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1

Numbers

Natural Numbers:

Natural numbers are the numbers that we use to count, 1, 2, 3, 4, 5.....this set is infinite, since you can always add 1 to obtain the next number. It is important to note that, for example, $12/2 = 6$ is a natural number, or $\sqrt{4}$, which is 2 is also a natural number.

Prime Number:

A number (greater than 1) which has only two factors, one of which is the number itself and the other one is unity, i.e. 1.

For example: 2, 3, 5, 11, 19, 23, 29, 31, 37.... etc.

Note:

- Unity (1) is not a prime number
- The lowest prime number is 2
- The only even prime number is 2
- Any prime number greater than 3 can be expressed as $6k \pm 1$, where k is an integer.

For example:

$$\begin{aligned}5 &= 6 \times 1 - 1 \\19 &= 6 \times 3 + 1 \\101 &= 6 \times 17 - 1, \text{ etc.}\end{aligned}$$

To check whether a number is Prime or not:

In order to find out that a given number (N) is prime or not do the following check.

- ❖ Find \sqrt{N} and round off the square root to the immediate lower integer. Let's call this number X.
- ❖ Now check for the divisibility of the number N by all prime numbers up to X. If the number is not divisible by any of the prime numbers up to X then N is prime.

Example:

Find out whether 241 is prime or not.

Solution:

$$(15)^2 = 225$$

$$(16)^2 = 256$$

$\Rightarrow \sqrt{241}$ will lie between 15 and 16

\Rightarrow To find out whether 241 is prime or not we have to check the divisibility of 241 by prime numbers till 15. Now 241 is not divisible by 2, 3, 5, 7, 11, and 13 this means 241 is prime.

Composite Numbers:

Composite Numbers are those numbers that have got at least three factors i.e. at least one factor extra than the number itself and unity.

For example: $24 = 2^3 \times 3$, factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

Whole Numbers:

It consists of all the natural numbers (1, 2, 3.....) and the number zero.

$$W = \{0, 1, 2, 3, 4, \dots\}$$

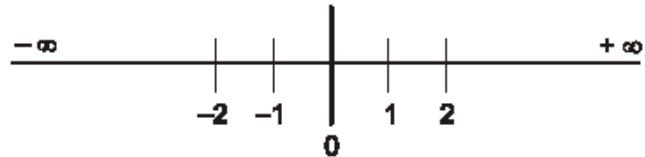
Whole numbers are also called non negative integers.

Integers:

The set of integers is defined as consisting of natural numbers, zero and negative integers

$$I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\},$$

Number Line:



Number line is a straight line that represents real numbers from negative infinity to positive infinity.

Real Numbers:

All those numbers that can be represented on the Number line are called real Numbers.

The operation of addition, subtraction, multiplication and division are valid for Real numbers.

For any real or whole numbers a, b, and c

We have:

- (1) Commutative property of addition: $a + b = b + a$
- (2) Associative property of addition: $(a + b) + c = a + (b + c)$
- (3) Commutative property of multiplication: $a.b = b.a$
- (4) Associative property of multiplication: $(a.b).c = a . (b . c)$
- (5) Distributive property of multiplication with respect to addition: $(a + b)c = ac + bc$

Rational Numbers:

A Rational Number is defined as a number that can be represented in the form of a/b where 'a' and 'b' are integers and $b \neq 0$. It means any number which can be represented in the form of ratio of two integers whose denominator is not equal to 0 is a rational number.

The set of Rational Numbers consist of integers and fractions.

Decimal representations of rational numbers either terminate or repeat:

- (1) Terminating (or finite) decimal fractions:

For example: $\frac{91}{4} = 22.75$ $\frac{44}{5} = 8.8$

- (2) Non terminating repetitive fractions:

These are non-terminating decimal fractions of the type:-

$\frac{17}{3}$

For example: $\frac{17}{3} = 5.6666\ldots$

which means any non - terminating repetitive decimal is a rational number and therefore can be expressed as a ratio of two integers (denominator is not equal to zero).

Example:

Let $x = 34.14141414141414\ldots$

Then $100x = 3414.1414141414\ldots$

Now $100x - x = (3414 - 34)$

or, $99x = 3380$

Hence $x = 3380 / 99$

- ❖ Non-terminating non-repetitive fractions:

These are of the type $x.a_1a_2a_3\ldots a_n b_1b_2b_3\ldots b_n$

For example: $3.237568132579\ldots$

Irrational Numbers:

Irrational numbers are those numbers that are non-terminating, non-repetitive decimals.

For example π , $\sqrt{2}$, $\sqrt{3}$, $0.14114111411114\ldots$ etc.

Factorial:

Factorial of a number N is defined as the product of all integers from 1 to N.

$$N! = 1 \times 2 \times 3 \times 4 \times \dots \times N.$$

$$\text{And } 0! = 1$$

$$7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$$

DIVISIBILITY RULES

A number is divisible by:

- ❖ 2, 4 & 8 when the numbers formed by the last, last two, last three digits are divisible by 2, 4 & 8 respectively.

For example:

1376547898732124 is divisible by 4 as the number made by last two digits is 24, which is divisible by 4.

567890345678901168 is divisible by 8 as the number made by last three digits is 168, which is divisible by 8.

- ❖ 3 & 9 when the sum of the digits of the number is divisible by 3 & 9 respectively

For example:

1234567812345678 is divisible by 3 and 9 as $2 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 72$, which is divisible by 3 & 9.

- ❖ 5 if the last digit is 5 or 0.

- ❖ 6, 12 & 15 when it is divisible by 2 and 3, 3 and 4 & 3 and 5 respectively.

- ❖ 7, if the number of tens added to five times the number of units is divisible by 7. Apply this rule over and over again as necessary.

Example: 826. Twice 6 is 12. So subtract 12 from the truncated 82. Now $82 - 12 = 70$.

This is divisible by 7, so 826 is divisible by 7 also.

- ❖ **11** When the difference between the sum of the digits in the odd places and of those in even places is 0 or a multiple of 11.

Example: $19151 \Rightarrow (9+5) - (1+1+1) = 11$, so 19151 is divisible by 11

- ❖ **13**, if the number of tens added to four times the number of units is divisible by 13. Apply this rule over and over again as necessary.

Example: $50661 \Rightarrow 5066 + 4 = 5070 \Rightarrow 507 + 0 = 507 \Rightarrow 50 + 28 = 78$ and 78 is 6×13 , so 50661 is divisible by 13.

- ❖ **17**, Subtract five times the last digit from the remaining leading truncated number. If the result is divisible by 17, then so was the first number. Apply this rule over and over again as necessary.

Example: $3978 \Rightarrow 397 - 5 \times 8 = 357 \Rightarrow 35 - 5 \times 7 = 0$. So 3978 is divisible by 17.

- ❖ **19**, if the number of tens added to twice the number of units is divisible by 19. Apply this rule over and over again as necessary.

Example: $101156 \Rightarrow 10115 + 2 \times 6 = 10127 \Rightarrow 1012 + 2 \times 7 = 1026 \Rightarrow 102 + 2 \times 6 = 114$ and $114 \Rightarrow 6 \times 19$, so 101156 is divisible by 19.

HCF and LCM

FACTOR:

A factor of a given number is every number that divides exactly into that number.

Example:

Write down all factors of 10.

$10 = 2 \times 5$, so numbers 2 and 5 are factors of 10.

Also $10 = 10 \times 1$, so 10 and 1 are factors of 10.

The factors of 10 are 1, 2, 5, 10.

NOTE: Number 1 and the number itself are always factors of any number.

COMMON FACTORS:

When two (or more) numbers have the same factor, that factor is called a common factor.

Example:

Find all the common factors of 12 and 18.

Factors of 12 are 1, 2, 3, 4, 6, 12.

Factors of 18 are 1, 2, 3, 6, 18.

The common factors of 12 and 18 are 1, 2, 3 and 6.

HIGHEST COMMON FACTOR (H.C.F.):

The Highest Common Factor (H.C.F.) of two (or more) numbers is the largest number that divides evenly into both numbers. In other words the H.C.F. is the largest of all the common factors.

The common factors of 12 and 18 are 1, 2, 3 and 6.

The largest common factor is 6, so this is the H.C.F. of 12 and 18.

It is very easy to find the H.C.F. of small numbers, like 6 and 9 (it is 3) or 8 and 4 (it is 4).

The best way is to keep finding the factors of the smaller number, starting from the largest factor. The first factor of the smaller number that is also a factor of the larger number is the H.C.F.

The H.C.F. is useful when simplifying fractions.

FINDING THE H.C.F. OF BIG NUMBERS:

For larger numbers you can use the following method:

1. Find all prime factors of both numbers.
2. Write both numbers as a multiplication of prime numbers.
3. Find which factors are repeating in both numbers and multiply them to get H.C.F.

Example:

Find the Highest Common Factor (H.C.F.) of 240 and 924.

Solution:

Finding all prime factors of 240:

We will start with the smallest prime number and we will factorize 240 into it. Do the same with the result (or quotient), and we will keep dividing by prime numbers until we have 1 as a quotient. Each time we write the prime factor to the right and the

quotient below:

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

Finding all prime factors of 924:

$$924 = 2 \times 2 \times 3 \times 7 \times 11$$

Multiply the factors which repeat in both numbers to get the H.C.F.

The Highest Common Factor is $2 \times 2 \times 3 = 12$

Example:

What is the greatest length which can be used to measure exactly the following lengths: 20 ft, 13 ft 9 inches, 17 ft 6 inches and 21 ft 3 inches?

Solution:

We must express these lengths in the same denomination and find their HCF.

$$\begin{aligned} 20 \text{ ft} &= 240 \text{ inches} \\ 13 \text{ ft } 9 \text{ inches} &= 165 \text{ inches} \\ 17 \text{ ft } 6 \text{ inches} &= 210 \text{ inches and} \\ 21 \text{ ft } 3 \text{ inches} &= 255 \text{ inches.} \end{aligned}$$

Now, HCF of 240, 165, 210 and 255 = $15 = 1 \text{ ft } 3 \text{ inches}$

MULTIPLES, COMMON MULTIPLES:

When you multiply a given whole number by any other whole number, the result is a multiple of that number.

For example, 5 is the first multiple of 5 (because $5 \times 1 = 5$), 10 is the second multiple of 5, and so on.

Example:

Find common multiples of 3 and 5.

Solution:

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33,

Multiples of 5 are 5, 10, 15, 20, 25, 30, 35,

Common multiples of 3 and 5 are 15, 30,

LOWEST COMMON MULTIPLE (L.C.M.):

The Lowest Common Multiple (L.C.M.) is the smallest number that is a common multiple of two or more numbers

For example, the L.C.M. of 3 and 5 is 15.

The simple method of finding the L.C.M. of smaller numbers is to write down the multiples of the larger number until one of them is also a multiple of the smaller number.

Example:

Find the Lowest Common Multiple of 8 and 12.

Solution:

Multiples of 12 are 12, 24...

24 is also a multiple of 8, so the L.C.M. of 8 and 12 is 24.

FINDING L.C.M. OF BIGGER NUMBERS:

1. Find all the prime factors of both numbers.
2. Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included.

Example:

Find the Lowest Common Multiple (L.C.M.) of 240 and 924.

We know the prime factors of both numbers

$$924 = 2 \times 2 \times 3 \times 7 \times 11$$

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

The L.C.M. is $2 \times 2 \times 3 \times 7 \times 11 \times 2 \times 2 \times 5 = 924 \times 20 = 18,480$

Example:

Three bells A, B and C ring at 12:00 noon simultaneously. The bells A, B and C ring at intervals of 10 minutes 18 minutes and 15 minutes respectively. At what time after 12 O' clock bells A, B and C will again ring simultaneously for the first time?

Solution:

$$\text{LCM of } 10, 18 \text{ and } 15 = 90$$

Hence after 90 minutes at 1: 30 pm the three bells will again ring simultaneously.

Application of HCF & LCM:

- ❖ Product of two numbers = $\text{HCF} \times \text{LCM}$
(if the fractions are in most reduced form)-

$$\text{❖ HCF of fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

- ❖ LCM of fractions = $\frac{\text{LCM of Denominators}}{\text{HCF of Numerators}}$
- ❖ The GREATEST NUMBER that will exactly divide x, y, z.
Required number = H.C.F. of x, y, and z (greatest divisor).
- ❖ The GREATEST NUMBER that will divide x, y and z leaving remainders a, b and c respectively.
Required number (greatest divisor) = H.C.F. of $(x - a)$, $(y - b)$ and $(z - c)$.
- ❖ The LEAST NUMBER which is exactly divisible by x, y and z.
Required number = L.C.M. of x, y and z (least divided).
- ❖ The LEAST NUMBER which when divided by x, y and z leaves the remainders a, b and c respectively.
Then, it is always observed that $(x - a) = (z - b) = (z - c) = K$ (say).
 \therefore Required number = $(\text{L.C.M. of } x, y \text{ and } z) - K$.

Example:

Find the least number which when divided by

20,25,35,40 leaves remainders 14,19,29,34.

Solution:

Here we observe that $(20 - 14) = (25 - 19) = (35 - 29) = (40 - 34) = 6$

LCM of 20, 25, 35 and 40 = 1400

Required Number is $= 1400 - 6 = 1394$.

- ❖ The LEAST NUMBER which when divided by x, y and z leaves the same remainder 'r' in each case.

Required number = (L.C.M. of x, y and z) + r.

Example:

What is the smallest number which when divided by 2,3,5,6, gives remainder 1

Solution:

Required number is $= (\text{LCM of } 2, 3, 5, 6) + 1 = 31$

- ❖ The GREATEST NUMBER that will divide x, y and z leaving the same remainder in each case.

Required number = H.C.F of $(x - y)$, $(y - z)$ and $(z - x)$.

QUESTIONS

1. Find the value of $x + y$ if $119xy$ is divisible by 99?
 - a. 7
 - b. 16
 - c. Either 7 or 16
 - d. None of the above
2. Sum of 5 prime numbers is 11250. Find the smallest of these 5 prime numbers?
 - a. 2
 - b. 3
 - c. 5
 - d. 13
3. Product of a rational and an irrational number is a rational number then what is the product?
 - a. 0
 - b. 1
 - c. Can be any rational number
 - d. None of the above
4. Ramu and Kallu are playing a game of addition and subtraction. They have written first 100 numbers side by side and they need to put plus or minus signs in between and then final results will be calculated. If the final sum comes out to be an even number then Ramu will win and if the sum comes out to be an odd number then Kallu will win. Who will win the game? Assume that both of them are playing smartly.
 - a. Ramu
 - b. Kallu
 - c. Depends on who will have the last chance.
 - d. Depends on luck.
5. There are 100 eggs in a basket. Two players (Ramu and Kallu) take turns to remove these eggs from the basket. On each turn, a player must remove at least one egg, and may remove up to 5 eggs. The player picking the last egg(s) wins the game. If you are allowed to start first, what is the number of eggs you will decide to remove so that you end up on the winning side?
 - a. 1
 - b. 2
6. If you write all the numbers from 1 to 75 side by side, you will get a huge number. What is the remainder when you divide this number by 8
 - a. 1
 - b. 2
 - c. 3
 - d. 4
7. If $49a0b$ is divisible by 8 and 9, then what is the value of a ?
 - a. Either 1 or 5
 - b. Either 1 or 6
 - c. Either 5 or 6
 - d. Either 1 or 5 or 6
8. There are 48 apple trees, 72 banana trees and 120 papaya trees. These trees are needed to be put in certain rows in such a way that the number of trees in each and every row is constant and at the same time, one row will contain trees of only one type. What can be the minimum number of rows to be planted?
 - a. 5
 - b. 10
 - c. 13
 - d. 15
9. What is the second smallest multiple of 8 which when divided by 15 leaves a remainder of 12?
 - a. 72
 - b. 144
 - c. 132
 - d. None of the above
10. If N divides 449, 667 and 883 to leave remainder of 9, 7 and 3 respectively then which of the following can be the value of N ?
 - a. 22
 - b. 24
 - c. 30
 - d. 45
11. A bell rings every 12 seconds. A second bell rings every 15 seconds and a third one rings every 20 seconds. All the three ring together for the first time at the same time at 8 A.M. How many times these bells will ring together till 9 A.M. of the same day including the first and the last ring?
 - a. 58
 - b. 60

- c. 61 d. 59

12. Find the largest 3 digit number which gives a remainder of 7 when divided by 12 and 15 both.
a. 967 b. 987
c. 947 d. 997

13. Find the largest number that can divide 1485, 1947 and 2013 completely?
a. 21 b. 33
c. 39 d. 41

14. Find the LCM of 0.8, 0.75 and 1.35
a. 108 b. 1.08
c. 54 d. None of the above

15. Find the highest common factor of $\frac{21}{4}, \frac{33}{8}, \frac{42}{16}$
a. $3/8$ b. $3/16$
c. $3/4$ d. None of the above

16. If LCM of first 120 natural numbers is A then what will be the LCM of first 125 natural numbers?
a. A b. 11A
c. 5A d. 55A

17. LCM of two numbers is 82 and HCF of the same pair of numbers is 14. If one of the numbers is 28 then find the other number?
a. 41 b. 28
c. 52 d. Data inconsistent

18. Find the largest number that can divide the product of 5 consecutive odd numbers.
a. 15 b. 120
c. 945 d. None of the above

19. Find the largest number that will divide the product of 5 consecutive even numbers.
a. 15 b. 120
c. 945 d. None of the above

20. Which is the smallest number that must be multiplied with 0,abcabcabc....to convert it into a natural number irrespective of the values of a, b and c?

a. 1001 b. 1/999
c. 999 d. None of the above

21. What is the largest six digit number that will completely divided by 24, 52, 55 and 63?
a. 360360 b. 720720
c. 108108 d. 540540

22. If LCM of a and b is 42 and LCM of c and d is 63 then what is the LCM of a, b, c and d?
a. 21 b. 63
c. 126 d. 2646

23. Find the value of the following expression
$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \dots + \frac{1}{9900} = ?$$

a. $98/99$ b. $99/100$
c. $100/101$ d. $101/102$

24. If $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots + \frac{1}{\infty} = a$, then find value of the following expression
a. $a/2$ b. $a/4$
c. $3a/4$ d. None of the above

25. X leaves a remainder of 102 when divided by 228. What will be the remainder when the same number is divided by 57?
a. 25 b. 35
c. 45 d. 55

26. If a,b are positive even numbers and c is an positive odd number then which of the following will be surely odd positive number
a. $a + b - c$ b. $a - b - c$
c. $a + b + 2c$ d. $a + b + c$

27. For how many numbers $(n-1)!$ is not divisible by n? Assume n is less than 100 and more than 10.
a. 35 b. 25
c. 21 d. 19

28. What is the value of m+n if $m2467342n$ is divisible by both 8 and 11?
a. 7 b. 8

- a. 7 b. 8
 c. 9 d. 10

41. Solve: $0.\overline{67} + 0.\overline{35}$

- a. $\frac{33}{34}$ b. $\frac{35}{34}$
 c. $\frac{34}{33}$ d. $\frac{43}{44}$

$$\begin{array}{r} 12 \\ \hline 1+ \quad \quad 12 \\ \hline 1+ \quad \quad 12 \\ \hline 1+ \quad \quad 12 \\ \hline 1+..... \end{array}$$

42. Solve:

- a. 3 b. 4
 c. 5 d. Either 3 or 4

43. There is a number $x = 0.\overline{567}$, What number from the following options should we multiply with x to get an integer?

- a. 792 b. 19998
 c. 1998 d. 1188

44. 2465765984 is divisible by which of the following?

- a. 7 b. 13
 c. 9 d. 11

45. There is a number 'N' which when divided by 15, 18 and 21 gives a remainder of 11, 14 and 17 respectively. Find the smallest such number.

- a. 1256 b. 630
 c. 334 d. 626

46. A number 'N' which when divided by 16, 20 and 24 gives a remainder of 13, 17 and 21 respectively. Find the second smallest such number.

- a. 717 b. 477
 c. 957 d. 1032

47. There is a number N such that when it is divided by 11, 13 and 15 it gives a remainder of 5, 7 and 9 respectively. Find the smallest such number.

- a. 2139 b. 2500
 c. 2100 d. 2250

48. There is a number 'N' which when divided by 7, 9 and 12 gives a remainder of 5, 7 and 10 respectively. Find that number.

- a. 502 b. 754
 c. 250 d. 832

49. The last 3 digits of a number N are y25. For how many values of y can N be the square of an integer?

- a. 1 b. 2
 c. 3 d. 4

50. A two digit number is divided by the sum of its digits. The answer is 6. What is the product of the digits?

- a. 18 b. 20
 c. 24 d. 30

• • •

2

Averages

Median:

The **median** of a set of data values is the middle value of the data set when it has been arranged in ascending or descending order.

Example:

The marks of nine students in a geography test which had maximum possible marks of 50 are given below:

47 35 37 32 38 39 36 34
35

Find the median of this set of data values.

Solution:

Arrange the data values in order from the lowest value to the highest value:

32 34 35 35 36 37 38 39
47

The fifth data value, 36, is the middle value in this arrangement.

Hence Median = 36

- ❖ If the number of values in the data set is even, then the median is the average of the two middle values.

Example:

Find the median of the following data set:

12 18 16 21 10 13 17 19

Solution:

Arrange the data values in order from the lowest value to the highest value:

10 12 13 16 17 18 19 21

The number of values in the data set is 8, which is even. So, the median is the average of the two middle values.

Hence Median = $(4^{\text{th}} \text{ data value} + 5^{\text{th}} \text{ data value}) / 2 = (16 + 17) / 2 = 16.5$

- ❖ Half of the values in the data set lie below the median and half lie above the median.
- ❖ The median is the most commonly quoted figure used to measure property prices. The use of the median avoids the problem of the mean property price which is affected

by a few expensive properties that are not representative of the general property market.

Mode:

The **mode** of a set of data values is the value(s) that occurs most often.

The mode has applications in printing. For example, it is important to print more of the most popular books; because printing different books in equal numbers would cause a shortage of some books and an oversupply of others.

Likewise, the mode has applications in manufacturing. For example, it is important to manufacture more of the most popular shoes; because manufacturing different shoes in equal numbers would cause a shortage of some shoes and an oversupply of others.

Example:

Find the mode of the following data set:

48 44 48 45 42 49 48

Solution:

The mode is 48 since it occurs most often.

- ❖ It is possible for a set of data values to have more than one mode.
- ❖ If there are two data values that occur most frequently, we say that the set of data values is bimodal.
- ❖ If there is no data value or data values that occur most frequently, we say that the set of data values has no mode.

Analysing Data:

The mean, median and mode of a data set are collectively known as **measures of central tendency** as these three measure focus on where the data is centred or clustered. To analyse data using the mean, median and mode, we need to use the most appropriate measure of central tendency. The following points should be remembered:

- ❖ The mean is useful for predicting future results when there are no extreme values in the data set. However, the impact of extreme values on the mean may be important and should be considered. e.g. Impact of a stock market

- crash on average investment returns.
- ❖ The median may be more useful than the mean when there are extreme values in the data set as it is not affected by the extreme values.
 - ❖ The mode is useful when the most common item, characteristic or value of a data set is required.

Arithmetic Mean:

The arithmetic mean is the standard "average", often called the mean. The arithmetic mean of n numbers $x_1, x_2, x_3, \dots, x_n$ is denoted by \bar{x} and calculated as:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

The arithmetic mean of two quantities a and b is

$$\frac{a+b}{2}$$

Example:

The arithmetic mean of 178, 243, 198, 189, 207, 203 is $(178 + 243 + 198 + 189 + 207 + 203)/6 = 1218/6 = 203$.

Shortcut method:

In the above example we can use the shortcut technique to calculate the Arithmetic mean. This method is convenient to use when the number of items is large and the values are big.

An arithmetic mean is generally assumed and the deviations from the assumed mean are calculated, then arithmetic mean is calculated as

$$\bar{x} = a + \frac{d_1 + d_2 + d_3 + \dots + d_n}{n} = A + \frac{1}{n} \sum_{i=1}^n d_i$$

Here $d_1 = x_1 - a$, $d_2 = x_2 - a$, $d_3 = x_3 - a$, ..., $d_i = x_i - a$

In the above example let $a = 200$, then arithmetic mean =

$$\begin{aligned} & \frac{(178-200)+(243-200)+(198-200)}{6} \\ & + \frac{(189-200)+(207-200)+(203-200)}{6} \\ & = (200) + (-22+43-2-11+7+3)/6 \\ & = 200 + (18/3) = 203 \end{aligned}$$

Weighted Arithmetic Mean:

If the values $x_1, x_2, x_3, \dots, x_n$ are assigned weights $w_1, w_2, w_3, \dots, w_n$ respectively, then the weighted arithmetic mean denoted \bar{x}_w by is calculated as:

$$\bar{x}_w = \frac{x_1 w_1 + x_2 w_2 + x_3 w_3 + \dots + x_n w_n}{w_1 + w_2 + w_3 + \dots + w_n} = A + \frac{\sum_{i=1}^n x_i w_i}{\sum_{i=1}^n w_i}$$

Here the term w can be replaced by f in case of frequency distribution. Then f_i will simply denote the frequency of x_i i.e. number of times x_i has occurred in the frequency distribution.

Example:

In a residential colony, there are 30 men, 25 women and 15 children. The average weight of the men is 60 kg. that of the women is 50 kg, while that of children is 40 kg; Find (approximately) the average weight of the entire colony.

- (1) 52.14 kg
- (2) 63.64 kg
- (3) 49.22 kg
- (4) 50.00 kg

Solution:

	Number f_i	Avg. weight x_i	Total weight $x_i f_i$
Men	30	60	1800
Women	25	50	1250
Children	15	40	600
Total	70		3650

$$\therefore \text{Avg. weight of all} = \frac{\text{Total weight of all}}{\text{Total numbers}}$$

$$\begin{aligned} & = \frac{\sum x_i f_i}{\sum f_i} \\ & = \frac{3650}{70} = 52.14 \\ & \text{Hence [1]} \end{aligned}$$

Shortcut Method:

We calculate the deviation from the assumed mean A , then the weighted arithmetic mean is calculated as:

$$\bar{x}_w = A + \frac{x_1 d_1 + x_2 d_2 + x_3 d_3 + \dots + x_n d_n}{w_1 + w_2 + w_3 + \dots + w_n} = A + \frac{\sum_{i=1}^n x_i d_i}{\sum_{i=1}^n w_i}$$

Where d_i is the deviation from the assumed mean.

Example:

In a school test 30 boys have scored 50 marks, 25 girls have scored 60 marks and 20 boys have scored 40 marks. Then the average marks of all the students are:

$$\frac{30 \times 50 + 25 \times 60 + 20 \times 40}{30 + 25 + 20} = (3800/75) = (152/3) = 50.66$$

Geometric Mean:

The geometric mean of n numbers $x_1, x_2, x_3, \dots, x_n$ is the n th root of their product.

$$\text{i.e. GM} = \sqrt[n]{x_1 \times x_2 \times x_3 \times \dots \times x_n}$$

- ❖ The geometric mean of two quantities a and

- b will be \sqrt{ab}
- If geometric mean of one group of m numbers is x and that of another group of n numbers is y then the geometric mean of the combined group is $(x^m y^n)^{\frac{1}{m+n}}$

Harmonic Mean:

The harmonic mean of n numbers $x_1, x_2, x_3, \dots, x_n$ is calculated as:

$$H.M. = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

The harmonic mean of two numbers a and b is $\frac{2ab}{a+b}$

Weighted Harmonic Mean:

If a set of weights w_1, \dots, w_n is associated to the dataset x_1, \dots, x_n , the weighted harmonic mean is defined by

$$\text{Weighted H.M.} = \frac{\sum_{i=1}^n w_i}{\sum_{i=1}^n \frac{w_i}{x_i}}$$

The harmonic mean as defined is the special case where all of the weights are equal to 1, and is equivalent to any weighted harmonic mean where all weights are equal.

Application of means in calculating average speed:

- When different distances are covered in the same time, arithmetic mean is used to determine the average speed.*

Example:

A car covers a distance of 40 km in first 30 minutes, 50 km in next 30 minutes and 60 km in the last 30 minutes. What is the average speed of the car.

Solution:

Average speed of the car $= (40+50+60)/3 = 50$ km per 30 minutes, i.e. 25 kmph.

- When equal distances are covered at different*

speeds, harmonic mean is used to determine the average speed.

Example:

A car is running along the four sides of a square ground at speeds of 50, 100, 150 and 200 kmph. Find the average speed of the car.

Solution:

$$\text{Average speed} = \frac{4}{\frac{1}{50} + \frac{1}{100} + \frac{1}{150} + \frac{1}{200}} = \frac{4 \times 600}{12 + 6 + 4 + 3} = 96 \text{ kmph}$$

- When different distances are covered at different speeds, weighted harmonic mean is used to determine the average speed.

Examples:

$M = 6$ has four divisors: 1, 2, 3, and 6. $A_6 = (1 + 2 + 3 + 6)/4 = 3$. On the other hand,

$$H_6 = 4 / (1/1 + 1/2 + 1/3 + 1/6) = 4 / (6/6 + 3/6 + 2/6 + 1/6) = 24/12 = 2.$$

Therefore we indeed have $6 = 3 \times 2$.

$M = 8$ has also four divisors: 1, 2, 4, and 8.

$$A_8 = (1 + 2 + 4 + 8)/4 = 15/4.$$

$$H_8 = 4 / (1/1 + 1/2 + 1/4 + 1/8) = 4 / (8/8 + 4/8 + 2/8 + 1/8) = 32/15$$

Finally, $(15/4) \times (32/15) = 32/4 = 8$.

Relation between the three means i.e. A.M., G.M. & H.M.:

- The Arithmetic Mean of two numbers a and b, is $\frac{a+b}{2}$, their Geometric Mean is \sqrt{ab} and their Harmonic Mean is $\frac{2ab}{a+b}$

$$(GM)^2 = AM \times HM$$

- $AM \geq GM \geq HM$

And the equality holds if and only if $x_1 = x_2 = x_3 = x_4 = \dots = x_n$

\therefore The difference between these two digits = 4
Hence [2]

QUESTIONS

1. Average income of 10 managers and 5 operators in an organization is Rs 433.33. If average income of all the managers is Rs 500 then what is the average income of the operators?
 a. Rs 200 b. Rs 250
 c. Rs 300 d. Rs 350
2. Average age of 20 boys of a class increases by 1 year when 4 boys of average age of 30 years join them. What is the new average age of the complete group?
 a. 23 years b. 24 years
 c. 25 years d. 26 years
3. Average temperature of 7 days of a week is 35°C . If average temperature of Sunday, Monday and Tuesday was 32°C and average temperature of Thursday, Friday and Saturday was 38°C then what was the average temperature on Wednesday?
 a. More than 35°C b. Less than 35°C
 c. Equal to 35°C d. None of the above
4. Average of 9 two digit numbers is 12. If we change the order of digits of one number then the average of these nine numbers will become
 13. Which of the following statements is true?
 a. Unit's place of the original two-digit number exceeds the ten's place by 1.
 b. Unit's place of the original two-digit number exceeds the ten's place by 2.
 c. Ten's place of the original two-digit number exceeds the unit's place by 1
 d. Ten's place of the original two-digit number exceeds the unit's place by 2.
5. Average age of 5 members of a family was 23 years 5 years back and it is 23 years today because one of the members of the family died of a heart attack today. What was the age of the person who died?
- a. 42 years b. 44 years
 c. 46 years d. 48 years
6. Average weight of 40 boys is 60 Kg and this weight increased by 1 Kg when Katiyal joins them. What is the weight of Katiyal?
 a. 101 b. 100
 c. 99 d. None of the above
7. Following table shows the average weight and number of wrestlers of 5 countries participating in world's wrestling championship. Find the approximate average weight of all the wrestlers participating in the prestigious competition?
- | Country | Number of
wrestlers | Average
(in Kg) |
|-----------|------------------------|--------------------|
| Japan | 60 | 50 |
| India | 40 | 60 |
| USA | 30 | 80 |
| Australia | 50 | 70 |
| China | 80 | 100 |
- a. 70 b. 72
 c. 74 d. 76
8. Average age of 9 boys increased by 1 year when a boy of 15 years was replaced by another boy. What was the age of this new boy?
 a. 22 years b. 24 years
 c. 26 years d. 28 years
9. Chaman has taken 300 wickets in his international cricket career at an average of 30 runs per wicket. In the next 4 matches, what is the minimum number of wickets he should take to reduce his average to less than 29 runs per wicket? Assume he gives 25 runs in every match.
 a. 10 b. 12
 c. 14 d. 16

10. Average age of children of Kats was 12 years till yesterday. Today, if Kats give birth to a new born baby and average age of her children is reduced to 10 years then Kats is now a proud mother of how many children?

- a. 4
- b. 5
- c. 6
- d. 7

11. In a class of 50 students, average weight of the students is 50 Kg. A student left the class and average weight of the class reduced to 49.5 Kg. What was the weight of that student?

- a. 74.5 kg
- b. 75 kg
- c. 75.5 kg
- d. None of the above

12. Average age of a group of 10 boys increased by 1 year when Ramu was replaced by Kallu in the group. If age of Ramu is 50 years then what is the age of Kallu

- a. 59 years
- b. 60 years
- c. 61 years
- d. None of the above

13. In a school, there are only three sections— A, B and C. Average marks per student in section A is 72. Average marks per student in section B is 77. Average marks per student in section C is 81. If the ratio of students in these three classes is 5:7:9, then what is the average marks per student in the complete school?

- a. 72.8
- b. 74.2
- c. 77.5
- d. 79.1

14. Average of 7 numbers is 13. Average of first four numbers is 15 and average of last four numbers is 11. What is the fourth number?

- a. 11
- b. 12
- c. 13
- d. 14

15. Average age of 30 students of a class is 23 years but if ages of two of the teachers are also included then average age of the students will increase by 1 year. What is the average age of both the teachers?

- a. 37 years
- b. 38 years
- c. 39 years
- d. 40 years

16. Average of 10 numbers is 14. If one is added to all the numbers, then what will be the new average?

- a. 14.1
- b. 14.5
- c. 14.8
- d. 15

17. If average of four consecutive even numbers is 43 then which of the following is not one of these four numbers?

- a. 40
- b. 44
- c. 46
- d. 48

18. Average weight of 4 mangoes is 45 grams. If a mango of 150 grams is added to these mangoes then what will be the new average weight of the mangoes.

- a. 56 grams
- b. 60 grams
- c. 62 grams
- d. 66 grams

19. Average age of all the members of a family was 40 years 5 years back. Just because a baby is born today, average age of all the members has again reduced down to 40. How many members are there in the family now?

- a. 7
- b. 8
- c. 9
- d. 10

20. What is harmonic mean of $\frac{1}{2}$ and $\frac{1}{8}$?

- a. $\frac{1}{3}$
- b. $\frac{1}{4}$
- c. $\frac{1}{5}$
- d. $\frac{1}{6}$

Directions for question 21 to 23: Read the following passage and answer the questions that follow.

In a family of five v, w, x, y, z, each and everyone loves one another very much. Their birthdays are in different months and on different dates. 'v' remembers that his birthday is between 25th and 30th, of 'w' it is between 20th and 25th, of 'x' it is between 10th and 20th, of 'y' it is between 5th and 10th, of 'z' it is between 1st and 5th of the month. The sum of the date of birth is defined as the addition of the date and the month, for example 12th January will be written as 12/1 and will add to a sum of the date of 13. (Between 25th and 30th includes both 25 and 30).

21. What may be the maximum average of their sum of the dates of birth?

- a. 24.6
- b. 15.2
- c. 28
- d. 32

22. What is the minimum average of their sum of the dates of births?
- 24.6
 - 15.2
 - 28
 - 32
23. If the date of birth of four of them are prime numbers, then find the maximum average of the sum of their dates of birth.
- 27.2
 - 16.4
 - 28
 - None of these
24. The average monthly sale for the first eleven months of the year of a certain salesman was Rs.12000, but due to his illness during the last month, the average monthly sales for the whole year came down to Rs 11375. What was the value of sales during the last month?
- 4500
 - 3700
 - 4100
 - 4700
25. The average weight of 10 men is increased by $1\frac{1}{2}$ kg when one of the men, who weighs 68 kg is replaced by a new man. Find the weight of the new man.
- 82 kg
 - 81 kg
 - 80 kg
 - 83 kg.
26. Find two numbers whose Arithmetic mean is 12.5 and Geometric mean is 10.
- 20 and 5
 - 22 and 3
 - 10 and 15
 - 18 and 7
27. The students of three classrooms, A, B and C, take a test. The results of the test are as follows. The average scores of students in classes A, B and C are 74, 84 and 78 respectively. The average scores of all the students of classes A and B together and B and C together are 78 and 81 respectively. Find the average score of all the three classes put together.
- 80
 - 78
 - 79.5
 - None of these
28. A set of several consecutive natural numbers beginning with 1 was written on the board. A

- number out of those numbers written was erased and the average of the remaining numbers was $28\frac{4}{9}$. Find the number that was erased.
- 3
 - 4
 - 5
 - 6
29. A is the average of four consecutive odd natural numbers. If $A/10$ is a perfect square, then one of the numbers cannot be:-
- 37
 - 93
 - both(a) and(b)
 - 25
30. Sanat's average expenditure for ten days is Rs 13, while that for first five days is Rs 10. Also the average expenditure for the last 3 days is Rs 16. Find the average expenditure for the sixth and seventh day.
- Rs 13
 - Rs 14
 - Rs 15
 - Rs 16
31. The speed of a sports car in three different practice runs on the same track are 24, 40 and 120 miles/hour. Find its average speed (approximately) in miles/hour
- 36
 - 40
 - 61.33
 - 76
32. The average of seven results is 9; average of first six is 7; average of last two is 18. Sixth result is
- 0
 - 15
 - 21
 - 13
33. The average of 10, two digits numbers was to be calculated. But for one of the numbers, the digits were swapped during calculation and hence the result was more than the actual average by 3.6. Find the difference between the digits of that number.
- 3
 - 4
 - 5
 - 6

Directions for question 34 to 36: Answer the questions based on the following information.

There are 60 students in a class. These students are divided into three group A, B and C of 15, 20 and 25 students each. The group A and C are combined

to form group D.

34. What is the average weight of the Students in group D?
- More than the average weight of A.
 - More than the average weight of C.
 - Less than the average weight of C .
 - Cannot be determined.
35. If one student is shifted from group A to group B, which of the following will be true?
- The average weight of both the groups increases.
 - The average weight of both the groups decreases.
 - The average weight of the class remains the same.
 - Cannot be determined.
36. If all the students of the class have the same weight, then which of the following is false.
- The average weight of all the three groups is the same.
 - The total weight of A and C is twice the total weight of B.
 - The average weight of D is greater than the average weight of A.
 - The average weight of all the groups remain the same even if a number of students are shifted from one group to another.
37. A boy travels half the distance from his house to the school at a speed of 10 km/hr. For exactly half of the remaining time he travelled at 15 km/hr and the rest of the time at 20km/hr. Find the average speed.
- 13 km/hr.
 - $12\frac{8}{11}$ km/hr.
 - 15 km/hr.
 - 18 km/hr.
38. There are 20 students in Mr. Mahindra's class. Once he conducted a test for a maximum of 100 marks. He then arranged the marks of all the students in the ascending order. He found that Rohit, who normally topped the class, had slipped to the tenth position. He found that the average of the last 11 students was 64 and that of the top 10 was 67. If the average marks of

all the students of his class was 65, how many marks did Rohit score?

- 64
- 67
- 72
- 74

39. Lekhakram, the renewed author, recently got his new novel released. To his utter dismay he found that for the total 1,007 pages there were on an average 2 mistakes on every page. While, in the first 612 pages there were only 434 mistakes, they seemed to increase for the subsequent pages. Find the average number of mistakes per page for the remaining pages.
- 6
 - 4
 - 2
 - 3

Directions for question 40 and 41: These questions are based on the following data.

Four persons A, B, C and D each have a certain sum of money with them. The average amount with them is Rs 300. If the amount with A and B increases by 100%, then the average for all the four increases by Rs 75. If the amount with C increases by 100% and that with D by 40% the average amount with C and D is Rs 690.

40. How much money did C have?
- Rs 100
 - Rs 150
 - Rs 200
 - Rs 400
41. If B gave A Rs 50, both of them would have the same amount. Which of the following is the amount that A had?
- Rs 50
 - Rs 100
 - Rs 200
 - Cannot be determined
42. A school has only four classes having 10, 20, 30 and 40 students respectively. The pass percentages of these classes are 20%, 30%, 60% and 100% respectively. Find the pass percentage of the entire school.
- 56%
 - 76%
 - 34%
 - 66%
43. $\sum_{r=1}^n (n+1)r$ where $r = n$.
- $\frac{(n-1)(n)(n+1)}{2}$
 - $\frac{n(n+1)^2}{2}$

- c. $\frac{n(n-1)^2}{2}$ d. $\frac{n^2}{2}$
44. Find the average of $f(x)$, $g(x)$, $h(x)$, $d(x)$ at $x = 10$. $f(x)$ is equal to $x^2 + 2$, $g(x) = 5x^2 - 3$, $h(x) = \log_{10}x^2$ and $d(x) = (4/5)x^2$
 a. 170 b. 170.25
 c. 70.25 d. 70
45. In a T20 cricket match, a team scores at the run rate of 7 per over in first 8 overs. At what average should they score in the remaining 12 overs to achieve a target of 176?
 A. 10
 B. 12
 C. 8
 D. None of the above
46. The average of 'n' numbers is z. If the number x is replaced by the number x_1 , then the average becomes z_1 , find the relation between n, z, z_1 , x and x_1 .
 a. $\left[\frac{z-z_1}{x_1-x} = \frac{1}{n} \right]$ b. $\left[\frac{x_1-x}{z_1} = \frac{1}{n} \right]$
 c. $\left[\frac{z-z_1}{x-x_1} = \frac{1}{n} \right]$ d. $\left[\frac{x-x_1}{z-z_1} = \frac{1}{n} \right]$
47. Three people x, y and z weigh themselves in a particular order. First x, y, z weigh themselves individually and then xy, xyz, zx and xyz together respectively. The recorded weight for the last measures 180 kgs. The average of the 7 measures is:
 a. 360 kgs b. 207 kgs
 c. 103 kgs d. Cannot be determined
48. The average weight of a group of 10 people increases by 1 kg when one of them weighing 58 kgs is replaced by a new person. The weight of the new person is:
 a. 62 kg b. 68 kg
 c. 66 kg d. None of these.
49. What is the average of the first five multiples of 495?
 a. 1188 b. 2475
 c. 1485 d. 505
50. In a class of 100 students, 48 students secured 40 marks in English, 32 secured 60 marks and the rest 80 marks. Find the average in English for the whole class?
 a. 60 b. 19.42
 c. 62.66 d. 54.40

3

Percentages

❖ Concept of Percentage:

By a certain percent, we mean that many hundredths.
Thus x percent means x hundredths, written as x%.

To express x % as a fraction:

We have, $x\% = \frac{x}{100}$

Thus, $20\% = \frac{20}{100} = \frac{1}{5}$; $48\% = \frac{48}{100} = \frac{12}{25}$, etc.

To express a / b as a percent:

We have, $\frac{a}{b} = \left[\left(\frac{a}{b} \right) \times 100 \right] \%$.

Thus, $\frac{1}{4} = \left[\left(\frac{1}{4} \right) \times 100 \right] = 25\%$;

$0.6 = \frac{6}{10} = \frac{3}{5} = \left[\frac{3}{5} \times 100 \right] \% = 60\%$.

❖ If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the

expenditure is $\left(\frac{R}{100 + R} \times 100 \right) \%$.

❖ If the price of the commodity decreases by R%, then the increase in consumption so as not to decrease

the expenditure is $\left(\frac{R}{100 - R} \times 100 \right) \%$.

❖ Results on Population:

Let the population of the town be P now and suppose it increases at the rate of R% per annum, then:

$$\text{Population after } n \text{ years} = P \left[1 + \frac{R}{100} \right]^n$$

$$\text{Population } n \text{ years ago} = \frac{P}{\left[1 + \left(\frac{R}{100} \right) \right]^n}$$

❖ Results on Depreciation:

Let the present value of a machine be P. Suppose it depreciates at the rate R% per annum. Then,

$$\text{Value of the machine after } n \text{ years} = P \left[1 - \frac{R}{100} \right]^n$$

$$\text{Value of the machine } n \text{ years ago} = \frac{P}{\left[1 - \left(\frac{R}{100} \right) \right]^n}$$

❖ If A is R% more than B, then B is less than A by $\left[\frac{R}{100 + R} \times 100 \right] \%$.

❖ If A is R% less than B, then B is more than A by $\left[\frac{R}{100 - R} \times 100 \right] \%$.

Percentage change in product of two quantities:

Consider a product of two quantities $A = a \times b$; If a & b change (increase or decrease) by a certain percentage say x & y respectively, then the overall percentage change in their product is given by the formula:

$$x + y + \frac{xy}{100}$$

This formula also holds true if there are successive changes as in the case of population increase or decrease. But when there are either more than 2 successive changes or there is a product of more than 2 quantities as in the case of volume, **then we have to apply the same formula twice**.

This formula can be used for following questions:

- If A is successively increased by X% and Y%, Then the % increase = $X + Y + \frac{xy}{100}$
- If there is successive discount of X% and Y%, The the % decrease = $-X - Y + \frac{xy}{100}$
- If there is X% increase and Y% decrease, = $X+Y+ \frac{xy}{100}$ the total change in $X-Y-\frac{xy}{100}$
- If the sides of a rectangle increase by X% and Y%, find then percentage increase in its

$$\text{area } X+Y+\frac{xy}{100}.$$

Mixture problems:

If x % of a quantity is taken by the first person, y % of the remaining quantity is taken by the second person, and z % of the remaining is taken by the

third person and if A is left, then initial quantity was

$$\frac{A \times 100 \times 100 \times 100}{(100 - x)(100 - y)(100 - z)}$$

The same concept we can use, if we add something, then the initial quantity was

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QUESTIONS

1. If A is 30% more than B then B is what percentage less than A?
 - a. 23%
 - b. 30%
 - c. 42%
 - d. None of the above
2. Fresh grapes contain 80% water by weight whereas once dried they will contain only 20% water by weight. Find the quantity of dried grapes that can be obtained from 20 kg of fresh grapes?
 - a. 2.5 kg.
 - b. 4 kg.
 - c. 4.8 kg.
 - d. 5 kg.
3. Chronicle has announced a scholarship of Rs 5,000 for all the students who score more than 50% in their scholarship test. Over and above this, an additional discount of 10 percent was admissible for all the students who enrol for their courses before 31st of December. If the current fee for their complete course is Rs 30,000, then what is the maximum percentage of discount being offered on their course?
 - a. 20 %
 - b. 25 %
 - c. 33.33 %
 - d. 40 %
4. In an exam, Ramu scored 35 % and failed by 24 marks and Kallu scored 40 % and got 6 marks over and above the passing marks. How much were the passing marks in the exam?
 - a. 210
 - b. 230
 - c. 234
 - d. 236
5. An employee of an organization is paid Rs 40 per hour for stipulated 8 hours a day. But at the same time, he gets 75 % above the basic pay as overtime if he works for more than 8 hours in a day. Find his average hourly income if he works 12 hours a day.
 - a. Rs 48 per hour
 - b. Rs 50 per hour
 - c. Rs 44 per hour
 - d. Rs 60 per hour
6. Chaman, a shrewd shopkeeper, increases prices of all his commodities by certain percentage and then provides a discount of same percentage. Which of the following statements can be deduced on his operational activities?
 - a. He sells his goods at a higher price than initially fixed price.
 - b. He sells his goods at a lower price than initially fixed price.
 - c. He sells his goods at the same price which was fixed before the increase.
 - d. None of the above.
7. In an election, there were only two candidates. Winning candidate got 20 % more votes than what losing candidate got. What percentage of votes was received by the winning candidate?
 - a. 60%
 - b. 40%
 - c. 54.54%
 - d. 55%
8. Price of petrol increase by 20 % but Ramu can increase his expenditure on petrol only by 10%. What should be the reduction in his petrol consumption?
 - a. 32%
 - b. 28%
 - c. 8.33%
 - d. 12%
9. Ramu spends 10% of his pocket money on food, 20% of the remaining on transportation and 25% of the balance on clothing. If he saves Rs 270 every month then how much does he spend on clothing?
 - a. Rs 80
 - b. Rs 90
 - c. Rs 100
 - d. Rs 120
10. Population of a town is estimated to be 3456000 after three years. Rate of increase of population is 20 % per annum. What is the current population of the town?
 - a. 1800000
 - b. 2000000
 - c. 2400000
 - d. 3000000

11. What is the percentage increase of population of a bacteria per month if it triples itself in 3 months?
- 100%
 - 66%
 - 44%
 - 10%
12. A vegetable vendor sold 50% of his vegetables and then threw away 30% of the remaining vegetables. Next day he sold 60% of his remaining vegetables and then threw away all the remaining vegetables. How much of his vegetables were thrown away?
- 30%
 - 29%
 - 25%
 - 22%
13. Raman increased prices at his shop by 8% and thus his daily sales (numbers) decreased by 5%. As an overall result, his net sales (revenues) increased by Rs 5200. What are his new daily sales?
- Rs 2,00,000
 - Rs 2,05,200
 - Rs 1,94,800
 - None of the above
14. If a shopkeeper provides two successive discounts of 10 % and 12 % then find the overall discount that this shopkeeper offers to his customers?
- 0.8 %
 - 3.2 %
 - 20.8 %
 - 23.2 %
15. In a class there are 75 % boys and remaining are girls. If one third of the boys in the class and 20% of the girls smoke then what percentage of the whole class smoke?
- 25 %
 - 28 %
 - 30 %
 - 35 %
16. A shopkeeper provides a discount of 20 % and then sells a gift item for Rs 840. What was the marked price of that gift item?
- Rs 1008
 - Rs 1020
 - Rs 1050
 - Rs 1080
17. Ramu got an increment of 20 % in his monthly salary and hence his monthly salary increased by Rs 120 per month. What is his monthly salary after the increase?
- Rs 600
 - Rs 720
 - Rs 660
 - Rs 840
18. If ratio of salaries of Ramu and Kallu is $\frac{5}{6} : \frac{2}{5}$ then by what percentage, Ramu's salary is less than that of Kallu?
- 20%
 - 21.8%
 - 25%
 - 28%
19. Length of a rectangle increases by 8% and breadth decreases by 9%. What will be the percentage change in the area of the rectangle?
- Decrease by 1.72%
 - Decrease by 1 %
 - Decrease by 0.9%
 - Decrease by 2 %
20. If price of sugar increases by 30%, then by what percentage consumption of sugar should be decreased if budget for sugar cannot be increased by more than 10%?
- 12.8%
 - 15.4%
 - 17.2%
 - None of the above
21. A shopkeeper sells his goods at a profit of 20%. What is the margin of profit in his sales? Margin of profit is defined as the profit as a percentage of selling price and profit percentage is defined as the profit as a percentage of cost price.
- 16.66%
 - 20%
 - 25%
 - None of the above
22. Chaman's salary is 20% more than what it was last year. And it will increase by 10% next year. If in next year he will get an increment of Rs 12,000, then what was the increase in his salary this year over the last year?
- 20,000
 - 1,00,000
 - 1,20,000
 - 1,32,000
23. Prices of all the pizzas at pizza home are increased by 8% and as a result number of customers get reduced by 3%. As an overall result, revenues of pizza home have increased by Rs. 14,280. What will be the new revenue for pizza home?
- Rs 3,14,280
 - Rs 2,64,280
 - Rs 2,50,000
 - Rs 3,00,000

24. There are 50 boys in a class of 80 students. 10 new boys joined the class with effect from the new session. What is the increase in percentage composition of boys of the class in terms of percentage points?
- 12.5 %
 - 8.2 %
 - 6.8%
 - 4.2 %
25. Total cars sold in India during 2009 were 70,00,000 and Marti had a 45 % share in the same. During 2010, Total cars sold were 90,00,000 and Marti had a share of 56 % in it. What is the percentage increase in the sales of Marti in 2010 over 2009?
- 40 %
 - 50 %
 - 60 %
 - 70 %
26. 30% of Indian men are Chaman and 20 % of the Indian women are Chaman. Which of the following can be the percentage of Chaman among all Indians? Assume all Indians are either men or women.
- 12%
 - 24 %
 - 42 %
 - 50 %
27. A thief has stolen certain number of diamonds. After stealing, he realizes that two of the heaviest diamonds weigh 25% of the total weight of the diamonds and 5 of the lightest diamonds weigh 45% of the total weight of the diamonds. How many diamonds did the thief steal?
- 8
 - 9
 - 10
 - 11
28. Iraq has 40,000 gallons of crude oil in their reserves. It added 5,000 gallons of the crude oil to their reserves this year. What percentage of their total reserve crude oil was added this year? Assume, no crude oil was taken out from their reserves this year?
- 14.28%
 - 12.50%
 - 11.11%
 - 10%
29. Kallu had to increase a number by 10 % but instead he decreased the number by 8 %. If the difference between the desired answer and

- answer that Kallu got was 45 then what should have been the correct answer?
- 250
 - 230
 - 270
 - 275
30. In an election, Ram Prasad got 20 % more votes than what he got last time and thus won by double the votes by which he won last time. There were only two candidates in this election as well as the previous one and number of voters were same in both the elections as well. What percentage of total votes, Ram Prasad got this time?
- 60 %
 - 62.5 %
 - 75 %
 - 80 %
31. A college raised 60 % of the fund required by collecting an average of Rs 600 per head from 75 % of the staff. What is the average money that remaining 25 % of the staff members should give in order to fulfil the fund requirements?
- Rs 800
 - Rs 1,000
 - Rs 1,200
 - Rs 1,400
32. If 25 % of Ramu's salary is 20 % more than 40% of Kallu's salary then what is the ratio of Ramu's salary to Kallu's salary?
- 25:48
 - 48:25
 - 54:46
 - 46:54
33. A manufacturer increases price of a nut and bolt by 20% while selling it to the distributors. The distributors increases the price of the same nut and bolts by 20% while selling it to dealers. The dealers further increase the price of same nut and bolts by 50% while selling it to shopkeeper who in turn sells it at a profit of 50%. If a customer pays Rs 648 for 100 nuts and bolts then how much does the manufacturer earns while selling one set of nut and bolt?
- 30 paisa
 - 40 paisa
 - 50 paisa
 - 60 paisa
34. As per the 2010 budget, prices of food commodities have increased by 10 %, prices of clothes have increased by 20 % and other miscellaneous goods have increased by 5

- %. Monthly expenditure of Lolita was 50 % on food, 30 % on clothes and 20 % on miscellaneous items. What is the percentage increase in monthly expenditure of Lolita?
- 8%
 - 10%
 - 12 %
 - 14%
35. Radius of a cylinder is decreased by 10 %. What is the percentage increase in the height of the cylinder in order to maintain the same volume?
- 9 %
 - 11%
 - 21%
 - 23%
36. A shopkeeper buys some pens at the rate of Rs. 10 for 3 and twice these pens at the rate of Rs. 13 for 4. He sells all the pens at the rate of Rs 59 per dozen. What is his profit per cent?
- 16.667%
 - 33.333%
 - 50%
 - 40%
37. 110% of a number X is 33. What is 33% of X?
- 100
 - 10
 - 11
 - 9.9
38. A trader bought rice for Rs 1350. He had to sell a third of it at a loss of 10%. Find the % profit at which he should sell the rest so that he makes an overall profit of 10% on the whole.
- 15%
 - 20%
 - 30%
 - 10%
39. The amount of a work increased by 80%. By what percent is it necessary to increase the number of workers to complete the amount of work in previously planned time, if the labour productivity of the additional labour is 20% more?
- 80%
 - 50%
 - 60%
 - 67%
40. An alloy contains 50% copper, 20% tin and the rest impurities. A second alloy contains 30% copper, 40% tin, and the remaining impurities. The alloys are melted and mixed in the ratio of 1:3. If the impurities are removed, find the percentage of copper?

- 40%
 - 48%
 - 50%
 - 57%
41. In an examination, 1100 were boys and 900 were girls. 50% of the boys and 40% of the girls passed the examination. The percentage of candidates who failed is:
- 45%
 - 45.5%
 - 54.5%
 - 59.2%
42. A discount series of 10%, 20% and 40% is equal to a single discount of:
- 50%
 - 56.80%
 - 60%
 - 70.28%
43. The price of cooking oil has increased by 25%. The percentage of reduction, that a family should effect in the use of cooking oil so as not to change its expenditure on cooking oil, is:
- 15%
 - 20%
 - 25%
 - 30%
44. A worker gets a 5% raise in salary. A year later, the worker receives a 2.5% cut in pay, and now his salary is 22702.68. What was his salary in the beginning?
- Rs 22000
 - Rs 22176
 - Rs 25000
 - Rs 22193
45. A trader wants 10% profit on selling price of a product whereas his expenses amount to 15% on sales. What should his rate of mark-up on an article be?
- 20%
 - 25%
 - 30%
 - 38%
46. Shatabdi Express has a capacity of 500 seats of which 10% is in the Executive Class, rest being Chair Car. During one journey the train was booked to 85% of its capacity. If Executive Class was booked to 96% of its capacity, how many Chair Car seats were empty during that journey?
- 75
 - 73
 - 71
 - None of these

47. If $a\%$ of x is equal to $b\%$ of y , then $c\%$ of y is what % of x ?
a. $(c) \%$ b. $(ac / b) \%$
c. $(bc / a) \%$ d. $(abc) \%$
48. In a class, 40% of the boys is same as of the girls and there are 20 girls. Total number of students in the class is:
a. 70 b. 45
c. 35 d. 25

49. Ravi's salary is 150% of Amit's salary. Amit's salary is 80% of Ram's salary. What is the ratio of Ram's salary to Ravi's salary?
a. 1 to 2 b. 2 to 3
c. 5 to 6 d. 6 to 5
50. $x\%$ of y is $y\%$ of
a. x b. $y/100$
c. $x/100$ d. $100x$

4

Simple & Compound Interest

The amount of sum that is lent by one person to another is called **Principal (P)**. The person to whom money is given is called the debtor and the person who gives the money is called the creditor.

The time for which the money is borrowed is called period (T). The interest that is paid per 100 rupees in a year is called rate percent per annum (R). The interest (I) accrued when added to the principal (P) is called the Amount (A).

$$A = P + I$$

Simple Interest (S.I.)

When interest is charged on the original sum (Principal) borrowed throughout the loan period, it is called simple interest.

$$S.I. = \frac{P \times T \times R}{100}$$

Example:

Find the interest to be owed on a sum of Rs 2500 at 5% p.a. for a period of 3 years

Solution:

$$S.I. = \frac{P \times T \times R}{100} = \frac{2500 \times 3 \times 5}{100} = 375$$

Example:

An amount given on S.I. becomes three times of itself in 8 years Find the rate of interest charged.

Solution:

$$A = P + I \Rightarrow 3P = P + I \Rightarrow I = 2P$$

$$\text{And also, } S.I. = \frac{PRT}{100}$$

$$\Rightarrow 2P = \frac{P \times R \times 8}{100}$$

$$\Rightarrow \frac{2P \times 100}{8 \times P} = 25\%$$

Compound Interest (C.I.)

When the interest that becomes due is added to the principal and the interest for the next period is calculated on the new principal, then such scheme of interest is called compound interest.

Second year's principal = First year's principal + First year's interest.

$$A = P \left(1 + \frac{r}{100}\right)^n$$

When interest is payable annually.

$$A = P \left(1 + \frac{r}{2 \times 100}\right)^{2 \times n}$$

When interest is payable half-yearly.

- When rates are different for different years, say R₁, R₂, R₃ percent for the first, second and third year respectively then

$$\text{Amount} = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right)$$

- In case the time is a fraction of a year, say $\frac{b}{c}$ years, then

$$\text{Amount} = P \left(1 + \frac{r}{100}\right)^a \times \left(1 \times \frac{\frac{b}{c} \times r}{100}\right)$$

Example:

Find the C.I. on a principal of Rs 1200, for two years when rate of interest for first years is 2% and for the second year is 5%.

Solution:

$$\begin{aligned} A &= P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \\ &= 1200 \left(1 + \frac{2}{100}\right) \left(1 + \frac{5}{100}\right) \\ &= 1200 \times \frac{51}{50} \times \frac{21}{20} = 1285.2 \end{aligned}$$

$$\Rightarrow C.I = A - P = 1285.2 - 1200 = \text{Rs } 85.2$$

Population Formula:

The population of a town is P and annual rate of increase of population is $R\%$, then the population of

the town n years is given by $P \left(1 + \frac{R}{100}\right)^n$.

If the rate of decrease of population is $R\%$ the

population after n years is given by: $P \left(1 - \frac{R}{100}\right)^n$.

This is also called the **Depreciation formula**.

Example:

If the annual increase in the population of a town is 4% and the present population is 16625 what will be its population after 3 years (Approximate).

Solution:

Population after 3 years

$$= P \left(1 + \frac{R}{100}\right)^3 = 16625 \times \left(1 + \frac{4}{100}\right)^3$$
$$= 16625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} = 18700$$

CHRONICLE
IAS ACADEMY

QUESTIONS

1. Ramu invests Rs.8,000 in two banks. Bank 1 offers 5% simple interest per annum and Bank 2 offers 9% simple interest per annum. Interest for three years from these two investments come out to be Rs. 1440. Then what is the amount invested at 5 % per annum?
 - a. Rs 2,000
 - b. Rs 3,000
 - c. Rs 5,000
 - d. Rs 6,000
2. Population of certain cancer bacteria grows at a rate of 20 % per annum and this cancer can be cured if the bacteria are detected when grown to 1, 00,000. If a person starts feeling the symptoms of these bacteria when he has 50,000 bacteria, at what time a person has to start his treatment from the time he first notices the symptoms of these bacteria?
 - a. 4 years
 - b. Less than 4 years
 - c. More than 4 years
 - d. None of the above
3. If the difference between SI and CI on a constant principal is Rs 305 for a period of 3 years and rate of interest in both the cases is 5 % per annum then what was the principal amount invested in both cases.
 - a. 30000
 - b. 40000
 - c. 50000
 - d. 60000
4. What will be the interest earned on Rs 2,000 at 10% compound interest compounded half yearly for 2 years?
 - a. Rs 400
 - b. Rs 420
 - c. Rs 431
 - d. Rs 440
5. The simple interest on certain sum of money for 3 years comes out to be Rs 3000 and simple interest on the same sum of money in a different scheme of SI for 2 years comes out to be Rs 2250. Find the amount of money invested? Assume rate of interest per annum in both the cases to be single digit integers.
6. Ramu had to invest some money for three years He had two options open with him. In the first option, bank was offering a Simple Interest of 10 % and in the second option, bank was offering compound interest of 8 % per annum compounded annually. Which of the two options is a better deal?
 - a. First Option
 - b. Second Option
 - c. Both the options are equally good
 - d. None of the above
7. If ratio of SI and CI for two years is 20:21 then what is the rate of interest offered in both the cases? Assume principal money invested and rate of interest offered is same in both the cases.
 - a. 5 %
 - b. 10 %
 - c. 20 %
 - d. Cannot be determined
8. Ramu started investing in a recurring deposit wherein he need to pay an amount of Rs 10,000 on every 1st of January for 5 years and he will get the total amount due at the end of 5th year. If bank pays a compound interest of 10 percent per annum then how much money should Ramu get at the end of 5th year?
 - a. 59,267
 - b. 67,157
 - c. 74,124
 - d. 81,134
9. Ramu purchased a TV set for Rs 50,000 and decided to pay the total amount in 5 equal monthly instalments. If rate of interest charged by the bank was 2.5% per month then what will be the approximate value of that monthly instalment? Assume that the first monthly installment is to be paid at the time of purchase of TV.

- a. 10,000 b. 11,050
c. 12,050 d. 13,000
10. Difference between Simple Interest and Compound Interest for 2 years on a certain sum of money @ 8% per annum is Rs 8,000. What was the Principal amount invested?
a. 10,00,000 b. 12,50,000
c. 15,00,000 d. 17,50,000
11. Value of machinery depreciates at a rate of 10% every year. If value of three year old machinery is Rs 10935 then what was the price of the machinery when it was purchased?
a. Rs 12,000 b. Rs 15,000
c. Rs 18,000 d. Rs 20,000
12. I invested some amount of money at certain rate of interest at SI and I realized that my money has increased to four times in four years. In how many years, it will increase to 16 times?
a. 12 years b. 16 years
c. 20 years d. 24 years
13. At certain rate of compound interest, it was realized that amount at the end of 3 years will be Rs 5,750 and amount at the end of 6 years will be Rs 6,612.50. What was the principal invested in the deal?
a. Rs 4,500 b. Rs 5,000
c. Rs 5,500 d. None of the above
14. At a certain rate of compound interest, my money triples in 6 years. In how many years it will increase to 27 times the original principal invested?
a. 54 years b. 27 years
c. 18 years d. 45 years
15. Spirit evaporates at a rate of 10 % every minute if left open in air. One fine day, Ramu by mistake left the spirit jar open in air. After how much time, volume of the jar will be less than or equal to 60 % of the original volume?
a. 4 min b. 5 min
c. 6 min d. 7 min
16. Chaman has purchased a bungalow in Jhumritalaiya. Cost of the land on which the flat is built is Rs 81,00,000 and price of the building is Rs 1,21,00,000. Price of the land appreciates

- at a rate of 10% per annum whereas the cost of the building depreciates at a rate of 10 % per annum. After how many years, cost of the land and building will be equal?
a. 1 year b. 2 years
c. 3 years d. 4 years
17. What will be the approximate compound interest on Rs 20,000 for nine months at 20 % per annum compounded quarterly?
a. Rs 3,000 b. Rs 3,125
c. Rs 3,150 d. Rs 3,500
18. Ramu invested Rs 10,000 in a scheme of compound interest wherein rate of interest for first year was 5%, for next couple of years it was 10% and then onwards it was 20%. What will be the net amount that Ramu will receive at the end of 5 years if money was compounded annually?
a. Rs 18,295.2 b. Rs 17,465.2
c. Rs 18,467.8 d. Rs 18,269.8
19. Population of a town increases at a rate of 15% per annum. What was the population of this town two years back if the current population of the town is 29,09,500?
a. 20,00,000 b. 21,00,000
c. 22,00,000 d. 23,00,000
20. Billu borrowed Rs 10,000 partly from Kallu and partly from Lallu. He agreed to repay the money to Kallu at simple interest of 5% per annum for three years and to Lallu at simple interest of 8% per annum for two years If Billu repaid a total amount of Rs 11,560 in the complete transaction then find the money borrowed from Kallu.
a. Rs 4,000 b. Rs 5,000
c. Rs 6,000 d. Cannot be determined
21. In XYZ Ltd., every machine depreciates at a rate of 8% per annum. Find the cost of the machine two years hence, if the current cost of the machine is Rs 8,000.
a. Rs 5,120 b. Rs 6,858
c. Rs 9,452 d. Rs 6,772
22. Current population of Jhumritalaiya is 1,50,000. Male population of Jhumritalaiya increases at a rate of 5% per annum and female popula-

- tion increases at a rate of 10 % per annum. If the population of Jhumritalaiya is expected to be 1, 75, 050 two years hence, then what percentage of the current population is male?

a. 40% b. 60%
c. 90% d. 10%

23. Compound Interest on a certain sum of money for two years at 10 % per annum is Rs 1,680. Find the simple interest on the same sum of money at the same rate?

a. Rs 1,550 b. Rs 1,665
c. Rs 1,600 d. Rs 1,650

24. The simple interest on a certain sum of money will be Rs 200 after 4 years. For the next 4 years if the principal is trebled, then what will be the additional interest in next four year because of the increase in principal?

a. 300 b. 450
c. 400 d. 500

25. A sum was given at simple interest at a certain rate for 3 years. It would have fetched Rs 90 more had it been charged interest at 3% higher rate. Find the Sum?

a. 1100 b. 1000
c. 1200 d. 500

26. A sum of Rs 800 amounts to Rs 960 in 4 years at simple interest. What will be the amount if the rate of interest is increased by 4%?

a. 1088 b. 1188
c. 188 d. 1888

27. If a certain sum of money becomes double at simple interest in 10 years, what would be the rate of simple interest per annum?

a. 12 b. 10
c. 11 d. 12.5

28. A sum of money becomes $\frac{9}{4}$ of itself in 5 years. Find the rate of simple interest charged?

a. 20 b. 30
c. 25 d. 45

29. A sum of money at simple interest becomes four times of itself in 16 years. Find in how many years it will become 16 times at the same rate?

a. 40 b. 45

c. 80 d. 90

30. The population of Dalsing Sarai is 300000. If the annual birth rate is 6% and death rate is 4%, then calculate the population of Dalsing Sarai after 2 years?

a. 220000 b. 320000
c. 12100 d. 312120

31. Find the compound interest on Rs 128000 for 1 year at the rate of 10% per annum compounded quarterly.

a. 1300 b. 1100
c. 13288 d. 1200

32. If the annual increase in the population of a town is 5% and the present population be 92610000, what was the population three years ago?

a. 8×10^7 b. 8×10^6
c. 8×10^5 d. 7×10^7

33. The simple interest on a sum is $\frac{1}{4}$ th of the principal and the rate of interest per annum was equal to the number of years, find the rate per cent.

a. 6 b. 9
c. 5 d. 8

34. Niraj lent Rs 1200 to Nitin for 2 years, and Rs 1500 to Raghunath for 4 years and received Rs 420 as interest from both of them. Find the rate of interest if rate of interest in both the cases is same is calculated as Simple Interest.

a. 5 b. 6
c. 4 d. 10

35. The compound interest on a certain sum of money for a period of 2 years is Rs 80.40 and the simple interest is Rs 80. Find the sum of money invested.

a. 5000 b. 4000
c. 4500 d. 6000

36. A sum of Rs 400 is lent for 4 years at 5% per annum. Find the simple interest and the amount.

a. 80 b. 100
c. 200 d. 50

37. In what time, at simple interest will sum of money treble itself at 20%.

a. 12 b. 10

- c. 15 d. 20
38. The value of a machinery decreases every year at the rate of 15% on its value at the beginning of that year. If its present value be Rs 4913, what was its value 3 years ago?
 a. 7000 b. 9000
 c. 8000 d. 4500
39. A certain sum of money amounts to Rs 960 in 2 years and to Rs 1200 in 5 years at certain rate of Simple Interest. Find the rate of interest per annum?
 a. 10 b. 12
 c. 12.5 d. 15
40. A certain sum of money becomes 4 times of the principal in 12 years. Find the rate of simple interest per annum.
 a. 20 b. 25
 c. 35 d. 40
41. The rate of simple interest for the first 4 years is 8% per annum. For the next 3 years, 6% per annum. If a man lent out a sum of 1600 for 7 years, find the total interest accrued?
 a. 900 b. 400
 c. 800 d. 825
42. Rs 4800 is lent at compound interest of 5% per annum for 3 years Find the amount after 3 years
 a. 5550 b. 5556.6
 c. 1200 d. 1300
43. What is the difference between CI and SI for the sum of Rs 4000 over a period 2 years if CI is calculated at 10% and SI is calculated 12%.
 a. 120 b. 240
 c. 360 d. 450
44. Find the principal if the interest compounded at the rate of 20% per annum for two years is Rs 880.
 a. 2000 b. 2500
- c. 1500 d. 750
45. Amit and Nitin invest Rs 6000 and Rs 8000 respectively in a business. If Amit triples his capital after 6 months, then in what proportion should Amit and Nitin divide that years profits?
 a. 4:7 b. 3:2
 c. 1:5 d. 8:5
46. A sum of Rs 4600 is lent out on SI in two parts in such a way that the interest on one part at 5% for 6 years is equal to that on another part at 2% for 8 years Find the sum lent out at 5%:
 a. 1600 b. 2000
 c. 1800 d. 2400
47. If Rs 1200 amounts to Rs 1500 in 5 years, then what sum of money will it amount to in 9 years at the same rate of interest? Assume mode of interest to be Simple Interest.
 a. 1800 b. 1640
 c. 1740 d. 1960
48. A sum of Rs 1500 is given on CI for a period of $2\frac{1}{4}$ years The rate of interest charged is 10% p.a.compounded annually. Find the sum that amounts to:
 a. 1950 b. 1800
 c. 2000 d. 1500
49. The present worth of a car is Rs 64000. The annual depreciation of its value occurs at 5% per annum. Find the value of the car after three years.
 a. 59852 b. 54872
 c. 60452 d. 52785
50. A man invests Rs 5000 for 3 years at 5% p.a. compound interest reckoned yearly. Income tax at the rate of 20% on the interest earned is deducted at the end of each year. Find the amount at the end of the third year.
 a. Rs 5624.32 b. Rs 5630.50
 c. Rs 5788.125 d. Rs 5627.20



Profit, Loss & Discount

Some Important Concepts

Cost Price (CP):

The amount which is spent to make the product is its cost price. It also includes the cost of raw material.

Sales Price (SP):

It is the price at which the product is sold.

Marked Price:

It is the price which is marked on the product.

Profit:

If the sale price is more than cost price then the difference is realized as profit.

Loss:

If the cost price is more than the sale price, then the person makes a loss.

Discount:

Discount is the rebate offered on the selling price or marked price of the product.

Types of costs:

1. Direct Costs or Variable Costs:

These cost are directly associated with the direct selling of the product. In other words, that part of cost that varies directly with every additional unit of the product sold is called the Direct cost or Variable cost.

For example, if the variable cost for selling a T-shirt for Rs 100 is Rs 10, then the variable cost for selling 10 units of the same T-shirt is $10 \times 10 = \text{Rs } 100$.

2. Indirect Costs (Overhead Costs) or Fixed Costs:

The costs which have to be incurred irrespective of the number of items sold are called Indirect or Fixed Costs for example, rent of the office.

3. Semi Variable Cost:

These costs behave as fixed cost under normal conditions but have to be increased when a certain level of sales figure is reached. For example if sales increase to such an extent that additional space is required to accommodate the increase in work due to increase in sales, so rent of the office will be a part of semi variable cost.

The concept of margin

The difference between the value of the selling price and the variable cost for a product is known as the margin or contribution of the product.

Break Even:

The Break Even point is defined as the volume of sale at which there is no profit or loss. This point is also called the Break Even sales.

For example:

A shop owner has to pay a rent of Rs 10000 per month and salaries of Rs 50000 to the staff. He sells jeans of only one variety for Rs. 1000 each. The direct cost in making one jeans is Rs. 500 per jeans.

Then the margin is $1000 - 500 = \text{Rs } 500$ per jeans

Hence, Break Even sales = fixed cost / margin per unit = $60000 / 500 = 120$. Hence the shop owner has to sell 120 jeans to achieve the break even.

