Turbulent Non-Premixed Combustion

Combustion Summer School

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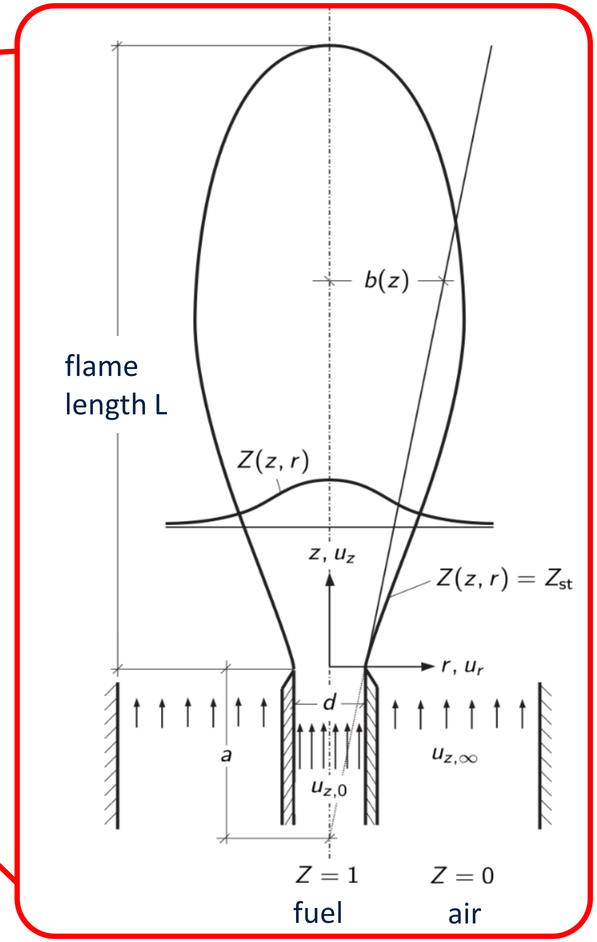
Course Overview

Part II: Turbulent Combustion

- Turbulence
- Turbulent Premixed Combustion
- Turbulent Non-Premixed Combustion →
  - Laminar Jet Diffusion Flames
  - Turbulent Jet Diffusion Flames
- Turbulent Combustion Modeling
- Applications

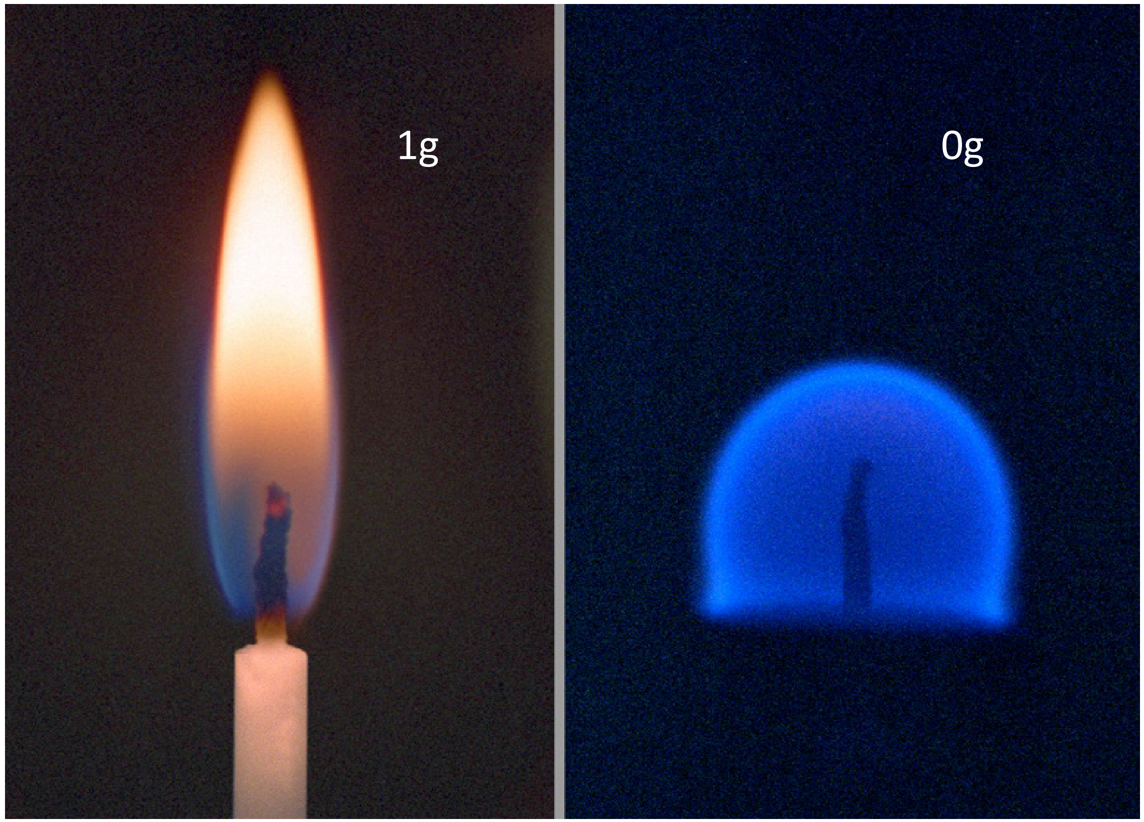
Laminar Jet Diffusion Flames



Laminar Jet Diffusion Flame

- Fuel enters into the combustion chamber as a round jet
- Forming mixture is ignited
- Example: Flame of a gas lighter
  - Only stable if dimensions are small
  - Dimensions too large: flickering due to influence of gravity
  - Increasing the jet momentum → Reduction of the relative importance of gravity (buoyancy) in favor of momentum forces
  - At high velocities, hydrodynamic instabilities gain increasing importance: laminar-turbulent transition

Laminar Diffusion Fame: Influence of Gravity



Laminar Jet Diffusion Flame (Governing Equations)

- Starting point: Conservation equations for stationary axisymmetric boundary layer flow without buoyancy
- Continuity:

