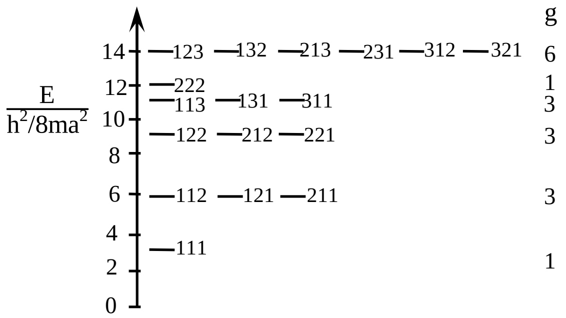
Particle in a 3-Dimensional Box

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\Psi + \frac{\partial^2}{\partial y^2}\Psi + \frac{\partial^2}{\partial z^2}\Psi\right) + V(x,y,z)\Psi = E\Psi$$

$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(x,y,z)\Psi = E\Psi \ 1$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\Psi + \frac{\partial^2}{\partial y^2}\Psi + \frac{\partial^2}{\partial z^2}\Psi\right) = E\,\Psi$$



For Separable Potentials, $\Psi(x,y,z)$ is a Product of One-Dimensional Wave Functions:

$\Psi(x,y,z) = X(x)Y(y)Z(z)$ for $0 \le x \le a$, $0 \le y \le b$, and $0 \le z \le c$

$$\frac{\partial^2}{\partial x^2}\Psi = Y(y)Z(z)\,\frac{d^2X(x)}{dx^2}$$

$$-\frac{\hbar^2}{2m}\left(Y(y)Z(z)\,\frac{d^2X(x)}{dx^2} + X(x)Z(z)\,\frac{d^2Y(y)}{dy^2} + X(x)Y(y)\,\frac{d^2Z(z)}{dz^2}\right) = E\,X(x)Y(y)Z(z)$$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} + \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + \frac{1}{Z(z)}\frac{d^2Z(z)}{dz^2}\right) = E = ?$$

$$-\frac{\hbar^2}{2m}\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = E_x \qquad -\frac{\hbar^2}{2m}\frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} = E_y \qquad -\frac{\hbar^2}{2m}\frac{1}{Z(y)}\frac{d^2Z(z)}{dz^2} = E_z$$

$$-\frac{\hbar^2}{2m}\frac{d^2X(x)}{dx^2} = E_x\,X(x) \qquad -\frac{\hbar^2}{2m}\frac{d^2Y(y)}{dy^2} = E_y\,Y(y) \qquad -\frac{\hbar^2}{2m}\frac{d^2Z(z)}{dz^2} = E_z\,Z(z)$$

$$X(x) = \left(\frac{2}{a}\right)^{1/2}\sin\frac{n_x\pi x}{a} \qquad Y(y) = \left(\frac{2}{b}\right)^{1/2}\sin\frac{n_y\pi y}{b} \qquad Z(z) = \left(\frac{2}{c}\right)^{1/2}\sin\frac{n_z\pi z}{c}$$

$$E_x = \frac{h^2}{8m}\frac{n_x^2}{a^2} \qquad E_y = \frac{h^2}{8m}\frac{n_y^2}{b^2} \qquad E_z = \frac{h^2}{8m}\frac{n_z^2}{c^2}$$

$$\Psi(x,y,z) = \left(\frac{4}{abc}\right)^{1/2}\sin\frac{n_x\pi x}{a}\sin\frac{n_y\pi y}{b}\sin\frac{n_z\pi z}{c}$$

$$E = E_x + E_y + E_z = \frac{h^2}{8m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)$$

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square box with side length a:

$$E = \frac{h^2}{8ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$