Formula Sheet:
Electricity and Magnetism

Coulomb's law

$$F = k\frac{qQ}{r^2}$$

Electric Field

$$\vec{E} = \frac{\vec{F}}{q}$$

Field of a point charge

$$E = k\frac{Q}{r^2}$$

Electric field inside a capacitor

$$E = \frac{\eta}{\varepsilon_0}$$

Principle of superposition

$$\vec{E}_{net} = \sum_{i=1}^{N} \vec{E}_i$$

Electric flux

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Gauss's law

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Electric potential

$$V = \frac{U}{q}$$

$$\Delta \mathrm{V} = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

For a point charge $V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

For a paralle-plate capacitor

$$V = Es$$

Potential Energy

$$U = qV$$

Two point charges

$$U = k\frac{qQ}{r}$$

Capacitors

$$C = \frac{Q}{\Delta V}$$

Parallel-plate $C = \varepsilon_0 \frac{A}{d}$

Capacitors connected in parallel

$$C_{eq} = C_1 + C_2 + \cdots$$

Capacitors connected in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Energy stored in a capacitor $U = \frac{Q^2}{2C}$

Ohm's law

$$V = IR$$

$$I = \frac{dQ}{dt}$$

$$R = \rho\frac{l}{A}$$

$$\sum I_{in} = \sum I_{out}$$

$$\sum \Delta V_i = 0$$

Power

$$P = IV$$

Resistors connected in series

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Resistors connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

The potential difference across a charging capacitor in RC circuit

$$V(t) = \varepsilon(1 - e^{-t/RC})$$

A magnetic field exerts a force

$$\overrightarrow{dF} = I\overrightarrow{dl} \times \vec{B}$$

$$\vec{F} = I\vec{l} \times \vec{B}$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

The Biot-Savart Law

$$\vec{B} = \frac{\mu_0 q\vec{v} \times \hat{r}}{4\pi r^2}$$

$$d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi r^2}$$

The magnetic field of:

A straight line wire

$$B = \frac{\mu_0 I}{2\pi r}$$

A solenoid

$$B = \mu_0 n I$$

Magnetic flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Inductance

$$L = \frac{\Phi_B}{I}$$

$$L = \frac{\mu_0 N^2 A}{l}$$

$$\varepsilon = -L\frac{dI}{dt}$$

Energy stored in an inductor

$$U = L\frac{I^2}{2}$$

"Discharged" LR circuit

$$I = I_0 e^{-t/\tau};\ \tau = L/R$$

Maxwell's equations

$$\oint \vec{E} \cdot \overrightarrow{dA} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{B} \cdot \overrightarrow{dA} = 0$$

$$\varepsilon = \oint \vec{E} \cdot \overrightarrow{ds} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot \overrightarrow{ds} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

The Poynting vector

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

Malus's Law

$$I = I cos^2\theta$$

Traveling Wave

$$y(x,t) = Asin(kx - \omega t + \varphi_0)$$

$$k = \frac{2\pi}{\lambda};\ \omega = \frac{2\pi}{T};\ v = \lambda f$$

Interference

$$\Delta\varphi = 2\pi\frac{\Delta r}{\lambda} + \Delta\varphi_0 = m2\pi\ (constr)$$

$$\Delta\varphi = 2\pi\frac{\Delta r}{\lambda} + \Delta\varphi_0$$

$$= (m + \frac{1}{2})2\pi\ (destr)$$

$$A = \left|2\text{acos}(\frac{\Delta\varphi}{2})\right|$$

Standing Waves

$$A(x)=2aSin(kx)$$

$$\lambda_m = \frac{2L}{m}; \;\; f_m = m\frac{v}{2L}$$

Double Slit

$$y_m = \frac{m\lambda L}{d}$$

Diffraction grating

$$d\sin\theta_m = m\lambda$$

$$y_m = L\tan\theta_m$$

Thin-lens equation:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$m = -\frac{s'}{s}; \quad |m| = \frac{h'}{h}$$

Snell's Law:

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\text{TIR:} \;\; \sin\theta_c = \frac{n_2}{n_1}$$

Constants

Charge on electron

$e = 1.60 \cdot 10^{-19}\ C$

Electron mass $m = 9.11 \cdot 10^{-31}\ kg$

Proton mass $m = 1.67 \cdot 10^{-27}\ kg$

Permittivity of free space

$$\varepsilon_0 = 8.85 \cdot 10^{-12}\ C^2/Nm^2$$

Permeability of free space

$$\mu_0 = 4\pi \cdot 10^{-7}\ Tm/A$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \cdot 10^9\ Nm^2/C^2$$

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3.0 \cdot 10^8 m/s$$

Kinematic eq-ns with const. Acc.:

$v(t) = v_{0x} + at$

$x(t) = x_0 + v_{0x}t + (1/2)\,at^2$

$v^2 = v_{0x}^2 + 2a(x - x_0)$

Centripetal acceleration $a_R = v^2/r$

$L = 2\pi R$

$A = \pi R^2$

$V = (4/3)\pi R^3$

EEC 130A : Formula Sheet

~~Up to~~ Midterm 2

Updated: Mar. 7th 2012

1 Electrostatics

Force on a point charge q inside a static electric field

$$\mathbf{F} = q\mathbf{E}$$

Gauss's law

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q \quad \text{or} \quad \nabla \cdot \mathbf{D} = \rho$$

Electrostatic fields are conservative

$$\nabla \times \mathbf{E} = 0 \quad \text{or} \quad \oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

Electric field produced by a point charge q in free space

$$\mathbf{E} = \frac{q\left(\mathbf{R} - \mathbf{R_i}\right)}{4\pi\epsilon_0 \left|\mathbf{R} - \mathbf{R_i}\right|^3}$$

Electric field produced by a volume charge distribution

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{V'} \hat{\mathbf{R}}' \frac{\rho_v \, dV'}{R'^2}$$

Electric field produced by a surface charge distribution

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{S'} \hat{\mathbf{R}}' \frac{\rho_s \, ds'}{R'^2}$$

Electric field produced by a line charge distribution

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_l \, dl'}{R'^2}$$

Electric field produced by an infinite sheet of charge

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_s}{2\epsilon}$$

Electric field produced by an infinite line of charge

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon} = \hat{\mathbf{r}} \frac{D_r}{\epsilon} = \hat{\mathbf{r}} \frac{\rho_l}{2\pi\epsilon r}$$

Electric field - scalar potential relationship

$$\mathbf{E} = -\nabla V \quad \text{or} \quad V_2 - V_1 = -\int_{P1}^{P2} \mathbf{E} \cdot d\mathbf{l}$$

Electric potential due to a point charge (with infinity chosen as the reference)

$$V = \frac{q}{4\pi\epsilon_0 \left|\mathbf{R} - \mathbf{R_i}\right|}$$

Poisson's equation

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

Constitutive relationship in dielectric materials

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

where $\mathbf{P}$ is the polarization.

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

Electrostatic energy density

$$w_e = \frac{1}{2}\epsilon E^2$$

Boundary conditions

$$E_{1t} = E_{2t} \quad \text{or} \quad \hat{\mathbf{n}} \times (\mathbf{E_1} - \mathbf{E_2}) = 0$$

$$D_{1n} - D_{2n} = \rho_s \quad \text{or} \quad \hat{\mathbf{n}} \cdot (\mathbf{D_1} - \mathbf{D_2}) = \rho_s$$

Ohm's law

$$\mathbf{J} = \sigma \mathbf{E}$$

Conductivity

$$\sigma = \rho_v \mu$$

where μ stands for charge mobility.

Joule's law

$$P = \int \mathbf{E} \cdot \mathbf{J} \; dv$$

2 Magnetostatics

Force on a moving charge q inside a magnetic field

$$\mathbf{F} = q\mathbf{u} \times \mathbf{B}$$

Force on an infinitesimally small current element $Id\mathbf{l}$ inside a magnetic field

$$d\mathbf{F}_m = Id\mathbf{l} \times \mathbf{B}$$

Torque on a N-turn loop carrying current I inside a uniform magnetic field

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

where $\mathbf{m} = \hat{\mathbf{n}} NIA$.

