



# Internal Combustion Engines I: Fundamentals and Performance Metrics

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2018 Princeton-Combustion Institute  
Summer School on Combustion  
Course Length: 9 hrs

(Mon.- Wed., June 25-27)

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### **Short course outline:**

Internal Combustion (IC) engine fundamentals and performance metrics, computer modeling supported by in-depth understanding of fundamental engine processes and detailed experiments in engine design optimization.

#### **Day 1 (Engine fundamentals)**

Hour 1: IC Engine Review, Thermodynamics and 0-D modeling

Hour 2: 1-D modeling, Charge Preparation

Hour 3: Engine Performance Metrics, 3-D flow modeling

#### **Day 2 (Computer modeling/engine processes)**

Hour 4: Engine combustion physics and chemistry

Hour 5: Premixed Charge Spark-ignited engines

Hour 6: Spray modeling

#### **Day 3 (Engine Applications and Optimization)**

Hour 7: Heat transfer and Spray Combustion Research

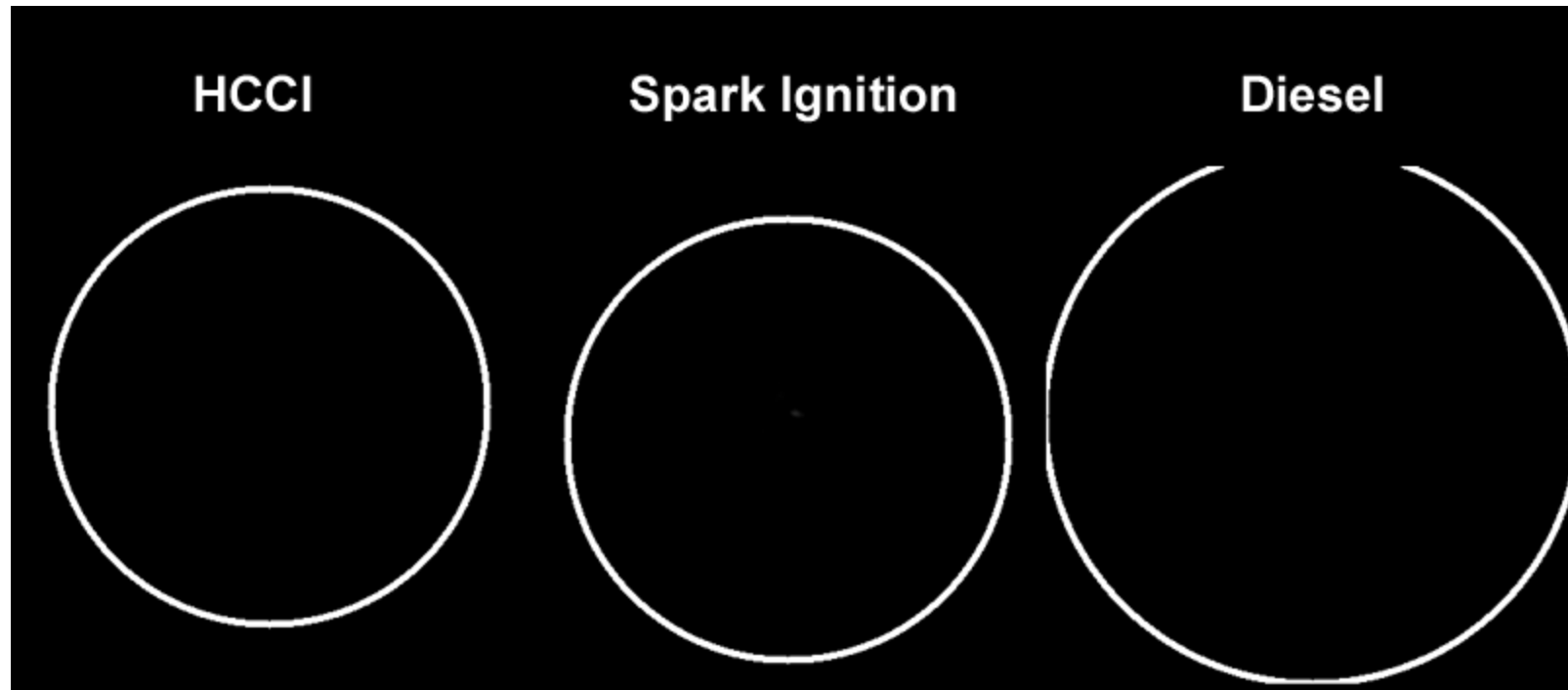
Hour 8: Diesel Combustion modeling

Hour 9: Optimization and Low Temperature Combustion



### Modes of engine combustion

<http://www.erc.wisc.edu/combustion.php>



HCCI uses a hybrid combustion strategy. Premixed fuel and air is inducted, but instead of igniting with a spark as in a SI engine, the high temperature from compression causes the mixture to spontaneously react, like in a diesel engine. Ignition occurs at slightly different times at different locations in the chamber.

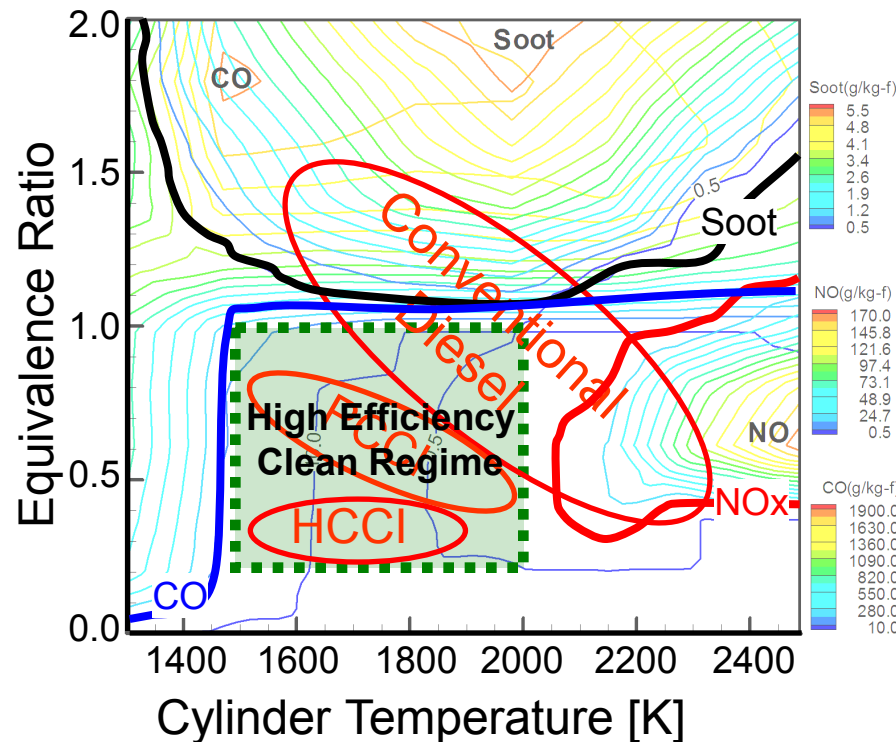
One feature of HCCI combustion is how quickly the fuel is consumed.





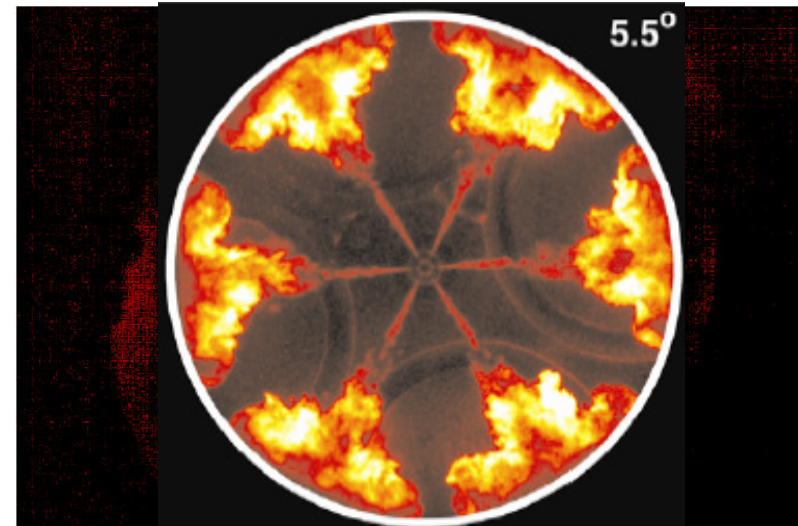
## IC Engine combustion regimes

Kamimoto plot

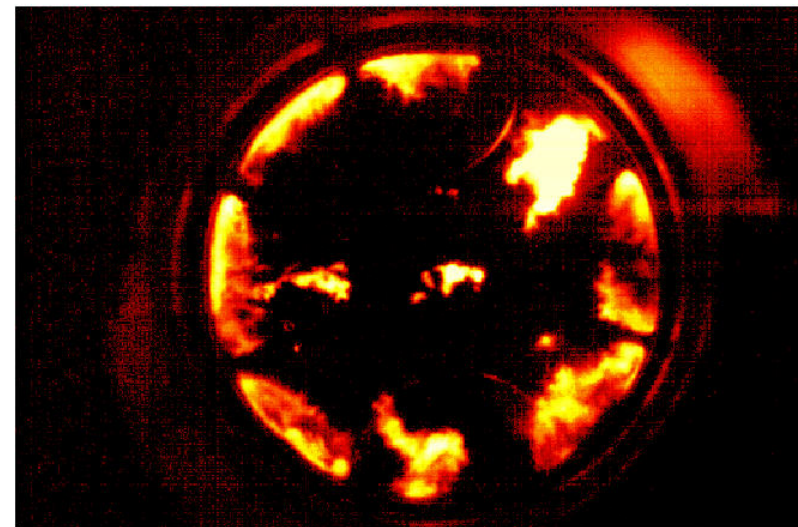


PCCI – Premixed Charge Compression Ignition  
HCCI – Homogeneous Charge Compression Ignition

