<u>Postulates of Quantum Mechanics</u>[1]

I. The physical state of the system is described by a wave function as completely as possible. The wave function is derived from an orthonormal set of eigenfunctions of the Hamiltonian.

Example: Particle-in-a-Box: $\Psi_n(x) = (2/a)^{½} \sin(n\pi x/a)$ are the solutions to $\mathcal{H}\Psi = E\Psi$

II. Any observable may be represented by a linear operator. The results should be a real number. The <u>least</u> restrictive requirement is that the operator must be Hermitian:

$$\int \Psi_j^* \hat{o}\, \Psi_i\, dx = \int \Psi_i\, (\hat{o}\, \Psi_j)^*\, dx = \int \Psi_i\, \hat{o}^*\, \Psi_j^*\, dx$$

The observable operator is constructed from the following table:

<u>Classical</u>	<u>Quantum Operator</u>
x	$\hat{x}$
p	$\frac{\hbar}{i}\left(\frac{\partial}{\partial x}\right) = -i\hbar\left(\frac{\partial}{\partial x}\right)$
t	$\hat{t}$
E vs. time	$\hat{E} = i\hbar\left(\frac{\partial}{\partial t}\right)$

III. If the wave function is an eigenfunction of the observable, then repeated measurements of the observable always give the same result, the eigenvalue:

if $\hat{o}\,\Psi = o\,\Psi$ then each measurement gives the result, o.

Example: Free Particle: e^{ikx} is an eigenfunction of the momentum operator $\frac{\hbar}{i}\left(\frac{d}{dx}\right)$, so the momentum of the particle is constant, $p = \hbar k$.

IV. If the wave function is not an eigenfunction of the observable, the measurement gives a different result each time. So, you need to find the "average," or expectation value.

$$\langle o \rangle = \frac{\int_{-\infty}^{\infty} \Psi^*\, \hat{o}\, \Psi\, dx}{\int_{-\infty}^{\infty} \Psi^*\, \Psi\, dx}$$

Example: Particle-in-a-Box: $\Psi_n = (2/a)^{½} \sin(n\pi x/a)$ is not an eigenfunction of the momentum operator, so to find the average momentum you must find the expectation value:

$$\langle p_x \rangle = \frac{\int_o^a \Psi_n^*\, \hat{p}_x\, \Psi_n\, dx}{\int_o^a \Psi_n^*\, \Psi_n\, dx} = \int_0^a \Psi_n \frac{\hbar}{i}\left(\frac{d}{dx}\right) \Psi_n\, dx = \left(\frac{2}{a}\right) \int_0^a \sin(n\pi x/a)\, \frac{\hbar}{i}\left(\frac{d}{dx}\right) \sin(n\pi x/a)\, dx$$

$$= \frac{\hbar}{i}\left(\frac{2}{a}\right)\left(\frac{n\pi}{a}\right) \int_0^a \sin(n\pi x/a) \cos(n\pi x/a)\, dx = 0$$

V. The wave function evolves in time according to $\mathcal{H}\Psi(x,t) = i\hbar\left(\frac{\partial \Psi(x,t)}{\partial t}\right)$

II '. Commuting observables can be simultaneously specified with arbitrary precision.

$$\sigma_A\, \sigma_B \geq \left| \frac{1}{2i} \int \Psi^* [\hat{A},\hat{B}]\, \Psi\, dx \right| \qquad \text{where } |\;| \text{ is the absolute value}$$

Example: Position and momentum: $[\hat{x}, \hat{p}_x] = -(\hbar/i)$, giving $\sigma_x\, \sigma_p \geq \hbar/2$.

1. See Hanna Section 3-3, McQuarrie & Simon Ch. 4, Winn Section 11.2