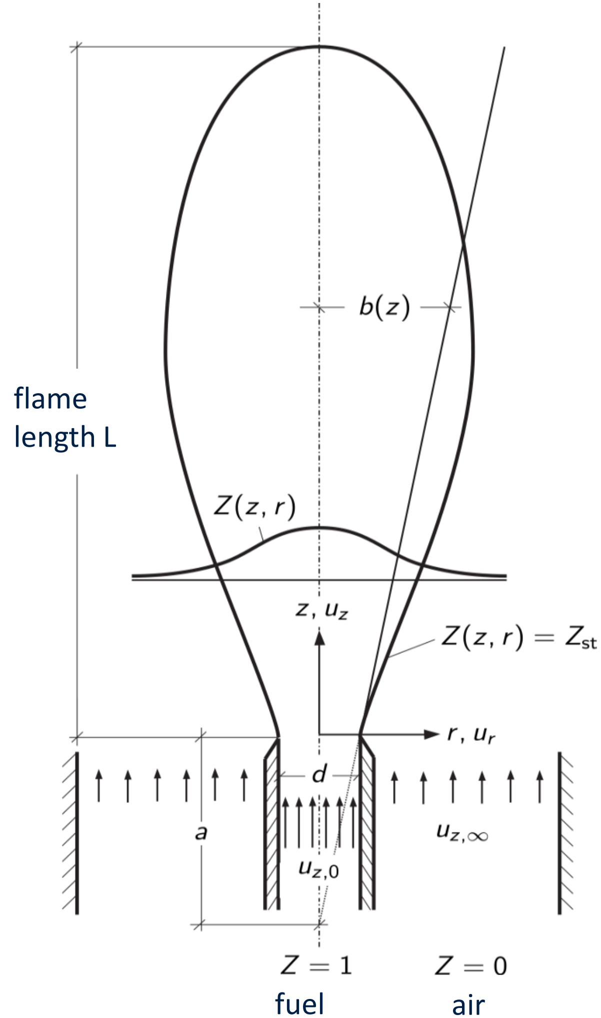
$$\frac{\partial(\rho u_z r)}{\partial z} + \frac{\partial(\rho u_r r)}{\partial r} = 0$$

- Momentum equation in *z*-direction

$$\rho u_z r \frac{\partial u_z}{\partial z} + \rho u_r r \frac{\partial u_z}{\partial r} = -r \frac{\partial p}{\partial z} + \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_z}{\partial r}\right)$$

- Mixture fraction

$$\rho u_z r \frac{\partial Z}{\partial z} + \rho u_r r \frac{\partial Z}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\mu}{Sc} r \frac{\partial Z}{\partial r}\right)$$



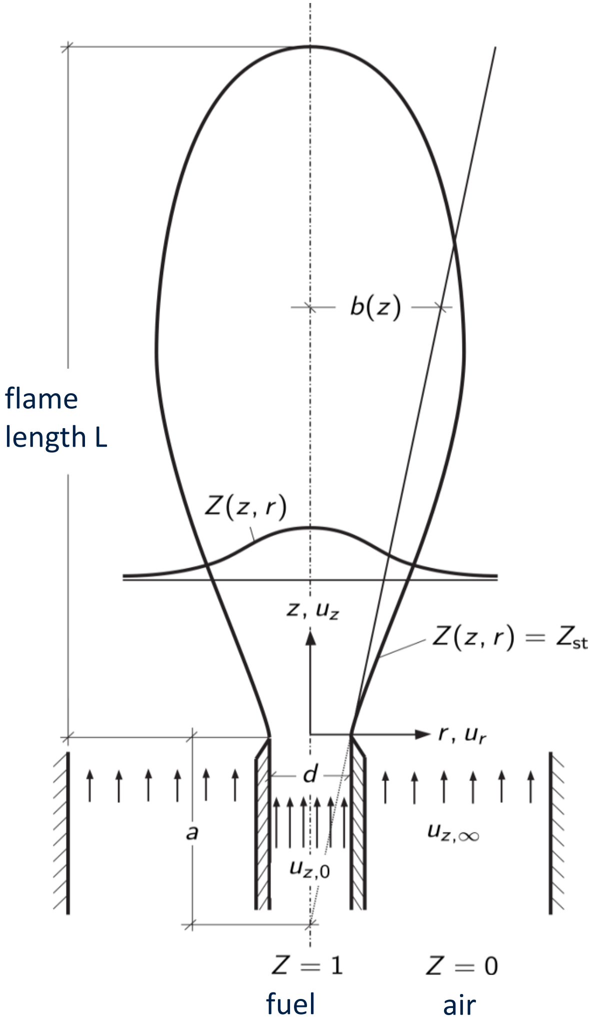
Laminar Jet Diffusion Flame (Assumptions + BC)

- Schmidt number $Sc = \mu/\rho D$
- Farfield area
  - $r \rightarrow \infty$: $u_z = u_r = 0$
  - From z-momentum equation → $dp/dz = 0$
- Boundary layer flow:

$$\rho u_z r \frac{\partial u_z}{\partial z} + \rho u_r r \frac{\partial u_z}{\partial r} = -r \underbrace{\cancel{\frac{\partial p}{\partial z}}}_{0} + \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_z}{\partial r}\right)$$

- Incompressible round jet
  - Quiescent ambient
  - Constant density
  - No buoyancy

  → Similarity solution
- Simularity coordinate $\eta = r/z$
  (Schlichting, „Boundary Layer Theory“)



Laminar Jet Diffusion Flame (Similarity Coordinates)

