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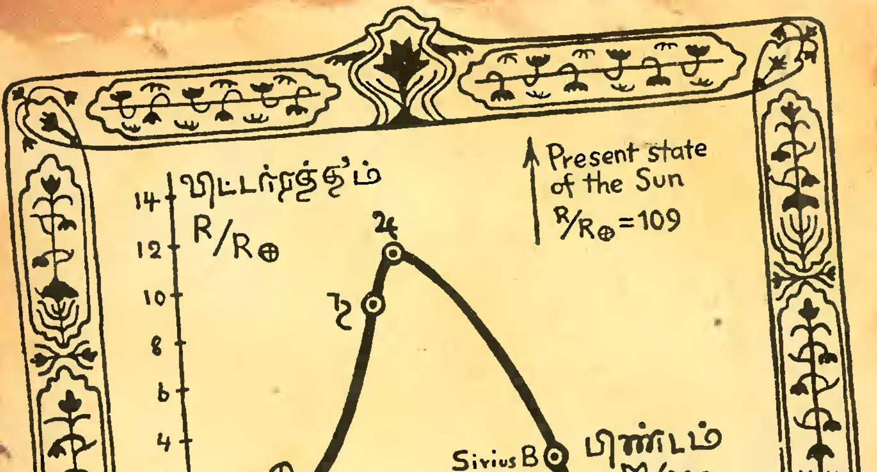
G. VENKATARAMAN

CHANDRASEKHAR AND HIS LIMIT



Chandrasekhar and His Limit

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Chandrasekhar and His Limit

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Cover: The drawing is based on a figure in Gamow's popular book the Birth and Death of the Sun (Viking Press: New York, 1941). Gamow says that the curve illustrates "the relationship between the radii and the masses of cold stellar bodies, according to the calculations of the Indian astrophysicists Chandrasekhar and Kothari." Gamow adds, "The words of mass and radius are in Dr. Chandrasekhar's original Tamil." While Gamow is correct as far as the word radius is concerned, the other Tamil word is better translated as dead matter.

Frontispiece: Subrahmanyam Chandrasekhar

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Preface

To the adult reader

This book and others in this series written by me are inspired by the memory of my son Suresh who left this world soon after completing school. Suresh and I often used to discuss physics. It was then that I introduced him to the celebrated *Feynman Lectures.*

Hans Bethe has described Feynman as the most original scientist of this century. To that perhaps may be added the statement that Feynman was also the most scintillating teacher of physics in this century.

The Feynman Lectures are great but they are at the textbook level and meant for serious reading. It seemed to me that there was scope for small books on diverse topics in physics which would stimulate interest, making at least some of the young students take up later a serious study of physics and reach for the Feynman as well as the Landau classics.

Small books inevitably remind me of Gamow's famous volumes. They were wonderful, and stimulated me to no small extent. Times have changed, physics has grown and we clearly need other books, though written in the same spirit.

In attempting these volumes, I have chosen a style of my own. I have come across many books on popular science where elaborate sentences often tend to obscure the scientific essence. I have therefore opted for simple English, and I don't make any apologies for it. If a simple style was good enough for the great Enrico Fermi, it is also good enough for me. I have also employed at times a chatty style. This is deliberate. Feynman uses this with consummate skill, and I have decided to follow in his footsteps (whether I have succeeded or not, is for readers to say). This book is meant to be read for fun and excitement. It is a book you can even lie down in bed and read. Without going to sleep I hope!

Naturally I have some basic objectives, the most important of which is to stimulate the curiosity of the reader. Here and there the reader may fail to grasp some details, and in fact I have deliberately pitched things a bit high on occasions. But if the reader is able to experience at least in some small measure the *excitement* of science, then my purpose would have been achieved. Apart from excitement, I have also tried to convey that although we might draw boundaries and try to compartmentalise Nature into different subjects, she herself knows no such boundaries. So we can always start anywhere, take a random walk and catch a good glimpse of Nature's glory. Where she is concerned, all topics are 'fashionable'. There is today an unnecessary polarization of the young towards subjects that are supposed to be fashionable. To my mind this is unhealthy, and I have tried to counter it.

This series is essentially meant for the curious. With humility, I would like to regard it as some sort of a 'Junior Feynman Series', if one might call it that. With much love, and sadness, it is dedicated to the memory of Suresh who inspired it.

To the young reader

This book is about a very famous discovery made by a very famous man. Of course, when he actually made this very, very important discovery, he was not so famous. He was quite young actually, and, what is more important, he had to struggle a lot—I really mean a lot—to establish his discovery. It is a heart-warming and a very inspiring story. His name is Subrahmanyam Chandrasekhar, and it is no exaggeration to say that he is one of the most distinguished mathematical physicists this century has produced.

In a long and remarkable career, Chandrasekhar has done many outstanding things but in this book I shall concentrate mostly on one of them, namely, the discovery of the *Chandrasekhar limit.* As long as there are stars, and as long as there are people to talk about stars, the Chandrasekhar limit will not be forgotten!

Acknowledgements

I would like to express my gratitude to Professors V. Balakrishnan and N. Mukunda, and to Dr. M.V. Atre for a careful scrutiny of the manuscript and for making many valuable suggestions. Mr. A. Ratnakar and Mr. J.D. Vincent offered much assistance in collecting source material. As always, Mrs. Naga Nirmala rendered invaluable help with the preparation of the manuscript. The editorial assistance and friendly cooperation received from Universities Press is also thankfully acknowledged.

G. VENKATARAMAN

1 *The Young Chandra*

The period 1920 to 1930 may be called the golden decade of physics for it was then that quantum mechanics was born. People often think that stuff like quantum mechanics is for eggheads to worry about and that it has very little to do with the world we live in. Not true! Without quantum mechanics, we cannot have the transistor and all the modern marvels produced by this innocent-looking device and its even more successful descendent, the silicon chip.

You must have heard about superconductivity—it has been so much in the news lately. One of these days there might be long-distance trains running on the basis of superconductivity. This is not a fanciful idea for there *is* already an experimental train running in Japan over a short track of about 10 km or so. And superconductivity also is a phenomenon which cannot be understood without the help of quantum mechanics.

Around this period when epoch-making discoveries were being made in Europe, four major discoveries in physics and astrophysics were made by Indians. Everyone of them is a landmark, and they are, in the order of their discovery: (i) the Saha ionization formula (1920), (ii) Bose statistics (1924), (iii) the Raman effect (1928), and (iv) the Chandrasekhar limit (1934/1935). The first three discoveries were made entirely in India, whereas the fourth one was made by Chandrasekhar while he was a doctoral student in England. However, he began thinking about the problem even while he was a student here in India.

Subrahmanyam Chandrasekhar was born in Lahore on October 19, 1910, a date which is easy to remember—19/10/1910. His father C. Subrahmanya Iyer, popularly known as C.S. Iyer, was in Government service, and his job took him to various places—that is how he was in Lahore when Chandra was born. By the way, Lahore was then a part of India or British India as some historians might like to say. It was only after the Partition in 1947 that Lahore became a part of Pakistan.

Mr. C.S. Iyer's father was one Mr. Chandrasekhara Iyer who served as a Professor in the Mrs. A.V. Narasimha Rao College in Vishakapatnam (in those days it was called Vizagapatam). Mr C.S. Iyer's younger brother was C.V. Raman, famous for his discovery of the Raman effect, and for which discovery he won the Nobel Prize in 1930. Roughly fifty years later, Subrahmanyam Chandrasekhar (named after his grandfather) repeated the feat of his uncle; so that makes it two in the family!

Chandra was one of a large family—there were three boys and six sisters. Though he was born in north India, Chandra grew up mainly in the south, in Madras in fact. Grandpa was fond of reading and music (Carnatic music). These tastes were picked up by the sons and the grandsons of Mr. Chandrasekhara Iyer, including Subrahmanyam Chandrasekhar.

Young Chandra did not attend kindergarten, elementary school and things like that. All of Mr. Iyer's children received their primary education at home. How about that! After a few years of such private tuition, Chandra was straightaway admitted to the second form (equivalent of today's sixth standard) in the Hindu High School in Madras. Chandra excelled in studies, giving a clear indication of things to come. Mathematics was a favourite subject, and about this Chandra's uncle C. Ramaswamy has this to say:

> Chandrasekhar's performance at school specially in mathematics was at least three years ahead of the class. His classmates were aware of his potentialities and recognised he was a genius among them.

The famous mathematician Srinivasa Ramanujan died while Chandra was a young boy. About this incident Chandra recalls:

> I remember very well a day in April 1920 (when I was hardly ten years old), my mother drawing my attention to an article in a newspaper on the day of the death of Ramanujan with the comment that a very great Indian had died in his prime.

After school, Chandra entered the Presidency College in Madras, where his uncle Raman had studied two decades earlier. Like his uncle Chandra studied physics, and once again like Raman, his interests were not confined to what he learnt in the class. He realised the importance of mathematics to physics, and took pains to meet Professors of mathematics and learn many things from them outside the prescribed syllabus. He told me this himself, when I met him some years ago in Madras. As a student he wrote an essay on "Band Spectra" (this refers to the spectra of molecules) which is preserved to this day by the Madras University.

Chandrasekhar told me a story about Raman which is amusing to recall. At the time the incident happened, Chandra was about twelve years old. Uncle Raman was visiting their family in Madras. Those days Raman used to be the Palit Professor of Physics in the Calcutta University. In 1922 he made his first visit abroad (to England actually), and soon after returning from his foreign trip he visited his brother Mr. C.S. I· r and his family in Madras. By the way, in those days one had to travel by ship and it was while returning from England that Raman was so struck by the blue colour of the Mediterranean sea that he became interested in the scattering of light. That interest eventually led him to discover the Raman effect, but all that is another story. Let me get back to the one Chandrasekhar told me.

Going abroad was an extremely rare event in those days, and whenever any one did, the others would be very eager to know about the experiences abroad. And so when Raman came to Madras, a small crowd collected in Mr. C.S. Iyer's house to hear from Raman about his experiences. Raman was a lively narrator and he kept his little audience roaring with laughter. And then suddenly a young boy in the crowd asked, "Did you not find it embarrassing to go around London wearing a turban?" What this young lad meant was that while Englishmen went around in hats, did not Raman feel conspicuous in his turban? Raman looked at this young questioner and said:

> Young man, I will tell you about a little incident that happened while I was in London. One evening I went to the Royal Institution in London to hear a lecture by Lord Rutherford. I arrived a little late and by that time the lecture had started. And so I quietly slipped into one of the back rows and sat there. Suddenly Rutherford looked at me and said, "Professor Raman, why are you sitting there all alone in the back row? Come up here to the front." I then went and sat in the front row with all the famous British scientists. After the lecture I went to Rutherford and asked him, "Professor Rutherford, how did you recognise me? This is my first trip to England, and we have never met so far." Rutherford replied, "Well, I have read your papers and when I saw a person in the audience wearing a Madrasi turban, I knew it must be you." So young man, you tell me what is wrong with a Madrasi turban?

Raman discovered the Raman effect in Calcutta in February 1928. In early March he travelled south in order to give a lecture on his discovery in Bangalore. This lecture has since become famous. On the way to Bangalore, Raman stopped at his brother's place in Madras, and naturally he was full of news about his momentous discovery. About this visit Chandrasekhar recalls:

> I have a vivid recollection of a day in early March of 1928, when Professor Raman visited our home in Madras on his way to Bangalore where, on the 16th of March, he was to give the address announcing the discovery of what was soon to be called the Raman effect. I remember well his showing slides of the first Raman spectra ever taken and the state of euphoria he was in. On that occasion someone drew attention to the discovery of the Compton effect some few years earlier, and Raman responded with, "Ah, but my effect will play a very great role for chemistry and molecular structure!" That statement was indeed prophetic.

In the summer of 1928, young Chandra, then a student of the fourth year Honours class, went to Calcutta to spend two months at the Indian Association for the Cultivation of Science (see Box 1.1). This is where

> **Box 1.1** The Indian Association for the Cultivation of Science (IACS) is largely the creation of one man namely, Mahendralal Sircar. Sircar was born in Calcutta in 1833. He qualified as a doctor, but his interests were wide. He was deeply nationalistic and felt that the problems of the country could be solved only through science. So he worked hard to set up a centre for scientific research similar to the Royal Institution in London. The latter, by the way, is the place where Faraday did pioneering research. After much effort by Sircar, the IACS finally came into existence in 1876.
>
> The Association soon became a meeting place for all interested in science, but nothing much happened by way of research. All that happened was that some people gave lectures and others listened to them. Nobody was keen on doing research at the Association and Sircar died a disappointed man.
>
> Sircar's effort was not all in vain, for, three years after his death, Raman, then in Calcutta as a Government officer, came to know about the Association and became a member. He spent all his spare time there doing research and brought fame both to himself and the Association. And it was in the Association that he discovered the Raman effect (in 1928), and for which he later received the Nobel Prize (in 1930).
>
> The Association was originally located in Bow Bazar Street. In the fifties, it was moved to Jadavpur where it is presently located. A monument has been erected at the Bow Bazar Street location to commemorate the discovery of the Raman effect.

Raman did his research, and it was a sort of Mecca for Indian physicists at that time. About this visit, Chandrasekhar recalled many years later:

> During the summer of 1928 I spent two months at Raman's laboratory, where, at that time, there were many young men who together with Raman were pursuing the new discovery. Among them were several who were later to become leaders of Indian science: Professor K.S. Krishnan (see Box 1.2) who later became the Director of the National Physical Laboratory; Professor S. Bhagavantam (see Box 1.3) who was to occupy the responsible position of the Scientific Adviser to the Defence Minister; Dr. S. Venkateswaran (see Box 1.4), later the Registrar of Trade Marks and Patents; Dr. L.A. Ramdas, later the Director of Agricultural Meteorology; and a host of others. You can imagine what a marvellous experience it must have been for a young man to have witnessed at such close quarters a group of enthusiastic scientists caught in the wake of a great discovery.

Chandrasekhar also recalls a meeting with Professor Meghnad Saha (see Box 1.5) and how Saha invited him to dinner along with many leading scientists of the day, even though he (Chandra) was but a mere student.

Box 1.2 K.S. Krishnan is one of the eminent physicists produced by the country in the pre-war era. He was born in Watrap, Tamil Nadu in 1898, and received his education in Madurai and Madras. For a brief period, he worked as a demonstrator after obtaining the Master's degree. Attracted by Raman's fame, Krishnan went to Calcutta to study and work under Raman. And thanks to his talent, Krishnan soon became a favourite of Raman. In fact, he closely worked with Raman during the final stages before the discovery of the Raman effect. After obtaining his doctorate degree, he spent a few years working in the Physics Department of the Dacca University where Satyen Bose was also teaching. Though he was closely associated with the discovery of the Raman effect, Krishnan switched to another topic while at Dacca. Here it was that he did his work on diamagnetism which later won him fame. From Dacca, Krishnan went back to Calcutta to become the Mahendralal Sircar Professor at the IACS. From Calcutta he went to Allahabad and then finally to Delhi as the Founder-Director of the National Physical Laboratory (NPL).

Krishnan was a great scholar, and besides physics he enjoyed Tamil, Sanskrit and English literature. He was widely respected. Jawaharlal Nehru, an admirer of Krishnan, once said that he would rather be the Director of NPL than the Prime Minister! For his achievements in science, Krishnan was elected a Fellow of the Royal Society and a member of the U.S. National Academy of Sciences. Krishnan passed away in 1961.

Box 1.3 Suri Bhagavantam was born in 1909 in Andhra Pradesh. Like Krishnan, he was attracted by Raman's fame and headed for Calcutta. And he arrived in Raman's lab at the right time, i.e.. just when the Raman effect was being discovered. Later in 1930 when the Nobel Prize was awarded to Raman, it was Bhagavantam who broke the news to Raman.

When Raman moved to the Indian Institute of Science in Bangalore in 1933, he tried to get Bhagavantam appointed as his Scientific Assistant but failed. Bhagavantam then went to the Andhra University where he spent fifteen years. There he wrote two books which became quite famous. These are: *Scattering of Light and the Raman Effect and The Theory of Groups and its Application to Physical Problems* (which he co-authored with T. Venkatrayudu). Later he served as the Vice-Chancellor of the Osmania University in Hyderabad, as the Director of the Indian Institute of Science and as the Scientific Adviser to the Defence Minister. He maintained a strong interest in research until he retired. He was an excellent teacher. He passed away in 1989.

In Calcutta Chandra was not idling. He was engaged in research of his own, even though he was still a student! And how was he able to do this? Not merely because he was good at studies and maths and things like

Box 1.4 S. Venkateswaran was also one of Raman's "students"; but he was a student with a difference. Venkateswaran was working in the Test House in Calcutta in the twenties. His basic degree was in physics, and he was interested in research. So, like Raman, he spent all his spare time at the IACS doing research under Raman's guidance.

Many of Raman's students worked on the scattering of light by liquids, K.S. Krishnan was one of those, Some of them reported a mysterious type of scattering which they all referred to as *feeble fluorescence*. In December 1927, Venkateswaran was studying the scattering of light by glycerine, and he found that the usually feeble effect was rather strong here, Raman immediately suspected this was not fluorescence at all but some other new phenomenon, He then dropped all that he was doing and started working on the subject himself (along with Krishnan), So it was Venkateswaran's experiments which provided the trigger for Raman, Raman generously acknowledged this in his Nobel lecture,

Box 1.5 Meghnad Saha was born in a village near Dacca in 1893, Young Saha was keen on studies and won a scholarship to study in school, But in 1905 when the Governor of Bengal came to Dacca, Saha joined in a protest march (remember the British were ruling us then, and there were always protests), So Saha lost the scholarship and faced many difficulties in completing school, In 1911, Saha joined the Presidency College in Calcutta, After college, he tried for a Government job but was disqualified because he had participated in the nationalist movement. Luckily, he managed to become a lecturer in the Calcutta University. Here Saha began his research and did his famous work on the *Saha ionization formula*, now a cornerstone of astrophysics, In 1923, Saha became the Professor of Physics in the Allahabad University, In 1933 he returned to Calcutta where he became the Palit Professor of Physics in the University. He also maintained close links with the IACS, In addition, he founded the Saha Institute of Nuclear Physics, In later years, Saha became deeply involved in national work, He was a member of the Lok Sabha for many years till he died in 1956, Saha won many honours including that of being elected to the Royal Society,

that. He was also in touch with the research that was going on in the laboratories of the world. And how did he keep in touch? By reading scientific journals. When I met him in Madras in 1986, he told me how foreign mail would come on a particular day of the month (this depended on the arrival of ships in the Madras harbour), and how he would eagerly go to the library on that day to look up the new issues of scientific journals like *Nature, Philosophical Magazine*, etc.

At the Association, Chandra studied a certain problem connected with the interior of stars—you can see that already his mind was getting set on astrophysics (see Box 1.6). In January 1929, The Indian Science Congress held its annual meeting in Madras. Raman, fresh from the triumph of his great discovery, was the General President of the Congress. Professor S.N. Bose already famous for his Bose statistics, was the President of the Physics Section. And in that Section, young Chandra presented a paper (perhaps the first time he had done so). It was a glorious moment in the history of Indian physics.

Box 1.6 First page of the paper published by Chandrasekhar in the *Indian Journal of Physics*, based on the work he did at the IACS in 1928.

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Thermodynamics of the Compton Effect with Reference to the Interior of the Stars

BY

S. CHANDRASEKHAR.

ABSTRACT.

This paper deals with the statistical equilibrium of a system containing (1) electrons and quanta and (2) electrons, ionised atoms and quanta. Expressions are derived for the ratio of the probability coefficients connected with the 'Compton-type' collisions and 'Reversed-Compton type' collisions. This is nearly unity in the first case and nearly $3 \cdot 3 \times 10^{17}$ in the second case. This enormous difference in the results in the two cases is explained as due to the fact that the Reversed Compton Effect in the case of bound electrons is one of a triple collision and that with free electrons is an ordinary double collision. On this basis, the softening of high frequency radiation created in the interior of a star is attributed to the Compton-scattering with bound electrons.

1. *Compton Scattering with Free Electrons.*

Let a quantum of energy $h\nu(=E_2)$ collide with an electron of energy E_1. After the collision let the recoil electron go with energy E_3 and the scattered quantum with $E_4(=h\nu')$. If the collision is of the Compton-type the electron goes with increased energy and the quantum with reduced frequency

(*i.e.*) $h\nu > h\nu'$ and $E_3 > E_1$

[But $h\nu + E_1 = h\nu' + E_3$]

As a student Chandra had received as a prize, Eddington's famous book, *The Internal Constitution of the Stars*. This perhaps was partly responsible for turning Chandra's mind towards astrophysics. A new type of star called the *white dwarf* had been discovered around that time. As I shall explain later, a white dwarf is really a "dead" star. The white dwarf was something of a puzzle, and young Chandra began to think about it. A man called Fowler in England seemed to have explained the puzzle, but it required quantum mechanics. In 1928, the great German mathematical physicist Arnold Sommerfeld (Box 1.7) visited India. One

Box 1.7 Arnold Sommerfeld was one of the great names of classical physics. He was born in Germany in 1868. He was always interested in mathematics but studied many other subjects as well, even economics. He also took much interest in sword fighting (fencing)! In the University, his instructor in mathematics was the great David Hilbert. For a while he also served in the army. Towards the end of the last century, Sommerfeld moved to Gottingen where he worked for a while with the great mathematician Felix Klein. Here it was that he did his first major piece of work which was to solve *exactly* a difficult diffraction problem. This solution immediately attracted attention, including that of the well-known French mathematician Poincare. This soon became Sommerfeld's hallmark—evaluation of complex integrals. Sommerfeld was a great teacher and not only produced outstanding students like Pauli, Bethe, Heisenberg and Debye (all of whom won the Nobel Prize), but also wrote a series of books on theoretical physics. In those days, this was the series to learn from (now of course we have a more modern series written by Landau, and revised by his students). Physicists always think of Sommerfeld as an outstanding theoretical physicist which of course he was, but few know he has done much work in engineering also. The vibration of bridges, problems of locomotive construction and the lubrication of machines are some examples.

In 1928, Sommerfeld came to India. He was actually on his way to America and instead of going there by crossing the Atlantic, decided to go via the east, stopping in India for nearly a month. He had earlier met Saha in Europe, and knew of Raman from his various researches. In Calcutta Sommerfeld spent several days at the IACS, and the notes of his lectures (prepared by Krishnan) were later published by the Calcutta University. One place he visited on his Indian tour was Madras. How he influenced Chandrasekhar by his lectures there is described in the text.

of his stops was the Presidency College, Madras, where he gave a lecture on the exciting new developments in physics. I don't know if all the members of the audience followed what Sommerfeld said, but certainly one person in the audience namely, Chandra, hung on to every word the great master uttered. After the lecture, Chandra had some very useful discussions with Sommerfeld. This was a major turning point, but for the full story you have to wait till Chapter 6. I think this is enough to show how Chandra got started off, and how his interests were shaped.

After he passed the Honours exam (winning all kinds of prizes of course), Chandra was asked by his father Mr. C.S. Iyer to join Government service. Chandra flatly refused, saying that his mind was in science. There was much pressure, but Chandra was firm—he would stay with science, and he would continue his studies in Cambridge. And there it was that he made the discovery which is the subject of this book and which also made him famous. But the thinking had already started while he was in Madras. More about all this in a later chapter.

Let me now get back to the point with which I started this chapter. At a time when great discoveries in physics were being made elsewhere in the world, important discoveries were being made in India too. Unfortunately, this sort of thing has not happened later. So it is all the more important to understand why this unique phenomenon occurred. Let Chandrasekhar himself speak on the subject. He says:

> The twenties was a time when scientists of the highest stature and calibre walked the corridors of Science; and it raises an interesting historical question. Here was a period of time that was not unique to Science in any way: it was also the time when great national leaders like Mahatma Gandhi, Motilal and Jawaharlal Nehru, Sardar Patel, Rajendra Prasad, Sarojini Naidu and many others of comparable stature were launching India's struggle for Independence. And of course, there was also Rabindranath Tagore, who by his writings and by his songs inspired entire generations of Indians. I could go on adding to the list. The early decades of this century was a time when the air was more bracing and the wind was more fresh than it ever was or has ever been. The national spirit was *high at all levels of human activity*; and the development of Science was only a part of that activity.

There is much in this statement. If you think of the Renaissance, for example, the great scientific discoveries of that period were really a part of an overall revolution in human thought. Similarly, if today America seems to excel in Science, that appears to be a part of the general excitement prevailing in that society.

You may or may not agree with all this. Anyway, that is not very important. What is more important is the story of Chandra's great discovery. In the next few chapters, I shall be switching from biographical story-telling to astrophysics and physics, till I reach Chapter 6 where I shall describe the discovery itself. And, to spice it up, I shall also narrate in some detail the drama accompanying the discovery. You might be tempted to jump and read that chapter straightaway! I advise you not to. Not only would it spoil the fun (it would be like reading the last page of a murder mystery first), but more important, you will appreciate Chandra's discovery only if you have all the background. So please be patient, go carefully through the next few chapters, and then get ready for the exciting climax!

2 *The Birth And Death Of A Star*

2.1 Introduction

We all know the nursery rhyme starting with the lines,

> *Twinkle, twinkle, little star,*
> *How I wonder what you are!...*

Honestly, how many of us really wonder? It is a pity most of us don't, for, if we did, we would be in for some wonderful surprises. This chapter is intended to give you a glimpse of what our heavens contain. Actually, if one takes this job seriously, one would need an encyclopaedia rather than a mere chapter. So I shall be somewhat choosy in what I am going to say. I shall say just as much as is needed for understanding the work which made Chandrasekhar famous. The work itself I shall slowly unfold in the following chapters.

2.2 How big is our Universe?

When we look at the sky at night, we see the stars. On a clear night, we can see several thousands of them. You and I would not have the patience to count them but there have been people from ancient times who have painstakingly made all kinds of observations. When the telescope was invented, one could see many more stars, and as the telescopes began to get better and better, the number of stars one could see became very large indeed.

Do you have any idea how vast the "sky" is? Pretty big really, and let me now give you some sort of a feeling for it in different ways. First of all, let us use the distance between the Earth and the Sun as a unit. This distance (which is about 1.5×10^8 km) is called an *astronomical unit* or AU for short. In terms of this unit, the distance from the Sun to Pluto (which is the farthest planet of our solar system) is about 40 AU. The distance from the solar system to the *nearest* star is about 300,000 AU; mind you, this is the distance to the nearest star.

On a clear night you would be able to see the famous Milky Way stretching across the sky, like a beautiful white cloud. The Milky Way is nothing but a collection of a very large number of stars—millions of them in fact! Our Sun is a modest star belonging to the Milky Way, occupying a position somewhat to one side. And how big is the Milky Way? To describe that, I need a new unit of distance which is even bigger than the AU, and that is the *light year*. A light year is the distance light travels in one year; this is a very large distance because in *one second,* light travels 300,000 km. So a light year is about 9.5×10^{12} km or 60,000 AU, which is quite a large distance. In terms of light years, the Milky Way has a diameter of about **100,000** light years (or 6,000,000,000 AU).

The Milky Way is called a *galaxy*, and it is not the only galaxy in the Universe; there are in fact millions and millions of them. So you see our Universe is really very big. Let me try to emphasise this in two more ways. Figure 2.1 is one of these, and if this does not impress you, let me add finally that the distance between us and the farthest star in the Universe as we know it today is several *billion* light years!

2.3 What is a star?

A star is nothing but a great big gaseous cloud—actually a mass of *burning* gas, but we shall come to the burning part a little later. Clouds are nothing new. We see them all the time in the sky. These are clouds of water vapour, and one thing about them is that they rapidly *diffuse* away. Stars obviously don't do that for we know that they have been around for *at least* a few thousand years, i.e., from the time people first started observing them seriously and recording their observations. And now scientists tell us that stars can live for millions if not billions of years (more about this later). So obviously, stellar clouds do not diffuse away in a jiffy like the water vapour clouds do. How come, and what holds stellar clouds together? The force that holds them together is *gravity*.

As you know, gravity is an attractive force. It is actually a very feeble force compared to its companions (see Box 2.1), but the reason why it clicks in a star is simply because stars are *very massive* (compared to the Earth, for example). What really happens is that one part of the star attracts another, and in this way all the different parts stick or hang together, so to speak. Now there is a problem with this picture. If the star is really held together by gravity, then why does not gravity crush the star to a point? How come stars have finite radii? What prevents the star from collapsing completely? The answer to all these questions is *outward pressure*.

So the star is held in equilibrium basically under the action of two opposing forces—one is gravity which tries to crush the star, and the other is outward pressure which tends to distend the star. If gravity wins, the star collapses, and I would have more to say about that sort of thing

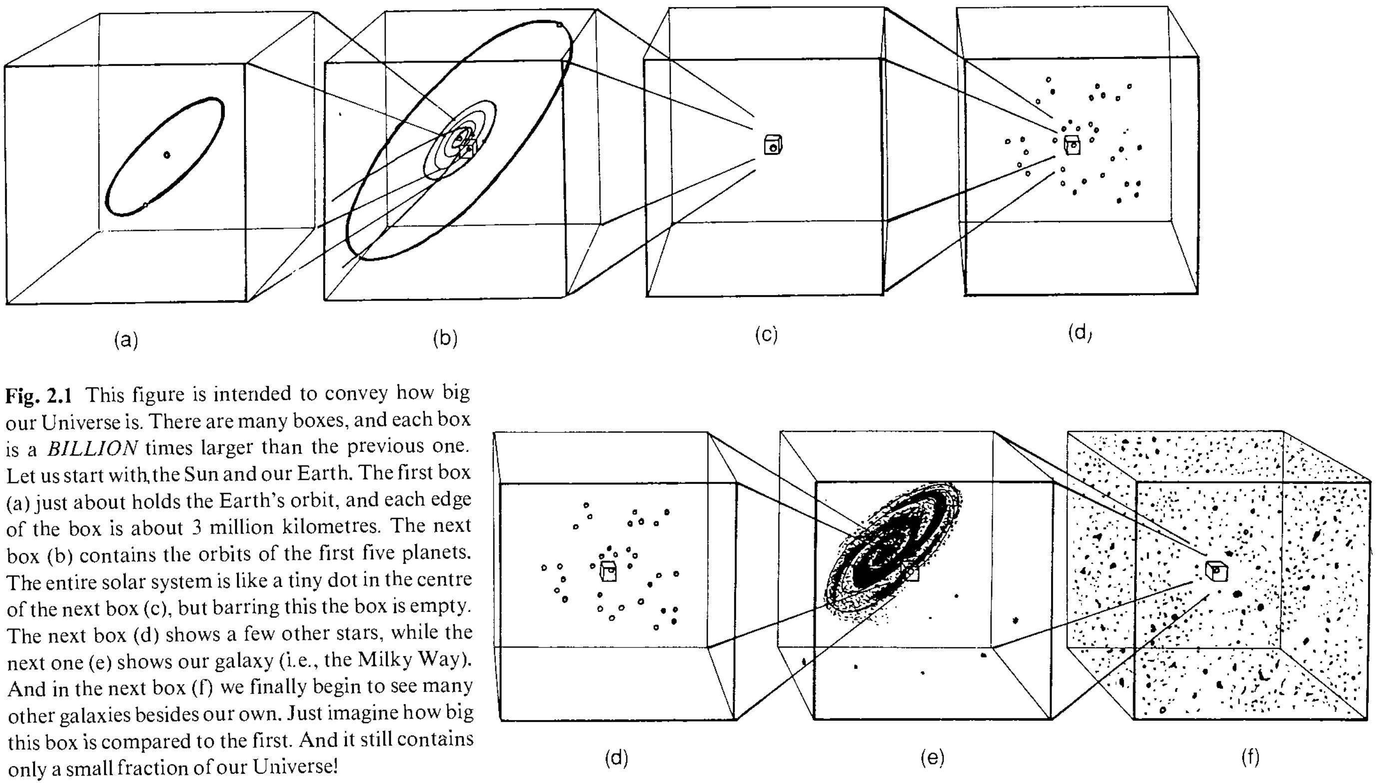
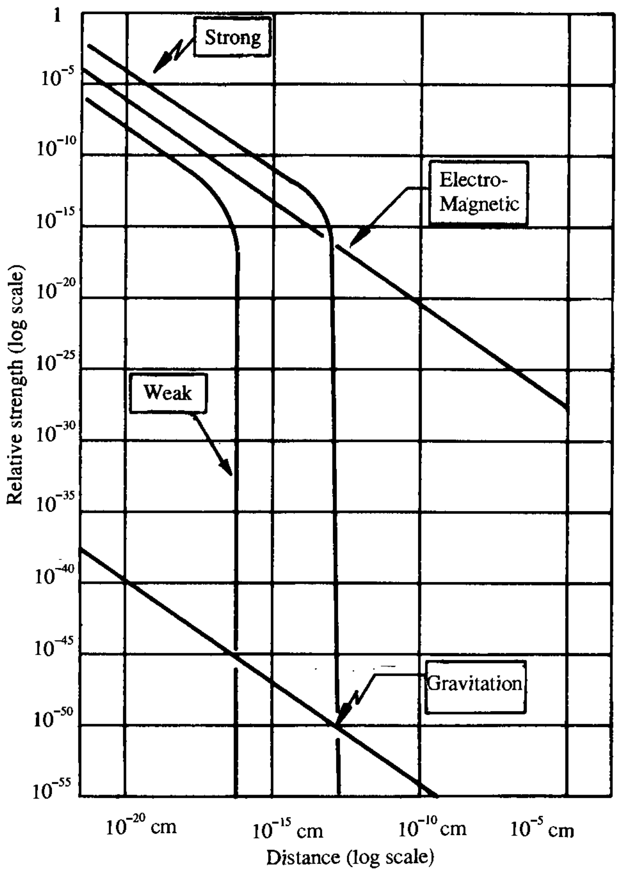


Fig. 2.1 This figure is intended to convey how big our Universe is. There are many boxes, and each box is a *BILLION* times larger than the previous one. Let us start with the Sun and our Earth. The first box (a) just about holds the Earth's orbit, and each edge of the box is about 3 million kilometres. The next box (b) contains the orbits of the first five planets. The entire solar system is like a tiny dot in the centre of the next box (c), but barring this the box is empty. The next box (d) shows a few other stars, while the next one (e) shows our galaxy (i.e., the Milky Way). And in the next box (f) we finally begin to see many other galaxies besides our own. Just imagine how big this box is compared to the first. And it still contains only a small fraction of our Universe!

Box 2.1 The four basic forces in Nature are:

1. Gravitational force
2. Weak force
3. Electromagnetic force
4. Strong force

In the above, the forces have been arranged according to their strength, the feeblest being at the top of the list. As you know, the law concerning the gravitational force was discovered by Newton, while our understanding of the electromagnetic force came out of the work of Faraday, Ampere and Maxwell, to name a few. Both these forces are long-ranged, meaning that they can act over distances as large as the Universe itself. Also both were discovered before this century. The remaining two were discovered only during this century while people were studying nuclear physics. Both these forces operate over very very short distances—see figure (a).



(a)

Physicists now believe that in the early Universe, all these forces were united into one super force and that they became separated when the Universe started expanding. People have been trying to understand how exactly these forces are united. Einstein tried to unite the old familiar forces namely, the gravitational and the electromagnetic. More recently, Salam, Weinberg and Glashow succeeded in unifying the electromagnetic and the weak force into what is called the *electro-weak* force. This electro-weak fellow has still to be tied to the strong and then finally to the gravitational force. A reference to this problem of unification of forces may be found in the companion volume *The Many Phases of Matter*.

later. On the other hand if pressure wins, the star starts expanding; sometimes when the pressure increase is sudden and great, there is even an explosion. But if the two forces balance each other, then the star is in *equilibrium*. In short, a star is a cosmic gaseous cloud held in equilibrium under the action of two opposing forces, gravity and pressure—see Fig. 2.2.

No star is in equilibrium for ever; but if equilibrium is established it lasts for quite a while, and for the moment we shall be concerned only with stars already in equilibrium.

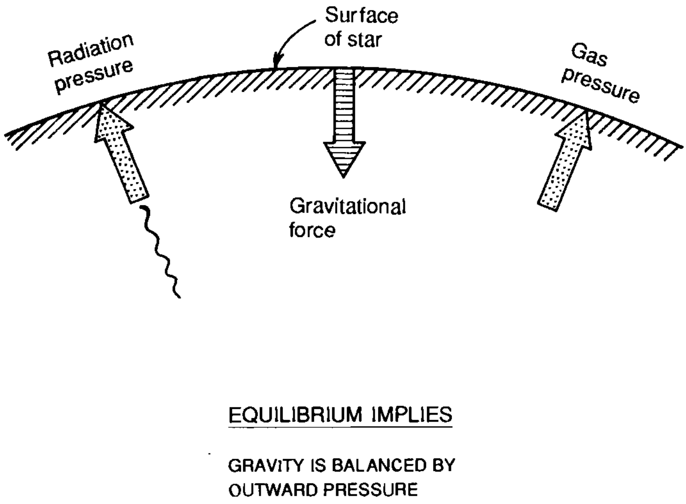


Fig. 2.2 This figure illustrates schematically the competition between pressure and gravity. In a live star, the pressure is mainly due to thermonuclear burning and has two components namely, gas pressure and radiation pressure. In a "dead" star on the other hand, the pressure arises mainly due to a quantum mechanical effect. All this is explained in the text.