Gauss's law for magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \text{or} \quad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

Ampere's law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \text{or} \quad \oint_C \mathbf{H} \cdot d\mathbf{l} = I$$

Magnetic flux density — magnetic vector potential relationship

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Magnetic potential produced by a current distribution

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}}{R'} dV'$$

Vector Poisson's Equation

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Magnetic field intensity produced by an infinitesimally small current element (Biot-Savart law)

$$d\mathbf{H} = \frac{I}{4\pi} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2}$$

Magnetic field produced by an infinitely long wire of current in the z-direction

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}$$

Magnetic field produced by a circular loop of current in the ϕ-direction

$$\mathbf{H} = \hat{\mathbf{z}} \frac{Ia^2}{2(a^2 + z^2)^{3/2}}$$

Constitutive relationship in magnetic materials

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$

Magnetization

$$\mathbf{M} = \chi_m \mathbf{H}$$

Boundary conditions

$$B_{1n} = B_{2n} \quad \text{or} \quad \hat{\mathbf{n}} \cdot (\mathbf{B_1} - \mathbf{B_2}) = 0$$

$$H_{1t} - H_{2t} = J_s \quad \text{or} \quad \hat{\mathbf{n}} \times (\mathbf{H_1} - \mathbf{H_2}) = \mathbf{J_s}$$

Magnetostatic energy density

$$w_m = \frac{1}{2}\mu H^2$$

3 Useful Integrals

$$\int \frac{dx}{\sqrt{x^2+c^2}} = \ln(x + \sqrt{x^2+c^2})$$

$$\int \frac{dx}{x^2+c^2} = \frac{1}{c}\tan^{-1}\frac{x}{c}$$

$$\int \frac{dx}{\left(x^2+c^2\right)^{3/2}} = \frac{1}{c^2}\frac{x}{\sqrt{x^2+c^2}}$$

$$\int \frac{x\ dx}{\sqrt{x^2+c^2}} = \sqrt{x^2+c^2}$$

$$\int \frac{x\ dx}{x^2+c^2} = \frac{1}{2}\ln\left(x^2+c^2\right)$$

$$\int \frac{x\ dx}{(x^2+c^2)^{3/2}} = -\frac{1}{\sqrt{x^2+c^2}}$$

$$\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

4 Constants

Free space permittivity

$$\epsilon_0 = 8.85 \times 10^{-12} \qquad \text{F/m}$$

Free space permeability

$$\mu_0 = 4\pi \times 10^{-7} \qquad \text{H/m}$$

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1,$ for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1,$ for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1,$ for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}}R & \hat{\boldsymbol{\phi}}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Mechanical Reps Inc.

Austin
3901 Woodbury Drive
P.O. Box 41869
Austin, Texas 78704
Tel: 512.444.1835
Fax: 512.444.5522
www.mechreps.com

San Antonio
4710 Perrin Creek #300
San Antonio, Texas 78217
Tel: 210.650.9005
Toll Free: 800.650.6507
Fax: 210.590.1645
www.mechreps.com

Rio Grande Valley
1409 N. Stuart Place Road
Suite E
Harlingen, Texas 78552
Tel: 956.412.1110
Fax: 956.412.1350
www.mechreps.com

FORMULAS & CONVERSIONS

$$KW = \frac{BHP \; x \; 0.746}{Motor \; Eff.}$$

$$BTUH_{(Latent)} = 4700 \; x \; CFM \; x \; \Delta W_{(\Delta \; humidity \; ratio)}$$

$$Mixed \; Air \; Temp = \frac{(OAT)(outside \; air \; CFM) + (RAT)(return \; air \; CFM)}{(outside \; air \; CFM) + (return \; air \; CFM)}$$

$$Boiler \; HP = 33{,}475 \; BTU/HR$$

$$Sensible \; Heat, BTU/HR = CFM \; x \; 1.08 \; x \; \Delta T \; (D.B.°F)$$

$$Latent \; Heat, BTU/HR = CFM \; x \; 0.68 \; x \; \Delta g \; (grains)$$

$$Total \; Heat, BTU/HR = CFM \; x \; 4.5 \; x \; \Delta h \; (enthalpy)$$

$$GPM = \frac{BTU/hr}{500 \; x \; \Delta T \; (°F)}$$

$$Cooling \; Tower \; GPM \; (compression) = \frac{Tons \; x \; 30}{\Delta T \; (°F)}$$

$$Chiller \; GPM = \frac{Tons \; x \; 24}{\Delta T \; (°F)}$$

Lbs. Per Hour of Cooling:

$$Coil \; Condensate = \frac{4.5 \; x \; CFM \; x \; \Delta W_{(\Delta \; humidity \; ratio)}}{7000}$$

For Pumps:

$$HP = \frac{GPM \; x \; Head \; in \; Feet \; x \; Specific \; Gravity}{3960 \; x \; Efficiency \; of \; pump}$$

For Fans and Blowers:

$$HP = \frac{CFM \; x \; Pressure \; (in \; H_2O)}{6350 \; x \; Efficiency}$$

CONVERSION FACTORS

To Convert	To	Multiply By
Feet of Water	Inches of Mercury	0.8826
Feet of Water	Pounds/ Sq. Inch	0.4335
Gallons of Water	Pounds of Water	8.3452
Gallons of # 2 Oil	BTU's	140,000
Horsepower	Kilowatts	0.7457
Horsepower	BTU / Hr.	2545
Horsepower (boiler)	BTU / Hr.	33479
Horsepower (boiler)	Kilowatts	9.803
Inches of Mercury	Feet of Water	1.133
Inches of Mercury	Pounds/ Sq. Inch	0.4912
Kilowatts	BTU/ Hr.	3413
Kilowatts	Watts	1000
Pounds of Water	Cubic Feet	0.01602
Pounds of Water	Gallons	0.1198
Pounds/ Sq.Inch	Feet of Water	2.307
Therms of Gas	BTU's	100,000
Tons (refrigeration)	BTU/ Hr.	12,000
Watts	BTU/ Hr.	3.413

Electrical Formulas

To Find	Single Phase	Two Phase-Four Wire	Three Phase
Amps when "HP" is known	HP x 746 / V x %EFF x PF	HP x 746 / V x %EFF x PF x 2	HP x 746 / V x %EFF x PF x 1.73
Amps when "KW"is known	KW x 1000 / V x PF	KW x 1000 / V x PF x 2	KW x 1000 / V x PF x 1.73
Amps when "kVA"is known	kVA x 1000 / V	kVA x 1000 / V x 2	kVA x 1000 / V x 1.73
Kilowatts	V x A x PF / 1000	V x A x PF x 2 / 1000	V x A x PF x 1.73 / 1000
Kilovolt-Amps "kVA"	V x A / 1000	V x A x 2 / 1000	V x A x 1.73 / 1000
HP	V x A x %EFF x PF / 746	V x A x %EFF x PF x 2 / 746	V x A x %EFF x PF x 1.73 / 746

V=VOLTS A=AMPS EFF=EFFICIENCY PF=POWER FACTOR

MOTOR FULL-LOAD AMPERES

Horse-power	Three-Phase AC Squirrel-Cage & Wound-Rotor			Single Phase AC		
	208 V	230 V	460 V	115 V	200 V	230 V
1/6				4.4	2.5	2.2
1/4				5.8	3.3	2.9
1/2	2.3	2	1	9.8	5.6	4.9
3/4	3.2	2.8	1.4	13.8	7.9	6.9
1	4.1	3.6	1.8	16	9.2	8
1.5	5.7	5.2	2.6	20	11.5	10
2	7.5	6.8	3.4	24	13.8	12
3	11	9.6	4.8	34	19.6	17
5	16.7	15.2	7.6	56	32.2	28
7.5	24	22	11	80	46	40
10	31	28	14	100	57.5	50
15	46	42	21			
20	59	54	27			
25	75	68	34			
30	88	80	40			
40	114	104	52			
50	143	130	65			
60	170	154	77			
75	211	192	96			
100	273	248	124			
125	342	312	156			
150	396	360	180			
200	528	480	240			

