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Abduction and styles of scientific thinking

Mariana Vitti Rodrigues[1] · Claus Emmeche[1]

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Abstract
In philosophy of science, the literature on abduction and the literature on styles of thinking have existed almost totally in parallel. Here, for the first time, we bring them together and explore their mutual relevance. What is the consequence of the existence of several styles of scientific thinking for abduction? Can abduction, as a general creative mode of inference, have distinct characteristic forms within each style? To investigate this, firstly, we present the concept of abduction; secondly we analyze what is understood by styles of thinking; thirdly, we give some comments on abduction and styles of thinking by analyzing examples of scientific discovery or innovation within each style. We develop a case-based comparative investigation of creative aspects of abductive reasoning with examples drawn from different styles of scientific thinking and doing as understood by the Crombie/Hacking tradition. We argue that abduction, as a general mode of reasoning, can have a variety of specific expressions enabled and constrained by the styles of scientific thinking. Finally, we draw some conclusions on the relationship between abduction and styles of thinking suggesting that scientific discovery is a dynamical goal-directed activity within the scientific community that benefits from distinct styles of thinking and doing research.

Keywords Abduction · Discovery · Conceptual innovation · Styles of thinking · Peirce · Hacking

1 Introduction

What is the consequence of the existence of distinct scientific styles of thinking for abductive reasoning? Can abduction, as a general creative mode of inference, have distinct characteristic forms within each style? It is well-known that abduction is a mode of inference that is not confined to scientific investigation and not even to

✉ Mariana Vitti Rodrigues
mvittirodrigues@gmail.com

[1] Department of Science Education, University of Copenhagen, Øster Voldgade 3, Copenhagen, Denmark

the human realm (Magnani 2009; Hoffmeyer 2008; Vitti-Rodrigues and Emmeche 2017). In this sense, abduction can be understood as a broad kind of reasoning that integrates scientific as well as non-scientific thinking in the context of problem solving. In this paper, we claim that abduction performs different roles in scientific practice that can be investigated in the light of different styles of scientific thinking and doing.

Peirce, in his later writings, proposes the concept of abduction as the first phase of scientific investigation within what he calls "The method of methods" (CP 7.59). According to Peirce, the method of methods should answer "[…] how, with a given expenditure of money, time, and energy, to obtain the most valuable addition to our knowledge." (CP. 7.140). This method consists in attentive observation of some strange phenomena within the context of a scientific community (Peirce CP 5.331). It presupposes that real things exist and can be accessed by scientific investigation through the dynamic interrelationships between abduction, deduction and induction aiming at the constitution of solid habits. According to Peirce (CP 5.397), habit can be characterized as a rule of action: "First, it is something that we are aware of; second, it appeases the irritation of doubt; and, third, it involves the establishment in our nature of a rule of action, or, say for short, a *habit*". The scientific method for habit acquisition lies in the core of Peircean realism, undermining relativistic accounts of scientific practice. In this sense, Peirce's account of scientific investigation strengthens the argument against strong forms of relativism; however, the present paper will not address debates about relativism versus anti-relativism.[1]

As a first phase of scientific investigation, the aim of abduction is to incorporate a strange fact into a consolidated network of habits by suggesting hypotheses. The second phase relies on deduction, which focuses on the necessary consequences of the general hypothesis suggested by abduction, allowing it to become testable. Finally, induction enables the testing of the consequences deduced from the hypothesis generated via abduction. Once the truth of the hypothesis is confirmed, the original surprise, generated by the observation of a strange fact, is dissipated (CP 5.171). From the suggestion of explanatory hypotheses, abductive reasoning allows the growth of knowledge. According to Peirce, knowledge about an object amounts to the conceivable practical consequences that the object could cause in one's conduct (CP 5.402). In this context, we investigate how different styles of thinking can unveil different consequences of the object, disclosing some of its multiple layers. Peirce's analysis of abduction (presented in detail below) is famous and has generated its own literature of commentary and criticism, but attempts to use and confront the notion of abduction with examples from the history of science have been scarce, and few (if any) have explored abductive reasoning in relation to the well-known literature on distinct styles of thinking.

In 1994, Crombie issued a monumental work on styles of scientific thinking in the European tradition. The author proposes six styles of thinking: (1) Axiomatic—this

[1] The metaphysical and pragmaticist realism of Peirce differs from Hacking's so-called entity realism, though also Hacking (e.g., Hacking 1992a) claims a version of metaphysical realism. It is not, however, our aim to analyze their different versions.

style regards the postulation of statements. Postulation can be characterized in two ways: (1.1) the demonstrative power of geometry and arithmetic; and (1.2) the demonstrative power of logic through the Aristotelian syllogisms. (2) Experimental: This style is used to explore and control nature through observation and measurement of puzzling phenomena. (3) Hypothetical-Analogical: The purpose of this style is to elucidate unknown characteristics of the studied object through simulation, analogy and modeling of the phenomenon. (4) Taxonomy: comprises the logic of classification, whose goal is to establish differences and similarities between objects as being placed within a specific order of the phenomenal world. (5) Probability or statistical analysis: this style is based on the logic of decision in the investigation of patterns and regularities, such as the establishment of medical diagnoses. (6) Genealogy or historical derivation: this style of thinking integrates the analysis of the development of events, and in general, this method pursue patterns through the observation of historical facts (for example, historical, geological, or medical evidence) to postulate their possible sequence and the causes. According to Crombie, the first three styles focus on the study of individual regularities and the last three emphasize the study of populations of objects. Hacking (2002a, b, 2012), interpreting Crombie's work, proposes a seventh style: Laboratory. This style consists in building an apparatus that can generate new phenomena in order to help the scientific investigation. Generally Hacking emphasizes the intervening aspects of research and talk about Styles of Thinking and Doing (SoT&D). It is an open question to which extent styles are different among each other and if there are more styles of reasoning apart from Crombie's list.[2]

In this paper, we analyze the role of abductive reasoning within each style by using illustrative examples of scientific discovery and innovation. The literature within the history and philosophy of science (H&PS) on styles of thinking has tended to discuss these styles at a general level. Here, we expand the literature by drawing attention to the abductive aspects of inquiry by analyzing cases of discovery and innovation from each style. Both for Peirce scholars and for the H&PS community it is relevant to explore the creative aspects of scientific change and innovation. We hope to indicate how an abductive account inspired by Peirce's pragmatism clarifies contemporary practice within different styles of reasoning. For scholars familiar with Peirce's account on scientific investigation, it is not surprising to find abductive reasoning throughout the styles of thinking and doing. However, many of philosophers of science, following Popper (1959), disregard abduction as playing a role in scientific discovery. For this reason, we find it important to stress not only the inferential character of abduction but also its role in the generation of new hypotheses within each style of thinking.

[2] One part of Kusch (2010) criticism goes in the direction of questioning to what extent Crombie's list is a relevant starting point to the philosophical discussion of styles. Kusch claims that "Hacking has not offered a satisfactory rationale for individuating styles;" (ibid., p. 164) and "Sometimes he writes as if Crombie's list were definitive, sometimes he adds the laboratory style, sometimes he leaves open the possibility that any number of further styles might be discovered." (ibid., p. 170). It is beyond the scope of this paper to address this issue further.

Although Hacking is aware of the existence of abductive reasoning, he disregards its role in SoT&D. For instance, he claims that "Induction and deduction are not styles of reasoning: induction and deduction *preserve* truth, styles of reasoning *create the possibility* of truth values (2002a, p. 168, our highlights)". We believe that the creation of possibilities of truth values is allowed by abductive reasoning. In this context, SoT&D would be seen as constraints on the possibilities given by abduction in the constitution of meaningful hypotheses; however, at the same time, SoT&D enable abduction to take place in scientific investigation. In the following, we present the concept of abduction; in the third section, we analyze what is understood by styles of thinking and doing; in the fourth section, we provide seven illustrative examples of scientific discovery or innovation indicating the role of abductive reasoning within each style. Finally, we draw some conclusions pointing out future questions to be discussed.

2 Abductive reasoning

This section presents the main characteristics of abductive reasoning as proposed by Peirce and interpreted by Hintikka (1998), Paavola (2004a, 2004b, 2014) and Minnameier (2004, 2017).[3] Peirce, in his later writings, proposes the concept of abduction as the first phase of scientific investigation (CP 7.59). According to Peirce, abduction is a process that consists in attentive observation of some strange phenomenon within the context of a scientific community in order to generate explanatory hypotheses (Peirce CP 5.331).

