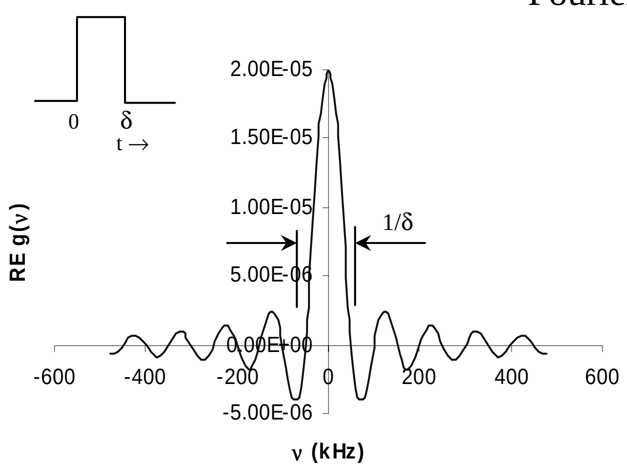
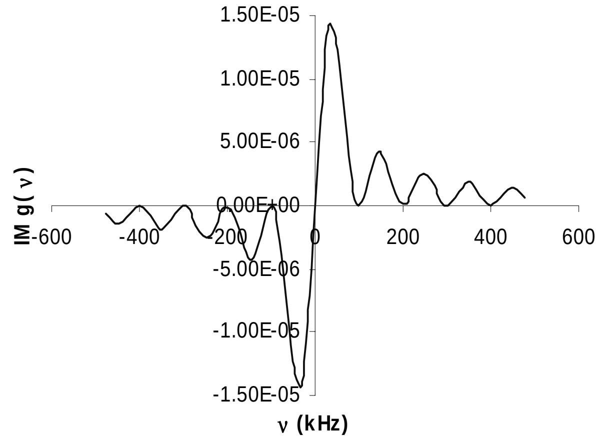
Fourier Integrals





Fourier series: $f(t) = \sum_{n=0}^{\infty} A_n \cos(2\pi n \nu_o t) + \sum_{n=0}^{\infty} B_n \sin(2\pi n \nu_o t)$ $\qquad \nu_o = 1/L$

$A_n = 2 \int_0^L f(t) \cos(2\pi n \nu_o t)\, dt \qquad B_n = 2 \int_0^L f(t) \sin(2\pi n \nu_o t)\, dt$

Let $L \rightarrow \infty$ then $2\pi n \nu_o t \rightarrow 2\pi \nu t$ with ν a continuous variable:

$A(\nu) = 2 \int_0^\infty f(t) \cos(2\pi \nu t)\, dt \qquad B(\nu) = 2 \int_0^\infty f(t) \sin(2\pi \nu t)\, dt$

Square pulse of length δ: $f(t) = 1$ for $t=0$ to δ and $f(t) = 0$ after:

$A(\nu) = 2 \int_0^\delta \cos(2\pi \nu t)\, dt \qquad B(\nu) = 2 \int_0^\delta \sin(2\pi \nu t)\, dt$

$A(\nu) = 2 \left. \frac{\sin(2\pi \nu t)}{2\pi \nu} \right|_0^\delta \qquad B(\nu) = 2 \left. \frac{-\cos(2\pi \nu t)}{2\pi \nu} \right|_0^\delta$

$A(\nu) = 2 \frac{\sin(2\pi \nu \delta)}{2\pi \nu} \qquad B(\nu) = 2 \frac{1-\cos(2\pi \nu \delta)}{2\pi \nu}$

$A(\nu)$ = absorption spectrum $\qquad B(\nu)$ = dispersion $\qquad$ full width to first nulls $= \frac{1}{\delta}$

Combine : $e^{i2\pi \nu t} = \cos(2\pi \nu t) + i \sin(2\pi \nu t)$ $\qquad$ Absorption = RE[g(ν)]

$f(t) = \int_{-\infty}^{\infty} g(\nu) e^{-i2\pi \nu t}\, d\nu \qquad g(\nu) = 2 \int_0^\infty f(t) e^{i2\pi \nu t}\, dt$ $\qquad$ Dispersion = IM[g(ν)]