After that with every additional piece of jeans sold will lead him to the profit.

$$\text{Profit} = (\text{Actual sales} - \text{break even sales}) \times \text{margin}$$

$$\text{Loss} = (\text{Break Even sales} - \text{actual sales}) \times \text{margin}$$

Formulae:

$$1. \text{ Profit (P)} = \text{SP} - \text{CP} \dots\dots\dots (\text{SP} > \text{CP})$$

$$2. \text{ Loss (L)} = \text{CP} - \text{SP} \quad (\text{CP} > \text{SP})$$

$$3. \text{ Percentage Profit} = \frac{P}{CP} \times 100$$

$$4. \text{ Percentage Loss} = \frac{L}{CP} \times 100$$

$$5. \text{ SP} = CP + \frac{r}{100} \times CP = CP \left[1 + \frac{r}{100} \right]; \text{ where } r \text{ is percentage profit.}$$

$$6. \text{ CP} = \frac{SP \times 100}{100+r}; \text{ where } r \text{ is percentage profit.}$$

$$7. \text{ SP} = CP - \frac{r}{100} \times CP = CP \left[1 - \frac{r}{100} \right]; \text{ where } r \text{ is percentage loss.}$$

$$8. \text{ CP} = \frac{SP \times 100}{100-r}; \text{ where } r \text{ is percentage loss.}$$

$$9. \text{ Percentage discount} = \frac{\text{Discount}}{\text{Market Price}} \times 100$$

Examples:

1. A shopkeeper buys ball pens in bulk for Rs 15 each. He sells them for Rs 40 each. Calculate the profit on each ball pen in Rs and as a percentage of the cost price.

Given:

Cost price = 15, Selling price = 40

$$\text{Profit} = \text{selling price} - \text{cost price} = 40 - 15 = 25$$

Expressing the profit as a percentage of the cost price:

$$\text{Profit percentage} = \frac{P}{CP} \times 100 = \frac{25}{15} \times 100 = 166.7\%$$

2. A school bookshop sells an old book for Rs 50, making a loss of 6%. What is the cost price of the book, and what is the cash value of the loss?

Given: selling price = 50, loss = 6% of cost price

But, (cost price) = (selling price) + loss

Expressing percentages of the cost price: (cost price) = (selling price) + loss

$$\text{i.e. } 100\% = x\% + 6\%$$

This means that the selling price is $(100-6) = 94\%$ of the cost price.

$$\text{Selling price} = \frac{94}{100} \times CP$$

$$\text{Cost Price} = \frac{100}{94} \times SP = \frac{100}{94} \times 50 = \text{Rs. } 53.19$$

$$\text{Loss} = (\text{cost price}) - (\text{selling price}) = 53.19 - 50 = \text{Rs } 3.19$$

Some important points:

- ❖ If two articles are sold at same selling price, say Rs x, one at a profit of p% and the other at a loss of p %, then there is always an overall loss given by .

Example:

Two goods are sold at the same price one at a loss of 15 % and other at a profit of 15 %. Find the overall profit/loss.

Solution:

$$\text{Overall loss } \frac{p^2}{100}\% = \frac{15^2}{100}\% = 2.25\%$$

- ❖ By using a false weight, if a substance is sold at cost price, the overall gain percentage is given by:

$$\text{Gain Percentage}$$

$$= \frac{\text{True Weight} - \text{False Weight}}{\text{False Weight}} \times 100$$

Example:

A dishonest shopkeeper professes to sell his articles at cost price but uses a faulty weight of 920 grams instead of 1 kg. What is his overall gain percentage?

Solution:

The shopkeeper gets price of 1000 grams on the sale of 920 grams. Hence overall gain percentage is:

$$\frac{1000 - 920}{920} \times 100 = \frac{80}{920} \times 100 = \frac{200}{23}\%$$

- ❖ Buy m get n free, i.e. if (m + n) articles are sold at a cost price of m articles, then the percentage discount

$$= \frac{m}{m+n} \times 100$$



QUESTIONS

1. If cost price of 8 articles is same as the selling price of 10 articles then find the profit or loss percentage?
 a. 20 %, profit b. 25 %, profit
 c. 20 %, loss d. 25 %, loss
2. If I gain from Cost Price of 7 articles by selling 21 articles then what is my net profit percentage?
 a. 33.33 % b. 50 %
 c. 25 % d. 20 %
3. If I gain from Selling Price of 7 articles by selling 21 articles then what is my net profit percentage?
 a. 33.33% b. 50%
 c. 25% d. 20%
4. Ramu sold two horses for Rs 19200 each, one at a profit of 20 % and the other at a loss of 20 %. Find his net profit or loss
 a. 2% loss b. 4% loss
 c. 4% profit d. No loss no gain
5. Ramu sold an article at a loss of 3%. Had he been able to sell it for Rs 100 more, he would have gained 2 %. Find the price at which he sold that article?
 a. Rs 2000 b. Rs 1940
 c. Rs 2040 d. Rs 3880
6. By selling bananas at Rs 30 per dozen, Ramu loses 10 % of his cost price. At what price, he should sell bananas in order to gain 20 % on the cost price of bananas?
 a. Rs.35 per dozen b. Rs.38 per dozen
 c. Rs.40 per dozen d. Rs.50 per dozen
7. A shopkeeper uses a weight of 900 grams for 1 kg. If he claims to purchase and sell his goods at the same price, then what is his profit or loss percentage?
 a. 11.11%, profit b. 11.11%, loss
 c. 10%, profit d. 10%, loss
8. Profit earned while selling an article for Rs 9600 is 20 % more than the loss incurred for selling the same article for Rs 8500. Find the cost price of that article.
 a. Rs 8,800 b. Rs 9,000
 c. Rs 9,200 d. Rs 9,400
9. At a particular shop, a scheme is running which says "buy four get one free". What is the discount percentage in offering?
 a. 33.33 % b. 25 %
 c. 20 % d. 16.66%
10. Ramu purchased certain shares of Rs 100 each at a 10% discount and sold them at 20% premium. Find the net profit percentage gained by Ramu in the complete transaction if he invested Rs 4500 in the deal and assume there is no transaction cost.
 a. 16.66 % b. 20 %
 c. 25 % d. 33.33%
11. Ramu claims to sell fruits at 10 % profit, but he uses a weight of 900 grams for 1 Kg. Find the net profit percentage that he makes on the deal.
 a. 20% b. 21%
 c. 22.22% d. 25%
12. Ramu sells milk in such a fashion that 20 % of the solution that he has contains water. If he increases the price of milk by 10 % over the cost price then what is the net profit percentage that he makes.
 a. 30 % b. 37.5 %
 c. 35 % d. 33.33 %
13. A garment shopkeeper runs a scheme of selling one free garment on purchase of every 3 garments but he has doubled the prices of all the garments. What is the profit percentage that he makes on the complete deal?
 a. 33.33 % b. 50 %
 c. 66.66 % d. 80 %

14. A man sold an article at 20 % profit. Had he purchased the article for Rs 50 less and sold for 10 % more then he would have gained 50 %. What was the original cost price of the article?
- Rs 405
 - Rs 415
 - Rs 455
 - Rs 485
15. There are two shopkeepers, first one provides three successive discounts of 10%, 10% and 10% whereas the second one provides three successive percentage discounts of 20 %, 5 % and 5 %. Which of the two is providing a higher discount?
- First shopkeeper
 - Second shopkeeper
 - Both of them are giving equal discounts
 - None of the above
16. A shopkeeper purchased 100 eggs at 50 paisa per egg. 10 of these eggs were broken during transportation. If this shopkeeper sells eggs with a discount of 33.33 % and wants to gain 20 % profit then what should be the marked price of these eggs?
- 80 paisa per egg
 - 90 paisa per egg
 - Re 1 per egg
 - Rs 1.1 per egg
17. Ramu purchased mangoes at 7 Kg for Rs 32 and sold them at 5 Kg for Rs 32. If he earned a total amount of Rs 160 by selling the complete lot of mangoes then how many mangoes did he sell?
- 80 Kg
 - 82.5 Kg
 - 85 Kg
 - 87.5 Kg
18. Ramu increases price of all his commodities to a level which is 20 % more than the cost price. Then he provides a discount of 10 % on cash payment. If half of his goods are sold in cash and half of his goods are sold by credit card, then what is the profit percentage he makes in total?
- 15 %
 - 14 %
 - 16 %
 - 18%
19. Ramu used to sell his goods at 20 % profit. If cost price of all his goods is decreased by 10 % and he increased all his selling prices by 10

% then what is his new profit percentage?

- 36.66%
- 43.33%
- 46.66%
- 53.33%

20. A shopkeeper fixed the price of sugar in such a way that despite giving a discount of 10 % this person is able to make a 20 % profit. If he purchases sugar at Rs 30 per Kg then approximately how much should be his marked price?
- Rs 35 per kg
 - Rs 38 per kg
 - Rs 40 per kg
 - Rs 45 per kg

Directions for questions 21 and 22: These questions are based on the information given below.

Karan makes and sells tables at a certain price. He sells each table at Rs 300. The cost, in rupees, of producing Y tables, per day is $2600 + py + 2y^2$ where P and Q are constant. When he raises the daily production from 10 to 20, his daily production cost increases by 25%, while if he raises it from 20 to 30, his daily production cost again increases by 25%. There are no restrictions on the demand for his chairs or his sales. He aims to maximize his net profit.

21. Find the number of tables Karan must produce daily.
- 125
 - 120
 - 130
 - 140
22. The maximum daily profit, (in Rs), that Karan can realize is approximately:
- Rs 13,000
 - Rs 13,050
 - Rs 13,200
 - Rs 13,025
23. In a particular business Ajay, Navjot and Vikas invested in the ratio of 2:3:6 for the same period of time. Find the difference in the shares of Ajay and Vikas if that between Ajay and Navjot is Rs. 500.
- Rs 2000
 - Rs 1000
 - Rs 500
 - Cannot be determined
24. Ankit adds 20% water to some quantity of pure milk and sells the mixture at the same rate at which he had bought the pure milk. After selling a few litres of the mixture he is caught and has to pay a fine of Rs. 200. After paying the fine he realises that he has suffered a loss of Rs 100. How many litres of the mixture did he

- sell if he bought pure milk for Rs 10 per litre?
- 100 litres
 - 85 litres
 - 50 litres
 - 60 litres
25. A dealer buys dry fruits at Rs 100, Rs 80 and Rs 60 per kilogram. He mixes them in the ratio 3:4:5 by weight, and sells at a profit of 50%. At what price per kilogram does he sell the dry fruit?
- Rs. 80
 - Rs. 100
 - Rs. 95
 - None of these
26. Instead of a metre, of cloth, a cloth merchant uses a 120 cm scale while buying, but uses a 80 cm scale while selling the same cloth. If he offers a discount of 20% on cash payment, what is his overall profit percentage?
- 20%
 - 25%
 - 40%
 - 15%
27. Two partners invested Rs 100000 and Rs 80000 respectively in a business. They decided to donate 40% of their profit to old age homes and then share the balance profit in the ratio of their investments. If one partner receives Rs 1200 more than the other. Find the total profit
- Rs. 1080
 - Rs. 18000
 - Rs. 1800
 - Rs. 1.8 Lakh
28. Shamvav and Tanya entered into a joint venture, investment in the ratio 5:6. After 3 months, Shamvav withdraws 1/5th of his money and Tanya withdraws 1/6th of her money. If the profit at the end of the year was Rs. 22800, what was Tanya's share?
- 12600
 - 63000
 - 11400
 - None of these
29. A dealer offers a cash discount of 20% and still makes a profit of 20% when he further allows 16 articles to a dozen to a particularly sticky bargainer. How much percent above the cost price were his wares listed?
- 100%
 - 80%
 - 75%
 - $66\frac{2}{3}\%$
30. The ABC magazine prints 5000 copies for Rs 5,00,000 every month. In the July issue of the magazine, ABC distributed 500 copies free.

Besides it was able to sell 2/3 of the remaining magazines at 20% discount. Besides, the remaining magazines were sold at the printed price of the magazine (which was Rs 200). Find the percentage profit of ABC in the magazine ventures in the month of July.

- 56%
 - 24.8%
 - 28.5%
 - 22.6%
31. A stockist wants to make some profit by selling sugar. He contemplates about various methods. Which of the following would maximize his profit?
- Sell sugar at 10% profit
 - Use 900g of weight instead of 1 kg.
 - Mix 10% impurities in sugar and selling sugar at cost price
 - Increase the price by 5% and reduce weights by 5%
- I or III
 - II
 - II, III and IV
 - Profits are same
32. A merchant makes a profit of 20% by selling an article. What would be the percentage change in the profit percent had he paid 10% less for it and the customer paid 10% more for it?
- 120%
 - 125%
 - 133.33%
 - 150%
33. A fruit seller buys 50 water melons at the rate of Rs. 15 per kg and average weight of water melon is 4kg. Due to the hot and dry weather 25% of the weight content evaporates after two days of transportation. Now the fruit seller wants to gain 10% profit so at what price should he sell a water melon per kg?
- Rs. 22
 - Rs. 16.5
 - Rs. 20
 - Rs. 17.5
34. A tradesman fined his selling prices of goods at 30% above the cost price. He sells half the stock at this price, one quarter of his stock at a discount of 15% on the original selling price and rest at a discount of 30% on the original selling Price. Find the gain percent altogether.

- a. 14.875% b. 15.375%
 c. 15.575% d. 16.5%
35. A manufacturer marks his goods in advance at 80 percent more than the cost price, but he allows 15 articles to the dozen and also 10 percent discount for cash. What rate of profit on his outlay does he obtain from a customer who pays cash?
 a. 18.2% b. 25%
 c. 29.6% d. Cannot be determined
36. Manish sells a microwave at a discount on its market price of Rs 2500. But in a clearance sale he gives an additional discount of 20% thus selling it for Rs 1500. Find the first discount.
 a. 20% b. 25%
 c. 12% d. 30%
37. Ankur a dealer marks up the price of his goods by 20% and gives a discount of 10% to the customer, besides he also cheats both his supplier and his buyer by 100gms while buying or selling 1 kilogram. Find the percentage profit earned by the shopkeeper
 a. 20% b. 25%
 c. 32% d. 27.5%
38. A trader man marks an article at Rs205 more than the cost price. He allows a discount of 10% on the marked price. Find the profit percent if the cost price is Rs x
- a. $\left[\frac{x}{18450} - 100 \right]$
 b. $\left[(18450) - 10x \right]$
 c. $\left[\frac{x}{18450} - 100 \right]$
 d. $\left[\frac{18450}{x} - 100 \right]$
39. By selling 5 dozen mangoes for Rs156 it was found that $\frac{3}{10}$ th of the outlay was gained. What should the retail price per mango be in order to gain 60%?
 a. Rs 4 b. Rs 2
 c. Rs 3.2 d. Rs 4.2
40. A sports goods shop owner has 6 packets of balls each containing the same number of balls. If he sells each ball at Rs8 he gains Rs96. If there was 1 ball less in each packet and he sells each ball at Rs 9 then he gains Rs 90 in the transaction. Assume that the cost price of each packet of balls remained the same. Find the cost price of each packet of balls
 a. Rs 36 b. Rs 48
 c. Rs 64 d. Rs 52
41. Saket travels from A to B to buy goods which he can get 10% cheaper in B than in A. If the expenses of the journey are Rs15 and he makes a clear saving of Rs10, how much does he pay for the goods?
 a. Rs 225 b. Rs 200
 c. Rs 150 d. Cannot be determined
42. The cost price of 10 articles is equal to the selling price of 9 articles. Find the profit percent.
 a. 10.33% b. 11.11%
 c. 13.1% d. 9%
43. If cost price of 10 articles is equal to selling price of 8 articles. How much profit does the shopkeeper earn?
 a. 10% b. 12.5%
 c. 25% d. 0%
- Directions for questions 44:** Read the following and answer the questions that follow.
- Aryan and Suman decided to sell their cars each at Rs 36000. While Aryan decided to give a discount of 8% on the first Rs 8000, 5% on next Rs 12000 and 3% on the rest to the buyer Shakshi, Suman decided to give a discount of 7% on the first Rs 12,000 and 6% on the next Rs 8,000 and 5% on the rest to the buyer Rajesh.
44. If each of them got the payments on time what is the approximate of discounts offered by the two?
 a. 43:53 b. 53:63
 c. 63:73 d. 73:83
45. fan working at 100% energy efficient consumes 3 units in a day. An office has 30 fans which

are consuming 4 units per day per fan. After maintenance they become 100% energy efficient. Taking cost of one unit of electricity equivalent to Rs. 5, how much profit % can office earn per day?

- a. 15%
- b. 20%
- c. 25%
- d. 30%

46. 1 Kg of tea and 4 kg of sugar cost Rs6.28. But if the cost of sugar rose by 50% and that of tea by 10%, the same quantity of tea and sugar would cost Rs7.42. Find the prices per kg of tea and sugar respectively.

- a. Rs 1.25 and Rs 1.28
- b. Rs 1.42 and Rs 1.25
- c. Rs 1.28 and Rs 1.42
- d. Rs 1.28 and Rs 1.25

47. If goods are purchased for Rs120, and one-third of them are sold at a loss of 5%, then at what profit percentage should the rest be sold to obtain an overall profit percentage of 5% ?

- a. 20%
- b. 10%

- c. 15
- d. 20%

48. A trader marks his product 15% above the cost price and offers a discount of 20%. Find the loss percentage incurred by the trader.

- a. 16%
- b. 13%
- c. 8%
- d. 4%

49. The Price of diesel was increased by 30% in the budget. Due to pressures from the opposition, the Finance Minister had to reduce the price of diesel by 15%. What is the effective percentage increase in the price of diesel?

- a. 5.25%
- b. 7%
- c. 10.5%
- d. 15%

50. Amnish invests Rs 28200 in certain stocks at Rs. 94. He then sells it when it is quoted at Rs. 106. Find the overall profit percentage on the transaction.

- a. 13 %
- b. 12.50 %
- c. 12 %
- d. 12.75 %

6

Ratio & Proportion

Ratio and Proportion:

The ratio of two quantities is the fraction that one is of another i.e. the ratio of 2 to 3 is $2/3$. The first term of a ratio is called antecedent and the second term is known as consequent.

The equality of two ratios is called Proportion.

Since $1:4 = 2:8$, we write $1:4::2:8$ and say that 1, 4, 2 and 8 are in proportion. Again, we have
4th Proportion = $(2\text{nd} \times 3\text{rd}) / 1\text{st}$
3rd Proportion = $(1\text{st} \times 4\text{th}) / 2\text{nd}$.

Continued Proportion:

Three quantities a, b, c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

Similarly if a, b, c, d are in continued proportion

$$\text{then } \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Operations on Proportions:

If we have $\frac{a}{b} = \frac{c}{d}$

Then (i) $\frac{a}{c} = \frac{b}{d}$ (Alternendo)

(ii) $\frac{b}{a} = \frac{d}{c}$ (Invertendo)

(iii) $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

(iv) $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

(v) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = \frac{a+k_1 + c+k_2 + e+k_3 + \dots}{b+k_1 + d+k_2 + f+k_3 + \dots}$

Where k_1, k_2, k_3, \dots are any real numbers except all of them being simultaneously equal to zero.

❖ Let x: y be a ratio, which can also be written

as x/y . We will try to find out what will happen when a constant a is added both to the numerator and denominator.

CASE I ($x/y < 1$):

If $x/y < 1$, then addition of a constant positive number to numerator and denominator leads to a bigger ratio than the ratio itself, i.e. $\frac{x+a}{y+a} > \frac{x}{y}$ for $\frac{x}{y} < 1$

Where a is a constant positive number.

For Example:

$1/2$ is less than 1 and when we add 2 to both numerator and denominator we get $3/4$ and $3/4$ is greater than $1/2$. Similarly, Subtraction leads to a Smaller ratio.

$$\frac{x}{y} < \frac{x-a}{y-a} \text{ for } \frac{x}{y} < 1$$

Let's consider a fraction $5/10$, if 5 is subtracted from numerator as well as from denominator, we get 0 and it is less than $5/10$ (i.e. $1/2$) Thus the rule for the case of subtraction is the reverse of the case of addition, as can be easily seen by the given example.

CASE II ($x/y > 1$):

The above rule gets totally and exactly reversed for

$x/y > 1$. Therefore, $\frac{x+a}{y+a} > \frac{x}{y}$ for $\frac{x}{y} > 1$

And $\frac{x}{y} < \frac{x-a}{y-a}$ for $\frac{x}{y} > 1$

Direct and Indirect Variation:

Variation, in general, will concern two variables; say height and weight of a person, and how, when one of these changes, the other might be expected to change.

- We have **direct** variation if the two variables change in the same sense; i.e. if one increases, so does the other.

- We have **indirect** variation if one going up causes the other to go down. An example of this might be speed and time to do a particular journey; so the higher the speed, the shorter the time.

Direct Variation:

When x and y are directly proportional, then doubling x will double the value of y ; and if we divide these variables we get a constant result.

Example:

Given that y and x are directly proportional, and $y = 2$ when $x = 5$, find the value of y when $x = 15$.

We first find value of k , using $\frac{y}{x} = k$ i.e. $\frac{2}{5} = k$

Now use this constant value in the equation $y = kx$ for situation when $x = 15$.

$$y = \frac{2}{5} \times 15 \rightarrow = \frac{30}{5} = 6$$

Indirect or Inverse Variation:

We know that 'the higher the speed, the shorter the time.'

$$xy = k \text{ or } y = \frac{k}{x}$$

In this case, if we double the speed, we halve the time. So the product, speed \times time = constant.

In general, if x and y are inversely proportional, then the product $x \times y$ will be constant.

Example:

If it takes 4 hours at an average speed of 90 km/hour to do a certain journey, how long would it take at 120 km/hour?

$k = \text{speed} \times \text{time} = 90 \times 4 = 360$ (k in this case is the distance.)

$$\text{Then time} = \frac{k}{\text{speed}} = \frac{360}{120} = 3 \text{ hours.}$$

Some important results of proportions:

- ❖ If $a \propto b$ and $b \propto c$, then $a \propto c$
- ❖ If $a \propto c$ and $b \propto c$, then $a + b \propto c$ and $\sqrt{ab} \propto c$
- ❖ If $a \propto bc$, then $b \propto \frac{a}{c}$ and $c \propto \frac{a}{b}$
- ❖ If $a \propto b$ and $c \propto d$, then $ac \propto bd$

- ❖ If $a \propto b$, then $a^n \propto b^n$
- ❖ If $a \propto b$ then $ap \propto bp$, where p is any quantity variable or constant.

Application of ratios:

Partnership:

Partnership is an association of two or more persons who invest their money in order to carry on a certain business.

- ❖ A partnership is called a **simple partnership**, if the capitals of the partners are invested for the same time period.

In case of simple partnership, the profit or loss is divided in the ratio of their investments.

If A and B are partners in a business, then:

$$\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}}$$

$$\text{or } \frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

If there are three partners A, B and C, then:

Investment of A: Investment of B: Investment of C = Profit of A: Profit of B: Profit of C

OR

Loss of A: Loss of B: Loss of C

Example:

Three partners A, B and C invest Rs. 3200, Rs. 3600 and Rs. 4600 respectively in a business. The total profit is Rs. 798.

Then profit is to be divided in the ratio 32:36:46 or 16:18:23

Now, A's share of profit = $\frac{16}{16+18+23} \times 798 = \text{Rs. 224}$

B's share of profit = $\frac{18}{16+18+23} \times 798 = \text{Rs. 252}$

C's share of profit = $\frac{23}{16+18+23} \times 798 = \text{Rs. 322}$

- ❖ A partnership is called a compound partnership, if the capitals of the partners are invested for different time periods.

In this case we calculate the product of the capital invested and the period for which it is invested for all the partners, this product is called monthly equivalent investment. Now the profit or loss is divided

in the ratio of their monthly equivalent investment.

If A and B are partners in a business, then:

$$\frac{\text{Monthly equivalent investment of A}}{\text{Monthly equivalent investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}}$$

$$= \frac{\text{investment of A} \times \text{Period of Investment of A}}{\text{investment of B} \times \text{Period of Investment of B}}$$

Or

$$\frac{\text{Monthly equivalent investment of A}}{\text{Monthly equivalent investment of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

$$= \frac{\text{investment of A} \times \text{Period of Investment of A}}{\text{investment of B} \times \text{Period of Investment of B}}$$

Example:

A, B, C enter into a partnership. A invests Rs. 2400 for 4 months, B invests Rs. 2800 for 8 months and C invests Rs. 2000 for 10 months. They gain Rs. 1170 altogether.

In this case Monthly equivalent investment of A = $2400 \times 4 = 9600$

Monthly equivalent investment of B = $2800 \times 8 = 22400$

Monthly equivalent investment of C = $2000 \times 10 = 20000$

The profit is divided in the ratio 96: 224: 200 i.e. 12:28:25

Now, A's share of profit = $\frac{12}{65} \times 1170 = \text{Rs. } 216$

B's share of profit = $\frac{28}{65} \times 1170 = \text{Rs. } 504$

C's share of profit = $\frac{25}{65} \times 1170 = \text{Rs. } 450$

Races:

The concepts of ratio are also used in solving problems on races. For example: The statement "In a race of 1000 meters, A can give B a start of 100 meters" means when A runs 1000 meters, B runs 900 meters. Thus the ratio of the speeds of A and B is 10:9.

Time and Work:

Example:

A can do a piece of work in 21 days, B is 40 % more efficient than A. In how many days B will complete the same work?

Solution:

Ratios of the efficiencies of A and B = 100: 140 or 5:7

Since efficiency is inversely proportional to the number of days, the ratio of days taken to complete the job is 7: 5

So number of days taken by B = $(5/7) \times 21 = 15$ days.

Unitary Method:

Consider the following example:

9 dozens of bananas cost Rs. 540, what will be the cost of 4 dozens of bananas? Here cost of 9 dozens of bananas is given. This part of question is called a statement. From this the cost of 1 dozen of bananas is to be obtained by compound division and from this value of one dozen of bananas the cost of 4 dozens of bananas is obtained by compound multiplication.

This method in which first the value of one unit is to be found is called the unitary method.

Example:

12 men or 18 women can reap a field in 7 days, in what time can 4 men and 8 women can reap the field.

Solution:

12 men = 18 women

1 man = $(18/12) = (3/2)$ women

4 men = $4 \times (3/2)$ women = 6 women

4 men + 8 women = $(6 + 8) = 14$ women

The more the women less the number of days, i.e. indirect proportion

Hence 18: 14 :: x: 7

i.e. $x = (18/14) \times 7 = 9$ days (Answer)

Example:

If 25 elephants consume 18 quintals in 36 days, how long will 28 quintals last for 30 elephants?

Solution:

Let required number of days be m

More elephants less the number of days (indirect proportion)

More food more the number of days (direct proportion)

Hence Elephant 30: 25

:: 36: m

Food 18: 28

Hence $30 \times 18 \times m = 25 \times 28 \times 36$

Or $m = \frac{46}{3}^2$ days (Answer)

Example:

If the wages of 5 men for 12 days be Rs. 6000 then what will be the wages of 6 men for 20 days?

Solution:

Let the required wages be m

More men more wages (direct proportion)

More days more wages (direct proportion)

Hence Men 5: 6

:: 6000: m

Days 12:20

Hence $5 \times 12 \times m = 6 \times 20 \times 6000$

Or $m = 12,000$ (Answer)

QUESTIONS

1. A and B are in a ratio 5:7 and sum of these two numbers is 108. Find the difference of these two numbers?
- 18
 - 8
 - 9
 - 12
2. Raja, Raju and Ram have invested Rs. 5,00,000 Rs. 7,00,000 and Rs. 9,00,000 respectively in a project for 4 months, 5 months and 6 months respectively. If the total income at the end of complete project was Rs. 2,18,000 then find the share of Raju?
- Rs. 40,000
 - Rs. 70,000
 - Rs. 1,08,000
 - None of the above
3. If $a:b = 4:5$ and $b:c = 3:4$, then find the value of $a:b:c$?
- 8:10:15
 - 6:9:12
 - 12:15:20
 - 12:15:22
4. If $a:b = 4:7$ then b is how much per cent more than a?
- 42 %
 - 75 %
 - 27 %
 - 100 %
5. If $n:4 = 9:n$, then what is the value of n?
- 6
 - $1/6$
 - 8
 - $1/8$
6. Two dozen of glasses were being transferred from one place to another when a few of these glasses fell on the floor and broke down. Which of the following cannot be the ratio of broken and unbroken glasses?
- 1:2
 - 1:3
 - 1:4
 - 1:5
7. If price of sugar increases by 25 %, then by what percentage sugar consumption should be decreased to maintain the same expenditure?
- 20 %
 - 25 %
 - 33.33 %
 - None of the above
8. While organizing a party, average contribution per head comes out to be Rs 100 when there are exactly 100 boys attending the party. But the average contribution increases to Rs 120 when there were 80 boys attending the party. What shall be the approximate average contribution if exactly 120 boys are attending the party?
- Rs 80
 - Rs 85
 - Rs 70
 - Rs 75
9. In an exam, Ramu and Kallu have scored in the ratio of 1:2. But had both of them scored 3 more marks each, then they would have scored in the ratio of 3:5. How much marks did Ramu score?
- 6
 - 9
 - 12
 - 15
10. If $a:b = 3:5$, then find the value of $\frac{3a+4b}{4b-3a}$:
- 3:5
 - $11/29$
 - $29/11$
 - $9/20$
11. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \dots = \frac{y}{z} = \frac{1}{2}$, then find $\frac{a+b+c+d+\dots+y}{b+c+d+e+\dots+z}$?
- 1
 - $1/2$
 - $1/3$
 - $1/4$
12. There is a fruit juice which contains 20 % fruit pulp. How much pure pulp should be added to 1 kg of juice to increase the concentration of fruit pulp to 25 %?
- 33.33 grams
 - 50 grams
 - 66.66 grams
 - 100 grams
13. Price of a piece of diamond is directly proportional to the square of the weight of the

- diamond. If a diamond of Rs. 5,000 breaks into two equal pieces then find the loss incurred?
- Rs 1,250
 - Rs 2,500
 - Rs 3,750
 - None of the above
14. Two gears are working with each other in such a way that first gear has 80 cogs and second gear has 16 cogs. If first gear takes 100 revolutions in 1 minute then second gear will take how many revolutions in 15 seconds?
- 5
 - 125
 - 2000
 - 80
15. There are two shopkeepers one of whom is giving a discount of 20 % and the second one is giving 20 % extra on each and every purchase. Which of these is a better deal to choose from if both claim the same price?
- First shopkeeper
 - Second shopkeeper
 - Both are equally good
 - None of the above
16. What is the mean proportion of 8 and 162?
- 24
 - 36
 - 54
 - 72
17. In a map, if 20 cm represents 400 km. What will be actual area of a country which is represented by 40 square cm on this map?
- 16000 sq Km
 - 1600 sq Km
 - 8000 sq Km
 - 80000 sq Km
18. A group of students decided to go for a sea trip whose cost is constant. If the number of students decrease by 25 then averages contribution per student increases by Rs 20. What will be the further increase in average contribution if 20 more students back out from the trip?
- Rs 25
 - Rs 29
 - Rs 15
 - Cannot be determined
19. Chaman, Raman and Karan started a business by investing Rs 4,500, Rs 6,750 and Rs 10,125 respectively. If their return on complete investment was in the ratio of 1: 2: 3 then find the ratio of time for which they invested their money.
- 3:4:4
 - 4:3:3
 - 4:3:3
 - 3:4:3
20. Ramu, Kallu and Billu started a business in partnership which required an investment of Rs 1,00,000 for one full year. So on 1st of January, Ramu invested Rs 50,000, Kallu invested Rs 30,000 and Billu invested Rs 20,000 in the year but on 1st of April, when Ramu required Rs 20,000, both Kallu and Billu gave him Rs 10,000 each and again on 1st of July Ramu required another 20,000 and again Kallu and Billu gave him Rs 10,000 each to rescue him. Finally on 1st of October Ramu gave back Rs 10,000 to both Kallu and Billu. Find the ratio in which they should divide their return on the investment?
- 3:3:4
 - 3:4:3
 - 4:3:3
 - 2:3:4
21. In what ratio should a 20% sugar solution be mixed with a 50% sugar solution so that the resultant solution has 40% sugar in it?
- 1 : 2
 - 2 : 1
 - 1 : 3
 - 3 : 1
22. 50 men were hired to complete a job in 50 days. But after 10 days, it was realized that only 10 % of the job is finished. How many men should be hired with effect from 11th day to finish the job at least 5 days in advance?
- 78
 - 79
 - 128
 - 129
23. Ramu, Kallu and Billu started a business by investing some money in the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. They had invested for some time in the ratio of $\frac{1}{4}:\frac{1}{3}:\frac{1}{2}$. If net income from this investment was Rs 1950, then what is the share of Kallu in the profit?
- Rs 650
 - Rs 600
 - Rs. 702
 - None of the above
24. If price of sugar increases by 20 %, then Ramu can purchase 4 Kg less sugar for Rs 120. What was the original price of sugar per Kg.?
- Rs 4 per Kg
 - Rs 5 per Kg

- c. Rs 6 per kg d. Rs 7 per Kg
25. Ramu can complete a piece of work in 120 days. Kallu is 20 % more efficient than Ramu. In how many days, Kallu can complete the same work?
 a. 100 days b. 96 days
 c. 90 days d. 80 days
26. Increase in height of a person is in proportion with the square root of his age till he attains the age of 16 years. And after 16 years of age, he stops growing. If a kid was 4 ft tall when he was 4 years old and 5 ft when he was 9 years old then what will be the final height that he will attain?
 a. 5 ft and 4 inches
 b. 5 ft and 8 inches
 c. 6 ft
 d. 6 ft ad 4 inches
27. If $x = a + b + c$, then what is the ratio of $a:b:c$ if $x - a:x - b:x - c = 4:5:6$?
 a. 5:3:1 b. 7:5:3
 c. 9:7:5 d. 11:9:7
28. If 5 is added to two numbers in the ratio 1:3, they will become 1:2. Which of the following is one of those two numbers?
 a. 3 b. 4
 c. 5 d. 6
29. If $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$, then what will be the value of $a:b:c$?
 a. $yz : xz : xy$ b. $x : y : z$
 c. $x^2 : y^2 : z^2$ d. $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$
30. If $\frac{a}{b} = \frac{1}{2}, \frac{b}{c} = \frac{1}{3}, \frac{c}{d} = \frac{1}{4}, \frac{d}{e} = \frac{4}{3}, \frac{e}{f} = \frac{4}{3}, \frac{f}{g} = 6$, then what is the value of $\frac{abc}{def}$?
 a. 1/144 b. 1/96
 c. 1/108 d. 1/72
31. 10 bottles of "Fumes Up" cost Rs. 55 if you return the bottles after emptying. If you want to keep the empty bottles 5 bottles of "Fumes

"Up" will cost Rs. 35. Pooja wants to buy 6 bottles. Had she promised to return 4 of the empty bottles, how much should she pay?

- a. 55 b. 36
 c. 42 d. 44
32. The ratio of the density of oil : petrol : diesel is 9 : 7 : 5. The density of diesel is 5 gm/cc. They are mixed in the ratio of 12 : 10 : 8 by weight. If a litre of petrol costs Rs. 25.2, then what is the value of petrol in 300 kg of the mixture?
 a. Rs. 326 b. Rs. 375
 c. Rs. 360 d. Rs. 355
33. Pipe A is connected to Vessel A and Pipe B is connected to Vessel B. If the volumes of vessel A : volume of vessel B :: 26 : 9 and they get filled in 2 minutes and 13 minutes respectively, find the ratio of the diameter of pipe A : pipe B, assuming that volume of water flowing through a pipe is proportional to the square of its diameter.
 a. 3 : 9 b. 13 : 6
 c. 19 : 9 d. 13 : 3
34. Three persons A, B and C agree to pay their hotel bill in the ratio 4 : 5 : 6. A pays the first day's bill which amounts to Rs. 45.75, B the second day's bill which amounts to Rs. 59.60 and C the third day's bill which amounts to Rs. 74.65. When they settle their accounts, which of the following is going to happen?
 a. C will pay A Rs. 2.25
 b. B will pay C Rs. 0.40
 c. A will pay C Rs. 1.35
 d. B will pay A Rs. 1.60

Directions (Questions 35–36) based on the following information.

A monkey distributed ladoos to two cats, cheating them of some and eating these himself. At the end of the distribution, if the black cat were to give some ladoos to the white cat, the white cat would have 5 times as many whole ladoos as the black cat. If the white cat were to give the same number of ladoos to

the black cat, the white cat would have 3 times the number of ladoos as the black cat.

35. What is the ratio of ladoos initially distributed to the white cat and black cat respectively?
- a. 10 : 3
 - b. 19 : 5
 - c. 5 : 21
 - d. Cannot be determined

36. If the total number of ladoos was thirty, what was the number of ladoos eaten by the monkey?
- a. 19
 - b. 5
 - c. 6
 - d. Cannot be determined

37. The salaries of three persons are in the ratio $a : b : c$; their expenses are in the ratio $p : q : r$. If the first person earns Rs. x and saves Rs. y , find the ratio of their savings, if $a = p$, $b = q$ and $c = r$.
- a. $ax : q(x - y) : r(x - y)$
 - b. $1 : q/p : r/p$
 - c. $(x - y) : q(x - y) : r(x - y)$
 - d. $(a - p) : (b - q) : (c - r)$

38. Let a, b, c, d , and e be integers such that $a = 6b = 12c$, and $2b = 9d = 12e$. Then which of the following pairs contains a number that is not an integer?
- a. $\left(\frac{a}{27}, \frac{b}{e}\right)$
 - b. $\left(\frac{a}{36}, \frac{c}{e}\right)$
 - c. $\left(\frac{a}{12}, \frac{bd}{18}\right)$
 - d. $\left(\frac{a}{6}, \frac{c}{d}\right)$

39. A person spent half of the money he had. Now he finds that he has just as many paisa as he had rupees and half as many rupees as he had paisa in the beginning. If 1% error is allowed what should be your nearest guess about his money in the beginning?
- a. Rs. 50
 - b. Rs. 80
 - c. Rs. 90
 - d. Rs. 100

40. 1/3rd of the contents of a container evaporated on the 1st day. 3/4th of the remaining contents of the container evaporated on the second day. What part of the contents of the container is left at the end of the second day?
- a. 1/4
 - b. 1/2

- c. 1/18
- d. 1/6

41. Mohan ate half a pizza on Monday. He ate half of what was left on Tuesday and so on. He followed this pattern for one week. How much of the pizza would he have eaten during the seven days of the week?

- a. 99.22%
- b. 95%
- c. 98.22%
- d. 100%

42. In a mixture of 45 litres, the ratio of milk and water is 4 : 1. How much water must be added to make the mixture ratio 3 : 2?
- a. 72 litres
 - b. 24 litres
 - c. 25 litres
 - d. 15 litres

43. If one of the diagonals of a rhombus is equal to its side, the diagonals of the rhombus are in the ratio
- a. $\sqrt{3} : 1$
 - b. $\sqrt{2} : 1$
 - c. 3 : 1
 - d. 2 : 1

44. Rs. 1360 have been divided among A, B, C such that A gets $(2/3)$ of what B gets and B gets $(1/4)$ of what C gets, then B's share is
- a. Rs. 120
 - b. Rs. 160
 - c. Rs. 240
 - d. Rs. 300

45. The Binary Ice Cream Shoppe sells two flavours, vanilla and chocolate. On Friday, the ratio of vanilla cones sold to chocolate cones sold was 2 : 3. If the store had sold 4 more vanilla cones, the ratio of vanilla cones sold to chocolate cones sold would have been 3 : 4. How many vanilla cones did the store sell on Friday?
- a. 32
 - b. 35
 - c. 42
 - d. 48

46. After reading $3/5$ of the biology homework on Monday night, Sanjay read $1/3$ of the remaining homework on Tuesday night. What fraction of the original homework would Sanjay have to read on Wednesday night to complete the biology assignment?
- a. 1/15
 - b. 2/15
 - c. 4/15
 - d. 2/5

47. If $\frac{1}{3}A = \frac{1}{4}B = \frac{1}{5}C$, the A : B : C is
a. 4 : 3 : 5 b. 5 : 4 : 3
c. 3 : 4 : 5 d. 20 : 15 : 12
48. Visitors to a show were charged Rs. 15 each on the first day, Rs. 7.50 on the second day and Rs. 2.50 on the third day and the attendance on the three days was in the ratio 2:5:13. The average charge per person for the whole show was
a. Rs. 6.33 b. Rs. 9
c. Rs. 5 d. Rs. 7.50
49. In what proportion must water be added to spirit to gain 20% by selling it at the cost price?
a. 2 : 5 b. 1 : 5
c. 3 : 5 d. 4 : 5
50. Salaries of A, B and C were in the ratio 3 : 5 : 7, respectively. If their salaries were increased by 50%, 60% and 50% respectively, what will be the new ratio of their respective salaries?
a. 4 : 5 : 7 b. 3 : 6 : 7
c. 4 : 15 : 18 d. 9 : 16 : 21

7

Allegations

Allegation is a rule that enables us to find the ratio in which the two or more ingredients at different prices must be mixed to find a mixture of the desired price. The cost price of a unit quantity of the mixture is called the **Mean Price**.

Let's consider the following problem:

Example:

In what ratio should two qualities of Custard Powder having the rates of Rs. 47 per kg and Rs. 32 per kg be mixed in order to get a mixture that would have a rate of Rs. 37 per kg?

Solution:

The conventional method is:

Let x kilograms of custard powder of Rs. 47 per kg be mixed with y kilograms of custard powder of Rs. 32 per kg.

$$\text{Hence } x(47) + y(32) = (x+y)(37)$$

$$(47 - 37)x = (37 - 32)y$$

$$\frac{x}{y} = \frac{37-32}{47-32} = \frac{\text{Mean Price - Price of cheaper}}{\text{Price of dearer - Mean price}}$$

$$\frac{x}{y} = \frac{1}{2}$$

Allegations Rule

Let Q_c = Cheaper quantity

Q_d = Dearer quantity

M = Mean price or Price of one unit of mixture

P_c = Cost price of one unit of cheaper variety

P_d = Cost price of one unit of dearer variety hen

$$\frac{Q_c}{Q_d} = \frac{P_d - M}{M - P_c}$$

- Mean price of one unit of mixture cannot be more than the cost price of dearer item and cannot be less than the cost price of one unit of cheaper item

Mixing a pure component to a solution:

Example:

A jar contains a mixture of two liquids A and B in the ratio 5:1. When 10 litres of liquid B is poured into the jar the ratio becomes 3:2. How many litres of liquid A were contained in the jar?

Solution:

Conventional Method:

Let the quantities of A and B in the original mixture be $5x$ and x litres, then

$$\frac{5x}{x+10} = \frac{3}{2}$$

$$10x = 3x + 30$$

$$x = (30 / 7) \text{ liters}$$

Hence quantity of A in the original mixture = $(150 / 7)$ liters

By Allegation Rule:

The average composition of B in the first mixture = $(1/6)$

The average composition of B in the second mixture = 1

The average composition of B in the final mixture = $(2/5)$

B in First	B in Second
$1/6$	1
\	/
$2/5$	
/	\
$(1-2/5)$	$(2/5-1/6)$
$=3/5$	$=7/30$

Mixture of Mixtures:

Example:

Two jars A and B contain mixture of water and milk in the ratio 1:2 and 3:2 respectively. In what ratio one must mix these two mixtures in order to get a

final mixture which contains water and milk in the ratio 1:1.

Solution:

Average composition of milk in jar A = $\frac{1}{3}$

Average composition of milk in jar B = $\frac{3}{5}$

Average composition of milk in final mixture = $\frac{1}{2}$

$$\begin{array}{ccc}
 & 1/3 & 3/5 \\
 & \swarrow & \searrow \\
 & 1/2 & \\
 & \swarrow & \searrow \\
 (3/5 - 1/2) & & (1/2 - 1/3) \\
 = 1/10 & & = 1/6 \\
 = 3/30 & & = 5/30
 \end{array}$$

So one must mix these two mixtures in the ratio 3:5

Case of more than two varieties:

- When a mixture of three ingredients P, Q & R is given, take any two ingredients such that cost of the mixture is between the costs of the two chosen ones and find the ratio. Once again, take two more ingredients and find their ratio. Then find the combined ratio. This will give an infinite number of solutions.

Example:

Let there be varieties of rice A, B & C priced at Rs. 10 per kg, Rs. 15 per kg & Rs. 20 per kg respectively. In what ratio these three varieties should be mixed to get a mixture which would have a rate of Rs. 16 per kg?

Solution:

We can get a mixture of Rs. 16 per kg

(i) By mixing the varieties of Rs. 10 per kg and Rs. 20 per kg.

(ii) By mixing the varieties of Rs. 15 per kg and Rs. 20 per kg.

In the first case:

$$\begin{array}{ccc}
 & A & C \\
 & 10 & 20 \\
 & \swarrow & \searrow \\
 & 16 & \\
 & \swarrow & \searrow \\
 (20 - 16 = 4) & & (16 - 10 = 6)
 \end{array}$$

Hence we should mix A and C in the ratio 4:6 or 2:3

In the second case:

$$\begin{array}{ccc}
 & B & C \\
 & 15 & 20 \\
 & \swarrow & \searrow \\
 & 16 & \\
 & \swarrow & \searrow \\
 (20 - 16 = 4) & & (16 - 15 = 1)
 \end{array}$$

Hence we should mix B and C in the ratio of 4:1

Now if we mix these two mixtures in any ratio the cost price of one unit of the final mixture will be Rs 16 in either case

Suppose we take 5 kg of first mixture and 5 kg of second mixture, then in the final mixture:

Quantity of A = 2 kg

Quantity of B = 4 kg

& Quantity of C = $(1 + 3) = 4$ kg

Hence we should mix A, B and C in the ratio 2:4:4 or 1:2:2



QUESTIONS

1. If x litres of 20 % milk solution is added to 50 liters of 50 % milk solution then we get a 40 % milk solution. What is the value of x ?
a. 25 litres b. 75 litres
c. 100 litres d. 150 litres
2. We have 20 litres of 50% spirit solution. How much pure spirit should be added to this to make it a 75% spirit solution?
a. 15 litres b. 20 litres
c. 25 litres d. 30 litres
3. An unpuffed cigarette burns completely in 15 minutes. But once puffed, rate of burning increases to thrice the original speed. If Ramu smokes a complete cigarette in 12 minutes then what part of the cigarette was puffed?
a. 30% b. 70%
c. 60% d. 50%
4. In the previous question, cigarette was puffed for how much time?
a. 1.5 minutes b. 4.5 minutes
c. 3 minutes d. 3.6 minutes
5. Petrol costs Rs 50 per litres and kerosene cost Rs 20 per litres. In what ratio petrol and kerosene should be mixed to obtain a solution that is Rs 32 per litres?
a. 2::3 b. 3:2
c. 1:2 d. 2:1
6. A shopkeeper purchased goods worth Rs 6,900. Part of these goods are sold at a profit of 20 % and another part at a loss of 10 %. Finally, it was realized that in the complete process, shopkeeper made a profit of 12 %. What was the selling price of all the quantities which were sold at a profit of 20%?
a. Rs 5060 b. Rs 6072
c. Rs 1840 d. Rs 1656
7. Raju covers 625 Km in 25 hours partly by train and partly by bus. If the speed of bus is 15 Km/Hr and speed of train is 40 Km/Hr then find the distance travelled by bus?
a. 400 km b. 225 km
c. 250 km d. 325 km
8. Peanuts can be purchased for Rs 20 per Kg. Cashew nuts can be purchased for Rs 200 per Kg. In what ratio these two should be mixed in order to gain 10 % on selling it for Rs 154 per Kg?
a. 1:2 b. 1:3
c. 2:3 d. 3:5
9. Ramu has two solutions, first one contain 5% spirit and second contain 10% spirit. In what ratio these two should be mixed so that the resultant solution contains 8% spirit?
a. 2:3 b. 3:2
c. 1:4 d. 4:1
10. Milk can be purchased at Rs 20 per liter. How much water can be added to 11 liters of this milk solution so that milkman can get 20 % profit by selling this solution at Rs 16.50 per litres?
a. 2 litres b. 3 litres
c. 4 litres d. 5 litres
11. There are three types of rice available in the market. The first cost Rs 12 per Kg, second cost Rs 18 per Kg and the third one cost Rs. 24 per Kg. In what ratio, these three should be mixed to ensure an average price of Rs 20 per Kg.
a. 8:2:4 b. 4:2:8
c. 3:4:5 d. Can be more than one solution
12. Find the ratio in which two milk solutions of milk to water ratio 1:3 and 1:5 should be mixed to get milk to water ratio of 1:4

- a. 1:1 b. 3:5
 c. 2:3 d. None of the above
13. What part of a milk solution having milk water ratio of 1:2 should be replaced with water to get milk water ratio of 1:3?
 a. One third b. One Fourth
 c. One fifth d. One sixth
14. Ramu purchases two shirts for Rs 1000, he sold the first one at a loss of 5 % and the other at a profit of 15 %. If he neither gains nor loses on the complete transaction, what was the Cost price of the first shirt?
 a. Rs 250 b. Rs 500
 c. Rs 750 d. Rs 600
15. Ramu takes some loan from Bank 1 and some more loan from Bank 2. Bank 1 charges 6 % rate of interest per annum and Bank 2 charges 8.8 % rate of interest per annum. If Ramu ends up paying an interest of 7 % on the total amount of loan then what is the ratio of loan that he has taken from bank 1 and bank 2?
 a. 5:9 b. 9:5
 c. 7:9 d. 9:7
16. A total amount of Rs 270 was collected from 90 students. If all the boys contributed Rs 5 per head and all the girls contributed Rs 2 each than how many girls are there among these 90 students?
 a. 30 b. 40
 c. 50 d. 60
17. How much water should be added to 80 litres of 80% milk solution to reduce the milk concentration to 60 %?
 a. 23 litres b. 26 litres
 c. 33 litres d. 36 litres
18. Average marks per student of class A is 79 and average marks per student of class B is 87. If the average marks per student of both the classes combine in an integer then which of the following cannot be the ratio of the students of both the classes?
 a. 1:7 b. 2:3
 c. 3:5 d. 4:6
- c. 1:3 d. 1:1
19. There are two pegs of whiskey, first one contains 20 % whiskey and the second one contains 30 % whiskey. In what ratio these two pegs should be mixed in order to ensure 28 % whiskey in the final mixture?
 a. 1:5 b. 1:4
 c. 4:1 d. 3:2
20. A waiter stole whiskey from a bottle of whiskey which contained 40 % whiskey and replaced it with another solution which contained 16 % whiskey. Finally 24 % of the bottle was whiskey. What proportion of the bottle was stolen by that waiter?
 a. One third b. Two third
 c. One fourth d. Three fourth
21. Two vessels contain mixtures of whisky and soda in the ratio of 4 : 9 in the first vessel and in the ratio of 2 : 7 in the second. In what ratio should the contents of these two vessels be mixed such that the resultant mixture has whisky and soda in the ratio of 2 : 5?
 a. 31 : 25 b. 9 : 26
 c. 26 : 63 d. 26 : 9
22. A shopkeeper mixes 10 kg of sugar worth Rs.20 per kg with 15 kg of sugar worth Rs. x per kg so that after selling the whole at Rs. 30 per kg he gains a profit of 20%. What is the value of x?
 a. 25 b. 28.33
 c. 30 d. 35
23. In what ratio should Ramu mix two varieties of Wheat costing Rs. 10 and Rs. 15 per kg so that by selling it at Rs. 18 per kg he makes a profit of 50%?
 a. 1:2 b. 2:3
 c. 3:2 d. 4:5
24. Vessel 'P' contains milk and water in the ratio 4:5 and vessel 'Q' contains the same in the ratio 4:1. In what proportion should quantities be taken from P and Q to form a mixture in which milk & water are in ratio 5:4?

- a. 11:5 b. 12:5
c. 2:1 d. 3:2
25. A merchant has 100 kg of wheat, part of which he sells at 4% gain and the rest at 9% gain. He gains 7% on the whole. Find the quantity sold at 9% profit.
a. 45 Kg b. 60 Kg
c. 75 Kg d. 55 Kg
26. How many litres of 20% alcohol solution should be added to 40 litres of a 50% alcohol solution to make a 30% solution?
a. 100 b. 80
c. 90 d. 95
27. John wants to make a 100 ml of 5% alcohol solution mixing a quantity of a 2% alcohol solution with a 7% alcohol solution. What are the quantities of each of the two solutions (2% and 7%) he has to use?
a. 40, 60 b. 30, 70
c. 20, 80 d. 55, 45
28. Sterling Silver is 92.5% pure silver. How many grams of Sterling Silver must be mixed to certain amount of 90% silver alloy to obtain a 500g of a 91% silver alloy?
a. 200 gm b. 250 gm
c. 300 gm d. 150 gm
29. How many kg. of Pure water is to be added to 100 kg. of a 30% saline solution to make it a 10% saline solution?
a. 250 b. 200
c. 198 d. 144
30. In 70 litres of a mixture of Alcohol and soda, the quantity of Soda is 20%. How much Soda should be added so that the new mixture may contain 25% of what?
a. $14/3$ b. 5
c. 3 d. 6
31. The ratio of milk and water in a mixture of 50 litres is 4:1. How much water must be added to the mixture so that the ratio of milk and water be 3:2?
- a. 16.66 b. 18
c. 20 d. 15
32. Dheeraj adds 2 litre of water to 10 litres of a 20% solution, of alcohol in water. What is the final strength of alcohol?
a. $16\frac{2}{3}$ b. $16\frac{1}{3}$
c. 15 d. 12
33. If Manoj decides to travel 80 Kilometers in 8 hours partly by foot and partly on a bicycle, his speed on foot being 8 km/h. and that on bicycle being 16 km/hr. What distance would he travel on foot?
a. 12 Km b. 18 Km
c. 48 Km d. 42 Km.
34. A mixture of 40 litres of brandy and water contains 20% water. How much water should be added to it to increase the percentage of water to 50%?
a. 18 b. 12
c. 24 d. 48
35. How many kilograms of sugar costing Rs 42 per kg must a man mix with 25 kg of sugar costing Rs 24 per kg, such that on selling the mixture at Rs. 40 per kg, there is a gain of 25% on the outlay?
a. 20 b. 25
c. 16 d. 12
36. A jar contains a mixture of two liquids P and Q in the ratio 4:1. When 10L of the mixture is replaced with liquid Q, the ratio becomes 2:3. How many litres of liquid P was present in the jar earlier.
a. 16 b. 24
c. 8 d. 5
37. If Ram makes a profit of 15% on one fourth quantity sold and a loss of 20% on the rest, then what is his average (net) profit or loss?
a. 11.25 b. -11.25
c. 12 d. 15
38. 27 L of Banana juice contains 312 calories and 9 L of orange juice contains 54 calories. If 18 L of mixture of both Banana and orange juice contains 118 calories, find the proportion of Banana juice in the mixture?

- a. 1/10 b. 3/10
 c. 1/2 d. 1/3
39. Flask A contains a 20% solution of sulphuric acid and flask B contains a 25% solution of sulphuric acid. Some part of solutions are taken from both the flasks to make 6 litres of a 21% solution of Sulphuric Acid. How much solution was taken from the second flask?
 a. 1.2 l b. 1.5 l
 c. 2 l d. 3 l
40. Nigam combines 20 litres of a 25% acid solution with 50 litres of a 32% acid solution. What is the strength (in percentage) of the acid in the mixture?
 a. 25% b. 35%
 c. 22% d. 30%
41. A merchant has 36 kg of sugar, part of which he sells at 12% gain and the rest at 16% gain. He gains 15% on the whole. Find the quantity sold at 16% profit?
 a. 27 kg b. 18 kg
 c. 36 kg d. 25 kg
42. A chemist has 16 litres of a 25% glycol syrup. How much glycol must be added so that the resulting syrup contains 75% glycol?
 a. 32 b. 36
 c. 30 d. 27
43. A solution of milk and water in the ratio 4:1 mixes with another solution (of same volume) of milk and water in the ratio of 3:2. Find the concentration of milk in the resultant mixture?
 a. 80 b. 90
 c. 75 d. 70
44. A Shopkeeper mixes 44 kg of tea costing Rs 22 per kg with 26 kg tea costing Rs. 26 per kg. What is the cost of the mixture per kg?
 a. $11\frac{1}{3}$ b. $12\frac{2}{3}$
 c. $15\frac{1}{3}$ d. $10\frac{2}{3}$
- a. 22.5 b. 23.5
 c. 42 d. 21
45. A solution of volume 60 L contains milk and water in the ratio 5:1. If 5L water is added to the mixture, what will be the ratio of milk and water in the resulting mixture?
 a. 8:5 b. 6:5
 c. 10:3 d. 2:3
46. A shopkeeper mixes 10 kg of tea costing Rs. 5/Kg with 20 kg of tea costing Rs 7/kg. What is the cost of the mixture per kg?
 a. 6.33 b. 6
 c. 5 d. 6.5
47. A person mixes 5 kg of rice costing Rs. 3/kg with 15 kg of another rice costing Rs. 5/kg. What is the cost of the final mixture per kg?
 a. 4.5 b. 3.5
 c. 4.2 d. 5.5
48. Two varieties of Sugar costing Rs. 15/kg and 18/kg are mixed together in such a way that the price of the resulting mixture is Rs. 16/kg. Find the quantity of Rs 18/kg sugar in a mixture of 90 kg.
 a. 30 b. 25
 c. 32 d. 40
49. A shopkeeper sells $\frac{2}{3}rd$ of the shirts he has at a profit of 75% and the remaining at a loss of 25%. Find his overall loss or profit per cent?
 a. $41\frac{1}{3}$ b. $41\frac{2}{3}$
 c. 41 d. 41.5
50. How many litres of a 20% alcohol solution should be mixed with certain quantity of 4% alcohol solution to create 40 litres of a 16% alcohol solution?
 a. 11 l b. 12 l
 c. 15 l d. 10 l

8

Mixtures & Solutions

When two or more ingredients are mixed together in a certain ratio, a mixture is formed.

Simple Mixture:

When two different ingredients are mixed in a certain ratio a Simple Mixture is formed.

Compound Mixture:

When two or more simple mixtures are mixed together, they form a Compound Mixture.

Mixing quantities of different cost:

Example:

How many kilograms of chocolate worth Rs. 1.20 a kilogram must be mixed with 10 kilograms of chocolate worth 90 paisa a kilogram to produce a mixture worth Rs. 1.00 a kilogram?

Solution:

Step 1: Set up a table for different types of chocolate.

	Original	Added	Result
Cost			
Amount			

Step 2: Fill in the table with information given in the question.

Let x = amount of chocolate added.

	Original	Added	Result
Cost	0.9	1.2	1
Amount	10	x	$x+10$

Step 3: Multiply down each column.

	Original	Added	Result
Cost	0.9	1.2	1
Amount	10	x	$x+10$
Multiply	0.9×10	$1.2 \times x$	$1 \times (x+10)$

Step 4: original + added = result

$$0.9 \times 10 + 1.2 \times x = 1 \times (x + 10)$$

$$9 + 1.2x = x + 10$$

Isolate variable x

$$1.2x - x = 10 - 9$$

$$0.2x = 1$$

$$x = 5$$

5 kilograms of the Rs. 1.20 per kilogram chocolate needs to be added.

Adding to the Solution:

Example:

Atul has 20 kilograms of a 20% of salt solution, how much salt should he add to make it a 25% solution?

Solution:

Step 1: Set up a table for salt.

	Original	Added	Result
Concentration			
Amount			

Step 2: Fill in the table with information given in the question.

The salt added is 100% salt, which is 1 in decimal. Change all the percent to decimals.

Let x = amount of salt added. The result would be $20+x$.

	Original	Added	Result
Concentration	0.2	1	0.25
Amount	20	x	$20+x$

Step 3: Multiply down each column.

	Original	Added	Result
Concentration	0.2	1	0.25
Amount	20	x	$20+x$
Multiply	0.2×20	$1 \times x$	$0.25(20+x)$

Step 4: original + added = result

$$0.2 \times 20 + 1 \times x = 0.25(20 + x)$$

$$4 + x = 5 + 0.25x$$

Isolate variable x

$$x - 0.25x = 5 - 4$$

$$0.75x = 1$$

$$x = \frac{1}{0.75} = \frac{1}{3/4} = \frac{4}{3}$$

He should add $\frac{4}{3}$ kilogram of salt.

Removing from the Solution:

Example:

Atul has 20 kilograms of a 20% of salt solution. How much water should be evaporated to make it a 30% solution?

Solution:

Step 1: Set up a table for water. The water is removed from the original.

	Original	Removed	Result
Concentration			
Amount			

Step 2: Fill in the table with information given in the question.

The original concentration of water is $100\% - 20\% = 80\%$.

The resulted concentration of water is $100\% - 30\% = 70\%$.

The water evaporated is 100% water, which is 1 in decimal.

Change all the per cent to decimals.

Let x = amount of water evaporated. The result would be $20 - x$.

	Original	Removed	Result
Concentration	0.8	1	0.7
Amount	20	x	$20 - x$

Step 3: Multiply down each column.

	Original	Removed	Result
Concentration	0.8	1	0.7
Amount	20	x	$20 - x$
Multiply	0.8×20	$1 \times x$	$0.70(20 - x)$

Step 4: Since the water is removed, we need to subtract

Original – removed = result

$$0.8 \times 20 - 1 \times x = 0.70(20 - x)$$

$$16 - x = 14 - 0.7x$$

Isolate variable x

$$x - 0.7x = 16 - 14$$

$$0.3x = 2$$

$$x = \frac{2}{0.3} \approx 6.67$$

He should evaporate 6.67 kilograms of water.

Replacing the Solution:

Problem:

A tank has a capacity of 10 gallons. When it is full, it contains 15% alcohol. How many gallons must be replaced by an 80% alcohol solution to give 10 gallons of 70% solution?

Solution:

Step 1: Set up a table for alcohol. The alcohol is replaced i.e. removed and added.

Original	Removed	Added	Result
Concentration			
Amount			

Step 2: Fill in the table with information given in the question.

Change all the per cent to decimals.

Let x = amount of alcohol solution replaced.

	Original	Removed	Added	Result
Concentration	0.15	0.15	0.8	0.7
Amount	10	x	x	10

Step 3: Multiply down each column.

	Original	Removed	Added	Result
Concentration	0.15	0.15	0.8	0.7
Amount	10	x	x	10
Multiply	0.15×10	$0.15 \times x$	$0.8 \times x$	0.7×10

Step 4: Since the alcohol solution is replaced, we need to subtract and add.

Original – removed + added = result

$$0.15 \times 10 - 0.15x + 0.8x = 0.7 \times 10$$

$$1.5 - 0.15x + 0.8x = 7$$

Isolate variable x

$$0.8x - 0.15x = 7 - 1.5$$

$$0.65x = 5.5$$

$$x = \frac{5.5}{0.65} \approx 8.46$$

Hence 8.46 gallons of alcohol solution needs to be replaced.

Some Important Results:

- When two or more ingredients ($I_1, I_2, I_3, \dots, I_n$) are mixed such that the mixture contains x_1 quantity of ingredient I_1 of unit cost C_1 , x_2 quantity of ingredient I_2 of unit cost C_2 and so on then the average Mean Cost C_m is defined as

$$C_m = \frac{C_1 x_1 + C_2 x_2 + \dots + C_n x_n}{x_1 + x_2 + \dots + x_n}$$

- When two mixtures M_1 and M_2 , each containing ingredients A and B in the ratio $a:b$ and $x:y$ respectively, are mixed, the proportion of the ingredients A and B i.e., $Q_a : Q_b$, in the compound mixture is given by:

$$\frac{Q_a}{Q_b} = \frac{M_1 \times \left(\frac{a}{a+b} \right) + M_2 \times \left(\frac{x}{x+y} \right)}{M_1 \times \left(\frac{b}{a+b} \right) + M_2 \times \left(\frac{y}{x+y} \right)}$$

- And, the quantity in which M_1 and M_2 is to be mixed when the quantity of A and B i.e. Q_a and Q_b in the compound mixture is known is given by:

$$\frac{\text{Quantity of } M_1}{\text{Quantity of } M_2} = \frac{\left(\frac{x}{x+y} \right) - \left(\frac{Q_a}{Q_a+Q_b} \right)}{\left(\frac{Q_a}{Q_a+Q_b} \right) - \left(\frac{a}{a+b} \right)}$$

- If a vessel contains 'x' litres of liquid A, and 'y' liters be withdrawn and replaced by liquid B, then if 'b' litres of mixture be withdrawn and replaced by liquid B, and the operation is repeated 'n' times in all then:

$$\frac{\text{Liquid A left in the vessel after } n^{\text{th}} \text{ operation}}{\text{Initial quantity of liquid A in vessel}} = \left(\frac{x-y}{x} \right)^n$$

$$\frac{\text{Liquid A left in the vessel after } n^{\text{th}} \text{ operation}}{\text{Liquid B left in the vessel after } n^{\text{th}} \text{ operation}}$$

$$= \frac{\left(\frac{x-y}{x} \right)^n}{1 - \left(\frac{x-y}{x} \right)^n}$$

Example:

From a vessel containing 64 litres of milk, 8 litres are drawn out and the vessel is filled up with water. If the same operation is repeated a second time then a third time, how many liters of milk will be left in the cask? What will be the proportion of milk to water in the resulting mixture?

Solution:

$$\frac{\text{Milk left in the vessel after } 3^{\text{rd}} \text{ operation}}{\text{Initial quantity of Milk in the vessel}} = \left(\frac{64-8}{64} \right)^3 = \left(\frac{56}{64} \right)^3 = \left(\frac{7}{8} \right)^3 = \left(\frac{343}{512} \right)$$

$$\text{Proportion of milk to water} = \frac{\left(\frac{343}{512} \right)}{1 - \left(\frac{343}{512} \right)} = \frac{343}{169}$$

■ ■ ■

QUESTIONS

1. From 100 litres of milk water solution, we remove 10 litres of the solution and then we add 10 liters of water. Final milk water ratio comes out to be 3:7. Find the initial milk water ratio?
 - a. 1:2
 - b. 11:23
 - c. 22:45
 - d. 33:67
2. From 100 liters of milk water ratio, we add 10 liters of water and then we remove 10 liters of solution and final milk water ratio comes out to be 3:7. What was the initial milk water ratio?
 - a. 1:2
 - b. 11:23
 - c. 22:45
 - d. 33:67
3. We have two beakers of equal capacity. First beaker contains pure milk and second beaker contains pure water. We transfer half the content of the first beaker into second beaker followed by transferring the same quantity from the second beaker to the first beaker. And then the same process was repeated. What will be final ratio of milk and water in the second beaker?
 - a. 4:5
 - b. 5:4
 - c. 9:5
 - d. 5:9
4. There are two beakers, first one containing 4 litres of milk and second one containing 12 litres of water. Some quantity of milk was transferred from first beaker to second beaker and simultaneously an equal amount of water was transferred to first beaker from second beaker. Finally it was realized that now both the beakers contain same ratio of milk and water. What was the quantity transferred?
 - a. 1 litres
 - b. 2 litres
 - c. 3 litres
 - d. None of the above
5. We have 100 litres of pure milk solution. 10 litres of the solution replaced by water thrice. What is the final concentration of milk in the solution?
 - a. 70 %
 - b. 72.9 %
 - c. 81 %
 - d. 67 %
6. There are two beakers with 100 ml of milk in first beaker and 100 ml of water in the second beaker. Now 10 ml is transferred from first beaker to second beaker followed by transferring of 10 ml from second beaker to the first beaker. This is followed by transferring 20 ml from first beaker to second beaker followed by transferring 20 ml from second beaker to the first beaker. Then which of the following statements is true?
 - a. Milk to water ratio in first beaker will be more than water to milk ratio in the second beaker
 - b. Milk to water ratio in first beaker will be less than water to milk ratio in the second beaker
 - c. Milk to water ratio in first beaker will be equal to water to milk ratio in the second beaker
 - d. None of the above
7. A substance is 99% water. Some water evaporates, leaving a substance that is 98% water. How much of the water evaporated?
 - a. 1 %
 - b. 10 %
 - c. 5 %
 - d. 50 %
8. There is 100 litres of 50 % milk solution. Half of the solution is removed and replaced with milk. Then one third of the solution is removed and replaced with milk. Finally one fourth of the solution was removed and replaced with milk. Now, what is the concentration of milk in the final solution?
 - a. 25 %
 - b. 75 %
 - c. 87.5 %
 - d. 12.5 %
9. We have a 100 litres solution of 50 % milk solution. How many times 10 litres of this

- solution can be replaced with water to ensure at least 30% milk solution?

 - 3
 - 4
 - 5
 - 6

10. We had a 750 ml pure alcohol solution, X ml of this solution was replaced with water twice and finally this solution had only 607.5 ml of alcohol. What is the value of x?

 - 25 ml
 - 50 ml
 - 71.25 ml
 - 75 ml

11. In a sodium water solution, there was 20 % sodium and 80 % water. If 2 KG of water has evaporated with time and 2 KG of sodium is added to the solution, concentration of sodium has increased to 40%. Initially, what was the total weight of the solution?

 - 8 KG
 - 10 KG
 - 12 KG
 - 15 KG

12. 10 ml of a pure whiskey solution was replaced by water and then this process was repeated. Finally water to whiskey ratio in the peg was 9:16. What was the amount of complete peg?

 - 50 ml
 - 100 ml
 - 150 ml
 - 200 ml

13. A milk man mixes 10 litres of water in 90 litres of milk. If he sells one third of these solutions and replenishes it with water then what part of the complete solution is milk now?

 - 30 %
 - 50 %
 - 60 %
 - 80 %

14. From a container of 800 litres containing pure milk, 80 litres of milk is replaced with water and this process is repeated twice. What will be the final concentration of milk in the solution?

 - 81%
 - 72.9%
 - 64.8%
 - 58.32%

15. Two mixtures of milk and water in the ratio 1:2 and 1:3 are mixed in equal quantity. What will be milk water ratio in this final mixture?

 - 2:5
 - 5:14

c. 7:17 d. 9:23

16. How much water should be added to 1 litre of 2% acid concentration to decrease the acid concentration to 1%?

 - 0.1 litre
 - 0.5 litre
 - 0.9 litre
 - 1 litre

17. What percentage of a milk solution was water if a milk man makes 10 % profit by selling milk at his cost price?

 - 10 %
 - 9.09 %
 - 11.11%
 - None of the above

18. What part of a 3:2 milk water solution should be replaced with water in order to reverse the ratio of milk and water?

 - Half
 - One third
 - One fourth
 - One fifth

19. Chaman purchased two types of apples: first at Rs 40 per Kg and second at Rs 60 per Kg. He mixed both the apples and sold all of them at Rs 50 per Kg. What will be his net profit or loss if he spends equal amount of money on both of them?

 - No profit, no loss
 - 4 %, profit
 - 4.16 %, profit
 - 3.84 %, profit

20. Chaman purchased two types of apples: first at Rs 40 per Kg and second at Rs 60 per Kg. He mixed both the apples and sold all of them at Rs 50 per Kg. What will be his net profit or loss if he purchased equal quantity of both types of apples?

 - No profit, no loss
 - 4 %, profit
 - 4.16 %, profit
 - 3.84 %, profit

21. Two vessels contain mixtures of whisky and soda in the ratio of 4 : 9 in the first vessel and in the ratio of 2 : 7 in the second. In what ratio should the contents of these two vessels be mixed such that the resultant mixture has whisky and soda in the ratio of 2 : 5?

 - 31 : 25
 - 9 : 26

- c. 26 : 63 d. 26 : 9
22. A vessel is completely filled with beer. 10 litres are drawn from the vessel and replaced with water. 10 litres of this mixture is then drawn and replaced with water. Find the capacity of the vessel if the ratio of beer to water is 16:9 at the end.
 a. 60 ltrs. b. 36 ltrs.
 c. 50 ltrs. d. 46 ltrs.
23. Sixty litres of wine is drawn from 600 litres of wine and replaced by water. 60 ltrs of mixture is again drawn and replaced with water and this process is repeated once more. Find the present quantity of wine in the mixture.
 a. 420 ltrs. b. 393.66 ltrs.
 c. 486 ltrs. d. 437.4 ltrs.
24. A vessel has 90 litres of a mixture of milk and water containing 10% water. If x litres of water is added to the vessel, the ratio of milk and water would be reversed. Instead, if y litres of milk is removed from the vessel (assume that it is possible to separate milk and water in a solution), the ratio of milk and water would be reversed. Find $(x-y)$.
 a. 720 b. 625
 c. 640 d. 80
25. Two varieties of rice – A and B are mixed in the ratio of 3 : 2. The mixture is sold at Rs. 100 per kg at 100% profit. If variety A costs Rs. 10 more than variety B, what is the cost of variety B per kg?
 a. Rs. 34 b. Rs. 54
 c. Rs. 44 d. Rs. 64
26. The price per kg of mangoes has trebled. If Dheeraj wants to keep his expenditure same, by what percentage should his consumption decrease.
 a. 100% b. 66.67%
 c. 33.33% d. 200%
27. 750 candidates appeared in an examination. 70% of the girls and 65% of the boys passed the examination. If the total pass percentage was 67%, find the number of boys that appeared in the examination.
 a. 350 b. 420
 c. 300 d. 450
28. A 20 litre mixture of milk and water contains milk and water in the ratio 3 : 2. 10 litres of the mixture is removed and replaced with pure milk and the operation is repeated once more. At the end of the two replacements, what is the ratio of milk and water in the resultant mixture?
 a. 17 : 3 b. 9 : 1
 c. 3 : 17 d. 5 : 3
29. In which of the following ratios must a person mix three kinds of tea costing Rs. 60/kg, Rs. 75/kg and Rs. 100/kg so that the resultant mixture when sold at Rs. 96/kg yields a profit of 20%?
 a. 1 : 2 : 4 b. 3 : 7 : 6
 c. 1 : 4 : 2 d. None of these
30. A merchant mixes three varieties of rice costing Rs.20/kg, Rs.24/kg and Rs.30/kg and sells the mixture at a profit of 20% at Rs.30/kg. How many kgs of the second variety will be in the mixture if 2 kgs of the third variety is there in the mixture?
 a. 1 kg b. 5 kgs
 c. 3 kgs d. Cannot be determined
31. How many litres of water should be added to a 30 litre mixture of milk and water containing milk and water in the ratio of 7 : 3 such that the resultant mixture has 40% water in it?
 a. 7 litres b. 10 litres
 c. 5 litres d. None of these
32. How many kgs of Basmati rice costing Rs.42/kg should a shopkeeper mix with 25 kgs of ordinary rice costing Rs.24 per kg so that he makes a profit of 25% on selling the mixture at Rs.40/kg?
 a. 20 kgs b. 12.5 kgs

- c. 16 kgs d. 200 kgs
33. How many litres of a 12 litre mixture containing milk and water in the ratio of 2: 3 be replaced with pure milk so that the resultant mixture contains milk and water in equal proportion?
 a. 4 litres b. 2 litres
 c. 2.5 litres d. 3 litres
34. A sample of x litres from a container having a 60 litre mixture of milk and water containing milk and water in the ratio of 2 : 3 is replaced with pure milk so that the container will have milk and water in equal proportions. What is the value of x ?
 a. 6 litres b. 10 litres
 c. 30 litres d. None of these
35. A zookeeper counted the heads of the animals in a zoo and found them to be 80. When he counted the legs of the animals he found them to be 260. If the zoo had either pigeons or horses, how many horses were there in the zoo?
 a. 60 b. 20
 c. 50 d. 70
36. From a cask of milk containing 30 litres, 6 litres are drawn out and the cask is filled up with water. If the same process is repeated a second, then a third time, what will be the number of litres of milk left in the cask?
 a. 0.512 litres b. 13 litres
 c. 15 litres d. 15.36 litres
37. A bucket having a capacity of 100 litres is full of milk. 10 litres of milk is removed and replaced with water. This process is repeated 2 more times. Find the final quantity of milk removed from the bucket?
 a. 27.1 lts. b. 32 lts.
 c. 25 lts. d. 72.9 lts.
38. A flask is full of milk. 10 litres of milk is removed from the flask and replaced with water. This is done one more time. The proportion of milk in the bucket becomes $\frac{9}{25}$. Find the capacity of bucket?
 a. 25 b. 45
- c. 20 d. 24
39. One alloy of brass is composed of tin, copper and zinc in the ratio of 2:15: 5. Another alloy of brass contains these metals in the ratio of 7 : 23 : 3. Equal weights of these alloys are taken and fused together. What is the ratio of these ingredients in the resulting alloy?
 a. 20:91:21 b. 19:20:12
 c. 91:20:21 d. 10:20:31
40. Three containers A, B and C are having mixtures of milk and water in the ratio of 1 : 5, 3 : 5 and 5 : 7 respectively. If the capacities of the containers are in the ratio 5: 4: 5, find the ratio of milk to water, if the mixtures of all the three containers are mixed together.
 a. 45 : 56 b. 51 : 110
 c. 53 : 115 d. 70 : 71
41. Two boxes A and B were filled with coffee and chicory mixed in A in the ratio of 5 : 3, and in B in the ratio of 7 : 3. What quantity must be taken from each to form a mixture which shall contain 6 kg of coffee and 3 kg of chicory (in kg respectively)?
 a. 4, 6 b. 5, 6
 c. 4, 5 d. 6, 7
42. A mixture contains milk & water in the ratio 5:1 on adding 5 litres of water, the ratio of milk and water becomes 5 : 2. Find the quantity of milk in the original mixture.
 a. 20 lt b. 25 lt
 c. 30 lt d. 28 lt
43. A 50 ml after-shave lotion containing 30% alcohol is mixed with 30 ml of pure water. What is the percentage of alcohol in the new solution?
 a. 20% b. 22%
 c. 15% d. 18.75%
44. You add x ml of a 25% alcohol solution to a 200 ml of a 10% alcohol solution to obtain another solution. Find x so that the final solution has a alcohol percentage of 15%.

