

Turbulent Premixed Combustion

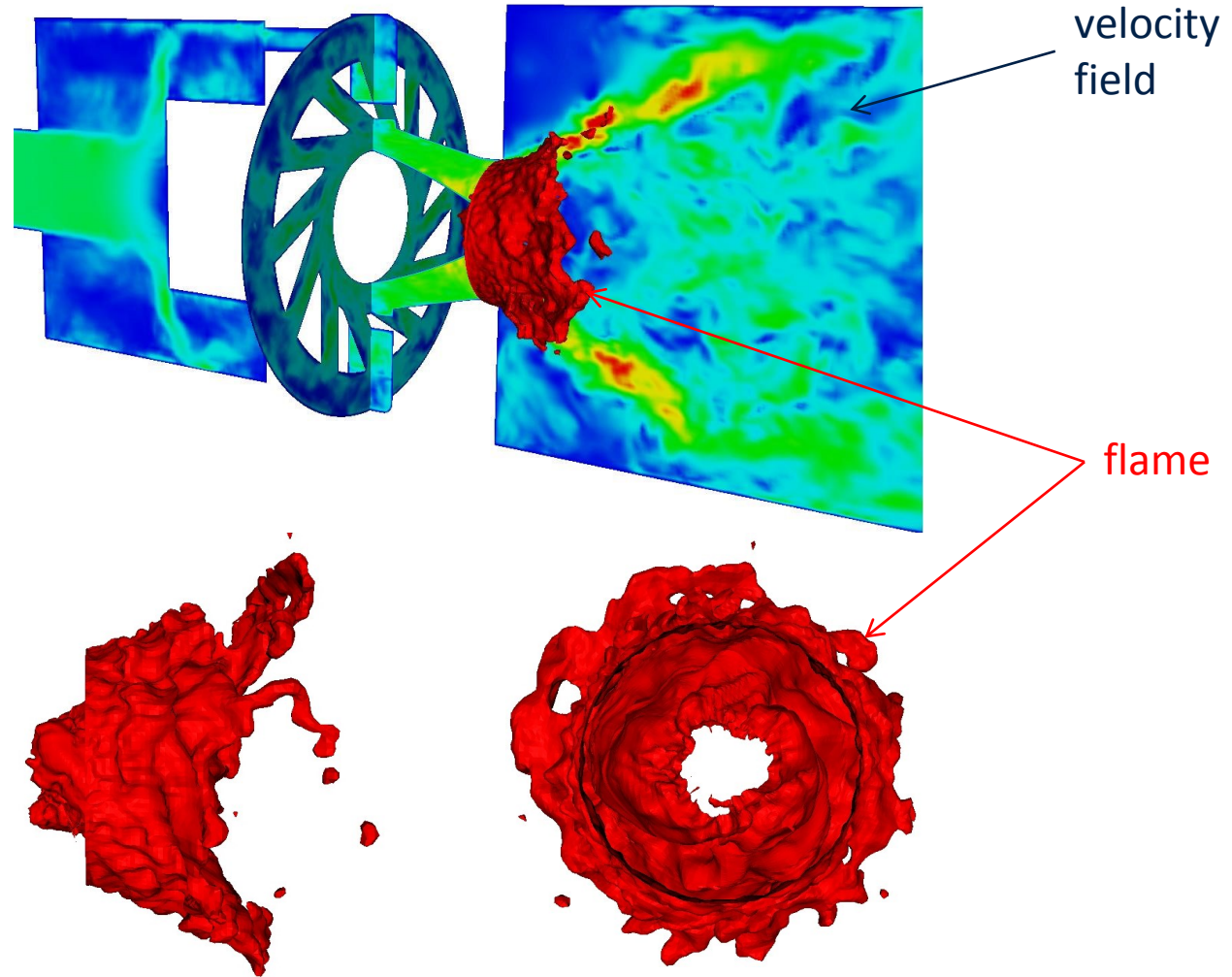
Combustion Summer School

2018

Prof. Dr.-Ing. Heinz Pitsch




Example: LES of a stationary gas turbine



Course Overview

Part II: Turbulent Combustion

- Turbulence
 - **Turbulent Premixed Combustion** 
 - Turbulent Non-Premixed Combustion
 - Turbulent Combustion Modeling
 - Applications
- **Scales of Turbulent Premixed Combustion**
 - Regime-Diagram
 - Turbulent Burning Velocity

Scales of Turbulent Premixed Combustion

- Integral turbulent scales

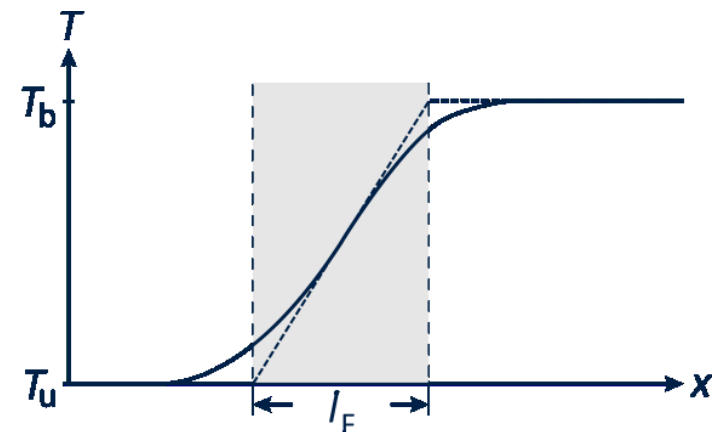
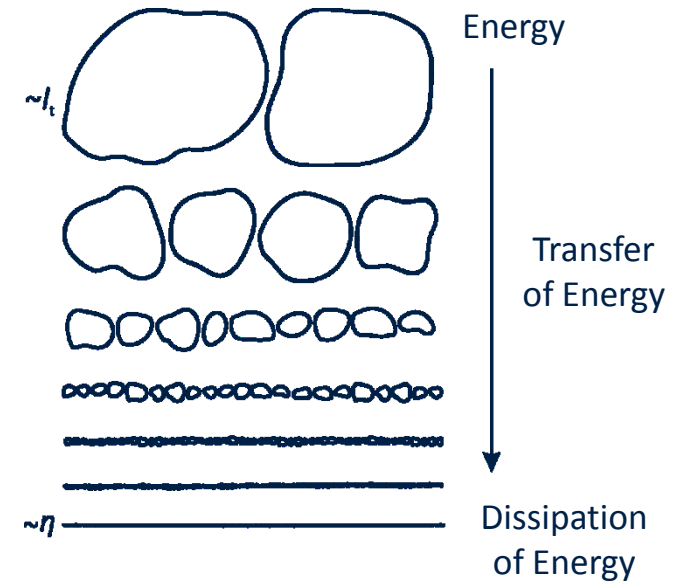
$$l_t = c_1 \frac{\bar{k}^{3/2}}{\bar{\epsilon}}, \quad u' = \sqrt{\frac{2}{3} \bar{k}}, \quad \tau = \frac{l_t}{u'} \sim \frac{\bar{k}}{\bar{\epsilon}}$$

- Smallest turbulent scales/Kolmogorov scales

$$\eta = \left(\frac{\nu^3}{\bar{\epsilon}} \right)^{1/4}, \quad u_\eta = (\nu \bar{\epsilon})^{1/4}, \quad t_\eta = \left(\frac{\nu}{\bar{\epsilon}} \right)^{1/2}$$

- Flame thickness and time, reaction zone thickness

$$l_F = \frac{D}{s_L} = \frac{\lambda_b}{\rho_u c_p s_L}, \quad t_F = \frac{l_F}{s_L} = \frac{D}{s_L^2}, \quad l_\delta \ll l_F$$



- Turbulent Reynolds number

$$Sc = \frac{\nu}{D} = 1 \quad \rightarrow \quad Re_t = \frac{l_t}{l_F} \frac{u'}{s_L}$$

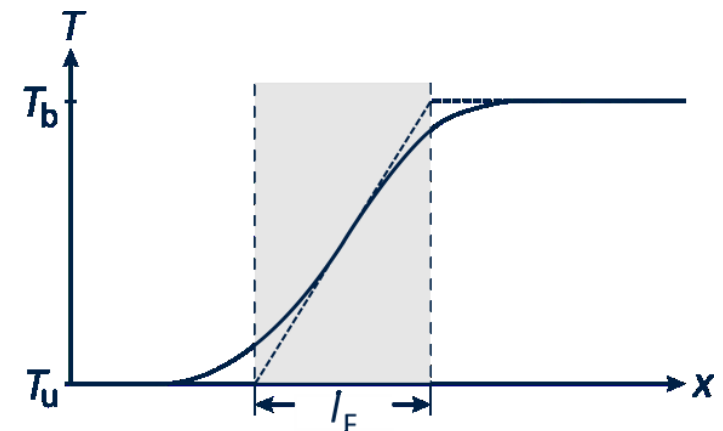
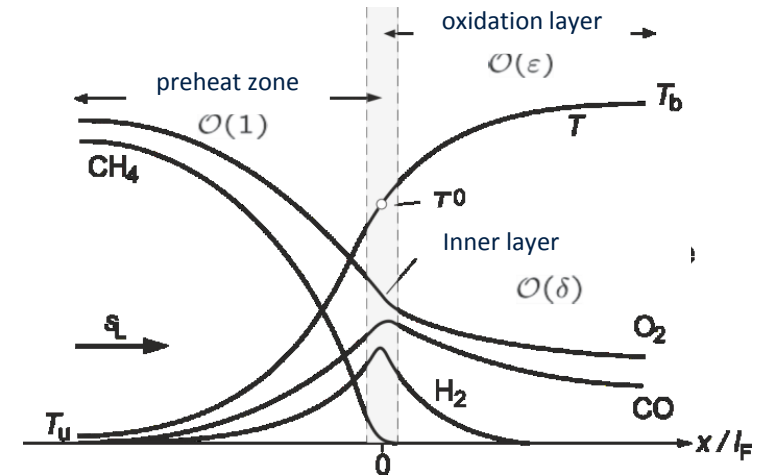
- Turbulent Damköhler number

$$Da_t = \frac{\tau}{t_F} = \frac{l_t}{l_F} \frac{s_L}{u'}$$

- Karlovitz number** (interaction of small-scale turbulence with the flame)

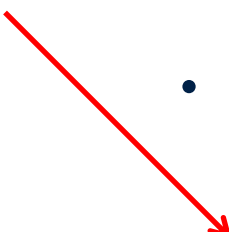
$$Ka = \frac{t_F}{t_\eta} = \frac{l_F^2}{\eta^2} = \sqrt{\frac{l_F}{l_t} \left(\frac{u'}{s_L} \right)^3} \quad \text{und} \quad Ka_\delta = \frac{l_\delta^2}{\eta^2} = \delta^2 Ka$$