In a famous quote, Peirce says that "Abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea" (CP 5.171). If abduction introduces new ideas by means of generation of new explanatory hypotheses, the role of deductive reasoning is to draw the conceivable consequences of a hypothesis given by abduction to be tested by induction: "Induction consists in starting from a theory, deducing from it predictions of phenomena, and observing those phenomena in order to see ***how nearly*** they agree with the theory." (Peirce CP 5.170). Peirce insists that abduction is a mode of inference that generates plausible hypotheses that, if true, may explain surprising facts. Accordingly, the logical form of an abductive inference can be express from this structure:

The surprising fact, C, is observed;
But if A were true, C would be a matter of course,
Hence, there is reason to suspect that A is true.

[3] It is common to find in the literature on philosophy of science the concept of abduction as synonymous with Inference to the Best Explanation (IBE) as well as attempts to formalize abductive reasoning (for a brief historical summary of these attempts see Aliseda 2017, p. 219). It is important to highlight that our analysis of abduction is based on Peirce's pragmaticism and as such we differentiate abduction from IBE (to be understood as a kind of induction—more precisely qualitative induction in Peirce's vocabulary). However, it is not the aim of this paper to discuss the difference between abduction and IBE (for an account of this difference see Minnameier 2004, 2017; Paavola 2006; and Campos 2011).

(Peirce CP 5.189)

It is well-known that the canonical structure of abductive reasoning brings many objections such as comparing abductive inference to the fallacy of affirming the consequent; or put into question the validity of abduction. Is abduction an inference or an insight? Is it a rational process or just blind guesses? How can we validate abduction? (Paavola 2004a, b; Anderson 1986, 1987; Hintikka 1998; Burks 1946; Fann 1970). In the present work, we do not discuss the details involved in the scholarly discussion of abductive reasoning. Here, inspired by Paavola, Hintikka and Minnameier, we present abduction as a method intrinsic to Peircean pragmatism, based on strategies when clues and constraints play a fundamental role.

In his methodological writings, Peirce proposes three claims about abduction intrinsically related to his pragmatism that operate as constraints on the scientific investigation. First, an abduced hypothesis should be subject to experimental test, according to Peircean pragmatism "the entire meaning of a hypothesis lies in its conditional experiential predictions: if all its predictions are true, the hypothesis is wholly true" (CP 7.203). Second, the hypothesis generated by abduction must be able to explain the strange, curious or surprising fact, avoiding future surprises (CP 5.197). The third claim is about the economy of research and comes as a question about "how, with a given expenditure of money, time, and energy, to obtain the most valuable addition to our knowledge?" (CP 7.140).

Within this third claim, Peirce emphasizes that a hypothesis should be simple, precise, and it should be potentially extended to a set of subjects where it might be applied. The precision of a hypothesis can be achieved through the known method of analysis and synthesis, and through a game of questions and answers that aims to constrain possible ways to solve a given problem (Peirce CP 7.220; Hintikka 1998; Paavola 2004b). The extension of a possible hypothesis to other domains may increase the breadth of a concept or term. A possible idea is to add the notion of *depth* as a fourth element of the "intrinsic value of a hypothesis". Could an abductively created new hypothesis about an object also increase the depth of the previous knowledge of properties attributed to the object? We think so, as by abduction we are searching for new properties of a given object, by means of constraining the possible characteristics attributed to it. If successful, this increases the depth of the concept about its properties.

Thus, we use the terms of breadth and depth according to a Peircean account of information: "By the ***informed breadth*** of a term, I shall mean all the real things of which it is predicable, with logical truth on the whole in a supposed state of information" (CP 2.407); "By the ***informed depth*** of a term, I mean all the real characters (in contradistinction to mere names) which can be predicated of it (with logical truth, on the whole) in a supposed state of information" (CP 2.408).

Inspired by Peirce, Hintikka, Paavola and Minnameier, we understand abductive reasoning as a dynamic process that involves deductive and inductive reasoning in the interior of a scientific community with its established rules and methods. This process takes into account the enabling power of strategic rules, the attentive observation of patterns, as well as the establishment and adjustment of previous constraints.

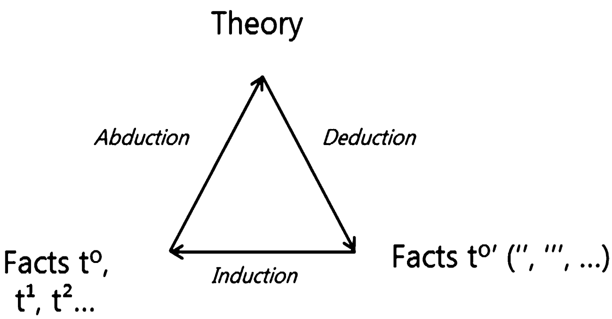

Fig. 1 Adapted from Minnameier (2004, p. 81)

According to Paavola, abduction is a goal-directed activity performed by a scientist who tries to anticipate phenomena or answer questions according to the constraints given by the background theory (Paavola 2004a, p. 270). In this context, the author states that "[…] in strategies, the reasoner tries to anticipate the counter-arguments, and to take into account all the relevant information, and this rules out very 'wild' hypotheses, except, when there is no other available, or alternatively, when these are presented simply as 'wild guesses'." (Paavola 2004a, p. 272). In the context of problem solving, Paavola stresses that, the existence of strategies implies that "[…] various ways of searching, anticipating, and combining "moves" of the inference are highlighted. This means that clues, constraints and search for connections are used while searching fertile hypotheses abductively (…)".

In this context, abductive reasoning can be understood as a strategic process that, through previous restrictions, directs the researcher in the formulation of reasonable hypotheses, i.e., claims that are genuine candidates for truth or falsehood within a given style of thinking (see next section). In this process, the elaboration of relevant questions can constitute a first constraint in the establishment of hypotheses that restricts the domain of explanation for a puzzling event. Although the establishment of constraints is essential to the generation of explanatory hypotheses, this may hinder the creative process of unraveling unknown patterns or account for what seems to be recalcitrant anomalies within the current conceptual framework. Thus, it would be interesting to develop a "meta-constraint" that evaluates the suggestion of hypotheses taking into account the restrictions used at first place. Such restrictions will be related to the style of thinking used, or to the combination of conceptual resources provided by several styles. We understand that the establishment of these two constraints may be the key to generate reasonable explanatory hypotheses that allow the anticipation of the possible effects of the phenomenon by broadening the domain of explanation of a theory within a scientific community and its experimental apparatus.

Minnameier (2004, 2017) offers a dynamic account of scientific investigation with some interesting insights to understand abduction as a method of generating new hypotheses. According to the author, the scientific process is recursive; it means that, "whenever the theory is applied, the whole process of its original discovery recurs, at least in principle, and by acting accordingly (or simply watching what happens), the theory is re-evaluated." (Minnameier 2004, p. 76). In Fig. 1 (adapted from Minnameier 2004, p. 81) expresses the dynamic process of scientific inquiry.

This diagram represents the process of scientific investigation; *"t^o"* stands for the original cycle of a scientific investigation, i.e., the observation of a surprising or curious phenomenon; *"$t^{o\prime}$ (´´, ´´, …)"* stands for the cases tested by induction; and *"t^1, t^2…"* represents the anticipated future situations based on inductive generalizations that may or may not recur to start a new process of investigation. According to Minnameier (2004, p. 80), "all three inferences together yield a complete and dynamical method for the acquisition of (scientific) knowledge as well as its further application and evolution.". Triggered by a strange phenomenon and based in some previous knowledge (also within a community in its dynamics of embodied, situated and partly distributed cognition) abduction creates explanatory candidates forming a set of hypotheses of different kinds that constitutes a possible theory or complements an existing theory. From the abduced hypothesis generated in accordance with the theory, one deduces its possible consequences. Induction, in turn, confirms or disconfirms the expectations produced by the abduced hypothesis in its deducible consequences. Note that when an accepted hypothesis is applied in accordance with an established theory, this hypothesis is not simply being applied, but is constantly re-evaluated (Minnameier 2017, p. 180).

In this context, Minnameier reminds us that "[…] knowledge acquisition and knowledge application are fundamentally tied together and follow the same inferential principles". Within a historical continuum of scientific investigation it may sound arbitrary to focus on a particular start or end point of inquiry. However, Peirce stresses that an inquiry always starts with the irritation of a doubt, following by abductive reasoning. Minnameier highlights that even what seems to be an inductive reasoning has passed through abduction first: "In the beans example, having taken out the first two beans, we would perhaps remind ourselves that beans normally are sorted (and *abduce* that this might be so in the present case), from which it *deductively* follows that if two beans are white, the rest must also be white. To be (almost) certain, we could take out a few more beans to test our hypothesis and thus come to a warranted *inductive* judgement." (Minnameier 2004, p. 82, our emphasis). When describing the process of scientific investigation we make simplifications that may dismiss the interrelation of the three forms of reasoning in a small time frame, as the author (ibid.) points out: "The point is simply that in obvious cases we may be certain even on a very small empirical basis, so that abduction, deduction, and induction may, in the extreme, be merged into one single and immediate conscious judgement".

