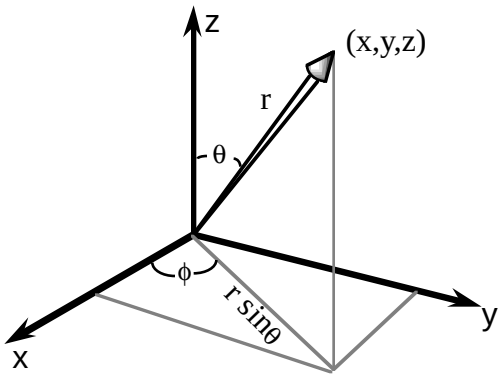
Spherical Polar Coordinates

$x = r \sin\theta \cos\phi$
$y = r \sin\theta \sin\phi$
$z = r \cos\theta$

θ: $0 \le \theta \le \pi$
ϕ: $0 \le \phi \le 2\pi$
r: $0 \le r \le \infty$
$r^2 = x^2 + y^2 + z^2$

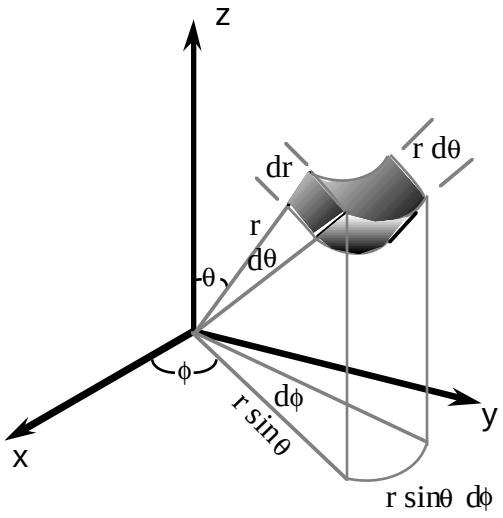


Volume Element in Spherical Polar Coordinates

$dV = dx\, dy\, dz = dr\ r\, d\theta\ r \sin\theta\, d\phi$

$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$\int_0^{2\pi} d\phi = 2\pi \qquad \int_0^{\pi} \sin\theta\, d\theta = 2$$



Total Solid Angle

$$\int_0^{\pi} \int_0^{2\pi} \sin\theta\, d\theta\, d\phi = 4\pi$$

Volume of a Sphere

$$V = \int_0^{r_0} \int_0^{\pi} \int_0^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi =$$

Surface Area of a Sphere

$$\text{Surface area} = \int_0^{\pi} \int_0^{2\pi} r_o^2 \sin\theta\, d\theta\, d\phi =$$