- a. 100 b. 150
c. 300 d. 200
45. In what ratio must milk be added with water so as to gain 20% by selling the mixture at cost price of pure milk?
a. 1 : 4 b. 1 : 3
c. 1 : 5 d. 1 : 6
46. There is a milk water solution with milk water ratio as 1:4. What is the minimum number of times, that 10 % of the solution should be removed and replaced with water in order to reduce the milk water ratio to less than 1:8?
a. 5 b. 6
c. 7 d. 8
47. A beaker contains milk and water in the ratio 4:1. 10 litres of this solution is replaced with water and the milk water ratio becomes 3:2. What will be the final volume of the beaker?
a. 40 litres b. 50 litres
c. 75 litres d. 100 litres
48. There is a beaker with 80 liters of 20% milk solution. How much pure milk should be added to the solution to make it 25% milk solution?
a. 4 ltr. b. 5 ltr.
c. 5.33 ltr. d. 5.66 ltr.
49. There is a holy beaker of spirit which contains 100 ltr. of 50% spirit solution. Every week 10% of the spirit in the solution is evaporated and this need to be made up by adding 5 liters of pure spirit to the solution every Sunday. On particular Sunday, pure spirit was not added to the solution then how much spirit should be added next week to make up for the evaporated spirit during the course of two weeks?
a. 8 ltr. b. 9.5 ltr.
c. 10 ltr. d. 10.5 ltr.
50. 5 litres of a pure water solution is replaced with pure milk twice and finally water milk ratio becomes 49:576. Find the total quantity of the solution.
a. 250 ltr. b. 100 ltr.
c. 125 ltr. d. None of the above

Time, Speed & Distance

If one has to travel a constant distance with different speeds, he/she would observe that greater the speed, lesser the time taken. Therefore speed is inversely proportional to the time. If one has to travel different distances in equal time intervals, he/she would observe that greater the speed, greater the distance travelled. Therefore speed is directly proportional to the distance travelled.

$$\text{Speed} \propto \frac{1}{\text{Time}} \text{ and Speed} \propto \text{Distance}$$

Definition of Speed:

Speed is nothing but the rate at which the distance is covered.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Example:

Anubhav leaves Delhi for Jaipur in his car and at the same time Deepika leaves Jaipur for Delhi in her car. Both of them meet at Midway at a distance of 150 km from Delhi. Given that distance between Delhi & Jaipur is 400 Km.. What is the ratio of their speeds?

Solution:

Ratio of their speeds = ratio of distance covered

$$\frac{\text{Speed Anubhav}}{\text{Speed Deepika}} = \frac{150}{400 - 150} = \frac{150}{250} = \frac{3}{5}$$

- ❖ When time is constant, distance covered is directly proportional to the speed.

Example:

Anant leaves home for office at 7:00 am and reaches office at 9:00 am. While in the evening he leaves office at 6:00 pm and drives slowly because of dense fog, and reaches home at 9:00 pm. What is the ratio of speed of Anant in the morning to the speed in the evening?

Solution:

$$\frac{\text{Speed in morning}}{\text{Speed in evening}} = \frac{2}{3}$$

- ❖ When distance is the same, speed is inversely proportional to time.

Example:

If Anubhav travels at $\frac{3}{4}$ of his usual speed, he reaches his office 6 minutes late. What is the usual duration of time he takes to reach office?

Solution:

As $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$ i.e. Time taken is inversely proportional to the speed.

Hence while travelling at $\frac{3}{4}$ of the usual speed he will take $\frac{4}{3}$ of the usual time T.

$$\text{i.e. } \frac{4}{3} T = T + 6$$

$$\text{Hence } T = 18 \text{ minutes}$$

Example:

If Anant drives at 45 kmph he reaches his office 15 minutes late, and if he drives at 60 kmph he reaches his office 15 minutes early. What should be Anant's speed to reach his office at usual time?

Solution:

Let the distance be D km and the usual time be T

$$\text{hour, then Usual speed} = \frac{D}{T} \text{ kmph}$$

$$\text{Also } \frac{D}{45} = T + \frac{15}{60} \quad \dots(i)$$

$$\& \frac{D}{60} = T - \frac{15}{60} \quad \dots(ii)$$

Equation (i) – Equation (ii), gives

$$\frac{D}{45} - \frac{D}{60} = \frac{1}{2} \text{ i.e. } \frac{D}{180} = \frac{1}{2} \text{ or } D = 60 \text{ km or } D = 90 \text{ km}$$

Now, substituting $D = 90$ in equation (i) {or in equation (ii)}, we get

$$T = \frac{90}{45} - \frac{15}{60} = \frac{7}{4} \text{ hour} = 1 \text{ hour } 45 \text{ minutes (Usual Time)}$$

And hence usual speed = $\frac{90}{\frac{7}{4}} = \frac{360}{7} \text{ kmph} = 51.43 \text{ kmph approx.}$

Average Speed:

If a person travels different distances $D_1, D_2, D_3, \dots, D_n$ with speeds $S_1, S_2, S_3, \dots, S_n$ in different time intervals $T_1, T_2, T_3, \dots, T_n$ respectively, then the average speed of the person for the entire journey is:

$$\begin{aligned} \text{Or, } & \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ & \frac{D_1 + D_2 + D_3 + \dots + D_n}{T_1 + T_2 + T_3 + \dots + T_n} \\ &= \frac{S_1 T_1 + S_2 T_2 + S_3 T_3 + \dots + S_n T_n}{T_1 + T_2 + T_3 + \dots + T_n} \\ &= \frac{(D_1 + D_2 + D_3 + \dots + D_n)}{\frac{D_1}{S_1} + \frac{D_2}{S_2} + \frac{D_3}{S_3} + \dots + \frac{D_n}{S_n}} \end{aligned}$$

Example:

Anubhav is planning to move to Lucknow from Delhi in his car. The distance between Lucknow and Delhi is 900 km. Anubhav covers the first 200 km in 4 hours, next 300 km in 6 hours and the last 400 km in 8 hours. What is his average speed for the entire journey?

Solution:

Total distance travelled = 900 km

And total time taken = $4 + 6 + 8 = 18$ hour

Hence average speed = $900/18 = 50$ kmph

Example:

Anant while going to his office from home drives his car at 60 kmph and while returning from his office to home drives at 90 kmph. He uses the same way for both the journeys, then what is his average speed.

Solution:

Let the distance between office and home be D km.

Then time taken while going to office = $D/60$ hours.

And the time taken while returning from the office = $D/90$ hours.

$$\begin{aligned} \text{Hence average speed} &= \frac{2D}{\frac{D}{60} + \frac{D}{90}} = \frac{2 \times 60 \times 90}{60 + 90} = \frac{10800}{150} \\ &= 72 \text{ kmph.} \end{aligned}$$

- If a person travels from A to B with speed 'a' and returns from B to A with speed 'b', then the average speed for "to & fro" journey is $2ab/(a+b)$ kmph which is the harmonic mean of a and b.

Also, if T_1 and T_2 is the time taken to travel from A to B and B to A, respectively, the distance 'D' from A to B is given by:

$$D = (T_1 + T_2) \left(\frac{ab}{a + b} \right) = (T_1 - T_2)$$

$$\left(\frac{ab}{a - b} \right) = (a - b) \left(\frac{T_1 T_2}{T_1 - T_2} \right)$$

If two persons A and B start at the same time in opposite directions from two points P and Q respectively, and arrive at Q and P respectively after having met, then:

$$\frac{\text{Speed of A}}{\text{Speed of B}} = \frac{\sqrt{b}}{\sqrt{a}}$$

Example:

Anant and Anubhav start from two different points A and B at 7:00 am. They meet at a point C between A and B at 2:00 pm. Anant reaches B at 06:00 pm and Anubhav reaches A at 11 pm. What is the ratio of speed of Anubhav to Anant.

Solution:

Anubhav takes 9 hours after having met Anant at C, to reach at point A. (2:00 pm to 11:00 pm)

Anant takes 4 hours after having met Anubhav at C, to reach at point B. (2:00 pm to 6:00 pm)

$$\text{Hence } \frac{\text{Speed Anubhav}}{\text{Speed Anant}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

Relative Speed

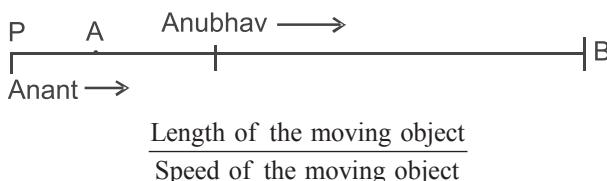
When moving in opposite direction:

Consider two persons, Anant and Anubhav, standing

at A and B respectively. Now if they start walking towards each other in a straight line, they would meet sooner than had one of them been stationary and the relative speed will be the sum of their speeds.

- ❖ Relative speed of two objects moving in opposite direction = Sum of their speeds

When moving in same direction:



Consider a person Anubhav walking from point A to point B in a straight line. Now if another person Anant walks from point P which is behind A at a speed greater than Anubhav's speed, they would meet later than had Anubhav been stationary at point A. Their relative speed is the difference of their speeds.

- ❖ Relative speed of two objects moving in same direction = Difference of their speeds

Important points:

Time taken by a moving object of length 'L' at a speed 'S' in passing a stationary object of negligible length:

$$\frac{\text{Length of the moving object}}{\text{Speed of the moving object}}$$

Example:

A 150 meter long train is running at a speed 90 kmph crosses a man standing on a platform. Calculate the time taken by the train in crossing the man?

Solution:

$$\text{Speed of train} = 90 \text{ kmph} = 90 \times \frac{5}{18} = 25 \text{ meter/second}$$

$$\text{Hence total time taken in crossing the man } \frac{150}{25} = 6 \text{ Second}$$

- ❖ Time taken by a moving object of length 'L1' at a speed 'S' in passing a stationary object of length 'L2'.

$$= \frac{\text{Length of the moving object} + \text{length of the stationary object}}{\text{Speed of the moving object}} \\ = \frac{L_1 + L_2}{S}$$

Example:

A 300 meter long train is running at a speed of 90 kmph crosses a platform which is 150 metre long. Calculate the time taken by the train in crossing the platform?

Solution:

$$\text{Speed of train} = 90 \text{ kmph} = 90 \times \frac{5}{18} = 25 \text{ meter/second}$$

Hence total time taken

$$\frac{\text{Length of train} + \text{Length of platform}}{\text{Speed of the train}} = \frac{300 + 150}{25} \\ = \frac{450}{25} = 18 \text{ Second}$$

- ❖ If two objects of length 'L1' and 'L2' respectively, are moving in opposite direction at speeds 'S1' and 'S2' respectively, then the time taken to cross each other:

$$\frac{\text{Sum of Lengths of the moving object}}{\text{Sum of Speeds of the moving object}} = \frac{L_1 + L_2}{S_1 + S_2}$$

Example:

Two trains of lengths 300 metre and 150 metre are running in opposite direction at speeds 72 kmph and 90 kmph respectively. Calculate the time taken by the two trains in crossing each other.

Solution:

$$\text{Sum of speeds (Relative speed) of the two trains} = 72 + 90 = 162 \text{ kmph}$$

$$= 162 \times \frac{5}{18} = 45 \text{ metre/second}$$

Time taken in crossing each other

$$= \frac{\text{Sum of lengths of two trains}}{\text{Sum of speeds of two trains}} = \frac{300 + 150}{45} = \frac{450}{45}$$

$$= 10 \text{ seconds}$$

- ❖ If two objects of length 'L1' and 'L2' respectively, are moving in same direction at speeds 'S1' and 'S2' respectively, then the time taken to cross each other:

$$\frac{\text{Sum of Lengths of the moving object}}{\text{Sum of Speeds of the moving object}} = \frac{L_1 + L_2}{S_1 - S_2} \text{ if } S_1 > S_2$$

$$\frac{L_1 + L_2}{S_2 - S_1} > S_2 \text{ or } = 30 \times \frac{5}{18} = \frac{25}{3} \text{ if } S_2 > S_1$$

Example:

How long does a train 150 metre long running at 90 kmph take to cross another train of same length running in the same direction at 60 kmph?

Solution:

Relative speed of the two trains = $90 - 60 = 30$ kmph

$$= 30 \times \frac{5}{18} \times \frac{25}{3} \text{ metre/second}$$

Time taken to cross each other

$$= \frac{\text{Sum of lengths of two trains}}{\text{Difference of speed of the two trains}} = \frac{150 + 150}{\frac{25}{3}} = \frac{900}{25} = 36 \text{ seconds}$$

Boats and Stream

If the speed of a boat in still water is 'a' kmph and the speed of the stream is 'b' kmph, then:

Downstream speed = Speed while travelling with the stream = $(a + b)$ kmph

Upstream speed = Speed while travelling against the stream = $(a - b)$ kmph

Also, Downstream speed + Upstream speed = $(a + b) + (a - b) = 2a$

Hence $\frac{1}{2}$ (Downstream speed + Upstream Speed) = a = Speed of boat in still water

Similarly, Downstream speed – upstream speed = $(a + b) - (a - b) = 2b$

Hence $\frac{1}{2}$ (Downstream speed – upstream speed) = b = Speed of stream

Example:

A man rows 45 km downstream and 36 km upstream taking 9 hours each time. Find his speed in still stream and the speed at which the stream flows.

Solution:

$$\text{Downstream speed} = 45/9 = 5 \text{ kmph}$$

$$\text{Upstream speed} = 36/9 = 4 \text{ kmph}$$

Hence, speed of man in still water = $(\text{downstream speed} + \text{upstream speed})/2 = (5 + 4)/2 = 4.5 \text{ kmph}$

And speed of stream = $(\text{downstream speed} - \text{upstream speed})/2 = (5 - 4)/2 = 0.5 \text{ kmph}$

QUESTIONS

1. Ramu takes 20 minutes more to reach his office if he travels at 20 Km/hr instead of travelling at 30 Km/hr. Find the distance between his office and home.
 - a. 15Km
 - b. 20 Km
 - c. 25 Km
 - d. 30 Km
2. If I travel from Delhi to Lucknow at a speed of 30 Km/Hr and return at a speed of 20 Km/hr then find the average speed for the entire journey.
 - a. 22 Km/hr
 - b. 24 Km/hr
 - c. 25 Km/hr
 - d. 26 Km/hr
3. A train overtakes a man travelling at 18 Km/hr in 30 seconds and a platform in 45 seconds. Find the length of platform if speed of train is 72 Km/hr and train and the man were travelling in the same direction.
 - a. 900 m
 - b. 600 m
 - c. 450 m
 - d. 300 m
4. Ramu and Kallu started from two ends of a river towards each other. They crossed each other to reach their respective ends and then, they turned back to meet each other. If the distance between two ends is 30 km, both Ramu and Kallu travel at 5 Km/hr and speed of the river is 1 Km/hr, find the total time taken by them to meet for the second time.
 - a. 7.5 hours
 - b. 8.5 hours
 - c. 9.5 hours
 - d. 10.5 hours
5. Ramu and Kallu are travelling towards each other at a speed of 10 Km/hr and 5 Km/hr respectively. How much time before they collide, they will be 1 km away from each other
 - a. 2 min
 - b. 3min
 - c. 4 min
 - d. 5 min
6. Ramu can beat Kallu by 20 points in a game of pool of 60 points. In a similar game, if Kallu scores 100, then what shall be the score of Ramu?
7. Convert 36 km/hr in m/sec.
 - a. 5 m/sec
 - b. 10 m/sec
 - c. 15 m/sec
 - d. 20 m/sec
8. A train moving at a speed of 72 Km/hr passes a platform in 50 seconds and a stationary man standing on the same platform in 15 seconds. Find the length of the platform.
 - a. 500 m
 - b. 600 m
 - c. 700 m
 - d. 800 m
9. Rajdhani express left Delhi for Lucknow at 8 AM and is destined to reach its destination in 6 hours. Same day, Gomti Express left Lucknow for Delhi at 6 AM and is destined to reach its destination in 10 hours. At what time, these two trains will meet each other?
 - a. 10:45 AM
 - b. 11:00 AM
 - c. 11:15 AM
 - d. 11:30 AM
10. Ramu and Kallu start running on a circular track from the same point in opposite directions. Ramu can complete one full round in 8 hours and Kallu can run one full round in 12 hours. After how much time, they should meet for the second time.
 - a. 4 hours and 48 minutes
 - b. 7 hours and 12 minutes
 - c. 9 hours and 36 minutes
 - d. 24 hours
11. Ramu and Kallu start a race of swimming of 1 km in a swimming pool of length 100 m. Both of them will start from the same end and will reach the other end and then they will turn back and this will continue till any one of the two complete 1 km. If ratio of their speeds is 3:5, then how many times, they will meet each other in between start and end?

- a. 7 b. 8
 c. 9 d. 10
12. Chaman will reach his destination 15 minutes late if he travels at 10 Km/hr and he will reach his destination 30 minutes late if he travels at 5 Km/hr. At what speed, he should travel if he wants to reach his destination on time?
 a. 20 Km/hr
 b. 25 Km/ hr
 c. 100 Km/hr
 d. He cannot reach his destination on time
13. Dhoni and Yuvraj started running on a circular track from two diametrically opposite points in opposite directions and met for the second time after 45 minutes. If Dhoni can run one full round of that circular track in 45 minutes then find the time taken by Yuvraj to complete one full round of the same track.
 a. 60 minutes b. 75 minutes
 c. 90 minutes d. 120 minutes
14. I have two candles. First candle can burn in 15 minutes while second one takes 20 minutes to burn completely. Length of the first candle is 20 cm and length of the second candle is 15 cm. if both of them are lit together then after how much time, both of them will have equal lengths?
 a. $(60/5)$ minutes b. $(60/7)$ minutes
 c. $(60/9)$ minutes d. $(60/11)$ minutes
15. A thief somehow managed to escape from a jail at 6 AM and started running at 10 Km/hr. Police after realizing the complete situation started following the thief at 9 AM. Police was running at 15 Km/hr. There was a police dog which started running with police and kept on running between police and thief till the time, police caught the thief. If the dog was running at 25 Km/hr then find the total distance travelled by the dog.
 a. 90 Km b. 120 Km
 c. 150 Km d. 180 Km
16. Refer question number 15; what time police will be able to catch the thief?
 a. 12 PM b. 2 PM
 c. 3 PM d. 6 PM
17. Every hour one bus leaves from Lucknow and Delhi, for Delhi and Lucknow respectively. The complete journey takes 10 hours from either side and all the buses follow the same route. How many buses will be encountered by a bus leaving Delhi at 9 AM in its entire journey, including the first and the last bus?
 a. 10 b. 11
 c. 20 d. 21
- Directions for questions number 18 to 19:** Two non-stop trains left from Delhi and Lucknow for Lucknow and Delhi respectively at 7 AM and both take 12 hours to reach their respective destinations.
18. One fine day, the train from Delhi met with an accident and thus could not move for 1 hour. And thereafter it moved at half of the original speed. As a result, this train reached Lucknow at 4 AM. On that particular day, what time will these two trains will meet each other?
 a. 1:30 PM b. 1:45 PM
 c. 2:00 PM d. 2:15 PM
19. One fine day, the train from Delhi met with an accident and thus could not move for 1 hour. And thereafter it moved at half of the original speed. And eventually these two trains met at 3 PM of the same day. What time will this train reach Lucknow?
 a. 5 AM b. 6 AM
 c. 7 AM d. 8 AM
20. Ramu can row in still water at 20 Km/hr, if speed of the river is 5 Km/hr. What is the average speed of Ramu in going from Point A of river to Point B of the same river and returning back?
 a. 20 Km/hr b. 18.75 Km/hr
 c. 17.25 Km/hr d. 21.25 Km/hr
21. Ramu and Kallu were 100 Km apart and started moving towards each other at a speed of 5 Km/hr each. After every hour, Ramu increased his speed by 1 Km/hr and Kallu reduced his speed by 1 Km/hr and kept on doing so till they meet each other. Approximately, at what time will they meet each other? Assuming after reaching a speed of

zero, Kallu will stop moving and will stand at that point only.

- a. 9 hours
 - b. 9 hours and 15 minutes
 - c. 9 hours and 45 minutes
 - d. 10 hours
22. Chaman is 20 % faster than Raman and thus can give a lead of 20 min to Raman in a race. Find the time taken by Chaman to complete the race?
- a. 80 minutes
 - b. 100 minutes
 - c. 120 minutes
 - d. 140 minutes
23. Raman had to travel from Delhi to Jumritalaiya in 5 hours and started at a speed of 50 Km/hr. After three hours he realized that he has travelled only 40 % of the total distance. At what speed, he should cover the remaining distance to reach his destination in time?
- a. 100 Km/hr
 - b. 112.5 Km/hr
 - c. 125 Km/hr
 - d. 137.5 Km/hr
24. Chaman decided to go from Lucknow to Delhi. Distance between Lucknow and Delhi is 420 Km and he decided to complete his journey in 7 hours. But towards the end of the journey, he got stuck in traffic and thus wasted an hour in the jam and then he increased his speed to 80 Km/hr to finally reach his destination. After how many hours after starting, he was forced to increase his speed if he just managed to reach in time.
- a. 2 hours
 - b. 3 hours
 - c. 4 hours
 - d. 5 hours
25. Ramu travelled first one third of the journey at 40 Km/hr, next one third of the journey at 50 Km/hr and last one third of the journey at 80 Km/hr. Find the approximate average speed of Ramu for the entire journey?
- a. 50 Km/hr
 - b. 52 Km/hr
 - c. 54 Km/hr
 - d. 56 Km/hr
26. Ramu and Kallu were typing a book of 368 pages. Ramu started typing from the first page at a speed of 3 pages per minute and Kallu started typing from the last page at a speed of 5 pages per minute. Both of them kept on typing till the

complete book was completed. Find the number of the last page typed by Ramu.

- a. 137
 - b. 138
 - c. 139
 - d. 140
27. Ramu and Kallu started swimming from two opposite ends of a swimming pool in such a way that they will go to opposite ends and then they will reverse the direction. First time they met at 50 m from the shallow end and next time at 40 m from the deep end. Find the length of the swimming pool.
- a. 90 m
 - b. 100 m
 - c. 110 m
 - d. 120 m
28. Travelling at a speed of $\frac{3}{4}$ th of the original speed, Chaman takes 20 minutes more than what he would have taken otherwise. Find the time saved if Chaman would have travelled at $\frac{4}{3}$ rd of the original speed.
- a. 15 min
 - b. 20 min
 - c. 25 min
 - d. None of the above
29. Chaman and Raman started from point A and point B respectively towards point B and point A respectively. After crossing each other they took 4 minutes and 9 minutes to reach their respective destinations. Find the ratio of their speeds.
- a. 3:2
 - b. 2:3
 - c. 4:9
 - d. 9:4
30. A goods train travelling at 54 Km/hr takes 20 seconds to cross Aligarh Junction railway station and the same train takes 25 seconds to cross New Delhi railway station. Find the differences between the lengths of Aligarh Junction railway station and New Delhi railway station.
- a. 75 m
 - b. 300 m
 - c. 375 m
 - d. 450 m
31. A passenger train and a goods train left the same yard at a gap of 2 hours with passenger train leaving first. Speed of the goods train is 50 Km/hr and it overtakes passenger train exactly after travelling 3 hours. Find the speed of passenger train?
- a. 20 Km/hr
 - b. 25 Km/hr
 - c. 30 Km/hr
 - d. 40 Km/hr

32. A boat takes 4 hours to go from A to B and takes 5 hours to go from point B to A. Find the time taken by the same boat to go from A to B if speed of the river is doubled.

- a. 3 hrs
- b. 3.63 hrs
- c. 3.46 hrs
- d. 3.87 hrs

33. Ramu can travel 50 Km upstream and 72 Km downstream in 9 hours and 70 Km upstream and 90 Km downstream in 12 hours. Find the time taken by Ramu to travel 100 Km upstream.

- a. 9 hours
- b. 10 hours
- c. 12 hours
- d. 15 hours

34. Jalgaon to Jalsamadhi is a downstream journey on a river which flows at 5 Km/hr. Two boats start from Jalgaon and Jalsamadhi for the opposite destinations. Boat from Jalgaon has a still water speed of 25 Km/hr and boat from Jalsamadhi has a still water speed of 15 Km/hr. At what distance from Jalgaon these boats will meet each other if distance between these two places is 300 Km.

- a. 225 Km
- b. 250 Km
- c. 200 Km
- d. None of the above

35. Ramu went from point A of a river to point B of the same river and returned back in 20 hours. Total distance between the two places is 48 Km and the speed of the river is 1 Km/hr. Find the speed of the Ramu in still water.

- a. 3 Km/hr
- b. 4 Km/hr
- c. 5 Km/hr
- d. 6 Km/hr

36. Raman went to his aunt's place at a speed of 40 Km/hr and returned at a speed of 60 Km/hr. Find the average speed of Raman for the entire journey.

- a. 45 Km/hr
- b. 48 Km/hr
- c. 50 Km/hr
- d. 52 Km/hr

37. Speed of a bus excluding all the stoppages is 50 Km/hr and including all the stoppages is 40 Km/hr. Find the average stoppage time of the bus per hour.

- a. 8 minutes
- b. 10 minutes
- c. 12 minutes
- d. 15 minutes

38. Ramu covers 30 % of his journey in 50% of the total permissible time. By how much percentage he should increase his speed in order to complete the journey in time?

- a. 133.33 %
- b. 100 %
- c. 66.66 %
- d. 33.33 %

39. Ramu beats Kallu by 50 m in a race of 1000 m and beat Billu by 100 m in a race of 1000 m. Find the lead that Kallu can give to Billu in a race of 500 m.

- a. 52 m
- b. 50 m
- c. 26.3 m
- d. 25 m

40. Ramu can beat Kallu by 100 m in a race of 1000 m or he can give a lead of 90 seconds to Kallu. Find the time taken by Kallu to complete 1000m.

- a. 450 seconds
- b. 750 seconds
- c. 900 seconds
- d. 1000 seconds

41. Ramu can beat Kallu by 20 m and Billu by 30 m in a race. If Kallu can beat Billu by 12 m in the same race, then find the length of the race.

- a. 100 m
- b. 120 m
- c. 140 m
- d. 160 m

42. Ramu takes 3 hours to go from A to B in a river and takes 6 hours to go from B to A in the same river. What is the ratio of speed of Ramu in still water and speed of stream?

- a. 2:1
- b. 3:1
- c. 4:1
- d. 5:1

43. A man got stuck in some office work and thus got delayed by 20 minutes. By what percentage he should increase the speed of his car to reach his home at usual time? Assume he used to take exactly 60 minutes to reach his home on other days.

- a. 20%
- b. 33.33%
- c. 40%
- d. 50%

44. Two cyclists start cycling on a circular track of 300m from the same point. If the speed of these two cyclists are 25 m/sec and 15 m/sec respectively then after how much time they should meet for the first time? Assume both of them are moving in the same direction.

- a. 7.5 seconds
- b. 15 seconds
- c. 30 seconds
- d. None of the above

45. At what time in between 3 o'clock and 4 o'clock, hour's hand and minute's hand of the clock are perpendicular to each other?
- a. 3:30
 - b. 3:33
 - c. 3:35
 - d. 3:01
46. A man going on a highway in his car realized that it took him 30 seconds to come from 200 m behind a truck to 100 m ahead of the truck. If speed of the car was 100 Km/hr then what is the speed of the truck? Assume length of car and truck to be negligible.
- a. 36 Km/hr
 - b. 64 Km/hr
 - c. 136 km/hr
 - d. None of the above
47. If area covered by hour's hand of a clock in a day is 3 % of the area covered by the minute's hand during the same day. Then what is the ratio of length of the hour's hand and minute's hand of the same clock?
- a. 3:5
 - b. 2:5
 - c. 3:8
 - d. 5:8
48. A train crosses two platforms of 250 m and 150 m in 40 seconds and 30 seconds respectively. What is the speed of the train?
- a. 18 Km/hr
 - b. 36 Km/hr
 - c. 72 Km/hr
 - d. 108 Km/hr
49. A train crosses a telephone post in 50 seconds. How much will it take to cross the same post if it increases its speed by 20 %?
- a. 60 Seconds
 - b. 41.66 seconds
 - c. 40 seconds
 - d. 62.5 Seconds
50. A person covers first 100 km of his journey at 80 Km/hr, next 150 km of his journey at 60 Km/hr and last 150 km of his journey at 40 Km/Hr. find his average speed for the entire journey?
- a. 50 Km/hr
 - b. 53 Km/hr
 - c. 56 Km/hr
 - d. 60 Km/hr

...
...

10

Time and Work

Time and work related problems are very much similar to time speed and distance related problems, where in speed is replaced with rate of doing the work.

Thus Total time taken to complete a work

$$= \frac{\text{total work done}}{\text{total time taken}}$$

Prob: If Ramu can complete a work in 6 days and Kallu can complete the same work in 8 days, then working together in how many days, they can finish the work?

Sol: If Ramu can finish the work in 6 days then his one day's work = $1/6$ of the total work

If Kallu can finish the work in 8 days then his one day's work = $1/8$ of the total work

If both of them start working together, then their one day's work = $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$

Now if they can finish $\frac{7}{24}$ part of the work in 1 day then they can finish total work in $\frac{40}{10 \times 4}$ days.

Alternate way: Let us assume total work to be 24 units (which is nothing but LCM of the two numbers)

This means that Ramu completes 24 units in 6 days completing 4 units a day and Kallu complete 24 units in 8 days completing 3 units a day. Thus they complete 7 units per day and thus they will take days to complete the work.

Prob: If Ramu can construct a work in 6 days and Kallu being a demolisher can destroy the work in 8 days, then starting together in how many days, the work will complete?

Sol: If Ramu can finish the work in 6 days then his one day's work = $1/6$ of the total work

If Kallu can destroy the work in 8 days then his one day's destruction = $1/8$ of the total work

If both of them start working together, then their one day's work =

Now if they can finish part of the work in 1 day then they can finish total work in days.

Alternate way: Let us assume total work to be 24 units (which is nothing but LCM of the two numbers)

This means that Ramu completes 24 units in 6 days completing 4 units a day and Kallu destroy 24 units in 8 days destroying 3 units a day. Thus they complete 1 unit per day and thus they will take days to complete the work.

Points to be considered:

- If rate of working for some men is given then rate of working is inversely proportional to time taken by them
- Time taken to complete a work is always the first incident when the work was finished.
- Above used concept can be used if and only if rate of working is constant throughout the time duration.

Problem:

Ramu can complete a work in 6 days and Kallu can destroy it in 8 days. Both of them have started working on alternate days then in how many days with obviously Ramu working on day 1. Then in how many days work will be completed?

Sol: If Ramu can finish the work in 6 days then his one day's work = $1/6$ of the total work

If Kallu can destroy the work in 8 days then his one day's destruction = $1/8$ of the total work

Then their two day's work =

So, their 40 days work =

And on last day, Ramu will complete $1/6^{\text{th}}$ of the work to finish the job in 41 days.

Pipes and Cistern: Questions on Pipe and cistern also use the same concept of time and work wherein inlet pipe can be compared with constructor and outlet pipe will be compared with destroyer.

Prob: A tank is fixed with two inlet pipes which can fill the tank in 8 hours and 10 hours respectively and there is an outlet pipe which can empty the tank in 4 hours. In a totally filled tank, if all the three pipes are opened together then in how much time, the tank will be emptied?

Sol: First pipe can fill the tank in 8 hours filling $1/8^{\text{th}}$ of the tank every hour whereas second inlet will fill $1/10^{\text{th}}$ of the tank every hour.

Outlet pipe will empty the complete tank in 4 hours emptying $1/4^{\text{th}}$ of the tank every hour.

So net amount of water leaked every hour =
of the complete tank

If $1/40^{\text{th}}$ part of the tank is emptied every hour then it means that complete tank will be emptied in 40 hours.

Alternate way: Let us assume that total capacity of the tank to be 40 Ltrs.

Then first inlet will fill 5 ltrs every hour and second inlet will fill 4 ltrs every hour whereas outlet will leak out 10 ltrs every hour thus emptying 1 ltr every hour.

And thus complete tank will be emptied in 40 hours.

Prob: An inlet pipe can fill a cistern in 7 hours but because of a leakage, it takes 9 hours to fill that particular cistern. In how much time, this leakage can empty the complete cistern?

Sol: Let us assume the capacity of cistern to be 63 ltrs. Then Inlet can fill 9 Ltrs per hour of itself but because of the leakage, it will fill only 7 ltrs per hour. This means that there is a leakage of 2 ltrs per hour and complete cistern will be empties in 31.5 hours.

QUESTIONS

1. If Ramu can complete a task in 8 days and Kallu can complete the same task in 12 days then working together in how many days they can finish off the same task?
 - a. 4.2 days
 - b. 4.4 days
 - c. 4.6 days
 - d. 4.8 days
2. If Sheela can type 400 words per hour and Munni can type 600 words per hour. A typing job was given to both of them and they completed the job together in 3 days. If they were given Rs 4800 for the job then what will be Sheela share in the total amount.
 - a. Rs 480
 - b. Rs 960
 - c. Rs 1920
 - d. Rs 1640
3. Chaurasia can complete digging a well in 8 days. His son is half as efficient as he is and his father is one third as efficient as he is. They got a contract to dig the same well and all three of them started to dig the same. Chaurasia's father stopped working after one day and his son stopped working after two days. In how many more days, Chaurasia will be able to finish off the work?
 - a. 3 days
 - b. 4 days
 - c. 5 days
 - d. 6 days
4. Raju and Ramu start race of 10 km on a circular track of 1 Km. Ratio of their speeds is 1:4 and they meet for the first time exactly after 5 minutes. In how much time, the race will be finished?
 - a. 37.5 minutes
 - b. 50 minutes
 - c. 25 minutes
 - d. None of the above
5. An inlet pipe can fill a tank in 8 hours. Because of a leak, it takes 10 hours to fill the tank. In how much time, this leak will empty the complete tank?
 - a. 20 hours
 - b. 40 hours
 - c. 60 hours
 - d. 80 hours
6. Ram and Shyam working together can finish a job in 6 hours. Ram and Dhyan working together can finish the same job in 8 hours. Ram and Nyan working together can finish the same job in 24 hours. If all four, Ram, Shyam, Dhyan and Nyan are working together the work can be finished in 3 hours. Ram alone can finish the job in how many days?
 - a. 48 hours
 - b. 72 hours
 - c. 108 hours
 - d. None of the above
7. Ramu and Kallu are running on a circular track in opposite directions. Ramu can run one complete round in 6 minutes and Kallu can run one full round in 8 minutes. If they started together at 8:00 am from the same point then they will be together for the 21st time at what time?
 - a. 8:24 am
 - b. 9:12 am
 - c. 10:48 am
 - d. 5:36 pm
8. In the previous question, at how many distinct points will they meet each other?
 - a. 7
 - b. 20
 - c. 21
 - d. 22
9. Refer question 7, at what time both of them will be diametrically opposite to each other for the first time?
 - a. 8:07
 - b. 8:12
 - c. 8:21
 - d. 8:24
10. If Ramu finishes a job in 20 days and Kallu finishes the same job in 60 days. If they decided to work on alternate days, then in how many days, they will be able to finish off the work?
 - a. 15 days
 - b. 30 days
 - c. 40 days
 - d. Depends on who starts the work
11. How many times does hours hand and minutes hand meet in a day of a standard clock?
 - a. 21
 - b. 22
 - c. 23
 - d. 24
12. Machine A can complete a work in 10 days. Machine B is 25 % more efficient than the first one. In how many days, these two machines can complete the work if working together.

- a. $40/7$ days b. $40/8$ days
 c. $40/9$ days d. $40/10$ days
13. Ramu, Kallu and Billu can complete a job in 6 days, 8 days and 12 days respectively working 12 hours a day. If they started working 8 hours a day, and Ramu leaves after 2 days, Kallu leaves after 4 days. In how many more days, Billu will be able to finish the complete work?
 a. 2 days b. 3 days
 c. 4 days d. 5 days
14. In a cistern, there are three pipes connected: A, B and C. A and B are inlet pipes and can fill the cistern in 12 hours and 18 hours respectively, whereas pipe C is an outlet pipe which can empty the cistern in 15 hours. If pipe A was opened for 4 hours following by pipe C for 2 hours and 30 minutes and then finally complete tank was filled by pipe B. How much time was taken in the complete process to fill the cistern?
 a. 18 hours b. 20.5 hours
 c. 21.5 hours d. 22.5 hours

Directions for questions 15 to 17: Ramu and Kallu start running on a circular track in opposite directions starting from the same point at the same time. Ramu can complete one full round in 8 hours and Kallu can complete one full round in 10 hours.

15. After how much time, they will meet for the first time?
 a. $40/9$ hours b. 40 hours
 c. $80/9$ hours d. 80 hours
16. After how much time, they will meet for the first time at the starting point?
 a. $40/9$ hours b. 40 hours
 c. $80/9$ hours d. 80 hours
17. How many times, they will meet if they continue to run for 50 hours?
 a. 9 times b. 10 times
 c. 11 times d. 12 times

Directions for questions 18 to 20: Ramu and Kallu start running on a circular track in same direction starting from the same point at the same time. Ramu can complete one full round in 7 hours and Kallu can complete one full round in 11 hours.

18. After how much time, they will meet for the first time?
 a. 77 hours b. 3.85 hours
 c. 19.25 hours d. 39.5 hours
19. After how much time, they will meet for the first time at the starting point?
 a. 77 hours b. 3.85 hours
 c. 19.25 hours d. 39.5 hours
20. How many times, they will meet if they continue to run for 50 hours?
 a. 1 times b. 2 times
 c. 11 times d. 3 times
21. 6 men can complete a work in 8 days. One woman is half as efficient as a man and a child is half as efficient as a woman. 4 children, 6 women and 4 men will take how many days to complete the work?
 a. 4 days b. 6 days
 c. 8 days d. Cannot be determined
22. A and B can complete a work in 10 days, B and C can complete the same work in 15 days and C and A can complete the same work in 18 days. All three of them will be able to complete the work in how many days?
 a. 7 days b. 8 days
 c. 9 days d. 10 days
23. Ramu being a constructor can complete a work in 8 days. Kallu being a destroyer can destroy the complete work in 12 days. If Ramu and Kallu start working on a job working on alternate days, then how many days will be required to complete the work?
 a. 41 days b. 43 days
 c. 47 days d. 48 days
24. Ramu can complete a job in 16 days, he starts the work and work on the same for 4 days, and then Kallu complete the remaining work in 18 days. Ramu could have completed the job alone in how many days?
 a. 20 days b. 22 days
 c. 24 days d. 26 days
25. Raman is 20 % more efficient than Chaman and Karan is 25 % more efficient than Raman. If all of them can complete a piece of work in 10 days, then how many days will be taken by Chaman to complete the work?
 a. 33 days b. 35 days
 c. 37 days d. 39 days

26. To complete a work, Ramu takes twice as much time as taken by Kallu and Billu together. Kallu takes thrice as much time as taken by Ramu and Billu together. If working together, all of them can complete the job in 5 days, then Ramu can complete the work in how many days?
- 10 days
 - 12 days
 - 15 days
 - 20 days
27. 9 men and 12 women can complete a work in 4 days. 4 men and 16 women can complete the same work in 6 days. How much time will be taken by 24 women and 6 men to complete the work?
- 3 days
 - 4 days
 - 5 days
 - 6 days
28. A certain number of men can complete a job in 40 days. Had there been 5 more men, number of days required would have been 10 less. Find the number of men required to finish the job in 60 days.
- 10 men
 - 15 men
 - 20 men
 - 25 men
29. A alone can finish a job in 12 days and B alone can finish the job in 18 days. C takes twice as many days as taken by A and B together. If all of them are working together, then in how many days, they shall be able to complete the job?
- 4 days
 - 4.4 days
 - 4.8 days
 - 5.2 days
30. How many times, hour's hand and minute's hand of a clock are perpendicular to each other in a day?
- 42
 - 44
 - 46
 - 48
31. What is the angle between hours hand and minutes hand at 7:45
- 37.5°
 - 60°
 - 10°
 - 45°
32. What time, in between 4 o'clock and 5 o'clock, hours hand and minutes hand will meet each other?
- 4:20
 - 4:21:12
 - 4: 21:48
 - 4:22:16
33. If 20 men can weave 30 m in 40 days working 9 hours a day then how many more men are required to weave 50 m in 50 days working 6 hours a day?
- a. 10 men b. 20 men
 c. 30 men d. 40 men
34. If 10 men eat 10 rotis in 10 days then 50 men will eat how many rotis in 50 days?
- 50
 - 100
 - 250
 - 2500
35. Ramu, Kallu and Billu can finish painting the fence in 2 hours. If Ramu does the job alone he can finish it in 5 hours. If Kallu does the job alone he can finish it in 6 hours. How long will it take for Billu to finish the job alone?
- 6 hours
 - 7.5 hours
 - 9 hours
 - 10 hours
36. 34 men can complete $\frac{2}{5}$ th of a work in 7 days. How many additional men are required to complete the remaining work in another 7 days?
- 13 men
 - 15 men
 - 17 men
 - 19 men
37. 5 women or 10 men can complete a work in 10 days. How many days will be required by 4 men and 8 women to complete the work?
- 2 days
 - 5 days
 - 6 days
 - 8 days
38. Ramu can complete a job in 8 days working 7 hours a day and Kallu can complete the same work in 7 days working 6 hours a day. If both of them start on the same job together working 8 hours a day, then how many days are required to finish the job?
- 2 days
 - 3 days
 - 4 days
 - 5 days
39. Chaman can make a software program in 12 days and Raman is 20 % less efficient than Chaman. But working together their efficiencies increases by 20 %. In how many days working together, they shall be able to complete a software program?
- 5 days
 - 5.55 days
 - 6 days
 - 6.25 days
40. Ramu is twice as fast as Kallu is and Kallu is thrice as fast as Billu is. If all of them can complete a job in 10 days working together then find the number of days taken by Billu to complete the same job?
- 100 days
 - 33.33 days
 - 16.66 days
 - 25 days

41. There are two gears working with each other. First gear with 84 teeth take 40 revolutions in 50 seconds. Find the number of revolutions taken by second gear with 42 teeth in 90 seconds
 a. 120 b. 36
 c. 144 d. None of the above
42. If ratio of efficiency of Ramu, Kallu and Billu is 1:2:3. Then find the ratio of time taken by them to complete a given work?
 a. 3:2:1 b. 6:3:1
 c. 6:3:2 d. None of the above
42. An inlet pipe can fill a tank in 5 minutes, but because of a leak it takes 8 minutes to fill the tank. Find the time taken by the leak to empty a filled tank?
 a. 40 minutes b. 20 minutes
 c. 13.33 minutes d. 10 minutes
44. A ration of 40 kg is sufficient for 10 men for 4 days. One woman eats half as much as a man and a child eat half as much as a woman. Then a ration of 50 kg should be sufficient for 12 boys, 6 women and 4 men for how many days?
 a. 2 days b. 3 days
 c. 4 days d. 5 days
45. Ramu can complete a work in 8 days. Kallu is 20 % more efficient than Ramu. Find the number of days required by Kallu to finish a job which is 50 % more than the initial work.
 a. 8 days b. 10 days
 c. 12 days d. 15 days
46. There is a leak in a tank which can empty the tank in 30 hours. This tank is initially 80 % filled. Find the time required to fill the tank with the help of an inlet pipe which can fill the tank in 12 hours
 a. 3 hours b. 4 hours
 c. 5 hours d. 6 hours
47. There are three pipes A, B and C which can fill a tank in 10, 12 and 15 minutes respectively. If all the inlets are opened initially and after 2 minutes first and third inlets are closed. Then find the total time taken to fill the tank completely.
 a. 6 minutes b. 7 minutes
 c. 8 minutes d. 9 minutes
48. 4men and 8 women can dig an area of 42 sq m in 70 days. Find the number of days required by 8 men and 16 women to dig an area of 120 sq m.
 a. 80 days b. 100 days
 c. 120 days d. Cannot be determined
49. Chaman can do a work in 12 days and Raman in 20 days. If they work on it together for 5 days, then what fraction of the work that is left?
 a. $1/3$ b. $2/3$
 c. $1/4$ d. $3/4$
50. Ramu is thrice as good as workman as B and therefore is able to finish a job in 60 days less than B. Working together, how many days will be required to complete the job?
 a. 45 days b. 22.5 days
 c. 67.5 days d. Cannot be calculated

...
 ...

LINEAR EQUATIONS

An equation in which the maximum power of any variable is unity (one) is called a linear equation.

A linear equation is of the form:

$$ax + by + c = 0$$

Where X and Y are variables and 'a' and 'b' are their respective coefficients and c is a constant.

The graph of a linear equation is a straight line on the X – Y Cartesian plane.

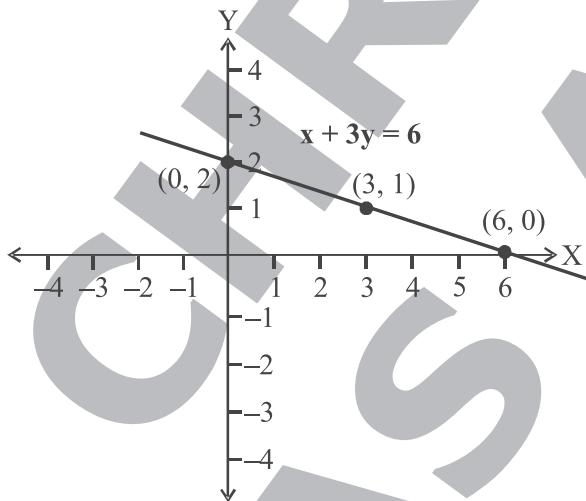
Example:

Suppose if you want to draw the graph of a linear equation:

$$x + 3y = 6$$

Then first we have to find few points that satisfy the above equation—

X	0	3	6
Y	2	1	0



If we take any other solution of the equation $x + 3y = 6$ then we can see that all those solutions will lie on the above drawn line.

Simultaneous Linear Equation:

Equations often arise in which there is more than one unknown quantity. When this is the case there

will usually be more than one equation involved. For example in the two linear equations $7x + y = 9$; $-3x + 2y = 1$ there are two unknowns: x and y. In order to solve the equations we must find values for x and y that satisfy both of the equations simultaneously. The two equations are called simultaneous equations. You should verify that the solution of these equations is $x = 1$, $y = 2$ because by substituting these values into both equations, the left-hand and right-hand sides are equal.

Solving two linear equations in two unknown:

Let there be two linear equations in two unknown.

$$a_1x + b_1y = K_1 \quad \text{and} \quad a_2x + b_2y = K_2$$

A pair of numbers (n_1 , n_2) which satisfies both the equations is called a solution of the system of equations.

One way of solving simultaneous equations is by elimination. As the name implies, elimination, involves removing one of the unknowns. Note that if both sides of an equation are multiplied or divided by a non-zero number an equivalent equation results.

Example:

Solve the simultaneous equations

$$3x + 5y = 31 \quad (1)$$

$$2x + 3y = 20 \quad (2)$$

Solution:

We first try to modify each equation so that the coefficient of x is the same in both equations. This can be achieved if the Equation (1) is multiplied by 2 and the Equation (2) is multiplied by 3. This gives

$$6x + 10y = 62$$

$$6x + 9y = 60$$

Now the unknown x can be removed (or eliminated) if the second equation is subtracted from the first:

$$6x + 10y = 62$$

$$\text{Subtract } 6x + 9y = 60$$

$$\text{Which gives: } 0x + 1y = 2$$

The result implies that $1y = 2$ and we see immediately that y must equal 2. To find x we substitute the value found for y into either of the given equations (1) or (2). For example, using Equation (1),

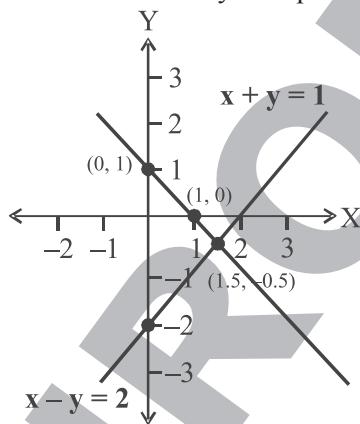
$$3x + 5(2) = 31 \rightarrow 3x = 21 \rightarrow x = 7$$

Thus the solution of the given equations is $x = 7$, $y = 2$.

There are three possible cases in the case of two linear equations.

1. The system has exactly one (unique) solution:

In this case in the graph of the two linear equations, the lines will intersect at only one point.



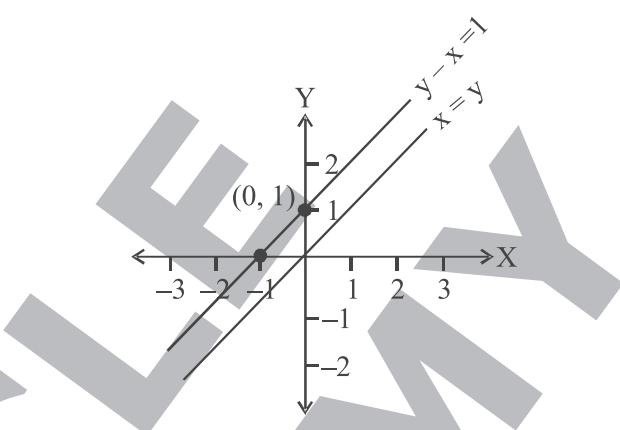
❖ The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(As in this case lines intersect at a single point.)

2. The system of equations has no solution:

In this case the lines are parallel to each other and they have the same slope.

$$\begin{aligned} Y - X &= 1 \\ X &= Y \end{aligned}$$



The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Example:

The following equations have no solution.

$$10x - 2y = -3 \quad \dots(A)$$

$$-5x + y = 1 \quad \dots(B)$$

We say that equations (A) and (B) are inconsistent and they have no solution.

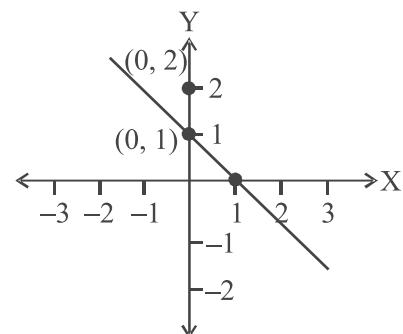
(As in this case the lines become parallel)

3. The system of infinite number of solutions:

In this case the two equations are coinciding with each other.

$$X + Y = 1$$

$$3X + 3Y = 3$$



The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has infinite number of solutions, if

$$= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(As in this case the lines overlap)

Example:

The following equations have infinite number of solutions:

$$2x + y = 8 \quad (\text{A})$$

$$4x + 2y = 16 \quad (\text{B})$$

Consistent System:

A system of two or more linear equations is said to be consistent, if it has at least one solution.

Inconsistent System:

A system of linear equations is said to be inconsistent if it does not have any solution.

For Example: System of linear equations $x + y = 4$ & $3x + 3y = 9$ is inconsistent.

In the above case there is no value of X and Y which simultaneously satisfy the given equations. So the system is inconsistent.

Quadratic Equations

A quadratic equation in X is an equation, in which the highest power of X is 2, and the equation is generally satisfied by the values of X, but these values may or may not be equal to each other.

$AX^2+BX+C=0$ is the general form of a quadratic equation, where $A \neq 0$, and A, B and C are all constants.

For example $X^2 + 4X - 780 = 0$, $3X^2 + 4X + 1 = 0$ are quadratic equations.

The constants A, B and C are called respectively, the quadratic coefficient, the linear coefficient and the constant term respectively.

Solution of Quadratic Equations:

There are two most commonly used methods to solve a quadratic equation.

Factorization Method

Quadratic Formula Method

Factorization Method:

In this method we split the middle term of the quadratic equation into two parts such that the sum of the terms

with coefficient X is equal to the middle term of the quadratic equation and their product should be equal to the product of the terms on the extreme.

Then we factorize the equation into two factors either of them is equal to zero and so we can find the value of X.

Example:

Find the value of X for which $3X^2 + 4X + 1 = 0$

Solution:

The given equation can be written as $3X^2 + 3X + X + 1 = 0$

$$[(3) + (1)] = 4 \text{ and } (3) \cdot (1) = 3 \text{ d } 3X(X + 1) + (X + 1) = 0 \quad (3X + 1)(X + 1) = 0$$

$$X = -1$$

Quadratic Formula Method:

A general solution to $AX^2+BX+C=0$ is:

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Example:

Solve the equation $X^2 - 4X + 13 = 0$

Solution:

Here we have $A = 1$, $B = -4$, $C = 13$

$$\Rightarrow X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2 \times 1}$$

$$\Rightarrow X = \frac{4 \pm \sqrt{-36}}{2 \times 1} \Rightarrow X = \frac{4 \pm 6i}{2 \times 1}$$

[Where $i = \sqrt{-1}$ $\Rightarrow X = 2 \pm 3i$

Discriminant of a Quadratic Equation:

The quantity $D = B^2 - 4AC$, is known as the Discriminant of the equation $AX^2 + BX + C = 0$.

Nature of Roots:

The nature of the roots depends on the expression $B^2 - 4AC$. The nature of the roots will be Real and Distinct if and only if $D = B^2 - 4AC > 0$.

Real and Equal if and only if $D = B^2 - 4AC = 0$.

Complex conjugate numbers of the form $a \pm ib, a \neq 0, b \neq 0$,

If and only if $D = B^2 - 4AC > 0$.

Rational and Unequal if $D = B^2 - 4AC > 0$. and only if D is a perfect square.

Irrational and Unequal if and only if $D = B^2 - 4AC = 0$. and D is not a perfect square.

Example:

Find the nature of the roots of the following equations.

- $X^2 - 4X + 13 = 0$
- $X^2 + X - 6 = 0$
- $2X^2 - 8X + 1 = 0$
- $3X^2 - 8X + 1 = 0$
- $4X^2 - 12X + 9 = 0$

Solution:

Quadratic Equation	$D = B^2 - 4AC$	Nature of Roots
$X^2 - 4X + 13 = 0$	-36	Complex Conjugate
$X^2 + X - 6 = 0$	25	Rational and Unequal
$2X^2 - 8X + 3 = 0$	40	Irrational and Unequal
$3X^2 - 8X + 1 = 0$	52	Irrational and Unequal
$4X^2 - 12X + 9 = 0$	0	Real and Equal

Relation between Roots and Coefficient of a Quadratic Equation:

If α and β are the roots of a Quadratic Equation $AX^2 + BX + C$, then $(x - \alpha)(x - \beta)$ is the required equation

$$\Rightarrow (x - \alpha)(x - \beta) = AX^2 + BX + C = 0$$

$$\Rightarrow x^2(x - \beta) + (\alpha.\beta) = x^2 + \frac{B}{A}X + \frac{C}{A} = 0$$

Which gives $(\alpha + \beta)$

$$= -\frac{B}{A} \text{ and } \alpha.\beta = \frac{\text{Coefficient of } X}{\text{Coefficient } X^2}$$

$$\text{Sum of roots} = \frac{-B}{A} = -\frac{\text{Coefficient of } X}{\text{Coefficient } X^2}$$

$$\text{Product of roots} = \frac{C}{A} = \frac{\text{constant term}}{\text{coefficient of } X^2}$$

Also, the equation is:

$$x^2 - (\text{sum of roots})x + (\text{Product of roots}) = 0$$

If $C = a$, the roots are reciprocal of each other.

If $B = 0$, the roots are of equal magnitude and opposite in sign

If $C = 0$, one of the roots of the equation will be zero.

If one of the roots of the quadratic equation is $a + \sqrt{b}$ then the other root will be $a - \sqrt{b}$ and vice versa.

If one of the roots be imaginary $a + i b$, then the other root will be $a - i b$ and vice versa.

Progression

Sequence:

A succession of numbers formed and arranged in a definite order according to a certain definite rule is called a sequence.

The number occurring at the nth place of a sequence is called its nth term or the general term, to be denoted by T_n . A sequence is said to be finite or infinite depending on the number of distinct terms is finite or infinite.

Series:

By adding the terms of a sequence, we obtain a series. A series is said to be finite or infinite depending on the number of terms added is finite or infinite.

Progressions:

Sequences which follow certain patterns are called progressions. If the terms of a series are added up to certain terms it forms a series.

Arithmetic Progression:

Arithmetic progression is the sequence in which the difference between any term and the preceding one is same throughout the series. For example:

- $a, a+d, a+2d, a+3d, \dots$ are in A.P. as a is the first term and d is the common difference in the sequence.
- n^{th} term i.e. $T_n = a+(n-1)d$
- Sum of first n terms $S = \frac{n}{2}[2a + (n-1)d]$
or $S = \frac{n}{2}\left(\frac{a+L}{2}\right)$, where L is the last term.
- Arithmetic mean between two quantities a and b = $\frac{a+b}{2}$
- n^{th} term T_n = Sum of first n terms – Sum of first $(n-1)$ terms $S_n - S_{n-1}$
- If three numbers are in A.P., we can take the three terms to be $(a-d), a$ and $(a+d)$
- If four numbers are in A.P., we can take the four terms to be $(a-3d), (a-d), (a+d)$ and $(a+3d)$

Geometric Progression:

Geometric progression is the sequence in which the ratio of every two consecutive terms is always the same and this constant ratio is called the common ratio. For example:

- a, ar, ar^2, ar^3, \dots here a is the first term and r is the common ratio.
- n^{th} term $T_n = ar^{n-1}$
- Sum of n terms i.e. $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$
or $S_n = \frac{a(r^n-1)}{r-1}$ if $r > 1$
- If three terms are in geometric progression then the middle term is a geometric Mean of the other two terms, i.e., if a, b and c are in G.P., then b is the geometric mean of the three terms and $b^2 = ac$
- When there are three terms in geometric progression, we can take the three terms to be $a/r, a$ and ar .

Example 1:

Find the 10th term of the series 4, 8, 12, 16, 20.

Solution:

Here $a = 4$ and $d = 4$

Hence

$$10^{\text{th}} \text{ term} = T_{10} = a + (n-1)d = 4 + (10-1)4 = 40$$

Example 2:

Determine the 25th term of an A.P. whose 9th term is –6 and common difference is $5/4$.

Solution:

$$T_9 = a + (n-1)d$$

$$-6 = a + (9-1)(5/4)$$

$$-6 = a + 20$$

$$a = -26$$

$$\text{Now, } T_{25} = -26 + (25-1)(5/4) = -26 + 30 = 4$$

Example 3:

The fourth term of an A.P. is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. Find the first term and the common difference.

Solution:

Let a be the first term and d be the common difference.

$$\text{Now } a+3d = 3a \quad \{\text{since } T_4 = 3T_1\} \quad \dots \dots (1)$$

$$\text{And } a+6d = 2(a+2d)+1 \quad \{\text{since } T_7 = 2T_3 + 1\} \quad \dots \dots (2)$$

$$\text{From (1) we get } a = \left(\frac{3}{2}\right)d$$

Putting $a = (3/2)d$ in (2) we get

$$\frac{3}{2}d + 6d = 2 \times \frac{3}{2}d + 4d + 1$$

$$\text{Or, } d = 2, \text{ hence } a = (3/2) \times 2 = 3$$

The first term is 3 and the common difference is 2.

Example 4:

Determine the sum of the first 35 terms of an A.P., if $T_2 = 2$ and $T_7 = 22$

Solution:

Let 'a' be the first term and 'd' the common difference.

Then

$$a + d = 2 \text{ and } a + 6d = 22$$

Solving these two we get, $d = 4$ and $a = -2$

$$\text{Now, } S = \frac{n}{2} [2a + (n-1)d]$$

$$S_{35} = \frac{35}{2} [2 \times (-2) + (35-1)4] = 35 \times 66 = 2310$$

Example 5:

The 2nd and the 5th term of a series in G.P. are 96 and 324. Find the 7th term.

Solution:

Let 'a' be the first term and 'r' the common ratio, then

$$T_2 = ar = 96 \quad \dots\dots \text{(i)}$$

$$T_5 = ar^4 = 324 \quad \dots\dots \text{(ii)}$$

Dividing (ii) by (i), we get $r^3 = 324/96 = 27/8$ Hence,
 $r = (27/8)^{1/3} = 3/2$

Hence $ar = a(3/2) = 96$, which gives, $a = 64$

$$\text{Hence } T_7 = ar^6 = 64 \times (3/2)^6 = 64 \times 3^6 / 64 = 729$$

QUESTIONS

1. If $x + \frac{1}{x} = 5$ then find $x^2 + \frac{1}{x^2}$
- 25
 - 27
 - 23
 - 10
2. If $x + \frac{1}{x} = 5$, then find $x^3 + \frac{1}{x^3}$
- 125
 - 130
 - 120
 - 110
3. If $x = 124$, $y = 126$ and $z = 128$, then find $x^3 + y^3 + z^3 - 3xyz$
- 4356
 - 4536
 - 4635
 - 4653
4. While solving a quadratic equation, Ramu miswrote the constant part and hence got the roots as 4 and 5. Kallu while solving the same quadratic equation miswrote the coefficient of x and hence got the roots as 2 and 4. Find the correct roots of the quadratic equation?
- 1 & 4
 - 4 & 9
 - 1 & 8
 - 9 & 8
5. Billu went to the market to purchase certain tables and chairs for Rs 1000. With the total money he has, he could only purchase 4 chairs and 5 tables. But if he decides to purchase 5 chairs and 4 tables, he will fall short by 20% of the total bill. What is the cost of one table and one chair?
- Rs. 150
 - Rs. 250
 - Rs. 350
 - Rs. 450
6. In an Arithmetic Progression, fifth term of the progression is twice the ninth term. Then which of the following statements is true?
- Tenth term of the series is zero.
 - Eleventh term of the series is zero.
 - Twelfth term of the series is zero.
 - Thirteenth term of the series is zero.
7. What is the minimum number of numbers that I should select out of first 15 natural numbers to ensure that there is at least one pair of numbers which will add to 16?
- 6
 - 7
 - 8
 - 9
8. What is the remainder if $4x^4 - 3x^3 - 8x^2 + 7x + 2$ is divided by $(x+3)$?
- 314
 - 30
 - 310
 - None of these
9. If $(x-2)^2 = 16$ and $(y-3)^2 = 25$ then find the maximum possible value of $\frac{x}{y}$.
- 3
 - 1/4
 - 0.75
 - 1
10. Find the value of $m+n$ if one of the roots of the quadratic equation $x^2 + mx + n = 0$ is $4+3i$.
- 29
 - 17
 - 25
 - 16
11. What is the value of m if $3x^3 - 4x^2 - 2x + m$ is divisible by $(x-2)$
- 4
 - 4
 - 2
 - 5
12. If roots of a quadratic equation $ax^2 + bx + c$ are α and β then find the equation whose roots are $(3\alpha + 1)$ and $(3\beta + 1)$
- $ax^2 + (2a-3b)x + 9c - 3b + a = 0$
 - $ax^2 - (2a - 3b)x - 9c + 3b - a = 0$
 - $ax^2 + (3b-2a)x + 9c - 3b + a = 0$
 - $ax^2 + (2a-3b)x - 9c + 3b - a = 0$

Directions for questions 13 to 14: Function F and G are defined as follows

$$F(x) = 3x + 2 \quad G(x) = x^2 + 5$$

13. Find $F[F(3)]$.

- a. 154
- b. 126
- c. 44
- d. 25

14. Find the number of real number solutions for $F(x) = G(x)$?

- a. 0
- b. 1
- c. 2
- d. 3

15. Which of the followings is same as:

$$(x-y)^3 + (y-z)^3 + (z-x)^3$$

- a. $3xyz$
- b. $3(x+y+z)(xy+xz+yz)$
- c. $3(x-y)(y-z)(z-x)$
- d. $(x-y)^2 + (y-z)^2 + (z-x)^2$

16. In a zoo cage, there are certain number of rabbits and hen. When the number of total legs was counted, it came out to be 110 and when the number of total heads was counted, it came out to be 40. Find the number of hens?

- a. 5
- b. 15
- c. 25
- d. 35

17. Product of 3rd and 9th term of a geometric progression is 4. Find the product of first eleven terms of the series?

- a. 1024
- b. 2048
- c. 4096
- d. None of the above

18. Third term of a Geometric Progression is 42 and sixth term of the same progression is 210. What will be the ninth term of the progression?

- a. 2100
- b. 5
- c. 1050
- d. None of these

19. In an Arithmetic Progression, first term of the progression is 2 and the last term is 238. If sum of all the terms is 4800 then what is the number of terms in the complete series?

- a. 20
- b. 10
- c. 50
- d. 40

20. Sum of an arithmetic progression of four terms is 68. If product of second and third term of the series exceed the product of first and fourth

term of the same series by 32, then which of the following can be first term of the series?

- a. 9
- b. 11
- c. 13
- d. 15

21. A ball is dropped from a 100 m high building. It rebounds to 50 % of its previous height. What is the total distance travelled by the ball till it gets to rest?

- a. 150m
- b. 200m
- c. 300m
- d. Infinite

22. First three terms of a geometric progression are 3a-8, 3a-7 and 3a-5. What will be the sixth term of the series.

- a. 3
- b. 4
- c. 8
- d. 32

23. What will be the sum of first ten terms of the series:

$$1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + 4 \times 5^2 + 5 \times 6^2 + 6 \times 7^2 \dots$$

- a. 3850
- b. 3580
- c. 3085
- d. 3855

24. Series S1 is defined as 1, 7, 13, 19, 25, and series S2 is defined as 1, 6, 11, 16, 21, How many numbers less than 500 will be common to these two series?

- a. 1
- b. 16
- c. 17
- d. 18

25. There are two numbers whose arithmetic mean is 10 and geometric mean is 8. What is the smaller of these two numbers?

- a. 1
- b. 2
- c. 4
- d. 6

26. What is the value of x if

$$1 + \frac{2}{x} + \frac{4}{x^2} + \frac{8}{x^3} + \frac{16}{x^4} + \frac{32}{x^5} + \dots \dots \dots \infty = 3$$

- a. 2
- b. 3
- c. 4
- d. 5

27. What will be the value of

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}}$$

- a. 1 b. 2
c. 3 d. 4

28. If difference between the squares of two numbers is 77 then which of the following cannot be either of the two numbers

- a. 2 b. 9
c. 11 d. 38

29. Find the number of real roots of a quadratic equation $4x^2 + 7x + 5 = 0$?

- a. 0 b. 1
c. 2 d. 3

30. What is the minimum value of $a + \frac{1}{a}$ if a is a positive real number.

- a. 0 b. 1
c. 2 d. 4

31. If $a = \sqrt{b} = \sqrt[3]{c} = \sqrt[4]{d} = \sqrt[5]{e} = \sqrt[6]{f}$, then what is the value of $a \times b \times c \times d \times e \times f$

- a. a^{21} b. a^{120}
c. a^{127} d. a^{60}

32. What will be the ratio of sum of cubes of first 10 natural numbers to sum of squares of first 10 natural numbers to sum of first 10 natural numbers?

- a. 55 : 7 : 1 b. 45 : 7 : 1
c. 55 : 9 : 1 d. 45 : 9 : 1

33. If $\frac{x^3 + 3x}{3x^2 + 1} = \frac{341}{91}$, then what is the value of x?

- a. 7 b. 9
c. 11 d. 13

34. If $\frac{5^x + 100}{5^x + 75} = \frac{5}{4}$, then what is the value of x?

- a. 1 b. 2
c. 3 d. 4

35. If $\frac{1}{2}(a^x - a^{-x}) = \frac{1}{4}(a^x + a^{-x})$, then which of the following can be the values of a and x?

- a. $a = 2, x = 1$ b. $a = 2, x = 0.5$

- c. $a = 3, x = 1$

- d. $a = 3, x = 0.5$

36. If $\frac{\log A}{x-y} = \frac{\log B}{y-z} = \frac{\log C}{z-x}$, then what is the value

of $A \times B \times C$

- a. xyz b. 0
c. 1 d. $x+y+z$

37. If $\log 2 = 0.3010$, then how many digits are there in 2^{40} ?

- a. 11 b. 12
c. 13 d. 14

38. What will be the sum of first 100 terms of the series $1 \times 2 \times 4 + 2 \times 3 \times 6 + 3 \times 4 \times 8 + 4 \times 5 \times 10 \dots$

- a. 52368499 b. 52368500
c. 52368502 d. 52368502

39. If sum of first 15 terms of an Arithmetic series is same as the sum of first 25 terms of the series, then what is the sum of first 40 terms of the series?

- a. -1 b. 1
c. 0 d. Cannot be determined

40. If $x = 0.60$, then what is the value of $(1+x-x^2) + \frac{x^3}{(1+x)}$

- a. 11/8 b. 12/8
c. 9/8 d. 13/8

41. For how many integral values of x, $3x^3 + 4x^2 - 2x + 7 - \frac{48}{x}$ is an integer

- a. 48 b. 47
c. 10 d. 20

42. If $3^x - 3^{x-3} = 702$, then what is the value of x?

- a. 3 b. 4
c. 5 d. 6

43. If $1 \times 5 \times 5^2 \times 5^3 \times 5^4 \times \dots \times 5^x = 5^{55}$, then what is the value of x?

- a. 9 b. 10
c. 11 d. 12

44. Find the number of natural number solutions for x and y in the equation $4x + 5y = 120$.

- a. 4 b. 5
c. 6 d. 7

45. What will be the sum of coefficient of the expansion of $(2x + y + z)^5$?
a. 1024 b. 625
c. 243 d. None of the above

46. If one red rose cost Re 1, one yellow rose cost Rs 10 and a white rose cost Rs 100. Find the number of ways in which 100 rose sticks can be purchased spending exactly Rs 1000. Assume, he needs to purchase at least one stick of each type of rose.
a. 0 b. 1
c. 9 d. 10

47. If x and y are real numbers and $x > y$ then which of the following is Surely true?
a. $x^2 > xy$ b. $x^2y > xy^2$
c. $x^3y^2 > x^2y^3$ d. $\frac{x}{y} > 1$

48. Sum of tenth and seventh term of an AP is same as the eleventh term of the same AP. There is particular term of the series which is equal to zero. Which term of the series is zero?
a. Fourth b. Fifth
c. Sixth d. Seventh

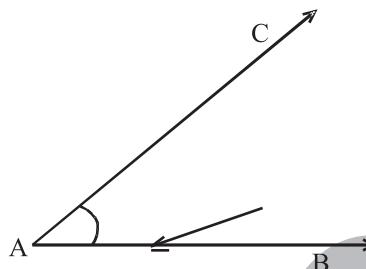
49. Six pens and three pencils cost Rs 100 which is 25 % more than the cost of three pens and six pencils. Find the cost of one pen and one pencil.
a. 19.44 b. 20
c. 22.22 d. 25

50. Ramu started tearing pages of a book in anger. First page that he tore away was 546. After tearing certain pages, his anger calmed a bit and he stopped tearing the pages. Which of the following could be the last page that he has torn?
a. 645 b. 654
c. 465 d. Both a and c are possible

Angles

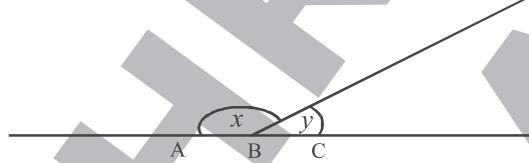
An angle is formed by two rays with the same initial point.

The two rays forming an angle are called the "arms" of the angle and common initial point is called the "vertex" of the angle.



Type of angles and some properties:

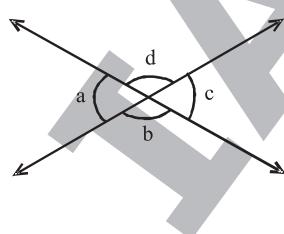
(A) Sum of the angles in a straight line = 180°



Here $\angle x + \angle y = 180^\circ$

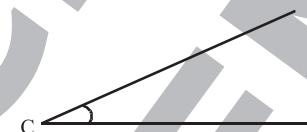
(B) Vertically opposite angles are congruent (equal)

$\angle a$ and $\angle c$ are vertically opposite angles, $\angle a = \angle c$ Similarly $\angle b$ and $\angle d$ are vertically opposite angles, $\angle b = \angle d$



(C) Acute angles:

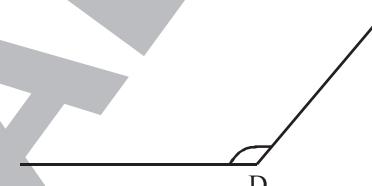
Acute angles are the angles whose measure is less than 90° .



$\angle C$ is an acute angle

(D) Obtuse Angles:

Obtuse angles are the angles whose measure is greater than 90° and less than 180°



$\angle D$ is an obtuse angle.

(E) Reflex Angles:

An angle with measure more than 180° and less than 360° is called a reflex angle, $180^\circ < \angle a < 360^\circ$

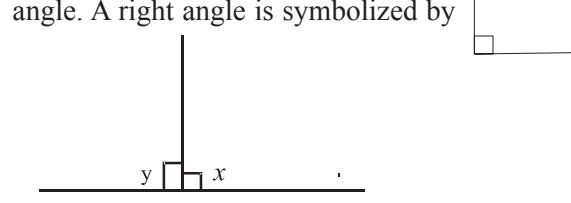
(F) Supplementary Angles:

If the sum of two angles is 180° , the angles are called supplementary.

