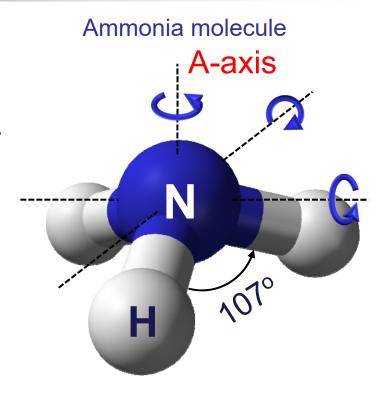
### Quantitative Laser Diagnostics for Combustion Chemistry and Propulsion



### **Lecture 4: Polyatomic Spectra**

- 1. From diatomic to polyatomic
- 2. Classification of polyatomic molecules
- Rotational spectra of polyatomic molecules
- 4. Vibrational bands, vibrational spectra





### 1. From diatomic to polyatomic

Rotation – Diatomics

Recall: For diatomic molecules

Energy: 
$$F(J), cm^{-1} = BJ(J+1) - DJ^{2}(J+1)^{2}$$

Centrifugal distortion constant

Rotational constant: 
$$B, cm^{-1} = \frac{h}{8\pi^2 Ic}$$

Selection Rule: 
$$J' = J'' + 1 \rightarrow \Delta J = +1$$

Line position: 
$$\overline{v}_{J''+1 \leftarrow J''} = 2B(J''+1) - 4D(J''+1)^3$$

Notes:

1. D is small, i.e., 
$$D/B = 4(B/\overline{\nu}_{vib})^2 << 1$$

2. E.g., for NO, 
$$\left(\frac{D}{B}\right)_{NO} = 4\left(\frac{B}{\omega_e}\right)^2 \approx 4\left(\frac{1.7}{1900}\right)^2 \approx 3 \times 10^{-6}$$

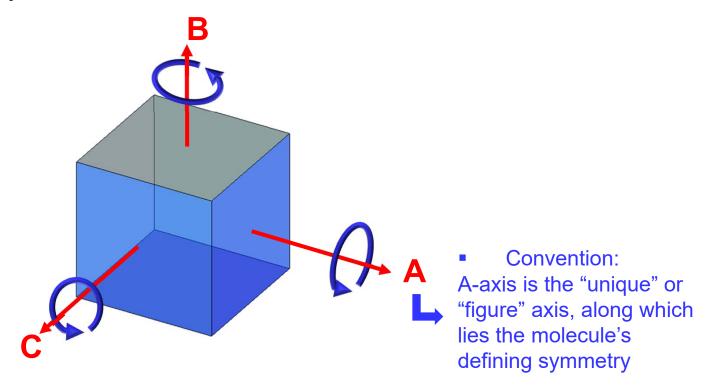
→ Even @ J=60, 
$$D/B \cdot J^2 \sim 0.01$$

What about polyatomics (≥3 atoms)?



### 1. From diatomic to polyatomic

3D-body rotation



- 3 principal axes (orthogonal): A, B, C
- 3 principal moments of inertia: I<sub>A</sub>, I<sub>B</sub>, I<sub>C</sub>
- Molecules are classified in terms of the relative values of I<sub>A</sub>, I<sub>B</sub>, I<sub>C</sub>



Types of molecules

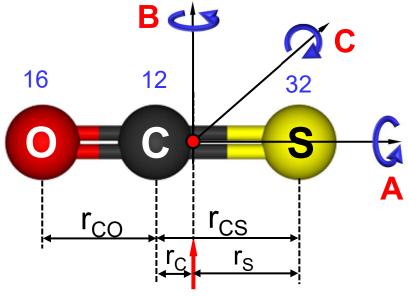
<i>y</i> ,				
Туре	Linear Molecules	Symmetric Tops	Spherical Tops	Asymmetric Rotors
Relative magnitudes of I <sub>A,B,C</sub>	I <sub>B</sub> =I <sub>C</sub> ; I <sub>A</sub> ≈0*	I <sub>B</sub> =I <sub>C</sub> ≠I <sub>A</sub> I <sub>A</sub> ≠0	I <sub>A</sub> =I <sub>B</sub> =I <sub>C</sub>	I <sub>A</sub> ≠I <sub>B</sub> ≠I <sub>C</sub>
Examples	CO <sub>2</sub> C <sub>2</sub> H <sub>2</sub> Acetylene OCS Carbon oxysulfid		CH <sub>4</sub> oron ichloride	H <sub>2</sub> O NO <sub>2</sub>
	Relative	ely simple	No dipole moment	0

<sup>\*</sup>Actually finite, but quantized momentum means it is in lowest state of rotation



#### Linear molecules

E.g., Carbon oxy-sulfide (OCS)

