Disturbance Propagation and Generation in Reacting Flows

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A) Introduction and Outlook

B) Flame Aerodynamics and Flashback

C) Flame Stretch, Edge Flames, and Flame Stabilization Concepts

D) Disturbance Propagation and Generation in Reacting Flows

E) Flame Response to Harmonic Excitation

- Introduction
- Decomposition of Disturbances into Fundamental Disturbance Modes
- Disturbance Energy
- Nonlinear Behavior
- Acoustic Wave Propagation Primer
- Unsteady Heat Release Effects and Thermoacoustic Instability

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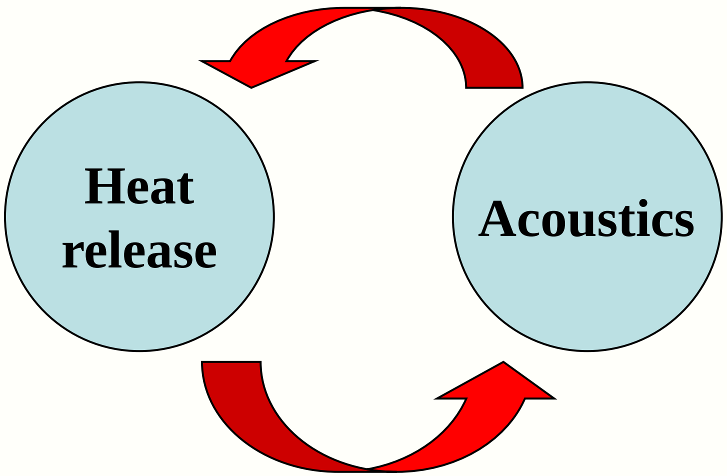
Motivation – Combustion Instabilities

- Large amplitude acoustic oscillations driven by heat release oscillations
- Oscillations occur at specific frequencies, associated with resonant modes of combustor



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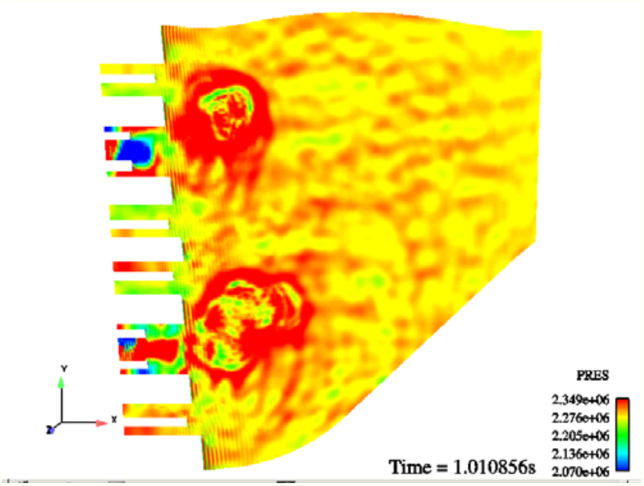

Video courtesy of S. Menon

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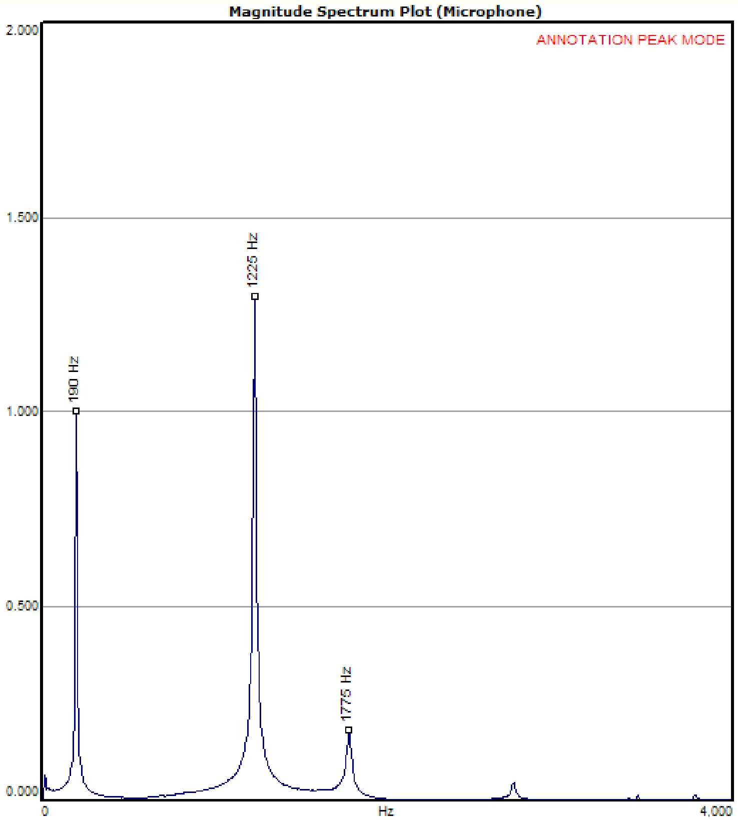
Resonant Modes – You can try this at home

- Helmholtz Mode
  - 190 Hz
- Longitudinal Modes
  - 1,225 Hz
  - 1,775 Hz
- Transverse Modes
  - 3,719 Hz
  - 10,661 Hz



Slide courtesy of R. Mihata, Alta Solutions

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Key Problem: Flame Sensitive to Acoustic Waves

Video from Ecole Centrale – 75 Hz, Courtesy of S. Candel

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Rubens Tube

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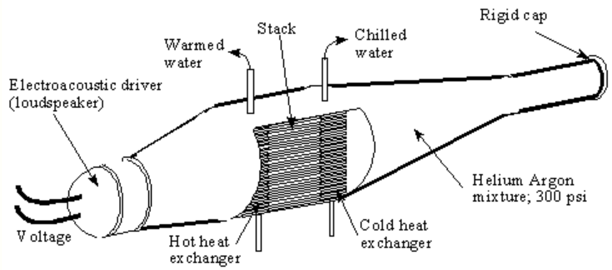
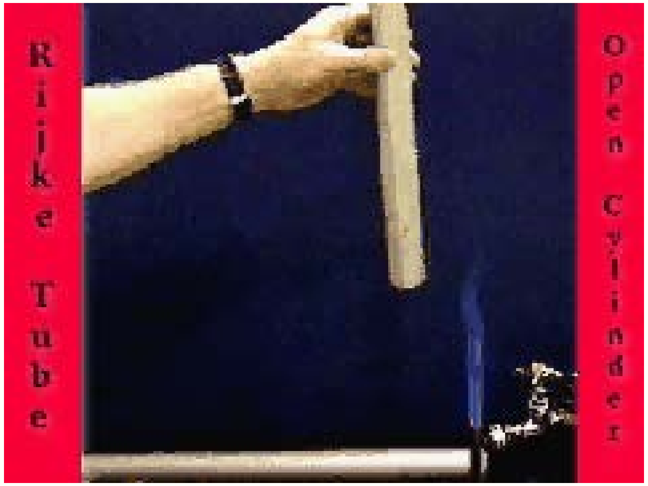
- Rijke Tube (heated gauze in tube)
- Self-excited oscillations in cryogenic tubes
- Thermo-acoustic refrigerators/heat pumps



Purdue's Thermoacoustic Refrigerator



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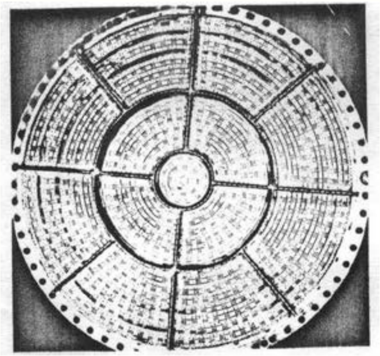
- F-1 Engine
  - Used on Saturn V
  - Largest thrust engine developed by U.S
  - Problem overcome with over 2000 (out of 3200) full scale tests

Injector face destroyed by combustion instability, Source: D. Talley



From Liquid Propellant Rocket Combustion Instability, Ed. Harrje and Reardon, NASA Publication SP-194

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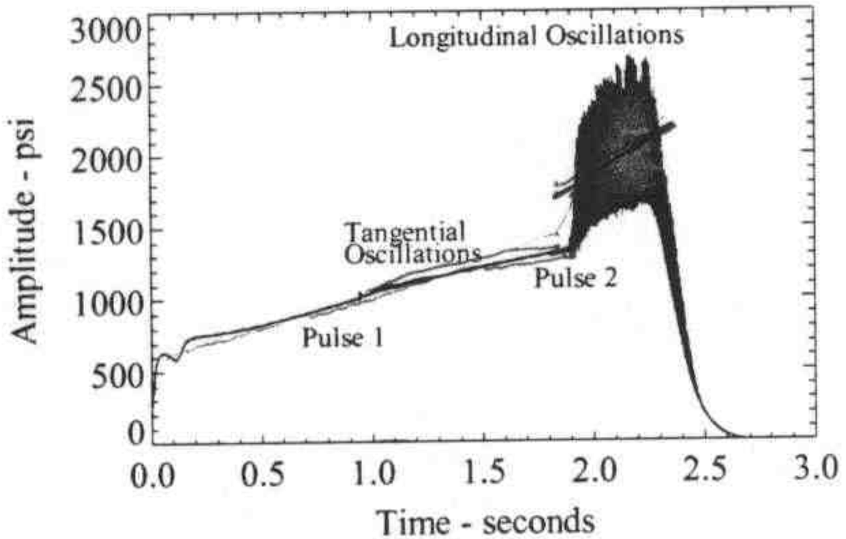
- Examples:
  - Space shuttle booster- 1-3 psi oscillations (1 psi = 33,000 pounds of thrust)

Adverse effects –thrust oscillations, mean pressure changes, changes in burning rates



From Blomshield, AIAA Paper #2001-3875

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- Assume infinitesimal perturbations superposed upon a spatially homogenous background flow.

- We will also introduce two additional assumptions:

  (1) The gas is non-reacting and calorically perfect, implying that the specific heats are constants.

  (2) Neglect viscous and thermal transport.

Decomposition Approach

- Decompose variables into the sum of a base and fluctuating component; e.g.,

$$
\begin{aligned}
p(x,t) &= p_0 + p_1(x,t) \\
\rho(x,t) &= \rho_0 + \rho_1(x,t) \qquad (2.9) \\
\vec{u}(x,t) &= \vec{u}_0 + \vec{u}_1(x,t)
\end{aligned}
$$

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Perturbing the Continuity Equation

- Consider the continuity equation:

$$-\frac{1}{\rho}\frac{D\rho}{Dt}=\frac{1}{c_p}\frac{Ds}{Dt}-\frac{1}{\gamma p}\frac{Dp}{Dt}=\nabla\cdot\vec{u} \qquad (2.10)$$

- *Ds/Dt* is identically zero because of the neglect of molecular transport terms and chemical reaction. Therefore …

$$-\frac{1}{\gamma p}\frac{Dp}{Dt}=\nabla\cdot\vec{u} \qquad (2.11)$$

- Expanding each variable into base and fluctuating components yields:

$$-\frac{1}{\gamma\left(p_0+p_1\right)}\left(\frac{\partial p_1}{\partial t}+\left(\vec{u}_0+\vec{u}_1\right)\cdot\nabla p_1\right)=\nabla\cdot\vec{u}_1 \qquad (2.12)$$

- Continuity Equation Expanded:

$$-\frac{1}{\gamma}\left(\frac{\partial p_1}{\partial t}+\vec{u}_0\cdot\nabla p_1+\vec{u}_1\cdot\nabla p_1\right)=p_0\nabla\cdot\vec{u}_1+p_1\nabla\cdot\vec{u}_1 \qquad (2.13)$$

- Linearize by neglecting products of perturbations. If perturbations are small in amplitude then products of perturbations are negligible.

$$-\frac{1}{\gamma p_0}\frac{D_0 p_1}{Dt}=\nabla\cdot\vec{u}_1\equiv\Lambda_1 \qquad (2.14)$$

- Definition of the substantial derivative, D_0/Dt.

$$\frac{D_0}{Dt}(\quad)=\frac{\partial}{\partial t}(\quad)+\vec{u}_0\cdot\nabla(\quad) \qquad (2.15)$$

Summary of Disturbance Equations

- Vorticity: $$\frac{D_0 \vec{\Omega}_1}{Dt} = 0 \qquad (2.18)$$

- Acoustic: $$\frac{D_0^2 p_1}{Dt^2} - c_0^2 \nabla^2 p_1 = 0 \qquad (2.20)$$

- Entropy: $$\frac{D_0 s_1}{Dt} = 0 \qquad (2.27)$$

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- Further decompose perturbations by their origin

$$\vec{\Omega}_1 = \vec{\Omega}_{1\Lambda} + \vec{\Omega}_{1s} + \vec{\Omega}_{1\Omega}$$

$$s_1 = s_{1\Lambda} + s_{1s} + s_{1\Omega} \tag{2.28}$$

$$p_1 = p_{1\Lambda} + p_{1s} + p_{1\Omega}$$

- Examples:
  - $\vec{\Omega}_{1s}$ vorticity fluctuations induced by entropy fluctuations.
  - $p_{1\Omega}$ pressure fluctuations induced by vorticty fluctuations.
- Dynamical equations are linear and can be decomposed into subsystems

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Oscillations from Vorticity

- *Oscillations associated with vorticity mode:*

$$\frac{D_0 \vec{\Omega}_{1\Omega}}{Dt} = 0 \tag{2.29}$$

$$p_{1\Omega} = s_{1\Omega} = T_{1\Omega} = \rho_{1\Omega} = 0 \tag{2.30}$$

$$\frac{D_0 \vec{u}_{1\Omega}}{Dt} = 0 \tag{2.31}$$

  - Vorticity, and induced velocity, fluctuations are convected by the mean flow.
  - Vorticity fluctuations induce no fluctuations in pressure, entropy, temperature, density, or dilatation.

