbolicity.

So we could ask: Does our notion of predictability in the Universe need any modifications? Currently, this means that the initial data at the 'present' moment fix the entire past and future evolution 'once and for all', and there is no possibility for any 'new creations' in future.

The question is, even if we hide naked singularities in horizons, what shall we do with all the other kinds of 'non-predictable' situations that arise in general relativity? Therefore, in our view, the predictability argument does not help much either for or against censorship or naked singularity, and must be viewed essentially separately. Such an unpredictability being quite common in gravitation theory, the real solution may be the removal of the singularity itself.

It is worth noting that the general theory of relativity admits several features that are considered quite unusual for a classical theory. The causality violation is, of course, a predictability breakdown in itself, but there are wormholes, non-trivial spacetime topologies, a very natural formation of Cauchy horizons, and other such features which do not respect global hyperbolicity. What this means is that naked singularities are not special here as far as 'predictability' or global hyperbolicity violation is concerned. They are as good or as bad as any of these other features of general relativity, and we need to accept them as such in order to make further progress.

The predictability requirement, as used today in general relativity, rules out any topology change for the spacetime as it evolves toward the future. But we know that topology changes do occur, for example when two black holes collide and merge. If we believe that galaxies have black holes at their centers, then predictability is necessarily violated whenever galaxy collisions occur. Also, at very small scales, where quantum effects are to be included, a favorite picture is that of 'spacetime foam', which is a continually changing spacetime topology. Again, this is ruled out if we insist on 'predictability' as above and impose no topology change.

Singularities and Predictability

Spacetime singularities in cosmology or collapse are inevitable features in Einstein's gravity. While the Big Bang singularity which created the Universe and all of us is visible, the singularities arising from the collapse of massive stars can be either covered within event horizons of gravity, which are black holes, or without horizons, thus being naked singularities. One of the arguments against naked singularities is that these are undesirable as they break predictability in the Universe.

As we noted earlier, while naked singularities do cause Cauchy horizons, they are not the only ones to do so. Cauchy horizons arise in many physically reasonable circumstances, as we discussed earlier, so they seem more natural and common than global hyperbolicity, which is a rather too strong extra condition to be assumed on a general relativistic spacetime. Such a predictability loss happens in many physically reasonable situations, so even in the absence of naked singularities, global hyperbolicity is far from guaranteed.

Naked singularities are super-ultra-high-energy regions where quantum effects are dominant, and so are the most likely places in which quantum gravity effects might arise. Then we would have a chance of observing signals due to quantum gravity processes.

It has been suggested that if all singularities were hidden within black holes, then predictability would be restored. However, we recall that black holes create a set of fundamental paradoxes, such as 'information loss'. In our view, the real problem is the classical singularities themselves, whether hidden in black holes or visible. What we really need is a resolution and better understanding of singularities, probably using quantum gravity. Assuming cosmic censorship and hiding the singularities may not be the real solution to these basic problems, because cosmic censorship may be only a partial solution to the key issues, or just shifting the problem elsewhere, manifesting as current major paradoxes in black hole physics.

In our view, the real breakdown of predictability is the occurrence of the spacetime singularity itself, which indicates the true limitation of classical gravity theory. It does not matter whether it is hidden within an event horizon. The real solution of the problem would be the resolution of the singularity, either through a quantum theory of gravity or in some way at the classical level itself. Thus, it may be useful to study the physics of the visible super-ultra-density regions indicated by naked singularities, which may offer possible removal of the singularity through quantum gravity and possibly also predictability, if the physics of these regions is understood better.

How does this 'unpredictability' manifest itself? Basically it is in our inability to predict the future fully from the physical data and quantities given at the current epoch of time.

But then it is useful to note that over the past years strong arguments have been given already against assuming this sort of 'predictability' for the Universe. A good example is the argument by Stephen Hawking to justify his general singularity theorem. When Roger Penrose provided the first singularity theorem in 1965, one of the conditions assumed was that spacetime is globally hyperbolic; that is, it admits a Cauchy surface and is predictable in the sense we have discussed above. But Hawking argued that global hyperbolicity is too strong a condition, that it is not physical or something which can be assumed easily. He pointed out that:

1. If there was a Cauchy surface for spacetime, we would be able to predict the state of the Universe any time in the past or future if we knew the physical data on the surface. However, how is it really possible to know such data unless we are in the future of every point of the surface? That is impossible in most physical cases, so there is no physically compelling reason to believe that the Universe admits a Cauchy surface, and that it is predictable in the above sense.
2. There are a number of most useful solutions to Einstein's equations which do not admit a Cauchy surface. Among them are anti-de Sitter, plane wave solutions, Taub-NUT space, and the Reissner–Nordstrom charged black hole case. Thus, there could

> be extra information coming from infinity or a singularity which would upset the predictions made simply on the basis of the data on the surface.

For these reasons Hawking argued that Penrose's theorem, which assumes global hyperbolicity, is not physical enough, and that the theorem must therefore be generalized so that it holds in actual physically realistic conditions. It was, in fact, argued that the real weakness of the theorem was the requirement that spacetime has a Cauchy surface, or that assuming predictability or determinism is a physical weakness. Then Hawking assumed a much weaker condition, namely the causality only of spacetime, and the more general Hawking–Penrose theorem was proved in 1970.

The above discussion brings to light the useful point that there is really no argument against naked singularities on the count that they violate global hyperbolicity. Because in any case we have agreed that the Universe does not have to be globally hyperbolic, and that it is too restrictive a condition to physically assume.

Actually our ability to predict the future breaks down as soon as the spacetime singularity occurs, whether it is covered or visible. Because at the singularity, the observer is suddenly destroyed, so we can no longer evolve the spacetime beyond it. So the thing to focus on is the removal or resolution of the singularity itself, which is the real breakdown of predictability, if any. Therefore, it is hardly unnatural or something to be alarmed about if a visible singularity arises out of a physically realistic collapse. It is just another reason for the spacetime not to be globally hyperbolic.

In some sense, we could say that not having global hyperbolicity means that the Universe has reserved its own right and freedom to do what it wants in the future, independently of our current knowledge of it at this moment of time. So the issues here are profound and interesting, and with richness in fundamental physics. They may also be important for science and philosophy in general where a considerable discussion continues.

Rabbits Popping out of a Hat?

If naked singularities occur, do we have a 'rabbit out of a hat' model? If future events are determined by completely arbitrary boundary conditions at singularities, does it not sound virtually like magic? Does the indeterminism emanate from the naked singularity, or from infinity? Is it really the case that something might 'pop' out of the singularity like a rabbit out of a hat? Do we envisage that a quantum gravity theory will eliminate it and restore determinism everywhere in spacetime? The question is, what precisely does it mean for the evolution not to be deterministic, and whether an element of randomness intrudes, and if so, how does it manifest itself?

To clarify this, we can ask the following. Suppose we are sitting in a non-globally-hyperbolic spacetime. Where exactly does our ability to predict events to happen fail? That is, fail 'in principle', because this ability always fails 'in practice' as we lack sufficiently precise measurements. Does this introduce an element of magic into the world, namely that things happen for no reason? If so, it would be disturbing to see this in a classical theory. What we ask is the key physical question: namely, does an element of randomness intrude, and if so how does it manifest?

As we noted earlier, the spacetimes which are non-globally hyperbolic are as fine and acceptable in general relativity as those which actually are. There are many ways in which a universe may not be 'deterministic' or 'predictable' in the above sense. What really happens in the naked singularity case is that the information on the present spacelike surface does not determine physics at all points in the future in such models. What is needed then is for us to understand and study physical processes in the ultra-super-dense regions close to the singularity. Once we do that, we may be able to tell what is likely to come out of such regions, whereas the singularity itself should be resolved through quantum gravity.

On determinism and predictability, we can say that indeterminism is not just a problem for quantum physics, but also for classical systems, such as chaos and nonlinearity. Perhaps what we call determinism, in terms of 'predicting' the future and past using first-order differential equations, may need a change. The point is there are good examples in general relativity where traditional global determinism breaks down, and the occurrence of naked singularities is only one such example.

Restoring the Predictability

Sometimes we hear the argument that naked singularities are not desirable because if they existed, then whenever a massive star collapsed in the faraway Universe, a massive burst of radiation and high-energy particles will be emitted and wipe out planets such as Earth from existence. But there is no reason for such a fear, as by conservation of energy, a star will not radiate away more than the total mass-energy density it has. So if such a fireball happened in the faraway Universe because of a massive star's collapse, the total radiation reaching us will be minuscule, or on a similar order of what we see in gamma ray bursts today that take place in the faraway Universe. Even if a naked singularity happened in our own galaxy, there is no such danger; the total radiation in our direction would be less than a massive solar flare pointed to Earth. In that sense, naked singularities do not break predictability in any serious physical way.

It may be useful to note that researchers found an interesting behavior near Cauchy horizons, namely that particles collide with very high energies and that Planck-scale physics could possibly occur there. In a way this creates a singularity at the Cauchy horizon, marking an end of spacetime. Then spacetime terminates at the Cauchy horizon itself whenever a naked singularity happens in gravitational collapse. Since these mark the boundary of spacetime, predictability is restored, because the rest of spacetime is predictable before the Cauchy horizon formed. This would be an interesting possibility to probe further.

General relativity is the best theory we have today for describing gravity at the classical level and it has been quite successful in experimental tests. But we know that it is not complete, in that it must have spacetime singularities, covered and naked. At such singularities predictability breaks down and so does the theory itself. In a way the theory is predicting its own breakdown. Other theories of classical gravity are no better, but for different reasons.

One approach could be that we accept that general relativity is not complete as a classical theory, which would be valid only till quantum gravity effects take over in super-high-density regions. We need to study and understand the physics and quantum gravity processes in these regions indicated by naked singularity, which may resolve the singularity, and that way also possibly restore the predictability once a better understanding is obtained.

As we understand gravity physics better, one important point emerging is that black holes are no better than naked singularities as far as predictability is concerned. We may hide the singularity within a horizon, but as many recent results have shown, event horizons and black holes have their own problems, such as information loss, causality paradox, and the infinite-density singularity sitting at the center. In fact, the debate going on currently about the stability of the black hole horizon, in connection with the firewall proposal or the throwing in of charges or spinning particles in a hole raises doubt about the black hole's stability. At the classical level, even if it formed in collapse, the event horizon could break up and the singularity become visible.

