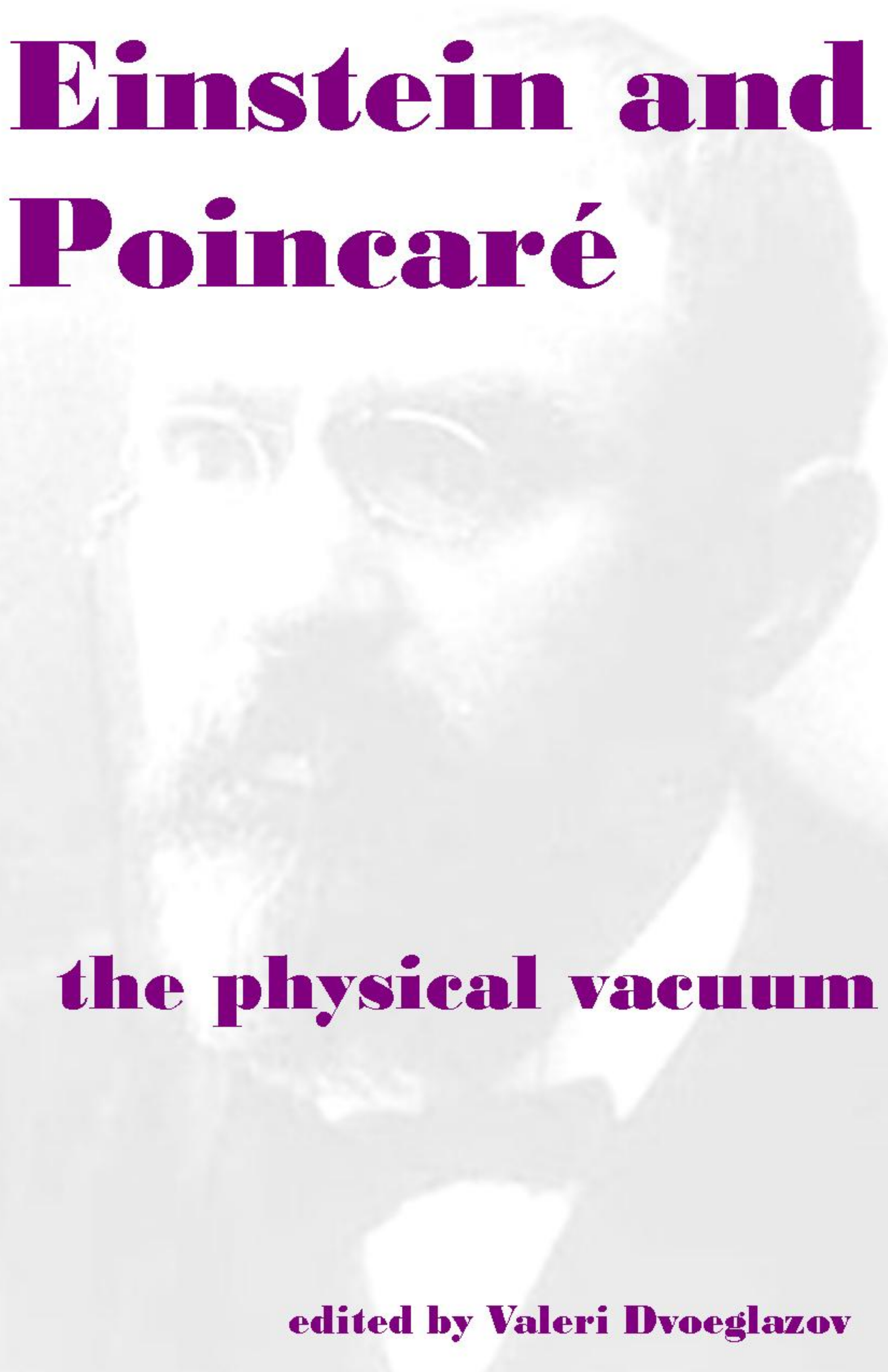

Einstein and Poincaré

the physical vacuum

edited by Valeri Dvoeglazov

Einstein and Poincaré:
The Physical Vacuum

Edited by Valeri V. Dvoeglazov

Apeiron
Montreal

Published by C. Roy Keys Inc.
4405, rue St-Dominique
Montreal, Quebec H2W 2B2 Canada
http://redshift.vif.com

First Published 2006

Library and Archives Canada Cataloguing in Publication

Einstein and Poincaré : the physical vacuum / edited by Valeri V. Dvoeglazov.

Includes bibliographical references.
ISBN 0-9732911-3-3

1. Einstein, Albert, 1879-1955. 2. Poincaré, Henri, 1854-1912. 3. Relativity (Physics). 4. Ether (Space) I. Dvoeglazov, Valeri V.

QC173.6.E45 2006 530.11092'2 C2006-903323-4

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Editorial Introduction

This book is dedicated to significant contributions made to physics by Poincaré and Einstein 100 years ago [1,2] (see also [3]). As we celebrate the International Year of Physics, we might ask: Is Physics in good health? Everyone has his own answer to this simple question. Consequently, we decided to publish this book under the intriguing title “Einstein and Poincaré: the physical vacuum.” The reader may be puzzled by the choice of theme, which would seem to have been fully studied years ago. We fully sympathize with him: it was and is indeed astonishing for us that after more than 100 years of research the topic is still full of surprises. In a way, it has a life of its own. The highly respected authors of this volume appear to have agreed with us in presenting their thoughtful pieces of research. These are Profs. Vigier, Duffy, Selleri, Kholmetskii, Barbosa, Sidharth, Munera, Onoochin and Weber, Pierseaux, Cahill, Krasnoholovets, Martin and Roscoe. Special thanks to them all! While their papers speak for themselves, I feel compelled to say few words for myself.

The posthumous paper by Vigier opens the volume. The unification of the gravitational and electromagnetic interactions and the Dirac “aether” concept [4] have always been the main concerns of this notable scientist. His intuition is impressive indeed.

The Duffy paper presents an historical review following from a comparison of the original Lorentz-Poincaré [5] and Einstein-Minkowski traditions to modern ideas (the extensive list of references supports our statement made in the previous paragraphs). Particular attention has been given to the recent works of Cavalleri, Dmitriev and Winterberg (“vortex-sponge ether analogue”), *e.g.*, Ref. [6], who for various reasons were not able to contribute to this volume, and to the concept of the Dirac ether. Recalling Dirac’s ideas, Duffy repeats: “Dirac regarded electric potential and velocity field as physially real,” an idea closely akin to Vigier’s works. Moreover, Duffy several times stresses that the “physical vacuum” is nothing more than the development of the old idea of ether (“...many modern theorists ... use alternative expressions, such as vacuum field, physical vacuum, or cosmological plenum rather than the obvious term”). Unfortunately, there are almost no insights into the obvious relations between the ether concept and the recently discovered “dark matter” and “dark energy.” Finally, an important point brought up by Duffy is that several theorists state that ether drift in the Michelson-Morley-Miller-type experiment is in principle unobservable, and suggest different experiments.

The Selleri paper continues the book, a culmination of many years of research. The most attractive part for us personally is the part where he discusses the class of equivalent transformations with e_1, the synchronization parameter.

"...Clock synchronization in inertial systems is conventional and the choice of the invariance of the one way velocity of light made in [special relativity] is only based on simplicity," see the Eq. (5.1) of his paper. Thus, as opposed to the general opinion, the special relativity does *not* reject the ether; it is simply unobservable within this class of theories. This is the most important point! Next, Selleri discusses the well-known twin paradox, and Einstein's misunderstandings in its explanation. In my personal opinion, the twin paradox can be explained within the SRT too, provided that we agree which of the twins turns around.

The Kholmetskii paper discusses the question of the experimental distinguishability of special relativity (SRT) [1] and Lorentz ether theories [5], since for "many years two alternative physical theories were considered to be mathematically equivalent each other." The paper contains some statements which are difficult to agree with (for instance, the author thinks that "SRT is not, in general, a consequence of the GRP [general relativity principle]"; "Fitzgerald-Lorentz contraction is not observable," *etc*.). However, the main idea of "covariant ether theories" may be quite valid (I would still suggest that the author not make a distinction between "physical" and "measurable," as no one else does).

L.C. Barbosa tries to find a new interpretation of the Hubble's constant basing his insights on light dispersion in the interstellar ether. "Based on this idea, the model of the universe is static, lacking expansion" (precisely what "Apeiron" signifies). Unfortunately, the next paper by B.G. Sidharth frequently refers to previous papers by the same author. As a result, the reader must work hard to understand his paper. On the other hand, the zero-point field (ZPF) may indeed be a good candidate for both dark matter and ether. Thus, the reader may certainly find grain of truth in it.

H. Múnera then argues that no evidence of the Lorentz-Fitzgerald contraction exists. However, careful analysis of the Michelson-Morley-Miller (MMM) experiment and an impressive list of references diminishes the significance of any possible debatable points. I paid particular attention to the discussion of the statement: "reflection from the mirrors in motion would necessarily lead to a negative result in the MMM experiment" in view of another, not very old paper [7].

V. Onoochin and S. von Weber also consider the contraction of bodies in motion. They believe that certain terms have been omitted in explanations of the MMM experiment.

Y. Pierseaux puts forward a more complex question. Is "the image by the Lorentz transformation of a spherical light wave, emitted by a moving source," also spherical or ellipsoidal? The answer to the question may have far-going consequences, because it is normally assumed that light propagates in *any* reference frame with the same velocity c. It is helpful to recall Einstein's and Poincaré's viewpoints on this subject.

The Head of the School of Physics at Flinders University (Australia), R. T. Cahill states categorically: “So the Einstein postulates have had an enormous negative influence on the development of physics, and it could be argued that they have resulted essentially in a 100-year period of stagnation of physics, despite many other exciting and valid developments....” It is difficult to agree with him, but it is also difficult to agree with editors who do not permit criticisms of the explanation of the MMM experiment. Let us be free, at least, in science.

Some new concepts are introduced into physics by V. Krasnoholovets. I should point out that he is not a physicist by training. We should be open to consider multidisciplinary studies undertaken by scientists trained in different fields.

The Martin paper is written in an “old-fashioned style.” “A gas, composed of particles moving in all directions, is assumed to pervade the entire Universe...” Yet some of the insights into relations between thermodynamics and gravitation would appear be useful, at least for future developments.

Lastly, D. Roscoe continues the quest for the “massive photon” [8]. After long (and perhaps unnecessarily complicated) calculations he seeks to prove that “[the Maxwell field] cannot exist in isolation, but must always be associated with an additional massive vector field.” I believe the reader can place some credence in this statement, as it is closely linked with my own research [9].

For my part I can say the following. I agree with Duffy that it is not helpful to use a lot of words to denote the same thing in science. My own preference lies with papers on gauge fields [10] (some day I shall comment on them more extensively). Secondly, the theoretical possibility of additional scalar and/or 4-vector fields in fundamental physics (the Maxwell-like electrodynamics) and astrophysics has been proven [11]. This is *not* the final word.

* * * * *

Therefore, in the hope of some success with this book, and in view of the above, we extend a call for papers for future regular issues of the *Apeiron* Journal. We intend to continue our policy of publishing high-quality papers on the problems of modern gauge theories, gravitation and cosmology.

In conclusion, we again remind the reader that in recent years we have been able to see that new breaches have appeared in the fortress of modern physics (just think about the “compatibility” of modern SRT with modern astrophysics—more on this another time). Even though the present-day education system prepares ever more new defenders, it is inevitable that a new beautiful edifice of Physics will be built. And this is something from which we all stand to gain. Finally, we extend our special thanks to our publisher, our authors, our referees, and our friends.

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December 2005.

Interactions of Internal Inertial and Phase Space Motions of Extended Particle Elements Moving in Dirac's Real "Aether" Model*

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One analyses in this work new possible physical properties of extended Quantum particles considered to be sets of vortices built (with their associated real "pilot" waves) with a tight combination of localized "holes" (gravitational) and "bumps" (electromagnetic) propagating within a moving, real, stochastic physical Dirac-type "aether" within an infinite three-dimensional flat-space. In this model the corresponding interaction forces are mediated by new vector bosons.

Introduction

One of the unsolved problems of Q.E.D. is the observed numerical value of $\alpha = e^2/\hbar c$, which relates the three constants e, $\hbar$, and c determined by [better: "each representing", Eds.] different observed physical quantities. The most remarkable attempt to solve it is the introduction, by D. Batchelor [1][†] of a semiclassical model to determine virtual antiparticle pairs. The aim of the present work is to develop this line of research within the frame of the new model of unification of gravitation and electromagnetism recently proposed by Van Flandern and the author. [2]

In that model spinning massive extended electrons-positrons and photons (and the elements of their pilot waves) are built with e^+ and e^- charges rotating around a centre of mass O, on the extremities of a diameter, with velocity $\pm c$.[‡]

[*] Some pages may be missing from the manuscript. We have tried to preserve the text as close to the original manuscript as possible. Editors may agree or may disagree with the author's statements and calculations. (Eds.)

[†] The numbering of bibliographic entries has been corrected in some places to make it consecutive. (Eds.)

[‡] For a recent review of composite models of vector bosons, see V.V. Dvoeglazov, "Speculations on the Neutrino Theory of Light," *Annales de la Fondation de Louis de Broglie*, 24, No. 1-4, pp. 111-128 (1999); physics/9807013, (Eds.)

In photons both charges are attracted by strong spin-orbit and spin-spin gravitational forces.

To unify gravitation and electromagnetism, two models have been developed which differ in basic respects.

I. The first (by Einstein *et al*. [5] considers matter and fields as densities within a single curved Riemannian space-time with torsion, etc.
II. The second (by Lorentz [6]) considers them as two different types of perturbations containing closed regions (vortices, etc.) with internal motions, both moving in an external medium (the aether) which moves in an infinite three-dimensional space.

In both cases the problem is to unify macroscopic and microscopic gravitational and electromagnetic forces, and to relate them to the observed quantum microscopic (nuclear) forces [4] interpreted in the Copenhagen or the de-Broglie-Bohm framework.

In this work (since this has not been done until now in Einstein's line or research) we shall explore Lorentz's point of view only. As will be discussed in the next sections of this work, we will deal with some possible predictions of Lorentz's model, which rests on the following assumptions.

a) The "aether" is a real continuous chaotic medium moving in an infinite flat three-dimensional space.
b) It has variable local properties, *i.e.*, density, velocity, etc., and can carry localized metastable moving bumps and holes endowed with internal motions.
c) Such local density holes and bumps can be associated with basic gravitational and electromagnetic elements. They can move together (since they are related by the surrounding aether fluid) and thus are part of the collective waves piloting different types of solitons defined by average properties satisfying special relativity theory.
d) In Lorentz's point of view,* the aether can be associated [with] a specific average Lorentz frame Σ_0 in space (in which it is isotropic on average).
e) One can attribute to each (aether-built) observer [corresponding, Eds.] aether-built measuring devices, performing [better: "which perform", Eds.] real measurements with rods and clocks associated with variable Weyl units relating neighbouring aether elements. Their measurement (results) are thus different from the real space-time, intervals of the flat aether-carrying Galilean space. They are locally defined by the Lorentz-Weyl laws of Special Relativity. These transformations Σ correspond to aether-built observing system, *i.e.*, local $\Sigma_0 \rightarrow \Sigma$ transformations.

* We note that since Mach proposed to use the distant galaxies as a basic reference frame, this relates him (indirectly) to this point of view.

1. Aether interpretation of particle inertia

In the case of electrons (or positrons), one can evaluate the contribution of the extended charges $e^{\pm}$ in extended particle models and [of the] surrounding extended gravitational wave element. This has already been done in recent work by Klyushin [3] in an Electron Dynamic aether which we shall first briefly discuss.

Klyushin starts from an extension of the usual electromagnetic relation of the point-like charge*

$$\vec{F} = m\vec{a} + q\vec{V} \tag{2}$$

where q is the electron's charge and V its velocity. He introduces, in the surrounding aether, its inertial resistance in the form [3] $1/\varepsilon_0 c$, where ε_0 is the aether's density, *i.e.*, the dielectric constant. He thus writes for the force $\vec{F}$ on an isolated electron

$$\vec{F} = m\vec{a} + q\vec{V} - \frac{qV^2\hat{V}}{2c} \tag{3}$$

where $\hat{V}$ is a unit vector in the direction of the velocity $\vec{V}$.

In scalar form, for a projection of (2),† one has

$$\frac{d\vec{V}}{dt} = \frac{\vec{F}}{m} - \frac{q\vec{V}}{m} + \frac{qV^2\hat{V}}{2mc} \tag{4}$$

With $\vec{a} = \vec{F}/q$, $b = -1$, and $\vec{p} = \hat{V}/2c$, relation (3) reduces to

$$\frac{d\vec{V}}{dt} = \frac{q}{m}\left[\vec{a} + b\cdot\vec{V} + \vec{p}\cdot V^2\right] \tag{5}$$

[This is a vector equation which can be solved component-wise: thus, for each component on the rhs, Eds.] which, [satisfies] for $a + bV_0 + pV_0^2 \neq 0$ [the equation] has solutions (w.r.t. V) satisfying

$$\int_{V_0}^{V} \frac{dV}{a + bV + pV^2} = \frac{q}{m}\int_{t_0}^{t} dt \tag{6}$$

If $a + bV_0 + pV_0^2 = 0$ [for every rhs component, Eds.], [then] the straight line $V = V_0$ is a solution [of (5), Eds.]. These solutions of (6) are real if

$$1 - \frac{2\vec{F}}{q\cdot c} \geq 0 \tag{7}$$

They conserve $\vec{V}_0$ so that an electron moving with average velocity $\vec{V}_0$ with respect to external space will preserve that velocity. When $\vec{F}° = 0$, [then] [it is] the centre of mass's velocity $\vec{V}$, *i.e.*,

---

* The system of units in which the dimension of qV is equal to the dimension of the force is not defined (Eds.)

† Obviously, it is not yet in "scalar form." See below. It would be better to write: "This latter equation can be expressed as…"

$$\vec{V} = \frac{2 \cdot c \cdot \vec{V}_0}{\left\{V_0 + \left(2c - V_0\right)\exp\left[\left(t - t_0\right)q \cdot m^{-1}\right]\right\}} \tag{8}$$

which decreases exponentially.

If $V_0 = t_0 = 0$, the solution of relation (5) becomes:

$$\vec{V} = \frac{2\vec{F}\left[\exp\left\{t \cdot \frac{q}{m}\sqrt{1 - 2\vec{F}/qc}\right\} - 1\right]}{q\left[\left(-1 + \sqrt{1 - 2\vec{F}/qc} + \exp\left\{t \cdot \frac{q}{m}\sqrt{1 - 2\vec{F}/qc}\right\} \cdot \left\{1 + \sqrt{1 - 2\vec{F}/qc}\right\}\right)\right]} \tag{9}$$

when $\vec{F}$ is given a speed $\vec{U} = 2\vec{F}/q$. The speed $\vec{V}$ of the charge is proportional to U [and has an asymptotic limit, as t tends to ∞, of $\vec{V} = \vec{U}\Big/\sqrt{(1 - \left|\vec{V}\right|/c}$), Eds.] The time gained from 0 to U is thus also the time of the accelerated electron movement. Relation (9) thus implies that

$$\frac{2\vec{F}}{q} = \vec{U} = 0 \tag{10}$$

when $\vec{V} = c$, *i.e.*, $2\vec{F}/q = \vec{U} = c$.

One can also have $1 < 2\vec{F}/qc$. In that case $\vec{V}$ oscillates around $\vec{U} = 2F/q$.*

This model implies that electrons, photons, and the collective motions of other particles experience a certain drag, even when moving steadily, as a direct consequence of the fact that these extended particle models contain internal motions of the electromagnetic charge and internal gravitational field really correlated with their external motions in the real physical aether, and this also applies to extended photon elements, which contain e^+ and e^- charges. This interpretation of inertia as resulting from aether friction is not new in the literature. It evidently also applies to light if one introduces extended wave elements containing e^- or e^+ or e^+e^- pairs.

2. Lorentz's aether model

When one compares two different cosmological models, such as Einstein's (and his successors) [5] based on point-like singularities (particles) moving in curved Riemannian space-time with Lorentz's model [6], *i.e.*, a continuous four-dimensional aether fluid carrying extended particle-like solitons moving on flat three-dimensional space, one must fix alongside different experimental predictions their different theoretical observable criteria. In this paper we start from the assumption that the "sum of all forces of any nature (gravitational, electric, magnetic, elastic, nuclear and aether) acting on any existing extended body is always zero in all real physical frames of reference," a statement which

* One of the editors disagrees with this passage.

includes the results of all measurements with real rods and clocks in both models.*

For example, as suggested by Pendleton [7] Fresnel's formula for the velocity of light in a moving medium should be combined with Lorentz's and Poincaré's formulation of relativity theory to reanalyze the data from Dayton Miller's aether drag experiments.[7b] This yields a velocity of the solar system relative to the local preferred rest-frame Σ_0 (in which the 2.7 K microwave background is isotropic).

In Lorentz's model, if one considers in Σ_0 the real set of nonzero mass photons of the 2.7 K microwave background electromagnetic field as a static distribution of extended nonlinear particles surrounded by real pilot waves, also with nonzero mass, one can consider them as perturbations within the total electromagnetic field distribution of extended particle-like photons surrounded by linear waves satisfying Maxwell's equations with nonzero photon mass, which fills and moves within the unlimited basic three-dimensional space. These waves can be considered as collective wave packets of extended photons with a local average density and internal spin *h*.

3. Interactions between extended photon-elements moving in flat space which appear in collective motions

When two elements collide, these interactions imply cross-sections with each other in space, depend on the orientations of their external four-momenta and/or their internal motions. These interactions preserve the total external and internal momenta and energy, when one neglects aether contributions. They join in compound quanta and are dissolved within one cycle of their existence, and thus preserve the total energy-momentum of the vacuum (aether) contribution of the element's system.

One can derive two consequences from these assumptions:

A. The first is that if one describes the real aether as a continuous fluid moving in flat space (described by its element's total density and impulsion) this fluid can carry moving closed vortices of different sizes, *i.e.*, contain different types of internal motions connected (in different ways) to the surrounding aether.
B. The second is that the introduction of different types of such collective vortex motions described by collective parameters (vortex density, etc.) which characterize their local properties including the relations of their individual internal vortex motions with the surrounding aether. In other words, in this type of aether one can describe definite types of vortex motions, *i.e.*, in terms of different chaotic isotropic fluids of vortices which carry collective linear wave motions (of vortices) guid-

* This applies to all measurements, including the forces between condenser plates in vacuum in the Casimir effect.[4]

ing (piloting) nonlinear extended vortices, a description which recovers the deBroglie-Bohm description of individual quantum particles described as real pilot waves guiding extended particles. [2]

4. Average internal element values in extended elements

In such a model, the corresponding average values within a complete rotation of the extended charge are evidently given by their values for $r = r_m$, *i.e.*, (in its rest frame)

$$\left|E_m^l\right| = \left|E_m^r\right| = \left|H_m\right| = \frac{e}{r_m^2} \tag{32}$$

At each point X_μ there is an equilibrium (zero total values) of the sum of all varying electromagnetic fields. Following a suggestion by Schönfeld [10] one can write

$$m_0 c^2 = \left(1 + \frac{\alpha}{\sqrt{2}} + 2\alpha^2\right) m_m c^2$$

so that $\alpha^- \cong \pi^4 \sqrt{2}\left(m_m / m_0\right)$ is a good approximation when one neglects vacuum fluctuations.

The external magnetic field of electrons and photons is thus not the field of a circular current or a permanent dipole since one needs (FAPP) a point-like electromagnetic charge motion distribution to explain observed scattering average values.*

This implies three consequences.

A. In this type of model one can assume (following Ghosh [8]) that one should add to the distance-dependent gravitational force between two bodies (with masses m_1 and m_2), $F = Gm_1m_2/r^2$, an additional acceleration-dependent term $\Delta\vec{F} = \left(Gm_1m_2/c^2r\right)\vec{a}$, *i.e.*, an inertial induction, where $\vec{a}$ denotes the relative acceleration.† If the Universe is considered isotropic in the large sale, only the acceleration dependent term remains. The action of all external masses is thus given by a relation which expresses the identity between inertial and gravitational masses. This relation (33) does not diverge:
a) if G decays exponentially with distance, and
b) if mass increases with distances.

It is given by

* It is apparently not possible to explain the existence of the transversal electromagnetic field of the electron (for example) by a classical model which includes a static dipole of momentum M_z, and one needs real internal physical motions to explain the observed facts.

† Modification of Coulomb (Newton) forces has been discussed extensively in the literature recently. (Eds.)

$$\vec{F} = \int_0^R \frac{Gm\left(\rho \cdot 4\pi r'^2 dr'\right)}{c^2 r'} dr'$$
$$= \frac{2\pi G \rho R^2}{c^2} \cdot m\vec{a} \tag{33}$$
$$\cong m\vec{a}$$

which yields (if R represents the distance from the particle):

$$G = \frac{Ac^2}{4\pi R}\exp\left(-MR\right)$$

where A ($\cong 1.4\ \mathrm{m}^2\ \mathrm{kg}^{-1}$) is the universal parameter for the interface between the inertial mass and extended gravitational terms.*

B. On the periphery of the circle of rest radius r_m around Y_μ, one can also write.

$$r_m = \frac{\hbar}{m_{mc}} = \frac{\hbar c}{e^2} \cdot \frac{e^2}{m_m} c^2 = \frac{1}{\alpha} \cdot \frac{e^2}{m_m c^2} \tag{33'}$$

i.e.,

$$\alpha = \frac{e^2}{\hbar c} = \frac{e^2}{r_m \cdot m_m c^2} \tag{34}$$

and

$$r_m \cdot m_m c = \hbar \tag{35}$$

During two rotations of the point charge on the limiting rest circle, one has for the magnetic moment of the *Zitterbewegung*

$$\left|M\right| = er_m = \frac{e\hbar}{2m_m c} \tag{35}$$

When one leaves aside the rotation of the point-like charge on itself. This yields

$$m_0 c^2 = m_m c^2 - e\left|A_m^t\right| + \frac{e^2}{m_m c^2}\left|A_m^t\right|^2 + \\ + \frac{1}{8\pi}\int_0^\infty \int_0^\pi \int_0^{2\pi} \left(\varepsilon_V E^2 + m_V H^2\right) r^2 dr \sin\theta d\theta d\phi \tag{35}$$

where the first term is the kinetic energy of the electron, the second term is its potential energy, the third term is the kinetic energy, the fourth term is the total local electromagnetic energy.

The sum of the last three terms is the effective electromagnetic energy $(m_0 - m_m)c^2$.

C. Since particle cores can be represented by bilocal pairs of points Y_μ and X_μ in space-time, one must introduce two types of termperature to

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* This parameter and its value are adopted from H. Broberg, "Mass and Gravitation in a Machian Universe," in: *Mach's principle and the origin of inertia*, M. Sachs and A. Roy, eds., Apeiron (Montreal: 2003).

describe Y_μ motions. For Y_μ the "locking" of internal orbital core rotations (*i.e.*, of their orbital spin) suggests that one must use Boltzmann's formula to connect the variation of the action S and the heat Q_0 of a periodic system characterized by its frequency, $\nu = 1/\tau$, so that in Dirac's thermostat the resonance mechanism of extended oscillation implies that one can write:

$$\delta S = -\tau S Q_0 = -\tau S M_0 c^2 \tag{38}$$

so that

$$\rho \delta S = T \delta S_c \tag{34}$$

where S_c denotes the entropy and T the temperature of the surrounding heat bath. Since internal extended oscillations imply on the average a temperature in a four-volume ΔW, the phase velocity of neighbouring elements, mediated by the pilot wave, implies:

$$m_0 c^2 = h\nu = kT \tag{39}$$

where k is Boltzman's constant.

We thus see that

$$\frac{S_c}{k} = \frac{S}{h} \text{ with } \frac{\delta S}{h} = -\frac{\delta Q_0}{m_0 c^2} \tag{40}$$

In other words, the local variation of the total mass M_0 corresponds to an exchange of heat with the hidden thermostat occurs around the temperature T which corresponds to the resonance energy associated with the equality of the local variation of the resonance energy of the internal core oscillation: the energy present corresponding to part of the local aether energy.

Conclusion

We conclude this preliminary description of some localized extended aether motions with five remarks on the way one can connect internal core motions (*i.e.*, their collective motions) to the corresponding causal stochastic interpretation of Quantum Mechanics to define the gravitational and electromagnetic behaviour.

I. The first remark is on the way one can introduce local internal extended core motions within waves describing associated collective motions. As stated before, one can, following Boltzmann's description of a gas of extended elements, assume that one can introduce at each point Y_μ functions $F(Y_\mu)$ representing local average internal properties within an extended four-volume element ΔW around Y_μ. The choice of ΔW is important, since it determines the choice of the significant internal values and suppresses (through averaging within ΔW) some detailed submicroscopic internal fluctuations of the chosen internal variables. In other words, $F(Y_\mu)$ represents average internal motions within ΔW and around Y_μ, in the corresponding field description, and one should define what terms correspond to internal (within the ΔW centred on Y_μ) and to collective (between different neighbouring ΔW) motions in terms of the $F(Y_\mu)$ field expressions.

II. The second remark is that the terms connecting internal and collective motions should be precisely defined in this type of description, since they correspond to basic assumptions of the corresponding model. For example, in the description of a fluid-built field with extended molecules spinning on themselves one must distinguish within a global field description individual spin motions from global, local vortex motions, and one should define separately their corresponding laws of motion.

III. This model also implies some consequences which can be tested experimentally. If we follow a particle-like extended element moving in a straight line, *i.e.*, piloted by a small distribution of plane collective comoving waves, the particle and the plane-wave's piloting individual elements will interact with the background aether (a friction-type process) so that their velocity will decrease, along their paths, through elastic or inelastic collisions. This explains, in the individual particle and pilot wave elements, the observed friction (redshift) effect. If one assumes that during one frequency beat of the observed moving particle, it absorbs $N(1,2,N,N)$ vacuum objects in phase with a factor $1/N$. The cross section for a particle moving at velocity $\vec{\upsilon}$ (*i.e.*, with mass $m_p = m_\gamma \Big/ \sqrt{1-\upsilon^2/c^2}$ can thus be written $\sigma_p = A_p m_p$, where A_p in the case of the photon varies slowly with υ. Denoting the momentum in the Σ_0 frame as $p(\upsilon) = m_p \upsilon$, for two bodies interacting in Σ_0 the forces are given by:

$$\begin{cases} \vec{F}_{21} - \phi m_{g1} \vec{a}_{1f} = 0 \\ \vec{F}_{12} - \phi m_{g2} \vec{a}_{2f} = 0 \end{cases} \tag{13}$$

IV Assuming Weber's proposal [9] (equivalent in this case to Lorentz's assumptions), one finds that Newton's laws should be modified. If the kinetic energy of extended structures is given in S0 by $m_0 c^2 \Big/ \sqrt{1-\upsilon^2/c^2} - 1$ instead of $m\upsilon^2/2$, we get

$$U_{12} = -3G\frac{m_{g1}m_{g2}}{r_{12}} + 2G\frac{m_{g1}m_{g2}}{r_{12}}\frac{1}{\left(1-r_{12}^2/c^2\right)^{3/2}} \tag{14}$$

the last term representing a gravitational potential energy. One then recovers Newtonian mechanics in the case of circular orbits around the centre of mass if one has

$$\frac{1}{4\pi}H_0^2 = -\rho_0 G \tag{14}$$

a relation confirmed by observational values of the independent measured quantities H_0, ρ_0 and G.

V. The last remark is on the observed experimental evidence of the permanent presence of the aether in Newton's bucket. If we change instantaneously the angular velocity of such a Newton bucket (or the period of a pendulum), one observes an instantaneous variation of the water's curvature and of the pendulum's frequency, which implies the permanent local character of Newton's faster-than-light gravitational field.

Of course one can add to these new internal oscillations other types of periodic internal motions which will introduce new possible types of interactions associated with new vectors describing internal rotations. Some of them will be discussed in a subsequent paper. Let us just mention here that since the velocity of X_μ is $\cong c$, only two such vectors (*i.e.*, $Y_\mu = X_\mu \cong B_\mu^{1,2}$) are likely to appear in this model, both associated with higher internal mass and quantum numbers of the corresponding intermediate vector bosons. In other words, if one limits oneself to the three-dimensional character of $Y_\mu - X_\mu$ and to the flattened character of the electric charge in nucleons, one is (FAPP) limited to two orthogonal type four-dimensional internal oscillations of $Y_\mu - X_\mu$, both associated with intermediate vector bosons. In other words, this model leads to the prediction of a future discovery of new stronger vector interactions, which mediate observable periodic internal oscillations of X_μ around Y_μ in space-time.

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The Ether Concept in Modern Physics

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The ether in modern physics interprets the formal structure of Relativity (Special and General), and suggests ways of unifying Relativity with Quantum Mechanics. The modern ether is a relativistic medium, compatible with geometrical, non-classical formulations of physics. It serves as a disclosing model, indicating the relationship between quasi-classical Poincare-Lorentz Relativity—couched in terms of a Lorentzian ether—and Relativity expressed in the geometrized tradition established by Einstein and Minkowski. Poincare-Lorentz Relativity is a limited sub-group of interpretations in a wider class of Einstein-Minkowski expressions. The vortex-sponge mechanical analogue, which removes long-standing methodological objections to Poincare-Lorentz Relativity, is the most promising analogue. When geometrized it provides an equivalent to Einstein's space-time of General Relativity. The vortex-sponge resembles a Dirac ether. On the smallest scale it can be regarded as a chaotic medium, generating phenomena interpreted by dynamic algebras, or nilpotent theories. Use of the concept does not imply an adverse attitude to the Einstein-Minkowski tradition, and the long-standing ether–relativity polemic is a sterile misconception.

The "Ether Question" 1905-2005

The ether in 21st C physics is a continuum theory, generally non-classical, which interprets fundamental activity in terms of space-time geometry or action in a medium. For full development, it requires a mechanical analogue to supplement the geometrical interpretation. There are too many particular ether theories to review. This paper concentrates on the comprehensive ether theories which cover special and general relativity, and which promise to unify relativity with quantum theory, electrodynamics and cosmology. The modern ether is the consequence of three development programmes, two associated with Relativity, one with Quantum Mechanics. In the context of Relativity, one is the Poincare-Lorentz programme; the other is the Einstein-Minkowski programme (Prokhovnik, 1973b, Zahar, 1973). Both interpret the accepted formal structure of relativity, and are practically indistinguishable. Once regarded as mutually exclusive ways of interpreting relativity, they are now regarded as alternative ways of interpreting the formal structure which can be "mapped" or translated into each other Philosophy and history of science are needed to correct misconceptions about ether and relativity which originated between 1910-1920 and which still confuse the "ether question." A small number of ether theorists link the term to an anti-Einstein, anti-relativity polemic and thereby discredit its use, but modern ether theory builds on the positive achievements of Einstein's relativity, Minkowski's geometry, non-classical physics, multi-dimensional geometrization, geometrodynamics, and relativistic cosmology. The ether-versus-relativity polemic (Gratzer, 2000) limited acceptance of the ether concept in modern physics. The rise of Einstein's Special Relativity, with its abandonment of the then-prevailing notion of ether, caused the concept to be set aside as redundant (Hirosige, 1967). The rise of Einstein's General Relativity, with its transformation of long-standing concepts of space and time, suggested that the classical Newtonian ether was incapable of interpreting 20th C physics. The legitimacy of the ether concept has been an enduring question in physics since 1920. Many misconceptions abound concerning "absolute" and "relative" definitions of ether, and its many roles. The ether concept has a complex history (Cantor & Hodge, 1981; Psillos, 1992; Whittaker, 1953), as have other fundamental terms like mass, space, time, energy. Too many ether theorists fail to clarify which are the essential features of an ether, and whether the several kinds of ether, found in contemporary physics, are radically different from each other, or whether they are all aspects of one fundamental medium. In the early 19th C, ether was a subtle medium made up of fine-scale matter, but this gave way to ether as a non-ponderable medium in which matter was a configuration. The evolving concept of modern ether, and the new concept of the electron played major roles in the evolution of early relativity between 1890-1910 (Goldberg, 1969; McCormmach, 1970). The main early expositions of special relativity were Poincare-Lorentz relativity using a classical ether, and Einstein-Minkowski relativity expressed in terms of geometrized Space-Time. The Poincare-Lorentz exposition of relativity has continued to this day as a minority programme, and much modern ether theory has come out of it (Prokhovnik, 1973b). Because of this there is a tendency to associate the term ether with a narrow definition (absolutist, classical, Newtonian) linked to the Poincare-Larmor-Lorentz programme, which was developed by Ives, Builder, Prokhovnik, *et al.* Most unfortunate was the association of the ether with a polemicists' campaign against Einstein's Relativity, the geometrization of physics, and the use of non-Euclidean geometry (Gratzer, 2000). This polemicists' campaign has, at various times, been linked to anti-semitism, and pseudo-science made to serve political, metaphysical and theological interests (Craig, 2000; Lodge, 1933; Turner & Hazelett, 1979). This polemic, continued by an active minority, brought the concept into disrepute, and keeps alive misconceptions about ether which are completely unjustified. The other great source of modern ether theory is quantum theory (see below).

Ether, Geometrized Physics & Non-Euclidean Space-Time

The geometrizing of physics, associated with the Einstein- Minkowski exposition of special relativity took place for good reasons. Unresolved difficulties faced the Lorentz theory of electrons, the electromagnetic world view, and the undetected ether. Failure to provide a mechanical interpretation of the ether robbed matter (and thereby instruments) of any satisfying mechanical underpinning at a time when matter was increasingly thought of as an ethereal state. The conceptual and methodological impasse was overcome when Minkowski geometrized the Lorentz theory of electrons, and fused it with Einstein's early special relativity (Minkowski, 1908). Einstein later developed the "Einstein-Minkowski" geometrized exposition to interpet gravitation, using non-Euclidean geometry (Einstein, 1914, 1922). Non-classical general relativity, and relativistic cosmology, became exemplary physical theories. Geometrized physics became normative after the success of General Relativity in the 1920s, and led to the later development of Geometrodynamics (Graves, 1969). Later, interpretations of general relativity and cosmology were devized in classical terms, using the evolved Poincare-Lorentz programme, but these followed Einstein, and reinterpreted what he did (Kox, 1988). They could not replace him, let alone establish that his methods were in error. In fact, the ether was recast in relativistic terms, though the Poincare-Lorentz classical ether remained as an optional element in a quasi-classical subgroup within a larger body of relativistic ethers.

The major concepts of ether developed within two main classes, though there were many other types. One main class contains ethers defined as "Space with Physical Properties" (Einstein,1920; Kostro, 2000; Whittaker, 1953), or "Ether as Field" (Dirac, 1951, 1954; De Haas, 2004). These were compatible with general relativity and non-classical theories. They enjoyed a long history, and in the 19th C were discussed by Clifford, Riemann and Pearson. A second main class includes the classical ethers, serving to provide a background Euclidean space in which Newtonian absolute time prevailed, and energy and momentum were conserved (Erlichson, 1973). Because they interpret relativistic effects from a classical base, they are termed "quasi-classical" or "pseudo-classical" theories. This is the ether of Poincare-Lorentz Relativity, as presented by Builder (1958a, 1958b) Ives (Turner & Hazelett, 1979), and Prokhovnik (1967, 1973a, 1973b). Generally, this ether lacked a mechanical analogue, which was essential for a stronger conceptual and methodological foundation. Failure to detect this ether by universally repeated experiment remains a major conceptual flaw in these Lorentzian theories because their most vital feature remains undetected by science. Between 1920 and 1950 there emerged a transformed ether theory out of which came the ether of present day physics. The Lorentz Theory of Electrons gave birth to the exposition of special relativity in terms of observations conducted with rods and clocks, subjected to specific synchronization techniques and slow instrument transport velocities. This Rod-Contraction; Clock-Retardation Ether Theory (Erlichson, 1973) is associated with Broad (1923), Builder (1958), Ives (Turner & Hazelett, 1979), Janossy (1971), Larmor (1900), Levy (1996, 2002), Lorentz (1915), Mansouri and Sexl (1977), Prokhovnik (1967, 1973), and many present day advocates of a Poincare-Lorentz interpretation of the agreed relativistic formal structure. A comprehensive, non-ad hoc derivation of Special Relativity was established and extended to cover General Relativity and Cosmology. The following four papers, taken together, typify Lorentzian general relativity in the 1960s and 1970s (d'E Atkinson, 1963; Clube, 1977; Cornish, 1963; Rongved, 1966). These quasi-classical interpretations of general relativity and cosmology were openly recognized by eminent relativists like Eddington as valid, consistent alternative presentations of relativistic physics but there was no pressing reason why the geometrized, non-classical Einstein formulation should not remain the norm (Rindler, 2001) Einstein's theory was better understood and was conceptually and methodologically superior.

Hartley, Kelly & the Vortex Sponge: The Kelvin-Larmor Dynamic Ether Analogue

In the 1950s, Hartley, a colleague of Ives at Bell Laboratories, developed the Kelvin-Larmor vortex-sponge mechanical ether analogue to provide a new dynamic interpretation of matter, ether, space and time (Hartley, 1950a, 1950b, 1957, 1959). He used the time dependency of its rotational stiffness to set up "wave-particles" representative of matter, exploiting a property not recognized by previous ether modellers. This modified vortex-sponge was developed in later years, to the present time, by E M Kelly (1963, 1964, 1976, 1990, 1996, 2003) Dmitriyev (1992, 1993a, 1993b, 1998, 1999) Winterberg (1992, 2002) *et al.* to interpret quantum mechanical and other fundamental phenomena. The vortex-sponge provides a mechanical analogue which underpins the Poincare-Lorentz theories and removes conceptual and methodological weaknesses by remechanizing the Lorentzian world-view (Duffy, 1978, 2004). If this is not done, geometrization remains the obvious way of expressing relativity (Einstein, 1920, 1922). The mechanism was imagined as a classical array of gyrostats held in a frictionless framework, which could take different equivalent forms, including hydrodynamical analogues (Hartley, 1950a, 1950b, 1957; Lorentz, 1927). The vortex-sponge put a classical rod and clock at each point in a classical, Euclidean space, with Newtonian clock time regulated throughout. The passive, featureless Lorentzian ether gave way to a dynamical modified Kelvin-Larmor ether which filled or defined space and time. Material particles were represented as spherical standing waves, in a random atmosphere of vortex rings. These wave-particles are equivalent to miniature Langevin clocks, idealized interferometers, and the combined rods-and-clocks used by Builder (1958), Prokhovnik (1967, 1973a, 1973b), Jennison (1978, 1986, 1988) and Clube (1977, 2002) to extend the Poincare-Lorentz ether-based theories to cosmology, and to set up a space-time metric. A relatively "large-scale; long-time" view of phenomena, in which the wave-particles retained their identity, provides a complete interpretation in Lorentzian terms of special and general relativity and much of cosmology. The imaginary surveying operations and measurements with rods and clocks can be referred to an ultimate set of standards motionless in the urether which define a Lorentzian background frame.

However, a non-Lorentzian interpretation was equally available (see below) following Einstein and Minkowski.

If a view is taken of phenomena over much smaller length scales, and shorter time intervals, the instruments become disordered by the turbulence of the vortex-sponge, and wave-particles used as ultimate measuring instruments encounter intrinsic lower limits to meaningful measurement of spatial intervals and clock-time ordering. There are random dislocations of measuring rods, and clock action. Instrument activity becomes discontinuous. The dynamic pulsations of the vortex-sponge are described in terms of higher-order geometries and the classical scheme is no longer adequate (Duffy, 1979). Energy exchange between wave-particle and ether is in fixed units or quanta, and a way is suggested of incorporating quantum mechanics and relativity into the same analogue. At the present time, V Dimitiyev, E M Kelly, and F Winterberg continue to extend the range of the vortex sponge. Donnelly (University of Oregon) has produced laboratory-scale vortex-sponges using supercooled liquids, which exhibit quantum mechanical behaviour (Donnelly, 1988; Donnelly & Swanson, 1986), and this research has provided a growing body of vortex-sponge theory applied in aeronautics, hydrodynamics, meteorology, and cryogenics (Mantarrese 1985; Schwartz, 1988; Tough, 1987). It is a quantum-mechanical medium, able to bear wave-particles which obey the principles of general relativity (Volovik, 1998). It discloses relationships between the Poincare-Lorentz classical exposition of relativity, and the normative Einstein geometrical exposition which shows them to be aspects of the same theory, rather than mutually exclusive rivals.

Einstein's Ether & the Vortex-Sponge

Space and time measurements using a wave-particle as a combined rod-and-clock, in a vortex-sponge ether, in a gravitational field, as viewed from different platforms, gives the familiar "Einstein-General Relativity" relation for the space-time interval. Ives (1939b) developed an equivalent relation (the "chronotopic interval") within the context of Poincare-Lorentz Relativity, without using a mechanical analogue. This relation can be given a classical, or a non-classical interpretation. Definitions concerning basic rods and clocks (rigid or distortable?) determine whether or not the chronotopic interval is given a classical interpretation (referred to a background Lorentzian frame defined by classically rigid rods and clocks) or a non-classical formulation, in terms of non-Euclidean space-time geometry in the gravitational case. Both space-time geometries are practically equivalent. One can map back and forth between them. They are the space-time counterpart of two maps, drawn to different projections, used for navigating between two places on earth. Each map projection (e.g. Mercator's, or the "equal-area" projection, or various cylindrical, conical or polar projections) has its own convention for calculating direction and distance at each latitude, because the metric scales differ. A navigator uses the simplest projection or geometry adequate for the task facing him. The appropriate geometry might be decided by the coarse or fine scale nature of the job. No projection or geometry is more natural, or more real than any other, because the different metrics are not possessed by the earth or object surveyed– they are conventions for enabling direction and distance to be computed. This is exactly the same case with the quasi-classical and the non-classical geometries, projections and metrics by which the Poincare-Lorentz and the Einstein-Minkowski programmes survey and compute space-time intervals associated with the same physical problem.

The vortex-sponge, originally presented as a classical mechanism of gyrostats and linkages, or a hydrodynamical equivalent, can be geometrized and interpreted in non-classical terms. Kron's geometrization of any mechanical and electromagnetic ensemble shows a general technique for achieving this (Kron, 1934, 1938, 1963; Hoffman, 1955). The vortex-sponge geometrized is a world-ether, equivalent to the space-time of Einstein's geometrized general relativity, and possessing the characteristics of the fundamental plenum of geometrodynamics depending on whether a coarse or fine scale view is taken. The "static" or geometric presentation of space-time, can be split into the "Frame-Space" perspectives in which laboratory measurements take place. Measurements in these laboratory frames can be correlated and interpreted using the non-classical Einstein-Minkowski tradition, or the Poincare-Lorentz pseudo-classical tradition. It is a matter of choice and convention in defining ultimate measuring units as Ives realized, though he preferred the Lorentzian interpretation.

Kostro's recent research (2000) into Einstein's later writings identifies a concept equivalent to ether, which Kostro terms "Einstein's Ether" This is an example of "Ether as Field," or "Ether as Space-Time," proposed after 1920 by engineers and physicists (Dirac, 1951, 1954; Einstein, 1920; Whittaker, 1953). Kostro refers to a dynamic and a static image. In the dynamic image, the motion of reference spaces is studied in the (clock) time of the laboratory reference frames. In this frame space perspective, the position of the reference spaces changes in time. The vortex-sponge, represented by a classical array of gyrostats or the hydrodynamical equivalents, can model frame-space phenomena and it provides an underpinning for the Poincare-Lorentz ether theory. The static or geometric image presents space and time fused together into the space-time continuum, and the reference space-times are composed of world-lines and instantaneous spaces. Kostro argues that this relativistic ether, or world ether, is not composed of world-lines or instantaneous spaces, but is a four-dimensional continuum made up of events. Geometrizing the wave-particle of the vortex-sponge into an event-particle and surveying space-time with it, leads to the same result. Einstein's Ether, and the geometrized vortex-sponge "World-Ether" are the same. Modern ether theory is not incompatible with Einstein's Relativity, or developments such as Geometrodynamics, which deal with a fine-scale perspective. The misconceived "ether versus relativity" and "Lorentz versus Einstein" polemics, have delayed a fruitful unification of two main programmes in Relativity, that of Einstein and Minkowski, and that of Poincare and Lorentz (Prokhovnik, 1973b, Zahar, 1973). They have hindered the subsequent development of a unified relativistic ether theory, and associated the term “ether” with archaic ideas and naïve Realism.

At present there has been no unequivocal experimental distinction drawn between these two programmes, which have practically the same formal structure. Undisputed detection of ether drift would favour the Poincare-Lorentz programme in the context of relativity, but would by no means destroy the validity of geometrized, non-classical relativity. Present day ether drift experiments have not yet amassed sufficient evidence to favour the claims of the Poincare-Lorentz exposition, despite careful and persistent work carried out with modified Michelson-Morley apparatus, double Fizeau toothed wheels, and the Sagnac apparatus (Selleri, 1998). Individual experiments may raise a question mark, but these tests need to be repeated many times, by disinterested experimenters, to separate misinterpreted results (given wide publicity by over-enthusiastic polemicists) from the genuine observations of colleagues whose work deserves consideration. Sometimes unsubstantiated claims that ether drift has been detected are acclaimed uncritically by ether theorists looking for supportive experimental evidence. Over-eagerness to accept the results of single experimenters, working unseen, based on relatively few tests, is a betrayal of scientific caution and has done much to bring the ether hypothesis into still deeper disrepute. It will require hundreds of undisputed detections of ether drift, carried out by impartial investigators in first class laboratories, all over the world, with impartial witnesses, and publication of meticulous records, before the normative status of Einstein’s relativity is called in question. The question of drift is vital. It must be addressed—but it is up to ether theorists to beware the unjustified claims of anti-relativity polemicists if they are to win a fair hearing for their ideas.

Ether, Cosmology & Gravitation

Einstein’s relativity became the norm when it was extended from the Special Theory to cover gravitation and relativistic cosmology (Eddington, 1920; Einstein, 1914). Any comprehensive ether theory must cover the same ground, and a considerable body of present-day ether theory does so. The most important group is expressed in terms of Poincare-Lorentz theory, using the techniques of the rod-contraction; clock-retardation exposition. S. J. Prokhovnik (1973b, 1985, 1990) interprets cosmology using Poincare-Lorentz relativity, with an ether as reference frame, and taking rod-contraction and clock-retardation as real phenomena. Prokhovnik does not advocate ether as a hidden mechanism, but he recommends the absolute reference frame of Poincare and Lorentz, and he uses the “instrument transport” techniques of Langevin, Ives and Builder for synchronizing clocks in inertial frames. Prokhovnik (1990) stresses the causal significance of absolute velocities, and defines absolute as “relative to the universe.” He treats the galaxies as the fundamental “particles” of the universe at large, which define an expanding frame filled with background radiation. Distribution of "particles" is seen as homogeneous by an observer on any one galaxy-particle. The universe expands according to Hubble's law, and the cosmological principle holds. Prokhovnik shows that the Robertson-Walker metric applies and defines a unique, observable, cosmological reference frame in which light is propagated in all directions with a speed always measured as "c." In thought experiments one can refer to non-isotropy in the speed of light with respect to a moving body in the frame, but this is not observable because rod contractions and clock retardations make the as-measured speed "c." This is a common feature of the entire group of Poincare-Lorentz ether theories, and is the source of much criticism. Prokhovnik claims that astronomy has revealed a unique, fundamental frame, within which moving bodies are effected by motion. Velocity with respect to this expanding reference frame can be estimated from the 2.7K microwave background radiation. The expansion of the frame provides a measure of cosmic time, which advocates claim enables a clear, paradox-free exposition of relativity to be presented. The work of Prokhovnik and Builder has been developed further by Paparadopoulos. Wegener (2000) argues that the "spray substratum" of Milne meets the requirements set out by Builder and Prokhovnik for a universal reference frame.

Clube (1977, 2002) presents theories of gravity and cosmology, along Poincare-Lorentz lines, using an ether described as a superfluid or material vacuum. Clube develops the de Sitter-Atkinson gravitational theory as an approximation to a more fundamental Lorentz-Dicke gravitational theory. It models the production of particle pairs in the physical vacuum (ether) and relates a wide range of astrophysical and cosmological phenomena within a Lorentzian theory of gravity, couched in terms of a static model of the universe. Its chief features include redshift arising from vacuum processes; baryonic and non-baryonic matter formation in the ether; and the suggestion that phase-locking of fundamental particles may involve a principle more important than relativity. It should be compared with the work of Surdin.

The theories of Arminjon, Broekaert, Podlaha and Sjodin form a consistent group, which can be related to work by Builder, Clube, Cornish, d'Atkinson, Ives, Prokhovnik, Rongved, Roscoe, and Surdin. The vortex-sponge analogue, though not forming an essential part of these theories, can provide a compatible mechanical underpinning. Arminjon's work typifies this school of thought. Gravitation is modelled after Euler as an Archimidean thrust in a fluid ether. Particles of ponderable matter are localised flows in the ether, and creation and annihilation is represented in fluid terms—an idea, developed to a high order by Karl Pearson and Schuster in the 1880s. Gravitation is a "smoothed-out" macro-force, in which ether pressure plays a crucial role. This is compatible with Hartley's vortex-sponge interpretation. The ether fills the homogenous space of special relativity, and the effects are interpreted by rod contractions and "Larmor clock slowing." Gravitation is due to an apparent variation in ether density, or heterogeneity of space, with gravitational rod-contraction and clock slowing. These theories constitute a major development of the Poincare-Fitzgerald-Larmor-Lorentz-Ives programme. Interpretations of space-time employ two metrics. There is a flat background metric (against which rod-contractions and clock slowings are defined), and a physical metric, which is curved in the gravitational case. This is the fundamental characteristic of the Poincare-Lorentz programme, which has the “Ives Group” of theories as perhaps its most representative aspect. The ether is identified with the background metric. The “uncorrected” readings of rods and clocks defines the “physical metric.” The quasi-classical expositions of relativity (d’Atkinson, Cornish, Clube, Rongved) refer all instrument readings back to the ether. This is a legitimate way of interpreting the relativistic formal structure which a minority prefer to use. The majority argue that until the background ether is detected by universally repeated experiment in a clear and undisputed manner, the instrument readings should be taken as “read-off,” and not reduced into an undetected ether-state. This is the orthodox relativistic position, which is superior on methodological grounds. The two approaches are equivalent. One can map from one to the other. They are by no means mutually exclusive. Ives’ “chronotopic interval” paper (1939b) is worth consulting in this respect.

Arminjon and his colleagues explore various options for developing these theories. Rod contraction can be treated as anisotropic, lying along the ether pressure gradient and line of gravitational acceleration (Arminjon), or as being isotropic (Podlaha and Sjodin). The anisotropic assumption gives the Schwarzschild exterior metric in the static case with spherical symmetry, giving the same observable results (light ray behaviour) as does general relativity starting with the Schwarzschild metric. Problems are encountered with the weak equivalence principle, but Arminjon argues that these will occur with general relativity also, with anisotropic metrics. These problems are not encountered with the isotropic case.

Broekaert offers an alternative scalar interpretation of general relativity, following geometrical conventions introduced by Poincare, and presenting gravitationally modifed Lorentz transforms. Depending on which isotropic scaling functions are applied, one set of Lorentz transforms distinguish between a “natural geometry” which is affected by gravitation, and a co-ordinate geometry which is not affected by gravitation. A spatially varied speed of light is suggested. Different scaling functions give a different set of transforms giving the invariant (locally observed) speed of light and the local Minkowski metric. This group of theories contains work by Sjodin on gravitation and determination of one-way velocity of light; and by Podlaha, which presents a comprehensive ether model of the physical vacuum and wave-particle which is similar in many ways to the vortex-sponge and wave-particles of Hartley.

Grand Comprehensive Theories

Several comprehensive ether theories cover a wide range of phenomena from the quantum to the cosmic scale. Examples include the theories of Cavalleri

(2002), Dimitriyev (1992, 1998), and Winterberg (2002). In a brief review all that can be done is to summarize a few major features and direct the reader to the references.

