Histories of Particles in the Dark Universe

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H.13 "A Measurement of Excess Antenna Temperature at 4080 Mc/s" by A.A. Penzias and R.W. Wilson, 1965 601
H.14 "The redshift of extragalactic nebulae" by F. Zwicky, 1933 and "Rotation of the Andromeda nebula from a spectroscopic survey of emission regions" by V. Rubin and W. Ford, 1969. 603
H.15 "A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May 29, 1919" by F.W. Dyson, A.S. Eddington and C. Davidson, 1919 607

I Glossary **609**

'There is no difference between Time and any of the three dimensions of Space except that our consciousness moves along it'

H.G. Wells, *The Time Machine*, 1895

'*If you are doing everything well, you are not doing enough*', Howard Georgi.

0.1 Foreword

I wrote this book from a frustration. Several excellent works and reviews are on the market. I cite some of them at the beginning of the appendix with their particularity. However, a vast majority of them has been made by astrophysicists and I always needed the equivalent of the Particle Data Booklet to compute cross section in specific process, especially fighting on the 2π factors. On the other hands, particle physics textbook were very light when discussing thermodynamic of the Universe or radiative processes especially in astrophysics framework. In other words, I was mainly always juggling between the *Kolb and Turner* book to solve subtleties in Boltzmann equation, the *Jackson* to clarify the radiative effect of a model in the Galactic scale, the *Jungmann and Kaminowski* for some amplitude computations and *Shifmann* papers for details in the nucleus composition. This work is an attempt of a particle physicist to propose a textbook which can be useful for any phenomenologist who is interested a minimum in astroparticle physics, especially its dark matter aspects.

I also tried to unify units. In such a vast field as dark matter, we have to deal with cosmological scales (when solving Boltzmann equation or computing reheating temperatures), astrophysical scales (when looking at propagation of cosmic rays or indirect detection of dark matter) and of course microscopic scale when computing any interaction cross section. It is always possible to convert Jansky to Joule, $\mathrm{cm}^2\mathrm{s}^{-1}$ to GeV^{-2}... However, as a particle physicist by formation, I decided to work in a unified GeV-framework. From this point of view, comparisons between energy loss, expansion rates or annihilation cross section is straightforward. In any case, all conversion factors are given in Appendix and I always gives the results in the more "natural" units for people more used with their own scale-related units.

I want to prevent the reader from passing too fast on the historical biographies and historical articles. Due to a lack of space I included the complete breakthrough articles in the companion manuscript of this book ("Et Fiat Lux"). I urge the reader to read some of them, and the historical insights I propose before each of them. It is very instructive to understand how the more revolutionary ideas of our field was perceived when they went out.

I have included in this book a fundamental part (part V) in which I derive the main fundamental concepts developed in the book from an historical perspective. It is, for me, the most important part of this long manuscript, and I urge the reader not to bypass it. Useful for undergraduate as graduate students, these chapters are more than just historical perspectives, but the real roots and blood that made us understand the Universe as a whole nowadays. These sections can also be taken individually by teachers to illustrate their lectures with the original articles highlighting how the discoveries has been made, and how the ideas have evolved with time.

I also prevent the reader to read the manuscript from A to Z, but more to watch at the content, or even better, the index, picking a word he would like to understand better and read the corresponding section. This book should touch the curiosity of the reader more than to teach him a complete history of the Universe. I also advice the reader to keep this opuscule in a place easily accessible: anytime he reads an article related to dark matter, or physics beyond the Standard Model he should find the answer, or at least a hint of his answer in this book. If not, he should not hesitate to send me an email concerning a subject no treated or too lightly treated here. This textbook is far to be completed and will evolve

thanks to the comments of all the readers.
To appreciate at its best the reading of the manuscript, some symbol * and ** are added at the beginning of each section of chapter depending on its technical difficulty. So we advice the readers to skip them for a first lecture.
I want to thank especially all the people who contributed directly or indirectly to the production of this manuscript.

Chapter 1

Introduction

The XXth century studies evidenced the existence of a new form of matter that have inspired interest in modern physics scenario. It has been named "Dark Matter" (DM), exotic name but with a clear meaning: a component of matter that does not emit luminous radiation. Beginning from study presented by Zwicky in 1933 [see his article in appendix] who analyzed the motion of the galaxies in the cluster Coma, subsequently other observations have indicated the presence of dark matter from the kinematics of virally systems and rotating spiral galaxies, the effects of gravitational lensing of background objects, various evidences among which the recent observations of the Bullet cluster, until recent observations from the PLANCK satellite. Furthermore, the dark matter appears to have an important role in the formation of the structures, in the evolution of galaxies and also has effects on non-uniformity observed cosmological microwave of background radiation. Before going into detailed analyses of each step structuring the dark matter presence and interaction in our Universe, we will first introduce in this chapter the most important evidence, explaining where the dark matter may intervene to resolve the oddities observed and we then list the general features of dark matter particles.

1.1 Local Dark Matter

The dynamical density of matter in the Solar vicinity can be estimated using vertical oscillations of stars around the galactic plane. The orbital motions of stars around the galactic center play a much smaller role in determining the local density. Oort in 1932 indicated, in his analysis, as members of a "star atmosphere", a statistical ensemble in which the density of stars and their velocity dispersion defines a "temperature" from which one obtains the gravitational potential. The result contradicted grossly the expectations: the potential provided by the known stars was not sufficient to keep the stars bound to the Galactic disk because the density of visible stellar populations by a factor of up to 2, and so the Galaxy should rapidly be losing stars [1]. Since the Galaxy appeared to be stable there had to be some missing matter near the Galactic plane, Oort thought, exerting gravitational attraction. This limit is often called the Oort limit. This used to be counted as the first indication for the possible presence of dark matter in our Galaxy: the amount of invisible matter in the Solar vicinity should be approximately equal to the amount of visible matter. A modern calculation of the local relic abundance using several observable can be found in

the work of R. Catena and P. Ullio in [2]. Their result gives a local amount of dark matter of $\rho_0 \simeq 0.39$ GeV/cm^3 which corresponds mainly to 100 000 particles passing through us per cm^2 per seconds.
It is important to notice that such a value for local amount of dark matter (0.39 GeV/cm^3) is very small. To collect one gram of dark matter we would have to put together all the particles contained in a volume comparable to that occupied by the entire Earth! Indeed, the dark matter is much more present near the Galactic Center. That also explains why we can safely use the Newton laws in the solar system. Of course, when one look at larger scale, the situation is completely different: between the Sun and the next star for instance, the void is huge. A quick calculation shows that the density of the planets ins the solar system in a sphere that extends halfway to the next star, Proxima Centauri, is about the same as the density of dark matter!

1.2 Anomalies in rotation curves of galaxies

Radio radiation from interstellar gas, in particular that of neutral hydrogen, is not strongly absorbed or scattered by interstellar dust [3]. It can therefore be used to map and to study the motion of neutral hydrogen clouds concentrated in spiral arms. Hence it can be determined angular velocity of gas for various distance from the galactic centre and corresponding rotation curve $v = v(r)$, the circular velocity as a function of galactic radius. The most convincing and direct evidence for dark matter on galactic scales comes mainly from the observations of the rotation curves of galaxies. Rotation curves are usually obtained by combining observations of the 21cm line with optical surface photometry: if θ is the angle between the velocity of the star and the line of sight, the velocity components can be written $v_r = v\cos\theta$ and $v_t = v\sin\theta$. The tangential velocity v_t results in the proper motion, which can be measured by taking photos at intervals of several years or decades. The radial velocity v_r can be measured from Doppler shift of the stellar spectrum, in which the spectral lines are often displaced towards the blue or red. The blueshift means that the star is approaching, while the redshift indicates that it is receding. From 1940 (Oort) numerous observations in spiral galaxies showed, in outer regions of galaxies, an anomaly in rotation velocity that can be translated in an high M/L, mass-luminosity ratio. Observed rotation curves usually exhibit that the central part of the galaxy rotates like a rigid body, i.e. $v \propto r$ and then the velocity reach a maximum value, or plateau. At this point we expected a decreasing velocity outwards, as the third Kepler law suggests, instead there is a characteristic flat behavior until edges of galaxies where few light is emitted. We consider for simplicity a spherical distribution of matter in the galaxy. In Newtonian dynamics, the viral theorem determine the circular velocity which is expected to be

$$\frac{v^2}{r} = \frac{GM(r)}{r^2} \quad \Rightarrow \quad v = \sqrt{\frac{GM(r)}{r}} \tag{1.1}$$

where $M(r) = 4\pi \int_0^r \rho(a) a^2 da$ is the mass of the matter of density $\rho(a)$ contained in a sphere of radius r. From Eq.(1.1), $v(r)$ should fall following $1/\sqrt{(}r)$ outside the optical disk where M should be constant without the presence of dark matter. The fact that the observation gave $v(r) \simeq$ constant implies the existence of an halo with $M(r) \propto r$ and then

Figure 1.1: Rotation curve of NGC 3198 with the contributions of gas, disk and dark matter compared with the measured value [4] .

$\rho(r) \propto 1/r^2$ confirming that a large part of the mass is present in the outer part of the galaxy and not in the visible disk. This is well illustrated in Fig.(1.1). The mass distribution is obtained from rotational curves, determined from light distribution of luminous components in the galaxy. By photometry the estimated mass in our Galaxy between the Galactic Center (GC, Sagitarius A*) and the sun (at a distance $r_\odot = 8$ kiloparsec (kpc) from the GC) is $M = 9 \times 10^{10} M_\odot$, while for outer edge of galaxy, where the luminosity decreases exponentially, the component of luminous is negligible. At this distance from the galactic center the rotational velocities verify approximatively the Keplerian law as the visible matter dominate the matter distribution at such distances from the GC.

Below $r_\odot$ some discrepancies in the rotational velocity can be explain by the presence of an invisible mass halo, called dark matter halo or dark halo, around our Galaxy. This dark component of matter would be spherically distributed in a halo extended until 230 kpc from galactic center and having a density profile following

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{a}\right)\left(1+\frac{r}{a}\right)^2} \tag{1.2}$$

a being a typical scale of the halo depending on the galaxy. With this profile the galaxy behaves like $1/r$ at the center ($r \ll a$) and $1/r^3$ in the edges $r \gg a$. With this calculation the mass of halo of dark matter must be $5.4 \times 10^{11} M_\odot$ within 50 kpc and $2.5 \times 10^{12} M_\odot$ at 230 kpc [5]. The profile proposed in Eq.(1.2) is called NFW profiles and was proposed by Navarro Frenk and White in [6], whereas the first measurement of the rotation curve was made by Ford and Rubin in [7] (see the appendix for the history).

1.3 Cluster Dark Matter

Another mass discrepancy was found by Zwicky in 1933 [8]. He measured redshifts of galaxies in the Coma cluster and found that the velocities of individual galaxies with respect to the cluster mean velocity are much larger than those expected from the estimated total mass of the cluster, calculated from masses of individual galaxies. His article was in fact more a review on the status of observational cosmology in 1933. It is just at the end of its article that he conclude on the remark concerning the probable presence of dark matter in the Coma cluster. You can have a look at the appendix for the historical section. Stars move in galaxies and galaxies in clusters along their orbits, those are virialy bound systems: the orbital velocities are balanced by the total gravity of the system, similar to the orbital velocities of planets moving around the Sun in its gravitational field. In the simplest dynamical framework one treats clusters of galaxies as statistically steady, spherical, self-gravitating systems of N objects of average mass m and average orbital velocity v. The total kinetic energy E of such a system is then

$$E = \frac{1}{2}Nmv^2$$

If the average separation is r, the potential energy of $N(N-1)/2$ pairings is

$$U = \frac{1}{2}N(N-1)\frac{Gm^2}{r}$$

The virial theorem states that for such a system

$$E = -\frac{U}{2}$$

The total dynamic mass M can then be estimated from v, and r from the cluster volume

$$M = Nm = \frac{2rv^2}{G}$$

Zwicky was the first to use the virial theorem to infer the existence of unseen matter. He found that the orbital velocities are almost a factor of ten larger than expected from the summed mass of all galaxies belonging to the clusters, and this implies that the average mass of galaxies within the cluster has a value about 400 times greater than expected from their luminosity: the gravity of the visible galaxies in the cluster would be far too small for such fast orbits, so something extra was required. This is known as the "missing mass problem" and he proposed that most part of the missing matter was a dark, non-visible form of matter which would provide enough of the mass and gravity to hold the cluster together.
It exists another method to determinate the mass of cluster: the temperature of the hot intracluster gas, like the galaxy motion, traces the cluster mass. Indeed, hot gas inside the clusters emits X-ray radiation through bremsstrahlung process (see the section (4.5.4) for more details). Observations show that the gas is in hydrodynamic equilibrium ($dF_{grav} = dF_{press} = \frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$ with M_r the inner total mass inside the radius r) and it moves in the gravitational field of cluster in orbits with velocities depending on the mass of the cluster. Through spectroscopic analysis of hot gas we can obtain density and temperature of gas in function of galactic distance r. With these parameters we can get mass distribution of

Figure 1.2: Rotation curve for the galaxy NGC6503 with the different component extracted from observations : gas, visible disk and dark halo [9] .

cluster. For example gas mass of Coma cluster is $M_{gas} = 1.05 \times 10^{14} M_{\odot}$ which is larger than the visible mass $M_{vis} = 1.5 \times 10^{13} M_{\odot}$ but not sufficiently to explain the value extracted from the virial theorem that is $M_{vir} = 3.3 \times 10^{15} M_{\odot}$. We illustrate in Fig.(1.2) the different component of the galactic structure and their relative abundance as function of their distance from galactic center in the case of the galaxy NGC6503 [9].

From all the observations quoted above and N-body simulations which also give information on the possible distribution of dark matter in our galaxy. Indeed, this is a daunting task especially when one considers that even the three–body problem - the problem of describing the orbits of three celestial bodies under their reciprocal gravitational attraction- is extraordinarily difficult, and can only be solved in some simplified cases. How can we therefore hope to solve the problem of computing the reciprocal interactions of *all particles in the universe?*.

The problem is that, in principle, one has to calculate for each particle the attraction of every other particle in the universe. Eric Holmberg, a, ingenious Swedish scientist, found an original solution to the problem in 1941. He decided to simulate the intersection of two galaxies using 74 lightbulbs, together with photocells and galvanometers, using the fact that light follows the same inverse square law as the gravitational force. He then calculated the amount of light received by each cell, and manually moved the lightbulbs in the direction that received the most intense light.

The first application of computer calculations to gravitational systems was probably by John Pasta and Stanislaw Ulam in 1953. Their numerical experiments were performed on the Los Alamos computer, which by then had already been applied to a variety of other problems, including early attempts to decode DNA sequences and the first chess–playing program.

Two young astrophysicists, the Toomre brothers, had access in the early 1970's to one of NASA's two IBM 360-95 computers, complete with high–resolution graphics workstations and auxiliary graphics–rendering machines (computing facilities far in advances of any other astrophysics laboratory. They set up a series of simulations of galaxy grazings and collisions using a simple code that described the galaxies as two massive points surrounded by a disk of test particles. The outcome of the analysis was a very influential paper, published in 1972, that contained a detailed discussion of the role of collisions in the formation of galaxies. Together with their paper, the Toomre brothers also created a beautiful 16mm microfilm movie "Galactic Bridges and Tails" that you can find at http://kinotonik.net/mindcine/toomre. A photo of the simulation is represented in the Fig.(1.3)
Numerical simulations have improved immensely since these pioneering attempts, thanks to a dramatic increase in computing power. Modern supercomputers allow us to simulate entire universes by approximating their constituents with up to 10 billion particles, as in the case of the Millenium simulation shown in Fig.(1.3) which was run at the Max Planck Society's Supercomputing Center in Garching, Germany. This has a computing power ten million times larger than the old IBM 360-95 used by the Toomre brothers.
We usually parametrize our "ignorance" of the exact distribution obtained by the N-body simulation systems with an empirical formula for the dark matter halo in the cluster or in our Milky Way:

$$\rho_{DM}(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)^{\alpha}\left(1+\frac{r}{r_s}\right)^{3-\alpha}}$$

where r_s is a typical scale (usually the distance from the sun to the galactic center) and ρ_0 is the normalization constant, determined from the observation of the local density of dark matter in the vicinity of the sun. α is fitted with observations or after running simulations and ordinary lies between the bounds $0 \leq \alpha \leq 3/2$. For a more detailed analysis on the profile structure and how one obtains them, have a look at the section (4.7.2). It is however important to notice that N-body simulations deal with "particle" mass of the order of several solar mass and that one should extrapolate their result down to electroweak masses which can infer some errors. Moreover, the smaller dimension scale that these simulations can reach is of the order of the kilo parsec, so all spatial extrapolations around the parsec galactic center can be dubious. Especially when we do not know exactly which kind of ingredient and initial conditions are taken by the simulations. That explains the order of magnitude existing between prediction in indirect detection of dark matter when most of the annihilation process occur around the galactic center, where the precisions of the simulations are the worst, without real visible observables to test them.

1.4 Gravitational lensing

The theory of general relativity teaches us that gravitational field curves the space-time metric, whereas the particles or photons travel in geodetic trajectory. An observable consequence of this effect is the gravitational lensing: a photon in a gravitational field moves as if it possessed mass, and light rays therefore bend around gravitating masses. Thus celestial bodies can serve as gravitational lenses probing the gravitational field, whether baryonic or

Figure 1.3: N-body simulation in 1972 with 120 particles (left) and in 2013 with 10 billions of particles (right) .

dark without distinction, and thus can probe the dark massive component of any celestial object. If we consider that a trajectory of light ray in a gravitational filed with a spheric symmetry (r, θ, ϕ) is represented as

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = 3\frac{GM}{r^2}$$

The solution of this equation can be though as a perturbation of special relativity (without gravitational field) and the deflection angle is derived in the Appendix, Eq.(A.9) :

$$\delta = 4\frac{GM}{r_0 c^2}$$

r_0 being the closest distance from the light ray to the massive body causing the deflection. The deflexion provided for a light ray that enters in gravitational field of Sun is $\delta \simeq 1.75$" and has been measured by Eddington in 1917 (see the historical section for more details). Since photons are neither emitted nor absorbed in the process of gravitational light deflection, the surface brightness of lensed sources remains unchanged. Changing the size of the cross-section of a light bundle only changes the flux observed from a source and magnifies it at fixed surface brightness level. We can categorized three classes of gravitational lensing :

- Strong lensing, the photons move along geodesics in a strong gravitational potential which distorts space as well as time, causing larger deflection angles and requiring the full theory of general relativity. The images in the observer plane can then become quite complicated because there may be more than one null geodesic connecting source and observer. Strong lensing is a tool for testing the distribution of mass in the lens rather than purely a tool for testing general relativity. The masses of clusters of galaxies determined using this method confirm the results obtained by the virial theorem and the X-ray data.

Figure 1.4: Left: effect of gravitational lensing of a cluster of galaxies acting on there light emitted from a galaxy on its way to earth. Right : distorted images due to the lensing in the galaxy cluster Abell 2218 ($NASA$).

- Weak Lensing, refers to deflection through a small angle when the light ray can be treated as a straight line, and the deflection as if it occurred discontinuously at the point of closest approach (the thin-lens approximation in optics). One then only invokes special theory of relativity which accounts for the distortion of clock rates. This kind of lensing allows to determine the distribution of dark matter in clusters as well as in superclusters: the lensing mass estimate is almost twice as high as that determined from X-ray data.

- Microlensing, if the mass of the lensing object is very small, one merely observe a magnification of the brightness of the lensed object. Microlensing of distant quasars by compact lensing objects (stars, planets) has also been observed and used for estimating the mass distribution of the lens-quasar systems. A fraction of the invisible baryonic matter can lie in small compact object. To find the fraction of these object in the cosmic balance of matter, special studies have been initiated, based on the microlensing effect. This process is used to find Massive Compact Halo Object (MACHOs), small baryonic objects as planets, dead stars or brown dwarfs, which emit so little radiation that they are invisible most of time. A MACHOs may be detected when it passes in fron of a star and the MACHOs gravity bends the light, causing the star to appear brighter. Some authors claimed that up to 20% of dark matter in our Galaxy can be in low-mass stars.

1.5 Bullet Cluster

The bullet cluster (1E 0657-558) consists of two colliding clusters of galaxies. It is at a co-moving radial distance of 1.141 Gpc (3.721 Gly). Gravitational lensing studies of the Bullet Cluster are claimed to provide the best evidence to date for the existence of dark matter [10]. At a statistical significance of 8σ, it was found that the spatial offset of the

Figure 1.5: Bullet Cluster photo in X-ray superimposed with the gravitational lensing isocurves (green), exposition time about 140 hours and megaparsec scale [11].

center of the total mass from the center of the baryonic mass peaks cannot be explained with an alteration of the gravitational force law. In Fig.(1.5) we show the Chandra Xray observatory image of this cluster taken in 2004 [11]. This cluster was formed after the collision of two large clusters of galaxies. Hot gas detected by Chandra in X-rays is seen as two pink clumps in the image and contains most of the "normal" or baryonic, matter in the two clusters. The bullet-shaped clump on the right is the hot gas from one cluster, which passed through the hot gas from the other larger cluster during the collision. An optical image from Magellan and the Hubble Space Telescope shows the galaxies in orange and white. The blue areas in this image show where astronomers find most of the mass in the clusters. The concentration of mass is determined using the effect of gravitational lensing (green lines in the figure), where light from the distant objects is distorted by intervening matter. Most of the ordinary visible matter in the clusters is clearly separate from the matter responsible of the gravitational lensing, giving direct evidence that nearly all of the matter in the clusters is dark: the hot gas in each cluster was slowed down by a force like air resistance whereas the dark matter was not slowed by the impact because it does not directly interact with the gas itself or if not through gravity. Therefore, during the collision the lumps of dark matter from the two clusters moved ahead of the hot gas, producing the separation of dark matter and baryonic matter. If hot gas was the most massive component in the clusters, as proposed by alternative theories of gravity, this effect would not be seen: this result shows that dark matter is required at least in the bullet cluster.

1.6 Compared of three matter abundance

From all the measurements described above, one can estimate the relative mass contribution of the galaxies $\Omega_g = \frac{\rho_g}{\rho_{crit}}$, ρ_g being the density of mass in the galaxies and ρ_{crit} the critical

density of the Universe (see section 2.2.3 for details) to be $\Omega_g \simeq 0.03 - 0.07$. This estimation comes from combining the mean luminosity per unit volume produced by galaxies $\mathcal{L}_g$ and the mean mass to light ratio $\langle M/L \rangle$

$$\rho_g = \mathcal{L}_g \times \langle \frac{M}{L} \rangle \simeq 6 \times 10^{-31} h^2 \text{ g cm}^3$$

which gives[1] $\Omega_g \simeq 0.03 - 0.07$.
At the level of the cluster of galaxies, estimation of the gravitational lens produced by Abell 2218 or Abell 1689 give a similar result , with a mean $\langle M/L \rangle$ around 10 times the one in galaxies (almost constant for scales above the Mpc [12]), and coherent with other observations, which gives $\Omega_{cluster} \simeq 0.2 - 0.4$. The discrepancy between the three values of Ω_i can then also be attributed to the presence of non-luminous dark matter which may play an important role in structure formation. For a mass scale $R > 1.5\ h^{-1}$ Mpc (typical galaxy radius) the mass-to-light ratio of superclusters of galaxies confirm that do not exist a additional quantity of dark matter at higher scale, $R = 6\ h^{-1}$ Mpc.
We can also estimate the contribution from baryonic material by comparing the observed abundances of light elements (deuterium, ^{3}He, ^{4}He and ^{7}Li) with the predictions of primordial nucleosynthesis computations, that give us $\Omega_b \simeq 0.04$. These values are obtained from the standard Big Bang nucleosynthesis and (except from the Lithium "problem") correspond to the recent observations. For the curious people, they can go to have a look at the section (2.3) for more details. In few words, according to the Big Bang model, the Universe began in an extremely hot and dense state. For the first second it was so hot that atomic nuclei could not form, space was filled with a hot soup of protons, neutrons, electrons, photons and other short, lived particles. Occasionally a proton and a neutron collided and sticked together to form a nucleus of deuterium (a heavy isotope of hydrogen), but at such high temperatures they were broken immediately by high-energy photons. When the Universe cooled off, these high-energy photons became rare enough that it became possible for deuterium to survive, deuterium bottleneck. These deuterium nuclei could keep sticking to more protons and neutrons, forming nuclei of ^{3}He, ^{4}He, lithium, and beryllium. This process of element-formation is called nucleosynthesis. The denser proton and neutron gas is at this time, the more of these light elements will be formed. As the Universe expands, however, the density of protons and neutrons decreases and the process slows down. Neutrons are unstable (with a lifetime of about 15 minutes) unless they are bound up inside a nucleus. After a few minutes, therefore, the free neutrons will be gone and nucleosynthesis will stop. There is only a small window of time in which nucleosynthesis can take place, and the relationship between the expansion rate of the Universe (related to the total matter density) and the density of protons and neutrons (the baryonic matter density) determines how much of each of these light elements are formed in the early Universe. The Fig.(1.6) shows the computed abundance of deuterium D (^{2}H), ^{3}He, ^{4}He and ^{7}Li (compared with H hydrogen). The abundances are all shown as a function of η_b, the baryon-to-photon ratio which is related to Ω_b by $\Omega_b = 0.004\ h^{-2} \frac{\eta_b}{10^{-10}}$. The estimates of the primordial values of the relative abundances of these elements obtained by WMAP [13] and PLANCK [14] appear[2] to be in agreement

[1] $\rho_c = 1.78 \times 10^{-29} h^2$, see section 2.2.3
[2] $\eta_b = 6.19 \times 10^{-10}$ measured by WMAP

Figure 1.6: Relative abundance of light elements (relative to hydrogen) as function of the ordinary matter relative to photon (η_b).

with nucleosynthesis predictions, but only if the density parameter in baryonic material is $\Omega_b h^2 = 0.02$.

1.7 Cosmic Microwave Background (CMB)

The relic abundance of dark matter can also be extracted from the analysis of the Cosmic Microwave Background (CMB), radiation originating from the propagation of photons in the early Universe once they decoupled from matter, recombination era (see section 2.2.11 for more details). In 1965 this radiation was detected by Penzias and Wilson (see section H.13) and this discovery was a powerful confirmation of the Big Bang theory. WMAP [13] and more recently the european PLANCK satellite [14] gave us the more precise photo of the Universe and their ingredient. After many decades of experimental efforts, the CMB is known to be isotropic but having minimum temperature fluctuations called anisotropy with amplitude of order $10^{-3} - 10^{-5}$ and it follows with extraordinary precision the spectrum of a black body corresponding to a temperature $T = 2.726$ K. The variations of temperature of CMB can be expressed as sum of spherical harmonic Y_{lm}

Figure 1.7: The sky observed by PLANCK satellite [14].

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi) \tag{1.3}$$

a_{lm} gives us the variance $C_l = \langle |a_{lm}|^2 \rangle = \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$. If the temperature fluctuations are assumed to be Gaussian, as it appears to be the case, all of the information contained in CMB maps can be compressed into the power spectrum, essentially giving the behavior of $l(l+1)C_l/2\pi$ as a function of l. WMAP and PLANCK data could map universal fluctuations after removing the dipole anisotropy ($l = 1$) and galactic and extragalactic contaminations. To extract information from CMB we must consider a cosmological model with fixed number parameters. The spectrum profile is represented in Fig.(1.7). comparing each position of the peak and its height, one can deduce the deep composition of the Universe. We represent the result of PLANCK in figure 1.7. This graphics was be fitted by a model that consider a Universe with a cosmological constant Λ and a cold dark component of matter (Λ_{CDM}). The position of the first peak determines $\Omega_m h^2$. Combining the first 2-years PLANCK measurements of the temperature power spectrum with determinations of the Hubble constant h, the PLANCK team finds the total mass density parameter $\Omega_m = 0.3175$. The ratio of amplitudes of the second-to-first Doppler peaks determines the baryonic density parameter $\Omega_b = 0.048$; the dark matter component is then $\Omega_{DM} = 0.2695$.
We will now go more deeply into the standard history of our Universe to understand each step of its formation and the influence on dark matter existence.

1.8 Alternatives

To be complete, we can notice that it exists some alternatives to the dark matter scenario. Indeed, all the evidences of dark matter that we have discussed above relies on the strong

assumption that we know the law of gravity at all scales. Is it possible that by changing the laws of gravity we could rid of this mysterious component of the universe?
In 1983, Mordehai Milgrom proposed to get rid of dark matter altogether and to replace the known laws of gravitation with the so called MoND paradigm, short for "modified Newtonian dynamics". The price to be paid was to abandon general relativity, a theory that is particularly appealing to many physicists because of its elegance and formal beauty. But there is no dogma in physics, and history has taught us that our theories can always be refined and improved. Milgrom proposed that the law of gravity is modified below a certain *acceleration*, that is, when the gravitational force becomes very weak.
This proposal is very clever, because it bypasses one of the main difficulties of theories of modified gravity: the easiest way to construct them is to introduce a new distance scale, above which the gravitational force is modified from its characteristic inverse square law. But observations tell us that modifications of gravity (or, alternatively, the presence of dark matter) are observed on different scales in different systems.
MoND is surprisingly accurate on the scale of galaxies, and it even addresses some mysterious correlations between properties of galaxies that find no explanation in the standard dark matter paradigm. A few years later, Jacob Bekenstein even embedded Milgrom's proposal into a more relativistic theory called TeVeS, for "tensor-vector-scalar" theory, promoting it from a phenomenological model to a more fundamental theory. It is without doubt an interesting proposal, and attracted and is still attracting substantial interest.
There is a problem with these theories, however: as soon as we move from the scale of galaxies to the scale of galaxy clusters, they fail to reproduce the observational data. Perhaps the biggest challenge to MoND–like theories, and one of the most direct proofs of the existence of dark matter, is provided by systems like the so–called Bullet Cluster discussed previously in Fig.(1.5), the system of two clusters of galaxies that have recently collided, with one of the two passing through the bigger one like a bullet (hence the name). To explain this observation with MoND–like theories, one has to postulate the existence of additional matter, in the form of massive neutrinos, for instance. But there is a ensign between the properties required of the neutrinos and current data, and, in general, it is not very appealing to require at the same time a modification of gravity *and* the presence of some form of dark matter.
It is still perfectly possible that it is through a modification of the laws of gravity that we will be able to explain the motion and the shape of cosmic structures, and it is important that part of the research effort of the scientific community should focus on this possibility. Fortunatley, as Bekenstein says:

The increasing sophistication of the measurements in [gravitational lensing and cosmology] should eventually clearly distinguish between the various modified gravity theories, and between each other and General relativity

The dark matter paradigm will remain a conjecture until we finally put our hands on the particles by measuring their properties in our laboratories. Before describing the techniques that have been devised so far to detect dark matter however, we need to understand a little bit the thermal history of the universe and the behavior of the dark matter in this primordial bath.

Part I

The primordial Universe

Chapter 2

A thermal history of the Universe

2.1 Introduction

During the early 1930s, Fritz Zwicky, a professor of astronomy at the California Institute of Technology (a famously caustic scientist whose appreciation for symmetry led him to call his colleagues spherical bastards because, he explained, they were bastards any way you looked at them), realized that the outlying galaxies in the Coma cluster, a collection of thousands of galaxies some 370 million light-years from earth, were moving too quickly for their visible matter to muster an adequate gravitational force to keep them tethered to the group. Instead, his analysis showed that many of the fastest-moving galaxies should be flung clear of the cluster, like water droplets thrown off a spinning bicycle tire. And yet none were. Zwicky conjectured that there might be additional matter permeating the cluster that did not give off light but supplied the additional gravitational pull necessary to hold the cluster together. His calculations showed that if this explanation was right, the vast majority of the cluster's mass would comprise this non–luminous material. By 1936, corroborating evidence was found by Sinclair Smith of the Mount Wilson observatory, who was studying the Virgo cluster and came to a similar conclusion. But since both men's observations, as well as a subsequent others, had various uncertainties, many remained unconvinced that there was voluminous unseen matter whose gravitational pull was keeping the groups of galaxies together.

Over the next thirty years observational evidence for non–luminous matter continued to mount, but it was the work of the astronomer Vera Rubin from the Carnegie Institution of Washington, together with Kent Ford and others, that really clinched the case. Rubin and her collaborators studied the movements of stars within numerous spinning galaxies and concluded that if what you see is what there is, then many of the galaxy's stars should be routinely flung outward. Their observations showed conclusively that the visible galactic matter could not exert a gravitational grip anywhere near strong enough to keep the fastest–moving stars from breaking free. However, their detailed analyses also showed that the stars *would* remain gravitationally tethered if the galaxies they inhabited were immersed in a giant ball of non–luminous matter, as in Fig.(2.1), whose total mass far exceeded that of the galaxy's luminous material. And so, like an audience that infers the presence of a dark–robed mime even though it sees only his white–gloved hands flitting to and fro on the unlit stage, astronomers concluded that the Universe must be suffused with *dark matter* –matter

Figure 2.1: A galaxy immersed in a ball of dark matter (artist view).

that does not clump together in stars and hence does not give off light, and that thus exerts a gravitational pull without revealing itself visibly. The Universe's luminous constituent –stars– were revealed as but floating beacons in a giant ocean of matter.

2.2 A brief thermal story of the Universe in some dates

The story of the Universe is rich and complex and depends strongly on its temperature. In this section, we will gives a quick snapshot of the thermal history of the Universe that will be developed through this chapter. Each phase will be studied in detail later on. Let us begin by recalling some temperatures, possibly realized in the hot universe, the related cosmic times, and the connection with microscopic physics at the corresponding energies:

- $T \simeq 0.1\text{eV}$ [$t \simeq 10^{13}$ s $\simeq 380000$ years]

  Light nuclei and electrons form neutral atoms and the universe becomes transparent to photons. They decouple from the plasma (see section 2.2.11) and are observable today as cosmic microwave background (CMB).

- $T \simeq 0.1 - 10$ MeV [$t \simeq 10^2 - 10^{-2}$ s]

  Light nuclei are formed from protons and neutrons (primordial nucleosynthesis, BBN) and neutrinos decouple from the plasma (see section 5.6).

- $T \simeq 10$ GeV [$t \simeq 10^{-8}$ s]

  Weakly interacting massive particles (WIMPs), the most popular dark matter candidates, decouple from the plasma (see section 2.4).

Figure 2.2: Epochs of the hot early universe, their cosmic times scale (top) and temperatures (bottom).

- $T \simeq 100$ GeV [$t \simeq 10^{-10}$ s]

  The Higgs vacuum expectation value forms, and all Standard Model particles becomes massive. Baryon and lepton number changing 'sphaleron process' are no longer in thermal equilibrium.

- $T \simeq 10^8 - 10^{11}$ GeV [$t \simeq 10^{-22} - 10^{-28}$ s]

  Baryogenesis via leptogenesis takes place and gravitino dark matter or particles coupling via GUT interactions to the Standard Mode (see section 2.6) can be thermally produced.

- $T \simeq 10^{12}$ GeV [$t \simeq 10^{-30}$ s]

  In supersymmetric theories with extra dimensions one expects that the present 'vacuum' of the universe is metastable. To avoid a rapid transition to a supersymmetric flat higher dimensional ground state, the reheating temperature cannot exceed a maximal reheating temperature.

This little thermal history is summarized in Figs.(2.2) and (2.3).
Nowadays, the universe is mainly composed by baryonic matters[1] (of density $\rho_m(T) \simeq \frac{10^{11}\text{solar mass}}{Mpc^3} \simeq 0.5$ GeV/m^3), photons ($\rho_\gamma(T)$ of temperature $T_\gamma = T = 2.7$K) and relativistic neutrino ($\rho_\nu(T)$ of temperature $T_\nu = (\frac{4}{11})^{1/3} T_\gamma = 1.96$K, see section [5.6]). From this present measured photon temperature, one thus has deduced the neutrino temperature, and one can deduce the present radiation and neutrino density from the Eq.(2.36) taking $g_\gamma = 2$ and $g_\nu = \frac{7}{8} \times 2 \times 2 = \frac{7}{2}$, one obtains

[1]By default when one talks about temperature, T represents the photon temperature T_γ

Figure 2.3: The little thermal history of the Universe with each processes described in the following sections.

$$\rho_\gamma(T) = 4.1 \text{ keV/m}^3(\frac{T_\gamma}{1K})^4 \quad \text{and} \quad \rho(T_\nu) = 7.1 \text{ keV/m}^3(\frac{T_\nu}{1K})^4 = 1.9 \text{ keV/m}^3(\frac{T_\gamma}{1K})^4 \quad (2.1)$$

2.2.1 The Hubble law: a Newtonian approach

According to inflationary theory, the Universe is homogeneous and isotropic over distances of the order Gpc, but becomes highly inhomogeneous for scale below 100 Mpc. It is also an observed fact, that the Universe expands (and cools down) following the Hubble law since around 13.8 bilions of years. The Hubble law which governs the expansion of the Universe is fundamental in many aspects. Indeed, it governs the physics in the primordial Universe, and the decoupling of particles from the thermal bath, acts on the structure formation of galaxies and clusters of galaxies and determines the mean free path of relativistic particles in our present Universe. Understanding this law is then a critical step in the understanding of the whole process. In this section, we will show how that the basic Newtonian concepts give a relatively clear picture of the phenomena.

In an expanding, homogeneous and isotropic Universe, the relative velocities of observers obey the *Hubble law* : the velocity of observer **B** with respect to **A** is

$$\vec{v}_{AB} = H(t)\vec{r}_{AB} \quad (2.2)$$

Homogeneous and isotropic

Homogeneity and isotropy are fundamental because it means that our physics, and the results we obtain from our computation, are valid in the whole Universe. In other word, cosmology as a science can exist[a]. We can then distinguish an: *Homogeneous Universe* which is the same wherever you are, and an *Isotropic Universe* which looks the same on every angle you look at (see Fig.2.4).
It is possible to show that **the unique law which respects an homogeneous and isotropic Universe is in fact the Hubble law**. Indeed suppose three observers **A**, **B** and **C** (Fig. 2.5). Let define the velocity function of an observer i , $v^i = f^i(r,t)$. The observer **A** sees the point **C** moving away following $v_C^A = f^A(r_{AC},t)$, whereas the observer **B** sees **C** receding at a velocity $v_C^B = f^B(r_{BC},t)$. The observer **A** then *believes* that **B** sees **C** moving following its own law. Then for **A**, the velocity of **C** observed by **B** is the velocity of **C** observed by himself (**A**) minus the velocity of **B** relative to himself : $v_C^B = f^A(r_{AC},t) - f^A(r_{AB},t)$. If the Universe is homogeneous and isotropic, **A** and **B** should observe the same law in any direction, one can then write $f^B = f^A$, and so, equalizing the velocity for both observers, $v_C^B = f(r_{BC},t) = f(r_{AC},t) - f(r_{AB},t)$. For **C**= **A**, on obtains $f(r_{BA},t) = -f(r_{AB},t)$. Noting $r_{BC} = r_{BA} - r_{CA}$ one deduces $f(r_{BA} - r_{CA},t) = f(r_{BA},t) - f(r_{CA},t)$. Developing $f(x) = \sum_i H_i(t)x^i$ it is straightforward to see that only the linear term survives this condition. We just have proven that the homogeneity and isotropy of the Universe imposes f to be a linear function of r, $f(r,t) = v(r,t) = H(t)r$. The Hubble law is indeed the unique law which respect an homogeneous and isotropic Universe.

[a]Hubble never had the Nobel prize because cosmology was not recognized as a scientific discipline until his death in 1953, see section (G.6)

This law means that for an object **B** situated at a distance twice larger than the object **C** from the observer **A**, the velocity of **B** is twice larger than the one of **C**. Going backward, if one considers constant velocity, it will take the same time for **C** to join **A** than **B** because even if being further away, its velocity compensates its larger distance. Measuring the Hubble

Figure 2.4: *Left* : An homogeneous but not isotropic Universe: the dot and the cross see the same universe, but not isotropic ones because one direction is preferred. *Right*: An isotropic but non homogeneous Universe: The dot sees the same Universe whatever direction he looks at, but the cross does not see the same Universe than the dot.

Figure 2.5: Illustration of the Hubble law (see text for details). *Left*: Universe with expansion parameter a(t), and comoving coordinate χ. *Right*: Velocity of a point **C** viewed by and observer **A** and **B**

parameter nowadays ($H_0 \simeq 67$ km/s/Mpc) we can compute the time t_0 needed for **B** and **C** to join **A**, $t_0 \simeq 1/H_0 \simeq 14.2$ billions years, which as a good approximation corresponds to the age of the Universe (13.8 billions years). One can also understand why the Hubble law is valid only at scales above 100 Mpc. Indeed, at such distances, the velocity due to the expansion is of the order of 6700 km/s, which is larger than the peculiar velocities of galaxies (of the order of hundreds of km/s). For instance, the Andromeda galaxy, which is at a distance of 0.7 Mpc from the Milky way (see section 17.1.4), possesses a recession velocity: Andromeda falls *toward us* at a velocity of around 300 km/s : at such low distance, the Hubble expansion has few influence on the Newton law of attraction.

The computation is in fact a little bit more subtile as the Hubble parameter is not constant and depends on the Universe content (matter, radiation or cosmological constant). This law is easy to interpret if one consider points on a radius of an expanding sphere of radius $a(t)$ as illustrated in Fig.(2.5). One can define $r_{AB}(t) = \chi_{AB} a(t)$, then $v_{AB} = \chi_{AB}\dot{a}(t)$. χ_{AB} is called the *comoving coordinate*. It does not evolve with time, and can be considered as an initial condition. Then

$$v_{AB} = H(t) r_{AB} \quad \Rightarrow H(t) = \frac{\dot{a}(t)}{a(t)} \tag{2.3}$$

It is then interesting to solve the equation of motion in an homogeneous and isotropic space in expansion dominated by matter (dust dominated) *ie* respecting the Hubble law. If one define ρ the mass density that we suppose constant between **A** and **B** one can write

$$M = \frac{4\pi}{3} r^3 \rho(t), \quad \Rightarrow \rho(t) = \rho_0 \left(\frac{a_0}{a(t)}\right)^3 \tag{2.4}$$

where $r = r_{AB} = \chi a(t)$, and M is the mass inside the radius r. On the other hand, the Newtonian equation of motion is for the observer **B** of mass m_B

$$m_B \ddot{r} = m_B \chi \ddot{a}(t) = -\frac{G m_B M}{\chi^2 a^2(t)}, \quad \Rightarrow \ddot{a}(t) = -\frac{4\pi G}{3}\rho(t) a(t) = -\frac{4\pi G}{3}\rho_0 \frac{a_0^3}{a^2(t)}. \tag{2.5}$$

Multiplying by $\dot{a}$ both sides of Eq.(2.5) and integrating with constant of integration k one obtains

$$\frac{1}{2}\dot{a}^2 = \frac{4\pi G}{3}\rho_0 \frac{a_0^3}{a(t)} + k, \Rightarrow \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = H^2(t) = \frac{8\pi G}{3}\rho(t) + \frac{k}{a^2(t)} \tag{2.6}$$

where we used $H(t) = \dot{a}(t)/a(t)$ (Eq. 2.3). This is the Friedmann equation that we obtained in a pure Newtonian formalism and which determines entirely the physics of an expanding mass dominated (dust) Universe (up to the cosmological constant). We will see that the equation is still valid in the general relativity framework, ρ being the density of energy. In fact, Eq.(2.6) can be seen as the conservation of energy equation, k being the initial energy. A positive k means that at the beginning of the expansion, the kinetic energy was larger then the potential attraction, then the Universe will evolve in a continuous expansion with time. On the opposite, a negative k drives the Universe to a future collapse on itself to to

the dominance of the gravitational attraction. k will in fact corresponds to the curvature of the Universe in a Friedmann-Robertson-Walker metric.

It is interesting to compute another useful cosmological parameter called the *deceleration* parameter $q(t)$

$$q(t) = -\frac{1}{H^2(t)}\frac{\ddot{a}(t)}{a(t)}. \tag{2.7}$$

Historically speaking, this was the main source of information on the matter content of the Universe before the precise measurement of the CMB and its anisotropies. From Eq.(2.5) one obtains

$$q(t) = \frac{4\pi G}{3H^2(t)}\rho(t) = \frac{1}{2}\frac{\rho(t)}{\rho_{cr}(t)} = \frac{1}{2}\Omega_m(t) \tag{2.8}$$

where we have defined the critical density $\rho_{cr}(t) = \frac{3H^2(t)}{8\pi G}$, corresponding to the density of a flat Universe ($k = 0$). In the early 60's until the 80's, the matter content of the present Universe was limited by the measurement of $q_0 = q(t_0) \lesssim 2.5$ from the observation of the deceleration of nearby clusters of galaxies. this limit gives $\Omega_m^0 \lesssim 5$, far above the present limit given by the measurement of the anisotropies of the CMB ($\Omega_m^0 \lesssim 0.3$).

2.2.2 The Friedmann- Robertson-Walker metric

generalities

The construction of a general space-time metric of the Universe is based on the hypothesis that the Universe is homogenous and isotropic. This is *cosmological principle* and is empirically justified on scales larger than 100 Mpc. So one should first build a 3D spatial metric plunged in a 4D space. Using Cartesian coordinates (x, y, z, w) but replacing (x, y, z) by spherical polars (ρ, θ, ϕ), we have for the infinite space interval dl

$$dl^2 = d\rho^2 + \rho^2 d\Omega^2 + dw^2,$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is short-hand for the solid angle. For a positive curvature one also can write

$$x^2 + y^2 + z^2 + w^2 = \rho^2 + w^2 = R^2$$

where R is the curvature radius, independent of the position (x, y, z) by the hypothesis of homogeneity. We then have

$$\rho d\rho + w dw = 0$$

Therefore

$$dw^2 = \frac{\rho^2}{w^2}d\rho^2 = \frac{\rho^2}{R^2 - \rho^2}d\rho^2$$

and so

$$dl^2 = d\rho^2 + \frac{\rho^2 d\rho^2}{R^2 - \rho^2} + \rho^2 d\Omega^2$$

giving

$$dl^2 = \frac{d\rho^2}{1 - (\rho/R)^2} + \rho^2 d\Omega^2$$

This is a homogeneous, isotropic 3D space of (positive) curvature $1/R^2$. Setting $\rho = Rr$ we can express

$$dl^2 = R^2 \left(\frac{dr^2}{1 - r^2} + r^2 d\Omega^2 \right)$$

Negative and zero curvature is also possible. For the negative case, the radius condition should be written

$$dl^2 = d\rho^2 + \rho^2 d\Omega^2 - dw^2 \quad \text{and} \quad \rho^2 - w^2 = -R^2$$

This is the equivalent of the hyperboloid surfaces in 2D. We then obtain

$$dl^2 = R^2 \left(\frac{dr^2}{1 + r^2} + r^2 d\Omega^2 \right)$$

We can then combine all the possible curvature with the generic expression

$$dl^2 = R^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

with $k = +1$ for a positive curvature, $k = 0$ for a flat one and $k = -1$ for a negative curvature. In general we must allow for R to be an arbitrary function of time $R(t)$ (not position since that would destroy homogeneity), thus we arrive at

$$\boxed{ds^2 = c^2 dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right)}$$

This is the Friedmann-Robertson-Walker metric. It was first derived by Freidmann in 1922, and then more generally by Robertson and Walker in 1935. It applies to *any* metric theory of gravity, not just general relativity.

Geometry of the Universe

We have then three cases

- $k = 1$: positive curvature, closed Universe
- $k = 0$: zero curvature, flat Universe (flat space, not flat space-time
- $k = -1$: negative curvature, open Universe.

An alternative form of the metric is often useful. for $k = 1$, setting $r = \sin\chi$, the interval becomes

$$ds^2 = c^2dt^2 - R^2(t)(d\chi^2 + \sin^2\chi d\Omega^2)$$

which can be written by generalisation

$$ds^2 = c^2dt^2 - R^2(t)(d\chi^2 + S_k^2(\chi)d\Omega^2)$$

where

$$S_1(\chi) = \sin\chi, \quad S_0(\chi) = \chi, \quad S_{-1}(\chi) = \sinh\chi.$$

Redshift

This convention of variables (t, χ, Ω) is less obvious to understand the physics lying beyond the equation, but much practical to deal with formal calculation. This becomes clear when one needs to compute processes like the redshift for example. Indeed, the wavelength of light from astronomical sources is a crucial, easily measured observable. Consider two pulses of light emitted at times $t = t_e$ and $t = t_e + \delta t_e$ by an object at χ towards an observer at the origin who picks them up at $t = t_0$ and $t = t_0 + \delta t_0$. The light is emitted in a solid angle dΩ=0 and follow a geodesic ($ds = 0$).
For photons travelling towards the origin, since $ds = 0$

$$cdt = -R(t)d\chi$$

Therefore, because χ represent a comoving coordinate, which is fixed and independent of time

$$\chi = \int_{t_e}^{t_0} \frac{cdt}{R(t)} = \int_{t_e+\delta t_e}^{t_0+\delta t_0} \frac{cdt}{R(t)}$$

Subtracting the first integral from the second:

$$\int_{t_0}^{t_0+\delta t_0} \frac{cdt}{R(t)} - \int_{t_e}^{t_e+\delta t_e} \frac{cdt}{R(t)} = 0.$$

For small intervals $R(t)$ is almost constant[2], so

$$\frac{\delta t_0}{R(t_0)} = \frac{\delta t_e}{R(t_e)}$$

Therefore the redshift z, which is defined as the relative difference between the observed wavelength and the emitted one ($z = \frac{\lambda_0 - \lambda_e}{\lambda_e}$) is given by

$$1 + z = \frac{\lambda_0}{\lambda_e} = \frac{\nu_e}{\nu_0} = \frac{\delta t_0}{\delta t_e} = \frac{R(t_0)}{R(t_e)}$$

[2]Whereas t_e and t_0 are usually very spaced : $R(t_e) \neq R(t_0)$

Hubble law

The universal "fluid" (=galaxies) is at rest in *comoving* coordinates r or χ, θ and ϕ. Expansion of the Universe is contained in the size factor $R(t)$.
Consider the proper distance to a galaxy at radius χ

$$d_P = \int_0^{\chi} R(t) d\chi = R(t)\chi$$

Since χ is fixed, the rate of recession of the galaxy is

$$v = \frac{d}{dt}(d_P) = \dot{R}\chi = \frac{\dot{R}}{R} d_P.$$

identifying

$$H(t) = \frac{\dot{R}}{R},$$

we have

$$\boxed{v = H(t) d_P}$$

which is the Hubble's law, while $H(t)$ is the Hubble's "constant" $= H(t_0) = H_0$ today. Hubble's law is thus a direct outcome of homogeneity and isotropy.

2.2.3 Hubble expansion for dummies

We present here the main results useful for us concerning the evolution of the scale factor in the Universe. For more details in the computations and results, see the appendix B.1. The Friedmann equation can be written

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{2.9}$$

with a being the scaling factor, $a = 1/(1+z)$ (a=1 today), k the curvature factor $(0, \pm 1)$, $G = 1/M_P^2$ the gravitational coupling (M_P is the Planck mass $= 1.22 \times 10^{22}$ GeV) and Λ the cosmological constant. Compared to the formulae of the preceding section, a is defined as $R(t) = R_0 * a(t)$, R_0 being the present time radius of the universe (and then $a \leq 1$).
The presence of the cosmological constant Λ which appeared in the 90's through observational measurements, changed completely the fate of the Universe. Indeed, without it, the Universe had 3 options:

- If k=-1, the expansion slows down all the time, but without stopping
- If k=0, the expansion slows down and stops at t=∞
- If k = +1, the expansion slows down, stops, then turns over to contraction.

All these different destiny was related by a slowing down Universe. Indeed, in all case, the ρ ($\propto 1/a^3$ if matter dominated and $1/a^4$ if radiation dominated, see section 2.3.7) term or curvature terms were *decreasing* as a is increasing. The presence of a positive Λ term inverts all the process, giving a nowadays an accelerating expanding Universe.

If the universe is matter-dominated, then the mass density of the universe ρ can just be taken to include matter so $\rho = \rho_0/a^3$, ρ_0 being the density of matter today. We know for non–relativistic particles that their mass density decreases proportional to the inverse volume of the universe so the equation above must be true. We can also define

$$\rho_c = \frac{3H^2}{8\pi G}, \qquad \Omega_M = \frac{\rho_0}{\rho_c}, \qquad \Omega_R = \frac{-kc^2}{H_0^2} \qquad \text{and} \qquad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2} \tag{2.10}$$

ρ_c is the critical density. It corresponds to the density we would expect if the Universe is flat ($k = 0$). Any deviation on ρ_c can be interpreted as a measurement of the space curvature. Nowadays, for a value of $H_0 = 100h$ km/s/Mpc $= 2.13 \times 10^{-42}$ h GeV (corresponding to the speed of a galaxy 1Mpc away, h being measured to be in 2012: $h = 74.3 \pm 2.1$ km/s/Mpc) one obtains, reminding that $G = 1/M_P^2$

$$\rho_c^0 = \frac{3H_0^2}{8\pi G} = 10^{-5}h^2 \text{ GeV cm}^{-3} = 1.88h^2 \times 10^{-29} \text{ g cm}^{-3} \tag{2.11}$$

The CMB measurements as well as the type Ia supernovae observations seems to favorize a flat Universe with a matter component composing a fraction $\Omega_M = \rho_M/\rho_c^0 = 0.3$ of the critical density. One can then compute which fraction of this matter is composed from baryonic ($\simeq 1$ GeV as it is mainly proton/hydrogenic clouds) matter from the data given by WMAP of the ratio of baryon to photon number density [13]

$$\eta = \frac{n_B}{n_\gamma} = 6.19 \times 10^{-10} \tag{2.12}$$

knowing the temperature of the photon nowadays (2.7 K) one can deduce the number density of photon (2.50) : 394 cm^{-3}, and so the number density of baryon, $n_b = \eta \times n_\gamma = 2.4 \times 10^{-7}$cm^{-3}. Considering 1 GeV baryon, and $h = 0.71$, from the value of ρ_c^0 computed in (2.11) we deduce

$$\Omega_b = \frac{1 \text{ GeV} \times n_b}{\rho_c^0} \simeq 0.047 \tag{2.13}$$

As we notice, this value is far to be sufficient to account for the matter component of the Universe ($\Omega_M = 0.3$) which implies the need for a dark component.
Substituting all of this in into the Friedman equation Eq.(2.9) at the start of this section and replacing a with $a = 1/(1+z)$ gives

$$\boxed{H^2(z) = H_0^2 \left[\Omega_M^0(1+z)^3 + \Omega_R^0(1+z)^2 + \Omega_\Lambda^0\right] \qquad (2.14)}$$

2.2.4 The deceleration parameter q

In a lot of papers before we had the WMAP [13] or PLANCK [14] data, physicists did not have the value of the cosmological constant or the matter content of the Universe. They were using the deceleration parameter $q = \ddot{R}R/\dot{R} = \ddot{R}/H^2$.

indeed, the equation we used Eq.(2.9) is coming from the first Einstein equation, involving the velocity. The second equation, involving acceleration, can be written

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{2.15}$$

The curvature does not appear in the equation, but the pressure yes. However, nowadays, $p_0 \ll \rho_0$ so we can write

$$H \tag{2.16}$$

where the subscript "0" stands for present values (redshift $z = 0$).

2.2.5 The comoving distance or codistance

Sometimes, when computing a physical process, one needs to know the phenomena characteristic (spectra, lifetime, temperature..) at a given distance and given time t (or equivalently redshift z). At that redshift z, the Universe was quite different than today, and its metric was also reduced by a factor $1/(1+z)$. One should then be careful when computing the signal observed now, from a source producing it at a high redshift.
For instance, let's think about the spectrum of a source, which emits a monochromatic signal continuously in the Universe. One has to take account that the energy emitted at the distance $d(z)$ as been redshifted along its way from the source to the earth. As a consequence, the observed signal on earth is not monochromatic anymore, but a sum of redshifted signal, on a distance which evolves also with time (as z evaluates with time). Let's put is in numbers. The first step is to compute the distance of the source from the earth at a redshift z, that we call the comoving distance χ. In other word, what is the distance x that a photon has crossed if it has been produced at a redshift z. For a massless particle like the photon, we can write

$$dt^2 - a^2(t)dx^2 = 0 \quad \Rightarrow \quad dx = \frac{dt}{a(t)}$$

In the meantime, we know that by definition, $H(t) = \frac{da(t)/dt}{a(t)}$ which implies

$$dx = \frac{da(t)}{a^2(t)H(t)}, \quad a = \frac{a_0}{(1+z)} \quad \Rightarrow \quad dx = -\frac{dz}{a_0 H(t)}$$

where we kept $a_0(=1)$ for a better understanding[3]. The codistance $d\chi$ $from$ the earth can then be written $d\chi = -dx$ or

$$\frac{d\chi}{dz} = \frac{1}{a_0 H(z)}$$

We now need to find the expression of $H(z)$ which obliviously depends on the redshift as, if the cosmological constant stays the same, the *density* of matter evolves with time due to the expansion of the metric. Using Eq.(2.14) one obtains

[3]we remind the reader that by convention, the radius of the universe at a time t is written $R(t) = a(t)R_0$ where R_0 is the present radius of the Universe, of the order of 28 Gpc.

H	0.71
He	0.28
C	0.34×10^{-2}
N	0.99×10^{-3}
O	0.96×10^{-2}
Ne	0.18×10^{-2}
Na	0.35×10^{-4}
Mg	0.66×10^{-3}
Al	0.56×10^{-4}
Si	0.70×10^{-3}
S	0.3×10^{-3}
Cl	0.47×10^{-5}
Ar	0.11×10^{-3}
Ca	0.65×10^{-4}
Cr	0.18×10^{-4}
Fe	0.13×10^{-2}
Co	0.36×10^{-5}
Ni	0.73×10^{-4}

Table 2.1: Mass fraction of different elements in the Solar neighbourhood.

$$H(z) = H_0\sqrt{(1+z)^3\Omega_M^0 + \Omega_R^0(1+z)^2 + \Omega_\Lambda^0} \quad \Rightarrow \tag{2.17}$$
$$\frac{d\chi}{dz} = \frac{1}{a_0 H_0 (1+z)^3\sqrt{\Omega_M^0 + \Omega_R^0/(1+z)^{3/2} + \Omega_\Lambda/(1+z)^3}}$$

Depending on the problem one then can integrate from $z_0 = 0$ to any redshift z to compute the consequences of an event on earth which is occurring regularly since the redshift z, like a decaying dark matter for instance, see section (4.13).

2.2.6 And what about the statistics of gas and radiation?

Most of the matter in the Universe exists in gaseous form. As we already discussed, a fraction ($\sim 10\%$) of it is baryonic matter, like what we are familiar with in everyday experience, and a larger fraction ($\sim 90\%$) is non-baryonic dark matter, the nature of which we try to identify in this essay. The dark matter makes itself felt only through its gravitational interaction, all visible matter is therefore still baryonic.
The baryonic matter in the universe is composed mainly of Hydrogen (about three quarter of the mass) and Helium (about a quarter). There is a small fraction of heavier elements which, collectively, are referred to as metals. The abundance of metals in the solar neighbourhood is about 2 % by mass, while the Hydrogen abundance is 71 %. Table 2.1 shows the mass fraction of several elements in the solar neighbourhood.

The equation of state of a gas represents a relation connecting the state variables such as pressure, density, temperature or internal energy for instance. We will first concentrate on the case of a gas where the energy of mutual interaction between particles is negligible.
A gas will have particles with a distribution of kinetic energies of random motion, and a kinetic energy E would be associated with a momentum of the particle where the magnitude of the momentum $p(E)$ is determined by the energy. The pressure is defined as the rate of momentum transfer in a given direction through an unit of area per unit time, and since the direction of the momentum is randomly distributed in 3 dimensions, the pressure P is given by

$$P = \frac{1}{3}\int_0^\infty f(E)p(E)v(E)dE \tag{2.18}$$

where $f(E)dE$ is the number of particles with energy between E and $E+dE$ in an unit volume of the gas, and $v(E)$ is the speed associated with the kinetic energy E. For simplification, we assume that the particles of the gas in question have no internal degrees of freedom (like spin or charges), and the kinetic energy is entirely that of their random translational motion[4].
If the random motion in the gas is non–relativistic, then we can write $v = p/m$ and $E = p^2/2m$, resulting in

$$P = \frac{2}{3}\int_0^\infty f(E)EdE = \frac{2}{3}\rho \tag{2.19}$$

where ρ is the internal energy density (kinetic energy of random motion per unit volume).
If the gas is relativistic, $v = c$ and $E = pc$ giving

$$P = \frac{1}{3}\int_0^\infty f(E)EdE = \frac{1}{3}\rho \tag{2.20}$$

The proportionality constant between P and ρ has a physical significance: it is equal to $\gamma - 1$, where γ, called the "adiabatic index" (Eq. B.32) , equals the ratio of specific heats for a thermal gas. Reversible adiabatic processes yield PV^γ =constant[5], (Eq. B.31) where V is the volume of the gas. Using this, one finds that for a specific adiabatic expansion the total energy $U = \rho V$ of a gas drops as R^{-2} for a non-relativistic gas and as R^{-1} for a relativistic gas ($V \propto R^3$)[6].
At this point, let us remind ourselves what we mean by a "thermal gas". For a classical gas, this means that all energy levels of the gas, both discrete and continuous, are occupied according to the Boltzmann distribution:

$$N(E) \propto g(E)e^{-E/kT}$$

where $g(E)$ is the so called "density of states". In case of quantum statistics, the corresponding distribution is:

[4]Hypothesis which is obviously not valid in the case of particles of the Standard Model which all possess internal degrees of freedom

[5]See section B.4.1 for the demonstration of this law called Laplace's law

[6]Another way to obtain the same result is to note that the energy of a relativistic species is $U_{rel} = pc$ and scales as R^{-1} whereas the energy of non-relativistic species is $U_{non-rel} = \frac{1}{2}mv^2$ scales as R^{-2}.

$$N(E) \propto \frac{g(E)}{e^{(E-\mu)/kT} \pm 1}$$

where the positive sign in the denominator corresponds to a Fermi gas and the negative sign to a Bose gas. Quantum statistics comes into play only when the number of particles per phase space cell of volume h^3 is of order unity. For dilute gases, as encountered in most astrophysical situations, classical description is quite adequate whereas in the case of the dense primordial plasma in the early Universe, a complete details quantum analysis is necessary.

2.2.7 And what about thermodynamics?

Distribution functions and thermodynamics quantities

From elementary quantum mechanics on can remember the "particle in the box". Let this be a cubic box of length L (volume $V = L^3$) with periodic boundary condition. Solving the Schrodinger equation to determine the energy and momentum eigenstates, we obtain a discrete set of momentum eigenvalues

$$\vec{p} = \frac{h}{L}(n_x\vec{i} + n_y\vec{j} + n_z\vec{k}), \quad n_i = 0 \pm 1, \pm 2, ... \tag{2.21}$$

where $h = 4.14 \times 10^{-24}$ GeV is the Planck's constant. The density of states in momentum space $\vec{p}$ (i.e. the number of states per $\Delta p_x \Delta p_y \Delta p_z$) is thus[7]

$$\frac{L^3}{h^3} = \frac{V}{h^3} \ ,$$

and the state density in phase space $\vec{x}$, $\vec{p}$ is $1/h^3$. If the particle has g internal degrees of freedom (e.g. spin) then the density of states becomes

$$\frac{g}{h^3} = \frac{g}{(2\pi)^3}$$

where in the second equality we used natural units with $\hbar = h/(2\pi) = 1$. To obtain the number density of a gas of particles we need to know how the particles are distributed amongst the momentum eigenstates. This information is contained in the phase space distribution function $f(\vec{x}, \vec{p}, t)$. Because of homogeneity, the distribution function should, in fact, be independent of the position $\vec{x}$. Moreover, isotropy requires that the momentum dependance is only in terms of the magnitude of the momentum $p = |\vec{p}|$. We will typically leave the time dependance implicit: it will manifest itself in terms of the temperature dependance of the distribution function. The particle density in phase space is then the density of states times the distribution function

[7] Another way to understand it is to imagine a square box made of sides corresponding to $n_i = 0$ and $n_i = 1$. Each side of the box has a size of h/L, so the box has a volume of h^3/L^3. However, each of the eight corners of the box corresponds to a state, pertaining to 8 boxes surrounding it, so counting as 1/8 of states par box. Then, a box of 8 corners contains $8 \times 1/8 = 1$ state in its h^3/L^3 volume. In other words, $\times \Delta p_x \Delta p_y \Delta p_z/(h^3/L^3)$ represents the number of states [number of boxes] having momentum $\vec{p}$ in the range (Δp_x, Δp_y, Δp_z).

Figure 2.6: Pressure in a weakly interacting gas of particles.

$$\frac{g}{(2\pi)^3} \times f(\vec{p}) \tag{2.22}$$

The *number density* of particles (in real space) is found by integrating (2.22) over momentum

$$n = \frac{g}{(2\pi)^3} \int d^3p f(p). \tag{2.23}$$

To obtain the energy density of the gas of particles, we have to weight each momentum eigenstates by its energy. To a good approximation, the particles in the early universe were *weakly interacting*. This allows us to ignore the interaction energies between the particles and write the energy of a particle of mass m and momentum p simply as

$$E(p) = \sqrt{m^2 + p^2}. \tag{2.24}$$

Integrating the product of (2.22) and (2.24) over momentum then gives the *energy density*

$$\rho = \frac{g}{(2\pi)^3} \int d^3p f(p) E(p) \tag{2.25}$$

And similarly as we saw in section (2.2.6), we define the pressure

$$P = \frac{g}{(2\pi)^3} \int d^3p f(p) \frac{p^2}{3E} \tag{2.26}$$

where we recover the expressions (2.19) and (2.20) for $E = p$ and $E = p^2/2m$ respectively.

Let remind where the $p^2/3E$ factor in (2.26) comes from. Consider a small area element of size dA, with unit normal vector $\vec{n}$ (see figure 2.6). All particles with velocity $|\vec{v}|$, striking this area element in the time interval between t and $t+dt$ where located at t in a spherical shell of radius $R = |v|t$ and width $|v|dt$. A solid angle $d\Omega$ of this shell defines the volume $dV = R^2|v|dtd\Omega$. Multiplying the phase space density (2.22) by dV gives the number of particles in the volume (per unit volume in momentum space) with energy $E(|v|)$,

$$dN = \frac{g}{(2\pi)^3} f(E) \times R^2|v|dtd\Omega. \tag{2.27}$$

Not all particles in dV reach the target, only those with velocities directed to the area element. Taking into account the isotropy of the velocity distribution, we find that the total number of particles striking the area element $dA\ \hat{n}$ with velocity $\vec{v} = |v|\hat{v}$ is

$$dN_A = \frac{|\hat{v}.\hat{n}|dA}{4\pi R^2} \times dN = \frac{g}{(2\pi)^3} f(E) \times \frac{|\hat{v}.\hat{n}|}{4\pi} dAdtd\Omega \tag{2.28}$$

where $\hat{v}.\hat{n} < 0$. If these particles are reflected elastically, each transfer momentum $2|\vec{p}.\hat{n}|$ to the target. Therefore, the contribution of particles with velocity $|v|$ too the pressure is

$$dP(|v|) = \int \frac{2|\vec{p}.\hat{n}|}{dAdt} dN_A = \frac{g}{(2\pi)^3} f(E) \times \frac{p^2}{2\pi E} \int \cos^2\theta \sin\theta d\theta d\phi = \frac{g}{(2\pi)^3} \times f(E) \frac{p^2}{3E} \tag{2.29}$$

where we have used $|v| = |p|/E$ and integrated over the hemisphere defined by $\hat{v}.\hat{n} = -\cos\theta < 0$ (i.e. integrating only over par tiles moving towards dA). Integrating over the energy E (or momentum p) we obtain (2.26)

General framework

The original gas, denoted as "Ylem" by Gamow himself exists under the form of a complete ionized plasma of elementary particles. As far as their kinetic energy is dominant on their distance potential energy, we can suppose that the primordial gas is "perfect". It is important to notice that this hypothesis will not be valid anymore when we will treat about the quark-hadrons phase transition where the strong interactions confine quarks inside protons, neutrons or pions. That is the subject of the next section.
The primordial plasma is at the same time in thermal *and* chemical equilibrium.

- The plasma is in thermal equilibrium under collisional effect of the type:

$$e + \gamma \to e + \gamma. \tag{2.30}$$

- The chemical equilibrium is realized through reactions like

$$3\gamma \leftrightarrow e^+e^- \leftrightarrow 2\gamma. \tag{2.31}$$

from the last reaction, one can immediately deduce that the chemical potential of photon is null ($3\mu_\gamma = 2\mu_\gamma$) and that the chemical potential of positron and electrons are opposite ($\mu_{e^-} = -\mu_{e^+}$). Moreover, from the very tiny ratio of the baryon to photon ratio of today ($n_B/n_\gamma \simeq 7 \times 10^{-10}$), one can deduce that $\mu_{e^-} = \mu_{e^+}$. The two preceding hypothesis imply that $\mu_{e^-} = \mu_{e^+} = 0$ and one can describe the primordial plasma as a group of bosonic and fermionic population at temperature **T** with **null** chemical potential. In this case, for a particle A, in a more general context its statistic (homogen) distribution is given by :

$$f_A(\vec{p}) = \frac{g_A}{e^{(E-\mu_A)/kT} \pm 1} \tag{2.32}$$

with $E^2 = m^2c^4 + p^2c^2$ and ± 1 correspond to fermionic (+) or bosonic (-) statistic. g_A is the spinorial/helicity degree of freedom of the particle A (2 for a fermion, 3 for a massive vector, 2 for a photon..). The number density $n_A(T)$ and energy density $\rho_A(T)$ of a specie **A** is thus given by :

$$\begin{aligned} n_A(T) &= \frac{g_A}{(2\pi)^3} \int f_A(\vec{p}) d^3p = \frac{g_A}{2\pi^2} \int_{m_A}^{\infty} \frac{(E^2 - m_A^2)^{1/2}}{e^{(E-\mu_A)/kT} \pm 1} E dE \\ &= \frac{g_A}{2\pi^2} \int_{m_A}^{\infty} \frac{p^2 dp}{e^{(\sqrt{p^2+m_A^2}-\mu_A)/kT} \pm 1} \end{aligned} \tag{2.33}$$

$$\begin{aligned} \rho_A(T) &= \frac{g_A}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3p = \frac{g_A}{2\pi^2} \int_{m_A}^{\infty} \frac{(E^2 - m_A^2)^{1/2}}{e^{(E-\mu_A)/kT} \pm 1} E^2 dE \\ &= \frac{g_A}{2\pi^2} \int_{m_A}^{\infty} \frac{(\sqrt{p^2 + m_A^2})\, p^2 dp}{e^{(\sqrt{p^2+m_A^2}-\mu_A)/kT} \pm 1} \end{aligned} \tag{2.34}$$

In the relativistic limit ($T \gg m$) and for negligible chemical potential ($T \gg \mu_A$) on can have an analytical expression of 2.34 using the relation

$$\int_0^\infty dx \left(\frac{x^n}{e^x - \delta} \right) = \Gamma(n+1)\zeta(n+1)y(\delta) \tag{2.35}$$

with $y(\delta) = 1$ if $\delta = 1$ and $1 - \frac{1}{2^n}$ if $\delta = -1$. Γ is the Euler function ($\Gamma(z+1) = z\Gamma(z)$) and $\zeta(s)$ is the zeta function defined by $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ [$\zeta(0) = -1/2; \zeta(1) = \infty; \zeta(2) = \pi^2/6; \zeta(3) \simeq 1.2; \zeta(4) = \pi^4/90; ..$]. Integrating Eq.(2.34) gives

$$\begin{aligned} \rho_A(T)_{T \gg m,\mu} &= \frac{\pi^2}{30} g_A T^4 \quad \text{(bosons)} \\ \rho_A(T)_{T \gg m,\mu} &= \frac{7}{8}\frac{\pi^2}{30} g_A T^4 \quad \text{(fermions)} \end{aligned} \tag{2.36}$$

$$\begin{aligned} n_A(T)_{T \gg m,\mu} &= \frac{\zeta(3)}{\pi^2} g_A T^3 \quad \text{(bosons)} \\ n_A(T)_{T \gg m,\mu} &= \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_A T^3 \quad \text{(fermions)} \end{aligned} \tag{2.37}$$

For non-relativistic species (when the temperature of the plasma approaches the mass of a particle **A**), the Boltzmann factor dominates the denominator in Eq.(2.34) so bosonic and

fermionic distribution are identical. Developing (2.33) in expansion of p^2/m_A^2 and using $\int_0^\infty x^2 e^{-ax^2} = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$ we obtain

$$n_A(T)_{T\ll m_A} = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} e^{-(m_A-\mu_A)/T}$$
$$\rho_A(T)_{T\ll m_A} = g_A m_A \left(\frac{m_A T}{2\pi}\right)^{3/2} e^{-(m_A-\mu_A)/T} \tag{2.38}$$

One can ask what is the mean energy *per particle* for non–degenerate relativistic species, $\langle E\rangle = \frac{\rho}{n}$:

$$\langle E\rangle^{per\ particle}_{T\gg m_A} = \frac{\pi^4}{30\zeta(3)} T \simeq 2.7\ T \quad \text{(bosons)}$$
$$\langle E\rangle^{per\ particle}_{T\gg m_A} = \frac{7\pi^4}{180\zeta(3)} T \simeq 3.2\ T \quad \text{(fermions)} \tag{2.39}$$

Putting some numbers, one can make an idea of the energy density of a relativistic gas at a temperature T. For that we need to reintroduce the lost unities and multiply the expression (2.36) by $\frac{k^4}{\hbar^3 c^3} \simeq 1.15\times 10^{-21}\text{Jcm}^{-3}\text{K}^{-4}$ for the energy density and $(k/\hbar c)^3$ (remembering that $g_\gamma = 2$)

$$\rho_\gamma(T) \simeq 9\times 10^{40}\ \text{GeVcm}^{-3}\left(\frac{T}{\text{GeV}}\right)^4; \quad n_\gamma(T) \simeq 20\left(\frac{T}{\text{K}}\right)^3 \simeq 3.3\times 10^{40}\left(\frac{T}{\text{GeV}}\right)^3 \tag{2.40}$$

When the particle is non-relativistic, but still having some kinetic energy it is an intermediate regime. If we develop $E = \sqrt{m^2+p^2} \simeq m + p^2/2m$ in the non relativistic limit, integrating on $\mathbf{p}$ and using $\int_0^\infty x^4 e^{-ax^2} = \frac{3}{2^3 a^2}\sqrt{\frac{\pi}{a}}$, we can obtain the kinetic energy of a gaz of non-relativistic particles :

$$\langle E_c\rangle_{T\ll m_A} = \frac{3g_A}{16}\left(\frac{2m_A T}{\pi}\right)^{3/2} T\ e^{-m_A/T} \quad\Rightarrow\quad \langle E_c\rangle^{per\ particle}_{T\ll m_A} = \frac{\langle E_c\rangle}{n_A(T)_{T\ll m_A}} = \frac{3}{2}T. \tag{2.41}$$

Gathering the mass and kinetic energy we finally obtain :

$$\langle E\rangle^{per\ particle}_{T\ll m_A} = m + \frac{3}{2}T, \tag{2.42}$$

which is obviously the same for a boson or a fermion because the exponential suppression is the dominant contributor to the density and it washes up the spin-statistics differences. In another word, one can say that, in average, a relativistic particle has only kinetic energy (by definition) of 2.7 T if it is a boson, or 3.2 T if it is a fermion. On the other hand, if the particle is non-relativistic, its average kinetic energy is 1.5 T (fermion as boson).

Another possibility (especially if one needs to use analytical approximate solutions) is to use the Boltzmann distribution instead of the Fermi-Dirac or Bose-Einstein one. Indeed, in

regimes where $T \ll E, \mu_A,\ f_A(\vec{p}) \simeq g_A e^{-\frac{E}{T}} = f_{\text{Boltzmann}}$. In this case, a look at Eq.(B.30) gave us

$$n_A^{\text{Boltzmann}} = g_A \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} = g_A \frac{m_A^2 T}{2\pi^2} K_2\left(\frac{m_A}{T}\right) \tag{2.43}$$

$K_2(x)$ being the modified Bessel function of second order. In fact, we can show that this expression is still valid at the order of 20 % to 25% for $T \gg E$. Indeed, comparing Eq.(2.37) and Eq.(2.43), a 100 GeV majorana dark matter ($g_A = 2$) at a temperature of 1 000 TeV has a density of 2.4×10^{23} GeV^3 whereas the Boltzmann approximation (2.43) gives $2.\times10^{23}$ GeV^3

Degrees of freedom : computation of g(T)

So far we have considered plasma of specific species. The primordial plasma is a mix between different kind of particles, some are relativistic, some no. However, since the energy density of relativistic species is much greater than that of non relativistic species (Boltzmann suppressed), it suffices to include the relativistic species only[8]. The energy density can thus can be written

$$\rho(T) = \frac{\pi^2}{30} T^4 \sum_{i=all\ \ species} \frac{30}{\pi^2}\left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2} \int_{x_i}^{\infty} \frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T} \pm 1} = g_\rho(T)\frac{\pi^2}{30}T^4 \tag{2.44}$$

with $x_i = m_i/T$ and

$$g_\rho(T) = \frac{30}{\pi^2} \sum_{i=all\ \ species} \frac{g_i}{2\pi^2}\left(\int_{x_i}^{\infty} \frac{(\epsilon^2 - x_i^2)^{1/2}\epsilon^2 d\epsilon}{e^{\epsilon-\mu_i/T} \pm 1}\right)\left(\frac{T_i}{T}\right)^4 \tag{2.45}$$

which gives with a pretty good approximation, approximating the Boltzmann factor as a step function:

$$g_\rho(T) \simeq \sum_{b=bosons} g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8} \sum_{f=fermions} g_f \left(\frac{T_f}{T}\right)^4 \tag{2.46}$$

The relative factor of 7/8 account for the difference in Fermi and Bose statistics. The individual temperatures T_b and T_f are equal to the photon temperature of the bath ($T_\gamma = T$) as soon as they are relativistic in equilibrium in the bath. One can interpret g_ρ as being the degree of freedom of a gas made only of relativistic bosons of degree 1. The degrees of freedom of all the other relativistic species are integrated in g_{eff}. As an example we can compute the total degree of freedom of a gas in the $SU(3) \times SU(2) \times U(1)$ standard model. The contents is : 6 dirac (4) quarks colored (3) $[6 \times 4 \times 3]$ plus 3 dirac (4) leptons $[3 \times 4]$ plus 3 majorana (2) neutrino $[3 \times 2]$ plus 3 massive (3) vecor fields Z^0 $W^\pm$ $[3 \times 3]$ nine non massive vector fields (2) (1 photon and 8 gluon) $[9 \times 2]$ and finally one rela scalar (1) Higgs field [1] which gives

[8]This is true in the early universe, but not at a later times when eventually the rest masses of the particles left over from annihilation begin to dominate and we enter a matter-dominated era, see section (2.3.7)

Figure 2.7: Effective degree of freedom of the primordial plasma as function of the temperature

$$g_\rho^{SM} = (3 \times 3 + 9 \times 2 + 1) + \frac{7}{8}(6 \times 4 \times 3 + 3 \times 4 + 3 \times 2) = 106.75 \tag{2.47}$$

If one add a dirac dark matter candidate, $g_\rho^{SM+DM} = g_\rho^{SM} + \frac{7}{8} \times 4 = 110.25$. We represent in fig.2.7 the evolution of the degree of freedom as function of the temperature. We observed that the degree of freedom decrease especially at around 200 MeV where the quark/hadrons transition absorbs the up-type, down-type and gluonic degrees of freedom to form the first nucleus (see the Big Bang Nucleosynthesis section for more details) passing from 61.75 to 10.75. below 511 keV, we enter in a zone where the neutrino decoupling plays a role (see the corresponding section for more details).
We can compute the value of g_ρ nowadays, noticing that, a priori the 2 relativistic species still living in the bath are the photon and the neutrino, even if the neutrino has a lower temperature T_ν than the photon due to the entropy given by the electron to the photon when they decoupled (see section 5.6) $T_\nu = (4/11)^{1/3} T_\gamma = (4/11)^{1/3} \times 2.725 \approx 1.95$ K. What is left is thus 3 families of neutrino (fermions) with 2 degrees of freedom (SU(2)) and a temperature $T_\nu = 1.95$ K and a photon (boson) with 2 degrees of freedom and a temperature $T_\gamma = T_0 = 2.725$ K. Thus we can write Eq.(2.46)

$$g_\rho^{\text{today}} = 2 + \frac{7}{8} \times 2 \times 3 \times \left(\frac{4}{11}\right)^{4/3} = 3.36 \tag{2.48}$$

which implies

$$\rho_{\text{rel}} = \frac{\pi^2}{30} g_\rho T^4 = 8.1 \times 10^{-34} \text{ g cm}^{-3} \tag{2.49}$$

and

$$n_\gamma^{\text{today}} = \frac{2\zeta(3)}{\pi^2} T^3 = 394 \text{ cm}^{-3} \tag{2.50}$$

QCD (quark–hadrons) phase transition

Before the strange quark had time to annihilate, something else happens: matter undergoes the *QCD phase transition* ()also called quark-hadron transition). This takes place at $T \sim 150$ MeV. While quarks are *asymptotically free* (i.e. weakly interacting) at high energies, below 150 MeV, the strong interactions between the quarks and the gluons become important. The quarks and gluons then form bound three-quark systems, called *baryons*, and quark-antiquark pairs, called *mesons* We should treat these as our new degrees of freedom[9].

The lightest baryons are the proton and the neutron. The lightest mesons are the pions $\pi^\pm$, π^0. Baryons are fermions mesons are bosons. There are many different species of baryons and mesons, but all except the pions are non relativistic below the temperature of the QCD phase transition. Thus, the only particle species left in large number are the puions, electrons, muons, neutrinos and the photons. The three pions (spin 0) corresponds to $g = 3$ internal degrees of freedom. We therefore get $g_\rho = 2 + 3 + \frac{7}{8} \times (4 + 4 + 6) = 17.25$. Soon after the QCD phase transition, the pions and muons annihilate and $g_\rho = 17.25 - 3 - \frac{7}{8} \times 4 = 10.75$. Next electrons and positrons annihilate (see section 5.6).

A little history of $g_\rho(T)$: summary

As a summary, we can give in the table 2.2.7 the different values of $g_\rho(T)$ after the annihilation is over assuming the next annihilation would not have begun yet. In reality they overlap in many cases. The temperature on the left is the approximate mass of the particle in question and indicates roughly when the annihilation begins. The temperature is much smaller when the annihilation ends. Therefore top annihilation is placed after the electroweak transition. The top quark receives its mass in the electroweak transition, so annihilation only begins after the transition.

The entropy

To compute the entropy of the Universe, one needs to remember the second law of thermodynamics. The first law was used to compute the relation between pressure and volume in the section (B.4.1) in the case of adiabatic transformations. The second principle of thermodynamics establishes the link between entropy, pressure, temperature and internal energy. It can be written

$$TdS = dU + PdV = \delta Q \tag{2.51}$$

where dS is the variation of entropy at a temperature T under a pressure P, corresponding to an exchange of heat δQ. If one define ρ as the energy density ($U = \rho V$) and s the density of entropy ($S = sV$) , one obtains

[9]The assumption of a weakly interacting gas of particles still holds for the baryons and the mesons, but not for the individual quarks and gluons.

Table 2.2: History of $g_\rho(T)$.

T	particles	$g_\rho(T)$
200 GeV	all present	106.75
100 GeV	Electroweak transition	(no effect)
$<$ 170 GeV	top annihilation	96.25
$<$ 80 GeV	$W^\pm, Z^0, H^0$	86.25
$<$ 4 GeV	bottom	75.75
$<$ 1 GeV	charm, τ^-	61.75
$<$ 150 MeV	QCD transition	17.25
$<$ 100 MeV	$\pi^\pm, \pi^0, \mu^-$	10.75
$<$ 500 keV	e^- annihilation	(7.25) $2 + 5.25(4/11)^{4/3} = 3.36$

$$d\rho - Tds = (Ts - \rho - P)\frac{dV}{V}. \tag{2.52}$$

In equilibrium, the entropy density, energy density and pressure are intensive quantities that can be written as functions only of the temperature $\rho(T)$, $s(T)$, $P(T)$, so that $d\rho - Tds \propto dT$. The coefficients in front of dT and dV are then independent and must vanish separately because one is intensive (independant of volume) and the other one is extensive (depends on the size of the system). This relates the entropy density to the energy density and pressure

$$s = \frac{\rho + P}{T} \tag{2.53}$$

using (2.20) and (2.36) one has

$$s = \frac{2\pi^2}{45} g_s T^3 \tag{2.54}$$

$$g_s = \sum_{b=bosons} g_b \left(\frac{T_b}{T}\right)^3 + \frac{7}{8} \sum_{f=fermions} \left(\frac{T_f}{T}\right)^3 \tag{2.55}$$

Except in the case where the decoupling of some particles transfers its entropy to the photon and not to other relativistic particles like neutrinos which have already decoupled from the bath (case of the decoupling of the electrons/positrons), all particles have the same temperature and we can reasonably approximate $g_s \simeq g_\rho$.
One can also notice that $s_\gamma \simeq 3.6\ n_\gamma$, and constant entropy impose $s \propto 1/R$. Then, working with comoving frame densities ($Y_i = n_i/s = n_i \times R^3$) is equivalent to working with particle/photon ratio density $Y_i \simeq n_i/n_\gamma$. It is also important to notice that, like $n(T)$, $s(T)$ evolve as T^3. However, the degrees of freedom (especially the statistical factors is 7/8 and not 3/4 as in $n(T)$) are the ones of a statistic in T^4, as the entropy is proportional to $\rho(T)/T$ and not $n(T)$. Conservation of entropy is then directly linked to the conservation of energy: $\rho(T_i)/T_i = \rho(T_f)/T_f$.

As in the case of the density energy degrees of freedom, one can compute the entropy effective degree of freedom today, different from g_ρ as the decoupling of the neutrino has created a difference between the neutrino and photon temperature (see section 5.6). On obtain[10]

$$g_s^{\text{today}} = 2 + \frac{7}{8} \times 2 \times 3 \times \frac{4}{11} = 3.91 \tag{2.56}$$

from that number one can deduce the present entropy s_0, which will be very useful to compute relic abundance of stable species

$$s_0 = \frac{2\pi^2}{45} g_s^{\text{today}} T_0^3 = 34.71 \text{ K}^3 = 2.1 \times 10^{-38} \text{ GeV}^3 = 2787 \text{ cm}^{-3} \tag{2.57}$$

The constance of the entropy $S = sR^3$ of the Universe have several consequences. First, from Eq.(2.54) the temperature *of the photons* follows a law $T \propto R^{-1}$. The other consequence is that, every time a particle decouple from the thermal bath, this decoupling happens at constant entropy. It means that this particle "gives" its entropy before leaving to the relativistic particles still present in the bath. This information is in fact encoded in the degrees of freedom g^s_{eff}: after the decoupling of a specie i the effective degrees of freedom in the bath decrease : $g^s_{eff} \to g_s - g_i$. the entropy being constant (and so its density as the universe do not have time to evoluate during this process) $g_s T^3$=cst implies that the decoupling of a specie increase the temperature of the bath (ie of the photons) following $T_\gamma^{\text{after}} = T_\gamma^{\text{before}} \times (g_s^{\text{before}}/g_s^{\text{after}})^{1/3}$. After this heating of the bath, the temperature of the plasma follows the R^{-1} law.

On the other hand, the particle which has decoupled from the plasma follows different laws if it is a massless or massive particle. Indeed, after decoupling the energy of each massless particle is redshifted by the expansion of the Universe $E(t) = E(t_{dec})R(t_{dec})/R(t) = E_{dec}R_{dec}/R = p(t)$ in the relativistic case. As n(t) decrease proportionally to R^{-3} because the density is "frozen" (no any way of producing it thermally nor destroying it after decoupling time) the distribution function $f(\vec{p}) = d^3n/d^3p$ is constant during the expansion. This imply $e^{E/T} = e^{E_{dec}/T_{dec}} = e^{ER/R_{dec}T_{dec}} \Rightarrow T = T_{dec}R_{dec}/R \propto R^{-1}$. The relativistic species thus follows the R^{-1} evolution after they decoupled from the plasma but are not affected by the $(g_s)^{1/3}$ enhancement that affects the photons and relativistic species still living in the bath.

The massive non-relativistic species ($m \gg T_{dec}$) follows another law after being decoupled. Their kinetic momentum follows the classical redshift $p = p_{dec}R_{dec}/R$ from which it follows that the kinetic energy of each particle red shifts as R^{-2} : $E_c = E_c^{dec}R_{dec}^2/R^2$. Conservation of the density function thus imply $e^{p_{dec}^2/2mT_{dec}} = e^{p^2/2mT} \Rightarrow T = T_{dec}R_{dec}^2/R^2 \propto R^{-2}$. As a conclusion, after the decoupling of a specie "i"

$$T_i \propto R^{-1} \quad \text{relativistic} \qquad T_i \propto R^{-2} \quad \text{nonrelativistic} \tag{2.58}$$

We have summarized this effect in the Fig.2.8

[10]The fact of taking into account three species of neutrino, even if it is probable from the measurement of Δm_{atm} and Δm_{sol} that some are non-relativistic nowadays comes from the fact that, once decoupled, the massive neutrinos still follows a classical relativistic distribution function (see section 2.2.8 for more details)

Figure 2.8: Evolution of the temperature of different species (relativistic/massless and nonrelativistic/massive) after their decoupling from the thermal bath. It can be for instance the dark matter decoupling followed by the neutrino decoupling.

The Hubble constant

The evolution of the Hubble parameter as function of the temperature is easily derived through the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho(T) \tag{2.59}$$

At the temperature we consider, the Universe is totaly dominated by the relativistic particles (radiation dominated) and $\rho(T) = g_\rho \frac{\pi^2}{30} T^4$ (Eq.2.44). replacing the expression in Eq.(2.59), we obtains

$$H(T) = \sqrt{g_\rho}\sqrt{\frac{4\pi^3}{45}}\sqrt{G}T^2 = \sqrt{g_\rho}\sqrt{\frac{4\pi^3}{45}}\frac{T^2}{M_P} \simeq 1.66 g_\rho^{1/2}\frac{T^2}{M_P} \tag{2.60}$$

What is decoupling?

Suppose a particle A interacting in the bath with a rate per particle $\Gamma = n\langle\sigma v\rangle$, n being the density of the target particle, and $\langle\sigma v\rangle$ the average cross section times the relative velocity. $\Delta t = 1/\Gamma$ represents the mean time between two collisions. During this time Δt the universe has expanded by a factor ΔR such as $\Delta R/R = H\Delta t = H/\Gamma$. In another word, when $H \simeq \Gamma$, the size of the Universe has doubled and the density n of the target has been divided by 8, as the interaction rate $\Gamma \propto n$. In another word, the time (temperature) of the Universe when $H \simeq \Gamma$ is the epoch where the particles decouple from the bath and their interaction rates with the plasma decrease exponentially. We illustrate it in Fig.2.9. The exact way to treat

Figure 2.9: Illustrative example of the decoupling epoch when the number of interaction is divided by 2 during a time Δt due to the dilution of the target. The volume necessary to have 2 collision (R_{before}) is now just sufficient to give one collision (R_{after})

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the decoupling problem is to solve the Boltzmann equation. However, the approximation $H \simeq \Gamma$ to obtain the decoupling time of particles is usually quite accurate. We will give two specific example to understand how the nature of the interaction can change drastically the temperature of decoupling of species. We will consider first i) interactions mediated by a massless gauge boson (like the photon) and ii) interactions mediated by a massive gauge boson (Z or Z').

The exchange of a massless gauge field between two particles S and $\tilde{S}$ can be parameterized in a case of bosonic particles[11] by a lagrangian of the form $\mathcal{L} = (D_\mu S)(D^\mu S)^\dagger + (D_\nu \tilde{S})(D^\nu \tilde{S})^\dagger$ which includes the terms of interactions $ig\, p^\mu A_\mu S S^\dagger + ig\, \tilde{p}^\nu A_\nu \tilde{S}\tilde{S}^\dagger$, A_μ being the massless vectorial field (photon for instance) and g its coupling to the particles in the bath. On can then compute the amplitude of the interaction (see appendix C) $\mathcal{M} = g^2 p_\mu \eta^{\mu\nu} \tilde{p}_\nu / p^2 = g^2 p.\tilde{p}/p^2 \simeq E\tilde{E}(1-\cos\theta)/E^2$, $\cos\theta$ being the diffused angle between S and $\tilde{S}$ which implies $|\mathcal{M}|^2 \simeq g^4 E\tilde{E}(1-\cos\theta)/E^2$. Using Eq.C.77 ($\sigma \simeq |\mathcal{M}|^2/64\pi^2 s$), considering that the particles are relativistic in the plasma and using Eq.(2.39) ($m_{S,\tilde{S}} \ll T \Rightarrow E \simeq \tilde{E} \simeq T$) one can deduce $\sigma \simeq g^4/T^2$ implying when combined with Eq.(2.33) $\Gamma = n_S \langle \sigma v \rangle = n_S \langle \sigma c \rangle \simeq g^4 T$. The particle S will thus be decoupled from the primordial plasma when

$$\frac{\Gamma}{H} \lesssim 1 \Rightarrow \frac{g^4 M_P}{T} \lesssim 1 \Rightarrow T \gtrsim g^4 M_p \simeq 10^{15}\text{GeV} \qquad [\text{non} - \text{massive gauge boson exchange}] \tag{2.61}$$

It is important to notice that T in this case is an upper bound, which means that as soon a $T \lesssim 10^{15}$ GeV the eletromagnetics interactions will always be sufficient to maintain charged relativistic particles in equilibrium in the bath: the decoupling will appears only when the temperature will reach m_S, where the density will be exponentially suppressed by the Boltzmann factor (the term n_S in the expression of Γ).

In the case of the exchange of a massive gauge boson Z' ($M_{Z'} \gtrsim T$) the amplitude of the reaction can be written $\mathcal{M} \simeq g^2 E^2/M_{Z'}^2 \simeq g^2 T^2/M_{Z'}^2 \Rightarrow \Gamma \simeq g^4 T^5/M_{Z'}^4$. The decoupling temperature will then be given by the usual condition $\Gamma/H \lesssim 1$:

[11]The analysis does not depend on the nature -fermionic/bosonic- of the particles we consider..

$$\frac{\Gamma}{H} \lesssim 1 \Rightarrow \frac{g^4 M_P T^3}{M_{Z'}^4} \lesssim 1 \Rightarrow T \lesssim \left(\frac{M_{Z'}}{g}\right)^{4/3} M_p^{-1/3} \simeq 0.1 \left(\frac{M_{Z'}}{1\ \text{TeV}}\right)^{4/3} \text{MeV} \tag{2.62}$$

In this case we clearly see that the decoupling of particles charged only under $SU(2)$ (Z boson exchange) will decouple quite late in the history of the universe (around 1 MeV) which is precisely the case of the neutrino (that we will study more in detail in another section).
We represent in Fig.2.10 the evolution of $H(T)$ and $\Gamma(T)$ in different cases (taking $g = 0.1$)

$$\log H \simeq -2\log 1/T - 19;\ \ \log \Gamma_\gamma \simeq -\log 1/T - 4;\ \ \log \Gamma_{M_{Z'}} = -5\log 1/T - 4\log M_{Z'} - 4 \tag{2.63}$$

2.2.8 Chemical decoupling or kinetic/thermal decoupling?

The main idea

In fact, to be precise, all the discussion above concerned mainly the "chemical" decoupling, i.e. the temperature for which the production rate of dark matter is too small to maintain it in equilibrium with the thermal bath. In other words the thermal bath cannot change anymore the *number* of dark species, so the name "chemical" decoupling in analogy with chemical reactions. It is very important to realize that even though dark matter is now a chemically distinct particle species, it is still a constituent of the local heat bath. Indeed, the relevant target density for elastic scattering processes is provided by the number density of relativistic SM particles and thus only decreasing at T^3, and not exponentially as the "self" target, the dark matter itself. Eventually, at a temperature T_k, the elastic scattering (and not annihilating) rate[12] $\Gamma_{scat} = n_{SM}^{eq}\langle\sigma_{scat}\rangle$ cannot compete with that of the expansion of the Universe, and the dark matter particles start to decouple from kinetic equilibrium. Elastic scattering processes cease. In fact, to be precise, even before the last scattering occurs, the temperature of the dark matter has no time to be relaxed because of the Hubble expansion. It is thus the relaxation time which will determine the kinetic decoupling of the dark matter, i.e. the time when the temperature of the dark matter is not maintained anymore to the plasma one through scattering and drops quickly with the scale factor a as a^{-2} instead of a^{-1}, the temperatures of the dark matter particles and the radiation background are approximately related by $T_\chi = \langle|\mathbf{p}|^2/2m_\chi\rangle \sim T^2/T_k$ [15]

Approximate solution, to have the feeling

Let us estimate the relaxation time for a WIMP of mass m_χ living in a thermal bath of temperature T. Let suppose that after each shock, the SM particles transfer an impulsion $\Delta\mathbf{p}$ (tri–vector), roughly given by its energy $|\Delta\mathbf{p}| \sim T$ to the dark matter corresponding to a velocity $\Delta\mathbf{v}$. After the number of collision N_{coll}, the velocity of the dark matter is $\mathbf{v}_{N_{coll}} = \mathbf{v}_{N_{coll}-1} + \Delta\mathbf{v}$, implying $|\mathbf{v}_{N_{coll}}|^2 = |\mathbf{v}_{N_{coll}-1}|^2 + 2|\mathbf{v}_{N_{coll}-1}||\Delta v|\cos\theta + |\delta v|^2$, θ being

[12] Notice that $\sigma_{scat} v_{SM} = \sigma_{scat}$ as the SM particles are still largely relativistic in the thermal bath : $v_{SM} = c$.

Figure 2.10: Evolution of $H(T)$, $\Gamma_\gamma(T)$ and $\Gamma_{M_{Z'}}(T)$ for different masses of Z' and relativistic species. The black dots show the temperature when the decoupling occurs ($\Gamma/H \simeq 1$).

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the angle between the two colliding (dark matter and SM) particles. We can thus deduce $\langle v_{N_{coll}}^2 \rangle = \langle v_0^2 \rangle + N_{coll}(\Delta v)^2$. In other words, after N_{coll} collisions, the velocity (and thus impulsion) gained by the dark matter (if we suppose it at rest for instance at the beginning, $v_0 = 0$) is $\Delta p_{N_{coll}} = N_{coll}^{1/2} \Delta p$ which should be equal to $(2m_\chi E_c)^{1/2} \sim (m_\chi T)^{1/2}$ for a dark matter particle at the temperature T. Replacing $\Delta p \sim T$ we then can compute the number of collision needed to keep the dark matter in kinetic equilibrium :

$$N_{coll} \sim \frac{m_\chi}{T} \gg 1. \tag{2.64}$$

The relaxation time (time needed to still keep the kinetic equilibrium, i.e. to obtain N_{coll} collisions) with a scattering Γ_{scat} is

$$\tau_r \sim \frac{N_{coll}}{\Gamma_{scat}} = \frac{m_\chi}{T} \frac{1}{n_{SM}^{eq} \sigma_{scat}} \sim \frac{m_\chi}{T^4 \sigma_{scat}} \sim \frac{10^9 \ m_\chi}{T^4 \left(\frac{\sigma_{scat}}{10^{-9}}\right)} \tag{2.65}$$

Then, kinetic decoupling occurs approximately at the temperature T_k, for which $\tau_r(T_k) = H^{-1}(T_k)$ which gives with $H = 1.66 g_\rho^{1/2} \frac{T^2}{M_P}$

$$T_k \sim 10^{-4} g_\rho^{1/4} \left(\frac{\sigma_{scat}}{10^{-9} \text{ GeV}^{-2}}\right)^{1/2} \left(\frac{m_\chi}{100 \text{ GeV}}\right)^{1/2} \text{ GeV} \tag{2.66}$$

corresponding roughly to the MeV scale for a 100 GeV dark matter and an electroweak–like scattering cross-section ($\sigma_{scat} \simeq \sigma_{EW} \sim 10^{-9}$ GeV^{-2}). When the temperature of the plasma drops below T_k, the number of collisions needed to still keep the dark matter in kinetic equilibrium is such that the time to reach it becomes larger that the Hubble expansion time: the dark matter reaches the kinetic decoupling.
This effect can play an important role when the velocity appears in processes like Sommerfeld enhancement for instance [16]. A more detailed analysis and analytical solutions of the kinetic decoupling temperature can be found in [15]. The mechanism to obtain kinetic equilibrium in the primaveal Universe is described more in detail in section (2.12) whereas the thermalization of the dark bath is explained in section (2.6.6).

What is happening after the decoupling?

Once Γ (scattering or decay) falls below the expansion rate H, the particles decouple from the plasma and propagate freely along geodesics of the spacetime. **The form of the distribution function $f(p)$ is conserved**, while the momentum redshifts as $p(t) \propto 1/a$, implying $p(t) = p(t_{\rm dec})a(t_{\rm dec})/a(t)$. The form of the distribution function is indeed conserved as soon as the particle is not in thermal bath with another dark sector (which can also have decoupled from the primordial plasma before) because of the conservation of energy/momentum of the decoupled system: there is no source (thermal bath of collisions) that can modify the distribution in energy of the decoupled gas. It follows that the distribution function for any $t > t_{\rm dec}$ is given by

$$f(p, t > t_{\rm dec}) = f(\frac{a(t)}{a(t_{\rm dec})} p, t_{\rm dec}) \quad . \tag{2.67}$$

We see that the distribution function of the decoupled particles is simply a rescaled version of the distribution function at decoupling.

- *Decoupling while relativistic*: if a particle of mass m decouples when relativistic, $T_{\rm dec} \gg m$, then the distribution function (2.67) takes the form

$$f(p, t > t_{\rm dec}) = \frac{1}{e^{E/\tilde{T}(t)} \pm 1}, \quad \text{with} \quad \tilde{T}(t) = T_{\rm dec} \frac{a(t_{\rm dec})}{a(t)} \ . \tag{2.68}$$

  So the temperature of a decoupled relativistic species falls strictly as a^{-1}. If the particle later becomes non-relativistic, we have $E \sim m$ but the distribution function keeps the form (2.68). This is an important point, especially when computing the relativistic degrees of freedom for the present entropy for instance. What is important is that the particle was relativistic during the decoupling time. Once it is decoupled, it is as if the thermal bath gave them a punch, and they continue their way in space without any interaction at the same velocity, just redshifted by the scale factor. The distribution is a relativistic one, respecting $n(\tilde{T}) \propto \tilde{T}^3$ (2.37) even in $m > T$ (no exponential suppression). In other words, all particles that were relativistic during their decoupling time count as relativistic degrees of freedom in the Universe nowadays.

  Notice that the conservation of the entropy, in the radiation dominated era, impose $S = sR^3 = sa^3R_0^3$=cte, using (2.54), one deduce $T \propto a^{-1}$. This means that even after decoupling, a gas of relativistic particle keep a "virtual" temperature $\tilde{T}$ following the one of the thermal bath.

- *Decoupling while non-relativistic*: if, on the other hand, a particle is non-relativisitic : if, on the other hand, a particle is non–relativistic at the time of decoupling, $T_{\rm dec}$, then $E \simeq m + p^2/2m$ and the distribution function is given by

$$f(p, t > t_{\rm dec}) = e^{-m/T_{\rm dec}} e^{-p^2/2m\tilde{T}}, \quad \tilde{T}(t) = T_{\rm dec} \left(\frac{a(t_{\rm dec})}{a(t)} \right)^2 \ . \tag{2.69}$$

  So the temperature of decoupled non–relativistic species falls as a^{-2}, which means much faster than the relativistic ones. Their number density thus decreases *not exponentially* but as T^2 in a radiation dominated Universe.

It is important to emphasize that, despite the different scaling behavior of the temperature for relativistic and non-relativistic species after decoupling, in both cases the equilibrium distribution is maintained. The particles are *not* anymore in thermal equilibrium but the *shape* of the distribution is maintained around a *virtual* temperature $\tilde{T}$.
We illustrate the behavior of the two cases (relativistic and non relativistic) in the Figure (2.11) for $T = 100$ GeV and two masses of particles :10 and 200 GeV for the relativistic and non–relativistic case respectively.

2.2.9 The neutrino decoupling

Earlier in time, the neutrino decoupled from the plasma. Indeed, the neutrino is the only particle of the Standard Model which interacts only weakly and not electromagnetically nor strongly. Because of that, the interaction rate $\Gamma_\nu = n_\nu \sigma v$ from the scattering reaction

Figure 2.11: Distribution function $h(p) = \frac{p^2}{2\pi^2} \times f(p)$ for $T = 100$ GeV for different value of the scale factor $a(t)$ in the relativistic fermion (left) and non–relativistic fermion (right).

$e\nu \to e\nu$ or annihilation $\nu\nu \to ee$ becomes smaller than the Hubble expansion rate $H(T)$ much earlier than for the other particles of the Standard model in the Plasma. We can easily compute this rate combining Eq.(C.94) and Eq.(B.28) :

$$\Gamma_\nu \simeq n_\nu \frac{G_F^2}{8\pi} 9.93 \times T^2, \quad \text{with} \quad G_F = 1.1664 \times 10^{-5}\text{GeV}^{-2} \quad \text{and} \quad n_\nu = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_\nu T^3 \tag{2.70}$$

At such earlier time, the Universe is radiation dominated, which means $H(T) = 1.66 g_*^{1/2} T^2/M_P$, see Eq.(2.59). The condition $\Gamma_\nu(T_{\nu dec}) \lesssim H(T_{\nu dec})$ gives

$$\frac{3}{4}\frac{\zeta(3)}{\pi^2} 2 \frac{9.93}{8\pi} \left(1.16 \times 10^{-5}\right)^2 T_{\nu dec}^5 \lesssim 1.66 (10.75)^{1/2} \frac{T_{\nu dec}^2}{M_P l} \Rightarrow T_{\nu dec} \lesssim 3.58 \text{ MeV} \tag{2.71}$$

From 3.6 MeV, the neutrino density is frozen and will not increase even when $T \lesssim m_e =$ 511 keV because the process $ee \to \nu\nu$ is too weak compare to the interaction with the only light particles left in the plasma at such low temperature : the entropy in $e^\pm$ pair is transfered to the photon through the stronger electromagnetic interaction (Thomson scattering), but not to the neutrino through a "too much" weakly interaction.

When the temperature is below 3 MeV, the neutrinos are decoupled, which means they are "invisible" in the thermal plasma where only electrons, positrons and photons survive. However, even if decoupled, their temperature followed the classical law $T_\nu^3 R^3 =$ cte as which is the translation to the fact that the effective number of neutrino is fixed because they escaped from the plasma. The photons follows the same law, which means than even after the neutrino-decoupling, $T_\nu = T_\gamma$: there is no "transfer" of the neutrino degrees of freedom to the temperature of the plasma from entropy conservation, as the neutrino still participate at the universe entropy. They decoupled from the photon but still keep the same temperature and evolution. When the temperature drops below 511 keV, the electrons and positrons decouple from the plasma. they gives all their kinetic energy to the photons and increase

Figure 2.12: A brief history of the neutrino

their temperature by this way. This transfer of energy is made with constant entropy, which means $g_*^{e+e-\gamma}(T_\gamma^{e+e-\gamma})^3R^3 = g_*^\gamma(T_\gamma^\gamma)^3R^3$ with $g_*^{e+e-\gamma} = 2+4*(7/8) = 11/2$ and $g_*^\gamma = 2$ and T_γ^γ the photons temperature after the electrons decoupling and $T_\gamma^{e+e-\gamma}$ the temperature before the electrons decoupling[13] . We then obtain $T_\gamma^\gamma = (11/4)^{1/3}T_\gamma^{e+e-\gamma} = (11/4)^{1/3}T_\nu$. As the present photon temperature in the CMB is 2.725 K on deduce that the nowadays neutrino temperature $T_\nu = (11/4)^{-1/3} \times 2.725 = 1.95$ K. The processes are summarized in Fig.2.12

As a final notes we want to stress that there is finally 2 ways of obtaining a freeze out condition : $n_i(T) \times \langle\sigma v\rangle < H(T)$. Or the density decrease due to the exponential suppression of the Boltzmann factor $e^{-E_i/T}$ (*kinetic* freeze out) when $T \lesssim m_i$ (case of the quarks, gauge bosons or dark matter candidate), or the particle is light and still relativistic, but the expansion rate $H(T)$ begin greater than the annihilation cross section $\langle\sigma v\rangle$ because of weak couplings (like in the case of the neutrino, *chemical* freeze out). In the case of the recombination temperature (decoupling of the photon) we will see that it occured not at $T = 13.6$ eV which is the binding energy of the atom of hydrogen but a little bit later, at at 0.3 eV (see Fig.2.13) because what is important is not only that the photons have a temperature below the binding energy B_H but than $n_\gamma\langle\sigma v\rangle$ is below the rate to destroy and forbid the hydrogen atom formation. In other words, even at $T = T_\gamma = 1$ eV, there are still some photons present with $T = 13.6$ eV by quantum fluctuations able to destroy the structure of the atom.

Another interesting point is the measurement of the relic abundance of neutrinos nowadays. From $T_\nu = 1.95$ K one can compute from Eq.(2.37) the neutrino density to be $n_\nu = 109$ cm^3 implying[14] $\rho_\nu^0 = m_\nu\nu_\nu^0$

$$\Omega_\nu = \frac{\rho_\nu^0}{\rho_c^0} = \frac{m_\nu\ 109\ \text{cm}^{-3}}{10^{-5}h^2\ \text{GeV cm}^{-3}} \approx \frac{m_\nu}{92\ h^2\ \text{eV}} \quad \Rightarrow \quad \Omega_\nu h^2 \approx \frac{m_\nu}{92\ \text{eV}} \tag{2.72}$$

Asking for the neutrino abundance to be less than the CMB bound on $\Omega_M \sim 0.3$ and taking $h \simeq 0.7$ we can derive the Mac–Lelland limit on the mass of a stable neutrino:

[13] 2 degrees of freedom for the photon, 2 fermionic states for the electron, plus 2 for the positrons

[14] Notice that the mass density of a massive neutrino does not follows the boltzmann suppressed evolution of classical massive particles as neutrino have already decoupled from the thermal bath, and there density thus always follows the T^3 evolution by number of particle conservation ($n_\nu(t) \times a(t) \propto n(t) \times T^{-3} = cte$), $a(t)$ being the scale factor of the Universe.

$$\sum_{\nu} m_\nu \leq 14 \text{ eV}. \tag{2.73}$$

This cosmological bound to the mass of a stable, light neutrino species is often referred to as the Cowsik-McClelland bound.

2.2.10 Historical perspective on the neutrino bound

The limit computed in the previous section is usually called "Cowsik Mac-Clelland" bound. However, these authors were not the first to propose the computation.

2.2.11 The recombination: decoupling of the photons

This period of the Universe happens quite late ($\sim$ 380 000 years after the Big Bang) and is at the origin of the cosmic microwave background (CMB). Indeed, at this time, 2 phenomenon took place : the temperature of the plasma in equilibrium decreased such that the photon living in the plasma did not have anymore sufficient kinetic energy to destroy the forming atom of hydrogen. The would-be-free electrons begin to form bound state (binding energy of the hydrogen is 13.6 eV) with the proton from the plasma to compose an electromagnetic neutral atom of hydrogen (recombination era). As a consequence of this process, the population of free electrons drastically decreases opening the way to the photons which do not scatter anymore on them and begin their veryyy long journey toward us (to create the CMB, decoupling era). In this section, we will rederivate "a la Gamow" the temperature of these two processes and show that they happened in a very short period of time. As soon as the first atoms of hydrogen were formed, the photons were liberate from the plasma.

