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Numerical Solutions for Poisson Image Blending Problem using 4-EDGSOR Iteration

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Abstract: Poisson image blending is an operation in image processing to generate a new image by using Poisson partial differential equation. In this paper, 4-EDGSOR iteration is used to solve the Poisson image blending problem and its efficiency in solving the proposed problem is illustrated. The approximation Poisson equation is formed by applying a finite difference method. Then, the rotated Laplacian operator which is constructed by a rotated finite difference scheme is used in this paper. Then a linear system is formed and solved using the 4-EDGSOR iterative method. The performance of the 4-EDGSOR iterative method in solving the proposed problem is compared to the SOR and 4-EGSOR iterative methods. The results obtained from numerical solutions showed that the 4-EDGSOR iterative method required lesser time and number of iterations to blend an image. From quality point of view, all images obtained the same natural look.

Keywords: Poisson image blending, 4-EDGSOR iteration, rotated Laplacian operator.

1. Introduction

The Poisson equation has a great application in the field of image processing. For instance, it is applied in image completion, image matting, and image blending. This paper examines the application of Poisson equation to overcome an image blending problem. The technique that is employed in Poisson image blending process can be considered as a gradient domain technique. This technique manipulates the gradient in the image which allows it to blend the image more gracefully compared to the use of image pixels.

The concept of applying a Poisson equation to solve an image blending problem was inspired by [1]. A finite difference discretization scheme is used to form the Poisson approximation equation, then the Gauss-Seidel, Successive Over Relaxation (SOR) and V-cycle multigrid method are used to solve the linear system in [1]. Further details about the Poisson image blending concept are presented in the next section.

The inspiration to apply Poisson equation in solving image editing problems had motivated researchers to formulate the efficient ways to generate a well-blended image. For instance, the author in [2] suggested the implementation of the Fourier solver in solving the problem of Poisson image blending and the results obtained are satisfied. While the researchers in [3] identified some issues caused by the Poisson image blending, which are the bleeding artifacts and color bleeding. Thus, they modified the implementation from being solely dependent on the target boundary pixels to depending on both the target and source boundary pixels.

Besides, an idea was recommended by [4] to subdivide the image into smaller pieces to ensure the process of blending becomes faster. It is an efficient technique where the composing time is shortened yet the output obtained is similar to other methods.

More recently, researchers from [5] proposed a new exploration that applied generative adversarial networks (GAN) to deliver a realistic and high-resolution image blending. They worked on the fusion of Gaussian-Poisson equation and GAN (GP-GAN) for image blending. The final images obtained were more realistic with high-resolution.

In this paper, we used a numerical approach to solve image blending problem. The Four Point Explicit Decoupled Group Successive Over Relaxation (4-EDGSOR) iterative method via rotated the five-point Laplacian operator was selected as an efficient method to solve the proposed problem. The 4-EDGSOR iterative method had been proven as an efficient method to solve the Poisson equation in [6]. However, in this paper, we applied this method to solve the Poisson image blending, and its performance was evaluated.

Applying 4-EDGSOR iterative method in Poisson image blending is the first such attempt in the image editing field. Previous studies in [7, 8] used SOR and MSOR iterative methods via the standard five-point Laplacian operator. Thus, the full-sweep SOR (FSSOR) and Four Point Explicit Group SOR (4-EGSOR) [9] iterative methods were used to compare the performance of the 4-EDGSOR iterative method in terms of the number of iterations and composing time taken.

Other than image blending, other image processing operations such as image analysis also has a great contribution to the medical [10, 11] and agriculture [12, 13] fields. Furthermore, robot path planning problem also utilizes the potential of Laplace equation [14, 15].

2. Poisson Image Blending

In 2003, researchers in [1] proposed a simple technique for image blending where the new images produced are well blended. It is a simple idea to blend a desired image gracefully. Firstly, the desired region D is selected from a source image s. Then, the desired region is blended into the destination image t^* to form a new output image t. This image blending process can be defined as a minimization problem as follows,

$$min_t \iint_D |\nabla t - \mathbf{v}|^2 \ with\ t|_{\partial D} = t^*|_{\partial D}. \quad (1)$$

Equation (1) is defined to calculate a new set of intensity values t. This is to ensure that the differences between the vector field $\mathbf{v}$ and gradient of the output image are minimized.

