

# Teaching the Difference between Constant and Variable via Productive Pedagogies

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**Abstract:** This paper aims to help teachers come up with effective strategies in teaching the distinction between constants and variables, as well as pattern recognition. This is achieved through a Lesson Study and the notions of Productive Pedagogies. Lesson study as a methodology is an effective way to maximize student learning inside the classroom as it enables teachers to effectively design and implement lessons. Post-lesson discussion yields comments and suggestions that may be classified under two general ideas: (1) Foster a student-centered environment and (2) Evaluate the learning of students by giving them Higher Order Thinking Skills (HOTS) questions. Based on the four dimensions of productive pedagogies, the lesson is successfully implemented by the teacher, although it is evident that most of the students have difficulty transitioning from Math expression to verbal or English expression. The comments and suggestions given by the observers really help the teacher to execute lessons better, especially in terms of utilizing various innovative strategies and techniques suited to the level and interest of the students while maintaining higher order thinking skills.

**Keywords:** constant variables, lesson study, productive pedagogies.

## 1. Introduction

Several studies have stated the importance of learning Algebra. It can be defined as a language used to solve different kinds of numerical problems [1]. Considering it as a language shows the importance of learning Algebra to understand mathematics [2]. Teachers' experiences and some studies show that students have difficulties in understanding algebra. For example, students face intellectual struggles as Wang suggested [3]. According to [4], cognitive demands are placed on Algebra, such as 1) treating symbolic representations, which have a little or no semantic content, as mathematical objects and operate on these objects through processes that usually do not produce numerical solutions; 2) modifying their former experiences in arithmetic context and represent the relationships in word problems with inverse operations used in arithmetic context. These two cognitive demands, operations and representations, are the intellectual struggles for students in learning Algebra [3].

One of the most important difficulties in learning Algebra is the failure in understanding the concept of variables [5]. A study conducted by Sengul & Edrohan in 2014 revealed that 6th grade students have low procedural knowledge in the field of Algebra [6]. They continue by saying that this may mean that students have difficulty in understanding the concept of variables and other terms related to algebra, and solving algebraic problems. One of the reasons that make the concept of variable difficult is the frequent use of letters and verbal

symbols [5]. The ambiguous transition of arithmetic to algebra must be strongly supported by a strong foundation. It is important for students to have a strong mathematical foundation and educators must assess students' understanding before beginning new instruction and integrate manipulatives in the classroom for deeper understanding of concept [7]. The study of Aniban, Chua, Garcia, and Elipane emphasize on the significance of variables by making use of sequence and patterns as a lesson to aid transition from arithmetic to algebra, and they found out that the strategy shows a clear link of generalizing a set of numbers into an algebraic expression [8]. Moreover, encouraging students to talk about mathematics and helping them to develop appropriate vocabulary and phraseology to do so is an important part of learning it [9].

This paper discusses the introduction of algebra to students, particularly the concept of variables and constants. It aims to contribute to the discussion on the transition from arithmetic to algebra. More so, this hopes to enrich the skills and broaden the techniques of a teacher through a process of teachers learning together via a professional development model called Lesson Study [10-12].

## 2. Theoretical Framework

Constants and variables are two important concepts that students should know when dealing with algebra. Skemp stressed that a variable is a key concept in algebra [13]. He describes variable as an unspecified element of a given set; and constant is a number with a value that is always the same [13]. The teachers' understanding of teaching of algebra, as well as their understanding of algebra as content for teaching, both influence their ways of teaching it and the students' ways of coping with it [14].

Many researchers have been concerned with the move of the students from arithmetic to algebra and the cognitive gap that exists between the two [15].

As teachers of mathematics, we understand that the role of variables and constants is essential for developing mathematical reasoning and understanding in algebra. It is required to manipulate algebraic expressions and enables students "to express mathematical relations in different ways, and know more about them" [16].

According to Kieran (as cited in [17]), there are two aspects which are crucial in the transition of arithmetic to algebra [4]. These are, first, the use of letters to represent numbers and, second, the explicit awareness of the mathematical method that is being symbolized using both numbers and letters.