The recombination

To simplify the picture, we will suppose that the plasma is made of electron (e), proton (p) photon (γ) and hydrogen (H) in thermal equilibrium:

$$p + e \leftrightarrow H\gamma \tag{2.74}$$

The universe being electrically neutral, the electron and proton density are equal: $n_e = n_p$. The density of each species can thus be written

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{\frac{\mu_i - m_i}{T}} \tag{2.75}$$

and the condition of chemical equilibrium imposes $\mu_e + \mu_p = \mu_H$ implying

$$n_H = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\frac{B_H}{T}} \tag{2.76}$$

B_H being the binding energy of the hydrogen, $B_H = m_e + m_p - m_H$. For $T \gg B_H$, one have $n_H \propto n_e \times n_p$ which can be understand as the probability of forming an hydrogen is proportional to the probability of presence of an electron multiplied by the probability of

Figure 2.13: Ionization rate X_e as function of the temperature (Eq.2.77). The Hydrogen begins to be formed for $T \simeq 0.3$ eV. The recombination time is defined when only 10% of the electrons are still free.

presence of a proton. Once the temperature approaches the hydrogen binding energy $T \sim B_H$, the exponential factor dominates the hydrogen density : the photons of the backgrounds are not able to split the atom of hydrogen, and its formation is exponential. hydrogen (binding energy)

If one defines the ionization fraction $X_e = \frac{n_p}{n_B} = \frac{n_p}{n_p+n_H} = \frac{n_e}{n_B}$, and knowing that $n_B/n_\gamma = \eta\Omega_B h^2$ with $\eta = 2.68 \times 10^{-8}$ and $n_\gamma = 2\frac{\zeta(3)}{\pi^2}T^3$ (Eq.2.13 and B.27), on can simplify Eq.(2.76) to[15]

$$1 - X_e = X_e^2 \frac{2.68 \times 10^{-8} 4\sqrt{2}\zeta(3)}{\sqrt{\pi}} \left(\frac{T}{m_e}\right)^{3/2} e^{\frac{B_H}{T}} \ \Omega_B h^2 \tag{2.77}$$

which is a second order equation that we can solve straightforwardly. The result is shown in Fig.(2.13) with $\Omega_B = 0.047$ (Eq.2.13) where we plotted the ionization fraction as function of the temperature. The exponential nature of the formation of the atom of Hydrogen is clearly visible for $T \simeq 0.3$ eV. If one defines the recombination temperature as the temperature where only 10% of the electrons are still free. Solving $X_e(T_{rec}) = 0.1$ we obtained $T_{rec} = 0.297$ eV.

Decoupling of the photons: the last scattering surface

After the formation of the atoms of hydrogen, electromagnetically neutral, the free path of the photons increases dramatically as there are very few electrons left in the plasma to interact. We compute in this section the temperature for which the free path of the photons becomes larger than the horizon of the Universe (Hubble distance), the decoupling temperature. Photons interact with electron through the Thomson scattering process (Fig. 2.14). The interaction rate can be written (see the appendix G.2 for more details)

[15] with $g_e = g_p = 2$ (spin 1/2) and $g_H = 4$ (combination of two spins 1/2)

Figure 2.14: Thomson scattering of a photon off an electron

$$\Gamma_\gamma = \sigma_T c n_e \quad [\mathrm{s}^{-1}] \tag{2.78}$$
$$\text{with} \quad \sigma_T = \frac{8\pi}{3}\left(\frac{q^2}{4\pi\epsilon_0 m_e c^2}\right)^2 \simeq 6.65\times10^{-25}\mathrm{cm}^2 = 0.665 \text{ barn} \simeq 2500\ [\mathrm{GeV}^{-2}]$$

Γ_γ^{-1} corresponds to the mean time between two collision, thus Γ_γ^{-1}/c is the distance that a photon will go through before colliding an electron. From Eq.(2.79), the photon will decouple at the temperature for which its mean path equals the Hubble horizon $H(T)$. In a matter dominated universe (which is already the Universe 70 000 years after the Big Bang) we have (Eq.2.14)

$$H(T) = H_0\sqrt{\Omega_M^0}\left(\frac{T}{2.37\times10^{-4}}\right)^{3/2} = 2.13\times10^{-42}h\sqrt{\Omega_M^0}\left(\frac{T}{2.37\times10^{-4}}\right)^{3/2}\text{GeV} \tag{2.79}$$

Asking for $c\Gamma_\gamma^{-1} = H^{-1}T$ one obtains $H(T) = n_e\sigma_T$ which can be written

$$2500 n_e(T_{dec}) = 2.13\times10^{-42}\sqrt{\Omega_M^0 h^2}\left(\frac{T_{dec}}{2.37\times10^{-4}}\right)^{3/2} \tag{2.80}$$

Combining $n_e(T) = X_e(T)n_B$, Eqs.(B.29) and (2.77) one can solve numerically the condition Eq.(2.80). We obtained $T_{dec} = 0.234$ eV : just after de recombination, the photon become free. This results from the exponential formation of the atoms of Hydrogen: very quickly n_e decreases and no electron are available anymore for any Thomson scattering[16].

The dark ages, or re-ionization

There is also another epoch called "dark ages" or "re-ionization time" which runs from the decoupling/recombination time (380 000 years, see section 2.2.11) till 100 millions or 1 billions years. At the recombination time, the rate of recombination of electrons and protons to form neutral hydrogen was higher the the reionization rate. Universe was opaque due to

[16]The Thomson scaterring is the low energy limit of the Compton scattering. Both effects is a diffusion a the electromagnetic field by a charged particle. In the CMB, the Thompson limit is a valid one.

the large rate of scattering of photons on electrons, and then became transparent once the first atom of hydrogen formed. The dark ages of the Universe start at that point, because there were no light sources other than the gradually redshifting cosmic background radiation. The second phase change occurred once objects started to condense in the early Universe that were energetic enough to re-ionize neutral hydrogen. As these objects formed and radiated energy, the Universe reverted from being neutral, to once again being an ionized plasma. this occurred between 150 million and one billion years after the Big Bang. At that time however, matter had been diffused by the expansion of the Universe, and the scattering interactions of photons and electrons were much less frequent than before electron-proton recombination. Thus, a Universe full of low density ionized hydrogen will remain transparent, as is the case today.

2.3 The Big-Bang Nucleosynthesis

2.3.1 Generalities

The very early Universe was hot and dense, resulting in particle interactions occurring much more frequently than today. For example, we saw that while photon can today traverse the entire Universe without interacting (deflection or capture), resulting in a mea–free path greater than 10^{28} cm (the horizon), the mean–free path of a photon when the Universe was 1 second old was about the size of an atom. This resulted in a large number of interactions which kept the interacting constituents of the Universe in equilibrium. As the Universe expanded, the mean–free path of particles increased (thus decreasing the rates of interactions) to the point where these could no longer maintain equilibrium conditions. Different constituents of the Universe decoupled (fell out of equilibrium with the rest of the Universe) at different times, which determined their abundance.
We have previously seen the history of the decoupling of the dark matter and the photon. In this chapter, we will concentrate on the decoupling of the baryons. This decoupling is a little bit more complicated (in a sense) that the neutrino. The main reason is that this decoupling occurs around the same time (1 MeV) but, if the neutrino is a fundamental "free" particle, unable to form bound state with particle surrounding it, it is not the case of the quarks and gluons (which are the *partons*, main constituents of the baryons) which begins to form bound states : proton and neutron first. Then, proton and neutron begins to form the first nucleus (deuterium, Helium and Lithium). Why 1 MeV? As the $\simeq 3$ MeV scale corresponds to the decoupling of the neutrino which cannot "talk" anymore with the thermal bath due to the weakness of its interaction, 511 keV is the decoupling scale for the electron due to its underproduction for a thermal bath with $T \lesssim 511$ keV (see Fig.2.12), 1 MeV corresponds to the energy at which the photons are not able anymore to forbid nuclear bounded state through its scattering processes. MeV is indeed the scale of nuclear boundary energies. A nice summary of the mechanisms in play in these different processes can be found in [38].
As we just describe in the previous chapters, when the temperature of the Universe reaches 1 MeV, the cosmic plasma consists of:

- **Relativistic particles in equilibrium: photons, electrons and positrons.** These interact among themselves via electromagnetic interaction $e^+e^- \leftrightarrow \gamma\gamma$. The abundances of these constituents are given by FERMI-Dirac and Bose-Einstein statistics.

- **Decoupled relativistic particles the neutrinos.** It decoupled at around 3 MeV due to its weak coupling with the photons and electrons (which couple together quite "strongly" through the electromagnetic interaction). Even if decoupled from the photon bath, they continue to be present in the Universe, and relativistic.

- **Nonrelativistic parcels : the baryons.** The baryon are present thanks to the baryonic asymmetry in the Universe. Indeed, if there was as many baryon (b) as antibaryon ($\bar{b}$), after being frozen out they should annihilate $b + \bar{b} \to \gamma\gamma$ (through a t-channel b exchange for instance) which is clearly not the case as we are made of baryons.

What happened is that there was an initial asymmetry between baryons and anti baryons

$$\frac{n_b - n_{\bar{b}}}{s} \simeq 10^{-10}, \tag{2.81}$$

throughout the early history of the Universe, until the anti baryons were annihilated away at about $T \simeq 1$ MeV. We can express the ratio baryon density to photon density η_b at 1 MeV as function of the present ratio (n_b^0/n_γ^0) which gives

$$\begin{aligned}\eta_b \equiv \frac{n_b}{n_\gamma}(1\text{ MeV}) = \frac{n_b^0}{n_\gamma^0} = \frac{\frac{\rho_b^0}{m}}{n_\gamma} = \frac{\frac{\rho_c \Omega_b}{m}}{n_\gamma} = \Omega_b \left(\frac{1\text{ GeV}}{m}\right) \frac{10^{-5} h^2\text{ cm}^{-3}}{394\text{ cm}^{-3}} \\ = 2.5 \times 10^{-8} \Omega_b h^2 \left(\frac{1\text{ GeV}}{m}\right) = 5 \times 10^{-10} \left(\frac{1\text{ GeV}}{m}\right) \left(\frac{\Omega_b h^2}{0.02}\right),\end{aligned} \tag{2.82}$$

where we used the observed values given in section (F.1.1). By a simple argument one can understand why the ratio n_b/n_γ is constant with respect to the time after the decoupling from the thermal plasma. Just from the Boltzmann equation (see section 2.4 for a detailed study of the equation) $\frac{dn_b}{dt} = -3Hn_b$, with $H = \frac{\dot{R}}{R}$. Imposing a constant entropy, $S = R^3 s \propto R^3 T^3$, $\frac{dS}{dt} = 0 \;\Rightarrow\; \frac{\dot{R}}{R} = -\frac{1}{T}\frac{dT}{dt}$ implying $\frac{dn_b}{dt} = -\frac{3}{T}\frac{dT}{T} \;\Rightarrow\; \frac{n_b}{T^3} \propto \frac{n_b}{n_\gamma}$ =constant, where we have used the formulae (2.37) and (2.54) for n_γ and s respectively.
Therefore, there are orders of magnitude more relativistic particles than baryons at about $T \simeq 1$ MeV (and nowadays of course). The key-point of this observation is that baryons were frozen-out ($t \sim 10^{-4}$ sec), the number of photons and baryons/anti-baryon pairs must have been nearly equal, since they were constantly exchanging (via pair production/annihilation). Indeed, the matter-antimatter pairs must also have been equal, i.e. the Universe was equal parts matter and anti-matter. At freeze-out, there must have been a slight excess of matter over anti-matter (1 part in 10^9), so the present Universe is dominated by matter.
The aim of this chapter is to understand how the baryons combine themselves from 1 MeV town to keV scale to form the first light nucleus. This process bare the name "Big Bang Nucleosynthesis" (or BBN) in contrast with the Solar Nucleosynthesis developed by Hoyle and Fowler some years after (the first idea of BBN genesis was the Gamow work in 1948 [39], sees the historical appendix for details). If the baryons were living in thermal equilibrium thorough the history of the Universe, they will naturally produced lead which is the more stable nuclear element (the higher binding energy). However, it is far to be the case as the number of interactions between baryons decreases as the Universe evolve (as soon as $H(T) \gtrsim \langle \sigma v \rangle_{equilibrium}$) letting baryon living with themselves but out of any equilibrium. At that moment, the binding energy plays a role. Indeed, baryons will tend naturally to form binding state, which lower their ground energetic state as soon as the photons, scattering around, are not able to interfere in the binding process by destroying newly formed bounded state.

2.3.2 The Deuterium formation

The first reaction is the binding of a neutron and a proton into a nucleus of deuterium (D):

$$n + p \to D + \gamma \tag{2.83}$$

The binding energy of deuterium is $B_D = 2.2$ MeV (see appendix F.2), which means that $m_D = m_p + m_n - B_D = 1877.62$ MeV. As soon as the temperature of the Universe falls below 2.2 MeV, the photons from the thermal bath are not sufficiently energetic to destroy a "molecule" of deuterium. So, naively, one would expect that the formation process of the first deuterium appears at around 1-2 MeV. It happens in fact much later due to the very weak concentration of baryons (protons and neutrons) at this temperature, reduced by the measured baryon to photon ratio (around 10^{-10}) from the particle/antiparticle asymmetry (see above). Let's check it quantitatively. At $T \simeq 1$ MeV, we are clearly in the regime $m_i \gg T$ and can apply the formulae (2.38) and compute, not forgetting that deuterium has $g_D = 3$ because of 3 spin states of D and $g_p = g_n = 2$:

$$\begin{aligned}\frac{n_D}{n_n n_p} &= \frac{g_D \left(\frac{m_D T}{2\pi}\right)^{3/2} e^{-m_D/T}}{g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} e^{-m_n/T} g_p \left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-m_p/T}} \\ &= \frac{g_D}{g_n g_p}\left(\frac{T}{2\pi}\right)^{-3/2}\left(\frac{m_D}{m_n m_p}\right)^{3/2} e^{-(m_D - m_n - m_p)/T} = \frac{3}{4}\left(\frac{2\pi m_D}{m_n m_p T}\right)^{3/2} e^{B_D/T}\end{aligned} \tag{2.84}$$

because $B_D = m_n + m_p - m_D$. With reasonable approximations one can write

$$\frac{n_D}{n_n n_p} \simeq \frac{3}{4}\left(\frac{4\pi}{m_p T}\right)^{3/2} e^{B_D/T} \tag{2.85}$$

Noticing that at 1 MeV $n_p \simeq n_n \simeq n_b/2$ (the only baryons at 1 MeV are mainly protons and neutron), we can rewrite Eq.(2.85)

$$\frac{n_D}{n_b} \simeq \eta_b n_\gamma \frac{3}{4}\left(\frac{4\pi}{m_p T}\right)^{3/2} e^{B_D/T} = \frac{12\zeta(3)}{\sqrt{\pi}}\eta_b \left(\frac{T}{m_p}\right)^{3/2} e^{B_D/T} \tag{2.86}$$

where we have used $n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3$ from Eq.(2.37). Having a look to this result, we understand easily that as $\eta_b \ll 1$ and $m_p \ll 1$ MeV, the temperature at which the density of the deuterium is of the order of magnitude of the proton one is below 1 MeV. A simple solution of Eq.(2.86) gives

$$\log\left(\frac{n_D}{n_b}\right) \simeq -23 + \frac{B_D}{T} \quad \Rightarrow \left(\frac{B_D}{T}\right)_{n_D \simeq n_b} \simeq 23. \tag{2.87}$$

We then expect the beginning of the BBN to occurs at around 50 keV which is around the minute scale if we look at the relation (2.98).
Small baryon-to-photon ratio thus inhibits nuclei production until the temperature drops well beneath the binding energy ($T \ll B_D$). This is why at temperatures $T > 0.1$ MeV virtually all baryons are in the form of neutrons and protons. Around this temperature, the production of deuterium and helium starts, but the reaction rates are too low to produce heavier elements. The heavier elements are formed in stars (triple alpha process):

$$^4He +^4 He +^4 He \rightarrow^1 2C, \tag{2.88}$$

but that is only much later. The early Universe is not sufficiently dense for this reaction to take place, *i.e.* for three helium nuclei to find another on relevant timescales. The three alpha process has been found to take place in stars by Burbidge, Burbidge, Fowler and Hoyle in 1957 [40].

2.3.3 The neutron abundance

2.3.4 The Lithium problem

We now discuss the relative abundance of the light elements. The primary activity of BBN took place in the era associated with photon temperatures between $T \simeq$ few MeV and $T \simeq 10$ keV, in the cosmic time windows $t \simeq (0.1 - 10^4)$ s. BBN produce the bulk of ^{4}He and D as well as good fractions of ^{3}He and ^{7}Li observed in the current Universe. It is assumed that the Universe is spatially homogeneous and isotropic and that its energy density is completely dominated by radiation so that the density energy ρ scale like $\rho \propto R^{-4}$, and from Friedman equation, $H = \frac{\dot{R}}{R} \propto \sqrt{\rho}$ implying $\frac{\dot{R}}{R} \propto R^{-2} \Rightarrow RdR \propto dt \Rightarrow R(t) \propto t^{1/2}$, and $H \propto \frac{1}{2t}$ (see appendix (B.1) for completeness).
the standard field content implies that there was an "uneventful" cosmological period between the time of BBN and the epoch of hydrogen recombination with electrons which implies that the baryon-to-entropy density ratio measured at the period ofCMB-formation directly translates into the one at BBN :

$$\frac{n_b}{s}(t_{BBN}) = \frac{n_b}{s}(t_{CMB}) \tag{2.89}$$

Indeed the measurement of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite have allowed us to pinpoint the baryon density for a standard ΛCDM cosmology, exposed in terms of the baryon-to-photon ratio,

$$\eta_b(t_{CMB}) = \frac{n_b}{n_\gamma}(t_{CMB}) = (6.23 \pm 0.17) \times 10^{-10}. \tag{2.90}$$

Standard Big Bang Nucleosynthesis (SBBN) comprises the set of first-order Boltzmann equation on the abundances of the different species

$$\frac{dY_i}{dt} = -H(T)T\frac{dY_i}{dT} = \sum(\Gamma_{ij}Y_j + \Gamma_{ikl}Y_kY_l + ...) \tag{2.91}$$

Figure 2.15: Time and temperature evolution of all standard big bang nucleosyntesis relevant nuclear abundances. The vertical arrow indicates the moment at $T \simeq 8.5 \times 10^{10}$ at which most of the helium nuclei are synthetized. the gray vertical bands indicate main BBN stages. From left to right : neutrino decoupling, electron-positron annihilation and n/p freeze-out, D bottleneck, and freeze-out of all nuclear reactions. Proton (H) and neutrons (N) are given relative to n_b whereas Y_p denotes the ^4He mass fraction.

Assuming thermal distribution and the expression of $H(T)$ given in Eq.(2.59) one can solve the system of equations (2.91).

Very generally, the BBN bounds require that unstable relics previously present in the Universe, decay with lifetime smaller than 100 s, unless the abundance of these particles is very small or only a tiny fraction of these particles decay with energetic hadrons in the final state. Among the MSSM particles, the latter condition is satisfied by the lightest sneutrino or the lightest stau, hence its parameter space allows for $\tau_{\tilde{\tau}_1,\tilde{\nu}} > 100$ s.

REFERENCES

Several papers has studied the Big Bang Nucleosynthesis, as well as from a model independant point of view as for more specific models. A nice and clear introduction of the subject can be find in [38]. Obviously, historically speaking, the idea and first calculation of relic abundance of nuclei was done by Alpher, Herman and Gamow in [39]. The Fig.2.15 is extracted from [38].

2.3.5 Abundance of the light elements

2.3.6 The Lithium7 problem

2.3.7 Time and temperature

From all the relations computed above and from nowadays data, we can extract the evolution of temperature, density number and energy density of the Universe as a function of time[17]. Indeed, from the measurement of the CMB background (2.7 ^{0}K $= 2.3 \times 10^{-4}$ eV) we can deduce the present energy density of the photons with Eq.(2.37), $\rho_\gamma^0 = \frac{\pi^2}{30} \times 2 \times T^4 = 2.4 \times 10^{-10}$ GeV/cm^3. We saw in the previous section that the neutrino temperature is 1.91 ^{0}K (in the absence of any kind of "non-standard" phenomena that could increase the temperature or the effective degrees of freedom of the neutrino, see section 2.7 for some exceptions) which give us for the density of radiation of 3 neutrino, $\rho_\nu = \frac{\pi^2}{30} \times \frac{7}{8} \times 2 \times 3 \times T^4 = 1.5 \times 10^{-10}$ GeV/cm^3. Finally, the present day radiation is thus

$$\rho_R = \rho_\gamma + \rho_\nu = 2.4 \times 10^{-10} + 1.5 \times 10^{-10} = 3.7 \times 10^{-10} \text{ GeV/cm}^3 \tag{2.92}$$

For the matter content, we measured around 10^{11} solar mass per megaparsec which implies a matter density

$$\rho_{\text{matter}} = 10^{11} \text{ solar masses/Mpc}^3 \simeq 4.1 \times 10^{-6} \text{ GeV/cm}^3 \tag{2.93}$$

in relatively good accordance with WMAP data which gives

$$\rho_{\text{baryons+dark matter}} \simeq 1.5 \times 10^{-6} \text{ GeV/cm}^3; \ \rho_{\text{baryons+dark matter+dark energy}} \simeq 5.9 \times 10^{-6} \text{GeV/cm}^3 \tag{2.94}$$

from which we can deduce $\rho_{\text{dark energy}} \simeq 4.4 \times 10^{-6}$ GeV/cm^3.
From Eq.(2.11), $\rho_c^0 = 10^{-5} h^2$ GeV cm^{-3} we can also write

$$\Omega^0_{matter} h^2 = \frac{\rho_{matter}}{\rho_c^0} \simeq 0.15 \qquad \Omega^0_{radiation} h^2 = \frac{\rho_{radiation}}{\rho_c^0} \simeq 3.7 \times 10^{-5}. \tag{2.95}$$

As we can see, the Universe today is clearly largely dominated by matter. We are in the so–called "matter domination" era. Presently Universe density of matter is 4 000 times greater than radiation density[18]. However, it was not always the case. In the early universe, all Standard Model particles were relativistic and contributed to the radiation. The evolution in time of matter and radiation are indeed very different. The density of matter changes because the volume V of the Universe changes, while no new matter is created: $\rho_m \sim 1/V \sim 1/a^3$ where a is the distance scale between galaxies. For the radiation, the best is to think about photons. Its *number* density changes $\sim 1/V$ as well, however, ever each photon wavelength is redshifted, so energy of individual photon also changes as $E_\gamma \sim 1/a$. Therefore the energy

[17]For an equivalent analysis but the scale parameter a as function of time, see section (B.1).

[18]Be careful to not be confused with the ratio $\eta_b = n_b/n_\gamma \simeq 5 \times 10^{-10}$. Indeed, a photon energy is around 10^{-4} eV at present time whereas a baryon mass is 1 GeV. $\rho_b/\rho_\gamma \simeq 1000$ gives you $n_b/n_\gamma \simeq 10^{-10}$. The number density of the photon is much larger than the number density of the baryons nowadays, this is the baryogenesis concept.

Figure 2.16: Evolution of the different components of the Universe density as function of the temperature of the photons computed from Eq.(2.96).

density (expressed as mass density) of radiation evolves as $1/a^4$. We can then deduce from Eq.(2.37) that the scale factor evolves as $a \propto 1/T$ and thus the matter density evolves as $\rho_m \propto T^3$. From the actual value obtained by WMAP one can deduce

$$\rho_m = 1.5 \times 10^{-6} \left[\frac{T}{T_0}\right]^3 \text{GeV/cm}^3; \qquad \rho_R = 3.7 \times 10^{-10} \left[\frac{T}{T_0}\right]^4 \text{GeV/cm}^3 \tag{2.96}$$

with T_0 is the present temperature of the Universe $T_0 = 2.34 \times 10^{-13}$ GeV The result is shown in Figure (2.16). We can see that the matter has begun to dominate at a temperature of around 1 eV, corresponding to the decoupling time, see section (2.2.11).

Hubble parameter and time

We can have a simple expression of the Hubble parameter $H(t)$ for the two cases of Universe matter dominated and radiation dominated. Indeed, neglecting the cuvature and cosmological constant (approximation completely valid in the early Universe) the Friedmann equation can be written

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \tag{2.97}$$

with $\rho = \rho_M = n \times M \propto a^{-3}M$ for a Universe dominated by a massive field of mass M, and $\rho = \rho_R = n \times E_R \propto a^{-3}a^{-1}$ for a radiation dominated Universe (where the energy is redshifted by a factor a^{-1} due to the expansion). In this context, solving the Friedmann equation (2.97) for $a(t)$ gives $H(t) = \frac{2}{3t}$ for a matter dominated Universe and $H(t) = \frac{1}{2t}$ for a radiation dominated Universe, justifying the common approximation $H \simeq \frac{1}{t}$

If one need to obtain the expression of the densities as function of times instead of temperature, one needs first to deduce from Hubble expansion the link between time and temperature. As a first approximation, on can write $H : \dot{a}/a \simeq 1/t$. Combining Eq.(2.59) and Eq.(2.37) we then deduce the time t_R in the radiation dominated era[19] :

$$H = \frac{\dot{a}}{a} \simeq \frac{1}{t_R} = \sqrt{\frac{8\pi}{3M_P^2}}\sqrt{\rho_R} = \sqrt{g_\rho}\sqrt{\frac{4\pi^3}{45}}\frac{T^2}{M_P} \tag{2.98}$$

$$\Rightarrow t_R \simeq 4.9 \times 10^{-6}\frac{1}{\sqrt{g_\rho}}\left(\frac{\text{GeV}}{T}\right)^2 \text{ seconds} \qquad \text{[Radiation dominated era : T > 1 eV].}$$

Later on, when the matter begins to dominate the Universe, the density is dominated by ρ_m given in Eq.(2.96) and the time dependance now becomes t_m:

$$\frac{1}{t_m} = H(T) = \sqrt{\frac{8\pi}{3M_P^2}}\sqrt{\rho_m} \tag{2.99}$$

$$\Rightarrow \quad t_m \simeq 0.09 \times \left(\frac{1 \text{ GeV}}{T}\right)^{3/2} \text{ seconds} \qquad \text{[Matter dominated era : T < 1 eV].}$$

We can then inverse the preceding relations to obtain the evolution of the temperature of the Universe as function of time in the radiation era $[T_R(t)]$ and the matter era $[T_m(t)]$.

$$T_R(t) = \left[\frac{M_P}{t}\sqrt{\frac{45}{4\pi^3 g_{eff}(t)}}\right]^{1/2} \simeq 1.3 \times 10^{-3}\left(\frac{1 \text{ sec}}{t}\right)^{1/2} \text{ GeV}$$

$$T_m(t) \simeq 0.21 \times \left(\frac{1 \text{ sec}}{t}\right)^{2/3} \tag{2.100}$$

We represent the evolution of the temperature as function of the time in the Figure 2.17 where we can notice the kink at t=100 seconds, corresponding to a temperature $T \simeq 0.2$ MeV. This is the BBN epoch when the quarks/colors degrees of freedom are converted to heat the photon of the plasma following the entropy conservation. There is a second kink at around t=10^{13} seconds corresponding to the recombination epoch (last scattering surface) when the universe begins to be matter dominated : the slop of $T = f(t)$ is thus changing from -1/2 to -2/3 Eq.(2.100).

2.4 The Boltzmann equation

2.4.1 State of the arts

The Boltzmann equation formalize the interaction of a particle interacting with the photons (and other relativistic species) in the thermal bath before they decoupled from it. it can be written :

[19] see section B.1 for the exact link between the Hubble parameter and the time. The expression $H \simeq 1/t$ is a good approximation to understand the process.

Figure 2.17: Evolution of the temperature as function of the time computed from Eq. (2.100).

Evolution of particle A per unit of volume
=
particle A created by annihilation of photon like particles B minus self annihilation of A

Putting this in equation, we obtain:

$$\frac{1}{R^3}\frac{d\left(n_A R^3\right)}{dt} = \langle\sigma v\rangle_{B\to A} n_B^2 - \langle\sigma v\rangle_{A\to B} n_A^2 \tag{2.101}$$

$$\Rightarrow \quad \dot{n}_A - 3Hn_A = \langle\sigma v\rangle_{B\to A} n_B^2 - \langle\sigma v\rangle_{A\to B} n_A^2 \tag{2.102}$$

Using $H = \frac{1}{R}\frac{dR}{dt}$ and knowing that by definition, when particle A is at equilibrium, its number per comoving frame is constant[20] and writing the evolution as function of the temperature from the relations $T^3 \times R^3$ =cte (constance of the entropie) $\Rightarrow dR/R = -dT/T$ and $H = \frac{dR/dt}{R}$ one can deduce $\frac{d}{dt} = \frac{dT}{dt}\frac{d}{dT} = -HT\frac{d}{dT}$ and write (forgetting from now on the index A)

$$-\frac{dn}{dT} = -3\frac{n}{T} + \frac{\langle\sigma v\rangle}{HT}\left(n_{eq}^2 - n^2\right) \tag{2.103}$$

We can distinguish 3 regimes before giving an explicit solution of the equation.

A) When the temperature of the plasma is large compare to the mass of the particle A ($T \gg m_A = m$) and the Hubble expansion is negligible, the particle A is relativistic and stays in thermal equilibrium with its other relativistic companions, like the photons,

[20] $d[n_A \times R^3]/dt = 0$ when $n_A = n_A^{eq}$.

and their number density is given by Eq.(2.37): the Boltzmann equation (2.103) can be simplified as $n_A \simeq n_{eq} \propto T^3$

B) When the temperature of the plasma reaches the value $T \lesssim m$, then the value of $n_{eq} \propto e^{-m/T}$ decreases very steeply, almost like a step function Eq.(2.38) and the **production rate** of Eq.(2.103) proportional to n_{eq}^2 is almost completely suppressed. We can interpret it as if the photons in the plasma do not possess sufficiently kinetic energy ($\propto T$) to produce particles **A** onshell. We are left with a bath of particles **A** which does not "discuss" anymore with the bath of the photons[21] , and can only annihilate. Their density decreases thus, but not as steeply as $e^{-m/T}$ as they have decoupled from the bath : they just follow the annihilation rate $n^2\langle\sigma v\rangle$ which behavior as function of the temperature depends strongly on the model and on σv. Up to now, we are still not in the "frozen" regime but in a semi–decoupled one (the Hubble parameter H is still negligible compared to $n\langle\sigma v\rangle$). The Eq.(2.103) can then be approximated by $\frac{dn}{dT} \simeq \frac{n^2\langle\sigma v\rangle}{HT}$

C) After a while, the Hubble parameter H dominates on the annihilation rate $\Gamma = n\langle\sigma v\rangle$ and we recover the decoupling limit ("freeze out") we discussed previously $\Gamma/H \lesssim 1$. In this regime, the number of particles **A** per comoving frame is constant and frozen and the Boltzmann equation (2.103) can be simply approximated by $dn/dT \simeq 3n/T \Rightarrow n \propto T^3$. The law has the same behavior than a relativistic particle in thermal bath but for a different reason (expansion rate in this case). In fact, fundamentally speaking, the reason is not so different. The keypoint being the conservation of the entropy which, at the same time is responsible for the thermal equilibrium *and* for the evolution of the space metric as function of the temperature: asking for a constant number of particle per comoving frame gives similar evolution for a relativistic particle in thermal equilibrium with the photons than for a particle out of equilibrium which density evolve solely due to the expansion. These 3 regimes A), B) and C) are sumarized in Fig.(2.18)

2.4.2 Overview

Neglecting the evolution of $g_s(T)$ (see [**?**] for a more complete treatment of the equation taking into account this correction) we can write (2.103) in a more workable way. Indeed, introducing $Y = n/s$, s being the entropy density, and supposing adiabatic processes, we can deduce $sR^3 = S =$cste $\Rightarrow\ \dot{s}/s = -3\dot{R}/R = -3H \Rightarrow\ \dot{Y}/Y = \dot{n}/n - \dot{s}/s = \dot{n}/n - 3H$. We then deduce

$$\dot{n} - 3Hn = s\dot{Y} = -H\ s\ T \times \frac{dY}{dT} \tag{2.104}$$

where we have used as previously $\frac{d}{dt} = \frac{dT}{dt}\frac{d}{dT} = -HT\frac{d}{dT}$. We can then write

$$\frac{dY}{dx} = \frac{\langle\sigma v\rangle s}{Hx}\left(Y_{eq}^2 - Y^2\right) \simeq \frac{6.6\times 10^9}{x^2}\frac{g_s}{g_\rho^{1/2}}\left(\frac{m}{\text{GeV}}\right)\left(\frac{\langle\sigma v\rangle}{3\times 10^{-26}\text{cm}^3\text{s}^{-1}}\right)\left(Y_{eq}^2 - Y^2\right) \tag{2.105}$$

[21] More precisely, the particles **A** talk to the photons but cannot hear them.

Figure 2.18: Schematically evolution of a massive particle **A** as a function of the temperature following the Boltzmann equation (red-full) in comparison with the photon thermal bath density (brown-dotted) and the equilibrium state (blue-dashed) i the regimes A, B and C (see the text for details). Kinetic equilibrium is equivalent to thermal equilibrium.

with $x = m/T$, $s = (2\pi^2/45)g_s m^3/x^3$, $Y_{eq} = n_{eq}/s = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g}{g_s}x^{3/2}e^{-x} \simeq 0.14\, g/g_s x^{3/2}e^{-x}$ $(x \gg 3)$ $\simeq 0.278 g_{eff}/g_s$ $(x \ll 3)$ $[g_{eff} = g$ (bosons), $g_{eff} = 3/4g$ (fermions)$]$ and $H = 1.66\frac{m^2}{M_P x^2}g_\rho^{1/2}$
To quantify the above argument concerning the transition between the thermal equilibrium and the freeze out time (the region B). Defining the deviation from the equilibrium Δ such as $Y = (1+\Delta)Y_{eq}$, after neglecting Δ^2, $d\Delta/dx \ll \Delta$ Eq.(2.105) becomes

$$\Delta \simeq \frac{H}{2\Gamma_{eq}}\left(x - \frac{3}{2}\right) \simeq 5.5\times 10^{10}\frac{g_s}{g}\frac{(x-3/2)}{x^{1/2}}e^x\left(\frac{1\text{ GeV}}{m}\right)\left(\frac{3\times 10^{-26}\text{ cm}^3\text{s}^{-1}}{\langle\sigma v\rangle}\right) \tag{2.106}$$

We can easily understand from Eq.(2.106) that the yield Y will departure from Y_{eq} when $\Gamma_{eq} = n_{eq}\langle\sigma v\rangle$ begins to be small. For $m = 100$ GeV and $\langle\sigma v\rangle = 3\times 10^{-26}$ cm^{3}s^{-1} one obtains

$$\Delta \simeq 1 \text{ (decoupling condition)} \Rightarrow x = \frac{m}{T} \simeq 26. \tag{2.107}$$

which means that the decoupling does not begin when $T \lesssim m$ but much later, when $T \lesssim m/26$. At this temperature, $Y \simeq 2Y_{eq} > Y_{eq}$ as we described already.
Another way of understanding the decoupling phenomena is by writing Eq.(2.105) as

$$\frac{dY}{dx} = \frac{\Gamma_{eq}}{H}\frac{1}{xY_{eq}}\left(Y_{eq}^2 - Y^2\right) \tag{2.108}$$

It is then easy to understand that in early universe, as long as $\Gamma \gg H\ Y_{eq}$, the number density Y try to track the equilibrium value Y_{eq}. This is because Y wants to change to match Y_{eq}. However, Γ_{eq} is decreasing. Eventually $\Gamma \simeq H$ at some 'time' x_f. From that point on, dY/dx becomes small and Y does not want to change anymore. We are left with $Y(x) \simeq Y(x_f)$, so that the number of particles per comoving volume has frozen out. For neutrinos this occurs while the species are still relativistic (see above) whereas for WIMPs, this occurs when the particles are already non-relativistic (see below).

That is also a **fundamental point** to underline. We made the hyposthesis, in the neutrino case as in the WIMP case, that the dark matter, or the neutrino, are in thermal equilibrium with the bath before decoupling from it. In one case (neutrino) the decoupling comes from the fact that its coupling to the bath is too weak to counterbalance the expansion, whereas in the dark matter case, it comes from the density, reduced by the Boltzmann suppression factor $e^{m/T}$. It exist a third way, whihc is the production rom the thermal bath while *not being* in equilibrium with it. In this case, all the particles are relativistic, the initial ones (bath) as the final ones (dark matter) and the treatment above is not valid anymore as we will see in Section (2.6.2).

2.4.3 Solving the equation

Before solving the equation (2.105), let's write it in a convenient way

$$\frac{dY}{dx} = \sqrt{\frac{\pi}{45}}\frac{g_s}{\sqrt{g_\rho}} m M_P \frac{\langle \sigma v \rangle}{x^2}\left(Y_{eq}^2 - Y^2\right) = \sqrt{\frac{\pi}{45}}\frac{g_s}{\sqrt{g_\rho}} m M_P \frac{\langle \sigma v \rangle}{x^2} = \frac{\lambda}{x^2}\left(Y_{eq}^2 - Y^2\right) \tag{2.109}$$

with $\lambda = \langle \sigma v \rangle m M_P \sqrt{\frac{\pi}{45}}\ \frac{g_s}{\sqrt{g_\rho}}$ and $g_\rho = g_s$ (which is the case at such temperature as non-massive degrees of freedom like neutrino has not decoupled yet. A typical electroweak cross section

$$\sigma_{EW} \simeq |\mathcal{M}|^2/64\pi^2 s \simeq g^2/64\pi^2 (100\ \text{GeV})^2 \simeq 10^{-9}\ \text{GeV}^{-2} \tag{2.110}$$

implying $\lambda \simeq \sigma m M_P \simeq 10^{12} \gg 1$ for a 100 GeV particle ($m = 100$ GeV). As a consequence, such a large value of λ depletes very quickly the density of the particle (even if much slower than the equilibrium density which is Boltzmann suppressed).

s-wave

The equation (2.109) is a type of Riccati equation with no analytic solution. Despite not being exactly solvable, we can still see this through by invoking some physics intuition. We know that all happens around $x \sim 1$. In this region we can see that the left hand side of (2.109) is $\mathcal{O}(Y)$ while the right hand side is $\mathcal{O}(\lambda Y)$. We just understood that $\lambda \gg 1$, so the right hand side must have a cancelation in the $Y^2 - Y_{eq}^2$ term.

After freeze out, Y_{eq} will continue to decrease according to the thermal suppression $e^{-m/T}$, so that $Y \gg Y_{eq}$. This happens at late times $x \gg 1$ where the Boltzmann equation reduces to

$$\frac{dY}{dx} \approx -\frac{\lambda(x)}{x^2} Y^2 \tag{2.111}$$

s-wave annihilation is characterized by $\langle \sigma v \rangle$ = cte. In this case, the resolution of the Eq.(2.109), integrating between the freeze out time x_f and the present time x_0, one obtains (noticing $x_0 \gg x_f$ and $Y_0 \ll Y_f$)

$$\frac{1}{Y_0} - \frac{1}{Y_f} = \lambda(\frac{1}{x_f} - \frac{1}{x_0}) \quad \Rightarrow \quad Y_0 \simeq \frac{x_f}{\lambda} \tag{2.112}$$

with x_f, time of freeze out obtained as a first approximation by solving

$$n_{eq}(x_f)\langle \sigma v \rangle = H(x_f) \tag{2.113}$$

which, for a weakly cross section, gives $x_f \simeq 20$ (a more precise solution will be given in the following section). From Y_0 one can deduce the massive relic abundance of the particle A of mass m with

$$\Omega_A = \frac{\rho_A}{\rho_c^0} = \frac{m n_A}{\rho_c^0} = \frac{m Y_0 s_0}{\rho_c^0} = \frac{m\ x_f}{\lambda} \frac{2\pi^2 g_s^{\text{today}} T_0^3}{45 \rho_c^0} \tag{2.114}$$

with $\rho_c^0 = 3H_0^2/8\pi G = 1.88 h^2$ g cm^{-3} = $10^{-5} h^2$ GeV cm^{-3} as we computed in Eq.(2.11) and the value of $g_s^{\text{today}} = 3.91$ as we found in Eq.(2.56) we obtain

$$\Omega_A = \frac{x_f}{\frac{\langle \sigma v \rangle}{(10^{-9}\text{GeV}^{-2})}} \frac{8.8 \times 10^{-7}}{\sqrt{g_\rho} \rho_c^0} \text{ GeV cm}^{-3} \tag{2.115}$$

Taking $g_\rho \simeq 100$ (see Fig.(2.7)), $x_f \sim 20$ (Eq.(2.125)) and the value of ρ_c^0 of Eq.(2.11) we can write[22]

$$\Omega_A h^2 \simeq \frac{0.17}{\frac{\langle \sigma v \rangle}{(1.2 \times 10^{-26}\ \text{cm}^3\ \text{s}^{-1})}} \tag{2.116}$$

This is oftenly called "WIMP miracle". Indeed, we see that for a typical electroweak cross section the relic abundance Ω_A reach $0.17/h^2 \simeq 0.3$ which is the measured value of the matter content in the Universe. Some corrections has to be taken into account: the velocity at decoupling time is not c, the value of x_f should be computed iteratively (see next section for a more complete calculation) and the dependance on the effective degree of freedom or mass of dark matter should be looked carefully. However, this approximation is surprisingly quite accurate in any models with s-wave dominated annihilation process.

General solution

Now that we understood how to compute the relic abundance in a specific case, we can now apply the same method in the generic case, developing $\sigma v = a + bv^2$, v being the relative

[22] $\langle \sigma v \rangle$ has been normalized to a typical electroweak cross section for a 100 GeV particle: 10^{-9} GeV^{-2} = 1.2×10^{-26}cm^3 s^{-1}, Eq.(2.110).

velocity between the two annihilating particles[23]. Notice that in the In the he Boltzmann equation, it is not σv which enters in the definition of λ in Eq.(2.109) but the thermal averaged cross section $\langle\sigma v\rangle$. At the temperature of interest at freeze out ($x_f = m/T_f \approx 20$ as we will compute more in detail later on) we can consider that the annihilating particles "1" and "2" is non-relativistic and thus their Boltzmann distributions (2.32) can be approximate by $f_i \simeq e^{-E_i/T} \simeq e^{-(m+p_i^2/2m)/T}$. One thus can write

$$\langle\sigma v\rangle = \frac{\int_1\int_2 d^3p_1 d^3p_2 (a+bv^2) e^{-E_1/T} e^{-E_2/T}}{\int_1\int_2 d^3p_1 d^3p_2 e^{-E_1/T} e^{-E_2/T}} \tag{2.117}$$

We can thus deduce $\langle a+bv^2\rangle = a+b\langle v^2\rangle$ with in the non-relativistic limit $v = |\mathbf{p_2}-\mathbf{p_1}|/m$ We then have $v^2 = (|\mathbf{p_1}|^2 + |\mathbf{p_2}|^2 - 2\mathbf{p_1}\mathbf{p_2}\cos\theta)/m^2$, θ being the angle between the two colliding particles. Noticing by symmetry that $\langle\cos\theta\rangle = 0$ and $\langle|\mathbf{p_1}|^2\rangle = \langle|\mathbf{p_2}|^2\rangle$, using Eq.(B.39) we then can write[24]

$$\langle v^2\rangle = 2\frac{\int_1 p_1^2 dp_1 (p_1^2/m^2) e^{-p_1^2/2mT}}{\int_1 p_1^2 dp_1 e^{-p_1^2/2mT}} = \frac{2}{m^2}\frac{3/8\sqrt{\pi}(2mT)^{5/2}}{1/4\sqrt{\pi}(2mT)^{3/2}} = 6\frac{T}{m} = 6x^{-1} \tag{2.118}$$

giving

$$\langle\sigma v\rangle = a + \frac{6b}{x} \tag{2.119}$$

We can now solve the Boltzmann equation (2.109) generically in the two steps we already discussed: in the regime $x \gg x_f$ in the first regime to obtain $Y_0 = Y(x_0)$ nowadays. In this regime, defining $\Delta = Y - Y_{eq}$ and neglecting Y_{eq} and dY_{eq}/dx, negligible due to the Boltzmann suppression at such late times. Expressing $\lambda = \langle\sigma v\rangle\lambda_0$ with $\lambda_0 = \sqrt{\pi g_\rho/45} m M_P$ Eq.(2.109) can be rewritten

$$\frac{d\Delta}{dx} \simeq -\frac{\left(a+\frac{6b}{x}\right)\lambda_0}{x^2}\Delta^2 \tag{2.120}$$

After integration from $x_f \to x_0$ and noticing $\Delta_f = \Delta(x_f) \ll \Delta(x_0) = \Delta_0$ due to the larger value of λ_0, using $(g_\rho(x_f) = g_s(x_f))$

$$\lambda_0 = \sqrt{\frac{\pi g_\rho(x_f)}{45}} m M_P, \quad s_0 = \frac{2\pi^2}{45} g_s^0 T_0^3, \quad \text{and} \quad \rho_c^0 = \frac{3(H_0)^2 M_P^2}{8\pi} \tag{2.121}$$

one obtains

$$\begin{aligned} Y_0 \simeq \Delta_0 = \frac{x_f}{\lambda_0}\frac{1}{\left(a+3\frac{b}{x_f}\right)} \quad &\Rightarrow \quad \Omega_0 = \frac{mY_0 s_0}{\rho_c^0} = mY_0\frac{g_s^0 16\pi^3 (T_0)^3}{135 M_P^2 (H_0)^2} \\ = \frac{g_s^0}{\sqrt{g_\rho(x_f)}}\frac{16\pi^{5/2}(T_0)^3}{3\sqrt{45}M_P^3(H_0)^2}\frac{x_f}{(a+3b/x_f)} \quad &\approx \quad \frac{4\times 10^{-2}}{h^2\sqrt{g_\rho(x_f)}}\frac{x_f}{\frac{(a+3b/x_f)}{3\times 10^{-26}\ \mathrm{cm^3 s^{-1}}}} \end{aligned} \tag{2.122}$$

[23]We define the relative velocity between two particles i and j by $v_{ij} = \frac{\sqrt{(p_i.p_j)^2 - m_i^2 m_j^2}}{E_i E_j}$, with p_i and E_i being four-momentum and energy of particle i.

[24]See the section (2.5.1) for another way to lead the integration for the mean $\langle\sigma v\rangle$.

To complete the solution we need to compute a more precise value of x_f. For that we can "define" x_f such that the temperature when the deviation of the density from the equilibrium value is of the order of the equilibrium density itself. Putting in equation, one need to solve $\Delta(x_f) = cY_{eq}(x_f)$, c being of the order of unity and can be calculated exactly numerically. We will keep it as a free parameter during all our calculation. However, in this regime, we cannot neglect anymore Y_{eq} and dY_{eq}/dx as around x_f the evolutions of the densities are very large and supposing the evolution of $\Delta(x)$ negligible with respect to the evolution of Y_{eq} ($d\Delta/dx \ll dY_{eq}/dx$). We can then write (2.109)

$$\frac{d\Delta}{dx} = -\frac{dY_{eq}}{dx} - \frac{\lambda_0\left(a + \frac{6b}{x}\right)}{x^2}\Delta\left(2Y_{eq} + \Delta\right) \tag{2.123}$$

with

$$Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_{eff}}{g_s}x^{3/2}e^{-x} = \alpha x^{3/2}e^{-x} \qquad \left[\mathrm{g_{eff}} = \mathrm{g(boson)};\ \frac{3}{4}\mathrm{g(fermions)}\right]$$

which gives (considering $x_f \gg 1$)

$$e^{x_f} = \frac{\lambda_0\alpha}{\sqrt{x_f}}\left(a + \frac{6b}{x_f}\right)c(c+2) \tag{2.124}$$

which gives using (2.121)

$$x_f = \ln\left[c(c{+}2)\sqrt{\frac{45}{8}}\frac{1}{2\pi^3}\frac{g_{eff}M_P m(a + 6b/x_f)}{\sqrt{x_f}\sqrt{g_s(x_f)}}\right] \approx \ln\left[2c(c{+}2)\times 10^8\frac{m\ g_{eff}}{\sqrt{g_s(x_f)}}\frac{(a + 6b/x_f)}{3\times 10^{-26}\mathrm{cm^3s^{-1}}}\right] \tag{2.125}$$

g is the internal degree of freedom of the annihilating particle (WIMP for instance), c is an order unity parameter which is normally determined numerically in the somution of the Boltzmann equation. It is usually set equal to 0.5 for analytical approximation. Eq.(2.125) gives $x_f \simeq 23$ for a 100 GeV mass particle which confirms our approximation $x_f \gg 1$.

Hot dark matter

It can be interesting to see what's happening with a particle which decoupled while still relativistic (a neutrino-like particle, but decoupling at a freeze out time x_f). In this case,

$$Y_{eq}(x) = \frac{n_{eq}}{s} = \frac{45}{2\pi^4}\frac{g_{eff}}{g_s(x)}\zeta(3) \tag{2.126}$$

Applying the same reasoning than above, we can easy show that as $dY_{eq}/dx = 0$ around the freeze out time, contrarily in the cold/massive case where $dY_{eq}/dx \propto Y_{eq}$. As a consequence, $\Delta(x) \approx 0$ and so Y follows Y_{eq} and stay constant after the freeze out:

$$Y_0 = Y_{eq}(x_f) = \frac{45}{2\pi^4}\frac{g_{eff}}{g_s(x_f)}\zeta(3) \quad\Rightarrow\quad \Omega_0 = \frac{g_{eff}\ g_s^0}{g_s(x_f)}\frac{8m(T_0)^3\zeta(3)}{3\pi(H_0)^2M_P^2} \approx \frac{9.6\times 10^{-2}}{h^2}\frac{g_{eff}}{g_s(x_f)}\left(\frac{m}{\mathrm{eV}}\right) \tag{2.127}$$

From this result, we can obtain an upper limit for a hot dark matter (right-handed neutrino like for instance with $g_{eff} = 2 \times 3/4$ and $g_s(x_f) \simeq 10$) from the condition $\Omega h^2 \lesssim 0.1$ we obtain $m \lesssim 7$ eV.

Another approach to average the annihilation cross section

We just saw that the thermal average of the annihilation cross section times velocity can be done by expanding the cross section at low relative velocity , $\langle \sigma v \rangle = a + \frac{6b}{x}$ from $\sigma v = a + bv^2$, v being the relative velocity between the two colliding particles. However, there is cases where the approximation is not valid anymore or even divergent In this case an integral formulation of the solution becomes useful.
We thus have to consider the case of two dark matter particles with energy E_1 and E_2 and impulsion $\mathbf{p_1}$ and $\mathbf{p_2}$ collide with an angle $\cos\theta$. It is important to note that in this case, one should make the computation in the gas rest frame and not in the mass frame of the two colliding particles, because one should take into account a statistical population of colliding particles with a statistical distribution of impulsion and angles. First of all, one should compute the cross secction of two particles "1" and "2" of masses M_1 and M_2 into two particles "3" and "4" of mass m_3 and m_4 representing the relativistic particles in the plasma. From the equation C.77, taking into account that $\sqrt{(P_1.P_2)^2 - M_1^2 M_2^2} = E_1 E_2 v_{12}$, v_{12} being the relative velocity between the particle 1 colliding the particle 2 (from now denoted simply by v) we deduce

$$\frac{d\sigma}{d\Omega} = \frac{|\bar{\mathcal{M}}|^2}{128\pi^2 s} \frac{\sqrt{s^2 - 2m_3^2 s - 2m_4 s + (m_3 - m_4)^2}}{E_1 E_2 v} \tag{2.128}$$

$|\bar{\mathcal{M}}|^2$ being the squared mean of the amplitude over the initial spin states. As a good approximation, for the temperature of interest, we can relatively safely consider that the particles living in the bath in thermal equilibrium with the massive particles 1 and 2 are massless (they are electrons, neutrinos, quarks and gauge bosons mainly) as if they are massive, they already had decoupled from the bath (Boltzmann suppression) very quickly and do not discuss anymore with the other particles in thermal equilibrium. Moreover, we will concentrate on annihilation of dark matter (the generalization to coannihilation is straightforward) and we will thus consider $M_1 = M_2 = M$. Note that during all the demonstration, we will consider annihilation of dark matter $1 + 2 \to 3 + 4$, the computation would be the same if one consider the production process $3 + 4 \to 1 + 2$.

The mean of the annihilations cross section per solid angle[25] can then be written (considering $m_2 = m_3 \ll s$) :

$$\langle v \; d\sigma \rangle = \frac{\int f_1(E_1) f_2(E_2) \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{|\bar{\mathcal{M}}|^2}{128\pi^2 E_1 E_2} d\Omega}{\int f_1(E_1) f_2(E_2) \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3}} = \frac{A}{n_{eq}^2} \tag{2.129}$$

n_{eq} being the density of the colliding particles at equilibrium (the dark matter candidate in our case). the phase space $d^3p_1 d^3p_2$ can be rewritten $4\pi|p_1|E_1 dE_1 4\pi|p_2|E_2 dE_2 \frac{1}{2} d\cos\theta$ giving

[25] be careful, the solid angle $d\Omega$ is the one the outgoing particles m_3 and m_4 in the center of mass of the colliding particles, to not confound with the solid angle of the colliding particles $\cos\theta$ on which we perform the statistical mean.

$$A = \int f_1 f_2 |p_1| dE_1 |p_2| dE_2 d\cos\theta \frac{|\bar{\mathcal{M}}|^2}{(2\pi)^4 64\pi^2} d\Omega \tag{2.130}$$

As $\mathcal{M}$ usually is expressed as function of the center of mass energy it would be easier to make a benefit change of variables :

$$\begin{aligned} E_+ &= E_1 + E_2 \\ E_- &= E_2 - E_1 \\ s &= (P_1 + P_2)^2 = 2M^2 + 2E_1E_2 - 2|p_1||p_2|\cos\theta \end{aligned} \tag{2.131}$$

Using the Jacobian and the new limit on integration given in the appendix B.5.5 on obtains

$$A = \int_{s-4M^2}^{\infty} \int_{E_+=\sqrt{s}}^{\infty} \int_{E_-=-\sqrt{E_+^2-s}\sqrt{1-4M^2/s}}^{E_-=+\sqrt{E_+^2-s}\sqrt{1-4M^2/s}} \frac{f_1 f_2}{(2\pi)^4} \frac{|\bar{\mathcal{M}}|^2}{256\pi^2} d\Omega dE_+ dE_- ds \tag{2.132}$$

For the regime where $T \simeq M$ (in other words when the dark matter candidate is not relativistic at the freeze out temperature) on can approximate the Fermi-Dirac or Bose-Einstein statistical distribution f_1 and f_2 by the Boltzman one $= f \simeq e^{-E_1/T}$; $f_2 \simeq e^{-E_2/T}$. We let the reader have a look at the section 2.6 to see how this change in the case of non thermal production, when the approximation $T \lesssim M$ is not valid anymore. In the present case, $f_1 f_2 = e^{E_+/T}$ and one can integrate on E_- and E_+ directly using the definition of modified Bessel function $K_1(z)$ of the appendix B.5.3 :

$$\begin{aligned} A &= \int_{s=4M^2}^{\infty} \int_{E_+=\sqrt{s}}^{\infty} \frac{e^{-\frac{E_+}{T}} \sqrt{s-4M^2}\sqrt{E_+^2 - s}}{(2\pi)^6 32\sqrt{s}} dE_+ ds d\Omega \\ &= \frac{T}{32(2\pi)^6} \int_{4M^2}^{\infty} \sqrt{s-4M^2} K_1(\frac{\sqrt{s}}{T}) |\bar{\mathcal{M}}|^2 ds d\Omega = n_{eq}^2 \langle v d\sigma \rangle \end{aligned} \tag{2.133}$$

This result is one of the more important of the chapter and will be very useful to solve the Boltzmann equation analytically. One can also compute n_{eq} as it is done in Appendix B.4

$$\begin{aligned} \langle v d\sigma \rangle &= \frac{\frac{T}{32(2\pi)^6} \int_{4M^2}^{\infty} \sqrt{s-4M^2} |\bar{\mathcal{M}}|^2 K_1\left(\frac{\sqrt{s}}{T}\right) ds d\Omega}{\left[\frac{gm^2T}{2\pi^2} K_2\left(\frac{m}{T}\right)\right]^2} \\ &= \frac{1}{256\pi^2 T M^4 g^2 K_2^2(m/T)} \int_{4M^2}^{\infty} \sqrt{s-4M^2} |\bar{\mathcal{M}}|^2 K_1\left(\frac{\sqrt{s}}{T}\right) ds d\Omega \end{aligned} \tag{2.134}$$

It is important to notice that if we would have looked at the production process $3+4 \to 1+2$ the result would have been the same. Indeed, keeping $m_3 = m_4 = M$ in the Eq.(2.128), this time the final state being the dark matter, one would have obtain the $\sqrt{s-4M^2}$ obtained in the $1+2 \to 3+4$ case from the integration on E_-, Eq.(2.134). This equation is thus valid for the annihilation as for the production process. This fact can be of interest when we will look at freeze in mechanism, where the production process is suppressed compare to the annihilation one.

2.4.4 The Lee-Weinberg bound

For any kind of cold dark matter, we can apply Eq.(2.116) and find the minimal mass for a typical WIMP (heavy neutrino) still in accordance with WMAP limits. Indeed, in the case of a heavy neutrino ν of mass m_ν, the Lagrangian needed to compute the annihilation cross section is

$$\mathcal{L}_\nu = Z_\mu \left[\frac{g}{2\cos\theta_W} \bar{\nu}\gamma^\mu (c_V^\nu - c_A^\nu \gamma^5)\nu + \frac{g}{2\cos\theta_W} \bar{f}\gamma^\mu (c_V^f - c_A^f \gamma^5) f \right] \tag{2.135}$$

which gives, with the approximation $M_Z \gg m_\nu$ we can write the Z_μ propagator as $i\frac{\eta_{\mu\nu}+P_\mu^Z P_\nu^Z/M_Z^2}{P_Z^2-M_Z^2} \approx \frac{i}{M_Z^2}$ we obtain

$$\mathcal{M} = i\frac{g^2}{4\cos^2\theta_W M_Z^2}\ \bar{v}(p_{\bar{\nu}})\gamma^\mu(c_V^\nu - c_A^\nu\gamma^5)u(p_\nu)\ \bar{u}(p_f)\gamma_\mu(c_V^f - c_A^f\gamma^5)v(p_{\bar{f}}) \tag{2.136}$$

with $c_V^f = T_{3L}^f - 2q_f \sin^2\theta_W$ and $c_A^f = T_{3L}^f$, T_{3L}^f being the isospin of the particle f. We keep the c_i^ν as free parameter for our calculation. Computing $|\mathcal{M}|^2$

$$|\mathcal{M}|^2 = 32\left(\frac{g^2}{4\cos^2\theta_W M_Z^2}\right)^2 \sum_f c_f\ [(c_V^f)^2 + (c_A^f)^2] \quad \Big[[(c_V^\nu)^2 + (c_A^\nu)^2]\left(p_f.p_\nu\ p_{\bar{f}}.p_{\bar{\nu}} + p_f.p_{\bar{\nu}}\ p_{\bar{f}}.p_\nu\right) + 2m_\nu^2[(c_V^\nu)^2 - (c_A^\nu)^2]\ p_f.p_{\bar{f}}\Big] \tag{2.137}$$

with c_f being the color factor of $SU(3)$ charged particles. At the limit of zero velocity, $E_\nu \simeq m_\nu$ and Eq.(2.137) can be written

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{8g^4 m_\nu^4}{\cos^4\theta_W M_Z^4}(c_V^\nu)^2 \sum_f c_f\ [(c_V^f)^2 + (c_A^f)^2] \\ \Rightarrow \sigma v &\simeq \frac{1}{4}\frac{|\mathcal{M}|^2}{32\pi^2}\frac{4\pi}{s} = \frac{g^4 m_\nu^2}{16\pi\cos^4\theta_W M_Z^4}\ (c_V^\nu)^2 \sum_f c_f\ [(c_V^f)^2 + (c_A^f)^2] \end{aligned} \tag{2.138}$$

For light masses, $\sum_f c_f\ [(c_V^f)^2 + (c_A^f)^2] \simeq 6.2$. If we impose $\Omega h^2 \lesssim 0.1$, Eq.(2.116) gives $\sigma v \gtrsim 2\times 10^{-26}\text{cm}^3\text{s}^{-1} = 1.7\times 10^{-9}\text{GeV}^{-2}$ we then obtain

$$m_\nu \gtrsim 4.1\ \text{GeV} \quad \text{if} \quad \Omega_\nu h^2 \lesssim 0.1 \tag{2.139}$$

Althought it is often called the Lee–Weinberg bound [18], it was discovered independently by a number of people and was recalculated more precisely in [19]. A viable way of building light dark matter models avoiding the Lee–Weinberg bound is by postulating new light bosons. This increases the annihilation cross section and reduces the coupling of dark matter particles to the Standard Model making them consistent with accelerator experiments [20].
This limit was used during a long time in several publication as it justified relatively heavy ($\sim$ 100 GeV, or electroweak scale) dark matter. However, since the nineties, several lighter

candidates appeared and when one looks in more details the solution to the Boltzmann equation, one found several exceptions to this bound. The main point is that the expression 2.136 is somewhat too simplistic as 1) we cannot decouple the Z boson so easily every time (especially near its pole mass), and 2) maybe the particle exchanged is not the Z boson, but a Z' or a supersymmetric particle. In any case, it is important to keep in mind that the Lee-Weinberg bound is only valid for a dark matter candidate coupling only to the Z boson, far from its pole and with electroweak like strength coupling. We will review some exceptions to this bound in the following discussion.

2.5 3 exceptions to the Boltzmann equation

The three exceptions we will discuss in this section are the pole region, the kinematic threshold and the coannihilation channel. You can find a detailed analysis of such regime in [21].

2.5.1 The pole region

Computing the annihilation cross section of a particle χ through an intermediate particle exchanged of mass M_{ex} gives

$$\sigma v = \frac{\alpha_D \alpha_V s}{(s - M_{ex}^2)^2 + M_{ex}^2 \Gamma_{ex}^2} \tag{2.140}$$

where $\alpha_D, \alpha_V \sim 0.01$ corresponds to the coupling of the exchange particle to the dark sector (D) and the visible one (V). Γ_{ex} is the total width of the exchanged particle and $s = \frac{4m_\chi^2}{1-v^2/4}$ (see Eq.(C.78) and discussion below for details and the expression (C.107) for a concrete example of Z' exchanged). If one define

$$r = \left(\frac{2m_\chi}{M_{ex}}\right)^2 \quad \text{and} \quad \epsilon = \left(\frac{\Gamma_{ex}}{M_{ex}}\right)^2$$

one obtains

$$\sigma v = \frac{\alpha_D \alpha_V}{M_{ex}^2} \frac{\frac{r}{1-v^2/4}}{\left[1 - \frac{r}{1-v^2/4}\right]^2 + \epsilon} \tag{2.141}$$

In the case of a Z exchange, $\epsilon =\simeq (2.5/91.2)^2 \simeq 7.5 \times 10^{-4}$, and one can understand easily why the velocity in Eq.(2.141) should be treated with care. At zero relative velocity, the pole occurs at $r = 1$ while in general it occurs at $r = 1 - v^2/4$. The value of the cross section is then

$$\sigma v_{pole} = \frac{\alpha_D \alpha_V}{M_{ex} \epsilon}. \tag{2.142}$$

However, as we saw in the previous section, in relic abundance calculation, the cross section must be thermal averaged. The standard method was to Taylor expand it to first order in v^2 and then substitute $\langle v^2 \rangle = 6\frac{T}{m_\chi} = 6/x$ (see Eq.(2.118) and below). However, for small

ϵ the expansion in v^2 breaks down near the pole and the standard method gives extremely poor results, even allowing the cross section to become negative!
Two methods have usually been used to treat the pole region, with quantitatively bad results. A common approach has been to factorize the "pole factor" $P(v^2) = \left(\left[1 - \frac{r}{1-v^2/4}\right]^2 + \epsilon\right)^{-1}$ before making the Taylor expansion and then multiply the result by the pole factor at zero velocity which would give

$$\langle \sigma v \rangle_1 = \frac{\alpha_D \alpha_V r}{M_{ex}^2}\left[1 + \frac{3}{2x}\right] P(0) = \frac{\alpha_D \alpha_V r}{M_{ex}^2}\left[1 + \frac{3}{2x}\right]\frac{1}{(1-r)^2 + \epsilon} \tag{2.143}$$

Another possibility is to approximate $\langle \sigma v \rangle$ by just substituting $v^2 \to 6/x$ in σv

$$\langle \sigma v \rangle_2 = \sigma v(v^2 = \frac{6}{x}) = \frac{\alpha_D \alpha_V r}{M_{ex}^2}\frac{\frac{1}{1-3/2x}}{(1 - \frac{r}{1-3/2x})^2 + \epsilon} \tag{2.144}$$

Both approximations can be valid far from the pole, but a real numerical treatment should be used when approaching it. The numerical thermalized cross section $\langle \sigma v \rangle$ is found from the calculation of the mean of σv in the center of mass frame ($\vec{p}_1 = -\vec{p}_2$)

$$\langle \sigma v \rangle = \frac{\int_1 \int_2 d^3p_1 d^3p_2 \delta^3(\vec{p}_1 - \vec{p}_2)(\sigma v) e^{-E_1/T} e^{-E_2/T}}{\int_1 \int_2 d^3p_1 d^3p_2 \delta^3(\vec{p}_1 - \vec{p}_2) e^{-E_1/T} e^{-E_2/T}} \tag{2.145}$$

in the non-relativistic limit, one can approximate $E_1 \simeq E_2 \simeq m + \frac{p^2}{2m}$ with $|p_1| = |p_2| = |p| = \gamma(v_1) m v_1 = \gamma(v/2) m v/2$, v being the relative velocity between particle 1 and particle 2 (see discussions after Eq.(C.78) for this point) and $\gamma(v_1) = 1/\sqrt{1 - v_1^2}$. We then obtain with $x = m/T$ and the help of the functions (B.39)

$$\langle \sigma v \rangle = \langle \sigma v \rangle_{num} = \frac{\int p^2 (\sigma v) e^{-p^2/mT} dp}{\int p^2 e^{-p^2/mT} dp} = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 (\sigma v) e^{-xv^2/4} \tag{2.146}$$

The results are shown in Fig.(2.19) where we plotted $\langle \sigma v \rangle$ as function of r for $x = 25$ and $M_{ex} = 91$ GeV, $\alpha_D = \alpha_V = 0.01$. As expected, in the first method, the pole appear at $r = 1$, whereas in the second one appears at $r = 1 - \frac{3}{2x} = 0.94$. We also show the behavior of the two asymptotic values $\sigma v = \frac{\alpha^2 r}{M_{ex}^2}$ and $\sigma v = \frac{\alpha^2}{r M_{ex}^2}$corresponding to the average cross section far from the pole (no dependance on the velocity, so $\langle \sigma v \rangle = \sigma v$ in this case).

2.5.2 Kinematic threshold

Suppose a channel where $2m_\chi \lesssim m_1 + m_2$, m_1 and m_2 being the mass of the two Standard Model–like particles The annihilation channel $\chi\,\chi \to 1\ 2$ should then be forbidden, except in the early universe where the temperature at the decoupling time $T \simeq m\chi/25$ can be sufficient for s to reach the threshold $m_1 + m_2$. Since the χ particles are Boltzmann suppressed, the annihilation takes place at a certain rate. If the masses of the annihilation products are not too much greater, this kind of annihilation can dominate the cross section.

Figure 2.19: Numerical thermal averaged cross section $\langle\sigma v\rangle_{num}$ near a pole as function of $r = \frac{2m_X}{M_{ex}}$ in comparison with the two asymptotic behaviors (left) and with the the two approximation (called "1" and "2" in the text).

2.5.3 Coannihilation

2.6 Non-thermal production of dark matter

2.6.1 The idea

The preceding discussion was based on a fundamental hypothesis: all the particles, including the dark matter are thermal: they are produced, at the reheating temperature in equilibrium with the photons. There is another possibility that we should take into account: if some particles, including the dark matter, is not produced after the reheating time with the thermal bath, their evolution follows a complete different way. In this case, one should neglect in Eq.(2.109) the first term corresponding to the annihilation of the dark matter, because its number density Y is too small, and only consider the production rate from the particles living in the bath. The evolution of the number density Y is then given by

$$\frac{dY}{dx} \simeq \frac{\lambda}{x^2} Y_{eq}^2 = \sqrt{\frac{\pi}{45}} \frac{g_s}{\sqrt{g_\rho}} m M_P \frac{\langle\sigma v\rangle}{x^2} Y_{eq}^2 = \sqrt{\frac{\pi}{45}} \frac{g_s}{\sqrt{g_\rho}} \frac{m M_P}{x^2 \mathbf{s^2}} \langle\sigma v\rangle n_{eq}^2 \tag{2.147}$$

$\mathbf{s}$ being the entropy of the system at the temperature T and $x = m/T$. Using Eq.(2.134) ($A = \langle\sigma v\rangle n_{eq}^2$) we obtain

$$\frac{dY}{dx} = \sqrt{\frac{\pi}{45}} \frac{g_s}{\sqrt{g_\rho}} \frac{m M_P}{x^2} \frac{T^4}{16(2\pi)^6 \mathbf{s^2}} \int_{\frac{2m}{T}}^{\infty} z\sqrt{z^2 - 4x^2} K_1(z) |\mathcal{M}|^2 dz d\Omega \tag{2.148}$$

with $z = \sqrt{s}/T$. Developing $\mathbf{s} = \frac{2\pi^2}{45} g_s T^3$ we obtain

Figure 2.20: Feynman diagram representing the process of production of dark matter from annihilation of the standard model particles living in the bath through the exchange of a mediator H.

$$\frac{dY}{dx} = \left(\frac{45}{\pi}\right)^{3/2} \frac{M_P}{m} \frac{1}{g_s\sqrt{g_\rho}16(2\pi)^8} \int_{2x}^{\infty} z\sqrt{z^2-4x^2}K_1(z)|\mathcal{M}|^2 dz d\Omega \tag{2.149}$$

We can find solutions to this equations for canonical regimes. Before studying a microscopic definite model, let's first suppose a simplified model with a particle mediator H coupling to the SM and dark matter with a strength α'. The amplitude squared can then be approximated (to some numerical factors of the order of unity) by

$$|\mathcal{M}|^2 \simeq \frac{(\alpha')^2 s^2}{(s-M_H^2)^2} \quad \text{with} \quad \alpha' = \frac{(g')^2}{4\pi} \tag{2.150}$$

The temperature of interest (around the reheating time) we can safely neglect the mass of the dark matter compared to the center of mass energy s : $m \ll \sqrt{s}$. We then have to consider 2 cases : very heavy mediators, with $M_H \gg T_{RH}$ and thus $M_H \gg \sqrt{s}$ or light mediator, $M_H \ll \sqrt{s}$. We will distinguish the two cases in the following

Case a : heavy mediator : $M_H \gg T_{RH}$: Heavy Mediated Dark Matter (HMDM)

In this case, $|\mathcal{M}|^2$ can be approximated by $|\mathcal{M}|^2 \approx (\alpha')^2 s^2/M_H^4$. At the temperature of interest, we can consider that $g_\rho = g_s$. The Eq.(2.149) then becomes

$$\frac{dY}{dx} = \frac{48(\alpha')^2}{(2\pi)^7}\left(\frac{45}{\pi g_s}\right)^{3/2}\frac{M_P m^3}{x^4 M_H^4} \Rightarrow Y(T) = \frac{4}{(2\pi)^9}\left(\frac{45}{\pi g_s}\right)^{3/2}\frac{M_P}{(M_H/g')^4}\left[T_{RH}^3 - T^3\right] \tag{2.151}$$

T_{RH} being the reheating temperature. From Eq.(2.151) we can see that all the dark matter is produced around the reheating temperature. Very quickly Y reach its maximum value $Y_\infty = Y(T \to 0)$. Physically speaking, it means that only the photon and relativistic particles which posses large energy (around T_{RH}) can contribute to the production of the dark matter candidate. As soon as the temperature decrease below T_{RH} this population disappear quickly and thus in the meantime the production process. From Eq.(2.151) one can also deduce the relation between the heavy particle mediator and the temperature of

reheating to produce sufficiently dark matter to respect WMAP bound, $\Omega h^2 = 0.1$. We can indeed write

$$Y_\infty = \frac{\Omega \rho_c^0}{m\ s_0} = \left(\frac{\Omega}{0.1}\right)\left(\frac{100\ \text{GeV}}{m}\right)\frac{135\times 10^{-3} H_0^2 M_P^2}{16\pi^3 g_s^0 T_0^3} \quad \Rightarrow \tag{2.152}$$

$$M_H^4 = (g')^4 \sqrt{\frac{45}{\pi}}\frac{g_s(T_0)}{g_s^{3/2}(T_{RH})}\left(\frac{16\times 10^3}{3(2\pi)^7}\right)\left(\frac{T_{RH}^3 T_0^3}{H_0^2 M_P}\right)\left(\frac{0.1}{\Omega}\right)\left(\frac{m}{100\ \text{GeV}}\right)$$

$$\Rightarrow \quad M_H \simeq 4.6\times 10^5\ g'\left(\frac{m}{100\ \text{GeV}}\right)^{1/4}\left(\frac{0.1}{\Omega h^2}\right)^{1/4} T_{RH}^{3/4} \tag{2.153}$$

The last expression[26] gives us the minimum value required by M_H for each reheating temperature to avoid the overclosure of the Universe ($\Omega h^2 < 0.1$) :

$$M_H^{\min} = 4.6\times 10^5\ g'\left(\frac{m}{100\ \text{GeV}}\right)^{1/4}\ T_{RH}^{3/4}. \tag{2.154}$$

The results is shown in figure (2.21).
It is important to notice several key-points at this stage. First of all, the approximation concerning the distribution (Maxwell-Boltzmann approximation) we used to obtain Eq.(2.149) is not valid anymore for marge temperature. Strictly speaking we should take the Fermi-Dirac expression of the distributions f_1 and f_2. A complete numerical analysis gave us a 44% of differences between the two treatments. Secondly, α' is in fact the interaction strength multiplied by the charges of the interacting particles, so factor of unities can also enter in the results: $\alpha' = q_{DM}^2 q_{SM}^2 \alpha'_{real}$. Finally our condition is valid in the approximation $M_H > T_{RH}$ (which was our hypothesis) which correspond, from Eq.(2.153)

$$g' > 2.1\times 10^{-6}\ T_{RH}^{1/4}\ \left(\frac{100\ \text{GeV}}{m}\right)^{1/4}\left(\frac{\Omega h^2}{0.1}\right)^{1/4}, \tag{2.155}$$

lower bound which stay perturbative in any cases. There is however other scenario possible if the mediator is light (let suppose massless for a good approximation). We will study them in the next section. The process is depicted in Fig.(2.21).

Case b : light mediator : $M_H \ll s$, weak-like coupling : $\alpha' \simeq \alpha_{EW}$

When the particle exchanged between the dark sector and the visible world is very light or even massless, the thermal history is quite different. Depending on the value of the coupling α', the dark matter can reach its thermal equilibrium value before that the temperature of the plasma drops below the mass of the dark matter candidate, $m$or after. The former case correspond to relatively large value of α' (of the order of electroweak coupling) and we recover the classical WIMP-like scenario, even if the dark matter was not produced with the SM ones at the reheating time, but progressively. In the former case however, the coupling is so feeble that the dark matter abundance reach the WMAP critical value before reaching

[26] As an indication, $Y_\infty \simeq 3.3\times 10^{-12}\left(\frac{100\ \text{GeV}}{m}\right)\left(\frac{\Omega h^2}{0.1}\right)$

Figure 2.21: 3 different ways to produce dark matter from SM bath and obtain the correct critical relic abundance $Y_\infty \simeq 3.3 \times 10^{-12} \left(\frac{100\ \text{GeV}}{m}\right) \left(\frac{\Omega h^2}{0.1}\right)$: Thermal WIMP, Heavy Mediated Dark Matter (HMDM) and Feebly Interacting Dark Matter (FIMP).

the thermal bath equilibrium. Approximating $M_H \ll \sqrt{s}$, we can approximate $|\mathcal{M}|^2 \approx (\alpha')^2$. The equation (2.149) becomes

$$\frac{dY}{dx} = \left(\frac{45}{\pi}\right)^{3/2} \frac{(\alpha')^2}{g_s\sqrt{g_\rho}4(2\pi)^7} \frac{M_P}{m} \tag{2.156}$$
$$\Rightarrow \quad Y(T) = \left(\frac{45}{\pi}\right)^{3/2} \frac{(\alpha')^2}{g_s\sqrt{g_\rho}4(2\pi)^7} \left(\frac{M_P}{T}\right) \simeq (\alpha')^2 \frac{4.2 \times 10^{11}\ \text{GeV}}{T}$$

contrarily to the heavy mediator case, the evolution is very fast with decreasing temperature. For a relativistic particle, one can compute the temperature for which the dark matter number density reach its equilibrium number :

$$Y_{eq} = \frac{n_{eq}(T)}{s(T)} = \frac{45\zeta(3)g_{eff}}{2\pi^4 g_s(T)} \approx 2.6 \times 10^{-3}\ g_{eff} \tag{2.157}$$

giving $Y_{eq} \simeq 7.8 \times 10^{-3}$ for a dirac fermion. If we suppose a classical coupling ($\alpha' \sim 10^{-3}$) and using Eq.(2.156) on can see that the equilibrium is reached for $T \simeq 5 \times 10^7$ GeV so very quickly after the reheating. Once $Y = Y_{eq}$, $\frac{dY}{dx} \simeq 0$ from Eq.(2.109), and we recover the classical WIMP thermal history with freeze out scenario. In fact, for weakly–like coupling dark matter, there is no real difference if the candidate is produced at the reheating time

at the same time that the SM particles (through the decay of the inflaton for instance) or if it is produced by SM particles annihilation because in the later case it reaches its thermal value vary fast. The process is depicted in Fig.(2.21).

Case c : light mediator : $M_H \ll s$, feebly-like coupling : $\alpha' \ll \alpha_{EW}$

In this scenario, the coupling is so feeble that the dark matter particle is not sufficiently produced by SM annihilation: when the temperature of the SM-thermal bath reach $T \simeq m$, the production is completely frozen "in". The particles produced by this mechanism are called FIMP as Freeze In (or Feebly Interacting) Massive Particle. The evolution follows the Eq.(2.156) but with very tiny values of α'. The value of the coupling needed to respect WMAP can be obtain asking for the number density to reach the relic abundance density just before the decoupling temperature ($T_f \simeq m/20$, Eq. 2.125) where the SM particles of the plasma do not have sufficient energy to produce the dark matter species (exponential suppression). Strictly speaking, we cannot really talk about "decoupling temperature" or "decoupling time" as the particle was never coupled to the plasma. This happens when $Y(T_f \approx m/20)Y_\infty$ which gives

$$Y\left(\frac{m}{20}\right) = Y_\infty \quad \Rightarrow \quad (\alpha')^2 = \left(\frac{\Omega}{0.1}\right)\frac{g_s^2}{\sqrt{g_\rho}}\, 3\times 10^{-1}(2\pi)^4\frac{M_P H_0^2}{T_0^3} \simeq 10^{-22}$$
$$\Rightarrow \alpha' = \alpha_{FIMP} \approx 10^{-11} \tag{2.158}$$

this scenario is summarized in Fig.(2.21).

2.6.2 Generalisation : the FIMP case

The Boltzmann equation

One can then make a more systematic study of the non-thermal production of dark matter (or any species) through the thermal bath. This mechanism is called "Freeze -in", giving the name of "FIMP" (Freeze-In Massive Particle) to the candidates produced through this mechanism. The meaning is quite clear: the particle is frozen by the expansion while being still *in* the process of reaching thermal equilibrium: it is on its way that the density is fixed, contrarily to the freeze out process, where the density is frozen while the particle is *outing* the thermal bath (see section 2.4.2). We can compute easily the production rate from the scattering of particles $1+2$ in the thermal bath, Following Eq.(2.103) we can write

$$\frac{dn}{dt} = -3H(t)n(t) + n_{eq}^1 n_{eq}^2\langle\sigma v\rangle = -3H(t)n(t) + R(t) \tag{2.159}$$

with $R(t) = n_{eq}^1 n_{eq}^2\langle\sigma v\rangle$ is the interaction rate (number of interactions per unit of time and per unit of volume) and where n_{eq}^i is the local density of thermalized particles i. As you notice, compared to Eq.(2.103), the last term on right hand side is absent because the density of dark matter, if produced by freeze in, is by definition much smaller than the one of the thermalized particles in the plasma as they did not yet reached the thermal equilibrium. Transforming the Boltzmann equation for n(t) into the one for the Yields $Y(t) = \frac{n}{s}$ (s being the entropy density Eq. 2.54) one obtains

$$\frac{dY}{dt} = \frac{R}{s} = \frac{n^1_{eq} n^2_{eq} \langle \sigma v \rangle}{s}$$

Writing the equation as function of temperature, noticing that the entropy $S = s \times a^3$ is constant in a universe of radius a, and $s \propto T^3$, we obtain

$$H = \frac{\dot{a}}{a} = -\frac{\dot{T}}{T} \quad \Rightarrow \quad \frac{dT}{dt} = -H\ T \quad \rightarrow \quad \frac{d}{dt} = \frac{dT}{dt}\frac{d}{dT} = -H\ T\frac{d}{dT} \tag{2.160}$$

giving

$$\frac{dY}{dT} = -\frac{R}{H\ T\ s} \tag{2.161}$$

Computing the rate R

The rate $R = n^1_{eq} n^2_{eq} \langle \sigma v \rangle$ can be computed directly from the expressions (C.54) and (C.62). Indeed, the interaction rate (number of interactions per unit of time) dw_{fi} is given by (C.53)

$$dw_{fi} = V \times (2\pi)^4 \delta^{(4)}(p_f - p_i) |\mathcal{T}_{fi}|^2 \prod_{f=1}^{n_f} \frac{V}{(2\pi)^3} d^3 p_f$$

To understand quickly each terms of the equation above, notice that $\mathcal{T}_{fi}$ as been defined from the $\mathcal{S}$ matrix element $\mathcal{S}_{fi} = 1 + (2\pi)^4 \delta^{(4)}(p_f - p_i)\mathcal{T}_{fi}$, and $w_{fi} = \frac{|\mathcal{S}_{fi}|^2}{T}$, which gives, in a phase space of volume[27] $V \times d^3p$, $dw_{fi} = w_{fi} \times \prod_{f=1}^{n_f} \frac{V\ d^3 p_f}{(2\pi)^3}$ (and using the relation $[(2\pi)^4 \delta^{(4)}(p_f - p_i)]^2 = V \times T \times (2\pi)^4 \delta^{(4)}(p_f - p_i)$).

However, the precedent reasoning was made in a non-covariant form (quantum mechanics). As explained in section C.7.1 one need to normalize the wave function by $\psi_i \rightarrow \frac{1}{\sqrt{2E_i V}}\psi_i$ to render the theory Lorentz invariant (normalization corresponding to $2E_i$ particles i in volume V). We then define the matrix element $\mathcal{M}_{fi}$ by (see Eq.C.54)

$$\mathcal{T}_{fi} = \left(\prod_{i=1}^{n_i} \frac{1}{\sqrt{2E_i V}} \right) \left(\prod_{f=1}^{n_f} \frac{1}{\sqrt{2E_f V}} \right) \mathcal{M}_{fi}$$

The rate R (interaction par unit of time for a given initial state) is then given by eq.(C.55)

$$R = dw_{fi} = \frac{V^{1-n_i}}{(2\pi)^{3n_f}} (2\pi)^4 \delta^{(4)}(p_f - p_i) |\mathcal{M}_{fi}|^2 \prod_{i=1}^{n_i} \frac{1}{2E_i} \prod_{f=1}^{n_f} \frac{d^3 p_f}{2E_f} \tag{2.162}$$

However, in the situation where the momentum of initial state is not definite, but follow a statistical law (as it is the case in the early Universe) one should multiply this rate by the number of quantum states of the initial particles (1 and 2) weighted by their distribution functions (Fermi-Dirac or Bose-Einstein, see Eq.2.32) f_1 and f_2 respectively:

[27] To see it easily, one can use uncertainty principle of Heinsenberg for instance, where the number of states N_f is $\frac{d^3 x d^3 p}{h^3}$, and $\hbar = 1 \ \Rightarrow h = 2\pi$, which implies $N_f = \frac{V\ d^3 p}{(2\pi)^3}$

$$d\langle\tilde{R}\rangle = \prod_{i=1}^{n_i} f_i \frac{V}{(2\pi)^3} d^3p_i \times R \tag{2.163}$$

which gives, combining with Eq.(2.164)

$$\tilde{R} = V \int \prod_{i=1}^{n_i} f_i \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^{(4)}(p_f - p_i) |\mathcal{M}_{fi}|^2 \prod_{f=1}^{n_f} \frac{d^3p_f}{(2\pi)^3 2E_f} \tag{2.164}$$

with $\tilde{R}$ the rate of interaction, per un it of time, in the volume V. To obtain the rate $R(t)$ one needs to solve the Boltzmann equation (2.159), one just needs to divide $\tilde{R}$ by V which gives

$$R(t) = \frac{\tilde{R}}{V} = \int \prod_{i=1}^{n_i} f_i \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^{(4)}(p_f - p_i) |\mathcal{M}_{fi}|^2 \prod_{f=1}^{n_f} \frac{d^3p_f}{(2\pi)^3 2E_f} \tag{2.165}$$

A little remark here: We notice that $\mathcal{T}_{fi}$ has a dimension of $(Energy)^4$ and then $\mathcal{M}_{fi}$ has dimension $(Energy)^{4-n_i-n_f}$, which is obviously adimensional for $2 \to 2$ scattering.
In our case of interest ($2 \to 2$ scattering, $1 + 2 \to 3 + 4$), using the formula for the 2-body phase-space final state Eq.(C.67)

$$\frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} \delta^{(4)}(p_f - p_i) = \frac{d\Omega_{34}}{512\pi^6} \tag{2.166}$$

with $d\Omega_{34}$ being the solid angle between particles 3 and 4. In the relativistic case for the initial particles[28], $p_i = E_i$. We can then write

$$\frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} = \frac{4\pi \times 2\pi \; d\cos\theta_{12} \; E_1 dE_1 E_2 dE_2}{4} \tag{2.167}$$

where $\cos\theta_{12}$ is the angle between particle 1 and 2 *in laboratory frame*. Combing with Eq.(2.165) one obtains

$$R(t) = \int f_1 f_2 \frac{E_1 E_2 dE_1 dE_2 \; d\cos\theta_{12}}{1024\pi^6} \int |\mathcal{M}_{fi}|^2 d\Omega_{34} \tag{2.168}$$

Some examples

It exists a lot of cases where the production of dark matter can be more challenging than the SM particles. Indeed, as long as the visible sector and dark sector are sequestred (by a very massive mediator or very feebly coupling), the way of producing dark matter is complicated. We will present in this section different UV construction where the freeze in mechanism is the keypoint to compute the relic abundance. We will suppose a generic process $\gamma\gamma \to dm\ dm$, where γ represents the SM particles of the thermal bath, and χ our dark matter candidate. We will also consider a mediator M, which can be scalar, vectorial or even fermionic, of mass M_M and a coupling of χ to M, $f_{\chi M}$ and we will denote $f_{\gamma M}$ for the coupling between the SM bath and the mediator. The process can be summarized by the figure 2.22

[28] which is the case in freeze-in as the photons and/or Standard Model particles are the 1 and 2 particles, much lighter than the energy at the reheating time, and then massless

Figure 2.22: Feynman diagram of a particle A in thermal bath decaying into 2 dark matter candidates χ.

The light mediator scenario : $M_M \ll m_{dm}, T_{RH}$

2.6.3 The special case of the gravitino

$M_{SUSY} < T_{RH}$

If the SUSY spectrum lies below the gravitino mass, the gravitino is produced also from the annihilation of standard model particles from the bath, mainly the ones charged inder $SU(3)$. The more common process is the annihilation of gluons g into gluino $\tilde{g}$ of mass M_3 and gravitino $\tilde{G}$ of mass $m_{3/2}$ through the exchange of a gluon : $g+g \to g \to \tilde{g}+\tilde{G}$. The sum of all the scattering process has been calculated in [17] and gives, with a good approximation :

$$\langle \sigma v \rangle_{3/2} n_{eq}^2 \simeq \frac{4 g_3^2 T^6}{\pi^2 M_P^2} \left(1 + \frac{M_3^2}{3 m_{3/2}^2}\right) \tag{2.169}$$

$$\Rightarrow \quad \frac{dY}{dx} = \frac{1}{4 g_s \sqrt{g_\rho}} \left(\frac{45}{\pi}\right)^{3/2} \frac{m M_P}{x^2 T^6} \langle \sigma v \rangle_{3/2} n_{eq}^2 \simeq \left(\frac{45}{g_s \pi}\right)^{3/2} \frac{m g_3^2}{x^2 \pi^2 M_P} \left(1 + \frac{M_3^2}{3 m_{3/2}^2}\right)$$

which implies after integrating on x

$$Y(T) = \left(\frac{45}{g_s \pi}\right)^{3/2} \frac{g_3^2}{M_P \pi^2} \left(1 + \frac{M_3^2}{3 m_{3/2}^2}\right) \left[T_{RH} - T\right] \tag{2.170}$$

The evolution of the population of gravitino is thus proportional to the temperature and its rate of production is softer than in the case of the heavy mediator we discussed in the previous section. Moreover, the ratio of the gluino to gravitino mass becomes important in this case. In specific scenarios where $m_{3/2} \gg M_3$ (anomaly mediation models for instance) one obtain ($g_3^2 \sim 0.5 \sim (g_3^{GUT})^2$ at such high temperatures)

$$Y_\infty \simeq 2 \times 10^{-12} \left(\frac{T_{RH}}{10^{10}\ \text{GeV}}\right) \tag{2.171}$$

Figure 2.23: Feynman diagram of a particle A in thermal bath decaying into 2 dark matter candidates χ.

giving a reheating temperature of $T_{RH} \simeq 10^8$ GeV for a 10 TeV gravitino[29]. We have represented the evolution of the gravitino production mechanism in Fig.2.21.

$M_{SUSY} > T_{RH}$

2.6.4 Non-thermal production through decays

Generalities

Another possibility is that heavy particles like scalars, Z' or moduli fields can decay into the dark matter candidate to populate the Universe with it. Let's consider a particle A in thermal equilibrium in the bath, and decaying into 2 dark matter candidates χ of impulsions p_1 and p_2 : $2m_\chi \lesssim M_A$. Supposing that initially the dark matter is not produced in the thermal bath: it can happen if the dark matter is not coupled with the inflaton (through symmetries for instance). In this case, the number of particles produced per second from the decay of the parent A is $dn_\chi/dt = \Gamma_A n_A$, with Γ_A being the width of A. We can then rewrite Eq.(2.103) neglecting the annihilation rates (as the annihilation process is an order of coupling weaker than the decay)

$$\frac{dn_\chi}{dt} + 3Hn = g_A n_A \langle \Gamma_A \rangle, \quad \Rightarrow \frac{dY_\chi}{dT} = -g_A \frac{\langle \Gamma_A \rangle}{HT} Y_A \quad \Rightarrow \frac{dY_\chi}{dx} = g_A \frac{\langle \Gamma_A \rangle}{Hx} Y_A \tag{2.172}$$

with $x = m_\chi/T$, g_A the number of degrees of freedom of A and $Y_i = n_i/s$, s being the entropy density given by Eq.(2.54), $s = 2\pi^2/45 g_s T^3$. It is important to notice that $\langle \Gamma_A \rangle$ is the *mean* of the width of the parent A and not *pure* width. We need to stop some time on this issue. Indeed, there is a little subtelty in the Boltzmann equation that is not obvious at the first glance[30]. The real Boltzmann equation (2.172) should in fact be written

[29] after using $Y_\infty \simeq 3.3 \times 10^{-12} \left(\frac{100 \text{ GeV}}{m}\right) \left(\frac{\Omega h^2}{0.1}\right)$ from Eq.(2.152)

[30] This remark is also valid in the case where the dominant process is the annihilation one

$$\frac{dn_\chi}{dt} + 3Hn = g_A \int \frac{d^3p_A}{(2\pi)^3 2E_A} f_A \int\int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_A - p_1 - p_2)|\mathcal{M}|^2$$
$$= g_A \int \frac{d^3p_A}{(2\pi)^3} f_A \frac{M_A}{E_A} \Gamma_A = g_A n_A \langle \Gamma_A \rangle = S\frac{dY}{dt} = -HTs\frac{dY}{dT} = Hxs\frac{dY}{dx} \tag{2.173}$$

f_A being the statistical distribution function of A in the thermal bath and $x = m_A/T$. Comparing with the general formula Eq.(C.111) we note that the presence of the Lorentz invariant phase space in the right hand side of (2.173). This is similar in the case of the annihilation. Indeed, whereas the "$2E_A$" factor necessary to satisfy Lorentz invariance in Eq.(2.173) is absorbed in the "$2M_A/2E_A$" factor, the "$2E_1 2E_2$" factor in annihilating case is absorbed in the "v" of σv from the formula

$$4\sqrt{(P_1.P_2)^2 - M_1^2 M_2^2} = 2E_1 2E_2 v \tag{2.174}$$

that one find in Eq.(C.73). We can then solve step by step the evolution equation for Y_χ supposing that A is in thermal equilibrium and that all the physics is happening for temperature of the plasma T much above M_A, the thermal distribution can be approximated by the Maxwellian one from $T = M_A$ to the reheating temperature T_{RH}, $f_A \simeq e^{-E_A/T}$ we then can write

$$Hxs\frac{dY}{dx} = M_A \Gamma_A \int_{M_A}^{T_{RH}} e^{-E_A/T} \mathbf{p}_A \frac{dE_A}{(2\pi)^3} 4\pi = \frac{M_A \Gamma_A}{2\pi^2} \int_{M_A}^{\infty} e^{-E_A/T} \sqrt{E_A^2 - M_A^2} dE_A$$
$$= \frac{TM_A^2 \Gamma_A}{2\pi^2} K_1\left(\frac{M_A}{T}\right), \quad \Rightarrow \quad \frac{dY}{dx} = \frac{M_A^3 \Gamma_A}{Hx^2 s 2\pi^2} K_1(x). \tag{2.175}$$

K_1 being the first modified Bessel function of the second kind that the reader can find in section B.5.3. Using the common relations (2.37) for $s(x)$ and (2.60) for $H(x)$

$$s(x) = g_s \frac{2\pi^2}{45} \frac{M_A^3}{x^3}, \quad H(x) = \sqrt{g_\rho} \sqrt{\frac{4\pi^3}{45}} \frac{M_A^2}{x^2 M_P}$$

we finally obtain the equation

$$Y_0 = Y(x \to \infty) = \left(\frac{45}{\pi}\right)^{3/2} \frac{1}{g_s \sqrt{g_\rho}} \frac{M_P \Gamma_A g_A}{8\pi^4 M_A^2} \int_{\frac{M_A}{T_{RH}}}^{\infty} x^3 K_1(x) dx \tag{2.176}$$

To have an idea of the order of magnitude of the yield of the dark matter produced from the decay of a massive particle of mass M_A, we can approximate $\Gamma_A \simeq g_D^2 M_A/(16\pi)$, g_D being a gauge coupling in the case of a vectorial particle A, a Yukawa–type coupling in the case of a scalar one[31]. Noticing that $g_S \simeq g_\rho$ at the energy of interest, we can write

$$Y_0 \simeq \left(\frac{45}{\pi}\right)^{3/2} \frac{g_D^2 g_A M_P}{128\pi^5 M_A} \int_{\frac{M_A}{T_{RH}}}^{\infty} x^3 K_1(x) dx \tag{2.177}$$

[31] See Eq.(C.124) and (C.127) for the exact expressions

We then need to distinguish two cases: 1) $M_A < T_{RH}$, A is in thermal equilibrium with the bath and $\int_{\frac{M_A}{T_{RH}}}^{\infty} x^3 K_1(x)dx \simeq \int_0^{\infty} x^3 K_1(x)dx = 4.7$ and 2) $M_A > T_{RH}$. In this case, using the equations (B.39) and the approximation of $K_1(x)$ given in section (B.5.3) we have

$$\int_X^{\infty} x^3 K_1(x) \simeq \sqrt{2\pi} \int_{\sqrt{\frac{M_A}{T_{RH}}}}^{\infty} t^6 e^{-t^2} dt \simeq \sqrt{\frac{\pi}{2}} e^{-\frac{M_A}{T_{RH}}} \left(\frac{M_A}{T_{RH}}\right)^{5/2} \tag{2.178}$$

Applying then Eq.(2.122)

$$\Omega_0 = \frac{mY_0 s_0}{\rho_c^0} = m_\chi Y_0 \frac{g_s^0 16\pi^3 (T_0)^3}{135 M_P^2 (H_0)^2} \Rightarrow \Omega_0 h^2 \simeq 2.8 \times 10^8 \; m_\chi \; Y_0 \tag{2.179}$$

which gives

$$\begin{aligned} \Omega_0 h^2 &\simeq 2 \times 10^{22} \; g_D^2 \; \frac{m_\chi}{M_A} && [M_A < T_{RH}], \\ \Omega_0 h^2 &\simeq 5 \times 10^{21} \; g_D^2 \; \frac{m_\chi}{M_A} e^{-\frac{M_A}{T_{RH}}} \left(\frac{M_A}{T_{RH}}\right)^{3/2} && [M_A > T_{RH}]. \end{aligned} \tag{2.180}$$

To respect WMAP one thus need $g_D \simeq 10^{-11}$ if A is a TeV scale dark matter in the plasma (first case) or $M_A \simeq 4 \times 10^{10}$, for $T_{RH} = 10^9$ GeV and a 100 GeV weakly interacting particle ($g_D \simeq 10^{-1}$) in the second case.

An example : decay of the gravitino to populate dark matter

In several minimal supersymmetry extension of the Standard Model, the dark matter candidate is a *wino*, fermionic partner of the W^a gauge vector. the wino being charged under SU(2), the coupling to the Standard Model bath (added to the coannhinilation process with its charged supersymmetric partner which have the same mass at tree level) underproduced dark matter. Typical relic abundance for a wino dark matter is around $\Omega h^2 \simeq 10^{-3}$ [22]. There are different ways to overcome this issue. One solution is to populate the universe through the decay of the gravitino $\tilde{G} \to \chi\chi$ of mass $m_{3/2}$ [23,24]. As we discussed earlier on , the gravitino is super–weakly interacting and decouple from the plasma at an early stage when the plasma has temperature T_d, around the reheating temperature T_{RH}, $T_d \simeq T_{RH}$. Its density is then fixed by T_{RH} by Eq.(2.171) $Y_{\tilde{G}} \simeq 2 \times 10^{-12} (T_{RH}/10^{10}$ GeV). The equilibrium condition can be written $\Gamma_{\tilde{G}} \lesssim H(T)$.
To understand physically speaking what is happening in the plasma, it is useful to express Eq.(2.175) as function of $\Gamma_{\tilde{G}}$ and H only:

$$\frac{dY_\chi}{dx} \propto \frac{\Gamma_{\tilde{G}}}{H} x K_1(x). \tag{2.181}$$

As we did when we try to understand the Boltzmann equation and the decoupling effect for annihilating dark matter, we see that a similar phenomenon exists for a decaying particle into dark matter. Indeed, when the temperature of the plasma reach $\Gamma_{\tilde{G}} \lesssim H(T = T_{3/2})$, $\frac{dY_\chi}{dx} \simeq 0$. In other word, the number of dark matter particle per covolume (Y_χ) resulting

from the annihilation of the gravitino is constant. Physically speaking on can understand it as during the doubling Hubble time $t = 1/H$, the decay of $\tilde{G}$ populate the universe with 2 particles : the density is thus constant. We need to impose that this decay occurs before the Big Bang Nucleosynthesis (BBN) time to avoid any disturbance in the nucleosynthesis process ($T_{BBN} \simeq 1$ MeV). Indeed, at least one possible channel of decay must include either a photon, a charged lepton or a meson, each of which would be energetic enough to destroy a nucleus if it strikes one. One can show that enough such energetic particles will be created in the decay as to destroy almost all the nuclei created in the era of nucleosynthesis, in contrast with observations. In fact, in such a case the universe would have been made of hydrogen alone, and star formation would probably be impossible.
From the expression of $\Gamma_{\tilde{G}}$ [23]

$$\Gamma_{\tilde{G}} = \frac{1}{4}\left(n_v + \frac{n_m}{12}\right)\frac{m_{3/2}^3}{M_P^2} = 24\left(\frac{10\text{ TeV}}{m_{3/2}}\right)^{3/2}\text{ sec} \tag{2.182}$$

where $n_v = 12$ and $n_m = 49$ are the number of vector and chiral matter multiplets, respectively. The condition $H(T_{3/2}) = \Gamma_{\tilde{G}}$ can then be written in the radiation dominated era

$$\begin{aligned} T_{3/2} &= \left(\frac{45}{4g_\rho\pi^3}\right)^{1/4}\sqrt{M_P\Gamma_{\tilde{G}}} = \left(\frac{45}{4g_\rho\pi^3 M_P^2}\right)^{1/4}\left(\frac{n_v + \frac{n_m}{12}}{32\pi}\right)^{1/2} m_{3/2}^{3/2} \\ &\simeq 0.24\left(\frac{10.75}{g_\rho(T_{3/2})}\right)^{1/4}\left(\frac{m_{3/2}}{10\text{ TeV}}\right)^{3/2}\text{ MeV} \end{aligned} \tag{2.183}$$

We thus can observe that for a gravitino lighter than 10 TeV, its decay happens at a temperature below T_{BBN}. This is what is called as the gravitino problem. As the result of the late time decays of the gravitino, the wino (or any supersymmetric particle which can play the role of the dark matter) is non–thermally produced at around $T_{3/2}$ which is lower than the freeze-out temperature of the wino. After the decay of all the gravitino, the resultant relic density for χ is given by the gravitino yield, Eq.(2.171) applied to the formula (2.180) which gives us

$$\Omega_\chi^{decay} h^2 \simeq 0.168 \times \left(\frac{m_\chi}{300\text{ GeV}}\right)\left(\frac{T_{RH}}{10^{10}\text{ GeV}}\right). \tag{2.184}$$

All together, the relic abundance of a wino (or any non-thermally produced dark matter from a decay) is the sum of the thermal production plus the non-thermal one

$$\Omega_\chi h^2 = \Omega_\chi^{thermal} h^2 + \Omega_\chi^{decay} h^2. \tag{2.185}$$

As a conclusion, the thermal history of the universe in the presence of a non-stable gravitino begins by the decoupling of the gravitino itself at a temperature T_d around T_{RH} when $H(T_d) \simeq \langle\sigma v\rangle_{SM\ SM\to\tilde{G}\ \tilde{G}}$. Then the dark matter candidate χ decouples (freezes out) at a temperature around $T_\chi = m_\chi/20$ which can give $\Omega_\chi h^2 \ll 0.1$, corresponding to $H(T_\chi) = \langle\sigma v\rangle_{SM\ SM\to\chi\ \chi}$. The population of dark matter is however still populated by the gravitino decay. Finally the end of the decay (kind of freeze out too) of the gravitino $\tilde{G} \to \chi\chi$ at $T_{3/2}$,

Figure 2.24: Evolution of the Yields of a wino dark matter and through decay of the gravitino

stabilized to obtain $\Omega_\chi h^2 \simeq 0.1$ at a temperature $T_{3/2}$ respecting $H(T_{3/2}) = \Gamma_{\tilde{G}\to\chi\chi}$. Several comments are in order. First, it should be noted that the entropy produced by the decay of the gravitino is negligible since the energy density of the gravitino at the decay time is subdominant. Second, it should be also noted that the annihilation of the wino after the non-thermal production is negligible, since the yield of the non-thermally produced wino is small enough. Indeed,

$$Y_\chi^{decay} \ll Y_\chi^{annihilation} \simeq \frac{H(T_{3/2})}{s\langle\sigma v\rangle} \simeq 10^{-9} \times \left(\frac{10^{-24}\ \mathrm{cm^3 s^{-1}}}{\langle\sigma v\rangle}\right)\left(\frac{1\ \mathrm{MeV}}{T_{3/2}}\right)$$

The dark matter is thus largely diluted and not able to annihilate anymore (no return to the thermal bath). A summary is shown in Fig.(2.24)

Example of the moduli decay in supersymmetry

It can happen that a heavy scalar field present in many supergravity extensions of the standard model decays into gravitino. In this case, there is teh danger of overproduction of gravitino. However, the decay of the modulus into gravitino also increase considerably the entropy of the Universe, diluting $Y3/2 = n_{3/2}/s$.

Example of the decay of a heavy state after inflation

2.6.5 A concrete example : heavy Z' in SO(10) models**

Introduction

The Standard Model (SM) of particle physics is now more than ever motivated by the recent discovery of the Higgs boson at both the ATLAS [25] and CMS [26] detectors. The SM, however, contains many free parameters, and the gauge couplings do not unify. Among the most elegant approaches to understand some of these parameters is the idea of a grand unified theory (GUT) in which the three gauge couplings $\alpha_{1,2,3}$ originate from a single gauge coupling associated to a grand unified gauge group [27]. This idea is supported by the fact that quantum numbers of quarks and leptons in the SM nicely fill representations of a GUT symmetry, e.g., the **10** and $\bar{\mathbf{5}}$ of $SU(5)$ or **16** of $SO(10)$.

Another issue concerning the SM is the lack of a candidate to account for Dark Matter (DM) which consists of 22 % of the energy density of our universe. Stable Weakly Interacting Massive Particles (WIMPs) are among the most popular candidates for DM. In most models, such as popular supersymmetric extensions of the SM [29], the annihilation of WIMPs in thermal equilibrium in the early universe determined the relic abundance of DM.

In this letter, we will show that GUT gauge groups such as E_6 or SO(10) which contain additional U(1) gauge subgroups and are broken at an intermediate scale, can easily lead to gauge coupling unification [30] and may contain a new dark matter candidate which is charged under the extra U(1). However, unlike the standard equilibrium annihilation process, or complimentary process of freeze-in [31], we propose an alternative mechanism for producing dark matter through interactions which are mediated by the heavy gauge bosons associated with the extra U(1). While being produced from the thermal bath, these dark matter particles never reach equilibrium. We will refer to dark matter produced with this mechanism as Non-Equilibrium Thermal Dark Matter or NETDM. The final relic abundance of NETDM is obtained shortly after the inflationary reheating epoch. This mechanism is fundamentally different from other non-thermal DM production mechanisms in the literature (to our knowledge). Assuming that none of the dark matter particles are directly produced by the decays of the inflaton during reheating, we compute the production of dark matter and relate the inflationary reheat temperature to the choice of the gauge group and the intermediate scale needed for gauge coupling unification. As an added benefit, the model naturally possesses the capability of producing a baryon asymmetry through leptogenesis, although that lies beyond of the scope of this work.
The letter is organized as follows. After a summary of the unified models under consideration in section II, we show how the presence of an intermediate scale allows for the possibility of producing a dark matter candidate which respects the WMAP constraint [13] and apply it to an explicit scenario in section III. A discussion of our main result is found in section IV before concluding in section V.

Unification in SO(10) models

The prototype of grand unification is based on the SU(5) gauge group. In an extension of SU(5) one can introduce SU(5) singlets as potential dark matter candidates. The simplest

extension in which singlets are automatically incorporated is that of SO(10). There are, however, many ways to break SO(10) down to $SU(3) \times SU(2) \times U(1)$. This may happen in multiple stages, but here we are mainly concerned with the breaking of an additional U(1) (or SU(2)) factor at an intermediate scale M_{int}. Here, we will not go into the details of the breaking, but take some specific, well-known examples when needed. Assuming gauge coupling unification, the GUT mass scale, M_{GUT}, and the intermediate scale M_{int} can be predicted from the low–energy coupling constants with the use of the renormalisation group equation,

$$\mu \frac{d\alpha_i}{d\mu} = -b_i \alpha_i^2 \ . \tag{2.186}$$

The evolution of the three running coupling constants α_1, α_2 and α_3 from M_Z to the intermediate scale M_{int} is obtained from Eq.(2.186) using the β–functions of the Standard Model: $b_{1,2,3} = (-41/10, 19/6, 7)/2\pi$. We note that the gauge coupling, g_D, associated with $U'(1)$ is related at the GUT scale to g_1 of $U(1)_Y$ by $g_D = \sqrt{\frac{5}{3}} g_1$ and $\alpha_i = g_i^2/4\pi$. Between M_{int} and M_{GUT} (both to be determined) the running coupling constants are again obtained from Eq.(2.186), now using β–functions associated with the intermediate scale gauge group, which we will label $\tilde{b}_i$. The matching condition between the two different runnings at M_{int} can be written:

$$(\alpha_i^0)^{-1} + b_i(t_{int} - t_Z) = \alpha^{-1} + \tilde{b}_i(t_{int} - t_{GUT}) \tag{2.187}$$

with $t_{int} = \ln M_{int}$, $t_Z = \ln M_Z$, $t_{GUT} = \ln M_{GUT}$, $\alpha_i^0 = \alpha_i(M_Z)$ which is measured, and $\alpha = \alpha_i(M_{GUT})$ is the unified coupling constant at the GUT scale. This gives us a system of 3 equations, for 3 unknown parameters: α, t_{int}, t_{GUT}. Solving the Eq.(2.187), we obtain

$$t_{int} = \frac{1}{b_{32} - b_{21}} \left[\frac{(\alpha_3^0)^{-1} - (\alpha_2^0)^{-1}}{\tilde{b}_2 - \tilde{b}_3} - \frac{(\alpha_2^0)^{-1} - (\alpha_1^0)^{-1}}{\tilde{b}_1 - \tilde{b}_2} \right.$$
$$\left. + (b_{32} - b_{21}) t_Z \right] , \tag{2.188}$$

where $b_{ij} \equiv (b_i - b_j)/(\tilde{b}_i - \tilde{b}_j)$.

To be concrete, we will consider a specific example to derive numerical results for the case of the breaking of $SO(10)$: $SO(10) \to SU(4) \times SU(2)_L \times U(1)_R \to_{M_{int}} SU(3)_C \times SU(2)_L \times U(1)_Y \to_{M_{EW}} SU(3)_C \times U(1)_{em}$. When the intermediate symmetry is broken by a **16** of Higgs bosons, the $\tilde{b}_i$ functions are given by $\tilde{b}_{1,2,3} = (5/2, 19/6, 63/6)/2\pi$. For this case, we obtain $M_{int} = 7.8 \times 10^{12}$ GeV and $M_{GUT} = 1.3 \times 10^{15}$ GeV using $(\alpha_{1,2,3}^0)^{-1} \simeq (59.47, 29.81, 8.45)$. The evolution of the gauge couplings for this example is shown in Fig. 2.25.

Heavy Z' and dark matter

It has been shown in [32] and [33] that a stable dark matter candidate may arise in SO(10) models from an unbroken Z_2^{B-L} symmetry. If the dark matter is a fermion (scalar) it should belong to a $3(B-L)$ even (odd) representation of SO(10). For example, the **126** or **144** contains a stable component χ which is neutral under the SM, yet charged under the extra U(1). As we have seen, to explain the unification of the gauge couplings in SO(10) one needs an intermediate scale M_{int} of order 10^{10} GeV. The dark matter candidate, χ, can be

Figure 2.25: Example of the running of the SM gauge couplings for $SO(10) \to SU(4) \times SU(2)_L \times U(1)_R$.

produced in the early Universe through s-channel Z' exchange: $SM\ SM \to Z' \to \chi\ \chi$. Since $M_{Z'} = \frac{5}{\sqrt{3}} g_D\ M_{int}$, the exchanged particle is so heavy (above the reheating scale, as we show below) that the DM production rate is very slow, and we can neglect the self annihilation process in the Boltzmann equation. Thus while the dark matter is produced from the thermal bath, we have a non–equilibrium production mechanism for dark matter, hence NETDM.

The evolution of the yield of χ, $Y_\chi = n_\chi/\mathbf{s}$ follows

$$\frac{dY_\chi}{dx} = \sqrt{\frac{\pi}{45}} \frac{g_s}{\sqrt{g_\rho}} m_\chi M_P \frac{\langle \sigma v \rangle}{x^2} Y_{eq}^2 \tag{2.189}$$

where n_χ is the number density of χ and $\mathbf{s}$ the entropy of the universe, g_ρ, g_s are the effective degrees of freedom for energy density and entropy, respectively; $x = m_\chi/T$, m_χ being the dark matter mass, M_P the Planck mass and

$$\langle \sigma v \rangle n_{eq}^2 \approx \frac{\kappa^2\ T}{2048\pi^6} \int_{4m_\chi^2}^{\infty} d\Omega ds \sqrt{s - 4m_\chi^2} |\mathcal{M}|^2 K_1(\sqrt{s}/T)\ . \tag{2.190}$$

Here n_{eq} is the equilibrium number density of the initial state (SM) particles; and K_1 is the first order modified Bessel function and κ the effective degrees of freedom of incoming particles.

