Flame Stretch, Edge Flames, and Flame Stabilization

Natarajan *et al.* Combustion and Flame 2007

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Course Outline

A) Introduction and Outlook

B) Flame Aerodynamics and Flashback

C) Flame Stretch, Edge Flames, and Flame Stabilization Concepts

D) Disturbance Propagation and Generation in Reacting Flows

E) Flame Response to Harmonic Excitation

- Introductory Concepts
- Flame Stretch
- Edge Flames
- Flame Stabilization in Shear Layers
- Flame Stabilization by Stagnation Points

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Flame Stabilization and Blowoff

- Flame stabilization requires:
  - A point where the local flame speed and flow velocity match: $s_u^d = |\vec{u}|$
    - Typically found in regions with:
      - Low flow velocity
        - » Aerodynamically decelerated regions (VBB)
      - High shear
        - » Locations of flow separation (ISL & OSL)

Figures:
Natarajan *et al*. Combustion and Flame 2007
Petersson *et al*. Applied Optics 2007

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Premixed Flame Stabilization: Basic Effects

- Flame stabilization:
  - Balance combustion wave propagation with flow velocity
    - Burning velocity, edge speed, autoignition front?

- Suggests that stable flames are rare
  - However, flames have self-stabilizing mechanisms
    - Shear layer stabilized: Upstream flame propagation increases wall quenching
    - Aerodynamic stabilization – velocity profiles

Lewis & Von Elbe 1987

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Flame Anchoring Locations

- Complex flows can provide multiple anchor locations

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Flame Stabilization and Blowoff

- Stabilization locations determine location/spatial distribution of flame
  - Flame Shape
  - Flame Length

- Combustor operability, durability, and emissions directly tied to these fundamental characteristics affecting:
  - Heat loadings to combustor hardware
  - Combustion instability boundaries
  - Blowoff limits

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Review of the Idealized Premixed Flame

- Simplest possible premixed flame configuration
  - 1-dimensional, planar
  - Adiabatic
- $T_b^0 = T_{ad} \rightarrow$ Adiabatic Flame Temperature
- $\rho_u u_u = \rho_u S_L^0 = \rho_b u_b \rightarrow$ Mass Burning Flux
- What are the controlling parameters?
  - Thermal and mass diffusivities
  - Reaction rates
  - Temperature of reactants
  - Pressure
  - Exothermicity of fuel/oxidizer
- $S_L^0 \rightarrow$ Fundamental property of fuel/oxidizer mixture

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What Happens if a Flame isn't Flat?

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What Happens if Flow Field isn't 1-D?

- Common theme to 2 problems:
  misaligned convective and diffusive fluxes

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Overview of Flame Stretch

- Consider a **stationary flame**; focus on a C.V. intersection of a streamtube, and the flame.
- Steady state energy balance
- (no viscous effects, no body forces)

$$\underbrace{\rho\vec{u}\cdot\nabla h_T}_{\text{Enthalpy Convection}} = \underbrace{-\nabla\cdot\vec{q}}_{\text{Enthalpy Diffusion}}$$

- Constitutive relation of enthalpy flux (Fickian diffusion, no radiative heat transfer, no Soret or DuFour effects)

$$\vec{q} = \underbrace{-k_T\nabla T}_{\text{Heat Flux}} \underbrace{-\rho\sum_{i=1}^{N} h_i \mathscr{D}_i \nabla Y_i}_{\text{Mass Flux}}$$

Curved flame
Straight streamlines

Flat flame
Curved streamlines

Control Volume
Heat flux
Mass flux

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Overview of Flame Stretch

- **Flame stretch effects: is there a net enthalpy loss/gain or change in composition inside the C.V. because of diffusion fluxes through its lateral surface?**
- Two mechanisms:
  - <u>Lewis Number effects</u> $Le = \alpha / \mathscr{D}$

    Diffusion of mass and of heat are unbalanced.
  - <u>Differential diffusion effects</u> $\mathscr{D}_{Fuel} / \mathscr{D}_{ox}$

    Lighter species diffuse faster than heavier species: equivalence ratio or diluent/reactant ratio inside the C.V. can change.