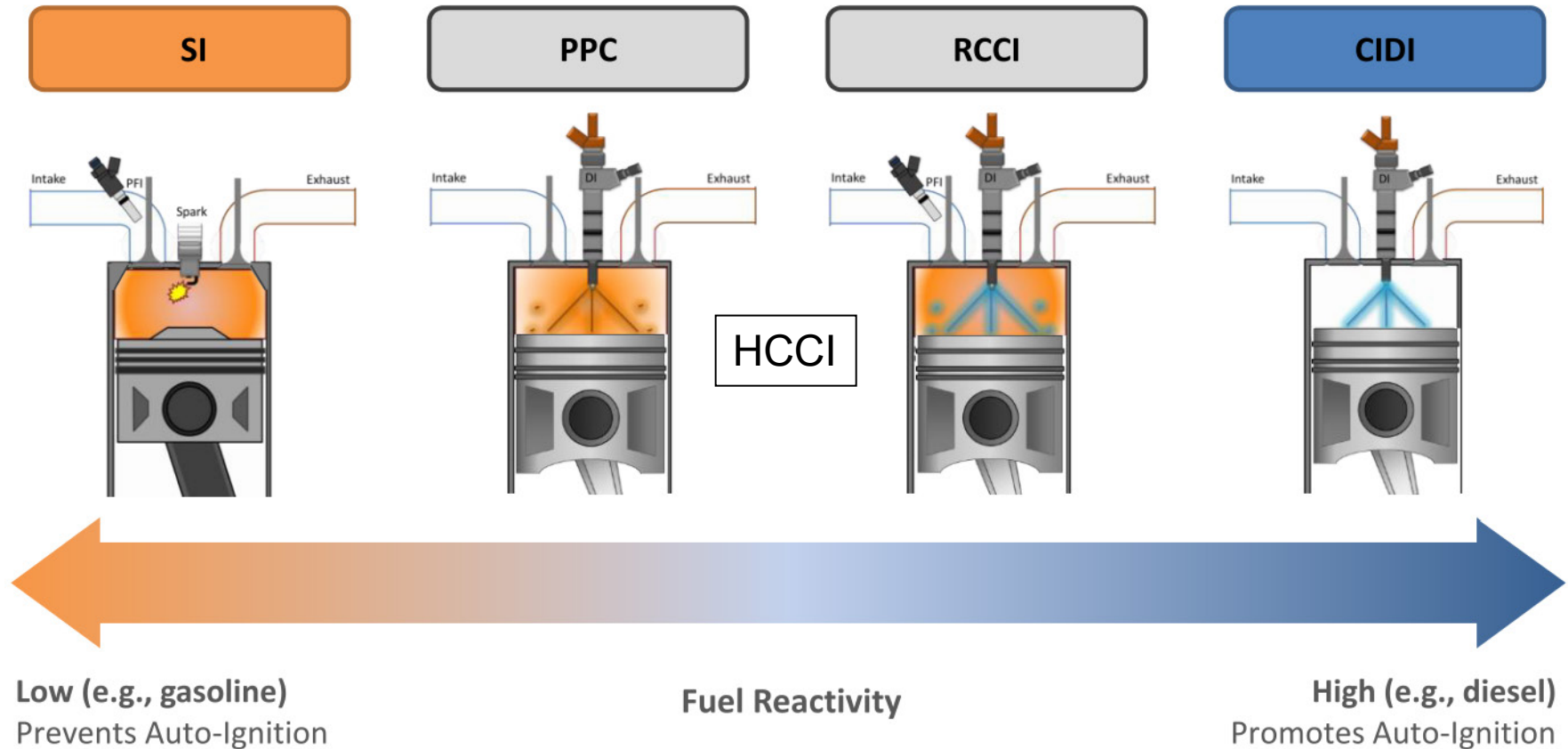
Conventional diesel



Early injection PCCI



## Advanced combustion regimes



SI – Spark Ignition (Homogeneous Charge)  
PPC – Partially Premixed Charge  
RCCI – Reactivity Controlled Compression Ignition  
CIDI – Compression Ignition, Direct Injection



### Lessons from history (1910-20) – “the Mayflower”

Ignitability affects engine operating regime - limits compression ratio (CR).

Early Spark Ignition (SI) engines were plagued by “spark knock”, CR ~ 4:1.

Cylinder pressure measurements by Midgley and Kettering at DELCO/GM showed different fuels had different knock tendency

e.g., kerosene worse than gasoline

Volatility differences were thought to be the explanation.

Guided by the “Mayflower,” they added a red dye (iodine) to kerosene and knock tendency was greatly reduced!

Unfortunately, tests with other red dyes did not inhibit knock, disproving the theory.

But, finding powerful antiknock additives was a **major serendipitous discovery!**



Mayflower – Trailing Arbutus Jane in early spring

Boyd T (1950) Pathfinding in Fuels and Engines. SAE 500175, 4(2) 182-195.



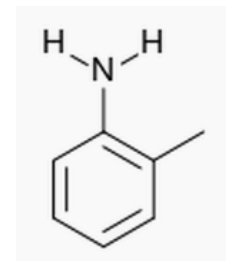




### Lessons from history (1920-30) – Amines and TEL

Research after WW-I was motivated by national security

- Improved fuel efficiency with higher CRs made possible the first non-stop airplane flight from New York to San Diego in the 1920's.



GM and US Army studied hundreds of additives

- found aromatic amines to be effective knock suppressors.

1920 experimental GM car driven on gasoline with toluidine with CR ~7:1

- 40% better fuel consumption than 4:1.

Engine exhaust plagued by unpleasant odors - “the goat”!



Much research was devoted to find acceptable additives,

- finally leading to tetraethyl lead (TEL)

But, TEL caused solid deposits, damaged exhaust valves and spark plugs.

Scavenger additives with bromine and chlorine corrected the problem.

- Partnership with Ethyl-Dow and DuPont to extract compounds from sea water
- 10 tons of sea water needed to provide 1 lb of bromine!

**WW-II aviation engines used iso-heptane (triptane: 2,2,3-trimethyl butane)**  
- allowed CR as high as 16:1.



## Lessons from history (1930-70) – ~~TEL~~ and the future

Lead poisoning was an early concern

- In 1926 US Surgeon General determined that TEL poses no health hazards.
- Use of lead in automotive fuels has been called “The mistake of the 20th century”

1950: Dr. Arie Haagen-Smit - cause of smog in LA to be HC/NO

- Cars were the largest source of UHC/NO<sub>x</sub>

1950: Eugene Houdry - developed catalytic converter for auto exhaust.

- But, lead was found to poison catalytic converters.

20 years later: US EPA announces gas stations must offer "unleaded" gasoline,

- Based accumulated evidence of negative effects of lead on human health.
- Leaded gasoline was still tolerated in certain applications (e.g., aircraft), but was permanently banned in the US in 1996, in Europe since 2000

*World Wars & national security* played a major role to define automotive fuels.

Today's engines and their fuels would not have been developed without close collaboration between engine OEMs, energy and chemical companies!

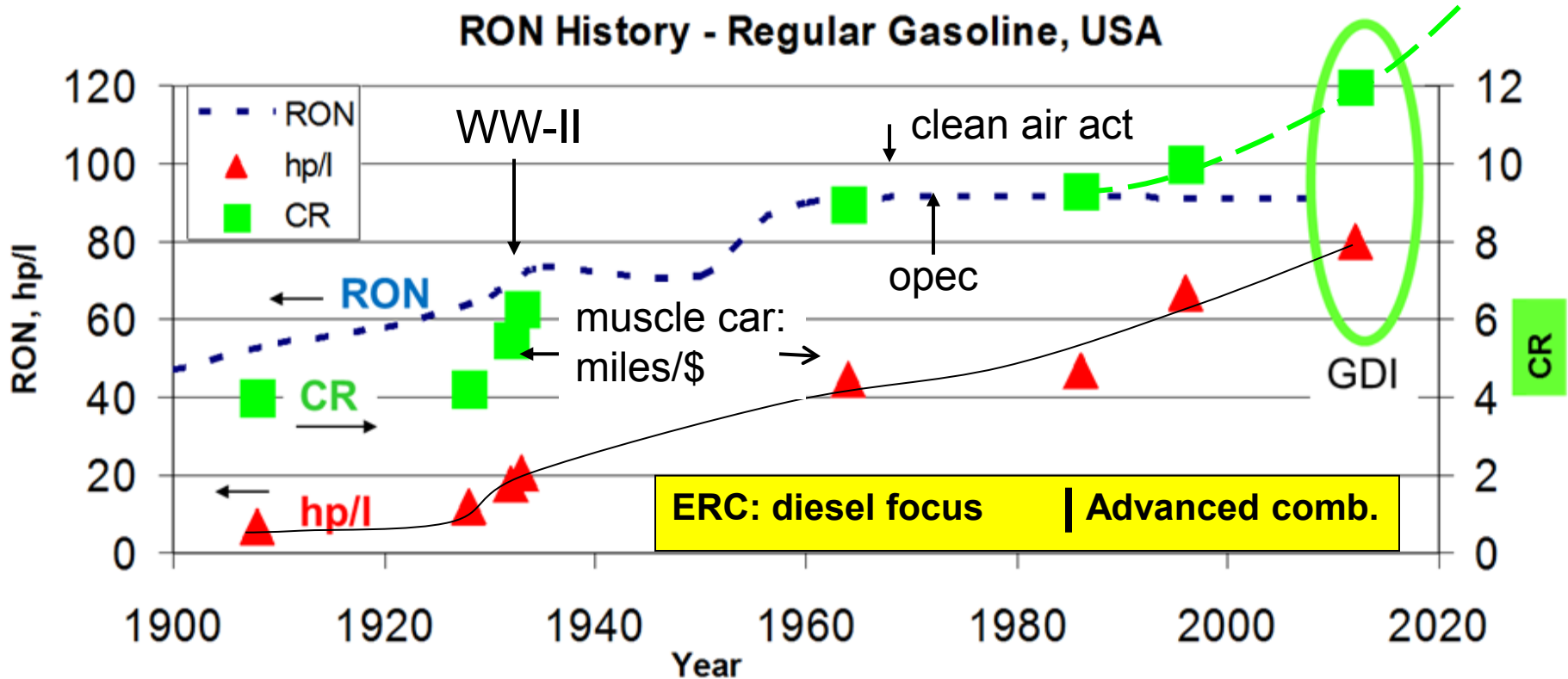
A consequence of collaboration between “big” engine and “big” oil is that transformative changes in transportation systems will not occur easily.

→ A new concept engine must be able to use available fuels,  
A new fuel must run in existing engines.



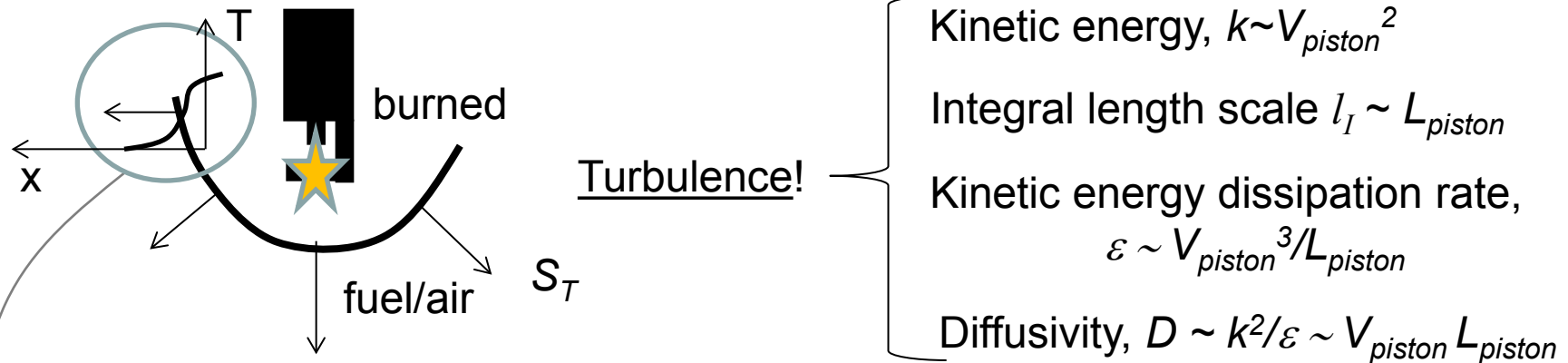
# “Race between compression ratio and octane number”

Curtis, 2013



## Basic combustion concepts – Spark Ignition (SI)

How can SI engines operate with engine speeds from 100 to 20,000 rev/min?