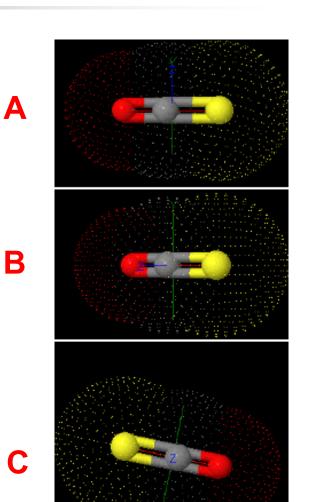


Center of mass

$$r_{CO} = 1.165 \text{Å}$$
  
 $r_{CS} = 1.558 \text{Å}$ 

$$I_{B}=I_{C}; I_{A}\approx 0$$

$$B, cm^{-1} = \frac{h}{8\pi^{2}I_{B}c}$$





Symmetric tops

### $I_B = I_C \neq I_A$ ; $I_A \neq 0$

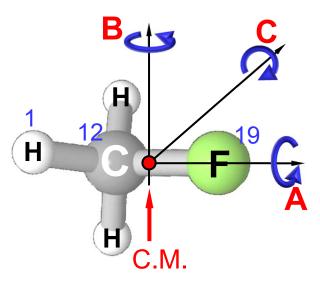
$$A, cm^{-1} = \frac{h}{8\pi^2 I_A c}$$

$$B, cm^{-1} = \frac{h}{8\pi^2 I_B c}$$

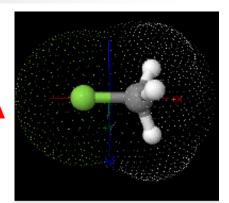
$$C, cm^{-1} = \frac{h}{8\pi^2 I_C c}$$

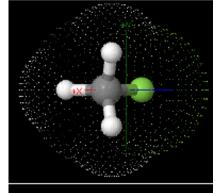
#### **Prolate**

 $I_A < I_B = I_C$ , A > B = CE.g.,  $CH_3F$ 

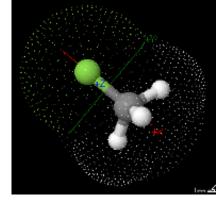


**Tripod-like (tetrahedral bonding)** 





B



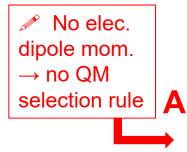


Symmetric tops

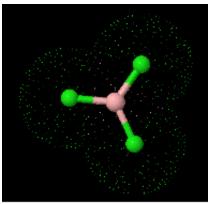
### **Oblate**

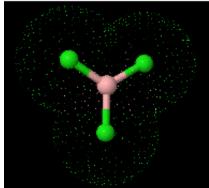
 $I_A > I_B = I_C$ , A < B = C

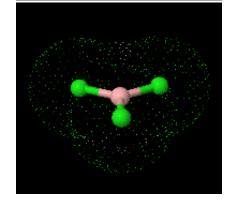
E.g., BCl<sub>3</sub> (Planar)

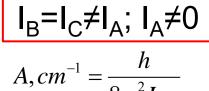


B



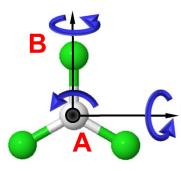


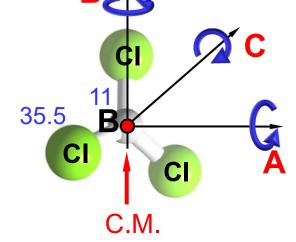




$$B, cm^{-1} = \frac{h}{8\pi^2 I_B c}$$

$$C, cm^{-1} = \frac{h}{8\pi^2 I_C c}$$





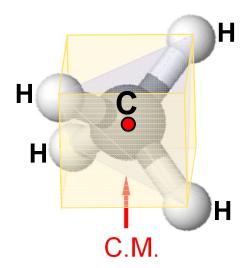




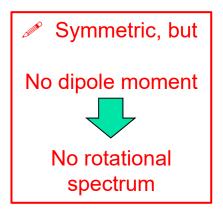
Spherical tops

$$I_A = I_B = I_C$$

E.g., CH<sub>4</sub> (methane)

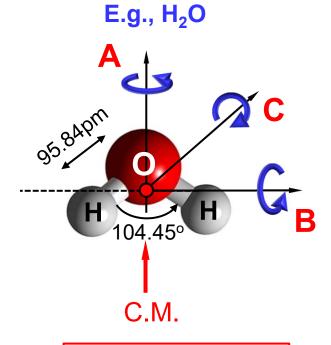


Cube w/ C at center and H at diagonal corners



Asymmetric rotors





Complex and not addressed here



- Linear molecules (I<sub>B</sub>=I<sub>C</sub>; I<sub>A</sub>≈0)
  - Examples

OCS HCN CO<sub>2</sub> C<sub>2</sub>H<sub>2</sub> HC<sub>2</sub>Cl

Symmetric, no dipole moment



Must be asymmetric to have electric dipole moment (isotopic substitution doesn't change this as bond lengths remain fixed)

Energies and line positions

Can treat like diatomic (1 value of I) → same spectrum

$$F(J) = BJ(J+1) - DJ^{2}(J+1)^{2}$$

$$\overline{V}(J) = 2B(J+1) - 4D(J+1)^{3}$$

$$\uparrow$$
Rotational Centrifugal const. distortion const.