Since the production of DM occurs mainly at $T_{RH} \gg m_\chi$, we can neglect m_χ in estimating the amplitude for production. In this case, assuming that both χ and the initial state, f, are fermions, we obtain

$$|\mathcal{M}_\chi|^2 \approx \frac{g_D^4 q_\chi^2 q_f^2 N_c^f}{(s - M_{Z'}^2)^2} \left[s^2 (1 + \cos^2\theta) \right] \tag{2.191}$$

where θ is the angle between the two outgoing DM particles, N_c^f is number of colors of the particle f, and q_i is the charge of the particle i under $U'(1)$ with a gauge coupling g_D. Here, q is an effective coupling which will ultimately depend on the specific intermediate gauge group chosen. With the approximations $m_\chi, m_f \ll \sqrt{s}$ and $M_{Z'} \gg T_{RH}$, and after integration over θ and sum over all incoming SM fermions in the thermal bath, we obtain

$$\frac{dY_\chi}{dx} = \sum_f \frac{g_D^4 q_\chi^2 q_f^2 N_c^f}{x^4} \left(\frac{45}{\pi}\right)^{3/2} \frac{1}{g_s\sqrt{g_\rho}} \frac{m_\chi^3 M_P}{M_{Z'}^4} \frac{\kappa_f^2}{2\pi^7} \tag{2.192}$$

Solving Eq.(2.192) between the reheating temperature and a temperature T gives

$$Y_\chi(T) = \sum_f q_\chi^2 q_f^2 N_c^f \left(\frac{45}{g_s\pi}\right)^{3/2} \frac{M_P}{M_{int}^4} \frac{3\ \kappa_f^2}{1250\pi^7} \left[T_{RH}^3 - T^3\right] \tag{2.193}$$

where we replaced the mass of the Z' by $M_{Z'} = \frac{5}{\sqrt{3}} g_D M_{int}$ and made the approximation $g_\rho = g_s$. We note that the effect of Z' decay on the abundance of χ is completely negligible due to its Boltzmann suppression in the Universe: the Z' is largely decoupled from the thermal bath already at the time of reheating.

We note several interesting features from Eq.(2.193). First of all, the number density of the dark matter does not depend at all on the strength of the $U'(1)$ coupling g_D but rather on the intermediate scale (that is determined by requiring gauge coupling unification as we demonstrated in the previous section). Second, the production of dark matter is mainly achieved at reheating. Thirdly, once the relic abundance is obtained, the number density per comoving frame (Y) is fixed, never having reached thermal equilibrium with the bath. And finally, upon applying the WMAP determination for the DM abundance, we obtain a tight constraint on T_{RH} once the pattern of $SO(10)$ breaking is known (and thus M_{int} fixed).

Thus, given a scheme of $SO(10)$ breaking we can determine the reheating temperature very precisely from the relic abundance constraint in the Universe. From

$$Y_0 = \frac{\Omega}{m_\chi} \frac{\rho_0^{crit}}{\mathbf{s}_0} = \left(\frac{\Omega}{0.1}\right) \frac{13.5}{16\pi^3} \frac{H_0^2 M_P^2}{g_s^0 T_0^3 m_\chi} \tag{2.194}$$

where H is the Hubble parameter and the index "0" corresponds to present-day values. Combining Eq.(2.193) and Eq.(2.194) we find

$$T_{RH}^3 = \frac{5625\ \pi^4}{16 q_\chi^2 \sum_f \kappa_f^2 q_f^2 N_c^f} \left(\frac{\Omega}{0.1}\right) \left(\frac{g_s\pi}{45}\right)^{3/2} \frac{M_P H_0^2}{T_0^3 m_\chi g_s^0} M_{int}^4 \tag{2.195}$$

or

$$T_{RH} \simeq 2\times 10^{-8}\text{GeV} \left(\frac{\Omega h^2}{0.1}\right)^{1/3} \left(\frac{100\ \text{GeV}}{m_\chi}\right)^{1/3} \left(\frac{M_{int}}{1\ \text{GeV}}\right)^{4/3} \tag{2.196}$$

where we took for illustration $q_\chi^2 \sum_f \kappa_f^2 q_f^2 N_c^f = 1$. We show in Fig.(2.26) the evolution of T_{RH} as function of M_{int} for different values of the dark matter mass m_χ. We can thus determine the reheating temperature predicted by different symmetry breaking patterns[32]. We summarize them in Table 10.1, where the values of T_{RH} are given for $m_\chi = 100$GeV.

Figure 2.26: Reheating temperature as function of the SO(10) breaking scale for different mass of dark matter : 10, 100 and 1000 GeV

Table 2.3: Possible breaking schemes of SO(10).

	$SO(10) \to \mathcal{G}\times$ [Higgs]	M_{int}(GeV)	T_{RH}(GeV)
A	$4 \times 2_L \times 1_R$ [**16**]	$10^{12.9}$	3×10^9
A	$4 \times 2_L \times 1_R$ [**126**]	$10^{11.8}$	1×10^8
B	$4 \times 2_L \times 2_R$ [**16**]	$10^{14.4}$	3×10^{11}
B	$4 \times 2_L \times 2_R$ [**126**]	$10^{13.8}$	5×10^{10}
C	$3_C \times 2_L \times 2_R \times 1_{B-L}$ [**16**]	$10^{10.6}$	3×10^6
C	$3_C \times 2_L \times 2_R \times 1_{B-L}$ [**126**]	$10^{8.6}$	6×10^3

Finally, we must specify the identity of the NETDM candidate in the context described above. The DM can be in the **126** or **144** representations of SO(10). There are several mechanisms to render the DM mass light [33], one of which is through a fine-tuning of the SO(10) couplings contributing with different Clebsh-Gordan coefficients (see for example, [**?**]) to the masses of the various **126** components. For example, for the group $\mathcal{G}_A$:

$$\overline{\mathbf{126}}(M + y_{45}\mathbf{45}_H + y_{210}\mathbf{210}_H)\mathbf{126} \tag{2.197}$$

where $M \sim M_{GUT}$, and a $\mathcal{G}_A$ singlet in a linear combination of $\mathbf{210}_H$ and $\mathbf{45}_H$ has a vev at the GUT scale. m_χ is then given by a linear combination of M and the vev and can be tuned to small values, while all other particles inside the **126** live close to M_{GUT}.

[32]We note that the value obtained for the intermediate scale in different $SO(10)$ breaking schemes is not modified by the presence of a dark matter particle which is not charged under the SM gauge group.

Discussion

Unfortunately, the chance of detection (direct or indirect) of NETDM with a massive mediator Z' is nearly hopeless. Indeed, the diagram for the direct detection process, measuring the elastic scattering off a nucleus, proceeds through the $t-$channel exchange of the Z' boson, and is proportional to $1/M_{Z'}^4$ yielding a negligible cross-section. In addition, due to the present low velocity of dark matter in our galaxy ($\simeq$ 200 km/s), the indirect detection prospects from $s-$channel Z' annihilation $\chi\chi \to Z' \to ff$ proportional to $s^2/M_{Z'}^4$ is also negligible.

As we have seen in Eq.2.193, the production of dark matter occurs in the very early Universe at the epoch of reheating. A similar mechanism (though fundamentally completely different) where a dark matter candidate is produced at the reheating time is the case of the gravitino [29,34]. Indeed, in both cases equilibrium is never reached and the relic abundance is produced from the thermal background to attain the decoupling value Γ/H, with H the Hubble constant and $\Gamma = \langle\sigma v\rangle n_f$ the production rate. However, in the case of SO(10), the cross section decreases with the temperature like $\langle\sigma v\rangle_{Z'} \propto T^2/M_{Z'}^4$, in the case of the gravitino the cross section is constant $\langle\sigma v\rangle_{3/2} \propto 1/M_P^2$ implying $Y(T) \propto T_{RH}$.

Finally, we note that cases B and C (in Table I) predict reheating temperatures which are larger (B) or smaller (C) than the case under consideration. Case A would also be compatible with successful thermal leptogenesis with a zero initial state abundance of right–handed neutrino [35, 36]. However in the cases B and C, the persistence of the $\mathrm{SU}(2)_R$ symmetry would imply that the cancelation in Eq. 2.197 would leave behind a light $\mathrm{SU}(2)_R$ triplet (for DM inside a **126**) or doublet (for DM inside a **144**). These would affect somewhat the beta functions for the RGE's but more importantly leave behind a test of the model. In the triplet (doublet) case, we would expect three (at least two) nearly degenerate states with charge 0, being the DM candidate, and also states with electric charge ±1 and ±2 (or ±1 in the doublet case).

Conclusion

In this work, we have shown that it is possible to produce dark matter through non–equilibrium thermal processes in the context of $SO(10)$ models which respect the WMAP constraints. Insisting on gauge coupling unification, we have demonstrated that there exists a tight link between the reheating temperature and the scheme of the $SO(10)$ breaking to the SM gauge group. Interestingly, the numerical values we obtained are quite high and very compatible with inflationary and leptogenesis-like models.

2.6.6 Transfer of energy and thermalization

Generalities

There exist a second Boltzmann equation which concern the transfer of energy. If a set of particles i are all produced by the same mechanism and the scattering is more efficient than the expansion of the Universe, they naturally reach a common thermal temperature (the one of the photon), $T = T_i$. However, in scenario where the a dark sector is not produced with the standard model ones or very feebly coupled with the visible sector, the temperature of

Figure 2.27: Illustration of the boltzmann equation describing the transfer of energy from the Standard Mode bath to the dark bath.

the dark sector increase slowly due to the transfer of the energy from the thermal bath to the dark bath of energy density ρ' and pression P'. If the density of dark particles is very small at the reheating time, the boltzmann equation for the transfer from the thermal bath energy density ρ to ρ' from $1+2 \to 3+4$ reaction can be written

$$\frac{d\rho'}{dt} + 3H(\rho' + P') = n_{EQ}^2 \langle \sigma v (E_1 + E_2) \rangle \tag{2.198}$$

with n_{EQ} being the density of particles in the bath. The phenomenon is illustrated in Fig.(2.27). The right hand side of Eq.(2.198) should also contents terms of proportional to n_{dark}^2 for the inverse transfer process $\rho'(3+4) \to \rho(1+2)$ but as we considered the dark bath composed of particles of mass M with feeble interactions with the thermal bath, this processes can be neglected. It is very similar than in the case of FIMP or heavy mediators discussed is section 2.6. With the hypothesis that the thermal bath is at equilibrium, combining Eq.(2.198), Eq.(2.33) and the fact that in the relativistic case[33] $P' = \rho'/3$ and $\frac{d}{dt} = -HT\frac{d}{dT}$

$$\rho \frac{d\,(\rho'/\rho)}{dT} = \frac{d\rho'}{dT} - 4H\rho' \quad \Rightarrow \quad \frac{d(\rho'/\rho)}{dT} = -\frac{n_{EQ}^2}{\rho H T} \langle \sigma v (E_1 + E_2) \rangle \tag{2.199}$$

with

[33]Notice that we made the supposition in this section that the dark matter bath is in thermal equilibrium with itself (with a temperature T') or from the self–scattering of the dark matter on itself or from the scattering of other particles of the dark sector on themselves, i.e. $\Gamma_{\text{scatter}}(T) > H(T)$. For a more detailed description of the mechanism of thermalization, see the next section.

$$\langle \sigma v(E_1+E_2)\rangle = \frac{1}{n_{EQ}^2}\int f_1(E_1)f_2(E_2)\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{|\bar{\mathcal{M}}|^2}{128\pi^2 E_1E_2}(E_1+E_2)d\Omega \tag{2.200}$$

$\bar{\mathcal{M}}$ being the mean of the amplitude squared on the spin of the initial particles and f_i their statistical distribution. Following the procedure to obtain Eq.(2.132) we can write

$$\begin{aligned}\langle \sigma v(E_1+E_2)\rangle &= \frac{g_1g_2}{32(2\pi)^6}\int_{4M^2}^{\infty}\sqrt{s-4M^2}|\bar{\mathcal{M}}|^2 s\ dsd\Omega\int_1^{\infty} t\sqrt{t^2-1}e^{-t\frac{\sqrt{s}}{T}}dt\\ \frac{d(\rho'/\rho)}{dT} &= -15\sqrt{\frac{45}{\pi}}\frac{g_1g_2\ M_P}{1024\pi^9T^3g_\rho^{3/2}}\int_{2\frac{M}{T}}^{\infty}|\bar{\mathcal{M}}|^2x^2\sqrt{x^2T^2-4M^2}\ K_2(x)\ dxd\Omega\end{aligned} \tag{2.201}$$

g_1 and g_2 being the internal degrees of freedom of particles 1 and 2. We can then divide the integral (2.201) in two regimes: The first one the particle can be considered as relativisitic ($M \ll T$), and the second one when the temperature drops below the mass of the dark matter ($T \ll M$) where the integral in Eq.(2.201) tends to 0:

$$\begin{aligned}\frac{d(\rho'/\rho)}{dT} &= -15\sqrt{\frac{45}{\pi}}\frac{g_1g_2\ M_P}{1024\pi^9T^2g_\rho^{3/2}}\int_0^{\infty}|\bar{\mathcal{M}}|^2x^3\ K_2(x)\ dxd\Omega && [\mathrm{M} \ll \mathrm{T}]\\ \frac{d(\rho'/\rho)}{dT} &= 0 && [\mathrm{T} \ll \mathrm{M}]\end{aligned} \tag{2.202}$$

A specific case: exchanged of a massless gauge boson

To illustrate the general result obtained in Eq.(2.201) we will take a simple toy model, based on mirror dark matter, where the dark matter, a dirac fermion of mass M_χ, is charged under the electromagnetic charge with a coupling δ. this happens for instance in the case of the presence of a kinetik mixing between an extra U(1)[dark photon] and photon. This is also called "millicharged dark matter" as it is equivalent to suppose that the dark matter is very feebly coupled to the electromagnetic fields (coupling of the order $\delta \ll 1$). The amplitude can then be written

$$\mathcal{M} = \delta\bar{\chi}\gamma^\mu f\frac{\eta_{\mu\nu}}{(p_1+p_2)^2}q_f e\bar{f}\gamma^\nu f \tag{2.203}$$

p_i being the quadrivector of the incoming particles. We than can compute

$$\begin{aligned}|\bar{\mathcal{M}}|^2 = 16\ q_f^2e^2\delta^2\left[(1+\cos^2\theta)+\frac{M_\chi^2}{s}(1-\cos^2\theta)\right. &\Rightarrow \int|\bar{\mathcal{M}}|^2\ d\Omega = 128\pi\left[\frac{2}{3}+\frac{1}{3}\frac{M_\chi^2}{s}\right]\\ \Rightarrow\ \frac{d(\rho'/\rho)}{dT} = -\frac{5}{8}\sqrt{\frac{45}{\pi}}\frac{g_1g_2M_Pq_f^2e^2\delta^2}{\pi^8T^3g_\rho^{3/2}}&\int_{2\frac{M_\chi}{T}}^{\infty}\left(2+\frac{M_\chi^2}{T^2}\right)x^2\sqrt{x^2T^2-4M_\chi^2}\ K_2(x)\ dx\end{aligned}$$

Noticing that $\int x^3K_2(x)dx = 8$, after integration on T we obtain, in the regime $T \gg M_\chi$ for dirac annihilating particles ($g_1 = g_2 = 4$) and $q_f = 1$ as an example

$$\frac{d(\rho'/\rho)}{dT} = -2560\sqrt{\frac{45}{\pi}}\frac{\alpha\alpha_D M_P}{\pi^6 T^2 g_\rho^{3/2}} \quad \Rightarrow \quad \left(\frac{\rho'}{\rho}\right) = 2560\sqrt{\frac{45}{\pi}}\frac{\alpha\alpha_D M_P}{\pi^6 T g_\rho^{3/2}} \tag{2.204}$$

with $\alpha = e^2/4\pi$ and $\alpha_D = \delta^2/4\pi$. We made the hypothesis that $\rho' \simeq 0$ at the beginning of the transfers, and integrated from T_{RH} until T considering $T_{RH} \gg T$.

We have to notice that we neglected in the second term of Eq.(2.200) a second term in the right hand side corresponding to the inverse transfer from the dark bath to the Standard Model (photons) bath. This comes from the fact we supposed that the dark system has not been populated by dark particles from the beginning of the thermal history (no coupling to the inflaton for instance). Once the density of dark particles reach (through the boltzmann equation describing the evolution of the dark thermal density of particles) an equilibrium appears between ρ' and ρ similar to the one for the Yields Y_i: $d(\rho'/\rho)/dT \simeq 0 \;\Rightarrow\; \rho' \simeq \rho$ and ρ' follows then the evolution of ρ as function of the temperature T. As the we supposed the dark bath is in thermal equilibrium ($\Gamma_{\text{scatter}}(T) > H(T)$), we can use Eq.(2.33) and write $\frac{\rho'}{\rho} = \left(\frac{T'}{T}\right)^4$. Eq.(2.204) then becomes

$$T' = (\alpha\alpha_D)^{1/4}(2560)^{1/4}\left(\frac{45}{\pi}\right)^{1/8}\frac{M_P^{1/4}T^{3/4}}{\pi^{3/2}g_\rho^{3/8}} \;\Rightarrow\; \left(\frac{T'}{1\text{ GeV}}\right) \simeq\; 3000\,\sqrt{\delta}\left(\frac{T}{1\text{ GeV}}\right)^{3/4} \tag{2.205}$$

The numerical result is illustrated in Fig.(2.28) where we plotted $\frac{T'}{T}$ as a function of T for $M_\chi = 100$ GeV and different values of δ from 10^{-6} down to 10^{-9}. We clearly see the two regimes $T \ll M_\chi$ and $T \gg M_\chi$ with the two behaviors given by Eqs.(2.202) and (2.205) respectively. We also notice that tha value $\delta \simeq 10^{-6}$ is the limit value for which the dark bath do not enter in thermal equilibrium with the thermal bath. Indeed, for any values of δ larger than 10^{-6}, the teperature T' (and thus the density of energy ρ') will reach T (ρ respectively) and then form a common thermal bath. Similar work can be find in [37].

Thermalization

In the previous sections, we always made the supposition (explicitly or implicitly) that our system(s) we were studying were in thermal equilibrium. We can check when this condition is realized and, if not realized, how the dark bath reach equilibrium through a (somewhat) complex thermalization process. The condition for the dark bath to be in thermal equilibrium is realized when $\Gamma_{scatter/dark\leftrightarrow dark}(T) > H(T)$. It means that before the time the Universe can double, we already had (at least) one scattering. We show the different scattering processes in Fig.(2.29). The scattering is present to keep the distribution function of the systems of particles in the equilibrium (Boltzmann, or Fermi-Dirac or Bose-Einstein) state.

The processes to maintain equilibrium is the scattering process. This processes depicted in Fig.(2.29) are equivalent to the Bhabha scattering computed in the section (C.7.6), replacing the electromagnetic coupling e by the dark one g_D. To a good approximation, one can keep the s–channel exchange diagram and applying Eq.(2.134)

Figure 2.28: Evolution of T', temperature in the dark bath as function of T for different values of the coupling δ and for $M_\chi = 100$ GeV.

Figure 2.29: Feynman diagrams of th $\chi\chi \to \chi\chi$ scattering which contributes to the thermalisation in the dark sector of temperature T'

Figure 2.30: Region of the parameter space (T, δ), T being the temperature of the Standard Model bath, where we can consider that the dark bath of temperature T' is in equlibrium with itself through scattering.

$$|\bar{\mathcal{M}}|^2 \simeq g_D^4(1+\cos^2\theta) \quad \Rightarrow \quad \langle\sigma v\rangle n_\chi^2 = \frac{T' g_\chi^2}{32(2\pi)^6}\int \sqrt{s}K_1\left(\frac{\sqrt{s}}{T'}\right)|\bar{\mathcal{M}}|^2 ds d\Omega = \frac{g_D^4 T'^4 g_\chi^2}{3(2\pi)^6} \tag{2.206}$$

Taking the hypothesis that the dark matter is in equilibrium at a temperature T'. We want to know for which temperature T the condition of equilibrium $\Gamma_{scat}(T) => H(T)$. We then obtain at equilibrium using Eq.(2.37), $n_\chi = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_\chi T'^3$:

$$\Gamma_{scat}(T') = n_\chi \langle\sigma v\rangle_{scat} \simeq \frac{g_D^4 T' g_\chi}{9\zeta(3)(2\pi)^4} \tag{2.207}$$

Using the expression of $H(T)$ given by Eq.(2.60) and the relation between T' and T we obtained[34] in Eq.(2.205)

$$\frac{\Gamma_{scat}(T')}{H(T)} = \frac{g_D^4 g_\chi}{9\zeta(3)(2\pi)^5}\sqrt{\frac{45}{\pi}}\frac{M_P}{T}\left(\frac{T'}{T}\right) \simeq 2.6\times 10^{14}\left(\frac{g_D}{0.1}\right)^4 \sqrt{\delta}\left(\frac{1\text{ GeV}}{T}\right)^{5/4} \tag{2.208}$$

We show in Fig.(2.30) the region of the parameter space where thermal equilibrium is satisfied. We notice that the temperature T of the Standard Model bath from which the thermal equilibrium of the dark bath of temperature T' is attained is smaller for small values of δ

[34] We took dirac fermions for the dark matter and particles 1 and 2: $g_1 = g_2 = g_\chi = 4$

and of g_D. The reasons are different for both cases. Indeed, small delta imply a few energy transfer between the two systems, and thus a low temperature T', and as a consequence, a low density of dark particles: the scattering is thus not sufficiently efficient to bring the new created dark matter from annihilation of the Standard Model in the thermal bath into a new dark equilibrium state. A small value of g_D renders the scattering less efficient also but due from the too weak coupling of the self interaction of the dark matter on itself. We can also remark that the condition of thermal equilibrium we supposed to obtain Fig.(2.28) is valid. We summarize in Fig.(2.31) the process of thermalization: the dark mater χ is produced from the thermal (Standard Model) bath with a distribution of energy dn_χ/dE_γ which follows the distribution of energy of the photons from the thermal bath, by conservation of energy : $E_\gamma = E_\chi$, so $(T_\chi)_{after\ production} \simeq T_\gamma = T$. However, the interaction between the dark matter and the photon is very weak (proportional to δ) and thus, the scattering of the photons on the dark matter is not sufficient to maintain the thermal equilibrium of χ. However, other χ particles already produced before and in dark equilibrium at a temperature T' between themselves can scatter on the newly produced χ to brings it into the dark bath and to follow the new distribution of energies dn_χ/dE_χ which defines the temperature T'. But in the meantime, there was a small transfer of energy between the thermal bath to the dark bath which has increased a little bit $T' \to T' + \epsilon$. Then the process loops until T' reaches T and then the dark matter enters in equilibrium with the thermal bath, or until the the temperature of the thermal bath T drops below the dark matter mass M_χ: the photons are then not sufficiently energetic anymore to heat the dark bath.

γ' entering in the dance

After some time the γ' will become sufficiently abundant to enter in thermal, and chemical equilibrium with the dark matter χ. When it was only the mediator of the self–scattering process, its density was not of real interest for the evolution of T' or n_χ. However, once it enters in the dark bath ($\Gamma_{\chi\chi\leftrightarrow\gamma'\gamma'}(T) > H(T)$), its effect on T' will be equivalent to the inverse process of the neutrino decoupling (see section 5.6). Whereas neutrino gives its degrees of freedom to heat the bath of photons when it decouple, in this case, from conservation of entropy, the inclusion of the γ' in the thermal bath will *decrease* the temperature T' from entropy conservation in the dark system, S' :

$$S'_{\text{before}} \propto g_\chi (T'_{\text{before}})^3; \quad S'_{\text{after}} \propto (g_\chi + g_{\gamma'})(T'_{\text{after}})^3 \;\Rightarrow\; T'_{\text{after}} = T'_{\text{before}} \times \left(\frac{g_\chi}{g_\chi + g_{\gamma'}}\right)^{1/3} \tag{2.209}$$

This result can also be observed from the Boltzmann equation where a new term expressing the effect of γ' on the abundance of n_χ from its annihilation enters in the game.

Conclusion

We understood in this section how important is the process of thermalization, and how complicated can be the mechanism if it involved several species of particles. 3 kind of equilibrium can co-exist in the primordial plasma: kinetic equilibrium, where all the particles share the same distribution of energy, chemical equilibrium where the number density of particles are

Figure 2.31: Summarize of the different steps leading to thermalization of a dark bath.

maintained constant from annihilation/production processes and thermal equilibrium which is the state of kinetic + chemical equilibrium.

2.7 Dark radiation

A lot of work have been published focussing on increasing the effective number of neutrino species in the Universe N_{eff}, trying to solve the "issue" observed by spectrum of WMAP and ACT telescopes. Indeed, the relativistic degrees of freedom in [] has been measured $N_{eff} = 3.7^{+0.8}_{-0.7}(2\sigma)$ from their inferred abundance of primordial ^{4}He. The Atacama Cosmology Telescope (ACT) [] and the South ole Telescope [] have measured a damping tail of the CMB spectrum and, in combination with data from WMAP [], Baryon acoustic oscillation (BAO) [] and the Hubble constant H_0 [] also find a preference for a value of N_{eff} greater than three, obtaining $N_{eff} = 4.6 \pm 0.8$ (1σ) and $N_{eff} = 3.9 \pm 0.4$ (1σ) respectively.
The simpler solution is to introduce new particles, which decouple at a temperature above the one of the Standard Model neutrino decoupling. The cosmic radiation content is usually expressed in term of the effective number of thermally excited neutrino species, N_{eff} with its standard value being $N_{eff} = 3.046$ The number of relativistic degrees of freedom of a plasma containing the standard relativistic particles (photon γ and neutrino ν) with a new candidate $\tilde{\nu}$ can be written from Eq.(2.37) :

$$\begin{aligned}\rho_R = \rho_\gamma + \rho_\nu + \rho_{\tilde{\nu}} &= \frac{\pi^2}{30}(g_\gamma T_\gamma^4 + 3 \times \frac{7}{8} g_\nu T_\nu^4 + N_{\tilde{\nu}} \times \frac{7}{8} g_{\tilde{\nu}} T_{\tilde{\nu}}^4) \\ &= \frac{\pi^2}{30}\left(g_\gamma T_\gamma^4 + \frac{7}{8} g_\nu T_\nu^4 \left[3 + N_{\tilde{\nu}} \frac{g_{\tilde{\nu}}}{g_\nu}\left(\frac{T_{\tilde{\nu}}}{T_\nu}\right)^4\right]\right) = \frac{\pi^2}{30}\left(g_\gamma T_\gamma^4 + \frac{7}{8} g_\nu T_\nu^4 N_{eff}\right)\end{aligned} \tag{2.210}$$

with g_i the internal degree of freedom of particle i (2 for photons and neutrino) and $N_{\tilde{\nu}}$ the number of types (generations) of particles $\tilde{\nu}$ and

$$N_{eff} = 3 + N_{\tilde{\nu}} \frac{g_{\tilde{\nu}}}{g_\nu}\left(\frac{T_{\tilde{\nu}}}{T_\nu}\right)^4 = 3 + \Delta N_{eff} \tag{2.211}$$

Following the procedure described in the section (5.6) for the decoupling of the standard neutrino, we need to impose the conservation of entropy $g_s^i T_i^3 =$ constant :

$$g_s^{\gamma\nu\tilde{\nu}} T_{\tilde{\nu}}^3 = g_s^{\gamma\nu} T_\nu^3, \qquad g_s^{\gamma\nu} T_\nu^3 = g_s^\gamma T_\gamma^3 \quad \Rightarrow \quad \frac{T_{\tilde{\nu}}}{T_\nu} = \left(\frac{g_s^{\gamma\nu}}{g_s^{\gamma\nu\tilde{\nu}}}\right)^{1/3} \quad \Rightarrow \Delta N_{eff} = N_{\tilde{\nu}} \frac{g_{\tilde{\nu}}}{g_\nu}\left(\frac{g_s^{\gamma\nu}}{g_s^{\gamma\nu\tilde{\nu}}}\right)^{4/3}$$

with the notation $g_s^{\gamma\nu\tilde{\nu}}$ being the number of degrees of freedom while γ, ν and $\tilde{\nu}$ are still coupled while $g_s^{\gamma\nu}$ corresponds to the number of degrees of freedom when only γ and ν are coupled. This is equivalent to the phenomena of the electron decoupling transferring its degrees of freedom to the temperature of the photon. Indeed the $\tilde{\nu}$ particle is massive and couple to the neutrino and photons (through the exchange of a Z' for instance) and, as it is massive, when it decouples, $\tilde{\nu}$ disappears and thus heat the plasma. As an example we can imagine a family of right handed neutrino ($N_{\tilde{\nu}} = 3$, $g_{\tilde{\nu}} = 2$) decoupling at a temperature

of 3 GeV ($g_s^{\gamma\nu\tilde{\nu}}(3\text{ GeV}) = 86$), remembering that[35] $g_s^{\gamma\nu}(3\text{ MeV}) = 43/4$ we obtain $\Delta N_{eff} = 3 \times (1/8)^{4/3} \simeq 0.2$. A study in the SO(10) framework can be found in [41].

2.7.1 One exemple to increase N_{eff}

In [42], the authors proposed another solution. Indeed, we showed that introducing a new relativistic particle increase the temperature of the plasma when decoupling and thus The other possibility is to introduce a new particle χ which can play for the neutrino the same role than the electron for the photon. In other words, if a χ of mass $m_\chi \sim 10$ MeV couples *only* with neutrino, it decouple from the plasma *after* the neutrino decoupling temperature of 1 MeV ($m_\chi/20 \sim 0.5$ MeV) it will reheat only the neutrino temperature without modifying the photon one, thus increasing N_{eff}. The entropy can thus be decomposed, at the temperature of the neutrino system decoupling T_D into 2 different entropies S^γ and S^ν. Each entropy are thus constant after T_D

$$S = S^\gamma + S^\nu; \quad S^\gamma = g_s^\gamma T_\gamma^3 a^3; \quad S^\nu = g_s^\nu T_\nu^3 a^3 \Rightarrow \frac{T_\nu^3}{T_\gamma^3} = \frac{S^\nu}{S^\gamma}\frac{g_s^\gamma}{g_s^\nu} \tag{2.212}$$

we can determine $\frac{S^\nu}{S^\gamma}$ noticing that at $T = T_D$, $T_\gamma = T_\nu$ we then can write

$$\frac{T_\nu^3}{T_\gamma^3} = \frac{g_s^\nu|_{T_D}}{g_s^\gamma|_{T_D}}\frac{g_s^\gamma}{g_s^\nu} \quad \Rightarrow \quad \frac{T_\nu}{T_\gamma} = \left(\frac{g_s^\nu|_{T_D}}{g_s^\gamma|_{T_D}}\frac{g_s^\gamma}{g_s^\nu}\right)^{1/3}. \tag{2.213}$$

We can notice indeed that if the neutrino is alone in its own plasma, $g_s^\nu|_{T_D} = g_s^\nu$ as neutrino of course cannot decouple from itself!! and we recover the result $\frac{T_\nu}{T_\gamma} = (4/11)^{1/3}$. Looking at Eq.(2.213) it became obvious that if neutrino is in equilibrium in a system isolated from the thermal plasma, if one particle decouples from the neutrino plasma, the neutrino temperature increases, and this the effective degree of freedom: increasing the temperature of the neutrino is equivalent to believe that the neutrino temperature is still 1.91 K but with a higher N_{eff}.

2.7.2 Another example: the case of the mirror dark matter

The mirror dark matter arises in models where you suppose a completely "mirrored" world from the standard model like q' quarks or ν' neutrinos, even mirrored hydrogen H' or oxygen O'. The interaction between the mirror world and our visible Universe is only due to gravity (except if one allows for some kinetic mixing between the photon γ and γ'). For a complete review of such a construction see [43]. Both world are supposed to be simultaneously in thermal equilibrium but not in thermal equilibrium between them. On can then define two different temperatures (and entropies) T and S for the visible thermal bath, and T', S' for the mirror bath. Both entropy being constant, and the volume being obviously the same, we can deduce that

$$\frac{S'}{S} = \frac{a^3 s'}{a^3 s} = \frac{s'}{s} = \frac{g_s(T')T'^3}{g_s(T)T^3} \equiv x^3 = \text{constant} \quad \Rightarrow \quad \frac{T'}{T} = x \times \left(\frac{g_s(T)}{g_s(T')}\right)^{1/3} \tag{2.214}$$

[35] corresponding to 3 active neutrinos, $e^\pm$ and photons.

One can then deduce the total radiation density $\bar{\rho}_R$ including the mirror bath

$$\bar{\rho}_R = \frac{\pi^2}{30}(g_\rho T^4 + g'_\rho T'^4) = \frac{\pi^2}{30} T^4 g_\rho \left[1 + \frac{g'_\rho(T')}{g_\rho(T)} \left(\frac{g_s(T)}{g'_s(T')}\right)^{4/3} x^4\right] = \frac{\pi}{30} g_\rho T^4 (1 + a x^4)$$

At the temperature we consider, above 1 MeV, $a \simeq 1$. The deviation from g_ρ can be interpreted as a new neutrino effective degree of freedom. g_ρ with 3 neutrinos (2 degrees of freedom each), 1 photon (2 degrees of freedom) and 1 electron (4 degrees of freedom) gives

$$g_\rho = 3 \times 2 \times \frac{7}{8} + 2 + 4 \times \frac{7}{8} = \frac{43}{4} = 10.75.$$

The difference between $\bar{g}_\rho = g_\rho(1+x^4)$ and g_ρ can be written as $\Delta N_{eff} \times 2 \times \frac{7}{8} = 1.75 \Delta N_{eff}$. Thus we have

$$\Delta N_{eff} = 6.14\ x^4. \tag{2.215}$$

Imposing the recent constraints from PLANCK satellite [14], one obtains

$$\Delta N_{eff} < 1 \quad \Rightarrow \quad x < 0.70. \tag{2.216}$$

This means that the mirror matters should have an equilibrium temperature lower than the observable one in our Universe. It is interesting to note that the mirror dark matter is just a motivated framework. In any model where two different thermal baths are present, the constraints from N_{eff} on its temperature T' should be extracted from Eq.(2.216)

2.8 Structure formation constraints

The formation of large scale structures (LSS) is a fundamental keypoint in any strategy to unveil the nature of the dark matter. Indeed, like clouds of smokes in a room, energetic movements of the hands will destroy any formation of small structures, the dark matter, if too hot, will prevent the formation of the first structures. This constraint on the temperature of the dark matter is for instance one of the main argument which excludes light neutrino as the main component of the universe. It is often say that "a dark matter mass below 1 leV is excluded by LSS", we will recover this result in this section.

2.8.1 Correlation function

Two–points correlation function of galaxies is the simplest description of the galaxies distribution at large scale. The two–point correlation function quantify the excess of probability to find a galaxy 2 at a distance r from a galaxy 1 selected by chance, compared to a mean uniform distribution. For instance, if the galaxies are distributed aleatory following a Poisson distribution with a mean density ρ_0, de probability $dN(r)$ to find a galaxy in a volume dV_2 at a distance r from the galaxy 1 in the volume dV_1 is the fraction of galaxies in $dV_1 \times$ the fraction of galaxies in dV_2:

$$dN(r) = \rho_0^2 dV_1 dV_2 \tag{2.217}$$

When we write *Poissonian* distribution we mean aleatory stationary process (identical mean properties whatever is the size or the place of the sample considered) i.e. the galaxies are placed by chance and independantly one with respect with each others, with an homogenous spatial probability. The spectrum of such a distribution is thus flat: this is the distribution of a "white noise". If the distribution of of the galaxies changes lightly for a poissonian distribution, the probablility will be

$$dN(r) = \rho_0^2(1 + \zeta(r))dV_1 dV_2 \tag{2.218}$$

where $\zeta(r)$ is the two–points correlation function of the galaxies. We used r instead of $\vec{r}$ with the hypothesis of the isotropy of the distribution. $\zeta(r) > 1$ indicate a surdensity , $\zeta(r) = 0$ indicates a galaxy density equal to the mean density and $\zeta(r) < 0$ indicate an underdensity. The probability of finding a galaxy at a distance r from another one can thus be written with Eq.(2.217)

$$dN(x, r) = \rho(x)dV1\ \rho(x + r)dV_2 \tag{2.219}$$

We can then introduce the contrast of density δ_ρ

$$\delta_\rho(x) = \frac{\rho(x) - \rho_0}{\rho_0} \quad \Rightarrow \quad \rho(x) = \rho_0(1 + \delta_\rho(x))$$
$$\Rightarrow \quad dN(r) = \langle dN(x, r)\rangle = \rho_0^2(1 + \langle\delta_\rho(x)\delta_\rho(x + r)\rangle)dV_1 dV_2 = \rho_0^2(1 + \zeta(r))dV_1 dV_2$$

with $\zeta(r) = \langle\delta_\rho(x)\delta_\rho(x + r)\rangle$ is called the function of **autocorrelation of the matter distribution**.

2.8.2 Power spectrum P(k)

We can define a power spectrum function $P(k)$ by

$$\zeta(r) = \frac{V}{(2\pi)^3}\int e^{k.r}P(k)d^3k \tag{2.220}$$

$P(k)$ is then the Fourier transform of the two–point correlation function $\zeta(r)$. In other word, $P(k)$ represents the probability of having distribution of matter periodically distributed in the close universe with a wave vector $k \simeq 1/\lambda$. In the case isotrope, Eq.(2.220) can be written

$$\zeta(r) = \frac{V}{2\pi^2}\int_0^\infty P(k)\frac{\sin(kr)}{kr}k^2 dk \tag{2.221}$$

r represent a comoving scale (or wavelength) corresponding to the wave number $k = \frac{2\pi}{r}$. The function $\frac{\sin(kr)}{kr}$ acts like a windows letting only the modes with $k < \frac{2\pi}{r}$ to contribute to the fluctuation at the scale r. For these reasons, fluctuations with $k > \frac{2\pi}{r}$ (i.e. small scales compared to r) does not contribute at the scale r. As a conclusion the correlation function at a scale r will be sensible only to power spectrum corresponding to scales $> r$.
To have an idea, recent measurements with some simulation hyptothesis gives

$$\zeta(r) \simeq \left(\frac{r}{r_0}\right)^{-\gamma} \tag{2.222}$$

with $\gamma \simeq 1.77$ and $r_0 \simeq 5h^{-1}$ Mpc for scales from 0.1 to 10 h^{-1} Mpc. Higher is r_0 larger is $\zeta(r)$ and so the structure features. r_0 quantify the clustering.

2.8.3 Free streaming

If dark matter is composed by collisionless particles (like neutrino for instance), after they are kinematicaly decoupled from the bath (see section 2.2.8) they are subject to Landau damping , also known as collisionless phase mixing or free streaming. Until perturbations become Jeans unstable and begin to grow after the decoupling time, collisionless particles can stream out of overdense regions and into underdense regions, in the process smoothing out inhomogeneities. We will estimate the scale of collisionless damping in this section[36].
Once a species decouples from te plasma, it simply travels in free fall in the expanding Universe. We thus may choose the particle motion to be along $d\phi = d\theta = 0$ so that the motion of a freely propagating particle is simply given by $a(t)dr = v(t)dt$. The distance λ_{FS} traversed by a free streaming particle at a time t can then be written

$$\lambda_{FS}(t) = \int_0^t dr = \int_0^t \frac{v(t')}{a(t')} dt' \tag{2.223}$$

While the particles are relativistic, $v = c$, and later, $v \propto a^{-1}$ because the momentum is redshifted. We thus need to introduce a new scale, the scale factor a_{nr} where the dark matter particles become non–relativistic. It is defined by the condition $kT_{nr} = a_{nr}^{-1} kT_0$. Following Eq.(B.14) we have the link between temperature and scaling factor : $T \propto a^{-1}$. We thus can deduce the temperature when a particle χ becomes non relativistic : $m_\chi \simeq T_{nr} \;\Rightarrow\; a_{nr} \simeq \frac{T_0}{m_\chi}$. In the case of the neutrino $\chi = \nu$, Eq.(2.72) gave us $m_\nu \approx \Omega_\nu h^2 \times 92$ eV implying

$$a_{nr} = \frac{T_0}{m_\nu} = \frac{2.35 \times 10^{-4}}{92} \left(\Omega_\nu h^2\right)^{-1} = 2.6 \times 10^{-6} \left(\Omega_\nu h^2\right)^{-1} \tag{2.224}$$

We will then integrate the free streaming equation in the 3 regimes independently : t between 0 and t_{nr}, between t_{nr} and t_{EQ} and finally between t_{EQ} and t_0.

- Between $t = 0$ and $t = t_{nr}$, the neutrino is relativistic and thus $v = c$ and the expansion factor is proportional to $t^{1/2}$ [Eq.(B.16)], $a(t) = \alpha\ t^{1/2}$

$$\lambda_{FS}^{0-nr}(t) = \int_0^t \frac{dt'}{a(t')} = \frac{2}{\alpha} t^{1/2} = 2\frac{a(t)t_{nr}}{a_{nr}^2} \qquad [0 < \mathrm{t} < \mathrm{t_{nr}}]. \tag{2.225}$$

[36] In order to take this effect into account properly, one must integrate the Boltzmann equation that describes the collisionless component

- Between t_{nr} and t_{EQ}, the velocity of the particle $v(t)$ is proportional to $a^{-1}(t)$ by redshift, which means $v(t) = c\frac{a_{nr}}{a(t)} = \frac{a_{nr}}{a(t)}$ which gives

$$\lambda_{FS}^{nr-EQ}(t) - \lambda_{FS}^{nr-EQ}(t_{nr}) = \lambda_{FS}^{nr-EQ}(t) - \lambda_{FS}^{0-nr}(t_{nr}) = \int_{t_{nr}}^{t} \frac{a_{nr}dt'}{a^2(t')} = \frac{t_{nr}}{a_{nr}} \ln\left(\frac{t}{t_{nr}}\right)$$
$$\Rightarrow \quad \lambda_{FS}^{nr-EQ}(t) = 2\frac{t_{nr}}{a_{nr}} + 2\frac{t_{nr}}{a_{nr}} \ln\left(\frac{a(t)}{a_{nr}}\right) = 2\frac{t_{nr}}{a_{nr}}\left[1 + \ln\left(\frac{a(t)}{a_{nr}}\right)\right] \qquad [\mathrm{t_{nr}} < \mathrm{t} < \mathrm{t_{EQ}}]. \tag{2.226}$$

- Between $t = t_{EQ}$ and $t = t_0$ the dependance on the velocity is of course still redshifted, but the relation between time and scale factor from Eq.(B.16) is $a(t) = \alpha\ t^{2/3}$. Remarking $t_{nr} = t_{EQ}(a_{nr}/a_{EQ})^2$ we then obtain

$$\lambda_{FS}^{EQ-t_0}(t) = \lambda_{FS}^{nr-EQ}(t_{EQ}) + a_{nr}\int_{t_{EQ}}^{t} \frac{dt'}{a^2(t')} = 2\frac{t_{nr}}{a_{nr}}\left[1 + \ln\left(\frac{a_{EQ}}{a_{nr}}\right)\right] + \frac{3t_{nr}}{a_{nr}}\left[1 - \left(\frac{a_{EQ}}{a(t)}\right)^{1/2}\right]$$
$$\Rightarrow \quad \lambda_{FS}^{EQ-t_0}(t) = \frac{2t_{nr}}{a_{nr}}\left[\frac{5}{2} + \ln\left(\frac{a_{EQ}}{a_{nr}}\right) - \frac{3}{2}\left(\frac{a_{EQ}}{a(t)}\right)^{1/2}\right] \qquad [\mathrm{t_{EQ}} < \mathrm{t} < \mathrm{t_0}]. \tag{2.227}$$

We can see that $\lambda_{FS}(t)$ quickly reaches and asymptotic value λ_{FS}^{∞}. With the values obtained in Eq.(2.98) and Eq.(B.11-B.16) we can deduce t_{nr} and a_{EQ} and thus compute the asymptotic limit λ_{FS}^{∞}:

$$\lambda_{FS}^{\infty} = \frac{M_P}{T_0 T_{nr}}\sqrt{\frac{45}{4\pi^3 g_\rho}}\left[\frac{5}{2} + \ln\left(\frac{\rho_0^R T_{nr}}{\rho_0^M T_0}\right)\right] \approx 70\ \mathrm{Mpc}\frac{1\ \mathrm{eV}}{T_{nr}} \approx 210\ \mathrm{Mpc}\frac{1\ \mathrm{eV}}{m_\nu} \tag{2.228}$$

if we suppose $T_{nr} \simeq m_\nu/3$ (an exact solution of the boltzmann solution is needed here). It means that structures of larger scales than 70 Mpc should have been destroyed by neutrino of masses $m_\nu < 1$ eV. if they were the main matter constituent of the density of the Universe. We plotted a numerical solution of λ_{FS} in Fig.(2.32).
The limit $m_\nu = 1$ keV is known as the warm dark matter limit . Indeed, the free streaming of such particle is around 1 Mpc, which is the typical size of protogalaxies. Hot dark matter are particles still relativistic at the recombination time ($m_\nu \lesssim 10$ eV as we can see from Fig.2.32). In fact, more generically speaking, candidate particles can be grouped into three categories on the basis of their effect on the fluctuation spectrum. If the dark matter is composed of abundant light particles which remain relativistic until shortly before recombination, then it may be termed "hot". The best candidate for hot dark matter is a neutrino. A second possibility is for the dark matter particles to interact more weakly than neutrinos, to be less abundant, and to have a mass of order 1 keV. Such particles are termed "warm dark matter", because they have lower thermal velocities than massive neutrinos . There are at present few candidate particles which fit this description. Gravitinos and photinos have been suggested. Any particles which became nonrelativistic very early, and so were able to diffuse a negligible distance, are termed "cold" dark matter (CDM) .

Figure 2.32: Free streaming distance (in Mpc) as function of the scaling parameter $a(t)$ for different masses of neutrino (from 1 eV to 1 keV)

2.9 The CMB

2.9.1 Generalities

2.9.2 Accoustic oscilations

2.9.3 WMAP/PLANCK results

2.10 Baryogenesis, leptogenesis

2.11 The Sunyaev-Zeldovich effect

2.12 Reheating

2.12.1 Generalities and first approach

We discussed in the section 2.6.6 the general process of thermalization of a dark bath heated by the standard model plasma. In this section we will concentrate on the processes happening *before* this epoch: the thermalization of the standard model bath in itself. In other words, we will describe how a bath of standard model particles produced by the decay of an inflaton (see section 2.13). The spectrum of the particles in the plasma at the production can be flat or monochromatic, but the equilibrium is then established in two phases: kinetic equilibrium first where the scattering (elastic or inelastic) or annihilation $2 \to 2$ processes redistribute the

temperature to reach a kinetic equilibrium. However, elastic scattering alone cannot bring the decay products of the inflaton into thermal (chemical) equilibrium because the number densities in general will not be correct. Annihilation which does not change the number of particles will neither be of any help. It was shown in [45] that chemical equilibrium is reached after kinetic equilibrium when $2 \to 3$ collisions become operative (see section 2.2.8 for the difference between kinetic and chemical decoupling and section 2.6.6 for some insights on the thermalization process).

Supposing that the thermal bath is populated by standard model particles $f,\ \bar{f}$ produced of the inflaton decay, one needs first to approximate at which time (and thus at which temperature) the decay occurs. A first approximation is that the decay of the inflaton I begins to be effective at a time t_d corresponding to a temperature T_d respecting $\Gamma_I = H(T_d)$, Γ_I being the width of the inflaton. Indeed, as we saw in section 2.2.7, the Hubble constant $H(t)$ corresponds to the doubling time of the Universe: during the time $t = 1/H(t)$ the radius of the Universe has doubled his size by doubling the scaling factor[37] a. The condition $\Gamma_I = H(T_d)$ corresponds to compute the time t_d at which, for every decay $I \to f\ \bar{f}$ the Universe has doubled his size and the (physical) density remains constant: the Universe, after being matter dominated by the massive presence of the inflaton becomes radiatively dominated by (effectively) massless particles $f,\ \bar{f}$.

Before that time, particles produced in the bath were diluted by the expansion (see Fig.(2.33) for an illustration of the process). Solving $\Gamma_I = H(T_d)$ with the expression (2.60) and approximating at a first step $T_d \simeq T_{\rm RH}$ we obtain

$$\Gamma_I = H(T_d) \quad \Rightarrow \quad T_d \simeq T_{\rm RH} = \sqrt{M_P \Gamma_I}. \tag{2.229}$$

In the case of yukawa–type coupling h_f of the inflation to the particles of the bath : $\mathcal{L}_I = h_f I \bar{f} f$ we have $\Gamma_I \sim \frac{h_f^2}{16\pi} m_I$ (C.124) which gives finally

$$T_{\rm RH} \sim \sqrt{M_P m_I} \tag{2.230}$$

The mass of the inflaton being extracted from WMAP/PLANCK data for different kind of inflationary models (see section 2.13 for more details) a typical mass of inflaton of 10^{15} GeV implies a reheating temperature of $T_{\rm RH}$ of 10^{11} GeV. This is the typical scale one can expect for a reheating temperature from inflationary models constructions.

[37] $H(t) = \frac{\dot{a}}{a} \Rightarrow da = \frac{a}{H(t)} dt$ so during $dt = H(t), da = a$: the radius of the Universe has doubled is size.

Figure 2.33: Illustration of the principle of dilution of the decay products of the inflaton fields. We can easily notice that it is when the Universe reaches the temperature obeying $\Gamma_d \simeq H(T_d)$ that the mechansm of population becomes efficient.

Instantaneous reheating and instantaneous thermalization: computing T_{RH}

Once the inflaton settled at its minimum, it will decay with a decay rate Γ_I. One usually can consider that the thermalization process in the early Universe is instantaneous. It means that during a doubling time of the Universe, the new "photons" (by photon we means all relativistic particles entering in the primordial plasma) produced by the decay of the inflaton, enter automatically in equilibrium with the existing plasma $[n_\gamma \langle \sigma v \rangle_{scattering}(T) > H(T)]$. This is called *instantaneous thermalization*. However, one can treat the inflaton as a particle decaying instantaneously (*instantaneous reheating*) or with a time delay. In the instantaneous reheating, on can consider that the inflaton decay when $H(t) = \Gamma_I$ as we did in this section. The interpretation being that the Universe is dominated by the radiation once the doubling time of the Universe dominates the decay rate, see Fig.(2.33). The other possibility is to consider that the time of reheating t_{RH} can be approximated by Γ_I^{-1}, the inflaton lifetime. This is justified by the fact that the radiation dominate the energy budget of the Universe if the inflaton decays instantaneously at the time Γ_I^{-1}. In this case, the Friedmann equation (B.1) should be written in a *matter* dominated Universe, as the inflaton is the only source of energy of density $\rho_I = n_I \times m_I$:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\ \rho_I}{3M_{Pl}^2} = \frac{\rho_I}{3M_P^2} = \frac{n_I\ m_I}{3M_P^2} \quad \Rightarrow \quad a(t) = \left(\frac{3}{2}\sqrt{\frac{n_I\ m_I}{3M_P^2}}\right)^{2/3}\ t^{2/3}$$
$$\Rightarrow \quad H = \frac{2}{3}\frac{1}{t} \quad \Rightarrow \quad H(t_{RH}) = \frac{2}{3}\ \Gamma_I. \tag{2.231}$$

To compute T_{RH}, one supposes that just after t_{RH}, the Universe is dominated by a radiation, of energy density $\rho_R = \frac{g_{RH}\pi^2}{30} T_{RH}^4$, with g_{RH} the relativistic degrees of freedom of the thermal bath at t_{RH}. We then obtain

$$H(T_{RH}) = \frac{2}{3}\Gamma_I = \frac{\rho_R(T_{RH})}{3M_P^2} = \frac{g_{RH}\ \pi^2}{90M_P^2} T_{RH}^4 \quad \Rightarrow \quad T_{RH} = (\Gamma_I \times M_P)^{1/2}\ \left(\frac{40}{g_{RH}\pi^2}\right)^{1/4}. \tag{2.232}$$

2.12.2 Before the reheating

The reheating temperature discussed above is calculated assuming an instantaneouss conversion of the energy density in the inflaton field into radiation when the decay width if the inflation energy Γ_I, is equal to H, the expansion of the Universe. The reheating temperature is best regarded as the temperature below which the Universe expands as a radiation dominated Universe, with the scale factor decreasing as $g_*^{-1/3}T^{-1}$. In this regard it has a limited meaning. For instance, T_{RH} *should not* be used as the maximum temperature obtained by the Universe during reheating. The maximum temperature is, in fact, much larger than T_{RH}. One implication of this is that it is incorrect to assume that the maximum abundance of a massive particle species of mass M, produced after inflation is suppressed by a factor $e^{-M/T_{RH}}$. We will show in this section, how to calculate the evolution of the temperature in a hot Universe, in presence of massive states.

Let us consider a model Universe with three components: inflaton field energy ρ_I, radiation energy density ρ_R. We will assume that the decay rate of the inflaton field energy density into light degrees of freedom, generically referred to as radiation, is Γ_I. We will also assume that the light degrees of freedom are in local thermal equilibrium.

With the above assumption, the Boltzmann equations describing the redshift and interchange in the energy density among the different component is

$$\dot{\rho}_I + 3H\rho_I + \Gamma_I\rho_I = 0$$
$$\dot{\rho}_R + 4H\rho_R - \Gamma_I\rho_I = 0 \tag{2.233}$$

where dot denotes time derivative. It is convenient to work with dimensionless quantities that can absorb the effect of expansion of the Universe. This may be accomplished with the definitions

$$\Phi \equiv \rho_I M_I^{-1} a^3 \ ; \quad R \equiv \rho_R a^4 \tag{2.234}$$

It is also convenient to use the scale factor, rather than time, for the independent variable. With this choice, the system of equations can be written as (prime denotes d/da)

$$\Phi' = -\frac{\Gamma_I}{H}\frac{\Phi}{a} \ , \qquad R' = \frac{\Gamma_I M_I}{H}\Phi \ . \tag{2.235}$$

Under the hypothesis that, at the beginning, the inflaton decay dominates on the expansion rate ($\Gamma_I \ll H$), the system (2.235) can be solve exactly, which gives

$$R = \frac{2}{5}\Gamma_I\sqrt{\frac{3M_I}{8\pi}}M_p(a^{5/2} - a_I^{5/2})\sqrt{\Phi_I} \tag{2.236}$$

where Φ_I is the inflaton energy (almost constant) during this preheating phase, and a_I the scale factor at the end of the inflation (at the beginning of the preheating phase, instal condition). Using (2.36) to define the temperature of the plasma of relativistic particles, one can then write

$$T = \left(\frac{12M_p\Gamma_I}{g_*}\right)^{1/4}\left(\frac{3\rho_I}{8\pi^5}\right)^{1/8}\left[\left(\frac{a}{a_I}\right)^{-3/2} - \left(\frac{a}{a_I}\right)^{-4}\right]^{1/4} \tag{2.237}$$

Using the definition of the reheating temperature ($H(T_R H) = \Gamma_I$, with $H(T_R H)$ computed in the radiation dominated epoch) and introducing H_I, the Hubble parameter at the end of inflation we have

$$\Gamma_I = \sqrt{\frac{8\pi^3 g_*}{90}}\frac{T_{RH}^2}{M_p}, \quad \text{and} \quad \rho_I = \frac{3M_p^2}{8\pi}H_I^2 \ , \tag{2.238}$$

which gives

$$T = \left(\frac{9}{5g_*\pi^3}\right)^{1/8}\sqrt{T_{RH}}(H_I M_p)^{1/4}\left[\left(\frac{a}{a_I}\right)^{-3/2} - \left(\frac{a}{a_I}\right)^{-4}\right]^{1/4} \tag{2.239}$$

We illustrate the behavior of the temperature as function of the reduced scale factor a/a_I on Fig.(2.34). We clearly see that the temperature first increase, due to the dominance of the decay rate of the inflaton on the expansion rate of the universe. The plasma is heated by the decay process. However, when the Hubble expansion rate begin to dominate, the plasma reaches a classical thermal equilibrium, without the introduction of new particles from the inflaton decay (or at least, their influence diminishes). The temperature then follows the

Figure 2.34: Evolution of the temperature of the universe as function of the scaling parameter a/a_I for different values of reheating temperature. One clearly see the maximum occurring at $x = (\frac{8}{3})^{2/5} = 1.48$, and then the classical evolution of a radiation dominated Universe $T \propto a^{-1}$.

classic $T \propto 1/a$ law. We can compute the value $a_{\max}$ for which the temperature of the plasma reaches is maximum by derivate Eq.(2.239), which gives

$$x_{\max} = \frac{a_{\max}}{a_I} = \left(\frac{8}{3}\right)^{2/5} \simeq 1.5 \tag{2.240}$$

which is independent on the initial conditions. It means that when the Universe was roughly 1.5 times larger than after the inflation phases, the temperature begins to decreases toward a reheating temperature. The maximal temperature can be computed injecting $x_{\max}$ in (2.239), we obtain

$$T_{\max} = \left(\frac{5}{3}\right)^{1/4} \left(\frac{8}{3}\right)^{-2/5} \left(\frac{9}{5 g_* \pi^3}\right)^{1/8} \sqrt{T_{RH}} (H_I M_p)^{1/4} \quad . \tag{2.241}$$

We can consider for illustration, the simplest model of chaotic inflation, $H_I \simeq M_I$ with $M_I \simeq 10^{13}$ GeV, which leads to $T_{\max} \simeq 10^3 (200/g_*)^{1/8} T_{RH}$ for $T_{RH} = 10^9$ GeV. We can see from (2.239) that in the early regime T scales as $a^{-3/8}$, which implies that entropy is created in the early-time regime. So if one is producing a massive particle during reheating, it is necessary to take into account the fact that the maximum temperature is greater than T_{RH}, and that during the early-time evolution, $T \propto a^{-3/8}$. This is exactly because Universe can reach so high temperature that very heavy (WIMZILLA) [44] particles can be produced and still be the main dark matter component of the Universe.

2.13 Inflation

2.13.1 The flatness problem

2.13.2 The horizon problem

2.13.3 The inflation as a natural solution

2.14 Other candidates

2.14.1 Axionic dark matter

2.14.2 Assymetric dark matter

2.14.3 Sterile neutrino

2.15 Summay: cosmological relic density of particles

We will summarize in this section the methodology to compute the relic abundance of light and heavy dark matter candidates we just developed above. We saw that the dark matter problem is a significant challenge for the particle physics community in that any candidate must satisfy stringent constraints. From a particle physics point of view, finding a candidate having a lifetime much longer than the Hubble time ~ 10 Gyr, with a cosmological density compatible with the observed dark matter density is challenging. The current measured value of the dark matter density is $\Omega h^2 = 0.135 \pm 0.009$, where Ω is the density relative to the present critical density $\Omega = \rho/\rho_c^0$ with $\rho_c^0 = 3H_0^2/(8\pi G)$ and $h = 0.71 \pm 0.04$ is the scaled Hubble constant, $H_0 = 100h$ km s^{-1} Mpc^{-1} (see section 2.2.3). In particle physics units, $\rho_c \simeq 10^{-5}\ h^2$ GeV cm^{-3}. Accounting for the possibility of a significant neutrino density, we take a 95 % confidence region $0.087 < \Omega h^2 < 0.129$.

The simplest possibility would be for a candidate to have the correct density from thermal processes alone. Computing the relic density proceeds as follows: the species is assumed to be in thermal equilibrium at early enough times, i.e. when the universe was essentially a radiation dominated plasma. The production and annihilation rates are tracked as the universe expands, and when these rates become longer than the (current) Hubble time, the species is said to be "frozen out" with a comoving density that no longer changes. This statement is expressed in the Boltzmann equation,

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2)$$

where n and n_{eq} are the particle density and its equilibrium value, H is the Hubble parameter and $\langle \sigma v \rangle$ is annihilation cross section, appropriately thermally averaged. We immediately notice that in the absence of interactions, the number density evolves exactly as expected, $n \propto a^{-3}$, where a is the cosmic scale factor and $H = \dot{a}/a$. The relativistic gas has a temperature evolving as $T \propto a^{-1}$ (redshift due to the conservation of the entropy), thus $n \propto T^3$. The details of freeze out now depend on wether the relic is relativistic or not.

2.15.1 Relativistic thermal freeze out

For relativistic partciles, $n_{eq} \propto T^3$, thus a species that has frozen out will still track the equilibrium density. The relic density of such particles is thus insensitive to the details of freeze–out. The only relevant parameter is the effective number of degrees of freedom at freeze–out, g_s (see Eq. 2.55 and section 2.2.7). This can change as a function of the temperature, thus the density of a frozen out species need not track its equilibrium density, but the scaling wityh temperature remains as long as the particles are relativistic. The well known example of neutrinos freeze–out with $g_s \simeq 10.75$, prior to the freeze–out of electrons and positrons (which lowers g_s). Thus the (massless) neutrino temperature is lower than the photon temperature in the present universe. The relic density of any relativistic species is given by (see section 5.6):

$$\Omega h^2 \simeq \frac{m}{100 \text{ eV}} \left(\frac{g_*}{10}\right)^{-1} \tag{2.242}$$

2.15.2 Non–relativistic thermal freeze–out

Non–relativistic particles have an exponentially suppressed equilibrium density, $n_{eq} \propto (mT)^{3/2} e^{-m/T}$. A frozen–out species would thus have a density much larger than its equilibrium density. We can simply derive the scaling of relic density with annihilation cross section as follows. In a radiation dominated universe, $H \propto T^2$ (since $H^2 \propto \rho$ according to the Friedman equation). The freeze–out condition can then be expressed simply as $H(T_{FO}) \sim \sigma n_{eq}(T_{FO})$. We find that $T_{FO} \propto m$, with only logarithmic corrections (this is due to the exponential Boltzmann factor). We can then derive the current mass density as $\rho \sim m n_{eq}(T_{fr})(T/T_{FO})^3 \sim T^3(m/T_{FO})/\sigma$. Thus we find the crucial result that $\rho \propto 1/\sigma$, with only logarithmic dependance on mass since m/T_{FO} is roughly constant. this means that smaller annihilation cross section yields larger relic densities, which makes sense, as less efficient annihilations should allow more particles to survive. Taking the simple scaling, that $\sigma \propto m^{-2}$, we find the approximate relic density

$$\Omega h^2 \sim \left(\frac{m}{1 \text{ TeV}}\right)^2 \tag{2.243}$$

This is a very interesting result in that a stable particle at the weak interaction scale of several hundred GeV would give the proper relic density to be dark matter. Any weakly interacting massive particle (WIMP) might thus be a compelling dark matter candidate.

2.15.3 Pedagogical example

Chapter 3

Direct detection of Dark Matter

Another way of detecting dark matter is through its direct interaction on a nucleus on earth. Direct detection first demonstrated by Goodman and Witten (yes, the same Witten) [46]. The idea is to study the scattering of dark matter from the halo on targets on earth to extract informations about their mass and coupling to the Standard Model. Because of dark matter is weakly interacting with matters, such experiments require large detectors (from 10 kilos for the first version to 1 ton for the future projects). This is similar to neutrino detection experiment in a sense, except that dark matter is supposed to be quite massive, and the method are thus rather different. While neutrino generates Cerenkov shower, a WIMP bumps into a heavy nuclei and experimentalists try to measure. A nice and pedagogical review can be found in [47].

3.1 Generality

The general strategy is to measure the energy of recoil of a target nucleus produced by its elastic collisions with a dark matter candidate, distinguish it from the background, before comparing it with theoretical predictions (parametrized by the dark matter mass and its coupling to matter). The interaction on a nucleus acts on three steps: first the interaction on the partons (quarks and gluons) is given by the Lagrangian and the calculation of Feynman amplitudes. The second step is to deduce from the parton interaction, the nucleon interaction which depends strongly on the structure of the nucleons. Results from lattice QCD or chiral perturbation theories $\chi PT$are the fundamental tool to extract the component of the nucleons (and often both approaches disagree). The third and last step is to pass from the nucleon level to the nuclear level. At this stage, the distribution function of the nucleon inside the nucleus is crucial. Different form factors (Woods–Saxon for instance) then enter in the game. We will first concentrate on each steps one by one. The physical quantity connecting experiments to theoreticians community is the elastic nuclear recoil spectrum dR/dE_R, where R is the recoil event rate and E_R is the energy of the recoiling nucleus. The majority of experiments are based on ionization, scintillation, now phonon techniques, or some combination of these. They have in common the same basic theoretical interpretation: the differential energy spectrum of such nuclear recoils is expected to be featurless and smoothly decreasing, with (for the simplest case of a detector stationary in the Galaxy) the typical form:

$$\frac{dR}{dE_R} = \frac{R_0}{E_0 r} e^{-E_R/E_0 r} \tag{3.1}$$

where E_R is the recoil energy, E_0 is the more probable incident kinetic energy of a dark matter particle of mass m_χ, r is a kinematic factor $4m_\chi M_T/(m_\chi + M_T)^2$ for a target nucleus of mass M_T, R is the event rate per unit mass, and R_0 the total event rate. Since Galactic velocities are of order $v \simeq 10^{-3}c$, values of m_χ in the 10-1000 GeVc^{-2} range would give typical recoil energies in the range 5-500 keV ($\sim \frac{1}{2}m_\chi v^2$). All the experimental efforts lie on discriminating in Eq.(3.1) the signal to the background composed of neutrons produced in cosmic ray muons, or to develop methods to distinguish between nuclear and electron recoils which permit to reject gamma and beta–decay background.
More generically, one can deduce the differential rate R per energy of recoil E_R, dR/dE_R for the interaction of an incoming particle of mass m_χ and of velocity v_χ with a distribution $f(v_\chi)$ on a nucleus of mass $M_N \simeq Am_p$ per kilogram of target :

$$dR = \frac{N_0}{A}\frac{\rho_0}{m_\chi}\left(v_\chi \frac{d\sigma}{d|\mathbf{q}|^2}\right)\ d|\mathbf{q}|^2\ f(\mathbf{v}_\chi)d^3\mathbf{v}_\chi \tag{3.2}$$

N_0 being the Avogadro number[1] (6.02×10^{26} kg^{-1}) and $\mathbf{q}$ the impulsion transferred from the WIMP to the nucleus. We will compute each of the terms of this equation in the following sections.
In this section we will reconstruct step by step the Eq.(3.1) and shows its limitation. We will also correct it, adding the dependence on the earth velocity, escape velocity or form factor of the nucleus. We will then go into the details of the experimental strategies settled nowadays to detect such events. The unit for differential events or background rates are conventionally expressed as 1 event keV^{-1} kg^{-1} d^{-1} called '*differential rate unit*' (*dru*). When integrated over the recoil energy, on can talk about '*total rate unit*' (*tru*) expressed in events kg^{-1} d^{-1}. When integrated between two energies (threshold energy and escape velocity for instance) we usually refer to *integrated rate unit* (*iru*) also expressed in events kg^{-1} d^{-1}.

3.2 Velocity distribution of dark matter : $f(v)$

The differential particle density, number of particles with velocities on earth (hitting the detector) comprised between $\mathbf{v} \pm d\mathbf{v}$ can be written

$$dn = \frac{n_0}{C} f(\mathbf{v}, \mathbf{v_E}) d^3\mathbf{v} \tag{3.3}$$

where C is a normalization constant such that $\int_0^{v_{esc}} dn = n_0$, in other words,

$$C = \int_0^{2\pi} d\phi \int_{-1}^{+1} d\cos\theta \int_0^{v_{esc}} f(\mathbf{v}, \mathbf{v}_E) v^2 dv \tag{3.4}$$

n_0 is the mean dark matter particle number density (ρ_0/m_χ, ρ_0 being the local dark matter density), $\mathbf{v} = \mathbf{v}_{\chi/Earth}$ the velocity of the dark matter in the referential of the experiment,

[1] As a reminder, the Avogadro number 6.02×10^{23} g^{-1} corresponds by definition to the number of Carbon 12 in 12 grams of Carbon, or in other word, the number of mass (nucleons) per gram of material.

Figure 3.1: Example of distribution function $\frac{v^2}{(\pi v_0^2)^{3/2}} e^{-v^2/v_0^2}$ for different values of v_0.

the velocity which is *effectively* measured by the collaboration (through the recoil of the nucleus for instance), $\mathbf{v}_E = \mathbf{v}_{Earth/Halo}$ the Earth velocity relative to the halo (see section (4.1.3) for details) and v_{esc} is the *local* escape velocity. As a good approximation one can represent the annual modulation of the orbit of the Earth around the Sun by

$$v_E = 232 \ + \ 15 \ \cos\left(2\pi \frac{t - 152.5 \text{ days}}{365.25 \text{ days}}\right) \ \text{km s}^{-1}. \tag{3.5}$$

Considering that the dark matter statistical distribution of its velocity in the halo follows a Maxwellian distribution with a more probable velocity v_0[2], and observing that $\mathbf{v}_{\chi/Halo} = \mathbf{v}_{\chi/Earth} + \mathbf{v}_{Earth/Halo} = \mathbf{v} + \mathbf{v}_E$ one obtains[3] for $v_{esc} = \infty$

$$\begin{aligned} f(\mathbf{v}, \mathbf{v}_E) &= e^{-\frac{1}{2} m_\chi v_{\chi/Halo}^2/kT} = e^{-v_{\chi/Halo}^2/v_0^2} = e^{-(\mathbf{v}+\mathbf{v}_E)^2/v_0^2} \\ \Rightarrow \quad C_{v_{esc}=\infty} &= (\pi v_0^2)^{3/2} \quad \Rightarrow dn = \frac{\rho_0}{m_\chi} \frac{e^{-(\mathbf{v}+\mathbf{v}_E)^2/v_0^2}}{(\pi v_0^2)^{3/2}} d\phi d\cos\theta v^2 dv \end{aligned} \tag{3.6}$$

As we could have expected, the number density of dark matter ($n_0 = \int dn$) is independent of the Earth velocity. The distribution $v^2 f(\mathbf{v}, \mathbf{v}_E) = \frac{v^2}{(\pi v_0^2)^{3/2}} \ e^{-(\mathbf{v}+\mathbf{v}_E)^2/v_0^2}$ is maximum for $v = v_0$. Considering that the earth is moving with a velocity $v_E \simeq v_0$ around the galactic center, it corresponds to an absolute dark matter velocity $\mathbf{v}_{\chi/Halo} = \mathbf{v} + \mathbf{v}_E \simeq 0$: a static halo. We show in Fig.3.1 some example of distributions for different values of v_0.

If one would have taken v_{esc} is not ∞, the results one would have obtained is

$$C_{v_{esc}\neq\infty} = (\pi v_0^2)^{3/2} \left[\text{erf}\left(\frac{v_{esc}}{v_0}\right) - \frac{2}{\pi^{1/2}} \frac{v_{esc}}{v_0} e^{-v_{esc}^0/v_0^2} \right] \tag{3.7}$$

with erf the Gauss error function (see appendix for details). If we take $v_0 = 230$ km s^{-1} and $v_{esc} = 600$ km s^{-1} we obtain $C_{v_{esc}=\infty}/C_{v_{esc}=600} \simeq 0.9965$ which can justify in the majority of

[2]Notice that v_0 is the more probable velocity, whereas the mean velocity is $\langle v^2 \rangle = \frac{3}{2} v_0^2$

[3]Noticing that $(\mathbf{v} + \mathbf{v}_E)^2 = v^2 + v_E^2 + 2 v v_E \cos\theta$ and using Eq.(B.39)

Figure 3.2: Collision between the nucleus and the WIMP in the center of mass frame.

cases the approximation $v_{esc} = \infty$. For our calculation, we made the assumption that the dark matter in the halo has a 'sufficiently' Maxwellian velocity distribution. The Maxwell-Boltzmann distribution $\propto e^{m_\chi v^2/2kT}$ describes the velocities of particles which move freely up to short collisions. We assumed that the WIMPs are isothermal and isotropically distributed in phase space. It is important to notice that this assumption is not exact and there can exist deviation from this Maxwellian distribution. Noticing that

$$\frac{1}{2}m_\chi v_0^2 = kT \quad \Rightarrow \quad v_0 \simeq 220 \text{ km/s} \simeq 0.75 \times 10^{-3}c. \tag{3.8}$$

which shows that dark matter is highly non relativistic in the Milky Way (and justifies using Maxwell-Boltzmann distribution instead of Fermi-Dirac or Bose-Einstein one. For a more general discussion of how the distribution can be affected, see for instance [48, 49]
Usually, estimation of the local density ρ_0 is in the range 0.2 GeV cm$^{-3} < \rho_0 < 0.4$ GeV cm^{-3} leading to the adoption of $\rho_0 = 0.3$ GeV cm^{-3} as the central value. However the main result of a more recent analysis [2] have given a novel determination of the local dark matter halo density which, assuming spherical symmetry and either an Einasto or an NFW density profile is found to be around 0.39 GeV cm^{-3} with a 1σ error bar of about 7%; more precisely they found $\rho_0 = 0.385 \pm 0.027$ GeV cm^{-3} for the Einasto profile and $\rho_0 = 0.389 \pm 0.025$ GeV cm^{-3} for the NFW. This is in contrast to the standard assumption is that ρ_0 is about 0.3 GeV cm^{-3} with an uncertainty of a factor of 2 to 3.

3.3 Measuring a differential rate : $\frac{d\sigma}{d|\mathbf{q}|^2}$

3.3.1 Kinematics

First one should compute the energy transfered to a nucleon at rest of mass M_N from a collision with a dark matter of mass m_χ hitting the nucleus with a velocity $\mathbf{v}_\chi$. We first compute in the center of mass frame with velocity (see Fig.3.2 for illustration)

$$\mathbf{v}_{CM} = \frac{m_\chi\ \mathbf{v}_\chi + \mathbf{0}}{m_\chi + M_N} = \frac{\mu}{M_N}\mathbf{v}_\chi \Rightarrow \mathbf{V}_N = \mathbf{0} - \mathbf{v}_{CM} = -\frac{\mu}{M_N}\mathbf{v}_\chi;\ \ \mathbf{V}_\chi = \mathbf{v}_\chi - \mathbf{v}_{CM} = \frac{\mu}{m_\chi}\mathbf{v}_\chi \quad (3.9)$$

with $\mu = \frac{m_\chi M_N}{m_\chi + M_N}$ is the reduced mass and $\mathbf{V}_i$ ($\mathbf{V'}_i$) the velocity of the nucleus before (after) the collision in the rest frame. In this frame, $\mathbf{P}_\chi + \mathbf{P}_N = \mathbf{0} = \mathbf{P'}_\chi + \mathbf{P'}_N$, the last equality being the conservation of impulsion implying $\mathbf{V'}_\chi = -\frac{M_N}{m_\chi}\mathbf{V'}_N$. The kinetic energy conservation gives

$$\frac{1}{2}m_\chi \mathbf{V}_\chi^2 + \frac{1}{2}M_N\mathbf{V}_N^2 = \frac{\mu}{2}v_\chi^2 = \frac{1}{2}m_\chi \mathbf{V'}_\chi^2 + \frac{1}{2}M_N\mathbf{V'}_N^2 = \frac{M_N}{m_\chi}[m_\chi + M_N]\mathbf{V'}_N^2$$
$$\Rightarrow \quad (V'_N)^2 = \frac{\mu^2}{M_N^2}v_\chi^2 \quad \Rightarrow (v'_N)^2 = |\mathbf{V'}_2 - \mathbf{v}_{CM}|^2 = 2v_\chi^2\frac{\mu^2}{M_N^2}(1-\cos\theta^*) \quad (3.10)$$

with θ^* the angle of scattering in the rest frame. From Eq.(3.10) one can deduce the energy of recoil of the nucleus,

$$E_R = \frac{|\mathbf{p'_N}|^2}{2M_N} = \frac{|M_N\ v'_N|^2}{2M_N} = \frac{\mu^2 v_\chi^2}{M_N}(1-\cos\theta^*) \quad (3.11)$$

The maximal recoil energy that can be generated by the elastic collision is when the nucleus moves backward after the hit ($\cos\theta^* = -1$) and is $E_R^{max} = \frac{2\mu^2 v_\chi^2}{M_N}$. Another way of understanding the phenomenon is to observe that for a fixed (measured) energy of recoil E_R the minimum velocity able to deposit E_R correspond to $\cos\theta^* = -1$ which gives

$$v_\chi^{\min} = \sqrt{\frac{E_R M_N}{2\mu^2}} \quad (3.12)$$

Usually, each experiment possess a threshold energy E_{th}, i.e. a minimum energy that it can measure. So the *absolute* minimum velocity required to measure an event is $v_\chi^{min} = \sqrt{\frac{E_{th}M_N}{2\mu^2}}$. As a summary, we obtained

$$\mathbf{P'}_N = -\mu\mathbf{v}_\chi\ ;\ \ E'_N = E_R = \frac{\mu^2 v_\chi^2}{M_N}(1-\cos\theta^*)\ ;\ \ E'_{tot} = \frac{\mu}{2}v_\chi^2\ ;\ \ |\mathbf{q}|^2 = 2\mu^2 v_\chi^2(1-\cos\theta^*) \quad (3.13)$$

One can write Eq.(3.13) as $E_R = 2\frac{m_\chi M_N}{(m_\chi+M_N)^2}E_\chi(1-\cos\theta^*)$ with $E_\chi = \frac{1}{2}m_\chi v_\chi^2$ the initial energy of the collision. The function $\frac{m_\chi M_N}{(m_\chi+M_N)^2}$ is maximum (for a given m_χ) for $M_N = m_\chi$ (classical ping-pong ball versus bowling ball situation). It means that for a 100 GeV WIMP, the energy of recoil will be maximized if the nucleus is also of the order of $\simeq 100$ GeV. This explain the strategy of the present experiment for the detection of WIMPs, to use relatively heavy nuclear targets.

3.3.2 Differential rate

For a given energy of recoil, one needs to compute $\frac{d\sigma}{dE_R}$. Noticing that we are in non-relativistic conditions, one can apply the Fermi's Golden Rule:

$$d\Gamma_{if} = 2\pi \ |\mathcal{M}_{fi}|^2 \ \rho(E_f) \tag{3.14}$$

with

$$\rho(E_f) = \frac{dn}{dP'_N}\frac{dP'_N}{dE_f} \quad \text{and} \quad dn = \frac{d^3\mathbf{P'_N}}{(2\pi)^3} = \frac{(P'_N)^2 dP'_N d\Omega}{(2\pi)^3}$$

with Γ_{fi} is the number of transition[4] per unit of time from initial state $|i\rangle$ to final state $|f\rangle$, $\mathcal{M}_{fi}$ the transition matrix element and $\rho(E_f)$ the density of final state, $E_f = E'_N + E'_\chi$. From Eq.(3.13) one deduce $d\Omega = 2\pi d(\cos\theta^*) = \frac{\pi \ d|\mathbf{q}|^2}{\mu^2 \ v^2}$, $P'_N = M_N V'_N = \mu v_\chi$, $E_f = \frac{\mu}{2} v_\chi^2 = \frac{(P'_N)^2}{2\mu}$ and so $dE_f = \mu^{-1} P'_N dP'_N$. Combining all these relations with Eq.(3.14) we obtain

$$d\Gamma_{if} = \frac{|\mathcal{M}_{fi}|^2 d|\mathbf{q}|^2}{4\pi \ v} \tag{3.15}$$

$d\Gamma_{if}$ being the rate of the interaction, it equals the cross section times the velocity :

$$d\Gamma_{if} = v \times d\sigma \quad \Rightarrow \quad \frac{d\sigma}{d|\mathbf{q}|^2} = \frac{|\mathcal{M}_{fi}|^2}{4\pi \ v^2}, \tag{3.16}$$

which is a less conventional (but more practical in our case) form of the Fermi's Golden Rule. A quantum field theory treatment can be found in Eq.(C.108).

3.4 Structure Function of the nucleus : $F(q)$

One have to keep in mind, when using Eq.(3.16) that the transition matrix element should be taken between two *nucleus* final state. However, a WIMP hits protons and neutrons of the nucleus (in fact quarks at a more microscopical level as we will explain later on). For a nucleus of nuclear mass number A with a coupling WIMP-nucleons f and nucleons wave functions $\psi_{i,f}(r) = e^{-i\mathbf{p_{i,f}}\cdot\mathbf{r}}$, one should thus write it

$$\begin{aligned} \mathcal{M}_{fi} &= \int d^3r \psi_f^*(r) V(r) \psi_i(r) \quad \text{with} \quad V(r) = A\rho(r) f \\ \Rightarrow \mathcal{M}_{fi} &= Af \int d^3r \rho(\mathbf{r}) e^{-i\mathbf{q}.\mathbf{r}} = AfF(q) \end{aligned} \tag{3.17}$$

with $\mathbf{q} = \mathbf{p}_f - \mathbf{p}_i$ and $F(q)$ the form factor of the nucleus, defined as the Fourier transform of wavefunction of the target nucleus, normalized to unity when no impulsion is transferred,

[4]Due to the very low velocities in consideration for direct detection processes ($v_\chi \simeq 200$ km s^{-1}) we will work in classical quantum mechanics framework. We could work in relativistic limits with the changes $|\mathcal{M}|^2 \to |\mathcal{M}|^2/2E_\chi 2E_N$ and $2\pi d^3P/(2\pi)^3 \to (2\pi)^4 d^3P'_\chi d^3P'_N/(2\pi)^6 2E'_\chi 2E'_N$

$$F(q) = \int \rho(\mathbf{r}) e^{i\mathbf{q}.\mathbf{r}} d^3\mathbf{r} = \frac{4\pi}{q} \int_0^\infty r \sin(qr)\rho(r) dr. \tag{3.18}$$

If protons and neutrons have different coupling coefficients, the transition matrix element has two parts and can be written as

$$\mathcal{M}_{fi} = \Big[Z f_p + (A - Z) f_n \Big] \; F(q). \tag{3.19}$$

The form factor represent the effect of the coherence of the interaction on the nucleon. When the momentum transfer $|\mathbf{q}| = \sqrt{2M_N E_R}$ becomes large enough that the wavelength becomes similar to the size of the nucleus $r_N = (a_N A^{1/3} + b_N)$, the structure of the nucleus becomes important. In another word when

$$q\, r_N \text{ dimensionless } \simeq (2AE_R)^{1/2}(a_N A^{1/3} + b_N) \simeq 6.92 \times 10^{-3} A^{1/2} E_R^{1/2} (a_N A^{1/3} + b_N) \tag{3.20}$$

with E_R in keV, a_N and b_N expressed in fermi. To find this form factor, one needs to know the distribution function $\rho(\mathbf{r})$ of the nucleons in the nucleus. There exists different options.

- The simplest option is to consider a sum of delta–dirac function, assuming point–like nucleons $\rho(\mathbf{r}) = \sum_i \delta^3(\mathbf{r} - \mathbf{r_i})$.
- A more (intermediate) realistic distribution is a thin shell of effective radius r_N : $\rho(\mathbf{r}) = \frac{\delta(\mathbf{r} - \mathbf{r_N})}{4\pi r^2}$. In this case, it is equivalent to imagine that the nucleons are homogeneously dispersed around the nucleus. Implementing the density in Eq.(3.18) we then obtain

$$F(qr_N) = \frac{\sin(qr_N)}{qr_N}. \tag{3.21}$$

- Another commonly used approximation is

$$F(qr_N) = e^{-(qr_n)^2/10} \tag{3.22}$$

  which is the exact form factor for a Gaussian density distribution $\rho(\mathbf{r})$ with mean square radius $\sqrt{(3/5)}\; r_N$, $r_N = 1.2A^{1/3}$ fm [52].
- Helm found a more realistic form of the nucleons in the nucleus after studying the scattering of electrons from different nuclear targets [50]. Another parametrization proposed by Engel can be found in [51, 52]. Helm has approximated the nucleus as a solid core with a nearly constant density and a surface of thickness s :

$$\rho(\mathbf{r}) = \int d^3\mathbf{r}' \; \rho_0(\mathbf{r}') \; \rho_1(\mathbf{r} - \mathbf{r}'). \tag{3.23}$$

  ρ_0 defines the radius of the target nuclei and is given by a constant inside a sphere of radius $r_N \simeq 1.2A^{1/3}$ fm, and is zero outside the sphere. ρ_1 defines the surface

thickness of the order of 1 fm by $\rho_1(\mathbf{r}) = e^{-|\mathbf{r}|^2/2s^2}$. ρ_0 and ρ_1 are normalized such that $\int d^3\mathbf{r}'\rho_{0,1}(\mathbf{r}') = 1$. After implementing the density function in Eq.(3.18) one obtains the Wood-Saxon form

$$F(q) = \frac{3j_1(qr_N)}{qr_N} e^{-(qs)^2/2}, \tag{3.24}$$

where j_1 is a spherical Bessel function of the first kind. A least squares fit [47] from muon scattering data gives

$$r_N^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2, \quad c \simeq 1.23A^{1/3} - 0.60 \text{ fm}, \quad a \simeq 0.52 \text{ fm}, \; s \simeq 0.9 \text{ fm}.$$

This result is quite similar to the Fermi distribution:

$$\rho_{Fermi}(r) = \rho_0 \left[1 + e^{(r-c)/a}\right]^{-1}. \tag{3.25}$$

We show in Fig.(3.3) the evolution of the coherence factor F as function of the recoil energy $E_R = q^2/2M_N$ for different nucleus used in different direct detection experiments (Sodium for DAMA, Germanium for CDMS and Xenon for XENON100). We notice the effect of the form factor becomes stronger for higher energies of recoil, where the condition $qr_N \lesssim 1$ becomes not valid anymore. The exchanged wavelength begins to reach the size of the nucleus. We also clearly see the influence of the number of nucleons in the process: the larger is the mass number A, the larger is the radius of the nucleus and the stronger becomes the decoherence effect. For a Xenon nucleus, the form factor already vanished for an energy of recoil of 50-100 keV.

3.5 Computing a rate

After defining the differential cross section at zero momentum transfer σ_0 and remembering that F(q=0) = 1,

$$\sigma_0 = \int_0^{q_{max}=4\mu v_\chi^2} \frac{d\sigma}{d|\mathbf{q}|^2} d|\mathbf{q}|^2 = \frac{(\mu A f)^2}{\pi} \quad \Rightarrow \quad \frac{d\sigma}{d|\mathbf{q}|^2} = \sigma_0 \frac{F^2(E_R)}{4\mu^2 v_\chi^2}, \tag{3.26}$$

when we combine Eq.(3.2), Eq.(3.16) and Eq.(3.17) one can write

$$\frac{dR}{dE_R} = \frac{N_0 M_N}{A} \frac{\rho_0}{m_\chi} \frac{\sigma_0}{2\mu^2 v} F^2(E_R) f(\mathbf{v}, \mathbf{v_E}) d^3\mathbf{v} \tag{3.27}$$

To have an idea of the evolution of the rate as function of the recoil energy, one can build a toy model with $v_E = 0$, $v_{esc} = \infty$. In this case, assuming a Maxwellian velocity distribution and integrating on the WIMP velocity from $v_{min} = \sqrt{\frac{E_R M_N}{2\mu^2}}$ to ∞ using Eq.(B.39) one obtains

Figure 3.3: Form factor as function of the energy of recoil of the nucleus for several types of nucleus used in different experiments: Silicium (blue-dotted), Germanium (green–dashed) and Xenon (red–full).

$$f(v) = \frac{1}{(v_0^2\pi)^{3/2}} e^{-v^2/v_0^2} \quad \Rightarrow \quad \frac{dR}{dE_R} = \frac{N_0 M_N}{A} \frac{\rho_0 \sigma_0}{m_\chi \mu^2} \frac{F^2(E_R)}{v_0 \pi^{1/2}} e^{-E_R M_N/2\mu^2 v_0^2} \equiv \frac{R_0}{E_0 r} e^{-E_R/E_0 r} \tag{3.28}$$

which is the form of the Eq;(3.1) that we have reconstructed. If moreover we neglect the influence of the form factor ($F(E_R) \simeq 1$), we can compute an analytical expression of the total rate after integrating on the recoil energy E_R from the threshold energy to an energy of recoil E_R we have

$$\begin{aligned} R(E_R) &= \frac{2}{\pi^{1/2}} \frac{N_0}{A m_\chi} \rho_0 \sigma_0 v_0 \left[e^{-E_{th} M_N/2\mu^2 v_0^2} - e^{-E_R M_N/2\mu^2 v_0^2} \right] \\ &= R_0 \left[e^{-E_{th} M_N/2\mu^2 v_0^2} - e^{-E_R M_N/2\mu^2 v_0^2} \right] \quad \text{with} \\ R_0 &\simeq \frac{5 \times 10^{-6}}{A} \left(\frac{100 \text{ GeV}}{m_\chi} \right) \left(\frac{\rho_0}{0.4 \text{ GeVcm}^{-3}} \right) \left(\frac{\sigma_0}{10^{-42} \text{ cm}^2} \right) \left(\frac{v_0}{220 \text{ kms}^{-1}} \right) \frac{\text{events}}{\text{day kg}} \end{aligned} \tag{3.29}$$

R_0 corresponding to the total number of event expected (integration on E_R from 0 to ∞). In fact, the majority of the events appears at low energy of recoil due to the Boltzmann suppression. Lowering threshold energy is thus a fundamental keypoint in the experimental process to increase the number of events, especially for light dark matter candidates. A first glance at this number shows that the rate is *very* weak and the detection is thus a difficult task from the experimentally front. If we take as an example a 100 GeV WIMP hitting a 100 kg experience of Xenon ($A = 131$) with a cross section of 1 pb (10^{-36} cm^2) we expect

Figure 3.4: Example of limits obtained from Eq.(3.29) if we suppose 5 events observation on 100 kg of Xenon for different threshold energies : 5 keV (dotted blue), 20 keV (dashed green) and 50 keV (full red).

around 5 events per day. An important point is that σ_0 is the WIMP-*nucleus* cross section. However, to be able to compare different experiments, the interesting cross section is the WIMP-*nucleon* one (proton or neutron) that we will note by σ_0^n. We can easily show that by rescaling

$$\sigma_0 = \frac{\mu^2}{\mu_n^2} A^2 \sigma_0^n \tag{3.30}$$

with $\mu_n = M_n m_\chi/(M_n + m_\chi)$, '$n$' standing forn $n$ucleon. A 1pb cross section on a nucleus of Xenon correspond to a 1.7×10^{-6} pb cross section on a nucleon. The exclusion limits given by the different experiences is based on a number of events expected but finally not seen. We can plot the (total) iso-events expected from a collision with a nucleus of mass A, above a threshold energy E_{th} -taking $E_R = \infty$ in Eq.(3.29)- in a plane (μ_χ, σ_0^n) values of A and E_{th}. the result is presented in Fig.(3.4): all the point lying on the line predict 5 events for a 100 kg Xenon–like experiment. Even if the model we were working on is a "toy" model (without taking into account the velocity of the earth or the form factor of the nucleus) it is interesting to see that the behavior and the order of magnitude we obtained are quite reasonable. The behavior for large mass of dark matter is mainly governed by R_0 in Eq.(3.29) as the exponential factor is suppressed. The rate thus evolves proportionally to $1/m_\chi$ as we can see in Fig.(3.4). For light dark matter mass however, the Boltzmann suppression becomes dominant. It is indeed more difficult to find high velocities able to generate sufficient kinetic energy and compensate the low mass of the dark matter. In this region, the queue of the Boltzmann distribution and the different effects of the earth

Figure 3.5: Examples of differential fluxes obtained for a 10 GeV WIMP (left) and a 100 GeV WIMP (right) hitting different nuclei : Silicium (blue), Germanium (green) and Xenon (red). The effect of the form factor is shown in the right panel where the differential rate is shown without taking into account the form factor in dotted lines.

velocities become important.

3.6 Being more realistic

3.6.1 Taking into account the earth velocity

We now have developed all the necessary tools to compute the differential rate taking into account the form factor of the nucleus and the kinematic of the earth in the Galactic plane. If we do not neglect the velocity of the earth, we can have a more precise analytical expression of the differential rate dR/dE_R noticing that $|\mathbf{v}_\chi + \mathbf{v}_E|^2 = v_\chi^2 + v_E^2 + 2v_\chi v_E \cos\theta$

$$\begin{aligned}\frac{dR}{dE_R} &= \frac{N_0 M_N}{A}\frac{\rho_0\sigma_0 F^2(E_R)}{2m_\chi\mu^2 v_0 v_E\sqrt{\pi}}\int_{v_\chi^{min}}^{\infty}\left(e^{(v-v_E)^2/v_0^2} - e^{-(v+v_E)^2/v_0^2}\right)dv \\ &= \frac{N_0 M_N}{A}\frac{\rho_0\sigma_0 F^2(E_R)}{4m_\chi\mu^2 v_E}\left[\mathrm{erf}\left(\frac{v_\chi^{min}+v_E}{v_0}\right) - \mathrm{erf}\left(\frac{v_\chi^{min}-v_E}{v_0}\right)\right] \end{aligned} \tag{3.31}$$

where v_χ^{min}, given by Eq.(3.12) is the minimum WIMP velocity able to induce a nuclear recoil energy above the threshold. We show in Fig.(3.5) the differential rate as function of the energy of recoil for different mass of dark matter (10 and 100 GeV). We notice that a 10 GeV dark matter gives a measurable spectrum at very low energy of recoil ($\lesssim$ 10 keV) whereas the spectrum is harder for heavier WIMPs. For light WIMP, the shape of the spectrum is mainly independent of the form factor $F(E_R)$ because at the energy considered ($\lesssim$ 10 keV) $F(E_R) \simeq 1$. The differential rate depends mainly on the factor $\left[\mathrm{erf}\left(\frac{v_\chi^{min}+v_E}{v_0}\right) - \mathrm{erf}\left(\frac{v_\chi^{min}-v_E}{v_0}\right)\right]$ of the Eq.(3.31) which converges rapidly to 0 as $v_{min} \gg v_E$ for $E_R \gtrsim 10$ keV. Physically speaking it means that the quantity of WIMPS able to give a 10 keV energy of recoil to the nucleus is very rare because of the low mass of the dark matter: the population of WIMP with high velocity is boltzmann–suppressed. We do not see such a strong phenomenon in the case of a 100 GeV WIMP on the right panel of Fig.(3.5). We also

Figure 3.6: Left : Rotation of the earth around the galactic center. Right : Spectrum as a function of the energy of recoil on a Germanium target for a 10 GeV WIMP in June and December.

plotted the differential rate *without* taking into account the form factor. As expected, the effect of $F(E_R)$ is much stronger for heavier nuclear targets (due to the decoherence effect $q\ r_N \lesssim 1$ on "bigger" targets) and is even the dominant process in the case of the Xenon nucleus above $\simeq 20$ keV whereas the effect can be neglected for Silicium targets.

3.6.2 Annual modulation of the signal

Up to now we did not take into account that the earth is turning annually around the sun. As we can see in the left panel of Fig.(3.6) and using Eq.(3.5) the absolute velocity of the earth with respect to the Local Standard of Rest -see section (4.1.3)- vary from 217 km/s to 247 km/s being maximum on the 2nd of June. This variation induces a modulation of $\simeq 3\%$ in the signal measurable by experiments like DAMA [498]. We show the example in the case of a Germanium target in the right panel of Fig.(3.6).

3.7 Influence of the structure of the nucleons

Since several years it is known that the uncertainties generated by the quark contents of the nucleons can be as important (if not more) than astrophysical uncertainties. Some authors pointed out this issue and applied it to supersymmetric models [55, 56], in effective operator approach [57] or even in the scalar extension of the SM [687], but rarely taking into account the latest lattice results [56]. Indeed, due to its large Yukawa coupling, the strange quark and its content in the nucleon is of particular interest in the elastic scattering of the dark matter on the proton. We learnt in the previous section that the spin independent part of the cross section can be written

$$\sigma_0 = \frac{\mu^2}{\pi}\left[Z f_p + (A - Z) f_n\right]^2 \tag{3.32}$$

The couplings needed in Eq.(3.32) are the effective couplings of the dark matter to the nucleons whereas we compute at a first step the dark matter coupling to the partons (quarks or gluons). Due to the low energy scale of the interaction in the elastic scattering (the velocity of the dark matter χ is around 300 km/s) we can approximate the dark matter quark couplings as $f_q \bar{q} q \bar{\chi} \chi$, f_q being a dimensionful coefficient proportional to $1/m_h^2$ in the case of t–channel Higgs exchange or $1/m_{\tilde{q}}^2$ in the case of s-channel squark exchange. The mean interaction of the dark matter on the nucleon N of mass m_N can then be written $f_q \langle N|\bar{q}q\bar{\chi}\chi|N\rangle = f_q \langle N|\bar{q}q|N\rangle \bar{\chi}\chi$. If one defines $\langle N|\bar{q}q|N\rangle = \frac{M_N}{m_q} f_q^N$, the effective quark nucleon coupling becomes $f_{\chi N}^{eff} = \frac{M_N}{m_q} f_q f_q^N$. We then can write[5]

$$f_N = M_N \left(\sum_{q=u,d,s} f_q^N \frac{\mathcal{A}_q}{m_q} + \frac{2}{27} f_H^N \sum_{q=c,b,t} \frac{\mathcal{A}_q}{m_q} \right) \tag{3.33}$$

with $\mathcal{A}_q$ the scattering amplitude on a single quark q and $f_q^N = (m_q/M_N)\langle N|\bar{q}q|N\rangle$ is the reduced (dimensionless) sigma terms of the nucleon N, and $f_H^N = 1 - \sum_{q=u,d,s} f_q^N$ is the heavy quark content of the nucleon (through loop interaction with the gluon content of the nucleon). [58, 59].

There are different ways of extracting the reduced dimensionless nucleon (N) sigma terms $f_q^N \equiv (m_q/M_N)\langle N|\bar{q}q|p\rangle$. This sigma terms can be derived by phenomenological estimates of the $\pi - N$ scattering $\Sigma_{\pi N}$ (see [60] and references therein for a review): $\Sigma_{\pi N} \equiv M_N f_l = m_l \langle N|\bar{u}u + \bar{d}d|N\rangle$ with $m_l = (m_u + m_d)/2$. While an early experimental extraction [61] gave $\Sigma_{\pi N} = 45 \pm 8$ MeV, a more recent determination [62] obtained $\Sigma_{\pi N} = 64 \pm 7$ MeV.

On the other hand, the study of the breaking of $SU(3)$ within the baryon octet and the observation of the spectrum leads to derive a constraint on the non-singlet combination $\sigma_0 = m_l \langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$. Chiral effective field theory lead to a value $\sigma_0 = 36 \pm 7$ MeV. Following [63] by introducing $z = (\langle N|\bar{u}u - \bar{s}s|N\rangle)/(\langle N|\bar{d}d - \bar{s}s|N\rangle) = 1.49$ one obtains

$$\begin{aligned} f_d &= \frac{m_d}{M_N} \frac{\Sigma_{\pi N}}{m_u + m_d} \frac{y(z-1)+2}{1+z} \\ f_u &= \frac{m_u}{M_N} \frac{\Sigma_{\pi N}}{m_u + m_d} \frac{y(1-z)+2z}{1+z} \\ f_s &= \frac{m_s}{M_N} \frac{\Sigma_{\pi N}}{m_u + m_d} \quad y \end{aligned} \tag{3.34}$$

where $y = 2\langle N|\bar{s}s|N\rangle / \langle N|\bar{u}u + \bar{d}d|N\rangle = 1 - \sigma_0/\Sigma_{\pi N}$ represents the strange fraction in the nucleon. We show in Fig.3.7 the dependance of f_q as function of $\Sigma_{\pi N}$. The two extreme values are obtained with the lower bound of $\Sigma_{\pi N}$ at 1σ extracted from [61] (37 MeV) and the higher bounds from [62] (71 MeV) which gives for $(m_u, m_d, m_s, m_c, m_b, m_t, m_p) =$

[5]The partons content of the nucleon can be derived from computations of anomalies of the trace of energy–momentum tensor in QCD : $M_N \langle N|N\rangle = \langle N| \sum_{q=u,d,s} m_q \bar{q}q - \frac{9}{8\pi}\alpha_s G_{\mu\nu}G^{\mu\nu}|N\rangle$, $G_{\mu\nu}$ being the gluon tensor [58].

Figure 3.7: Sigma commutators of the proton with two different phenomenological measurements of $\Sigma_{\pi N} = 45\pm8$ MeV [61] and 64 ± 7 MeV [62]. We also showed the mean evaluation from more recent lattice results [717] (left).

$(2.76, 5., 94.5, 1250, 4200, 171400, 938.3)$ [MeV] :

$$\begin{aligned} f_u^{min} &= 0.016 \qquad f_u^{max} = 0.030 \\ f_d^{min} &= 0.020 \qquad f_d^{max} = 0.044 \\ f_s^{min} &= 0.013 \qquad f_s^{max} = 0.454 \end{aligned} \tag{3.35}$$

These limitations on the phenomenological estimation of the strange structure of the nucleon clearly open the way for lattice QCD to offer significant improvement. Using the Feynman–Hellman relation $f_q = (m_q/M_N)\partial M_N/\partial m_q$ different authors have extracted the light–quark and strangeness sigma terms (see [60] for a clear review). The last results obtained by the MILC collaboration [718] and by the authors of (labeled "Young" from now on) [717] provide stringent new limits on the strange quark sigma-terms. The modern lattice results for f_s agree that the size is substantially smaller than has been previously thought :

$$f_s^{\text{Young}} = 0.033 \pm 0.022 \qquad f_s^{\text{MILC}} = 0.069 \pm 0.016 \tag{3.36}$$

These two results are marginally consistent, although there may be differences in how the derivative with respect to m_s is taken. Moreover, they tend to favor the smaller phenomenological evaluation of $\Sigma_{\pi N}$. In the following, we will consider the central values of f_q extracted from the Young et al analysis and referred it to the "lattice" one : $f_u = f_d = f_l = 0.050$, and $f_s = 0.033$, and the maximum and minimum values for f_q given by phenomenological references [61] and [62] (Eq.3.35). We adapted the code micrOMEGAs [716] to the different values of f_q depending on the model we used, and modified it to include the new couplings/spectrum/interactions induced by the singlet scalar extension of the SM[6].
As we can see in Fig.3.9, these uncertainties have a strong impact on the direct detection cross section, up to one order of magnitude. We also plotted the value of σ_{SI} obtained by

[6]We want to thank warmly S. Pukhov for his help to solve technical problems related to the code.

Figure 3.8: Example of spin independent elastic scattering cross section as function of the pion-nucleon sigma-term $\Sigma_{\pi N}$ for a scalar dark matter in tha Higgs portal model (cf section 9.1) respecting WMAP constraint: $m_s = 90$ GeV, $\lambda_{HS} = 0.2$, $m_h = 130$ GeV giving $\Omega_S h^2 = 0.102$. We also represented the central values of the cross section for the lattice simulations [718] and [717] labelled "MILC" and "Young" respectively.

the two lattice groups that we took into consideration in our analysis, corresponding to the central values $(\Sigma_{\pi N}, \sigma_{SI}) = (26 \text{ MeV}, 2.84\times10^{-9} \text{ pb})$ [718] and $(47 \text{ MeV}; 2.95\times10^{-9} \text{ pb})$ [717]. We clearly see that the lattice results are in much more accordance with the lower bound on $\Sigma_{\pi N}$: $\sigma_{SI}^{min}(\Sigma_{\pi N} = 37 \text{ MeV}) = 1.93 \times 10^{-9}$ pb, whereas $\sigma_{SI}^{max}(\Sigma_{\pi N} = 71 \text{ MeV}) = 1.05 \times 10^{-8}$ pb. We compiled all the necessary values of f_i in the following table (see section 3.9.3 for a detailed calculation of f_H and $f_N = \frac{7}{9}\sum f_l + \frac{2}{9}$)

f_i	Lattice	Min	Max
f_u	0.050	0.016	0.030
f_d	0.050	0.020	0.044
f_s	0.033	0.012	0.454
$f_N \sum f_l + 3 \times f_H$	0.326	0.260	0.629

In the rest of the paper, we will always present our results with the evaluation of f_s given by the maximum and minimum allowed value for $\Sigma_{\pi N}$ and the lattice extraction of Young et al. (which gives quite similar cross section to the one obtained by the MILC group as we concluded above).

3.8 Spinorial effect

When the dark matter have a spin, there is possibility to measure spin-spin interaction between the dark matter and the nucleus[7]. Indeed, the experimentalist cannot control

[7]One of the clearer presentation of the phenomena is present in the first article proposing the direct detection principle [46] which is also a very pedagogical work.

the energy spectrum of the incident WIMPs, but can control the target material. The choice of the target nuclei has a major impact on the type of WIMP scatters allowed. Indeed, the interaction can be spin–dependent or spin–independent. The spin independent interaction occurs when a WIMP scatter off *all* the nucleons in a nucleus. For a spin–dependent interaction, the incident WIMP *must* have a spin, and the target nucleus must *also* have net spin from an unpaired nucleon. The unpaired nucleon and the WIMP interact. As we already discussed, the spin–independent interaction will be obviously enhanced by a coherence factor of A^2 if $qR \lesssim 1$ (Heisenberg uncertainty principle, in other words when the wavelength of the exchanged energy is smaller than the size of the nucleus -not nucleons of course), which is not the case in spin–dependent interactions if we suppose that the dark matter interact only with the unpaired nucleons (the *independent single–particle shell model*). In this section, we will extract the spin dependent interaction of the dark matter on the nucleus step by step : from the partons level to the nucleons one and then the nucleus level. At the partons level, the operator responsible for spin–dependent dark matter–nucleus $(\chi - N)$ interactions is generated by the Lagrangian

$$\mathcal{L}_{SD} = \sum_{q=u,d,s} \alpha_q \; (\bar{\chi}\gamma^\mu\gamma^5\chi)(\bar{q}\gamma_\mu\gamma^5 q) \tag{3.37}$$

where α_q is the coupling determined by the model under study. As we can deduce from the section (C.3.1) we can write

$$\langle \mathbf{n}|\bar{q}\gamma_\mu\gamma^5 q|\mathbf{n}\rangle = 2s_\mu^{(\mathbf{n})}\Delta q^{(\mathbf{n})} \tag{3.38}$$

$\mathbf{n}$ standing for $\mathbf{n}$ucleon (p or n). and where $s_\mu^{(\mathbf{n})}$ is the spin of the nucleon and the $\Delta q^{(\mathbf{n})}$ are extracted from polarized deep inelastic scattering. They represent the fractional spin of the proton carried by quarks of type q. Assuming flavor SU(3) symmetry, one can extract two independent combinations of three δq, denoted by F and D, from the observed semileptonic decay rates of baryon in the lowest octet. Fits to all the decays fix F and D at [67]

$$\begin{aligned} F &= \frac{1}{2}(\Delta u - \Delta s) = 0.47 \pm 0.04, \\ D &= \frac{1}{2}(\Delta u - 2\Delta d + \Delta s) = 0.81 \pm 0.03. \end{aligned} \tag{3.39}$$

A third combination, $(1/9)(4\Delta u + \Delta d + \Delta s) = 0.175 \pm 0.018$ has been measured in polarized electron and muon scattering experiments [68, 69] but this result should be regarded with caution. Combining the three measurements gives us

$$\begin{aligned} \Delta u^{(p)} &= \Delta d^{(n)} = 0.78 \pm 0.08, \\ \Delta d^{(p)} &= \Delta u^{(n)} = -0.50 \pm 0.08, \\ \Delta s^{(p)} &= \Delta s^{(n)} = -0.16 \pm 0.08. \end{aligned} \tag{3.40}$$

The conventional nonrelativistic QCD partons model however predicts instead $\Delta u = 0.97$, $\Delta d = -0.28$, $\Delta s = 0$, though with reasonable modification [70] the model can be made

consistent with the scattering data giving the results of Eq.(3.40). Recent global WCD analysis for the g_1 structure function [71], including $O(\alpha_s^3)$ corrections, corresponds to the following values of spin nucleon parameters [72]

$$\Delta u^{(p)} = \Delta d^{(n)} = 0.78 \pm 0.02, \quad \Delta d^{(p)} = \Delta u^{(n)} = -0.48 \pm 0.02, \quad \Delta s^{(p)} = \Delta s^{(n)} = -0.15 \pm 0.02 \tag{3.41}$$

The coefficient of the effective dark matter–nucleon interaction can thus be written

$$\mathcal{L} = 2\sqrt{2}G_N \; a_{\mathbf{n}}(\bar{\chi}\gamma^\mu\gamma^5\chi)(\bar{\mathbf{n}}s_\mu^{(\mathbf{n})}\mathbf{n}) \quad \text{with} \quad a_{\mathbf{n}} = \sum_{q=u,d,s} \frac{\alpha_q}{\sqrt{2}G_N} \; \Delta q^{(\mathbf{n})}. \tag{3.42}$$

After summing on all the nucleons $\mathbf{n}$ composing the nucleus N and noticing that $\bar{\chi}\gamma^\mu\gamma^5\chi = 2s_\chi^\mu \; \mathbf{s}_\chi$ being the spin of the dark matter, we obtain

$$\mathcal{M} = 4\sqrt{2}G_N \; \mathbf{s}_\chi.\langle N||a_p\mathbf{S}^p + a_n\mathbf{S}^n||N\rangle \tag{3.43}$$

with $\mathbf{S}^{p,n} = \sum_i \mathbf{s}^{p_i,n_i}$. After applying the Wigner–Eckardt theorem to the reduced matrix element of the spin $\langle N||\mathbf{S}||N\rangle$

$$\langle N; J||\mathbf{S}||N; J\rangle = \frac{\sqrt{J(2J+1)(J+1)}}{M_J}\langle N; J, M_J|\mathbf{S}|N; J, M_J\rangle \tag{3.44}$$

and in accordance with the convention where the z component of the angular momentum J and spin operators S are evaluated in the maximal M_J state e.g. $\langle\mathbf{S}\rangle = \langle N|\mathbf{S}|N\rangle = \langle J, M_J = J|S_z|J, M_J = J\rangle$ we finally can write

$$\mathcal{M} = 4\sqrt{2}G_N \; \mathbf{s}_\chi.\sqrt{\frac{(2J+1)(J+1)}{J}}\,(a_p\langle N|\mathbf{S}^p|N\rangle + a_n\langle N|\mathbf{S}^n|N\rangle) \tag{3.45}$$

from the Fermi golden rule and after a mean on the initial spin/momentum

$$\begin{aligned}\frac{d\sigma_{SD}}{dE_R} &= 2M_N\frac{d\sigma}{d|q|^2} = \frac{2M_N|\mathcal{M}|^2}{\pi v^2(2J+1)} = \frac{16M_N G_F^2 J(J+1)}{\pi\; v^2}\left[a_p\frac{\langle N|\mathbf{S}^p|N\rangle}{J} + a_n\frac{\langle N|\mathbf{S}^n|N\rangle}{J}\right]^2 \\ &= \frac{16M_N G_F^2 J(J+1)}{\pi\; v^2}\Lambda^2 = 2M_N\frac{\sigma_{SD}^0}{4\mu^2 v^2}\end{aligned} \tag{3.46}$$

with

$$\Lambda = a_p\frac{\langle N|\mathbf{S}^p|N\rangle}{J} + a_n\frac{\langle N|\mathbf{S}^n|N\rangle}{J} \quad \text{and} \quad \sigma_{SD}^0 = \frac{32G_F^2}{\pi}\mu^2 J(J+1)\Lambda^2 \tag{3.47}$$

A typical cross section for weakly interacting particles is 10^{-4} pb. From the experimental point of view, the PICASSO (Project in Canada to Search for Supersymmetric Objects) experiment is a direct dark matter search experiment that is located at SNOLAB in Canada. It uses bubble detectors with Freon as the active mass. PICASSO is predominantly sensitive to spin-dependent interactions of WIMPs with the fluorine atoms in the Freon.

