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Quantum Squeezing in Multichannel Nonlinear Coupler

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***Abstract*:** Occurrences of the first order squeezing in a nonlinear coupler composed of multichannel Kerr waveguides is simulated by integrating the Langevin stochastic equations derived in the Positive-*P* representation semi-analytically. We show that the system exhibits squeezed light in the framework of first-order single-mode and enhanced compound-modes for a suitable choice of interaction parameters.

***Keywords*:** quantum optics, phase-space representation, quantum computers**.**

1. Introduction

Exponential improvement of computer technology in accordance with Moore's Law [1] has already started to falter. In an attempt to increase the processor performance, over 400 million transistors are currently being jammed together on a limited spaced silicon chips and the number is doubling in capacity for every two years [1]. In less than a decade away, silicon-based microprocessors are expected to reach their peak capabilities. Skimming down the scale to make up more room for additional circuitry will become extremely difficult for the reason that such scheme requires circuit features of an atomic scale where classical measure is futile. Quantum computer [2] offers a way of overcoming such limitation with the potential to deliver factoring calculations and processing tasks much faster than conventional computers. Squeezed light is one of the most important fundamental elements in making the quantum computer into realization as reported recently [3], by means of reducing the quantum noise to a significant degree. Quantum noise is a consequence of the smallest energy quanta of light statistics. Due to the inherent existence of indeterminacy, the resolution of many light-based applications is limited by the standard quantum limit. Having a low noise property, quantum squeezing of light [4] is central in surpassing the limitation. In recent years, squeezed light has also emerged as an essential element in various applications especially in precision measurement [5], communication [6] and teleportation [7]. In achieving aims of promising modern quantum, technologies such as ultrafast quantum computer [2], developing a strong and well understood squeezed light is of vital importance. Light interaction in an optical coupler with nonlinear media as a medium of interaction holds a special property for squeezing diverse development. Among many simplest directional couplers [8-10] as possible quantum squeezing generators, much attention has been given to directional coupler operating by third-order Kerr nonlinearity. The optical Kerr nonlinearity is well known for its efficiency in providing a good mechanism for squeezing [11]. Kerr squeezing of the conventional two-mode coupler has been studied via both analytical [12] and numerical [11] approaches respectively. An extension of phase mismatched [13] and contra propagation mechanism [14] have been reported as well. Robust Kerr squeezing is achievable by direct manipulation of the number of fields mode involved in multimode interaction [15]. To promote multimode fields interaction further, three-mode two-channel coupler and three-mode three-channel coupler versions have been developed [15, 16].

While much effort has been given on production of squeezed light using optical coupler in terms of first order basis, different regimes for generation and transmission of quantum light is possible via higher order squeezed light superposition [17, 18]. Many theoretical results indicate that strong-higher order squeezing due to the superposition of the states is achievable, which as far as one can tell from the literature, has not been studied yet in the multichannel Kerr nonlinear directional coupler (NLDC). The aim of this paper is to study the occurrences of compound modes squeezing in multichannel Kerr NLDC via phase-space representation implementing positive-*P* functions. The system is arranged in such a way that a more flexible coupling between the evanescent fields mode is possible. A well-defined high reflecting mirrors at both ends of the coupler channels are employed to provide natural confinement for nonlinearity amplification. Such mechanism allows for the possibilities of robust compound-mode squeezing of various forms to emerge.

2. Methodology

2.1 Model Formalism and Mathematical Solution

In order to study the compound-mode squeezing, we consider four-identical optical fibers placed in close proximity to each other. The medium of each channel is characterized by third-order Kerr nonlinearity. Each fiber channel is traversed by one mode and the modes are interacting with each other via evanescent overlaps of the modes. A schematic representation of the device is presented in Figure 1. Introducing a cavity setup on Kerr NLDC will resonantly amplify nonlinearities of the media. The use of cavity setup for studying various quantum behaviors has been applied previously in studying bright entanglement [19], Gaussian steering [20] as well as squeezing [21].

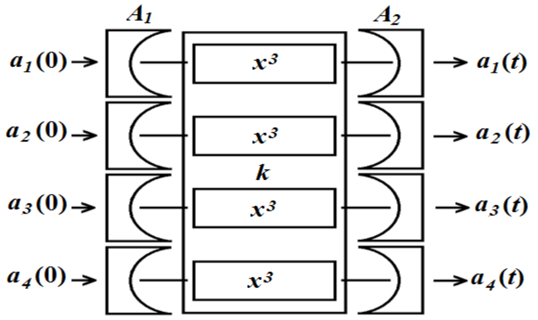

Figure 1. Schematic Representation of the Multichannel Nonlinear Coupler with Cavity Setup.

