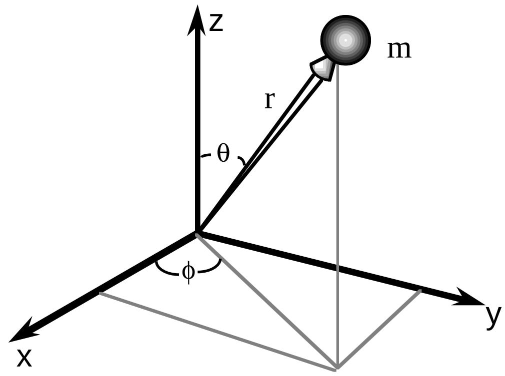
Rigid Rotor - Rotation in Three Dimensions

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$$

$$V = 0$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$



$$\nabla^2 = \frac{1}{r}\left(\frac{\partial^2}{\partial r^2}\right) r + \left(\frac{1}{r^2}\right)\Lambda^2$$ (angular momentum operator)$^2 = -\hbar^2\Lambda^2$

$$\Lambda^2 = \frac{1}{\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right) + \left(\frac{1}{\sin\theta}\right)\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}\right)$$

$$\frac{-\hbar^2}{2mr^2}\Lambda^2\Psi = E\Psi$$

$$\frac{-\hbar^2}{2I}\Lambda^2\Psi = E\Psi$$

$$\Psi = \Theta(\theta)\ \Phi(\phi)$$

$$\Phi(\phi) = \left(\frac{1}{2\pi}\right)^{1/2} e^{\,i m_l \phi}$$ $m_l = 0, \pm1. \pm2, \pm3, ...$

$$\Psi = Y_{l,m_l} = \Theta(\theta)\ \Phi(\phi)$$

$$-\hbar^2\Lambda^2 Y_{l,m_l} = l(l+1)\hbar^2 Y_{l,m_l}$$

l	m_l	Y_{l,m_l}
0	0	$(1/4\pi)^{1/2}$
1	0 ±1	$(3/4\pi)^{1/2}\cos\theta$ $\pm(3/8\pi)^{1/2}\sin\theta\, e^{\pm i\phi}$
2	0 ±1 ±2	$(5/16\pi)^{1/2}(3\cos^2\theta - 1)$ $\pm(15/8\pi)^{1/2}\cos\theta\sin\theta\ e^{\pm i\phi}$ $\pm(15/32\pi)^{1/2}\sin^2\theta\, e^{\pm i2\phi}$

$$\frac{-\hbar^2}{2I}\Lambda^2 Y_{l,m_l} = E\, Y_{l,m_l}$$

$$E = \frac{\hbar^2}{2I}\, l(l+1)$$

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magnetude of total angular momentum = $\hbar\sqrt{l(l+1)}$