2.4 Wherefrom pressure?

Where does the outward pressure come from? Now of course the very fact that the star is a gas cloud means that there must be some pressure associated with it, just as our atmosphere has pressure associated with it. But that kind of pressure would be peanuts and unable to fight gravity. To counter gravity, one needs a significant amount of outward pressure, which comes from burning. So the star is really a mass of *burning* gas, held in equilibrium by gravity on the one hand and gas pressure generated by burning on the other. Pressure can arise due to other reasons as well, but I shall come to that later.

2.5 Stellar burning

The burning we are used to is due to *chemical combustion*. Consider, for example, the burning of hydrogen. Hydrogen combines with oxygen to form water vapour, releasing a lot of energy (explosively in fact). A chemist would represent this burning by the equation

$$2\,H_2 + O_2 \rightarrow 2\,H_2O + \text{energy}.$$

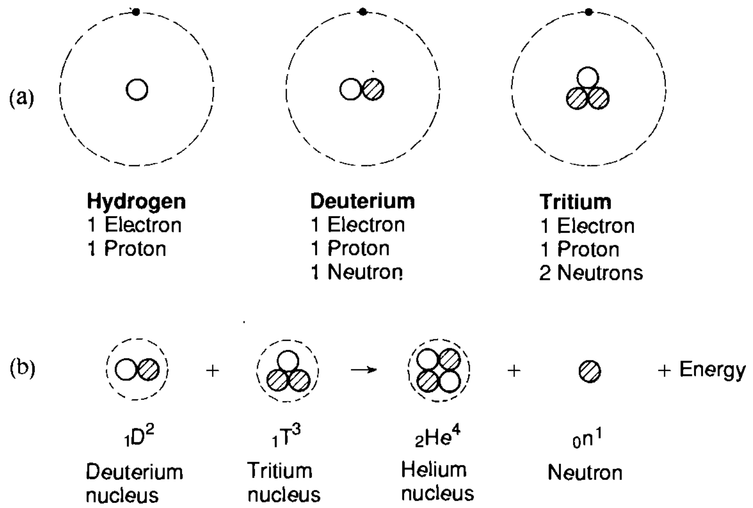
This equation represents a *chemical* reaction. Chemical reactions occur because electrons in the outer shells of the atoms like to get readjusted, in the process breaking up some molecules and joining up others.

The burning in stars is due to *nuclear reactions*. These are similar to chemical reactions but with two important differences. Firstly, instead of atoms it is nuclei which combine or break up as the case may be. Secondly, the energy released per reaction is about a *million* times greater than in the case of the chemical reaction. See Box 2.2 for a primer on nuclear reactions.

2.6 The very first stars

The Universe hasn't always been the way it appears to us at present. Scientists now believe that the Universe began with a mighty big explosion referred to as the Big Bang which occurred about 15 billion years ago. At birth the Universe was tiny but then it began to expand; it is still expanding. A few minutes after the Universe was born, it was filled almost entirely with hydrogen. In course of time, blobs of gas formed in this hydrogen atmosphere, which then began to shrink under the influence of gravity as shown in Fig. 2.3. A stage then came when the core of the gas became so hot as to trigger nuclear reactions—this is called *thermonuclear burning*, i.e., burning of the nuclear type, caused by excessive

Box 2.2 Nuclear reactions are very similar to chemical reactions, except that the participants are nuclei rather than atoms or molecules, and the forces involved are nuclear forces rather than electromagnetic forces. As an illustration, let us consider a nuclear reaction involving some isotopes of hydrogen. If you don't remember what an isotope is, figure (a) should remind you. It



shows the different isotopes of hydrogen. All of them contain just one proton. Because of this, they have only one electron orbiting around them. As a result, the *chemical* properties of these isotopes are very similar. However, their nuclei differ because they contain different numbers of neutrons. So the *nuclear* properties are different.

A typical nuclear reaction is (see figure b)

$$_1D^2 + {_1T^3} \rightarrow {_2He^4} + {_0n^1} + \text{energy}$$

i.e.,

deuteron + triton → alpha particle + neutron + energy

Typically, the energy released in a nuclear reaction is about a million times the energy released in a chemical reaction. The nuclear reaction described above is made use of in the hydrogen bomb.

heat. The thermonuclear reactions in the first stars belonged to the p–p cycle:

$$_1H^1 + {_1H^1} \rightarrow {_1D^2} + e^+ + \text{energy},$$
$$_1D^2 + {_1H^1} \rightarrow {_2He^3} + \text{energy},$$
$$_2He^3 + {_2He^3} \rightarrow {_2He^4} + 2\,{_1H^1}.$$

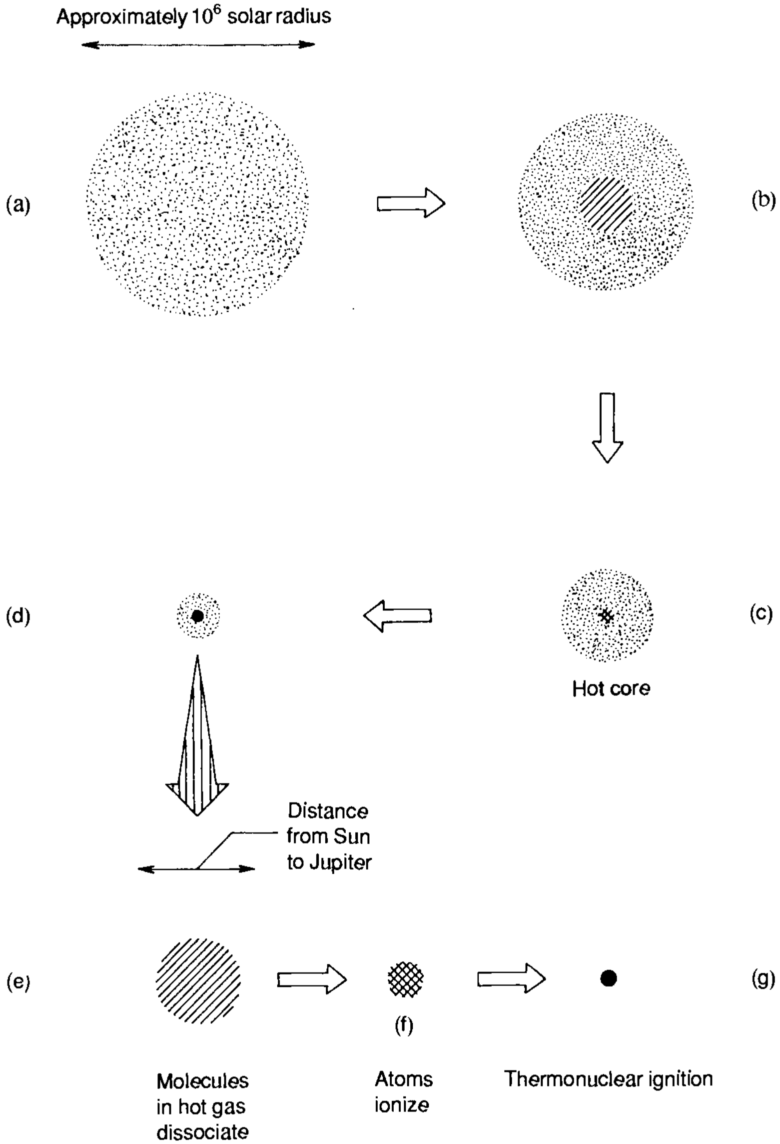


Fig. 2.3 This figure illustrates how the very first stars were formed. First a great big globule of hydrogen was formed as in (a). This sphere then started to contract under gravity, causing an increase of both the pressure as well as the temperature in the core—see (b)–(f). The core is shown separately in (e), (f) and (g). When the core became sufficiently hot, it ignited and started undergoing a thermonuclear burn via the p–p cycle described in the text—see (g).

Here, $_1H^1$ refers to the proton and $_1D^2$ to the deuteron. The positron (or the "anti electron") is denoted by the symbol e^+. $_2He^3$ and $_2He^4$ are the two isotopes of helium. Of these, the latter nucleus is often referred to as the *alpha particle.* Notice carefully what happens in the cycle. It starts off with just protons but as the reactions proceed, some of the protons disappear and in their place alpha particles appear—in short the light element hydrogen gets converted into the slightly heavier element helium.

Over a period of time, most if not all the hydrogen in the core of a burning star would get converted by the p–p cycle into helium. At this stage, the thermonuclear burning comes to a stop which means that energy production also comes to a halt. The story does not end here for from this residue is born the next generation star.

2.7 The next generations

When the p–p cycle comes to an end, the gas pressure in the star starts coming down. Gravity now gains the upper hand, and the star starts shrinking. This now heats up a shell of hydrogen surrounding the "burnt up" core. And due to this heating, the shell now gets ignited, leading to thermonuclear burning (in the shell, that is). This shell-burning stage is short and it leads to an expansion of the outer mantle. Meanwhile, the core contracts while the outside expands to a huge size making the star appear like a giant. The glow of the giant star is dull; usually it is red in colour. As far as the core is concerned, as it contracts it also becomes hot and a stage comes when a *new* thermonuclear cycle starts operating—this time it involves helium.

There are in fact many nuclear-burning cycles possible, and these have been carefully studied by astrophysicists and nuclear physicists working together. What these people do is to take data from carefully-performed laboratory experiments on nuclear reactions, and then figure out what happens in the stars. Thus Weizacker and Bethe have concluded that energy production occurs in our Sun due to the burning of hydrogen, but assisted by carbon, nitrogen and oxygen. This is sometimes called the *Carbon–Nitrogen cycle* and is illustrated in Fig. 2.4. One might say that carbon, nitrogen and oxygen play the role of catalysts.

Let us briefly summarise. Stars are great big gaseous clouds. A star is held in equilibrium by two opposing forces—gravity and pressure. Pressure is generated by the burning that goes on inside the star, thermonuclear burning actually. Each burning cycle involves several steps, essentially leading to the conversion of light elements into slightly heavier ones. This combining of light elements into heavy elements is called nuclear *fusion.* When the elements feeding a particular fusion reaction are nearly

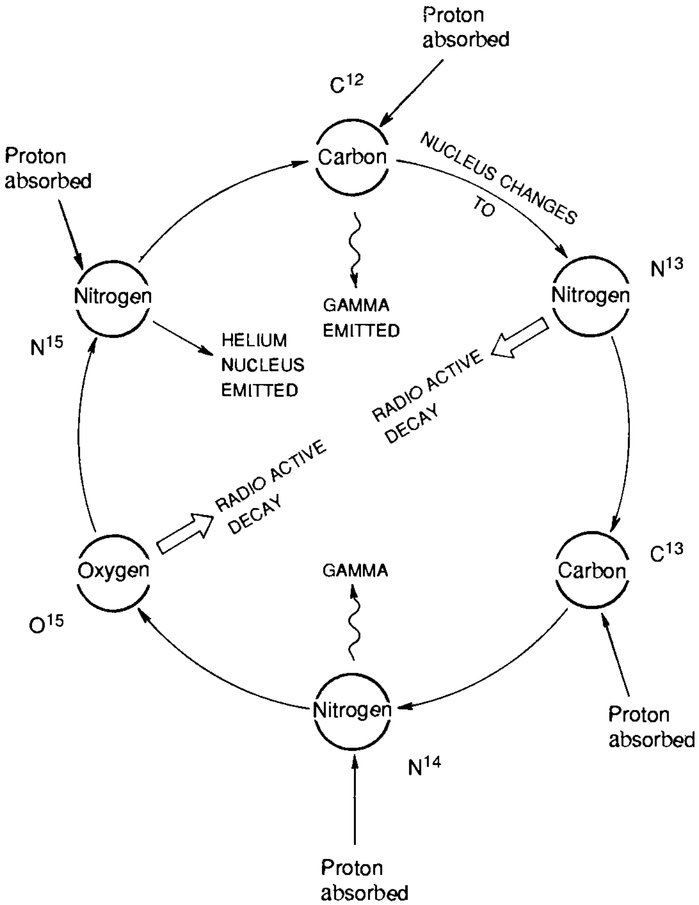
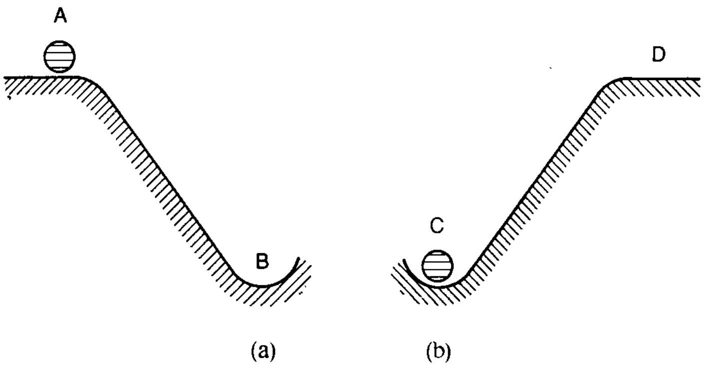


Fig. 2.4 The C–N cycle which is believed to be the source of energy production in our Sun.

exhausted, the burning ceases, and the core of the star begins to shrink under the influence of gravity. The collapse is stopped when the next cycle of thermonuclear reactions gets triggered. This goes on repeatedly.

Is there an end to this business? Yes, when the core becomes iron, and the reason for this is explained in Box 2.3. In a nutshell, stars produce energy (i.e., heat and light) via thermonuclear reactions, and, depending upon their age, could be operating on different nuclear cycles. We shall come back to this evolution business later.

Box 2.3 Consider figures (a) and (b). Both show balls placed near a slope. A slight push, and the ball in (a) would roll over from position A to B. This happens because B has a lower gravitational energy than A. On the other hand, if in figure (b) the ball is to be moved from C to D, then some energy must be supplied to do work against the force of gravity.



The same with chemical and nuclear reactions. Such reactions are of two types—*exothermic* and *endothermic*. The former need merely a trigger; once that is provided, the reaction proceeds releasing energy. The burning of petrol (which is an oxidation reaction) is an example. The process is similar to figure (a) because the reaction products are energetically more stable; and the excess energy (which in the case of the rolling ball appears as kinetic energy) is released in the chemical reaction as heat. The D–T reaction discussed in Box 2.2 is exothermic. There is an energy advantage for Nature to go through such a reaction.

The D–T reaction is a *fusion* reaction, i.e., light nuclei join to become slightly heavier nucleus (here the alpha particle is fused out of the lighter nuclei D and T). Light nuclei favour exothermic fusion nuclear reactions. However, once iron is reached, such fusion is not possible because the situation is now like in figure (b), i.e., there is an energy barrier against the occurrence of fusion. And so it is that thermonuclear burning in stars stops once the core becomes iron. Then how come we have heavy elements like mercury, gold and uranium? Ah! There is a mystery for you. Try to find the answer. I will give you one clue—supernova.

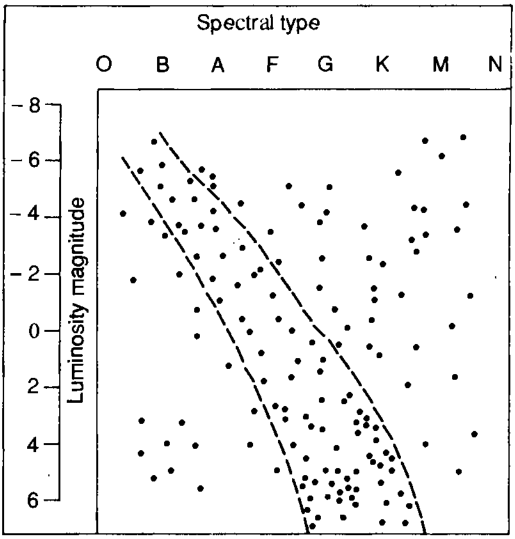
2.8 Stellar sequences

As I mentioned earlier, there are billions of stars—big stars, small stars, bright stars, feeble stars, very hot stars and not-so-hot stars. People have been patiently observing all these types for years and years, and quite independently of astro- and nuclear physicists, they have been accumulating a vast amount of *observational* data.

In science, whenever a large amount of data has been accumulated, people try to see if there is some kind of a pattern behind all that has been observed. If such an enquiry is carried out intelligently, then there *are* rewards. You may remember that Kepler discovered the laws of planetary motion by analysing the voluminous data patiently collected by Tycho Brahe for *decades*. In the case of stars, some kind of a pattern emerged about sixty years ago, thanks to the work, carried out independently, by the Danish scientist Ejnar Hertzsprung and the American astronomer Henry Norris Russell. What they did was to plot all the data as a graph, a plot which is now called the *Hertzsprung–Russell* plot or H–R diagram (see Box 2.4).

Box 2.4 Henry Norris Russel was born in America in 1877. He was educated at the Princeton University where he obtained a doctorate degree in astronomy. He then spent some time in England and returned to Princeton in 1905 to join the faculty there.

Russell got to work on the H–R diagram as the result of a suggestion by Edward Pickering of the Harvard College Observatory. The observatory had accumulated a lot of data on the brightness of stars and on their spectra. Pickering asked Russell to examine if there was a connection between the two.



(a)

Before looking at the spectrum of the stars, Russell decided to consider first only their colour, and see if there was a relation between the brightness and the colour. Stars had been classified according to their colour using a series of letters—O, B, A, F, G, K, M, N. The O stars were the bluest while the M and the N stars were red. Blue colour meant the surface temperature of the star was very high while red meant the temperature was comparatively lower. While the colour is related to surface temperature, the brightness of a star is related to its luminosity. This connection is explained in Box 2.5

Russell's first plot of brightness versus colour (or luminosity versus spectral type) looked somewhat like figure (a). Meanwhile, in 1920 Saha published his now famous ionization formula which enabled people to calculate the surface temperature of the star from data on the spectrum of the star. Russell quickly recognised the importance of Saha's discovery, and started determining the surface temperature of the stars from spectroscopic data. Saha himself wanted to do this, but he could not get the data. In America there was plenty of data, and Russell had no problem. As a result, the horizontal axis of the H–R diagram could now be marked in terms of the surface temperature of the star. From his study of the spectra of stars, Russell was also able to discover an important rule about atomic spectra themselves. He died in 1957.

Observe the clustering between the two dashed lines. The significance of this band is explained later.

The H–R diagram is very important and requires some explanation. It is basically a plot of the *luminosity* of stars versus their *surface temperatures*. Let us take these things slowly. First we start with luminosity. The luminosity of a star is the total amount of radiation energy it emits from its surface into outer space. Now stars emit radiation at all sorts of wavelengths, and when we talk of luminosity, we must take account of radiation emitted at *all* wavelengths. Luminosity is measured in terms of ergs/sec.

How do we, sitting here on earth, find the luminosity? This is not easy but can be done, though somewhat indirectly. Basically, we look at the star through a telescope, and then put a light meter at the place from where we look. The meter reading is a measure of the light intensity of the star, which in turn is a measure of its luminosity. There are some details, and if you are interested, you should look into Box 2.5.

Box 2.5 In the beginning, people could see and estimate the brightness of stars only with their eyes. They would, for example, say, "That star is twice as bright as this one," and so on. In this way, they established an intensity magnitude scale 0, 1, 2, 3, ... All these magnitudes were with reference to a particular star whose magnitude was taken as 0. Later people used light meters to actually measure the light intensity and found that a magnitude 5 star was really 100 times brighter or more intense than a magnitude 0 star.

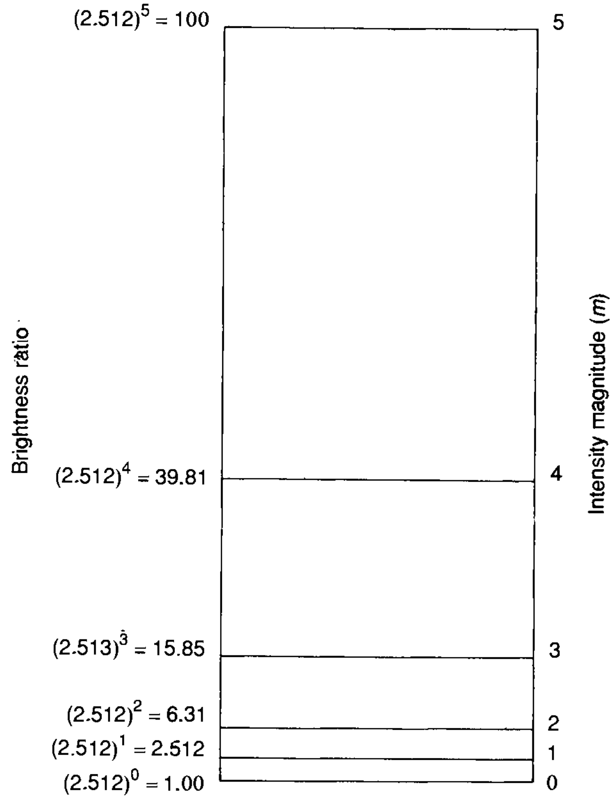
Let $I(m)$ denote the actual intensity of a star whose magnitude is taken as m on the scale 0, 1, 2, ... It was found that

$$I(1)/I(0) = I(2)/I(1) = I(3)/I(2) = \ldots = 2.512$$

In other words, what the eye notes as equal intervals of brightness are actually equal ratios. One can thus draw up an intensity scale as in figure (a). Note that

$$I(5)/I(0) = (2.512)^5 = 100,\ I(4)/I(0) = (2.512)^4 = 39.81, \text{etc.}$$

To make the H–R plot, one must first determine the luminosity of the stars. Experimentally, one measures an intensity with a light meter attached to the telescope. Say, we get some reading I. This may be a small number but from this we cannot straightaway say that the star is feeble. It might appear feeble because it is very far away. If, however, we know the distance of the star from the Earth (and there are methods for finding this distance), then we can ask:



(a)

"The star is actually at a distance D, and its intensity is measured as I. What intensity would we measure if the star were at some other distance S"? This is easy to calculate since the intensity of light from a source decreases as the square of the distance from the source. In this way, we can calculate the intensity I_S corresponding to some *standard* distance S, knowing I and D.

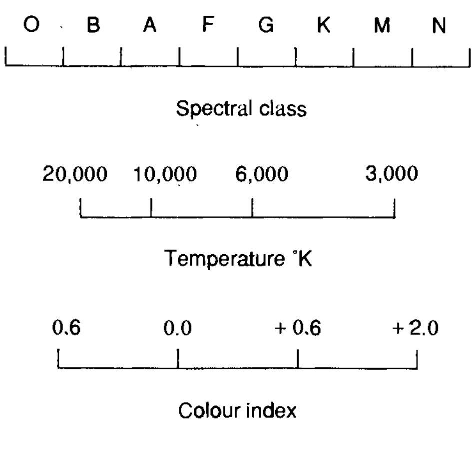
Having obtained I_s, one now converts it into a magnitude m_s using the conversion diagram (a). The number m_s one thus gets is called the *absolute magnitude* of the star. The y-axis of the H—R diagram usually shows m_s. Note that m_s can be negative since the star might be weaker than the standard reference star.

Luminosity alone is not enough for making the H–R plot. One must also know the surface temperature. A simple method of measuring the surface temperature is to determine the colour of the star. If you stick a rod of iron into a fire and allow it to get hot, you know that as the rod gets hotter, its glow changes from dull red to orange to yellow. In the same way, the colour of a star is a measure of its surface temperature.

OK, so we now look at thousands of stars through the telescope and measure their luminosities as well as their surface temperatures. Next we make a plot of all the data, using the vertical or the y-axis to show the absolute luminosity, and the x-axis to show the surface temperature. Other equivalent ways of using the x-axis are discussed in Box 2.6.

If a plot is made as described above, then the graph would look quite disappointing at first sight for one would see nothing but a big scatter of points. However, if one is patient—and the pioneer astronomers certainly

Box 2.6 The H–R diagram is essentially a plot of luminosity versus the surface temperature. As explained in Box 2.5, luminosity is usually given in terms of the magnitude m_s. As far as the horizontal axis is concerned, different people use different scales—see figure (a). Historically, Russell used the spectral scale first. As mentioned in the text, O is near the blue end while M is near the red end. The Saha ionization formula enables a better classification in terms of the surface temperature. The colour index scale is a quantitative version of the O, B, ... scale.



(a)

were—one can see patterns. It turns out that many stars lie on a broad band as it were, as shown in Fig. 2.5. This band (to which attention was first called in Box 2.4) is known as the *main sequence*. Other stars fall on *branches* to the main sequence. Quite apart from this, there is an island all by itself towards the left-hand corner. All these features are important, but soon our attention would be focussed on this island.

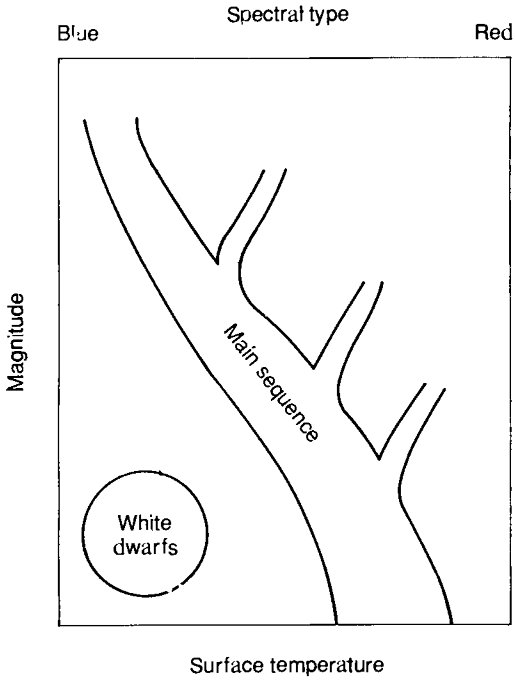


Fig.2.5 H–R plot (schematic) showing the main sequence, the side branches, and the island corresponding to the white dwarfs.

2.9 The main sequence

Most of the familiar stars (including the Sun), lie on the main sequence. Depending upon their initial mass, composition, etc., different stars occupy different positions on this sequence. An important feature is that they all burn hydrogen, although some like our Sun might be using catalysts (recall that the Sun uses C, N and O).

To understand the main sequence better, let us consider the birth of a star and how it gets onto the main sequence. The star typically starts off as a big dark cloud. The cloud would have some luminosity and also some surface temperature; so we could represent the cloud as a point in the H–R diagram and it would appear as in Fig. 2.6. Under the influence

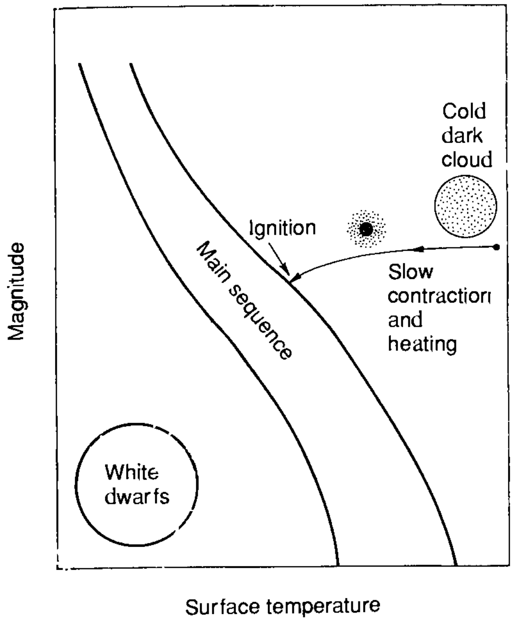


Fig. 2.6 This shows the trajectory during the formation period of a star, born as described in Fig. 2.3. By the time ignition occurs, the tip of the trajectory is in the main sequence region. The star's position then remains more or less fixed while the burn is going on, which may last millions of years. Generally speaking, light stars live longer than heavy stars. What happens after the burn stops is shown in the next figure.

of gravity, the cloud would then begin to shrink, tracing a line in the H–R diagram as shown in that figure. Eventually (as already explained earlier), the core of the gas cloud would become sufficiently hot for thermonuclear burn to occur, and at this stage the track would hit the main sequence band.

The main sequence is sometimes referred to as the *zero-age* main sequence because this is where newly formed stars are to be found. What happens after the burn stops? The burnt-out star goes through some complicated manoeuvres before it fires up again, with another nuclear-burning cycle of course. In between what happens is that while the core contracts, the outer layers expand. In the process, the star becomes a *red giant*, so-called on account of its colour. For example, if our Sun were to become a red giant (and it will one day), its radius would be about the radius of the orbit of Jupiter. So you can imagine how big a red giant is! Of course, the star does not stay as a giant for ever, and after a while the core starts shrinking and the trajectory on the H–R plot moves towards the main sequence again—see Fig. 2.7. Once again there is a core burn, a burnout, then an expansion, then a new-giant stage, a core contraction ... you get the general idea I suppose. Generally speaking, after they leave the main sequence, stars sweep complex trajectories in the H–R diagram.

Please note that I am giving a highly simplified picture of the formation

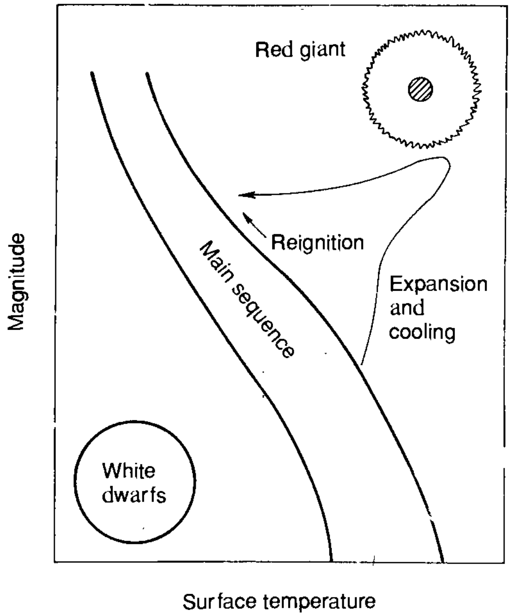


Fig. 2.7 This shows the trajectory of a star after it has gone through one thermonuclear burn cycle. The star then cools down and expands, entering the red-giant phase. Correspondingly, the trajectory wanders off from the main sequence. Sometimes the trajectory can be quite complex. The expansion-cooling phase is followed by a contraction-heating phase. Under suitable conditions, a thermonuclear burn can occur again, this time according to a new cycle. The trajectory then returns to the main sequence.

of stars and their subsequent evolution but that is OK for our present purpose

2.10 White dwarfs

Do all stars start off from the p–p cycle and go through *all* the nuclear burn cycles ending up finally with an iron core? Not necessarily. It all depends on the initial mass of the gas cloud, and perhaps also on the initial composition. But the point is that all stars, at some stage or the other, for some reason or the other, may get tired of burning and quit! What happens then?

One thing that could happen then is the formation of a *white dwarf*. Figure 2.8 shows the trajectory leading to the formation of the white dwarf. Notice the position of the white dwarfs in the H–R diagram—bottom left-hand corner. When the star finally runs out of fuel and stops

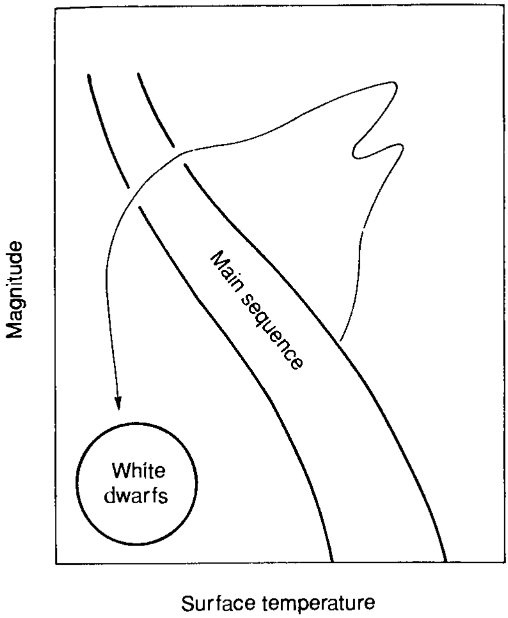


Fig. 2.8 This shows the trajectory of a spentout star which ends up as a white dwarf.

Fig. 2.8 This shows the trajectory of a spentout star which ends up as a white dwarf.

burning, large-scale heat production inside also ceases. So the gas pressure comes down whence gravity starts gaining the upper hand. The star now starts shrinking, and the question is: for how long? Since gas pressure has practically disappeared and since no more burning is possible, one would think that the dead star has no option but to go on shrinking and shrinking till it becomes a geometrical point! Imagine that—all the mass of a star like the Sun shrunk to a point! This sounds absurd does it not? We know this could not quite happen, and yet what is it that prevents this endless collapse? Nobody knew the answer, until in 1927 an astrophysicist in England named Fowler said that when a star's corpse is crushed to a small size, a new kind of pressure due to quantum mechanical effects is produced called *degeneracy* pressure. I shall tell you more about it in later chapters. For the moment, let us simply say that according to Fowler, dead stars do not shrink endlessly to disappear into a point but the shrinking stops much earlier. What results then is a *white dwarf.* No doubt the radius of such a shrunk, dead star would be quite small, and since stars have fairly large masses to start with (compared to that of the Earth, I mean), the density of the white dwarf is expected to be quite large—something like several thousands of kilograms/cc; see Fig. 2.9.

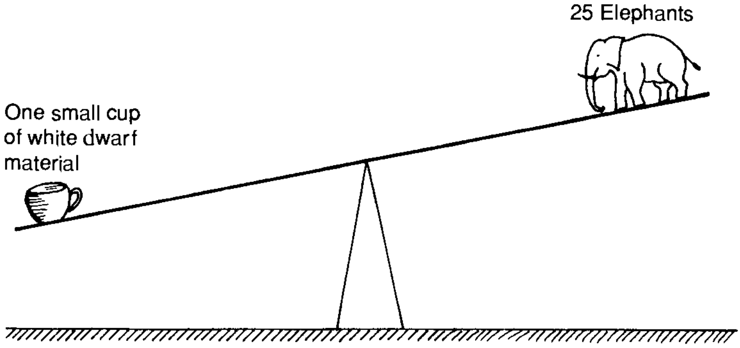


Fig. 2.9 Schematic drawing whose main purpose is to make you remember that white dwarfs are very dense!

The white dwarf is small and is therefore not easy to detect. However, people have perfected techniques for doing that. The first white dwarf to be detected is the so-called companion of Sirius. Now Sirius is the brightest star visible to the naked eye. People who observed it during the last century found that instead of moving through space on a smooth curve as most stars did, Sirius went on a wobbly path—see Fig. 2.10.

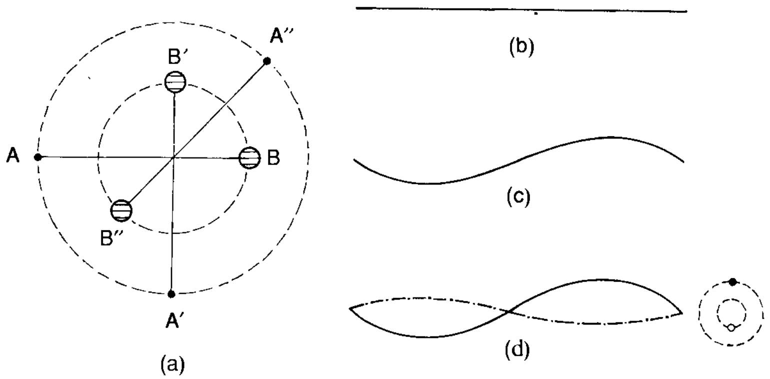


Fig. 2.10 (a) shows a binary star, i.e., a pair of stars revolving about their common centre of mass. (b) shows the trajectory of a single star in the sky. (c) shows schematically a wobbly trajectory as observed in the case of Sirius. This made people suspect that Sirius had a companion, which was later found. (d) shows the wobbly trajectory of the pair. Notice the centre of mass moves as in (b).

Astronomers then figured out that the motion was wobbly because Sirius had a companion, and that the two stars were revolving around each other. In other words, Sirius was one member of a *binary star*. From the wobble of Sirius, people could estimate that the companion had a mass roughly equal to that of our Sun. All this was known by 1844. Thereafter people tried to see this companion in the sky, but being small it took a lot of effort and finally was seen for the first time in 1863. Much later, around 1925 or so, the American astronomer Walter Adams identified this companion as a white dwarf. Since then a large number of white dwarfs have been discovered, all clustering together in the bottom left-hand corner of the H–R plot.