As for general relativity, there are many different ways in which predictability breaks down; singularities are not the only cause. For example, the Gödel universe is geodesically complete and singularity free, but it still violates global hyperbolicity in a bad way by admitting causality violation. Actually, general relativity does admit many features which may be considered disturbing for a classical theory. Apart from causality violation, which is a predictability breakdown, there can be worm holes, weird spacetime topology, and so on. These violate global hyperbolicity in different ways, and at a classical level. So there is no special offense to be taken on naked singularities on that count. In fact, if we so desire, we may take the breakdown of global hyperbolicity as the breakdown of classical physics, and then study quantum gravity to do better. That is what we do when we study quantum effects near naked singularity, as we will discuss in the next chapter.

9

A Lab for Quantum Gravity

If naked singularities do indeed arise in the Universe as gravitational collapse final states of massive stars, then clearly they will have their own quantum aspects which would be of much interest. As we emphasized earlier, these are the regions where quantum gravity effects will be important and dominating. When these are able to propagate away to faraway and external observers in the Universe, we can wonder whether they will provide us with a possible cosmic laboratory for testing and examining how quantum gravity works. If so, this could be an exciting prospect and a novel direction for our search for a quantum theory of gravity and towards the unification of the forces of nature.

It is widely believed that when we have a reasonable and complete quantum theory of gravity available, all spacetime singularities, whether naked or inside black holes, will be resolved and smeared away. However, this has not been realized so far, and as of now it remains an open question whether quantum gravity will remove naked singularities. After all, the occurrence of spacetime singularities could be a purely classical phenomenon. On the other hand, there are also arguments that singularities may not actually go away in quantum gravity.

In any case, the important issue is whether the extreme strong gravity regions formed due to gravitational collapse are visible to faraway observers. It is quite clear that the collapse could certainly proceed classically, at least till the quantum gravity starts governing and dominating the dynamical evolution at Planck-scale lengths, that is, till the extreme gravity configurations have developed due to collapse. The key point is that it is the visibility or otherwise of such ultra-dense regions, and whether they are classical or quantum in nature, that is under discussion.

What is important is that classical gravity implies the existence of ultra-strong-gravity regions, where both classical and quantum gravity aspects come into their own. In fact, if naked singularities do develop during gravitational collapse, then in a literal sense we come face to face with the laws of quantum gravity.

In this way, the gravitational collapse phenomenon has the potential to provide us with a possibility of actually testing the laws of quantum gravity. In the case of a black hole developing during the collapse of a finite-sized object such as a massive star, such strong-gravity regions are necessarily hidden behind an event horizon of gravity, which would be well before the physical conditions become extreme near the singularity. In that case, the quantum effects, even if they cause qualitative changes closer to the singularity, will be of no physical consequence. This is because no causal communications are allowed from such regions. On the other hand, if the causal structure were of a naked singularity, then the communications from such a quantum gravity-dominated extreme-curvature ball would be visible in principle. This will be so either through direct physical processes near a strong-curvature naked singularity or via the secondary effects, such as the shocks produced in the surrounding medium.

We will elaborate on some of these aspects here and future perspectives are indicated, mentioning possible directions toward resolving the cosmic puzzles as created by recent discoveries on gravitational collapse and black holes.

The Quest for Quantum Gravity

Whenever we discuss spacetime singularities, a consideration of quantum effects in the vicinity of the singularity becomes important, because when the curvatures grow arbitrarily high near a singularity, then the classical general relativity can be relied upon less and less and the quantum effects must become important to dominate. However, despite serious attempts over the past half-century, we have no quantum theory of gravity available which can be used to examine the quantum effects near a classical singularity, or to study whether the singularity can be avoided in a quantum gravity theory.

The key issue toward a quantum theory of gravity is that all familiar methods we use in quantizing other physical fields, and which work very well for them, fail and run into serious problems when applied to the gravitational field. Actually, these difficulties seem to be related to the fact that global issues become important as soon as we try to describe the gravitational field. So novel ideas and techniques are needed and also there are several profound conceptual issues which need serious attention.

Why do we need to quantize gravity and why should we go over to a quantum theory of gravitation? There are compelling arguments for this. The fundamental forces of nature known to us, namely the electromagnetic field, and the weak and strong nuclear forces, with the exception of gravity, have been quantized. So if we want a unified theory, then gravity must also be quantized. Also, the idea that all fundamental interactions in nature are described by a single, unified conceptual framework, being the manifestation of a single basic field, is rather appealing. The unification of the electric and magnetic forces was achieved by Maxwell in the nineteenth century with the theory of electromagnetic fields. The Weinberg–Salam theory then provided a unified quantum treatment for the electromagnetic and weak interactions, namely the electoweak theory. Then the grand unified theories included the strong nuclear forces in this scheme. Therefore, the aesthetic motivation to include gravity also is rather compelling. As is known, Einstein himself spent considerable effort trying to unify gravitation with electromagnetic theory and other forces of nature.

General relativity theory and quantum theory have been very successful in their own domains, namely the macroscopic Universe governed by gravity and the microscopic world of elementary particles. These theories explain the observed phenomena and make predictions which are verified to a high degree of accuracy. However, their conceptual frameworks have no similarities and we have no clue as to any common underlying structure.

General relativity assumes the arena of a spacetime as the basic framework, assuming a continuum differentiable manifold structure where a metric tensor defines the causal structure with light cones determining the propagation of particles. Such a metric is composed of ten independent functions which obey Einstein's equations that involve the second derivatives of the metric tensor. Typically, for any given physical system, the components of the metric tensor are obtained by solving Einstein's equations. On the other hand, quantum theory is based on a Hilbert space of the state vectors describing any given state of the quantum mechanical system. Then physical fields are represented by linear operators acting on these vectors to produce real numbers which are interpreted as the results of measurements.

The explanation for such a diversity in approach could be that gravity, compared to other forces of nature, is extremely weak. The strength of the gravitational force between two elementary particles is much weaker than the other forces between them. The scale at which a classical description breaks down for a particular theory depends on the masses and charges of the particles involved and the values of the fundamental constants involved. For the quantum theory of gravity, the length scale at which quantum effects will become important is determined by the values of the velocity of light, c, the Planck constant, $\hbar$, and the Newtonian constant of gravity, G. A unique length scale is decided by these three constants as given by $\ell_p = (G\hbar/c^3)^{1/2}$, which is called the *Planck length*. In cgs units it has the value of about 10^{-33} cm, which is the fundamental length scale when quantum gravity effects will be important. As it happens, the values of fundamental constants are such that the Planck length is very small and the corresponding Planck energy very high compared to the laboratory scales at which we normally operate.

This allows us to use general relativity for the purposes of astrophysics and cosmology, and quantum theory for atomic and subatomic physics. Even though they look different at the laboratory scale, experience so far has shown us that both frameworks must be only different approximations of the same quantum gravity theory, when suitable limits are taken. Such a unified quantum gravity theory should become important at the Planck length scales and the Planck energies.

Several approaches for quantizing general relativity have been tried. These include covariant quantization, the canonical method, the path integral approach, and other similar methods. Attempts have also been made to modify general relativity, such as higher-order gravity. None of these ideas has worked successfully and they run into one or more formidable difficulties. The main reason behind such difficulties appears to be the rather unconventional structure of general relativity compared to other fundamental theories of physics. In general relativity, the metric tensor plays a dual role. The metric functions determine the kinematic arena, in that the metric determines the causality, light cone structure, and spacelike surfaces of spacetime. On the other hand, it is also a dynamical quantity playing the role equivalent to the Newtonian potential. Thus, there is no a priori background given in general relativity in which one propagates the physical fields.

In contrast, for example, to the Maxwell theory, the background is the Minkowski spacetime in which electromagnetic fields are characterized by the field tensor. Here the flat spacetime provides a kinematic framework in which the Minkowski flat metric defines the light cones and the spacelike surfaces in which the field propagates. Given the values of the field on one spacelike surface, the values on a future spacelike surface are determined. But in the general theory of relativity, no background spacetime is given a priori. It must be constructed by solving Einstein's equations for the metric potentials. These potentials then determine spacetime as a kinematic arena and play the role of dynamical variables also. Such a dual role basically is a characterization of the equivalence principle, giving Einstein's theory its elegance. But it also raises conceptual difficulties when we try to quantize the gravitational force.

Covariant Approach

The covariant approach attempts to solve the quantization problem by mimicking the method for electromagnetic fields. Here the metric of general relativity is broken into two parts. These are the flat metric and an additional part that contains the rest of the nonlinearities of the metric tensor. There is much conceptual simplicity in this method in that to first order in the perturbed part this gives a free spin-2 field theory in the Minkowski spacetime. Including the full nonlinear part reduces the general relativity to a self-interacting spin-2 field theory in the flat Minkowski spacetime. This allows us to use the familiar perturbation methods of quantum field theories to obtain a formal perturbation series. But the major trouble is that the perturbation theory generated this way is not renormalizable. The squared Planck length, ℓ_p^2, enters the perturbation series as an expansion parameter in this case. To achieve finiteness it is required that the series be finite in each order but there are many difficulties in achieving this goal.

The recent supergravity theories have been an initiative in a similar spirit. The idea was to couple gravity with suitably chosen matter fields. The hope is that the infinities of the bosonic fields, including those of gravity, will be cancelled by those of the fermionic fields to get a renormalized theory of gravity interacting with matter. The most refined theory here, which is called the $N = 8$ supergravity, has several good features such as some cancellation of infinities. The Hamiltonian is manifestly positive and the theory is unitary as well. The drawback, however, is that again the theory is not renormalizable. Despite many fresh attempts, there is no solution to these issues in sight.

Loop Quantum Gravity

The other major quantization effort, namely the canonical quantization, can be used for a theory cast in the Hamiltonian form. This is possible for the general theory of relativity. The state of the system is described by wave functions of the configuration variables and the time evolution of the system is determined by the Schrödinger equation that uses the Hamiltonian operator derived from the classical Hamiltonian of the theory. The serious issue here has been a constraint equation which must be solved in order to reduce to the variables which represent only the true dynamical degrees of freedom. Several attempts have been made to get rid of this issue, the most successful being the loop quantum gravity approach by Abhay Ashtekar and others. This defines a set of new variables to deal with the equations of canonical quantum gravity.