$\delta \approx 0,1$



Course Overview

Part II: Turbulent Combustion

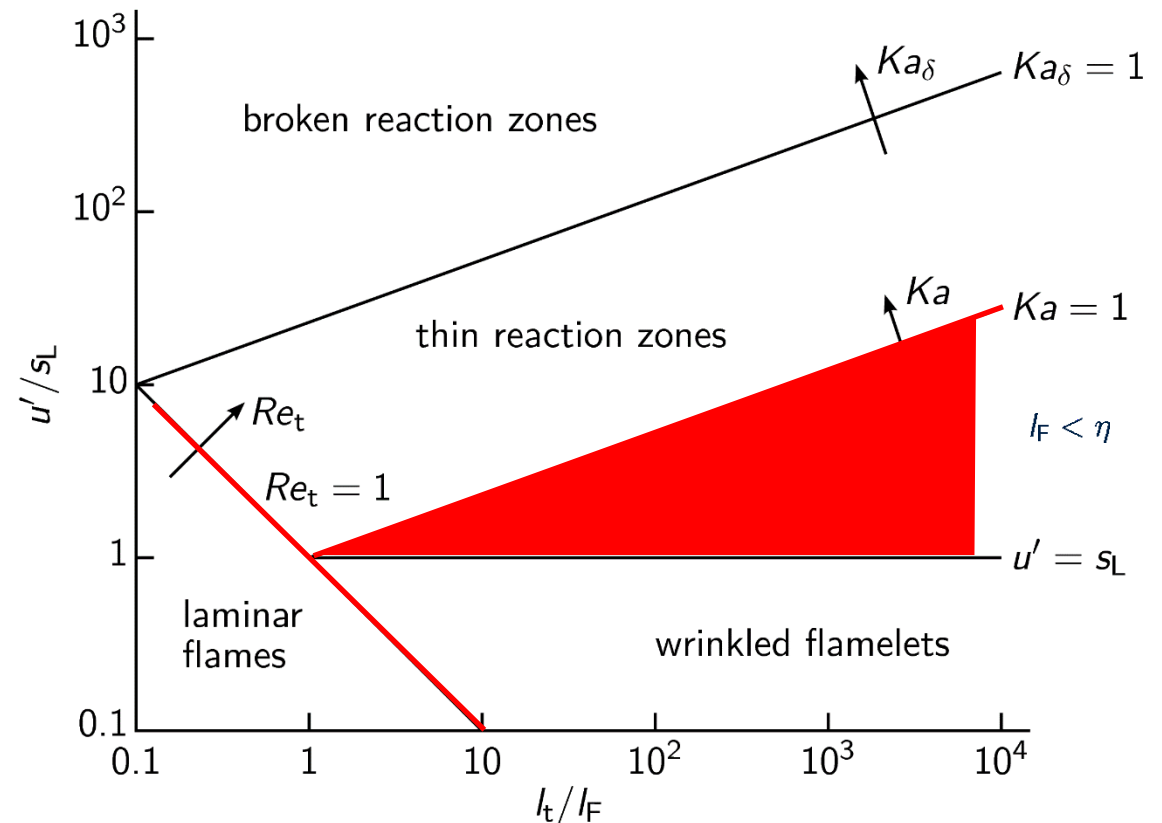
- Turbulence
 - **Turbulent Premixed Combustion**
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 - **Regime-Diagram**
 - Turbulent Burning Velocity
- 

Corrugated Flamelet Regime

$$Re_t = \frac{l_t u'}{l_F s_L}$$

$$Ka^2 = \left(\frac{l_t}{l_F}\right)^{-1} \left(\frac{u'}{s_L}\right)^3$$

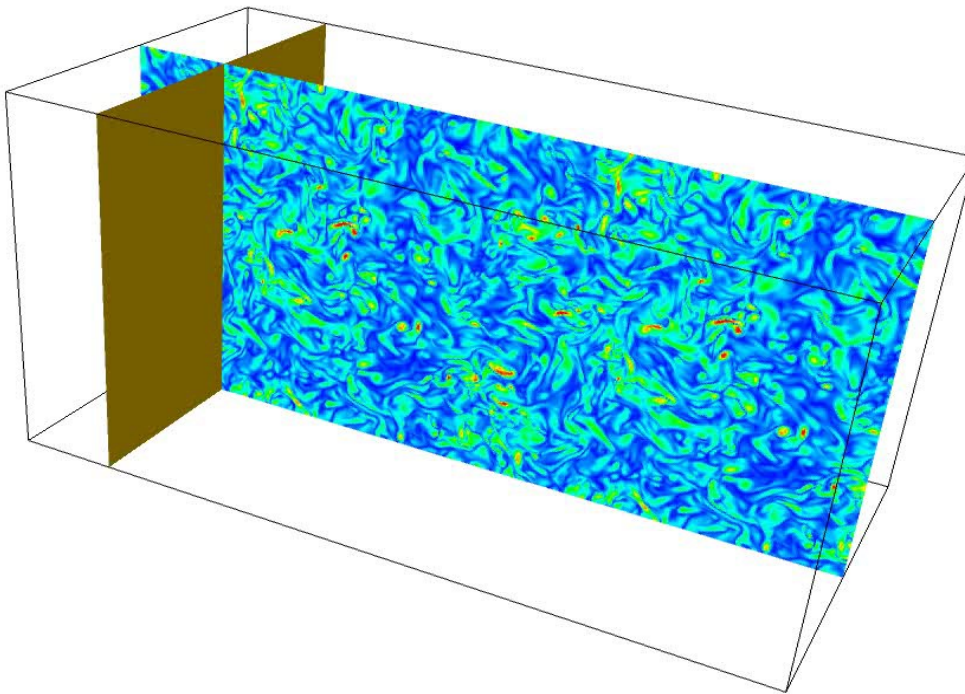
$$Da_t = \frac{l_t}{l_F} \left(\frac{u'}{s_L}\right)^{-1}$$



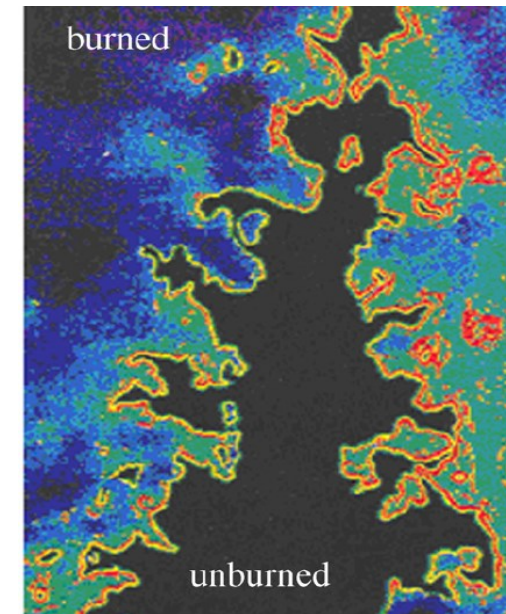
Regime Diagram: Corrugated Flamelets

- $Ka < 1 \rightarrow \eta > l_f$
 - Interaction of a very **thin flame** with a turbulent flow
 - Assumption: **infinitely thin flame** (compared to turbulent scales)

premixed flame in isotropic turbulence



OH-radical-distribution in a turbulent premixed flame



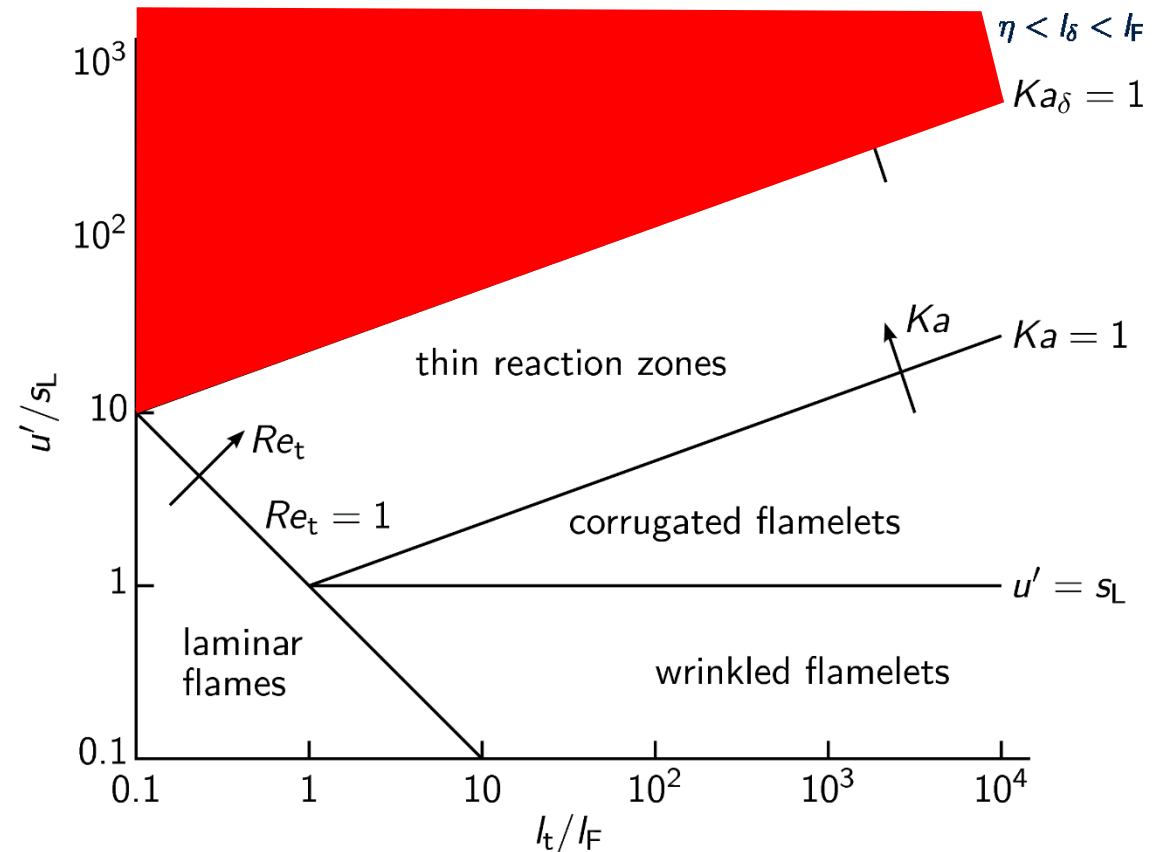
Buschmann (1996)

