# The Eigenvalues for Quantum Mechanical Operators are Real

Hermitian operator: $\int \Psi_j^* \hat{o}\, \Psi_i\, d\tau = \int \Psi_i\, (\hat{o}\, \Psi_j)^*\, d\tau = \int \Psi_i\, \hat{o}^*\, \Psi_j^*\, d\tau$

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Hermitian operator $\hat{o}$ and one of its eigenfunctions $\Psi_n$:

I. $\hat{o}\, \Psi_n = o\, \Psi_n$

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Show that $o^* = o$

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Multiplication of I from the left by $\Psi_n^*$:

$\int \Psi_n^*\, \hat{o}\, \Psi_n\, dx = \int \Psi_n^*\, o\, \Psi_n\, dx = o \int \Psi_n^*\, \Psi_n\, dx$

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Complex conjugate of I:

II. $\hat{o}^*\, \Psi_n^* = o^*\, \Psi_n^*$

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Multiplication of II from the left by $\Psi_n$ :

$\int \Psi_n\, \hat{o}^*\, \Psi_n^*\, dx = \int \Psi_n\, o^*\, \Psi_n^*\, dx = o^* \int \Psi_n\, \Psi_n^*\, dx$

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$\hat{o}$ is Hermitian:
$\int \Psi_n^*\, \hat{o}\, \Psi_n\, dx = \int \Psi_n\, \hat{o}^*\, \Psi_n^*\, dx$

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$o \int \Psi_n^*\, \Psi_n\, dx = o^* \int \Psi_n\, \Psi_n^*\, dx$

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Just functions: $\int \Psi_n^*\, \Psi_n\, dx = \int \Psi_n\, \Psi_n^*\, dx$.

$o = o^*$

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The eigenvalues of Hermitian operators are real, therefore the eigenvalues for quantum mechanical observables are real.