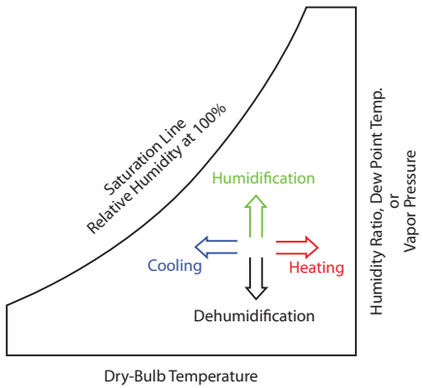
Basic Fan Laws

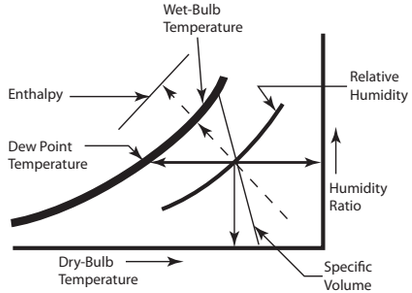
Variation	Fan Speed Change	Density Change
Volume	Varies Directly with Speed Ratio $CFM_2 = CFM_1 \left(\frac{RPM_2}{RPM_1}\right)$	No Change
Pressure	Varies with SQUARE of Speed Ratio $P_2 = P_1 \left(\frac{RPM_2}{RPM_1}\right)^2$	Varies DIRECTLY with Density Ratio $P_2 = P_1 \left(\frac{D_2}{D_1}\right)$
Horse Power	Varies with CUBE of Speed Ratio $HP_2 = HP_1 \left(\frac{RPM_2}{RPM_1}\right)^3$	Varies DIRECTLY with Density Ratio $HP_2 = HP_1 \left(\frac{D_2}{D_1}\right)$

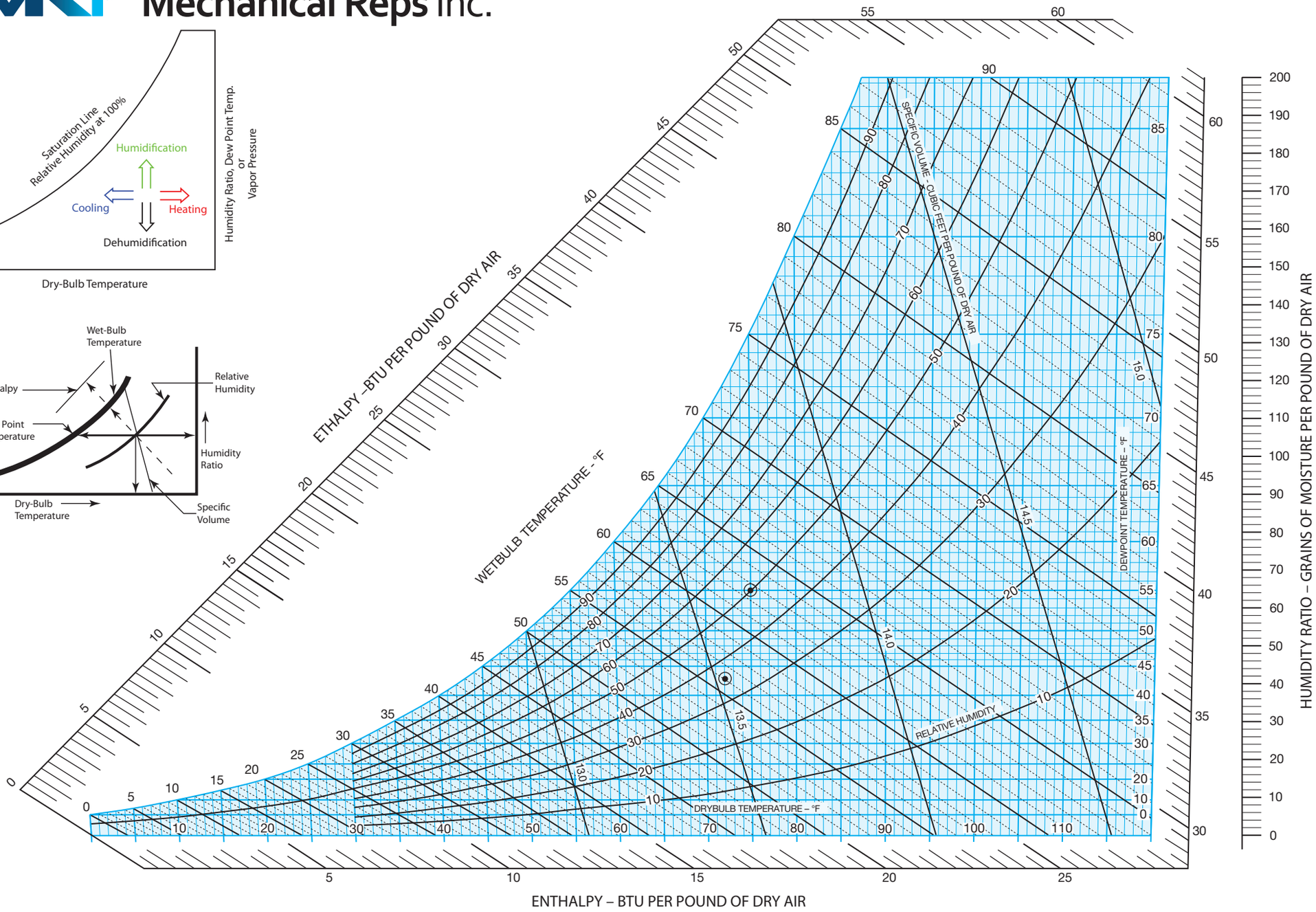
Armstrong • BAC • Berner • Car-Mon • Cemline • ClimateMaster • Dri-Steem • Energy Labs • FabricAir
Fulton • Greenheck • Lochinvar • Price • Pro Hydronic • VMC/Amber Booth • Yaskawa

Mechanical Reps Inc.

PSYCHROMETRIC CHART







Formulas for IPhO

Version: July 4, 2018

I Mathematics

1. Taylor series (truncate for approximations):

$$F(x) = F(x_0) + \sum F^{(n)}(x_0)(x-x_0)^n/n!$$

Special case — linear approximation:

$$F(x) \approx F(x_0) + F'(x_0)(x-x_0)$$

Some examples for $|x| \ll 1$:

$$\sin x \approx x,\ \cos x \approx 1 - x^2/2,\ e^x \approx 1 + x$$
$$\ln(1+x) \approx x,\ (1+x)^n \approx 1 + nx$$

2. Perturbation method: find the solution iteratively using the solution to the "non-perturbed" (directly solvable) problem as the 0th approximation; corrections for the next approximation are calculated on the basis on the previous one.

3. Solution of the linear differential equation with constant coefficients $ay'' + by' + cy = 0$:

$$y = A\exp(\lambda_1 x) + B\exp(\lambda_2 x),$$

where $\lambda_{1,2}$ is the solution of the characteristic equation $a\lambda^2 + b\lambda + c = 0$ if $\lambda_1 \neq \lambda_2$. If the solution of the characteristic equation is complex, while a, b and c are real numbers, then $\lambda_{1,2} = \gamma \pm i\omega$ and

$$y = Ce^{\gamma x}\sin(\omega x + \varphi_0).$$

4. Complex numbers

$$z = a + b\mathrm{i} = |z|e^{\mathrm{i}\varphi},\ \bar{z} = a - \mathrm{i}b = |z|e^{-\mathrm{i}\varphi}$$
$$|z|^2 = z\bar{z} = a^2 + b^2,\ \varphi = \arg z = \arcsin\frac{b}{|z|}$$
$$\mathrm{Re}z = (z+\bar{z})/2,\ \mathrm{Im}z = (z-\bar{z})/2$$
$$|z_1 z_2| = |z_1||z_2|,\ \arg z_1 z_2 = \arg z_1 + \arg z_2$$
$$e^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\sin\varphi$$
$$\cos\varphi = \tfrac{e^{\mathrm{i}\varphi}+e^{-\mathrm{i}\varphi}}{2},\ \sin\varphi = \tfrac{e^{\mathrm{i}\varphi}-e^{-\mathrm{i}\varphi}}{2i}$$