To recapitulate, white dwarfs are the dying embers, so to speak, of stars that have stopped burning. They are compact and therefore very dense. This leads to degeneracy pressure, which is due to a quantum mechanical effect. It is this degeneracy pressure which prevents the total collapse of the dead star and gives it a finite radius. Unlike gas pressure, degeneracy pressure can exist even at the absolute zero of temperature. Depending on its history, the core of the white dwarf would not only have hydrogen but also other nuclei like helium, carbon, etc.; but nothing heavier than iron.

To sum up:

- Stars generally form from great big blobs of hydrogen gas. The gas blob shrinks heating the core and a stage comes when the core ignites and undergoes a thermonuclear burn. A star is now born.
- The very first stars went through the p–p cycle.
- Most of the familiar stars are "young" by cosmic standards. They all burn hydrogen and lie on the main sequence.
- When the hydrogen burning is over, the outer layer of the star expands and cools off, while the core shrinks. The star then presents a dull-red appearance and is called a red giant.
- When the core of a giant becomes sufficiently hot due to contraction, the helium there gets ignited and undergoes a thermonuclear burn.
- Different kinds of thermonuclear burn cycles are possible. The common point about all of them is that at the end of each cycle, light elements become converted to slightly heavier elements (through fusion reactions).
- The larger the number of nuclear-burning stages a star goes through, the greater is the variety of elements produced in the interior (see Fig. 2.11).

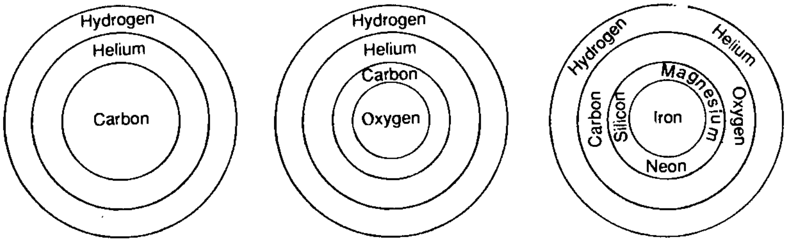


Fig. 2.11 Internal structure of stars which have undergone varying numbers of thermonuclear burn cycles. As explained in the text, elements heavier than iron cannot be formed in this manner.

- The fusion of light elements into heavier elements proceeds till iron is formed in the core. Thermonuclear reactions are not possible thereafter on account of the nuclear properties of the heavier elements.
- When all the possible nuclear burning is over, the gas pressure comes down, whereupon the dead star starts shrinking.
- There are many scenarios for a dead star, as I shall explain later. For the present, I shall concentrate on one of these. In this, the shrinking is arrested at a certain stage by a new pressure called *degeneracy* pressure which is produced when the "stellar ash" becomes very dense. This pressure is due to a *quantum mechanical* effect. It stops further contraction and the object now becomes a white dwarf. *Degeneracy pressure can exist even when the object is at the absolute zero of temperature*!

All that I have said above, was sort of known by 1930. People then believed that they not only knew how stars are born and how they evolve, but also how they die—they become white dwarfs. And then came young Chandrasekhar. He analysed the problem much more carefully than anybody else had previously, and came to the conclusion that if a star had a mass of 1.4 times the solar mass at the time it started shrinking to become a white dwarf, then the resulting white dwarf would have *zero radius*! This created a sensation. Degeneracy pressure was invoked by Fowler to save the (dead) star from shrinking to a geometric point, and here now was a young and unknown student from India who gave them the licence to do so, though under certain conditions. This was too much for some people, and they started an argument which then led to high drama! All that comes later—read on.

3 *Stellar Physics—A First Look*

3.1 Introduction

If we really start looking at all the details, then stars are pretty complicated objects to study and analyse. We must worry about where the energy comes from, how the energy production is sustained, how radiation escapes outwards from the interior, whether all the different parts of the star have the same density, pressure, etc., or whether they are different in different regions and so on. In addition we would have to worry about the rotation of the star, the disturbances to it from other stars and a number of other things like that. You and I might say, "Whew! This is too much!" and go off to do something much simpler. Big shots don't operate that way, and as far as stellar physics is concerned, Chandra is a superstar. Let us retrace the road followed by him nearly sixty years ago.

As I mentioned earlier, in this book I am going to concentrate mainly on Chandra's first major piece of work, which incidentally also made him famous. Chandra starts from the fact that people have been looking at stars for ages, and have accumulated a wealth of data—all this is the outcome of *observational astronomy*. He says:

> We shall restrict ourselves to the consideration of stars which are in equilibrium and in a steady state. Such an equilibrium configuration can be characterised by three parameters: its mass M; its radius R; and its luminosity L (L being defined as the amount of radiant energy, expressed in ergs, radiated by the star per second to the space outside)... We shall assume that we have sets of values of these quantities for a number of stars. Stellar structure deals with these results of observational astronomy.

You see what Chandra is doing? Instead of trying to tackle all the problems at once, he is focussing on one clearly-defined and self-consistent question namely, "What is the relation between M, R, and L for a star in equilibrium?" Chandra spent about five to six years studying this problem in all its aspects, and when he was finished, he put everything into a book which has since become a classic. He then went on to the next set of unanswered questions, spent a few years on them, again wrote a book when he had all the answers, then went on to the next set of questions, found the answers, wrote a book ... Get the idea? This is what he has been doing during the last sixty years or so. If you go to a (good!) library, you will see all his works (provided of course they have not been borrowed). I suggest you do this and spend a few minutes browsing through the volumes even if you don't understand a word. You can at least say that you have seen great masterpieces; and, after reading this little volume, I hope you will get some idea of what the first of these great works contains. By the way, this first book of Chandra is called *An Introduction to the Study of Stellar Structure*. The present volume is a guide to the above classic.

3.2 Problem formulation

Getting back to our theme, I have already told you what Chandra's global objective is, which is to establish "some relation between all the three parameters, L, M, and R". He says:

> We can hope to make some progress toward the solution of this problem in the following way. When we observe a star, we see that in a prescribed spherical volume of radius R an amount of material of total mass M is enclosed; we also know that through this mass there occurs a continual streaming-out of a certain mean flux of radiant energy specified by the luminosity, L. By hypothesis the star is in a steady state. The question we can then ask is: "How is it that a certain specified march of the net flux of radiant energy is able to support (against the gravitational attraction) an amount of mass equal to M inside a spherical volume of precisely the radius R?"

Are you scared?! Don't think Chandra is trying to use fancy words; he is just being very careful and precise—that is his style. What is more important, he is precise and careful in his analysis as well, which is why he keeps making major discoveries, especially some missed by others.

So the problem has been reduced to that of finding a relation between the three quantities L, M, and R. Even though the question is simple, the answer does not come all that easy. One has to proceed step by step, as I shall presently describe.

3.3 Types of equilibrium

I have already pointed out that a star is held in equilibrium under the action of two competing forces namely, gravity and pressure. From the fact that the star radiates energy, we know that there is energy production inside, and that each gram of the stellar material releases an average amount of energy $\bar{\varepsilon} = L/M$. Actually, this energy production would not be uniform throughout the star. We expect it to be maximum at the centre and decrease somewhat as one approaches the periphery. In short, $\varepsilon(r)$ the rate of energy release at the point r would vary with r; the quantity $\bar{\varepsilon}$ is the average of this variable $\varepsilon(r)$ over the whole volume.

We are now talking about heat production, which means that the equilibrium of the star is really a case of *thermodynamic* equilibrium. If radiation also plays an important role (i.e., contributes to the pressure—see Box 3.1), then one talks of *radiative* equilibrium. Either way, i.e., with or without radiation, one must clearly bring thermodynamics into the picture. See also Box 3.2.

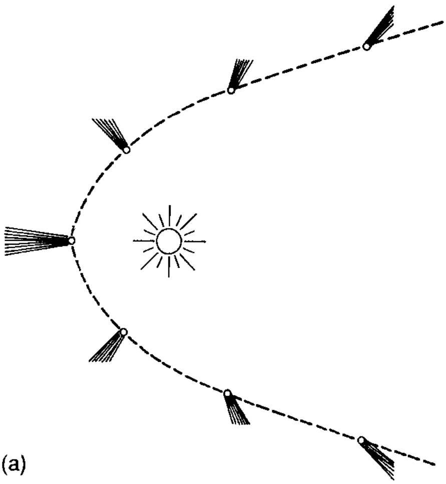
Box 3.1 Radiation pressure refers to the pressure exerted by light. Let us first ask ourselves: What is the meaning of the statement that a gas exerts pressure? Let us say we have some gas in a container. The gas is made up of molecules flying about in all sorts of directions with all sorts of velocities. Consider now one of the walls of the container. This wall would be constantly bombarded by molecules. After the collision, the molecules would simply bounce back like tennis balls. Every such bounce-back would impart momentum to the wall. The total averaged effect of all this momentum transferred to the wall by the numerous bounce-backs is what is called pressure.

Suppose we now have radiation trapped in a cavity. Radiation energy, as you probably know, is carried in "packets" called *photons*. Momentum is transferred to the walls of the cavity when photons are bounced back or absorbed. This leads to radiation pressure. Even the light coming from a torch light exerts pressure but it is very very weak and is therefore not felt. However, experiments have been done to demonstrate the existence of this pressure. One case where the effect of radiation pressure is quite visible is the tilting of a comet's tail. As you know, comets have tails. When they approach the Sun, the tail turns around considerably as shown in the sketch (a). This is due to the pressure of the radiation coming from the Sun. Solar radiation pressure also affects the solar panels of artificial satellites.

The pressure in a star is made up of gas pressure and radiation pressure. The famous astronomer Eddington (about whom you will hear a lot later on) once did a calculation along the following lines: Let us say there is a scientist who has never heard of stars but knows about gas pressure and radiation pressure. This fellow visualises gas globes or spheres of various masses—10 gm, 100 gm, 1000 gm, ... and so on. Thus the nth globe contains 10^n gm. He then calculates the ratio of the radiation pressure and the gas pressure in each of these globes. Part of his results are shown in the table. Staring at the numbers, our friend then concludes that what goes on in these spheres is a tussle between gas pressure and radiation pressure. In most of the spheres, either the one or the other dominates—i.e., the contest is one-sided. But for globes in the range 33–35, something may be expected to happen. As Eddington says, "What 'happens' is the stars."

Ratio of radiation pressure to gas pressure

No. of globes	Radiation pressure	Gas pressure
32	0.0016	0.9984
33	0.106	0.894
34	0.570	0.430
35	0.850	0.150
36	0.951	0.049
37	0.984	0.016
38	0.9951	0.0049
39	0.9984	0.0016



(a)

Box 3.2 Thermodynamics is mainly concerned with the exchange of energy (i.e., the conversion of heat energy into work and vice versa) and equilibrium properties. There is no reference to the structure of matter, i.e., one does not bother about the fact that matter is made up of atoms and molecules. The laws of thermodynamics are famous, especially the second law.

The subject is ideal for mathematical development, and some books specialise in presenting an *axiomatic* treatment. Chandrasekhar begins his book on stellar structure with a chapter on thermodynamics. Naturally, he opts for a treatment that is mathematical, and he selects the approach given by Caratheodory. Chandra gives two reasons for doing so: "First, there exists no treatise in English which gives Caratheodory's theory; and second, ... Caratheodory's theory is not merely an alternative, but also an elegant approach to thermodynamics and is the only physically correct approach to the second law." Of course, Chandra also cites the "logical rigour and the beauty of Caratheodory's theory" as additional reasons.

3.4 Equation of state

It is time to introduce the *equation of state*. Matter (as I have described in the volume *The Many Phases of Matter*) can exist in many phases (such as solid, liquid and gas), and the equation of state is a sort of shorthand description of the properties of the phase concerned. Our real target is an equation of this type for stellar material, but for the moment we shall restrict attention to a perfect gas.

Remember Boyle's law PV = constant? Well, that is one of the properties of a perfect gas. As far as thermodynamics is concerned, if we know the pressure P of the gas, its temperature T and the volume V it is occupying, we know all there is to know about the gas. And when it is in (thermodynamic) equilibrium, there is a relationship between the values these quantities must have. To illustrate, suppose we have M grams of a perfect gas, and we make it occupy a volume V and hold the temperature at T °K. Then its pressure P cannot have any arbitrary value if the gas is to be in thermodynamic equilibrium; in fact, the value of the pressure is fixed by the equation of state.

A gas in equilibrium can obviously have many equilibrium states, each defined by appropriate values for the three quantities P, V, and T. If we plot all these values, we would obtain a surface as in Fig. 3.1(a). Points like A, B, ...etc., with coordinates (P_A, V_A, T_A), (P_B, V_B, T_B), ...etc., represent various possible equilibrium states. For a perfect gas, the surface of Fig. 3.1(a) is described by the equation

$$PV = RT \tag{3.1}$$

where R is the gas constant. Equation (3.1) is the equation of state for a perfect gas.

3.5 Types of changes

One could of course always change the physical conditions of a gas. If one makes these changes very, very slowly and in such a manner that the

(a)

(b)

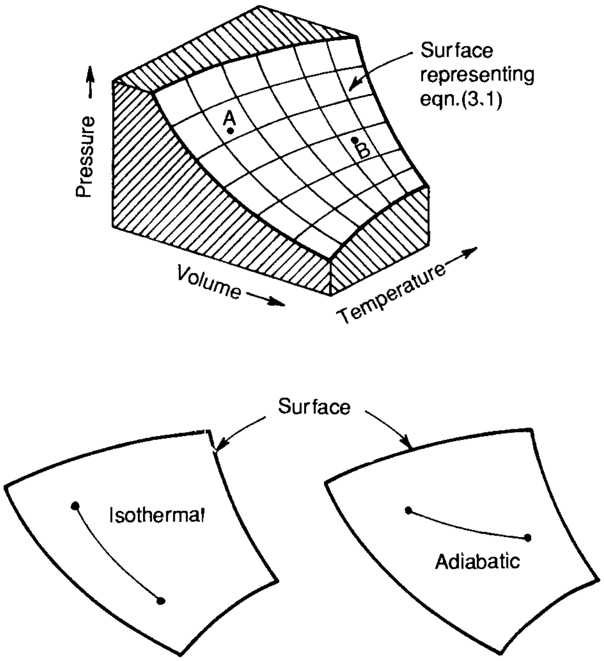


Fig. 3.1 (a) Schematic illustration of the surface corresponding to equation (3.1). This refers to a perfect gas. A and B are two typical equilibrium states. Real gases can condense into a liquid, which in turn can freeze into a solid. These complications are ignored here because they are not relevant; but they are discussed in the companion volume *The Many Phases of Matter*. (b) Illustration of isothermal and adiabatic trajectories.

system is always in thermodynamic equilibrium, then the point describing the state of the system would always remain on the surface, defined by equation (3.1), and trace a line or a trajectory on it; see Fig. 3.1(b) for some examples. Some tracks are of special interest. Lines along which

$$PV = \text{constant} \tag{3.2}$$

are called *isothermals*, while those along which

$$PV^{\gamma} = \text{constant} \tag{3.3}$$

are called *adiabatics*. Here γ is a constant which has the value 4/3 for a perfect gas. If the state of the gas is changed so as to take it along an isothermal, then such a change is called an *isothermal* change. Likewise, one can talk of an *adiabatic* change. In an isothermal change the temperature remains constant. This is because the system is allowed to be in contact with the external world which is at temperature T, and which can either supply heat energy to or accept heat energy from the system undergoing the change. In an adiabatic process the system is thermally *insulated* from its surroundings. The energy in the system therefore remains fixed, and changes caused to the system show up, among other things, as a change in the temperature. Actually, both isothermal and adiabatic changes are special cases of what is called a *polytropic* change, and the trajectory associated with such a change is called a *polytropic*. Along it, one has,

$$PV^{\gamma'} = \text{constant} \tag{3.4}$$

When $\gamma' = 1$, the polytropic becomes an isothermal while $\gamma' = \gamma$ gives us an adiabatic. Real gases are not perfect, i.e., their equations of state are *NOT* given by equation (3.1). However, one can still talk of isothermals, adiabatics and polytropics.

Usually, mathematical analysis becomes more useful when one deals with the general rather than the particular case. No wonder nineteenth century pundits like Kelvin, Ritter, Lane and Emden preferred to view a gas as "polytropic gas of index n" or simply a "polytrope of index n", where n is defined by

$$n = \frac{1}{\gamma' - 1} \tag{3.5}$$

One advantage of considering a polytropic gas of general index n is that one can deal with gases which are not perfect in the sense of (3.1). This is useful when dealing with stars, as we shall see later.

A sample result quoted by Chandra which comes out of such an analysis of polytropes is:

> The configuration resulting from the uniform expansion of a polytropic gas sphere is again another polytropic gas sphere belonging to the same index.

3.6 A bit more about equilibrium configurations

The gas which we are now considering is surely not the same stuff as stars are made of; but a study of it could provide a clue about the conditions inside the star—that at least is the hope. To start with, we shall assume that our gas is like the perfect gas of kinetic theory, i.e., it obeys the law

$$PV = RT$$

which can also be written in the form

$$P = \text{constant} \cdot \rho \cdot T \tag{3.6}$$

where ρ is the density of the gas. Later we shall consider other situations where P varies as some *power* of the density ρ.

Now it is all very well to say that the gas sphere has a mass M, that it is in equilibrium under the action of competing forces, and that the value of the radius is determined by this equilibrium. However, one can have various spheres, all meeting the requirement of equilibrium, and they would all have different radii.

Consider now one such sphere. If we now draw another sphere of radius r within this sphere as illustrated in Fig. 3.2, and if $\bar{\rho}(r)$ is mean density within this sphere, it is reasonable to suppose that $\bar{\rho}(r)$ would not

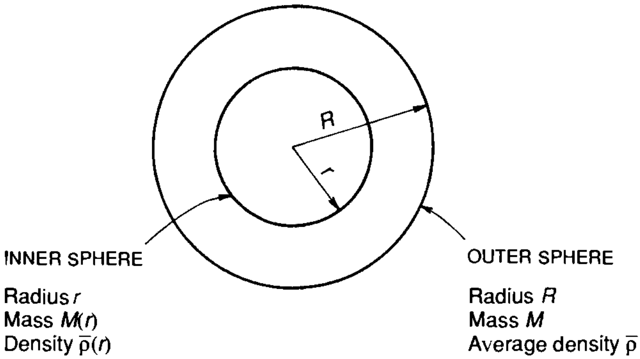
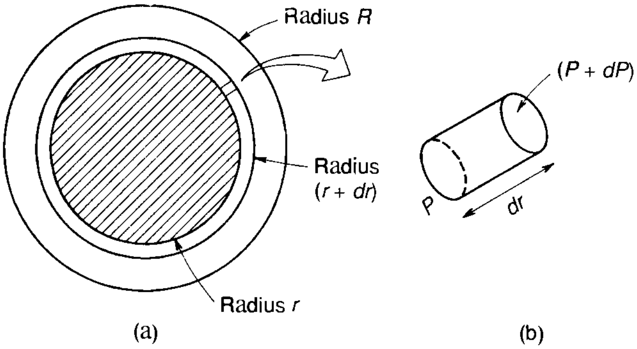


Fig. 3.2 Figure to illustrate the meaning of $\bar{\rho}(r)$. The outer sphere of radius R represents the star. It has an average density $\bar{\rho} = \dfrac{M}{(4/3)\,\pi R^3}$, where M is the mass of the gas. The density need not be uniform and have the same value everywhere. The average value in a smaller sphere of radius r is given by $\bar{\rho}(r) = \dfrac{M(r)}{(4/3)\pi r^3}$, where $M(r)$ is the mass of the sphere of radius r.

increase as r increases from 0 to R. This is a slightly technical way of saying that the density in the star basically does not increase as one moves outwards from the centre. Shortly I shall make sketches to show how the density could vary. Chandra makes this simple and reasonable assumption, namely that $\bar{\rho}(r)$ does not increase with r, and then examines how much one can infer about the polytropic gas sphere, given that it is in equilibrium.

The meaning of equilibrium must now be made precise, which means one must use some maths. If you are game for it, you could take a peep at Box 3.3. Otherwise, carry on! OK, so I was telling you about drawing some inferences. What one actually deduces are *inequalities*. If the

Box 3.3 The basic equation describing gravitational equilibrium is quite simple if you know a bit of calculus. This box can be easily read by those who do. Others need not bother! They can feel happy that there exists an equation which can be solved for answers.



To start with, let us pretend that our gas cloud has the shape of a perfect sphere of radius R. Consider now two shells inside of radii r and $r + dr$ respectively— see (a). Next consider a cylinder of length dr between the two shells which is shown separately in figure (b). Let us say the cylinder has a cross-section area of 1 sq.cm and that the pressures on the two sides are P and $P + dP$ respectively. Physically we know that the magnitude of the pressure decreases as the radius increases.

The mass of gas inside the cylinder is acted on by two forces. One of these is the force $P - (P + dP) = -dP$, acting outwards. The other is the gravitational pull acting inwards. This pull is exerted by the mass inside the sphere of radius r, shown shaded in figure (a). We denote this mass by $M(r)$. The mass of the infinitesimal cylinder is

$$\rho \cdot 1 \cdot dr = \rho\, dr \tag{1}$$

Using this, the pull on the cylinder is

$$G \cdot M(r) \cdot \rho \cdot dr/r^2$$

Equating the gas pressure and the gravitational pull, we get

$$-dP = G \cdot M(r) \cdot \rho \cdot dr/r^2$$

or,

$$dP/dr = -G \cdot M(r) \cdot \rho\,/\,r^2 \tag{2}$$

Using (2), the result

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho \tag{3}$$

and doing a bit of algebra, we then get

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho \tag{4}$$

This equation is repeatedly used by Chandrasekhar for analysing stars on the main sequence.

quantity on the left-hand side of an inequality relation is greater than that on the right, we use the symbol >; in the opposite case, we use the symbol <. We now need some notations. We introduce:

$\bar{\rho}$ = average density = mass of gas/volume of gas

$\bar{P}$ = average pressure

$\bar{T}$ = average temperature

ρ_c = central density

P_c = central pressure

T_c = central temperature

Given below are some sample results deduced by Chandra:

$$\begin{aligned} P_c &> 4.5 \times 10^8 \quad (M/M_\odot)^2 \cdot (R_\odot/R)^4 \text{ atmospheres} \\ T_c &> 3.84 \times 10^6 \quad (M/M_\odot) \cdot (R_\odot/R)^\circ\text{K} \end{aligned} \tag{3.7}$$

Here $M_\odot$ and $R_\odot$ are respectively the mass and the radius of the Sun. It is common in stellar physics to relate quantities like L, M, and R to those for the Sun. The atmosphere is a unit of pressure, and one atmosphere is the pressure at sea level—if you prefer to be more precise, it is equal to the pressure that can hold up a column of mercury 760 mm in height at sea level. μ is the molecular weight of the gas. So what do these inequalities tell us? If, for example, we put $M = M_\odot$ and $R = R_\odot$ in (3.7), we find that the central pressure in the gas sphere must be at least around several million atmospheres. Similarly, assuming that the gas is hydrogen, the core temperature would be at least around several million degrees.

Chandra also deduces many theorems whose essence can be understood by referring to Fig. 3.3. As I told you earlier, given a mass M, one can construct many spheres that are in equilibrium; two samples are sketched in Figs. 3.3 (a) and (b). In both, the density is constant, having

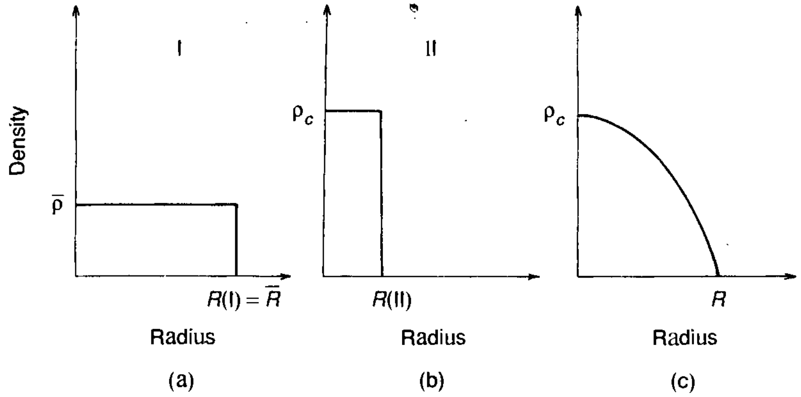


Fig. 3.3 (a) and (b) show two possibilities for equilibrium for a gas of mass M. In both, the density is a constant. While in (a) it has the value $\bar{\rho}$, in (b) it has the value ρ_c which is the centre density. The actual density distribution is somewhere in between, as in (c).

the values shown. It is clear that R(I) must equal $\bar{R}$. Now what do the theorems of Chandra say? In effect they say that $\bar{P}$ and $\bar{T}$ must obey the inequalities

$$\begin{aligned} \bar{P}(\mathrm{II}) &> \bar{P} > \bar{P}(\mathrm{I}) \\ \bar{T}(\mathrm{II}) &> \bar{T} > \bar{T}(\mathrm{I}) \end{aligned} \tag{3.8}$$

It stands to reason that the real configuration is somewhere between I and II (as schematically illustrated in Fig. 3.3(c)).

Let us pause for a moment and reflect. All we have done so far is to figure out what could happen in a gas sphere which is so large that gravitational effects are very important. As yet we are not talking about a real star—just a great big gas sphere. Why? Because such a discussion will give us some idea of the sizes, densities, pressures and temperatures we may expect in real stars. And how do we go about it? Essentially by exploiting a simple, classical equilibrium equation given in Box 3.3, supplemented by a very reasonable assumption namely, that ρ (r) does not increase outward from the centre. Notice that thermodynamics has not yet got into the act; it will shortly.

Some of you might be disappointed that I am skipping the mathematical details. Well, there are several reasons why I am doing so, the first being that they are a bit complicated. Secondly, the maths can be found in many books. Instead, I have chosen to constantly call attention to the strategy being employed, something which books often skip leaving the job to the reader! Notice how much is being deduced with just a very general statement of the problem and with very few assumptions. This, incidentally, is the method favoured by expert theoretical physicists.

3.7 Gravitational equilibrium of a polytropic gas

So far, we have considered a (classical) perfect gas whose equation of state is described by (3.1). An important feature of such a gas is that its pressure is proportional to the density. The gases of which stars are made, are not perfect and cannot therefore be described by (3.1). Indeed, as we shall see in Chapters 5 and 6, quantum mechanical effects must also be allowed for in addition. The first step in this business is to consider, instead of a perfect gas (described by (3.1)), a polytropic gas of general index n.

The history of this subject goes back to Lord Kelvin who in 1862 analysed how temperature varied with height in our atmosphere. Some of this history is briefly recalled in Box 3.4.

In a polytropic gas, pressure is related to density ρ by

$$P = K\rho^{(n+1)/n} \tag{3.9}$$

where K is a constant and n has been defined in (3.5). Notice that P no longer varies linearly with ρ but as some power of ρ. Later we shall see

Box 3.4 Chandrasekhar gives interesting historical information about how ideas concerning the gravitational equilibrium of gases evolved during the nineteenth century.

The starting point is the work of Lord Kelvin done in 1862 in which he analysed what happens in our atmosphere. The atmospheric gas experiences the pull of the Earth. Inside the atmosphere, the hot air rises while the cold air sinks. When a steady state is reached, there is *convective equilibrium*—this is what Kelvin described.

Seven years later, Homer Lane in America analysed the temperature and the density at the surface of the Sun in his paper *On the Theoretical Temperature of the Sun under the Hypothesis of a Gaseous Mass Maintaining its Volume by its Internal Heat and Depending on the Laws of Gases known to Terrestrial Experiment* (wow!). In his analysis, Lane followed Kelvin and essentially considered a polytropic gas with $\gamma = \gamma' = 5/3$. Lane was not interested in the gravitational equilibrium of a gas as such, but results relating to this were derived by him. In 1876, Lord Kelvin visited America, and some results obtained by Lane were mentioned to him by Mr. Newcomb, a friend of Lane. Kelvin did not believe in what he heard, and Newcomb could not satisfactorily explain Lane's work to Kelvin. Later Lane sent a proof of his results to

Lord Kelvin. Kelvin not only accepted them, but used the results to publish a paper of his own on the gravitational equilibrium of a gas. Incidentally, Kelvin's interest in this problem was aroused by a question set for the Ferguson Scholarship exam of 1885! The question is as follows:

> Assuming Boyle's law for all pressures, form the equation of the equilibrium-density at any distance from the centre of a spherically attracting mass, placed in an infinite space filled originally with air. Find the integral which depends on a power of the distance from the centre of the sphere alone.

Twenty years later, Kelvin wrote another paper entitled *The Problem of a Spherical Gaseous Nebula*, which was published after his death.

We now come to the German mathematician/astronomer Ritter. He did monumental work and published eighteen papers during the period 1878–1889. Ritter derived many results which had already been derived earlier by others, but that was not known to him. Chandrasekhar says that the so-called Lane–Emden equation should really be called the Lane–Ritter equation.

Finally, Emden in true Germanic style systematised all the earlier work, besides giving a thorough discussion of the general solutions to the problem. He also painstakingly computed numerous tables which would help those making numerical calculations.

some specific examples of this nonlinearity. This is also the place where the story gets a bit more technical and mathematical, and so I shall skip the details. But if you would like to know what the equation governing the equilibrium looks like in the present case, then you should consult Box 3.5. In effect, if one has the patience and the ability (!), one could crank the handle, so to speak, and pull out all kinds of interesting results out of the Lane–Emden equation described in Box 3.5. Later I shall

Box 3.5 The Lane–Emden equation is the equilibrium equation of the previous box specialised to the case of a polytropic gas. For deducing this, one must first make some changes of variables in equation (4) of Box 3.3. These are:

$\lambda\ \theta^n$ instead of ρ (λ is an arbitrary constant)

$a\xi$ instead of r (a is a suitable constant)

One must also remember that pressure is now given by

$$P = K \cdot \rho^{[1+(1/n)]} = K \cdot \lambda^{[1+(1/n)]} \cdot \theta^{n+1}$$

With all these changes, the equilibrium equation becomes,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n \quad (1)$$

The above equation is referred to as the Lane–Emden equation of index n. The problem now is one of solving this equation under suitable boundary conditions. Finding a solution means finding the function $\theta_n(\xi)$ which satisfies (1) and the assumed boundary conditions. Explicit formulae can be written down for

θ_0, θ_1, and θ_5, e.g.,

$$\theta_5(\xi) = \frac{1}{(1 + 1/3\xi^2)^{1/2}}$$

In general, the solution is given by the series

$$\theta_n(\xi) = 1 - \frac{1}{6}\xi^2 + \frac{n}{120}\xi^4 - \dots$$

A study of this reveals the properties of polytropic gases in gravitational equilibrium.

discuss how meaningful such results are. In passing we might note that the (classical) perfect gas described by (3.1) corresponds to the case n tending to infinity.

3.8 Radiation equilibrium

A few words now about radiation equilibrium. First we must get acquainted with some basic concepts. I know some of this stuff may sound a bit boring but bear with me for just a little. I promise that later the story would get very interesting.

Specific Intensity Let us visualise a star as a sphere in Fig. 3.4(a). Inside the star, energy is being produced all over the place, especially deep in the interior. Energy production also means emission of radiation and hence, from every point inside, radiation would be flowing or streaming in all directions. Consider a sphere of radius r inside the big sphere of radius R. Let P be a point on this inner sphere. We now consider a small area on the inner sphere around P as shown in Fig. 3.4(b). Radiation will be crossing this area both from the inside to the outside as well as from the outside to the inside. If the patch about P has an area of 1 sq. cm, then the *specific intensity* I_s is the intensity crossing this unit area in the direction s—see Fig. 3.4(c).

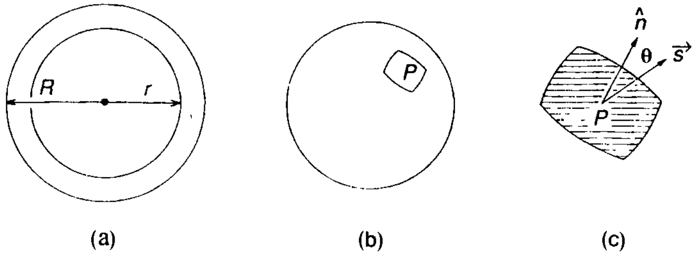


Fig. 3.4 (a) This shows the gas sphere of radius R and an inner sphere of radius r. To define the specific intensity I_s, we consider a small area as in (b) across which radiation is streaming both ways. I_s is the intensity streaming in the direction s making an angle θ with the normal $\hat{n}$—see (c).

Isotropy Consider Fig. 3.5. If the specific intensity at the point P is the same in all directions, then one says that the radiation is *isotropic* at P.

Homogeneity If I does not depend on the direction at P and is further not dependent on the *position* of P as well, then one says that the specific intensity is *homogeneous*.

Flux Consider a patch as in Fig. 3.6, and let us say its area is 1 sq. cm. The total amount of radiation crossing this area from one side to the other is called the *flux*.

Frequency distribution As you know, an electromagnetic wave has a frequency. The specific intensity $I(\omega)$ associated with a particular radiation frequency ω is called the *monochromatic intensity*. In general, inside a star, all frequencies would be present. The distribution of radiation intensity amongst these frequencies is called the *frequency distribution*. If we add, or better integrate, the monochromatic intensity at all the various frequencies, then we get the *integrated* intensity. Mathematically,

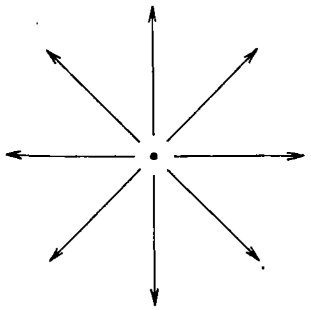


Fig. 3.5 Let us denote the specific intensity I_s at the point P in the direction s by an arrow of a certain length. If the specific intensity at P in all possible directions can be represented by vectors of the *same* length, then we say that the radiation is isotropic at P.

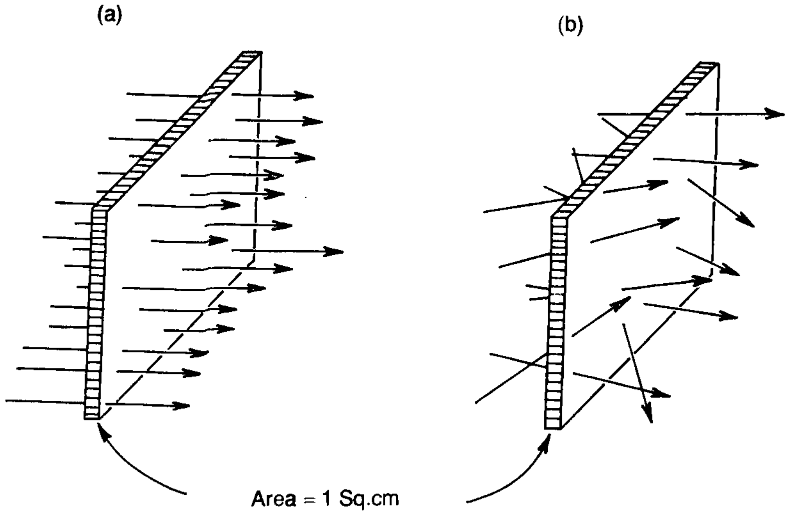


Fig. 3.6 This figure explains the meaning of the term flux. Say we have a surface of unit area as shown, and that radiation is crossing this surface. Flux is the amount of radiation crossing this surface in unit time. If radiation is streaming in all sorts of directions as in (b), then we must be a bit more careful in the definition.

$$I = \text{integrated intensity} = \int_0^\infty I(\omega)d\omega \qquad (3.10)$$

If you know calculus, you can make sense of this equation; otherwise, don't bother! The equation merely says "via a formula" what I earlier said in words.