Scientific discovery has often been said to occur by a combination of aim-directed and chance processes, as for example, to bump into some strange phenomena, by serendipity discovering some principle, rule or effect, to meet a colleague with useful suggestions, to participate in a crucial discussion, and so on. To see something as strange demands, of course, a lot of background information about what we know and about our ignorance (Firestein 2012). Having an attentive eye to capture the new aspects of a surprising phenomenon is essential to the production of knowledge. Furthermore, it is crucial to base the research in a rich methodology that guides the scientific community in the unveiling of new properties of the studied object, figuring out the possible effects of the object in conduct. Here, other norms of science (apart from the epistemic ones governing each style) play a role; like doing science with responsibility and high integrity when searching for informational and causal relationships, without 'data fishing' or 'p-hacking', i.e., using data mining to uncover patterns in data that can be presented as statistically significant, without first devising a specific hypothesis as to the underlying causality.

In this context, we argue for a processual account of abductive reasoning, understood as a general method of scientific investigation which generates explanatory hypotheses that can be generalized by induction into a new law (or a postulate on some regularity) that feedbacks to the previous knowledge by means of unveiling the properties of a given object, establishing constraints that guides action and predict possible effects (cf. Peirce's pragmatic maxim). We emphasize that the whole process of scientific investigation involves a scientific community using one or more specific styles of reasoning (see below) including conceptual and instrumental apparatuses, laboratories, computers, etc.

When we analyze some cases of the use of abduction in the different styles of thinking one may conceive of such styles as just special methodological aspects of scientific investigation in general. The question whether abduction may have distinct characteristic forms within each style or whether the styles simply serve as cognitive strategic resources (providing enablements and constraints) for particular cases of abduction, cannot be fully addressed in the brief treatment we give each case. Our primary aim is to use the cases to highlight the hitherto neglected interrelationship between abduction and styles of scientific thinking. We claim that abduction, in the context of scientific practice, operates contextually within the different cases according to each SoT&D. Furthermore, we emphasize that scientific innovation takes time and that some discoveries take form of slow realizations done by a community of researchers as they restructure how we can conceive the complexities of nature. In this process a specific discovery or a conceptual innovation may be the result of the combination of several styles. In contrast to paradigms, styles can overlap or interweave within a single research tradition (cf. Winther 2012).

3 Styles of scientific thinking and doing

> The styles flourish in a complex web of interactions whose evolution they help determine (Hacking 2012, p. 605)

In the context of general history of science, the monumental work of Crombie (1994) on styles of thinking in the European tradition built up a memorable contribution to history and philosophy of science by identifying six different styles of thinking through the history of western science. According to Crombie (1994, vol. 1, p. 85): "Each style defined the questions to be put to its subject-matter and these yielded answers within that style. Each introduced new objects of scientific inquiry with appropriate scientific methods and arguments and kinds of evidence, demonstration and explanation that would provide answers acceptable within that style". Crombie's work has received a growing attention in the recent years (e.g., Gayon 1999; Bensaude-Vicent 2009; Kusch 2010; Bueno 2012; Ruphy 2011; Winther 2012; Sciortino 2017), primarily due to Hacking's use of Crombie in his own styles project (Hacking 1992a, 1992b, 2002a, 2002b, 2012) and work on the statistical style (Hacking 1990, 1992a). We will therefore give a brief presentation of Hacking's notion of style. We are aware of the criticism regarding the very idea of styles of scientific thinking (Kusch 2010), however, the present paper does not aim to address this debate.

Bringing Crombie's historical work to philosophy of science, Hacking develops the concept of *styles of scientific thinking and doing in the European tradition*. In "Language, Truth, and Reason" (2002a), Hacking proposes a contextual notion of what is true-or-false; namely that nothing like *p,* being a hypothesis or sentence within a SoT&D, is intrinsically true-or-false, but the way in which *p* points to truth or falsehood depends on the style appropriate to *p*.

According to Hacking, being true-or-false depends on the style of thinking for which being a candidate for truth or falsehood will apply: "[…] the very candidates for truth or falsehood have no existence independent of the styles of reasoning that settle what it is to be true or false in their domain" (2002a, p. 161). Because of this kind of statement, some readers place Hacking's work in a strong relativist shelf of philosophy. To avoid this placement, Hacking argues that, even if the sentences depend on a style of thinking to be candidates for truth or falsehood, the reasons to accept these sentences do not depend on the context, they are objectively tested in the outside world (cf. Hacking 1992a, p. 135: "what is true in no way depends upon the style of reasoning. The truth does not depend on how we think"). Hacking (2002a, p. 164) stresses that: "Styles of reasoning have histories, and some emerged sooner than others. Humankind has got better at reasoning. What ground for relativism could there be in all that?". In this sense, the proper way we reason is subject to modification and development through time and community.

Hacking proposes, according to our understanding, five main characteristics of the SoT&D that can be summarized in the following: (1) They are self-authenticating, "To say that these styles of thinking and doing are self-authenticating is to say that they are autonomous: they do not answer to some other, higher, or deeper, standard of truth and reason than their own." (2012, p. 605); (2) they have self-stabilization, in Hacking's words "[a] proposed account of self-stabilizing techniques begins by observing that a style becomes autonomous of the local microsocial incidents that brought it into being". (2002b, p. 196). (3) They have an 'ecological history': "The styles are adapted to their environment, but like any creature they also change the environment to their advantage" (2012, p. 607). (4) They crystallize along time and some died out. Hacking stresses that "[t]here are different styles of reasoning. Many of these are discernible in our own history. They emerge at definite points and have distinct trajectories. Some die out, others are still going strong." (Hacking 2002a, p. 175). And (5) they introduce novelties including new types of: objects, evidence, sentences, laws, and possibilities[4] (Hacking 2002b, p. 189).

[4] Here we agree with Ruphy (2011 p. 1219, emphasis by the author) that a SoT&D do not create new objects but enrich ontologically the studied objects: "I suggest that the introduction of new kinds of entities gives rise to an *ontological enrichment* of the objects studied by science, to the extent that the use in

As stressed by Gayon (1999) a SoT&D is "a nonexclusive line of exploration and substantiation of objectivity. In the history of science, such lines of approach can be seen to appear and become rigid. Some, perhaps many, fade away, but the fact that some continue to exist is particularly impressive and renders somewhat ridiculous the most radically relativist interpretations of science. Yet what is remarkable in "styles of scientific thinking" is not their durability, but also their plurality, their openness at the scale of vast historical periods" (Gayon 1999, p. 224; see also Sciortino 2017). This openness is seen in Crombie's (1994, vol. 3) detailed exploration of the history of Darwin's theory of evolution. In this case, as noted by Hall (1995, p. 418), "four styles—the historical, the statistical, model-making and the taxonomic" works together in union. Crombie did not discuss neo- or post-Darwinian examples, but we will give three cases of the taxonomic, the statistical and the historical style from contemporary biology.

We believe that the contextualism developed by Hacking does not contradict Peirce's realism regarding the notion of truth if we understand SoT&D as strategical steps that enable and constrain the generation of hypotheses. As we have seen, the creative character of abduction affords the generation of possible explanatory hypotheses that may be true. In a Peircean sense, once the necessary consequences of an abduced hypothesis have been deduced, one needs inductive test that shows, in the long run, its truth value. In this context, an abduced hypothesis generated within a SoT&D, enabled by its particular way of scientific thinking and doing, needs experimental test to state its truth value. As Ruphy (2011 p. 1216) states: "[…] *the style does not fix what the truth value of these sentences is; the world does*" (Ruphy's emphasis, cf. Hacking (1992a, p. 135): "the actual truth value of those sentences is external to the style"). Moreover, the Peircean approach is incompatible with positivistic verificationism, as a hypothesis cannot be verified by any single experiment, but it may be tested in the long run through the course of time.

4 On abduction and SoT&D: examples of scientific discoveries

To support the analysis of the role of abductive reasoning in each SoT&D, we use the methodological model of abduction provided by Peirce, highlighting the distinct aspects presented in each SoT&D in the context of scientific discovery. For each discovery, to the extent it is relevant we comment upon: (1) its relation to experimental test; (2) the suggested explanation of the phenomenon that caused surprise; (3) the economy of research in relation to: (3.1) money, energy, time; (3.2) intrinsic value of the hypothesis in terms of (3.2.1) simplicity; (3.2.2) caution; (3.2.3) extension; (3.2.4) depth. We discuss the process of scientific discovery by presenting some of the major steps for the generation of new explanatory hypotheses, taking into account its recursiveness, its strategic moves, constraints and the observation

Footnote 4 (continued)
scientific practice of different styles of reasoning widens and diversifies the classes of propositions that can be true or false about them".

of patterns within its distinct style of thinking. We are aware of the complexity involved in each example; however, it is beyond the scope of this paper to give a detailed account of each discovery.