5. Cross and dot products of vectors are distributive: $a(b+c) = ab + ac$.

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a} = a_x b_x + a_y b_y + \ldots = ab\cos\varphi$$
$$|\vec{a}\times\vec{b}| = ab\sin\varphi;\ \ \vec{a}\times\vec{b} = -\vec{b}\times\vec{a} \perp \vec{a}, \vec{b}$$
$$\vec{a}\times\vec{b} = (a_y b_z - b_y a_z)\vec{e}_x + (a_z b_x - b_z a_x)\vec{e}_y + \ldots$$
$$\vec{a}\times[\vec{b}\times\vec{c}] = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b}).$$

Mixed prod. (volume of parallelep. def. by 3 vec.):

$$(\vec{a},\vec{b},\vec{c}) \equiv (\vec{a}\cdot[\vec{b}\times\vec{c}]) = ([\vec{a}\times\vec{b}]\cdot\vec{c}) = (\vec{b},\vec{c},\vec{a}).$$

6. Cosine and sine laws:

$$c^2 = a^2 + b^2 - 2ab\cos\varphi$$
$$a/\sin\alpha = b/\sin\beta = 2R$$

7. $\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\tan(\alpha\pm\beta) = (\tan\alpha + \tan\beta)/(1 \mp \tan\alpha\tan\beta)$
$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$, $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$
$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}, \ldots$
$\cos\alpha + \cos\beta = 2(\cos\frac{\alpha+\beta}{2} + \cos\frac{\alpha-\beta}{2}), \ldots.$

8. An angle inscribed in a circle is half of the central angle that subtends the same arc on the circle. *Conclusions:* hypotenuse of a right triangle is the diameter of its circumcircle; if the angles of a quadrilateral are supplementary, it is a cyclic quadrilateral.

9. Surface area of a triangle $= \frac{1}{2}ah_a = pr = \sqrt{p(p-a)(p-b)(p-c)} = abc/4R$.

10. Triangle's centroid: intersection point of medians, divides medians to 2:1.

11*. Vector approach to geometry problems.

12. Taking derivatives:

$$(fg)' = fg' + f'g,\ f[g(x)]' = f'[g(x)]g'$$
$$(\sin x)' = \cos x,\ (\cos x)' = -\sin x$$
$$(e^x)' = e^x,\ (\ln x)' = 1/x,\ (x^n)' = nx^{n-1}$$
$$(\arctan x)' = 1/(1+x^2)$$
$$(\arcsin x)' = -(\arccos x)' = 1/\sqrt{1-x^2}$$

13. Integration: the formulas are the same as for derivatives, but with swapped left-hand-side and rhs. (inverse operation!), e.g.

$$\int x^n \mathrm{d}x = x^{n+1}/(n+1).$$

Special case of the substitution method:

$$\int f(ax+b)\mathrm{d}x = F(ax+b)/a.$$

14. Conic sections: $a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + a_1 x + a_2 y + a_0 = 0$ with $a_{11} = a_{22}$ — circle; with $a_{11}\cdot(a_{11}a_{22} - a_{12}^2) > 0$ — ellipse, $\ldots < 0$ — hyperbola, with $a_{11}a_{22} = a_{12}^2$ — parabola. Ellipse: $l_1 + l_2 = 2a$, $\alpha_1 = \alpha_2$, $A = \pi ab$; hyperbola: $l_1 - l_2 = 2a$, $\alpha_1 + \alpha_2 = 0$; parabola: $l + h = \text{const}$, $\alpha_1 = \alpha_2$.

15. *Numerical methods.* Newton's iterative method for finding roots $f(x) = 0$:

$$x_{n+1} = x_n - f(x_n)/f'(x_n).$$

Trapezoidal rule for approximate integration:

$$\int_a^b f(x)\mathrm{d}x \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)]$$

16. Derivatives and integrals of vectors: differentiate/integrate each component; alternatively differentiate by applying the triangle rule for the difference of two infinitesimally close vectors.

II General recommendations

1. Check all formulas for veracity: a) examine dimensions; b) test simple special cases (two parameters are equal, one param. tends to 0 or ∞); c) verify the plausibility of solution's qualitative behaviour.

2. If there is an extraordinary coincidence in the problem text (e.g. two things are equal) then the key to the solution might be there.

3. Read carefully the recommendations in the problem's text. Pay attention to the problem's formulation — insignificant details may carry vital information. If you have solved for some time unsuccessfully, then read the text again — perhaps you misunderstood the problem.

4. Postpone long and time-consuming mathematical calculations to the very end (when everything else is done) while writing down all the initial equations which need to be simplified.

5. If the problem seems to be hopelessly difficult, it has usually a very simple solution (and a simple answer). This is valid only for Olympiad problems, which are definitely solvable.

6. In experiments a) sketch the experimental scheme even if you don't have time for measurements; b) think, how to increase the precision of the results; c) write down (as a table) all your direct measurements.

III Kinematics

1. For a point or for a translational motion of a rigid body (integral $\to$ area under a graph):

$$\vec{v} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t},\ \vec{x} = \int \vec{v}\mathrm{d}t \quad (x = \int v_x \mathrm{d}t \text{ etc.})$$
$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{x}}{\mathrm{d}t^2},\ \vec{v} = \int \vec{a}\mathrm{d}t$$
$$t = \int v_x^{-1}\mathrm{d}x = \int a_x^{-1}\mathrm{d}v_x,\ x = \int \frac{v_x}{a_x}\mathrm{d}v_x$$

If $a = \text{Const.}$, then previous integrals can be found easily, e.g.

$$x = v_0 t + at^2/2 = (v^2 - v_0^2)/2a.$$

2. Rotational motion — analogous to the translational one: $\omega = \mathrm{d}\varphi/\mathrm{d}t$, $\varepsilon = \mathrm{d}\omega/\mathrm{d}t$;

$$\vec{a} = \vec{\tau}\mathrm{d}v/\mathrm{d}t + \vec{n}v^2/R$$

3. Curvilinear motion — same as point 1, but vectors are to be replaced by linear velocities, accelerations and path lengths.

4. *Motion of a rigid body.* **a)** $v_A\cos\alpha = v_B\cos\beta$; $\vec{v}_A$, $\vec{v}_B$ — velocities of pts. A and B; α, β — angles formed by $\vec{v}_A$, $\vec{v}_B$ with line AB. **b)** The instantaneous center of rotation ($\neq$ center of curvature of material pt. trajectories!) can be found as the intersection pt. of perpendiculars to $\vec{v}_A$ and $\vec{v}_B$, or (if $\vec{v}_A, \vec{v}_B \perp AB$) as the intersection pt. of AB with the line connecting endpoints of $\vec{v}_A$ and $\vec{v}_B$.

5. Non-inertial reference frames:

$$\vec{v}_2 = \vec{v}_0 + \vec{v}_1,\ \vec{a}_2 = \vec{a}_0 + \vec{a}_1 + \omega^2\vec{R} + \vec{a}_{Cor}$$

Note: $\vec{a}_{Cor} \perp \vec{v}_1, \vec{\omega}$; $\vec{a}_{Cor} = 0$ if $\vec{v}_1 = 0$.

6*. Ballistic problem: reachable region

$$y \le v_0^2/(2g) - gx^2/2v_0^2.$$

For an optimal ballistic trajectory, initial and final velocities are perpendicular.

7. For finding fastest paths, Fermat's and Huygens's principles can be used.

8. To find a vector (velocity, acceleration), it is enough to find its direction and a projection to a single (possibly inclined) axes.

IV Mechanics

1. For a 2D equilibrium of a rigid body: 2 eqns. for force, 1 eq. for torque. 1 (2) eq. for force can be substituted with 1 (2) for torque. Torque is often better — "boring" forces can be eliminated by a proper choice of origin. If forces are applied only to 2 points, the (net) force application lines coincide; for 3 points, the 3 lines meet at a single point.

2. Normal force and friction force can be combined into a single force, applied to the contact point under angle $\arctan\mu$ with respect to the normal force.