Radiation pressure The idea of radiation pressure has already been introduced earlier. Radiation pressure p_r depends on I, and in some stars it is significant. Where it is, it must be added to the gas pressure p_g to obtain the total pressure P, i.e.,

$$P = p_g + p_r \qquad (3.11)$$

It is P so obtained which must be used in the analysis of the equilibrium configuration of the star. Sometimes, p_r is expressed as a fraction of P using the following definitions. Let

$$(p_g/P) = \beta, \quad \text{or} \quad p_g = \beta P \qquad (3.12)$$

Then,

$$(p_r/P) = 1 - \beta, \quad \text{or} \quad p_r = (1 - \beta)P \qquad (3.13)$$

Luminosity I have already introduced you to the idea of luminosity; it is the total amount of energy radiated by the star in one second to the space outside. Now this L refers to the energy radiated from the outer surface of the star. Likewise, we can also think of the energy going outwards from a sphere of radius r drawn inside the sphere of radius R—see Fig. 3.7. $L(r)$ is the energy going outwards from such a sphere. If F_r is the energy flux crossing this surface (remember flux is the energy crossing unit area), then, since the surface area is $4\pi r^2$, we have

$$4\pi r^2 \cdot F_r = L(r) \qquad \text{or} \qquad F_r = L(r)/4\pi r^2 \tag{3.14}$$

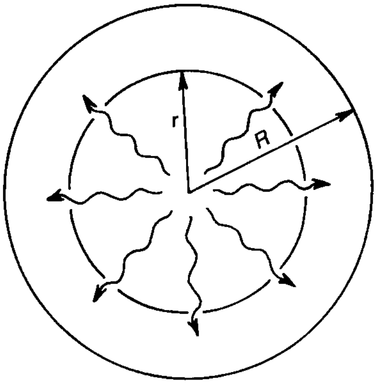


Fig.3.7 r is the radius of a sphere inscribed inside a larger sphere of radius R. The two spheres are concentric. $L(r)$ is the total amount of energy crossing the surface of the smaller sphere in the outward direction. Dividing $L(r)$ by the surface area, we get the outward flux F_r.

Luminosity and radiation pressure As is to be expected, luminosity and radiation pressure are related. The relationship can be qualitatively understood as follows. Consider a small surface S as in Fig. 3.8. For convenience we shall assume S has an area of 1 sq. cm. Let us say there is a flux F of radiation crossing S as shown. S' is another surface at a distance dr from S. If some of the flux crossing S is absorbed between S and S', then naturally less flux will leak across S'. Let the radiation pressure on the two surfaces be as shown. It turns out that this pressure difference is related to the energy absorbed in the prism. The volume clearly is $1 \times dr = dr$. If ρ is the density of the gas in the prism, then the mass of gas in the prism $= \rho$ dr. The absorption of radiation is usually described by a quantity κ called the *mass absorption coefficient*. κ is the energy absorbed per unit mass per unit flux. So clearly the energy absorbed by the prism is

$$F \cdot \kappa \cdot \rho \cdot dr \tag{3.15}$$

Absorption of this energy produces a kick on S'. Using more respectable language, the momentum communicated by radiation in the direction of

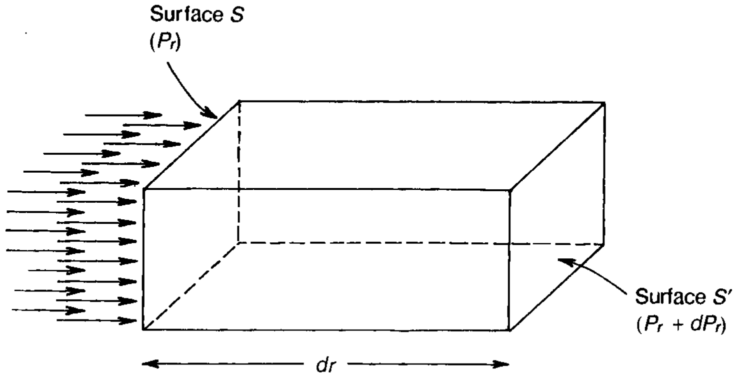


Fig.3.8 Shown here is a rectangular prism. Radiation enters via the surface S and leaves via the surface S'. Part of the radiation is absorbed, which then produces a momentum. It is this which shows up as the difference in the pressures acting on the two sides.

the flux is obtained by dividing (3.15) by c (the velocity of light). This is equal to the difference in pressures acting on the surfaces S and S', i.e.,

$$p_r - (p_r + dp_r) = -dp_r = F\left(\frac{\kappa\rho}{c}\right)dr \tag{3.16}$$

This is a schematic argument but hopefully, it would enable you to understand the following result applicable to the gas sphere we have been considering.

Consider Fig. 3.9. Using similar arguments as above we can write

$$-dp_r = F_r \cdot \left(\frac{\kappa\rho}{c}\right) \cdot dr \tag{3.17}$$

We now substitute for F_r from (3.13) which then gives

$$-dp_r = \frac{L(r)}{4\pi r^2}\left(\frac{\kappa\rho}{c}\right) \cdot dr \tag{3.18}$$

We thus have a tie-up between radiation pressure and luminosity.

3.9 The standard model

All that I have been discussing so far is really a prelude to the so-called *standard model*. This is the name given to a model for stars which was invented by Eddington (about whom you will hear a lot more in a later chapter). Models are big business in physics. A model is something one

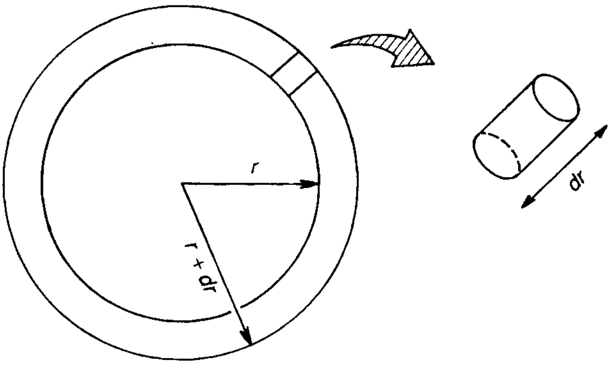


Fig. 3.9 This shows a prism between two spheres of radii r and $r + dr$. The difference in pressure is related once again to the energy absorbed in the prism and the momentum generated thereby.

tries to construct when one has to describe a complicated situation. A model is therefore an approximate description of reality and invariably involves many simplifying assumptions. Why does one invent a model? Basically to make calculations using it, and also to make *predictions*. These calculations and predictions are then checked out by performing suitable experiments, measurements, etc. If the result of the model disagrees with observations, then tough luck—out goes the model! If the observations agree with predictions, then one-up for the model. If the agreement is only partial, then obviously the model needs some improvement. This is the way the model game is played. Pundits like Chandra try to produce models which can be solved *exactly* rather than approximately. Then there is absolutely no doubt about the solutions given by the model. Of course, it is no use having an *exactly solvable model* if its predictions are way off from reality. This is where skill in formulating the model comes in. If the model can be solved exactly and if the results of the model agree with experiment, then one has hit the jackpot!

One thing we must always remember—models are convenient idealisations. For example, our model might take the star to be a perfect sphere whereas in reality the star is likely to have a ragged profile as in Fig. 3.10. But if the fuzziness in the radius is small compared to the average value of the radius, then there is no harm in making a simplified assumption about the radius having a constant value.

So much for a general introduction on models. Let us move on to Eddington's *standard model*. Eddington first proposed this model in 1917 and later described it in his famous book *The Internal Constitution*

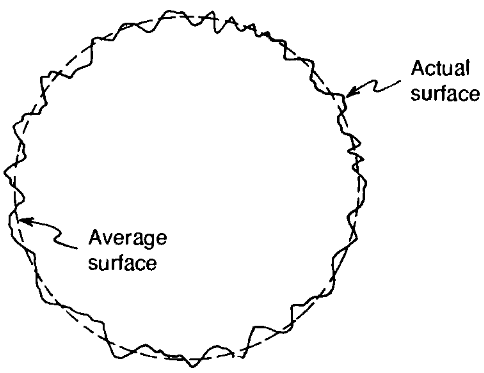


Fig. 3.10 When we imagine a star to be a perfect sphere of radius R, we are actually making a model. In reality, the surface of the star would be rough. However, if the fluctuations in the radius are small compared to the average radius itself, we can ignore these fluctuations and pretend that the star is a perfect sphere with a radius equal to the average radius.

of Stars which was published in 1926. Interestingly, Chandra received a copy of this book as a prize while he was in college. Later Chandra met Eddington, and there arose a serious disagreement. That story comes in Chapter-6. To get back to the standard model, we start by pretending that the star is a gas sphere of mass M and radius R and that the pressure in the interior is due both to the gas itself and to radiation. Together these pressures balance gravity. The equation governing equilibrium is

$$\frac{d}{d}(p_g + p_r) = \frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} \tag{3.19}$$

which has been explained earlier. On the other hand, equation (3.18) tells us how radiation pressure is related to luminosity: we have

$$\frac{dp_r}{dr} = -\left(\frac{\kappa\rho}{c}\right) \cdot \frac{L(r)}{4\pi r^2} \tag{3.20}$$

If we divide (3.20) by (3.19), we get

$$\frac{dp}{dP} = \frac{\kappa}{c} \cdot \frac{L(r)}{4\pi} \cdot \frac{1}{GM(r)} \tag{3.21}$$

This result above essentially tells us that the manner in which the radiation pressure varies with the total pressure P depends on how strong luminosity and absorption are compared to the gravitational crushing. Remember what the game is? It is to find the relationship between L, M, and R. So what do we do? We try to find an equation which describes the delicate balance between radiation, pressure, luminosity and gravity from point to point. This is what equations (3.19), (3.20) and (3.21) do; if you don't quite follow them, doesn't matter. At least you have the general idea. OK, to get back to these equations, if they are carefully analysed and if one studies this delicate balance from point to point in a gas of mass M, then one could in principle pull out the desired L–M–R relation one is after.

As yet the problem is still a bit general, and we haven't come to the standard model yet. For that we need one more simplifying assumption. Let us introduce a quantity $\eta(r)$ which is the ratio of the average rate of liberation of the ("heat") energy $\bar{\varepsilon}(r)$ inside the sphere of radius r, to the corresponding average for the whole star, i.e.,

$$\eta(r) = \frac{\bar{\varepsilon}(r)}{\bar{\varepsilon}} \tag{3.22}$$

As you can see, the value of the ratio $\eta(r)$ could in principle depend on r, i.e., vary from point to point. Likewise, the mass absorption coefficient could also be dependent on r—I mean there could be different species of atoms present in the different regions of the star, causing $\kappa(r)$ to vary. The standard model assumes that the product $\eta(r) \cdot \kappa(r)$ is the *same throughout*, i.e., is independent of r. It then turns out that the pressure is related to the density by the relation

$$P = K \cdot \rho^{4/3} \tag{3.23}$$

where K is a suitable constant. If you remember what I said before, you will immediately recognise that equation (3.23) is nothing but the equation for a polytrope of index 3. And now we can go to town and pull out all the results we want by a polytropic analysis, including the one between L, M, and R.

Analysis of the standard model is by no means easy, and one look at Chapter 6 of Chandra's book should convince you of that. Meanwhile, let me give you a sample of the kind of things one can pull out.

$$L = \frac{\pi^3}{4^{3+s}} \left(\frac{GH}{k}\right)^{7+s} \frac{ac}{3} \frac{\xi_1^s}{[{}_0W_3]^{4+s}} \cdot \frac{1}{K_0\eta_c} \cdot \frac{M^{5+s}}{R^s} \cdot (\mu\beta)^{7+s} \tag{3.24}$$

Looks rather complicated doesn't it? I am not going to explain the meanings of the various strange-looking symbols occurring above because that would be taking me too far away—in any case, we don't need them. What I want to draw your attention to is that (3.24) offers a relation between L, M, and R. To drive home the point, let us consider the special case when the quantity s takes the value 1/2. Chandra simplifies the result for this case to

$$\frac{L}{L_\odot} = \text{STUFF} \cdot \left(\frac{M}{M_\odot}\right)^{5.5} \cdot \left(\frac{R_\odot}{R}\right)^{0.5} \tag{3.25}$$

where STUFF essentially contains some numbers and constants. I guess I do not have to explain the meaning of $L_\odot$.

So at last, we are close to the sort of things we set ourselves as the goal. How good is the standard model? This is the subject of the next chapter.

To sum up:

- Astronomers have accumulated a lot of results by painstaking observations.
- A theoretical astrophysicist's job is to explain the observations.
- Problems of stellar physics are many, and it does not make sense to tackle all of them at once.
- Chandra sets himself the limited task of relating L, M, and R, and comparing theory with observations.
- For deriving a theoretical relationship between these three quantities, one needs a model.
- A model is a simplified/idealised description of a complicated, real-life scenario. The model adopted in quest of the desired L–M–R relation is:
  * The star is a gas sphere of mass M.
  * There is energy production throughout.
  * There is also absorption throughout.
  * There is gas pressure as well as radiation pressure.
  * Density decreases outwards in the sense described.
  * The gas can be viewed as a polytrope of index n.
- The standard model (due to Eddington) makes a few additional assumptions. The net result is that the gas is now a polytrope of index 3.
- *Formulating* a model might be relatively easy but its *analysis* need not be! Chandra's book shows that.
- Caution: So far, only classical physics is used, especially in describing the behaviour of the gas. Under certain conditions, quantum effects are important. They are so in white dwarfs, but we shall discuss that later.

4 *Polytropic Analysis And The H–R Diagram*

In the last chapter I discussed in broad terms the luminosity–mass–radius (L,M,R) relation. It was assumed that the stellar medium could be described by polytrope equations, and the standard model of Eddington was used for analysis and the derivations of the equations. The assumptions of the standard model might not always hold for the particular star we are interested in. Even so, the above type of analysis can be applied provided it is suitably modified to take care of the deviations from the standard model. This type of extension involves what is called *perturbation theory*, and Chandra gives a crisp discussion of it in his book.

What does one do with all this? As I told you earlier people have accumulated a lot of data about stars by painstaking observations, and an important objective of theory is to explain available experimental facts (—another objective, by the way, is to make predictions). Naturally, Eddington who proposed the standard model also had some ideas about how the model should be used—I shall come to that shortly. However, Chandra has a larger goal which is: Can the polytropic analysis enable us to understand the structure of the H–R diagram? An attempt in this direction had already been made by the Swedish astronomer Stromgren. Chandra carries Stromgren's analysis further, pegging it firmly to the polytropic study described earlier.

This chapter is a brief description of Chandra's work on the interpretation of the H–R diagram. You will get a glimpse of how, using results derived by him (and others) by patient mathematical analysis, we can get a feel for *why* there is a band in the H–R diagram associated with the main sequence. We will also see how all this impressive analysis fails to explain the occurrence of the white dwarfs. And it is this failure of the standard model (and its perturbative extensions) which impelled Chandra to study white dwarfs; but that comes later. For the moment, let us see how Chandra explains for us the occurrence of the main sequence.

We start with the formula

$$L = 7.17 \times 10^{24} \times \frac{M^{5.5}}{R^{0.5}} \times \frac{1}{\kappa_0} \times (\mu)^{7.5} \times \text{other factors} \tag{4.1}$$

This is an adaptation of the formula (3.25) which we have already met. We don't have to bother too much about the details. Let us just note a few pertinent facts. Firstly, M is the mass of the star in terms of $M_\odot$; similarly R is the ratio of the radius of the star to $R_\odot$. κ_0 is a constant related to the absorption coefficient introduced in the last chapter. The details are rather complicated but fortunately for our purposes, we can skip all that. Let us just say that κ_0 and a related quantity κ^* denote a physical property of the star; and, as we shall shortly see, this quantity has an interesting role to play.

A word now about μ, for this also is a very important quantity. Briefly, it denotes the *mean molecular weight* of the star. As we already know, several elements would generally be present in a star, although hydrogen, and to a slightly lesser extent, helium would dominate. Further, because the interior of the star would be at a high temperature, the elements would be in various stages of ionization. What it means is that varying numbers of electrons in the outer shells of the atom would have been torn apart and rendered free. The quantity μ depends in an intricate way on the chemical composition as well as the state of the ionization of the atoms.

Before we proceed further, there is one more thing we must take note of. Let 1 gram of stellar material contain X grams of hydrogen (clearly, X must be a fraction). In the theory, there occurs a parameter X_0 which is related to X. So giving the value of X_0 is like giving the value of X which in turn is related to μ. Keep all this in mind.

In his work on stars (done prior to Chandra's work) Eddington assumed that all stars have the same mean molecular weight ($\mu \sim 2$; the symbol $\sim$ means *about* or *of the order of*). In turn, this implies that hydrogen abundance is low. Chandra points out that this view has to be abandoned, and he gives an argument by considering the example of the star Capella. For this star, it is known from observations that

$$L = 120 L_\odot,\ M = 4.18 M_\odot,\ R = 15.8 R_\odot$$

If one follows Eddington and uses formula (4.1) to calculate κ^*, one gets

$$\kappa^*\ (\text{theory}) = 3.9 \times 10^{25}$$

This is obtained assuming $\mu \sim 2$ and that hydrogen is *NOT* present. On the other hand, from experiments it is found that for Capella,

$$\kappa^*(\text{observed}) = 8.8 \times 10^{26}$$

The two values of κ^* differ by a factor of about 23. This disagreement is referred to as the *opacity discrepancy*.

Eddington was not bothered too much by this discrepancy. He said (in effect), "Well, let us use κ^* (observed) for stars together with formula (4.1), to predict the luminosity (knowing of course, M, R and a few other things that go into the formula)." Chandra does not accept this approach since it tends to take too simplistic a view of stars. Based on facts known to him, Chandra asserts (i) that one cannot assume the same value of μ for all stars, and that (ii) the dominant presence of light elements, especially hydrogen, must be recognised. He goes further and says that one should in fact use observational data on L, M, R for stars to try and understand why a band like the main sequence occurs on the H–R diagram.

The way Chandra goes about doing all this is roughly as follows. The details are tedious and intricate and I shall therefore suppress them. However, sometime you should browse through Chapter 7 of Chandra's book (for it is the content of that which I am trying now to summarise), to get a feel for the phenomenal slogging he has done! To get back to the story, the steps involved are as follows:

1. Start with observational data on L, M, R for stars.
2. Use observational data together with formula (4.1) to obtain μ and therefore X_0. (This is more easily said than done!).
3. Take a graph paper and mark M and R axes as in Fig. 4.1(a).
4. Using observational data, mark every star as a dot and, along with it, show the computed value of X_0 (obtained as in step two above).

If all this is done, we would get a plot as in Fig. 4.1(a). We can now draw smooth curves linking dots with nearly the same value for X_0. In this way, we get the family of curves in Fig. 4.1(b). One can now suppress the dots and collect the curves together as in Fig. 4.1(c). Each curve in the figure describes stars of different masses and different radii, but corresponding to the same value of X_0 (and therefore same hydrogen content). Note that these curves are the result of combining experimental data with theoretical analysis. And in performing the latter, Chandra was more careful than Stromgren was earlier, especially in evaluating X_0.

What next? How does Fig. 4.1(c) tell us something about the structure of the H–R diagram? For this some more work is needed. First, we use the observational data on L and R to obtain the surface temperature of the star. This is done using a law of physics called *Stefan's law*. Using this law, we can write

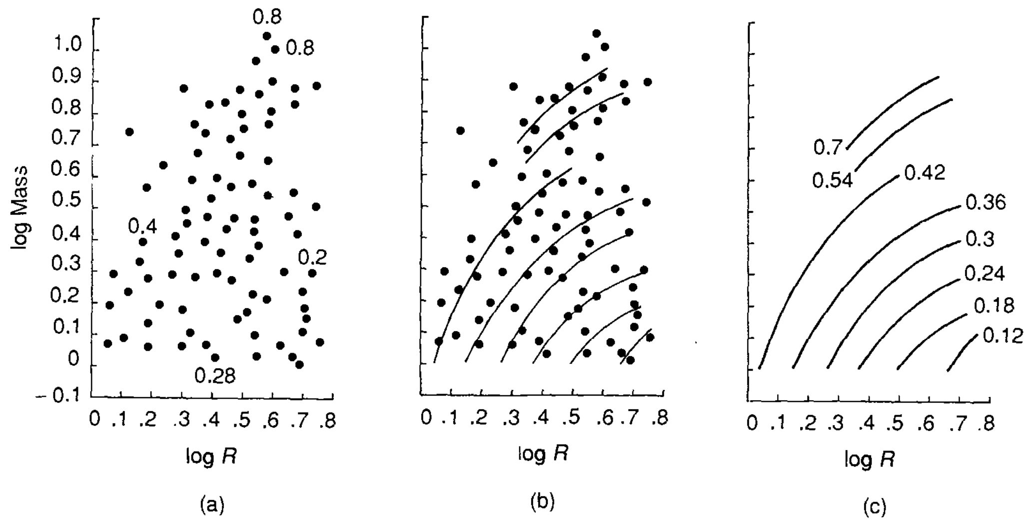


Fig.4.1 Pictorial outline of Stromgren-type analysis. (a) shows a schematic plot of the "raw" data of the massand the radii of stars, as determined from observations. Each dot (star) is labelled by its X_0 value (for convenience, only some dots have been labelled here). Smooth curves are now drawn (as in (b)) through this cluster, linking points having nearly the same value of X_0. The result of this exercise would appear as in (c) where the different curves correspond to different values of X_0.

$$\sigma T^4 = \frac{L}{4\pi R^2} \tag{4.2}$$

where σ is a constant called *Stefan's constant* and T is the surface temperature. We already know how to convert luminosity to absolute magnitude. So we can now prepare a new graph with axes as in Fig. 4.2. We now go back to Fig. 4.1(c), and take a point on one of the curves there. For this point, we know not only M and R but also X_0. Because of the L–M–R relation, we also know the L for this point, which means we know both the absolute magnitude as well as T. So we plot this point in our new graph, and once again write down the corresponding value of X_0. We patiently do this for various points on all the curves of Fig. 4.1(c). Having completed this transfer of data to our new graph, we can, like before, link or join points corresponding to the same value of X_0. In this way, we get the various solid curves in Fig. 4.2. The dashed lines correspond to constant M.

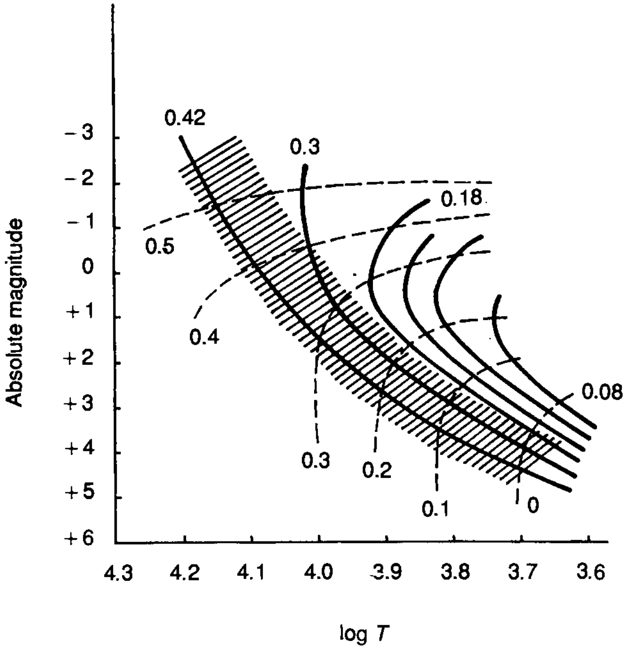


Fig. 4.2 As described in the text, one can obtain the luminosity L corresponding to points on the various curves of the previous figure. The associated temperatures can also be calculated. Using these new results, brightness versus temperature curves can now be plotted. The resulting "theoretical" H–R diagram would look like the above. The shaded patch indicates the region corresponding to the main sequence.

We can now group those with large X_0 and band them as shown. This would represent the region of hydrogen-rich stars. And now observe that this band looks like the main sequence band sketched earlier. In short, using the standard model and its perturbative extensions, together with a very careful analysis of experimental data and a rejection of previously made unwarranted assumptions (like μ being the same for all stars), Chandra shows how pages and pages of mathematical analysis can be made to work and deliver the goods! Notice, however that all this analysis does *NOT* explain the white dwarfs. The explanation of that follows in subsequent chapters.

5 *Quantum Effects*

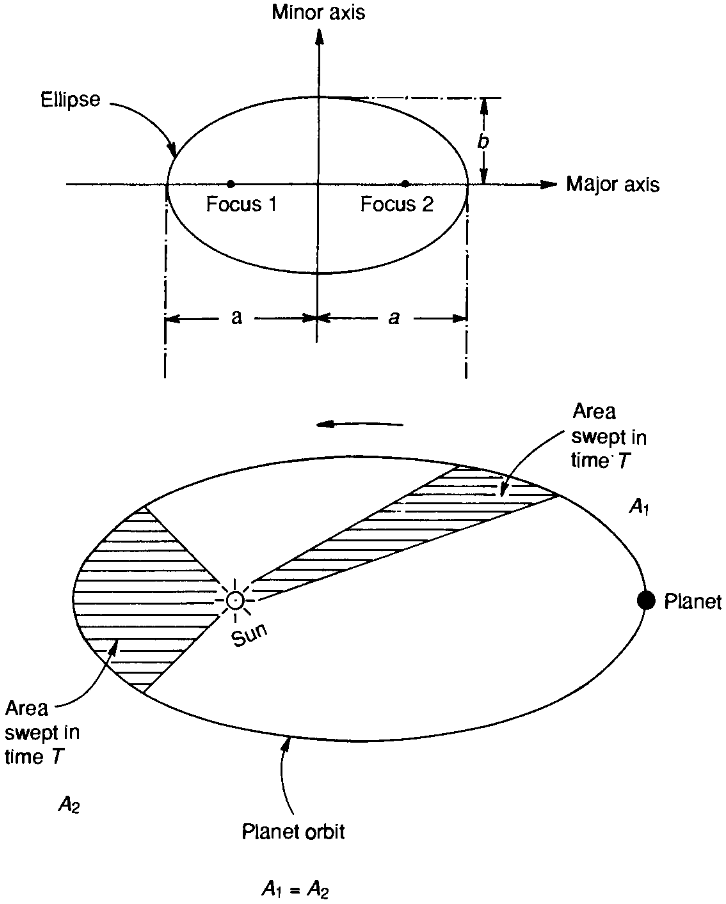
In the previous chapters, we have been concerned mainly with a *classical* gas, i.e., a gas the particles of which obey *classical mechanics*. As I shall explain in the next chapter, one can get away with this sort of thing in the case of many stars but not so in the case of *white dwarfs*. This chapter is a primer on *quantum gases*, i.e., gases, the particles of which obey *quantum mechanics*. Quantum gases help us to understand the behaviour of white dwarfs.

5.1 Classical mechanics

Let us start with classical mechanics before we go on to quantum mechanics and then to quantum gases. For many centuries, people have been interested in the motion of material bodies, like planets, bullets, flying arrows, and even stones. As you know, motion of material bodies can be produced when a force is applied. Before Newton, people did not have a clue as to how precisely force influences motion. It was Newton who first stated that the application of a force causes a velocity change proportional to the force applied. This is Newton's Second Law of Motion, and its discovery was the starting point of a new subject—*mechanics*. It was also a turning point in physics. As Richard Feynman says:

> Before Newton's time, the motion of things like the planets was a complete mystery, but after Newton there was complete understanding. Even the slight deviations from Kepler's laws (see Box 5.1) due to the perturbation of the planets, were computable. The motion of pendulums, oscillators with springs and weights in them, and so on, could all be analysed completely after Newton's laws were enunciated.

Box 5.1 I wonder whether you have heard about Kepler's laws; don't worry if you have not for I am going to tell you something about them. I must start with the Danish astronomer named Tycho Brahe who lived between 1546 and 1601. Tycho seems to have been a very patient man for he spent years and years watching the sky at night and recording the positions of various heavenly bodies, including the planets. Think of all the sleep he must have missed!



Brahe's assistant Johannes Kepler (1571–1630) was made of different stuff. He did not care to make more observations, lose his sleep, and merely add to the pile of data already left behind by Tycho. Instead he said: "Let me analyse Tycho's data and see if there is something in there." And sure enough there was, and what he found is now taught to us as *Kepler's laws of planetary motion.* They are:

1. Every planet moves around the Sun in an elliptic orbit with the Sun at one focus.
2. The radius vector from the Sun to the planet sweeps out equal areas in equal intervals of time.

3. The squares of the periods of any two planets are proportional to the cubes of the semi-major axes of their respective orbits, i.e.,

   $T \propto a^{3/2}$ (symbol $\propto$ means *proportional to*).

Newton followed Kepler, and his greatness was that he could derive the laws of Kepier from his law of gravitation. This was a major feat for one could now analyse not only the motion of planets in our own solar system but of stars and planets anywhere. Of course, lately we have realised that we must be a bit more careful in this business and use the theory of gravitation as given by Einstein (in his *General Theory of Relativity*) rather than the theory given by Newton that is, if we are dealing with the cosmos, black holes, etc. Otherwise, Newton's law of gravitation works fine.

After Newton came Lagrange (see Box 5.2), Hamilton (see Box 5.3), Euler (see Box 5.4) and many others, and thanks to all these people, mechanics flowered in the eighteenth and nineteenth centuries. Today when we talk of classical mechanics, we refer to mechanics as developed by these pioneers. Of course, in their time the subject was simply called mechanics; the adjective *classical* was added later when quantum

Box 5.2 Joseph Louis Lagrange was born in 1736. He has made important contributions to mathematics, mechanics and astronomy. He spent the first thirty years in Turin, Italy, where he was born. Then he moved to Berlin where he spent twenty years. The last phase of his life was spent in Paris.

Lagrange's father wanted him to study law and Lagrange agreed. But while he was studying physics and geometry as a student, he liked the subjects so much that he later decided to ignore law!

As I said earlier, Lagrange has made many well-known contributions. Not so well-known are his participations in the competitions of the French Academy. In those days, the French Academy used to pose problems once every few years. Often the problems dealt with astronomy. For example, in 1762 it asked "Whether it can be explained by any physical reason why the moon presents almost the same face to us ..." The solution was due in 1764. Lagrange sent one which partly solved the problem but he did not get the prize. The questions for 1766 involved the satellites of Jupiter, and Lagrange's essay won the prize. The question for 1768 again involved the moon. Lagrange was not keen to compete but his King (in Turin) compelled him. Finally, he could not participate due to illness. But he participated in the competition of 1772—the subject was still the moon! This time he shared the prize with Euler. This went on till around 1780. Thanks to all this, Lagrange was able to write a treatise on celestial mechanics which appeared in 1786. He died in 1813.

Box 5.3 William Rowan Hamilton was born in Ireland 1805. A , a young lad, he demonstrated a remarkable capacity to master languages. Besides the traditional classical languages namely, Greek, Latin and Hebrew, Hamilton also learnt Persian, Arabic, Sanskrit, Chaldee, Syriac, Hindustani, Bengali, Marathi, and Malay! Hamilton also showed a talent for mathematics, though it was his calculating abilities which caught attention first. In 1827, Hamilton was appointed as an astronomer. As a practical astronomer Hamilton was a failure since he did very little observation. But he had plenty of time for maths and physics.

Hamilton's major contributions were in optics, dynamics and a mathematical topic called quaternions. He made good use of Fermat's principle that light rays chose paths governed by the *principle of least time.* Later he applied similar ideas to dynamics, where he introduced a quantity later called the *Hamiltonian function* or simply the *Hamiltonian.* With the advent of quantum mechanics, the Hamiltonian rose in importance.

Hamilton's papers are supposed to be very difficult to read. Nevertheless, his greatness was realised later. His personal life was painful. He went through a failed love affair which nearly drove him to suicide. He also became an alcoholic but later overcame this problem. He was a good friend of the poets Coleridge and Wordsworth. The latter advised Hamilton that his (i.e., Hamilton's) natural calling was mathematics and not poetry, as Hamilton believed was the case! Hamilton died in 1865.

Box 5.4 Leonhard Euler was a famous Swiss mathematician who, however, did a good bit of his work in Russia. He was born in Basel, Switzerland in 1707. His father was a priest but was interested in mathematics, an interest later on picked up by the son. While in the University, young Leonhard met the famous Bernoulli and requested him to give private tuition in mathematics. Bernoulli flatly refused but gave permission to the young lad to visit him every Saturday afternoon to get his doubts cleared.

Like his father, Leonhard also was supposed to become a priest but his keen interest in mathematics made him give up this idea. Like Lagrange, Euler too won many prizes from the Paris Academy. Around 1725, the St. Petersburg Academy of Sciences in Russia was looking for new talent. Euler was invited to serve as adjunct of physiology (!) and so he began to study this subject with the idea of applying mathematics to it!! However, he was soon allowed to work in mathematics, his favourite subject.

In St. Petersberg, Euler made brilliant discoveries in areas such as analysis, theory of numbers and mechanics. Euler became famous, and his former teacher Bernoulli referred to him as the "most famous and wisest mathematician".

Euler had a phenomenal memory, and at the age of seventy, he could not only recite completely Virgil's *Aeneid,* but even recall the first and the last line of every page of that book which he had studied as a student! He died in 1783.

mechanics came into the picture and had to be distinguished from what existed earlier.

As I said before, in mechanics one deals with the motion of particles. If you like pictures, then the motion of particles can be represented by trajectories. Of course, before drawing the trajectories, one must first *calculate* the trajectory. For this, one must use the appropriate *equations of motion*. What these equations do for us is essentially the following: Let us say we have a particle at a certain position at a certain time, and moving with a certain velocity. Knowing the velocity is the same thing as knowing the momentum, since momentum = mass × velocity. OK, so we have this particle whose position we know and whose momentum also we know, both corresponding to the same instant of time. Using this data as the input and the equation of motion, we can then calculate the position and the momentum the particle would have at the next moment. Using this data as a fresh input, we can then again use the equation of motion to calculate the position and the momentum of the particle at a still later instant. In this way, step by step, we can trace the motion of the particle and build up its trajectory. Actually, this is not quite the way one would figure out the trajectory if one were working with pencil and paper. One would try to *solve* the equations completely and then plot the solutions. But the *spirit* of the game is exactly as I said, and in fact if one were to *simulate* the trajectory using a computer, then one would (in certain situations at least), proceed like I have just described.

You might have noticed that this whole business of constructing the trajectory step by step depends crucially on our knowing the starting position and the starting momentum of the particle. I *must* know *both* these things; and if I don't know them, I assume that I can do an experiment and *simultaneously* measure with *infinite accuracy*, the position as well as the momentum. To put it in a nutshell, classical mechanics assumes that one can *simultaneously* measure with *infinite* accuracy (—a Pundit would say "specify with an arbitrarily high degree of accuracy"), both the position as well as the momentum of a particle.

It is this which enables one to think of precise trajectories for moving particles, bodies, etc.

5.2 Classical mechanics and heat

Physics is not all mechanics. I mean take a thing like heat; that also is a part of physics. Now you may ask: "Does mechanics have anything to do with heat?" Many people asked this question during the last century, and thanks to the work of Maxwell (see Box 5.5) and Boltzmann (see Box

Box 5.5 James Clerk Maxwell was born in Scotland in 1831. As a young boy, he took great delight in reflecting sunlight into various corners of his room, using a polished tin plate. Perhaps this was a foretaste of the great contribution he was to make later to the theory of light. Noticing his interest in science, his father began to take him to the meetings of the Royal Society of Edinburgh and at the age of fifteen, Maxwell communicated a paper to the Society on the construction of a perfect oval.

In 1850, Maxwell went to Cambridge for higher studies and stayed there till 1856. He then went back to Scotland to serve as a Professor there. In 1870, a special laboratory for experimental physics was set up in Cambridge University named Cavendish laboratory. Maxwell was appointed as the first Cavendish Professor.

Maxwell is of course famous for his *electromagnetic theory of light*. As Richard Feynman observes, ten thousand years from now, people would still remember Maxwell's theory as the greatest event of the nineteenth century, forgetting everything else. Maxwell's other famous contribution is the *kinetic theory of gases*.

But there are also many other contributions he made which are not so well known. For example, in 1860 he showed by experiments that any colour can be obtained by suitably mixing three *primary* colours. Also that colour blindness is caused when the eye becomes insensitive to one of these three primary colours. For this work, Maxwell received the Rumford Medal of the Royal Society of London. Maxwell also showed by careful experiments that Ohm's law is obeyed to one part in 200,000 over a wide range of the variables.

It is little known that Maxwell also helped experimental physicists in America. At that time, the only reputed journal for science in America was the *American Journal of Science*, and it mostly carried articles on botany, zoology, geology and the like. A young American scientist named Henry Rowland did some research in magnetism but his paper was rejected by the *American Journal of Science* as it was mathematical! Knowing the great reputation enjoyed by Maxwell, Rowland sent his paper to the former who got it published in the British journal *Philosophical Magazine*. Rowland's reputation immediately shot up and he was offered a professorship in Johns Hopkins University. Incidentally, Rowland has made important contributions to optics, besides which he also made Johns Hopkins a well-known centre for optical research.

Maxwell died in 1879, at a rather young age one must say.

5.6), we now have the answer. We know that heat is essentially the random motion of atoms in the system.