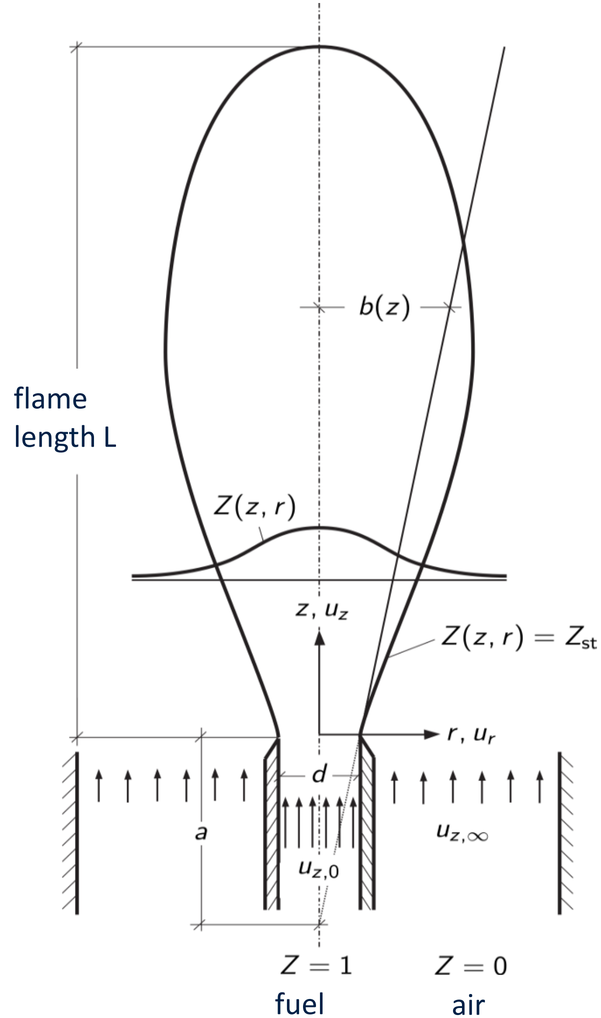
- If density not constant
  → Transformation

$$\zeta = z + a, \qquad \eta = \frac{\sqrt{2\int\limits_0^r \frac{\rho}{\rho_\infty} r dr}}{\zeta}$$

- a: Distance of the virtual origin of the jet from the nozzle exit
- For ρ = const. und $a \to 0$

$$\zeta = z, \qquad \eta = \frac{r}{z}$$

- Implies linear spreading of the roung jet



Laminar Jet Diffusion Flame (Stream Function)

- Introduction of a stream function Ψ

$$\rho u_z r = \frac{\partial \Psi}{\partial r}, \quad \rho u_r r = -\frac{\partial \Psi}{\partial z}$$

→ Continuity equation identically satisfied

- Applying the transformation rules

$$\zeta = z + a, \quad \eta = \frac{\sqrt{2\int\limits_0^r \frac{\rho}{\rho_\infty} r \mathrm{d}r}}{\zeta} \quad \rightarrow \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \zeta} + \frac{\partial \eta}{\partial z}\frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r}\frac{\partial}{\partial \eta}$$

to the convective terms in the momentum and mixture fraction equations yields

$$\rho u_z r \frac{\partial}{\partial z} + \rho u_r r \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r}\left(\frac{\partial \Psi}{\partial \eta}\frac{\partial}{\partial \zeta} - \frac{\partial \Psi}{\partial \zeta}\frac{\partial}{\partial \eta}\right)$$

Laminar Jet Diffusion Flame (Transformation Rules)

- Applying the transformation rules

$$\zeta = z + a, \quad \eta = \frac{\sqrt{2\int\limits_0^r \frac{\rho}{\rho_\infty} r\mathrm{d}r}}{\zeta} \quad \rightarrow \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \zeta} + \frac{\partial \eta}{\partial z}\frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r}\frac{\partial}{\partial \eta}$$

to the convective terms in the momentum and mixture fraction equations yields

$$\rho u_z r \frac{\partial}{\partial z} + \rho u_r r \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r}\left(\frac{\partial \Psi}{\partial \eta}\frac{\partial}{\partial \zeta} - \frac{\partial \Psi}{\partial \zeta}\frac{\partial}{\partial \eta}\right)$$

- The diffusive terms become

$$\frac{\partial}{\partial r}\left(\mu r \frac{\partial}{\partial r}\right) = \mu_\infty \frac{\partial \eta}{\partial r}\frac{\partial}{\partial \eta}\left(C\eta \frac{\partial}{\partial \eta}\right)$$

- C: Chapman-Rubesin-Parameter $\quad C = \frac{\rho \mu r^2}{2\mu_\infty \int\limits_0^r \rho r\mathrm{d}r}$

- For constant density (with $\eta = r/\zeta$ and $\mu = \mu_\infty$): $C = 1$

Laminar Jet Diffusion Flame (Non-dim. Stream Func.)

- Formal transformation of the momentum and concentration equations and assumption that $C = f(\zeta,\eta)$
- With ansatz for non-dimensional stream function F

$$\Psi = \mu_\infty \zeta F(\zeta, \eta)$$

for the velocities follows

$$u_z = \frac{\partial F / \partial \eta}{\eta} \frac{\mu_\infty}{\rho_\infty \zeta}, \qquad \rho u_r r = -\mu_\infty \left(\zeta \frac{\partial F}{\partial \zeta} + F - \eta \frac{\partial F}{\partial \eta} \right)$$

- u_z und u_r can be expressed as a function of the nondimensional stream function F and its derivatives

Laminar Jet Diffusion Flame (Transformation)

- From the momentum equation

$$\rho u_z r \frac{\partial u_z}{\partial z} + \rho u_r r \frac{\partial u_z}{\partial r} = \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_z}{\partial r}\right)$$

→

$$\zeta \left(\frac{\partial F/\partial \eta}{\eta} \frac{\partial}{\partial \zeta} \frac{\partial F}{\partial \eta} - \frac{\partial F}{\partial \zeta} \frac{\partial}{\partial \eta} \frac{\partial F/\partial \eta}{\eta} \right) - \frac{\partial}{\partial \eta}\left(F \frac{\partial F/\partial \eta}{\eta} \right) = \frac{\partial}{\partial \eta}\left(C\eta \frac{\partial}{\partial \eta} \frac{\partial F/\partial \eta}{\eta} \right)$$