4.1 Postulation: the consolidation of the abstract concept of group in mathematics

Crombie (1988, p. 10) characterizes the axiomatic style as "[…] the simple method of postulation exemplified by the Greek mathematical sciences originated within the common Greek search for the rational principles alike of the perceptible world and of human reasoning". One could argue that in mathematics or other pure deductive sciences there would be no space for abductive reasoning. Happily, many other researchers and mathematicians see postulation and axiomatic reasoning as a creative activity (see for example Campos 2010). Discussing the creative role of axiomatics, Schlimm (2011, p. 49) stresses that "[…] the axiomatic method is not only a way of systematizing our knowledge of specific domains, but that it can be—and has been—used as a fruitful tool for discovering and introducing new mathematical notions".

To illustrate the role of abductive reasoning in the axiomatic style of thinking we describe the consolidation of a new mathematical entity: the abstract concept of group in mathematics. For space constrains we focus here on the final step, made by Heinrich Martin Weber, that allowed the consolidation of the concept of abstract group as we know it today. During the nineteenth century, mathematicians were trying to solve problems through developing statements and postulating characterizations that could be applied in several domains, such as permutations in algebra, symmetry in geometry, substitutions in different collections, binary forms in number theory and so on (Wussing 1984). The first usages of the word "group" were not precise; being applied by Galois with these two different connotations: regarding a collection of permutation and a collection of substitution (see Neumann 1999, p. 288). With the work of Weber (1895/1896), the concept of group was consolidated; once Weber's axiomatization does not refer to specifics objects, the concept of group is understood as *abstract*.

Before Weber's postulation of the inverse property, the concept of group had three axioms (encompassing the properties of closure, identity and associativity) that allowed this concept to be applied to finite sets. Developing the concept of finite groups could also be analyzed as a result of abductive reasoning, but here we focus on the next step of its development. During the late nineteenth century, infinite groups started to be mathematically relevant. In this scenario, Weber (1895/1896, vol. 2, p. 4) postulated a new axiom that extended the domain of application of the group concept to infinite sets: "Usually, in finite groups the inverse compound can be inferred. However, to apply the concept of group in an infinite domain they had to postulate the *inverse*." (Neumann 1999, p. 290). As an outcome, a new mathematical entity was created (i.e., an algebraic structure consisting of a set of elements equipped with an operation that combines any two elements to form a third element and that satisfies the four axioms, namely closure, identity, inverses and associativity) feeding-back to other mathematical application and innovation, such as the concepts of *abelian* groups and cyclic groups, among others.

According to our understanding, the described abductive step, abstracted from a long ongoing process of collective research, is the postulation of the fourth axiom (the inverse property) that allows the abstract group concept to encompass not only a finite set of numbers, but also infinite sets of elements. Remember that in any axiomatic system, the implicit meaning of the involved formal symbols changes when new axioms are added (think of a game of chess as formal system governed by rules; this would become a different game if just a single rule for allowed moves were changed). The postulation of a fourth axiom increases not just the depth of the abstract concept of group, including a newly stated property (inverses), but it also increases the extension of the theory that now encompasses infinite sets as well. It conforms to some of the methodological characteristics of abductive reasoning: (1) it is subject to a mathematical experimental test; (2) as a mathematical postulation it avoids future surprises based on its intrinsic deductive aspect, that extend from postulated axioms to the deduced theorems; (3) regarding the economy of research and the intrinsic value of a hypothesis: (3.1) it is parsimonious because it encompasses the fewest number of axioms to account finite and infinite sets; (3.3) it increases the extension of the theory once it encompasses finite and infinite sets and (3.4) it increases the depth of the concept of group creating a new mathematical entity. In short, the creative step was to postulate the fourth axiom that allowed the theory to expand its mathematical domain of application.

In this example, the community of mathematicians, not simply a single inventor, created a new entity—the concept of abstract group—that solved not just specific problems but also opened up the way to the solution of other problems feeding-back upon other branches of mathematics. Thus, this example illustrates a creative activity within the axiomatic style of postulation, an activity that led to the innovation of the abstract concept of group. This would not have been possible without abductive reasoning. Thus, within the postulation/axiomatic SoT&D, abduction plays a crucial role. It supports the capacity by the community of mathematics to come up with relevant, adequate, and fruitful generalizations, invent new rules of the mathematical game and refine them to cohere with and extend the existing ontology of mathematical structures.

4.2 Experimental: do ravens have a theory of mind?

According to Crombie, "[t]he deployment of experiment, both to control postulation and to explore by observation and measurement, was required by the scientific search for principles in the observable relations of more complex subject-matter. […] The scientific experimental method derived from the union of these practical habits with the logic of control, with further quantification through new techniques of instrumentation and mathematical calculation" (Crombie 1988, p. 10–11). Experimental style requires building up a set of rules and apparatuses to control and manipulate a specific phenomenon, to quantify the variables within the strategic creation of an experiment that aims to make replicable a given

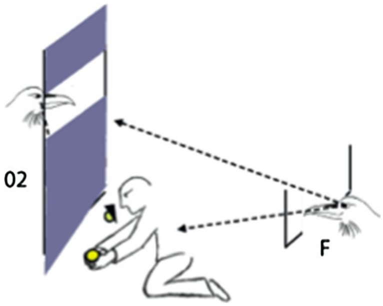

Fig. 2 Sketch of the modified experimental set-up designed to dissociate the focal subject's view from the view of its competitors (Bugnyar 2013, p. 5)

situation to be explained. In other words, this style is used to explore and control nature through observation, controlled intervention and measurement of natural phenomena. Designing experiments is in no way trivial and demands the experimenter to come up with some ways of relating interesting or surprising observations to their possible explanations; ways that would confirm or disconfirm such explanations. Standard accounts of the role of experimentation in science (like the hypothetical-deductive accounts) tend to trivialize the creation of an experimental set-up, reducing an experiment to be a set of observations that may confirm or falsify a hypothesis being the outcome of observable test-implications deduced from a higher-level hypothesis or theory. In contrast, we would like to draw attention to the creative aspect of experimental design, for instance, in testing an already conceived hypothesis.

Our example of the experimental style comes from the findings of Bugnyar (2013) in social cognition in ravens. By designing a controlled experiment, Bugnyar explores the ravens' ability to catch and pilfer by observing and deceiving their conspecifics. The context for his experiments is previous research on social cognition in different animal species, and discussions about the extent to which an animal is able to take the perspective of another animal [implicitly assuming that the other conspecific individual knows things about the world, i.e., implying that it has some kind of 'Theory of Mind' (ToM)].The experiment was built to analyze if (a) "ravens can remember who was visually present at which catching events and that they can relate the presence at catching with a high risk of subsequent pilfering" (2013, p. 3); and if (b) "ravens base their pilfer decisions on their own perspective or, to some extent, also on the other's perspective" (2013, p. 3). Portraying an ongoing research, this example illustrates the role of abduction in the creative development of an experimental set-up to test an existent hypothesis according to which non-human animals may have the ability to read others' mind.

Bugnyar's general strategy is to design a test that would show if ravens behave differently when other ravens are watching a human experimenter placing food in a certain position. It would show that the "ravens base their choices not only on the memory of their own perspective ('whom they could see at catching') but they seemingly also take aspects of the other's perspective into account ('what the other could see at catching')" (2013, p. 4). In Fig. 2, we can see a part of the designed experiment:

In this experiment, a focal subject (F) is able to see the caching being made by a human, as well as the possible competitor for pilfering the catches. It is interesting that this specific set-up allows the focal subject to observe that its potential competitor is not able to see the catches being made. Based on this information, the focal subject can make its decision by considering its competitor's perspective.

Designing the experiment this way allowed Bugnyar in a controlled and repeatable manner to manipulate the interaction between two ravens in order to test the hypothesis according to which ravens possess some kind of a Theory of Mind (i.e., knowing that conspecifics have a mind being able to observe and memorize events in order to help a raven save food safely when nobody else is present). In the author's words: "[t]hese findings support the assumption that maneuvers seen at pilfering are based on advanced socio-cognitive skills and may represent the first building blocks of abilities that go together with a 'Theory of Mind' in humans" (Bugnyar 2013, p. 6). The reasoning allowing Bugnyar to posit this conclusion crucially relies upon the experimental style and its enablement of a researcher to explore and compare situations with different causal outcomes, due to different causal factors influencing the resulting process under investigation. The ravens' possession of some kind of ToM is the simplest way to explain the experimental outcome of behaviors produced during the experimental exploration. Of course, the test of the consequences of a hypothesis occurs in the long run, encompassing a vast amount of comparative studies of socio-cognitive abilities in corvids (Taylor 2014), but for each test, an experiment has to be designed, and for each such design, abductive reasoning is a crucial step.