*Because turbulent flame speed,  $S_T$ , scales with rpm!*

### Characteristic Time Combustion (CTC) model

Reitz & Bracco, 1983; Abraham, 1985

Species conversion rate ( $Y_i$ , species mass fraction, \* local equilibrium solution)

$$\frac{dY_i}{dt} = -\frac{Y_i - Y_i^*}{\tau_c} \quad ; \quad \tau_c \sim k/\varepsilon \sim L_{piston} / V_{piston}$$

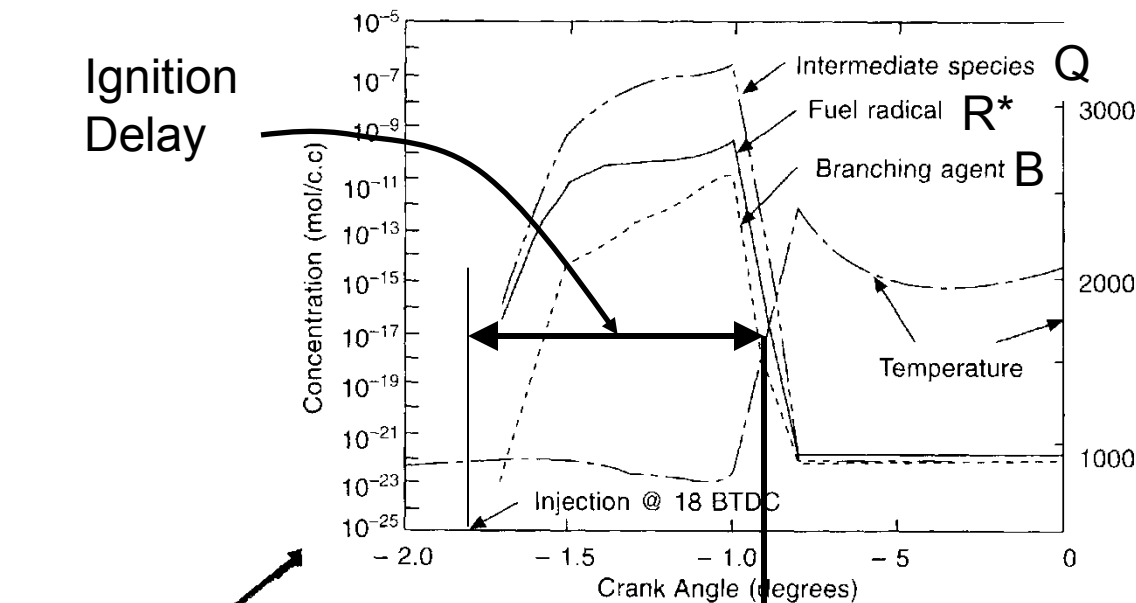
Mallard-Le Chatelier propagating wave speed:

Glassman, 1996

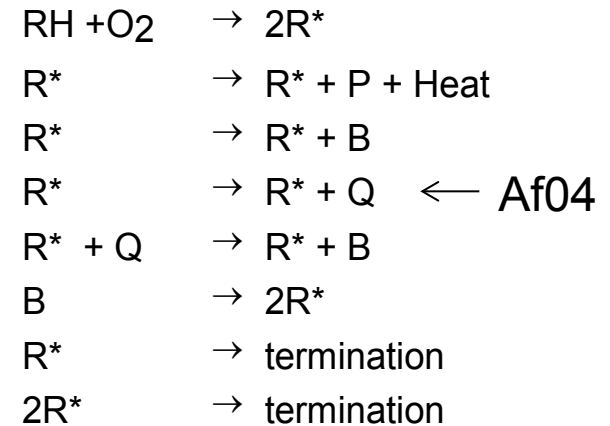
$$S_T = \sqrt{D \frac{dY_i}{dt}} \sim V_{piston}$$



## Basic combustion concepts – Diesel (CI)



### Shell Ignition Model

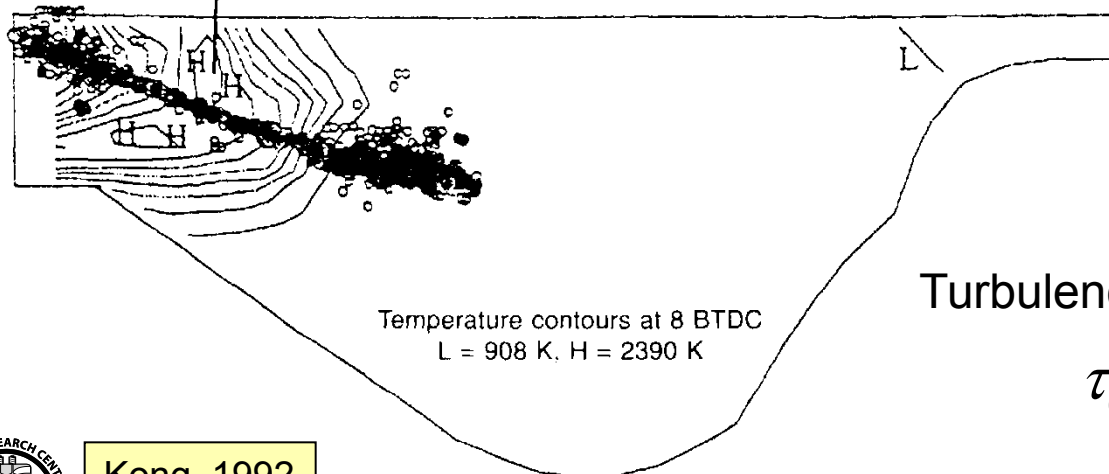


Switch to Characteristic Time Combustion model

$$\frac{dY_i}{dt} = -\frac{Y_i - Y_i^*}{\tau_c}$$

Turbulence generated by fuel injection

$$\tau_c \sim k/\varepsilon \sim L_{\text{nozzle}} / V_{\text{nozzle}}$$

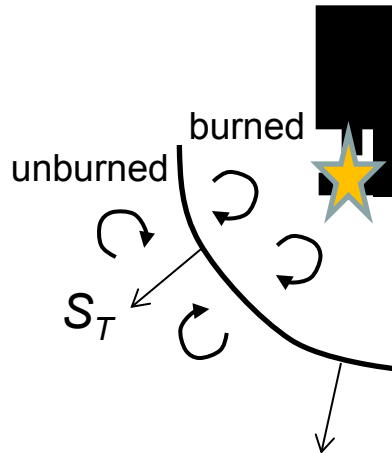


Kong, 1992



## Turbulent mixing

### Spark-ignition

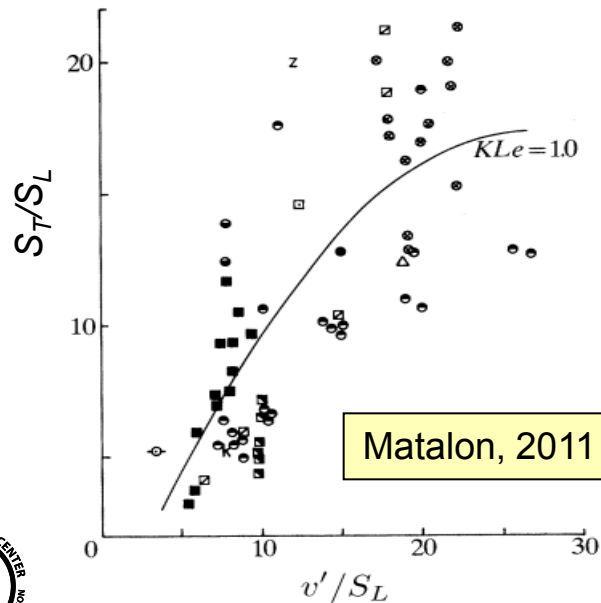
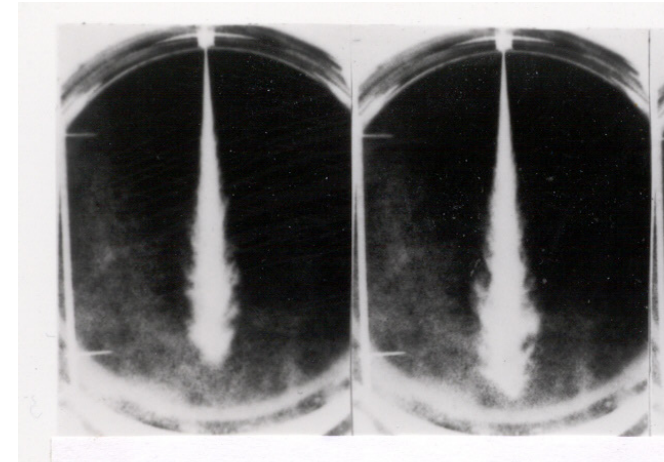


Hot products with  
Cold reactants

$$\tau \sim k/\varepsilon$$

$$\sim L_{piston} / V_{piston}$$

High turbulence  
- faster combustion



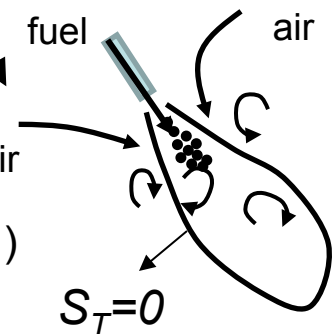
Injected fuel with  
entrained air

$$\tau \sim k/\varepsilon$$

$$\sim L_{nozzle} / V_{nozzle}$$

Delayed ignition (PCCI)  
- better mixing

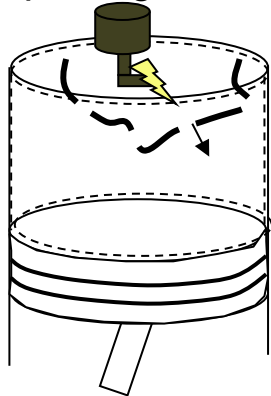
### Diesel



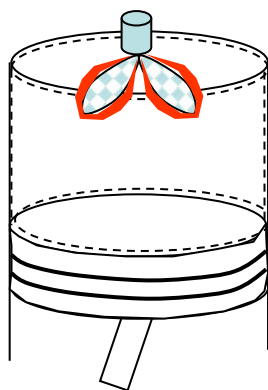
## Summary of combustion regimes

- Gasoline engine spark-ignition with flame propagation:  
High turbulence for high flame speed  $\rightarrow$  heat losses. Issues: NO<sub>x</sub> and UHC/CO, knock (CR, fuels), throttling losses  $\rightarrow$  low thermal efficiency TE  $\sim$  25%
- Diesel engine with spray (diffusion) combustion:  
Rich mixtures (soot) & high temperatures (NO<sub>x</sub>)  $\rightarrow$  higher TE  $\sim$  45%
- H/Premixed Charge Compression Ignition – LTC, chemistry controlled (CR):  
Sensitive to fuel, poor combustion/load control, low NO<sub>x</sub>-soot  $\rightarrow$  TE  $\sim$  50%

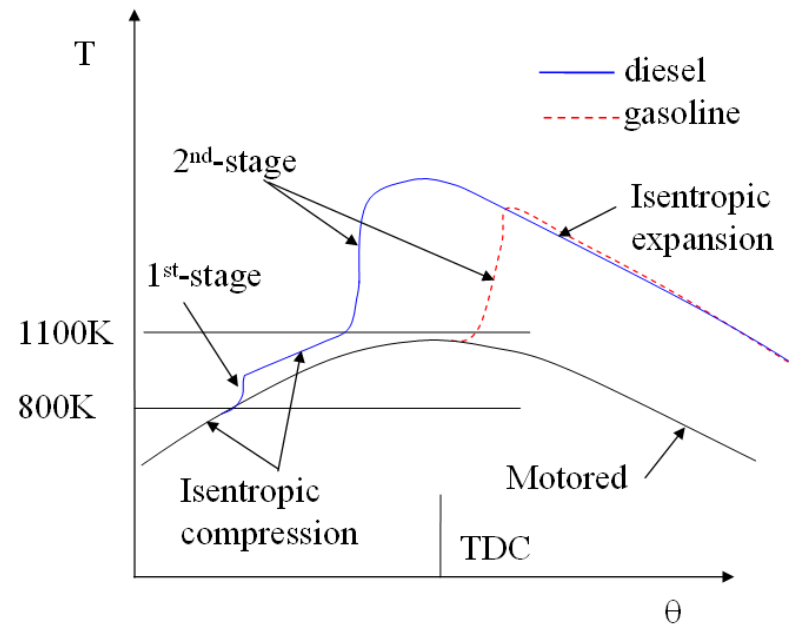
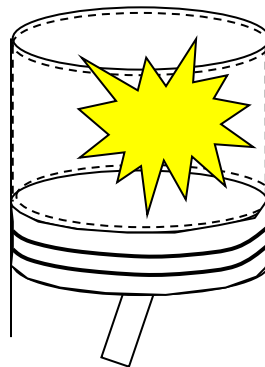
spark-ignition



diesel



H/PCCI



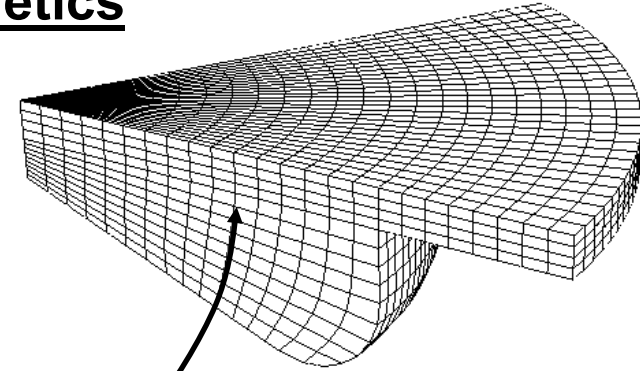


# Premixed volumetric combustion & chemical kinetics

Species and energy conservation equations

$$\left( \frac{\partial \rho_i}{\partial t} \right) + \nabla \cdot (\rho_i \mathbf{u}) = \nabla \cdot \left[ \rho D \nabla \left( \frac{\rho_i}{\rho} \right) \right] + \dot{\rho}_i^c + \dot{\rho}_i^s$$

$$\left( \frac{\partial (\rho I)}{\partial t} \right) + \nabla \cdot (\rho \mathbf{u} I) = -p \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{J} + \rho \varepsilon + \dot{Q}^c + \dot{Q}^s + \dot{Q}^r$$