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Oscillations from Acoustics

- *Oscillations associated with acoustic mode:*

$$\frac{D_0^2 p_{1\Lambda}}{Dt^2} - c_0^2 \nabla^2 p_{1\Lambda} = 0 \qquad (2.32)$$

$$\vec{\Omega}_{1\Lambda} = s_{1\Lambda} = 0 \qquad (2.33)$$

$$\rho_{1\Lambda} = \frac{p_{1\Lambda}}{c_0^2} = \frac{\rho_0 c_p}{c_0^2} T_{1\Lambda} \qquad (2.34)$$

$$\rho_0 \frac{D\vec{u}_{1\Lambda}}{Dt} = -\nabla p_{1\Lambda} \qquad (2.35)$$

- The density, temperature, and pressure fluctuations are locally and algebraically related through their isentropic relations

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Oscillations from Entropy

- *Oscillations associated with entropy mode:*

$$\frac{D_0 s_{1s}}{Dt} = p_{1s} = \vec{\Omega}_{1s} = \vec{u}_{1s} = 0 \tag{2.37}$$

$$\rho_{1s} = -\frac{\rho_0}{c_p} s_{1s} = -\frac{\rho_0}{T_0} T_{1s} \tag{2.38}$$

- Entropy oscillations do not excite vorticity, velocity, pressure, or dilatational disturbances

- They do excite density and temperature perturbations

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Comments on the Decomposition

- In a homogeneous, uniform flow, these three disturbance modes propagate independently in the linear approximation.
- three modes are decoupled within the approximations of this analysis - vortical, entropy, and acoustically induced fluctuations are completely independent of each other.
- For example, velocity fluctuations induced by vorticity and acoustic disturbances, $\vec{u}_{1\Omega}$ and $\vec{u}_{1\Lambda}$ are independent of each other and each propagates as if the other were not there.
- Moreover, there are no sources or sinks of any of these disturbance modes. Once created, they propagate with constant amplitude.

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Comments on Length Scales

- *A*coustic disturbances propagate at the speed of sound.
- Vorticity and entropy disturbances convect at bulk flow velocity, $\overline{u}_0$.
- Acoustic properties vary over an acoustic length scale, given by $\lambda_\Lambda = c_0 / f$
- Entropy and vorticity modes vary over a convective length scale, given by $\lambda_c = u_0 / f$
- Entropy and vortical mode "wavelength" is shorter than the acoustic wavelength by a factor equal to the mean flow Mach number.

$$\lambda_c / \lambda_\Lambda = u_0 / c_0 = M$$

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Comments on Acoustic Disturbances

- Acoustic disturbances, being true waves, reflect off boundaries, are refracted at property changes, and diffract around obstacles.

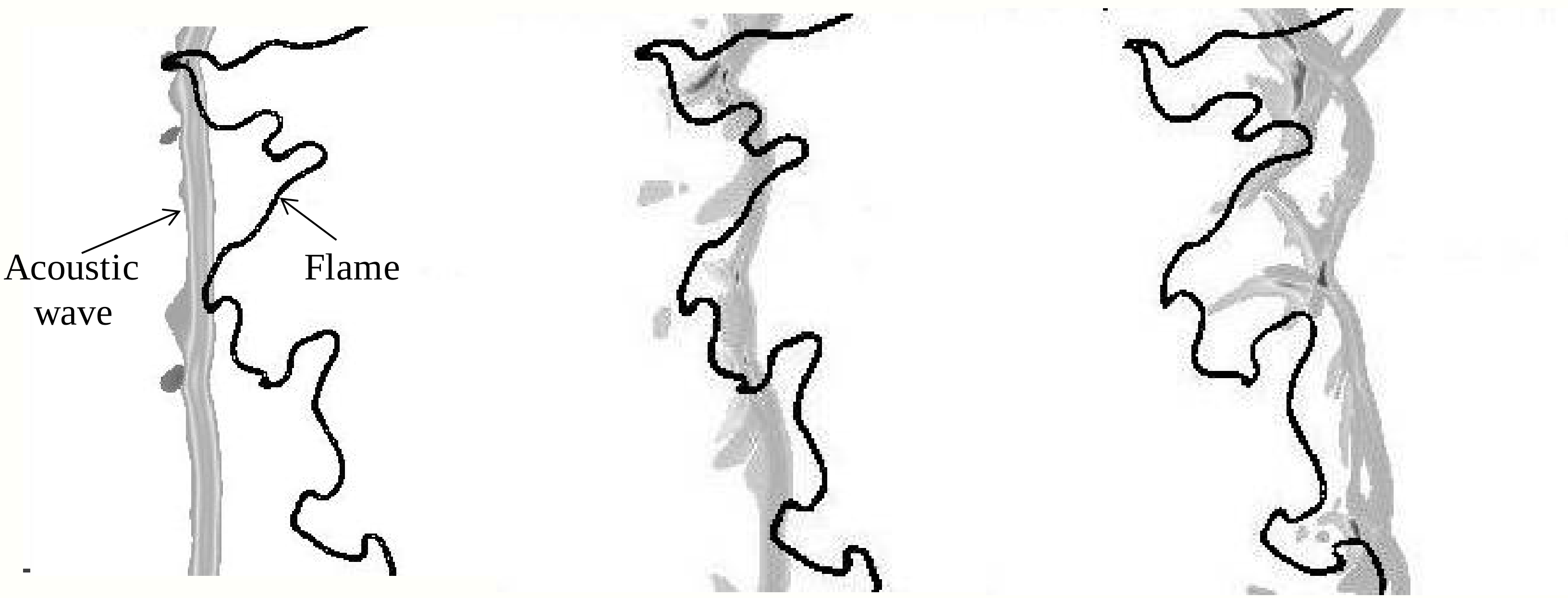

Image of instantaneous pressure field and flame front of a sound wave incident upon a turbulent flame from the left at three successive times. Courtesy of D. Thévenin.

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- PIV data shows example of
  - refraction in a combustor environment
  - Simultaneous presence of acoustic and vortical disturbances

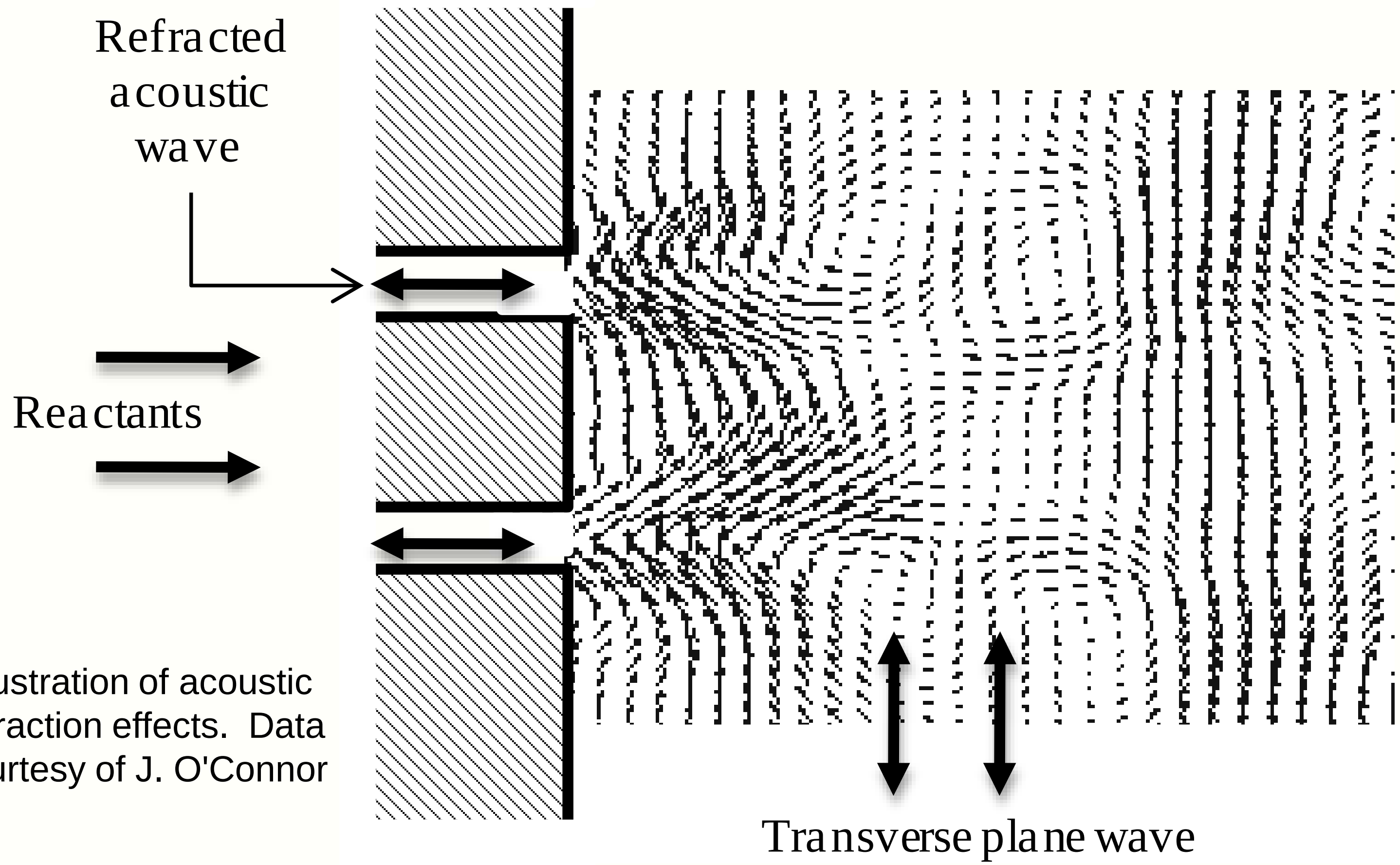

Illustration of acoustic refraction effects. Data courtesy of J. O'Connor

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Summary

	Acoustic	Entropy	Vorticity
Pressure	X		
Velocity	X		X
Density	X	X	
Temperature	X	X	
Vorticity			X
Entropy		X	

Example: Effects of Simultaneous Acoustic and Vortical Disturbances

- Consider superposition of two disturbances with different phase speeds:

$$u_1(x,t) = u_{1,\Lambda} + u_{1,\Omega} = \mathcal{A}_\Lambda \cos\left(\omega t - \frac{x}{c_0}\right) + \mathcal{A}_\Omega \cos\left(\omega t - \frac{x}{u_{x,0}}\right)$$

- For simplicity, assume $\mathcal{A} = \mathcal{A}_\Lambda = \mathcal{A}_\Omega$

$$u_1(x,t) = 2\mathcal{A} \cos\left(\frac{c_0 - u_0}{2c_0 u_0} x\right) \cos\left(\omega t - \frac{c_0 - u_0}{2c_0 u_0} x\right)$$

- Velocity field oscillates harmonically at each point as cos(ωt)
- Amplitude of these oscillations varies spatially as $\cos\left(\frac{c_0 - u_0}{2c_0 u_0} x\right)$ due to interference

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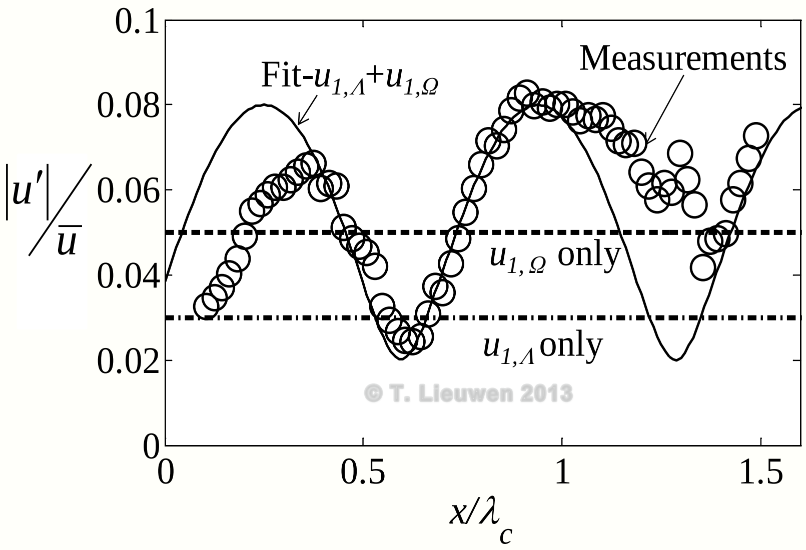
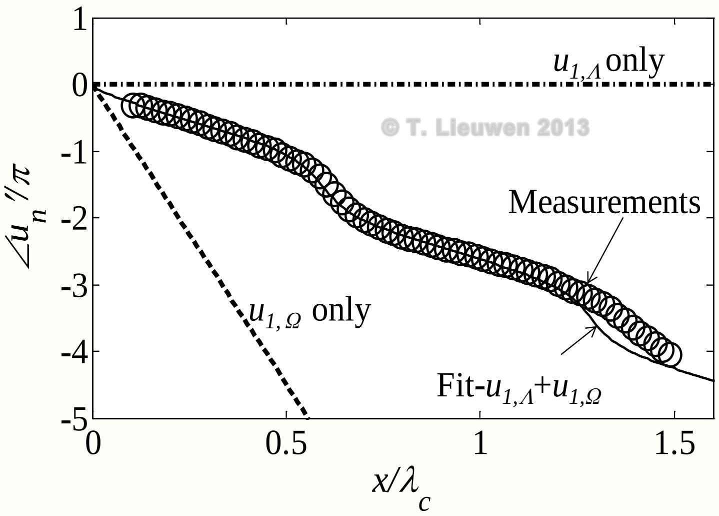
Some Data Illustrating This Effect