Regime Diagramm: Broken Reaction Zones Regime

$$Re_t = \frac{l_t}{l_F} \frac{u'}{s_L}$$

$$Ka^2 = \left(\frac{l_t}{l_F}\right)^{-1} \left(\frac{u'}{s_L}\right)^3$$

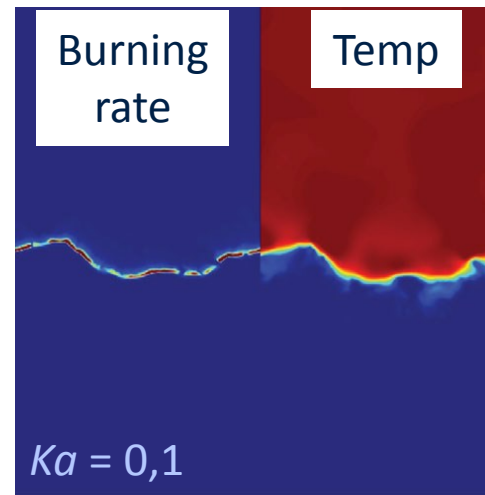
$$Da_t = \frac{l_t}{l_F} \left(\frac{u'}{s_L}\right)^{-1}$$



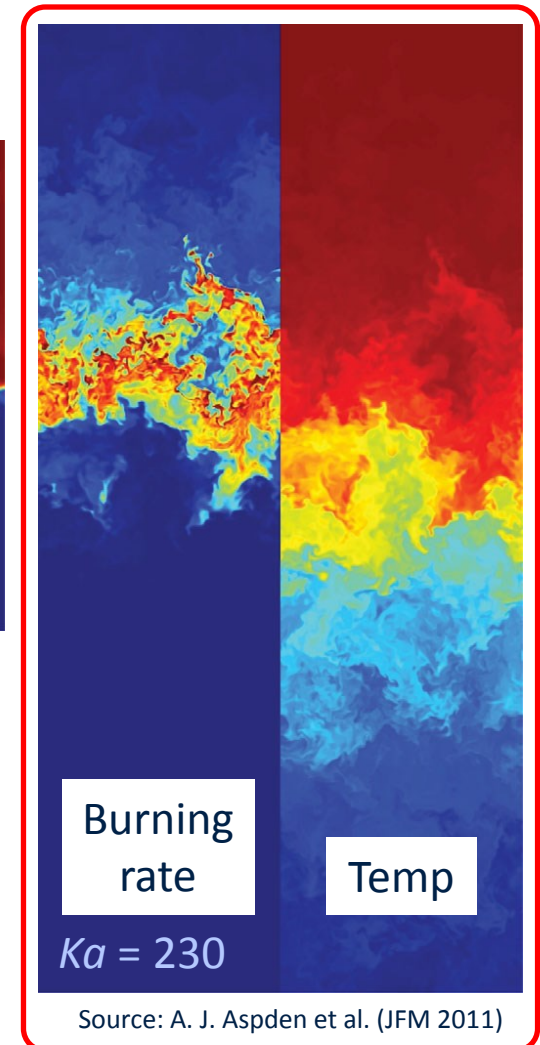
Regime Diagramm: Broken Reaction Zones Regime

- $Ka_\delta > 1 \rightarrow \eta < l_\delta$
 - Smallest **turbulent eddies** enter **the reaction zones**
 - Turbulent transport \rightarrow **radicals** are removed **from reaction zone**
 - Local **extinction** in the inner reaction zone possible
 - Can lead to global flame extinction

Example: Supernovae flames with transport mechanisms very different from normal flames



Two-dimensional slices from three-dimensional simulations

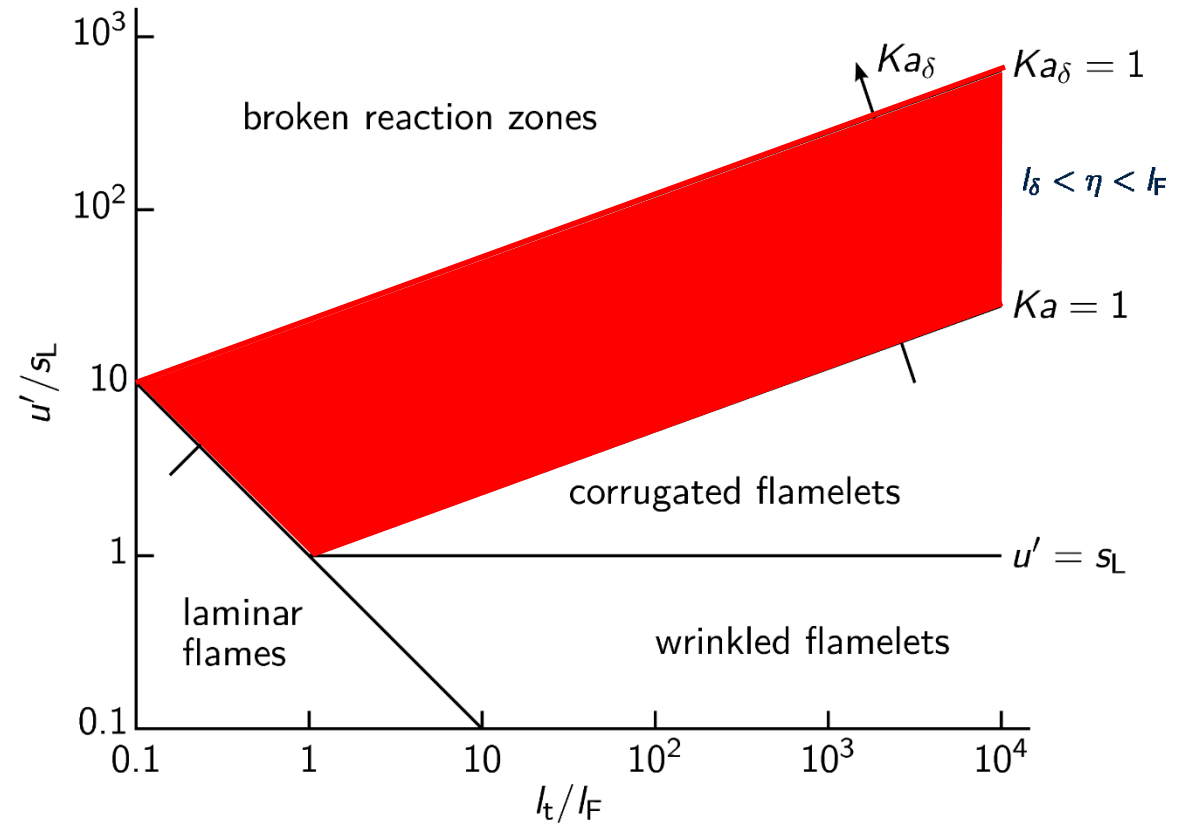


Regime Diagramm: Thin Reaction Zones Regime

$$Re_t = \frac{l_t}{l_F} \frac{u'}{s_L}$$

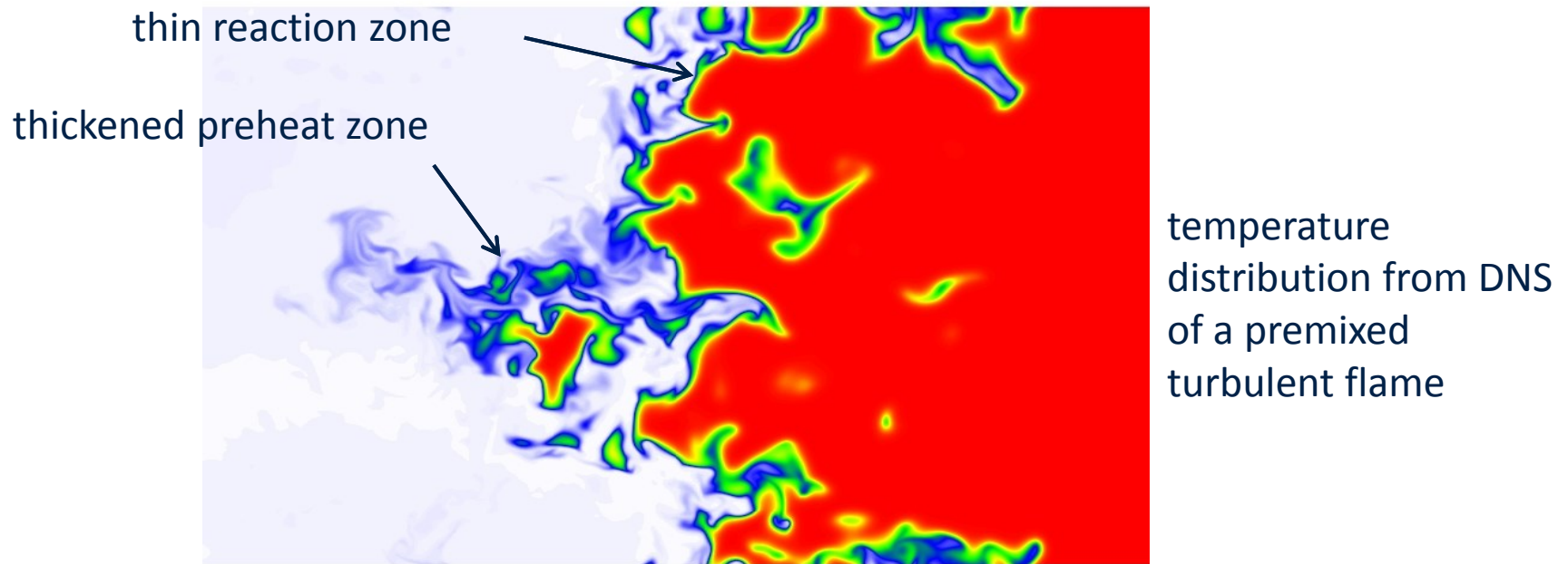
$$Ka^2 = \left(\frac{l_t}{l_F}\right)^{-1} \left(\frac{u'}{s_L}\right)^3$$

$$Da_t = \frac{l_t}{l_F} \left(\frac{u'}{s_L}\right)^{-1}$$



Regime Diagramm: Thin Reaction Zones Regime

- $Ka > 1$ und $Ka_\delta < 1 \rightarrow l_\delta < \eta < l_F$
 - With $l_\delta \approx 0,1 l_F \rightarrow Ka \approx 100 Ka_\delta$
 - Turbulent mixing inside preheat zone
 - Assumption: **infinitely thin reaction zone** (compared to turbulent scales)

