Narrative sculptures: graph theory, topology and new perspectives in narratology

There have been many attempts to model narratives from a structural point of view. From these numerous models we want to preserve a macroscopic vision that allows a quick and simultaneous understanding of various important elements of the story, which we call, following Labov's and Wilensky's definitions, narrative points[1]. Models mapping the general structure of the story can be found, for instance, in the work of Marie Laure Ryan where both diegetic and possible events are represented and where narrative points are related by vectors. In order to preserve this telescopic view and superpose its logic with McCloud's notion of infinite canvas[2], which will be defined in the body of this text, an option is to start with the notion of a parametric curve. Before doing so, an overview of the pragmatic motivation that led to this research is needed.

The motivation behind this exploration is taken from an interest in mathematics and an increasing amount of narratives using complex time structures and story representations. Movies like *Primer* (2005) by Shane Carruth lead to the construction of various charts in attempts to understand the hidden time structure[3]. *Source Code* (Jones, 2011) and *Looper* (Johnson, 2012) are other examples that created the need for such macroscopic representation and many other films, like *Cronocrimenes*, *Triangle* (Smith, 2009) and the *Terminator* suite are cases that could have led to similar practices. In the case of the movie *Looper*, a three dimensional version of the chart has been produced, bringing to light wider possibilities (Figure 1).

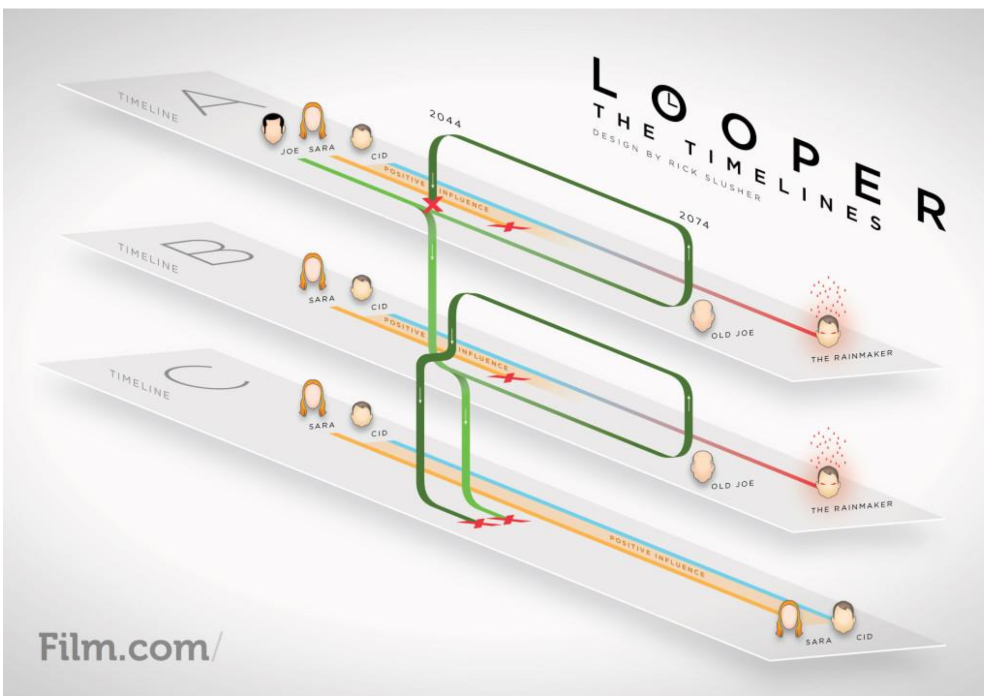

Figure 1: Movie chart for the movie Looper by Rich Slusher.

[1] Ryan, p. 150-151
[2] McCloud, 2000.
[3] An example can be found at http://movies.yahoo.com/blogs/the-projector/incredibly-detailed-primer-timeline-210027548.html

Comic artists have explored this path in some isolated cases, either in the use of bigger expositional space[4], or as the juxtaposition of various three dimensional objects, like the booklets in Chris Ware's *Building Stories* (2012). This work constitutes a box containing many booklets that can be read in different order. This creates multiple combinations for the reader exploring the diegetic world. Another interesting example, comparable to a mutoscope or other early cinematic devices, is the three dimensional cyclical structure of *Julius Coretin Acquefaques, prisonnier des rêves: Le décalage* by Marc-Antoine Mathieu. In this case, when leaving the story at the end of the comic, they actually enter the story again to loop the cycle.