- Similarity solution only exists, if $F \neq f(\zeta)$
- Then, u_z is proportional to $1/\zeta$ (see previous slide)
  → velocity decreases linearly with $1/(z + a)$
- Prerequesites: Boundary conditions and C are independent of z
  (e. g. $u_z = 0$ and $u_r = 0$ for $\eta \to 0$)

Laminar Jet Diffusion Flame (Resulting Equations)

- Equation for the nondimensional stream function

$$\boxed{-\frac{\partial}{\partial \eta}\left(F\frac{\partial F/\partial \eta}{\eta}\right) = \frac{\partial}{\partial \eta}\left(C\eta\frac{\partial}{\partial \eta}\frac{\partial F/\partial \eta}{\eta}\right)}$$

- Let $\omega = Z(z,r)/Z_a(z)$, ratio of the mixture fraction $Z_a(z)$ to its value at $r = 0$
- Applying the same transformations to the ω-equation yields

$$\zeta\left(\frac{\partial F}{\partial \eta}\frac{\partial \omega}{\partial \zeta} - \frac{\partial F}{\partial \zeta}\frac{\partial \omega}{\partial \eta}\right) + \zeta\frac{\partial F}{\partial \eta}\omega\frac{\partial \ln(Z_a)}{\partial \zeta} - F\frac{\partial \omega}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{C}{Sc}\eta\frac{\partial \omega}{\partial \eta}\right)$$

- In case of a similarity solution

$$\boxed{-F\frac{\partial \omega}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{C}{Sc}\eta\frac{\partial \omega}{\partial \eta}\right)}$$

Laminar Jet Diffusion Flame (Analytic Solution)

- Integration for C = const. yields:

$$F = \frac{C(\gamma\eta)^2}{1+(\gamma\eta)^2/4}, \qquad \omega = \left(\frac{1}{1+(\gamma\eta)^2/4}\right)^{2Sc}$$

where γ is integration constant

- The assumption C = const. Holds if

$$C = \frac{\rho\mu r^2}{2\mu_\infty \int\limits_0^r \rho r dr} \quad \rightarrow \quad C = \frac{\rho\mu}{\rho_m \mu_\infty}$$

and $\rho\mu/\rho_m\mu_\infty$ = const.

- C = const. Often not a good assumption, since $\mu \sim T^{0,7}$ und $\rho \sim T^{-1}$

Laminar Jet Diffusion Flame (Integration Constant γ)

- Constant of integration γ can be determined from the condition that the jet momentum is independent of ζ
- Substitution of the solution into the momentum balance

$$\int_0^\infty \rho u_z^2 r \mathrm{d} r = \rho_0 u_{z,0}^2 \frac{d^2}{8}$$

yields

$$\gamma^2 = \frac{3}{64} \frac{\rho_0}{\rho_\infty} \frac{Re^2}{C^2}$$

- ρ_0: density of the fuel stream
- Reynolds number $Re = u_{z,0} d / \nu_\infty$

Laminar Jet Diffusion Flame (Centerline Mixt. Fraction)

- Analogously for the mixture fraction (with $Z_0 = 1$)

$$\int_0^\infty \rho u_z Z \, r \mathrm{d}r = \rho_0 u_{z,0} \frac{d^2}{8}$$

→ Mixture fraction on the centerline $Z_\mathrm{a}(z) = Z(z, r=0)$:

$$Z_\mathrm{a}(z) = \frac{1 + 2Sc}{32} \frac{\rho_0}{\rho_\infty} \frac{Re}{C} \frac{d}{\zeta}$$

→ Z_a decreases with $1/\zeta$ (as the velocity)

Laminar Jet Diffusion Flame (Flame Length)

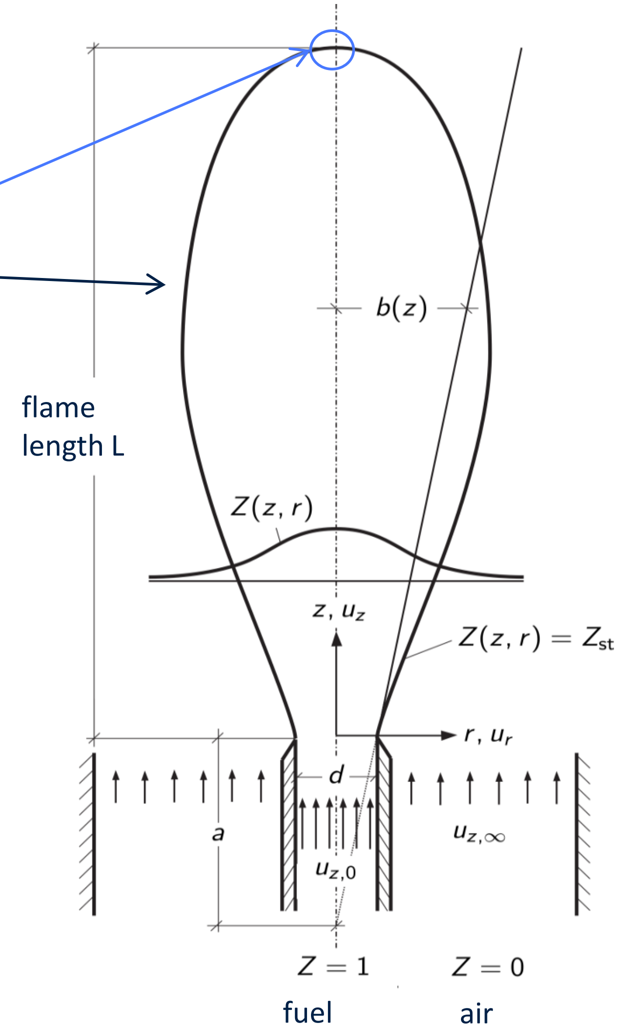
- Determination of the flame contour r as function of z from the condition

$$Z(z, r) = Z_a \omega(\eta) = Z_{st}$$

- Flame contour intersects centerline, $r = 0$, if $Z_a = Z_{st}$
- Corresponding value of z defines the flame length