In Figure 1, the waveguide are placed in close proximity to each other to allow the periodical energy transfer through the Kerr like media. The high reflecting input mirror A_1 and mirror A_2 are placed at each end of the waveguide for optical confinements. The fundamental Mode a_1 is prepared in a coherent state while the other modes a_2, a_3 and a_4 are initially prepared in vacuum states. We, furthermore, assume that the evanescent waves are linked to each other through linear coupling strength k and energy exchanged is restricted between the main waveguide where energy will be launched at first, with the initial vacuum waveguides only. All of the initial vacuum states are preserved to be isolated from each other for all cases. The effective Hamiltonian controlling the above system may be written as

$$\hat{H} = \hbar \begin{pmatrix} \omega\sum\limits_{i=1}^{4} \hat{a}_i \hat{a}_i^\dagger + g\sum\limits_{i=1}^{4} \hat{a}_i^{\dagger 2}\hat{a}_i^2 + k(\sum\limits_{i=2}^{4} \hat{a}_1 \hat{a}_i^\dagger + \sum\limits_{i=2}^{4} \hat{a}_i \hat{a}_1^\dagger) \\ + i(E(t)e^{-i\omega_l t}\hat{a}_1^\dagger - E^*(t)e^{i\omega_l t}\hat{a}_1) + \sum\limits_{i=1}^{4}\left(\hat{a}_i \Gamma_{ci}^\dagger + \hat{a}_i^\dagger \Gamma_{ci}\right) \end{pmatrix} \cdot \quad (1)$$

In particular, Equation (1) gives the characteristics of continuous wave fields in nonlinear coupler in coupled-mode approximation. The dynamical behavior can be used as a generalization to several coupler systems such as those found in some studies [19, 22]. Without pumping classical force, the system Hamiltonian is expected to show behavior identical to two-mode [11] and three-mode [16] Kerr NLDC accordingly. By analogy, the system Hamiltonian also holds a close resemblance to several models of Bose-Einstein condensation [23]. The first term of Equation (1) gives the harmonic Hamiltonian of the system itself with $\hat{a}_i \hat{a}_i^\dagger$ being the creation and annihilation operators for the interacting modes respectively. The second term is responsible for the self-action Kerr coupling providing a third order nonlinear mechanism for the interaction. The linear coupling Hamiltonian permitting the flux transfer between waveguides is given in the third term. All modes are modeled to propagate in their respective waveguides after being launched and we assumed that exchanged of periodical evanescent energy occurs linearly via coupling k. The fourth term represents the classical pump amplitude with $E(t)$ and $E^*(t)$ as the magnitude of the driving field. Consider ω_l to be the carrier frequency of the cavity mode, the pumping amplitude in the reference system is given by $E(t)e^{-i\omega_l t} \to E(t)$ and $E^*(t)e^{-i\omega_l t} \to E^*(t)$. The fifth term is responsible for the cavity damping with $\Gamma_i = 2\gamma_i n_i^{th}$ being the reservoir operator for losses with i = 1, 2, 3 and 4 for each mode. Coupling of the damping term has been included through the Liouvillian terms [21] characterizing the field losses γ_i and the thermal fields n_i^{th}. We should also stress here that the focus of attention is on the system behavior driven by quantum fluctuation alone. Hence, the thermal fluctuation should be eliminated. On that account, since $\hbar\omega \gg k_B T$ for $n_i^{th} = (\exp(\hbar\omega_i / k_B T) - 1)^{-1}$, hence, at T = 0, the thermal fields n_i^{th} may be ignored [21] from the system equation in accordance with Markov and Born approximations. The time evolution of Equation (1) follows Liouville von Neumann equation of motion in the following form