G. Cavalleri has developed a comprehensive ether theory based on a stochastic medium made up of vortex elements for interpreting QED and a wide range of electronic and electromagnetic phenomena. Cavalleri proposes that the zero point field of QED is caused by the classical EM radiation of all the particles in the universe, emitted since the "big bang," and that it may be regarded as a real ether. Motion of rods and clocks through this ether produces the length contraction and clock retardation of the Poincare-Lorentz or Ives group of theories. Cavalleri shows that the power spectral density is Lorentz invariant, and that experiments of the Michelson-Morley type, and their equivalent, cannot detect ether drift. However, if the power spectral density is limited to prevent infinite energy density in a zero point field regarded as real, the relativistic invariance is lost, and there should be a privileged observer for which the zero point field ether is isotropic. Cavalleri suggests that this ether might be detected by accelerating a charged hydrogen atom in a synchrotron. An energy of 20Tev would be needed to detect an effect interpreted as "friction in vacuo." Cavalleri, like Winterberg, rejects the metaphysics of the Copenhagen school, and aims for realism. He regards the zitterbewegung of Schroedinger as a real phenomenon located in the physical vacuum and not an illusory effect due to uncertainty. He regards the zero point field as the producer of fluctuations in velocity direction which amplify fluctuations in electron position, and generate quantum mechanical effects. His ether is a space filling stochastic medium reacting with matter within the framework of Poincare-Lorentz relativity.

V.P.Dmitriyev proposes a comprehensive theory covering a wide range of fundamental physical phenomena from quantum mechanics to general relativity based on mechanical ether analogues developed from the models of Kelvin, Hicks, McCullogh and Larmor (Lorentz, 1927). He shows that solid, liquid and mechanical ether analogues are equivalent, and unusually presents many of his findings in terms of a solid elastic continuum analogue (Dmitriyev, 1992, 1993a). Much of his work is also expressed using the vortex-sponge (Dmitriyev, 1993b). Dmitriyev remarks that the Yang-Mills theory of physical fields can be used to create a complete theory in terms of a solid body with singularities, but he reverses this procedure to devize a theory of physical fields and particles using an elastic solid ether. There is no empty space. All space is occupied by physical vacuum or ether through which all interactions are transmitted, including electromagnetic waves and gravity waves. Beginning with a linear-elastic substratum, Dmitriyev shows that this is Lorentz-invariant because it is practically incompressible, and that the universe is practically static. Material particles are approximately modelled by localised energetic excitations (solitons) in the ether, which ideally should be modelled in non-linear terms. Models of quantum particles, gravitation and general relativity are provided. General relativity is treated as a sub-algebra in the non-linear theory of elasticity. The solid continuum model, provided with internal rotation is equivalent to the vortex-sponge, which Dmitriyev uses, along with dipole models, to interpet microphenomena and asymmetry in the macroscopic world. As with Winterberg's model, Dmitriyev's system is hierarchical, having six "levels" of phenomena, which are classical mechanics; general relativity and quantum mechanics; the solid substratum; the solid substratum with internal rotation; the vortex-sponge regarded as chaotic turbulence in the primary medium; and the primary medium. It would be better if the whole picture could be expressed in terms of the vortex-sponge alone.

F. Winterberg , like most modern ether theorists, accepts the formal structure of relativity but is critical of attempts to develop the geometrical theories by adding dimensions of space-time, starting with the adding of a fifth dimension by Kaluza and Klein to unify gravity and electromagnetism, and ending with the multidimensional space-times of superstring theory. He believes (as do many supporters of Poincare-Lorentz ether theory) that physical reality is 4-d space-time which must be made the foundation of any theory which avoids "physical impossibilities or absurdities" like infinite stresses in zero diameter strings. Winterberg works within the Poincare-Lorentz framework, with rod-contraction and clock retardation being real phenomena caused by motion through a substratum. Insisting that all modern ether theory must be within a quantum mechanical framework, Winterberg (2002) models the substratum as a superfluid ether full of quantized vortices. The superfluid ether is made up of an equal number of positive and negative masses called Planckions, densely packed together which preserve the zero-point energy fluctuations of the physical vacuum, but make the average vanish. The analogue interprets vector gauge bosons, charge and charge quantization; special relativity as a dynamic symmetry; gauge invariance; Dirac spinors; and elementary particle mass as a function of Planck mass. For many of its functions, Winterberg's ether resembles the vortex-sponge. Indeed in considering its energy spectrum, Winterberg used results from liquid helium theory (phonon-roton structure) presented by Feynman in 1954, and supported by recent laboratory investigations on vortex-sponges using low temperature helium (Donnelly & Swanson, 1986; Volovik, 1998).

Winterberg's model is hierarchical. Elementary particles are bound states in the superfluid quantum mechanical ether. The model provides a classical description of Schroedinger's zitterbewegung derived from Dirac's equation. A quantum mechanical interpretation is obtained compatible with relativity. The model analyses wave function collapse in a "realistic, objective" manner, rejecting the Copenhagen interpretation. Like Prokhovnik, Winterberg regards very large systems of galaxies as defining a privileged frame of reference at rest in the ether. This ether is Heisenberg's fundamental field, admitting wave modes of superluminal velocity. Winterberg argues that an absolute space-time structure allows for a realistic interpretation of wave-function collapse. He regards the Minkowski space-time continuum, and the Riemannian manifold, as illusions caused by true physical distortions which should be interpreted in terms of Poincare-Lorentz theory using a real, absolute ether. This reflects the attitude of many ether theorists.

Dirac Ether & the Physical Vacuum

An ether was proposed by Dirac to unify electrodynamics and quantum mechanics in a manner different from that found in the quantum electrodynamics programme (Dirac (1951, 1954). E.M. Kelly (1963, 1964, 1976, 1990, 1996, 2003) used the vortex-sponge to effect this unification, but others have tried to develop Dirac's ether, to achieve the same result, without reference to the dynamical analogue. De Haas's recent efforts are important contributions (De Haas, 2004). De Haas defines Dirac's ether as a revived Maxwell's ether, which requires that the non-gauge invariant stress-energy tensor has non-zero, non-symmetric magnitude in space-time. If so, the Lorentz force and Poincare force can be obtained, with Poincare force represented by a translation pressure, or "something rotating in the Maxwellian operational ether." (In the vortex-sponge, these are forces resulting from ether pressure due to the atmosphere of fine-scale vortex rings between the wave-particles). Dirac regarded electric potential and velocity field as "physically real." Maxwell's operational ether required that its electromagnetic stress-tensor was zero, and it was overtaken as a concept by Lorentz's ether, in the early days of relativity which was defined in terms of motion (special relativity—not the accelerations of the general theory) and not by the "real" existence of Maxwellian ether-stresses. In the Maxwellian ether, magentism was due to inner rotations of space; in Einstein's space-time (general relativity) rotations were replaced by geodesic movements in curved space, with the gravitational part of the stress-energy tensor being zero. A minority of physicists continued to develop the Maxwellian approach. The vortex-sponge might effect a reconciliation of these several approaches (Einstein, Lorentz, Maxwell). De Haas accepts that general relativity allows for an operational ether, though Einstein never incorporated electrodynamics or quantum mechanics into general relativity. The operational ether of Dirac, though a "real, physical ether" with charge flow velocity being an ether velocity, is not a substantial ether fixed to absolute space. It must be distinguished from an "Aristotelian substance connected to Euclidean, absolute space"—which is the obsolete ether concept of so many anti-relativity, anti-Einstein "dissidents" who advocate it to the detriment of ether theory in general. De Haas identifies the problem central to modern Dirac ether theory as being formulation of the stress-energy tensor. Maxwell interpreted magnetic stress as arising from rotation in the ether, but Einstein-Minkowski flat space time (special relativity) and the curved space-time of general relativity contain no rotations by definition, and their spaces and time are orthogonal. Dirac's ether can contain non-orthogonalities in space and time, as required by Maxwellian electromagentism, by using an antisymmetric tensor or 6-vector following a path indicated by Minkowski. Developing a 6-vector Lorentz Transformation matrix yields a 6-d electromagnetic space-time additional to 4-d Einstein-Minkowski space-time. De Haas points out that Dirac explored a Poincare-Lorentz programme (spherical electron held together with ether pressure), and an Einstein-Minkowski programme (point charge without extension; no ether pressure; an ether free metric) before cultivating the neo-Maxwellian ether of his later years. Referring to von Laue' work unifying Newtonian mechanics with special relativity (1911) and Minkowski's work of fusing Maxwell's electrodynamics and special relativity (1908), De Haas presents his own development of a Dirac ether which all contemporary ether theorists should consider.

Another promising line of development is indicated by S Bell, who quantizes general relativity using quantum electrodynamics in an analysis which (like that of Carroll) draws on techniques from information science, computing, systems processing and signal analysis. She shows how inverse square law of force, and Bohr's quantization of angular momentum can be derived from vortex theory and special relativity. Though she does not use the ether concept, this work, together with that of Carroll (2002, 2004) and Rowlands (1996, 2003), shows how theories of the physical vacuum (or ether) might be greatly extended and unification achieved.

Cavalleri, Puthoff, Winterberg and Surdin have developed comprehensive theories of the physical vacuum, based on the zero point field, and related them to large and small scale phenomena. Cavalleri's and Winterberg's work has been summarized above. Surdin assumes that all the laws of physics originate in the ZPF and he sought the cosmology which best suited stochastic electrodynamics. Early stochastic electrodynamics used steady-state cosmology and was criticised because it could not explain 2.7K background radiation. Surdin's ambitious programme sought to use a classical model of the physical world, to show that quantum phenomena are a consequence of stochastic electrodynamics, to obtain a cosmology compatible with the large numbers hypothesis, to relate gravitational and electromagnetic forces, and to explain cosmic background radiation and other cosmological effects. Surdin employs Newtonian Mechanics to derive the well-known general relativity effects of gravitational redshift, perihelion advance, and bending of light by massive bodies. He takes the same quasi-classical, Lorentzian approach found in Ives, Builder, Prokhovnik and Clube. In his later work, Surdin attempts to unify general relativity with stochastic electrodynamics and develops a steady-state cosmology with an expanding closed universe, with matter density constant. He proposes a mechanism for particle creation. He argues that the existence of a real zero point field or ether overcomes objections levelled against standard steady-state theory because it accounts for cosmic background radiation and continuous creation of matter. Surdin provides an example of a comprehensive theory, encompassing cosmology, general relativity, particle creation, electrodynamics and microphenomena. Puthoff is another example of a theorist who treats the zero point field as a dynamic ether in which gravity is an induced effect caused by the field being loaded by large scale ponderable matter. These theories based on the ether as zero point field or physical vacuum take the ether concept beyond the point reached by Lorentz, Ives, Builder and Prokhovnik by giving it a dynamical character. There are considerable differences between the various comprehensive theories and more effort should be directed to synthesising a unified theory from them. The Poincare-Lorentz programme is the foundation of most of these theories.

Ether & Matter

The structure of material particles and the mechanism of particle creation are vital elements in comprehensive ether theories because fundamental particles provide the ultimate combined-rod-and-clock systems for surveying space-time. The wave-particle in Hartley's vortex-sponge is one example. Similar wave particles are suggested by Podlaha who models ponderable matter as being made up of spherical standing waves, sent out from a centre, reflected at an envelope, and returned simultaneously to centre (Podlaha, 1977, 1979, 1999; Podlaha & Sjodin, 1984). These can be treated as "idealized interferometers" and used to set up a Poincare-Lorentz interpretation of special and general relativity. Podlaha postulates the existence of matter waves of another kind, which spread with the velocity of light and are essential for particle stability, possibly playing the same role as the vortex-ring atmosphere in the vortex-sponge which holds the wave-particle together by ether pressure. Jennison's theories of material particles is based on theoretical and experimental work at Canterbury, where he created an imaginary fundamental particle serving as a combined rod-and-clock system (Jennison, 1978, 1983, 1988, 1989). Jennison does not specifically mention the ether, and he accepts the geometrical formulations of relativity which offend some classically minded supporters of the Poincare-Lorentz programme. Jennison studied perfectly lossless entrapment of monochromatic radiation in confined spaces, termed "phase-locked cavities" which obey Newton's first and second laws; exhibit quantized momentum at microscopic level; and are fit to serve as proper rods and clocks. Jennison stresses the need to identify the rods and clocks which are the best instruments (and methods) for calibrating the space-time metric. Jennison and others argue that phase-locked cavities fulfill this role. Mackinnon applied phase-locking modelling to de Broglie matter-waves and devised the 'soliton' with waves phase-locked at the centre. Jennison devised a rotating matter wave particle, and derived a stable three dimensional system to serve as a combined proper clock and relativistically rigid measuring rod (in which velocity of sound is the speed of light). These instruments are used to calibrate the space-time metric. Clock time is "very real" and "cannot be assumed to exist where matter itself cannot exist"—such as in an environment where there is entirely free radiation without rest mass. Electrons and protons can be used as clocks. Pair production can produce proper clocks whose proper time starts from the moment of formation. Fundamental questions are raised concerning the meaningfulness of time and space measure beyond the province of the best available rod and clock—such as within the minimum measurable intervals of the vortex sponge. Jennison explains how to construct macroscopic measuring rods and clocks using laser light and microwave radiation, for practical experimental investigations. Other important discussions of the nature of fundamental rods and clocks are given by Kostro and Prokhovnik. Kostro's papers on the three-wave hypothesis, and the soliton, are important discussions on the inter-relation between ultimate particles, particle creation, and the nature of fundamental "best" rods and clocks (Kostro, 1985a, 1985b). Simon (1990) has considered the role of the electron in this role, within the context of Eddington's analysis of general relativity and quantum mechanics. Eddington regarded a particle as a conceptual carrier of measurable properties subject to probability in space-time. He considered replacing the physical reference system of objects by an ideal standardized reference object which was "a fluid, permeating all space like an ether" This was defined mathematically, in non-classical terms. These are a few examples of studies of how reference particles are defined, and how postulated mechanisms of particle creation are related to particular theories of ether, ZPF or physical vacuum.

Present Situation for Modern Ether Theory

Modern ether theory is relativistic because the measuring operations which it defines are described by the accepted formal structure of Relativity. Any departure from Relativity theory is beyond experimental detection by current techniques, though the Sagnac experiment continues to raise questions which require answers. The supposed detections of ether drift will require confirmation by multiple, independent findings by disinterested parties before they are accepted. Most ether theories are practically identical to Einstein's Relativity. Most modern ether theory is couched in non-classical terms (e.g. Ellis, 1973), with multi-dimensional spaces and times, though much of it can be given a "quasi-classical" interpretation as leading relativists (Eddington) have acknowledged since 1920. The ether is an abstract construct, a "disclosing model," for facilitating the classification and analysis of observations. Its use is justified by its ability to solve problems efficiently, checked by experiment. It is ontologically non-Realistic. It is a provisional instrument which must evolve and incorporate new discoveries which transform understanding of the ether. Ether theory is not anti-relativity, nor anti-Einstein's relativity. This must be clearly established or the concept will be rejected. Destructive misconceptions are spread by naïve Realists who argue that a classical ether, within the Poincare-Lorentz programme, is more rooted in objective reality because it employs Newtonian 3-d space and absolute time. This is a totally unjustified claim. A classical, Newtonian ether is as much an imaginary construct of the mathematicians as is a superstring or brane. Polemicists have used the classical ether concept to support an absolutist metaphysic for religious reasons (Turner & Hazelett, 1979), whilst condemning Einstein's relativity as the source of subversive irrationalism (Gratzer, 2000). This unjustified misuse of the ether concept is the result of bad history, bad philosophy and bad science. It delays acceptance of the ether concept by the community of Science, and causes many modern ether theorists to use alternative expressions, such as “vacuum field,” “physical vacuum” or “cosmological plenum” rather than the obvious term. Redundant concepts of ether must be set aside, and the futile ether-versus-relativity polemic should end. Ether theorists should concentrate much more than they do on integrating the range of present day theories into a comprehensive whole. Leaving a disarray of isolated, unrelated theories will not serve. Re-inventing Ives and Lorentz in different guises will not do. Modern ether theory must do more than show that it can interpret General Relativity and Cosmology, as geometrized physics has done. This is merely the basic qualification for being taken seriously because it is necessary to correct the assumption that relativity made the ether concept untenable after 1920. It has been done, and must be followed by an integrated ether interpretation of the full range of major physical theories, from which new and creative insights can be obtained.

Future Developments in Ether Theory

Modern ether theory has several promising development programmes which break new ground. The following trends promise to take ether theories into a new phase and are compatible with the comprehensive theories reviewed above. Ether can be treated as a seat of symmetry breaking mechanisms. Vortex theory is finding a key role in symmetry breaking analysis, and in creating new analogues of general relativity, some of them derived from the vortex-sponge, or from studies of superfluid helium and turbulent superfluids (Pismen, 1999; Volovik, 1998).

Mathematical formulations of ether have always been important (Clifford, Eddington, McLaren) and today one finds concepts of ether as a “generator” of mathematical systems: hypercomplex numbers, dynamic algebras, and tesselated spaces (Trell, 1998, 2003; Kassendrov, 2004; Santilli). These mathematical formulations of ether are often the result of techniques borrowed from communications theory, information science, fractals, or chaos theory.

Ether as the physical vacuum is a vital area of growth. The earlier ether theories postulating mechanisms for “creation out of nothing” or out of the vacuum or ZPF, have been joined (for example) by neo-quaternion theory, and nilpotent theory (P Rowlands, 1996, 2003). Dr Rowlands (Liverpool) has developed an interpretation of the physical vacuum which emerges as a mathematical property of the Dirac nilpotent operator, and for any individual fermion, the vacuum represents the rest of the universe. Three vacuum operators leave this original operator unchanged. These correspond to the vacuum mediated by weak, strong and electric charges. The fermion state vector expressed as a four-component spinor specifies the fermion and its three vacuum “reflections.” These partitions of the vacuum are discrete, but the combined “gravitational” or “total vacuum” is not. This vacuum is the carrier of nonlocality because it directly expresses Pauli exclusion. The continuous vacuum is connected with irreversible time, the Higgs mechanism, renormalization, zero-point energy, the Casimir effect and thermodynamics. The crossover between discrete nilpotent and continuous vacuum emerges as inertia, and if treated as a "gravity plus inertia (nilpotent) theory," the general theory of relativity avoids singularities, nonlinearity and non-renormalizability. It can be quantized and yields accelerating cosmological redshift and background radiation. The nilpotent operator incorporates proper time, and hence causality, whereas Einstein's theory excludes it as a separate parameter and defines the space-time scalar product as an invariant. In the restricted cases Einstein considered, with causality introduced *ad hoc,* quantum and the vacuum can be avoided and left undiscussed. This is a limited instance, which requires an artificial concept of simultaneity, unknown in a quantum context, which leads to the many difficulties (conceptual and methodological) which adverse critics associate with the special theory. Rowlands argues that these disappear in all physically realistic cases. An examination of the proper times shows that the "twin paradox" involves an asymmetry which makes the apparent simultaneity the result of first order approximation. Dr Rowlands argues that the exclusion of proper time from the special theory of relativity allows the absolute frame and absolute time to be ignored in the theory's fields of application, though these are required outside the immediate confines of the theory to preserve Einstein's concept of causality. This suggests that there must therefore be some form of absolute frame of reference, as well as "absolute birthordering of all quantum events." Advocates of ether theory should heed Rowland's warning that it is an entirely different question whether or not this absolute frame and vacuum, which is quantum rather than classical in origin, can be derived from the Poincare-Lorentz version of the ether. Larmor, Lodge, Lorentz and Poincare define the boundaries of the special theory of relativity, but the full quantum theory of the Dirac state, goes far beyond the supposed 4-dimensionality of space and time and takes physics to an order of understanding which lies beyond the confines of the Poincare-Lorentz-Einstein-Minkowski dispute.

The information theoretical aspects of the physical vacuum (ether) and matter is receiving much attention. Information science, computer science, systems processing, and signal analysis are being used to interpret the physical vacuum. Non-classical, geometrized ether theories, generating new mathematical interpretations, are developing out of the dynamical interpretations (vortex sponge) and the geometrodynamical equivalents. Communications signal theory was always fundamental to dynamical ether theory, and played a central role in the mechanism of wave-particle creation postulated by Hartley and Jennison. Recent work by S Bell and J Carroll, show how vortex theory, and communications signal theory reveal the hidden mechanisms which determine particle structures, particle stability, and the appropriate space-time metric. This "correlation technique" provides a method for developing the vortex-sponge from a "large-scale; long-time period" model, to one which interprets the short-period, small-scale phenomena which become manifest at the "minimum measurable intervals" scale. Wave particle dynamics is then unstable and three spatial dimensions are no longer adequate for interpretations. The ether is then a correllating mechanism, linking Relativity, and Quantum Mechanics. Its activity finds expression in Clifford algebra, dynamic algebras, nilpotent theories, and space-time vortices. The work of J E Carroll (2002, 2004) represents current work in this field, though he does not use the word “ether” to identify the physical vacuum. He develops correlation theory to relate special relativity, Clifford algebra and quantum theory, splitting scalar signals into even and odd to give sense of direction in space and time co-ordinates. Scalar signals “hide” structures which find expression in multivectors, geometries of different orders, and Clifford algebra. The correlation technique (hailed by some as heralding a revolution in the way physics is defined and understood) gives rise to models of space-time which include orthogonal vectors, spinors, and differences in chirality. Three dimensions at least are required for an observable isotropic space. The fourth dimension cannot be equivalent to one of these 3 dimensions of isotropic space and the natural metric for a 4-d space is that of special relativity in space-time, following Einstein and Minkowski. Note how the analysis establishes the Lorentzian programme of ether, particles as configurations in ether, and operations with rods and clocks, as the “other face” of geometrized special relativity. The route from signals in ether to the Einstein-Minkowski metric is direct, and of course, one can reverse the derivation. Once more the claims of the anti-Einstein polemical ether theorists stand revealed as invalid. The correlation technique can be developed by embedding (3+1) space time in 6-d space time to gain insight into the issues governing observations where signals are randomly fluctuating. It is interesting that correlation technique, and Clifford algebra, applied in the technology of image processing, now finds application in interpreting gravitation, relativity and quantum theory. Correlation analysis of 4-d space-time shows that a +ve metric leads to contradictions, but the relativity metric gives sensible results. Mermin shows that reliable, meaningful observations are the outcome of correlations, and that correlations in space-time cannot exist between two objects moving at speeds greater than that of light. Any modern ether theorist should pay great attention to correlation theory which relates the physical theories (signals in physical vacuum, observations, wave-particle structure) to geometrized formulations and mathematical expositions.

Conclusion

The above developments favour the vortex-sponge ether analogue, which is a quantum-mechanical entity, and which requires orders of space-time in excess of 4-d on the microscale and for fullest exposition. It is only for certain interpretations that a 4-d expression will suffice. Several promising lines of development for modern ether theory are evident. The sterile ether-versus-relativity polemic must be set aside before anything positive can be accomplished. Next, a greater degree of unification must be sought within the comprehensive theories which interpret relativity (Special and General), effects of physical vacuum, cosmology, space-time metric and Quantum Mechanics. Providing an equivalent and alternative "second interpetation" to the formal structure of General Relativity, already provided by the "Einstein school," isn't enough to justify ether theory. Ether theorists must show that the concept points towards creative theories in the future. The Einstein-Minkowski and the Poincare-Lorentz programmes are both valid and the geometrized vortex-sponge is equivalent to the space-time continuum of general relativity. When a very small scale perspective is taken, the vortex-sponge becomes the foam-like "punctured, fluctuating" continuum of geometrodynamics, requiring multi-dimensional geometrical interpretations. The ether can be regarded as a generator of mathematical decriptions (or first interpretations) after the example of Rowlands, Santilli, or Trell. This is a promising line of development. Equally fruitful is analysis of the ether in terms of information science and signal theory, following the example of Carroll's correlation technique. The rapidly growing studies of chaotic media, fractals, and symmetry-breaking mechanisms suggest what the ether is in the 21st C. It is an increasingly important class of unifying disclosing models, and associated theories, usually presented in non-Euclidean, multi-dimensional terms, which contains a quasi-classical "Newtonian sub-group" which can be identified with the Poincare-Lorentz programme, the Ives group of theories and the Lorentzian ether. It is a "scale-dependent" model. Large-scale, long-clocktime observations can be correlated with a simpler geometry than much smaller-scale measurements. The purely mathematical formalisms (first interpretations) generated by the ether require more study. They may resemble the dynamic algebras, and the nilpotent expositions of contemporary theories (Kassendrov 2004; Rowlands, 1996, 2003). This modern ether owes as much to Clifford, Riemann, Einstein, Minkowski, Planck, Dirac, and Eddington, as it does to Newton, Euler, Kelvin, Poincare, Lorentz, Larmor, Hartley and Ives.

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Absolute Velocity Resolution of the Clock Paradox

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The differential retardation effect between separating and reuniting clocks ("clock paradox") is discussed. A variational method is used to show, both in the theory of relativity and in more general theories, that among all possible trajectories of a clock connecting two given points at two given times the rectilinear uniform motion requires the longest proper time. A complete resolution of the clock paradox is obtained by giving an exhaustive unified description of all possible situations. Velocity (and nothing else) is thus seen to be responsible for the differential retardation effect. We conclude that hidden behind relativism there is in all cases a physically active inertial background.

1. The Clock Retardation Formula

In the paper on the theory of relativity (Einstein, 1905) the clock retardation prediction was presented as follows: Imagine one of the clocks which mark the time t_0 when at rest relatively to the "stationary" inertial system S_0, and the time t when at rest relatively to the "moving" inertial system S, to be located at the origin of the coordinates of S, where it marks the time t. What is the rate of this clock, if viewed from the stationary system? The quantities x_0, t_0 , and t, which refer to the position of the clock, satisfy $x_0 = vt_0$ and the Lorentz transformation of time

$$t = \frac{1}{R}\left(t_0 - {vx_0}\big/{c^2}\right)$$

where $R = \sqrt{1 - v^2 / c^2}$. Therefore

$$t = t_0\, R = t_0 - \left(1 - R\right) t_0$$

whence it follows that the time t marked by the clock is slow by $1 - R$ seconds per second with respect to the S_0 time t_0.

From this Einstein deduced another consequence, which has become famous as "clock paradox." If at the points A and B of S_0 there are two synchronous stationary clocks; and if the clock at A is moved with the velocity v along the line AB to B, then on its arrival at B the two clocks no longer show the same time, but the clock moved from A to B lags behind the other which has remained at B by $(1 - R)\, t_0$, t_0 being the duration of the journey from A to B.

Einstein considered evident that this result still holds good if the clock moves from A to B in any polygonal line, and also when the points A and B coincide. Assuming that the result obtained for a polygonal line holds also for a continuously curved line, he concluded: "If one of two synchronous clocks at A is moved on a closed curve with constant velocity until it returns to A, the journey lasting t_0 seconds, then by the clock which has remained at rest the travelled clock on its arrival at A will be $(1-R)\,t_0$ second slow."

That is all. Clearly Einstein went beyond what he could say by considering only inertial systems, as he introduced also accelerated motions (in the vertices of the polygonal line and along the continuous curve). He did so in an intuitive way, without solid foundations, and yet he was much closer to the correct result than with the 1918 paper (as we shall see).

Today, the retardation of moving clocks and its independence of acceleration are well established empirical facts. In a CERN experiment (Bailey *et al.*, 1977) muons with a velocity of 0.9994 c, corresponding to $R = 0.0341$, were circling in a ring with diameter of 14 m, with a centripetal acceleration $10^{18}\,g$. The lifetime τ_0 of the circling muons, measured in the laboratory, was in agreement with the formula $\tau = \tau_0 R$, where τ is the lifetime measured in the muon rest system. No effect of the huge acceleration on the lifetime was observed.

Another experiment (Hafele and Keating, 1972) compared six synchronised caesium atomic clocks. Two were carried by ordinary commercial jets in an eastbound tour around the planet; another two were carried in a westbound tour; the last two remained on the ground. It was observed that with respect to the time t_0 shown by the latter clocks, those of the westbound trip had lost 59 ± 10 ns, while those on the eastbound trip had advanced 273 ± 7 ns. These results were in agreement with the usual formula $t = t_0 R$ if one used three different R's for the three pairs of clocks. The largest (smallest) R was that of eastward (westward) clocks, for which the Earth rotation velocity added to (subtracted from) the jet velocity. One had to include the effect of the Earth gravitational potential, variable with altitude, which modifies the rates of travelling clocks differently from those on the ground. No effect of the acceleration on the curved paths was detected on the clock readings.

Similar conclusions have been obtained with the GPS (*Global Positioning System*) network of 24 satellites (Van Flandern, 1998). With an orbital radius of about four Earth radii and an orbital speeds of about 3.9 km/s, each satellite has on board four atomic clocks marking time with an error of a few ns/day. The gravitational effect implies that the atomic clocks on board the satellites tick faster by about 45.900 ns/day because they are in a larger gravitational potential than atomic clocks on the Earth surface. The velocity effect makes atomic clocks moving at GPS orbital speeds tick slower by about 7.200 ns/day. Therefore the global prediction is a gain of about 38.700 ns/day. Rather than having clocks with such large rate differences, the satellite clocks were reset in rate before launch slowing them down by 38.700 ns/day. The rich data show that the on board atomic clock rates do indeed agree with ground clock rates. Once more the orbital acceleration does not seem to have any effect on the clock rates.

The experimental evidence, in full agreement with Einstein's 1905 statements, points to the validity of the following clock retardation formula. We assume that if a clock U, marking the time t_0 when at rest in a certain isotropic inertial system S_0, is set in motion with arbitrarily oriented and possibly variable velocity $u(t_0)$ relative to S_0, the rate of the time marked by U at S_0 time t_0 is given by

$$d\tau = dt_0 \sqrt{1 - u^2(t_0)/c^2} \tag{1.1}$$

This τ is exactly what an observer travelling with U finds by time reading on the clock itself. Therefore $\mathrm{d}\tau$ is the "proper time" variation of U. The idea behind Eq. (1) is that only the instantaneous velocity (and not the acceleration) fixes the rate of the clock, in agreement with the observations.

Let us check that this conclusion about $\mathrm{d}\tau$ is fully correct. In physics one can recognize the cause of a phenomenon by varying it and verifying the existence of corresponding variations of the effect. Viceversa, if arbitrary variations of a physical quantity Q do not modify the effect E, one can exclude that Q is among the causes of E. Let us apply this criterion to Eq. (1.1). If $u(t_0)$ is varied a corresponding variation of the proper time rate arises: therefore velocity can be claimed to be a cause of the proper time rate variation. On the contrary, if the acceleration is modified at time t_0 while $u(t_0)$ remains the same $\mathrm{d}\tau$ will not change. Therefore the acceleration has no effect on $\mathrm{d}\tau$ and cannot be counted among the causes of the variation of its rate. This type of reasoning is important for determining the real physical roots of the"clock paradox." Such a phenomenon being completely deducible from Eq. (1.1), we will conclude that velocity and nothing else has a causal connection with the differential retardation effect.

2. Builder's Resolution of the Clock Paradox

It has been stressed (Prokhovnik, 1979) that the resolution of the clock paradox in terms of absolute motion was found by the Australian physicist G. Builder (Builder, 1957) who showed that the differential retardation effect between two clocks which separate and reunite can be validly considered in respect to a single inertial reference frame. Such an effect (read on the clocks) appears obviously to be the same to observers in all states of motion, and in this sense is absolute ("invariant"). Builder concluded that the emergence of an absolute effect consequence of velocity implies the existence of a privileged inertial frame, in the sense that motion relative to this frame assumes an absolute significance and is associated with absolute effects.

Builder's paper is mostly qualitative and in some points difficult to understand. My hopefully clearer reformulation of his argument is based on two simple assumptions:

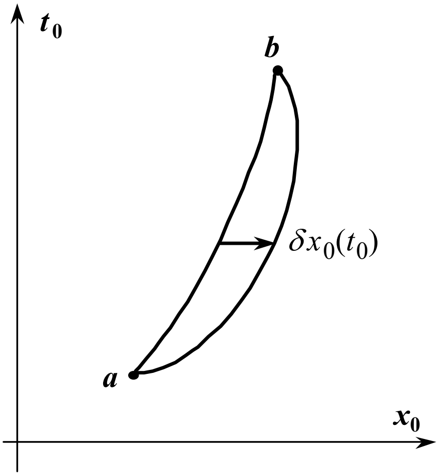

Figure 1. A space and time diagram showing a "trajectory" between two points, and a second varied trajectory between the same points.

A1. The velocity of light relative to an inertial system, S_0, is "c" in all directions, so that clocks can be synchronized in S_0 with the Einstein method and one way velocities relative to S_0 can be measured;

A2. A clock moving with speed $u(t_0)$ relative to S_0 during the S_0 time interval dt_0 marks a (proper) time increase $d\tau$ given by Eq. (1.1).

The theory of special relativity (TSR) is well known to satisfy the above assumptions in all inertial systems. The theory of the "equivalent transformations" (Selleri, 1996) accepts S_0 as the privileged system relative to which A1 and A2 are satisfied as well. Therefore all the consequences deduced below from A1 and A2 are valid both in the TSR and in the theory of the equivalent transformations.

In this section we consider only one spatial dimension. According to A2 the proper time increase T marked by a clock moving from the point a at time t_{0a} to the point b at time t_{0b}, both fixed in S_0, (see Fig. 1) is

$$T = \int_{t_{0a}}^{t_{0b}} dt_0 \sqrt{1 - u^2(t_0)/c^2} \tag{2.1}$$

where

$$u(t_0) = \frac{dx_0}{dt_0} \tag{2.2}$$

and $x_0 = x_0(t_0)$ is the equation of motion of the clock on some "trajectory" in the (x_0, t_0) plane connecting the points a,b of Fig. 1. We consider a second ("varied") trajectory, very near to the original one, as follows

$$x_0(t_0) \rightarrow x_0(t_0) + \delta x_0(t_0) \tag{2.3}$$

with

$$\delta x_0\left(t_{0a}\right)=\delta x_0\left(t_{0b}\right)=0 \tag{2.4}$$

From Eq. (2.4) we see that the clock on the varied trajectory occupies the points a and b at the same times t_{0a} and t_{0b} at which it occupies them on the unvaried trajectory. These conditions clearly correspond to the case of two clocks separating at point a at time t_{0a}, following two different trajectories and reuniting again in point b at time t_{0b}.

Correspondingly, also the velocity undergoes a variation

$$u\left(t_0\right)\rightarrow u\left(t_0\right)+\delta u\left(t_0\right) \tag{2.5}$$

with $u\left(t_0\right)$ given by Eq. (2.2) and

$$\delta u\left(t_0\right)\equiv\frac{d}{dt_0}\delta x_0\left(t_0\right) \tag{2.6}$$

The proper time integral will correspondingly become

$$T+\delta T=\int_{t_{0a}}^{t_{0b}} dt_0\sqrt{1-\left[u\left(t_0\right)+\delta u\left(t_0\right)\right]^2\Big/c^2} \tag{2.7}$$

From (2.6) and (2.7) it follows, for small variations

$$\delta T=-\frac{1}{c^2}\int_{t_{0a}}^{t_{0b}} dt_0\frac{u\left(t_0\right)}{\sqrt{1-u\left(t_0\right)^2\Big/c^2}}\frac{d}{dt_0}\delta x_0(t_0) \tag{2.8}$$

Integrating by parts one gets

$$\delta T=-\frac{1}{c^2}\left\{\left[\frac{u\left(t_0\right)\delta x_0(t_0)}{\sqrt{1-u\left(t_0\right)^2\Big/c^2}}\right]_{t_{0a}}^{t_{0b}}-\int_{t_{0a}}^{t_{0b}} dt_0\frac{d}{dt_0}\left[\frac{u\left(t_0\right)}{\sqrt{1-u\left(t_0\right)^2\Big/c^2}}\right]\delta x_0(t_0)\right\} \tag{2.9}$$

Due to (2.4) the first term in the right hand side vanishes. The derivative in the second term gives

$$\delta T=\frac{1}{c^2}\int_{t_{0a}}^{t_{0b}} dt_0\frac{\dot{u}\left(t_0\right)}{\left[1-u\left(t_0\right)^2\Big/c^2\right]^{3/2}}\delta x_0(t_0) \tag{2.10}$$

Clearly, $\delta T=0$ for arbitrary $\delta x_0(t_0)$ satisfying (2.4) if and only if $\dot{u}\left(t_0\right)=0$ at all times. This is like saying, as far as the proper time T is concerned, that the extremum of all motions is the uniform one with the constant velocity

$$u_1 = \frac{x_{0b}-x_{0a}}{t_{0b}-t_{0a}} \tag{2.11}$$

for which the proper time integral (2.1) takes the value

$$T_1 = \left(t_{0b}-t_{0a}\right)\sqrt{1-u_1^2/c^2} \tag{2.12}$$

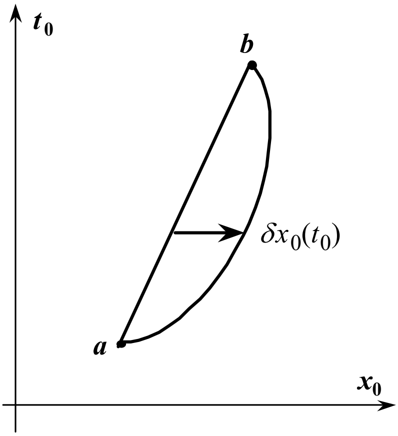

Figure 2. Space and time diagram showing a constant velocity connection between two points a and b, and a second varied trajectory between the same points.

Among all motions connecting a and b this extremum is unique, as it can be obtained for $\dot{u}(t_0)=0$ only. Therefore it gives either the maximum or the minimum proper time of all possible motions from a to b, not only of those obtained with an infinitesimal deformation of the straight line.

That the extremum is actually a maximum can be seen as follows. One has

$$\sqrt{1-\frac{(u_1+\delta u)^2}{c^2}} \cong \sqrt{1-u_1^2/c^2} - \frac{1}{c^2}\frac{u_1\delta u}{\sqrt{1-u_1^2/c^2}} - \frac{1}{2c^2}\frac{(\delta u)^2}{\left(1-u_1^2/c^2\right)^{3/2}} \tag{2.13}$$

But u_1 is a constant and δu satisfies (2.6) so that, after integration, the first order variation of T_1 arising from (2.13) vanishes due to (2.4). The second order variation is

$$\delta^2 T = -\frac{1}{2c^2}\int_{t_{0a}}^{t_{0b}} dt_0 \frac{\left[\delta u(t_0)\right]^2}{\left[1-\frac{u_1^2}{c^2}\right]^{3/2}} \tag{2.14}$$

Clearly, one has $\delta^2 T<0$ for all possible velocity variations. Therefore, moving away from the constant velocity line of Fig. 2 to a different line connecting a and b implies in all cases a decrease of the elapsed proper time. Then the found extremum is a maximum. Comparing the motion with velocity u_1 with any different motion from a to b with velocity $u_2(t_0)$ one has

$$\Delta T = T_1 - T_2 = \int_{t_{0a}}^{t_{0b}} dt_0 \left[\sqrt{1-\frac{u_1^2}{c^2}} - \sqrt{1-\frac{u_2(t_0)^2}{c^2}}\right] > 0 \tag{2.15}$$

As a difference of proper times, ΔT is exactly what two observers who traveled with the clocks find by direct comparison of the clocks readings.

Notice that the special theory of relativity satisfies the assumptions A1 and A2. Therefore the previous argument can be taken to describe all possible "clock paradox" situations from the point of view of a particular inertial system. But the principle of relativity, the metrics of Minkowski space and/or the gravitational potential of the fictitious forces do not seem to have anything to do with the essence of the matter. Only velocities are able to influence the rhythm of the physical phenomenon on which the working of a clock is based.

3. Arbitrary Clock Retardation Formula

We now generalize the obtained results by substituting assumption A2 with the following one: If during the S_0 time interval dt_0 a clock is moving with velocity $u(t_0)$ relative to S_0, it marks a (proper) time increase $d\tau$ given by

$$d\tau = dt_0 F\left[u\left(t_0\right)\right] \tag{3.1}$$

In Eq. (3.1) F is an arbitrary function of its argument. Thus the proper time T spent by a clock moving between any two points *a,b* fixed in S_0 (see Fig. 1) is given by

$$T = \int_{t_{0a}}^{t_{0b}} dt_0 F\left[u\left(t_0\right)\right] \tag{3.2}$$

where

$$u\left(t_0\right) = \frac{dx_0}{dt_0} \tag{3.3}$$

and $x_0 = x_0\left(t_0\right)$ is the equation of motion of the clock on some "trajectory" connecting the points a,b of Fig. 1 in the $\left(x_0, t_0\right)$ plane. We consider a variation of this trajectory as in eq. (2.3), with the conditions (2.4) satisfied. Using the notation of Section 2, the proper time integral becomes

$$T + \delta T = \int_{t_{0a}}^{t_{0b}} dt_0 F\left[u\left(t_0\right) + \delta u\left(t_0\right)\right] \tag{3.4}$$

From (2.6) and (3.4) it follows, for small variations

$$\delta T = \int_{t_{0a}}^{t_{0b}} dt_0 F'\left[u\left(t_0\right)\right] \frac{d}{dt_0} \delta x_0(t_0) \tag{3.5}$$

where F' indicates the u derivative of F. An integration by parts now gives

$$\delta T = \left\{F'\left[u\left(t_0\right)\right] \delta x_0\left(t_0\right)\right\}_{t_{0a}}^{t_{0b}} - \int_{t_{0a}}^{t_{0b}} dt_0 \frac{d}{dt_0} \left\{F'\left[u\left(t_0\right)\right]\right\} \delta x_0\left(t_0\right) \tag{3.6}$$

Due to (2.4) the first term in the right hand side vanishes and from the second term one gets

$$\delta T = -\int_{t_{0a}}^{t_{0b}} dt_0 F''\left[u\left(t_0\right)\right] \dot{u}\left(t_0\right) \delta x_0\left(t_0\right) \tag{3.7}$$

Clearly, $\delta T = 0$ for arbitrary $\delta x_0\left(t_0\right)$ if, at all times t_0

$$F''\left[u\left(t_0\right)\right]\dot{u}\left(t_0\right)=0 \tag{3.8}$$

Thus we see that at least one of the possible solutions is $\dot{u}\left(t_0\right)=0$ at all times, the uniform motion. Whether it corresponds to a maximum or to a minimum has to be seen case by case. Many functions F give rise to a maximum of the proper time integral for $\dot{u}\left(t_0\right)=0$, *e.g.*

$$F\left[u\left(t_0\right)\right]=\exp\left\{-\frac{1}{2}\frac{u^2}{c^2}\right\} \tag{3.9}$$

Therefore there is nothing typical of a relativistic theory in the maximum of the proper time integral provided by the rectilinear uniform motion.

4. Three Dimensional Generalization

We consider again the proper time formula, Eq. (2.1), for a three dimensional generalization of the argument given in section 2. Now we assume

$$u^2\left(t_0\right)=u_x^2\left(t_0\right)+u_y^2\left(t_0\right)+u_z^2\left(t_0\right) \tag{4.1}$$

where

$$u_x\left(t_0\right)=\frac{dx_0}{dt_0};u_y\left(t_0\right)=\frac{dy_0}{dt_0};u_z\left(t_0\right)=\frac{dz_0}{dt_0} \tag{4.2}$$

and $x_0=x_0\left(t_0\right)$, $y_0=y_0\left(t_0\right)$, $z_0=z_0\left(t_0\right)$ are the equations of motion of the clock on some "trajectory" connecting the points a,b in the $\left(x_0,y_0,z_0,t_0\right)$ space. We consider a variation of this trajectory, as follows

$$\begin{aligned}x_0\left(t_0\right)&\to x_0\left(t_0\right)+\delta x_0\left(t_0\right)\\ y_0\left(t_0\right)&\to y_0\left(t_0\right)+\delta y_0\left(t_0\right)\\ z_0\left(t_0\right)&\to z_0\left(t_0\right)+\delta z_0\left(t_0\right)\end{aligned} \tag{4.3}$$

with

$$\begin{aligned}\delta x_0\left(t_{0a}\right)&=\delta x_0\left(t_{0b}\right)=0\\ \delta y_0\left(t_{0a}\right)&=\delta y_0\left(t_{0b}\right)=0\\ \delta z_0\left(t_{0a}\right)&=\delta z_0\left(t_{0b}\right)=0\end{aligned} \tag{4.4}$$

Correspondingly, also the velocity undergoes a variation

$$\begin{aligned}u_x\left(t_0\right)&\to u_x\left(t_0\right)+\delta u_x\left(t_0\right)\\ u_y\left(t_0\right)&\to u_y\left(t_0\right)+\delta u_y\left(t_0\right)\\ u_z\left(t_0\right)&\to u_z\left(t_0\right)+\delta u_z\left(t_0\right)\end{aligned} \tag{4.5}$$

with $u_x\left(t_0\right)\equiv\dot{x}_0\left(t_0\right)$, $u_y\left(t_0\right)\equiv\dot{y}_0\left(t_0\right)$, $u_z\left(t_0\right)\equiv\dot{z}_0\left(t_0\right)$ and

$$\delta u_x\left(t_0\right)\equiv\frac{d}{dt_0}\delta x_0(t_0),\delta u_y\left(t_0\right)\equiv\frac{d}{dt_0}\delta y_0\left(t_0\right),\delta u_z\left(t_0\right)\equiv\frac{d}{dt_0}\delta z_0\left(t_0\right) \tag{4.6}$$

The proper time integral will become

$$T+\delta T=\int_{t_{0a}}^{t_{0b}} dt_0\sqrt{1-\frac{\left[u_x+\delta u_x\right]^2}{c^2}-\frac{\left[u_y+\delta u_y\right]^2}{c^2}-\frac{\left[u_z+\delta u_z\right]^2}{c^2}} \tag{4.7}$$

From (2.1) and (4.7) it follows, for small variations

$$\delta T=-\frac{1}{c^2}\int_{t_{0a}}^{t_{0b}} \frac{dt_0}{\sqrt{1-\frac{u\left(t_0\right)^2}{c^2}}}\left[u_x\delta u_x+u_y\delta u_y+u_z\delta u_z\right] \tag{4.8}$$

which, due to (4.6), can also be written

$$\delta T=-\frac{1}{c^2}\int_{t_{0a}}^{t_{0b}} \frac{dt_0}{\sqrt{1-\frac{u\left(t_0\right)^2}{c^2}}}\left[u_x\frac{d}{dt_0}\delta x_0+u_y\frac{d}{dt_0}\delta y_0+u_z\frac{d}{dt_0}\delta z_0\right] \tag{4.9}$$

An integration by parts now gives

$$\begin{aligned}\delta T=&-\frac{1}{c^2}\left[\frac{u_x\delta x_0+u_y\delta y_0+u_z\delta z_0}{\sqrt{1-\frac{u^2}{c^2}}}\right]_{t_{0a}}^{t_{0b}}\\&+\frac{1}{c^2}\int_{t_{0a}}^{t_{0b}} dt_0\left\{\frac{d}{dt_0}\left[\frac{u_x}{\sqrt{1-\frac{u^2}{c^2}}}\right]\delta x_0+\frac{d}{dt_0}\left[\frac{u_y}{\sqrt{1-\frac{u^2}{c^2}}}\right]\delta y_0+\frac{d}{dt_0}\left[\frac{u_z}{\sqrt{1-\frac{u^2}{c^2}}}\right]\delta z_0\right\}\end{aligned} \tag{4.10}$$

Due to (4.4) the first term in the right hand side vanishes. From the condition that also the second term should vanish for arbitrary $\delta x_0\left(t_0\right)$, $\delta y_0\left(t_0\right)$, $\delta z_0\left(t_0\right)$ one gets

$$\frac{d}{dt_0}\left[\frac{u_x}{\sqrt{1-\frac{u^2}{c^2}}}\right]=\frac{d}{dt_0}\left[\frac{u_y}{\sqrt{1-\frac{u^2}{c^2}}}\right]=\frac{d}{dt_0}\left[\frac{u_z}{\sqrt{1-\frac{u^2}{c^2}}}\right]=0 \tag{4.11}$$

The three quantities, which are constant according to (4.11), form a vector $\vec{V}$ having the same direction as $\vec{u}$. This vector is constant :

$$\vec{V}\equiv\frac{\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}}=\text{constant} \tag{4.12}$$

Obviously then, also the modulus of $\vec{V}$ has to be constant. Therefore:

$$\frac{d}{dt_0}\left[\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right]=\frac{\dot{u}}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}=0 \tag{4.13}$$

whence

$$\dot{u}=0 \tag{4.14}$$

Therefore $\vec{u}$ has to be constant in direction and modulus. This is like saying, as far as the elapsed proper time is concerned, that the extremum of all motions is the rectilinear uniform one with velocity components

$$u_{1x} = \frac{x_{0b} - x_{0a}}{t_{0b} - t_{0a}};\ u_{1y} = \frac{y_{0b} - y_{0a}}{t_{0b} - t_{0a}};\ u_{1z} = \frac{z_{0b} - z_{0a}}{t_{0b} - t_{0a}} \tag{4.15}$$

for which the proper time integral takes the value (2.12) with

$$u_1^2 = u_{1x}^2 + u_{1y}^2 + u_{1z}^2 \tag{4.16}$$

Among all motions connecting a and b this extremum is unique, as it is obtained for $\dot{u}(t_0) = 0$ only. Therefore it gives either the maximum or the minimum proper time of all possible motions connecting a at time t_{0a} with b at time t_{0b}, not only of those obtained with an infinitesimal deformation of the straight line.

The extremum is actually a maximum, as can be seen by giving variations to the coordinates, as in (4.4), and therefore [*via* (4.6)] corresponding variations δu_x, δu_y, δu_z to the constant velocity components (4.15). If we write the square root entering in the proper time integral (2.1) as

$$R\left(u_x, u_y, u_z\right) = \sqrt{1 - \left(u_x^2 + u_y^2 + u_z^2\right)\Big/ c^2} \tag{4.17}$$

we have, up to the second order of approximation

$$R\left(u_x + \delta u_x,\ u_y + \delta u_y,\ u_z + \delta u_z\right) = R + \delta R + \delta^2 R \tag{4.18}$$

Correspondingly, we have a variation of the proper time integral

$$T\left(u_x + \delta u_x,\ u_y + \delta u_y,\ u_z + \delta u_z\right) = T + \delta T + \delta^2 T \tag{4.19}$$

From (4.17) it follows

$$\delta R = -\frac{u_x \delta u_x + u_y \delta u_y + u_z \delta u_z}{c^2 R} \tag{4.20}$$

so that

$$\delta T = -\frac{1}{c^2} \int_{t_{0a}}^{t_{0b}} dt_0 \ \frac{u_x \delta u_x + u_y \delta u_y + u_z \delta u_z}{R} \tag{4.21}$$

If u_x, u_y, u_z are given the constant values (4.15) δT vanishes, simply because the variations δu_x, δu_y, δu_z are time derivatives and (4.4) has to be applied.

For the second order variation a straightforward calculation gives

$$\delta^2 R = -\frac{1}{c^2}\frac{1}{R^3}\Big[\delta u_x^2 + \delta u_y^2 + \delta u_z^2$$
$$+ 2\frac{u_x u_y}{c^2}\,\delta u_x \delta u_y + 2\frac{u_x u_z}{c^2}\,\delta u_x \delta u_z + 2\frac{u_y u_z}{c^2}\,\delta u_y \delta u_z\Big]$$

We can increase the right hand side of the previous equation multiplying the first three (certainly not positive) terms by factors positive and less than unity. Given that $u_x^2, u_y^2, u_z^2 < c^2$, we have

$$\delta^2 R < -\frac{1}{c^2}\frac{1}{R^3}\left[\frac{u_x^2}{c^2}\delta u_x^2 + \frac{u_y^2}{c^2}\delta u_y^2 + \frac{u_z^2}{c^2}\delta u_z^2 \right.$$

$$\left. + 2\frac{u_x u_y}{c^2}\delta u_x \delta u_y + 2\frac{u_x u_z}{c^2}\delta u_x \delta u_z + 2\frac{u_y u_z}{c^2}\delta u_y \delta u_z \right]$$

Now the quantity within brackets simplifies and we get:

$$\delta^2 R < -\frac{1}{c^2}\frac{1}{R^3}\left[\frac{u_x}{c}\delta u_x + \frac{u_y}{c}\delta u_y + \frac{u_z}{c}\delta u_z\right]^2 \tag{4.22}$$

The right hand side of (4.22) is negative. Therefore $\delta^2 R < 0$ and

$$\delta^2 T = \int_{t_{0a}}^{t_{0b}} dt_0 \delta^2 R < 0 \tag{4.23}$$

for all possible choices of $\delta u_x, \delta u_y, \delta u_z$. Therefore, moving away from the rectilinear uniform motion to a different motion connecting a at time t_{0a} with b at time t_{0b} implies in all cases a decrease of the elapsed proper time. This means that the found extremum is a maximum. Comparing the motion with velocity (4.15) with a different motion with velocity $u_2(t_0)$ one obtains Eq. (2.15) also in the present case.

The clock moving with rectilinear uniform motion can be considered at rest in a different inertial system. Therefore the previous argument can be taken to describe all possible "clock paradox" situations from the point of view of the particular inertial system S_0 we have considered. The description is the same in the TSR and in all other theories satisfying the assumptions A1 and A2 of section 2.

5. The Equivalent Transformations

According to some authors (Poincaré, 1898), (Reichenbach, 1958), (Jammer, 1979), (Mansouri and Sexl, 1977), clock synchronization in inertial systems is conventional and the choice of the invariance of the one way velocity of light made in the TSR is only based on simplicity. Following the same line of thought, I introduced a suitable parameter e_1 describing synchronizations in the transformations of the time variables (Selleri, 1996). The TSR is obtained for a particular nonzero value of e_1. In this way an infinite set of theories empirically equivalent to the TSR was developed. Every element of the set satisfies the assumptions A1 and A2 and therefore all the consequences deduced from them. We can thus say that the "clock paradox" situation is not peculiar to the TSR, but appears in the same way in all the theories "equivalent" to the TSR. This matter will be discussed in the next section.

Given the inertial frames S_0 and S one can set up Cartesian coordinates and make the following standard assumptions:

i. Space is homogeneous and isotropic and time homogeneous, at least if judged by observers at rest in S_0;

ii. In the isotropic system S_0 the velocity of light is *"c"* in all directions, so that clocks can be synchronized in S_0 and one way velocities relative to S_0 can be measured;
iii. The origin of S, observed from S_0, is seen to move with velocity $v<c$ parallel to the $+x_0$ axis, that is according to the equation $x_0 = v\,t_0$;
iv. The axes of S and S_0 coincide for $t = t_0 = 0$;

The system S_0 turns out to have a privileged status in all theories satisfying the assumptions (i) and (ii), with the exception of the TSR. Two further assumptions based on direct experimental evidence can be added:

v. The two way velocity of light is the same in all directions and in all inertial systems;
vi. Clock retardation takes place with the usual velocity dependent factor when clocks move with respect to S_0. This assumption is the same as A2 of the second section.

These conditions were shown to lead to the following transformations of the space and time variables from S_0 to S

$$\begin{cases} x = \dfrac{x_0 - v\,t_0}{R} \\ y = y_0 \qquad ; \qquad z = z_0 \\ t = R\,t_0 + e_1\left(x_0 - v\,t_0\right) \end{cases} \tag{5.1}$$

where

$$R = \sqrt{1 - v^2/c^2} \tag{5.2}$$

and e_1 is the synchronization parameter. By using again (ii) and Eq. (5.1) one can find the one way velocity of light relative to the moving system S for light propagating at an angle θ from the velocity $\vec{v}$ of S relative to S_0:

$$c_1\left(\theta\right) = \frac{c}{1 + \Gamma\cos\theta} \tag{5.3}$$

with

$$\Gamma = e_1 R\,c + \frac{v}{c} \tag{5.4}$$

The TSR is a particular case, obtained for

$$e_1 = -\frac{v}{c^2 R} \tag{5.5}$$

giving $\Gamma = 0$ and $c_1\left(\theta\right) = c$ and reducing (5.1) to their Lorentz form.