Constant volume combustion – Well-Stirred-Reactor (WSR)

$$\frac{dY_i}{dt} = \frac{W_i}{\rho} \sum_{k=1}^{n_r} (\nu''_{k,i} - \nu'_{k,i}) \omega_k(\mathbf{Y}, T), \quad i = 1, \dots, n_s$$

$$\sum_{i=1}^{n_s} \nu'_{k,i} M_i = \sum_{i=1}^{n_s} \nu''_{k,i} M_i, \quad k = 1, \dots, n_r$$

$$\omega_k(\mathbf{Y}, T) = \kappa_{f,k} \prod_{i=1}^{n_s} \left( \frac{\rho Y_i}{W_i} \right)^{\nu'_{k,i}} - \kappa_{b,k} \prod_{i=1}^{n_s} \left( \frac{\rho Y_i}{W_i} \right)^{\nu''_{k,i}}$$

$$\kappa_{f,k}(T) = A_k T^{b_k} \exp \left( -\frac{E_k}{RT} \right) ; \quad \kappa_{b,k}(T) = \kappa_{f,k}(T) / K_{eq,k}(T)$$

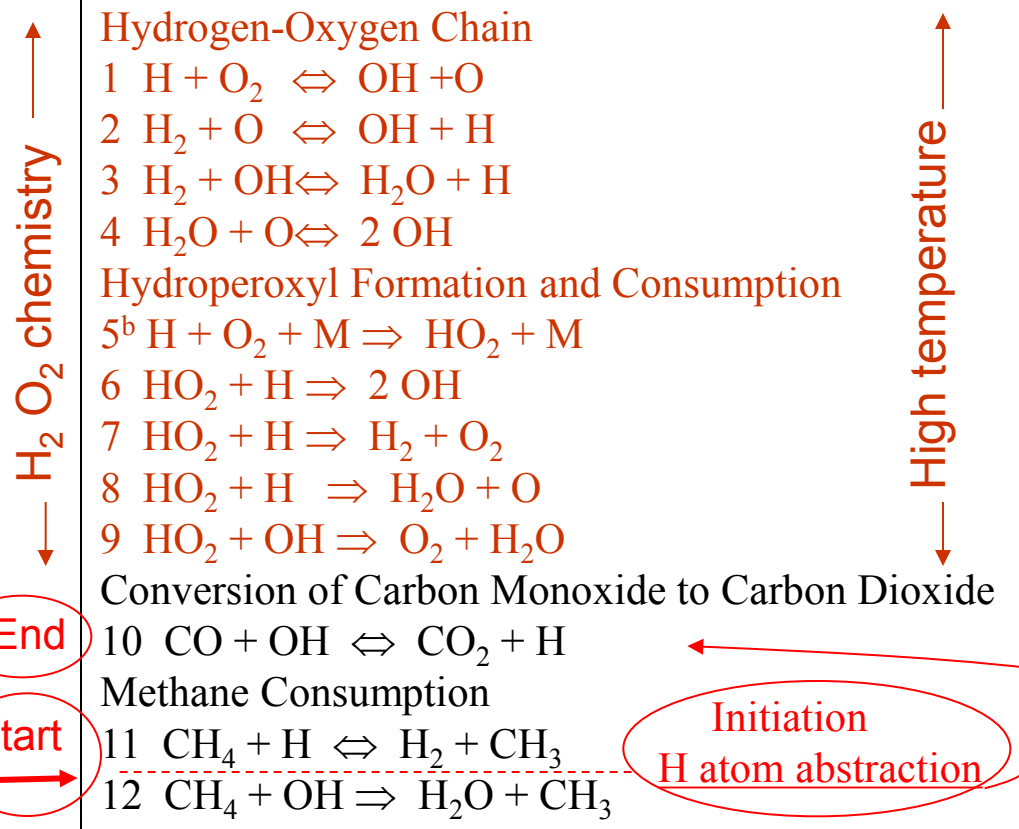
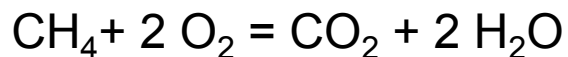
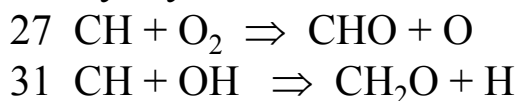
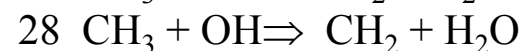
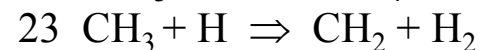
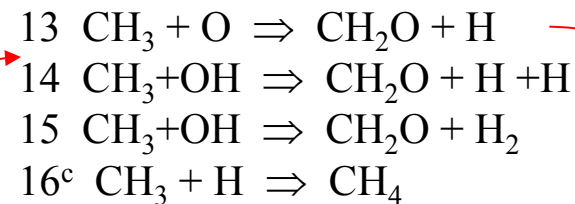
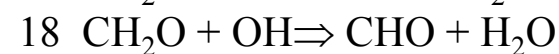
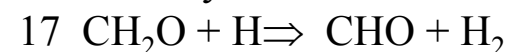
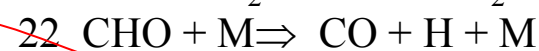
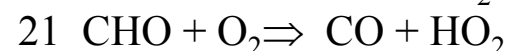
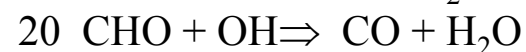
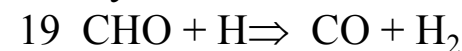
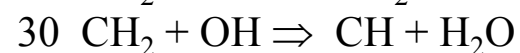
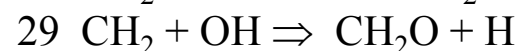
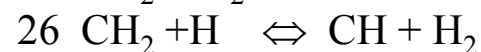
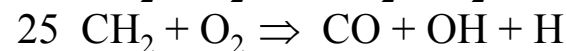
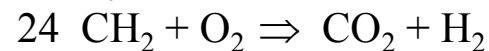
$$K_{eq,k}(T) = \exp(-\Delta g_k^0) \left( \frac{p_{atm}}{RT} \right)^{\sum_{i=1}^{n_s} (\nu''_{k,i} - \nu'_{k,i})}$$

$$\frac{dT}{dt}(\mathbf{Y}, T) = -\frac{1}{\bar{c}_v(\mathbf{Y}, T)} \sum_{i=1}^{n_s} \left( \frac{e_i(T)}{W_i} \frac{dY_i}{dt}(\mathbf{Y}, T) \right)$$

$I$  specific internal energy  
 $M_i$  chemical label  
 $n_r$  reactions  
 $n_s$  species  
 $\nu'_{k,i}, \nu''_{k,i}$  reactant/product stoichiometric coefficients  
 $Y_i$  mass fraction  
 $W_i$  molecular weight  
 $e_i$  species energy

$$e_i = R_{mol} \left[ (a_i - 1) T + \frac{b_i}{2} T^2 + \frac{c_i}{3} T^3 + \frac{d_i}{4} T^4 + \frac{e_i}{5} T^5 + f_i \right]$$



**Combustion chemistry models** – CH<sub>4</sub> (15 spec, 31 react.)**Methylidyne Reactions****Methyl Reactions****Formaldehyde Reactions****Formyl Reactions****Methylene Reactions**

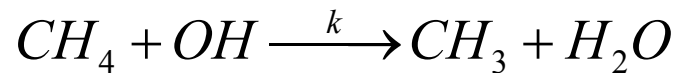
Conversion to products by sequential fragmentation by H abstraction

## Homogeneous charge: no spatial gradients

$$\frac{\partial Y_i}{\partial t} = \omega_i / \rho$$

$$\frac{\partial T}{\partial t} = - \sum_{i=1}^{n_s} \frac{\Delta h_{f,i}^0 \omega_i}{\rho c_p}$$

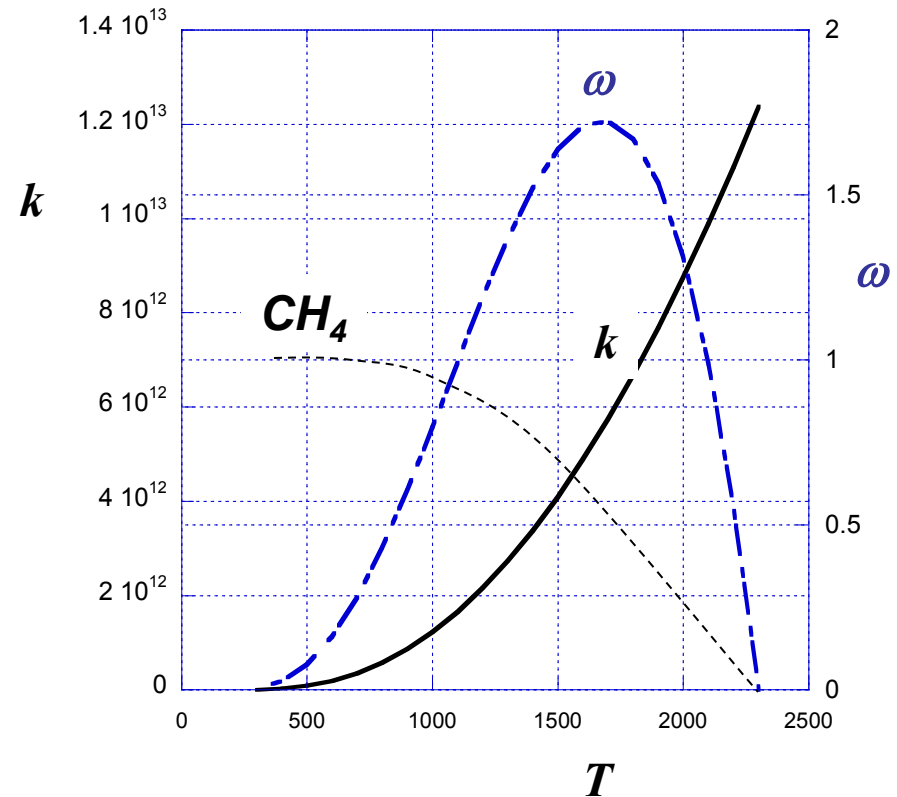
Consider single overall reaction



$$k = AT^b \exp(-E / RT)$$

$$\frac{d[CH_4]}{dt} = -k[CH_4][OH] = \frac{\omega_{CH_4}}{\rho W_{CH_4}}$$

$$A = 1.6 \cdot 10^7 \text{ (cm, mol, s)}, b = 1.83, E = 11.6 \text{ (kJ / mol)}$$





## HCCI: Ignition delay

$$\frac{\partial Y_i}{\partial t} = \omega_i / \rho$$

$$\frac{\partial T}{\partial t} = - \sum_{i=1}^{n_s} \frac{\Delta h_{f,i}^0 \omega_i}{\rho c_p}$$