3.9 More about the effective approach

3.9.1 Validity of the approach

To compute a differential or absolute rate in the section (3.7) we took an effective approach where we integrated out the massive fields exchanged in the interaction process. This is only possible if the energy Q transfered from the dark matter to the nucleus in the interaction is negligible compared the the mass scale of the intermediate particle. We computed in Eq.(3.11) the maximum recoiled energy transfered, $E_R^{max} = \frac{2\mu^2 v_\chi^2}{M_N}$ which gives for the maximum impulsion of the intermediate state

$$Q^{max} = \sqrt{2M_N \times E_R^{max}} = 2\mu v_\chi = 2v_\chi \frac{M_N m_\chi}{M_N + m_\chi} \simeq m_\chi v \simeq 100 \text{ MeV} \tag{3.48}$$

where we took $M_N = m_\chi = 100$ GeV and $v_\chi = 300$ km/s for a numerical approximation. We thus see that any mediator M at the electroweak scale ($M_M \simeq 100$ GeV like a Z or Higgs boson for instance) is sufficiently heavy to justify the approximation in the propagator $(Q^2 - M_M^2)^2 \simeq M_M^4$: the effective approximation is valid. We can also notice that the energy of recoil of a 100 GeV nucleus is then of the order of

$$E_R^{max} \simeq \frac{1}{2} m_\chi v_\chi^2 \simeq \frac{1}{2}\left(\frac{m_\chi}{1 \text{ GeV}}\right) \text{ keV} \tag{3.49}$$

which means about 50 keV for a 100 GeV dark matter (still considering $v_\chi \simeq 300$ km/s), justifying the low threshold needed in direct detection experiments (around the keV scale)

3.9.2 Effective operators

Generalities

It is possible to write the more general form of a Lagrangian for a fermion field in the effective approach

$$\mathcal{L}_F = S_F\ \bar{\chi}\chi\bar{f}f + V_F\ \bar{\chi}\gamma^\mu\chi\bar{f}\gamma_\mu f + A_F\ \bar{\chi}\gamma^\mu\gamma^5\chi\bar{f}\gamma_\mu\gamma^5 f - \frac{1}{2}C_F\ \bar{\chi}\sigma^{\mu\nu}\chi\bar{f}\sigma_{\mu\nu}f \tag{3.50}$$

f being the Standard Model fermions (quarks and leptons), χ the dark matter field and $\sigma^{\mu\nu} = \gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu$ is the antisymmetric current tensor. A lot of operators are absent because they disappear in the limit of null momentum transfer. Indeed, $\bar{u}(p_2)\gamma^5 u(p_1)$ is proportional to $p_2 - p_1$ and so is negligible in direct detection prospects. Let's demonstrate it with the use of Eq.(C.22)

$$\begin{aligned}\bar{u}(p_2)\gamma^5 u(p_1) &= \bar{u}(p_2)\frac{\not{p}_2}{2m}\gamma^5 u(p_1) + \bar{u}(p_2)\gamma^5\frac{\not{p}_1}{2m}u(p_1) \\ &= \frac{(p_2 - p_1)_\nu}{2m}\bar{u}(p_2)\gamma^\nu\gamma^5 u(p_1) \propto (p2 - p1)_\nu \simeq 0\end{aligned} \tag{3.51}$$

With the same argument we can show that $\bar{u}(p_2)\gamma^\mu u(p_1)$ is not-null only for the time component ($\mu = 0$) whereas $\bar{u}(p_2)\gamma^\mu\gamma^5 u(p_1)$ is null for the time component so avoiding mixing terms of the form $\bar{u}(p_2)\gamma^\mu\gamma^5 u(p_1) \times \bar{u}(p_2)\gamma_\mu u(p_1)$. Let's demonstrate it:

$$\begin{aligned}
&\bar{u}(p_2)\gamma^\mu u(p_1) = \bar{u}(p_2)\frac{\not{p}_2}{2m}\gamma^\mu u(p_1) + \bar{u}(p_2)\gamma^\mu\frac{\not{p}_1}{2m}u(p_1) = \frac{(p_2)_\nu}{2m}\bar{u}(p_2)\gamma^\nu\gamma^\mu u(p_1) + \frac{(p_1)_\nu}{2m}\bar{u}(p_2)\gamma^\mu\gamma^\nu u(p_1)\\
&= \frac{(p_2)_\nu}{2m}\bar{u}(p_2)(-\gamma^\mu\gamma^\nu + 2\eta^{\mu\nu})u(p_1) + \frac{(p_1)_\nu}{2m}\bar{u}(p_2)\gamma^\mu\gamma^\nu u(p_1)\\
&= \frac{(p_1-p_2)_\nu}{2m}\bar{u}(p_2)\gamma^\mu\gamma^\nu u(p_1) + \frac{(p_2)_\mu}{m}\bar{u}(p_2)u(p_1) \simeq \frac{(p_2)_\mu}{m}\bar{u}(p_2)u(p_1)
\end{aligned} \tag{3.52}$$

Only the time component is non-zero in the non-relativistic case ($p_i \ll E_i$). In the case of the operator $\bar{u}(p_2)\gamma^\mu\gamma^5 u(p_1)$ one can write

$$\begin{aligned}
&\bar{u}(p_2)\gamma^\mu\gamma^5 u(p_1) = [..] = \frac{(p_1+p_2)_\nu}{2m}\bar{u}(p_2)\gamma^\mu\gamma^5\gamma^\nu u(p_1) + \frac{(p1+p_2)^\mu}{2m}\bar{u}(p_2)\gamma^5 u(p_1)\\
&\simeq \frac{E_1+E_2}{2m}\bar{u}(p_2)\gamma^\mu\gamma^5\gamma^0 u(p_1) \simeq -\bar{u}(p_2)\gamma^\mu\gamma^0\gamma^5 u(p_1)
\end{aligned} \tag{3.53}$$

which vanish in the zero-momentum transfer limit for the time component $\mu = 0$. This result shows that terms of the form $\bar{u}(p_2)\gamma^\mu\gamma^5 u(p_1) \times \bar{u}(p_2)\gamma^\mu u(p_1)$ can be neglected in the computation of direct detection processes.

Scalar coefficient: generalities

The scalar coefficient of the equation Eq.(3.50) $S_F\bar{\chi}\bar{f}f$ is in fact a sum or partonic interactions

$$\mathcal{L}_S = S_F\ \bar{\chi}\chi\bar{f}f = \sum_q \lambda_q\bar{\chi}\chi\bar{q}q \Rightarrow \langle N|\mathcal{L}_S|N\rangle = \sum_q \lambda_q\langle N|\bar{q}q|N\rangle\bar{\chi}\chi = \sum_q \lambda_q\frac{M_N}{m_q}f_q\ \bar{N}N\bar{\chi}\chi \tag{3.54}$$

with $f_q = \frac{m_q}{M_N}\langle N|\bar{q}q|N\rangle = \frac{\sigma_q}{M_N}$ that we will compute[8] in section 3.9.3. We can then compare Eq.(3.54) to Eq.(3.50) to express the effective scalar coupling to the nucleon $S_F = \sum_q \lambda_q\frac{M_N}{m_q}f_q$. From this effective Lagrangian, we can compute the amplitude square $|\mathcal{M}_N|^2$ and the cross scattering cross section of χ on the nucleon N :

$$\begin{aligned}
&|\mathcal{M}_N|^2 = Tr\Big[(\not{p}_N + M_N)(\not{p}_N + M_N)\Big]Tr\Big[(\not{p}_\chi + m_\chi)(\not{p}_\chi + m_\chi)\Big] \times \left(\sum_q \lambda_q f_q\frac{M_N}{m_q}\right)^2\\
&= 64\left(\sum_q \lambda_q f_q\frac{M_N}{m_q}\right)^2 M_N^2 m_\chi^2
\end{aligned} \tag{3.55}$$

[8]The physical interpretation of $\langle N|\bar{q}q|N\rangle$ is the creation of a quark-antiquark pair from the nucleons N. This measure in fact the "number" or probability of presence of the sea quarks in the nucleons and does not take into account the valence quarks (contrarily to the vectorial interaction as we will see later).

The cross section is then (using Eq. C.108)

$$\sigma_{\chi N}^{SI} = \frac{|\mathcal{M}|^2}{4}\frac{1}{16\pi(M_N+m_\chi)^2} = \left(\sum_q \lambda_q f_q \frac{M_N}{m_q}\right)^2 \frac{M_N^2 m_\chi^2}{\pi(M_N+m_\chi)^2} \tag{3.56}$$

If the particle is a Majorana, on need to multiply the amplitude by 2 (and thus the cross section by 4) because one need to add the two symmetric diagrams by the exchange of ingoing/outgoing dark matter particle, the factor 1/4 comes from the mean on the ingoing spin particles.
To obtain the scattering amplitude on a nucleus made of Z protons p and $A-Z$ neutron n, on obtains (considering $m_p \simeq m_n = M_N$)

$$|\mathcal{M}|^2 = 64 m_\chi^2 M_N^2 \left(\lambda_p Z + \lambda_n (A-Z)\right)^2 \tag{3.57}$$

with $\lambda_i = \sum_q \lambda_q f_q^i \frac{M_N}{m_q}$ and $f_q^i = \frac{m_q}{M_N}\langle i|\bar{q}q|i\rangle$.

Scalar coefficient: application

We can apply the previous calculation to compute the cross section in models with Higgs (scalar) portal in the case of a spin 0 or spin 1/2 dark matter. If one include the Higgs propagator in an effective Lagrangian at the quark level can be written, after the breaking of the electroweak symmetry[9]

$$\begin{aligned}
&\mathcal{L}_S = y_\chi H\bar{\chi}\chi + \sum_q y_q H\bar{q}q \quad \rightarrow \quad \mathcal{L}_S^{\text{eff}} = \sum_q \frac{y_q y_\chi}{(p^2-m_H^2)}\bar{\chi}\chi\bar{q}q \quad \simeq \quad -\sum_q \frac{y_q y_\chi}{m_H^2}\bar{\chi}\chi\bar{q}q \\
&\Rightarrow \quad \mathcal{L}_S^N = -\sum_q \frac{y_q y_\chi}{m_H^2}\frac{M_N}{m_q} f_q\ \bar{\chi}\chi\bar{N}N
\end{aligned} \tag{3.58}$$

with y_i are the yukawa-like couplings[10] and $p = p_q' - p_q$ the transferred energy in the process, negligible compared to the Higgs mass. We can consider the Higgs as a general extra-scalar particle. In the case of a standard model Higgs, we know that $y_q = \frac{m_q}{v} = \frac{g m_q}{2M_W}$ ($\langle H\rangle = v/\sqrt{2}$). We will compute the elastic diffusion cross section for a scalar and a fermionic dark matter by first compute the amplitude squared in the scalar case

$$|\mathcal{M}_\mathcal{S}|^2 = (4)\times\frac{y_\chi^2 M_N^2}{m_H^4} Tr\left[(\not{p}_N + M_N)(\not{p}_N' + M_N)\right]\left(\sum_q \frac{y_q}{m_q} f_q\right)^2 = (4)\times\frac{8M_N^4 y_\chi^2}{m_H^4}\left(\sum_q \frac{y_q}{m_q} f_q\right)^2 \tag{3.59}$$

[9]Be careful, in our notation H is the *physical* Higgs field after the symmetry breaking, which means $H \rightarrow (H+v)/\sqrt{2}$, compared with the definition of the Yukawa coupling. The y_q is the yukawa coupling divided by $\sqrt{2}$!!

[10]Notice the factor $\sqrt{2}$. The Standard Model yukawa coupling is $\frac{y_q}{\sqrt{2}}H\bar{q}q$ due to the Higgs normalization field after the electroweak breaking.

in the limit $\not{p}_N - \not{p}'_N \ll M_N$. The first factor (4) is a symmetry factor that should be present if $\bar{\chi} = \chi$ (a factor 2 from the amplitude if χ is real). From Eq.(C.108) we can compute the spin independent cross section on the nucleon N

$$\sigma^{\rm SI}_{\chi N} = \frac{1}{2}\frac{|\mathcal{M}_S|^2}{16\pi(M_N+m_\chi)^2} = \frac{(4)\times M_N^4 y_\chi^2}{4\pi m_H^4 (M_N+m_\chi)^2}\left(\sum_q \frac{y_q}{m_q} f_q\right)^2 \tag{3.60}$$

the first 1/2 factor comes from the mean on the nucleon spin.
In a microscopic (gauge invariant) model, the effective coupling $y_\chi H\bar{\chi}\chi$ is generated through the invariant term (before the symmetry breaking) $-\frac{1}{4}\lambda_{hSS}H^\dagger H\chi^2$ which gives (after the symmetry breaking $H \to (v+H)/\sqrt{2} \to \ -\frac{1}{4}v\lambda_{hSS}H\chi^2 = -\frac{M_W}{2g}\lambda_{hSS}H\chi^2$ after the $SU(2)\times U(1)$ breaking. We then obtain using $m_q = y_q v$

$$\begin{aligned}
\sigma^{\rm SI}_{\chi N} &= \frac{(4)\times M_N^4\lambda_{hSS}^2}{64\pi(M_N+m_\chi)^2 m_H^4}\left(\sum_q f_q\right)^2 \\
\Rightarrow \quad \sigma^{\rm SI}_{A\chi} &= \frac{(4)\times M_N^4\lambda_{hSS}^2}{64\pi(M_N+m_\chi)^2 m_H^4}(f_p Z + (A-Z)f_n)^2
\end{aligned} \tag{3.61}$$

where $\sigma^{\rm SI}_{A\chi}$ represents the cross section on the nucleus and $f_{i=p,n} = \sum_q f_q^i = \sum_q \frac{m_q}{M_i}\langle i|\bar{q}q|i\rangle$. You can see another computation in section C.7.5. For a 100 GeV dark matter and a typical coupling $\lambda_{hSS} = 10^{-1}$, we obtain a nucleon cross section of the order of $\sigma^{\rm SI}_{N\chi} \simeq 10^{-16}$ GeV$^{-2} \simeq 4\times 10^{-20}$ barns $= 10^{-8}$ pb which is in the limits of actual experiments.
In the case of a fermionic dark matter we have

$$\begin{aligned}
|\mathcal{M}_F|^2 &= (4)\times\frac{y_\chi^2 M_N^2}{m_H^4} Tr\left[(\not{p}_\chi + m_\chi)(\not{p}'_\chi + m_\chi)\right]\times Tr\left[(\not{p}_N + m_N)(\not{p}'_N + m_N)\right]\left(\sum_q \frac{y_q}{m_q} f_q\right)^2 \\
&= (4)\times 64\frac{y_\chi^2 M_N^4 m_\chi^2}{m_H^4}\left(\sum_q \frac{y_q}{m_q} f_q\right)^2 \\
\Rightarrow \quad \sigma^{\rm SI}_{N\chi} &= \frac{(4)}{4}\frac{|\mathcal{M}_F|^2}{16\pi(M_N+m_\chi)^2} = (4)\times\frac{y_\chi^2 M_N^4 m_\chi^2}{\pi m_H^4 (M_N+m_\chi)^2}\left(\sum_q \frac{y_q}{m_q} f_q\right)^2
\end{aligned} \tag{3.62}$$

the factor (4)$\times$ being present if we have a Majorana dark matter. In the case of a microscopic model, using $y_q = \frac{g m_q}{2M_W}$ and $y_\chi = -\frac{1}{4}\frac{\lambda hff}{\Lambda}\frac{2M_W}{g}$ generated by the gauge invariant Lagrangian $-\frac{\lambda_{hff}}{4\Lambda}H^\dagger H\bar{\chi}\chi$, with Λ an effective BSM scale we obtain

$$\sigma^{\rm SI}_{\chi N} = (4)\times\frac{\lambda_{hff}^2}{16\pi\Lambda^2}\frac{M_N^4 m_\chi^2}{m_H^4(M_N+m_\chi)^2}\left(\sum_q f_q\right)^2 \tag{3.63}$$

The sum $\sum f_q$ is on all the heavy and light quark and is explicitly computed in section 3.9.3, Eq.(3.81).

Vector coefficient: generalities

The operator $\bar{\chi}\gamma^\mu\chi$ is odd under the charge conjugaison and this is not present if the dark matter is a Majorana particle (see section 3.9.2) However if the dark ammter is a Dirac one the operator $\bar{\chi}\gamma^\mu\chi$ is not nulle and contribute to the direct detection process. It is typically the cas for the exchange of a vectorial particle (Z or Z'). Indeed, if the particle is not a Majorana particle, its antimatter is not itself and the detector detect both signals without distinguishing them. The operators $\bar{\chi}\gamma^\mu\chi$ or $\bar{q}\gamma^\mu q$ are in fact proportional to the impulsion. Indeed, using the Dirac equation (C.22) and $\gamma^0(\gamma^\nu)^\dagger\gamma^0 = \gamma^\nu$. We have seen in Eq.(3.52) that only the time component is no-null. we can then write $\bar{q}\gamma^0 q = q^\dagger q$. After the decomposition on creation and annihilation operators (see the Peskin and Schroeder pages 60-62 for details)

$$q(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s (a_p^s u^s(p)e^{-ip.x} + (b_p^s)^\dagger v^s(p)e^{ip.x}) \tag{3.64}$$

$$\Rightarrow \quad \int d^3x q^\dagger(x)q(x) = \int \frac{d^3p}{(2\pi)^3}\sum_s((a_p^s)^\dagger a_p^s + b_{-p}^s(b_{-p}^s)^\dagger) = \int \frac{d^3p}{(2\pi)^3}\sum_s((a_p^s)^\dagger a_p^s - (b_p^s)^\dagger b_p^s)$$

the minus sign in the last equation is coming from the anticommutation relation between two fermionic states. a_p^s $[(a_p^s)^\dagger]$ being the annihilation [creation] operator of a particle of impulsion p and spin s, b_p^s $[(b_p^s)^\dagger]$ the annihilation [creation] operator of an *anti*particle of impulsion p and spin s. For simplification in the following, we will forget the sum on the spin and note a_p^q (b_p^q) the annihilation operator of (anti)quark q $(\bar{q})$ of impulsion p. We can then define a nucleon state as a normalized sum of states composed by valence quarks, valence quarks plus a pair of quark-antiquark of the sea, valence quarks plus 2 pairs of quark-antiquark... We will take the case of a proton and up-type quarks in the sea. The generalization for a neutron and a complete sea of quarks is straightforward. If we write[11]

$$|P\rangle = |u_{p_1}u_{p_2}d_{p_3}\rangle\,(1 + |\bar{u}_{p_4}u_{p_5}\rangle + |\bar{u}_{p_4}u_{p_5}\bar{u}_{p_6}u_{p_7}\rangle + ...) \quad \text{with} \quad \langle P|P\rangle = 1 \tag{3.65}$$

where the first state represent the valence quarks states and the states in parenthesis are the sea quarks,

$$|u_{p_1}u_{p_2}d_{p_3}\rangle = (a_{p_1}^u)^\dagger(a_{p_2}^u)^\dagger(a_{p_3}^d)^\dagger|0\rangle \quad \text{and} \quad |\bar{u}_{p_4}u_{p_5}\rangle = (b_{p_4}^u)^\dagger(a_{p_5}^u)^\dagger|0\rangle \tag{3.66}$$

Using the anticommutation relation $a_p^u(a_{p_i}^u)^\dagger = -(a_{p_i}^u)^\dagger a_p^u + (2\pi)^3\delta(p-p_i)$ we can easily show (do not forget there is an integration on the impulsion on the Eq.(3.64) eliminating the δ function and using anticommutation relation to reorder p_1, p_2, p_3

$$\begin{aligned}
&\int \frac{d^3p}{(2\pi)^3}(a_p^u)^\dagger a_p^u(a_{p_1}^u)^\dagger(a_{p_2}^u)^\dagger(a_{p_3}^d)^\dagger|0\rangle = (a_{p_1}^u)^\dagger(a_{p_2}^u)^\dagger(a_{p_3}^d)^\dagger|0\rangle - \int \frac{d^3p}{(2\pi)^3}(a_p^u)^\dagger(a_{p_1}^u)^\dagger(a_p^u)^\dagger(a_{p_2}^u)^\dagger(a_{p_3}^d)^\dagger|0\rangle \\
&= |u_{p_1}u_{p_2}d_{p_3}\rangle - (a_{p2}^u)^\dagger(a_{p_1}^u)^\dagger(a_{p_3}^d)^\dagger|0\rangle - 0 = 2\,|u_{p_1}u_{p_2}d_{p_3}\rangle \\
&\Rightarrow \int \frac{d^3p}{(2\pi)^3}\left[(a_p^u)^\dagger a_p^u - (b_p^u)^\dagger b_p^u\right]|u_{p_1}u_{p_2}d_{p_3}\rangle = 2|u_{p_1}u_{p_2}d_{p_3}\rangle
\end{aligned} \tag{3.67}$$

[11] We will not include the gluonic contribution in the expression of the proton state as the creation/annihilation operators in $q^\dagger q$ do not create or annihilate gluons.

With the same method, we can easily show that

$$\begin{aligned}
&\int \frac{d^3p}{(2\pi)^3}(a_p^u)^\dagger a_p^u|u_{p_1}u_{p_2}d_{p_3}\rangle|\bar{u}_{p_4}u_{p_5}\rangle = 2|u_{p_1}u_{p_2}d_{p_3}\rangle|\bar{u}_{p_4}u_{p_5}\rangle + |u_{p_1}u_{p_2}d_{p_3}\rangle|\bar{u}_{p_4}u_{p_5}\rangle \\
&\int \frac{d^3p}{(2\pi)^3}(b_p^u)^\dagger b_p^u|u_{p_1}u_{p_2}d_{p_3}\rangle|\bar{u}_{p_4}u_{p_5}\rangle = |u_{p_1}u_{p_2}d_{p_3}\rangle|\bar{u}_{p_4}u_{p_5}\rangle \\
&\Rightarrow \int \frac{d^3p}{(2\pi)^3}\left[(a_p^u)^\dagger a_p^u - (b_p^u)^\dagger b_p^u\right]|u_{p_1}u_{p_2}d_{p_3}\rangle|\bar{u}_{p_4}u_{p_5}\rangle = 2|u_{p_1}u_{p_2}d_{p_3}\rangle|\bar{u}_{p_4}u_{p_5}\rangle
\end{aligned} \tag{3.68}$$

Combining the previous result, we can deduce

$$\int \frac{d^3p}{(2\pi)^3}\left[(a_p^u)^\dagger a_p^u - (b_p^u)^\dagger b_p^u\right]|P\rangle = 2|P\rangle \quad \Rightarrow \quad \int d^3x\langle P|u^\dagger u|P\rangle = 2 \tag{3.69}$$

We can then obviously prove following the same procedure that

$$\int d^3x\langle P|d^\dagger d|P\rangle = 1, \quad \int d^3x\langle Ne|u^\dagger u|Ne\rangle = 1, \quad \int d^3x\langle Ne|d^\dagger d|Ne\rangle = 2 \tag{3.70}$$

$|Ne\rangle$ being the neutron state. As a conclusion, we have proved that the operator $\langle N|q^\dagger q|N\rangle$ measure the contents of the *valence* quark q in the nucleon N. We can then deduce the effective dark matter-nucleon coupling from the dark matter-quark coupling

$$\begin{aligned}
&\mathcal{L}_V^\chi = \sum_q f_q\ \bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q \quad \Rightarrow \quad \langle N|\mathcal{L}_V|N\rangle = \sum_q f_q\langle N|\bar{q}\gamma^\mu q|N\rangle\bar{\chi}\gamma_\mu\chi \\
&= \sum_q f_q f_q^N\ \bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N, \quad \text{with} \ \ f_u^p = f_d^n = 2 \ \ \text{and} \ \ f_d^p = f_u^n = 1.
\end{aligned} \tag{3.71}$$

The particle χ being a Dirac particle, its antimatter $\tilde{\chi}$ has also interaction to the nucleus. However, contrarily to the scalar interaction, we can easily show using Eq.(C.33) that the operator $\bar{\tilde{\chi}}\gamma^\mu\tilde{\chi} = -\bar{\chi}\gamma^\mu\chi$

$$\mathcal{L}_V^{\tilde{\chi}} = \sum_q f_q f_q^N\ \bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N \tag{3.72}$$

We notice that the interaction Lagrangian for the anti-dark matter particle is the opposite to the one with the dark matter interaction. However, these two processes are independents without interference between them. When one computes an interaction cross section one should add both processes, proportional to the local density of (anti)dark matter respectively. This happens typically for asymmetric dark matter where the density of the anti-dark matter is much less important that the dark matter one.

Vector coefficient: application

We can also apply the previous calculation to compute the effective Lagrangian in the case of the exchange of a massive vectorial particle V_μ

$$\mathcal{L} = y_\chi V_\mu \bar{\chi}\gamma^\mu\chi + \sum_q y_q V_\nu \bar{q}\gamma^\nu q \quad \Rightarrow \quad \mathcal{L}^{\text{eff}} = \frac{y_\chi y_q}{M_V^2}\bar{\chi}\gamma^\mu\chi\ \bar{q}\gamma_\mu q \tag{3.73}$$

We will concentrate on a fermionic dark matter, we let the reader make the computation for a scalar dark matter. The amplitude squared is then (we let the reader check it with the trace formulae in the appendix)

$$|\mathcal{M}|^2 = |\bar{\chi}\gamma^\mu\chi\ \bar{N}\gamma_\mu N|^2 = 64\ m_\chi^2 M_N^2 \tag{3.74}$$

which is the same amplitude squared than in the case of the exchange of a massive scalar we computed in Eq.(3.62). We can then use the results already obtained.

Axial-vector coefficient

$\sigma_{\mu\nu}$ coefficient

Majorana case

In the case of Majorana dark matters, the antimatter field $\tilde{\chi} = \chi$. The expressions are even simpler as some operators vanish. The operator $\bar{\chi}\gamma^\mu\chi$ for instance can be written

$$\bar{\chi}\gamma^\mu\chi = \frac{1}{2}\left(\bar{\chi}\gamma^\mu\chi + \bar{\tilde{\chi}}\gamma^\mu\tilde{\chi}\right) = \frac{1}{2}\left(\bar{\chi}\gamma^\mu\chi - \bar{\chi}\gamma^\mu\chi\right) = 0$$

where we used the relations Eq.(C.33). We also obtain a null contribution for the $\sigma_{\mu\nu}$ term. The only operators that we need to take into consideration are then

$$S_F\bar{\chi}\chi\bar{f}f + A_F\bar{\chi}\gamma^\mu\gamma^5\chi\bar{f}\gamma^\mu\gamma^5 f. \tag{3.75}$$

which gives respectively the spin independent and spin dependent amplitude.

3.9.3 Gluon and heavy quarks contribution

We can go into a more detailed study of the quark content of the nucleon. Indeed, in the equation (3.50) the fermion f entering in the interaction should be the nucleon. However, the effective interaction at a microscopic level is on the partons (quarks and gluons) and not directly on the nucleon. Concentrating in the scalar case first, one need to compute

$$\begin{aligned}\mathcal{L}_{eff} &= \sum_f \lambda_f \langle N;\chi|\bar{\chi}\chi\bar{f}f|\chi;N\rangle = \sum_f \lambda_f \langle\chi|\bar{\chi}\chi|\chi\rangle\langle N|\bar{f}f|N\rangle \\ &= \sum_q \lambda_q\bar{\chi}\chi \sum_{q=u,d,s}\langle N|\bar{q}q|N\rangle + \sum_Q \lambda_Q\bar{\chi}\chi\sum_{Q=c,b,t}\langle N|\bar{Q}Q|N\rangle\end{aligned} \tag{3.76}$$

where we have separated light (q=u, d, s) and heavy (Q=c, b, t) quark states.The values of $\langle N|\bar{q}q|N\rangle$ can be deduced directly by lattice QCD or perturbative QCD approaches. By these

methods one can usually extract the quantity $\sigma_q = m_q\langle N|\bar{q}q|N\rangle$ (in MeV) or equivalently $f_q = \frac{m_q}{M_N}\langle N|\bar{q}q|N\rangle$ (dimensionless). However, there are no equivalent mean to compute $\langle N|\bar{Q}Q|N\rangle$. In order to obtain such a quantity, one needs first to compute the trace of the energy-momentum tensor $\theta_{\mu\nu}$ in the nucleon state [58] :

$$
\begin{aligned}
M_N\langle N|N\rangle = \langle N|\theta_{\mu\mu}|N\rangle &= \langle N|\sum_{q=u,d,s} m_q\bar{q}q + \sum_{Q=c,b,t} m_Q\bar{Q}Q + \frac{\beta(\alpha_s)}{4\alpha_s}G^a_{\mu\nu}G^{\mu\nu}_a|N\rangle \\
&= \langle N|\sum_{q=u,d,s} m_q\bar{q}q + \sum_{Q=c,b,t} m_Q\bar{Q}Q - \frac{\alpha_s}{8\pi}\left[11 - \frac{2}{3}n_{f=q+Q}\right]G^a_{\mu\nu}G^{\mu\nu}_a|N\rangle
\end{aligned}
\tag{3.77}
$$

for n_f quark family, where $G^a_{\mu\nu}$ is the gluonic field tensor and $\beta(\alpha_s) = \mu\frac{\partial\alpha_s}{\partial\mu} = -\frac{\alpha^2}{2\pi}\left[11-\frac{2}{3}n_f\right]$ is the β-function of the strong $SU(3)$ interaction, μ being the energy scale. The contribution proportional to $G^a_{\mu\nu}G^{\mu\nu}_a$ in Eq.(3.77) comes from the triangle anomalies in $\theta_{\mu\mu}$ [58]. We will separate the energy momentum into the heavy quark states and the light quark states. Indeed, one can make one step further and get rid of the heavy quarks in Eq.(3.77) because there are no vamence heavy quarks in the nucleon. they can enter only via a virtual state, at short distances of order $\frac{1}{2m_Q}$. It is equivalent to develop a quark propagator $\frac{1}{\not{P}-m_Q}$ when $m_Q \gg P$. The heavy quark term at the heavy mass limit gives (at the first order) [73]

$$
\sum_{Q=c,b,t} m_Q\bar{Q}Q \to -\frac{2}{3}\frac{\alpha_s}{8\pi}n_Q G^a_{\mu\nu}G^{\mu\nu}_a + \mathcal{O}(\alpha_s^2) \tag{3.78}
$$

n_Q being the number of heavy quarks. As we can see, the limit obtained in Eq.(3.78) cancels exactly the anomaly contribution of the heavy quarks in the energy-momentum tensor in Eq.(3.77) and is independant of the mass of the quark (at first order). We are then left with only the light quark contribution, in other words

$$
M_N\langle N|N\rangle = M_N = \langle N|\sum_{q=u,d,s} m_q\bar{q}q - \frac{9\alpha_s}{8\pi}G^a_{\mu\nu}G^{\mu\nu}_a|N\rangle \tag{3.79}
$$

Combining Eqs.(3.78)and (3.79) we deduce

$$
\begin{aligned}
&\langle N|\sum_{Q=c,b,t} m_Q\bar{Q}Q|N\rangle = \frac{2n_Q M_N}{27} - \frac{2n_Q}{27}\langle N|\sum_{q=u,d,s} m_q\bar{q}q|N\rangle \\
&\Rightarrow \quad f_Q = \frac{m_Q}{M_N}\langle N|\bar{Q}Q|N\rangle = \frac{2}{27}\left[1 - \sum_{q=u,d,s} f_q\right]
\end{aligned}
\tag{3.80}
$$

which is the same contribution for any heavy quarks as their mass never appear in the limit $m_Q \to \infty$. We can also understand it as their contribution are generated by triangle anomalies which are mass-independent (topological) quantities.
The quantity useful in the computation of the direct detection cross section is $\sum f_q$ (see Eq.3.62 for instance). we then have

$$f_N = \sum_{q=u;d;s} f_q + \sum_{Q=c,b,t} f_Q = \sum_q f_q + 3 \times \frac{2}{27}\left[1 - \sum_q f_q\right] = \frac{7}{9}\sum_q f_q + \frac{2}{9}. \tag{3.81}$$

3.9.4 A concrete example: the exchange of a Z'.

3.9.5 The saga of the strange content of the nucleon

3.10 The experiments

3.10.1 The XENON experiment

Principle

XENON is a direct dark matter detection experiment using liquid xenon as the detector medium. The goal is to detect the small charge and light signal after a dark matter particle interaction with a xenon nucleus in a liquid-gas two-phase time projection chamber (TPC): the prompt light signal (S1) is detected with two array of photomultipliers. The ionization electrons are separated from the Xe ions and drifted upwards by a strong electric field. A second electric field extracts the charges from the liquid into the gas phase where they generate secondary scintillation light (S2) which is proportional to the charge signal. The secondary signal – delayed to the S1 by the electron drift time – is detected with the same two photodetector arrays. The TPC design allows the precise 3-dimensional reconstruction of the interaction vertex which can be used to reduce the background contamination by fiducial volume cuts. Furthermore, the ratio S2/S1 has a different value for electron recoils (background) and nuclear recoils (signal) and can be used for background discrimination.

the XENON100 detector

The XENON100 detector is a position-sensitive XeTPC, with the sensitive LXe volume viewed by two arrays of total 178 photomultiplier tubes (PMTs), to detect simultaneously the primary scintillation signal (S1) and the ionization signal via the proportional scintillation mechanism (S2). The active target is enclosed in a PTFE cylinder of 15 cm radius and 30 cm height. This PTFE cylinder reflects scintillation light with high efficiency, and optically separates the LXe target from the surrounding LXe which is necessary to separate the TPC with its electric field from the walls of the vessel. 64 PMTs turn this outer LXe volume into an active LXe veto, with a total mass of 105 kg, including LXe layers above the top and below the bottom PMT arrays.

3.10.2 the LUX experiment

The Large Underground Xenon experiment (LUX) is a 370 kg liquid xenon detector, located at round 1.5 kms underground at the Sanford Underground Laboratory (formerly the Deep Underground Science and Engineering Laboratory, or DUSEL) in the Homestake Mine (South Dakota) in Lead, South Dakota, USA. Their first results were released on the 30th

Figure 3.9: respectively.

of October 2013 and were, a that time, the best limits on spin independent scattering cross section of weakly interacting particles. The LUX cryostat was filled for the first time in the underground laboratory in February 2013.

The LUX detector

The apparatus is a cylindrical volume of liquid and gaseous xenon (hence "two-phase"). A uniform axial electric field is maintained using voltage rings, which are evenly spaced along the length of the detector. There are two arrays of photomultiplier tubes (phototubes), situated with one array at the top and the other at the bottom. The interior walls are designed to reflect as much light as possible, while trapping as few drifting electrons as possible, to maximize the detection of both.

Principle

Particle interactions in liquid xenon generate both 175 nm ultraviolet photons and electrons. The photons are immediately detected by two arrays of 61 photomultiplier tubes located at the top and bottom of the detector. These prompt photons are called the S1 signal. The electrons generated by the particle interaction are drifted upwards towards the xenon gas phase with an electric field in the liquid xenon. The electrons are extracted out of the liquid and into the gas by using a stronger electric field at the liquid xenon surface. In the gas, the electrons produce electroluminescence photons that are detected by the photomultiplier tubes as the S2 signal. The pair of an S1 and a subsequent S2 signal constitutes an individual particle interaction in the liquid xenon.

The LUX detector is a time-projection chamber (TPC) since it uses the time between the S1 and the S2 signals to find the depth of a given particle interaction. Because the electrons

Figure 3.10: LUX detector and principle (left) and its latest results released on October the 30th, 2013 (right) [561] corresponding to the exclusion regions for spin independent scattering.

move with a constant velocity in the liquid xenon (around 1 to 2 km.s^{-1}, depending on the strength of the electric field), the time delay of the S2 signal relative to the S1 signal can be converted into a physical depth (z-coordinate). The x-y coordinate of the event can be inferred by looking at the distribution of detected electroluminescence photons in the top PMT array. Since the S2 signal is generated about 5 cm below the top PMT array, the PMT nearest to the S2 electron bunch will receive the most photons. Statistical techniques (such as a Monte Carlo simulation and a maximum likelihood estimation) can be used to find the x-y location of a particle interaction from the top PMT photon hit pattern with a resolution smaller than 1 cm. We illustrate the principle of the LUX detection in Fig.(3.10)[left].

Results

The results from an initial unblinded data run of the LUX experiment were announced on October 30, 2013. In an 85-live-day run with a 118 kg fiducial volume, LUX obtained 160 events passing the data analysis selection criteria, all consistent with electron recoil backgrounds. A profile likelihood statistical approach used by the collaboration shows that this result is consistent with the background-only hypothesis (no WIMP interactions) with a p-value of 0.35. The 85-day LUX result is the most sensitive dark matter direct detection result in the world, and rules out low-mass WIMP signal hints such as those from CoGeNT and CDMS-II Si. Their data are consistent with the background-only hypothesis, allowing 90% confidence limits to be set on spin-independent WIMP-nucleon elastic scattering with a minimum upper limit on the cross section of 7.6×10^{-46} cm^2 at a WIMP mass of 33 GeV/c^2 [561].

3.10.3 Summary plot

To summarize the direct detection experimental status, we present on the Fig.(3.11) the state of the arts of the present and future prospects in the field. Sensitivity projections are subject to uncertainties from many factors, including technical issues with the experiments,

Figure 3.11: A compilation of WIMP-nucleon spin-independent cross section limits (solid curves), hints for WIMP signals (shaded closed contours) and projections (dot and dot-dashed curves) for direct detection experiments that are expected to operate over the next decade. Also shown is a band indicating the cross sections where WIMP experiments will be sensitive to backgrounds from solar, atmospheric, and diffuse supernovae neutrinos.

the appearance of unexpected backgrounds, and delays in funding. Despite these uncertainties, the history of the field gives us confidence that progress will continue unabated through the next decade. There is, however, an irreducible background caused by solar, atmospheric, and supernova neutrinos. The ^8B component of the solar neutrinos provides the dominant neutrino coherent scattering rate at low recoil energies. The ^8B spectrum can mimic a WIMP with a mass in the range of $5 - 10\text{GeV/c}^2$ (depending on the target) and a nucleon cross section of $5 \times 10^{-45}\text{cm}^2$. For experiments targeted at larger WIMP masses the solar neutrinos give way to the more energetic atmospheric neutrinos and diffuse supernovae background. The flux of these neutrinos is much lower, and exposures with sensitivities to WIMP-nucleon cross sections of 10^{-48} cm^2 are required to be sensitive to this neutrino component. Depending on the particular WIMP mass under consideration, these neutrino backgrounds can have a recoil spectrum that is very similar to an authentic WIMP signal. Given the Poisson fluctuations from the neutrino signal and their relatively large total flux uncertainties, this creates a challenge to improving the sensitivity of WIMP searches much beyond such cross sections. Figure **??** shows not only the current landscape, but also the projected sensitivities of proposed experiments superimposed on the neutrino background, where coherent neutrino scattering will begin to limit WIMP sensitivity. This will eventually require either background subtraction or techniques such as directional or annual modulation to press beyond this background in the absence of a positive WIMP sighting

3.11 A concrete example :the kinetic dark-mixing in the light of CoGENT and XENON100**

This section is mainly based on the reference [499].

3.11.1 Introduction

Neutral gauge sectors with an additional dark $U(1)_D$ symmetry in addition to the Standard Model (SM) hypercharge $U(1)_Y$ and an associated Z_D are among the best motivated extensions of the SM, and give the possibility that a dark matter candidate lies within this new gauge sector of the theory. Extra gauge symmetries are predicted in most Grand Unified Theories (GUTs) and appear systematically in string constructions. Larger groups than $SU(5)$ or $SO(10)$, like E_6 allows the SM gauge group to be embedded into them. Brane–world $U(1)'$s are special compared to GUT $U(1)'$s because there is no reason for the SM particle to be charged under them. For a review of the phenomenology of the extra $U(1)'s$ generated in such scenarios see e.g. [592]. On the other hand, recent anomalies in cosmic rays and direct detection experiments have motivated the exploration of new gauge interactions in a putative dark sector [**?**, **?**, **?**, 631]. The new vector boson Z_D can interact with the SM, even if no SM fermions are directly charged under the additional gauge symmetry. This interaction occurs via mixed kinetic terms between the SM's hypercharge field strength and the new abelian field strength [481, 483–486, 552, 593]. Very recently, a possibility of effective higgs couplings to the dark sector generated through a triangular loop of Z and/or Z_D has been analyzed in [501] and string scenarios can lead to naturally light hidden photons [597].

Other important consequences and clear dark matter signatures in satellite telescopes are studied in [506, 507, 604]

Our objective is triple: to know if, taking into account the last data and analysis from CDMSII, CoGENT and XENON100, there is still part of the parameter space allowed by all constraints, especially WMAP and the electroweak precision tests. Secondly, considering CoGENT, CDMSII or DAMA as *signal* events, we compute the kinetic mixing and Z_D mass required to fit the excesses. Finally, if we consider that the last XENON100 results exclude CoGENT and DAMA excesses, we project the future sensitivity needed to explore the remaining part of the parameter space. The paper is organized as follows: after an introduction to the model and its motivations, we look in details the cosmological and accelerator constraints we should apply for our study. We then look at the parameter space already reached out by XENON100 and CDMS-Si and fit the last data released by CoGENT, DAMA and CRESST. We conclude with some prospects for the XENON100 experiment.

3.11.2 The model

The matter content of any *dark* $U(1)_D$ extension of the SM can be decomposed into three families of particles:

- The *Visible sector* is made of particles which are charged under the SM gauge group $SU(3) \times SU(2) \times U(1)_Y$ but not charged under $U(1)_D$ (hence the *dark* denomination for this gauge group)
- the *Dark sector* is composed by the particles charged under $U(1)_D$ but neutral with respect of the SM gauge symmetries. The dark matter (ψ_0) candidate is the lightest particle of the *dark sector*
- The *Hybrid sector* contains states with SM *and* $U(1)_D$ quantum numbers. These states are fundamental because they act as a portal between the two previous sector through the kinetic mixing they induce at loop order.

From these considerations, it is easy to build the effective lagrangian generated at one loop :

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\rm SM} - \frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{\delta}{2}\tilde{B}_{\mu\nu}\tilde{X}^{\mu\nu} \\ &+ i\sum_i \psi_i\gamma^\mu D_\mu\psi_i + i\sum_j \Psi_j\gamma^\mu D_\mu\Psi_j \end{aligned} \tag{3.82}$$

B_μ being the gauge field for the hypercharge, X_μ the gauge field of $U(1)_D$ and ψ_i the particles from the hidden sector, Ψ_j the particles from the hybrid sector, $D_\mu = \partial_\mu - i(q_Y\tilde{g}_Y\tilde{B}_\mu + q_D\tilde{g}_D\tilde{X}_\mu + gT^aW^a_\mu)$, T^a being the $SU(2)$ generators, and

$$\delta = \frac{\tilde{g}_Y\tilde{g}_D}{16\pi^2}\sum_j q^j_Y q^j_D \log\left(\frac{m_j^2}{M_j^2}\right) \tag{3.83}$$

with m_j and M_j being hybrid mass states [513] .

Notice that the sum is on all the hybrid states, as they are the only ones which can contribute to the $Y_\mu X_\mu$ propagator. After diagonalization of the current eigenstates that makes the gauge kinetic terms of Eq.D.13 diagonal and canonical, we can write after the $SU(2)_L \times U(1)_Y$ breaking[12] :

$$\begin{aligned} A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \\ Z_\mu &= \cos\phi(\cos\theta_W W^3_\mu - \sin\theta_W B_\mu) - \sin\phi X_\mu \\ (Z_D)_\mu &= \sin\phi(\cos\theta_W W^3_\mu - \sin\theta_W B_\mu) + \cos\phi X_\mu \end{aligned} \tag{3.84}$$

with, at the first order in δ:

$$\begin{aligned} \cos\phi &= \frac{\alpha}{\sqrt{\alpha^2 + 4\delta^2\sin^2\theta_W}} \quad \sin\phi = \frac{2\delta\sin\theta_W}{\sqrt{\alpha^2 + 4\delta^2\sin^2\theta_W}} \\ \alpha &= 1 - M^2_{Z_D}/M^2_Z - \delta^2\sin^2\theta_W \\ &\pm \sqrt{1 - M^2_{Z_D}/M^2_Z + 4\delta^2\sin^2\theta_W} \end{aligned} \tag{3.85}$$

and + (-) sign if $M_{Z_D} < (>) M_Z$. The kinetic mixing parameter δ generates an effective coupling of SM states $\psi_{\rm SM}$ to Z_D, and a coupling of ψ_0 to the SM Z boson which induces an interaction on nucleons. Developing the covariant derivative on SM and ψ_0 fermions state, we computed the effective $\psi_{\rm SM}\psi_{\rm SM}Z_D$ and $\psi_0\psi_0 Z$ couplings at first order in δ. One can find other implications of such construction in [**?**, 513, 600]

3.11.3 The constraints

The cosmological constraint

The abundance of a thermal relic dark matter candidate ψ_0 is controlled by its annihilation cross section into SM particles mediated by the exchange of a Z_D gauge boson through $s-$channel or $t-$channel $Z_D Z_D$ final state (see [506, 604] for a detailed study of the relic abundance constraints). We modified the micrOMEGAs2.2.CPC code[13] [716] in order to calculate the relic abundance of ψ_0. We show in Fig.3.12 the points that fulfill the WMAP 5σ bound [13] on $\Omega_{\rm DM}$ for $m_{\psi_0} = 10$ GeV (left) and 5 GeV (right) in the (M_{Z_D}, δ) plane. One can clearly sees the Z_D-pole region when $M_{Z_D} \sim m_{\psi_0}$. One important point is that for a given M_{Z_D} and m_{ψ_0}, there exists a unique solution δ (up to the very small uncertainties at 5σ) fulfilling WMAP constraints : from 3 parameters $(m_{\psi_0}, M_{Z_D}, \delta)$, the WMAP constraints reduce it to two (M_{Z_D}, δ).

The electroweak precision constraints

Concerning the electroweak symmetry breaking, the mixing between $\tilde{X}_\mu$ and $\tilde{B}_\mu$ generates new contributions to precision electroweak observables. However, none of the particle of

[12] Our notation for the gauge fields are $(\tilde{B}^\mu, \tilde{X}^\mu)$ before the diagonalization, (B^μ, X^μ) after diagonalization and (Z^μ, Z^μ_D) after the electroweak breaking.

[13] The author wants to thank particularly G. Belanger and S. Pukhov for their help to address this issue.

Figure 3.12: Two examples of allowed parameter space for $m_{\psi_0} = 10$ GeV (left) and $m_{\psi_0} = 5$ GeV (right). The points between the full-red region respect the 5σ WMAP constraint, the points below the dashed-black line do not exceed accelerator data on precision tests, and the points above the dotted-green line are excluded by XENON100 data.

the SM has any $U(1)_D$ charges: the $U(1)_D$ can be considered has a $lepto - hadrophobic$ Z_D. Other authors in [?, ?, 599] or [598] have looked at hidden-valley like models or milli–charged dark matter but concentrating their study to relatively heavy Z_D and large mixing angle. The authors of [509] have computed the observables from effective Peskin–Takeuchi parameters [517], and found

$$\begin{aligned} \Delta m_W &= (17\text{MeV})\ \zeta \\ \Delta\Gamma_{l+l-} &= -(8\text{keV})\ \zeta \\ \Delta \sin^2\theta_W^{eff} &= -(0.00033)\ \zeta \end{aligned} \tag{3.86}$$

where

$$\zeta \equiv \left(\frac{\delta}{0.1}\right)^2 \left(\frac{250\text{GeV}}{m_X}\right)^2 \tag{3.87}$$

Different electroweak measurements from LEP give $|\zeta| \lesssim 1$. These constraints are represented by the black line in Fig.3.12. A new analysis was made more recently in [?] and [?] but they confirmed that in models with extra $U(1)'s$ which does not couple at tree-level with SM particles (like a leptophobic Z_D for instance) or with the higgses, the constraints on the mass of the gauge boson are very weak. The only case where one can put some strongest constraints is if a non-trivial higgs sector acts as a portal between the Dark sector and the SM one at tree level (like in Supersymmetry for instance).

However, for the mass range of interest in this work ($m_{Z_D} \sim 10$ GeV) we needed to look at the search of production/decay of hidden bosons at low energy $e + e-$ colliders [517].

Indeed, over a large range of parameters, the cross sections for the production of dark–sector particles scale as

$$\sigma \sim \frac{g g_D \delta^2}{16\pi^2 E_{\rm cm}^2} \tag{3.88}$$

where E_{cm} is the center-of-mass energy of the collider. The search sensitivity of a given e^+e^- machine above mass threshold scales as the ratio of integrated luminosity over squared center-of-mass energy, $\mathcal{L}_{\rm int}/E_{\rm cm}^2$. LEP and Tevatron are much less sensitive to direct production of low mass dark sectors than the B-factories.

While writing the present article, the authors of [518] have published an extensive model independent analysis in the energy range of interest in our study. They bounded the kinetic mixing by $\delta \lesssim 0.03$ for 10 GeV $< M_{Z_D} <$ 200 GeV which is in complete agreement with the constraints given by Eq.8.7 plotted in Fig.3.12. Their strongest upper limit ($\delta \lesssim 0.03$) come from a *wide* Z_D whose dark decay is maximized. For $M_{Z_D} \lesssim 10$ GeV, they also computed a model dependent exclusion from BaBar searches and obtained $\delta \lesssim 3 \times 10^{-3}$ which is also in agreement with Eqs.8.7.

The XENON/ CDMS constraints

In recent months, there have been new data releases from many experiments that have engendered a great deal of excitement (see section 3.11.4 for a discussion and references). The XENON100 collaboration has recently released new dark matter limits [**?**], placing particular emphasis on their impact on searches known to be sensitive to light-mass ($\sim$ 10 GeV) WIMPs. The existing bounds set by the XENON10 [**?**], and the recognition that the effect of channeling in NaI(Tl) crystal is less important than previously assumed [**?**] can be combined by the full data set released recently by CDMS [**?**] to obtain tighter bounds to the elastic cross section. In our work, we will use the analysis made by the authors of [**?**], whereas a similar analysis can be found in [**?**]. In practice, the differences between CDMS and XENON constraints appear when $m_{\psi_0} \lesssim$ 10GeV, where CDMS-Si is more sensitive than XENON100 [up to the renormalization used for the calculation of the XENON100 efficiency discussed in the section 3.11.4]. We show in Fig.3.12 two examples of points, one excluded (left) and the other one allowed (right) by XENON100.

3.11.4 Results

Combining all the constraints

We show in Fig.14.4 the parameter space still allowed after applying all the constraints described above. The red points respect WMAP constraints after a scan on m_{ψ_0}, and the ones below the black lines are not excluded by electroweak precision tests. The points with black circle are excluded by the last data released by the XENON experiment. We observe that a region with $M_{Z_D} \lesssim 40$ GeV and $10^{-4} \lesssim \delta \lesssim 10^{-3}$ is still open. We can understand easily why for increasing values of the kinetic mixing the XENON constraints seem to weaken: for a fixed M_{Z_D}, higher values of δ increase the annihilation cross section, and decrease the relic density. To fulfill WMAP, one needs to find a point with m_{ψ_0} far away from the pole ($M_{Z_D}/2$), and therefore lighter. This is a region that XENON has difficulties to exclude:

Figure 3.13: Constraints coming from WMAP (red boxes), electroweak data (black line), and recent direct detection analysis of XENON100 after correction of their efficiency factor [**?**]

the sensitivity of a direct detection experiments decreases for light dark matter candidate as their efficiencies are worst for low-energy nuclear recoil. For instance, for $M_{Z_D} = 20.6$ GeV and $\delta_1 = 10^{-4}$, WMAP is fulfilled for $m_{\psi_0} = 10.5$ GeV, which is a point lying exactly in the Z_D-pole region. The spin independent elastic scattering on the proton is in this case $\sigma^p_{SI} = 7 \times 10^{-40}$ cm^2 which is already excluded by XENON and CDMS-Si. However, for $\delta_2 = 4 \times 10^{-3}$, WMAP is fulfilled for $m_{\psi_0} = 4.04$ GeV, quite avay from the Z_D pole, generating a higher cross section $\sigma^p_{SI} = 10^{-38}$ cm^2 ($\delta_2 > \delta_1$) but which is not yet excluded by XENON whose sensitivity is 3.5×10^{-38} cm^2 for such a light ψ_0.

Signals from COGENT, CRESST or DAMA?

The DAMA collaboration has provided strong evidence for an annually modulated signal in the scintillation light from sodium iodine detectors. The combined data from DAMA/NaI [**?**] (7 annual cycles) and DAMA/LIBRA [**?**] (4 annual cycles) with a total exposure of 0.82 ton yrs shows a modulation signal with 8.2σ significance. The phase of this modulation agrees with the assumption that the signal is due to the scattering of a WIMP.

Recently, the CoGeNT collaboration has announced the observation of an excess of low energy events relative to expected background [497]. This excess, if interpreted as dark matter, implies the dark matter particles possess a mass in the range of 5-15 GeV and an elastic scattering cross section with nucleons of the order of 10^{-4} pb. Moreover, recently, a series of analysis and comments have been released concerning the effective value of the XENON100 efficiency at low energy (L_{eff}). We will not enter into all the technical details here, a complete analysis of the computation of L_{eff} and its consequence on the constraints that we can derive from the XENON experiment can be found in [**?**, **?**, **?**, **?**, **?**]. The main conclusion (until now) is that it is not yet clear if the DAMA/LIBRA and CoGeNT regions are excluded by XENON100 (see [502] for a model independent analysis concerning light dark matter scenario). The main discussion concerns the extrapolation of L_{eff} and its interpretation in the detection of $S1$ light from low-energy nuclear recoil. To be as conservative as possible, we explore in this section the possibility to interpret these excesses with a dark sector with a kinetic mixing portal.

Figure 3.14: Parameter space allowed within 90 % of C.L. for the CoGeNT signal (blue), DAMA without channeling (red), with channeling (green), CRESST (black), and the exclusion region depending on the hypothesis concerning L_{eff} (se the text for details).

We show in Fig.3.14 the points respecting WMAP, and the DAMA/LIBRA (with and without channeling) CoGeNT and CRESST[14] results at 90 % of CL. In performing our fits, we have used the 13 DAMA/LIBRA bins below 8.5 keVee and the 28 CoGeNT bins between 0.4 and 1.8 keVee. The data at higher energies will not include any events from dark matter particles in the mass range considered here, and the inclusion of higher energy bins would not affect our results in any significant way. Concerning the CRESST result, it is important to emphasize that some fraction of the events observed in the oxygen band could be spillage from CRESST's alpha or tungsten bands, neutron backgrounds, or be the result of radioactive backgrounds. Further information from the CRESST collaboration will be essential for understanding these results. All the constraints have been calculated for a standard Maxwellian velocity distribution (with mean velocity $v_0 = 230$ km/s and an escape velocity $v_{esc} = 600$ km/s). One can observe in Fig.3.14 that for all experiments, the regions are quite surprisingly near and correspond to 10 GeV $\lesssim M_{Z_D} \lesssim$ 30 GeV and $10^{-4} \lesssim \delta \lesssim 10^{-3}$, which is in complete agreement with the measurement of electroweak precision tests. Moreover, such values of δ are typical of one loop-order corrections and can easily be generated by heavy-fermions loops in the $Z - Z_D$ propagator.

[14]For the CRESST estimation, we used an extrapolation given in the talk of T. Schwetz and the CRESST collaboration [496].

We show in Fig.3.15 the points respecting the accelerator, cosmological, and the more severe direct detection constraints in the plane $(m_{\psi_0}; \sigma^p_{SI})$ in comparison with XENON100 and CDMS-Si sensitivity. To take into account the uncertainties on L_{eff}, we plotted 3 exclusion limit for XENON corresponding to the best fit set by XENON100 in [**?**], which give $L_{eff} \simeq 0.12$ (L_{effMed}) at small nuclear recoil energy E_{nr}. A more conservative choice (L_{effMin}, corresponding to a lower 90 % C.L. fit to the data) gives a L_{eff} which decreases monotonically with E_{nr} and vanishes at $E_{nr} < 1$ keV. The ZEPLIN experiment (also a Xe experiment) uses a different L_{eff}, which is essentially zero below 6-7 keV (L_{effZep} here). For dark matter masses below ~ 10 GeV, the CDMS-II silicon detectors provide very stringent constraints [**?**] du to the favorable kinematics of the lighter target nucleus. However, the observed CDMS-II silicon nuclear recoil quenching is not reproduced by Linhard theory [**?**]. This discrepancy could indicate a $\sim 20 - 30$ % error in the low energy calibration [**?**]. The uncorrected exclusion curve is also presented in Fig.3.15. We also took into account the exclusion limit at 90% of C.L. from CRESST-I experiment with sapphire-based cryogenic detector at a threshold of 600 eV [**?**]. We see that a large region is still to be explored. It corresponds to dark matter masses between 1 and 10 GeV, a range of masses which could be difficult but far from impossible to probe in a near future experiment.
During the completion of this work, the authors of [**?**] showed that even without channeling but when taking into account uncertainties in the relevant quenching factors, a dark matter candidate with a mass of approximately ~ 7 GeV and a cross section with nucleons of $\sigma_p^{\rm SI} \sim 2 \times 10^{-40}$ cm^2 could account for both these observations. Even if the values they used for the Na quenching factor can be considered extreme to some extent, these results correspond to a dark gauge boson mass $M_{Z_D} \sim 15$ GeV and $\delta \sim 2 \times 10^{-4}$, which is in the range of interest for our present study. Other interesting constraints to check would be the antimatter production/detection as computed in specific final states cases in [**?**], galactic gamma-ray or from the isotropic diffuse emission [504] and colliders perspectives [**?**].

3.11.5 Conclusion

We showed that the existence of a *dark* $U(1)_D$ gauge sector which interacts with the Standard Model only through its kinetic mixing possesses a valid dark matter candidate respecting accelerator, cosmological and the more recent direct detection constraints. Moreover, considering the latest results of DAMA/LIBRA, CoGENT and CRESST, we demonstrated that a specific range of the kinetic mixing ($\delta \sim 10^{-4} - 10^{-3}$) can explain all these excesses for a dark boson mass $M_{Z_D} \sim 10 - 20$ GeV. Such a value of kinetic mixing is intriguingly in agreement with the value one can expect if the mixing is generated by heavy hybrid-fermions loop corrections. Other models have similar specificities ([503] for instance) : the diagram for annihilation is the same than the one leading the scattering process (Z_D exchange in the former case, h exchange in the latter). We also showed that the region of the parameter space still allowed by all constraints will be difficult but far from impossible to probe in a near future.

Figure 3.15: Points still allowed by electroweak, cosmological and direct detection constraints in the plane $(M_{\psi_0}; \sigma^p_{SI})$. The green region corresponds to CoGeNT (minimum χ^2, with contours at 90 and 99.9% C.L.), assuming a constant background contamination [497]. The DAMA regions (goodness-of-fit, also at 90 and 99.9 % C.L.) are given both with (upper black region) and without (lower black region) channelling [**?**]. The black dashed line is the 90 % C.L. exclusion limit for the CDMS-Si [**?**] and the brown full line the 90% C.L. exclusion limit for the CRESST I experiment [**?**]. The blue dotted lines corresponds to the 90% C.L. exclusion limit from the XENON100 experiment corresponding respectively to the LeffMed (left), Leffmin (middle) and LeffZep (right) scintillation efficiency -see text for details.

Chapter 4

Particles in the galaxies

The indirect detection of any kind of particles in the Universe is strongly linked with its density and its propagation in the interstellar medium. We will describe in the first sections of this chapter the different elements one should take into consideration when studying indirect detection of dark matter, before entering in more details into the possible detection modes and strategy set up to detect indirectly dark matter.

4.1 The anatomy of the Milky Way

4.1.1 Internal characteristics

The unit of distance commonly used in astronomy is the parsec. The parsec (parallax of one arcsecond; symbol: pc) is a unit of length, equal to just under 31 trillion kilometres (30.857×10^{15} meters), or about 3.26 light-years[1]. The parsec measurement unit is used in astronomy. It is defined as the length of the adjacent side of an imaginary right triangle in space. The two dimensions that specify this triangle are the parallax angle (defined as 1 arcsecond) and the opposite side (which is defined as 1 astronomical unit (AU), the distance from the Earth to the Sun). Given these two measurements, along with the rules of trigonometry, the length of the adjacent side (the parsec) can be found.

The stellar disk of the Milky Way Galaxy is approximately 100 000 light-years (30 kpc, 9.5×10^{17} km) in diameter, and is considered to be, on average, about 1 000 ly (0.3 kpc, 9.5×10^{15} km) thick. It is estimated to contain at least 200 billion stars and possibly up to 400 billion stars, the exact figure depending on the number of very low-mass stars, which is highly uncertain. The Sun (and therefore the Earth and the Solar System) lies close to the inner rim of the Milky Way Galaxy's Orion Arm at a hypothesized distance of 7.5-8.5 kpc (25 000-28 000 lightyears) from the Galactic Center. The orbital speed of the Solar System about the center of the Galaxy is approximately 251 km/s. All the features of the Milky Way are summarized in Fig.(4.1)

[1] A light-year or light year (symbol: ly) is a unit of length, equal to just under 10 trillion (i.e. 10^13) kilometres. As defined by the International Astronomical Union (IAU), a light-year is the distance that light travels in a vacuum in one Julian year

Figure 4.1: Anatomy of the Milky Way.

4.1.2 The color of the sky : the diffuse gamma ray background

When we look up on a clear night, we see the beautiful twinkling of stars and planets amid a black sky. If we could see the same sight with X-ray or gamma ray eyes, we'd still see bright points, but the sky would no longer be dark. Instead, it would glow faintly. This is the diffuse high-energy background: X-ray and gamma-ray light from all over the sky. By looking at different wavelengths of X-rays and gamma rays, which correspond to different energies, we can find out what causes this background glow.

X-ray Diffuse Background

At low X-ray energies (about 1/4 keV), the sky glows with radiation from hot gas filling some of the space between the stars. This gas has a temperature of about 1 million degrees and is heated in two ways: by supernovae, which leave shining remnants of hot gas behind; and by the hot winds of massive young stars, which heat surrounding gas and form stellar wind bubbles. At higher X-ray energies (above 1/2 keV), the source of the diffuse background changes considerably. While emission from supernova remnants and stellar wind bubbles is still visible, it is less dominant than at lower energies. Much of the background radiation becomes isotropic (that is, it looks the same in all directions). Scientists believe the radiation comes from outside our Milky Way Galaxy, since radiation from within the Galaxy would be brighter in some places and dimmer in others, due to our galaxy's shape.

Above 1 keV, most of the "diffuse" background is not truly diffuse in origin at all, but comes from many distant extragalactic objects. We know this from "deep" observations of the diffuse X-ray background. In astronomy, a "deep" observation means that a detector looks at a given point in space for a very long time. Using the deepest ROSAT observations, over 60 % of the 1 - 2 keV diffuse background has been resolved into very distant, separate sources, typically quasars.

Gamma-ray Diffuse Background

While there are many individual point sources of gamma rays, there is also a significant amount of gamma-ray light from gas in our own Milky Way Galaxy. Gamma-ray light from this gas stretches out in a band across the sky, comprising much of the diffuse gamma-ray background. The remaining gamma-ray background light we see comes from far outside our galaxy: it's the faint glow of the rest of the Universe, covering all the sky beyond the Milky Way.

Yet another source of diffuse gamma-ray radiation in our galaxy is the violent interaction of matter and anti-matter. Certain galactic phenomena produce positrons. When positrons and electrons come into contact, they annihilate abd release a burst of gamma-ray energy of exactly 511 keV. These gamma rays are part of the diffuse gamma-ray background in our galaxy.

The diffuse gamma-ray emission from outside our galaxy may be due to the combined light of far-away individual objects. The objects are most likely active galaxies (AGN) like blazars, starburst galaxies and millisecond pulsar. These are very similar to quasars, the main source of the X-ray background outside our galaxy. The diffuse gamma ray background was forecast to be one of the more robust constraints of annihilating WIMP dark matter., and determining its spectrum/amplitude is a challenge to restrict the dark matter parameter space.

4.1.3 Galactic coordinates, velocity of the Sun and of the Earth

To observe a position in the sky, or when a satellite like COBE, WMAP, FERMI or PLANCK gives a MAP of the sky, they usually work in galactic coordinates : galactic longitude (l) and galactic latitude (b). The longitude measures the angular distance of an object from the sun eastward along the galactic equator from the galactic center whereas the latitude measures the angular distance of an object perpendicular to the galactic equator, positive to the north, negative to the south. This is well illustrated if Fig.(4.2). The Milky way is at his densest near Sagittarius as it is where the galactic center lies. The center of Sagitarius, a black hole called Sagittarius A^* is often considered as the real galactic center. Its coordinates are in fact[2] : ($l = 359^0\ 56^{'}\ 39.5^{"}$, $b = -0^0\ 2^{'}\ 46.3^{"}$) which is in fact a little bit on the south-east of the galactic center. We list in the following table (4.1) some coordinates of most common objects, especially dwarf galaxies important source of observation for dark matter candidates due to their few amount of baryonic matter.

The velocity of the sun is define from the Local Standard of Rest (LSR) and is usually designated by U, V, and W, given in km/s, with U positive in the direction of the Galactic center, V positive in the direction of galactic rotation, and W positive in the direction of the North Galactic Pole. If one define a point in space that is moving on a perfectly circular orbit around the center of the galaxy at the Sun's galactocentric distance, all velocities of stars are described relative to this point, which is known as the Local Standard of Rest. The velocity of the Local Standard of Rest is then given by ($U_{LSR} = 0$; $V_{LSR} = 0$; $W_{LSR} = 0$). The peculiar motion of the Sun with respect to the LSR is then measured and give (U, V, W) = (10.00 ± 0.36, 5.23 ± 0.62, 7.17 ± 0.38) km/s. We show an illustration of this system of

[2] $1^0 = 60^{'}$ (arc minutes) $= 3600^{"}$ (arc seconds)

Object	longitude (l)	lattitude (b)	distance (from the sun/kpc)
Carina	260.1	-22.2	103 ± 4
Draco	86.4	34.7	84 ± 8
Fornax	237.1	-65.7	138 ± 9
LeoI	226.0	49.1	247 ± 19
LeoII	220.2	67.2	216 ± 9
Sculptor	287.5	-83.2	87 ± 5
Sextans	243.5	42.3	88 ± 4
Ursa Minor	105.0	44.8	74 ± 12

Table 4.1: Galactic coordinates of some Dwarf Spheroidal galaxies used to constrain dark matter interaction.

Figure 4.2: The galactic coordinates use the Sun as the origin. Galactic longitude (l) is measured with primary direction from the Sun to the center of the galaxy in the galactic plane, while the galactic latitude (b) measures the angle of the object above the galactic plane.

Figure 4.3: (U, V, W) system of coordinates to define the peculiar motion of the sun and stars relatively to the Local Standard of Rest (LSR). Sometimes U is defined pointing out from the galactic center (given negative values for the sun velocity U–component).

coordinates in Fig.(4.3). The sun is thus deriving slowly toward the galactic center, moving a little bit faster than the entire disk, with some components toward the north pole of the Milky Way.
We can then deduce the absolute velocity of the Earth in the Milky Way:

$$\mathbf{v}_E = \mathbf{v}_r + \mathbf{v}_S + \mathbf{v}_e \tag{4.1}$$

with, in galactic coordinates

- the galactic rotation
  $\mathbf{v}_r = (0, 230, 0)$ km s^{-1};
- the Sun's proper motion (motion relative to nearby stars)
  $\mathbf{v}_S = (10, 5, 7)$ km s^{-1};
- the Earth's orbital velocity relative to the Sun.
  $\mathbf{v}_e = 15 \cos\left(2\pi \frac{t-152.5 \text{ days}}{365.25 \text{ days}}\right)$ km s^{-1}

4.2 Computation of a flux

The different profiles that have been proposed in the literature can be parameterised as [75]

$$\rho(r) = \frac{\rho_0[1 + (R_0/a)^\alpha]^{(\beta-\gamma)/\alpha}}{(r/R_0)^\gamma[1 + (r/a)^\alpha]^{(\beta-\gamma)/\alpha}} \ , \tag{4.2}$$

where ρ_0 is the local (solar neighborhood) halo density ($\simeq 0.3$ GeV/cm^3), R_0 the sun to Galactic center distance ($\simeq 8.5$ kpc) and a is a characteristic length. The calculation of the flux necessitates to know the value of the integral $J(\psi) = \frac{1}{8.5\text{kpc}} \left(\frac{1}{0.3\text{GeVcm}^{-3}}\right)^2 \int_{line\ of\ sight} \rho^2(r(l,\psi))dl$ in a solid angle Ω. From fig.(4.4), one can write $r = \sqrt{l^2 + R_0^2 - 2R_0 l \cos\psi}$ re-expressing

Figure 4.4: Topology for the measurement of a flux from the galactic halo.

Eq.(4.2) as $\rho(l, \cos\psi)$ and, for a solid angle Ω one has $d\Omega = d\phi \sin\psi d\psi \Rightarrow \cos\psi_0 = 1 - \Omega/2\pi$ which gives

$$\int_{line\ of\ sight} \rho^2(r) dl = \tag{4.3}$$
$$\int_{\cos(\psi+\psi_0)}^{\cos\psi} d\cos\psi' \int_0^{R_0 - 10^{-5}} dl \rho^2(l, \cos\psi') + \int_{\cos(\psi+\psi_0)}^{\cos\psi} d\cos\psi' \int_{R_0+10^{-5}\mathrm{pc}}^{100\mathrm{pc}} dl \rho^2(l, \cos\psi')$$

10^{-5} pc being the limit radius of the integral, where one can not say anything about the pure galactic center where lies a possible black hole (for $\psi = 0$), and the upper bound of the integral 100 pc correspond to the end of the halo core. Here you can find an example of mathematica code in the case of the galactic center in the NFW model and in a solid angle of 10^{-3}.

```
rho0 = 0.3;
alpha = 1;
beta = 3;
gamma = 1;
a = 20.;
R0 = 8.;
r = Sqrt[l^2 + R0^2 - 2  R0 l cpsi];
fr = rho0 *(1 + (R0/a)^alpha)^((beta - gamma)/alpha)/((r/R0)^
       gamma  (1 + (r/a)^alpha)^((beta - gamma)/alpha));
fr /. r -> Sqrt[l^2 + R0^2 - 2  R0 l cpsi];
fl = %
```

	a (kpc)	α	β	γ	$\bar{J}(10^{-3}\text{sr})$	$\bar{J}(10^{-5}\text{sr})$
NFW	20	1	3	1	1.214×10^3	1.264×10^4
NFW_c	20	0.8	2.7	1.45	1.755×10^5	1.205×10^7
Moore et al.	28	1.5	3	1.5	1.603×10^5	5.531×10^6
Moore_c	28	0.8	2.7	1.65	1.242×10^7	5.262×10^8

Table 4.2: NFW and Moore et al. density profiles without and with adiabatic compression (NFW_c and Moore_c respectively) with the corresponding parameters, and values of $\bar{J}(\Delta\Omega)$ from [82].

```
rhol[l_, cpsi_] = fl
dOmega = 0.001;
cpsi0 = 1 - dOmega/(2 Pi);
ftointegrate = fl^2/(1 - cpsi0)/8.5*(1/0.3)^2;
Jave = NIntegrate[
   ftointegrate, {cpsi, cpsi0, 1.}, {l, 0, R0 - 0.00001}] +
  NIntegrate[
   ftointegrate, {cpsi, cpsi0, 1.}, {l, R0 + 0.00001, 100}] +
  0.00002*rhor[0.00001, 0]

 Out =  1223.31
```

the third contribution in the integral is just the center part supposing that the density is constant in the Galactic center region. This hypothesis is not really important as the 2×10^{-5} pc region is very small.

4.3 Radiative processes in Astrophysics part I : the non-relativistic case

We present in this appendix the main processes involved in the lost of energy due to the interaction of a charged particle or a radiation on a medium (also composed of charged particles or radiation). The typical example is of course the interaction of a cosmic ray -charged particles- on the CMB or its radiation when crossing a Galactic magnetic field. We will first described the phenomena in the non–relativistic approximation ($v \ll c$) and then applied the formulae in the relativistic limit. We refer the reader to the very complete Jackson or Longair books for more details. For nucleon or electron propagation in the interstellar medium neutral matter (90% H and 10 % He) the relevant energy losses are due to electromagnetic and nuclear effects according to the type of interaction. The most relevant processes of the first group are ionization and Coulomb scattering while for the second we have spallation, fragmentation and radioactive decay. For electromagnetic processes that involve electrons, even bremsstrahlung in the neutral and ionized medium, as well as Compton and synchrotron losses became important. Although all these processes are well–known and are often explained during academic courses, the formulae for the different cases are rather scattered throughout the literature. We try to present in this appendix the main results concerning the energy losses in the interstellar medium and give formulae that can be used in any study concerning the subject.

4.3.1 Maxwell equations

4 linear equations describes the space-time evolution of a set of electromagnetic fields $\vec{E}$ and $\vec{B}$:

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0} \quad [\text{Maxwell} - \text{Gauss}] \; ; \quad \vec{\nabla}.\vec{B} = 0 \quad [\text{Maxwell} - \text{Thomson}] \tag{4.4}$$
$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad [\text{Maxwell} - \text{Faraday}] \; ; \quad \vec{\nabla} \wedge \vec{B} = \mu_0 \, \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad [\text{Maxwell} - \text{Ampere}]$$

with $c^2 = 1/\mu_0\epsilon_0$, ρ the density of charges and $\vec{j} = \sum e_i \vec{v}_i$, the density of current. Combining cleverly the equations, one can write

$$\vec{j}.\vec{E} + \frac{\partial}{\partial t}\left(\frac{\epsilon_0}{2}|\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2\right) = -\vec{\nabla}.\left(\frac{\vec{E} \wedge \vec{B}}{\mu_0}\right) = -\vec{\nabla}.\vec{\Pi} \tag{4.5}$$

with $\vec{\Pi} = (\vec{E} \wedge \vec{B})/\mu_0$ is the Poynting vector whose module $|\vec{\Pi}|$ gives the power flux (energy emitted per second per m^2). $\vec{j}.\vec{E} = \sum e_i \vec{v}_i.\vec{E}$ is the density of mechanical energy (energy produced by a moving charged particle in an electric field $\vec{E}$) whereas $\frac{\epsilon_0}{2}|\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$ is the density of energy of the radiated electric and magnetic field.
These equations are written in the SI system of units (International System). Sometimes one can find expressions of the fields in another popular system of units, the Gaussian units, that is part of the CGS (Centimeter-Gram-Second) system. When using CGS units

it is conventional to use a slightly different definition of electric field $\vec{E}_{CGS} = c^{-1}\ \vec{E}_{SI}$. This implies that the modified electric and magnetic fields have the same units (in the SI convention this is not the case). Then it uses a unit of charge defined in such a way that the permititivity of the vacuum $\epsilon_0 = 1/(4\pi c)$, hence $\mu_0 = 4\pi/c$. Using these different conventions, the Maxwell equations become:

$$\begin{aligned} &\vec{\nabla}.\vec{E} = 4\pi\rho\ ; \quad \vec{\nabla}.\vec{B} = 0 \\ &\vec{\nabla}\wedge\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}\ ; \quad \vec{\nabla}\wedge\vec{B} = \frac{1}{c}\left(4\pi\vec{j} + \frac{\partial\vec{E}}{\partial t}\right) \end{aligned} \tag{4.6}$$

4.3.2 Loss of energy of a moving charged particle

Larmor's formula

We will not deep too much into the detail study of radiative fields (see the Jackson for more details). A particle of charge e and velocity $\vec{v}$ radiate an electric field $\vec{E}$ and $\vec{B}$

$$\begin{aligned} &\vec{E}(\vec{r},t) = \frac{e}{4\pi\epsilon_0}\left[\frac{(\vec{n}-\vec{\beta})(1-\beta^2)}{\kappa^3 R^2}\right] + \frac{e}{4\pi\epsilon_0 c}\left[\frac{\vec{n}}{\kappa^3 R}\wedge[(\vec{n}-\vec{\beta})\wedge\dot{\vec{\beta}}]\right] = \vec{E}_r + \vec{E}_\theta \\ &\vec{B}(\vec{r},t) = \frac{1}{c}\left[\vec{n}\wedge\vec{E}\right], \end{aligned} \tag{4.7}$$

with $\vec{n} = \vec{R}/R$, $\vec{\beta} \equiv \vec{v}/c$ and $\kappa \equiv 1 - \vec{n}.\vec{\beta}$ (see Fig. 4.5) . The electric field can be divided into the first term, the *velocity field*, or radial field $\vec{E}_r$ which falls off as $1/R^2$ and is just the generalization of the Coulomb law to moving particles. The second term is the *acceleration field* $\vec{E}_\theta$ which falls off as $1/R$, is proportional to the particle's acceleration and is perpendicular to $\vec{n}$. This electric field, along with the corresponding magnetic field, constitutes the *radiation field*. For large values of R, the dominant term is $\vec{E}_\theta$, the acceleration field. If we look at the radiated field in a region during a time Δt at a distance $R \gg v \times \Delta t$, we can neglect the effect of the retarded emission, and consider that the retarded effects are negligible : $R \simeq$ constant during the process.
In the limit $\beta \ll 1$ we can simplify Eq.(4.7) to

$$\begin{aligned} &\vec{E}_{rad} = \frac{e}{4\pi R\epsilon_0 c^2}\ \vec{n}\wedge(\vec{n}\wedge\dot{\vec{v}}) \\ &\vec{B}_{rad} = \frac{1}{c}\ \vec{n}\wedge E_{rad} \end{aligned} \tag{4.8}$$

The energy W radiated during a time dt is then the density of energy radiated by the charge multiply by the volume dV of the radiation covered during the time dt, $dV = R^2 d\Omega \times cdt$, $d\Omega$ being the solid angle from where is seen the radiating particle at the retarded time (see Fig. 4.5) is :

$$d^2W = \left(\frac{\epsilon_0}{2}|\vec{E}_{rad}|^2 + \frac{1}{2\mu_0}|\vec{B}_{rad}|^2\right) \times R^2 d\Omega\ cdt \tag{4.9}$$

Figure 4.5: Example of the electric field $E = E_1 + E_2$ radiated by a charged particle at a distance R from the position of the particle a the retarded time.

Figure 4.6: Radiated field strength of an accelerated particle.

Figure 4.7: Geometry for polarization decomposition of radiation emitted by a circulating charge. The unitary vectors $\vec{a}_1$ and $\vec{a}_2$ are in the plane perpendicular to $\vec{n}$ and form an orthonormal basis with $\vec{n}$ ($\vec{a}_1 = -\vec{i}$, $\vec{a}_2 = -\cos\theta\vec{j} + \sin\theta\ \vec{k}$, $\vec{n} = \sin\theta\ \vec{j} + \cos\theta\ \vec{k}$)

which gives using Eq.(4.8)

$$\frac{d^2W}{dtd\Omega} = \frac{e^2\sin^2\theta|\dot{\vec{v}}|^2}{16\pi^2\epsilon_0 c^3} \tag{4.10}$$

θ being the angle between $\dot{\vec{v}}$ and $\vec{n}$. After integration over $d\Omega = 2\pi d\cos\theta$ one obtains the power P emitted by the radiating charge :

$$P = \frac{dW}{dt} = \frac{e^2|\dot{\vec{v}}|^2}{6\pi\epsilon_0 c^3}, \tag{4.11}$$

which is known under the name of Larmor's formula. The acceleration is the proper acceleration of the particle and the loss rate is measured in its instantaneous rest frame. The electric field strength varies as $\sin\theta$ and the power radiated per solid angle is proportional to $\sin^2\theta$, where θ is the angle with respect to the acceleration vector of the particle. Notice that there is no radiation along the acceleration vector and the field strength is greater at right angle to the acceleration vector. The radiation is polarised with the electric field vector lying in the direction of the acceleration vector of the particle, as projected onto a sphere at a distance r from the charged particle, see Fig.(4.6).

In the CGS units system we would have obtained

$$P_{CGS} = \frac{2e^2|\dot{\vec{v}}|^2}{3c^3} \tag{4.12}$$

Figure 4.8: Collision of a high energy particle of charge Ze with an electron at rest.

Case of a circulating particle

We can also compute the frequency of the radiated field E_{rad} in a specific example of a charged particle rotating around an axe. With the convention of Fig.(4.7) and using Eq.(4.8) one can express

$$\begin{aligned} r = a(\cos\omega t\ \vec{i} + \sin\omega t\ \vec{j}) \quad &\Rightarrow \quad \dot{\vec{v}} = -v\ \omega(\cos\omega t\ \vec{i} + \sin\omega t\ \vec{j}) \\ \Rightarrow \quad E_{rad} = -\frac{ev\omega}{4\pi R\epsilon_0 c^2}&(\cos\omega t\ \vec{a_1} + \cos\theta\sin\omega t\ \vec{a_2}) \end{aligned} \tag{4.13}$$

We then can conclude (which is an important and no so-intuitive result) that a circulating particle will radiate a *monochromatic* electromagnetic field of the same frequency that the one of the circulating charged particle. This will be a useful result when studying cyclotron and synchrotron radiation. This result is not exact anymore when radiation of higher order than dipole is included.

4.3.3 Coulomb and Ionization losses

When a high energy charged particle pass through a medium or near other charged particles like electrons bounded in atoms, it transfers part of its kinetic energy to the electrons, causing ionization or excitation of the atoms. Indeed, the electromagnetic force of the traveling particle can give to the electron orbiting on atoms a sufficient impulsion to extract it from its orbit (or makes it pass to an upper layer) and ionize the gas (or excites it). From an historical perspective, in 1895 the passing of electrons (called "cathod rays" at this time) through a tube of Crookes was the source of the first X-ray observed by Rontgen through the ionization created on the target. In this section we will concentrate on the influence on the propagation of a charged particle in mediums made of electron, and its energy lost along his path. The result we obtained is quite general and can be applied to several type of radiative processes. Technically speaking, one call Coulomb scattering when the interaction is on free electron, and ionization losses when the interaction takes place on bound electrons, the binding energy corresponding to the ionization potential.

Figure 4.9: Moving particle in an interstellar medium of density N_e.

Non-relativistic treatment

Consider first the collision of a high energy nucleus of mass M, charge Ze and velocity v with a stationary electron of mass m_e. Only a small fraction of the nucleus kinetic energy is transfer to the electron. We can understand it easily, remembering that the maximal velocity of an electron after being hit by the nucleus if one consider solid spheres collision is $v_e^{max} = \frac{2M}{M+m_e}v$ which tends to $2v$ for $m_e \ll M$. The fractional kinetic energy loss is then less than $\frac{1}{2}m_e(2v)^2/\frac{1}{2}Mv^2 = 4m_e/M$. We can then consider that the trajectory of the hitting nucleus is not influenced by the presence of the electron.
To compute the energy transferred to the electron, we compute first the impulsion received by the electron bounded in an atom from the electromagnetic field created by the traveling particle. The impulsion can be repulsive or attractive depending on the charge of the moving particle, it does not affect the result as in both cases, the electron is expelled from the atom. From the Fig.(4.8) and remarking that by symmetry only the component perpendicular to the motion contribute to the electronic impulsion

$$\Delta p = \int_{-\infty}^{+\infty} |F| dt = \int_0^{-\pi} \frac{Ze^2}{4\pi\epsilon_0 (b\sin\theta)^2} \sin\theta \, \frac{1}{v} d\left(\frac{b}{\tan\theta}\right) = \frac{Ze^2}{2\pi\epsilon_0 bv} \tag{4.14}$$

which gives for the kinetic energy transferred to the electron

$$\Delta E_c = \frac{p^2}{2m_e} = \frac{Z^2e^4}{8\pi^2\epsilon_0^2 b^2 v^2 m_e} \tag{4.15}$$

The moving particle lost the equivalent energy. Of course, when traveling in a medium like the interstellar medium, one has to sum on all the atoms in the medium. The number of electron in a gas of density N_e at a distance b of the axe of the moving particle is (see Fig.4.9) $2\pi b db dx \times N_e$

$$\frac{dE_{Ze}^{\text{loss}}}{dx} = -\frac{Z^2e^4N_e}{4\pi\epsilon_0^2 v^2 m_e} \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{Z^2e^4N_e}{4\pi\epsilon_0^2 v^2 m_e} \ln\left(\frac{b_{max}}{b_{min}}\right), \tag{4.16}$$

the minus sign representing the fact that the energy is lost by the traveling particle. The computing of b_{min} and b_{max} requires a special treatment. b_{max} represents the distance at which the influence of the traveling particle on the electron is negligible. It corresponds roughly to the time when the orbital period is lower than the typical interaction time. In other words, if the electron takes more time to move around the nucleus than to interact with the moving particle, the electromagnetic influence of the later becomes weak. If one write τ the interacting time and ν_0 the frequency of the rotating electron in the atom ($\nu_0 = \omega_0/2\pi$), it corresponds to

$$\tau \simeq \frac{2b}{v} < \frac{1}{\nu_0} \quad \Rightarrow \quad b < \frac{v}{2\nu_0} = b_{max} \tag{4.17}$$

The lower limit b_{min} can be obtained if we suppose by a quantum treatment and the application of the uncertainty principle the maximum energy transfer being $\Delta p_{max} = 2m_e v$ (because as we discussed earlier, the maximum velocity transferred to the electron is $2v$) from $\Delta p \delta x \lesssim \hbar$ (Heisenberg principle) we have $\Delta x \gtrsim \hbar/2m_e v$. We can then write

$$b_{min} = \frac{\hbar}{2m_e v}. \tag{4.18}$$

which implies

$$\frac{dE}{dx} = -\frac{z^2 e^4 N_e}{4\pi\epsilon_0^2 v^2 m_e} \ln\left(\frac{2\pi m_e v^2}{\hbar\omega_0}\right) = -\frac{z^2 e^4 N_e}{4\pi\epsilon_0^2 v^2 m_e} \ln\left(\frac{\pi m_e v^2}{I}\right) \tag{4.19}$$

where I is the ground state binding energy of the electron in the atom ($I = \frac{1}{2}\hbar\omega_0$). But the system being more complex, it contents different atoms with different kind of excited level, so one should take into account a mean ionization potential $\bar{I}$ instead of I. Integrating the π factor and a factor 2 for physical interpretation (see below) in the definition of I we finally obtain

$$\frac{dE}{dx} = -\frac{z^2 e^4 N_e}{4\pi\epsilon_0^2 v^2 m_e} \ln\left(\frac{2m_e v^2}{\bar{I}}\right) \quad \Rightarrow \quad \langle P \rangle = \frac{dE}{dt} = v\frac{dE}{dx} = -\frac{z^2 e^4 N_e}{4\pi\epsilon_0^2 v\ m_e} \ln\left(\frac{2m_e v^2}{\bar{I}}\right) \tag{4.20}$$

The value of $\bar{I}$ cannot be calculated explicitly (except for very simple models of atoms) and should thus be measured by experiment. Typical values for the logarithmic factor, called Coulomb logarithm is around 20. We recognize in the equation 4.20 the ratio maximum kinetic energy ($2m_e v^2$) on the binding energy, justifying the normalization of $\bar{I}$ A careful inspection of Eq.(4.20) reveals the the energy loss of an energetic moving particle depends on the charge and the velocity of the particle, but not on its mass (except for the Coulomb logarithm). Also, low–energy particles loose their energy more rapidly than high–energy particle.

4.3.4 Thomson scattering

The Thomson scattering is the process in which a free charge radiates in response to an incident electromagnetic wave. If the charge oscillates at non–relativistic velocities, $v \ll c$, then we may directly apply the Larmor's formula computed in the previous section. This

Figure 4.10: Thomson scattering of a polarized radiation by a charged particle.

effect arise in astroparticle typically when photon radiated by annihilation of dark matter for instance interacts with cosmic rays. If one supposes an incident *linearly polarized wave*, the force of the electromagnetic wave $\vec{F}$ on the charge particle is

$$\vec{F} = e\ \vec{\epsilon}\ E_0 \sin\omega_0 t \quad \Rightarrow \quad m\dot{\vec{v}} = e\ E_0 \sin\omega_0 t \times \vec{\epsilon} \quad \Rightarrow \langle P \rangle = \frac{e^4 E_0^2}{12\pi\epsilon_0\ m^2 c^3} \tag{4.21}$$

where $\vec{\epsilon}$ is the normalized $\vec{E}$-field direction (see Fig. 4.10). We used the fact that $\langle|\dot{\vec{v}}|^2\rangle = \frac{e^2}{2m^2}E_0^2$. Defining the Thomson cross section σ_T:

$$\langle P \rangle = \sigma_T \times [\text{initial flux of energy}] = \sigma_T \left(c\frac{\epsilon_0}{2}|E_0|^2\right) = \sigma_T\ cU_{rad} \tag{4.22}$$

where $U_{rad} = \frac{\epsilon_0}{2}|E_0|^2$ is the energy density of radiation in which the electron is located. Combining with Eq.(4.21) we obtain

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 mc^2}\right)^2 = \frac{8\pi}{3}r_0^2 = 6.65\times10^{-29}\ \text{m}^2 = 1714\ \text{GeV}^{-2} \tag{4.23}$$

with $r_0 = \frac{e^2}{4\pi\epsilon_0\ mc^2}$ is the classical radius of the charged particle[3]. σ_T is the Thomson scattering cross section and represents the fraction of the initial wave scattered by the charged particle.

Thomson scattering is one of the most important processes which impedes the escape of photons from any region. Indeed, notice that there is no change in the energy of the photons in the process: the energy is first transfer to the electron, which then by its acceleration radiates back the energy it received. After the scattering, the photons perform a random walk, each step between a mean free path depending on the density of the electrons. The electrons then prevent the photons to escape from their own plasma. The Thomson free path is then $\lambda_T = (\sigma_T N_e)^{-1}$, N_e being the number of electrons per unit volume.

[3] r_0 is defined by $mc^2 = V = \frac{e^2}{4\pi\epsilon_0 r_0}$

4.3.5 Cyclotron radiation

Cyclotron radiation corresponds to the radiation emitted by a non–relativistic charged particle trapped in a magnetic field B. Indeed, the Lorentz force acting perpendicular to the motion of the particle, creating an acceleration which in turn transforms into loss of energy by radiation. Cyclotron effect becomes a synchrotron effect when the particle becomes relativistic (see Fig.4.13). The fundamental principle of dynamics applied to the charged particle of mass m and charge e, at a distance r from the magnetic field gives us

$$\frac{mv^2}{r} = eB\,v \;\Rightarrow\; \omega_c = \frac{eB}{m} \;\Rightarrow\; \nu_c = \frac{eB}{2\pi m} \simeq 14\,\text{MHz}\left(\frac{e}{1.6\times10^{-19}}\right)\left(\frac{B}{1\text{ Gauss}}\right)\left(\frac{1\text{ GeV}}{m}\right) \tag{4.24}$$

ω_c (ν_c) being the cyclotron pulsation (frequency). Notice that the frequency is independent of the position of the particle or its velocity. Moreover, as we saw in Eq.(4.13), the radiated field is monochromatic and have the same frequency that the moving particle itself. As all the particle in rotation around $\vec{B}$ possess the *same* frequency, independently of their rotation radius, they will all contribute to the electromagnetic field at the same frequency. For an electron in a typical 10 μG magnetic field from Galactic Center, we expect a frequency around the MHz, which is the range of measurement of several experiments. In the relativistic case treated below, we should add the Doppler effect given by Eq.(4.42) with a mean on $\cos\theta$, which gives a frequency multiply by the factor γ. In other words, a relativistic electron of 1 GeV would give a synchrotron radiation at a frequency around 2000 times larger than in the non–relativistic case, so around the 100 MHz-1 GHz.
We can then compute, combining Eq.(4.24) withe Eq.(4.11) the energy lost per second by a particle of mass m and charge $q\times e$ turning around a magnetic field of intensity B, noticing that in the non–relativistic case, $E = \frac{1}{2}mv^2$:

$$\frac{dE}{dt} = -\frac{e^4B^2E}{3\pi\epsilon_0c^3m^3} \quad\Rightarrow\quad E = E_0e^{-t/\tau} \quad \text{, with } \tau = 3\pi\epsilon_0\frac{c^3m^3}{e^4B^2}$$

Implementing some more natural units for particle physicists, we can express

$$\begin{aligned}\tau &= 3\pi(3\times10^8)^3\left(\frac{1}{q\times1.6\times10^{-19}}\right)^4\left(\frac{1\text{ Tesla}}{B}\right)^2\left(\frac{1.88\times10^{-27}m}{1\text{ GeV}}\right)^3\\ &\simeq \frac{2\times10^{18}}{q^4}\left(\frac{1\text{ Gauss}}{B}\right)^2\left(\frac{m}{1\text{ GeV}}\right)^3 \text{ seconds}\end{aligned}$$

To give an example, an electron in a typical local magnetic fields of a galaxy (around 1 μG, see section 4.11.4 for a more detailed modelization) we obtain $\tau \simeq 8\times10^{12}$ years, much larger than the age of the Universe. For a proton, the lifetime is even greater. In other words, cyclotron radiation is not a significant cooling mechanism for interstellar gas.

4.3.6 Bremsstrahlung radiation

In1927, Carl Anderson found that the ionization rate given by the Bethe-Bloch formula (4.48) underestimates the energy loss rate for relativistic electrons. The additional loss

was associated to the acceleration of the electron in the electromagnetic field. Indeed, we computed in Eq.(4.11) the energy lost by an accelerated particle. Then, a particle passing through the magnetic field generated by a gas of ionized particle will lose energy through the impulsion it will transfer to the constituents of the gas (Coulomb/ionization scattering) but also from its own acceleration. This phenomena was in fact already observed by Nikola Tesla in 1880 in another context, and was called *bremsstrahlung*. The process is identical to what is called $free - free\ emission$ in the language of atomic physics, in the sense that the radiation corresponds to transitions between unbound states of the electron in the field of the nucleus. In 1934, computations of the spectrum of non–relativistic and relativistic bremsstrahlung were carried out by Bethe and Heitler.
The bremsstrahlung, from the german *bremsen* "to brake" and *Sthralung* "radiation" i.e. "braking radiation" or "deceleration radiation" is the electromagnetic radiation produced by the deceleration of a charged particle when deflected by another charged particle, typically an electron by an atomic nucleus (see Fig.4.13). The moving particle loses kinetic energy, which is converted into a photon because energy is conserved. Strictly speaking braking radiation is any radiation due to the acceleration of charged particles. However, the term is frequently used in the more narrow sense of radiation from electrons slowing in matter.

4.4 Notions of relativity

4.4.1 Main idea

In a lot of astrophysical processes, the charged particle radiating electromagnetic field is produced with a relativistic velocity. It is for instance the case for GeV–like dark matter annihilating into leptons. We would like to apply our preceding results to such an energetic particle. The main idea is to move to an instantanenous rest frame K' such that the particle has zero velocity at a certain time and we then can apply all the previous results we obtained. During an infinitesimal lapse of time the particle moves non-relativistically and we can use the Larmor's formula.

4.4.2 Lorentz transformations

We recall in this section the main results concerning the transformations induced by special relativity. Let us consider two frames K and K' with a relative velocity v along the x axis. The origins are assumed to coincide at $t = 0$. If a pulse of light is emitted at $t = 0$ each observer will see an expanding sphere centered on his own origin :

$$x^2 + y^2 + z^2 - c^2t^2 = 0, \qquad x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

The actual relations between (x, y, z, t) and (x', y', z', t') are called the *Lorentz transformations*:

$$x' = \gamma(x - vt); \quad y' = y, \quad z' = z, \quad t' = \gamma\left(t - \frac{v}{c^2}x\right) \quad \text{with} \ \ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4.25}$$

The inverse relations $x_i = f(x'_i)$ have obviously the same form with $v \to -v$.

Deriving of the Lorentz transformations

Suppose a light signal proceeding through the x-axis in the reference (lab) frame K, one should have

$$x - ct = 0 \tag{4.26}$$

On the other hand, the hypothesis of constant velocity of light, c in any frame should impose, in a system K' moving with a velocity v relative to K

$$x' - ct' = 0 \tag{4.27}$$

Combining the two previous equations, and adding the situation of a light traveling toward negative x-axis one can write

$$x' - ct' = \lambda(x - ct), \quad x' + ct' = \mu(x + ct) \tag{4.28}$$

which gives by combination

$$x' = ax - bct, \qquad ct' = act - bx \tag{4.29}$$

with $a = \frac{\lambda+\mu}{2}$ and $b = \frac{\lambda-\mu}{2}$. For the origin of the reference frame K', we always have $x' = 0$ which gives $x = \frac{b}{a}ct$. Remembering that K' travels with a velocity v relative to K, this gives $v = \frac{b}{a}c$. Now, if one makes a snapshot at $t = 0$ of a unit length $\Delta x' = 1$, the observatory in the lab frame will observe $\Delta x = \frac{\Delta x'}{a} = \frac{1}{a}$. On the other hand, the unit length $\Delta x = 1$ will be observed in the moving frame at $t' = 0$ as $\Delta x' = \Delta x\left(a - \frac{b^2}{a}\right) = a\Delta x\left(1 - \frac{v^2}{c^2}\right) = a\left(1 - \frac{v^2}{c^2}\right)$. Imposing the principle of relativity (the observed length is the same, independent of the reference frame, $\Delta x = \Delta x'$) we obtain

$$a = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \text{and then} \quad b = \frac{\frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4.30}$$

The transformations (4.29) then becomes the Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4.31}$$

We can then easily check that $x'^2 - ct'^2 = x^2 - ct^2$ confirming that the velocity of light is the same in both frame.

Writing the Lorentz transformation into a differential form, one can deduce the transformation laws for velocities:

Figure 4.11: Doppler effect and aberration.

$$
\begin{aligned}
&dx = \gamma(dx' + vdt'); \quad dy = dy', \quad dz = dz', \quad dt = \gamma\left(dt' + \frac{v}{c^2}dx'\right) \quad \Rightarrow \\
&u_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma(dt' + \frac{v}{c^2}dx')} = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}}, \\
&u_y = \frac{u'_y}{\gamma(1 + \frac{vu'_x}{c^2})}, \\
&u_z = \frac{u'_z}{\gamma(1 + \frac{vu'_x}{c^2})}.
\end{aligned} \tag{4.32}
$$

The generalization of such transformations to an axis not parallel to the x axis can be written by decomposing u into its parallel ($u_\parallel$) and perpendicular ($u_\perp$) components with respect to $\vec{v}$:

$$
u_\parallel = \frac{u'_\parallel + v}{1 + \frac{vu'_\parallel}{c^2}}, \quad u_\perp = \frac{u'_\perp}{\gamma(1 + \frac{vu'_\parallel}{c^2})} \tag{4.33}
$$

For $u' = c$ (light emitted in K') we can notice

$$
\tan\theta = \frac{u_\perp}{u_\parallel} = \frac{c}{\gamma v} \tag{4.34}
$$

which means that if the distribution of light is isotropic in the K' frame, the observer, for highly relativistic speeds ($\gamma \gg 1$) will see half of the photons lying with a cone of half angle $1/\gamma$. This is called the *beaming effect*.
We can also compute explicitly the angle (see Fig.4.11) θ' under which is seen an incident from an electron with a velocity $\vec{v}$ in its rest frame as function of the same angle θ measured in the galactic (laboratory) frame:

$$
\sin\theta' = \frac{v'_y}{c} = \frac{\sin\theta}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)}, \qquad \cos\theta' = \frac{v'_x}{c} = \frac{\cos\theta - \frac{v}{c}}{(1 - \frac{v}{c}\cos\theta)} \tag{4.35}
$$

with $\sin\theta = v_y/c$, $\cos\theta = v_x/c$ don't forget that $v \to -v$ in the formula compared to Eq.(4.32 because we express the measured value in S' as function of the ones in S).
For $u' = c$ (light emitted in K') we can notice

$$\tan\theta = \frac{u_\perp}{u_\parallel} = \frac{c}{\gamma v} \tag{4.36}$$

which means that if the distribution of light is isotropic in the K' frame, the observer, for highly relativistic speeds ($\gamma \gg 1$) will see half of the photons lying with a cone of half angle $1/\gamma$. This is called the *beaming effect*.
It is then straightforward to deduce the Lorentz transformations for the acceleration:

$$\begin{aligned}
a_x &= \frac{a'_x}{\gamma^3(1+\frac{vu_x}{c^2})^3} \\
a_y &= \frac{a'_y}{\gamma^2(1+\frac{vu_x}{c^2})^2} - \frac{u'_y v}{c^2}\frac{a'_x}{\gamma^2(1+\frac{vu_x}{c^2})^3} \\
a_z &= \frac{a'_z}{\gamma^2(1+\frac{vu_x}{c^2})^2} - \frac{u'_z v}{c^2}\frac{a'_x}{\gamma^2(1+\frac{vu_x}{c^2})^3}
\end{aligned} \tag{4.37}$$

If the particle is at rest in K' we obtain

$$a'_\parallel = \gamma^3 a_\parallel, \qquad a'_\perp = \gamma^2 a_\perp. \tag{4.38}$$

The Lorentz transformation for the electromagnetic fields

$$\begin{aligned}
E'_\parallel &= E_\parallel, & B'_\parallel &= B_\parallel \\
E'_\perp &= \gamma(E_\perp + \frac{1}{c}\vec{v}\wedge\vec{B}), & B'_\perp &= \gamma(B_\perp - \frac{1}{c}\vec{v}\wedge\vec{E})
\end{aligned} \tag{4.39}$$

4.4.3 Relativistic Larmor's formula

From Eq.(4.38) one can then deduce the lost of energy of a relativistic particle in the laboratory frame: We apply the Larmor's formula (4.11) in the rest frame of the particle and then make a Lorentz boost. We obtain

$$P = \frac{e^2\gamma^4}{6\pi\epsilon_0 c^3}(|a_\perp|^2 + \gamma^2|a_\parallel|^2) \tag{4.40}$$

4.4.4 Doppler effect

Any periodic phenomenon in the moving frame K' will appear to have a longer period by a factor γ when viewed by local observers in frame K. We also have to add the additional effect on the period due to the delay time for light propagation between the journey $1 \to$ observer and $2 \to$ observer. The joint effect is called Doppler effect.
Suppose that the moving source emits one period of radiation as it moves from point 1 to point 2 at velocity v. If the frequency of the radiation in the rest frame of the source is ω'

then the time to move from point 1 to point 2 in the observer's frame is given by the time dilatation effect

$$\Delta t = \frac{2\pi\gamma}{\omega'} \tag{4.41}$$

Adding the classical Doppler effect taking into account the fact that the journey of the light is shorter by a time d/c with $d = v\Delta t\cos\theta$, θ being the angle between the gamma-ray and $\vec{v}$ in the observer referential at the time of *emission*, one obtains

$$\omega = \frac{\omega'}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)}. \tag{4.42}$$

This is the relativistic Doppler formula, where ω is the frequency observed by the fixed observer. Notice that if it is the observer who moves at a velocity v, like in the case of inverse Compton scattering, $v \to -v$.

4.4.5 Transformations on the energies

Another quadrivector that we can build and which respect the Lorentz transformation (4.25) is the energy-momentum one, $p_\mu = (E/c, \vec{p})$. In this case the transformation can be written:

$$p'_x = \gamma(p_x + \frac{v}{c}\frac{E}{c}); \quad p'_y = p_y, \quad p'_z = p_z, \quad \frac{E'}{c} = \gamma\left(\frac{E}{c} - \frac{v}{c}p_x\right) \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{4.43}$$

with $p = 0$ and $E = mc^2$. Notice that comparing to (4.25), the transformation corresponds to a reference frame with velocity $-v$ compared to the frame where the particle is at rest ($p = 0$). We then deduce the energy and momentum of a particle of velocity $+v$ compared to the laboratory frame:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mc^2, \qquad p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mv. \tag{4.44}$$

We remark that developing for velocities $v \ll c$, one obtains

$$E \sim mc^2 + \frac{1}{2}mv^2$$

and we find back the expression of the energy as potential (mass) energy plus kinetic energy. Note also that if a particle of rest mass m, moves with a velocity v and lose or gain the energy E_0 (through absorption or emission of a photon for instance), its energy in the lab frame is increased by $E_0/\sqrt{1-\frac{v^2}{c^2}}$ which means that the total energy in the laboratory frame is now $(m + \frac{E_0}{c^2})c^2/\sqrt{1-\frac{v^2}{c^2}}$. Its behavior is then equivalent of a particle of mass $m + E_0/c^2$. This is the famous formulae proposed by Einstein in an addenda of his article published in october 1905 [included in the appendix]: a particle at velocity v receiving (loosing) an amount of energy E_0 without altering its velocity receives a mass contribution $\Delta m = \pm E_0/c^2$

(the famous formula "$E = mc^2$"). This way of "proving" relativity was the one preferred by Einstein himself but one has to wait until the work of Cockcroft and Walton in 1932 who observed the first transmutation mechanism.

4.4.6 Fizeau experiment

One of the first proof of special relativity was made by Fizeau in 1851 when he measures the relative speeds of light in moving water. Fizeau used a special interferometer arrangement to measure the effect of movement of a medium upon the speed of light. For a light of velocity $w = c/n$, n being the index of refraction of water, the law of composition should give the velocity in the lab frame u (4.32):

$$u = \frac{w+v}{1+\frac{w.v}{c^2}} \simeq (w+v)\left(1-\frac{w.v}{c^2}\right) \simeq w+v\left(1-\frac{w^2}{c^2}\right) = w+v\left(1-\frac{1}{n^2}\right) \tag{4.45}$$

which is exactly the experimental result measured by Fizeau in his 1851 experiment.

4.5 Radiative processes in Astrophysics part II : the relativistic case

4.5.1 Relativistic Coulomb scattering or ionization losses

Ionization loss

For the treatment of relativistic Coulomb loss we use the equations 4.32 and 4.39. We then compute the transfer of impulsion from the moving frame (rest frame of the moving particle where the treatment is non relativistic) to the rest frame of the gaz. If we write $\gamma = E/m$ We thus have rest frame

$$\Delta p = F_\perp dt = zeE_\perp dt = zeE'_\perp/\gamma \times (\gamma dt') = F'_\perp dt'. \tag{4.46}$$

The impulsion is thus the same in the relativistic frame than in the rest frame. The Eq.(4.20) is valid even for a relativistic moving particle. The difference is then due to the factors b_{min} and b_{max}. b_{max} is greater by a factor γ because the duration of the impulse is shorter by this factor. In the case of b_{min}, the transverse momentum of the electron is greater by a factor γ and hence, because of the Heisenberg uncertainty principle,

$$\Delta x \simeq b_{min} = \frac{\hbar}{\Delta p} \propto \gamma^{-1}. \tag{4.47}$$

where we used the Lorentz transformation for the impulsion (velocity) obtained in Eq.(4.33). Thus, we expect the logarithmic term to have the form $\ln(2\gamma^2 m_e v^2/\bar{I})$. A more precise computation taking into account relativistic quantum theory effects is given by the Bethe-Bloch formula

$$\frac{dE}{dx} = -\frac{z^2 e^4 N_e}{4\pi\epsilon_0^2 v^2 m_e}\left[\ln\left(\frac{2\gamma^2 m_e v^2}{\bar{I}}\right) - \frac{v^2}{c^2}\right]$$

We derived all the expression above, except for the factor v^2/c^2 which is always practically very small compared to $\ln(2\gamma^2 m_e v^2/\bar{I})$ which is $\gtrsim 20$ in the majority of physical process in the galactic medium. As discussed earlier, $\bar{I}$ is a parameter that should be obtain experimentally. We usually express the energy loss as function of the density of mass of the material crossed by the high energy particle using[4]

$$\frac{dE}{dt} = -\frac{4\pi z^2 r_0^2 c^3 m_e N_A Z}{\beta A}\left[\ln\left(\frac{2\gamma^2 m_e c^2 \beta^2}{\bar{I}}\right) - \beta^2\right] \times \rho$$

$r_0 = e^2/4\pi\epsilon_0 m_e c^2$ being the classical radius of the electron. The exact treatment of the ionization energy loss of heavy particles, taking into account that the interstellar medium is mainly composed of hydrogen and helium, leads to

$$\begin{aligned}\frac{dE}{dt} &= -\frac{4\pi z^2 r_0^2 c^3 m_e N_A Z}{\beta A}\sum_{i=H,He}\left[\frac{1}{2}\ln\left(\frac{2\gamma^2 m_e c^2 \beta^2 E_{max}}{\bar{I}_i^2}\right) - \beta^2 - \frac{\delta}{2}\right] \times \rho \\ &= -\frac{4\pi z^2 r_0^2 c^3 m_e Z N_s}{\beta}\sum_{i=H,He}\left[\frac{1}{2}\ln\left(\frac{2\gamma^2 m_e c^2 \beta^2 E_{max}}{\bar{I}_i^2}\right) - \beta^2 - \frac{\delta}{2}\right] \qquad (4.48)\end{aligned}$$

where δ represent a screening effect due to the density of atoms of the middle crossed by the particle and $E_{max} = 2m_e c^2\beta^2\gamma^2/(1 + 2\gamma m_e/M + (m_e/M)^2)$, M being the nucleon mass, is the maximum kinetic energy transferred to one atom and N_s the number density of the medium. The density correction can be neglected in our case, and the values of $I_H = 19$ eV and $I_{He} = 44$ eV are commonly used. A more complete treatment in quantum field theory gives the Bethe Bloch formula for the ionization loss

$$\left(\frac{dE}{dt}\right)_I = -2\pi r_0^2 m_e c^3 \frac{1}{\beta}\sum_{s=H,He} Z_s n_s\left[\ln\left(\frac{(\gamma-1)\beta^2 E^2}{2I_s^2}\right) + \frac{1}{8}\right] \qquad (4.49)$$

Coulomb scattering

The Coulomb collisions in a completely ionized plasma are dominated by scattering off the thermal electrons. The corresponding energy losses are given by

$$\begin{aligned}\left(\frac{dE}{dt}\right)_{Coul} &= -2\pi r_0^2 m_e c^3 Z n \frac{1}{\beta}\left[\ln\left(\frac{E m_e c^2}{4\pi r_0 \hbar^2 c^2 Z n}\right) - \frac{3}{4}\right] \qquad (4.50) \\ &\simeq -5.5\times 10^{-16}\left(\frac{Zn}{1\ \mathrm{cm}^{-3}}\right)\left(\frac{1}{1-(mc^2/E)^2}\right)\left[1 + \frac{1}{74}\ln\left(\frac{E/m_e c^2}{Zn/(1\ \mathrm{cm}^{-3})}\right)\right]\ \mathrm{GeV\ s}^{-1}\end{aligned}$$

n being the density of ions of charge Z in the ionized plasma and m the mass of the traveling particle. The values of the densities n_e, n_H or n_{He} depends on the position in the galactic plane. As the sun reference point (8 kpc) they are $n_e = 0.0024$ cm^{-3}, $n_H = 0.6$ cm^{-3} and $n_{He} = 0.144$ cm^{-3}.

[4] remembering that the Avogadro number N_A is by definition the number of carbon12 atoms in 12 grams of carbon, or equivalently, N_A/A represents the number of atoms of nuclei A in one gram, thus $Z \times N_A/A$ represent the number of protons (and then electrons) per gram of material. We can then deduce $N_e = \rho Z N_A/A$, ρ being the density of mass of material A.

Figure 4.12: Inverse Compton scattering geometry.

4.5.2 Inverse-Compton scattering

The inverse Compton interaction corresponds to the relativistic limit of the Thomson scattering (see Fig.4.13). The strategy to compute it is quite straightforward: to place ourself in the framework where the charged particle is at rest, and applying a Lorentz transformation to obtain the result in the galactic (observer) coordinate frame, the energy loss being Lorentz invariant : $P' = dE'/dt' = dE/dt = P$. We then need to compute $dE'/dt' = \sigma_T c U'_{rad}$ (Eq. 4.22). For that we need to compute $U'_{rad} = N\hbar\omega'$ as function of $U_{rad} = N\hbar\omega$, N being the density of photons. The frequency $\omega' \to \omega$ follows the classical Doppler effect described through Eq.(4.42). However, there is another subtlety, which is equivalent to a second Doppler effect in a sense: U'_{rad} is a $flux$ of energy. One then need to compute the interval of time $\Delta t'$ between the arrival of two impulse on the electron in S' as function of the interval of the same event Δt measured in S. After a look at Fig.(4.12) we can write

$$\Delta t = t_2 - t_1 + \frac{x_2 - x_1}{c}\cos\theta = (t'_2 - t'_1)\gamma[1 + (v/c)\cos\theta] \tag{4.51}$$

One obtain, exactly as for the Doppler effect, $\Delta t = \Delta t' \times \gamma[1 + (v/c)\cos\theta]$ that is, the time interval between the arrival of photons from the direction θ is shorter by a factor $\gamma[1 + (v/c)\cos\theta]$ in S' than in S. Thus, the rate of arrival of photons, and correspondingly their number density, is greater by this factor. Then, as observed in S', the energy of the beam is therefore

$$U'_{rad} = \gamma^2[1 + (v/c)\cos\theta]^2 \times U_{rad} \tag{4.52}$$

After a mean on the angle θ and using the relation $\gamma^2 - 1 = (v/c)^2\,\gamma^2$ we can write

$$U'_{rad} = \frac{1}{4\pi}\int_0^\pi \gamma^2[1+(v/c)\cos\theta]^2\ 2\pi\sin\theta d\theta = \frac{4}{3}U_{rad}(\gamma^2 - \frac{1}{4}) \tag{4.53}$$
$$\Rightarrow \langle P\rangle_{IC} = \langle P'\rangle = \frac{4}{3}\sigma_T c U_{rad}\left(\frac{v^2}{c^2}\right)\gamma^2 \simeq 2.56\times 10^{-17}\left(\frac{E}{1\ \text{GeV}}\right)^2\left(\frac{511\ \text{keV}}{m}\right)^2$$

where we have used $v \simeq c$, σ_T form (4.23) and $U_{rad} = 2.4\times 10^{-10}$ GeV cm^{-3} from Eq.(2.92).