These results are enough for the needs of the present paper. We should mention, however, that we also found that the choice $e_1 = 0$ is the only one allowing for a treatment of accelerations rationally connected with the physics of inertial systems S_0. The theory was applied to the rotating platform and to the

Sagnac effect (Sagnac, 1914) with the result that only the choice $e_1 = 0$ could give a satisfactory explanation (Selleri, 2004). A consequence of this research was the discovery of a relativistic discontinuity between inertial systems and slowly accelerated systems. The discontinuity disappears only if $e_1 = 0$. Its existence in the TSR is the root of the difficulties met in explaining the physics on a rotating platform (Langevin, 1937), (Post, 1967), (Landau and Lifschitz, 1996), (Anandan, 1981).
There are several other good reasons to adopt $e_1 = 0$:

(a) Einstein in his 1905 article assumed that aberration depends on the relative velocity star-Earth, but this relativistic description works poorly, given that empirically the aberration angle is the same for all stars in the sky (Puccini and Selleri, 2002).
(b) The growing evidence for the existence of superluminal signals can easily be accommodated in the theory with $e_1 = 0$, while it is incompatible with relativity due to the presence of a causal paradox in which events belonging to the future of an observer can actively modify the past of the same observer (Selleri, 2002).
(c) Practically all paradoxes of the special theory of relativity disappear in a theory based on the inertial transformations.

For these reasons a satisfactory theory of the physics of space and time has to be based on absolute simultaneity ($e_1 = 0$).

Assuming the inertial transformations, the S_0 system is initially considered to be privileged, and the velocity of light relative to it isotropic. Other inertial systems are described as "moving" and relative to them the observers detect an anisotropic velocity of light given by equations like (5.3). In a recent paper (Selleri, 2005) I described a resynchronization of clocks (ROC) and showed that it is uniquely determined by the new inertial frame S chosen to replace S_0 as "privileged" and by the requirement that absolute simultaneity should be preserved. One can add that from the point of view of the inertial transformations the validity of relativity appears accidental, more than fundamental. It would be enough to discover a very small noninvariance of the two way speed of light to make the whole game of ROC impossible.

6. Invariance of the Proper Time

We will now consider the clock paradox by referring all motions to the moving inertial system S and applying the equivalent transformations. This calculation would not be strictly needed, as all possible situations were already discussed relatively to the privileged system S_0, but it will give us a consistency test of the more general framework provided by the equivalent transformations. The trasformations inverse of (5.1) are

$$\begin{cases} x_0 = \left(R - e_1 v\right) x + \dfrac{v\,t}{R} \\ y_0 = y \qquad ; \qquad z_0 = z \\ t_0 = \dfrac{t - R\,e_1\,x}{R} \end{cases} \tag{6.1}$$

Let $\vec{u}_0$ and $\vec{u}$ be the velocity of a clock with respect to S_0 and S respectively. From (6.1) one gets the (inverse) inertial transformations of velocities

$$\begin{gathered} u_{0x} = \frac{R\left(R - e_1 v\right) u_x + v}{1 - R\,e_1 u_x} \\ u_{0y} = \frac{R\,u_y}{1 - R\,e_1 u_x} \qquad ; \qquad u_{0z} = \frac{R\,u_z}{1 - R\,e_1 u_x} \end{gathered} \tag{6.2}$$

From these equations it follows

$$\sqrt{1 - {u_0^2}/{c^2}} = R\frac{\sqrt{\left[1 - {\left(R\,e_1 c + v/c\right) u_x}/{c^2}\right]^2 - \dfrac{u^2}{c^2}}}{1 - R\,e_1 u_x} \tag{6.3}$$

The clock moving with velocity $\vec{u}$ satisfies $x = u_x t$. Therefore substituting in the transformation of time (6.1), we get

$$t_0 = \frac{1 - R\,e_1\,u_x}{R}\,t \tag{6.4}$$

In the general situation we are considering the clock velocity could be time dependent. Therefore it is useful to write (6.4) for infinitesimal time intervals

$$dt_0 = \frac{1 - R\,e_1\,u_x}{R}\,dt \tag{6.5}$$

By multiplying side by side Eqs. (6.3) and (6.5) and writing $u_x = u\cos\theta$ it follows

$$dt_0\,\sqrt{1 - \frac{u_0^2}{c^2}} = dt\sqrt{\left[1 - \frac{\left(R\,e_1 c + v/c\right) u\,\cos\theta}{c}\right]^2 - \frac{u^2}{c^2}} \tag{6.6}$$

Eq. (6.6) represents the invariance under equivalent transformations of the clock proper time interval $d\tau$. It can also be written

$$d\tau = dt\sqrt{\left[1 - \frac{\left(R\,e_1 c + {v}/{c}\right) u\,\cos\theta}{c}\right]^2 - \frac{u^2}{c^2}} \tag{6.7}$$

This $d\tau$ is the proper time increase during the clock propagation. Notice that $d\tau$ depends both on the velocity of the clock relative to S, u, and on the absolute velocity of S, u. As the calculation leading to (6.6) holds for an arbitrary S, the quantity in the right hand side of (6.7) is the same in all reference frames. Furthermore, given the left hand side of (6.6), $d\tau$ is independent of the synchronization chosen in the moving frames, so that $d\tau$ (in spite of appearence) does not depend on e_1, this being possible because the one way velocity u has an implicit dependence on e_1 canceling the explicit dependence in (6.7).

The total proper time needed for going from the point a at time t_{0a} to the point b at time t_{0b} is

$$T = \int_{t_a}^{t_b} dt \sqrt{\left[1 - \frac{\left(R\,e_1 c + {}^{v}\!/_{c}\right) u \cos\theta}{c}\right]^2 - \frac{u^2}{c^2}} \tag{6.8}$$

The right hand side is again the same in all reference frames. There is an interesting “relativistic” property of this expression: consider the clock going from the point a at time t_{0a} to the point b at time t_{0b} along a straight line with a constant velocity *u*. If we write $dt = d\ell / u$ and integrate between two points separated by a distance ℓ_{ba}, Eq. (6.8) gives

$$T = \frac{\ell_{ba}}{u} \sqrt{\left[1 - \frac{\left(R\,e_1 c + {}^{v}\!/_{c}\right) u \cos\theta}{c}\right]^2 - \frac{u^2}{c^2}} \tag{6.9}$$

Given its invariance, Eq. (6.9) has a clear meaning. A clock moving on a straight line with constant velocity u relative to the frame S, from the point a to the point b, both belonging to the frame S and separated by a distance ℓ_{ba}, marks a proper time increase that is the same in all inertial reference frames S independently of the chosen synchronization.

7. The Clock Paradox Again

Let $x = x(t)$ be the equation of motion of the clock on an arbitrary “trajectory” in the (x,t) plane of a moving inertial reference frame S, connecting the point a at time t_a with the point b at time t_b. We consider a variation of this trajectory, as follows

$$x(t) \to x(t) + \delta x(t) \tag{7.1}$$

with

$$\delta x(t_a) = \delta x(t_b) = 0 \tag{7.2}$$

Correspondingly, also the velocity undergoes a variation

$$u(t) \to u(t) + \delta u(t) \tag{7.3}$$

with $u(t) \equiv \dot{x}(t)$ and

$$\delta u(t) \equiv \frac{d}{dt}\delta x(t) \tag{7.4}$$

Due to the variation δu of velocity there is a variation also of the proper time integral (6.8), which for $\theta = 0$ can easily be shown to be

$$\delta T = -\int_{t_a}^{t_b} dt\, G(u,v) \frac{d}{dt}\delta x(t) \tag{7.5}$$

where

$$G(u,v)=\frac{\left(1-uRe_1-{vu}/{c^2}\right)\left(Re_1+{v}/{c^2}\right)+{u}/{c^2}}{\sqrt{\left(1-uRe_1-{vu}/{c^2}\right)^2-{u^2}/{c^2}}} \tag{7.6}$$

Integrating by parts in (7.5) it follows

$$\delta T=-\left[G(u,v)\,\delta x(t)\right]_{t_a}^{t_b}+\int_{t_a}^{t_b}dt\frac{dG(u,v)}{dt}\delta x(t)$$

The first term in the right hand side vanishes due to Eq. (7.2). The derivative in the second term can be calculated and gives the simple result

$$\delta T=\int_{t_a}^{t_b}dt\frac{{\dot{u}}/{c^2}}{\left[\left(1-uRe_1-{vu}/{c^2}\right)^2-{u^2}/{c^2}\right]^{3/2}}\delta x(t) \tag{7.7}$$

Clearly, $\delta T=0$ for arbitrary $\delta x(t)$ satisfying (7.2) if and only if $\dot{u}(t)=0$ at all times t. This is like saying, as far as the elapsed proper time is concerned, that the extremum of all possible motions is uniform with the constant velocity

$$u=u_1\equiv\frac{x_b-x_a}{t_b-t_a} \tag{7.8}$$

for which the proper time integral (6.8) takes the form

$$T=T_1\equiv\left(t_b-t_a\right)\sqrt{\left(1-u_1Re_1-{vu_1}/{c^2}\right)^2-{u_1^2}/{c^2}} \tag{7.9}$$

Among all motions connecting a and b this extremum is unique, as it is obtained only for $\dot{u}(t)=0$. Therefore it gives either the maximum or the minimum proper time of all possible motions from a to b, not only of those obtained with an infinitesimal deformation of the straight line.

Thus we conclude that the differential retardation of separating and reuniting clocks is predicted equally by all “equivalent” theories as a velocity effect. There is no need to consider the situation from the point of view of the nonuniformly moving clock by invoking the equivalence principle and the gravitational potential of the inertial forces. In the next section we will furthermore see that the general relativistic approach to the clock paradox is more a game of appearances than a representation of reality and can be dropped altogether.

8. Clock Paradox and General Relativity

The 1905 formulation of the clock paradox had an implication that probably Einstein did not like. The delay is an absolute effect, as all observers agree that the clock moving with variable velocity marks a smaller time. They disagree, however, on the numerical value of this variable velocity. In relativity all potential observers (forming an infinite set) are completely equivalent, so that, in a sense, one can say that the clock velocity assumes at any time all conceivable values. But a quantity having at the same time infinitely many values is totally undefined. In this way the presumed cause of the differential retardation would seem to vanish into nothingness. But, no, this is not physically reasonable, obviously the cause of a real physical effect should be concrete as well, in spite of the evasive description coming from the theory. Therefore causality implies that velocity itself should be well defined, that is, relative to a physically active reference background (ether) which defines at the same time the privileged inertial reference background.

It is no surprise, then, that to escape from such conclusions the original formulation was completed with a later one based on the theory of general relativity (TGR) (Einstein, 1918), whose essential points we will now review. Let S be an inertial reference system. Further, let U_1 and U_2 be two exactly similar clocks working at the same rate when at rest relatively to S. If one of the clocks—let us say U_2—is in a state of uniform translatory motion relative to S, then, according to the TSR it works more slowly than U_1, which is at rest in S.

At this point Einstein adds an interesting remark: "This result seems odd in itself. It gives rise to serious doubts when one imagines the following thought esperiment." In the thought experiment A is the origin of S, and B a different point of the positive x-axis. The two clocks are initially at rest at A, so that they work at the same rate and their readings are the same. Next, a constant velocity in the direction $+x$ is imparted to U_2, so that it moves towards A. At B the velocity is reversed, so that U_2 returns towards A. When it arrives at A its motion is stopped, so that it is again at rest near U_1. Since U_2 works more slowly than U_1 during its motion along the line AB, U_2 must be behind U_1 on its return.

Now comes the problem, says Einstein. According to the principle of relativity the whole process must surely take place in exactly the same way if it is considered in a reference frame S′ sharing the movement of U_2. Relatively to S′ it is U_1 that executes the to-and-fro movement while U_2 remains at rest throughout. From this it would seem to follow that, at the end of the process, U_1 must be behind U_2, which contradicts the former result.

But, Einstein adds, the TSR is inapplicable to the second case, as it deals only with inertial reference frames, while S is at times accelerated. Only the TGR deals with accelerated frames. From the point of view of the TGR, one can use the coordinate system S′ just as well as S. But in describing the whole process, S and S′ are not equivalent as the following comparison of the movements shows.

S Reference System

1. The clock U_2 is accelerated by an external force in the direction $+x$ until it reaches the velocity v. U_1 is at rest, now as in all the subsequent steps.
2. U_2 moves with constant velocity v to the point B on the $+x$-axis.
3. U_2 is accelerated by an external force in the direction $-x$ until it reaches the velocity v in the direction $-x$.

4. U_2 moves with constant velocity v in the direction $-x$ back to the neighbourhood of U_1.
5. U_2 is brought to rest by an external force very near to U_1.

S′ Reference System

1. A gravitational field, oriented along $-x$, appears, in which the clock U_1 falls with an accelerated motion until it reaches the velocity v. When U_1 has reached the velocity v the gravitational field vanishes. An external force applied to U_2 prevents U_2 from being moved by the gravitational field.
2. U_1 moves with constant velocity v to a point B' on the $-x$-axis. U_2 remains at rest.
3. A homogeneous gravitational field in the direction $+x$ appears, under the influence of which U_1 is accelerated in the direction $+x$ until it reaches the velocity v, whereupon the gravitational field vanishes. U_2 is kept at rest by an external force.
4. U_1 moves with constant velocity v in the direction $+x$ into the neighbourhood of U_2. U_2 remains at rest.
5. A gravitational field in the direction $-x$ appears, which brings U_1 to rest. The gravitational field then vanishes. U_2 is kept at rest by an external force.

The second description is based on the principle of equivalence between fictitious and gravitational forces. According to both descriptions, at the end of the process the clock U_2 is retarded by a definite amount compared with U_1. With reference to S′ this is explained by noticing that during the stages 2 and 4, the clock U_1, moving with velocity v, works more slowly than U_2, which is at rest. But this retardation is overcome by the quicker working of U_1 during stage 3. For, according to the GTR, a clock works the faster the higher the gravitational potential in the point where it is placed, and during stage 3 U_1 is indeed placed in a region of higher gravitational potential than U_2. A calculation made with instantaneous acceleration shows that the consequent advancement amounts to exactly twice as much as the retardation during stages 2 and 4 (Einstein, 1918). Arrived at this conclusion Einstein states: "This completely clears up the paradox."

As it is well known, a clock works faster the larger is the gravitational potential ϕ in the region in which it is placed. This prediction of the TGR is fully confirmed by the experiments performed in the gravitational field of the Earth, so that at first sight the 1918 reasoning could seem to be a consequence of the empirical facts. The mathematical treatment of the clock paradox situation given by the TGR is thought to lead to the right result by describing the retardation of U_2 as a consequence of ϕ (Møller, 1957). Yet the theory shows its weakness in several ways.

Einstein describes the gravitational potential as active in modifying the rate of U_1. But it is unreasonable that something happens to U_1, as the effect would be an objective phenomenon and during the (short) times of acceleration some change should be observed also by some independent observers.

For the sake of clarity let us compare two different experiments, E1 and E2 below, both starting in the same way, as follows. Let A be a point of the inertial system S. Let U_1 be the clock constantly at rest in A and U_2 the mobile clock, initially at rest in A. At time $t = 0$ we give U_2 a constant acceleration in the direction $+x$ until it reaches the constant velocity v as in Einstein's thought experiment. After this U_2 moves with velocity v until, at time $t = t_1$, two alternative developments can start.

E1. In the time interval (t_1, t_2), with $t_2 > t_1 > 0$, U_2 experiences a constant acceleration, exactly reversing its velocity. The 1918 formulation introduces a gravitational potential ϕ in the rest system of U_2, during (t_1, t_2). The role of ϕ is very important as it must give rise to a variation in the time marked by U_1 opposite and twice as large as that arising in the long stretches of uniform motion during which U_1 is delayed with respect to U_2.

E2. In the time interval (t_1, t_2) U_2 continues its rectilinear and uniform motion in the direction $+x$. The two clocks do not reunite anymore. No gravitational potential can arise, as there is no acceleration. Therefore in this second case there is no retardation of U_1 with respect to U_2.

In comparing E1 and E2, we note that the observers at rest with respect to U_1 see in both cases the same identical situation, namely no modification of the rate of time keeping of U_1. This can also be checked from a distance by observers in arbitrary states of motion, *e.g.*, by monitoring U_1 on a TV screen. The fundamental fact is always the same: nothing *ever* happens to U_1, in particular nothing happens in the time interval (t_1, t_2), when U_2 accelerates. Therefore ϕ cannot be the cause of any change of U_1, as in passing from E1 to E2 the presumed cause varies from a position dependent ϕ to a constant ϕ without any variation of the effect.

The criticism becomes even stronger if one considers not just one but several clocks U_1, U_1', U_1'',… at rest in different points of the line AB of the inertial reference frame S. When U_2 accelerates these clocks should be influenced differently, as ϕ depends on distance from U_2. But in reality nothing happens: observers can check that the delay with which a light signal originated near U_1 touches U_1', U_1'',… is the same before and after the time interval during which U_2 was accelerated.

We can conclude that the gravitational potential of the fictitious forces exerts no action on the clocks, contrary to Einstein's 1918 opinion. The gravitational fields in the accelerated systems are not ordinary static fields like that of the Earth, but arise from the accelerations of bodies (Sciama, 1953), (Ghosh, 2000). Einstein assumed that these fields had the same effect on clocks as ordinary fields, but we can now conclude that on this particular point he was not right, in spite of the very probable correctness of the general idea of equivalence between fictitious and gravitational forces. One finds an analogy in the magnetic field, which can be considered a dynamical manifestation of the electric field, but has quite different interaction properties.

We can claim, finally, that concerning the nature of the differential retardation of separating and reuniting clocks Einstein was much nearer to the truth in 1905 than in 1918.

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Empty Space-Time and the General Relativity Principle

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We investigate the properties of empty space-time proceeding from the general relativity principle. An infinite number of so-called covariant ether theories (CETs) have been found, which, like special relativity theory (SRT), satisfy all known experimental facts in the physics of empty space-time. A new approach to the problem of experimental testing of SRT is discussed. In particular, we show that covariant ether theories predict a dependence of Thomas-Wigner angle on the "absolute" velocity of an observer's reference frame. Hence, a measurement of this dependence is capable of distinguishing between SRT and CETs. It has been shown that the Lorentz ether theory is one of the CETs, corresponding to the admissible Galilean transformations in physical space-time. Hence, we conclude that SRT and Lorentz ether theory can be distinguished experimentally, at least in principle. A crucial experiment, based on the Mössbauer effect, is proposed.

1. Introduction

Modern physics accepts two relativity principles: the special relativity principle (SRP), which asserts that the equations of fundamental physics do not change (they are form-invariant) under transformations between inertial reference frames in an empty space, and the general relativity principle (GRP), which states that fundamental physical equations do not change their form (they are covariant) under transformations between any frames of reference. Strictly speaking, the GRP requires a covariance of physical equations with respect to "admissible" space-time transformations, which keep the requirements $g_{00}>0$ (when this metric coefficient is defined as positive), $g_{ij}dx^i dx^j<0$, where g is the metric tensor, and i, j=1...3.

Both relativity principles were introduced by Einstein at the beginning of 20th century: the SRP was the basis of special relativity, while the GRP in combination with the equivalence principle gave rise to general relativity theory.

A fundamental physical consequence of SRP is that it is impossible to discover the absolute velocity of an inertial reference frame. Einstein simply assumed that an absolute reference frame is absent in nature, and all inertial frames are equivalent to each other. On the other hand, Lorentz adopted an ether theory, where an absolute frame (non-entrained ether) does exist, though an absolute velocity cannot be detected due to some general properties of this ether (Lorentz ether theory). Henri Poincaré, admitting both approaches, focused his research on the symmetry properties of space-time, and together with Lorentz, developed a mathematical formalism of relativistic kinematics and dynamics, which was physically interpreted by Einstein on the basis of SRP.

For many years the two alternative physical theories (special relativity and Lorentz ether theory) were considered mathematically equivalent. In this contribution we look closer at this problem, applying the general relativity principle to explore the properties of empty space-time.

The GRP is one of the deepest principles of physics: it states that any phenomenon can be described from any reference frame that can be realized in nature (Møller, 1972; Logunov, 1987). The problem of an experimental test of GRP means nothing more than an experimental test of space-time homogeneity, causality principle, *etc.* These principles constitute the cornerstones of modern knowledge, and we simply accept their validity: otherwise the whole of modern physics would be destroyed.

There is a widespread opinion that the SRP is a direct consequence of the GRP in the case of inertial motion in an empty space. If it were actually so, there would be no meaning to testing the SRP experimentally. Indeed, in this case an experimental test of the SRP would mean simultaneously a test of GRP, which would seem to be impractical. However, the SRP is not, in general, a consequence of the GRP; it represents an independent physical assumption. Only in the case when empty space-time has a pseudo-Euclidean geometry with a Minkowskian metric* in any inertial frame (which special relativity theory demands) can we derive a form-invariance of physical equations with respect to the Lorentz transforms as the special inference from the covariance principle. At the same time, from the viewpoint of formal logic, we may adduce ether theories, where a metric in an arbitrary inertial frame depends on its velocity in the ether. If, nevertheless, the metric coefficients in such a coordinate geometry continue to be "admissible," the theory would be in agreement with the GRP, but contradict the SRP.

The aim of the present paper is to inspect the relationship between the SRP and the GRP more closely, as well as to analyse the experimental facts from this point of view. Section 2 derives some important consequences of the GRP applied to empty space. Section 3 investigates the properties of a hypothetical empty space-time with metrics that differ from the Minkowskian, and Section 4 describes the kinematics of covariant ether theories (CETs), which are developed on the basis of the GRP and symmetries of an empty space-time. In section 5 we propose an experiment which can distinguish CETs from SRT. Finally, Section 6 contains some conclusions.

* By definition, the Minkowskian metric tensor has the form $g_{00}=1, g_{11}=g_{22}=g_{33}=-1$, and all others $g_{\alpha\beta}=0$.

2. Consequences of GRP for the case of inertial motion in empty space

Let us write a space-time transformation between two inertial reference frames in the general form

$$x_{\alpha} = A_{\alpha\beta} x'^{\beta}, \tag{1}$$

where α, $\beta = 0...3$, x, x' are four-vectors in the inertial frames. It is known that the principle of space-time homogeneity ensures the linearity of this transformation (Fock, 1955). The GRP requires that the transformations **A** constitute a Lie group with ten parameters: four initial space-time coordinates, three Eulerian angles, and three projections of a relative velocity (Fock, 1955). Further, let us exclude the trivial space translations and rotations. In this case, the transformation depends upon a single vectorial parameter—relative velocity $\vec{v}$, *i.e.*

$$x_{\alpha} = \mathbf{A}_{\alpha\beta}(\vec{v}) x'^{\beta}. \tag{2}$$

The GRP also requires validity of the reciprocity principle (Terletskii, 1968): the mutual velocities of two inertial reference frames should differ only by sign

$$\mathbf{A}^{-1}(\vec{v}) = \mathbf{A}(-\vec{v}). \tag{3}$$

In its turn, the reciprocity principle ensures that det**A** = 1 (Terletskii, 1968). Thus, the transformations **A** are special orthogonal.

In fact, this is all that we can say about the properties of the matrix **A** proceeding from the GRP. In order to determine **A** in closed form, it is necessary to define a model of an inertial reference frame and make some additional physical assumptions. (For example, under Einstein's postulates, the matrix **A** is equal to the Lorentz matrix **L** in Cartesian inertial reference frames). For these reasons no one has pursued an analysis of the properties of empty space-time within the GRP. However, as we will see below, the limited information already obtained on the basis of the GRP about the matrix **A** is, nevertheless, sufficient to determine a number of general laws of inertial motion. This analysis would appear important for a better understanding of the experimental basis of SRT. Indeed, experiments dealing with inertial motion in empty space are often unambiguously considered tests of the SRP. However, if we show that the result of this or that experiment can be explained by the GRP alone, that it is a test of the GRP, not the SRP, then the experiment becomes useless for physics.

In order to continue this investigation, let us take a hypothetical assumption about the existence of an "absolute space" that has pseudo-Euclidean geometry with a Minkowskian metric. We designate a preferred frame attached to the "absolute space" K_0. We look for the possibility of measuring the absolute velocity $\vec{v}$ within a moving frame K for space-time transformation **A** in Eq. (2) with the properties defined above.

Consider two inertial frames K_1 and K_2 initially both at rest in the absolute frame K_0. The frame K_1 contains a device D to measure the absolute velocity of that frame (by means of internal measurement procedures). Initially,

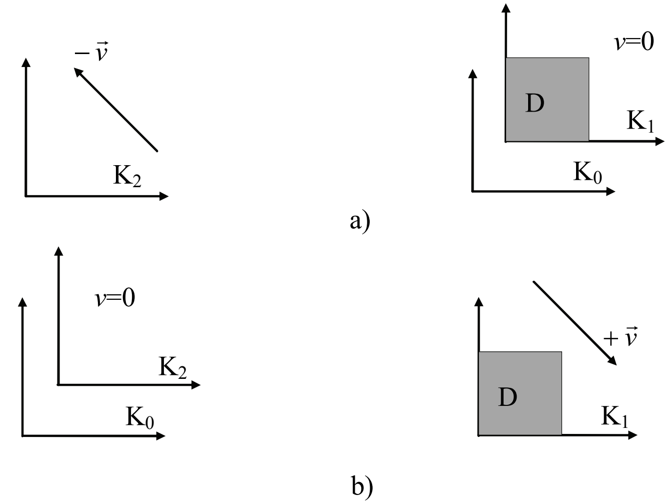

Fig. 1: a – the frame K_1 remains at rest in K_0, while the frame K_2 acquires a constant velocity $-\vec{v}$ in K_0; b – the frame K_2 rests in K_0, and K_1 moves at constant "absolute" velocity $+\vec{v}$ in K_0.

$v=0$, and hence, the device D stays in a state corresponding to $v=0$. Let us imagine that the frame K_2 acquires a constant absolute velocity $-\vec{v}$ (Fig. 1, a). This operation does not influence our device D in K_1, and thus, it remains in its original state. According to Eqs. (2), (3) we can denote a transformation from K_1 to K_2 $\mathbf{A}^{-1}(-\vec{v})=\mathbf{A}(\vec{v})$ and conclude that $\mathbf{A}(\vec{v})$ has no effect on the device D.

We now consider a different case, namely, where the frame K_2 remains at rest in the absolute frame K_0, while the frame K_1 containing the device D acquires a constant velocity $+\vec{v}$ in the frame K_0 (Fig. 1, b). For this case, a transformation from K_1 to K_2 (and K_0 as well) takes the same form, $\mathbf{A}(\vec{v})$, as for Fig. 1, a. On the other hand, it was found that the $\mathbf{A}(\vec{v})$ has no effect on the state of the device D. Thus, according to the GRP, an absolute velocity is not observable in this kind of experiment. It can easily be seen that this kind of experiment corresponds to a case where all inertial parts of the device D are at rest in the (laboratory) frame K_1.

Thus, we find the *first general implication of the GRP* with respect to inertial motion in an empty space: *no absolute motion with constant velocity can be detected by a device whose inertial parts are all at rest with respect to one another***.** This theorem explains the null results of all interference experiments that search for "ether wind," beginning with Michelson-Morley.

Let us now consider a device whose inertial parts moving at constant nonzero relative velocities with respect to one another in the frame K_1. Because each part has a different constant velocity $\vec{u}_i$ in K_1 it can be attached to its own proper inertial frame K_i. Then for the first motion diagram (K_1 at rest in K_0, K_2 moving at the constant velocity $-\vec{v}$ in the absolute frame K_0, Fig. 2, a), a transformation from each K_i to K_2 takes on the form $\mathbf{A}^{-1}(-\vec{v})\mathbf{A}(\vec{u}_i)=\mathbf{A}(\vec{v})\mathbf{A}(\vec{u}_i)$.

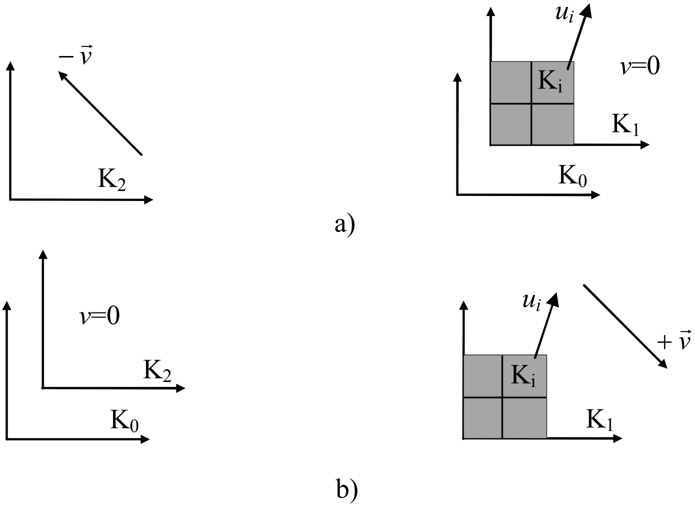

Fig. 2: a – the frame K_1 rests in K_0, and the frame K_2 acquires a constant "absolute" velocity $-\vec{v}$ in K_0; b – the frame K_2 remains at rest in K_0, while K_1 moves at a constant "absolute" velocity $+\vec{v}$ in K_0. A measuring instrument D contains the moving elements to be attached to the inertial reference frames K_i.

Due to the fact that motion of the frame K_2 has no effect on D (which belongs to K_1), we can conclude that this transformation leaves all parts of the device D in the original state, regardless of the value of the index *i*, details of the transformation itself, and particular construction of the device. In order to find the indication of D for the second motion diagram (K_2 at rest in K_0, K_1 moving at the constant velocity $+\vec{v}$ in K_0, Fig. 2, b), we can assume, in general, two kinds of transformations from each K_i to K_2 (and K_0 as well): $\mathbf{A}(\vec{v})\mathbf{A}(\vec{u}_i)$ and $\mathbf{A}(\vec{v}\oplus\vec{u}_i)$. These transformations, generally, are not equal to each other, since orthogonal transformations are not commutative (the group of space-time transformations is non-Abelian). We proved above that the first $\mathbf{A}(\vec{v})\mathbf{A}(\vec{u}_i)$ transformation does not change the state of D. Simultaneously we conclude that the other transformation $\mathbf{A}(\vec{v}\oplus\vec{u}_i)$, being different from $\mathbf{A}(\vec{v})\mathbf{A}(\vec{u}_i)$, changes the state of the measuring device D. Therefore, it is able to describe the difference of readings of D under "absolute rest" and "absolute motion" of the inertial reference frame K_1. This again shows that the GRP and SRP (where such a situation is impossible) represent two independent physical assumptions: generally speaking, the GRP does not forbid the existence of absolute space.

There is only one particular case ($\vec{v}$ collinear to all $\vec{u}_i$) where $\mathbf{A}(\vec{v})\mathbf{A}(\vec{u}_i)=\mathbf{A}(\vec{v}\oplus\vec{u}_i)$, and the state of the device D does not depend on the absolute velocity $\vec{v}$ of frame K_1 regardless of the particular construction of D.

Hence, we obtain the *second general implication of GRP* with respect to inertial motion in empty space: *no absolute motion with constant velocity can be detected by a device whose inertial parts move parallel (or opposite) to the absolute velocity*.

One can show that the two general implications of the GRP obtained above, taken together, explain the null results of all experiments that search for "ether wind" velocity performed up to date (Kholmetskii, 1997).

We stress that the first and second implications of the GRP were obtained in quite general form, and they do not depend on any specific construction of the device D or a specific choice of the transformation **A**.

The independence of these inferences from any specific structure of the device D is of primary importance. It means that a device D can be constructed on the basis of any known form of interaction, and it can be either macroscopic or microscopic. Hence, both inferences obtained above belong to relativistic kinematics, and also remain valid for any other area of physics.

The independence of these inferences from a particular choice of the transformation A in Eq. (2) appears to contradict the mainstream opinion that the Lorentz transformations **L** exclusively describe phenomena with non-observable absolute velocity in an empty space-time. However, our consideration of general motion diagrams in Figs. 1 and 2 indicates that this opinion is erroneous. Further we will actually show that all available experimental facts in the physics of empty space can be explained under any admissible transformation **A**. A concurrent problem is to understand the physical meaning of ether space-time theories with different transformations **A**, as well as different successive their actions for the motion diagram in Fig. 2, b. In order to solve these problems, it is necessary to analyse more closely the properties of empty space-time under an ether hypothesis.

3. Pseudo-Euclidean empty space-time with oblique-angled metrics

We will further analyse ether theories which adopt pseudo-Euclidean geometry with Minkowskian metrics for "absolute space." Since the motion of an arbitrary inertial frame does not influence the geometry of absolute space, it continues to be pseudo-Euclidean for any moving inertial observer. However, due to the possible dependence of space and time intervals on absolute velocity admitted in ether theories, the metric tensor g in moving frames is no longer Minkowskian. This means that physical space-time four-vectors in an arbitrary inertial frame should be linear functions of Minkowskian four-vectors x_L :

$$\left(x_{\mathrm{ph}}\right)_{\alpha} = B_{\alpha\beta}\left(x_{\mathrm{L}}\right)^{\beta}, \tag{4}$$

where the coefficients $B_{\alpha\beta}$ do not depend on space-time coordinates of a moving inertial frame; they depend only on its absolute velocity $\vec{v}$. This statement follows from space-time homogeneity (Kholmetskii, 1997). Such a pseudo-Euclidean geometry has the so-called oblique-angled metric. Here x_{L} obey the Lorentz transformation **L**:

$$x_{\mathrm{L}\alpha} = \mathbf{L}_{\alpha\beta} x_{\mathrm{L}}^{\prime\beta} . \tag{5}$$

In any analysis of space-time with an oblique-angled metric, one essential methodological feature has to be taken into account. Although this feature was stressed many years ago by Reichenbach (1920), present-day ether theories do not take it into account explicitly.

It seems natural to believe that in any inertial reference frame we may construct a method for measurement of space and time intervals such that the measurement result directly gives the physical magnitude of the corresponding interval. But strictly speaking, this is a property exclusive to pseudo-Euclidean geometry with Minkowskian metrics. Only in this kind of geometry can we omit a distinction between physical space-time four-vectors and four-vectors obtained via measurements (Logunov, 1987; Kholmetskii, 1997; Kholmetskii, 2003a; Kholmetskii, 2003b; Kholmetskii, 2004a). That is, in general, we have "measured" x_{ex} and physical x_{ph} four-vectors, and only in pseudo-Euclidean geometry with Minkowskian metrics do we have

$$x_{\text{ph}} = x_{\text{ex}} = x_{\text{L}} . \tag{6}$$

The essential property of space-time with oblique-angled metrics is the difference between measured and physical space-time four-vectors in arbitrary inertial reference frames: $x_{\text{ph}} \neq x_{\text{ex}}$. The need to distinguish them can be easily demonstrated with the Fitzgerald-Lorentz contraction hypothesis, which was first invoked to explain the null result of the Michelson-Morley experiment. According to this hypothesis, if a rod initially at rest in the absolute frame has the length l, then under motion at a constant absolute speed v along its axis, the length of the rod becomes $l\sqrt{1-v^2/c^2}$. However, due to proportional contraction of the unit scale in an attached inertial reference frame, an experimenter in this frame measures the same length l as in the case $v = 0$: Fitzgerald-Lorentz contraction is not observable. Thus, we see that the length of the rod in physical space-time is $l_{\text{ph}} = l\sqrt{1-v^2/c^2}$, while the measured length is equal to $l_{\text{ex}} = l$, and $l_{\text{ph}} \neq l_{\text{ex}}$. One can easily demonstrate that the same conclusion is valid for time intervals in an empty space-time with the oblique-angled metrics: $t_{\text{ph}} \neq t_{\text{ex}}$. Thus, the four-vectors in physical space-time (hereinafter "physical" four-vectors) are not equal to the four-vectors whose components were obtained *via* a measurement of corresponding space and time intervals (hereinafter "measured" four-vectors). The measurements may involve, for example, unit scales and standard clocks synchronized by light exchange (the Einstein model of inertial reference frame). By definition, the indications of these instruments are taken as "measured" spatial and time intervals. However, only in the absolute frame are these "measured" values equal to the corresponding physical quantities. Such equality is no longer valid in an arbitrary inertial frame with oblique-angled metrics, and the "physical" four-vectors become unobservable. However, this fact does not yet mean that they should be excluded from the ether space-time theory. Hence, in any alternative to SRT theory we will derive the transformation rules for both kinds of four-vectors, when the condition (6) remains valid only for absolute space:

$$\left(x'_{\mathrm{ph}}\right)_{\alpha} \doteq \left(x'_{\mathrm{ex}}\right)_{\alpha} \doteq \left(x'_{\mathrm{L}}\right)_{\alpha} . \tag{7}$$

(Hereinafter the primed four-vectors belong to the absolute frame.) This problem will be considered in the next Section.

4. Covariant ether theories

First of all, we notice that Eq. (4) under the condition (7) means that the matrix **B** becomes equal to the unit matrix when $v = 0$, and

$$\left(x_{\mathrm{L}}\right)_{\alpha} = \left(x_{\mathrm{ph}}\right)_{\alpha} (v = 0) . \tag{8}$$

This allows one to rewrite Eq. (4) in the form

$$\left(x_{\mathrm{ph}}\right)_{\alpha} (\vec{v}) = B_{\alpha\beta}(\vec{v}) \left(x_{\mathrm{ph}}\right)^{\beta} (v = 0) , \tag{9}$$

which clearly indicates a physical meaning for the matrix **B**: it describes the dependence of physical space and time intervals in a moving inertial frame on its absolute velocity $\vec{v}$.

Further, let us write a relationship between time components of the four-vectors x_{ph} and x_{L}, proceeding from Eq. (4):

$$\left(x_{\mathrm{ph}}\right)_{0} = B_{00} \left(x_{\mathrm{L}}\right)^{0} + B_{0i} \left(x_{\mathrm{L}}\right)^{i} \tag{10}$$

($i = 1..3$). For two events at a fixed spatial point $\left(x_{\mathrm{ph}}\right)^{i} = 0$

$$\left(x_{\mathrm{ph}}\right)_{0} (\vec{v}) = B_{00} \left(x_{\mathrm{L}}\right)^{0} = B_{00} \left(x_{\mathrm{ph}}\right)^{0} (v = 0) . \tag{11}$$

Hence, the coefficient B_{00} describes the change of clock rate under its motion at the constant absolute velocity $\vec{v}$. The change takes place for both standard and physical time intervals. Therefore, the measured time interval at a fixed spatial point is

$$\left(x_{\mathrm{ex}}\right)_{0} = \left(x_{\mathrm{ph}}\right)^{0} \big/ B_{00} . \tag{12}$$

For time intervals at two different spatial points, separated by the distance $\left(x_{\mathrm{ph}}\right)_{i}$, we must write

$$\left(x_{\mathrm{ex}}\right)_{0} = \left[\left(x_{\mathrm{ph}}\right)_{0} + \Delta\left(x_{\mathrm{ph}}\right)_{0}\right] \big/ B_{00} . \tag{13}$$

where $\Delta\left(x_{\mathrm{ph}}\right)_{0}$ is the error of synchronization of clocks separated by the distance $\left(x_{\mathrm{ph}}\right)_{i}$ in oblique-angled space-time. (It appears due to possible anisotropy of light velocity in different directions under Einstein's synchronization of distant clocks.) The value of $\Delta\left(x_{\mathrm{ph}}\right)_{0}$ can be found from the equality

$$\left(x_{\mathrm{ph}}\right)_{02} = \left(x_{\mathrm{ph}}\right)_{01} \big/ 2 \tag{14}$$

(Einstein's synchronization method), where $\left(x_{\mathrm{ph}}\right)_{01}$ stands for the time of light propagation from the first clock Cl_1 (at the origin of coordinates) to the second clock Cl_2 (at the point $\left(x_{\mathrm{ph}}\right)_{i}$) and back according to Cl_1, while $\left(x_{\mathrm{ph}}\right)_{02}$ is the reading of Cl_2 at the moment of arrival of the light pulse. For oblique-angled space-time the propagation time of light from Cl_1 to Cl_2 $\left(x_{\mathrm{ph}}\right)_{0+}$ is not equal, in general, to the propagation time in the reverse direction $\left(x_{\mathrm{ph}}\right)_{0-}$. Thus, implementation of the equality (14) is possible only in the case where the readings of both clocks at the initial moment of time differ by the value $\Delta\left(x_{\mathrm{ph}}\right)_0$, and

$$\left(x_{\mathrm{ph}}\right)_{01}=\left[\left(x_{\mathrm{ph}}\right)_{0+}+\left(x_{\mathrm{ph}}\right)_{0-}\right],\ \left(x_{\mathrm{ph}}\right)_{02}=\left[\left(x_{\mathrm{ph}}\right)_{0+}+\Delta\left(x_{\mathrm{ph}}\right)_{0}\right], \tag{15}$$

Taking Eq. (14) into account, we obtain:

$$\Delta\left(x_{\mathrm{ph}}\right)_0=\left[\left(x_{\mathrm{ph}}\right)_{0-}-\left(x_{\mathrm{ph}}\right)_{0+}\right]/2\,. \tag{16}$$

Expressions for $\left(x_{\mathrm{ph}}\right)_{0+}$ and $\left(x_{\mathrm{ph}}\right)_{0-}$ can be found from Eq. (4):

$$\left(x_{\mathrm{ph}}\right)_{0+}=B_{00}\left(x_{\mathrm{L}}\right)^0+B_{0i}\left(x_{\mathrm{L}}\right)^i,\ \left(x_{\mathrm{ph}}\right)_{0-}=B_{00}\left(x_{\mathrm{L}}\right)^0-B_{0i}\left(x_{\mathrm{L}}\right)^i. \tag{17}$$

Substituting Eq. (17) into Eq. (16), we obtain:

$$\Delta\left(x_{\mathrm{ph}}\right)_0=-B_{0i}\left(x_{\mathrm{L}}\right)^i. \tag{18}$$

Further substitution of Eqs. (18) and (10) into Eq. (13) gives:

$$\left(x_{\mathrm{ex}}\right)_0=\left(x_{\mathrm{L}}\right)_0. \tag{19}$$

Thus, we have derived an important result: for any ether theory: when a Minkowskian metric of absolute space is adopted, the measured time intervals always obey the Lorentzian transformations.

When examining Eq. (19), we may ask the following question: does this equality continue to be valid for space intervals, too? In another words, would we get the equality

$$\left(x_{\mathrm{ex}}\right)_i=\left(x_{\mathrm{L}}\right)_i \tag{20}$$

for arbitrary admissible matrix **B** in Eq. (4)? In general, this is not the case. Let us show that Eq. (20) is realized only in the case where the coefficients B_{i0}=0. Indeed, write a relationship between space components of the four-vectors x_{ph} and x_{L}, proceeding from Eq. (4):

$$\left(x_{\mathrm{ph}}\right)_i=B_{i\alpha}\left(x_{\mathrm{L}}\right)^\alpha=B_{i0}\left(x_{\mathrm{L}}\right)^0+B_{ij}\left(x_{\mathrm{L}}\right)^j. \tag{21}$$

Introducing a unit scale $\left(x_{\mathrm{phu}}\right)_i$ in physical space-time, we can write the similar relation:

$$\left(x_{\mathrm{phu}}\right)_i=B_{i0}\left(x_{\mathrm{L}}\right)^0+B_{ij}\left(x_{\mathrm{Lu}}\right)^j, \tag{22}$$

where $\left(x_{\mathrm{Lu}}\right)^i$ is the corresponding unit scale in Minkowskian space. Dividing (21) by (22), we obtain:

$$\frac{\left(x_{\mathrm{ph}}\right)_i}{\left(x_{\mathrm{phu}}\right)_i}=\frac{B_{i0}\left(x_{\mathrm{L}}\right)^0+B_{ij}\left(x_{\mathrm{L}}\right)^j}{B_{i0}\left(x_{\mathrm{L}}\right)^0+B_{ik}\left(x_{\mathrm{Lu}}\right)^k}.$$

Taking into account the obvious equality for Minkowskian space

$$\left(x_{\mathrm{L}}\right)^j/\left(x_{\mathrm{L}}\right)^i=\left(x_{\mathrm{Lu}}\right)^j/\left(x_{\mathrm{Lu}}\right)^i,$$

we obtain, after straightforward manipulations:

$$\frac{x_{\mathrm{ph}i}}{x_{\mathrm{phu}i}}=\frac{\left(x_{\mathrm{L}}\right)_i}{\left(x_{\mathrm{Lu}}\right)_i}\left[\frac{1+B_{i0}\left(x_{\mathrm{L}}\right)^0/\left(x_{\mathrm{L}}\right)_i B_{ik}\left(x_{\mathrm{L}}\right)^k}{1+B_{i0}\left(x_{\mathrm{L}}\right)^0/\left(x_{\mathrm{Lu}}\right)_i B_{ij}\left(x_{\mathrm{L}}\right)^j}\right]. \tag{23}$$

This Eq. (23) proves our statement. Indeed, under B_{i0}=0 it transforms to

$$x_{\text{ph}i}/x_{\text{phu}i} = x_{\text{L}i}/x_{\text{Lu}i} ,$$

which is equivalent to Eq. (20): the measured scale in oblique-angled space-time coincides with its value in Minkowskian space-time. Then Eqs. (19), (20) are written simultaneously as

$$\left(x_{\text{ex}}\right)^{\alpha} = \left(x_{\text{L}}\right)^{\alpha}, \tag{24}$$

which means that a distinction of oblique-angled metrics in moving inertial frames from Minkowskian metrics is not experimentally observable. In another words, an observer in any inertial frame moving in absolute space sees the world, almost as in SRT, for an infinite set of ether space-time theories with B_{i0}=0. (However, this does not yet mean that SRT and all ether theories cannot be distinguished experimentally. This problem will be analysed below.)

Now let us determine a physical meaning of the equality B_{i0}=0 in Eq. (4). For this purpose we combine Eqs. (1), (5), (7) and derive:

$$\left(x_{\text{ph}}\right)_{\alpha} = A_{\alpha}^{\gamma}\left(\vec{v}\right)L_{\gamma\beta}^{-1}\left(\vec{v}\right)\left(x_{\text{L}}\right)^{\beta}. \tag{25}$$

Comparing Eq. (25) with Eq. (4), we find

$$B_{\alpha\beta}(\vec{v}) = A_{\alpha}^{\gamma}\left(\vec{v}\right)L_{\gamma\beta}^{-1}\left(\vec{v}\right). \tag{26}$$

Substituting into Eq. (26) the known form of the matrix **L** (Møller, 1972), denoting $\gamma = 1/\sqrt{1-v^2/c^2}$, and using Eq. (3), we get:

$$B_{00} = \gamma/A_{00} , \tag{27a}$$

$$B_{i0} = 0, \tag{27b}$$

$$B_{0i} = A_{0i} + A_{00} \times \left[\frac{v_i}{c^2}\gamma + \left(\frac{1}{A_{00}^2} - 1\right)\frac{v_i}{v^2}\left(\gamma - 1\right)\right], \tag{27c}$$

$$B_{ij} = A_{ij} + A_{i0}\frac{v_j}{v^2}\left(1 - 1/\gamma\right). \tag{27d}$$

We see that the coefficient B_{i0} is equal to zero for any matrix **A.** It can be shown that the reciprocity principle (3) ensures this result.

Thus, we conclude that the equality B_{i0} *= 0 represents an equivalent form of the reciprocity principle, resulting from the GRP.*

Therefore, for any admissible transformation **B** satisfying the GRP, an experimenter will not detect a distinction between the oblique-angled metrics of his coordinate geometry and Minkowskian metrics. This is why all the experiments to verify the Lorentz transforms (beginning with the Michelson-Morley experiment and ending with modern experiments that search for ether velocity (Champeney *et al.*, 1963; Chialdea, 1963; Nikolenko *et al.*, 1979)) find an infinite number of alternative explanations of their results. This conclusion is in full accordance with the implications of the GRP obtained above for the case of inertial motion in empty space-time.

We will call acceptable theories “covariant ether theories” (CETs). The exclusive place of SRT among all such CETs is defined by the fact that it directly asserts the equality of measured and physical space-time four-vectors, *i.e.*, the equality of the matrices **A** and **L**, which means a Minkowskian metric of physical space-time in any inertial reference frame. In the alternative assumption $\mathbf{A} \neq \mathbf{L}$, the metric of physical space-time is, in general, oblique-angled, and we must distinguish space-time transformations for physical and measured four-vectors:

$$\left(x_{\mathrm{ph}}\right)_{\alpha} = A_{\alpha\beta}\left(x'_{\mathrm{ph}}\right)^{\beta},\ \left(x_{\mathrm{ex}}\right)_{\alpha} = L_{\alpha\beta}\left(x'_{\mathrm{ex}}\right)^{\beta}, \tag{28}$$

where the primed four-vectors, as before, belong to the absolute frame K_0. This means that these transformations do not yet solve the main kinematical problem (determination of space-time transformations between two arbitrary inertial frames): they act only in the special case where one of the frames is absolute. In order to find a transformation between two arbitrary inertial frames K and K″, we must write

$$\left(x_{\mathrm{ex}}\right)_{\alpha} = L_{\alpha\beta}\left(\vec{v}_1\right)\left(x'_{\mathrm{ex}}\right)^{\beta};\left(x''_{\mathrm{ex}}\right)_{\alpha} = L_{\alpha\beta}\left(\vec{v}_2\right)\left(x'_{\mathrm{ex}}\right)^{\beta}, \tag{29}$$

$$\left(x_{\mathrm{ph}}\right)_{\alpha} = A_{\alpha\beta}\left(\vec{v}_1\right)\left(x'_{\mathrm{ph}}\right)^{\beta};\left(x''_{\mathrm{ph}}\right)_{\alpha} = A_{\alpha\beta}\left(\vec{v}_2\right)\left(x'_{\mathrm{ph}}\right)^{\beta}, \tag{30}$$

where $\vec{v}_1, \vec{v}_2$ are the absolute velocities of the frames K and K″, respectively. Eliminating four-vector x'^{β}_{ex} from Eqs. (29), and x'^{β}_{ph} from Eq. (30), we obtain general transformations for measured and physical space-time four-vectors in two arbitrary inertial frames:

$$\left(x_{\mathrm{ex}}\right)_{\alpha} = L_{\alpha\beta}\left(\vec{v}_1\right)\left[L^{-1}\left(\vec{v}_2\right)\right]^{\beta\gamma}\left(x''_{\mathrm{ex}}\right)_{\gamma}, \tag{31}$$

$$\left(x_{\mathrm{ph}}\right)_{\alpha} = A_{\alpha\beta}\left(\vec{v}_1\right)\left[A^{-1}\left(\vec{v}_2\right)\right]^{\beta\gamma}\left(x''_{\mathrm{ph}}\right)_{\gamma}, \tag{32}$$

where the matrix **A** can be taken in arbitrary admissible form. Thus in contrast to SRT, under the hypothesis $\mathbf{A} \neq \mathbf{L}$, nature does not “know” a direct relative velocity of two arbitrary inertial frames K and K″: it is always composed as a sum $\vec{v}_1 \oplus \vec{v}_2$, where $\vec{v}_1$ and $\vec{v}_2$ are the corresponding velocities of K and K″ in the absolute frame K_0. This means, in particular, that a direct rotation-free Lorentz transformation between measured space-time four-vectors in K and K″ is impossible: by the general group properties of these transformations, an additional rotation of the coordinate axes of the frames K and K″ appears at the Thomas-Wigner angle Ω, depending on $\vec{v}_1$ and $\vec{v}_2$. It is quite important that such a rotation occurs in measured space-time coordinates: *i.e.*, it can actually be detected. It defines a prime possibility to experimentally distinguish the hypotheses $\mathbf{A} = \mathbf{L}$ and $\mathbf{A} \neq \mathbf{L}$. We also notice that for collinear $\vec{v}_1$ and $\vec{v}_2$, $\Omega=0$, and the absolute velocity is not observable. This result corresponds to the second implication of GRP, obtained in Section 2. Hence, in corresponding experiments to test CETs, these velocities should be non-collinear.

Among admissible space-time theories that assume $\mathbf{A} \neq \mathbf{L}$, the simplest case corresponds to the choice $\mathbf{A} = \mathbf{G}$, where **G** is the matrix of the Galilean transformation: $G_{\alpha\alpha} = 1$, $G_{i0} = -v_i$, and all others $G_{\alpha\beta} = 0$. Inserting matrix **G** in place of matrix **A** in Eqs. (27), we get the following coefficients of **B**:

$$B_{00}=\gamma, B_{i0}=0, B_{0i}=\frac{v_i}{c^2}\gamma, B_{ij}=\delta_{ij}\frac{v_i v_j}{v^2}(1-1/\gamma) \tag{33}$$

where δ_{ij} is the Kronecker symbol. Further substitution of Eq. (33) into Eq (9) allows us to determine a dependence of physical space-time four-vectors on the absolute velocity $\vec{v}$ of some arbitrary inertial reference frame K:

$$\vec{r}_{\text{ph}}(\vec{v})=\vec{r}_{\text{ph}}(v=0)+\frac{\vec{v}\left[\vec{r}_{\text{ph}}(v=0),\vec{v}\right]}{v^2}\left[\sqrt{1-\left(v^2/c^2\right)}-1\right], \tag{34}$$

$$t_{\text{ph}}(\vec{v})=\frac{t_{\text{ph}}(v=0)}{\sqrt{1-\left(v^2/c^2\right)}}+\frac{\vec{r}_{\text{ph}}(v=0)\vec{v}}{c^2\sqrt{1-\left(v^2/c^2\right)}}. \tag{35}$$

For the time interval in a fixed spatial point of the frame K ($r_{\text{ph}}=0$), we obtain a dependence of t_{ph} on $\vec{v}$ (see, Eq. (35)):

$$t_{\text{ph}}(\vec{v})=t_{\text{ph}}(v=0)/\sqrt{1-\left(v^2/c^2\right)}, \tag{36}$$

that means an absolute dilation of time by factor $\sqrt{1-\left(v^2/c^2\right)}$. Furthermore, one obtains from Eq. (34):

$$\left[\vec{r}_{\text{ph}}(\vec{v})\cdot\vec{v}\right]=\left[\vec{r}_{\text{ph}}(v=0)\cdot\vec{v}\right]\sqrt{1-v^2/c^2},\ \left[\vec{r}_{\text{ph}}(\vec{v})\times\vec{v}\right]=\left[\vec{r}_{\text{ph}}(v=0)\times\vec{v}\right], \tag{37}$$

that means an absolute contraction of moving scale along a vector of absolute velocity by factor $\sqrt{1-\left(v^2/c^2\right)}$ (Fitzgerald-Lorentz hypothesis). Finally, transformation (1) (under **A**=**G**)

$$\left(x_{\text{ph}}\right)_\alpha=\left[G_{\alpha\beta}\left(\vec{v}_1-\vec{v}_2\right)\right]\left(x''_{\text{ph}}\right)^\beta$$

leads to the Galilean law of speed addition for the physical light velocity c_{ph}.