- Fit parameter: $\mathscr{A}_\Lambda / \mathscr{A}_\Omega = 0.6$





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- We just showed that the small amplitude canonical disturbance modes propagate independently within the fluid domain in a homogeneous, inviscid flow.
- These modes couple with each other from:
  - Boundaries
    - e.g., acoustic wave impinging obliquely on wall excites vorticity and entropy
  - Regions of flow inhomogeneity
    - e.g., acoustic wave propagating through shear flow generates vorticity
    - Accelerating an entropy disturbance generates an acoustic wave
  - Nonlinearities
    - e.g., large amplitude vortical disturbances generate acoustic waves (jet noise)

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- Although the time average of the disturbance fields may be zero, they nonetheless contain non-zero time averaged energy and lead to energy flux whose time average is also non-zero.
  - Ex:

$$E_{kin} = \frac{1}{2}\rho\left(\vec{u}_1 \cdot \vec{u}_1\right)$$

- Consider the energy equation.

$$\frac{\partial E}{\partial t} + \nabla \cdot \vec{I} = \Phi \qquad (2.50)$$

$$\frac{d}{dt}\iiint_V E dV + \oiint_A \vec{I} \cdot \vec{n} dA = \iiint_V \Phi d\mathrm{V} \qquad (2.51)$$

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Acoustic Energy Equation

- Consider the acoustic energy equation, assuming:
  - Entropy and vorticity fluctuations are zero
  - Zero mean velocity, homogenous flow.
  - Combustion process is isomolar

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_0\left(\vec{u}_1\cdot\vec{u}_1\right)+\frac{1}{2}\frac{p_1^2}{\rho_0 c_0^2}\right)+\nabla\cdot\left(p_1\vec{u}_1\right)=\frac{\left(\gamma-1\right)}{\gamma p_0}p_1\dot{q}_1 \qquad (2.52)$$

$$E_\Lambda=\frac{1}{2}\rho_0\left(\vec{u}_1\cdot\vec{u}_1\right)+\frac{1}{2}\frac{p_1^2}{\rho_0 c_0^2} \qquad (2.53)$$

$$\vec{I}_\Lambda=p_1\vec{u}_1 \qquad (2.54)$$

$$\Phi_\Lambda=\frac{\left(\gamma-1\right)}{\gamma p_0}p_1\dot{q}_1 \qquad (2.55)$$

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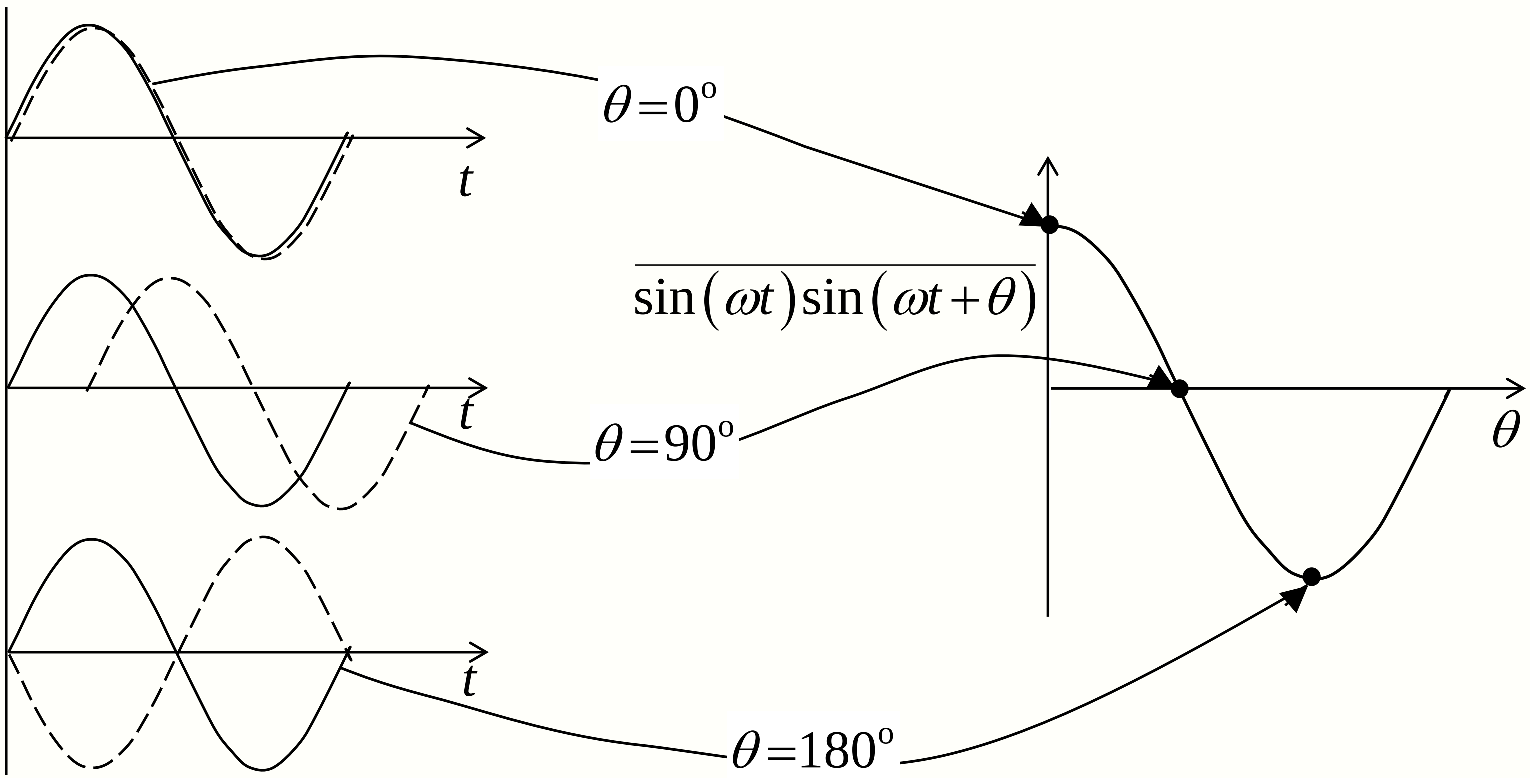
Time Average of Products

- Time average of product of two fluctuating quantities depends on their relative phasing
- Example $\overline{\sin(\omega t)\sin(\omega t+\theta)} = \frac{1}{2}\cos\theta$



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Comments on Acoustic Energy

- The energy density, E_Λ , is a linear superposition of the kinetic energy associated with unsteady motions, and potential energy associated with the isentropic compression of an elastic gas.

- The flux term, $p_1\vec{u}_1$, reflects the familiar “pumping work” done by pressure forces on a system.

- The source term, Φ_Λ, shows that unsteady heat release can add or remove energy from the acoustic field, depending upon its phasing with the acoustic pressure.

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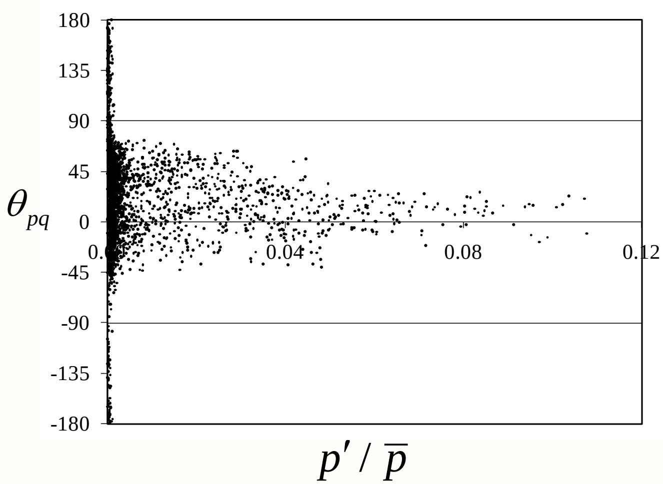
- Rayleigh's Criterion states that unsteady heat addition locally adds energy to the acoustic field when the phases between the pressure and heat release oscillations is within ninety degrees of each other.
- Conversely, when these oscillations are out of phase, the heat addition oscillations damp the acoustic field.
- Figure illustrates that the highest pressure amplitudes are observed at conditions where the pressure and heat release are in phase.

Data courtesy of K. Kim and D. Santavicca.



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- The disturbances which have been analyzed arise because of underlying instabilities, either in the local flow profile or to the coupled flame-combustor acoustic systems (such as thermoacoustic instabilities).

- Consider a more general study of stability concepts by considering the time evolution of a disturbance

$$\frac{d\mathscr{A}(t)}{dt} = F_A - F_D \tag{2.63}$$

F_A and F_D denote processes responsible for amplification and damping of the disturbance, respectively.

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Linear Stable/Unstable Systems

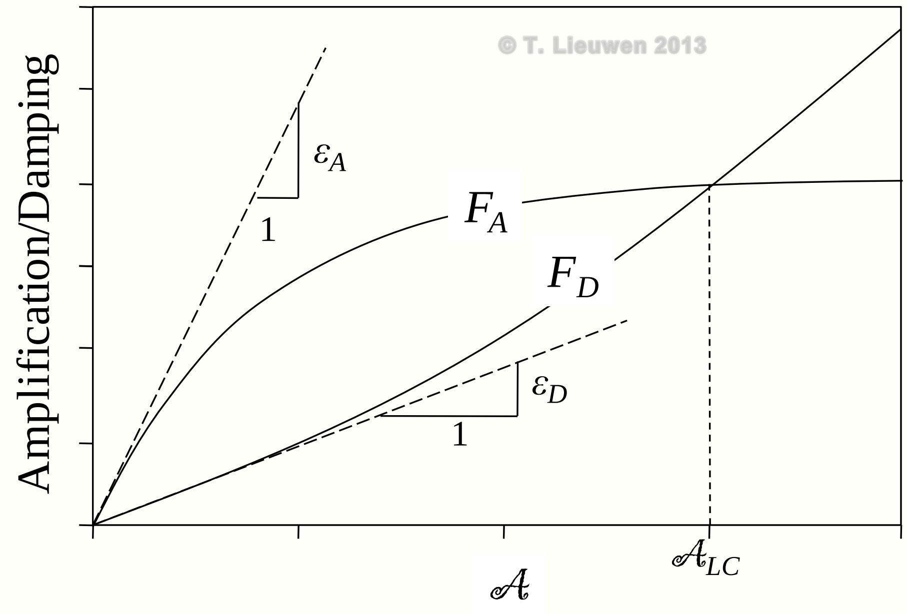
- In a linearly stable/unstable system, infinitesimally small disturbances decay/grow, respectively.

- To illustrate these points, we can expand the functions F_A and F_D around their $\mathscr{A}$=0 values in a Taylor's series:

$$F_A = \varepsilon_A \mathscr{A} + F_{A,NL} \qquad (2.64)$$

$$F_D = \varepsilon_D \mathscr{A} + F_{D,NL} \qquad (2.65)$$

$$\varepsilon_A = \left.\frac{\partial F_A}{\partial \mathscr{A}}\right|_{\mathscr{A}=0} \qquad (2.66)$$



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Comments on Linear Behavior

- The amplification and damping curves intersect at the origin, indicating that a zero amplitude oscillation is a potential equilibrium point.
- However, this equilibrium point is unstable, since $\varepsilon_A > \varepsilon_D$ and any small disturbance makes F_A larger than F_D resulting in further growth of the disturbance.
- If $\varepsilon_A < \varepsilon_D$ the $\mathscr{A}$=0 point is an example of an “attractor” in that disturbances are drawn toward it
- Linearized solution:

$$\mathscr{A}_1(t) = \mathscr{A}(t=0)\exp((\varepsilon_A - \varepsilon_D)t) \tag{2.67}$$

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- This linearized solution may be a reasonable approximation to the system dynamics if the system is linearly stable (unless the initial excitation, $\mathscr{A}(t=0)$ is large).

- However, it is only valid for small time intervals when the system is unstable, as disturbance amplitudes cannot increase indefinitely.