$\angle x + \angle y = 180^\circ \Rightarrow \angle x$ and $\angle y$ are supplementary angles.

(G) Right angles:

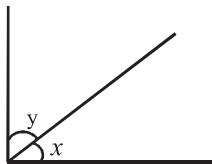
If the measure of an angle is 90° , it is called a right angle. A right angle is symbolized by



$\angle x$ and $\angle y$ are right angles.

(H) Complementary angles:

If the sum of two angles is 90° , the angles are complementary and each angle is the complement of the other.



$$\angle x + \angle y = 90^\circ$$

$\angle x$ and $\angle y$ are complementary angles.

Parallel Lines

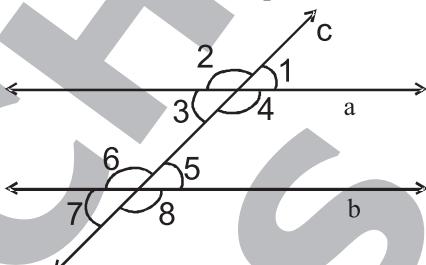
Two lines in a plane that do not intersect no matter how far they have extended are said to be parallel. In the adjoining figure 'a & b' are parallel lines, we use ' $a \parallel b$ ' to represent a and b as parallel lines.



Transversal: It is a straight line, which intersects two parallel lines.

In the adjoining figure ' $a \parallel b$ ' and line 'c' is a transversal to parallel lines 'a' and 'b'.

A transversal intersects two parallel lines then:



(A) Corresponding angles are equal i.e

$$\angle 1 = \angle 5; \angle 2 = \angle 6; \angle 4 = \angle 8 \text{ and } \angle 3 = \angle 7$$

(B) Alternate interior angles are equal i.e.

$$\angle 4 = \angle 6 \text{ and } \angle 3 = \angle 5$$

(C) Alternate exterior angles are equal i.e.

$$\angle 1 = \angle 7 \text{ and } \angle 2 = \angle 8$$

(D) Interior angles on the same side of transversal add up to 180° i.e. $\angle 4 + \angle 5 = \angle 3 + \angle 6 = 180^\circ$

(E) Exterior angles on the same side of transversal add up to 180° i.e. $\angle 1 + \angle 8 = \angle 2 + \angle 7 = 180^\circ$

Polygons

A polygon is traditionally a plane figure that is bounded by a closed path or circuit, composed of a finite sequence of straight lines. For examples triangles, quadrilaterals, hexagons, etc.

Polygons can broadly be classified into two types:

Regular Polygon: A polygon, in which, all its sides and angles are equal is called a regular polygon.

Irregular Polygon: A polygon that is not a regular polygon, i.e., whose sides do not have the same length or whose interior angles do not have the same measure.

Polygons can also be classified depending on the number of sides of the polygon.

<u>Number of sides</u>	<u>Polygon</u>
3 sides	----- Triangle
4 sides	----- Quadrilateral
5 sides	----- Pentagon
6 sides	----- Hexagon
7 sides	----- Heptagon
8 sides	----- Octagon
9 sides	----- Nonagon
10 sides	----- Decagon

Properties of Polygons:

- Sum of all interior angles of a regular polygon with 'n' sides can be calculated using $(n - 2) \times 180^\circ$

- The internal angles of a regular polygon with 'n' sides can be calculated using

$$\frac{(n - 2) \times 180^\circ}{n}$$

For example the interior angles of an octagon

$$(8 \text{ sides}) \text{ is: } \frac{(8-2) \times 180^\circ}{8} = 135^\circ$$

- Sum of all the external angles of a regular polygon is 360° .

- So each external angle of a regular polygon with 'n' sides is $\frac{360^\circ}{n}$.

For example each exterior angle of a hexagon

$$\text{is } \frac{360^\circ}{6} = 60^\circ.$$

- Polygons (except the triangle) have diagonals.

$$\text{The number of diagonals is equal to } \frac{n(n-3)}{2}.$$

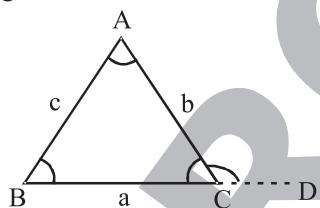
$$\text{For example, an octagon has } \frac{8(8-3)}{2} = 20.$$

Triangles:

A triangle is a polygon with three sides. The symbol for a triangle is Δ ?

In the adjoining figure ABC is a triangle.

In triangle ABC, AB = c, BC = a, CA = b.



Some properties of triangles:

- Sum of interior angle = 180°
- Measure of exterior angle = sum of remote interior angles. i.e., $\angle ACD = \angle BAC + \angle ABC$.
- Interior angle + corresponding exterior angle = 180° i.e., $\angle ACD + \angle ACB = 180^\circ$.
- Sum of two sides of a triangle is always greater than the third side.
- A triangle must have at least two acute angles.
- Triangles on equal bases and between the same parallel have equal areas.
- If a, b, c are the lengths of the sides of a triangle, then

If $c^2 < a^2 + b^2$, Triangle is acute-angled

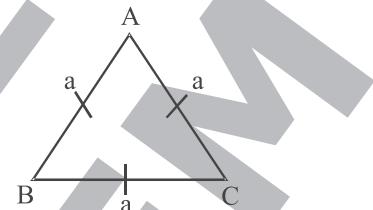
If $c^2 = a^2 + b^2$, Triangle is right-angled

If $c^2 > a^2 + b^2$, Triangle is obtuse-angled

Types of Triangles:

Equilateral Triangle:

Equilateral Triangles have equal sides and equal angles. Each angle measures 60° .

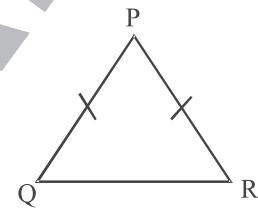


Here $AB = BC = CA = a$ 'unit'

And $\angle A = \angle B = \angle C = 60^\circ$

Isosceles Triangle:

The triangle with two equal sides is called an Isosceles Triangle.



The two equal angles are sometimes called the base angles and the third angle is called the vertex angle.

Here $PQ = PR$

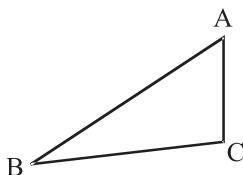
$\angle P$ is vertex angles.

$\angle Q = \angle R$, $\angle Q$ and $\angle R$ are base angle.

Scalene Triangle:

A triangle with all three sides of different lengths and all angles of different measures is called a scalene triangle.

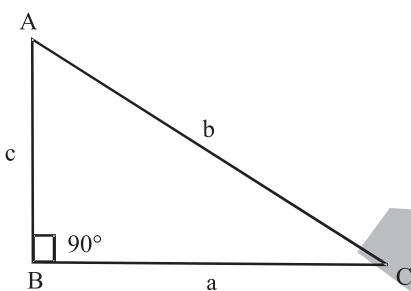
The shortest side is opposite the angle of smallest measure and the longest side is opposite the angle of greatest measure.



If $AB > BC > CA$ then $\angle C > \angle A < \angle B$

Right-angled Triangle:

Right-angled triangles contain one right-angle, and therefore the other two angles are complementary. The side triangle opposite to the right-angle is called hypotenuse, the other two sides are called base and perpendicular.



Pythagoras Theorem:

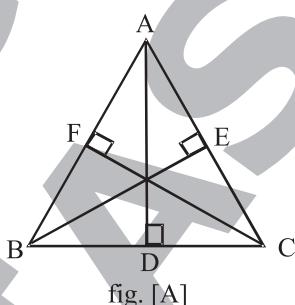
The Pythagoras theorem states that the square of the length of the hypotenuse is equal to the sum of the squares of the length of the other two sides.

In $\triangle ABC$, $AB^2 + BC^2$ or $b^2 = c^2 + a^2$

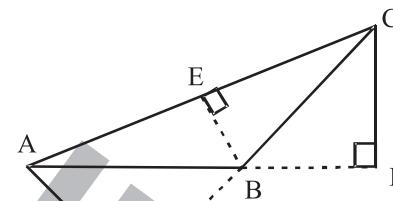
Altitude, Median and Angle Bisector to a Triangle:

Altitude: An Altitude is a perpendicular drawn from one vertex to the opposite side of the triangle.

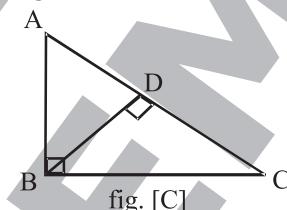
For an acute triangle all the altitudes are present in the triangle. In figure [A] AD, BE and CF are altitudes of $\triangle ABC$.



In obtuse-angled triangle, altitudes drawn from the vertices corresponding to two other acute-angles cut the extension of corresponding opposite side.



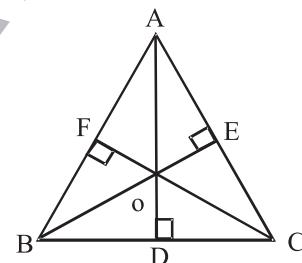
For a right triangle two of the altitudes lie on the sides of the triangle.



Orthocenter:

The altitudes of a triangle meet a point, called orthocenter of the triangle.

In the adjoining figure 'O' is the orthocenter of triangle ABC.



Also $\angle BOC = 180^\circ - \angle A$

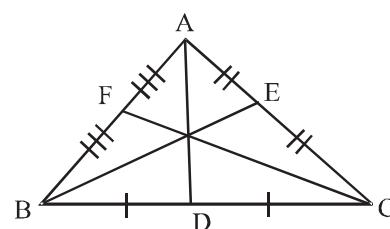
$\angle COA = 180^\circ - \angle B$ and $\angle AOB = 180^\circ - \angle C$

Area of triangle

$$= \frac{1}{2} \times \text{Base} \times \text{Altitude} = \frac{1}{2} \times BC \times AD \times AE = \frac{1}{2} \times AB \times CF.$$

Median:

The line drawn from a vertex of a triangle to the midpoint of the opposite side is called the median of the triangle. A median bisects the area of the triangles.



i.e., Area($\triangle ABD$) = Area($\triangle ABE$) =

$$= \text{Area}(\triangle CFA) = \frac{1}{2}(\triangle ABC)$$

Centroid:

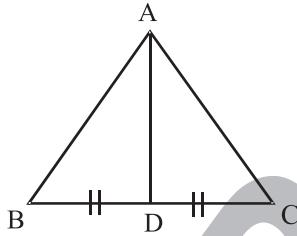
The point of intersection of the medians is called the centroid of the triangle. The Centroid divides any median in the ratio 2 : 1.

Apollonius Theorem:

AD is a median of triangle ABC. Apollonius theorem states that the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and the square of half the third side. i.e.,

$$AB^2 + AC^2 = 2(AD^2 + DC^2) = 2(AD^2 + BD^2)$$

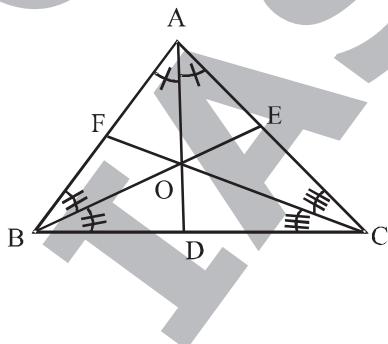
$$\therefore BD = DC$$



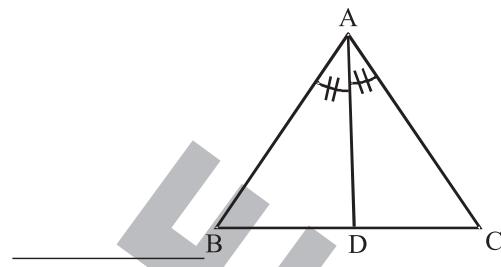
Angle Bisector and in centre of a Triangle:

A line drawn from a vertex of a triangle to the opposite side such that it bisects the angle formed by the other two sides is called the angle bisector.

The point of intersection figures 'O' is the **in center** of the $\triangle ABC$. Perpendicular drawn from 'O' to any of the side is **in radius** of the in circle of $\triangle ABC$.



Interior angle Bisector:

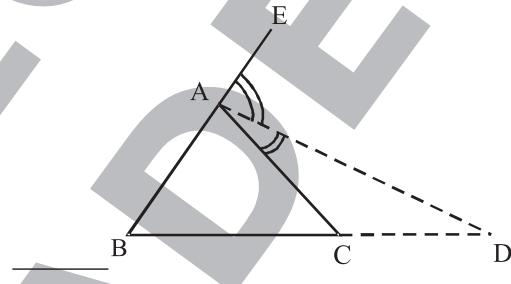


In $\triangle ABC$, AD is the angle bisector of $\angle A$, then

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\text{And } AB \times AC - BD \times DC = AD^2$$

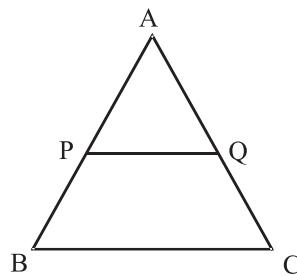
Exterior angle Bisector:



$$\frac{AB}{AC} = \frac{BD}{DC}$$

Mid-Point Theorem:

In a triangle, the line joining the mid points of two sides is parallel to the third side and is half of the length of the third side.

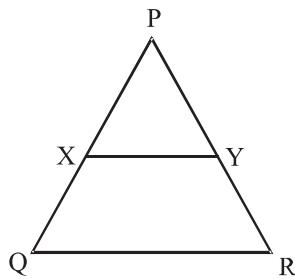


P and Q are the mid-point of sides AB, and AC of $\triangle ABC$ respectively, then

$$PQ \parallel BC \text{ and } PQ = \frac{1}{2}BC$$

Basic Proportionality Theorem:

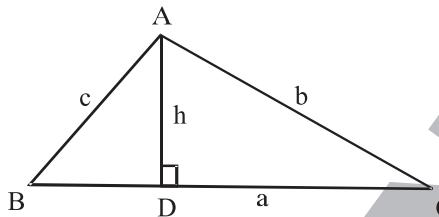
A line parallel to any one side of a triangle divides the other two sides Proportionally.



If XY is parallel to QR, then

$$\frac{PX}{XQ} = \frac{PY}{YR}, \frac{PQ}{PY} = \frac{PR}{XY} \text{ And } \frac{PX}{XY} = \frac{PQ}{QR}$$

Area of a triangle:



in $\triangle ABC$, $AB = c$, $BC = a$ and $AC = b$.

in $\triangle ABC$, $AB = c$, $BC = a$ and $AC = b$

AD is an altitude and $AD = h$

$$\text{Area} = \frac{1}{2} \times a \times h = \frac{1}{2} \times \text{base} \times \text{perpendicular to base from opposite vertex.}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

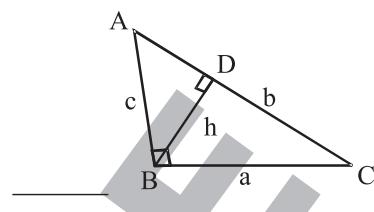
$$\text{where } s = \text{semi perimeter} = \frac{a+b+c}{2}.$$

Area = where $s = \text{semi perimeter} = .$

Area = $r \times s$, where r is the in radius of the triangle.

Area = $\frac{abc}{4R}$, where R is the circum radius of the triangle. The point of intersection of the perpendicular bisectors of a triangle meets at a point which is called the **circum centre** of the triangle.

Right-Angled Triangle:



$$\text{Area} = \frac{1}{2} \times AB \times BC = \frac{1}{2} ac$$

Also Area = $r \times S$

$$\text{i.e. } \frac{1}{2} ac = r \left(\frac{a+b+c}{2} \right)$$

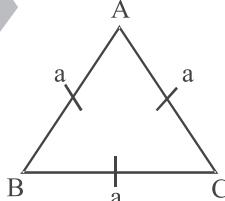
$$\text{In radius} = \frac{ac}{a+b+c}$$

$$\text{Also Area} = \frac{abc}{4R}$$

$$\therefore \text{Circum Radius } R = \frac{abc}{4(\text{area})} = \frac{abc}{Q4 \times \frac{1}{2} ac} = \frac{C}{2}$$

$$\text{Circum Radius} = \frac{1}{2} \times \text{Hypotenuse}$$

Equilateral Triangle:



$\triangle PQR$ is an equilateral triangle.

$PQ = QR = PR = a$ (say),

$$\text{then } h = \frac{\sqrt{3}}{2} a, \text{ And Area} = \frac{\sqrt{3}}{4} a^2$$

Also Area = $r \times s$,

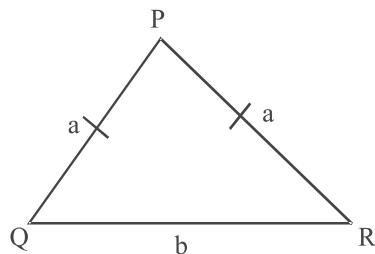
$$\text{i.e. } r \left(\frac{a+a+a}{2} \right) = \frac{\sqrt{3}}{4} a^2$$

$$\text{In radius} = a/2\sqrt{3}$$

$$\text{Again area} = \frac{abc}{4R}$$

$$R = \frac{abc}{4(\text{area})} = \frac{a^3}{4\sqrt{3}\frac{4}{4}a^2} \text{ or, Circum radius } = a/\sqrt{3}$$

Isosceles Triangle:

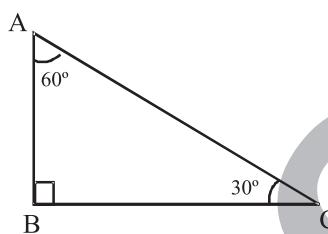


$\triangle PQR$ is an isosceles triangle.

$PQ = PR = a$, and $QR = b$.

$$\text{Then area } (\triangle PQR) = \frac{b}{4} \sqrt{4a^2 - b^2}$$

$30^\circ - 60^\circ - 90^\circ$ triangle:

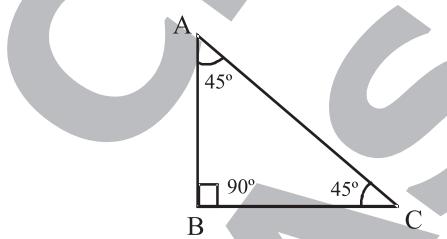


$\triangle ABC$, $\angle A = 60^\circ$, $\angle B = 90^\circ$ and $\angle C = 30^\circ$, then

$$\text{Side opposite to } \angle 30^\circ = \frac{\text{Hypotenuse}}{2}$$

$$\text{And Side opposite to } \angle 60^\circ = \frac{\sqrt{3}}{2} \times \text{Hypotenuse.}$$

$45^\circ - 45^\circ = 90^\circ$ triangle

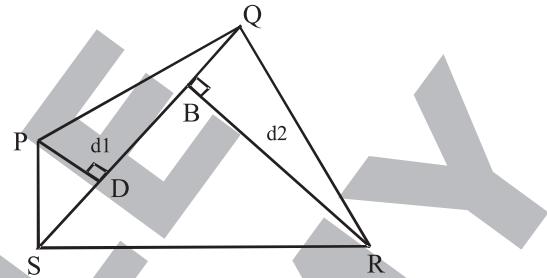


In $\triangle ABC$, $\angle A = 45^\circ$, $\angle C$ and $\angle B = 90^\circ$ then:

$$AB = BC = \frac{1}{2} \times \text{Hypotenuse}$$

Quadrilaterals:

A figure bounded by four sides is called a quadrilateral.



In $\square PQRS$, diagonal $SQ = d$, $PA = d_1$ and $RB = d_2$

$$\text{Then Area } (\square PQRS) = \frac{1}{2} \times (d_1 + d_2)$$

OR, Area of a quadrilateral

$$= \frac{1}{2} \times \text{one diagonal} \times (\text{sum of perpendiculars to it from opposite vertices})$$

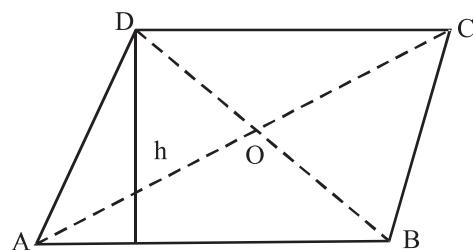
If $\square PQRS$ is a cyclic quadrilateral, then Area of a cyclic quadrilateral $= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$

Where a, b, c and d are the sides of quadrilateral and $S = \text{semi' perimeter} = \frac{a+b+c+d}{2}$.

Parallelogram:

In a parallelogram

- Opposite sides are parallel and congruent.
- Opposite angles are congruent.
- Diagonals bisect each other.



$\square ABCD$ is a parallelogram, then

(I) $AB \parallel CD$ and $BC \parallel AD$

(II) $AB = CD$ and $BC = AD$

(III) $\angle A = \angle C$ and $\angle B = \angle D$

(IV) $\angle A = \angle D = \angle B + \angle C = 180^\circ$

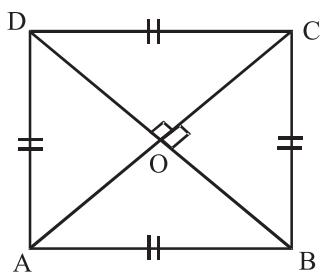
(V) $AO = OC$ and $BO = OD$

$\square ABCD$ Area of $ABCD = \text{Base} \times \text{height} = \frac{1}{2} AB \times h$.

Rhombus:

If in a parallelogram, all sides are equal it is a rhombus.

In a rhombus

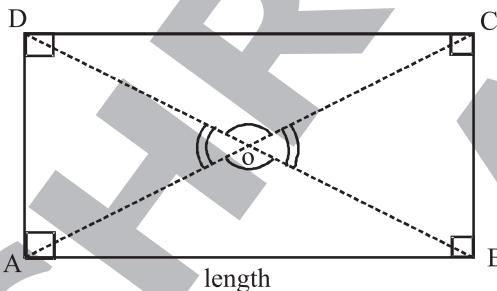


- Opposite sides are parallel and all sides are equal. i.e. $AB \parallel CD$ & $BC \parallel AD$ and $AB = BC = CD = DA$.
- Opposite angles are congruent.
- Diagonals bisect each other at 90° .

Area of rhombus = $\frac{1}{2} \times \text{product of diagonals}$.

Rectangle:

In a rectangle

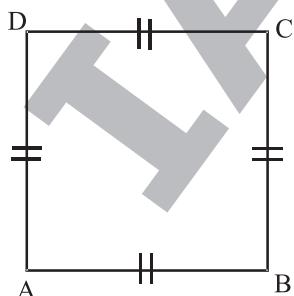


- Opposite sides are equal, each angle = 90°
- Diagonals bisect each other (not at 90°).

Area of rectangle = length \times breadth.

Square:

In a square

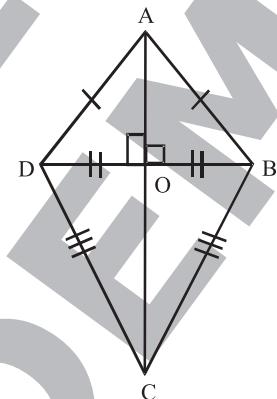


- All sides are congruent and each angle = 90° .
- Diagonals are congruent and bisect each other at 90° .
- Length of diagonal = $\sqrt{2} \times (\text{side})$.

Area of square

$$= (\text{side})^2 = \frac{1}{2} \times \text{product of diagonals.}$$

Kite:



$\square ABCD$ is a kite, then

Two pairs of adjacent sides are congruent
i.e. $AB = AD$ and $BC = CD$.

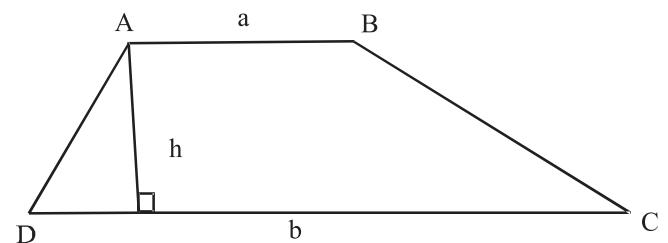
Diagonals bisect each other at 90° and longer diagonal bisect shorter diagonal.

i.e. $BO = OD$ and $\angle AOB = \angle AOD = 90^\circ$.

Area of a kite = $\frac{1}{2} \times \text{product of diagonals}$.

Trapezium:

A quadrilateral in which only one pair of opposite is parallel is a trapezium.



$\square ABCD$ is a trapezium,

in which $AB \parallel CD$, $AB = a$ and $CD = b$

Area: $S = 6s^2$

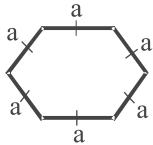
OR,

Area of Trapezium = $\frac{1}{2} \times (\text{sum of parallel sides})$

= (distance between parallel sides)

Regular Hexagon:

ABCDEF is a regular hexagon,



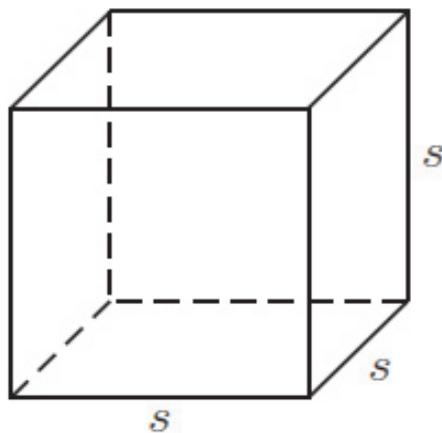
- AB = BC = CD = DE = EF = FA = a (say).
- Sum of interior angles = 720°
- Each interior angle = 120°
- Each exterior angle = 60°

$$\text{Area} = 6 \times \frac{3\sqrt{3}}{4} (a^2) = \frac{3\sqrt{3}}{2} (\text{side})^2$$

[∴ Six equilateral triangles are formed by joining the opposite vertices of the hexagon].

Area and Volume of 3 - dimensional solids:

CUBE:

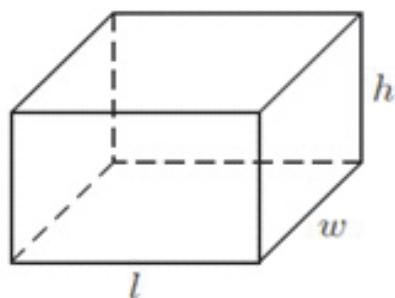


s = side

$$\text{Volume: } V = s^3$$

$$\text{Surface Area: } S = 6s^2$$

RECTANGULAR SOLID:



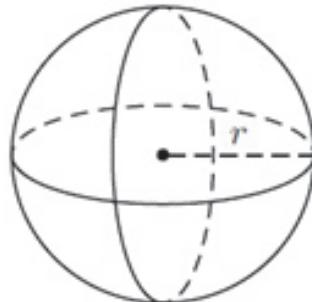
l = length, w = width,

h = height

$$\text{Volume: } V = l \times w \times h$$

$$\text{Surface Area: } S = 2lw + 2lh + 2wh$$

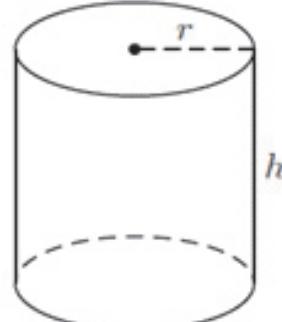
SPHERE:



r = radius

$$\text{Volume: } V = \frac{4}{3}\pi r^3, \quad \text{Surface Area: } S = \frac{4}{3}\pi r^3$$

RIGHT CIRCULAR CYLINDER:

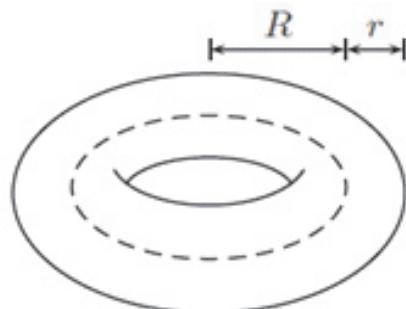


r = radius, h = height

$$\text{Volume: } V = \pi r^2 h,$$

$$\text{Surface Area: } S = 2\pi rh + 2\pi r^2$$

TORUS:



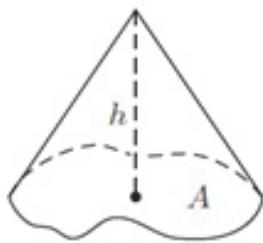
r = tube radius,

R = torus radius

$$\text{Volume: } V = 2\pi^2 r^2 R^2$$

$$\text{Surface Area: } S = 4\pi^2 r R$$

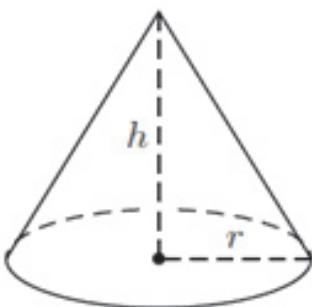
GENERAL CONE OR PYRAMID:



A = area of base,
h = height

$$\text{Volume: } V = \frac{1}{3} Ah$$

RIGHT CIRCULAR CONE:

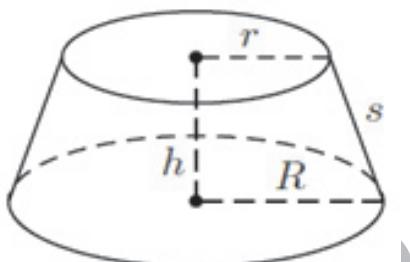


r = radius, h = height

$$\text{Volume: } V = \frac{1}{3} \pi r^2 h$$

$$\text{Surface Area: } S = \pi r \sqrt{r^2 + h^2} + \pi r^2$$

FRUSTUM OF A CONE:



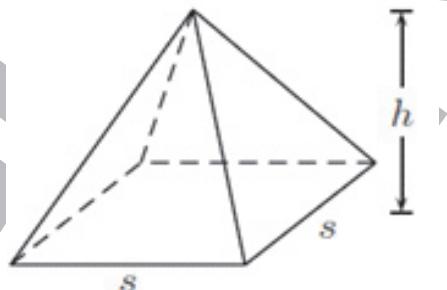
r = top radius, R = base radius,

h = height, s = slant height

$$\text{Volume: } V = \frac{\pi}{3} (r^2 + rR + R^2) h$$

$$\text{Surface Area: } S = ps(R+r) + pr^2 + pR^2$$

SQUARE PYRAMID:

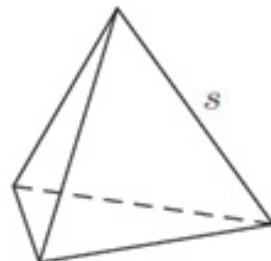


s = side, h = height

$$\text{Volume: } V = \frac{1}{3} S^2 h$$

$$\text{Surface Area: } S = s(4 + \sqrt{s^2 + 4h^2})$$

REGULAR TETRAHEDRON:



s = side

$$\text{Volume: } V = \frac{1}{12} \sqrt{2} s^3$$

$$\text{Surface Area: } S = \sqrt{3}s^2$$

■ ■ ■

QUESTIONS

1. There is a rhombus of side 10m which can circumscribe a circle of diameter 8 m. What will be area of this rhombus?
 - a. 16
 - b. 32
 - c. 24
 - d. 48

2. There is a cube of dimensions 5 by 5 by 5. This cube is originally painted red. This cube is divided into 125 equal cubes. How many of these cubes will have exactly 1 painted side?
 - a. 54
 - b. 25
 - c. 36
 - d. 48

3. Refer to question 2, how many of these cubes will have exactly 2 painted sides?
 - a. 54
 - b. 25
 - c. 36
 - d. 48

4. Following figure shows a game of balancing playing cards. Ramu had 26 cards of 10 cm height and negligible thickness. Find the height of tower that can be prepared by using these 26 cards.
 

$\text{a. } 20\sqrt{3}$

$\text{b. } 30\sqrt{3}$

$\text{c. } 15\sqrt{3}$

$\text{d. } 18\sqrt{3}$

5. ABCD is a rectangle with AD=6 and AB=8. If AD and AB are extended to P and Q in such a way that PQ passes through C, then what is the length of PQ?
 - a. 5
 - b. 10
 - c. 20
 - d. Can't be determined

6. A circular table is adjusted to right angled corner of a room in such a way that this table touches the walls of the room at two distinct points. If

there is one point on the circumference of the table which is at a distance of 2m and 4 m from two walls of the room then what is the radius of the circular table?

- $\text{a. } 10 \text{ m}$
- $\text{b. } 2 \text{ m}$
- $\text{c. Either } 10 \text{ m or } 2 \text{ m}$
- $\text{d. } 12 \text{ m}$

7. Area of a triangle is 80 sq units. If the largest side of the triangle is 20 and the smallest side is 10, then what is the length of the third side?
 - a. $5\sqrt{26}$
 - b. $2\sqrt{65}$
 - c. $2\sqrt{65}$
 - d. 60.

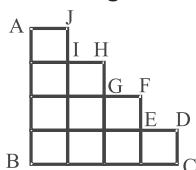
8. There is a square ABCD of side 10 cm and an equilateral triangle ABE inside the square. What will be the square of the distance between E and D?
 - a. $100(2-\sqrt{3})$
 - b. $50(2-\sqrt{3})$
 - c. $100(\sqrt{3}-1)$
 - d. $100(2-\sqrt{2})$

9. There are two circles of radius 2 cm and 1 cm touching each other at point A. There are two tangents to the larger circle, first one at A and second one passing through the centre of the smaller circle. Find the distance between A and point of intersection of these two tangents.
 - a. 1.5cm
 - b. 2cm
 - c. 3cm
 - d. 1cm

10. There is a room of dimension 10m by 20m by 30m. Find the distance that an insect need to crawl in order to reach from one of the vertices to the diagonally opposite vertex?
 - a. $5\sqrt{26}$
 - b. $10\sqrt{13}$
 - c. $10\sqrt{13}$
 - d. 60

11. In the given figure,

if $AJ=JI=IH=HG=GF=FE=ED=DC=10$ m, then find the area of the given figure?



- a. 100 b. 500
c. 1000 d. 2000

12. There are two similar triangles; length of the perpendicular from the opposite vertex on the largest side is 10 cm and 8 cm. If area of one of the triangles is 100 cm² then find the area of the other triangle?

- a. 64 sq units b. 156.25 sq units
c. Neither (a) nor (b) d. Either (a) or (b)

13. What will be area of largest triangle that can be inscribed in a semi-circle of radius 10 cm?

- a. 200 b. 100
c. 50 d. $100\sqrt{2}$

14. In a square ABCD of side 10 cm. Diagonal AC is joined and then from B, a line is drawn to meet AC at E in such a way that $\angle CBE = 30^\circ$. What is the area of $\triangle ABE$?

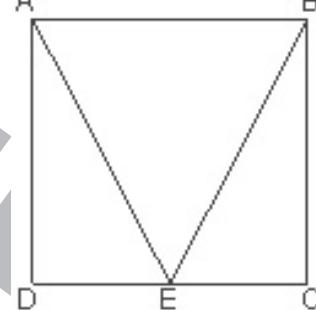
- a. $\frac{100}{\sqrt{3}+1}$ b. $\frac{50}{\sqrt{3}-1}$
c. $\frac{50\sqrt{3}}{\sqrt{3}+1}$ d. $\frac{50}{\sqrt{3}+1}$

15. In $\triangle ABC$, There are two points D and E on AB and AC respectively in such a way that $AD=6$, $BD=3$, $AE=3$ and $\angle DEC = 140^\circ$. What is the length of EC?

- a. 18 b. 15
c. 3 d. 9

16. ABCD is a four equal sided figure with AB parallel to CD and AD parallel to BC. ADC is a right angle. If E is mid-point of CD and the perimeter of ABE is 6 cm, the area of the figure ABCD is:

A B



- a. $\frac{36}{21-4\sqrt{5}}$ b. $\frac{9}{6+2\sqrt{5}}$
c. $\frac{18}{21+4\sqrt{5}}$ d. $\frac{18}{21-4\sqrt{5}}$

17. A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.

- a. 2 cm b. 3 cm
c. 4 cm d. 5 cm

18. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, the altitude of the triangle is

- a. 100m b. 200 m
c. $100\sqrt{2}m$ d. $10\sqrt{2}m$

19. If one of the diagonals of a rhombus is equal to its side, the diagonals of the rhombus are in the ratio:

- a. $\sqrt{3}:1$ b. $\sqrt{2}:1$
c. 3 : 1 d. 2 : 1

20. A rhombus OABC is drawn inside a circle whose centre is at O in such a way that the vertices A, B and C of the rhombus are on the circle. If the area of the rhombus is $32\sqrt{3} \text{ m}^2$, the radius of the circle is

- a. 64 m b. 8 m
c. 32 m d. 46 m

21. A metallic sheet is of rectangular shape with dimensions 48 m by 36 m. From each one of its corners, a square is cut off so as to make an

- open box. The volume of the box is Xm^3 when length of the square is 8 m. The value of X is:
- 5120
 - 8960
 - 4830
 - 51.20
22. The sum of length, breadth and height of a room is 19 m. The length of the diagonal is 11 m. The cost of painting the total surface area of the room at the rate of Rs. 10 per m^2 is:
- Rs. 240
 - Rs. 2400
 - Rs. 420
 - Rs. 4200
23. A storage tank consists of a circular cylinder with a hemisphere adjoined on either side. If the external diameter of cylinder be 14m and its length be 50m, what will be the cost of painting it at Rs. 10 per sq. m ?
- Rs. 25080
 - Rs. 28160
 - Rs. 39160
 - None of these
24. A hemispherical bowl is made of steel 0.5 cm thick. The inside radius of the bowl is 4 cm. The volume of steel used in making the bowl is
- 55.83 cm^3
 - 56.83 cm^3
 - 57.83 cm^3
 - 58.83 cm^3
25. A cylindrical bucket of height 36 cm and of radius 21 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed. The height of conical heap is 12 cm. The radius of the heap at the base is:
- 63 cm
 - 53 cm
 - 56 cm
 - 66 cm
26. How many squares are there in a 5 inch by 5 inch square grid, if the grid is made up of one inch by one inch squares?
- 50
 - 150
 - 55
 - 25
27. A horse is placed for grazing inside a rectangular field of 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze?
- 386.5 m^2
 - 325.5 m^2
 - 346.5 m^2
 - 246.5 m^2
28. In a triangle ABC, the lengths of the sides AB, AC and BC are 3, 5 and 6 cm respectively. A point D on BC is drawn such that the line AD

- bisects the angle A internally. What is the length of BD?
- 2 cm
 - 2.25 cm
 - 2.5 cm
 - 3 cm
29. Sixteen cylindrical cans, each with a radius of 1 unit, are placed inside a cardboard box four in a row. If the cans touch adjacent cans and/or the walls of the box, which of the following could be the interior area of the bottom of the box in square units?
- 16
 - 32
 - 64
 - 128
30. The length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq meters is:
- 56 m
 - 64 m
 - 88 m
 - 168 m
31. How many bricks each measuring 24 cm by 11.5cm by 8 cm will be needed to construct a wall 8m long, 6m high and 23cm wide, while 5% of the total wall contains mortar?
- 345
 - 465
 - 365
 - 475
32. The inner dimensions of an open box are 52by 40cm by 29cm. Its thickness is 2cm. If 1 cu cm of metal used in the box weighs 0.5g, the weight of the box is:
- 10.5 kg
 - 11.8 kg
 - 12 kg
 - 13.1 kg
33. How many metres of cloth 2.5m wide will be needed to make a conical tent with base radius 7m and height 24m?
- 120 m
 - 180 m
 - 220 m
 - 550 m
34. The two sides of a right triangle containing the right angle measure 3 cm and 4 cm. The radius of the incircle of the triangle is:
- 3.5 cm
 - 1.75 cm
 - 1 cm
 - 0.875 cm
35. The distance between the tops of two trees 20 m and 28 m high is 17 m. The horizontal distance between the trees is:

- a. 9 m b. 11 m
 c. 15 m d. 31 m
36. A 25 m long ladder is placed against a vertical wall inside a room such that the foot of the ladder is 7 m from the foot of the wall. If the top of the ladder slides 4 m downwards, then the foot of the ladder will slide by:
 a. 2 m b. 4 m
 c. 8 m d. 16 m
37. If two diameters of a circle intersect each other at right angles, then the quadrilateral formed by joining their end points is a:
 a. rhombus b. rectangle
 c. square d. parallelogram
38. Of all the chords of a circle passing through a given point in it, the smallest is that which:
 a. is trisected at the point
 b. is bisected at the point
 c. passes through the centre
 d. None of these
39. A cuboid of dimensions $84 \times 42 \times 42$ cm³ is cut into two equal cubes. From one of the cubes a sphere of maximum volume is cut. The waste material from the cube is used to make a square sheet of 5 cm thickness. Find the side of the square sheet.
 a. 84 cm b. 98 cm
 c. 72 cm d. None of these
40. ABC is a triangle whose area is 30 sq. cm. The mid-points of sides AB and AC are joined to form another triangle ADE. Again the mid-points of sides AD and AE are joined to form another triangle AFG. This process is repeated infinitely. Find the ratio of the area of ABC to the area of the other triangles taken together.
 a. 3 : 1 b. 4 : 1
 c. 2 : 1 d. 1 : 1
41. The same string, when wound on the exterior four walls of a cube of side n cms, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string, in cms, is
-
- a. $\sqrt{2}n$ b. $\sqrt{17}n$
 c. n d. $\sqrt{13}n$
42. Let ABCDEF be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF?
 a. 3 : 4 b. 4 : 5
 c. 2 : 3 d. 1 : 2
43. In the figure (not drawn to scale) given below, if AD = CD = BC, and BCE = 96°, how much is DBC?
-
- a. 32° b. 84°
 c. 64° d. Can't be determined
44. A trapezium is inscribed in a circle. One of the parallel sides of the trapezium is the diameter of the circle and the other side is 16 cm. The height of the trapezium is 3 cm. Find the area of the circle.
 a. 73π b. 74π
 c. 30π d. 25π
45. In a circle, drawn on different sides of the centre, are parallel chords equal to 36 mm and 48 mm; the distance between them is 42 mm. Determine the radius of the circle.
 a. 30 mm b. 48 mm
 c. 36 mm d. 37 mm
- Directions for (Questions 46-47):** A regular cone is cut and spread out to form a semicircle.

46. What is the ratio of the area of the semicircle to the lateral surface area of the cone?
- a. 1 : 1
 - b. 2 : 1
 - c. 3 : 1
 - d. data insufficient
47. The semicircle is now cut into two identical parts. Each one is folded to form a cone. What is the ratio of the base radius of this cone to that of the original cone?
- a. 5 : 2
 - b. 1 : 5
 - c. 1: 2
 - d. data insufficient
48. The area of square A is equal to the area of rectangle R. If the perimeter of A is 24 and the breadth of R is 4, what is the perimeter of R?
- a. 18
 - b. 14
 - c. 24
 - d. 26
49. The angle of elevation of a tower's top from a distance 'b' from its foot is 60° . What is the distance from its foot so that the angle of elevation becomes 30° ?
- a. $2b$
 - b. b
 - c. $3b$
 - d. $b + \text{height of the tower}$
50. One of the following is correct.
- a. The perpendicular to a chord from the center of the circle divides the chord in the ratio 1 : 2.
 - b. The opposite angles of a cyclic quadrilateral are supplementary.
 - c. The area of a parallelogram is equal to 1.6 times the area of a rectangle of the same base & same altitude.
 - d. The centroid bisects the median of a triangle.

Permutation & Combination

Permutations and Combinations Basics Fundamental Principle of Counting

MULTIPLICATION PRINCIPLE:

Consider the 3 letter words that can be made from the letters WORD if no letter is repeated. There are:

4 ways of choosing the 1st letter

3 ways of choosing the 2nd letter

2 ways of choosing the 3rd letter

Number of words = $4 \times 3 \times 2 = 24$

This is an illustration of the multiplication principle i.e. if several operations are carried out in a certain order, then the number of ways of performing all the operations is the product of the numbers of ways of performing each operation.

If an operation can be performed in 'm' different ways and another operation in 'n' different ways then these two operations can be performed one after the other in 'm n' ways.

ADDITION PRINCIPLE:

Consider the 3 letter words starting or finishing with O that can be made from the letters WORD if no letter is repeated. Now words starting or finishing with O are mutually exclusive i.e. they do not overlap. Therefore we can find the number starting with O and the number finishing with O and add the two numbers.

Starting with O:

1st letter	2nd letter	3rd letter	
1 ×	3 ×	2	= 6

Finishing with O:

1st letter	2nd letter	3rd letter	
3 ×	2 ×	1	= 6

Number of words starting or finishing with

$$O = 6 + 6 = 12$$

This is an illustration of the addition principle i.e. if two operations are mutually exclusive (i.e. they do not overlap), then the number of ways of performing one operation or the other is the sum of the numbers of ways of performing each operation.

If an operation can be performed in 'm' different ways and another operation in 'n' different ways then either of these two operations can be performed in 'm + n' ways. (Provided only one has to be done)

This principle can be extended to any number of operations.

FACTORIAL 'n':

The continuous product of the first 'n' natural numbers is called factorial n and is denoted by n!.

$$\text{i.e., } n! = 1 \times 2 \times 3 \times \dots \times (n-1) \times n.$$

PERMUTATION:

An arrangement that can be formed by taking some or all of a finite set of things (or objects) is called a *Permutation*.

Order of the things is very important in case of permutation.

A permutation is said to be a *Linear Permutation* if the objects are arranged in a line. A linear permutation is simply called as a permutation.

A permutation is said to be a *Circular Permutation* if the objects are arranged in the form of a circle.

The number of (linear) permutations that can be formed by taking r things at a time from a set of n distinct things is denoted by ${}^n P_r$ or $P(n, r)$.

$${}^n P_r = n(n-1)(n-2)(n-3) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

NUMBER OF PERMUTATIONS UNDER CERTAIN CONDITIONS:

- Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement is: ${}^{n-a}P_r$
- Number of permutations of n different things, taken r at a time, when a particular thing is never taken in each arrangement is: ${}^{n-a}P_{r-a}$
- "TOGETHER" ARRANGEMENTS: In this type of problem, we need to count arrangements where some of the objects must remain together. The multiplication principle applies and we use a "treat as one" technique.

For example:

3 science, 4 mathematics and 5 history books are to be arranged on a shelf. How many arrangements are possible if the books from each subject are to be together?

Solution:

Treat the books for each subject as one book:

$$\text{Number of arrangements} = 3!$$

$$\text{Number of ways of arranging the science books} = 3!$$

$$\text{Number of ways of arranging the mathematics books} = 4!$$

$$\text{Number of ways of arranging the history books} = 5!$$

$$\text{Total number of arrangements} = 3! \times 3! \times 4! \times 5! = 103680$$

- Number of permutations of n different things, taken all at a time, when m specified things always come together is: $m!(n-m+1)!$
- Number of permutations of n different things, taken all at a time when m specified never come together is: $n! - (m!(n-m+1)!)$
- The number of permutations of n dissimilar things taken r at a time when $k (< r)$ particular things always occur is: $\left[{}^{n-k}P_{r-k} \right] \cdot \left[{}^rP_k \right]$

- The number of permutations of n dissimilar things taken r at a time when k particular things never occur is: ${}^{n-k}P_r$

- The number of permutations of n dissimilar things taken r at a time when repetition of things is allowed any number of times is: n^r
- The number of permutations of n different things, taken not more than r at a time, when each thing may occur any number of times is:

$$n + n^2 + n^3 + \dots + n^r = \frac{n(n^r - 1)}{n - 1}$$

- The number of permutations of n different things taken not more than r at a time:

$${}^nP_1 + {}^nP_2 + {}^nP_3 + \dots + {}^nP_r$$

PERMUTATIONS OF SIMILAR THINGS:

Consider the number of arrangements of the letters EMPLOYEE. If all 8 letters were different, then the number of arrangements would be $8!$ but this number involves counting arrangements more than once.

The $8!$ Arrangements include 6 versions of EEMPLOY:

Similarly every arrangement occurs 6 times in the total of $8!$

(6 is the number of arrangements of the 3 E's i.e. $3!$).

$$\text{Number of distinct arrangements} = 3! \times 8!$$

This idea can be extended to problems where more than one type of object is repeated:

Number of distinct arrangements of the letters

$$\text{MISSISSIPPI} = \frac{11!}{4 \times 4 \times 2!} = 34650$$

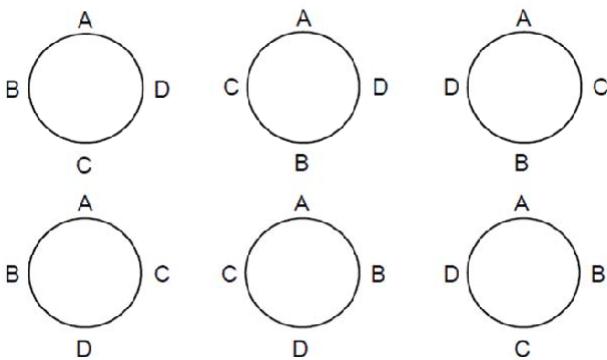
- The number of permutations of n things taken all at a time when p of them are all alike and

the rest are all different is: $\frac{n!}{p!}$

- If p things are alike of one type, q things are alike of other type, r things are alike of another type then the number of permutations with $p + q + r$ things is: $\frac{(p+q+r)!}{p!q!r!}$

CIRCULAR PERMUTATIONS:

Consider the different arrangements when 4 people sit in a circle. There are $3!$ arrangements (not $4!$ as might be expected) as it does not matter where the first object is placed.



In general, n objects can be arranged in a circle in $(n-1)!$ ways.

- The number of circular permutations of n dissimilar things taken all at a time is: $(n-1)!$
- The number of circular permutations of n things taken r at a time in one direction is: $({}^n P_r / 2r)$
- The number of circular permutations of n dissimilar things in clockwise direction = Number of permutations in anti - clock - wise direction = $\frac{(n-1)!}{2}$

COMBINATION:

A selection that can be formed by taking some or all of a finite set of things (or objects) is called a **Combination**.

The number of combinations of n dissimilar things

taken r at a time is denoted by: ${}^n C_r = \frac{n!}{r!(n-r)!}$

- ${}^n C_r = {}^n C_{n-r}$
- ${}^n C_r = {}^n C_{r-1} = {}^{n+1} C_r$
- If ${}^n C_r = {}^n C_s$, Then $r = s$ or $n = r + s$

The number of combinations of n things taken r at a time in which

- s particular things will always occur is: ${}^{n-s} C_{r-s}$

- s particular things will never occur is: ${}^{n-s} C_r$
- s particular things always occurs and p particular things never occur is: ${}^{n-p-s} C_{r-s}$

DISTRIBUTION OF THINGS INTO GROUPS:

- Number of ways in which $(m+n)$ items can be divided into two unequal groups containing m and n items is: ${}^{m+n} C_m = \frac{(m+n)!}{m!n!}$
- The number of ways in which $m \times n$ different items can be divided equally into m groups, each containing n objects and the order of the groups is not important is: $\frac{(mn)!}{(n!)^m} \times \frac{1}{m!}$

IMPORTANT RESULTS TO REMEMBER:

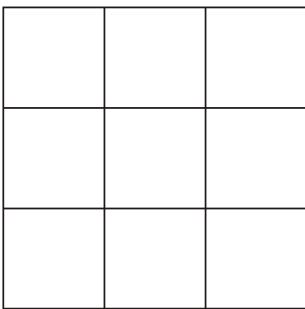
In a plane if there are n points of which no three are collinear, then

- The number of straight lines that can be formed by joining them is = ${}^n C_2$
- The number of triangles that can be formed by joining them is = ${}^n C_3$
- The number of polygons with k sides that can be formed by joining them is = ${}^n C_k$

In a plane if there are n points out of which m points are collinear, then

- The number of straight lines that can be formed by joining them is = ${}^n C_2 - {}^m C_2 + 1$
- The number of triangles that can be formed by joining them is = ${}^n C_3 - {}^m C_3$
- The number of polygons with k sides that can be formed by joining them is = ${}^n C_k - {}^m C_k$
- Number of rectangles of any size in a square of $n \times n = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$
- Number of squares of any size in a square of $n \times n = 1^2 + 2^2 + 3^2 + \dots + n^2 = \left[\frac{n(n+1)(2n+1)}{6} \right]$

EXAMPLE:

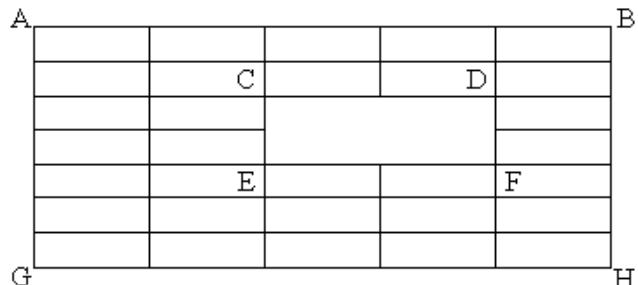


In the above figure, a 3×3 square is given, number of squares in the square of $3 \times 3 = 1^2 + 2^2 + 3^2 = 14$.

- Number of rectangles in the square of $3 \times 3 = 1^3 + 2^3 + 3^3 = 36$
- In a rectangle of $p \times q$ ($p < q$) number of rectangles of any size is $= \frac{pq}{.} (p+1)(q+1)$
- n straight lines were drawn in the plane such that no two lines are parallel and no three lines are concurrent. Then the number of parts into which these lines divide the plane is equal to $= 1 + \frac{n(n+1)}{2}$.

CHRONICLE
IAS ACADEMY

QUESTIONS



- c. 1512 d. 504
15. There are total of 15 stations excluding starting and last terminals for EMU running between Aligarh and New Delhi railway station. What should be number of types of tickets that should be printed in order to justify the need of all the commuters on this train? Assume that a passenger can board any of these stations and can get down any of these stations.
- a. 136 b. 272
c. 408 d. 544
16. In how many ways, 16 boys can be seated 16 chairs on a square table with 4 chairs per side?
- a. $\frac{16!}{2}$ b. $\frac{16!}{16}$
c. $\frac{16!}{4}$ d. $16!$
17. A necklace is to be prepared with 16 beads. 5 out of these beads are similar and 3 of the other beads are similar. How many distinct necklaces can be prepared with the help of these beads?
- a. $\frac{1}{16} \times \frac{1}{2} \times \frac{16!}{5!3!}$ b. $\frac{1}{16} \times \frac{16!}{5!3!}$
c. $\frac{1}{2} \times \frac{16!}{5!3!}$ d. $\frac{1}{16} \times \frac{1}{2} \times 16!$
18. How many distinct five flag signals can be created with the help of 8 flags of different colours?
- a. 5C_3 b. 8^3
c. 5^8 d. None of the above
19. A team of 5 basketball players including a captain need to be formed out of 13 basketball players. Find the number of ways in which this job can be done?
- a. $13 \times {}^{13}C_5$ b. $13 \times {}^{12}C_5$
c. $5 \times {}^{13}C_5$ d. Either b or c
20. Ramu has 3 pairs of similar black socks and 4 similar pairs of blue socks. What is the minimum number of socks that he must pick out in order to ensure one full pair of similar colored socks?
- a. 3 b. 7
c. 9 d. 13
21. There are 15 points on a piece of paper. 5 out of these 15 points are collinear and no other 3 points are collinear. Find the number of distinct lines that can be drawn on this paper by joining any pair of points.
- a. 90 b. 91
c. 105 d. 96
22. There are 15 points on a piece of paper out of which 5 points are collinear and no other three points are collinear. Find the number of distinct triangles that can be formed by joining these points.
- a. 455 b. 435
c. 445 d. 425
23. Find the number of three digit numbers which contain 5 as at least one of the digits.
- a. 252 b. 255
c. 258 d. 261
24. Ramu was planning for his new movie, Phookh-10. He has plan for the music director, lyricist and singer for the songs of the movie. He has three music directors and 5 lyricists in his mind for the movie. While finalizing singers, he realized that there should be 4 songs to be sung by different singers. All the songs have to be duet and there are 7 male and 8 female singers in contention. Find the number of ways in which he can finalize the complete music artists?
- a. $15 \times 4! \times 4!$ b. $7! \times 15$
c. $15 \times 6!$ d. $8! \times 15$
25. In how many ways letters of the word UNION can be arranged such that N's are not together?
- a. 36 b. 24
c. 84 d. 60
26. Find the number of ways in which five different books can be put on a shelf one over the other such that English book is above the Hindi book?
- a. 120 b. 60
c. 24 d. 48
27. If all the words using the alphabets of the word RAHUL are formed and arranged in their increasing alphabetical order then what will be the rank of word RAHUL in that list?
- a. 72 b. 73
c. 74 d. 75
28. In an examination, a student is supposed to answer 7 out of 10 questions in such a way that he needs to answer at least 3 out of first four. Then find the number of ways, in which a student can answer

- the paper?
- 20
 - 60
 - 80
 - 100
29. How many distinct colors can be formed with the help of 7 different colors by mixing any number of colors in equal quantity?
- 128
 - 127
 - 28
 - 56
30. In how many ways one chairman, 1 president and 2 vice presidents can be selected from 50 delegates of different countries?
- 2763600
 - 2764800
 - 2763400
 - 2769200
31. One fine day, 6 single men, 7 single ladies and 8 married couples decided to attend a party. How many handshakes will take place if all of them shake hand with each other except for the married couples among themselves when only men & woman hand shake is allowed?
- 210
 - 206
 - 202
 - 198
32. In how many ways, at least one fruit can be selected from 6 types of fruits if 4 fruits of each type are available for selection?
- $5^6 - 1$
 - $4^6 - 1$
 - $6^4 - 1$
 - $6^5 - 1$
33. Find the maximum number of words (with or without meaning) that can be formed by using any number of alphabets of the word CAMPUS
- 1296
 - 64
 - 720
 - 1956
34. In how many ways, a cookie flavor can be selected from 5 types of fruits; wherein a flavour can be a combination of any number of fruits?
- 120
 - 32
 - 31
 - 5
35. A wine can be distinguished by quality of grapes, age of grapes and age of wine. Quality of grapes can be good, medium or bad. Age of grapes can be either more than 10 years, in between 1 year and 10 years or fresh and age of wine can be either more than 20 years, between 10 and 20 years or less than 10 years. Considering all these parameters, how many types of wine are possible?
- 3
 - 9
 - 27
 - 26
36. In how many ways a superhero can climb a staircase of 10 steps?
- 1024
 - 10
 - 10!
 - 512
37. How many outcomes of tosses of 10 coins are possible if third and fourth coins have resulted in heads and ninth and tenth coins have resulted in tails?
- 32
 - 64
 - 128
 - 256
38. There are ten coins on a paper with all of them showing heads upfront. If fifteen coins are flipped one after the other (any coin can be flipped any number of times), which of the following is a possible outcome at the end of complete exercise?
- There are 7 heads and 3 tails upfront
 - There are 6 heads and 4 tails upfront
 - All of them are showing head upfront
 - None of the above is a possibility
39. Chaman forgot the mobile number of his close friend Raman. Although he remembers that the total number of digits in his mobile number was 10 with first two digit of the number being 9 and 8 in any particular order. Moreover he remembers that the third to sixth digits were in an decreasing order and so were seventh to tenth. He also remembers that in Raman's mobile number all the digits were used from 0 to 9. Find the maximum number of attempts that Chaman need to identifying mobile number of Chaman.
- $2 \times {}^8C_4 \times 4! \times 4!$
 - $2 \times {}^8P_4 \times 4!$
 - $2 \times {}^8C_4$
 - $4 \times {}^8C_4$
40. In how many ways, five speakers can address a press conference if a particular speaker need to speak before another particular speaker?
- 24
 - 48
 - 60
 - 120
41. In how many ways, 5 distinct letters can be posted in 7 letterboxes?
- 5^7
 - 7^5
 - 7C_5
 - 7P_5
42. In how many ways, Ramu, Kallu and Billu can share 12 bottles of beer among themselves if only one of them will consume exactly 5 bottles? Assume that all the bottles of beer to be similar and all of them will take at least one bottle.

each and every side of the table?

PROBABILITY

The concepts of probability play an important role in many problems of everyday life. It has very extensive applications in the development of physical sciences, economics, commerce, etc.

Sample space and event:

The set S of all possible outcomes of a random experiment is called the sample space or probability space. Every subset of a sample space is an event.

For example:

On throwing a dice, the sample space

$$S = \{1, 2, 3, 4, 5, 6\} \text{ and } |S| = 6$$

$E_1 = \{1, 3, 5\}$ is a subset of S . So E_1 is an event and $n(E_1) = 3$.

The event $E_1 = \{1, 3, 5\}$ is also expressed as the event of getting an odd number in throwing a dice.

Probability of an Event:

If the sample space S is finite, then the probability

$P(E)$ of the event is given by:

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

The problems of restricted permutation or combination are convertible into problems of probability.

For such problems,

$$P(E) = \frac{\text{number of ways under restriction}}{\text{number of ways without restriction}} = \frac{n(E)}{n(S)}$$

For example:

How many numbers of four distinct digits can be made with the digits 1, 2, 3 and 4 which begin with 4? It is a problem on restricted permutation. A similar

problem in probability can be as following:

"A four digit number of distinct digits is written down at random by using digits 1, 2, 3 and 4. What is the probability (chance) that the number begins with the digit 4?"

Here $n(S) =$ total number of numbers of four distinct digits that can be made with 1, 2, 3 and 4 (without restriction) $= {}^4P_4 = 4!$

$n(E) =$ the number of numbers of four distinct digits beginning with 4 that can be made with 1, 2, 3 and 4 $= 1 \times {}^3P_3 = 3!$

$$\text{Hence, required probability} = \frac{n(E)}{n(S)} = \frac{3!}{4!} = \frac{1}{4}$$

If $n(S)$ and $n(E)$ are both infinite, the probability can be estimated by geometrical method as below:

$$P(E) = \frac{\text{Favourable length}}{\text{Total length}} \text{ or } P(E) = \frac{\text{Favourable area}}{\text{Total area}} \text{ or } P(E) = \frac{\text{Favourable volume}}{\text{Total volume}}$$

respectively as the sample points (outcomes) are distributed over a length (one dimensional) or an area (two dimensional) or a volume (three dimensional)

Example:

A hockey match is played from 4 PM to 6 PM. A boy arrives to see the match (not before the match starts). What is the probability that he will miss the only goal of the match which takes place at the 15th minute of the match?

Solution:

To arrive at the match the boy has to reach between 4 PM to 6 PM. So, the total possible length of time = 2 hours = 120 minutes.

He will miss the goal if he arrives between 4.15 PM and 6 PM. So, the favourable length of time = 1 hour 45 minutes = 105 minutes.

$$\text{Hence the required probability } = \frac{105}{120} = \frac{7}{8}$$

Impossible and Sure Event:

Null set ' \emptyset ' and Sample space 'S', are also subsets of S. For example, on throwing of a dice event.

$S = \{1, 2, 3, 4, 5, 6\}$ is sure to happen, hence its probability will be 1. And probability of getting any number other than $\{1, 2, 3, 4, 5, 6\}$ on its face is impossible to occur, hence its probability will be zero.

Also, $0 \leq p(E) \leq 1$

Complementary Event:

In throwing of a dice consider a particular event

$$E = \{1, 3, 5\} = \{\text{getting an odd number on the face}\}$$

The complimentary event of E, represented by E' is $= \{2, 4, 6\} = \text{not happening of event E.}$

$$P(E) = 1 - P(E) \text{ or } P(E) + P(E') = 1$$

Odds in favour and Odds against:

$$\text{The odds in favour of the event } E = \frac{P(E)}{P(E')}$$

$$\text{The odds against the event } E = \frac{P(E')}{P(E)}$$

If odds in favour of the event E = a : b then,

$$P(E) = \frac{a}{a+b}$$

$$\text{If odds against the event E = a : b then, } P(E) = \frac{b}{a+b}$$

Some Critical types of 'event':

Equally likely event:

When the possibilities of two events to occur are equal then the events are called equally likely.

For Example:

Result of drawing a card from a well shuffled pack such that any card may appear in a draw, so that 52

cards turn to 52 different events which are likely equally.

Mutually Exclusive Events:

If the events have nothing in common in their occurrences, they are mutually exclusive events.

For example:

Tossing a coin, there are two events viz. turning up a head or a tail. But both the two cannot occur simultaneously so they are mutually exclusive.

Example:

Three groups of children have 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. What are the chances that the three selected consist of 1 girl and 2 boys?

Solution:

The selection from the respective groups can be made in the following manner:

$$(A) \text{ Boy, boy, girl: Probability } = \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{32}$$

$$(B) \text{ Boy, girl, boy: Probability } = \frac{1}{4} \times \frac{1}{2} \times \frac{3}{4} = \frac{3}{32}$$

$$(C) \text{ Girl, boy, boy: Probability } = \frac{3}{4} \times \frac{1}{2} \times \frac{3}{4} = \frac{9}{32}$$

As the probability of a boy being selected from the first group is $\frac{1}{4}$ and that of a girl is $\frac{3}{4}$ etc. Since the above three events are mutually exclusive, by using addition theorem of probability, the chance that any one of these events happens

$$= \frac{1}{32} + \frac{3}{32} + \frac{9}{32} = \frac{13}{32}$$

Independent Dependent Events:

Two events are independent if the occurrence of one does not affect that of other, if it is so, then they are dependent.

Independent:

Two successive tosses of a coin.

Dependent:

Drawing a red card from a pack of cards in second attempt. As this event will depend on what was

done in 1st attempt. If in first attempt a red card is drawn and it is not replaced then it is an example of dependent event. In case, 1st card is placed in the pack, the events will be independent.

So, for dependent events,

$$P(E_1 E_2) = P(A).P(B/A) = P(B).P(A/B)$$

Here, $P(A/B)$ = probability of occurrence of A given B has already been occurred.

And $P(B/A)$ = probability of occurrence of B given A has already been occurred.

Example:

The probability that a boy will pass an examination is $\frac{3}{5}$ and that for a girl it is $\frac{2}{5}$. What is the probability that at least one of them will pass the examination?

Solution:

The probability that the boy and girl will fail in the examination are $1 - \frac{3}{5}$ and $1 - \frac{2}{5}$, i.e., $\frac{2}{5}$ and $\frac{3}{5}$ respectively.

Since the boy and girl appear in the examination independently, the probability that none of them will pass the examination is $(\frac{2}{5}) \times (\frac{3}{5}) = \frac{6}{25}$.

Hence the probability that at least one of the boy and girl will pass the examination $= 1 - \frac{6}{25} = \frac{19}{25}$.

Example:

A problem in statistics is given to three students whose probabilities of solving it are $\frac{1}{2}$; $\frac{1}{3}$ and $\frac{1}{4}$. What

is the probability that the problem will be solved?

Solution:

Since the three students solve the problem independently, the probability that it will not be solved by any of them is $= (1/2) \times (2/3) \times (3/4) = \frac{1}{4}$

Hence the probability that the problem will be solved $= 1 - \frac{1}{4} = \frac{3}{4}$

Expectation:

This is one more concept that should be covered with above concepts. Expectation of an individual is defined as:

$$\text{Probability of winning} \times \text{reward of winning}$$

Example:

A man buys 10 out of 100 lottery tickets. The reward for winning ticket is Rs.5000. Then find the expectation of the man.

Solution:

Expectation

$$= \text{Probability of winning} \times \text{reward of winning}$$

$$= \frac{10}{100} \times 5000 = \text{Rs.}500$$

If the values of random variable x have the probabilities $p_1, p_2, p_3, \dots, p_n$ respectively then the expectation of x , is given by $E(x) = \sum_{i=1}^n x_i p_i$



QUESTIONS

1. Raj tossed ten times an unbaised coin and in all the results he got head. What is the probability of getting a head in his next toss?
- 1
 - $\frac{1}{2}$
 - $\frac{1}{11}$
 - $\frac{10}{11}$
2. Probability that India will win a match against Australia is $\frac{1}{4}$. Same day there is another match between Pakistan and South Africa. Probability that South Africa will win this match is $\frac{1}{4}$. What is the probability that both India and Pakistan will win their respective matches? Assume that there is no possibility of a tie match.
- $\frac{9}{20}$
 - $\frac{1}{5}$
 - $\frac{2}{5}$
 - $\frac{1}{4}$
3. In throw of 5 dices, what is the probability that the final sum of all the outcomes is a multiple of 6?
- $\frac{1}{3}$
 - $\frac{1}{4}$
 - $\frac{1}{5}$
 - $\frac{1}{6}$
4. Ramu had a well shuffled pack of 52 cards. One fine day, Kallu stole 6 of these playing cards. Next day when Ramu checked the cards, what is the probability that the top most card is ace of spade?
- $\frac{1}{52}$
 - $\frac{1}{46}$
 - $\frac{3}{52}$
 - None of the above
5. In a betting machine, one customer can bet on any of the ten numbers from 0 to 9. And there will be exactly one outcome and if this outcome is same as the number on which customer has bet, then customer will get 9 times the money he has bet. If the chances of selection of all the numbers is equally likely then what is the most probable daily income from this betting machine if there is a daily bet of Rs 1, 00, 000 on this machine and assume that there is no operational cost involved.
- a. Rs. 90,000
b. Rs. 50,000
c. Rs. 20,000
d. Rs. 10,000
6. Of all the three digit numbers, what is the probability that the unit's place is more than the ten's place?
- 0.4
 - 0.45
 - 0.5
 - 0.55
7. There are 9 places to be filled with 9 digits from 1 to 9. What is the probability that 6 will get filled before 4?
- $\frac{1}{2}$
 - $\frac{1}{4}$
 - $\frac{1}{6}$
 - $\frac{1}{10}$
8. Probability that Ramu will speak truth is $\frac{2}{5}$ and probability that Kallu will speak truth is $\frac{1}{5}$. What is the probability that both of them will give same answer when asked "whether it rained today or not"?
- $\frac{14}{25}$
 - $\frac{12}{25}$
 - $\frac{11}{25}$
 - $\frac{13}{25}$
9. If A and B are two positive integers then what is the probability that $A+B$ is an even number?
- $\frac{1}{4}$
 - $\frac{1}{2}$
 - $\frac{1}{3}$
 - $\frac{3}{4}$
10. What is the probability of getting at least one 1 in throw of two dices provided the total sum of the outcomes of these two dices is 7?
- $\frac{1}{3}$
 - $\frac{1}{6}$
 - $\frac{1}{4}$
 - $\frac{1}{5}$
11. If we select three numbers out of first 100 natural numbers, then what is the probability that product of these three numbers is even?
- $1 - \frac{{}^50C_3}{{}^{100}C_3}$
 - $\frac{{}^50C_3}{{}^{100}C_3}$
 - $\frac{{}^{50}C_2 \times {}^{50}C_1}{{}^{100}C_3}$
 - $1 - \frac{{}^{50}C_2 \times {}^{50}C_1}{{}^{100}C_3}$

12. If five dices are rolled then what is the most probable sum of all the outcomes of these dices?

 - 17
 - 18
 - Neither 17 nor 18
 - Either 17 or 18

13. Three squares of a chess board are selected at random. What is the probability that exactly two of these three are of the same color?

 - $\frac{^{32}C_2 \times ^{32}C_1}{^{64}C_3}$
 - $\frac{2 \times ^{32}C_2 \times ^{32}C_1}{^{64}C_3}$
 - $1 - \frac{^{32}C_2 \times ^{32}C_1}{^{64}C_3}$
 - $\frac{3 \times ^{32}C_2 \times ^{32}C_1}{^{64}C_3}$

14. Probability that Ramu will live for another ten years is $\frac{1}{4}$ and probability that Rani, Ramu's wife will live for another 10 years is $\frac{1}{3}$. What is the probability that at least one of these two will live for next 10 years?

 - $\frac{1}{2}$
 - $\frac{1}{4}$
 - $\frac{3}{4}$
 - None of the above

15. One dice is rolled three times. What is the probability that the same number will appear every time?

 - 1/6
 - 1/36
 - 1/216
 - 7/216

16. There are two coins, first coin is normal and have one head and one tail, while the other coin contains both head. If one coin was tossed result is a head if then what is the probability that the coin was tossed at random?

 - 1/2
 - 1/3
 - 2/3
 - None of the above

17. If all the words of letter CHRONICLE are arranged then what is the probability that both the C's of the word will come together?

 - 1/8
 - 1/9
 - 2/9
 - 2/7

18. Probability that Chaman will move one step forward is 0.4 and probability that he will move one step backward is 0.6. What is the probability that Chaman will be at his original position after moving two steps?