Note: Larger I, smaller B (& line spacing) than diatomics

(" is suppressed, i.e. J=J")

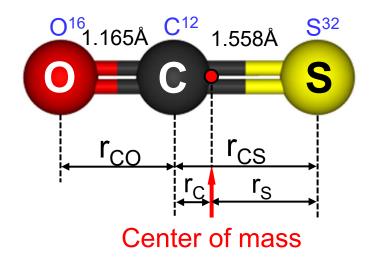


- Linear molecules (I<sub>B</sub>=I<sub>C</sub>; I<sub>A</sub>≈0)
  - Bond lengths

N atoms N-1 bond lengths to be found

- Abs./Emis. spectra ⇒ B ⇒ 1 value of I<sub>B</sub>
- Use N-1 isotopes → N-1 values of I<sub>B</sub>

#### Example: OCS (carbon oxy-sulfide)



#### Use 2 isotopes for 2 equations:

$$I_{^{16}O^{12}C^{32}S} = \mathcal{F}(masses, r_{CO}, r_{CS})$$
 $I_{^{18}O^{12}C^{32}S} = \mathcal{F}(masses, r_{CO}, r_{CS})$ 



Solve for  $r_{CO}$ ,  $r_{CS}$ 



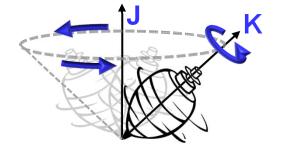
- Symmetric tops (I<sub>B</sub>=I<sub>C</sub>≠I<sub>A</sub>; I<sub>A</sub>≠0)
  - 2 main directions of rotation → 2 quantum numbers
    - **J** (total angular momentum): 0, 1, 2, ...
    - **K** (angular momentum about A): J, J-1, ..., 1, 0, -1, ... -J
    - + & allowed, w/o change in energy

⇒ 2J+1 possibilities of K for each J

Quantized angular momentum

As before: 
$$I_A^2 \omega_A^2 + I_B^2 \omega_B^2 + I_C^2 \omega_C^2 = J(J+1)\hbar^2$$

Plus new:  $I_A^2 \omega_A^2 = K^2 \hbar^2$ 



Energy levels

$$E_{J,K} = \frac{1}{2} \sum_{i} I_{i} \omega_{i}^{2}$$

$$F(J,K) = BJ(J+1) + (A-B)K^{2}$$

Note degeneracy, i.e., independent of sign of K



- Symmetric tops (I<sub>B</sub>=I<sub>C</sub>≠I<sub>A</sub>; I<sub>A</sub>≠0)
  - Q.M. Selection rules

• 
$$\Delta J = +1$$
 Remember that  $\Delta J = J' - J''$ 

 $\Delta K = 0$ 

No dipole moment for rotation about A-axis No change in K will occur with abs./emis.

Line positions

$$\overline{v}_{J.K} = F(J+1,K) - F(J,K) = 2B (J+1) [cm^{-1}]$$

Note: Independent of K for a rigid rotor Same as rigid diatomic! K-dependence introduced for non-rigid rotation



- Symmetric tops (I<sub>B</sub>=I<sub>C</sub>≠I<sub>A</sub>; I<sub>A</sub>≠0)
  - Non-rigid rotation

Effect of extending bond lengths (w/ changes in K)



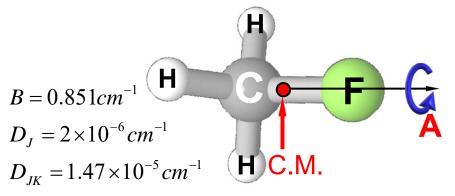
Change energies of rotation



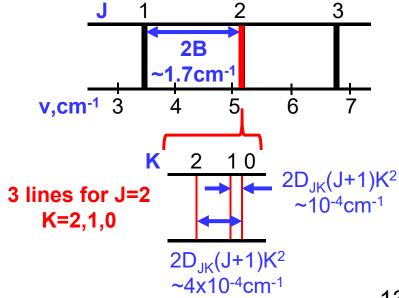
Centrifugal distortion const. D<sub>J</sub>, D<sub>K</sub>, D<sub>JK</sub>

$$F(J,K) = BJ(J+1) + (A-B)K^{2} - D_{J}J^{2}(J+1)^{2}$$
$$-D_{JK}J(J+1)K^{2} - D_{K}K^{4}$$
$$-\frac{1}{V_{J.K}} = 2(J+1)[B-2D_{J}(J+1)^{2} - D_{JK}K^{2}] [cm^{-1}]$$

Note: Each J has 2J+1 components, but only J+1 frequencies E.g., CH<sub>3</sub>F, Methyl Fluoride



If J≈20, J<sup>2</sup>≈400, 2DJ<sup>2</sup>≈1.6x10<sup>-3</sup>, 2DJ<sup>2</sup>/B≈.2%