Thus, we obtain the full set of Lorentz ether postulates in case **A** = **G**.* However, the physical space-time in the Lorentz ether theory is not observable in an arbitrary inertial reference frame, while the measured four-vectors x_{ex} obey the Lorentz transformations in the form of (31). (This important circumstance concerning a difference of physical and measured four-vectors for oblique-angled metrics of space-time was dropped by Lorentz and his successors). Therefore, we may regard the Lorentz ether theory (LET) as one of the CETs defined above, and the simplest among them. Due to this fact, the Lorentz ether postulates have always been successfully applied to explain "null" results of all experiments that search for "ether wind speed." At the same time, we may now we proceed not only from the Lorentz ether postulates, but from the complete kinematics of LET. Its full description is given by the following equations, obtained above:

* Let us recall the postulates of Lorentz ether theory in its modern form (see, e.g. (Mansouri and Sexl, 1977)):
1) There is an absolute reference frame K_0, wherein light velocity is isotropic and equal to c. **2)** In an arbitrary reference frame K, moving at constant velocity $\vec{v}$ in K_0, the velocity of light is equal to $\vec{c}'=\vec{c}-\vec{v}$. **3)** In this reference frame K time is dilated by $\sqrt{1-v^2/c^2}$ times. **4)** In this reference frame K a linear scale is contracted by $\sqrt{1-v^2/c^2}$ times along the vector $\vec{v}$.

$$\left(x'_{\text{ph}}\right)_\alpha \doteq \left(x'_{\text{ex}}\right)_\alpha \text{ (for the absolute frame),} \tag{38}$$

$$\left(x_{\text{ex}}\right)_\alpha = L_{\alpha\beta}\left(\vec{v}_1\right)\left[L^{-1}\left(\vec{v}_2\right)\right]^{\beta\gamma}\left(x''_{\text{ex}}\right)_\gamma \tag{39}$$

$$\left(x_{\text{ph}}\right)_\alpha = \left[G_{\alpha\beta}\left(\vec{v}_1 - \vec{v}_2\right)\right]\left(x''_{\text{ph}}\right)^\beta \tag{40}$$

$$\vec{r}_{\text{ph}}\left(\vec{v}\right) = \vec{r}_{\text{ph}}\left(v=0\right) + \vec{v}\left[\vec{r}_{\text{ph}}\left(v=0\right), \vec{v}\right]\left[\sqrt{1-\left(v^2/c^2\right)} - 1\right] \Big/ v^2 , \tag{41}$$

$$t_{\text{ph}}\left(\vec{v}\right) = \frac{t_{\text{ph}}\left(v=0\right)}{\sqrt{1-\left(v^2/c^2\right)}} + \frac{\vec{r}_{\text{ph}}\left(v=0\right)\vec{v}}{c^2\sqrt{1-\left(v^2/c^2\right)}} . \tag{42}$$

Note that the Galilean transformation itself does not restrict the value of limited velocity; it can be infinite. However, in case of LET a restriction is established by the equality $\left(x'_{\text{ph}}\right)_\alpha \doteq \left(x'_{\text{L}}\right)_\alpha$, which simultaneously means that the Galilean transformations in LET are "admissible," and they act within pseudo-Euclidean geometry.

The most important physical consequence of the kinematics of LET, expressed by Eqs. (38)-(42), is the possibility of experimentally detecting absolute velocity of an inertial reference frame, due to the above-mentioned dependence of the Thomas-Wigner angle Ω on absolute velocity $\vec{v}$. It can be seen in the problem of diametrical synchronization of distant clocks by a moving rod, as follows.

Let two clocks Cl_1 and Cl_2 be placed upon the x-axis of some inertial reference frame K at rest in the absolute frame K_0. The distance between Cl_1 and Cl_2 is equal to L. Let some rod with a proper length L move along the y-axis at a constant velocity $\vec{u}$. The axis of the rod is parallel to the x-axis, and the coordinates of its opposite ends on the x-axis coincide with the respective coordinates of Cl_1 and Cl_2. So at the instant when the rod intersects the x-axis, it is simultaneously touching Cl_1 and Cl_2. We assume that, at the moment of touching, both clocks emit a short light pulse towards the time analyzer (TA) placed between them. Thus, the reading of TA is $\Delta t = 0$.

Now consider the same problem when the frame K moves at the constant absolute velocity $\vec{v}$ along the x-axis (Fig. 3). It is necessary to find an indication Δt of TA in the laboratory frame K.

We attach the frame K_r to the moving rod. The velocity of K_r with respect to K is equal to u along the y axis, while the velocity of K with respect to K_0 is equal to v along the axis x. In order to calculate the value Δt within SRT, one should apply a special Lorentz transformation from K_r to K. Hence, we get $\Delta t = 0$, as in the case $v = 0$.

Let us calculate the value Δt for Fig. 3 in the Lorentz ether theory ($\mathbf{A} = \mathbf{G}$). In this case, the measured space-time coordinates x_{ex} are subjected to the transformation (31), according to which we should apply the series $K_r \to K_0 \to K$ with the velocities $\vec{V} = \vec{v} \oplus \vec{u}$ and $\vec{v}$, respectively. This transformation for the x_{ex} coordinates entails a relative rotation of K_r and K coordi-

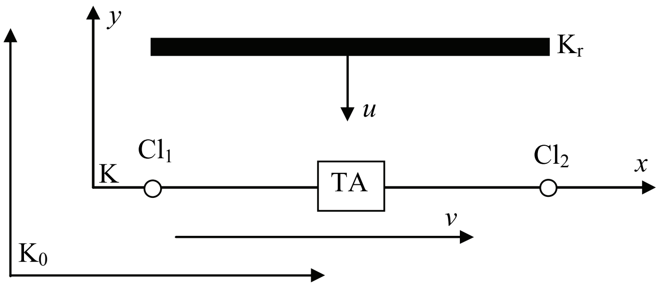

Fig. 3. The scheme of 'diametrical' synchronization of distant clocks by moving 'ideal' rod.

nate axes at the angle $\Omega \approx uv/2c^2$. Hence, at the instant when the left end of rod touches the Cl_1, its right end has a non-zero coordinate $\Omega L \approx Luv/2c^2$ on the axis y. From this

$$\Delta t \approx \Omega L/u = Lv/2c^2 \ . \tag{43}$$

Since the measured light velocity is isotropic, Eq. (43) directly gives the reading of TA. However, Eq. (43) has no physical interpretation in the measured space-time coordinates. At the same time, this equation does have a clear physical meaning in physical space-time. Indeed, here we get an absolute contraction of a moving rod in the direction of its resultant absolute velocity $\vec{V} = \vec{v} \oplus \vec{u}$. According to Eq. (41), the projection of the rod perpendicular to $\vec{V}$ remains unchanged. Let us denote it as $L\sin\alpha$, where α is the angle between $\vec{L}$ and $\vec{V}$. A projection of the rod, which is parallel to $\vec{V}$, becomes equal to $L\sqrt{1-V^2/c^2}\cos\alpha$. As a result, the axis of the rod turns with respect to the x-axis through the angle $\varphi \approx uv/2c^2$ in comparison with the case $v = 0$ (to order of approximation c^{-2}). Further, the physical light velocity along the x-axis of the laboratory frame K is equal to $c_+ = c - v$, and in the opposite direction $c_- = c + v$. Hence, the reading of TA is

$$\Delta t \approx \frac{\varphi L}{u} + \frac{L}{2(c+v)} - \frac{L}{2(c-v)} \approx \frac{Lv}{2c^2} \ .$$

This coincides with Eq. (43). Thus, Eq. (43) can be interpreted as the real appearance of the properties of physical space-time, in spite of the impossibility of directly measuring x_{ph} . In particular, the calculations presented show that Eq. (43) implies an absolute contraction of the rod as well as anisotropy of physical light speed c_{ph} in the moving laboratory frame K. From a formal viewpoint, this result follows from the dependence of Ω on $\vec{v}$ in experimentally measured coordinates x_{ex} caused by the general transformation rule (39). Thus, a formal application of the transformation (39) for Minkowskian four-vectors x_{L} (leading to the measurable dependence of Ω on $\vec{v}$) finds a physical interpretation only in the x_{ph} coordinates, despite the impossibility of observing the x_{ph} four-vectors experimentally.

Lastly, we may add that the solution (43) can also be produced by conventional relativistic calculations when the relative velocity of K and K_0 is equal to

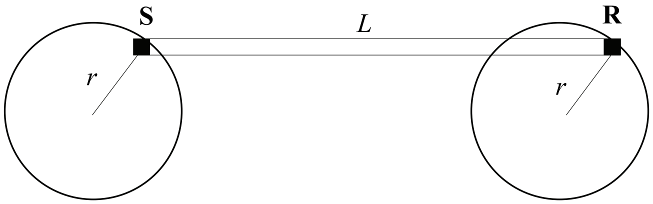

Fig. 4. Schematic of the experiment for measurement of an absolute velocity

$\vec{v}$, while the relative velocity of K_r and K_0 is equal to $\vec{V} = \vec{v} \oplus \vec{u}$. However, the motion diagram of the frames K_r, K in K_0 differs from motion diagram in Fig. 3. Thus, the solution (43) and the solution $\Delta t = 0$ within SRT correspond to different physical problems, while for LET these solutions reflect a dependence of Ω on $\vec{v}$ for a fixed experimental instrument (Cl_1, Cl_2 + moving rod).

5. Proposed experiment to test CETs

The possibility of distinguishing SRT from LET (as one of the CETs) experimentally can be realized in the following experiment. Let there be two rotors with equal radius r, lying on the same plane and separated by distance L. By synchronous rotation at the angular frequency ω these rotors drive a rod of length L, as shown in Fig. 4. The source of electromagnetic radiation S and receiver R are fixed on the opposite sides of the rod. In this geometry we measure a relative Doppler shift between the emitted and absorbed radiation, which, as we will see below, is a function of the "absolute" velocity of Earth $\vec{v}$. In order to obtain this function in an explicit form, let us consider a diagram of source and receiver velocities in the absolute frame (Fig. 5). In this diagram the vector of tangential velocity $\vec{u}$ rotates clockwise at the angular frequency ω for both source and receiver. For simplicity we can imagine these vectors as "clock arrows," and within LET, where the Galilean transformations are valid, the physical directions of both "clock arrows" coincide at any fixed instant. (A similar diagram drawing according to SRT gives a corresponding retardation for the right "clock arrow.") However, during the time of light propagation from the first clock to the second clock, the right "arrow" has time to turn to an angle $\Delta\varphi$, which causes a corresponding linear Doppler shift between emission and absorption lines. It is obvious that the time of light beam propagation from S to R (and hence, the value of $\Delta\varphi$) depends on the angle between the vector of "absolute" velocity $\vec{v}$ and the line S-R. Therefore, the same dependence should be detected when the relative frequency shift between the emitted and absorbed radiation is measured. Indeed, when the vector $\vec{v}$ is parallel to the x-axis, straightforward calculations give the following expression for the frequency of absorbed radiation:

$$\nu_a = \nu_0 \left(1 - \frac{u\omega L \sin \omega t}{c^2} - \frac{uv\omega L \sin \omega t}{c^3} \right), \tag{44}$$

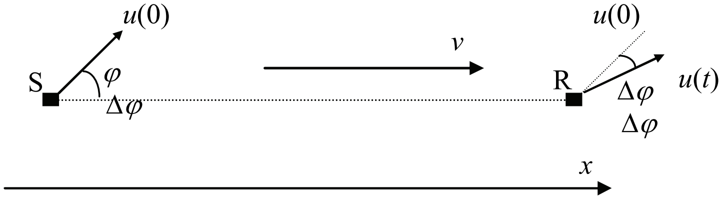

Fig. 5. Diagram of the velocities of source S and receiver R in the experiment of Fig. 4.

where ν_0 is the proper frequency. The second term in brackets on the *rhs* of Eq. (44) signifies "gravitational shift" of frequency in the non-inertial reference frame attached to the source and receiver. Indeed, in accordance with the equivalence principle, an oscillating gravitational field with the potential $u\omega\sin\omega t$ appears in this frame. And the measured frequency shift depends on the potential difference between S and R, according to known result of general relativity. The shift is vanishing for $\varphi = 0$, *i.e.*, for the case when the acceleration is perpendicular to the line S-R.

The most interesting is the third term in brackets of Eq. (44), which is proportional to the "absolute" velocity v. We see that for $\varphi = 0$ (*i.e.*, when the momentary velocities of the source and receiver are collinear to $\vec{v}$), this term becomes equal to zero. This result directly reflects the fact that under addition of Lorentz boosts with collinear velocities, the Lorentz operators commute with each other and an absolute velocity is not observable in CETs. Let us estimate this term numerically for $\sin\varphi = 1$. We adopt $L = 1$ m, the rotation frequency $\nu = 200$ Hz ($\omega \approx 1250$ Hz); the rotor's radius $r = 10$ cm ($u = \omega r = 125$ m/s); $v/c = 10^{-3}$. Then the relative frequency shift is equal to $\left(\nu_a - \nu_0\right)/\nu_0 =$ 1.7×10^{-15}. Such a shift can be measured by means of Mössbauer spectroscopy, when the method of resonant detection of gamma-quanta is applied (Kholmetskii and Missevitch, 1992).

Other schemes for experimental testing of LET have been proposed elsewhere (Kholmetskii, 2000; Kholmetskii, 2003a; Kholmetskii, 2004b).

6. Conclusions

(1) Consideration of all hypothetical ether theories of empty space-time that admit pseudo-Euclidean geometry with oblique-angled metrics in arbitrary inertial frames must be based on a distinction between physical and measured space-time four-vectors. A general analysis of the properties of admissible space-time transformations shows that in any theory which adopts the general relativity principle and space-time symmetries, the measured space and time intervals always obey the Lorentz transformation, regardless of the concrete choice of physical space-time transformation. The latter circumstance makes it possible to explain all known experimental results in the physics of empty space-time within an infinite number of admissible space-time theories, called "covariant ether theories."

(2) SRT is unique among admissible theories of empty space-time because it directly asserts the equality of measured and physical four-vectors under optimal measurements. The adoption of this equality defines the possibility of direct rotation-free Lorentz transformation between two arbitrary inertial frames. This is impossible in all other admissible space-time theories, termed covariant ether theories. An absolute motion of inertial frame in CETs actually induces an admissible coordinate transformation, which depends on absolute velocity. We stress that this coordinate transformation in CETs is an objective property of nature, in contrast to purely mathematical coordinate transformations in SRT. That is why CETs lead to a physical alternative to SRT. In particular, CETs predict a dependence of the Thomas-Wigner angle Ω on an "absolute" velocity $\vec{v}$, resulting from the transformation (31).

(3) It has been shown that the choice of Galilean transformation in physical space-time within covariant ether theories leads to the Lorentz ether theory. The kinematics of LET is then described by Eqs. (38)-(42). These show the possibility of measuring an absolute velocity experimentally. Therefore, an ether formulation of the special relativity principle, proposed by Lorentz within his LET, is not possible. In contrast, we arrive at the following conclusion: if the existence of an absolute frame is assumed, then an absolute velocity can be detected experimentally. One of the possible experiments to test LET, based on the Mössbauer effect, is proposed here.

Acknowledgement

The author thanks the editor for his kind invitation to contribute a paper to this book, Thomas E. Phipps, Jr., and anonymous referees for a careful reading of the manuscript and for their useful remarks, which were taken into account in the final version of the paper.

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Temporal Light Dispersion in Intergalactic Space

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An analogy between temporal dispersion and gravitational dispersion of light as observed by E. Fermi is combined with an idea proposed by F. Arago, who noted in 1857 that light from variable stars could show a difference in speed between components, to give an new model for the cosmological redshift and a new interpretation of Hubble's constant. This model shows that Hubble's constant depends on three parameters: the variation of the speed of light, the dispersive power of the environment, and the spectral width of the source. This approach eliminates the need to postulate the existence of dark matter and gives a new interpretation for microwave background radiation.

Keywords: Redshift, alternative cosmology, Hubble's constant

Introduction

As noted by E. Fermi[1] there is a strong analogy between the gravitational potential of Mechanics and the refractive index of Optics. For the metrics this analogy between gravitation and refraction is illustrated in Figures 1(a) and 1(b), and taking the analogy of these metrics into consideration, we can apply the ideas of light dispersion in dielectric media to develop a new interpretation of the cosmological redshift.

Light dispersion in interstellar space is not a new notion. It was postulated initially by Newton[2] in a letter to Flamsteed in 1691, where he discussed the issue of light rays with different wavelengths that propagated with exactly the same speed in interstellar space. Based on this, light dispersion in the interstellar environment might or might not exist. However, F. Arago[3] was the first to place the matter on a firm basis for interstellar space when he argued that variable stars could show a difference in speed between violet and blue. In 1881, through interstellar space measurements and using Fizeau's method, Young found differences in speed between red and blue rays of light, recording a 1% excess speed between the blue and red light[4]. This result was immediately contested by Rayleigh[5] There was a renewed interest in this field in 1908-

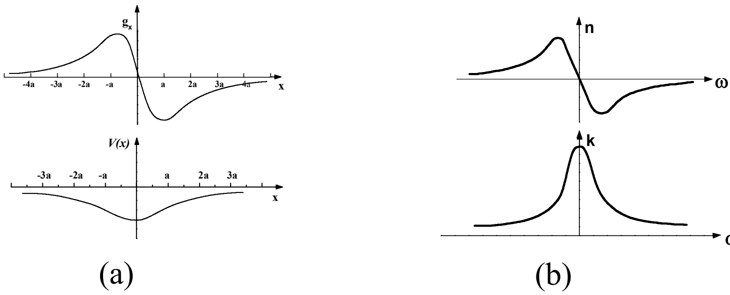


Figure 1 Analogy between optics and mechanics (a) Gravitational field and gravitational potential between two bodies with mass m. (b) Refractive index and extinction coefficient of a dielectric medium.

1909, when Nordmann[6,7] and Tikhoff [8] detected light dispersion in space when studying the light coming from the Cepheids. These results were completely rejected by Lebedew [9,10] and this type of study disappeared completely from the literature. Recently, Narlikar *et al.* [11], Amoroso *et al.*[12], and Vigier[13] have been involved with similar research but with a different approach.

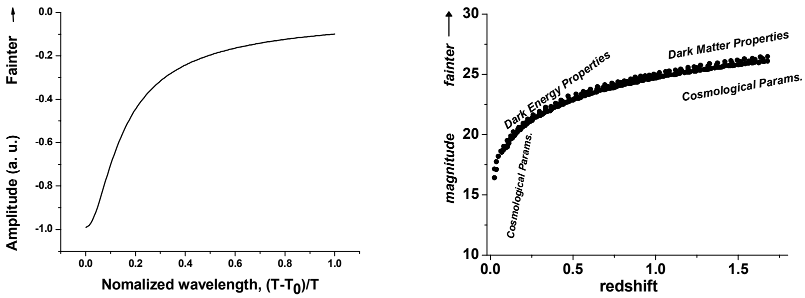
Light Dispersion in a Dielectric Medium

In optics, light dispersion means that there is a variation of the refractive index in relation to the wavelength as measured by $dn/d\lambda$. However, in the field of modern optical fiber communications, dispersion means the second derivative of the refractive index in relation to the wavelength $dn^2/d\lambda^2$, is important. This is called the temporal material dispersion, a phenomenon that occurs in the core of optical fibers due to the dependence of light speeds with wavelengths in dielectric environments[14,15]. Physically this implies that the phase velocity of a plane wave traveling in the dielectric varies nonlinearly with the wavelength and consequently, a light pulse will broaden as it travels through it.

In the optical fiber core, two rays of light travel at different speeds and there is a time delay between them given by:

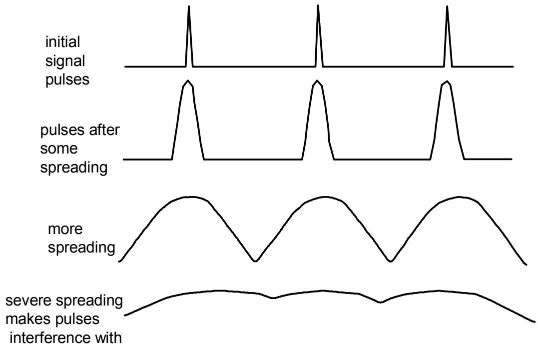
$$\Delta t = \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}\Delta\lambda L\,, \tag{1}$$

where c = speed of light, λ = wavelength, $\Delta\lambda$ = spectral width, $d^2n/d\lambda^2$ = the second derivative of the refractive index due to the wavelength, and L is the distance traveled by the light. It is noted that the delay is directly proportional to the distance traveled by light. When traveling through the fiber core, the light pulse broadens with time, while its amplitude decreases. Figures 2(a) and (c) show this phenomenon in a simple and well-known manner in the field of optical fibers. This phenomenon is very similar to the Doppler effect. In equation (1), if both terms are multiplied by a speed variation Δv, we have:



(a) (b)

Figure 2. (a) Curve of the amplitude versus red shift of the pulses when pass in the core of an optical fiber with length L, fainter in this figure means more optical fiber length (b) Hubble diagram given by Knop et al [18] that show the strong analogy with the light dispersion that occur in the core of optical fiber core. We can note from this analysis that the dark energy according this paper don't exist,



(c)

Figure 2(c) Mechanism of the temporal material dispersion of a train of pulses as they travel along of optical fiber. The longer the distance, the more the pulses stretch, until they overlap.

$$\Delta t \Delta \nu = \frac{\lambda}{c}\left(\frac{d^2 n}{d\lambda^2}\Delta \nu \Delta \lambda\right)L \tag{2}$$

Naming the term:

$$H = \left(\frac{d^2 n}{d\lambda^2}\Delta \nu \Delta \lambda\right) \tag{3}$$

and the term:

$$\Delta\lambda = \Delta t \Delta \nu \tag{4}$$

we have exactly Hubble's Law[16], which is:

$$\Delta\lambda = \frac{\lambda}{c} HL \tag{5}$$

or

$$\frac{\Delta\lambda}{\lambda}c = HL \tag{6}$$

It is noteworthy that the H constant depends on three parameters: Δv is the variation of the speed of light, $d^2n/d\lambda^2$ is the dispersion power of the environment, and $\Delta\lambda$ is the spectral width of the source, and is therefore not a constant.

Gravitational Light Dispersion

In 1911, Einstein[17] showed that a ray of light passing close to a celestial body undergoes deflection on the side where the gravitational potential decreases, that is, on the side facing the celestial body, whose value is:

$$\alpha = \frac{2GM}{c^2 r} \tag{7}$$

where G is the gravitational constant, M is the mass of the celestial body and r is the distance of the ray of light to the center of the celestial body. He also showed that the speed of this ray of light varies according to

$$c = c_o\left(1 + \frac{\phi}{c^2}\right) \tag{8}$$

where ϕ is the gravitational potential, and c_o is the speed of light in a vacuum. We infer that it is possible to have two rays of light very close to each other at different speeds, and that therefore the phenomenon of temporal material dispersion might occur.

Temporal Gravitational Dispersion

Since space is transparent and free of matter, we will temporarily call this phenomenon temporal gravitational dispersion of light in space. Considering the form of the metrics, and due to the analogy between Optics and Mechanics, the term $d^2n/d\lambda^2$ is better defined as d^2n/dr^2. Thus, expression (3) becomes:

$$H = \left(\frac{d^2n}{dr^2}\Delta v \Delta\lambda\right) \tag{9}$$

which then corresponds to the redshift of light in intergalactic space. We observe that this "constant" is governed by three new parameters. Light traveling throught intergalactic space undergoes redshift due to this mechanism, while light amplitude decreases within time, and the wavelength always increases, thus producing the same type of behavior given by Hubble's Law. We therefore conclude from this model that the dark energy issue is non-existent, there is only the broader phenomenon of light in intergalactic space, and so the behavior of the curve produced by this dispersion is analogous to the current Hubble curve given by Knop *et al.* [18] as shown in figure 2(b).

This model also explains the work of H. Arp [19,20], where galaxies might exist with different redshifts, even though located at practically the same distance in relation to the observer.

In addition, we can explain the observed redshift phenomenon without the assumption of any dark energy. Since the stars in the arms of the galaxies are hotter than those at the core, their spectral widths are greater because they are hotter, and, in addition, they are located on the periphery of the galaxy, where the terms d^2n/dr^2 and Δv are large. As a result, the light pulses are emitted into space with a greater wavelength than those of the stars in the core of the galaxy. This shift of wavelength can be interpretated as a Doppler shift, and an apparent speed is therefore assumed. The Doppler effect is represented by the formula:

$$\frac{\lambda}{\lambda_o} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \tag{10}$$

So if stars are in motion with respect to the observer, the wavelengths of the emitted light are changed. Reversing this argument, a shift in wavelength means a relative motion. However, our model suggests that not all shifted wavelengths are necessarily due to relative motion. There might be other sources of shifted wavelengths which are assumed to be due to the Doppler shift, but really due to phenomena as in our model.

The microwave background radiation can also be explained by our model. It is attributed to light coming from galaxies located at infinite distances from the Earth, where the pulses overlap and transform into thermalized pulses of background radiation.

Conclusion

This presents a new approach to the old controversy surrounding the redshift phenomenon which occurs in galaxies in the universe. This phenomenon is not produced by the Doppler effect, but by temporal gravitational dispersion of light in its path through the intergalactic space. Therefore, issues such as dark energy and dark matter cease to be relevant, and the introduction of cosmological constants and other assumptions of the standard models of the universe are not needed. Based on this model, the universe would be stationary.

Acknowledgments

The author acknowledges Prof. Dr. André K. Assis for discussions of this paper.

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The Mysterious Dark Energy

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The concept of an all pervading aether is age old, and contrary to popular belief, it has survived through the twentieth century, though with different nuances. Using this concept of a background Quantum Vacuum, the author in 1997 proposed a cosmological model with some resemblance to the Dirac cosmology, which correctly predicted a dark-energy driven accelerating universe with a small cosmological constant, as was subsequently confirmed by observation in 1998. Moreover the so called Large Number coincidences, including the mysterious Weinberg formula, are deduced in this theory, rather than being *ad hoc*. We examine the concept of aether in this context and indicate how this dark energy may be harnessed.

Introduction

In 1997, when the ruling paradigm was dark matter and a decelerating universe, the author proposed a model based on the Zero Point Field dark energy and an accelerating Universe with a small cosmological constant. Moreover, in this theory, the empirically well-known, but otherwise inexplicable, so called Large Number relations, including the mysterious Weinberg formula were deduced as consequences, rather than being *ad hoc* [1,2,3,4,5]. In 1998, the observations of Perlmutter and others on distant supernovae confirmed the above scenario—this work was in fact the Breakthrough of the Year 1998 of the American Association for Advancement of Science's Science Magazine [6,7,8]. Subsequently observations by the Wilkinson Microwave Anisotropy Probe (WMAP) and the Sloan Digital Sky Survey confirmed the predominance of the new paradigmatic dark energy—this was the Breakthrough of the Year 2003 [9].

We first observe that the concept of a Zero Point Field (ZPF) or Quantum Vacuum (or aether) is an idea whose origin can be traced back to Max Planck himself. Quantum Field Theory attributes the ZPF to the virtual Quantum effects of an already present electromagnetic field [10]. What is the mysterious energy of the supposedly empty vacuum?

It may sound contradictory to attribute energy or density to the vacuum. After all vacuum is a total void. However, over the past four hundred years, it has been realized that it may be necessary to replace the vacuum by a medium with some specific physical properties. For instance Descartes, the seventeenth century French philosopher mathematician proclaimed that the so-called empty space above the mercury column in a Torricelli tube, that is, what is called the Torricelli vacuum, is not a vacuum at all. Rather, he said, it was something which was neither mercury nor air, something he called aether.

The seventeenth century Dutch Physicist, Christian Huygens required a non intrusive medium like aether, to enable light waves to propagate, rather like the ripple waves on the surface of a pond. Hence the word luminiferous aether. In the nineteenth century the aether was re-invoked. First in a very intuitive way, Faraday conceived of magnetic effects in vacuum in connection with his experiments on induction. Based on this, the aether was used for the propagation of electromagnetic waves in Maxwell's Theory of electromagnetism, which infact laid the stage for Special Relativity. This aether was a homogenous, invariable, non-intrusive, material medium which could be used as an absolute frame of reference, at least for certain chosen observers. However, the experiments of Michelson and Morley toward the end of the nineteenth century led to its downfall, and thus was born Einstein's Special Theory of Relativity in which there is no such absolute frame of reference. The aether lay shattered once again.

Very shortly thereafter the advent of Quantum Mechanics led to its rebirth in a new and unexpected avatar. Essentially there were two new ingredients in what is today called the Quantum Vacuum. The first was a realization that Classical Physics had allowed an assumption to slip in unnoticed: In a source or charge free "vacuum," one solution of Maxwell's Equations of electromagnetic radiation is no doubt the zero solution. But there is also a more realistic non-zero solution. That is, the electromagnetic radiation does not necessarily vanish in empty space.

The second ingredient was the mysterious prescription of Quantum Mechanics, the Heisenberg Uncertainty Principle, according to which it would be impossible to precisely assign momentum and energy on the one hand and space-time location on the other. Clearly the location of a vacuum with no energy or momentum cannot be specified in space-time.

This leads to what is called a Zero Point Field. For instance a Harmonic Oscillator, a swinging pendulum for example, according to classical ideas has zero energy and momentum in its lowest position. But the Heisenberg Uncertainty endows it with a fluctuating energy. This fact was recognized by Einstein himself way back in 1913 who, contrary to popular belief, retained the concept of aether though from a different perspective [11]. It also provides an understanding of the fluctuating electromagnetic field in vacuum.

From another point of view, according to classical ideas, at the absolute zero of temperature, there should not be any motion. After all the zero is when all thermodynamic motion ceases. But as Nernst, father of the third law of Thermodynamics himself noted, experimentally this is not so. There is the well known superfluidity due to Quantum Mechanical—and not thermodynamic—

effects. This is the situation where supercooled Helium moves in a spooky fashion [10].

This mysterious Zero Point Field or Quantum Vacuum energy has since been experimentally confirmed in effects like the Casimir effect which demonstrates a force between uncharged parallel plates separated by a charge free medium, the Lamb shift which demonstrates a minute oscillation of an electron orbiting the nucleus in an atom-as if it was being buffetted by the Zero Point Field—the anomalous Quantum Mechanical gyromagnetic ratio $g = 2$ and so on [12]-[17],[18].

The Quantum Vacuum is a far cry however, from the passive aether of olden days. It is a violent medium in which charged particles like electrons and positrons are constantly being created and destroyed, almost instantly, within the limits permitted by the Heisenberg Uncertainty Principle for the violation of energy conservation. One might call the Quantum Vacuum a new state of matter, a compromise between something and nothingness. Something which corresponds to what the Rig Veda described thousands of years ago: “Neither existence, nor non-existence.”

The Quantum Vacuum can be considered to be the lowest state of any Quantum field, having zero momentum and zero energy. The properties of the Quantum Vacuum can, under certain conditions, be altered, which was not the case with the erstwhile aether. In modern Particle Physics, the Quantum Vacuum is responsible for phenomena like Quark confinement, a property whereby it would be impossible to observe an independent or free Quark, the spontaneous breaking of symmetry of the electroweak theory, vacuum polarization wherein charges like electrons are surrounded by a cloud of other oppositely charged particles tending to mask the main charge and so on. There could be regions of vacuum fluctuations comparable to the domain structures of feromagnets. In a ferromagnet, all elementary electron-magnets are aligned with their spins in a certain direction. However there could be special regions wherein the spins are aligned differently.

A Quantum Vacuum can be a source of cosmic repulsion, as pointed out by Zeldovich and others [19,20]. However a difficulty in this approach has been that the value of the cosmological constant turns out to be huge, far beyond what is observed. This has been called the cosmological constant problem [21].

There is another approach, sometimes called Stochastic Electrodynamics which treats the ZPF as primary and attributes to it Quantum Mechanical effects [22, 23]. It may be re-emphasized that the ZPF results in the well known experimentally verified Casimir effect [24, 25]. We would also like to point out that contrary to popular belief, the concept of aether has survived over the decades through the works of Dirac, Vigier, Prigogine, String Theoriests like Wilzeck and others [26]-[31]. As mentioned above, it appears that Einstein himself continued to believe in this concept [33].

We would first like to observe that the energy of the fluctuations in the background electromagnetic field can lead to the formation of elementary particles. Indeed, this was Einstein's belief. As he observed as early as 1920 [32], "... according to our present conceptions, the elementary particles are... but condensations of the electromagnetic field."

In the words of Wilzeck, [30], "Einstein was not satisfied with the dualism. He wanted to regard the fields, or aethers, as primary. In his later work, he tried to find a unified field theory, in which electrons (and of course protons, and all other particles) would emerge as solutions in which energy was especially concentrated, perhaps as singularities. But his efforts in this direction did not lead to any tangible success."

The Zero Point Field

Let us see how this can be realized. In the words of Wheeler [18], "From the zero-point fluctuations of a single oscillator to the fluctuations of the electromagnetic field to geometrodynamic fluctuations is a natural order of progression..."

Following Wheeler, let us consider a harmonic oscillator in its ground state. The probability amplitude is

$$\varphi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-(m\omega/2\hbar)x^2}$$

for displacement a distance x from its position of classical equilibrium. So the oscillator fluctuates over an interval

$$\Delta x \sim (\hbar/m\omega)^{1/2}$$

The electromagnetic field is an infinite collection of independent oscillators, with amplitudes X_1, X_2 *etc.* The probability for the various oscillators to have amplitudes X_1, X_2 and so on is the product of individual oscillator amplitudes:

$$\varphi(X_1, X_2 ...) = \exp\left[-\left(X_1^2 + X_2^2 + ...\right)\right]$$

wherein there would be a suitable normalization factor. This expression gives the probability amplitude φ for a configuration $B(x, y, z)$ of the magnetic field that is described by the Fourier coefficients $X_1, X_2, ...$ or directly in terms of the magnetic field configuration itself we have

$$\varphi(B(x, y, z)) = P \exp\left(-\int\int \frac{B(x_1).B(x_2)}{16\pi^3\hbar c r_{12}^2} d^3x_1 d^3x_2\right)$$

P being a normalization factor. Let us consider a configuration where the magnetic field is everywhere zero except in a region of dimension l, where it is of the order of $\sim \Delta B$. The probability amplitude for this configuration would be proportional to

$$\exp[-(\Delta B)^2 l^4 / \hbar c)]$$

So the energy of fluctuation in a region of length l is given finally by [18, 34, 35]

$$B^2 \sim \frac{\hbar c}{l^4}. \quad (1)$$

We next argue that l, the mean length of fluctuations, will be the Compton length. We note that as is well-known: a background ZPF of the kind we have been considering can explain the Quantum Mechanical spin half as well as the anomalous $g = 2$ factor for an otherwise purely classical electron [36, 37, 38]. The key point here is [36] that the classical angular momentum $\vec{r} \times m\vec{v}$ does not satisfy the Quantum Mechanical commutation rule for the angular momentum $\vec{J}$. However, when we introduce the background Zero Point Field, the momentum now becomes

$$\vec{J} = \vec{r} \times m\vec{v} = (e/2c)\vec{r} \times (\vec{B} \times \vec{r}) + (e/c)\vec{r} \times \vec{A}^0, \quad (2)$$

where $\vec{A}^0$ is the vector potential associated with the ZPF—for example if the electric part of the ZPF is $\vec{E}^0$, this is usually considered to be a Gaussian random process and $\vec{A}^0$ is related to $\vec{E}^0$ by the usual Maxwell equation. $\vec{B}$ is an external magnetic field introduced merely for convenience, and which can be made vanishingly small.

It can be shown that $\vec{J}$ in (2) satisfies the Quantum Mechanical commutation relation for $\vec{J} \times \vec{J}$. At the same time we can deduce from (2)

$$\langle J_z \rangle = -\frac{1}{2}\hbar\omega_0 / |\omega_0| \quad (3)$$

Relation (3) gives the correct Quantum Mechanical results referred to above.

From (2) we can extend the arguments and also deduce that

$$l = \langle r^2 \rangle^{\frac{1}{2}} = \left(\frac{\hbar}{mc}\right). \quad (4)$$

Eq. (4) shows that the mean dimension of the region in which the fluctuation contributes is of the order of the Compton wavelength of the electron. By relativistic covariance [36], the corresponding time scale is at the Compton scale.

In (1) above, if l is taken to be the Compton wavelength of a typical elementary particle, then we recover its energy mc^2, as can easily be verified. As mentioned, Einstein himself had believed that the electron was a result of such condensation from the background electromagnetic field [39, 20].

We now very briefly indicate the cosmology referred to in the introduction [1]-[5], [40, 41]. Elementary particles are created from the ZPF as above. If there are N elementary particles, then by fluctuation a net $\sqrt{N}$ particles are created within the Compton time τ [1]-[5], [40], so that

$$\frac{dN}{dt} = \frac{\sqrt{N}}{\tau} \quad (5)$$

We also use the well known facts that

$$M = Nm \quad (6)$$

and

$$R = GM/c^2. \quad (7)$$

In (6), M is the mass of the Universe, m the mass of a typical elementary particle like the pion, $N \sim 10^{80}$ the number of elementary particles in the Universe and R its radius.

Differentiation of (7) and use of (6) and (5) then leads to a host of consistent relations,

$$v = \dot{R} = HR, \quad H = \frac{c}{l} . \frac{1}{\sqrt{N}}, \tag{8}$$

$$G\rho_{vac} = \Lambda < 0(H^2), \; R = \sqrt{N} l, T = \sqrt{N}\tau, \; \rho_{vac} = \rho / \sqrt{N} \tag{9}$$

$$m = \left(\frac{H\hbar^2}{Gc} \right)^{1/3}, \quad \frac{e^2}{Gm^2} \approx \frac{1}{\sqrt{N}} \tag{10}$$

and so on.

In (8) above, H is the Hubble constant, l the pion Compton length, while in (9) ρ the average density, Λ the cosmological constant and ρ_{vac} the vacuum density. The second relation of (9) is the empirically known so called Eddington formula. The first and second relations of (10) are respectively, the Weinberg formula and the well known (but otherwise ad hoc) electromagnetism - gravitational coupling constant.

It may also be mentioned that all this can be interpreted elegantly in terms of underlying Planck oscillators in the Quantum Vacuum [42, 43].

Finally, it may be mentioned that (10) shows that both Λ and $H \to 0$ as $N \to \infty$, as indeed is the current belief.

Harnessing the ZPF?

Two of the earliest realizations of the ZPF as mentioned were in the form of the Lamb shift and the Casimir effect.

In the case of the Lamb shift, as is well known, the motion of an orbiting electron is affected by the background ZPF. Effectively there is an additional field, over and above that of the nucleus. This additional potential, as is well known, is given by [44]

$$\Delta V(\vec{r}) = \frac{1}{2} \left\langle (\Delta r)^2 \right\rangle \nabla^2 V(\vec{r})$$

The additional energy

$$\Delta E = \left\langle \Delta V(\vec{r}) \right\rangle$$

contributes to the observed Lamb shift which is $\sim 1000 mc/\sec$.

The essential idea of the Casimir effect is that the interaction between the ZPF and matter leads to macroscopic consequences. For example if we consider two parallel metallic plates in a conducting box, then we should have a Casimir force given by [45]

$$F = \frac{-\pi^2}{240} \frac{\hbar c A}{l^4}$$

where A is the area of the plates and l is the distance between them. More generally, the Casimir force is a result of the boundedness or deviation from a Euclidean topology in the Quantum Vaccuum. These Casimir forces have been experimentally demonstrated [46, 47, 48, 49].

Let us return to equation (1). The ZPF fluctuations typically take place within the time τ, a typical elementary particle Compton time as suggested by (4). This begs the question whether such ubiquitous fields can be tapped for terrestrial applications or not.

We now invoke the well known result from macroscopic physics that the current in a coil is given by

$$i = \frac{nBA}{r\Delta t} \tag{11}$$

where n is the number of turns of the coil, A is its area and r the resistance.

Introducing (1) into (11) we deduce that a coil in the ZPF would have a fluctuating electric current given by

$$i = \frac{nA}{r}\frac{e}{l^2\tau} \tag{12}$$

Of course, this would be a small effect. But in principle it should be possible to harness the current (12) using advanced technologies, possibly superconducting coils to minimize r.

Conclusion

We have noted that the concept of an aether has survived in one form or the other through the twentieth century. In the post Quantum Theory era, it has been the Quantum Vacuum, or the Zero Point Field, necessitated by the Heisenberg Uncertainty Principle. It leads to a space-time cut-off at the Compton scale, and in particular the Planck scale as a special case. Together with fluctuations, these ideas lead to a cosmology which is consistent with the latest observations of a dark energy driven accelerating universe with a small cosmological constant. Moreover it may be possible to technologically harness the Zero Point Field.

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The Evidence for Length Contraction at the Turn of the 20th Century: Non-existent

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This paper briefly reviews some old criticisms of the original 1887 Michelson-Morley experiment, and formulates two new, in our opinion very strong, criticisms. The first one is based on a direct calculation of the missing error bounds in the original experiment. Contrary to Michelson-Morley's conclusion, the results of the experiment were consistent with their initial expectations. The second cirticism demonstrates that the design of the Michelson-Morley experiment could not possibly measure the magnitude of Earth's velocity relative to a preferred frame. This is achieved by calculating the response of the Michelson-Morley interferometer, taking into account the full motion of the laboratory with respect to the frame of cosmic background radiation (place, dates, time of day, and rotation rate as in the original experiment). The retrodicted fringe-shift curves show the expected harmonic variation with a large amplitude of several wavelengths. Since Michelson and Morley incorrectly expected an amplitude of less than one wavelength, they designed their experiment to read only fractions of a wavelength. The response of the interferometer to Michelson-Morley's incorrect data gathering procedure is also calculated. The incorrect fringe-shift curves are approximately periodic, with amplitudes that are much lower than the true variation. This reduced amplitude immediately leads to the small speed reported in all Michelson-Morley-type experiments.

I. Brief review of criticisms of the Michelson-Morley experiment

According to Michelson and Morley (MM, henceforth), the experiment that they carried out in 1887 at Cleveland (USA) failed to yield the expected value of Earth's speed in her motion relative to a preferred frame of reference (the luminiferous aether, presumably at rest, in MM days).[1] This result is conventionally interpreted as negative, in the sense that the velocity of light relative to an observer does not change with the motion of the earth.

Given the profound implications of MM negative result, it is rather surprising that so few *critical* papers on the experiment have ever been published. In order to put our new criticisms in context, three groups of criticisms are briefly reviewed.

A. The theory of the MM apparatus

Hicks [2] published in 1902 a detailed paper noting various experimental and theoretical aspects that were not duly considered by MM[1], for instance the width of the fringes, and the exact location for the formation of the interefer-ence pattern. Related papers were also published [3-5].

By far the most cited theoretical question is kinematical reflection upon the moving mirrors of the interferometer. Indeed, in the supplement to the original paper MM state that kinematical reflection could be "another method for multiplying the square of the aberration sufficiently to bring it within the range of observation, which has presented itself during the preparation of this paper" [1, pp. 341-342]. Obviously, for MM kinematical reflection was an afterthought, which means that it did not play any rôle in the design and analysis of the original MM experiment. Righi [6-8] maintained that reflection from the mirrors in motion would necessarily lead to a negative result in the MM experiment, while Villey [9] had the oppossite view. The Vatican Astronomer Stein [10] sided with Righi, while Kennard and Richmond [11] sided with Villey. Instead, kinematical reflection has recently been suggested as the correct mechanism to explain the ray diagram used by MM for the transversal ray in the interferometer [12-16]. We concur with the latter opinions.

Additionally, Miranda *et al.* [16] noted that when the fringes are observed on one of the interferometer mirrors (as in the MM experiment) there are no first-order effects (*i.e.*, only second order effects as in the MM analysis), but first order dependences may be present when the interference pattern is observed on a screen located on one side of the beam splitter. This novel aspect of kinematical reflection may be relevant for the interpretation of our own ongoing experiment in Bogotá. (See section IV.)

B. The data reduction in the MM experiment

Hicks [2] also noted that the different sessions in the MM experiment could correspond to different calibrations of the apparatus, so that they could not be averaged together (as MM did). In our opinion [17], Hicks's criticism is completely valid.

Additionally, Hicks suggested that the monotonic trend observed by MM could be due to thermal drift, and suggested a linear correction that was incorporated by Miller [18, pp. 213-214] in his analyses of the MM experiment and his own data. In the present writer's opinion, [19, pp. 193-194] at least part of the monotonic trend may be ascribed to Earth's motion. However, for the determination of the periodic variations, the correction suggested by Hicks remains appropriate.

Múnera [17] has also noted other weaknesses in the data reduction process. For instance, (a) Since solar motion is missing in the MM analysis, the projection of Earth's motion on the interferometer plane does not coincide in general with the initial direction (to the local east) of one of the interferometer's arms. Hence, MM did not look for the position of the maxima and minima of their data at angles other than their expectations. (b) The statistical error associated with the individual points of the MM final curve were not reported. Section II below explicitly calculates the missing error band.

C. The work of Dayton C. Miller

Miller devoted his scientific career from 1902 to around 1930 to repeating the MM experiment. He always claimed that both MM and his own results were not negative, although they were smaller than expected [18]. In an analysis of Miller's work, Shankland and collaborators [20] acknowledged that Miller′s variations were not random, but surprisingly concluded that Miller's periodic variations were caused by thermal (presumably periodic) variations. This particular issue was empirically checked many years before by Miller during the 1922-1924 tests, who concluded that periodic thermal effects could not possibly appear in his experimental setup [18, p. 220].

Since the 1990's, there has been a renewed interest in Miller's work; several authors independently consider that Miller's results are real and not mere experimental artefacts [17,19,21-23]. In particular, Allais [23] has uncovered new annual periodicities in Miller's data.

If Miller's results were real, what then is the origin of the small amplitude (hence, small laboratory velocity) observed by MM and by Miller? This is the remaining puzzle in the whole story. In our earlier work [19, pp. 192-193; 24, pp. 474-475] a clue was offered: the MM and the Miller experiments were designed to measure fractions of wavelength, and larger variations were recalibrated away. For the first time (to our knowledge), Section III presents an explicit explanation for the small amplitude in the MM-type experiments. A final Section IV closes this paper.

II. The missing error bound in the MM experiment

In this section, the data obtained by MM is statistically analysed to obtain the missing error bounds. We adhere strictly to MM's data reduction procedures, as explicitly (or tacitly, but unambiguosly) explained by MM in their paper [1]. Therefore, in this paper we do not calculate error bounds for the MM final curves, because the original MM paper does not offer any clue as to how they were constructed (see A.1 below). The present author has calculated the error bounds elsewhere [25]. Despite the critical remarks below, the values and the methods of MM are taken at face value for the calculations.

A. The analysis by Michelson and Morley

MM presented their experimental data in two tables for noon and evening observations,[1, page 340] that are reproduced here as rows 1 through 8 in Table 1 (A and B). Immediately below their tables, MM stated: "The results of the observations are expressed graphically in fig. 6. The upper is the curve for the observations at noon, and the lower that for the evening observations. The dotted curves represent *one-eighth* of the theoretical displacements. It seems fair to conclude from the figure that if there is any displacement due to the relative motion of Earth and the luminiferous ether, this cannot be much greater than 0.01 of the distance between the fringes" (emphasis in the original, page 340). In a more recent and easily accessible paper, Handschy [26] presents facsimiles of MM's tables plus figure 6.

The paragraph quoted above contains the whole discussion of results and error analysis for the MM experiment. In retrospect, it is almost unbelievable that such a simple and qualitative analysis could have served as the empirical basis for proposing a novel explanation: the FitzGerald-Lorentz length contraction.

Before continuining with the description of the MM experiment, it is worth calling attention to three facts in MM's figure 6 and in the paragraph just quoted:

1. The curves are not a *direct* plot of the tables, as MM seem to imply in the quoted paragraph. (See more below.) To the best of our knowledge, Handschy [26, p.987, second column] is the only previous author to have noticed this fact. It is quite surprising that neither Hicks, [2] nor Miller [18] noticed this inconsistency.
2. There are no error bars associated with each of the points in Figure 6. This absence is more notorious because MM compared each empirical point to a theoretical point ("the dotted curves"), and concluded that they were different. From our current standpoint, MM's conclusion means *a fortiori* that the distance between the empirical and the theoretical points was several standard deviations. Hence, MM's estimate for the value of the standard deviation associated with each point was surely small. It is not clear whether "0.01 of the distance between the fringes" is an estimate for the standard deviation, or for the amplitude A of the empirical curve, or for both, or for what (?).
3. The scale for the negative part of the vertical axis in Figure 6 is not shown. Although this is a minor point, the absence of the scale gives the visual impression that all values of the empirical curve are close to zero, and, additionally, makes it difficult to estimate the amplitude of the empirical curve, which varies mostly towards *negative* values. For instance, the noon curve varies from a maximum of +0.007 wavelengths (wl, henceforth) to a minimum of –0.023 wl. These values can be read from MM's Figure 6 [1, p. 340], or from the zoomed-in Handschy Figure 4 [26, p. 989].

Returning to the description of MM experiment, the position of a reference fringe was read with a telescope having a scale with 100 divisions. The calibration varied from 40 to 60 divisions per wavelength, so that there were between 0.017 and 0.025 wavelengths per division. MM used an intermediate conversion factor C = 0.02 wavelengths per division [1, pp.339-340]. For rigour, an error band should be assigned to C; however, we will follow MM in using C as a deterministic exact factor.

In each session of the MM experiment there were six slow turns of the interferometer (one revolution in six minutes, *i.e.* angular speed $\omega = 0.01745$ radians/second). The position of the reference fringe was recorded at various values of the angle θ between one of the arms and the initial direction, that apparently was the local north, N. Although, MM were completely silent in the text about the initial direction, their Figure 6 implies that position $\theta = 0^\circ$ corresponds to N. Specifically, MM made one reading every 22.5º, with the interferometer in motion, which amounts to 17 readings from $\theta = 0^\circ$ to $\theta = 360^\circ$, which we will label $j = 0, 1,2,...,16$. Note that MM referred to position 0 as 16, so 16 opens and ends their series. MM stated that the interferometer was rotated counter-clockwise during the noon observations, but clockwise in the evening observations [1, p. 340]. This statement misled Hicks,[2, p. 38] into believing that the order of one of the series should be reversed for the data reduction process. However, Morley himself advised Hicks that the data in their tables was already reversed [27].

There were three sessions numbered here from 1 to 3 (session 1 = July 8, session 2 = July 9, session 3 = July 11/1887) for the noon observations, and three similar sessions on July 8, 9, 12/1887 for the evening observations. The readings for the six turns in each session were averaged and shown by MM in their tables [1, p. 340]; these values are reproduced here as rows 1 through 6 in Table 1 (sections A and B). We note that MM's values are not "raw" data, but averages of six experimental readings. Strictly speaking, error bounds—that were not provided by MM—are required for these data. Since we do not have any estimate for the error, as a first approximation, the individual points given by MM will be taken here as *bona fide* raw data.

The value associated with each interferometer position j is represented here by X_j. According to MM expectations, for a given j, the variable X_j represents a well defined physical process. For different sessions the value of X_j may change due to unavoidable experimental errors, that we assume to be normally distributed. MM also expected that the physical process underlying X_j would be different for different j, as discussed next.

MM correctly expected that their observations would reveal a harmonic variation on 2θ (*i.e.*, with $2j$), although they were wrong regarding the phase[17] and the magnitude (see section III below). Hence, from each session MM obtained two sets of data (A and B), one for the interferometer at positions 0 through 8, another for positions 8 through 16. For instance, session 1A in Table 1 corresponds to measurements on July 8, with interferometer readings from 0 to 180º, while session 1B also corresponds to July 8, with readings from 180 to 360º, and so on. Let m_j be the average value of X_j for the six sessions 1A, 1B, 2A, 2B, 3A, and 3B; for each j the calculated m_j (in divisions) is shown as row 7 of Table 1. MM's "final mean" is shown as row 8; this is the same m_j expressed in wavelengths, wl (row 7 multiplied by the conversion factor C, defined above).

Table 1. Data reduction for the original MM values

Description		A. Noon observations								
Position, j		0	1	2	3	4	5	6	7	8
Row	Item\□º	0	22.5	45	67.5	90	112.5	135	157.5	180
1	1A	44.7	44.0	43.5	39.7	35.2	34.7	34.3	32.5	28.2
2	1B	28.2	26.2	23.8	23.2	20.3	18.7	17.5	16.8	13.7
3	2A	57.4	57.3	58.2	59.2	58.7	60.2	60.8	62.0	61.5
4	2B	61.5	63.3	65.8	67.3	69.7	70.7	73.0	70.2	72.2
5	3A	27.3	23.5	22.0	19.3	19.2	19.3	18.7	18.8	16.2
6	3B	16.2	14.3	13.3	12.8	13.3	12.3	10.2	7.3	6.5
7	mj, div	39.22	38.10	37.77	36.92	36.07	35.98	35.75	34.60	33.05
8	mj, ẉl	0.784	0.762	0.755	0.738	0.721	0.720	0.715	0.692	0.661
9	sj, div	18.17	19.80	21.35	22.39	23.22	24.21	25.67	25.82	27.31
10	sj, ẉl	0.363	0.396	0.427	0.448	0.464	0.484	0.513	0.516	0.546
11	SEj, wl	0.148	0.162	0.174	0.183	0.190	0.198	0.210	0.211	0.223
12	tSEj,wl	0.381	0.416	0.448	0.470	0.487	0.508	0.539	0.542	0.573
13	Expect	0.784	1.067	1.184	1.067	0.784	0.501	0.384	0.501	0.784
Description		B. Evening observations								
Position, j		0	1	2	3	4	5	6	7	8
Row	Item\□º	0	22.5	45	67.5	90	112.5	135	157.5	180
1	1A	61.2	63.3	63.3	68.2	67.7	69.3	70.3	69.8	69.0
2	1B	69.0	71.3	71.3	70.5	71.2	71.2	70.5	72.5	75.7
3	2A	26.0	26.0	28.2	29.2	31.5	32.0	31.3	31.7	33.0
4	2B	33.0	35.8	36.5	37.3	38.8	41.0	42.7	43.7	44.0
5	3A	66.8	66.5	66.0	64.3	62.2	61.0	61.3	59.7	58.2
6	3B	58.2	55.7	53.7	54.7	55.0	58.2	58.5	57.0	56.0
7	mj, div	52.37	53.10	53.17	54.03	54.40	55.45	55.77	55.73	55.98
8	mj, wl	1.047	1.062	1.063	1.081	1.088	1.109	1.115	1.115	1.120
9	sj, div	18.26	18.20	17.31	17.17	16.05	15.73	15.72	15.61	15.72
10	sj, ẉl	0.365	0.364	0.346	0.343	0.321	0.315	0.314	0.312	0.314
11	SEj, wl	0.149	0.149	0.141	0.140	0.131	0.128	0.128	0.127	0.128
12	tSEj, wl	0.383	0.382	0.363	0.360	0.337	0.330	0.330	0.328	0.330
13	Expect.	1.047	1.330	1.447	1.330	1.047	0.764	0.647	0.764	1.047

B. Missing statistical analysis

It is mentioned in passing that MM did not check (at least, it is not reported) whether the two sets of data A and B in each session were compatible with the hypothesis that they were drawn from the same data set (*i.e.*, of representing the same physical process). The present author has carried out the statistical comparison elsewhere: they may in fact be added [25].

Figures 1A and 1B show MM's raw data in graphical form (rows 1 through 6 in Table 1). Similar graphs were prepared by Hicks ([2], plate between pages 8 and 9) for each daily session. There are two striking features: (a) the monotonic trend of each individual curve, noted by Hicks, and (b) the curves for each group of observations (noon and evening) are widely spread, which was not noted by Hicks because he plotted the curves in separate graphs without ordinates. This is the subject matter of our first new criticism.

The *direct* graphical representation of MM's data (row 8 in Table 1) is shown in Figures 2A and 2B; these figures are the same Figure 2 shown by Handschy on the rare ocassion where MM's data reduction process has been discussed [26, p. 988]. The curves depict the well-known monotonic trends

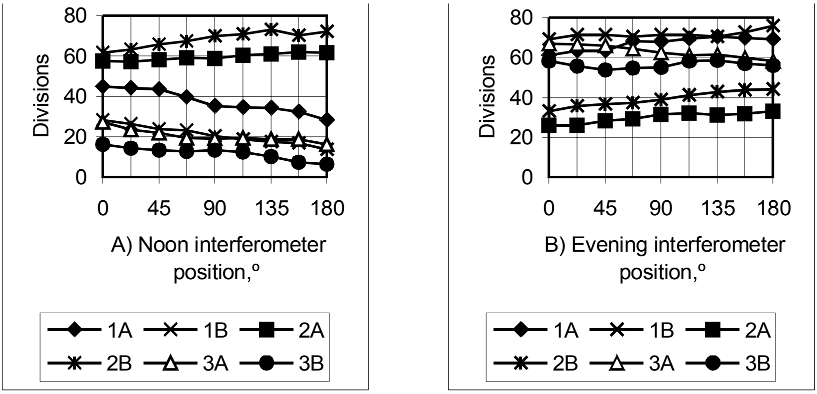


Figure 1. "Raw" data in the 1887 MM experiment. Direct plot of values given by MM in the original paper.[1] Each point is the average for six turns of the apparatus.