The model presented in this paper is a first exploration in the variety of different surfaces that might be used in further narrative experimentations as well an attempt to establish the basis of a formal narrative tool for academics and artists. Therefore, the author wishes to open discussions in defining narratives and hopes to inspire artists in exploring the challenges offer by this model.

One of the key elements of our model is the use of curves with the continuous stretch of time maintained across them. Even if it seems natural nowadays to represent time with a line, its extensive use in various models results from many different traditions. In our model, these influences are mainly the following: history charts, the construction of the real number line based on Dedekind and Cantor's work, and the use of parametric curves with time as the general parameter. We will discuss these three influences briefly.

For most of the Middle Ages, time was mainly represented on timetables. Around the beginning of the 19th century, time flux started to be embedded within natural metaphors like lightning and rivers[5]. These two examples are important since they allow the time frame to branch out simultaneously. Various time lines could be traced out of single elements. In mathematical terms, these structures are equivalent to oriented graphs, and more precisely to oriented trees, since cycles do not exist in these structures.

For its part, the concept of continuity led to multiple complications and was not well defined until the topology of the real line was properly described. We owe much to the work of Weirstrass, Dedekind and Cantor for this definition and understanding. This dense continuous line of values serves as well in defining parametric curves, curves based on a continuous parameter, usually the time. These curves can be used to represent various types of motion, for instance, the movement of particles in space.

The first trick to make use of mathematical models to represent time frames is to base diegetic time on parametric curves. As a building strategy, this enables various constructions of diegetic time structures. First of all, it allows the concatenation of many line segments as it happens in the time charts discussed above, therefore constructing structures like tree graphs. A simple example of a narrative based on that idea is Griffith's movie *Intolerance*, in which different independent stories flow separately[6]. Examples can also be found in the work of artists like Chris Ware or Jason Shiga, or in the hypercomics based on McCloud's infinite canvas such as Daniel Merlin Goodrey's work[7].

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[4] Gravett, p. 136-137
[5] Rosenberg and Grafton, p.143-149.
[6] Eisenstein, p. 397
[7] http://e-merl.com/

The concatenation of various time segments allows the construction of multi-cyclic time structures as well. This kind of structure is not in itself a novelty; in some mythologies, cyclic time is accepted as the general topology of time frames, and some even make use of many intricate cyclic times as in the Tzolkin and Vedic time constructions. In extending parametric curves into graph theoretical frameworks, we can obtain infinity of cyclic graphs where cycles may be intersecting or independent. This application naturally allows a wide variety of already proven theorems to apply to narratology. For instance, observing the underlying structure of a graph might allow us to determine the number of possible cycles, each of them being a possible reading path.

Because cycles are naturally embedded on a flat surface, some considerations about the implied spaces become important. The Jordan curve theorem states that any simple closed curve separates space in exactly two sections, the interior and exterior of the closed curve, or equivalently, of a cycle. As a result, constructing a cyclical story leads to the creation of these inside and outside spaces that might be used later for a semantic purpose.

In *Reinventing Comics*, Scott McCloud coined the term *infinite canvas* to represent the possibility of extending comics infinitely in all directions of a plane. His website specifies that it provides the perfect conditions for a type of comic he names hypercomics. Looking back at mathematical definitions of planes and surfaces, it seems clear that various types of infinities are involved in the notion of an extended version of the infinite canvas.

First, in terms of the continuum defined by Cantor, a plane is dense since it follows from the product of two continuous axes. This implies that infinite zooms are possible at any point on a plane, and as such, on any compact surface[8]. To understand this implication, we have to look at a category of curves called space-filling curves, or Peano curves after Giuseppe Peano who first proposed such an example. Space-filling curves are iterated curves that, at their limits, fill a whole part of the plane. (Figure 2) Indeed, many other examples have been provided by other mathematicians in order to provide extra characteristics, as for instance Moore's curve that is a closed space-filling curve. The density of the plane implies that the breakdown of iterated narrative into infinitely smaller scales is possible. This density leads to possible infinite zoom, fractal-like, construction as found in Marc-Antoine Mathieu's first and third tomes of his Julius Corentin Acquefaques serie.