In the usual set of canonical variables, the canonical equations are non-polynomial and cannot be solved in the full quantum gravity. With the introduction of new variables, all such equations become polynomials and this offers a range of possibilities for addressing traditional questions which could not be solved earlier. Particularly interesting here has been the loop space representation developed by Lee Smolin and others, where the quantum states are expressed as functionals of closed loops on a spatial three-dimensional manifold. This allows one to generate an infinite-dimensional space of solutions for all of the quantum constraints which includes the Fock space of states in the weak field limit.

Path Integrals

The path integral approach has been tried out to quantize gravity. The major problem here is the definition of measure on the space of all paths in consideration. We could foliate the spacetime and consider all possible three-geometries on each spacelike surface. The amplitude in question is that of going from a given spatial metric on the initial hypersurface to another given spatial geometry at the final spacelike hypersurface. Each of these amplitudes is given by the action functional and the path integral is the sum over all such possible paths. However, the foliation given for the spacetime is not unique but is quite arbitrary.

Therefore, the initial and final times given here are rather arbitrary. So the physical meaning of the amplitude here is far from clear, apart from the question of measure mentioned above.

String Theories

An effort toward quantum gravity has been the development of string theories in particle physics. Here, instead of point particles and associated fields, one-dimensional objects, called strings, are considered as fundamental physical objects. In that case, familiar physical particles, including the zero rest mass spin-2 gravitons, arise as different components of these string excitations. The theory can have only one free parameter and hence it needs a very minimal external input, with the coupling fixed automatically. So this theory could have been the theory of everything, containing all physical information. The string model would be unitary and it is believed it is finite order by order in the perturbation theory. The trouble, however, is when summed, the perturbation series diverges quite powerfully. Of course, such divergences are also there in theories such as quantum electrodynamics. But quantum electrodynamics already assumes the flat Minkowski background, continuum spacetime structure, and so on. So we could blame such infinities on these external factors. On the other hand, string theory leaves very little outside the scope of itself and must solve the problem of infinities within its own parameters.

Other Approaches and the Outlook

In addition to the methods just described, other approaches to achieving the quantization of the gravitation field have also been tried. They include the Euclidean path integral approach of Stephen Hawking, and the twistor approach initiated by Roger Penrose.

Researchers have also tried to modify Einstein's equations so that the resulting quantum gravity turns out to be renormalizable via the covariant approach. Here the Lagrangian is modified by adding terms quadratic in curvature which means the field equations involve the fourth derivatives of the metric tensor. It turns out that these approaches have their own difficulties of one kind or another.

The present state of quantizing the gravitational field and the history of efforts to do so indicate that the usual perturbative approach that worked so well in particle physics and quantum field theories may not produce the desired results. It looks clear that before we can formulate a working theory of quantum gravity, profound conceptual issues and problems involving global and topological concepts must be addressed. These may concern the very fundamental nature and structure of spacetime itself. It also appears necessary to understand better the non-perturbative aspects of gravity as implied by the basic approach in general relativity and the role these may play in quantum gravity.

In fact, we should emphasize that the attempts toward quantization of the gravitational force very probably involve non-perturbative and global questions concerning the structure of spacetime itself. Without such an effort, progress here does not appear very likely. It is possible that in string theory, the non-perturbative and global methods might lead to a solution of the current problems.

All the same, we note that despite major issues of quantum gravity remaining unsolved, the theory of a quantized free matter field in a fixed curved background is a well-defined issue which has been studied in detail. This is only an approximation to the full quantum gravity with quantum matter fields, but it gives an idea of the possible quantum effects in the full theory. Such a theory predicts the creation of particles by the gravitational field, and when applied to the case of a black hole geometry, this particle creation implies a thermal spectrum of emission with a temperature proportional to the surface gravity of the black hole as was shown by Hawking in 1975. Such an effect also shows an interesting relationship between black holes and thermodynamics.

In summary, at present, we have no mechanism or complete theory to deal with both quantum effects and the intense force of gravity together. However, a quantum gravity theory should take over from purely classical general relativity near spacetime singularities and in the very advanced stages of the gravitational collapse of massive stars. Spacetime singularities may indicate the incompleteness of the classical gravity theory. It is possible that when quantum effects are combined with the gravitational force, the classical singularity of Einstein's gravity may be resolved.

Need for Observational Data

In view of the scenario mentioned above and taking into account the enormous difficulties that we have been facing in constructing a quantum gravity theory through purely theoretical means, it is clear that some kind of observational inputs will be essential.

We need to ask where the quantum gravity effects will be most important in the Universe. Two possibilities that emerge are the early Universe, and the late stages of a massive star's gravitational collapse. In each of these cases we have the situation where the densities and spacetime curvatures grow to extreme limits so quantum effects will become very important. The early Universe is the vicinity of the Big Bang singularity, and as for collapse, it would create a singularity either within a black hole or as a naked singularity. Thus, rather natural candidates for the regions where quantum gravity effects will be important are the vicinity of the spacetime singularities in the Universe.

As such, any quantum gravity theory should have certain observational implications in a natural manner. This is because, first, the Planck energies are only a few orders of magnitude higher than the energies involved in the grand unified theories. So the possibility is not ruled out that quantum gravity could have useful implications for the grand unified theories of weak and strong interactions. Second, general relativity itself predicts the existence of spacetime singularities where curvatures and densities become unboundedly high, as we pointed out. Near the singularity and near the Planck length scale, quantum gravity will be important and the physics close to the singularity will be described by quantum gravity only. In this way, for example, we need quantum gravity to describe the very early Universe near the Big Bang singularity which was the origin of the Universe. The behavior of physical fields in this phase will have observational implications for the later evolution of the Universe and the observations which are subsequently made.

Similarly, another important physical issue is whether the extreme gravity regions formed during a massive star's gravitational collapse are visible to external observers in the Universe. An affirmative answer means that such a collapse provides a good laboratory for studying quantum gravity effects in the Cosmos. This may generate clues for an as-yet unknown theory of quantum gravity. Quantum gravity theories in the making, such as string theory or loop quantum gravity, are very much in need of some kind of observational data and inputs. Without that, it appears nearly impossible to constrain the plethora of possibilities.

The key point is to try and observe regions in the Universe where quantum gravity effects are important and dominate the physics. If we can get observable signals and data from such regions, that would possibly provide us with quite useful clues toward constructing a proper and complete quantum theory of gravity.

Singularity Resolution in Quantum Gravity

Supposing that we do make a revolutionary breakthrough and finally construct a quantum theory of gravity, a question that is frequently asked is whether such a theory will have no singularities.

As for the spacetime singularities of Einstein gravity, in fact, the very existence of singularity could become questionable when quantum effects are taken into account. There have been several proposals for possible singularity resolution when quantum effects are incorporated. For example, Bryce DeWitt suggested in the 1960s the possibility for singularity avoidance within a quantum framework by imposing a specific condition related to the vanishing of the wave function at the singular boundary, where the state functional for the Universe vanishes at the singularity.

There have been similar attempts. As there is no quantum theory of gravity available today, several different approaches have been tried out to study this problem. These include quantizing the homogeneous and isotropic Friedmann models, or quantizing only part of the degrees of freedom of the metric tensor. The basic difficulty here is that in the absence of a consistent and complete quantum gravity theory, there is no unique scheme to apply. As was anticipated by Dieter Brill in 1975, there are a variety of ways of viewing the singularity and the quantum theory of the Universe, and depending upon the procedure used, the singularity may or may not be avoided. Hence, we must try out various alternatives as well as possible model calculations which may eventually lead to a better insight on the status of singularities in quantum gravity.

It was in this spirit that an approach to study the quantum effects near a spacetime singularity was initiated and developed by Jayant Narlikar and collaborators, where a limited range of perturbations of the metric tensor were quantized, namely the conformal degree of freedom of the spacetime metric. Using both the path integral and operator methods we can see in this case that the quantum effects must diverge in the vicinity of a classical spacetime singularity. This offers the possibility that the singularity can be avoided when the quantum effects are taken into account. No doubt it remains an open question as to how the results will be changed when the quantization of other degrees of freedom is taken into account; thus, the approach is not complete. However, it has the merit of deducing definite conclusions and to that extent it provides insight into the quantum effects near a singularity. It would seem that even when the quantization of other degrees of freedom is included, several basic features may continue to be relevant. This may be so in a semi-classical approximation where the propagation of quantum matter fields is considered on a fixed classical background. So such an approach of quantization of the conformal factor can also be viewed as the quantization of a scalar field on a fixed spacetime background.

The point above is that despite the elegance of the path integral approach, it is beset with problems such as the question of measure on the space of all paths. On the other hand, the operator approach in quantum theory is more direct and has an intuitive appeal. This is used here to find the time evolution of quantum effects, and a wide range of spacetimes can be covered to examine such effects near the singularity. We can also consider the measure of singular geometries in the space of all allowed geometries when the quantum effects are taken into account. Both the cosmological context and the homogeneous gravitational collapse can be considered for quantum effects near singularities, and the same near a Schwarzschild singularity within a black hole is also examined.

There are also views, however, that a quantum theory of gravity will not resolve singularities, suggesting that singularities are, in fact, physical attributes of nature rather than mere theoretical artifacts. The question here is whether singularities are mathematical requirements only, or are they physical? We have many cases where gravitational tidal forces diverge at the singularity. In order to understand this, we need to ask what will be the physical processes that will cause or lead to the formation of such singularities. We need to know whether such processes will be necessarily possible physically. In fact, this is one of the main reasons why we have all along preferred to study a series of explicit collapse models to find out what happens as the final state and we then try to understand the nature of the singularities that result.

In this connection, Gary Horowitz and collaborators have argued in recent years that singularities would exist in all theories of quantum gravity as well, in order to regulate these theories. The argument is that singularities must exist in such theories if these are to have stable ground states. In other words, if we argue that all singularities will be necessarily resolved in quantum gravity theories, then the theory will fail to have a stable ground state. This is how we argue for the need for singularities in the theory.

One of the main reasons why it has been argued many times that quantum gravity may be singularity free is that there will be terms in the action which are very small at low curvatures and small energies, but will become important at high curvatures. These will create violations of the positivity of energy conditions that are needed to prove the singularity theorems. But researchers have argued that certain classes of timelike singularities, which are visible singularities, cannot be resolved in this way, because if we try to resolve and remove the singularities, then the theory will have no stable ground state. So it is argued that all reasonable theories must admit timelike singularities. Thus, the claim is that for a broad class of theories this must happen. However, we do not know for sure about singularities in quantum gravity theories because their structure is not clear.