The Lesson Study that was used in this research is another way to create a strong foundation of concepts of algebra to the students. The four dimensions of Productive Pedagogies are used as an evaluation tool in this lesson study. The Productive Pedagogies as a framework to enhance teaching and learning, developed by Lingard et al in 2001 (as cited in [18]), consist of twenty pedagogies that have been broken down into four dimensions- intellectual quality, connectedness, supportive classroom and environment, and working with and valuing differences. According to the study of Lingard, Hayes, & Mills in 2003, the Productive Pedagogies model seeks to provide a lens through which educators can see existing teaching practices, with a view to reconceptualizing them in ways that increase the academic and social outcomes for all students [18].

As teachers engage in the process of Lesson Study, they are collectively examining practice; they are functioning as communities of practice. In Lesson Study, collaboration entails the collaborative planning, observing, and debriefing of lessons. Similarly, the collaborative nature of Lesson Study can help teachers entrenched notions of instruction and result in better student learning [19].

### 3. Methodology

The purpose of this Lesson Study is to design a lesson on constants and variables and to gain in-depth understanding on the topic. The researchers are a group of graduate students of Math Ed 701 (Theories of Learning) from the Philippine Normal University (PNU). To achieve the goal of the lesson study, several steps are taken, focusing on three different phases: Planning, Implementation, and Post-Lesson Discussion.

#### 3.1 Selecting the Lesson Topic

The researchers chose constants and variables to be the topic for the research lesson. The group agreed to execute the research lesson to the 7th grade - since most of the researchers are presently Grade 7 Teachers. The topic selected is based from the Department of Education (DepEd) K to 12 Mathematics curriculum as well as the advice of the current grade 7 teacher from the school. There are twenty (20) Grade 7 students from the School of Everlasting Pearl, Inc., who are seated as the participants. The composition of the class is heterogeneous.

#### 3.2 Lesson Planning

The lesson planning is a collaborative effort of the researchers where they discuss the DepEd competencies and specific activities to be conducted on the research lesson. The research lesson teacher is chosen based on the length of experience with teaching junior high school mathematics.

#### 3.3 Implementation of the Research Lesson

The research lesson is implemented on November 15, 2017 at the School of Everlasting Pearl in Antipolo. The participants are the grade 7 students belonging to one section, numbering twenty (20) in all. The observers are the PNU graduate students taking up the course Math Ed 701 Theories of Learning as well as the section's current Mathematics teacher.

The research lesson teacher is Mrs. Dhalia Maglacas, a grade 7 teacher from Gumaca National High School. The research lesson takes more than an hour to implement and videos and pictures are taken to document the lesson for future reference. A post-lesson discussion is immediately held after the lesson implementation.

#### 3.4 Post-Lesson Discussion

The four dimensions of Productive Pedagogies are used as a design and evaluation tool of the lesson. The details in the discussion are analyzed through the different aspects of productive pedagogies. Comments and suggestions are given by the listeners to come up in a more clear way of discussing variable and constant. The four main dimensions serve as the basis of the audience to review the activities and teaching strategies made by the teacher during the discussion. Previous studies indicate that high intellectual quality classroom helps students perform well academically [20]. Connectedness attempts to connect students' lives with schooling (curriculum and content) or school studies to be more 'relevant' that will provide them with more meaningful experiences. The supportive classroom dimension is needed to support high intellectual quality and connectedness to ensure that students can achieve the learning objectives. In addition, Hayes et al. assert that teachers should give more emphasis and need to recognize the identities and narratives of students, especially those with different cultural backgrounds and beliefs. This is the dimension on Recognition of Difference [18].

### 4. Result and Discussion

A post-lesson discussion is immediately done on the same day of the research lesson. The comments and suggestions of the presenting group, as well as the other groups, including the professor, are conferred, crystallized, and documented. These comments and suggestions are summarized and classified below based on the four (4) major classifications or dimensions of productive pedagogies.

#### 4.1 Intellectual Quality

The first dimension of productive pedagogies is the intellectual quality which consists of higher order thinking skills, deep knowledge, deep understanding, and substantive conversation [21]. These elements ensure that manipulation of information given to students lead to the interpretation of new meanings and understandings that allow students to solve problems analytically and critically, as well as apply abstract concepts to real-life situations.

The teacher starts the lesson by defining what a constant is, followed by a variable.

