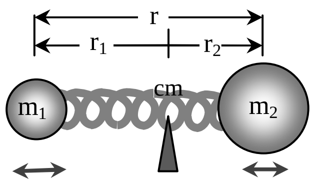
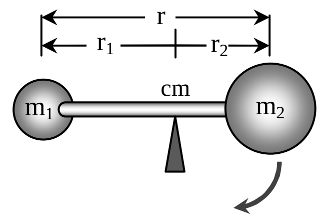
Classical Vibration and Rotation of Diatomic Molecules





vibration $\nu_o = {}^{1}/_{2\pi}\sqrt{k/\mu}$ rotation $I = \mu r^2$

Center of mass:

(I). $m_1 r_1 = m_2 r_2$ with $r = r_1 + r_2$ equal leverage

Add $m_2 r_1$ to both sides of (I): $m_1 r_1 + m_2 r_1 = m_2 r_2 + m_2 r_1$ or $(m_1 + m_2) r_1 = m_2(r_1 + r_2)$

(II). $r_1 = \dfrac{m_2}{m_1 + m_2} r$

Add $m_1 r_2$ to both sides of (I). $m_1 r_1 + m_1 r_2 = m_2 r_2 + m_1 r_2$ or $m_1(r_1 + r_2) = (m_1 + m_2) r_2$

(III). $r_2 = \dfrac{m_1}{m_1 + m_2} r$

$E_k = ½\, m_1\left(\dfrac{dr_1}{dt}\right)^2 + ½\, m_2\left(\dfrac{dr_2}{dt}\right)^2$ in reference to the center of mass

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Substitute (I). and (III).:

$$E_k = ½\,\frac{m_1 m_2^2}{(m_1 + m_2)^2}\left(\frac{dr}{dt}\right)^2 + ½\,\frac{m_1^2 m_2}{(m_1 + m_2)^2}\left(\frac{dr}{dt}\right)^2$$

$$E_k = ½\,\frac{m_1 m_2^2 + m_1^2 m_2}{(m_1 + m_2)^2}\left(\frac{dr}{dt}\right)^2 = ½\,\frac{m_1 m_2(m_2 + m_1)}{(m_1 + m_2)^2}\left(\frac{dr}{dt}\right)^2$$

$E_k = ½\,\dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{dr}{dt}\right)^2 = ½\,\mu\left(\dfrac{dr}{dt}\right)^2$ with $\mu \equiv \dfrac{m_1 m_2}{m_1 + m_2}$

Extension: $x \equiv r - r_o$ $\dfrac{dx}{dt} = \dfrac{d(r - r_o)}{dt} = \dfrac{dr}{dt}$ $E_k = ½\,\mu\left(\dfrac{dx}{dt}\right)^2$

$L = I\,\dfrac{d\phi}{dt}$ $E_k = \dfrac{L^2}{2I}$ $I = \displaystyle\sum_{i=1}^{n} m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2$

Multiply (I) by r_1: $m_1 r_1^2 = m_2 r_1 r_2$ or $m_2 r_1 r_2 = m_1 r_1^2$

Multiply (I) by r_2: $m_1 r_1 r_2 = m_2 r_2^2$

$(m_1 + m_2)\, r_1 r_2 = m_1 r_1^2 + m_2 r_2^2$

$I = (m_1 + m_2)\, r_1 r_2$

Substitute (II) and (III): $I = (m_1 + m_2)\left(\dfrac{m_2}{m_1 + m_2} r\right)\left(\dfrac{m_1}{m_1 + m_2} r\right) = \dfrac{m_1 m_2}{m_1 + m_2} r^2 = \mu r^2$