(downwards at noon, upwards in the evening), that have been independently noted by several authors [2,16,17,26]. It is mentioned that Hicks's average curves for the noon and evening sessions (bottom curves in the plate cited above) have a different shape from our Figures 2. This is due to an additional data reduction by Hicks [2] to eliminate the monotonic trend, and to his inversion of the order in one of the series. (Recall the beginning of this section.)

Obviously, the central average curves in Figures 2A and 2B do not correspond to the famous Figure 6 in MM paper. There is a hidden additional data reduction that was not even mentioned by MM in their paper. Handschy [26] was the first author to notice this inconsistency.

Additionally, Figures 2A and 2B show, for the first time ever, the error bound associated with m_j for the noon and evening observations. The statistical estimator for the standard deviation of each X_j is s_j, calculated directly from the sample with $n = 6$ (shown as rows 9 in Table 1), using

$$s_j^2 = \frac{1}{n-1}\sum_i (X_{ij} - m_j)^2,\ i = 1A, 1B, \ldots, 3B;\ j = 0, 1, \ldots, 8 \tag{1}$$

The value of s_j in wavelengths (wl) is shown as row 10 (row 9 multiplied

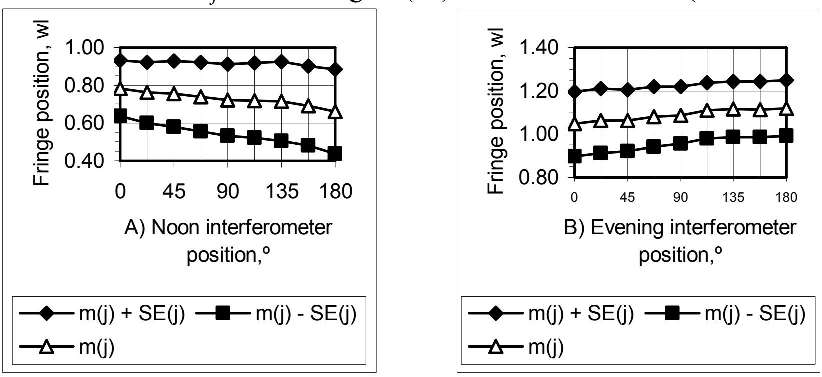


Figure 2. Average of raw data shown in figure 1 with one standard error band. Divisions converted to wavelength by multiplying by 0.02 (see text).

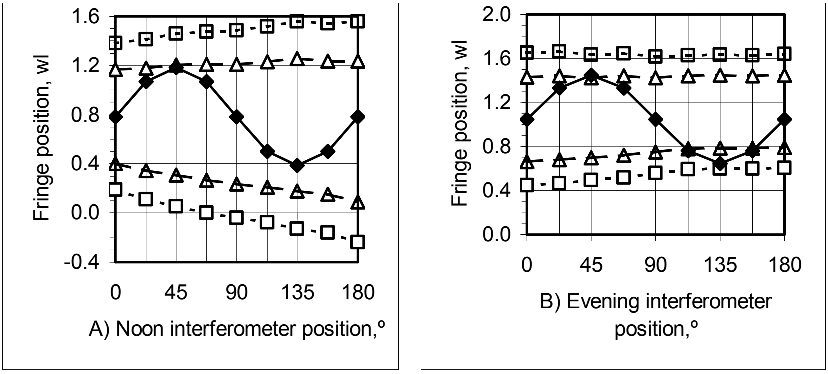


Figure 3. Consistency of MM expectations with their data. The limits of the 99% confidence band is represented by short dashes and squares, the limits for the 95% confidence band are in long dashes and triangles. The variation expected by MM is the sinusoidal continuous curve. Only one point from the evening observation is not within the 95% band (it is within the 99% band).

by factor C). The error band appearing in Figures 2 corresponds to the standard error SE_j associated with the mean m_j, obtained from [28]

$$SE_j^2 = \frac{1}{n} s_j^2 \,. \tag{2}$$

The value of SE_j (in wl) is row 11 in Table 1. It may be stressed that the standard error just calculated is not overestimated. On the contrary, we have deliberately left out two sources of variation, namely: (a) The conversion factor $C = 0.02$ was treated as an exact constant, and (b) The "raw" points also are averages with unknown dispersion.

At a confidence level of 95%, the two-sided interval containing the population mean has a width of $t_{0.975}SE(_j)$, where $t_{0.975}$ is obtained from Student's t distribution with $\nu = n - 1 = 5$ degrees of freedom. In our case, $t_{0.975} = 2.571$. At a confidence level of 99%, $t_{0.995} = 4.032$ [28]. Row 12 shows the 95% error bandwidth in wavelengths (wl).

Figures 3A and 3B show the 95% and 99% confidence level error bands associated with m_j. This is compared in the same figure to MM's expectations: a sinusoidal curve having an amplitude of 0.4 wl, with respect to the initial fringe-shift [1, page 341]. Values for this expectation are in rows 13 of our Table 1.

From our Figures 3, it can be seen that the sinusoidal curve expected by MM is within the error bands of their published data (without any manipulation). Hence, at a confidence level of 95%, the results of the 1887 MM experiment were consistent with MM's original expectations. Note that this is the full 0.4 wl amplitude, not "one-eighth" as in the incorrect qualitative analysis of MM's. This is in stark contrast to MM's claims. In fact, MM drew their conclusions from their Figure 6 where they plotted some data (reduced by an unspecified process), *without any error bars*. The amplitude of MM's mean curve was certainly smaller than MM's expected curve (see the reason in next section), but the extremely large magnitudes of the *unreported* statistical errors immediately void MM's unwarranted conclusion.

It can be noted that the error bands in Figure 3 are associated with the "final mean" m_j, whereas MM's Figure 6 contains an *additional reduction process to remove the trend* from the "final mean." Since this is done using the same empirical data, the unreported error bands associated with MM's Figure 6 *must* be larger than those in Figure 3 above [25]. Defenders of the correctness of MM's interpetation argue that the standard deviation of the MM data cannot be calculated directly with eq. (1) above. They claim that the data must be corrected first for systematic variations, and that the standard deviation should be calculated on the residuals. We concede that it may be so, but counter-argue that MM did not explicitly mention any correction for systematic trends in their paper, and that the overall error decreases if, and only if, there are no errors associated with the systematic error correction (*i.e.*, if the corrections are treated as deterministic). This is discussed elsewhere [25].

As mentioned before, it is very remarkable that neither Hicks [2], nor Miller [18] noted MM's hidden data reduction. At a first glance, a possible explanation could be that such reduction processes were standard at that time. However, this explanation is unlikely because Hicks proposes a method to eliminate the monotonic trend, and Miller implements it.

III. The small amplitude of MM's curves explained

Let us imagine that MM could have had at their disposal a respectable estimate for the velocity of the sun $\mathbf{V}_S$ relative to a preferred frame, say the cosmic background radiation [29]. In this frame, the center of mass of the earth moves with a net velocity $\mathbf{V}$ equal to the vector addition of $\mathbf{V}_S$ and $\mathbf{V}_O$, the orbital velocity of earth around the sun. The very pertinent question is: what changes, if any, would the design of the MM experiment require?

This question is answered here by re-calculating MM's expectations for the net $\mathbf{V}$. It immediately follows that the data gathering process would necessarily be different. Thence, we calculate the response of the interferometer to MM's actual design of the experiment, and obtain, for the first time, *a pre-relativistic explanation* for the small magnitude of the observed amplitudes in all MM-type experiments.

A. The effect of solar motion on the MM experiment

Consider local horizon coordinates with Cartesian axes labelled east (E), north (N), zenith (Z). The MM apparatus (arms L_1 and L_2) is contained in the horizontal plane E-N. At arbitrary time t, velocity $\mathbf{V}$ is decomposed into two orthogonal components: V_I on the interferometer plane, and V_Z perpendicular to it. The projection V_I makes angle γ with the E-axis. Evidently, there are diurnal and annual cycles on $\mathbf{V}$ and its components. Let λ and c be the wavelength and speed of light relative to the preferred frame, and $\beta_I = V_I/c$, $\beta = V/c$. The orientation of the interferometer is given by θ, the angle between L_1 and the E-axis.

At initial time $t = t_0$, $\theta = 0$ and the other parameters are identified by subindex 0.

The interference pattern at t depends on N, the number of fringes that have shifted relative to the initial patern. Up to second order in β, N is given by

$$\begin{aligned} N = {} & \frac{L}{\lambda}\left[\beta_I^2 \cos 2(\gamma - \theta) - \beta_I^2(t_0)\cos 2\gamma_0\right] \\ & + \frac{\Delta L}{\lambda}\left[\beta_I^2 - \beta_I^2(t_0) + 2\left(\beta^2 - \beta^2(t_0)\right)\right] \end{aligned} \tag{3}$$

where $L = 0.5(L_1 + L_2)$ is the average arm length, and $\Delta L = 0.5(L_1 - L_2)$ [17,24]. In the conventional MM design, it is assumed that both arms are exactly equal so that the second term on the right-hand side disappears.

Note that N depends on both the magnitude and the direction of the projection of Earth's velocity on the plane of the apparatus. The direction γ changes quickly during the day. On the contrary, MM assumed that $\gamma = 0$, and only considered Earth's orbital velocity [17]. However, from equation (3) it is quite evident that, at MM's epoch, the response of the apparatus would depend on the solar motion, independently of whether or not they knew the magnitude and direction of its velocity. For the sake of discussion, let us assume that solar motion relative to the preferred frame has constant speed $V_S = 390 \pm 60$ km/sec, in the direction of right ascension 11±0.6 hours, and declination 6º±10º [29]. Any other reasonable velocity may be used, it will not change the main results of this section. Indeed, a different right ascension will simply lead to a change in the position of the minimum, while a different speed will simply change the amplitude of the variation.

B. Retrodiction of MM's observations

In the MM apparatus, $L = 11$m and $\lambda \approx 580$ nanometers. The terrestrial orbit is assumed to be circular in the plane of the ecliptic. The geographical coordinates of MM's laboratory at Cleveland were assumed to be 41–º30' N, and 81º–39' W.

For our calculations, it was assumed that noon sessions started at 12 midday, and evening sessions at 6 p.m., local Cleveland time, and that the six turns in each noon session continuously ran from 12:00 to 12:36 (each turn in six minutes)—and similarly for the evening observations.

Figure 4A shows the expected fringe shifts for the first (run 1) and last turns (run 6) during the noon session of July 8, 1887. The average over the six turns for each position j is also shown in the graph. This value is our retrodiction for the "raw" data X_j, the position of the reference fringe that MM *intended* to read (recall section II).

Obviously, there is an error band associated with X_j, that will not be calculated here because MM did not report it. Please note that N varies over more than 30 wavelengths. The amplitude of the fringe-shift variation is $A = 0.5(N_{max} - N_{min}) = 0.5\,[29.7 - (-3.6)] = 16.7$ fringes for the average of the six turns on July 8, 1887.

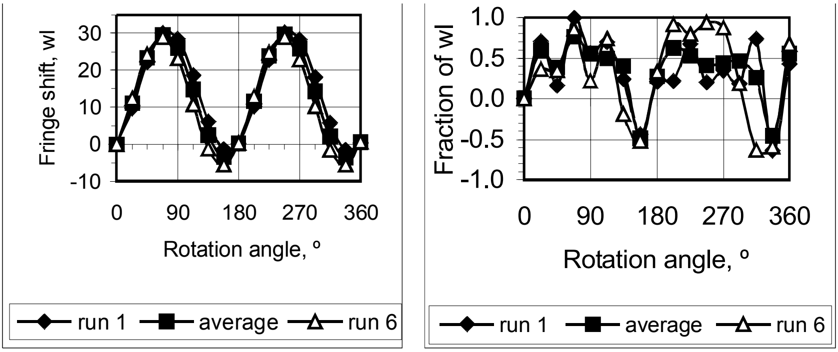


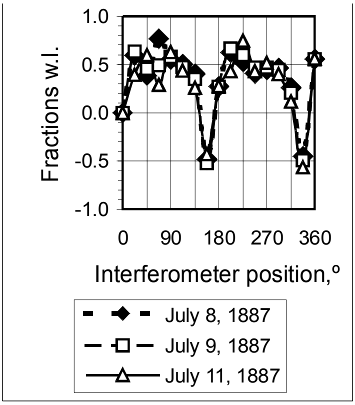
Figure 4. Expected fringe-shift for the noon session of July 8, 1887 with the solar velocity given by Smoot el al.[29]. (A) The left graph shows the fringe shift due to total terrestrial motion. (B) The right graph shows the fractional part of individual readings in (A). The latter curves correspond to the incorrect expectations of MM.

On the contrary, MM incorrectly expected variations smaller than one wavelength. Hence, they only measured the fraction of a wavelength above the corresponding integer number of fringes. Figure 4B shows our retrodiction for MM's expectations. It is obtained directly from the individual curves in Figure 4A by substracting the smaller integer from each reading. The session average is obtained from the individual incorrect readings.

Similar calculations for the other two noon sessions on July 9 and 11 produce fringe shift curves that are practically identical to the average in Figure 4A. The calculations for the three evening sessions also produce indistinguishable fringeshift curves, thus vindicating MM's basic expectation that data from the three days could be averaged. Also, the curves from 0-180º and from 180º-360º are indistinguishable, again vindicating MM's basic procedure of obtaining six sets of data from three sessions (recall section II). These remarks refer, of course, to the correct data as in Figure 4A, not to the incorrect data gathering actually employed by MM (as in Figure 4B).

Although MM's procedure fails to yield the correct amplitude for the fringeshift variation, it is very remarkable that the position of the *minima* remains approximately correct at 157º and 337º. As discussed by the author elsewhere [24], these angles are closely related to the right ascension of solar motion, which in this case is 11 h = 165º. A little reflection will convince the reader that the minima will be visible at the correct position in approximately 90% of cases. Indeed, let the value of fringeshift at a minimum be $n + f$, n being an integer and f the fraction, the minimum will still be visible at the same position with a depth of 0.1 fringes, even if $f = 0.9$.

The averages for the incorrect data gathering procedure used by MM in the three noon sessions of the experiment are shown in Figure 5A. Note the variations for MM's readings on different days. These variations do not appear in the correct readings of Figure 4A.



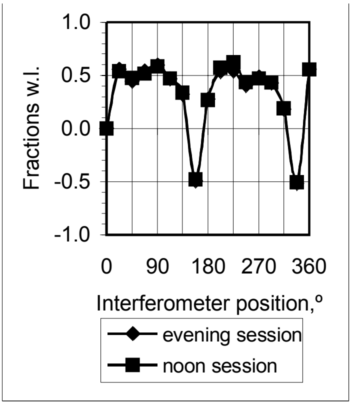


Figure 5. (A) Retrodicted averages for the 3 noon sessions of MM experiment (incorrect data gathering). Note the variations for MM readings in different days. (B) Overall averages for the three sessions of the noon and evening observations are indistinguishable. Note that the second half of graphs (180-360º) is slightly different from the first half (0-180º).

The global average for the three noon sessions is shown in Figure 5B. As expected, the averaging procedure smooths out the variations and further decreases the amplitude of the final curve. The global average for the evening observations is indistinguishable from the noon case. From Figure 5B, the amplitude for the average noon curve according to MM's data gathering procedure is $A_{MM} = 0.5\ [0.58–(–0.48)] = 0.53$ fringes, while the amplitude of the actual fringe-shift variation obtained from the average of the three noon sessions is 16.6 fringes (recall Figure 4A). Hence, the ratio between MM's amplitude and the correct amplitude is $A_{MM}/A_{CORRECT} = 0.53/16.6 = 0.032$. This means that, for a solar motion similar to the velocity used in this paper [29], the original experiment of MM could only detect 3.2% of the amplitude that they intended to measure!

Since Earth's calculated net speed depends on the amplitude of the fringe-shift curve (recall equation 3), it is evident that all MM-type experiments up to 1930 [17]—which typically used the same incorrect data gathering procedure—were bound to yield speeds much lower than expected.

In retrospect, it is rather surprising that Miller [18] did not recognize the need to revise his data gathering procedures. However, he knew that the amplitude that he was obtaining was much lower than required for a correct calculation of the total speed of earth. To account for this unexplained phenomenon he introduced a reduction factor $k = 0.0514$ (see Miller's Table V and pages 234-235). He noted that "until the physical nature of this reduction factor is understood, it need not be assumed that it should be constant for all epochs." [18, p. 235] It is our contention that the argument advanced here clearly explains for the first time the origin of Miller's adjustment. It may be noted that the order of magnitude of Miller's k is similar to our $A_{MM}/A_{CORRECT} = 0.032$ (although our calculation is for MM's interferometer).

As a toy exercise, let us use our correction factor $k = 0.032$ for the MM experiment, and apply Miller's procedure to MM's data. From MM's Figure 6, the amplitude is $A_{MM} = 0.5\ [0.007-(-0.023)] = 0.015$ [26, Fig. 4]. Note that this value—taken from MM's own graph—is 50% larger than the "0.01 of the distance between the fringes" mentioned by MM without any explanation [1, p. 340].

The corrected amplitude for noon observations is then 0.015/0.032 = 0.47. From equation 1, the projection of Earth's velocity on the interferometer plane is then $V_I = 47$ km/s. The error bounds are large. For the purpose of this toy illustration let us use the standard error associated with the mean $m6$ at the position of minimum amplitude in MM's Figure 6 ($\theta = 315°$, $j = 6$). From Table 1A above: SE6 = 0.211 w.l. (row 11). The corrected standard error is then 0.211/0.032 = 6.6 wl. From eq. 3, this corresponds to 177 km/s. Hence, within one standard error, the projection of the velocity on the plane of the interferometer is 47±177 km/s, *i.e.* V_I is in the range 0 to 224 km/s, within one-sigma; for a higher confidence level the interval grows significantly. From here, the terrestrial speed may be found if the declination of Earth's motion is known. (See Múnera [24] for ways of obtaining this datum.)

Note two differences with the MM results: (a) the velocity obtained is the projection on the plane of the interferometer, while MM incorectly expected the orbital speed, (b) the magnitude of the speed on the interferometer plane is a large interval, that has zero at one end, while MM incorrectly reported a null value, without any error band.

IV. Concluding remarks

To put our two new criticisms in context, a short survey of the standard criticisms to the MM experiment was presented in Section I. The first novel criticism is the notorious absence of error bounds in the final curve presented by MM as Fig. 6 [1, p. 340], which serves as a qualitative basis for MM's negative interpretation of the results of their experiment. MM's Figure 6 has been reproduced without any critical comment by most books on classical and relativistic physics during the 20th century; in particular, they do not mention the absence of statistical error bounds. Handschy noted the absence of error bounds, but finds it justifiable because "statistics were not as a standard a part of the scientist's repertoire in 1887 as they are now." [26, p. 987] It is noteworthy that most books that ignore MM's omission of error bounds emphasize the extraordinary sensitivity of MM's apparatus, which was not given in MM's original paper.

As shown here in Section II, the average standard error associated with MM's "final mean" for the noon observations was 0.19 wl, and for the evening observations 0.14 wl (calculated from row 11, Table 1). As a consequence of our new analysis, the results of the 1887 experiment were compatible with MM's original expectations at a 95% confidence level. The present quantitative conclusion is contrary to MM's qualitative conclusion.

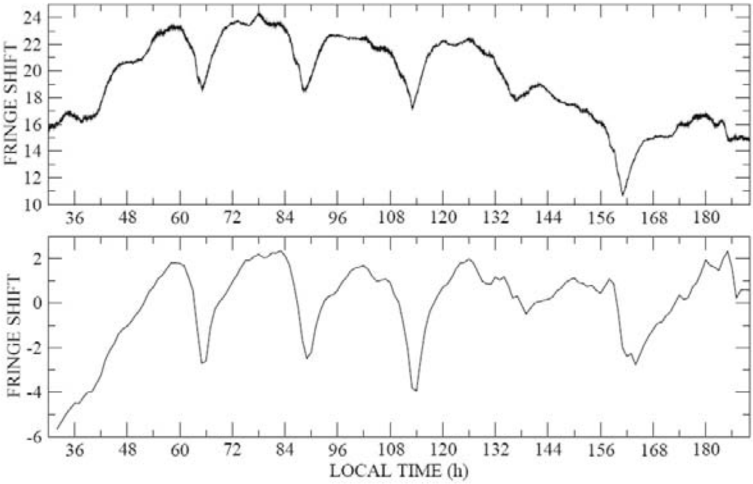


Figure 6. A typical fringeshift curve obtained in the ongoing MM experiment at CIF, Bogotá, fColombia. The raw curve observed during September 2-8, 2003 is shown above. The residual curve (below) after correcting for relative humidity and pressure exhibits a periodicity around 24 hours, and an amplitude of 4 fringes. This residual curve is no longer correlated with humidity and pressure. Correlation with ambient temperature was not clear.

Secondly, in Section III it was argued for the first time that the MM experiment was not properly designed to measure the magnitude of the velocity of Earth relative to a preferred frame. Indeed, MM measured fractions of a wavelength shift, instead of the whole fringeshift. For a modern value of the solar velocity [29], MM's data gathering design produces an apparent amplitude that is 3.2% of the real one.

The same incorrect data gathering procedure was followed in other repetitions of the MM experiment, including those by Miller. For instance, Miller [18, p.213] describes the procedure for calibrating away fringeshifts of one or more wavelengths. Hence, all MM-type experiments up to 1930 that used the same incorrect data gathering process were bound to obtain apparent earth speeds that were too low. Starting with the Kennedy-Thorndyke experiment, modern "MM-type experiments" assume that Lorentz-contraction exists and check whether time dilation ensues [30, p. 73]. Unfortunately, modern papers do not report the raw data, as MM did, thus pre-empting any effort toward an alternative interpretation of the results.

As argued here, in order to calculate earth's speed from an MM-type experiment, it is necessary to register the full amplitude of the fringe-shift curve. Since December 2002, an MM-type experiment has been underway using a stationary interferometer at the International Center for Physics (CIF) in Bogotá (Colombia). Readings are taken every minute which amounts to an Earth rotation of 0.25º between readings. This is an improvement by a factor of 90 with respect to the 22.5º between readings in the original MM experiment. Figure 6 shows typical fringeshift curves obtained in our experiment [31]. These curves are analogous to the theoretically expected curve shown in Figure 4A (making allowance for Bogota's latitude). This is a definite empirical support for the novel criticisms advanced in the present paper.

In a letter to Lord Rayleigh on August 18/1892, Lorentz asked: "Can there be some point in the theory of Mr. Michelson's experiment which has as yet been overlooked?" [32, p. 32] With a delay of 113 years we have identified two such points: large experimental errors, and an incorrect data gathering procedure. As a consequence, at the turn of the 20th century there was no empirical evidence in favour of the FitzGerald-Lorentz length contraction hypothesis, and, hence, no empirical evidence for the similar prediction from Einstein's special theory of relativity.

The majority of writers agree that Einstein did not know enough about the results of the MM experiment, and, hence, did not use this empirical basis as input in *formulating* his theory of special relativity [33], although there are dissenting voices [34]. However, Einstein himself acknowleged that the apparently null result of the MM experiment was instrumental for the *acceptance* of his theory. In Einstein's words: "The successes of Lorentz's theory were so significant that the physicists would have abandoned the principle of relativity without qualms, had it not been for the availability of an important experimental result of which we must now speak, namely Michelson's experiment." [33, p. 145]

One may then wonder how physics might have developed in the 20th century if MM had reported their error bounds, and if Michelson and Morley had designed their experiment correctly.

Acknowledgements

The author is indebted to the editor for his kind invitation to contribute a paper to this volume, and to an anonymous referee for a careful reading of the manuscript and for the opportunity to include Figure 6.

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On the Size of Moving Rigid Bodies Determined from Conditions of Equilibrium of Ions in a Crystalline Lattice

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We consider changes in the sizes of rigid bodies moving uniformly in a given direction. The macroscopic change in the size of the body is caused by a displacement of the points of equilibrium of the ions in the crystalline lattice due to a change in the convection potential. Some applications for the findings are considered in an interpretation of the Michelson-Morley experiment.

Keywords: Michelson-Morley experiment, Lorentz transformation, cross contraction, convection potential

1. Introduction

Recent research in physics and cosmology has produced many new ideas and cosmological models. Some of the most frequently used terms are Quantum Vacuum [1], Brane [2, 3], Domain Wall [4], Soliton [5] or Supermembrane [6, 7]. Common to all these theories is the understanding that space is something with properties. These properties may be energy, tension, fine structure, metric—to name but a few. If these properties are reconsidered, we have to think anew about such simple questions as the propagation of electromagnetic waves, and we have to think about the existence of a stationary medium through which the waves propagate [8, 9]. This is underlined by the discovery of the CBR frame by Penzias and Wilson [10]. A stationary medium is not a contradiction to special or general relativity [11, 12], nor is it a contradiction to the constancy of the speed of light. But the attempt to connect the QED and GR raises new questions concerning models of space. One of these questions is the reality of length contraction of rigid bodies demanded by SR. Here we will show that in considering the relativistic contraction of the moving bodies, for example in the Michelson-Morley (MM) experiment, one factor is omitted. This factor is the effect of the convection potential created by the ions in the lattice of a crystal* uniformly moving relative to the surrounding ions, which results in re-distribution of the ions of the lattice to other points of electrostatic equilibrium. The total contraction of the moving body must be greater than predicted by both special relativity [11, 12] and Lorentz's concept of ether [13, 14]. The authors introduce a cross contraction and an enhanced length contraction. This double contraction is not in contradiction to special relativity, since the quotient of the length and the cross contraction complies with the contraction quotient of special relativity.

This paper is set out as follows. In Section 2, we consider how the conditions of electrostatic equilibrium of ions in the lattice of a crystal body change if this body begins to move uniformly. We calculate the distances between the new points of equilibrium of the ions, and how this results in changes in the size of the whole body. In Section 3, we apply the results of Section 2 to interpret the measurement data from the MM experiment. In Section 4, we numerically calculate the potential of an ionic crystal lattice. Section 5 contains some conclusions.

2. Contraction of moving bodies due to the convection potential

Let us consider the motion of a rigid body composed of an ionic crystal (for example, a NaCl crystal). Our choice of material of the body is due to our intention to reduce the analysis of its solid state behaviour to that of the electrostatic forces providing equilibrium in the crystalline lattice. Actually, one could calculate the change in the points of equilibrium of the ions in the lattice of some perfect metal. In this case, the distribution of the conductive electrons in the lattice would have to be considered. The dominant factor determining the locations of the ions in the lattice is the electrostatic repulsion force between the ions. This force is shielded by a spatial negative charge due to the conductive electrons. So even in this case, the task can be reduced to an electrostatic analysis. It can be established by considering the Hamiltonian of the crystal ([15] Ch. 1.3, Eq. (1.3.1))

$$H = \frac{1}{2}\sum_{l}\frac{\mathbf{p}^{2}}{m} + U\left(\mathbf{R}_{1},\mathbf{R}_{2},..\mathbf{R}_{l}...\right), \tag{1}$$

where the potential energy of the crystal is only a function of the distances between the atoms of the lattice (Eq. 1.3.3 of Ref. 15). So in the equilibrium configuration, when the atoms are located exactly in the sites of the lattice, we have

* The type of material that the rigid body is composed of is not significant. It can be either a crystalline or an amorphous material; what is significant here is that the ions of the material are located at points of equilibrium and that these points are determined by electrostatic forces. The only difference in the types of material of the rigid body that the body is composed of concerns the order of the crystalline lattice. The calculation of the points of equilibrium of the ions in a lattice is much easier than for amorphous materials.

$$\frac{\partial U}{\partial \mathbf{r}_l} = 0 \qquad \text{where } \mathbf{r}_1 = \mathbf{r}_2 = \ldots \mathbf{r}_N = 0\,, \tag{2}$$

for all $\mathbf{r}_l = \mathbf{R}_l - la$. In solid state problems that consider the dynamic properties of the crystal, the potential energy is represented by

$$U\left(\mathbf{r}_1, \mathbf{r}_2, ..\mathbf{r}_l ...\right) = U_0 + \frac{1}{2}\sum_{ll'} \mathbf{r}_l \mathbf{r}_{l'} \frac{\partial^2 U}{\partial \mathbf{r}_l \partial \mathbf{r}_{l'}}\,, \tag{3}$$

This added constant in the energy U_0 is not essential in studying the dynamic properties of the crystal, so it is omitted from solid state problems. But we will be interested in this added constant, *i.e.* how it changes if the forces acting on the atoms change too. Obviously, U_0 obeys Eq. (2) and we will analyze this equation below.

When each ion moves with the whole lattice, we are able to consider the fields of one elementary cell. For NaCl, the lattice is of the cubic type and it is sufficient to consider how the points of equilibrium change in the longitudinal and transversal directions relative to the motion of the body (we suggest that one axis of symmetry of the lattice is oriented in the direction of motion and the other two axes are directed transversally). For uniformly moving charges, the EM fields created by these charges are stationary, which means that they become static in the co-moving frame. We find the magnitude of the interaction force between the ions [16],

$$\boldsymbol{F} = e\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right), \tag{4}$$

where $\boldsymbol{v}$ is the velocity of uniform motion of the lattice, $\boldsymbol{E}$ and $\boldsymbol{B}$ the electric and magnetic fields created by one ion. The values of these EM fields can be found from the expressions for the Liennard-Wiechert potentials written in the 'present time' coordinates. The force is given by

$$\begin{gathered} \boldsymbol{F} = e^2\left[-\nabla\left(\frac{1}{s}\right) + \left(\boldsymbol{v}\cdot\nabla\right)\frac{\boldsymbol{v}}{c^2 s} + \frac{\boldsymbol{v}}{c^2}\times\left(\nabla\times\frac{\boldsymbol{v}}{s}\right)\right] \\ s = \sqrt{\left(x-x'\right)^2 + \left(1 - v^2/c^2\right)\left[\left(y-y'\right)^2 + \left(z-z'\right)^2\right]} \end{gathered}\,. \tag{5}$$

Here, x, y, z and x', y', z' are the coordinates of the interacting ions, and the x-axis is assumed to be parallel to v, without any restriction of generality. Further, we assume that charges do not change with speed v. Neglecting the sign, Eq. (5) can be rewritten as

$$\begin{gathered} F = e^2\nabla\left(\frac{\left(1 - v^2/c^2\right)}{s}\right) = e^2\nabla\Psi \\ \Psi = \frac{\left(1 - v^2/c^2\right)}{\sqrt{\left(x-x'\right)^2 + \left(1 - v^2/c^2\right)\left[\left(y-y'\right)^2 + \left(z-z'\right)^2\right]}} \end{gathered}\,, \tag{6}$$

where the scalar function Ψ is called the *convection potential*. So we see from Eq. (6) that for a system of the moving charges, the electrostatic potential is changed to the convection potential. Because the electrostatic potential determines the location of the ions where they are in equilibrium when the lattice is at rest, we can suggest that if the lattice moves, the convection potential alone must determine the equilibrium points of the ions in the moving lattice. It follows from Eq. (6) that if the velocity of motion of the lattice changes, the magnitude of the convection potential changes too. Therefore, the location points of the ions must change too. Below, we find how the new points of equilibrium depend on the velocity.

It is reasonable to assume that if the magnitude of the potential that determines the equilibrium of the lattice changes, the ions tend to displace in such a way that the change in the potential conserves the total energy. Let us assume that when the lattice is at rest, the distance between two neighbour ions is d. The energy of the lattice is the potential electrostatic energy of the ions

$$\begin{aligned} W_{rest} &= -\sum_{l,m,n,k=0}^{\infty} \frac{(-1)^{l+m+n} e^2}{\sqrt{(ld)^2 + (md)^2 + (nd)^2}} \\ &= -\frac{e^2}{d} \sum_{l,m,n,k=0}^{\infty} \frac{(-1)^{l+m+n}}{\sqrt{l^2 + m^2 + n^2}} \end{aligned}, \tag{7}$$

where summation over l corresponds to the summation of the ions along the x-axis, summation over m along the y-axis, and summation over n along the z-axis; the indices $l = m = n = 0$ are excluded.

When the lattice moves, because of changes in the convection potential, the electrostatic energy of the ions changes with velocity as

$$W_{mov} = -\sum_{l,m,n,k=0}^{\infty} \frac{(-1)^{l+m+n} \left(1 - v^2/c^2\right) e^2}{\sqrt{(ld)^2 + \left(1 - v^2/c^2\right)\left[(md)^2 + (nd)^2\right]}} . \tag{8}$$

The terms $1 - v^2/c^2$ in the numerator and in the denominator, of Eq. (8) arise from the finite speed c of propagation of electromagnetic forces. The case of two moving charges (charge 1 and charge 2) with their distance perpendicular to speed v is easier to explain (see Fig. 1b). If md is such a perpendicular distance, then the traveling distance of the electromagnetic force in the resting frame is enlarged, by analogy with the hypotenuse of a right angle triangle with adjacent side vt and opposite side ct, by the factor $(1 + v^2/c^2)^{1/2}$, or $(1 - v^2/c^2)^{-1/2}$ if we are considering both terms, in the denominator, and in the numerator, respectively. Traveling time for the distance $(1 + v^2/c^2)^{1/2}$ is t.

To explain the parallel case, one must assume that the action and reaction of the electromagnetic force need to be twice the distance—from charge 1 to charge 2 and back. If ld is the parallel distance, we find the traveling time t_l of the action from charge 1 to charge 2 to be $t_1 = (ld + t_1 v)/c$ or $t_1 = ld/(c - v)$. The reverse trip, t_2, is $t_2 = (ld - t_2 v)/c$ or $t_2 = ld/(c + v)$. The averaged simple traveling distance is then $c(t_1 + t_2)/2 = ld/(1 - v^2/c^2)$, *i.e.*, exactly the factor of the distance ld in Eq. (8).

Since there is no internal motion of the ions, the total internal energy is equal to the electrostatic energy. Comparison of Eq. (7) to Eq. (8) shows that the total internal energy of the lattice increases with the velocity. This means that the ions are not located at the points of equilibrium, and it is necessary to find the new location points of the ions in such a way that the magnitude of W_{mov} will be equal to the value of W_{rest}. To do this, we consider the partial sums over *m, n* and *l,* separately. We have for Eqs. (7) and (8) at $m = n = 0$

$$W_{rest}^{\|} = -\frac{e^2}{d}\sum_{l,k=0}^{\infty}\frac{(-1)^l}{l} \quad ; \quad W_{mov}^{\|} = -\frac{e^2}{d}\sum_{l,k=0}^{\infty}\frac{(-1)^l\left(1-v^2/c^2\right)}{l} \quad . \tag{9}$$

The total energy does not change if the magnitudes of $W_{rest}{}^{\|}$ and $W_{mov}{}^{\|}$ are equal. Because the only parameter, which can change in Eq. (9), is the interatomic distance *d*, the equivalence of these quantities is provided by changing the interatomic distance (in *x* direction, length contraction due to von Weber [8]); that is,

$$d_{rest}^{\|} = \frac{d_{mov}^{\|}}{1-v^2/c^2} \quad \Rightarrow \quad d_{mov}^{\|} = \left(1-v^2/c^2\right)d_{rest}^{\|} \quad . \tag{10}$$

Accordingly, analysis of the partial sums at $l = 0$ gives

$$\begin{aligned} W_{rest}^{\perp} &= -\frac{e^2}{d}\sum_{m,n,k=0}^{\infty}\frac{\delta_{m+n;2k+1}}{\sqrt{m^2+n^2}} \quad ; \\ W_{mov}^{\perp} &= -\frac{e^2}{d}\sum_{m,n,k=0}^{\infty}\frac{\sqrt{1-v^2/c^2}\,\delta_{m+n;2k+1}}{\sqrt{m^2+n^2}} \quad ; \end{aligned} \tag{11}$$

and the interatomic distances, which are transverse to the direction of motion of the lattice at rest and the moving lattice, are connected (due to cross contraction) by

$$d_{rest}^{\perp} = \frac{d_{mov}^{\perp}}{\sqrt{1-v^2/c^2}} \quad \Rightarrow \quad d_{mov}^{\perp} = \sqrt{1-v^2/c^2}\,d_{rest}^{\perp} \quad ; \tag{12}$$

Using conditions (10) and (12), one can evaluate changes in the distance between two arbitrary ions separated by sites *l, m* and *n* ($d_{rest}^{\|} = d_{rest}^{\perp} = d$)

$$\begin{aligned} d_{mov}(l,m,n) &= \sqrt{l^2\left(d_{mov}^{\|}\right)^2 + \left[m^2+n^2\right]\left(d_{mov}^{\perp}\right)^2} = \\ &\sqrt{\left(1-v^2/c^2\right)^2(ld)^2 + \left(1-v^2/c^2\right)\left[(md)^2+(nd)^2\right]} \quad . \end{aligned} \tag{13}$$

Using Eq. (9), we calculate the electrostatic energy of the moving lattice, taking into account that the transverse component of the interatomic distance should enter into the formula *via* the factor $\left(1-v^2/c^2\right)$

$$W_{mov} = -\sum_{l,m,n,k=0}^{\infty} \frac{\left(1-v^2/c^2\right)e^2\delta_{l+m+n;2k+1}}{\sqrt{\left[d_{mov}^{\parallel}\right]^2 + \left(1-v^2/c^2\right)\left[d_{mov}^{\perp}\right]^2}} =$$

$$-\sum_{l,m,n,k=0}^{\infty} \frac{\left(1-v^2/c^2\right)e^2\delta_{l+m+n;2k+1}}{\sqrt{\left(1-v^2/c^2\right)^2(ld)^2 + \left(1-v^2/c^2\right)^2\left[(md)^2+(nd)^2\right]}} \tag{14}$$

$$= -\frac{e^2}{d}\sum_{l,m,n,k=0}^{\infty} \frac{\delta_{l+m+n;2k+1}}{\sqrt{l^2+m^2+n^2}} = W_{rest} \quad .$$

If the conditions (10) and (12) are fulfilled, the total electrostatic energy of the lattice does not change with velocity. (We consider steady-state motion only.)

This is an important result in the analysis of the electrodynamics of moving bodies. It can easily be shown that any other changes in the interatomic distances of the lattice do not provide the minimum electrostatic energy when the body is in motion. For example, if we assume that only relativistic contraction of the bodies occurs, *i.e.* the interatomic distance changes only in the direction of motion,

$$d_{mov}^{\parallel} = \sqrt{\left(1-v^2/c^2\right)}d_{rest}^{\parallel} \quad ; \quad d_{mov}^{\perp} = d_{rest}^{\perp} \, ; \tag{15}$$

we have for W_{mov} taking into account Eqs. (8) and (11)

$$W_{mov} = -\frac{e^2}{d_{rest}^{rel}}\sum_{l,m,n,k=0}^{\infty} \frac{\left(1-v^2/c^2\right)\delta_{l+m+n;2k+1}}{\sqrt{\left(1-v^2/c^2\right)l^2 + \left(1-v^2/c^2\right)\left[m^2+n^2\right]}} =$$

$$-\frac{\sqrt{1-v^2/c^2}\,e^2}{d_{rest}^{rel}}\sum_{l,m,n,k=0}^{\infty} \frac{\delta_{l+m+n;2k+1}}{\sqrt{l^2+m^2+n^2}} = \sqrt{1-v^2/c^2}\,W_{rest} \tag{16}$$

and the electrostatic energy of the lattice does not reach its minimum. If the interatomic distances change in accordance with Eqs. (10) and (12), the overall size of the moving body must change as

$$L_{mov}^{\parallel} = \left(1-v^2/c^2\right)L_{rest}^{\parallel} \quad ; \quad L_{mov}^{\perp} = \sqrt{1-v^2/c^2}\,L_{mov}^{\perp} \, . \tag{17}$$

We should note that the relativistic transformation of the fields is included in our calculations (the expression for the convection potential), but it follows from a more sophisticated consideration of the change in the electrostatic energy, and, therefore, the equilibrium conditions of the ions forming the crystalline lattice demand that some additional contraction of the moving bodies must occur.

3. Application of the above results to the interpretation of the Michelson-Morley experiment

Now we show that the contraction of a moving body described by Eq. (11) can be used to explain the *null results* of the Michelson-Morley (MM) experiments [17, 18] (the scheme of this experiment is given in Fig. 1). First, we note that

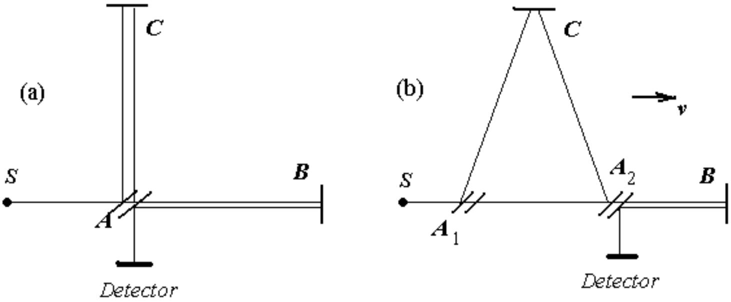

Fig. 1 - Scheme of the Michelson- Morley experiments in rest (a) and moved (b)

the optical paths of the light beam in the Michelson interferometer are determined by the lengths of the arms ***AB*** and ***AC*** supporting the mirrors. The arms are made of a solid material and, therefore, they should contract, in accordance to Eq. (13). We denote the inertial frame of reference in which the Michelson interferometer moves (with the Earth) as the CBR frame, and the inertial frame where the interferometer is at rest as the laboratory frame. Let us assume that the length of the path ***AB*** for the Michelson interferometer is at rest and is $L^{\|}_{rest}$ * and that the length of the path ***AC*** is $L^{\perp}_{rest}$.
Then the optical paths are

$$P^{\|}_{rest} = 2L^{\|}_{rest} \quad ; \quad P^{\perp}_{rest} = 2L^{\perp}_{rest} \ , \tag{18}$$

and the difference in the optical paths is

$$\Delta P_{rest} = P^{\|}_{rest} - P^{\perp}_{rest} = 2\left[L^{\|}_{rest} - L^{\perp}_{rest} \right], \tag{19}$$

so a difference in the traveling times of the light beam in this frame is

$$\Delta t_{rest} = \frac{2\left[L^{\|}_{rest} - L^{\perp}_{rest} \right]}{c}, \tag{20}$$

Now we go to the CBR frame where the interferometer moves. When the arm $L^{\|}$ is oriented parallel to the light beam ***A →B→A,*** the traveling time of the light is

$$\begin{aligned} t_{ABA} &= t_{AB} + t_{BA} = \frac{L^{\|}_{mov}}{c-v} + \frac{L^{\|}_{mov}}{c+v} = \frac{2cL^{\|}_{mov}}{c^2 - v^2} \\ &= \frac{2c\left(1 - v^2/c^2\right)L^{\|}_{rest}}{c^2 - v^2} = \frac{2L^{\|}_{rest}}{c} \end{aligned} \tag{21}$$

When the arm $L^{\perp}$ is oriented perpendicular to direction of motion, the traveling time of the light is

$$t_{ACA} = t_{AC} + t_{CA} = \frac{2L^{\perp}_{mov}}{\sqrt{c^2 - v^2}} = \frac{2\sqrt{1 - v^2/c^2}\,L^{\perp}_{rest}}{\sqrt{c^2 - v^2}} = \frac{2L^{\perp}_{rest}}{c}, \tag{22}$$

* The arm ***AB*** is directed along the velocity vector of the laboratory frame with respect to the CBR frame of reference.

so the difference in the traveling times of the light beam in the laboratory frame is

$$\Delta t_{mov} = t_{ABA} - t_{ACA} = \frac{2\left[L^{\parallel}_{rest} - L^{\perp}_{rest}\right]}{c} . \tag{23}$$

Because the difference in the optical paths is

$$\Delta P_{mov} = c\Delta t_{mov} = \left[L^{\parallel}_{rest} - L^{\perp}_{rest}\right], \tag{24}$$

we see from Eqs. (19) and (24) that the type of contraction that the moving body (13) undergoes provides conservation of the optical paths of the light beam in the Michelson interferometer, independent of the frame of reference. Because the difference in the optical paths determines the interference pattern distribution, we conclude that this type of contraction explains the *null result* of the MM experiment.

4. The numerical calculation of the potential of an ionic crystal lattice

When the lattice moves, the electrostatic energy of the ions changes because of changes in the conventional potential with increasing the velocity according to

$$W_{mov} = -\sum_{l,m,n,k=0}^{\infty} \frac{\left(1 - v^2/c^2\right) e^2 \delta_{l+m+n;2k+1}}{\sqrt{(ld)^2 + \left(1 - v^2/c^2\right)\left[(md)^2 + (nd)^2\right]}} . \tag{25}$$

Since there is no internal motion of the ions, the total internal energy is equal to the electrostatic energy. Comparing Eq. (3) to Eq. (25) shows that the total internal energy of the lattice increases with the velocity. This means that the ions are not located at the points of equilibrium and it is necessary to find the new location points of the ions in such a way that the magnitude of W_{mov} will be equal to the value of W_{rest}.

The volume of the two possible space cells of our NaCl type crystal is $(2d)^3$. The nearest distance between two ions of the same kind is $2d$. One space cell consists of 8 ions of the first kind positioned at the corners of a cube, positive for example, and one central ion of the other kind, that is negative. The second possible space cell is built similarly, but the ions interchanged.

Let us consider spherical shells surrounding a central ion. The number $N(r)$ of ions in a shell with thickness dr and radius r is proportional to $4\pi r^2 dr$. With greater r, we can consider the number $N(r)$ as defined statistically. Using the Bernoulli equations, we assume a statistical error of $N^{-1/2}$. Since positive and negative ions are mixed up in a shell, the number of positive acting ions is $N_+ - N_-$ with a statistical error of about $(2N_+)^{-1/2}$ or $N_o r$. The constant factor N_o contains the number of ions per spatial unit, and additionally contains the constants $4\pi dr$. The potential of the $N_+ - N_-$ ions acting on the central ion is then statistically averaged $P_o\, r/r = P_o$. This means that the potential of each shell is zero with a statistical error of constant variance across all radii. Integrating the

Table. 1 - Relative potentials of the central ion in an ionic crystal lattice for different β and different contraction formulae

β	P_1/P_0	P_2/P_0	P_3/P_0
0.000	1.0000000	1.0000000	1.0000000
0.001	0.9999993	0.9999995	1.0000000
0.002	0.9999973	0.9999980	1.0000000
0.003	0.9999940	0.9999955	1.0000000
0.004	0.9999893	0.9999920	1.0000000
0.005	0.9999833	0.9999875	1.0000000
0.006	0.9999760	0.9999820	1.0000000

contribution of the shells, we get a divergent integral similar to the sine or cosine integral.

The Laplace transform shows a way to handle such divergent integrals. We introduce the damping factor $\exp(-\delta r)$ for the integrand with $r = \sqrt{(ld)^2 + (md)^2 + (nd)^2}$ and calculate the sum according to Eq. (25) using this damping factor. The practical computation written in C-language uses a spherical body of ions with radius $r_{\max} = 500$ ion distances d. The damping factor δ was chosen so that for $r = r_{\max}$ we got $\exp(-\delta r) = 10^{-5}$. The thickness of a shell was $\mathrm{d}r = d/150$. Table 1 shows the results of the numerical calculation.

Here P_o is the potential at $\beta = 0$. P_1 is the potential calculated without any contraction of the crystal. P_2 is the potential calculated with Lorentz contraction, *i.e.*, contraction only in the x-direction. P_3 is the potential calculated with the length- and cross-contraction introduced above. Since the absolute value is not of interest, Table 1 shows the relative change of the calculated potentials with increasing β.

The relative potential $P_{1/}\, P_0$ changes as $(1-\beta^2)^{2/3}$. The relative potential $P_{2/}\, P_0$ changes as$(1-\beta^2)^{1/2}$. The relative potential $P_{3/}\, P_0$ does not change in value as a result of changing β. Any decrease in the potential energy of the crystal lattice is physically not explicable. Consequently, case 3 with cross- and length-contraction is the only case with non-decreasing potential.

Another important question is the influence of the relativistic increase in mass with velocity on the potential energy. Here we quote Richard Feynman [19], who said that the gravitational potential of two charged particles is about 40 orders of magnitude less than the electrostatic potential. An increase in mass plays an important role if we are considering ion speeds near the speed of light.

Based on the above numerical result, we consider the process of displacement of the ions in the lattice while the velocity of the crystal increases. Because the absolute values of the relativistic length contraction for the lattice cell are too small, we conclude that the shift of the ions in each cell of the lattice occurs in both directions due to the above defined cross- and length-contraction. Because the quotient of the length- and the cross-contraction $(1-\beta^2)/(1-\beta^2)^{1/2} = (1-\beta^2)^{1/2}$ is the same as the SR or Lorentzian length contraction $(1-\beta^2)^{1/2}$, we get the same null result for the MM experiment as the SR or the Lorentzian ether theory.

The authors have showed, starting from basics, how the sizes of moving rigid bodies are determined from the conditions of equilibrium of the ions in a lattice. It appears, however, that the other type of contraction of relativistically moving bodies yields the same effects as SR predicts. Actually, if we calculate the difference in the optical paths in the moving interferometer, we obtain the same value as given by Eq. (25). The SR states that, when the interferometer moves, the arms contract as

$$L^{\|}_{mov} = \sqrt{1-v^2/c^2}\,L^{\|}_{rest} \quad ; \quad L^{\perp}_{mov} = L^{\perp}_{rest} \,. \tag{26}$$

Then, instead of Eqs. (25) and (26) we have

$$\begin{aligned} t'_{ABA} &= t'_{AB} + t'_{BA} = \frac{L^{\|}_{mov}}{c-v} + \frac{L^{\|}_{mov}}{c+v} = \frac{2cL^{\|}_{mov}}{c^2-v^2} \\ &= \frac{2c\sqrt{1-v^2/c^2}\,L^{\|}_{rest}}{c^2-v^2} = \frac{2L^{\|}_{rest}}{c\sqrt{1-v^2/c^2}} \end{aligned} \,. \tag{27}$$

and

$$t'_{ACA} = t_{AC} + t'_{CA} = \frac{2L^{\perp}_{mov}}{\sqrt{c^2-v^2}} = \frac{2L^{\perp}_{rest}}{c\sqrt{1-v^2/c^2}} \,. \tag{28}$$

One obtains the difference in the traveling times from Eqs. (27) and (28)

$$\Delta t'_{mov} = \frac{2L^{\|}_{rest} - 2L^{\perp}_{rest}}{c\sqrt{1-v^2/c^2}} \,. \tag{29}$$

In the SR, the factor $1\big/\sqrt{1-v^2/c^2}$ is eliminated by the time dilation $\Delta t'_{\mathrm{mov}} \to \Delta t'_{\mathrm{mov}}/(1-v^2/c^2)^{0.5}$, so Eq. (29) should be transformed to

$$\frac{\Delta t'_{mov}}{\sqrt{1-v^2/c^2}} = \frac{2L^{\|}_{rest} - 2L^{\perp}_{rest}}{\sqrt{1-v^2/c^2}} \,. \tag{30}$$

Hence the difference in the optical paths conserves

$$\Delta P^{SR}_{mov} = c\Delta t'_{mov} = 2\left[L^{\|}_{rest} - L^{\perp}_{rest}\right] = \Delta P_{rest} \,, \tag{31}$$

which yields a null result. We find the same time dilation in the Lorentz-Fitzgerald theory yielding the null result too.

However, although both types of contraction of the bodies, *i.e.* relativistic and those considered above, predict no change in the interference picture when the velocity of the interferometer, with respect to the cosmic background radiation frame, changes, there is one difference between these types of contraction. Below we analyze it in more detail:

1. The magnitude of the contraction we considered above (WO contraction) is stronger that the magnitude of the SR contraction. Because we derived the WO contraction from the Maxwell equations, which are primary with respect to the SR*, we should conclude that the rigid bodies mostly contract while they move in accordance to the WO contraction. The optical

* The SR was derived from the symmetries of the Maxwell equations.

path $\boldsymbol{A}\rightarrow\boldsymbol{B}\rightarrow\boldsymbol{A}$ of the light beam in the interferometer can be found, in the CBR frame, either from Eq. (25) or from geometric consideration of Fig. 1

$$P_{mov}^{\|}=\frac{L_{mov}^{\|}}{1-v/c}+\frac{L_{mov}^{\|}}{1+v/c}=\frac{2L_{mov}^{\|}}{1-v^2/c^2}=2L_{rest}^{\|}. \tag{32}$$

2. We will measure this optical path in terms of the wavelengths λ, *i.e.*, the number of the crests of the EM field distribution in the resonance cavity of the Michelson interferometer. Because observation of the interference pattern picture requires a long time compared to propagation time of the light in the interferometer, we should consider a stationary EM field distribution in this cavity and, therefore, the factor $\exp(i\omega t)$ in the term describing the EM wave can be neglected.
3. Because the light source is linked to the laboratory frame, we should transform λ' to the CBR frame. This is done in accordance with Eq. (11.19) or Eq. (11.22) of [20]

$$\lambda=\frac{\lambda'}{\sqrt{1-v^2/c^2}}. \tag{33}$$

And the number of the crests in the resonance cavity is

$$N=\frac{P_{mov}^{\|}}{\lambda}=\frac{2\sqrt{1-v^2/c^2}}{\lambda'}L_{rest}^{\|}. \tag{34}$$

4. Now we use the fact that the quantity N is *invariant,* because the invariant quantity is the phase of the EM wave (Sec. 11.4 of [20]). This means that if we count the number of crests of the EM wave passing some distance in two frames, we must obtain the same value of crests. Thus, if we count this quantity from the above equation in the laboratory frame, we should obtain the correct result. All quantities in the *rhs* of Eq. (34) are now defined in the laboratory frame, so this formula can serve to count the crests of the EM field distribution in the resonance cavity of the interferometer.
5. Counting the number of crests of the EM wave of the light beam in the Michelson interferometer is a very difficult, but in principle, solvable experimental task. Actually, one needs to solve a simpler technical task. It can be seen from Eq. (34) that the number of crests depends on the velocity v of the laboratory frame with respect to the CBR frame and evaluation of Eq. (34) for the values $L_{rest}^{\|}\approx 1$ meter, $\lambda'\approx 1$ μm, and that if the velocity changes from 300 km/sec (when the main axis of the resonator is directed along the vector $\boldsymbol{v}$) to 0 (when the main axis is directed transversally to $\boldsymbol{v}$) it gives

$$\Delta N=N(v)-N(0)\approx 1. \tag{35}$$

Only this quantity ΔN needs to be detected in the experiments.
6. But if we analyze the above statement, we find we have a paradox, from the viewpoint of the SR results, *i.e.*, formally, we can detect a change in the number of crests when the velocity of the laboratory frame changes. This contradicts the first postulate of the SR, *i.e.*, complete equivalence of

the inertial frames. Thus we find that the problem of contraction of lengths and the time dilation of bodies moving with relativistic velocities is still an open problem and requires more sophisticated investigation.

Acknowledgements

The authors would like to express their gratitude to the following individuals for their support and assistance in developing this paper: Alexander von Eye, Michigan State University, Alexandr Kholmetski, Belarus State University, and Tom Anderson Smith, FH Furtwangen.

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Einstein's Spherical Wavefronts *versus* Poincaré's Ellipsoidal Wavefronts

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We show that the image produced by the Lorentz Transformation of a spherical light wave emitted by a moving source is not a spherical light wave but an ellipsoidal light wave. Poincaré's elongated ellipse is the direct geometrical representation of Poincaré's relativity of simultaneity. Einstein's circles are the direct geometrical representation of Einstein's convention of synchronisation. Poincaré's ellipse supposes another convention for the definition of space-time units whereby the Lorentz Transformation of a unit of length is *directly proportional* to the Lorentz Transformation of a unit of time. The historical problem of priorities is therefore scientifically resolved because Einstein's explicit kinematics and Poincaré's implicit kinematics are not the same.