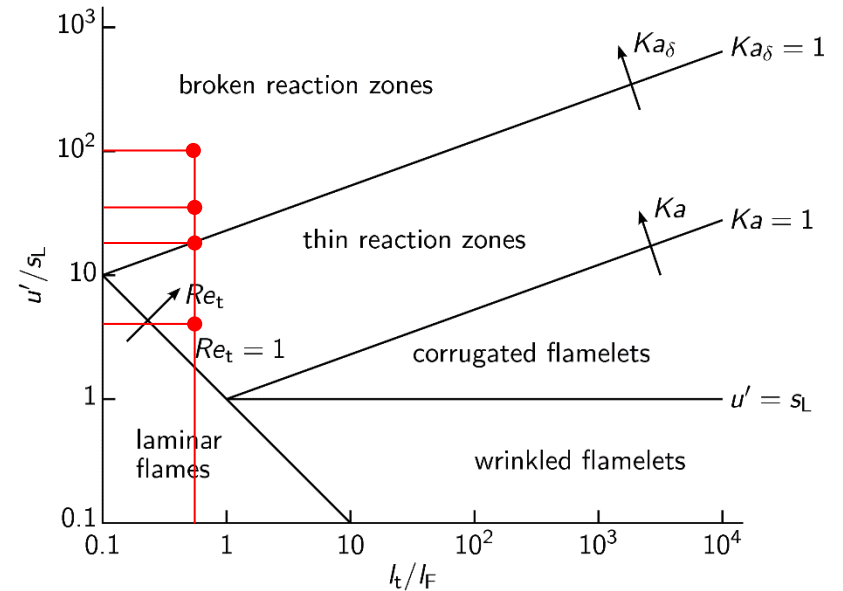


Regime Diagram: Summary

Case	A40	B40	C40	D40
Equivalence ratio (ϕ)	0.40	0.40	0.40	0.40
→ Flame speed (s_L) (m s^{-1})	2.24×10^{-1}	2.24×10^{-1}	2.24×10^{-1}	2.24×10^{-1}
→ Flame width (l_L) (m)	6.29×10^{-4}	6.29×10^{-4}	6.29×10^{-4}	6.29×10^{-4}
Domain width (L) (m)	3.14×10^{-3}	3.14×10^{-3}	3.14×10^{-3}	3.14×10^{-3}
Domain height (H) (m)	2.512×10^{-2}	2.512×10^{-2}	2.512×10^{-2}	2.512×10^{-2}
→ Integral length scale (l) (m)	3.14×10^{-4}	3.14×10^{-4}	3.14×10^{-4}	3.14×10^{-4}
→ RMS velocity (\check{u}) (m s^{-1})	0.825	3.83	7.34	23.9
Damköhler number (Da_L)	1.36×10^{-1}	2.92×10^{-2}	1.52×10^{-2}	4.68×10^{-3}
Levels of refinement	1	1	1	2
Effective resolution (N)	$128^2 \times 1024$	$128^2 \times 1024$	$128^2 \times 1024$	$256^2 \times 2048$
Cell width (Δx) (m)	2.45×10^{-5}	2.45×10^{-5}	2.45×10^{-5}	1.23×10^{-5}
Kolmogorov length (η) (m)	4.33×10^{-5}	1.37×10^{-5}	8.41×10^{-6}	3.47×10^{-6}
Cell Kolmogorov length ($\eta_{\Delta x}$) (m)	7.36×10^{-6}	7.36×10^{-6}	7.36×10^{-6}	3.68×10^{-6}
Effective Kolmogorov length (η_e) (m)	4.33×10^{-5}	1.51×10^{-5}	11.2×10^{-6}	5.12×10^{-6}

TABLE 2. Turbulent flame properties for the four simulations at equivalence ratio $\phi = 0.40$.

Source: A. J. Aspden et al. (JFM 2011)



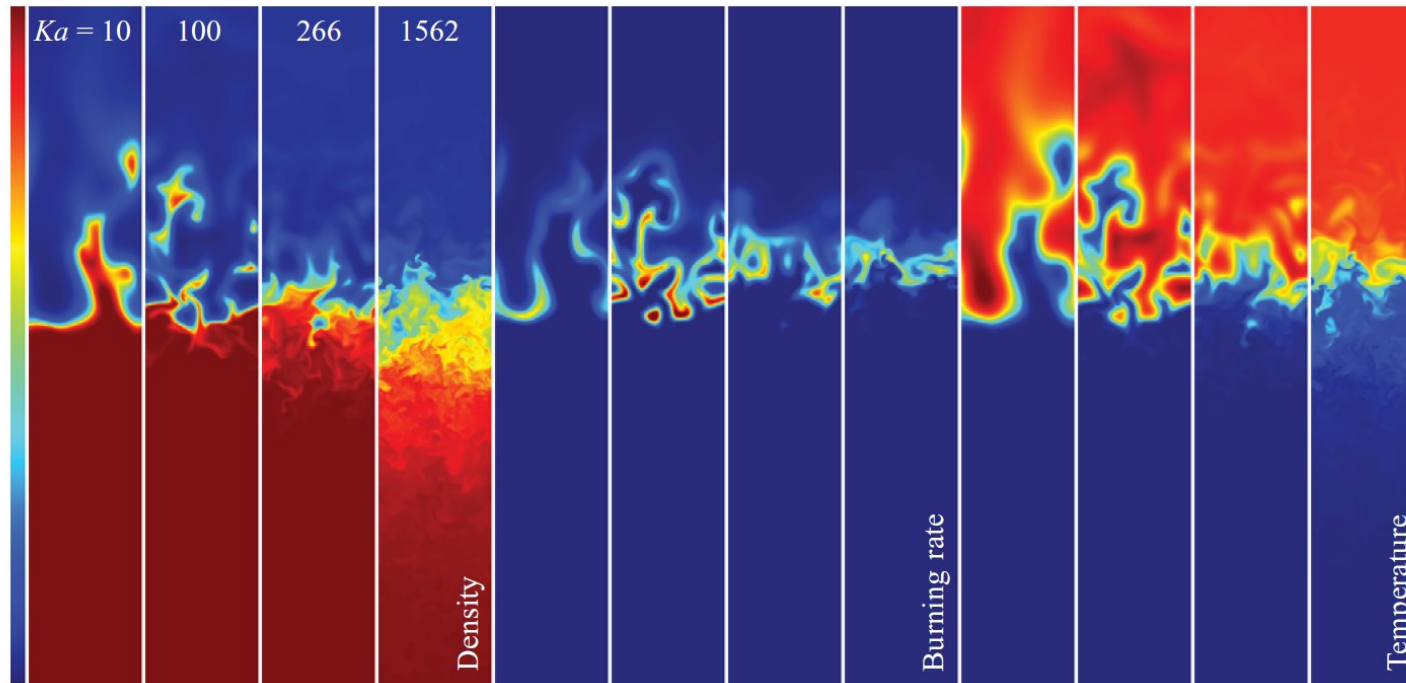


FIGURE 8. Two-dimensional vertical slices through three-dimensional simulations showing density, burning rate and temperature at $\varphi = 0.40$, respectively. The density, burning rate and temperature ranges are $[0.2, 1.02] \text{ kg m}^{-3}$, $[0, 64] \text{ kg m}^{-3} \text{ s}^{-1}$ and $[298, 1600] \text{ K}$, respectively.

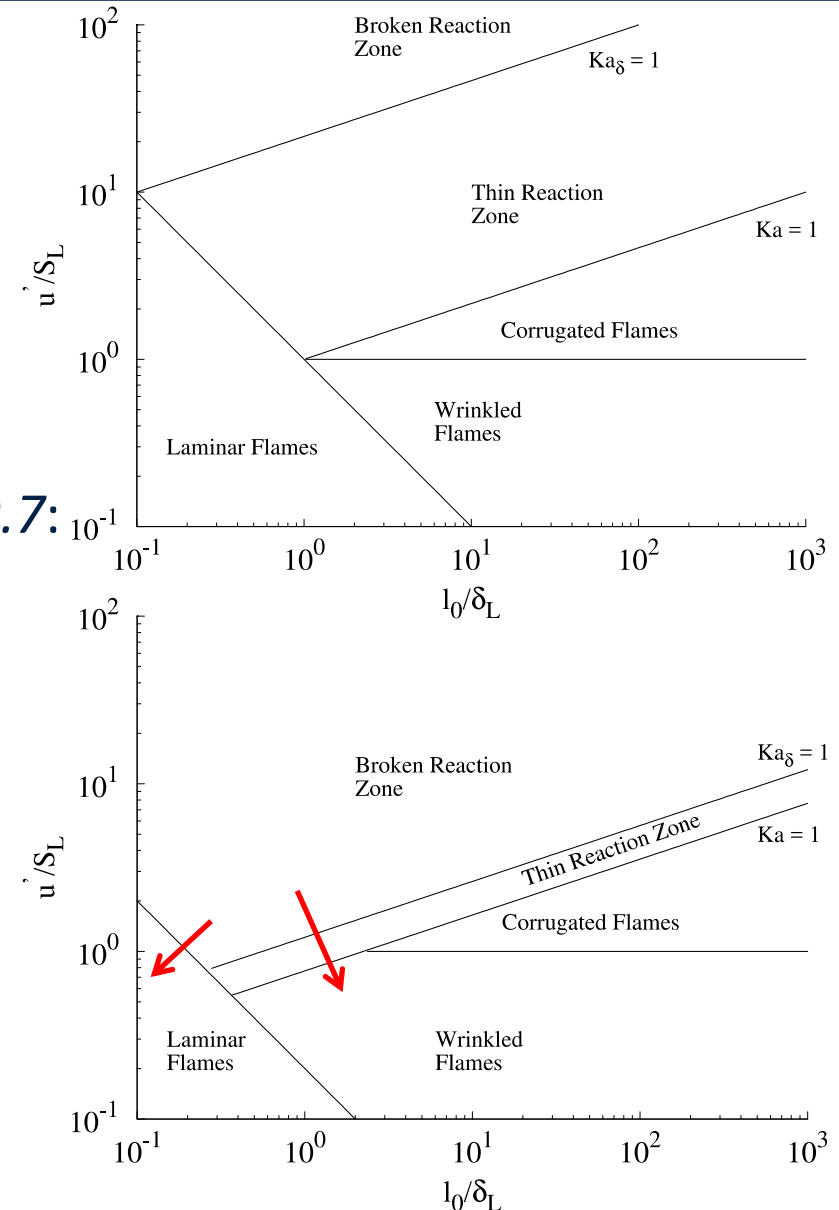
Source: A. J. Aspden et al. (JFM 2011)