In general, the issue, namely whether singularities are merely 'mathematical', as opposed to being 'physical', is very interesting indeed. At the classical level, gravity theory certainly admits spacetime singularities, and that is a mathematically proven fact. All the same, if singularities are visible to faraway observers, then they would surely have physical consequences as well, in that the super-ultra-dense regions in the spacetime will be observable to those far away in the Universe.

Naked Singularity and Quantum Gravity

For very massive stars with tens of solar masses, which far exceed the Chandrasekhar limit which is about 1.4 solar masses, a total gravitational collapse is inevitable as it would appear quite unlikely for the star to throw off almost all its mass to become light enough to achieve stability in the form of a neutron star or a white dwarf. The final fate of collapse is then a spacetime singularity of infinite curvature and density. Quantum gravity effects will be important in the advanced stages of such a collapse at the scales of Planck length. When taken into account, these may avoid the singularity, but any definite formulation of this idea is far from achieved and calls for an as-yet non-existent quantum gravity theory.

In any case, a region of matter as compact as on the order of Planck lengths where quantum gravity effects become important may be considered a spacetime singularity for all practical purposes. If such a singularity is always covered by an event horizon and hidden within a black hole, as happens for the spherically symmetric homogeneous dust collapse, the censorship holds. On the other hand, the occurrence of a naked singularity gives us the possibility of observing super-ultra-dense regions in the Universe and the physics taking place there. For both possible outcomes of collapse of a dying massive star, namely black hole or naked singularity, we find the formation of a singularity, a wad of matter so dense that the laws of physics break down and we need new laws to describe it. Anything that hits the singularity is destroyed.

In the case of a black hole forming, the singularity is 'clothed' by the event horizon. Nothing that falls through this surface can ever get back out, and no information escapes. On the other hand, objects that fall toward a naked singularity, visible to outside observers, can in principle reverse course right up to the moment of impact. Conventional wisdom has it that a large star eventually collapses to a black hole, but new theoretical models that we have discussed here suggest that it might instead become a naked singularity. Sorting out what happens is one of the most important unresolved problems in astrophysics and cosmology. The discovery of naked singularities would transform the search for a unified theory of physics, not least by providing direct observational tests of such a theory.

As discussed above, it is possible that when we have a reasonable and complete quantum theory of gravity available, spacetime singularities, whether naked or hidden inside black holes, will be resolved. As of now, it remains an open question whether quantum gravity will remove singularities. The occurrence of spacetime singularities could be a purely classical phenomenon, and whether they are naked or covered should not be relevant, because quantum gravity will possibly remove them all.

In any case, the important and real issue is whether the extreme strong-gravity regions formed due to gravitational collapse are visible to faraway observers. It is quite clear that the gravitational collapse would certainly proceed classically, at least till quantum gravity starts governing and dominating the dynamical evolution at the scales on the order of the Planck length, i.e. till the extreme gravity configurations have already developed due to collapse. The point is, it is the visibility or

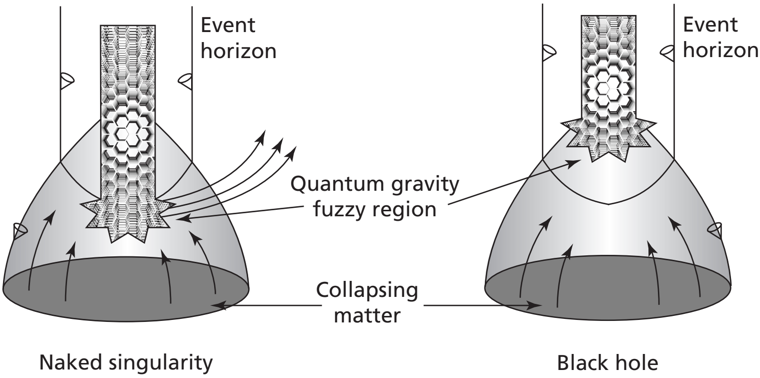

Figure 9.1 Even if a naked singularity is resolved by the quantum gravity effects which are inevitable in super-high-energy domains, the ultra-strong-gravity region that develops during gravitational collapse will still be visible to external observers in the Universe. This is quite opposite to the black hole scenario where such a possibility can never be realized.

otherwise of such ultra-dense regions that is under discussion, whether they be classical or quantum (see Fig. 9.1).

What is important is that classical gravity implies the existence of ultra-strong gravity regions, where both classical and quantum gravity come into their own. In fact, if naked singularities do develop during gravitational collapse, then in a literal sense we come face to face with the laws of quantum gravity, whenever such an event occurs in the Universe. In this way, the gravitational collapse phenomenon has the potential of providing us with a possibility of actually testing the laws of quantum gravity. In the case of a black hole developing during the collapse of a finite-sized object such as a massive star, such strong-gravity regions are necessarily hidden behind an event horizon of gravity, and this would be well before the physical conditions became extreme near the spacetime singularity. In that case, quantum effects, even if they caused qualitative changes closer to the singularity, will be of no physical consequence, as no causal communications are then allowed from such regions.

On the other hand, if the causal structure were that of a naked singularity, then the communications from such a quantum gravity-dominated extreme-curvature ball would be visible in principle. This will be so either through direct physical processes near a strong-curvature naked singularity or via secondary effects, such as the shocks produced in the surrounding medium.

In view of recent results on gravitational collapse, and various problems with the black hole paradigm, a possibility worth considering is the delay or avoidance of horizon formation as the star evolves while collapsing under gravity. This happens when collapse into a naked singularity takes place, where the horizon does not form early enough or is avoided. In such a case, in the late stages of collapse, a star radiating away most of its mass might offer a way out of the black hole conundrums, while also resolving the singularity issue, because now there is no mass left to form the curvature singularity. The purpose is to resolve the black hole paradoxes and avoid the singularity, either visible or within a black hole, which actually indicates the breakdown of physical theory. The current work on gravitational collapse suggests interesting possibilities in this direction.

Quantum Stars?

We could say quite realistically that a laboratory similar to that provided by the early Universe is created during the collapse of a massive star. However, the Big Bang, which is also a naked singularity in that it is in principle visible to all observers, happened only once in the life of the Universe and is therefore a unique event. But a naked singularity of gravitational collapse could offer an opportunity to explore and observe the quantum gravity effects every time a massive star in the Universe ends its life.

The important questions we could ask are: If in realistic astrophysical situations the star terminates as a naked singularity, would there be any observable consequences which reflect the quantum gravity signatures in the ultra-strong-gravity region? Do naked singularities have physical properties different from those of a black hole? Such questions underlie our study of gravitational collapse.

Let us consider the scenario when a collapsing star terminates as a naked singularity, making the ultra-super-strong-gravity regions visible to external observers. In this context, we have considered a cloud that has collapsed into a naked singularity final state, and introduced quantum gravity effects as suggested by the loop quantum gravity formalism. What we found was that the quantum effects generated an extremely powerful repulsive force within the cloud. Classically the cloud would have terminated as a naked singularity, but quantum effects caused a burstlike emission of matter in the very last phases of collapse, thus dispersing away the star and dissolving the naked singularity. The density remained finite and the singularity was avoided. We could possibly expect this to be a basic feature of other quantum gravity theories as well.

For a realistic star, its catastrophic collapse at the end of its life-cycle takes place in a matter of seconds. A star that has lived millions of years collapses in only tens of seconds. It is shown by the above results that in the very last fraction of a microsecond, almost a quarter of its total mass must be emitted due to quantum effects. Therefore this would appear like a massive abrupt burst to an external observer far away. Typically, such a burst would also carry with it specific signatures of quantum effects taking place in such ultra-dense regions. In our case, these included a sudden dip in the intensity of emission, just before the final burstlike evaporation due to quantum gravity.

The question is whether such unique astrophysical signatures may be detected by modern experiments and observations. In that case we may ask what they would reveal about quantum gravity, and whether there are any new insights into other aspects of cosmology and fundamental theories of physics such as string theory.

What we have found is that the stage at which the burst occurs during the gravitational collapse and the energy emitted depend on a quantization parameter in the theory. It is interesting to note that for all theoretically favored choices of this parameter the energy emitted in the radiation would be extreme. The emission mechanism of the burst must be further investigated. The constituents of such a burst may include extreme-energy gamma rays, cosmic rays, and neutrinos. This raises the interesting possibility of using upcoming experiments such as the Extreme Universe Space Observatory (EUSO) hosted by the JEM space station, which may have the needed sensitivity to provide us with a test of this prediction. These experiments could provide us with a proof or signature of quantum gravity. Future astronomical experiments could then constrain the parameters in quantum gravity in the same way as particle accelerators at CERN and Fermilab constrain the parameters for the standard model.

Interestingly, these experiments, which may constrain the value of the quantization parameter in the theory, would also have consequences for cosmology. This is because loop quantum cosmology on which our work is based also changes the picture of cosmological dynamics in the very early Universe. For example, it can change the way inflation occurs and it has been shown that such a change has observable signatures in the cosmic microwave background radiation (CMBR), in the form of a suppression of power at large scales, as observed by the Wilkinson Microwave Anisotropy Probe (WMAP). Observable effects of quantum gravity in CMBR are also controlled by the same quantization parameter which determines the details of energy emission in the burst we have considered, and the way a dying star would dim before the burst occurs. Thus, any constraints on the quantization parameters would have direct consequences for cosmology as well as astrophysics.

The key point here is that the very final ultra-dense regions of the star are no longer hidden within a horizon as in the black hole case. Therefore, the exciting possibility of observing these quantum effects arises now, independently of the quantum gravity theory used. An astrophysical connection to extreme high energy phenomena in the Universe, such as gamma ray bursts that have defied any explanation so far, may not be ruled out. Japanese researchers have also examined the possible generation of gravity waves from such ultra-strong-gravity regions.

We note that the naked singularity is resolved once the quantum gravity effects are introduced. Such a resolution of naked singularity through quantum gravity would be a philosophically satisfying possibility. Then, when a massive star undergoes a gravitational collapse, this might create a laboratory for quantum gravity in the form of what we may call a *quantum star*, which we may possibly access, also suggesting intriguing connections to high energy astrophysical phenomena in the Universe. The present situation poses one of the most interesting challenges which have emerged through work on gravitational collapse.