4.5.3 Synchrotron radiation

Synchrotron radiation is the relativistic limit of the cyclotron mechanism (see Fig.4.13). We will compute the synchrotron emission with both methods: first directly in the observer (fixed) frame using the relativistic Larmor's formula. We will then recover this result computing the emission in the particle frame (particle will be at rest) but feeling the electric field generated by the relativist boost.

From the observer point of view

To obtain the formulation for the cyclotron radiation produced by a relativistic particle (called synchrotron radiation) we first need to compute the acceleration of the particle. The relativistic dynamic principle gives us

$$\frac{d}{dt}\gamma m v = e(\vec{v}\wedge\vec{B}) \tag{4.54}$$

We then should recover the same result than for the synchrotron replacing $m \to \gamma m$ and applying the relativistic Larmor's formula, noticing that there is no acceleration parallel to $\vec{B}$ and that the acceleration is perpendicular to the velocity vector (Eq. 4.54), we deduce $|a_\parallel|^2 = 0$. We then obtain

$$P = \frac{e^4\gamma^2 B^2 v_\perp^2}{6\pi\epsilon_0 m^2 c^3} = \frac{e^4\gamma^2 B^2 v^2\sin^2\theta}{6\pi\epsilon_0 m^2 c^3} \tag{4.55}$$

If one averages on the velocity angle :

$$\langle v_\perp^2\rangle = \frac{v^2}{4\pi}\int \sin^2\theta d\Omega = \frac{2}{3}v^2 \quad\Rightarrow\quad \langle P\rangle = \frac{e^4\gamma^2 B^2 v^2}{9\pi\epsilon_0 m^2 c^3} \tag{4.56}$$

If the particle is relativist, one can consider $E = pc$ with $p = \gamma m v$ which implies $v^2 = E^2/(\gamma^2 m^2 c^2)$ and

$$\frac{\langle P\rangle}{\text{GeV s}^{-1}} = -\frac{dE}{dt} = \frac{e^4 B^2 E^2}{9\pi\epsilon_0 m^4 c^5} = \frac{2e^4 E^2}{9\pi\epsilon_0^2 m^4 c^7}\left(\frac{B^2}{2\mu_0}\right) \simeq 2.51\times 10^{-18}\left(\frac{E}{\text{GeV}}\right)^2\left(\frac{B}{\mu\text{G}}\right)\left(\frac{511\ \text{keV}}{m}\right)^4 \tag{4.57}$$

From the particle point of view

In the rest frame of the particle, the fundamental principle of dynamic can be written

$$\frac{d}{dt'}m\vec{v'} = e(E' + \vec{v'} \wedge \vec{B'}) = eE' = \frac{e}{c}\gamma v B \sin\theta \tag{4.58}$$

because the particle is at rest, $\vec{v'} = \vec{0}$, θ being the angle between $\vec{v'}$ and $\vec{B'}$. We then obtain

$$\begin{aligned} P &= -\left(\frac{dE}{dt}\right)' = \frac{e^2|\dot{\vec{v'}}|^2}{6\pi\epsilon_0 c^3} = \frac{e^4\gamma^2 B^2 v^2 \sin^2\theta}{6\pi\epsilon_0 m^2 c^3} = 2\sigma_T c U_B \left(\frac{v}{c}\right)^2 \gamma^2 \sin^2\theta \\ \langle P\rangle_{synch} &= \frac{4}{3}\sigma_T c U_B \left(\frac{v}{c}\right)^2 \gamma^2 \end{aligned} \tag{4.59}$$

with $U_B = B^2/2\mu_0$, recovering Eq.(4.55).

4.5.4 Relativistic Bremsstrahlung

4.5.5 Photoelectric effect

4.5.6 Energy losses : summary

as a summary, we can write the total energy loss for an electron in a galactic medium from synchrotron (synch) [Eq.(4.59)], inverse Compton (IC) [Eq.(4.54)] Coulomb (C), ionization (I) and bremsstrahlung (brem) radiation in the relativistic case ($v \simeq c$) summarized in Fig.(4.13) :

$$\langle P\rangle = \langle P\rangle_{synch} + \langle P\rangle_{IC} + \langle P\rangle_{Coul} + \langle P\rangle_{I} + \langle P\rangle_{brem} \tag{4.60}$$

$$\begin{aligned} &= \frac{4}{3}\sigma_T c U_B + \frac{4}{3}\sigma_T c U_{rad} + 2\pi r_0^2 m_e c^3 Zn \frac{1}{\beta}\left[\ln\left(\frac{E m_e c^2}{4\pi r_0 \hbar^2 c^2 n Z}\right) - \frac{3}{4}\right] \\ &+2\pi r_0^2 m_e c^3 \frac{1}{\beta}\sum_{s=H,He} Z_s n_s \left[\ln\left(\frac{(\gamma-1)\beta^2 E^2}{2I_s^2}\right) + \frac{1}{8}\right] + cE\sum_{s=H,He}\frac{n_s M_s}{T_s} \\ &= 2.51\times10^{-18}\left(\frac{E}{\text{GeV}}\right)^2\left(\frac{B}{\mu\text{G}}\right)\left(\frac{511\text{ keV}}{m}\right)^4 + 2.56\times10^{-17}\left(\frac{E}{\text{GeV}}\right)^2\left(\frac{511\text{ keV}}{m}\right)^2 \\ &+5.5\times10^{-16}\left(\frac{Zn}{1\text{ cm}^{-3}}\right)\left(\frac{1}{\beta}\right)\left[1 + \frac{1}{74}\ln\left(\frac{E/m_e c^2}{Zn/(1\text{ cm}^{-3})}\right)\right] \\ &+7.4\times10^{-18}\left(\frac{1}{\beta}\right)\sum_{s=H,He}\left(\frac{Z_s n_s}{1\text{ cm}^{-3}}\right)\left[\ln\left(\frac{(\gamma-1)\beta^2 E^2}{2I_s^2}\right) + \frac{1}{8}\right] \\ &+3\times10^{10}\left(\frac{E}{\text{GeV}}\right)\sum_{s=H,He}\frac{n_s M_s}{T_s}\text{ GeV s}^{-1} \end{aligned} \tag{4.61}$$

M_s being the atomic mass of species s and T_s its radiation length ($T_H \simeq 62.8$ g cm^{-2}, $T_{He} \simeq 93.1$ g cm^{-2}). The local density of different species at a certain distance from the Milky Way center can be found in the following table (4.3). The electron species can be identified with the ionized hydrogen HII specie). We show in Fig.(4.14)

Figure 4.13: Summary of energy loss of a charged particle in an astrophysical medium.

R (kpc)	n_{HI} (atom cm^{-3})	$2n_{H2}$ (mol cm^{-3})	n_{HII} (atom cm^{-3})	n_{He} (atom cm^{-3})
0	0.16411	0.	0.0275	0.01805
1	0.16411	0.06211	0.04226	0.02488
2	0.20349	0.12422	0.08674	0.03605
3	0.24616	0.47352	0.15635	0.07917
4	0.36104	0.76053	0.19357	0.12337
5	0.51694	1.04366	0.15542	0.17167
6	0.51694	0.99151	0.08488	0.16593
7	0.54976	0.88814	0.03951	0.15817
8	0.60720	0.71078	0.02390	0.14498
9	0.50280	0.39278	0.02024	0.09851
10	0.65116	0.10128	0.01901	0.08277
11	0.40338	0.	0.01802	0.04437
12	0.57451	0.	0.01701	0.06320
13	0.48365	0.	0.01598	0.05320
14	0.22138	0.	0.01494	0.02435
15	0.14515	0.	0.01389	0.01597
16	0.09448	0.	0.01286	0.01039
17	0.06782	0.	0.01184	0.00746
18	0.04867	0.	0.01085	0.00535
19	0.03491	0.	0.00989	0.00384
20	0.02504	0.	0.00897	0.00275

Table 4.3: Number densities of atomic hydrogen (H_I), mollecular hydrogen (H_2), ionized hydrogen (H_{II}) and helium (He) as function of the galactic radius R at $k = 0$ kpc.

Figure 4.14: Combining the energy loss mechanisms bor $B = 100\mu$G.

4.6 Ultra High Energetic (UHE) processes

If a very high energetic particles like a proton for instance interacts with the photons of the Cosmological Microwave Background (CMB), pions or other particles can be created by reactions. As a consequence, it exists a maximum energy (and thus free–path) above which these particles (protons or photons) cannot propagate in the universe anymore. Their spectrum should then possess a typical cut which should be observable by dedicated experiences (called GZK cut in the case of protons). in other words, if ultra high energy events are observed by such experiments, the origin would be local (like dark matter origin for instance).

4.6.1 Cosmic rays case

Photons of the CMB acts on accelerated protons through the reactions

$$\begin{aligned}
&\gamma + p \to \Delta^+ \to n + \pi^+ \to n + \mu^+ + \nu_\mu \\
&\gamma + p \to \Delta^+ \to p + \pi^0 \to p + \gamma + \gamma \\
&\gamma + p \to p + N\pi,
\end{aligned} \tag{4.62}$$

the charged pions decaying creating ultra high energy muons and muons neutrinos[5]. These

[5]For the reader not used to the particle physics contents, the pions are composed of $\pi^0 = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$, $\pi^+ = \bar{d}u$ and $\pi^- = d\bar{u}$. The Delta baryons Δs also call *Delta resonances* are made of $\Delta^{++} = uuu$, $\Delta^+ = uud$, $\Delta^0 = udd$ and $\Delta^- = ddd$. their masses are about 1.232 GeV, and can be considered (in the case of Δ^+ and Δ^0 as excited states of the proton and neutron respectively.

high energy neutrino can be detected by ground based detectors like Icecube (see section 5.5.3). Computing the maximum energy allowed by the proton before the reaction occurs is equivalent to compute the energy necessary for the reaction to occurs in the rest frame of the proton and then to make a Lorentz transformation back in the galactic (laboratory) frame.
In the proton rest frame R, the threshold energy E_γ^R of the photon is obviously the mass of the pion $E_\gamma^R > 200$ MeV. Using the formulae computed for the doppler effect (Eq. 4.42) we deduce for the energy of the photon in the laboratory frame E_γ^0

$$200 \text{ MeV} \lesssim E_\gamma^R = E_\gamma^0 \gamma \left(1 + \frac{v}{c}\cos\theta\right) < 2\gamma E_\gamma^0 \tag{4.63}$$

with $E_\gamma^0 = 6 \times 10^{-4}$ eV (see section F.1.1) which gives

$$\gamma \gtrsim 1.7 \times 10^{11} \quad \Rightarrow \quad E_p^0 = \gamma E_p^R = \gamma m_p c^2 \gtrsim 1.7 \times 10^{20} \text{ eV} = E_p^{max}. \tag{4.64}$$

A more detailed treatment, taking into account the integration over the Planck spectrum of the CMB decreases this value to $E_p^{max} = 5 \times 10^{19}$ eV. Any proton of energy $E_p \gtrsim 5 \times 10^{19}$ eV reacts then with the CMB to produce pions and then muons and muon neutrinos. This limit is called the *GZK cutoff* computed for the first time by Greisen and independently by Kuz'min and Zatsepin in 1966.
The mean free path λ for a single scattering is then given by

$$\lambda \times \sigma_{p\pi} n_\gamma^0 = 1 \quad \Rightarrow \quad \lambda = (\sigma_{p\pi} n_\gamma^0)^{-1} \simeq 10^{23}\text{m} = 3 \text{ Mpc} \tag{4.65}$$

$\sigma_{p\pi} \simeq 2.5 \times 10^{-4}$ barns being the interaction cross section of the reactions (4.62) and $n_\gamma = 394$ cm^{-3} (Eq. 2.50) the density of photons of the CMB. After each reaction described in (4.62), the proton in the galactic frame lose the kinetic energy $\Delta E_p = \gamma m_\pi c^2$. Its initial energy being $E_p^0 \gamma m_p c^2$, the relative loss of energy $\Delta E_p / E_p^0 \simeq 1/10$. As a consequence, if the highest energy cosmic rays are protons, they cannot have originated from further than about 30 Mpc from our galaxy.

4.6.2 Photons case

4.6.3 Neutrino cases

4.7 Indirect Detection of gamma–ray

4.7.1 The principle

For the continuum of gamma rays, the observed differential flux at the Earth coming from a direction forming an angle ψ with respect to the galactic center is

$$\Phi_\gamma(E_\gamma, \psi) = \sum_i \frac{dN_\gamma^i}{dE_\gamma} \langle \sigma_i v \rangle \frac{1}{8\pi m_\chi^2} \int_{line\ of\ sight} \rho^2 \, dl \ , \tag{4.66}$$

where the discrete sum is over all dark matter annihilation channels, dN_γ^i/dE_γ is the differential gamma-ray yield, $\langle \sigma_i v \rangle$ is the annihilation cross section averaged over its velocity distribution, m_χ is the mass of the dark matter particle, and ρ is the dark matter density. Assuming

a spherical halo, one has $\rho = \rho(r)$ with the galactocentric distance $r^2 = l^2 + R_0^2 - 2lR_0 \cos\psi$, where R_0 is the solar distance to the galactic center ($\simeq$ 8 kpc). It is customary to separate in the above equation the particle physics part from the halo model dependence introducing the (dimensionless) quantity

$$J(\psi) = \frac{1}{8.5 \text{ kpc}} \left(\frac{1}{0.3 \text{ GeV/cm}^3} \right)^2 \int_{line\ of\ sight} \rho^2(r(l,\psi))\ dl\ . \tag{4.67}$$

Thus one can write

$$\begin{aligned} \Phi_\gamma(E_\gamma, \psi) &= 0.94 \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1} \text{ GeV}^{-1} \text{ sr}^{-1} \\ &\times \sum_i \frac{dN_\gamma^i}{dE_\gamma} \left(\frac{\langle \sigma_i v \rangle}{10^{-29} \text{cm}^3 \text{s}^{-1}} \right) \left(\frac{100 \text{ GeV}}{m_\chi} \right)^2 J(\psi)\ . \end{aligned} \tag{4.68}$$

Actually, when comparing to experimental data one must consider the integral of $J(\psi)$ over the spherical region of solid angle $\Delta\Omega$ given by the angular acceptance of the detector which is pointing towards the galactic center, i.e. the quantity $\bar{J}(\Delta\Omega)$ with

$$\bar{J}(\Delta\Omega) \equiv \frac{1}{\Delta\Omega} \int_{\Delta\Omega} J(\psi)\ d\Omega\ , \tag{4.69}$$

must be used in the above equation. For example, for EGRET $\Delta\Omega$ is about 10^{-3} sr whereas for FERMI, MAGIC or HESS it is 10^{-5} sr.

The gamma-ray flux can now be expressed as

$$\begin{aligned} \Phi_\gamma(E_{thr}) &= 0.94 \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1} \\ &\times \sum_i \int_{E_{thr}}^{m_\chi} dE_\gamma \frac{dN_\gamma^i}{dE_\gamma} \left(\frac{\langle \sigma_i v \rangle}{10^{-29} \text{cm}^3 \text{s}^{-1}} \right) \left(\frac{100 \text{ GeV}}{m_\chi} \right)^2 \bar{J}(\Delta\Omega)\Delta\Omega\ , \end{aligned} \tag{4.70}$$

where E_{thr} is the lower threshold energy of the detector, Concerning the upper limit of the integral, note that the dark matter moves at galactic velocity and therefore its annihilation occurs at rest.

4.7.2 Galactic Halo

A crucial ingredient for the calculation of $\bar{J}$ in (4.69), and therefore for the calculation of the flux of gamma rays, is the dark matter density profile of our galaxy. The different profiles that have been proposed in the literature can be parameterised as [75]

$$\rho(r) = \frac{\rho_0 [1 + (R_0/a)^\alpha]^{(\beta-\gamma)/\alpha}}{(r/R_0)^\gamma [1 + (r/a)^\alpha]^{(\beta-\gamma)/\alpha}}\ , \tag{4.71}$$

where ρ_0 is the local (solar neighborhood) halo density and a is a characteristic length. Although we will use $\rho_0 = 0.3$ GeV/cm^3 throughout the paper, since this is just a scaling factor in the analysis, modifications to its value can be straightforwardly taken into account in the results.

4.7.3 Adiabatic compression mechanism

As we discussed above, highly cusped profiles are deduced from N-body simulations[6]. In particular, NFW [**?**] obtained a profile with a behaviour $\rho(r) \propto r^{-1}$ at small distances. A more singular behaviour, $\rho(r) \propto r^{-1.5}$, was obtained by Moore et al. [76]. However, these predictions are valid only for halos without baryons. One can improve simulations in a more realistic way by taking into account the effect of the normal gas (baryons). This loses its energy through radiative processes falling to the central region of forming galaxy. As a consequence of this redistribution of mass, the resulting gravitational potential is deeper, and the dark matter must move closer to the center increasing its density.

This increase in the dark matter density is often treated using adiabatic invariants. The present form of the adiabatic compression model was numerically and analytically studied by Blumental et al. [78]. This model assumes spherical symmetry, circular orbit for the particles, and conservation of the angular momentum $M(r)r = \text{const.}$, where $M(r)$ is the total mass enclosed within radius r. The mass distributions in the initial (r) and final (r_f) configurations are therefore related by $[M(r) + M_b(r)]r = [M_b(r_f) + M(r)]r_f$, where $M(r)$ and $M_b(r)$ are respectively the mass profile of the galactic halo before the cooling of the baryons at a distance r (obtained through N-body simulations), and the baryonic composition of the Milky Way observed now. From this expression, one can deduce r_f as function of r and then the new distribution of dark matter. This approximation was tested in numerical simulations [79, 80]. Nevertheless, a more precise approximation can be obtained including the possibility of elongated orbits [90]. In this case, the mass inside the orbit, $M(r)$, is smaller than the real mass, the one the particle 'feels' during its revolution around the galactic center. As a consequence, the modified compression model is based on the conservation of the product $M(\overline{r})r$, where $\overline{r}$ is the averaged radius of the orbit. The time average radii is given by $\overline{x} \sim 1.72x^{0.82}/(1+5x)^{0.085}$, with $x \equiv r/r_s$, and r_s the characteristic radius of the assumed approximation.

The models and constraints one can use for the Milky Way can be found in Table I of Ref. [90]. In [82], the authors have fitted the resulting data of that work with the power-law parameterisation of eq. (4.71) and applied it to SUSY models. The results is summarized in Table 4.4. There we label the resulting NFW and Moore et al. profiles with adiabatic compression by NFW_c and Moore_c, respectively. As one can see, at small r the dark matter density profile following the adiabatic cooling of the baryonic fraction is a steep power law $\rho \propto r^{-\gamma_c}$ with $\gamma_c \approx 1.45(1.65)$ for a $\text{NFW}_\text{c}(\text{Moore}_\text{c})$ compressed model[7]. We observe for example that the effect of the adiabatic compression on a NFW profile is basically to transform it into a Moore et al. one (see Table 4.4).

Of course, these results have important implications for the computation of the gamma-ray fluxes from the galactic center. In particular, in Table 4.4 we see that for $\Delta\Omega = 10^{-3}$ sr one has $\bar{J}_{\text{NFW}_\text{c}}/\bar{J}_{\text{NFW}} \simeq 145$ and $\bar{J}_{\text{Moore}_\text{c}}/\bar{J}_{\text{Moore et al.}} \simeq 77$. Thus the effect of the adiabatic compression is very strong, increasing the gamma-ray fluxes about two orders of magnitude. Similarly, for $\Delta\Omega = 10^{-5}$ sr one obtains $\bar{J}_{\text{NFW}_\text{c}}/\bar{J}_{\text{NFW}} \simeq 953$ and $\bar{J}_{\text{Moore}_\text{c}}/\bar{J}_{\text{Moore et al.}} \simeq 95$.

Let us finally remark that the $\bar{J}$ calculation has been regulated by assuming a constant density for $r < 10^{-5}$ kpc. This procedure has consequences essentially for divergent $\bar{J}$ when

[6]For analytical derivations see e.g. the recent work [77], and references therein.

[7]It is worth noticing that we obtain the same order of magnitude as the authors of Ref. [84].

	a (kpc)	α	β	γ	$\bar{J}(10^{-3}\text{sr})$	$\bar{J}(10^{-5}\text{sr})$
NFW	20	1	3	1	1.214×10^3	1.264×10^4
NFW_c	20	0.8	2.7	1.45	1.755×10^5	1.205×10^7
Moore et al.	28	1.5	3	1.5	1.603×10^5	5.531×10^6
Moore_c	28	0.8	2.7	1.65	1.242×10^7	5.262×10^8

Table 4.4: NFW and Moore et al. density profiles without and with adiabatic compression (NFW_c and Moore_c respectively) with the corresponding parameters, and values of $\bar{J}(\Delta\Omega)$ from [82].

$\gamma \geq 1.5$. To check our results we have also calculated $\bar{J}$ in the following way. We used a cut off in density, $\rho(r < R_s) = 0$, R_s being the Schwarchild radius of a $3 \times 10^6 M_\odot$ black hole. In addition we also considered dark matter annihilation and estimate its effect by requiring an upper value for dark matter density, $\rho(r < r_{max}) = \rho(r_{max}) = m_\chi/(\langle\sigma v\rangle.t_{BH})$ ($t_{BH} \sim 10^{10}$ yr, being the age of the black hole). We also applied the same procedure with a more realistic cut-off value $\rho(r < 10^{-6}$ pc$) = 0$ suggested by dark matter particle scatterings on stars [83]. This kind of effects can be significant only for the Moore_c halo model we consider in this paper and increase our $\bar{J}$ value by less than an order of magnitude for all the $\langle\sigma v\rangle$ values corresponding to our wide SUSY parameter range. In this sense our analysis is conservative. More refine treatments of dark matter annihilation effects in the innermost region of the galaxy can be found in [84, 85].

4.8 The tricky case of the Galactic Center

4.8.1 The idea

Astrophysical searches for dark matter (DM) are a fundamental part of the experimental efforts to explore the dark sector. The strategy is to search for DM annihilation products in preferred regions of the sky, i.e., those with the highest expected DM concentrations and still close enough to yield high DM-induced fluxes at the Earth. For that reason, the Galactic Center (GC), nearby dwarf spheroidal galaxy (dSphs) satellites of the Milky Way, as well as local galaxy clusters are thought to be among the most promising objects for DM searches. In particular, dSphs represent very attractive targets because they are highly DM-dominated systems and are expected to be free from any other astrophysical gamma-ray emitters that might contaminate any potential DM signal. Although the expected signal cannot be as large as that from the GC, dSphs may produce a larger signal-to-noise (S/N) ratio. This fact allows us to place very competitive upper limits on the gamma-ray signal from DM annihilation [86–88], using data collected by the Large Area Telescope (LAT) onboard the Fermi gamma-ray observatory [89]. These are often referred to as the most stringent limits on DM annihilation cross-section obtained so far.

Despite these interesting limits derived from dSphs, the GC is still expected to be the brightest source of DM annihilations in the gamma-ray sky by several orders of magnitude.

Although several astrophysical processes at work in the crowded GC region make it extremely difficult to disentangle the DM signal from conventional emissions, the DM-induced gamma-ray emission is expected to be so large there that the search is still worthwhile. Furthermore, the DM density in the GC may be larger than what is typically obtained in N-body cosmological simulations. Ordinary matter (baryons) dominates the central region of our Galaxy [90]. Thus, baryons may significantly affect the DM distribution. As baryons collapse and move to the center they increase the gravitational potential, which in turn forces the DM to contract and increase its density. This is a known and qualitatively well understood physical process [91–93]. It is also observed in many cosmological simulations that include hydrodynamics and star formation [94–99]. If this is the only effect of baryons, then the expected annihilation signal will substantially increase [90, 100].

In this work, we analyze in detail the constraints that can be obtained for generic DM candidates from *Fermi*-LAT inner Galaxy gamma-ray measurements assuming some specific (and well motivated) DM distributions. The approach is conservative, requiring simply that the expected DM signal does not exceed the gamma-ray emission observed by the *Fermi*-LAT in an optimized region around the GC. The region is chosen in such a way that the S/N ratio is maximized. This kind of analysis, without modeling of the astrophysical background, was also carried out by the *Fermi*-LAT collaboration to constrain DM models from Galactic halo observations [101].

This section is organized as follows. In Section 4.8.2 we discuss DM density profiles such as Navarro, Frenk and White (NFW) [102, 103], Einasto [104, 105] and Burkert [106], paying special attention to the effect of baryonic compression. In Section 4.141 we model the gamma-ray flux from DM annihilation, taking into account the contributions from prompt photons and photons induced via inverse Compton scattering (ICS). The latter is specially relevant for the $\mu^+\mu^-$ channel. We will see that compressed profiles significantly increase the gamma-ray flux in the inner regions of the Galaxy. In Section 4.8.4 we analyze the gamma-ray flux from *Fermi*-LAT measurements. For that we use an optimized region around the GC, which will depend on the particular DM density profile considered. Taking into account these results, in Section 4.8.5 we are able to obtain significant limits on the annihilation cross-section for a generic DM candidate annihilating to $b\bar{b}$, $\tau^+\tau^-$, $\mu^+\mu^-$ or W^+W^-, in the case that the collapse of baryons to the GC is accompanied by the contraction of the DM. In general, the upper limits on the annihilation cross section of DM particles are two orders of magnitude stronger than without contraction, where the thermal cross section is not excluded. Finally, the conclusions are left for Section 4.8.6.

4.8.2 Dark matter density profiles

Cosmological N-body simulations provide important results regarding the expected DM density in the central region of our Galaxy. Simulations suggest the existence of a universal DM density profile, valid for all masses and cosmological epochs. It is convenient to use the following parametrization for the DM halo density [107], which covers different approximations for DM density:

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)^{\gamma}\left[1+\left(\frac{r}{r_s}\right)^{\alpha}\right]^{\frac{\beta-\gamma}{\alpha}}} \ , \tag{4.72}$$

where ρ_s and r_s represent a characteristic density and a scale radius, respectively. The NFW density profile [102, 103], with $(\alpha,\beta,\gamma) = (1,3,1)$, is by far the most widely used in the literature. Another approximation is the so-called Einasto profile [104, 105]

$$\rho_{\rm Ein}(r) = \rho_s \exp\left\{-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]\right\} \ , \tag{4.73}$$

which provides a better fit than NFW to numerical results [105, 108]. Finally, we will also consider DM density profiles that possess a core at the center, such as the purely phenomenologically motivated Burkert profile [106]:

$$\rho_{\rm Burkert}(r) = \frac{\rho_s \ r_s^3}{(r + r_s) \ (r^2 + r_s^2)} \ . \tag{4.74}$$

Early results on the central slopes of the DM profiles showed some significant disagreement between the estimates, with values ranging from $\gamma = 1.5$ [109] to $\gamma = 1$ [102, 103]. As the accuracy of the simulations improved, the disagreement became smaller. For the Via Lactea II (VLII) simulation the slope in Ref. [110] was estimated to be $\gamma = 1.24$. A re-analysis of the VLII simulation and new simulations performed by the same group give the slope $\gamma = 0.8 - 1.0$ [111], which is consistent with the Aquarius simulation [112]. Another improvement comes from the fact that the simulations now resolve the cusp down to a radius of ~ 100 pc, which means that less extrapolation is required for the density of the central region.

Yet, there is an additional ingredient that is expected to play a prominent role in the centers of DM halos: baryons. Although only a very small fraction of the total matter content in the Universe is due to baryons, they represent the dominant component at the very centers of galaxies like the Milky Way. Actually, the fact that current N-body simulations do not resolve the innermost regions of the halos is a minor consideration relative to the uncertainties due to the interplay between baryons and DM.

The baryons lose energy through radiative processes and fall into the central regions of a forming galaxy. As a consequence of this redistribution of mass, the resulting gravitational potential is deeper, and the DM must move closer to the center, increasing its density. This *compression* of DM halos due to baryonic infall was first studied in Ref. [91] for a spherically symmetric DM halo using simple simulations and adiabatic invariants. A convenient analytical approximation was provided in Ref. [92]. The model was later modified [93] to account for the eccentricity of orbits of DM particles. The effect seems to be confirmed by recent hydrodynamic simulations (see e.g. Refs. [94–99].). In Ref. [97], for instance, the authors ran a set of high-resolution hydrodynamic simulations that self-consistently included complex baryonic physics such as gas dissipation, star formation and supernova feedback. They all showed a clear steepening of the inner DM density profiles with respect to DM-only simulations. Indeed, it is argued by the authors that such effect should be always included in order to correctly model the mass distribution in galaxies and galaxy clusters.

As pointed out in Ref. [90], the effect of the baryonic adiabatic compression might be crucial for indirect DM searches, as it increases by several orders of magnitude the gamma-ray flux from DM annihilation in the inner regions, and therefore the DM detectability. In Ref. [100], this effect was used to study the detection of supersymmetric DM by the *Fermi*-LAT, with the conclusion that fluxes from the GC would be largely reachable in significant

Profile	α	β	γ	ρ_s [GeV cm^{-3}]	r_s [kpc]
Burkert	$--$	$--$	$--$	37.76	2
Einasto	0.22	$--$	$--$	0.08	19.7
NFW	1	3	1	0.14	23.8
NFW$_c$	0.76	3.3	1.37	0.23	18.5

Table 4.5: DM density profiles used in this work, following the notation of Eqs. (4.72-4.74).

regions of the supersymmetric parameter space. The effect of compression on galaxy clusters was recently studied in Ref. [113].

There is however another possible effect related to baryons that tends to decrease the DM density and flatten the DM cusp [114–116]. The mechanism relies on numerous episodes of baryon infall followed by a strong burst of star formation, which expels the baryons. At the beginning of each episode the baryons dominate the gravitational potential. The DM contracts to respond to the changed potential. A sudden onset of star formation drives the baryons out. The DM also moves out because of the shallower potential. Each episode produces a relatively small effect on the DM, but a large number of them results in a significant decline of the DM density. Indeed, cosmological simulations that implement this process show a strong decline of the DM density [117, 118]. Whether the process happens in reality is still unclear. Simulations with the cycles of infall-burst-expansion process require that the gas during the burst stage does not lose energy through radiation, which is not realistic. Still, the strong energy release needed by the mechanism may be provided by other processes and the flattening of the DM cusp may occur. If this happened to our Galaxy, then the DM density within the central ~ 500 pc may become constant [118]. This would reduce the annihilation signal by orders of magnitude. We note that this mechanism would wipe out the DM cusp also in centers of dwarf galaxies [117]. Yet, a recent work that also includes stellar feedback offers a much more complicated picture in which galaxies may retain or not their DM cusps depending on the ratio between their stellar-to-halo masses [119].

As discussed in the Introduction, in this work we pay special attention to those scenarios where the DM cusp is not flattened. However, in order to quantify the uncertainty in the DM density profile, we will use three well-motivated models: Einasto, NFW and a compressed NFW (NFW$_c$), whose parameters have been constrained from observational data of the Milky Way, as well as a cored Burkert profile, also compatible with current constraints. We have followed Ref. [90] to choose the parameters of both the NFW and the NFW$_c$. We have fitted the resulting data of that work with the power-law parametrization of Eq. (4.72). The results for both profiles are listed in Table 4.5. The effect of baryonic adiabatic compression is clearly noticed at small r as a steep power law $\rho \propto 1/r^\gamma$ with $\gamma = 1.37$ for NFW$_c$, which is in contrast to the standard NFW value, $\gamma = 1$. We note that a value of $\gamma = 1.37$ is indeed perfectly consistent with what has been found in recent hydrodynamic simulations [97] and it is also compatible with current observational constraints (mainly derived from microlensing and dynamics) on the slope of the DM density profile in the central regions of the Milky Way [120]. These studies actually allow for even steeper adiabatically contracted profiles. Concerning the Einasto profile we will use the parameters provided in Ref. [2]. Finally, for the Burkert profile, we decided to choose a core radius of 2 kpc. This core size is in line

with previous works [101, 121] and with that suggested by recent hydrodynamic simulations of Milky Way size halos [122]. For the normalization of the profile we chose the value of the local density suggested in Ref. [2] for Milky Way Burkert-like profiles. The resulting profile is also compatible with current observational constraints [120]. Note, however, that a recent work favors a substantially larger core radius and a slightly higher normalization for Burkert-like profiles [123]. All the profile parameters are summarized in Table 4.5 and the four profiles are shown in the left panel of Figure 4.15.

Let us finally point out that there are other possible effects driven by baryons that might steepen the DM density profiles in the centers of DM halos, such as central black holes (see e.g. Refs. [124–126]), that we will not consider here.

4.8.3 Gamma-ray flux from dark matter annihilation

The gamma-ray flux from DM annihilation in the Galactic halo has two main contributions [127]: prompt photons and photons induced via ICS. The former are produced indirectly through hadronization, fragmentation and decays of the DM annihilation products or by internal bremsstrahlung, or directly through one-loop processes (but these are typically suppressed in most DM models). The second contribution originates from electrons and positrons produced in DM annihilations, via ICS off the ambient photon background. The other two possible contributions to the gamma-ray flux from DM annihilation can be neglected in our analysis: radiation from bremsstrahlung is expected to be sub-dominant with respect to ICS in the energy range considered (1 - 100 GeV) and a few degrees off the Galactic plane (see Fig. 14 in Ref. [197]), and synchrotron radiation is only relevant at radio frequencies, below the *Fermi*-LAT threshold. Thus the gamma-ray differential flux from DM annihilation from a given observational region $\Delta\Omega$ in the Galactic halo can be written as follows:

$$\frac{d\Phi_\gamma}{dE_\gamma}(E_\gamma, \Delta\Omega) = \left(\frac{d\Phi_\gamma}{dE_\gamma}\right)_{prompt} + \left(\frac{d\Phi_\gamma}{dE_\gamma}\right)_{ICS} . \tag{4.75}$$

We discuss in detail both components in the next subsections.

Prompt gamma rays

A continuous spectrum of gamma rays is produced mainly by the decays of π^0's generated in the cascading of annihilation products and also by internal bremsstrahlung. While the former process is completely determined for each given final state of annihilation (we will study $b\bar{b}$, $\tau^+\tau^-$, $\mu^+\mu^-$ and W^+W^- channels), the latter depends in general on the details of the DM model such as the DM particle spin and the properties of the mediating particle. Nevertheless, it is known that internal bremsstrahlung always includes much model-independent final state radiation (FSR), which is emitted directly from charged particles in the external legs [129, 130]. In our analysis of generic DM models, we only consider these FSR components of the internal bremsstrahlung. It is a safe choice for the conservative approach that we follow, since the inclusion of model-dependent emission from virtual charged mediators would make constraints only stronger [130, 131].

As we will consider throughout this work the case of self-conjugated DM particles, the prompt contribution can be written as

$$\left(\frac{d\Phi_\gamma}{dE_\gamma}\right)_{prompt} = \sum_i \frac{dN_\gamma^i}{dE_\gamma} \frac{\langle\sigma_i v\rangle}{8\pi m_{DM}^2} \bar{J}(\Delta\Omega)\Delta\Omega \ . \tag{4.76}$$

This equation has to be multiplied by an additional factor of 1/2 if the DM particle studied is not its own anti-particle. The discrete sum is over all DM annihilation channels. dN_γ^i/dE_γ is the differential gamma-ray yield[8], $\langle\sigma_i v\rangle$ is the annihilation cross-section averaged over the velocity distribution of the DM particles, m_{DM} is the mass of the DM particle, and the quantity $\bar{J}(\Delta\Omega)$ (commonly known as the *J-factor*) is defined as

$$\bar{J}(\Delta\Omega) \equiv \frac{1}{\Delta\Omega} \int \ d\Omega \ \int_{l.o.s.} \rho^2(r(l,\Psi)) \ dl \ . \tag{4.77}$$

The J-factor accounts for both the DM distribution and the geometry of the system[9]. The integral of the square of the DM density ρ^2 in the direction of observation Ψ is along the line of sight ($l.o.s$), and r and l represent the galactocentric distance and the distance to the Earth, respectively. Indeed, in Eq. (4.76), all the dependence on astrophysical parameters is encoded in the J-factor itself, whereas the rest of the terms encode the particle physics input[10]. The most crucial aspect in the calculation of $\bar{J}(\Delta\Omega)\Delta\Omega$ is related to the modeling of the DM distribution in the GC.

In the right panel of Figure 4.15, the $\bar{J}(\Delta\Omega)\Delta\Omega$ quantity corresponding to each of the four profiles discussed in Section 4.8.2 is shown as a function of the angle Ψ from the GC. The associated observational regions $\Delta\Omega$ to each Ψ are taken around the GC. The angular integration is over a ring with inner radius of 0.5° and external radius of Ψ. We have assumed a $r = 0.1$ pc constant density core for both NFW and NFW$_c$, although as discussed e.g. in Refs. [100, 138] the results are almost insensitive to any core size below ∼1 pc (or even larger given the *Fermi*-LAT PSF). Remarkably, the adiabatic compression increases the DM annihilation flux by several orders of magnitude in the inner regions, i.e., the regions of interest in the present study. This effect will turn out to be especially relevant when deriving limits on the DM annihilation cross section. We also note that for the Burkert profile the value of $\bar{J}(\Delta\Omega)\Delta\Omega$ is larger than for the NFW and Einasto profiles. This is so because of the relative high normalization used for this profile compared to the others and, especially, due to the annular region around the GC where we are focusing our studies, which excludes

[8]For the spectra of gamma rays we use pre-evaluated tables in [217], which are generated using `PYTHIA` [135] and thus containing FSR properly.

[9]Although in principle the point-spread function (PSF) should be included in this formula (see e.g., Refs. [90, 132, 133]), it turns out to be not relevant in our study mainly for two reasons: i) we deal with fluxes integrated in large regions of the sky, much larger than the PSF, and ii) we avoid the very center of the Galaxy, where the PSF would artificially smear out the cusps expected from some of the DM density profiles. See section 4.8.4 for more details.

[10]Strictly speaking, both terms are not completely independent of each other, as the minimum predicted mass for DM halos is set by the properties of the DM particle and is expected to play an important role also in the J-factor when substructures are taken into account. In our work, we do not consider the effect of substructures on the annihilation flux, as large substructure boosts are only expected for the outskirts of DM halos [136, 137], and thus they should have a very small impact on inner Galaxy studies.

Figure 4.15: Left panel: DM density profiles used in this work, with the parameters given in Table 4.5. Right panel: The $\bar{J}(\Delta\Omega)\Delta\Omega$ quantity integrated on a ring with inner radius of 0.5° ($\sim$ 0.07 kpc) and external radius of Ψ ($R_\odot \tan\Psi$) for the DM density profiles given in Table 4.5. Blue (solid), red (long-dashed), green (short-dashed) and yellow (dot-dashed) lines correspond to NFW, NFW$_c$, Einasto and Burkert profiles, respectively. The four DM density profiles are compatible with current observational data.

the GC itself (where such cored profiles would certainly give much less annihilation flux compared to cuspy profiles, see left panel of Figure 4.15). We note, however, that the use of another Burkert-like profile with a larger DM core than the one used here, as e.g., the one recently proposed in Ref. [123], may lead to substantially lower $\bar{J}(\Delta\Omega)\Delta\Omega$ values, and thus to weaker DM constraints. In particular, we checked for the profile in Ref. [123] that the values of $\bar{J}(\Delta\Omega)\Delta\Omega$ in the region shown in Figure 4.15 are always smaller than about 10^{22} GeV^2 cm^{-5} sr. Notice finally that the NFW$_c$ profile reaches a constant value of $\bar{J}(\Delta\Omega)\Delta\Omega$ for a value of Ψ smaller than the other profiles. This is relevant for our discussion below for optimization of the region of interest for DM searches, since we see that for NFW$_c$ a larger region of analysis will not increase the DM flux significantly as for NFW, Einasto and Burkert profiles.

Gamma rays from Inverse Compton Scattering

Electron and positron ($e^\pm$) fluxes are generated in DM annihilations mainly through the hadronization, fragmentation and decays of the annihilation products, since direct production of e^+e^- is supressed by small couplings in most DM models. These $e^\pm$ propagate in the Galaxy and produce high-energy gamma rays via ICS off the ambient photon background. The differential flux produced by ICS from a given observational region $\Delta\Omega$ in the Galactic halo is given by [217]

$$\frac{d\Phi_\gamma^{ICS}}{dE_\gamma} = \sum_i \frac{\langle\sigma_i v\rangle}{8\pi m_{DM}^2} \int_{m_e}^{m_{DM}} \frac{dE_I}{E_\gamma} \frac{dN_{e^\pm}^i}{dE_e}(E_I) \int d\Omega \, \frac{I_{IC}(E_\gamma, E_I; \Psi)}{E_\gamma} \,, \tag{4.78}$$

where E_I is the $e^{\pm}$ injection energy, Ψ corresponds to the angular position where the ICS gamma rays are produced, and the function $I_{IC}(E_\gamma, E_I; \Psi)$ is given by

$$I_{IC}(E_\gamma, E_I; \Psi) \;=\; 2E_\gamma \int_{l.o.s.} dl \int_{m_e}^{E_I} dE_e \; \frac{P_{IC}(E_\gamma, E_e; \mathbf{x})}{b_T(E_e; \mathbf{x})} \; \tilde{I}(E_e, E_I; \mathbf{x}) \; . \tag{4.79}$$

Here $\mathbf{x} = (l, \Psi)$ and $b_T \propto E^2$ is the energy-loss rate of the electron in the Thomson limit. The function P_{IC} is the photon emission power for ICS, and it depends on the interstellar radiation (ISR) densities for each of the species composing the photon background. It is known that the ISR in the inner Galactic region can be well modeled as a sum of separate black body radiation components corresponding to star-light (SL), infrared radiation (IR), and cosmic microwave background (CMB) [139]. In this work we have used the interstellar radiation field provided by `GALPROP` [140] to calculate the normalization and the temperature for each of these three components. For the injection spectra of $e^{\pm}$, we utilize pre-evaluated tables in [217].

The last ingredient in Eq. (4.79) is the $\tilde{I}(E_e, E_I; \mathbf{x})$ function, which can be given in terms of the well- known halo function [217], $I(E, E_I; \mathbf{x}) = \tilde{I}(E, E_I; \mathbf{x})[(b_T(E)/b(E, \mathbf{x}))(\rho(\mathbf{x})/\rho_\odot)^2]^{-1}$, where $\rho_\odot$ is the DM density at Sun's position and $b(E, \mathbf{x})$ encodes the energy loss of the $e^{\pm}$. The $\tilde{I}(E_e, E_I; \mathbf{x})$ function obeys the diffusion loss equation [217],

$$\nabla^2 \tilde{I}(E_e, E_I; \mathbf{x}) + \frac{E_e^2}{K(E_e; \mathbf{x})} \frac{\partial}{\partial E_e} \left[\frac{b(E_e; \mathbf{x})}{E_e^2} \tilde{I}(E_e, E_I; \mathbf{x}) \right] = 0 \; , \tag{4.80}$$

and is commonly solved by modeling the diffusion region as a cylinder with radius $R_{\max}$ =20 kpc, height z equal to $2L$ and vanishing boundary conditions. Also the diffusion coefficient $K(E; \mathbf{x})$ has been taken as homogeneous inside the cylinder with an energy dependence following a power law $K(E) = K_0(E/1\text{GeV})^\delta$. For these three parameters L, K_0 and δ, the so called diffusion coefficient, we have adopted three sets referred to as MIN, MED and MAX models [208], which account for the degeneracy given by the local observations of the cosmic rays at the Earth including the boron to carbon ratio, B/C [218]. We take them as our benchmark points, although we note that MIN and MAX models do not imply minimal or maximal expected gamma-ray signal, respectively. To solve this equation under the described conditions, we have used `BoxLib` [143] which is a general purpose partial differential equation solver with an adaptive mesh refinement method. We show in Section 4.8.5 that the use of the different diffusion models, MIN, MED, MAX, does not introduce a large variation in the DM constraints.

Let us finally remark about the importance of the energy loss function $b(E; \mathbf{x})$. The two main energy loss mechanisms of $e^{\pm}$ in the Galaxy are the ICS and synchrotron radiation produced by interaction with the Galactic magnetic field. The former is the only contribution to the energy losses that is usually considered, since it is the most important one in studies of sources far from the GC. But when the $e^{\pm}$ energy reaches several hundreds of GeV, synchrotron radiation can dominate the energy loss rate due to the suppression factor in the ICS contribution in the Klein-Nishina regime. By contrast, synchrotron radiation losses do not have this suppression, and are driven by the magnetic field energy density $u_B(\mathbf{x}) = B^2/2$. Although the strength and exact shape of the Galactic magnetic field is not well known, in

Figure 4.16: Schematic view of our choice of the ROI. The gray area represents the masked region.

the literature it is broadly described by the from [217],

$$B(r,z) = B_0 \exp\left(-\frac{r - 8.5 \text{ kpc}}{10 \text{ kpc}} - \frac{z}{2 \text{ kpc}}\right) , \tag{4.81}$$

normalized with the strength of the magnetic field around the solar system, B_0, which is known to be in the range of 1 to 10 μG [140]. This field grows towards the GC and therefore one should expect that the energy losses are dominated by synchrotron radiation in the inner part of the galaxy [217]. On the other hand, we can expect that when the magnetic field is stronger, the energy of the injected $e^{\pm}$ is more efficiently liberated in the form of synchrotron emission resulting in a softer spectrum, and producing therefore smaller constraints on the DM annihilation cross-section. We will check this in Section 4.8.5. There we will also check that the ICS effect becomes relevant only for the $\mu^+\mu^-$ annihilation channel, since the contribution of the prompt gamma rays is less important than in other channels with hadronic decays.

4.8.4 Gamma-ray flux from *Fermi*-LAT measurements

Data selection and processing

The Fermi satellite was launched on June 11, 2008. Its main instrument, the Large Area Telescope (LAT) [89], collects high energy gamma rays ($\sim$ 20 MeV to $>$ 300 GeV) with a large effective area ($\sim$ 6200 cm^2 above 1 GeV for P7CLEAN_V6 at normal incidence [144]) and a large field of view (2.4 sr). Further details on the LAT can be found in Ref. [89, 144].

In our analysis we use the LAT photon data measured between August 4, 2008, and June 15, 2012, in the energy range between 1 GeV and 100 GeV. Events with zenith angles $< 100°$ are selected to reduce the contamination by gamma-ray emission coming from cosmic-ray interactions in the atmosphere. We select events from the P7ULTRACLEAN_V6::FRONT class. This choice reduces the cosmic-ray background contamination and takes advantage of a narrower PSF as compared to back-converting events. We make a reasonable assumption on systematic uncertainty extending it from Source and Clean classes. The systematic uncertainty of the effective area for both Source and Clean class events is quoted as 10% at 100 MeV, decreasing to 5% at 560 MeV, and increasing to 20% at 10 GeV and above [144].

Maps of flux for different energy ranges from a region of 30° around the GC is made using version V9r28 of the LAT Science Tools [145]. As we will show below, we can use a single flux map (built summing up the flux maps for the different energy ranges) to select the Region Of Interest (ROI) with the aim of maximizing the S/N ratio for each individual DM profile under study.

Optimization of the region of interest for dark matter searches

An important step in our analysis is the optimization of the ROI using a data-driven procedure that maximizes the S/N ratio. In order to do so, we follow a procedure similar to the one presented in the appendix A of [131]:

1. We produce $40° \times 40°$ maps centered on the GC of the quantity $\bar{J}(\Delta\Omega)\Delta\Omega$ for the four DM density profiles considered (i.e., Einasto, NFW, NFWc and Burkert) and use them as signal. Each pixel i has an area of $0.2° \times 0.2°$ and contains a J-factor value J_i calculated with Eq. (4.77).

2. We use as noise the square root of the observed photon flux integrated in the energy range 1-100 GeV. We use a single map, instead of a different one for each energy bin since the morphology of the background does not exhibit strong variations in energy. The flux in pixel i is labelled as F_i.

3. A mask, defined by three angles θ_1, θ_2 and $|b|$ as shown in Figure 4.16, is introduced to cover the GC, the Galactic plane, and the high galactic latitude regions, where the statistical fluctuations of the Galactic foreground dominate over the expected DM signal.

4. The optimization procedure consists of finding the set of angles that mask a region such that the S/N ratio is the largest for each DM profile considered. What we technically do is to minimize the inverse of the following quantity

$$S/N = \frac{\sum_i J_i}{\sqrt{\sum_i F_i}} \tag{4.82}$$

   with i running over unmasked pixels, varying masks. We use the numerical routine `Fmin` of the Python module `scipy.optimize` [11], which minimizes a function using the downhill simplex algorithm. We end up with four masks characterized by those angles given in Table 4.6. In the same Table, we also show the values of $\Delta\Omega$ and $\bar{J}(\Delta\Omega)\Delta\Omega$ for each profile. As expected, $\theta_1 = |b|$ for all the profiles, since the broadest emission in the Galactic plane is the one in the GC.

Figure 4.17 shows the ROI that we have obtained for each DM density profile in Table 4.5. Clearly, the NFW_c ROI is the smallest one. This can be easily understood by inspecting the right panel of Figure 4.15: the $\bar{J}(\Delta\Omega)\Delta\Omega$ quantity for NFW_c becomes almost constant beyond only 5°, whereas for the other profiles this quantity becomes flat at much larger radii.

[11] http://docs.scipy.org/doc/scipy/reference/optimize.html#

Profile	θ_1 [°]	θ_2 [°]	$\lvert b \rvert$ [°]	$\Delta\Omega$ [sr]	$\bar{J}(\Delta\Omega)\,\Delta\Omega$ [$\times 10^{22}$ GeV^2 cm^{-5} sr]	Flux ($1-100$ GeV) [$\times 10^{-7}$ cm^{-2}s^{-1}]
Burkert	0.8	15.9	0.8	0.225	41.9	32.1 ± 0.3
Einasto	0.7	15.6	0.7	0.217	5.1	31.4 ± 0.3
NFW	0.6	16.7	0.6	0.253	3.3	38.0 ± 0.3
NFW$_c$	1.0	3.0	1.0	0.005	86.8	2.2 ± 0.1

Table 4.6: The optimized regions for the DM density profiles studied, defined by the angles shown in Figure 4.16. The corresponding values for $\Delta\Omega$, $\bar{J}(\Delta\Omega)\Delta\Omega$, and observed flux with statistical errors only (in the energy range between $1-100$ GeV) are also shown.

Therefore, in the case of the NFW$_c$ profile, increasing the angular aperture by a few more degrees does not increase the S/N.

Note that the usual quantities to calculate S/N ratios are observed counts and expected DM-induced counts but in this work, instead, we use observed gamma-ray flux (rather than counts) and J-factors (formally proportional to the expected DM-induced gamma-ray flux). Nevertheless, we checked that the use of observed fluxes and predicted J-factors turns out to be a very good approximation, which leads to similar optimized ROIs. We performed the following test. Using the gtobssim tool and assuming the NFW profile, we simulated the events that different DM models could produce in the LAT after 46 months of observation. We used the same IRFs, cuts and procedure to select events as those used for the real observations. Instead of using a fixed $1-100$ GeV energy range we further optimize this quantity choosing an energy range centered around the DM emission peak. We then compare the simulated DM counts maps and the actually observed counts map in the given energy range to re-derive the optimized S/N region. As anticipated above we find that the derived ROIs parameters using counts maps do not change significantly for different DM models from those calculated using observed fluxes and J-factors and a fixed energy range.

Flux measurement

Following the analysis described above, we show in Figure 4.17 the flux observed by the *Fermi*-LAT, and the ROIs corresponding to each of the DM profiles considered. The value of this flux integrated in the energy range $1-100$ GeV can be found in the last column of Table 4.6. The energy spectra from the ROI associated to each profile are shown in Figure 11.9. We limit the energy range of the analysis to be below 100 GeV in order to have a small statistical uncertainty in each bin, falling generally below the systematic uncertainty. In this way we remove the possibility for the upper limits to be accidentally dominated by a large downward fluctuation in the energy bins close to the peak of the gamma-ray emission from DM annhilation, which is the most constraining point when comparing to the measured flux.

To set constraints we require that the DM-induced gamma-ray flux does not exceed the flux upper limit (UL) evaluated as follows. We set 99.98% UL signal counts using the Bayesian approach presented in Ref. [146], for the case of absence of background with systematic uncertainties not included, which correctly takes into account the Poisson limit (i.e. the case of low counts). Using exposure maps calculated with the gtexpcube2 tool of

the Science Tools we are able to convert UL signal counts into the UL signal flux needed to set constraints.

4.8.5 Limits on the dark matter annihilation cross-section

As already discussed, we adopt a conservative approach in the analysis of the limits on the DM annihilation cross section, simply requiring that the integrated gamma-ray flux of the

Figure 4.17: Maps of the observed flux by the *Fermi*-LAT in the energy range $1-100$ GeV, in units of photons cm^{-2} s^{-1}, for the four DM profiles studied. Upper left: Einasto, upper right: NFW, bottom left: NFW$_c$, and bottom right: Burkert. For each profile, the ROI is the region inside the circle excluding the band on the Galactic plane. Color scale is logarithmic, yellow, red and blue correspond to 3.6×10^{-9}, 6.4×10^{-10} and 1.2×10^{-10} photons cm^{-2} s^{-1}, respectively. These values also correspond to black contours. In order to reduce statistical noise and to bring up finer features in the inner galaxy the map is smoothed with a 0.2° FWHM Gaussian function.

Figure 4.18: Energy spectra extracted from *Fermi*-LAT data for the optimized regions that are shown in Figure 4.17. Data are shown as points and the vertical error bars represent the statistical errors. The latter are in many cases smaller than the point size. The boxes represent the systematic error in the *Fermi*-LAT effective area.

expected DM signal for each energy bin does not exceed the upper limit signal flux evaluated following the Bayesian procedure in Ref. [146][12]. We do not subtract any astrophysical background.

We present the results in Figure 4.19, where the constraints obtained are shown for different final states. There we also illustrate the case $\langle\sigma v\rangle = 3\times10^{-26}$ cm^3 s^{-1}, which corresponds to the value of the annihilation cross-section associated to the correct thermal relic abundance for a WIMP whose annihilation is dominated by the s-wave (velocity-independent) contribution and thus, $\Omega_{DM}\ h^2 \approx 3\times10^{-27}$ cm^3 s^{-1} $\langle\sigma v\rangle^{-1} \approx 0.1$ [147]. For comparison, the constraints are given considering only the contribution from prompt gamma rays and the total contribution from prompt plus ICS gamma rays.

First, it is worth noting that if the DM density follows an Einasto, NFW or Burkert profile, the upper limits on the annihilation cross section are above the value of the thermal cross-section for any annihilation channel. Nevertheless, the situation is drastically different when we consider the DM compression due to baryonic infall in the inner region of the Galaxy. Indeed, by adopting the NFW$_c$ profile and for a $b\bar{b}$, $\tau^+\tau^-$ and W^+W^- channel, the thermal annihilation cross-section is already reached for a DM mass of 680, 530 and 490 GeV, respectively. For the $\mu^+\mu^-$ channel the effect of the prompt gamma rays is less important since generally fewer photons are produced in the FSR compared to the hadronic decays of the other channels. (For the W^+W^- which is open when $m_{DM} \gtrsim 90$ GeV, the $W^\pm$

[12]It is worth noting that even though we optimize the ROIs based on both, DM and observed distributions, to set limits on DM annihilation cross section we perform a spectral analysis. It would be interesting for a future work to check that at the upper limit cross sections derived here, the implied spatial distribution of the gamma-ray signal intensity does not significantly exceed the data anywhere within the ROI at any energy.

Figure 4.19: 3σ upper limits on the annihilation cross-section of models in which DM annihilates into $b\bar{b}$, $\mu^+\mu^-$ (upper panel), $\tau^+\tau^-$ or W^+W^- (lower panel), for the four DM density profiles discussed in the text. Upper limits set without including the ICS component in the computation are also given as dashed curves (prompt) for comparison. The uncertainty in the diffusion model is shown as the thickness of the solid curves (from top to bottom: MIN, MED, MAX) while the lighter shaded regions represent the impact of the different strengths of the Galactic magnetic field with lower(higher) values of the cross-section corresponding to $B_0 = 1\ \mu$G($B_0 = 10\ \mu$G). The horizontal line corresponds to the expected value of the thermal cross-section for a generic WIMP candidate.

decays produce a large number of photons, especially at high energy). Notice that the lower bound associated with prompt gamma rays for $\mu^+\mu^-$ is 100 GeV compared to about 500–700 GeV in the other channels. Thus the ICS is important in this case, also due to the relatively harder $e^\pm$ spectrum [129]. We can see that for $B_0 = 1\ \mu$G the lower bound on the DM mass turns out to be 358 GeV and for $B_0 = 10\ \mu$G the bound is 157 GeV, using the MIN diffusion model. For MED and MAX diffusion models the values turn out to be 404, 171 GeV and 439, 179 GeV, respectively. As discussed in Subsection 4.8.3, when the magnetic field is stronger the energy of the injected $e^\pm$ is more efficiently liberated in the form of microwaves, resulting

in a softer gamma-ray spectrum, and producing therefore lower constraints . Therefore, we have shown that in those cases in which the ICS component is dominant (for heavy WIMP masses in general), the variation of the magnetic field can significantly alter the expected gamma-ray fluxes from the inner regions of the Galaxy.

Although the above results can be interpreted in general as implying that vanilla WIMP models and contracted DM profiles are incompatible with the Fermi data, one should keep in mind that if one works in the framework of a specific particle physics model this conclusion might in principle be avoided in some regions of the parameter space. For example, the final state can be a combination of the annihilation channels presented here, as in supersymmetry where the lightest neutralino annihilation modes are 70% $\bar{b}b$ − 30% $\bar{\tau}\tau$ for a Bino DM, and 100% W^+W^- for a Wino DM (or for a Higgs-portal model). More importantly, the value of the annihilation cross section in the Galactic halo might be smaller than 3×10^{-26} cm^3 s^{-1} for a DM candidate that is thermally produced. For example, in the early Universe coannihilation channels can also contribute to $\langle \sigma v \rangle$. Also, DM particles whose annihilation in the early Universe is dominated by p-wave (velocity-dependent) contributions would have a smaller value of $\langle \sigma v \rangle$ in the Galactic halo, where the DM velocity is much smaller than at the time of freeze-out, and can therefore escape the constraints derived in this work. These two effects can in fact occur in some regions of the parameter space of well motivated models for particle DM, such as the neutralino. In this sense, the results derived above for pure annihilation channels can be interpreted as limiting cases that give an idea of what can happen in realistic scenarios.

Let us remark that the upper limits on the annihilation cross-section that we have obtained for the cases of NFW, Einasto and Burkert profiles are comparable to the ones previously reported by the *Fermi*-LAT collaboration [101], after a similar analysis of the Galactic halo without modeling of the astrophysical background (similar results were also obtained in Ref. [148, 149]). Modeling of the background was also considered in Ref. [101], and the results are competitive with those from dSphs [86–88], where the upper limit of the annihilation cross section is below the thermal one for DM masses smaller than 27 and 37 GeV assuming a $b\bar{b}$ and a $\tau^+\tau^-$ channel, respectively. Remarkably, when we take into account the baryonic infall in our conservative analysis, forcing the DM to contract in those inner regions of the Galaxy, we obtain much stronger limits. In particular, as discussed above, using our compressed DM density profile, NFW$_c$, the thermal cross section is excluded for a DM mass smaller than 680 and 530 GeV in the $b\bar{b}$ and $\tau^+\tau^-$ channel, respectively, thus improving those limits obtained from dSphs [86–88], and also those obtained from galaxy clusters [150]. In the latter, DM masses smaller than about 100 GeV are constrained provided that DM subhalos significantly contribute to boost the DM signal.

In general, the upper limits on the DM annihilation cross section are two orders of magnitude stronger than without contraction. In recent works on the GC a similar analysis was carried out but subtracting the emission from known point sources and from the Galactic disk [121, 151].

4.8.6 Conclusions about the Galactic Center

We derived constraints on the parameter space of generic DM candidates using *Fermi*-LAT inner Galaxy measurements. We considered well motivated DM density profiles, such as

Burkert, Einasto and NFW, which are perfectly compatible with current observational data of the Milky Way. We then selected optimal regions around the GC, such that the S/N ratio is maximized. When the effect of contraction in the DM halo due to baryons is included in the computation [90, 100], the constraints turn out to be very strong. In particular, a compressed DM density profile allows us to place upper limits on the DM annihilation cross section that exclude the thermal cross section for a broad range of DM masses, as shown in Figure 4.19. This is the case for masses smaller than 680, 530 and 490 GeV for $b\bar{b}$, $\tau^+\tau^-$ and W^+W^- channels, respectively. For the $\mu^+\mu^-$ channel, where the ICS effect is important, the exclusion of the thermal cross-section is for a mass smaller than about 150 to 400 GeV, depending on models of the Galactic magnetic field. Alternatively, one may interpret these results as implying that vanilla WIMP models and contracted DM profiles are incompatible with the *Fermi* data.

Although the constraints are very strong, the analysis is conservative since we require that the expected DM signal does not exceed the gamma-ray emission observed by the *Fermi*-LAT, and modeling of the astrophysical background is not carried out. The latter would only lead to better constraints on the DM annihilation cross section.

4.9 Indirect detection of antimatter

4.9.1 The principle

4.9.2 Astrophysical Backgrounds

In the conventional background model, positrons are produced in the interactions between cosmic-ray nuclei and the interstellar medium. The expected positron flux on earth can be written as [226]

$$\Phi_{e^+}^{conv} = \frac{4.5E^{0.7}}{1 + 650E^{2.3} + 1500E^{4.2}}\,\mathrm{GeV}^{-1}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1} \tag{4.83}$$

where E is given in GeV. In contrast to the antiproton case, this conventional background is not compatible with the recent data from PAMELA [185] and Fermi-LAT [186]. Even after taking into account the possible uncertainties due to cosmic ray propagation, the data reveals a clear excess over this background at high energies, $E \gtrsim 10$ GeV. Hence, a new source of high energy positrons is necessary to explain the data. .

The other main interpretation for the observed positron excess is that it is generated by astrophysical sources, such as pulsars [189]. If that is the case, such astrophysical positrons constitute an additional background to the positrons from dark matter annihilation. And they must be taken into account when studying the sensitivity of future experiments to a dark matter positron signal, as pointed out in [190]. Here, we will closely follow their approach. That is, we use as background the positron flux obtained from the pulsar interpretation of PAMELA and Fermi-LAT data [191]–see figure 2 in [190]. That background is compatible with the absolute positron flux derived in [192].

4.9.3 How to treat the Background?

PAMELA observed a clear excess in the positron spectrum in the end of 2008. The main astrophysical interpretation for the observed positron excess is that it is generated by astrophysical sources, such as pulsars [189]. If that is the case, such astrophysical positrons constitute an additional background to the positrons from dark matter annihilation. And they must be taken into account when studying the sensitivity of future experiments to a dark matter positron signal, as pointed out in [190]. Here, we will closely follow their approach. That is, we use as background the positron flux obtained from the pulsar interpretation of PAMELA and Fermi-LAT data [191]–see figure 2 in [190]. That background is compatible with the absolute positron flux derived in [192].

4.10 A concrete example: the Higgs-portal case**

4.10.1 Introduction

One of the simplest extensions of the Standard Model that can explain the dark matter is the addition of a real scalar singlet and an unbroken Z_2 symmetry under which the singlet is odd while all other fields are even. Such a model, known as the singlet scalar model of dark matter, has been studied several times in the literature [163, 673, 675, 677, 684, 690]. Predictivity is undoubtedly its most salient feature. In contrast with other common scenarios that explain the dark matter, the singlet model contains only one additional field, the singlet scalar, and two new parameters: the singlet mass and the coupling between the singlet and the higgs boson –the only standard model field that couples to it. The singlet relic density as well as its direct and indirect detections rates depend additionally on the higgs mass, which however cannot vary in a wide range. After imposing the dark matter constraint, the viable parameter space gets reduced simply to the singlet mass and the higgs mass. The implications of the model can then be studied as a function of the singlet mass for a few representative values of the higgs mass. The singlet scalar model, therefore, provides a compelling and predictive scenario to explain the dark matter.

Recently, it was shown [690] that the singlet may have the right relic density to explain the observed dark matter abundance, that its direct detection cross section is large enough to be probed by present and planned experiments, and that the gamma rays from the annihilation of singlet scalar dark matter will likely be observed by the Fermi satellite [165]. To complete the analysis of this model, in this paper we study the indirect detection of singlet scalar dark matter through positrons and antiprotons.

The indirect detection of dark matter is one of the promising avenues towards its identification. Among them, antimatter searches in cosmic rays play an essential role. Present experiments such as Pamela [166] and Fermi [165] are already measuring the cosmic ray spectrum with unprecedented precision. In fact they have already challenged our knowledge of the positron spectrum. Future experiments such as AMS-02 [167], which is scheduled to be launched next year, will measure the antiproton and the positron spectrum in a wider energy range and with significantly better statistics. Hence, the chances of observing an exotic component from dark matter annihilation in the positron or the antiproton spectrum are now higher than ever. It is therefore crucial to study the expected antiproton and positron

fluxes in well-motivated scenarios for physics beyond the standard model that account for the dark matter of the Universe.

The antiproton and positron fluxes from singlet annihilation not only depend on the dark matter model but also on a number of astrophysical factors that affect the production and propagation of such particles throughout the Galaxy. To take these effects into account, we consider different propagation models for positrons and antiprotons, and we study the possible role of substructures in the dark matter halo. Our goals are to find out whether any region of the parameter space is already ruled out by present data from Pamela, and to identify those regions that are within the sensitivity of the AMS-02 experiment. In addition, by comparing the corresponding regions for positrons and antiprotons, we expect to determine which of these two signals offers better perspectives to probe the singlet scalar model.

In the next section the main features of the singlet scalar model of dark matter will be briefly reviewed. Then, in section 4.10.3, we present a detailed analysis of the antiproton signal in this model. In particular, we determine the excluded and detectable regions in the plane $(m_S, \langle\sigma v\rangle)$ for different sets of parameters accounting for propagation and substructure effects. In section 4.10.4 we present an analogous analysis for the positron channel. A brief discussion of our main results is given in section 4.10.5, which is followed by our conclusions.

4.10.2 The singlet scalar model of dark matter

The Lagrangian that describes the singlet scalar model of dark matter is

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{m_0^2}{2} S^2 - \frac{\lambda_S}{4} S^4 - \lambda S^2 H^\dagger H \,, \tag{4.84}$$

where $\mathcal{L}_{SM}$ denotes the Standard Model Lagrangian, H is the higgs doublet, and S is the singlet scalar field. This is the most general renormalizable Lagrangian that is compatible with the $SU(3) \times SU(2) \times U(1)$ gauge invariance and with the symmetry $S \to -S$. Notice, from the Lagrangian, that the higgs boson is the only standard model field that directly couples to the singlet. This extension of the standard model contains, therefore, two new phenomenologically relevant parameters: m_0 and λ. Instead of m_0, it is useful to consider the physical mass of the singlet

$$m_S = \sqrt{m_0^2 + \lambda v_{EW}^2} \tag{4.85}$$

as a free parameter of the model. The other free parameter is λ, which determines the trilinear $(S^2 h)$ and quartic $(S^2 h^2)$ coupling between the singlet and the physical higgs boson. In addition to m_S and λ, the dependence on the higgs mass –a SM parameter– should also be also taken into account, as it affects the dark matter phenomenology. To accurately compute the singlet relic density we use the micrOMEGAs package [168], which can calculate the relic density in a generic dark matter model.

Singlets can annihilate through s-channel higgs boson exchange into a variety of final states: $f\bar{f}$, W^+W^-, Z^0Z^0, and hh. Additionally, they can also annihilate into hh either directly or through singlet exchange. Depending on the singlet mass, two annihilation regions can be clearly distinguished. A light singlet, $m_S < m_W$, will annihilate mainly into the

Figure 4.20: Annihilation branching fractions as a function of the dark matter mass for the singlet model. The higgs mass was set to 120 GeV while the value of λ was obtained by imposing the dark matter constraint.

$b\bar{b}$ final state, with subdominant contributions from other light fermions. To obtain the correct relic density, such light singlets require a relatively large value of λ ($\gtrsim 0.1$), and are consequently constrained by direct detection experiments. Singlets masses below 50 GeV, for instance, are already ruled out by current measurements [690]. A heavy singlet, $m_S > m_W$, on the other hand, will annihilate mainly into W^+W^-, with additional contributions from Z^0Z^0, hh and $t\bar{t}$. In this region the required value of λ is typically smaller (~ 0.01) and present direct detection constrains are ineffective. Figure 4.20 displays the annihilation branching fractions as a function of the singlet mass for $m_h = 120$ GeV. The sharp contrast between the light singlet region and the heavy one is clearly observed.

Being a scalar field, the singlet annihilation rate today is not suppressed with respect to that of the early Universe (s-wave annihilation). Moreover, throughout most of the viable parameter space $\langle\sigma v\rangle$ is constant and equal to the typical annihilation rate, $\langle\sigma v\rangle \sim 2.3 \times 10^{-26}\text{cm}^3\text{s}^{-1}$ [690]. The only two places where $\langle\sigma v\rangle$ is smaller are at the W threshold ($m_S = m_W - \epsilon$) and at the higgs resonance ($m_h \sim 2m_S$). In fact, the main effect of the higgs mass is to determine the position where this resonance lies. In the following analysis we will set m_h to 120 GeV and consider only the remaining dependence on the singlet mass. For the sake of definiteness, singlet masses below 600 GeV will be considered.

4.10.3 Antiprotons

Antiprotons can be produced either by dark matter annihilations or through different kinds of astrophysical mechanisms. After being produced they propagate in the Galaxy and reach

the Earth, where they can be detected as exotic components in cosmic rays. The PAMELA experiment reported recently [169] the measurement of the antiproton to proton flux ratio up to 100 GeV, which is in agreement with that predicted by the conventional background model. In the near future, the AMS-02 experiment, to be launched next year, will measure the antiproton flux with even higher precision, increasing the odds of finding an additional component due to dark matter annihilations.

Propagation

Once produced, antiprotons propagate throughout the galaxy undergoing several processes:

- They scatter on irregularities of the galactic magnetic field –Alfvén waves. These scatterings constitute, in fact, a space diffusion process with a diffusion coefficient given by

$$K(E_{\rm kin}) = K_0 \beta_{\bar{p}} \left(\frac{p_{\bar{p}}}{\rm GeV} \right)^{\delta} , \tag{4.86}$$

  where $E_{\rm kin}$ is the antiproton kinetic energy, $p_{\bar{p}} = (E_{\rm kin}^2 + 2m_{\bar{p}} E_{\rm kin})^{1/2}$ is the antiproton momentum, and $\beta_{\bar{p}} = \left(1 - \frac{m_{\bar{p}}^2}{(E_{\rm kin}+m_{\bar{p}})^2}\right)^{1/2}$. K_0 and δ are free parameters of the propagation model that are constrained by a combination of theoretical predictions and astrophysical data.

- They undergo a second order Fermi mechanism reacceleration, due to the motion of the scattering centers with a velocity of $V_a \approx (20 - 100)$ km/sec. This reacceleration process is described by the coefficient

$$K_{EE} = \frac{2}{9} V_a^2 \frac{E^2 \beta^4}{K(E)} . \tag{4.87}$$

- They lose energy either adiabatically, or through Coulomb scattering, or by ionizing the interstellar (IS) medium. The total energy loss rate is denoted by b and depends on the antiproton energy.

- They are wiped away from the galactic disk through convection, with a velocity $V_c \approx (5 - 15)$ km/sec. In the following, this velocity will be taken to be completely vertical to the disk: $V_c = V_c \mathrm{sign}(z)$.

- They can annihilate upon scattering on the IS medium. In this study, we shall consider the two primary components of the medium, namely Hydrogen and Helium. The annihilation cross-sections for $\bar{p} - \mathrm{H}$ and $\bar{p} - \mathrm{He}$ scattering have been taken from [170], where the well-known Tan&Ng parametrization [171] is used.

The propagation of antiprotons in the interstellar medium, taking into account all of the above processes, is described by a diffusion-convection equation of the form

$$\partial_z (V_c \psi) - K \nabla \psi + \partial_E \left[b(E) \psi - K_{EE}(E) \partial_E \psi \right] = q , \tag{4.88}$$

where we denote by $\psi = dn/dE$ the energy density of the antiprotons, and by q the source term –see equation (4.92). Actually, this equation is rather generic and can be applied to other particles species propagating in the galaxy. Only the values of the different parameters, but not the equation itself, will vary depending on the propagating particle. We will see in section 4.10.4, for instance, that this same equation describes the propagation of positrons in the interstellar medium. Let us now see how to solve this equation for antiprotons.

The primary flux

To solve equation (4.88), we shall use the method proposed, for example, in [172, 173, 176]. The idea of this method is to adopt a simpler version of equation (4.88) for which the Green function can be calculated analytically. To achieve such simplification, certain processes contributing to the final antiproton spectrum have to be neglected. Specifically, all energy redistributions in the initial (injection) spectrum –energy losses, reacceleration, as well as "tertiary" contributions (i.e. contributions from secondary antiprotons produced upon inelastic scattering with the IS medium)– are ignored. Whether these redistributions are important or not depends mainly on the antiproton energy. For GeV energies, the results may deviate up to 50% from those obtained with the (more complete) Bessel function treatment –in [177] a comparison between the two methods can be found (see figure 2). But for energies around 10 GeV, the accuracy of the method improves dramatically, yielding essentially indistinguishable results at slightly higher energies. Since the $\bar{p}$ energy region we shall consider begins at 10 GeV, we can safely use this simplified approach. Apart from its clearer physical interpretation, the main advantage of this method is that astrophysical effects can be separated from particle physics ones. As a result, it is a well-suited method for scans in parameter spaces, as those we are going to carry out in the following.

If we denote by $\Gamma^{\rm ann}_{\bar{p}} = \sum_{\rm ISM} n_{\rm ISM} v \sigma^{\rm ann}_{\bar{p}\ \rm ISM}$ the destruction rate of antiprotons in the ISM, where ISM = H, He, and implementing the aforementioned simplifications, the transport equation for a point source (which actually defines the propagator G) is:

$$\left[-K\nabla + V_c \frac{\partial}{\partial z} + 2h\Gamma_{\rm tot}\delta(z)\right] G = \delta(\vec{r} - \vec{r}') \,. \tag{4.89}$$

The antiproton propagator can then be written as

$$G^{\odot}_{\bar{p}}(r,z) = \frac{e^{-k_v z}}{2\pi K L} \sum_{n=0}^{\infty} c_n^{-1} K_0(r\sqrt{k_n^2 + k_v^2}) \sin(k_n L) \sin(k_n(L-z)) \,, \tag{4.90}$$

where K_0 is a modified Bessel function of the second kind and

$$\begin{aligned} c_n &= 1 - \frac{\sin(k_n L)\cos(k_n L)}{k_n L} \,, \\ k_v &= V_c/(2K) \,, \\ k_d &= 2h\Gamma^{\rm ann}_{\bar{p}}/K + 2k_v \,, \end{aligned} \tag{4.91}$$

with $h = 100$ pc being the half thickness of the galactic disc, and L being the half-thickness of the diffusive zone. k_n is obtained as the solution of the equation

$$n\pi - k_n L - \arctan(2k_n/k_d) = 0, \quad n \in \mathbb{N} \,.$$

	L (kpc)	K_0(kpc^2/Myr)	δ	V_c(km/s)
MIN	1	0.0016	0.85	13.5
MED	4	0.0112	0.70	12.0
MAX	15	0.0765	0.46	5.0

Table 4.7: Values of propagation parameters widely used in the literature and providing minimal and maximal antiproton fluxes, or consitute the best fit to the B/C data.

Then, in order to compute the flux expected on earth, we should convolute the Green function (4.90) with the source distribution $q(\vec{r}, E)$. For dark matter annihilations in the galactic halo, the source term is given by

$$q(\vec{r}, E) = \frac{1}{2}\left(\frac{\rho(\vec{x})}{m_\chi}\right)^2 \sum_i \left(\langle\sigma v\rangle \frac{dN^i_{\bar{p}}}{dE_{\bar{p}}}\right) , \tag{4.92}$$

where the index i runs over all possible annihilation final states. The decay of SM particles into antiprotons can be calculated with programs like PYTHIA [180]. In the singlet scalar model of dark matter, antiprotons may originate in $W^\pm$, Z^0, h, and t decays. Regarding the distribution of dark matter in the Galaxy, $\rho(\vec{x})$, we assume a NFW profile with a local density of 0.3 GeVcm^3. The final expression for the antiproton flux on the Earth takes the form

$$\Phi^{\bar{p}}_{\odot}(E_{\text{kin}}) = \frac{c\beta}{4\pi}\frac{\langle\sigma v\rangle}{2}\left(\frac{\rho(\vec{x}_\odot)}{m_\chi}\right)^2 \frac{dN}{dE}(E_{\text{kin}}) \int_{DZ}\left(\frac{\rho(\vec{x}_s)}{\rho(\vec{x}_\odot)}\right)^2 G^{\odot}_{\bar{p}}(\vec{x}_s) d^3x \, , \tag{4.93}$$

where none of the integrated quantities depends on the antiproton energy. This feature demonstrates one of the virtues of the Green function method applied to antiprotons: The integral in equation (4.93), which we compute using a VEGAS Monte-Carlo algorithm, needs to be calculated only once for each value of the injection energy, for it is the same as the detection energy.

Regarding the propagation parameters L, K_0, δ, and V_c, we take their values from the well-established MIN, MAX and MED models –see table 4.7. The former two models correspond to the minimal and maximal antiprotons fluxes that are compatible with the B/C data. The MED model, on the other hand, correspond to the parameters that best fit the B/C data.

Influence of substructures

N-body simulations reveal that galactic halos are not completely smooth; they also contain a significant number of substructures (clumps). Such substructures have been studied repeatedly in the literature as a possible way to enhance the dark matter annihilation rate. It has been claimed though, that in realistic scenarios it is rather improbable to expect large enhancements from dark matter clumps [172]. Here, we will study rather qualitatively the possible effect of substructures on the antiproton (and positron) signal.

To that end, we closely follow the approach outlined in [173]. The enhancement due to dark matter substructure is described by an energy-dependent function known as the boost factor B. Because the distribution of dark matter clumps in the Galaxy is unknown, B

cannot be computed from first principles; it can only be studied from a statistical point of view. For simplicity, we will limit our discussion to the effective boost factor, B_{eff}, defined as the average value of B over a large number of realizations of the Galactic halo. It must be kept in mind, however, that in some exceptional cases –for example when there is a large dark matter clump very close to the Earth– B_{eff} can deviate significantly from B. Since the probability of such an event is quite small [175], we will not consider those cases in this work.

The energy-dependent effective boost factor, B_{eff}, can be written, under certain assumptions (see [173]), as

$$B_{\rm eff} \equiv \frac{\langle \phi \rangle}{\phi_{\rm sm}} = (1-f)^2 + f B_c \frac{\mathcal{I}_1}{\mathcal{I}_2}\,, \tag{4.94}$$

where $\langle \phi \rangle$ is the average flux coming from the clumpy DM distribution, $\phi_{\rm sm}$ is the flux that we would expect if the whole halo were smooth, f is the fraction of DM in clumps, and B_c is the boost factor (assumed constant with energy) for an individual clump. In this study, this constant boost factor is supposed to be universal for all clumps. Finally, $\mathcal{I}_{n=1,2}$ are given by

$$\mathcal{I}_n = \int_{\rm DM\ halo} G(\vec{x}, E) \left(\frac{\rho_{\rm sm}(\vec{x})}{\rho_0} \right)^n d^3\vec{x}\,. \tag{4.95}$$

The effective boost factor, then, depends on f and B_c. When invoking clumpiness, we will follow [173] and use $f = 0.2$ as a representative value (see e.g. [178, 179]). Regarding the constant boost factor, B_c, it could vary from just a few up to two orders of magnitude [172, 174, 179]. We will use $B_c = 3, 10, 100$, which give rise to effective boost factors in the approximate ranges $(1, 2)$, $(3, 5)$ and $(10, 40)$ respectively. This last range roughly coincides with the upper limit for the boost factor found in [172], for the case of a NFW smooth halo and clumps with a Moore et al internal profile.

Astrophysical backgrounds

So far, our main concern has been the antiproton signal from dark matter annihilations. But, to determine whether a model is detectable or whether it is compatible with present data, we also need to know the antiproton background. The most well-known treatment of the astrophysical antiproton background is that of Strong and Moskalenko [170], which has recently received additional experimental support. In fact, the antiproton data [169] from the PAMELA experiment is perfectly compatible with their predictions. We thus have a relatively simple antiproton background model that is able to explain the data.

In the following, when trying to examine whether the model is excluded by the PAMELA data, we shall be employing the theoretical background predictions, normalizing them to the low-energy PAMELA data. More specifically, we take advantage of the fact that predictions demonstrate a relatively constant background with respect to the propagation model, and use the simple parametrization (properly normalized) presented in [177]. When studying the detectability of the model in AMS-02, on the other hand, we shall use the PAMELA measurements themselves as background. More concretely, we borrow the fit performed by Cirelli et al in [181], which provides a sufficiently good fit to theoretical predictions but also to the recent PAMELA data. We pay special attention at reproducing the good

Figure 4.21: Regions of the parameter space that are excluded by the recent antiproton data from the PAMELA experiment. The area above the MIN, MED, and MAX lines is excluded for the given propagation model. The solid (black) line shows the viable parameter space of the singlet scalar model of dark matter.

background normalization at low energies, so as to stay as close as possible to the PAMELA measurements.

Finally, let us point out that since the antiproton background does not vary significantly with the propagation parameters [177], it is reasonable to consider a global background, irrespective of the propagation model. We will do so in the following.

Results

In this section we present our results for antiprotons. First, we combine the predicted flux and the expected background to determine the regions in the plane $(m_S, \langle\sigma v\rangle)$ that are already excluded by present antiproton data from PAMELA. Then, we determine the regions that are detectable by AMS-02.

In the singlet scalar model of dark matter, the singlet mass and the higgs mass are the only free parameters left after imposing the relic density constraint. Since we take $m_h = 120$ GeV throughout this work, the annihilation branching ratios and the total annihilation rate, $\langle\sigma v\rangle_{singlet}$, depend only on m_S. To obtain the excluded regions in the plane $(m_S, \langle\sigma v\rangle)$, we first compute, for a model with the same branching ratios as the singlet scalar model (see figure 4.20), the value of $\langle\sigma v\rangle$ required to exclude the model, $\langle\sigma v\rangle_{excl}$. By comparing $\langle\sigma v\rangle_{singlet}$ with $\langle\sigma v\rangle_{excl}$ we can then determine whether the model is excluded or not at a given singlet mass. An analogous procedure is followed to determine the detectable regions.

The condition we impose for exclusion is that the sum of the signal prediction and the (properly normalized) background prediction exceed the measurement by the PAMELA collaboration by more than 3σ. Figure 4.21 shows the excluded region for the MIN, MED, and MAX propagation models. The area above the lines is already excluded by present data. Also shown as a solid (black) line is the region along which the relic density constraint is satisfied –the viable parameter space of the model. Because that line is entirely below the exclusion lines, we conclude that, in the absence of substructure enhancements, current antiproton data do not yet constrain the singlet scalar model of dark matter. This situation

Figure 4.22: Regions of the parameter space that are within the sensitivity of the AMS-2 experiment. The area above the MIN, MED, and MAX lines is detectable by AMS-02. The solid (black) line shows the prediction of the singlet model. Notice that for MED and MAX, essentially the whole parameter space is detectable.

may change soon, however, for the MAX exclusion line lies just above the prediction of the model for masses around 100 GeV. Future data from PAMELA may start constraining that region of the parameter space.

To assess the sensitivity of AMS-02 to an antiproton signal from singlet dark matter annihilations, we compute the annihilation rate needed to distinguish the signal from the background at the 95% confidence level. To do so, we first calculate the χ^2 as

$$\chi^2 = \sum_{n=0}^{N} \frac{(\phi_n^{tot} - \phi_n^{bkg})^2}{(\phi_n^{tot})} A \cdot T \ , \tag{4.96}$$

where ϕ^{tot} is the total antiproton flux, ϕ^{bkg} is the background flux, N is the number of energy bins considered, A is the geometrical acceptance of the experiment, and T is the data acquisition time. AMS-02 is expected to take data for three years and features an antiproton geometrical acceptance of 330 cm^{2}sr [182]. We consider 20 energy bins evenly distributed in logarithmic scale between 10 and 300 GeV. A 95% confidence level corresponds then to $\chi^2 > 31$.

The annihilation rates needed for a detection at AMS-02 are shown in figure 4.22 for the three propagation models. The singlet model is detectable whenever its annihilation cross-section (solid line) is larger than the exclusion lines at AMS-02 for a given propagation model. We see that for the MIN propagation model only a small region around 60 GeV is within the AMS-02 reach. For the MED and MAX propagation models, on the other hand, essentially the whole parameter space is detectable at AMS-02. Only in the Higgs resonance region, $2m_S \sim m_h$, and at the W-threshold does the model's annihilation rate fall below the exclusion lines. Hence, for the MED and MAX models, a singlet scalar with a mass below 600 GeV should be easily detected at AMS-02.

We now consider the possible effect of dark matter substructures on the exclusion and the detectable regions. Figure 4.23 shows the exclusion lines for the MIN (upper figure),

MED (middle figure), and MAX (lower figure) propagation models for the different values of B_c we investigate. As before, the solid (black) line shows the prediction of the singlet model. Notice that the MIN propagation model is nowhere excluded, not even for the most optimistic enhancement. For the MED propagation model, $B_c = 100$ is excluded for singlet masses below 360 GeV. The other values of B_c are all compatible with present data. For the MAX propagation model, $B_c = 100$ is ruled out in the whole mass range while $B_c = 10$ is at the limits of exclusion for masses around 100 GeV. Hence, we see that, once the possible enhancement due to dark matter substructures is included, current antiproton data already constrain some combinations of astrophysical and particle physics parameters.

The prospects for the detection of an antiproton signal at AMS-02 were already good without any additional boost from dark matter clumps, as we saw in figure 4.22. The additional enhancement from dark matter substructures only improves such prospects, as clearly seen in figure 4.24. For the MIN model, for instance, the detectable region increases from a small region around $m_S \sim 60$ GeV without boost factor to $m_S \lesssim 200$ GeV for a moderate boost factor, covering the whole mass range for $B_c = 100$.

To summarize, we have seen that the antiproton signal from singlet annihilation is a promising way to indirectly detect singlet scalar dark matter. Current antiproton data from PAMELA already constraint some scenarios and future data from AMS-02 is likely to reveal a significant excess over the expected background.

4.10.4 Positrons

We now discuss the positron signal from singlet dark matter annihilation, carrying out an analysis similar to that presented for antiprotons in the previous section.

Propagation

There have been many treatments on the propagation of positrons throughout the galactic medium. In this paper, we adopt the two-zone diffusion model and solution described in [173]. For completeness, we review here the main points of this approach.

Being a diffusive process, positron propagation is governed by the same general equation, (4.88), that describes antiproton propagation. The approximations and the parameters that were used for antiprotons, however, are not the same as for positrons. Indeed, contrary to the $\overline{p}$ case, where energy redistribution processes become inefficient above a few GeV, energy loss –through either synchrotron radiation or inverse Compton scattering on stellar light and CMB photons– is the main process involved in positron propagation. These processes lead to an energy loss rate that can be written as

$$b(E) = \frac{E^2}{E_0 \tau_E} \,, \tag{4.97}$$

where E is the positron energy, E_0 is a reference energy (which we take to be 1 GeV) and $\tau_E = 10^{16}$s is the characteristic energy-loss time.

The other important difference with respect to antiprotons is that for positron propagation the effect of the galactic convective wind, as well as reacceleration and annihilation

Figure 4.23: Regions excluded by the antiproton data from PAMELA including the possible effect of substructures in the DM halo. From top to bottom the figures correspond to the MIN, MED, and MAX propagation models. The solid (black) line shows the prediction of the singlet model. The area above the lines is excluded for the corresponding parameter values.

Figure 4.24: Detectable regions at AMS-02 including the possible effect of substructures. From top to bottom the figures correspond to the MIN, MED, and MAX propagation models. The solid (black) line shows the prediction of the singlet model. The area above the lines is detectable for the corresponding parameter values.

	L (kpc)	K_0(kpc^2/Myr)	α
MIN	1	0.00595	0.55
MED	4	0.0112	0.70
MAX	15	0.0765	0.46

Table 4.8: Values of propagation parameters widely used in the literature and roughly providing minimal and maximal positron fluxes, or constitute the best fit to the B/C data.

processes can all be neglected. After these simplifications are taken into account, we are left with the following equation

$$\partial_t \psi - \nabla \left[K(x,E) \nabla \psi \right] - \partial_E \left[b(E) \psi \right] = q(x,E) \,, \tag{4.98}$$

where K is the space diffusion coefficient –steady state is assumed. This coefficient is taken to be constant in space but depends on the energy as

$$K(E) = K_0 \left(\frac{E}{E_0} \right)^{\alpha} . \tag{4.99}$$

Here the diffusion constant, K_0, and the spectral index, α, are propagation parameters. Then, the master equation for positron propagation gets simplified to its final form

$$K_0 \epsilon^{\alpha} \triangle \psi + \frac{\partial}{\partial \epsilon} \left(\frac{\epsilon^2}{\tau_E} \psi \right) + q = 0, \tag{4.100}$$

where $\epsilon = E/E_0$. This is the equation we solve to calculate the effects of positron propagation on a signal produced at some point in the galaxy.

As in the case of antiprotons, a crucial factor in the treatment of positron propagation is the adopted propagation model. In this case there are mainly 3 relevant parameters, namely L, K_0 and α; that is, the half-thickness of the cylindrical diffusive zone, the diffusion constant and the spectral index, respectively. For their values we use the three models described in table 4.8.

The primary flux

The resulting positron flux from DM annihilations can be written as (see [183] for details)

$$\Phi_{e^+}(E) = \frac{\beta_{e^+}}{4\pi} \frac{\langle \sigma v \rangle}{2} \left(\frac{\rho(\vec{x}_\odot)}{m_\chi} \right)^2 \frac{\tau_E}{E^2} \int_E^{m_\chi} f(E_s) \tilde{I}(\lambda_D) dE_s \,, \tag{4.101}$$

where the detection and the production energy are denoted respectively by E and E_s. $f(E_s)$ is the production spectrum for positrons, $f(E_s) = \sum_i dN^i_{e^+}/dE_s$, with i running over all possible annihilation channels. The diffusion length, λ_D, is defined by

$$\lambda_D^2 = 4 K_0 \tau_E \left(\frac{E^{\alpha-1} - E_s^{\alpha-1}}{1-\alpha} \right) . \tag{4.102}$$

The so-called halo function, $\tilde{I}$, contains all the dependence on astrophysical factors. It is given as

$$\tilde{I}(\lambda_D) = \int_{DZ} d^3\vec{x}_s \tilde{G}(\vec{x}_\odot, E \to \vec{x}_s, E_s) \left(\frac{\rho(\vec{x}_s)}{\rho(\vec{x}_\odot)} \right) . \tag{4.103}$$

The modified Green function $\tilde{G}$ is in its turn defined by

$$\tilde{G} = \frac{1}{4\pi K_0 \tilde{\tau}} e^{-(z_\odot - z_s)^2/4K_0\tilde{\tau}} \tilde{V} , \tag{4.104}$$

with $\tilde{V}$ depending on the value of the characteristic parameter $\zeta = \frac{L^2}{K_0\tilde{\tau}}$. When $\zeta > 1$ – when the diffusion time is small– boundary conditions can be ignored and the propagation equation can be treated as a 1-D Schroedinger equation. In that case

$$\tilde{V} = \frac{1}{\sqrt{4\pi K_0 \tilde{\tau}}} \exp\left(-\frac{(z - z_s)^2}{4\pi K_0 \tilde{\tau}} \right) . \tag{4.105}$$

When ζ is small this approximation no longer holds but we can express $\tilde{V}$ as

$$\tilde{V} = \sum_{n=1}^{\infty} \frac{1}{L} \left[e^{-\lambda_n \tilde{\tau}} \phi_n(z_s)\phi_n(z) + e^{-\lambda'_n \tilde{\tau}} \phi'_n(z_s)\phi'_n(z) \right] \tag{4.106}$$

where

$$\phi_n(z) = \sin(k_n(L - |z|)) , \qquad k_n = \left(n - \frac{1}{2} \right) \frac{\pi}{L} \tag{4.107}$$

$$\phi'_n(z) = \sin(k'_n(L - z)) , \qquad k'_n = n\frac{\pi}{L} . \tag{4.108}$$

We now have all the necessary ingredients for the computation of the expected positron flux from singlet scalar dark matter annihilation. Next we discuss the positron background.

Astrophysical Backgrounds

In the conventional background model, positrons are produced in the interactions between cosmic-ray nuclei and the interstellar medium. The expected positron flux on earth can be written as [226]

$$\Phi_{e^+}^{conv} = \frac{4.5E^{0.7}}{1 + 650E^{2.3} + 1500E^{4.2}} \, \mathrm{GeV}^{-1}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1} \tag{4.109}$$

where E is given in GeV. In contrast to the antiproton case, this conventional background is not compatible with the recent data from PAMELA [185] and Fermi-LAT [186]. Even after taking into account the possible uncertainties due to cosmic ray propagation, the data reveals a clear excess over this background at high energies, $E \gtrsim 10$ GeV. Hence, a new source of high energy positrons is necessary to explain the data.

Two main interpretations of this positron excess have been considered in the literature. One is the annihilation of dark matter particles. Though viable, such models are rather contrived. They require multi-TeV masses, annihilation into leptonic final states, and an

Figure 4.25: Regions of the parameter space that are excluded by the recent positron data from the PAMELA experiment. The area above the lines is excluded for the corresponding propagation model. Notice that no region of the viable parameter space is currently ruled out.

annihilation rate several orders of magnitude larger than that expected for a thermal relic [187]. Moreover, they are tightly constrained by experimental data –gamma rays, radio emission, CMB, BBN [210]. Given that, in addition, the singlet model cannot explain the positron excess (at least in the energy region we examine and for reasonable astrophysical assumptions), we will not consider this dark matter interpretation any further.

The other main interpretation for the observed positron excess is that it is generated by astrophysical sources, such as pulsars [189]. If that is the case, such astrophysical positrons constitute an additional background to the positrons from dark matter annihilation. And they must be taken into account when studying the sensitivity of future experiments to a dark matter positron signal, as pointed out in [190]. Here, we will closely follow their approach. That is, we use as background the positron flux obtained from the pulsar interpretation of PAMELA and Fermi-LAT data [191]–see figure 2 in [190]. That background is compatible with the absolute positron flux derived in [192].

Results

For positrons we follow a procedure analogous to that for antiprotons. We first find the regions that are already excluded by present data and then determine those that will be detectable in AMS-02. When using equation (4.96), it must be taken into account that the geometrical acceptance of AMS-02 for positrons is 420cm^2 sr [182].

Figure 4.25 displays the regions that are already excluded for the MIN, MED, and MAX propagation models. The solid (black) line shows, instead, the prediction of the singlet scalar model. Notice that the variation due to the propagation model is very small, certainly much smaller than for antiprotons. From the figure we also see that the exclusion lines all lie well above the model's prediction. We checked that a similar result is found even in the optimistic scenario for substructure enhancement. At present, henceforth, no region of the viable parameter space is even close to being excluded via the positron signal.

The AMS-02 detectable regions are shown in figure 4.26 for the three propagation models.

Figure 4.26: Regions of the parameter space that give a positron signal within the sensitivity of the AMS-02 experiment. The lines corresponding to the MIN, MED and MAX propagation models are shown. They must be compared to the actual prediction (solid line) of the singlet scalar model.

Again, not much difference is observed between the MIN, MED and MAX lines. Moreover, only a small region around $m_S \sim 200$ GeV is within the AMS-02 reach. Thus, without additional boost factors, singlet masses above 200 GeV do not produce a detectable positron signal at AMS-02.

Once the effect of dark matter substructures is included, the detectable region extends to higher singlet masses, as illustrated in figure 4.27. For the optimistic scenario, $B_c = 100$, the whole mass range becomes detectable independently of the propagation model. For the moderate scenario, $B_c = 10$, the AMS-02 reach extends to singlet masses of 100 GeV and 200 GeV for the MIN and MAX models respectively. Thus, a substructure enhancement is required to probe the singlet model beyond $m_S \sim 200$ GeV at AMS-02.

4.10.5 Discussion

Our results clearly indicate that in the singlet scalar model the indirect detection of dark matter via antimatter is feasible, particularly through the antiproton channel. The positron channel also gives rise to an observable signal at AMS-02, but only for light singlets. The antiproton signal, on the other hand, should be visible over most of the viable parameter space.

To identify the singlet scalar model as the correct model of dark matter, one would need to measure m_h and m_S, the only two parameters of the model, and to confirm some of the model predictions. Such a task would certainly require input from different experiments: accelerator searches, direct detection experiments, and indirect searches through several channels. Searches at the LHC, for instance, could provide a measurement of the higgs mass, but they are not expected to say much about the singlet mass. It is mainly through direct and indirect detection searches that the singlet mass could be determined.

In [690], it was shown that the singlet direct detection rate is within the reach of current and future experiments and that the gamma ray signal from singlet dark matter annihilation

Figure 4.27: Detectable regions for the MIN, MED, and MAX propagation models in the presence of dark matter substructures. The region above the lines is detectable for the corresponding parameters. The solid (black) line shows the prediction of the singlet scalar model.

is likely to be observed by the Fermi satellite. In this paper, we have found that also the antiproton signal is observable at AMS-02. By combining the signals from these three experiments the singlet mass, m_S, might be determined. Once m_h and m_S are known, the singlet model of dark matter is completely specified, at least within the standard cosmological scenario.

4.10.6 Conclusions

The singlet scalar model provides a simple and predictive scenario to account for the dark matter. In this paper, we studied in detail the antiproton and positron signals expected from the annihilation of singlet scalar dark matter. Particular attention was paid to the role of the propagation model and to a possible enhancement in the annihilation rate due to dark matter substructures in the Galaxy. We used the recent measurements, from PAMELA and Fermi-LAT, of the positron and antiproton flux to find the regions in the plane $(m_S, \langle\sigma v\rangle)$ that are already excluded by the data. For antiprotons, an optimistic enhancement, $B_c = 100$, due to dark matter clumps is essentially excluded for both the MED and the MAX propagation models in the entire mass range we considered. For positrons, on the contrary, no region is excluded independently of the propagation model or the possible effect of substructures. We also analyzed the perspectives for detecting antimatter signals in future experiments. In AMS-02, the positron signal, in the absence of additional boosts, was found to be detectable but only for light singlets. Antiprotons, on the other hand, offer excellent perspectives to be detected at AMS-02. Even without boosts, the whole mass range is within the sensitivity of AMS-02 for the MED and the MAX propagation models. For the MIN model, a moderate enhancement from substructures, $B_c = 10$, would allow to probe the mass range below 200 GeV. Antiprotons, therefore, seem to be a promising way to constrain or detect the singlet scalar model of dark matter.

4.11 Synchrotron radiation

The diffusion–loss equation can be written :

$$\frac{\partial}{\partial_t}\frac{\mathrm{dn_e}}{\mathrm{d}E_e} = \frac{\partial}{\partial_{E_e}}\left[b(E_e,\mathbf{r})\frac{\mathrm{d}n_e}{\mathrm{d}E_e}\right] + Q(E_e,\mathbf{r}) \tag{4.110}$$

where

$$Q(E_e,r) = \frac{1}{2}\left(\frac{\rho(r)}{m_\chi}\right)^2 \langle\sigma v\rangle \frac{\mathrm{d}N_e}{\mathrm{d}E_e} \tag{4.111}$$

which is the electrons injected by dark matter annihilation at a constant rate (per second per GeV).

In this section, we will try to explain and analyse this equation step by step.

4.11.1 Neglecting diffusion

$n_e(E_e) = \int_0^{E_e} \frac{dn_e}{dE_e}$ represent the number of electrons with energy lower than E_e. During 1 second, the electrons of energy $E(\gtrsim E_e)$ loose an amount of energy $\frac{dE}{dt}(E) = b(E)$: $b(E)$ represents the energy loss rate, ie the energy lost per electrons in 1 second at the energy E. One can thus deduce that all electrons having an energy between E_e and $E_e + b(E_e)$ will have their energy *below* E_e after 1 second. This number is $\frac{dn_e}{dE} \times \frac{dE}{dt} = b(E_e)\frac{dn_e}{dE}$ In another words, $N(E_e) \to N(E_e) + b(E_e)dt$ during the time dt. Per GeV, the particles lost at energy E_e is $\partial_{E_e}\left[b(E_e)\frac{dn_e}{dE_e}\right]$ which represent the loss-energy term in Eq.(4.110)

If we suppose that the transport timescale of the particle (which will fly away from its production point) is much longer that the cooling timescale responsible of the lost of energy, a steady state equation ("steady state" in the sense that we supposed that the galaxy has reached an equilibrium) can be written: Number of particles created between E_e and E_{max} + number of particle lost by cooling effects= 0, or after integration of Eq.(4.110) and reminding that $dN_e/dE_e(\infty, \mathbf{r}) = 0$.

$$\int_{E_e}^{\infty} dE'_e Q(E'_e, \mathbf{r}) + \left[0 - \frac{dE_e(E_e)}{dt}\frac{dN_e}{dE_e}(E_e, \mathbf{r})\right] = \int_{E_e}^{m_\chi c^2} dE'_e Q(E'_e, \mathbf{r}) - b(E_e)\frac{dN_e}{dE_e}(E_e, \mathbf{r}) = 0 \tag{4.112}$$

implying

$$\frac{dN_e}{dE_e}(E_e, \mathbf{r}) = \frac{\int_{E_e}^{m_\chi c^2} dE'_e Q(E'_e, \mathbf{r})}{b(E_e)} \tag{4.113}$$

b(E) represents the loss energy rate, and can have different origins : Synchrotron radiation (discussed below), Inverse Compton, brehmstralung or Coulomb. We will note them b_{syn}, b_{IC}, b_{brem} and b_{com} respectively : $b(E) = b_{syn}(E) + b_{IC}(E) + b_{brem}(E) + b_{com}(E)$

4.11.2 Synchrotron loss of energy

As we just saw, among the indirect dark matter detection channels, gamma–ray emission represents one of the most promising opportunity due to the very low attenuation in the interstellar medium and to its high detection efficiency. Positrons and protons strongly interact with gas, radiation and magnetic field in the galaxy and thus the expected signal sensibly depends on the assumed propagation model. However, during the process of thermalization in the galactic medium the high energy e^+ and e^- release secondary low energy radiation, especially in the radio and X-ray band, that, hence, can represent a chance to look for dark matter annihilation. A synchrotron emission is the relativistic limit of the cyclotron radiation given by the Larmor's formula (see appendix 4.3.2 for more details concerning radiative loss of energy) which gives the total power radiated by a non relativistic point charge as it accelerates (or decelerates):

$$P^{SI}_{Larmor} = \frac{e^2}{6\pi\epsilon_0 c^3}|\mathbf{a}|^2; \quad P^{CGS}_{Larmor} = \frac{2}{3}\frac{e^2}{c^3}|\mathbf{a}|^2 \tag{4.114}$$

where $\mathbf{a}$ stands for $\dot{\mathbf{v}}$ and SI or CGS (Centimeter-Gram-Second system) refers to the metric system (Longair versus Rybicki convention). We will give the results in both conventions as different authors use freely different type of conventions through their works. One can express the Larmor's formula as function of the B field after applying the equation of motion

$$\begin{aligned} m\frac{d\mathbf{v}}{dt} &= e\mathbf{v}\wedge\mathbf{B} \quad \Rightarrow \quad \mathbf{v}_{\parallel} = \text{cste}; \quad \dot{\mathbf{v}}_{\perp} = a_{\perp} = \frac{ev_{\perp}B}{m} \qquad \text{[SI]} \\ m\frac{\mathbf{v}}{dt} &= \frac{e}{c}\mathbf{v}\wedge\mathbf{B} \quad \Rightarrow \quad \mathbf{v}_{\parallel} = \text{cste}; \quad \dot{\mathbf{v}}_{\perp} = a_{\perp} = \frac{ev_{\perp}B}{mc} \qquad \text{[CGS]} \end{aligned} \tag{4.115}$$

Implementing Eq.(4.115) in Eq.(4.116) one obtains

$$P^{SI}_{\text{Larmor}} = \frac{e^4B^2v_{\perp}^2}{6\pi\epsilon_0 m^2c^3}; \qquad P^{CGS}_{\text{Larmor}} = \frac{2}{3}\frac{e^4B^2v_{\perp}^2}{m^2c^5} \tag{4.116}$$

The cyclotron Larmor's formula transforms into a synchrotron radiation for very relativistic electrons, with the replacement[13]

$$m \quad \rightarrow \gamma m \; ; \qquad (|a_{\parallel}|^2 + |a_{\perp}|^2) \rightarrow \gamma^4(\gamma^2|a_{\parallel}|^2 + |a_{\perp}|^2) \tag{4.117}$$

Implemented in Eq.(4.116) on can write

$$\begin{aligned} P &= \frac{e^4B^2\gamma^2v_{\perp}^2}{6\pi\epsilon_0 m^2c^3} = \sigma_T^{SI}\epsilon_0\gamma^2\beta_{\perp}^2B^2c^3 = 2\sigma_T^{SI}\gamma^2\beta_{\perp}^2c\left(\frac{B^2}{2\mu_0}\right) \qquad \text{[SI]} \\ \Rightarrow \langle P\rangle &= \frac{4}{3}\sigma_T^{SI}c\gamma^2\beta^2U_B^{SI} \end{aligned} \tag{4.118}$$

and

$$\begin{aligned} P &= \frac{2e^4B^2\gamma^2v_{\perp}^2}{3m^2c^5} = \frac{1}{4\pi}\sigma_T^{CGS}c\gamma^2B^2v_{\perp}^2 = 2\sigma_T^{CGS}\gamma^2\beta_{\perp}^2c\left(\frac{B^2}{8\pi}\right) \qquad \text{[CGS]} \\ \Rightarrow \langle P\rangle &= \frac{4}{3}\sigma_T^{CGS}c\gamma^2\beta^2U_B^{CGS} \end{aligned} \tag{4.119}$$

which at the end can be finally written :

$$\langle P\rangle = \frac{4}{3}\sigma_T^{i}c\gamma^2\beta^2U_B^{i} \tag{4.120}$$

with $\sigma_T^{SI} = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 mc^2}\right)^2$, $\sigma_T^{CGS} = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$, $U_B^{SI} = B/2\mu_0$ and $U_B^{CGS} = B/8\pi$. σ_T is the Thompson cross section (see section 2.2.11), equal to $0.665\times10^{-24}\text{cm}^2$ for electrons. In a third step, one needs to transform Eq.(4.118) as function of the energy of the electron. If it is considered as relativistic, his energy can be written $E^2 = p^2c^2$, with $p = \gamma mv$, one obtain $E^2 = \gamma^2m^2v^2c^2$ which gives us

[13]Remember that in special relativity the lorentz transformation on accelerations gives $a'_{\parallel} = \gamma^3a_{\parallel}$ and $a'_{\perp} = \gamma^2a_{\perp}$

$$P = \frac{e^4 B^2 E_\perp^2}{6\pi\epsilon_0 m^4 c^5} \Rightarrow \langle P \rangle = \frac{2}{3}\frac{e^4 B^2 E^2}{6\pi\epsilon_0 m^4 c^5} = P_0 \left(\frac{B}{\mu\mathrm{G}}\right)^2 \left(\frac{E}{\mathrm{GeV}}\right)^2 \mathrm{GeV\ s^{-1}} \tag{4.121}$$

with $P_0 = 2.51 \times 10^{-18}$. One can also write Eq.(4.121) with a more explicit dependance on the energy density of the magnetic field B:

$$\langle P \rangle = \frac{4}{3}\frac{e^4}{6\pi\epsilon_0^2}\frac{E^2}{m_e^4 c^7}\left(\frac{B^2}{2\mu_0}\right) = \frac{4}{3}\frac{e^4}{6\pi\epsilon_0^2}\frac{E^2}{m_e^4 c^7} U_B = \tilde{P}_0 \left(\frac{E}{\mathrm{GeV}}\right)^2 \left(\frac{U_B}{\mathrm{ev/cm^3}}\right) \quad \mathrm{GeV\ s^{-1}} \tag{4.122}$$

with $\tilde{P}_0 = 1.02 \times 10^{-16}\mathrm{GeV\ s^{-1}}$. On can apply Eq.(4.122) to the inverse Compton loss of energy process. Indeed, this process is generated by the interaction of an electron on a photon from the background of energy density U_{rad}. One can then write $b_{IC}(E) = \tilde{P}_0 \left(\frac{E}{\mathrm{GeV}}\right)^2 \left(\frac{U_{rad}}{\mathrm{ev/cm^3}}\right)$ GeV s^{-1}. The relevant radiation background for the Inverse Compton Scattering is given by an extragalactic uniform contribution consisting of the CMB, with $U_{CMB} = 8\pi^5 (kT)^4/15(hc)^3 \simeq 0.26$ eV/cm^3, the optical/infrared extragalactic background and the analogous spatially varying galactic contribution, the Interstellar Radiation Field (ISRF). The ISRF intensity near the solar position is about 5 eV/cm^3, and reaches values as large as 50 eV/cm^3 in the inner kpc's. Combining the synchrotron and inverse Compton loss of energy processes, and forgetting in the first approximation the brehmstralung and Compton effects, on obtain

$$b(E) = b_{syn}(E) + b_{IC}(E) = 2.51\times10^{-18}\left(\frac{E}{\mathrm{GeV}}\right)^2\left(\frac{B}{\mu\mathrm{G}}\right)^2 + 1.02\times10^{-16}\left(\frac{E}{\mathrm{GeV}}\right)^2\left(\frac{U_{rad}}{\mathrm{eV/cm^3}}\right) \tag{4.123}$$

in GeV s^{-1}. One can see from Eq.(4.123) that the inverse Compton scattering dominate the loss energy process up to a magnetic filed $B \lesssim 50\mu$G.

