The Momentum Operator is Hermitian

Hermitian:

$$\int \Psi_j^* \hat{o}\, \Psi_i \, dx = \int \Psi_i (\hat{o}\, \Psi_j)^* \, dx = \int \Psi_i \, \hat{o}^* \, \Psi_j^* \, dx$$

$$\hat{p} = \left(-\, i\hbar \frac{d}{dx} \right)$$

Show:

$$\int_{-\infty}^{\infty} \Psi_j^* \left(-\, i\hbar \frac{d}{dx} \right) \Psi_i \, dx = \int_{-\infty}^{\infty} \Psi_i \left(-\, i\hbar \frac{d}{dx} \right)^* \Psi_j^* \, dx$$

$$\frac{d\Psi_i}{dx} dx = d\Psi_i$$

$$\int_{-\infty}^{\infty} \Psi_j^* \left(-\, i\hbar \frac{d}{dx} \right) \Psi_i \, dx = -\, i\hbar \int_{-\infty}^{\infty} \Psi_j^* \, d\Psi_i$$

Integration by parts: $\int u \, dv = uv - \int v \, du$

with $u = \Psi_j^*$ and $dv = d\Psi_i$

$$-\, i\hbar \int_{-\infty}^{\infty} \Psi_j^* \, d\Psi_i = -\, i\hbar \left[\Psi_j^* \Psi_i \Big|_{x = -\infty}^{x = \infty} - \int_{-\infty}^{\infty} \Psi_i \, d\Psi_j^* \right]$$

For a confined particle: the product $\Psi_j^* \Psi_i$ goes to zero at each endpoint, since the wave function approaches zero for long distances

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$$\int_{-\infty}^{\infty} \Psi_j^* \left(-\, i\hbar \frac{d}{dx} \right) \Psi_i \, dx = i\hbar \int_{-\infty}^{\infty} \Psi_i \, d\Psi_j^* = \int_{-\infty}^{\infty} \Psi_i \left(i\hbar \frac{d}{dx} \right) \Psi_j^* \, dx$$

$$(-\, i\hbar \, d/dx)^* = (i\hbar \, d/dx)$$