- In this situation, the amplitude dependence of system amplification/damping is needed to describe the system dynamics.

- The steady state amplitude is stable at $\mathscr{A}_{LC}=0$ because amplitude perturbations to the left (right) causes F_A to become larger (smaller) than F_D, thus causing the amplitude to increase (decrease).

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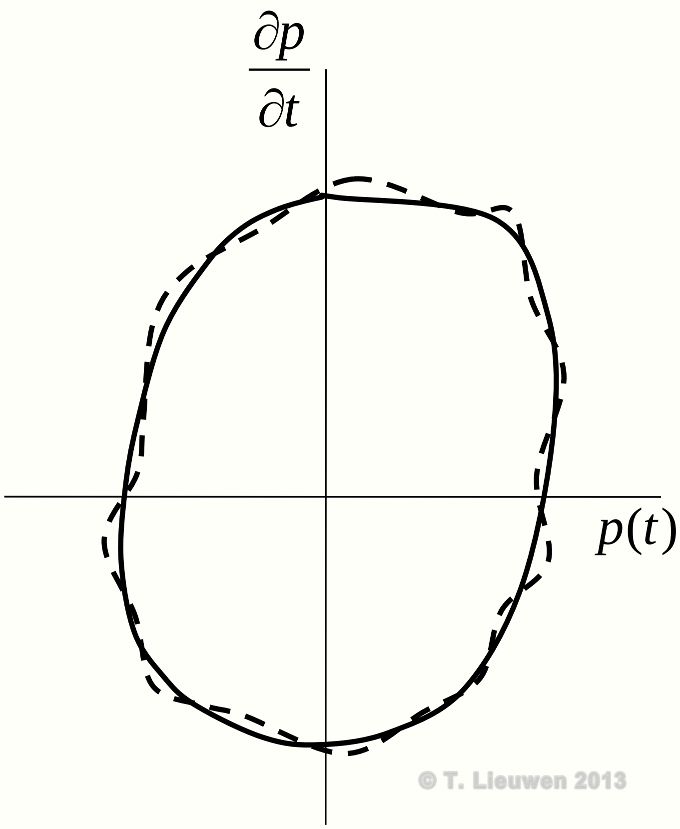
Example of Stable Orbits: Limit Cycles

- In many other problems, $\mathcal{A}(t)$ is used to describe the amplitude of a fluctuating disturbance; for example,

- Limit cycle is example of orbit that encircles an unstable fixed point
  - A stable limit cycle will be pulled back into this attracting, periodic orbit even when it is slightly perturbed.

$$p(t)-p_0=\mathcal{A}(t)\cos(\omega t) \quad (2.69)$$



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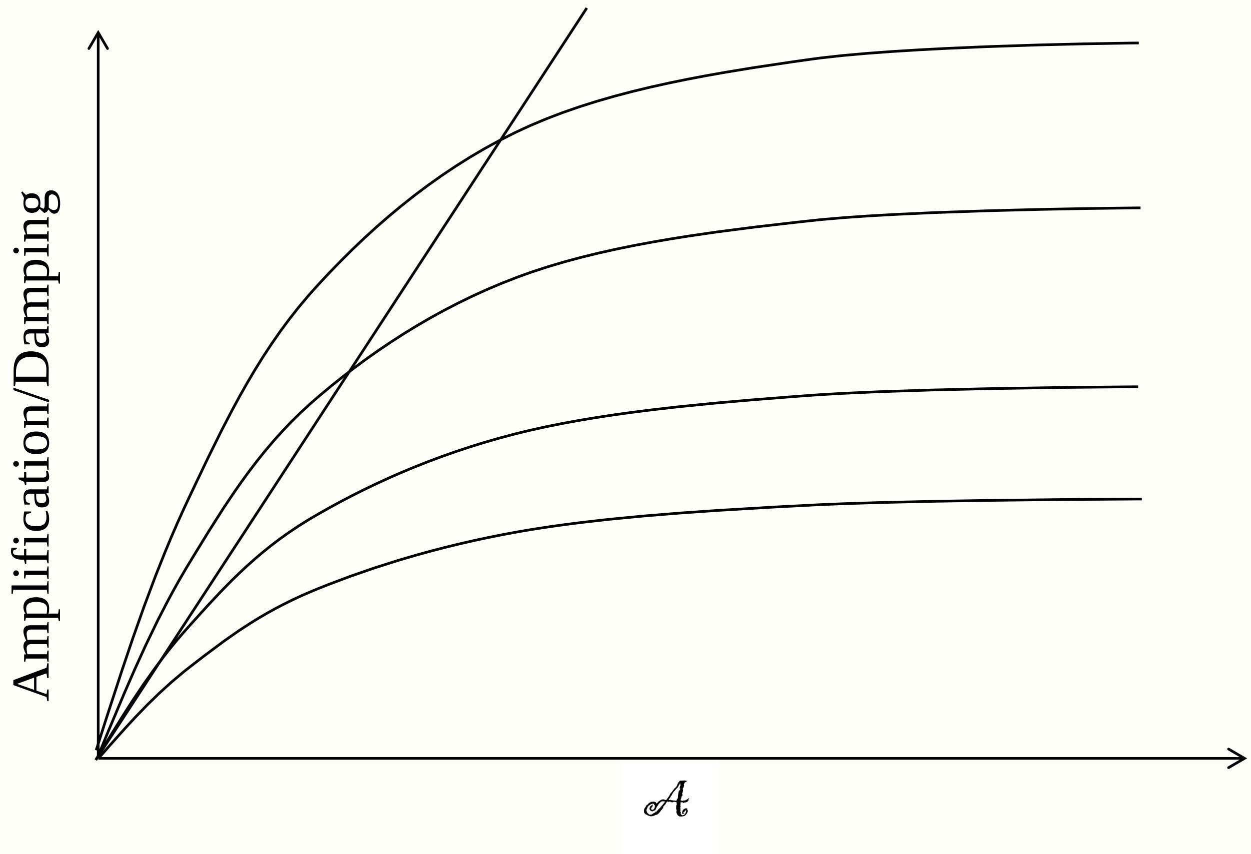
Variation in System Behavior with Change in Parameter

- Consider a situation where some combustor parameter is systematically varied in such a way that ε_A increases while ε_D remains constant.



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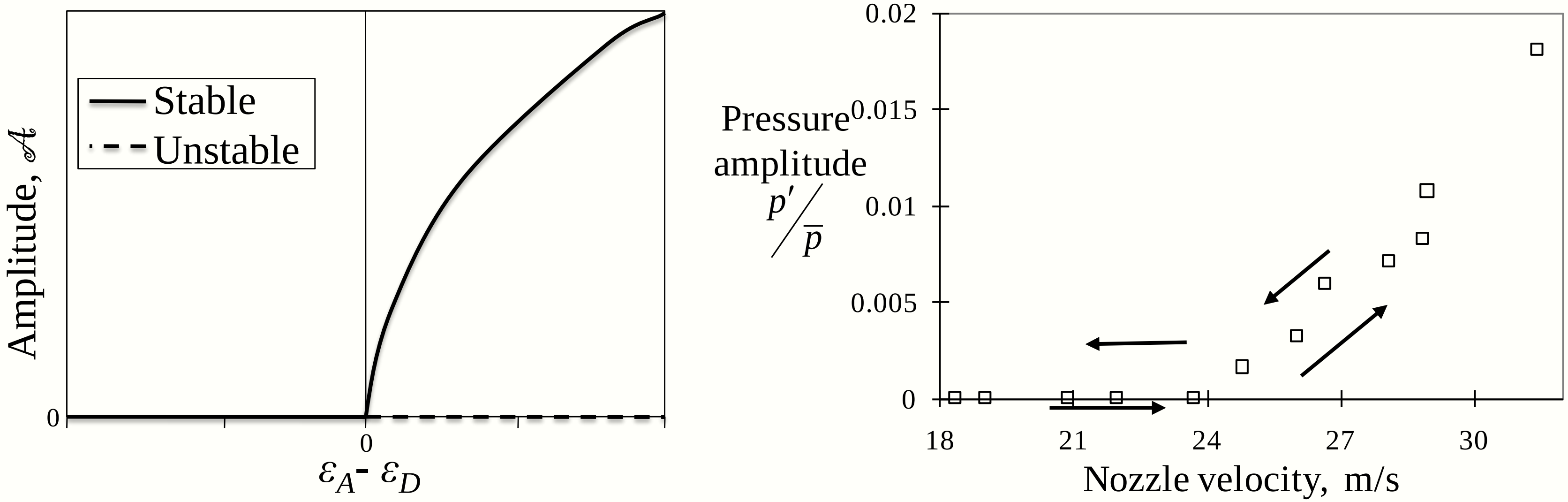
- The $\varepsilon_A = \varepsilon_D$ condition separates two regions of fundamentally different dynamics and is referred to as a *supercritical bifurcation point*.
  - Note smooth monotonic variation of limit cycle amplitude with parameter



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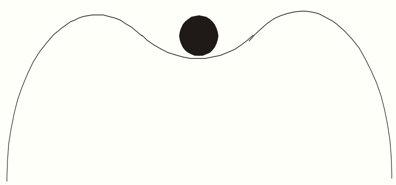
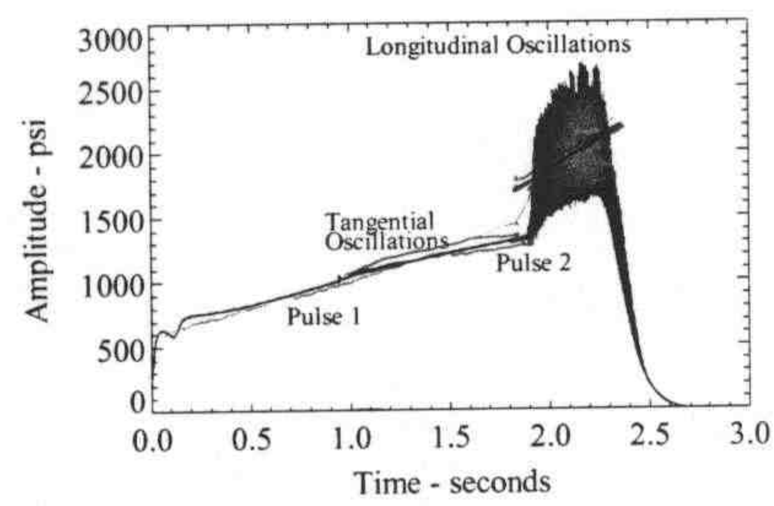
Nonlinearly Unstable Systems

- Small amplitude disturbances decay in a nonlinearly unstable system, but disturbances with amplitudes exceeding a critical value, $\mathscr{A}_c$, will grow.



- This type of instability is sometimes referred to as *subcritical*.

- Other examples:
  - hydrodynamic stability of shear flows without inflection points
  - Certain kinds of thermoacoustic instabilities in combustors.
    - historically referred to as “triggering” in rocket instabilities



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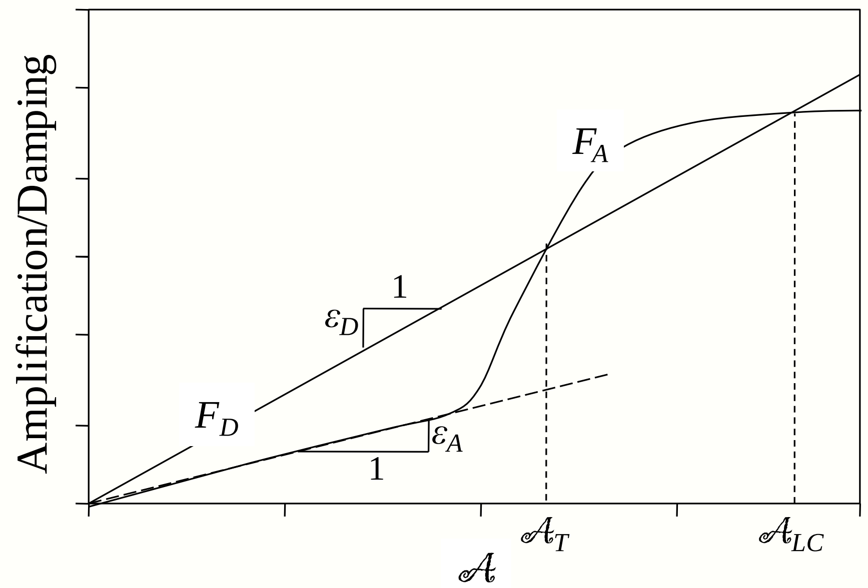
Subcritical Instability

- Figure provides example of amplitude dependence of F_A and F_D that produces *subcritical* instability.

- In this case, the system has three equilibrium points where the amplification and dissipation curves intersect.

- All disturbances with amplitudes $\mathcal{A}<\mathcal{A}_T$ return to the stable solution $\mathcal{A}=0$ and disturbances with amplitudes $\mathcal{A}>\mathcal{A}_T$ grow until their amplitude attains the value $\mathcal{A}=\mathcal{A}_{LC}$.

- Consequently, two stable solutions exist at this operating condition. The one observed at any point in time will depend upon the history of the system.