Regime Diagram: Corrections from Ideal Scaling

- Usual assumptions:
 - $Sc = 1 \rightarrow \nu = D$
 - $S_L l_F / \nu \approx 1$
 - $l_\delta \approx 0,1 l_F \rightarrow Ka \approx 100 Ka_\delta$
- Example: Methane/air flame, $T_u = 800K$, $\varphi = 0.7$:
 - $Sc \approx 1 \rightarrow \nu \approx D$
 - but
 - $S_L l_F / \nu \approx 5$
 - $l_\delta \approx 0,5 l_F \rightarrow Ka \approx 4 Ka_\delta$

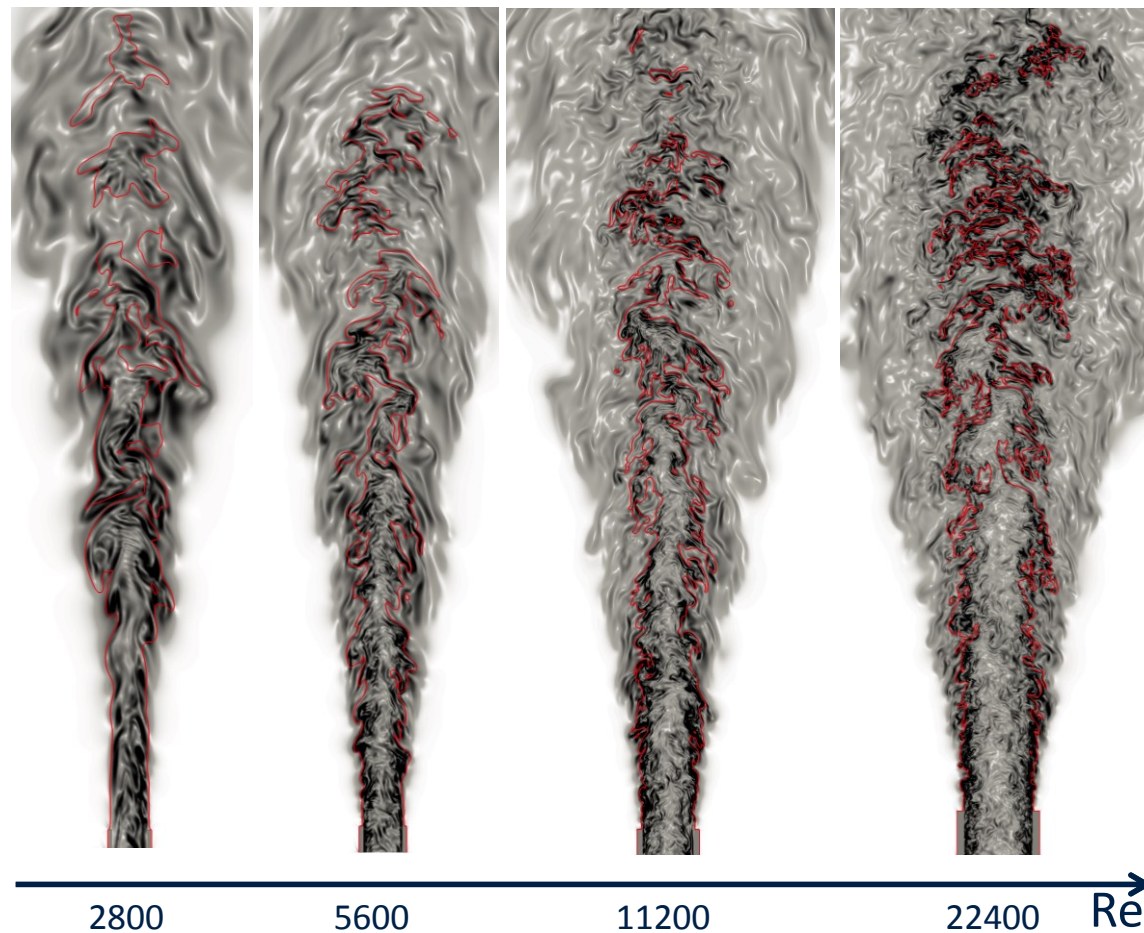


Lines only for scaling,
be careful with absolute values



DNS at Constant Ka for Various Re

- Lean methane flame $T_u=800K$, $\phi=0.7$ ($S_L=1m/s$)
- Re variation: constant u' and increased $I_t \rightarrow$ constant Karlovitz (approximately)**



Re	2800	5600	11200	22400
Ka	40	40	40	40
U_{bulk}	100 m/s	100 m/s	100 m/s	100 m/s
u'	10 m/s	10 m/s	10 m/s	10 m/s
Jet widths	0.6 mm	1.2 mm	2.4 mm	4.8 mm
Grid points	88 Million	350 Million	2.8 Billion	22 Billion

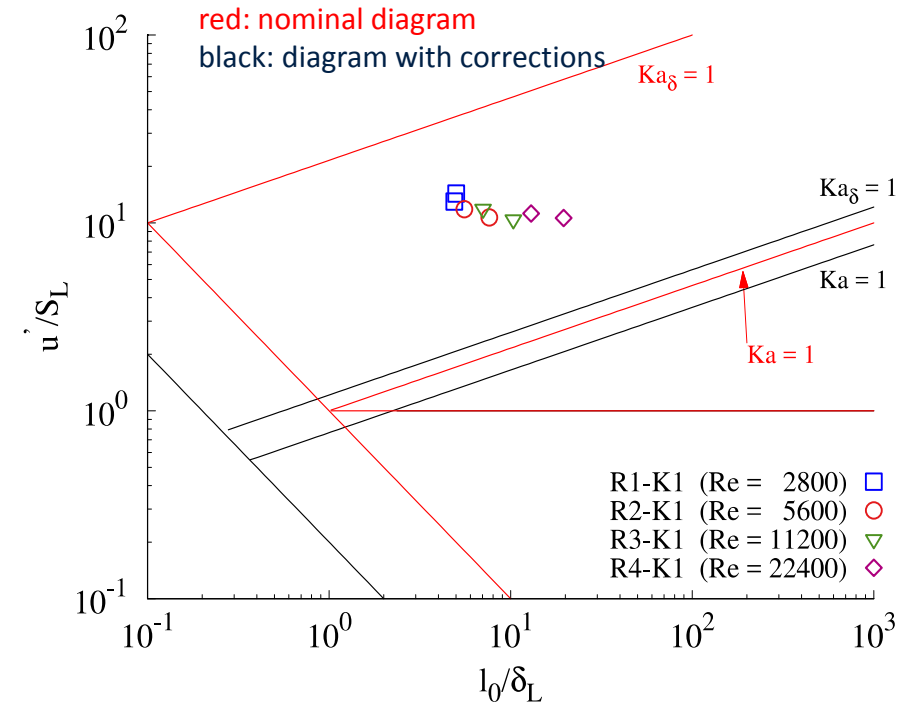
(from A. Attili et al, 2017)

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- Reynolds number changed by jet width H
- $L_t \sim H$**
- $\eta = l_t Re_t^{-3/4} \sim l_t^{1/4}$, hence η increases slightly with increasing H

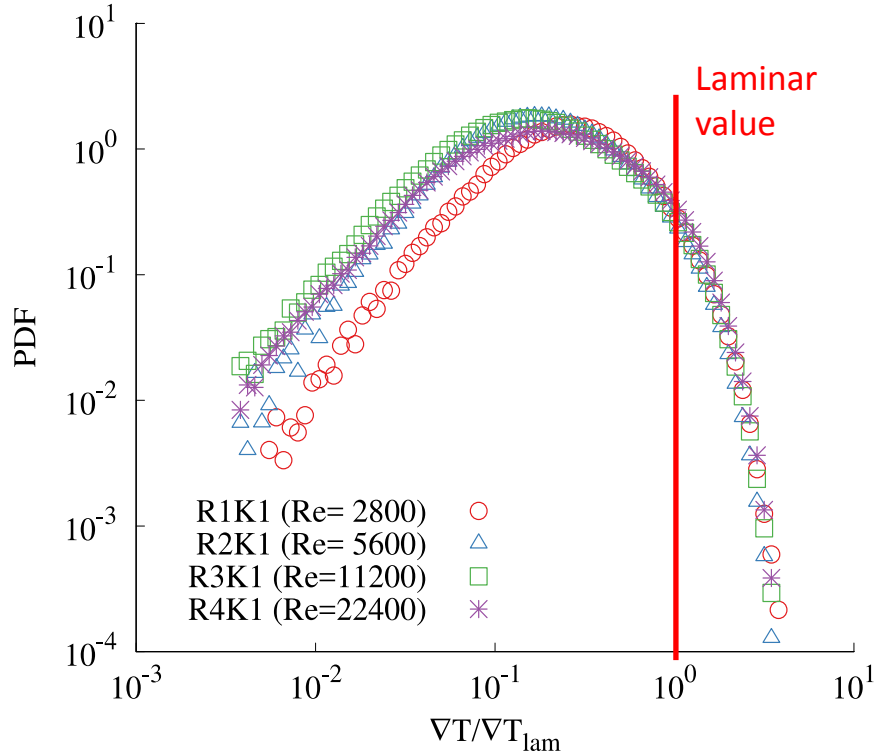


Not clear in which regime the flames are

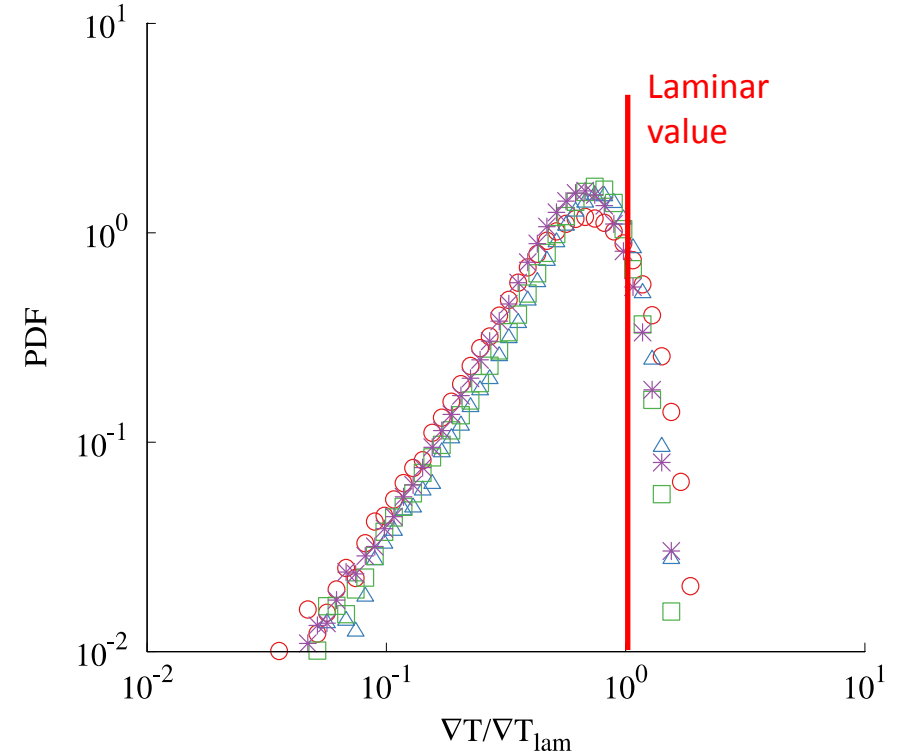
- Thin reaction zone
- Broken reaction zone