Consider single component system

$$U = \frac{T - T_{unburned}}{T_{burned} - T_{unburned}}$$

Example:

$$\frac{dU}{dt} = F(U) = \beta U^{m+1} (1-U)^m$$

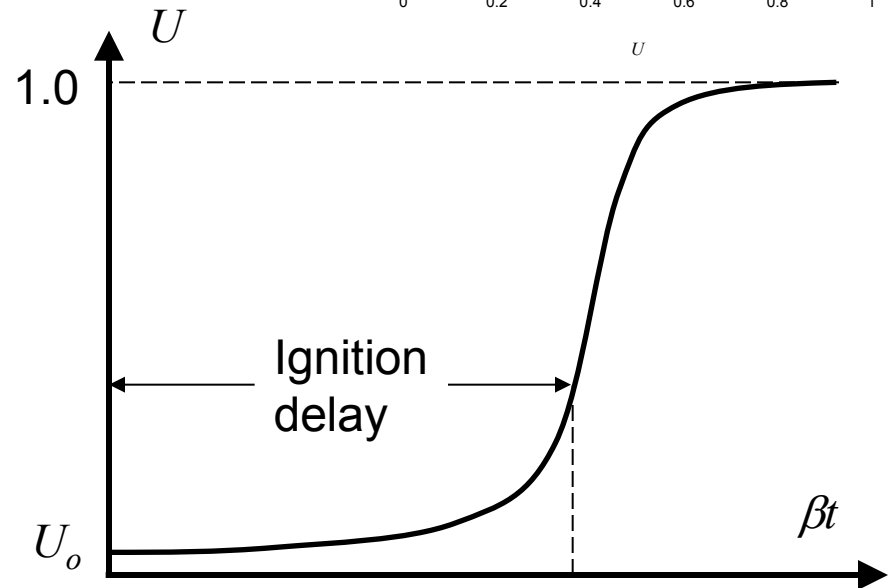
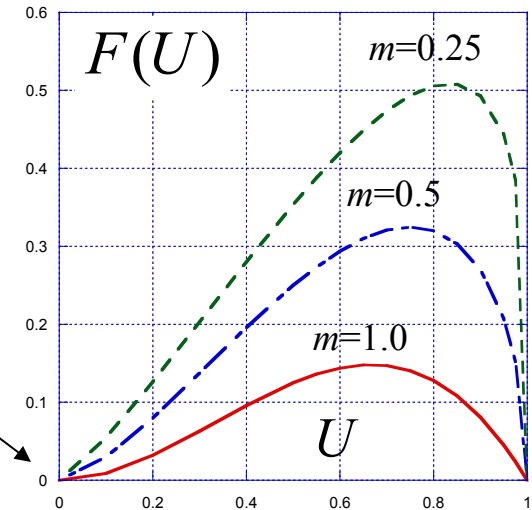
$$\text{For } U \rightarrow U_0: \quad m\beta t = \frac{1}{U_0^m} - \frac{1}{U^m}$$

So, time to reach, say,  $5U_0$  :

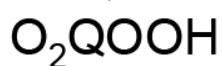
Ignition delay:

$$\beta t = \frac{4}{5mU_0^m}$$

Cold boundary  
difficulty

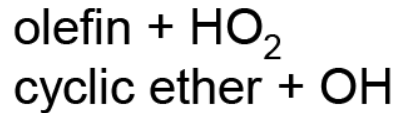


$$m\beta t = \text{Const.} - \frac{1}{U^m} {}_2F_1(-m, m; (1-m); U)$$

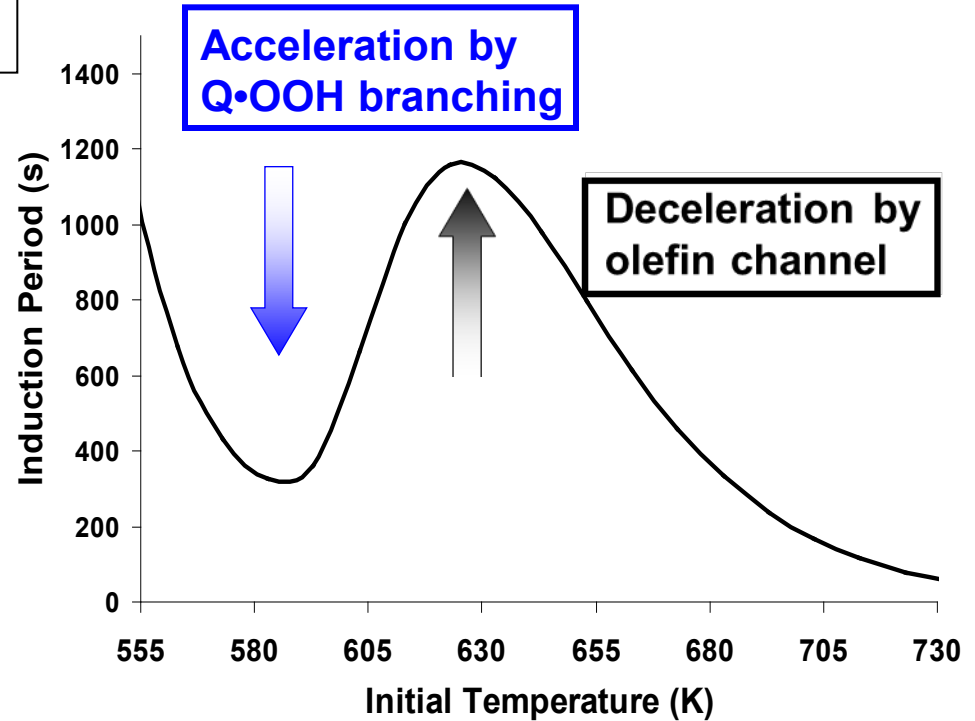
$$\text{H}_2\text{O}_2 = \text{OH} + \text{OH}$$


deceleration

acceleration



(low T branching)

