Differentiation Formulas

$$\frac{d}{dx} k = 0 \tag{1}$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x) \tag{2}$$

$$\frac{d}{dx}[k \cdot f(x)] = k \cdot f'(x) \tag{3}$$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \tag{4}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \tag{5}$$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \tag{6}$$

$$\frac{d}{dx} x^n = nx^{n-1} \tag{7}$$

$$\frac{d}{dx} \sin x = \cos x \tag{8}$$

$$\frac{d}{dx} \cos x = -\sin x \tag{9}$$

$$\frac{d}{dx} \tan x = \sec^2 x \tag{10}$$

$$\frac{d}{dx} \cot x = -\csc^2 x \tag{11}$$

$$\frac{d}{dx} \sec x = \sec x \tan x \tag{12}$$

$$\frac{d}{dx} \csc x = -\csc x \cot x \tag{13}$$

$$\frac{d}{dx} e^x = e^x \tag{14}$$

$$\frac{d}{dx} a^x = a^x \ln a \tag{15}$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x} \tag{16}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \tag{17}$$

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \tag{18}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1} \tag{19}$$

$$\frac{d}{dx} \cot^{-1} x = \frac{-1}{x^2+1} \tag{20}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}} \tag{21}$$

$$\frac{d}{dx} \csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}} \tag{22}$$

Integration Formulas

$$\int dx = x + C \tag{1}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \tag{2}$$

$$\int \frac{dx}{x} = \ln|x| + C \tag{3}$$

$$\int e^x \, dx = e^x + C \tag{4}$$

$$\int a^x \, dx = \frac{1}{\ln a} a^x + C \tag{5}$$

$$\int \ln x \, dx = x \ln x - x + C \tag{6}$$

$$\int \sin x \, dx = -\cos x + C \tag{7}$$

$$\int \cos x \, dx = \sin x + C \tag{8}$$

$$\int \tan x \, dx = -\ln|\cos x| + C \tag{9}$$

$$\int \cot x \, dx = \ln|\sin x| + C \tag{10}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \tag{11}$$

$$\int \csc x \, dx = -\ln|\csc x + \cot x| + C \tag{12}$$

$$\int \sec^2 x \, dx = \tan x + C \tag{13}$$

$$\int \csc^2 x \, dx = -\cot x + C \tag{14}$$

$$\int \sec x \tan x \, dx = \sec x + C \tag{15}$$

$$\int \csc x \cot x \, dx = -\csc x + C \tag{16}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C \tag{17}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \tag{18}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{|x|}{a} + C \tag{19}$$

G Formulas

G.1 Differentiation and Integration Formulas

■ Use differentiation and integration tables to supplement differentiation and integration techniques.

Differentiation Formulas

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
9. $\frac{d}{dx}[e^u] = e^u u'$
10. $\frac{d}{dx}[\sin u] = (\cos u)u'$
11. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
12. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
13. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
14. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
15. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

Integration Formulas

Forms Involving u^n

1. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
2. $\int \frac{1}{u}\,du = \ln|u| + C$

Forms Involving $a + bu$

3. $\int \frac{u}{a+bu}\,du = \frac{1}{b^2}(bu - a\ln|a+bu|) + C$
4. $\int \frac{u}{(a+bu)^2}\,du = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$
5. $\int \frac{u}{(a+bu)^n}\,du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} + \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C, \quad n \neq 1, 2$
6. $\int \frac{u^2}{a+bu}\,du = \frac{1}{b^3}\left[-\frac{bu}{2}(2a - bu) + a^2\ln|a+bu|\right] + C$
7. $\int \frac{u^2}{(a+bu)^2}\,du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$
8. $\int \frac{u^2}{(a+bu)^3}\,du = \frac{1}{b^3}\left[\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right] + C$
9. $\int \frac{u^2}{(a+bu)^n}\,du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C, \quad n \neq 1, 2, 3$
10. $\int \frac{1}{u(a+bu)}\,du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

Integration Formulas (continued)