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] \quad (2)$$

where $\hat{\rho}$ gives the density operator by analogy to the classical distribution function. A classical description of the quantum mechanical master equation may be obtained in phase-space if one introduces the quasiprobability function. The operator algebra rule [24] allows the derivation of the Fokker-Planck equation in the positive-P representation. The resulting equation is in the standard form thus permitting swift conversion to the Langevin stochastic differential equations as the following;

$$\begin{aligned}
\frac{\partial\alpha_1}{\partial\tau_s} &= -(i\delta_{s1} + \gamma_{s1})\alpha_1 - 2ig_s\beta_1\alpha_1^2 - ik_{s1}\alpha_2 - ik_{s2}\alpha_3 - ik_{s3}\alpha_4 + E_s + \sqrt{-2ig_s}\alpha_1\eta_1(\tau_s) \\
\frac{\partial\beta_1}{\partial\tau_s} &= (i\delta_{s1} + \gamma_{s1})\beta_1 + 2ig_s\alpha_1\beta_1^2 + ik_{s1}\beta_2 + ik_{s2}\beta_3 + ik_{s3}\beta_4 + E_s^* + \sqrt{2ig_s}\beta_1\eta_5(\tau_s) \\
\frac{\partial\alpha_2}{\partial\tau_s} &= -(i\delta_{s2} + \gamma_{s2})\alpha_2 - 2ig_s\beta_2\alpha_2^2 - ik_{s1}\alpha_1 + \sqrt{-2ig_s}\alpha_2\eta_2(\tau_s) \\
\frac{\partial\beta_2}{\partial\tau_s} &= (i\delta_{s2} - \gamma_{s2})\beta_2 + 2ig_s\alpha_2\beta_2^2 + ik_{s1}\beta_1 + \sqrt{2ig_s}\beta_2\eta_6(\tau_s) \\
\frac{\partial\alpha_3}{\partial\tau_s} &= -(i\delta_{s3} + \gamma_{s3})\alpha_3 - 2ig_s\beta_3\alpha_3^2 - ik_{s2}\alpha_1 + \sqrt{-2ig_s}\alpha_3\eta_3(\tau_s) \\
\frac{\partial\beta_3}{\partial\tau_s} &= (i\delta_{s3} - \gamma_{s3})\beta_3 + 2ig_s\alpha_3\beta_3^2 + ik_{s2}\beta_1 + \sqrt{2ig_s}\beta_3\eta_7(\tau_s) \\
\frac{\partial\alpha_4}{\partial\tau_s} &= -(i\delta_{s4} + \gamma_{s4})\alpha_4 - 2ig_s\beta_4\alpha_4^2 - ik_{s3}\alpha_1 + \sqrt{-2ig_s}\alpha_4\eta_4(\tau_s) \\
\frac{\partial\beta_4}{\partial\tau_s} &= (i\delta_{s4} - \gamma_{s4})\beta_4 + 2ig_s\alpha_4\beta_4^2 + ik_{s3}\beta_1 + \sqrt{2ig_s}\beta_4\eta_8(\tau_s)
\end{aligned} \quad (3)$$

where γ_s represents the input loss rate of the optical field from the cavity, δ_s gives the detuning between the beams and cavity mode, τ_s is the evolution parameter representing the coordinate of the normalized interaction length, k_s represents the evanescent linear coupling and g_s as the nonlinear coupling. The subscript s means that all parameters are in dimensionless form for numerical stability purpose. In the above system equations, the quantum creation and annihilation operators are replaced by c-number variables $\alpha_i \beta_i$ and submited to the following correspondence relationship $(\alpha_1\beta_1, \alpha_2\beta_2, \alpha_3\beta_3) = \langle \alpha_1\alpha_1^\dagger, \alpha_2\alpha_2^\dagger, \alpha_3\alpha_3^\dagger \rangle$. The variables $\alpha_i \beta_i$ are not complex conjugate to each other except in the mean fields due to the uncorrelated real Gaussian noise terms $\eta_i(\tau_s)$ resulting from the second quantization of the Hamiltonian.

2.2 Squeezing Criteria

Equation (3) enables one to predict the occurrence of squeezing in the form of normal ordered operator product. Squeezed light is a concept of squeezing the fluctuating radiation field to decrease the noise in one field quadrature. The process is obtainable at the cost of excess noise in the other field quadrature, so that the requirements of the uncertainty relation are preserved. In the most elementary case, the electric field of the propagating radiation mode can be described as [25];

$$\hat{E}(t) = 2\lambda[\hat{E}_1 \sin(\omega t - \vec{k}\cdot\vec{x}) - \hat{E}_2 \cos(\omega t - \vec{k}\cdot\vec{x})] \quad (4)$$

