

Thermodynamic Functions from Partition Functions

$$U - U(0) = -\frac{1}{Q} \left(\frac{\partial Q}{\partial \beta} \right)_V \quad S = k \ln Q + \frac{U - U(0)}{T}$$

$$H = U + PV \quad \text{at } T = 0, \quad H(0) = U(0)$$

for ideal gases: $PV = nRT$

$$H - H(0) = U - U(0) + nRT$$

$$A = U - TS \quad \text{at } T = 0, \quad A(0) = U(0)$$

$$A - A(0) = U - U(0) - TS$$

$$A - A(0) = U - U(0) - kT \ln Q - (U - U(0))$$

$$A - A(0) = -kT \ln Q$$

$$P = -\left(\frac{\partial A}{\partial V} \right)_T = kT \left(\frac{\partial \ln Q}{\partial V} \right)_T$$

$$G = A + PV \quad \text{at } T=0, \quad G(0) = A(0)$$

$$G - G(0) = A - A(0) + PV$$

$$G - G(0) = -kT \ln Q + kTV \left(\frac{\partial \ln Q}{\partial V} \right)_T$$

for ideal gases: $PV = nRT$ $G - G(0) = -kT \ln Q + nRT$

$$Q = \frac{q^N}{N!} \quad G - G(0) = -kT \ln q^N + kT \ln N! + nRT$$

$$\ln N! = N \ln N - N \quad G - G(0) = -NkT \ln q + NkT \ln N - NkT + nRT$$

$$Nk = nR \quad G - G(0) = -nRT \ln q + nRT \ln N - nRT + nRT$$

$$G - G(0) = -nRT \ln \left(\frac{q}{N} \right)$$

separate out translational and internal degrees of freedom:

$$G - G(0) = -nRT \ln \left(\frac{q_t}{N} \right) - nRT \ln (q_r q_v q_e)$$