To compare our prediction to the experiment, one needs to convert the power spectrum which is a function of the energy of the electron, to a spectrum in frequencies ν. We should so consider that the number of events in a width $d\nu$ is equal to the number of event in a width dE, which should be written $P(\nu)d\nu = n(E,r)P(E)dE$, or $P(\nu) = n(E,r)P(E)dE/d\nu$. $P(\nu)$ is called the emissivity and $n(E,r) = dN/dE$ is the density number of particles with energy comprised between E and $E+dE$. We thus see that we need to know the "conversion" factor $dE/d\nu$ to compute the emissivity, in other word, we need to express the frequency of the synchrotron radiation as function of the electron energy. In fact, the synchrotron spectrum of an electron gyrating in a magnetic field has its prominent peak at the resonance frequency ν_c

$$\nu_c = \frac{3eE_e^2 B}{4\pi m_e^3 c^4} = \nu_0 \left(\frac{B}{\mu\mathrm{G}}\right)\left(\frac{E_e}{\mathrm{GeV}}\right)^2 \Rightarrow E(\nu) = \sqrt{\frac{\nu}{\nu_0}}\left(\frac{B}{\mu\mathrm{G}}\right)^{-1/2} \mathrm{GeV} \tag{4.124}$$

with $\nu_0 = 16.02 \times 10^6$ Hz. This implies that, in practice, a $\delta-$approximation around the peaks works extremely well. To compute the total fluxes (Watt per meter square per second

per Hertz), one needs to integrate the expression 4.113 along the line of sight (los) and the solid angle $d\Omega$:

$$\mathcal{F}_\nu = P(E_e) \times \frac{dE_e}{d\nu} \times \frac{1}{4\pi} \int d\Omega \int_{los} ds \; \frac{dN_e}{dE_e}(r) \tag{4.125}$$

Combining Eqs. (4.113), (4.121), (4.124) and (4.125) one obtains for the flux, expressed in Jansky (10^{-26} Wm^{-2}Hz^{-1}):

$$\mathcal{F}_\nu = 2.92 \times 10^7 \text{Jy} \times \left(\frac{1}{1 + \frac{\tilde{P}_0}{P_0} \left(\frac{B}{\mu\text{G}}\right)^2} \right) \times \left(\frac{\nu}{\text{GHz}}\right)^{-1/2} \times \left(\frac{B}{\mu\text{G}}\right)^{-1/2} \times \int_{E_\nu}^{m_{DM}} \frac{dN_{\text{source}}}{dE}$$
$$\times \left(\frac{\sigma v}{3.10^{-26}\text{cm}^3\text{s}^{-1}}\right) \times \left(\frac{m_{DM}}{\text{GeV}}\right)^{-2} \times \left(\frac{\rho_0}{\text{GeV/cm}^3}\right)^2 \times \frac{1}{4\pi} \int d\Omega \int \frac{ds(r,\theta)}{\text{kpc}} f^2(r) \tag{4.126}$$

where we have defined $\rho(r) = \rho_0 \times f(r)$. The integral on the solid angle and line of sight can be developed: $\frac{1}{4\pi}\int d\Omega \int ds(r,\theta) = \frac{1}{2}\int_0^\theta \sin\theta' d\theta' \int_0^{D(1-\sin\theta')} dl$ where $D = 8.5$ kpc is the distance from the observer (on earth) to the galactic center. The angle θ is the one formed between the direction of D and the line of sight, l is related to r through the relation $r = \sqrt{D^2 + l^2 - 2Dl\cos\theta}$.

Concerning the loss of energy, we only considered the synchrotron and inverse Compton process. If one take into account Coulomb losses and Bremsstrahlung, one would obtain :

$$b(E) = P_0 \left(\frac{E}{\text{GeV}}\right)^2 \left(\frac{B}{\mu\text{G}}\right)^2 + \tilde{P}_0 \left(\frac{E}{\text{GeV}}\right)^2 \tag{4.127}$$
$$+ b_0^{Coul} n(1 + \log(\gamma/n)/75) + b_0^{Brem} n(0.36 + \log(\gamma/n)) \tag{4.128}$$

where n is the mean number density of electrons in cm^{-3}, the average over space gives about $n \simeq 1.3 \times 10^{-3}$, $\gamma = E/m_e$ and $b_0^{Coul} \simeq 6.13 \times 10^{-16}$, $b_0^{Brem} \simeq 1.51 \times 10^{-16}$ in GeVs^{-1} units.

4.11.3 Semi-analytical approach

If one takes into account the space diffusion, the equation (4.110) becomes

$$\frac{\partial n(x,E)}{\partial t} = \nabla \cdot [K(x,E)\; \nabla n(x,E)] + \frac{\partial}{\partial E}[b(x,E)n(x,E)] = q(x,E) \;, \tag{4.129}$$

where $b(x,E)$ encodes the energy loss rate, $n(x,E) = dN/dE$ is the number density of electron per energy and $q(x,E)$ the source term per GeV. As we discussed above, cosmic ray electrons loose energy mainly through synchrotron radiation and Inverse Compton scattering (IC), with a rate $b(x,E)$ which at the galactic medium is typically of the order 10^{-16} GeV·s^{-1}. Additional bremsstrahlung losses of electron energies on the interstellar medium are neglected in this semi-analytical approach, although they may have a very small effect (see for example

Fig.**??**), since for the synchrotron frequency we work with, the electron energies of interest are around 1 GeV.

Assuming a steady state, Eq. (4.110) can be re-expressed as

$$\frac{\partial \tilde{n}(x,E)}{\partial \tilde{t}} - K_0 \; \Delta \tilde{n}(x,E) = \tilde{q}(x,E) \tag{4.130}$$

in which the derivative with respect to energy has been parametrized in terms of the parameter $\tilde{t} \equiv -\int dE(E^\delta/b(x,E))$. If at the level of propagation one considers that energy losses have an average value over all the diffusion region, i.e. $b(x,E) \approx b(E) = E^2/\tau$ GeV·s^{-1} (see Eq. (4.145)), then in (4.130) $\tilde{n}(x,E) = E^2 n(x,E)$ and $\tilde{q}(x,E) = E^{2-\delta} q(x,E)$. The solution of this equation can be found in the Green function formalism to be

$$\begin{aligned} n(x,E) &= \frac{1}{b(E)} \int_E^\infty dE_s \\ &\times \int_{\rm DZ} d^3x G(x,E \leftarrow x_s,E_s) q(x_s,E_s) \end{aligned} \tag{4.131}$$

where the volume integral is over the diffusion zone (DZ). The Green function $G(x,E \leftarrow x_s,E_s)$ gives the probability for an electron injected at x_s with energy E_s to reach x with energy $E < E_s$, and has a general solution of the form

$$G(x,E \leftarrow x_s,E_s) = \frac{\tau}{E^2} G(x,\tilde{t} \leftarrow x_s,\tilde{t}_s) \tag{4.132}$$

$$G(x,\tilde{t} \leftarrow x_s,\tilde{t}_s) = \left(\frac{1}{4\pi K_0 \Delta\tilde{t}}\right)^{3/2} e^{-\frac{(\Delta x)^2}{4K_0\Delta\tilde{t}}} \; , \tag{4.133}$$

where $\Delta\tilde{t} = \tilde{t} - \tilde{t}_s$ and $(\Delta x)^2 = (\mathbf{x} - \mathbf{x}_s)^2$. The unique argument of the Green function is actually the diffusion length $\lambda = 4K_0\Delta\tilde{t}$, because the energy dependence enters only in this combination. This is the characteristic length of an electron traveling during its propagation. Here we are going to focus on diffusion models for which the half-thickness $L_h \sim 4$ kpc is small compared to the radius of the disk R_h, such that in practice the radial boundary has negligible effect on propagation [208]. The Green function for which $n(x,E)$ vanishes at $z = \pm L_h$ may be expressed as [207]

$$\tilde{G}(\lambda,L) = \frac{1}{(\sqrt{\pi}\lambda)^3} \sum_{n=-\infty}^{n=\infty} (-1)^n \; \exp^{-(2nL+(-1)^n z_s)^2/\lambda^2} \; . \tag{4.134}$$

As for the source term, in the case of production from DM annihilation with cross-section $\langle \sigma v \rangle$, it can be expressed as

$$q(x,E) = \eta \; \langle \sigma v \rangle \left\{ \frac{\rho(x)}{m_{DM}} \right\}^2 \frac{dN}{dE} \; . \tag{4.135}$$

Here $\rho(x)$ is the DM profile, given in units of GeV/cm^3, and $\eta = 1/4$ or $\eta = 1/2$ depending on the Dirac or Majorana nature of DM. The injection spectrum of electrons is given by dN/dE.

With all these ingredients we can express the electron number density as

$$n(x,E) = \eta \, \langle \sigma v \rangle \left(\frac{\rho_\odot}{m_{DM}} \right)^2 \frac{1}{b(E)} \int_E^{m_{DM}} dE_s \frac{dN}{dE_s} I_{\rm halo} \ . \tag{4.136}$$

Here $I(\lambda)$ is the so-called halo function, which is defined has

$$I_{\rm halo} = \int_{\rm DZ} d^3 x_s \tilde{G}(x, E \leftarrow x_s, E_s) \left\{ \frac{\rho(x_s)}{\rho_\odot} \right\}^2 . \tag{4.137}$$

When electrons (and positrons) are created in the Galaxy, they are accelerated by the local magnetic field and produce synchrotron radiation with an energy flux per unit frequency ν per solid angle (or spectral energy distribution) given by

$$F_\nu = \frac{1}{4\pi} \int_{\rm los} dl \int_{m_e}^{m_{DM}} dE \ P(x,\nu,E) \ n(x,E) \tag{4.138}$$

where the integration are performed along the line of sight (los) and on the electron energies. In this relation $P(x,\nu,E)$ is the synchrotron power (per unit frequency) emitted at ν by an electron of energy E which, for an energy distribution of electrons $n(x,E)$, must be integrated over all the electron energies E which lead to synchrotron radiation at the same frequency ν. In practice it can be shown (see *e.g.* [209]) that the radiation power as a function of ν is strongly peaked near a so-called critical frequency ν_c, defined as

$$\nu_c = \frac{3eBE^2}{4\pi m_e^3} \equiv 16{\rm MHz} \left(\frac{E}{\rm GeV} \right)^2 \left(\frac{B}{\mu G} \right) \tag{4.139}$$

in natural units, so that there is a near one-to-one correspondence between ν and E which, following [210], we take to be such that

$$P(\nu,E) \approx \frac{4}{27} \frac{e^3 B}{m_e} \delta \left(\frac{\nu}{\nu_c(E)} - 0.29 \right) . \tag{4.140}$$

Using this handy approximation, which is in one form or another is often adopted in the literature (see also [211, 534]), the flux of Eq.(4.138) takes a simple form,

$$F_\nu = \frac{1}{4\pi} \int_{\rm los} dl \, \frac{E}{2\nu} \, P_{\rm sync}(E) \, n(x,E) \tag{4.141}$$

where

$$E \equiv E(\nu) \approx \left(\frac{\nu}{4.7{\rm MHz}} \right)^{1/2} \left(\frac{\mu G}{B} \right)^{1/2} {\rm GeV} \tag{4.142}$$

which stems from $\nu = 0.29 \, \nu_c(E)$, and where $P_{\rm sync}$ is the total synchrotron energy loss rate of an electron of energy E,[14]

$$\begin{aligned} P_{sync}(E) &\hat{=} \int d\nu P(\nu,E) = \frac{e^4 B^2 E^2}{9\pi m_e^4} \\ &\simeq 2.5 \cdot 10^{-18} \left(\frac{E}{\rm GeV} \right)^2 \left(\frac{B}{\mu {\rm G}} \right)^2 {\rm GeV} \cdot {\rm s}^{-1} \end{aligned} \tag{4.143}$$

[14] In these expressions, we are a bit loose regarding the definition of the magnetic field. In principle B should be the effective magnetic field felt by the electron, $B_\perp$. In practice, assuming an isotropic distribution of electron velocities, $\langle B_\perp^2 \rangle = 2/3 B^2$. This is explicit in Eq.(4.143) (see for instance Eq.(6.7) in Ref.([209])). For convenience, this factor of 2/3 has been included in the definition of Eq.(4.140), as in [210].

Finally, we may express the synchrotron energy flux as

$$
\begin{aligned}
F_\nu &= 1.21\times 10^8 \frac{\mathrm{Jy}}{\mathrm{sr}} \left\{ \frac{\eta}{2} \left(\frac{\langle \sigma v \rangle}{3.1\times 10^{-26}\mathrm{cm}^3/s} \right) \right. \\
&\times \left(\frac{1\mathrm{GeV}}{m_{DM}} \right)^2 \left(\frac{\rho_\odot}{\mathrm{GeV/cm}^3} \right)^2 \left(\frac{\mu\mathrm{G}}{B} \right)^{1/2} \\
&\times \left(\frac{\mathrm{GHz}}{\nu} \right)^{1/2} \frac{P_{\mathrm{sync}}(E)}{b(E)} \frac{1}{4\pi} \int \frac{dl}{\mathrm{kpc}} \int_E^{m_{DM}} dE_s \frac{dN}{dE_s} I_{\mathrm{halo}} \ .
\end{aligned}
\tag{4.144}
$$

using jansky flux units (Jy), 1 Jy $= 10^{-26}$ W·m^{-2}·Hz^{-1}.[15] The total energy loss assumed here is given by

$$
b(E) = P_{\mathrm{sync}}(E)(1 + r_{\mathrm{IC/sync}}) \tag{4.145}
$$

where

$$
r_{\mathrm{IC/sync}} = \frac{2}{3} \frac{U_{\mathrm{rad}}}{B^2/2} \simeq 2 \left(\frac{U_{\mathrm{rad}}}{8\ \mathrm{eV/cm}^3} \right) \left(\frac{B}{10\ \mu\mathrm{G}} \right)^{-2} \tag{4.146}
$$

is the ratio between IC and synchrotron energy loss and U_{rad} is the total radiation density.

4.11.4 General astrophysical setup

Astrophysical uncertainties

Studies coming from N-body simulations have led to popular expressions for the distribution of DM in the Galactic halo, like the Navarro-Frenk-White (NFW) density profile [6]

$$
\rho_{\mathrm{NFW}}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2} \ , \tag{4.147}
$$

or the Einasto profile [212, 213]

$$
\rho_{\mathrm{EIN}}(r) = \rho_s \exp\left\{ -\frac{2}{\alpha} \left(\left(\frac{r}{r_s} \right)^\alpha - 1 \right) \right\} \ , \tag{4.148}
$$

where r the radial distance from the centre of the DM halo. On the other hand, observations of galactic rotation curves as well some of the simulations which include also baryonic feedback on DM density (for a recent example see [214]) find DM density profiles which are more cored towards the inner regions of the Galaxy. One example of such profile is the isothermal profile [215, 216]

$$
\rho_{\mathrm{iso}}(r) = \rho_s \ \frac{r_s^2}{r^2 + r_s^2} \tag{4.149}
$$

or modified Einasto profile (in this case the parameter α is smaller when compared to the parameters found in simulations which contain only DM component, [217]). While the parameter α for the Einasto profile is fixed from a fit to the simulations, the values of

[15] In the following, we will express the flux F_ν in terms of the brightness temperature of the radiation, T, $F_\nu = 2\nu^2 T/\exp(2\pi\nu/T) - 1$, using the blackbody relation.

Model	L_h [kpc]	K_0 [cm^2 s^{-1}]	δ	v_a [km s^{-1}]
MIN	1	$4.8\,10^{26}$	0.85	0
MED	4	$3.4\,10^{27}$	0.70	0
MAX	15	$2.3\,10^{28}$	0.46	0
1a	4	$6.6\,10^{28}$	0.26	34.2
1b	4	$6.6\,10^{28}$	0.35	42.7
2a	10	$1.2\,10^{29}$	0.3	39.2
2b	10	$1.05\,10^{29}$	0.3	39.2
PD	4	$3.4\,10^{28}$	0.5	0

Table 4.9: Upper three raws: parameter sets derived using a semi-analytical approach, to lead to MIN/MED/MAX anti-proton fluxes at Earth from an exotic Galactic component [196]. Lower four rows: parameters from [198], consistent with [197], derived using `GALPROP` code in a fit to CR data, and shown to reproduce the gamma-ray diffuse data well. Last row: plain diffusion model, shown to be consistent with the radio data at 22 MHz – 94 GHz frequencies, [199].

parameters ρ_s, a typical scale density, and r_s, a typical scale radius for the Milky Way are determined from astrophysical observations (e.g. local stellar surface brightness, stellar rotational curves, total Milky Way mass within a given distance..., see e.g. [217, 579]). As we will see in section 4.11.4, the ratio of the synchrotron signal calculated with DM density of these three profiles gets smaller at higher latitudes, as at those distances (closer to the Solar position) DM density is better constrained. As the rotational curve measurements are poor at distances smaller than 2 kpc (or $\sim 10°$) from the Galacic Center and those regions of the Galaxy are baryon dominated, we choose the Region Of Interest (ROI) which spans $|b| \in (10 \pm 3)^\circ$ in Galactic latitudes, and $|l| \lesssim 3°$ in Galactic longitudes.

Together with DM density profile, the CR propagation parameters pose one of the main uncertainties in prediction of the synchrotron signal. In [**?**], the three models called MIN/MED/MAX [196] and featured in Table 4.9 are used to probe the uncertainty in CR propagation parameters. Originally, those parameters were derived to produce the maximal, median and minimal anti-proton flux from dark matter, while being compatible with the CR secondary to primary B/C ratio measurement [218,219]. Therefore, by construction, they do not necessarily capture the uncertainty in the electron fluxes in the inner Galaxy, which is of interest here. In section 4.11.4 we will comment in more detail on the impact of a choice of a CR parameters, exploring additional sets consistent with the CR data and which were i) derived numerically and ii) shown to reproduce the observed whole sky gamma-ray or radio emission, therefore probing more directly the signals in the inner Galaxy, [197–199].

The Galactic magnetic field (GMF) is considered possibly the most important ingredient when dealing with synchrotron radiation. In the diffuse interstellar medium it has a large-scale regular component as well as a small-scale random part, both having a strength of order micro-Gauss. The best available constraints in determining the *large-scale* GMF are Faraday rotation measures and polarized synchrotron radiation, see e.g. [220–223], while *random* component is deduced mainly based on the synchrotron emission.

Several 3D models of the large scale magnetic field are implemented in `GALPROP` (e.g.,

[224, 225]). However, as our ROI is away from the Galactic disk and the small-scale random component is expected to dominate the total value of the magnetic field, we use a simple parametrisation for a *total* magnetic field as customary in literature:

$$B(\rho, z) \propto \exp\left(-\frac{r - r_\odot}{R_m} - \frac{|z|}{L_m}\right) \ . \tag{4.150}$$

The parameters R_m and L_m should in principle depend on the diffusion model assumed (or *vice versa*), since the propagation in the Galactic medium is intimately related with the magnetic field. We follow [**?**] in taking $L_m = \delta \cdot L_h$ and $R_m = \delta \cdot R_h$. It was shown in that work that an actual extent of L_m and R_m does not play a critical role (the difference between a constant magnetic field and the exponential form defined above is $\lesssim 30\%$), as long as the field extends into our ROI (i.e. $L_m \gtrsim 1$ kpc). The normalisation at Sun's position $B_\odot$ is more or less well constrained, and we take the value of 6 μG [226], consistent with the measurements [220–223]. However the value of the field in the inner Galaxy is considerably less known and we rewrite the normalisation of (4.150) as [16]

$$B_0 \equiv B_\odot[1 + K\ \Theta(R_{IG} - r)] \tag{4.151}$$

where $\Theta(R_{IG} - r)$ is the unit step function as a function of $r = \sqrt{\rho^2 + z^2}$. With this change, while leaving $B_\odot$ unchanged *locally* we allow for the magnetic field to have a higher effective normalization $B_\odot\ (1 + K)$ in the inner Galaxy. We arbitrary set $R_{IG} = 2$ kpc, in order to cover our ROI. We therefore fix the spatial dependence of the B field and explore the impact of overall normalisation $B_\odot\ (1 + K)$ in the inner Galaxy, on synchrotron fluxes in section 4.11.4.

Contributions to the energy losses for electrons are assumed to come only from synchrotron and IC processes in our semi-analytical approach, as commented above. In principle, the radiation density $U_{\rm rad}$ (see Eq. (4.146)) has a spatial profile, which affects the synchrotron flux estimations. However, in the semi-analytical estimations used here we take $U_{\rm rad}$ to be constant, which turns out to be a good approximation for the particular study performed in this work, as is justified in the next section. For the value of $U_{\rm rad}$ we take 8 eV cm^{-3}, which is approximately the value read off the ISRF maps used by `GALPROP` in our ROI. [17]

Concerning the frequency of observation we focus on the data taken at 45 MHz. In [**?**] it was shown that in a wide range of frequencies (22 to 1420 MHz) the change in the measured synchrotron flux is very small, while going to lower frequencies maximises a synchrotron signal of low mass WIMPs. The RMS temperature noise of this survey is 3500 K, which is generally subdominant with respect to the systematics errors involved in theoretical modeling.

[16] following [227].

[17] Note however that the conditions in the inner Galaxy might be quite different from a simple CR propagation setup assumed here. In particular, observations of the bubbles like structures Centerd at the Galactic centre and extending to 50 deg in latitudes in gamma rays [228, 229] and WMAP haze at microwave frequencies, observed by WMAP [230] and Planck [231], witness of possibly more complicated configuration of the magnetic fields and CR propagation parameters in that region. While bubble-like structures appear sub-dominant with respect to the standard components of the diffuse emission in our ROI and electron energies of interest, we caution that before their origin is understood, the actual structure of magnetic fields or the CR propagation conditions, cannot be reliably modeled.

Synchrotron signal for different choices of DM density profile

In the remainder of the text we will focus on two DM profiles, NFW, eq. (4.147) and isothermal (ISO) Eq. (4.149), using the following values of parameters, consistent with observations: $\rho_s = 0.31$ GeV cm^{-3}, $r_s = 21$ kpc, for an NFW profile, and $\rho_s = 1.53$ GeV cm^{-3} and $r_s = 5$ kpc for Isothermal profile[18], [**?**]. However, in this section we also show the prediction for the synchrotron signal in the case of a modified Einasto profile. In particular, this profile is modified to have a shallower inner slope than the usual Einasto profile found in DM-only simulations and it describes better results of simulations which include baryonic feedback (parameters we use are $\alpha = 0.11$, $r_s = 35.24$ kpc, $\rho_s = 0.041$ GeV cm^{-3} [217]). As this profile is 'bulkier' at distances 1 kpc from the GC, the DM signals are generally higher than those of NFW in that region.

In the parameter sets we chose, the local value of DM density is set to $\rho_\odot = 0.43$ GeV cm^{-3}. One should also keep in mind that the overall normalisation of DM distribution $\rho_\odot$ is uncertain, being in the $(0.43 \pm 0.113 \pm 0.096)$ GeV cm^{-3} range[19] [232]. Therefore, in addition to the differences in the signal caused by the DM profile shape, and shown in Figure 4.28, synchrotron signals scale with $\rho_\odot^2$.

Figure 4.28: Comparison between synchrotron signals for a DM mass of 10 GeV annihilating to muons (top) and b quarks (bottom figure) for three DM density profiles: modified Einasto, NFW and ISO thermal profile. $\rho_\odot = 0.43$ GeV cm^{-3} is assumed for this plot (see text for the remaining parameters) and propagation of electrons is done using a CR propagation setup as shown in Table 4.9.

Synchrotron signal for different choices of CR parameters

As discussed above, MED/MIN/MAX sets of CR propagation parameters were derived using a semi-analytical description of CR propagation, and a fit to B/C measurement, with a requirement to produce minimal, medium, and maximal DM generated anti-proton fluxes, at a Solar position. These models were recently reanalysed in [233], where whole sky radio data together with B/C measurements where considered, testing therefore impact of these

[18]Note that we make a conservative choice by choosing a rather extended core. Smaller values of r_s would result in fluxes more similar to those obtained with the NFW profile.

[19]However, depending on the analysis, the uncertainty window on this value can vary in the $0.2 - 0.8$ GeV cm^{-3} range.

parameters on electron population, and its synchrotron emission in various regions in the Galaxy. This work concludes that both MIN and MAX models are disfavored by radio data towards the galactic anti-centre. Demanding consistency with B/C nuclei data, it is found that small halo sizes $L_h \sim 1$ kpc are essentially excluded (see also [199]), but also large values $L_h \gtrsim 15$ kpc show some tension with radio data, arguing that MIN/MAX sets of parameters present somewhat extreme choices when compared to observations complementary to CR nuclei data.

In parallel to the above mentioned analysis based on semi-analytical approaches, we also use results of three analysis based on a numerical calculation with the `GALPROP` code [197–199]. In [197] the `GALPROP` code is used to derive a set of CR parameters which provide a good fit to the CR data, and at the same time reproduce the gamma-ray data well. In a parallel work, [198], a full Bayesian analysis of a fit of CR models is made using `GALPROP` to confront CR data and to derive the best fit model and its scatter. It is shown that gamma-ray predictions of such model are consistent with the *Fermi* LAT gamma-ray measurement. We will use the best fit model from [198], and few models within a 2 sigma scatter of the Bayesian analysis, see Table 4.9. We show a comparison of synchrotron signal calculated with these choices of CR propagation parameters in Fig.4.29. The `GALPROP` based parameter sets in general have lower values of parameters δ and consequently higher values K_0 are obtained in a fit to B/C data. That in turn means that diffusion is higher in this set of models, *i.e.* electrons diffuse out of our ROI, and contribute instead more at higher latitudes, as seen in Fig. 4.29.

A study in [199] has shown, however that some models with re-acceleration ($v_a \neq 0$, similar to the case 1a above) are in tension with the large set of radio data. The main reason for this is that production of CR secondary electrons is enhanced in this case and produces too strong radio emission at low frequencies. We therefore consider also a plain diffusion model (PD) often used in literature and shown to be consistent with the radio data in [199].

Figure 4.29: Comparison between synchrotron signals for a DM mass of 10 GeV annihilating to muons (top) and b quarks (bottom figure) for different CR propagation setups, detailed in Table 4.9. NFW DM profile is assumed here.

In the remainder of the text we will adopt the MED model together with the NFW profile and $\rho_\odot = 0.43$ GeV cm^{-3} as an moderately optimistic scenario, and the '1a', `GALPROP` based CR propagation model, together with the Isothermal profile and $\rho_\odot = 0.3$ GeV cm^{-3} as a

moderately conservative setup.

Synchrotron signal for different choices of magnetic field

In this section we study how DM limits change depending on the assumptions of the overall normalisation of the magnetic field in our ROI. It has already been noticed (see e.g. [534]) that for a fixed electron injection spectrum there exists an optimum value for the magnetic field which maximises the synchrotron flux at a given synchrotron frequency. However in this section we want to understand this fact in more detail.

As can be seen in Eq. 4.144, the magnetic field influences the flux through the energy losses, and through the electron energy. In principle, the electron spectrum $dN(E_s)/dE$ affects the maximisation of the flux with respect to the magnetic field B. A way to see this is the following: in the assumption of considering a one-to-one correspondence between the emitted synchrotron frequency ν and the electron energy E_c, (see Eq. 4.142), for a given value of the magnetic field B, the correspondent E_c could be disfavored (or not available) by the electron spectrum produced by the DM. However, as in this study we are going to work with synchrotron data with frequency of 45 MHz, and magnetic fields $\gtrsim 6$ μG, the electron energies producing those desired frequencies are always smaller than 1 GeV, which can perfectly be produced by our DM candidates in the range [1-200] GeV. So in practice, for our analysis, the only dependence on B that matters are encoded in the energy losses. Indeed, given a specific frequency ν, and a given annihilation channel, it can be expressed in the following form:

$$F(\nu, B) \propto \left(\frac{B^2}{\alpha + B^2}\right) \frac{1}{\sqrt{B}} \ , \tag{4.152}$$

where α represents here the rest of energy-losses, here assumed to be only IC. Note that since both synchrotron and IC losses scale with energy as E^2, the energy dependence cancels in this particular analysis. From (4.152) one observes two extremal cases: one in which synchrotron loss is negligible with respect to the rest of energy losses, for which the flux scales with B as $F \sim B^{3/2}$, thus increasing as B increases, and the other, in which synchrotron is actually the dominant energy loss, after which the flux scales as $F \sim 1/\sqrt{B}$, thus decreasing as B increases. In other words, there will be an intermediate value of the magnetic field for which synchrotron becomes the dominant energy loss, and this value is actually the one maximising the flux.

Figure 4.30 shows the shape of synchrotron flux as a function of the value of magnetic field at GC. Taking into account only IC (apart from synchrotron of course), one can have an idea about the maximum of the flux already by direct differentiation of (4.152), assuming the values of α correspondent to this case. The value of B for which the flux is maximal scales as $B_{\rm GC}^{\rm max} \propto \sqrt{U_{\rm rad}}$. For $U_{\rm rad} = 8$ eV/cm^3, $B_{\rm GC}^{\rm max} \simeq 26$ μG.

In the following we will show DM limits using two cases for magnetic field configurations. The first one, a standard $B_{\rm GC} \simeq 10$ μG value in our region of interest is assumed, and the second case will be the normalisation which maximises the synchrotron signal, *i.e.* $B_{\rm GC} \simeq 26$ μG.

Figure 4.30: Flux predicted at 10 deg off the GC, as a function of the value of magnetic field at the GC (roughly, at our ROI, we have $B_{\rm ROI} \approx 0.5 B_{\rm GC}$). Lines represent the results in the Semi-Analytical approach. We assume a DM annihilating directly to electrons, using the NFW profile and the MED diffusion model

4.11.5 Applying synchrotron constraints on dark matter models**

Effective operator approach

The more effective approach to study dark matter coupling to the SM particles is to introduce generic operators with a effective scales Λ_i (see section 7 for details) Introducing new physics scales Λ_f one can define effective couplings between (*e.g.* fermionic) DM to SM fermions f by

$$\mathcal{O}_i^f = \frac{1}{\Lambda_f^2}(\bar{\chi}\Gamma_i\chi)(\bar{f}\Gamma^i f) \ , \tag{4.153}$$

where Γ_i =$(1; \gamma_\mu;\ \gamma_5\gamma_\mu)$ for scalar ($\mathcal{O}_S^f$), vector ($\mathcal{O}_V^f$) or axial ($\mathcal{O}_A^f$) operators respectively. An upper limit on the synchrotron flux is thus giving a lower limit on the Λ_f scales which can be understood as the scale "beyond the standard model" in a sense. As an illustration, we show in Fig. 4.31 the limits we obtained on the leptonic and hadronic effective scales, Λ_l and Λ_q, obtained by LEP and CMS as function of the DM mass m_χ in the case of scalar/vectorial interaction with universal leptonic/hadronic coupling (see section 7.1 for details). We notice that the bounds on $\Lambda_{f=l,q}$ become weaker as DM mass increases because of the phase space reduction, up to some point in which it is kinematically impossible - given the energies of the experiment- to produce the pair of DM particles (the threshold is of course higher in LHC experiments than at LEP). The lower bounds on $\Lambda_l(q)$ is around 300 GeV (600 GeV) and relatively independent of the DM mass as one measures missing energy. It becomes then interesting to compare this limit to the one derived from synchrotron

Figure 4.31: Lower bounds on the effective scales $\Lambda_{l,q}$ from some collider studies, as well as from synchrotron data for a certain choice of parameters (see text for details). "CMS(had)" means CMS bounds to quark operators. Solid lines are bounds from synchrotron radiation data, while dashed lines are bounds from LHC (CMS) or LEP. Red lines correspond to quark operators while black lines correspond to leptonic ones.

emission. These bounds obtained by the LEP and CMS can also be converted directly into upper bounds on the leptonic/hadronic annihilation cross-section $\langle \sigma v \rangle_{ee/qq}$ from the inverse processes ($\bar{\chi}\chi \to \bar{e}e/qq$), generated by the operators defined in Eq. 4.153
The bounds on Λ_i has been obtained from synchrotron data at 45 MHz with a 26 and 10 μG magnetic field in our ROI, with the hypothesis that the dark matter signal lies within 5% of uncertainty of the background, and using the NFW+MED set-up. We clearly see that the synchrotron radiation, with the present data, can already give stronger limit on the effective scales, independently on the value of the magnetic field, and is complementary to the bounds from accelerator searches. Indeed, whereas there are no "threshold effect" for synchrotron radiation at large DM mass, one notice that for low masses, in the hadronic channel the bounds becomes weaker.

Higgs portal

Higgs portal is a model where the particle communicating between the dark matter candidate and the standard model ones is the Higgs boson (see section **??** for more detail on the model). To be conservative, we concentrate in this section, as an illustrative example, on the case with scalar dark matter (vectorial or fermionic species would give even stronger constraints) In this case, the mediator is the Higgs boson itself, and the model is usually called the "Higgs portal". Although it is logically possible to generalize this scenario to more than one singlet, the simplest case of one singlet already provides a very useful framework to analyse

the generic implications of an augmented scalar sector. The most general renormalisable potential involving the SM Higgs doublet H and the singlet S is

$$\begin{aligned}\mathcal{L} \supset & -\frac{\mu_S^2}{2}S^2 - \frac{\lambda_S}{4}S^4 - \frac{\lambda_{HS}}{4}S^2 H^\dagger H \\ & - \frac{\kappa_1}{2}H^\dagger H S - \frac{\kappa_3}{3}S^3 \end{aligned} \tag{4.154}$$

We show in Figure 4.32 the synchrotron flux in our ROI expected for the Higgs-portal model for two different representative values of the magnetic fields (10 μG and 26 μG) as a function of the dark matter mass, for a Higgs of 125 GeV and candidates that respect the WMAP constraint (at 5σ). We also represented the limits we obtain based on the 45 MHz radio data in the two cases discussed above: when we supposed that all the signal is due to synchrotron radiation (labelled "no backgr", extremely conservative case A), and when we allowed the signal to lie within the 5% of uncertainties due to modelling of an astrophysical background of standard origin (labelled "5% background, less conservative case B). We do this for two typical astrophysical setups, as studied above in the paper: a NFW dark matter profile, with MED diffusion model and a local normalisation of a DM density of $\rho_\odot = 0.43$ GeVcm^{-3}, and on the other hand, an Isothermal profile, with diffusion model "1a" [20] (see Table I) and a local normalisation of a DM density of $\rho_\odot = 0.3$ GeVcm^{-3}.

For the (NFW + MED) setup, we observe that, in the more conservative case, dark matter masses below $\simeq 20$ GeV are already close to be excluded for small values of the magnetic fields ($B = 10$ μG), and even DM masses below 30 GeV are excluded for $B = 26$ μG. This result is better that the one can obtain based on the *Fermi* LAT gamma–ray and PAMELA positron data [252, 253], which is one of the important results of our analysis. Indeed, dwarf galaxies study exclude only $m_s \lesssim 10$ GeV [254]. Also, with the same assumption on the CR propagation model (MED), one can see that the current synchrotron constraints already reach a higher level of precision than those based on the PAMELA positron data (see Fig. 2 of [253]) and gives even better result than the projected AMS–02 sensitivity (see Fig. 3 of [253]) if the DM signal lies in the 10% uncertainty of the background. The exclusion by synchrotron emission is also similar to the one sets by the *Fermi* LAT data on galactic gamma–rays [252].

In case B, allowing the DM signal to stay within 5% uncertainty, we observe that mainly all the parameter space of the model is excluded for masses lighter than 100 GeV, except in two narrow regions, namely near the Higgs pole ($2m_s \simeq m_H$), and just above the $W^\pm$ threshold ($m_S \simeq 80$ GeV). Indeed, in a region around the pole, the enhancement of the cross section due to the Breit-Wigner form of the amplitude implies that a very low value of $\mathcal{C}_{HSS}$ is required to match the WMAP constraint, $\langle \sigma v \rangle \sim 3 \times 10^{-26}$ cm^{3}s^{-1}, as the amplitude is proportional to

$$\mathcal{M} \propto \frac{\mathcal{C}_{HSS}}{E_{CM}^2 - m_H^2 + i\Gamma_H m_H}$$

with E_{CM} being the CM energy of the annihilating S pair. However, the synchrotron radiation produced in the Galaxy comes from S essentially at rest, whereas its kinetic energy at

[20] Those models, where Alfvén velocity can not be neglected – at least for our energies of interest-, can not be studied directly by the Semi-Analytical approach followed here. Instead, we use the full numerical analysis to estimate suppression factors in every annihilation channel, and from there we derive the new bounds.

Figure 4.32: Synchrotron flux at 45 MHz produced from the Higgs-portal model. For every $m_{\rm DM}$, the coupling $\lambda_{\rm HSS}$ is scanned while requiring that the relic abundance respects WMAP constraint at 5σ. We presented the result for a magnetic field profile normalised to 26μG (red, orange) and 10μG (green, light green). Dashed line shows bound coming from data, assuming no background; whereas dotted line shows the bound assuming a full background with uncertainties of 5% (see text for details). Red and green bands are for NFW+MED+$\rho_\odot = 0.43$ GeV cm^{-3} astrophysical setup, while light-green and orange correspond to ISO+1a model+$\rho_\odot = 0.3$ GeV cm^{-3}.

freeze-out was about $T \simeq m/25$. Consequently, for annihilation in the Galaxy, the points respecting WMAP are away from the pole and the enhancement is not sufficient to counterbalance the small value of $\mathcal{C}_{HSS}$: the annihilation cross section is $\langle \sigma v \rangle_{v \simeq 200 \rm kms^{-1}} \simeq 10^{-29} \rm cm^3 s^{-1}$. The synchrotron flux is thus largely reduced in this region of the parameter space, explaining the first dip seen in Fig. 4.32.

The second dip seen in Fig. 4.32 is also due for kinetic reasons. Indeed, as soon as the $W^\pm$ channel is open, the $W^\pm$ final state is by far the dominant one due to the large gauge coupling of the W's to the Higgs. Thus, for singlet candidate with a mass slightly below $M_{W^\pm}$, its kinetic energy at the freeze-out temperature allows its annihilation into $W^\pm$, even for small values of $\mathcal{C}_{HSS}$, as the process is dominated by the gauge interaction and not the Yukawa ones (the $b\bar{b}$ final state is the dominant one for $m_s \lesssim M_{W\pm}$). However, nowadays, such slightly lighter singlet cannot annihilate into $W^\pm$, as its kinetic energy has dropped below the threshold. Consequently, its main decay in the Galaxy is into $b\bar{b}$ pairs, but with a small value of $\mathcal{C}_{HSS}$, again giving a very low synchrotron emission. These two narrow regions

are thus very difficult to observe with this type of signal. We should perhaps emphasize that the arguments we gave above are valid for any kind of, indirect detection prospect (γ or antimatter) and is not specific to synchrotron.

Z' portal

Neutral gauge sectors with an additional dark $U'(1)$ symmetry in addition to the Standard Model (SM) hypercharge $U(1)_Y$ and an associated Z' are among the best motivated extensions of the SM, and give the possibility that a dark matter candidate lies within this new gauge sector of the theory. Extra gauge symmetries are predicted in most Grand Unified Theories (GUTs) and appear systematically in string constructions. Larger groups than $SU(5)$ or $SO(10)$ allow the SM gauge group and $U(1)'$ to be embedded into bigger GUT groups. Brane–world $U'(1)$s are special compared to GUT $U'(1)$'s because there is no reason for the SM particles to be charged under them; for a review of the phenomenology of the extra $U'(1)$s generated in such scenarios see e.g. [592]. In such framework, the extra Z' gauge boson would act as a portal between the "dark world" (particles not charged under the SM gauge group) and the "visible" sector. See section **??** for more details.

We show in Figure 4.33 the synchrotron flux emitted at 45 MHz for different values of Z' masses after a scan on $(m_{DM} = m_{\psi_0}; \delta)$ for points respecting the WMAP and electroweak precision tests (including Z width, ρ parameter $g-2$ and atomic parity violation constraint). We do this exercise for the astrophysical NFW+MED setup. We first observe a similar behaviour to that in the Higgs portal around two poles, M_Z and $M_{Z'}$. Indeed, the two channels giving a good relic density are the $s-$channel exchange of the Z' bosons for $m_{DM} \simeq M_{Z'}/2$. We have restricted the fluxes for each $M_{Z'}$ in windows to avoid juxtapositions of the fluxes. However, when $M_{Z'} \simeq 200$ GeV the value of the flux begins to be quite weak for regions of other parameter space far from the pole region, due to the reduced factor in the amplitude square of the $s-$channel process $\bar{\psi}_0\ \psi_0 \to Z' \to SM\ SM$ $|\mathcal{M}|^2 \propto 1/M_{Z'}^4$ which is almost a factor 1/6 compared to a 125 GeV Higgs exchange, factor that we recover in the synchrotron fluxes. In this case one needs a precision better than 5% to be able to distinguish a dark matter signal if $m_{DM} \gtrsim 120$ GeV.
One observes that the conclusions we have obtained for the Higgs portal are also quite valid for the Z' portal too, except that there are different fine-tuned pole regions for each mass of the Z'. The fluxes in both cases lie in a region of parameter space between 5% to 100% of the measured flux and could thus be probed in a near future.

4.12 Sommerfeld enhancement

4.12.1 Generalities

The Sommerfeld enhancement is an elementary effect in nonrelativistic quantum mechanics, which account for the effect of a potential on the interaction cross section. The Sommerfeld enhancement is named after Sommerfeld who proposed it in 1931 [261]. More technical details are developped in section (C.9). The application of this effect has been used extensively in the dark matter sector in [262], mainly to explain the positron excess observed by PAMELA.

Figure 4.33: Top) Scatter plot of predictions for synchrotron flux coming from a Z'-portal model defined in (D.13). For every $m_{\rm DM}$, the coupling δ is scanned while requiring good relic abundance, for different Z' masses, $m_{Z'}$. A frequency of 45 MHz was used, assuming a magnetic field profile normalised to 26μG (dark colours) and 10μG (light colours). Dashed line shows bound coming from data, assuming no background; whereas dotted line shows the bound assuming a full background with uncertainties of 5% at 95%CL.

We can find a gravitational classical analogy of this quantum effect. Indeed, if a meteor of mass m approach the earth of mass M with a velocity v, the lower is the velocity, the more chance it has to enter into collision with it as the gravitational attraction will compensate the kinetic energy of the meteor as we can see in the Fig.(4.34). Without any force of gravity, the cross section is simply $\sigma_0 = \pi R$, R being the radius of the earth. By conservation of angular momentum we can write $L_i = L_f$ with

$$L_i = |\vec{OM} \wedge m\vec{v}| = mhv, \quad |\vec{OM}_1 \wedge m\vec{v_1}| = mRv_1 \quad \Rightarrow \quad v_1 = \frac{vh}{R} \tag{4.155}$$

where h is the impact parameter and R the radius of the earth (see the figure). The conservation of energy gives

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 - \frac{G_N M m}{R} \quad \Rightarrow \quad v_1^2 = v^2 + \frac{2G_N M}{R} \tag{4.156}$$

G_N being the newton gravitational constant. Combining Eqs.(4.155) and (4.156) we obtain

Figure 4.34: Classical manifestation of the Sommerfeld enhancement in the case of a meteor hitting the earth.

$$h^2 = R^2\left(1 + \frac{2G_N M}{v^2 R}\right) \quad \Rightarrow \quad \sigma = \pi h^2 = \sigma_0\left(1 + \frac{2G_N M}{v^2 R}\right) \tag{4.157}$$

For very small values of v, there is a large enhancement of the cross section due to the gravity. Even if the correction vanishes as gravity is switched off ($G_N \to 0$), the expansion parameter is $2G_N M/(Rv^2)$ which can become large at small velocity.

4.12.2 Solving the Schrodinger's equation

Before reading this paragraph, we advice the reader to read the section (C.9) to remind some basic structures of the solutions of Schrodinger's equation. We are considering the solution to Schrodinger's equation for scattering of an incoming plane wave in the z-direction $e^{ikz} = e^{ikr\cos\theta}$ by a potential localized in a region near the origin, so that the total wave function beyond the range of the potential has the form[21]

$$\psi_k = e^{ikr\cos\theta} + f(\theta)\frac{e^{ikr}}{r}. \tag{4.158}$$

The overall normalization is of no concern, we are only interested in the *fraction* of the ingoing wave that is scattered. Since the interaction or the annihilation is taking place locally, near $r = 0$, the only effect of a perturbation V to the Schrodinger's equation described in section (C.9) is to change the value of the modulus of the wave–function at the origin relative to its unperturbed value. Then we can write

$$\sigma = \sigma_0 S_k \tag{4.159}$$

[21]The symmetry around the azymuthal axis imposes $f(\theta, \phi) = f(\theta)$.

where the Sommerfeld enhancement factor S is simply

$$S_k = \frac{|\psi_k(0)|^2}{|\psi_k^{(0)}(0)|^2} = |\psi_k(0)|^2 \tag{4.160}$$

One can wonder why using ingoing plane wave following the z-axis and not directly a spherical plane wave. Indeed, many potentials in nature are spherically symmetric, or nearly so, and from a theorists point of view it would be nice if the experimentalists could exploit this symmetry by arranging to send in spherical waves corresponding to different angular momenta rather than breaking the symmetry by choosing a particular direction. Unfortunately, this is difficult to arrange, and we must be satisfied with the remaining azimuthal symmetry of rotations about the ingoing beam direction. In fact, though, a full analysis of the outgoing scattered waves from an ingoing plane wave yields the same information as would spherical wave scattering. This is because a generic wave can actually be written as a *sum over spherical waves*:

$$\psi_k = \sum_l A_l \; P_l(\cos\theta) \; R_{kl}(r) \tag{4.161}$$

Note that the A_l coefficients are independent on ϕ as they correspond to the $m = 0$ solution of the $Y_{lm}(\theta, \phi)$ eigenvector of the $\hat{l}$ operator (see section (C.9) for more details). To determine the coefficients A_l, we use the fact that the combination $A_l \times P_l(\cos\theta)$ is independent of the radius r. Thus, we can compute them for large values of r where the influence of the potential $V(r)$ is negligible and $\psi_k \simeq e^{ikz}$ is a plane wave. We will then look at the influence of $V(r)$ for small values of the radius r, keeping the values of A_l obtained for $r \to \infty$. We use the asymptotic expansion of e^{ikz} and indentify it with the asymptotic Schrodinger solution for ψ_k with a null potential[22] for $R_{kl}(r) = R^0_{kl}(r)$ obtained[23] in Eq.(C.150) and using the fact that for large r, $\delta_l \ll kr$

$$\begin{aligned} e^{ikz} &= \frac{1}{2ikr} \sum_l (2l+1) P_l(\cos\theta)[e^{ikr} - (-1)^l e^{-ikr}] \simeq \sum_l A_l \; P_l(\cos\theta) \; \frac{1}{r} \sin(kr - l\frac{\pi}{2} + \delta_l) \\ \Rightarrow \quad A_l &= \frac{1}{k} i^l (2l+1) e^{-i\delta_l}. \end{aligned} \tag{4.162}$$

It is now very simple to determine $\psi_k(0)$ and then the Sommerfeld factor (4.160) If the potential $V(r)$ does not blow faster than $1/r$ near $r \simeq 0$, then we can ignore it relative to the kinetic terms, and we have $R_{kl}(r) \propto r^l \to 0$ as we showed in Eq.(C.148). As a consequence, all but the term with a null momentum $l = 0$ vanish at the origin.

$$S_k = \left| \frac{R_{k,0}(0)}{k} \right|^2 \tag{4.163}$$

with $R_{k0}(r) = \chi_k(r)/r$ solution of Eq.(C.151) with $l = 0$

[22] supposing the potential $V(r) \to 0$ when $r \to \infty$.

[23] Note the change or normalization by multiplying R^0_{kl} by the momentum k.

Figure 4.35: Sommerfeld enhancement as function of v/c in the case of an attractive Coulomb potential $V(r) = -\frac{\alpha}{r}$ for different values of α.

$$\frac{d^2\chi_k(r)}{dr^2} + \Big[k^2 - 2MV(r)\Big]\chi_k(r) = 0. \tag{4.164}$$

with the normalization condition $\chi_k(r) \to \sin(kr+\delta_0) \simeq \sin(kr)$ when $r \to \infty$, corresponding to the solution of a free particle obtained in Eq.(C.146) as $V(r)$ has no influence anymore when $r \to \infty$. As we noticed, $R_{k0} \to$ constant when $r \to 0$, one deduce $\chi_k(r) \to 0$ when $r \to 0$, implying $\chi_k(r) \simeq r\frac{d\chi_k(r)}{dr}$ when $r \simeq 0$. We then can express the Sommerfeld factor

$$S_k = \left|\frac{\frac{d\chi_k}{dr}(0)}{k}\right|^2 \tag{4.165}$$

with the boundary conditions $\chi(r) \simeq \sin kr,\ r \to \infty$ and $\chi(0) = 0$. We can check that for a null potential, $\chi_k^0 = A\sin(kr)$, with $A = 1$ by boundary condition. Then $(\chi_k^0)' = k$, and $S_k = 1$ as we expected.

4.12.3 The Coulomb potential

Let us begin to compute the Sommerfeld enhancement for the attractive Coulomb potential $V(r) = -\frac{\alpha}{r}$. The equation 4.164 can then be written

$$\frac{d^2\chi}{dx^2} + \frac{v^2}{\alpha^2}\,\chi + \frac{2}{x}\,\chi - \frac{l(l+1)}{x^2}\,\chi = 0 \tag{4.166}$$

Figure 4.36: Sommerfeld enhancement as function of the mass of the dark matter M in the case of an attractive Yukawa potential $V(r) = -\frac{\alpha e^{-m_\phi r}}{r}$ for $\alpha = 0.1$ and $m_\phi = 10$ GeV vor different values of v/c.

with $x = r \times \alpha M$. The solutions for this equation are usually expressed in terms of confluent hypergeometric functions. Its solution is the regular Coulomb Wave function which play an important role in various problems in quantum mechanics and can be expressed, for $l = 0$:

$$\chi(x) = \frac{-2\pi\frac{\alpha}{v}}{(e^{-2\pi\frac{\alpha}{v}} - 1)}\frac{v}{\alpha}x\left(1 + \sum_i A_i x^i\right) \tag{4.167}$$

A_i being coefficients of the order of unity, which does not have influence for the Sommerfeld enhancement as $x,\ r \to 0$. Applying the solution of (4.167) to Eq.(4.165) we obtain

$$S_k = \left|\frac{2\pi\frac{\alpha}{v}}{1 - e^{-2\pi\frac{\alpha}{v}}}\right|^2 \tag{4.168}$$

Note that as $v \to \infty$, $S_k \to 1$, which means that the high velocity regime does not affect the interaction cross section. However, as $v \to 0$ $S_k \to 2\pi\frac{\alpha}{v}$ and what is named Sommerfeld enhancement. We show in Fig.(4.36) the evolution of the Sommerfeld enhancement in the case of the attractive Coulomb interaction for different values of the coupling α. We clearly see that the effect can enhance the interaction or annihilation cross section of orders of magnitude. We also notice that the enhancement is independent on the mass of the interacting particle.

4.12.4 The Yukawa interaction

In the precedding Coulomb potential, generated by electromagnetic force, the particles exchanged are massless photons. In general, the force exchanging particles ϕ can be massive. This interaction is described by a Yukawa potential (see sections G.13 and H.6 for historical insights) :

$$V(r) = -\frac{\alpha}{r} e^{-m_\phi r} \tag{4.169}$$

where m_ϕ is the mass of the exchanged particle. The Yukawa potential is clearly a generalization of the Coulomb potential that we recover for $m_\phi = 0$. The Schrodinger equation then becomes

$$\frac{d^2\chi}{dx^2} + \frac{v^2}{\alpha^2}\,\chi + \frac{2e^{-\frac{m_\phi x}{\alpha M}}}{x}\,\chi - \frac{l(l+1)}{x^2}\,\chi = 0 \tag{4.170}$$

The above equation (4.170) does not possess analytical solution. A numerical solution exists and has been studied extensively in the literature. However, it is possible to give an approximate analytical expression if one approximate the Yukawa potential by the Hulthen potential :

$$V(r) \simeq V_H(r) = \frac{\alpha\delta e^{-\delta r}}{1 - e^{-\delta r}} \tag{4.171}$$

One can find the best choice of δ to best reproduce the Yukawa potential. It was found in [263] that the more reasonable value for δ is

$$\delta = \frac{\pi^2 m_\phi}{6} \tag{4.172}$$

Approximating also $\frac{l(l+1)}{Mr^2}$ by $\frac{l(l+1)}{M}\frac{\delta^2 e^{-\delta r}}{(1-e^{-\delta r})^2}$ we can derivate an analytical solution of Eq.(4.170) :

$$S_k \simeq \left(\frac{\pi\alpha}{v}\right) \frac{\sinh\left(\frac{12vM}{\pi m_\phi}\right)}{\cosh\left(\frac{12vM}{\pi m_\phi}\right) - \cos\left(2\pi\sqrt{\frac{6\alpha M}{\pi^2 m_\phi} - \frac{36v^2M^2}{\pi^4 m_\phi^2}}\right)} \tag{4.173}$$

We want to underline that the obtention of such result is far from being straightforward and need a solid algebraic work. We refer the readers to [263] for the detailed derivation of the approximate formula. The advantage of having expressed the Eq.(4.173) analytically is that its utilization is straightforward and avoid any computing consuming time. We can notice that we recover the Coulomb limit in the case $m_\phi \to 0$. Another detailed analysis of the solution in more generic cases can be found in [264].

4.13 Decaying dark matter

4.13.1 Introduction

We studied in details in the the previous section the case of annihilating dark matter, *ie* stable candidates. However, it is important to notice that there exist some possibilities of non-stable decaying dark matter. Naively speaking, one would say that if a dark matter possesses a lifetime larger than the age of the Universe (10^{17} seconds) it could fit in the picture. However, things are not so simple. First of all..

4.13.2 An exemple: decay of an Axion Like Particle (ALP)

Pion decay

An interesting exemple, to understand the phenomena, is the pion decay into two photons. We showed in section (C.3.5) that the neutral pion π^0 is a pseudoscalar meson, *ie* of an odd parity[24].

Case of an ALP dark matter

[24] we urge the reader to jump on section (C.3.5) to remind some subtleties of the parity operator.

Chapter 5

Neutrino physics

Neutrino have been around, literally, since the beginning of time. In the sweltering moments following the Big Bang, neutrinos were among the first particles to emerge from the primordial sea. A minute later, the universe had cooled enough for protons and neutrons to bind together and form atomic nuclei. Ten to twenty billion years later -today- the universe still teems with these ancient neutrinos, which outnumber protons and neutrons by roughly a billion to one. Stars such as the sun churn out more; Wolfgang Pauli himself was unknowingly awash in trillions of solar neutrinos while he was drafting his "desperate remedy" as he called himself.

We tend to think of neutrinos as transients, interacting only through the weak force and gravity and tracing long, lonely trajectories across the universe. But what they lack in strength they make up in number. Even if neutrinos were to have a mass as small as one billionth of that of a proton or neutron, their cumulative tug would be enormous, affecting the gravitational evolution of the universe as much as the normal matter we observe every day.

Concerning its mass, we saw in Eq.(2.73) that a neutrino mass of would cause our universe to contract and eventually collapse because of gravitational forces if $\sum m_\nu > 14$ eV. More recently, cosmological analysis even affirm that $\sum m_\nu < 0.3$ eV [461]. It may seem unusual to have such low values for masses of neutrinos, when all other particles like electrons, muons or quarks are much heavier, with their masses relatively closely grouped. Given that particles get mass via the Higgs mechanism, why, for example, should the electron neutrino be 10^5 times or more lighter than the electron, up and down quarks. That is, why would the coupling to the Higgs field be so many orders of magnitude less? One might not be too surprised if the Higgs coupling were zero, giving rise to zero mass. One might likewise not be too surprised if the coupling resulted in masses on the order of the Higgs, or even the GUT, symmetry breaking scale. Consider the quite reasonable possibility that after symmetry breaking, two types of neutrino exist, with one having zero mass (no Higgs coupling) and the other having (large) mass of the symmetry breaking scale. As we will see, it turns out that reasonable superpositions of these fields can result in light neutrinos (like those observed) and a very heavy neutrino (of symmetry breaking scale, and unobserved).

5.1 Historical context

5.1.1 From Chadwick to Fermi

After the discovery of the mysterious X-rays by Rontgen in 1895, a large community of physicists tried to understand the nature of the radioactivity, from Becquerel and the Curies to Rutherford. However, after that three different types of radiation were distinguished (alpha, beta and gamma), physicists did find difficulties to balance the energy account in the beta decay case. Indeed, during beta decay, a nucleus appeared to release a single particle, an electron. As the british physicists James Chadwick found in 1914, the trouble was that the energy of the electron was not always the same. Sometimes the electron emerged with little energy, while at tother times it came with a whole lot. In fact, Chadwick's laboratory measurements, confirmed by others, showed that electrons emerged with a continuous range of energies up to a certain maximum value. How could that be? If energy conservation is obeyed, the electron emitted in beta decay should have the same energy every time. Some scientists wondered wether the maximum value represented the true energy released in beta decay, but a varying fraction of it somehow vanished through an unknown process.

The problem with beta decay was so persistent and so severe that even Niels Bohr considered abandoning the hallowed principle of energy conservation altogether. As an awkward way out of the crisis, he proposed in 1930 that the conservation law might not apply strictly in the subatomic realm, but only in some average statistical sense. As he explained in a lecture in London "We may say that we have no argument, either empirical or theoretical, for upholding the energy principle in the case of beta-ray disintegration, and are even led to complications and difficulties in trying to do so." Even as Bohr acknowledged that "a radical departure from this principle would imply strange consequences," he argued that "in atomic theory, notwithstanding all the recent progress, we must still be prepared for new surprises."

Pauli and many others were skeptical of Bohr's suggestion, as they were not prepared to give up on energy conservation. "Do you intend to mistreat the poor energy law further?" Pauli prodded Bohr in a letter. True to form, he didn't stop there, teasing Bohr on another occasion. "What if someone owed you a great deal of money and offered to pay it back in installments, but each time the agreed–upon installment was not met? Would you consider this to be a statistical error or that something was missing?"

Indeed, even as he trapped with personal setbacks, Pauli was thinking long and hard about the predicament that physicists faced with beta decay. Finally he came to a bold resolution. Perhaps something was indeed unaccounted for: perhaps it all had to do with a ghostly particle that fled the scene of beta decay carrying the "missing" energy with it. To balance the books on energy and electric charge during beta decay, he worked out that such a particle would have to be lighter than an electron and neutral. Excited about his insight, Pauli wanted to share it with a group of top European physicists due to meet in the German city of Tubingen in early december of 1930. He was not willing to miss a winter ball in Zurich so he decided to send a letter to his colleague instead (letter reproduced in the historical section).

In the famous letter beginning by 'Dear Radioactive Ladies and Gentlemen," Pauli declared that he had "hit upon a desperate remedy" to salvage the law of energy conservation

in beta decay. He proposed that electrically neutral particles might exist in the nucleus, and referred to them as "neutrons." Pauli elaborated: "The continuous beta spectrum would then be understandable on the assumption that in beta decay, along with the electron, a neutron is emitted as well, in such a way that the sum of energies of the neutron and the electron is constant." Surprisingly, the usually cocky Pauli harbored some doubts about what he had sowed in the field of subatomic physics. "I have done a terrible thing. I have postulated a particle that cannot be detected. That is something no theorist should ever do," he confessed to the German astronomer Walter Baade.

In Rome, after a meeting with Pauli, Fermi concentrated on the mystery of beta decay. In the Solvay conference of 1933, he was able to formulate a clear mathematical description of beta decay within the framework of quantum mechanics, introducing the famous Fermi constant G_F. Fermi made it clear that the neutrino did not reside in the nucleus to begin with, but was created during beta decay. He compared the results of his theoretical calculations with experimental data and concluded that "the rest mass of the neutrino is either zero, or in any case very small with respect to the mass of the electron". This was the introduction of the weak force. Fermi submitted his paper to the review *Nature* in 1934. It was rejected by the editors claiming that "it contained speculations too remote from reality to be of interest to the reader". The article was finally published in the Italian review *Nuevo Cimento*.

After some years, as a result of discouraging theoretical estimates and the failure of experimental attempts, a growing chorus of skeptics doubted that the neutrino would ever be detected. Among them was the Nobel laureate Paul Dirac, who predicted the existence of antimatter. He was favorably inclined toward neutrinos at first, and praised FERMI's theory of beta decay in a letter to a colleague in 1934, in which he noted that "neutrinos seem to provide the only escape from non–conservation of energy and until something else turns up one should not be unsympathetic to them". Barely two years later, he seemed to have had a change of heart, dismissing the neutrino as an "unobservable particle".

5.1.2 From Fermi to Cowan

The next chapter of the neutrino saga was connected intimately to the harnessing of nuclear energy during the Second World War and its aftermath. Many of the physicists involved in the Manhattan Project returned to University after the tragic events. Among the few who remained at Los Alamos in the early 1950's was a thirty-three-year-old theorist by the name of Fred Reines. In 1951, Reines asked his boss at Los Alamos for a research leave to focus on fundamental physics. It occurred to him that a nuclear bomb should produce a prodigious burst of neutrinos, and could therefore come in handy for detecting these spooky particles. On average, each fission event results in six neutrinos, so the upshot is a tremendous burst of them that is released at once.

As it happened, Enrico Fermi was visiting Los Alamos in the summer of 1951, and Reines summoned up the courage to speak with him. They agreed that nuclear bomb should be the best source to detect neutrino, but could not face the technical challenge to imagine a doable experience. Later in 1951, Reines was on his way to Princeton when his plane was grounded at the Kansas City airport because of engine trouble. His Los Alamos colleague Clyde Cowan was with him. As the two stranded scientists passed the time, wandering about

the airport, they decided to work together on a problem of physics that they both found challenging. Reines proposed they focus on neutrino, and Cowan agreed.

They decided to concentrate on a particular reaction in their quest: when a proton absorbs a neutron, Fermi's theory predicts, it turns into a neutron while emitting an anti-electron, also called positron. They knew that if they detected this positron, they could establish that a neutrino was present at the scene. What they needed was a way to register the positron. Luckily for them, researchers had recently identified organic liquids that scintillate (or produce tiny flash of light) when a charged particle passes through them. So Reines and Cowan planned to fill a large vat with scintillator fluid and mount several photomultiplier tubes on the inside of its wall to record the flashes resulting from the positrons. They intended to suspend this detector from the top of a vertical shaft dug into the ground, just 130 feet from a tower on which they will explode a 20-kiloton fission bomb. As Reines later admitted in his lecture at the Nobel Prize ceremony, "the idea that such a sensitive detector could be operated in the close proximity... of the most violent explosion produced by man was somewhat bizarre, but we had worked with bombs and felt we could design an appropriate system".

With this design, Reines and Cowan could allow the detector to fall freely in a vacuum inside the shaft for a few seconds while the shock wave from the explosion passed by. Then the vat would land on a thick layer of foam rubber and feathers, which would soften its touchdown, and register the positrons emitted by the bomb's fission fragments as the fireball launched into the sky. As outlandish as their scheme might sound today, it was a clever setup, and the director of the Los Alamos laboratory gave it the go-ahead.

After Reines and Cowan had arranged for workers to come to the site and dig a massive hole and had started to build the detector, their colleague Hans Bethe asked them how they expected to distinguish a guanine neutrino event from the other radiation that he knew would be emitted by the bomb. While they tried to come up with answers to this question, Reines and Cowan arrived at an even better way to do the experiment: instead of a one–off bomb explosion, they could use a controlled nuclear reactor as the neutrino source.

Instead of registering only the positron, Reines and Cowan realized they could also measure the neutron that was produced as a result of a neutrino interacting with matter. They knew that the positron would hit an electron in the scintillator fluid, annihilating both almost instantly and producing gamma rays that the photomultiplier tubes could record. The neutron, on the other hand, would zigzag through the fluid, like a drunkard staggering through a crowd, losing their energy as it collided with one nucleus and then another, until it was eventually absorbed. The nucleus that captured the neutron would emit the excess energy as gamma rays. As reines and Cowan were aware, there is a characteristic time for the neutron's random walk: 5 microseconds. That means there is a precise delay between the two gamma ray bursts, the first one from the positron's annihilation and the second from the neutron's absorption. If their experiment recorded two flashes coming precisely 5 microseconds apart, it would constitute the unmistakable signal of a neutrino. Reines and Cowan could then conclude that flashes due to other particles hitting the detector at random times were simply background noise.

They decided to make their experience at the Savannah River Site in South Carolina. the apparatus, completed in late 1955, weighed in at some ten tons. Located in a basement below the nuclear reactor, it was shielded from neutrons produced by the reactor and from

Figure 5.1: Left : apparatus of Cowan and Reines at the Savannah River Site in 1956; Right : first detection of a neutrino measured by Cowan and Reines in 1956, with the two pulses (positron then neutron) separated by 5 microseconds.

cosmic rays (see Fig.5.1). Over five months, the scientists recorded hundreds of hours of data with the reactor turned on and off for comparison. The apparatus registered five times as many pair of flashes separated by a few microseconds when the reactor was on than when it was off. By the summer of 1956, after extensive tests and checks, the team was convinced that they had definitively detected neutrino.

Reines and Cowan then immediately telegraphed Pauli the following news: "We are happy to inform you that we have definitely detected neutrinos from fission fragments ...". The message reached Pauli at a conference in Geneva. He interrupted the proceedings and read the telegram out loud. Pauli wrote back to Reines and Cowan the next day, and, true to his character, he quipped: "Everything comes to him who knows how to wait". Reines was awarded a share of the 1995 Physics Nobel prize. Cowan had died twenty-one years earlier, so he missed out on the honor.

5.2 Then endpoint and the measurement of neutrino mass

5.2.1 The continuous spectrum

5.2.2 The KATRIN experiment

5.3 The cosmological neutrino

5.4 Astrophysical and cosmological sources of neutrino

The neutrino arriving on earth have different origins. Some are produced in the sun (*solar neutrino*) whereas other are produced via the interaction of cosmic rays, mainly composed of ultra–energetic proton, on the atmosphere (*atmospheric neutrinos*). Others can be directly

generates via ultra high energy (UHE) sources like blazars or Active Galactic Nuclei (AGN). These UHE neutrino propagates and diffuse in the Universe, pinpointing their source clearly because, contrarily to the cosmic rays produced by the same objects, neutrino are not affected by magnetic fields or the interstellar medium (ISM). Another production mechanism of neutrino is the interaction of the cosmic rays on the photons from the CMB *cosmogenic neutrino* : $p + \gamma_{\rm CMB} \to \Delta \to n + \pi^+$, and then $\pi^+ \to \mu^+ \nu_\mu$ and $\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$. This source is one of the dominant one, and produce 2 μ-types neutrino for 1 e-type one. The flavor ratio at the production is then (1:2:0), which will be an important point to distinguish them on earth. Finally, the last source of neutrino, not yet detected, is obviously the relic ones from the decoupling of the primordial plasma. They have a temperature of 1.95 K and a density $n_\nu = 109$ cm^{-3}, see section (5.6), compared to the 2.5 K and $n_\gamma = 394$ cm^{-3} of the photons from the CMB (see Eq.2.50). We will look into the details of each production and detection mechanism in this section.

5.4.1 Solar neutrinos

Electron neutrinos are produced in the Sun as a product of nuclear fusion. By far the largest fraction of neutrinos passing through the Earth are Solar neutrinos. The main contribution comes from the proton-proton reaction. The reaction is:

$$p + p \to d + e^+ + \nu_e$$

From this reaction, 86% of all solar neutrinos are produced. The deuterium will then fuse with another proton to create a ^{3}He atom and a gamma ray. This reaction can be seen as:

$$d + p \to \ ^3He + \gamma \tag{5.1}$$

The isotope ^{4}He is then produced using two ^{3}He from the previous reaction

$$^3He + \ ^3He \to \ ^4He + 2p \tag{5.2}$$

With both helium-3 and helium-4 in the system now, beryllium can be fused by the reaction of one of each helium atom as seen in the reaction:

$$^3He + \ ^4He \to \ ^7Be + \gamma \tag{5.3}$$

Since there are four protons and only three neutrons, the beryllium can go down two different paths from here. The beryllium could capture an electron and produce a lithium-7 atom and an electron neutrino. It can also capture a proton due to the abundance in a star. This will create boron-8. Both reactions are as seen below respectfully:

$$^7Be + e^- \to \ ^7Li + \nu_e \tag{5.4}$$

This reaction produces 14% of the solar neutrinos. The lithium-7 will combine with a proton to produce 2 atoms of helium-4. On the other hand, the beryllium will capture a proton to form the boron-8 following

$$^7Be + p \to \ ^8B + \gamma \tag{5.5}$$

Figure 5.2: Solar neutrinos (proton-proton chain) in the Standard Solar Model

Then, the Boron-8 will beta decay into beryllium-8 due to the extra proton which can be seen below:

$$^{8}B \rightarrow \ ^{8}Be + e^{+} + \nu_{e} \tag{5.6}$$

The reaction produces about 0.02% of the solar neutrinos, but these few solar neutrinos have the larger energies. The highest flux of solar neutrinos come directly from the proton-proton interaction, and have a low energy, up to 400 keV. From the Earth, the amount of neutrino flux at Earth is enormous (around 7×10^{10} particles $\mathrm{cm}^{-2}\mathrm{s}^{-1}$). Moreover, since neutrinos are insignificantly absorbed by the mass of the Earth, the surface area on the side of the Earth opposite the Sun receives about the same number of neutrinos as the side facing the Sun.We show the combined process from the proton-proton chain in Fig.(5.2).
One should notice that the first solar neutrino generated by proton proton fusion (constituting 99.77 % of the solar neutrinos) was observed in august 2014 by the Borexino experiment [465]. The Borexino experiment was a joint collaboration of several European (Italy, Germany, France, Poland), USA and Russian institutions, installed in the Gran Sasso Laboratory in central Italy. The reaction was harder to observe than the one generated by the Boron because of the very low energy of the escaping neutrino (around the keV compared to $\simeq 10$ MeV for the ^{8}B ones). The fusion reactions occur deep within the Sun, and which then pass right through the Sun, taking just over eight minutes to reach the Earth. On the other hand, all solar energy measurements before were based on light from the Sun's photosphere (the familiar sunlight which lights up our skies and warms the Earth). But the

Figure 5.3: The Borexino experiment (left) and the neutrino spectrum it obtains (right) combined with other neutrino experiments.

energy carried by this sunlight was produced in solar fusion reactions about 100,000 years ago[1]. Comparison between the Borexino measurement and those of the Sun's radiant energy reveals that the solar power has not changed in quite a long time. The collaboration reported spectral observations of *pp* neutrinos, demonstrating that about 99 % of the power of the Sun, (3.84×10^{33} ergs per second), is generated by the proton-proton fusion process.
We show in Fig.(5.4) the Borexino apparatus and the spectrum it observes. Notice the low energy of the *pp* generated neutrino compared to the ^{8}B ones.

5.4.2 Atmospheric neutrinos

Atmospheric neutrinos result from the interaction of cosmic rays with atomic nuclei in the Earth's atmosphere, creating showers of particles, many of which are unstable and produce neutrinos when they decay. A collaboration of particle physicists from Tata Institute of Fundamental Research (India), Osaka City University (Japan) and Durham University (UK) recorded the first cosmic ray neutrino interaction in an underground laboratory in Kolar Gold Fields in India in 1965.

5.5 Ultra High energetic neutrinos

In the galaxy, or outer space, there are different sources of ultra High Energy neutrinos (> 1000 TeV $= 1$ PeV). It ranges from cosmic rays interaction with the Inter Stellar Medium (ISM) to direct production from Super Nova remnants. In any cases, these Ultra High energy neutrino interacts with the CMB or the earth atmosphere, producing specific phenomena, like a GZK-cut or the Glashow resonance. We will develop these two effects in this section and show how one can observe them experimentally.

[1]the average time for energy to percolate from the central regions of the Sun and reach its surface

5.5.1 The GZK cut for the neutrino

5.5.2 The Glashow resonance

5.5.3 An example of detection: Icecube

5.6 Neutrino mass

5.6.1 Dirac mass

To give mass to neutrino, we face the same "issue" that we have concerning the mass term of "up" type quark. Indeed, the Lagrangian $\mathcal{L}_{down} = -\lambda_d \bar{L}\Phi d_R + h.c.$, where L represent the left doublet $(u,d)_L^T$, Φ the Higgs field and d_R the right handed part of the spinor $(d_L, d_R)^T$ [Weyl convention see section C.3.1]. However, has the Higgs field develop a *vev* v, $\Phi = (0, (v+\phi)\sqrt{2})^T$, which gives mass to down type particles like the bottom quarks, strange ones or the electrons or taus, but not to the top quarks or neutrino that are of the "up" type $SU(2)_L$. The idea is to introduce the charge conjugate of the Higgs field Φ, $\tilde{\Phi}^c = i\sigma_2\Phi^*$, σ_i being the Pauli matrices (see C.3.5). With this new field, one easily sees that the coupling $-\lambda_u \bar{L}\tilde{\Phi}^c u_R + h.c.$ generates a mass term for the up-type spinors once the Higgs develops a *vev*. The expression of the neutrino mass is then

$$\mathcal{L}_\nu^{Dirac} = -m_D\bar{\nu}\nu = -m_D(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L), \quad \text{with} \quad \nu = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \tag{5.7}$$

and $m_D = \lambda_\nu v\sqrt{2}$. We know today that neutrinos (at least some of the neutrinos) have masses ($\lesssim 0.1$ eV). We can just introduce these very small masses via the Dirac mechanism, but of course this leaves completely unexplained the smallness of such masses with respect to other fermion masses. Alternatively, one could introduce the so-called Majorana mass term as we will see below.

5.6.2 Majorana mass

Without right–handed neutrino

It is also possible to generate mass to the left handed neutrino through a Majorana mass term, without needing to ever introduce the right handed degrees of freedom. Let us first recall that a fermion field ψ possess a charge conjugate counterpart $\psi^c = C\bar{\psi}^T$ where the role of particles and antiparticles are exchanged[2]. A Majorana fermion is one where $\psi^c = \psi$, *i.e.* a Majorana fermion is its own antiparticle. Let us assume that le left handed neutrinos are particles of this kind. This is, in principle possible as they are not charged[3]. Thus let us define

$$\nu_L^c = C\bar{\nu}_L^T \tag{5.8}$$

[2]See section C.3.5 for more details.

[3]It is important to underline here that this Majorana mass term can only exist for neutrino as it is the only neutral fermion in the Standard Model, which is invariant under the electromagnetic $U(1)$ transformation. Indeed, a term of the form $\frac{1}{2}m_e^M(\bar{e}e^c + \bar{e}^c e)$ clearly violates the electric charge conservation.

PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1, 1960

Resonant Scattering of Antineutrinos

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(Received October 26, 1959)

The hypothesis of an unstable charged boson to mediate muon decay radically affects the cross section for the process $\bar{\nu}+e\rightarrow\bar{\nu}+\mu^-$ near the energy at which the intermediary may be produced. If the boson is assumed to have K-meson mass, the resonance occurs at an incident antineutrino energy of $\sim 2\times 10^{12}$ ev. The flux of energetic antineutrinos produced in association with cosmic-ray muons will then produce two muon counts per day per square meter of detector, independently of the depth and the orientation at which the experiment is performed.

THE interaction responsible for muon decay also permits an inelastic scattering of antineutrinos by electrons,

$$\bar{\nu}+e\rightarrow\bar{\nu}+\mu^-.$$

With the conventional four-Fermion form of decay interaction, the cross section for this process is

$$\sigma_0=(E/m_e)1.5\times 10^{-45}\ \text{cm}^2,$$

where E is the energy of an antineutrino incident upon a stationary electron. However, if muon decay is mediated by a charged, unstable boson, this cross section becomes radically altered. The process will occur by the sequence

$$\bar{\nu}+e\rightarrow Z^-\rightarrow\bar{\nu}+\mu^-,$$

and at some antineutrino energy there will be a resonance, occasioned by the real production of an intermediary boson. The cross section, in this case, assumes a typical resonance form,

$$\sigma=\sigma_0\frac{E_0^2}{(E-E_0)^2+\Gamma^2},$$

in which the incident antineutrino energy at the resonance is $E_0=m_Z^2/2m_e$ and Γ denotes its width, $\Gamma=(m_Z/m_e)(1/\tau_Z)$ in terms of the lifetime, τ_Z, of the Z meson. Although σ_0 is proportional to the fourth power of the coupling constant of Z mesons to leptons, the average cross section near the resonance,

$$\frac{1}{2\Delta}\int_{E_0-\Delta}^{E_0+\Delta}\sigma(E)dE\cong\frac{\pi}{4}\left(\frac{E_0}{\Delta}\right)\left(\frac{E_0}{\Gamma}\right)\sigma_0,$$

depends only upon its square. If the Z-meson mass is not much greater than that of the nucleon, this enhanced cross section is not necessarily beyond experimental reach. We shall consider only values of the Z-meson mass such that $m_K\leqslant m_Z\leqslant m_N$, since smaller values of m_Z would prohibit the use of the Z meson to mediate K-meson decays.

The principal decay modes of the Z meson are expected to be $Z^-\rightarrow e+\bar{\nu}$ and $Z^-\rightarrow\mu^-+\bar{\nu}$. With coupling strengths of the Z meson to muon and electron currents chosen equal (in accordance with universality) and of magnitude determined by the muon lifetime, we find

$$\tau_Z=(m_N/m_Z)^3 10^6 m_N^{-1}\hbar c^2\ \text{sec}.$$

With $m_Z=m_N$, the energy of the incident antineutrino energy at the resonance is 9×10^{11} ev and the width of the resonance is 2×10^6 ev, while with $m_Z=m_K$, $E_0=2.3\times 10^{11}$ ev and $\Gamma=1.5\times 10^5$ ev.

Although the natural width of the resonance is quite small, a significant broadening is produced by the spread in velocity of the target electrons. In a collision with an electron of velocity βc along the direction of incidence, the resonance occurs at the antineutrino energy

$$E_0'=(1+\beta)^{-1}E_0.$$

Thus the experimental width of the resonance will be approximately $(\delta/137)E_0$, where δ is the mean atomic number of the target material. Upon earth, antineutrinos of energies within the resonance should have a mean free path of some hundreds of kilometers, corresponding to a cross section of 10^{-32} cm^2.

The only known source of antineutrinos of sufficiently great energies is the decay of cosmic-ray pions and K mesons. Practically all such antineutrinos are produced in association with muons, consequently their intensity and energy spectrum may be deduced from the known sea-level flux of muons.[1] We estimate that at 9×10^{11} ev the antineutrino flux is 10^{-11} cm^{-2} sec^{-1} Bev^{-1}, and at 2.3×10^{11} ev it is 10^{-8} cm^{-2} sec^{-1} Bev^{-1}. Exposed to these antineutrino fluxes, each target electron will act as a source of 4×10^{-40} muon per second if $m_Z=m_N$, or 10^{-38} muon per second at the lower value of $m_Z=m_K$.

With a muon-sensitive area of one square meter, placed underground, the experimenter might anticipate a counting rate of two per day (at $m_Z=m_K$) or of 0.1 per day (at $m_Z=m_N$) independently of the depth at which the experiment is performed. The counting rate should be relatively insensitive to the orientation of the experimental apparatus with respect to the vertical, since the muons should be produced isotropically in the

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[1] A. Subramanian and S. D. Verma, Nuovo cimento 8, 572 (1959).

Figure 5.4: The original paper of Glashow (1959) computing the resonant production of a Z from neutrinos-electrons interaction.

We cannot impose $\nu_L = \nu_L^c$ for the simple reason that ν_L^c is actually a right–handed field, but we can assume that ν_L and ν_L^c actually describe the left–handed and right–handed degrees of freedom of one fermion that is identical to its antifermion, respectively. We can then construct the Lorentz invariant mass term

$$-\frac{1}{2}m_M^L \bar{\nu}_L \nu_L^c + h.c. = -\frac{1}{2}m_M^L \bar{\nu}\nu \quad \text{with} \quad \nu = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \nu^c \tag{5.9}$$

This is the so–called Majorana mass term. It is not possible to make this gauge invariant with a dimension 4 operator. Indeed, this term changes weak hypercharge by 2 units - not possible with the standard Higgs interaction, requiring the Higgs field to be extended -to include an extra triplet with weak hypercharge for instance - whereas for right-chirality neutrinos as we will see, no Higgs extensions are necessary. For both left and right chirality cases, Majorana terms violate lepton number, but possibly at a level beyond the current sensitivity of experiments to detect such violations.
It is also interesting to notice that this minimal scheme to give mass to neutrino does not advocate right handed states: is it indeed possible to give mass to neutrino (to the price of non–conserving lepton number and a Majorana nature for the neutrino). This is *the minimal scheme* which was the point of view adopted by Gribov and Pontecorvo [464].
Note that Majorana mass terms have nothing to do with the Majorana representation in spinor space. One can use any representation for the fields of which Majorana and Dirac mass terms are composed. Neither do Majorana mass terms imply the associated particles/fields are Majorana fermions, of which you may have heard. Majorana fermions are their own anti-particles. More on this in section (C.3.5). For now, we will assume that both Dirac and Majorana mass terms contain only Dirac type particles.

With right–handed neutrino

We just saw that we can write a Majorana mass term for the left handed neutrino, at the price to work in dimension 6 operators. However, if one introduce the right handed neutrino from the equation (5.7), we clearly understand that nothing forbid us to write gauge invariant dimension 4 mass operators with the right handed neutrino of the form:

$$-\frac{1}{2}m_M^R \left(\bar{\nu}_R \nu_R^c + \bar{\nu}_R^c \nu_R\right) \tag{5.10}$$

Note that in the expression (5.10) , the second term destroy a right handed particle and create a right–handed antiparticle (equivalent to a left–handed particle) as in the case of Eq.(5.7). The lepton number is however not conserved because two neutrino are created from the vacuum whereas their Dirac coupling attribute to them a leptonic number. Adding the three contributions (5.7,5.9,5.10) just discussed above one obtains

$$\begin{aligned} \mathcal{L}_\nu &= -m_D(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) - \frac{1}{2}m_M^L\left(\bar{\nu}_L\nu_L^c + \bar{\nu}_L^c\nu_L\right) - \frac{1}{2}m_M^R\left(\bar{\nu}_R\nu_R^c + \bar{\nu}_R^c\nu_R\right) \\ &= -\frac{1}{2}\left(\bar{\nu}_L^c \bar{\nu}_R\right)\mathcal{M}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c. \quad \text{with} \quad \mathcal{M} = \begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix}. \end{aligned} \tag{5.11}$$

where we used $\bar{\nu}_L\nu_R = \bar{\nu}_R^c\nu_L^c$ (Eq. C.37).

5.7 The see-saw mechanism

From the equation (5.11) one can easily diagonalize the mass matrix in the (ν_L^c, ν_R) space and then study the different possibilities or hierarchy between m_D, m_M^L and m_M^R. However, to really understand the subtleties of the mechanism one should first study simple cases.

5.7.1 A simple example

Let's suppose that the Higgs does not couple to the neutrino, in this case, the Dirac mass is absent of the potential and one can write $m_D = 0$ in (5.11). One then need to express the mass egeinstates (ν, N) as function of ν_L, ν_L^c, ν_R, ν_R^c from

$$-\frac{1}{2}(\bar{\nu}_L^c, \bar{\nu}_R) \begin{pmatrix} m_M^L & 0 \\ 0 & m_M^R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c. = -\frac{1}{2}(\bar{\nu}, \bar{N}) \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix} \tag{5.12}$$

which gives (noticing that $\bar{\nu}_R \nu_R = 0$)

$$\nu = (\nu_L + \nu_L^c) = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix}; \qquad N = (\nu_R + \nu_R^c) = \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}. \tag{5.13}$$

where we have used the Dirac representation of the spinors ν and N which are 4–dimensional objects. As we can notice, in this case both ν and N are Majorana particles ($\nu^c = \nu$ and $N^c = N$), which can justify the term "Majorana masses" for m_M^L and m_M^R. Another important point is concerning the notation (which is sometimes quite difficult to follow in the literature on the market) the notation $\begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix}$ should be read $\nu_L \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \nu_L^c \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, ν_L and ν_L^c being 4 dimensional spinors, even if as a left–handed spinor (ν_L) and right–handed spinor (ν_L^c), each of them possesses effectively only 2 degrees of freedom. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the base vectors in the plane of the P_L and P_R projectors. So, in a sense, it is formally correct to right $\nu_L = \begin{pmatrix} \nu_L \\ 0 \end{pmatrix}$, but on the left side, one should understand ν_L as a two–dimensional Weyl spinor.

5.7.2 Generalization

Now, if we assume that in the mass term enter flavor left–handed fields ν_L, right–handed sterile neutrino ν_R and the lepton number L is not conserved, we come to the most general Dirac and Majorana mass term (5.11) which can be written

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\bar{n}_L \, \mathcal{M} \, n_L^c + h.c. \quad \text{with} \quad n_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad \text{and} \quad \mathcal{M} = \begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix}.$$

We denoted n_L as it is a 2 dimensional vector (in the mass basis) only formed by left–handed spinors. Noticing that a left–chiral spinor cannot mix with another left–chiral spinor ($\nu_L \nu_L = 0$), we can write

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\bar{n}\ \mathcal{M}\ n \tag{5.14}$$

with $n = n_L + n_L^c$ which is a Majorana vector in the mass basis, which components sum of left and right chirality spinors. One then have reduced the problem of finding eigenstate into the one of diagonalizing $\mathcal{M}$ into the basis $n = \begin{pmatrix} \nu_L + \nu_L^c \\ \nu_R + \nu_R^c \end{pmatrix} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$. M being a complex symmetrical matrix, it can be diagonalized wight he help of *one unitary matrix U*. We have $\mathcal{M} = UmU^T$ with

$$m = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \tag{5.15}$$

We skip the computation of the eigenvalues which are obtained solving the characteristic equation for the matrix $\mathcal{M}$: $|\mathcal{M} - \lambda Id| = 0$, given 2 values for the second order equation[4] for λ, m_1 and m_2:

$$\begin{aligned} m_1 &= \frac{1}{2}\left[(m_M^L + m_L^R) + \sqrt{(m_M^L + m_M^R)^2 + 4(m_D^2 - m_M^L m_M^R)}\right] \\ m_2 &= \frac{1}{2}\left[(m_M^L + m_L^R) - \sqrt{(m_M^L + m_M^R)^2 + 4(m_D^2 - m_M^L m_M^R)}\right] \end{aligned} \tag{5.16}$$

5.7.3 The specific case $m_M^L = 0$

The eigenvalues

We have seen in the preceding section that the majorana mass term for the left handed neutrino should appear only at two loops order, or generated by dimension 6 operator, and so should naturally be considered as much smaller than the right handed majorana term m_M^R which can be naturally present in the lagrangian by gauge invariance. The mass values m_1 and m_1 then becomes

$$\begin{aligned} m_1 &= \frac{1}{2}\left[m_L^R + \sqrt{(m_M^R)^2 + 4m_D^2}\right] \simeq m_M^R \\ m_2 &= \frac{1}{2}\left[m_L^R - \sqrt{(m_M^R)^2 + 4m_D^2}\right] \simeq \frac{m_D^2}{m_M^R} \end{aligned} \tag{5.17}$$

where we have developed at the first order in m_D/m_M^R, supposing $m_D \ll m_M^R$. This approximation is justified in a lot of extensions of the standard model (but not all) and can be understood as the fact that the majorana mass term m_M^R being a free mass term, its natural scale should be the GUT or PLANCK scale (or at least an intermediate scale) whereas $m_D = y_{LR}v_H/\sqrt{2}$ ($v_H = 246$ GeV being the Higgs field *vev* and y_{LR} the Yukawa coupling between ν_L and ν_R, see below) is boded by the electroweak scale.

[4]Notice that one eigenvalue is negative, which is not an issue as the field can be redefined by a global phase transformation.

We then have a clear view on the famous the see-saw mechanism (and on its name). Indeed, a quick glance at Eq.(5.17) shows us that a hierarchy between m_D and m_M^R will *naturally* drive one eigenvalue toward zero, keeping the second one relatively high. In other words, after diagonalization, the left handed part of the multiplet (ν_L) will rotate and acquire a small admixture of ν_R which will generate its mass term, proportional to the admixture.

The eigenvectors

If finding the mass terms m_1 and m_2 is an easy task, one should be careful when treating the compositions on the new states N_1 and N_2, mass eigenstates of the Lagrangian and thus, the *physical* states, the only one having really a meaning in any observable computation. To find them, one has to write down the equations

$$(\bar{N}_1\ \bar{N}_2)\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = (\bar{n}_1\ \bar{n}_2)\begin{pmatrix} 0 & m_D \\ m_D & m_M^R \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{5.18}$$

with

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \mathcal{R}_\theta \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{5.19}$$

Solving the equation (5.18) with the definition (5.19) we obtain

$$\tan 2\theta = \frac{2m_D}{m_M^R} \quad \Rightarrow \quad \theta \simeq \sin\theta \simeq \frac{m_D}{m_M^R} \tag{5.20}$$

which gives

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} \simeq \begin{pmatrix} n_1 - \theta\ n_2 \\ n_2 + \theta\ n_1 \end{pmatrix} = \begin{pmatrix} \nu_L + \nu_L^c - \theta\ (\nu_R + \nu_R^c) \\ \nu_R + \nu_R^c + \theta\ (\nu_L + \nu_L^c) \end{pmatrix} \tag{5.21}$$

As we can notice, N_1 and N_2 are majorana particles ($N_i = N_i^c$). N_1 represents the *physical* neutrino of the Standard Model, whose mass is constraint to be below 1 eV, whereas N_2 would be the (non-observed) heavy states responsible of the see-saw mechanism.

5.7.4 An application: coupling to a scalar field (Majoron)

It is always possible to work in a dynamical framework where the masses are given by the breaking of a symmetry "a la Higgs", with a new heavy scalar field (majoron field S) whose *vev* generates mass to the right–handed neutrino. The Lagrangian can be written, after the breaking of symmetry (assuming typical Higgs potential in the heavy scalar sector)

$$\begin{aligned}
\mathcal{L} &= -\frac{h}{\sqrt{2}}\bar{\nu}_R^c(v_S + S)\nu_R - \frac{y_{LR}}{\sqrt{2}}\bar{\nu}_L(v_H + h)\nu_R \ + \ h.c. \\
&= -\frac{h}{\sqrt{2}}(\bar{\nu}_R^c + \bar{\nu}_R)(v_S + S)(\nu_R + \nu_R^c) - \frac{y_{LR}}{2\sqrt{2}}(\bar{\nu}_L + \bar{\nu}_L^c)(v_H + h)(\nu_R + \nu_R^c) \\
&-\frac{y_{LR}}{2\sqrt{2}}(\bar{\nu}_R + \bar{\nu}_R^c)(v_H + h)(\nu_L + \nu_L^c),
\end{aligned} \tag{5.22}$$

that we can express as function of the *physical* states N_1 and N_2

$$\begin{aligned}\mathcal{L} = & - \frac{h_1}{\sqrt{2}} S \bar{N}_2 N_2 - \frac{h_1 \theta^2}{\sqrt{2}} S \bar{N}_1 N_1 + \frac{h_1 \theta}{\sqrt{2}} S (\bar{N}_1 N_2 + \bar{N}_2 N_1) \\ & - \frac{y_{LR}}{2\sqrt{2}} h (\bar{N}_1 N_2 + \bar{N}_2 N_1) + \frac{y_{LR}\theta}{2\sqrt{2}} (\bar{N}_2 N_2 + \bar{N}_1 N_1) - m_1 \bar{N}_1 N_1 - m_2 \bar{N}_2 N_2\end{aligned} \tag{5.23}$$

with $m_1 \simeq \frac{y_{LR}^2 v_H^2}{2\sqrt{2} h_1 v_S}$ and $m_2 \simeq \sqrt{2} h_1 v_S$.
Such a dynamical model has a lot of interesting characteristics, like for instance the decay of the heavy scalar S into two (very energetic) light states N_1 which can be measured by the Icecube telescope for instance.

5.8 Neutrino oscillations

Chapter 6

Group theory beyond the Standard Model

6.1 Introduction

6.1.1 Definition

The direct product $G \otimes H$ of two groups G (elements x) and H (elements y) also form a group. The elements of such a group can be combined in a new representation (x, y) with the operation $(x_1, y_1).(x_2, y_2) = (x_1.x_2, y_1.y_2)$.

6.1.2 Lie group

6.2 Unification

6.2.1 Introduction

6.2.2 Quantization of the charge

Another problem inherent to the Standard Model is the quantization of the charge. Indeed, one notices that the hypercharge in the Standard Model is quantized (see the table F.2 in appendix). There are no reason at all that for an abelian $U(1)$ interaction, the value of the charge is quantized. Indeed, the $U(1)$ transformation of parameter θ, $e^{iQ\theta}$ transformation admit any real continuous charge Q. The situation is completely different for a non-abelian case $SU(N)$ for instance. The conserved charge Q should respect the commutation relation $[Q, T_i] = 0$ (Noether theorem) with T_i the generator of the transformation $SU(N)$. Q should then be one of the Cartan matrices. In $SU(N)$, there exist $N-1$ commuting matrices with the generators (σ_3 for $SU(2)$ and λ_3 and λ_8 in the case of $SU(3)$ Eq.6.2). It is then natural to expect that the abelian hypercharge gauge group $U(1)_Y$ is generated by the breaking of a non-abelian group $SU(N)$ or $SO(N)$, justifying the quantization of the hypercharge Y, and then as a consequence, the quantization of the electric charge $Q = T^3 - Y/2$, T^3 being the isospin eigenvalue.

6.2.3 History

Pati-Salam, Glashow, Minkowski

6.3 SU(N)

6.3.1 SU(2)

$SU(n)$ is the set of all complex $n\times n$ matrices (the n of $SU(\mathbf{n})$) M that are unitary ($M^\dagger M = 1$, the U of $S\mathbf{U}(n)$) and have determinant one (called Special matrices, the S of $\mathbf{S}U(n)$). The group operation is matrix multiplication.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & \text{-i} \\ \text{i} & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & \text{-1} \end{pmatrix} \tag{6.1}$$

6.3.2 SU(3)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & \text{-i} & 0 \\ \text{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{-1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{6.2}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & \text{-i} \\ 0 & 0 & 0 \\ \text{i} & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \text{-i} \\ 0 & \text{i} & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \text{-2} \end{pmatrix}$$

6.3.3 SU(5)

6.4 SO(N)

SO(N) is the group of orthogonal real $N \times N$ matrices, $O^T O = 1$ with $\det O = 1$. These are the matrices of the rotations. The fundamental representation is formed by matrices O themselves. The infinitesimal transformations are

$$O^{ij} = \delta^{ij} + \omega^{ij} \tag{6.3}$$

from the orthogonality condition $O^{ij}O^{kj} = \delta^{ik}$ we get $\omega^{ij} = -\omega^{ji}$, so we have $\frac{N(N-1)}{2}$ independent parameters of the group. We can easily find that number by counting the number of free places in a $N \times N$ matrix above the central diagonal : $N-1$, then $N-2$, then $N-3$... until 1, which gives the sum $\frac{N(N-1)}{2}$.
Another point of view is to consider the conservation of the length, which is the case for any rotation matrices. That means, if we define a vector $(x_1, x_2, ...x_N)$ in the basis of the generators $(\Gamma_1, \Gamma_2, ..., \Gamma_N)$, the length is define as $x_1^2 + x_2^2 + ... + x_N^2$. Asking for the constant length in the transformation, we are lead to the condition

$$x_1^2 + x_2^2 + .. + x_N^2 = (x_1\Gamma_1 + x_2\Gamma_2 + ... + x_N\Gamma_N)^2 \quad \Rightarrow \{\Gamma_a, \Gamma_b\} = \Gamma_a\Gamma_b + \Gamma_b\Gamma_a = 2\delta_{ab} \tag{6.4}$$

In other words, the requirement that the sum of the squares is equal to the square of the sums will restrict the gamma matrices to satisfy the Clifford Algebra which is obviously fulfilled by the Pauli matrices for the $SO(3)$ case as well. Theses Γ_i will generate the spinorial representation Σ_{ab} of SO(N).

6.4.1 SO(10)

6.4.2 The Yukawa sector

One needs two **10**s representations to ensure than m_u, m_ν are different from m_d, m_e (Yanagida page 137).

6.5 Discrete Z_N symmetries and dark matter stability

In the vast majority of extensions of the Standard Model, one invokes a Z_2 symmetry to justify the stability of the dark matter candidate. However, it can be proved that a discrete Z_N symmetry can be seen as the residual of a local (gauged) $U(1)$ symmetry.

6.5.1 Introductory example

In order to get a notion of how to break a continuous symmetry down to a discrete symmetry in a physical theory, the following intuitive simplest example is helpful. Consider two complex scalar fields ψ and ϕ of different charges under a common $U(1)$ gauge symmetry

$$\psi \mapsto e^{i\alpha(x)}\psi \tag{6.5}$$

$$\phi \mapsto e^{iq\alpha(x)}\phi. \tag{6.6}$$

Besides the standard renormalizable Lagrangian for both of the scalars (and a kinetic term for the gauge fields), we can write down the following gauge invariant interaction terms with coupling constant α, γ, δ

$$\beta\phi\phi^*\psi\psi^*, \quad \gamma\psi^q\phi^*, \quad \delta\psi^{*q}\phi \tag{6.7}$$

and powers of these. Now, assume that the $U(1)$ symmetry is spontaneously broken due to a vacuum expectation value $\langle\phi\rangle = v$ developed by the q charged field ϕ,

$$\phi(x) \xrightarrow{SSB} v + \phi'(x). \tag{6.8}$$

As a consequence, the Lagrangian of the broken theory contains the term $v\psi^q$ and $v\psi^{*q}$, which are obviously no more $U(1)$ invariant, though, they still are manifestly invariant under the transformation

$$\psi \to e^{\frac{2i\pi}{q}m}\psi, \quad m \in \mathbb{Z} \tag{6.9}$$

which corresponds to the residual discrete Abelian symmetry $\mathbb{Z}_q$.

For a low energy observer having no knowledge of the field ϕ, Eq.(6.9) appears to be an ordinary global discrete symmetry. It is its local origin why it is called discrete gauge symmetry.

6.5.2 Systematical approach

We will now put this mechanism, which we understood by looking at the explicit coupling structure so far, on a neat theoretical footing. Let us reconsider the simple case of a single $U(1)$ local theory from above. We have to imagine that the *vev* of the field ϕ has indeed been realized through the breaking of a gauged $U(1)$ symmetry. let's suppose that the charge of the ϕ field, which can be identified by a Higgs field in the Standard Model physics, is $Q_\phi = N \times q$ (or let say 0 modulo q), q being an integer $\geqslant 2$. Asking for the invariance of the *vev* of ϕ, $\langle\phi\rangle$ after the breaking of $U(1)$ is in fact equivalent to finding for the specific angle (parameter α) of the $U(1)$ rotation that was leaving the ϕ field invariant at the scale where $U(1)$ was unbroken. Indeed, the coupling of the other fields ψ to the *vev* of ϕ in the broken phase are the same that the coupling of the ψ fields to the ϕ in the unbroken phase. This angle α is easy to find :

$$\langle\phi\rangle \xrightarrow{U(1)} \langle\phi\rangle \quad \Rightarrow \quad e^{iQ_\phi\alpha}\langle\phi\rangle = e^{iNq\alpha}\langle\phi\rangle = \langle\phi\rangle \quad \Rightarrow \quad \alpha = \frac{2\pi}{q} \tag{6.10}$$

For this value of α, the Higgs field was invariant in the unbroken phase, and then invisible for the other fields ψ after this transformation. In other words, for the same angle, the part of the Lagrangian which was not containing the Higgs was also invariant, as the total Lagrangian was invariant under any phase transformation of the $U(1)$ gauged group. It results that, this "out-of-the-Higgs" part of the Lagrangian, which is the untouched part (the ψ fields of charges p) is still invariant under the discrete transformation under this phase α in the broken phase, where the *vev* is also, by definition, invisible. The Lagrangian is then invariant under the operation

$$\psi \to e^{i\alpha p}\psi = e^{i\frac{2\pi}{q}p} \tag{6.11}$$

Considering p as integer (we can always redefine the charges of a model invariant under a $U(1)$ transformation asking all of them to be positif integer) one obtains that the broken Lagrangian exhibit a $e^{i\frac{2\pi}{q}} \equiv \mathbb{Z}_q$ symmetry. This $\mathbb{Z}_q$ symmetry is the remnant of the unbroken $U(1)$ gauge symmetry in the broken phase.
To have an image of what is happening, one can have a look at the figure 6.1. In this figure, we represented a circle on the left side. This circle is obviously invariant under the $U(1)$ transformation consisting of a rotation around its center. Now, suppose that we break this symmetry, spontaneously, by flattening it on its diameter as you can see on the right of the figure 6.1. As you can see, there is still a symmetry which is preserved which is the mirror symmetry around the vertical axe, which is indeed a $\mathbb{Z}_2$ symmetry.

Figure 6.1: Representation of a circle, obeying a U(1) symmetry (rotation) transformed into a $\mathbb{Z}_2$ symmetry after flatten it.

Part II

Non–supersymmetric candidates

Chapter 7

Effective operator approach*

7.1 The model

Microscopic models of particle physics are usually described by the exchange of new or existing particles playing the role of interactions between the invisible world (dark matter) and the visible sector (Standard Model particles). The particle exchanged can be a gauge boson (Z_μ or Z'_μ for instance), the Higgs boson or another type of field (supersymmetry..). We will discuss in detail several of this "portal" models in the next chapters. However, a more pragmatic way to describe and analyze the phenomenological aspect of the dark matter interaction with the SM is to build effective couplings. Indeed, below a certain scale Λ_i, the exchanged particle decouple from the interaction and the couplings is equivalent to a Fermi constant G_F if the four-fermion interaction model. Depending on the spin of the now-decoupled exchanged particle, one thus can build 4 types of interactions consistent with the requirement of Lorentz invariance (and strongly constrained by LEP analysis). This enables us to describe the interaction between WIMPs and standard model fermions in terms of an effective field theory, in which we keep only the first term in the expansion of the heavy propagator. Several work in the past have been done in this field [**?**]. However, contrarily to the preceding descriptions (see [462] for instance) we generalize the analysis taking into account 1) the neutrino couplings and 2) the possibility of hadronic tree level couplings. This implies the introduction of a second effective scale, Λ_h. Indeed there is no reason for the effective hadronic breaking scale to be the same than the leptonic one Λ_l. We will thus introduce hadronic and leptonic coupling constants g_h and g_l, such as

$$\frac{1}{\Lambda_l} \equiv \frac{\sqrt{g_l}}{\Lambda}; \qquad \frac{1}{\Lambda_h} \equiv \frac{\sqrt{g_h}}{\Lambda} \tag{7.1}$$

For a dark matter candidate χ we can then consider the set of operators

$$
\begin{aligned}
&\text{Vector}: \mathcal{L}_V = \sum_i \frac{g_l^i}{\Lambda^2}(\bar{l}^i\gamma^\mu l^i)(\bar{\chi}\gamma_\mu\chi) + \sum_i \frac{g_h^i}{\Lambda^2}(\bar{q}^i\gamma^\mu q^i)(\bar{\chi}\gamma_\mu\chi) \\
&\text{Scalar, s} - \text{channel}: \mathcal{L}_S = \sum_i \frac{g_l^i}{\Lambda^2}(\bar{l}^i l^i)(\bar{\chi}\chi) + \sum_i \frac{g_h^i}{\Lambda^2}(\bar{q}^i q^i)(\bar{\chi}\chi) \\
&\text{Axial}: \mathcal{L}_A = \sum_i \frac{g_l^i}{\Lambda^2}(\bar{l}^i\gamma^\mu\gamma^5 l^i)(\bar{\chi}\gamma_\mu\gamma^5\chi) \\
&\qquad\qquad + \sum_i \frac{g_h^i}{\Lambda^2}(\bar{q}^i\gamma^\mu\gamma^5 q^i)(\bar{\chi}\gamma_\mu\gamma^5\chi) \\
&\text{Scalar, t} - \text{channel}\ \mathcal{L}_t = \sum_i \frac{g_l^i}{\Lambda^2}(\bar{l}^i\chi)(\bar{\chi}l^i) + \sum_i \frac{g_h^i}{\Lambda^2}(\bar{q}^i\chi)(\bar{\chi}q^i)
\end{aligned}
\tag{7.2}
$$

Throughout this chapter, we will assume that the dark matter particle χ is a Dirac fermion (except in section **??**, where we consider a real scalar DM candidate). A vectorial interaction is motivated by the exchange of a Z'_μ [599] whereas scalar interaction is motivated by Higgs-portal like models [711].

As a good set of examples, we will consider 3 kinds of models which are representative of UV completion:

- Electrophilic couplings (model A): $g_l^e = g_e$, $g_l^{i=\mu,\tau,\nu_i} = 0$
- Charged lepton couplings (model B) : $g_l^{i=e,\mu,\tau} = g_l$, $g_l^{i=\nu_i} = 0$
- Universal lepton couplings (model C) : $g_l^{i=e,\mu,\tau,\nu_i} = g_l$

As we are interested in the ratio of the hadronic to the leptonic final states in the DM annihilation, we will consider without loss of generality an universal generation/family coupling in the hadronic sector: $g_h^{i=u,d,c,s,b,t} = g_h$. Note that we assumed lepton flavor to be conserved in the dark matter interaction.

The authors of [462] made an analysis with relatively little model dependance, by pair production of pair of dark matter particles in association with a hard photon. The LEP experiments have searched for anomalous single–photon events in their data sets, but have found no discrepancy from the prediction of the standard model. They used the single–photon spectrum from the DELPHI experiment to place upper bound to $1/\Lambda_e^2 = g_e/\Lambda^2$. We reproduce interpolated functions of this result in Fig.(7.1). We then translated this limit on Λ_e to a limit on the ratio of hadronic to leptonic channel, $\mathrm{Br_h}/\mathrm{Br_l}$, taking into account the relic density constraints.

Figure 7.1: DELPHI lower limit on $\Lambda_e \equiv \Lambda/\sqrt{g_e}$ as a function of the dark matter mass for the different types of couplings : vector (red dashed), scalar (blue dashed-dotted), axial (green full–line) and t-channel scalar (magenta dotted).

7.2 Constraints from the thermal relic to the hadronic branching ratio

7.2.1 The fermionic case

The LEP lower bound on the scale $\Lambda_e = \Lambda/\sqrt{g_e}$ can be converted in a upper bound to the dark matter annihilation into e^+e^- (case A), into charged leptons pair (case B) or into general leptons pair (case C). Moreover, if dark matter is a thermal relic, asking for the density to respect the upper bound given by WMAP [13] $\Omega_\chi h^2 \lesssim 0.1$, one needs to impose[1] $\langle\sigma v\rangle \gtrsim 3\times 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1} \simeq 1$ pb to avoid an overclosed universe (but letting for the possibility of having another dark matter candidate). We computed the annihilation cross section which is given for a final state with particles masses m_3, m_4 by

$$\frac{d\sigma_{\mathrm{I}}}{d\Omega} = \frac{|\mathcal{M}_{\mathrm{I}}|^2}{64\pi^2 s}\frac{\sqrt{s-2m_3^2-2m_4^2+\frac{(m_3^2-m_4^2)^2}{s}}}{\sqrt{s-4m_\chi^2}} \tag{7.3}$$

with I=V, S, A, t. We then substitute $s = 4m^2/(1-v^2/4) \simeq 4m_\chi^4 + m_\chi^2 v^2$ in Eq.(7.3) and expanding in powers of the relative velocity between two annihilating WIMPs up to order v^2 for each type of couplings. We find

$$\sigma_I^J v = g_l^2 \sum_{l=e,\mu,\tau,\nu} \sigma_{I,l}^J v + c\, g_h^2 \sum_{h=u,d,c,s,t,b} \sigma_{I,h}^J v \tag{7.4}$$

[1] in the absence of resonances or coannihilation.

where I=V, S, A, t represents the nature of the coupling (vectorial, scalar, axial or t-scalar) and J=A, B, C the type of coupling (electronic, charged leptonic or universal leptonic), c the color factor and

$$\begin{aligned}
&\sigma^J_{Ik} v = \sigma_{I,k} v \times \theta^J_k(m_\chi) \\
&\theta^{J=A,B,C}_{h=u,d,c,s,t,b}(m_\chi) = \Theta_H(m_\chi - m_h) \\
&\theta^A_e(m_\chi) = \Theta_H(m_\chi - m_e), \quad \theta^A_{l=\mu,\tau,\nu}(m_\chi) = 0 \\
&\theta^B_{l=e,\mu,\tau}(m_\chi) = \Theta_H(m_\chi - m_l), \quad \theta^B_{l=\nu}(m_{m_\chi}) = 0 \\
&\theta^C_{l=e,\mu,\tau,\nu}(m_\chi) = \Theta_H(m_\chi - m_l)
\end{aligned} \tag{7.5}$$

Θ_H being the classical heaviside function ($\Theta_H(x) = 1$ if $x > 0$, and 0 otherwise) and $\sigma_{I,k}$ is given by:

$$\begin{aligned}
&\sigma_{V,k} v = 4 g_\Lambda \left(24(2m_\chi^2 + m_k^2) + \frac{8m_\chi^4 - 4m_\chi^2 m_k^2 + 5m_k^4}{m_\chi^2 - m_l^2} v^2 \right) \\
&\sigma_{S,k} v = 24 g_\Lambda (m_\chi^2 - m_k^2) v^2 \\
&\sigma_{A,k} v = 4 g_\Lambda \left(24 m_k^2 + \frac{8m_\chi^4 - 22m_\chi^2 m_k^2 + 17m_k^4}{m_\chi^2 - m_k^2} v^2 \right) \\
&\sigma_{t,k} v = g_\Lambda \left(24(m_\chi + m_k)^2 \right. \\
&\left. + \frac{(m_\chi + m_k)^2 (8m_\chi^2 - 16 m_\chi m_k + 11 m_k^2)}{m_\chi^2 - m_k^2} v^2 \right)
\end{aligned} \tag{7.6}$$

with $g_\Lambda = \frac{\sqrt{1 - m_k^2/m_\chi^2}}{192\pi\Lambda^4}$.
The LEP constraint on $\Lambda_e = \Lambda/\sqrt{g_e}$ gives a maximum value for the leptonic annihilation cross section σ_l^{max} for each type of couplings we considered (A, B and C, see Fig.7.1). This maximum value of the leptonic cross–section give a lower bound on $\Omega_\chi h^2$: one thus can calculate the hadronic contribution needed to satisfy WMAP upper bound limit ($\Omega_\chi h^2 \lesssim 0.1$) corresponding to the thermal condition $\sigma v \gtrsim 3 \times 10^{-26} \mathrm{cm}^3/\mathrm{s}$. This can be summarize by:

$$\sigma_l^{max} v + \sigma_h^{max} v \gtrsim 3 \times 10^{-26} \mathrm{cm}^3 \mathrm{s}^{-1} \simeq 2.5 \times 10^{-9} \ \mathrm{GeV}^{-2} \tag{7.7}$$

As an example, we can analytically evaluate the order of magnitude for the hadronic branching ratio Br_h/Br_l we expect for a dark matter mass $m_\chi \simeq 5$ GeV in the case of an electronic (case A) vector–like coupling ($\mathcal{L}_V$). We combined the condition given in Eq.(7.7) to the value of $\sigma_V v$ computed through Eq.(7.6) with the value of Λ_e obtained by LEP (see Fig.7.1): $\Lambda_{eV}^{max} \simeq 480$ GeV for $m_\chi \simeq 5$ GeV. Neglecting $m_{l,h} \ll m_\chi$, in the electronic–type coupling, one can simplify $\sigma_V v \simeq \frac{m_\chi^2}{\pi \Lambda_e^4} (1 + Br_h/Br_l) \gtrsim 2.5 \times 10^{-9}$ which gives $Br_h/Br_l \gtrsim \frac{(\Lambda_e^{max})^4 \pi}{m_\chi^2} 2.5 \times 10^{-9} \simeq 16$. This corresponds to a 94% annihilation rate to hadronic states. Of course, we ran the analysis with the complete formulation for the cross sections and the results are shown in Fig.7.2. One can see that whatever is the nature of the coupling (electronic, charged-leptonic or universal leptonic), a dark matter of mass $m_\chi \simeq 10$ GeV has

Figure 7.2: Minimum hadronic branching ratio needed to respect WMAP upper bound in the case of electronic couplings (model A, top), charged-leptonic couplings (model B, middle) and universal-leptonic couplings (model C, bottom) with 4 different types of interactions: vector (red dashed), scalar (blue dashed-dotted), axial (green full–line) and t-channel scalar (magenta dotted). Bounds coming from LEP constraints on leptonic couplings.

a very strong hadronic component in its annihilation final state, in the case of scalar and axial interactions (above 90%). On the other hand, for vector and t-scalar interactions, the nature of the coupling plays an important role, being an hadronic component as large as 80% in one case (electronic coupling), or a 0% (i.e. no need of hadronic channel) in other case (universal leptonic), for a vector interaction for example. These behaviors can be understood from expressions (7.6), where the scalar and axial interactions are suppressed by the velocity and the leptonic masses, respectively. As a consequence one needs a much larger hadronic contribution to ensure a relic abundance below the WMAP limit and avoid the over–closure of the Universe. However for the vector and t-scalar interactions there is no such suppression, so leading to possible large contributions coming only from the leptonic couplings. One can notice the presence of small "kinks" in the branching fraction corresponding to the kinetic opening/closing of the different annihilation channels.