where $\hat{E}_1$ and $\hat{E}_2$ give the real and imaginary amplitude field quadratures respectively and λ is a constant number. The amplitude field quadratures differ in phase by 90° and both can be represented in term of the bosonic creation and annihilation operator to account for a normal squeezing of the first order as [15];

$$\hat{E}_{1,n} = \frac{1}{2}\sum_{j=1}^{n}(\hat{a}_j + \hat{a}_j^\dagger),$$
$$\hat{E}_{2,n} = \frac{1}{2i}\sum_{j=1}^{n}(\hat{a}_j - \hat{a}_j^\dagger). \quad (5)$$

with *n* being the mode of squeezing. In single-mode basis (*n*=1), fluctuation of the uncertainty in the radiation field variances submits to the Heisenberg uncertainty principle of the form

$$\left\langle (\Delta\hat{E}_{1,1})^2 \right\rangle \left\langle (\Delta\hat{E}_{2,1})^2 \right\rangle^{\frac{1}{2}} \geq \frac{1}{4} \quad (6)$$

where the variance in Equation (6) is characterized for an observable $\hat{E}$ as

$$\left\langle (\Delta\hat{E}_{h,1})^2 \right\rangle = \left\langle \hat{E}_{h,1}^2 \right\rangle - \left\langle \hat{E}_{h,1} \right\rangle^2 \quad (7)$$

with *h*=1,2 giving the definition of quadrature numbers. The system is generating squeezed light if any of the field quadrature variances fluctuates below the standard quantum limit so that,

$$\left\langle (\Delta\hat{E}_{k,1})^2 \right\rangle < \frac{1}{4}. \quad (8)$$

Equation (8) specifies the standard quantum limit in which the uncertainty is distributed equally between the quadratures field of the coherent state. For compound-mode squeezing, the uncertainty relation yields

$$\langle (\Delta\hat{E}_{1,j})^2 \rangle \langle (\Delta\hat{E}_{2,j})^2 \rangle \geq \frac{|c_j|^2}{16} \quad (9)$$

where *j* takes the value of 2, 3 and 4 for two-mode, three-mode, and four-mode respectively. Rearrange Equation (9) gives the squeezing expressions in the following form [15];

$$S_{1,j} = 4\langle (\Delta\hat{E}_{1,j})^2 \rangle - |c_j| \leq 0, S_{2,j} = 4\langle (\Delta\hat{E}_{2,j})^2 \rangle - |c_j| \leq 0 \quad (10)$$

3. Results and Discussion

Simulating the quantum trajectories to ascertain the possible existence of squeezing required one to specify at least one of the radiation states is coherent. In what follows, the dynamical simulation of Equation (3) is performed with consistent values of parameter combination; the initial state $\alpha_1 = 1, \alpha_2 = \alpha_3 = \alpha_4 = 0,$ the classical field amplitude $E_s = 1,$ the nonlinear self-action coupling coefficient $g_s = 0.01,$ the linear evanescent strength $k_s = 1$ and the cavity-input losses $\gamma_i = 0.001$ for all cases. Such combination connects the system with real physical quantities, derived in our previous work [11, 22]. The detuning δ_s depends on several design parameters such as the cavity resonance frequency, the separation between the cavity mirrors and the number of supported modes. For practical systems, we may safely assume δ_s to vary within a range of -100 to +100. In discussing the evolution of squeezing however, we shall restrict ourselves within three main scenarios where the impact is significant: (*i*) when the detuning value between the cavity and the driving field is small, $\delta_s = 1$, (*ii*) the detuning value is high, $\delta_s = 10$ and (*iii*) when it is zero, $\delta_s = 0.$

3.1 Single-Mode Squeezing

In Figure 2(a), we plot the dynamic of the quadrature variances in the initial coherent channel as a function of their normalized interaction length. At a small value of detuning, both quadrature evolution satisfied Equation (10) for single mode squeezing, *n* = 1. Although the quadrature squeezing is not portrayed for the other channels, the initial vacuum state actually exhibits an identical pattern of squeezing with maximal amplitude slightly below those observed in the coherent mode. As a direct consequences of Equation (6), the distribution of the real Gaussian noise between the two quadrature components is in symmetrical order. For long interaction length, the steady state behavior of the quadrature oscillation is observed with no recurring of squeezing.