Symmetric tops  $(I_B=I_C\neq I_A;I_A\neq 0) \rightarrow \text{gets complex fast!}$ 

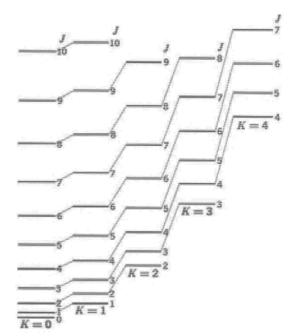
#### **Prolate**

$$I_A < I_B = I_C$$
,  $A > B = C$ 

$$F(J,K) = BJ(J+1) + (A-B)K^{2}$$

$$A - B = \frac{h}{8\pi^{2}c} \left(\frac{1}{I_{A}} - \frac{1}{I_{B}}\right) > 0$$

### Example energy levels

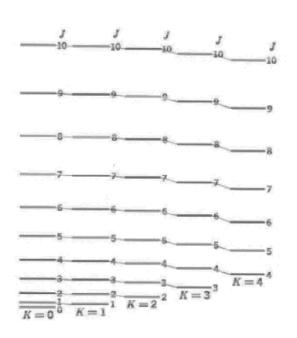


#### **Oblate**

$$I_A > I_B = I_C$$
,  $A < B = C$ 

$$(J,K) = BJ(J+1) + (A-B)K^{2} F(J,K) = BJ(J+1) + (A-B)K^{2}$$

$$A - B = \frac{h}{8\pi^{2}c} \left(\frac{1}{I_{A}} - \frac{1}{I_{B}}\right) > 0 A - B = \frac{h}{8\pi^{2}c} \left(\frac{1}{I_{A}} - \frac{1}{I_{B}}\right) < 0$$





### Rotational partition function

Linear	Symmetric top	Spherical top	Asymmetric rotor	
B=C; I <sub>A</sub> ≈0	B=C≠A; I <sub>A</sub> ≠0	A=B=C	A≠B≠C	
$Q_{rot} = \frac{kT}{\sigma h c B}$	$Q_{rot} = \frac{1}{\sigma} \sqrt{\frac{\pi}{AB^2} \left(\frac{kT}{hc}\right)^3}$	$Q_{rot} = \frac{1}{\sigma} \sqrt{\frac{\pi}{B^3} \left(\frac{kT}{hc}\right)^3}$	$Q_{rot} = \frac{1}{\sigma} \sqrt{\frac{\pi}{ABC} \left(\frac{kT}{hc}\right)^3}$	

$$A, cm^{-1} = \frac{h}{8\pi^2 I_A c}$$

$$B, cm^{-1} = \frac{h}{8\pi^2 I_B c}$$

$$C, cm^{-1} = \frac{h}{8\pi^2 I_C c}$$

### σ – molecule-dependent symmetry factor

Molecule	σ	Molecule Type
CO <sub>2</sub>	2	Linear
NH <sub>3</sub>	3	Symmetric Top
CH <sub>4</sub>	12	Spherical Top
H <sub>2</sub> O	2	Asymmetric Rotor

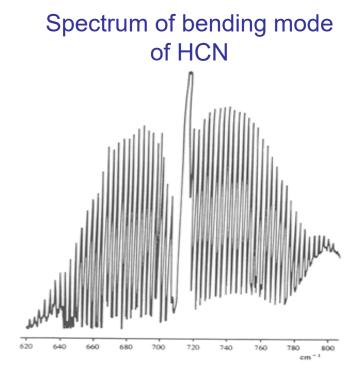


- Linear (diatomic & polyatomic) and symmetric top molecules give similar (equal spacing) spectra at rigid rotor level
- High resolution needed to detect corrections / splittings
- Spectra → microscopic parameters (r<sub>e</sub>, angles)
- Isotopes useful for spectral studies



### 4. Vibrational Bands, Rovibrational Spectra

- 1. Number of vibrational modes
- 2. Types of bands
  - Parallel and perpendicular
  - Fundamental, overtones, combination and difference bands
- 3. Relative strengths
- Rovibrational spectra of polyatomic molecules
  - Linear molecules
  - Symmetric tops





### 1. Number of vibrational modes

N-atom molecule

**3N** dynamical coordinates needed to specify instantaneous <u>location</u> and <u>orientation</u>

Total: 3N

**Center of Mass: 3** coordinates (3 translational modes)

**Rotation:** Linear molecules Nonlinear molecules

2 angular coordinates3 angular coordinates

(rot. modes)

(rot. modes)

Vibration: Linear molecules Nonlinear molecules

**3N-5** vibrational coordinates **3N-6** vibrational coordinates

(vib. modes) (vib. modes)