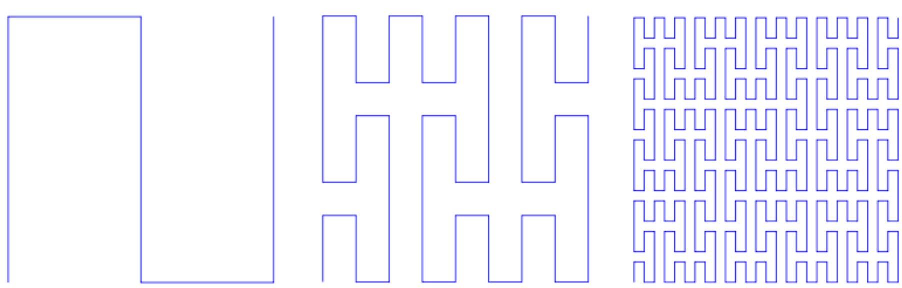

Figure 2: Peano curve. Source: Wikipedia

[8] It should simply be understand in this case of surfaces of finite area.

The second way in which the canvas is infinite arises first when we allow the plane to be infinite in all directions. In mathematical term, it means the surface is not compact because it would be impossible to cover a plane with a finite amount of bounded sets. From a representative point of view it means it could never be entirely seen, in particular, not in a finite amount of images. In this case, this is why McCloud claims that the infinite canvas naturally supports digital comics. Although true, we suggest the infinite canvas presents even more value with the infinite amount of shapes we can allow the canvas to have.

Also, the canvas does not have to be contained simply in the plane. For instance, as suggested visually in McCloud[9] and in the diegetic world of French author Marc-Antoine Mathieu[10], comics could be presented on spheres[11]. The use of different properties of the sphere can lead to a variety of narratological compositions in link with the intrinsic properties of the sphere: the presence of loxodromes, the covering groups different from the wallpaper groups and so on.

In addition, as proposed by many artists, from Alan Moore in *Promethea* to Jim Woodring in a side project[12] passing by members of the OuBaPo collective, the use of a Möbius strip as the canvas leads to interesting constructions. These can be used as objects existing within the diegetic world as in Moore, or directly as a support inducing a specific topology within the diegesis as in Woodring's case.

Indeed, any sculptural surface may offer interesting options for narrations and a complete survey of such an approach should be done. In our case, we would like to focus on surfaces that have been studied from a mathematical point of view. The reason is that many theorems shed light on hidden properties that enable us to imagine interesting narratives and limiting ourselves to a sculptural point of view would have prevent us from finding and using these properties. The variety of surfaces is infinite and a list of inspiring surfaces can be found in the fields of differential geometry, differential topology, and knot theory. For instance, as a result of their definition, minimal surfaces seem pleasing to embed stories. It involves the possibility of working on some surfaces of infinite area spreading in different axes, as with Sherk's surface and Costa's surface (Figure 3), or even with self-intersecting sections, as in the case of Henneberg's surface.

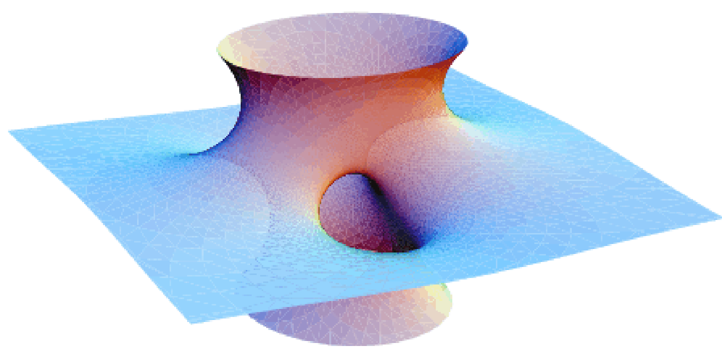

Figure 3: Costa's surface. Source: Mathworld.com

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[9] McCloud, 1993
[10] Mathieu, 2004
[11] McCloud also suggest writing on the cube in *Reinventing Comics*.
[12] https://www.fantagraphics.com/rarities-and-miscellany-by-various-artists/moebius-strip-comic-by-jim-woodring-video-photo-animation.html

Compact surfaces also lead to interesting possibilities. In topology, the study of surfaces is bound to the analysis of characteristics which are preserved when surfaces are torn and stretched. Such invariants are coined topological invariants. An example is the number of holes present in the surface. For instance, the sphere contains no holes, but the torus has one; therefore the two surfaces are fundamentally different. On the other hand, the sphere and the cube are classified as the same surface since they both have no holes. This argument leads to a classification for compact surfaces depending on the number of holes involved. As it turns out, all compact orientable surfaces are torus of genus n, meaning a torus with *n* holes, for *n* a positive integer These will become useful in the next section.