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Lewis number effects

- If ***Le* = 1** then $\alpha = \mathscr{D}$: no net enthalpy loss through the lateral surface

$$\text{Energy Eq.:}\quad \rho\vec{u}\cdot\nabla h_T = \nabla\cdot\left(\rho\mathscr{D}\nabla h_T\right)$$

- If ***Le* > 1** then $\alpha > \mathscr{D}$: Heat flux > Mass flux
  - Positive stretch: net enthalpy flux *out of* the C.V.
  - Negative stretch: net enthalpy flux *into* the C.V.

- If ***Le* < 1** then $\alpha < \mathscr{D}$: Heat flux < Mass flux
  - Positive stretch: net enthalpy flux *into* the C.V.
  - Negative stretch: net enthalpy flux *out of* the C.V.

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Differential diffusion effects

Consider the temperature at the tip of a Bunsen Flame T_{tip} :

- **CH_4/Air**: CH_4 is lighter than O_2, thus diffuses faster ($\mathscr{D}_{Fuel} > \mathscr{D}_{Ox}$) and its concentration inside the C.V. decreases.
  - If overall equivalence ratio is lean, then locally in the C.V. ϕ is made leaner: T_{tip} is lower then T_{ad} calculated at the overall ϕ.
  - If overall equivalence ratio is rich, then locally in the C.V. ϕ pulled toward stoichiometric: T_{tip} is then higher then T_{ad} calculated at the overall ϕ.
- **C_3H_8/Air**: C_3H_8 is heavier than O_2, thus diffuses more slowly ($\mathscr{D}_{Fuel} < \mathscr{D}_{Ox}$) and its concentration inside the C.V. increases.
  - The dependence of T_{tip} on the overall ϕ is opposite to that of CH_4/Air.

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Bunsen Tip Flame Temperature Data

From C.K. Law,
"Combustion Physics"

Figure 10.5.7. Comparison of measured flame tip temperatures and calculated adiabatic flame temperatures of methane–air, ethylene–air and propane–air Bunsen flames with various equivalence ratios.

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Example: Tips of Bunsen Flames

Propane(C_3H_8) **Methane(CH_4)**

d=10mm

$\phi =$	**1.38**	**0.53**	**1.52**	**0.58**

Ref: Mizomoto, Asaka, Ikai and Law, Proc. Combust. Inst. 20, 1933 (1984)

Slide courtesy of J. Seitzman

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Stretched Flames: Non-unity Lewis Numbers

Φ=0.27 Φ=0.27 Φ=0.37

T_u=298K
P=1atm

Le_i>1 Le_i≈1 Le_i<<1

Bell et al. *Proceedings of the Combustion Institute* (2007)

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Mathematical expressions of stretch κ

$$\kappa \equiv \frac{1}{A}\frac{dA}{dt}$$

Williams (1975)

- Flame stretch rate is defined as the normalized differential change with respect to time of an infinitesimal flame surface area element
- Flame stretch rate quantifies the degree of stretch imposed on a differential flame surface element
  - Lagrangian quantity
  - Units of flame stretch are 1/s – i.e. an inverse time scale

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Mathematical expressions of stretch κ

- Expression for κ in terms of flow velocity, u, and flame sheet velocity, v_F :

$$\kappa = \frac{\partial u_{t1}}{\partial t_1} + \frac{\partial u_{t2}}{\partial t_2} + (\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n}) = \underbrace{\nabla_t \cdot u_t}_{\kappa_a} + \underbrace{(\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n})}_{\kappa_b}$$

- **Hydrodynamic stretch κ_a**: variation of tangential flow velocity in the tangential direction (t_1, t_2) or, equivalently (by continuity), variation of normal flow velocity in the direction normal (n) to the flame.

- **Unsteady Curvature stretch κ_b**: non-stationary flames
  - **Positive κ**: divergent tangential velocities or expanding flame → flame area increases
  - **Negative κ**: convergent tangential velocity or contracting flame → flame area decreases

$$\kappa = \frac{1}{A}\frac{dA}{dt}$$

Hydrodynamic stretch

Unsteady curvature stretch

$$\nabla \cdot \vec{n} = \begin{cases} \pm 1/R & \text{cylynder} \\ \pm 2/R & \text{sphere} \end{cases}$$

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Mathematical expressions of stretch κ

- Stationary flames

$$\vec{v}_F = 0 \;\rightarrow\; \kappa = \nabla_{\not t} \cdot u_{\not t} = -\vec{n} \times \nabla \cdot \left(\vec{n} \times \vec{u}\right)$$

$$\text{since} \quad u_{\not t} = \vec{n} \times \left(\vec{u} \times \vec{n}\right)$$

Hydrodynamic stretch can be interpreted as a variation of the angle between flow velocity and flame normal or, equivalently, as a variation in tangential flow magnitude along the flame surface.