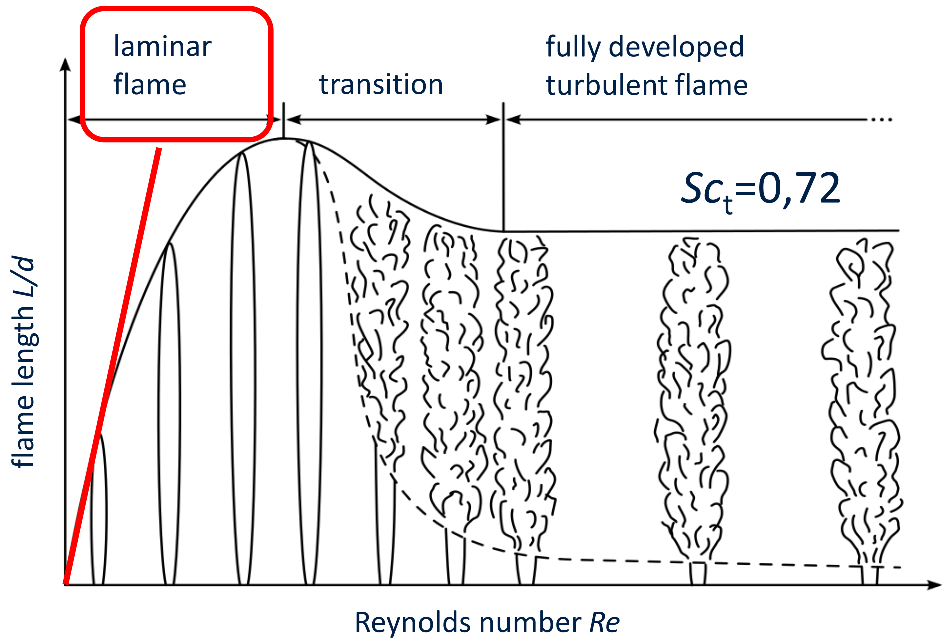
$$Z_a(z) = \frac{1+2Sc}{32} \frac{\rho_0}{\rho_\infty} \frac{Re}{C} \frac{d}{\zeta} \quad \rightarrow \quad L = \frac{1+2Sc}{32 Z_{st}} \frac{\rho_0}{\rho_\infty C} \frac{u_0 d^2}{\nu} - a$$

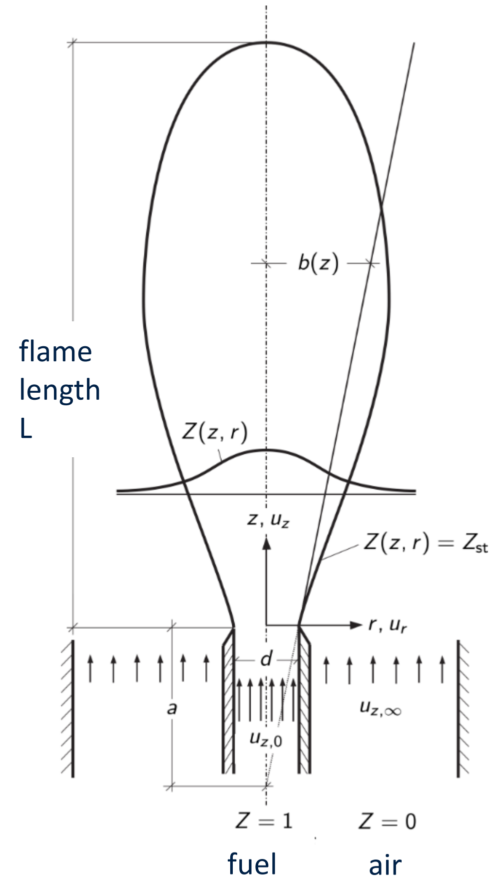
- Valid for laminar jet flames without buoyancy



Laminar Jet Diffusion Flame

- For a given nozzle diameter,
  L increases linearly with the Reynolds number *Re*





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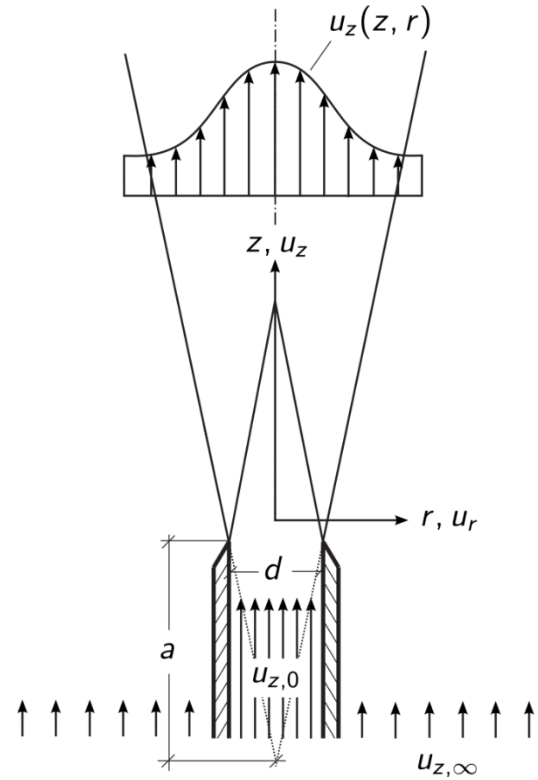
Turbulent Jet Diffusion Flame

- Shear flow at nozzle exit
- Flow instabilities
  (Kelvin-Helmholtz-instabilities) → laminar-turbulent transition
- Ring shaped turbulent shear layer propagates in radial direction
- Merging after 10 to 15 nozzle diameters downstream
- Streamlines are parallel in potential core
- Velocity profile reaches self similar state after 20-30 nozzle diameters

