Internal Energy and q

$$\begin{array}{c|c} & & & \varepsilon_5 \\ & & \varepsilon_4 & \text{increase T} \\ & & \varepsilon_3 & \\ & & \varepsilon_2 & \\ & & \varepsilon_2 & \\ & & \varepsilon_1 & \\ \end{array}$$

$$U - U(0) = -\frac{1}{Q} \left(\frac{\partial Q}{\partial \beta} \right)_{V}$$

$$\epsilon = \begin{bmatrix} \epsilon_5 \\ -\epsilon_4 \\ -\epsilon_4 \\ -\epsilon_3 \\ -\epsilon_2 \\ -\epsilon_2 \\ -\epsilon_1 \end{bmatrix} \qquad U-U(0) = \frac{\sum n_i E_i}{\sum n_i}$$

$$Q = \frac{q^N}{N!}$$

$$\overline{ \left(\frac{\partial \mathbf{Q}}{\partial \mathbf{B}} \right)_{\!\!\!\! V} } = \frac{1}{\mathbf{N}!} \left(\frac{\partial \mathbf{q}^{\mathbf{N}}}{\partial \mathbf{B}} \right) = \frac{\mathbf{N} \ \mathbf{q}^{\mathbf{N}-1}}{\mathbf{N}!} \left(\frac{\partial \mathbf{q}}{\partial \mathbf{B}} \right)$$

$$U - U(0) = \frac{-N}{q} \left(\frac{\partial q}{\partial \beta} \right)_{V}$$

since
$$\left(\frac{\partial \ln q}{\partial \beta}\right)_V = \frac{1}{q} \left(\frac{\partial q}{\partial \beta}\right)_V$$

$$U - U(0) = -N \left(\frac{\partial \ln q}{\partial \beta} \right)_{V}$$

$$U - U(0) = -\left(\frac{\partial \ln Q}{\partial \beta}\right)_V$$

$$U - U(0) = \frac{-N}{q} \left(\frac{\partial q}{\partial T} \right) V \frac{\partial T}{\partial \beta}$$

$$\frac{\partial \mathbf{B}}{\partial \mathbf{T}} = \frac{\partial \left(\frac{1}{\mathbf{k}\mathbf{T}}\right)}{\partial \mathbf{T}} = -\frac{1}{\mathbf{k}\mathbf{T}^2}$$

$$\frac{\partial T}{\partial \beta} = -kT^2$$

$$U - U(0) = \frac{NkT^2}{q} \left(\frac{\partial q}{\partial T}\right) V$$

$$U - U(0) = \frac{kT^2}{Q} \left(\frac{\partial Q}{\partial T} \right) V$$

$$U - U(0) = \frac{nRT^2}{q} \left(\frac{\partial q}{\partial T}\right) V$$

$$Nk = nR$$

$$U - U(0) = NkT^2 \left(\frac{\partial \ln q}{\partial T}\right)_V$$

$$U - U(0) = kT^2 \left(\frac{\partial \ln Q}{\partial T} \right)_V$$