 - 0.48
 - 0.52
 - 0.16
 - 0.36

19. Probability that Ramu will be able to solve a question is $\frac{1}{4}$, probability that Kallu will be able to solve the question is $\frac{1}{5}$ and probability that Billu will be able to solve the question is $\frac{1}{6}$. If a question is given to all of them then what is the probability that the question will be solved?

a. $\frac{1}{2}$ b. $\frac{1}{4}$
c. $\frac{37}{60}$ d. None of the above

20. Probability that Inspector Chaman will be able to kill a thief with one bullet while chasing is $\frac{1}{2}$. One fine day, Inspector Chaman was chasing a thief and he had total of 3 bullets. What is the probability that the thief will be killed?

a. $\frac{1}{2}$ b. $\frac{1}{8}$
c. $\frac{7}{8}$ d. None of the above

21. Four points of a regular hexagon are joined. What is the probability that these four points when joined will form a parallelogram?

a. $\frac{1}{2}$ b. $\frac{1}{3}$
c. $\frac{1}{4}$ d. $\frac{1}{5}$

22. If three points of a regular hexagon are joined then what is the probability that the final figure obtained will be an equilateral triangle?

a. $\frac{1}{10}$ b. $\frac{1}{20}$
c. $\frac{1}{30}$ d. $\frac{1}{40}$

23. There is a rectangle of 4 cm by 5 cm. One point is selected at random. What is the probability that minimum distance of this point from any of the four sides of the rectangle is less than 1 cm?

a. 0.3 b. 0.5
c. 0.7 d. 0.8

24. If a thread is cut from a point, then what is the probability that the larger part is at least three times the smaller one?

a. $\frac{1}{3}$ b. $\frac{2}{3}$
c. $\frac{1}{2}$ d. $\frac{1}{4}$

25. One unbiased dice is thrown six times, what is the probability that all the outcomes of the dices were different?

a. $\frac{1}{6!}$ b. $\frac{6!}{6^6}$
c. $\frac{1}{6^6}$ d. None of the above

26. In toss of 6 coins what is the probability of getting 3 heads and 3 tails?
- $\frac{1}{2}/2$
 - $\frac{1}{2^6}$
 - $\frac{1}{2^6} \times \frac{6!}{3! \times 3!}$
 - $\frac{1}{2^6} \times \frac{3! \times 3!}{6!}$
27. Write drawing 3 cards from a well shuffled pack of cards, and then what is probability of getting 1 picture card and two non picture cards?
- $\frac{^{12}C_1 \times ^{40}C_2}{^{52}C_3}$
 - $\frac{^{12}C_1 \times ^{40}C_2}{^{52}C_3} \times 3$
 - $\frac{^{16}C_1 \times ^{36}C_2}{^{52}C_3}$
 - $\frac{^{16}C_1 \times ^{36}C_2}{^{52}C_3} \times 3$
28. In throw of a dice and toss of a coin, what is the probability of getting either 6 on the dice or tail on the coin or both?
- $5/12$
 - $2/3$
 - $7/12$
 - $3/4$
29. There are two boxes first one with 6 red and 5 black balls and second one with 5 red and 6 black balls. Firstly, one ball is transferred from box 1 to box 2 and then a ball was drawn from second box. What is the probability that the ball drawn from the second box was black if the ball transferred from the first box was black?
- $6/11$
 - $1/2$
 - $7/12$
 - $5/12$
30. We have a bucket of 50 liters capacity filled with water which can be emptied with the help of 10 smaller equal sized buckets all full except the last one which had at least one liter of water. What is the probability that the size of smaller buckets were more than 5.25 liters?
- $7/16$
 - $9/16$
 - $3/8$
 - $5/8$
31. In a colony of 200 men, 50 had blood group B+, 30 had blood group B-, 20 had blood group A+ 50 had blood group A- and 50 had blood group AB+. What is the probability that if two of them, going for a party had same blood group?
- $43/199$
 - $43/201$
 - $43/197$
 - $43/195$
32. Five letters were to be put in five corresponding addressed envelopes. If each letter was put in one of the envelopes then what is the probability that each letter will be put in correct envelope?
- a. $\frac{1}{2^5}$
b. $\frac{1}{5!}$
c. $\frac{1}{5C_2}$
d. None of the above
33. There are 3 dairy milk chocolates in a bag which is to be distributed among Ramu, Kallu and Billu then what is the probability that all of them will get exactly one chocolate?
- $1/3$
 - $2/3$
 - $2/9$
 - $3/5$
34. In an exam, there are two sections, Section A consists of 5 questions and Section B consists of 8 questions. There are total of 10 questions to be attempted. What is the probability that Ramu attempted exactly 4 questions from Section A if it is known that he has attempted all the questions?
- $\frac{5 \times ^8C_6}{^{13}C_{10}}$
 - $\frac{^8C_6}{^{13}C_{10}}$
 - $\frac{^8C_4}{^{13}C_{10}}$
 - $\frac{^8C_5}{^{13}C_{10}}$
35. Probability that Ramu will live for next 10 years is 0.5 and probability that Rani will live for next 10 years is 0.4. What is the probability that exactly one of these two will be alive after 10 years?
- 0.5
 - 0.3
 - 0.2
 - None of the above
36. If all the letters of word CHRONICLE are arranged then what is the probability that the word will start with C?
- $1/9$
 - $2/9$
 - $1/6$
 - None of the above
37. A particular software program is approved if and only if it passes through three stages of the software validations. Probability that a particular program will pass through any of these three stages is 0.5. What is the probability that this particular program will be approved?
- $1/125$
 - $124/125$
 - $7/8$
 - $1/8$
38. In a company there are 30 managers and 70 operators. Probability that an employee will take a leave on a day is 0.033. Probability that a manager will take a leave is 0.04. What is the probability that an operator will take a leave on a particular day? Assume that company has only these two types of employees.
- 0.028
 - 0.30

- c. 0.031 d. 0.032

39. Two mangoes were taken out from a lot of 10 mangoes, out of which 4 were rotten. If one of the two mangoes was checked and it was found to be good then what is the probability that the other one is also good?
a. $\frac{1}{5}$ b. $\frac{5}{9}$
c. $\frac{4}{9}$ d. $\frac{4}{5}$

40. In a multiple choice questions test, there were 100 questions, each with 4 options, a, b, c, d. Out of these 4 options, there was only one correct option, while the other options were wrong. Each student is awarded 4 marks for each correct answer and is penalized with 1 mark for each wrong answer. One student marked all the answers as 'c'. What is the most probable score that this student should get?
a. 0 b. 25
c. -25 d. 100

41. In a multiple choice questions test, there were 100 questions, each with 4 options, a, b, c, d. out of these 4 options, there was only one correct option, while the other options were wrong. Each student is awarded 4 marks for each correct answer and is penalized with 1 mark for each wrong answer. What is the maximum number of students in the class if all of them have scored different scores?
a. 501 b. 401
c. 495 d. 395

42. In a bag there are 4 red, 5 black and 6 yellow balls. What is the minimum number of balls that must be picked in order to ensure at least two balls of same color?
a. 3
b. 4
c. 12
d. 13

43. Refer question number 42, what is the minimum number of balls that must be picked in order to ensure all the balls of at least one color?
a. 3
b. 4

c. 12 d. 13

c. 12 d. 13

44. Refer question number 42, what is the minimum number of balls that must be picked in order to ensure at least one ball of each color?
a. 3
b. 4
c. 12
d. 13

45. There are 25 tickets numbered from 10 to 34. What is the probability that one particular ticket picked, will be either an even numbered ticket or have a multiple of 5 but not both?
a. $\frac{3}{5}$ b. $\frac{2}{5}$
c. $\frac{18}{25}$ d. $\frac{12}{25}$

46. What is the probability of getting a sum of 9 in throw of two dices?
a. $\frac{1}{9}$ b. $\frac{1}{12}$
c. $\frac{5}{36}$ d. None of the above

47. Three unbiased coins are tossed. What is the probability of getting at least two heads?
a. $\frac{1}{8}$ b. $\frac{3}{8}$
c. $\frac{1}{2}$ d. $\frac{7}{8}$

48. Two numbers are chosen at random. What is the probability that either both of them are even or both of them are odd?
a. $\frac{1}{4}$ b. $\frac{1}{2}$
c. $\frac{3}{4}$ d. None of the above

49. Probability that a student will score distinction in his exams is 0.3, probability that he will get first division is 0.4, probability that he will score second division is 0.1 and probability that he will score third division is 0.1. What is the probability that this student will fail?
a. 0
b. 0.1
c. 0.2
d. 0.3

50. There are two bags, first one with 4 red and 6 white balls and the second with 1 red and 5 white balls. If one ball is picked from either of the two bags, then what is the probability that the ball picked is red?
a. 0.28
b. 0.31
c. 0.4
d. 0.2



Logarithm and Indices

Logarithm and indices are two very closely related concepts. Both of these concepts are very much formula driven and we need to be comfortable with each and every formulae of the same.

Let us take indices firstly. When a particular number is multiplied to itself a certain number of times then this number of times becomes the power of that particular number. For example: if we multiply 6 by itself seven times, then it becomes 6^7 , in this case 7 is the power of 6 or we can read this number as 6 raised to the power of 7. Value of 6^7 is $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 279936$.

Following are certain formula on which all the questions of Indices are based.

- $a^0=1$
- $a^x \times a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $a^{-x} = \frac{1}{a^x}$
- $a^{\frac{x}{y}} = \sqrt[y]{a^x}$
- $(a \times b)^x = a^x \times b^x$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Example: find the value of a if $\left(\frac{a^4 \times 3^{1/2} \times 9^{3/4}}{2^4 \times 8^{5/3} \times 5^{-1}}\right) = 360$

Solution:

$$\begin{aligned} \left(\frac{a^4 \times 3^{1/2} \times 9^{3/4}}{2^4 \times 8^{5/3} \times 5^{-1}}\right) &= \frac{a^4 \times 3^{1/2} \times 3^{3/2} \times 5^1}{2^4 \times 2^5} \\ &= 360 = 2^3 \times 3^2 \times 5 \end{aligned}$$

Now comparing each and every power, we will realize that powers of 3 and 5 are equal on either sides but power of 2 differ from right hand side to left hand side. Power of 2 on right hand side is -9 and on left hand side, it is 3 which means that a^4 should be same as 2^{12} which is same as $(2^3)^4 = 8^4$. Hence, $a=8$.

Example: Solve $\left(\frac{a^{4/3} \times b^{7/2} \times c^{5/4}}{a^{7/3} \times b^{3/2} \times c^{1/4}}\right)$

$$\text{Solution: } \left(\frac{a^{4/3} \times b^{7/2} \times c^{5/4}}{a^{7/3} \times b^{3/2} \times c^{1/4}}\right) = \frac{\frac{7-3}{2} \times c^{\frac{5-1}{4}}}{\frac{7-4}{3}} = \frac{b^2 c}{a^3}$$

Example: Solve for $\left(\frac{6^6 \times 8^8 \times 12^{12}}{24^{24}}\right)$

$$\begin{aligned} \text{Solution: } \left(\frac{6^6 \times 8^8 \times 12^{12}}{24^{24}}\right) &= \frac{2^6 \times 3^6 \times 2^{24} \times 2^{24} \times 3^{12}}{2^{72} \times 3^{24}} \\ &= 2^{6+24+24-72} \times 3^{6+12-24} = 2^{-18} \times 3^{-6} \\ &= \frac{1}{2^{18} \times 3^6} = \frac{1}{24^6} \end{aligned}$$

Logarithm is just an extension of indices and it is defined as follows

If $a^x = b$, then it can also be written as $\log_a b = x$, wherein a will be called the base of the log.

Points to remember

- Log is defined only for positive values and even base. log can only take a positive value. Log of negative values and zero is not defined and even base can neither be zero nor negative. Although the value of log may come out to be a negative value.
- If base is not given in any log then it is taken as 10 by default; so Log 1=0, Log 10=1, Log 100=2, Log 1000=3 and so on and similarly Log 0.1=-1

Log 0.01=-2 and so on and so forth. Reasons for these values are $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, $10^{-1} = 0.1$, $10^{-2} = 0.01$ and so on.

Some basic formulae which are helpful in solving all the questions of logarithm are as follows:

- $\log_x 1 = 0$
- $\log_x x = 1$
- $\log_x(a \times b) = \log_x a + \log_x b$
- $\log_x\left(\frac{a}{b}\right) = \log_x a - \log_x b$
- $\log_x(a^n) = n \times \log_x a$
- $\log_x(a^{1/n}) = \log\left(\sqrt[n]{a}\right) = \frac{1}{n} \times \log_x a$
- $\log_{x^n} a = \log_{\sqrt[n]{x}} a = \frac{1}{n} \times \log_x a$
- $\log_x a = \frac{\log a}{\log x}$
- $\log_{x^n} a^m = \frac{m}{n} \log_x a$
- $a^{(\log_a x)} = x$

Another commonly used base in questions related to log is "e" which is an irrational number whose value is close to 2.718. Log to the base of e is also called natural logarithm and is represented by Ln.

Some common mistakes

- $\log(a+b) \neq \log a + \log b$
- $\log(a+b) \neq \log a \times \log b$
- $\log(a-b) \neq \log a - \log b$
- $\log(a-b) \neq \frac{\log a}{\log b}$

Example:

Calculate $\log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25$

Solution:

$$\begin{aligned} &= \log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25 \\ &= \frac{\log 10}{\log 5} \times \frac{\log 15}{\log 10} \times \frac{\log 20}{\log 15} \times \frac{\log 25}{\log 20} \end{aligned}$$

$$= \frac{\log 25}{\log 5} = \frac{\log 25}{\log 5} = \frac{2 \times \log 5}{\log 5} = 2$$

Example: If $\log(x^2 - 25x + 100) = 2$, then what is the value of x?

Solution: $2 = \log 100$

Hence, we can say that

$$\log(x^2 - 25x + 100) = 2 = \log 100$$

Now, removing Log from the either side, we get

$$(x^2 - 25x + 100) = 100$$

Thus $(x^2 - 25x) = 0$, which means x equals either zero or 25

Since, $(x^2 - 25x + 100)$ is a positive number for both values of x, so x equals to either 0 or 25.

Example: If $\log 2 = 0.301$, then how many digits are there in 2^{672} ?

Solution: In such questions, we need to calculate $\log 2^{672}$.

Since base is not given the question, we will assume that base is 10. Hence we get $\log_{10} 2 = 0.301$.

$$\begin{aligned} \text{Thus, } \log_{10} 2^{672} &= 672 \times \log_{10} 2 \\ &= 672 \times 0.301 = 202.3 \end{aligned}$$

Now, we know that,

$\log 10^{202} = 202$ and $\log 10^{203} = 203$, thus we can say that 2^{672} will lie in between 10^{202} and 10^{203} . Thus 2^{672} will be a 203 digits number.

Shares and Stock

It is one of the easiest topics which form certain questions in many aptitude tests. Just like many other topics from arithmetic, this topic is also an extension of basics of percentages.

Assume if a company need to raise its fund value then it will issue certain number of shares in the market which can be purchased by public at certain rate defined by the company and in return, company promises to pay certain money in return depending upon the pre decided policies.

Now the cost of the shares is called the par value of the share and the return which is generally prescribed in the percentage form is called dividend. For example if a question says the company issues a share of Rs 100 which pays a dividend of 15 %, then it means that one share of the company can be purchased for Rs 100 or par value of the share is Rs 100 and company will pay Rs 15 for every share or every Rs 100 invested in the company.

But depending upon, whether a particular share is performing very good or very bad and depending upon the reputation of the company, a share can be sold at a value higher or lower than its par value. This value at which a share is being sold is called market value of the share. If a share is sold for more than Rs 100, then it is said to be above par or sold at a premium. For example, if a company is selling its Rs 100 share, Rs 15 above par or at a premium of Rs 15, then it means that the par value of the share is Rs 100 but it can be purchased at Rs 115 and its market value is Rs 115 only which means that the dividend will be calculated on Rs 100 but a person need to invest more than Rs 100 to get the same dividend.

In a similar way, a particular share can be sold for even less than it's par value which is known as below par or at discount. For example if a question says that a particular Rs 100 share is being sold at a discount of Rs 15 or Rs 15 below par, then it means that market price of the share is Rs 85 although the par value of the share is Rs 100 and the dividend will be calculated on Rs 100 only.

Some Basic definitions:

- Face Value/ Par Value is defined as the price of the share as printed on the share on which dividend is calculated and this also known as the actual value of the share.
- Market Value is the price at which the share is being sold in the market. It may or may not be same as the face value of the share.
 - ◆ If market value of the share is more than the face value then it is said to be above par or at premium.

- ◆ If market value of the share is less than the face value then is said to be below par or at discount.
- ◆ If market value of the share is same as the face value then it is said to be sold at par.
- Stock is another name used for shares. It is the total amount of shares present with a person. For example if we say that someone has stock of Rs 50,000 then it means that the total par value of shares present with that person is Rs 50,000.
- Brokerage is the charges of the agent who sells the shares It is the additional amount that a person needs to pay in order to purchase certain shares. This amount will be a part of the investment but not a part of stock or face value or market value.

Now, in case some person has invested some money in certain share or stock, then his sole concern is return on investment. Return on investment is calculated by dividing the net returns by total investment done for the gain. Let us take the given example to understand this concept.

Example: Raju invested Rs. 1,00,000 in a Rs 100 share whose market value was Rs 125. Find the return on investment for Raju if company has decided to pay a dividend of 12% on the share.

Solution:

$$\text{Number of shares purchased} = \frac{\text{Rs } 1,00,000}{\text{Rs } 125} = 800$$

$$\text{Total value of the stock} = 800 \times \text{Rs } 100 = \text{Rs. } 80,000$$

$$\text{Dividend paid} = \text{Rs } 80,000 \times 12.5\% = \text{Rs. } 10,000$$

Return on investment

$$= \frac{\text{Return}}{\text{Investment}} = \frac{\text{Rs } 10,000}{\text{Rs } 1,00,000} = 10\%$$

Example: Raju has Rs. 10,00,000 stocks paying a dividend of 10% whose market value was Rs 125. He sold all his stock and invested in another stock at Rs 80 paying a 12% dividend. Find the increase in his annual income because of this investment.

Solution: If price of a share is not given then we will assume it to be Rs 100 by default.

So, his initial number of shares was

$$= \frac{\text{Rs.} 10,00,000}{\text{Rs.} 100} = 10,000 \text{ shares}$$

After selling, he will receive

$$\text{Rs.} 10,000 \times \text{Rs.} 125 = \text{Rs.} 12,50,000$$

Number of new shares that he will purchase is

$$\frac{\text{Rs.} 12,50,000}{\text{Rs.} 80} = 15,625 \text{ shares}$$

New Value of stock = $15,625 \times \text{Rs.} 100 = \text{Rs.} 15,62,500$

Now, Initial income was

$$= \text{Rs.} 10,00,000 \times 10\% = \text{Rs.} 1,00,000$$

New Income is = $\text{Rs.} 15,62,500 \times 12\% = \text{Rs.} 1,87,500$

So, increase in income

$$= \text{Rs.} 1,87,500 - \text{Rs.} 1,00,000 = \text{Rs.} 87,500$$

Now, if a person invests Rs. P in a Rs. F share selling at Rs. M in the market and the company pays D% dividend on its share then

$$\text{Number of shares purchased} = \frac{P}{F}$$

$$\text{Total value of the stock} = \frac{P}{F} \times F$$

$$\text{Income through these stocks} = \frac{P}{M} \times F \times \frac{D}{100}$$

$$\text{So, return on investment} = \frac{\frac{P}{M} \times F \times \frac{D}{100}}{P} \times 100 = \frac{FD}{M}$$

Thus, we can say that return on investment while investing in a particular share does not depend on the total investment; rather it depends on face value of the share, market value of the share and percentage of dividend paid by the company. And as a matter of fact, whenever, a person invests in any particular share then, he is only concerned about the face value and market value of the share along with the dividends paid by the company.

Trigonometry

Trigonometry is a Latin word which means measure

of three angles. This is a branch of mathematics, which deals with the ratios of sides of right angled triangles with reference to either of the acute angles.

Some basic conversions

Measure of the Angle	Circular Measure
30°	$\pi/6$
45°	$\pi/4$
60°	$\pi/3$
90°	$\pi/2$
120°	$2\pi/3$
180°	π
360°	2π

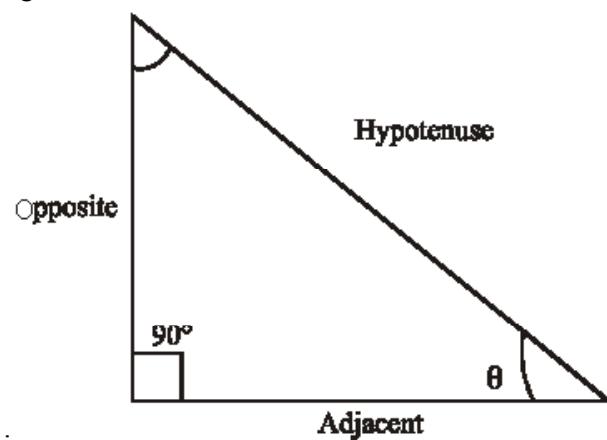
In a right angled triangle, there are three sides which are named as hypotenuse, opposite (also called height) and adjacent (also called base) which are defined as follows

Hypotenuse is the side opposite to right angle

Opposite is the side opposite to angle in question (θ in the given case)

Adjacent is the side adjacent to the angle in question which joins angle θ and right angle as shown in the figure.

All these values and nomenclature are shown in the figure below



As per this nomenclature, there are six ratios which are defined as follows

- $\sin\theta = \text{Opposite}/\text{Hypotenuse}$
- $\cos\theta = \text{Adjacent}/\text{Hypotenuse}$

- $\tan\theta = \text{Opposite}/\text{Adjacent}$
- $\cot\theta = \text{Adjacent}/\text{Opposite}$
- $\sec\theta = \text{Hypotenuse}/\text{Adjacent}$
- $\csc\theta = \text{Hypotenuse}/\text{Opposite}$

From these formulae, we can observe that

- $\sin\theta = 1/\csc\theta$
- $\cos\theta = 1/\sec\theta$
- $\tan\theta = 1/\cot\theta$

To excel in all the questions of trigonometry, following values of these ratios for different angles should be grasped.

	0°	30°	45°	60°	90°
\sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
\cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
\tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
\cot	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
\sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
\csc	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

There are certain more formulae which can be derived from the basic definitions used for trigonometry, which are as follows

- $\sin^2\theta + \cos^2\theta = 1$
- $\csc^2\theta - \cot^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\sin(90^\circ - \theta) = \cos\theta$
- $\tan(90^\circ - \theta) = \cot\theta$

Periodicity of these trigonometric ratios is defined as follows

$$\sin\theta = \sin(180^\circ - \theta) = -\sin(180^\circ + \theta) = -\sin(-\theta)$$

$$\cos\theta = -\cos(180^\circ - \theta) = -\cos(180^\circ + \theta) = \cos(-\theta)$$

$$\tan\theta = \tan(180^\circ - \theta) = \tan(180^\circ + \theta) = -\tan(-\theta)$$

Some other standard results, which are basically defined for each and every triangle

- Sine Rule: In case of any triangle, where a, b, c are the sides of the triangle and A, B and C are the angles opposite to these sides. Then the sides and angles bear a simple ratio which is as follows

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- Cosine Rule: In case of any triangle, where a, b, c are the sides of the triangle and A, B and C are the angles opposite to these sides. Then the sides and angles bear a simple relation which is as follows

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

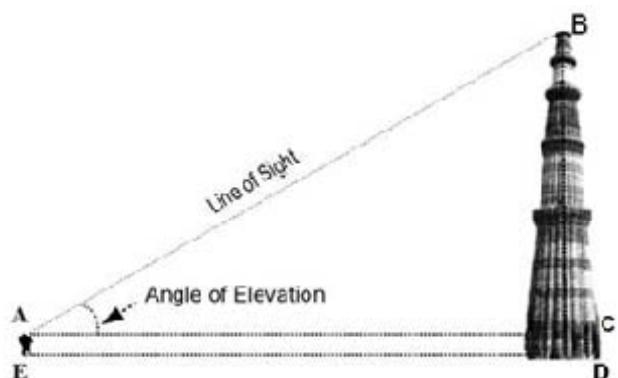
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- Area of a triangle can also be calculated with the help of these trigonometric ratios by the given formulae:

$$\text{Area of a triangle} = \frac{1}{2} \times a \times b \times \sin\theta$$

Where a and b are the sides of the triangle and θ is the angle between these two sides.

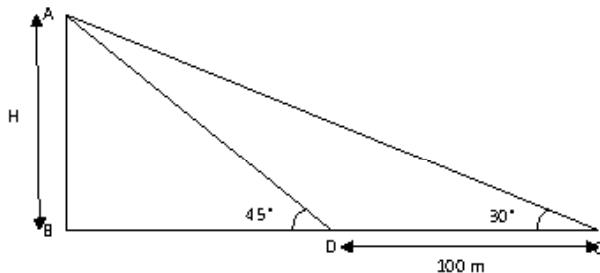
Height and Distance: These are typical trigonometric questions which require use of these ratios. First of all, we need to understand the term angle of elevation (also known as angle of depression). This is the angle formed by joining the point in question with the top of any building or tower as per the question. This is depicted by the figure given below.



Now, as a short-cut, we should remember that if there is a right angled triangle with angles as $30^\circ - 60^\circ - 90^\circ$ then the ratio of sides opposite to these angles will

be $1:\sqrt{3}:2$. In a similar way, if there is a triangle with angles as $45^\circ-45^\circ-90^\circ$, then the ratio of sides opposite to these angles will be $1:1:\sqrt{2}$. These ratios can also be obtained with the help of trigonometric ratios of Sin, Cos, Tan of 45° and 30° . Let us try to understand this with the help of given example.

Example: After travelling 100 m, a person realized that the angle of elevation of the top of a building has reduced from 60° to 45° . Find the height of the building.



Solution:

In the given figure,

Assume, $AB = H$ (that we need to calculate)

So, $BD = H$ (as $\triangle ABD$ is a $45^\circ-45^\circ-90^\circ$ triangle)

Similarly, $BC = \sqrt{3}H$ (as $\triangle ABC$ is a $30^\circ-60^\circ-90^\circ$ triangle)

And moreover, $DC = BC - BD$

Thus, $100 = \sqrt{3}H - H$

$$\text{Hence, } H = \frac{100}{\sqrt{3}-1} = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 50(\sqrt{3}+1)$$

COORDINATE GEOMETRY

Introduction

Coordinate geometry is a branch of mathematics that uses the principles of algebra to study geometry. Figures like lines and circles can be represented using algebraic equations and their properties can be studied using these equations. Coordinate geometry also helps us understand the behaviour of functions.

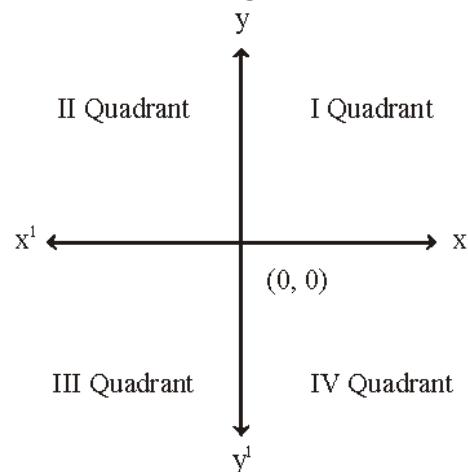
A one-dimensional coordinate system, used to represent points, is the number line similar to the one that we have studied in Number Systems.

A two-dimensional coordinate system is used to represent two-dimensional figures like lines, circles and other curves.

The Coordinate System

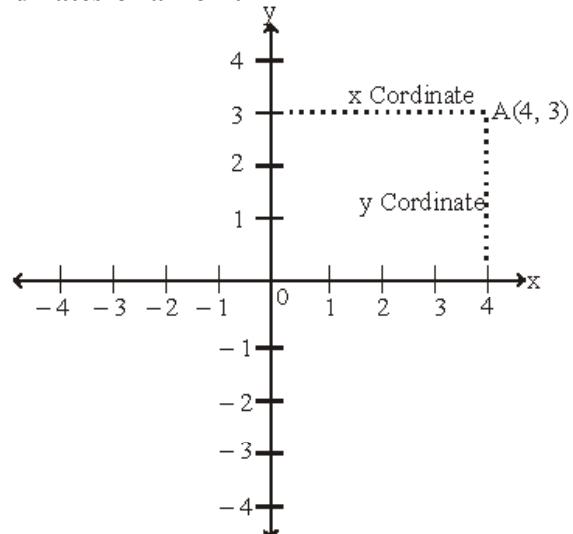
Two perpendicular number lines, XOX' and YOY' , intersecting at O form the *coordinate system*.

Quadrants, Axes and Origin



The line XOX' is called the x-axis and the line YOY' is called the y-axis. The x-axis and the y-axis are called the coordinate axes. The point O is called the origin. The axes divide the plane on which they are drawn into four parts known as quadrants and the plane is called the x-y plane.

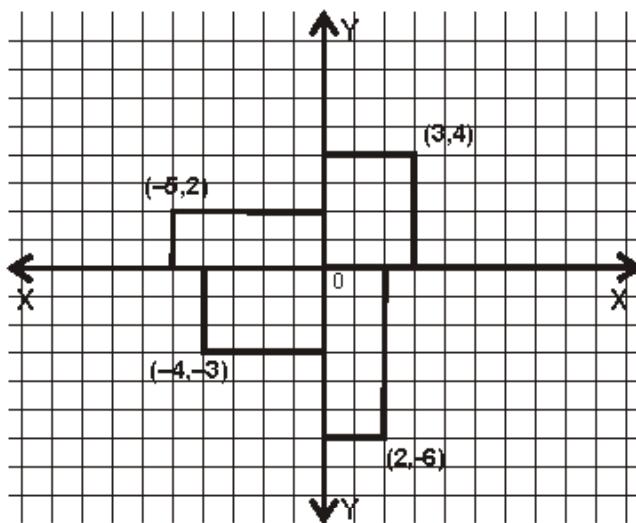
Coordinates of a Point



Any point on the x-y plane can be identified by coordinates. The x-coordinate is the distance of the point from the y-axis and the y-coordinate is the distance of the point from the x-axis. The x-coordinate is also known as the abscissa and the y-coordinate

is also known as the ordinate. Any point can be represented by an ordered pair of x and y coordinates as (x, y). For example, a point A with x-coordinate 4 and y-coordinate 3 can be represented as A (4, 3). The distance of this point A is 4 units from the y-axis and 3 units from the x-axis.

The signs of the x and y-coordinates of a point change depending on the quadrant in which it lies.



Quadrant	x-coordinate	y-coordinate
I	Positive	Positive
II	Negative	Positive
III	Negative	Negative
IV	Positive	Negative

Remember:

The points lying on the axes are not considered to be in any quadrant.

Distance Formula and Section Formula

The Distance Formula

The distance between any two points lying on a line parallel to the x-axis and having x-coordinates x_1 and x_2 is $|x_2 - x_1|$.

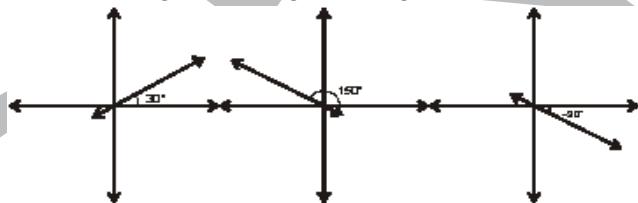
Similarly, the distance between any two points lying on a line parallel to the y-axis and having y-coordinates y_1 and y_2 is $|y_2 - y_1|$.

Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Lines

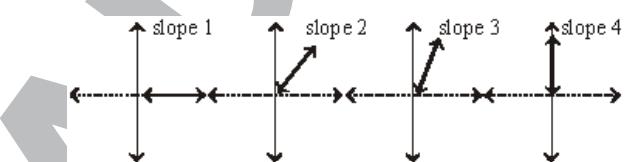
Positive and Negative Angles

The angles measured anti-clockwise from the positive direction of the x-axis are considered to be positive. In the following figure, the first two angles are positive. The angles measured clockwise from the positive direction of the x-axis are considered to be negative. The third angle is a negative angle.



Slope of a Line

The slope of a line, generally denoted by m , is the slant of a line. The slope of a horizontal line is 0. Its magnitude goes on increasing as we gradually make the line vertical as shown.



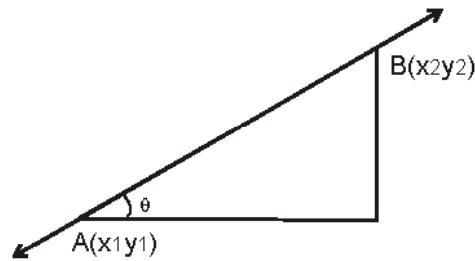
Clearly, the slope depends on the angle that it makes with the horizontal. It can be calculated in two ways.

- If the angle made by the line in the positive direction of the x-axis is known, and is say θ , the slope is given by: $m = \tan \theta$

θ is called the inclination of the line θ .

Slopes can be positive or negative depending on θ . This is illustrated in the following table.

θ	Slope	θ	Slope
0°	$\tan 0^\circ = 0$	$120^\circ = -60^\circ$	$\tan -60^\circ = -\sqrt{3}$
30°	$\tan 30^\circ = \frac{1}{\sqrt{3}}$	$135^\circ = -145^\circ$	$\tan -45^\circ = -1$
45°	$\tan 45^\circ = 1$	$150^\circ = -30^\circ$	$\tan -30^\circ = -\frac{1}{\sqrt{3}}$
60°	$\tan 60^\circ = \sqrt{3}$	180°	$\tan 180^\circ = 0$
90°	$\tan 90^\circ = \infty$	$270^\circ = -90^\circ$	$\tan -90^\circ = -\infty$



2. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are any two points on a line, the slope is given by: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slopes of Parallel Lines

Parallel lines make equal angles with the positive direction of the x-axis. Hence, the slopes of parallel lines are always equal.

Slopes of Perpendicular Lines

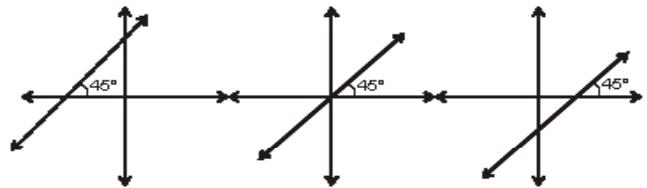
If there are two perpendicular lines, then the product of their slopes will be -1.

Remember: Slope of the x-axis is 0 and the slope of the y-axis is ∞ .

Equation of a Line

As stated earlier, coordinate geometry helps us study geometry using algebraic principles. We can describe a line completely using an algebraic equation. Now let us first try to understand how a line is described.

We have seen the concept of slope. It denotes the slant of a line. If we consider a line with slope 1, we know that it makes an angle of $\tan^{-1}(1) = 45^\circ$ with the positive direction of the x-axis.



All the above lines have slope 1. But all the lines are different. If we know any one point on the line in addition to the slope, we will know exactly where the line lies. The equation of a line can be stated in various forms as described below.

Slope-point Form

The equation of a line having slope m and passing through the point $P(x_1, y_1)$ is $(y - y_1) = m(x - x_1)$.

Remember: Any point on a line always satisfies the equation of the line.

Two Point Form

As stated earlier, we need a point on a line and the slope to write the equation of the line. When we have two points, we can find the slope. Using this slope and any one of the given points, the equation of the line can be found.

The equation of a line passing through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ when $x_1 \neq x_2$, is given by

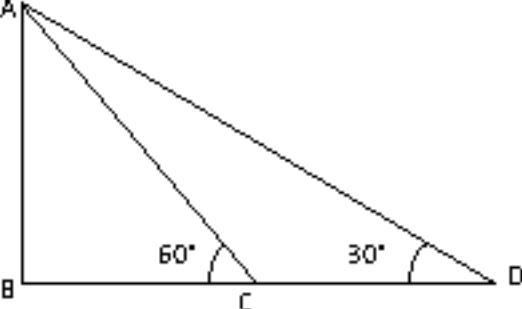
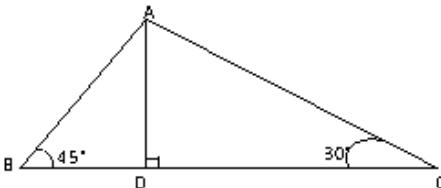
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

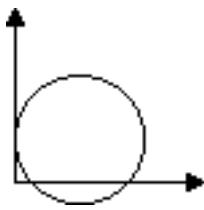


QUESTIONS

1. What is half of 4^{100} ?
 - 2^{100}
 - 4^{99}
 - 2^{199}
 - None of the above
2. If $6^{a+b} \times 4^{2a} \times 9^b = 144$, then, what is the value of a ?
 - $9/10$
 - $5/7$
 - $2/3$
 - Cannot be determined
3. Which of the given numbers is largest?
 $\sqrt{2}, \sqrt[3]{3}, \sqrt[5]{5}, \sqrt[10]{10}$
 - $\sqrt{2}$
 - $\sqrt[3]{3}$
 - $\sqrt[5]{5}$
 - $\sqrt[10]{10}$
4. Which of the given values are negative?
 $\log_5 8, \log_5 3, \log_{0.5} 0.8, \log_{0.5} 0.2$
 - $\log_5 8$ and $\log_{0.5} 0.2$
 - $\log_{0.5} 0.8$ and $\log_5 3$
 - $\log_{0.5} 0.8$ and $\log_{0.5} 0.2$
 - None of the above
5. Which of the given numbers is larger: 2^{300} or 3^{200}
 - 2^{300}
 - 3^{200}
 - Both are equal
 - None of the above
6. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then what is the value of $a \times b \times c$?
 - 0
 - 1
 - abc
 - none of the above
7. What is the value of $2^{\log_2 5}$?
 - 2
 - 5
 - 1
 - 0
8. Find the number of digits in 3^{32} if $\log 3 = 0.477$
 - 12
 - 14
 - 15
 - 16
9. If $\log 2 = a, \log 3 = b$ then what is the value of $\log 1.2$?
 - $2a+b$
 - $2a-b$
 - $2a+b+1$
 - $2a+b-1$
10. If $\log_3(2^x - 5)$ is the arithmetic mean of $\log_3 2$ and $\log_3(2^x - 7/2)$, then what is the value of x ?
 - 2
 - 3
 - 4
 - Either 2 or 3
11. Find the value of x if $\log_{10} x = x^{-1}$.
 - 10
 - 1
 - $\sqrt{10}$
 - None of the above
12. If x, y and z are in GP, then $\log_x 10, \log_y 10$ and $\log_z 10$ are in
 - AP
 - GP
 - HP
 - None of the above
13. If $\log x + \log(x+3) = 1$, then what is the value of x ?
 - 2
 - 3
 - 5
 - Either 2 or 5
14. Which of the given value is higher: $(2^3)^4$ or 2^{3^4}
 - $(2^3)^4$
 - 2^{3^4}
 - Both of these numbers are equal
 - Cannot be determined
15. Find the percentage increase in income of a person who decided to sell his Rs 100 shares being sold at a premium of 20% and paying 12% dividend and reinvest his money in another Rs 100 share sold at a discount of 20% paying a 10% dividend.
 - 12 %
 - 25 %
 - 20%
 - 22%
16. Find the return on investment for a person who has invested all his savings in Rs 10 share whose market value was Rs 40 paying a dividend of 25%.
 - 25%
 - 12.5%
 - 6.25%
 - 5%
17. Find the annual income of a person who has invested Rs 1,00,000 in a Rs 10 share selling at Rs 12.5 if the dividend percentage on the share

- is 12 %.
- Rs 9000
 - Rs 9,200
 - Rs. 9,600
 - Rs 9,800
18. A person earns Rs 1,200 through his investments on a share paying Rs 6 per share and the market price for the share is Rs 120. How much did he invest in the shares?
- Rs 12,000
 - Rs 10,000
 - Rs 8,000
 - Rs 5,000
19. Ramu sold all his Rs 100 shares at Rs 120 and invested all his money in another Rs 10 share selling at Rs 12. Find the percentage change in his income if initially he was earning Rs 1000 and both the shares are paying a dividend of 10%.
- His income will increase
 - His income will decrease
 - His income will remain same
 - Depend upon the investment
20. Yadav ji had to raise funds for his tabela. He decided to float shares for his tabela at Rs 100 but sold these shares at Rs 120. He needed exactly Rs 96000 for his tabela. Find how much he needs to pay per annum as dividend if he raised exactly the same amount of money which he needed and has already declared a payout of 12 %.
- Rs 9,200
 - Rs 9,600
 - Rs 9,800
 - Rs 10,000
21. Sheela and Munni invested Rs 1,00,000 and Rs 80,000 respectively in two shares. Ratio of dividend percentages was a:b and ratio of their market price was b:a. Find the ratio of the face value of the shares if both of them were earning same amount of money from their respective investments.
- $\frac{5b^2}{4a^2}$
 - $\frac{4a^2}{5b^2}$
 - $\frac{4b^2}{5a^2}$
 - $\frac{5a^2}{4b^2}$
22. Calculate $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \times \tan b}$
- $1 + \sqrt{3}$
 - $1 + \sqrt{3}$
 - $1 + 2\sqrt{3}$
 - None of these
23. A particular angle is given in degree. Which of the given factors should be multiplied to the given angle to convert them into radians ?
- $1/2\pi$
 - $180/\pi$
 - 2π
 - $\pi/180$
24. Angle of elevation of the top of a building from a 100 m far point was 60° . Find the height of the building.
- 100
 - $100\sqrt{3}$
 - $100/\sqrt{3}$
 - 50
25. After moving 100 m angle of elevation of top of building decreased from 45° to 30° . What could be the height of the building? Assume that initially person was moving towards the building and after reaching building he started moving away from the building and he moved in a straight line only.
- $\frac{100}{\sqrt{3}-1}$
 - $\frac{100}{\sqrt{3}+1}$
 - $100\sqrt{3}$
 - None of these
26. There are six circles of equal radius inscribed in a triangle as shown in the figure. If radius of the circle is 1 cm then find the length of the side of the triangle?
-
- $4 + 2\sqrt{3}$
 - $2 + 4\sqrt{3}$
 - $4 + \sqrt{3}$
 - $2 + \sqrt{3}$
27. Find the length of the thread required to tie three cylinders of radius 10 m.
- $10\pi + 40$
 - $20\pi + 80$
 - $20\pi + 40$
 - $60\pi + 80$
28. Find the value of $\tan \theta$ if $\sin \theta = 0.8$.
- $3/5$
 - $4/3$
 - $5/3$
 - $5/4$
29. Which of the given relationship is true
- $\frac{\sin \theta}{\operatorname{cosec} \theta} = \cot \theta$
 - $\frac{\operatorname{cosec} \theta}{\sin \theta} = \cos \theta$

- c. $\frac{\tan \theta}{\cot \theta} = \sin \theta$ d. $\frac{\sin \theta}{\cos \theta} = \tan \theta$
30. Which of the following is an odd function
 a. $\sin x + \cos x$ b. $\cos x + \tan x$
 c. $\sin x + \tan x$ d. None of the above
31. If $\sin x : \cos x = 4 : 7$; then find $\frac{\sin x + \cos x}{\sin x - \cos x}$.
 a. $11/3$ b. $-11/3$
 c. $3/11$ d. $-3/11$
32. Shadow of a 200 m high building on a sunny day was 150 m. What will be the length of the shadow of a 15m tall pole?
 a. 10 m b. 12m
 c. 13.5 m d. 11.25 m
33. In the given figure if AB = 150 m, then what is the length of CD?

 a. $100/\sqrt{3}$ b. $100\sqrt{3}$
 c. $50\sqrt{3}$ d. 100
34. Refer to the figure given in question number 33, if CD = 150 m then what is the value of AB ?
 a. $100/\sqrt{3}$ b. $100\sqrt{3}$
 c. $75\sqrt{3}$ d. 100
35. Refer to the figure given in question number 33, if BC = 150 m then what is the length of CD?
 a. $100/\sqrt{3}$ b. $100\sqrt{3}$
 c. 100 d. 300
36. In the given figure, if AD = 100 m, then what is the length of BC?

 a. $100/\sqrt{3}$ b. $100\sqrt{3}$
 c. 100 d. 300
- a. $100(\sqrt{3} + 1)$ b. $100(\sqrt{3} - 1)$
 c. 300 d. 250
37. Refer to figure given in question number 36, if BC = 100 m, then what is the length of AD?
 a. $100\sqrt{3}/(\sqrt{3}-1)$ b. $100\sqrt{3}/(\sqrt{3}+1)$
 c. $100\sqrt{3}$ d. 150
38. Refer to figure given in question number 36, if BD = 100 m, then what is the length of DC?
 a. 100 b. 150
 c. $100\sqrt{3}$ d. $100/\sqrt{3}$
39. Find the equation of a line passing through point (3, 4) which makes an angle of 60° with the x-axis.
 a. $3x + 4y = 5$ b. $\sqrt{3}x + y = 3\sqrt{3} - 4$
 c. $\sqrt{3}x - y = 3\sqrt{3} - 4$ d. $x - \sqrt{3}y = 3\sqrt{3} - 4$
40. Find the equation of a line which passes through points (3, 5) and (4, 4)?
 a. $3x + 4y = 5$ b. $x + y = 5$
 c. $x + y = 8$ d. $3x + 5y = 16$
41. Find the point which divides the line joining point (6, 8) and (-3, -7) in a ratio 2 : 1.
 a. (0, -2) b. (2, 0)
 c. (2, 5) d. (1, -2)
42. Find the equation of a circle with center as (3,4) and radius of the circle as 3 units.
 a. $9x^2 + 16y^2 = 9$ b. $9x^2 + 16y^2 + 12xy = 9$
 c. $x^2 + y^2 - 6x - 8y + 16 = 0$ d. $x^2 + y^2 + 6x + 8y + 16 = 0$
43. Find the value of a + b if (6, 9) is the mid-point of (4, 7) and (a, b)
 a. 17 b. 19
 c. 21 d. 23
44. Which of the following can be the equation of the circle given in the figure if the coordinates of the center of the circle are (5, 4)?



- a. $(x-5)^2 + (y-4)^2 = 25$
b. $(x-4)^2 + (y-5)^2 = 25$
c. $4x^2 + 5y^2 = 16$
d. $(x-4)^2 + (y-5)^2 = 16$
45. Line $3x+5y=7$ will not pass through which of the given points?
a. (11, 8) b. (-11, 8)
c. (8, 11) d. (-8, 11)
46. Which of the following statements are true regarding the line whose equation is $4x=7y-87$?
i. If value of x increases, then value of y also increases.
ii. If value of x increases, then value of y decreases.
iii. If value of x decreases, then value of y also increases.

iv. If value of x decreases, then value of y also decreases.

- a. (i) and (iv) b. (ii) and (iii)
c. (i) and (ii) d. (iii) and (iv)

47. $y = x^2 + 5x - 3$ is equation of a _____

- a. Circle b. Parabola
c. Ellipse d. Hyperbola

48. Find the area of a triangle whose vertices are (x, x) , $(x+1, x+2)$ and $(x+7, x+2)$.

- a. x^2 b. 6
c. $6x^2$ d. Cannot be determined

49. What will be the coordinates of the circumcenter of a triangle whose vertices are $(0, 0)$, $(0, 8)$ and $(10, 0)$?

- a. (5, 6) b. (2, 3)
c. (5, 4) d. (3, 5)

50. Does ellipse has a constant radius?

- a. Yes
b. No
c. Depends on the axis
d. None of the above



Answers Key

CHAPTER-1

1.	b	6.	c	11.	c	16.	d	21.	b	26.	d	31.	a	36.	a	41.	c	46.	b
2.	a	7.	b	12.	a	17.	d	22.	c	27.	c	32.	c	37.	a	42.	a	47.	a
3.	a	8.	b	13.	b	18.	a	23.	b	28.	c	33.	c	38.	d	43.	c	48.	c
4.	a	9.	d	14.	a	19.	d	24.	c	29.	c	34.	b	39.	b	44.	d	49.	b
5.	d	10.	a	15.	a	20.	c	25.	c	30.	c	35.	a	40.	a	45.	d	50.	b

CHAPTER-2

1.	c	6.	a	11.	a	16.	d	21.	c	26.	a	31.	b	36.	c	41.	b	46.	c
2.	c	7.	c	12.	b	17.	d	22.	b	27.	b	32.	b	37.	b	42.	d	47.	c
3.	c	8.	b	13.	c	18.	d	23.	a	28.	b	33.	b	38.	d	43.	b	48.	b
4.	a	9.	c	14.	c	19.	c	24.	a	29.	d	34.	d	39.	b	44.	b	49.	c
5.	d	10.	c	15.	c	20.	c	25.	d	30.	d	35.	c	40.	c	45.	a	50.	d

CHAPTER-3

1.	a	6.	b	11.	c	16.	c	21.	a	26.	b	31.	c	36.	c	41.	c	46.	b
2.	d	7.	c	12.	b	17.	b	22.	a	27.	c	32.	b	37.	d	42.	b	47.	b
3.	b	8.	c	13.	b	18.	b	23.	a	28.	c	33.	b	38.	b	43.	b	48.	a
4.	c	9.	b	14.	c	19.	a	24.	d	29.	d	34.	c	39.	d	44.	b	49.	c
5.	b	10.	b	15.	c	20.	b	25.	c	30.	c	35.	d	40.	c	45.	d	50.	a

CHAPTER-4

1.	d	6.	a	11.	b	16.	b	21.	d	26.	a	31.	c	36.	a	41.	c	46.	a
2.	b	7.	b	12.	c	17.	c	22.	a	27.	b	32.	a	37.	b	42.	b	47.	c
3.	b	8.	b	13.	b	18.	a	23.	c	28.	c	33.	c	38.	c	43.	a	48.	a
4.	c	9.	b	14.	c	19.	c	24.	c	29.	c	34.	a	39.	a	44.	a	49.	b
5.	b	10.	b	15.	b	20.	a	25.	b	30.	d	35.	b	40.	b	45.	b	50.	a

CHAPTER-5

1.	c	6.	c	11.	c	16.	c	21.	a	26.	a	31.	b	36.	b	41.	a	46.	d
2.	a	7.	a	12.	b	17.	d	22.	d	27.	b	32.	c	37.	c	42.	b	47.	b
3.	b	8.	b	13.	b	18.	b	23.	a	28.	a	33.	a	38.	b	43.	c	48.	c
4.	b	9.	c	14.	b	19.	c	24.	d	29.	a	34.	b	39.	c	44.	a	49.	c
5.	b	10.	d	15.	b	20.	c	25.	d	30.	a	35.	c	40.	b	45.	c	50.	d

CHAPTER-6

1.	a	6.	c	11.	b	16.	b	21.	b	26.	c	31.	b	36.	c	41.	a	46.	c
2.	b	7.	a	12.	c	17.	a	22.	b	27.	b	32.	c	37.	b	42.	d	47.	c
3.	c	8.	b	13.	b	18.	d	23.	b	28.	c	33.	d	38.	d	43.	a	48.	c
4.	b	9.	a	14.	b	19.	a	24.	b	29.	b	34.	b	39.	d	44.	c	49.	b
5.	a	10.	c	15.	a	20.	b	25.	a	30.	c	35.	b	40.	d	45.	a	50.	d

CHAPTER-7

1.	a	6.	b	11.	d	16.	d	21.	d	26.	b	31.	a	36.	a	41.	a	46.	a
2.	b	7.	b	12.	c	17.	b	22.	b	27.	a	32.	a	37.	b	42.	a	47.	a
3.	a	8.	a	13.	b	18.	b	23.	c	28.	a	33.	c	38.	a	43.	d	48.	a
4.	a	9.	a	14.	c	19.	b	24.	a	29.	b	34.	c	39.	a	44.	b	49.	b
5.	a	10.	d	15.	b	20.	b	25.	b	30.	a	35.	a	40.	d	45.	c	50.	d

CHAPTER-8

1.	a	6.	c	11.	b	16.	d	21.	d	26.	b	31.	c	36.	d	41.	c	46.	b
2.	d	7.	d	12.	a	17.	b	22.	c	27.	d	32.	a	37.	a	42.	b	47.	a
3.	a	8.	c	13.	c	18.	b	23.	d	28.	b	33.	b	38.	a	43.	d	48.	c
4.	c	9.	b	14.	b	19.	c	24.	c	29.	c	34.	b	39.	a	44.	a	49.	b
5.	b	10.	d	15.	c	20.	a	25.	c	30.	d	35.	c	40.	c	45.	c	50.	c

CHAPTER-9

1.	b	6.	d	11.	a	16.	c	21.	b	26.	b	31.	c	36.	b	41.	b	46.	b
2.	b	7.	b	12.	d	17.	b	22.	b	27.	c	32.	b	37.	c	42.	b	47.	a
3.	c	8.	c	13.	c	18.	c	23.	b	28.	a	33.	b	38.	a	43.	d	48.	b
4.	c	9.	b	14.	b	19.	c	24.	c	29.	a	34.	a	39.	c	44.	c	49.	b
5.	c	10.	c	15.	c	20.	b	25.	b	30.	a	35.	c	40.	c	45.	b	50.	b

CHAPTER-10

1.	d	6.	d	11.	b	16.	b	21.	b	26.	c	31.	a	36.	c	41.	c	46.	b
2.	c	7.	b	12.	c	17.	c	22.	c	27.	b	32.	c	37.	b	42.	c	47.	c
3.	c	8.	a	13.	c	18.	c	23.	b	28.	a	33.	b	38.	b	43.	c	48.	b
4.	a	9.	b	14.	c	19.	a	24.	c	29.	c	34.	c	39.	b	44.	d	49.	a
5.	b	10.	b	15.	a	20.	b	25.	c	30.	b	35.	b	40.	a	45.	b	50.	b

CHAPTER-11

1.	c	6.	d	11.	b	16.	c	21.	c	26.	b	31.	a	36.	c	41.	d	46.	c
2.	d	7.	d	12.	a	17.	b	22.	d	27.	b	32.	a	37.	c	42.	d	47.	c
3.	b	8.	a	13.	c	18.	c	23.	a	28.	c	33.	c	38.	b	43.	b	48.	c
4.	c	9.	d	14.	a	19.	d	24.	c	29.	a	34.	b	39.	c	44.	b	49.	b
5.	b	10.	b	15.	c	20.	b	25.	c	30.	c	35.	d	40.	a	45.	a	50.	a

CHAPTER-12

1.	b	6.	c	11.	c	16.	b	21.	a	26.	d	31.	d	36.	c	41.	b	46.	a
2.	a	7.	b	12.	d	17.	b	22.	b	27.	c	32.	a	37.	c	42.	d	47.	c
3.	c	8.	a	13.	b	18.	b	23.	a	28.	b	33.	c	38.	b	43.	c	48.	d
4.	a	9.	a	14.	c	19.	a	24.	b	29.	c	34.	c	39.	a	44.	a	49.	c
5.	d	10.	c	15.	b	20.	b	25.	a	30.	a	35.	c	40.	a	45.	a	50.	b

CHAPTER-13

1.	d	6.	b	11.	c	16.	c	21.	d	26.	b	31.	c	36.	d	41.	b	46.	b
2.	b	7.	a	12.	b	17.	a	22.	c	27.	c	32.	a	37.	b	42.	b	47.	c
3.	a	8.	b	13.	a	18.	d	23.	a	28.	c	33.	d	38.	a	43.	b	48.	b
4.	c	9.	c	14.	b	19.	c	24.	d	29.	b	34.	c	39.	c	44.	b	49.	d
5.	a	10.	d	15.	b	20.	a	25.	a	30.	a	35.	c	40.	c	45.	a	50.	d

CHAPTER-14

1.	b	6.	b	11.	a	16.	c	21.	d	26.	c	31.	a	36.	b	41.	c	46.	a
2.	b	7.	a	12.	d	17.	c	22.	a	27.	a	32.	b	37.	d	42.	b	47.	c
3.	d	8.	a	13.	b	18.	a	23.	c	28.	c	33.	c	38.	b	43.	d	48.	b
4.	a	9.	b	14.	a	19.	a	24.	c	29.	c	34.	a	39.	b	44.	c	49.	b
5.	d	10.	a	15.	b	20.	c	25.	b	30.	a	35.	a	40.	b	45.	a	50.	a

CHAPTER-15

1.	c	6.	b	11.	d	16.	c	21.	d	26.	a	31.	b	36.	a	41.	a	46.	a
2.	b	7.	b	12.	c	17.	c	22.	c	27.	b	32.	d	37.	b	42.	c	47.	b
3.	b	8.	d	13.	a	18.	a	23.	d	28.	b	33.	b	38.	c	43.	b	48.	b
4.	d	9.	d	14.	b	19.	c	24.	b	29.	d	34.	c	39.	c	44.	b	49.	c
5.	b	10.	b	15.	b	20.	b	25.	b	30.	c	35.	d	40.	c	45.	b	50.	b

SOLUTIONS

Chapter 1

1. Since, 119xy is divisible by 99, it means that 119xy should also be divisible by 9 and 11.

The divisibility rule for 9 is that the sum of all the digits should be divisible by 9 and for 11 is that the difference of sum of alternate digits of the number should be divisible by 11.

Now, for divisibility by 9, the possible sum of x and y can be 7 or 16 x = 7, & y = 9. But, for divisibility by 11, the possible sum of x and y is 16. Hence, $x + y = 16$.

2. Since, the sum of all the 5 prime numbers is even. All prime numbers except 2 are odd.

Sum of 4 odd numbers is even. Hence, there should be a prime number that is even so that the sum of 5 prime numbers becomes even. Hence, the smallest prime number is 2.

3. The product of a rational number and an irrational number is always an irrational number except if the rational number is 0. Hence, the product will be 0.

4. Since, there are 50 odd numbers and 50 even numbers any operation (addition or subtraction) on these 50 odd and 50 even numbers will result in even number. Hence, Ramu will win.

5. If I pick up 4 eggs then there will be 96 eggs remaining. Now, to win the picking must go in a sequence of 5. Suppose if, Kallu picks up 3 eggs then I'll pick up 2, if Kallu picks 5 then I'll also pick 5, so that in the end only 1 egg remains that I'll pick up and win.

6. To find the remainder of a number when divided by 8, we have to find the remainder of last three digits of the number by 8. The last three digits of the number are 475 the remainder of which when divided by 8 comes out to be 3. Hence, the remainder is 3.

7. For divisibility by 9 the sum of a+b should be 5 or 14. For divisibility by 8, b should be an even number and a0b must be divisible by 8. The values of a and b satisfying both the above conditions are a = 1, b = 4 and a = 6, b = 8. Hence, a can take the values either 1 or 6.

8. The number of trees in each row should be the HCF of 48, 72 and 120 which comes out to be 24.

So, the number of rows = $2 + 3 + 5 = 10$

9. The smallest multiple of 8 when divided by 15 gives a remainder of 12 is 72. The other options are either not a multiple of 8 or do not give the remainder as 12 when divided by 15. Hence. [d]

10. The numbers 449, 667 and 883 gives the remainders 9, 7 and 3 when divided by N. It means that 440, 660 and 880 are perfectly divisible by N. Hence, N can be 22.

11. They will ring together again after the LCM of 12, 15 and 20 seconds, which is after 60 seconds. Hence, they will ring together once every minute and a total of 60 times in 60 minutes. Including the first ring also, there will be a total of 61 rings till 9 A.M.

12. Let the number be $(N + 7)$. So, N should be divisible by 12 and 15, which means that N should be divisible by 60. Going, through the options, 960 is divisible by 60. Hence, 967 is the required number

13. The largest number dividing these three numbers is the HCF of these 3 numbers which is 33. We can also go through the options and find out the number

14. Reduce the number in Fractions $.8 = \frac{4}{5}$, $.75 = \frac{3}{4}$ & $1.35 = \frac{27}{20}$

$$\text{required LCM} = \frac{\text{LCM of } (4, 3, 27)}{\text{HCF of } (5, 4, 20)} = \frac{108}{1} = 108$$

15. Reduce the numbers to the simplest forms.

The HCF of Fraction =

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}} = \frac{\text{HCF of } 21, 33, 21}{\text{LCM of } 4, 8, 8} = \frac{3}{8}$$

16. The LCM of numbers from 1 to 120 will contain all the factors of 122, 123 and 124. There will be

one 11 and two 5's. Hence, the LCM of all the numbers from 1 to 125 = $11 \times 5 \times A = 55A$.

17. Since one number is 28 and the LCM of both the numbers is 82. But 82 is not divisible by 28. Hence, data is inconsistent.

18. Five consecutive odd numbers will always contain a multiple of 3 and 5. Hence, the largest number that can divide the product of 5 consecutive odd numbers is $3 \times 5 = 15$

19. Five consecutive even numbers will always contain a multiple of 1, 2, 3, 4 and 5. Moreover all the numbers will be multiples of 2 as well. Hence, the smallest number that will always divide the product of 5 consecutive even numbers will be $5! \times 2^5 = 120 \times 32 = 3840$

20. Since, it is a three digit recurring number, Hence, it must be multiplied by 999 to make it a natural number

21. The number should be divisible by 2, 5, 7, 9, 11 and 13. Going through the options, option [c] is straight away out. All other options satisfy the criteria. Hence, the largest number is option [b]

22. The LCM of $42 = 2 \times 3 \times 7$ and $63 = 3^2 \times 7$ will be the LCM of a, b, c and d which is $2 \times 3^2 \times 7 = 126$.

23. The series can be also written as:

$$\begin{aligned} &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{99} - \frac{1}{100} \\ &= \frac{1}{1} - \frac{1}{100} = \frac{99}{100} \end{aligned}$$

$$24. a = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \dots$$

$$\frac{a}{4} = \frac{1}{4} + \frac{1}{16} + \frac{1}{36} + \dots$$

$$a - \frac{a}{4} = \frac{1}{1} + \frac{1}{9} + \frac{1}{25} + \dots = \frac{3a}{4}$$

25. The number is of the form of $x = 228n - 102$

When this number is divided by 57; 228 is divisible by 57, hence the remainder of 228n divided by 57 is 0. And the remainder of 108 divided by 57 is 45. Hence, the remainder is 45.

26. The sum of two positive even number will always be even number. The sum of an even and an odd number is always odd. Hence, a + b + c will always be an odd number

27. This is only possible if n is a prime number. Number of prime numbers between 10 and 100 is 21. Hence, there are 21 such values of n.

28. For the number to be divisible by 8 the last 3 digits need to be divisible by 8. Hence, n = 4

For the number to be divisible by 11, the difference of sum of alternate numbers need to be divisible by 11. Hence, m = 5. So, m+n = 9.

29. Statement (i) is the necessary condition. Hence, true
Statement (ii) is also true.

Statement (iii) is not necessarily true.

Hence, option [c]

30. Let there be n students. Each student can be put in any one of the classes. So, the number of ways = 2^n . But, it also includes a case when all the students are in class 1 and class 2 is empty and also a case when all the students are in class 2 and class 1 is empty. Hence, total number of ways = $2^n - 2 = 1022$. But in this complete calculations, each and every pair of arrangements will counted twice. so, to adjust for the repetition, number of types need to be divided by 2. Hence, n=10.

31. The number of 3 digit numbers that contain 7 = Total 3 digit numbers – Number of numbers that contain 7

Number of numbers that do not contain any 7 = $8 \times 9 \times 9 = 648$

Number of 3 digit numbers that contain 7 = 900 – 648 = 252

32. Let the number be $100a + 10b + c$. Reverse = $100c + 10b + a$.

Difference = $99a - 99c = 297$

$a - c = 3$. Hence, the difference between the Hundred's digit and Unit's digit = 3

33. Let the number be x.

From the question, $74x - 47x = 1485$

Hence, $x = 55$.

34. The divisor is the difference between the two numbers = $37653 - 28465 = 9188$.

Now, on dividing both the numbers by 9188, the remainder is 901.

35. Number of 4 digit numbers = 1. Number of digits = 4

Number of 3 digit numbers = 900. Number of digits = 2700.

Number of 2 digit numbers = 90. Number of digits = 180

Number of 1 digit numbers = 9. Number of digits used = 9

$$\text{Total digits used} = 4 + 2700 + 180 + 9 = 2893$$

36. Going through the options, we get that Rs 10 postage cannot be paid using any number of stamps of given denominations.

37. There are 3 coloured balls. So, one must pick at least 4 balls to ensure two balls of 1 colour.

38. There are 7 yellow coloured balls and 6 black coloured balls. So , if someone picked 7 balls which were of yellow colours and next 6 which were black. So, one must pick at least 14 balls in order to confirm one ball of each colour.

39. The following weights are required:

1, 3, 9, 27, 81. Hence, a total of 5 weights need to be carried.

40. He needs to carry the envelopes of following denominations.

1, 2, 4, 8, 16, 32, 64. Hence, a total of 7 envelopes need to be carried.

41. Let $x = 0.676767\dots$ and $y = 0.353535\dots$

$$\Rightarrow 100n = 67.6767\dots \text{ and } 100y = 35.3535\dots$$

$$\Rightarrow 99n = 67 \quad \text{and} \quad 99y = 35.$$

$$\Rightarrow x = \frac{67}{99} \quad \text{and} \quad y = \frac{35}{99}$$

$$\Rightarrow x + y = \frac{67}{99} + \frac{35}{99} = \frac{102}{99} = \frac{34}{33}$$

42. Let the given expression be equal to X.

$$\text{then:- } x = \frac{12}{1+x}$$

$$\Rightarrow x^2 + x - 12 = 0.$$

$$\Rightarrow x + 4x - 3x - 12 = 0.$$

$$\Rightarrow (x+4)(x-3) = 0.$$

$$\Rightarrow x = -4 \text{ or } 3$$

But 'x' cannot be negative so, $x = 3$

43. We can see that ' x ' = 0.567567....

$$1000x = 567.567567\dots$$

$$999x = 1000x - x = 567$$

$$\Rightarrow x = \frac{567}{999}$$

So if we multiply ' x ' with 999 or its multiple we will get an integer. Hence, option [c]

44. It is divisible by 11 as we can see that (sum of

digits in odd places) – (sum of digits in even places) = 0. [i.e. $(2 + 6 + 7 + 5 + 8) - (4 + 5 + 6 + 9 + 4) = 0$.]

45. L.C.M. of 15, 18, 21 = 3 | 15, 18, 21

$$\begin{array}{r} 3 \\ | \\ 5, 6, 7 \\ | \\ 5, 2, 7 \end{array}$$

$$= 3 \times 3 \times 5 \times 2 \times 7 = 630.$$

$$\text{the number } N = 630 - 4 = 626$$

46. L.C.M. of 16, 20, 24 =

$$\begin{array}{r} 2 \\ | \\ 16, 20, 24 \end{array}$$

$$\begin{array}{r} 2 \\ | \\ 8, 10, 12 \end{array}$$

$$\begin{array}{r} 2 \\ | \\ 4, 5, 6 \end{array}$$

$$\begin{array}{r} | \\ 2, 5, 3 \end{array}$$

$$= 2 \times 2 \times 2 \times 2 \times 5 \times 3$$

$$= 240.$$

And the difference between the divisors and remainder is of 3.

So the second smallest such number = $237 + 240 = 477$

47. L.C.M. of 11, 13 and 15 = $11 \times 13 \times 15 = 2145$

$$\text{The number } N = 2145 - 6 = 2139$$

48. We can see that the difference between the divisor and the remainder in each of the cases is same i.e. 2.

$$\begin{array}{r} 3 \\ | \\ 7, 9, 12 \end{array}$$

$$\begin{array}{r} 2 \\ | \\ 7, 3, 4 \end{array}$$

$$\begin{array}{r} | \\ 7, 3, 2 \end{array}$$

L.C.M. of 7, 9, 12 =

$$= 3 \times 2 \times 7 \times 3 \times 2 = 252$$

$$\text{Required number } N = 252 - 2 = 250$$

49. N can be a square only when $y = 2$ or 6 so y can take only 2 values.

50. Let the number be $10a + b$.

$$\frac{10a + b}{a + b} = 6.$$

Given that,

$$\Rightarrow 10a + b = 6a + 6b$$

$$\Rightarrow 4a = 5b.$$

$$\Rightarrow \frac{a}{b} = \frac{5}{4}$$

So the number is 54 and product of digits is 20.

Chapter 2

1. Total Income of all the employees = $433.33 \times 15 = 6500$

Total income of all the managers = $500 \times 10 = 5000$

Total income of all the operators = $6500 - 5000 = 1500$

Average income of each operator = $1500/5 = \text{Rs}300$

2. Let the average of 20 students be x .

Total age = $20x$

Total Increase in age of the class = $30 \times 4 = 120$ years

New total age = $24(x + 1) = 20x + 120$

$x = 24$.

New average = 25 years

3. Sum of temp. of the week = $35 \times 7 = 245$

Sum of temp. of Sunday, Monday, Tuesday, Thursday, Friday, Saturday = $32 \times 3 + 38 \times 3 = 96 + 114 = 210$. So Temperature of wednesday = 35

4. On reversing the digits of one number the average of 9 numbers increases by 9. Hence, the Unit's place exceeds Ten's place by 1, so on reversing the digit, the difference between the two nos. will be increased by 9 and so the average increases by 1.

5. Total age of 5 members 5 years back = $23 \times 5 = 115$

Total age of 5 members now = $115 + 25 = 140$

Total age of 4 members now = $23 \times 4 = 92$

Difference = 48 yrs = The age of the man who died.

6. Katiyal added 40 kgs to the total age of all the persons and 61 kgs more as to maintain the average to 61. Hence, the weight of Katiyal = 101 kgs.

7. Avg. weight = $(60 \times 50 + 40 \times 60 + 30 \times 80 + 50 \times 70 + 80 \times 100)/(60 + 40 + 30 + 50 + 80) = 74$

8. The average increased by 1, means the boy added 9 more over the weight of the boy who was replaced. Hence, his weight = $15 + 9 = 24$ kgs

9. Total runs he has given before these 4 matches = $300 \times 30 = 9000$

Minimum total runs he will give after these 4 matches = 9100

To reduce his average to less than 29 he should take till this match = $9100/29 = 313.79$

He must take atleast 14 wickets in the next four matches.

10. Let the no. of children Kats has be x

Total age = $12x$

New avg. = $12x/(x + 1) = 10$

$x = 5$

\therefore Now she has, $x+1 = 6$ children.

11. The weight of the boy added the weight of 0.5kg to the average of each of the 49 students in the class.

Since, the average weight was 50 kg and he added 24.5 kg more, so his weight was 74.5 kg

12. Kallu increased the weight of 10 students by 1 kg each and he replaced a 50 kg boy. So, his weight will be 60 kg.

13. Let the no. of students in section A, B and C be $5x$, $7x$ and $9x$

Average of complete school

$$\frac{(72 \times 5x) + (77 \times 7x) + (81 \times 9x)}{5x + 7x + 9x} = \frac{1628x}{21x} = 77.5 \text{ years}$$

14. Total sum of 7 numbers = 91

Total sum of first four numbers. = 60

Total sum of last four numbers. = 44

4th number = $60 + 44 - 91 = 13$

15. The two teachers increased the age of 32 people by 1 year each in addition to the average of the class. Hence, the sum of their ages = $23 + 23 + 32 = 78$ years

- Their average = 39 years.
16. If 1 is added to all the nos. the average will also get increased by 1. Hence, new average = 15.
17. Since, the average of 4 even consecutive nos. will be the mean of those 4 nos., hence, the nos. can have a maximum difference of 3 from the average. Hence, 48 cannot be one of those four numbers.
18. This one mango of 150 gm. will add $150 - 45 = 105$ gm. to the total weight of the mangoes in addition to their average weight. Hence, new average will be increased by $105/5 = 21$ gm.
Hence, new average will be $45 + 21 = 66$ gm.
19. Let the no. of people 5 years back be x.
Total age 5 years back = $40x$
Total age today = $40x + 5x = 45x$
Average age = $45x/(x + 1) = 40$
 $x = 8$.
Hence, there are 9 members in the family now.
20. Harmonic mean = $\frac{2ab}{a+b} = \frac{2 \times \frac{1}{2} \times \frac{1}{8}}{\frac{1}{2} + \frac{1}{8}} = \frac{1}{5}$
21. The maximum average will occur when the maximum possible values are used. Thus:
'v' should have been born on 30th, 'w' on 25th, 'x' on 20th, 'y' on 10th and 'z' on 5th. Further, the months of births in random order will have to be between August to December to maximize the average. Hence the total will be $30 + 25 + 20 + 10 + 5 + 12 + 11 + 10 + 9 + 8 = 140$.
Hence average is 28.
22. The minimum average will be when we have $1 + 5 + 10 + 20 + 25 + 1 + 2 + 3 + 4 + 5 = 76$.
Hence average is 15.2
23. The prime dates must be 29th, 23rd, 19th and 5th. Hence, the maximum possible average will reduce by $4/5 = 0.8$. Hence, answer will be 27.2
24. Total sales for the first 11 months
 $= 12,000 \times 11 = \text{Rs } 132000$
Total sales for the whole year = $\text{Rs } 11375 \times 12$
 $= \text{Rs } 136500$
- Values of sales during the last month
 $= 136500 - 132000 = \text{Rs } 4500$
25. Total weight increases by $10 \times 1.5 = 15$ kg
 \therefore Weight of the new man = $68 + 15 = 83$ kg
26. Let the two numbers be a and b.
 $\therefore \frac{a+b}{2} = 12.5$ i.e., $a + b = 25$
 $\sqrt{ab} = 10$ i.e., $ab = 100$
 $\therefore a = 20, b = 5$
27. The number of students in class A and B are in the ratio of 3:2, since the average of A & B together is 4 marks more than average of A and 6 marks less than the average of B. Similarly the ratio of number of students in B & C is in the ratio 1:1
 $\therefore A:B:C = 3:2:2$
 \therefore Average of A, B and is
- $$\frac{3 \times 74 + 2 \times 84 + 2 \times 78}{7} = 78$$
28. Let the largest natural number written be n and let the erased number be x.
$$\frac{\frac{n(n+1)}{2} - x}{n-1} = 28 \frac{4}{9} = \frac{256}{9}$$

 $\Rightarrow (n-1)$ is a multiple of 9
The average of n natural numbers is $\left(\frac{n+1}{2}\right)$
 $\therefore 28 \frac{4}{9}$ Should be close to $\left(\frac{n+1}{2}\right)$ as $(n-1)$ is divisible by 9,
 $\therefore n$ is of the form $9k + 1$ close to $(2 \times 28) = 56$
Only value of 'n' satisfying this is $n = 55$
 $\therefore n = 55$
When $n - 1 = 54 \Rightarrow n = 55$
- $$\frac{\frac{55 \times 56}{2} - x}{54} = \frac{256}{9}$$
- $$\Rightarrow x = 1540 - 1536 = 4$$
29. Let the four consecutive odd numbers be n, n + 2, n + 4 and n + 6
The average is n + 3 $\therefore A = n + 3$

Given $\frac{A}{10}$ i.e., $\frac{n+3}{10}$ is a perfect square.
 $\therefore n + 3 = 10 K$ where K is a perfect square
i.e., $K = \{1, 4, 9, 16, \dots\}$

If $K = 1$, then $n + 3 = 10 \Rightarrow n = 7$

The four consecutive odd numbers are 7, 9, 11, 13. If $K=4$, then $n+3 = 40$.

$n = 37$. The four consecutive odd no.s are 37, 39, 41, 43

Similarly if we take $K = 9$, we will get a set of consecutive odd numbers which contain 93.

From the given choices 25 is not the possible value.

30. Sanat's average expenditure for ten days is Rs 13.

\therefore His total expenditure for ten days = Rs 130. Also his total expenditure for first five days = Rs 50. and that for the last three days = Rs 48. Now, let his expenditure for the sixth and seventh day be x and y respectively.

$$130 = 50 + x + y + 48$$

$$x + y = 32$$

Average expenditure = Rs 16

31. Average speed is the harmonic mean of 24, 40 and 120.

\therefore Average speed

$$= \frac{3}{\frac{1}{24} + \frac{1}{40} + \frac{1}{120}} = \frac{3}{\frac{1}{8} \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{15} \right)} = 40$$

32. Sum of all results = 63

Sum of first six results = 42

Hence last result is 21

Sum of last two results = 18

So, sixth result is 15

33. Let the two digits number whose digits were swapped be $(ab)10$

If the actual average is A , then

$$10a + b + T = 10a \quad \dots(1)$$

$$10b + a + T = 10(A + 3.6) \quad \dots(2)$$

where T denotes the sum of the remaining 9 numbers

Subtracting (1) from (2) we get

$$9b - 9a = 36$$

$$\Rightarrow b - a = 4$$

\therefore The difference between these two digits = 4

34. Number of students in group D is maximum. But no information about the weight of the students is given, hence no comparison of weight of group D can be made with that of the other group.

35. If one student from group A is shifted to group B, the number of students in the class and total weight of class still remains the same and it does not affect the average weight of the class.

36. Suppose the weight of each student be 10 kg then average weight of class A, B, C, D would be $\frac{150}{15} = 10$, $\frac{200}{20} = 10$, $\frac{250}{25} = 10$ and $\frac{400}{40} = 10$.

Hence (a) is true

Total weight of A and C = $(150 + 250) = 400$ kg. Which is 2×200 . Hence (b) is true.

In option (a) we have seen that average weight of all the four groups is same, hence average weight of D cannot be greater than the average weight of A. Hence, Choice (c) is false.

37. The boy travels half the distance at 10 km/hr. The average speed for the other part = $(15+20)/2 = 17.5$ km/hr.

Therefore, the average for the entire trip is

$$\frac{2(10)(17.5)}{10+17.5} \text{ km/hr.}$$

$$= \frac{350}{27.5} \text{ km/hr.} = \frac{140}{11} \text{ km/hr} = 12\frac{8}{11} \text{ km/hr}$$

38. Total marks obtained by the top 10 students = 10 (67) = 670

Total marks obtained by the last 11 (from 10th to the 20th) students = 11 (64) = 704

If we add these two, Rohit's score is getting added twice. Now, total marks scored by the 20 students = $20(65) = 1300$

$$\therefore \text{Rohits score} = 670 + 704 - 1300 = 74$$

39. Total number of mistakes = 2 (1,007) = 2,014

\therefore The number of mistakes in the last 1007–612 or 395 pages = $2,014 - 434 = 1,580$

\therefore Average number of mistakes per page for the last 395 pages $\frac{1580}{395} = 4$

Solution for 40 and 41:

Let the amounts with A, B, C and D be Rs x, Rs y, Rs z, and Rs w. respectively $x + y + z + w = 1200$

$$\text{Also, } 2x + 2y + z + w = 1500 \quad \dots(1)$$

$$\Rightarrow x + y = 300 \quad \dots(2)$$

It is also given that $2z + 1.4w = 2(690)$

$$\Rightarrow 2z + 1.4w = 1380 \quad \dots(3)$$

But $2x$ (1) – (2)

$$\Rightarrow z + w = 900 \quad \dots(4)$$

Solving (3) and (4) we get $z = 200$ and $w = 700$

40. C had Rs z = Rs 200

$$41. x + y = 200 \quad \dots(1)$$

$$x + 50 = y - 50 \Rightarrow y - x = 100 \quad \dots(2)$$

Solving (1) and (2), we get, $x = 100$

42. The number of pass candidates are $2 + 6 + 18 + 40 = 66$ out of a total of 100.

43. Solve through options. Option [b] will be the answer.

44. Put $x = 10$ in the given equations and find the average of the resultant options.

45. Runs scored in first 8 overs = 56

Runs to score in next 12 overs = $176 - 56 = 120$

So required average is 10.