3. Newton's 2nd law for transl. and rot. motion:

$$\vec{F} = m\vec{a},\ \vec{M} = I\vec{\varepsilon} \quad (\vec{M} = \vec{r}\times\vec{F}).$$

For 2D geometry $\vec{M}$ and $\vec{\varepsilon}$ are essentially scalars and $M = Fl = F_t r$, where l is the arm of a force.

4. *Generalized coordinates.* Let the system's state be defined by a single parameter ξ and its time derivative $\dot{\xi}$ so that the pot. energy $\Pi = \Pi(\xi)$ and kin. en. $K = \mu\dot{\xi}^2/2$; then $\mu\ddot{\xi} = -\mathrm{d}\Pi(\xi)/\mathrm{d}\xi$. (Hence for transl. motion: force is the derivative of pot. en.)

5. If the system consists of mass points m_i:

$$\vec{r}_c = \sum m_i\vec{r}_i / \sum m_j,\ \vec{P} = \sum m_i\vec{v}_i$$
$$\vec{L} = \sum m_i\vec{r}_i\times\vec{v}_i,\ \ K = \sum m_i v_i^2/2$$

$$I_z = \sum m_i(x_i^2 + y_i^2) = \int (x^2 + y^2)\mathrm{d}m.$$

6. In a frame where the mass center's velocity is $\vec{v}_c$ (index c denotes quantities rel. to the mass center):

$$\vec{L} = \vec{L}_c + M_\Sigma \vec{R}_c \times \vec{v}_c,\ K = K_c + M_\Sigma v_c^2/2$$
$$\vec{P} = \vec{P}_c + M_\Sigma \vec{v}_c$$

7. Steiner's theorem is analogous (b — distance of the mass center from rot. axis): $I = I_c + mb^2$.

8. With $\vec{P}$ and $\vec{L}$ from pt. 5, Newton's 2nd law:

$$\vec{F}_\Sigma = \mathrm{d}\vec{P}/\mathrm{d}t,\quad \vec{M}_\Sigma = \mathrm{d}\vec{L}/\mathrm{d}t$$

9*. Additionally to pt. 5, the mom. of inertia rel. to the z-axis through the mass center can be found as $I_{z0} = \sum_{i,j} m_i m_j[(x_i - x_j)^2 + (y_i - y_j)^2]/2M_\Sigma$.

10. Mom. of inertia rel. to the origin $\theta = \sum m_i \vec{r}_i^2$ is useful for calculating I_z of 2D bodies or bodies with central symmetry using $2\theta = I_x + I_y + I_z$.

11. Physical pendulum with a reduced length $\tilde{l}$:

$$\omega^2(l) = g/(l + I/ml),$$
$$\omega(l) = \omega(\tilde{l} - l) = \sqrt{g/\tilde{l}},\quad \tilde{l} = l + I/ml$$

12. Coefficients for the momenta of inertia: cylinder $\frac{1}{2}$, solid sphere $\frac{2}{5}$, thin spherical shell $\frac{2}{3}$, rod $\frac{1}{12}$ (rel. to endpoint $\frac{1}{3}$), square $\frac{1}{6}$.

13. Often applicable conservation laws:
energy (elastic bodies, no friction),
momentum (no net external force; can hold only along one axis),
angular momentum (no net ext. torque, e.g. the arms of ext. forces are 0 (can be written rel. to 2 or 3 pts., then substitutes conservation of lin. mom.).

14. Additional forces in non-inertial frames of ref.: inertial force $-m\vec{a}$, centrifugal force $m\omega^2\vec{R}$ and Coriolis force* $2m\vec{v} \times \vec{\Omega}$ (better to avoid it; being $\perp$ to the velocity, it does not create any work).

15. Tilted coordinates: for a motion on an inclined plane, it is often practical to align axes along and $\perp$ to the plane; gravit. acceleration has then both x- and y- components. Axes may also be oblique (not $\perp$ to each other), but then with $\vec{v} = v_x\vec{e}_x + v_y\vec{e}_y$, $v_x \neq$ to the x-projection of $\vec{v}$.

16. Collision of 2 bodies: conserved are a) net momentum, b) net angular mom., c) *angular mom. of one of the bodies with respect to the impact point*, d) total energy (for elastic collisions); in case of friction, kin. en. is conserved only along the axis $\perp$ to the friction force. Also: e) if the sliding stops during the impact, the final velocities of the contact points will have equal projections to the contact plane; f) if sliding doesn't stop, the momentum delivered from one body to the other forms angle $\arctan\mu$ with the normal of the contact plane.

17. Every motion of a rigid body can be represented as a rotation around the instantaneous center of rotation C (in terms of velocities of the body points). NB! Distance of a body point P from $C \neq$ to the radius of curvature of the trajectory of P.

18. Tension in a string: for a massive hanging string, tension's horizontal component is constant and vertical changes according to the string's mass underneath. Pressure force (per unit length) of a string resting on a smooth surface is determined by its radius of curvature and tension: $N = T/R$. Analogy: surface tension pressure $p = 2\sigma/R$; to derive, study the pressure force along the diameter.

19. Liquid surface takes equipot. shape (neglecting σ); in incompr. liquid, $p = p_0 - w$, w—vol. dens. of pot. en. (for a gas, see X-6).

20. Bernoulli law for incompr. fluid:

$$p + \frac{1}{2}\rho v^2 + \rho\varphi = \text{const};$$

in homog. field, the gravit. potential $\varphi = gh$. For gas of specific heat c_p [J/kg],

$$\frac{1}{2}v^2 + c_pT = \text{const}.$$

21*. Momentum continuity by straight streamlines: $p + \rho v^2 = \text{const}$.

22*. Adiabatic invariant: if the relative change of the parameters of an oscillating system is small during one period, the area of the loop drawn on the phase plane (ie. in p-x coordinates) is conserved with a very high accuracy.

23. For studying stability use a) principle of minimum potential energy or b) principle of small virtual displacement.

24*. Virial theorem for finite movement:
a) If $F \propto |\vec{r}|$, then $\langle K\rangle = \langle\Pi\rangle$ (time averages);
b) If $F \propto |\vec{r}|^{-2}$, then $2\langle K\rangle = -\langle\Pi\rangle$.

25. Tsiolkovsky rocket equation $\Delta v = u\ln\frac{M}{m}$.

V Oscillations and waves

1. Damped oscillator:

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0\ (\gamma < \omega_0).$$

Solution of this equation is (cf. I.2.):

$$x = x_0 e^{-\gamma t}\sin(t\sqrt{\omega_0^2 - \gamma^2} - \varphi_0).$$

2. Eq. of motion for a system of coupled oscillators: $\ddot{x}_i = \sum_j a_{ij}x_j$.

3. A system of N coupled oscillators has N different eigenmodes when all the oscillators oscillate with the same frequency ω_i, $x_j = x_{j0}\sin(\omega_i t + \varphi_{ij})$, and N eigenfrequencies ω_i (which can be multiple, $\omega_i = \omega_j$). General solution (with $2N$ integration constants X_i and ϕ_i) is a superposition of all the eigenmotions :

$$x_j = \sum_i X_i x_{j0}\sin(\omega_i t + \varphi_{ij} + \phi_i)$$

4. If a system described with a generalized coordinate ξ (cf IV-2) and $K = \mu\dot{\xi}^2/2$ has an equilibrium state at $\xi = 0$, for small oscillations $\Pi(\xi) \approx \kappa\xi^2/2$ [where $\kappa = \Pi''(0)$] so that $\omega^2 = \kappa/\mu$.

5. The phase of a wave at pt. x, t is $\varphi = kx - \omega t + \varphi_0$, where $k = 2\pi/\lambda$ is a wave vector. The value at x, t is $a_0\cos\varphi = \Re a_0 e^{i\varphi}$. The phase velocity is $v_f = \nu\lambda = \omega/k$ and group velocity $v_g = d\omega/dk$.