Different Re and constant Ka DNS: regime assessment

Preheat zone $T = 900K$



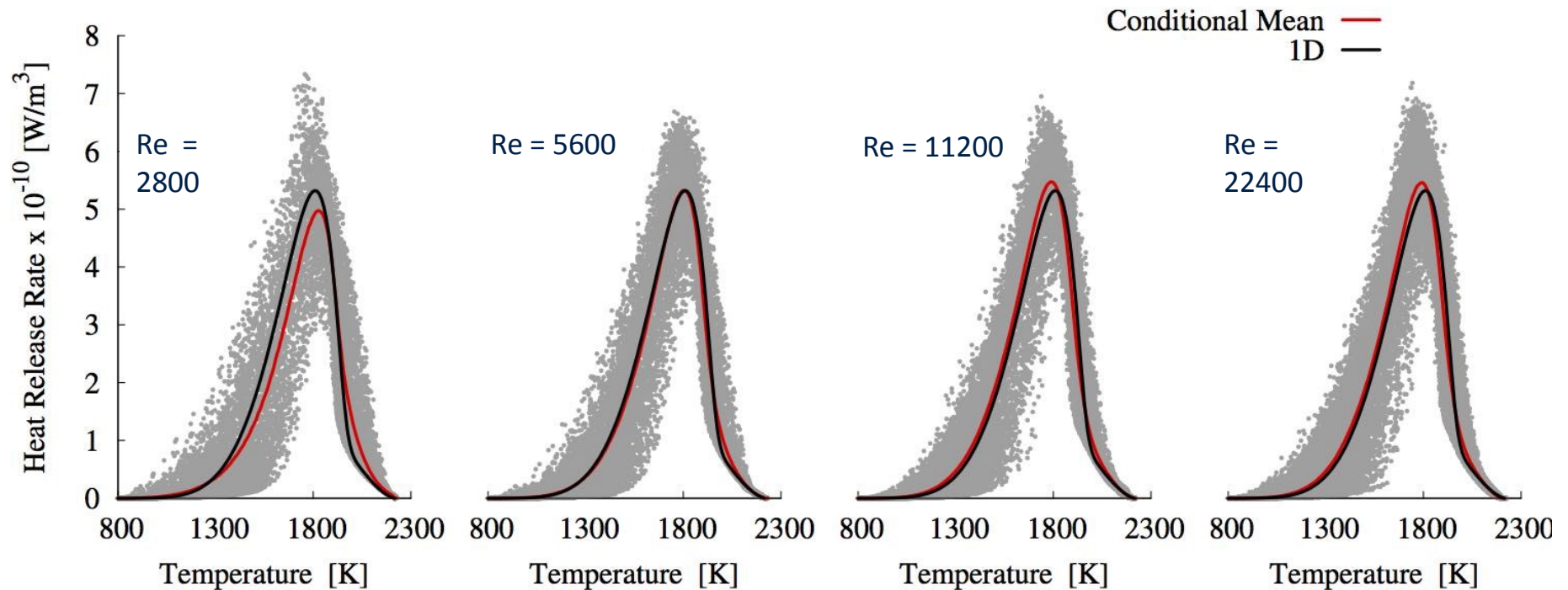
Reaction zone $T = 1800K$



- Strong turbulent mixing in the preheat zone
 - gradient PDF is wide
 - PDF close to log normal (typical for gradients in turbulence)
 - far from the gradient in a laminar 1D flame
- Reaction zone not affected by change in turbulence
 - gradient PDF is narrow
 - close to the gradient in laminar unstretched 1D flame

The flames are in the thin reaction zone regime

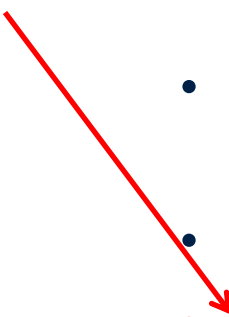
Different Re and constant Ka DNS: flame structure



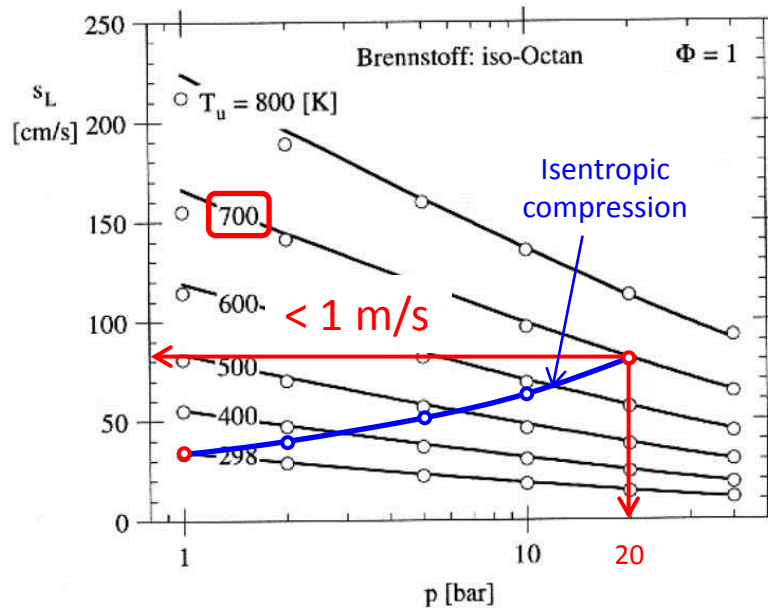
- Flame structure very similar to 1D laminar flame
 - Conditional mean from DNS agrees well with 1D flame profile
 - Small scatter
- **Reynolds number effects** are related to **different transport in the preheat zone**, not to modifications of the flame structure

Course Overview

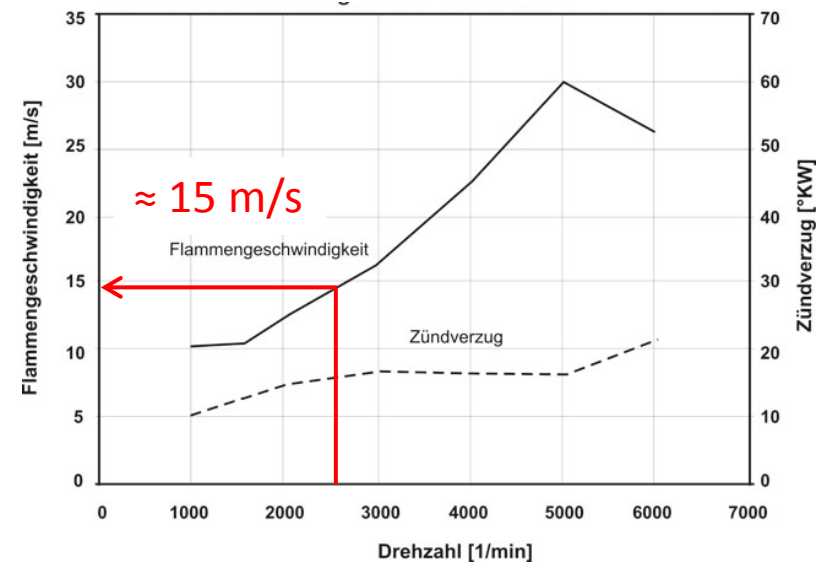
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 - **Turbulent Burning Velocity**

Comparison: Laminar/Measured Burning Velocity

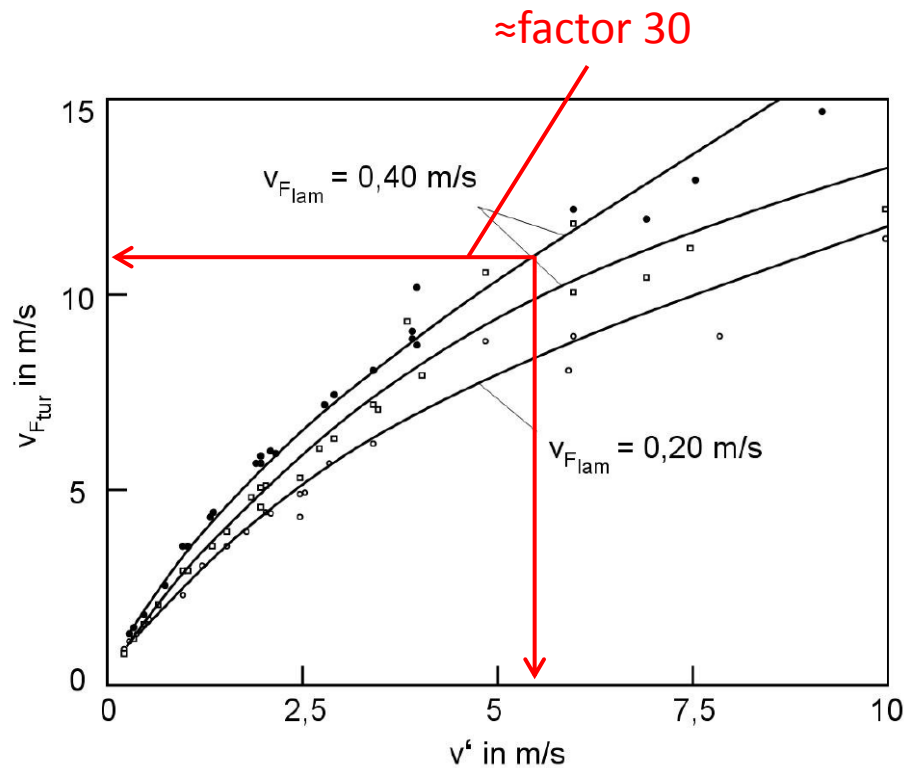


Laminar burning velocity
of iso-octane



Exemplary measurements in gasoline engine with
tumble generator of flame velocity
at spark plug position during full load (Source:
Merker, „Grundlagen Verbrennungsmotoren“)