11. $\displaystyle\int \frac{1}{u(a+bu)^2}\,du = \frac{1}{a}\left(\frac{1}{a+bu} + \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

12. $\displaystyle\int \frac{1}{u^2(a+bu)}\,du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

13. $\displaystyle\int \frac{1}{u^2(a+bu)^2}\,du = -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} + \frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right] + C$

Forms Involving $\sqrt{a+bu}$

14. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$

15. $\displaystyle\int \frac{1}{u\sqrt{a+bu}}\,du = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \quad a > 0$

16. $\displaystyle\int \frac{1}{u^n\sqrt{a+bu}}\,du = \frac{-1}{a(n-1)}\left[\frac{\sqrt{a+bu}}{u^{n-1}} + \frac{(2n-3)b}{2}\int\frac{1}{u^{n-1}\sqrt{a+bu}}\,du\right], \quad n \neq 1$

17. $\displaystyle\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int\frac{1}{u\sqrt{a+bu}}\,du$

18. $\displaystyle\int \frac{\sqrt{a+bu}}{u^n}\,du = \frac{-1}{a(n-1)}\left[\frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2}\int\frac{\sqrt{a+bu}}{u^{n-1}}\,du\right], \quad n \neq 1$

19. $\displaystyle\int \frac{u}{\sqrt{a+bu}}\,du = -\frac{2(2a-bu)}{3b^2}\sqrt{a+bu} + C$

20. $\displaystyle\int \frac{u^n}{\sqrt{a+bu}}\,du = \frac{2}{(2n+1)b}\left(u^n\sqrt{a+bu} - na\int\frac{u^{n-1}}{\sqrt{a+bu}}\,du\right)$

Forms Involving $u^2 - a^2,\ a > 0$

21. $\displaystyle\int \frac{1}{u^2-a^2}\,du = -\int\frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

22. $\displaystyle\int \frac{1}{(u^2-a^2)^n}\,du = \frac{-1}{2a^2(n-1)}\left[\frac{u}{(u^2-a^2)^{n-1}} + (2n-3)\int\frac{1}{(u^2-a^2)^{n-1}}\,du\right], \quad n \neq 1$

Forms Involving $\sqrt{u^2 \pm a^2},\ a > 0$

23. $\displaystyle\int \sqrt{u^2 \pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \pm a^2\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

24. $\displaystyle\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{1}{8}\left[u(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - a^4\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right] + C$

25. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u}\,du = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

26. $\displaystyle\int \frac{\sqrt{u^2 \pm a^2}}{u^2}\,du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

27. $\displaystyle\int \frac{1}{\sqrt{u^2 \pm a^2}}\,du = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

28. $\displaystyle\int \frac{1}{u\sqrt{u^2+a^2}}\,du = \frac{-1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

29. $\displaystyle\int \frac{u^2}{\sqrt{u^2 \pm a^2}}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2 \ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

30. $\displaystyle\int \frac{1}{u^2\sqrt{u^2 \pm a^2}}\,du = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$

31. $\displaystyle\int \frac{1}{(u^2 \pm a^2)^{3/2}}\,du = \frac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$

Forms Involving $\sqrt{a^2 - u^2}$, $a > 0$

32. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}\,du = \sqrt{a^2 - u^2} - a \ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

33. $\displaystyle\int \frac{1}{u\sqrt{a^2 - u^2}}\,du = \frac{-1}{a} \ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

34. $\displaystyle\int \frac{1}{u^2\sqrt{a^2 - u^2}}\,du = \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$

35. $\displaystyle\int \frac{1}{(a^2 - u^2)^{3/2}}\,du = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

Forms Involving e^u

36. $\displaystyle\int e^u\,du = e^u + C$

37. $\displaystyle\int ue^u\,du = (u - 1)e^u + C$

38. $\displaystyle\int u^n e^u\,du = u^n e^u - n\int u^{n-1}e^u\,du$

39. $\displaystyle\int \frac{1}{1 + e^u}\,du = u - \ln(1 + e^u) + C$

40. $\displaystyle\int \frac{1}{1 + e^{nu}}\,du = u - \frac{1}{n}\ln(1 + e^{nu}) + C$