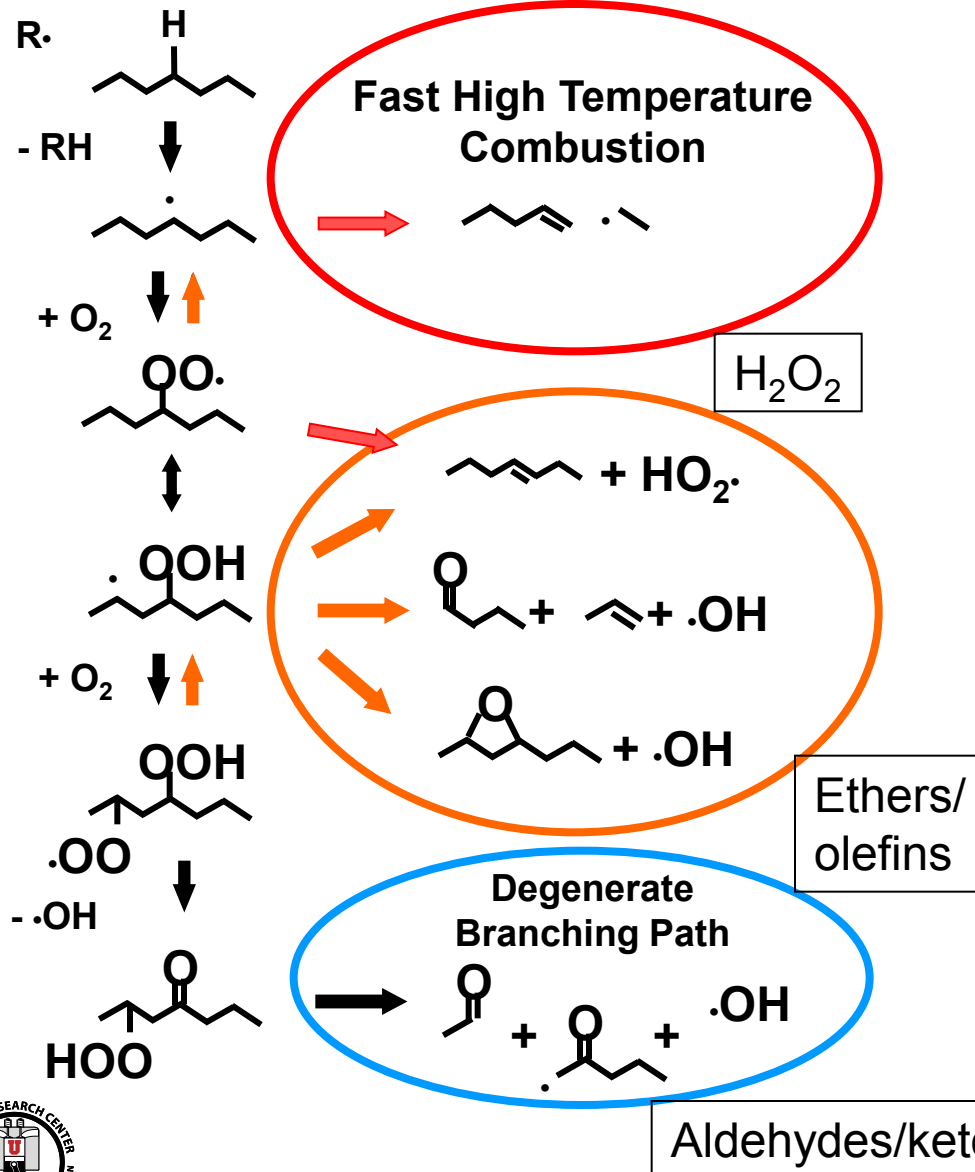


## First Stage Ignition

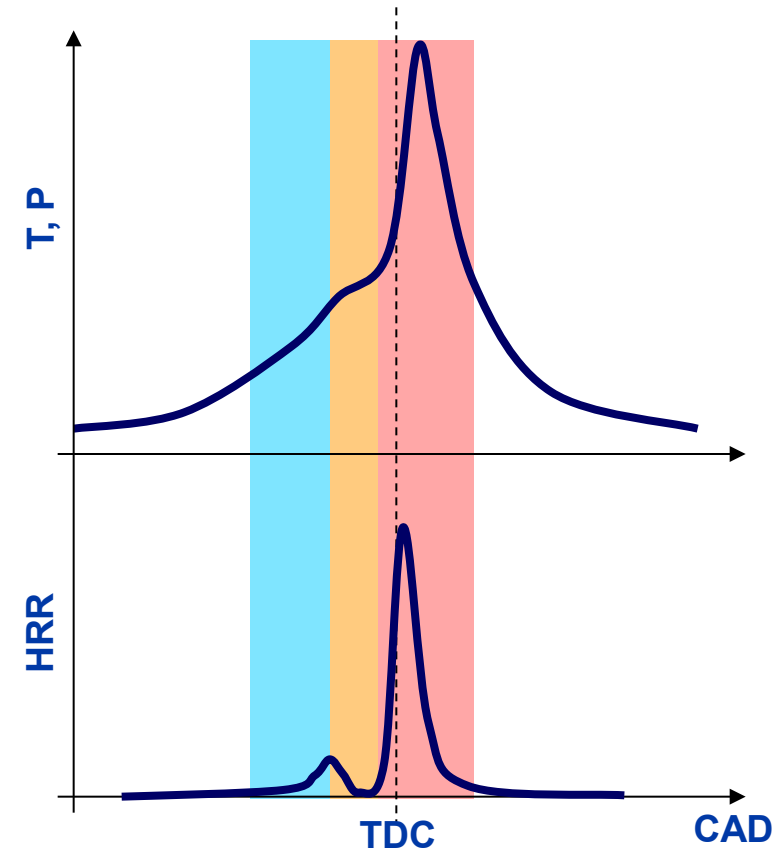
### Isomerization steps



## HCCI combustion kinetics



## Typical HCCI Combustion Temperature and Heat Release Rate profiles

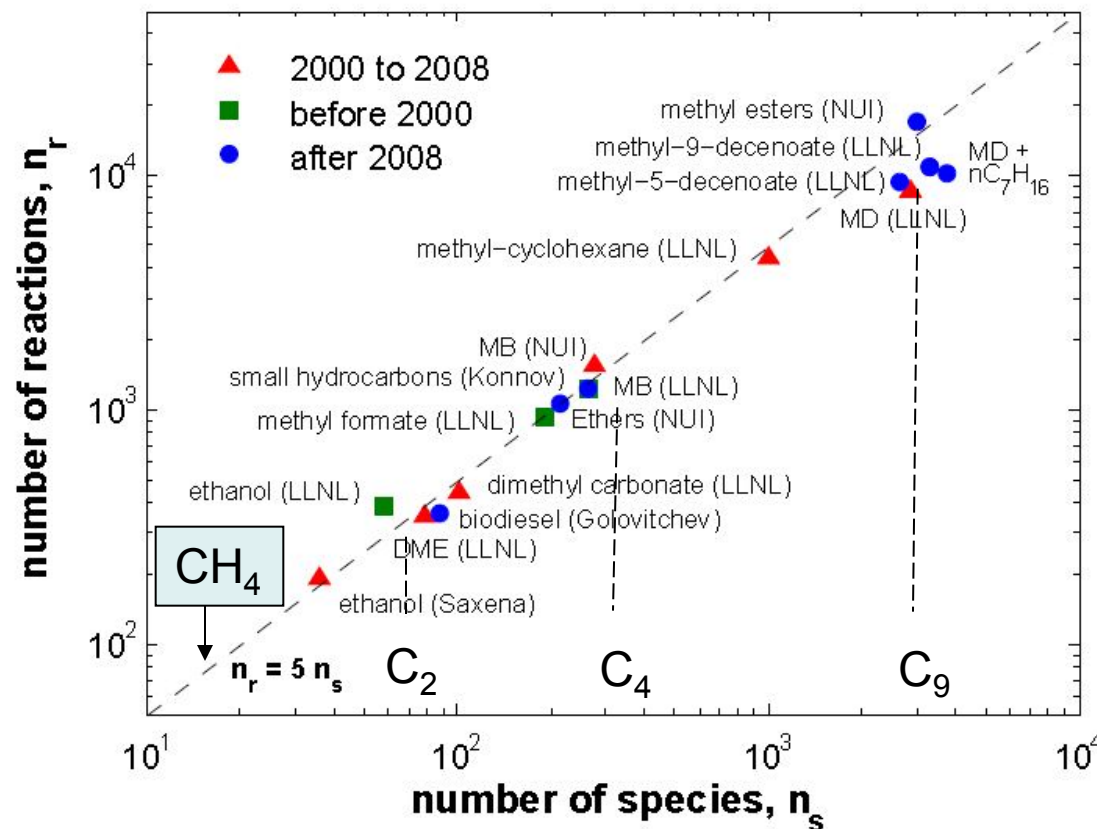




## Chemical kinetic mechanisms for engine simulations

Requirements for mechanisms for practical engine simulations:

- Size can not be too large due to CPU time limitation ~ 100 species
- Capable of predicting auto-ignition delay time accurately
- Contain proper reactions for pollutant formation precursors



### Biodiesel surrogates

- Significant mechanism reduction is required.

### Soy biodiesel - Methyl:

- palmitate (C16:0)
- stearate (C18:0)
- oleate (C18:1)
- linoleate (C18:2)
- linolenate (C18:3)





## ERC-MultiChem: Primary Reference Fuel (PRF)

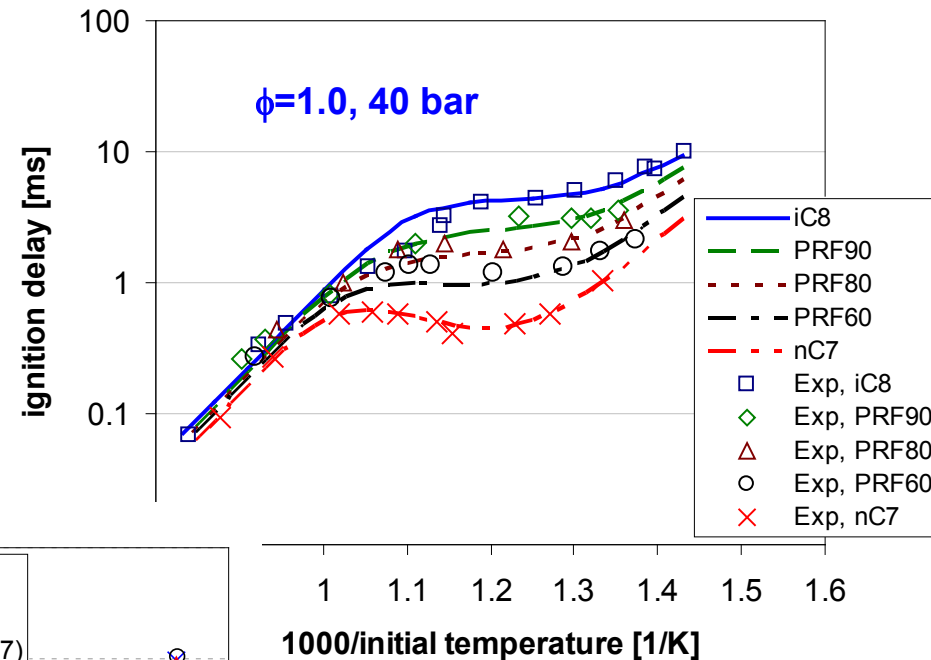
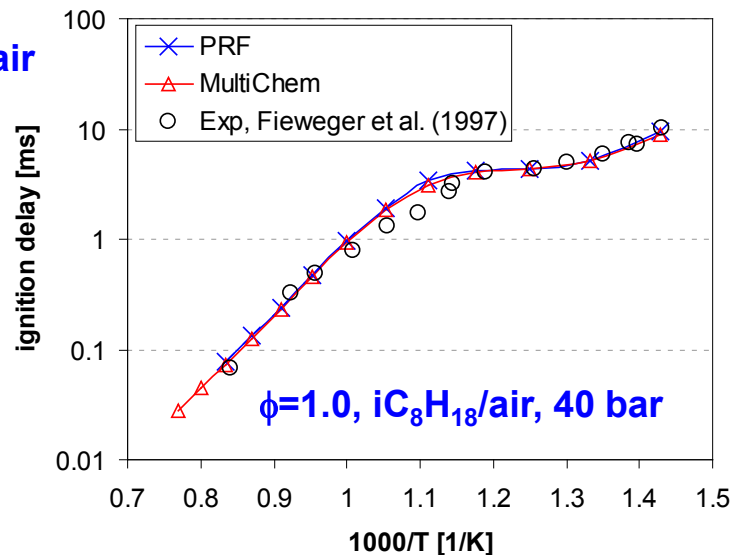
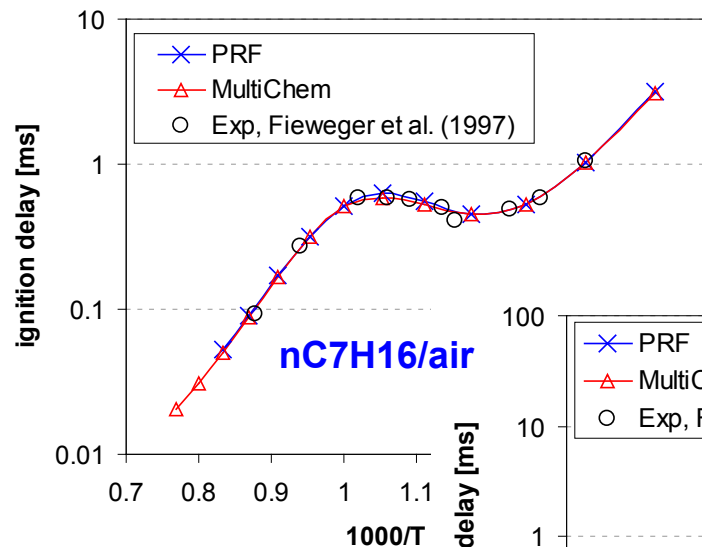
41 species, 158 reactions → base mechanism

Source mechanisms: LLNL n-heptane

(560 species; 2,539 reactions), isooctane

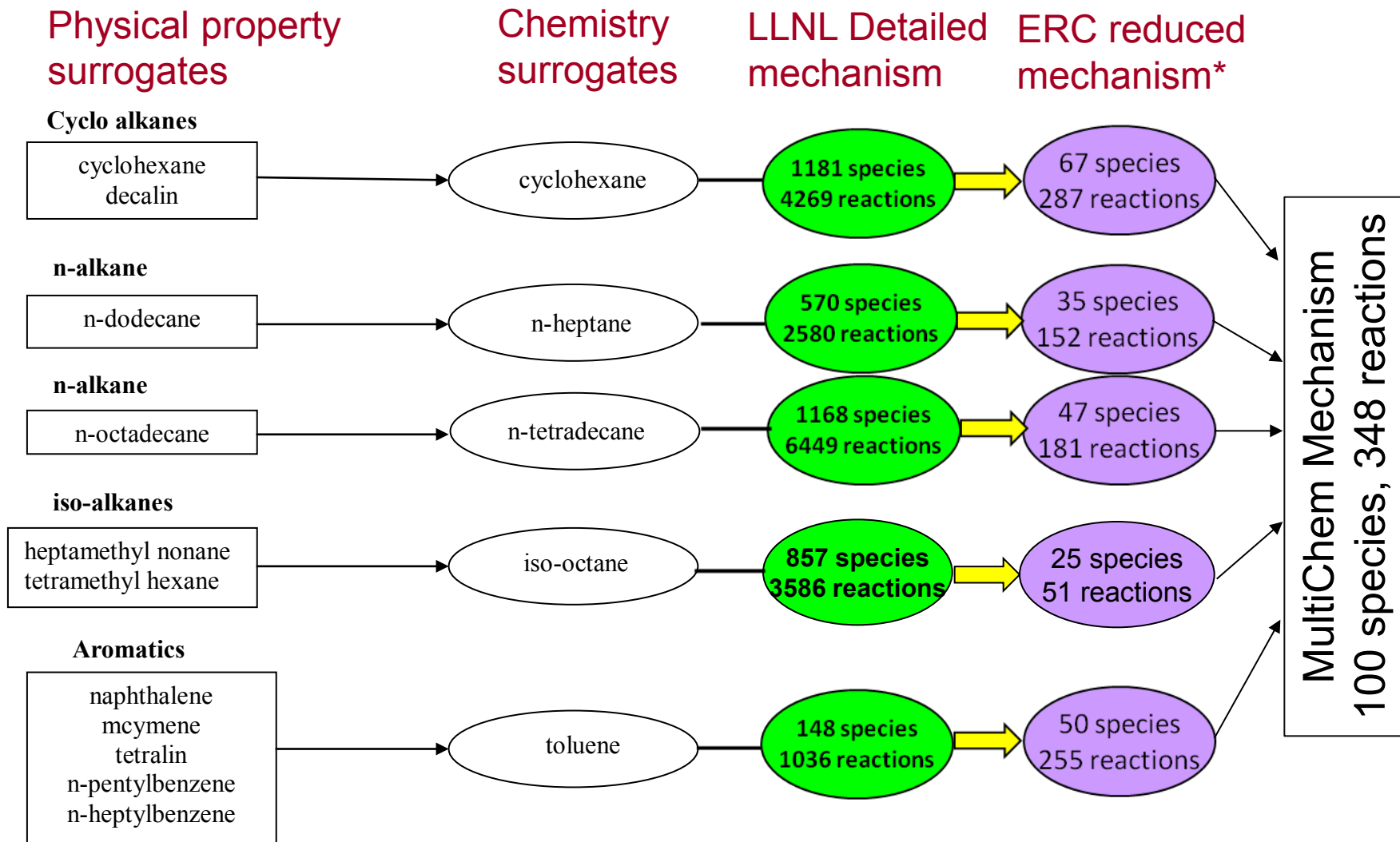
(857 species; 3,606 reactions),

ERC n-heptane (29 species; 52 reactions)