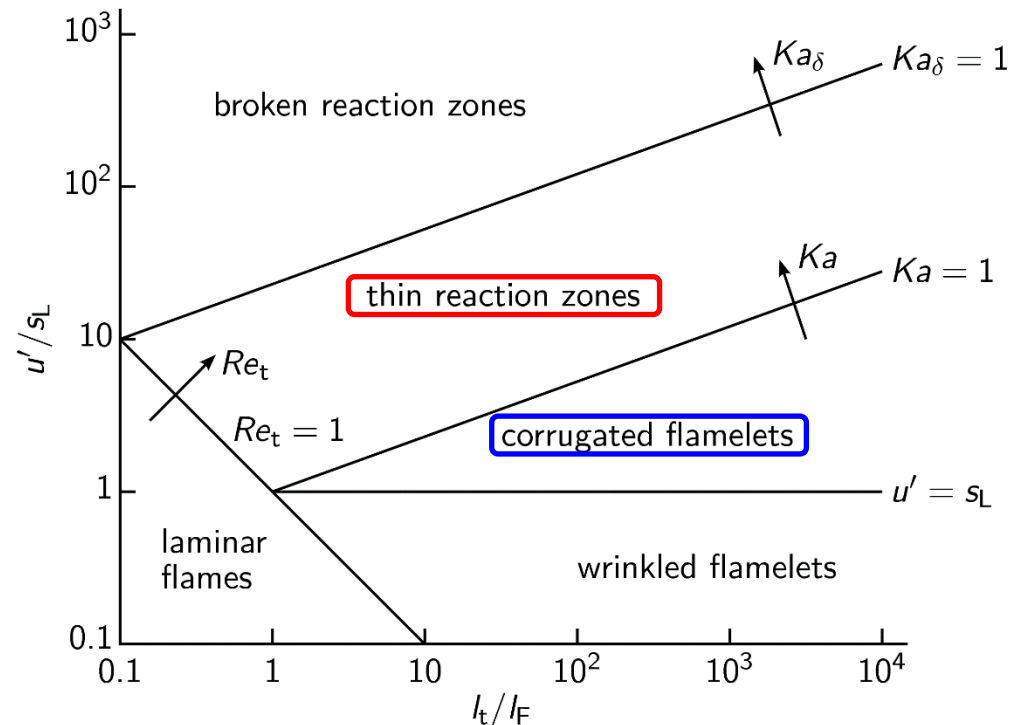
Comparison: Laminar/Measured Burning Velocity



Experimental data of s_T vs. wrinkled laminar-flame theories of turbulent flame propagation
(data from Turns 2000)

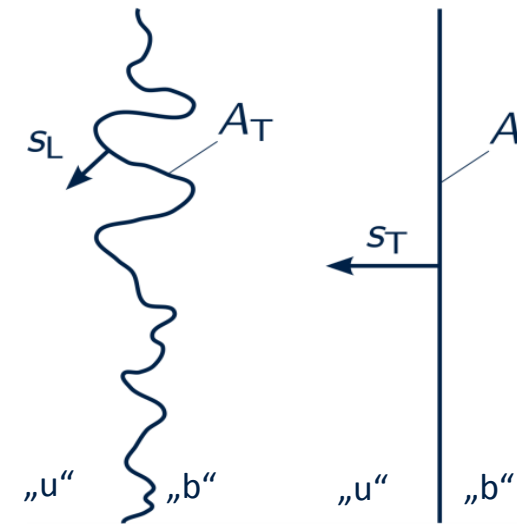
Turbulent Burning Velocity

- Main problem for turbulent premixed combustion:
Quantification of turbulent burning velocity s_T
- s_T : Velocity which quantifies the propagation of the **turbulent flame front into unburnt mixture**
- Distinction of two limiting cases by Damköhler (1940)
 1. Large scale turbulence \leftrightarrow **corrugated flamelets**
 2. Small scale turbulence \leftrightarrow **thin reaction zones**



Turbulent Burning Velocity: Corrugated Flamelets

- Instantaneous flame front
 - Flame surface area A_T
 - Propagates locally with **laminar burning velocity** s_L into unburnt mixture
- Mean flame front
 - Mean flame surface area A
 - Propagates with **turbulent burning velocity** s_T



Turbulent Burning Velocity: Corrugated Flamelets

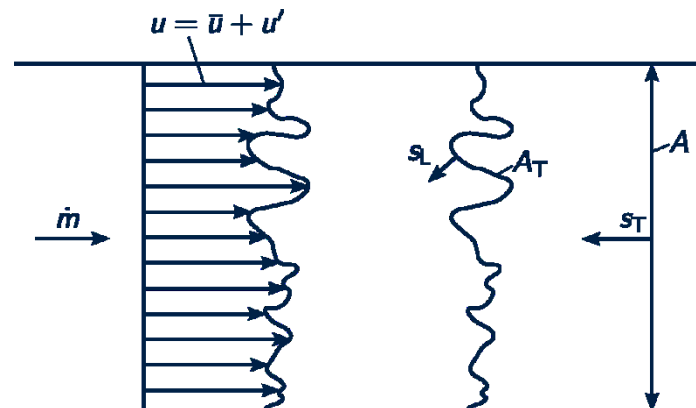
- With the **mass flux** through A and A_T

$$\dot{m} = \rho_u s_L A_T = \bar{\rho}_u s_T A$$

- Assume constant density in the unburnt mixture (assumption) $(\rho_u = \bar{\rho}_u)$

$$\frac{s_T}{s_L} = \frac{A_T}{A}$$

- Wrinkling of the laminar flame ($A_T \uparrow$) \rightarrow increase of s_T



Turbulent Burning Velocity: Corrugated Flamelets

- Turbulence \rightarrow flame surface area \uparrow
- Using an analogy with a Bunsen flame

$$s_L = u_u \sin \alpha \xrightarrow{\text{hier}} \sin \alpha = \frac{s_L}{u'} \Rightarrow \frac{A_T}{A} \sim \frac{d / \sin \alpha}{d} = \frac{u'}{s_L}$$

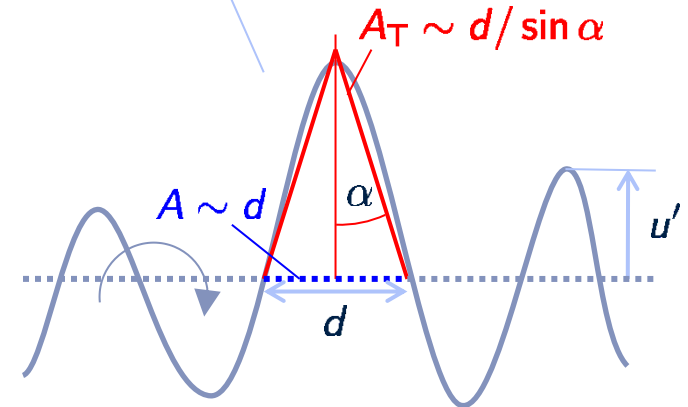
- Limit for $u' \rightarrow 0$

$$\frac{s_T}{s_L} = \frac{A_T}{A} = 1 + \frac{u'}{s_L}$$

- Internal combustion engine:
 - Engine speed $n \uparrow \rightarrow$ burning velocity $s_T \uparrow$ due to

$$u' \sim u_{\text{piston}} \sim n$$

\rightarrow High engine speed achievable

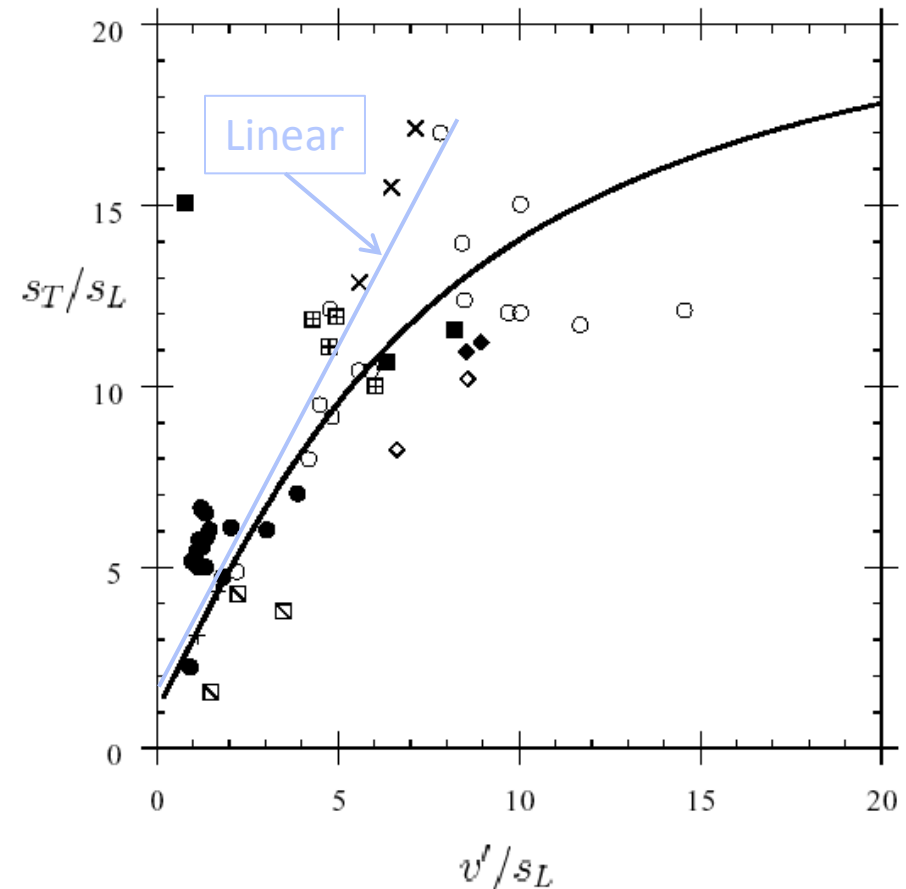


Turbulent Burning Velocity: large-scale turbulence

- In experiments often used empirical relation

$$\frac{s_T}{s_L} = 1 + C \left(\frac{u'}{s_L} \right)^n$$

- Constant C experimentally determined
- Typical values: $0.5 < n < 1.0$
- From experimental data →
 - For small u' , $s_T \sim u'$ applies
 - Consistent with Damköhler theory
 - Increase of turbulent intensity
 - s_T grows linearly
 - With further increase less than linear



