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A New Aggregation Operator Based on Intuitionistic Fuzzy Choquet Integral

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Abstract: There are many aggregation techniques have been introduced to intuitionistic fuzzy sets (IFS). Most of the existing aggregation techniques are considered all the elements are independent. Usually, in some situation, these methods cannot be used to deal with it as in the real situation involves interdependent among the criteria. For this reason, traditional aggregation is not suitable to apply to aggregate since there have not considered the interaction between criteria when making decisions. Thus, the appropriate method to exhibit the interdependence between criteria is Choquet integral. The Choquet integral is relies on its fuzzy measure. The established fuzzy measure is lambda-measure. However, lambda-meausre has a single solution. In this paper, we have evaluated the Choquet integral by considering maximized L-measure and Delta-measure that based on intuitionistic fuzzy sets. An illustrative example is provided by showing step by step of the proposed model. It show that it is effective and applicable in the decision-making environment.

Keywords: choquet integral, intuitionistic Fuzzy sets, Fuzzy measures, multi-criteria decision-making.

1. Introduction

Fuzzy set theory has been extensively accomplished in conducting fuzzy decision making environments. The pioneer to the fuzzy sets is Zadeh [1]. The characterization of the fuzzy set theory is, it has a membership degree. A fuzzy set is an important tool as it broadly employed to model the uncertainty, ambiguity and imprecise situation in multi-criteria decision-making (MCDM), fuzzy logic, approximate reasoning and pattern recognition. However, fuzzy sets have its limitation which is it considered single value henceforth cannot deliver information accurate according to the real data. Furthermore, the information regarding alternatives which based on the fuzzy set possibly incomplete. As a consequence of this, the fuzzy sets theory fails to apply when lacking knowledge and understanding of membership degrees. Atanassov [2] extended the fuzzy set theory to intuitionistic fuzzy sets (IFS). The elements contains in IFS are membership, non-membership and indeterminacy. Consequently, it provides us the alternative tool to handle the real problems which involve uncertainty.

The intuitionistic fuzzy sets have been used in various MCDM problems process including aggregations. As a result, many scholars have applied IFS to the aggregation. For example, Xu and Yager [3] proposed weighted geometric (WG) and the ordered weighted geometric (OWG) for IFS. Liu and Wang [4] employed IFS in fuzzy point operators and provide a new definition of score series. In 2007, Xu [5] developed aggregation operators by utilizing the IFS to the averaging method. In spite of that, the above-mentioned aggregation operator has solely assumed that the criteria are independent of one another. In many practical MCDM problems, there exists interdependent among the criteria[6]. Therefore, a lot of methods being developed to model the interdependence between the criteria. One of them is Choquet integral [7].

Choquet integral is one of the aggregation operators which is used to aggregate and calculate the global score. This method considered the importance of the weight of criteria which denoted by fuzzy measure. The popular fuzzy measure is lambda (λ) measure. However, this fuzzy measure just considered the additive measure and yet to be fitted with other fuzzy measure such that sub-additive, additive or super-additive. Due to the drawback, this paper propose Choquet integral operator by taking into account of maximized L-measure and delta-measure as a fuzzy measure that based on intuitionistic fuzzy set. These two multivalent fuzzy measures are considered because they have infinitely many solutions and suitable apply in the Choquet integral in MCDM problems. The plan of this work goes as follows. In section 2, we reviewed the basic notations and preliminaries to build our approach. An algorithm is conferred on to solve the Choquet integral based on IFS presented in section 3. Meanwhile in section 4, an illustrative example is given to understand the employment of the method completely. Finally, conclusion is made in the Section 5.

2. Fuzzy Measure and Choquet Integral

Definition 1. A non-additive or fuzzy measures on X is a set function $\mu : S(X) \to [0,1]$ if it satisfies the following properties[8]:

(1) $\mu(\phi) = 0$ and $\mu(X) = 1$ (boundary conditions)

(2) $A \subseteq B \Rightarrow \mu(A) \leq \mu(B)$ (monotonicity)

Definition 2. A function $s : X \to [0,1]$ with nonempty finite set X is called fuzzy density function of μ if it satisfying [8]:

$$s(x) = \mu(\{x\}), x \in X$$

Definition 3. $L_{M\delta}$-measure [9] $s(x)$ indicates the fuzzy density of singleton x.