- Alternative flame stretch expression

$$\kappa = \underbrace{-\vec{n}\vec{n} : \nabla\vec{u} + \nabla \cdot \vec{u}}_{\kappa_S} - \underbrace{s^u (\nabla \cdot \vec{n})}_{\kappa_{curv}} = -\vec{n} \cdot \underline{\underline{S}} \cdot \vec{n} + \nabla \cdot \vec{u} - s^u (\nabla \cdot \vec{n})$$

$$\underline{\underline{S}} = \frac{1}{2}\left(\nabla\vec{u} + \nabla\vec{u}^T\right) \text{ Flow Strain}, \; s^u \cdot \vec{n} = \left(\vec{v}_F - \vec{u}\right)$$

  - κ_s: stretch due to flow non uniformities
  - κ_{curv}: stretch of a curved flame in a uniform approach flow

κ can be non zero also when the flow strain is zero

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Unsteady Effects – Motion of Curved Flames

- Curvature is present but flow velocities align with flame surface normal
- Stationary spherical flame would be stretchless

$$\kappa = \frac{\partial u_{t1}}{\partial t_1} + \frac{\partial u_{t2}}{\partial t_2} + (\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n}) = \underbrace{\nabla_t \cdot u_t}_{\kappa_a} + \underbrace{(\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n})}_{\kappa_b}$$

Combustion Physics by C.K. Law (Cambridge University Press, 2006)

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Weak stretch effects

- Asymptotic analysis shows that in the linear limit of weak stretch the effect of various type of stretch (κ, κ_a, κ_b, κ_S, κ_{curv}) on flame characteristics (s^u, δ_F,…) is the same.
- For the flame speed measured in a reference frame attached to the unburned gases, s^u, we can write:

$$s^u = s^u(\kappa) \sim s^u\big|_{\kappa=0} + \frac{\partial s^u}{\partial \kappa}\Big|_{\kappa=0}\kappa = s^{u,0} - \delta_M^u \kappa$$

δ_M^u Markstein length

- Definition of Markstein length is not unique but depends on the isosurface used to define it; e.g. for the flame speed measured in a reference frame attached to the burned gasses s^b we have:

$$s^b \sim s^{b,0} - \delta_M^b \kappa \qquad \delta_M^u \neq \delta_M^b$$

- Dimensionless quantities
  - Markstein number Ma
  - Karlovitz number Ka

$$Ma^u = \frac{\delta_M^u}{\delta_F^0} \qquad Ka = \frac{\delta_F^0 \kappa}{s^{u,0}} \quad \rightarrow \quad \frac{s^u}{s^{u,0}} = 1 - Ma^u Ka$$

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Weak stretch effects

- The Markstein number contains all the stretch effects described previously

$$Ma = Ma(Le, \mathscr{D}_{Fuel}/\mathscr{D}_{Ox}, \phi, \ldots)$$

- For lean mixtures of fuels lighter than air (e.g. H_2, CH_4) and rich mixtures of fuels heavier than air (e.g. C_3H_8), $Ma < 0$
  - Conversely, for lean mixtures of heavier than air fuels and rich mixtures of lighter than air fuels, $Ma > 0$

- *Ma* values are sensitive to the position at which flow and flame speeds are measured
  - For a specific mixture, measured *Ma* numbers in the literature can vary widely among different investigators.