46. $nz - x + x^1 = nz^1$, Simplify to get option (c) correct.

47. The order of measures is $x, y, z, x+y, y+z, z+x, x+y+z$.

Given $x + y + z = 130$ (approx)

Hence, average of the 7 measures.

$$= \left[\frac{(x) + (y) + (z) + (x+y) + (y+z) + (z+x) + (x+y+z)}{7} \right]$$

$$\frac{4}{7}(x+y+z) = \frac{4}{7} \times 130 = \frac{720}{7} \text{ kgs.} = 103 \text{ (approx)}$$

48. The average weight of 10 people is increased by 1 kg. Therefore, the total weight is increased by $10 \times 1 = 10$ kg.

Hence the weight of the new person is $58 + 10 = 68$ kg

49. The sum of the first five multiples of 495

$$= 495 \times 1 + 495 \times 2 + 495 \times 3 + 495 \times 4 + 495 \times 5 \\ = 495 (1 + 2 + 3 + 4 + 5) = 495 \times 15$$

$$\text{Required Average} = \frac{495 \times 15}{5} = 495 \times 3 = 1485.$$

50. Average

$$= \frac{48 \times 40 + 32 \times 60 + 80 \times 20}{100} = \frac{1920 + 1920 + 1600}{100} \\ = \frac{5440}{100} = 54.40$$

Chapter 3

1. A is 30% more than B. It means $A = 1.3 B$. So,
 $B = \frac{1}{1.3} A = 0.769A$. It means that B is 23% less
than A.

2. 20 kg grapes contains 80% of 20 = 16 kg water.
So, ratio of Water : Matter in fresh grapes = 4 : 1
Ratio of Water : Matter in dried grapes = 1 : 4

$$\therefore \frac{16}{4} = \frac{4}{x}$$

$$\rightarrow x = 1$$

Hence, the total weight = $4 + 1 = 5$ kg.

3. After discount of 5000 the course fee = 25000.
After additional 10% discount, the course fee =
22500. Total discount = 7500.

$$\% \text{ Discount} = \frac{7500}{30000} \times 100 = 25\%$$

4. Let total marks be x

Passing marks = 35% of $x + 24 = 40\% \text{ of } x - 6$

$$x = 600$$

$$\text{Passing marks} = 40\% \text{ of } 600 - 6 = 240 - 6 = 234$$

5. Overtime paid per hour = $40 + 75\% \text{ of } 40 =$
Rs. 70

$$\text{Total income per day} = 40 \times 8 + 70 \times 4 = \text{Rs } 600$$

$$\text{Avg. income per hour} = 600/12 = \text{Rs } 50$$

6. Since, the discount is applied on the increased value, so, the value of discount is more than the value of price added. Hence, he sells his goods at a price lower than the initially fixed price.

7. Suppose the losing person gets 100 votes.

The winning person gets 120 votes. Total votes = 220

% Votes received by Winning person

$$= \frac{120}{220} \times 100 = 54.54\%$$

8. Let the price of petrol be Rs 100 per litre

Let his expenditure be = Rs 1000. Original consumption = 10 litre

New price = Rs 120

New expenditure be = Rs 1100

New Consumption = 9.16 l

% Decrease in consumption

$$= \frac{10 - 9.16}{10} \times 100 = 8.4\%$$

9. Let his pocket money be Rs. 100x

Money remaining after spending on food = $90x$

Money remaining after spending on transportation = $90x - 18x = 72x$

Money remaining after spending on clothing
 $= 72x - 18x = 54x = 270$

$$x = 5.$$

Money spent on clothing = $18x = \text{Rs } 90$

10. Let the current population be x

$$\begin{aligned} \text{Population after 3 years} &= x \left(1 + \frac{20}{100}\right)^3 \\ &= x \times 1.2^3 = 3456000 \\ \rightarrow x &= \frac{3456000}{1.2^3} = 20,00,000 \end{aligned}$$

11. Let the initial no. of bacteria be 100

After 3 months, no. of bacteria = 300

$$\begin{aligned} \text{So, } 300 &= 100 \left(1 + \frac{x}{100}\right)^3 \\ \rightarrow x &= 44\% \end{aligned}$$

12. Let the initial amount of vegetables be 100 kgs.

Vegetables thrown on 1st day = 30% of 50 kg = 15 kg.

Remaining vegetables = 35kg.

Vegetables thrown the next day = 40% of 35 kg = 14 kg

Total vegetables thrown = $15 + 14 = 29$ kgs. = 29%

13. Let the original sales be x .

According to the question,

$$1.08 \times 0.95 \times x = x + 5200 \rightarrow x = 200,000$$

$$\text{New Sales} = 2,00,000 + 5,200 = \text{Rs } 2,05,200$$

14. Let the price of the commodity be Rs 100

After 10% discount, price = Rs 90

$$\text{After 12% discount, price} = 90 \times 0.88 = 79.2$$

$$\text{Total Discount} = 20.8\%$$

15. 1/3rd of 75% is 25%

And 20% of 25% = 5%. So, a total of 30% of the class smokes.

16. 80% of the price = 840

$$100\% \text{ of the price} = \text{Rs } 1050$$

17. 20% of his salary = Rs 120

$$100\% \text{ of his salary} = \text{Rs } 600$$

$$\text{Increased salary} = \text{Rs } 720$$

18. If Ramu's salary is Rs 5, then Kallu's salary = Rs 6.4

This means that Ramu's salary =

$$\left(\frac{5}{6.4} \times 100 \right)\% = 78.2\% \text{ of Kallu's salary}$$

Hence, Ramu's salary is 21.8% less than that of Kallu's salary.

19. Let the original area be $l \times b$

$$\text{New area} = 1.08l \times 0.91b = 0.9828 l \times b$$

$$\text{Total \% change} = -9 + 8 - \frac{9 \times 8}{100} = -1.72$$

$$\% \text{ Decrease} = 1.72\%$$

20. Let the price be Rs 100

Consumption = 10 kgs

New Price = Rs 130

New Expenditure = Rs 1100

New Consumption = 8.461

% Change = 15.4%

21. Let CP be Rs 100

SP = Rs 120

$$\text{Margin} = \frac{20}{120} \times 100 = 16.66\%$$

22. If 10% increment = Rs 12000

Then 100% salary = 120,000

$$\frac{120000}{1.2} = 100000$$

Last year's salary =

Increment = Rs 20,000

23. Let the original revenue be Rs x

$$\text{As per question, } 1.08 \times 0.97 x = x + 14280$$

$$x = 3,00,000$$

Hence, new revenue = Rs 3,14,280

24. Percentage of boys in the class earlier =

$$\frac{50}{80} \times 100 = 62.5\%$$

Percentage of Boys in the class now = 66.66%

Increase in percentage points = 4.2%

25. Share of Marti in 2009 = 45% of 70,00,000

$$= 31,50,000$$

Share of Marti in 2010 = 56% of 90,00,000

$$= 50,40,000$$

$$\% \text{ Increase} = \frac{5040000 - 3150000}{3150000} \times 100 = 60\%$$

26. Let the percentage of men be x .

Percentage of women = $100 - x$

Percentage of men Chaman = $0.3x$

Percentage of women Chaman = $0.2 \times (100 - x)$

Total Chaman = $0.3x + 20 - 0.2x = 0.1x + 20$

$$\text{Percentage of Chaman} = \frac{0.1x + 20}{100} \times 100 \\ = 0.1x + 20\%$$

Since, x cannot be greater than 100 and cannot be less than 0, so, the possible value is 24.

27. Let the total weight of the diamonds stolen by the thief be 100gms.

Weight of heaviest two = 25g. So, each heavy diamond weighs = 12.5g.

Weight of the lightest 5 = 45g. So, each of the lightest diamond weighs = 9 g.

Weight of the 7 diamonds = 70g.

Since, the remaining diamonds weigh more than 9g and less than 12.5g, so, the total weight of the

remaining diamonds = 30g. So, only 3 diamonds can together weigh 30g in the range of 9g. and 12.5g

Hence, he stole 10 diamonds.

28. A total of 5000 gallons were added to 40000 gallon oil. So, their current reserve = 45000

Percentage of Crude oil added =

$$\frac{5000}{45000} \times 100 = 11.11\%$$

29. Let the original no. be x .

According to the question, $1.1x - 0.92x = 45$

$$\rightarrow x = \frac{45}{0.18} = 250$$

The correct answer = $1.1 \times 250 = 275$

30. Assume total number of votes polled both the years to be 100 and votes received by Ram Prakash in the first year to be x and next year to be $1.2x$ and votes received by the opposition are $100-x$ first year $100 - 1.2x$ in the second year . Hence

$$2 \times (x - (100 - x)) = 1.2x - (100 - 1.2x)$$

$$\Rightarrow 1.2x = 75\%$$

31. Let the no. of staff be 100

Fund raised by 75% staff = $75 \times 600 = 45000$

This amount is 60% of required funds.

$$\text{So, required funds} = \frac{45000}{60} \times 100 = 75,000$$

The amount to be paid by 25% = 30000

Avg. money to be paid = $30000/25 = \text{Rs } 1200$

32. Let Kallu's salary be Rs 100

40% of Kallu's salary = Rs 40

25% of Ramu's salary = $1.2 \times 40 = \text{Rs } 48$

Ramu's salary = $48 \times 4 = 192$

Ratio of Ramu and Kallu's salary = 192:100 = 48:25

33. Let the original price be Rs x per nut and bolt set.

New price = $1.2 \times 1.2 \times 1.5 \times 1.5 \times x \times 100 = 648$

$$x = 2$$

The profit of manufacturer = $2 \times 0.2 = 0.4 = 40$ paisa.

34. Let her monthly expenses be Rs 100

$$\begin{aligned} \text{New monthly expenses} &= (50 \times 1.1) + (30 \times 1.2) \\ &\quad + (20 \times 1.05) = 112 \end{aligned}$$

Percentage increase in monthly expenses = 12%

$$35. \frac{\text{Volume of cylinder earlier}}{\text{Volume of cylinder now}} = \frac{r^2 H}{(0.9r)^2 h} = 1$$

$$\frac{h}{H} = 1.23$$

Hence, the height of the new cylinder should be increased by 23%.

36. [Mention that the ratio of mixing the 2 pens is 1:2]

No. of pens he buys at Rs 10 for 3 be 12

Total cost for 12 pens = Rs 40

No. of pens he bought at the Rate of 13 for 4 = 24

Cost of 24 pens = 78

Total cost = $40 + 78 = 118$

Total Pens = 36

$$\text{Cost per dozen} = \frac{118}{36} \times 12 = 39.33$$

SP per dozen = Rs 59

$$\% \text{ Profit} = \frac{19.66}{39.33} \times 100 = 50\%$$

37. Let total rice be 3 kg.

$\frac{1}{3}$ of rice is lost at 10 % less so price of 1 kg = 1350

$$1350 - 135 = 1215$$

Total profit = 10%

$$\text{So total SP} = 1350 \times 3 \times \frac{110}{100} = 4455$$

$$\text{Now } 2 \times 1215 = 4450$$

$$x = 1620$$

$$\% \text{ increase} = \frac{1620 - 1350}{1350} \times 100$$

$$= 20\%$$

38. Let x be the profit $\frac{1}{3} \times -10 + \frac{2}{3} \times x = 10$.

Solving, $x = 20\%$

39. Earlier, 100 units work was done by people but now additional 80% i.e. 80 units will be done by labour which is 20% more productive additional labour = $80/1.2 = 66.66\%$

40. Assume, we have taken 1 Kg of both the alloys.

Hence, total copper in the final alloy will be $0.5+0.9=1.4$ Kg. And total weight of the final alloy will be $0.5+0.2+(0.4+0.3) 3 = 2.8$ kg.

Hence, percentage of copper will be

$$\frac{1.4}{2.8} \times 100 = 50\%$$

41. 50% boys + 40% girls = 50% (1100) + 40% (900) = $550 + 360 = 910$.

Failed candidates = $2000 - 910 = 1090$.

Required percentage = $1090/2000 = 54.5\%$.

42. Starting with 100, we get

Amount after first discount = $100 - 10 = 90$

Amount after second discount = $90 - 18 = 72$

Amount after third discount = $72 - 28.80 = 43.20$

Total discount = $100 - 43.20 = 56.80\%$.

$$43. \text{ Reduction} = \frac{100}{100+25} = \frac{4}{5}$$

Quantity after Reduction = 80%

so reduction = $100-80$

= 20%

$$44. \text{ Required salary} = 22702.68 \times \frac{100}{105} \times \frac{100}{97.5} = \text{Rs } 22176.$$

45. Let the selling price be

Rs. 100

Expenditure = 15% so total expense = 115

Profit = 10%

$$\text{S.P} = 115 \times \frac{110}{100} = 138$$

$$\text{Mark up price} = 138 - 100 = 38$$

Thus, mark-up 38%

46. Total No. of seats of executive class

$$= 500 \times \frac{10}{100} = 50$$

$$\text{Total No. of seats of chair car} = 500 - 50 = 450$$

$$\text{Total No. of booked seats} = 500 \times \frac{85}{100} = 425$$

$$\text{Total No. of executive class booked seats}$$

$$= 50 \times \frac{96}{100} = 48$$

$$\text{Total No. of empty chair car } 450 - (425 - 48) = 450 - 377 = 73.$$

47. $a\%$ of $x = b\%$ of y . Hence, $\frac{a}{100} \times x = \frac{b}{100} \times y$

$$\rightarrow y = \frac{ax}{b}$$

$$\text{Now, } c\% \text{ of } y = \frac{c}{100} \times y = \frac{c}{100} \times \frac{ax}{b}$$

$$= \frac{ac}{b \times 100} \times x = \frac{ac}{b} \% \text{ of } x.$$

48. 40% of Boys = 20

100% Boys = 50. Total students = $50 + 20 = 70$

49. Let Ram's salary = Rs x

$$\text{Amit's salary} = x \times \frac{80}{100} = \frac{4x}{5} \text{ Rs.}$$

$$\text{Ravi salary} = \frac{4x}{5} \times \frac{150}{100} = \frac{6x}{5} \text{ Rs.}$$

Ratio of Ram's salary to Ravi's salary =

$$\frac{\frac{x}{5}}{\frac{6x}{5}} = \frac{5x}{6x} = \frac{5}{6}$$

50. The answer is $\frac{x}{100} \times y = \frac{y}{100} \times x$

Hence, $x\%$ of y is $y\%$ of x .

Chapter 4

1. Let the amount invested in Bank 1 be Rs x

$$\text{Interest from Bank 1} = \frac{x \times 5 \times 3}{100} = \frac{15x}{100}$$

$$\text{Interest earned from Bank 2} = \frac{(8000-x) \times 9 \times 3}{100} = \frac{216000 - 27x}{100}$$

$$\text{Total Interest} = \frac{15x}{100} + \frac{216000 - 27x}{100} = 1440$$

$$\rightarrow x = 6000$$

Hence, amount invested at 5% per annum = Rs 6000

2. This is a case of Compound Interest

$$\text{Hence, } 1,00,000 = 50,000 \left(1 + \frac{20}{100}\right)^x$$

$$\rightarrow 2 = (1.2)^x$$

$$\rightarrow x < 4$$

Hence, the treatment can be started before 4 years

3. Let the principal be Rs x

$$SI = \frac{x \times 3 \times 5}{100} = \frac{15x}{100}$$

$$CI = x \times \left(1 + \frac{5}{100}\right)^3 - x$$

$$CI - SI = (1.05)^3 x - x - 0.15x = 305$$

$$\rightarrow x = 40000$$

4. For interest payable half yearly,

$$A = P \left(1 + \frac{r}{2 \times 100}\right)^{2 \times n}$$

$$= 2000 \left(1 + \frac{10}{2 \times 100}\right)^{2 \times 2} = 2431$$

Interest = Rs 431

5. Let the amount invested be Rs p

$$\frac{SI \text{ for 3 years}}{SI \text{ for 2 years}} = \frac{p \times r_1 \times 3}{p \times r_2 \times 2} = \frac{3000}{2250}$$

$$\frac{r_1}{r_2} = \frac{4 \times 2}{3 \times 3} = \frac{8}{9}$$

$$\text{Hence, } \frac{p \times 8 \times 3}{100} = 3000$$

$$\rightarrow p = 12500$$

6. SI for 3 years for an amount x at 10%

$$= \frac{x \times 3 \times 10}{100} = 0.3x$$

CI for 3 years for an amount x at 8%

$$= \frac{x \times \left(1 + \frac{8}{100}\right)^3}{100} - x = 0.26x$$

Hence, the option of investing in SI is a good option.

7.

$$\frac{SI \text{ for 2 years}}{CI \text{ for 2 years}} = \frac{p \times r \times 2}{p \left(\left(1 + \frac{r}{100}\right)^2 - 1\right)} = \frac{20}{21}$$

on solving the expression we will get $r = 10\%$
another way is to going through options in the above expression.

8. Amount after 5 years = 1000

$$\left[\left(1 + \frac{10}{100}\right)^5 + \left(1 + \frac{10}{100}\right)^4 + \left(1 + \frac{10}{100}\right)^3 + \left(1 + \frac{10}{100}\right)^2 + \left(1 + \frac{10}{100}\right)^1 \right] = 67157$$

9. $P = \text{Rs } 50,000, R = 2.5\%, T = 5$

$$SI = \frac{50000 \times 2.5 \times 5}{100} = 6250$$

Amount = 56250

Monthly Instalments = $56250/5 = \text{Rs } 11050$

10. SI for 2 years for an amount x at 8%

$$= \frac{x \times 2 \times 8}{100} = 0.16x$$

CI for 2 years for an amount x at 8%

$$= x \times \left(1 + \frac{8}{100}\right)^2 - x = 0.1664x$$

Difference = $0.0064x = 8000$

$$x = 12,50,000$$

11. Let the value of machine be Rs x

$$\text{Value after 3 years} = x - \left(\frac{1-10}{100} \right)^3 = .729x$$

$$\rightarrow x = 15000$$

$$12. \text{ SI after 4 years} = \frac{x \times r \times 4}{100} = 3x \rightarrow r = \frac{300}{4}$$

$$\text{SI after } y \text{ years} = \frac{x \times \frac{300}{4} \times y}{100} = 15x \rightarrow y = 20 \text{ yrs.}$$

So, after 20 years, his money will become 16 times.

13. Amount at the end of 3 years

$$= x \times \left(1 + \frac{r}{100} \right)^3 = 5750 \quad \dots(1)$$

Amount at the end of 6 years

$$= x \times \left(1 + \frac{r}{100} \right)^6 = 6612.5 \quad \dots(2)$$

On dividing eqn. (1) by (2), we get,

$$= \frac{1}{\left(1 + \frac{r}{100} \right)^3} = \frac{5750}{6612.5} \quad \dots(3)$$

Put the value of (3) in (1)

So, the amount invested = Rs 5000

14. After 6 years, amount = $x \times \left(1 + \frac{r}{100} \right)^6 = 3x$

$$\rightarrow \left(1 + \frac{r}{100} \right)^6 = 3^{\left(\frac{1}{6} \right)}$$

$$\text{After } y \text{ years, amount} = x \times \left(1 + \frac{r}{100} \right)^y = 27x$$

$$\rightarrow 3^{\left(\frac{y}{6} \right)} = 27 \rightarrow y = 18 \text{ years}$$

15. This is a case of Compound Interest.

$$\text{Amount after } n \text{ minutes} = x \times \left(1 - \frac{10}{100} \right)^y \leq 0.6x$$

$$\rightarrow y \leq 5 \text{ min}$$

16. Let the time be t years

$$\text{After } t \text{ years, Cost of land} = 8100000 \times \left(1 + \frac{10}{100} \right)^t$$

After t years, Cost of Building

$$= 12100000 \times \left(1 - \frac{10}{100} \right)^t$$

As per question,

$$8100000 \times \left(1 + \frac{10}{100} \right)^t = 12100000 \times \left(1 - \frac{10}{100} \right)^t$$

$$\rightarrow t = 2 \text{ years}$$

17. Amount after 9 months

$$= 20000 \times \left(1 + \frac{20}{4 \times 100} \right)^{4 \times \frac{9}{12}} = 23152$$

Hence, the amount of interest = Rs 3152

18. The amount received after 5 years will be

$$10000 \times \left(1 + \frac{5}{100} \right) \times \left(1 + \frac{10}{100} \right)^2 \times \left(1 + \frac{20}{100} \right)^2$$

$$= 10000 \times 1.05 \times 1.21 \times 1.44 = \text{Rs } 18295.2$$

19. Let the population two years ago be x

$$\text{Current population} = x \times \left(1 + \frac{15}{100} \right)^2 = 2909500$$

$$\rightarrow x = 220000$$

20. Let the money borrowed from Kallu be Rs x

$$\text{Interest to be paid to Kallu} = \frac{x \times 5 \times 3}{100} = 0.15x$$

Interest to be paid to Lallu

$$= \frac{(10000-x) \times 8 \times 2}{100} = 1600 - 0.16x$$

$$\text{Total Interest Paid} = 0.15x + 1600 - 0.16x = 1560$$

$$\rightarrow 0.01x = 40 \rightarrow x = 4000$$

$$21. \text{ Cost of the machine after 2 years} = 8000 \times \left(1 - \frac{8}{100} \right)^2$$

$$= 8000 \times 0.8484 = \text{Rs } 6772$$

22. Let $x\%$ of population be male.

Population of males after 2 years

$$= x \times \left(1 + \frac{5}{100} \right)^2 = 1.1025x$$

Population of females after 2 years

$$= (150000 - x) \times \left(1 + \frac{10}{100} \right)^2 = 181500 - 1.21x$$

$$\text{Total Population} = 1.1025x + 181500 - 1.21x$$

$$= 175050$$

$$\rightarrow x = 60000$$

$$\text{Hence, \% of Male Population} = \frac{60000}{150000} \times 100 = 40\%$$

23. Let Principal be Rs x

On applying CI, amount after 2 years

$$= x \left(1 + \frac{10}{100} \right)^2 - x = 1680$$

$$1.21x - x = 1680$$

$$x = \frac{1680}{.21}$$

$$x = 8000$$

$$SI = \frac{8000 \times 10 \times 2}{100} = 1600$$

$$24. S.I. \text{ for one year} = \frac{200}{4} = 50$$

In 8 years the principal is tripled

$$\Rightarrow S.I. = 3P - P = 2P$$

$$\Rightarrow 2P = 8 \times 50$$

$$P = 200$$

$$\text{Interest} = 2P = 2 \times 200 = 400$$

25. If the sum is invested at 3% p. a. for 3 years then it would have fetched an interest of Rs 90.

$$\Rightarrow 90 = \frac{P \times 3 \times 3}{100}$$

$$= \frac{90 \times 100}{9} = 1000.$$

$$26. S.I. \text{ in the years} = \frac{960-800}{4} = \frac{160}{4} = 40$$

$$\text{Rate of interest} = \frac{40}{800} \times 100 = 5\% \text{ p.a.}$$

If interest is charged at 9%

$$S.I. = \frac{800 \times 9 \times 4}{100} = 288$$

$$\text{Amount} = 800 + 288 = 1088$$

$$27. S.I. = \frac{P \times r \times 10}{100}$$

$$\Rightarrow r = \frac{100}{10} = 10\% \text{ p.a.}$$

$$28. \text{Amount} = \frac{9}{4}P$$

$\Rightarrow S.I. = \text{Amount} - \text{Principal}$

$$= \frac{9}{4}P - P = \frac{5P}{4}$$

$$\Rightarrow \frac{5P}{4} = \frac{P \times r \times 5}{100}$$

$$r = \frac{100}{4} = 25\%$$

29. Let the principal be 'P'

Then Amount after 16 years = 4P

$$\Rightarrow S.I. = 4P - P = 3P = \frac{P \times r \times 16}{100}$$

$$\Rightarrow r = \frac{3 \times 100}{16} = \frac{75}{4}\%.$$

In the second case the amount becomes 16P

$$S.I. = 16P - P = 15P$$

$$15P = P \times \frac{75}{4} \times \frac{t}{100}$$

$$t = \frac{15 \times 100 \times 4}{75} = 80 \text{ years}$$

30. Net rate of increase in population

= birth rate - death rate

$$= (6 - 4)\%$$

$$= 2\%.$$

$$\text{Population after 2 years} = 300000 \left(1 + \frac{2}{100}\right)^2$$

$$= 300000 \times \frac{102 \times 102}{100 \times 100}$$

$$= 30 \times 102 \times 102 = 312120$$

$$31. A = 12800 \left(1 + \frac{10}{100 \times 4}\right)^{1 \times 4}$$

$$= 12800 \left(\frac{41}{40}\right)^4$$

$$= \frac{12800 \times 41 \times 41 \times 41 \times 41}{40 \times 40 \times 40 \times 40}$$

$$\Rightarrow \text{Interest} = \text{Amount} - \text{Principal} = 13288$$

$$32. \text{Population 3 year ago} \times \left(1 + \frac{5}{100}\right)^3 = 92610000$$

$$\text{Population 3 years ago} \times \left(\frac{21}{20}\right)^3 = 92610000$$

$$\text{Population 3 years ago} = \frac{20 \times 20 \times 20}{21 \times 21 \times 21} \times 9261000$$

$$= 8000 \times 10000 = 8 \times 10^7$$

33. Principal = P, Time = n years, Rate percent = n,

$$\text{Then } S.I. = \frac{P \times R \times T}{100}$$

$$\Rightarrow \frac{P}{4} = \frac{P \times n \times n}{100}$$

$$\Rightarrow n^2 = \frac{100}{4} = 25$$

$$\Rightarrow n = \sqrt{25} = 5$$

34. Let the rate of interest be r.

$$\Rightarrow \frac{1200 \times r \times 2}{100} + \frac{1500 \times r \times 4}{100} = 420$$

$$\Rightarrow 24r + 60r = 420$$

$$\Rightarrow 84r = 420$$

$$\Rightarrow r = \frac{420}{84} = 5$$

35. \Rightarrow Interest for 1st years $= \frac{80}{2} = 40$

$$\Rightarrow$$
 Interest on Rs 40 for 1 year $= 80.40 - 80 = 0.4$

$$\Rightarrow r = \frac{40 \times r \times 1}{100} = 0.4$$

$$r = \frac{0.4 \times 10}{4} = 1\% \text{ per annum.}$$

$$\text{And also } \frac{P \times 1 \times 1}{100} = 40 \Rightarrow P = 4000$$

36. $S.I. = \frac{400 \times 5 \times 4}{100} = 80$

\Rightarrow Amount = Principal + S.I.

$$\Rightarrow 400 + 80 = 480$$

37. Let the principal be P.

Then the amount becomes 3P.

$$S.I. = 3P - P = 2P = \frac{P \times 20 \times T}{100}$$

$$T = \frac{2P \times 100}{P \times 20} = 10 \text{ years}$$

38. Let the value 3 years ago be x.

$$\Rightarrow x \times \left(1 - \frac{15}{100}\right)^3 = 4913$$

$$\Rightarrow x \left(\frac{85}{100}\right)^3 = 4913$$

$$\Rightarrow x = \frac{4913 \times 20 \times 20 \times 20}{17 \times 17 \times 17} = 800$$

39. Interest per annum $= \frac{240}{3} = 80$.

$$\Rightarrow$$
 Interest in 2 years $= 80 \times 2 = 160$

$$\Rightarrow$$
 Principal $= 960 - 160 = 800$

$$\text{And } = \frac{800 \times r \times 2}{100} = 160$$

$$\Rightarrow r = \frac{160 \times 100}{800 \times 2} = 10\%$$

40. Let the principal be = P

$$\Rightarrow$$
 Amount $= 4P$

$$\Rightarrow$$
 Interest $= 4P - P = 3P$

$$\Rightarrow 3P = \frac{P \times r \times 12}{100}$$

$$\Rightarrow r = \frac{3P \times 100}{P \times 12} = 25\%$$

41. Total interest $= \frac{1600 \times 8 \times 4}{100} + \frac{1600 \times 6 \times 3}{100}$

$$= 512 \times 288 = 800$$

$$42. A = P \left(1 + \frac{r}{100}\right)^n$$

$$= 4800 \times \left(1 + \frac{5}{100}\right)^3 = \frac{4800 \times 21 \times 21 \times 21}{20 \times 20 \times 20}$$

$$= \frac{6}{10} \times 21 \times 21 \times 21 = \frac{55566}{10} = 5556.6$$

$$43. SI = \frac{4000 \times 12 \times 2}{100} = 40 \times 24 = 960.$$

$$CI = 4000 \left(1 + \frac{10}{100}\right)^2 - 4000 = 4000 \left[\frac{121}{100} - 1\right]$$

$$= \frac{4000 \times 21}{100} = 40 \times 21 = 840$$

So the difference is of $960 - 840 = 120$

44. Let the principal be P

$$\Rightarrow P \left(1 + \frac{20}{100}\right)^2 - P = 880$$

$$\Rightarrow P \left[\left(\frac{6}{5}\right)^2 - 1\right] = 880$$

$$\Rightarrow P = \frac{880 \times 25}{11} = 80 \times 25 = 2000$$

45. Amit's investment = 6000 for 12 months and 12000 for 6 months.

Nitin's investment = 8000 for 12 months.

Ratio of profit sharing Amit and Nitin will be:-

$$= \frac{6000 \times 12 + 12000 \times 6}{8000 \times 12}$$

$$= \frac{144000}{96000} = \frac{3}{2}$$

46. Let the principal be x and y.

$$\Rightarrow \frac{5 \times x \times 6}{100} = \frac{2 \times y \times 8}{100}$$

$$\Rightarrow 30x = 16y$$

$$\Rightarrow 15x = 8y$$

and also $x + y = 4600$.

$$\Rightarrow x + \frac{15x}{8} = 4600$$

$$\Rightarrow \frac{23}{8}x = 4600 = 4600$$

$$x = \frac{4600 \times 8}{23} = 200 \times 8 = 1600$$

47. S.I. accrued = $1500 - 1200 = 300$.

$$\text{Interest for 1 year} = \frac{300}{5} = 60$$

For 9 years simple interest = $60 \times 9 = 540$

Amount after 9 years = $1200 + 540 = 1740$

$$48. A = 1500 \left(1 + \frac{10}{100}\right)^2 \times \left(1 + \frac{\frac{3}{4} \times 10}{100}\right)$$

$$= 1500 \times \frac{11}{10} \times \frac{11}{10} \times \frac{43}{40} = 1951.125$$

$$49. \text{Depreciation} = P \left(1 - \frac{r}{100}\right)^n$$

$$\text{Value of car after 3 years} = 64000 \left(1 - \frac{5}{100}\right)^3$$

$$= 64000 \times \frac{19}{20} \times \frac{19}{20} \times \frac{19}{20} = 54872$$

50. Amount after one year

$$= 5000 + 5000 \times \frac{5}{100} \times \frac{80}{100}$$

= Rs 5200

Amount after two years

$$= 5200 + 5200 \times \frac{5}{100} \times \frac{80}{100}$$

= Rs 5408

Amount after three years

$$= 5408 + 5408 \times \frac{5}{100} \times \frac{80}{100}$$

= Rs 5624.32

Chapter 5

1. Let the selling price of each article be Rs. 10

Selling price of 8 articles = Rs. 80

Cost Price of 8 articles = Rs. 100

Loss = Rs. 20

$$\% \text{ Loss} = \frac{20}{100} \times 100 = 20\%$$

2. Let the cost price of one article be Rs. 10

C.P. of 21 articles = 210

Profit = 70

$$\% \text{ Profit} = \frac{70}{210} \times 100 = 33.33\%$$

3. Let the selling price of one article be Rs. 10

S.P. of 21 articles = 210

Gain = 70

C.P. of 21 articles = 140

$$\% \text{ Profit} = \frac{70}{140} \times 100 = 50\%$$

4. S.P. of each horse = 19200

C.P. of 1st horse = $19200/1.2 = 16000$

C.P. of 2nd horse = $19200/0.8 = 24000$

Total C.P. = 40000

Total S.P. = 38400

Loss = 1600

$$\% \text{ Loss} = \frac{1600}{40000} \times 100 = 4\%$$

5. By selling the article for Rs. 100 more the change in percentage is 5%.

So, 5% of the price of the article = 100

100% of the price of article = $100 \times 20 = \text{Rs. } 2000$

S.P. = $2000 \times 0.97 = \text{Rs. } 1940$

6. S.P. = Rs. 30

Loss = C.P. - 30

$$\% \text{ Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{\text{C.P.} - 30}{\text{C.P.}} \times 100 = 10$$

C.P. = 33.33

Now, Gain = SP - CP = SP - 33.33

% Gain =

S.P. = 40

Hence, by selling the bananas at Rs. 40 per dozen, he'll gain 20% profit.

7. Let the C.P. be Rs. 100 per kg.

C.P. of 900g = Rs. 90

S.P. of 900g = Rs. 100

Profit = Rs. 10

$$\% \text{ Profit} = \frac{10}{90} \times 11.11\%$$

8. Let the CP be x .

Loss = $x - 8500$

Gain = $9600 - x$

As per question, $9600 - x = 1.2(x - 8500)$

$$x = 9000$$

9. Buy 4 get 1 free means get 5 at the cost of 4.

Let MP of each be Rs. 100

Total MP = Rs. 500

SP = 400

Discount = 100

$$\% \text{ Discount} = \frac{100}{500} \times 100 = 20\%$$

10. He bought 50 shares of Rs. 90 each. Sold them at Rs. 120 each with a profit of Rs. 30

So total profit = $50 \times 30 = 1500$

$$\% \text{ Profit} = \frac{1500}{4500} \times 100 = 33.33\%$$

11. Let the CP be Rs. 100 per kg.

C.P. of 900g = Rs. 90

S.P. of 900g = Rs. 110

Profit = 20

$$\% \text{ Profit} = \frac{20}{90} \times 100 = 22.22\%$$

12. Let the CP of milk be Rs. 100 per litre

He sells 800ml milk and 200ml water at Rs. 110.

C.P. of 800ml = Rs. 80

S.P. of 800ml = Rs. 110

Profit = Rs. 30

$$\% \text{ Profit} = \frac{30}{80} \times 100 = 37.5\%$$

13. He is giving a discount of 25% on garments.

Let the CP be Rs. 100

MP = Rs. 200

S.P. = 150

Profit = 50

$$\% \text{ Profit} = \frac{50}{100} \times 100 = 50\%$$

14. Let CP be Rs. x

$$SP = 1.2x$$

As per question, New CP = $x - 50$

$$\text{New SP} = 1.32x$$

$$\text{Profit} = 1.32x - (x - 50) = 0.32x + 50$$

$$\% \text{ Profit} = \frac{0.32x + 50}{x - 50} \times 100 = 50$$

$x = 416.66$. Hence, option (b)

15. Let the MP be Rs. 100

After three successive discounts of 10%, the SP = 72.9

After three successive discounts of 20%, 5% and 5%, SP = 72.2

Hence, the 2nd shopkeeper is giving more discount.

16. CP of 100 eggs = Rs. 50

He wants to make a profit of 20%.

Then, SP = Rs. 60

Since, he gave a discount of 33.33% in the MP.

Then, $MP \times 0.66 = SP$

MP = 90

MP of 90 eggs is Rs. 90.

MP of 1 egg = Re. 1 per egg.

17. CP of 1kg Mangoes = 32/7

SP of 1kg Mangoes = 32/5

$$\text{Profit on 1kg Mangoes is } \frac{32}{5} - \frac{32}{7} = \frac{64}{35}$$

No. of Mangoes sold if Profit is 160

$$= 160 \times \frac{35}{64} = 87.5 \text{ kg}$$

18. Let CP be Rs. 100 each of 10 articles

MP = Rs. 120 each

SP of 5 articles = Rs. 108 each

SP of other 5 articles = Rs. 120 each

$$\text{Total SP} = 540 + 600 = 1140$$

$$\text{Total CP} = 1000$$

$$\% \text{ Profit} = \frac{140}{1000} \times 100 = 14\%$$

19. Let the original CP be Rs. 100

Original SP = Rs. 120

New CP = 90

New SP = 132

$$\% \text{ Gain} = \frac{132 - 90}{90} \times 100 = 46.66\%$$

20. CP = Rs. 30

SP = Rs. 36

$$MP \times 0.9 = SP$$

$$MP = \frac{36}{0.9} = 40 \text{ per kg}$$

Solutions for questions 21 and 22.

As per given information, $2600 + py + qy^2$

$$2600 + 20p + 400q - (2600 + 10p + 100q) = 25\% \text{ of } (2600 + 10p + 100q)$$

$$\rightarrow 3p + 110q = 260 \dots (1)$$

$$2600 + 30p + 900q - (2600 + 20p + 400q) = 25\% \text{ of } (2600 + 20p + 400q)$$

$$\rightarrow p + 80q = 130 \dots (2)$$

From (1) and (2), we get that

$$p = 50 \text{ and } q = 1$$

Hence, the equation becomes $\rightarrow 2600 + 50y + y^2$

21. Let us check the answer with the help of options
If $y = 120$, then CP = 23,000, SP = 36,000 and profits = 13,000
If $y = 125$, then CP = 24,425, SP = 37,500 and profits = 13,025
If $y = 130$, then CP = 26,000, SP = 39,000 and profits = 13,000
If $y = 140$, then CP = 29,200, SP = 42,000 and profits = 12,800
Hence, option (a) is the right choice.

22. Maximum possible profits were Rs 13,025
23. Let the profit of Ajay, Navjot and Vikas be $2x$, $3x$ and $3x$ respectively.

Difference in shares of profit of Ajay and Navjot
 $= 3x - 2x = 500$
 $\therefore x = \text{Rs. } 500$

Now difference in shares of Ajay and Vikas $= 6x - 2x$
 $= 4x = 4 \times 500 = \text{Rs. } 2000$

24. CP of 120 litres of mixture = Rs 1000
SP of 120 litres of mixture = Rs. 1200

He gains Rs. $\frac{20}{12}$ per litre for diluted milk sold
Now suppose Ankit sells x litres of diluted milk
He gains Rs $\frac{20}{12}x$
After paying the fine of Rs 200, loss = Rs 100
So before paying the fine, profit = Rs 100

$$\frac{20}{12}x = 100$$

$$x = 60$$

25. Cost price of $(3 + 4 + 5) = 12\text{kg}$ of fruits
 $= \text{Rs. } (300 + 320 + 300) = \text{Rs. } 920$
S.P at a profit of 50% = Rs. 1380
S.P of fruits per kg $= \frac{1380}{12}$ Rs. 115.

26. Let price of 100 cm of cloth be Rs. 100. The merchant gets 120 cm of cloth for Rs 100. Hence,

his actual cost for 1 cm $= \frac{10}{12}$

Now, instead of selling 100 cm, by cheating he sells 80 cm of cloth at the cost price of 120 cm of cloth $= \frac{5}{6} \times 80 = \text{Rs. } 66.66$

Selling price of 80 cm. of cloth is actually 100 cm for the buyer, at a discount of 20%
 $= 100 \times 0.8 = \text{Rs. } 80$

$$\text{Profit \%} = \frac{80 - 66.66}{66.66} \times 100 = 20.01\%$$

or 20% approximately.

27. Let the total profit be Rs. x .

$$\text{Distributable profit} = \text{Rs. } \frac{3x}{5} (\text{60\% of } x)$$

Now ratio of investments

$$= 10000 : 80000 = 5:4$$

One partner gets $\frac{5}{9} \times \frac{3x}{5}$ i.e. $\text{Rs. } \frac{x}{3}$

And the other partner gets $\frac{4}{9} \times \frac{3x}{5}$ i.e. $\frac{4x}{15}$

$$\text{Difference} = \frac{x}{15} = 1200 \Rightarrow x = 18000$$

28. Suppose Shamvav invested Rs. $5x$ and Tanya invested Rs $6x$ after 3 months.

Shamvav's share for entire period $= 4x + 9 + 5x \times 3 = 51$

Shamvav withdrew Rs x and Tanya withdrew Rs x
 \therefore Tongais capital for the entire period.

$$\therefore 5x \times 9 + 6x \times 3 = 63x$$

$$\therefore \text{Tanya share } \frac{63x}{114x} \times 22800 = 12600$$

29. Let the CP of the articles be Rs x , since he earns a profit of 20%, hence SP = $1.2x$. It is given that he incurs loss by selling 16 articles at the cost

of 12 articles $\left(\text{loss} = \frac{16-12}{16} \times 100 = 25\% \right)$

\therefore His selling price = $\text{SP} \times 0.75$

$$\text{Now } \text{SP} \times 0.75 = 1.2x$$

$$\Rightarrow SP = \frac{1.2}{0.75} x = 1.6x$$

This SP is arrived after giving a discount of 20% on MP

Hence, MP = $\frac{1.6x}{0.8} = 2x$. It means that the article has been marked 100% above the cost price.

30. Sales of 3000 (2/3 of 4500) copies for Rs 160 and 1500 copies for Rs 200 each. So the selling price = $3000 \times 160 + 1500 \times 200 = 7,80,000$

Hence profit = 7,80,000 - 5,00,000 = 280,000

Profit % = $2,80,000 / 5,00,000 \times 100 = 56\%$

Hence the answer is (a).

31. Let the CP of 1 kg of sugar be Rs. 100

$$\text{Then CP of 900g of sugar} = \frac{100}{100} \times 900 = \text{Rs. 90}$$

$$\text{Hence, profit \% in case II} = \frac{100 - 90}{90} \times 100 = 11.11\%$$

$$\text{If he adds 10\% impurity then his CP for 1 Kg} = \frac{100}{1100} \times 1000 = \text{Rs. 90.90}$$

Hence, Profit% in case III

$$= \frac{100 - 90.90}{90.90} \times 100 = 10.01\%$$

If he reduces weight by 5%

$$\text{Then cost price of 950g.} = \frac{100}{1000} \times 950 = \text{Rs. 95 and SP} = \text{Rs}105$$

Hence, Profit % in case IV

Hence, method II maximizes his Profit.

32. If initial cost price is 100, then initial selling price will be 120. then, the new cost price is 90, while the new selling price is 132

$$\text{New profit \%} = \frac{132 - 90}{90} \times 100 = 46.66\%$$

33. C.P. of one water melon = Rs. 60

In order to get profit of 10% fruit seller should sell it at Rs. 66

Now 25% of the content evaporates, so net weight = 3kg

So per kg it would be Rs. 22

35. Let CP be Rs.x

$$MP = 1.8x$$

$$\text{SP of 15 articles} = 12 \times 1.8x = 21.6x$$

$$\text{Discount of } 0.9 \times 21.6x = 19.44x$$

$$\text{Profit on 15 articles} = (19.44 - 15)x = 4.44x$$

$$\text{Profit \%} = \frac{4.44x}{15x} \times 100 = 29.6\%$$

36. Let the first discount be x %

$$\frac{(100 - 20)}{100} \times \frac{100 - x}{100} \times 2500 = 1500$$

$$\frac{80}{100} \times \frac{100 - x}{100} \times 2500 = 1500$$

$$100 - x = \frac{1500 \times 100 \times 100}{80 \times 2500}$$

$$100 - x = 75$$

$$x = 25\%$$

37. While buying he buys 1100gm instead of 1000 grams (due to his cheating).

Suppose he bought 1100 gm for Rs 1000.

While selling:

he sells only 900 gm. while he takes the money for 1 Kg.

Now according to the problem he sells at 8% profit (20% mark up and 10% discount).

Hence his selling price is Rs 1080 for 900 gram.

To calculate profit percentage, we either equate the goods or the money.

In this case, let us equate the money as follows.

Buying;

1100 gram for Rs1000

Hence: 1188 grams for Rs1080

Selling: 900 grams for Rs1080

Hence, profit% = $288/900 = 32\%$

(Using goods left by goods sold formula)

38. If CP = x, the marked price = x + 205

and Selling Price = $0.9x + 184.5$

$$\text{Profit} = \frac{184.5 - 0.1x}{x} \times 100$$

$$\text{Profit \%} = \frac{1845 - 10x}{x}$$

39. Let CP of 5 dozen mangoes be Rs x. SP = Rs 156 and gain = 0.3x

$$\therefore 156 - x = 0.3x$$

$$\therefore x = 120$$

$$\therefore \text{SP of 60 mangoes} = 120 \times 1.6 = \text{Rs} 192$$

$$\therefore \text{SP per mango} = 3.2$$

40. Let there be n number of balls in each packet.

$$\therefore 6n \times 8 = \text{CP} + 96$$

$$48n = \text{CP} + 96$$

$$\text{Also, } 6(n - 1) \times 9 = \text{CP} + 90$$

$$54n = \text{CP} + 144 \quad \dots\text{(ii)}$$

Solving (i) & (ii), we get, n = 8

$$\therefore \text{CP} = 48 \times 8 - 96 = 384 - 96 = \text{Rs} 288$$

Each packet cost = Rs 48.

41. Let CP at A be Rs. x

$$\therefore \text{CP at B} = \text{Rs } 0.9x$$

$$\therefore 0.9x + 15 + 10 = x$$

$$0.1x = 25$$

$$x = 250$$

$$\therefore \text{Price at B} = 250 \times 0.9 = \text{Rs. } 225$$

42. Let the cost price of 1 article be Rs 1

cost of 10 articles is Rs 10

$$\text{Selling price of 10 articles} = \frac{10 \times 10}{9} = \text{Rs } \frac{100}{9}$$

$$\text{Gain on Rs } 10 = \text{Rs } \frac{100}{9} - \text{Rs } 10 = \text{Rs } \frac{10}{9}$$

$$\text{Gain on Rs } 100 = \text{Rs } \frac{100}{9} = \text{Rs } 11\frac{1}{9}$$

Profit percent it $11\frac{1}{9}\%$

43. Let the cost price of 10 articles and selling price of 8 articles be Rs. 100

C.P. of 1 article = Rs. 10

S.P. of 1 article = Rs. 12.5

Profit = Rs. 2.5

Profit \% = 25%

44. Aryan's total discount =

8% on 8000 = Rs. 640

5% on 12000 = Rs. 600

3% on 16000 = Rs. 480

Total discount = Rs. 1720 on Rs. 36000

Suman's total discount:

7% on 12000 = 840

6% on 8000 = 480

5% on 16000 = 800

Total Discount = Rs. 2120 on Rs. 36000

Hence, ratio of discounts = 43 : 53

45. Consumption before maintenance = 4 X 30 X 5 = Rs. 600

Consumption after maintenance = 3 X 30 X 5 = Rs. 450

Total profit = Rs. 150

Profit% = 25%

46. Assume cost of 1 kg of tea and 1 kg of sugar to be x and y respectively. Hence

$$x + 4y = 6.28$$

$$1.5x + 4.4y = 7.42$$

Hence x = 1.28 and y = 1.25

47. Cost price of one-third of goods

$$= \frac{1}{3}(120) = \text{Rs. } 40$$

Selling price of these goods at 5% loss = (40)(0.95) = Rs 38

Let selling price of the rest of the goods be x.

5% profit on Rs. 120 gives S.P as Rs. 126

$$\Rightarrow 126 = x + 38 \Rightarrow x = \text{Rs} 126$$

As the cost price of the remaining good is Rs 80,

$$\text{required profit\%} = \frac{88 - 80}{80} \times 100 = 10\%$$

48. Let the cost price be Rs 100. then marked price = Rs 115

Since a discount of 20% is allowed on Rs 115,

$$\text{Selling Price} = (80\% \text{ of } 100) \times 115 = \text{Rs. } 92$$

$$\text{Loss\%} = \frac{100 - 92}{100} \times 100 = 8\%$$

49. Let initial price be Rs.100

Increase of 30% would have made it 1.3×100

i.e. Rs130

Reduction of 15% would make it $0.85 \times 130 =$
Rs. 110.5

$$\therefore \text{Net effective increase} = \frac{110.5 - 100}{100} \times 100 = 10.5\%$$

50. Cost Price per share was Rs 94 and Selling Price per share was Rs 106

Hence, profit per share was Rs 12 and profit percentage will be $\frac{12}{94} \times 100 = 12.75\%$

IAS ACADEMY
CHRONICLE

Chapter 6

1. Let the nos. be $5x$ and $7x$

$$\text{Sum} = 12x = 108$$

$$\text{Hence, } x = 9$$

$$\text{Difference} = 2x = 18$$

2. Their contributions are in the ratio $20 : 35 : 54$.

Raju's contribution = $35/109$ of 218000 = Rs. 70,000

$$3. \frac{a}{b} = \frac{4}{5}, \quad \frac{b}{c} = \frac{3}{4}$$

Now making b similar in both the ratios,

$$\frac{a}{b} = \frac{3 \times 5}{3 \times 5} \& \frac{4b}{c} = \frac{5 \times 3}{5 \times 4}$$

$$\text{Hence, } a:b:c = 12:15:20$$

$$4. a:b = 4:7 = 4k:7k$$

$$a = 4k, b = 7k$$

$$b - a = 3k$$

$$\% \text{ More} = \frac{3k}{4k} \times 100 = 75\%$$

$$5. \frac{n}{4} = \frac{9}{n} \rightarrow n^2 = 36, \quad \rightarrow n = 6$$

6. The sum of the ratio should be a factor of 24. All the options except [c] is a factor of 24. Hence, [c] cannot be the ratio.

7. Let the price be Rs. 100/kg and consumption be 10kg.

$$\text{Total Exp} = \text{Rs. } 1000$$

$$\text{New Price} = \text{Rs. } 125$$

$$\text{New consumption} = 1000/125 = 8 \text{ kg.}$$

So, the consumption decreased by 20%.

8. Total Expenditure when 100 boys are coming = 10000

Total expenditure when 80 boys are coming = 96000

When 120 students are coming, the expenditure will rise. Going through the options, only the option [b] increases the total expenditure. Hence, [b]

9. Suppose Ramu scored x marks, then Kallu scored $= 2x$ marks.

If Ramu scored = $x + 3$, and Kallu scored $= 2x + 3$

$$\text{Then, ratio} = \frac{x+3}{2x+3} = \frac{3}{5}$$

$$\rightarrow x = 6. \text{ Hence, Ramu scored 6 marks.}$$

$$10. \text{ Let } a:b = 3:5 = k$$

$$\text{Then, } a = 3k, b = 5k$$

$$\frac{3a+4b}{4b-3a} = \frac{9k+20k}{20k-9k} = \frac{29}{11}$$

$$11. \text{ If } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \dots = \frac{y}{z} = \frac{1}{2}$$

$$\frac{a+b+c+\dots+y}{b+c+d+\dots+z} = \frac{1}{2}$$

$$12. \text{ Amount of Pulp in 1 kg Juice} = 200\text{g}$$

$$\text{Amount of Pure Juice} = 800\text{g}$$

$$\text{Ratio of Pulp to Juice} = 1:4$$

To increase the concentration to 25%, the ratio = 1:3

$$\frac{x}{800} = \frac{1}{3}, \quad \rightarrow x = 266.66$$

So, the amount of pure pulp to be added = 66.66g

13. Let P_1 be the price of Original diamond and P_2 be the price of one piece of broken diamond.

$$\frac{P_1}{P_2} = \frac{W_1^2}{W_2^2} \rightarrow \frac{5000}{P_2} = \frac{x^2}{\left(\frac{x}{2}\right)^2}$$

$$\frac{P_1}{P_2} = \frac{W_1^2}{W_2^2} \rightarrow \frac{5000}{P_2} = \frac{x^2}{\left(\frac{x}{2}\right)^2} \rightarrow P_2 = 1250$$

$$\rightarrow P_2 = 1250$$

Hence, there is a total loss of Rs. 2500

14. The no. of cogs and the no. of revolutions per minute are inversely proportional to each other. Let the no. of revolution of gear with 16 cogs be x.

$$\frac{80}{16} = \frac{x}{100} \rightarrow x = 500$$

Hence, no. of revolutions in 15 secs. = $500/4 = 125$

15. Let the MP be Rs. 100

SP of Shopkeeper 1 = Rs. 80

MP for Shopkeeper 2 = Rs. 120 (Since he is giving 20% extra)

SP for Shopkeeper 2 = Rs. 100

Ratio of Discounts to MP of Shopkeeper 1 = 1/5

Ratio of Discounts to MP of Shopkeeper 2 = 1/6

Hence, shopkeeper 1 is giving more discounts.
So, the first shopkeeper is giving a better deal by giving more discounts.

16. Let the mean proportion be x

$$\frac{8}{x} = \frac{x}{162} \rightarrow x^2 = 8 \times 162 \rightarrow x = 36$$

17. Ratio of squares of length = Ratio of areas

$$\frac{(20)^2}{(400)^2} = \frac{40}{x} \rightarrow x = 40 \times 400 \rightarrow x = 16000$$

18. Since, the ratio of number of students and amount is not given. Hence, it cannot be determined.

19. Suppose they invested the money for x : y : z period respectively.

$$\text{Ratio of their returns} = 4500x : 6750y : 10125z \\ = 1 : 2 : 3$$

Hence, x : y : z = 3 : 4 : 4

20. Amount invested by Ramu

$$= \frac{50000 \times 3 + 30000 \times 3 + 10000 \times 3 + 30000 \times 3}{12} \\ = \frac{360000}{12} = 30000$$

Amount invested by Kallu =

$$\frac{30000 \times 3 + 40000 \times 3 + 50000 \times 3 + 40000 \times 3}{12}$$

$$= \frac{480000}{12} = 40000$$

Amount invested by Billu

$$= \text{Rs. } 100000 - 70000 = \text{Rs. } 30000$$

Ratio of their investments = 3:4:3

So, ratio of their returns = 3:4:3.

21. Let there be 1 litre of the solution after mixing 20% sugar and 50% sugar..

If the concentration of sugar in it is 40%, then 0.4 litres of the resultant mixture is sugar.

Let x litres of the solution containing 20% sugar be mixed with (1 - x) litres of the solution containing 50% sugar to get 1 litre of the solution containing 40% sugar.

X litres of 20% sugar solution will contain 20% of x = 0.2x litres of sugar in it.

(1 - x) litres of 50% sugar solution will contain 50% of (1 - x) = 0.5(1 - x) litres of sugar.

The sum of these quantities of sugars added up to the total of 0.4 litres in the resultant mixture.

Therefore, $0.2x + 0.5(1 - x) = 0.4$ litres

$$0.2x + 0.5 - 0.5x = 0.4$$

$$0.5 - 0.4 = 0.5x - 0.2x$$

$$x = \frac{0.1}{0.3} = \frac{1}{3} \text{ litres}$$

So, the two solutions are mixed in the ratio of 1:2.

22. Suppose a man can do 1 unit of work in 1 day.

In 10 days work done = 500 units which was 10% of the total work to be done and 90% of the work is yet to be completed.

Remaining work = 4500 units.

To complete 4500 units of work in 35 days, no.

$$\text{of workers needed} = \frac{4500}{35} = 128.57 \\ = 129 \text{ workers.}$$

So, no. of extra men required = $129 - 50 = 79$.

23. Ratio of investment

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = \frac{12}{2} : \frac{12}{3} : \frac{12}{4} = 6 : 4 : 3$$

Ratio of time period for investment

$$= \frac{1}{4} : \frac{1}{3} : \frac{1}{2} = \frac{12}{4} : \frac{12}{3} : \frac{12}{2} = 3 : 4 : 6$$

Total Ratio of investment = 18:16:18 = 9:8:9

$$\text{Share of Kallu} = \frac{8}{26} \times 1950 = 600$$

24. Ratio of Prices = 5:6

Ratio of Consumption = 6:5

If he buys 4 kgs less, it means he used to buy 24 kgs earlier and 20 kgs now.

Price of 20 kg = Rs. 120. Price of 1 kg = Rs. 6

Original Price = Rs. 5

25. Ratio of efficiency is inversely proportional to time taken to complete the work

$$\frac{100}{120} = \frac{x}{120} \rightarrow x = 100$$

Hence, Kallu can complete the work in 100 days.

26. Ratio of Increase in height = $\sqrt{\text{Ratio of ages}}$

Increase in height = 1, when age is 4

Increase in height = x , when age = 16

$$\text{Ratio} = \sqrt{\text{Ratio of ages}} \frac{1}{x} = \frac{\sqrt{4}}{\sqrt{16}} = \frac{1}{2} \rightarrow x = 2$$

Hence, his final height will be 6ft.

27. $x-a : x-b : x-c = b+c : a+c : a+b = 4 : 5 : 6$

$$= k$$

$$\rightarrow b+c=4k, a+c=5k, a+b=6k$$

$$\rightarrow a = 3.5k, b = 2.5k, c = 1.5k$$

28. Let the nos. be x and $3x$.

$$\frac{x+5}{3x+5} = \frac{1}{2} \rightarrow 2x + 10 = 3x + 5$$

$$\rightarrow x = 5$$

Hence, one of the two nos. is 5.

$$\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = k$$

29. Let

$$\text{Then, } a = xk, b = yk, c = zk$$

Hence, $a : b : c = x : y : z$

$$30. \frac{a}{b} = \frac{1}{2} \rightarrow 2a = b \rightarrow b = 2a$$

$$\frac{b}{c} = \frac{1}{3} \rightarrow 3b = c \rightarrow c = 6a$$

$$\frac{c}{d} = \frac{1}{4} \rightarrow 4c = d \rightarrow d = 24a$$

$$\frac{d}{e} = \frac{4}{3} \rightarrow 3d = 4e \rightarrow e = 18a$$

$$\frac{e}{f} = 6 \rightarrow e = 6f \rightarrow f = 3a$$

$$\text{Hence, } \frac{abc}{def} = \frac{ax \times 2a \times 6a}{24a \times 18a \times 3a} = \frac{1}{108}$$

31. Price of a "Fumes Up" bottle without returning the empty bottle = Rs. 7

Price of "Fumes Up" after returning bottle = Rs. 5.5

Price of an empty bottle = Rs. 1.5

She buys 6 bottles of "Fumes Up" and also pays the price of 2 empty bottles.

Hence, total price = $5.5 \times 6 + 2 \times 1.5 = 33 + 3 =$ Rs. 36.

32. Density of diesel = 5 gm/c

Density of oil = 9 cm/c

Density of petrol = 7 gm/cc

In 300 kg petrol's weight = $300 \times 10/30 = 100$ kg Volume of petrol in 300 kg = $100/7 = 100/7$ litres Cost = $100/7 \times 25.2 =$ Rs. 360. You should know that (Weight = Volume \times Density), hence after calculating weight divide it by density to find the volume.

33. Let the volume of vessel A be $26x$, vessel B = $9x$

Volume of water flowing through pipe A per unit time = $\frac{26x}{2}$

Volume of water flowing through pipe B per unit time = $\frac{9x}{13}$

$$\text{Ratio of Diameters} \Rightarrow \frac{D_A}{D_B} = \frac{\sqrt{VA}}{\sqrt{VB}} = \sqrt{\frac{(13)}{\left(\frac{9}{13}\right)}} = \frac{13}{3}$$

34. The total they paid was $45.75 + 59.60 + 74.65 = 180$. It was to be shared in the ratio of $4 : 5 : 6$, their shares will be 48, 60 and 72. Hence A will pay 2.25 and B will pay Rs. 0.40 to C, hence, option [b].

35. At the end of distribution let black cat have x and white cat have y ladoos.

If black cat gives ' k ' ladoos to white cat.

From (1) and (2) :

$$x = 5k$$

$$y = 19k$$

$$y:x = 19:5$$

36. Total no. Of ladoos = 30

\Rightarrow white cat had 19 and black cat had 5

\Rightarrow Monkey ate = $30 - (19+5) = 6$ ladors.

37. Let the salaries be ak , bk and ck . Let the expenses be pl , ql and rl ; i.e. al , bl and rl . So the savings will be $ak - al$, $bk - bl$, $ck - cl$; and as "k" and "l" are constants, the savings will be in the ratio of $a : b : c$, i.e. $p : q : r$, i.e. $1 : q/p : r/p$.

38. If we check option (d).

$$\frac{a}{b} = \frac{6b}{6} = b \quad [a = 6b \text{ given}]$$

$$\frac{c}{d} = \frac{12c}{12d} = \frac{6b}{12d} = \frac{3(2b)}{12d} = \frac{3 \times 9d}{12d} = \frac{9}{4}$$

Which is not an integer so, option (d).

39. Suppose he has x rupees and y paisa. Then $(100x + y)$ paisa is his total worth. Now he $\frac{1}{2}$ spent

$(100x + y)$ \Rightarrow he is left with $\frac{1}{2}(100x + y)$ paisa.

$$\text{Then } \frac{1}{2}(100x + y) = \left(\frac{y}{2}\right)100 + x \Rightarrow 100x + y = 100y + 2x$$

$\Rightarrow x/y = 99/98$. So, Answer is Rs. 100.

40. Contents left after first day = $1 - \frac{1}{3} = \frac{2}{3}$

Evaporation on second day = $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

\therefore contents left after second day = $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

41. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} = \frac{127}{128}$ required % = 99.22%.

42. Milk in mixture = $\frac{4}{5} \times 45 = 36$ litres

Water in mixture = $\frac{1}{5} \times 45 = 9$ litres

Now, let x litre water be added

$$\text{Then, } \frac{36}{9+x} = \frac{3}{2}$$

$$\Rightarrow x = 15 \text{ litres.}$$

43. Let the diagonals of the rhombus be x and y respectively.

$$\therefore \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = x^2$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{4} = x^2$$

$$\Rightarrow \frac{y^2}{4} = \frac{3x^2}{4}$$

$$\Rightarrow \frac{y^2}{x^2} = \frac{3}{1}$$

$$\therefore \frac{y}{x} = \sqrt{3} : 1$$

44. $A + B + C = 1360$

$$A = \frac{2}{3}B, B = \frac{1}{4}C \text{ or } C = 4B$$

$$\text{then, } \frac{2}{3}B + B + 4B = 1360$$

$$\text{or } 2B + 3B + 12B = 1360 \times 3$$

$$\text{or } B = \frac{1360 \times 3}{17} = \text{Rs. 240.}$$

45. Let the no. of Vanilla and chocolate flavour sold be $2x$ and $3x$ respectively.

$$\text{Then, } \frac{2x+4}{3x} = \frac{3}{4} \Rightarrow x = 16$$

$$\text{Vanilla cones sold} = 16 \times 2 = 32$$

46. Homework left for Tuesday night

$$\therefore = 1 - \frac{3}{5} = \frac{2}{5}$$

$$\text{Tuesday's reading} = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$$

$$\text{Homework left for wednesday night} = \frac{2}{5} - \frac{2}{15} = \frac{4}{15}$$

$$\frac{A}{3} = \frac{B}{4} = \frac{C}{5} = K$$

$$A=3K, B=4K, C=5K$$

$$A : B : C = 3K : 4K : 5K = 3 : 4 : 5$$

48. Let the no. of people were 2, 5 and 13 on the respective days.

Total charge = $30 + 37.5 + 32.5 = 100$

Total people = 20

Average Charge per person = Rs. 5

49. To gain 20%, the Selling Price should be 20% more than Cost Price. Hence, 20% water must be added. So, the ratio of Water and Spirit should be 1:5

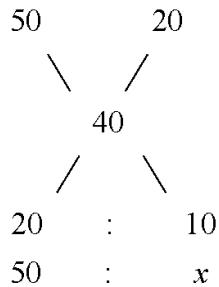
$$50. \text{ New ratio} \Rightarrow 3x \frac{150}{100} : 5x \frac{160}{100} : 7x \frac{150}{100}$$

$$= \frac{9}{2} : 8 : \frac{21}{2} = 9 : 16 : 21$$

IAS ACADEMY
CHRONICLE

Chapter 7

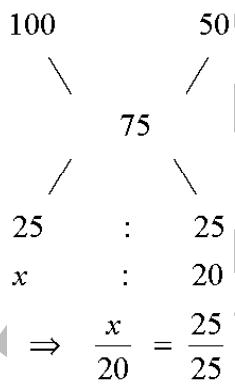
1. Applying allegation rule



$$\text{So, } \frac{50}{x} = \frac{20}{10}$$

$$\Rightarrow x = 25$$

2. Applying allegation rule



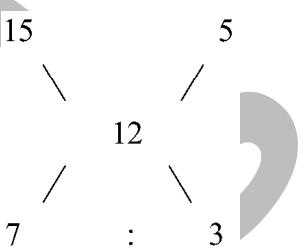
$$\Rightarrow \frac{x}{20} = \frac{25}{25}$$

$$\Rightarrow x = 20 \text{ liters}$$

3. Unpuffed cigarette burns in 15 minutes

Puffed cigarette burns in 5 minutes

By applying allegation rule



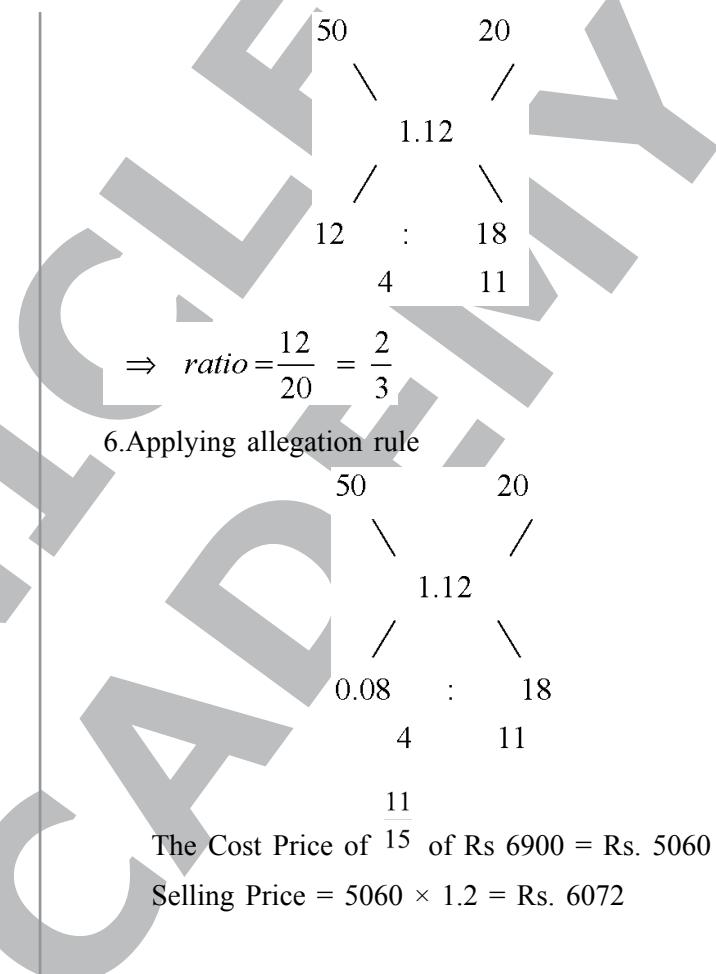
Ratio of Unpuffed : Puffed = 7 : 3

Hence, 30% of the cigarette was puffed.

4. If 100% Puffed cigarette burns in 5 minutes.

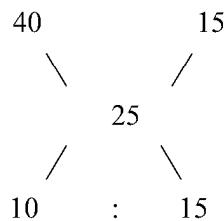
30% will burn in 30% of 5 = 1.5 minutes.

5. By applying allegation rule



The Cost Price of $\frac{11}{15}$ of Rs 6900 = Rs. 5060
Selling Price = $5060 \times 1.2 = \text{Rs. } 6072$

7. By applying allegation rule



$$\text{Ratio of time} = \frac{10}{15} = \frac{2}{3}$$

So, he travelled $\frac{3}{5}$ of the time by bus

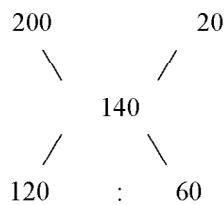
$$= \frac{3}{5} \times 25 = 15 \text{ hours.}$$

$$\text{Distance travelled} = 15 \times 15 = 225 \text{ km}$$

8. SP = 154

$$\text{CP} = \frac{154}{1.1} = \text{Rs. } 140$$

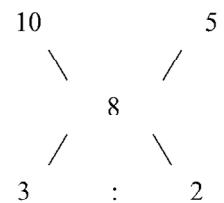
By applying allegation rule



$$\text{Ratio of Cashew: Peanuts} = \frac{120}{60} = 2 : 1$$

$$\text{Peanuts: Cashew} = 1 : 2$$

9. By applying allegation rule



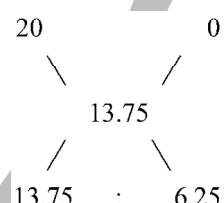
$$\text{The ratio of } 10\% \text{ to } 5\% = 3 : 2$$

$$\text{Required ratio} = 2 : 3$$

10. SP = Rs. 16.50 / litre

$$\text{CP} = \frac{16.50}{1.2} = 13.75 \text{ / litre}$$

Now applying allegation rule



$$\text{Ratio} = \frac{13.75}{6.25} = \frac{11}{5}$$

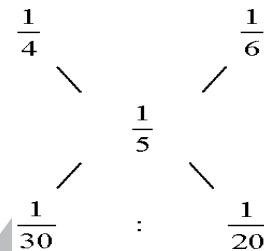
So, if 11 litres of pure milk is added then 5 litres of water must we added = 5 litres.

11. Since there are three types of rices there are many possible solutions to this question as any of the two can be added only. Hence, there is no unique answer to this question.

12. Concentration of milks available are $\frac{1}{4}$ and $\frac{1}{6}$

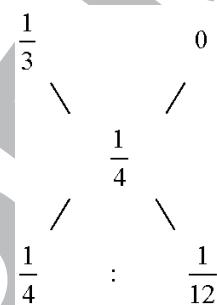
and concentration of milk required $\frac{1}{5}$

By applying allegation rule,



$$\text{Hence, ratio} = 20 : 30 = 2 : 3$$

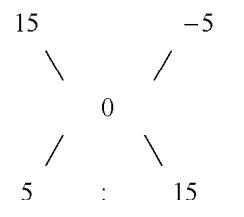
13. Concentration of original milk is $\frac{1}{3}$ and final concentration of milk required is $\frac{1}{4}$. Applying allegation rule,



$$\text{Required ratio} = 12 : 4 = 3 : 1$$

It means that one fourth of the solution should be replaced with water.

14. By applying allegation rule

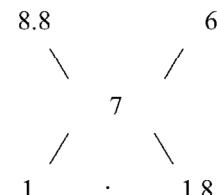


$$\text{Ratio of Prices} = \frac{x}{1000-x} = \frac{5}{15}$$

$$\Rightarrow x = 250$$

Cost Price of shirt sold at 5% loss = $1000 - 250$
= Rs. 750

15. By applying allegation rule

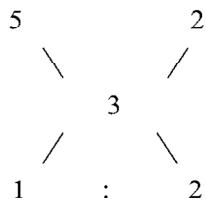


$$\text{Ratio} = \frac{1}{1.8} = \frac{5}{9}$$

Ratio of loan from bank 1 to bank 2 = 9 : 5

$$16. \text{ Average collection} = \frac{270}{90} = \text{Rs. } 3 \text{ per head}$$

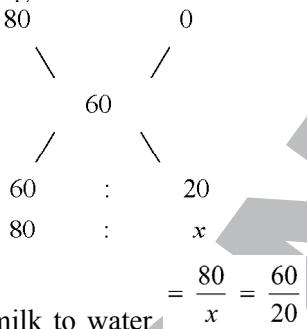
Applying allegation



Boys: Girls = 1 : 2

$$\text{No. of Girls} = \frac{2}{3} \times 90 = 60$$

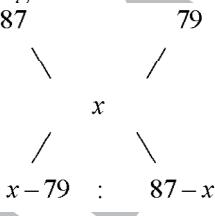
17. By applying allegation rule



$$\text{Ratio of 80% milk to water} = \frac{80}{x} = \frac{60}{20}$$

$$\Rightarrow x = 26 \text{ litres}$$

18. By applying allegation rule

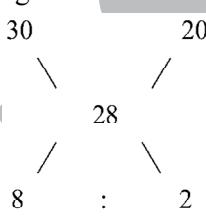


$$\text{Ratio} = \frac{87-x}{x-79}$$

Now, checking through the options, option [b] does not give x as an integer.

Hence 2 : 3 cannot be the required ratio.

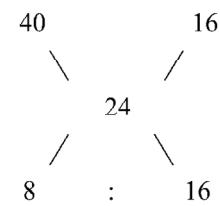
19. By applying allegation rule



$$\text{Ratio} = 4 : 1$$

$$\text{Required ratio} = 1 : 4$$

20. By applying allegation rule



The ratio in which they are mixed = 8 : 16 = 1 : 2

Proportion of solution stolen = $\frac{2}{3}$.