10

The Frontiers

The work of recent years in gravitation theory and on collapse and black holes brings out several most intriguing and exciting prospects and possibilities, at both the theoretical and observational levels. We discuss here some of these emerging directions.

As an example, an important question is, if naked singularities which emerge as plausible astrophysical objects do occur in nature, then how do we distinguish them from their black hole counterparts? Could major observational missions, the existing ones as well as those coming up, help in this direction? Recent work points to an answer in this connection and we will consider the developments here as to how to distinguish black holes and naked singularities observationally.

Other important evolving directions include an increasing attention to the black hole paradoxes in recent years. While black holes form during collapse in gravitation theory, it is being realized that the existence of event horizons causes some rather fundamental and formidable difficulties, such as the information paradox as well as the teleological paradoxes. We discuss some of these issues and point out how recent work on gravitational collapse points to possible resolutions.

Observational Frontiers

It is clear that black holes and naked singularities, as outcomes of a massive star's gravitational collapse, are radically different from each other in their theoretical and basic properties. So their observational properties should also be quite different. If naked singularities exist in nature, would they be observationally distinguishable from their black hole counterparts? Will they offer any unique observational signatures which we can use to possibly detect them?

Considering the observational aspects of the Universe, the centers of galaxies are expected to harbor supermassive black holes, or in any case a huge central massive core. The active galactic nuclei (AGNs) may be powered by a black hole central engine. It is also believed that powerful gamma ray bursts originate from the collapse of massive stars. We consider here the implications of recent developments such as these, and inquire into the possible observational signatures of naked singularities if they exist in nature.

As for the work on the gravitational collapse of massive stars, its relevance and impact in astrophysics and cosmology should be highlighted. The main physics excitement is the possible observational consequences of the physical processes in the super-ultra-dense regions that may form as a result of a star's gravitational collapse, and which may be visible to external observers in the Universe. Apart from this, at a theoretical level the entire issue is crucial to black hole physics and relativistic astrophysics, as is widely agreed upon today.

As for the astronomical missions, the basic issue is one of sensitivity. How accurately and precisely can we measure and determine the concerned parameters such as rotation? A number of present and future astronomical missions could be of help here. One of these is the square-kilometer array (SKA) radio telescope. It has a collecting area exceeding a factor of hundred compared to that of other radio telescopes. SKA astronomers have pointed out they will have the desired sensitivity to determine the vital fundamental issues in gravitation physics, such as cosmic censorship and no-hair theorems, and to decide on their validity.

Other missions that could, in principle, provide considerable observational data are those currently hunting for gravitational waves. Gravitational wave astronomy will soon probably claim its first detection of waves. In coming years the first observations should be made by experiments such as LIGO and VIRGO, which are currently just below the threshold for observation. Then gravitational wave astronomy will become an active field with possibly large amounts of data to be checked against theoretical predictions, which will almost certainly have a strong impact on open theoretical issues such as the cosmic censorship problem.

Testing Censorship using Astronomical Observations

With many high power technology missions to observe the Cosmos, can we not just observe the skies carefully to determine the validity of cosmic censorship? What could astronomical observations possibly tell us about censorship and its validity?

To answer these questions, first, we need to distinguish the two main candidates for naked singularities, which are quite different from each other and bear rather different observational signatures. The first, which has been discussed in detail already, is the naked singularity that results at the end of a spherical collapse. When we are dealing with a star's collapse, a naked singularity will happen at the center of the collapsing cloud and eventually will most likely be covered by the event horizon at the end of the collapse. This is a transient event and so its observational signature may be in the form of a short-lived explosive event. Similar phenomena have been observed in the Universe, for example gamma ray bursts, which are believed to originate from the core collapse of massive stars. However, a link between such events and the possible existence of a naked singularity at the core of the collapse has not been thoroughly investigated.

On the other side, we have scenarios such as the super-spinning Kerr solution or some axially symmetric vacuum metrics. These are derived as exact solutions of Einstein's field equations and they generally present naked singularities. We do not know yet whether such configurations arise from a dynamical process such as collapse. So one could ask whether these metrics could represent some real object existing in the Universe. How these kind of objects form from the collapse of an initially regular star is still unanswered and little is known about the behavior of a realistic source during the final stages of collapse. However, recently researchers have investigated these vacuum solutions in order to understand what observational properties they might have and to see whether they could be detected by current observations.

The point is that if we see objects with sufficiently high angular momentum compared to their mass, then the possibility of the nonexistence of an event horizon increases drastically, and we may be able to say whether naked singularities exist in astrophysical reality. The prime survey targets here include the innermost regions of our galaxy and globular clusters, which are vast collections of the oldest stars.

Super-Kerr Geometries

The rotating Kerr metric is a solution for a source with angular momentum. Here the nature of a compact object, or the super-massive central object at the center in a galaxy, is decided by the ratio of its mass and the spin angular momentum per unit mass. For a rotating object described by the Kerr metric, if the ratio of angular momentum to mass is smaller than 1, then this is a black hole, but for a larger ratio it is a naked singularity, as the event horizon then does not exist. Such a Kerr naked singularity could form from three different processes. The first is the complete collapse of a rotating star with mass exceeding the neutron star limit. The possibility that collapse of such an object forms a super-spinning Kerr metric has been suggested by researchers and has been also investigated in some numerical simulations. Also, a Kerr naked singularity can form by the inflow of angular momentum due to accreting particles or the merger of rapidly spinning compact objects.

We could measure the mass and spin ratios for compact objects and for the galactic center. There have been suggestions to use the shadow cast by the compact object to test this ratio in stellar-mass objects, or using the X-ray energy spectrum emitted by the accretion disk might also be useful. It is also possible to use certain observable properties of gravitational lensing that depend upon the rate of spin. A number of proposals to measure this carefully have been made by Andrzej Krolak using pulsar observations, gravitational waves, and the spectra of X-ray binaries.

Could such naked singularities form in some realistic physical processes? From the measurement of X-ray binary systems there are indications that black holes with a mass-to-spin ratio very close to unity exist in the Universe. Furthermore, compact objects such as neutron stars with very high spin can also exist in nature. It would also be interesting to study how the spin of an object is affected when it undergoes an inflow of mass and angular momentum due to an accretion disk. Such inflows have been studied in order to understand whether the particles falling onto a black hole can contribute to increasing the angular momentum, thus reaching the critical limit and removing the horizon.

The idea of over-spinning a black hole can be traced back to a thought experiment proposed by Robert Wald and it has important implications for cosmic censorship. These are some of the astrophysical possibilities by which a Kerr singularity, either covered or naked, could be obtained. If such a process proved to be physically viable, then the investigation of the observational properties of Kerr naked singularities would become important for astrophysics. Similarly, the inverse procedure has also been investigated, amounting to slowing down a Kerr naked singularity to a black hole by means of infalling counter-rotating particles, which has a greater efficiency than the opposite process.

Studies have been carried out to understand how the process of angular momentum transfer works during the merger of two compact objects such as neutron stars with high spin. If the angular momentum is not dissipated away during the merger, these events could give rise to a final configuration in the form of a super-spinning Kerr singularity. Numerical simulations by Luciano Rezzolla have examined such possibilities and these indicate that the 'sub-Kerr' models are more likely to happen in such processes.

The Kerr metric is, of course, only one of the vacuum exact solutions with angular momentum. There are other such solutions too, and the Kerr is only one of the members of this family. Cosimo Bambi recently tried to study the observational properties of some of these metrics. But at present it is not clear whether the rotating compact objects in the Universe are represented by the Kerr metric or some more complicated axially symmetric spacetimes.

Observable Signatures of Naked Singularities

As we discussed, the black hole has a sibling, the naked singularity. Physicists long thought and hoped that it did not exist. But the stars may have a different story to tell. What we may really need is to check for the observable signatures of naked singularities, if these actually exist in nature.

We noted that naked singularities that appear in solutions of Einstein's equations can take various forms. Mainly, they could be 'long-lived' as in the case of super-Kerr solutions, or 'short-lived' as in the case of those forming in certain classes of gravitational collapse models. So possibly their observational features might be different as well. If such objects do exist, then it is important in astrophysical considerations to study how they might interact with the surrounding environment. This will help us understand whether they can be observed and how.

So the key question when considering observational aspects of naked singularities is, would there be any observational signatures emerging from their vicinity, and if so, can we distinguish the signature from that of other astrophysical objects, such as black holes?

We have noted that the singularity itself is not a part of the spacetime, so we are actually considering the ultra-dense regions near the singularity where general relativity breaks down and quantum gravitational effects become operational. So we can argue that quantum gravity theory is needed to resolve the quantities that diverge and make predictions on the observational signature of such objects. But a naked singularity may present some observable features at the classical level in view of its distinct causal structure. To check the observable features of a naked singularity, we can consider relativistic effects at extremely high densities, or study effects due to modified theories of gravity for interiors with very strong fields, or model the possible quantum gravity effects either in the semiclassical approximation or by adapting theories such as loop quantum gravity or string gravity, at least for some toy models.

Where could the observational signatures of naked singularities lie? If we consider naked singularities that appear as collapse end-states, we need to consider explosive and high energy events in the Universe. These models expose the ultra-high-density region at the time of formation of the singularity while the outer shells are still falling inward. In such a case, powerful shockwaves emanating from the super-dense region at scales smaller than the Schwarzschild radius, possibly due to quantum effects or repulsive classical effects, and collisions of particles near the Cauchy horizon could have effects on the outer layers. These would be quite different from the black hole formation case, where the most dense regions are confined within the horizon and do not communicate with the exterior.

On the other hand, naked singularities such as those of the super-spinning Kerr solution will have different kinds of observational signatures. The most prominent features would be how the singularity could affect incoming particles, in the form of light bending in gravitational lensing, in particle collisions close to the singularity, or in the properties of accretion disks, as we will discuss later.

A short-lived naked singularity should look more like a transient event. The search for cosmic point sources and transient events has been, of course, going on for many years. For example, the construction of the ice cube experiment at the Amundsen–Scott South Pole station was completed some time ago. The detector has thousands of photomultipliers deployed and buried in ice at about two kilometers. One of the main goals of the ice cube experiment is to detect astrophysical sources of high energy neutrinos and to eventually identify the sources of high energy cosmic radiation. Also, using timing and directional information from other telescopes, more focused searches for neutrinos in coincidence with observed events such as gamma ray bursts (GRBs), active galactic nuclei (AGN), pulsars, and other sources could be made. Important information on the nature of sources might emerge from such analysis.