In order to solve the equation (1), the Poisson equation with Dirichlet boundary condition is defined as follows:

$$\Delta t = \Delta s \text{ at } \mathrm{D} \text{ with } t|_{\partial \mathrm{D}} = t^*|_{\partial D} \tag{2}$$

where the vector field is set to be equal to the gradient of the source image. From [1], the unique solution of the Poisson equation (2) is equivalent to the minimization problem (1). Therefore, in this paper, the finite difference discretization scheme is used to form the Poisson approximation equation and then solved using the 4-EDGSOR iterative method.

2.1 Implementation of 4-EDGSOR Iterative Method

The concept of EDG was proposed by [16] in order to reduce the computational complexity when solving partial differential equations. In this paper, the concept of EDG is combined with SOR to become the 4-EDGSOR iterative method which is a block iterative method. This iterative method is considered a group of four points in one iteration, as shown in Figure 1.

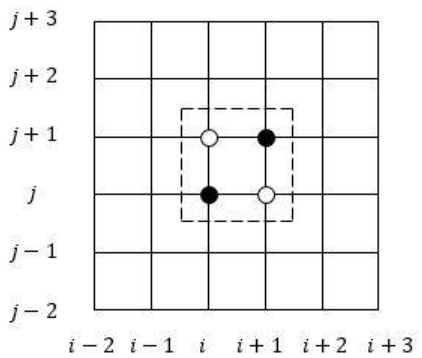

Figure 1. A group of four points iteration

There are two types of nodes in Figure 1, the ● and ○. The implementation of 4-EDGSOR only involved one of the two types of nodes. In this paper, we implemented the 4-EDGSOR iterative method based on the nodes ●.

According to Figure 1 using a finite difference discretization scheme via the rotated five-point Laplacian operator, the four approximation equations based on the Poisson equation (2) are derived as follows:

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} t_{i,j} \\ t_{i+1,j+1} \\ t_{i+1,j} \\ t_{i,j+1} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}. \tag{3}$$

Then, the inverse of equation (3) is determined,

$$\begin{bmatrix} t_{i,j} \\ t_{i+1,j+1} \\ t_{i+1,j} \\ t_{i,j+1} \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}. \tag{4}$$

Equation (4) can be independently decoupled into two (2×2) systems, where the first two equations are based on nodes ● while the last two equations are based on nodes ○. Since, in this paper, we implemented the 4-EDGSOR iterative method based on nodes ●. Therefore, we used the first two equations in Equation (4), as follows:

$$\begin{bmatrix} t_{i,j} \\ t_{i+1,j+1} \end{bmatrix}^{(k+1)} = \frac{1}{15} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{5}$$

with

$$I_1 = -2h^2 s_{i,j} + t_{i-1,j-1}{}^{(k+1)} + t_{i-1,j+1}{}^{(k+1)} + t_{i+1,j-1}{}^{(k+1)}$$

$$I_2 = -2h^2 s_{i+1,j+1} + t_{i+2,j+2}{}^{(k)} + t_{i,j+2}{}^{(k)} + t_{i+2,j}{}^{(k)} \tag{6}$$

The equation (5) is the standard 4-EDG iterative method. By adding one weighter parameter ω, the 4-EDGSOR iterative method [6] is derived as follows:

$$\begin{bmatrix} t_{i,j} \\ t_{i+1,j+1} \end{bmatrix}^{(k+1)} = \frac{\omega}{15} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + (1-\omega) \begin{bmatrix} t_{i,j} \\ t_{i+1,j+1} \end{bmatrix}^{(k)} \tag{7}$$

After computing all the nodes ●, the remaining ○ is directly determined by implementing the standard five-point formulation. Thus, the computational complexity is reduced by approximately 50%.

