Hydrogen Atomic Orbitals

$$\Psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/a_o}$$

$$\Psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_o}\right)^{3/2}\left(2 - \frac{Zr}{a_o}\right) e^{-Zr/2a_o}$$

$$\Psi_{2pz} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/2a_o}\,\frac{Zr}{a_o}\cos\theta$$

$$\Psi_{211} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/2a_o}\,\frac{Zr}{a_o}\sin\theta\, e^{i\phi} \qquad l_z = \hbar$$

$$\Psi_{21-1} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/2a_o}\,\frac{Zr}{a_o}\sin\theta\, e^{-i\phi} \qquad l_z = -\hbar$$

$$\frac{e^{i\phi} + e^{-i\phi}}{2} = \frac{(\cos\phi + i\sin\phi) + (\cos\phi - i\sin\phi)}{2} = \cos\phi$$

$$\frac{e^{i\phi} - e^{-i\phi}}{2i} = \frac{(\cos\phi + i\sin\phi) - (\cos\phi - i\sin\phi)}{2i} = \sin\phi$$

$$\Psi_{2px} = \frac{\Psi_{211} + \Psi_{21-1}}{2} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/2a_o}\,\frac{Zr}{a_o}\sin\theta\cos\phi$$

$$\Psi_{2py} = \frac{\Psi_{211} - \Psi_{21-1}}{2i} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_o}\right)^{3/2} e^{-Zr/2a_o}\,\frac{Zr}{a_o}\sin\theta\sin\phi$$

Angular portion of the d-orbitals

Notice that:

$2p_z \Rightarrow r\cos\theta = z$ $\qquad$ $2p_x \Rightarrow r\sin\theta\cos\phi = x$ $\qquad$ $2p_y \Rightarrow r\sin\theta\sin\phi = y$

$d_{xz} \Rightarrow xz = \sin\theta\cos\phi\cos\theta$

$d_{yz} \Rightarrow yz = \sin\theta\sin\phi\cos\theta$

$d_{xy} \Rightarrow xy = \sin\theta\cos\phi\,\sin\theta\sin\phi = \sin\theta^2\cos\phi\,\sin\phi$

$d_{x^2-y^2} \Rightarrow x^2 - y^2 = \sin^2\theta\cos^2\phi - \sin^2\theta\sin^2\phi$

$d_{z^2} = d_{z^2-1} \Rightarrow 3\cos^2\theta - 1$

needed to make a sphere from superposition of all d orbitals