Supposed that the composed measure of maximized L-measure and δ-measure be $g_{L_{M\delta}}$ is a fuzzy measure on a finite set X, $|X| = n,$ satisfying [8]:

1) $L \in [-1, \infty), \sum_{x \in X} s(x) = 1$

2) $g_{L_{M\delta}}(\phi) = 0, g_{L_{M\delta}}(X) = 1$

3) $$g_{L_{M\delta}}(A) = \begin{cases} \max_{x \in A} s(x) & if \quad L = -1 \\ \dfrac{(1+L)\sum_{x \in A} s(x)\left[1 + L \max_{x \in A} s(x)\right]}{1 + L\sum_{x \in A} s(x)} - L \max_{x \in A} s(x) & if \quad L \in (-1, 0] \\ \dfrac{L(|A|-1)\sum_{x \in A} s(x)\left[1 - \sum_{x \in A} s(x)\right]}{(n-|A|)\sum_{x \in X-A} s(x) + L(|A|-1)\sum_{x \in A} s(x)} + \sum_{x \in A} s(x) & if \quad L \in (0, \infty) \end{cases}$$

Definition 4. Let μ be a non-additive measure (fuzzy measure) on X. Then, the Choquet integral [10] of f based on non-additive measure μ is represented as followings:

$$C_\mu(f) = \sum_{i=1}^{n}\left[\mu(x_{(i)}) - \mu(x_{(i+1)})\right] f_{(i)} \qquad (1)$$

where $(.)$ $f_{(1)} \le \cdots \le f_{(n)}, A_{(i)} = \{i, \ldots, n\}$, and $A_{(n+1)} = \phi$.

Aggregation Operators: Choquet Integral based on IFS

An IFS deals with the two degrees of membership and non-membership respectively which satisfied with the property that the sum of its membership is not greater than one. The IFS is:

Definition 5. An IFS A on X is defined as [9]:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\}, \qquad (2)$$

Definition 6. The score function [11] of $\tilde{a}$ is mathematically represented in the following form.

$$S(\tilde{a}) = t_{\tilde{a}} - f_{\tilde{a}} \qquad (3)$$

The accuracy function [10] of $\tilde{a}$ is mathematically represented as following expression.

$$H(\tilde{a}) = t_{\tilde{a}} + f_{\tilde{a}}$$
(4)

Choquet integral based on intuitionistic fuzzy set

G. Choquet in 1954 [7] proposed a method and named it as Choquet integral. This integral is proposed to solve the interrelationship among the criteria. It is treated as an effective method to settle these problems regarding the properties and weight of criteria [12]. The Choquet integral definition is:

Definition 7. Let $\tilde{a} = t_{\tilde{a}i}, f_{\tilde{a}i}$ $(i = 1,2,\ldots,n)$ be an ordered pair of of intuitionistic fuzzy values, and μ be a non-additive measure. The Choquet integral of $\tilde{a}_i$ with respect to μ for intuitionistic fuzzy number can be defined as

$$IFC_\mu(\tilde{a}_1, \ldots \tilde{a}_n) = \tilde{a}_{(1)}(\mu(A_{(1)}) - \mu(A_{(2)})) \oplus \tilde{a}_{(2)}(\mu(A_{(2)}) - \mu(A_{(3)})) \oplus \cdots \oplus \tilde{a}_{(n)}(\mu(A_n) - \mu(A_{(n+1)}))$$

$$= \sum_{i=1}^{n} \oplus \tilde{a}_{(i)}\left(\mu(A_{(i)}) - \mu(A_{((i+1)})\right) \qquad (5)$$

where $(.)$ $\tilde{a}_{(1)} \le \tilde{a}_{(2)} \le \cdots \tilde{a}_{(n)}$. And $A_{(i)} = (i, \ldots, n), A_{(n+1)} = \phi$.