21. Fraction of Whiskey in vessel

$$= \frac{4}{13}$$

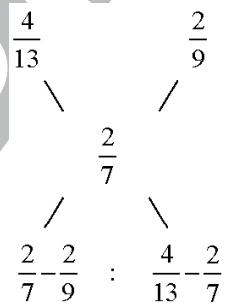
Fraction of Whiskey in vessel

$$= \frac{2}{9}$$

Fraction of Whiskey mixture

$$= \frac{2}{7}$$

Now, applying allegation,



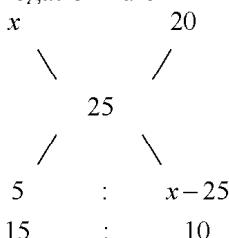
$$\text{Required ratio} = \frac{\left(\frac{2}{7} - \frac{2}{9}\right)}{\left(\frac{4}{13} - \frac{2}{7}\right)}$$

$$= 26 : 9$$

22. SP = Rs. 30

$$\text{CP} = \frac{30}{1.2} = 25$$

By applying allegation rule



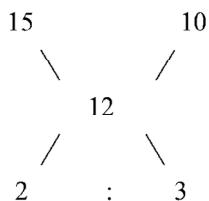
$$\Rightarrow \frac{15}{10} = \frac{5}{x-25}$$

$$\Rightarrow x = 28.33$$

23. SP = Rs. 18

$$\frac{18}{1.5} = 12$$

Applying allegation rule

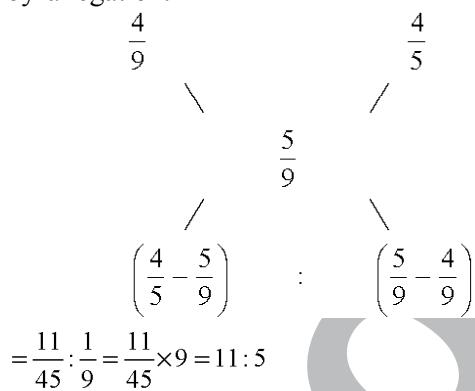


Ratio is 3 : 2

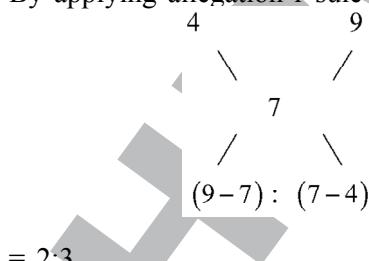
Milk in vessel P = 4/8

Milk in vessel Q = 4/3

24. by allegation:-



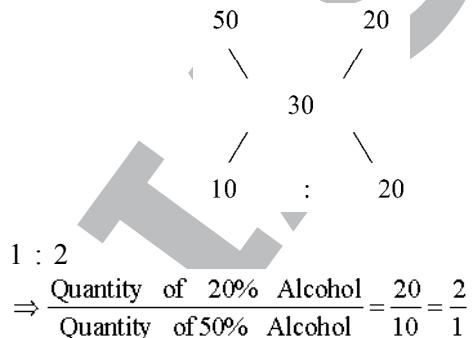
25. By applying allegation rule



$$\text{quantity sold at } 9\% \text{ gain} = \frac{100 \times 3}{5}$$

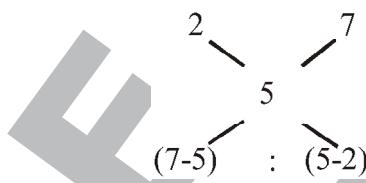
$$= 60 \text{ kg}$$

26. Solution By allegation:-



$$\Rightarrow \frac{x}{40} = \frac{2}{1} = x = 80$$

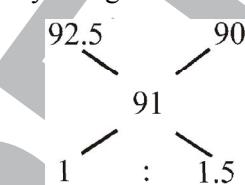
27. Solution By allegation:



the ratio of 2% alcohol solution to 7% alcohol solution has to be 2 : 3

\Rightarrow In 100 ml such solution their quantities will be 40 and 60

28. Solution By Allegation:-

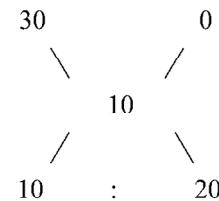


The ratio of Sterling silver and 90% silver alloy = 1 : 1.5 = 2 : 3

So amount of Sterling silver in 500 gm of above mixture

$$= \frac{2}{5} \times 500 = 200 \text{ gm}$$

29. Solution By Allegation:

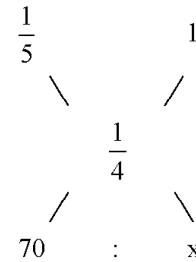


The ratio of 30% saline water to pure water = 1:2

So if saline solution = 100kg, Pure water = 200kg

30. By Allegation Rule:-

For Soda:-

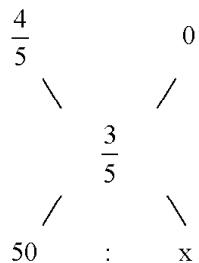


$$\Rightarrow \frac{1 - \frac{1}{4}}{\frac{1}{4} - \frac{1}{5}} = \frac{x}{70}$$

$$\Rightarrow x = \frac{14}{3} \text{ liters}$$

31. By Allegation Rule:-

For Milk:-

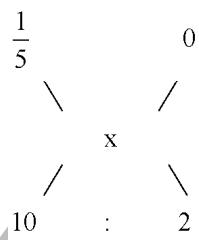


[we take ratio of Milk as 0 in 2nd solution as only water is added]

[x is the volume of water added]

32. By Allegation Rule:-

For Alcohol:-



(x' is the strength of alcohol in final solution)

$$\frac{10}{2} = \frac{x - 0}{\frac{1}{5} - x}$$

$$\Rightarrow 2 - 10x = 2x$$

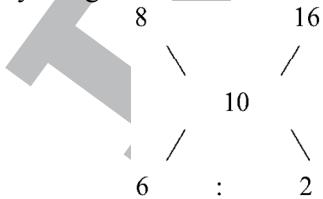
$$\Rightarrow x = \frac{2}{12} = \frac{1}{6}$$

Alcohol is $\frac{1}{6} \times 100\% = 16\frac{2}{3}\%$

33. His net average Speed would be

$$= \frac{\text{Total Distance}}{\text{Total time}} = \frac{80}{8} = 10 \text{ km/hr.}$$

Now by allegation.



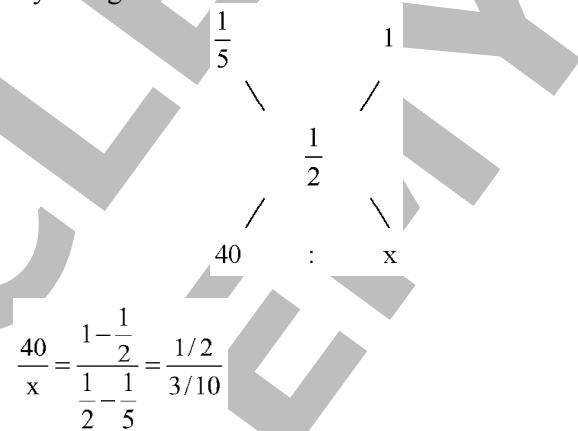
i.e. the ratio of time is 6:2 or 3:1.

\Rightarrow He walks $\frac{3}{4}$ of the total time on foot

i.e. $\frac{3}{4} \times 8 = 6$ hrs. on foot.

Distance = $6 \times 8 = 48$ km (Ans).

34. By Allegation:



$$\frac{40}{x} = \frac{1 - \frac{1}{2}}{\frac{1}{2} - \frac{1}{5}} = \frac{1/2}{3/10}$$

Now:

$$\Rightarrow \frac{40}{x} = \frac{10}{6}$$

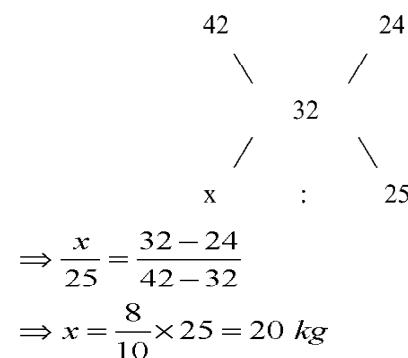
$$\Rightarrow n = \frac{40 \times 6}{10} = 24 \text{ liters}$$

35. S.P. = 40

$$\frac{40}{C.P.} = 32$$

$$C.P. = \frac{40}{32} = 1.25$$

Now by Allegation Rule:

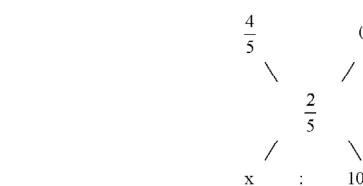


$$\Rightarrow \frac{x}{25} = \frac{32 - 24}{42 - 32}$$

$$\Rightarrow x = \frac{8}{10} \times 25 = 20 \text{ kg}$$

36. By allegation Rule:-

taking liquid 'P':-



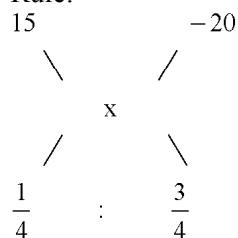
$$\Rightarrow \frac{x}{10} = \frac{(2/5 - 0)}{(4/5 - 2/5)}$$

$$\Rightarrow x = 10 (1) = 10 \text{ liters}$$

\Rightarrow original Solution was $10 + 10 = 20$ litres
(As 10 litres was replaced)

$$\text{quantity of 'P'} = \frac{4}{5} \times 20 = 16 \text{ liters}$$

37. By Allegation Rule:



(where x is his net profit or loss)

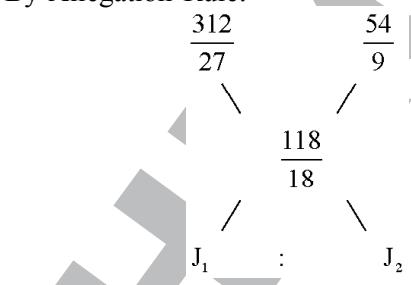
$$\Rightarrow \frac{1}{3} = \frac{x+20}{15-x}$$

$$\Rightarrow 15 - x = 3x + 60$$

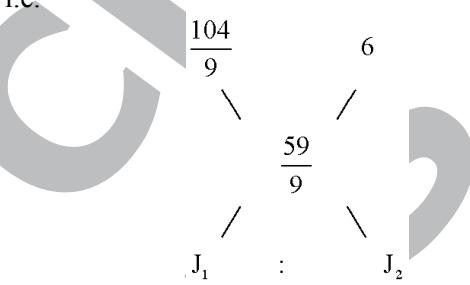
$$\Rightarrow 4x = -45$$

$$\Rightarrow x = \frac{-45}{4} = -11.25\% \text{ loss}$$

38. By Allegation Rule:



i.e.

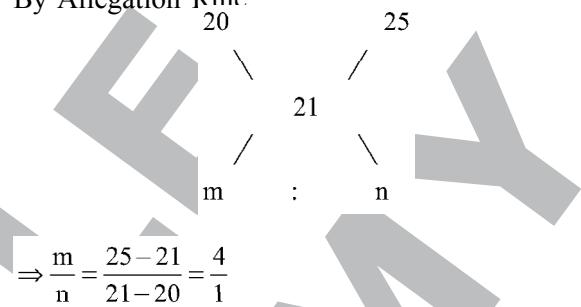


$$\Rightarrow \frac{J_1}{J_2} = \frac{5/9}{45/9}$$

$$\Rightarrow \frac{J_1}{J_2} = \frac{1}{9} \text{ or } 1:9$$

Proportion of Banana Juice = $\frac{1}{10}$ (= Proportion of J_1)

39. By Allegation Rule:



$$\Rightarrow \frac{m}{n} = \frac{25-21}{21-20} = \frac{4}{1}$$

i.e. the ratio of two solutions will be 4:1

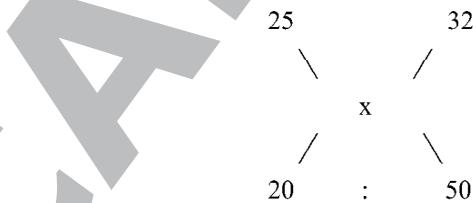
The final solution is of 6 litres.

$$\text{So amount of solution P in it} = \frac{4}{5} \times 6 = \frac{24}{5} = 4.8 \text{ liters}$$

and amount of Solution Q in it = $(6 - 4.8)$ liters = 1.2 liters

40. Let the strength be x.

By allegation:

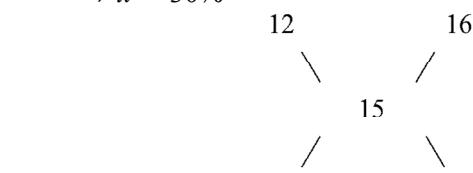


$$\Rightarrow \frac{20}{50} = \frac{32-x}{x-25}$$

$$\Rightarrow 2x - 50 = 160 - 5x$$

$$\Rightarrow 7x = 210$$

$$\Rightarrow x = 30\%$$



41.

$$\frac{m}{n} = \frac{16-15}{15-12}$$

[where m is quantity sold at 12% gain & n is quantity sold at 16% gain]

$$\Rightarrow \frac{m}{n} = \frac{1}{3}$$

now quantity sold at 16% gain (i.e. n)

$$= \frac{3}{4} \times 36 = 27 \text{ kg}$$

42. By allegation method:

let glycol added be x litre, then:

$$\begin{array}{ccc} \frac{1}{4} & & 1 \\ & \backslash & / \\ & \frac{3}{4} & \\ / & & \backslash \\ \frac{1}{4} & : & \frac{1}{2} \end{array}$$

$$\text{the ratio is } \frac{1}{4} : \frac{1}{2} = 1 : 2$$

now 1 part is 16 litres

$$\text{so } x = 2 \times 16 = 32 \text{ L}$$

43. Let the proportion of milk be ' x ' in the final mixture, then by Allegation:

for milk:

$$\begin{array}{ccc} \frac{4}{5} & & \frac{3}{5} \\ & \backslash & / \\ & x & \\ / & & \backslash \\ 1 & : & 1 \end{array}$$

(Ratio is 1:1 as volume is equal given)

$$\Rightarrow \frac{1}{1} = \frac{x - \frac{3}{5}}{\frac{4}{5} - x}$$

$$4 - 5x = 5x - 3 \\ 10x = 7 \\ x = \frac{7}{10}$$

Milk is 70%

44. let the price be x :

by Allegation:

$$\begin{array}{ccc} 22 & & 26 \\ & \backslash & / \\ & x & \\ / & & \backslash \\ 44 & : & 26 \end{array}$$

$$\Rightarrow \frac{44}{26} = \frac{26-x}{x-22}$$

$$\Rightarrow 22(x-22) = 13(26-x)$$

$$\Rightarrow 22x - 484 = 338 - 13x.$$

$$\Rightarrow 35x = 822$$

$$\Rightarrow \frac{822}{35} = 23.5$$

45. For Milk:

$$\begin{array}{ccc} \frac{5}{6} & & 0 \\ & \backslash & / \\ & x & \\ / & & \backslash \\ 60 & : & 5 \end{array}$$

$$\Rightarrow \frac{60}{5} = \frac{x-0}{\frac{5}{6}-x}$$

$$\Rightarrow 12 = \frac{6x}{5-6x}$$

$$\Rightarrow 60 - 72x = 6x$$

$$\Rightarrow 78x = 60$$

$$\Rightarrow x = \frac{60}{78} = \frac{30}{39} = \frac{10}{13}$$

$$\text{Proportion of Water} = 1 - \frac{10}{13} = \frac{3}{13}$$

$$\text{Ratio of Milk and Water} = \frac{10}{13} : \frac{3}{13} = 10:3$$

46. By allegation:

let the cost per kg be ' x '.

$$\begin{array}{ccc} 5 & & 7 \\ & \backslash & / \\ & x & \\ / & & \backslash \\ 10 & : & 20 \end{array}$$

$$\Rightarrow \frac{10}{20} = \frac{7-x}{x-5}$$

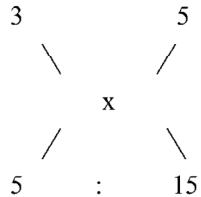
$$\Rightarrow x - 5 = 2(7 - x) = 14 - 2x$$

$$\Rightarrow 3x = 19$$

$$\Rightarrow \frac{19}{3} = \text{Rs } 6\frac{1}{3}/\text{kg.}$$

47. Let the final price be x .

By Allegation:



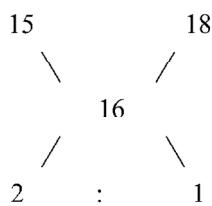
$$\Rightarrow \frac{5}{15} = \frac{5-x}{x-3}$$

$$\Rightarrow x - 3 = 15 - 3x$$

$$\Rightarrow 4x = 18$$

$$\Rightarrow x = \text{Rs. } 4.5/\text{kg}$$

48. By Allegation:



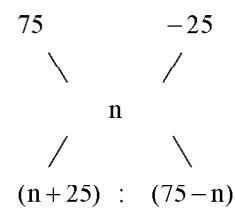
\Rightarrow the two varieties are mixed in a ratio of 2:1

\Rightarrow quantity of Rs 18/kg in 90 kg mixture =

$$\frac{1}{3} \times 90 = 30 \text{ kg}$$

49. Let his net loss or profit per cent be x

By Allegation:



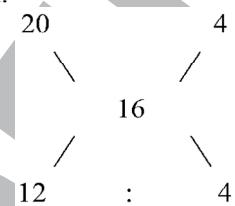
$$\text{given that } \frac{n+25}{75-n} = \frac{2/3}{1/3} = \frac{2}{1}$$

$$\Rightarrow n + 25 = 150 - 2n$$

$$\Rightarrow 3n = 125$$

$$\Rightarrow n = \frac{125}{3} = 41\frac{2}{3}\% \text{ Profit}$$

50. By Allegation:



$$\Rightarrow \frac{12}{4} = 3:1$$

\Rightarrow i.e. they should be mixed in the ratio 3:1.

$$\Rightarrow \text{volume of A} = \frac{3}{4} \times 40 = 30 \text{ L.}$$

Volume of B = 10 L.

Chapter 8

1. After mixing water, the final quantity of Water = 70 litres and Milk = 30 litres.

Before mixing quantity of water = 60 litres, Milk = 30 litres

Ratio of Milk to water = 1:2

2. Finally, the quantity of Water = 70, Milk = 30

Before taking 10 litres of solution, the ratio is same. Hence, the amount of Milk and Water = 33 and 77.

Before adding 10 litres of Water, the quantity of water = $77 - 10 = 67$ litres.

Hence, ratio of Milk and Water = 33:67

3. Let the first beaker contain 100 litres of milk and 2nd beaker contain 100 litres of water.

After first transfer, 2nd beaker has Milk:Water = 1:2

After second transfer, 1st beaker has milk: water = 2:1

After third transfer, 2nd beaker has Milk : Water=4:5

Hence, the final ratio of Milk:Water in 2nd beaker=4:5

4.

	Beaker 1		Beaker 2	
	Milk	Water	Milk	Water
Initially	4	0	0	12
Step 1	$4-x$	x	x	$12-x$

$$\frac{4-x}{x} = \frac{x}{12-x} \Rightarrow 48 - 16x + x^2 = x^2$$

$$\Rightarrow x = 3$$

Hence, the quantity transferred = 3 litres

5.

	Milk	Water
Initially	100	0
Step 1	90	10
Step 2	81	19
Step 3	72.9	27.1

Since, every time we take a solution, the ratio in which the Milk and Water was present in the solution also gets reduced in the same ratio. Final concentration of Milk = 72.9%

6. The third statement is obvious as the ratio of Milk to Water in first beaker will be same as the ratio of Water to Milk in second beaker as nothing is poured out or added to the both.

7. Ratio of Water to Substance before evaporation = 99 : 1

Ratio of Water to Substance after evaporation = 98:2 = 49:1. It means that the quantity of water evaporated = 50%

8.

	Milk	Water
Initially	50	50
Step 1	75	25
Step 2	83.33	16.66
Step 3	87.5	12.5

Hence, final concentration is 87.5%

9.

	Milk	Water
Initially	50	50
Step 1	45	55
Step 2	40.5	59.5
Step 3	36.45	63.55
Step 4	32.805	67.195

Any further step will reduce the conc. to less than 30%

No. of times it can be done = 4

10. Since, 142.5 ml of alcohol had been replaced in two times. So, the value of x must be more than 71.25. When the first time x ml of alcohol was replaced with water, the ratio of alcohol to water is $750-x:x$. In the second time, the amount of alcohol replaced will be less than the amount replaced in first time, as this time the solution has alcohol and water which will be replaced in the same ratio as in the solution. Hence, the value of x must be more than 71.25. Hence, going through the options, $x=75$.

11. Let the amount of Sodium in the initial solution be $2x$ kg. Water = $8x$ kg.

Amount of Water remains after evaporation = $8x - 2$. Amount of Sodium after addition = $2x + 2$

$$\text{Final ratio} = \frac{2x+2}{8x-2} = \frac{40}{60} \Rightarrow x=1$$

Hence, total amount of solution = 10 kg.

12. Let the total content in the peg be V ml.

According to the formula,

$$\text{Final conc.} = \text{Initial conc.} \left(1 - \frac{\text{volume replaced}}{\text{total volume}} \right)^n$$

$$\frac{16}{25} = 1 \left(1 - \frac{10}{V} \right)^2 \Rightarrow \left(1 - \frac{10}{V} \right)^2 = \frac{4}{5}$$

$$\Rightarrow V = 50$$

13. After selling one third, the quantity of milk and water is 60 and 6.66 liters respectively. He then adds 33.33 liters of water. So total water = 40 liters and Milk = 60 liters. So, the milk is 60% of the complete solution.

14. 10% of Milk is replaced with water in the original solution. Then, the solution contains 90% Milk and 10% Water. In second step the milk replaced will be 90% of 720 liters and 10% of 80 litres water. Now, the conc. of milk is 81%. In the final step 81% of the 81% milk is replaced by water. Hence, the final concentration of milk = 72.9%.

15. Let there be 12 litres of first solution and 12 litres of second solution.

So, Milk in first solution = 4 liters and in second solution = 3 liters.

So, total quantity of milk in 24 liters solution = 7 liters. Ratio = 7:17

16. Amount of water in 1 liter of 2% Acid solution = 980 ml and amount of Acid = 20 ml

Amount of Acid in 1 liter of 1% Acid solution = 10 ml. Since, we cannot take the acid out. Hence, For 20 ml of Acid to be 1%, the solution has to be 2 liters. So, amount of Water needed in 1% Acid Solution = 1980 ml. Hence, amount of water to be added = 1 liter.

17. Let the CP of Pure milk be R.s 100 per litres.

SP of 1 litre Solution = Rs. 100

Profit = 10%

Hence, CP of 1 litres solution = Rs. 90.91

Hence, the solution had 90.91% Milk and 9.09% water.

18. Initial Conc. of Milk is 60%.

Final Conc. of Milk is 40%.

It means that in 100 litres, 20 liters of Milk should be taken out, which means that one-third of the solution must be replaced by water in order to get the ratio as 2:3.

19. Suppose he spent Rs. 120 on each of both types of apples.

Total apples he bought = 5 kg. Total Money Spent = Rs. 240

On selling, he sold it at Rs. 50 per kg. So, total money earned = Rs. 250

Profit = Rs. 10

$$\% \text{ Profit} = \frac{10}{240} \times 100 = 4.16\%$$

20. Suppose he bought 1 kg each of both the types of apples.

Total apples he bought = 2 kg. Total Money Spent = Rs. 100

While selling, he sold it at Rs. 50 per kg. So, total money earned = Rs. 100

Hence, there is no profit and no loss.

21. The concentration of Whiskey in first vessel is $\frac{4}{13}$ and that in second vessel is $\frac{2}{9}$.

The final concentration of Whiskey required = $\frac{2}{7}$.

$$\text{So, the ratio of Whiskey to Soda} = \frac{\frac{2}{7} - \frac{2}{9}}{\frac{4}{13} - \frac{2}{7}} = \frac{\frac{4}{9}}{\frac{2}{13}} = \frac{26}{9}$$

22. Initial Conc. = 1, Final Conc. = 16:25

As per the formula,

$$\frac{16}{25} = 1 \left(1 - \frac{10}{V}\right)^2 \Rightarrow \left(1 - \frac{10}{V}\right) = \frac{4}{5}$$

$$\Rightarrow V = 50 \text{ ltr.}$$

23. Initial quantity of wine = 600 liters.

Final quantity of wine = ?

As per the formula, final quantity=

$$600 \left(1 - \frac{60}{600}\right)^3 = 437.4$$

Hence, the quantity of wine in the mixture = 437.4 liters.

24. **Case 1:** Initial conc. = 9:1

Final conc. = 1:9

$$\text{Hence, } \frac{81}{9+x} = \frac{1}{9} \Rightarrow x = 720$$

Case 2: Initial conc. 9:1

Final conc. = 1:9

$$\text{Hence, } \frac{81-y}{9} = \frac{1}{9} \Rightarrow y = 80$$

$$\text{Hence, } x-y = 640$$

25. SP of mixture = Rs 100 per kg

Profit = 100%

Hence, CP = Rs. 50 per kg

Suppose he mixed 3 kg of A and 2 kg of B.

Total price = Rs. 250

$$3A + 2B = 250$$

$$\text{Also, } A = B + 10$$

$$\text{Hence, } 5B + 30 = 250$$

$$B = \text{Rs. } 44$$

The price of B is Rs. 44 per kg

26. The price has trebled. Hence, if the expenditure has to remain the same, the quantity bought must become $\frac{1}{3}$ rd of the original amount. Since the price is inversely proportional to the quantity bought. So, the drop in percentage terms is 66.67 %.

27. Apply the Rule of Allegation:

$$\frac{\text{No. of boys}}{\text{No. of girls}} = \frac{(70-65)}{(67-65)} = \frac{3}{2}$$

$$\text{Number of boys} = 450.$$

28. The 20 litre mixture contains milk and water in the ratio 3:2. Therefore, there will be 12 litres of milk in the mixture and 8 litres of water in the mixture.

Step 1. When 10 litres of the mixture is removed, 6 litres of milk is removed and 4 litres of water is removed. Therefore, there will be 6 litres of milk and 4 litres of water left in the container. It is then replaced with pure milk of 10 litres. Now the container will have 16 litres of milk and 4 litres of water.

Step 2. When 10 litres of the new mixture is removed, 8 litres of milk and 2 litres of water is removed. The container will have 8 litres of milk and 2 litres of water in it. Now 10 litres of pure milk is added. Therefore, the container will have 18 litres of milk and 2 litres of water in it at the end of the second step.

Therefore, the ratio of milk and water is 18 : 2 or 9 : 1.

29. The resultant mixture is sold at a profit of 20% at Rs. 96/kg.

$$\text{i.e. } 1.2 \text{ (Cost)} = \text{Rs. } 96 \Rightarrow \text{Cost} = \frac{96}{1.2} = 80/\text{kg.}$$

Check by options:-

The resultant ratio QA: QB : QC :: 1 : 4 : 2.

1 kg of variety A at Rs.60 is mixed with 4 kgs of variety B at Rs.75 and 2 kgs of variety C at Rs.100.

The total cost for the 7 kgs = $60 + (4 \times 75) + (2 \times 100) = 60 + 300 + 200 = 560$.

Cost per kg of the mixture = $560/7 = 80$

30. Since, we have to mix more than two qualities of rice, there will be more than one unique solution and thus all the options are possible.

31. 30 litres of the mixture has milk and water in the ratio 7 : 3. i.e. the solution has 21 litres of milk and 9 litres of water.

When you add more water, the amount of milk in the mixture remains constant at 21 litres. In the first case, before addition of further water, 21 litres of milk accounts for 70% by volume. After water is added, the new mixture contains 60% milk and 40% water.

Therefore, the 21 litres of milk accounts for 60% by volume.

Hence, 100% volume = $21/0.6 = 35$ litres.

We started with 30 litres and ended up with 35 litres. Therefore, 5 litres of water was added.

32. Let the amount of Basmati rice being mixed be x kgs. As the trader makes 25% profit by selling the mixture at Rs.40/kg, his cost per kg of the mixture = Rs.32/kg.

$$\text{i.e. } (x \times 42) + (25 \times 24) = 32(x + 25)$$

$$\Rightarrow 42x + 600 = 32x + 800$$

$$\Rightarrow 10x = 200 \text{ or } x = 20 \text{ kgs.}$$

33. The mixture contains 40% milk and 60% water in it. That is 4.8 litres of milk and 7.2 litres of water.

Now we are replacing the mixture with pure milk so that the amount of milk and water in the mixture is 50% and 50%. That is we will end up with 6 litres of milk and 6 litres of water.

Water gets reduced by 1.2 litres.

To remove 1.2 litres of water from the original mixture containing 60% water, we need to remove $1.2 / 0.6$ litres of the mixture = 2 litres.

34. The best way to solve this problem is to go from the answer choices.

The mixture of 60 litres has in it 24 litres of milk and 36 litres of water. (2 : 3 :: milk : water)

When you remove x litres from it, you will remove $0.4x$ litres of milk and $0.6x$ litres of water from it.

Take choice (2). According to this choice, $x = 10$.

So, when one removes, 10 litres of the mixture, one is removing 4 litres of milk and 6 litres of water.

Therefore, there will be 20 litres of milk and 30 litres of water in the container.

Now, when you add 10 litres of milk, you will have 30 litres of milk and 30 litres of water – i.e. milk and water are in equal proportion.

35. Let the number of horses = x

Then the number of pigeons = $80 - x$.

Each pigeon has 2 legs and each horse has 4 legs.

Therefore, total number of legs = $4x + 2(80 - x) = 260$

$$\Rightarrow 4x + 160 - 2x = 260$$

$$\Rightarrow 2x = 100$$

$$\Rightarrow x = 50.$$

36. The problem can be solved by the traditional method but it is cumbersome process to do that. The problem is simple if its solution is simpler. Hence we will go for a simpler solution for this kind of problem.

There is a short cut method to find the Quantity of milk left after nth operation

It is given by $\left(\frac{x-y}{x}\right)^n$ of the whole quantity, where x is initial quantity of milk in the cask y is the quantity of milk withdrawn in each process and n is the number of process.

Hence from the above rule it can be said that Quantity of milk left after the 3rd operation = $[(30-6)/30]3 \times 30 = 15.36$ ltr.

37. Find quantity of milk in bucket

$$= 100 \left(1 - \frac{10}{100}\right)^3 \\ = 100 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = 72.9l.$$

Milk removed = $(100 - 72.9)l = 27.1$ liters

38. Let the volume of bucket be V = volume of milk initially.

$$\text{Final quantity of milk} = V \left(1 - \frac{10}{V}\right)^2 \\ = \frac{V(V-10)^2}{V^2} = \frac{(V-10)^2}{V}$$

Proportion of milk

$$= \frac{\text{quantity of Milk}}{\text{total volume}} = \frac{(V-10)^2/V}{V} \\ = \frac{(V-10)^2}{V^2} = \frac{9}{25} \Rightarrow \frac{V-10}{V} = \frac{3}{5}$$

$$5V - 50 = 3V, \text{ thus } V = 25.$$

39. Let 66 grams of both alloys be mixed together. Then, in the final mixture:-

$$\text{Amount of tin} = 6 + 14 = 20$$

$$\text{Amount of Copper} = 45 + 46 = 91$$

$$\text{Amount of Zinc} = 15 + 6 = 21$$

So, the final ratio is 20 : 91 : 21

40. Assume that there are 500, 400 and 500 litres respectively in the 3 containers. Then we have, 83.33, 150 and 208.33 litres of milk in each of the three containers. Thus, the total milk is 441.66 litres. Hence, the amount of water in the mixture is $1400 - 441.66 = 958.33$ litres.

Hence, the ratio of milk to water is 441.66 : 958.33 = 53 : 115.

41. Suppose X kg are taken from A; then $(9 - X)$ kg are taken from B.

$\frac{5}{8}$ of the mixture in A and $\frac{7}{10}$ of the mixture in B is coffee.

$$\frac{5}{8}X + \frac{7}{10}(9 - X) = 6$$

Hence $X = 4$

4 kg must be taken from A and 5 kg from B.

42. Let the quantity of milk be $5X$ & water X .

Then $5X / (X + 5) = 5/2$ or $X = 5$

Quantity of milk = $5X = 25$ litres

43. The amount of the final mixture is given by 50 ml + 30 ml = 80 ml.

$$\text{Alcohol content} = \frac{30 \times 50}{100} \text{ ml} = 15 \text{ ml.}$$

$$\text{Percentage of Alcohol in new solution} = \frac{15}{80} \times 100\% \\ = 18.75\%$$

44. Let us first find the amount of alcohol in the 10% solution of 200 ml.

$$200 \times 10\% = 20 \text{ ml}$$

The amount of alcohol in the x ml of 25% solution is given by

$$25\%x = 0.25x$$

The total amount of alcohol in the final solution is given by $20 + 0.25x$

The ratio of alcohol in the final solution to the total amount of the solution is given by

$$\frac{(20 + 0.25x)}{(x + 200)} = 15\% = 0.15$$

Solve the above equation to get $x = 100 \text{ ml}$

45. Let S.P. of pure milk be 120, then C.P. = S.P. = 120 (as has been given) profit = 20%

$$\Rightarrow \text{Actual cost of milk in solution} = \frac{120}{1.2} = 100$$

$$\Rightarrow \text{Proportion of milk in solution} = \frac{100}{120} = \frac{5}{6}$$

$$\text{Proportion of water} = 1 - \frac{5}{6} = \frac{1}{6}$$

$$\text{Ratio of Water:Milk} = (1/6 : 5/6)$$

$$= 1 : 5$$

46. Initial conc. = 1:5

Final Conc. = 1:9

$$\frac{1}{9} = \frac{1}{5} \left(1 - \frac{1}{10}\right)^n \Rightarrow \frac{5}{9} = \left(\frac{9}{10}\right)^n \Rightarrow n = 6$$

As per formula,

$$\frac{3}{5} = \frac{4}{5} \left(1 - \frac{10}{x}\right)^4 \Rightarrow \frac{3}{4} = \frac{x-10}{x} \Rightarrow x = 40$$

48. Milk in 80% of solution = 16 ltr. Total water = 64 ltr.

For 75% of Water to be 64 liters, total solution = 85.33 liters

So, total milk = $85.33 - 64 = 21.33$

Total milk added = $21.33 - 16 = 5.33$ liters.

49. 50% of 100 ltr of spirit solution contains 50 ltr of pure spirit.

Amount of spirit evaporated every week = 5 ltr.

Amount after one week = 45 ltr.

Next week since the spirit is not added, the amount

of spirit that evaporated = 10% of 45 = 4.5 ltr.

Amount of spirit remaining = 40.5 ltr.

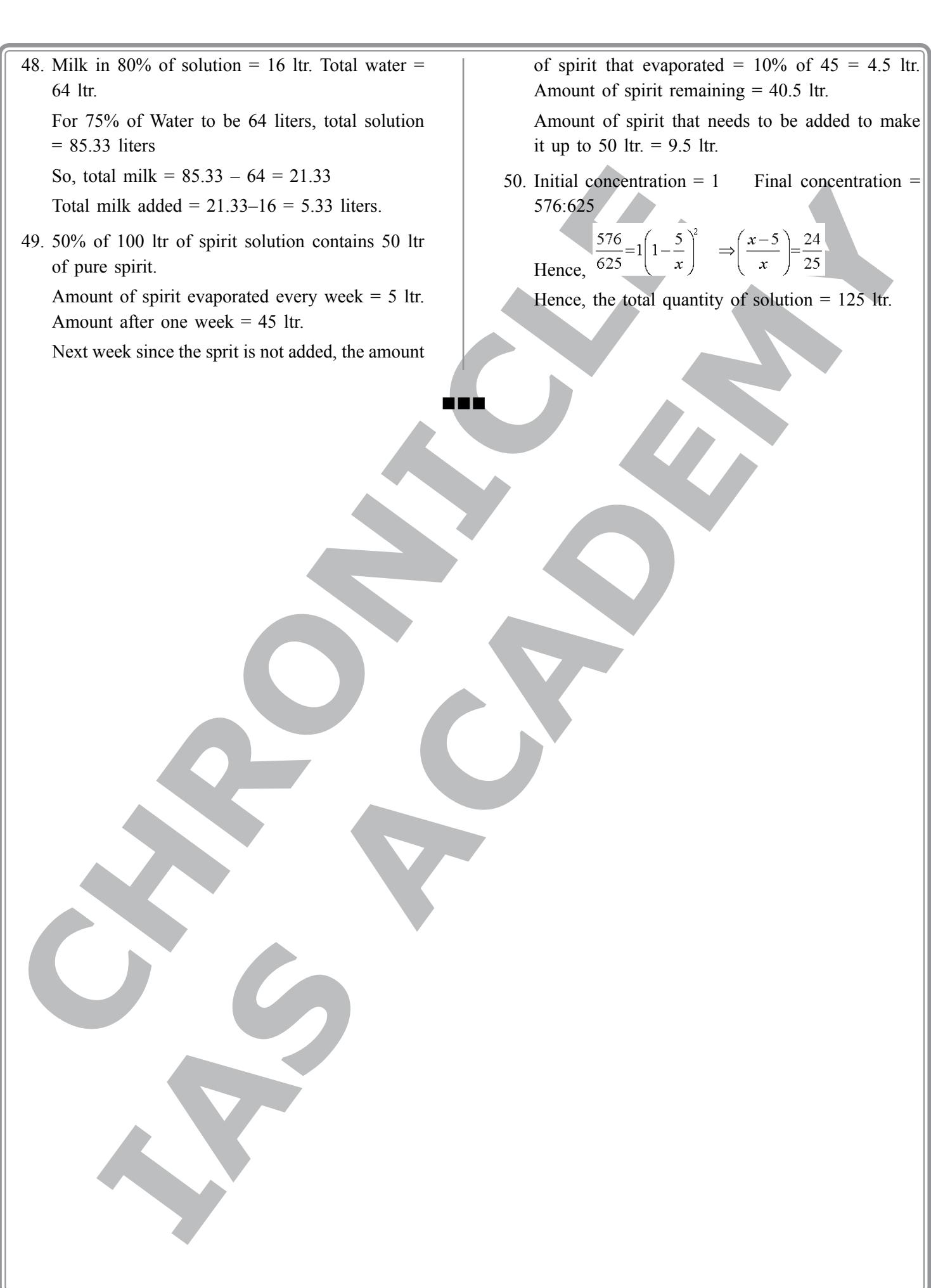
Amount of spirit that needs to be added to make it up to 50 ltr. = 9.5 ltr.

50. Initial concentration = 1 Final concentration = 576:625

$$\text{Hence, } \frac{576}{625} = 1 \left(1 - \frac{5}{x}\right)^2 \Rightarrow \left(\frac{x-5}{x}\right) = \frac{24}{25}$$

Hence, the total quantity of solution = 125 ltr.

■ ■ ■



Chapter 9

1. Let the distance be x kms.

Time taken when he travelled at 20km/hr = $\frac{x}{20}$ hrs.

Time taken when he travelled at 30km/hr = $\frac{x}{30}$ hrs.

$$\text{Difference} = \frac{x}{20} - \frac{x}{30} = \frac{20}{60} \rightarrow x = 20 \text{ kms}$$

2. When distance is same,

$$\begin{aligned} \text{Average Speed} &= \frac{2 s_1 s_2}{s_1 + s_2} \\ &= \frac{2 \times 30 \times 20}{20 + 30} = 24 \text{ km/hr.} \end{aligned}$$

3. Let the length of platform be x kms.

$$\text{Length of train} = (72 - 18) \times \frac{5}{18} \times 30 = 450 \text{ mtrs.}$$

Distance travelled by train in crossing the platform
= Length of Train + Length of platform = x +

$$450 = 72 \times \frac{5}{18} \times 45 \rightarrow x = 450 \text{ mtrs.}$$

4. Ramu will reach at one end in $\frac{30}{4} = 7.5$ hours

Kallu will reach the other end in $\frac{30}{6} = 5$ hours

Thus, after 7.5 hours, distance between the two
will be $30 - 2.5 \times 4 = 20$ Km

Now their relative speed will be 10 Km/Hr and
they will take another 2 hours to meet each other.
Hence, total time required will be 9.5 Hours.

5. Relative Speed = 15 Km/Hr.

Distance = 1 Km

Time taken to cover 1 Km = $1/15$ hours = 4 minutes.

So, they will take 4 mintues to collide from the
movement they were 1 Km apart.

6. Ramu's Score = 60

Kallu's Score = 40

When Kallu scores 100 (i.e. 40×2.5) then Ramu
will score = $60 \times 2.5 = 150$ points.

$$7. 36 \text{ km/hr} = \frac{36 \times 1000}{60 \times 60} = 10 \text{ m/s}$$

8. Speed of the train = 72km/hr = 20 m/s

Distance covered by train while crossing a
stationary man = Length of the train = $20 \times 15 = 300$ m

Let the length of platform be x.

Distance covered by train while crossing the
platform = Length of train + Length of platform
= $300 + x$

$$\begin{aligned} \text{So, } 300 + x &= 50 \times 20 \\ &\rightarrow = 700 \text{ m.} \end{aligned}$$

9. Let the distance between Delhi and Lucknow be
60 kms.

Speed of Gomti express = 6 km/hr

Speed of Rajdhani express = 10 km/hr

Distace covered by Gomti till 8 AM = 12 km.

Now, the distance between Gomti and Rajdhani
express at 8 AM = $60 - 12 = 48$ kms

Relative Speed = $10 + 6 = 16$ km/hr

$$\text{Time taken to cover this distance} = \frac{48}{16} = 3 \text{ hrs.}$$

So, they will meet at 11 AM.

10. Let the total distance be 24 kms.

Ramu's speed = 3 km/hr

Kallu's speed = 2 km/hr

Relative speed = 5 km/hr

To meet for the second time, they should cover
a total distance of 48 kms, for which time taken
= $48/5 = 9.6$ hrs = 9 hours 36 mins.

11. If Ramu can swim 300 m then Kallu can swim
500 m. They will meet 3 times in between and
once at the end of 500m. Now, again Ramu will
cover 3 rounds and Kallu will cover 5 rounds
and meet 3 times. So, they will meet a total of
7 times.

12. On doubling the speed, his time gets halved. But if we keep on doubling the speed, the late time will never become zero. Hence, he cannot reach his office on time.

13. Let the length of the track be 90m. They meet for the second time in 45min.

$$\text{Distance travelled by both to meet for the second time} = 90 + 45 = 135\text{m}$$

$$\text{Relative Speed} = 3\text{m/min.}$$

$$\text{Speed of Dhoni} = 90/45 = 2\text{m/min}$$

$$\text{Speed of Yuvraj} = 1\text{ m/min.}$$

So, Yuvraj can complete one round in 90 mins.

14. Speed of burning of first candle = $20/15$ cm/min
Speed of burning of second candle = $15/20$ cm/min

After x mins;

$$1\text{st candle burned} = \frac{20x}{15}$$

$$2\text{nd candle burned} = \frac{15x}{20}$$

$$\text{Candle remaining is same; } 20 - \frac{20x}{15} = 15 - \frac{15x}{20}$$

$$\rightarrow x = \frac{60}{7} \text{ mins.}$$

15. Distance covered by thief till 9 AM = 30 km.

$$\text{Relative speed of Police and thief} = 5 \text{ km/hr}$$

Time taken to cover this distance = 6 hrs.

Distance covered by dog in 6 hours at the speed of 25 km/hr = 150 km

16. Police will catch him after 6 hours after 9 AM, i.e. at 3 PM.

17. At 9 AM the bus that started 10 hours ago will meet this bus. And henceforth a bus that started in between that 10 hours will meet. Also, 10 buses that start from 9 AM to 10 hours of journey will meet this bus, and a bus that is just going to start when this bus will be reaching will also meet this bus. Hence, a total of 21 buses will meet this bus.

18. Let the distance be 120 kms. Speed of each Train = 10 km/hr.

Let the distance travelled by Train before accident be x kms.

Time taken to travel this distance = $x/10$ hrs.

$$\begin{aligned}\text{Total time} &= \frac{x}{10} + 1 + \frac{120-x}{5} = 21 \\ \rightarrow x &= 40\end{aligned}$$

Time taken by train to cover this distance = 4 hrs.
By 11 AM, distance covered by both the trains = 80kms.

By 12 AM distance covered by both the trains = 90 kms.

Remaining distance = 30 km

Relative Speed = 15 km/hr

Time taken = 2 hrs. So, they will meet at 2 PM

19. If they moved with their original speeds, they would have met after 6 hours.

Let the distance travelled by Train before accident be x kms.

$$\begin{aligned}\text{Total Time} &= \frac{2x}{20} + 1 + \frac{110-2x}{15} = 8 \\ \rightarrow x &= 10\end{aligned}$$

Time taken by each train to travel 10 kms = 1 hrs.

By 8 AM distance covered by the 1st train = 10kms

By 9 AM distance covered by the 1st train = 10 kms

Now, the speed = 5 km/hr

Remaining Distance = 110km

$$\text{Time taken} = 110/5 = 22 \text{ hrs.}$$

So, the train will reach Lucknow at 7 AM, the next day.

20. B = 20 kmph

S = 5 kmph

Let the distance be 150 km

Time taken to travel upstream = $150/15 = 10$ hrs

Time taken to travel downstream = $150/25 = 6$ hrs

$$\text{Avg. speed} = \frac{150+150}{10+6} = \frac{300}{16} = 18.75 \text{ kmph}$$

21. Their relative speed remains the same as one of them increases the speed by 1 kmph and the other decreases the speed by 1 kmph until one of them reaches its speed to 0.

After 6 hrs, distance travelled by both = $10 \times 6 = 60$ kms.

Remaining distance = 40 kms

Speed keeps on increasing every hour by 1.
 After 7 hours distance remaining = $40 - 11 = 29$.
 After 8 hours distance remaining = $29 - 12 = 17$.
 After 9 hours distance remaining = $17 - 13 = 4$.
 Now, the speed is 14km/hr and distance is 4, so the time taken is approx 15 mins.
 So, the total time taken = 9 hours and 15 mins.

22. Chaman is 20% faster than Raman.
 So, the time taken by Raman will be 20% more than Chaman to complete the race.
 Let the time taken by Chaman to complete the Race = 100 min
 Time taken by Raman to complete the Race = 120 min
 As, per question lead given by Chaman = 20 min which is the same in this case.
 Hence, our assumption was correct. The time taken by Chaman to complete the race is 100 mins.

23. Distance travelled in 3 hrs = 150 km
 This distance is 40% of total distance.
 Hence, total distance = $150 \times 2.5 = 375$ kms.
 Distance remaining = 225km and time remaining is 2 hrs.
 Speed = $225/2 = 112.5$ km/hr

24. Speed at which he should go = $420/7 = 60$ km/hr
 Let the time for which he travelled at 60 km/hr be x.
 $60x + 80(6 - x) = 420$
 $\rightarrow x = 3$

Hence, after 4 hours ($3 + 1$ hour in traffic) he was forced to increase his speed.

25. Let the distance be 120 kms.

$$\text{Avg. speed} = \frac{\frac{120}{40} + \frac{40}{40} + \frac{40}{80}}{3} \approx 52 \text{ km/hr}$$

26. Relative typing speed = 8 pages/min
 Time taken to complete the book = $368/8 = 46$ min.
 No. of pages typed by Ramu in 46 min = $46 \times 3 = 138$.
 Hence, the last page typed by Ramu is page no. 138.

27. Let the length of the swimming pool be x mts.
 Let the ratio of their speeds be a : b.
 Time taken by Ramu to cover $x - 50$ is equal to time taken by Kallu to cover 50.

$$\text{Hence, } \frac{x - 50}{a} = \frac{50}{b}$$

Also, when they meet second time

$$\frac{2x - 40}{a} = \frac{x + 40}{b}$$

On solving we get, $x = 110$ mts.

28. Let the original speed be x.
 Let original time taken = 1 hr
 So distance = x

$$\text{Time taken} = \frac{4x}{3} = 1 \text{ hr } 20 \text{ mins}$$

 Time taken if travelled by $4/3$ of the speed = $\frac{3}{4}$ = 45 mins
 Time saved = 15 mins.

29. Let the time after they meet be x.
 According to the question, Distance travelled by Raman in x mins is covered by Chaman in 4 mins Also, the Distance travelled by Raman in 9 mins is covered by Chaman in x mins

$$\text{Hence, } \frac{x}{4} = \frac{9}{x} \rightarrow x = 6$$

Ratio of speeds = 6:4 = 3:2

30. Speed of the train = 54 km/hr = 15m/s
 Distance travelled in 20 sec = 300m
 Distance travelled in 25 sec = 375m
 Difference between the two distances is the difference in the length of the platforms = 75m.

31. Distance travelled by goods train in 3 hours = 150 km

The same distance is travelled by the passenger train in 5 hours. Hence, the speed of the passenger train = $150/5 = 30$ km/hr.

32. Let the distance be 20 km
 Speed of Boat downstream = 5 km/hr
 Speed of Boat upstream = 4 km/hr
 Speed of the Boat in still water = 4.5 km/hr
 Speed of the Stream = 0.5 km/hr.
 If the speed of the stream is doubled. Relative speed downstream = 5.5 km/hr
 Time = $20/5.5 = 3.63$ hrs.

33. Let the speed upstream be u and speed downstream be d .

$$\frac{50}{u} + \frac{72}{d} = 9$$

$$\frac{70}{u} + \frac{90}{d} = 12$$

$$\rightarrow u = 10, d = 18$$

Hence, the time taken to travel 100 km upstream = 10 hours.

34. The net speed of the boat from Jalgaon = $25 + 5 = 30$ km/hr.

The net speed of the boat from Jalsamadhi = $15 - 5 = 10$ km/hr.

Time taken to meet = 7.5 hrs

Hence, both the boats will meet at 225 kms from Jalgaon

35. Let the speed of the boat be x km/hr

Speed upstream = $x - 1$

Speed downstream = $x + 1$

$$\text{Time} = \frac{48}{x-1} + \frac{48}{x+1} = 20$$

$$\rightarrow x = 5 \text{ km/hr}$$

36. Let the distance be 120 kms

$$\text{Avg. speed} = \frac{\frac{240}{120} + \frac{120}{60}}{40} = 48 \text{ km/hr}$$

37. Let the distance be 200 kms.

Time taken without stoppages = 4 hrs

Time taken with stoppages = 5 hrs

So a stoppage of 1 hour is there in every 5 hours.
i.e., a stoppage of 12 mins every hour.

38. Let the distance be 100 kms and time be 10 hrs.

He covers 30 km in 5 hrs, i.e. with a speed of 6 km/hr.

He has to cover 70 km in 5 hours with a speed of 14 km/hr

$$\text{So, percentage increase} = \frac{14-6}{6} \times 100 = 133.33\%$$

39. At the same time,

Distance covered by Ramu = 1000 m

Distance covered by Kallu = 950 m

Distance covered by Billu = 900 m.

When Kallu covers 1000 m then distance covered

$$\text{by Billu} = \frac{900 \times 1000}{950} = 947.3 \text{ m}$$

When Kallu covers 500 m then Billu covers = 473.7 m

So, Kallu leads Billu by 26.3 m in a 500 m race.

40. Kallu can be given a lead of 100m or 90 secs.

It means that he can cover a distance of 100 m in 90 secs. Hence, he can cover a distance of 1000m in 900 secs.

41. Let the length of the race be x km.

Beat Distance of Kallu = $x - 20$

Beat Distance of Billu = $x - 30$

$$\text{So, } \frac{x-20}{x-30} = \frac{x}{x-12}$$

Hence, $x = 120$ m

42. Let the speed of Boat be u and that of River be v .

So according to question,

$$3(u+v) = 6(u-v)$$

$$\text{Hence, } \frac{u}{v} = \frac{3}{1}$$

Hence, the ratio of speeds = 3 : 1

43. Let the distance be 60 km.

His usual speed = 1 km/min

Increased speed = $60/40 = 1.5$ km/min

% increase = 50%

44. Relative Speed = $25 - 15 = 10$ m/sec

Time taken to cover one more round (i.e. 300m) to meet for the first time = $300/10 = 30$ secs.

45. The difference in angle at 3 o'clock = 90° ?

To cover a difference of 90° again, the minute hand has to gain 180° over the hour hand.

The relative speed of minute and the hour hand is 5.5° per minute.

To cover a difference of 180° , time taken = $180/5.5 = 32.72$ minutes i.e. approx 33 minutes.

Hence, the hour hand and minute hand will be perpendicular to each other at 3:33.

46. Total distance travelled = 300 m.

Total time taken = 30 sec

Relative Speed = 10m/sec = 36 km/hr

Relative Speed = Speed of the Car – Speed of the truck.

Hence, the speed of the Truck = Speed of the car – Relative Speed = $100 - 36 = 64$ km/hr

$$47. \frac{\text{Area covered by Hour hand}}{\text{Area covered by Minute Hand}} = \frac{2 \times \pi R^2}{24 \times \pi r^2} = \frac{3}{100}$$

$$\text{Hence, } \frac{R}{r} = \frac{3}{5}$$

48. Let the length of the train be x m.

$$\begin{aligned} \text{Total Distance covered while crossing 1st platform} \\ = x + 250 \end{aligned}$$

$$\text{Time taken} = 40 \text{ sec}$$

$$\text{Speed} = \frac{x + 250}{40}$$

$$\text{Total Distance covered while crossing 2nd platform} = x + 150.$$

$$\text{Time taken} = 30 \text{ sec}$$

$$\text{Speed} = \frac{x + 150}{30}$$

$$\frac{x + 250}{40} = \frac{x + 150}{30}$$

$$\rightarrow x = 150$$

$$\text{Speed of the train} = 400/40 = 10 \text{m/sec} = 36 \text{km/hr}$$

49. Let the speed be 10m/sec

$$\text{Then, length of train} = 500 \text{ m}$$

If Speed increases by 20%, then Speed = 12 m/s

$$\text{Time} = 500/12 = 41.66 \text{ secs.}$$

$$50. \text{Average Speed} = \frac{400}{\frac{100}{80} + \frac{150}{60} + \frac{150}{40}} = 53.33 \text{ km/hr}$$

Chapter 10

1. Ramu's 1 day work = $\frac{1}{8}$

Kallu's 1 day work = $\frac{1}{12}$

Ramu's and Kallu's 1 day work = $\frac{1}{8} + \frac{1}{12} = \frac{20}{96} = \frac{1}{4.8}$

So, together they will complete the work in = 4.8 days.

2. Ratio of speeds of Sheela and Munni = 400 : 600 = 2 : 3

So the ratio of share of Sheela and Munni is 2 : 3.

Hence, $2x + 3x = 4800$

$x = 960$,

Hence, Sheela's share = $2 \times 960 = \text{Rs. } 1920$

3. Suppose the total work is 48 units.

Chaurasia's 1 day's work = 6 units.

His son's 1 day work = 3 units.

His father's 1 day's work = 2 units.

Work done by all the three in 1 day = 11 units.

Work Remaining = 37 units.

Work done by him and his son in 2nd day = 9 units.

Remaining Work = 28 units.

Time taken by Chaurasia to complete the remaining work = $4\frac{2}{3}$ days.

So he will complete the work in 5 days.

4. Ratio of their speeds = 1 : 4.

So they will meet for the third time at starting point when Raju had completed 1 km and Ramu had completed 4 kms.

Time taken = 15 mins.

So, Time taken by Ramu (the faster runner) to complete 10 km = $\frac{15 \times 10}{4}$ mins = 37.5 mins.

5. Suppose the total capacity of tank be 80 ltrs.

Tank filled by inlet pipe in 1 hr = 10 ltrs.

Net Tank filled by inlet pipe due to leak in 1 hr = 8 ltrs.

Amount of leak per hour = 2 ltrs/hr.

Total time taken to empty the complete tank = 40 hrs.

Let the total work be 48 units.

Work done by Ram & Shyam in 1 hr = 8 units.

Work done by Ram & Dhyan in 1 hr = 6 units.

Work done by Ram & Nyan in 1 hr = 2 units.

Work done by Ram, Shyam, Dhyan & Nyan in hr = 16 units.

So,

$$(Ram + Shyam) + (Ram + Dhyan) + (Ram + Nyan) - () = (8 + 6 + 2 - 16) = 0.$$

$$2(Ram) = 0$$

Hence, Work Done by Ram = 0 units.

7. Time taken by Ramu and Kallu to meet for the first time = $\frac{6 \times 8}{14} = \frac{24}{7}$ mins.

Time taken by them to meet for the 21st time = $\frac{24}{7} \times 21 = 72$ mins.

Hence, they meet for the 21st time at 9:12 am.

8. Let the length of the circular track be 24 units.

They will meet for the first time after $\frac{24}{7}$ mins. and covered a distance of $\frac{24}{7}$ mins.

So they will meet at 7 distinct points on the track.

9. Let the length of the circular track be 24 units.

They will be diametrically opposite for the first time if at the same time the distance between them is 12 units.

At the end of 12 mins Ramu has covered = 12 units

And Kallu has covered = 24 units.

Hence they will be diametrically opposite at the end of 12 mins which is at 8:12 am.

10. Let the total work to be done = 60 units.

Ramu's 1 day's work = 3 units

Kallu's 1 day's work = 1 unit.

If they work on alternate days then at the end of 2 days; work done = 4 units.

Time taken to complete 60 units of work = 30 days.

11. The hour's and minute's hand coincide after every $65\frac{5}{11}$ minutes.

In 24 hours they will meet = $\frac{11}{720} \times 24 \times 60 = 22$ times.

12. Let total work be 80 units.

Work done by A in 1 day = 8 units.

Work done by B in 1 day = 10 units.

Working done by (A + B) in 1 day = 18 units.

Total time taken to complete 80 units work = $\frac{80}{18} = \frac{40}{9}$ days.

13. Let the total job be 72 units.

Work done by Ramu in 12 hrs = 12 units.

Work done by Kallu in 12 hrs = 9 units.

Work done by Billu in 12 hrs = 6 units.

They started working together for 8 hrs a day.

Work done by Ramu in 8 hrs = 8 units.

Work done by Kallu in 8 hrs = 6 units.

Work done by Billu in 8 hrs = 4 units.

Work done by (Ramu + Kallu + Billu) in 8 hrs = 18 units.

After 2 days(16 hrs); Work done = $18 \times 2 = 36$

units.

Work done by Kallu and Billu in 2 days(16 hrs) = $10 \times 2 = 20$ units.

At the end of 4 days; Work done = 56 units.

Work remaining = 16 units.

Billu can do 16 units work in 4 days.

14. Let the capacity of the Cistern be 360 ltrs.

Work done by A in 1 hr = 30 ltrs.

Work done by B in 1 hr = 20 ltrs.

Work done by C in 1 hr = -24 ltrs.

After 4 hrs; Tank filled = 120 ltrs.

After 6 hrs and 30 mins; Tank filled = $120 - 60 = 60$ ltrs.

Remaining Work = 300 ltrs.

Time taken to completely fill the tank = 6 hr 30 mins + 15 hrs. = 21 hrs and 30 mins.

15. Time taken to meet for the first time = $\frac{8 \times 10}{18} = \frac{40}{9}$ hours.

16. They will meet at the starting point after LCM(8, 10) = 40 hours.

17. They meet after every $\frac{40}{9}$ hours.

So, after 50 hours they will meet = $\frac{50 \times 9}{40} = 11$ times.

18. They are running in same direction.

So they will meet after every $\frac{7 \times 11}{11 - 7} = 19.25$ hours.

19. They will meet at the starting point for the first time after LCM(7, 11) = 77 hours.

20. They will meet every time after 19.25 hrs.

So after 50 hrs they will meet = $\frac{50}{19.25} = 2$ times.

21. Let a man can do 4 units of work in a day.

Working capacity of a women = 2

Working Capacity of a child = 1.

Work done by 6 men in 8 days = $6 \times 8 \times 4 = 192$ units.

- Work done by 4 children, 6 women and 4 men in 1 day = $4 \times 1 \times 6 \times 2 \times 4 \times 4 = 32$ units.
- No of days required to complete the work = $\frac{192}{32} = 6$ days.
22. Let the total work be 90 units.
- Work done by (A + B) in 1 day = 9 units.
- Work done by (B + C) in 1 day = 6 units.
- Work done by (C + A) in 1 day = 5 units.
- Work done by $2(A + B + C)$ in 1 day = 20 units.
- Work done by (A + B + C) in 1 day = 10 units.
- No of days required to complete 90 units of work = $\frac{90}{10} = 9$ days
23. Let the total work be 48 units.
- Work done by Ramu = 6 units.
- Work done by Kallu = -4 units.
- Work done after 2 days = 2 units.
- Work done after 42 days = 42 units.
- On 43rd day Ramu will do the remaining 6 units of work and the job will be completed.
- Hence total no of days required = 43 days.
24. Let the total work be 144 units.
- Work done by Ramu per day = 9 units.
- Work done by Ramu in 4 days = 36 units.
- Work remaining = 106 units.
- Time taken by Kallu to do 106 units = 18 days.
- Work done by Kallu per day = $\frac{106}{6} = 24$ days.
- Total time taken by Kallu to do 144 units = $\frac{144}{6} = 24$ days
25. Let the amount of work Chaman can do in 1 day = x units.
- Amount of work Raman can do in 1 day = y units.
- Amount of work Karan can do in 1 day = z units.
- Total work done by all three of them = $3.7x$ units.
- Work done by all of them in 10 days = $37x$.
- Time required by Chaman to do $\frac{37x}{x} = 37$ days
26. Let the total work be 60 units.
- Work done by all three together per day = 12 units.
- Work done by Ramu = $\frac{1}{2}$ Work done by (billu + Kallu)
- Hence, Ramu did $\frac{1}{3}$ of the work per day.
- So, Work done by Ramu per day = 4 units.
- Time taken by Ramu to complete the work alone = 15 days.
27. Let the work done by a Man in 1 day = x units.
- Let the work done by a Woman in 1 day = y units.
- $9x + 12y = \frac{1}{4}$ and $4x + 16y = \frac{1}{6}$
- On solving, we get $x = \frac{1}{48}$ and $y = \frac{1}{192}$
- Thus, 24 Women and 6 Men can do $= 24 \times \frac{1}{192} + 6 \times \frac{1}{48} = \frac{1}{4}$ units of work per day.
- Total time taken to complete the work = 4 days.
3. Let the no of men be x .
- Work done in 40 days = $40x$.
- If 5 men were added;
- $30(x+5) = 40x$
- $x = 15$.
- If 15 men can do a work in 40 days, then no of men required to complete the same job in 60 days = $\frac{15 \times 40}{60} = 10$ men
4. Let the amount of work = 36 units.
- Work done by A in 1 day = 3 units
- Work done by B in 1 day = 2 units.
- Work done by (A + B) in 1 day = 5 units.

Work done by C in 1 day = $\frac{1}{2} \times 5 = 2.5$ units

Total work done by all the three = $(3+2+2.5) = 7.5$ units.

Total time taken = $\frac{36}{7.5} = 4.8$ days.

5. The hour's and minute's hand are perpendicular to each other twice every $65\frac{5}{11}$ minutes.
So, they will be perpendicular 44 times every day.
6. At 6 o'clock the angle between hours hand and minute's hand = 180°

The minute hand gains $5\frac{1}{2}$ per minute over the hour's hand.

In 105 minutes the minute hand gains = $\frac{105 \times 11}{2} = 577.5^\circ$

Total difference = $577.5 + 180 = 757.5^\circ$

Difference in angle = $757.5 - 360 = 37.5^\circ$

7. At 4 o' clock the hands are 20 minutes apart.
So, the minute hand must gain 20 minutes before they coincide. But the minute hand gains 55 minutes in 1 hour.

So, it will gain 20 minutes in: $20 \times \frac{60}{55} = 21\frac{9}{11}$ minutes past 4, which is at 4:21:48.

8. $M_1 = 20, W_1 = 30, D_1 = 40, T_1 = 9$
 $M_2 = ?, W_2 = 50, D_2 = 50, T_2 = 6$
By the formula; $M_1 \times D_1 \times T_1 \times W_1 = M_2 \times D_2 \times T_2 \times W_2$
 $20 \times 40 \times 9 \times 30 = M_2 \times 50 \times 6 \times 50$

Hence, $M_2 = 40$.

9. If 10 men eat 10 rotis in 10 days, then 50 men will eat 50 rotis in 10 days.
So, 50 men will eat $50 \times 5 = 250$ rotis in 50 days.

10. Let the total job be 30 units.

Work done by (Ramu + Kallu + Billu) in 1 hour = 15 units.

Work done by Ramu in 1 hour = 6 units.

Work done by Kallu in 1 hour = 5 units.

Work done by Billu in 1 hour = $15 - (6 + 5) = 4$ units.

Time taken to complete the job by Billu = $\frac{30}{4} = 7.5$ hrs.

11. Let the work done by 34 men in 7 day = 34 units.

Hence, Total Work = 85 units.

Remaining Work = 51 units.

Since no. of days remains the same, the no of men required to complete the remaining work = 51

Hence, No of extra men required = $51 - 34 = 17$ men.

12. Let the total work be 100 units.

Work done by a man per day = 1 units.

Work done by a woman per day = 2 units.

Work done by (4 men + 8 women) in a day = $4 + 16 = 20$ units.

No of days required to complete the work = 5 days.

13. No of hours required by Ramu to complete the job = 56 hrs.

No of hours required by Kallu to complete the job = 42 hrs.

Let the total work be 168 units (LCM of 56 and 42).

Work done by Ramu per hour = 3 units.

Work done by Kallu per hour = 4 units.

Work done by both of them together per hour = 7 units.

Total time required = 24 hours = 3 days working 8 hours a day.

14. Let the total work be 120 units.

Work done by Chaman per day = 10 units.

Work done by Raman per day = 8 units.

Work done by both of them together =

$$\text{Total time required} = \frac{120}{21.6} = 5.55 \text{ days.}$$

15. Let the amount of work done by Billu per day is 1 unit.

Work done by Kallu = 3 units

Work done by Ramu = 6 units.

Total work done by all of them together per day = 10 units.

So, Total work = 100 units.

Time taken by billu to complete the job = 100 days.

16. This type of question is done by the formula:

$$\frac{T_1 \times R_1}{t_1} = \frac{T_2 \times R_2}{t_2},$$

Where T₁ and T₂ are the no. of teeths of 1st and 2nd gear respectively. and t₁ and t₂ are time taken by 1st and 2nd gear respectively.

Here T₁ = 84, R₁ = 40, t₁ = 50

And T₂ = 42, R₂ = ?, t₂ = 90.

So, R₂ = 144.

17. Let the total work be 6 units.

Work done by Ramu per day = 1 unit.

Work done by Kallu per day = 2 units.

Work done by Billu per day = 3 units.

Time Required By Ramu, Kallu and Billu to complete the work = 6, 3 and 2 days respectively.

Ratio of time taken = 6 : 3 : 2

18. Let the total capacity be 40 ltrs.

Work done by inlet pipe per minute = 8 ltrs.

Work done by inlet pipe due to leak per minute = 5 ltrs.

Amount of water leaked per minute = 3 ltrs

Total time required to empty the tank using

$$\text{leak} = \frac{40}{3} = 13.33 \text{ minutes.}$$

19. Ration a man requires per day = $\frac{40}{10 \times 4} = 1 \text{ kg.}$

Ration required by a woman per day = 0.5 kg

Ration required by a child per day = 0.25 kg

Ration required for (12 boys + 6 women + 4 men) per day = $(12 \times 0.25 + 6 \times 0.5 + 4 \times 1) = 10 \text{ kg.}$

No of days 50 kg ration will last = 5 days.

20. Let the total amount of work be 80 units.

Work done by Ramu per day = 10 units.

Work done by Kallu per day = 12 units.

Total Work to complete = $80 \times 1.5 = 120 \text{ units.}$

Total time required by Kallu = $\frac{120}{12} = 10 \text{ days.}$

21. Let the total capacity of tank be 120 ltrs. Amount of water leaking per hour = 4 ltrs.

Amount of water filled by the inlet per hour = 10 ltrs.

Net amount of water added per hour = 6 ltrs.

Amount of water to be filled = $120 = 80\% \text{ of } 120 = 24 \text{ ltrs.}$

Time required to fill = 4 hrs.

22. Let the total capacity of tank be 60 ltrs.

Work done by pipes A, B and C is 6, 5 and 4 ltrs per minute respectively.

Amount of water added in 2 minutes =

Remaining work = 30 ltrs.

Time required by inlet B alone to fill the remaining tank = $\frac{30}{5} = 6 \text{ mins.}$

23. The no. of workmen has doubled. So the time required by 8 men and 16 women to dig 42 square mtrs will be halved = 35 min.

Time required to dig 120 m = $120 \times \frac{35}{42} = 100 \text{ days.}$

24. Let the amount of work be 60 units.

Work done by Chaman per day = 5 units.

Work done by Raman per day = 3 units.

Work done by both in 5 days = $5 \times (5 + 3) = 40$ units.

Amount of work remaining = 20 units.

Fraction of work remaining = $1/3$.

25. Let the amount of work done by Ramu = 3 units and time taken by him is x days.

Work done by B = 1 unit.

Total Work = $3x$

Time required by B to complete the work = $3x$ days.

So, $3x - x = 60$

$x = 30$.

Total Work = 90 units.

Total time taken by both = $\frac{90}{(3+1)} = 22.5$ days.

Chapter 11

1. $\left(x + \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x}\right)^2 + 2x\frac{1}{x} = 25$

$$x^2 + \left(\frac{1}{x}\right)^2 = 25 - 2 = 23$$

2. $\left(x + \frac{1}{x}\right)^3 = x^3 + \left(\frac{1}{x}\right)^3 + 3x\frac{1}{x}\left(x + \frac{1}{x}\right) = 125$

$$x^3 + \left(\frac{1}{x}\right)^3 = 125 - (3 \times 5) = 110$$

3. $x = (126 - 2), y = 126, z = (126 + 2)$

$$x^3 + y^3 + z^3 = 3 \times 126^3 + 3 \times (126 \times 2)(128 - 124)$$

$$3xyz = 3 \times 124 \times 126 \times 128$$

$$x^3 + y^3 + z^3 - 3xyz = 3 \times 126 [126^2 + 8 - 124 \times 128] = 3 \times 126 \times 12 = 4536$$

4. Since, the constant part was wrong, the sum of the roots would have not been affected. Hence, the sum of the roots is $4 + 5 = 9$.

While, in when the coefficient of x was wrong, the product of the roots would have not been affected. Hence, the product of the roots is 8.

$$\text{Hence, } \frac{-b}{a} = 9; \quad \frac{c}{a} = 8$$

$$\text{Hence, } a = 1, b = -9 \text{ and } c = 8$$

$$\text{The equation is } x^2 - 9x + 8 = 0$$

The roots are 1 and 8.

5. Let the cost of 1 chair be x and cost of 1 table be y.

$$4x + 5y = 1000$$

$$\text{Also, } 5x + 4y = 1250$$

$$\text{Hence, } x = 250, y = 0$$

Cost of 1 chair and 1 table = Rs. 250

$$T_s = a + 4d, \quad T_9 = a + 8d$$

$$T_s = 2T_9 \Rightarrow a + 4d = 2a + 16d$$

$$\Rightarrow a + 12d = 0$$

Hence, the thirteenth term of the series will be 0.

7. To ensure the sum to be 16 we will have to select the first 9 natural numbers.

8. $\frac{4x^3 - 5x^2 + 37x - 104}{(x+3)(4x^4 - 3x^3 - 8x^2 + 7x + 2)}$

The remainder will be 314.

9. $(x-2)^2 = 16 \Rightarrow x-2 = \pm 4 \Rightarrow x = -2, 6$
 $(y-3)^2 = 25 \Rightarrow y-3 = \pm 5 \Rightarrow y = -2, 8$

Maximum possible value of $\frac{x}{y} = \frac{-2}{-2} = 1$

10. If one of the roots of the equation is $4 + 3i$, then the other root will be $4 - 3i$
 $\Rightarrow m = -8, n = 25$

Hence, $m + n = 17$

11. For the equation to be completely divisible by $(x-2)$ the value of m has to be -4.

12. Sum of the roots = $\alpha + \beta = \frac{-b}{a}$

Product of the roots = $\alpha \times \beta = \frac{c}{a}$

For the roots $(3\alpha + 1)$ and $(3\beta + 1)$, the sum of roots = $3(\alpha + \beta) + 2 = \frac{-3b + 2a}{a}$

Product of the roots =

$$9\alpha\beta + 3(\alpha + \beta) + 1 = \frac{9c - 3b + a}{a}$$

The new equation =

$$ax^2 + (2a - 3b)x + 9c - 3b + a = 0$$

13. $G(3) = 9 + 5 = 14$

$$F(G(3)) = 3(14) + 2 = 44$$

14. $F(x) = G(x)$

$$3x + 2 = x^2 + 5 \Rightarrow x^2 - 3x + 3 = 0$$

There is no real solution for x. Hence, option [a]

15. It is of the form $a^3 + b^3 + c^3$, also here $a + b + c = 0$

$$\text{Hence, } a^3 + b^3 + c^3 = 3abc$$

$$3(x-y)(y-z)(z-x)$$

16. Let the total no. of Rabbits be x and total no. of hens be y.

$$4x+2y=110$$

$$x+y=40$$

$$\Rightarrow x=15, y=25$$

Hence, the no. of hens = 25

$$17. T_3=ar^2, \quad T_9=ar^8$$

$$T_3 T_8 = a^2 r^{10} = (ar^5)^2 = 4 \Rightarrow ar^5 = 2$$

Now, product of first eleven terms

$$= a^{11} r^{55} = (ar^5)^{11} = 2^{11} = 2048$$

$$18. T_3=ar^2=42, \quad T_6=ar^5=210$$

$$\frac{T_6}{T_3} = \frac{ar^5}{ar^2} = r^3 = \frac{210}{42} = 5$$

Let T_9 be x

$$\frac{T_9}{T_6} = \frac{x}{210} = \frac{ar^8}{ar^5} \Rightarrow x = 210 \times 5 = 1050$$

$$19. \text{First term} \Rightarrow a = 2$$

$$\text{Last term} \Rightarrow l = 238$$

$$\text{Sum of all the terms} = \left(\frac{a+l}{2} \right) \times n = 4800$$

$$\Rightarrow \left(\frac{2+238}{2} \right) \times n = 4800$$

$$\Rightarrow n = 40$$

$$20. \text{Sum of all the 4 terms} \Rightarrow 4a + 6d = 68$$

$$\text{Also, } \Rightarrow (a+d)(a+2d) = a(a+3d) + 32$$

$$\Rightarrow 2d^2 = 32 \Rightarrow d = 4$$

$$\text{So, } 4a + (6 \times 4) = 68$$

Hence, the first term of the series = 11

21. It forms a GP since, every time the height is reduced to 50%.

So, total distance

$$= 100 + 2 \left(\frac{50}{1-0.5} \right) = 100 + 200 = 300m$$

22. Let the first term be x and ratio be r .

$$x = 3a - 8$$

$$xr = 3a - 7$$

$$xr^2 = 3a - 5$$

$$\text{So, } (3a-7)^2 = (3a-5)(3a-8)$$

$$\Rightarrow a = 3$$

Hence, $x = 1$ and $r = 2$

$$\text{So, } T_6 = xr^5 = 2^5 = 32$$

23. The n^{th} term will be $n(n+1)^2$

So the sum of the series will be

$$\sum n(n+1)^2 = \sum (n^3 + 2n^2 + n)$$

$$\Rightarrow \left(\frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right)$$

$$\Rightarrow \frac{n(n+1)}{2} \left(\frac{n(n+1)}{2} + \frac{2(2n+1)}{3} + 1 \right)$$

$$\Rightarrow \frac{10 \times 11}{2} \left(\frac{10 \times 11}{2} + \frac{42}{3} + 1 \right)$$

$$\Rightarrow 55(55 + 14 + 1) = 3850$$

24. The common difference in series S_1 is 6 and that in series S_2 is 5. So, there will be a common term after every 30 numbers. So in 500 numbers, the no. of common terms = 17, since, the first term is also same.

$$25. \text{Arithmetic mean} \Rightarrow \frac{a+b}{2} = 10 \Rightarrow a+b=20$$

$$\text{Geometric Mean} \Rightarrow \sqrt{ab} = 8 \Rightarrow ab = 64$$

Hence, the two numbers are 4 and 16 and the smaller number is 4.

26. This is a Geometric Progression with $a=1$ and $r = 2/x$

$$\text{So, Sum of the series} \Rightarrow \left(\frac{1}{1 - \frac{2}{x}} \right) = 3$$

$$\Rightarrow \left(\frac{x}{x-2} \right) = 3 \Rightarrow x = 3$$

27. Let the value of the series be x .

$$x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}$$

$$\Rightarrow x^2 - 2 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$$

$$\Rightarrow x^2 - 2 = x \Rightarrow x = -1, 2$$

-1 is neglected as the sum of positive numbers cannot be a negative number. Hence, $x = 2$

28. The difference of the squares of two nos. x and y is 77.

$$\text{Hence, } x^2 - y^2 = 77$$

$$(x-y)(x+y) = 7 \times 11$$

$$\text{Hence, } x-y=7 \text{ and } x+y=11$$

$$x=9, y=2$$

$$\text{Or, } x^2 - y^2 = 77$$

$$(x-y)(x+y) = 1 \times 77$$

$$\text{Hence, } x-y=1 \text{ and } x+y=77$$

$$x=39, y=38$$

Hence, none of the numbers can be 11.