## Chemical class grouping: “MultiChem” skeletal mechanism

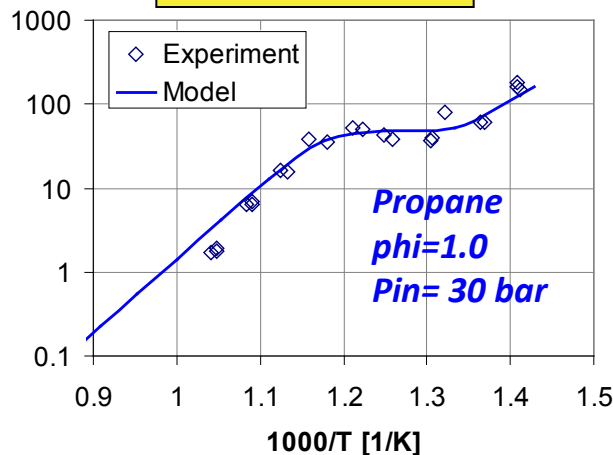




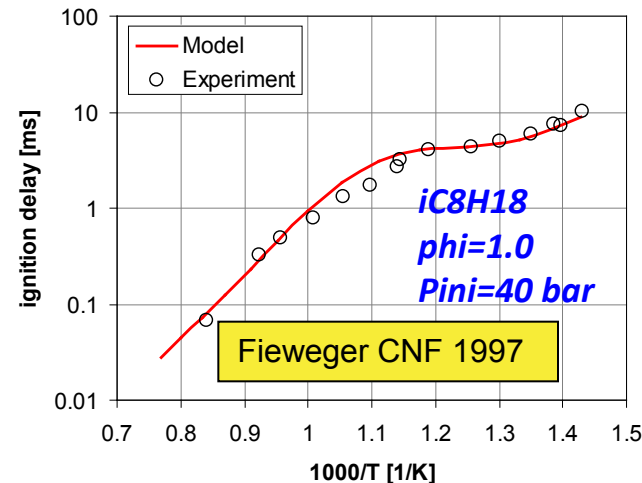
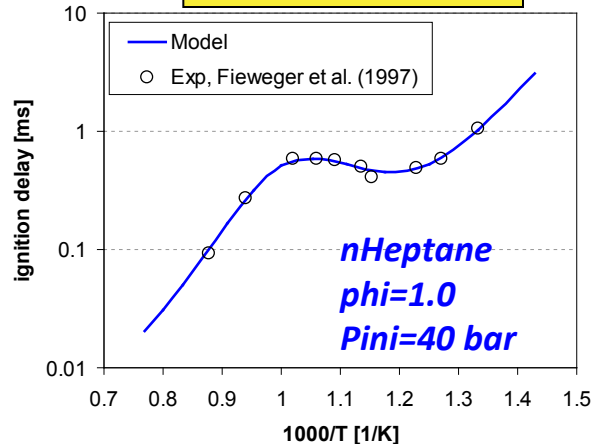
## Ignition delay validations - "MultiChem"

8 Surrogate fuels: *n*-heptane, *iso*-octane, tetradecane, cyclohexane, toluene, decalin, ethanol, MB/D.....

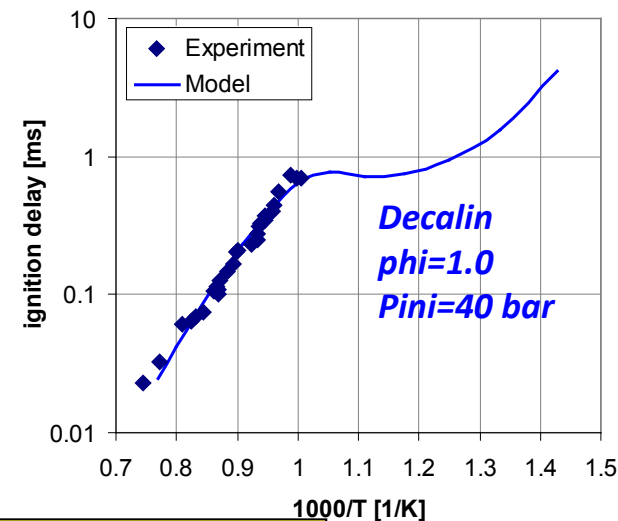
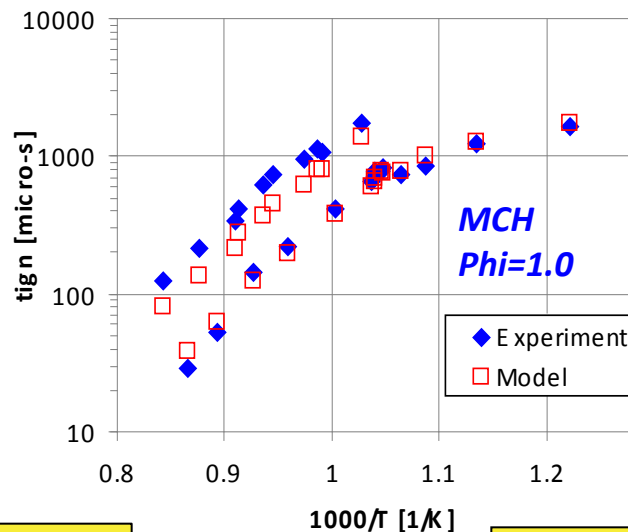
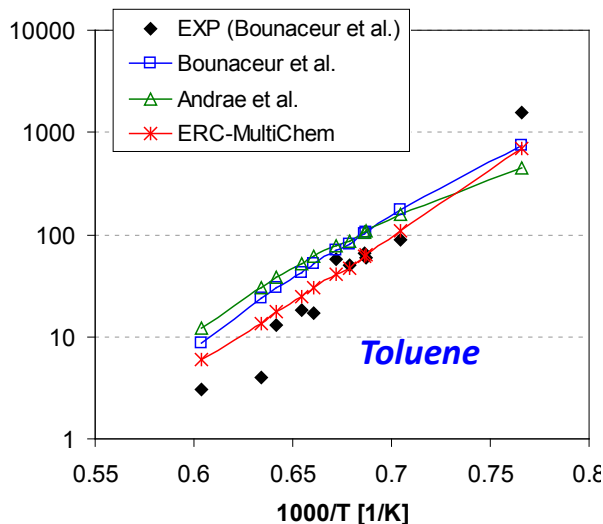
Gauthier CNF 2004



Fieweger CNF 1997



Fieweger CNF 1997



Bounaceur IJCK 2005; Andrae CNF 2005

Shen, Energy &amp; Fuels, 2009



### 3-Dimensional models

Solve conservation equations on (moving) numerical mesh

Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \dot{\rho}^s$$

spray source terms

Species

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{u}) = \nabla \cdot \left[ \rho D \nabla \left( \frac{\rho_m}{\rho} \right) \right] + \dot{\rho}_m^c + \dot{\rho}_m^s$$

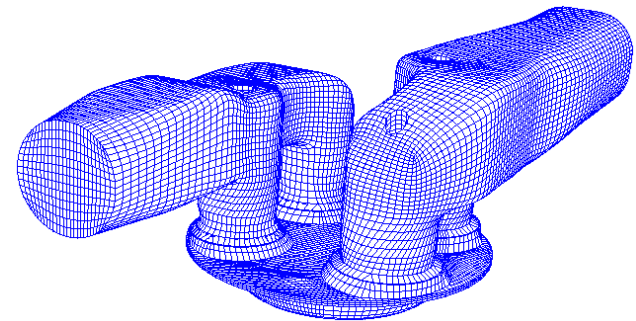
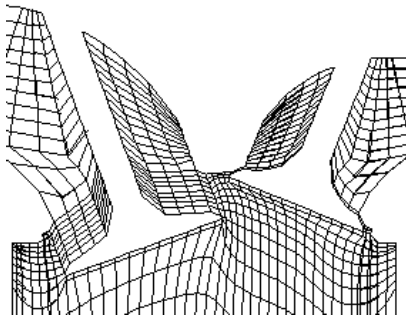
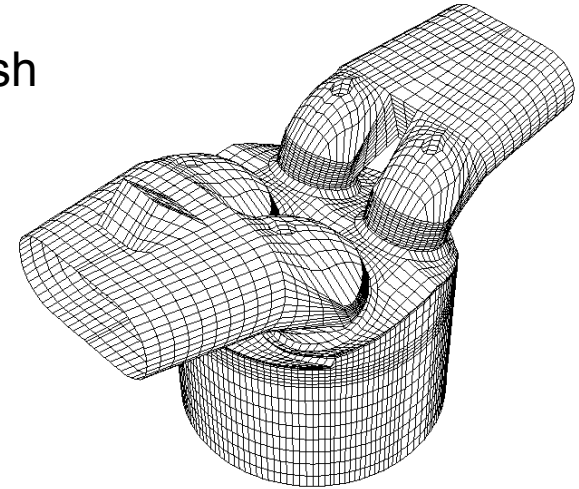
Momentum

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \rho \mathbf{g} + \mathbf{F}^s - \nabla p + \nabla \cdot \bar{\sigma}$$

combustion source terms

Energy

$$\frac{\partial (\rho I)}{\partial t} + \nabla \cdot (\rho \mathbf{u} I) = -\nabla \cdot \mathbf{J} + \dot{Q}^c + \dot{Q}^s - p \nabla \cdot \mathbf{u} + \bar{\sigma} : \nabla \mathbf{u}$$





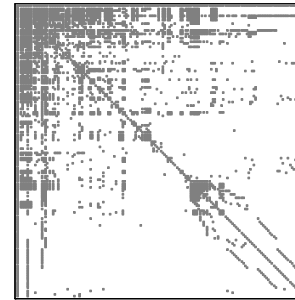
### 3-D CFD: Improved solver numerics

Sparse analytical Jacobian  
formulation

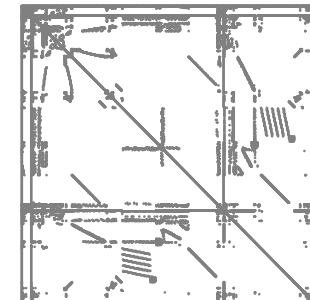
Sparsity of hydrocarbon  
fuel mechanisms increases  
with size



47 (62.7%)



160 (86.2%)



2878 (99.7%)

$n_s$

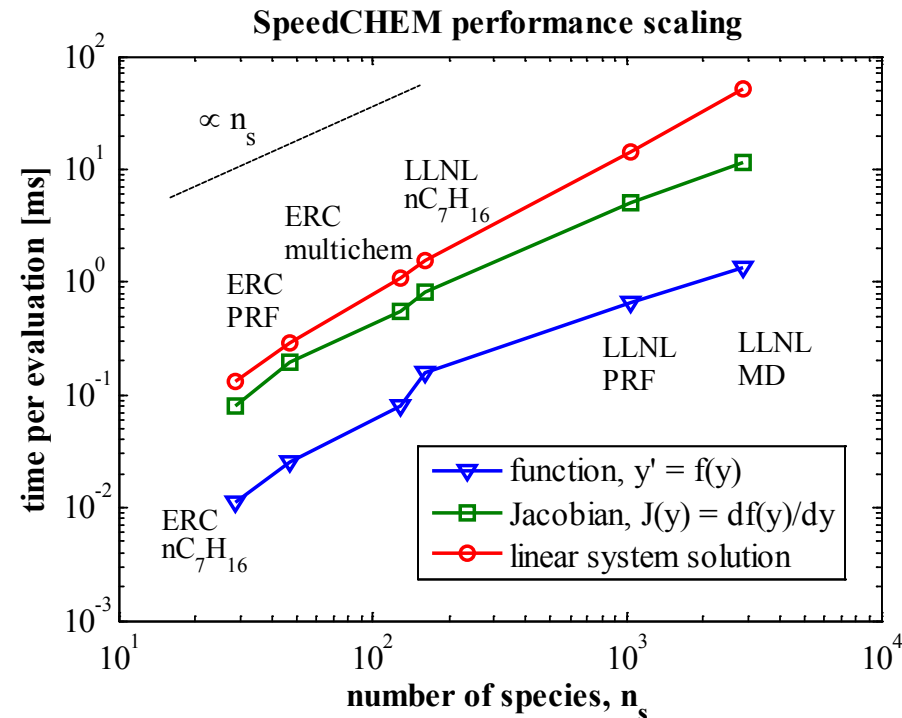
$$\frac{\partial Y^i}{\partial t} = \omega_i / \rho$$

$$\frac{\partial T}{\partial t} = - \sum_{i=1}^{n_s} \frac{\Delta h_{f,i}^0 \omega_i}{\rho c_p}$$

$$\mathbf{J} = \begin{array}{|c|c|} \hline \textcircled{4} \frac{\partial \dot{T}}{\partial T} & \frac{\partial \dot{T}}{\partial Y_j} \textcircled{2} \\ \hline \frac{\partial \dot{Y}_i}{\partial T} \textcircled{3} & \frac{\partial \dot{Y}_i}{\partial Y_j} \textcircled{1} \\ \hline \end{array}$$