To get a feel for this statement, consider Fig. 5.1(a). This shows a rectangular box containing marbles or balls say. You might even visualise it as a billiards table if you wish. Let us say that when we start off, these particles are moving in the various directions shown by the arrows, with

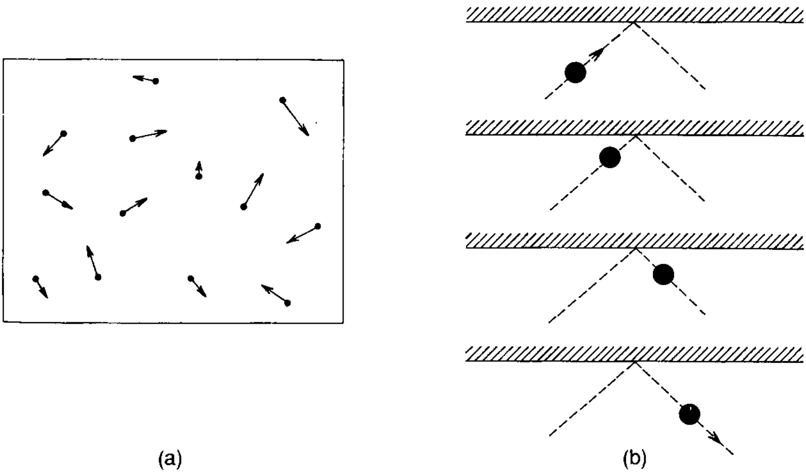
Box 5.6 Ludwig Boltzmann was born in Vienna in 1844. He did his doctorate in the University of Vienna under the guidance of Joseph Stefan, famous for *Stefan's law*. Boltzmann spent a lifetime trying to understand the Second Law of Thermodynamics in terms of the motion of atoms. In this of course, he was inspired by the earlier work of Maxwell. Maxwell's work on the kinetic theory of gases was generalised by Boltzmann.

Now heat is essentially disordered energy. Boltzmann showed how this disorder is related to a quantity called *entropy*. In fact, there is a famous formula for entropy namely,

$$S = k_B \log W$$

where S is the entropy and W a quantity related to the random motion of the atoms. k_B is a constant called the *Boltzmann constant*. This formula is engraved on the tombstone standing over Boltzmann's grave.

Interestingly, in the latter part of the last century, most physicists in Europe did not believe in the atom (!), in contrast to their counterparts in Britain. So Boltzmann's theories were attacked, which upset him very much. Finally, in 1906 he ended his life by jumping into the Adriatic sea. Not far from the place where he committed suicide, stands the International Centre for Theoretical Physics (ICTP), founded by Abdus Salam. It is ironical that just about the time Boltzmann was feeling frustrated, Einstein had published his famous paper on *Brownian motion*. Shortly thereafter, careful experiments on Brownian motion gave direct evidence for the existence of atoms.



(a) (b)

Fig. 5.1 Consider a box containing a number of particles which are moving about in different directions with all sorts of velocities. (a) shows the positions of these particles at some instant of time. The arrows point in the directions in which the particles are moving and the lengths of the arrows denote the speeds of the particles. (b) shows a particle bouncing off an edge.

different velocities. We also assume that there is no net energy loss during collisions; for example, if a particle hits the edge, it bounces back with the same speed as shown schematically in Fig. 5.1(b). Let us allow the particles to bounce back and forth and rattle around in the box for a long while. During this process, the particles will collide with each other many many times, and eventually their movements would become completely random. I can make this statement fancier, more precise, more mathematical and all that, but the simple fact is that what we usually call heat is nothing but the random motion of the atoms in the hot substance. The higher the temperature of the substance, the more rapidly do the particles move. I can even say that the motion is "more random" in a sense. I can make it more precise by introducing a concept called *entropy*, but I shall not go into all that here.

One can actually make a computer movie of these random motions; people have done so, and all they needed to use were Newton's equations and simple rules about the conservation of energy and of momentum—in short, classical mechanics. In turn, this means that particle trajectories can be precisely traced, if one has the patience!

5.3 Enter quantum mechanics

This is not the place to narrate the full story of the birth of quantum mechanics, much as I would like to. Maybe some other time; but I shall give a quick summary such as we need for the present. It all started with people trying to build models for the internal structure of atoms. Initially physicists did not bother about atoms. However, later they accepted that there were atoms but preferred to look upon them like billiard balls or hard spheres, so to speak. And then in 1896, J.J. Thomson discovered the electron. It now became clear that the atom *did* have something inside it and was not *indivisible* as people had thought earlier. It was also evident that since the atom as a whole is electrically neutral, there also had to be positive charge inside the atom. For many years, people were puzzled as to how the positive and the negative charges were distributed within the atom. Rutherford's brilliant experiments performed around 1910 showed that all the positive charge inside the atom was concentrated in a tiny region called the *nucleus*. The nucleus was really very small, its radius being about 10^{-13} cm. By contrast, the atom was much bigger, its radius being around 10^{-8} cm. Armed with these facts, people then tried to visualise models of the atom. An obvious thing to try was to suppose that the atom was like a mini-solar system i.e., imagine that the nucleus was sitting in the centre like the Sun, and that all the electrons were going round it. This picture is no doubt very attractive but straightaway there was a problem. It was noted that as the electrons went round and round the nucleus like planets do around the Sun, they would radiate energy since they would be experiencing acceleration. (According to Maxwell's electromagnetic theory, an accelerated charge must radiate.) If electrons radiate energy, then it means they would *lose* energy which in turn means that their orbits would start shrinking. This process would go on and on till all the electrons fell into the nucleus. The atom would then be completely wiped out and in fact this collapse would occur very rapidly, before you could even say Jack Robinson. In that case, there would be no atoms, you and I would not be there and so on! Obviously something is wrong with this picture somewhere, but at that time nobody knew what was wrong.

In 1913, Niels Bohr (see Box 5.7) solved the problem in a clever way. He said (in effect): "Listen you guys, I tell you the electron will not radiate and lose energy like you say it will, provided the electron sticks to some special or privileged orbits." You can't just say something like that and hope to get away with it. Nor did Bohr try any such stunt. He in

Box 5.7 Niels Bohr was born in Copenhagen, Denmark in 1885. His father was a Professor of Physiology and his brother a mathematician (besides being a very good football player). While still a student, Bohr won a prize in a science contest. The results were later published in the *Transactions of the Royal Society*.

In 1911, Bohr went to England. He first spent a year at Cambridge with J.J. Thomson and then went to Manchester to work with Rutherford. It was here that Bohr developed his famous atom model, for which he later won the Nobel Prize. In 1916, Bohr returned to Copenhagen to become a Professor. From 1920 till his death, Bohr was the head of the Institute for Theoretical Physics which was created by the University specially for him. The Institute became a famous place, with leading scientists from all over the world visiting and exchanging ideas.

In addition to the atom model, Bohr has done important work on the passage of radiation through matter and on nuclear fission. He also played a key role in the philosophical interpretation of quantum mechanics. Einstein who himself played a key role in the birth of quantum mechanics, would not accept Bohr's interpretation. For years Bohr tried to convince Einstein, but in vain.

During the War, Bohr escaped to Sweden from where he later went to America to work in the atom bomb project. His code name there was Nicholas Baker. Bohr visited India in the late fifties. Among other things, he lectured at the Tata Institute of Fundamental Research (TIFR), where he spoke on his debate with Einstein. At the end of the lecture, Bohr broke down and wept, still unhappy that his good friend and he could not come to an agreement. Bohr died in 1962.

fact gave a rule for identifying such privileged orbits, now called *Bohr orbits*. What Bohr said was that these are orbits wherein the angular momentum of the electron has some relationship to the Planck constant.

I now have to digress a bit and say a few words about the Planck constant. Actually, this is a story in itself, and I have narrated a part of it in the volume *Bose and His Statistics*. So over here I shall shorten the tale. In 1900 Planck made the monumental discovery that the energy of an oscillator is quantised. Prior to Planck, people believed that an oscillator could be thought of as capable of having any energy one pleased, but Planck said, "No Sir, an oscillator of frequency can have only the energies 0, $h\nu$, $2h\nu$, $3h\nu$, $4h\nu$,..." Here h is a constant which we now refer to as the *Planck constant*. Bohr recognised that in the microscopic world, not only is energy quantised, but so is *angular momentum*. What is more important, the same Planck constant governs the quantisation of the angular momentum as well, i.e., that the angular momentum can assume only the values $(h/2\pi)$, $(2h/2\pi)$, $(3h/2\pi)$... etc. Of course, Bohr did not *prove* that angular momentum is quantised; he merely postulated it. But once he did that, he was home. By this I mean that he could explain a lot of facts, including the beautiful order in the frequencies of the spectral lines of the hydrogen atom (see Box 5.8). Everybody was very happy and Bohr was awarded the Nobel Prize.

Box 5.8 As you know, when light emitted by a light source like a discharge lamp is examined in a spectroscope, one sees sharp lines. Some of these occur in the red region, others in the green region, blue region and so on. During the last century, many people observed the spectral lines emitted by hydrogen and measured their frequencies. Interesting patterns were then discovered. Balmer found that the frequency ν of lines in the visible region could be described by the formula

$$\nu = 3.29 \times 10^{15} \left(\frac{1}{2^2} - \frac{1}{b^2}\right) \quad b = 3, 4, \ldots$$

These spectral lines are referred to as the *Balmer series*. Lyman similarly found a series in the ultraviolet which could be described by the formula

$$\nu = 3.29 \times 10^{15} \left(\frac{1}{1} - \frac{1}{b^2}\right) \quad b = 2, 3, 4, \ldots$$

Similarly Paschen found a series in the infrared and so on. Why did the spectral lines of hydrogen obey these little formulae? It was Bohr who first gave the answer with his famous atom model. Using his model he showed that the frequency could in general be described by the formula

$$\mathrm{v} = Rc\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Here c is the velocity of light and R a quantity called the Rydberg constant. Bohr showed by calculation that $Rc = 3.29 \times 10^{15}$. Also n can take on integer values 1, 2, 3 ... For a given n, m takes on the values $(n + 1)$, $(n + 2)$, ... In short, Bohr's formula explains the occurrence of the Lyman, Balmer, Paschen and other series.

But soon there was trouble because the Bohr model of the atom could not explain many observed facts. By the early twenties it became clear that the mechanics of Newton, Hamilton, etc., does *NOT* work for electrons in atoms for that is what people were generally trying, along with Bohr's quantisation rule of course. The signals were already there when people tried to make a solar-system model of the atom. Bohr fixed the problem by sort of artificially bringing in the quantisation rule. This was clearly a step in the right direction, but was not enough. Suddenly between 1924 and 1928, all the pieces of the puzzle fell together in place as it were. It was realised that it was meaningless to think of electrons moving nicely in precise orbits like planets do, and that they obeyed Newton's equations of motion. The people who discovered this and restored order into the scene which was becoming more and more confusing were Heisenberg (see Box 5.9), Schroedinger (see Box 5.10), Bohr himself, Max Born (see Box 5.11) and Dirac (see Box 5.12). Actually many others also were involved but I have shortened my list to the leading figures.

Box 5.9 Werner Heisenberg was born in Germany in 1901. His father was a Professor of Greek in the University of Munich. After completing school, young Werner entered the Munich University to study physics under the great Arnold Sommerfeld. In 1923, he went to Gottingen to work under Max Born. Heisenberg then went to Copenhagen to spend some time in Niels Bohr's Institute. While there, he had an attack of hay fever, and went to a place called Helgoland to recuperate. It was there he wrote his first paper on quantum mechanics. He tackled the problem of calculating the energy levels of atomic oscillators. His method was daringly original and yielded good results but Heisenberg was not sure if it meant anything and wondered if he should publish his results or "throw it into the flames". Luckily, he showed his manuscript to Born who recognising it to be important, got it published in the German journal *Physikalische Zeitschrift*. Eight days later, Born noted that Heisenberg's so-called new rules corresponded to matrix algebra. Thus the mechanics introduced by Heisenberg was called *matrix mechanics*. Shortly thereafter, Heisenberg discovered the uncertainty principle. For all this, he received the Nobel Prize (in 1932). During the Second World War, Heisenberg worked on problems of nuclear energy in Germany and so he was wanted by the American army. When American troops entered Germany, Heisenberg was picked up and interned in England for about a year. When he was released, Heisenberg returned to Germany and helped to rebuild German physics again. Although Heisenberg is best remembered for his work on quantum mechanics, it must be mentioned that he is also the father of the quantum mechanical theory of magnetism.

Box 5.10 Erwin Schroedinger was the only child of his parents. His father owned an oil-cloth factory, but was more interested in chemistry, botany and painting. Erwin was born in Vienna in 1887. His early education was by private tuition at home but according to Schroedinger he learnt more from his father than from his tutor. Later he entered the Gymnasium (equivalent of the high school) where he learnt more of Latin and Greek than science. But Schroedinger discovered the pleasures of mathematics which of course influenced him very much later. After Gymnasium it was the University, and he received his doctorate in 1910. He served in the army as an officer during World War I but even so, found time to read Einstein's papers. After the war, he returned to the lab and did some experimental work. Soon after marriage in 1920, he moved to Zurich in Switzerland to become Professor there. This post was earlier held by Einstein. In Zurich, Schroedinger worked on problems connected with the statistical theory of heat. This is not surprising because Schroedinger was influenced by another great Austrian, Ludwig Boltzmann who had contributed much to the subject.

In 1924, de Broglie suggested that matter could behave both as waves as well as particles. This got Schroedinger thinking and led him to discover *wave mechanics*. Later Schroedinger proved that his wave mechanics and Heisenberg's matrix mechanics were really one and the same thing. Today, we refer to it all simply as quantum mechanics.

In 1927, Schroedinger went to Berlin to succeed Max Planck as the Professor in the University there. But life became difficult after Hitler came to power in the early thirties. So in 1933, Schroedinger left for England. It was in that year he was awarded the Nobel Prize, which he shared with Dirac. In 1939, he went to Ireland where he spent seventeen years. After the Second World War, Austria tried very hard to persuade Schroedinger to return to his native land but he refused. He finally returned only in 1956, after the Soviet troops left Austria. Schroedinger died in 1961.

5.4 The uncertainty principle

All these people whose names I have just rattled off created, so to speak, a new kind of mechanics called *quantum mechanics*. The rules of the game

Box 5.11 Max Born was born in 1882 in Germany. He studied in Gottingen under the great astronomer Karl Swarzchild and obtained a doctorate in applied mathematics (studying the stability of elastic wires and tapes). Born served with the military during World War 1 but he spent most of his time working on problems in crystal physics. It was during this period that Born, together with von Karman (who later became famous for his work on aerodynamics) did the well-known work on lattice dynamics.

Born was a good friend of Einstein, and wrote a book on relativity. Born's student Heisenberg played a key role in discovering quantum mechanics but it was Born who pointed out that Heisenberg's "strange rules" for calculating spectra were really rules of matrix algebra. Later when Schroedinger developed wave mechanics, it was Born who gave the (probabilistic) interpretation. For this Born received the Nobel Prize, but only thirty years later! He was very much upset by this delay.

In the early thirties, he left Germany along with many other German scientists since they all disliked Hitler. Born first went to Cambridge and then came to India to spend six months at the Indian Institute of Science (I.I.Sc.), Bangalore. Born came on Raman's invitation, and in fact Raman wanted to make Born a Professor in the Institute. But there was much opposition to Raman's idea and Born went back to England. Here in India, Raman was forced to resign as the Director of I.I.Sc., because, among other things, he had tried to make Max Born a Professor. Later Born and Raman had a long quarrel about theories of lattice dynamics.

When Born grew old, he decided to go back to Germany. By now the War was over and Hitler was forgotten. So Born felt it was OK to go back. But Einstein was very upset, and he stopped writing to Born! Born died in 1970.

Box 5.12 Paul Adrian Maurice Dirac was born in England in 1902, his father being Swiss and his mother English. Dirac studied in England, and after matriculation qualified as an electrical engineer. But he became interested in mathematics and went to Cambridge to do a doctorate, which he got in 1926. During this period, Heisenberg visited Cambridge to lecture on the work he was doing. After returning to Germany, he sent to Fowler (you will run into Fowler again in the next chapter) a copy of his manuscript (not yet in the final form). Fowler showed it to Dirac who then developed the ideas further, giving Heisenberg's discovery a firm mathematical basis. Dirac then burst forth, producing many papers of great importance, one of which was on what has now become famous as the *Dirac equation*.

Dirac received many honours, including of course the Nobel Prize. Many Universities were keen to award him an honorary doctorate but Dirac refused them all! Dirac was a very quiet and soft spoken person but *very clear* in his speech and in his writings. No wonder he was unhappy with Bohr who always seemed so confusing! The story is told that after a lecture by Dirac, someone in the audience made a very long remark. No one quite knew what this person was trying to say, and everyone was rather annoyed. Dirac gave a patient hearing and at the end simply asked: "Was that a question or a comment?" The audience roared with laughter.

In the fifties, Dirac visited India and lectured at the TIFR. The notes of those lectures are still available at the Institute; I have a copy which I treasure. While on a visit to Moscow in 1955, Dirac gave a lecture at the university there. During the lecture he wrote on the board:

PHYSICAL LAWS SHOULD HAVE MATHEMATICAL BEAUTY

I believe those words are still preserved! Dirac passed away in 1984.

were completely new and quite bewildering at first. By now of course we have got used to the whole "funny business". It was found that in quantum mechanics one could not talk of things one took for granted in the old mechanics of Newton, Hamilton and others. Incidentally, it is at this point that people began to refer to the old mechanics as classical mechanics.

The *uncertainty principle* is one of the surprises provided by quantum mechanics. This principle was enunciated by Heisenberg when he was just twenty-five. What the principle says is roughly the following: If a measurement of the position of a particle is made with an accuracy Δx and if a measurement of the momentum is *simultaneously* made with an accuracy Δp, then the product of the two accuracies *can never be made smaller than h*. In other words,

$$\Delta p \quad \Delta x \gtrsim h \tag{5.1}$$

Let us try to understand what the above relation really means. Suppose we have an electron. The uncertainty principle says that no experiment can measure both these quantities (i.e., the position coordinates and the momentum) simultaneously with accuracies greater than that implied by (5.1). The crucial word here is *simultaneously*—remember that. If we try to be very very careful and measure the position very accurately, we would end up getting a very bad answer for the momentum. It has nothing to do with our measuring skill! Conversely, if we do a good job of measuring the momentum, we would get a lousy answer for the position; no way to beat (5.1). Incidentally, the question whether the electron "really" has both an exact position and momentum is irrelevant and indeed meaningless. What is important is how we can observe and describe them. At least, this was the view introduced by Heisenberg, and we continue to hold that view even today.

People often think that the development of quantum mechanics started with the discovery of the uncertainty principle. Not true! What happened was that Heisenberg found a new rule which fixed some of the problems which were plaguing physicists at that time. This was a very strange and weird rule he had stumbled upon. Heisenberg then thought deeply about it and concluded that it all happened because we could not ever measure simultaneously with infinite precision, both the position and the momentum of the particle. As Feynman says, "The Uncertainty Principle 'protects' quantum mechanics!"

5.5 Fuzzy trajectories

The uncertainty principle straightaway knocks out the idea of precise trajectories. Remember that to draw precise trajectories, we have to know both the position as well as the momentum of the particle at start. That means we should be able to measure both the position and the momentum simultaneously and with infinite accuracy. But Mr. Heisenberg will not allow us to do that. And so out goes the idea of precise trajectories.

This is so important an idea that maybe I should say a few more words by way of further explanation. Consider Fig. 5.2(a) where we have two particles colliding with each other. In classical mechanics one can precisely say which particle is going where. This is because we can (at least in principle) draw precise trajectories. In quantum mechanics, thanks to Heisenberg, we don't quite know the positions and the momenta of the two particles at start. The result is that the particles seem quite fuzzy. And if they are fuzzy, we cannot say which particle went where during a collision. There are in fact two options as shown in Fig. 5.2(c). Quantum mechanics says that both options are possible and if we want to predict what happens in the collision, we must in fact *allow for both the possibilities, each with its own probability amplitude.*

5.6 Quantum statistical mechanics

Remember what I said earlier about heat being random motion of particles? You might wonder whatever happens to all that if the particle trajectories are fuzzy all over. Well, even if you have not wondered, others have and the results are as follows.

The first thing that people decided was not to worry about trajectories—they are fuzzy anyway. Let the collisions take place anyway they like—

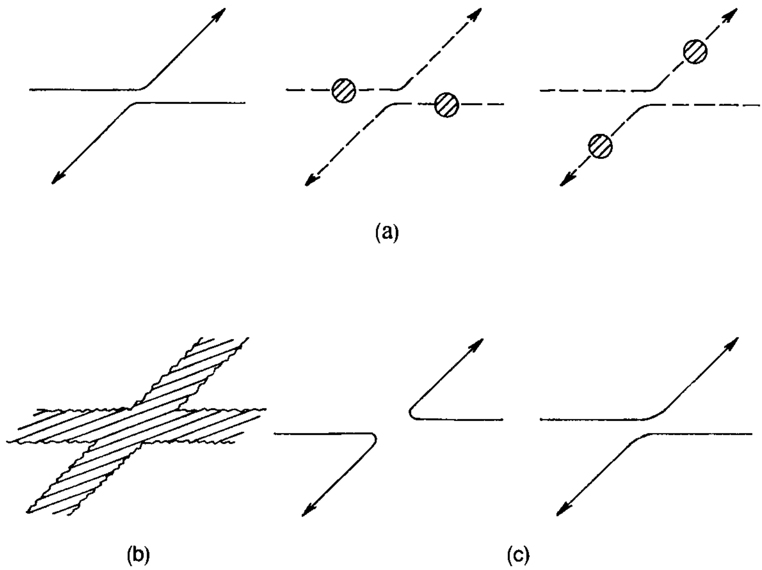


Fig.5.2 (a) shows the classical trajectories of two colliding balls, along with some "snapshots" of the collision taken at different instants. Thanks to the uncertainty principle, the trajectories are fuzzy in quantum mechanics as shown in (b). The net result is that the two balls could have gone in the two different ways shown in (c). We can't say which one actually happened. In fact, quantum mechanics teaches us not to ask certain questions! Instead, we should accept that both events are possible, each with its own well-defined and calculable probability amplitude. When we perform calculations about collisions, we must allow for these various possibilities.

fuzzily or otherwise. What we have to worry about is how the energy is distributed amongst the particles. This is something we must always keep track of, quantum mechanics or no quantum mechanics.

Now it is a good rule of the thumb that if there are N particles in a gas at a temperature T, then the total energy of the gas is approximately of the order Nk_BT. Here k_B is a constant called the *Boltzmann constant*. I have explained it elsewhere. Here let me just say that when the temperature T is multiplied by k_B, we get the temperature in energy units. Now the fact that the total energy is Nk_BT does not of course mean that all particles have the same amount of energy, i.e., k_BT. In fact, some would have more and some would have less; only the *average* would be k_BT. Indeed, we would like to know the *distribution* of energies.

This energy distribution business can be made clearer with a few pictures. Consider Fig. 5.3. Let us say that the energy levels available to a particle in a gas are as shown in Fig. 5.3(a). Now let us suppose that all

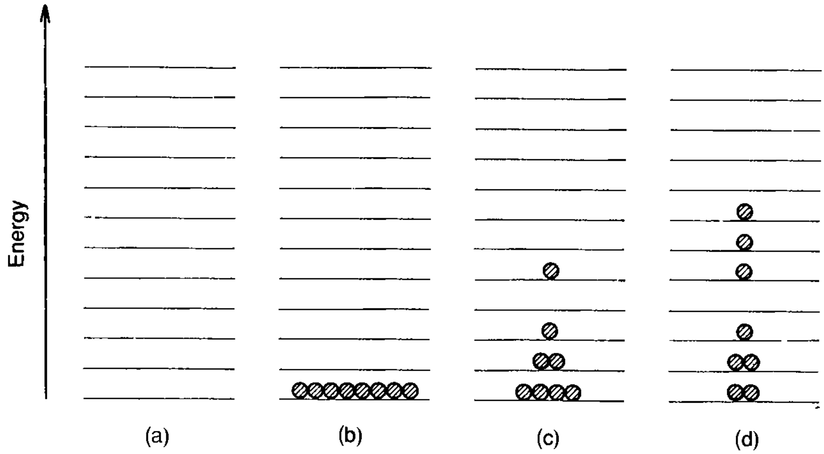


Fig. 5.3 In statistical mechanics, we have the problem of distributing particles amongst the various energy levels available. Say the levels available are as in (a), and say we have eight particles. These could be distributed in various ways, some of which are shown in (b), (c) and (d). In short, at a given temperature, there are various ways of distributing say, N particles. Statistical mechanics is concerned with the *average* number of particles occupying energy level i (say), at temperature T. This quantity denoted $\langle n_i(T)\rangle$ is different for Bosons, Fermions and Boltzmann particles.

the particles in the gas have the same set of levels available to them. The problem of energy distribution now becomes a bit clearer, but as yet we do not have full information to figure out the distribution we want. We still need a rule which would tell us how many particles can occupy a given energy level of Fig. 5.3. That rule is given to us by *quantum statistical mechanics.* Once we have such a rule, we can figure out how the particles are distributed amongst the various energy levels, and many other things can be calculated.

What is this statistical mechanics I am talking about? Remember my remark that heat is nothing but random motion of molecules? Well, how does one analyse these random motions and figure out what the temperature of a collection of particles moving about chaotically, as in Fig. 5.1, is? Statistical mechanics is the subject which provides the answer. In ordinary mechanics, we usually deal with collisions between two or three particles. In statistical mechanics, we deal with billions and billions of collisions. Obviously it is a hopeless task to analyse each and every one of these collisions. Nobody ever does, and we do not *need* to. Thanks to Boltzmann, we look for averages and things like that, i.e., *statistical answers*—and these are the ones directly related to the physical properties of the system which we can probe experimentally.

The statistical mechanics that existed before quantum statistical mechanics was developed is called *classical statistical mechanics.* It was developed largely by Boltzmann. His idea was that even though in a gas all the atoms look alike, we can, if we have the patience, keep track of which atom is doing what by following their trajectories—and remember, classical mechanics allows us to do this. In short, it is as good as giving names to the atoms, so to speak. One could say that Boltzmann regarded the particles of his gas to be *distinguishable*; the rest of classical statistical mechanics follows from that.

5.7 Bosons and Fermions

Quantum mechanics created problems concerning distinguishability. Suppose we have two identical particles like two electrons or two neutrons colliding. As we saw earlier, thanks to the uncertainty principle, we can no longer regard these particles as distinguishable in the sense Boltzmann would have regarded them. So what does one do now? That became clear from the work of Satyen Bosé and Einstein on the one hand, and Fermi and Dirac on the other. In case you are interested in some of the details, I suggest you look up the companion volume in this series entitled *Bose and His Statistics.* In a nutshell, people realised that *all* particles in Nature belong to one of two types: *Bosons or Fermions.* Bosons obey one kind of occupancy rule while Fermions obey another kind. The statistical distributions that result are called the *Bose–Einstein* (BE) and the *Fermi–Dirac* (FD) distributions respectively. A gas made up of Bose particles is often referred to as a Bose gas; likewise, a gas of Fermions is sometimes referred to as a Fermi gas. Bose gases obey BE statistics and Fermi gases obey FD statistics.

The rule for Fermion occupancy is quite easy to understand, for it says that in each quantum level there can at the most be only one Fermion—like in a cinema theatre, only one person can sit in each seat. Bosons are not shy, and they can crowd together so that in principle, one quantum level can hold all of them! Of course, this does not happen in practice except in a special situation, but that is a different matter. The rule that only one Fermion can occupy a given quantum level was first stated by Wolfgang Pauli, and is called the *Pauli exclusion principle.* It is called the exclusion principle because the rule excludes other Fermions, once one of them has got into a given quantum level. Soon after Pauli made this discovery, Fermi in Italy and Dirac in England figured out how the rule affects the statistical mechanics of Fermions. This is how the Fermi–Dirac statistics was discovered.

I must now say a word about what happens to all the results painstakingly calculated by Boltzmann during the last century. It turns out that if the temperature of a gas is high, then irrespective of whether the particles of the gas are Fermions or Bosons, they behave like Boltzmann said the particles of a gas would. To use slightly more formal language, both the Fermi gas and the Bose gas behave like a Boltzmann gas at sufficiently high temperatures. Or in other words, both Bose–Einstein and the Fermi–Dirac distributions tend towards the Boltzmann distribution at high temperatures. So Boltzmann's work does have a place. I have explained all this in more detail in the companion volume *Bose and His Statistics*.

Figure 5.4 gives a schematic picture of the energy distribution business under various conditions. Electrons are Fermions, and our concern is with a Fermi gas for that is what the core of a white dwarf is. Let us therefore study this case a bit more carefully. To start with, we see from

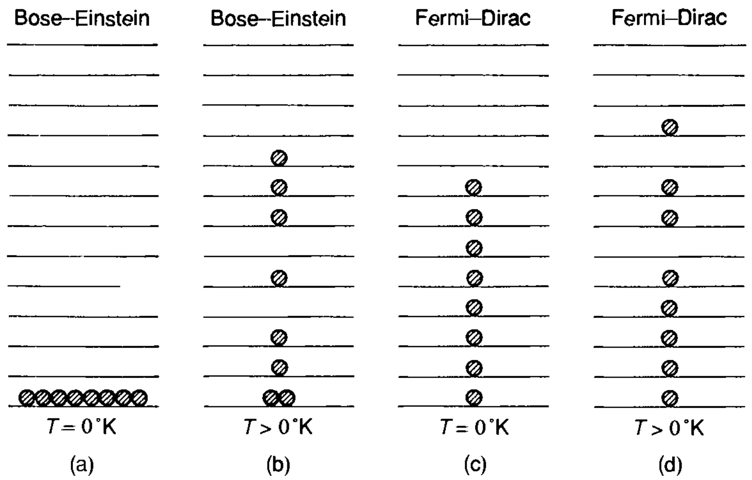


Fig. 5.4 This figure brings out the important difference between Bosons and Fermions. Compare (a) and (c). These show the occupation of the energy levels at the absolute zero of temperature. The basic rule of the game is that at $T = 0$ °K, the total energy of the system must be a rock-bottom minimum. Since Bosons don't mind crowding, they achieve this minimum by packing into the lowest level (see (a)). Fermions too try to keep the energy to a minimum, but they are not willing to sacrifice their comfort and insist on single occupancy. This gives rise to (c). When the temperature is greater than zero, the Bosons spread themselves and explore higher energy states. So do the Fermions, maintaining, however, the rule of one Fermion per quantum state.

Fig. 5.5(a) how $\langle n_i(T)\rangle$, the average number of Fermions occupying level i (with energy E_i), at a given temperature varies with energy. At zero temperature, there is occupancy upto the *Fermi energy* E_F and another above—see also Fig. 5.4(c). Next let us compare the Fermi gas with the Boltzmann gas by considering momentum distribution rather than energy distribution. In Fig. 5.5(b) we have what is usually called the Maxwell–Boltzmann distribution. You might be familiar with it for it is usually taught as a part of the kinetic theory of gases. Figure 5.5(c) shows the corresponding result for Fermions.

Many things become evident from these graphs. For example,

1. In a Boltzmann gas, all the particles have zero momentum at the absolute zero of temperature. By contrast, in a Fermi gas, the particles can have non-zero momenta even at absolute zero temperature, upto a maximum p_F (called the *Fermi momentum*).

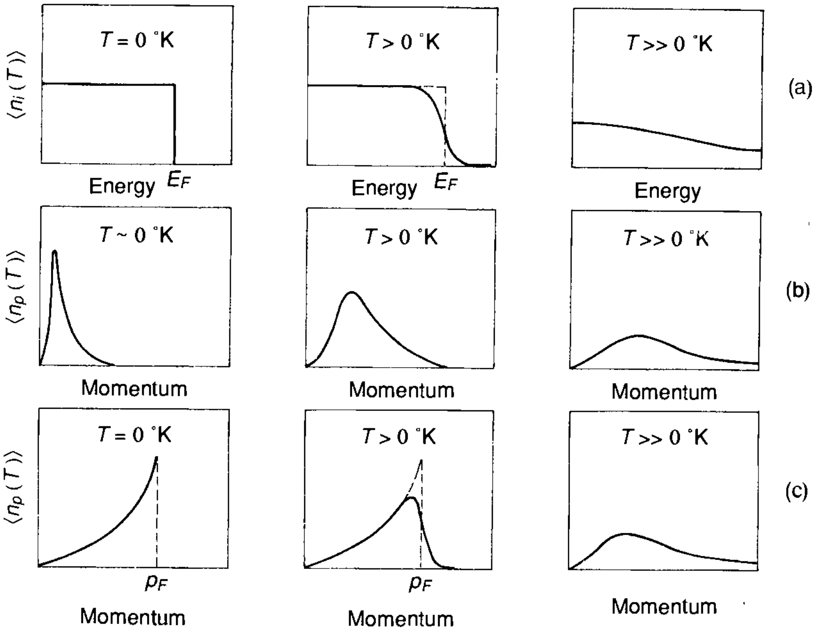


Fig. 5.5 Earlier we considered $\langle n_i(T)\rangle$, the average number of particles in quantum energy level i at temperature T. Graphs of this are shown in (a) for Fermions. We now consider the *momentum distribution* $\langle n_p(T)\rangle$ which describes the average number of particles having a momentum of magnitude p. (b) shows the results for Boltzmann particles, while (c) does the same for Fermions. Notice that even at $T = 0$ °K, there are Fermions with varying momenta, upto a maximum value of p_F. Chandra noted that in white dwarfs, p_F is so large that relativistic effects become important.

2. At high temperatures, the momentum distribution for a Fermi gas tends towards that for a Boltzmann gas.

5.8 About degeneracy

Let us now consider a Fermi gas at the absolute zero of temperature. The energy of the gas would be the lowest possible, and this happens because only the lowest energy levels are occupied (recall Fig. 5.4(c)). Such a gas is said to be a *degenerate Fermi gas.* The word degeneracy is used in more than one sense in physics but here we use it in the same sense as Chandra who says that a "completely degenerate electron gas is one in which all the lowest quantum states are occupied."

We are focussing on degeneracy because that is the name of the game in a white dwarf. We already know that a white dwarf could have a layered structure—recall Fig. 2.11. For the moment let us suppose that our white dwarf contains only hydrogen. However, this hydrogen will not be in the form we are normally used to. Here on earth, we usually deal with hydrogen gas which is made up of hydrogen molecules. In the white dwarf, not only is the molecule ripped apart, but even the atom is. In other words, what one has are protons and electrons—two seas of them, mixed up together. Technically, such a system is called an *electron gas.* The electron gas inside the white dwarf is very very dense. The matter is in a degenerate state, and what one is looking for is an equation of state for such degenerate matter. The story does not change if the core of the white dwarf contains other elements. Their atoms are also ripped apart, and one has essentially two "fluids" one made up of the electrons and the other made up of all the nuclei. The focus again is on the electrons, and they are in a highly degenerate state.

The study of degenerate matter started almost immediately after the discovery of the Fermi–Dirac statistics. Arnold Sommerfeld and his students in Germany applied the idea to electrons in metals and obtained many new and interesting results. But the electron gas in a metal is not at the high pressure nor at the high temperature one has in a white dwarf. So one cannot apply the results for metals directly to a white dwarf; true one must proceed along similar lines, but one must carefully keep track of the differences. And so what are the different cases one can have? Chandra identifies four.

1. Non-degenerate, non-relativistic
2. Non-degenerate, relativistic
3. Degenerate, non-relativistic
4. Degenerate, relativistic

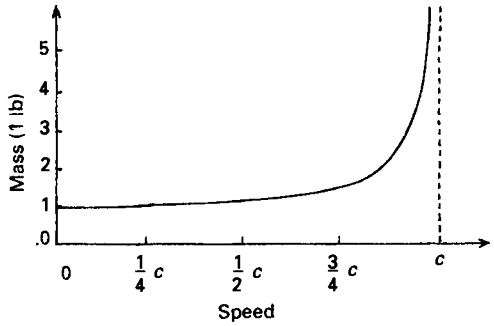
For a primer on relativity, see Box 5.13; for more details, see the companion volume *At the Speed of Light.*

Box 5.13 The theory of relativity is one of the finest achievements of the human mind. It was unfolded in two stages—first came the Special Theory, followed then by the General Theory. Both are due to the great Albert Einstein. Let me now give you a capsule summary of the Special Theory.