Theorem 1. [6] Let $\tilde{a} = t_{\tilde{a}i}, f_{\tilde{a}i}$ $(i = 1,2,\ldots,n)$ be an ordered value of IFS and μ be a non-additive measure. The IFC_μ is represented as:

$$IFC_u(\tilde{a}_1,\ldots,\tilde{a}_n)=\left(1-\prod_{i=1}^{n}\left(1-t_{\tilde{a}(i)}\right)^{\mu(A_{(i)})-\mu\left(A_{(i+1)}\right)},\prod_{i=1}^{n}\left(f_{\tilde{a}(i)}\right)^{\mu(A_{(i)})-\mu\left(A_{(i+1)}\right)}\right) \quad (6)$$

where $(.)$ $\tilde{a}_{(1)} \le \tilde{a}_{(2)} \le \cdots \tilde{a}_{(n)}$. And $A_{(i)} = (i,\ldots,n), A_{(n+1)} = \phi$.

3. Algorithm to Solve Intuitionistic Fuzzy Choquet Integral

In this part, we show the step by step of the procedure of the proposed fuzzy measure to the Choquet integral for IFS [6].

Some experts are asked to give their opinion based on alternatives (A_1, A_2, … ,A_m), and criteria (C_1, C_2, … ,C_n). The step by step of the proposed method described as below.

Step 1: Present the ranking of the k[th] expert in intuitionistic fuzzy number (IFN) $\tilde{e}_k = (t_{\tilde{e}_k}, f_{\tilde{e}_k}, \pi_{\tilde{e}_k})$. See Table 2 below.

Step 2: The importance weight λ_k of the k[th] experts are determined:

$$\lambda_k = \frac{\left(t_{\tilde{e}_k} + \left(\dfrac{t_{\tilde{e}_k}}{t_{\tilde{e}_k} + f_{\tilde{e}_k}}\right)\right)}{\sum_{k=1}^{E}\left(t_{\tilde{e}_k} + \left(\dfrac{t_{\tilde{e}_k}}{t_{\tilde{e}_k} + f_{\tilde{e}_k}}\right)\right)} \quad (7)$$

E denotes the number of experts

$$\sum_{k=1}^{E}\lambda_k = 1, \lambda_k \in [0,1].$$

Step 3: The ranking L is presented in the linguistic variables, namely I, II, III, …,X based on the IFNs is constructed. The example is displayed in Table 3.

Step 4: The experts are assigned to evaluate the alternatives against criteria based on the linguistic ranking as shown in the list L. The decision of the experts is presented into evaluation matrix which rows refer to the alternatives and columns refer to the criteria.

Step 5: The matrix, $\left[\tilde{P}_{ij}^{(k)}\right]_{mxn}$ for k[th] expert based on the linguistic ranking of IFNs.

Step 6: The opinion of the experts are fused to get the intuitionistic fuzzy integrated decision matrix $\tilde{D}$. The opinion of the experts are aggregated using IFWA operators (8).

$$\tilde{a}_{ij} = IFWA_\lambda\left(\tilde{p}_{ij}^{(1)}, \tilde{p}_{ij}^{(2)},\ldots,\tilde{p}_{ij}^{(k)},\ldots,\tilde{p}_{ij}^{(E)}\right)$$

$$= \left(\lambda_1\tilde{p}_{ij}^{(1)} \oplus \lambda_2\tilde{p}_{ij}^{(2)},\ldots,\oplus\lambda_k\tilde{p}_{ij}^{(k)}\oplus,\ldots,\oplus\lambda_s\tilde{p}_{ij}^{(E)}\right) \quad (8)$$

Step 7: The fuzzy measure is expressed for all criteria. The $\mu(C_i,\ldots,C_j)$ denotes the fuzzy measure of the criteria.

Step 8: The component in every row of the decision matrix, $\tilde{D}$ needs to be reordered before proceed to the Choquet integral, $\tilde{a}_{i(\sigma(j))} \le \tilde{a}_{i(\sigma(j+1))}$. The score function is used to reorder this matrix in Definition 6. The reordered matrix is obtained, such in Table 10.

