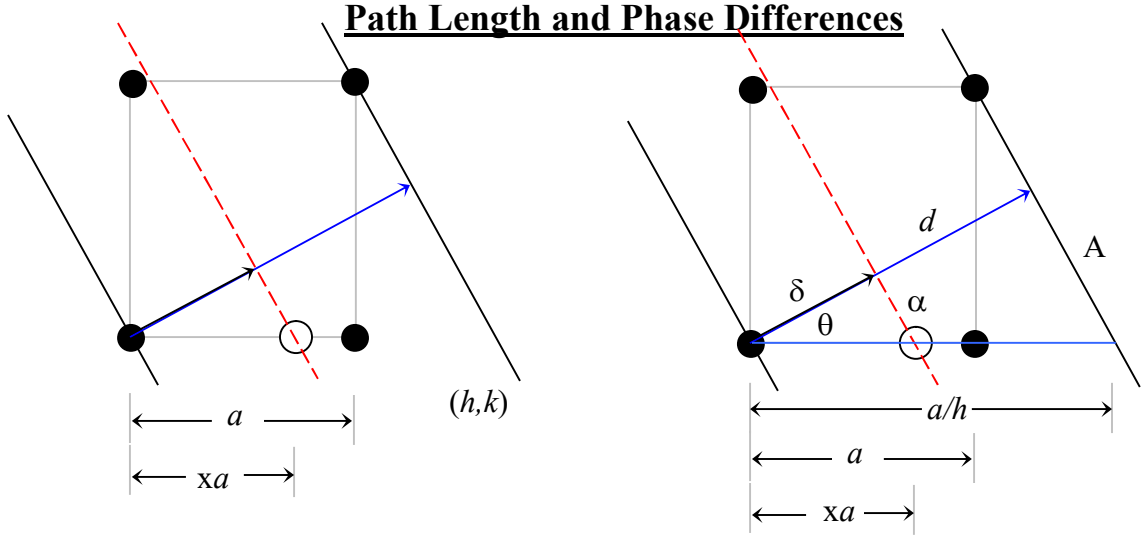
# Path Length and Phase Differences




Unit cell side = a  Set of diffraction planes = $(h,k)$

New atom in unit cell at position = x$a$

Path length difference for rays reflecting from old and new planes = δ

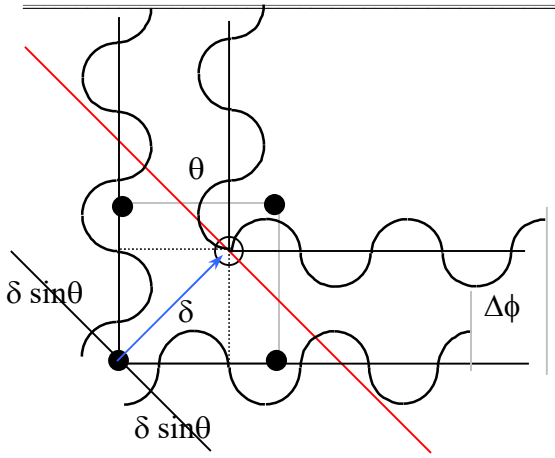
for large Δ: $d^2 = \frac{a^2}{h^2} - A^2$

for small Δ: $\delta^2 = x^2a^2 - \alpha^2$

$\alpha = xa \sin\theta$  $A = \frac{a}{h}\sin\theta$

$\left.\begin{aligned}\delta^2 &= x^2a^2(1-\sin^2\theta)\\ d^2 &= \frac{a^2}{h^2}(1-\sin^2\theta)\end{aligned}\right\}$  $\frac{\delta}{d} = \frac{xa}{\frac{a}{h}} = hx$

$\delta = (hx)\,d$  in general: $\delta = (hx + ky + lz)\,d$




diffraction angle: $2\,d\sin\theta = \lambda$

$\sin\theta = \lambda/2d$

extra path length: $\Delta x = 2\,\delta\sin\theta = \delta\lambda/d$

phase difference: $\Delta x\,\frac{2\pi}{\lambda} = 2\pi\,\frac{\delta}{d}$

phase difference: $\Delta\phi = 2\pi\,(hx + ky + lz)$

plane wave amplitude: $e^{2\pi i\,(hx + ky + lz)}$