One may still wonder if the design of an experiment is part of an inductive phase for testing a pre-given hypothesis. We believe that consecutive tests of a given hypothesis in the long run may involve inductive reasoning. However, as noted above, abduction plays an essential role in the creative and strategic development of an experiment. Moreover, we would like to emphasize the recursive aspect of scientific investigation. In the present example, Bugnyar re-establishes a hypothesis according to which ravens possess ToM. He does not create this hypothesis; however, he revises the existing hypothesis by the development of an experiment which recurs to the whole process of scientific investigation of social cognition.

According to abductive methodology: (1) the hypothesis that ravens possess ToM is subject to experimental test; (2) the hypothesis, together with the theory of social cognition in ravens, explains the phenomena observed in catching/pilfering situation; (3.1.1) Simplicity: having or not having a ToM is the most simple explanation of the differences between raven behavior in such situations and the behavior of less intelligent birds; (3.2.2) Caution: Bugnyar asks different questions in order to restrict the scope of analysis to understand social cognition in ravens. (3.2.3) The hypothesis that claims that ravens possess a ToM, together with the designed experiment, can be extended to other corvids or even other non-avian species. (3.2.4) This experiment increases the depth of the concept of ravens, because it includes a new property that can be now attributed to them: the ability of reading others' mind.

4.3 Hypothetical-analogical: the cadaveric agent

Crombie named the hypothetical-analogical style of thinking as "Hypothetical Modeling" stating that it "was developed in a sophisticated form first in application to early modern perspective painting and to engineering, and was then transported from art into science as likewise a method of analysis and synthesis by the construction of analogies" (1988, p. 11). As we have seen, the generation of new hypotheses is the core of abductive reasoning. However, we should ask: what is the relationship between abductive reasoning and analogy? Concerning this question Peirce says that "Analogy…combines the characters of Induction and Retroduction [or abductive reasoning]" (CP 1.65). To understand this relationship, we draw briefly on Minnameier's (2010) interpretation of the concept of analogy in the interior of Peirce's philosophy.

According to Minnameier (2010, p. 107–108), analogical reasoning involves the relationship between abduction and induction in the generation of a hypothesis by connecting the target domain—"the domain where a problem has to be solved"——and the source domain—"the domain from which the analogy is drawn".

As common in abduction, the reasoning starts with a problem to be solved or an explanation or action seeking clarification. The abductive step within the analogical style of thinking lies in finding the deep structured source domain to be compared with the target structure. In this step, scientists discover a similar situation to the one seeking for explanation that can potentially solve their problem or answer their question. The inductive step draws the deep similarities from the discovered source domain to the target domain, completing the generation of an explanatory hypothesis. Once a hypothesis is generated, the researcher deductively draws the possible consequences of this hypothesis to be subject to experimental test. Finally, s/he proceeds with the inductive phase in order to confirm, deny, or see how far the hypothesis corresponds to the solution of the given problem.

Following this interpretation, we present the discovery of the 'cadaveric agent' made by Semmelweis in the Vienna General Hospital. The scenario of this discovery is a maternity hospital with two clinics, where women and newborn babies were dying from a mysterious childbed fever. In this context, Semmelweis' problem was to explain and prevent the deaths caused by childbed fever. A curious fact was that the first clinic, ran by Semmelweis and his students who also performed autopsies, presented more cases of childbed fever than the second one, ran by midwives. Immersed in this context, one of the main strategies used by Semmelweis to understand the cause of deaths was to compare the two clinics looking for differences and similarities in their practice and environment.

Searching for plausible explanations, Semmelweis conjectured some explanatory hypotheses and tested them without success (for a detailed account see: Semmelweis 1983 and Paavola 2006). His discovery happened when his colleague died of blood poisoning from a wound on his finger while doing an autopsy. At this point, Semmelweis found the appropriated source domain that could be linked to his target domain generating a reasonable hypothesis: "Day and night I was haunted by the image of Kolletschka's disease and was forced to recognize, ever more decisively, that the disease from with Kolletschka died was identical to that from which so many maternity patients died." (Semmelweis [1983] 2008, p. 6). The comparison and then realization of the *identical* effects of an unknown cause of the death in his colleague and in the women and babies of the maternity was the core comparison that made possible the analogical reasoning: the transference from the source domain, Kolletschka's disease effects, to the target domain, women and babies affected by childbed fever.

The abductive step enabled within the analogical style was to realize that women and babies as well as his colleague Kolletschka suffered from the same symptoms that lead them to death. This abductive step—finding the source domain—plus the inductive step, extrapolating from the source domain to the target domain—led Semmelweis to the hypothesis that he and his students were transporting through their hands a cadaveric agent that causes childbed fever. Following these two steps, Semmelweis traced the conceivable consequences of his hypothesis through deduction: If my hypothesis is correct, washing the hands with the appropriate soap would decrease the number of deaths in the hospital. Finally, there was the inductive test based on washing hands and observing the decreasing number of deaths.

According to our suggestions, finding the adequate source domain was the abductive step of the discovery by analogy that lead to the generation of the explanatory hypothesis. The inductive step was toextrapolate the abduced source structure to the target structure. This discovery attends the following methodological criteria of abduction: (1) Semmelweis' hypothesis was subject to experimental test; (2) Although it did not explain entirely the phenomena as it would be clear with Pasteur's theory of germs, Semmelweis' hypothesis explains enough to avoid, to some extent, the surprise caused by the different incidents of childbed fever in the two clinics; (3.2.1) It was parsimonious; (3.2.2) The hypothesis was precise enough to draw its consequences allowing experimental test; (3.2.3) His hypothesis extended the domain in which childbed fever was believed to be able to manifest itself, i.e., not just for women and their babies but everyone that could be contaminated through blood. (3.2.4) It also extended the depth of the explanation pointing out a cause (even being still an unknown 'cadaveric agent') suggesting a prevention that resulted in the decrease of deaths.

4.4 Taxonomy: the discovery of a new class of protozoans (*Bodo designis*)

According to Crombie's exposition of the history of natural science and its styles,

> Taxonomy emerged first in Greek thought as a logical method of ordering variety in any subject-matter by comparison and difference. The elaboration of taxonomic methods and of their theoretical foundations may be attributed to the need to accommodate the vast expansion of known varieties of plants and animals and diseases following European exploration overseas, with attempts to relate diagnostic signs and symptoms to their causes and to discover the natural system that would express real affinities. (Crombie 1988, p. 11)

Generally, taxonomy is understood as an organizational process that classifies organisms and other entities (e.g., animals, plants, rocks, protozoans, diseases). It differs from postulation by relying upon empirical resources, and aiming at mapping the patterns discerned in a complex world. It does not necessarily take into account the ancestry of the entities or constitute a historical derivation, as it is the case in biological cladistics (see below). To illustrate this style of thinking, we discuss the discovery of a new structure of a group of protozoans through the reclassification of *Bodo designis,* formerly thought to be a single biological species.

At the beginning of their research, Koch and Ekelund (2005) were aware that different strains of *Bodo designis* shared the same morphological aspects, what usually means that they belong to the same species (2005, p. 98). However, the ability to survive in many different environments made the authors suspect that sharing the same morphology would not be enough to claim that the strains pertain to the same species. Their hypothesis was that "*B. designis* isolates from similar environments would cluster out in monophyletic groups" (Koch and Ekelund 2005, p. 98).

The strategy used by the authors to provide an adequate taxonomic tree to solve this puzzle was to analyze the classification of *Bodo designis* using morphological, phylogenetic and physiological methodologies. Through these three different analyses, the authors discovered that *Bodo designis* should be considered as a group of organisms that did not belong to a single species as it was considered before. Although the strains share the same morphological aspects, they differ in relation to phylogenetic and physiological properties. In this context, the authors highlight that "[i]f our phylogenetic trees provide a realistic picture of the relationship, then it is not meaningful to consider *B. designis* as a species, but rather, *B designis* should be conceived as a group of organisms which shares a common external morphology" (Koch and Ekelund 2005, p. 107). Such a statement is only a candidate for being true or false within the taxonomic style of thinking used in present-day biology. Though this style aims "to discover the natural system that would express real affinities" (Crombie, quoted above), and here, "real" affinities mean more than just morphological similarities (as in pre-Darwinian taxonomy); they have to reflect common descent, i.e., also be phylogenetic affinities.