1. Introduction: Einstein's Spherical Wavefront and Poincaré's Ellipsoidal Wavefront

Einstein writes in the third section of his famous 1905 paper:

> At the time $t = \tau = 0$, when the origin of the two coordinates [K and k] is common to the two systems, let a *spherical wave* be emitted therefrom, and be propagated with the velocity c in system K. If (x, y, z) be a point just attained by this wave, then
>
> $$x^2 + y^2 + z^2 = c^2 t^2 \tag{1}$$
>
> Transforming this equation with our equations of transformation, we obtain after a simple calculation
>
> $$\xi^2 + \eta^2 + \zeta^2 = c^2 \tau^2 \tag{2}$$
>
> The wave under consideration is therefore no less a *spherical wave* with velocity of propagation c when viewed in the moving system k. [3]

Poincaré writes in 1908 in his second paper on *La dynamique de l'électron* with the subtitle *Le principe de relativité*:

> Imagine an observer and a source involved together in the transposition. The wave surfaces emanating for the source will be *spheres*, having as centre the successive positions of the source. The distance of this centre from the present position of the source will be proportional to the time elapsed since the emission—that is to say, to the radius of the sphere. But for our observer, on

account of the contraction, all these spheres will appear as *elongated ellipsoids*. The compensation is now exact, and this is explained by Michelson's experiments. [18]

We can further find in Poincaré's text the equation (two dimensions) of an elongated light ellipse. The observer at rest (let us call him: O) is situated at the centre C and the source S (with "our observer," let us call him O′) in motion is situated at the focus F of the ellipse.

The contrast between both great relativists, Einstein and Poincaré, about an experiment that appears to be the same (*the image of a spherical wave emitted by a moving source*) is very clear: according to Einstein, the image of a spherical wave is a spherical wave, while according to Poincaré the image of a spherical wave is an ellipsoidal wave. Was Poincaré not aware of the invariance of the quadratic form? Not at all, because with the group structure he demonstrates in his first paper on *La dynamique de l'électron* [14], that the Lorentz Transformation (LT) "does not modify the quadratic form $x^2+y^2+z^2-c^2t^2$." It must be noted that Poincaré's lengthened light waves have been almost completely ignored for a whole century by the scientific community*. We note also that Poincaré does not use the LT in the previous quotation, and deduces the ellipsoidal shape of the light wave directly from the principle of contraction of (the unit) of length (*cf.* Conclusion).

Who is right: Einstein or Poincaré? The best way to solve this dilemma is to apply a LT to a spherical wavefront.

2. Image produced by LT of the "Circular Wave"

What is the image (shape) in the system K of a spherical wave emitted at $t'=t=0$ by a source S at rest at the origin O′ of the system K′? The LT defined by Poincaré (K′ is in uniform translation with respect to K) is:

$$x'=k(x-\varepsilon t) \qquad y'=y \qquad t'=k(t-\varepsilon x) \tag{3}$$

We keep Poincaré's notations where ε, k correspond to Einstein-Planck's notations β, γ because, according to Poincaré in his 1905 work on the theory of relativity, *"I shall choose the units of length and of time in such a way that the velocity of light is equal to unity.*" [14] The deeper meaning of Poincaré's choice of space-time units *with* $c=1$ will be indicated in the Conclusion. In order to have *a single* wavefront, we have to define a time t' as unit of time $1_{t'}$. The equation of the circular wave front in K′ (the geometrical locus of the object-points in K′) in $t'=1$ is:

$$x'^2+y'^2=t'^2=1_{t'} \tag{4}$$

* Poincaré's ellipsoidal wavefront was mentioned in his 1905-1906 course *Les limites de la loi de Newton* [16]. We also find them in *La Mécanique Nouvelle* (1909) [19]. In fact it was in 1904, at a talk in Saint Louis, that Poincaré first introduced the elongated ellipsoidal wave as an *alternative* (non-relativistic alternative, developped by Guillaume[19], Leroux[6] and Dive[2]) and not as a *consequence* of the contraction of the unit of length [13]. Poincaré's ellipsoidal wavefront is also connected with the problem of the ellipsoid of observation (Abramson, 23) and the problem of the relativistic appearance of moving object (Penrose, 8). See end of paragraph 3.1 (Ellipse I).

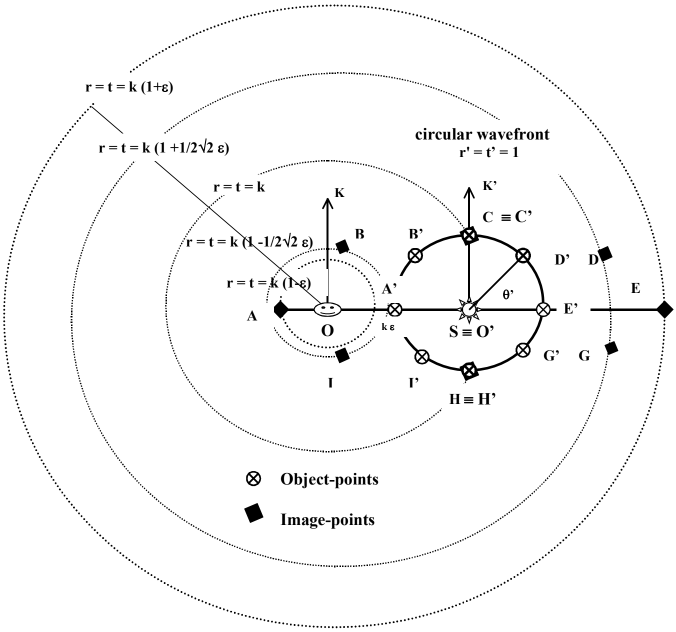

Figure 1 - Image-Points (A, B, C...) in K of the Object-Points (A′, B′, C′...) of a spherical wavefront emitted by a source in K′ (ε ~ 0.9, k ~ 2.3 *and* kε ~ 2.

The unprimed coordinates of the image-points are given by the inverse LT:

$$x = k(x' + \varepsilon t') \qquad y = y' \qquad t = k(t' + \varepsilon x') \tag{5}$$

The coordinates $(0,0,1)$ in K of the source in $t' = 1$ are $(k\varepsilon, 0, k)$ and $(k\varepsilon t', 0, kt')$ in $t' \neq 1$.

Let us determine the images (x, y, t) in K of different object-points in $t' = 1$: $(1,0,1)$, $(-1,0,1), (0,1,1), (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1)$ *etc.* (Figure 1).

- The image-point E, $k(1+\varepsilon)$, 0, $k(1+\varepsilon)$, is on the large dotted circle $x^2 + y^2 = t^2$ with radius $r = t = k(1+\varepsilon)$.
- The image-point A, $k(\varepsilon - 1)$, 0, $k(1-\varepsilon)$, is on the small dotted circle $x^2 + y^2 = t^2$ with radius $r = t = k(1-\varepsilon)$.
- The image-point C, $(k\varepsilon, 1, k)$, is on the dotted circle $x^2 + y^2 = t^2$ with the radius k. The image-point D, $k(\frac{\sqrt{2}}{2} + \varepsilon), \frac{\sqrt{2}}{2}, k(1 + \frac{\sqrt{2}}{2}\varepsilon)$, is on the dotted circle $x^2 + y^2 = t^2$ with radius $r = t = k(1 + \frac{\sqrt{2}}{2}\varepsilon)$, *etc.*

Contrary to what one might expect, the images produced by the LT (5) of the points are not situated on one circular wavefront but, *given the invariance of the quadratic form* (the dotted circles $x^2 + y^2 = t^2$), in a circular ring between $k(1-\varepsilon) \leq r \leq k(1+\varepsilon)$ (Figure 1).

We will now show that all the image-points of the circular wavefront in K′ are on an elliptical wavefront. By introducing, in the system K′, the angle

θ' determined by both the radius vector $\mathbf{r}'$ and the Ox$'$ axes, we have $x' = r'\cos\theta'$ and $y' = r'\sin\theta'$. So with $r' = t' \neq 1$ we have:

$$t = kt'\left(1 + \varepsilon cos\theta'\right) \tag{6}$$

which is the temporal LT (5), $t = k(t' + \varepsilon x')$, with $x' = r'cos\theta' = t'cos\theta'$. We can also write $(r = t)$ the locus of the images-points:

$$r = kr'(1 + \varepsilon cos\theta') \tag{7}$$

If $r' = t' = 1$ (Figure 1), we then have:

$$t = r = k(1 + \varepsilon cos\theta') \tag{8}$$

We will show next that this "temporal equation" (6) or "radial equation"(7) is the equation of an Ellipse In polar coordinates if we define the polar angle θ (Figure 2) as the relativistic transformation of the angle θ' (Section 3).

3. Poincaré's Elongated Ellipse deduced from LT

3.1 Poincaré's ellipse (I) and the Relativity of Simultaneity

We first define Poincaré's elongated Ellipse In Cartesian coordinates. We are interested in the spatial shape of the wavefront $t' = 1$ in K, given the invariance of the quadratic form:

$$x^2 + y^2 = t^2 \tag{9}$$

If the time t were *fixed* (Section 4 on Einstein synchronisation), we would obviously have a circular wavefront; but via the LT t depends on x'. If t were written as a function of x', we would not have the image of the wave in K. We must write *t as a function of x*. By using the first and the third (x and t) LT (5), we have, if $r' = t' \neq 1$ or $r' = t' = 1$:

$$t = k^{-1}t' + \varepsilon x \qquad \text{or} \qquad t = k^{-1} + \varepsilon x \tag{10}$$

We immediately obtain the Cartesian equation of Poincaré's elongated Ellipse If $r' = t' \neq 1$ or $r' = t' = 1$ (Figure2):

$$x^2 + y^2 = (k^{-1}t' + \varepsilon x)^2 \qquad \text{or} \qquad x^2 + y^2 = (k^{-1} + \varepsilon x)^2 \tag{11}$$

We point out that if t is not fixed, t' is fixed. If the primed coordinate $t' = t'_0$ then appears in the first equation and not in the second equation (11), this is a simple problem of normalization of the constant t'_0. The image in coordinates of K (x, y, and t) of the circle in K$'$ is *elliptical* because t depends via the LT on x. One circular wavefront with constant radius t' in K$'$ corresponds, via the LT, not to one circular wavefront with constant radius t in K (Section 4) but to an infinity of circles with the continuously variable radius $t(x)$ in K.

Poincaré's ellipse $t(x,\ y)$ can be also written $(y' = y)$ by replacing x' as a function of x (5):

$$x' = k^{-1}x - \varepsilon t' \qquad \text{or} \qquad x' = k^{-1}x - \varepsilon t \tag{12}$$

in $x'^2 + y'^2 = t'^2 = 1_{t'}$ (4), if $r' = t' = t'_0$ or $r' = t' = 1$ (normalization):

$$(k^{-1}x - \varepsilon t'_0)^2 + y^2 = t_0'^2 \qquad \text{or} \qquad (k^{-1}x - \varepsilon)^2 + y^2 = 1 \tag{13}$$

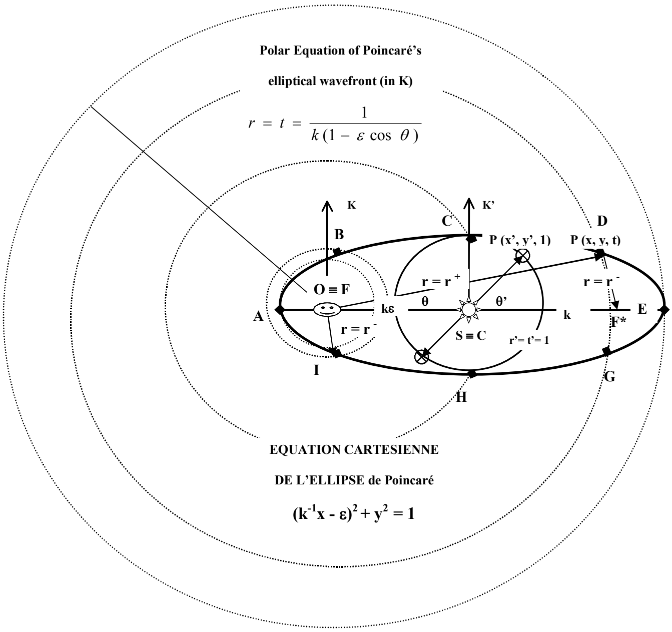

Figure 2 - Poincaré's elongated elliptical (*normalized*) wavefront and the relativity of simultaneity $(t' = r' = 1,\ \ t^{+} = r^{+} \neq t^{-} = r^{-})$.

At once we check that $(11) \equiv (13)$. The image-points are situated on Poincaré's elongated ellipse (for a spatial shape, t is eliminated, see 10), with Observer O at the focus F and source S at the centre C. *One circular wavefront in K′ corresponds, by LT, to one elliptical wavefront in K* (Figure 2, Poincaré's normalized ellipse).

The eccentricity of the ellipse is $\varepsilon = \frac{k\varepsilon}{k}$ where k is the length of the large axis. (We choose, in Figure 2, the small axis of the ellipse $r' = t' = 1$.) The equation of Poincaré's *normalized* ellipse can be written in polar coordinates with pole O, focus F and the polar angle θ defined *in K* (with both standard parameters of the ellipse e, p and with the small axis of the ellipse $b = r' = 1$):

$$r = \frac{p}{1 - e\cos\theta} \tag{14}$$

with

$$p = a(1-\varepsilon^2) = ak^{-2} = kk^{-2} = k^{-1}$$

we immediately deduce the polar equation of Poincaré's ellipse:

$$r = \frac{\sqrt{1-\varepsilon^2}}{1-\varepsilon\cos\theta} = \frac{1}{k(1-\varepsilon\cos\theta)} \tag{15}$$

with eccentricity $e = \frac{f}{a} = \frac{k\varepsilon}{k} = \varepsilon$ and with the two standard parameters of special relativity $\varepsilon,\ k$:

$$a^2 - f^2 = b^2 \qquad\qquad k^2 - \varepsilon^2 k^2 = 1$$

If $r' = r'_0 \neq 1$, we have the equation of the ellipse:

$$r = \frac{r'_0}{k(1-\varepsilon\cos\theta)} \tag{16}$$

with $r_0'^2(k^2 - \varepsilon^2 k^2) = r_0'^2$.

It should be remembered that the "radial equation" (7) of the ellipse is

$$r = kr'_0(1+\varepsilon cos\theta') \tag{17}$$

Thus we obtain from (16 and 7) the formula for relativistic transformation of angles:

$$\cos\theta = \frac{\cos\theta + \varepsilon}{1+\varepsilon\cos\theta'} \tag{18}$$

It has now been fully demonstrated that Poincaré is right and that *the geometrical image produced by the LT of a circular wavefront is an elongated ellipse*, its polar equation being (16) and its Cartesian equation being (13). Poincaré's ellipse gives the other formulae of aberration, in particular:

$$\sin\theta = \frac{\sqrt{1-\varepsilon^2}}{1+\varepsilon\cos\theta'}\sin\theta' \tag{19}$$

What is the *physical interpretation* of Poincaré's elongated ellipse? We wish to stress two points at this stage. (For the relativistic Doppler effect, see the conclusion.)

1) Poincaré's elongated ellipse, is the direct translation of the *relativity of simultaneity*: the *set* of simultaneous events in K′ of the spherical wavefront at time $t' = 1_{t'}$ is *not a set* of simultaneous events in K at time *t*. In particular, if the two events $(1_{t'}, 0, 1_{t'})$ and $(-1_{t'}, 0, 1_{t'})$ are simultaneous in K′, they are *not* simultaneous events in K.

2) Poincaré's elongated ellipse is the direct translation of the "headlight effect" (18, 19): the isotropic emission of a moving source is not isotropic seen from the rest system (relativistic transformation of angles θ into θ). In three dimensions of space the reduction of the angle of aperture of the emission cone of a moving source is physically (synchrotron radiation, *bremsstrahlung*, *etc.*) represented, *generally* (from any angle), by the ellipsoidal shape of the wave-front. We note here that the relativistic formulae for transformation of angles is the core of Poincaré's implicit kinematics while these formulae appear in the second part (application of his explicit kinematics) of Einstein's famous 1905 work. These two fundamental points corroborate the fact that Poincaré's ellipse is not only a geometrical *image* but also the physical *shape* of the wavefront.*

* This matter is disputed in he scientific literature. On the basis of Einstein's kinematics, according to Abramson, the sphere of observation is transformed into an ellipsoid of observation [23] whilst according to Penrose and Terrel the sphere of observation remains a sphere of observation (because of Einstein's contraction). According to Trempe [24] and Martin [26], the ellipse is developed in Galilean space-time where "the length and time intervals remain the same in all reference frames." (See Section 4.) Poincaré's space-time wavefronts (contraction of unit of length) are not in this sense in a Galilean space-time. Poincaré's relativistic definition of contraction is not compatible with Einstein's definition of contraction. (Section 5) See also Born *versus* Keswani [25]. The main difference between our ap-

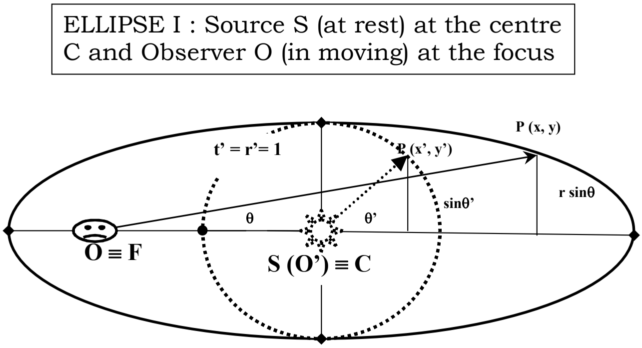

Figure 3 - Elliptical image in the system of the observer O (relativity of simultaneity).

3.2 Poincaré's ellipse (II) and Michelson Experiment

It is now essential to interpret the historical case (*cf.* Introduction) considered by Poincaré. (In connection with Michelson's experiment where the *source is on the Earth,* see Introduction and Conclusion.) Until now we have actually defined the case (Ellipse I) where the observer O is in motion relative to the source S.

We now show that the (normalized) Ellipse II is immediately obtained from (normalized) Ellipse I by inversion. We have the radial, polar, and Cartesian equations of Ellipse I (Figure 3):

$$r = k(1+\varepsilon cos\theta') \qquad r = \frac{1}{k(1-\varepsilon\cos\theta)} \qquad (k^{-1}x-\varepsilon)^2 + y^2 = 1_{t'} \tag{20}$$

We now invert primed and unprimed and change ε to $-\varepsilon$. We obtain (Figure 4):

$$r' = k(1-\varepsilon\cos\theta) \qquad r' = \frac{1}{k(1+\varepsilon\cos\theta')} \qquad (k^{-1}x'+\varepsilon)^2 + y'^2 = 1_t \tag{21}$$

with

$$\cos\theta = \frac{\cos\theta' + \varepsilon}{1+\varepsilon\cos\theta'} \qquad \cos\theta' = \frac{\cos\theta - \varepsilon}{1-\varepsilon\cos\theta} \tag{22}$$

Thus Equations (21) are the equations (respectively radial, polar and Cartesian) of Ellipse II in K′.

The roles of the source S and the observer O are reversed. The circular light wavefront is now developed around O: $r = t = r_0 = t_0 = 1$ (normalization). The image of the circular locus of points (determined now by θ) seen from O′ (where the source is at rest in K′, "system of the Earth") is Poincaré's Ellipse II.

Poincaré's elongated ellipse is the direct translation of Poincaré's completely relativistic ether: *setting the ether at rest in one (K) or in the other system (K′)* is *mathematically* equivalent to *defining the ellipse with the direct LT or the inverse LT*. So in Poincaré's own words: if t' is the true time ("circular"

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proach and the previous approaches is that we can deduce, from Poincaré's wavefronts, a relativistic Doppler formula that is not the same as Einstein's.

ELLIPSE II : Source S (in moving) at the focus F* and Observer O (at rest) at the centre C.

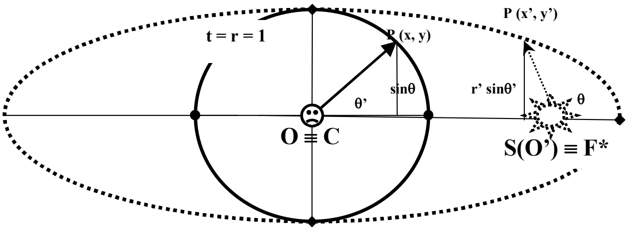

Figure 4 - Elliptical image in the system of the source and Michelson experiment (c = 1 in the two systems, see Conclusion for the space-time units).

time), then t is the local time ("elliptical" time), and inversely (*via* the LT), if t is the true time ("circular" time), then t' is the local time ("elliptical" time). This is completely relativistic, and Poincaré's elongated Ellipse II is physically "the immediate interpretation of the Michelson experimental result"($c = 1$; *cf.* conclusion for consequences of the choice of space-time units). Poincaré has a relativistic ether, and has *not abolished it* (as Einstein did), because it remains *the relativistic definition of the state of rest*: when we set the ether at rest in one system by definition (spheres or *true time*), it is *not at rest* in the other system (ellipsoids or *local time*).

Criterion: Objectively we have two possibilities to choose the criterion of the *relativistic state of rest of a system*: *the source of light or the medium of light*. In Einstein-Minkowski's relativistic *kinematics*, the criterion is clearly the *source* (*the proper system, cf. Section 4*). In Poincaré's relativistic *kinematics*, the criterion is clearly the *ether* ("circular waves").

In *Einstein-Minkowski's proper system (cf. Section 4)* we always have spherical waves by definition, or in other words, *equality of to and fro travel times*. This is not the case with Poincaré's definition of units, where we can have, without any contradiction, an elliptical *image* of the wavefront (a local time) *in the system of the source* (*cf.* Conclusion). Poincaré's relativistic duality between true time and local time *does not correspond* to Einstein-Minkowski's relativistic duality between proper time and improper time (Section 4).

4. Einstein's Kinematics: Identical Spheres, Identical Rigid Rods and Convention of Synchronisation

If according to Einstein, the object (1) and the image (2), are both spherical and *concentric* within the two systems, then two simultaneous events in K, for example (1, 0, 1) and (–1, 0, 1), must also be simultaneous in k.

This seems to be in contradiction not only with Poincaré's ellipse, but also with Einstein's well known definition of relativity of simultaneity. Therefore *the image produced by LT* in k of a spherical wave in K *cannot be a spherical* *wave* (Section 3). Thus, it could appear at this stage that *Poincaré is right* and *Einstein is wrong*.

However the question is: "What in Einstein's reasoning is true?" Let us return to Einstein's 1905 quotation (Section 1). The two quadratic forms $x^2+y^2+z^2=c^2t^2$ and $\xi^2+\eta^2+\zeta^2=c^2\tau^2$ are the *geometrical* equations of two spheres (two circles in two dimensions or two points equidistant from O and O′ in one dimension). We note that in the previous quotation, Einstein does not specify in *which system* the source is at rest. Hence, if we now consider that we have *two identical sources,* in O′ and O, which emit a light signal *simultaneously* at the time,

$$\tau=t=0 \tag{23}$$

the physical situation is perfectly identical in each system (Einstein's abolition of ether*). Therefore, we must have two identical spherical wavefronts—$x^2+y^2+z^2=c^2$ around O and $\xi^2+\eta^2+\varsigma^2=c^2$ around O′—simultaneously at the time:

$$\tau=t=1_t=1_\tau \tag{24}$$

It immediately follows from the latter choice of two *identical* units of time that we have two identical units of length: $1_x = 1_\xi = c1_t = c1_\tau$. It should be noted that the travel times of the circular wavefront to the right and to the left (on the x, ξ axis) are identical within the two systems. Einstein's definition of identical units of time is, therefore, completely consistent with Einstein's *identical rigid rods*. (On the other hand $1_t=1_\tau$ is incompatible with Poincaré's definition of units; see Conclusion). Accordingly, Einstein writes in 1905:

> Let there be given a stationary rigid rod; and let its length be L_0 as measured by a measuring-rod which is also stationary. In accordance with the principle of relativity *the length of the rod in the moving system*—must be equal to *the length L_0 of the stationary rod*. [3]

In this regard, M. Born is perhaps the only physicist who has stressed the fact that Einstein introduces a tacit assumption (1921):

> A fixed rod that is at rest in the system K and is of length 1 cm, will, of course, also have the length 1 cm, when it is at rest in the system k. We may call this *tacit assumption* of Einstein's theory the *principle of the physical identity of the units of measure*. [1]

Einstein's principle of identity (see also [25]) stipulates $L_0=1_x=1_\xi=c1_t=c1_\tau$ and therefore Equation (24) becomes

$$\tau=t=\frac{L_0}{c} \tag{25}$$

There are two Einstein spherical waves, and each spherical wave defines, in one dimension, two simultaneous events (–1, 0, 1) and (–1, 0, 1), within each system (K and k). In other words: *Einstein's rigid rod* $(2L_0)$ *is defined by two simultaneous events within each system (K and k)*. Einstein's two spherical waves are not in contradiction with "Einstein's relativity of simultaneity (after

* Einstein's abolition of the ether is inseparable from Einstein's photon (1905)[9].

application of the LT)" because it is "Einstein's convention of simultaneity (before application of the LT)," or in other words "Einstein's *convention of synchronisation of identical clocks in A and B* with the exchange of a signal of light in K (before application of the LT)." According to Einstein, "It is essential to define time by means of *stationary* clocks in a *stationary* system." Einstein's famous repetition of the concept "stationary" is essential because he notices about his second system k (ξ , η , ζ , τ):

> To do this [deduce LT] we have to express in equations that τ is nothing else than the set of data of clocks at rest in system k, which have been synchronized [A′B′] according to the rule given in paragraph 1 [AB] [3].

Without any loss of generality we make $A \equiv O$ (or $A' \equiv O'$) in Einstein's notations in his Section 1[3] (and then $t_A = \tau_{A'} = 0$). We have $2t_B = t_O^*$ in K ($2\tau_{B'} = \tau_{O'}^*$ in k) and $c = 2\frac{OB}{t_O^*}$ in K ($c = 2\frac{O'B'}{\tau_{O'}^*}$ in k), with $L_0 = OB = O'B'$, where $t_O = 0$ ($\tau_{O'} = 0$) is the time of emission of the light signal in K (in k) and t_O^* ($\tau_{O'}^*$) is the time of reception of the light signal in B in K (in B′in k).

$$t_O^* = \tau_{O'}^* = 2\frac{L_0}{c} = T_0 \tag{26}$$

Given that the to and fro travel times are identical with

$$t_O = \tau_{O'} = 0 \tag{27}$$

we have finally

$$t_B = \tau_{B'} = \frac{L_0}{c} = \frac{1}{2}T_0. \tag{28}$$

Einstein's interpretation of the invariant quadratic form as *two physical spherical wavefronts* (1 & 2) is therefore exactly the same concept (see equations 23 and 27, 25 and 28) as Einstein's 1905 *convention of synchronisation within the two systems* (*in one dimension* where the to wavefront becomes the to travel time, and the fro wavefront becomes the fro travel time). The proper time T_0 (index "zero" means "proper"), the duration between *two events at the same place*, in Einstein's notation is t_A^* ($\tau_{A'}^*$) or t_O^* ($\tau_{O'}^*$), with the identity between to travel time and fro travel time (factor $\frac{1}{2}$). This is Einstein's synchronisation *without contraction*: $OB = O'B'$. (Poincaré's convention of synchronisation with contraction is treated elsewhere: [21] and [9].) We point out that Einstein's convention is not based on the choice of only one unit of length in one system (*cf.* Conclusion), but on two identical units of length (B′ is not the image produced by LT of B) within each system.

5. Definition of Space-time Units in Poincaré's Relativistic Kinematics

Poincaré writes in 1911 in *L'espace et le temps* on the special theory of relativity:

> Today some physicists want to adopt a new convention. It is not that they have to do it; they consider that this convention is easier, nothing more; and

> those who have another opinion may legitimately keep the old assumption in order not to disturb their old habits. [20]

"Some physicists" is a clear allusion to Einstein and Minkowski. What is the difference, in Poincaré's view, between the "old convention" and the "new convention"? Let us examine Poincaré's old (tacit) assumption in detail. What happens if we place another source in the second system K in Poincaré's relativistic kinematics? Suppose that the relativistic ether is by definition at rest (spheres around O′) relative to the first source in K′. Poincaré's relativistic ether is then moving relative to the second source at O in K, and so we recover the second case with an ellipsoidal wave in the source system. And reciprocally, with inverse LT, the role of the ether (*the criterion of relativistic rest*) is inverted.

Logically in Poincaré's SR, with one source or two sources, we *always* have a *sphere* in one system and an *ellipsoid* in the other system, and *never* two spheres in the two systems (Section 4).

Whereas historically Poincaré deduced the ellipsoid directly from the contraction of units, we must now deduce the contraction of units from the ellipsoid provided directly by the LT. From the main property of an elongated ellipse $r^+ + r^- = 2kr'$ (Figure 2) the to distance r^+ and the fro distance r^- with respect to the second focus F*, or the to travel time t^+ (Figure 2) and the fro travel time t^- with respect to the second focus F*)* where M means "mean (average)," "round-trip" or "two-way," we obtain:

$$r_M = \frac{r^+ + r^-}{2} = kr' \qquad\qquad t_M = \frac{t^+ + t^-}{2} = kt' \tag{29}$$

We have by definition in the system K′ (*cf.* Section 2 for normalization): $t' = 1_{t'}$ and $r' = 1_{r'}$ (the choice of only one length unit). Thus if the elongated ellipse is an alternative definition of the unprimed units, we must be able to immediately deduce Poincaré's *"round-trip" units* in K. Indeed we have:

$$1_r = k1_{r'} \qquad\qquad 1_t = k1_{t'} \tag{30}$$

The unit of local time ("elliptical time") 1_t is always *dilated* relative to the unit of true time ("circular time") $1_{t'}$.

For the unit of space, we must first show that there is no transversal contraction:

$$1_y = 1_r \sin\theta \text{ and } 1_{y'} = 1_{r'} \sin\theta'$$

with $\theta' = \frac{\pi}{2}$, we have from (19) $\sin\theta = k^{-1}$ and thus

$$1_y = 1_{y'} \tag{31}$$

We immediately have $\cos\theta = \cos\theta' = 1$ for the longitudinal component, with $r^+ = k(1+\varepsilon)$ and $r^- = k(1-\varepsilon)$,

$$1_x = k1_{x'} \tag{32}$$

---

* Poincaré's *exact* synchronisation (to second order) is developped with Poincaré's elongated ellipsoid elsewhere [9] (1999). Poincaré's elongated ellipse ($t^+ \neq t^-$) *is* Poincaré's convention of synchronisation. We can also use the *tangent* to the ellipse, and thus obtain a complete theory of the *plane wave* that is utterly different from Einstein's [12].

Let us call $1_{x'}$ "the unit at (relativistic) rest" and 1_x "the unit in (relativistic) motion." So the unit at rest $1_{x'}$ is seen longitudinally elongated only (by a factor *k*) by the observer O in motion in K. This is an unusual language, but if we inverse the situation (if $A > B \Rightarrow B < A$) we then have

$$1_{x'} = k^{-1} 1_x \tag{33}$$

The unit 1_x in motion is seen longitudinally contracted (by a factor k^{-1}) by the observer at rest O′. This is a more usual language. We therefore recover Poincaré's initial postulate concerning contraction of a moving unit (*cf.* Introduction).

We stress the fact that Poincaré's deduction of *dilated units,* $1_t = k1_{t'}$ and $1_x = k1_{x'}$, is based, and *only* based, on the application of the LT (the "old convention"). Einstein-Minkowski's definition of identical units within both systems is clearly *beyond* the LT (the "new convention," *cf.* Section 4).

6. Conclusion

After deducing his elongated ellipse Poincaré writes:

> This hypothesis of Lorentz and FitzGerald will appear most extraordinary at first sight. All that can be said in its favour for the moment is that it is merely the immediate interpretation of Michelson's experimental result, if we *define* (in italics in the text) distances by the time taken by light to traverse them. [18]

With Poincaré's Ellipse II (Figure 4) we see immediately that the *two-way* time is the same (2k) in all directions (and therefore for Michelson's two perpendicular directions). Poincaré's historical ellipse is, therefore, the immediate interpretation of Michelson's null result (also the relativistic interpretation of Sagnac's *non*-null result), without the need to postulate that the source in the (proper) system of the Earth emits spherical waves. And this is not all: according to Poincaré, distances are defined by the dilated time taken by light to traverse them. We can deduce this fundamental point directly from the LT. The usual definition of the length of a rod implies that we consider the two ends of the rod *at the same time*. We therefore consider the two ends of the unit of length $1_{x'}$ in K′ at the same time $t' = 0$. (The primed coordinates are 0,0 and 1,0.) What is the length of the rod in the other system K (the moving system) according to Poincaré, *i.e.*, according to the LT? The calculation with Equation (5) gives immediately $1_x = k1_{x'}$ The elongation of the stationary rod in the moving system is a direct consequence of the fact that two simultaneous events in K′ are not simultaneous events in K (*cf.* Section 3).

We conclude by remarking that Poincaré's relativistic kinematics is based on a fundamental *space-time proportionality* (dilation by a factor *k*) in perfect accord with the invariance of the speed of light:

$$\frac{r_M}{t_M} = \frac{kr'}{kt'} = \frac{1_{r'}}{1_{t'}} = \frac{k1_{r'}}{k1_{t'}} = c = 1 \tag{34}$$

Poincaré's *direct space-time proportionality* (very strange in Einstein's kinematics, where the improper length and improper time are respectively contracted and dilated, *i.e.*, inverse proportionality) characterizes Poincaré's fundamental choice of space-time units in relativistic kinematics: "*I shall choose the units of length and of time in such a way that the velocity of light is equal to unity*." This is the reason why we have kept Poincaré's notations $\varepsilon(\beta)$ and $k(\gamma)$: behind Poincaré's notations, there is not only Poincaré's perfectly symmetrical representation of the LT in Equation (5), but also Poincaré's relativistic definition of the *metric* (in the sense of space-time units of measurement) underlying the invariance of quadratic form in SR. We have $c = 1$ in the second (moving) system "*if we define distances by the time taken by light to traverse them*." *This is Poincaré's expansion (dilation) of space*. The existence of a "fine structure" of SR (two very close but not fully merged theories) is therefore demonstrated [9]. Given that the velocity of light is isotropic (the same in all directions) in Einstein's two systems as well as Poincaré's two systems, there are two different relativistic theories of space-time.

Acknowledgements

I would like to thank Jean Reignier, Thomas Durt (TENA, Free University of Brussels) and Pierre Marage (IIHE, Free University of Brussels).

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The Einstein Postulates: 1905-2005 A Critical Review of the Evidence

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The Einstein postulates assert an invariance of the propagation speed of light in vacuum for any observer, which amounts to a presumed absence of any preferred frame. The postulates appear to be directly linked to relativistic effects which emerge from Einstein's *Special Theory of Relativity,* which is based on the concept of a flat spacetime ontology, and which then leads to the *General Theory of Relativity* with its curved spacetime model for gravity. While the relativistic effects are well established experimentally it is now known that numerous experiments, beginning with the Michelson-Morley experiment of 1887, have always shown that the postulates themselves are false, namely that there is a detectable local preferred frame of reference. This critique briefly reviews the experimental evidence regarding the failure of the postulates, and the implications for our understanding of fundamental physics, and in particular for our understanding of gravity. A new theory of gravity is seen to be necessary, and this results in an explanation of the 'dark matter' effect entailing the discovery that the fine structure constant is a 2nd gravitational constant.

Introduction

It is one hundred years since Einstein formulated his postulates for the invariant property of light, namely that the speed of light is always c ($\approx$ 300,000 km/s) for any uniformly moving observer, which is equivalent to the assertion that there is no preferred frame, that there is no detectable *space*, that a three-dimensional *space* has no physical existence.

Einstein postulates:

(1) The laws of physics have the same form in all inertial reference frames.
(2) Light propagates through empty space with a definite speed c independent of the speed of the observer (or source).
(3) In the limit of low speeds the gravity formalism should agree with Newtonian gravity.

The putative successes of the postulates lead to the almost universal acceptance of the Einstein Special Theory of Relativity, which is based on the concept of a flat spacetime ontology that replaces the older separate concepts of space and time, and then to the General Theory of Relativity with its curved spacetime model for gravity. While the relativistic effects are well established experimentally it is now belatedly understood in 2002 (Cahill and Kitto, Cahill

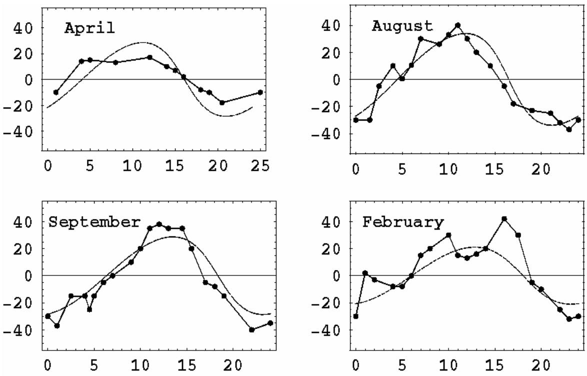

Fig.1 Azimuth ψ (deg), measured from south, from the Miller data, plotted against local sidereal time (hrs). Plots cross the local meridian at approx. 5hr and 17hr. The small monthly changes arise from the orbital motion of the earth about the sun. Miller used that effect to determine the value of k in (1), and which is now in agreement with the refractive index theory. Curves from theory (Cahill 2003a, 2004b)

2003a), that numerous experiments, beginning with the Michelson-Morley experiment of 1887, have always shown that postulates (1) and (2) (excepting the 2nd part) are false, namely that there is a detectable local frame of reference or ‘space’, and that the solar system has a large observed galactic velocity of some 420±30km/s in the direction (RA = 5.2hr, Dec = –67deg) through this space (Cahill 2003a, 2004b). This is different from the speed of 369km/s in the direction (RA = 11.20hr, Dec = –7.22deg) extracted from the Cosmic Microwave Background (CMB) anisotropy, and which describes a motion relative to the distant universe, but not relative to the local space. This critique briefly reviews the experimental evidence regarding the failure of the postulates, and the implications for our understanding of fundamental physics, and in particular for our understanding of gravity. A new theory of gravity is seen to be necessary, and this results in an explanation of the ‘dark matter’ effect, entailing the discovery that the fine structure constant is a 2nd gravitational constant (Cahill 2005a). This theory is a part of the information-theoretic modelling of reality known as *Process Physics*, which premises a non-geometric process model of time, as distinct from the current non-Process Physics, which is characterised by a geometrical model of time.

Over 100 years of Detecting Absolute Motion

The whole business of detecting absolute motion (motion relative to space itself) and so a preferred local frame of reference, came undone from the very beginning.

The Michelson and Morley air-mode interferometer fringe shift data revealed a speed of some 8km/s when analysed using the prevailing Newtonian theory, which has $k^2 = n^3 \approx 1$ in (1), where Δt is the difference between the light travel times for the two arms, effective length L, within the interferometer, and n is the refractive index of the gas present.

$$\Delta t = k^2 \frac{L v_P^2}{c^3} \cos\left[2(\theta - \psi)\right] \tag{1}$$

However, when we include the Fitzgerald-Lorentz dynamical contraction effect as well as the effect of the gas present, we find that $k^2 = n(n^2 - 1)$, giving $k^2 = 0.00058$ for air, which explains why the observed fringe shifts were so small. In the Einstein theory $k = 0$; absolute motion is to be undetectable in principle. In (1) θ is the azimuth of one arm relative to the local meridian, with ψ the azimuth of the projected absolute motion velocity v_P. Fig.1 shows ψ from the 1925/1926 Miller interferometer data for four different months of the year, from which the RA = 5.2hr is readily apparent. The orbital motion of the earth about the sun slightly affects the RA in each month, and Miller used this effect to determine the value of *k*; but the new theory of gravity required a re-analysis of his data. Two interferometer experiments used helium, enabling the refractive index effect to be confirmed (Cahill 2003a, b). Fig.2 shows the coaxial cable travel times measured by DeWitte in 1991, which also show the same RA. That these very different experiments show the same speed and RA of absolute motion is one of the most startling but suppressed discoveries of the twentieth century. So Postulates (1) and (2) are in disagreement with the experimental data. In all some seven experiments have detected this absolute motion. Modern interferometer experiments use vacuum with $n = 1$, and then from (1) $k = 0$, predicting no fringe shifts. In analyzing the data this is misinterpreted to imply the absence of absolute motion. As discussed in Cahill (2003a, 2005b), it is absolute motion which causes the dynamical effects of length contractions, time dilations and other relativistic effects, in accord with Lorentzian interpretation of relativity.

Gravity as Inhomogeneous and Time-Dependent Spatial Flows

We now come to postulate (3) for gravity. This postulate relates General Relativity to Newtonian gravity, and Newtonian gravity is now known to be seriously flawed, and so *ipso facto*, by using this postulate, Einstein and Hilbert inadvertently developed a flawed theory of gravity. Newtonian gravity was based upon Kepler's Laws for the planetary motions within the solar system and uses the acceleration field *g*,

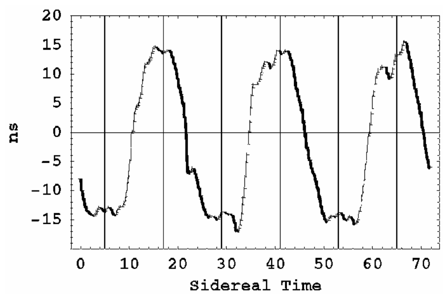

Fig.2 DeWitte 1991 RF travel-time variations, in ns, in a 1.5km NS coaxial cable, measured with atomic clocks, over three days and plotted against local sidereal time, showing that at approximately 5hr and 17hr the effect is largest (Cahill 2003a, 2004b).This remarkable agreement with the Miller interferometer experiment shows that the detection of absolute motion is one of the great suppressed discoveries in physics. At least six other interferometer or coaxial cable experiments are consistent with these observations .

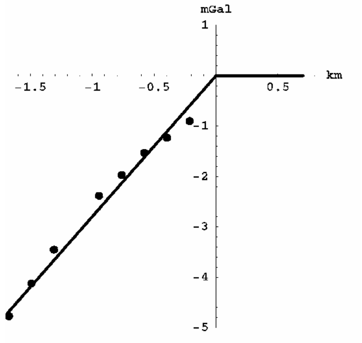

Fig.3 Gravity residuals from the Greenland bore hole anomaly data (Anders 1989). These are the differences in g between the measured g and that predicted by the Newtonian theory. According to (7) this difference only manifests within the earth, and so permits the value of α to be determined. The data shows that α is the fine structure constant, to within experimental error. This small value for α explains why the spiral galaxy rotation speed plots are so flat, and also explains the black holes masses at the centre of globular clusters (Cahill 2005a)..

$$\nabla.g = -4\pi G\rho \tag{2}$$

where G is Newton's universal gravitational constant, and ρ is the density of matter. However equally valid mathematically is a velocity field formulation

$$\frac{\partial}{\partial t}(\nabla.v) + \nabla.\left[(v.\nabla)v\right] = -4\pi G\rho \tag{3}$$

with g now given by the Euler 'fluid' convective acceleration

$$g = \frac{\partial v}{\partial t} + (v.\nabla)v = \frac{dv}{dt} \tag{4}$$

External to a spherical mass M a static velocity-field solution is

$$v(r) = -\sqrt{\frac{2GM}{r}}\hat{r} \tag{5}$$

which gives from (4) the usual inverse square law

$$g(r) = -\frac{GM}{r^2} \tag{6}$$

However (3) is not uniquely determined by Kepler's laws because

$$\frac{\partial}{\partial t}(\nabla.v) + \nabla.\left[(v.\nabla)v\right] + C(v) = -4\pi G\rho \tag{7}$$

where

$$C(v) = \frac{\alpha}{8}\left[(trD)^2 - tr\left(D^2\right)\right] \tag{8}$$

and

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{9}$$

also has the same external solution (5), as $C(v) = 0$ for the flow in (5). So the presence of the $C(v)$ dynamics would not have manifested in the special case of planets in orbit about the massive central sun. Here α is a dimensionless constant—a new additional gravitational constant. However inside a spherical mass $C(v) \neq 0$ and using the Greenland ice-shelf bore hole g anomaly data,

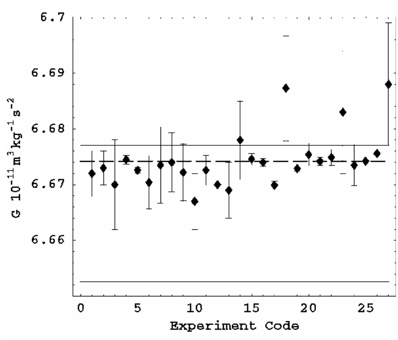

Fig.4 Results of precision measurements of G over the last sixty years in which the Newtonian theory of gravity was used to analyse the data. Shows systematic effect missing from the Newtonian theory, of fractional size $\approx\alpha/4$. For this reason G is the least accurately known fundamental constant. The upper horizontal line shows the value of G from an ocean Airy measurement (Zumberge 1991), while the dashed line shows the current CODATA value. The lower horizontal line shows the value of G after correcting for the 'dark matter' effect. Results imply that Cavendish laboratory experiments can measure α.

Fig.3, we find (Cahill, 2005a) that $\alpha^{-1} = 139 \pm 5$, which gives the fine structure constant $\alpha = 1/137$ to within experimental error.

From (7) and (8) we can introduce an additional effective 'matter density'

$$\rho_{DM} = \frac{\alpha}{32\pi G}\left[(trD)^2 - tr\left(D^2\right)\right] \tag{10}$$

as a phenomenological treatment of the new space dynamics within the velocity formulation of Newtonian gravity in (3). It is this spatial dynamics that has been misinterpreted as the 'dark matter' effect. This 'dark matter' dynamical effect also appears to explain the long-standing problems in measuring G in Cavendish-type experiments, as shown in Fig.4.

Eqn.(7) has novel black hole solutions where the in-flow is given by

$$v(r) = K\left[\frac{1}{r} + \frac{1}{R_S}\left(\frac{R_S}{r}\right)^{\frac{\alpha}{2}}\right] \tag{11}$$

where the key feature is the α-dependent term in addition to the usual 'Newtonian' in-flow in (5). For the in-flow in (11) the centripetal acceleration relation $v_O = \sqrt{rg(r)}$ for circular orbits gives orbital rotation speeds of the form

$$v_O(r) = \frac{K}{2}\left[\frac{1}{r} + \frac{\alpha}{2R_S}\left(\frac{R_S}{r}\right)^{\frac{\alpha}{2}}\right]^{1/2} \tag{12}$$

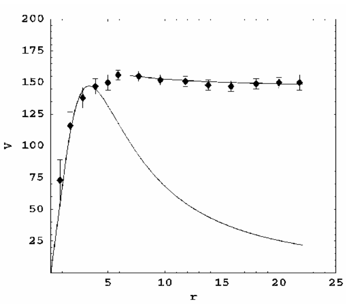

Fig.5 Data shows the non-Keplerian rotation-speed curve for the spiral galaxy NGC3198 in km/s plotted against radius in kpc/h. Complete curve is the rotation curve from the Newtonian theory or from General Relativity for an exponential disk, which decreases asymptotically like $1/\sqrt{r}$. This discrepancy is the origin of the 'dark matter' story. The upper curve shows the asymptotic form from (12), with the decrease determined by the small value of α. This asymptotic form is caused by the primordial black holes at the centres of spiral galaxies, and which play a critical role in their formation.

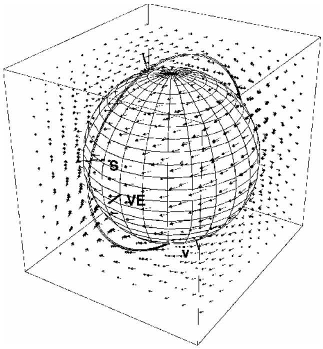

Fig.6 Shows the earth with absolute linear velocity V in the direction discovered by Miller in 1925/26. This motion causes a vorticity, shown by the vector field lines. A much smaller vorticity is generated by the rotation of the earth, known as the Lense-Thirring effect. In General Relativity only the earth-rotation induced vorticity is permitted, where it is known as a gravitomagnetic effect. Vorticity is a local rotation of space relative to more distant space. This rotation can be detected by observing the precession of a gyroscope, whose spin direction S remains fixed in the local space. VE is the vernal equinox. The Gravity Probe B satellite experiment is designed to detect these precessions.

which is characterised by their almost flat asymptotic limit. This rotation curve explains the 'dark matter' effect as seen in spiral galaxies, as shown in Fig.5.

Eqn.(3) is only applicable to a zero vorticity flow. The vorticity is given by

$$\nabla \times (\nabla \times v) = \frac{8\pi G\rho}{c^2} v_R \tag{13}$$

where v_R is the absolute velocity of the matter relative to the local space. See Cahill (2005a, b) for the more general form of (7) that includes vorticity. Fig.6 shows the vorticity field $\nabla \times v$ induced by the earth's absolute linear motion. Eqn.(13) explains the Lense-Thirring 'frame-dragging' effect in terms of vorticity in the flow field, but makes predictions very different from General Relativity. These conflicting predictions will soon be tested by the Gravity Probe B gyroscope precession satellite experiment. However the smaller component of the frame-dragging effect caused by the earth's absolute rotation component of v_R has been determined from the laser-ranged satellites LAGEOS(NASA) and LAGEOS2(NASA-ASI) (Ciufolini and Pavlis) and the data implies the indicated coefficient on the RHS of (13) to ±10%. However that experiment cannot detect the larger component of the 'frame-dragging' or vorticity induced by the absolute linear motion component of the earth, as that effect is not cumulative, while the rotation induced component is cumulative. The GP-B gyroscope spin precessions caused by the earth's absolute linear motion are shown in Fig.7. Both (3) and (7) have wavelike aspects to their time-dependent solutions, with the time-dependence being the rule rather than the exception. Such wave behaviour has been detected in most absolute motion experiments, as seen in the DeWitte data in Fig.2 (Cahill 2003a, 2004b).

For (3) these waves do not produce a gravitational force effect via (4), but with the inclusion of the 'dark matter' spatial dynamics in (7) such waves do produce a gravitational force effect, and there are various experimental 'anomalies' which are probably manifestations of this effect. As well such waves affect the vorticity from (13), and in principle could be detected by the GP-B experiment. General Relativity predicts a very different kind of gravitational wave, but these have never been seen, despite extensive searches. The new theory of gravity implies that these waves do not exist.

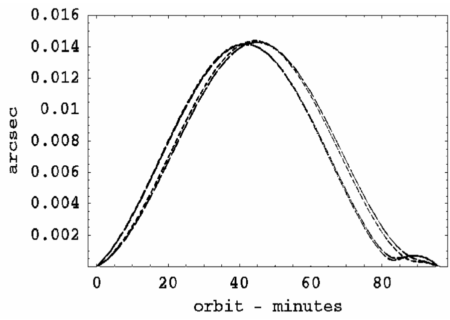

Fig.7 Shows predicted spin precession angle for the Gravity Probe B satellite experiment over one orbit, with the orbi shown in Fig.6, caused by the vorticity arising from the absolute linear motion of the earth. Plots for four different months. This particular precession is not cumulative, compared to the precession from the earth-rotation induced vorticity component. The GP-B experiment is optimized to detect this cumulative spin precession. General Relativity has earth-rotation induced vorticity but not linear motion induced vorticity (Cahill 2004d,e).

The trajectory of test particles in the differentially flowing space are determined by extremising the proper time

$$\tau[r_O]=\int dt\left(1-\frac{v_R^2}{c^2}\right)^{1/2} \tag{14}$$

where $v_R = v_O - v$, with v_O the velocity of the object relative to an observer frame of reference, which gives from (14) an acceleration independent of the test mass, in accord with the equivalence principle,

$$\frac{dv_O}{dt}=\left(\frac{\partial v}{\partial t}+(v.\nabla)v\right)+(\nabla\times v)\times v_R-\frac{v_R}{1-\frac{v_R^2}{c^2}}\frac{1}{2}\frac{d}{dt}\left(\frac{v_R^2}{c^2}\right) \tag{15}$$

where the 1st term is the Euler 'fluid' convective acceleration in (4), the 2nd term is the vorticity-induced Helmholtz acceleration, and the last is a relativistic effect leading to the 'geodesic' effects. It is significant that the time-dilation effect in (14) leads to well known 'fluid' accelerations, revealing a close link between the spatial flow phenomena and relativistic effects.

General Relativity Flow-Metrics

We saw that Newtonian gravity failed because it was expressed in the limited formalism of the gravitational acceleration field g. As soon as we introduce the velocity field formalism together with its 'dark matter' generalisation we see that numerous gravitational anomalies are explained. General Relativity was constructed to agree with Newtonian gravity, and so it is flawed by this connection. So it is interesting to understand why General Relativity (GR) is supposed to have passed key observational and experimental tests. GR uses the Einstein tensor

$$G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\frac{8\pi G}{c^2}T_{\mu\nu} \tag{16}$$

In this formalism the trajectories of test objects are also determined by extremising (14) which, after a general change of coordinates, gives the acceleration in (15) in terms of the usual affine connection.

$$\Gamma^{\lambda}_{\mu\upsilon}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}+\frac{d^2x^{\lambda}}{d\tau^2}=0 \tag{17}$$

In the case of a spherically symmetric mass M the well known solution of (16) outside of that mass is the external-Schwarzschild metric

$$d\tau^2=\left(1-\frac{2GM}{c^2r}\right)dt^2-\frac{r^2}{c^2}\left(d\theta^2+\sin^2(\theta)d\varphi^2\right)-\frac{dr^2}{c^2\left(1-\frac{2GM}{c^2r}\right)} \tag{18}$$

This solution is the basis of various experimental checks of General Relativity in which the spherically symmetric mass is either the sun or the earth. The four tests are: the gravitational redshift, the bending of light, the precession of the perihelion of Mercury, and the time delay of radar signals. However, the solution (18) is in fact completely equivalent to the in-flow interpretation of Newtonian gravity. Making the change of variables $t\to t'$ and $r\to r'=r$ with

$$t'=t+\frac{2}{c}\sqrt{\frac{2GMr}{c^2}}-\frac{4GM}{c^2}\tanh^{-1}\sqrt{\frac{2GM}{c^2r}} \tag{19}$$

the Schwarzschild solution (18) takes the form

$$d\tau^2=dt'^2-\frac{1}{c^2}\left(dr'+\sqrt{\frac{2GM}{r'}}dt'\right)^2-\frac{r'^2}{c^2}\left(d\theta'^2+\sin^2(\theta')d\varphi'^2\right) \tag{20}$$

which is exactly the differential form of (14) for the velocity field given by the Newtonian form in (5). This choice of coordinates corresponds to a particular frame of reference in which the test object has velocity v_R relative to local space. This result shows that the Schwarzschild metric in GR is completely equivalent to Newton's inverse square law: GR in this case is nothing more than Newtonian gravity in disguise. So the so-called 'tests' of GR were nothing more than a test of the 'geodesic' equation (14), where most simply this is seen to determine the motion of an object relative to an observable and observed absolute local frame of reference. These tests were merely confirming the in-flow formalism, and have nothing to do with a Schwarzschild spacetime ontology.