Forms Involving $\ln u$

41. $\displaystyle\int \ln u\,du = u(-1 + \ln u) + C$

42. $\displaystyle\int u \ln u\,du = \frac{u^2}{4}(-1 + 2\ln u) + C$

43. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}\left[-1 + (n+1)\ln u\right] + C, \quad n \neq -1$

44. $\displaystyle\int (\ln u)^2\,du = u\left[2 - 2\ln u + (\ln u)^2\right] + C$

45. $\displaystyle\int (\ln u)^n\,du = u(\ln u)^n - n\int (\ln u)^{n-1}\,du$

Forms Involving $\sin u$ or $\cos u$

46. $\displaystyle\int \sin u\,du = -\cos u + C$

47. $\displaystyle\int \cos u\,du = \sin u + C$

48. $\displaystyle\int \sin^2 u\,du = \frac{1}{2}(u - \sin u \cos u) + C$

49. $\displaystyle\int \cos^2 u\,du = \frac{1}{2}(u + \sin u \cos u) + C$

50. $\displaystyle\int \sin^n u\,du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n}\int \sin^{n-2} u\,du$

51. $\displaystyle\int \cos^n u\,du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n}\int \cos^{n-2} u\,du$

52. $\displaystyle\int u \sin u\,du = \sin u - u\cos u + C$

53. $\displaystyle\int u \cos u\,du = \cos u + u\sin u + C$

54. $\displaystyle\int u^n \sin u\,du = -u^n \cos u + n\int u^{n-1}\cos u\,du$

Integration Formulas (continued)

55. $\displaystyle\int u^n \cos u\, du = u^n \sin u - n\int u^{n-1} \sin u\, du$

56. $\displaystyle\int \frac{1}{1 \pm \sin u}\, du = \tan u \mp \sec u + C$

57. $\displaystyle\int \frac{1}{1 \pm \cos u}\, du = -\cot u \pm \csc u + C$

58. $\displaystyle\int \frac{1}{\sin u \cos u}\, du = \ln|\tan u| + C$

Forms Involving tan *u*, cot *u*, sec *u*, or csc *u*

59. $\displaystyle\int \tan u\, du = -\ln|\cos u| + C$

60. $\displaystyle\int \cot u\, du = \ln|\sin u| + C$

61. $\displaystyle\int \sec u\, du = \ln|\sec u + \tan u| + C$

62. $\displaystyle\int \csc u\, du = \ln|\csc u - \cot u| + C$

63. $\displaystyle\int \tan^2 u\, du = -u + \tan u + C$

64. $\displaystyle\int \cot^2 u\, du = -u - \cot u + C$

65. $\displaystyle\int \sec^2 u\, du = \tan u + C$

66. $\displaystyle\int \csc^2 u\, du = -\cot u + C$

67. $\displaystyle\int \tan^n u\, du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u\, du, \quad n \neq 1$

68. $\displaystyle\int \cot^n u\, du = -\frac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u\, du, \quad n \neq 1$

69. $\displaystyle\int \sec^n u\, du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} u\, du, \quad n \neq 1$

70. $\displaystyle\int \csc^n u\, du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} u\, du, \quad n \neq 1$

71. $\displaystyle\int \frac{1}{1 \pm \tan u}\, du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$

72. $\displaystyle\int \frac{1}{1 \pm \cot u}\, du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$

73. $\displaystyle\int \frac{1}{1 \pm \sec u}\, du = u + \cot u \mp \csc u + C$

74. $\displaystyle\int \frac{1}{1 \pm \csc u}\, du = u - \tan u \pm \sec u + C$

G.2 Formulas from Business and Finance

■ Summary of business and finance formulas

Formulas from Business

Basic Terms

x = number of units produced (or sold)