As usual in abductive reasoning, the first phase in this discovery was to notice an intriguing phenomenon, namely, how could these kinds of protozoans, sharing the same morphology, be part of the same species if they live in so many different environments? This puzzle was followed by the authors' strategic hypothesis, enabled by the taxonomic style, that if they isolate the different strains and investigate their phylogenetic relationships and physiological properties, the individual members of *Bodo designis* would eventually cluster out and form a more varied group of protozoans than a single species. With this hypothesis at hand, they deduced the possible consequences (e.g., if different strains represent different species, they should present different degrees of tolerance to salinity). Finally they tested the hypothesis based on measuring some physiological and phylogenetic properties to see how far the strains are functionally and genetically alike.

If we analyze this discovery as a case of abductive reasoning we have a common taxonomic assumption in biology that shared morphology implies shared membership of a single species. The puzzling phenomenon in the present case is that many individuals assigned to the same species (*B. designis*) were "found in highly contrasting environments" (*ibid*., p. 98). An abductive hypothesis according to which

B. designis cluster out the phylogenetic tree was subject to experimental test (1) that was done by measuring the different properties of the various strains of *B. designis.* This hypothesis avoid future surprises (2) by implying the existence of an incomplete classification; the authors proposed the suspicion that *B. designis* was just a morphospecies and not a genuine phylogenetic (monophyletic) species. Regarding extension (3.2.3) and depth (3.2.4), one could claim that Koch and Ekelund extended the notion of a morphospecies by showing that *B. designis* is one such instance. More interestingly, as they suggest that the different strains of *B. designis* should be assigned to different species (in the phylogenetic sense), they increased the depth of both a) the concept of *B. designis* as being not a phylogenetic but a morphological species (a group of organisms sharing a common external morphology) and b) the concept of a phylogenetic species as distinct from a set of morphological traits. It dissolves the often taken for granted identity between a morphospecies and a phylogenetic species, being a first step to an alternative classification of protozoan organisms through a reconstruction of their taxonomy.

4.5 Probability or statistical: the founder effect in evolutionary biology

According to Crombie (1988, p. 11), "[t]he statistical style and probabilistic analysis of expectation and choice developed in early modern Europe again took the same forms whether in estimating the outcome of a disease, of legal process, of commercial enterprise, or natural selection, or reasonableness of assent to a scientific theory". On the statistical style, Hacking (1992a, p. 146) points out that "[…] law-like sentences about disease rates did not exist [before the statistical style of thinking], and had even, by some of the most scrupulous observers, been excluded as stemming from a false analogy between disease and death". In general, this style aims at searching for patterns of correlational regularities between different factors and variables (that may or may not reflect law-like cause-effect relationship) based on the number of occurrence of such instances, and it uses statistical notions, like that of average and probability, to describe properties of real populations (Hacking 1990).

Our illustrative case is the discovery of a special effect studied in evolutionary biology and biogeography called the *founder effect,* often seen as an important causal factor in speciation processes. The concept dates back to the work of Ernst Mayr in the 1940s, building upon his observations and theorizing on the origins of species diversity, and on the conceptual innovations of population genetics emerging as part of a neo-Darwinian research program in the twentieth century, especially in the work of Sewall Wright in the 1930s and 1940s. Briefly summarized, Mayr proposed the notion of a founder effect (or 'founder principle') for the establishment of a new population by few original founding individuals (in extreme cases, a single fertilized female) which carry only a small fraction of the total genetic variation of the parental population (Mayr 1942, 1963). The variation in gene frequency between the original population and a newly founded colony may lead to a significant divergence between the two groups over the course of generations. As the genetic distance increases, the two separated populations may become distinctively different, genetically and phenotypically, and together with other evolutionary factors (like general genetic drift, natural selection, and mutation) the increased genetic divergence may effectively form a new species.

Population genetics can be mathematically intricate but the founder effect is easy to understand for non-specialists. In contrast, the discovery of *random genetic drift*, which may be seen as the more general evolutionary phenomenon of which the founder effect is a special instance (Dobzhansky 1970), has a more complicated history of discovery that has Sewall Wright as its key scientist (random drift is sometimes called the Sewall Wright effect). Also early remarks by Charles Darwin should count, and especially the work on Hawaiian land snails by the naturalist John Thomas Gulick (1872) forms parts of this history (Beatty 1992). The founder effect as presented by Mayr (together with random drift) represents a discovery of a real phenomenon in nature and a conceptual innovation that would have been very difficult or impossible to arrive at abductively without the statistical style of thinking—a style that characterizes a very large part of population genetics and evolutionary biology. Today, as exposure to basic statistic notions is very common, it is easy to see that the concept of "sampling error" (the error in a statistical analysis arising from the unrepresentativeness of the sample taken) is a very close analogue to the real process of splitting one population of organisms into two by some indiscriminate process. One can think of this, as Mayr did, as a kind of "natural sampling", like when a storm blows off a few founding individuals of flies or birds to an isolated distant island. Though the genetic composition of the founding colony may represent that of the original population, its representativeness is likely to decrease with decreasing sizes of the founding "sample", due to the stochastic phenomenon of sampling error.

The study of random divergence of a group split-off from its ancestral population goes back to Gulick who studied more than two hundred species of the land snail family Achatinellidae on the island of Oahu. He was interested in adaptive evolution by natural selection as a possible driving force in the creation of species variety. However, he found no apparent environmental differences correlated with the differences between the many snail species; a surprising observation in the context of Darwin's theory that he knew. Gulick began to speculate about nonadaptive modes of evolution. Could indiscriminate (with respect to the inheritable variation) divisions of breeding groups, together with indiscriminate causes of survival and reproduction, be used to account for these differences? Gulick (1872) thought so, and thus became one of the first to propose the idea that some evolutionary changes may be the result of chance variation, due to chance divisions of breeding groups, which were not caused by the directional forces of natural selection related to survival and the reproductive success of a species.

Based on what was at that time the recent advances in population genetics, Mayr (1963) could locate the role of a founder effect more precisely among other causal factors of evolutionary speciation within the expanding neo-Darwinist theory of evolution (for details see Beatty 1992; Mayr 1982). If the newly founded colony escapes extinction, natural selection and mutation may then work to restore its adaptedness to a new environment, and Mayr thought its gene pool could undergo drastic reorganizations, contributing to the generation of a new species. Dobzhanky (1970) reviewed the experimental work on fruit flies in the lab that followed in order to test the existence of a founder effect and explore its genetic mechanisms.

Thus, according to the methodological characteristics of abduction, we see here the generation of a hypothesis by abductively hypothesizing a kind of natural "sampling error" (the founder effect) to account for a surprising observation, namely the lack of environmental causes that would otherwise have led, by natural selection, to the divergent evolution of a population. The statistical style of thinking enables the researcher to conceive of population dynamics in statistical terms, involving processes known from statistical analyses, like the method of measuring a sample of individuals to estimate collective properties of a whole population, and the extent to which sampling errors may cause the result to diverge from the real value. We see that this hypothesis (1) was subject to experimental tests (sampling fruit fly populations in the lab); (2) gave a simple explanation of the observed pattern of populations; (3.1) was conducive to efficient use of research resources by providing (3.2.1) simplicity; (3.2.2) caution (in being just a possible factor of evolution and not ubiquitous); (3.2.3) it helped extend the catalogue of evolutionary factors involved in evolution and speciation; (3.2.4) and it increased the depth of the concept of speciation by showing how partly random processes can be decisive for the route taken by divergent populations.

4.6 Historical derivation: are birds reptiles? from disjunction to part-whole relation

Crombie (1988, p. 12) states that "[t]he subject-matter of historical derivation was defined by the diagnosis, from the common characteristics of diverse existing things, of a common source earlier in time, followed by postulation of causes to account for the diversification from that source". This style provides a map (which may be a time line of events, a narrative, or a temporal classification) that mirrors the processes of generation of a current state of the object by tracing back its origin. Here, abduction lies in the generation of hypothesis that explains (or connects) current objects (situations, organisms) with their derivative history or ancestry.

To illustrate the role of abduction in the historical derivation style of thinking, we provide the example of a conceptual change involved in the classification of the class Reptilia through phylogenetic analysis, changing our notion of birds and reptiles. This re-classification is based on a discovery (that took long time and had the form of a slow realization) that reptiles as we normally conceive of as vertebrate animals including extant snakes, lizards, crocodiles, turtles, and extinct dinosaurs (but excluding fish, amphibians, birds, and mammals), do not constitute a natural class (i.e., they are not a monophyletic group, to use the terminology of phylogenetic cladistics). It means that what we used to understand by reptiles does not mirror the natural order of the current *Reptilia* clade,[5] because extant birds belong to this clade, so birds as a group became a part of reptiles.