You must have learnt that the mass of an object is a constant and that only its weight varies as the object is taken to the Moon, Mars, etc. Relativity produced a surprise for it said that the *mass of an object varies with velocity*, the variation being

$$m = m_0 / \sqrt{(1 - v^2/c^2)};$$

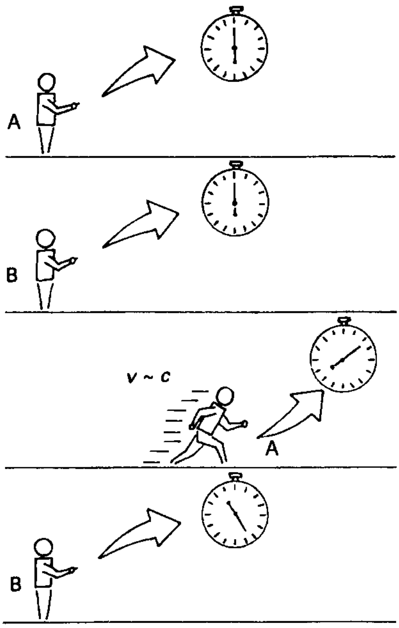
see also figure (a). We notice that the mass variation is really not serious until the velocity v approaches that of light. Incidentally, v cannot be greater than c for then the mass would become imaginary, which is not allowed. So the velocity of light is a ceiling—nothing can travel at a speed exceeding that. The quantity m_0 is called the *rest mass*, and is the mass the object has at zero velocity.



(a)

The second important effect predicted by the Special Theory is the *slowing down of clocks*. Suppose you had a stop watch and ran with it at nearly the speed of light. Your watch would then slow down. What it means is that when your watch ticks off one second on the dial, a man who is standing still and is watching you run will find that a stop watch he is carrying would have ticked off many more seconds—see figure (b).

Finally, there is the famous mass–energy relation $E = mc^2$. There was once a cartoon showing Einstein scribbling formulae on the black board. He first writes $E = ma^2$ and then cancels it. Next he tries $E = mb^2$ and scores that out too. He then comes up with $E = mc^2$ and shouts Eureka! What this famous



(b)

formula says is that mass (even rest mass) can be converted into energy and that energy can be converted back into mass. Without this formula, there wouldn't have been any atomic bomb! The most common example of the reverse process namely, energy becoming mass is the so-called *pair production*. This refers to the disappearance of a photon (light quantum) and the appearance of an electron–positron pair. For this to happen, the photon must have an energy of at least 1 MeV which is the energy equivalent of the rest mass of an electron and a positron.

Of the above four categories, the first two are not of interest to us now. Fowler was the first one to regard matter in white dwarfs from a quantum mechanical point of view, and he considered case three. I shall have more to say about this in the next chapter. Chandra noted that electrons in a white dwarf would be moving with very high velocities, and so suggested that case four is more relevant to white dwarfs than case three considered earlier by Fowler. By the way, remember that in a Fermi gas electrons can have momentum even at zero temperatures. What Chandra argued was that in white dwarfs the Fermi momentum p_F is sufficiently high and that relativistic considerations *must* be taken into account. He did this *very* carefully, and obtained sensational results, so sensational that many people, including some very important ones, would not believe them at first. That story comes in the next chapter. Here let me just say that the starting point of Chandra's work on white dwarfs was the equation for the pressure of a degenerate, relativistic Fermi gas.

May be I have gone a bit fast, and perhaps I should explain again what quantum mechanics has got to do with white dwarfs. If you recall, in Chapter 2 I said that when stars "die", gravitational collapse sets in. So the burnt-out star starts shrinking in size. People knew that this shrinking did not go on for ever but stopped at some stage, but what was the source of outward pressure which prevented the collapse to a point? Pressure from burning was ruled out since burning had stopped. Fowler solved the mystery by pointing out that there was a new form of pressure namely degeneracy pressure. This pressure is due to the Pauli principle, and in fact there is a simple way to understand its existence. Suppose you take a piece of iron and try to compress it. Obviously we cannot do it with our hands. So may be we should put it into a mechanical press and apply pressure. Even then, it is very difficult to squeeze; certainly, iron is not soft like sponge. Have you wondered why? Due to the Pauli principle!

Matter inside the core of a white dwarf can be thought of as an electron gas. Since electrons are Fermions, an electron gas must be described by statistical mechanics due to Fermi and Dirac. The pressure in such a gas is of quantum mechanical origin, and in fact due to the Pauli principle. In short, the degeneracy pressure that saves a white dwarf from collapsing into a point is due to the Pauli principle, and to describe this pressure in full and obtain the equation of state, one needs quantum statistical mechanics as applicable to Fermions. Obviously, a very high degeneracy pressure is needed to hold a white dwarf from total collapse. Such high pressures occur when matter is dense, which is the case with white dwarfs.

To sum up:

- Boltzmann described gases by assuming that the atoms moved on precise trajectories governed by the equations of classical mechanics. Today, we would refer to such an approach as classical statistical mechanics.
- The Bohr model of atom, while able to explain the spectrum of hydrogen, ran into difficulties in more complex cases. It became clear from this that electrons in atoms could not be described by the equations of classical mechanics.
- With the birth of quantum mechanics, one had to give up concepts like precise trajectories, distinguishability, etc.
- Particles were now classified as either Bosons or Fermions, and new rules were developed to describe their statistical mechanics.

- While a Bose gas is described by BE statistics, a Fermi gas is described by FD statistics.
- Fowler regarded the core of a white dwarf as a Fermi gas. The pressure operating in such a gas is called degeneracy pressure, and has its origins in the Pauli principle.
- According to Fowler, it was degeneracy pressure which arrested the total collapse of a white dwarf, i.e., the dwarf shrinking to a point.
- While applying FD statistics to the white dwarf, Fowler assumed that the electrons were non-relativistic.

On now to the next chapter, where I describe how Chandrasekhar questioned Fowler's work and got into trouble!

6 *All About White Dwarfs*

This is the most important chapter in this book, for I am going to describe the work that led to the famous *Chandrasekhar limit*. I have already introduced you to the white dwarf earlier. Chandra describes his theory of the white dwarf in Chapter 11 of his book, the chapter being entitled *Degenerate Stellar Matter and the Theory of the White Dwarf.* The present chapter is a sort of summary of the above-mentioned chapter, and besides including some background material I shall also tell you the exciting story of the discovery itself.

6.1 The first crack at the problem

We already know that white dwarfs are quite different from all the usual stars. As Chandra himself puts it, it is "plausible the white dwarfs differ from other stars in some fundamental way". What it all boils down to is that one can't play around with an ordinary polytropic gas as before, and pull out reliable results. The first person to draw attention to the fact that the white dwarfs are quite different was R.H. Fowler. He said that the electron gas in the star is sufficiently dense for quantum mechanics to apply. In other words, Fowler recognised that the electron gas in the white dwarf must be highly degenerate, at least in the interior.

Fowler's work was the starting point of a series of investigations which culminated in the great discovery made by Chandrasekhar. It is interesting that Fowler applied quantum mechanical ideas to stars, almost immediately after physicists had discovered the basic principles of quantum mechanics. Normally, ideas take some time to diffuse or percolate from one field (in this case, physics) to another (here, astrophysics). Fowler was able to borrow ideas easily because he was in an ideal place namely, Cambridge. Remember, we are talking about the late twenties and early thirties. Lord Rutherford was the presiding deity for physics in Cambridge, and all sorts of far-reaching discoveries were being made in his laboratory. In turn this led to a steady stream of visitors from elsewhere, who brought with them the latest news about developments and discoveries in other research centres. And let us not forget that Dirac was around in Cambridge at that time. What I am saying is that though ideas normally take time to penetrate into other fields, even if they are neighbouring ones, the climate in Cambridge was very different; no wonder Fowler was quick to grasp an interesting possibility.

Chandra was attracted by Fowler's work, and being already familiar with the theory of polytropes, he promptly applied that theory to the case considered by Fowler. Chandra did this work around 1930–1931, but he wasn't quite satisfied. Why? Because at the densities prevailing in a white dwarf, the Fermi momentum of the electrons would be so high (i.e., the velocity would be so high) that relativistic effects must be taken into account. Neither Fowler nor Chandra himself had so far done so. Clearly, more work was needed. This was the starting point of the trouble for Chandra! You would have to wait a bit more for that story.

6.2 Enter relativity

Chandra worked for several years, studying what happens if one considers relativistic degenerate matter instead of non-relativistic degenerate matter (considered earlier by Fowler). Let us start at the beginning. Remember that equilibrium basically means that pressure must balance gravity. If you would prefer an equation, then that equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho \tag{6.1}$$

already introduced to you in Box 3.3. It does not matter if you don't quite follow this equation; it simply makes more precise the idea that pressure and gravity must balance each other. We know how to describe pressure if we have a classical, perfect gas. The question now is how do we describe the pressure in the present case—remember "present case" means a relativistic, Fermi gas. Chandra analyses the problem and, based on quantum statistical mechanics, writes the pressure and density of a relativistic degenerate gas compactly as

$$P = A \cdot f(x), \qquad \rho = B \cdot x^3 \tag{6.2}$$

Here A and B are suitable constants, while x is the Fermi momentum measured in units of $m_e c$, i.e.,

$$x = p_F / m_e c \tag{6.3}$$

where p_F is the Fermi momentum introduced in the last chapter, and m_e is the rest mass of the electron. The function $f(x)$ is a bit complicated, but if you are curious, here it is:

$$f(x) = x(2x^2 - 3) \cdot (x^2 + 1)^{1/2} - 3 \sinh^{-1} x \tag{6.4}$$

When the expressions for P etc., are plugged into (6.1) and one grinds through the algebra, one obtains

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\phi}{d\eta}\right) = -\left(\phi^2 - \frac{1}{y_0^2}\right)^{3/2} \tag{6.5}$$

All sorts of new quantities like η, ϕ and y_0 have now appeared, but we need not bother about all these details. As far as we are concerned, broadly speaking, the new equation looks somewhat like the one we had before. I know we are not going to make detailed use of equation (6.5) but nevertheless I thought you should at least have a look at it for this is the handle which has to be cranked if one is interested in results for stars with a relativistic, degenerate, electron-gas core. If you like, (6.5) is supposed to deliver for white dwarfs, what the Lane–Emden equation (see Box 3.5) did for stars on the main sequence—and Chandra milked this equation dry!

6.3 A check and a limit

One thing that one usually does in physics when one replaces an old model by a new model is to check if the results of the old model can be recovered in a suitable limit, i.e., when the new model can be regarded as an extension or a generalisation of the old. In the present case, what it means is that from (6.5) which holds for degenerate and relativistic matter, one should be able to deduce the results earlier obtained (by Chandra himself) for the non-relativistic case via a polytropic analysis. All one has to do is to say that the velocity is small, and take due advantage of that. Chandra did precisely that, and got back all his old results. More specifically, Chandra found that in the small mass limit $P \propto \rho^{5/3}$. If you compare this with equation (3.9), it suggests that the core behaves like a polytropic gas of index $n = 3/2$. Please note that I am *NOT* saying that the earlier classical polytropic analysis is correct. All I am saying is that there is nothing *mathematically* wrong with that analysis, and one would like to make sure that the new, improved equation delivers the familiar old results when all the suitable approximations are made. That is a normally done thing, and it was done here also.

While getting back the old result in a suitable limit was expected, there also was a surprise. This came when Chandra began to examine the *high-density* limit instead of the low-density limit. Of course, it is only in the high-density limit that one must consider the relativistic effects, but what Chandra was now trying to do was to find out what happens when the density becomes infinite. Mathematically, this means going to the limit $y_0 \to \infty$ in equation (6.5). You might argue that this is crazy; how could there ever be infinite density in a star? Hold on! For a minute forget stars and concentrate on the maths. As far as mathematics is concerned, there is nothing to prevent one from considering such a limit. So why not find out what happens?

Chandra did precisely that and came up with a strange result. He found that for such a system $P \propto \rho^{4/3}$, meaning that the core has now decided to act as if it were a gas of index $n = 3$. In other words, if we continue to think in the polytrope language, there is a *crossover* in behaviour from $n = 3/2$ for small masses to $n = 3$ for large masses. More about this crossover shortly.

Chandra also found that such a system (i.e., one for which $y_0 \to \infty$) would have a finite mass which he denoted by the symbol M_3. Let us digest the meaning of this result. First of all, let us recall that density means mass per unit volume. Next let us remember from elementary maths that if a finite number is divided by zero, the answer is infinity. Putting these two things together, if the density of a star is infinite and at the same time its mass is finite, it could only mean that the star has *zero radius*! This is absolutely amazing, I mean who ever heard of a star with zero radius? And remember, the star, i.e., the white dwarf, had to have a mass of only M_3 to achieve zero radius. In case you are wondering how much M_3 really amounts to, an estimate has been made, and it is ~ 1.4 $M_\odot$. In other words, if a star slightly more massive than our Sun were to run out of fuel and start collapsing (into a white dwarf that is), it would go on collapsing till its radius shrinks to zero!

This is an incredible result, and no wonder many people (including some very famous ones as I shall shortly describe) could not believe this. Chandra himself was quite sure though, because his mathematics was impeccable. Nevertheless, he still had to convince the skeptics, and this is where there was trouble. That story comes a little later.

6.4 On to an exact theory

In the previous section we have discussed the results given by equation (6.5) in two limiting cases. One of these was the high-density limit which led to the result that a white dwarf of mass M_3 would have zero radius. Many people did not believe in this result, and so between 1931 and 1935, Chandra was busy carrying out an *exact* analysis of equation (6.5), for *all* values of the mass. This is just the sort of thing in which Chandra excels. From the basic equation, he pulled out all kinds of complicated looking expressions, but to compare with observational data, one needs numbers rather than mere formulae, and numbers come from doing arithmetic based on the theoretical formulae. Today, such arithmetic is easily done using computers, but in those days there were no computers, nor even pocket calculators. Chandra did all the calculations patiently by hand and his final result is shown in Fig. 6.1(a) in its original form. I did not feel like scribbling notes on it, and so I have sketched the curve again in Fig. 6.1(b) so that I could add some comments.

Let us take the various curves in Fig. 6.1(b) one by one. The dashed line is the result of the simple Fowler–Chandrasekhar polytrope analysis. This is obtained by assuming that white dwarf matter is degenerate but non-relativistic. The dotted curve is the result of the limiting analysis that

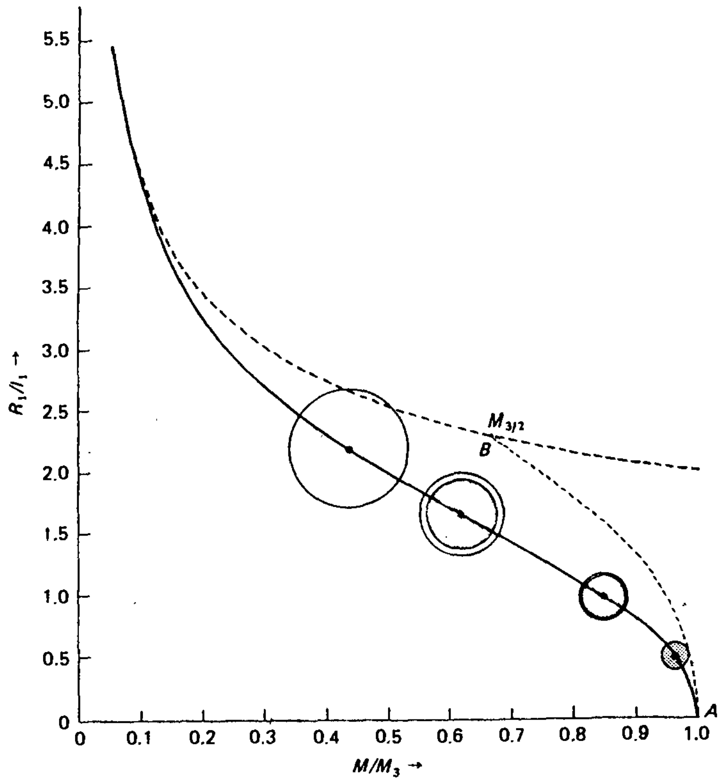


Fig. 6.1(a) This is the famous figure published by Chandrasekhar. The radius is measured in terms of a certain unit $l_1 \doteq 7.72 \times \mu^{-1} \times 10^8$ cm, while the mass is measured in terms of the limiting mass M_3. The dotted curve is based on the Lane–Emden polytrope of index 3/2. In this analysis, the momentum p_0 of the electron equals $m_e c$ at the point B. This may therefore be regarded as the crossover point. Along the exact curve, circles are shown. Wherever there are two circles, the inner circle shows the region where the electrons are relativistic.

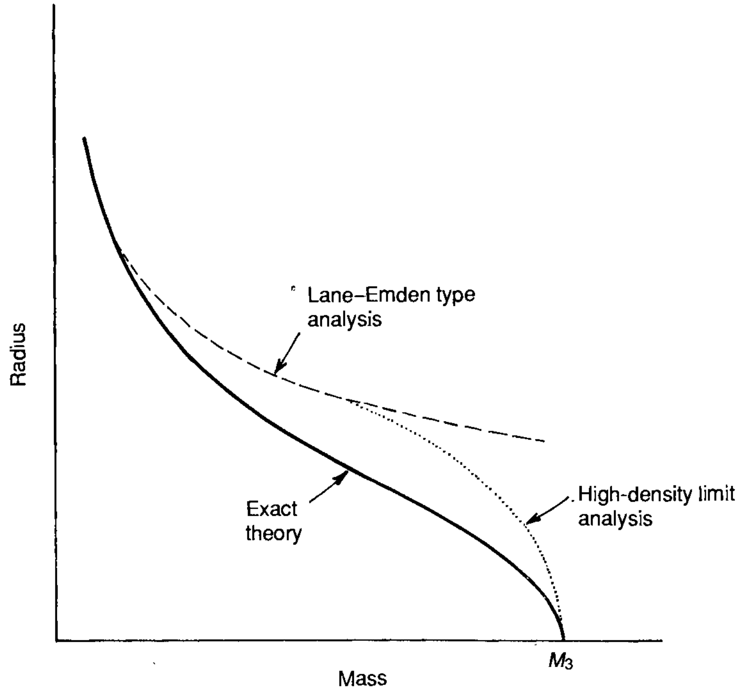


Fig. 6.1(b) Offers a commentary on the results obtained by Chandrasekhar. The explanations are in the text.

first revealed the existence of the limit M_3. Remember that in this latter analysis, the gas is completely degenerate *and* relativistic. The full line is the result of the exact analysis—no *limit* business here, but the entire works.

The full curve puts everything into place. For small masses we expect the gravitational crush to be small and so high densities cannot be expected, which is another way of saying that the relativistic effects would be small. Understandably, in this limit the exact theory should predict what the approximate Fowler–Chandrasekhar theory did earlier. Turning to the other extreme, as $M \to M_3$, the exact theory agrees (as it should) with the limiting analysis performed earlier by Chandrasekhar and which first revealed the existence of a limiting mass. In short, the exact theory provided a nice bridge between the various special cases considered earlier in section 6.2. One could no longer take exception to the existence of the limiting mass M_3. This limit is now called the *Chandrasekhar limit*, and quite appropriately so.

Let us pause for a minute and absorb what Chandra did. If you recall, the astrophysical problem that people were initially concerned about was to understand the occurrence of the main sequence. This could be done with a polytropic analysis of a classical gas (i.e., no quantum mechanics and no relativity). However, such an analysis could not explain the occurrence of white dwarfs. It was known that white dwarfs are "dead" stars, i.e., stars in which thermonuclear burn had ceased. Gravity dominates in such a star, and, according to classical physics, there is nothing to oppose gravity which should therefore crush the star, literally to a point. One knew, however, that such a thing did not happen but one did not know what the mysterious pressure was which arrested gravitational collapse to a point. In 1927, Fowler pointed out (thanks to the Pauli exclusion principle) that there is a repulsive force which effectively acts like an outward pressure. This is the pressure that was earlier referred to as degeneracy pressure, and it comes into play when the white dwarf becomes very dense. It is this pressure which arrests the total collapse of the star. Fowler applied standard non-relativistic quantum statistical mechanics (FD statistics) to white dwarf matter to derive the equation of state etc. Chandrasekhar questioned this since, in his view, the density is so high as to make relativistic effects important. And when he studied the equation of state for relativistic, degenerate matter (consisting of Fermions), he found that in a certain limiting case ($M = M_3$), *even degeneracy pressure cannot arrest collapse to a point.*

You might wonder if everything would change if radiation pressure is taken into account. Being a careful person, Chandra considered that case also and the outcome is shown in Fig. 6.2, also taken from his book. Basically, the R–M curve has the same shape; more important, there is a limiting mass in this case also. Only, the value of the limiting mass is slightly different. Anyway, radiation pressure is not important for many stars.

6.5 A feel for the result

One would like to get a feel for why the exact result is the way it is. Chandrasekhar discussed this during a lecture he delivered in Ahmedabad in 1982, and I shall summarise that here. Basically, it all depends on how the pressure varies with density. For a classical perfect gas, i.e., one which obeys the law $PV = RT$, it is easily seen that

$$P \propto \rho T \tag{6.6}$$

that is, if the temperature is kept constant, pressure is proportional to the density ρ. (The symbol $\propto$ means *is proportional to.*) Figure 6.3(a)

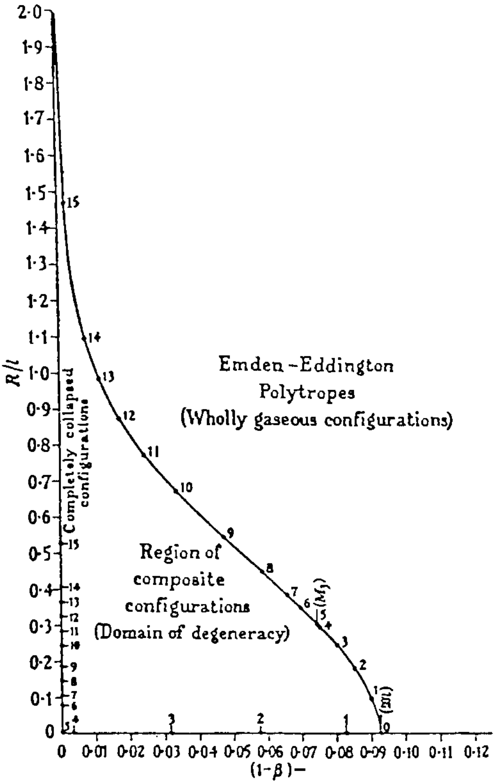


Fig. 6.2 Mass–radius relation with the effects of radiation pressure taken into account. Observe that once again there is a limiting mass.

shows some examples of the pressure–density curves of this nature. Notice that as the temperature increases, the lines become steeper. Of course, real gases consist of atoms with electrons etc., and for such systems, the pressure versus density curve is nonlinear. As Chandra himself puts it, "if one takes the gas at a certain temperature and increases the density, the pressure increases linearly for a while; but then this increase becomes more rapid at a certain point and, eventually, the relation between the pressure and the density becomes independent of the temperature". Such a nonlinear curve is shown in Fig. 6.3(b). Two nonlinear curves of importance are those for which $P \propto \rho^{4/3}$ and $P \propto \rho^{5/3}$ shown schematically in Fig. 6.3(c). The 5/3 behaviour is appropriate for a degenerate but non-relativistic gas, while the 4/3 behaviour is for a degenerate and relativistic gas. So the *crossover* I talked about earlier with reference to the R versus M curve in Fig. 6.2, arises because pressure varies with density according to the 5/3 law for the low-mass

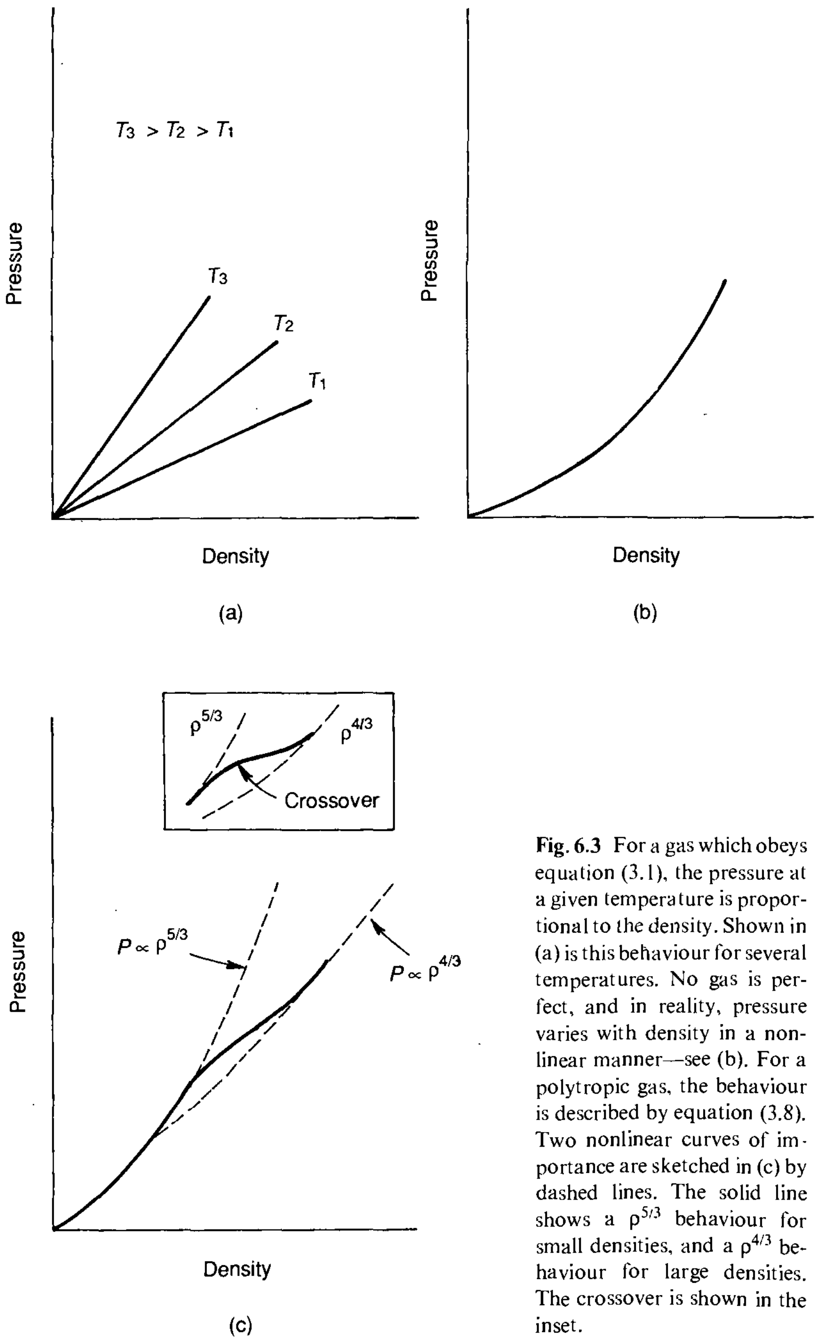


Fig. 6.3 For a gas which obeys equation (3.1), the pressure at a given temperature is proportional to the density. Shown in (a) is this behaviour for several temperatures. No gas is perfect, and in reality, pressure varies with density in a nonlinear manner—see (b). For a polytropic gas, the behaviour is described by equation (3.8). Two nonlinear curves of importance are sketched in (c) by dashed lines. The solid line shows a $\rho^{5/3}$ behaviour for small densities, and a $\rho^{4/3}$ behaviour for large densities. The crossover is shown in the inset.

systems and according to the 4/3 law for the high-mass systems. It is this crossover in the pressure–density behaviour that causes the $R-M$ curve of Chandrasekhar to swing away from the Fowler result and head towards the limiting mass M_3.

6.6 How true are the results?

All these results are fine; but how realistic are they? Do they actually describe the situation in the white dwarfs one sees out there in the sky? Today of course there is ample evidence about the correctness of Chandra's theory. But, at the time he first produced the results, Chandra himself checked them out.

Not much experimental information was available at that time, and Chandra had to make do with whatever little was known. What was striking about the white dwarfs was that in them "we encounter densities of the order of 10^6 and even 10^8 gm cm^{-3}. It is this characteristic which is generally emphasised...". So Chandra began to see if he could account for such densities.

The test was made by him on a white dwarf discovered by Kuiper and called *Kuiper's white dwarf*. For this star, Kuiper had given the experimental values,

$$\operatorname{Log} L = -1.76, \ \operatorname{Log} R = -2.38 \tag{6.7}$$

L and R being expressed in solar units. From this one can derive

$$\bar{\rho} = 19{,}600{,}000\,(M/M_\odot)\,\mathrm{gm.cm^{-3}} \tag{6.8}$$

All this is from experimental evidence. The *actual* density of the white dwarf would of course depend on what its mass M is, and this is where the R–M relation comes into play. Using his exact theory and a reasonable guess for μ, Chandra estimated the mass of Kuiper's white dwarf and obtained an average density roughly equal to that obtained via observation. On the other hand, using the same experimental data and the results of non-relativistic analysis, the density of the star comes out to be over twenty times higher. The estimate could be even higher, depending on what value one uses for the mean molecular weight of the stellar gas. The point simply is that the density comes out to be *unacceptably high*. We can easily understand this by referring to Fig. 6.4. For the same R, the approximate theory predicts a very high mass (especially when R is very small) and a very high mass automatically means a correspondingly very high density. Chandra therefore rules out the validity of the earlier, simplified polytropic analysis, which in other words means that relativistic effects *are* important, in addition to those of degeneracy.

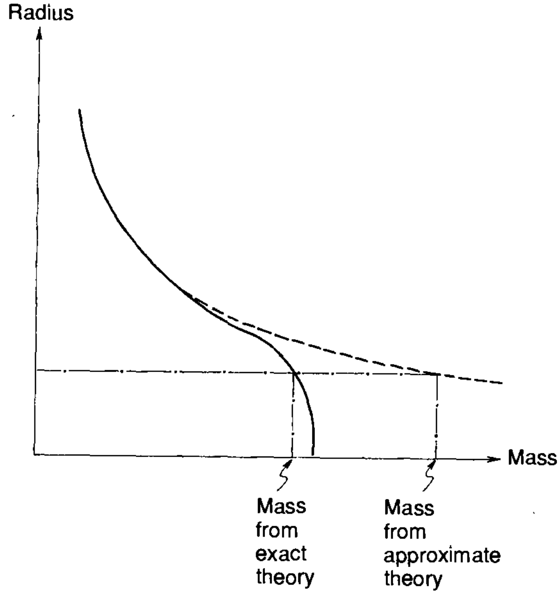


Fig. 6.4 Schematic figure to illustrate how the density of a white dwarf is estimated knowing its radius. To obtain the density one must, besides the volume, also know the mass. As can be seen from the above, Chandrasekhar's exact theory leads to a lower mass than Fowler's approximate theory and therefore also to a lower density. Such an estimate made by him gave an answer for Kuiper's star in agreement with experiment.

One might say that one example does not suffice. But Chandra is not bothered! He merely observes:

> Since the theory is such a straightforward consequence of the quantum mechanics and, further, uses Dirac's theory of the electron only in that phase of its application which has been confirmed by laboratory experiments (Klein–Nishina formula, production of cosmic ray showers, etc.), there can be little doubt that it is essentially correct.

You might think that Chandra is being a bit over confident, but the fact is that subsequent observations, much more detailed in fact, have proved him 100 per cent correct. Incidentally, in the above analysis we have assumed that the white dwarf could be regarded as a sphere of degenerate matter of radius R. In actual fact, only the core of the star might be degenerate, and the outer layer might be non-degenerate. Chandrasekhar carefully analyses this possibility also, and concludes that as long as the mass of the star is high, i.e., close to M_3, there is no serious error.

It was soon realised that Chandra was not merely explaining white dwarfs but had in fact derived the equation of state for matter under extreme conditions of temperature and pressure, a new phase of matter as it were. I suggest you look up the companion volume *The Many Phases of Matter* if you wish to know about the different phases matter can exist in, but I could, in a couple of sentences, highlight what was so special about Chandra's discovery.

Normally we talk of gas, liquid and the solid as being the different phases of matter. These are the phases we are used to, can produce in the laboratory, and perform experiments on. But this does not mean that these are the only *phases* matter is capable of. We can simply ask: What happens if the temperature is raised to a few million degrees, and the pressure also raised to a few million atmospheres? It doesn't matter if we cannot produce such conditions in a sustained manner in the laboratory; that does not prevent us from asking the question. Using our knowledge of physics, we can say that at these temperatures and pressures, matter will not contain atoms as we know them. The atoms would all have been crushed, and there would be nothing but electrons and nuclei. There would be a very dense cloud of this very "unusual" material. One can now ask: What is the equation of state of matter in this peculiar state? And that precisely is the question Chandra answered. True, he did not pose the question this way but came to it via stars; however, from a physicist's point of view, this is what Chandra's work really means. Thanks to Chandra's work, we can now say that if we have a "drop" of this matter with a certain radius, it would have a mass of so much or rather a density of so much. Notice that the density changes with the size of the "drop"! This is because gravity plays a role in determining the equation of state, and the gravitational force is determined, among other things, by the mass of the sphere. Of course, such "drops" cannot be formed in the laboratory, but out there in the vast sky, Nature has kindly produced for our benefit and curiosity, many such "drops".

OK, the long and short of it all is that Chandra added an important chapter to the physics of matter under extreme conditions, and for this he was rewarded with the Nobel Prize in 1982, some fifty and odd years after he made the discovery. The announcement was made on October 19, Chandrasekhar's birthday. He therefore called it a nice birthday gift! I doubt if the Nobel Prize has ever been awarded after such a great delay. The fault is entirely that of those who give the awards. The world at large did not of course wait for the Nobel Committee to make up its mind. Once it became known that Chandra was on firm ground, recognition came to him from all directions, and he was showered with innumerable awards, degrees, medals and what have you. As someone said, by giving him the Nobel Prize, the stature of the Prize was enhanced, not that of Chandrasekhar! More important, Chandra's discovery led to other very important discoveries, but that I shall touch upon in the next chapter.

6.7 The Eddington–Chandrasekhar clash

I now come to the most dramatic part of the whole story, the human interest part you might say. The story starts around the late twenties, when Chandra, then a teenager, was a student in the Presidency College, Madras. He was getting keenly interested in mathematical physics, and was already reading much more than his textbooks. He also regularly scanned scientific journals to keep in touch with research going on elsewhere. He had received Eddington's well-known book as a prize, and knew all about the attempts then in progress to understand the structure of stars. In addition, thanks to his mathematical curiosity, he was also well up on all the nineteenth century stuff on polytropes.

In late 1928, the well-known German physicist Arnold Sommerfeld visited India (see Box 1.7), and one of his stops was Madras where he delivered a lecture in the Presidency College. The quantum revolution had just then broken out, and Sommerfeld was full of the latest scientific news. One does not know whether Sommerfeld's audience in Madras understood what was being said by the German master, but Chandra certainly did. Already Chandra was doing some serious thinking about the problem of stars, and in fact during the summer he had done some research on the subject while he was visiting the Indian Association for the Cultivation of Science at Calcutta (—recall Box 1.6), where Raman had, earlier in the year, discovered the Raman effect. Sommerfeld exposed Chandra to the new statistics of Fermi and Dirac, and also perhaps told him about Fowler's work.

Fowler had no doubt done some thinking about the newly-discovered white dwarfs, but in his view, the star was not like an ionized gas but rather like a giant molecule in its lowest quantum state. Fowler called this object a *black dwarf* because it had no energy to give off radiation. Chandra began to think intensely on the subject. It was clear that degeneracy effects must be considered as Fowler himself had suggested, but the black dwarf idea seemed to be wanting. After all, white dwarfs could be seen, which meant that they did radiate energy. So something better was called for than had been reported by Fowler. The obvious thing to try was a polytrope kind of analysis, with, however, degeneracy effects thrown in.

The year was 1930, and Chandra having obtained his master's degree from the Madras University, now sailed for England for pursuing further studies in Cambridge. In those days, there were no jet planes and things like that, and one had to travel by ship. The journey to England took about two weeks, and on board the ship, Chandra kept himself busy applying the theory of polytropes to a star with degenerate (but non-relativistic) matter. By the way, Chandra's uncle Raman was also a great one for doing research while travelling on a ship, and he even wrote papers while on board!

In Cambridge, Chandra at last had people with whom he could discuss his work. This, by the way, is very important for all those doing research. I have already described the work Chandra did during the period 1930 to 1935. It was then that Chandra began to extend his earlier polytropic analysis of a degenerate gas by including relativistic effects. Eddington was also in Cambridge at that time, and he was very famous. So Chandra had a chance to discuss his work with Eddington too, and this is where all the trouble started! You remember I said that Chandra first discovered the existence of the limiting mass M_3 before he started on the exact analysis. Well, soon after he isolated this limit, he showed his result to Eddington but the latter thought it was all wrong. One of Eddington's problems was: Whatever happened to stars with mass larger than 1.4 M ? Chandra was very sure of his results, and tried hard to convince Eddington, but the big man simply refused to accept Chandra's findings. "Oh yes," he said, "your mathematics may be all right but I don't think your physics is correct."