T: "According to Webster dictionary, Constant is a noun referring to something that stays the same: something that does not change; a quantity or number whose value does not change".

After that, the teacher asks the students to give examples of a constant.

T: "Can you think of something does not change?"

Student 1: A mother's love.

T: Okay, that's great! What else? (seconds passed)

How about this, can the number of the sun in the solar system change?

Students: No.

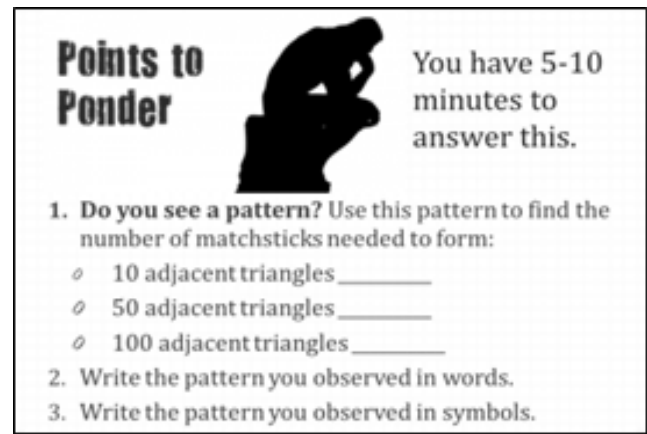
T: That is constant. Give another example. (Students were unable to give another example of constant.)

T: “Later, you might be able to think of more examples as we go along with our lesson.”

From the above, intellectual quality is still not evident from the strategy used by the teacher, which is, defining unknown terms using a dictionary. During the post-lesson discussion, an observer remarks that the lesson introduction could have been done in other ways. Instead of the teacher being the one to give dictionary definitions of the terms, the definition could have been extracted from the students. Another observer suggests that the teacher could have given mixed examples of constant and variable things and classify the examples into groups. The students could have been asked to orally express their justification to their groupings. The observer explains that it is possible for the students to give varied classifications, but in the end, through their justifications, they must realize which classifications fit the examples best. Since students have different levels of understanding, this technique may take some time but will yield more fruitful results on students’ comprehension.

Grouping the students and asking them to name their group allow for collaboration and ownership of ideas. Collaboration encourages dialogue between and among the members of the group (small group discussion), as well as the teacher, especially when she goes around the classroom facilitating but not intervening in the student work. A more in-depth discussion happens when the entire class discussed the activity with the teacher.

There are several opportunities during the research lesson when the students’ understanding could have been deepened further. During the construction activity, the teacher may deepen the students’ distinction between variable and constant by letting the students discover a pattern for the tasks. In this activity, students are grouped and given popsicle sticks, scotch tape, cartolina and marker and given several tasks. The first task is to form a triangle using the popsicle sticks and identify the number of sticks used to create a triangle. Each group post their work in a cartolina. In the second task, students are asked to form two adjacent triangles (the meaning of “adjacent” is given to the students) using the sticks and they are also asked how many sticks they used. Third, they are asked to form three adjacent triangles and to count how many sticks were used. The fourth task is to form four adjacent triangles and count how many sticks are used. The fifth task is to form five adjacent triangles and to count the sticks used. The last task is to answer the questions posted on the screen. Figure 1 shows the analysis questions in the last task of activity.



**Figure 1.** Post-construction activity questions

During the lesson planning, it is agreed that the tasks should be given one by one. However, during the actual lesson, the teacher gave all the tasks in one go. During the post-lesson discussion, the teacher mentions that she lacks time and there are still more tasks to be conducted. After the construction activity, the teacher draws a table on the whiteboard to summarize the number of sticks needed for each of the tasks. During the post-lesson discussion, an observer remarks that the summary table could have included on the worksheet provided so that the students will be the ones to continue the construction that will allow them to make deeper sense of what they have constructed with the popsicle sticks. Table 1 below shows the summary table of teacher.