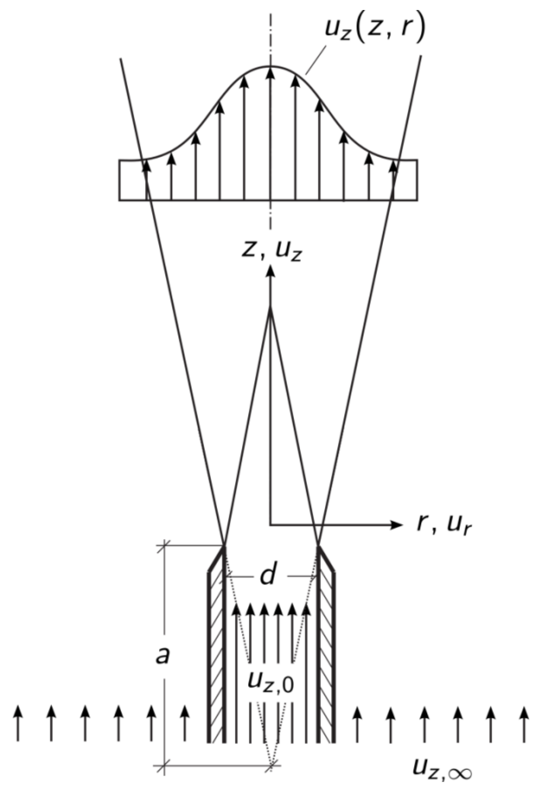


Round Turbulent Diffusion Flame

- Linear reduction of velocity along central axis
- Linear increase of jet width
- Assumption: fast chemical reaction

  ⟶ Scalar quantities such as temperature, concentration and density as function of mixture fraction Z
- Turbulent flow with variable density

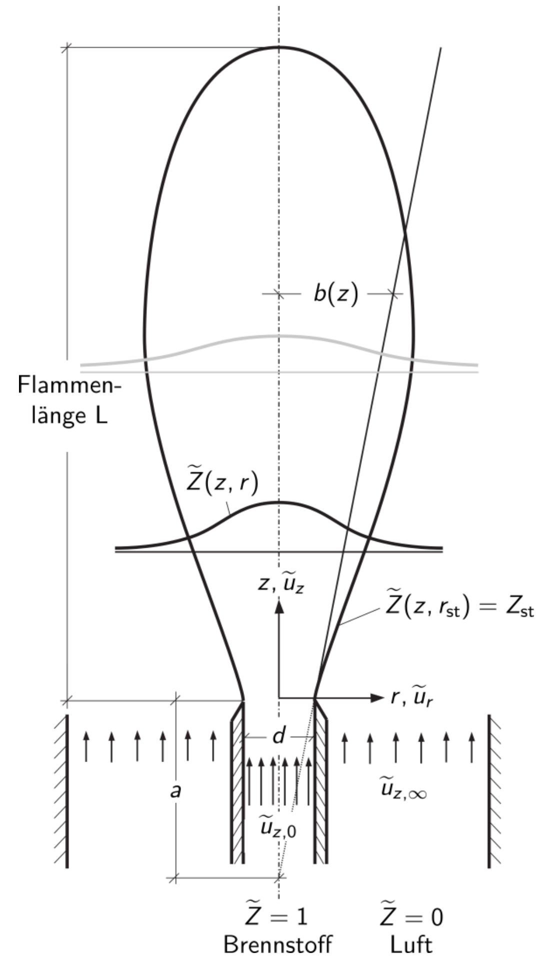
  ⟶ Favre-averaged boundary layer equations



Linear Propagation of (turbulent) Jet

Round Turbulent Diffusion Flame

- Assumptions:
  - Axisymmetric jet flame
  - Neglecting buoyancy
  - Neglecting molecular transport as compared to turbulent transport
  - Turbulent transport modeled by Gradient Transport model
  - $Sc_t = \nu_t / D_t$
- Using Favre averaging and the the boundary layer assumption we obtain a system of two-dimensional axisymmetric equations



Round Turbulent Diffusion Flame

- Continuity equation

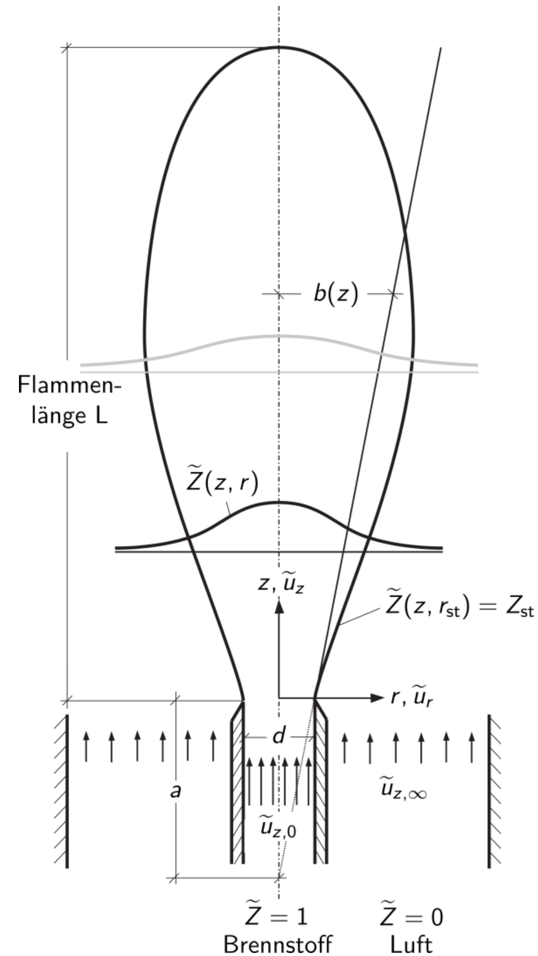
$$\frac{\partial\left(\overline{\rho}\widetilde{u}_z r\right)}{\partial z} + \frac{\partial\left(\overline{\rho}\widetilde{u}_r r\right)}{\partial r} = 0$$

- Momentum equation in *z*-direction

$$\overline{\rho}\widetilde{u}_z r\frac{\partial \widetilde{u}_z}{\partial z} + \overline{\rho}\widetilde{u}_r r\frac{\partial \widetilde{u}_z}{\partial r} = \frac{\partial}{\partial r}\left(\overline{\rho}\nu_\mathrm{t} r\frac{\partial \widetilde{u}_z}{\partial r}\right)$$