Orientability is another characteristic that helps refining surface classification. Orientable roughly means they possess an inside and an outside and it is impossible to move smoothly from the inside to the outside. For instance, it is impossible to move on the sphere and end up being inside the sphere without piercing a hole. The Möbius trip is a simple example of non-orientable surface since by smoothly moving along the surface it is possible to end up on the other side of the departure point. In constructing sculptures, non-orientable surfaces lead to some difficulties. For instance, the Klein bottle invented by German mathematician Felix Klein in 1882 cannot be embedded in our three-dimensional world without self-intersecting (Figure 4); it is only possible in four or more dimensions. This makes the visualisation of these surfaces more difficult, but a general classification is still possible.

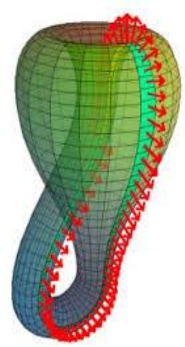

Figure 4: Klein bottle

The class of infinite compact non-orientable surfaces are all equivalent to spheres with a certain number of Möbius strips glued to holes in them (the edge of the Möbius strip is equivalent to a circle, therefore when cutting a circular hole on the sphere it becomes possible to glue the strip's edge along the edge of the hole). The more complex the non-orientable surface, the more dimensions one needs to avoid self-intersections. Even if it seems very hard to work on these surfaces as a possible infinite canvas, shortcuts exist. There is a way to represent any compact surface, orientable or not, with their fundamental polygons which can easily be represented on the plane. These polygons are simplified maps for these surfaces; to obtain a surface, it suffices to fold its edges by respecting so pair connections or edge directions. Indeed, the writing on non-orientable compact surfaces that aren't embeddable in three dimensions might be done in a virtual environment, or directly on the equivalent fundamental polygon. The figure below shows the construction of the Klein bottle from its fundamental polygon. (Figure 5)

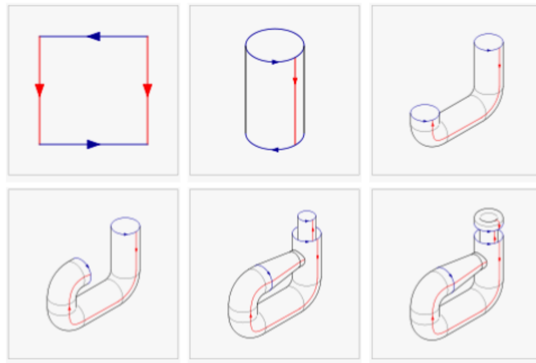

Figure 5: Klein bottle's fundamental polygon. Source: Wikipedia

As a result, the infinite canvas is infinite as well in the number of dimensions a non-orientable surface holding a story could ''naturally'' exist without self-intersecting. Indeed, the use of computers can be a handy tool in constructing such narratives.

The next question we need to address is the following: why would we want to work with parametric curves on this collection of surfaces? The answer comes from the field of topological graph theory. The Polish mathematician Kasimierz Kuratowski and the Russian mathematician Lev Pontryagin proved independently the necessary and sufficient conditions to be able to embed a graph on the plane without crossing edges. It states a graph is planar if and only if it does not contain the subgraphs $K_{3,3}$ or K_5. (Figure 6)

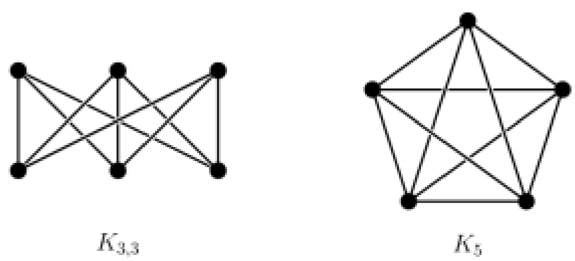

Figure 6: The obstruction set for the plane

In constructing comics on parametric curves based on graphs containing one of these would inevitably leads to edges crossovers. Indeed, such overlapping can always be dealt with, as in the case of Chris Ware diagram comics, but the point here is to explore the possibilities provided by restricting ourselves to planar embeddings. To give a pragmatic application, we know the two aforementioned graphs can be drawn on the torus or the Möbius strip without having edge overlapping, it means they have planar embedding for the torus. It follows that it is possible to draw planar stories on such graphs if we use the torus as the canvas. (Figure 7)