**Table 1.** Summary table showing number of triangles and number of popsicle sticks

Number of triangles	Number of popsicle sticks
1	3
2	5
3	7
4	9
5	11

Then, after constructing the summary table, the teacher redirects the students’ attention to the “Points to Ponder” part of the activity, particularly about how to find the number of popsicle sticks needed to form 10, 50, then 100 triangles. She mentions that by discovering a “pattern”, they won’t need to construct the triangles and that this pattern can be expressed via an algebraic expression. During the post-lesson discussion, an observer remarks that the teacher could have let the students express their patterns more verbally to activate their critical thinking. Each student could have been asked to explain more how he or she comes up with his/her pattern and let him/her realize that it is not the correct pattern via asking the right questions and extracting the correct answer. However, another observer commends the teacher’s patience in extracting and leading the students to the correct pattern. Another observer remarks that teachers must be aware of unintended mistakes that they may utter during a lesson, as what happened in this part of the lesson:

Student: the number of triangle will add by itself and plus

T: Ok, you mean that  $1+1+1$  is 3.

Student:  $2+2+1$  equals 5

T: Okay, what's next?

Student: 3 plus 3 plus 1 equals seven.

T: correct!

Student: 4 plus 4 plus 1 equals 9.

T: So if 10 adjacent triangles, 10 plus 10 plus 1 equals 21.

Is there another way to write this expression? What did you notice? The number is multiplied twice by itself.

The teacher commits a verbal expression error here. She should have said 'twice the number' (translated as  $2x$ ) and not 'the number is multiplied twice by itself' (translated as  $x2$ ) since these statements are not equivalent to each other.

## 4.2 Connectedness

The second dimension, connectedness, consists of four elements: background knowledge, knowledge integration, connectedness to the world and problem-based curriculum. These elements focus on the integration of the lesson to other areas and thus, allow the students to learn beyond the knowledge given by the teacher [21].

The research lesson shows connectedness in several instances. First is when a student gave the example of a mother's love when the teacher asks for an example of something that is constant. Second during the construction activity that called for the students' previous knowledge in constructing a triangle. From there, the lesson introduces the concept of adjacent triangles but still building on the previous knowledge of how to construct a basic triangle. Third is during the pattern recognition part of the lesson. The teacher is able to draw out the correct pattern from the students based on the construction activity and the students' understanding of constant and variable. There is integration of previous and new knowledge in this part of the lesson. Lastly, the construction tasks are given as problems to the students that they had to solve as a group, as well as looking for the pattern to answer how many popsicle sticks are needed to form any number of adjacent triangles without constructing the triangles.

During the post-lesson discussion, an observer remarks that the lesson shows limited integration with other subject areas. Students could have given more examples if they have seen the relevance of the topic to some areas. However, the observers agree that in general, the teacher shows connectedness to the world by presenting real life situations and examples. The different examples given mostly by the teacher except for "mother's love", like electric bill, allowance, number of days in a week and others are meant to make the lesson easy to comprehend. The group activity used by the teacher makes the class more engaged in the discussion which activates their critical thinking skills. The roles given to members of the group uplift their sense of responsibility. However, another observer remarks that it would have been more helpful for the students in identifying the pattern/rule if they have been given enough time to perform the activity. The observers also agree that problem-based discussion was used when the students were asked to determine the rule/pattern of the construction activity.

## 4.3 Supportive Classroom Environment

The third dimension of productive pedagogies, supportive classroom environment, consists of five elements: student direction, social support, academic management, explicit quality performance criteria and self-regulation. [21].

During the research lesson, the teacher has some difficulty in the manipulation of the PowerPoint Presentation such as activating the timer needed for the activities. The projector was accidentally turned off several times by a student which caused delays in the discussion. The limited classroom area makes it difficult to maximize the PowerPoint presentation since only a chair held the laptop and projector. Consequently, this also makes the students unable to maximize manipulation of their materials. However, despite all these undesirable happenings, the teacher is able to immediately establish rapport with the students by asking them how they are and by smiling at them constantly. That's establish a friendly, yet respectful atmosphere all throughout the lesson. The students are obviously eager to learn, and the teacher is equally excited to facilitate learning. The students show their participation by encouraging one another to answer and consoling those who gave incorrect answers.

The whiteboard/writing space is very limited, so the teacher is not able to keep important notes on the board, as she has to erase what she had previously written to make room for new things to write. Therefore, it is difficult for the students to organize their thoughts and recognize patterns since solutions are not shown properly and the important details have to be erased. It would have been better if important details are kept on the board so students could go back to any of it at any time to support his/her thinking. However, these do not affect the active participation of the class. With the teacher's encouragement and art of questioning, a student is still able to recognize and give the correct pattern. This is after a few students also tried to "guess" the pattern.

