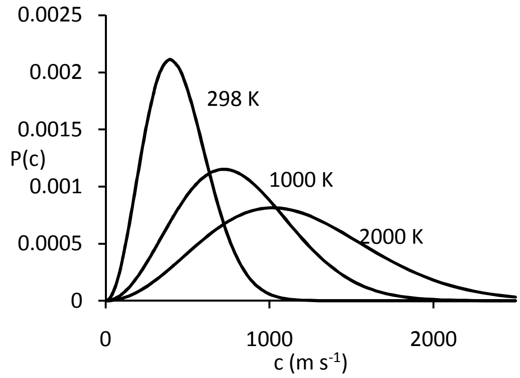
Average Molecular Speed

$$\frac{n_i}{n} = \frac{e^{-\varepsilon_x/kT}}{(2\pi mkT)^{1/2}\, a/h}$$

$$P(v_x)\, dv_x = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-mv_x^2/2kT}\, dv_x$$

$$\int_{-\infty}^{\infty} P(v_x)\, dv_x = \left(\frac{m}{2\pi kT}\right)^{1/2} \int_{-\infty}^{\infty} e^{-mv_x^2/2kT}\, dv_x = 1$$



$$P(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m(v_x^2+v_y^2+v_z^2)/2kT}\, dv_x\, dv_y\, dv_z$$

$$c^2 = v_x^2 + v_y^2 + v_z^2$$

$$P(c)\, dv_x\, dv_y\, dv_z = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mc^2/2kT}\, dv_x\, dv_y\, dv_z$$

$$P(c)\, dc = \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^{\pi}\int_0^{2\pi} e^{-mc^2/2kT}\, c^2 \sin\theta\, dc\, d\theta\, d\phi$$

Maxwell-Boltzmann Distribution of Molecular Speeds

$$P(c)\, dc = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mc^2/2kT}\, c^2\, dc$$

$$\bar{c} = \int_0^{\infty} c\, P(c)\, dc$$

$$\bar{c} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^{\infty} c^3\, e^{-mc^2/2kT}\, dc$$

$$\int_0^{\infty} x^3\, e^{-ax^2}\, dx$$

$$\int_0^{\infty} x^3\, e^{-ax^2}\, dx = \frac{1}{2a^2}$$

$$\bar{c} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{2} \left(\frac{m}{2kT}\right)^{-2} = \frac{2}{\pi^{1/2}} \left(\frac{m}{2kT}\right)^{-1/2}$$

$$\bar{c} = \left(\frac{8kT}{\pi m}\right)^{1/2}$$

$$\bar{c}_{rel} = \left(\frac{8kT}{\pi\mu}\right)^{1/2} \qquad \text{with } \mu = \frac{m_A\, m_B}{m_A + m_B} = \left(\frac{\mathcal{M}_A\, \mathcal{M}_B}{\mathcal{M}_A + \mathcal{M}_B}\right) \frac{1}{N_A}\, (1\ \text{kg}/1000\ \text{g})$$