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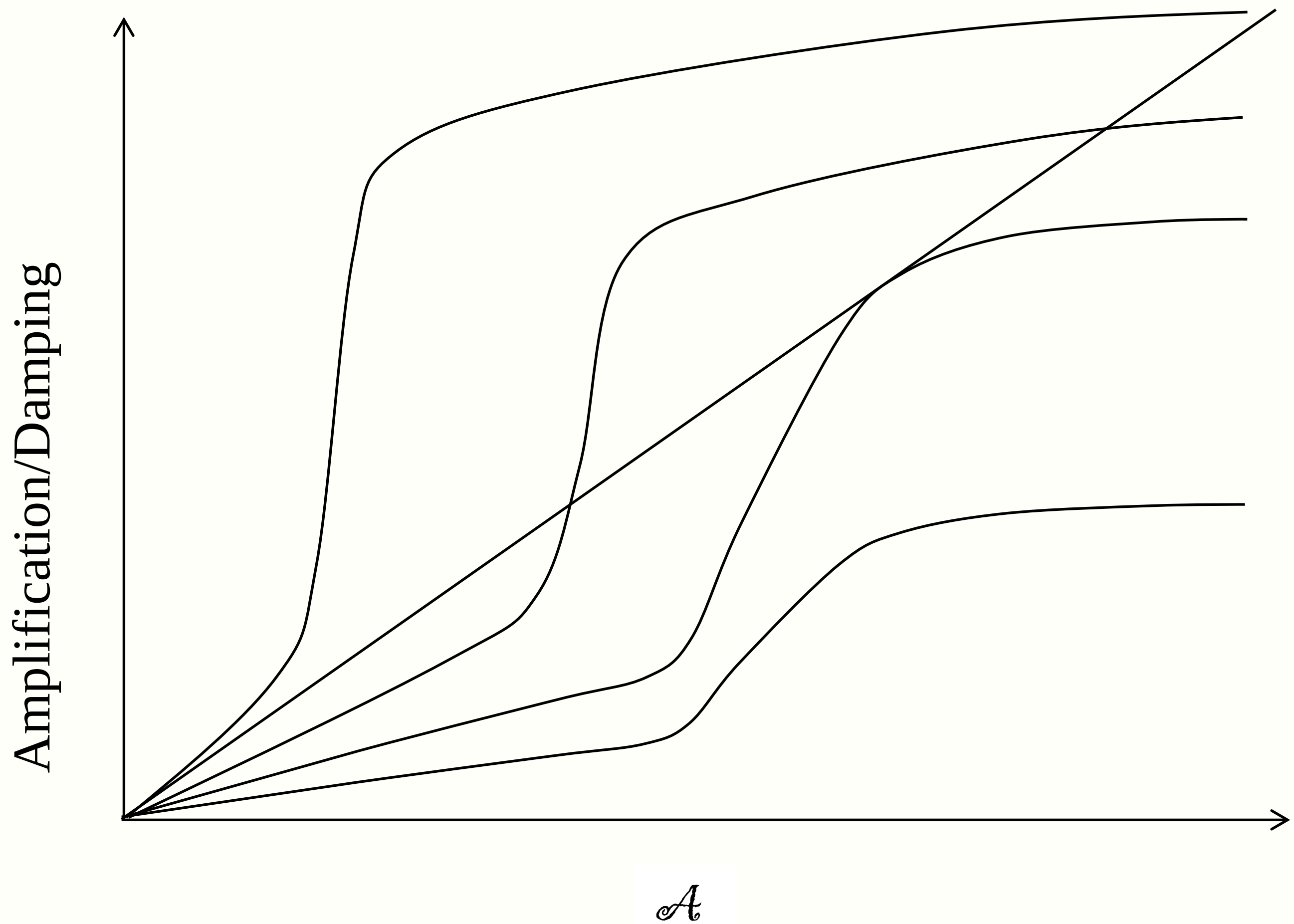
Variation in System Behavior with Change in Parameter



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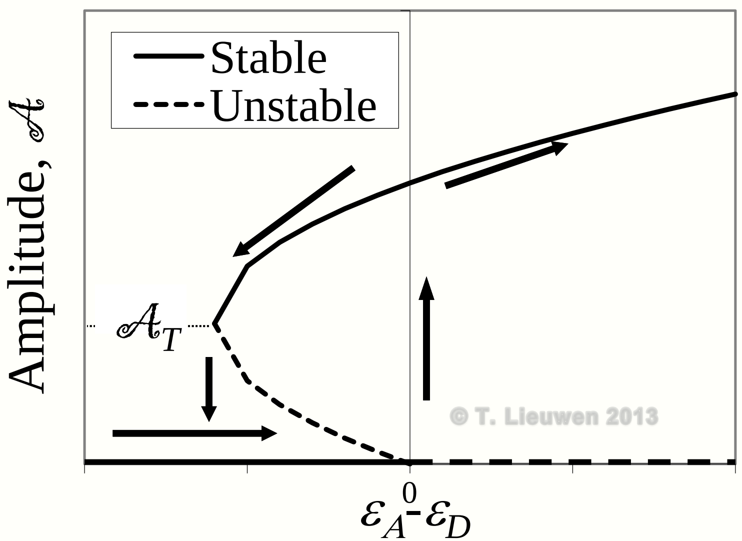
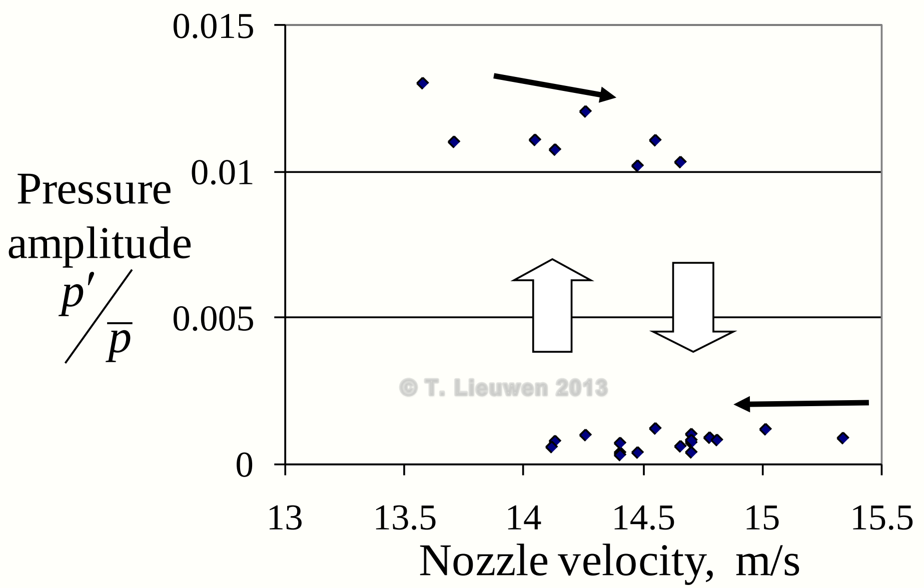
Subcritical Bifurcation Diagram

- If a system parameter is monotonically increased to change the sign of $\varepsilon_A - \varepsilon_D$ from a negative to a positive value, the system's amplitude jumps discontinuously from $\mathcal{A}$=0 to $\mathcal{A}$= $\mathcal{A}_{LC}$ at $\varepsilon_A - \varepsilon_D = 0$.

- Note hysteresis as well





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- Introduction
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- Flame Stabilization and Blowoff
- Combustion Instabilities
  - Motivation
  - Disturbance propagation, amplification, and stability
  - Acoustic Wave Propagation Primer
  - Unsteady Heat Release Effects and Thermoacoustic Instability
- Flame Dynamics

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- Acoustic Disturbances:
  - Propagate energy and information through the medium without requiring bulk advection of the actual flow particles.
  - Details of the time averaged flow has relatively minor influences on the acoustic wave field (except in higher Mach number flows).
  - Acoustic field largely controlled by the boundaries and sound speed field.

- Vortical disturbances
  - Propagate with the local flow field.
  - Highly sensitive to the flow details.
  - No analogue in the acoustic problem to the hydrodynamic stability problem.

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- Some distinctives of the acoustics problem:
  - Sound waves reflect off of boundaries and refract around bends or other obstacles.
    - Vortical and entropy disturbances advect out of the domain where they are excited.
  - An acoustic disturbance in any region of the system will make itself felt in every other region of the flow.
  - Wave reflections cause the system to have natural acoustic modes; oscillations at a multiplicity of discrete frequencies.

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- The acoustic wave equation for a homogeneous medium with no unsteady heat release is a linearized equation describing the propagation of small amplitude disturbances.

- We will first assume a one-dimensional domain and neglect mean flow, and therefore consider the wave equation:

$$\frac{\partial^2 p_1}{\partial t^2} - c_0^2 \frac{\partial^2 p_1}{\partial x^2} = 0 \tag{5.1}$$

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Harmonic Oscillations

- We will use complex notation for harmonic disturbances. In this case, we write the unsteady pressure and velocity as

$$p_1 = \mathrm{Real}\left(\hat{p}_1(x,y,z)\exp\left(-i\omega t\right)\right) \tag{5.13}$$

$$\vec{u}_1 = \mathrm{Real}\left(\hat{\vec{u}}_1(x,y,z)\exp\left(-i\omega t\right)\right) \tag{5.14}$$

- As such, the one-dimensional acoustic field is given by:

$$p_1 = \mathrm{Real}\left(\left(\mathscr{A}\exp\left(ikx\right)+\mathscr{B}\exp\left(-ikx\right)\right)\exp\left(-i\omega t\right)\right) \tag{5.15}$$

$$u_{x,1} = \frac{1}{\rho_0 c_0}\mathrm{Real}\left(\left(\mathscr{A}\exp\left(ikx\right)-\mathscr{B}\exp\left(-ikx\right)\right)\exp\left(-i\omega t\right)\right) \tag{5.16}$$

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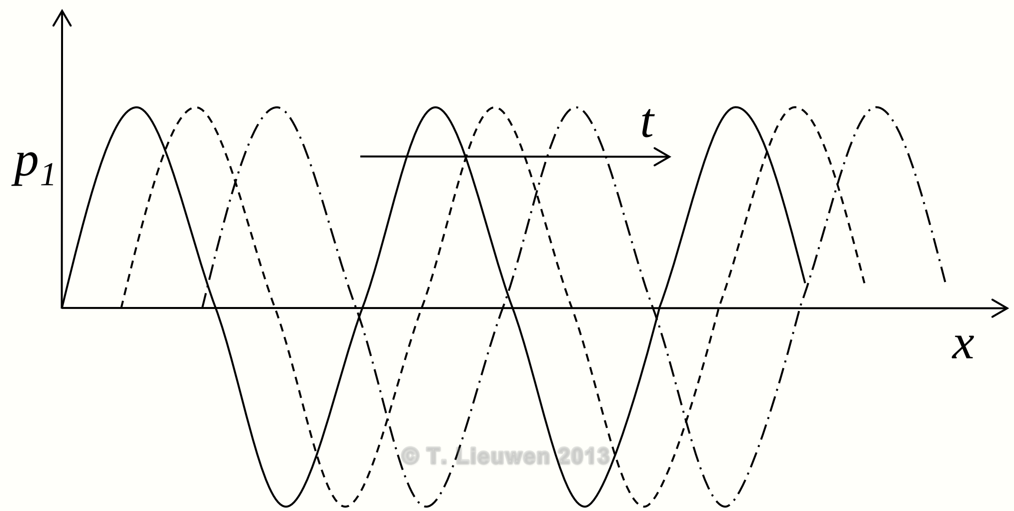
Time Response of Harmonically Oscillating Traveling Waves

- The disturbance field consists of space-time harmonic disturbances propagating with unchanged shape at the sound speed.
- An alternative way to visualize these results is to write the pressure in terms of amplitude and phase as:

$$\hat{p}_1(x) = \left|\hat{p}_1(x)\right| \exp\left(-i\theta(x)\right) \tag{5.17}$$

Temporal variation of harmonically varying pressure for rightward moving wave



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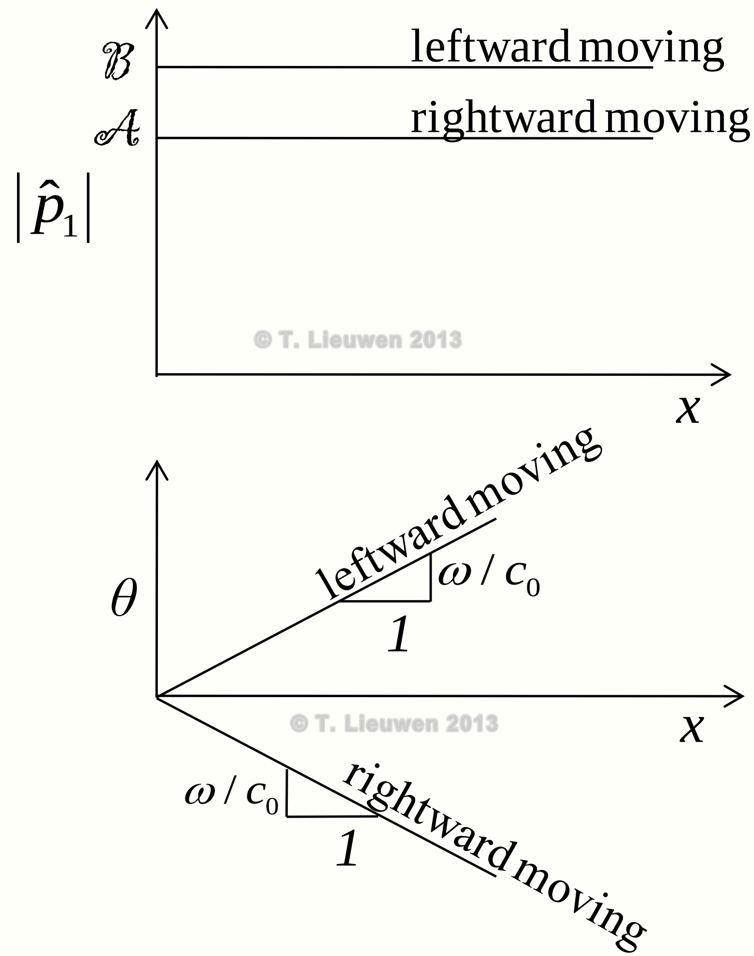
Amplitude and Phase of Traveling Wave Disturbances

- The magnitude of the disturbance stays constant but the phase decreases/increases linearly with axial distance.
- The slope of these lines are $\left|\frac{d\theta}{dx}\right| = \omega / c_0$.
- Harmonic disturbances propagating with a constant phase speed have a linearly varying axial phase dependence, whose slope is inversely proportional to the disturbance phase speed.