Quadrature squeezing with strong oscillatory behavior occurs if the system is launched at high detuning value. The influence of higher detuning resembles the leaf-revival-collapsed-like squeezing as shown in Figure 2(b). Such intense squeezing oscillation is due to the large reduction of the quadrature period, which mainly resulting from the phase mismatch between the laser and the cavity frequency. Despite having an equal magnitude of squeezing with Figure 2(a), a high detuning value enables the fluctuation to violate the standard limit. For very short interaction distance, abrupt transfer of quantum noise between the two quadrature variances occurs in turn repeatedly.

We further investigated the possible existence of a squeezed light by simulating the system equation with zero detuning. The quadrature evolution of the coherent channel is given in Figure 2(c). The disappearance of mismatch between the two operating frequencies contributes to the modification of the way squeezed light is generated. Intensity fluctuation of the coherent state shows that only the first quadrature variance display squeezing. As opposed to the case in Figure 2(a),(b), the whole oscillation of the first quadrature variance can evolute completely below the shot noise level for a certain range without violating the uncertainty principle. Squeezing is most pronounced if the amplitude is maximal and this is reflected in the case of zero detuning.

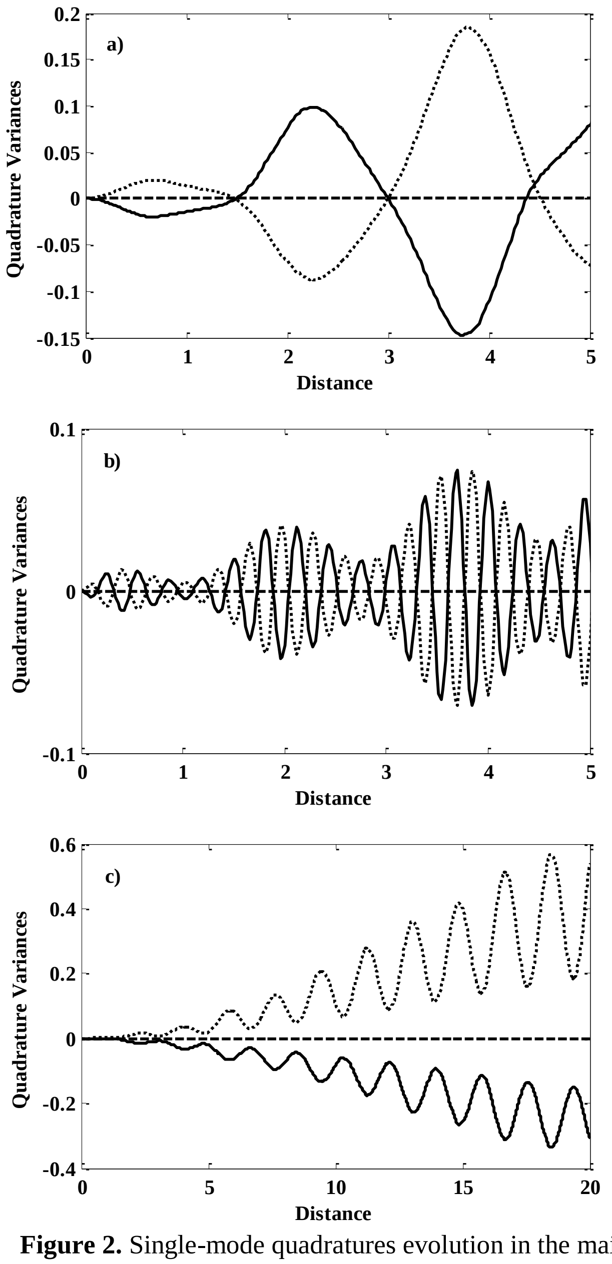

Figure 2. Single-mode quadratures evolution in the main channel for (a) $\delta_s = 1,$ (b) $\delta_s = 10,$ (c) $\delta_s = 0.$ The solid line is the first quadrature, while dotted line is the second quadrature and the dashed line is the shot noise level.

3.2 Compound-Mode Squeezing

The third order nonlinear optical interaction is one of the basic mechanisms required to generate squeezed light. To account for compound-mode squeezing, a multichannel system is introduced, so that possibilities for quantum correlation between the modes may exist which in turn enhances the generation of squeezed light. To see how the mechanism is manifested in the system, we focus on the highest compound-mode squeezing of the first order basis for a system with n = 4 in Equation (10). Also, for a fair comparison against the single-mode results, we have used identical controlling parameters. In Figure 3(a), we show the four-mode quadrature evolution of the main channel when the detuning value is small.