In January 1935, there was a meeting of the Royal Astronomical Society in London. The Society held meetings on the second Friday of every month during which time members reported their discoveries. Chandrasekhar was a member, having been introduced to the Society by Fowler in 1930. Members had to submit the papers they wished to pre- sent one week in advance; the Society then drew up a programme which it circulated to its members. Chandra had submitted a paper on his exact results for presentation at the January meeting. He recalls:

> On Thursday evening I got the programme and found that immediately after my paper Eddington was giving a paper on "Relativistic Degeneracy." I was really very annoyed because, here Eddington was coming to see me every day, and he never told me he was giving a paper.
>
> Then I went to dine in College and Eddington was there. Somehow I thought Eddington would come to talk with me, but I did not go and talk with him. After dinner I was standing by myself in the Combination Room where we used to have coffee, and Eddington came up to me and asked me, "I suppose you are going to London tomorrow?" I said, "Yes." He said, "You know your paper is very long. So I have asked Smart [the Secretary of the Society] to give a half hour for your presentation instead of the customary 15 minutes." I said, "That is very nice of you." And he still did not tell me. So I was a little nervous as to what the story was.

The day was January 11. Young Chandra arrived in London with great expectations and dreams. He was hoping that everyone would be impressed with his discovery and express appreciation. Let us pick up the story again from Chandra.

> There was tea before the meeting, and McCrea, who is a relativist, and I were standing together and Eddington came by. McCrea asked Eddington, "Well, Professor Eddington, what are we to understand by Relativistic Degeneracy?" Eddington turned to me and said, "That's a surprise for you," and walked away.

Chandra spoke as scheduled, and neatly explained his findings. He knew people would wonder about stars with masses greater than M_3, and so he added the remark, "A star of large mass cannot pass into the white dwarf stage, and one is left speculating on other possibilities." It was now Eddington's turn, and he said:

> I do not know whether I shall escape from this meeting alive, but the point of my paper is that there is no such thing as relativistic degeneracy (!)... Dr. Chandrasekhar had got this result before, but he has erased it in his last paper; and, when discussing it with him, I felt driven to the conclusion that this was almost a *reductio ad absurdum* of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way.

And so on he went, literally tearing young Chandra to pieces and pouring ridicule over him. Eddington cracked many jokes, at Chandra's expense of course, and the audience roared with laughter. Nobody cared much for the young Indian. After the meeting, Chandra had dinner in London with Plaskett, an astronomer from Oxford. Plaskett was silent through most of the dinner and Chandra wondered if Plaskett agreed with Eddington. After the dinner, Chandra went to the Paddington station to see Plaskett off to Oxford. At the station, he ran into Milne, a famous astronomer. Milne was very happy that Eddington had demolished Chandra's theory. Chandra recalls, "I was really very angry at that." He continues:

> I had gone to the meeting thinking that I would be proclaimed as having found something very important. Instead, Eddington made a fool of me. I was distraught. I didn't know whether to continue my career.
>
> I returned to Cambridge late that night, probably around one o'clock. I remember going into the common room. There was still a fire burning, and I remember standing in front of it and repeating to myself, "This is how the world ends, not with a bang, but with a whimper."

Next morning Chandra met Fowler and told him about the London meeting. Fowler offered sympathy; so did a few others, though privately.

Chandra did not give up. Eddington might be a big shot, but history is full of examples of big shots making mistakes. Chandra was sure of his maths as well as his physics. After all, there were many in Cambridge who were experts in various branches of physics. They all assured him that the physics behind his analysis was sound. Chandra now began corresponding with leading physicists outside England to find out if he was wrong in combining the exclusion principle of Pauli with relativity. He wrote to Rosenfeld, a close collaborator of Niels Bohr:

> Now that my work is completed, Eddington has started this "howler" and of course Milne is happy. My work has shown that Milne's ideas in many places are wrong... there is going to be a long period of stress and confusion and if somebody like Bohr can authoritatively make a pronouncement in the matter it will be of the greatest value for further progress..

Rosenfeld replied:

> Your letter was some surprise for me, for nobody had ever dreamt of questioning the equations, and Eddington's remark as reported in your letter is utterly obscure. So I think you had better cheer up and not be scared by the high priests...

In the next letter, Rosenfeld wrote:

> Bohr and I are absolutely unable to see any meaning in Eddington's statements ... If "Eddington's principle" had any sense at all, it would be different from Pauli's. Could you perhaps induce Eddington to state his views in terms more intelligible to humble mortals?

To put it in slightly plainer language, according to "Eddington's principle", several "high speed" electrons could be in one quantum level, in contrast to what the Pauli principle demands. In more technical terms, this is what McCrea says of this business:

> Basically, in order to obtain the relativistic form of any property, we require the fundamental equations of the subject to be *Lorentz invariant*, i.e., the same for all inertial frames [for more about Lorentz invariance and all that, see the companion volume *At the Speed of Light*] ... Eddington said that there can be no requirement of Lorentz-invariance in this case.

People would laugh if anybody says something like this today, but in those days relativity was new and quantum mechanics even newer; and so people did not laugh. In fact, McCrea, himself a relativist, once heard a lecture by Eddington on this business, and about that he says:

> When I listened to Eddington... I could not immediately weigh up all the implications of what he said, but my instinct seemed to tell me that he might be right...

He adds:

> I am ashamed not having got to the bottom of the sort of argument Eddington produced. Had anyone other than Eddington produced such arguments, I suppose I should have done so. But they were superficially satisfying to me, and since they satisfied Eddington, I confess that I was content to let go at that.

Eddington had prepared a manuscript describing his "principle", and Chandra managed to get hold of a copy and sent it to Rosenfeld for him and Bohr to see. In the covering letter, Chandra wrote:

> I have managed to get hold of Eddington's manuscript. He gave it to me and I am forwarding it to you for *you and Bohr alone* to read. I should be awfully glad if Bohr could be persuaded to interest himself in the matter...

Rosenfeld replied that Bohr was a bit tired and was proceeding on leave but was sending the Eddington manuscript to Pauli for comments. As far as Rosenfeld himself was concerned, he says:

> After having courageously read Eddington's paper twice, I have nothing to change in my previous statements; it is the wildest nonsense.

Pauli too dismissed Eddington's paper but would not come out in the open and say so. His point was that Eddington used all kinds of astrophysical arguments to refute the Pauli principle, and he [Pauli] was not interested in astrophysics. By now it was quite clear to Chandra that not only was he right (which of course he knew from the beginning), but also that all physicists agreed with him. And so he appealed to Milne, another leading astronomer but he too was hostile. Replying to Chandra, Milne said:

> Your marshalling of authorities such as Bohr, Pauli, Fowler, Wilson etc., very impressive as it is, leaves me cold. If the consequences of quantum mechanics contradict the obviously much more immediate considerations, then something must be very wrong with the principles underlying the equation of state derivation.

Amazing! Milne also thought that quantum mechanics was shady! The year was 1935, and by now not only physicists but even chemists had accepted quantum mechanics in toto. Somehow the old astronomers could not accept it because it went against their ideas of how stars would behave. And because they were all eminent men, they probably scared away many a young astrophysicist from trying out new and bold ideas.

Chandra might have received a hammering, but he stuck to his guns. Meanwhile, Eddington carried the campaign against Chandra to America. Speaking at the famous Harvard University in 1936 he said:

> I suppose that upto 1924 no one had given serious thought to abnormally dense matter; but just when it cropped up in astronomy, it cropped up in physics as well. Fowler showed that the newly discovered Fermi–Dirac statistics saved the star from the unfortunate fate which I had feared.
>
> Not content with letting that alone, physicists began to improve Fowler's formula. They pointed out that in white dwarf conditions the electrons would have speeds approaching the velocity of light, and there would be certain relativity effects which Fowler had neglected. Consequently, Fowler's formula, called the ordinary degeneracy formula, came to be superseded by a newer formula, called the relativistic degeneracy formula. All seemed well until certain researches by Chandrasekhar brought out the fact that the relativistic formula put the stars back in precisely the same difficulty from which Fowler had rescued them. The small stars could cool down all right, and end their days as dark stars in a reasonable way. But above a certain critical mass (two or three times that of the sun) the star could never cool down but must go on radiating and contracting until heaven knows what becomes of it. That did not worry Chandrasekhar; he seemed to like stars to behave that way, and believes that that is what really happens.

Eddington described the scene painted by Chandra as "stellar buffoonery."

By this time Chandra had obtained his doctorate degree from Cambridge, and he wanted to continue in England, doing research somewhere, while working as a lecturer perhaps. But his argument with Eddington ruined any chance he might have had of getting a job in that country. And so he left for America to join the University of Chicago, where he has been ever since! Soon after he arrived in America, Chandra decided to ignore the whole controversy. As he recalls:

> I had to make a decision. Am I going to continue the rest of my life fighting... or change to other areas of interest. I said, well, I will write a book and then change my interest. So I did.

This book was published in 1937 by the University of Chicago. It is that book which I am trying to describe in this volume. Soon this book became famous, and so did Chandra himself. People did not believe in Eddington's objection any more; it looked too silly. Quantum mechanics was right, and so was Chandra's application of it to the study of white dwarfs. Today, hundreds of white dwarfs are known, and they neatly fall on the curve calculated by Chandrasekhar—not a *single* exception. So Chandra's work is the triumph of pure thought!

Supposing Eddington had not opposed Chandra. Supposing he had praised Chandra in that fateful meeting in London. What would have been the outcome? Chandra himself has speculated on this, and he says:

> My position in science would have been radically altered as of that moment. Eddington's praise could make one very famous in astronomy.

But I really do not know how I would have reacted to the temptation, to the glamour. How many young men after being successful and famous have survived for long periods of time? Not many.

6.8 Eddington's problem

Why was Eddington so hostile to Chandrasekhar? You might think he was a horrible villain of some kind but that is not true. Eddington might have behaved arrogantly, but there *was* a *scientific* reason for Eddington to doubt Chandra's result.

It all goes back to the period before quantum mechanics. Now there is a theorem in classical mechanics called the *virial theorem* (see Box 6.1), and Eddington and Poincare (a famous French mathematical physicist) had adapted this theorem to a system of particles held together by gravitational force. One result which came out of this is the following: Consider a gas sphere (with certain properties which do not concern us here). Let us say the sphere contracts due to gravity and that its radius decreases by an amount dR. The gravitational potential energy of the sphere would come down on account of this contraction; say it comes down by the amount $d\Omega$. So the potential energy has decreased by a

Box 6.1 The *virial theorem* is a well- known result in mechanics. To understand it, we shall consider a collection of particles which, for simplicity move in one dimension. The position of the ith particle is x_i and the force acting on it is F_i. Define now

$$C = \langle \sum_i F_i \, x_i \rangle$$

where the symbol $\langle \ \rangle$ stands for the average value, as explained below. To obtain C, we multiply each x_i by the corresponding F_i and add them. Now x_i and F_i do not remain constant in time; they would vary. To allow for these, one now observes the system over a sufficiently long period of time, computes the sum described above for various instants during this period, and finds the average. The angular brackets $\langle \ \rangle$ denote such a *time-average*. Compute now $C/2$. This quantity is called the *virial of Clausius*.

What do we do with this virial? This is where the virial theorem comes in. It says that the average kinetic energy is equal to minus one-half the virial, i.e.,

$$\langle K \rangle = -C/2$$

For a gas in gravitational equilibrium, the forces F_i are due to gravitational attraction. So $\langle K \rangle$ can be related to the gravitational forces. This is a very useful result.

certain amount. But it can't simply disappear, and we must be able to account for it. Let us say a certain amount is radiated away by the sphere into outer space. This much of energy is lost totally. But all the energy donated by the shrinkage process is not lost in this manner. The remainder has to go somewhere. It has only one place to go, and that is to increase the store of the *internal* energy of the gas. If you want it in plain language, the energy of the molecules in the gas is increased. In essence, the gas would become hot.

All this is perfectly clear. But this is where the trouble starts! Let us say we have a great big star, and let us suppose that (thermonuclear) energy production has ceased. Then, according to what we know, the gas cloud must start collapsing. We can now start applying the above line of reasoning. The cloud shrinks, radiates away some of the energy but can't get rid of all the energy donated by the potential energy in this manner. Some of it is internally absorbed, and the gas becomes hotter. It also becomes denser. This thing keeps on happening, and there seems to be nothing to stop the star from continually shrinking and eventually collapsing to a point. Of course, the star could cool off. But it can't do that by radiation for if that were possible, it would have done it in the first place. The only other way to cool is by *expanding*. We know expansion can produce cooling; but in this case there is a big problem. If the gas has to expand, it must *do work* against gravity. To do work requires energy, and the question is: Where is this energy going to come from? You might say: Why not borrow it from the molecules? After all they have a storehouse called the internal energy, and haven't we been depositing energy into that account? True, but the amount of energy deposited into that kitty was only a *part of* what was shed as excess potential energy after contraction. In short, *there is not enough energy available for the star to expand back and cool*. As Eddington asked, "What on earth was the star to do if it ... had not enough energy to get cold?!" One knew that this sort of thing did not quite happen out there, but one did not know how the star was saved from endless collapsing and getting hotter and hotter all the time. This is where Fowler's important contribution comes in. Having learnt about Fermi–Dirac statistics, he immediately saw that when matter is heavily compressed the atoms would all be torn apart and that it would essentially be in a degenerate state. And in this state, Pauli pressure or degeneracy pressure would put an end to the collapse of the gaseous cloud. So, Fowler had as it were, rescued the star.

What was Chandra trying to do? He said that Fowler was not completely correct, and that relativistic effects *also* must be taken into account. And what happens when one does so? Well, stars of small mass escape crushing just like Fowler said, but a star of mass 1.4 $M_{\odot}$ will collapse totally! And this is what Eddington could not accept. I mean, here was this problem of collapse which had bugged people for several years, and thanks to Fowler it had been neatly solved. But along comes this young unknown Indian who kept on swearing by relativistic degeneracy, and all he was trying to do was to upset the apple cart, so to speak! (Recall Eddington's Harvard lecture.)

6.9 About Eddington

You must be wondering: "Who is this Eddington?" I guess I should now say a few words to satisfy your curiosity.

Arthur Stanley Eddington was born in England in 1882. His father was a schoolmaster, and he passed away when Arthur was only two years old. Even as a little boy, Eddington was attracted to big numbers; for example, he learnt the 24 × 24 multiplication table! And, on one occasion, he started counting all the words in the Bible!! This fascination for large numbers never left him.

Eddington studied first in Manchester, and later went to Cambridge. One of his teachers was Horace Lamb, famous for his treatise on hydrodynamics. Eddington liked Lamb very much, and later in life when he became a famous astronomer, he once said, "While I know what it is to be treated something like a lion, I would rather like to become something of a Lamb!"

In 1907, Eddington joined the famous Greenwich Observatory as the Chief Assistant. He served in this position for five years, and in 1912, went to Cambridge as the Plumian Professor. Cambridge became his home for the next thirty years. During this period, Eddington made many important contributions to astronomy and astrophysics. Perhaps he will be best remembered for the part he played in establishing that gravity does deflect light, exactly as Einstein had said it would in his General Theory of Relativity. I should mention here that in those days, not many people understood relativity but Eddington certainly did. The story is told that a press reporter once went to Eddington and asked him: "Sir, it is said that apart from Einstein, only two other persons understand the theory of relativity. Is this true?" Eddington looked at the reporter and said, "I wonder who the other person is." More seriously, Eddington wrote a book on relativity himself, called the *Mathematical Theory of Relativity*. As late as 1982, Chandrasekhar said that he still used this book as it was very good.

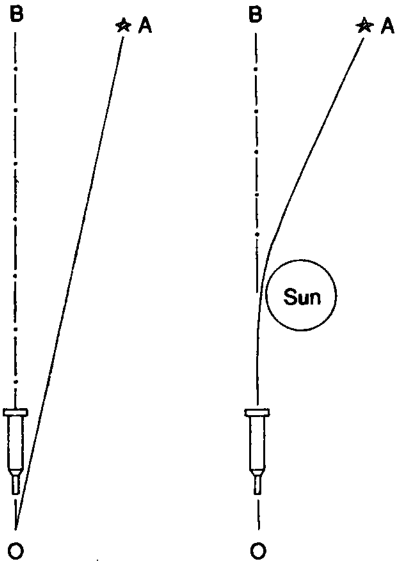
To get back to the light-bending story, Einstein made this prediction about light being deflected by gravity somewhere around 1915 when the

First World War was raging fiercely. For an idea of what this bending business is, see Box 6.2. From the box it should be clear that the only chance one has of observing this effect is during a total solar eclipse. In 1917, the Astronomer Royal of England, Sir Frank Dyson drew attention to the fact that a total solar eclipse would occur on May 29, 1919. It would not be visible in Europe, but certainly in Africa and in South America. This would be a good chance to verify Einstein's prediction.

Plans for such an experiment were made beginning in 1918, even though the war was still on. It was decided that the Greenwich Observatory would send two expeditions to observe the eclipse, one to Sobral in

Box 6.2 To understand the principle of the light-bending experiment, consider figure (a). Light from a star in position A would normally not reach observer O looking through a telescope along direction OB. However, if there is a massive object (like our Sun) in between, then light would bend and the star become visible as illustrated in figure (b). This is the way one detects the deflection of light.

Experiments to measure such a deflection cannot be done every day. Our Sun is so bright that stars in nearly the same line of sight as the Sun cannot be seen. But during a total solar eclipse the bright disc is covered by the moon, and the experiment becomes possible. At the time of the eclipse, the star would be "behind" the Sun, but nevertheless, from astronomical data one can calculate its position A and establish the direction OA. Observation would show the star to be along direction OB. In this way, the deflection can be measured. This measured value is then compared with that predicted by the General Theory of Relativity. The famous experiment of 1919 fully vindicated Einstein's theory. Interestingly, much before the expeditions to Africa and South America, Einstein wrote to his friend Besso, "I do not doubt any more the correctness of the whole system [i.e., the general theory], whether the observations of the solar eclipse succeeds or not." Since 1919, the experiment has been repeated many times. I believe it has also been repeated using satellites.



(a) (b)

Brazil, and the other to the island Principe near West Africa. Eddington went to Principe and he says:

> The eclipse day came with rain and a cloud-covered sky, which almost took away all hope. Near totality, the Sun began to show dimly; and we carried through the programme hoping that the conditions might not be so bad as they seemed. The cloud must have thinned before the end of the totality, because amid many failures, we obtained two plates showing the desired star images.

The other expedition to Sobral led by Crommelin was also successful, and on November 6, 1919, Sir Frank Dyson presented the results to a joint meeting of the Royal Society and the Royal Astronomical Society. The meeting was momentous for the world was told that Einstein was correct and that the Universe was warped space–time and all that. Overnight, Einstein became a world celebrity.

I have discussed how Eddington criticised Chandrasekhar. It is not that he singled out Chandra for attack. He did that sort of thing to many others as well, including some very well-known people. One of them was Sir James Jeans, himself a famous astronomer. Eddington often quoted original results obtained by Jeans but without referring to the work, i.e., without giving a proper reference, or even mentioning his name, something that is not normally done, nor is supposed to be done. No wonder Jeans became angry, and wrote as follows to the journal *Observatory*:

> So much work has been done on isothermal equilibrium that it is difficult to understand how Prof. Eddington can harbour the illusion that he is doing pioneer work in unexplored territory, yet his complete absence of reference to other theoretical workers (except for some numerical computations quoted from Emden) suggests that such is actually the case.

On another occasion Jeans wrote:

> May I conclude by assuring Prof. Eddington that it would give me great pleasure if he could remove a long-standing source of friction between us by abstaining in future from making wild attacks on my work which he cannot substantiate, and by making the usual acknowledgements whenever he finds that my previous work is of use to him? ...I find that some of the fruitful ideas which I have introduced into astronomical physics are by now generally attributed to Prof. Eddington.

Well, there you are!

Rather interesting that in spite of all that happened with respect to the discovery of the Chandrasekhar limit, Eddington and Chandrasekhar remained friends or at least in touch with each other after the latter left for America. They exchanged letters, and Eddington made frequent references to his long cycling trips. He really enjoyed these, and kept careful records of them. In one letter to Chandrasekhar he says: ·

> My cycling *n* is still 75. [*n* is Eddington's symbol for the number of trips he has made] I was rather unlucky this Easter as I did two rides, seventy-four and three-quarters miles, which do not count; I still need four more rides for the next quantum jump...

On another occasion he wrote:

> *n* is now 77. I think it was 75 when you were here. It made the last jump a few days ago when I took an eighty mile ride in the fern country. I have not been able to go on a cycling tour since 1940 because it is impossible to rely on obtaining accommodations for the night; so my record advances slowly.

When Eddington died (in 1944), Chandrasekhar delivered a memorial speech at the University of Chicago. In it he said:

> I believe that anyone who has known Eddington will agree that he was a man of the highest integrity and character. I do not believe for example, that he ever thought harshly of anyone. That was why it was so easy to disagree with him on scientific matters. You can always be certain that he would never misjudge you on that account. That cannot be said of others.

In 1982, Cambridge University invited Chandrasekhar to deliver a series of lectures on the occasion of Eddington's centenary. Chandrasekhar titled his lectures *Eddington: The Most Distinguished Astrophysicist of His Time.* Isn't it amazing that the very person who suffered most at the hands of Eddington was asked to give these lectures? But it is not surprising that Chandrasekhar praised Eddington highly. For him, the disappointment of the past was over a long, long time ago.

7 *Beyond The Limit*

It is natural to wonder what would happen if a star with a mass greater than 1.4 $M_\odot$ were to run out of fuel and start shrinking. Chandra's theory is of course silent about this, but such a thing could happen, and we should have an answer. In fact, this is precisely one of the questions that Eddington raised. Chandra was no doubt aware of all this and if you remember, he himself raised it during that eventful lecture in London. True, he did not come up with an instant answer, and now that we finally know the answer, we can definitely say that no one could have given it at that time. That is because many facts needed for giving the full answer were not known at that time. Well, what *is* the answer? That is what this chapter is all about.

7.1 Neutron stars

Let us say we have no theory for dying stars other than that given by the curve in Fig. 6.1. How do we use this curve to explain what could happen to stars with mass greater than the Chandrasekhar limit? We could simply say that when such a star (with mass greater than 1.4 $M_\odot$) starts collapsing, somewhere along the line it suddenly explodes and in the process loses a large amount of mass. If the remaining mass is less than 1.4 $M_\odot$, then we have no problem at all; Chandra's theory can then take care of everything. There is one problem though, which is that when the star explodes, it must remember to eject enough mass so that the remaining mass is less than the Chandrasekhar limit! Let us see how Chandrasekhar himself puts it. This is what he said in his Ahmedabad lecture (to which I have already made a reference):

> The question arises as to what happens to those stars which have large masses when they exhaust their source of energy? For them, is there a stage where the contraction can be arrested? One can argue that during the process of contraction it is possible that a star ejects a fraction of its mass—and ejection of mass during the course of evolution of stars is well known. Consequently, the question naturally arises: Will a star, of large mass, eject sufficient mass so that it comes below the limiting mass, i.e., 1.4 $M_\odot$? While for a star of 1.6 $M_\odot$, it is possible that the star could eject around 0.2 $M_\odot$, it

> does not appear reasonable to suppose, in the light of present knowledge, that stars of mass 5 $M_\odot$ could eject around 3.6 $M_\odot$ during their evolution. What will happen to such stars? Obviously, they will contract further. This contraction will proceed and must be arrested, if it is arrested at all, at some later stage.

And that later stage is the *neutron star*. Roughly speaking, what happens is the following: When a star more massive than 1.4 $M_\odot$ starts shrinking, initially everything happens as usual and the star reaches the white dwarf stage. The electrons then produce a degeneracy pressure but it is no longer sufficient to win over gravity. So the collapse goes on. The protons and the electrons now start coming very close to each other, and a stage is reached when they combine to form neutrons. Now neutrons are unstable, meaning that neutrons are radioactive and decay into protons, electrons and also neutrinos. There is a maximum energy which these electrons produced by neutron decay can have. So let us say the neutrons decay into protons and electrons. What happens to these electrons? They (i.e., the electrons) will of course start occupying the energy levels available to them. Remember now that in each quantum level, there can be only one electron. When all the available levels upto E_{max} (where E_{max} is the maximum energy for the electrons produced by neutron decay) are occupied, then the fresh electrons produced by neutron decay "have no place to go". At this stage, neutron decay inside the star stops. And so what do we have now? We have stellar material which is almost entirely made up of neutrons, and maybe some protons and some electrons, the latter two mostly in the outer regions—see Fig. 7.1. (Caution: This is what is called a "hand waving" argument, but will do for the present!)

Now neutrons being Fermions can exert degeneracy pressure, and so the collapse of this neutronic matter should stop at some stage due to

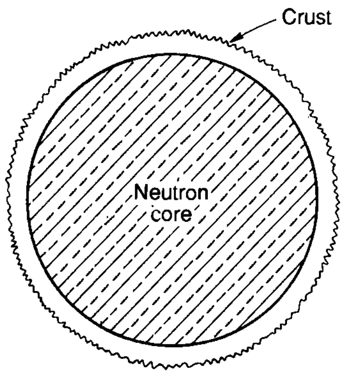


Fig. 7.1 A schematic section of a neutron star. The core contains neutrons, surrounding which is a crust.

such a pressure. And the object that is thereby formed is called a *neutron star*. The neutron was discovered by Chadwick (in Cambridge) in 1932. Two years later, Baade and Zwicky suggested the idea of a neutron star. In 1939, Oppenheimer and Volkoff took the idea further and made some calculations about neutron stars. Meanwhile the World War intervened. and people forgot about neutron stars. After the war, the interest again revived but now a problem surfaced. Calculations showed that a neutron star would be quite small—it would have a radius typically of the order of 20–30 km! Now how on earth does one try to spot a tiny, tiny star like this billions of kilometres away. Seems hopeless, doesn't it? Well, Nature has been kind to us, but before I tell you how, I must say a few words about radio astronomy.

7.2 Radio astronomy

We see stars by the light they emit, and light, if you remember, is nothing but an electromagnetic wave. If so, why can't stars emit radiation of frequencies other than those that correspond to visible light? They do, but to detect them, we must employ suitable methods. Based on these different methods, there are different types of astronomies like infrared astronomy, radio astronomy, X-ray astronomy and gamma-ray astronomy.

As far as radio astronomy is concerned, the star is something like a distant radio station. Only, the frequency of the "station" is somewhat high—of the order of several MHz or above. So all one must do (not quite so easy in practice!) is to have an antenna, point it in the right direction, and pick up the signals one is after. As you might have guessed, radio stars can be observed during daytime too—one does not have to wait for the night.

In the late sixties, Jocelene Bell of Cambridge one day observed very periodic signals being picked up by her radio telescope—see Box 7.1. There was much interest at that time about whether there was life elsewhere in the Universe and whether, if there were people elsewhere, they would try to send messages to other people in other places. Obviously, if they tried to send messages, it would consist of some kind of regular pulses—I mean who would think of transmitting random noise as a message? These people in other planets (wherever they be) were nicknamed "Little Green Men (LGM)", and when Bell picked up very periodic signals coming from outer space, she and her teacher wondered at first whether these were in fact signals being broadcast by LGM somewhere! After careful experiments they came to the conclusion that the signals were not man-made (even if the men were green), but emitted by a celestial object.

Box 7.1 Recalling the historic day when the first pulsar was discovered, Jocelene Bell observes:

> As the chart flowed under the pen [—radio signals from the stars cannot be photographed like light signals can be. Instead, radio astronomers use instruments in which a current is produced whose strength is proportional to the strength of the signal received. This current actuates a pen which produces a trace, rather like a trace is produced of ECG signals.—] I could see that the signal was a series of pulses, and my suspicion that they were equally spaced was confirmed as soon as I got the chart off the recorder. They were 1.33 seconds apart. I contacted Tony Hewish [Bell's supervisor] who was teaching in an undergraduate lab in Cambridge, and his first reaction was that they must be man-made. This was a very sensible response to the circumstances, but due to a remarkable depth of ignorance, I did not see why they could not be from a star.

And from a star they were, as detailed studies later showed!

Next question: "Why were the signals so periodic?" This was not difficult to answer. Ever seen a lighthouse? If you have, you might have noticed that as the light source rotates, the light beam flashes past the observer. So people said that what Bell had discovered was a heavenly lighthouse, emitting radio waves instead of light waves. That could explain the pulses.

This then leads to the question: "How come a celestial object behaves like a light house?" This is answered by saying that radiation comes out of a portion of the star's surface and that the star is also rotating at the same time. This way, one can readily explain a rotating beam. Almost all stars rotate, and so a rotating radio star is nothing unusual.

We now have to explain what kind of a star it is that emits radiation only in one direction? Remember, only if the beam is along one direction and if the object is also rotating, can one have the lighthouse effect. Such a thing can happen in a neutron star, and Fig. 7.2 briefly explains how. By the way, radio stars emitting signals in the form of pulses are called *pulsars*. What Fig. 7.2 says is that pulsars are nothing but neutron stars. Basically what happens is that the neutron star acts as a giant magnet, somewhat like what our Earth does too—only the strength of the magnetic fields produced by the neutron star is astronomically high. As in the case of the Earth, there is a magnetic north pole as well as a magnetic south pole. Electrons escaping from the crust of the neutron star spiral in the magnetic field and plunge towards the pole. In the process,

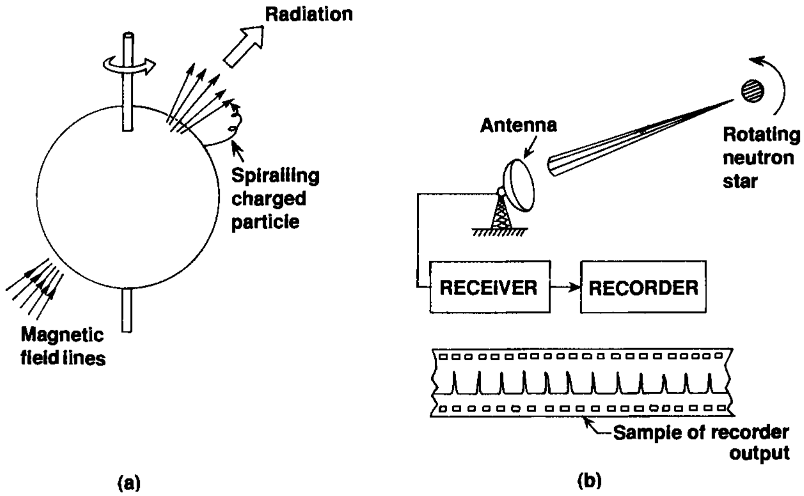


Fig. 7.2 Neutrons in the outer crust can disintegrate into protons and electrons. Now the neutron star acts like a giant magnet (somewhat like our Earth, but in a much more powerful way), producing strong magnetic fields and charged particles escaping from the surface spiral in this field and plunging towards the poles. In the process, they emit strong electromagnetic radiation—see (a). Since the neutron star is rapidly spinning, the electromagnetic beam sweeps around like the beacon from a lighthouse. An observer on Earth picks up a pulse whenever the radio beam sweeps across the antenna of his (radio) telescope—see (b).

they emit electromagnetic radiation. Now there are accelerators called *synchrotrons*, in which charged particles are made to go round and round at speeds close to the velocity of light. (A machine of this type named INDUS is getting ready at the Centre for Advanced Technology, Indore.) In the process, these particles emit electromagnetic radiation called *synchrotron radiation*. The electromagnetic radiation emitted by a neutron star is of the same nature.

It is remarkable that though all the radio emission takes place from a relatively small region around the two poles of the star (which itself is not more than about 20–50 km in diameter), we are still able to pick up the signals. This means that firstly the source is really very very powerful, and secondly, even though we receive a very tiny bit of that radiation here on Earth, our receivers are quite sensitive. Both are true.

Now how can we be sure that pulsars are nothing but neutron stars? Confirmation came when astronomers were able to link pulsars directly to *supernova explosions*. OK, so now I must tell you something about supernova explosions.

7.3 Supernova explosions

Earlier I said that when a massive star shrinks, it could shed some mass by explosion. The way such an explosion occurs is roughly as follows: When the star starts rapidly collapsing, the interior gets heavily compressed and therefore also very hot. If the compression is sudden and the heat generated is very high, then a violent explosion called *supernova explosion* could take place, tearing off the outer layers of the star and hurling them into space—see Fig. 7.3.

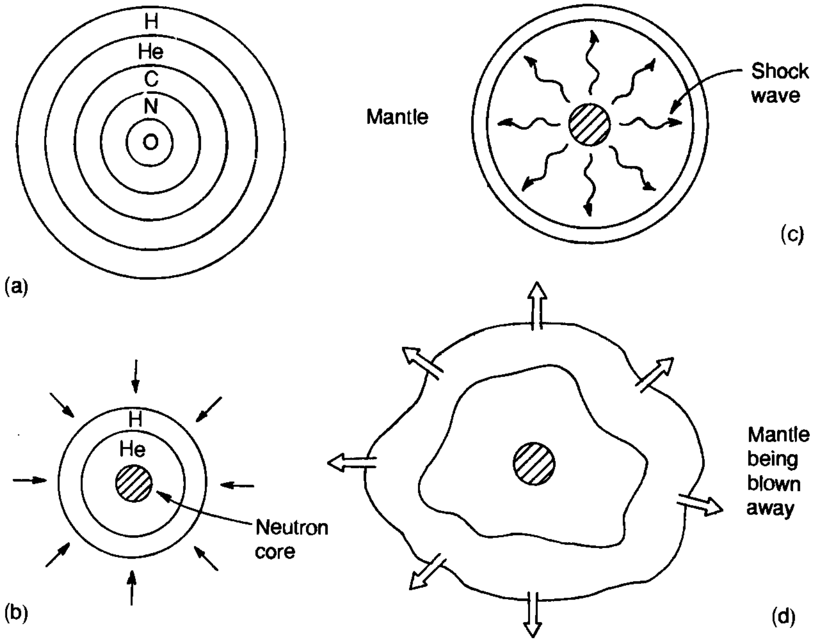


Fig. 7.3 A simplified story of supernova explosion. (a) shows a big star ready to collapse after running out of fuel. As a result of the collapse, the core becomes converted into neutrons as shown in (b). Violent heating during the core formation produces a shock wave as shown in (c). This then blows up the mantle (see (d)), and the resulting explosion is called a supernova explosion.

How do we know stars explode? Well, people have observed such explosions, and the most famous of these occurred in AD 1054. We find detailed record of that event made by many Chinese astronomers. Even American Indians have left a record in the form of cave paintings, but curiously, there are no records of that event in India. Could it be that the event was not seen in India because of the monsoon clouds?

The Chinese astronomers found that suddenly there appeared in the sky a bright new object—they called it a *guest star*—see Box 7.2. It was so bright, that for twenty-three days it could be seen even during daytime! After that, it could be seen at night for nearly two years; after that it disappeared.

Box 7.2 The supernova explosion leading later to the Crab Nebula was first sighted at 2 a.m. (local time) on July 4 1054 by Chinese astronomers. Four of them have left records, in addition to a Japanese observer. For twenty-three days after the event, the supernova remnant glowed so brilliantly that it was visible even during day time. When it was one week old, the remnant cloud was as big as the solar system and as bright as the Sun. After about four weeks, the cloud could be seen only at night. It kept getting fainter and fainter and, after 653 days could not be seen with the naked eye. The Chinese astronomer Yang Wei-te presented this report to the Emperor of China on August, 27:

> Prostrating before Your Majesty, I hereby report that a guest star has appeared; above the star in question there is a faint glow, yellow in colour. If one carefully examines the prognostications concerning the Emperor, the interpretation is as follows: The fact that the guest star does not tresspass against Pi the lunar mansion in the Taurus and its brightness is full means that there is a person of great wisdom and virtue in the country. I beg this to be handed over to the Bureau of Historiography.

In 1848, an Englishman peering through a telescope saw a gaseous stellar cloud; such a cloud is called a *nebula*, and since this particular cloud looked to him like a crab, he called it the *Crab nebula*. The first photograph of the Crab nebula was made in 1892, and in 1928, the famous American astronomer Hubble identified it as the gas cloud left behind by the explosion of 1054. It now became clear that the Crab was a *supernova remnant*.

If the Crab is indeed a supernova remnant, and if the *core* of a supernova contains a neutron star, and if the neutron star and the pulsar are one and the same thing, then there should be a pulsar sitting in the middle of the Crab. And so people looked for a pulsar there, and sure enough they found one—see Fig. 7.4. So one is now pretty sure that the pulsar is nothing but a neutron star. By the way, there are many other examples of supernova explosions known. Quite a few pulsars have also been identified.