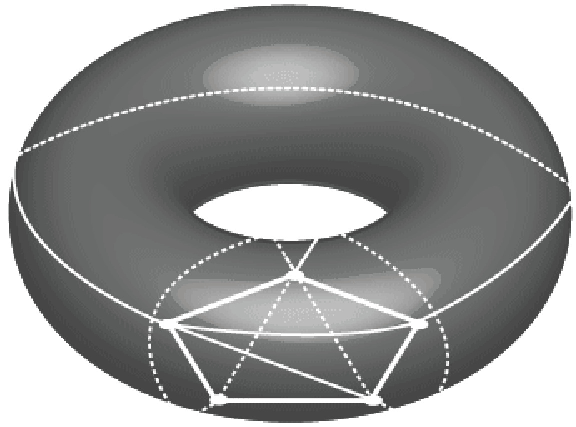

Figure 7:Toroidal embedding of K_5

The study of topological graph theory led to the discovery that different surfaces don't share the same obstruction groups, i.e. the set of graph making the planar embedding impossible, such as $K_{3,3}$, and the K_5, in the case of the plane. We know for instance that the Möbius plane has 35 such graphs (Archdeacon, 1980), and the Torus has more than 16 000! On the other hand every finite graph can find a planar embedding in some compact orientable surfaces with at least *n* holes for a certain *n* values, and same holds for non-orientable surfaces and a certain number of Möbius strip glued to the sphere.

Another result is that the presence of cycles leads to different amount of bounded spaces. In other words, if the Jordan curve theorem holds for the sphere, it is not true in general. Already in the case of the torus, construction of longitudinal and transversal cycles leads to a single bounded space; it does not hold for torus with *n* holes neither.

The construction of narrative on these extended infinite canvases, such as non-orientable surfaces, minimal surfaces and so on, is what we call *narrative sculptures* because their structures are deeply linked to the surfaces hosting them. The main goal in constructing narrative sculptures is the research for new narratological challenges. An optimised use of this involves considerations of the following distinctive properties of narrative sculptures: the possible use of complex multi-cyclic time curve constructions, the use of different spaces the cycles are bounding and the possible semantical implications in our world, or in a digital equivalent to it.

We present two examples, expressing challenges brought by simple constructions. The K_5 graph has a planar embedding on the torus. . It can as well be constructed by the union of two cycles by taking a cycle being the outside pentagon and the second one being the star shape in the middle. We could construct a highly ''twisted'' story as following. Through the double cycles, we could describe the interactions of two individuals at desynchronised moments of their life cycles. The complications and self-containing elements of the story could then be reinforced by presenting it on a trefoil knot, which is simply a torus but embedded differently in three dimensions. (Figure 8) Of course, many other options since the torus can find multiple embedding in four dimensions that could lead to interesting narrative sculptures[13].

[13] Séquin, 2012

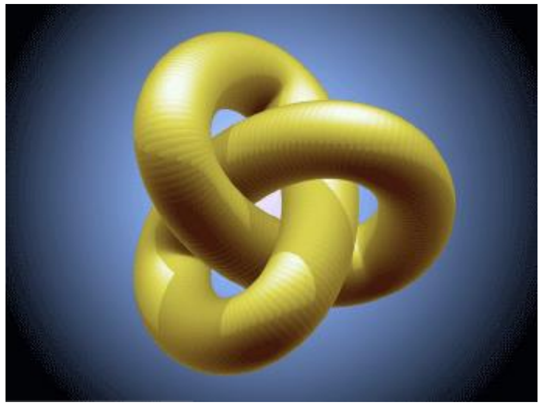

Figure 8: Trefoil knot by Jos Leys. Source: josleys.com

The graph K_5 also possesses a planar embedding on the Klein bottle. It would then possible to construct a complex science-fiction comic. First the multiple desynchronised elements present on the two cycles would bring an intricate time structure. Then, different bounded area could hold their proper images and symbolism related to the story. Finally, the Klein bottle canvas leads to a hyper-fictional statement since the canvas itself could not be properly constructed in our world. The same holds for the infinite collection of surfaces that aren't embeddable in three dimensions without self-intersecting.[14]

In conclusion, we have seen that by merging various paradigms and concepts from narrative theory, the infinite canvas and mathematical knowledge about surfaces and graphs, we can define highly complex narrative structures that we coined *narrative sculptures*. Such constructions not only leads to new narratological and artistic challenges, but it can bring new questioning about the way we first, understand stories, and secondly how we teach narratology. In the first case, experiments in cognition could help understanding the effect of dealing with highly complex but still visually clear narratives in our learning process. In the latter case, it evokes the possibility of including some mathematical notions in teaching narratology or even information design.

Félix Lambert, February 2015

[14] The same holds for the infinite collection of surfaces that aren't embeddable in three dimensions without self-intersecting.

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