Spatial amplitude/phase variation of harmonically varying acoustic disturbances.

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Standing Waves

- Consider next the superposition of a left and rightward traveling wave of equal amplitudes, $\mathscr{A}=\mathscr{B}$, assuming without loss of generality that $\mathscr{A}$ and $\mathscr{B}$ are real.

$$p_1(x,t) = 2\mathscr{A}\cos(kx)\cos(\omega t) \tag{5.18}$$

$$u_{x,1} = \frac{1}{\rho_0 c_0} 2\mathscr{A}\sin(kx)\sin(\omega t) \tag{5.19}$$

- Such a disturbance field is referred to as a “standing wave”.

- Observations:
  - amplitude of the oscillations is not spatially constant, as it was for a single traveling wave.
  - phase does not vary linearly with *x*, but has a constant phase, except across the nodes where it jumps 180 degrees.
  - pressure and velocity have a 90 degree phase difference, as opposed to being in-phase for a single plane wave.

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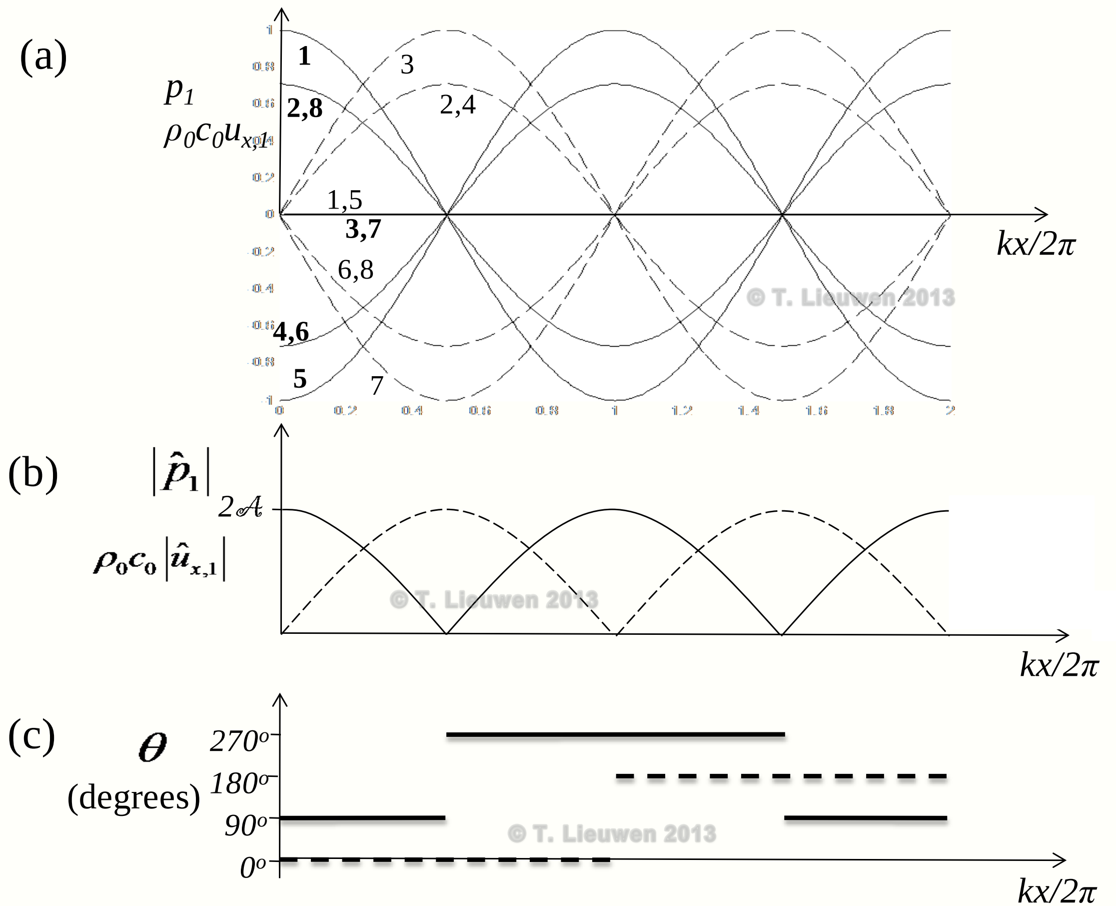

Spatial dependence of pressure (solid) and velocity (dashed) in a standing wave.

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One Dimensional Natural Modes

- Consider a duct of length *L* with rigid boundaries at both ends, $u_{x,1}(x=0,t)=u_{x,1}(x=L,t)=0$. Applying the boundary condition at x=0 implies that $\mathscr{A}=\mathscr{B}$, leading to:

$$u_{x,1}=\frac{1}{\rho_0 c_0}\mathrm{Real}\left(2i\mathscr{A}\sin(kx)\exp(-i\omega t)\right) \tag{5.62}$$

$$k=\frac{n\pi}{L} \tag{5.63}$$

$$f_n=\frac{\omega}{2\pi}=\frac{kc_0}{2\pi}=\frac{nc_0}{2L} \tag{5.64}$$

- These natural frequencies are integer multiples of each other, i.e., f_2=2f_1, f_3=3f_1, etc.

$$p_1(x,t)=2\mathscr{A}\cos\left(\frac{n\pi x}{L}\right)\cos(\omega t) \tag{5.65}$$

$$u_{x,1}(x,t)=\frac{2\mathscr{A}}{\rho_0 c_0}\sin\left(\frac{n\pi x}{L}\right)\sin(\omega t) \tag{5.66}$$

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Unsteady Heat Release Effects

- Oscillations in heat release generate acoustic waves.
  - For unconfined flames, this is manifested as broadband noise emitted by turbulent flames.
  - For confined flames, these oscillations generally manifest themselves as discrete tones at the natural acoustic modes of the system.

- The fundamental mechanism for sound generation is the unsteady gas expansion as the mixture reacts.

- To illustrate, consider the wave equation with unsteady heat release

$$\frac{\partial^2 p_1}{\partial t^2} - c_0^2 \nabla^2 p_1 = (\gamma - 1)\frac{\partial \dot{q}_1}{\partial t} + \gamma p_0 \left(\frac{\dot{n}}{n}\right)_1 \qquad (57)$$

- The two acoustic source terms describe sound wave production associated with unsteady gas expansion, due to either heat release (first term) or non isomolar combustion (second term).

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Unsteady Heat Release Effects – Confined Flames

- Unsteady heat release from an acoustically compact flame induces a jump in unsteady acoustic velocity, but no change in pressure:

$$u_{x,1b} - u_{x,1a} = \frac{1}{A}\frac{\gamma - 1}{\gamma p_0}\dot{Q}_1 \qquad (61)$$

$$p_{1b} = p_{1a} \qquad (62)$$

- Unsteady heat release causes both amplification/damping and shifts in phase of sound waves traversing the flame zone.
  - The relative significance of these two effects depends upon the relative phase of the unsteady pressure and heat release.
    - The first effect is typically the most important and can cause systems with unsteady heat release to exhibit self-excited oscillations.
    - The second effect causes shifts in natural frequencies of the system.

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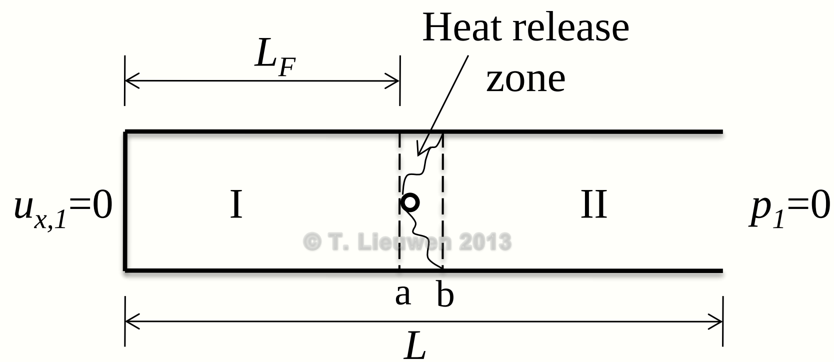
Combustor Stability Model Problem

- Assume rigid and pressure release boundary conditions at *x=0* and *x=L*, respectively, and that the flame is acoustically compact and located at $x=L_F$.



- Unsteady pressure and velocity in the two regions are given by:

Region I:

$$p_I(x,t)=\left(\mathcal{A}_I e^{ik_I(x-L_F)}+\mathcal{B}_I e^{-ik_I(x-L_F)}\right)e^{-i\omega t} \tag{63}$$

$$u_{x,1I}(x,t)=\frac{1}{\rho_{0I}c_{0I}}\left(\mathcal{A}_I e^{ik_I(x-L_F)}-\mathcal{B}_I e^{-ik_I(x-L_F)}\right)e^{-i\omega t}$$

Region II:

$$p_{II}(x,t)=\left(\mathcal{A}_{II} e^{ik_{II}(x-L_F)}+\mathcal{B}_{II} e^{-ik_{II}(x-L_F)}\right)e^{-i\omega t} \tag{64}$$

$$u_{II}(x,t)=\frac{1}{\rho_{0,II}c_{0,II}}\left(\mathcal{A}_{II} e^{ik_{II}(x-L_F)}-\mathcal{B}_{II} e^{-ik_{II}(x-L_F)}\right)e^{-i\omega t}$$

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Matching Conditions

- Applying the boundary/matching conditions leads to the following algebraic equations:

(Left BC) $$\mathscr{A}_I e^{-ik_I L_F} - \mathscr{B}_I e^{ik_I L_F} = 0$$

(Right BC) $$\mathscr{A}_{II} e^{ik_{II}(L-L_F)} + \mathscr{B}_{II} e^{-ik_{II}(L-L_F)} = 0$$

(Pressure Matching) $$\mathscr{A}_I + \mathscr{B}_I = \mathscr{A}_{II} + \mathscr{B}_{II}$$

(Velocity Matching) $$u_{II}\left(L_F^{\,+}, t\right) - u_I\left(L_F^{\,-}, t\right) = (\gamma - 1)\frac{\dot{Q}_1}{\rho_{0,I} c_{0,I}^{\;2}}$$

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Unsteady Heat Release Model

- Model for unsteady heat release is the most challenging aspect of combustion instability prediction

- We will used a “velocity coupled” flame response model

- Assumes that the unsteady heat release is proportional to the unsteady flow velocity, multiplied by the gain factor, n, and delayed in time by the time delay, τ.

$$\dot{Q}_1 = A \frac{\rho_{0,I} c_{0,I}^{\;2}}{\gamma - 1} n u_1 \left(x = L_F , t - \tau \right)$$

- This time delay could originate from, for example, the convection time associated with a vortex that is excited by the sound waves.

Summerfield, M., "A Theory of Unstable Combustion in Liquid Propellant Rocket Systems," Journal of the American Rocket Society, September 1951, Vol. 21 No. 5, pp. 108-114.

A Theory of Unstable Combustion in Liquid Propellant Rocket Systems

By MARTIN SUMMERFIELD

Member ARS, General Editor, Aeronautics Publication Program, Princeton University, Princeton, N. J.

On the basis of an hypothesis that low-frequency oscillations (chugging) sometimes observed in liquid propellant rocket engines, are the result of oscillatory propellant flow induced by a combustion time lag, conditions for the suppression of such oscillations are derived. It is found that stability can be achieved by increases in the length of feed line, the velocity of the propellant in the feed line, the ratio of feed pressure to chamber pressure, and the ratio of chamber volume to nozzle area. Equations are given for the frequencies of oscillation. Examination of the equation for stability indicates that self-igniting propellant combinations are likely to be more stable than non-self-igniting systems.