- Numbering (identification) convention of vibrational modes
  - Symmetry
  - Decreasing energy

	cm <sup>-1</sup> ]	
Symmetric Declining frequency	<ul> <li>V<sub>1</sub> Highest-frequency <u>symmetric</u> vibrational mode</li> <li>V<sub>2</sub> 2<sup>nd</sup> highest symmetric mode</li> </ul>	<b>;</b>
	<ul><li>ν<sub>i</sub> Lowest-frequency <u>symmetric</u> mode</li></ul>	
Asymmetric Declining frequency	<ul> <li>V<sub>i+1</sub> Highest-frequency <u>asymmetric</u> vibrational mod</li> <li>V<sub>i+2</sub> 2<sup>nd</sup> highest symmetric mode</li> <li></li> </ul>	de

Exception: the perpendicular vibration for linear  $XY_2$  and XYZ molecules is always called  $v_2$ 



Parallel and perpendicular modes

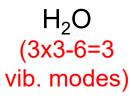
#### Examples:

### Parallel (||)

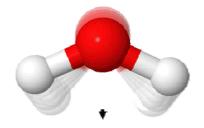
Dipole changes are || to the main axis of symmetry

#### Perpendicular (⊥)

Dipole changes are 1 to the main axis of symmetry

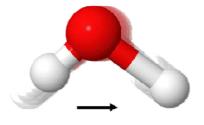




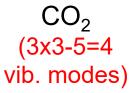


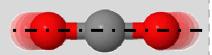
 $v_1 = 3652 \text{cm}^{-1}$ 

Symmetric stretch Symmetric bending  $v_2 = 1595 \text{cm}^{-1}$ 

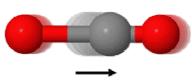


Asymmetric stretch  $v_3 = 3756 \text{cm}^{-1}$ 

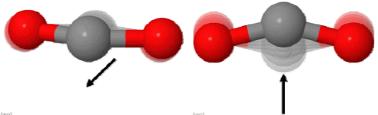




No dipole moment Not IR-active! Symmetric stretch  $v_1 = 1330 \text{ cm}^{-1}$ 



Asymmetric stretch  $v_3 = 2349 \text{cm}^{-1}$ 



Symmetric bending (2 degenerate)  $\nu_2 = 667 \text{ cm}^{-1}$ 



### Parallel and perpendicular modes

Symmetric molecules: vibrational modes are <u>either</u> IR-active <u>or</u> Raman-active

Vibrational modes of CO<sub>2</sub>

Mode	Frequency [cm <sup>-1</sup> ]	Туре	Description	IR	Raman
$\nu_1$	1388		Symmetric stretch	Not active	Active
$v_2$	667	Т	Symmetric bend (Degenerate)	Strong	Not active
$\nu_3$	2349		Asymmetric stretch	Very strong	Not active

#### Vibrational modes of HCN

Mode	Frequency [cm <sup>-1</sup> ]	Туре	Description	IR	Raman
$v_1$	3310		Symmetric stretch	Strong	Weak
$v_2$	715	Т	Symmetric bend (Degenerate)	Very strong	Weak
$\nu_3$	2097		Asymmetric stretch	Weak	Strong



Terminology for different types of vibrational bands

Fundamental Bands:  $v_i$ , the i<sup>th</sup> vibrational mode;  $\Delta v = v' - v'' = 1$  for the i<sup>th</sup> mode

1st Overtone:  $2v_i$ ;  $\Delta v = v' - v'' = 2$  for the i<sup>th</sup> mode

2<sup>nd</sup> Overtone:  $3v_i$ ;  $\Delta v = v' - v'' = 3$  for the i<sup>th</sup> mode

Combination bands: Changes in multiple quantum numbers, e.g.,

 $v_1+v_2$ ;  $\Delta v_1=\Delta v_2=1$ , i.e.,  $v_1$  and  $v_2$  both increase by 1 for

absorption or decrease by 1 for emission

 $2v_1+v_2$ ;  $\Delta v_1=2$  and  $\Delta v_2=1$ 

**<u>Difference bands</u>**: Quantum number changes with mixed sign

 $v_1$ - $v_2$ ;  $v_{1,\text{final}}$ - $v_{1,\text{initial}}$ =  $\pm 1$  and  $v_{2,\text{final}}$ - $v_{2,\text{initial}}$ =  $\mp 1$ , i.e., a

unit increase in  $v_1$  is accompanied by a unit

decrease in  $v_2$ , and vice-versa.