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Application – Stoichiometry Effects

C_3H_8/Air

p = 1 atm, T^u = 300 K

Tseng et *al.*, *Comb. Flame*, 95(2), 1993

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Application – Fuel Effects

n-alkanes/air

p =1atm, T^u=300K

Ref: Tseng et al., Comb. Flame, 95(2), 1993

n-C_8H_{18}/air

Ref: Halter et al. Comb and Flame, 157 (2010)

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Application – Pressure Effects

H_2/CO 30/70 (by vol.) in air, T^u = 300 K
Counterflow twin flame
ϕ adjusted at different p to maintain $s^{u,0}$ = 34 cm/s = const

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Strong Stretch Effects

PREMIXED FLAME CONCEPTS

FLAME STRETCH AND FLAME EXTINCTION

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Displacement Speed s_d and Consumption Speed s_c

- **Displacement speed** s_d: speed at which the flame is moving along its normal relative to the flow
- The value of s_d depends on the reference surface:
  - Low κ: the approach flow varies weakly upstream of the flame and the iso-surface choice is not to problematic
  - High κ: velocity gradients occurs on a scale comparable to the flame thickness; s_d definition becomes ambiguous
- Displacement speed can also become negative when diffusive fluxes are strong enough to contrast the bulk convection in the opposite direction

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Displacement Speed s_d and Consumption Speed s_c

- **Consumption speed** s_c: spatial integral of chemical rates
  - Can be obtained integrating along a streamline the heat production rate and normalizing by the total change in sensible enthalpy across the flame
  - Alternatively, the integrated quantity can be a reactant species consumption rate normalized by the total change in reactant species mass density

Example: 1D steady flame sensible enthalpy balance (no viscous and body forces)

$$\underbrace{\int_{-\infty}^{\infty} \frac{d(\rho u_x h_{sens})}{dx} dx}_{\substack{\rho^u u^u h_{sens,\infty} - \rho^b u^b h_{sens,-\infty} \\ \underset{\rho^u u^u = \rho^b u^b}{=} \rho^u s_c^u \left(h_{sens,\infty} - h_{sens,-\infty}\right)}} = \int_{-\infty}^{\infty} \dot{q} dx - \underbrace{\int_{-\infty}^{\infty} \left(\frac{dq_x}{dx} + \frac{d}{dx}\left(\rho \sum_{i=1}^{N} h_{f,i}^0 Y_i u_{x,D,i}\right)\right) dx}_{=0} \rightarrow \rho^u s_c^u = \rho^b s_c^b = \int_{-\infty}^{\infty} \dot{q} dx \Big/ \left(h_{sens,\infty} - h_{sens,-\infty}\right)$$

Similarly from species eq. $\quad s_c = \int_{-\infty}^{\infty} \dot{w}_i dx \Big/ \rho\left(Y_{i,\infty} - Y_{i,-\infty}\right)$

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Displacement Speed s_d and Consumption Speed s_c

$s_c^0 = s_d^0$ but their values differ at non-zero κ values

H_2/CO 30/70 (by vol.) in air
ϕ = 0.75, T^u = 300 K, p = 5 atm

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Extinction stretch rate κ_{ext}

κ_{ext}: maximum stretch that a flame can sustain before extinguishing

H_2/CO 30/70 (by vol.) in air
ϕ = 0.75, T^u = 300K

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Example: Pressure Effects

- Most of the available data are for steady symmetric opposed flow flames :
  - κ_{ext} depends on flame chemical time $\tau_{chem} = \delta_F / s^{u,0}$ (eg. p effects at fixed $s^{u,0}$)

H_2/CO 30/70 (by vol.) in air
T^u = 300 K
ϕ adjusted at different p to maintain $s^{u,0}$ = 34 cm/s = const

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Example: Fuel and Stoichiometry Effects

Ref: Jackson et al., Comb. Fl., 1994, 25(1)

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Example: Preheat Effects

- At high dilution/preheating levels, the flame does not "extinguish“
  - increases in reactant temperature are equivalent to a reduction in dimensionless activation energy
  - Example: calculation of CH_4/air flame stagnating against hot products, whose temperature is indicated on the plot

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Caveats on Stretch Sensitivity of Highly Stretched flames

Stretch sensitivities and κ_{ext} values are not intrinsic to mixture but also depend on manner in which stretch is applied

- Example: it depends also on flame geometry and configuration:
  - Velocity profile across the flame thickness (eg. κ_{ext} for opposed flow flames depends on jets distance)
  - Type of stretch (κ, κ_a, κ_b, κ_S, κ_{curv})
  - Length and time scale of flame/stretch interaction

H_2/air, ϕ = 0.37, T^u = 298 K, p = 1 atm

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Overview

- Real flames have edges
- structure is different than "continuous" flames previously considered:
  - Non-premixed flames do not propagate, but their edges do;
  - Premixed flame edge velocity is different from the laminar burning rate (e.g., can be negative)
- Applications
  - Stabilization of non-premixed (a) and premixed flames (b)
  - Propagation of an ignition front (c)
  - Flame propagation after local extinction (d)
- Edge flame can be advancing, retreating or stationary
  - Attention has to be paid to the observer reference frame.