However, in many cases it has been reported that the desired steady operation does not occur in actual test, and instead a condition variously described as “rough burning,” “chugging,” “screaming,” or simply unstable combustion may take place. Frequencies ranging from 10 cycles per sec to as much as 5000 cycles per sec have been observed in oscillographic chamber pressure traces, amplitudes from a few per cent to as much as 50 per cent of the mean chamber pressure, and in many cases, the oscillation was not truly periodic but seemed to be merely a series of random fluctuations. Not

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Heat Release Modeling Simplifications

- Solving the four boundary/matching conditions leads to:

$$\begin{pmatrix} \dfrac{\rho_{0,II}c_{0,II}}{\rho_{0,I}c_{0,I}}\cos\left(k_I L_F\right)\cos\left(k_{II}\left(L-L_F\right)\right) \\ -\left(1+\varkappa e^{i\omega\tau}\right)\sin\left(k_I L_F\right)\sin\left(k_{II}\left(L-L_F\right)\right) \end{pmatrix} = 0$$

- In order to obtain analytic solutions, we will next assume that the flame is located at the midpoint of the duct, i.e., $L=2L_F$ and that the temperature jump across the flame is negligible.

$$\cos kL = \varkappa e^{i\omega\tau}\sin^2\left(kL/2\right) \qquad (72)$$

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Approximate Solution for Small Gain Values

- We can expand the solution around the $\varkappa = 0$ solution by looking at a Taylor series in wavenumber k in the limit $\varkappa << 1$.

$$k = k_{\varkappa=0}\left(1 - \frac{\varkappa \exp\left(i\omega_{\varkappa=0}\tau\right)}{2k_{\varkappa=0}L \sin k_{\varkappa=0}L} + O\left(\varkappa^2\right)\right)$$

$$k_{\varkappa=0}L = \frac{(2n-1)\pi}{2}$$

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Complex Wavenumbers

- Wavenumber and frequency have an imaginary component,. Considering the time component and expanding ω:

$$\exp(-i\omega t) = \exp(-i\omega_r t)\exp(\omega_i t) \qquad (75)$$

- Real component is the frequency of combustor response.

$$\omega_r = \omega_{\varkappa=0}\left[1 + (-1)^n \frac{\varkappa\cos(\omega_{\varkappa=0}\tau)}{(2n-1)\pi}\right]$$

- Imaginary component corresponds to exponential growth or decay in time and space. Thus, ω_i>0 corresponds to a linearly unstable situation, referred to as combustion instability.

$$\omega_i = (-1)^n \frac{\varkappa c_0 \sin(\omega_{\varkappa=0}\tau)}{2L} \qquad (77)$$

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- *Rayleigh's criterion*: Energy is added to the acoustic field when the product of the time averaged unsteady pressure and heat release is greater than zero.

$$p_1\left(x=L/2,t\right)\dot{Q}_1(t)\propto\frac{\varkappa}{2}\sin\left(\frac{\left(2n-1\right)\pi}{2}\right)\cos\left(\omega_{\varkappa=0}t\right)\sin\left(\omega_{\varkappa=0}(t-\tau)\right)$$

$$\overline{p_1(x=L/2,t)\dot{Q}_1(t)}\propto\frac{\varkappa}{4}\sin\left(\frac{\left(2n-1\right)\pi}{2}\right)\sin\left(\omega_{\varkappa=0}\tau\right)$$

$$\overline{p_1(x=L/2,t)\dot{Q}_1(t)}>0\Rightarrow\left(-1\right)^{n+1}\sin\left(\omega_{\varkappa=0}\tau\right)>0$$

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Stability of Combustor Modes

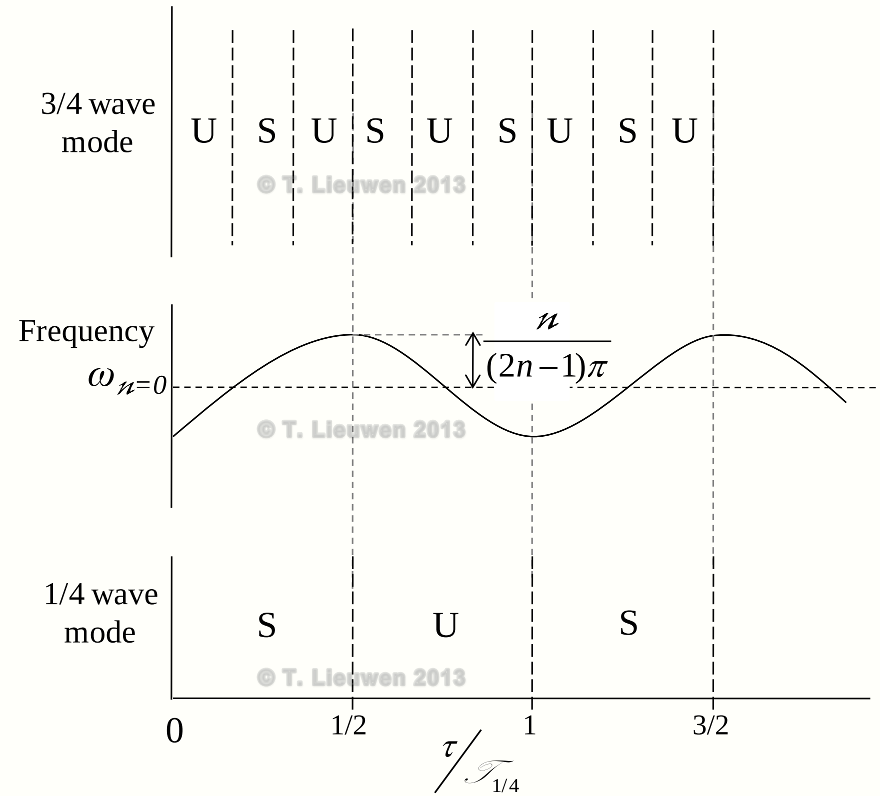
- Unstable 1/4 wave mode (n=1)

$$m - 1/2 < \frac{\tau}{\mathscr{T}_{1/4}} < m$$

- Unstable 3/4 wave mode (n=2)

$$\frac{m}{3} < \frac{\tau}{\mathscr{T}_{1/4}} < \frac{m + 1/2}{3}$$

- Sign of $\overline{p_1 \dot{Q}_1}$ alternates with time delay
  - Important implications on why instability prediction is so difficult- no monotonic dependence upon underlying parameters



- Largest frequency shifts occur at the values where oscillations are not amplified and that the center of instability bands coincides with points of no frequency shift.

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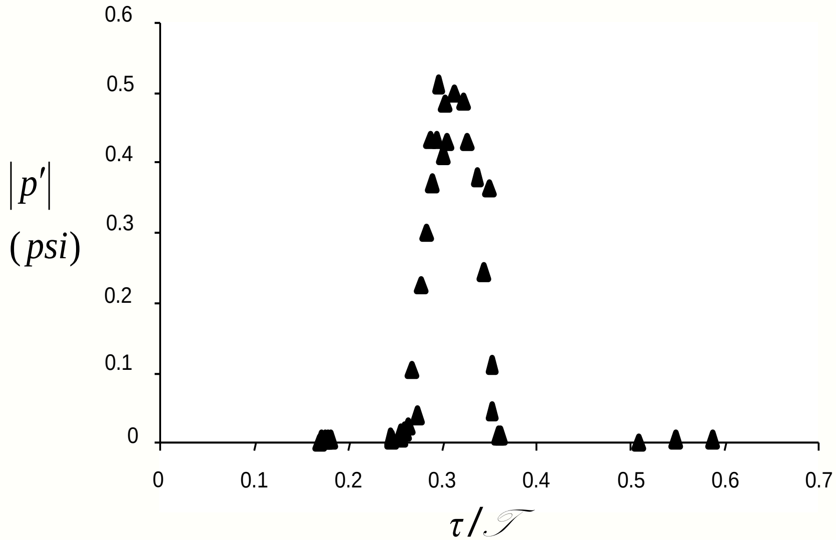
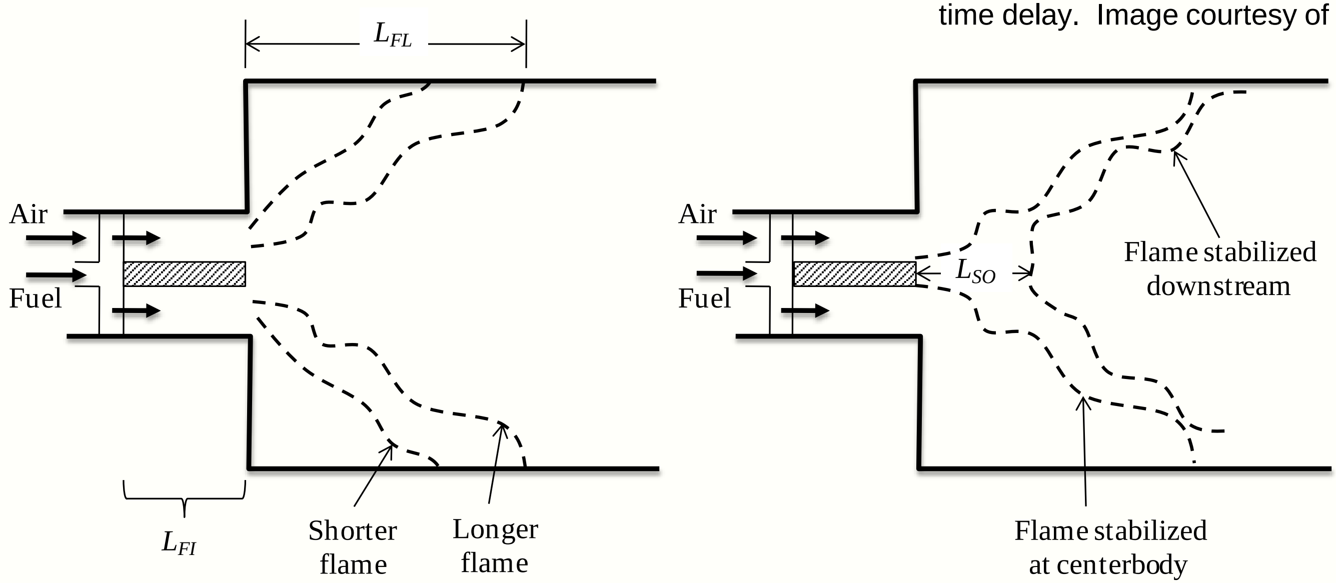
Thermo-acoustic Instability Trends

- Two parameters, the heat release time delay, τ, and acoustic period, $\mathscr{T}$, control instability conditions.
- Data clearly illustrate the non-monotonic variation of instability amplitude with $\tau/\mathscr{T}$.



Data illustrating variation of instability amplitude with normalized time delay. Image courtesy of D. Santavicca



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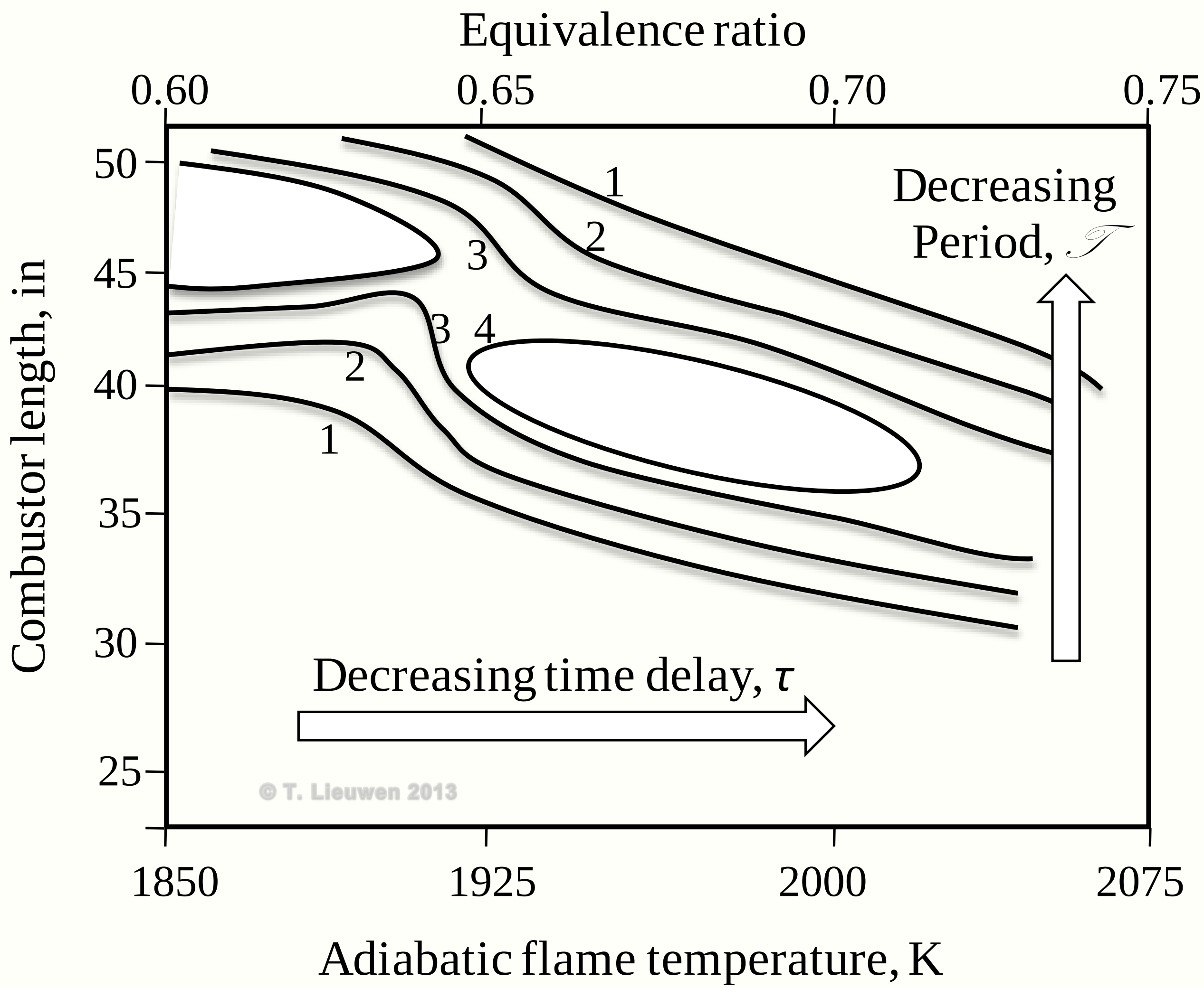
- Data from variable length combustor



Measured instability amplitude (in psi) of combustor as a function of fuel/air ratio and combustor length. Data courtesy of D. Santavicca.