Step 9: The elements in each row of Table 10 are aggregated using IFCA operators defined in (6). The value of i[th] in row is the aggregation evaluation of alternatives against criteria are constitute in the columns in Table 9.

$$\tilde{a}_i = IFC_\mu(\tilde{a}_{i1},\ldots,\tilde{a}_{in}) = \oplus_{i=1}^{n}\tilde{a}_{i\sigma(i)}\left[\mu(B_{\sigma(i)}) - \mu(B_{\sigma(i+1)})\right]$$

where $\tilde{a}_{ij}$ denotes the net evaluation for i[th] alternatives and j[th] criterion.

Step 10: Using Definition 6, the score function is obtained .

Score, $S(\tilde{a}_i) = t_{\tilde{a}i} - f_{\tilde{a}i}$; Accuracy $H(\tilde{a}_i) = t_{\tilde{a}i} + f_{\tilde{a}i}$

If $S(\tilde{a}_i) < S(\tilde{a}_j)$, the $rank(\tilde{a}_i) < rank(\tilde{a}_j)$;

If $S(\tilde{a}_i) = S(\tilde{a}_j)$, then

- If $H(\tilde{a}_i) = H(\tilde{a}_j)$, then the two IFNs are equal, denoted as $rank(\tilde{a}_i) = rank(\tilde{a}_j)$
- If $H(\tilde{a}_i) < H(\tilde{a}_j)$, then $rank(\tilde{a}_i) < rank(\tilde{a}_j)$

Step 11: The ranking of the alternatives are organized from higher to lower order based on the value obtained in Step 10.

4. An Illustrative Example

Three students have been selected in this study named student A, B, and C considering mathematics, physics and literature respectively to be evaluated. Then, three decision makers (DMs) are hired to evaluate the students' performance based on the different subjects. Table 1 exhibit the result of the students in the different subjects :

Table 1. Results of students' evaluation

	Mathematics	Physics	Literature
Student A	18	16	10
Student B	10	12	18
Student C	14	15	15

Step 1: The ranking in Table 2 is utilized to appraise the proficiency of the experts' in terms of intuitionistic fuzzy numbers.

Table 2. Importance/relative weight of experts

	Expert1	Expert2	Expert3
IFN	0.80, 0.10	0.60, 0.20	0.50, 0.30

Step 2: The importance of each experts' opinion is calculated using formula (7) that based on the ranking of the expert from Table 2. Then, the result is obtained:

$$\lambda_{E1} = \frac{\left(0.8+\left(\frac{0.8}{0.8+0.1}\right)\right)}{\sum_{k=1}^{3}\left(0.8+\left(\frac{0.8}{0.8+0.1}\right)\right)+\left(0.6+\left(\frac{0.6}{0.6+0.2}\right)\right)+\left(0.5+\left(\frac{0.5}{0.5+0.3}\right)\right)} = 0.405$$

$$\lambda_{E1} = 0.405 \qquad \lambda_{E2} = 0.324 \qquad \lambda_{E3} = 0.270$$

Step 3: The IFNs for the linguistic ranking such in Table 3 is decided by the committee of the organization.

Table 3. Linguistic ranking of intuitionistic fuzzy numbers.

Rank	I	II	III	IV	V	VI	VII	VIII	IX	X
IFN	(0.9,0.1)	(0.8,0.1)	(0.7,0.2)	(0.6,0.3)	(0.5,0.4)	(0.4,0.5)	(0.3,0.6)	(0.2,0.7)	(0.1,0.8)	(0.1,0.9)

Step 4: The decisions of all experts' are provided based on the ranking as given in Table 3. See in Table 4 to 6 in the appendix A .

Step 5: The matrix, $\left[\tilde{P}^{(k)}\right]_{3x3}$ for the k^th expert is constructed based on the linguistic ranking in Table 3.

Step 6: The opinions of the experts are aggregated to get the matrix, $\tilde{D}$. The entries $\left(\tilde{p}_{ij}^{(k)}\right)$ in each evaluation matrix are aggregated using (8). Table 7 presents the matrix, $\tilde{D}$.

Table 7. Aggregated IF decision matrix, $[\tilde{D}]$

0.635,0.262	0.810, 0.126	0.333, 0.583
0.306, 0.590	0.600, 0.300	0.442, 0.453
0.756, 0.189	0.813, 0.131	0.788, 0.170

Step 7: The fuzzy measures in Table 8 are provided by the organization. The Choquet integral with the fuzzy measure is applied in the aggregation process[13].

