Heisenberg Uncertainty and the Particle in a Box

Uncertainty: Standard Deviation

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x$$

$$\sigma_x^2 = \frac{\sum (x - \overline{x})^2}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x^2 - 2x\overline{x} + \overline{x}^2) \quad = \frac{1}{n}\sum_{i=1}^{n} x^2 \; - 2\,\overline{x}\,\frac{1}{n}\sum_{i=1}^{n} x + \frac{\overline{x}^2}{n}\sum_{i=1}^{n} 1$$

$$= \frac{1}{n}\sum x^2 - 2\,\overline{x}^2 + \overline{x}^2$$

$$\sigma_x^2 = \overline{x^2} - \overline{x}^2$$

$$\sigma_x^2 = \langle x^2 \rangle - (\langle x \rangle)^2$$

Particle in a Box

$\langle x \rangle = a/2$

$\langle p \rangle = 0$

$$E_n = \frac{\pi^2\,\hbar^2\,n^2}{2ma^2} = \frac{p^2}{2m} \qquad \langle p^2 \rangle = \frac{\pi^2\,\hbar^2\,n^2}{a^2}$$

$$\sigma_p^2 = \langle p^2 \rangle - (\langle p \rangle)^2 = \frac{\pi^2\,\hbar^2\,n^2}{a^2}$$

$$\sigma_p = \frac{\pi\hbar n}{a}$$

$$\sigma_x = \frac{a}{2\pi n}\left(\frac{\pi^2 n^2}{3} - 2\right)^{1/2}$$

$$\sigma_x\,\sigma_p = \frac{a}{2\pi n}\left(\frac{\pi^2 n^2}{3} - 2\right)^{1/2}\frac{\pi\hbar n}{a} = \frac{\hbar}{2}\left(\frac{\pi^2 n^2}{3} - 2\right)^{1/2} \qquad \left(\frac{\pi^2}{3} - 2\right)^{1/2} = 1.136$$

$$\sigma_x\,\sigma_p \geq \frac{\hbar}{2}$$