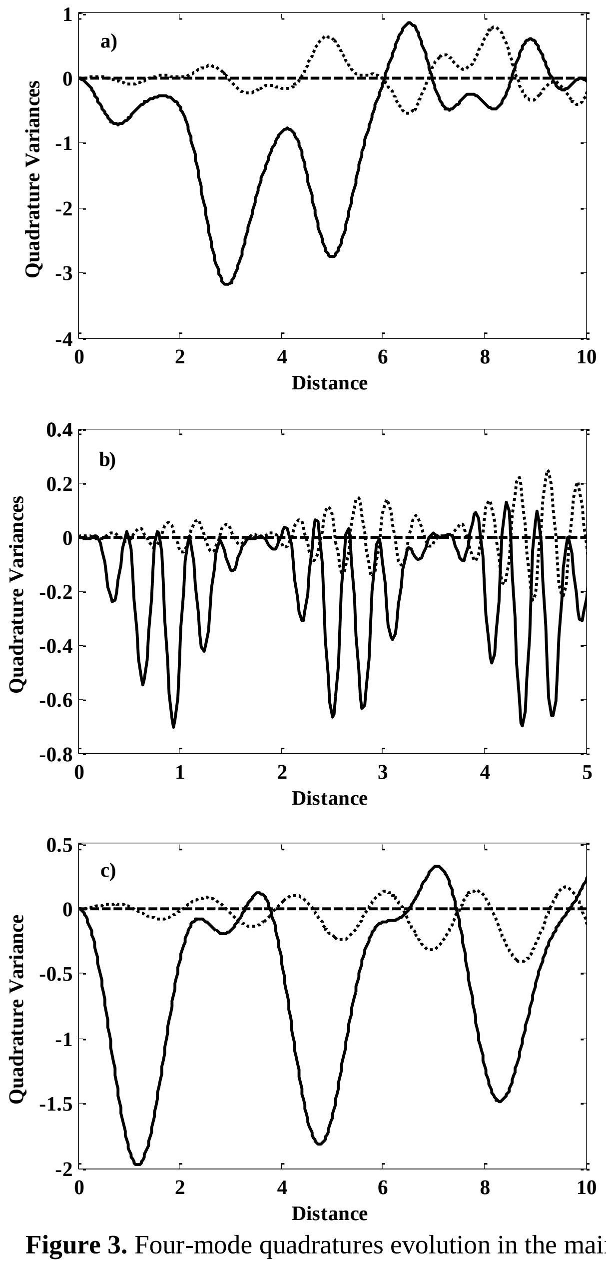

Figure 3. Four-mode quadratures evolution in the main channel for (a) $\delta_s = 1,$ (b) $\delta_s = 10,$ (c) $\delta_s = 0.$ The solid line is the first quadrature, the dotted line is the second quadrature and the dashed line is the shot noise level.

As one would expect, the multichannel system exemplifies strong violation of the standard quantum limit (dashed line) especially in the first quadrature (solid line). As opposed to the single-mode squeezing (Figure 2(a)), symmetrical noise distribution between the two quadrature is lacking in the four-mode version. Instead, the distribution of the noise appears to be major in the second quadrature, giving rise to the generation of stronger squeezing in the complementary quadrature as result.

For higher values of detuning parameter, Figure 3(b) reveals two apparent characters. Firstly, the four-mode quadrature variances retain the leaf-revival-collapse-like squeezing in the second quadrature with stronger violation of the standard limit. Secondly, the first quadrature of the four-mode squeezing breaks the single-mode pattern while showing the tendency of generating even stronger violation, giving a maximum squeezing consistently over the range of interaction length.

We next examine the four-mode squeezing when the detuning is fixed at zero. In Figure 3(c), the result for the four-mode squeezing suggests that, the quadrature variances may exhibit squeezing in both quadratures. Note that squeezing existed only in the first quadrature for the single-mode case. The spectrum of the second quadrature squeezing in four-mode basis is comparable to the single-mode squeezing. However, much better maximum noise reduction is observed in the first quadrature.

4. Conclusion

In conclusion, we have performed fully quantum analysis of squeezing up to a four-mode basis for a multichannel Kerr NLDC. We find that the squeezing properties for all cases are sensitive to the controlling detuning parameter. In general, a four-mode device displays a much better squeezing profile compared with the single-mode squeezing. Such criteria promote the system as an alternative device for amplifying quantum squeezing in coupler-based applications.

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