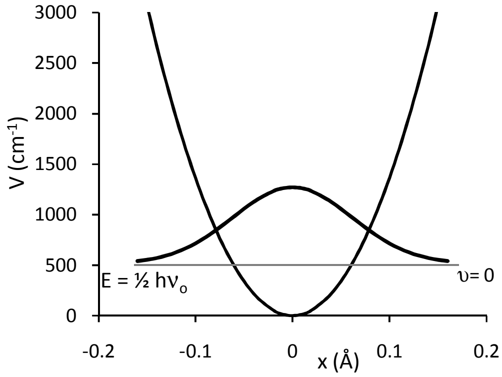
<u>Harmonic Oscillator-Ground State</u>

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V(x)\Psi = E\Psi$$

$$V(x) = ½\ k x^2$$

Guess: $\Psi(x) = A\ e^{-½\alpha^2x^2}$

$$\nu_o = \frac{1}{2\pi}\sqrt{k/m} \qquad \omega_o = 2\pi\nu_o \qquad \omega_o = \sqrt{k/m}$$



$$\frac{d\Psi}{dx} = A\ (-½\ \alpha^2)(2x)\ e^{-½\alpha^2x^2} = -A\ \alpha^2 x\ e^{-½\alpha^2x^2}$$

$$\frac{d^2\Psi}{dx^2} = -A\ \alpha^2\left(x(-½\ \alpha^2)(2x)\ e^{-½\alpha^2x^2} + e^{-½\alpha^2x^2}\right)$$

$$= A\ \alpha^4x^2\ e^{-½\alpha^2x^2} - A\ \alpha^2\ e^{-½\alpha^2x^2}$$

$$= \alpha^4x^2\ \Psi(x) - \alpha^2\ \Psi(x)$$

$$-\frac{\hbar^2}{2m}\left(\alpha^4x^2\ \Psi(x) - \alpha^2\ \Psi(x)\right) + ½\ k x^2 = E\Psi$$

$$-\frac{\hbar^2}{2m}\alpha^4x^2 + \frac{\hbar^2}{2m}\alpha^2 + ½\ k x^2 = E$$

$$E = \frac{\hbar^2\alpha^2}{2m} \qquad \text{or} \qquad \alpha = \frac{\sqrt{2mE}}{\hbar}$$

$$-\frac{\hbar^2}{2m}\alpha^4x^2 + ½\ k x^2 = 0 \qquad \text{or} \qquad -\frac{\hbar^2}{m}\alpha^4 + k = 0$$

$$\frac{\hbar^2}{m}\alpha^4 = k \qquad \alpha^2 = \frac{\sqrt{mk}}{\hbar} = \frac{m}{\hbar}\sqrt{\frac{k}{m}} = \frac{m\omega_o}{\hbar}$$

$$E = \frac{\hbar^2\alpha^2}{2m} = ½\ \hbar\omega_o = ½\ \frac{h}{2\pi}2\pi\nu_o = ½\ h\nu_o$$

quantum number υ: $E_\upsilon = h\nu_o\ (\upsilon + ½)$

$$\int_{-\infty}^{\infty}\Psi^2(x)\ dx = 1 \qquad A = \left(\frac{\alpha^2}{\pi}\right)^{¼}$$

$$\Psi(x) = \left(\frac{m\omega_o}{\hbar\pi}\right)^{¼} e^{-\frac{m\omega_o}{2\hbar}x^2}$$