Also, GRB astronomy is progressing well with instruments such as the Fermi Gamma Ray Space Telescope. GRBs are now known to be certainly the most powerful explosions in the Universe. They release an energy of something like 10^{51} ergs in tens of seconds in gamma rays. As of now, the most distant measured redshift for GRBs is close to 9 or even of order 10 for some galaxies. The detectability of GRBs at such high redshifts encourages us to believe that they may become important cosmological probes. The origin of the bursts lasting longer than a few seconds is now believed to be a core collapse supernova of a massive, rapidly rotating star. On the other hand, shorter events could be generated from neutron stars or black hole mergers.

While objects such as black holes are frequently invoked to explain the dynamics of GRBs, as yet no clear theoretical understanding of them is available. It is, however, clear that the gravitational collapse of massive stars at the end of their life-cycles is the root cause of the long GRBs. As black holes and naked singularities are the natural outcome of such a collapse, it is certainly interesting to ask whether GRB explosions exhibit the observational signatures of naked singularities. Although researchers have looked into this question in some detail, a more extensive analysis of different aspects of the problem is necessary.

As for the experimental and observational side, it would be good to know in the collapse ending in a naked singularity, whether we are able to make an estimate of the range of expected gamma and neutrino luminosities, and the corresponding spectra, as produced by such an ultra-dense fireball, which is the naked singularity. It would also be good to assess the star mass range that would lead to such a fireball, or what is even better, the proper event rate in time and a volume of space, at the present epoch, so that a flux can be estimated.

From an experimental perspective, the prediction of a jet of gamma rays and neutrinos not associated with a visible optical source could be possible proof of a naked singularity. Questions could be asked, such as what is the likely energy of the neutrinos? There have been suggestions from Govind Swarup and others for a search for ultra-high-energy neutrinos of $> 10^{21}$ eV, possibly using a multibeam system of the giant meterwave radio telescope at Pune, and also the Ooty radio telescope of TIFR. What would be the possibility of finding a naked singularity source then?

It is clear that having obtained the theoretical implications from general relativity, in the end we must see what the observations imply and teach us. So far our effort has been to see what Einstein gravity tells us on the problem of collapse, and the physicists have been busy exploring the models available within its framework. The theoretical findings so far and the observational prospects emerging could bring new young researchers closer to the problem of cosmic censorship.

Distinguishing Black Holes and Naked Singularities

The black hole candidates from observations belong to mainly two classes. These are stellar-mass black holes that form either from the collapse of very massive stars or from merger and accretion processes. When they have an orbiting companion, we can measure their mass from the orbital periods of the companion. Similar measurement can be made when the black hole is surrounded by a gas disk. Then there are supermassive black holes possibly at the center of galaxies. Their total mass can be estimated from the orbital period of nearby stars, which is typically in millions of solar masses. For both kinds of black hole candidates the angular momentum can be inferred from the analysis of the observed X-ray emission. But this measurement would rely on some assumptions and it is not clear whether Kerr naked singularities would present a similar spectrum.

The idea behind such investigations is to examine the physical reality of naked singularities. From works such as these, we see that naked singular spacetimes cannot be ruled out with the present knowledge that we have of these sources. These theoretical models provide a framework for testing observations coming from stellar and supermassive black hole candidates. In fact, some observations in the millimeter wavelength of the supermassive object at the galactic center suggest that the Kerr limit might be broken.

As for the naked singularity, the key question is what could be observed that would distinguish between black hole and naked singularity? While gravitational collapse has been investigated extensively over the past decades within the framework of gravitation theory, what needs to be explored are the observable signatures that would distinguish black holes from naked singularities, which are hypothetical astrophysical objects in nature predicted by the gravitation theory. We must then explore what special astrophysical consequences the latter may have.

We discussed earlier the existence, and then the genericity and stability of occurrence, of naked singularities in a gravitational collapse. As for singularities that result from collapse which may have observational signatures in the form of an explosive event, at present we do not know exactly how they would look different from an explosive event originating from collapse where the central ultra-high-dense region is a black hole.

It can be argued that gravity is attractive and nothing opposes gravity at the very last stages of collapse, so nothing could in principle come out of the singular central region that develops at the end of collapse. Also, the core of a collapsing star would be so dense that the mean-free path of a particle trying to escape would be extremely short, so perhaps it is unlikely that anything can escape from that region. On the other hand, at present we are not able to consider quantum effects that appear at the core when density reaches critical values. Possibly these will give rise to strong negative pressures that can disrupt collapse and create a shockwave that might propagate to the outer shells, thus dissipating away the mass of the star.

Also, the simulations of core collapse supernovae find it difficult to duplicate what happens at the last stages of collapse. They find some energy missing in such a way that the explosion cannot, in fact, take place. So it is possible, in principle, that this energy may come from a shockwave emanating from the central non-trapped ultra-dense region, and that naked singularities could, in fact, trigger the type II supernovae. At present, this is a matter of speculation and if we are to distinguish black holes from naked singularities observationally, it might help to consider axially symmetric naked singular solutions such as the Kerr spacetime.

In such a case, assuming that a Kerr naked singularity or some other object with a visible singularity exists in the Universe, we may ask: How can we distinguish it from a black hole or another compact object of the same mass? At present, there are three main observational possibilities that can be used to distinguish a naked singularity from a black hole.

Accretion Disk Properties

The first one is the study of accretion disks. It has been shown that the accretion properties of particles falling onto a naked singularity would be very different from those of a black hole of the same mass, and that the resulting accretion disks would be observationally very different in each case. The properties of accretion disks have been studied in terms of radiant energy, flux, and luminosity, in a Kerr-like geometry with a naked singularity and the differences from a black hole accretion disk have been investigated.

Also, a naked singularity gives rise to powerful repulsive forces that create an outflow of particles from the accretion disk on the equatorial plane. This outflow, which is not present in the black hole case, could be, in principle, distinguished from the jets of particles believed to be ejected from a black hole's polar region and which are due to strong electromagnetic fields. Further, when charged test particles are considered, the accretion disk properties for the naked singularity in the Reissner–Nördstrom spacetime are seen to be observationally different from those of black holes.

Gravitational Lensing

Another way of distinguishing black holes from naked singularities would be to use gravitational lensing. It has been argued by researchers that in many important cases the lensing features of light passing close to a naked singularity will be observationally different from those of a black hole. Several different configurations have been studied, including a Kerr-like solution of Einstein's equations for the center of galaxies and the lensing properties were studied. It was found that the behavior of the bending angle of light rays will be affected differently in each case. This would make these objects observationally different from black holes. The possibility is that if the center of a galaxy is rotating fast enough to harbor a naked singularity, the observational signatures will manifest in gravitational lensing.

Particle Collisions near a Naked Singularity

Finally, a third way of distinguishing black holes from naked singularities is from particle collisions and particle acceleration near the singularity. It is actually possible that the repulsive effects due to a singularity can deviate a class of infalling particles, making them outgoing eventually. These could then collide with some other ingoing particle and the energy of collision could be arbitrarily large. This energy depends on the impact parameter of the outgoing particle with respect to the singularity. The net effect is thus creating a very high-energy collision similar to that of an immense particle accelerator, which would be impossible in the vicinity of a Kerr black hole. In recent years such possibilities have been examined in detail.

Shockwaves near a Singularity?

We are investigating the question here as to how would an event that leads to a naked singularity be observationally distinguished from one that will be shrouded by a horizon. Can one be quantitative, even in the framework of a thought experiment?

Apart from what we discussed earlier, a possibility that may arise is would there be shocks generated within the cloud in later collapse stages? Let us consider a collapse event that goes to a naked singularity. Basically, this means that as the collapse progresses, there is no light or matter trapping developing. As a result, the super-ultra-dense regions forming in collapse are visible in principle and no longer wrapped under horizons. Now we know from our detailed collapse studies that basically this happens because of inhomogeneities in densities and pressure profiles close to the center of the cloud. As the current physics of shockwaves tell us, in the ultra-dense regions, such inhomogeneities would give rise to powerful shocks. Can these shocks be used to blow away most of the star during the very late stages of collapse? In this thought experiment, the star collapses, and in the very final stages, close to the visible singularity formation, a powerful shock develops due to inhomogeneity (see Fig. 10.1). So, in a matter of these final seconds, much of the star's mass is blown off, and we can probably estimate the mass and radiation thrown out to make it quantitative. Such a phenomenon also perhaps answers the question as to how the naked singularity event will be different from the black hole one, where no such phenomena can, in principle, take place.

We have several astrophysical images from the Hubble Space Telescope, and also other astronomical missions have obtained images which vividly display astrophysical shocks. Such shockwaves produce glowing filaments of gas, for example, and one of the brightest supernova of 2007 provided a very interesting picture. It would be interesting to examine the connection of these events to gravitational collapse.

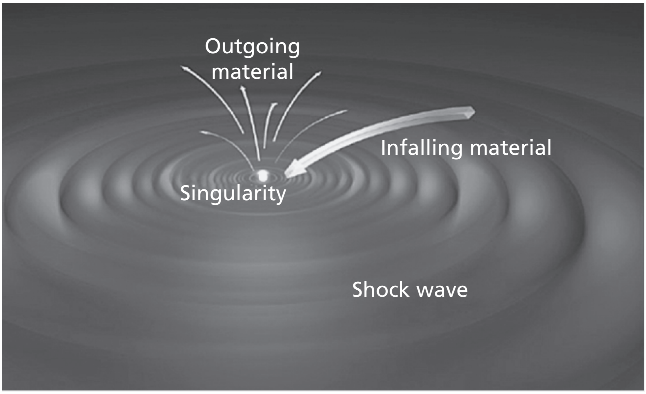

Figure 10.1 Shockwaves near a naked singularity, an artist's impression (*Scientific American*, Feb 2009; Kenn Brown, Mondolithic Studio).

What we need is somehow to get a picture of the geometry around a naked singularity using appropriate astrophysical phenomena as basic inputs. We need to make use of actual astrophysical observations of the compact gravitationally collapsing objects, and try to obtain a picture of a singularity's surroundings as it forms during a massive star's dynamical collapse. Is it possible that a key physical implication of a naked singularity could be very powerful shockwaves from its vicinity? This may characterize the geometry of the cloud around it. That some of the most powerful shocks that we observe today in the skies could be naked singularities may not be ruled out. While this is far from 'proved', there are hardly any very good explanations available for such shocks either.

