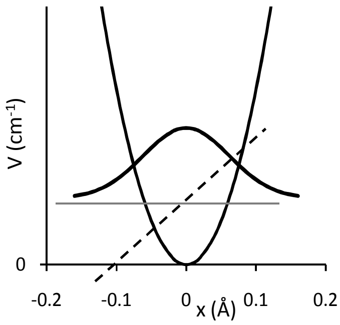
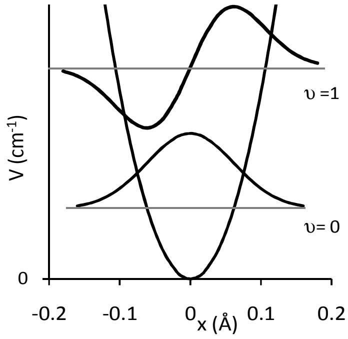
## Harmonic Oscillator-Excited States

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V(x)\Psi = E\Psi$$

$$V(x) = \tfrac{1}{2}\,k\,x^2$$

$$\Psi_o(x) = N_o\, e^{-\frac{1}{2}\alpha^2 x^2}$$

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$\Psi_1(x) = \text{polynomial} * \Psi_o(x)$ polynomial has one zero: $ax + b$

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$E = h\nu_o(\upsilon + \tfrac{1}{2})$

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$y = \alpha x$ $\Psi_\upsilon(y) = N_\upsilon H_\upsilon(y)\, e^{-\frac{1}{2}y^2}$

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polynomial = $H_\upsilon(y)$ $\dfrac{d^2H_\upsilon}{dy^2} - 2y\dfrac{dH_\upsilon}{dy} + 2\upsilon\, H_\upsilon = 0$ Hermite Polynomials

| $\upsilon$ | $H_\upsilon(y)$ | $H_\upsilon(\alpha x)$ | $\Psi_\upsilon(\alpha x)$ |
|---|---|---|---|
| 0 | 1 | 1 | $(\alpha/\pi^{1/2})^{1/2}\, e^{-\frac{1}{2}\alpha^2x^2}$ |
| 1 | $2y$ | $2\alpha x$ | $(\alpha/2\pi^{1/2})^{1/2}\,(2\alpha x)\, e^{-\frac{1}{2}\alpha^2x^2}$ |
| 2 | $4y^2 - 2$ | $4\alpha^2x^2 - 2$ | $(\alpha/8\pi^{1/2})^{1/2}\,(4\alpha^2x^2 - 2)e^{-\frac{1}{2}\alpha^2x^2}$ |
| 3 | $8y^3 - 12y$ | $8\alpha^3x^3 - 12\alpha x$ | $(\alpha/48\pi^{1/2})^{1/2}\,(8\alpha^3x^3 - 12\alpha x)\, e^{-\frac{1}{2}\alpha^2x^2}$ |

Generator: $H_{\upsilon+1} = 2y\, H_\upsilon - 2\upsilon\, H_{\upsilon-1}$ Recursion relation

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Example: $H_2 = 2y\, H_1 - 2\, H_o$ $H_2 = 2y(2y) - 2(1) = 4y^2 - 2$

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Orthogonality: $\int_{-\infty}^{\infty} H_{\upsilon'}\, e^{-\frac{1}{2}y^2}\, H_\upsilon\, e^{-\frac{1}{2}y^2}\, dy = 0$ if $\upsilon' \neq \upsilon$ $= \pi^{1/2}\, 2^\upsilon\, \upsilon!$ if $\upsilon' = \upsilon$

$$N_\upsilon = \left(\frac{\alpha}{\pi^{1/2}\, 2^\upsilon\, \upsilon!}\right)^{1/2}$$

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