Since GR has only been directly tested using the metric in (18) or (20), it is interesting to ask what particular form (16) then takes. To that end we substitute a special class of flow-metrics, involving an arbitrary time-dependent velocity flow-field, into (16)

$$d\tau^2=g_{\mu\upsilon}dx^{\mu}dx^{\upsilon}=dt^2-\frac{1}{c^2}\left(dr-v(r,t)dt\right)^2 \tag{21}$$

The various components of the Einstein tensor are then found to be

$$\begin{aligned}
G_{00} &= \sum v_i \bar{G}_{ij} v_j - c^2 \sum \bar{G}_{0j} v_j - c^2 \sum v_i \bar{G}_{i0} + c^2 \bar{G}_{00} \\
G_{i0} &= -\sum \bar{G}_{ij} v_j + c^2 \bar{G}_{io} \\
G_{ij} &= \bar{G}_{ij}
\end{aligned} \tag{22}$$

where the $\bar{G}_{\mu\upsilon}$ are given by

$$\bar{G}_{00} = \frac{1}{2}\left[\left(trD\right)^2 - tr\left(D^2\right)\right]$$

$$\bar{G}_{i0} = \bar{G}_{0i} = -\frac{1}{2}\left[\nabla \times \left(\nabla \times v\right)\right]_i$$

$$\bar{G}_{ij} = \frac{d}{dt}\left(D_{ij} - \delta_{ij} trD\right) + \left(D_{ij} - \frac{1}{2}\delta_{ij} trD\right) trD - \frac{1}{2}\delta_{ij} tr\left(D^2\right) - \left(D\Omega - \Omega D\right)_{ij} \tag{23}$$

and where

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right)$$

is the tensor form of the flow vorticity. In vacuum, with $T_{\mu\upsilon} = 0$, we find that $G_{\mu\upsilon} = 0$ implies that $\bar{G}_{\mu\upsilon} = 0$. This system of GR equations then demands that

$$\rho_{DM} = \frac{\alpha}{32\pi G}\left(\left(trD\right)^2 - tr\left(D^2\right)\right) = 0 \tag{24}$$

This simply corresponds to the fact that GR does not permit the 'dark matter' effect, and this happens because GR was forced to agree with Newtonian gravity, in the appropriate limits, and that theory also has no such effect. As well in GR the energy-momentum tensor $T_{\mu\upsilon}$ is not permitted to make any reference to absolute linear motion of the matter; only the relative motion of matter or absolute rotational motion is permitted. It is very significant to note that the above exposition of the GR formalism for the metrics in (21) is exact. Taking the trace of $\bar{G}_{ij}$ in (23) we obtain, also exactly, and in the case of zero vorticity and outside of matter, that

$$\frac{\partial}{\partial t}\left(\nabla . v\right) + \nabla .\left(\left(v.\nabla\right)v\right) = 0 \tag{25}$$

which is exactly the 'velocity field' formulation in (3) of Newtonian gravity outside of matter. This should have been expected as it corresponds to the previous observation that the 'Newtonian in-flow' velocity field is exactly equivalent to the external-Schwarzschild metric. There is in fact only one definitive confirmation of the GR formalism apart from the misleading external-Schwarzschild metric cases, namely the observed decay of the binary pulsar orbital motions, for only in this case is the metric non-Schwarzschild, and therefore non-Newtonian. However the new theory of gravity also leads to the decay of orbits, and on the grounds of dimensional analysis we would expect comparable predictions. It is also usually argued that the Global Positioning System (GPS) demonstrated the efficacy of General Relativity. However the new flow formalism of gravity also explains this system, and indeed gives a physical insight into the processes involved. In particular the relativistic speed and 'gravitational red-shift' effects now acquire a unified explanation (Cahill 2003b).

Discussion and Conclusions

The experimental evidence from at least seven observations of absolute linear motion, some using Michelson interferometers and some coaxial cable experiments, all showed that absolute linear motion is detectable, and indeed has been so ever since the 1887 Michelson-Morley experiment. Even Michelson and Morley reported a speed of 8km/s using the Newtonian theory for the instrument, but which becomes $\geq v_p = 300$ km/s when the Fitzgerald-Lorentz dynamical contraction effect and the refractive index effect are both taken into account. Essentially Michelson and Morley did not know that their interferomter only had 1/1700 the sensitivity that they had assumed. It then follows from the new 2002 theory for the interferometer that vacuum interferometer experiments will fail to detect that absolute motion, as is the case. We also understand that the various relativistic effects are caused by the absolute motion of systems through space, an idea that goes back to Lorentz. Elsewhere (Cahill 2003a, 2004a, 2005b) we have shown that both the Galilean and Lorentz transformations have a role in describing mappings of data between observers in relative motion, but that they apply to different forms of the data. So absolute motion is a necessary part of the explanation of relativistic effects, and indeed the Lorentz transformations and symmetry are consistent with absolute motion, contrary to current beliefs. On the contrary, the Einstein postulates and their apparent link to these relativistic effects have always been understood to imply that absolute motion is incompatible with these relativistic effects. It was then always erroneously argued that various observations of absolute motion over more than 100 years must have been flawed, since the relativistic effects had been confirmed in numerous experiments.

A major effect of the Einstein postulates was the development of a relativistic theory of gravity that was constrained to agree with Newtonian gravity in the non-relativistic limit. Evidence that Newtonian gravity was flawed has been growing for over 50 years, as evidenced by the numerous so-called 'gravitational anomalies', namely experimental observations of gravitational effects incompatible with Newtonian gravity. The most well known of these is the 'dark matter' effect, namely that spiral galaxies appear to require at least 10x the observed matter content in order to explain the high rotation speeds of stars and gas clouds in the outer regions. We now see that this effect is not caused by any form of matter, but rather by a non-Newtonian aspect to gravity. As well the Greenland bore hole g anomaly data has revealed that the dimensionless constant that determines the magnitude of this spatial dynamics is none other than the fine structure constant. The detection of absolute motion implies that space has some structure, for it is motion through that structure which is known as 'absolute motion', and which causes relativistic effects.

This means that the phenomena of gravity are described by two gravitational constant, G and α, and it is the small size of α that determines the asymptotic form of the orbital rotation speeds in spiral galaxies. As well it is α that determines the magnitude of the black hole masses at the centres of globular clusters, and the data from M15 and G1 confirms that $M_{BH} = \alpha M_{GC}/2$, in agreement with (7) (Cahill 2005a).

The detection of absolute motion and the failure of Newtonian gravity together imply that General Relativity is not a valid theory of gravity; and that it is necessary to develop a new theory. This has now been achieved, and the essential task of checking that theory against experiment and observation has now explained all the known effects that GR was supposed to have explained, but most significantly, has also explained the numerous 'anomalies' where GR was in manifest disagreement with the experimental or observational data. In particular, a component of the flow past the earth towards the sun has been extracted from the analysis of the yearly variations of the Miller data.

The putative successes of the Einstein postulates lead to the Minkowski-Einstein spacetime ontology that has dominated the mindset of physicists for 100 years. Spacetime was mandated by the misunderstanding that absolute motion had not been observed, and indeed that it was incompatible with the established relativistic effects. Of course it was always possible to have chosen one foliation of the spacetime construct as the actual one separating the geometrical model of time from the geometrical model of space, but that never happened, and that possibility became one of the banned concepts of physics.

We are now in the position of understanding that space is a different phenomenon from time, that they are not necessarily fused into some spacetime amalgam, and that the spacetime ontolgy has been one of the greatest blunders in physics. This must not be misunderstood to imply that the numerous uses of a mathematical spacetime, particularly in Quantum Field Theory, were invalid. What is invalid is the assertion that such a spacetime is a physical entity.

We may now ask, for the first time in essentially 100 years, about the nature of space. It apparently has 'structure' as evidenced by the fact that motion through it is detectable by various experimental techniques, and that its self-interaction is determined in part by the fine structure constant. One interpretation is that space is a quantum system undergoing classicalisation, and at a deep substratum level has the characteristics of a quantum foam. This quantum foam is in differential motion, and the inhomogeneities and time dependencies of this motion cause accelerations which we know of as gravity. This motion is not motion of something through a geometrical space, but an ongoing restructuring of that quantum foam. One theory for this quantum foam arises in an information-theoretic formulation of reality known as Process Physics, and one implication of this is that the quantum-foam system undergoes exponential growth, once the size of the quantum-foam system dominates over the matter part of the universe. This effect then appears to explain what is known as the 'dark energy' effect, although of course it is not an energy at all, just as 'dark matter' is not a form of matter. As well within this Process Physics we see a possible explanation for quantum matter, namely as topological defects embedded in the spatial quantum foam. That work gives the first insights into an explanation for the necessity of quantum behaviour and classicalisation.

This quantum-foam spatial system invites comparison with the much older concept of the 'aether', but it differs in that the aether was usually considered to be some form of matter residing within a geometrical space, which is not the case here with the quantum foam theory of space; for here the geometrical description of space is merely a coarse grained description. Nevertheless it would be uncharitable not to acknowledge that the quantum-foam system is a modern version of, and indeed a return to the aether concept, albeit a banned concept.

Physics is a science. This means that it must be based on (i) experiments that test its theories, and (ii) that its theories and reports of the analyses of experimental outcomes must be freely reported to the physics community. Regrettably, and much to its detriment, this has ceased to be the case for physics. Physics has been in an era of extreme censorship for a considerable time; Miller was attacked for his major discovery of absolute linear motion in the 1920's, while DeWitte was never permitted to report the data from his beautiful 1991 coaxial cable experiments. Amazingly, these experimenters were unknown to each other, yet their data was is in perfect agreement, for by different techniques they were detecting the same phenomenon, namely the absolute linear motion of the earth through space. All discussions of the experimental detections of absolute motion over the last 100 years are now banned from the mainstream physics publications. But using modern vacuum resonant cavity interferometer technology, and with a gas placed in the cavities, these devices could be used to perform superb experimental detections of absolute motion. Furthermore, the Miller and DeWitte data shows the presence of a wave phenomenon different from the waves argued to arise within GR theory, but which have not been detected, despite enormous costly efforts. It is now time to separate the genuine relativistic effects and their numerous manifestations from the flawed Einstein postulates, and to finally realise that they are caused by absolute motion of systems through a complex quantum system, which we know of as space. General Relativity turns out to have been a major blunder. Nevertheless, there is much evidence that a new theory of gravity has emerged, and this is to be exposed to critical analysis, and experimental and observational study.

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The Tessellattice of Mother-Space as a Source and Generator of Matter and Physical Laws

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Real physical space is derived from a mathematical space constructed as a tessellation lattice of primary balls, or superparticles. Mathematical characteristics, such as distance, surface and volume, generate in the fractal tessellation latttice the basic physical notions (mass, particle, the particle's de Broglie wavelength and so on) and the corresponding fundamental physical laws. Submicroscopic mechanics developed in the tessellattice easily results in the conventional quantum mechanical formalism at a larger atomic scale and Newton's gravitational law at a macroscopic scale.

1. Introduction

Although Poincaré (1905a) was the first to write the relativistic transformation law for charge density and velocity of motion, Einstein's (1905) special relativity article received wide recognition, perhaps due to his introduction of a radically new abstract approach to fundamentals, which then culminated in his famous theory of general relativity (Einstein, 1916). Due to its predictions, which were verified experimentally, abstract theoretical concepts took root in the minds of a majority of physicists. Einstein's approach resembled rather a generalized description that descended to particulars through a series of postulated axioms. His general relativity considers how matter and geometry, constructed in empty space, coexist and influence each other, though matter is not an intrinsic property of space.

Einstein's thoughts regarding an aether were expressed in his well-known lecture (Einstein, 1920). He noted that since space was endowed with physical qualities, an aether must exist. Then he mentioned that, according to general relativity, space without an aether is unthinkable (light would not propagate; there would not any space-time intervals in the physical sense, *etc.*). Nevertheless, Einstein stressed that this aether might not be thought of as endowed with quality characteristic of a ponderable medium, consisting of parts that might be tracked through time. However, the basic issue remained: Why could the aether not be associated with a substrate? This was never clarified by Einstein completely.

The hypothesis of an aether as a material substrate responsible for electromagnetic wave propagation has been tested by many researchers (Miller, 1933; Essen, 1955; Azjukowski, 1993). A new optical method of the first order was proposed and implemented by Galaev (2002) for measurements of the aether-drift velocity and kinematic viscosity of aether. Galaev's results correlate well with the results of other researchers quoted above.

Observability, reproducibility and repeatability of aether drift effects have been conducted in various geographical conditions using different methods of measurement and in various ranges of electromagnetic waves. Overall, this research strongly supports the idea that the aether is a subsrate responsible for propagation of electromagnetic waves. These studies shed light on the negative results of aether wind measurements by Michelson and Morley: Their tool had too low a sensitivity.

Other researchers demonstrated direct interaction of matter with a subquantum medium. In particular, the influence of a new "strange" physical field on test subjects has been shown by Baurov (2002), Benford (2002) and Urutskoev *et al.* (2002). Similar effects are described by Shipov (1997), though the changes in samples examined were associated with the so-called "torsion radiation" introduced by Shipov as a primary field that allegedly dominated over a vague physical vacuum long before its creation. One more incomprehensible phenomenon is the Kozyrev effect (Kozyrev and Nasonov, 1978) whereby a bolometer centrally located in the focal point of a telescope records a signal from a star much earlier than the light signal hits the focal point.

Let us briefly examine Poincaré's studies. His research was also highly abstract, especially his investigations in mathematics and mathematical physics. Nevertheless, in physics applications, he tried to hold to natural laws as closely as possible. In fact, Poincaré (1905b, 1906) believed any new success in science was further support of determinism. In his works, he tried to start from a few details, which should then disclose the problem as a whole. Poincaré (1905b) strongly supported the idea of an aether, as he considered the motion of a particle to be accompanied by an aether perturbation. The idea of perturbation of the aether by a moving object was predominant among leading mathematicians and physicists up to beginning of the 20th century.

Therefore, his idea deserves credit (if any kind of aether in fact exists). Poincaré treated particles as peculiar points in the aether, though he did not develop further ideas on its construction nor principles of the motion of material objects in it. Experimental facts were not abundant at that time, and theoretical notions of condensed matter physics, which would help one to look at a possible theory of aether in more detail, were lacking. Moreover, mathematical methods of description of space were also in an embryonic state at beginning of the 20th century, despite the fact it was Poincaré who proposed and developed new concepts and methods of the investigation of space as such. At that time, data were not so numerous as now, and this did not allow Poincaré to consolidate ideas on space and aether in a unified generalized concept of real space.

However, today when abundant data are available, we may try to look at the possibility of unifying mathematical and physical ideas to incorporate an aether into space in one unified object of comprehensive study.

2. The constitution of space

Many researchers are involved in the search for a theory of everything (TOE). However, do we yet have a "theory of something"? The problem was studied by Bounias (2000) on the basis of pure mathematical principles. He firmly believed the ultimate theory might be some mathematical principle.

Following Bounias (2000), and Bounias and Krasnoholovets (2002, 2003), we can explore the problem of the constitution of space in terms of topology, set theory and fractal geometry. Evidently, according to set theory, only the empty set (denoted $\varnothing$) can represent nearly nothing. If F is a part of E, then the remaining part of E, which does not contain F, is the complementary of F in E, which is denoted $\mathbf{C}_E(F)$. The empty set $\varnothing$ is contained in any set E, *i.e.* $\mathbf{C}_E(E)=\{\varnothing, E\}$, then $\mathbf{C}_E(E)=\varnothing$. This result, together with $\mathbf{C}_E(\varnothing)=E$, is known as the first law of Morgan. This allowed Bounias to conclude the complement of the empty set is the empty set: $\mathbf{C}_\varnothing(\varnothing)=\varnothing$. Following von Neumann, Bounias considered an ordered set, $\{\{\{\varnothing,\{\varnothing\}\},\{\varnothing,\{\varnothing,\{\varnothing\}\}$, and so on. By examining the set, one can count its members: $\varnothing=$ zero, $\{\{\varnothing\},\{\varnothing\}\}=$ one and *etc.* This is the empty set as long as it consists of empty members and parts. On the other hand, it has the same number of members as the set of natural integers, $N=\{0,\ 1,\ 2,\ ...,\ n\}$. Although it is proper that reality is not reduced to enumeration, empty sets give rise to mathematical space, which in turn brings about physical space. So, *something* can emerge from emptiness.

The empty set is contained in itself, hence it is a non-well-founded set, or hyperset, or empty hyperset. Any parts of the empty hyperset are identical, either a large part $(\varnothing)$ or the singleton $\{\varnothing\}$; the union of empty sets is also the same: $\varnothing\cup(\varnothing)\cup\{\varnothing\}\cup\{\varnothing,\{\varnothing\}\}\cup\ \ \{\varnothing,\{\varnothing\},\{\varnothing,\{\varnothing,\{\varnothing\}\}\}\cup...=\varnothing$. This is the major characteristic of a fractal structure, which means the self-similarity at all scales (in physical terms, from the elementary sub-atomic level to cosmic sizes). One empty set $\varnothing$ can be subdivided into to two others; two empty sets generate something $(\varnothing)\cup(\varnothing)$ that is larger than the initial element. Consequently, the coefficient of similarity is $\rho\in[\frac{1}{2},\ 1]$. In other words, ρ realizes fragmentation when it falls within the interval $]\frac{1}{2},\ 1[$ and union when ρ with the interval $]0,\ \frac{1}{2}]$ yields $]0,\ 1[$. The coefficient of similarity allows us to estimate the fractal dimension of the empty hyperset, which owing to the interval $]0,\ 1[$ becomes a fuzzy dimension.

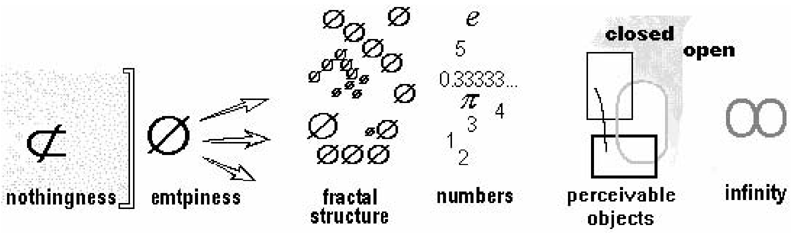

Figure 1. Range of things, from non-existence to something empty whose structure gives rise to something non-empty and up to infinity (from Bounias and Krasnoholovets, 2002).

Time can be called *nothing*, because it is a singleton that does not have parts (otherwise it will be in contradiction to the definition of time as such). The nothingness singleton ($\in$) is absolute unique. It is the lowest boundary of everything existing; this is the infimum of existence (Figure 1).

4-D mathematical spaces have parts in common with 3-D spaces, which yields 3-D closed structures. There are then parts in common with 2-D, 1-D and zero dimension (points). General topology indicates the origin of time, which should be treated as an assembly of sections S_i of open sets. Indeed, fractality of space generates fuzzy dimensions (Bounias and Krasnoholovets, 2003a), and hence a common part of a pair of open sets W_m and W_n with different dimensions m and n also accumulates points of the open space. If $m > n$, then those points, which belong to W_m and would not belong to the section of the given sets, cannot be included in a *x*-D object. Bounias and Krasnoholovets (2002) exemplified this in the following way: “You cannot put a pot into a sheet without changing the shape of the 2-D sheet into a 3-D packet. Only a 2-D slice of the pot can be a part of a sheet.” Therefore, infinitely many slices, *i.e.* a new subset of sections with dimensionality from 0 to 3, ensure the raw universe in its timeless form.

Thus a physical space is one that can be provided by closed intersections (timeless Poincaré sections) of abstract mathematical spaces. What happens to these sections S_i that all belong to an embedding 4-space? A series of sections S_i, S_{i+1}, S_{i+2}, *etc.* resembles the successive images of a movie, and only nothing does not move. Therefore, the difference of distribution of objects within two corresponding sections will mean a detectable increment of time. Hence time will emerge from order relations holding on these sections.

Two successive slices show a characteristic of mathematical objects from one to the next section. In other words, this is a mapping. The first section produces some x that then becomes $f(x)$ on the next one. The mapping between nearest sections can be treated in the framework of an indicatrix function $1(x)$ and Uryson’s theorem. By definition, $1(x)$ for any x state yields $1(x) = 0$ if x has one property and $1(x) = 1$ if x has an alternative property. A combination of $f(x)$ with $1(f(x))$ makes a demonstration whose result depends upon whether the variable x belongs to one part of the frame in S_i or belongs to the same part in S_{i+1}. The complete function is a composition of the variables with their distribution. That is, the function has the structure of a moment, called a "moment of junction" *MJ* by Bounias (2000; Bounias and Krasnoholovets 2003b). The function *MJ* describes the smallest increment of space. (One point is not at the same topological position for *MJ* to permit the change.) Such fine change of *MJ* also defines an increment in time—the minimal change. Since there is no thickness between two sections S_i and S_{i+1} the moment of junction *MJ* rigorously describes a differential element of space, which is also a differential element of time. This validates differential geometry from the description of the Universe.

3. Measure, distance, metric and objects

The concept of measure usually involves such particular features as existence of mappings and the indexation of collections of subsets on natural integers. Classically, a measure is a comparison of the measured object with some unit taken as a standard. The "unit used as a standard" is the part played by a gauge (*J*). A measure involves respective mappings on spaces, which must be provided with the rules $\cap$, $\cup$ and $\complement$. According to Bonaly and Bounias (1996), in spaces of the R^n type, tessellation by balls is involved, which again requires a distance to be available for measurement of diameters of intervals. Intervals can be replaced by topological balls, and therefore evaluation of their diameter still needs an appropriate general definition of a distance. More comprehensive determinations of measure, distance, metric and objects, which involve topology, set theory and fractal geometry, have been made by Bounias and Krasnoholovets (2003a).

In physics, a ruler is called a metric. As a rule, mathematical spaces including topological spaces have been treated as not endowed with a metric, and properties of metric spaces have not been the same as those of non-metric spaces. However, in 1994 Bounias (see, *e.g.*, Bonaly and Bounias, 1996) have shown that a non-metric topological space does not exist! Indeed, union and intersection allow the introduction of the symmetric difference between two sets A_i and A_j

$$\Delta(A_i, A_j) = \complement_{\cup\{A_i\}} \cup_{i \neq j} (A_i \cap A_j) \qquad (1)$$

i.e. the complementary of the intersection of these sets in their union. Symmetric difference satisfies the following properties: $\Delta(A_i, A_j) = 0$ if $A_i = A_j$, $\Delta(A_i, A_j) = \Delta(A_j, A_i)$ and $\Delta(A_i, A_j)$ is contained in union of $\Delta(A_i, A_j)$ and $\Delta(A_j, A_k)$. This means it is a true distance and can also be extended to the distance of three, four and *etc.* sets in one, namely, $\Delta(A_i, A_j, A_k, A_l, \ldots)$. Since the definition of a topology implies the definition such a set distance, every topological space is endowed with this set metric. The norm of the set metric is $\| A \| = \Delta(\varnothing, A)$. Therefore, all topological spaces are metric spaces, Δ- metric spaces, and they are measurable.

We now examine at the remaining part, *i.e.* the intersection of the sets. If they are of unequal dimensions, this intersection will be closed, *i.e.* the intersection in a closed space is closed, $\cup_{i \neq j}(A_i, A_j)$, which signifies the availability of physical objects. As distances Δ are the complementariness of objects, the system stands as a manifold of open and closed subparts. This procedure subdivides the Universe into two parts: the distances and the objects.

In general, we can imagine the universe as an immense drop containing N balls. Since measurement embraces such notions as length, surface and volume, we may represent ℓ—the loop distance of the universe (*i.e.,* the perimeter that can be measured with a ruler)—with parameters of the N balls. Indeed, let m be the measure of the balls (length, surface, or volume of dimension δ depending on what kind of the characteristics we are interested in). Inside a universe of dimension D we have N times m^{δ} approximately equal to ℓ^{D}, so that

$$D \sim (\delta \cdot \log m + \log N) / \log \ell. \qquad (2)$$

Thus if we know component parts of the universe, *i.e.* can describe sizes and shapes of the topological balls, we will be able to reconstruct the large unknown structure.

4. Tessellation lattice of primary balls

Let us now examine what space-time is in the approach proposed by Bounias and Krasnoholovets (2002, 2003). We started from the founding element. Namely, it is generally recognized in mathematics that some set does exist. A weaker form can be reduced to the existence of the empty set. If one provides the empty set $(\varnothing)$ with the combination rules $(\in, \subset)$ and the property of complementary $(\mathbf{C})$, a magma can be defined: The magma is a union of elements $(\varnothing)$, which act as the initiator polygon, and complementary $(\mathbf{C})$, which acts as the rule of construction; *i.e.*, the magma is the generator of the final structure. This allowed Bounias to formulate the following theorem: The magma $\varnothing^{\varnothing} = \{\varnothing, \mathbf{C}\}$ constructed with the empty hyperset and the axiom of availability is a fractal lattice. Writing $(\varnothing^{\varnothing})$ denotes the magma, and reflects the set of all self-mappings of $\varnothing$. The space, constructed with the empty set cells of the magma $\varnothing^{\varnothing}$, is a Boolean lattice, and this lattice $S(\varnothing)$ is provided with a topology of discrete space. A lattice of tessellation balls has been called a *tessellattice*, and hence the magma of empty hyperset becomes a fractal tessellattice.

Introduction of the lattice of empty sets ensures the existence of a physical-like space. In fact, the consequence of spaces (W_m), (W_n), ... formed as parts of the empty set $\varnothing$ shows that the intersections have non-equal dimensions, which gives rise to spaces containing all their accumulating points forming closed sets (Bounias and Krasnoholovets, 2003a). If morphisms are observed, then this enables the interpretation as a motion-like phenomenon, when one compares the state of a section with the state of mapped section. A space-time-like sequence of Poincaré sections is a non-linear convolution of morphisms. Our space-time then becomes one of the mathematically optimal morphisms, and time is an emergent parameter indexed on non-linear topological structures guaranteed by discrete sets. That is, the foundation of the concept of time is the existence of order relations in the sets of functions available in intersect sections.

Time is thus not a primary parameter, and the physical universe has no beginning: time is just related to ordered existence, not to existence itself. The topological space does not require any fundamental difference between reversible and steady-state phenomena, nor between reversible and irreversible process. Rather relations simply apply to non-linear;y distributed topologies, and from rough to finest topologies.

Such fundamental notions such as point, distance and similitude allow us to introduce relative scales in the empty-set lattice, *i.e.* the tessellattice; therefore space everywhere becomes quantized. Indeed, from mapping $G: N^D \mapsto Q$ of $(N \times N \times N \times ...)$ in Q we can identify a set of rational intervals. In this way, for n integers in each one of the 2-D spaces, $n \times n$, the pair $(1,\ n)$ yields fraction $1/n$ and the pair $(1,\ n-1)$ yields ratio $1/(n-1)$. Their distance is then the smaller interval, *i.e.* difference between these two fractions gives the smaller interval proportional to $1/n^2$, or more exactly, the interval $1/(n-1)-1/n$ $=1/(n(n-1))$ that denotes a special scale limit depending on the size of the considered space (this smaller interval formed by n^2 grains is constructed from $\varnothing$). In 3-D, we will have interval $1/(n^2(n-1))$.

Predictable orders of size from $x=1$ to $x=60$ are clusters/universes whose objects range from 1 (the Planck scale, *i.e.* the size of an elementary cell of the tessellattice), to $\sim 10^{10}$ elementary cells (roughly quark-like size), to about 10^{17} cells (atomic size), to 10^{21} cells (molecular size), to 10^{28} (human size), to 10^{40} cells (solar system size) up to 10^{56} cells (largest structures). The universe offers a quite different organization of matter at different scales.

5. Generation of matter

Nowadays quantum and particle physics are considered as the most fundamental disciplines. They study the behavior of quantum systems, such as interaction between particles in the presence of this or that potential, transformations of particles to the others, *etc.* However, the fundamental notions with which quantum physics operates (mass, wave ψ-function, wave-particle, de Broglie and Compton wavelengths, spin and others) lack comprehension of their nature and origin, inasmuch as these microscopic parameters are *a priori* treated as primordial. This makes it possible to raise questions concerning conceptual difficulties of quantum mechanics (Krasnoholovets, 2004). Are we able to develop deeper first principles and derive fundamental notions based on a submicroscopic concept? The “strangeness” of quantum mechanical behavior of

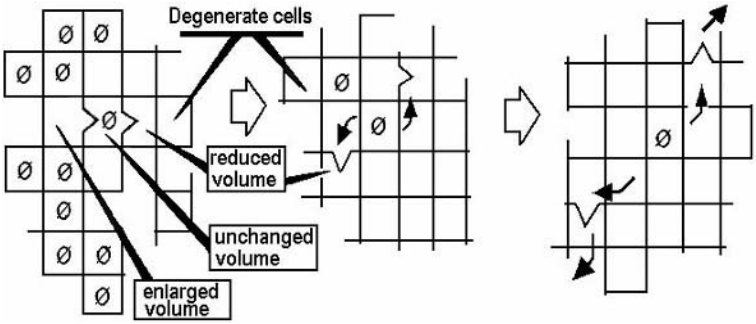

Figure 2. Volumetric fractality of cells as elementary deformations of the tessellattice. These deformations can occur with and without change in the volume of cells. Local deformations producing a reduction of the volume of cells are associated with the local generation of mass. These deformations can migrate in the tessellattice from cell to cell (from Bounias and Krasnoholovets, 2002).

particles must be completely clarified, owing to an inner determinism establishing specific links in quantum systems, which are hidden under the crude orthodox quantum formalism. In quantum electrodynamics, the electric and magnetic charges are not derived from first principles, and remain inconceivable observable points with special properties. In contrast, the sub-microscopic approach allows a rigorous mathematical study and clear definition of this notion (Krasnoholovets, 2003).

If we wish to provide insight into the structure of an abstract physical vacuum, we must rather assume that this substance is nothing, instead of complete emptiness. But nothing can be considered in terms of space, namely, topology, set theory and fractal geometry, which has just been demonstrated in the previous sections.

One of our starting points is the idea that organization of matter at the microscopic (atomic) level should reproduce submicroscopic space ordering. This means that the lattice of a crystal should be the reflection of the arrangement of real physical space. This space can be fully associated with the tessellattice of densely packed balls, or superparticles. And this is the degenerate space (one may associate it with an abstract physical vacuum). Superparticles constitute founding cells of the tessellattice and are stacked without any unfilled place between them, which refers to the nothingness singleton, discussed in Section 2.

Degeneration of a cell is removed when the cell receives several deformations, such that its volume may be reduced, while the equivalent volume is redistributed among other cells (or in terms of conventional physics, the deformed superparticle becomes a massive particle). The mass m of this particle is the product of a constant C for dimensional analysis by ratio of the volume V of a superparticle to that of our reduced superparticle (which is now called the particle),

$$m = C\, V_{\text{super}} / V_{\text{part}} \,. \tag{3}$$

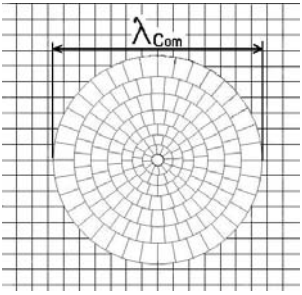

Figure 3. Particle as a local deformation of the tessellattice (the central cell) and the deformation coat that screens the particle from the degenerate tessellattice.

By analogy with the crystal lattice in which a foreign particle deforms the environment, we recognize that a deformation sheath emerges in the tessellattice around a canonical particle; the size of this sheath is associated with the particle's Compton wavelength (Figure 3).

Having established the particle, we may try to construct its mechanics in tessellation space, which immediately means development of physical laws and physics in general. Since the space should be densely packed with balls, any motion of a chosen (deformed) ball should be expressed in terms of interaction with other balls of the space. This brings about a radically new approach to the behavior of matter.

6. The submicroscopic mechanics

Thus, the real space exists in the form of the tessellattice, *i.e.* tessellation lattice of primary balls (superparticles, or cells) that densely pack the universe. The submicroscopic mechanics of particles has been developed by the author in a series of works (see, *e.g.* Krasnoholovets, 2002a). A particle cannot move without rubbing against superparticles of the tessellattice, and hence a packet of lattice deformations goes forward accompanying the particle. Elementary excitations migrating from cell to cell in fact represent a resistance, *i.e.*, inertia, of the space constructed as the tessellattice. These excitations, called *inertons*, are produced at collision-like phenomena: deformations (inertons) go from the particle to the surrounding space and then due to elastic properties of the tessellattice some come back to the particle. This motion can be described by the appropriate Lagrangian (simplified here)

$$L = m\dot{x}^2/2 + \mu\dot{\chi}^2/2 - \sqrt{m\mu}\,\dot{x}\chi/T \qquad (4)$$

where m, x and μ, χ are the mass and position of the particle and its inerton cloud, respectively; $1/T$ is the frequency of collisions between the particle and the cloud.

The Euler-Lagrange equations indicate periodicity in the behavior of the particle. Consequently, the particle velocity oscillates between the initial value υ and zero along each section λ of the particle path. This spatial amplitude is determined as follows: $\lambda = \upsilon T$. The same occurs for the cloud of inertons:

$\Lambda = cT$. These two amplitudes become connected by means of relationship $\Lambda = \lambda c / \upsilon$.

Furthermore, solutions to the equations of motion show that motion of the particle in the tessellattice is characterized by two de Broglie relationships for the particle: $E = h\nu$ and $\lambda = h/(m\upsilon)$ where $\nu = 1/(2T)$, and these allow the derivation of the Schrödinger equation. Therefore, at this stage submicroscopic mechanics passes into conventional quantum mechanics.

The amplitude of spatial oscillations of the particle λ appears in quantum mechanics as the de Broglie wavelength. The amplitude of the particle's cloud of inertons Λ becomes implicitly apparent through the availability of the wave ψ-function. Therefore, the physical meaning of the ψ-function becomes completely clear: it describes the range of space around the particle perturbed by its inertons.

The next step is that inertons transfer not only inertial, or quantum mechanical properties of particles, but also gravitational properties, because they transfer fragments of the deformation of space (*i.e.* mass) induced by the particle. Study (Krasnoholovets, 2002b) shows that the object's dynamic inertons allow the derivation of Newton's static gravitational law, as long as inertons spread as a standing spherical wave specified by the dependency $1/r$.

7. Concluding remarks

The mysteries of quantum mechanics are here explained in real space, and inertons have been experimentally detected in conditions predicted by the theory (see, *e.g.* Krasnoholovets, 2002a). The submicroscopic mechanics fully restores determinism. In addition, recently my colleagues and I have launched the project entitled "Inerton Astronomy," in conjunction with which we have built a special laboratory facility able to measure inerton waves. We can now record inerton signals along the West-East line at ~20 Hz, which is associated with proper rotation of the globe. From September to December, 2004, we recorded a flow of inertons at frequencies from 18 to 22 kHz coming from the northern sky in a universal time interval from 3 p.m. to 5 p.m.

The concept of the tessellattice of space replaces such uncertain notions as classical elastic aether and physical vacuum. This deeper concept makes it possible to uncover many inner details of the constitution and behavior of particles and physical fields, which have thus far eluded researchers.

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Gravitation in a Gaseous Ether

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Newtonian gravitation arises from the Clausius term added to the ideal gas equation to account for the space occupied by cosmons, fundamental particles of cosmonic gas. The gravitational force is explained as a pressure due to static pressure and cosmon concentration.

Introduction

A new model is proposed to explain known physical phenomena. A gas, composed of particles moving in all directions, is assumed to pervade the entire Universe, even the space between so-called elementary particles and within elementary particles themselves. The particles that make up this gas are called cosmons.

Cosmons have no moving parts, and hence no internal energy or rest mass, inertial or gravitational. The space between cosmons is absolute void, and there are no fields at this level. Since a cosmon has a definite diameter, it occupies a volume of space that is impenetrable to other cosmons. In vacuum, individual cosmon speed may vary from zero to indefinitely large, according to the Maxwell speed distribution. Cosmons have zero spin (no forces or friction).

Interactions with other cosmons occur only through encounters. Between encounters, cosmons move at constant velocity (speed and direction). In encounters with other cosmons, there is an exchange of the velocity component along the centre line. The velocity component normal to the line of centres remains with each cosmon. The midpoint of the line of two centres describes the same straight line before, during and after the encounter.

There are no other physical phenomena at the cosmon level. Physics is infact reduced to pure kinematics. The concepts of momentum and energy are not applicable at this level. But at the level of the gas, the classical concepts of mass, momentum, energy, *etc*. are statistical effects due to cosmon kinematics. Because the cosmons have a diameter, the cosmonic gas is non-ideal, and viscosity is present. The properties of electromagnetism and elementary particles are described by the mechanical properties of the cosmonic gas. (Martin 1994a)

Equation of state of cosmonic gas

An empirical equation of state for a molecular gas was given by Van der Waals:

$$\left(p+\frac{a}{V^2}\right)(V-b)=\mathrm{R}T\,,$$

where p is static gas pressure, V is total gas volume per mole, T is temperature, and R is the universal gas constant, with dimensions ergs per mole per degree. Both a and b vary with temperature in this equation: a varies due to the variation of the attractive and repulsive forces between molecules, and b due to the variation of volume of the molecules with temperature.

In the cosmonic gas, however, there are no potentials between cosmons, and thus the a term disappears. Meanwhile, b is proportional to the cosmon sphere of exclusion, but the cosmon volume is invariant, and b must therefore be a constant. With these assumptions, the Clausius equation is fully applicable to the cosmonic gas in the form:

$$p\left(1-\frac{b}{V}\right)=\frac{\mathrm{R}T}{V}=NkT=N\mu\frac{kT}{\mu}=\rho\frac{kT}{\mu}\,, \tag{1}$$

where N is the cosmon number density, k the Boltzman constant, μ the cosmon statistical interaction factor (or mass) and ρ the cosmonic gas mass density. From gas kinetic energy theory

$$\frac{b}{V}=\frac{\sigma}{L}=2\pi\frac{\sigma^3}{\sqrt{2}}N\,, \tag{2}$$

where σ is cosmon diameter. If $N_{\max}$ is cosmon number density at maximum compaction or density, the volume occupied by one cosmon, including its surrounding void, is

$$\frac{\sigma^3}{\sqrt{2}}=\frac{1}{N_{\max}}\,.$$

But the cosmon mean free path $L = 0$ when $N = N_{\max}$. Equation (2) can therefore be modified to

$$\frac{b}{V}=\frac{\sigma}{L}=\frac{2\pi N}{N_{\max}-N}\,,$$

and the Clausius equation for the cosmonic gas may then be written

$$p\left(1-\frac{2\pi N}{N_{\max}-N}\right)=NkT\,. \tag{3}$$

Ratios of cosmon concentrations

Following the model of the large numbers hypothesis of Eddington, Assis (2001) has proposed a Principle of Physical Proportions, according to which all physical laws must be reduced to ratios of like quantities, *i.e.*, dimensionless fractions. The laws of physics deduced from the Clausius equation in its cosmonic gas form can be shown to comply with this principle.

We know that the Newton gravitational potential energy of a source M is GM/r, while the gravitational potential energy of a mass m in the gravitational field of M is GMm/r; the gravitational potential energy of mass M in the gravitational field of m is likewise GMm/r. Thus, both masses have the same potential energy. Similarly, the interaction force on each mass is GMm/r^2 toward the other mass. We also know that $\sqrt{G}$ has dimensions e.s.u./mass, and when applied to a mass, transforms it into electrostatic units, such that the Newton formula has the same form as the Coulomb law of electrostatics (Qq/r for potential energy, and Qq/r^2 for the attractive force between the charges).

In classical electromagnetism, $Qq/r = e^2/r = m_E c^2$ is the total energy of electrostatic field. The total potential energy for two masses interacting gravitationally is analogously $\sqrt{G}M\sqrt{G}m/r$. The effect of a mass on its surrounding gravitational field is the potential energy $\phi = \sqrt{G}M\sqrt{G}/r$ (energy per mass), and the effect of a charge on its electrostatic field is potential energy $\psi = Q/r$ (energy per charge). But $M = (4/3)\pi r^3 \rho_0$ and $Q = (4/3)\pi r^3 \rho_E$. Hence we obtain a simple ratio

$$\frac{\phi}{\psi} = \frac{\sqrt{G}\rho_0}{\rho_E} = 1.52 \times 10^{-22},$$

where $G = 6.672 \times 10^{-8}$ dynes cm^{2}g^{-2} ($\sqrt{G} = 2.583 \times 10^{-4}$ e.s.u g^{-1}), and ρ_0 is vacuum mass density, and ρ_E vacuum charge density *at earth's surface*. A ratio of the same order of magnitude can be formed from the electron mass and charge:

$$\sqrt{G}m_E/e = 4.896 \times 10^{-22},$$

where standard values of $e = 4.803 \times 10^{-10}$ e.s.u. and $m_E = 9.10 \times 10^{-28}$ g are used.

In a companion paper (Martin 2005), values of mass density ρ_0 and electrical charge density ρ_E were calculated by an iterative method for the electron as follows:

$$\rho_0 = N_0\mu_0 = 4.62 \times 10^8 \text{ g cm}^{-3},$$

where μ_0 is the cosmon mass-energy equivalent ($= (5/2)kT_0/c^2$, $T_0 = 2.736$ K) and N_0 (vacuum cosmon density at or near Earth's surface) is obtained from pressure energy divided by pressure energy per particle, or $N_0 = p_0/kT_0$ ($= 4.4 \times 10^{44}$ cosmons cm^{-3}); and

$$\rho_E = \frac{3}{4\pi}\sum\frac{e_c}{a^3} = 7.85 \times 10^{26} \text{ e.s.u. cm}^{-3},$$

where e is the charge of a polytropic gas sphere and a its radius. When the cosmonic gas parameter $N_{\max}$ (concentration of cosmons at maximum compaction) is evaluated geometrically from the $N_{\max} = \sqrt{2}/\sigma^2$, using cosmon diameter $\sigma^2 = 1/\sqrt{2}\pi N_0 L_0$, L_0 being cosmon mean free path $h/\rho_0 c$, we obtain a value $N_{\max} = 1.18 \times 10^{67}$ cosmons cm^{-3}.

The value of the Clausius term in the cosmonic gas equation may then be calculated for vacuum at Earth's surface:

$$\frac{b}{V} = \frac{\sigma}{L} = \frac{2\pi N}{N_{\max} - N} = 2.34 \times 10^{-22}.$$

Considering the order of magnitude involved, the agreement obtained between ratios formed from experimentally known values G, m_E, e, on the one hand, and values derived from the description of the electron in terms of cosmonic gas ρ_0, ρ_E and N_0 and $N_{\max}$ (Martin 2005), we can state that the fundamental equations of physics, when expressed as ratios of like quantities, reduce to ratios of cosmon concentrations: $\psi/\phi = \sqrt{G}m_E/e = 2\pi N_0/(N_{\max} - N_0)$. The parameters of cosmonic gas theory would thus appear to be corroborated by experimentally known values.

The mechanism of gravity

In the cosmonic gas, gravitation is accounted for by this extra term added to the equation of state of an ideal gas by Clausius to account for the finite volume occupied by the particles, the cosmons in this instance. The added term is a negative dimensionless factor multiplying the static pressure in the ideal gas equation. The product of the static pressure and the Clausius term (simply the ratio of cosmon diameter to mean free path) will be called gravitational pressure, denoted $p_G = -p(2\pi N/[N_{\max} - N])$.

In the viewpoint of a contiguous fluid, a force is produced by a pressure gradient over an element of gas volume. For the case of gravitation, the force is due to the gradient of the ratio of gravitational pressure p_G to mass density (force/unit mass):

$$-\mathrm{grad}\left(\frac{p_G}{\rho}\right) \to -\mathrm{grad}\phi_M = -\frac{GM}{r^2} = \text{acceleration}\,.$$

Force density per gas volume element is thus $-(GM/r^2)\rho$, and the force on a test mass is just $-(GM/r^2)m$. The equations of gravitation, which have been used successfully since Newton, emerge from this fundamental relation. However, this does not elucidate the mechanism of gravitation, which must be explained from the properties of the cosmonic gas.

In the cosmonic gas, the gravitational pressure term p_G is a negative pressure—as is gravity—acting against the static pressure of the gas. Around each massive body, the difference of these two pressures must result in constant vacuum pressure p_0 in order to satisfy the equilibrium condition. As the mass increases, the static pressure p therefore increases to compensate for the increasing number density N, according to

$$p\left(1 - \frac{2\pi N}{N_{\max} - N}\right) = NkT = p_0 \text{ constant, or } p_G = p - p_0$$

We thus see that in the gravitational field of each mass, temperature T has to *decrease* as N *increases*, keeping vacuum pressure $p_0 = N_0kT_0$ constant. (There is no mass outside material bodies.)

Expressed in cosmonic gas terms, the gradient of gravitational potential would then take the form:

$$-\text{grad}\left(\frac{p_G}{\rho}\right) = -\text{grad}\frac{p}{\rho}\left(\frac{2\pi N}{N_{\max} - N}\right) + \frac{p}{\rho}\left(\text{grad}\frac{2\pi N}{N_{\max} - N}\right)$$

In other words, increase of cosmon density disturbs the equilibrium of the gravitational field of each body acting on each element of volume of the other, creating a cooling effect. This is a local phenomenon which transforms the internal energy of each element of volume into kinetic energy of free fall. The cooling is due to the fact that the body's internal energy is distributed over a greater number of particles as it moves into a region of higher cosmon density. The process is perfectly reversible; in Keplerian orbital motion, when masses move away from each other, kinetic energy is transformed back into heat as masses enter the warmer parts of gravitational fields. There is no transfer of energy or momentum between the two fields, as each mass remains constant during the whole orbit. Hence there is no need for the concept of gravity propagation speed. It can be treated as zero, in accordance with observation. (Van Flandern and Vigier, 2002) There is a change of mass only in non-elastic collisions of two bodies, as observed in astronomy.

Mass and inertia

The main difference between the behaviour of cosmons and the behaviour of molecules is due to the fact that cosmons travel in a void, while molecules travel in cosmonic gas, which is viscous. Mechanical resonances in the cosmonic gas (quantum conditions) neutralize viscosity effects, thus permitting the existence of long-lived phenomena. At the gas level, the definition of mass, applied to any kind of energy, is $m = E/c^2$.

As shown above, the gravitational pressure around each mass is equivalent to the static pressure of the cosmonic gas multiplied by the ratio of cosmon numerical concentration N to the maximum concentration $N_{\max}$. Inertia is likewise determined by cosmonic gas properties. Every moving fundamental particle is accompanied by a spherical vortex having energy equivalent to the kinetic energy of the particle. An applied force on a mass produces an acceleration. And when a mass is accelerated, an Euler reaction force is produced that is equal and opposite to the applied force.

At the cosmon level, however, the void surrounding each cosmon has no momentum or energy, and thus there is no gravitation potential or inertia. Consequently, the three conditions for mass are not present at the cosmon level. An individual cosmon therefore has no rest mass or dynamical mass, since these are phenomena at the cosmonic gas level. The step from a medium of continuous density (the cosmonic gas) to a particulate medium (the cosmon) requires division by particle concentration N, which yields a statistical mean cosmon mass μ, as required by the equations of kinetic gas theory.

No singularities

If we posit that the ratio of cosmon diameter to mean free path is equal to the ratio of a fundamental (minimal) radius R_1 to radius r, *i.e.*,

$$\frac{R_1}{r} = \frac{\sigma}{L} = \frac{2\pi N}{N_{\max} - N}$$

then a relation is obtained for the variation of N along the radius of a gravitational field:

$$N = \frac{N_{\max}}{\left[\left(2\pi r / R_1\right) + 1\right]}. \tag{4}$$

At $r = R_1$ the cosmon mean free path $L = \sigma$ (cosmon diameter), and the cosmon concentration becomes $N = 0.1373\ N_{\max}$, which nullifies the factor multiplying the static pressure in (3) above. Gravitational pressure (centripetal) therefore equilibrates static pressure (centrifugal) at $r = R_1$. Outside the fundamental radius, the negative gravitational term $p\sigma/L$ is always smaller than the static pressure. At the fundamental radius, it is equal to but of opposite sign to the static pressure term. The gravitational forces just counterbalance the static pressure forces. Inside the fundamental radius, gravity dominates and the cosmon density varies from $N_{\max}$ at the centre to $0.1373\ N_{\max}$ at the fundamental radius R_1.

The Schwarzschild solution to the Einstein field equations of General Relativity gives a relation between time t as observed from a great distance from a massive object and proper time τ as measured by a clock in free fall toward the object, including the effect of velocity $v = dr/dt$ as found in special relativity. According to Thorne *et al.* (1990) at $r = R_S = 2GM/c^2$, the observed time t becomes infinite while proper time τ remains finite all the way to the centre of the object, assuming a point mass. It is well known that the Schwarzschild solution is valid only outside R_S, as the Newtonian form of gravitational potential $\Phi = GM/r$ is valid only outside the radius of the mass. This could mean that the entire mass of a body must be within its Schwarzshild radius, with its density spherically distributed. In the case of the cosmonic gas, the radius R_1 is where the centrifugal and centripetal forces are equal and opposite, making the factor $(1 - 2\pi N/N_{\max} - N) = 0$, irrespective of the value of static pressure p. The resultant pressure $NkT = p_0 = N_0 kT_0$, would require an infinite static pressure p at $r = R_1$. Since at the centre of the mass, due to maximum compaction $N_{\max}$, the mean free path $L = 0$, temperature $T = 0$, root mean speed $C = 0$, static pressure $p = 0$, and mass density $\rho = 0$. At R_1 it becomes physically impossible to have p = infinite, especially in a gas.

The static pressure should vary smoothly for an observer entering into the mass from vacuum value p_0 at great distance from the body to a maximum value at the surface of the body, then decreasing to 0 as the observer approaches the mass centre. The variation of pressure from maximum to 0 inside the mass depends on the spherical distribution of mass density. In a gas this distribution cannot be other than smooth, thus producing a smooth variation through R_1, even for black holes. There are no singularities!

Discussion

Throughout most of space there are no solid surfaces in the cosmonic gas, except perhaps near the centres of black holes and high density concentrations. To define static pressure in the cosmonic gas, we must appeal to the definition of kinetic gas theory: static pressure $p = \rho C^2/3$ is pressure energy per unit volume, with C the root mean square of the random speed of cosmons. This is irrespective of encounters, the reason being that the root mean square of cosmon speed does not vary, whether there are encounters or not.

Static pressure is a scalar, since random speed is assumed equal in all directions. This argument rests on the fact that the energy of each particle in an element of volume—including heat energy $e = \rho C^2/2$—does not depend on volume, but only on temperature, *i.e.*, $p + e = NkT5/2$.

To determine cosmon parameters a zero point datum of the cosmonic gas is required. Due to particle-antiparticle charge symmetry, values at the centre of the electron-positron configuration can be taken as the zero point vacuum values for physical measurements. Vacuum properties that emerged from a study of electron structure in the cosmonic gas (Martin, 2005) are as follows: the cosmon mean free path is $L_0 = 2 \times 10^{-3}$ m; the cosmon diameter is $\sigma = 4.93 \times 10^{-25}$ m; the cosmon vacuum density is $N_0 = 4.4 \times 10^{50}$ m^{-3}, while $N_{\max} = 1.18 \times 10^{73}$ m^{-3}. As a result, cosmons rarely encounter other cosmons in the lighter fundamental particles with radius on the order of 10^{-15} m.

In electromagnetism, where the gradient of *total* pressure acts only on *electric charge density* (Martin 1994), whereas in gravitation the gradient of *gravitational* pressure (*static* pressure multiplied by the Clausius term) acts only on *material mass density* (of test particles).

In both gravitation and electrostatics, the potential is in the form $\int \rho 4\pi r^2 dr/r$. The potential around each individual source is obtained by integrating its mass and adding the potentials arithmetically for many sources. In certain cases the force field given by $-\text{grad}\phi$ for each source may be added to other fields vectorially to obtain the direction of the resultant force. The divergence of the force field is $\text{div}g = G4\pi\rho$ for gravitation, and $\text{div}E = 4\pi(\rho_E)$ for electrostatics, giving the gravitational and electrical densities respectively. It is noteworthy that electrostatics is fully described by replacing mass, pressure, gravitational potential, and gravitational force field with charge, total pressure, electrical potential, and electrical force field and setting the gravitational constant $G = 1$ in the same equations.

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Maxwell's Equations: New Light on Old Problems

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Maxwell's equations possess a certain generic structural property which is well-known, but rarely discussed. By considering this property as primary, we are able to derive the complete mathematical structure of Maxwell's equations described in terms of the orthogonality properties defined between certain spaces of linear operators. But, we find that the classical theory, whilst recovered intact here, is incomplete in the sense that the recovered Maxwell field is irreducibly associated with an additional massive vector field. In the overall context, this massive vector field can only be interpreted as a manifestation of a classical massive photon. One immediate consequence is that the Lorentz force law must be generalized and can be trivially made perfectly Newtonian once the massive vector field is accounted for.

1. Introduction

1.1. Historical Overview

Maxwell's equations, encapsulating as they do over one hundred years of observation and experimentation, arguably represent the ultimatne synthesis of the scientific age. For all engineers, and for some physicists, they are inevitably cast in Heavyside's vectorial form—the so-called Maxwell-Heavyside equations. The concept of *photon* plays no part in this theory—everything is arbitrated by the electromagnetic field in conjunction with the Lorentz force law.

During the first half of the 20th century, Maxwell's equations were given a new compact formulation—the canonical covariant formulation which expresses the electromagnetic field tensor in terms of the four-vector potential. At one level, this last step seemed to be no more than an advance of notation more suited to the requirements of theoretical physics than the Heavyside formulation. However, by the middle of the 20th century, with the development of quantum electrodynamics (qed), it became recognized that the electromagnetic field, when defined in terms of the four-vector potential, is the gauge field which must be introduced to guarantee invariance of a certain action under a local $U(1)$ gauge transformation—and along with this there came a corresponding gauge particle, the *massless photon*. In this way, the insights of mid-20th century theoretical physics were seen to validate and expand the insights of mid-19th century theoretical physics.

Notwithstanding the fact that qed requires photons to be massless and that there is no direct physical evidence that photons are anything other than massless, the idea of the massive photon is a persistent one that refuses to go away. At a fundamental level, for example, it is an implicit requirement of "pilot wave" interpretations of quantum mechanics and, as such, is primarily associated with the names of De Broglie [1,2], who originated the idea for single-particle systems and Bohm [3,4], who conceived it independently and subsequently extended it to multi-particle systems.

Of course, the photon idea plays no explicit role in either interpretation of non-relativistic quantum mechanics but, if the pilot wave interpretation is ever to receive a fully consistent qed-generalization (there has been much preliminary discussion: for example, see Bohm & Hiley [5], Holland [6], Bell [7] or Cushing *et al.* [8]) then the photon must be conceived as a massive extended particle. It then becomes problematic that there has hitherto been no *independent* theoretical imperative for introducing the idea of the massive photon—thus, if massive photons are required for the theory, then they must be "put in by hand", usually by constructing some variation of standard electromagnetic theory. A very recent example of this approach is provided by Vigier [9].

The present approach to the idea of the massive photon is distinguished from earlier approaches in the sense that, rather than modifying classical theory with *ad hoc* additions designed to give rise to some variety of the idea, we are able to show how an analysis based on certain generic properties of Maxwell's equations—rather than on the equations themselves—leads to the unavoidable conclusion that the classical Maxwell field is necessarily and irreducibly associated with a massive vector field (that is, that wherever the Maxwell field exists, then so does the massive vector field and *vice versa*). The irreducible nature of the association leads to the obvious identification of the massive vector field as the classical description of the massive photon.

1.2. Overview of present work

The considerations of this paper were not driven by any attempt to obtain an "improved" electrodynamics, nor to address any hypothetical shortcomings of the classical theory. They were driven, rather, by a spirit of curiosity concerning the general structure of Maxwell's equations: let us refer to this as *Property A*, defined below:

Property A: The equations of the canonical covariant Maxwell theory can be expressed as identities arising from the mutual orthogonality which exists between certain linear operator spaces. This is shown in detail in §2.1.

Subsequently, by accepting the central position of the Poincaré group within modern physics (*Property B* say), we set about the general problem of how to derive theories which possessed both *Property A & Property B*—the expectation was that, by definition, we must recover Maxwell's equations plus, perhaps, some other things. Maxwell's equations are indeed recovered intact, *but only in the context of being one-half of a bigger theory.* That is, the dual requirements of *Property A* plus *Property B* lead to the unavoidable conclusion that we *cannot* have the classical Maxwell field in isolation, but that it is irreducibly associated with an additional massive vector field.

1.3. Logical necessity and possible consequences

It is worth emphasizing the logical necessity of the foregoing:

- Maxwell's equations possess both *Property A* and *Property B;*
- A general search for theories possessing both *Property A* and *Property B* leads to the conclusion that the Maxwell field cannot exist in isolation—it is unavoidably and irreducibly associated with an additional massive vector field; where one is, the other is & *vice versa*.

Thus, the logical situation is that, if we accept the Maxwell field at all, then we must necessarily accept that an additional massive vector field is irreducibly associated with it.