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Edge flame examples

- **Piloted Bunsen flame**
  - **A retreating flame edge that is stationary in lab coordinate**

Rajaram et al., Comb. Sci.Tech ,(175) 2003

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Edge flame examples

- **Premixed bluff body stabilized flame near blow-off**

  Chaudhury et al., Comb. Flame (158), 2011

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Edge flame examples

- **Cabra burner**

Dunn et. al, Comb. Flame ,(151) 2007

Vitiated coflow
1500K, 0.8m/s

Balloon number	Description
1	Flashback over-pressure sensing port
2	Glass bead filled cavities
3	Cooling water coil
4	Coflow perforated baseplate
5	Pilot mixture feed exit
6	Central jet exit
7	Pilot perforated baseplate
8	Coflow collar

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Edge Flame Concepts

Illustrative Model Problem

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Buckmaster's Edge flame model problem

- Generalize the one dimensional non-premixed chambered flame ($z \rightarrow +\infty$).

$$\rho c_p \frac{\partial T}{\partial t} - k_T \frac{\partial^2 T}{\partial z^2} = \underbrace{\left[k_T \frac{\partial^2 T}{\partial x^2} - \rho c_p u_x \frac{\partial T}{\partial x} \right]}_{\text{Transverse fluxes}} - Q \qquad \underbrace{\dot{w}_{Fuel}}_{= -\mathscr{B} \frac{\rho Y_{Fuel}}{MW_{Fuel}} Y_{Ox} e^{-E_a/\mathscr{R}_u T}}$$

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Buckmaster's Edge flame model problem

- Generalize the one dimensional non-premixed chambered flame ($z \rightarrow +\infty$).

$$\rho c_p \frac{\partial T}{\partial t} - k_T \frac{\partial^2 T}{\partial z^2} = \underbrace{\left[k_T \frac{\partial^2 T}{\partial x^2} - \rho c_p u_x \frac{\partial T}{\partial x} \right]}_{\text{approximate as } k_T \frac{(T - T_b)}{L^2}} - Q \qquad \underbrace{\dot{w}_{Fuel}}_{= -\mathcal{B} \frac{\rho Y_{Fuel}}{MW_{Fuel}} Y_{Ox} e^{-E_a/\mathcal{R}_u T}}$$

-Approximate transverse flux terms by convective loss-like term

-L characterizes the scale of gradients normal to the flame, such as due to strain.

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Edge flame model problem

- Dimensionless equation ($Le=1$)

$$\tilde{E}=\frac{E_a}{\mathscr{R}_u T_b},\quad \tilde{z}=\frac{z}{L},\quad \tilde{T}=\frac{T}{T_b},\quad \tilde{Q}=\frac{Y_{Fuel,b}Q}{c_p T_b},\quad Da=\frac{\tau_{flow}}{\tau_{chem}}=\frac{\rho c_p L^2/k_T}{1/\mathscr{B}}$$

$$\boxed{\tilde{v}_F\frac{d\tilde{T}}{d\tilde{z}}-\frac{d^2\tilde{T}}{d\tilde{z}^2}=F(\tilde{T},Da)}$$

where $$F(\tilde{T},Da)=1-\tilde{T}+Y_{Fuel,b}Da\frac{(1-\tilde{T}+\tilde{Q})^2}{\tilde{Q}}\exp\left(-\frac{\tilde{E}}{\tilde{T}}\right)$$ and $$\tilde{v}_F=\frac{v_F\rho c_p L}{k_T}$$

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Solution Limit for Edge-less Flame

- Steady state solution with no z-direction variation: $F(\tilde{T}, Da) = 0$
  - the problem becomes the same as the steady well stirred reactor;
  - recover the same *S-curve* behavior.