We also observe that, paradoxically, the more electrophilic are the dark matter couplings (model A), the more hadrophilic it should also be. Indeed, because there are no possibility to fulfill the relic abundance constraints with charged lepton or neutrino channels and the hadronic final states become thus the dominant ones. In the charged leptonic and universal leptonic models (B and C), there exists a threshold mass with a null hadronic branching ratio: this corresponds to the mass for which the hadronic components of the annihilation rate are not anymore necessary (but can be present) to fulfill the relic density constraints. The leptonic channels are sufficient to avoid the relic overabundance for a DM mass above this threshold.

7.2.2 Scalar case

We also checked the case of a scalar dark matter. It could not be obvious at the first sight that we can apply the same analysis. In fact, we need to introduce a new scale Λ_S. We will consider a real scalar dark matter, which is produced via the following scalar-type effective operator:

$$\mathcal{L}_e^S = \frac{g_e}{\Lambda_S}\chi\chi\bar{e}e \ . \tag{7.8}$$

In an analogous procedure as the one done above for a fermionic candidate, we derive limits on the suppression scale Λ_S/g_e from single-photon signals in the DELPHI experiment at LEP. We assumed for simplicity that all data was taken at an energy of 100 GeV per beam. We used MadGraph/MadEvent [469] to simulate the distribution of number of events with photon energy E_γ. The background process $e^+e^- \to \gamma\nu\bar{\nu}$ was taken directly from the simulation done by [462]. On the other hand, the signal process $e^+e^- \to \gamma\chi\chi$ was studied assuming the following kinematical cuts: $E_\gamma > 6$ GeV, and a photon rapidity $\eta_\gamma > 2.5$. We realize that these constraints are less restrictive than in the fermionic case, so in principle the bounds on Λ_S could be different if using those more rigorous cuts. To quantify this difference, we reproduced the bounds on Λ_e coming from a signal due to a fermionic dark matter, and a vector-like effective operator, and compare it directly with the result shown in Fig.7.1. The result, shown in Fig.7.3-top, is a χ^2/d.o.f. $= 5.12/8$, which means a small difference of our case with respect to the more correct result of [462]. We include as an example of a scalar dark matter signal, the simulation of a $m_\chi = 10$ GeV case, with a suppression scale $\Lambda_S/g_e = 300$ GeV, using the DELPHI luminosity of 650 pb^{-1}, shown in Fig.7.3-bottom. We

Figure 7.3: (top). Lower limits on Λ_S/g_e (solid-red) coming from DELPHI experiment [463], at a 90% C.L. For reference, the resulting limits on $\Lambda/\sqrt{g_e}$ (fermionic dark matter) coming from a vector-like effective operator, using the same cuts as before, are shown (dashed-green), to be compared with the correspondent result shown in Fig.7.1, here in (dotted-blue). (bottom) Distribution of photon energies in single-photon events at DELPHI. The histogram shows the signal+background coming from a hypothetical scalar dark matter, as in (7.8), with mass $m_\chi = 10$ GeV, and a suppression scale $\Lambda_S/g_e = 300$ GeV. See body text.

can extract from the above analysis that, for example, a scalar dark matter of $m_X = 10$ GeV needs a suppression scale $\Lambda_S \gtrsim 520$ GeV, in order to be compatible with LEP bounds.
After computing the production cross section for the fermionic dark matter (σ_f^P) and bosonic one (σ_s^P) one can show that, in the limit of $s \gg m_\chi$ (which is the case in our analysis) we obtained

$$\sigma_f^P \sim \frac{g_e^2}{\Lambda^4}\frac{s}{16\pi} \qquad \text{and} \qquad \sigma_s^P \sim \frac{g_e^2}{\Lambda_S^2}\frac{1}{32\pi} \tag{7.9}$$

We thus observe that if we define $\Lambda_S \equiv \Lambda^2/\sqrt{2s}$ we can deduce the lower limit on Λ_S from the lower limit on Λ (see Fig.7.1). If imposing $\sigma_f^P \approx \sigma_s^P$, taking a dark matter candidate with mass of 10 GeV, we deduce from the above expressions the lower bound $\Lambda_S \gtrsim 815$ GeV. However from Fig.(7.3) we obtained $\Lambda_S \gtrsim 520$ GeV, which implies that in fact $\sigma_s^P \approx 2.45\sigma_f^P$.

With this bounds on Λ_S, one could in principle try to deduce bounds on the amount of hadronic channel from DM annihilation, as we did above for the fermionic case. The expression for the annihilation cross-section $\sigma_S^s v$ of a scalar DM with a scalar interaction, into an electron-positron pair is:

$$\sigma_{S,e}^s v \equiv g_e^2 \tilde{\sigma}_{S,e}^s v \simeq \frac{g_e^2}{4\pi\Lambda_S^2}\left(1 - \frac{m_e^2}{m_\chi^2}\right)^{3/2} + \frac{g_e^2}{32\pi\Lambda_S^2}v^2 \ . \tag{7.10}$$

Unfortunately, this single channel already gives, for $510 \lesssim \Lambda_S/g_e \lesssim 520$GeV and $1 \lesssim m_\chi \lesssim 20$GeV (as in Fig.7.3) a cross-section $\sigma_S^s v \simeq 10^{-24}\text{cm}^3/\text{s}$, with negligible dependence on m_χ. Being $\sigma_S^s v \gg 3 \times 10^{-26}$, there is in principle no need for hadronic channel. We conclude that LEP bounds are insufficient to constrain the nature of couplings in the case of scalar DM.

A similar conclusion holds for the Tevatron bounds, if considering the total cross-section

$$\sigma_S^s v = g_l^2 \sum_{l=e,\mu,\tau,\nu} \tilde{\sigma}_{S,l}^s v + c g_h^2 \sum_{h=u,d,c,s,t,b} \tilde{\sigma}_{S,h}^s v \tag{7.11}$$

and the lower limits shown in Fig.7.4. It turns out that the scale Λ_S/g_e above which $\sigma_S^s v$ starts to be of the order of the thermal relic one, is around 5 TeV. So in principle the LHC would be able to constrain the nature of couplings and interactions of a scalar DM candidate.

7.3 Complementarity with other experiments

7.3.1 Fitting with WMAP

To run a more precise analysis, we decided to implement the contact coupling lagrangian into CompHEP and micromegas [716] for the different type of interactions (vectorial, scalar and axial) in the fermionic DM case. We then applied the last 5σ constraint on the relic density from WMAP experiment [13], $\Omega_{\rm WMAP} h^2 = 0.1123 \pm 0.0175$ and ran a scan on the parameter space of the model (Λ_l, Λ_h, m_χ) keeping only the points respecting both astrophysical and accelerator constraints. We can understand easily that in order to respect WMAP upper bounds, the hadronic contribution depends on the type of interactions. After a look at Fig.7.2, we decided to consider the most and the less conservative cases, which are the universal-leptonic coupling for vector-like interaction, and the electronic coupling for scalar-like interaction, respectively. The results are shown in Figs.7.5 and 7.6. We see that WMAP forbids a dark matter with hadrophilic couplings ($g_h/g_e \gtrsim 10$) of $m_\chi \gtrsim 5$ GeV for a vector interaction, and $m_\chi \gtrsim 10$ GeV for a scalar interaction. For such values of hadronic couplings, $\Omega h^2 \lesssim \Omega_{\rm WMAP} h^2$. When combined with LEP analysis, a large part of the parameter space with small g_h/g_e is excluded because of the non–observation of mono–photon events at LEP (which implies an upper bound on g_e). Whereas it excludes a broad region of the parameter space for a dark matter mass $m_\chi \lesssim 11$ GeV in the vectorial case (in total agreement with Fig.6 of [462]) it completely exclude leptophilic ($g_h/g_e \lesssim 0.1$) dark matter with the scalar-like interaction. Combining these limits with the recent Tevatron analysis restricted even further the parameter space.

7.3.2 Tevatron constraints

Last year, the authors of [471,750] made a similar analysis searching for mono–jet events for the Tevatron. These non-discovery of any events of this kind can be translated into a lower bound on $\Lambda_h \equiv \Lambda/\sqrt{g_h}$ which depend on the nature of the coupling and is represented in Fig.7.4.

Contrarily to the LEP analysis, the center of mass energy does not limit the lower bound on Λ_h for $m_\chi \lesssim 100$ GeV. We have only plotted the limit on the up–type coupling, which is the one we used through the paper to stay as conservative as possible (limits of down or charm–type couplings on Λ_h are a factor 3 and 10 lower respectively [471]). We can easily understand how the Tevatron constraint imply some strong tensions when combined with WMAP and LEP analysis. Indeed, to reconciliate LEP constraints with WMAP we needed to increase

Figure 7.4: CDF lower limit on $\Lambda_h \equiv \Lambda/\sqrt{g_h}$ as a function of the dark matter mass for the different types of couplings : vector (red dashed), scalar (blue dashed-dotted) and axial (green full–line).

Figure 7.5: Hadronic ratio coupling for the annihilation of dark matter as function of the dark matter mass in the case of universal-leptonic couplings for a vector–like interaction after a scan on Λ_e and Λ_h. After applying the constraint of WMAP (top) mono–jet events from LEP, Tevatron and XENON100 constraint (bottom). See the text for details.

Figure 7.6: Hadronic ratio coupling for the annihilation of dark matter as function of the dark matter mass in the case of electronic couplings for a scalar–like interaction after a scan on Λ_e and Λ_h. After applying the constraint of mono–jet events from LEP (top) and Tevatron (bottom) we observe that no point of the parameter space respects both constraints (see the text for details).

the hadronic contribution (and thus, the coupling to quarks) in the annihilation process. This then enters in conflict with the limit from the non-observation of mono–jet excess at Tevatron. To keep the logic of the work and keep a conservative analysis, we considered universal leptonic couplings (implying a smaller hadronic contribution to respect WMAP upper bound). The results are shown in Figs.7.5 and 7.6. Confirming the conclusions of [471]: for a vector–like coupling the Tevatron bound is the more stringent for dark matter mass below 5 GeV. Indeed, for such low mass, the hadronic branching ratio needed to respect in the meantime WMAP and LEP would produce a clear excess in mono–jet events at Tevatron and would have been observed. The Tevatron constraints are even more impressive for a scalar–like couplings, where all the parameter space allowed by the combined WMAP and LEP analysis is excluded by Tevatron data (Fig.7.6).

7.3.3 XENON100 constraint

Recently, the XENON100 collaboration has released several analysis claiming for no detection signal of dark matter [727]. Their results are by far the more constraining one in the field of direct detection experiments. One can easily understand that XENON100 is adding new tensions when combined with WMAP, LEP and Tevatron bounds. Indeed, the hadronic branching fraction required to avoid the overproduction of dark matter in the early universe could enter in conflict not only with Tevatron results but also with XENON100 exclusion limits. Indeed, whereas the s–channel dark matter production $qq \to \chi\chi g$ is the process constraining Λ_h at Tevatron, the nuclear recoil gives bound to Λ_h (and thus g_h/g_e) through the t-channel process $\chi q \to \chi q$. As we can see in Fig.7.5, XENON100 restrict even a larger part of the parameter space for $m_\chi \gtrsim 6$ GeV (which would not have been the case if we took into account the previous XENON100 analysis [525]). Whereas the scalar–like interaction is already excluded without the XENON100 data, dark matter with vector–like coupling to the SM still survives in a narrow hadrophilic region of the parameter space with light dark

matter[2], and another region definitively hadrophobic for $m_\chi \gtrsim 12$ GeV.

7.4 Conclusion and prospect

Recently, several astrophysical data or would-be signals has been observed in different dark-matter oriented experiments. In each case, one could fit the data at the price of specific nature of the coupling between the Standard Model (SM) particles and a light Dark Matter candidate: hadrophilic or leptophilic. We computed the rate of hadronic coupling needed to respect WMAP combined with the LEP and Tevatron constraints from mono–jet events. We showed that a light fermionic dark matter ($\lesssim 10$ GeV) is mainly excluded whatever is its type of interaction, whereas heavier candidates ($\gtrsim 20$ GeV) should be largely hadrophobic for vectorial interaction, but excluded for scalar one. We also studied the special case of scalar dark matter, using the single–photon events to constraint the coupling of dark matter to electron with a complete simulation of DELPHI events and showed that LEP and Tevatron are not able to restrict the couplings. One of the main consequences is that models with light electrophilic couplings, explaining INTEGRAL data or constraints from synchrotron radiations are excluded by Tevatron/LEP analysis. One possibility to escape such strong conclusion would be to suppose that DM has no electronic coupling. In this case, LEP limits do not apply. Moreover, if at the same time the hadronic coupling is only to the bottom or charm quark, Tevatron XENON100 bounds are not applicable too. However such unnatural construction should be excluded by FERMI last analysis of dwarf galaxies [528].

[2]which strangely coincides with the CoGENT excess signal [499]

Chapter 8

Extra U(1) and kinetic mixing*

8.1 Introduction

Neutral gauge sectors with an additional dark $U_D(1)$ symmetry in addition to the Standard Model (SM) hypercharge $U(1)_Y$ and an associated Z_D ("D" standing for *Dark*) are among the best motivated extensions of the SM, and give the possibility that a dark matter candidate lies within this new gauge sector of the theory. Extra gauge symmetries are predicted in most Grand Unified Theories (GUTs) and appear systematically in string constructions. Larger groups than $SU(5)$ or $SO(10)$, like E_6 allows the SM gauge group to be embedded into them. Brane–world $U'(1)$s are special compared to GUT $U'(1)$s because there is no reason for the SM particle to be charged under them: for a review of the phenomenology of the extra $U'(1)$s generated in such scenarios see e.g. [592]. In such framework, the extra–gauge boson would act as a portal between the *dark* world (particles not charged under the SM gauge group) and the *visible* sector through its gauge invariant kinetic mixing $\delta/2F_Y^{\mu\nu}F_{\mu\nu}^D$ [481, 483–487, 552, 593, 599]. One of the first model of dark matter from the hidden sector with a massive additional $U'(1)$ through both mass and kinetic mixings, the so called dark force can be found in [594]. The Dark Matter (DM) candidate ψ_0 would be the lightest (and thus stable) particle of this secluded sector. Such a mixing has been justified in recent string constructions [595–598], but has also been studied with a model independent approach [494, 599–601].

On the other hand, recently the CRESST-II and CoGENT collaborations have reported the observation of low–energy events in excess of known backgrounds [496, 497]. This has encouraged the hypothesis that these signals -in addition to the long–standing DAMA [498] annual modulation signal – might arise from the scattering of a light (~ 10 GeV) dark matter particle. It was recently shown that models with extra $U'(1)$ can easily accommodate with such signals [499–502]. Similar models with singlet extension reached the same conclusion because they are very similar in the construction than the models with extra gauge boson, except that the role of the portal is played by the Higgs boson [503, 504]. Other *very* important astrophysical consequences from the tri–bosonic coupling Z_DYY generated by loop induced triangle diagrams are studied in [506, 507, 604] and in [605] in a more model independent way. Consequences of such anomaly-generated vertex are also well analyzed in [509, 621]. It is worth noticing that one can easily build a model where the Z_D becomes the DM candidate [696]: a clear summary of all these extensions can be found in [512].

Our objective in this work is to restrict the parameter space of an extra $U_D(1)$, taking into consideration the main detection modes, *i.e.* cosmological data, precision measurements, direct and indirect detection searches of dark matter. The paper is organized as followed: after a brief reminder of the model and its phenomenological characteristics, we analyze the experimental exclusion limits (in a conservative way) in different regimes ((very)light and (very)heavy DM), before presenting a general analysis and perspective for a XENON–like 1 ton detector. We then conclude.

8.2 The model

The matter content of any *dark* $U(1)_D$ extension of the SM can be decomposed into three families of particles:

- The *Visible sector* is made of particles which are charged under the SM gauge group $SU(3) \times SU(2) \times U_Y(1)$ but not charged under $U_D(1)$ (hence the *dark* denomination for this gauge group)
- the *Dark sector* is composed by the particles charged under $U_D(1)$ but neutral with respect of the SM gauge symmetries. The dark matter (ψ_0) candidate is the lightest particle of the *dark sector*
- The *Hybrid sector* contains states with SM *and* $U_D(1)$ quantum numbers. These states are fundamental because they act as a portal between the two previous sector through the kinetic mixing they induce at loop order.

From these considerations, it is easy to build the effective Lagrangian generated at one loop :

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\rm SM} - \frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{\delta}{2}\tilde{B}_{\mu\nu}\tilde{X}^{\mu\nu} \\ &+ i\sum_i \bar{\psi}_i\gamma^\mu D_\mu\psi_i + i\sum_j \bar{\Psi}_j\gamma^\mu D_\mu\Psi_j \end{aligned} \tag{8.1}$$

$\tilde{B}_\mu$ being the gauge field for the hypercharge, $\tilde{X}_\mu$ the gauge field of $U_D(1)$ and ψ_i the particles from the hidden sector, Ψ_j the particles from the hybrid sector, $D_\mu = \partial_\mu - i(q_Y\tilde{g}_Y\tilde{B}_\mu + q_D\tilde{g}_D\tilde{X}_\mu + gT^aW^a_\mu)$, T^a being the $SU(2)$ generators, and

$$\delta = \frac{\tilde{g}_Y\tilde{g}_D}{16\pi^2}\sum_j q_Y^j q_D^j \log\left(\frac{m_j^2}{M_j^2}\right) \tag{8.2}$$

with m_j and M_j being hybrid mass states [513] .

Notice that the sum is on all the hybrid states, as they are the only ones which can contribute to the $\tilde{B}_\mu\tilde{X}_\mu$ propagator. After diagonalization of the current eigenstates that

makes the gauge kinetic terms of Eq.(D.13) diagonal and canonical, we can write after the $SU(2)_L \times U(1)_Y$ breaking[1] :

$$\begin{aligned} A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \\ Z_\mu &= \cos\phi(\cos\theta_W W^3_\mu - \sin\theta_W B_\mu) - \sin\phi X_\mu \\ (Z_D)_\mu &= \sin\phi(\cos\theta_W W^3_\mu - \sin\theta_W B_\mu) + \cos\phi X_\mu \end{aligned} \tag{8.3}$$

with, at the first order in δ:

$$\begin{aligned} \cos\phi &= \frac{\alpha}{\sqrt{\alpha^2 + 4\delta^2\sin^2\theta_W}} \quad \sin\phi = \frac{2\delta\sin\theta_W}{\sqrt{\alpha^2 + 4\delta^2\sin^2\theta_W}} \\ \alpha &= 1 - M^2_{Z_D}/M^2_Z - \delta^2\sin^2\theta_W \\ &\pm \sqrt{(1 - M^2_{Z_D}/M^2_Z - \delta^2\sin^2\theta_W)^2 + 4\delta^2\sin^2\theta_W} \end{aligned} \tag{8.4}$$

and + (-) sign if $M_{Z_D} < (>) M_Z$. The kinetic mixing parameter δ generates an effective coupling of SM states $\psi_{\rm SM}$ to Z_D, and a coupling of ψ_0 to the SM Z boson which induces an interaction on nucleons. Developing the covariant derivative on SM and ψ_0 fermions state, we computed the effective $\psi_{\rm SM}\psi_{\rm SM}Z_D$ and $\psi_0\psi_0 Z$ couplings at first order[2] in δ and obtained

$$\mathcal{L} = q_D\tilde{g}_D(\cos\phi\ Z'_\mu\bar{\psi}_0\gamma^\mu\psi_0 + \sin\phi\ Z_\mu\bar{\psi}_0\gamma^\mu\psi_0). \tag{8.5}$$

We took $q_D\tilde{g}_D = 3$ trough our analysis, keeping in mind that our results stay completely general by a simple rescaling of the kinetic mixing[3] δ.

8.3 Constraints

We drove a complete analysis of the allowed parameter space for the kinetic mixing using the more recent searches in accelerator physics, direct and indirect detection experiments. Here follows a summary of the constraints we applied in our study:

8.3.1 Low energy and electroweak constraints

Concerning the electroweak symmetry breaking, the mixing between $\tilde{X}_\mu$ and $\tilde{B}_\mu$ generates new contributions to precision electroweak observables. However, none of the particle of the SM has any $U_D(1)$ charges: the $U_D(1)$ can be considered has a *lepto − hadrophobic* Z_D. Other authors in [599] or [598] have looked at hidden-valley like models or milli–charged dark matter but concentrating their study to relatively heavy Z_D and a large mixing angle. We summarize in this section the main constraints generated by such data.

[1] Our notation for the gauge fields are $(\tilde{B}^\mu, \tilde{X}^\mu)$ before the diagonalization, (B^μ, X^μ) after diagonalization and (Z^μ, Z^μ_D) after the electroweak breaking.

[2] One can find a detailed analysis of the spectrum and couplings of the model in the appendix of [494].

[3] The author would like to thank the referee for having pointed out this detail of the analysis.

Figure 8.1: Precision test, relic density, direct detection and indirect detection constraints in the plane (M_{Z_D}; δ) for different masses of Dark Matter (5, 25, 200 and 500 GeV)

- g-2 :

  The presence of a light Z_D can contribute to the anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)/2$. For our analysis we implemented the formulation of Ref. [514]. To compare with experimental data, we used the latest experimental value [515]

$$\delta a_\mu = (31.6 \pm 7.9) \times 10^{-10} \tag{8.6}$$

  We also checked that parity-violation effects [514] constrain a much smaller region of the parameter space.

- ρ parameter :

  The ρ parameter is defined as $\rho = m_W^2/M_Z^2 c_W^2 = 1$ in the Standard Model (SM). Any electroweak extension of the SM can generate a deviation to the ρ parameter, especially if $M_{Z_D} \simeq M_Z$, where the mixing is maximum. One can thus translate the global fit of the ρ parameter $\rho - 1 = 4^{+8}_{-4} \times 10^{-4}$ [516] to a constraint on the (M_{Z_D};δ) plane.

- Electroweak precision test :

  The authors of [509] have computed the observables from effective Peskin–Takeuchi parameters [517], and found

$$\begin{aligned} \Delta m_W &= (17\text{MeV})\ \zeta \\ \Delta\Gamma_{l^+l^-} &= -(8\text{keV})\ \zeta \\ \Delta \sin^2\theta_W^{eff} &= -(0.00033)\ \zeta \end{aligned} \tag{8.7}$$

  where

$$\zeta \equiv \left(\frac{\delta}{0.1}\right)^2 \left(\frac{250\text{GeV}}{M_{Z_D}}\right)^2 \tag{8.8}$$

  More recently, the authors of [518] have published an extensive model independent analysis in the energy range of interest in our study. They bounded the kinetic mixing by $\delta \lesssim 0.03$ for 10 GeV $< M_{Z_D} <$ 200 GeV which is in complete agreement with the constraints given by Eq.(8.7).

8.3.2 Relic Density

The abundance of a thermal relic dark matter candidate ψ_0 is controlled by its annihilation cross section into SM particles mediated by the exchange of a Z_D gauge boson through $s-$channel, or $t-$channel $Z_D Z_D$ final state (see [506, 604] for a detailed study of the relic abundance constraints). We modified the micrOMEGAs2.2.CPC code[4] [716] in order to calculate the relic abundance of ψ_0 and implemented the points that fulfill the WMAP 5σ

[4]The author wants to thank particularly G. Belanger and S. Pukhov for their help to address this issue.

bound [13] , $\Omega_{DM}h^2 = 0.1123 \pm 5 \times 0.0035$. One important point is that for a given M_{Z_D} and m_{ψ_0}, there exists a unique solution δ (up to the very small uncertainties at 5σ) fulfilling WMAP constraints : from 3 parameters $(m_{\psi_0}, M_{Z_D}, \delta)$, the WMAP constraints reduce it to two (M_{Z_D}, δ).
One want to stress that a zone with low relic abundance respecting WMAP is expected around the region where $M_{Z_D} \simeq M_Z$. This comes from the increase of the mixing angle $\sin\phi$ in this region of parameter space. Indeed, for a given δ, $\sin\phi$ increases by a factor of 10 when M_{Z_D} varies from 90 to 110 GeV (Eq.(D.16)), decreasing the relic abundance by a factor 100 (the $Z_D\bar{f}f$ coupling is proportional to $\sin\phi$).

8.3.3 Direct detection

There was recently a huge interest concerning the exclusion (or not) of the CoGENT/DAMA/CRESST signal region corresponding to light dark matter [496–498, 520]. Several authors tried to reconciliate the signal with minimal–like extensions of the standard model : scalar dark matter, extra U(1), .. Very recently, XENON experiment released a likelihood approach to the first results from XENON100 [521]. We took their 2σ limits on their spin–independent elastic WIMP–nucleon cross section. The XENON collaboration also recalculated the limit from CDMS assuming an escape velocity of 544 km/s which fits perfectly inside the 2σ limits they obtained, as the less conservative scenario on $\mathcal{L}_{eff}$. We also explored the possibility of a projected 1 ton XENON experiment [522] (more or less equivalent to a super CDMS extension of CDMS [523]). During the writing if the article, the XENON experiment published a more detailed analysis for light DM exclusion limit with XENON10 data in [524] and with XENON100 data [525]. Their results are compatible with the exclusion limit we used through the analysis.

8.3.4 Indirect detection

From the indirect detection data, we combined the very last analysis of different experimental groups.

- Observation of Milky Way dwarf spheroidal galaxies by FERMI : dwarf spheroidal galaxies (dSphs), the largest galactic substructures predicted by the CDM scenario, are ideal laboratories for indirect searches for dark matter for several reasons: the mass-to-light ratios in dSphs can be of order 100-1000 showing that they are largely dark matter dominated systems. In addition, dSphs are expected to be relatively free from γ-ray emission from other astrophysical sources as they have no detected neutral or ionized gas, and little or no recent star formation activity which make them very interesting objects for indirect dark matter searches, especially with the FERMI telescope. For our purpose, we will use the two last analysis of the FERMI collaboration: firstly, the study from the observations of 14 dwarf spheroidal galaxies published in [526], where we will use the data from DRACO observation as the representative milky way satellite, with an assumed Navarro-Frenk-White (NFW [527]) dark matter density profile; secondly, as it has been shown in [528], it is possible to increase the efficiency of the constraints using

a combined analysis. Even if preliminary, these results are quite robust[5]. It is useful to stress that for any indirect detection searches, experimental analysis are obtained for a given final state. As we wanted to be the more conservative, we considered their analysis with a $b\bar{b}$ final state. Indeed, this spectrum is representative of quark spectrum which is the dominant one (due to the color factor) in the decay of a Z_D in s or t channel. Moreover, the final states fraction containing $\mu^+\mu^-$ or $\tau^+\tau^-$ (never more than 5 percents in all our points respecting WMAP constraint) produces a hard γ-ray spectrum resulting in much more stringent constraints since they predict abundant photons fluxes at larger energies, where the diffuse background is lower (see [529, 691] for an analysis of these specific channels). For a quark final state, the resulting rays stem dominantly from the decay of neutral pions produced in the quark and antiquark hadronization chains, and do not crucially depend upon the specific quark flavor or mass; in fact, a very similar ray spectrum is produced by the (typically loop-suppressed) gluon-gluon final state. We thus stay conservative in our analysis. One can also read [531] for a clear review of the status and perspectives of the FERMI satellite.

- Searches from the Galactic Center with HESS: we also included in our analysis the very last searches for a very high γ-ray signal from DM annihilation in our galaxy in the region of 45 pc-150 pc after 112h of observations, excluding the Galactic plane done by the HESS collaboration [532]. These limits are among the best reported so far in this energy range. In this region of the halo, the dependance on the DM profile is quite low, and the *reflected background* technique allows for an intelligent subtraction of the background. The limits can shift by 30% due to both the uncertainty on the absolute flux measurement and the uncertainty of 15% on the energy scale, which does not affect our general conclusion. Note that to be consistent with the FERMI analysis, we adopted the NFW profile parameterization of the HESS results.

- Radio constraints : charged particles production in a halo is necessarily accompanied by radio waves. These signals are produced via the various energy loss processes that charged particles undergo, examples of which include synchrotron radiation or inverse Compton scattering of electrons from interstellar radiation field [533–536]. We applied the combined constraints derived using all energy loss processes, featuring only mild dependance on the astrophysical parameters. Radio data provide more severe restrictions for strong magnetic field B. Taking into account the very large uncertainty on the value of the field in the Galactic center, we studied the constraints for $B = 30\mu$G and 100μG, with the NFW profile to stay consistent with the HESS and FERMI searches. We followed the recent analysis given in [533].

- We also implemented the constraints derived from FERMI diffuse measurements [537]. However, these constraints were always lying between the constraints coming from the DRACO satellite and the ones from the combined dSphs analysis. We thus decided not to show them in the plots.

[5]Private communication with the author of [528]

8.4 Results

8.4.1 Where does hit each experiment?

We show in Fig.8.1 the electroweak, cosmological and astrophysical constraints in the plane (M_{Z_D};δ) for different DM masses (5, 25, 200 and 500 GeV). One can distinguish 4 regimes : very light DM ($\sim$ 5 GeV) light DM ($\sim$ 30 GeV), heavy ($\sim$ 100 GeV) and very heavy ($\gtrsim$ 500 GeV) DM. Depending on the regime, some constraints will dominate over other ones, making complementarity between all these detections mode fundamental for a general analysis of any $U'(1)$ extension of the SM. Another interesting region is the parameter space with maximal mixing, where $M_{Z_D} \sim M_Z$. However, this region when respecting WMAP constraints is totally excluded because of a large excess of the ρ parameter.

8.4.2 The (very)light Dark Matter regime

In the case of very light dark matter ($\lesssim$ 10 GeV), the XENON experiment is relatively inefficient. As one can check in Fig.8.1 (top) all the points respecting WMAP respect also the last data from XENON100. However, some other experiments like FERMI and especially radio constraints become more efficient as was already stressed by the authors of [534] where they showed that a DM mass between 1 and 10 GeV could be excluded by radio data from galactic center. One also notices that the electroweak constraints become the more effective constraints for masses of Z_D, $M_{Z_D} \gtrsim 55$ GeV where they exclude the region respecting WMAP. Indeed, these constraints are obviously independent of the DM mass, but are stringent when the $Z - Z_D$ mixing is important ($M_{Z_D} \sim M_Z$).
For 25GeV $\lesssim m_{\psi_0} \lesssim$ 200 GeV, one can see that the radio constraints become less effective, whereas the XENON experiment excludes a large part of the parameter space allowed by WMAP except near the pole. Indeed in the region (M_{Z_D};δ) where $2m_{\psi_0} \simeq M_{Z_D}$, the s-channel Z_D exchange cross section increases dramatically. Points respecting WMAP lie in region of parameter space with low values of δ (the "sea-gull" shape in Figs.8.1) : with such low values of δ, the direct detection (through t-channel Z_D exchange) cross section $\sigma_{p\psi_0}$ stays below the XENON100 exclusion limit. However, this Z_D-pole region is sensitive to indirect detection experiment like FERMI or HESS because of the high value of the annihilating rate $\psi_0\psi_0 \to Z_D \to \bar{f}f$.

8.4.3 The (very)heavy Dark Matter regime

In this regime, a XENON–like experiment becomes less effective (because of the mass suppression in the scattering cross section) whereas FERMI and HESS indirect detection data exclude the larger part of the parameter space around the pole. However, one can check in Fig.8.1(bottom) that a 500 GeV DM is not yet excluded, except in the region where $M_{Z_D} \lesssim m_{\psi_0}$, dominated by the $\psi_0\psi_0 \to Z_D Z_D$ annihilation final state. HESS limits become more efficient than FERMI for $m_{\psi_0} \gtrsim 300$ GeV. Contrary to the (very)light case, the electroweak observables are always less constraining than any astrophysical data given by direct or indirect detection. Whatever is the DM mass, one can check in Figs.8.1(all) that a XENON 1 ton experiment would be able to exclude all the region respecting WMAP except

in a very narrow region around the pole if $m_{\psi_0} \gtrsim 500$ GeV.

We also analyzed the allowed region in the (M_{Z_D};δ) parameter space after a complete scan on the DM mass 1GeV $< m_{\psi_0} <$ 1 TeV, taking into account the different natures of experimental exclusions. The results are presented in Figs.8.2. With such a scan, our results become independent of the nature, coupling or mass of the dark matter candidate, and one can compare directly the sensitivity of each type of experiment for specific regions of the parameter space.

It appears clearly that FERMI experiment excludes a region of the parameter space corresponding to $M_{ZD} \lesssim 100$ GeV which is the region where the satellite is the more effective. The radio constraints exclude a part of the parameter with lighter Z_D as expected [533, 534]. The ρ and EWPT constraints reach the region of the parameter space where $M_{Z_D} \simeq M_Z$ (maximal mixing). Even after taking into account the XENON100 exclusion region at $\pm 2\sigma$, a region respecting WMAP survives around $M_{Z_D} \simeq 10$ GeV where one can fit COGENT, CRESST or DAMA excesses [499,601] . One also observes that a 1 ton XENON–like experiment would exclude 99 % of the parameter space allowed by WMAP. It would be quite difficult to escape such constraints as the couplings of the Z_D to the quarks are fixed for a given value of δ .

We would like to say few words on the "hole" one observes in the parameter space -before taking into account the XENON100 constraints- in Figs.8.2(top). For a given M_{Z_D}, when δ increases, the Z_D pole "disappears" in the sense that the relic abundance stays below WMAP data between the Z and Z_D-pole: points that respect WMAP constraints lie *very* near from the Z-pole, especially because for such large values of δ, the $Z-$pole branch is extremely narrow. Indeed, $Z\psi_0\psi_0$ coupling is proportional to $\sin\phi$, Eq.(D.16), which means that the annihilation cross section $\langle\sigma v\rangle \propto \delta^2/\Gamma_Z^2$ (Γ_Z being the width of the Z boson) is very sensitive. When δ increases from 10^{-3} to 10^{-1}, the cross section gains 4 orders of magnitude! One needs an extremely fine tuning (at the 0.1% level) on the condition $m_{\psi_0} = M_Z/2$ to respect WMAP bounds. This extreme fine-tuning condition respects a law $\sin\phi$=cste $\Rightarrow \delta \propto M_{Z_D}$ (for $M_{Z_D} \gtrsim M_Z$, Eq.(D.16)) which is exactly the shape of the hole of Fig.8.2(top). For much higher values of δ, points do not need to be on the $Z-$pole anymore to respect WMAP (for $\delta = 0.8$ and $M_{Z_D} = 300$ GeV, one enters in WMAP bounds for $m_{\psi_0} \simeq 3$ GeV). In any case, this region of extreme fine-tuning is largely excluded by XENON100 experiment, even in the more conservative case.

8.4.4 Consequences on m_{ψ_0} and M_{Z_D}

One interesting analysis was to look at the consequences on m_{ψ_0} independently of δ. We show in Figs.8.3 the region of the parameter space allowed by the different type of experiments in the (m_{ψ_0};M_{Z_D}) plane after a scan on $10^{-4} < \delta < 0.8$. One observes that WMAP restricts the parameter space to a relatively broad region around the line $M_{Z_D} \sim 2m_{\psi_0}$. After taking into account the indirect detection constraints, the region of light mass is excluded as expected. The EWPT and ρ-parameter exclude the region near $M_{Z_D} \sim M_Z$ (maximal mixing) and further from the pole because the further one stays from the pole region, the higher the value of δ should be to respect WMAP[6], at the risk to be excluded by EWPT. Once we

[6]We also notice that even more stringent constraints from WMAP could have been applied [538], but we wanted to stay as conservative as possible in our study.

Figure 8.2: Parameter space allowed taking account different type of experiments in the $(m_{\psi_0};M_{Z_D})$ plane. The two plots taking into account the XENON constraints correspond to the $+2\sigma$ and -2σ zones of exclusion.

Figure 8.3: Parameter space allowed within 90 % of C.L. for the CoGeNT signal (blue), DAMA without channeling (red), with channeling (green), CRESST (black), and the exclusion region depending on the hypothesis concerning L_{eff} (se the text for details).

consider the exclusion limit by XENON100, the only points surviving are the ones lying exactly on the pole line $M_{Z_D} = 2m_{\psi_0}$: in this region, one needs on the contrary a very small value of δ ($\sim 10^{-3}$) to respect WMAP because of the high annihilation cross section of the process $\psi_0\psi_0 \to Z_D \to \bar{f}f$. Such small values for δ respect XENON100 exclusion limits. One also notices a vertical breaking step in the parameter space for $m_{\psi_0} \simeq 45$ GeV (before applying the XENON100 constraints) which corresponds to the $Z-$pole region. It is interesting to point out that there still exists a bulk region around $m_{\psi_0} \simeq 7$ GeV not yet excluded by XENON and which is able to explain the last CRESST/CoGENT data [499,500] . We should mention that this model has a common feature with the singlet extension of the SM studied in [503, 504] : the same diagram which gives the relic abundance is the one contributing to the nucleon-scattering (from $s-$ to $t-$ channel Z_D exchange). One can demonstrate that these sorts of models usually ensure a large direct detection rate.

8.5 Conclusion and Summary

We showed that the existence of a *dark* $U_D(1)$ gauge sector which interacts with the Standard Model only through its kinetic mixing possesses a valid dark matter candidate respecting accelerator, cosmological and the more recent direct detection constraints. We observed that when one combines direct and indirect detection searches, the restrictions are much more stringent that the ones extracted by precision test measurements. All these experiments are indeed complementary, excluding different parts of the parameter space allowed by WMAP. Moreover, once we take into account all these constraints, only a narrow region of the parameter space along the Z_D-pole region $M_{Z_D} = 2m_{\psi_0}$ survives. There is still a bulk region around $m_{\psi_0} \simeq 7$ GeV which can explain the recent excesses observed in some direct detection experiments. We also showed that a 1ton XENON–like experiment would be able to test more than 90% of the remaining parameter space, being able to exclude (discover?) robustly such models.

Chapter 9

Higgs portal

9.1 Scalar dark matter*

9.1.1 The Model

The simplest extension of the SM is the addition of a real singlet scalar field.Although it is possible to generalize to scenarios with more than one singlet, the simplest case of a single additional singlet scalar provides a useful framework to analyze the generic implications of an augmented scalar sector to the SM. The most general renormalizable potential involving the SM Higgs doublet H and the singlet S is

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{SM} + (D_\mu H)^\dagger (D^\mu H) + \frac{1}{2}\mu_H^2 H^\dagger H - \frac{1}{4}\lambda_h H^4 \\ &+ \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{\lambda_S}{4} S^4 - \frac{\mu_S^2}{2} S^2 - \frac{\lambda_{hSS}}{4} S^2 H^\dagger H \\ &- \frac{\kappa_1}{2} H^\dagger H S - \frac{\kappa_3}{3} S^3 - V_0 \end{aligned} \tag{9.1}$$

where D_μ represents the covariant derivative. We have eliminated a possible linear term in S by a constant shift, absorbing the resulting S-independent term in the vacuum energy V_0. We require that the minimum of the potential occur at $v = 246$ GeV[1]. Fluctuations around this vacuum expectation value are the SM Higgs boson. For the case of interest here for which S is stable and may be a dark matter candidate, we impose a Z_2 symmetry on the model, thereby eliminating the κ_1 and κ_3 terms. We also require that the true vacuum of the theory satisfies $\langle S \rangle = 0$, thereby precluding mixing of S and the SM Higgs boson and the existence of cosmologically problematic domain walls. In this case, the masses of the scalars are

$$m_h = \sqrt{2}\mu_H \qquad m_S = \sqrt{\mu_S^2 + \frac{\lambda_{HSS}}{\lambda_H}\frac{\mu_H^2}{2}} \tag{9.2}$$

and the HSS coupling generated is

[1] We took the convention $H = \frac{1}{\sqrt{2}}(v + h)$.

Figure 9.1: Feynman diagram for the annihilation cross section (left) direct detection scattering (center) and invisible width of the Higgs (right).

$$\mathcal{C}_{hSS}: \quad -\frac{\lambda_{HSS} M_W}{2g}. \tag{9.3}$$

We show in Fig.9.1 the Feynman diagrams relevant for our analysis[2]. Different aspects of scalar singlet extension of the SM has already been studied in [413, 663, 673–683, 742] whereas a nice preliminary analysis of its dark matter consequences can be found in [684]. Some authors also tried to explain the DAMA and/or COGENT excess [687,689,711] whereas other authors probed the model by indirect searches [690–692], or looked at the consequences of earlier XENON data [693–695]

9.1.2 Dark matter phenomenology

The dark matter phenomenology of such model is rich and easy to study as the number of parameters is very reduced. We will than concentrate on the analytical formula for the direct, indirect detection and accelerator constraints on the Higgs mass. More detailed studies can fe found in section(4.10) for indirect detection and (16.1) for accelerator physics and direct detection. We will concentrate more in this section to the formal aspect of the model.

9.1.3 Relic abundance

The annihilation channels responsible for the relic abundance computation and the indirect detection rates can be computed from the Feynman rules given in section (D.2). We can then check that the cross section of the different processes are given by

[2]The quartic coupling SSHH which can be efficient in the computation of the relic abundance if $m_s \gtrsim M_H$ is also present. We obviously took it into account in our numerical analysis but its contribution to the annihilation processes is always subdominant.

$$
\begin{aligned}
\langle \sigma v \rangle_{f\bar{f}} &= \frac{N_c \lambda_{hSS}^2 m_f^2 (M_S^2 - m_f^2)^{3/2}}{16\pi M_S^3 [(4M_S^2 - m_h^2)^2 + m_h^2 \Gamma_h^2]} \\
\langle \sigma v \rangle_{V\bar{V}} &= \frac{\lambda_{hSS}^2 \delta_V [2M_V^4 + (M_V^2 - 2M_S^2)^2](M_S^2 - M_V^2)^{1/2}}{32\pi M_S^3 [(4M_S^2 - m_h^2)^2 + m_h^2 \Gamma_h^2]} \\
\langle \sigma v \rangle_{hh} &= \left(\frac{\lambda_{hSS}}{2\lambda_h} \frac{\left[M_S^2 - \frac{1}{4} m_h^2 \right]^2 m_h^2}{MS^2 m_h^2 - 2M_S^2} + M_S^2 + \frac{m_h^2}{2} \right)^2 \frac{\lambda_{hSS}^2 (M_S^2 - m_h^2)^{1/2}}{16\pi M_S^2 [(4M_S^2 - m_h^2)^2 + m_h^2 \Gamma_h^2]}
\end{aligned}
\tag{9.4}
$$

with Γ_h the *total* width of the Higgs and $\delta_W = 1$ ($\delta_Z = 1/2$).

9.1.4 Direct detection

It is then possible to compute the direct detection rate on a nucleus from the interaction of the scalar dark matter at the parton level on the nucleons N:

$$
\sigma_{S-N} = \frac{\lambda_{hSS}^2}{16\pi m_h^4} \frac{m_N^4 f_N^4}{(M_S + m_N)^2} \tag{9.5}
$$

f_N being the effective coupling of the dark matter to the nucleon N which depends strongly, in the case of the Higgs portal, on the strange composition of the nucleon. More detailed on the subject is given in section (3.7).

9.1.5 Higgs physics

The width of the Higgs is easily computable and is given by

$$
\Gamma_{h \to SS} = \frac{\lambda_{hSS}^2 M_W^2}{32\pi g^2 m_h^2} (m_h^2 - 4M_S^2)^{1/2} \tag{9.6}
$$

Its consequences on the invisible Higgs searches at LHC are explored in section(16.1)

9.1.6 Indirect positron

The indirect detection rates of positron and photons can be deduced directly from formulae (9.4) and are studied in more details in section (4.10)

9.2 Vectorial Dark Matter*

The Higgs sector of the Standard Model offers a unique probe of the hidden sector. In this work, we explore the possibility of renormalizable Higgs couplings to the hidden sector vector fields which can constitute dark matter (DM). Abelian gauge sectors with minimal field content, necessary to render the gauge fields massive, have a natural Z_2 parity. This symmetry ensures stability of the vector fields making them viable dark matter candidates,

while evading the usual electroweak constraints. We illustrate this idea with the Stückelberg and Higgs mechanisms. Vector DM is consistent with the WMAP, XENON100, and LHC constraints, while it can affect significantly the invisible Higgs decay. Due to the enhanced branching ratio for the Higgs decay into the longitudinal components of the vector field, the vector Higgs portal provides an efficient way to hide the Higgs at the LHC. This could be the reason why the latest combined ATLAS/CMS data did not bring evidence for the existence of the Higgs boson.

9.2.1 Vector Higgs portal

The Higgs sector offers a unique opportunity to probe the hidden sector. The operator $\bar{H}H$ is the only dim-2 gauge and Lorentz invariant operator in the Standard model. Therefore renormalizable interactions of the form

$$\Delta\mathcal{L}_{\text{scalar}} = \frac{\lambda_{hs}}{4}\ \bar{H}H\ (S^2 + m_{hs}S) \tag{9.7}$$

are possible, where S is a hidden sector (real) scalar. These are known as the scalar "Higgs portal" (the name was coined in [663]). More generally, one can also have a dim-4 vector Higgs portal, which couples the Higgs doublet to a massive vector field X_μ from the hidden sector,

$$\begin{aligned}\Delta\mathcal{L}_{\text{vector}} &= \frac{\lambda_{hv}}{4}\ \bar{H}H\ X_\mu X^\mu + (\xi_1\ \bar{H}iD_\mu H\ X^\mu \\ &+ \ \xi_2\ \bar{H}H\ i\partial_\mu X^\mu + \text{h.c.})\ ,\end{aligned} \tag{9.8}$$

where the covariant derivative D_μ is taken with respect to the Standard Model (SM) gauge group.[3] X_μ can be associated with a hidden U(1). It becomes massive due to the Higgs or Stückelberg mechanism in the hidden sector.

A hidden vector field can be a good dark matter candidate. Suppose its decay into the hidden sector is kinematically forbidden and the only communication with the SM is given by Eq. 9.8. It is then stable if $\xi_{1,2}$ are zero, in which case the Higgs portal has a Z_2 symmetry $X_\mu \to -X_\mu$. In principle, X_μ can also be made long–lived by adjusting ξ_1 and ξ_2 to be very small. In particular, if there are fields charged under the U(1) apart from those providing longitudinal components of the gauge bosons, these terms are usually generated and it requires non–trivial engineering to make X_μ stable on the cosmological scale. In the absence of such charged fields, the Lagrangian enjoys a natural Z_2 parity which makes X_μ stable. This applies both to the Higgs and the Stückelberg mechanisms. Let us consider examples, starting with the less common Stückelberg dark matter.

Examples: Stückelberg and "Higgsed" dark matter

Consider a U(1) vector field which becomes massive through the Stückelberg mechanism (for a review, see [664]),

$$\mathcal{L}_{\text{St}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 X_\mu X^\mu\ , \tag{9.9}$$

[3]For generality, we have included the $\partial_\mu X^\mu$ term, which may be relevant in models with field–dependent mass terms.

where X_μ is composed of a vector potential X'_μ and a Stückelberg axion–like field ϕ with the following gauge transformation rule

$$\begin{aligned} X_\nu &\equiv X'_\nu + \frac{1}{\mu}\,\partial_\nu\phi\ , \\ \delta X'_\nu &= \partial_\nu\epsilon\ , \\ \delta\phi &= -\mu\epsilon\ . \end{aligned} \tag{9.10}$$

Here μ is a mass scale and ϵ is a gauge transfromation parameter. Note that the Lagrangian possesses a symmetry $X_\mu \to -X_\mu$, which in terms of the original variables reads $X'_\mu \to -X'_\mu$, $\phi \to -\phi$.[4] Taking $\mu = m$ corresponds to choosing a canonical kinetic term for the axion. In general, however, m^2 can be field–dependent, in which case the axion kinetic term involves a function of other fields of the system. Expressing

$$m^2 = \mu^2 f(S)\ , \tag{9.11}$$

with S being a scalar, we recover the axion kinetic term $1/2\ f(S)(\partial_\mu\phi)^2$ in the massless limit $\mu \to 0$. Consider 2 simple possibilities.

(i) One may allow for a Higgs–dependent mass term

$$m^2 = \mu^2\left(1 + \zeta\,\frac{\bar{H}H}{M^2}\right)\ , \tag{9.12}$$

where M is another scale. In this case, the Higgs portal coupling $\bar{H}HX_\mu X^\mu$ is generated with

$$\lambda_{hv} = 2\zeta\,\frac{\mu^2}{M^2}\ . \tag{9.13}$$

A priori neither of these scales is related to the EW scale, nor to each other so that λ_{hv} can be significant.

(ii) Consider another example, where $\bar{H}HX_\mu X^\mu$ is generated through the singlet Higgs portal. Take

$$m^2 = \mu^2\,\frac{S^2}{M^2}\ , \tag{9.14}$$

where S is a hidden sector scalar with a potential $V(S)$ and a Higgs portal coupling $(\lambda_{hs}/4)\bar{H}HS^2$. We have factored out μ^2 in order to have a smooth massless limit. Obviously, integrating out heavy S at tree level produces the required coupling with

$$\lambda_{hv} = 2\lambda_{hs}\,\frac{\mu^2\langle S\rangle^2}{M^2 m_S^2}\ , \tag{9.15}$$

where m_S is the mass of S. Note that λ_{hv} can also be written as $2\lambda_{hs}m_X^2/m_S^2$. The procedure of integrating S out is justified as long as $m_h^2, m_X^2 \ll m_S^2$. In practise, these quantities can differ by, say, an order of magnitude so that λ_{hv} can be as large as $\mathcal{O}(10^{-1} - 1)$. In the same fashion, self–interaction

$$\frac{1}{2}\,\frac{\mu^4\langle S\rangle^2}{M^4 m_S^2}\,(X_\mu X^\mu)^2 \tag{9.16}$$

[4]This symmetry is also preserved by the gauge fixing term.

is also induced. Note also that the physical Higgs boson mixes with S in this example, with the mixing angle being of order $v/\langle S\rangle$.

Furthermore, at one loop, both $\bar{H}H\ X_\mu X^\mu$ and $(X_\mu X^\mu)^2$ are generated with (formally) log–divergent coefficients. Keeping in mind that we are dealing with an effective theory, these are regularized by the cutoff which is expected to be of order M. This implies that λ_{hv} can be taken as a free parameter at low energies.

As the Higgs doublet develops a VEV, an additional mass term $\lambda_{hv}v^2/2$ for the vector is generated. It can be of either sign, as seen from the above examples. Therefore some cancellation between the original Stückelberg mass and the Higgs–induced one is allowed.

It is important that the linear in X_μ terms vanish in these examples,

$$\xi_1 = \xi_2 = 0\ . \tag{9.17}$$

This is because, in the absence of fields charged under U(1) (apart from the axion absorbed in X_μ), the Lagrangian possesses a natural parity

$$X_\mu \to -X_\mu\ . \tag{9.18}$$

Note that here we consider a U(1) "orthogonal" to the SM gauge group, so there is no mixing with hypercharge at tree level, nor is it induced radiatively. Thus, the new vector boson evades the usual electroweak constraints.

Some of the ingredients of this construction are well known in string theory. In particular, most of realistic models (see e.g. [665]) involve an "anomalous" U(1) whose anomaly is cancelled by the Green–Schwarz mechanism. In this process, the U(1) gauge boson becomes massive through the Stückelberg mechanism. The corresponding axion has a dilaton–dependent kinetic term, so that the vector mass term in Planck units is (see e.g. [666])

$$\frac{1}{4s^2}\ \left(\frac{1}{2}\delta\ X_\mu - \partial_\mu a\right)^2\ , \tag{9.19}$$

where a and s are the axion and the dilaton, respectively, and δ is a Green–Schwarz parameter. It is therefore quite natural to expect a field–dependent vector boson mass, although in typical string constructions this mass is close to the Planck scale.

(iii) Finally, one can equally well use the Higgs mechanism in the hidden sector. Suppose we have a U(1) gauge field and a charged complex scalar S, which has a scalar Higgs portal coupling,

$$\mathcal{L}_{\rm Higgs} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu S^* D^\mu S - V(S) + \frac{\lambda_{hs}}{4}\bar{H}HS^*S\ . \tag{9.20}$$

This system enjoys a symmetry

$$\begin{aligned} X'_\mu &\to -X'_\mu\ , \\ S &\to S^*\ , \end{aligned} \tag{9.21}$$

where X'_μ is the massless gauge field.[5] This symmetry is retained in the broken phase, that is, the massive gauge field can be reversed: $X_\mu \to -X_\mu$. The physical Higgs–like excitation

[5]Note that this parity does not generalize to the non–abelian case due to the triple gauge boson vertex.

has no interactions which are odd under $X_\mu \to -X_\mu$, so integrating it out in the unitary gauge will only produce even interactions and $\xi_{1,2} = 0$. On the other hand, $\bar{H}HX_\mu X^\mu$ is generated at both tree and loop level, the latter being log–divergent. Note that the vector field remains light as long as $g^2 \ll \lambda_S$, where g is the gauge coupling and λ_S is the singlet self–interaction. Otherwise, the analysis is very similar to that of the Stückelberg case.

It is important to remember that these examples are valid within effective field theory. Indeed, we either integrate out a heavy "hidden Higgs" or use higher dimensional operators for the field–dependent kinetic terms. The range of validity of the effective description can be estimated from unitarity considerations. The vector portal coupling together with the m^2 mass term from the hidden sector results in

$$\Delta\mathcal{L}_X = \frac{1}{2}m_X^2\Big(1 + a\,\frac{2h}{v} + b\,\frac{h^2}{v^2}\Big)X_\mu X^\mu \ , \tag{9.22}$$

where the Higgs field is decomposed as $H^0 = v + h/\sqrt{2}$; $m_X^2 = m^2 + \frac{1}{2}\lambda_{hv}v^2$ and $a/\sqrt{2} = b = \frac{\lambda_{hv}v^2}{4m_X^2}$. This form is useful for comparison with the Standard Model Higgs–vector scattering. The tree level amplitude for the process $hh \to XX$ (and $hX \to hX$) grows with energy as $\mathcal{A} \sim (a^2 - b)\frac{E^2}{v^2}$ for $E \gg m_X$. Note that in the Standard Model an analogous amplitude vanishes, whereas here it is proportional to the "non–Higgs" mass term m^2. Requiring the partial wave amplitude a_0 to be less than 1/2 (see e.g. [667]) and ignoring order 1 factors, we find that the unitarity cutoff is of order

$$E \sim \sqrt{\frac{16\pi}{\lambda_{hv}}}\frac{m_X^2}{m} \ . \tag{9.23}$$

Above this energy, additional states should be taken into account. Further, the amplitude for $XX \to XX$ proceeding through the Higgs exchange approaches a constant at $E \gg m_h, m_X$, which imposes $m_X > \lambda_{hv}v/\sqrt{8\pi}$. Finally, in the presence of X self–interaction, unitarity in $XX \to XX$ requires roughly $E < (8\pi/\lambda)^{1/4}m_X$, where λ is the X quartic coupling.

9.2.2 Phenomenology

The relevant Z_2–symmetric Lagrangian is given by

$$\mathcal{L} = \frac{\lambda_{hv}}{4}\ \bar{H}H\ X_\mu X^\mu + \frac{1}{2}m^2 X_\mu X^\mu + \frac{\lambda}{4}(X_\mu X^\mu)^2 \ . \tag{9.24}$$

Note that upon EW symmetry breaking the mass of the vector field is $m_X = \sqrt{m^2 + \lambda_{hv}v^2/2}$. In what follows, we will take an agnostic attitude to the origin of the various terms and will simply scan over λ_{hv} and m_X to see their phenomenological implications (see also [742]). We set $\lambda = 0$ for simplicity and allow for λ_{hv} up to order one, which is consistent with perturbativity up to 10 TeV or so. Since the relevant rates depend on λ_{hv}^2, in the plots we will only display the absolute value of λ_{hv}, whereas it is understood that its contribution to m_X can be of both signs. Finally, reliable computations can be made as long as the unitarity constraint (9.23) is satisfied. For light X, we require the theory to be valid up to the EW scale $\sim v$, whereas for heavy X we take the cutoff to be of order $2m_X$.

Figure 9.2: Feynman diagrams for DM annihilation (left), direct detection (center) and invisible Higgs decay (right).

In our numerical analysis, we have adapted the code Micromegas [716] to our model in order to compute the thermal WIMP relic abundance. We have also modified the direct detection cross section to take into account recent lattice results on the quark structure of the nucleon, which appear more reliable than those extracted from the pion–nucleon cross section. Concerning the Higgs physics, we use the code Hdecay [720] to compute the Higgs decay widths.

Direct detection and relic abundance constraints

The Higgs portal coupling allows the vectorial dark matter to annihilate into the light Standard Model species as well as to scatter off nuclei through the Higgs exchange. Fig.9.2 displays the relevant types of Feynman diagrams. (Numerically, we also take into account annihilation through the quartic vertex $X_\mu X^\mu hh$ which can be efficient at $m_X \gtrsim m_h$, although we always find it subdominant.)

Some aspects of the analysis are similar to those of the scalar singlet extension of the SM. The latter was studied in [672–686, 705, 742]. Using this framework, some authors have attempted to explain the DAMA and/or COGENT excess [687,689,711]. Others have studied its implications for indirect searches [690–692] and constraints from the earlier XENON data [693–695]. In what follows, we analyze the same constraints in the framework of vectorial dark matter. We note that some phenomenological aspects of the vector Higgs portal were studied in [742], while vector dark matter in somewhat different contexts was considered in [696], [697].

The relic abundance of DM is dictated by the s–channel annihilation through the Higgs exchange. For example, the cross section for annihilation into fermions is given by

$$\langle \sigma_{f\bar{f}} v \rangle = \frac{\lambda_{hv}^2 m_f^2}{48\pi} \, \frac{(1 - m_f^2/m_X^2)^{3/2}}{(4m_X^2 - m_h^2)^2} \, . \tag{9.25}$$

The calculational procedure is well known (see e.g. [742]) and here we will only quote the result. The 5σ WMAP–allowed [13] parameter space $\{\lambda_{hv}, m_X\}$ for $m_h = 150$ GeV is shown in Fig. 9.3 between the two "gull–shaped" curves. As in any Higgs portal model, DM annihilation becomes much more efficient around the Higgs pole, $m_h \simeq 2m_X$, which implies lower values of the coupling λ_{hv}. In Fig. 9.3, we also display the XENON100 constraint on

Figure 9.3: Parameter space allowed by WMAP (between the "gull–shaped" curves) and XENON100 for $m_h = 150$ GeV, and prospects for XENON1T.

the DM–nucleon interaction [727]. The spin–independent cross section is given by

$$\sigma^{SI}_{V-N} = \frac{\lambda_{hv}^2}{16\pi m_h^4} \frac{m_N^4 f_N^2}{(m_X + m_N)^2} \ , \tag{9.26}$$

where m_N is the nucleon mass and f_N parametrizes the Higgs–nucleon coupling. The latter subsumes contributions of the light quarks (f_L) and heavy quarks (f_H), $f_N = \sum f_L + 3\times\frac{2}{27} f_H$. There exist different estimations of this factor and in what follows we will use the lattice result $f_N = 0.326$ [717] as well as the MILC results [718] which provide the minimal value $f_N = 0.260$ and the maximal value $f_N = 0.629$. Since the cross section is rather sensitive to f_N, we dispay 3 curves corresponding to these benchmark values. The region above these curves is excluded by XENON100, so the maximal allowed value for λ_{hv} at $m_h = 150$ GeV is a few times 10^{-1}. This is still consistent with WMAP, especially around the resonant annihilation region. We also see that essentially the entire parameter space will be probed by XENON1T. Our results agree well with those of [742] (up to the XENON100 bound which was not available at the time of publication of [742]).

Finally, we have checked that the unitarity constraint (9.23) is satisfied in the entire WMAP–allowed region. For $\lambda_{hv} \sim 10^{-2}$, the cutoff is almost 2 orders of magnitude above m_X.

In what follows, we will use the XENON100 constraint based on the lattice evaluation of f_N.

Figure 9.4: Branching ratio for the Higgs invisible decay as a function of the Higgs mass (top) and the dark matter mass (bottom). The scan is over (m_h, m_X, λ_{hv}) and subject to the WMAP and XENON100 constraints.

Implications for the Higgs search

If the mass of the dark matter particle is less than a half of the Higgs boson mass, dark matter can efficiently be produced in the "invisible" Higgs decay $h \to XX$. This would affect the Higgs decay branching ratios, while leaving the production cross section unchanged. The partial width for the Higgs decay into vector dark matter can be read off from (9.22),

$$\begin{aligned} \Gamma^{\rm inv}_{h\to X_\mu X_\mu} &= \frac{\lambda_{hv}^2 v^2 m_h^3}{256\pi m_X^4}\left(1-4\frac{m_X^2}{m_h^2}+12\frac{m_X^4}{m_h^4}\right) \\ &\times \sqrt{1-4\frac{m_X^2}{m_h^2}}\,, \end{aligned} \tag{9.27}$$

where $v = 174$ GeV. We observe that this width grows fast with the Higgs mass, $\propto m_h^3$. This is because the decay rate into the longitudinal components of the vector field or, equivalently, the would–be Goldstone bosons, grows with energy due to the derivative couplings of the latter. This is to be contrasted with the decay rate into the usual scalars which decreases with the Higgs mass. Thus even a small coupling λ_{hv} can lead to a large invisible branching fraction for a light X. In Fig. 9.4, we show the result of our scan over (m_h, m_X, λ_{hv}) taking into account the WMAP and XENON100 constraints. We find that points with a large, up to 85%, invisible branching ratio survive for low Higgs masses. There is a sharp fall in BR$^{\rm inv}$ around 160 GeV, when the $h \to WW$ mode is in full force. The lightest m_X consistent with WMAP and XENON100 is close to $m_h/2$, in which case λ_{hv} is smaller than the gauge coupling and the invisible decay is subdominant. Nevertheless, BR$^{\rm inv}$ can still be as large as 20-30%.

LHC constraints

The invisible Higgs decay into dark matter can have a significant impact on the Higgs search at the LHC. Since the standard Higgs decay modes such as $\gamma\gamma$, $b\bar{b}$, etc. are not affected, the

Figure 9.5: Branching ratio for the invisible Higgs decay as a function of the Higgs mass (top) and the dark matter mass (bottom). The scan is over (m_h, m_X, λ_{hv}) and subject to the WMAP, XENON100 and ATLAS/CMS constraints.

effective branching ratio for a given SM decay $h \to \alpha\overline{\alpha}$ decreases by a factor of $\mathcal{R}$,

$$\mathrm{BR}(h \to \alpha\overline{\alpha})_{\mathrm{SM+X}_\mu} = \mathcal{R}\ \mathrm{Br}(h \to \alpha\overline{\alpha})_{\mathrm{SM}}\ , \tag{9.28}$$

where

$$\mathcal{R} = \frac{\Gamma^{\mathrm{tot}}_{\mathrm{SM}}}{\Gamma^{\mathrm{tot}}_{\mathrm{SM}} + \Gamma(h \to X_\mu X_\mu)}\ . \tag{9.29}$$

Since the production modes $qq/gg \to h$ remain the same, the "measured" cross section for the process $qq/gg \to h \to \alpha\overline{\alpha}$ is reduced by a factor of $\mathcal{R}$. Consequently, the event rates used in the ATLAS/CMS analysis [540, 541] would be overestimated by $1/\mathcal{R}$ and the actual exclusion limits would be weaker. For example, $m_h \gtrsim 141$ GeV is no longer ruled out by ATLAS/CMS.

Fig. 9.5 displays the result of our scan over m_X and λ_{hv} subject to the WMAP, XENON100 and ATLAS/CMS constraints. For each m_h, we find appropriate m_X and λ_{hv} such that the WMAP and XENON100 constraints are satisfied and recalculate the cross section for $qq/gg \to h \to \alpha\overline{\alpha}$. We find that the invisible decay branching ratio can be as large as 85% for m_h up to 150 GeV. For $m_h > 160$ GeV it drops to 20-30%, in which case the decay is almost SM–like and the corresponding Higgs mass range is largely excluded by ATLAS/CMS. Note, however, that the window $230 - 250$ GeV is still open and consistent with all the constraints.

The lower bound on m_X is determined roughly by $m_X \sim m_h/2$ for the minimal possible m_h, which is about 50 GeV. Higher masses $m_X > m_h/2$, up to TeV, are allowed by all the constraints. However, the invisible decay is inefficient in this case.

Finally, let us remark that if one gives up the assumption of thermally produced vector dark matter, $\mathrm{BR}^{\mathrm{inv}}$ can be even larger.

Comparison with scalar dark matter

Analogous analyses have recently appeared in the framework of scalar dark matter interacting with the Standard Model through the Higgs portal [545,700–703]. Although in many respects

the scalar and vector cases are similar, there is an important difference in the invisible decay of the Higgs. While the DM scattering cross section on nucleons is the same in both cases (for the same couplings and masses),

$$\sigma_{S-N}^{SI} = \frac{\lambda_{hs}^2}{16\pi m_h^4} \frac{m_N^4 f_N^2}{(m_S + m_N)^2} \ , \tag{9.30}$$

the invisible Higgs width for scalars is

$$\Gamma_{h\to SS}^{\rm inv} = \frac{\lambda_{hs}^2 v^2}{64\pi m_h} \sqrt{1 - 4\frac{m_S^2}{m_h^2}} \ , \tag{9.31}$$

where λ_{hs} is the analog of λ_{hv} (see Eq. 9.7). Unlike in the vector case (9.27), the decay rate decreases with the Higgs mass. As we mentioned before, this difference has to do with the nature of massive vector fields, which absorb the would–be Goldstone bosons. This results in a higher decay rate at higher energies, i.e. for a heavier Higgs. The decay rate into vectors is enhanced by m_h^4/m_X^4, which is a large factor for a light X. In the region of interest to us, $m_h \sim 2m_X$ and the enhancement is an order of magnitude. There are also other numerical differences in the scalar versus vector dark matter. For example, WMAP requires roughly twice as large λ_{hv} as λ_{hs}. This is because the annihilation cross section for scalars is

$$\langle \sigma_{f\bar{f}} v \rangle = \frac{\lambda_{hs}^2 m_f^2}{16\pi} \ \frac{(1 - m_f^2/m_S^2)^{3/2}}{(4m_S^2 - m_h^2)^2} \ , \tag{9.32}$$

which is triple that for vectors (due to averaging over polarizations)[6] with the same couplings and masses. Finally, the phase space coefficients in $\Gamma^{\rm inv}$ are different for vectors and scalars. We find that these factors compensate each other and the original esimate $\sim m_h^4/m_X^4$ gives approximately the right answer. To give an example, at $m_h = 170$ GeV the WMAP/XENON100 constraints are satisfied for $\lambda_{hs} = 0.13$ and $m_S = 71$ GeV. The corresponding $\mathrm{BR}^{\rm inv}$ is 2%. The vector counterpart of this example is $\lambda_{hv} = 0.21$ and $m_X = 72$ GeV, which gives $\mathrm{BR}^{\rm inv} = 25\%$. As expected, the invisible decay is an order of magnitude more efficient for the vectors.

Although $\Gamma^{\rm inv}$ for vectors is much larger than that for scalars, the effect on $\mathrm{BR}^{\rm inv} = \Gamma^{\rm inv}/(\Gamma^{\rm inv} + \Gamma^{\rm vis})$ is less significant if the Higgs decays predominantly into dark matter. At $m_h < 150$ GeV, the maximal $\mathrm{BR}^{\rm inv}$ for vectors is 85% while that for scalars is 65% (excluding the special case of very light ~ 5 GeV DM). We illustrate this in Fig. 9.6 which displays $\mathrm{BR}^{\rm inv}$ consistent with WMAP, XENON100 and ATLAS/CMS constraints for the scalar case.[7] Note that, up to the above reservations, the Higgs mass window $230 - 250$ GeV is not available since the invisible decay is inefficient for a heavy Higgs.

9.2.3 Conclusion

We have studied the possibility that dark matter is of vectorial nature and it communicates with the Standard Model through the Higgs portal. We find this possibility well motivated

[6] At low energies, the vector field can also be viewed as a set of 3 types of scalars which can only annihilate with scalars of the same type. This reduces the annihilation cross section by a factor of 3.

[7] The strip at the top corresponds to very light scalar DM studied in [711].

Figure 9.6: Branching ratio for the invisible Higgs decay into scalars as a function of the Higgs mass (top) and the dark matter mass (bottom). The scan is over (m_h, m_S, λ_{hs}) and subject to the WMAP, XENON100 and ATLAS/CMS constraints.

because abelian gauge theories with the minimal field content, which is necessary to render the vector fields massive, possess a natural Z_2 parity making dark matter stable. Vector dark matter is consistent with cosmological and direct detection constraints, and can lead to invisible Higgs decay. The latter is enhanced compared to the scalar dark matter case and provides an efficient way to hide the Higgs at the LHC. A combination of sensitive direct detection experiments (XENON1T) and further collider searches will probe most of parameter space of this scenario.

Chapter 10

Z' portal

The addition of extra $U(1)'$ gauge symmetries, associated to new Z' massive neutral gauge bosons, is among the well-motivated extensions of the Standard Model (SM) (for a review see e.g. [548, 592]). Extra $U(1)'$ are not only predicted by some Grand Unified Theories (GUT), like E_6 or higher rank simple groups, but also appear systematically in string-inspired braneworld constructions [549]. As the SM particles do not have to be charged under the extra $U(1)'$, it is possible that a dark matter (DM) candidate lies in the extended gauge sector. In such a framework, a Z' may act as messenger between the visible sector (which contains the SM particles) and a hidden sector (to which DM belongs), as for instance in the kinetic mixing portal scenario [552,593] (alternative scenarios are discussed, e.g, in [553,554]) . It has been shown that kinetic mixing is severely constrained by direct [555] or indirect detection [556] and that only some combination with scalar sector can relax the bounds [557].

The LHC experiments have set strong bounds on the mass of a Z' coupled to SM particles, $M_{Z'} \gtrsim 2.5$ TeV depending on the model [558, 559]. However, these studies consider only couplings to quark or leptons, and the LHC constraints would be (somewhat) relaxed if the Z' had non-negligible couplings to dark matter. At the same time the XENON100 [560] and, more recently, LUX [561] collaborations have set strong limits on spin-independent (SI) elastic cross section of dark matter with nucleons, whereas the PICASSO [562] and COUPP [563] collaborations are constraining spin-dependent (SD) interactions.

Clearly, if the Z' also couples to dark matter, there is an interesting complementarity between LHC searches of new Z' gauge bosons and direct searches for dark matter. Indeed, for a given Z' mass, the direct detection experiments set upper limits on its couplings to dark matter; the same couplings trigger invisible decay channels for the Z', which make the constraints from LHC experiments weaker. In the present paper we investigate to what extent the constraints from LHC and direct detection are compatible. Some LHC study of invisible Z' in some specific frameworks can be found in [564] whereas some direct detection analysis can be found in [565].

The paper is organized as follows. After a summary of the model under consideration and its effective low energy limit, we establish in section II the relation between the invisible branching ratio of a Z' and the direct detection cross sections (spin-dependent and spin-independent). We combine the exclusion limits set by the direct detection experiments with the LHC bounds on Z' mass and couplings in section III. Some prospects are discussed in the last section.

10.1 Invisible Z' and dark matter

10.1.1 Invisible Z' decay rates and direct detection cross sections

The model

While the existence of extra $U(1)'$ gauge symmetries may be motivated in various ways [592], we will try to be as generic as possible in the present work. To do so, we parametrize as follows the couplings of a Z' gauge bosons to the SM quarks and leptons on one hand and to a dark matter particle χ on the other hand, which we take to be fermionic for concreteness[1],

$$\begin{aligned} \Delta\mathcal{L} \supset \quad & g_D \bar{\chi}\gamma^\mu \left(V_D^\chi - A_D^\chi \gamma^5\right) \chi \; Z'_\mu \\ & + g_D \sum_f \bar{f}\gamma^\mu \left(V_D^f - A_D^f \gamma^5\right) f \; Z'_\mu. \end{aligned} \tag{10.1}$$

Here V_D^f and V_D^χ (A_D^f and A_D^χ) are the vectorial (axial) couplings for the SM fermions f and DM particle χ respectively ($V_D^\chi = 0$ if χ is a Majorana particle) and we have introduced an overall gauge coupling g_D. Since the gauge coupling g_D, as well as $V_D^{f,\chi}$ and $A^{f,\chi}$ are a priori free parameters, our formalism can then describe all the possible Z' realisations in terms of this five quantities. It is anyway possible to fix the combinations $g_D V_D^f$ and $g_D A_D^f$ by specifying the Z' model (e.g. SSM, E_6, B-L,...) as it is shown in table I. The partial widths of the Z' can be straightforwardly determined as function of these four fundamental parameters as:

$$\begin{aligned} \Gamma^i_{Z'} &= \frac{g_D^2 c_i}{12\pi} M_{Z'} \sqrt{1 - \frac{4m_i^2}{M_{Z'}^2}} \\ &\times \left[(V_D^i)^2 \left(1 + 2\frac{m_i^2}{M_{Z'}^2}\right) + (A_D^i)^2 \left(1 - 4\frac{m_i^2}{M_{Z'}^2}\right)\right] \end{aligned} \tag{10.2}$$

where $i = f$ or χ, c_f is the number of color of the SM fermion f and $c_\chi = 1$.

From the microscopic Lagrangian (10.1), we can deduce the effective low energy interaction of the dark matter to a nucleon. The XENON [560] or LUX [561] experiments are supposed to observe dark matter collisions on nuclei, which are translated as couplings to nucleons. These processes occur at velocities of the order of magnitude of that of the Sun in the galactic halo, i.e. $\simeq 300$ km/s so that the energy transferred in such collisions is below the MeV scale for typical dark matter masses. We can thus integrate out the Z' from the Lagrangian (10.1) to get the effective Lagrangian describing interactions between a χ and, say, a proton p,

$$\begin{aligned} \mathcal{L}_{\text{eff}} \supset \quad & \frac{g_D^2}{M_{Z'}^2} \left[V_D^\chi (2V_D^u + V_D^d) \; \bar{\chi}\gamma^\mu \chi \, \bar{p}\gamma_\mu p \right. \\ + \quad & \left. A_D^\chi (\Delta_u^p A_D^u + (\Delta_d^p + \Delta_s^p) A_D^d) \bar{\chi}\gamma^\mu\gamma^5 \chi \, \bar{p}\gamma_\mu\gamma^5 p \right], \end{aligned} \tag{10.3}$$

with a similar expression for the neutron. Here Δ_i^p is the spin content of the quark i in the proton, which may be extracted from lepton-proton scattering data. The values we use are

[1] Our results are similar for a scalar dark matter candidate and will be reported in a future work.

Figure 10.1: Feynman diagrams contributing to the width of the Z' boson and the scattering cross section of the dark matter on a nucleon.

taken from the latest determination of the light quark contributions [566], i.e.

$$\Delta_u^p = 0.842, \quad \Delta_d^p = -0.427, \quad \Delta_s^p = -0.085. \tag{10.4}$$

From Eq.(10.3), we can deduce the Lagrangian describing the interaction of DM with a neutron by isospin symmetry[2]: $2V_D^u + V_D^d \to V_D^u + 2V_D^d$ and $\Delta_u^n = \Delta_d^p$, $\Delta_d^n = \Delta_u^p$, $\Delta_s^n = \Delta_s^p$. As already discussed above, the combinations $g_D V_D^f$ and $g_D A_D^f$ are fixed in a given fundamental theory. For illustration, we report in Table I their values in some popular realizations of Z' models, while a more exhaustive list can be found e.g. in [548].

For the sake of concreteness, we will mainly in this paper refer to the so-called sequential model (noted SSM in the sequel) [592], for which the Z' has the same couplings to SM fermions as the SM Z boson. A possible realization of the SSM appears in constructions with extra dimensions at the weak scale, but in the present work we merely take it as a benchmark model. Other realizations like (B-L) or E_6 Z' do not change drastically our conclusion. Having set the Z' model, the two parameters left free are the $\chi - Z'$ couplings, namely V_D^χ and A_D^χ. We will show in the next section that V_D^χ is severely constrained by SI direct detection experiments, like XENON100 [560] or LUX [561]. On the contrary, A_D^χ can only be tested by SD direct detection experiments, like PICASSO [562], which are much less constraining. It is thus convenient to define a parameter

$$\alpha = A_D^\chi / V_D^\chi \tag{10.5}$$

which determines the nature of the coupling of the Z' with dark matter, $\alpha = 0$ corresponding to a purely vectorial coupling, whereas large α correspond to a mostly axial coupling. Although α is of order unity for a generic model, a vector-like DM candidate would have $\alpha = 0$ while $\alpha = \infty$ for a Majorana DM candidate. For concreteness we will present our results for some definite values, covering the range $0 \leq \alpha \leq 1000$.

[2]Another description of this formalism can be found in [567].

Table 10.1: Vectorial and axial couplings for some Z' models.

channels	$g_D\ V_D^f$ [SSM]	$g_D\ A_D^f$ [SSM]
$e^+\ e^-$	$\frac{g}{4\cos\theta_W}(4\sin^2\theta_W - 1)$	$-\frac{g}{4\cos\theta_W}$
$\nu\ \nu$	$\frac{g}{4\cos\theta_W}$	$\frac{g}{4\cos\theta_W}$
$u\ u$	$\frac{g}{4\cos\theta_W}(-\frac{8}{3}\sin^2\theta_W + 1)$	$\frac{g}{4\cos\theta_W}$
$d\ d$	$\frac{g}{4\cos\theta_W}(\frac{4}{3}\sin^2\theta_W - 1)$	$-\frac{g}{4\cos\theta_W}$
	$g_D\ V_D^f$ [B-L]	$g_D\ A_D^f$ [B-L]
$e^+\ e^-$	$-\sqrt{5/6}\tan\theta\, g$	0
$\nu\ \nu$	$-\sqrt{5/6}\tan\theta\, g$	0
$u\ u$	$\sqrt{5/6}\tan\theta\, g/3$	0
$d\ d$	$\sqrt{5/6}\tan\theta\, g/3$	0
	$g_D\ V_D^f[E_6]$	$g_D\ A_D^f[E_6]$
$e^+\ e^-$	0	$\sqrt{5}\tan\theta\, g$
$\nu\ \nu$	0	$\sqrt{5}\tan\theta\, g$
$u\ u$	0	$\sqrt{5}\tan\theta\, g$
$d\ d$	0	$\sqrt{5}\tan\theta\, g$

Direct detection

From (10.3) we can derive the elastic scattering cross section of χ with a nucleon N, which takes place through t-channel exchange of a Z', as illustrated in Fig.11.1. In the case of SI scattering we may write the effective nucleon scattering cross section as

$$\sigma_{\chi N}^{\rm SI} = \frac{4g_D^4 (V_D^\chi)^2 \mu_{\chi N}^2}{\pi M_{Z'}^4}\left[V_D^u\left(1+\frac{Z}{A}\right) + V_D^d\left(2-\frac{Z}{A}\right)\right]^2 \tag{10.6}$$

where Z and A are the charge number and mass number of of the nucleus that contains the nucleon and $\mu_{\chi N}$ is the nucleon-dark matter reduced mass. Notice that direct detection experiments give their limits on $\chi - N$ elastic scattering cross section assuming isopsin symmetry, $\sigma_{\chi p} = \sigma_{\chi n} = \sigma_{\chi N}$, where p, n stands for the proton and neutron respectively. This explains the presence in Eq.(10.6) of a factor that depends on the atomic number Z and mass number A of the given nucleus[3]. Similarly for Spin Dependent (SD) elastic scattering we obtain

$$\begin{aligned}\sigma_{\chi N}^{SD} = \quad & \frac{12 g_D^4 \mu_{\chi N}^2 |A_D^\chi|^2}{\pi M_{Z'}^4 (S_p^A + S_n^A)^2}\Big[A_D^u(\Delta_u^p S_p^A + \Delta_d^p S_n^A) \\ & + A_D^d\left((\Delta_d^p + \Delta_s^p)S_p^A + (\Delta_u^p + \Delta_s^p)S_n^A\right)\Big]^2\end{aligned} \tag{10.7}$$

S_p^A and S_n^A being the proton and neutron contribution to the nucleus spin.

[3]This implies that in general $\sigma_{\chi p} \neq \sigma_{\chi n} \neq \sigma_{\chi N}$, where the latter is defined in Eq.(10.6).

Combining Eqs.(10.2), (10.6) and (10.7) we can eliminate the parameters V_D^χ and A_D^χ, and express the invisible branching ratio of the Z' as function of physical observables, like the spin dependent and spin independent scattering cross sections:[4]

$$\begin{aligned} Br_\chi &= \frac{\Gamma^\chi_{Z'}}{\Gamma^\chi_{Z'} + \sum_f \Gamma^f_{Z'}} \\ &= \left[1 + \left(\frac{2g_D^2 \mu_{\chi N}}{M_{Z'}^2 \sqrt{\pi}}\right)^2 \frac{\sum_f c_f[|V_D^f|^2 + |A_D^f|^2]}{\sigma^{\rm SI}_{\chi N}/\alpha^{\rm SI}_{Z,A} + \sigma^{\rm SD}_{\chi N}/\alpha^{\rm SD}_{Z,A}}\right]^{-1} \end{aligned} \tag{10.8}$$

with

$$\begin{aligned} \alpha^{\rm SI}_{Z,A} &\equiv \left[V_D^u(1+\frac{Z}{A}) + V_D^d(2-\frac{Z}{A})\right]^2 \\ \alpha^{\rm SD}_{Z,A} &\equiv \frac{4}{(S_p^A + S_n^A)^2}\Big[A_D^u(\Delta_u^p S_p^A + \Delta_d^p S_n^A) \\ &+ A_D^d\left([\Delta_d^p + \Delta_s^p]S_p^A + [\Delta_u^p + \Delta_s^p]S_n^A\right)\Big]^2. \end{aligned} \tag{10.9}$$

It is convenient to re-express Br_χ in terms of α (cf. (10.5)),

$$Br_\chi = \left[1 + \left(\frac{2g_D^2 \mu_{\chi N}}{M_{Z'}^2}\right)^2 \frac{\tilde{c}_F \alpha^{\rm SI}_{Z,A}}{\pi(1+\alpha^2)\sigma^{\rm SI}_{\chi N}}\right]^{-1} \tag{10.10}$$

with $\tilde{c}_F = \sum_f c_f(|V_D^f|^2 + |A_D^f|^2)$. The expression (10.10), which establishes a relation between the direct detection cross section and the invisible branching ratio of the Z', is our main point. It implies that the exclusion limits on $\sigma^{\rm SI}_{\chi N}$ set by XENON100 or LUX give an upper limit on the dark coupling to the Z' and thus an upper limit to its invisible width. The model dependence is entirely encapsulated in V_D^f and A_D^f, which correspond to the SM fermions couplings to the extra $U(1)$ gauge group.

LHC limits

In this section we reexamine the current limits established at the LHC. They rely on searches of dileptons [558, 559] or dijet [568, 569] resonances, and monojet or single photons with missing transverse energy [570, 571]. In this work we will mainly rely on the limits actually set by the LHC collaborations. We are assuming a generic setup in which the Z' couples both to leptons and quarks; as a consequence, among resonance searches, the ones of dileptons turn to be the most constraining [5]. Regarding the searches of missing transverse energy, we notice that most of the analysis of the LHC collaborations are based on an effective

[4] In the expression (10.8) we have assumed $\{m_f, m_\chi\} \ll M_{Z'}$ for sake of simplicity. The approximation is mostly valid in the parameter space we are interested in, but the dependence on the mass of the SM fermions and of the DM is obviously taken into account in our numerical analysis.

[5] The are also realizations in which Z' does not couple to quarks, like e.g. [573]. Our analyis does not apply to such models.

operator approach, namely assuming that new heavy degrees of freedom (the Z' our case) are integrated out. This approximation may not be appropriate in general (see for instance [572]). In our analysis we have estimated possible modification of the limits when the effective approach is not valid while more quantitative results will require a dedicated study. We will notice anyway that in the example considered below, the monojet searches give subdominant constraints.

The existence of an invisible Z' branching ratio weakens the current LHC limits. Indeed, at the partonic level the cross section for $\sigma(pp \to Z' \to \ell\ell)$ has the following Breit-Wigner profile near the resonance,

$$\begin{aligned} \sigma(q\bar{q} \to Z' \to \ell\ell) &\approx \frac{g_D^4}{12\pi}(|V^q|^2 + |A^q|^2)(|V^\ell|^2 + |A^\ell|^2) \\ &\times \frac{s}{(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2}\ , \end{aligned} \tag{10.11}$$

where $\Gamma_{Z'}$ is defined in Eq. (10.2). For a narrow resonance, $\Gamma_{Z'} \ll M_{Z'}$, the Breit-Wigner peak can be approximated by a Dirac delta,

$$\lim_{\epsilon \to 0} \frac{\epsilon}{\epsilon^2 + a^2} = \pi\delta(a), \tag{10.12}$$

so that

$$\begin{aligned} \sigma(q\bar{q} \to Z' \to \ell\ell) &\approx \frac{g_D^4}{12\pi}(|V^q|^2 + |A^q|^2)(|V^\ell|^2 + |A^\ell|^2) \\ &\times \frac{M_{Z'}}{\Gamma_{Z'}}\pi\delta(s - M_{Z'}^2). \end{aligned} \tag{10.13}$$

As $\Gamma_{Z'} \propto g_D^2$ we see that the resonant cross-section scales as g_D^2. Now, the LHC exclusion limits do not take into account the possibility of decay of a Z' into some invisible sector, i.e. it is assumed that $Br_\chi = 0$. To implement an invisible Z' width, we first write the visible width as $\Gamma_{Z'}^{SM} = \sum_f \Gamma_{Z'}^f$, which is the quantity used by the experiments in reporting their limits. Then

$$\frac{M_{Z'}}{\Gamma_{Z'}} = \frac{M_{Z'}}{\Gamma_{Z'}^{SM}}\frac{\Gamma_{Z'}^{SM}}{\Gamma_{Z'}} = \frac{M_{Z'}}{\Gamma_{Z'}^{SM}}(1 - Br_\chi), \tag{10.14}$$

with $\Gamma_{Z'} = \Gamma_{Z'}^\chi + \Gamma_{Z'}^{SM}$. Thus, if all other things are kept constant, the resonant cross section into dileptons is diminished by a factor of $(1 - Br_\chi)$. We will furthermore assume that g_D may vary freely for a generic Z' model (while $g_D = g$ reproducing the ATLAS/CMS limits for the SSM). Taking into account the dependence of the cross section in g_D and a non-zero Br_χ in a generic Z' model, this implies a simple rescaling of the resonant cross section [6]

$$\sigma_{Z'll} \to \left(\frac{g_D}{g}\right)^2 \times (1 - Br_\chi) \times \sigma_{Z'll}. \tag{10.15}$$

[6]Notice that the narrow width approximation is not always appropriated for some Z' realizations, like in particular the SSM, since it may occur that the decay width of the Z' exceeds the detector resolution. In such a case the detector effect can be taken into account by convoluting the Breit-Wigner expression with a Gaussian factor [575], which depends on the width of the Z' and the detector resolution. As a consequence the relation (10.14) should be modified accordingly. We have taken into account this factor in our numerical analysis.

Figure 10.2: Maximum visible branching ratio allowed respecting the limit obtained by ATLAS in the case of the SSM, for $g_D = g$ (red,solid), 0.3 (magenta, dashed) and 0.1 (blue, dotted) as the function of $M_{Z'}/g_D$.

In Fig. (10.2) we have applied this scaling relation to reinterpret the ATLAS exclusion limit in presence of an invisible decay width of the Z'. The new limits have been reported as function of the Z' mass and for different values of g_D. The couplings of the Z' with SM fermions have been assumed to be the ones of the SSM since this is, at the moment, the most constrained Z' model. Thus our limits can be regarded as being conservative. For instance, we can see that, for $g_D = 0.3$, a Z' with a mass of 800 GeV is still allowed by the LHC data, provided its invisible branching ratio is large enough, $Br_\chi \gtrsim 40\%$.

10.1.2 Results

In order to confront the limits on an invisible Z' from resonant dilepton production at the LHC to direct searches for dark matter, we concentrate in this work on spin independent scattering and in particular on the results from the LUX collaboration [561]. The LUX detector, which is operated at the Sanford Underground Research Facility, is made of a dual–phase xenon time–projection chamber. With an exposure of 85.3 live–days the LUX collaboration has set the strongest bounds on the SI elastic scattering cross section of dark matter, $\sigma^{\mathrm{SI}}_{\chi N}$.

For a given dark matter candidate mass, we can translate the exclusion limit set by LUX experiment into an upper bound on the invisible branching ratio of the Z' using Eq.(10.10). We present the result as limits in the plane (m_χ, $M_{Z'}$) in figures (10.3) and (10.4), respectively for for a purely vectorial coupling ($\alpha = 0$) and for a mixture of vector and axial couplings ($\alpha = 10$).

Fixing $M_{Z'}$ and m_χ, the figures give the allowed range of invisible branching ratios. For instance, for $M_{Z'} = 5$ TeV and $m_\chi = 100$ GeV, the LUX limits forbid an invisible branching ratio of the Z' larger than 10 % for a purely vectorial coupling. Indeed, a larger invisible branching ratio would mean a larger coupling of the Z' with the DM and thus a larger $\sigma^{\mathrm{SI}}_{\chi N}$. Clearly, for a heavier Z', the invisible branching may be larger. Although the two plots

Figure 10.3: Minimum values of $M_{Z'}$ according to iso-contours of Br_χ as a function of DM mass, after applying LUX constraints in the case of a pure vectorial coupling ($\alpha = A_D^X/V_D^X = 0$, see the text for details).

share similar features, we notice that a non-zero axial couplings relaxes the limits on the constraints on the invisible branching ratio, since LUX is not very sensitive to such couplings which lead to spin dependent interactions. We further notice that very light dark matter candidate ($\lesssim 3$ GeV) can be consistent with an almost completely invisible light Z' because the constraints set by LUX (and direct detection experiments) are much weaker in this case. On the contrary, for heavier dark matter candidates, and in particular for masses between roughly 50 and 100 GeV, the limits on the invisible branching ratio are very severe, even for a quite heavy Z'. Indeed, this mass range corresponds to the region where LUX is the most sensitive. At the same time, if the invisible branching is small it should be easier to see the Z' at the LHC. It is thus of interest to combine both analyses.

We compare the exclusion limits from the ATLAS collaboration with the ones set by LUX, expressed in terms of the resonant dilepton production and of the Z' mass and for $g_D = g$ and $g_D = 0.3$ in figures (10.5) and (10.6) respectively. One clearly understands from these figures the complementarity between the LHC and direct detection searches: for a given Z' mass, the LHC sets an upper limit on the production cross section while, in our framework, LUX sets a lower limit. This is because a smaller dilepton production corresponds to larger invisible branching ratio, and thus larger coupling to DM. We then understand easily that the LUX constraint gets weaker if the coupling g_D is smaller. A similar conclusion holds if there is substantial axial coupling.

Figure 10.4: Same as Fig. 10.3 but with a large axial coupling, $\alpha = 10$.

The dependence of the direct detection exclusion limits on the dark mass is also illustrated in figures (10.5) and (10.6). For light dark matter (8 GeV for example), the constraints from direct detection are weaker, thus allowing more invisible branching, and a weaker limit on the Z' mass. On the contrary, for a 50 GeV dark matter, the LUX experiment substantially disfavors the presence of an invisible branching ratio, in the case of purely vectorial couplings, such that the LUX line practically overlaps with the prediction of the dilepton production cross section for the SSM without dark matter. In all cases, the allowed parameter range, for a given Z' realization and DM mass, corresponds to the region which is simultaneously below the LHC observational limit (black dashed line in the plots) and above the corresponding LUX exclusion line.

It is of interest to present the above constraints in the plane $\sigma_{SI} - m_\chi$, see figure (10.7). There we show again the fact that, for a given Z' mass, dilepton searches put a lower bound on the Z' coupling to dark matter (which is quantified by V_χ for fixed g_D). By the same token, a larger V_χ leads to a larger SI cross section, which may be excluded by direct searches. This is illustrated for two different ($M_{Z'}$,g_D) pairs. The features in this figure are pretty clear. Obviously there is no constraint from resonant production for $M_{Z'} > 2m_\chi$, since in this case the Z' is fully visible. The spike for $m_\chi \lesssim M_{Z'}/2$ is due to phase-space suppression, which requires a larger value of the coupling to dark matter to be compatible with dilepton searches (the values of V_χ quoted in the figure are for $m_\chi \ll M_{Z'}/2$). Of course there are no constraints from dileptons if the Z' is too heavy to be produced at the LHC (e.g. for

Figure 10.5: Observational limit (black dashed lines) with $1-2\sigma$ uncertainties (green-yellow band) on the production cross section times branching ratio of decay into leptons for a Z' prime particle as reported by ATLAS collaboration. The blue and red lines represent the LUX exclusion limits for $g_D = g$ and two values of the DM mass, namely $m_\chi = 8$ and 50 GeV, assuming pure vectorial (solid lines), namely $\alpha = 0$, and substantially pure axial (dashed lines), $\alpha = 10^3$. The black dash-dotted line represent the prediction of σB for the SSM. As discussed in the text this coincides with the LUX exclusion for $m_\chi = 50$ GeV and pure vectorial couplings since for this value of the mass only a very small invisible branching ratio is allowed.

$M_{Z'} \gtrsim 2.8$ TeV for $g_D = g$), but a lighter Z' is *a priori* possible provided it has a larger coupling to an invisible sector. However this coupling cannot be arbitrarily large. Imposing $g_D^2 V_D^2 \lesssim 4\pi$ gives a lower bound on the Z' mass, which we will refer to as a unitarity bound in the sequel.

Combining all our preceding results, we give in the figure (10.8) the parameter space allowed when combining ATLAS and LUX limits in the plane (m_χ, $M_{Z'}$) and in the case of the SSM. All the points below the lines are excluded by our analysis. We see that a large region does not survive the constraints, as expected from the discussion above. For instance one may conclude from this figure that it is excluded that a Z' lighter than about 2–3 TeV couples to a dark matter candidate with mass $\gtrsim 50$ GeV. The features in Fig.(10.8) are otherwise easy to understand. The plateau for $\alpha = 0$ corresponds to the current bound on $M_{Z'}$ set by the LHC experiments (here ATLAS), while the rise near $M_{Z'} \simeq 2m_\chi$ for the heaviest DM candidates is simply due to a threshold effect as $Br_\chi \to 0$. For light dark matter ($m_\chi \lesssim 10$ GeV) direct detection experiments allow for a larger invisible branching, and thus the LHC bound on the Z' mass can be much weaker. Similarly, for non-zero α, the constraints on invisible branching from LUX are weaker, and so the mass of the Z' may also be smaller.

In the region of the plots where the constraints from both direct detection and dileptons resonance are weak, one may expect that other limits become relevant. In the plot, we show the LEP limits on (non-resonant) dileptons production (dashed horizontal line), which for $g_D = g$ in the SSM model, corresponds to $M_{Z'} \gtrsim 1.7$ TeV [574, 592] (for other couplings, one

Figure 10.6: The same as Fig. (10.5) for $g_D = 0.3$.

may use the fact that this limits scales likes $g_D^4/M_{Z'}^4$). We have also reported the unitarity bound, $M_{Z'} \gtrsim 1.8$ TeV (solid horizontal line), which for the SSM is more constraining than the LEP limit[7]. Notice that these two constraints are particularly strong because we have chosen $g_D = g$ and could be relaxed by considering a weaker couplings of the Z' to leptons. We finally report our estimate of the limit from monojet searches, $M_{Z'} \gtrsim 0.4$ TeV (dot-dashed line) for $g_D^2 V_D^2 = 4\pi$. This bound is weaker than the two other bounds. However, because of the possibility of a resonant Z', notice that our estimate is stronger than the ones reported in *e.g.* [570,571], which rely on an effective operator analysis.[8] Moreover, the limit from monojet searches depends on the interplay between the coupling of the Z' with DM and leptons, so that the monojet constraint could be relevant in other realizations and should not be *a priori* neglected.

Conclusion and Prospects

We have considered a very generic scenario in which a Z' gauge boson couples both to the Standard Model fermions and to a dark matter candidate, which belongs to some invisible sector. We have encompassed many realizations existing in the literature by using a simple parametrization of the couplings of the Standard Model and dark sector to the Z'. We have studied the complementarity between the searches for Z' at LHC and those for dark matter by direct detection experiments. For the sake of illustration, we have confronted on one hand the constraints on the resonant production of dileptons at the LHC and on the other hand the exclusion limits on spin independent elastic scattering set by the LUX collaboration. Our main point is illustrated in Figs. (10.5-10.7), where we show how a Z' with substantial invisible branching ratio is constrained by the current direct detection experiments, and

[7]We remind that this unitarity limit comes from the dileptons analysis, see previous section.

[8]Analogous constraints come from mono-photons plus missing energy, which are however weaker than the monojet ones.

Figure 10.7: Constraints from dilepton searches shown in the plane $\sigma_{SI} - m_\chi$ together with the LUX exclusion limits. See text for details.

LUX in particular. The model dependent character of this statement is illustrated through the parametric dependence in an universal coupling parameter g_D, and a parameter noted α that parametrizes the sensitivity to spin independent scattering collisions in direct detection experiments (see Eq.10.5).

The final result of our analysis is shown in Fig.(10.8) in the case of a natural sequential model, in which all the Standard Model fermions have the same couplings as that to the Z boson. It shows that an electroweak scale dark matter candidate is essentially excluded in such a model, except for heavy Z' ($M_{Z'} \gtrsim 2$ TeV). We also remark that we have been agnostic regarding the mechanism at the origin of the abundance of dark matter. Standard thermal production through annihilations mediated by Z' leads in general to an overabundance of dark matter in the parameter space allowed by our limits. The correct value of the dark matter relic density might be achieved thanks to additional interactions in the hidden sector or due to a non-standard cosmological evolution. In a forthcoming work, we will extend the phenomenological analysis of partially invisible Z' models, including the dark matter abundance and several completions of the collider searches.

10.2 Axial Dark Matter

Models with an extra U(1) are among the simplest and most natural extensions of the Standard Model (SM). They enjoy both the top–down and bottom–up motivation. In particular, additional U(1)'s appear in many string constructions. From the low energy perspective, the coupling between an SM fermions f and a massive gauge boson Z′ [592]

$$\mathcal{L}_{\rm int} = g_f \; Z'_\mu \; \bar{f}\gamma^\mu(a + b\gamma^5)f \; , \tag{10.16}$$

Figure 10.8: Minimum values of $M_{Z'}$ for the SSM when combining LUX and ATLAS bounds in the case of the SSM for different nature of the dark matter coupling to the Z' ($\alpha = 0, 5$ and 10). Also shown is the bound from LEP (dashed horizontal line), the perturbativity bound on the coupling to dark matter (solid horizontal line) and the limit from monojet plus missing energy at the LHC. The points below the lines are excluded.

where g_f, a, b are some constants, represents one of the dimension-4 "portals" (see e.g. [577]) connecting the observable world to the SM–singlet sector. This is particularly important in the context of dark matter models [578]. If dark matter is charged under the extra U(1), the above coupling provides a DM annihilation channel into visible particles. As long as the Z′ has a TeV scale mass and the couplings are not too small, this framework fits the WIMP–miracle paradigm [579]. Recent LHC [558, 559] and direct DM detection constraints [561], however, put significant pressure on this idea since no traces of a Z′ were found in either direct collider searches or DM scattering on nuclei.

In this section, we argue that these negative results may be due to the axial nature of the Z′ and its stronger coupling to dark matter compared to g_f above. In this case, which we call "axial dark matter" (AxDM), DM scattering on nuclei is spin–dependent and weakly constrained. The LHC has limited sensitivity to such a Z′ due to the fact that it decays predominantly into dark matter, as in [580].[9] We thus find that all of the constraints can be satisfied, which adds some credibility to the WIMP paradigm.

[9]We allow a Z′ to couple universally to SM fermions, which distinguishes the model from the leptophobic scenarios (see e.g. [576]).

10.2.1 Axial Z′

In what follows, we consider the possibility that Z′ is purely axial, with the couplings[10]

$$\mathcal{L}_{\rm int}^{\rm eff} = \sum_f g_f \; Z'_\mu \; \bar{f}\gamma^\mu\gamma^5 f + g_\chi \; Z'_\mu \; \bar{\chi}\gamma^\mu\gamma^5\chi \; . \tag{10.17}$$

Here f represents the Standard Model (SM) fermions, χ is a Dirac fermion constituting dark matter and g_f, g_χ are the corresponding Z′ couplings. This Lagrangian represents an effective low energy interaction after heavy particles have been integrated out and the vector boson kinetic terms have been diagonalized. Clearly, the microscopic theory can be made anomaly–free by assigning appropriate charges to fermions (we do not exclude the possibility of having further heavy fermions coupled to Z′).

One may ask how natural it is to have a pure axial–vector interaction. In our opinion, this choice is quite natural given the fact that the photon interaction is purely vector and the axial case is just the other extreme. Also, our considerations hold in the presence of a small vector component of Z′, which may be generated through kinetic mixing [582].

To make our model as simple as possible, we will focus on the case of a universal coupling of Z′ to the SM fermions, g_f. (This assumption can of course be easily relaxed by inserting the fermion–dependent charges.) We then find that cosmological and accelerator constraints require

$$g_f \ll g_\chi \; , \tag{10.18}$$

by a factor of $\mathcal{O}(10)$ to $\mathcal{O}(10^3)$. One would be hesitant to attribute such a hierarchy to the difference in the observable and hidden charges. On the other hand, factors of this type can arise in the system of two U(1)'s mixing with each other. Consider the general Lagrangian describing two massive abelian gauge bosons,

$$\mathcal{L}_{AB} = -\frac{1}{4}(F_A^{\mu\nu})^2 - \frac{a}{2}F_A^{\mu\nu}F_{B\mu\nu} - \frac{1}{4}(F_B^{\mu\nu})^2 + \frac{1}{2}M_1^2 A_\mu^2 + \delta M^2 A_\mu B^\mu + \frac{1}{2}M_2^2 B_\mu^2 \; , \tag{10.19}$$

where A couples only to the dark sector with coupling g_A, while B couples only to the visible sector with coupling g_B. The lighter mass eigenstate would be a mixture of A and B, which couples to both sectors. The hierarchy (10.18) can then be recovered in various limits. For example, it can result from $M_1^2 \ll M_2^2$. For order one kinetic mixing, $a \sim 1$, the Z′ is composed mostly of A and

$$g_\chi = \mathcal{O}(g_A) \quad , \quad g_f = \mathcal{O}\left(\frac{M_1^2}{M_2^2}\, g_B\right) \; . \tag{10.20}$$

Another possibility is to attribute (10.18) to the hierarchy in the couplings, $g_A \gg g_B$. For a small kinetic mixing $a \sim 0$ and large mass mixing $M_1^2 \sim M_2^2 \sim \delta M^2$, the Z′ is a mixture of A and B with

$$g_\chi = \mathcal{O}(g_A) \quad , \quad g_f = \mathcal{O}\left(g_B\right) \; . \tag{10.21}$$

Note that for $M_1^2 \approx M_2^2 \approx \delta M^2$, the mixing is nearly maximal and the second mass eigenstate becomes heavy. In what follows, we will be agnostic as to the origin of the hierarchy (10.18) and will treat the two couplings as free parameters.

[10] An analysis of the axial DM coupling to the usual Z–boson has recently appeared in [581].

10.2.2 Axial dark matter and Z′ phenomenology

In this section, we provide a list of cosmological and accelerator constraints on the model. These set bounds on the two couplings g_f, g_χ and the Z′ mass $m_{Z'}$. In order to understand their qualitative behaviour and compatibility, we provide simple analytic approximations for the observables.

Planck/WMAP and DM annihilation. Suppose that DM is produced thermally, as in the traditional WIMP scenario. The main dark matter annihilation mechanism is the s-channel annihilation into SM fermion pairs. Although we will use the exact tree–level result in our numerical analysis, it is instructive to consider the heavy Z′, $m_{Z'}^2 \gg m_\chi^2$, and zero DM–velocity limit.[11] In this case, the cross section takes on a particularly simple form,

$$\langle \sigma v \rangle = \frac{g_f^2 g_\chi^2}{2\pi}\, c_f \sqrt{1 - \frac{m_f^2}{m_\chi^2}}\, \frac{m_f^2}{m_{Z'}^4} \ , \tag{10.22}$$

where c_f is the number of colors for quarks and 1 for leptons. We see that, for light final state fermions, the cross section is suppressed. The origin of the $m_f^2/m_{Z'}^2$ factor can be understood from (conserved) C-parity considerations. The C-parity of the initial state must be +1 to match that of Z′. Since for a fermion–antifermion pair it is given by $(-1)^{l+s}$ with l and s being the angular momentum and spin quantum numbers, the s-wave initial state ($v \to 0$) must then have $s = 0$. On the other hand, the helicities of the relativistic final state fermions add up to 1. Hence, a spin flip is required leading to the $m_f/m_{Z'}$ dependence. Note however that, for heavy fermions like the top quark, this factor does not lead to significant suppression of the amplitude.

Suppose that DM is sufficiently heavy such that its pair annihilation into top quarks is allowed. Then for $\sqrt{1 - m_t^2/m_\chi^2} \sim 1$, the canonical WIMP annihilation cross section $\sigma v = 3 \times 10^{-26}$ cm^{3}s^{-1} translates into

$$\frac{m_{Z'}}{\sqrt{g_f g_\chi}} \sim 1500 \text{ GeV} \ . \tag{10.23}$$

One should keep in mind that this figure indicates the ballpark of the result and the velocity- as well as m_χ–dependent contributions affect $\langle \sigma v \rangle$, while the definitive answer is given by our numerical analysis.

Direct DM detection. Tree level Z′ exchange leads to spin–dependent DM scattering on nuclei, which is constrained by a number of experiments. For an (approximately) universal Z′ coupling to quarks,

$$\sigma^{\rm SD} = \frac{12 g_f^2 g_\chi^2 m_N^2}{\pi m_{Z'}^4}\, (\Delta_u + \Delta_d + \Delta_s)^2 \ , \tag{10.24}$$

where m_N is the nucleon mass and Δ_i are the quark contributions to the proton spin: $\Delta_u = 0.84$, $\Delta_d = -0.43$, $\Delta_s = -0.09$. Taking 10^{-39} cm^2 as the benchmark bound on $\sigma^{\rm SD}$ for $m_\chi \sim 100$ GeV [**?**], one finds

$$\frac{m_{Z'}}{\sqrt{g_f g_\chi}} > 600 \text{ GeV} \ . \tag{10.25}$$

[11] Numerically, the velocity–independent terms dominate at relatively low $m_{Z'}$, while for a heavier Z′ velocity–dependent contributions are equally important. We choose the limit $v \to 0$ for transparency of our discussion, while using the full result in our numerical analysis.

This bound is satisfied automatically for thermally produced dark matter (see Eq. (10.23)).

Spin–independent DM scattering is generated at one loop with the corresponding amplitude being suppressed both by a loop factor and the quark masses required by a helicity flip. The resulting bound is weak [583].

We note that similar conclusions apply to the DM–nucleon interaction mediated by a pseudoscalar as recently studied in [584].

LEP bounds. Dark matter with $m_\chi > m_t$ cannot be produced on–shell at LEP. However, there are still significant constraints on Z′ due to the effective operators

$$\frac{g_f^2}{m_{Z'}^2}\ \bar{f}_i\gamma^\mu\gamma^5 f_i\ \ \bar{f}_j\gamma_\mu\gamma^5 f_j\ , \tag{10.26}$$

for various fermions f_i and f_j. These operators are constrained by the precise measurements of the cross sections and angular distributions of the final state fermions. In the axial case, the resulting bound is [574]

$$\frac{m_{Z'}}{g_f} > 5\ \text{TeV}\ . \tag{10.27}$$

Comparing this to Eq. (10.23), one finds that Z′ couples much stronger to DM than it does to SM fermions,

$$\alpha = g_\chi/g_f > 10\ . \tag{10.28}$$

Perturbativity. As is clear from the above discussion, the DM–Z′ coupling can become quite strong. Then, our approximation is controllable only if

$$\frac{g_\chi^2}{4\pi^2} < 1\ . \tag{10.29}$$

We do not impose further constraints on the position of the Landau pole of the coupling as we expect our model to be UV–completed already in the multi–TeV range.

Dilepton and monojet LHC bounds. At the LHC, both dark matter and Z′ can be produced on–shell, which leads to strong bounds from CMS and ATLAS. The most important constraint is due to searches for dileptons with a large invariant mass. We will use the CMS Z′ analysis of 3.6 fb^{-1}/8 TeV and 5 fb^{-1}/7 TeV [559] as our benchmark constraint. The result is summarized in Fig.2 (upper right panel) of that paper. For a sequential SM Z′ (SSM), that is having the same couplings as the Standard Model Z-boson, the exclusion limit is around 2.5 TeV. To adapt the results to our case, one must take into account the difference in the Z′ couplings as well as the reduced branching ratio for Z′ decay into visible fermions,

$$\sigma_{l^+l^-} \to \left(\frac{g_f}{g_Z}\right)^2 \text{BR}_{\text{vis}}\ \sigma_{l^+l^-} \tag{10.30}$$

For our estimates it suffices to approximate g_Z by its (universal) axial component, $g_2/(4\cos\theta_W)$. The branching ratio for Z′ decay into SM fermions is

$$\text{BR}_{\text{vis}} \simeq \frac{45 g_f^2}{45 g_f^2 + g_\chi^2\beta^3}\ , \tag{10.31}$$

where $\beta = \sqrt{1 - 4m_\chi^2/m_{Z'}^2}$ accounts for the kinematic suppression in an axial–vector decay (see e.g. [585]). These factors result in the dependence of the number of expected l^+l^- events on g_f and g_χ. The constraints on Z′ relax significantly as g_f decreases and $m_{Z'}$ as light as 500 GeV becomes allowed given it decays predominantly invisibly.

To estimate the resulting LHC bound on $m_{Z'}$, we analytically approximate the l^+l^- production cross section in Fig.2 of [559] and calculate how much it should be reduced to comply with its experimental bound. We find that the result can be cast in the form $m_{Z'} > m_0 + 0.55 \log_{10} [(g_f/0.17)^2 \, \mathrm{BR}_{\mathrm{vis}}]$, for $m_{Z'}$ in TeV and m_0 being an $m_{Z'}$–range dependent constant: $m_0 \simeq (2, 2.3, 2.5)$ TeV for $m_{Z'} \sim (0.5, 1, \geq 1.5)$ TeV. For instance, a 500 GeV Z′ becomes allowed if the l^+l^- cross section reduces by about 3 orders of magnitude, whereas a 2.5 TeV Z′ is allowed with no suppression required.

Z′ models are also constrained by monojet events with large missing energy, which is due to Z′ decay into dark matter. The ATLAS analysis of 10.5 fb^{-1}/8 TeV data [586] imposes the bound on the axial–vector interaction (D8–operator of [750]),

$$\frac{m_{Z'}}{\sqrt{g_f g_\chi}} > 600 - 700 \text{ GeV} \ , \tag{10.32}$$

for $m_\chi \sim 200$ GeV (and a weaker bound for heavier DM). Inclusion of on-shell effects does not make the constraint significantly stronger [580]. As a result, similarly to the direct DM detection constraint, it is satisfied when Eq. (10.23) is imposed.

Combined constraints. The above estimates serve to single out the most important constraints, whose compatibility is to be analyzed. We see that, once the correct DM relic abundance is imposed, the main factors restricting available parameter space are the LHC dilepton bound and perturbativity. Indeed, the LHC constraint can always be satisfied by decreasing g_f, which according to Eq. (10.23) increases g_χ until it hits the perturbative bound (10.29). We find that all of the constraints are in fact compatible. For instance, at $m_{Z'} \sim 500$ GeV, the allowed range of $\alpha = g_\chi/g_f$ spans about two orders of magnitude, from $\mathcal{O}(10)$ to $\mathcal{O}(10^3)$.