- Mean mixture fraction

$$\overline{\rho}\widetilde{u}_z r\frac{\partial \widetilde{Z}}{\partial z} + \overline{\rho}\widetilde{u}_r r\frac{\partial \widetilde{Z}}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\overline{\rho}\nu_\mathrm{t}}{Sc_\mathrm{t}} r\frac{\partial \widetilde{Z}}{\partial r}\right)$$



Round Turbulent Diffusion Flame

- Requires solving of equations for k and ε to determine ν_t
- Round turbulent jet: ν_t approximately constant
- Analogous for round laminar jet:

Laminar

$$\frac{\partial(\rho u_z r)}{\partial z} + \frac{\partial(\rho u_r r)}{\partial r} = 0$$

$$\rho u_z r \frac{\partial u_z}{\partial z} + \rho u_r r \frac{\partial u_z}{\partial r} = \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_z}{\partial r}\right)$$

$$\rho u_z r \frac{\partial Z}{\partial z} + \rho u_r r \frac{\partial Z}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\mu}{Sc} r \frac{\partial Z}{\partial r}\right)$$

Turbulent

$$\frac{\partial(\bar{\rho}\tilde{u}_z r)}{\partial z} + \frac{\partial(\bar{\rho}\tilde{u}_r r)}{\partial r} = 0$$

$$\bar{\rho}\tilde{u}_z r \frac{\partial \tilde{u}_z}{\partial z} + \bar{\rho}\tilde{u}_r r \frac{\partial \tilde{u}_z}{\partial r} = \frac{\partial}{\partial r}\left(\bar{\rho}\nu_t r \frac{\partial \tilde{u}_z}{\partial r}\right)$$

$$\bar{\rho}\tilde{u}_z r \frac{\partial \tilde{Z}}{\partial z} + \bar{\rho}\tilde{u}_r r \frac{\partial \tilde{Z}}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\bar{\rho}\nu_t}{Sc_t} r \frac{\partial \tilde{Z}}{\partial r}\right)$$

Round Turbulent Diffusion Flame

- Special case: Jet in quiescent ambient
  - Treatment of turbulent equations like those in a laminar round jet case
  - Using the laminar theory
- Similarity coordinate

Laminar

$$\eta = \frac{\sqrt{2\int\limits_0^r \frac{\rho}{\rho_\infty} r dr}}{z + a}$$

$$C = \frac{\rho \mu r^2}{2\mu_\infty \int\limits_0^r \rho r dr}$$

Turbulent

$$\eta = \frac{\sqrt{2\int\limits_0^r \frac{\bar{\rho}}{\rho_\infty} r dr}}{z + a}$$

$$C = \frac{\bar{\rho}^2 \nu_t r^2}{2\rho_\infty \nu_{t,\mathrm{ref}} \int\limits_0^r \bar{\rho} r dr}$$

- Chapman-Rubesin-Parameter

Round Turbulent Diffusion Flame

- Turbulent Chapman-Rubesin-Parameter approximately constant→

$$\widetilde{u}_z = \frac{2C\gamma^2 \nu_{\mathrm{t,ref}}}{\zeta \left(1 + (\gamma\eta)^2/4\right)^2}$$

- Integration constant γ, containing fuel density and reference viscosity

$$\gamma^2 = \frac{3}{64}\frac{\rho_0}{\rho_\infty C^2}\left(\frac{u_{z,0} d}{\nu_{\mathrm{t,ref}}}\right)^2 \qquad \left(\text{laminar:} \quad \gamma^2 = \frac{3}{64}\frac{\rho_0}{\rho_\infty}\frac{Re^2}{C^2}\right)$$

- The Favre-averaged velocity decreases proportional to $1/\zeta = 1/(z + a)$, just like in the laminar case

Round Turbulent Diffusion Flame

- Mean mixture fraction

$$\widetilde{Z} = \frac{\widetilde{Z}_\mathrm{a}}{\left(1 + (\gamma\eta)^2/4\right)^{2Sc_\mathrm{t}}}$$

with

$$\widetilde{Z}_\mathrm{a} = \frac{1+2Sc_\mathrm{t}}{32}\frac{\rho_0}{\rho_\infty C}\left(\frac{u_{z,0}d}{\nu_\mathrm{t,ref}}\right)\frac{d}{\zeta} \qquad \left(\text{laminar: } Z_\mathrm{a} = \frac{1+2Sc}{32}\frac{\rho_0}{\rho_\infty}\frac{Re}{C}\frac{d}{\zeta}\right)$$

→ Mixture fraction decreases proportional to $1/(z+a)$ on the jet axis

→ Progression of profiles along jet axis resembles those of the laminar case

- Also applies to the contour of the stoichiometric mixture

Round Turbulent Diffusion Flame

- Flame length L of round turbulent diffusion flame: Distance z from the nozzle, where the mean mixture fraction on the axis equals Z_{st}

$$\frac{L+a}{d} = \frac{1+2Sc_t}{32Z_{st}}\left(\frac{u_{z,0}d}{\nu_{t,ref}}\right)\frac{\rho_0}{\rho_\infty C}$$

- Comparison with experimental correlations (Hawthorne, Weddel and Hottel (1949))