Vibrational partition function

$$Q_{vib} = \prod_{i}^{\text{modes}} \left[ 1 - \exp\left(-\frac{hc\,\omega_{e,i}}{kT}\right) \right]^{-g_i}$$

E.g., 
$$NH_3$$
: 3N-6 = 6 vib. modes

E.g., 
$$\underline{\text{NH}}_{\underline{3}}$$
: 3N-6 = 6 vib. modes
$$Q_{vib} = \left[1 - \exp\left(-\frac{hc\omega_{e,1}}{kT}\right)\right]^{-1} \left[1 - \exp\left(-\frac{hc\omega_{e,2}}{kT}\right)\right]^{-1} \left[1 - \exp\left(-\frac{hc\omega_{e,3}}{kT}\right)\right]^{-2} \left[1 - \exp\left(-\frac{hc\omega_{e,4}}{kT}\right)\right]^{-2}$$

Vibration	Frequency [cm <sup>-1</sup> ]	Type	Description
$\nu_1$	3337	II	Symmetric stretch
$\nu_2$	950		Symmetric bend
$ u_3$	3444	Т	Asymmetric stretch (Degenerate)
$\nu_4$	1627	Т	Asymmetric bend (Degenerate)



### 4.3. Relative strength

- In general
  - Fundamental bands are much stronger than combination, difference, and overtone bands
- Fairly harmonic molecules
  - E.g., CO
  - Relative strength between fundamental and overtones ~ 10<sup>2</sup>
  - Closely SHO, overtone bands are *nearly* forbidden (low transition probabilities)
- Highly anharmonic molecules
  - E.g., NH<sub>3</sub>
  - Relative strength between fundamental and overtones ≤ 10
  - Overtone bands are less forbidden

#### <u>Exception – Fermi resonance</u>:

Accidental degeneracies (i.e., near resonances) can strengthen weak processes. Two vibrational modes strongly coupled by radiative and collisional exchanges. E.g.,  $2v_2 CO_2$  (i.e.  $2 \times 667 \text{ cm}^{-1} = 1334 \text{ cm}^{-1}$ )  $\approx v_{1, CO2}$ 



Linear polyatomic molecules

(limit consideration to fundamental transitions)

Energy: 
$$T(v_i, J) = G(v_i) + F(J)$$

Case I: Parallel bands (symmetric and asymmetric stretch)

Selection Rule:  $\Delta v_i = 1$ 

$$\Delta J = \pm 1$$

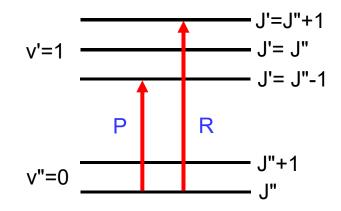
(R and P branches)

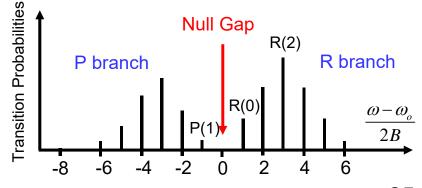
$$\Delta \mathbf{v}_{j} = 0, j \neq i$$

Absorption Spectrum: P & R branches only

Example:  $HCN(\nu_1, \nu_3)$ 



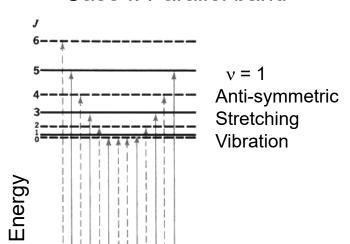




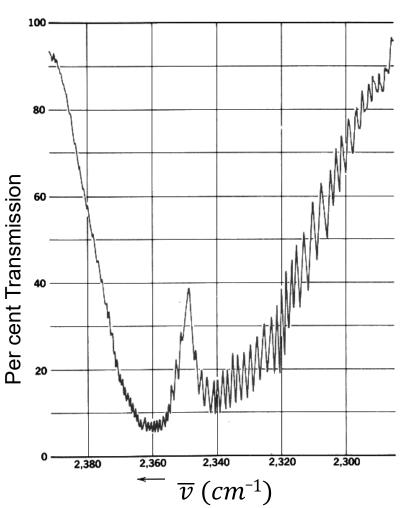


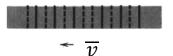
### Linear polyatomic molecules

Case I: Parallel band









Example-2: A parallel band  $(v_3)$  of the linear molecule  $CO_2$ 



Linear polyatomic molecules

(limit consideration to fundamental transitions)

Energy: 
$$T(v_i, J) = G(v_i) + F(J)$$

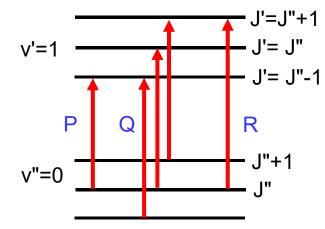
Case II: Perpendicular bands

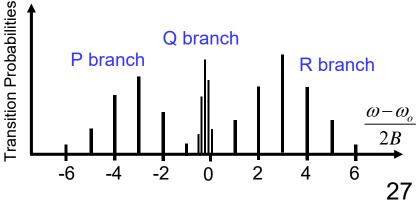
Selection Rule: 
$$\Delta v_i = 1$$
  
 $\Delta J = \pm 1,0$ 

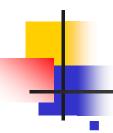
R, P and Q branches)