Table 8. Fuzzy measure, $g_{L_{M\delta}}$

Criteria	Fuzzy Measure
$g_{L_{M\delta}}$ (M)	0.4359
$g_{L_{M\delta}}$ (P)	0.4359
$g_{L_{M\delta}}$ (L)	0.2881

Example:
From Definition3, we obtained the fuzzy measure for $L_{M\delta}$-measure as follows:

Let L= 0.2,

$$g_{L_{M\delta}}(A)=\frac{0.2(0-1)(0.45)(1-0.45)}{(3-0)(1.2)+0.2(0-1)(0.45)}+0.45$$
$$=0.4359$$

Step 8: The component in every row of the matrix in the Table 7 needs to be reordered before applying the Choquet integral. The score function is used to reorder this matrix in Definition 6. The new matrix is obtained such in Table 10.

New ordering is determined using (3) and (4). Then, using the reordering matrix in Table 10, the Choquet integral is determined

Table 9. Reordering matrix with $\tilde{A}_{1(\sum(j))} \le \tilde{A}_{1(\sum(j+1))}$

$\tilde{a}_{13}$	$\tilde{a}_{11}$	$\tilde{a}_{12}$
$\tilde{a}_{23}$	$\tilde{a}_{21}$	$\tilde{a}_{22}$
$\tilde{a}_{31}$	$\tilde{a}_{33}$	$\tilde{a}_{32}$

Table 10 . IFNs in the reordering matrix

0.333, 0.583	0.635,0.262	0.810, 0.126
0.442, 0.453	0.306, 0.590	0.600, 0.300
0.756, 0.189	0.788, 0.170	0.813, 0.131

Step 9: The aggregated decision matrix is determined by using (6). The values in Table 10 are constructed using an aggregated weighted fuzzy decision matrix, given that the criteria are interactive.

$$\tilde{a}_i = IFCA_\mu\left(\tilde{a}_{i1},...,\tilde{a}_{ij},...,\tilde{a}_{in}\right)$$

$$\tilde{a}_i = \left(\left(1-\prod_{j=1}^{n}\left(1-t_{\tilde{a}ij}\right)^{\mu\left(B_{(j)}-\mu\left(B_{j+1}\right)\right)}\right),\left(\prod_{j=1}^{n}\left(f_{\tilde{a}ij}\right)^{\mu\left(B_{(j)}-\mu\left(B_{j+1}\right)\right)}\right)\right)$$

where $\tilde{a}_i$ represents i^{th} alternatives to be assessed.The experts' ranking is aggregated using IFCA operator and then, the alternative is evaluated using (6). Thus, we have

$$\tilde{a}_1 = IFCA_\mu\left(\tilde{a}_{11},\tilde{a}_{12},\tilde{a}_{13}\right)$$

Example:

$$= \left(\left(1-\prod_{j=1}^{3}\left(1-t_{\tilde{a}1j}\right)^{\mu\left(B_{(j)}-\mu\left(B_{j+1}\right)\right)}\right),\left(\prod_{j=1}^{3}\left(f_{\tilde{a}1j}\right)^{\mu\left(B_{(j)}-\mu\left(B_{j+1}\right)\right)}\right)\right)$$

$$= \left[\left\{1-(1-0.333)^{0.4359}(1-0.635)^{0.4359}(1-0.810)^{0.2881}\right\},\left\{(0.583)^{0.4359}(0.262)^{0.4359}(0.126)^{0.2881}\right\}\right]$$

$$= (0.6652{,}0.2427)$$

Similarly,

$$\tilde{a}_2 = (0.4921{,}0.3977)$$

$$\tilde{a}_3 = (0.8304{,}0.1244)$$

Step 10: Next, the values obtained in Step 9 is applied to the score function in order to rank the alternatives.