[5] A clade is an evolutionary 'branch'.

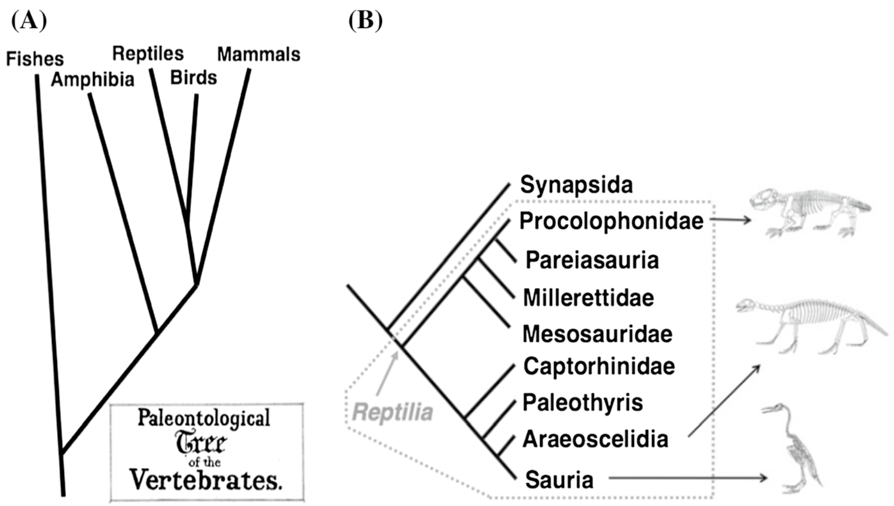

Fig. 3 **a** Ernst Haeckel's "Paleontological Tree of the Vertebrates", circa 1879 (redrawn from Haeckel's *The Evolution of Man*, fifth edition, London, 1910, plate no. XXI), exemplifying an older, yet evolutionary, notion of birds and reptiles as separate groups: birds are not reptiles. (B.) The cladogram for Reptilia, adapted from Modesto and Anderson (Modesto and Anderson 2004, p. 819). Notice that Synapsida corresponds to mammals, Sauria comprises birds, and the rest are covering a group of animals coextensive with the older meaning of the term of Reptiles: birds are one of the kinds of reptiles

This discovery, based on fossils record, took the form of a realization, based upon accumulating evidence (analyses of fossil data from new paleontological findings) and theoretical developments (in evolutionary cladistics), that showed that birds should be understood as living dinosaurs. According to Chiappe (2009), this research "related birds to groups of animals such as turtles, crocodiles, and their relatives, various primitive Triassic fossils (…), pterodactyls, and their kin, and the plant-eating ornithischians and the meat-eating theropod dinosaurs."

Figure 3 shows the contrasts between a traditional notion of Reptilia (A.) with the one provided by Modesto and Anderson (2004, p. 819) displaying Reptilia (B.) as a natural class that includes birds (belonging to the clade Sauria). This cladistic classification aims to mirror the evolutionary ancestry (the historical sequence of speciation events defining a natural order of these groups of animals) tracing back their common ancestor which led back to the dinosaurs.

Modesto and Anderson (2004) provide a precise classification of Reptilia, including the class of birds. According to the authors, their "new definition corrects an error created by the combination of the selection of a higher taxon (rather than a species) as a specifier, and an unexpected topology." (Modesto and Anderson 2004, p. 815). The authors take Gauthier's (1994) definition of Sauropsida which includes "reptiles plus all other amniotes more closely related to them than they are to mammals" (Modesto and Anderson 2004 p. 816) and re-named it as Reptilia, that includes birds and the well-known reptiles, because they "[…] believe that the use of the well-known Reptilia is preferable to the more obscure

Sauropsida because it captures the traditional meaning of the taxon being named, as the concept has evolved." (Modesto and Anderson 2004, p. 820).

One cannot attribute the discovery of birds being a kind of reptiles (and the innovation of the taxonomy to mirror this relation) to Modesto and Anderson, or any other specific author or paper. The realization that birds belong to Reptilia was made through time and it needed a community of scientists from different areas discovering how many anatomical features are shared by extant birds and extinct theropod dinosaurs (such as feathers, features of the skeleton, lungs and heart, and reproductive biology). Birds are structurally closer to some members of the traditional class of Reptilia, namely crocodiles, than the later are to other members of that class, such as lizards and snakes. What Modesto and Anderson claim is to name the class that includes reptiles and birds as Reptilia, not as Sauropisida, as claimed by Gathier (1994). This new concept of Reptilia can be said to have increased in depth (3.2.4), as it now includes a reference to the historical common genealogy of what we by ordinary language call birds and reptiles. Thus, the concept has also increased in extension, (3.2.3), comprising all the living beings that share the same ancestral, being the organism a reptile with or without feathers.[6] As for the other characteristics of abduction we already noticed how the new concept was based upon slowly cumulating observational evidence that can and will be put to test by future findings (corresponding to (1) 'experimental' test); the new concept conforms to present-day reconstructions of the evolutionary sequence leading to extant forms of reptiles (including birds and crocodiles) that better and more parsimoniously (3.2.1) explains the distribution of characters among these animals. The updated notion of reptiles thus represents a more developed evolutionary account with a higher (3.2) intrinsic value, also in terms of extension and depth.

4.7 Laboratory: the discovery of quasi-periodic crystals by Daniel Shechtman

> there is no such thing as quasi-crystals, only quasi-scientists[7]

The laboratory style is proposed by Hacking as a seventh style of scientific thinking that combines hypothetical-analogical and experimental styles. Although Hacking changes his mind concerning the existence of this seventh style,[8] we assume the laboratory style is grounded in the importance of instruments and apparatus for the development of laboratory sciences (e.g. chemistry). According to (early) Hacking:

[6] However, this new and enriched meaning of the term reptile, as discovered by paleontology and evolutionary biology, is not corresponding with the vernacular everyday meaning of reptiles. People may still refer to an extinct dinosaur or an extant snake or turtle without realizing that by calling this creature "reptile" they put it (in the scientific sense) into a group of organisms to which also birds belong.

[7] This discovery was not accepted by the scientific community of that time. Linus Pauling, an expert on the area and Nobel prize chemist, denied Shechtman' discovery stating that: "there is no such thing as quasi-crystals, only quasi-scientists" (Shechtman 2013, pp. 2 and 5).

[8] In "'Language, Truth and Reason' 30 years later" (2012, p. 603), Hacking abandons the laboratory style stressing that: "The advent of the laboratory is a radical eruption, but Occam's razor moves us to regard it as a crystallization within Crombie's second style [experimental], rather than a new style." (Hacking 2012, p. 603).

"It is my thesis that as a laboratory science matures, it develops a body of types of theory and types of apparatus and types of analysis that are mutually adjusted to each other" (1992b, p. 30). Hacking posits that the self-authentication of laboratory sciences depends on the apparatus and instruments used in the laboratory: "[…] laboratory sciences are those whose claims to truth answer primarily to work done in the laboratory. They study phenomena that seldom or never occur in a pure state before people have brought them under surveillance. Exaggerating a little, I say that the phenomena under study are created in the laboratory" (1992b, p. 33).

To illustrate this style we bring an example from the history of science when a laboratory apparatus—the electronic microscopy—was essential to the discovery of quasi-periodic crystals. In crystallography it is known that a crystal should present three main aspects: (1) order, (2) periodicity and (3) rotational symmetry. In this sense, a crystal was defined as "a solid composed of atoms arranged in a periodic pattern in three dimensions" (Shechtman 2013, p. 3). From these aspects, it was assumed, based on the mathematical model of lattices, that there is no crystal presenting five-fold rotation axis, what was corroborated by the fact that all the crystals analyzed until that time had two, three, four or six-fold rotational axis.

An important aspect of crystallography is that the standard apparatus to measure the diffraction pattern of crystals was X-ray machines. Differing from the standard procedure, the material engineer Daniel Shechtman was analyzing a chemical compound with an electronic microscope when he made his discovery. In his words:

> I was looking at an electron diffraction pattern of an aluminum manganese compound […] taken by an electron microscope. Electron diffractions contain the same information as X-ray diffractions. While looking at this pattern, I notice two very strange things: first, this compound had a 10 folded rotational axis and, second, it had no periodicity.[…] This crystal violated both laws of crystallography of the time: it had no periodicity, and it had a 10-fold rotational symmetry. (Shechtman 2013, p. 4).