3. Numerical Results and Discussion

Poisson image blending problem is solved using the 4-EDGSOR iterative method, and its performance is compared with the 4-EGSOR and FSSOR iterative methods. In this paper, three examples from [17] are used for the blending process, as shown in Figure 2.

Figure 2. (i) Source and (ii) target images

The number of iterations and blending time are used to compare the efficiency of the 4-EDGSOR iterative methods with 4-EGSOR and FSSOR iterative methods. The numerical results obtained are shown in Figure 3 and 4.

By referring to Figure 3, the numerical results have clearly shown that the 4-EDGSOR iterative method has significantly reduced the number of iterations. Compared to the FSSOR iterative method, the number of iterations has reduced by approximately 64.93% to 72.83% while the blending time taken has also improved approximately 46.14% to 72.64%, as shown in Figure 4.

Meanwhile, by comparing to the 4-EGSOR iterative method, the number of iterations used by the 4-EDGSOR iterative methods have been significantly reduced by 44.36% to 53.70% and the blending time taken has improved by approximately 26.18% to 40.00%.

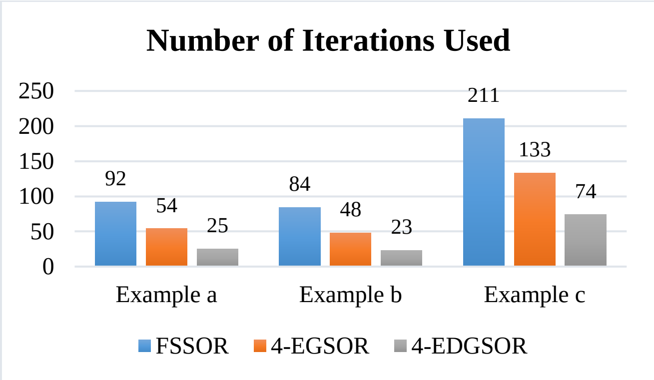

Figure 3. The number of iterations used

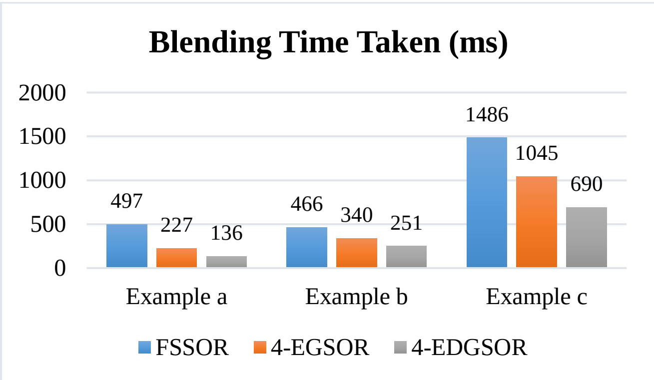

Figure 4. The blending time taken

The newly blended images are presented in Figure 5 to 7 accordingly.

From the numerical results obtained, the 4-EDGSOR iterative method used the least number of iterations and blending time. This is because its computational complexity had been reduced by 50% compared to the 4-EGSOR iterative method and it is more efficient than the FSSOR iterative method. This efficiency is caused by calculating a group of four points in each iteration. Meanwhile, all the newly formed images are well-blended without a noticeable seam.

4. Conclusion

The first attempt of using the 4-EDGSOR iterative method in overcoming an image blending problem is presented in this paper. Its potential in solving Poisson image blending problem is proven where it utilized the least number of iterations and composing time. Besides, all the newly formed images are gracefully blended.

(i)

(ii)

(iii)

Example a

Figure 5. New images formed by (i) FSSOR, (ii) 4-EGSOR and (iii) 4-EDGSOR iterative methods for Example A

(i)

(ii)

(iii)

Example b

Figure 6. New images formed by (i) FSSOR, (ii) 4-EGSOR and (iii) 4-EDGSOR iterative methods for Example B

(i)

(ii)

(iii)

Example c

Figure 7. New images formed by (i) FSSOR, (ii) 4-EGSOR and (iii) 4-EDGSOR iterative methods for Example C

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[17] Test examples are available in https://pixabay.com and http://www.publicdomainpictures.net.