To look at it another way, the very powerful relativistic shocks generated due to inhomogeneities and spacetime shear near a singularity can distort the nearby space enough to allow the matter and light to be thrown off to the exterior from regions close to the singularity due to the enormous energy of such shocks. Such a gravity typhoon has no event horizon cover and so the singularity may become a visible explosive event.

Black Hole Paradoxes

While black holes have been one of the most spectacular predictions of the general theory of relativity and play a central role in modern physics and astrophysics, it has been increasingly realized that they are plagued by fundamental paradoxes that remain unresolved to this day.

First, the black hole event horizon is teleological in nature. This means that we must know the entire future spacetime of the Universe to determine the current location of the horizon. This would be essentially impossible. Second, any information which is carried by the infalling matter is lost once the material falls through the event horizon. The black hole later evaporates by emitting Hawking radiation, but the lost information does not reappear, which has the rather disturbing consequence that quantum unitarity is violated.

The standard picture of black hole formation is motivated by the classic OSD model that studied the collapse of a dust cloud of uniform density. The key feature of this idealized collapse model, as shown in Fig. 3.3, is that the event horizon forms already at a spacetime point where local conditions in the infalling dust cloud are perfectly normal, and the density and curvature are finite. The singularity, where the curvature diverges, forms later in the future. The location of the horizon depends on the entire future history of infalling matter, including matter shells, which fall in at later times and whose very existence is unknown to the gas falling during collapse.

This illustrates the teleological nature of the event horizon, namely that its location is determined by the entire future history of spacetime, a profoundly paradoxical situation. In our view, the teleological paradox is essentially caused by the fact that the event horizon is causally cut off from the singularity, a consequence of the cosmic censorship assumption.

The second problem is that black holes run into a major conflict with quantum theory. A black hole swallows all information carried by matter falling in through the event horizon. When the hole subsequently evaporates by emitting Hawking radiation, the mass energy that was swallowed is returned to the external Universe, but in an uncorrelated mixed form that carries no information. Thus, a black hole takes in pure quantum states and converts them to mixed states. This violates quantum unitarity, which is a rather disturbing prospect and is dubbed the black hole information paradox.

The above paradoxes have attracted considerable attention over the years and various solutions have been proposed. A rather radical solution was suggested recently in which the event horizon is replaced by a firewall. According to this proposal, an infalling observer encounters a firewall of outgoing bolts of radiation at the horizon and is destroyed. Thus, the event horizon is replaced by a curvature singularity, which is the firewall associated with outgoing radiation at the horizon. The information carried in by the observer is absorbed at the firewall and is presumably returned via pure quantum states when the singularity radiates or the hole evaporates, thus solving the information paradox. Proponents of the firewall hypothesis present it as the most conservative resolution of the information paradox.

The firewall proposal, however, faces several objections, including the fact that CPT invariance (charge, parity and time reversal invariance) of quantum gravity rules out the model. In our view, the most serious problem is the fact that the firewall is by construction located at the event horizon, but the location of the horizon is determined teleologically. Somehow, the firewall singularity must sense the future spacetime and thereby decide where it ought to be located. There is no information, either under local conditions of the collapsing matter or in signals received from the past, that provides any indication that a firewall must form.

As an alternative to the firewall model, Stephen Hawking recently proposed that gravitational collapse produces only an apparent horizon but not a true event horizon, and that therefore no information needs to be lost in the collapse. In order to avoid the horizon, Hawking suggests that the region of the collapsed object inside the event horizon develops a chaotic metric and matter fields. Such a chaotic collapsed object would radiate chaotically but deterministically, so quantum unitarity is preserved. However, the chaos model again suffers from the teleological problem. Chaos must be generated at and inside the horizon, but the puzzle is how does the infalling material know that it should become chaotic when local conditions are perfectly normal and when the location of the horizon must be determined by all of the future. Moreover, for near-spherically symmetric collapse models such as those considered here, any chaos is likely to be restricted to regions close to the singularity, and the causal structure of the solution does not permit signals to propagate from this region out to the horizon.

It would seem, however, that the problems described above arise as a result of assuming CCC, which has been taken often as a foundational principle for black hole physics. The CCC, motivated by the OSD model, suggests that the singularity is always cut off from the external Universe. This has the profound consequence that it rules out any signals or communication from the singularity or its vicinity to the event horizon. In other words, no worldline at the horizon ever receives any information from the past about the singularity. The only way the horizon can find out about the existence of the singularity is by receiving information from the future, again causing the teleological problem.

We could suggest that this is possibly the root cause of the black hole paradoxes that we are facing today. Recently, Ramesh Narayan and I proposed that the above paradoxes are restricted to a particularly idealized class of collapse models and black holes in which the event horizon, which defines the boundary of the black hole, forms initially, and the singularity in the interior of the black hole forms at a later time. In contrast, gravitational collapse under more reasonable and physically more realistic initial conditions often leads to models in which the event horizon and the singularity form simultaneously. In such a case, we may mitigate the causality and teleological paradoxes and at the same time the two recently proposed solutions to the information paradox, namely the ‘firewall’ and ‘classical chaos’, are supported.

A reasonable alternative would emerge if we agree that any solution to the information paradox, such as firewall or chaos, requires information to be received at the horizon about the existence of a singularity in the spacetime, and moreover, that this information should be received via signals from the past. If such signals are able to reach the event horizon, they could carry information related to ‘new physics’ that might emerge in the vicinity of the singularity due to the extreme nature of all physical quantities there, and this information could potentially provide a causal trigger to generate either a firewall or chaos. By this reasoning, cosmic censorship and firewalls or chaos are mutually incompatible, since cosmic censorship requires the future of a worldline to determine its present behavior, which is a teleological communication.

As we have discussed here, it is now known that for large classes of physically reasonable and realistic gravitational collapse models cosmic censorship is transcended. In these solutions, the event horizon either is delayed or does not form at all, allowing a naked spacetime singularity to be visible to the external Universe. Moreover, such solutions are by no means fine-tuned. They occur under a wide range of physically reasonable initial conditions. For such more realistic collapse models, the causal structure is typically as shown in Fig. 5.3.

The key difference from the diagram in Fig. 3.3 for this scenario is that the event horizon now originates at the singularity itself, and the singularity becomes naked. Signals emitted at or near the singularity can have different histories. An infinite family of rays escapes to infinity, making the singularity at least partially visible. Some rays are trapped and fall back on the singularity, and a select few rays travel along the horizon, connecting the singularity to points on the horizon. The existence of these last rays, namely the null geodesics that emerge from the singularity and travel along the event horizon, is the key point we highlight here.

In this case, the singularity can communicate with the entire event horizon. So new physics, e.g. firewall or chaos, could thus be triggered on the horizon by signals from the singularity, i.e. from the past. Because of the presence of a naked singularity, this model violates cosmic censorship, which is rather restrictive and is confined to special models such as the OSD case. Correspondingly, there is no teleological paradox. A wide range of physically reasonable initial conditions of the cloud gives collapse with the causal structure shown here.

From the study of many realistic collapse models, it follows that the causal structure shown in Fig. 5.3 is as plausible as the standard OSD solution of Fig. 3.3, and possibly more realizable in realistic physical situations. The model in Fig. 5.3 produces a radically different scenario from that of the OSD picture of collapse, which has formed the basis of all discussions so far on the causal structure of black holes, the information paradox, cosmic censorship, etc. Fluid at the center of the cloud with an initial density peak no longer enters the horizon when it is physically very regular with modest density and curvature. Instead, by the time this matter reaches the horizon, it has already attained extremely high densities and has an arbitrarily large spacetime curvature. We expect the matter to behave very much like the hot Big Bang in reverse, and to become arbitrarily hot and radiation-dominated. Moreover, as the curvature approaches the Planck scale, or other appropriate scale, new physical phenomena associated with quantum gravity should emerge. Most importantly, signals from this ultra-dense region in the quantum gravity regime will flow out along the horizon, conceivably modifying physics throughout the horizon.

This could happen in two ways, namely, a causally consistent version of the firewall or the chaos model. Both models require some exotic phenomenon, a singular firewall or chaotic dynamics, to switch on suddenly at the horizon. To trigger such a behavior, a warning of some sort must reach fluid that is about to cross the horizon. Because of cosmic censorship, no such warning from the past is possible if the collapse behaves as in Fig. 3.3. In contrast, when the event horizon connects to the singularity in the past, as in the case of the naked singularity scenario shown in Fig. 5.3, signals from the singularity travel to all points on the horizon. Since the singularity is a region of extreme physical conditions corresponding to the quantum gravity regime, signals originating from here could, in principle, trigger the necessary behavior for the firewall and chaos models. We do not as yet know a specific trigger mechanism; the focus here is on establishing causality.

In one scenario, the quantum matter at the singularity is radiated away along outgoing rays. The maximum burst of radiation will arguably be along the event horizon because close to that surface and below the singularity the densities, pressures, and all other physical quantities attain their maximum and unbounded values. The firewall could then originate at the naked singularity and propagate as a singular wall of outgoing radiation. Material farther out in the cloud will approach the firewall and, even though its own local properties may be quite regular, when it hits the singularity at the firewall, it will be absorbed and will add to the firewall. We do not have any explanation of how the latter might happen, but neither was an explanation offered with the original firewall hypothesis. Our contribution here is to show that it is possible to have a firewall originate at a singular point and then evolve causally, without any need for a teleological connection to the future.

What we mean here is that a firewall or chaos model could potentially be causally consistent without having to invoke teleological properties. With reference to chaos from gravitational collapse, we know that the inner region of Kerr geometry is unstable and might be chaotic. If the collapse of a massive rotating star results in a configuration described by the Kerr metric on the outside, this may offer a way of producing chaos in the interior, as suggested by Hawking. But the unstable region does not extend outside the inner horizon and so chaos cannot propagate all the way to the outer horizon as needed to solve the information paradox. Thus the chaos model requires some other trigger to generate the necessary turbulence. Our proposal here is a possible solution.