$$\Delta \mathbf{v}_{j} = 0, j \neq i$$

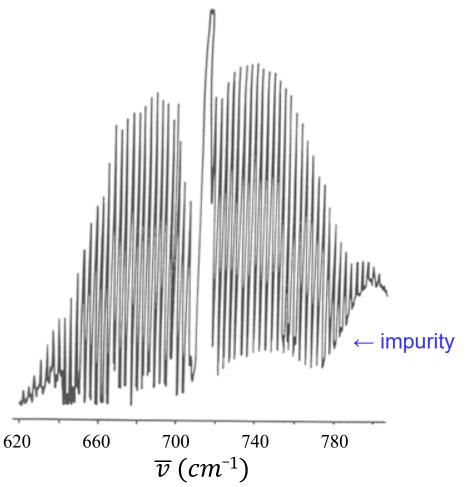
- 1. If B'=B", all Q branch lines occur at the same frequency
- 2. If B' $\neq$ B",  $Q(J") = \omega_0 + (B'-B")J"(J"+1)$ Q branch "degrades" to lower frequencies (i.e., to the "red" in wavelength)







- Linear polyatomic molecules
  - Case II: Perpendicular bands



#### Example:

Spectrum of the  $v_2$  bending mode of HCN, showing the PQR structure

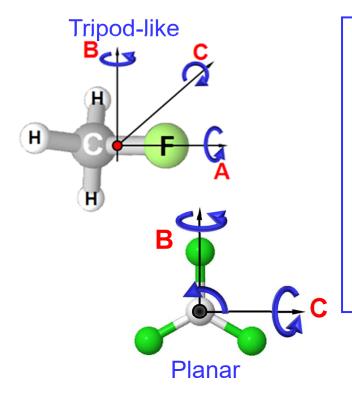


Symmetric top molecules (e.g., CH<sub>3</sub>F, BCl<sub>3</sub>)

Recall: K – quantum number for angular momentum around axis A

Energy: 
$$T(v_i, J, K) = G(v_i) + F(J, K)$$
  
=  $(v_i + 1/2)\omega_e^i - \omega_e^i x_e^i (v_i + 1/2)^2 + BJ(J+1) + (A-B)K^2$ 

Case I: Parallel bands

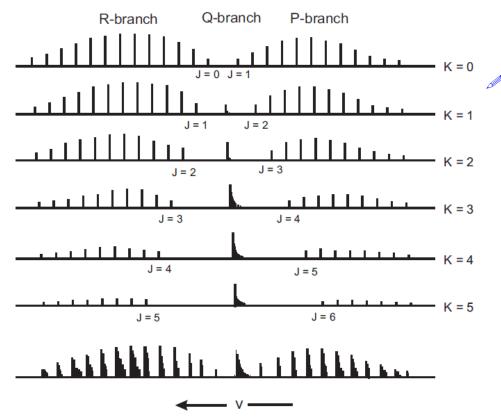


Selection Rule: 
$$\Delta v_i = 1$$
  
 $\Delta J = \pm 1,0 (P,Q,R \text{ branches})$   
 $\Delta K = 0$ 

- 1. 2J+1 values of K (K=J, J-1, ..., 0, ..., -J)
- 2. Intensity of Q branch is a function of  $(I_A/I_B)$
- 3. As  $(I_A/I_B) \rightarrow 0$ symmetric top  $\rightarrow$  linear molecule strength of Q branch  $\rightarrow 0$



- Symmetric top molecules (e.g., CH<sub>3</sub>F, BCl<sub>3</sub>)
  - Case I: Parallel bands



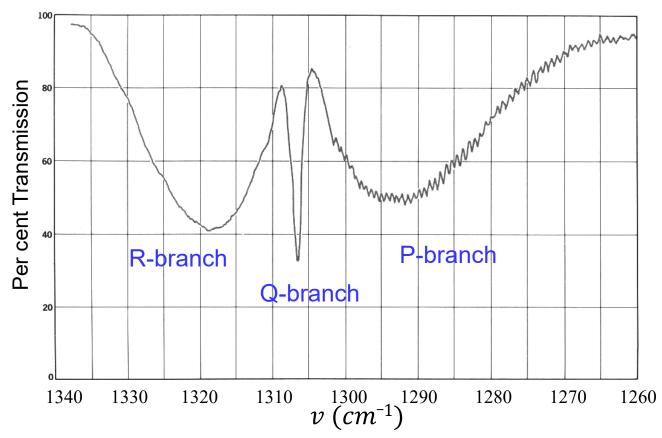
Resolved components of a parallel band showing contributions from each of the K levels of the v=0 state

#### Note:

- Splitting in P and R branch due to a difference in (A-B) in upper and lower vib. levels
- 2. Splitting in Q branch due to difference in B in upper and lower vib. levels
- 3. For K=0, spectrum reduces to that of linear molecules, no Q branch
- 4. K cannot exceed J



