Complex Waveforms-Euler Identity

We can prove that $e^{ix} = \cos x + i \sin x$ by finding the Taylor expansions for e^{ix}, sin x, and cos x.

Remember that : $$e^{x} = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \ldots + \frac{1}{n!}x^n$$

or $$e^{ix} = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \ldots + \frac{1}{n!}(ix)^n$$

Then expanding the trig functions:

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \ldots + (-1)^n \frac{1}{(2n+1)!}x^{2n+1}$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \ldots + (-1)^n \frac{1}{(2n)!}x^{2n}$$

Multiplying the sin expansion by i gives:

$$i \sin x = ix - \frac{1}{3!}ix^3 + \frac{1}{5!}ix^5 - \ldots + (-1)^n \frac{1}{(2n+1)!}ix^{2n+1}$$

Now noting that $i \cdot i = -1$,

$$i \sin x = ix + \frac{1}{3!}(ix)^3 + \frac{1}{5!}(ix)^5 + \ldots + \frac{1}{(2n+1)!}(ix)^{2n+1}$$

$$\cos x = 1 + \frac{1}{2!}(ix)^2 + \frac{1}{4!}(ix)^4 + \ldots + \frac{1}{(2n)!}(ix)^{2n}$$

Now combine the cos x and i sin x expansions:

$$\cos x + i \sin x = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \frac{1}{5!}(ix)^5 + \ldots$$

and you see that the result is the same expansion as for e^{ix}.

From trigonometric identities remember that sin(-x) = - sin(x) and cos(-x) = cos(x).
This is handy for finding the real and imaginary parts of wavefunctions.
To find the real part of a wavefunction note that you can take for the real part:

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix} \right)$$

and the imaginary part

$$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix} \right)$$