29. Here, the value of $b^2 - 4ac < 0$. Hence, there will be no real roots for the equation.

30. For $\left(a + \frac{1}{a}\right)$ to be maximum, a should be equal to $\frac{1}{a}$.

$$\text{Hence, } a=1. \text{ Max value of } \left(a + \frac{1}{a}\right) = 1+1=2$$

$$31. a=\sqrt{b}=\sqrt[3]{c}=\sqrt[4]{d}=\sqrt[5]{e}=\sqrt[6]{f}$$

$$\Rightarrow b=a^2, c=a^3, d=a^4, e=a^5, f=a^6$$

Hence,

$$a \times b \times c \times d \times e \times f = a \times a^2 \times a^3 \times a^4 \times a^5 \times a^6 = a^{21}$$

32. Sum of cubes of first n natural numbers =

$$\frac{n^2(n+1)^2}{4}$$

Sum of squares of first n natural numbers =

$$\frac{n(n+1)(2n+1)}{6}$$

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

$$\text{Ratio} = \frac{10^2 \times 11^2}{4} : \frac{10 \times 11 \times 21}{6} : \frac{10 \times 11}{2} = 55:7:1$$

$$33. \frac{x^3 + 3x}{3x^2 + 1} = \frac{341}{91}$$

Applying Componendo & Dividendo

$$\frac{x^3 + 3x + 3x^2 + 1}{x^3 + 3x - 3x^2 - 1} = \frac{341 + 91}{341 - 91}$$

$$\Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{432}{250} = \frac{216}{125} = \frac{6^3}{5^3} \Rightarrow \frac{(x+1)}{(x-1)} = \frac{6}{5}$$

Applying Componendo & Dividendo

$$\Rightarrow \frac{2x}{2} = 11 \Rightarrow x = 11$$

34. Let 5^x be y .

$$\text{Then, } \frac{5^x + 100}{5^x + 75} = \frac{y + 100}{y + 75} = \frac{5}{4}$$

$$\Rightarrow y = 25 \Rightarrow 5^x = 25$$

$$\text{Hence, } x = 2$$

$$35. \frac{1}{2}(a^x - a^{-x}) = \frac{1}{4}(a^x + a^{-x})$$

$$\Rightarrow 2\left(a^x - \frac{1}{a^x}\right) = \left(a^x + \frac{1}{a^x}\right)$$

Let a^x be y

$$\Rightarrow 2(y^2 - 1) = (y^2 + 1)$$

$$\Rightarrow y^2 = 3 \Rightarrow y = 3^{\frac{1}{2}}$$

$$\text{Hence, } a^x = 3^{0.5}$$

So, $a = 3$ and $x = 0.5$

$$36. \text{Let } \frac{\log A}{x-y} = \frac{\log B}{y-z} = \frac{\log C}{z-x} = k$$

Hence,

$$\log A = k(x-y), \log B = k(y-z), \log C = k(z-x)$$

$$\log A + \log B + \log C = \log(A \times B \times C) =$$

$$= kx - ky + ky - kz + kz - kx = 1$$

Since, $\log 1 = 0$. So, $A \times B \times C = 1$

$$37. \log 2 = 0.301$$

$$\log_2 x = 40$$

$$\text{Also, } \log_2 x = \frac{\log x}{\log 2} = 40$$

$$\Rightarrow \log x = 12.04$$

Hence, x has 13 digits.

38. The n^{th} term will be $n(n+1)2(n+1)$.

Hence, the sum will be

$$\sum n(n+1)(2n+2) = 2 \sum (n^3 + 2n^2 + n)$$

$$\Rightarrow 2 \left(\frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right)$$

$$\Rightarrow n(n+1) \left(\frac{n(n+1)}{2} + \frac{2(2n+1)}{3} + 1 \right)$$

$$\Rightarrow 100 \times 101 \left(\frac{100 \times 101}{2} + \frac{402}{3} + 1 \right)$$

$$\Rightarrow 10100(5050 + 134 + 1) = 52368500$$

39. $S_{15} = \frac{15}{2}(2a+14d)$

$$S_{25} = \frac{25}{2}(2a+24d)$$

Hence, $\frac{15}{2}(2a+14d) = \frac{25}{2}(2a+24d)$

$$\Rightarrow 3a + 21d = 5a + 60d$$

$$\Rightarrow 2a + 39d = 0$$

Sum of first 40 terms = $\frac{40}{2}(2a+39d) = 0$

40. $x = 0.60 = \frac{3}{5}$

$$(1+x-x^2) = 1 + \frac{3}{5} - \frac{9}{25} = \frac{31}{25}$$

$$\frac{x^3}{1+x} = \frac{27/125}{1+3/5} = \frac{27}{125} \times \frac{5}{8} = \frac{27}{200}$$

$$(1+x-x^2) + \frac{x^3}{1+x} = \frac{31}{25} + \frac{27}{200} = \frac{11}{8}$$

41. For the values of equation to be integer, the value of $48/x$ should be an integer. The value of $48/x$ will be an integer for all values of x that are positive and negative factors of 48. There are 10 positive factors of x and hence, there are 20 values of x for which the value of equation will be an integer.

42. $3^x - 3^{x-3} = 702$

$$\Rightarrow 3^x - 3^x \cdot 3^{-3} = 702$$

$$\Rightarrow 3^x \left(1 - \frac{1}{27} \right) = 702$$

$$\Rightarrow 3^x = \frac{702 \times 27}{26} = 27 \times 27 = 3^6$$

Hence, $x = 6$

43. $1 \times 5 \times 5^2 \times 5^3 \times 5^4 \times \dots \times 5^x = 5^{55}$

$$\Rightarrow 5^{(1+2+3+\dots+x)} = 5^{55}$$

$$\Rightarrow (1+2+3+\dots+x) = 55$$

$$\Rightarrow x = 10$$

44. For natural no. solutions the value of x and y cannot be 0.

For, $4x + 5y = 120$

There is a natural no. value for y for every multiple of $x=5$. So, the possible values of x for which y is a natural no. is $x = 5, 10, 15, 20, 25$. Hence, there are 5 natural number solution of the equation.

45. Put the value of x, y and z as 1. The sum of the coefficient of the expansion will be $4^5 = 1024$.

46. Let the no. of Red, Yellow and White roses be a, b and c respectively.

$$a + b + c = 100$$

Also, $a + 10b + 100c = 1000$

So, $9b + 99c = 900$

$$b + 11c = 100$$

So, c can vary from 0 to 9. Since, none of them can be 0. So there are a total of 9 ways in which the flowers can be selected.

47. On putting different values and eliminating the options we get that $x^3y^2 > x^2y^3$. The reason for this behaviour is that we have multiplied with squares of x and y on both the sides. Hence, the negative effect of the numbers did not affect the result.

48. $T_7 + T_{10} = T_{11}$

$$\Rightarrow a + 6d + a + 10d = a + 11d$$

$$\Rightarrow a + 5d = 0$$

Hence, T_6 or Sixth term of the series is 0.

49. Let the cost of 1 Pen be Rs. x and that of 1 pencil be Rs. y

$$\Rightarrow 6x + 3y = 100$$

$$\Rightarrow 3x + 6y = \frac{100}{1.25} = 80$$

Adding both the equations,

$$9x + 9y = 180$$

$$\Rightarrow x + y = 20$$

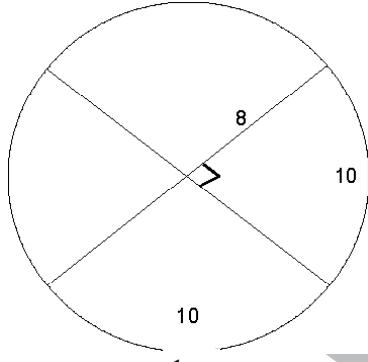
50. Since, the first page was an even no, so the last page should be an odd no. if he tears it after the page no. 546 and should be an even number if he tears the page previous to that of page no. 546.
Hence, the only possible page no. is 645.

■ ■ ■

Chapter 12

1. Since, the diagonals of the rhombus are perpendicular to each other and the length of the diagonal is equal to the diameter of the circle.

Hence,



$$\text{Area of Rhombus} = \frac{1}{2} \times \text{Product of diagonals}$$

$$\Rightarrow \frac{1}{2} \times 8 \times 8 = 32 \text{ sq.m.}$$

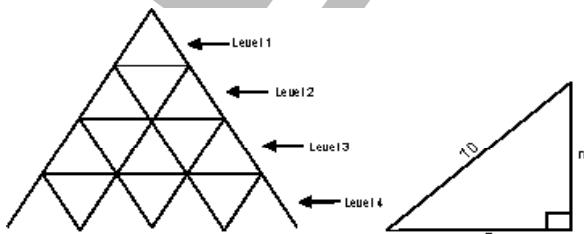
2. There will be 9 cubes on each face having painted on 1 side. Hence, there will be a total of 54 cubes painted on only one side.

3	2	2	2	3
2	1	1	1	2
2	1	1	1	2
2	1	1	1	2
3	2	2	2	3

3. There cubes numbered 2 will be painted on two sides. There are 12 cubes on one face but it also includes 3 cubes of each of the adjoining faces.

Hence, total no. of cubes
 $= 12 + 12 + 6 + 6 = 36$ cubes.

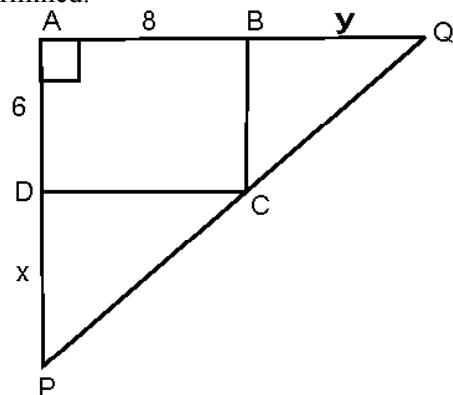
4. The number of cards required for the upper level = 3 Remaining cards = 23, 2nd level will contain 6 cards, so, remaining cards = 17. The third level will need 9 cards so, cards remaining = 8. The last level will require 8 cards since card for the base is not needed.



$$\text{Height of each level} = (10)^2 - (5)^2 = 5\sqrt{3}$$

$$\text{Total height} = 5\sqrt{3} \times 4 = 20\sqrt{3}$$

5. The line PQ passes through C, but we do not know the proportion in which the point C cuts the line PQ. So, there are many possibilities cannot be determined.



$$6. \quad \begin{aligned} & (r-4)^2 + (r-2)^2 = r^2 \\ & \Rightarrow r^2 + 16 - 8r + r^2 + 4 - 4r = r^2 \\ & \Rightarrow r^2 - 12r + 20 = 0 \\ & \Rightarrow r = 2, 10 \end{aligned}$$

Hence, the radius of the table can be either 2 or 10.

7. Consider the following figure.

The area is 80.

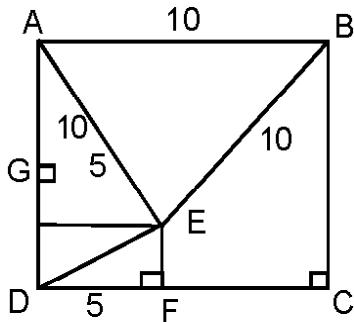
$$\Rightarrow \frac{1}{2} AC \times BD = 80$$

$$\Rightarrow BD = \frac{80 \times 2}{20} = 8 \text{ in } \triangle BDC$$

$$DC = \sqrt{(10^2) - (8^2)} = 6, AD = 14$$

$$\therefore X^2 = (14)^2 + 8^2 \Rightarrow \sqrt{260} = 2\sqrt{65}$$

8.



Join ED and draw

$EF \perp DC$ and $EG \perp AD$.

Now $DF = GE = 5 \text{ cm}$

$$\text{In } \triangle AGE, (AG)^2 = (10)^2 - 5^2 \Rightarrow AG = 5\sqrt{3}$$

$$\text{Hence, } GD = 10 - 5\sqrt{3}$$

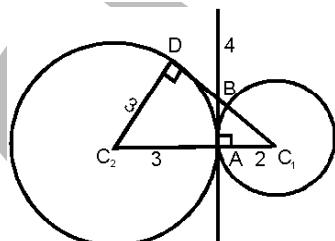
$$EF = 10 - 5\sqrt{3}$$

In $\triangle EFD$

$$(ED)^2 = (10 - 5\sqrt{3})^2 + (5)^2$$

$$ED^2 = 100(2 - \sqrt{3})$$

9.

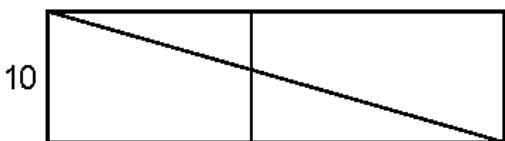


$\triangle C_1 C_2 D$

$$C_1 D = \sqrt{(5)^2 - (3)^2} = 4$$

$\triangle C_2 DC$, is similar to $\triangle BAC$,

$$\text{So, } \frac{C_2 D}{BA} = \frac{DC_1}{AC_1} \Rightarrow \frac{3}{BA} = \frac{4}{2} \Rightarrow BA = 1.5 \text{ cm}$$



10.

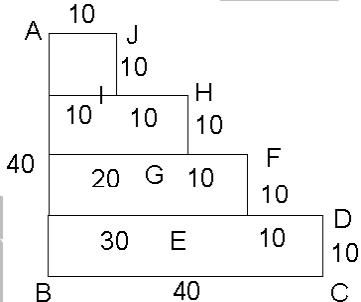
The distance travelled by insect

$$= \sqrt{30^2 + 30^2}$$

$$= \sqrt{1800}$$

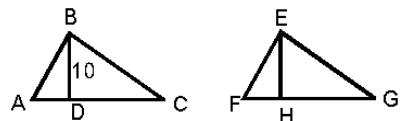
$$= 30\sqrt{2} \text{ m}$$

11. The length of the point of intersection of diagonals and point A is 1.5 cm.



$$\begin{aligned} \text{Area} &= 10 \times 10 + 20 \times 10 + 30 \times 10 + 40 \times 10 \\ &= 100 + 200 + 300 + 400 \\ &= 1000 \text{ m}^2 \end{aligned}$$

12. Consider the following figures



Since, ABC and EFG are similar,

$$\frac{BD}{EH} = \frac{AC}{FG}$$

$$\frac{AC}{FG} = \frac{5}{4}$$

$$\text{Let } AC = 5x, \text{ then } FG = 4x$$

If Area of bigger triangle

$$= \frac{1}{2} \times 10 \times 5x = 100 \Rightarrow x = 4$$

Area of other Triangle

$$= \frac{1}{2} \times 8 \times 4 \times 4 = 100$$

= 64 sq. units

If Area of smaller triangle

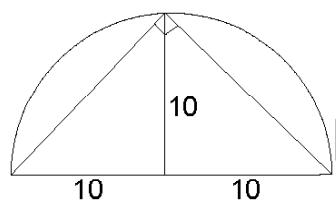
$$= \frac{1}{2} \times 4x \times 8 = 100$$

Area of other triangle

$$= \frac{1}{2} \times \frac{5 \times 100}{16} \times 10 = 156.25 \text{ sq. units}$$

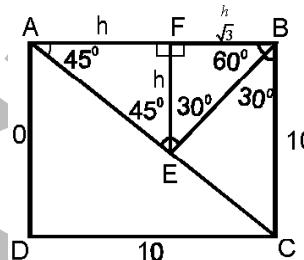
Hence both the answer are correct.

13. For area to be maximum, the largest side of the triangle should be the diameter of the semi-circle and it should be a right angled triangle.



$$\text{Hence, area of } \Delta = \frac{1}{2} \times 20 \times 10 = 100 \text{ sq. cm}$$

14. Draw a perpendicular from AB to E. Let the height of the perpendicular be h .



Since $\triangle AFE$ is an Isosceles triangle $AF = FE = h$

$$FB = h \tan 30^\circ = \frac{h}{\sqrt{3}}$$

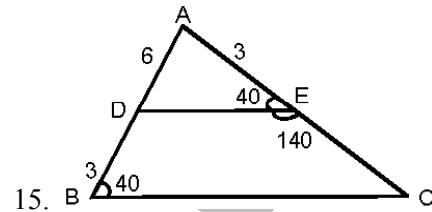
Also

$$\text{So, } h + \frac{h}{\sqrt{3}} = 10$$

$$(\sqrt{3} + 1)h = 10\sqrt{3}$$

$$\Rightarrow h = \frac{10\sqrt{3}}{\sqrt{3} + 1}$$

$$\text{Area of } \triangle ABE = \frac{1}{2} \times \frac{10\sqrt{3}}{\sqrt{3} + 1} \times 10 = \frac{50\sqrt{3}}{\sqrt{3} + 1}$$



Now, $\angle DEC = 140^\circ$.

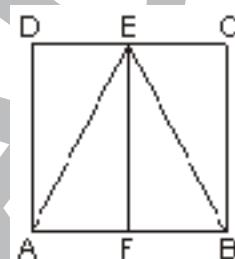
So, $\angle AED = 40^\circ$

So, $\triangle AED$ and $\triangle ABC$ are similar (A.A similarity)

$$\text{Hence, } \frac{AD}{AC} = \frac{AE}{AB}$$

$$\Rightarrow AC = \frac{6 \times 9}{3} = 18 \text{ cm.}$$

Hence, $EC = 15 \text{ cm.}$



16.

ABCD has to be a square and assume $AB = BC$

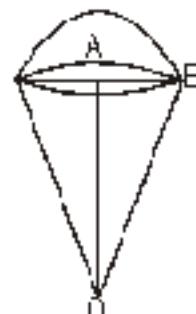
$= CD = DA = 2x$ and thus $DE = EC = x$. So,

$$AE = BE = \sqrt{(2x)^2 + x^2} = \sqrt{5x^2} = x\sqrt{5}$$

$$\text{Thus, } x\sqrt{5} + x\sqrt{5} + 2x = 6$$

$$\text{and } x = \frac{3}{\sqrt{5} + 1} \text{ and } x^2 = \frac{9}{6 + 2\sqrt{5}}$$

17. Let h be the height of conical portion OA and r be the radius of the cone. Then, $h = 4r$



Now

$$\pi \times 6^2 \times 15 = 10 \left(\frac{1}{3} \pi r^2 \cdot OA + \frac{2}{3} \pi r^3 \right)$$

$$\Rightarrow 540\pi = 10 \frac{\pi}{3} r^2 (4r + 2r)$$

$$\Rightarrow 540 = \frac{10}{3} r^2 (4r + 2r)$$

$$\Rightarrow 54 = \frac{1}{3} r^2 \cdot 6r$$

$$\Rightarrow r^3 = \frac{54}{2} \Rightarrow r = 3 \text{ cm.}$$

18. Let common base = xm .

Area of parallelogram = $x \cdot 100$

Now, the altitude of triangle = ym

$$\therefore x \cdot 100 = \frac{1}{2} \times x \times y$$

$$\Rightarrow y = 200 \text{ m.}$$

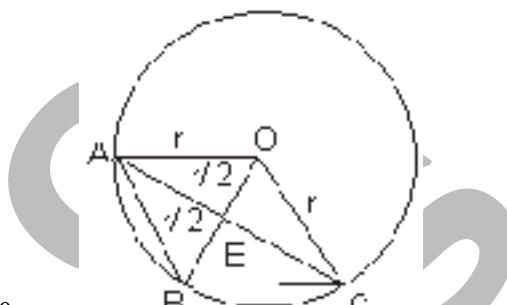
19. Let the diagonals of the rhombus be x and y

$$\therefore \left(\frac{x}{2} \right)^2 + \left(\frac{y}{2} \right)^2 = x^2$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{4} = x^2$$

$$\Rightarrow \frac{y^2}{4} = \frac{3x^2}{4}$$

$$\Rightarrow \frac{y^2}{x^2} = \frac{3}{1} \quad \therefore \frac{y}{x} = \sqrt{3} : 1$$



20.

$$EC^2 = r^2 - \frac{r^2}{4} = \frac{3r^2}{4}$$

$$\Rightarrow EC = \frac{\sqrt{3}r}{2}$$

$$\therefore 2EC = AC = \sqrt{3}r$$

$$\text{Now, } 32\sqrt{3} = \frac{1}{2} \times OB \times AC$$

$$\Rightarrow \frac{1}{2} \times r \times \sqrt{3}r$$

$$\Rightarrow 64 = r^2$$

$$\Rightarrow r = 8 \text{ m.}$$

21. Volume of Box

$$= [48 - (8 \times 2)][36 - (8 \times 2)] \times 8 = 5120$$

$$22. l + b + h = 19 \text{ m} \quad \dots(i)$$

$$\sqrt{l^2 + b^2 + h^2} = 11 \text{ m.}$$

$$\Rightarrow l^2 + b^2 + h^2 = 121 \text{ m.} \quad \dots(ii)$$

Now squaring equation (i)

$$(l + b + h)^2 = 19^2$$

$$\Rightarrow l^2 + b^2 + h^2 + 2lb + 2bh + 2lh = 361$$

$$\Rightarrow 121 + 2(lb + bh + lh) = 361$$

$$\Rightarrow 2(lb + bh + lh) = 361 - 121 = 240$$

\therefore cost of painting = 240×10

= Rs. 2400.

$$23. \text{ Total surface area} = 2\pi rh + (2\pi r^2)$$

$$\Rightarrow 2\pi r(h + r)$$

$$\Rightarrow 2 \times \frac{22}{7} \times 7 \times 57 = 2508 \text{ m}^2$$

\therefore amount required = 2508×10 = Rs. 25080

24. Required volume of steel

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} [(4.5)^3 - (4)^3]$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} [91.125 - 64]$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times 27.125$$

$$\Rightarrow 56.83 \text{ cm}^3$$

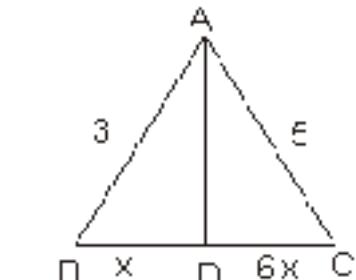
$$25. \text{ Required radius} = \sqrt{\frac{\frac{22}{7} \times 21 \times 21 \times 36}{\frac{1}{3} \times \frac{22}{7} \times 12}} = 63 \text{ cm.}$$

$$26. \text{ Number of squares} = \frac{5 \times 5}{1 \times 1} = 25$$

Width = $550/2.5 = 220$.

27. Required area = $\pi r^2 \times \frac{\theta}{360}$
 $= \frac{22}{7} \times 21 \times 21 \times \frac{90}{360}$
 $= 346.5 \text{ m}^2$

28. The bisector of a triangle divides the opposite side in the ratio of sides containing the angle.



$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{x}{6-x} = \frac{3}{5}$$

$$x = 2.25 \text{ cm.}$$

29. We have a box which holds 4 by 4 cans. Diameter of 1 can = 2. Hence one side = 8.

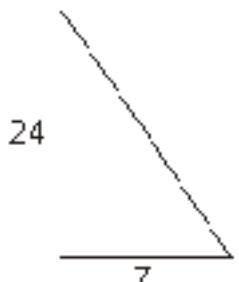
Area = 8 by 8 = 64.

30. $\pi r^2 = 9856, r = 56$.

31. Volume of the wall minus mortar = 95% of $(110400) = 1048800$

Divide this by vol. of one brick = (2208) to get the number of bricks. Answer 475.

32. Weight = 0.5 [Outer - Inner box]
 $= 0.5 [81312 - 60320]$
 Solving we get weight = 10.5 kg.

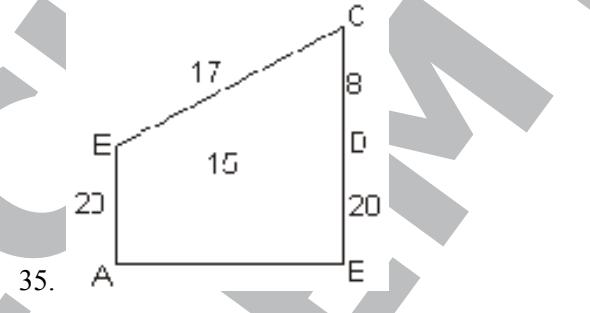


33. slant height = 25.
 Area of cloth required = $\pi rl = 550$.

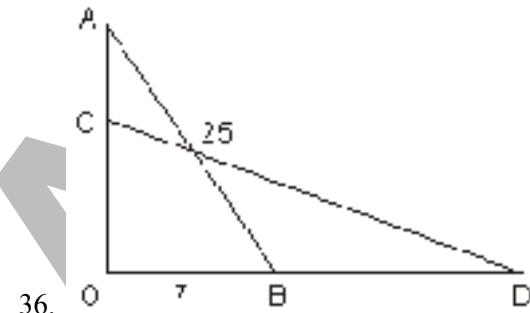
34. The sides of the Δ are 3, 4 and 5.

$$= \text{area} \Rightarrow r \times 6 = \frac{1}{2} \times (3 \times 4)$$

Then $r s$
 Hence $r = 1$



35. $BD = 20$, hence $DC = 8$. Hence $ED = 15$



$AB = 25$, $OB = 7$, hence $AO = 24$.

Now $CD = 25$, $CO = 20$ hence $OD = 15$.

This means that $BD = 8$.

37. If the diagonals intersect at right angles, then all the chords constituting the sides are equal.

38. Direct theorem

39. Dimension of each cube = $42 \times 42 \times 42$

$$\text{Radius of sphere} = \frac{42}{2} = 21 \text{ cm.}$$

Volume of waste material

$$= (42)^3 - \frac{4}{3}\pi \times (21)^3$$

$$= 74088 - 38808 = 35280 \text{ cm}^3$$

$$\therefore \text{side of square} = \sqrt{\frac{35280}{5}} = 84 \text{ cm.}$$

40. Sum of areas of other triangles

$$= \frac{30}{4} + \frac{30}{4^2} + \frac{30}{4^3} + \dots \infty$$

$$= \frac{30}{4} \left[1 + \frac{1}{4} + \frac{1}{4^2} + \dots \infty \right]$$

$$= \frac{30}{4} \left[\frac{1}{1 - \frac{1}{4}} \right] = \frac{30}{4} \times \frac{4}{3} = 10 \text{ cm}^2$$

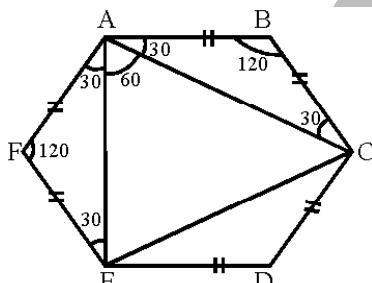
Required ratio = 30 : 10 = 3 : 1

41. $0^\circ = \frac{n}{4}$ [as string is making 4 turns]
 $OC = n$ [side of cube]

$$\Rightarrow (O'C)^2 = \left(\frac{n}{4}\right)^2 + n^2 = n^2 + \frac{n^2}{16} = \frac{17n^2}{16}$$

$$\Rightarrow O'C = \frac{\sqrt{17}}{4} n$$

$$\text{Length of string} = 4 \times \frac{\sqrt{17}}{4} n = \sqrt{17} n$$



42.

Every interior angle of a hexagon = 120°

In $\triangle ABC$:

$AB = BC$ and $\angle ABC = 120^\circ$

$$\Rightarrow \angle BAC = \angle BCA = \frac{180^\circ - 120^\circ}{2} = 30^\circ.$$

$$\Rightarrow \angle EAC = 120^\circ - (30^\circ + 30^\circ) = 60^\circ$$

$\triangle ACE$ is an equilateral triangle with side
 $= 2 \times x \cos 30^\circ$ [x = side of Hexagon]

$$\text{Area of } \triangle ACE = \frac{\sqrt{3}}{4} \text{ Side}^2 = \frac{\sqrt{3}}{4} \times (2x \cos 30^\circ)^2$$

$$= \sqrt{3} \left(x^2 \times \frac{3}{4} \right) = \frac{3\sqrt{3}}{4} x^2$$

$$\text{Area of Hexagon} = 6 \frac{\sqrt{3}}{4} x^2$$

$$\text{Ratio of Area} = \left(\frac{3\sqrt{3}x^2}{4} \right) : \left(6 \frac{\sqrt{3}}{4} x^2 \right)$$

43. Given $AD = CD = BC$

$$\Rightarrow \angle DAC = \angle ACD$$

[$\triangle ACD$ and $\triangle BCD$ are Isosceles]

$$\& \angle CDB = \angle CBD$$

$$\text{Let } \angle DAC = x = \angle ACD$$

$$\text{Let } \angle CDB = y = \angle DBC$$

$$\Rightarrow x + y = 96^\circ \dots (1)$$

[96° is the exterior angle of ABC and exterior angle = Sum of interior angles].

$$\& \angle BCD = 180 - 2y = 180 - (96 + x)$$

$$\Rightarrow 2y = 96 + x$$

$$\Rightarrow 2y - x = 96$$

$$\Rightarrow 2y - x = x + y$$

$$\Rightarrow y = 2x$$

Now putting values of $y = 2x$ in (1) we get
 $y = \angle DBC = 64^\circ$.

44. If the smaller side of the trapezium is 16 (i.e. half of it is 8), and the height is 3, then the radius 4 is given by $r^2 = 8^2 + 3^2 = 73$. Hence the area of the circles = $\pi r^2 = 73\pi$.

45. Using pythagoras, we get

$$r^2 = x^2 + 18^2 = 24^2 + (42 - x)^2$$

$$\Rightarrow \sqrt{(18)^2 + (24)^2} = 30$$

46. Area of semicircle = lateral surface area of cone.
So ratio = 1 : 1

47. Since circumference of the base of this new cone is reduced to half, so the radius of the base will also reduce to half.

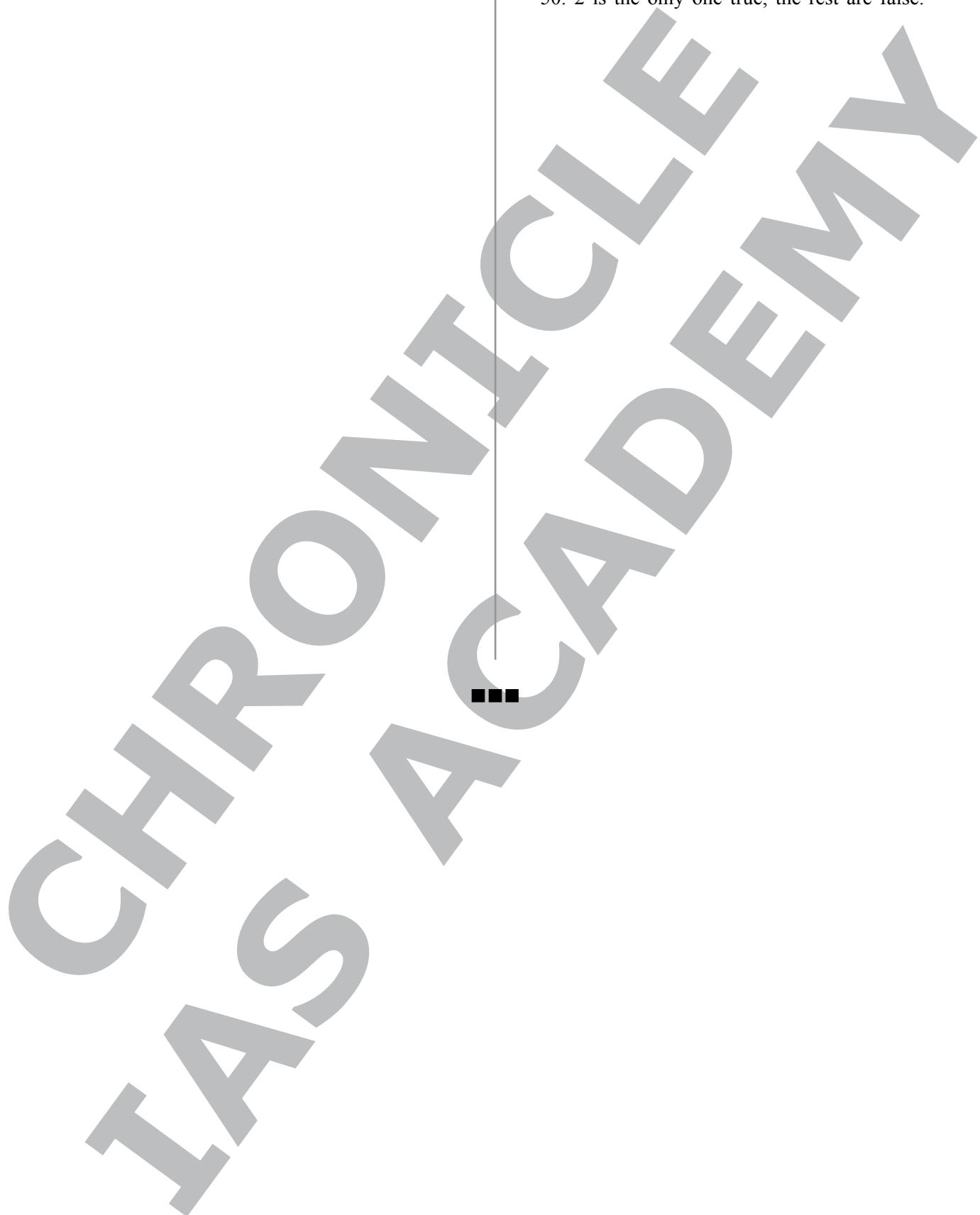
48. Side of square = 6.

Hence length is 9, because area is 36.

Perimeter = $18 + 8 = 26$.

49. Distance = $b \tan 60^\circ / \tan 30^\circ = 3b$

50. 2 is the only one true, the rest are false.



Chapter 13

- No. of ways of selecting five boys = ${}^{10}C_5$.
This arrangement includes equal no. identical arrangements.
Hence the required no. of ways
 $= {}^{10}C_5 / 2 = 126$
- Consider all the vowels together as one letter.
So, now there are 6 letters with two 'C' in it. It can be arranged in $\frac{7!}{2!}$.
Also, the three vowels can be arranged themselves in $3!$ ways.
Hence, No. of ways the letters can be arranged considering all vowels together $= \frac{7! \times 3!}{2!}$.
- Boys can sit on the circular table in $3!$ ways.
The girls can be seated in 4 places in $4!$ ways.
So, total no of ways $= 3! \times 4!$.
- There will be two cases; either the unit's place is zero or not.
Case-1: unit's place is zero: number of possible numbers $= 9 \times 8 \times 7 \times 6 = 3024$
Case-2: unit's place is not zero : number of possible numbers $= 8 \times 8 \times 7 \times 6 \times 4 = 10752$
Hence, total number of numbers are $3024 + 10752 = 13776$
- Number of selecting 5 balls with no conditions on colour of the ball $= 2^5 = 32$.
Out of these ways, there will be two cases when either all the balls are red or all the balls are black. Hence, these two cases are to be Subtracted.
Hence number of ways will be $32 - 2 = 30$
- Total possible outcomes $= 2^{10}$.
Since half the outcomes will have 3 outcome as head and half will have 3 outcome as tail.
So, total no of possible outcomes if 3rd outcome is Head $= 2^{10} / 2 = 2^9$

- First letter can be posted in 3 ways and second letter can be posted in 3 ways.
Similarly all the letters can be posted in three ways. So total no of ways $= 3^5$.
- Possible movements allowed:

1 st Step	2 nd Step	3 rd Step	4 th Step	5 th Step
1	2	2	2	2

In the 1st step he can move only forward. In 2nd he can move either forward or backwards, and so on.
So total possible ways $= 1 \times 2 \times 2 \times 2 \times 2 = 16$ ways.

- There are 32 Black and 32 White square boxes.
A black square can be selected in 32 ways.
There are 4 White squares in each row and each column which cannot be selected.
Hence, there are 24 White squares from which 1 can be selected in 24 ways.
So total no of ways of selection $= 32 \times 24 = 768$ ways.
- For selecting a rectangle we need to select two horizontal and two vertical lines which can be done in ${}^9C_2 \times {}^9C_2 = 1296$ ways.
- No. of ways in which any 3 boys can be selected $= {}^{20}C_3 = 1140$. Now these 3 boys can be arranged in $3!$ way hence

- Number of ways of reaching from A to D will be $\frac{6!}{4! \times 2!} = 15$

13. Number of ways of going from A to H if there

$$\text{was no gap would have been } \frac{12!}{7! \times 5!} = 792$$

Number of ways of going from A to D through

$$\text{the gap will be } \frac{6!}{3! \times 3!} \times \frac{6!}{4! \times 2!} = 300$$

Hence, number of required ways will be 492

14. The no. of digits that can be placed at the places are:

1 st digit	2 nd digit	3 rd digit	4 th digit
4	9	8	7

$$\text{Total such nos. } = 4 \times 9 \times 8 \times 7 = 2016$$

15. There are a total of 17 stations including starting and last terminals. Any one from any station can go to 16 different stations. So a total of $17 \times 16 = 272$ tickets need to be printed.

16. 16 boys can sit in $16!$ ways, but this includes repetition of each 4 times when they are sitting on 4 sides of a table. Hence the total no of ways

$$= \frac{16!}{4}.$$

17. A necklace of 16 beads can be arranged in $\frac{15!}{2}$ ways since a necklace is same when turned upside down. Now it has 5 and 3 similar beads. Hence the total no of distinct necklaces that can be made

$$\frac{15!}{2 \times 5! \times 3!}.$$

18. The no of 5 flag signals created by 8 different flags = 8P_5 .

19. A team of 5 basketball players can be selected from 13 players = ${}^{13}C_5$. Now there can be 5 different captains in each team. Hence the total no of ways = ${}^{13}C_5 \times 5$.

20. There are two colored socks. So, he must pick at least three in order to ensure one pair of similar colored socks.

21. From 15 points ${}^{15}C_2$ lines can be drawn. There are 5 collinear points so 5C_2 lines will give rise to a single line. So there will be ${}^{15}C_2 - {}^5C_2 + 1 = 96$ distinct lines.

22. No. of triangles that can be formed from 15 points is ${}^{15}C_3$. Since there are 5 collinear points so 5C_3 triangles cannot be formed. So total no of triangles that can be formed = $455 - 10 = 445$.

23. The simplest way to do is to find the nos with no 5 in it and subtract it from the total no of 3 digit nos.

The no. of digits that can be placed at the places are:

1 st digit	2 nd digit	3 rd digit
8	9	9

Total of no. of nos. with no 5 in it are $= 8 \times 9 \times 9 = 648$.

So total no. of nos. with at least one 900 - 648 = 252

24. Total number of ways will be

$$3 \times 5 \times {}^7C_4 \times {}^8C_4 \times 4! \times 4! = 8! \times 15$$

25. Let us first calculate the total number of possible words using these alphabets and then we will subtract the cases where N's are together.

Total number of possible words = $\frac{5!}{2!} = 60$

Words with N's together will be $4! = 24$.

Hence number of words satisfying the conditions will be 36

26. 5 books can be arranged in $5!$ ways.

Half the times the English book is above the Hindi book and half the times Hindi book is above the English book.

So, total no of ways in which the books can be

$$\text{arranged } = \frac{5!}{2} = 60 \text{ ways.}$$

27. Words starting with A = 4!
 Words starting with H = 4!
 Words starting with L = 4!
 Words starting with R, followed by A and H = 2.
 Since U appears first in RAHUL, so it will be the last word starting with RAH.
 So the total no of ways = $4! + 4! + 4! + 2 = 74$.

28. Case 1: Students answer 3 out of first 4 questions
 No. of ways = ${}^4C_3 \times {}^6C_4 = 60$.
 Case 2: Students answer 4 out of first 4 questions
 No. of ways = ${}^4C_4 \times {}^6C_3 = 20$
 Total no. of ways = $60 + 20 = 80$ ways.

29. There is a choice of adding or not adding a particular color with each of the color. So the 7 colors can be mixed in 2^7 ways which also includes not mixing any of the color. So total no of distinct colors that can be made = $128 - 1 = 127$.

30. 1 chairman can be selected from 50 delegates in ${}^{50}C_1$ ways.
 1 President can be selected from remaining 49 in ${}^{49}C_1$ ways.
 2 Vice - Presidents can be selected from remaining 48 in ${}^{48}C_2$ ways.
 Total no. of ways = ${}^{50}C_1 \times {}^{49}C_1 \times {}^{48}C_2 = 2763600$ ways.

31. 6 single men will shake hand with 15 ladies
 $6 \times 15 = 90$
 7 single ladies with shake hands with 8 married men (Case of single ladies shaking hand with single men is already considered in single men case) = 56
 8 married men shakes hands with 7 married ladies = 56
 Total no. of ways = $90 + 56 + 56 = 202$.

32. Number of possible selections out of every type of fruit is 5.

Hence, total number of possible selections are 5^6 . But, since at least one fruit is to be selected, number of ways will be $5^6 - 1$

33. Number of words will be

$${}^6C_1 \times 1! + {}^6C_2 \times 2! + {}^6C_3 \times 3! + {}^6C_4 \times 4! + {}^6C_5 \times 5! + {}^6C_6 \times 6! = 1956$$

34. A fruit can be added or not added to form a flavor. Hence the total no of flavors that can be made by using 5 fruits = $2^5 - 1 = 31$. (Since a flavor with no fruit cannot be made).

35. Types of grapes = 3
 Types of Age of grapes = 3
 Types of Age of wine = 3
 Total types of wines that can be made = $3 \times 3 \times 3 = 27$ ways.

36. There exactly two options with superhero with respect to each and every step except the last step. Thus, number of ways will be $2^9 = 512$

37. Total no of possible outcomes = 2^{10} .
 Since, the outcomes of 4 of the coins are fixed.

$$\text{Hence, total no of outcomes} = \frac{2^{10}}{2^4} = 64.$$

38. There are 15 flips and 10 coins. So, all the coins cannot face heads again. Hence, option (c) is eliminated.

Also, for 6 Heads and 4 Tails there has to be 2 flips of 6 Head facing coins and 1 flip of 4 Head facing coin which comes out to be 16. Hence, option (b) is also eliminated.

If 3 coins are flipped thrice and 3 coins are flipped twice then we get 7 Heads and 2 Tails.
 Hence, option (a).

39. The first two digits can be either of 9 or 8. Hence there are two possible ways of selecting first two digits. The next four digits can be selected in 8C_4 ways. The remaining 4 digits can be put in the remaining 4 places.

$$\text{So, no. of possible nos.} = 2 \times {}^8C_4.$$

40. 5 speakers can speak in $5!$ ways = 120 ways.
 Half of the times a particular speaker will address before the other particular speaker and half of the times he'll speak after him. So, the no of ways the conference can be addressed = $120/2 = 60$.
 41. 1st letter can be posted in 7 different ways. 2nd letter in 7 ways... 5th letter in 7 ways.

Hence, total no. of ways = $7 \times 7 \times 7 \times 7 \times 7 = 7^5$.

42. Possibilities

5	4	3
5	6	1

$\rightarrow 3! \text{ ways}$
 $\rightarrow 3! \text{ ways}$

Total no. of ways = $3! + 3! = 12$ ways.

43. 4 boys can be selected from 8 in 8C_4 ways. The other 4 boys will form another team. Since, in this each group of 4 is considered twice. eg. Suppose there are 8 boys 1 to 8. One time a group will be formed from 1-4 and other time a group will be formed from 5-8. Both these groups will form same two pairs of teams. Hence, the total no. of ways = ${}^8C_4 / 2$ ways.

44. No. of ways Asia 11 team can be formed will include case in which 3 players each are selected from 2 teams and 5 players from 3rd team and 3 players from one team and 4 players from two teams.

Case 1 : 3 players from two teams and 5 from one team

No. of ways = ${}^9C_3 \times {}^9C_3 \times {}^9C_5 \times {}^3C_1$

Case 2 : 4 players from two teams and 3 from one team
 No. of ways = ${}^9C_4 \times {}^9C_4 \times {}^9C_3 \times {}^3C_1$
 Total no. of ways
 $= {}^3C_1 ({}^9C_4 \times {}^9C_4 \times {}^9C_3 + {}^9C_3 \times {}^9C_3 \times {}^9C_5)$

45. Total no. of ways will be = $5 \times 8 \times 8 \times 7 = 2240$
 46. 16 boys can be seated in $16!/4$ ways around rectangular table having 4 chairs on each side.
 Total no. of ways = $16!/4 = 4 \times 15!$

47. Four boys can be selected amongst 12 in ${}^{12}C_4$ ways, and other 4 boys can be selected amongst remaining 8 in 8C_4 ways. Hence, there will be 3 groups of 4 boys each. Now, each group is counted 3! times.
 So the total no. of ways = ${}^{12}C_4 \times {}^8C_4 / 3!$ Ways.

48. Each flag can be used or cannot be used for flashing signals. Hence, there are two ways a flag can be used for signaling. Total no. of ways = $2^5 - 1 = 31$ (since 0 flags cannot be used for signaling).

49. The first letter can be posted in any of the 4 post boxes. Therefore, it has 4 choices.
 Similarly, the second, the third, the fourth, the fifth and the sixth letter can each be posted in any of the 4 post boxes.
 Therefore, the total number of ways the 6 letters can be posted in 4 boxes is

$$4 \times 4 \times 4 \times 4 \times 4 = 4^6$$

50. 5 boys can be seated in 7 chairs in ${}^7P_5 = {}^7C_5 \times 5!$ ways.

Hence, option [a] and [c] both are correct. Hence, answer is [d].



Chapter 14

1. As 11th toss is independent event so probability of getting a head = $1/2$

2. Prob. both will win the match $= \frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$

3. Irrespective of the sum of the outcomes of first four dices, there will be only one way in the last throw which can give us a multiple of 6. Hence, the required probability is $1/6$.

4. Since, we do not know that which are 6 stolen cards, the probability of picking a ace of spade will not change and it will remain $1/52$

5. The probability of winning a bet $= \frac{1}{10}$
If 1,00,000 bets of Re. 1 are made then there may be 10,000 winners winning Rs. 9 each.
Hence the total income $= 100000 - 90000 = 10000$.

6. There are 900, 3-digit nos.
Out of which 90 numbers have both unit's and ten's digit equal.
The remaining nos. have equal probability of ten's digit greater or less than unit's.

Hence 410 nos. will have unit's digit more than ten's digit.

$$\text{Probability} = \frac{410}{900} = 0.45$$

7. There is equal probability of 6 getting placed before or after 4.

$$\text{Probability} = \frac{1}{2}$$

8. The probability that both will speak true

$$= \frac{2}{5} \times \frac{1}{5} = \frac{2}{25}$$

The probability that both will speak lie

$$= \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$$

The probability that both will speak the same

$$\text{thing} = \frac{2}{25} + \frac{12}{25} = \frac{14}{25}$$

9. The sum of two odd is even and sum of two even is also even.

So probability of sum of two positive integers is even = $1/2$.

10. Number of ways in which sum of the outcomes of two dices is 7 are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1). Out of these six ways, there are exactly two favorable conditions and hence, the answer will be $1/3$.

11. The product will be odd if all three are odd whose probability $= \frac{50C_3}{100C_3}$

The probability that the product will be even $= 1 - \frac{50C_3}{100C_3}$

12. There is equal prob of getting any no. from 1 to 6 on all the dices.

So the average of all the possible outcome is the most probable sum.

So the outcomes will have five 1's, five 2's, five 3's, five 4's, five 5's and five 6's.

$$\text{Average} = \frac{105}{6} = 17.5$$

Hence probable sum can be either 17 or 18.

13. Probability that exactly two of the three are of same colour = Prob that two are white and one is black + Prob 2 is black and 1 is white.

$$= \frac{^{32}C_2 \times ^{32}C_1}{^{64}C_3} + \frac{^{32}C_2 \times ^{32}C_1}{^{64}C_3} = \frac{2 \times ^{32}C_2 \times ^{32}C_1}{^{64}C_3}$$

14. probability that at least any of them lives for another 10 years = $P(\text{Ramu lives}) \times P(\text{Rani Dies}) + P(\text{Ramu Dies}) \times P(\text{Rani Lives}) + P(\text{Ramu lives}) \times P(\text{Rani Lives})$

$$= \left(\frac{1}{4} \times \frac{2}{3} \right) + \left(\frac{3}{4} \times \frac{1}{3} \right) + \left(\frac{1}{4} \times \frac{1}{3} \right) = \frac{1}{2}$$

15. Prob of getting a no. in a single throw is $\frac{1}{6}$ and

$$\text{that in three throws } = \frac{1}{216}$$

Prob of getting any same no. three times(that no. can be any of the 6 no.) $= 6 \times \frac{1}{216} = \frac{1}{36}$

16. Total no. of heads in both the coins = 3

Number of heads in the second coin = 2

Prob of selecting second coin = 2/3

17. There are 2 C's in CHRONICLE. So on every 9 words formed there will be 2 words containing both C's together. Hence the prob $= \frac{2}{9}$.

18. The prob that he'll be at his original place = P(that he moves forward and then backwards) + P(that he moves backwards and then forward) $= 2 \times 0.4 \times 0.6 = 0.48$

19. $P(\text{Question is solved}) = 1 - P(\text{Question is not solved})$

$$= 1 - \left(\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \right) = \frac{1}{2}$$

20. $P(\text{Thief will be killed in 3 bullets}) = 1 - P(\text{Thief is not killed by all the 3 bullets})$

$$= 1 - \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) = \frac{7}{8}$$

21. No. of quadrilaterals formed by joining the 4 points of a Hexagon $= {}^6C_4 = 15$

No. of parallelograms possible = 3.

$$\text{Prob } = \frac{3}{15} = \frac{1}{5}.$$

22. No. of triangles formed by joining the 3 points of a Hexagon $= {}^6C_3 = 20$

No. of equilateral triangles possible = 2

$$\text{Prob } = \frac{2}{20} = \frac{1}{10}.$$

23. The point can be 1 cm away from all the four sides. Hence the difference of the area of original box and area of the box formed by reducing the sides by 2 cm i.e. (3×2) is the area where the point can be placed.

Probability

$$\frac{\text{Area of box}(5 \times 4) - \text{Area of box}(3 \times 2)}{\text{Area of box}(5 \times 4)} = \frac{20 - 6}{20} = 0.7$$

24. If the thread is cut, the cut can lie in any of the four regions ranging from 0 to $\frac{1}{4}$ th, $\frac{1}{4}$ th to $\frac{1}{2}$, $\frac{1}{2}$ to $\frac{3}{4}$ th or $\frac{3}{4}$ th to 1.

Now the length of the greater part will be always more than 3 times the smaller part if the cut is in the 1st or 4th part.

Hence Probability $= \frac{1}{2}$.

25. The no. of values, of outcomes of each dice can be so that all the outcomes are different are:

6	5	4	3	2	1
---	---	---	---	---	---

No. of possible outcomes $= 6^6$

$$\text{Prob } = \frac{6!}{6^6}.$$

26. 3 heads and 3 tails can be arranged in $\frac{6!}{3! \times 3!}$.

Prob of 3 heads and 3 tails

$$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) \times \frac{6!}{3! \times 3!} = \frac{1}{2^6} \times \frac{6!}{3! \times 3!}$$

27. No. of picture cards $= 4 \times 3 = 12$

$$\text{Prob } = \frac{{}^{12}C_1 \times {}^{40}C_2}{{}^{52}C_3}$$

28. $P(6 \text{ on the dice}) = \frac{1}{6}$

$$P(\text{Tails on the coin}) = \frac{1}{2}$$

$P(6 \text{ on the dice or Tails on the coin or both})$

$$= \frac{1}{6} \times \frac{1}{2} + \frac{5}{6} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{2} = \frac{7}{12}$$

29. If the ball transferred from the first box was black then the no. of Black balls in the second box becomes 7 and the total no. of balls becomes 12. Hence the probability of picking a black ball from the second box $= \frac{7}{12}$.

30. Minimum Capacity of the buckets = 5 ltrs.

$$\text{Maximum Capacity of the buckets} = \frac{49}{9} \text{ ltrs.}$$

$$\text{Range of capacity} = \frac{49}{9} - 5$$

Prob that the capacity is more than 5.25

$$= \frac{\frac{49}{9} - 5.25}{\frac{49}{9} - 5} = \frac{7}{16}$$

$$31. \text{Prob} = \frac{^{50}C_2 + ^{30}C_2 + ^{20}C_2 + ^{50}C_2 + ^{50}C_2}{^{100}C_2}$$

$$= \frac{3 \times ^{50}C_2 + ^{30}C_2 + ^{20}C_2}{^{100}C_2} = \frac{43}{199}$$

32. First letter can be put in 5 ways.

Second letter can be put in 4 ways.

And so on last letter can be put in only 1 way.

So no. of ways all letters can be put = 5!.

Prob that all the letters are put on correct envelope

$$= \frac{1}{5!}$$

33. Total number of ways, these three chocolates can be given to these three boys will be 3^3 .

Number of ways, these three chocolates are given in such a way that all of them receives exactly one chocolate each will be 3!.

$$\text{Hence, required probability will be } \frac{3!}{3^3} = \frac{2}{9}$$

34. No. of ways questions can be attempted = $^{13}C_{10}$

Probability of attempting exactly 4 questions from

$$\text{Section A} = \frac{^5C_4 \times ^8C_6}{^{13}C_{10}}$$

35. P(Ramu) = 0.5

P(Rani) = 0.4

P(Exactly one of them is alive)

$$= (0.5 \times 0.4) + (0.5 \times 0.6) = 0.5$$

36. If letters of the word CHRONICLE are arranged then 2 out of every 9 words will start from letter C.

Hence, the prob = 2/9

37. For the program to be approved it has to pass all the three stages, the probability of which is

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

38. No. of employee taking a leave = No. of Manager taking a leave + No. of Operator taking a leave

$$= 0.033 \times 100 = 30 \times 0.04 + 70 \times P(O)$$

$$\Rightarrow P(O) = 0.03$$

39. When the first mango is good then no. of good mangoes remaining = 5.

Total no. of mangoes remaining = 9

Prob(Good Mangoes) = 5/9

40. Out of 100 questions 25 questions can have 'c' as the correct option.

So he would have got = $25 \times 4 - 75 = 25$ marks.

41. The minimum possible score is -100 and maximum possible score is 400. Every integral score between -100 and 400 is possible except 399, 398, 397, 394, 393 and 389. Hence number of possible scores is $501 - 6 = 495$.

42. There are three types of balls. So at least 4 balls must be picked in order to ensure at least 2 balls of 2 colors.

43. If we look the worst case scenario, then we will realise that even after picking 12 balls, there is a possibility that we are left with three balls each of different colours. But once, we have picked the thirteenth ball, at least one ball of each colour is picked.

44. The worst case scenario is that first 6 balls are Yellow, then next 5 balls are Black.

So, to ensure at least one ball of each colour 12 balls must be fetched from the bag.

45. No. of even no. tickets = 13 (Since starting and ending tickets both are even numbered).

No. of tickets having a no. multiple of 5 = 5

No. of tickets having both an even no. as well as a no. multiple of 5 = 3.

Total no. tickets having either of Even no. or a multiple of 5 = $13 + 5 - 3 = 15$

$$\text{Probability of its selection} = \frac{15}{25} = \frac{3}{5}$$

46. Sum of 9 in a throw of two dices can be obtained by: 3 + 6, 4 + 5, 5 + 4, 6 + 3

No. of favourable outcomes = 4

No. of possible outcomes = 36

Probability = $4/36 = 1/9$

47. The possible outcomes are:-

H H H
H H T
H T H
H T T
T H H
T H T
T T H
T T T

No. of favourable outcomes = 4

No. of possible outcomes = 8

Probability = 1/2

48. The two nos. can be

Odd Odd

Odd Even

Even Odd

Even Even

No. of favourable outcomes = 2

Probability = 1/2

49. $P(\text{He'll fail}) = 1 - P(\text{He'll Pass})$

$$= 1 - (0.3 + 0.4 + 0.1 + 0.1)$$

$$= 0.1$$

50. Probability of selecting any bag is 0.5.

$$\text{Required Probability} = \frac{1}{2} \times \frac{4}{10} + \frac{1}{2} \times \frac{1}{6} = \frac{17}{60} = 0.28$$



Chapter 15

1. $4^{100} = 2^{200}$

Half of $2^{200} = 2^{200}/2 = 2^{199}$

2. $144 = 6^{a+b} \times 4^{2a} \times 9^b = 2^{(5a+b)} \times 3^{3b+a} = 2^4 \times 3^2$

$\rightarrow 5a+b=4, 3b+a=2$

$\rightarrow a=5/7$

3. The value of $\sqrt[3]{3}$ is highest.

4. The log values cannot be negative.

5. $2^3 = 8$

$3^2 = 9$

Since $2^3 < 3^2$, then it is obvious that $3^{200} > 2^{300}$.

6. Let $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$

$\Rightarrow \log a = k(b-c), \log b = k(c-a), \log c = k(a-b)$

$\Rightarrow \log a + \log b + \log c = k(b-c+c-a+a-b) = 0$

$\Rightarrow \log(abc) = 0$

$\Rightarrow abc = 1$

7. Using formula, $a^{(\log_a x)} = x$

Hence, $2^{\log_2 5} = 5$

8. $\log 3 = 0.477, \log_3 x = 32$

$\frac{\log x}{\log 3} = 32$

$\Rightarrow \log x = 32 \times \log 3 = 15.26$

Hence no. of digits = 16

9. $\log 1.2 = \log \left(\frac{12}{10} \right) = \log 12 - \log 10$

$= 2\log 2 + 3 - 1 \Rightarrow 2a + b - 1$

$= 2\log 2 + 3 - 1 \Rightarrow 2a + b - 1$

$= \log(3 \times 4) - 1 = \log 4 + \log 3 - 1 = \log 2^2 + \log 3 - 1$

10. $\Rightarrow \log_3 \left(2 \times \left(2^x - \frac{7}{2} \right) \right) = \log_3 (2^x - 5)^2$

$\Rightarrow \left(2 \times \left(2^x - \frac{7}{2} \right) \right) = (2^x - 5)^2$

$\Rightarrow (2^{x+1} - 7) = 2^{2x} + 25 - 5 \times 2^{x+1}$

$\Rightarrow 6 \times 2^{x+1} - 2^{2x} = 32$

$\Rightarrow 2^x(12 - 2^x) = 2^3(12 - 2^3)$

$\Rightarrow x = 3$

11. $\log_{10} x = x^{-1}$

$\Rightarrow 10^{\frac{1}{x}} = x^1$

Now, going through the options, none of the options satisfy the condition.

12. $\log_x 10 = \frac{\log 10}{\log x} = \frac{1}{\log x}, \log_y 10 = \frac{1}{\log y}$

$\log_z 10 = \frac{1}{\log z}$

If x, y, z are in GP

$\frac{y}{x} = \frac{z}{y} \Rightarrow 2\log(y) = \log(x \times z)$

$\Rightarrow 2\log y = \log x + \log z$

$\Rightarrow \frac{2}{\log_y 10} = \frac{1}{\log_x 10} + \frac{1}{\log_z 10}$

Hence, they are in HP.

13. $\log x + \log(x+3) = 1$

$\Rightarrow \log(x \times (x+3)) = \log 10$

$\Rightarrow x^2 + 3x = 10$

$\Rightarrow x = 2, -5$

-5 is neglected as Log of negative nos. doesn't exist. Hence, $x = 2$

14. $(2^3)^4 = 8^4 = 2^{12}$

$$2^{3^4} = 2^{81}$$

Hence, $2^{3^4} > (2^3)^4$

15. Assume initial number of shares to be 100.

Thus, money received by selling the shares will be

$$100 \times \text{Rs } 120 = \text{Rs } 12,000$$

Number of new shares purchased will be

$$\frac{\text{Rs } 12,000}{\text{Rs } 80} = 150$$

Thus, initial income is—

$$100 \times \text{Rs } 100 \times 12\% = \text{Rs } 1,200$$

New income is $150 \times \text{Rs } 100 \times 10\% = \text{Rs } 1,500$

Hence, % increase in the profit will be 25 %.

16. Assume his savings to be Rs 1000.

Hence, number of shares purchased will be

$$\frac{\text{Rs } 1000}{\text{Rs } 40} = 25$$

Hence, value of his shares will be

$$25 \times \text{Rs } 10 = \text{Rs } 250$$

His returns will be 25% of Rs 250 = Rs 62.5.

Hence return on investment will be

$$\frac{\text{Rs } 62.5}{\text{Rs } 1000} = 6.25\%$$

17. Number of shares purchased will be

$$\frac{\text{Rs } 1,00,000}{\text{Rs } 12.5} = 8,000$$

Hence value of shares will be

$$8,000 \times \text{Rs } 10 = \text{Rs } 80,000$$

Hence, his annual income will be 12 % of Rs 80,000 = Rs 9,600.

18. Number of shares with the man was

$$\frac{\text{Rs } 1,200}{\text{Rs } 12} = 100$$

Hence, his investment was $100 \times \text{Rs } 120 = \text{Rs } 12,000$.

19. There will not be any change in the income as the ratio of face value of the share and market value of the share is same in both the cases and moreover dividend paid by both the companies is same.

20. Number of shares sold was $\frac{\text{Rs } 96,000}{\text{Rs } 120} = 800$

Thus, total dividend to be paid is $800 \times \text{Rs } 100 \times 12\% = \text{Rs } 9600$

21. Earnings from shares=

$$\frac{\text{Investment}}{\text{Market value of share}} \times \frac{\text{Face value of share}}{\text{Percentage Dividend}} \times \frac{100}{100}$$

$$\text{Thus } \frac{I_s}{M_s} \times F_s \times \frac{D_s}{100} = \frac{I_m}{M_m} \times F_m \times \frac{D_m}{100}$$

Hence,

$$\frac{I_s}{I_m} \times \frac{M_m}{M_s} \times \frac{D_s}{D_m} = \frac{F_m}{F_s} = \frac{1,00,000}{80,000} \times \frac{a}{b} \times \frac{a}{b} = \frac{5a^2}{4b^2}$$

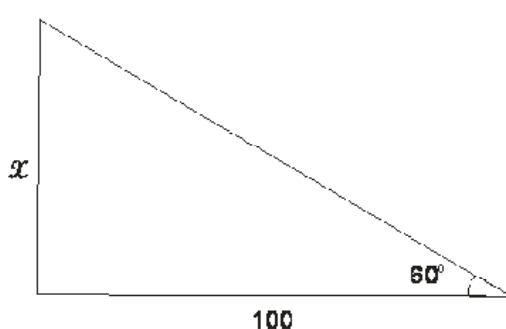
$$\tan 75^\circ = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \times \tan 30^\circ}$$

22. $1 \text{ radian} = \pi = 180^\circ$

So, $1^\circ = \pi/180$

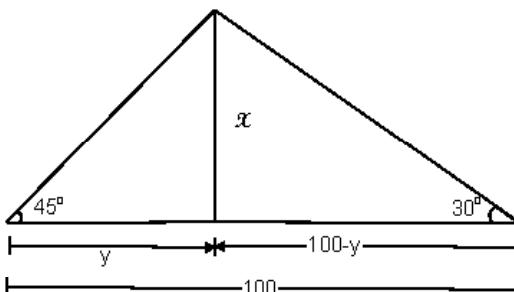
Hence, $\pi/180$ must be multiplied to given angle to calculate the radians.

24.



$$x = 100 \times \tan 60^\circ = 100\sqrt{3}$$

25.



$$\tan 45^\circ = \frac{x}{y}$$

$$\Rightarrow x = y$$

$$\tan 30^\circ = \frac{x}{100-y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{100-x}$$

$$\Rightarrow 100-x = \sqrt{3}x$$

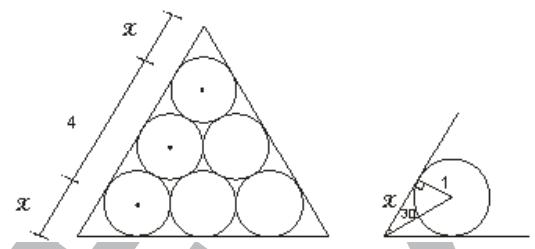
$$\Rightarrow (\sqrt{3}+1)x = 100$$

$$\Rightarrow x = \frac{100}{\sqrt{3}+1}$$

$$= \frac{100}{\sqrt{3}+1} \text{ m.}$$

Height of the building

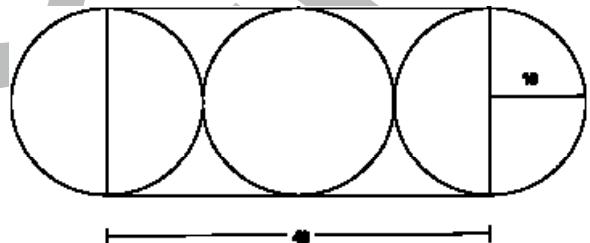
26.



$$\tan 30^\circ = \frac{1}{x}$$

Hence, length of the triangle = $4 + 2\sqrt{3}$

27.



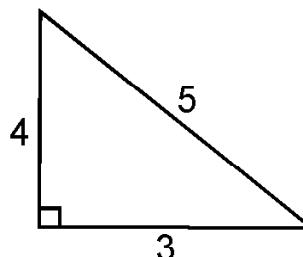
Length of the thread

$$= 2\pi r + 40 + 40$$

$$= 2\pi \times 10 + 80$$

$$= 20\pi + 80$$

28.



$$\sin \theta = \frac{p}{h} = 0.8 = \frac{4}{5}$$

$$\tan \theta = \frac{p}{b} = \frac{4}{3}$$

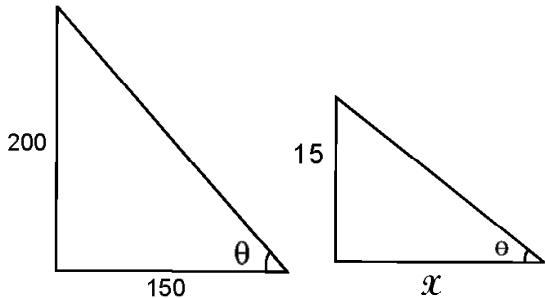
tan = sin / cos is the only one true.

30. $\sin x$ is an odd function and $\tan x$ is an odd function. So, the sum of two odd functions is an odd function. Hence, $(\sin x + \tan x)$ is an odd function

$$31. \frac{\sin x}{\cos x} = \frac{4}{7}$$

$$\frac{\sin x + \cos x}{\sin x - \cos x} = \frac{\frac{\sin x}{\cos x} + 1}{\frac{\sin x}{\cos x} - 1} = \frac{\frac{4}{7} + 1}{\frac{4}{7} - 1} = \frac{-11}{3}$$

32.

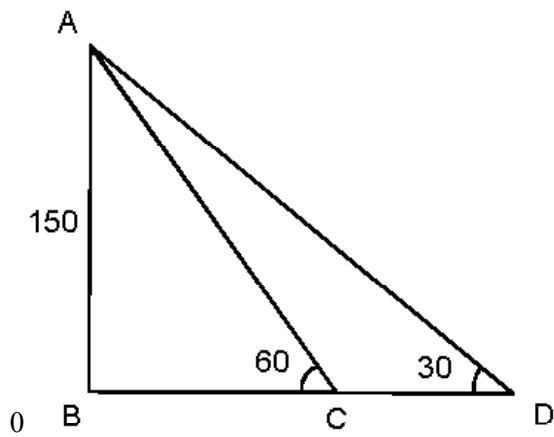


$$\tan \theta = \frac{200}{150}, \quad \tan \theta = \frac{15}{x}$$

$$\Rightarrow \frac{200}{150} = \frac{15}{x}$$

$$x = \frac{15 \times 150}{200} = 11.25 \text{ m.}$$

33.



$$\tan 30^\circ = \frac{150}{BD} \Rightarrow BD = 150\sqrt{3}$$

$$\tan 60^\circ = \frac{150}{BC} \Rightarrow BC = 150/\sqrt{3}$$

$$CD = BD - BC = 150 \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right)$$

$$= 150 \left(\frac{2}{\sqrt{3}} \right) \Rightarrow 100\sqrt{3} \text{ m.}$$

34. Let $AB = x$

$$\tan 30^\circ = \frac{x}{BD} \Rightarrow BD = \frac{x}{\tan 30^\circ}$$

$$\tan 60^\circ = \frac{x}{BC} \Rightarrow BC = \frac{x}{\tan 60^\circ}$$

$$CD = BD - BC = \sqrt{3}x - \frac{x}{\sqrt{3}} = 150$$

$$\Rightarrow \frac{2}{\sqrt{3}}x = 150$$

$$\Rightarrow x = 75\sqrt{3}$$

35. Let $AB = x$

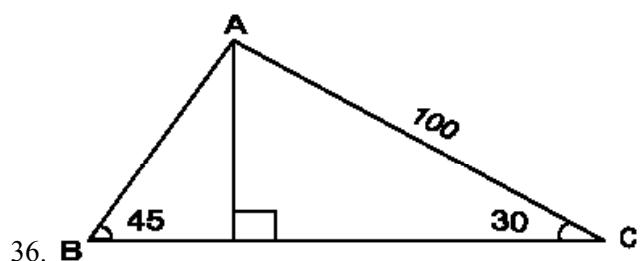
$$\tan 30^\circ = \frac{x}{BD} \Rightarrow BD = x\sqrt{3} \quad \dots\dots(i)$$

$$\tan 60^\circ = \frac{x}{150} \Rightarrow x = 150\sqrt{3} \quad \dots\dots(ii)$$

From (i) and (ii)

$$\Rightarrow BD = 150 \times 3 = 450$$

$$\text{Hence, } CD = 450 - 150 = 300 \text{ m.}$$



$$\tan 30^\circ = \frac{AD}{DC} \Rightarrow DC = 100 / \tan 30^\circ = 100\sqrt{3}$$

$$\tan 45^\circ = \frac{AD}{BD} \Rightarrow BD = AD = 100$$

$$\text{Hence, } BC = 100\sqrt{3} + 100 = 100(\sqrt{3} + 1)$$

$$37. BC = 100 \text{ m}$$

Let $AD = x$

$$\tan 45^\circ = \frac{AD}{BD} \Rightarrow BD = x$$

$$\tan 30^\circ = \frac{AD}{DC} \Rightarrow x = \frac{DC}{\sqrt{3}} = \frac{x}{\sqrt{3}}$$

$$\Rightarrow \frac{x}{\sqrt{3}} + x = 100$$

$$\Rightarrow x = \frac{100\sqrt{3}}{\sqrt{3} + 1}$$

38. $BD = 100m$

$$\tan 45^\circ = \frac{AD}{BD} \Rightarrow AD = 100m$$

$$\tan 30^\circ = \frac{AD}{DC} \Rightarrow DC = 100\sqrt{3}$$

39. Slope $= \tan \theta = \tan 60^\circ = \sqrt{3}$

$$m = \sqrt{3}$$

$$\text{Equation} \rightarrow (y - y_1) = m(x - x_1)$$

$$\Rightarrow (y - 4) = \sqrt{3}(x - 3)$$

$$\Rightarrow \sqrt{3}x - y = 3\sqrt{3} - 4$$

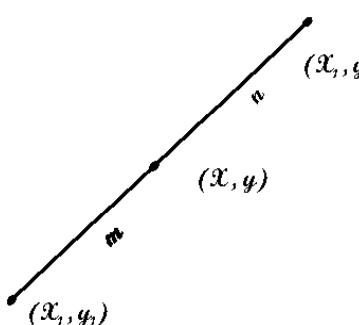
40. Equation of a line is $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$$\Rightarrow \frac{y - 3}{x - 5} = \frac{4 - 5}{4 - 3}$$

$$\Rightarrow (y - 3) = 5 - x$$

$$\Rightarrow x + y = 8$$

41.



The Section formula

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{2(-3) + 1(6)}{2+1}, \quad y = \frac{2(-7) + 1(8)}{2+1}$$

So, The point is $(0, -2)$.

42. Equation of a circle $= (x - a)^2 + (y - b)^2 = r^2$

$$\Rightarrow (x - 3)^2 + (y - 4)^2 = 3^2$$

$$\Rightarrow x^2 - y^2 - 6x - 8y + 16 = 0$$

43. $x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$

$$6 = \frac{4+a}{2}, \quad 9 = \frac{7+b}{2}$$

$$a = 8, \quad b = 11$$

$$a + b = 19$$

44. Since, the circumference is tangent to the y' axis, the radius of the circle is 5.

$$\text{So, eq. of the circle} = (x - 5)^2 + (y - 4)^2 = 5^2$$

$$= \frac{1}{2}[1(2y_1 - 7 \times 2)]$$

45. $3x + 5y = 7$

$$\text{Slope } m = \frac{-3}{5}$$

Using the options, and the equation,

$$y - y_1 = m(x - x_1)$$

The point $(-11, 8)$ satisfies the equation. Hence, $(-11, 8)$ passes through the line.

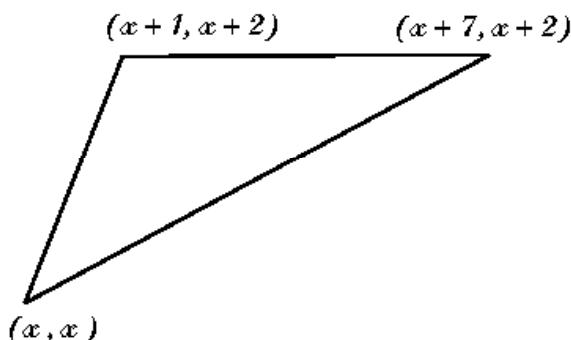
46. The statements (i) and (iv) are true as x is directly proportional to y . So, if the value of x increases the value of y also increases and vice versa.

47. The equation of a parabola is of the form of

$$y=ax^2+bx+c$$

So, the given equation is of parabola.

48.



$$\text{Area} = \frac{1}{2} \begin{vmatrix} x & x & 1 \\ x+1 & x+2 & 1 \\ x+7 & x+2 & 1 \end{vmatrix}$$

Applying property of determinant

$$R_2 \rightarrow R_2 - R_1 \text{ & } R_3 \rightarrow R_3 - R_1$$

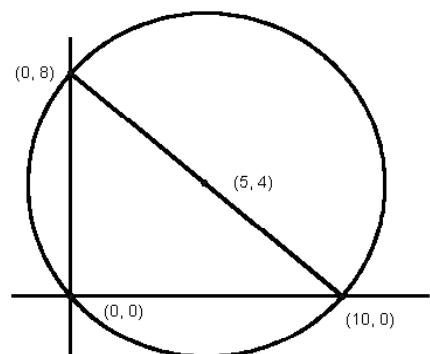
$$\text{Area} = \frac{1}{2} \begin{vmatrix} x & x & 1 \\ 1 & 2 & 0 \\ 7 & 2 & 0 \end{vmatrix} = \frac{1}{2} [1(2y_1 - 7 \times 2)]$$

the = -6

as area cannot be negative

so Area = 6 sq. unit

49.



The triangle is a right angled triangle. Hence, the circumcentre will lie on the midpoint of the hypotenuse. The mid point of hypotenuse is $(5, 4)$.

Hence, the co-ordinate of circumcentre is $(5, 4)$

50. Ellipse has a variable radius which is largest at the extreme ends and shortest at center.

