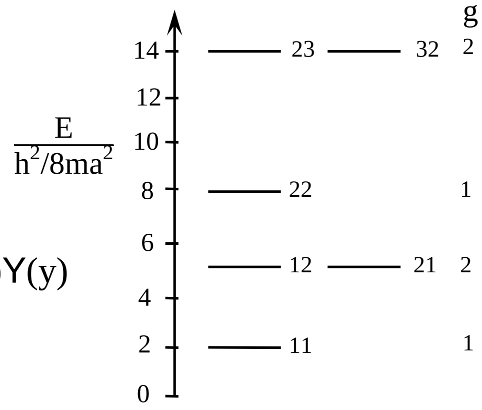
Particle in a 2-Dimensional Box

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\Psi + \frac{\partial^2}{\partial y^2}\Psi\right) + V(x,y)\Psi = E\Psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(x,y)\Psi = E\Psi$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\Psi + \frac{\partial^2}{\partial y^2}\Psi\right) = E\,\Psi \qquad \Psi(x,y) = X(x)Y(y)$$

for $0 \le x \le a$ and $0 \le y \le b$



$$\frac{\partial^2}{\partial x^2}\Psi = Y(y)\,\frac{d^2X(x)}{dx^2}$$

$$-\frac{\hbar^2}{2m}\left(Y(y)\,\frac{d^2X(x)}{dx^2} + X(x)\,\frac{d^2Y(y)}{dy^2}\right) = E\,X(x)Y(y)$$

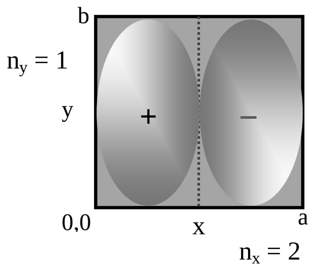
$$-\frac{\hbar^2}{2m}\left(\frac{1}{X(x)}\,\frac{d^2X(x)}{dx^2} + \frac{1}{Y(y)}\,\frac{d^2Y(y)}{dy^2}\right) = E = ?$$

$$-\frac{\hbar^2}{2m}\,\frac{1}{X(x)}\,\frac{d^2X(x)}{dx^2} = E_x \qquad -\frac{\hbar^2}{2m}\,\frac{1}{Y(y)}\,\frac{d^2Y(y)}{dy^2} = E_y$$

$$-\frac{\hbar^2}{2m}\,\frac{d^2X(x)}{dx^2} = E_x\,X(x) \qquad -\frac{\hbar^2}{2m}\,\frac{d^2Y(y)}{dy^2} = E_y\,Y(y)$$

$$X(x) = \left(\frac{2}{a}\right)^{1/2}\sin\frac{n_x\pi x}{a} \qquad E_x = \frac{h^2}{8m}\,\frac{n_x^2}{a^2} \qquad Y(y) = \left(\frac{2}{b}\right)^{1/2}\sin\frac{n_y\pi y}{b} \qquad E_y = \frac{h^2}{8m}\,\frac{n_y^2}{b^2}$$

$$\Psi(x,y) = \left(\frac{4}{ab}\right)^{1/2}\sin\frac{n_x\pi x}{a}\,\sin\frac{n_y\pi y}{b}$$



$$E = E_x + E_y = \frac{h^2}{8m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$

square box: $E = \frac{h^2}{8ma^2}\,(n_x^2 + n_y^2)$