Turbulent Burning Velocity: Thin Reaction Zones

- Reduced increase of turbulent burning velocity
→ second limiting case of Damköhler
- Thin reaction zones/small-scaled turbulence
- In analogy to

Damköhler uses

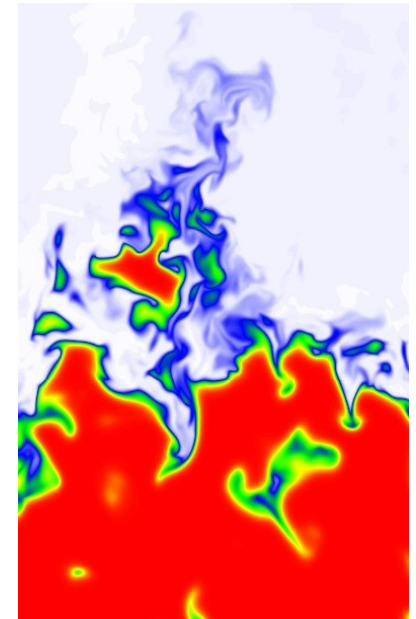
$$\left. \begin{aligned} s_L &= \sqrt{\frac{D}{t_c}} \\ s_T &= \sqrt{\frac{D_t}{t_c}} \end{aligned} \right\}$$

- t_c : chemical time scale
- Dimensional analysis $D_t \sim u' l_t$
- Constant of proportionality 0.78

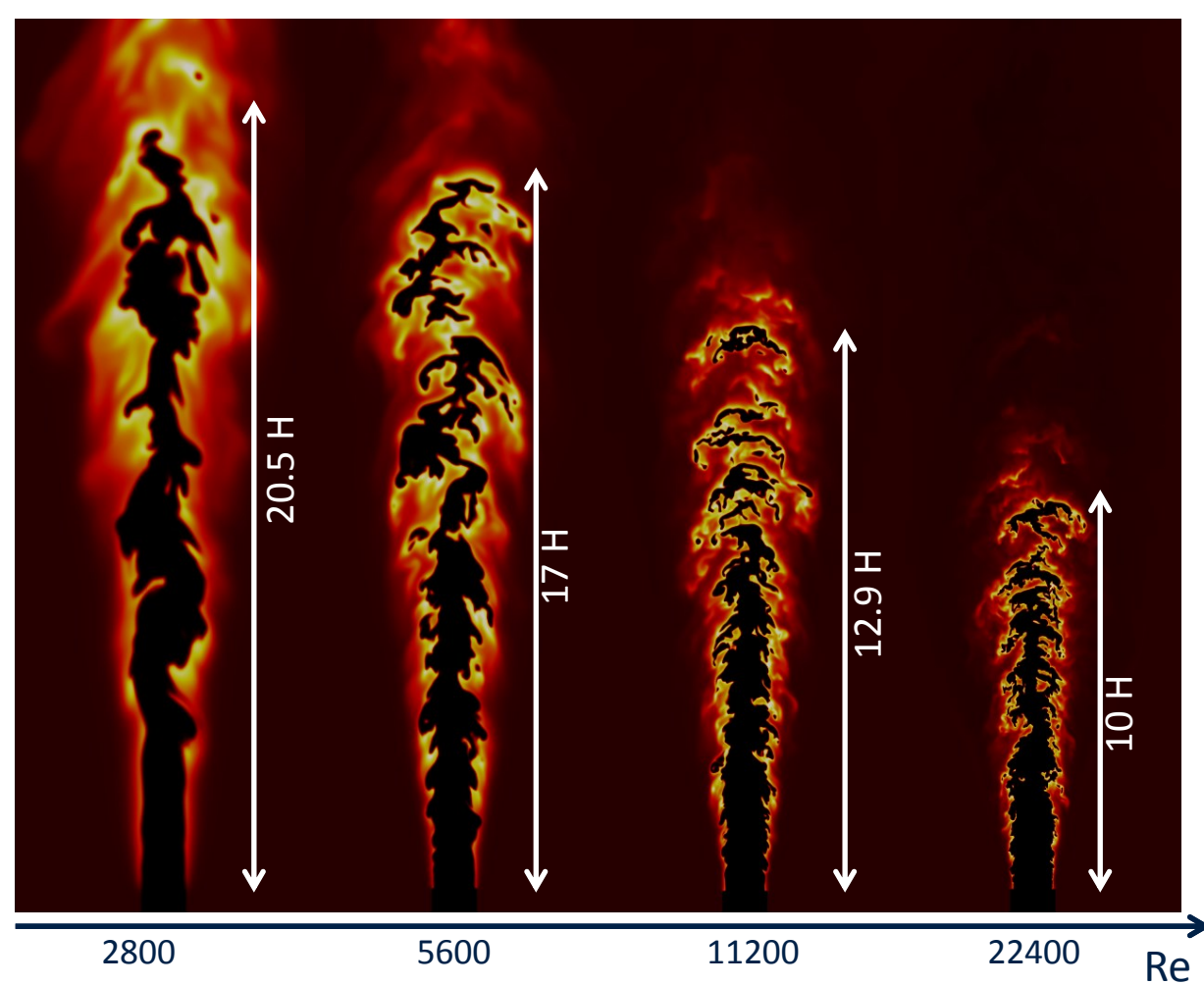
$$\frac{s_T}{s_L} = \sqrt{\frac{D_t}{D}} = \sqrt{\frac{0,78 u' l_t}{s_L l_F}}$$

$$s_T \sim \sqrt{u' l_t}$$

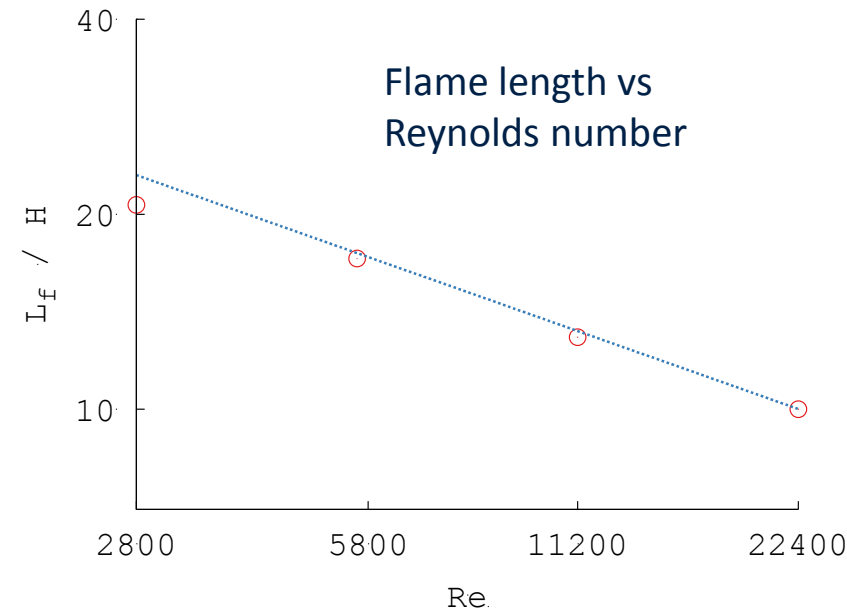
consistent with
experimental data



Turbulent Burning Velocity: Thin Reaction Zones



(from A. Attili et al, 2017)



- Decreased length → increased flame speed
- Turbulent flame speed increases with increasing Reynolds number
 - u' is constant
 - Increased flame speed due to increased integral scale

Turbulent Burning Velocity

- Damköhler-limits can be combined to a single formula (Peters, 1999):

$$\frac{s_T}{s_L} = 1 - \alpha \frac{l_t}{l_F} + \sqrt{\left(\alpha \frac{l_t}{l_F}\right)^2 + 4\alpha \frac{u' l_t}{s_L l_F}}$$

– constant $\alpha = 0,195$

- Low turbulence intensity →

$$\frac{s_T}{s_L} = 1 + 2 \frac{u'}{s_L}$$

- High turbulence intensity →

$$\frac{s_T}{s_L} = 1 + \sqrt{\frac{0,78 u' l_t}{s_L l_F}}$$

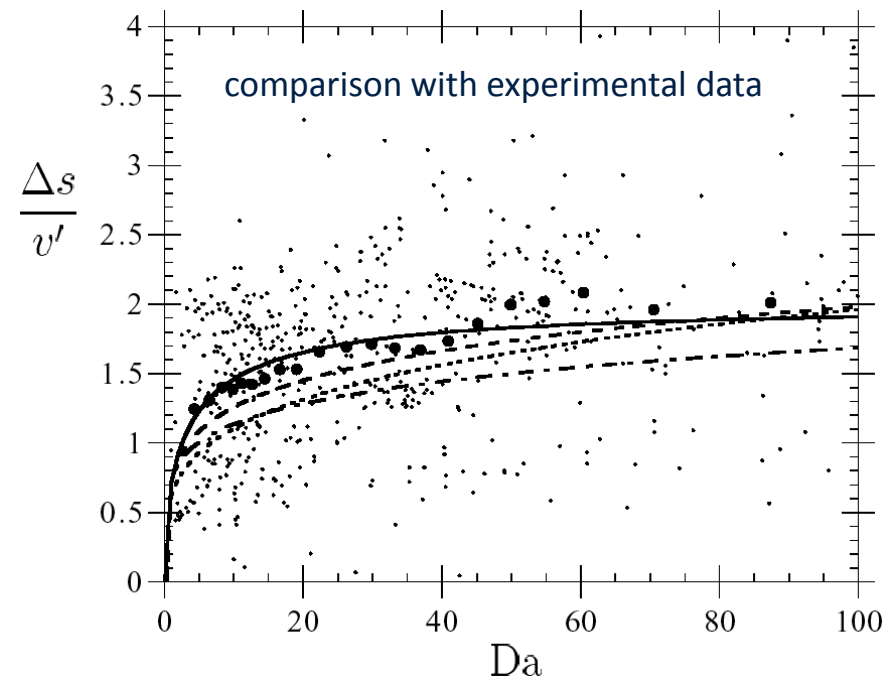
Turbulent Burning Velocity

- By rearranging this formula with $Da_t = (l_t s_L)/(l_F u') \rightarrow$

$$\frac{s_T - s_L}{u'} = -\alpha Da_t + \sqrt{\alpha^2 Da_t^2 + 4\alpha Da_t}$$

- Limit for high Damköhler number \rightarrow

$$\lim_{Da_t \rightarrow \infty} \frac{s_T - s_L}{u'} = 2$$



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