6. For linear waves (electromagn. w., small-amplit. sound- and water w.) any pulse can be considered as a superpos. of sinusoidal waves; a standing w. is the sum of two identical counter-propagating w.:

$$e^{i(kx-\omega t)} + e^{i(-kx-\omega t)} = 2e^{-i\omega t}\cos kx.$$

7. Speed of sound in a gas

$$c_s = \sqrt{(\partial p/\partial\rho)_{\text{adiab}}} = \sqrt{\gamma p/\rho} = \bar{v}\sqrt{\gamma/3}.$$

8. Speed of sound in elastic material $c_s = \sqrt{E/\rho}$.

9. Sp. of waves in shallow ($h \ll \lambda$) water: $v = \sqrt{gh}$; in a string: $v = \sqrt{T/\rho_{\text{lin}}}$.

10. Doppler's effect: $\nu = \nu_0\frac{1+v_\parallel/c_s}{1-u_\parallel/c_s}$.

11. Huygens' principle: wavefront can be constructed step by step, placing an imaginary wave source in every point of previous wave front. Results are curves separated by distance $\Delta x = c_s\Delta t$, where Δt is time step and c_s is the velocity in given point. Waves travel perpendicular to wavefront.

VI Geometrical optics. Photometry.

1. Fermat's principle: waves path from point A to point B is such that the wave travels the least time.

2. Snell's law:

$$\sin\alpha_1/\sin\alpha_2 = n_2/n_1 = v_1/v_2.$$

3. If refraction index changes continuously, then we imaginarily divide the media into layers of constant n and apply Snell's law. Light ray can travel along a layer of constant n, if the requirement of total internal reflection is marginally satisfied, $n' = n/r$ (where r is the curvature radius).

4. If refraction index depends only on z, the photon's mom. p_x , p_y, and en. are conserved:

$$k_x, k_y = \text{Const.},\ |\vec{k}|/n = \text{Const.}$$

5. The thin lens equation (pay attention to signs):

$$1/a + 1/b = 1/f \equiv D.$$

6. Newton's eq. (x_1, x_2 — distances of the image and the object from the focal planes): $x_1x_2 = f^2$.

7. Parallax method for finding the position of an image: find such a pos. for a pencil's tip that it wouldn't shift with resp. to the image when moving perpendicularly the position of your eye.

8. Geometrical constructions for finding the paths of light rays through lenses:
a) ray passing the lens center does not refract;
b) ray $\parallel$ to the optical axis passes through the focus;
c) after refr., initially $\parallel$ rays meet at the focal plane;
d) image of a plane is a plane; these two planes meet at the plane of the lens.

9. Luminous flux Φ [unit: lumen (lm)] measures the energy of light (emitted, passing a contour, etc), weighted according to the sensitivity of an eye. Luminous intensity [candela (cd)] is the luminous flux (emitted by a source) per solid angle: $I = \Phi/\Omega$. Illuminance [lux (lx)] is the luminous flux (falling onto a surface) per unit area: $E = \Phi/S$.

10. Gauss theorem for luminous flux: the flux through a closed surface surrounding the point

sources of intensity I_i is $\Phi = 4\pi \sum I_i$; single-source-case: at a distance r, $E = I/r^2$.

11. An experimental hint: if a grease stain on a paper is as bright as the surrounding paper, then the paper is equally illuminated from both sides.

VII Wave optics

1. Diffraction — method based on Huygens' principle: if obstacles cut the wavefront into fragment(s), the wavefront can be divided into small pieces which serve as imaginary point-like light sources; the wave amplitude at the observ. site will be the sum over the contributions of these sources.

2. Two slit interference (the slit width $d \ll a, \lambda$): angles of maxima $\varphi_{\max} = \arcsin(n\lambda/a)$, $n \in Z$; $I \propto \cos^2(k\frac{a}{2}\sin\varphi)$, where $k = 2\pi/\lambda$.

3. Single slit: angles of *minima* $\varphi_{\min} = \arcsin(n\lambda/d)$, $n \in Z$, $n \neq 0$. NB! the central maximum is double-wide. $I \propto \sin^2(k\frac{d}{2}\sin\varphi)/\varphi$.

4. Diffraction grating: the main maxima are the same as in pt. 2, the width of the main maxima — the same as for pt. 3 with d being the net grating length. Spectral resolving power $\frac{\lambda}{\Delta\lambda} = nN$, where n is the order number of the main max. and N — the number of slits.

5. Resolving power of a spectral device: $\frac{\lambda}{\Delta\lambda} = \frac{L}{\lambda}$, where L is the optical path difference between the shortest and longest beams.

6. Resolving power of a prism: $\frac{\lambda}{\Delta\lambda} = a\frac{dn}{d\lambda}$.

7. Angular distance when two pts. are resolved in an ideal telescope (lens): $\varphi = 1.22\lambda/d$. For that angle, the center of one point falls onto the first diffr. min. of the other point.

8. Bragg theory: a set of $\parallel$ ion planes of a crystal reflects X-rays if $2a\sin\alpha = k\lambda$; a — distance between neighb. planes, α — glancing angle.

9. Reflection from optically denser dielectric media: phase shift π. For semi-transparent thin films, $\phi_{\rightarrow} + \phi_{\leftarrow} = \pi$; $\phi_{\rightarrow}$ and $\phi_{\leftarrow}$ — phase shifts between reflected and trasmitted waves (arrows denote the direction of incidence).

10. Fabry-Pérot interferometer: two $\parallel$ semi-transp. mirrors with large reflectivity r ($1-r \ll 1$). Resolving power $\frac{\nu}{\Delta\nu} \approx \frac{2a}{\lambda(1-r)}$. Transmission spectrum can be found by introducing 5 plane waves (for left- and rightwards-propagating waves before the device, in the dev. and after the dev.) and tailoring these at the region boundaries.

11. Coherent electromagnetic waves: electric fields are added; vector diagram can be used, angle between vectors is the phase shift; NB! dispersion: $n = n(\omega) = \sqrt{\varepsilon(\omega)}$ (usually $\mu \cong 1$). Energy flux density (en. per unit area and time): $I = cn\varepsilon_0 E^2 = \frac{c}{n\mu_0}B^2$ (E, B — RMS values).

12. Malus' law: for linearly polarized light $I = I_0\cos^2\varphi$, where φ is the angle between the polarization planes.

13. $\lambda/4$-plate: phase shift $\pi/2$ between linearly polarized components.

14. Brewster's angle: reflected and refracted rays are $\perp$; reflected ray is completely polarized; incidence angle $\tan\varphi_B = n$.

15. Diffr. with optical elements: no need to calculate optical path lengths through lenses, prisms etc.: work simply with images. Particular conclusion: biprism gives the same diffr. as a double slit.

16*. Optical fibres: Mach-Zehnder interferometer is analogous to a double-slit diffraction; circular resonator — to Fabry-Pérot interferometer; Bragg filters work similarly to the X-ray case. Single-mode fibres: $\Delta n/n \approx \frac{1}{2}(\lambda/d)^2$.

VIII Circuits

1.
$$U = IR,\ P = UI$$
$$R_{\text{series}} = \sum R_i, \quad R_{\parallel}^{-1} = \sum R_i^{-1}$$

2. Kirchoff's laws:
$$\sum_{\text{node}} I = 0, \quad \sum_{\text{contour}} U = 0$$

3. To reduce the number of eqns. for pt 2: *method of node potentials; method of loop currents;* equivalent circuits (any 3-terminals $\Rightarrow \Delta$ or Y; 2-term. with emf $\Rightarrow r$ and $\mathcal{E}$ in series).

4. Resistance of infinite chain: use self-similarity; resistance between neighbour nodes of infinite grid: generalized method of electrical images.

5. AC: apply pts. 1–4 while substituting R with Z:
$$Z_R = R,\ Z_C = 1/i\omega C,\ Z_L = i\omega L;$$
$$\varphi = \arg Z,\ U_{\text{eff}} = |Z|I_{\text{eff}}$$
$$P = |U||I|\cos(\arg Z) = \sum I_i^2 R_i.$$

6. Characteristic times: $\tau_{RC} = RC$, $\tau_{LR} = L/R$, $\omega_{LC} = 1/\sqrt{LC}$. Relaxation to stationary current distribution exponential, $\propto e^{-t/\tau}$.

7. Energy conservation for electric circuits: $\Delta W + Q = Uq$, where q is charge which has crossed a potential drop U; work of emf is $A = \mathcal{E}q$.