$$F(\tilde{T}, Da) = 1 - \tilde{T} + Y_{Fuel,b} Da \frac{(1-\tilde{T}+\tilde{Q})^2}{\tilde{Q}} \exp\left(-\frac{\tilde{E}}{\tilde{T}}\right) = 0$$

- We will focus on $Da_I < Da < Da_{II}$ range, where propagating flame edges can occur
- Three possible behaviors:
  - propagates ($v_F > 0$)
  - retreats ($v_F < 0$)
  - stays stationary ($v_F = 0$)

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Edge flame model problem

- Edge flame velocity

$$\tilde{v}_F = \int_{\tilde{T}_{low}}^{\tilde{T}_{high}} F(\tilde{T}, Da)\, d\tilde{T} \Bigg/ \int_{-\infty}^{\infty} \left(\frac{d\tilde{T}}{d\tilde{z}} \right)^2 dz$$

$\tilde{E} = 14,\ \tilde{Q} = 3$

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Edge flame model problem

- Edge flame velocity

$$\tilde{v}_F = \int_{\tilde{T}_{low}}^{\tilde{T}_{high}} F(\tilde{T}, Da)\, d\tilde{T} \Bigg/ \int_{-\infty}^{\infty} \left(\frac{d\tilde{T}}{d\tilde{z}} \right)^2 dz$$

- v_F sign depends on the numerator: $v_F>0$ for $Da>Da_V$ and $v_F<0$ for $Da<Da_V$

- Physical meaning:
  - Bi-stable steady state region further divided into 2 regions if an edge exists
  - for $Da>Da_V$ the flame edge acts as an ignition source; if the flame locally develops an hole this will close;
  - for $Da<Da_V$ the unburned gasses quench the flame; if the flame develops an hole this will spread until the whole flame is extinguished.

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Edge Flame Concepts

Edge Structure and Velocity

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Edge flames structure

- **Non-premixed flames**:
  - Advancing (ignition front, $v_F>0$) have often a triple flame structure;
  - Retreating (extinction front, $v_F<0$) generally consist of a single edge.

$v_F>0$ $v_F\sim 0$ $v_F<0$

Computed Reaction rate contours
Daou et al., Proc. Comb. Inst.,(29) 2002

- **Premixed flames**: generally have a single edge but can have significant hook-like structures

Image counterflow burner
Liu et al., Comb. Sci. Tech.,(144) 1999

Computed Reaction rate contours
Verdarajan et al., Comb. Flame,(114) 1998

- Also stretch and heat losses influence the flame edge structure

Low Stretch

High Stretch

False color images
Cha et al., Comb. Flame,(146) 2006

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Flame edge velocity

Gas Expansion Effects

- Velocity of edge flames v_F depends on density ratio σ_ρ ($=\rho^b/\rho^u$) across the flame, Damkohler number Da, heat losses, Le, $\mathscr{D}_{Fuel}/\mathscr{D}_{Ox}$ and $\mathcal{Z}_{st}$.
- **Dependence on σ_ρ**: for a convex flame the deceleration Δu of the approach flow in front of the flame monotonically increases with σ_ρ (to be discussed later)
- Reutsch et al.'s nonpremixed flame scaling for $Da \rightarrow \infty$

$$v_F\Big|_{Da\rightarrow\infty} \propto s^u\sqrt{\sigma_\rho}$$

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Flame edge velocity

Heat Loss Effects

- Heat losses may be important process in edge flames stabilized near metal surfaces
- Heat losses can cause $v_F<0$ at high Da
- **Example**: Non-Premixed counterflow flame

Cha, Ronney, *Comb.* 2006, 146 (1-2)

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Conditions at the flame edge

- Consider a premixed flame subject to a spatially varying stretch rate κ at t=0.
  - Hole will form at points where $\kappa > \kappa_{ext}$
  - edges will retreat to points where $\kappa = \kappa(v_F = 0) = \kappa_{edge}$

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Conditions at the flame edge

- From edge flame model problem

$$\frac{\kappa_{edge}}{\kappa_{ext}} \cong \frac{e^2}{(1-T_0/T_{ad})} \frac{\mathscr{R}_u T_{ad}}{E_a}$$

  - For T_0/T_{ad}=0.2 and $E_a/\mathscr{R}_u T_{ad}$=10-20
  - $0.5<\kappa_{edge}/\kappa_{ext}<1$

- Conclusion: once flame hole opens, it grows larger than would be expected based on continuous flame concepts

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Conditions at the flame edge

- Experiments show additional physics:
  - Tangential flows of hot gases (e.g., in a lab stationary, retreating edge flame) can increase v_F and cause $\kappa_{edge} > \kappa_{ext}$
  - When a hole forms in a premixed flames, reactants and products can mix:
    - Mass burning rate increases because of presence of radicals and increased initial temperature. Peak heat release rate changes little across different dilution levels;
    - The flame looses its S-curve character.