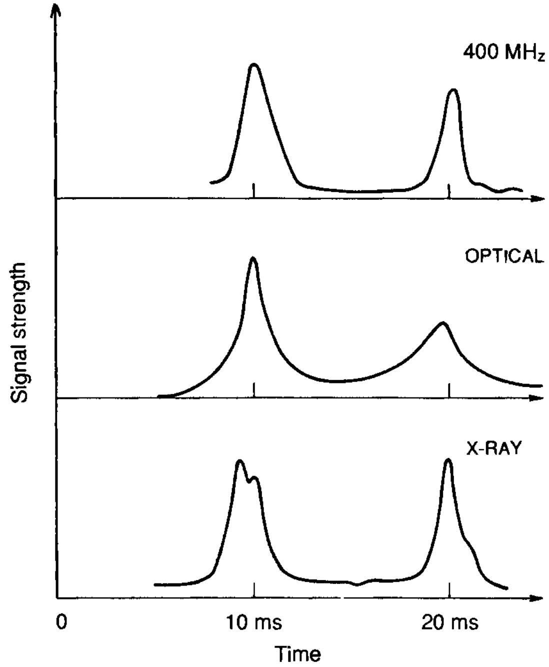


Fig.7.4 Various types of pulses observed in the Crab nebula. These indicate that a pulsar does reside in the Crab, and confirms the idea that supernova explosions result in the birth of neutron stars.

Not all massive stars end up as neutron stars. If the original mass of the dying star is greater than about, say, 5 or 10 $M_\odot$, it has a different fate—it ends up as a *black hole*!

7.4 Black hole

I think I shall let Chandrasekhar himself introduce the black hole. This is what he said (in his Ahmedabad lecture):

> What happens to stars with very large mass, say 10 $M_\odot$? This problem is of great theoretical significance. For such a star, the contraction cannot be arrested either at the white-dwarf stage or at the stage of the neutron star. They will contract further—so much in fact that gravitational field becomes strong enough that even light cannot escape. And when light

cannot escape, nothing else can escape from it and, consequently, it will become invisible. This is the so-called black hole.

The manner in which the dying/collapsing star suddenly becomes invisible is worth describing. Let us say that there is a collapsing star, which we observe from here on Earth. We suppose that somehow we have managed to put a friend on the surface of this distant star which is rapidly shrinking in size. This fellow is giving a running commentary, and we on Earth are picking it up on a radio and eagerly following the events. Let us call our friend as observer A and ourselves as observer B. I now hand you over to Chandrasekhar for a description of events:

> Observer A sends out signals to observer **B** at equal intervals of, say, I second. Initially, observer **B** will receive the signals at approximately the same intervals of time. But, as the collapse proceeds, the observer **B** will find that the interval between the successive signals begins to increase, and eventually, the increase will be exponential, i.e., every millisecond the interval will increase by a factor of ~ 2.5; and within a minute the interval will be elongated very considerably. At the same time, the wavelength of the received signals will also increase exponentially. Thus, *strictly speaking*, as far as the observer **B** is concerned, the process of collapse will take an indefinitely long time... in a minute it [the collapsing star] will become invisible for all practical purposes.

So we have lost our friend observer A! In practice of course, we cannot have an observer like A giving a running commentary. At best we can be observing the star from here. Let us now say we are looking through a telescope at such a collapse of a very massive star. Figure 7.5 shows schematically what one would see.

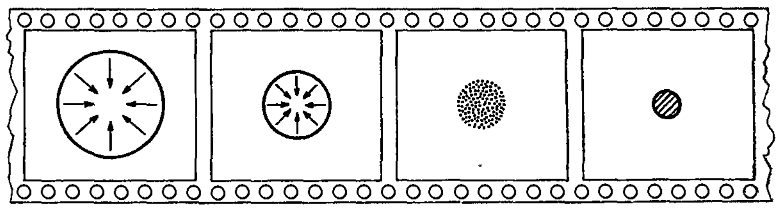
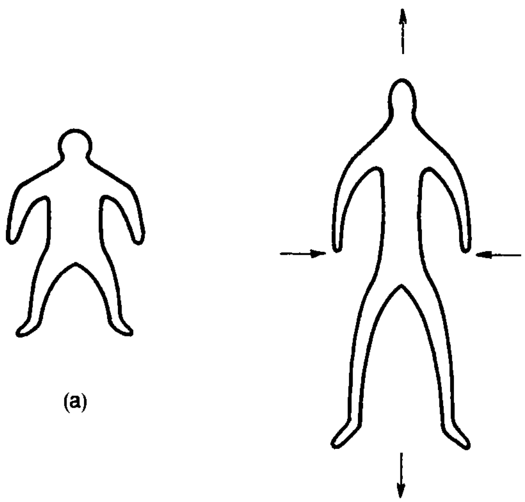


Fig.7.5 A movie showing the collapse of a massive star and its transformation into a black hole. As the star collapses to form a black hole, it becomes dim rapidly, even as the light gets redder and redder (due to what is called the *gravitational redshift*). Eventually, at a critical size, the star darkens into blackness. Thereafter, as far as external observers are concerned, the star appears to remain frozen in this state of collapse. In actual fact, however, the collapse continues. But this would be known only to observers inside the black hole.

For a moment, let us go back to our friend A. We can no longer see him or receive messages from him, but that does not mean he has vanished from the Universe. With our knowledge of physics, we can visualise what is happening to him. Actually, the poor fellow would be in deep trouble!

He would be subjected to what are called *tidal forces* (see Box 7.3),

Box 7.3 You must have heard of the high tide and the low tide on the seas, and how they arise. They are produced, for example, by the pull which the moon exerts on the Earth. Such forces act also on our body. For instance, since our feet are closer to the centre of the Earth, the pull on them is greater than that on the head. So, we should in fact be stretched and also feel some pain as a result. But this does not happen because the difference between the forces on the head and the feet is negligible. Near a black hole the story is different, and what could happen to a chap in the neighbourhood is sketched in figure (b); compare with (a) where tidal forces produce hardly any pull.

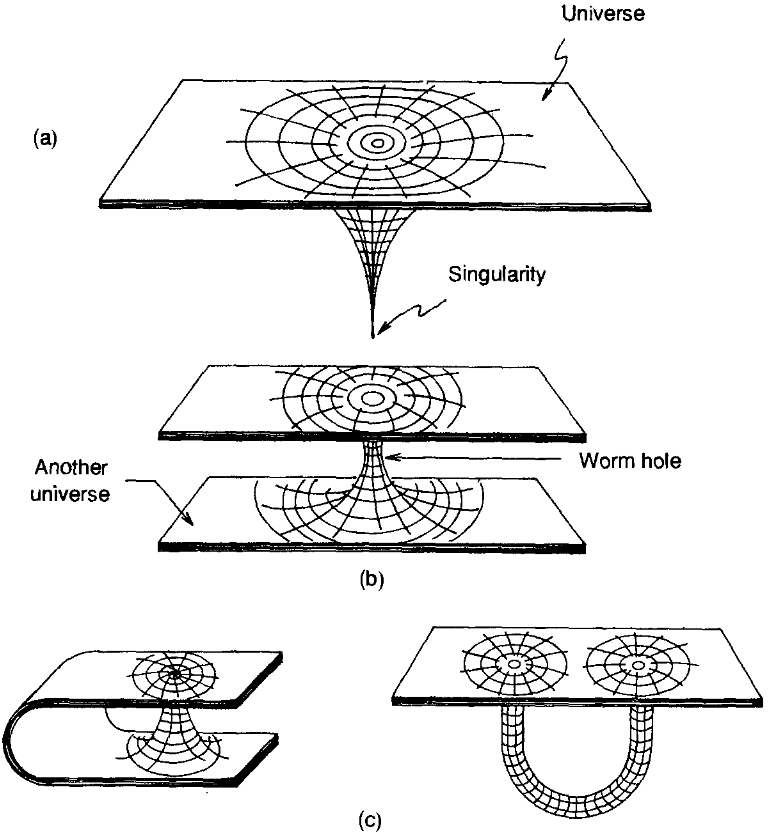


(a) (b)

and as a result he would begin to get elongated. Meanwhile, the star itself is shrinking, and so our poor friend is also hurling towards the centre. Eventually, he is crushed into nothingness, along with the star.

All this would sound very bizarre no doubt. You might ask: "Would a massive star really collapse into a geometric point? How do we know?" Well, I am not saying that the star would actually disappear into a strict point; many things could happen to save it from that fate—see Box 7.4. All these are problems to be analysed using Einstein's *General Theory of*

Box 7.4 Here is a sample of what could happen to save a star from being crushed to a geometric point. As I have said in the text, such matters have to be carefully analysed using Einstein's General Theory of Relativity. In this theory, the focus is on what is called *space–time* which, for our present purposes, can be visualised as a rubber sheet (see also Box 8.3). In the region of objects having mass, the sheet has a dimple but near a black hole, the distortion is severe. In fact, if the black hole got crushed to a point, the *warping* of space–time would be as schematically shown in figure (a). The sharp, pin-like feature is called a *singularity*. Physicists don't like singularities, and one way of avoiding them is to have a *worm hole* as in figure (b). This worm hole connects two *different* Universes, and our friend A trapped in the black hole will now not be crushed but will emerge through the worm hole into the other universe! If you don't like the idea of another universe, then we



could imagine that the worm hole is as in figure (c). In this case, our friend will reappear in another part of our own Universe! Sounds wild does it not? Believe me, all this is not science fiction but something which comes out of various possible solutions of Einstein's equations. Whether these solutions represent reality is a different matter.

Relativity. And, many people are doing precisely that. Chandrasekhar is himself one of them, and believe it or not, at the age seventy and odd years, he published a great big volume called the *Mathematical Theory of Black Holes!* As usual, this is a masterly volume. Chandrasekhar is now past eighty, and busy as ever. Imagine that!

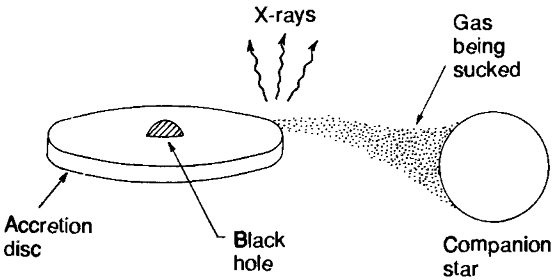
I am afraid I cannot go into the black-hole business here. It needs a book of its own, and maybe I shall write one someday! But let me tell you just this. Don't think that black holes are merely the inventions of theoretical astrophysicists. They exist out there, and people are trying to find evidence for their existence. Now how would you do that when you can't see them? Think about it. Perhaps that is an unfair question! So, let me give you a glimpse of how one hunts for black holes—see Box 7.5.

In summary, stellar corpses are basically of three types—the white dwarf, the neutron star and the black hole. In those days people thought that white dwarfs were exotic. And then came the neutron star which was even more exotic. Now we have the black hole which we can't even

Box 7.5 By definition, one cannot see a black hole with the eye. So there is no point in looking for it with an ordinary telescope, or even a radio telescope. Radio waves, infrared radiation, etc., are nothing but electromagnetic radiation, and the black hole sucks them all. However, there is one thing that one could still do. That depends on the fact that space–time near a black hole is *highly warped*. So the best way of probing a black hole is to examine its neighbourhood.

On the face of it, this might appear a silly idea. I mean we don't know in the first place where the black hole is (for we can't see it); besides, how are we going to get to its neighbourhood? In any case, even if we know where to probe, what do we do? Send a rocket there? This of course we cannot do for many reasons. But there is still a way out namely, make use of a celestial object.

This is not as crazy as it might sound. As I have told you earlier, the heavens contain many binary stars. Let us say that one of the stars in a binary is a black hole. Now we are getting somewhere. What happens is that on account of its strong gravitational pull, the black hole can suck matter out from its companion star (which is an ordinary star). In course of time, this matter accumulates around the black hole and forms what is known as the *accretion disc* (see figure). Strong beams of X-rays are produced when matter from the companion come close to the black hole and is sucked into the disc. So the trick is to look for X-rays coming from the region of (suspected) binary stars. Such X-ray emission has been observed, and one is now more or less sure that the X-ray source in Cygnus X–1 is associated with a black hole.



see (unlike the twinkling star of the nursery rhyme), and it is absolutely incredible! Isn't it all simply wonderful?

8 *Postscript*

You may be disappointed that I have given very little biographical information about Professor Chandrasekhar. Unfortunately, not many details were available to me about his personal life. But some about his professional life are known and I shall now try to make amends.

First of all, let me start with his career. As I mentioned earlier, he went to America in the mid-thirties and joined the University of Chicago; he has been there ever since. Once when a press reporter asked him about his career, he simply said, "You might say that I have been a professor for 37 years." Of course, he was being rather modest. In point of fact, Chandrasekhar started off as an Assistant Professor in 1937 and became a full Professor of Theoretical Astrophysics in 1947. In 1952, he was made the Morton D. Hull Distinguished Service Professor, a post he has been occupying ever since—thirty-eight years running now! I believe the University of Chicago has now named a professorship in honour of Chandrasekhar himself. Chandrasekhar has also been associated from the thirties with the Yerkes observatory in Wisconsin. This observatory is a couple of hundred miles from Chicago.

Chandrasekhar has won many awards, the first of which was his election as a Fellow of the Royal Society (of London) in 1944. This is indeed a rare honour, and among the very eminent (past) Indian mathematicians and physicists to be so honoured are Ramanujan, Harishchandra, Raman, Bose, Saha, K.S. Krishnan and Homi Bhabha. Then came the Bruce medal of the Astronomical Society of the Pacific in 1952, the gold medal of the Royal Astronomical Society of England in 1952, the Rumford medal of the American Academy of Arts and Sciences in 1957, the Srinivasa Ramanujan medal of the Indian National Science Academy in 1962, and the Royal medal of the Royal Society in the same year. In between, he was also elected to the National Academy of Sciences in America. In 1968 he was awarded the Padma Vibhushan title and in 1985 the Vainu Bappu medal of the Indian National Science Academy. The U.S. President has also honoured him.

The list of awards given above is just a sample but Chandrasekhar takes all these awards in his stride. For example, when he was told that he had been awarded the Nobel Prize, he simply said it was a birthday present (since the award was announced on October 19, his birthday; see Box 8.1). When he received his very first medal, he jokingly wrote to one of his sisters that he had hit the jackpot (see Box 8.2). And when he got the Rumford medal, he narrated a joke. It seems there was a Military General who had won many awards and medals. As you know, military officers wear their medals over the uniform; so did this General. A young lady came by and started admiring the medals. She then asked,

Box 8.1 The following are excerpts from an interview Professor Chandrasekhar gave to *India Today* in 1983, soon after the Nobel Prize award was announced.

You have won the Nobel Prize for Physics for the work you did in your twenties. How do you feel about it?

Well, I must say it was a birthday gift.

Can you say in a few words how your discovery has helped in improving our understanding of the universe?

I cannot do that in a few words. I can say that a number of astrophysical discoveries have resulted in the last 10 years. The enormous developments that have taken place in General Relativity have played a major role in the discovery of pulsars, X-ray stars and so on.

You are the second India-born American after Hargobind Khurana to win the Nobel Prize. Why do Indian scientists have to go to the U.S. to become worthy of the Nobel Prize?

I can only speak for myself. I left India soon after graduation and have been working abroad ever since. The first time I visited India was 16 years after I left the country. I have had a number of Indian students working with me. Some of them have in fact gone back to work in India.

At your age many scientists are retired or enjoy emeritus title. What is the secret of your professional longevity?

There is nothing secret about it. I can only say that I keep reading, studying and thinking about problems. Only a few months ago my latest book came out on black holes.

After your uncle, the late C.V. Raman, no scientist working in India has received a Nobel Prize. Is it because the scientific atmosphere in India is not conducive to research?

I was in India in 1961, in 1968 and again in 1982. I must say the scientific atmosphere in India has enormously improved between 1961 and 1982. There is no doubt about it.

Box 8.2 This is what Chandrasekhar wrote in October 1951 in a letter to one of his sisters, soon after he heard that he had been awarded the Bruce medal.

> Since the beginning of October I have been more busy than usual on account of my having to go to Chicago once a week for lectures. And during these last two weeks there have been additional excitements. First, about ten days ago a new idea regarding sunspots, and the idea, on working it out turned out to be surprisingly promising. I got the idea on a Sunday (Nov 25) and I worked out the theory far enough to describe it in my seminar the following day. All during the week I was thinking about the possibilities of the idea. On Thursday when I was in Chicago, I talked to Fermi about it. He was quite excited about it too. And when I get an idea like this, I can hardly sleep at nights. Then on Saturday morning when I left for the observatory very early in the morning (about 6 a.m.) I told Lalitha [Mrs. Chandrasekhar] that I had "hit the jackpot" with my work. And indeed, I had hit the jackpot that day. The morning mail brought the news that I had been awarded the Bruce Gold medal "for distinguished services to astronomy"...

"General, how did you win all these?" The General looked at her, pointed to a tiny medal in the middle and said, "Do you see this medal, my dear? I was awarded this by mistake, and after that all the others followed!"

Next I must mention Chandrasekhar's books. He has written many of them, and everyone of them is a classic. Typically, he works on a subject for about half-a-dozen years, and then writes a book on the subject. After that, the subject is cleaned up, so to speak. People who want to do anything new are usually asked to go and look for some other subject! Of course, all this means hard work, in fact, very hard work. Chandrasekhar starts very early, by about 6 a.m., and works for at least twelve hours. Often he does this seven days a week, and he has been going on like this for years and years.

Now getting back to his books (here I have been concentrating on his very first one), his second one published in 1942 was entitled *Principles of Stellar Dynamics*. In Chapter 2 I told you something about stellar equilibrium. Now the equilibrium of a star can be disturbed if another object, say, star, comes near it. In that case, the trajectory of the first star might wobble. Its shape and size might also be affected. If these perturbations die away after a while, the star would be out of trouble; otherwise, there could be problems. Chandrasekhar studied all these things carefully, and came out with a book. As he put it, he was formulating in the book, "certain abstract problems which appear to have an interest for the general dynamical theory even apart from the practical context in which they arise". His next book entitled *Radiative Transfer* came out in 1950. This book too has become very famous, and is used by people other than astrophysicists. We have already seen that inside a star, energy production goes on all over the place. Naturally, it is most intense at the centre. A part of the energy produced at every point in the star appears as radiation and moves away towards other regions. On the way, some of it could be absorbed. So there is absorption and emission occurring all over the inside. Eventually, some of this energy comes to the surface, from where it is radiated into outer space. It is this radiation which finally escapes that determines the luminosity of the star.

How does one describe the back and forth movement of radiation in the stellar atmosphere, against a background of both production and absorption? The problem had been formulated in a certain sense by Lord Rayleigh as early as 1871, and later in 1905, Schuster had taken it a bit further. But it was left to Chandrasekhar to really thrash out the problem. In the process, he developed novel mathematical methods based on what are referred to as *principles of invariance.* By the way, radiative transfer is of much interest to plasma physicists, and to those who design hydrogen bombs! I should also mention that a new branch of mathematics called *Invariant Imbedding* has blossomed, inspired by this book of Chandrasekhar.

About his approach to work and his style, Chandrasekhar says, "My motive has not been to solve a single problem, but to acquire a perspective of an entire area." He adds, "I work for my personal satisfaction on things generally outside of the scientific mainstream. Usually my work has become appreciated only after some length of time." An example is provided by his book *Ellipsoidal Figures of Equilibrium* which was published in 1969. In this he considered the stability of rotating fluids, a subject that had attracted many leading mathematicians in olden times. When Chandrasekhar started working on it, astrophysicists began to wonder whether anything useful relating to stars would come out of it. But Chandrasekhar was not bothered. To him, the subject "had been left in an incomplete state with many gaps and omissions and some plain errors and misconceptions. It seemed a pity that it should be allowed to remain in that state." And so he wrote a book. Fifteen years later, people found excellent use for this book, for it was needed to understand pulsars!

Chandrasekhar's latest book is called *The Mathematical Theory of Black Holes*. This is perhaps his best book to date and was published in 1983—he was seventy-three at that time! About this book he says:

I started studying black holes 11 years ago, particularly the question of what happens to a black hole when an object such as a star falls into it. There are pieces of this work that have attracted attention, but to me what is important is the final point of view I have of the subject. That is why I wrote the book—to see the subject as a whole. Obviously there are a number of problems in the area that I can still work on, but I don't feel inclined. If you make a sculpture, you finish it—you don't want to go on chipping it here and there.

Martin Schwarzchild an astrophysicist at the Princeton University says:

His [Chandrasekhar's] concentration is unbelievable. He combines sheer mathematical intelligence and phenomenal persistence. There is not one field he has worked in where we are not now daily using some of his results.

He adds:

Chandra's book on black holes is a *tour de force*—an example of will power conquering exhaustion. I really don't know what he can do after that.

Chandrasekhar is not yet a spent force. He is past eighty, and he says he is doing his best work now! And what is he working on? Certain problems in general relativity (see Box 8.3). He is very partial to general relativity, and says it is due to the "aesthetic beauty and the aesthetic sensibility" of the theory. Einstein once said that few people could escape the magic

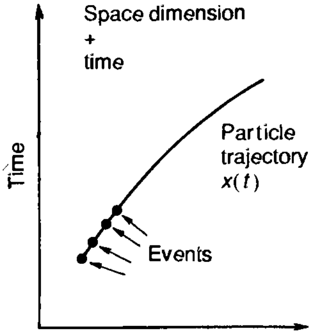
Box 8.3 In Box 5.3, I gave you a capsule of the Special Theory of Relativity. Einstein developed this in 1905. Roughly ten years later, he announced the General Theory of Relativity. If the Special Theory is difficult to describe within one box, the General Theory is even harder! Anyway, let me make the attempt.

The Special Theory deals basically with events. An event is described by (i) the coordinates of the point at which it occurs, and (ii) the instant of time at which it occurs. The motion of a particle is a succession of events which occurs in *space–time*—see figure (a). Let us say there are two observers O_1 and O_2, and that O_2 is moving with a *uniform velocity* with respect to O_1. Say both observe a moving particle. Special Theory is essentially concerned with relating the description of events provided by the two observers O_1 and O_2. Even prior to Einstein, people had proposed schemes for doing this but there were complications when people applied these rules to particles on the one hand and to light on the other. With his Special Theory, Einstein successfully sorted out all these difficulties.

I said that space–time is an arena in which events take place. In Special Theory, space–time is regarded as being "flat"; a convenient way to visualise this is by considering a rubber sheet as in figure (b). What happens if O_2 is not moving with uniform velocity with respect to O_1 but is accelerating? There are complications and it is precisely these which the General Theory takes care of. And the net result of acceleration is that space–time becomes "warped" as in figure (c).

Remember that gravity produces acceleration. As a result, space–time becomes warped due to mass. In flat space–time, the shortest distance between two points is a straight line. With warping present, this shortest path could be curved. The bending of light due to gravity (recall Box 6.2) can be understood along these lines.

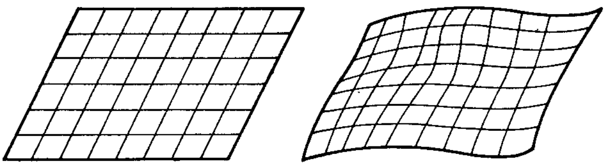
SPACE–TIME



(a)

Flat space–time

Warped space–time



(b) (c)

of the theory; certainly Chandrasekhar has been a captive, a willing one though. I should here mention that some years ago, Chandrasekhar suffered a heart attack and had to undergo bypass surgery. And all this hard work after that! Do you know what he did while he was "taking rest" after the operation? He was reading all the plays of Shakespeare, since he did not have time for it when he was active. He also read Newton's *Principia.*

What is it that drives him? He has discussed the motivations of scientists in several lectures he has given at various times, and these have in fact been collected together into a book entitled *Truth and Beauty—Aesthetics and Motivations in Science.* In a sense, he is in quest of beauty. It is not the beauty of the words that poets seek. Nor is it the beauty of the sound that the musician is after or the beauty of the form that sculptors concentrate on. Chandrasekhar is after what he calls mathematical beauty. May be you and I can't easily understand it. So to explain it to us he quotes the mathematician G.N. Watson who went into raptures after seeing some results derived by Ramanujan (see Box 8.4).

Talking of Ramanujan, Chandrasekhar has the highest regard for him. If you have seen a photograph of Ramanujan, you must thank Chandrasekhar for it. It all happened as follows: In 1936, the British mathematician G.H. Hardy who did much to help Ramanujan, was giving a series of lectures at the Harvard University in America on

Box 8.4 This is what the well-known mathematician G.N. Watson wrote about Ramanujan's work:

> The study of Ramanujan's work and the problems to which it gives rise, inevitably recalls to mind Lame's remark that when reading Hermite's papers on modular functions, *"on a la chair de poule"*. I would express my own attitude with more prolixity by saying that such a formula as
>
> $$\int_0^\infty e^{-3\pi \times 2} \frac{\sinh \pi x}{\sinh 3\pi x}\, dx = \frac{1}{e^{2\pi/3}\sqrt{3}}$$
>
> $$\sum_{n=0}^{\infty} e^{-2n(n+1)\pi} \cdot \frac{1}{(1+e^{-\pi})^2}$$
>
> $$\cdots \frac{1}{(1+e^{-(2n+1)\pi})^2}$$
>
> gives me a thrill which is indistinguishable from the thrill which I feel when I enter the Sagrestia Nuovo of Capelle Medicee and see before me the austere beauty of "Day", "Night", "Evening" and "Dawn" which Michelangelo has set over the tombs of Giuliano de Medici and Lorenzo de Medici...

Ramanujan and his work. The lectures were to be published, and Hardy wanted a photo of Ramanujan to include in the book. But no such photo was available. So Hardy talked to Chandrasekhar who was then in Cambridge. Soon after this Chandrasekhar visited India, and when in Madras met Mrs. Ramanujan. The latter had only her husband's latest passport photo. Chandrasekhar made three large enlargements of this and gave one copy to Mrs. Ramanujan, gave another to Hardy, and the third copy he kept in his office.

In 1976, an American mathematician named George Andrews discovered what has been described as Ramanujan's "lost notebook". This discovery made big news, and when Andrews visited Madras in 1981, he was interviewed by the newspaper *The Hindu*. Janaki Ammal, the widow of Ramanujan heard about this interview and said, "They said years ago that a statue would be erected in honour of my husband. Where is the statue?" Another American mathematician named Richard Askey heard about Janaki Ammal's complaint and decided that he would arrange to have a bust of Ramanujan made. He approached Chandrasekhar for the enlargement with him. Askey then met the sculptor Paul Granlund and asked him if he would make a bust. Granlund was willing, provided four copies of the bust would be bought. It turned out a few more were actually made and distributed. One of course is with Chandrasekhar. One was presented by him to the Indian Academy of Sciences, and one to Janaki Ammal.

Earlier I mentioned that while convalescing, Chandrasekhar read the *Principia*. These days he often lectures on Newton. In his opinion—and in that of a few others too—Newton is far above most of the scientists who have followed, including Einstein! As he says:

> It is only when we observe the scale of Newton's achievement that comparisons, which have sometimes been made with other men of science, appear altogether inappropriate both with respect to Newton and with respect to the others.

I am sure you must have wondered why Chandrasekhar never came back to India. Actually, he and his wife Lalitha (they got to know each other in college and got married in Madras in 1936) visit India periodically. But Chandrasekhar has preferred to stay in America and work there. In 1968 when Chandrasekhar came to Delhi to give the Jawaharlal Nehru Memorial Lecture (see Box 8.5), Indira Gandhi while welcoming him said, "Our country deserves the best, and the fact that we could not hold Dr. Khorana (Nobel Prize winner in medicine) and Dr. Chandrasekhar and cherish them is a matter for regret. Yet I wonder if they could have done their remarkable work had they remained in

Box 8.5 In 1968, Professor Chandrasekhar was invited to deliver the prestigeous Jawaharlal Nehru Memorial Lecture in Delhi. Welcoming him, Indira Gandhi who was then the Prime Minister said:

> We are honoured that Professor Chandrasekhar has come to deliver this memorial lecture... We welcome Dr. Chandrasekhar as a son of India and one of the greatest scientists to be born in this country...
>
> Our eminent lecturer today is an American citizen, but I am sure that that does not make him less of an Indian or diminish our admiration for him. He himself may perhaps be remembering some of the frustrations when he wanted to serve the land of his birth. And how can we forgive ourselves?
>
> The time has come, if it is not already too late, when we must make an all-out effort to break the bonds of defeatism which envelop science in India. Many of our scientists working within the country, and some working abroad, are rated amongst world leaders and can be the pride and adornment of any institution. We must do our utmost to give them the best deal by creating conditions where they can give their own best to the country and the world. There is no time to lose.

Chandrasekhar's lecture was entitled *Astronomy in Science and in Human Culture*. He began his lecture with the words:

> It is hardly necessary for me to say how deeply sensitive I am to the honour of giving this second lecture in this series founded in memory of the most illustrious name of independent modern India. As Pandit Jawaharlal Nehru has written, "The roots of an Indian grow deep into the ancient soil; and though the future beckons, the past holds back."

Chandrasekhar then traced the history of astronomy, a subject which had "expanded the realm of man's curiosity about his environment." He discussed Babylonian astronomy, Greek astronomy, Hindu astronomy and also Islamic astronomy. As far as Hindu astronomy was concerned, it was geometric in character (unlike Babylonian astronomy which was arithmetical), and was perhaps influenced by Greek astronomy. There was evidence that Hindu astronomy later influenced Western astronomy through Muslims, who also held sway over Spain.

Chandrasekhar then went on to describe the more recent developments, starting with Galileo and Newton and ending with the Big Bang. He concluded his lecture with the following words:

> With the discoveries I have described, astronomy appears to have justified the curiosity that man has felt about the origin of the universe, from the beginning of time.
>
> As I said at the outset, man's contemplation of the astronomical universe has provided us with the one continuous thread that connects

> us with antiquity. And I might add now that it has also inspired in him the best.

After Chandrasekhar concluded his lecture, Dr. Zakir Hussain, then the President of India, proposed the vote of thanks which he ended with the words:

> Wherever you [i.e., Chandrasekhar] may be, we shall always look upon you as one of us and as a great son of India who has tried his best to preserve the glorious traditions of this country and has brought renown and glory to its name.

India." I wonder what you have to say about that. Let us now hear wh Chandrasekhar himself has to say about that.

Somewhere in the early forties, Chandrasekhar's father Mr. C.S. Iyer sent his son in America, a copy of a convocation address delivered by Professor C.V. Raman. In reply to his father, Chandrasekhar wrote:

> I was glad to read C.V.R's convocation address. I was in general agreement with his depreciation of the craze for foreign degrees, but I think he is overlooking the obvious when he says that those who have benefited by going abroad would have "done infinitely better" by staying at home. I wonder how he can explain Ramanujan. After four years at Cambridge, and with Hardy, he [Ramanujan] lived to become the greatest name in mathematics in this century. Anyone who has even a passing acquaintance with Ramanujan's life will accede that he would have died unknown and unwept if he had continued the last precious five years of his life in India. Again, in a different plane, I can assert that I could not have done "infinitely better" had I continued in India; I am sure I would have done much worse. However, with his larger thesis that it is up to us Indians to improve our Universities and centres of education in India, I entirely agree. And, for my part, I hope that one day I shall contribute my small measure to this development. But this is looking too far ahead...

In the late sixties when the Tata Institute of Fundamental Research in Bombay (founded by Homi Bhabha) was celebrating its Silver Jubilee, Chandrasekhar was a distinguished invitee. I was in the audience, and I heard the then Governor of Maharashtra Ali Yavar Jung describe how he and Bhabha competed with each other to bring Chandrasekhar to India, he to the Osmania University in Hyderabad, and Bhabha to the Tata Institute. Of course, neither succeeded. Long before all this, Raman made a passionate appeal for developing astronomy in India. He wrote in the journal *Current Science* edited by him, and wrapped up his plea with the following words:

> The work of S. Chandrasekhar, now Professor of Astrophysics at the Chicago University is an indication of what could be accomplished in this country under favourable conditions. It would require an entire number of *Current Science* and not a paragraph or two to sketch the many fields of astrophysical research traversed by Chandrasekhar and the results obtained by him during the last fifteen years.

One wonders what Raman would have said were he to look at Chandrasekhar's publication record as it stands today! Raman was campaigning for observatories, telescopes and the like. I am happy to say that in the last twenty-five years or so things *have* been happening. There are radio telescopes at Ooty, Bangalore and a few other places. A very big radio telescope, the *Giant Metre Radio Telescope* (GMRT) is coming up near Pune. There is a big optical telescope in Kavalur, half-way between Madras and Bangalore, and there are gamma-ray telescopes in Pachmari and in Gulmarg. There is also a big centre for astronomy and astrophysics in Pune. So all in all, things are much better than they were before. It is time now for the country to produce more Chandrasekhars; who knows, you might be one of them!

I realise that I have told you very few anecdotes. Let me make up and narrate a few. A famous one relates to a class which Chandrasekhar was teaching in the late forties / early fifties period. At that time, he used to regularly share his time between Yerkes and Chicago. Once he was driving from Yerkes and was caught in a bad snow storm. The conditions were so bad that most people abandoned their driving and took shelter. But Chandrasekhar drove on so that he could be in time for his class. Everybody thought he was unwise, but he was determined. Later he told people that it was worth taking all the trouble on account of the students he had. And who were those students? None other than Lee and Yang who won the Nobel Prize for their discovery of the breakdown of the conservation of parity. Incidentally, here was a case where the students won the Prize before their master!

Professor Auluck of the Delhi University tells another story. Apparently, once when he was in Chicago, Auluck wanted to call on Chandrasekhar. So he telephoned, and to his surprise Chandrasekhar invited the visitor from India straightaway, even though it was quite late at night. So Auluck went there—it was close to midnight—and as he describes, "Sure enough he [Chandrasekhar] was there waiting for me, fresh and smiling."

Once a sister-in-law of his wanted to buy him a present. "Buy me a pencil" he said, adding "that is the only thing I use." During his visits to Madras, he often goes to the Presidency College quietly and without telling anyone. Dr. B.R. Pai, a former Professor of Chemistry recalls, "On coming out of my room one morning, I saw Dr. Chandrasekhar in the grounds. I went to him, introduced myself and requested him to step into my room. He was very much attached to the college." There is also a touching story about young Chandrasekhar and his mother. It appears that one day during the final exams (in Madras), Chandra rushed off to the exam hall without eating, ignoring his mother's request. On his return he found that his mother had also not eaten her food. He was deeply moved, and even today often narrates this incident.

And so Professor Chandrasekhar goes on and on—nonstop! Apparently, he has once wondered about retirement. He is reported to have said:

> One of the unfortunate facts about the pursuit of science the way I have done it is that I had to sacrifice other interests in life—literature, music, travelling. I wanted to read all the plays of Shakespeare very carefully, line by line, word by word. I know I could have been a different person had I done this. I don't know if regret is the right term for what I feel. But sooner or later one has to reconcile these losses.

From the point of view of science I can certainly say that it is good that Chandrasekhar has not had the time to read Shakespeare very carefully, to listen to music the way he would like and to travel to the places he wanted to. Thanks to him, we don't have to wonder so much about what the twinkling stars are; instead, we marvel at their structure, the way they produce energy, and the way they finally disappear!

Suggestions for further reading

After this volume was prepared for the press, there appeared the following, comprehensive biography of Professor Chandrasekar.

1. Wali, K. C. *Chandra*, Penguin Books India: New Delhi, 1990.

Chandrasekhar's books are rather technical and therefore I shall not refer to them here. Howerver, the following volume is worth reading. It is a collection of lectures given by Chandrasekhar on various occasions, about motivations in science.

2. Chandrasekhar, S. *Truth and Beauty*, University of Chicago Press: Chicago, 1987.

There are many popular books dealing with astronomy, astrophysics and cosmology. Here is a partial list.

3. Disney, M. *Hidden Universe*, MacMillan: New York, 1984.
4. *Frontiers in Astronomy—Readings from Scientific American*. W. H. Freeman: San Francisco, 1970.
5. Goldsmit, D. *Evolving Universe*, W. A. Benjamin: New York, 1981.
6. Goldsmit, D. and Levy, D. *From the Black Hole to the Infinite Universe*, Holden-Day: San Francisco, 1974.
7. Gribbin, J. (Editor) *Cosmology Today*, New Scientist, London, 1982.
8. Harrison, E. *Cosmology—The Science of the Universe*, Cambridge University Press: Cambridge, 1981.
9. Heldmann, J. *Extra Galactic Adventure—Our Strange Universe*, Cambridge University Press: Cambridge, 1982.
10. Hoyle, F. *Ten Faces of the Universe*, W. H. Freeman: San Francisco, 1977.
11. Murdin, L. and Murdin, P. *Supernovae*, Cambridge University Press: Cambridge, 1985.
12. Shipman, H. L. *Black Holes, Quasars and the Universe*, Houghton-Mifflin: Boston, 1975.
13. Silk, J. *Big Bang—Creation and Evolution of the Universe*, W. H. Freeman: San Francisco, 1980.

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