8. $W_C = CU^2/2$, $W_L = LI^2/2$.

9. $\mathcal{E} = -d\Phi/dt = -d(LI)/dt$, $\Phi = BS$.

10. Nonlinear elements: graphical method — find the solution in U-I coordinates as an intersection point of a nonlinear curve and a line representing Ohm/Kirchoff laws. In case of many intersection points study stability — some solutions are usually unstable.

11. Make use of short- and long-time limits. For $t_{\text{observation}} \gg \tau_{RC}$ or τ_{LR}, quasiequilibrium is reached: $I_C \approx 0$ (wire is "broken" near C) and $\mathcal{E}_L \approx 0$ (L is effectively short-circuited). For $t_{\text{observation}} \ll \tau_{RC}$ or τ_{LR}, the charge leakage of C and current drop in L are small, $\Delta Q \ll Q$ and $\Delta I \ll I$: C is "short-circuited" and L is "broken".

12. If $L \neq 0$, then $I(t)$ is a continuous function.

13. Through a superconducting contour, magnetic flux $\Phi = \text{Const}$. In particular, with no external B, $LI = \text{Const}$.

14. Mutual inductance: magnetic flux through a contour $\Phi_1 = L_1 I_1 + L_{12} I_2$ (I_2 — current in a second contour). Theorems: $L_{12} = L_{21} \equiv M$; $M \leq \sqrt{L_1 L_2}$.

IX Electromagnetism

1. $F = kq_1q_2/r^2$, $\Pi = kq_1q_2/r$ — Kepler's laws are applicable (Ch. XII).

2. Gauss's law: $\oint \vec{B}d\vec{S} = 0$,
$$\oint \varepsilon\varepsilon_0 \vec{E}d\vec{S} = Q,\ \oint \vec{g}d\vec{S} = -4\pi GM.$$

3. Circulation theorem
$$\oint \vec{E}d\vec{l} = 0\ (= \dot{\Phi}),\ \oint \frac{\vec{B}dl}{\mu\mu_0} = I,\ \oint \vec{g}d\vec{l} = 0.$$

4. Magnetic field caused by current element:
$$\mathrm{d}\vec{B} = \frac{\mu\mu_0 I}{4\pi}\frac{\mathrm{d}\vec{l} \times \vec{e}_r}{r^2};$$
hence, at the center of circular I: $B = \frac{\mu_0 I}{2r}$

5. $\vec{F} = e(\vec{v} \times \vec{B} + \vec{E})$, $\vec{F} = \vec{I} \times \vec{B}l$.

6. From the Gauss's and circulation laws: charged wire: $E = \frac{\sigma}{2\pi\varepsilon_0 r}$, DC: $B = \frac{I\mu_0}{2\pi r}$; charged surface $E = \frac{\sigma}{2\varepsilon_0}$, current sheet $B = \frac{\mu_0 j}{2}$;
inside a sphere (or infinite cylindrical surface) of homogeneous surface charge $E = 0$, inside a cylindrical surface current $\parallel$ to the axes $B = 0$, inside a ball ($d = 3$), cylinder ($d = 2$) or layer ($d = 1$) of homogeneous ρ or $\vec{j}$:
$$\vec{E} = \frac{\rho}{d\varepsilon_0}\vec{r};\ \ \vec{B} = \frac{1}{d\mu_0}\vec{j} \times \vec{r}$$
($\vec{r}$ — radius vector from the centre).

7. Long solenoid: inside $B = In\mu\mu_0$, outside 0; flux $\Phi = NBS$ and (with $n = \frac{N}{l}$) inductance $L = \Phi/I = Vn^2\mu\mu_0$. Short solenoid $B_{\parallel} = \frac{In\mu\mu_0\Omega}{4\pi}$ (Ω — solid angle).

8. Measuring magnetic field with a small coil and ballistic galvanometer: $q = \int \frac{\mathcal{E}}{R}\mathrm{d}t = NS\Delta B/R$.

9. Potential energy of a system of charges:
$$\Pi = k\sum_{i>j}\frac{q_i q_j}{r_{ij}} = \frac{1}{2}\int \varphi(\vec{r})\mathrm{d}q,\ \mathrm{d}q = \rho(\vec{r})\mathrm{d}V.$$

10. Force between parts of a uniformly charged sphere or cylindrical surface: substitute force due to charges with force due to hydrostatic pressure.

11. If all the charges are at the distance R (eg. at the center of an inhomogeneously charged sphere or ring), $\varphi = kQ/r$.

12. To find the net charge (or potential) induced by external charges, use the superpos. pr.: "smear" the charges to make the problem symmetric.

13. Conductor shields charges and electric fields, eg. charge distribution inside a hollow sphere cannot be seen from outside (it seems as if there is a conducting ball carrying a total charge Q)

14. Capacitances: $C = \varepsilon\varepsilon_0 S/d$ (plane), $4\pi\varepsilon\varepsilon_0 r$(sphere), $2\pi\varepsilon\varepsilon_0 l(\ln R/r)^{-1}$ (coaxial).

15. Dipole moment:
$$\vec{d}_e = \sum q_i\vec{r}_i = \vec{l}q,\ \vec{d}_\mu = I\vec{S}.$$

16. Energy and torque of a dipole:
$$W = -\vec{d}\cdot\vec{E}\ (\vec{B}),\ \ \vec{M} = \vec{d} \times \vec{E}\ (\vec{B}).$$

17. Dipole field: $\varphi = k\vec{d}\cdot\vec{e}_r/r^2$; $E, B \propto r^{-3}$.

18. Forces acting on a dipole: $F = (\vec{E}\vec{d}_e)'$, $F = (\vec{B}\vec{d}_\mu)'$; interaction between 2 dipoles: $F \propto r^{-4}$.

19. Point charge as a magn. dipole: $d_\mu \propto \Phi \propto v_\perp^2/B$ is an adiab. inv (see IV-20).

20. Electric and magnetic images: grounded (superconducting for magnets) planes act as mirrors. Field of a grounded (or isolated) sphere can be found as a field of one (or two) fictive charge(s) inside the sphere. The field in a planar waveguide (slit between metallic plates) can be obtained as a superposition of electromagnetic plane waves.

21. Ball's (cylinder's) polarization in homogeneous (electric) field: superpos. of homogeneously charged ($+\rho$ and $-\rho$) balls (cylinders), $d \propto E$.

22. Eddy currents: power dissipation density $\sim B^2v^2/\rho$; momentum given during a single pass: $F\tau \sim B^2a^3d/\rho$ (where d — thickness; a — size).

23. Inside a superconductor and for fast processes inside a conductor $B = 0$ and thus $I = 0$ (current flows in surface layer — skin effect).

24. Charge in homog. magnetic field $\vec{B} = B\vec{e}_z$ moves along a cycloid with drift speed $v = E/B = F/eB$; generalized mom. is conserved

$$p_x' = mv_x - Byq,\ p_y' = mv_y + Bxq,$$

as well as gen. angular mom. $L' = L + \frac{1}{2}Bqr^2$.

25. MHD generator (a — length along the direction of $\vec{E}$):

$$\mathcal{E} = vBa,\ r = \rho a/bc.$$

26. Hysteresis: S-shaped curve (loop) in B-H-coordinates (for a coil with core also B-I-coord.): the loop area gives the thermal energy dissipation density per one cycle).

27. Fields in matter: $\vec{D} = \varepsilon\varepsilon_0\vec{E} = \varepsilon_0\vec{E} + \vec{P}$, where $\vec{P}$ is dielectric polarization vector (volume density of dipole moment); $\vec{H} = \vec{B}/\mu\mu_0 = \vec{B}/\mu_0 - \vec{J}$, where $\vec{J}$ is magnetization vector (volume density of magnetic moment).