The materials provided and used by the students are not maximized, in terms of students being able to manipulate them freely and with ample time. The classroom is not spacious that students cannot move well during the group activity. It would have been better if they are able to sit around in a circle on the floor, especially since it takes time to tape the popsicle sticks on to the cartolina. The size of the popsicle sticks also matters (large size). It would have been better to have used either toothpicks, matchsticks or small-sized popsicle sticks. Due to safety concerns, popsicle sticks are used instead. However, these limitations do not stop the students from their active participation and the teacher was able to obtain her objectives for the lesson – for the students to distinguish between constant and variable and for them to recognize the pattern for the number of popsicle sticks needed to form a certain number of adjacent triangles. Teachers must be resourceful and creative enough to support and sustain student learning, despite of many classroom limitations.

## 4.4 Recognition of Difference

The last dimension, recognition of difference, also consists of five elements which are cultural knowledge, inclusivity, narrative, group identity and active citizenship [21].

It is a challenge to conduct the demonstration to a completely unknown class. The only thing the group knew

about the class was the grade level and Elipane's advice that the students of the School of Everlasting Pearl belong to the lower income bracket. In view of this, the group prepared activities that will cater to the class identity: the medium of instruction used is a mix of basic English and mostly Filipino. A comic strip ("Variable" by Davehohman, 20 November 2008) relevant to the lesson was translated not only into Filipino but also employing Filipino youth lingo such as 'lodi' (idol), 'werpa' (power) and 'petmalu' (awesome). The worksheet only used basic English and simple instructions. The examples presented are real-life examples that the students can easily relate to. At the end of the lesson, differentiated performance tasks (with rubrics) are assigned per group (to be checked by their current Math teacher). These tasks cater the varied interests and skills of the students in general.

Although the construction activity is done by group, each member of the group was assigned a specific role which mimics real life: carpenter (the one who led the construction), runner (the one who looked for and handed over the needed materials), timer (the one who monitored the time) and team captain (the one who supervised the group and made sure things were done). The responsibilities of each one was mentioned by the teacher before the conduct of the construction activity. During the lesson, the students took the specific role assigned to them seriously.

## 5. Conclusion

Based on the four dimensions of productive pedagogies, the lesson is successfully implemented by the teacher, especially in terms of piquing the curiosity of the grade 7 students of the School of Everlasting Pearl regarding constant and variable. The group informally interviewed some random students after the lesson if they understood the lesson and they answered positively. It is evident that most of the students had difficulty transitioning from Math expression to verbal or English expression, but this is understandable since the shift from arithmetic to algebra really requires thorough processing and explanations. It is their first time to encounter the terms constant and variable.

It is also evident that the students enjoyed the activities despite their first time to see the teacher and good rapport is immediately established between them due to the friendly approach of the teacher and the openness of the students. The teacher also has several teaching strategies, evidence of years of teaching experience, that made the discussion lively and interesting. Despite the limited space, the classroom is still conducive to learning since the students showed full cooperation and excitement during discussion. In general, the teacher serves her purpose as a facilitator to the students. She effectively motivated the students to become the protagonists of their own learning, being able to draw out the correct pattern for a relatively new lesson.

To further improve the teaching strategies exhibited in the research lesson, the following are a couple of recommendations given by the observers which the authors believe should be highlighted:

1. Foster a student-centered environment. Allow students to identify the lesson through the activities. Let the students discuss and reason-out their observations and learnings. Let them draw out their ideas and correct misconceptions. Acknowledge their potentials and strengthen their sense of belongingness.

2. Evaluate the learning of students by giving them HOTS (Higher Order Thinking Skills) questions. This allows the students to generate concepts and ideas and relate them to real life (application).

In conclusion, lesson study as a methodology is an effective way to maximize student learning, starting from the lesson planning down to the post-lesson discussion via collaboration. The comments and suggestions given by the observers really helped the teacher to execute lessons better, especially in terms of utilizing various innovative strategies and techniques suited to the level and interest of the students while maintaining higher order thinking skills. Truly, the success of the teacher in the implementation is also the success of a student in his/her learning.

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