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E) Flame Response to Harmonic Excitation

- Introductory Concepts
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Stretch Effects on Shear Layer Flames

- Flame will extinguish when flame stretch rate exceeds κ_{ext}
  - As expected, higher flow velocities result in flame extinction occurring at higher values of κ_{ext}

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Shear Layer Stabilized Flames

- In high speed flows, although locally low velocities exist within the shear layer, flame extinction typically leads to liftoff/blowoff
  - Limited by the amount of flame stretch which they can withstand before extinction
  - e.g, 50 m/s jet with 1 mm shear layer thickness shear~du_z/dx ~ 50×10^3 s^{-1}
    - Much greater than typical extinction strain rates, κ_{ext}
- How is *flame stretch* related to *flow strain* in a shear layer?

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Sources of Flame Stretch

Q. Zhang *et al.* J. Eng. for Gas Turbines & Power 2010

1. Flame curvature
2. Unsteadiness in flame and flow
3. Hydrodynamic strain:

$$\kappa_{s,strain} = \left(\delta_{ij} - n_i n_j\right)\frac{\partial u_i}{\partial x_j}$$

   – For reference, fluid strain rate given by tensor:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

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Flame Stretch Due to Fluid Strain

- General expression can be reduced by assuming 2-D flow and incompressibility (upstream of flame):

$$\kappa_s = \underbrace{(n_x^{\,2} - n_z^{\,2})\frac{\partial u_z^u}{\partial z}}_{\text{Normal Strain Contribution}} \underbrace{- n_x n_z \left(\frac{\partial u_x^u}{\partial z} + \frac{\partial u_z^u}{\partial x} \right)}_{\text{Shear Strain Contribution}}$$

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Flame Stretch Due to Fluid Strain

Shear Strain Contribution

$$\kappa_{s,shear} = -n_x n_z \left(\frac{\partial u_x^u}{\partial z} + \frac{\partial u_z^u}{\partial x} \right)$$

- Flame strain occurs due to variations in tangential velocity
  - Leads to positive stretch

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Flame Stretch Due to Fluid Strain

Normal Strain Contribution

$$\kappa_{s,normal} = \left(n_x^{\,2} - n_z^{\,2}\right)\frac{\partial u_z^u}{\partial z}$$

- Jet flows typically decelerate producing normal strain
  - Leads to negative stretch

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Flame Stretch Due to Fluid Strain

For high speed flows:

$$u_z^u \gg s_d^u$$

$$\therefore \theta \ll 1$$

$$\Rightarrow \boxed{n_x = -1 + O(\theta^2)} \quad \& \quad \boxed{n_z = \theta + O\left(\theta^3\right)}$$

Also assume:

$$\frac{\partial u_x^u}{\partial z} << \frac{\partial u_z^u}{\partial x}$$

Expressions for shear and normal stretch can be simplified as follows:

$$\kappa_{s,shear} = -n_x n_z \left(\frac{\partial u_x^u}{\partial z} + \frac{\partial u_z^u}{\partial x} \right)$$

$$\boxed{\kappa_{s,shear} \approx \theta \frac{\partial u_z^u}{\partial x}}$$

$$\kappa_{s,normal} = \left(n_x^{\,2} - n_z^{\,2} \right) \frac{\partial u_z^u}{\partial z}$$

$$\boxed{\kappa_{s,normal} \approx \frac{\partial u_z^u}{\partial z}}$$

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Stretch from Shear Strain

- $\kappa_{s,shear} \approx \theta \dfrac{\partial u_z^u}{\partial x}$

- If $\theta \approx \dfrac{s_d^u}{u_z^u}$ then $\kappa_{s,shear} \approx \dfrac{s_d^u}{u_z^u}\dfrac{\partial u_z^u}{\partial x}$

$$\rightarrow \kappa_{s,shear} \sim s_d^u / \delta_{sh}$$

Shearing flow velocity profile

$\Rightarrow$ ***Stretch rate due to shear ~indep. of u ?***

increasing u $\Rightarrow$ *shear* $\uparrow$ *but* $\theta \downarrow$

but *u can influence shear layer:* $\boldsymbol{\delta_{sh} \propto u^{-1/2}}$

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Stretch from Normal Strain

$$\kappa_{s,normal} \sim \frac{\partial u_z^u}{\partial z} \sim \frac{u_z^u}{L_{char}}$$