p = price per unit

R = total revenue from selling x units

C = total cost of producing x units

$\overline{C}$ = average cost per unit

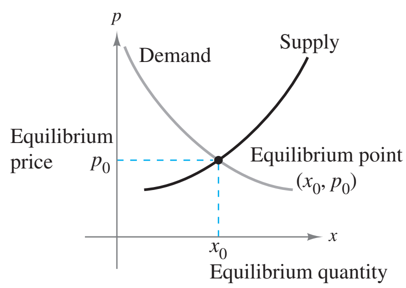
P = total profit from selling x units

Basic Equations

$$R = xp \qquad \overline{C} = \frac{C}{x} \qquad P = R - C$$

Typical Graphs of Supply and Demand Curves

Supply curves increase as price increases and demand curves decrease as price increases. The equilibrium point occurs when the supply and demand curves intersect.

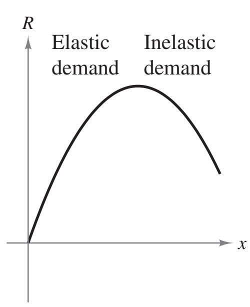


Demand Function: $p = f(x)$ = price required to sell x units

$$\eta = \frac{p/x}{dp/dx} = \text{price elasticity of demand}$$

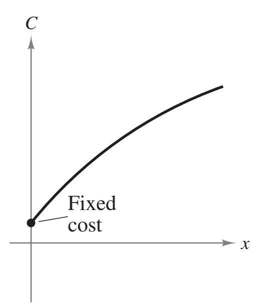
(When $|\eta| < 1$, the demand is inelastic. When $|\eta| > 1$, the demand is elastic.)

Typical Graphs of Revenue, Cost, and Profit Functions



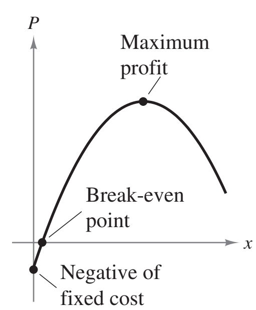
Revenue Function

The low prices required to sell more units eventually result in a decreasing revenue.



Cost Function

The total cost to produce x units includes the fixed cost.



Profit Function

The break-even point occurs when $R = C$.

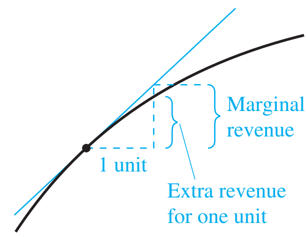
Formulas from Business (continued)

Marginals

$$\frac{dR}{dx} = \text{marginal revenue} \approx \text{the } \textit{extra} \text{ revenue from selling one additional unit}$$

$$\frac{dC}{dx} = \text{marginal cost} \approx \text{the } \textit{extra} \text{ cost of producing one additional unit}$$

$$\frac{dP}{dx} = \text{marginal profit} \approx \text{the } \textit{extra} \text{ profit from selling one additional unit}$$



Revenue Function

Formulas from Finance

Basic Terms

P = amount of deposit

r = interest rate

n = number of times interest is compounded per year

t = number of years

A = balance after t years

Compound Interest Formulas

1. Balance when interest is compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. Balance when interest is compounded continuously: $A = Pe^{rt}$

Effective Rate of Interest

$$r_{eff} = \left(1 + \frac{r}{n}\right)^{n} - 1$$

Present Value of a Future Investment

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Balance of an Increasing Annuity After *n* Deposits of *P* per Year for *t* Years

$$A = P\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]\left(1 + \frac{n}{r}\right)$$

Initial Deposit for a Decreasing Annuity with *n* Withdrawals of *W* per Year for *t* Years

$$P = W\left(\frac{n}{r}\right)\left\{1 - \left[\frac{1}{1 + (r/n)}\right]^{nt}\right\}$$

Monthly Installment *M* for a Loan of *P* Dollars over *t* Years at *r*% Interest

$$M = P\left\{\frac{r/12}{1 - \left[\frac{1}{1 + (r/12)}\right]^{12t}}\right\}$$

Amount of an Annuity

$$e^{rT}\int_0^T c(t)e^{-rt}\,dt$$

$c(t)$ is the continuous income function in dollars per year and T is the term of the annuity in years.