It is this latter property which is of particular interest: specifically, whilst the "photon as particle" can never be recovered from a purely classical theory such as the one considered here, the irreducible association of a massive vector field with the Maxwell field is entirely new. It is also fascinating because it suggests the immediate possibility that the structure of the Lorentz force law might also need generalizing to account for the irreducible presence of this massive vector field. As we shall see in §10, this turns out to be the case and the generalized form can easily be structured so that it becomes fully Newtonian.

1.4. Notation note

We use the convention that $\left(x^1, x^2, x^3\right)$ represent the spatial axes and $x^4 \equiv ict$ represents the temporal one with a correspondingly consistent notation for the four-vector current, J_a and the electromagnetic field tensor, F_{ab}.

2. Identities in Canonical Electromagnetic Theory

2.1. Basic observations

When expressed in terms of the field tensor, the microscopic Maxwell's equations in the presence of charge are conventionally written

$$\frac{\partial F_{ai}}{\partial x^i} = \frac{4\pi}{c} J_a, \tag{1}$$

for a conserved current $\mathbf{J} \equiv (\mathbf{j}, ic\rho)$, together with

$$\frac{\partial F_{st}}{\partial x^r} + \frac{\partial F_{tr}}{\partial x^s} + \frac{\partial F_{rs}}{\partial x^t} = 0. \tag{2}$$

It is well known that, when the four-vector potential $\Phi \equiv (\phi_1, \phi_2, \phi_3, \phi_4)$ is introduced and F_{ab} defined according to

$$F_{ab} \equiv \frac{\partial \phi_b}{\partial x^a} - \frac{\partial \phi_a}{\partial x^b}, \tag{3}$$

then (2) becomes identically satisfied. However, because $\mathbf{J}$ in (1) is conserved then, from (1), we have

$$\frac{\partial F_{ai}}{\partial x^i} = \frac{4\pi}{c} J_a \iff \frac{\partial^2 F_{ij}}{\partial x^i \partial x^j} = 0 \tag{4}$$

so that these last two equations are mutually equivalent. But since the second of these equations is also an identity under the definition (3), then we can say that the covariant formulation of Maxwell's equations can be reduced to a pair of identities, (2) and the second of (4). The physics, of course, comes in when the conserved current, $\mathbf{J}$, is identified with the flow of charge.

2.2. Interpretation in terms of orthogonal operators

If we now write (3) as

$$F_{ab} \equiv \frac{\partial \phi_b}{\partial x^a} - \frac{\partial \phi_a}{\partial x^b} \equiv P^k_{ab}\, \phi_k,$$

where the $P^k_{ab}, k = 1..4$ are linear differential operators, then the identities (2) and (4) can be formally expressed as

$$\frac{\partial F_{st}}{\partial x^r} + \frac{\partial F_{tr}}{\partial x^s} + \frac{\partial F_{rs}}{\partial x^t} \equiv R^{ij}_{rst}\, P^k_{ij}\, \phi_k = 0,$$

$$\frac{\partial^2 F_{ij}}{\partial x^i \partial x^j} \equiv Q^{ij}\, P^k_{ij}\, \phi_k = 0 \tag{5}$$

respectively, for linear differential operators Q^{ab} and R^{ab}_{rst}. Since an entirely *arbitrary* definition of $(\phi_1, \phi_2, \phi_3, \phi_4)$ satisfies (5), it follows that $Q^{ij} P^k_{ij} \equiv 0$ and $R^{ij}_{rst} P^k_{ij} \equiv 0$; that is, the canonical covariant form of Maxwell's equations can be considered based on algebraic orthogonality properties between sets of linear differential operators.

In the following, we use this insight into the nature of Maxwell's equations to write down a Lagrangian formulation of the most general theory possible that is defined over a two-index field and which leads to equations which are essentially identities in the above sense for Maxwell's equations.

3. A Lagrangian Density

We argue, in Appendix A, that the Lagrangian density which leads to the required theory must have the general structure:

$$L=\alpha_0\frac{\partial\Psi_{ij}}{\partial x^k}\frac{\partial\Psi_{ji}}{\partial x^k}+\alpha_1\frac{\partial\Psi_{ij}}{\partial x^k}\frac{\partial\Psi_{ij}}{\partial x^k}$$
$$+\beta_0\frac{\partial\Psi_{ik}}{\partial x^i}\frac{\partial\Psi_{kj}}{\partial x^j}+\beta_1\left(\frac{\partial\Psi_{ik}}{\partial x^i}\frac{\partial\Psi_{jk}}{\partial x^j}+\frac{\partial\Psi_{ki}}{\partial x^i}\frac{\partial\Psi_{kj}}{\partial x^j}\right)$$
$$+\gamma_0\frac{\partial\Psi_{ik}}{\partial x^j}\frac{\partial\Psi_{kj}}{\partial x^i}+\gamma_1\left(\frac{\partial\Psi_{ik}}{\partial x^j}\frac{\partial\Psi_{jk}}{\partial x^i}+\frac{\partial\Psi_{ki}}{\partial x^j}\frac{\partial\Psi_{kj}}{\partial x^i}\right),$$

where $(\alpha_0,\alpha_1,\beta_0,\beta_1,\gamma_0,\gamma_1)$ are arbitrary constants. However, it turns out that this density contains a large amount of redundancy; specifically, all the independent structure is retained if only one of (α_0,α_1) is non-zero and only one of $(\beta_0,\beta_1,\gamma_0,\gamma_1)$ is non-zero. Consequently, the working density can be assumed to be

$$L=-\lambda\frac{\partial\Psi_{ij}}{\partial x^k}\frac{\partial\Psi_{ji}}{\partial x^k}+\frac{\partial\Psi_{ik}}{\partial x^i}\frac{\partial\Psi_{kj}}{\partial x^j},\tag{6}$$

for free parameter $-\lambda$; the minus has been used for later convenience. From this, the corresponding Euler-Lagrange equations can be found as

$$-\lambda^2\Psi_{ab}+\frac{\partial}{\partial x^i}\left[\frac{\partial\Psi_{ai}}{\partial x^b}+\frac{\partial\Psi_{ib}}{\partial x^a}\right]=0,\quad \Box\equiv\frac{\partial^2}{\partial x^i\partial x^i},\tag{7}$$

which is a system of partial differential equations for Ψ_{ab}. Since no assumptions have been made concerning the structure of Ψ_{ab}, $a,b=1..4$, it can be assumed to contain sixteen degrees of freedom; suppose we represent these degrees of freedom as sixteen sufficiently differentiable functions, $\alpha_k(\mathbf{x},ct)$, $k=1..16$ which are, as yet, undetermined and then write

$$\Psi_{ab}(\mathbf{x},ct)=\sum_{k=1}^{16}U_{ab}^k\,\alpha_k(\mathbf{x},ct),\tag{8}$$

where, by analogy with $F_{ab}\equiv P_{ab}^k\phi_k$ defined in the previous section and in anticipation of the final result, the two-index objects U_{ab}^k are to be treated as undetermined linear differential operators. Substitution of (8) into (7) gives

$$\sum_{k=1}^{16}\left\{-\lambda^2U_{ab}^k+\frac{\partial}{\partial x^i}\left[\frac{\partial U_{ai}^k}{\partial x^b}+\frac{\partial U_{ib}^k}{\partial x^a}\right]\right\}\alpha_k(\mathbf{x},ct)=0\tag{21}$$

where the object $\{...\}$ is to be considered as a linear differential operator acting on $\alpha_k(\mathbf{x},ct)$. We now consider ways of satisfying this equation *identically*: Since λ is an undetermined parameter and the U_{ab}^k are undetermined differential operators, then they can be chosen to satisfy the manifestly Poincaré-invariant relations

$$\frac{\partial}{\partial x^i}\left[\frac{\partial U_{ai}^k}{\partial x^b}+\frac{\partial U_{ib}^k}{\partial x^a}\right]=\lambda^2\,U_{ab}^k,\;\; k=1..16.\tag{22}$$

Now define the notation $X_a\equiv\partial/\partial x^a$, $a=1..4$, $\mathbf{U}^k\equiv(U_{11}^k,U_{12}^k,U_{13}^k,...,U_{44}^k)^T$ and sixteen symmetric matrices σ_{mn}, $m,n=1..4$, each of dimension 16×16, according to (50) in Appendix B; then (22) can be written as

$$\sigma_{ij} X_i X_j \mathbf{U}^k = \lambda^2 \mathbf{U}^k, \; k = 1..16 \tag{11}$$

where summation is assumed over i and j. Noting that the eigenvalues of $\sigma_{ij} X_i X_j$ (treated as an algebraic matrix) must be simple multiples of the d'Alembertian, $\Box \equiv X_i X_i$, then (11) is seen to have the formal structure of an algebraic eigenvalue problem. Consequently, non-trivial solutions for $\mathbf{U}^k$ can only exist when λ^2 is an eigenvalue of $\sigma_{ij} X_i X_j$; in this case, $\mathbf{U}^k$ is the corresponding eigenvector and must have the structure of a column of differential operators.

Finally, we note how the symmetry of the matrices, σ_{ij} means that, if $\mathbf{U}^m$ and $\mathbf{U}^n$ are eigenvectors corresponding to distinct eigenvalues, then $U^m_{ij} U^n_{ij} = 0$, where summation over *i* and *j* is implied. It is these orthogonality relations which give rise, amongst other things, to the classical equations of electrodynamics.

4. The Eigensystem

Using (22), the eigensystem (11) can be written as

$$X_a X_i U^k_{ib} + X_b X_i U^k_{ai} = \lambda^2 U^k_{ab}, \quad k = 1...16,$$

and for which we find only five distinct eigenvalues corresponding to $\lambda = 2$, 1, 1, 0 and 0 respectively. The corresponding eigenspaces, which have dimensions one, three, three, three and six respectively, are denoted as $(R)_{sy,1}$, $(R)_{sk,3}$, $(R)_{sy,3}$, $(G)_{sk,3}$ and $(G)_{sy,6}$. We shall show that:

- classical electromagnetism arises from $(R)_{sk,3}$;
- the electromagnetic dual arises from $(G)_{sk,3}$;
- the equations of electromagnetism arise from the orthogonalities $(R)_{sy,1} \perp (R)_{sk,3}$ and $(G)_{sk,3} \perp (R)_{sk,3}$;
- $(R)_{sy,3}$ gives rise to a non-zero mass vector field which is irreducibly associated with the electromagnetic field. We shall argue that this vector field can only be sensibly interpreted as a classical representation of the massive photon.

Only the eigenspace $(G)_{sy,6}$ plays no obvious role in the present discussion. The eigenspaces are described as follows:

4.1. Eigenspace $(R)_{sy,1}$, $\lambda_1 = 2$

$(R)_{sy,1}$ is a one-dimensional subspace of eigenvectors associated with the eigenvalue $\lambda_1 = 2$ and the subspace is defined by the single operator

$$U^1_{ab} = X_a X_b \tag{12}$$

which is *symmetric* with respect to the indices (a,b). Consequently, when $\lambda = 2$ in (7), the solution (8) becomes

$$\Psi_{ab} = U^1_{ab}\, \alpha_1(\mathbf{x}, ct),$$

so that Ψ_{ab} is defined over a scalar field.

4.2. Eigenspace $(R)_{sk,3}$, $\lambda_2 = 1$

$(R)_{sk,3}$ is a three-dimensional subspace of eigenvectors associated with the eigenvalue $\lambda_2 = 1$ and a basis for the subspace is given by

$$U^k_{ab} = \left(X_a \delta_{rb} - X_b \delta_{ra}\right), \quad k = 2,3,4 \tag{13}$$

where, for $k = (2,3,4)$ then r takes any three distinct values from the set $(1,2,3,4)$; for example, $r = (1,2,3)$; these eigenvectors are *skew-symmetric* with respect to the indices (a,b). Consequently, when $\lambda = 1$ in (7), the solution (8) corresponding to $(R)_{sk,3}$ becomes

$$\Psi_{ab} = \sum_{k=2}^{4} U^k_{ab}\, \alpha_k(\mathbf{x}, ct)$$

so that Ψ_{ab} is defined over a vector field.

4.3. Eigenspace $(R)_{sy,3}$, $\lambda_3 = 1$

$(R)_{sy,3}$ is a three-dimensional subspace of eigenvectors associated with the eigenvalue $\lambda_3 = 1$ and a basis for the subspace is given by

$$U^k_{ab} = X_a \left(X_r \delta_{sb} - X_s \delta_{rb}\right) + X_b \left(X_r \delta_{sa} - X_s \delta_{ra}\right), \quad k = 5,6,7 \tag{14}$$

where for $k = (5,6,7)$, then (r,s) is three distinct pairs chosen from $(1,2,3,4)$. The basis is most conveniently chosen by picking any one of the four digits and pairing it with the remaining three: for example, $(r,s) = (1,4),(2,4),(3,4)$. These eigenvectors are *symmetric* with respect to the indices (a,b). Consequently, when $\lambda = 1$ in (7), the solution (8) corresponding to $(R)_{sy,3}$ becomes

$$\Psi_{ab} = \sum_{k=5}^{7} U^k_{ab}\, \alpha_k(\mathbf{x}, ct)$$

so that Ψ_{ab} is defined over a vector field.

4.4. Eigenspace $(G)_{sk,3}$, $\lambda_4 = 0$

$(G)_{sk,3}$ is a three-dimensional subspace of eigenvectors associated with the eigenvalue $\lambda_4 = 0$ and a basis for the subspace is given by

$$U^k_{ab} = \frac{X_r X_s X_t}{X_a X_b}\left(\left(\delta_{ra} - \delta_{sa}\right)\left(\delta_{sb} - \delta_{tb}\right) - \left(\delta_{rb} - \delta_{sb}\right)\left(\delta_{sa} - \delta_{ta}\right)\right); \quad k = 8,9,10 \tag{15}$$

where typically, for $k = (8,9,10)$ then $(r,s,t) = (2,3,4),(1,3,4),\ (1,2,4)$; these eigenvectors are *skew-symmetric* with respect to the indices (a,b). Consequently, when $\lambda = 0$ in (7), the solution (8) corresponding to $(G)_{sk,3}$ becomes

$$\Psi_{ab} = \sum_{k=8}^{10} U^k_{ab}\, \alpha_k(\mathbf{x}, ct)$$

so that Ψ_{ab} is defined over a vector field.

4.5. Eigenspace $(G)_{sy,6}$, $\lambda_5 = 0$

$(G)_{sy,6}$ is a six-dimensional subspace of eigenvectors associated with the eigenvalue $\lambda_5 = 0$ and a basis for the subspace is given by

$$U^k_{ab} = \left(X_r\delta_{sa} - X_s\delta_{ra}\right)\left(X_r\delta_{sb} - X_s\delta_{rb}\right); \quad k = 11...16 \tag{16}$$

where, typically, for $k = 11...16$ then $(r,s) = (1,2),(1,3)(1,4),(2,3), \ (2,4),(3,4)$ and δ_{ab} is the 4×4 unit matrix; these eigenvectors are *symmetric* with respect to the indices (a,b). Consequently, when $\lambda = 0$ in (7), the solution (8) corresponding to $(G)_{sy,6}$ becomes

$$\Psi_{ab} = \sum_{k=11}^{16} U^k_{ab}\,\alpha_k(\mathbf{x}, ct).$$

As we have already noted, and as we shall see in the following, $(G)_{sy,6}$ is the only solution of (22) which does not play any obvious part in the electromagnetic theory being discussed here.

5. The Electromagnetic Field From $(R)_{sk,3}$

5.1. The Orthogonality Relationships

In this section we show that Maxwell's equations arise naturally as a direct consequence of the orthogonality relations

$$U^E_{ij}U^D_{ij} \equiv 0, \ \mathbf{U}^E \in (R)_{sy,1}, \mathbf{U}^D \in (R)_{sk,3} \tag{17}$$

$$U^B_{ij}U^D_{ij} \equiv 0, \ \mathbf{U}^B \in (G)_{sk,3}, \mathbf{U}^D \in (R)_{sk,3} \tag{18}$$

given at (39) in Appendix B and where $(R)_{sy,1}$, $(G)_{sk,3}$ and $(R)_{sk,3}$ are defined at (12), (15) and (13) respectively.

The most general tensor which can be formed from $(R)_{sk,3}$ is given by

$$F_{ab} = \sum_{k=2}^{4} U^k_{ab}\,\alpha_k(\mathbf{x}, ct), \tag{19}$$

$$U^k_{ab} \equiv (X_a\delta_{rb} - X_b\delta_{ra}); \ k = 2,3,4$$

where r takes any three distinct values from $(1,2,3,4)$ and where, because of the skew-symmetry of U^k_{ab}, then F_{ab} is also skew-symmetric. Since $(R)_{sy,1}$ consists of the single operator X_aX_b, then (17) with (19) implies

$$X_iX_jF_{ij} \equiv \frac{\partial^2 F_{ij}}{\partial x^i \partial x^j} = 0, \tag{20}$$

from which it immediately follows

$$\frac{\partial F_{ai}}{\partial x^i} = J_a, \ \text{where} \ \frac{\partial J_i}{\partial x^i} = 0 \tag{21}$$

for some conserved current $\mathbf{J}$. Similarly, denoting the elements of $(G)_{sk,3}$ by Δ^{ab}_{rst}, the relation (6) together with (7) gives directly

$$\frac{1}{2}\Delta^{ij}_{rst}F_{ij} \equiv \frac{\partial F_{st}}{\partial x^r} + \frac{\partial F_{tr}}{\partial x^s} + \frac{\partial F_{rs}}{\partial x^t} = 0. \tag{22}$$

If $\mathbf{J}$ in (21) is interpreted as the 4-current density, then (21) and (22) are Maxwell's equations for the *electromagnetic field tensor*, F_{ab}. That is, Maxwell's equations are seen to arise as a direct consequence of the orthogonality between the invariant subspaces $(R)_{sk,3}$, $(R)_{sy,1}$ and $(G)_{sk,3}$. Consequently, in the form of (8) and (22), they impose *no* constraints (beyond differentiability) on the three-vector $\mathbf{A} \equiv (\alpha_{11}, \alpha_{12}, \alpha_{13})$ —this vector field can have arbitrary structure. Since (21) is in a one-to-one relationship with the identity (20) it must also be an identity and *not* a field equation as it is commonly interpreted. The practical use of (21), of course, arises from the identification of **J** with a measurable quantity—the four-vector current—which then allows the computations of the fields.

6. Recovery of the canonical four-vector formalism

Although one of (X_1, X_2, X_3, X_4) refers to the temporal axis and three refer to the spatial axes, specific associations of the indices with particular axes have not yet been made. In the following, we show how a simple transformation of $\mathbf{A} \equiv (\alpha_2, \alpha_3, \alpha_4)$:-

- allows the identification of the temporal axis;
- identifies $\mathbf{A} \equiv (\alpha_2, \alpha_3, \alpha_4)$ as the classical magnetic vector potential;
- reduces the presented formalism directly to the canonical formalism.

The expression (7) for the field tensor, F_{ab}, is unconventional insofar as it derives directly from a manifestly covariant treatment but is not expressed in terms of the usual four-vector potential; instead, it is expressed in terms of an uninterpreted three-vector $\mathbf{A} \equiv (\alpha_2, \alpha_3, \alpha_4)$. In the following, by showing how F_{ab}, defined at (19), can be transformed into the conventional four-vector formalism, we are able to identify $\mathbf{A}$ with the magnetic vector potential of the classical theory whilst, at the same time, identifying the temporal axis. We begin by noting, from (19), that

$$U^k_{ab} \equiv \left(X_a \delta_{rb} - X_b \delta_{ra}\right), k = 2,3,4$$

where r takes any three values from $(1,2,3,4)$. It is quite obvious that the action of picking three from four here has the effect of making the omitted integer special in some sense which is not yet immediately clear. It transpires that this process effectively associates the omitted index with the temporal axis and the three chosen indices with the spatial axes. For convenience, we begin by defining the object P^r_{ab} according to

$$P^r_{ab} \equiv \left(X_a \delta_{rb} - X_b \delta_{ra}\right), r = 1..4, \tag{23}$$

and similarly define the notation that any operand of P^r_{ab} is denoted by A_r, $r = 1..4$. In terms of this notation and after choosing (l,m,n) as any three from $(1,2,3,4)$, then (19) becomes

$$F_{ab} = P^l_{ab}\, A_l + P^m_{ab}\, A_m + P^n_{ab}\, A_n. \tag{24}$$

Additionally, from (23), we readily obtain the identity

$$P^r_{ab}\, X_r \equiv \left(X_a \delta_{rb} - X_b \delta_{ra}\right) X_r \equiv (X_a X_b - X_b X_a)$$

so that, under the assumption that the order of differentiation never matters, we can write the identity

$$P^1_{ab}\,X_1 + P^2_{ab}\,X_2 + P^3_{ab}\,X_3 + P^4_{ab}\,X_4 \equiv 0. \tag{25}$$

and, using this, define the three-vector $\mathbf{A}' \equiv (A'_l, A'_m, A'_n)$ according to

$$A'_r = A_r + \frac{X_r}{X_q} A'_q,\ r = l, m, n \tag{26}$$

where, since $X_q \equiv \partial/\partial x^q$, then $1/X_q$ indicates integration with respect to x^q. Equation (24) can now be rewritten as

$$F_{ab} = P^l_{ab}\left(A'_l - \frac{X_l}{X_q} A'_q\right) + P^m_{ab}\left(A'_m - \frac{X_m}{X_q} A'_q\right) + P^n_{ab}\left(A'_n - \frac{X_n}{X_q} A'_q\right).$$

Using the identity (25) then this last equation can be written as

$$\begin{aligned} F_{ab} &= P^l_{ab}\,A'_l + P^m_{ab}\,A'_m + P^n_{ab}\,A'_n + P^q_{ab}\,A'_q \\ &= P^1_{ab}\,A'_1 + P^2_{ab}\,A'_2 + P^3_{ab}\,A'_3 + P^4_{ab}\,A'_4 \end{aligned} \tag{27}$$

which, with (23), is easily shown to reduce to

$$F_{ab} \equiv X_a\,A'_b - X_b\,A'_a.$$

This latter expression is simply the standard form of F_{ab} in terms of the four-vector potential (A'_1, A'_2, A'_3, A'_4). It is now obvious that $\mathbf{A} \equiv (A_l, A_m, A_n)$, used at (1) to define $\mathbf{A}'$, is simply the magnetic vector potential of the classical theory and that the arbitrary scalar, A'_q, introduced at (26) is just the scalar potential usually associated with the electric field. This, in turn, implies that the index q is necessarily associated with the temporal axis and the indices (l, m, n) are necessarily associated with the spatial axes.

To summarize, the new formalism based upon $(R)_{sk,3}$ is expressed *entirely* in terms of the classical magnetic vector potential and the canonical formalism is recovered when an arbitrary scalar field is introduced via a certain linear transformation. We discuss the implications of this circumstance in the concluding section.

6.1. The explicit representation of F_{ab} in terms of $\mathbf{E}$ and $\mathbf{B}$.

In general, we have, from (19),

$$F_{ab} = \sum_{k=2}^{4} U^k_{ab}\,\alpha_k(\mathbf{x}, ct),$$

$$U^k_{ab} \equiv (X_a \delta_{rb} - X_b \delta_{ra}),\ k = 2, 3, 4,$$

where r is chosen as any three of $(1, 2, 3, 4)$. For convenience, we choose the basis $r \equiv (l, m, n) = (1, 2, 3)$ as $k = (2, 3, 4)$ so that, by the considerations of §6, the indices $r = (1, 2, 3)$ are associated with the spatial axes, the index $r = 4$ is associated with the temporal axis and $\mathbf{A} \equiv (\alpha_2, \alpha_3, \alpha_4)$ is identified as the magnetic vector potential. For the magnetic and electric fields, using the standard notation $\mathbf{B} \equiv (F_{23}, F_{31}, F_{12})$ and $-i\mathbf{E} \equiv (F_{14}, F_{24}, F_{34})$, we find

$$\mathbf{B} = (X_2 A_3 - X_3 A_2, X_3 A_1 - X_1 A_3, X_1 A_2 - X_2 A_1),$$

$$-i\mathbf{E} = (-X_4 A_1, -X_4 A_2, -X_4 A_3).$$

Thus, we see how, whilst the magnetic field takes its standard form in terms of the magnetic vector potential, the form of the electric field in the covariant $(R)_{sk,3}$ formalism differs from the conventional structure in the absence of the scalar potential component.

7. The Dual Field of Electrodynamics From $(G)_{sk,3}$

The most general tensor generated by $(G)_{sk,3}$ is given by

$$G_{ab} = \sum_{8}^{10} U_{ab}^{k}\, \alpha_k(\mathbf{x}, ct) \tag{28}$$

$$U_{ab}^{k} = \frac{X_r X_s X_t}{X_a X_b}\left(\left(\delta_{ra} - \delta_{sa}\right)\left(\delta_{sb} - \delta_{tb}\right) - \left(\delta_{rb} - \delta_{sb}\right)\left(\delta_{sa} - \delta_{ta}\right)\right); \quad k = 8, 9, 10$$

where, typically, for $k = (8, 9, 10)$ then $(r, s, t) = (2,3,4), (1,3,4), (1,2,4)$. If, for the sake of convenience, we define $(\alpha_8, \alpha_9, \alpha_{10}) \equiv (A_1, A_2, A_3)$, and use the given basis for (r, s, t), then it is easily found that

$$G_{ab} = \begin{pmatrix} 0 & -X_4 A_3 & X_4 A_2 & X_2 A_3 - X_3 A_2 \\ X_4 A_3 & 0 & -X_4 A_1 & X_3 A_1 - X_1 A_3 \\ -X_4 A_2 & X_4 A_1 & 0 & X_1 A_2 - X_2 A_1 \\ X_3 A_2 - X_2 A_3 & X_1 A_3 - X_1 A_2 & X_2 A_1 - X_1 A_2 & 0 \end{pmatrix}$$

A consideration of this skew-symmetric object soon shows that it is no more than a re-ordering of the terms of the electromagnetic field tensor—which suggests an electrodynamic interpretation of G_{ab}. In fact, it is easily shown that $G_{ab} = \varepsilon_{abmn} F_{mn}$ where ε_{abmn} is the Levi-Civita permutation tensor. Thus, G_{ab} is the dual of F_{ab}.

8. Wave Structures Supported by the Magnetic Vector Potential

It is shown that, according to the generalized electrodynamics, wavy solutions for the magnetic vector potential are composed of two distinct kinds of wave: the first kind is a propagating transverse wave, whilst the second kind, which is novel, is a stationary longitudinal wave. It is shown that the propagating transverse component corresponds identically to those solutions which arise in the conventional formalism when the Coulomb gauge is used. The stationary longitudinal component has no counterpart in the conventional formalism.

Using the notation $\mathbf{A} \equiv (A_1, A_2, A_3) \equiv (\alpha_2, \alpha_3, \alpha_4)$ in (19) and the basis $r = (1,2,3)$, then (21) can be written as

$$\sum_{r=1}^{3} X_i \left(X_a \delta_{ri} - X_i \delta_{ra}\right) A_r = J^a$$

which—upon remembering $X_a \equiv \partial / \partial x^a$ —can be written as the two equations

$$^2\mathbf{A} - \nabla\left(\nabla \cdot \mathbf{A}\right) = -\mathbf{J} \tag{29}$$

$$\frac{\partial}{\partial x^4}\left(\nabla \cdot \mathbf{A}\right) = J_4.$$

Given **J** and hence **A** via (4), the second of these equations provides a *definition* of J^4. Consider now, a wave given by

$$\mathbf{A}_{wave} = \mathbf{A}_0 \exp(i\mathbf{n}\cdot\mathbf{x}),$$

where $\mathbf{n} \equiv (n_1, n_2, n_3, n_4)$, $\mathbf{x} \equiv (x^1, x^2, x^3, x^4)$ and $\mathbf{A}_0$ is a constant three-vector. The requirement that $\mathbf{A}_{wave}$ satisfies (4) with $\mathbf{J} = 0$ leads to the system of equations

$$(\mathbf{n}\cdot\mathbf{n})\mathbf{A}_0 = (\hat{\mathbf{n}}\cdot\mathbf{A}_0)\hat{\mathbf{n}} \tag{30}$$

where $\hat{\mathbf{n}} \equiv (n_1, n_2, n_3)$ and, from this, we can form the scalar equation

$$(\mathbf{n}\cdot\mathbf{n})(\hat{\mathbf{n}}\cdot\mathbf{A}_0) = (\hat{\mathbf{n}}\cdot\mathbf{A}_0)(\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}). \tag{31}$$

This latter equation has two possible solutions which, together, form a basis for the homogeneous solutions of (29):

Case 1: The Transverse Wave: $\hat{\mathbf{n}}\cdot\mathbf{A}_0 = 0$

In this case, (30) only has a non-trivial solution if $\mathbf{n}\cdot\mathbf{n} = 0$. Consequently, this solution is given by

$$\mathbf{A}_T = \mathbf{A}_0 \exp(i\mathbf{n}\cdot\mathbf{x}),\ \ \mathbf{n}\cdot\mathbf{n} = 0,\ \ \hat{\mathbf{n}}\cdot\mathbf{A}_0 = 0, \tag{32}$$

which corresponds to a *transverse* wave propagating with speed c. Since, as noted in the previous section, there is no electric scalar potential in $(R)_{sk,3}$ and since $\hat{\mathbf{n}}\cdot\mathbf{A}_0 = 0 \rightarrow \nabla\cdot\mathbf{A}_T = 0$, then this component of the general solution of (4) corresponds *exactly* to those solutions which arise from the conventional formalism when the Coulomb gauge is chosen.

Case 2: The Longitudinal Wave: $\hat{\mathbf{n}}\cdot\mathbf{A}_0 \neq 0$

In this case, (31) gives $\mathbf{n}\cdot\mathbf{n} = \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}$ and this can only be true if $n_4 = 0$. From (30) we now get the equation

$$(\hat{\mathbf{n}}\cdot\hat{\mathbf{n}})\mathbf{A}_0 = (\hat{\mathbf{n}}\cdot\mathbf{A}_0)\hat{\mathbf{n}}$$

which is easily seen to have the solution $\mathbf{A}_0 = \alpha\hat{\mathbf{n}}$ for arbitrary α. To summarize, this solution is given by

$$\mathbf{A}_L = \alpha\hat{\mathbf{n}} \exp(i\hat{\mathbf{n}}\cdot\hat{\mathbf{x}}) \tag{33}$$

where $\hat{\mathbf{x}} = (x^1, x^2, x^3)$ and this corresponds to a *longitudinal* stationary wave. This wave is easily shown to give $\mathbf{E} = \mathbf{B} = 0$, so that a non-trivial magnetic vector potential can be associated with a zero electromagnetic field.

To summarize, we arrive at the conclusion that the magnetic vector potential supports two kinds of waves in free space: a propagating transverse wave (which corresponds exactly to the Coulomb gauge solutions of the conventional formalism) and a stationary longitudinal wave (which has no counterpart in the conventional formalism) so that the general wavy solution to the homogeneous form of (29) is given by

$$\mathbf{A}_{wave} = \mathbf{A}_T(\mathbf{x}, ct) + \mathbf{A}_L(\mathbf{x}),$$

where $\mathbf{A}_T$ is the transverse wave propagating with speed c and $\mathbf{A}_L$ is the stationary longitudinal wave. The component $\mathbf{A}_T$ gives rise to propagating transverse electromagnetic fields and the general phenomenology that, when such a field is created, any charged particle anywhere will eventually feel its effect. The component $\mathbf{A}_L$ gives rise to a *zero* electromagnetic field ($\mathbf{E} = \mathbf{B} = 0$) and so no electromagnetic effect at all is propagated; since the stationary wave cannot pass through an arbitrarily placed charged particle then the only way an effect can be observed is that the charged particle must pass through the stationary wave.

8.1. A Material Vacuum?

It has been shown how, according to the generalized electrodynamics, the magnetic vector potential supports a stationary longitudinal wave, as well as the conventional propagating transverse wave. The classical theory has managed to assimilate the idea of an electrodynamic wave propagating in the absence of any supporting medium—if only because there is, at least, a sense of *something* travelling and things can be conceived as travelling through empty space. However, the idea of stationary longitudinal waves in the absence of any kind of supporting medium presents an entirely new level of incomprehensibility since, now, *nothing* is travelling anywhere. In Appendix D, a possible solution to this perceived problem is obtained by showing how (29) supports a non-wavy $\mathbf{J} = 0$ solution which has a ready interpretation as a classical description of a fluctuating material vacuum. The magnetic vector potential waves discussed in §8 can then interpreted as *disturbances* of this material vacuum.

9. A Non-Zero Mass Photon From $(R)_{sy,3}$

In this section, we show that $(R)_{sy,3}$ implies the existence of a massive vector field constructed from the elements of $(R)_{sk,3}$, the electromagnetic field. By showing that this vector field is *irreducibly* associated with the electromagnetic field, we are led to the obvious interpretation that this massive vector field is a classical representation of the massive photon.

9.1. The Massive Vector Field

The basis for $(R)_{sy,3}$ is given at (14) and the most general field which can be formed from the operators lying in this subspace is given by

$$V_{ab} = \sum_{k=5}^{7} U_{ab}^{k}\, \alpha_k(\mathbf{x}). \tag{34}$$

If we define

$$P_{rs}^{t} = X_r \delta_{st} - X_s \delta_{rt} \tag{35}$$

then the basis for $(R)_{sy,3}$, given above, can be written as

$$U_{ab}^{k} = (X_a P_{rs}^{b} + X_b P_{rs}^{a}); \quad k = 5,6,7, \tag{36}$$

where for $k=(5,6,7)$, then (r,s) is chosen as in §4.3. With this notation and defining

$$V_a = \sum_{k=5}^{7} P_{rs}^{a}\,\alpha_k(\mathbf{x},ct), \quad V_b = \sum_{k=5}^{7} P_{rs}^{b}\,\alpha_k(\mathbf{x},ct), \tag{37}$$

then (21) can be expressed as

$$V_{ab} = \left(X_a V_b + X_b V_a\right). \tag{38}$$

Since the single element of $(R)_{sy,1}$ is orthogonal to every element of $(R)_{sy,3}$ and since V^{ab} is the most general field which can be formed by the operators lying within this latter subspace acting over a vector field, then operating $(R)_{sy,1}$ onto (34) gives immediately

$$X_i X_j V_{ij} \equiv \frac{\partial^2 V_{ij}}{\partial x^i \partial x^j} = 0,$$

from which it immediately follows

$$\frac{\partial V_{aj}}{\partial x^j} = J_a \quad \text{where} \quad \frac{\partial J_i}{\partial x^i} = 0,$$

for some unspecified current $\mathbf{J}$. Using (38) this latter equation can be expressed as

$$\frac{\partial}{\partial x^j}\left(\frac{\partial V_j}{\partial x^a} + \frac{\partial V_a}{\partial x^j}\right) = J_a. \tag{39}$$

However, since, from (35), $X_i P_{rs}^{i} \equiv 0$ then, from the definition of V_a at (37) and the definition of P_{rs}^{t} at (35), we can easily see that

$$X_j V_j \equiv \frac{\partial V_j}{\partial x^j} = 0, \tag{40}$$

so that (39) becomes

$$\Box^2 V_a = J_a. \tag{41}$$

However, since $\partial J_i/\partial x^i = 0$ and $\partial V_i/\partial x^i = 0$, we can write $J_a = mV_a + J_a^0$ for some constant m and conserved current J_a^0; finally, therefore, (41) can be written as

$$\Box^2 V_a = mV_a + J_a^0. \tag{42}$$

This latter equation implies that $(R)_{sy,3}$ is associated with a massive vector field.

9.2. Does the vector field represent a classical photon?

The first thing to notice, as reference to (23) shows, is that the operator P_{rs}^{t} defined at (35) and in terms of which the vector field is defined at (37), is the fundamental operator of $(R)_{sk,3}$—the operator space which acts over the magnetic vector potential to generate the electromagnetic field. Specifically, for the electromagnetic field tensor, we have

$$F_{ab} = \sum_{k=2}^{4} P_{ab}^{r}\,\alpha_k(\mathbf{x},ct) \tag{43}$$

where r varies with k whilst, for the massive vector field, we have

$$V_a = \sum_{k=5}^{7} P^a_{rs}\,\alpha_k(\mathbf{x},ct) \tag{44}$$

where (r,s) varies with k as in §4.3. Now form the inner product of V_a with the four components $A'_i, i=1..4$ defined at (26), to obtain

$$V_i A'_i = \sum_{k=5}^{7} P^i_{rs} A'_i \alpha_k(\mathbf{x},ct).$$

But, by (27), $F_{rs} = P^i_{rs} A'_i$ and so this latter equation becomes

$$V_i A'_i = \sum_{k=5}^{7} F_{rs}\,\alpha_k(\mathbf{x},ct). \tag{45}$$

First choice of basis: If we fix the basis of (36) by choosing $(r,s)=(1,4),(2,4),(3,4)$ and use the notation $\mathbf{a}\equiv(\alpha_5,\alpha_6,\alpha_7)$ and $(F_{14},F_{24},F_{34})=-i(E_1,E_2,E_3)$ then (7) gives

$$V_i A'_i = -i\mathbf{E}\cdot\mathbf{a}. \tag{46}$$

But, from (44), any constant $\mathbf{a}\equiv(\alpha_5,\alpha_6,\alpha_7)\neq 0$ implies $V_a=0$. Thus, either $\mathbf{E}=0$ or $\mathbf{E}$ is orthogonal to $\mathbf{a}$ in this particular case. However, since $\mathbf{a}$ and $\mathbf{E}$ are independent (*cf.* eqns (43) & (44)) then $\mathbf{a}$ can be chosen to have arbitrary orientation relative to $\mathbf{E}$ so that the possibility $\mathbf{E}\perp\mathbf{a}$ is excluded. Consequently, $V_a=0$ implies $\mathbf{E}=0$.

Second choice of basis: By contrast, if we fix the basis of (36) by choosing $(r,s)=(2,3),(3,1),(1,2)$ and use $(F_{23},F_{31},F_{12})\equiv(B_1,B_2,B_3)$ then, by similar arguments, $V_a=0$ implies $\mathbf{B}=0$ also.

To summarize: since $V_a=0$ implies $\mathbf{E}=0$ *and* $\mathbf{B}=0$, then the absence of the massive vector field implies the absence of the electromagnetic field. Consequently, the massive vector field is *always* present in the presence of the electromagnetic field. The only possible conclusion is that the massive vector field must necessarily be identified with a classical non-zero mass photon.

9.3. Constraints for the massive photon

The object

$$V_a = \sum_{k=5}^{7} P^a_{rs}\,\alpha_k(\mathbf{x},ct) \tag{47}$$

has been identified as a non-zero mass photon from which it is clear that it has only three degrees of freedom expressed in terms of the three functions $\mathbf{a}\equiv(\alpha_5,\alpha_6,\alpha_7)$. In the following, we show that each of the components of $\mathbf{a}$ must satisfy the Klein-Gordon equation.

With the basis $(r,s)=(1,4),(2,4),(3,4)$, then (21) gives

$$V_a = P^a_{14}\,\alpha_{14} + P^a_{24}\,\alpha_{15} + P^a_{34}\,\alpha_{16}$$

which, after expanding the operators P^a_{rs} gives

$$(V_1,V_2,V_3,V_4) = \left[-X_4(\alpha_{14},\alpha_{15},\alpha_{16}), \nabla\cdot(\alpha_{14},\alpha_{15},\alpha_{16})\right]$$

$$\downarrow$$

$$(\mathbf{V},V_4)=\left[-X_4\mathbf{a},\nabla\cdot\mathbf{a}\right]. \qquad (48)$$

Consequently, we find

$$\Box^2(\mathbf{V},V_4)=(-X_4^2\mathbf{a},\nabla\cdot^2\mathbf{a}). \qquad (49)$$

We now consider how $\mathbf{a}$ must constrained to ensure (49) assumes the form of (42). A consideration of (11) shows that the most simple possibility is given by the condition $\Box^2\mathbf{a}=m\mathbf{a}$ for some parameter m since then, use of (48) reduces (49) to

$$\Box^2(\mathbf{V},V_4)=m(\mathbf{V},V_4)$$

which is (4) without the conserved current. That is, the three functions, $\alpha_{14},\alpha_{15},\alpha_{16}$, which define the massive photon field V_a must each satisfy the Klein-Gordon equation.

10. Massive Photons and Mechanical Reaction

The eigenvalue $\lambda=1$ is associated with two distinct three-dimensional subspaces of eigenvectors, $R_{sk,3}$ and $R_{sy,3}$ of which the first has been identified with the electromagnetic field and the second with a field of classical massive mass photons. The general solutions associated with each of these subspaces are given by

$$F_{ab}=\sum_{k=2}^{4}U_{ab}^{k}\,\alpha_k(\mathbf{x},ct),$$

$$G_{ab}=\sum_{k=5}^{7}U_{ab}^{k}\,\alpha_k(\mathbf{x},ct),$$

respectively. However, since they are *both* associated with $\lambda=1$, then the *most* general solution associated with this particular eigenvalue is

$$\Psi_{ab}=F_{ab}+G_{ab}.$$

Now, according to the Lorentz force-law, the four-force generated by an electromagnetic field on a charged particle, e, with four-velocity $\mathbf{V}$ is given by $F_a=eV_iF_{ai}/c$. Thus, if the idea of the Lorentz force is to be generalized, then the most general solution associated with $\lambda=1$, $\Psi_{ab}\equiv F_{ab}+G_{ab}$ must give rise to a total system force of

$$F_a=\frac{e}{c}V_i\,F_{ai}+\frac{e}{c}V_i\,G_{ai}.$$

The natural question now is *what does* eV_iG_{ai}/c *represent?* It is well known that, according to the Lorentz force law of classical electrodynamics, the net electromagnetic forces generated by two charged particles on each other are not equal and opposite—that is, even in the case of non-relativistic motions, the classical electrodynamic description of a mutually interacting charged particle-pair does not satisfy Newtonian conservation principles. Consequently, *dynamical reactions*, and the freedom to include them, are missing from classical electrodynamics. Since we have already identified G_{ab} with an irreducible field of massive photons and since we know that an accelerated charged particle radiates electromagnetically, then the obvious interpretation is that the massive photons of the theory are these radiated photons and that $eV_i G_{ai}/c$ represents the reaction force associated with these accelerated photons; that is, it describes the *reaction* on the particle of charge e of its own action on the source of the field F_{ab}.

Thus, suppose that a non-relativistic system consists of just two charged particles, e_1 and e_2 with respective four-velocities $V_a^{(1)}$ and $V_a^{(2)}$, and that each particle generates electromagnetic fields $F_{ab}^{(1)}$ and $F_{ab}^{(2)}$ respectively, and generates a reaction to the action on itself through the reaction fields $G_{ab}^{(2)}$ and $G_{ab}^{(1)}$ respectively. Then, the respective forces acting in the vicinity of each particle are:

$$F_a^{(1)} = \frac{e_1}{c} V_i^{(1)} F_{ai}^{(2)} + \frac{e_1}{c} V_i^{(1)} G_{ai}^{(2)},$$

$$F_a^{(2)} = \frac{e_2}{c} V_i^{(2)} F_{ai}^{(1)} + \frac{e_2}{c} V_i^{(2)} G_{ai}^{(1)}.$$

If action and reaction are to be equal and opposite in this non-relativistic system, then we must have

$$F_a^{(1)} + F_a^{(2)} = 0$$

which, given the fields $F_{ab}^{(1)}$ and $F_{ab}^{(2)}$, represent a constraint on the reaction fields $G_{ab}^{(1)}$ and $G_{ab}^{(2)}$.

11. Conclusions

11.1. General comments

The work of this paper began by noting that classical electrodynamics possesses two generic properties—*Property A* and *Property B*—and then searched for general theories possessing these properties simultaneously (*cf.* §1.2, 1.3). The analysis recovered the Maxwell field, as expected and required, but also gave the surprising result that it cannot exist in isolation, but must always be associated with an additional massive vector field. The irreducible nature of this association led us to identify this massive vector field as the classical representation of the massive photon.

This approach opens the door on many possibilities, and we have briefly discussed one of them: specifically, that the difficulties associated with the fact that the Lorentz force law does not conform to Newtonian ideals are removed when this law is generalized to account for the presence of the new vector field.

11.2. Relationship with the canonical viewpoint

Classical electrodynamics has arisen, primarily, as the synthesis of laboratory-based experience and, in its covariant four-vector formulation, the electromagnetic field has a very beautiful interpretation as the $U(1)$-gauge field of the superbly successful quantum electrodynamics (qed) with the corresponding gauge particle being the *massless* photon. Thus, against the positive aspects of the ideas discussed herein, we must weigh the fact that the idea of the massive photon is radically at variance with classical qed. So, the question arises of whether it is possible to reconcile the results of this paper with qed and all that that theory represents. This author believes the answer to be positive, and argues as follows:

It was shown, in §6, that the new covariant formalism, based on the three-dimensional linear space of operators $(R)_{sk,3}$, is expressed purely in terms of the classical magnetic vector potential and makes no reference to the scalar potential. Thus, there can be no discussion of electrostatics in the $(R)_{sk,3}$ formalism. But the canonical theory, and hence the possibility of electrostatics, is recovered by the introduction of an arbitrary scalar field—identifiable as the classical scalar potential—into the $(R)_{sk,3}$ formalism. We now note that the theory of electrostatics assumes the existence of a charge distribution, Q say, which is at rest in some particular inertial frame. The theory then considers the effects of Q on some other charge, q say, under the assumption that q does *not* affect the state of motion of Q. Thus, in effect, the theory of electrostatics is a test-particle theory—it is an idealization that, in literal practice, can be approached but never attained. We can then conclude that any theory for which the scalar potential is an essential component (*e.g.*, the canonical covariant four-vector formalism) contains, at some fundamental level, the assumptions of a test-particle theory. Thus, we would argue that the covariant $(R)_{sk,3}$ formalism can be reconciled with the canonical four-vector formalism under the hypothesis that the transition from the former to the latter is the transition from a "real world" electromagnetism to its test-particle idealization.

11.3. Empirical evidence for non-zero mass photons?

The idea that photons might be massive is not new—Vigier [15], for example, argues that the long sequence of inferometer measurement made over several decades by Michelson, Morley and Miller [11,13,12,14] are actually consistent with a non-zero photon mass, and put an upper bound of about $10^{-68}\,kg$ on this mass. Vigier's interest in this is well known since, as a one-time student of DeBroglie, he has long recognized that the latter's interpretation of quantum mechanics probably requires photons to be massive—and *vice versa,* for if photons are found to have non-zero mass, then the standard interpretation of quantum mechanics—and with it, the whole standard model of particle physics—will have question marks raised over it.

11.4. Astrophysics

From an astrophysical point of view, the stakes are also high—the standard interpretation of cosmological redshift requires that it arise purely from expansion. But the only direct evidence supporting this interpretation comes from using the standard candles to verify Hubble's law—but this can only be done out to very modest distances—this direct evidence is actually extrapolated over many decades when it comes to discussing the physics of very high redshift objects. The standard argument against non-expansion mechanisms is that there is no conceivable alternative mechanism which would not broaden spectra, nor leave images unblurred. Since spectral lines are sharp and since images are remarkably un-blurred, it is inferred that redshift must be an expansion effect. However, such arguments are predicated directly upon the notion of the massless photon. Once massive photons are admitted, many different alternative mechanism become, at least in principle, possible.

Appendix A. The Lagrangian density

The required Lagrangian density was arrived at via the following considerations:

- The identities which occur in the canonical covariant formulation of electromagnetism arise because, when F^{ab} is defined as it is from the four-vector potential, the field equation and the Jacobi identity define self-cancelling sums of permutations of fixed-order differential operations on the arbitrarily defined four-vector potential. It is straightforward to see that, in the general case, a necessary condition for a differential expression to become an identity in this way is that the expression concerned must be homogeneous in the differential operators it contains (*differentially homogeneous*) and, when such expressions arise from variational principles, then the corresponding Lagrangian densities must also be differentially homogeneous.

  Since the necessary skew-symmetry of the electromagnetic field tensor in *any* covariant formulation ensures that the identity
  $$\frac{\partial F_{ij}}{\partial x^i \partial x^j} = 0$$
  is satisfied, then we are led to consider only those variational principles which give rise to equations which are second order in the field over which they are defined.
- Finally, in order to guarantee the algebraic orthogonality properties that we require, we must add in the general constraint that any variational principle must be invariant with respect to the interchange of any of its indices—this is also necessary to ensure that changing the labels of axes has no effect.

Putting these considerations together, the most general density is given by

$$\begin{aligned}
L = {} & \alpha_0 \frac{\partial \Psi_{ij}}{\partial x^k}\frac{\partial \Psi_{ji}}{\partial x^k} + \alpha_1 \frac{\partial \Psi_{ij}}{\partial x^k}\frac{\partial \Psi_{ij}}{\partial x^k} \\
& + \beta_0 \frac{\partial \Psi_{ik}}{\partial x^i}\frac{\partial \Psi_{kj}}{\partial x^j} + \beta_1 \left(\frac{\partial \Psi_{ik}}{\partial x^i}\frac{\partial \Psi_{jk}}{\partial x^j} + \frac{\partial \Psi_{ki}}{\partial x^i}\frac{\partial \Psi_{kj}}{\partial x^j} \right) \\
& + \gamma_0 \frac{\partial \Psi_{ik}}{\partial x^j}\frac{\partial \Psi_{kj}}{\partial x^i} + \gamma_1 \left(\frac{\partial \Psi_{ik}}{\partial x^j}\frac{\partial \Psi_{jk}}{\partial x^i} + \frac{\partial \Psi_{ki}}{\partial x^j}\frac{\partial \Psi_{kj}}{\partial x^i} \right),
\end{aligned}$$

where $(\alpha_0, \alpha_1, \beta_0, \beta_1, \gamma_0, \gamma_1)$ are arbitrary constants. This is easily shown to contain a large amount of redundancy.

Appendix B. The Orthogonality Relations

The orthogonality relations within the system are given because they provide an insight into the nature of Maxwell's equations, as we have seen in §9.

The algebraic eigensystem (11) is given by

$$\left(\sigma^{ij} X_i X_j\right)\mathbf{U}_k = \lambda \mathbf{U}_k$$

where $\mathbf{U}^k = (U_{11}^k, U_{12}^k, U_{13}^k, \ldots, U_{44}^k)$, where σ^{ab} $(a, b = 1\ldots4)$ are sixteen matrices each of dimension 16×16 whose elements in row (i, j) and column (r, s) are given by

$$\sigma_{ij:rs}^{ab} = \delta_{ia}\delta_{rb}\delta_{sj} + \delta_{ja}\delta_{sb}\delta_{ri}, \tag{B.1}$$

where the columns are taken in order $(1,1),(1,2),(1,3),(1,4),(2,1)\ldots(4,4)$ and where δ_{ab} is the 4×4 unit matrix. These matrices are easily shown to be symmetric so that, consequently, we have

$$\left(\sigma^{ij} X_i X_j\right)^T = \left(\sigma^{ij} X_i X_j\right).$$

We can conclude from this that eigenvectors lying in distinct subspaces of the eigenspace are orthogonal with respect to the ordinary vector scalar product; that is, if $\mathbf{U}_A$, $\mathbf{U}_B$, $\mathbf{U}_C$, $\mathbf{U}_D$ and $\mathbf{U}_E$ are such that

$$\mathbf{U}_A \in (G)_{sy,6};\ \mathbf{U}_B \in (G)_{sk,3};$$

$$\mathbf{U}_C \in (R)_{sy,3};\ \mathbf{U}_D \in (R)_{sk,3};\ \mathbf{U}_E \in (R)_{sy,1},$$

then

$$\begin{aligned}
\mathbf{U}_A^T\mathbf{U}_B = \mathbf{U}_A^T\mathbf{U}_C = \mathbf{U}_A^T\mathbf{U}_D = \mathbf{U}_A^T\mathbf{U}_E &= 0\\
\mathbf{U}_B^T\mathbf{U}_C = \mathbf{U}_B^T\mathbf{U}_D = \mathbf{U}_B^T\mathbf{U}_E &= 0\\
\mathbf{U}_C^T\mathbf{U}_D = \mathbf{U}_C^T\mathbf{U}_E &= 0\\
\mathbf{U}_D^T\mathbf{U}_E &= 0,
\end{aligned} \tag{B.2}$$

where, by $\mathbf{U}^T\mathbf{U}$, we effectively mean $U_{ij}U_{ij}$, with summation over the indices (i, j).

Naturally, these relations are directly verifiable by direct reference to the definitions of the eigenvectors given at (12), (16), (15), (13) and (14) respectively.

Appendix C. A Material Vacuum?

It has been shown how, according to Poincaré-invariant electrodynamics, the magnetic vector potential supports a stationary longitudinal wave, as well as the conventional propagating transverse wave. The classical theory has managed to assimilate the idea of an electrodynamic wave propagating in the absence of any supporting medium—if only because there is, at least, a sense of *something* travelling and things can be conceived as travelling through empty space. However, the idea of stationary longitudinal waves in the absence of any kind of supporting medium presents an entirely new level of incomprehensibility since, now, *nothing* is travelling anywhere. In Appendix D, a possible solution to this perceived problem is obtained by showing how (29) supports a non-wavy $\mathbf{J} = 0$ solution which has a ready interpretation as a classical description of a fluctuating material vacuum. The magnetic vector potential waves discussed in §8 can then interpreted as *disturbances* of this material vacuum.

Appendix D. The Material Vacuum

For $\mathbf{J} = 0$ and defining $\mathbf{x} \equiv (x_1, x_2, x_3)$, it is easily shown how (29) has a relativistically invariant non-radiated solution, given by

$$\mathbf{A} = \left(\Delta_0 - \Delta_1\right)\mathbf{A}_{01},$$
$$\Delta_0 \equiv (\mathbf{x} - \mathbf{x}_0)^2 - c^2(t - t_0)^2,$$
$$\Delta_1 \equiv (\mathbf{x} - \mathbf{x}_1)^2 - c^2(t - t_1)^2,$$

for an arbitrary constant vector $\mathbf{A}_{01}$ and origins $(\mathbf{x}_0, ct_0)$ and $(\mathbf{x}_1, ct_1)$ which satisfy $\Delta_0 > 0$ and $\Delta_1 > 0$ but which are otherwise arbitrary. Consequently, the general solution of this type is given by

$$\mathbf{A}_{vac} = \sum_{\mathbf{x}_0, ct_0} \sum_{\mathbf{x}_1, ct_1} \left(\Delta_0 - \Delta_1\right)\mathbf{A}_{01}(\mathbf{x}_0, ct_0, \mathbf{x}_1, ct_1) \tag{D.1}$$

where the summation is intended to be over all admissible spacetime origins, $(\mathbf{x}_0, ct_0)$ and $(\mathbf{x}_1, ct_1)$ and it has been assumed that the constant vector $\mathbf{A}_{01}$, which is a function of these origins, is such that the summation is uniformly convergent.

An understanding of the meaning of this solution can be had by considering the expanding surface

$$(\mathbf{x} - \mathbf{x}_0)^2 - c^2(t - t_0)^2 = k^2,$$

for k some real constant, generated by a single term in (D1). It is easily shown that the radial speed of such an expanding surface increases from 0 to c on the range $|k| \le |\mathbf{x} - \mathbf{x}_0| < \infty$. It follows that $\mathbf{A}_{vac}$, which is defined by (D.1) at the spacetime point $(\mathbf{x}, ct)$ by summing over all admissible origins $(\mathbf{x}_0, ct_0)$ and $(\mathbf{x}_1, ct_1)$, is a sum over an infinity of instantaneously intersecting surfaces expanding from all possible directions and at all possible subluminal speeds. In this way, we generate a classical image of a continually fluctuating relativistically invariant material vacuum; for example, see Dirac [10]. Consequently, it is suggested that the solution (D.1) can be interpreted as a classical model of the material vacuum within which magnetic vector potential waves can be interpreted as disturbances. It is to be noted that this material vacuum model is also applicable in the conventional electrodynamic theory.

Finally, it should be remarked that the non-radiated field $\mathbf{A}_{vac}$ should give rise to non-zero electromagnetic effects through (19). The reality, which is that such effects are only found at extremely low levels in the quantum fluctuations of the vacuum, puts constraints on the constants in (D.1).

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