- Obtain traditional flow time scaling approach

Decelerating flow velocity profile

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Total Stretch from Strain

- Total stretch due to strain

$$\kappa_s \approx \frac{\partial u_z^u}{\partial z} + \theta \frac{\partial u_z^u}{\partial x}$$

Shearing flow velocity profile

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$\Rightarrow$ ***Opposite signs if decelerating flow***

which term dominates - θ small ?

Decelerating flow velocity profile

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$$\text{even if } \frac{\partial u_x^u}{\partial z} << \frac{\partial u_z^u}{\partial x} \qquad \frac{\partial u_x^u}{\partial z} \text{ vs. } \theta \frac{\partial u_z^u}{\partial x} ??$$

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Flame Stretch Distribution in Swirl Flame

- Dimensionless value of 1= 4,125 1/s
- Shows dominance of deceleration term in first 10 mm
- Shear term dominates farther downstream

Zhang, Q., Shanbogue, S., Shreekrishna,O'Connor, J., Lieuwen, T., "Strain Characteristics Near the Flame Attachment Point in a Swirling Flow", *Combustion Science and Technology*, Vol. 83, 2011, 665-685.

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Piloting or Flow Recirculation Effects

- Flame stabilization can be enhanced through:
  - Pilot flames
  - Recirculation zones
    - Transport hot products to the attachment point of a flame

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Dilution/Liftoff Effects

- At high dilution/preheating levels, the flame does not "extinguish"
  - Increases in reactant temperature are equivalent to a reduction in dimensionless activation energy
  - Example: calculation of CH_4/air flame stagnating against hot products with indicated temperature

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Blowoff of Bluff Body Stabilized Flames

- Stages of blowoff
  - Stage 1: Flame is continuous but marked by local extinction events
  - Stage 2: Changes in wake dynamics as large scale structures become visible
  - Stage 3: Blowoff of flame
- Da Approach:
  - Scaling captures onset of flame extinction events
  - Ability to capture blowoff dependent on link between extinction events and blowoff physics

Nair J. Prop. Power 2007

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Flame Anchoring Locations and Flame Shapes in Swirling Flows

(c) IRZ & Outer Nozzle

(d) Centerbody & Outer Nozzle

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Flame Anchoring Locations and Flame Shapes in Annular Nozzle Geometries

- Flame stabilizes in front of stagnation point of vortex breakdown bubble
  - Stagnation point apparently precesses, probably also moves up and down
  - i.e., flame anchoring position highly unsteady, in contrast to stabilization at edges/corners
- Under what circumstances can such flames exist?
  - Not always observed; flames may blowoff directly without reverting to a "free floating" configuration
  - Flow must have interior stagnation point

From Kumar and Lieuwen

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Vortex Breakdown in Annular Geometries

- Nature of centerbody wake/ VBB changes with geometry, swirl #, and Reynolds #

Recirculation zone with vortex tube

Sheen *et al.*, Phys. Fluids, 1996

Recirculation zone with bubble-like breakdown above

Merged recirculation zones

Swirl number/ Centerbody Diameter

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Vortex Breakdown in Annular Geometries

- Nature of centerbody wake/ VBB changes with geometry, swirl #, heat release parameter, and Reynolds #

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Vortex Breakdown in Annular Geometries

- Nature of centerbody wake/ VBB changes with geometry, swirl #, and Reynolds #

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Flame Stabilization Influenced by Downstream Boundary Conditions

- Fluid rotation introduces inertial wave propagation mechanism
  - "sub-" and "supercritical" flow distinction
- Exit boundary condition has significant influence on vortex breakdown bubble topology

Emara *et. al 2009*

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