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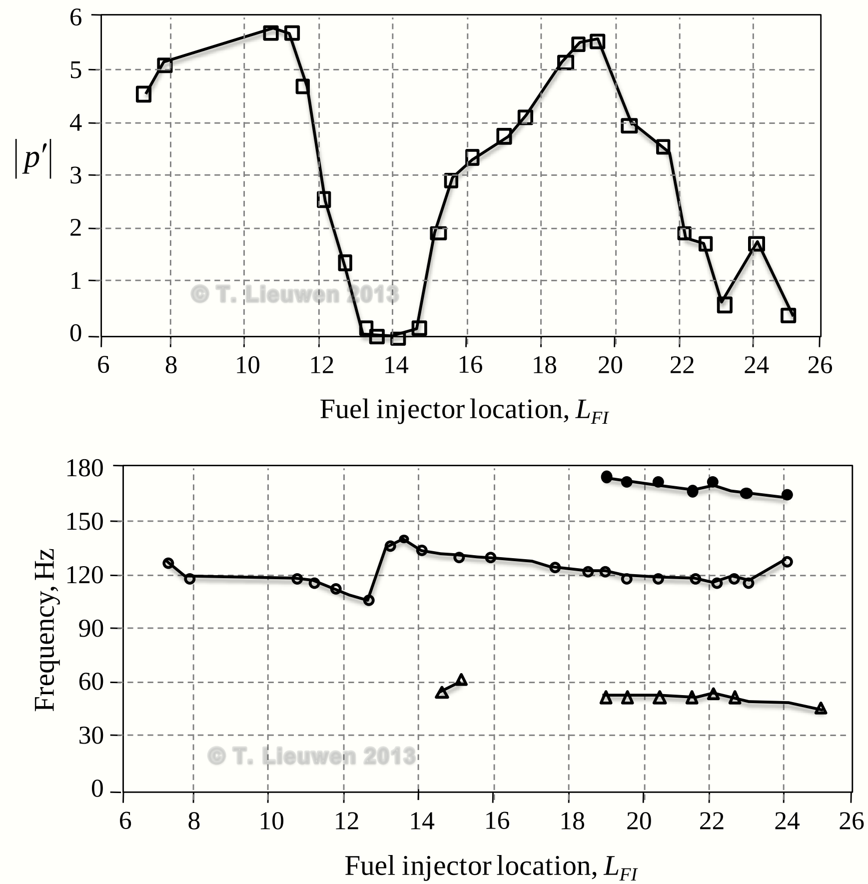
More Instability Trends 1 of 2

- Note non-monotonic variation of instability amplitude with axial injector location, due to the more fundamental variation of fuel convection time delay, τ.

- Biggest change in frequency is observed near the stability boundary.



Measured dependence of instability amplitude and frequency upon axial location of fuel injector. Data obtained from Lovett and Uznanski.

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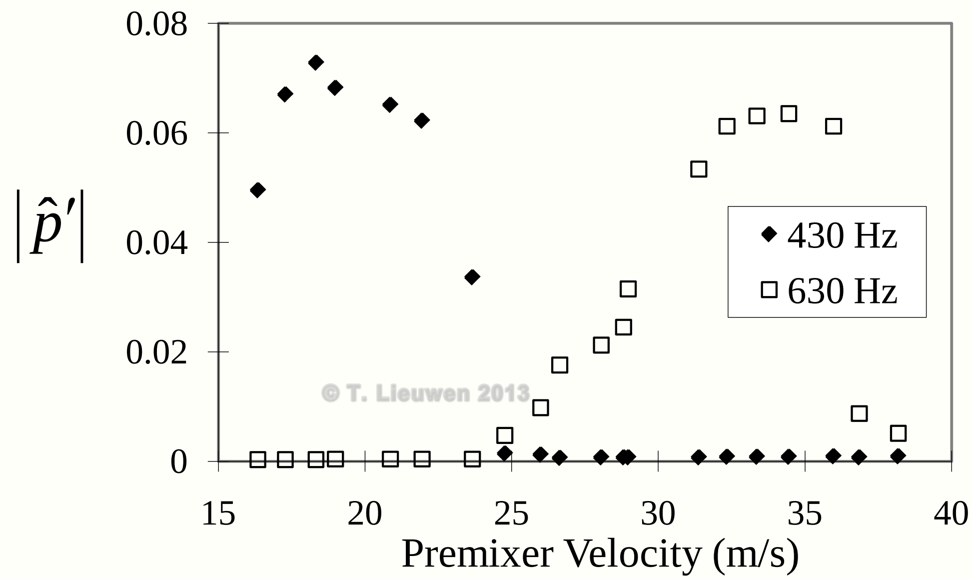

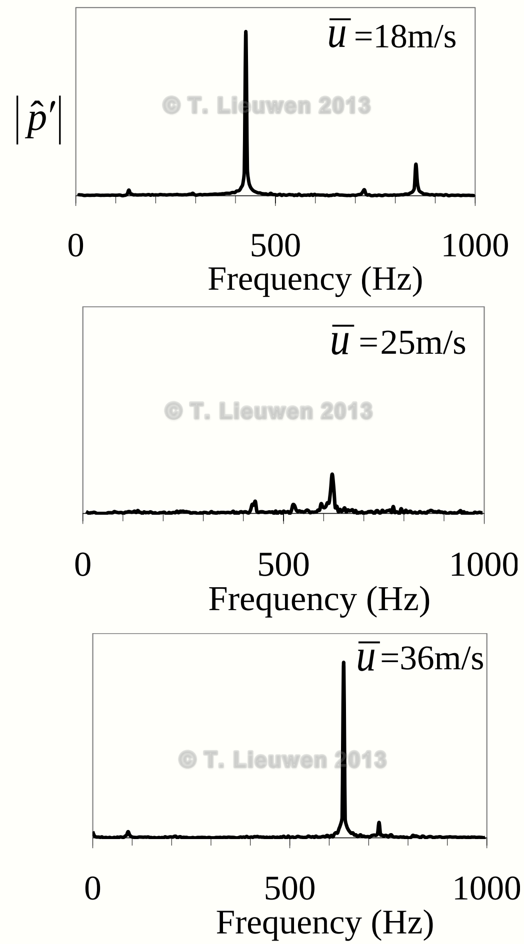

Measured dependence of the excited instability mode amplitude upon the mean velocity in the combustor inlet. Obtained from measurements by the author.

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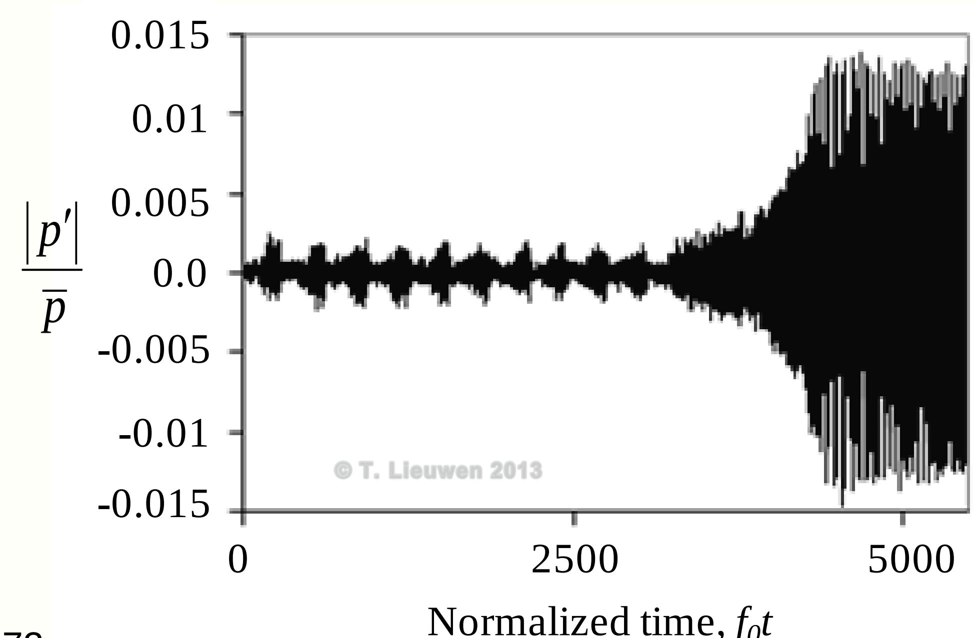
Limit Cycles and Nonlinear Behavior

- As amplitudes grow nonlinear effects grow in significance and the system is attracted to a new orbit in phase space, typically a limit cycle.

- This limit cycle oscillation can consist of relatively simple oscillations at some nearly constant amplitude, but in real combustors the amplitude more commonly "breathes" up and down in a somewhat random or quasi-periodic fashion.



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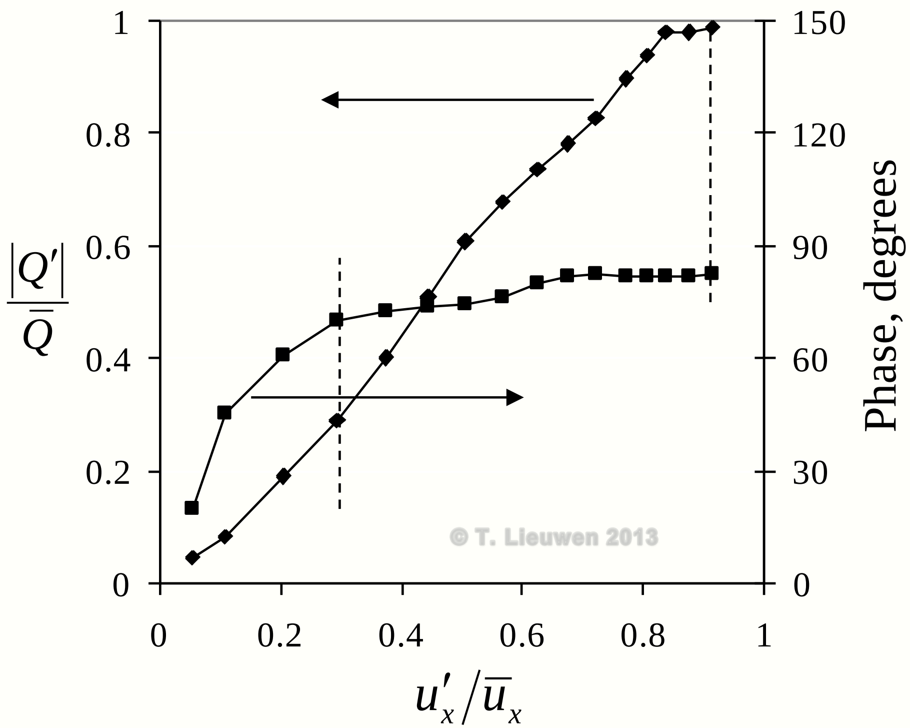
Combustion Process and Gas Dynamic Nonlinearities

- Gas dynamic nonlinearities introduced by nonlinearities present in Navier-Stokes equations
- Combustion process nonlinearities are introduced by the nonlinear dependence of the heat release oscillations upon the acoustic disturbance amplitude.



Dependence of unsteady heat release magnitude and phase upon velocity disturbance amplitude. Graph generated from data obtained by Bellows *et al*.

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Boundary Induced Nonlinearities

- The nonlinearities in processes that occur at or near the combustor boundaries also affect the combustor dynamics as they are introduced into the analysis of the problem through nonlinear boundary conditions.
- Such nonlinearities are caused by, e.g., flow separation at sharp edges or rapid expansions, which cause stagnation pressure losses and a corresponding transfer of acoustic energy into vorticity.
- These nonlinearities become significant when $u'/\bar{u} \sim \mathrm{O}(1)$.
- Also, wave reflection and transmission processes through choked and unchoked nozzles become amplitude dependent at large amplitudes.

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