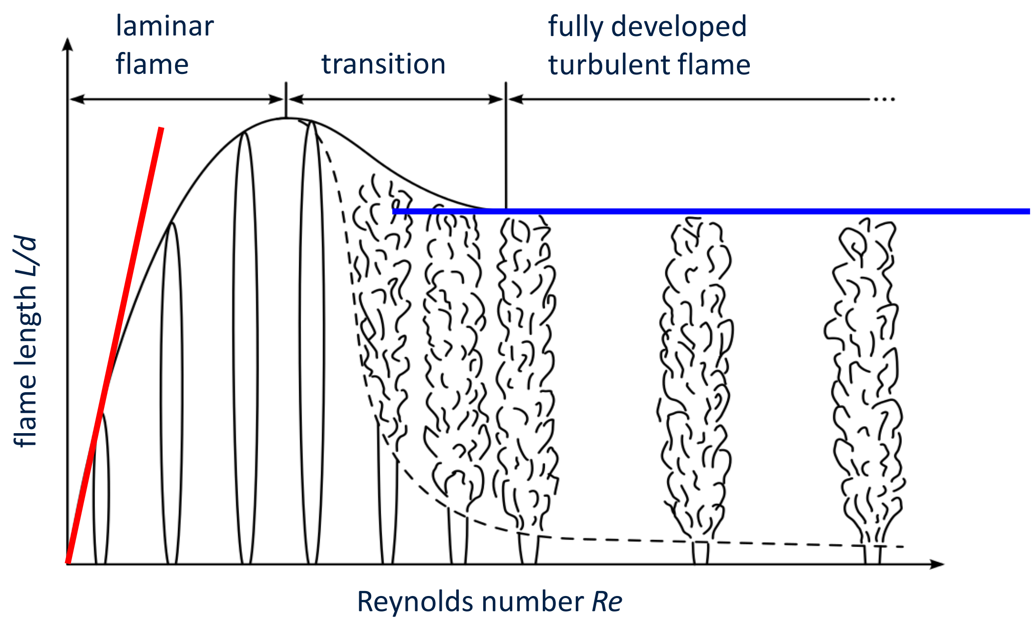
$$\frac{L+a}{d} = \frac{5{,}3}{Z_{st}}\sqrt{\frac{\rho_0}{\rho_\infty}}$$

  - With $u_{z,0}d/\nu_{t,ref}$ = 70 and Sc_t=0,72
  - Complete agreement for $C = (\rho_0\,\rho_{st})^{1/2}/\rho_\infty$

Round Turbulent Diffusion Flame

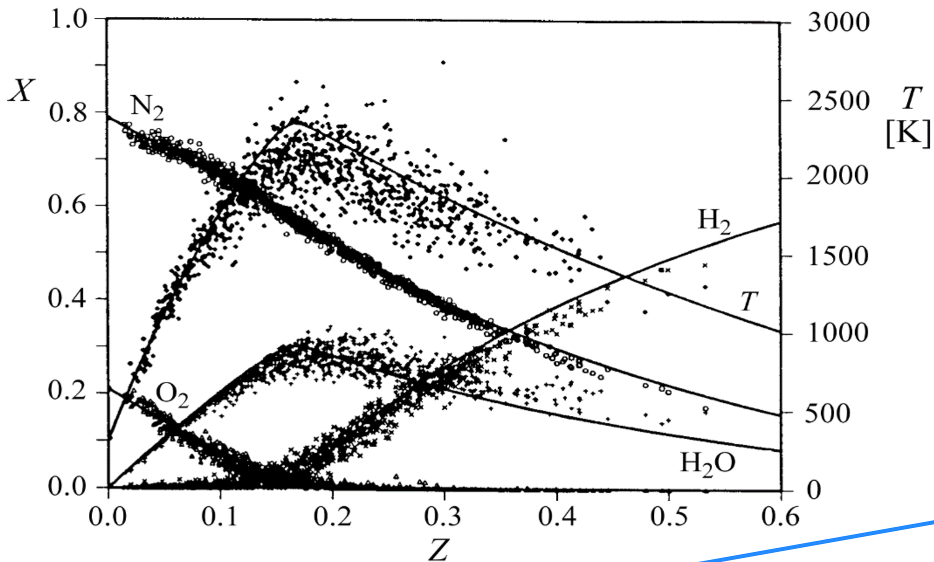
$$\frac{L+a}{d} = \frac{1+2Sc}{32Z_{\text{st}}} \frac{\rho_0}{\rho_\infty C} \underbrace{\overbrace{\boxed{\frac{u_0 d}{\nu}}}^{\text{linear}}}$$

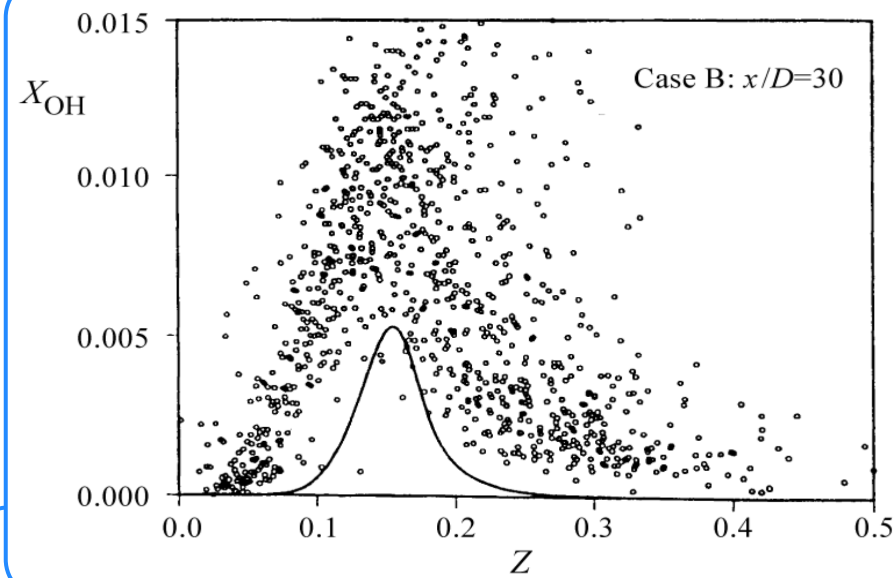
$$\frac{L+a}{d} = \frac{1+2Sc_{\text{t}}}{32Z_{\text{st}}} \frac{\rho_0}{\rho_\infty C} \underbrace{\overbrace{\boxed{\frac{u_0 d}{\nu_{\text{t,ref}}}}}^{\text{const.}}}_{\approx 70}$$



Experimental Data: Round Turbulent Diffusion Flame

- Comparison of experimental results and simulations with chemical equilibrium





- Concentration of radicals and emissions cannot be described by infinitely fast chemistry

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