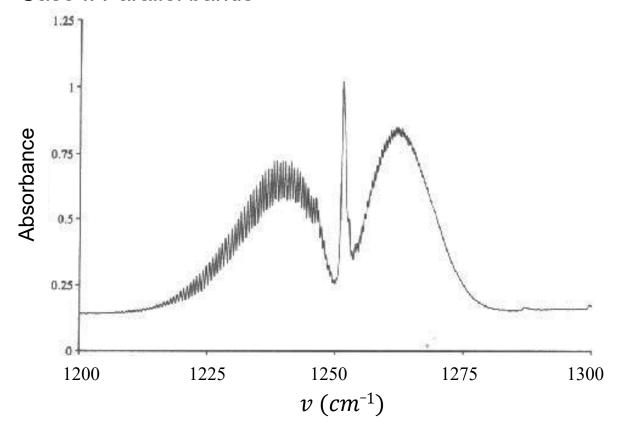
- Symmetric top molecules
  - Case I: Parallel bands



Example-1: A parallel absorption band of the symmetric top molecule CH<sub>3</sub>Br. The P branch is partly resolved, while only the contours of the R and Q branches is obtained



- Symmetric top molecules
  - Case I: Parallel bands



Example-2: The parallel stretching vibration, centered at 1251 cm<sup>-1</sup>, of the symmetric top molecule CH<sub>3</sub>I, showing the typical PQR contour.



- Symmetric top molecules (e.g., CH<sub>3</sub>F, BCl<sub>3</sub>)
  - Case II: Perpendicular bands

Selection Rule: 
$$\Delta v_i = 1$$
  
 $\Delta J = \pm 1,0 \ (P,Q,R \text{ branches})$   
 $\Delta K = \pm 1$   
R Branch:  $\Delta J = +1, \Delta K = \pm 1$   
 $v_R = \omega_o + 2B(J+1) + (A-B)(1 \pm 2K)$   
P Branch:  $\Delta J = -1, \Delta K = \pm 1$ 

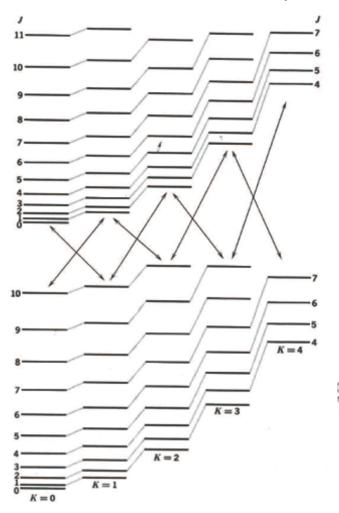
Q Branch: 
$$\Delta J = 0, \Delta K = \pm 1$$
  
 $v_Q = \omega_o + (A - B)(1 \pm 2K)$ 

Note: Two sets of R, P and Q branches for each lower state value of K

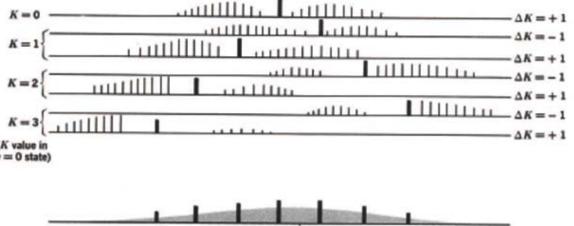
 $\bar{v}_{p} = \omega_{0} - 2BJ + (A - B)(1 \pm 2K)$ 



- Symmetric top molecules (e.g., CH<sub>3</sub>F, BCl<sub>3</sub>)
  - Case II: Perpendicular bands



- Energy levels of a symmetric top molecule showing transitions that are allowed for a perpendicular band
- Resulting spectrum, components of a perpendicular band showing the contributions from each K levels of the v=0 state

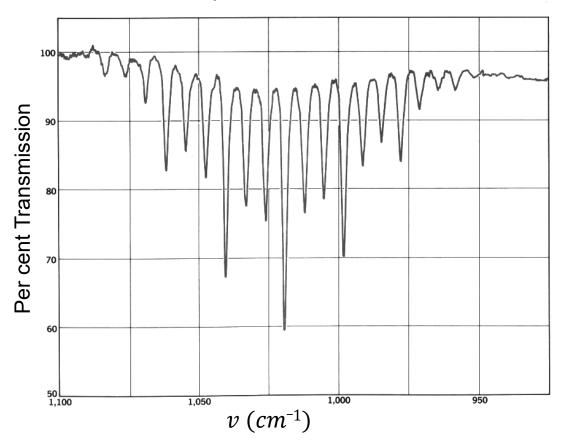


 $v_0$ 

 $\leftarrow v$ 

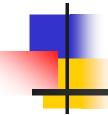


- Symmetric top molecules
  - Case II: Perpendicular bands



Note: Spacing of the Q branch lines in a perpendicular band can be identified with 2(A-B), and hence are observable if A-B is large enough

Example: The Q-branch of a perpendicular band, for the symmetric top molecule CH<sub>3</sub>Cl



### **Next: Quantitative Emission/Absorption**

- Spectral Absorptivity
- Eqn. of Radiative Transfer
- Einstein Coefficients/Theory
- Radiative Lifetime, Line Strength