His first move was to deny this discovery and try to find a way, within the current paradigm of crystallography, to explain the anomalous diffraction pattern.[9] Not being able to dismiss the problematic phenomenon within the laws of crystallography, Shechtman noticed that the aluminum manganese compound, instead of having a periodic distance between spots, had a pattern of distance representing a three-dimensional Fibonacci number, meaning that this specific compound had quasi-periodicity.

This discovery highlights the importance of an apparatus, because "the only tool that could have discovered them [the quasi-periodic crystals] is a transmission electron microscope" (Shechtman 2013, p. 6). At that time, the overwhelming majority

[9] Shechtman ran two experiments to eliminate his first hypothesis according to which the compound formed a twin-crystal, what, if true, could explain the anomaly without denying the laws of crystallography at that time. He does not describe this step in his 2013 paper, but he does so in the public lecture "The Discovery of Quasicrystals" held at Uppsala University December 13, 2011 (from minute 17 here: https://youtu.be/oa1GMwXuBwo), see also his Nobel Lecture, December 8, 2011 (here: https://www.nobelprize.org/prizes/chemistry/2011/shechtman/).

of crystallographers used X-rays to make the diffraction of these patterns. However, X-rays cannot diffract very small crystals such as in this case. It took 3 years to grow non-periodic crystals in a reasonable size that could be seen through X-ray instruments and, therefore, prove the hypothesis that states the possibility of quasi-periodic crystals (ibid., p. 5). It is important to emphasize that Shechtman's discovery opened up the development of new mathematical models to account for the new form of rotational symmetry of lattices (see Sørensen 2017).

The abduction starts with two puzzling observed phenomena: "first, this compound had a 10 folded rotational axis and, second, it had no periodicity". To explain these phenomena, Shechtman had to deny that these two laws of crystallography were sufficient, and proposed a complementary law (via abductive hypothesis) that includes quasi-periodic crystals, i.e., crystals that do not present periodicity. Based upon this hypothetical idea, he deduced that if one grows some aperiodic compound, slowly it would become big enough to be observable using the X-ray detection method, and thus, he, his colleagues and the general scientific community could confirm his hypothesis.

Accordingly, his hypothesis was (1) subject to experimental test; (2) the concept of quasi-periodic crystal explains the observed phenomenon, thus avoiding further surprises; (3.2.1) it is parsimonious; (3.2.2) it is precise; (3.2.3) it increases the extension of the concept of crystals, now being applied also to quasi-periodic crystals; (3.2.4) the abductive reasoning allowed the increase of the depth of the concept of crystal, including new properties such as five or ten-fold axis, and quasi-periodicity. Nowadays, the concept of crystal is defined as follows: "By crystal we mean any solid having an essentially discrete diffraction diagram, and by aperiodic crystal we mean any crystal in which three dimensional lattice periodicity can be considered to be absent" (Shechtman 2013, p. 6).

Why did Daniel Shechtman make this discovery? Because he was an expert looking at the element with the right apparatus: the electronic microscope. He also had courage to stand against the current rules of crystallography by defending his findings. A funny fact is that, as Shechtman tells us in his paper (2013), a student had previously discovered the anomaly that occasioned the concept of quasi-periodic crystals. However, the student did not show his results to his professor because "he would have been asked to stay on for two more years to research that discovery". As Peirce (W2: 107ff) addresses, time is essential in abductive reasoning, as well as energy!

5 Concluding remarks

Although the cases above could only be treated briefly while really deserving more detailed analysis, we try to explain some general ideas about the role of abduction within each SoT&D. Considering our first question on the consequence of the existence of several styles of scientific thinking and doing for abduction, we have shown that the existence of different SoT&D does not contradict the existence of abduction as a creative mode of inference. On the contrary, we have seen that abduction, as a broader kind of reasoning, pervades the process of scientific discovery performing different roles throughout the different SoT&D.

Regarding our second question, whether abduction, as a general creative mode of inference, may have distinct characteristic forms within each style, we have found it adequate to abstain from postulating special or distinct types or kinds of abduction and instead, our preliminary answer is that different SoT&D set-up different strategic contexts (cognitive enablements and constraints) for the abductive generation of new ideas and explanatory hypotheses. In other words, we have shown how abduction—traditionally conceived as one single special mode of reasoning aiming at the generation of new hypotheses—must be understood in the context of such enablements and constraints offered by each SoT&D. The distinct SoT&D applied within the various types of research practice, provide researchers with different resources for conceptual innovation that facilitate various theoretical developments to account for the observed phenomena.

Clearly, we can see aspects of abductive reasoning throughout different SoT&D in their methodological aspects. It is so, because abduction, together with deduction and induction, constitutes core processes of reasoning in investigations throughout modern scientific practice. But this does not undermine the fact that abductive reasoning may perform different roles in the diversity of SoT&D, and in the different ways scientific discovery occurs. For a philosophy of scientific practice that sees thinking and doing as always intertwined and situated in particular historical and material contexts, such a result should not come as a surprise, even though no analysis of abduction within different SoT&D has been made before.

How much have we learned about the different roles of abduction in each style of scientific thinking and doing? If we take abduction as a general creative mode of inference, we may understand SoT&D as strategic moves that enable the generation of new hypotheses by constraining the range of possibilities given by abduction. The notion that reasoning is constrained by the social-cognitive system within which some form of reasoning occurs is important, if not surprising. General modes of reasoning, being abductive, deductive or inductive, are used in different ways within the institutions of science, law, social planning, government, education, etc. These modes of reasoning may be constrained by the aims, scopes and social-cognitive procedures of these institutions. In the present paper, we have dealt with just one of these systems, namely, the institution of scientific research and its different styles. Within this scope, we have claimed that abduction as a general mode of creative reasoning can have a variety of specific expressions constrained and enabled by the different SoT&D within science and scientific practice. We are aware that a throughout analysis of the different modes of abductive reasoning in each style is preferable, but beyond the scope of this paper.

In addition to being an inference that allows the generation of new explanatory hypotheses, we argued that abduction can perform different roles in different styles of reasoning: (1) The postulation style, far from being purely deductive, embodies abductive reasoning enabled by its ability to create new concepts or mathematical entities by postulating axioms that can contribute to the solution of problems. In the (2) experimental style where thinking and doing are interwoven, we have seen how abduction acts as a strategic process in the design of an experiment that via controlled manipulation helps to reveal how a given phenomenon can be explained. The (3) hypothetical-analogical reasoning involves the relationship between abduction and induction in the generation of hypotheses by connecting the target domain (where a problem has to be solved) to the source domain (from which the analogy is drawn), in order to explain strange phenomena.

Besides the establishment of possible explanatory hypotheses, abduction can operate within (4) taxonomy, in the choice and application of methods to classify objects based on doubts and curiosity about similarities and differences. In (5) probability or statistics, an abductive hypothesis unveils patterns of correlational regularities between different factors and variables (that may or may not reflect law-like cause-effect relationship) based on the number of occurrence of such instances while acknowledging the role of chance. In (6) historical derivation, an abductive hypothesis incorporates a map (which may be a time line of events, a narrative, or a classification) that mirrors the processes of generation of a current state of the object by tracing back its origin. Finally, in the (7) laboratory style and within laboratory sciences, abductive reasoning not just generates new hypotheses that can be tested in the laboratory with its apparatuses, but it also allows the creation of new phenomena, and the generation of hypotheses to understand their properties.

We have seen the distributed character of abduction in the scientific innovation within the postulation, probability and historical derivation styles. One can always ask to which extent a discovery can be attributed to any singular researcher. In contemporary science, it is easy to see how often a discovery is just a small glimpse within a web of investigations that are spread out, even to different disciplines and institutions. Of course, there are some very visible scientific breakthroughs, made by single or a few individuals, that profoundly change the way we understand the universe or even the way we think about ourselves. Sometimes a discovery is promptly accepted; and sometimes the community does not welcome a new explanation of a strange phenomenon. But through time, based on continuing investigations, we can hope to solve some of the riddles of nature by interpreting odd patterns of observations through abductive reasoning within one or several SoT&D. To the extent that abduction is recursive, the effects of these activities feedback upon ongoing research and the wider society, enlightening the public about the world and contributing to the development of technology, social institutions and the human mind.

If we take into account that "just as we say that a body is in motion, and not that motion is in a body we ought to say that we are in thought and not that thoughts are in us" (CP 5.289), we should drop the idea of a solitary genius gifted with exceptional abductive powers, and instead understand abduction as a dynamic process of attentive observation of patterns based on strategic rules that constrain the possible predicates (properties) attributed to an object. This process depends on the community of inquirers, its institutions, society, materials, engagement, attentive observation, and even chance! It also depends on paper and pencil, energy and money, time and courage.

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