28. In an interface between two substances E_t, D_n ($= \varepsilon E_t$), H_t ($= B_t/\mu$) and B_n are continuous.

29. Energy density: $W = \frac{1}{2}(\varepsilon\varepsilon_0E^2 + B^2/\mu\mu_0)$.

30. For $\mu \gg 1$, fieldlines of B are attracted to the ferromagnetic (acts as a potential hole, cf. pt. 28).

31. Current density $\vec{j} = ne\vec{v} = \sigma\vec{E} = \vec{E}/\rho$.

32. Lenz's law: system responds so as to oppose to changes.

X Thermodynamics

1. $pV = \frac{m}{\mu}RT$

2. Internal energy of one mole $U = \frac{i}{2}RT$.

3. Volume of one mole at standard cond. is 22,4 l.

4. Adiabatic processes: slow as compared to sound speed, no heat exchange: $pV^\gamma = \text{Const.}$ (and $TV^{\gamma-1} = \text{Const.}$).

5. $\gamma = c_p/c_v = (i+2)/i$.

6. Boltzmann's distribution:

$$\rho = \rho_0 e^{-\mu gh/RT} = \rho_0 e^{-U/kT}.$$

7. Maxwell's distribution (how many molecules have speed v) $\propto e^{-mv^2/2kT}$.

8. Atm. pressure: if $\Delta p \ll p$, then $\Delta p = \rho g\Delta h$.

9. $p = \frac{1}{3}mn\bar{v}^2 = nkT$, $\bar{v} = \sqrt{3kT/m}$, $\nu = vnS$.

10. Carnot's cycle: 2 adiabats, 2 isotherms. $\eta = (T_1 - T_2)/T_1$; derive using S-T-coordinates.

11. Heat pump, inverse Carnot: $\eta = \frac{T_1}{T_1 - T_2}$.

12. Entropy: $dS = dQ/T$.

13. I law of thermodynamics: $\delta U = \delta Q + \delta A$

14. II law of thermodynamics: $\Delta S \geq 0$ (and $\eta_{\text{real}} \leq \eta_{\text{Carnot}}$).

15. Gas work (look also p. 10)

$$A = \int p\mathrm{d}V, \quad \text{adiabatic:} \quad A = \frac{i}{2}\Delta(pV)$$

16. Dalton's law: $p = \sum p_i$.

17. Boiling: pressure of saturated vapour $p_v = p_0$; at the interface betw. 2 liquids: $p_{v1} + p_{v2} = p_0$.

18. Heat flux $P = kS\Delta T/l$ (k — thermal conductivity); analogy to DC circuits (P corresponds to I, ΔT to U, k to $1/\rho$).

19. Heat capacity: $Q = \int c(T)\mathrm{d}T$. Solids: for low temperatures, $c \propto T^3$; for high T, $c = 3Nk$, where N — number of ions in crystal lattice.

20. Surface tension:

$$U = S\sigma,\ F = l\sigma,\ p = 2\sigma/R.$$

21. Stefan-Boltzmann law (gray body): $P = \varepsilon\sigma T^4$.

22. Wien's law: $f_{\max} = Ak_BT/h$ ($A \approx 2.8$), $\lambda_{\max} = hc/A'k_BT$ ($A' \approx 5$)

XI Quantum mechanics

1. $\vec{p} = \hbar\vec{k}$ ($|\vec{p}| = h/\lambda$), $E = \hbar\omega = h\nu$.

2. Interference: as in wave optics.

3. Uncertainty (as a math. theorem):

$$\Delta p\Delta x \geq \frac{\hbar}{2},\ \ \Delta E\Delta t \geq \frac{\hbar}{2},\ \ \Delta\omega\Delta t \geq \frac{1}{2}.$$

For qualitative estimates by non-smooth shapes, h serves better ($\Delta p\Delta x \approx h$ etc).

4. Spectra: $h\nu = E_n - E_m$; width of spectral lines is related to lifetime: $\Gamma\tau \approx \hbar$.

5. Oscillator's (eg. molecule) en. levels (with eigenfrequency ν_0): $E_n = (n + \frac{1}{2})h\nu_0$. For many eigenfrequencies: $E = \sum_i hn_i\nu_i$.

6. Tunnelling effect: barrier Γ with width l is easily penetrable, if $\Gamma\tau \approx \hbar$, where $\tau = l/\sqrt{\Gamma/m}$.

7. Bohr's model: $E_n \propto -1/n^2$. In a (classically calculated) circular orbit, there is an integer number of wavelengths $\lambda = h/mv$.

8. Compton effect — if photon is scattered from an electron, photon's $\Delta\lambda = \lambda_C(1 - \cos\theta)$.

9. Photoeffect: $A + mv^2/2 = h\nu$ (A - work of exit for electrons). I-U-graph: photocurrent starts at the counter-voltage $U = -(h\nu - A)/e$, saturates for large forward voltages.

10. Stefan-Boltzmann: $P = \sigma T^4$.

XII Kepler laws

1. $F = GMm/r^2$, $\Pi = -GMm/r$.

2. Gravitational interaction of 2 point masses (Kepler's I law): trajectory of each of them is an ellipse, parabola or hyperbola, with a focus at the center of mass of the system. Derive from R.-L. v. (pt 9).

3. Kepler's II law (conserv. of angular mom.): for a point mass in a central force field, radius vector covers equal areas in equal times.

4. Kepler's III law: for two point masses at elliptic orbits in r^{-2}-force field, revolution periods relate as the longer semiaxes to the power of $\frac{3}{2}$:

$$T_1^2/T_2^2 = a_1^3/a_2^3.$$

5. Full energy ($K + \Pi$) of a body in a gravity field:

$$E = -GMm/2a.$$

6. For small ellipticities $\varepsilon = d/a \ll 1$, trajectories can be considered as having a circular shapes, with shifted foci.

7. Properties of an ellipse: $l_1 + l_2 = 2a$ (l_1, l_2 — distances to the foci), $\alpha_1 = \alpha_2$ (light from one focus is reflected to the other), $S = \pi ab$.

8. A circle and an ellipse with a focus at the circle's center can touch each other only at the longer axis.

9*. Runge-Lenz vector (the ellipticity vector):

$$\vec{\varepsilon} = \frac{\vec{L} \times \vec{v}}{GMm} + \vec{e}_r = \text{Const.}$$

XIII Theory of relativity

1. Lorentz transforms (rotation of 4D space-time of Minkowski geometry), $\gamma = 1/\sqrt{1 - v^2/c^2}$:

$$x' = \gamma(x - vt),\ y' = y,\ t' = \gamma(t - vx/c^2)$$
$$p_x' = \gamma(p_x - mv),\ m' = \gamma(m - p_xv/c^2)$$

2. Length of 4-vector:

$$s^2 = c^2t^2 - x^2 - y^2 - z^2$$
$$m_0^2c^2 = m^2c^2 - p_x^2 - p_y^2 - p_z^2$$

3. Adding velocities:

$$w = (u + v)/(1 + uv/c^2).$$

4. Doppler effect:

$$\nu' = \nu_0\sqrt{(1 - v/c)/(1 + v/c)}.$$

5. Minkowski space can be made Euclidean if time is imaginary ($t \to ict$). Then, for rot. angle φ, $\tan\varphi = v/ic$. Express $\sin\varphi$, and $\cos\varphi$ via $\tan\varphi$, and apply the Euclidean geometry formulae.

6. Shortening of length: $l' = l_0/\gamma$.

7. Lengthening of time: $t' = t_0\gamma$.

8. Simultaneity is relative, $\Delta t = -\gamma v\Delta x/c^2$.

9. $\vec{F} = d\vec{p}/dt$ [$= \frac{d}{dt}(m\vec{v})$, where $m = m_0\gamma$].

10. Ultrarelativistic approximation: $v \approx c$, $p \approx mc$, $\sqrt{1 - v^2/c^2} \approx \sqrt{2(1 - v/c)}$.

11*. Lorentz tr. for E-B: $\vec{B}_{||}' = \vec{B}_{||}$, $\vec{E}_{||}' = \vec{E}_{||}$,

$$\vec{E}_\perp' = \gamma(\vec{E}_\perp + \vec{v} \times \vec{B}_\perp),\ \ \vec{B}_\perp' = \gamma(\vec{B}_\perp - \vec{v} \times \frac{\vec{E}_\perp}{c^2}).$$

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* marks an advanced material.

Corrections/suggestions ⇒ kalda@ioc.ee.

Composed by J. Kalda, translated by U. Visk and J.K.