All functions and equations are evaluated in  
matrix form

ODE system function, analytical Jacobian  
evaluation and linear system solution  
achieve linear scaling with  $n_s$





## Efficient chemistry solvers – cell clustering

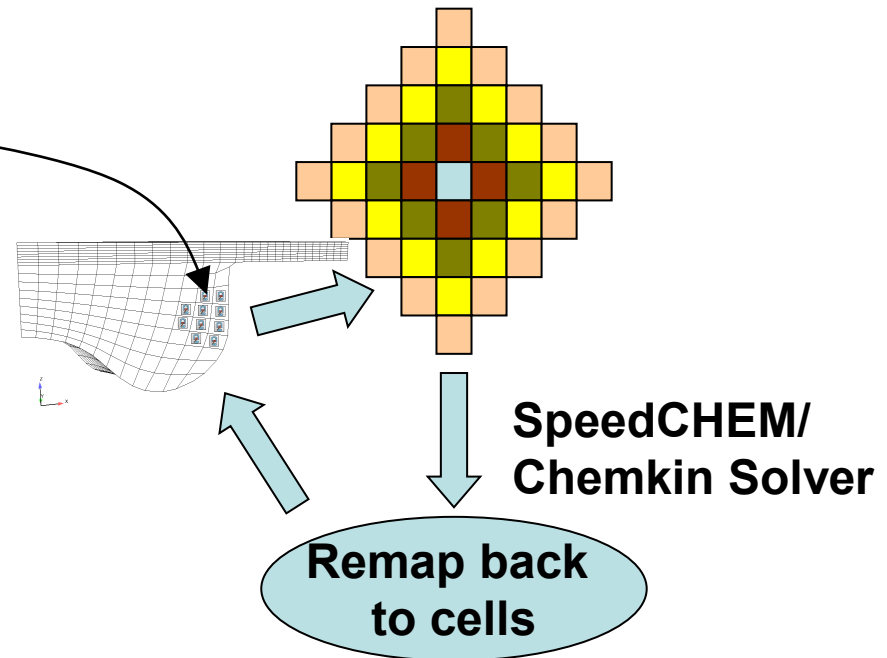
Group thermodynamically-similar cells to reduce the calling frequency to save computer time - Adaptive Mechanism Clustering (AMC) scheme

Extended dynamic adaptive chemistry (EDAC) scheme

Dynamically determine the size of fuel chemical mechanism based on the local and instantaneous thermal conditions of the cells

Thermodynamically similar cells  
(similar temperature, equivalence ratio  $\phi$ )

$$\phi = \frac{2C_{-CO_2}^{\#} + H_{-H_2O}^{\#}/2 - z'C_{-CO_2}^{\#}}{O_{-CO_2-H_2O}^{\#} - z'C_{-CO_2}^{\#}}$$



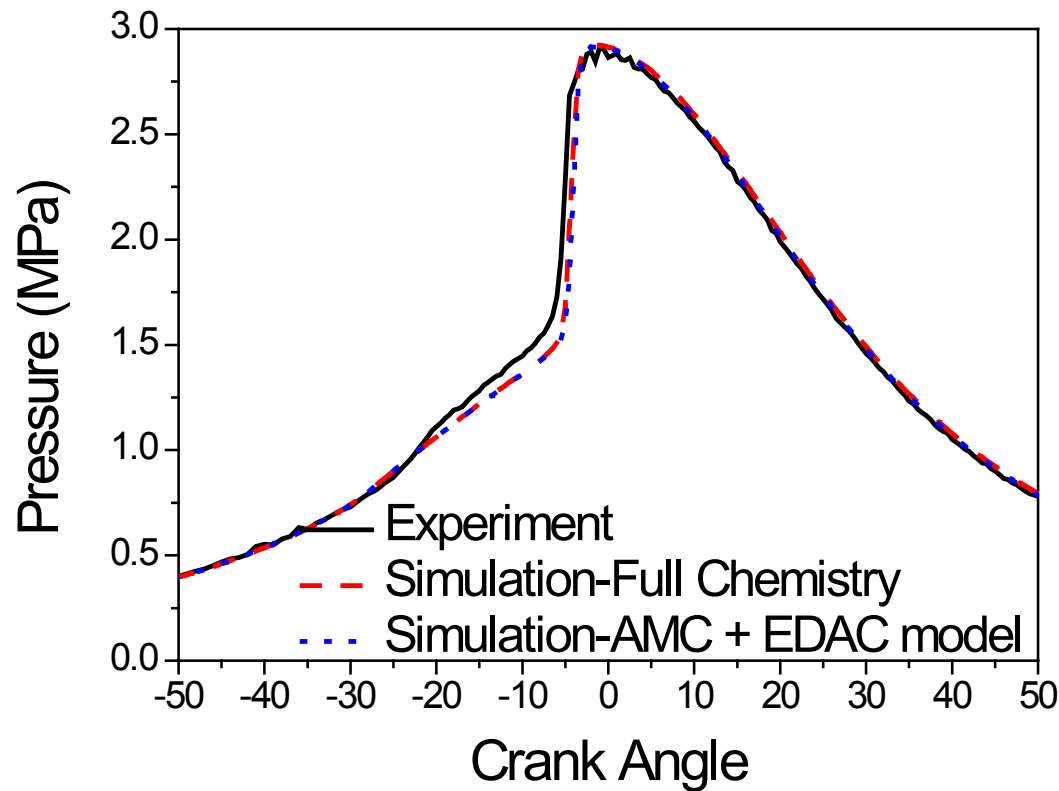




## HCCI engine validation

ERC PRF mech.  
(39 sp, 141 rxn)

Full	AMC	AMC+EDAC
48.27 hrs.	3.99 hrs.	2.88 hrs.





### **Engine emissions - transportation & toxic air pollutants**

Criteria air contaminants (CAC), or criteria pollutants

- air pollutants that cause smog, acid rain and other health hazards.

EPA sets standards on:

- 1.) Ozone (O<sub>3</sub>),
- 2.) Particulate Matter (soot):  
PM<sub>10</sub>, coarse particles: 2.5 micrometers (μm) to 10 μm in size  
PM<sub>2.5</sub>, fine particles: 2.5 μm in size or less
- 3.) Carbon monoxide (CO), 4.) Sulfur dioxide (SO<sub>2</sub>),
- 5.) Nitrogen oxides (NO<sub>x</sub>), 6.) Lead (Pb)

Toxic air pollutants - Hazardous Air Pollutants or HAPs known to cause or suspected of causing cancer or other serious health ailments.

- Clean Air Act Amendments of 1990 lists 188 HAPs from transportation.

In 2001, EPA issued Mobile Source Air Toxics Rule:

- identified 21 MSAT compounds.
- a subset of six identified having the greatest influence on health:  
benzene, 1,3-butadiene, formaldehyde, acrolein, acetaldehyde,  
and diesel particulate matter (DPM).

Harmful effects on the central nervous system:

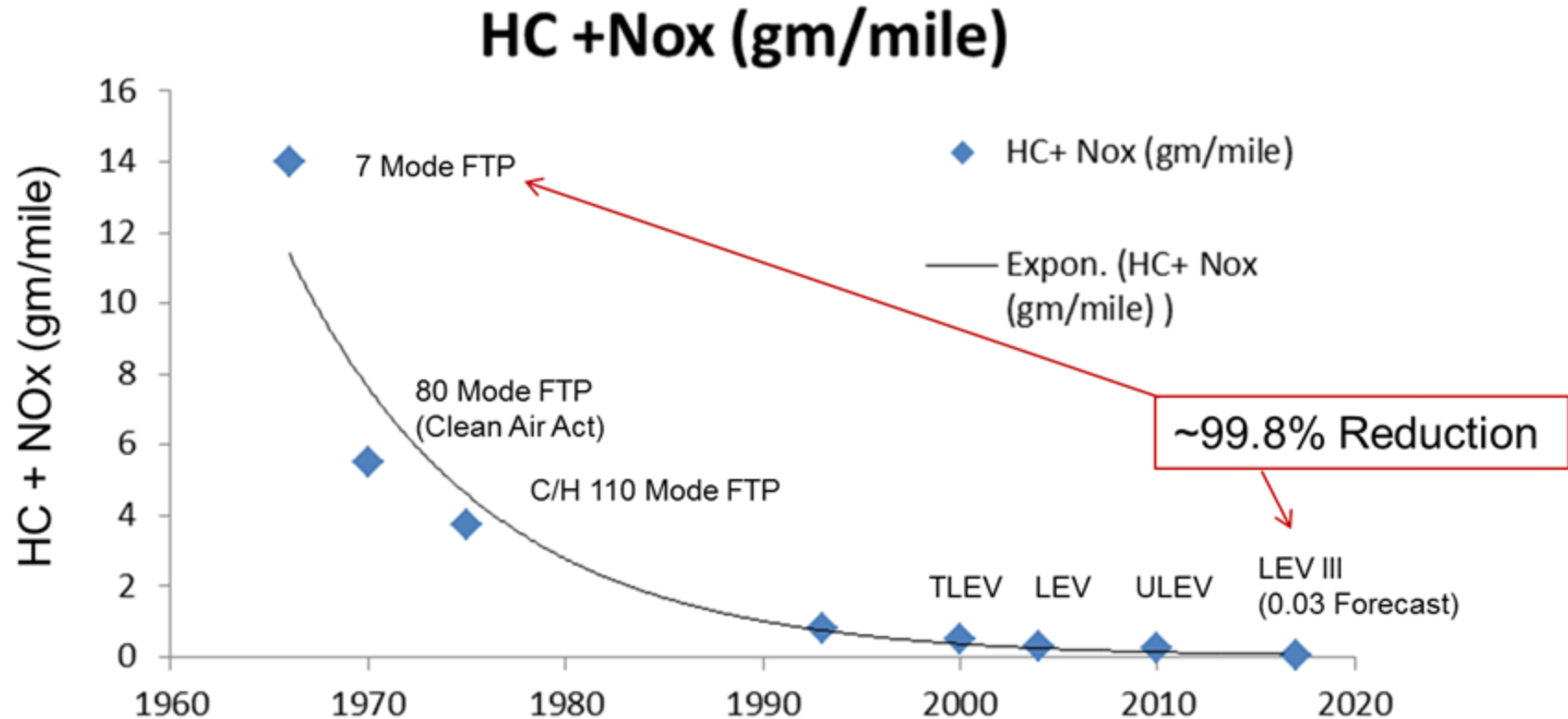
BTEX/N/S - benzene, toluene, ethylbenzene, xylenes, Naphthalene, Styrene





## Engine emissions - transportation & toxic air pollutants

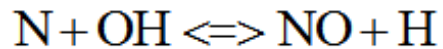
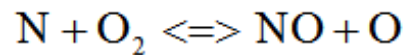
Future emissions standards will be a challenging constraint.





## NOx modeling

Zeldo'vich thermal NOx mechanism



ERC 12-step NOx model is based on GRI-Mech v3.11 and includes:

Thermal NOx Zeldovich, 1946

Prompt NOx around 1000 K. Fenimore, 1979

Extensions

NO can convert HCN and NH<sub>3</sub> Eberius, 1987

Interaction between NO and Soot Guo, 2007



### Summary

IC engine combustion physics depends critically on fuel chemistry

Much progress has been made in understanding and modeling combustion chemistry for realistic fuels

The various combustion regimes require different fuels, with differences in autoignition characteristics playing a major role

Meeting future toxic emissions regulations will require significant advances in engine research



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