$$S(\tilde{a}_1) = 0.4225 \qquad H(\tilde{a}_1) = 0.9079$$

$$S(\tilde{a}_2) = 0.0944 \qquad H(\tilde{a}_2) = 0.8898$$

$$S(\tilde{a}_3) = 0.706 \qquad H(\tilde{a}_3) = 0.9548$$

Step 11: Rank the alternatives from higher to lower value order based on the score function obtained in Step 10.

$$\tilde{a}_3 \succ \tilde{a}_1 \succ \tilde{a}_2$$

Hence,

Student C $\succ$ Student A $\succ$ Student B

5. Conclusion

To conclude this study, we proposed Choquet integral where the fuzzy measure is represented by L-measure and delta-measure under intuitionistic fuzzy environment. These fuzzy measures are proposed to the Choquet integral as it has infinitely many measures instead of additive measure. It can be considered as a generalization to the lambda measure. This method is proposed as it is appropriate to tackle MCDM problems. An algorithm and procedures are shown to give better understanding. Finally, an illustrative example is provided to portray the proposed method having feasibility and practicality rather than the traditional aggregation operators.

Furthermore, for future undertakings the Choquet integral with the new fuzzy measures can be applied in other fields as a new aggregation method in multi-criteria decision-making.

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References

[1] L. A. Zadeh, "Fuzzy Sets", *Information and Control*, vol. 8, no. 3, pp. 338-356, 1965.

[2] K. T. Atanassov, *Intuitionistic fuzzy sets: Theory and Applications*, Physica-Verlag, New York, 1986.

[3] Z. S. Xu, R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets", *International Journal of General Systems*, vol. 35, no. 4, pp. 417-433, 2006.

[4] H. W. Liu, G. J. Wang, "Multi-criteria decision making methods based on intuitionistic fuzzy sets", *European*

Journal of Operational Research, vol. 179, no. 1, pp. 220-233, 2007.

[5] Z. S. Xu, “Intuitionistic fuzzy aggregation operators”, *IEEE Transaction on Fuzzy Systems*, vol. 15, no. 6, pp. 1179-1187, 2007.

[6] C. Tan, X. Chen, “Intuitionistic fuzzy Choquet integral operator for multi-criteria decision making”, *Expert Systems with Applications*, vol. 37, no. 1, pp. 149-157, 2010.

[7] G. Choquet, “Theory of capacities”, *Annales de I’Institut Fourier*, vol. 5, pp. 131-295, 1953.

[8] H. C. Liu, “An extension signed lambda-fuzzy measure based on signed monotonicity”, *International Conference on Technologies and Applications of Artificial Intelligence*, pp. 47-52, 2010.

[9] H. C. Liu, H. C. Tsai, Y. D. Jheng, T. S. Liu, “Composed fuzzy measure of maximized L-measure and Delta measure”, *WSEAS Transactions on Information Science and Appplications*, vol. 7, no. 4, pp. 474-483, 2010.

[10] J. Wu, F. Chen, C. Nie, Q. Zhang, “Intuitionistic fuzzy-valued Choquet integral and its application in multicriteria decision making”, *Information Sciences*, vol. 222, pp. 509-527, 2013.

[11] M. Grabisch, T. Murofushi, M. Sugeno, *Fuzzy Measure and Integrals*, Physica-Verlag, New York, 2000.

[12] H. Zulzalil, A. A. A. Ghani, M. H. Selamat, R. Mahmod, “Using fuzzy integral to evaluate the web-based applications”, *Malaysian Software Engineering Interest Group (MSEIG)*, Malaysia, 2011.

[13] W.R.W. Mohd, L. Abdullah, “Choquet integral with respect to maximized L-measure and delta-measure”, *AIP Conference Proceeding*, vol. 1870, no. 1, 2017, http://doi.org/10.1063/1.4995836.

Appendix A

Table 4. Evaluation by expert 1

	Mathematics	**Physics**	**Literature**
Student A	IV	II	X
Student B	VII	IV	VII
Student C	III	III	I

Table 5. Evaluation by expert 2

	Mathematics	**Physics**	**Literature**
Student A	III	III	IV
Student B	IX	IV	IV
Student C	IV	I	III

Table 6. Evaluation by expert 3

	Mathematics	**Physics**	**Literature**
Student A	IV	I	III
Student B	V	IV	VI
Student C	I	II	IV