So it is likely that some of the problems that have plagued black hole physics might be the result of (i) relying on the classical OSD picture of gravitational collapse of a constant-density cloud, in which the event horizon forms much earlier than the singularity, and (ii) assuming that this model and its associated cosmic censorship describes the generic behavior of collapse. By making use of physically more realistic gravitational collapse models, e.g. those with initial density higher at the center, a very different picture of black hole formation through collapse emerges in which the horizon and the singularity generically form at the same epoch. Such models violate cosmic censorship and can potentially resolve the event horizon and information paradoxes.

Infall into a Black Hole versus Naked Singularity

What it would be like for an observer to fall into a black hole versus into a naked singularity, and what are the differences? As opposed to the experience of falling through an event horizon, what would one see and what tidal forces and other effects would be suffered? There have been many descriptions of the descent into a black hole and we would like to know similarly what a naked singularity descent would look like.

We will try to make a few speculative remarks here as to how the descent into a naked singularity would be different from that for a black hole. As is known, for the black hole case there is the 'redshift effect' as one goes closer and closer to the event horizon. As the observer nears the horizon, the light that she emits gets dimmer and dimmer, eventually going beyond any observable limit as the person falls into the black hole. After a finite time, she is simply out of vision for a faraway observer. There is no such effect near what would be the horizon in the naked singularity case. Thus, the observer would be able to convey all the information as she goes closer to the singularity, because even if bent, the light rays do manage to come out to faraway observers in this spacetime geometry. In particular, the quantum gravity effects and signatures happening closer to the singularity can, in principle, be conveyed to observers far away in the Universe. In both cases, however, there will be growing spacetime curvatures as the observer nears the singularity. Thus, at some finite distance away from the singularity, the curvatures and densities will be so strong that the observer will be ripped out of existence.

In the naked singularity case, we could send in an un-manned mission or a robot to descend into the singularity. Such a robot would be able to send out signals and information, however close it is to this object, as long as it is not destroyed ultimately in the limit very close to the singularity. But even then and in any case, the information sent out related to the ultra-strong-gravity regions and the quantum gravity signatures obtained will be simply far greater and immense than in the black hole case, where no information would be available once the observer is close enough to the horizon.

Suppose a spaceship moves toward a naked singularity. Once it is close enough, in the same way as usually happens in the black hole case, a warning message from the central computer of the ship would go off, namely that the spaceship had begun an unplanned acceleration, just as if it were close to a planet or a star. If there is a camera pointing in the direction of the flight, in the black hole case the camera would not show any such astronomical body close by. However, in the naked singularity case, the camera pointing the direction of flight would show a small but rather bright object far away. First, the observer might think she is approaching a bright star. However, a comparison with available stellar spectra in her computer would immediately reveal this is no ordinary star, because the quantum gravity effects very near the singularity should add very different and novel unexpected features to the spectrum of the object. The unusual kind of brightness of the object would be enough to create a suspicion that this is no ordinary star, but even if not at this stage, soon she would realize she is approaching a rather novel object.

From the measurements of the acceleration and other parameters, the scientific software of the central computer of the spaceship would measure the total mass of the object harboring the naked singularity at the center as it may be. As the mission approaches closer to the naked singularity, the spacetime curvatures and accelerations increase manifold, to millions of times the usual gravitational acceleration on Earth. The camera now sees a much brighter object at the center. This brightness or the object seen is not actually the naked singularity itself. What may be seen is the quantum resolved singularity, or the compact quantum gravity region, that we may call a *quantum star*. The brightness and luminosity seen is that of the bright accretion disk around this object, and emissions from the super-ultra-high-energy collisions of particles happening in the vicinity of the object. The classical 'naked singularity', as predicted by the Einstein gravity, by itself does not or need not emit light or energy directly.

The central computer of the spaceship would analyze the spectra of the light and emissions received from the vicinity of the naked singularity and would then find many remarkable new features of super-ultra-high-energy particle collisions near the naked singularity, a cosmic accelerator as yet unseen in terrestrial experiments. Also, it may be found that the spectrum contains many remarkable but as yet totally new and unseen features and signatures, which may be the characteristics of the quantum gravity processes taking place near the singularity. These data may contain clues for the construction of a quantum gravity theory. When the scientists on Earth receive these data, it would keep them busy for a number of years to devise appropriate theories which may possibly fulfill our dream of a unified theory for nature, and where gravity and the quantum forces come together.

As the spaceship approaches closer and closer, the brightness the camera sees increases, as the accretion disk gets much brighter and more luminous. The computer could possibly relay the message: 'Brighter than a thousand suns . . .', and unlike the black hole case, this message may reach the faraway observers in the Universe.

Emerging Perspective

Cosmic censorship has been the foundation for the laws of black holes such as the area theorem and others, and their astrophysical applications. But these are not free of major paradoxes. First, all the matter entering a black hole must collapse into a spacetime singularity of infinite density and curvatures, where all known laws of physics break down. This is some kind of instability at the classical level itself. This was a reason why many gravitation theorists of the 1940s and 1950s objected to black hole formation, and Einstein himself repeatedly argued against such a final fate of a collapsing star, writing a paper in 1939 to this effect. Second, as is well known and has been widely discussed in the past few years, a black hole, by potentially destroying information, appears to contradict the basic principles of quantum theory. In that sense, the very formation of a black hole with a singularity within it appears to come laden with inherent problems. It is far from clear how we would resolve these basic troubles even if censorship were correct.

No doubt, the biggest argument in support of censorship would be that it would justify and validate the extensive formalism and laws of black hole physics and its astrophysical applications so far. But without any rigorous formulation and in the absence of any proof, such an assumption by itself does not carry us too far.

In view of such problems with the black hole paradigm, a possibility worth considering is the delay or avoidance of horizon formation as the star collapses under gravity. This happens when collapse to a naked singularity takes place, namely, where the horizon does not form early enough or is avoided. In such a case, if the star could radiate away most of its mass in the late stages of collapse, this may offer a way out of the black hole conundrum, while also resolving the singularity issue, because now there is no mass left to form the singularity.

Our discussions so far have conveyed that the final fate of collapse is one of the most fundamental issues in current physics of black holes and cosmology, and there is a broad agreement on this by scientists. This is also a popular topic of great interest for experts as well as laypersons, even those who are not physics aficionados, and one may find that the breadth of material and the outreach of the issues is compelling, the issues treated being at the frontier, and some of the most exciting ones under current debate in modern physics for justified reasons. It generates new and basic thinking on these key problems of fundamental physics.

Black holes are fundamental to modern physics today. Their connections with both fundamental physics and modern cosmology and astrophysics are most intriguing. Our goal here has been to convey that the research in this arena is far from over, and is in fact getting more interesting by the day. Gravitational collapse of massive stars plays a key role here, and spacetime singularities may have many more hidden secrets.

We believe the considerations we have gone through here show that gravitational collapse, which is essentially the investigation of dynamical evolutions of matter fields under the force of gravity in a spacetime Universe, provides one of the most exciting research frontiers in gravitation physics and high energy astrophysics. As we have shown, there are issues here which have deep relevance for both theory and observational aspects in astrophysics. These problems are of importance for the basics of gravitation theory and quantum gravity, and have inspired a philosophical interest and inquiry into the nature and structure of spacetime, causality, and predictability in the Universe.

Active research is happening in many of these frontline areas as the discussion here has shown. If we are to list some of the most interesting topics, from our own personal perspective, these may include genericity and stability of collapse outcomes in terms of black holes and naked singularities, examining the quantum gravity effects near spacetime singularities, observational and astrophysical signatures of the collapse outcomes, and other related issues.

Specifically, one of the interesting and important questions is if naked singularities, which are hypothetical astrophysical objects, do actually form in nature, what distinct observational signatures would they present? In other words, how we would distinguish black holes from naked singularities is an important issue. There have already been some developments and efforts on this issue in recent years, as we indicated earlier. The point here is that there are already very high energy astrophysical phenomena being observed today, with several observational missions working from both ground and space. Black holes and naked singularities, which are logical consequences of the general theory of relativity as we consider the gravitational collapse of a massive star, would appear to be the leading candidates to explain these phenomena. Thus, the observational signatures that each of these present, and their astrophysical consequences, will naturally be of much interest for future theoretical as well as computational research and for their applications.

In our view, there is much scope for both theoretical and observational investigations in these frontier areas, which may have much to reveal about our quest on basic issues in quantum gravity, fundamental physics, and gravity theories, and toward the expanding frontiers of modern high energy astrophysical observations.

Bibliographic Notes

We give a few references here which may be useful to a reader who wants to know more and pursue these issues further, and we mention some of our work that has been used partly in the presentation here.

1. For several aspects of black holes, gravitational collapse, and related issues and the development of the subject, we refer to Kip S. Thorne: *Black Holes & Time Warps* (W. W. Norton, 1994). Many intricacies of gravitation theory come out beautifully in the discussions here.
2. Astrophysical aspects of collapsing stars are very well discussed by J. C. Wheeler: *Cosmic Catastrophes* (Cambridge University Press, 2000).
3. More detailed introductions to Einstein gravity can be found in several excellent texts, e.g. Jim Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Addison-Wesley, 2002); Bernard F. Schutz, *First Course in General Relativity* (Cambridge University Press, 2012); and R. Wald, *General Relativity* (University of Chicago Press, 2010).
4. Additional details including technical aspects of gravitational collapse, cosmic censorship, and various current debates in black hole physics can be found in the following and the references cited therein: (i) Pankaj S. Joshi and Ramesh Narayan, 'Black Holes, Firewalls and Chaos from Gravitation Collapse' (e-Print: arXiv:1402.3055 [hep-th]); (ii) Pankaj S. Joshi and Daniele Malafarina, 'Recent Developments in Gravitational Collapse and Spacetime Singularities', *IJMPD*, 20 (2011), 2641 (e-Print: arXiv:1201.3660 [gr-qc]); (iii) Pankaj S. Joshi, Daniele Malafarina, and Ravindra V. Saraykar, 'Genericity Aspects in Gravitational Collapse to Black Holes and Naked Singularities', *IJMPD*, 21 (2012), 1250066 (e-Print: arXiv:1107.3749 [gr-qc]).
5. Further results on global aspects of gravitation, spacetime singularities, and other related aspects can be obtained from S. W. Hawking and G. F. R. Ellis: *The Large Scale Structure of Spacetime* (Cambridge University Press, 1973), and Pankaj S. Joshi, *Global Aspects in Gravitation and Cosmology* (Oxford University Press, 1993), and *Gravitational Collapse and Spacetime Singularities* (Cambridge University Press, 2010).

Index

V

W

X