To go further, let us remind the reader that our estimate of the DM relic abundance constraint (10.23) is rather simplistic. It does not take into account resonant effects nor those due to thermal averaging. The correct treatment is provided by the numerical package micrOMEGAs [**?**]. Using this tool, we find the same qualitative conclusion: all of the constraints are compatible. In our numerical study, we impose the condition[12]

$$m_{Z'} > 2m_\chi \ , \tag{10.33}$$

which allows for the Z′ decay into dark matter and amounts to "invisibility" of the former. Two representative results are shown in Fig. 1. In the left panel, we set $m_\chi = 250$ GeV, $g_f = 0.005$ and scan parameter space $\{m_{Z'}, g_\chi\}$ satisfying the PLANCK constraint [587]. As $m_{Z'}$ increases from $2m_\chi$, so does g_χ. In this region, the effect of resonant annihilation is important. The resonance is quite broad, on the order of tens of GeV, due to the thermal smearing. Away from the resonance, g_χ quickly turns non–perturbative. The LHC constraint

[12]This condition may not be necessary if the coupling g_f is very small and the Z′ production cross section is suppressed altogether.

(a) (b)

Figure 10.9: Relic density (Planck) and direct search (CMS dileptons) constraints on Z′ for lighter (left panel) and heavier (right panel) dark matter. The area between the two red (solid) lines is consistent with Planck, while the area below the brown (dashed) line is excluded by CMS. Other constraints (LEP, direct DM detection, monojets) are satisfied automatically.

excludes part of the parameter space close to the threshold, where the invisible Z′ decay is inefficient. Further constraints from LEP, direct DM detection and monojets are satisfied automatically in this panel. The result is that the $m_{Z'}$ range 520-560 GeV is allowed, while g_χ varies by two orders of magnitude, from 0.05 to 5.

For heavier DM, the LHC bound becomes less severe and the coupling g_f is allowed to be larger. For example, in the right panel of Fig. 1, the CMS constraint is satisfied everywhere due to the suppressed Z′ production with $g_f = 0.05$. The $m_{Z'}$ range satisfying PLANCK extends over hundreds of GeV.

The pattern observed in this figure is quite general: the allowed parameter space is not far from the resonance region, $m_{Z'} \gtrsim 2m_\chi$, with the latter being relatively broad, $\sim 10 - 20\%\ m_{Z'}$. The LEP, direct DM detection and monojet constraints are satisfied automatically in the region of interest, while the dilepton LHC bound cuts out part of the parameter space. The dark matter candidate, AxDM, belongs to the general WIMP category as it has a TeV scale mass and couplings in the range $\mathcal{O}(10^{-2} - 1)$. A detailed scan of AxDM parameter space is reserved for a subsequent publication.

Conclusion

In this section, we have explored a very simple scenario in which dark matter couples to the SM fermions via an axial Z′ ("axial dark matter"). The model is characterized by 2 couplings as well as the Z′ and DM masses. If the Z′ couples much stronger to the dark sector compared to the visible sector, which may be due to a mixing of two U(1)'s, all the phenomenological constraints can be satisfied. In particular, the LHC constraints are loose due to invisible Z′ decay and allow for $m_{Z'}$ as low as 500 GeV, while DM scattering on nuclei is spin–dependent and thus weakly constrained. The correct DM relic density is obtained in regions not far from the resonance, $m_{Z'} \gtrsim 2m_\chi$. All in all, we find that AxDM is consistent with the thermal WIMP paradigm.

10.3 Conclusion

In the two examples presented in this section, we showed that the Z' portal, contrarily to the Higgs portal, is not yet excluded by combined analysis of direct versus accelerator searches. It mainly comes from the fact that the Z' mass is still a free parameter and has not yet been discovered. This degree of freedom allow to open a broad new region of the parameter space. However, all is not so allowed. The future searches at LHC will be fundamental in this direction, and the invisible Z' searches should be to my mind the next Graal for ATLAS or CMS.

Chapter 11

Anomalies and dark matter**

In April 2012 several authors [606, 607] after analyzing the data of the FERMI satellite claimed the existence of a monochromatic gamma-ray line signal above the background at an energy of about 130 GeV (renormalized to 135 then 133 GeV due to calibration corrections) corresponding to an annihilation rate of around $\langle \sigma v \rangle \simeq 10^{-27}$ cm^3 s^{-1}. There was then excitations in the particle physics community and hundreds of paper appears proposing models able to explain this feature in the spectrum. Why such an excitation? Mainly because the models on the market at this time were quite rare. Indeed, to generate a $\chi\chi\gamma\gamma$ coupling, one needs to pass through loops and then, there should exist a tree level annihilation process giving an annihilation cross section 100 times higher ($\simeq 4\pi^2/g^2$) than the $\gamma\gamma$ final state. If the dark matter is thermally produced, $\langle \sigma v \rangle_{\rm tree} \simeq 10^{-26}{\rm cm}^3{\rm s}^{-1}$ implying $\langle \sigma v \rangle_{\gamma\gamma} \simeq 10^{-28}{\rm cm}^3{\rm s}^{-1}$. This is independent of the model under study. Supersymmetry or minimal extension of the standard model face the same problem. The possibilities to address the issue could be decomposed into three categories:

1) Internal loops generated by charged scalar particles of the "slepton" types. However, to fit with such a high signal, one needs to consider charged scalar of the order of $\simeq 130$ GeV which is the upper bound of LEP data and could be excluded soon at ATLAS detectors.

2) The pole processes where a mediator (pseudoscalar or vectorial) generate the monochromatic line. Some models include NMSSM extensions [589] or Z' ones [590] that we will discuss more in detail below.

3) Internal bremsstrahlung which spectrum is similar to a monochromatic line in the experimental error bar [588] .

In this section, we will be concentrated on the exchange of a Z' which coupling is generated by the condition of anomalies cancelation in an effective approach in the first part and then in a more microscopic framework where the gauge invariance is restored through a Green Schwarz mechanism.

11.1 Extra U(1) as a natural source of monochromatic gamma–ray line

11.1.1 Introduction

One of the most important issues in particle physics phenomenology is the nature and properties of the dark matter in our universe. The observations made by the WMAP collaboration [13] show that the matter content of the universe is dark, making up about 85 % of the total amount of matter. On the other hand, the XENON collaboration recently released its constraints on direct detection of Dark Matter [727] excluding large regions of several extensions of the Standard Model. These constraints makes it plausible that dark matter sits in a different sector, communicating with our sector through new, weak enough interactions.

Neutral gauge sectors with an additional dark $U'(1)$ symmetry in addition to the Standard Model (SM) hypercharge $U(1)_Y$ and an associated Z' gauge boson are among the most natural extensions of the SM, and give the possibility that a dark matter candidate lies within this new gauge sector of the theory. Extra gauge symmetries are predicted in most Grand Unified Theories (GUTs) and appear systematically in string constructions. Larger groups than $SU(5)$ or $SO(10)$ allow the SM gauge group and $U(1)'$ to be embedded into bigger GUT groups. String theory and brane–world $U'(1)$s are special compared to GUT $U'(1)$'s; some of them are hidden, such that SM particles are uncharged under them. For a review of the phenomenology of the extra $U'(1)$s generated in such scenarios see e.g. [592]. In such a framework, the extra Z' gauge boson would act as a portal between the *dark* world (particles not charged under the SM gauge group) and the *visible* sector.

Several papers considered that the key of the portal could be the gauge invariant kinetic mixing $(\delta/2)F_Y^{\mu\nu}F'_{\mu\nu}$ [593]. One of the first models of dark matter from the hidden sector with a massive additional $U'(1)$, mixing with the SM hypercharge through both mass and kinetic mixings can be found in [594]. The Dark Matter (DM) candidate ψ_0 could be the lightest (and thus stable) particle of this secluded sector. Such a mixing has been justified in recent string constructions [595–598], but has also been studied within a model independent approach [599–601] or in a supersymmetric extension [602].

However, there exists another possibility for the Z' portal: the diagrams generated by the Chern-Simons terms, usually related to the mechanism of gauge anomaly cancelation. It has been shown [603, 604] that these vertices could generate a specific smoking-gun signal for dark matter searches: a monochromatic gamma ray line from the Galactic Center [605]. Moreover, the recent hint for such a line[1] [606, 607] raises the hope and interests for such theoretical extensions of the Standard Model (see also [609–611] for very recent discussions about this subject). The purpose of the present note is to prove that such models naturally accomodate both the WMAP data on dark matter and the generation of a monochromatic gamma ray line from the Galactic Center[2].

[1] At 3.3 σ if one takes into account the look elsewhere effect, but currently unconfirmed by FERMI collaboration.

[2] Very few models can achieve such a signal as its production is one–loop suppressed. However, this monochromatic ray can be enhanced in other scenarios like SUSY ones [612], extra-dimension constructions [613], singlet DM [614], decaying DM [615], including a neutrino sector [616], effective DM models [617] or inert Higgs doublet DM [618]. Internal Bremsstrahlung [619] can also exhibit a spectrum similar to the one

This note is organized as follows. After defining the model, we present the phenomenological consequences and study the parameter space which could respect WMAP and simultaneously explain a monochromatic gamma-ray line signal from the Galactic Center.

11.1.2 The model

Gauge invariance is a fundamental condition to ensure renormalizability and quantum consistency of any extension of the Standard Model. Triangle gauge anomalies cancelation can occur by a consistent field theory content or by cancelation of triangle loops by axionic couplings and Chern-Simons terms, via the string theory Green-Schwarz mechanism. At low-energy, remnants of the anomaly cancelation can lead to generalized Chern-Simon terms [639] containing new three gauge boson couplings. If one extend the Standard Model by an abelian gauge group $U'(1)$, at low-energy the particular Chern-Simons terms we are interested in is

$$\mathcal{L}_{CS} = \alpha_1 \ \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu F^Y_{\rho\sigma} + \alpha_2 \ \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu F'_{\rho\sigma} \ , \tag{11.1}$$

where CS stands for Chern-Simons, $F'_{\rho\sigma} = \partial_\rho Z'_\sigma - \partial_\sigma Z'_\rho$ and (α_1,α_2) are the coefficients (computable exactly once given the fermionic content of the model) generated by the triangle diagrams depicted in Fig.11.1. In the loops are running heavy fermions charged under both $U(1)_Y$ and $U'(1)$ (when heavy fermion masses are SM gauge invariant such diagrams are still generated by higher dimensional operators [603, 621]). Notice that the CS terms (11.1) are invariant under electromagnetism[3] $U(1)_A$. The electroweak symmetry breaking then generates $Z'ZZ$, $Z'Z\gamma$ and $Z'Z'Z$ vertices.

Figure 11.1: Triangle diagrams whose variation generate counter terms of the form Eq.11.1

From Eq.11.1 one can deduce the vertices and Feynman rules after the electroweak breaking:

produced by the emission of a monochromatic γ-ray line.

[3]The broken SM symmetries and the $U(1)'$ are realized in a Stueckelberg phase, as explained in [603]. The CS terms can be written in the manifestly gauge invariant way $\frac{i}{M^2}\epsilon^{\mu\nu\rho\sigma} D_\mu\theta(D_\nu H^\dagger H - H^\dagger D_\nu H)(c_1 F^Y_{\rho\sigma} + c_2 F'_{\rho\sigma})$, where $D_\mu\theta = \partial_\mu\theta - g_X Z'_\mu$, where θ is the Stueckelberg axion absorbed by the Z' gauge boson. Moreover M is a mass scale related to the mass of the heavy fermions. After electroweak symmetry breaking, we recover (11.1) with $\alpha_i \sim c_i v^2/M^2$. We notice here that CS terms are also generic in string constructions, where their gauge non-invariance is compensated by axionic couplings and triangle loops of light fermions charged under the extra $U(1)'$. If light fermions are present, the computations of three gauge boson vertices are changed qualitatively, see e.g. [639], [604], but we expect similar results to hold.

$$\Gamma^{\mu\nu\sigma}_{Z'ZZ}(p_3;p_1,p_2) = 2\alpha_1 s_W\ \epsilon^{\mu\nu\rho\sigma}(p_1-p_2)_\rho\ ,$$
$$\Gamma^{\mu\nu\sigma}_{Z'Z\gamma}(p_3;p_1,p_2) = 2\alpha_1 c_W\ \epsilon^{\mu\nu\rho\sigma}(p_2)_\rho\ ,$$
$$\Gamma^{\mu\nu\sigma}_{Z'ZZ'}(p_3;p_1,p_2) = 2\alpha_2\ \epsilon^{\mu\nu\rho\sigma}(p_2-p_3)_\rho\ , \tag{11.2}$$

with the obvious notation $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$.
If ψ is the lighter of the fermions charged under U'(1) (but not under the SM gauge group), coupling to Z' via the vertex

$$\Gamma^{Z'\bar\psi\psi}_\mu(p_3;p_1,p_2) = i\frac{g_D}{4}\gamma_\mu[(q_L+q_R)+(q_L-q_R)\gamma^5]\ , \tag{11.3}$$

where $q_L = q_R = 1$ in what follows, it can be considered to be a good dark matter candidate. The diagrams giving the annihilation rate contributing to the relic abundance are shown in Fig.11.2. Depending on the kinematics and values of the couplings, each of the two diagrams can dominate. Nowadays, the dark matter candidate being mainly at rest, the process $\bar\psi\psi \to Z' \to Z\gamma$ can generate a monochromatic $\gamma-$ray line observable by the FERMI telescope. Depending on the mass of ψ, we describe in detail each possibility in the following section.

Figure 11.2: Feynmam diagrams contributing to the dark matter annihilation and $\gamma-$ray line observable by FERMI telescope

11.1.3 The monochromatic $\gamma-$ray line

Recently, it has been argued that the FERMI telescope did observe a monochromatic γ-ray line from the galactic center around a region $E_\gamma \simeq 130$ GeV, with an annihilation cross section $\langle\sigma v\rangle \simeq 2\times10^{-27}\mathrm{cm}^3\mathrm{s}^{-1}$ [606], which would be a tantalizing smoking gun signal for new physics. Without discussing or anticipating the official analysis of the FERMI collaboration, we will try to check if such a clear signal can be produced by the CS terms generated in Eq.11.1. In what follows, we need to distinguish three different cases: $2m_\psi < M_{Z'} + M_Z$, $m_\psi < M_{Z'} < 2m_\psi - M_Z$ and $M_{Z'} < m_\psi$, for suitable values of the dark matter mass and couplings to fit the supposed γ-ray line. We will show how the first case is strongly disfavored by the astrophysical data and how the latter ones are compatible with very natural values

for the parameters of the model. Even if all the discussion in what follows is qualitative, the numerical analysis has been done using a version of Micromegas [716], adapted to include the new features of the model.

$M_{Z'} > 2m_\psi - M_Z$

In this case, the only annihilation process kinematically allowed is the $s-$channel exchange of a Z' (see the left Fig.11.2). However, the two only final states being ZZ and $Z\gamma$, from the values of the couplings in Eq.11.2 one can easily deduce $\frac{\langle\sigma v\rangle_{ZZ}}{\langle\sigma v\rangle_{Z\gamma}} \simeq 0.3$ which means that the $Z\gamma$ final state is always the dominant one. If one wants to fulfill WMAP constraints for a thermal relic ($\langle\sigma v\rangle \simeq 3\times10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}$), one should impose $\langle\sigma v\rangle_{Z\gamma} \simeq 2\times10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}$. Such a huge cross section would have produced a visible monochromatic line, which is already apriori excluded by the FERMI collaboration. Therefore, the mass range $2m_\psi < M_{Z'} + M_Z$ is disfavored.

$m_\psi < M_{Z'} < 2m_\psi - M_Z$

Figure 11.3: Combinations of Chern-Simon coefficients $g_D \times \alpha_1$ and $g_D \times \alpha_2$ respecting WMAP constraint and producing a monochromatic $\gamma-$ray line around 130 GeV with $\langle\sigma v\rangle_{Z\gamma} \simeq 2\times10^{-27}$ [606] for $M_{Z'} \lesssim 2m_\psi - M_Z$ (case B, see the text for details).

In this case, the opening of a new channel $\bar\psi\psi \to Z' \to Z'Z$ allows the possibility to obtain a relic density obeying WMAP constraint *and* a monochromatic $\gamma-$ray flux observable by FERMI at the same time. In fact, the main difference with the case discussed in the previous subsection consists in the fact that now the contribution to the $\bar\psi\psi \to Z'Z$ process is proportional to the second Chern Simons coefficient of the Eq.11.1. Therefore, it is possible to decouple the two different processes, the annihilation cross section to fulfill WMAP, proportional to α_2, and the annihilation cross section giving a monochromatic signal, proportional to α_1.

The result is shown in Fig.11.3, where we plotted the regions of the parameter space still allowed by WMAP and respecting $0.4 \times 10^{-27}\text{cm}^3\text{s}^{-1} < \langle \sigma v \rangle_{Z\gamma} < 1.09 \times 10^{-27}\text{cm}^3\text{s}^{-1}$ for a dark matter mass $m_\psi = 144.5$ GeV. One can see that the dependence on the parameters can be expressed in terms of the products $g_D \times \alpha_i$. At the same time, as the WMAP constraints require a relatively larger cross section than the one for the monochromatic line, we require a ratio $\simeq 10$ between α_2 and α_1. Therefore, if one consider reasonable values for standard coupling of the Chern Simons terms, $\alpha_2 \simeq 10^{-2}$, from Fig.11.3 one obtains $g_D \simeq 1$ and even stronger (non-perturbative) values for smaller values of α_2. This case is therefore compatible with the data, though a small hierarchy between α_1 and α_2 has to be assumed.

$M_{Z'} < m_\psi$

In this region the main contribution to the dark matter annihilation process comes from the t-channel ψ exchange depicted in the right panel of Fig.11.2. In this case, the relic density condition (independent on α_1 or α_2) is essentially decoupled from the $s-$channel diagram which produces the monochromatic line (Fig.11.2 left). We recall again that it was because the *same* diagram was responsible for the relic abundance and the monochromatic line that for example the region $M_{Z'} > 2m_\psi - M_Z$ has been excluded in the discussion above. Moreover, interestingly in this case, differently from the scenario in which $m_\psi < M_{Z'} < 2m_\psi - M_Z$, our analysis becomes independent on the parameter α_2 as soon as $\alpha_2 \lesssim g_D$.

Figure 11.4: Dark coupling g_D and Chern-Simon coefficient α_1 respecting WMAP constraint and producing a monochromatic $\gamma-$ray line around 130 GeV with $\langle \sigma v \rangle_{Z\gamma} \simeq 2 \times 10^{-27}$ [606] for $M_{Z'} \lesssim m_\psi$ (case C, see the text for details).

In more details, for a given monochromatic line (and so a given m_ψ), the WMAP condition fixes the coupling g_D: the cross-section for $\bar{\psi}\psi \to Z'Z'$ depends only weakly – through a phase space coefficient – on $M_{Z'}$. Then for a given value of the monochromatic annihilation cross section $\langle \sigma v \rangle_{Z\gamma}$, one can deduce the value of α_1 fitting the FERMI data. We made a scan on ($M_{Z'}$, g_D, α_1) and applied the 5σ constraint from WMAP and the annihilation cross section proposed by [606] in the case of an Einasto profile. The result is presented in

Figure 11.5: Example of spectrum observed from a cone within 0.1 radian from the Galactic Center, respecting WMAP and producing a monochromatic γ-ray line around 130 GeV with $\langle\sigma v\rangle_{Z\gamma} \simeq 2\times10^{-27}$.

Fig.11.4. In Fig. 11.5 we present an example of the gamma-ray spectrum obtained from the diagrams shown in Fig. 11.2, for a point in the parameter space respecting WMAP[4].
There are three striking features in Fig. 11.4: first, there is a weak dependence on $M_{Z'}$; secondly, the value of g_D takes natural value for a $U(1)$ coupling (we remind that $g_{EW} = 0.65$); and finally α_1 takes typical one-loop order values $\simeq 10^{-2}$, which are quite consistent with loop contributions generated by triangle diagrams of Fig.11.1.
This range of values is understandable. Indeed, we know that an annihilation cross section $\langle\sigma v\rangle \simeq 3\times10^{-26}\text{cm}^3\text{s}^{-1}$ leads to values of couplings of the order of the electroweak one for a WIMP mass of 100 GeV. As only the dark coupling g_D appears in the annihilation channel (Fig.11.2 right), one expect $g_D \simeq 0.6$ for a WIMP mass of $\sim$ 100 GeV independently on Z' mass (except around the threshold) as one can see in Fig.11.4. Now, if one imposes that $\langle\sigma v\rangle_{Z\gamma} \simeq 10^{-27}\text{cm}^3\text{s}^{-1}$, one can check semi-analitically that $\alpha_1 \simeq g_D/30$ which is effectively what we also observe in Fig.11.4.
It could be interesting to notice that in any of the cases discussed above, the direct detection rate is largely suppressed. Indeed, this rate can become important for a kinetic mixing around $\delta \simeq 10^{-3}$ [601] and could even explain DAMA/CoGENT excess with $\delta \simeq 10^{-2}$. However, in the model we are considering, the "portal" between the dark matter sector and the visible one does not go through this kinetic mixing, but through the tri-vectorial couplings generated in Eq.11.1. In this case, very low values of δ are still allowed, rendering the direct detection (through t-channel Z' exchange) very difficult to observe.
One can also remark that for $M_{Z'} \lesssim M_Z$, the $Z\gamma$ final state channel is kinematically closed. The main decay channel for the Z' is thus through the kinetic mixing with the Z. It is important to check that in this case the kinetic mixing should not be too small to disturb

[4]Micromegas was used for the calculation.

the Big Bang Nucleosynthesis (BBN) problem. A straightforward computation of the Z' lifetime leads to

$$\Gamma_{Z'\to q\bar{q}} \simeq \frac{e^2\delta^2\cos^2\theta_W M_{Z'}}{108\pi} \tag{11.4}$$
$$\Rightarrow \tau_{Z'} \simeq 10^{-22}\frac{8\pi}{e^2\delta^2\cos^2\theta_W}\left(\frac{M_{Z'}}{1GeV}\right) \text{ (seconds)} \quad .$$

One may deduce that for $\delta \gtrsim 10^{-11}$, the Z' decays before one minute and will not affect the BBN processes. Thus, the region $10^{-11} \lesssim \delta \lesssim 10^{-2}$ allows for a safely Z' decay from the BBN and electro-weak precision tests point of view. Moreover, notice that the natural one-loop contribution to the mixing, due to the diagrams induced by CS interaction terms, stays well inside that windows, for interesting (one-loop) values of the couplings α_i and reasonable assumptions[5] for the scale Λ where the theory is completed (for example, the mass M of the heavy fermions).
One should also notice that several constraints from the continuum photons should be taken into account, especially the ones coming from the dwarf galaxies measured by FERMI telescope[6] [623]. Indeed, one have checked that the photons fluxes generated by the subsequent decays of the ZZ' final state (case B) or $Z'Z'$ (the ZZ final state giving weak fluxes, reduced by a factor $\simeq \sin^2\theta_W$ compared to $Z\gamma$) does not exceed the constraints obtained by FERMI collaboration [623].

11.1.4 Conclusion

We have discussed an extension of the Standard Model with an extra $U'(1)$ abelian group, where a three gauge boson couplings $Z'Z\gamma$ is generated from Chern-Simons terms. We studied the different scenarios allowed by this model, for different values of the mass parameters, under the hypothesis that the dark matter candidate is charged only under the extra abelian group. Depending on the ratio between the mass of the dark matter and the mass of the mediator Z', WMAP data constrains, more or less severely, the gauge coupling of the group g_D, but always allowing it to have a very natural value comparable with the usual electroweak ones, independently of the absolute value of the Z' mass. At the same time, for $M_{Z'} < 2m_\psi - M_Z$ and $M_{Z'} < m_\psi$ the model can provide a monochromatic $\gamma-$ray line which can fit a 130 GeV signal at FERMI telescope for a dark matter candidate mass $m_\psi = 144.5$ GeV, again for rather natural values of the Chern-Simons couplings.
Obviously, the model presented is intended to be an effective theory, where the unique effects of the beyond the standard model physics are encoded in the trilinear vector bosons couplings and the presence of a fermionic dark matter candidate. Nonetheless, already at this level it is possible to have a good estimate of the constraints that the new physics should satisfy in order to fit with this dark matter scenario. For example, the CS term in (11.1) contributes, at the loop level, to the mass of the Z gauge boson, which is experimentally known with an accuracy of $2-3$ MeV. An order of magnitude estimate gives $\delta M_Z^2 \sim (\alpha_i^2/16\pi^2)\Lambda^2$, where again Λ is

[5] Stronger constraints on Λ are discussed in the conclusion.
[6] We are grateful to the referee for having pointing us this issue.

an UV cutoff naturally of the order of the fermions generating the CS term. For couplings of order $\alpha_i \sim 10^{-2}$, we find $\Lambda \leq 500$ GeV, which is marginally consistent with limits on vector-like fermions in the Standard Model. It would be interesting to compare the constraints we obtained on α_1 and g_D from our combined WMAP/FERMI analysis with the constraints one could find with the LEP searches through the process $e^+e^- \to Z \to \gamma Z' \to \gamma\gamma Z$ which is a 2γ plus Z final state signature. However, such analysis is beyond the scope of the present paper.

11.2 A clear Dark Matter gamma ray line generated by the Green-Schwarz mechanism

11.2.1 Introduction

Recent experiments have shown that dark matter (DM) makes up about 25% of the Universe's energy budget, but its nature is not yet understood [8,624–626]. A natural candidate is a class of weakly coupled massive particle (WIMP). Such a 100 GeV candidate would naturally give the right order of magnitude for the thermal relic abundance. One of the most studied extensions of the Standard Model (SM) is the Minimal Supersymmetric Model MSSM (mSUGRA in its local version) which extends the matter spectrum thanks to an extension of the space-time symmetries (see [627] and references therein). Alternative ideas have included attempts to study the dark matter consequences of the simplest gauge extension of the SM by adding a hidden sector charged under a new $U_X(1)$ symmetry [628, 629] and some specific signatures in gamma ray telescope were discussed. In [630] it was shown that the Stueckelberg Z' extension of the SM leads to a viable milli-charged dark matter candidate, whereas the authors of [631] invoke a multiplet of states and an extra non-abelian light gauge boson to explain recent experimental anomalies. In several works [631, 632], kinematical mixing is necessary to couple the dark sector to the visible one, and many of these models have exotic matter, including non-vectorlike matter that couples to both SM and hidden sector gauge group, the lighter of which would be the DM candidate. However, none of these works focussed on the consequences of the anomalies induced by these kind of spectra on DM detection. Recently, some works have studied extensively the LHC prospect of such a construction [633–635] whereas one dimension 6 effective operator approach leads to specific astrophysical signals [636]. In this work, we show that, even if the SM particles are not charged under the extra $U_X(1)$, anomalies generated by the heavy fermionic spectrum can generate through the Green-Schwarz mechanism, $XZ\gamma$ effective couplings. This coupling induces a clear γ ray line signal observable by GLAST at energy

$$E_\gamma = m_{DM}\left(1 - \frac{M_Z^2}{4m_{DM}^2}\right), \tag{11.5}$$

where m_{DM} is the WIMP mass. Line emission provides a feature that helps to discriminate against the background. The article is laid out as follows: we first review the motivations and construction of the effective vertex that couples the hidden sector X gauge boson to the visible sector. After this, we compute the relic density generated by the lightest hidden

fermion[7] ψ_{DM} and discuss the detection of the astrophysical signal and the related uncertainties, before concluding and comparing with LHC perspective and the predictions of different models.

11.2.2 Effective description of the model

It is well known that any extension of the SM which introduces chiral fermions with respect to gauge fields suffers from anomalies, a phenomenon of breaking of gauge symmetries of the classical theory at one-loop level. Anomalies are responsible for instance for a violation of unitarity and make a theory inconsistent [637, 638]. For this reason if any construction introduces a new fermionic sector to address the DM issue of the SM, it is vital to check the cancelation of anomalies and its consequences on the Lagrangian and couplings. In this letter, we concentrate on the Green-Schwarz mechanism which arises automatically in string theory settings. The idea is to add to the Lagrangian local gauge non-invariant terms in the effective action whose gauge variations cancel the anomalous triangle diagrams. There exist two kinds of term which can cancel the mixed $U_X(1) \times G_A^{\rm SM}$ anomalies, with $U_X(1)$ being the hidden sector gauge group and $G_A^{\rm SM}$ one of the SM gauge group $SU(3) \times SU(2) \times U_Y(1)$: the Chern Simons (CS) term which couples the $G_A^{\rm SM}$ to the $U_X(1)$ gauge boson, and the Peccei-Quinn (PQ, or Wess-Zumino (WZ)) term which couples the $G_A^{\rm SM}$ gauge boson to an axion. In the effective action, these terms are sometimes called Generalized Chern–Simons (GCS) terms [639]. In order to describe the relevant structure, we can separate the effective Lagrangian into a sum of classically gauge variant and gauge invariant terms[8] [633, 635, 639] :

$$\begin{aligned}
\mathcal{L}_{inv} &= -\frac{1}{4g'^2}F^{Y\mu\nu}F^Y_{\mu\nu} - \frac{1}{4g_X^2}F^{X\mu\nu}F^X_{\mu\nu} - \frac{1}{2}(\partial_\mu a_X - M_X X_\mu)^2 - i\overline{\psi}\gamma^\mu D_\mu\psi \\
\mathcal{L}_{var} &= \frac{C}{24\pi^2}a_X\epsilon^{\mu\nu\rho\sigma}F^Y_{\mu\nu}F^Y_{\rho\sigma} + \frac{E}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}X_\mu Y_\nu F^Y_{\rho\sigma}.
\end{aligned} \tag{11.6}$$

The Stueckelberg axion a_X ensures the gauge invariance of the effective Lagrangian and g_X and $F^X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ are the gauge coupling and field strength of $U_X(1)$. The axion has a shift transformation under $U_X(1)$

$$\delta X_\mu \;=\; \partial_\mu\alpha \quad , \quad \delta a_X \;=\; \alpha\; M_X. \tag{11.7}$$

Notice that our dark matter candidate is expected to be chiral with respect to the dark sector $U_X(1)$, with a mass of the order of M_X because its mass should be generated by the spontaneous breaking of $U_X(1)$. This differs considerably from the result obtained with a leptophylic dark sector [640] which considered vector-like dark matter with a Dirac mass term. A hierarchy between X and the lightest heavy fermion charged under $U_X(1)$ (our natural dark matter candidate) would imply a hierarchy between hidden sector Yukawa and gauge couplings. The hidden fermionic sector being chiral, looking for the effects of mixed anomalous diagrams on DM phenomenology is not an ad-hoc assumption, but a necessity.

[7] which is our natural dark matter candidate whose stability can be ensure by the $U_X(1)$ symmetry.

[8] We will consider $U_X(1) \times U_Y^2(1)$ mixed anomalies throughout the paper to simplify the formulae, the generalization to $U_X(1) \times SU^2(N)$ being straightforward.

Figure 11.6: Three vertices generated respectively by loops diagrams (**a**), Peccei-Quinn (**b**) and Chern–Simons (**c**) terms (Eq. 11.8).

From Eq.(11.6) and following [639], the triangle amplitude $[X_\mu(p_3)Y_\nu(p_1)Y_\rho(p_2)]$ can be decomposed according to

$$\begin{aligned}\Gamma^{ZYY}_{\mu\nu\rho} &= [A_1\ \epsilon_{\mu\nu\rho\sigma}p_2^\sigma + A_2\ \epsilon_{\mu\nu\rho\sigma}p_1^\sigma \\ &+ B_1\ p_{2\nu}\epsilon_{\mu\rho\sigma\tau}p_2^\sigma p_1^\tau + B_2\ p_{1\nu}\epsilon_{\mu\rho\sigma\tau}p_2^\sigma p_1^\tau + B_3\ p_{2\rho}\epsilon_{\mu\nu\sigma\tau}p_2^\sigma p_1^\tau + B_4\ p_{1\rho}\epsilon_{\mu\nu\sigma\tau}p_2^\sigma p_1^\tau \\ &+ C\ \frac{p_{3\nu}}{p_3^2}\epsilon_{\nu\rho\sigma\tau}p_2^\sigma p_1^\tau + E\ \epsilon_{\mu\nu\rho\sigma}(p_2^\sigma - p_1^\sigma)\Big] \qquad (11.8)\end{aligned}$$

where each contribution to the amplitude is as shown in Fig.11.6, and where B_i satisfy

$$\begin{aligned}B_1(p_1,p_2) &= -B_4(p_2,p_1) = -i\frac{g_X g'^2}{8\pi^2}Tr[Q_X Q_Y Q_Y]I_1 \\ B_2(p_1,p_2) &= -B_3(p_2,p_1) = -i\frac{g_X g'^2}{8\pi^2}Tr[Q_X Q_Y Q_Y]I_2 \qquad (11.9)\end{aligned}$$

with Q_i being the $U_i(1)$ charges of the hidden fermions running in the loop, and I_i being finite computable integrals (see the appendix of [639] for details). Whereas A_i are UV cutoff dependent, following [633] the Chern–Simons coefficient can be reabsorbed if we define the coupling $\tilde{A}_i = A_i + (-1)^{i+1}E$. The condition for the conservation of the 3 currents gives us 3 Ward identities

$$\begin{aligned}\tilde{A}_1 &= -p_1.p_2 B_1 - p_1^2 B_2 \\ \tilde{A}_2 &= -p_1.p_2 B_4 - p_2^2 B_3 \\ C &= \tilde{A}_1 - \tilde{A}_2. \qquad (11.10)\end{aligned}$$

We can now express our effective vertices in terms of finite integrals:

$$
\begin{aligned}
\Gamma^{XZZ}_{\mu\nu\rho} &= -2i\frac{\sin\theta_W^2 g_X g'^2 \mathrm{Tr}[\mathrm{Q_X Q_Y Q_Y}]}{8\pi^2 M^2}\Big([\tilde{I}_1\ p_1.p_2 + \tilde{I}_2\ p_1^2]\epsilon_{\mu\nu\rho\sigma}p_1^\sigma - [\tilde{I}_2\ p_2^2 + \tilde{I}_1\ p_1.p_2]\epsilon_{\mu\nu\rho\sigma}p_2^\sigma \\
&+ [\tilde{I}_1\ p_{2\nu} + \tilde{I}_2\ p_{1\nu}]\epsilon_{\mu\rho\sigma\tau}p_2^\sigma p_1^\tau - [\tilde{I}_1\ p_{1\rho} + \tilde{I}_2\ p_{2\rho}]\epsilon_{\mu\nu\sigma\tau}p_2^\sigma p_1^\tau \\
&+ \frac{1}{p_3^2}[\tilde{I}_1\ p_2^2 + \tilde{I}_2 p_1^2 + (\tilde{I}_1 + \tilde{I}_2)p_1.p_2]p_{3\mu}\epsilon_{\nu\rho\sigma\tau}p_2^\sigma p_1^\tau\Big) \qquad (11.11)
\end{aligned}
$$

$$
\Gamma^{XZ\gamma}_{\mu\nu\rho} = -2\left(\cos\theta_W/\sin\theta_W\right)\Gamma^{XZZ}_{\mu\nu\rho} \qquad (11.12)
$$

$$
\Gamma^{X\psi_{DM}\psi_{DM}} = \frac{g_X}{4}\gamma^\mu\left([q_X^R + q_X^L] + [q_X^R - q_X^L]\gamma^5\right) \qquad (11.13)
$$

where we defined $\tilde{I}_i = M^2 I_i$, $\tilde{I}_i$ being a dimensionless integral and M the $U_X(1)$ breaking scale (typically the masses of the hidden fermions running in the loops). This scale can be thought of as coming from effective derivative couplings as was explicitly shown in [635,636]. One might expect a $\gamma\gamma$ decay channel as well, but this decay is forbidden when both outgoing particles are massless. It is straightforward to generalize the formulae in the case of extra fermions charged under $SU(2)\times U_Y(1)\times U_X(1)$. New couplings of the type $\Gamma^{XW^+W^-}_{\mu\nu\rho}$ would be generated, but would not change the general conclusion of our study. Indeed, the annihilation channel $\psi_{DM}\psi_{DM} \to X \to W^+W^-$ would give a spectrum quite similar to the one generated by the annihilation into ZZ [636]. We made the analysis in a specific "next to minimal" $SU(2)\times U_Y(1)\times U_X(1)$ extension and found that generically the perspective of detection are even larger in such scenario than in the minimal $U_X(1)$ extension studied here. Our results are quite conservative in that sense.

Another interesting point is that, in order to generate dynamical masses from the $U_X(1)$ breaking, the hidden fermions (dark matter candidate included) should be chiral with respect to $U_X(1)$ with Weyl charges q_X^L and q_X^R. This situation is quite different from many other works concerning leptophylic dark matter [640]. Indeed, to deal with anomaly free models, the DM candidate is taken to be Dirac, with the Dirac masses having no dynamical origin. In our case, the Green–Schwarz mechanism allows for chiral fermions in the theory. In our specific example, whereas $q_X^L - q_X^R$ would be fixed by the coupling $S\psi_{DM}^L\psi_{DM}^R$, the value $q_X^L + q_X^R$ can be freely chosen. Without any lack of generality (the charges can be reabsorbed in the definition of g_X) we take $q_X^R = -2$ and $q_X^L = 1$ throughout the analysis.

Anomalies in QFT are of great interest for another reason: they could indirectly probe high energy physics and possibly even stringy effects. This is because anomalies can be thought of as both UV and IR effects. They are clearly visible in the limit of low–energy effective field theory which we expect to match to data, yet their UV character implies that the existence (and resolution) of these anomalies can often be tied to a stringy origin. This stringy origin makes the anomaly an interesting candidate for a stringy signature to be observed at colliders or in DM detection experiments. In particular, different string constructions do not have $U_X(1)$ couplings to SM fermions at tree level [641]. With this in mind, studying anomaly generated couplings and their consequences on the dark matter relic density is of fundamental importance. It is also important to mention that even if the spectrum is constructed so the model is anomaly free at the quantum level ($\mathrm{Tr}[\mathrm{Q_X Q_Y Q_Y}] = 0$),

Figure 11.7: Feynman diagrams contributing to the dark matter annihilation.

triangle diagrams anomalies can contribute in a non trivial way to the XYY vertices by higher dimensional operators (even dimension 4 for two extra $U(1)$) [635, 636, 638].

11.2.3 Dark matter phenomenology

Relic abundance

since ψ_{DM} is the lightest fermionic state that is charged under $U_X(1)$, the only allowed annihilation final states at tree level are[9] X or Y. Concerning the relic abundance of ψ_{DM}, we can easily deduce the different annihilation channels kinematically allowed (Fig.11.7) from the effective coupling generated in Eqs (11.11-11.13). Using the symmetries and current conservation invoked above, one can parameterize the whole model by 3 physical parameters : M_X, m_{DM} and $\Lambda_X = M\sqrt{8\pi^2/Tr[Q_X Q_Y Q_Y] g_X^2 g'^2(\tilde{I}_2 - \tilde{I}_1)}$. These low energy parameters would correspond to g_X, Y_h (Yukawa of the lightest fermions in the hidden sector) and $\langle S \rangle$ (vev of the higgs breaking $U_X(1)$) in a dynamical version of the model. Our goal is to outline the parameter space that can be probed by the FERMI telescope.

We modified the micrOMEGAs code [716] in order to calculate the relic abundance of ψ_{DM}. It is easy to fulfill the WMAP 5σ bound [13] on $\Omega_{\rm DM}$ for a wide range of the DM mass as shown in Fig.14.4 in the (M_X, Λ_X) plane for different values of the DM mass (100, 200 and 500 GeV). We can clearly see the s–channel resonance effect of $\psi_{DM}\psi_{DM} \to X \to YY$ around the region $M_X \simeq 2m_{DM}$. Within the band, the relic density respects WMAP constraints whereas below the pole, $\Omega_{DM} < \Omega_{WMAP}$. We also notice a region where the relic density is below WMAP data on the left of the plot, where $M_X \lesssim m_{DM}$. This region corresponds to the opening of the t-channel $\psi_{DM}\psi_{DM} \to XX$ which depends on g_X[10]. For heavier X ($M_X \gtrsim 1$

[9] We will suppose that the X boson is lighter than the chiral matter in the hidden sector. As was emphasized in [633] this is not an unreasonable hypothesis as the mass of the most hidden sector matter will be dominated by higher symmetry breaking scales, and will thus be heavier than X_μ.

[10] It is important to notice here that g_X has been absorbed in the definition of Λ_X to avoid any explicit dependance on the coupling. For simplicity, we took $g_X = 2$ throughout the analysis.

Figure 11.8: Scan on the mass of X (in logarithmic scale) versus the effective scale Λ_X. Colored lines represent the WMAP limits on the dark matter relic density for different values of the dark matter mass.

TeV, $m_{DM} \gtrsim 500$ GeV) there still exist regions of the parameter space respecting WMAP. This occurs for higher values of Λ_X in order to decrease the XYY coupling (proportional to $1/\Lambda_X$) to compensate the increasing of the relic density proportional to m_{DM}.

Indirect detection

The annihilation of WIMP into photons typically proceeds via a complicated set of processes with an upper cutoff at approximately the WIMP mass. A line due to the 2γ or $Z\gamma$ production channel could be observed as a feature in the astrophysical source spectrum [643]. In the most popular models, the branching ratio for the annihilation into lines is typically about 10^{-3} or less. However, some models exhibit specific signatures as high energy rise due to final state radiation [644, 645]. Other dark matter candidates (like the Inert Higgs Dark Matter (IHDM) [646, 647], extra-dimensional chiral square theories [648] or neutralino in SUSY models [649]) can give strong monochromatic signals from $\gamma\gamma$ or $Z\gamma$ final states. However, except in [636], none of these models exhibit only one γ ray line but two or three. In the Green–Schwarz mechanism, $\gamma\gamma$ final state is excluded by spin conservation, we are thus left with one monochromatic line, which would be a clear signature of the model. The spectrum of gamma–rays generated in dark matter annihilations and coming from a direction

forming an angle ψ with respect to the galactic center is

$$\Phi_\gamma(E_\gamma, \psi) = \sum_i \frac{dN_\gamma^i}{dE_\gamma} Br_i \langle \sigma v \rangle \frac{1}{8\pi m_\chi^2} \int_{line\ of\ sight} \rho^2\ dl\ , \tag{11.14}$$

where the discrete sum is over all dark matter annihilation channels, dN_γ^i/dE_γ is the differential gamma–ray yield, $\langle \sigma v \rangle$ is the annihilation cross-section averaged over its velocity distribution, Br_i is the branching ratio of annihilation into final state "i" , and ρ is the dark matter density.

We know that a line due to the $\gamma\gamma$ or Zγ channel could be observed in the astrophysical spectrum [643]. Such an observation is a "smoking gun" signal for WIMP DM as it is difficult to explain by a process other than WIMP annihilation or decay. Different models predicts different ratios for such processes. However, in SUSY or KK DM annihilation the γ line is typically loop suppressed as the main annihilation channel contributing to the thermal relic density dominates and gives a continuous γ spectrum. In some of the parameter space of the IHDM, for DM masses below the W mass, it has been shown in [647] that the two γ lines could be observed. A similar conclusion was found recently for the chiral square model [648]. The main reason is that in both models the $Z\gamma$ and the $\gamma\gamma$ are the only kinematically allowed annihilation channel, with a third line being generated by the KK final state in [648].

In our analysis we use diffuse-model simulated data from the centre annulus (r $\in [20^o, 35^o]$), excluding the region within 15° of the Galactic plane. It has recently been shown that it is possible in this case to minimize the contribution of the Galactic diffuse emission and could give a signal-to-noise ratio up to 12 times greater than at the Galactic Center (GC) [655]. A very interesting feature of excluding the GC in our analysis lies in the fact that our results are quite unsensitive to the different dark matter profile (Einasto [656], NFW [527] or Moore [657]) as their contributions differ largely within the parsec region around the GC. In addition, we use the LAT line energy sensitivity for 5σ detection calculated in [654]. The γ-ray spectrum is calculated using an adapted version[11] of micrOMEGAs [716].

Results and discussion

As an illustrative point, we show in the left panel of Fig.11.9 an example of spectrum from the centre annulus that could be observable by the FERMI telescope, generated by DM annihilation within the pole region respecting WMAP constraint($m_{DM} = 258$ GeV and $M_X = 591$ GeV). We can clearly distinguish a $\gamma-$ray line centered around $E_\gamma = m_{DM}\left(1 - \frac{M_Z^2}{4m_{DM}^2}\right)$ generated by the s-channel resonance $\psi_{DM}\psi_{DM} \to Z\gamma$ above the continuous flux produced by the annihilation process $\psi_{DM}\psi_{DM} \to ZZ/Z\gamma$. We calculated the fluxes generated by the Green-Schwarz mechanism and compared it with the expected sensitivity of FERMI after 5 years of data-taking [650, 654] for 50 energy bins logarithmically distributed between 1 and 300 GeV ($E_k = E_{min}e^{k(E_{max}-E_{min})}$) and a gamma-ray width of 12% of E_γ corresponding to the energy resolution of FERMI. The results are presented in the right panel of Fig.11.9. The Galactic diffuse model used in [654] is the GALPROP "conventional" model discussed in [651]. Indeed, the "optimized" model [652] seems to be disfavored by the non-confirmation

[11]The author wants to thank warmfuly G. Belanger and S. Pukhov for the precious help concerning the modification of the code.

Figure 11.9: Left: example of gamma–ray flux respecting WMAP constraint for a DM mass of 258 GeV. Right: monochromatic $\gamma-$ray fluxes generated by Green-Schwarz mechanism in comparison with expected 5σ and 95% CL sensitivity contours (5 years of FERMI operation) for the conventional background and unknown WIMP energy, for an effective scale $\Lambda_X = 1.5$ TeV

of EGRET excess by the first FERMI released data [653]. We used the results obtained by the full detector Monte Carlo simulation and reconstruction framework detailed in [654].

We clearly see in the right panel of Fig. 11.9 that for $\Lambda_X = 1.5$ TeV, all the parameter space would be observable by FERMI at 95% CL. Indeed, the points that respect the WMAP constraints lie around the pôle $M_X \sim 2m_{DM}$ (see left panel of Fig.14.1) where $\sim 60\%$ of the annihilation rate is dominated by the $Z\gamma$ final state. This proportion still holds for annihilating DM in the Galactic halo and gives a monochromatic line observable by FERMI. We also show what values of Λ_X are accessible by measurement of $\gamma-$ray for different values of DM masses (100, 200 and 300 GeV) in the right panel of Fig. 14.1, where all the points respect the WMAP relic abundance. We can see that for $m_{DM} \gtrsim 100$ GeV, $\Lambda_X \gtrsim 1$ TeV the model still gives a signal observable by FERMI at 95% CL. Obviously, for points lying away from the s-resonance we still have points which can respect WMAP constraint for lower values of Λ_X. All these points (black circles in the right panel of Fig. 14.1) would be observable at 5σ after 5 running years of FERMI.

Using gamma–ray data from a variety of experiments, the authors of [658] have calculated conservative upper limits on the dark matter annihilation cross section to gamma ray lines over a wide range of masses. Combining results from COMPTEL, EGRET, HESS and INTEGRAL from different regions of the sky, and requiring the signal to be as large as the full measured background in an energy bin, they found that, for a background spectrum $d\Phi/dE \sim 1/E^{\alpha}$, the upper limit to the cross section scales as $\langle\sigma v\rangle_{limit} \sim m_{\psi_{DM}}^{3-\alpha}\Delta(\ln E)$, $\Delta(\ln E)$ being the logarithmic energy bin. We checked that this constraint is respected for each point of our parameter space. It is important to emphasize that in the annulus region our result are independent of the halo profile because the lower boundary of this region is 15° from the Galactic Center, and the line of sight integral does not probe the cuspy region. Indeed, an isothermal-cored profile gives roughly the same result in this case.

We want to stress that because of the $\sim 10\%$ energy resolution of the FERMI telescope for 68% containment radius, the distinction of the two lines will be difficult for $m_{DM} \gtrsim 170$

Figure 11.10: Left: scan on (m_{DM}, M_X) plane and $\Lambda_X = 1.5$ TeV. Points respect WMAP constraints. Present experiments limits and FERMI sensitivity after 5 years of data taking are displayed. Right: monochromatic $\gamma-$ray fluxes generated by Green-Scharz mechanism in comparison with expected 5σ and 95% CL sensitivity contours (5 years of FERMI operation) for the conventional background and unknown WIMP energy, for an effective scale $\Lambda_X = 1.5$ TeV

GeV [659]. However, the distinction with other models with distinctive monochromatic $\gamma-$ray lines would still be possible[12]. In the chiral square model, a third line from $\gamma - KK$ excitation final state would be observable [648], whereas the $\gamma\gamma$ and $Z\gamma$ lines in the IHDM should be centered below M_Z, and thus distinguishable by FERMI [647]. The $\gamma-$spectrum generated from 3 body final states [644, 645] would be distinguishable from the $Z - \gamma$ line, whereas lines generated by a neutralino DM should induce a large diffuse spectrum easily observable by FERMI. To complete the analysis, we also checked that a $X - Z$ kinematic mixing respecting electroweak precision test [660] does not affect the $\gamma-$ray line signal, as was already discussed in [636].

We may also calculate the WIMP-nuclei elastic scattering cross section $\sigma_{\psi p}$ relevant for the direct detection of dark matter through its interaction with nuclei in a large detector. The points respecting WMAP constraint would give very low values for $\sigma_{\psi p}$, below the sensitivity of any future experiments. This comes from the fact that the only diagrams contributing to the scattering is the t-channel X exchange. As X is not directly coupled to the quarks, but couples through intermediate virtual Z-exchange, the amplitude is highly loop suppressed as we found in preliminary results. See [661] for a comparative study of other extensions of the SM.

Comparison with LHC

The Large Hadron Collider phenomenology of couplings generated by anomalous extra $U_X(1)$ has recently been studied in the framework of the Green-Schwarz mechanism [633] and higher dimensional operators [635]. The former computed the production cross-section of the X boson from vector boson fusion at the LHC. It was shown that, for the mass range $(M_X \sim 500 - 1000$ GeV), LHC could detect the new physics through $pp \to X \to ZZ \to 4l$

[12]We would like to thank A. Morselli for drawing our attention to this fact.

processes provided $\Lambda_X \sim 100 - 150$ GeV. In the case of $\gamma-$ray detection we showed that in the same framework, the FERMI satellite will be much more efficient and will be able to probe a scale $\Lambda_X \sim 100 - 4000$ GeV. The main reason is that the production of the X boson occurs through vector boson fusion and the qqX coupling is suppressed by a factor $\sim g_X/\Lambda_X^2$, whereas in the case of DM annihilation, the $\psi_{DM}\psi_{DM}X$ coupling is directly proportional to g_X.

11.2.4 Discussion and conclusion

We have studied the phenomenology of a $U_X(1)$ extension of the Standard Model where the SM particles are not charged under the new abelian group and where the Green-Schwarz mechanism ensures that the model is anomaly free. We showed that the dark gauge field X, even though difficult to produce at the LHC has however a clear signature in gamma-ray telescopes. This $U_X(1)$ extension has the unique feature that it generates a monochromatic $\gamma-$ray line from DM annihilation into $Z\gamma$. This is quite different from other models which predicts two lines [647] or three lines [648], or one line radiated by fermions [645]. Such a signature would be a smoking gun signal for these types of constructions[13] if the finite energy resolution of FERMI allows to separate the different lines ($m_{DM} \lesssim 170$ GeV depending on the incident angle). We investigated the scales reachable by the FERMI/GLAST telescope after 5 years of running and showed the a scale $\Lambda_X \sim 1.5$ TeV is easily observable. This scale can be interpreted as the string scale where the Green-Schwarz mechanism cancels the anomalies of the model. Alternatively, the absence of any signals would restrict the scale of such models to lie above $\gtrsim 500$ GeV. Such sensitivity is highly competitive with the LHC.

[13] One should notice that in a specific context with a decaying gravitino DM, there can exist an observable $\gamma-$ray line well below M_{W^+} [662]

Chapter 12

Axionic Dark Matter

Part III

Supersymmetric candidates

Chapter 13

String inspired scenario**

13.1 Supersymmetry Breaking due to Moduli Stabilization in String Theory

This section is mainly based on the work [326]

13.1.1 Introduction

Soon after the introduction of $N = 1$ supergravity as a phenomenological model [266–268] it was recognized that there is a cosmological moduli problem [327]. The problem is manifest whenever there is a weakly (gravitationally) coupled field with a weak scale $\sim \mathcal{O}(1)$ TeV mass and a Planck scale vacuum expectation value. This field may either be related to the mechanism of supersymmetry breaking as in the case of the Polonyi potential [266], or associated with one of many scalars in string theory such as compactification moduli.

In any attempt to resolve the moduli problem, one must first tackle the problem of moduli stabilization. An attractive and often studied mechanism for the vacuum stabilization of moduli is the KKLT mechanism [324]. The KKLT mechanism consists of two basic steps. First of all, one finds a stable supersymmetric anti-deSitter (AdS) vacuum, then this vacuum is uplifted to a de Sitter vacuum with an extremely small positive cosmological constant. One may interpret uplifting either as soft supersymmetry breaking induced by string theory effects [324], or as a D-term contribution [272, 273, 309], or an F-term contribution in N = 1 supergravity [347]. The phenomenology of this class of models has been heavily studied (see e.g. [275, 277–280, 342]). Soft supersymmetry breaking masses arise through a combination of modulus (gravity) and anomaly mediation leading to a pattern of masses with relatively heavy gravitino ($\sim$ 100 TeV) and a compressed scalar-gaugino spectrum around the TeV scale.

However, unless the moduli are very heavy, the cosmological problems associated with their late-time evolution persist. In addition, these models may suffer from a vacuum destabilization during inflation unless the Hubble parameter at the last stage of inflation is smaller than the gravitino mass [325]. In a generalization of the KKLT model based on a racetrack superpotential, also known as the KL model [325], the problem of destabilization is resolved as the moduli in this model are superheavy and allow for significantly higher scales for the

Hubble parameter and a successful implementation of the chaotic inflationary scenario in supergravity [330]. The KL model, however, leads to a very distinct pattern of soft supersymmetry breaking masses with gaugino masses typically much lighter than scalar masses as in models of split supersymmetry breaking [335]. We show that this result is a general consequence of any model in which the moduli are much heavier than the gravitino.

In what follows, we will first describe in Section II the basic ingredients of the KKLT and KL models. We will restrict our attention here to supersymmetry breaking via uplifting. We will also restrict our attention to a minimal Kähler potential for Standard Model (SM) fields, though we will comment on the consequences of generalizing to non-minimal models. In section III, we compare the contributions to soft masses in both the KKLT and KL models. We will show that in the simplest KKLT models, gaugino masses are proportional to the gravitino mass, while in KL models, they are proportional to $m_{3/2}^2/m_\sigma$. Here m_σ is the mass of the volume modulus, describing volume stabilization in string theory; $m_\sigma \gg m_{3/2}$ in the KL scenario. In both classes of models, soft scalar masses are set by $m_{3/2}$, but in the KL model the gaugino masses practically vanishes unless one takes into account quantum corrections. As a consequence, anomaly mediation becomes the first source of breaking terms in the gaugino sector of the theory. In an appendix, we show that this result is not restricted to the KL model, but is a general consequence of any model with a strongly stabilized vacuum, such that $m_\sigma \gg m_{3/2}$. Phenomenological implications of these results and our conclusions are summarized in section IV.

13.1.2 The KKLT and KL models

In this section, we will briefly review the KKLT and KL models. We will couple the KKLT(KL) sector to the SM through gravity and assume minimal $N = 1$ supergravity for the SM sector as a starting point. The KKLT (KL) sector consists of a single chiral field: the modulus ρ, and we will denote SM fields collectively as y. The scalar potential for uncharged chiral superfields in $\mathcal{N} = 1$ supergravity is

$$\begin{aligned} V &= e^K \left(K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2 \right) \\ &= e^G \left(G_i G^{i\bar{j}} G_{\bar{j}} - 3 \right), \end{aligned} \tag{13.1}$$

with

$$D_i W \equiv \partial_i W + K_i W \quad \text{and} \quad G = K + \log |W|^2. \tag{13.2}$$

In string theory, one must consider stabilization of the volume modulus ρ to explain why our universe is 4d rather than 10d. The simplest approach to this issue is based on the KKLT mechanism [324]. As we will see, in this theory, one first finds a stable supersymmetric vacuum with a negative vacuum energy density $V_{\rm AdS}$, and then uplifts it until its vacuum energy becomes positive but negligibly small, about 10^{-120} in Planck units. After uplifting, supersymmetry is broken, and the gravitino mass has a simple relation to the depth of the original AdS minimum [325]:

$$m_{3/2}^2 = |V_{\rm AdS}|/3 \ . \tag{13.3}$$

Thus the mechanism of supersymmetry breaking is built into the new generation of string theory models. One can add to it other mechanisms of supersymmetry breaking, such

the Polonyi mechanism [266], dynamical supersymmetry breaking [319], an O'Raifeartaigh mechanism [285,347] or something else. However, this would make the models more complicated. Therefore, in this paper we will concentrate on the string theory based mechanism of supersymmetry breaking.

For both the KKLT and KL models, we define a Kähler potential as

$$K = -3\log(\rho + \bar{\rho}) + y\bar{y} \ . \tag{13.4}$$

In each case we will assume the superpotential is separable so that

$$W = W(\rho) + W_{SM}(y) \tag{13.5}$$

where $W(\rho)$ is either the KKLT or KL superpotential and $W_{SM}(y)$ is the superpotential for the Standard Model (the subscript SM can be dropped without loss of clarity). From Eqs. (14.11) and (14.13), we can write the scalar potential as

$$V = e^K(|D_\rho W|^2 + |D_y W|^2 - 3|W(\rho) + W(y)|^2) \ . \tag{13.6}$$

Here $|D_\rho W|^2 = K^{\rho\bar{\rho}} D_\rho W \bar{D}_{\bar{\rho}} \bar{W}$. In the remainder of this section, we will ignore the SM contributions to the scalar potential. Along the direction $\sigma = \mathrm{Re}\,\rho$, we can write

$$\partial_\sigma V = \frac{D_\rho W}{3\sigma^2}(\sigma W_{\rho,\rho} - 2W_\rho) \ , \tag{13.7}$$

this equation being valid for any real superpotential W.

The KKLT model is specified by the choice of superpotential

$$W_{KKLT} = W_0 + Ae^{-a\rho} \ , \tag{13.8}$$

where W_0 and $a > 0$ are constants. In this theory, there is a supersymmetry preserving AdS minimum found by setting $D_\rho W = 0$. Since $K_\rho = -3/(2\sigma)$, we can use $D_\rho W = 0$ to solve for W_0 in terms of $\rho = \sigma_0$ corresponding to the minimum of the potential,

$$W_0 = -Ae^{-a\sigma_0}\left(1 + \frac{2}{3}a\sigma_0\right). \tag{13.9}$$

Furthermore, at the minimum,

$$W(\sigma_0) = -\frac{2a\sigma_0}{3}Ae^{-a\sigma_0} \qquad W_\rho(\sigma_0) = -aAe^{-a\sigma_0}, \tag{13.10}$$

and the scalar potential at the minimum becomes

$$V_{\mathrm{AdS}} = V(\sigma_0) = -\frac{a^2A^2e^{-2a\sigma_0}}{6\sigma_0}. \tag{13.11}$$

Using the relation $m_{3/2}^2 = e^G = e^K\,|W|^2$, one finds

$$m_{3/2} = \frac{aA}{3(2\sigma_0)^{1/2}}e^{-a\sigma_0} \ , \tag{13.12}$$

in agreement with Eq. (14.15). As we will see, this relation remains valid after the uplifting of the AdS minimum.

Note that the validity of the KKLT model requires that $\sigma_0 \gg 1$ and $a\sigma_0 > 1$ [324]. In the following, we will assume that $a\sigma_0 \gg 1$, to simplify the analytical calculations. In fact, this condition must be satisfied if we want to find the gravitino mass in the TeV range, which implies that $m_{3/2} \sim 10^{-15}$ in Planck mass units. Indeed, the parameters a and A of the KKLT construction are not expected to be exponentially small. Therefore, according to (13.12), the only way to have a hierarchically small gravitino mass $m_{3/2} \sim 10^{-15} \sim e^{-35}$ is to have $a\sigma_0 \gg 1$. For example, for $a \sim A \sim 1$ and $m_{3/2} \sim 10^{-15}$, Eq. (13.12) yields $\sigma_0 \sim a\sigma_0 \approx 30$.

It is useful to represent the expressions for $m_{3/2}$ in a different but equivalent way. Using (13.10), one finds, for $a\sigma_0 \gg 1$,

$$m_{3/2} = \frac{W_0}{(2\sigma_0)^{3/2}} \ . \tag{13.13}$$

In the example considered above one finds $W_0 \sim 10^{-12}$. This is the same degree of fine-tuning as the one required in the standard Polonyi model.

To find the relation between the mass of the volume modulus and the gravitino mass, one can can use Eq. (13.7), which yields

$$m_\sigma^2 = \frac{\partial_\sigma \partial_\sigma V}{2K_{\rho,\bar\rho}} = \frac{2}{9}(D_\rho W)_\sigma(\sigma W_{\rho,\rho} - 2W_\rho) \tag{13.14}$$

at the supersymmetric minimum. In this case, one can show that the mass of the volume modulus, as well as the mass of its imaginary (axionic) component, is given by

$$m_\sigma = \frac{\sqrt{2\sigma_0}}{3} W_{\rho,\rho} = 2a\sigma_0\, m_{3/2} \ . \tag{13.15}$$

As a result, the mass of the volume modulus is not much greater than the gravitino mass. And this means that the volume stabilization in the KKLT scenario is actually very soft [325].

As mentioned above, the AdS minimum at $\rho = \sigma_0$ must be uplifted. We illustrate the uplifting effect in the Fig. 13.1 in the phenomenologically reasonable case $A = 1$, $a = 1$ and $W_0 = 10^{-12}$ corresponding to a gravitino mass $m_{3/2} = 2 \times 10^{-15}$. The position of the minimum in this case is $\sigma_0 \approx 30.7$.

Uplifting of the AdS minimum induces supersymmetry breaking and is achieved by adding to the potential a term

$$\Delta V \approx |V_{\rm AdS}| \, \frac{\sigma_0^n}{\sigma^n} \ . \tag{13.16}$$

In the original KKLT construction it was assumed that $n = 3$ [324], but according to [286] $n = 2$ in the uplifting term, due to effects related to warping. One may have $n = 3$ if the uplifting occurs due to a D-term [272,273,309]. The choice of n will not affect our qualitative conclusions; for definiteness, we will choose $n = 2$.

Because of the dependence of the uplifting term on σ, the minimum after the uplifting shifts to greater values of σ. Let us denote the shift in σ after uplifting by $\Delta\sigma$. Once again, using Eq. (13.7), we can obtain $\Delta\sigma$ from

$$(D_\rho W)_\sigma(\sigma_0 W_{\rho,\rho} - 2W_\rho)\Delta\sigma = 6V_{\rm AdS}\sigma_0 \ . \tag{13.17}$$

Figure 13.1: Scalar potential of the KKLT model for the values of the parameters $A = 1$, $a = 1$ and $W_0 = 10^{-12}$ before and after uplifting. The potential has been multiplied by a factor of 10^{29} for clarity.

Then, for $a\sigma_0 \gg 1$, one can show that the relative value of the shift is small,

$$\frac{\Delta\sigma}{\sigma_0} = (a\sigma_0)^{-2} \ll 1. \tag{13.18}$$

This shift does not change much the values of the superpotential and the Kähler potential, which depend on σ. Therefore the gravitino mass after the uplifting is correctly represented by the expression (13.12). Similarly, the mass of the volume modulus (13.15) practically does not change during the uplifting.

In $N = 1$ supergravity, ignoring D-term and string theory effects leading to the uplifting, one could use the standard expression for the potential $V = e^K(|D_\rho W|^2 - 3|W|^2)$. Then the existence of the gravitino mass $m_{3/2}^2 = e^K|W|^2$ in a Minkowski vacuum with $V = 0$ would automatically imply that $|D_\rho W|^2 = 3e^{-K}m_{3/2}^2 \neq 0$.

However, because of uplifting, this relation is no longer valid. Since $D_\rho W = 0$ in the supersymmetric AdS vacuum, the value of $D_\rho W$ after uplifting is completely determined by the small shift $\Delta\sigma$. For $a\sigma_0 \gg 1$, it is given by

$$D_\rho W = (D_\rho W)_\sigma \Delta\sigma \simeq W_{\rho,\rho}\Delta\sigma = \frac{3\sqrt{2}}{a\sqrt{\sigma_0}}\, m_{3/2}\ . \tag{13.19}$$

As we see, all of the important parameters, such as W, W_ρ, $D_\rho W$, m_σ, have their scale determined by the gravitino mass, up to some combination of parameters a and σ_0. This fact will be important for us in the next section when we will discuss consequences of the KKLT mechanism for supersymmetry breaking in the observable sector.

The softness of the moduli stabilization in the simplest versions of the KKLT construction leads to a rather unusual problem: the Hubble constant during inflation cannot be much greater than the gravitino mass, $H \lesssim m_{3/2}$ [325]. The reason is that in the simplest KKLT models, the barrier separating the stabilized dS vacuum from the 10d Minkowski vacuum has a height proportional to $m_{3/2}^2$. When the inflationary potential is added to the system, it may lift the dS minimum above the barrier. If this happens, the universe decompactifies and becomes 10-dimensional. Thus, for $m_{3/2} \lesssim 1$ TeV, one must have a very low value of

the Hubble constant at the last stage of inflation in the KKLT based inflationary models. Special effort is required to build inflationary models of this type.

One can try to solve this problem in several different ways, see for example [288–290, 325, 331]. The simplest mechanism involves a slightly generalized KKLT model, which is sometimes called the KL model [325]. In this model, instead of the standard KKLT superpotential (13.8), one uses the racetrack superpotential

$$W_{\rm KL} = W_0 + Ae^{-a\rho} - Be^{-b\rho} \ . \tag{13.20}$$

In contrast to the KKLT case, the new degree of freedom offered by $Be^{-b\rho}$ allows the new model to have a supersymmetric Minkowski solution. Indeed, for the particular choice of W_0,

$$W_0 = -A\left(\frac{a\,A}{b\,B}\right)^{\frac{a}{b-a}} + B\left(\frac{a\,A}{b\,B}\right)^{\frac{b}{b-a}} , \tag{13.21}$$

the potential of the field σ has a supersymmetric minimum with $W(\sigma_0) = 0$, $D_\rho W(\sigma_0) = 0$, and $V(\sigma_0) = 0$, where $\sigma_0 = \frac{1}{a-b}\ln\left(\frac{a\,A}{b\,B}\right)$. In further contrast with the KKLT model, in the KL model there exist another minimum deeper than the supersymmetric one, implying that the SUSY minimum is metastable.

The gravitino mass in the supersymmetric Minkowski minimum vanishes, whereas the mass squared of the field σ at the minimum as well as the mass squared of the imaginary component of the field ρ, is given by [330]

$$m_\sigma^2 = \frac{2}{9}W_{\rho,\rho}^2\sigma_0 = \frac{2}{9}a\,A\,b\,B\,(a-b)\left(\frac{aA}{bB}\right)^{-\frac{a+b}{a-b}}\ln\left(\frac{aA}{bB}\right) \ . \tag{13.22}$$

To understand the implications of this result, let us consider a particular simple choice of parameters $A = B = 1$, $a = 0.1$, $b = 0.05$. For these parameters, one has $m_\sigma \sim 2\times 10^{-3}$, in Planck units, so the field σ is much heavier than the inflaton field, which, in the simplest model of chaotic inflation [291] has mass $m_\phi \sim 6\times 10^{-6}$. This hierarchy of mass scales is one of the necessary conditions which is required to ignore the dynamics of the volume modulus σ during inflation. What is most important here is that the mass m_σ, as well as the height of the barrier separating the Minkowski minimum from the AdS minimum in this model, does not have any relation to the gravitino mass. Therefore one can have inflation in this model for $H \gg m_{3/2}$ [290, 325, 330, 331].

In the KL model discussed so far, supersymmetry is unbroken in the vacuum state corresponding to the minimum of the potential with $V = 0$. The scale of supersymmetry breaking will be determined by a slight perturbation of the superpotential (14.14) by adding to it a small constant ΔW proportional to the weak scale, μ. Independent of the sign of ΔW, the constant shifts the minimum of the potential V from zero to a slightly negative value $V_{\rm AdS} < 0$ at $\sigma_0 + \delta\sigma$. Therefore $V_{\rm AdS}$ in the first approximation must be proportional to $-\Delta W^2$. At the shifted minimum, supersymmetry is preserved, and $D_\rho W(\sigma + \delta\sigma) = 0$. Since $W_\rho(\sigma_0) = 0$, we can write $W_\rho(\sigma_0 + \delta\sigma) = W_{\rho,\rho}(\sigma_0)\delta\sigma$. After some algebra, one finds that the position of the minimum shifts by $\delta\sigma = \frac{3\Delta W}{2\sigma_0 W_{\rho,\rho}}$, and the potential at the minimum becomes [330]

$$V_{\rm AdS}(\Delta W) = -\frac{3(\Delta W)^2}{8\sigma_0^3} = -\frac{3}{8}\left(\frac{a-b}{\ln\left(\frac{aA}{bB}\right)}\right)^3 (\Delta W)^2 \ . \tag{13.23}$$

In this minimum, the value of the superpotential (including the additional constant ΔW), is equal to ΔW up to small corrections $O(\Delta W)^2$. Supersymmetry in the minimum is still unbroken, $D_\rho W = 0$, whereas $W_\rho = \frac{3}{2\sigma_0}\Delta W$. Note that the only "large" quantities here (in Planck units) are σ_0 and $W_{\rho,\rho}$. Therefore, $\delta\sigma \propto \Delta W \propto m_{3/2}$.

Figure 13.2: Scalar potential of the KL model for the values of the parameters $A = B = 1$, $a = 0.1$, $b = 0.05$. The potential has been multiplied by a factor of 10^7 for clarity. The effect of uplifting is so small as compared to the height of the barrier in this model that one cannot distinguish an uplifted and non-uplifted potential on the scale of this figure.

As in the case of the KKLT model, we must add an uplifting potential. After uplifting to the present state with a nearly vanishing vacuum energy, the gravitino mass becomes

$$m_{3/2} = \sqrt{|V_{\rm AdS}|/3} = \frac{1}{2\sqrt{2}}\left(\frac{a-b}{\ln\left(\frac{aA}{bB}\right)}\right)^{3/2} |\Delta W| \ . \tag{13.24}$$

In particular, for $A = B = 1$, $a = 0.1$, $b = 0.05$, one has $m_{3/2} \sim 7 \times 10^{-3}|\Delta W| \sim 3.5\, m_\sigma|\Delta W| \ll m_\sigma$. Since we assume ΔW to be very small (see below), we have $m_{3/2}/m_\sigma \ll 1$. The shape of the potential, V, for this set of parameters is shown in Fig. 13.2. Note that the effect of uplifting is so tiny as compared to the height of the barrier in the KL model that one cannot distinguish an uplifted and non-uplifted potential on the scale of this figure. This helps to understand why the moduli stabilization in the KL model is so much stronger than in the simplest versions of the KKLT construction.

To have $m_{3/2} \sim 1$ TeV $\sim 0.4 \times 10^{-15}$ in Planck units in the model with $A = B = 1$, $a = 0.1$, $b = 0.05$, one should have $|\Delta W| \sim 6 \times 10^{-14}$. This means that to make the gravitino mass comparable to the electroweak scale, we must introduce a small parameter $\sim 10^{-13}$. This is the same degree of fine-tuning as the one required in the standard Polonyi model. Each of these models requires the presence of one very small number. In the KKLT model, it is W_0 that must be tuned small. In the KL model, we need approximately the same small number added to a number of O(1) in the superpotential. In the context of our paper, it is important that the required accuracy of fine-tuning of the superpotential in the KL model is the same as in the simplest KKLT model, and in both cases the goal of fine-tuning is the same: to achieve a very low level of supersymmetry breaking.

Thus it is hard to discriminate between the simplest KKLT models with the superpotential (13.8) and the KL models (14.14) on the basis of fine-tuning. It would be interesting to

check which one of these models is more probable in the landscape, but this would go beyond the scope of the present paper. The main advantage of the KL model is that the mass of the volume modulus can be many orders of magnitude greater than the gravitino mass. This strongly stabilizes the size of the compactified space and allows inflation with very high values of the Hubble constant, $H \gg m_{3/2}$ [290, 325, 330, 331]. Moreover, in the KL model, the presence of the light Polonyi moduli fields is not required, which solves the cosmological moduli problem. In addition, as we will see in the next section (see also [330]), this class of models provides a natural solution of the cosmological gravitino problem. Clearly, the KL model is not the only one where the strong moduli stabilization can be achieved. One may add some new terms to the racetrack superpotential of the KL model, or one may find another, totally different superpotential which allows strong stabilization with $m_\sigma \gg m_{3/2}$. All of these models share certain features to be discussed below.

Uplifting in the KL model induces a shift in the modulus field, which we will denote as $\Delta\sigma$ to distinguish it from the shift $\delta\sigma$ induced by adding the constant ΔW to the superpotential. Eq. (13.17) remains valid in the KL model, and we can use it to find $\Delta\sigma$. The left hand side of Eq. (13.17) should now be evaluated at $\sigma_0 + \delta\sigma$. Then since, $(D_\rho W)_\sigma \approx W_{\rho,\rho}$, we can easily solve for the uplifting shift $\Delta\sigma = \frac{6V_{\rm AdS}}{W^2_{\rho,\rho}}$. One can now show that for any choice of the superpotential which leads to strong volume modulus stabilization with the Kähler potential (14.12), the supersymmetry breaking term $D_\rho W$ in the minimum of the uplifted potential is given by

$$D_\rho W = (D_\rho W)_\sigma \Delta\sigma \simeq W_{\rho,\rho}\Delta\sigma = 6\sqrt{2\sigma_0}\,\frac{m_{3/2}}{m_\sigma}\; m_{3/2}. \tag{13.25}$$

We give the derivation of this general result in the Appendix.

The exact form of this equation will not be important for us. What is important is the following qualitative statement: In all models where the volume modulus can be strongly stabilized, i.e. $m_\sigma \gg m_{3/2}$, the supersymmetry breaking term $D_\rho W$ is strongly suppressed. The validity of this statement is very easy to understand: If the original AdS state is supersymmetric, the supersymmetry breaking term $D_\rho W$ vanishes before the uplifting. If the minimum is strongly stabilized (the potential sharply rises in the vicinity of the minimum), then adding a soft uplifting term $\Delta V \approx |V_{\rm AdS}|\,\frac{\sigma_0^n}{\sigma^n}$ barely affects the value of the field σ in the minimum. As a result, uplifting in theories with strong volume modulus stabilization will keep the term $D_\rho W$ vanishingly small. As we will see, this conclusion may have important implications for particle phenomenology in the context of models with strongly stabilized moduli.

13.1.3 Soft masses in the KKLT and KL models

We now return to Eqs. (14.12) - (13.6) to consider the scalar potential of SM matter fields. Inserting Eqs. (14.12) and (14.13) into (13.6), we can write the scalar potential as

$$\begin{aligned} V &= \frac{e^{y^2}}{8\sigma^3}\Big(-3|W(\rho)+W(y)|^2 + |\bar{y}(W(\rho)+W(y)) + W_y|^2 \\ &+ \frac{4\sigma^2}{3}(D_\rho W(\rho) - \frac{3}{2\sigma}W(y))(\bar{D}_{\bar\rho}\bar{W}(\bar\rho) - \frac{3}{2\sigma}\bar{W}(\bar{y}))\Big), \end{aligned} \tag{13.26}$$

assuming $\mathrm{Im}\,\rho = 0$. If we note first that we can remove contributions from $\frac{4\sigma^2}{3} D_\rho W(\rho)\bar{D}_{\bar\rho}\bar{W}(\bar\rho) - 3|W(\rho)|^2$ as they will be cancelled by $V_{\rm uplift}$ and then expand the remaining terms in (13.26) and take the low energy limit ($M_P \to \infty$), we obtain,

$$\begin{aligned} V &= \frac{1}{8\sigma^3}\left(|W_y|^2 + |W(\rho)|^2 y\bar{y}\right. \\ &\quad + \left((yW_y - 3W(y))\bar{W}(\rho)\right. \\ &\quad \left.\left. -2\sigma W(y)\bar{D}_{\bar\rho}\bar{W}(\bar\rho)) + h.c.\right)\right). \end{aligned} \tag{13.27}$$

If we rescale the superpotential by $W_{SM} \to 2\sqrt{2}\sigma^{3/2} W_{SM}$ (where σ to be evaluated at the near-Minkowski minimum), then we can write the potential in a more standard form

$$V_{SM} = \left|\frac{\partial W_{SM}}{\partial y^i}\right|^2 + m_0^2 y^i \bar{y}_i + (A_0 W_{SM} + h.c.) \tag{13.28}$$

where we have now defined the universal scalar mass as

$$m_0^2 = \frac{1}{8\sigma^3}|W(\rho)|^2 \equiv m_{3/2}^2, \tag{13.29}$$

and a universal A-term

$$A_0 W_{SM} = (yW_y - 3W(y))m_{3/2} - \frac{1}{\sqrt{2\sigma}}\bar{D}_{\bar\rho}\bar{W}(\bar\rho)W_{SM}. \tag{13.30}$$

Note that for tri-linears, the first term in Eq. (14.43) vanishes, leaving

$$A_0 = -\frac{1}{\sqrt{2\sigma}}\bar{D}_{\bar\rho}\bar{W}(\bar\rho) \tag{13.31}$$

while for bilinears (B-terms), it is $-m_{3/2}$ yielding the familiar supergravity relation $B_0 = A_0 - m_0$.

Gaugino masses require in addition a non-trivial dependence of the gauge kinetic function on the modulus, ρ. This dependence is generic in most of the models of $N = 1$ supergravity derived from extended supergravity and string theory [293]. The supergravity Lagrangian terms of interest include

$$\begin{aligned} &-\tfrac{1}{4}(\mathrm{Re}\,h_{\alpha\beta})F^\alpha_{\mu\nu}F^{\beta\mu\nu} + \tfrac{i}{4}(\mathrm{Im}\,h_{\alpha\beta})\epsilon^{\mu\nu\rho\sigma}F^\alpha_{\mu\nu}F^{\beta\rho\sigma} \\ &\quad + \left(\tfrac{1}{4}e^{G/2}h^*_{\alpha\beta\bar{n}}G^{k\bar{n}}G_k\lambda^\alpha\lambda^\beta + h.c.\right). \end{aligned} \tag{13.32}$$

For a suitable choice $h_{\alpha\beta} = h(\rho)\delta_{\alpha\beta}$, one can generate universal gaugino masses

$$m_{1/2} = \frac{\sqrt{2\sigma}}{6}D_\rho W(\rho)\ln(\mathrm{Re}\,h^*)_\rho\ . \tag{13.33}$$

In addition to the above expressions, loop contributions may also play a role (vital in the case of the KL model). These expressions have been derived in greater generality, e.g. when SM Yukawa couplings are also allowed to be moduli dependent, in [275, 342].

Next, we will compare the resulting soft supersymmetry breaking terms found in the KKLT and KL models. We note first that independent of our choice of superpotential, the soft scalar masses are always $m_0^2 = m_{3/2}^2$. That is, the chiral multiplets will always be split by the gravitino mass independent of the magnitude of $D_\rho W$.

In contrast, both A_0 and $m_{1/2}$ are proportional to $D_\rho W(\rho)$ and this differs greatly between the two classes of models:

$$\begin{aligned} D_\rho W(\rho) &= \frac{3\sqrt{2}}{a\sqrt{\sigma_0}}\, m_{3/2} && \text{KKLT} \\ D_\rho W(\rho) &= 6\sqrt{2\sigma_0}\,\frac{m_{3/2}}{m_\sigma}\, m_{3/2} && \text{KL} \end{aligned} \tag{13.34}$$

While the KKLT model could produce a pattern of soft scalar and gaugino masses all of order $m_{3/2}$ with acceptable phenomenologies [275, 277–280, 342], in all models with strong volume modulus stabilization, such as KL models, at the tree-level one finds that the gaugino masses and A-terms are suppressed by $m_{3/2}/m_\sigma$ and as such effectively vanish.

Recall that with $A = B = 1$, $a = 0.1$, $b = 0.05$, we have $m_{3/2}/m_\sigma \sim 10^{-13}$. As a result, in the strongly stabilized models, we are driven towards models resembling those mediated by anomalies [336], where the dominant contributions to gaugino masses and A-terms arise from loop corrections and give [342]

$$m_{1/2} = \frac{b_a g_a^2}{16\pi^2}\frac{F^C}{C_0} \tag{13.35}$$

and

$$A_{ijk} = -\frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2}\frac{F^C}{C_0}\ . \tag{13.36}$$

Here $b_a = 11, 1, -3$ for $a = 1, 2, 3$ are the one-loop beta function coefficients, γ_i are the anomalous dimensions of the matter fields y_i and

$$\frac{F^C}{C_0} = -\frac{1}{3}(\ln(\rho+\bar\rho)\sqrt{(\rho+\bar\rho)}D_\rho W)_\rho + m_{3/2} \tag{13.37}$$

is related to the conformal compensator. This is clearly of order $m_{3/2}$. Note that unlike the case of mixed modulus-anomaly mediation, there is no mirage unification in the KL model [278].

Because of the loop suppression factor in Eq. (14.50), in order to have a phenomenologically viable model in the context of KL stabilization, we are forced to consider relatively large ($\mathcal{O}(100)$ TeV) gravitino masses in order to have suitably large gaugino masses. Thus we are led to a rather unique pattern of sparticle masses. While anomaly mediation plays an important role in the pattern for gaugino masses, the large soft scalar masses generated from Eq. (13.29) yield a spectrum more reminiscent of split supersymmetry [335]. Indeed, the problem of tachyonic scalars normally associated with anomaly mediated models is absent here. While this problem is normally alleviated by adding a constant mass squared to the anomaly contribution, here we have a direct source and explanation of the this term. We also note that if the gravitino and scalar masses were pushed to very high values ($\gtrsim 10^{10}$ GeV) as in some models of split supersymmetry, anomaly mediation would be unnecessary.

Gravitinos in the mass range $\mathcal{O}(100)$ TeV usually do not pose significant cosmological problems because they decay early. In general, they could be harmful if their decays produce many light supersymmetric particles [294], but even this problem can be avoided in the simplest models of chaotic inflation based on the KL construction, where the reheating temperature is small, which strongly suppresses the gravitino production [330].

The heavy sparticle spectrum associated with split supersymmetry presents a challenge for detection of supersymmetry at the LHC. Nevertheless there is an upper limit on $m_0 = m_{3/2}$ of about 100 TeV stemming from the upper bound on the Higgs mass from the LHC [295]. An observation of a dark matter in direct detection experiments with no corresponding signal at the LHC could be an indication pointing to models such as the one described here.

We also note that the prediction that $B_0 \sim -m_{3/2}$ with the significantly smaller smaller A-terms can lead to difficulties in finding a consistent electroweak vacuum [296, 360]. Furthermore, the large mass scales associated with split supersymmetry also presents a challenge in finding a consistent electroweak vacuum, i.e. solutions with $\mu^2 > 0$, where μ is the Higgs mixing parameter. However, the former problem may be corrected in a variety of ways, e.g. by adding a mixing term to the Kähler potential [279, 299, 341], while the latter may be corrected in models containing a right handed neutrino [300, 301], in models where the input supersymmetry breaking scale is above the GUT scale [300, 365], or in models with non-universal Higgs masses [303]. We will return to these issues elsewhere.

The specific pattern of mass we obtained with the KL potential can also arise in models where the uplifting is provided by hidden sector F-terms [280, 304]. In this case, vacua with spontaneously broken supersymmetry, a small positive cosmological constant and a hierarchically small $m_{3/2}$ can be obtained. This procedure leads to "matter dominated" supersymmetry breaking, with the modulus contribution being suppressed. The resulting soft masses are also characterized by heavy scalars, though the gauginos are lighter than the scalars, they do receive contributions from the KKLT uplift in addition to AMSB and are not necessarily as light as we have found in the KL model. The split supersymmetry mass pattern may also appear in models with D-term uplifting, see e.g. [305], and in M-theory on manifolds of G2 holonomy [306].

A distinguishing feature of the models investigated in our paper is that we studied supersymmetry breaking which follows from the simplest versions of the KKLT/KL construction without any additional F-term or D-term contributions. We particularly emphasized the KL-type models with strongly stabilized internal dimensions, which ensures vacuum stability during inflation. In general, one can extend such models to achieve uplifting in the context of $N = 1$ supergravity, see e.g. [347]. Then the value of $D_\rho W$ will receive contributions of order the gravitino mass $m_{3/2}$. One may also introduce 'split uplifting' models, where part of the uplifting appears due to string theory effects in D = 10, and part is due to N = 1 supergravity. This could allow one to continuously interpolate between models with $D_\rho W = O(m_{3/2})$ and the split supersymmetry models discussed above. However, all such models would be more complicated than the models studied in our paper. Moreover, this would re-introduce the cosmological moduli problem. We were only able to resolve this problem because we did not need to include any light fields in the hidden sector of the models with strongly stabilized moduli.

13.1.4 Conclusions and prospective

In a Minkowski vacuum in N = 1 supergravity, in the absence of a D-term, supersymmetry breaking is determined by $D_\rho W$, or, equivalently, by the gravitino mass $m_{3/2}$. Meanwhile in phenomenological models inspired by string theory, $m_{3/2}$ and $D_\rho W$ are not directly linked to each other, because of the effects of uplifting. In the simplest versions of the KKLT model these two parameters are nevertheless of the same order of magnitude. However, as we have shown, the situation is very different in the KL-type models with strong stabilization of the volume modulus, where $D_\rho W \ll m_{3/2}$. These two classes of models have very different phenomenological consequences.

In both classes of models, chiral multiplets are split by the gravitino mass and $m_0^2 = m_{3/2}^2$. On the other hand, gaugino masses and A-terms are very different as they are both proportional to $D_\rho W$. In the simplest KKLT models, one can interpolate between modulus mediation and anomaly mediation to transmit supersymmetry breaking. The dominant contribution will depend on the details of the KKLT superpotential, e.g. on the value of $a\sigma_0$, smaller values of $a\sigma_0$ allowing modulus mediation, while larger values corresponding to anomaly mediation. In models with a heavy modulus, $D_\rho W$ is effectively vanishing and one is driven toward anomaly mediation to generate gaugino masses and A-terms.

However unlike typical anomaly mediation models, here we obtain a massive scalar sector. Therefore the common problem of tachyonic staus associated with anomaly mediation is absent here. Instead our spectrum resembles that of split supersymmetry with scalar masses approaching the PeV scale. Of course there are other immediate consequences of heavy moduli: There is no cosmological gravitino problem. Because we are forced to anomaly mediation to generate gaugino masses, the gravitino must be relatively heavy (approaching the PeV scale for scalar masses). These gravitinos decay harmlessly, well before nucleosynthesis. Moreover, their production can be strongly suppressed in the simplest models of chaotic inflation based on the KL scenario. There is no moduli or Polonyi problem because there is no need for a Polonyi field as supersymmetry breaking is encoded in the uplifting potential. The field ρ does not create any other cosmological difficulties. Indeed having a successful model of inflation in supergravity without requiring $H < m_{3/2}$ and free from the destabilizing effects led us toward heavy moduli in the first place.

To summarize, cosmological considerations, including the requirement of vacuum stability during inflation and the possibility of solving the cosmological moduli and gravitino problems in string theory and supergravity, point towards models with strongly stabilized internal dimensions, resulting in split supersymmetry with anomaly mediation.

Appendix: Generalized KL models with heavy moduli

Here we will give details of the derivation of Eq. (14.19) which are completely independent of the specific form of the superpotential.

Let return to the scalar potential for the modulus neglecting SM fields. In terms of the superpotential $W(\rho)$, we can write

$$V = \frac{1}{6\sigma^2} W_\rho(\sigma W_\rho - 3W) \tag{13.38}$$

for the potential along the $\mathrm{Re}\,\rho$ direction. The mass of the modulus can be easily computed

$$\begin{aligned} m_\sigma^2 &= \frac{1}{2}G^{\rho\bar{\rho}}\frac{\partial^2 V}{\partial\rho\partial\bar{\rho}} \\ &= \frac{1}{18\sigma^3}(\sigma^2 W_{\rho,\rho} - 3W)(4\sigma^2 W_{\rho,\rho} - 3W) \ . \end{aligned} \tag{13.39}$$

Recall also that at the minimum,

$$m_{3/2}^2 = \frac{1}{8\sigma^3}W^2 \ . \tag{13.40}$$

The limit $m_{3/2}^2 \ll m_\sigma^2$ amounts to

$$W^2 \ll \frac{4}{9}(\sigma^2 W_{\rho,\rho} - 3W)(4\sigma^2 W_{\rho,\rho} - 3W) \ , \tag{13.41}$$

which is equivalent to $W \ll \sigma^2 W_{\rho,\rho}$. Note that this was precisely the limit used to approximate Eq. (14.19), and in this limit

$$m_\sigma^2 = \frac{2}{9}\sigma W_{\rho,\rho}^2. \tag{13.42}$$

Finally, we can evaluate $D_\rho W$ from Eq. (14.19) inserting $\Delta\sigma$

$$D_\rho W = \frac{6V_{\mathrm{AdS}}}{W_{\rho,\rho}} = \frac{18m_{3/2}^2}{W_{\rho,\rho}} \ . \tag{13.43}$$

Substituting $W_{\rho,\rho}$ from Eq. (13.42) we arrive at

$$D_\rho W = 6\sqrt{2\sigma_0}m_{3/2}\frac{m_{3/2}}{m_\sigma}, \tag{13.44}$$

where σ is to be evaluated at the pre-uplifted minimum $(\sigma_0 + \delta\sigma)$ in the notation of the previous section. Thus, we have shown that in general,

$$D_\rho W \ll m_{3/2} \tag{13.45}$$

whenever the volume modulus is strongly stabilized, so that $m_\sigma \gg \sqrt{\sigma_0}m_{3/2}$. Our result is not specific to the KL model.

Chapter 14

The neutralino: an example of dark matter candidate in Gravity Mediation scenarios*

14.1 Elements of SUSY

Nowadays, Supersymmetry (SUSY) is an usual ingredient in theories beyond the Standard Model (SM). There are indeed strong theoretical reasons supporting the idea that SUSY is a good symmetry of nature in some region of scales.

Maybe the first motivation about its existence came from the idea of generalizing the Poincare symmetry, allowing for transformations that take fermions into bosons and vice-versa. The idea was improved in the context of String Theory, at its early stages. The easiest way to introduce fermions in the original bosonic string was through supersymmetry. From a more phenomenological perspective, a supersymmetric version of the Standard Model presents very attractive properties, as the gauge coupling unification at a high scale (not far from M_P) and the relaxation of the infamous hierarchy problem. The electroweak breaking requires the Higgs mass-term to be $\mathcal{O}(100\text{ GeV})$. However, the quadratic contributions to this mass-term arising from radiative corrections (e.g. from a loop of tops) are much higher if the cut-off is beyond a few TeV. This translates into a strong fine-tuning between the initial tree-level mass and the size of the radiative corrections. SUSY provides an automatic cancellation of the quadratic divergences, and thus an elegant way-out to this problem. Although there may be caveats about the interpretation of the fine-tuning, the hierarchy problem is still commonly accepted by the scientific community as a strong indication that there should exist new physics at $\mathcal{O}(\text{TeV})$.

Unfortunately, Supersymmetry has not been observed in experiments, which requires it to be spontaneously broken. The corresponding effective theory at low-scale consists of a supersymmetric Lagrangian plus a bunch of terms which break SUSY in an explicit, but "soft", fashion.

Concerning the supersymmetric Lagrangian, the minimal (N=1) supersymmetric extension of the Standard Model is the so-called Minimal Supersymmetric Standard Model (MSSM). The particle content of the MSSM is given in Table 14.1. It consists of the usual SM spectrum plus a supersymmetric partner of each SM particle. Besides, the Higgs sector

has to be extended, including two Higgs doublets, H_u, H_d. This is required [1] to keep the anomaly cancellation (since the fermonic components of the Higgs superfields contribute to the gauge anomalies) and also to provide masses to up- as well as down-type quarks and leptons.

Chiral super-multiplet fields						
Names		spin 0	spin 1/2	$SU(3)_C$,	$SU(2)_L$,	$U(1)_y$
squarks, quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\mathbf{3}$,	$\mathbf{2}$,	1/3
	$\bar{U}$	$\tilde{\bar{u}}_L = \tilde{u}_R^\dagger$	$\bar{u}_L = (u_R)^c$	$\bar{3}$,	$\mathbf{1}$,	-4/3
	$\bar{D}$	$\tilde{\bar{d}}_L = \tilde{d}_R^\dagger$	$\bar{d}_L = (d_R)^c$	$\bar{3}$,	$\mathbf{1}$,	2/3
sleptons, leptons	L	$(\tilde{\nu}_{eL}, \tilde{e}_L)$	(ν_{eL}, e_L)	$\mathbf{1}$,	$\mathbf{2}$,	-1
	$\bar{E}$	$\tilde{\bar{e}}_L = \tilde{e}_R^\dagger$	$\bar{e}_L = (e_R)^c$	$\mathbf{1}$,	$\mathbf{1}$,	2
Higgs, Higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1}$,	$\mathbf{2}$,	1
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\mathbf{1}$,	$\mathbf{2}$,	-1
Gauge super-multiplet fields						
		spin 1/2	spin 1	$SU(3)_C$,	$SU(2)_L$,	$U(1)_y$
gluinos, gluons		$\tilde{g}$	g	$\mathbf{8}$,	$\mathbf{1}$,	0
winos, W bosons		$\tilde{W}^\pm$, $\tilde{W}^0$	$W^\pm$, W^0	$\mathbf{1}$,	$\mathbf{3}$,	0
bino, B boson		$\tilde{B}$	B	$\mathbf{1}$,	$\mathbf{1}$,	0

Table 14.1: *Particle content of the Minimal Supersymmetric Standard Model.*

The superpotential of the MSSM is given by the most general renormalizable holomorphic function of the superfields consistent with all the symmetries of the theory (including some version of $R-$parity to prevent lepton- and baryon-number violation and thus proton decay),

$$W = y_u^{ij} \bar{U}_i Q_j \cdot H_u - y_d^{ij} \bar{D}_i Q_j \cdot H_d - y_e^{ij} \bar{E}_i L_j \cdot H_d + \mu H_u \cdot H_d, \tag{14.1}$$

where the dots denote SU(2)-invariant products of two doublets (colour indices are not shown), the y's are the Yukawa couplings and μ mass-term is an additional supersymmetric parameter. The corresponding supersymmetric Lagrangian can be derived using the standard rules. In particular, this Lagrangian contains the following pieces:

- *Yukawa-terms.* These arise from the general expression

$$\frac{1}{2} \sum_{nm} \frac{\partial^2 W(\phi)}{\partial \phi_n \partial \phi_m} \left(\psi_{nL}^T \epsilon \psi_{mL} \right),$$

  where the superpotential is expressed as a function of just the scalar components of all the superfields, ϕ_n; and ψ_n are the corresponding fermions. This part of the Lagrangian contains the usual SM Yukawa terms plus their supersymmetric counterparts. E.g. from the $u-$type Yukawa one gets

$$y_u \bar{u} Q \cdot H_u + y_u \tilde{\bar{u}} Q \cdot \tilde{H}_u + y_u \bar{u} \tilde{Q} \cdot \tilde{H}_u.$$

[1] From the more formal point of view, this comes from the need of building a holomorphic superpotential.

- *Scalar potential.* It arises from the general expression

$$V = \sum_n \left| \frac{\partial W(\phi)}{\partial \phi_n} \right|^2 \; + \; \left(\sum_{nm} \phi_n^* (t_A)_{nm} \phi_m \right)^2 .$$

  This includes terms like

$$|y_u|^2 |\tilde{Q} \cdot H_u|^2 + |y_u|^2 |\bar{\tilde{u}} \cdot H_u|^2 + |y_u|^2 |\bar{\tilde{u}} \tilde{Q}|^2 + |y_u \bar{\tilde{u}} \tilde{Q} + \mu H_d|^2$$

  and similar expressions for down-type quarks and leptons.

- Usual pure-gauge Standard Model interactions coming from the kinetic terms for ϕ and ψ_L plus additional gaugino – gauge-boson – gaugino vertices,

$$\bar{\lambda}_a \gamma^\mu C_{abc} A_{b\mu} \lambda_c$$

  where a, b, c are group-labels of the corresponding adjoint representation and C_{abc} is the group structure-function.

- *Matter-Gaugino interactions*: They have the general structure

$$i\sqrt{2} \sum_{Anm} \left(\bar{\psi}_{nL} (t_a)_{nm} \lambda_a \right) \phi_m \; + \; \text{h.c.} \; ,$$

  including terms like

$$(\bar{u} T_{SU(3)} \tilde{g}) \tilde{u} + (\bar{d} T_{SU(3)} \tilde{g}) \tilde{d} + (\bar{u} T_{SU(2)} \tilde{W}^+) \tilde{d} + (\bar{u} T_{SU(2)} \tilde{W}^0) \tilde{u} + (\bar{u} T_{U(1)} \tilde{B}^0) \tilde{u} + \cdots$$

On the other hand, the SUSY soft-breaking Lagrangian is parametrised by the soft terms, namely:

- Gaugino masses for each gauge group:

$$-\frac{1}{2} (M_3 \tilde{g}^a \cdot \tilde{g}^a + M_2 \tilde{W}^a \cdot \tilde{W}^a + M_1 \tilde{B} \cdot \tilde{B} + \text{h.c.}), \tag{14.2}$$

  where M_i are assumed to be real, are all real, so they do not introduce any new CP-violation. However they can be either positive or negative.

- Squark (mass)^2 terms:

$$-m^2_{Qij} \tilde{Q}_i^\dagger \cdot \tilde{Q}_j - m^2_{\bar{\tilde{u}}ij} \bar{\tilde{u}}^\dagger_{Li} \bar{\tilde{u}}_{Lj} - m^2_{\bar{\tilde{d}}ij} \bar{\tilde{d}}^\dagger_{Li} \bar{\tilde{d}}_{Lj}; \tag{14.3}$$

- Slepton (mass)^2 terms:

$$-m^2_{Lij} \tilde{L}_i^\dagger \cdot \tilde{L}_j - m^2_{\bar{\tilde{e}}ij} \bar{\tilde{e}}^\dagger_{Li} \bar{\tilde{e}}_{Lj}; \tag{14.4}$$

- Higgs (mass)^2 terms:

$$-m^2_{H_u} H_u^\dagger \cdot H_u - m^2_{H_d} H_d^\dagger \cdot H_d - (b H_u \cdot H_d + \text{h.c.}) \tag{14.5}$$

  where $H_u^\dagger \cdot H_u = |H_u^+|^2 + |H_u^0|^2$ (and similarly for H_d) and $H_u \cdot H_d = H_u^+ H_d^- - H_u^0 H_d^0$.

- Trilinear scalar couplings:

$$-a_u^{ij}\bar{\tilde{u}}_{Li}\tilde{Q}_j \cdot H_u + a_d^{ij}\bar{\tilde{d}}_{Li}\tilde{Q}_j \cdot H_d + a_e^{ij}\bar{\tilde{e}}_{Li}\tilde{L}_j \cdot H_d + \text{h.c.} \tag{14.6}$$

Admittedly, the soft terms introduce a huge amount of parameters in the theory. However, extensive regions of the parameter space are in fact excluded phenomenologically. In particular, generic values of most of the new parameters trigger processes of flavour changing neutral currents (FCNC) or CP-violation, at levels that are excluded by experiments. For example, if the matrix $\mathbf{m}^2_{\tilde{L}}$ in (14.4) has a non-suppressed off-diagonal term such as $(m^2_{\tilde{L}})_{e\mu}\tilde{e}^\dagger_L\tilde{\mu}_L$ (in the basis where charged-lepton masses are diagonal), then $\mu \to e + \gamma$ occurs at unacceptable rates. These kinds of restrictions partially justify the assumptions of the popular "constrained MSSM" (CMSSM), which amounts to start with complete universal soft terms at the unification scale (M_X)

$$M_3 = M_2 = M_1 = m_{1/2}; \tag{14.7}$$

$$\mathbf{m}^2_{\tilde{Q}} = \mathbf{m}^2_{\bar{\tilde{u}}} = \mathbf{m}^2_{\bar{\tilde{d}}} = \mathbf{m}^2_{\tilde{L}} = \mathbf{m}^2_{\bar{\tilde{e}}} = m_0^2\mathbf{1}, \tag{14.8}$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2; \tag{14.9}$$

and

$$\mathbf{a}_u = A_0\mathbf{y}_u, \quad \mathbf{a}_d = A_0\mathbf{y}_d, \quad \mathbf{a}_e = A_0\mathbf{y}_e. \tag{14.10}$$

Of course, such full universality is not required to get consistency with the previous phenomenological restrictions, but it is a possibility that can arise in some well-motivated scenarios of SUSY breaking and transmission to the observable sector, e.g. gravity mediation in minimal supergravity or dilation-dominance in string-inspired supergravity.

On the other hand, despite the initial diagonal mass matrices for squarks and leptons, they get off-diagonal entries along the RG evolution from the M_X to the electroweak scale, triggered by the various (flavour-violating) Yukawa couplings.

14.2 Introduction

One of the goals of this paper is to discuss an interesting interplay between string theory models with moduli stabilization, inflationary cosmology, phenomenological models of supergravity and the mass of the Higgs boson. Usually string theory is associated with an energy scale which is many orders of magnitude higher than the energies accessible at the LHC. This would make it extremely difficult to test various consequences of string theory. However, models of moduli stabilization in string theory such as KKLT [324] allows one to investigate string phenomenology, as well as string cosmology, from a new perspective. As we will see, that while string theory models with strongly stabilized moduli, provide natural solutions to several cosmological problems, they lead to a clear separation in scales in which the effects of string moduli can be tested in low energy experiments.

One of the results found in the simplest versions of the KKLT construction indicates that the mass of the volume modulus, which describes the "rigidity" of compactification, is of the same order of magnitude as the gravitino mass [325, 326]. If one then makes

the standard assumption that the gravitino mass is in the TeV range or below it, KKLT constructions bring the scale of supersymmetry breaking in string theory, as well as the masses of some of the the string theory moduli, down to the LHC energy range. This fact has an interesting phenomenological implication: Supersymmetry breaking in the standard model may be directly affected by details of the KKLT construction. Depending on one's point of view, this may be good news, if one tries to study properties of string theory compactification at LHC, or bad news, if one attempts to make predictions independent of the intricacies of string theory.

More importantly, this softness of string theory compactification in the simplest versions of the KKLT scenario leads to a specific cosmological problem: vacuum destabilization and decompactification of space if the Hubble constant during inflation was greater than the gravitino mass [325]. The requirement $H \leq \mathrm{O}(1)$ TeV is extremely restrictive; it would eliminate most (though not all) of the presently existing models of inflation. Moreover, a light volume modulus would lead to a novel version of the cosmological moduli problem, which has plagued supergravity cosmology for more than three decades [327]. In addition, one would still need to solve the cosmological gravitino problem, which is another long-standing problem of supergravity cosmology [328, 329].

A possible solution to the problem of vacuum destabilization was proposed back in 2004, in the same paper where the existence of this problem was expounded [325]. This solution is realized by adding an extra term to the superpotential of the KKLT scenario, as in the well-known racetrack potential. In this construction, the volume modulus mass can be made arbitrarily large, the barrier stabilizing the stringy vacuum can be made arbitrarily high, for any value of the gravitino mass, and the problem of the cosmological vacuum destabilization disappears. To distinguish this model from the original version of the KKLT scenario, we will refer to it as the KL model.

It has often been remarked that the KL model is very fine-tuned. However, a more detailed investigation of this issue in [326, 330] has demonstrated that the degree of fine-tuning of the parameters of this model is exactly the same as in the standard Polonyi model, or in the original version of the KKLT scenario: It is determined only by the postulated smallness of the gravitino mass. More specifically, in the KKLT model, the constant term in the superpotential, W_0, must be tuned small. In the KL model, we need approximately the same small number added to a number of O(1) in the superpotential. Recently, a set of supergravity inflationary models incorporating the KL scenario was proposed, which are very simple and nevertheless are general enough to describe *any* set of observational parameters n_s and r to be determined by the PLANCK satellite [14] [330–333]. The KL mechanism of vacuum stabilization can be used also in models of chaotic inflation in string theory as proposed in [334].

An interesting feature of this class of inflationary models is a controllably small value of the reheating temperature. The gravitino problem may be resolved by a suitably low reheat temperature or as we will see a large gravitino mass which is imposed by the resulting supersymmetric sparticle spectrum. As for the cosmological moduli problem, supersymmetry breaking is an unavoidable part of the KKLT and KL scenario, which is related to the mechanism of uplifting (see next section). As we shall see, if this mechanism is realized through F-term uplifting, no light Polonyi fields are required. This addresses the cosmological moduli problem in the KL scenario, where all moduli can be superheavy.

These advantages of the KL scenario prompted an investigation of its consequences for particle phenomenology [326, 330]. The results of this investigation appeared to be much more general than initially expected and apply to the KL model, as well as any other version of the KKLT scenario with strong modulus stabilization.

Because of the strong modulus stabilization, the KL scenario leads to some very specific predictions for supersymmetry breaking and particle phenomenology: It describes a certain version of split supersymmetry with anomaly mediation [326, 330]. Moreover, this prediction is stable with respect to various modifications of the KL model: The same type of supersymmetry breaking and the same pattern of particle masses appears in any version of the KKLT scenario with strong moduli stabilization, which makes the theory cosmologically consistent [326]. A more precise formulation and explanation of this statement is contained in Section 2, where we give a brief review of the KKLT and KL models.

Moreover, heavy scalars as predicted here are phenomenologically interesting for many reasons. Indeed, it is well known that heavy squarks can greatly improve the constraints coming from SUSY flavor and CP violating interactions. In addition, from a theoretical point of view, it is more likely that scalars have heavier masses than fermions, as fermions can be protected from large radiative corrections whereas scalar particles are generally not protected.

In this paper, we continue an investigation of the low-energy phenomenology of the KL model, as well as all other versions of the KKLT scenario with strong moduli stabilization. After a brief review of moduli stabilization and uplifts in Section 2 and establishing the sources of soft supersymmetry breaking masses in Section 3, we describe the procedure for consistently including radiative electroweak symmetry breaking in this theory. Because models with strong moduli stabilization require heavy scalars (as in split supersymmetry [335]) with relatively light gaugino masses (as in models with anomaly mediation [**?**]) there are difficulties in constructing a UV completion for this anomalously split supersymmetric model with the boundary conditions imposed by supergravity. These problems and possible solutions will be discussed in Section 4. In Section 5, we describe the sparticle mass spectrum in this theory as a function of gravitino mass. For obvious reasons, we concentrate on the predictions for the Higgs mass in such models. As we will see, to generate a chargino mass of at least 104 GeV (to be consistent with the LEP bound [337]) we need a gravitino mass $m_{3/2} \gtrsim 30$ TeV. At this value, the Higgs mass is $\simeq 125$ GeV, and rises slowly to ~ 130 GeV when $m_{3/2} \sim 1000$ TeV. In Section 6, we consider other phenomenological aspects of the model such as the role of 1 TeV gluinos and their detectabilility at the LHC. We also describe the prospect for dark matter in these models, as well as dark matter detection. Our conclusions are given in Section 7.

14.3 Moduli stabilization and uplifting: a brief review

14.3.1 KKLT versus KL

The KKLT (KL) sector consists of a single chiral field: the modulus ρ. We will denote SM fields collectively as ϕ. The scalar potential for uncharged chiral superfields in $\mathcal{N} = 1$

supergravity is [338]

$$V = e^K \left(K^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2 \right) , \tag{14.11}$$

where as usual we defined $D_a W = \partial_a W + K_a W$. We define a Kähler potential with a no-scale [339] structure in the moduli sector and kinetic terms in the matter sector depending in some unspecified way on the modulus ρ. This can be written as

$$K = -3\log(\rho + \bar{\rho}) + h_i^j(\rho, \bar{\rho})\phi^i \bar{\phi}_j + K(S^i, \bar{S}_i) + \Delta K(\phi^i, \bar{\phi}_i) + \cdots , \tag{14.12}$$

where $\cdots$ denote terms of higher-order in matter fields ϕ, irrelevant for our purposes. In a type IIB string theory setup orientifolded by $\Omega' = \Omega\, I_6 (-1)^{F_L}$, where I_6 denotes parity in the six internal dimensions and $(-1)^{F_L}$ is the left-handed fermion number [340], with D7 and D3 branes, the function $h(\rho, \bar{\rho})$ is a constant if matter fields originate from D7-D7 sector, it is given by $h_i^j(\rho, \bar{\rho}) = \delta_i^j/(\rho + \bar{\rho})$ for fields in the D3-D3 sector and has specific form for fields living at the intersection of various branes. We will discuss the fields S_i which are associated with F-term uplifting below and we specify ΔK in section 4 in connection with the Giudice-Masiero mechanism [341]. The important assumption in what follows is that the uplift fields S_i have a separable Kähler potential, that can be justified, for example, if the uplift fields arise as D7-D7 states. Indeed, we will be assuming that the S_i are not directly coupled to matter through either the Kähler potential, the superpotential, or gauge kinetic function. In each case we will assume the superpotential is separable in the uplift fields

$$W = W(\rho) + W_F(S^i) + g(\phi^i, \rho) , \tag{14.13}$$

where $W(\rho)$ is either the KKLT or KL superpotential, W_F is the superpotential associated with uplifting and $g(\phi^i, \rho)$ is the superpotential for the Standard Model, with $g(0, \rho) = 0$. The possible ρ dependence of the matter superpotential g is highly restricted by axionic symmetries and by the origin of matter fields (it is typically an exponential or a modular form of various weight). Provided that the vev's of matter fields are very small compared to the Planck scale, the results of the present paper are largely insensitive to the explicit form of the function $h(\rho, \bar{\rho})$ and the ρ dependence of $g(\phi^i, \rho)$.

The superpotential of the KKLT model is

$$W_{\rm KKLT} = W_0 + A e^{-a\rho} . \tag{14.14}$$

where W_0 and $a > 0$ are constants. In this theory, there is a supersymmetry preserving AdS minimum found by setting $D_\rho W = 0$. It occurs at $\mathrm{Im}\,\rho = 0$, and at a certain value σ_0 of the volume modulus $\sigma = \mathrm{Re}\,\rho$.

After the uplifting to the (nearly Minkowski) dS vacuum state, the gravitino mass becomes

$$m_{3/2} \approx \sqrt{|V_{\rm AdS}|/3} \approx \frac{aA}{3(2\sigma_0)^{1/2}} e^{-a\sigma_0} . \tag{14.15}$$

Furthermore, after uplifting

$$D_\rho W = \frac{3\sqrt{2}}{a\sqrt{\sigma_0}}\, m_{3/2} . \tag{14.16}$$

The conditions of applicability of the KKLT constructions are $a\sigma_0 > 1$ and $\sigma_0 \gg 1$. If one takes $a\sigma_0 \gg 1$, the gravitino mass becomes exponentially small. To have $m_{3/2}$ in the TeV range in the KKLT scenario, one should take $a\sigma_0 \sim 30$.

The mass of the volume modulus σ in the minimum, as well as the mass of its imaginary (axionic) component, is given by $m_\sigma = 2a\sigma_0\, m_{3/2}$ [326]. For $a\sigma_0 \sim 30$, one finds $m_\sigma \sim 60\, m_{3/2}$. As a result, the mass of the volume modulus is somewhat greater than the gravitino mass, but not by much. This means that the volume stabilization in the KKLT scenario describing light gravitinos is very soft; the mass of the volume modulus in this scenario is many orders of magnitude below the string scale or the Planck scale. It is this softness of the vacuum stabilization that leads to the catastrophic decompactification of extra dimensions during inflation with $H \gtrsim m_{3/2}$ [325, 326].

The simplest way to avoid this problem is to strongly stabilize the vacuum by making m_σ greater than $m_{3/2}$ by many orders of magnitude. This was achieved in the KL scenario by using the racetrack superpotential

$$W_{\rm KL} = W_0 + Ae^{-a\rho} - Be^{-b\rho}\ . \tag{14.17}$$

In contrast to the KKLT case, the new degree of freedom offered by $Be^{-b\rho}$ allows the new model to have a supersymmetric Minkowski solution. Indeed, for the particular choice of W_0,

$$W_0 = -A\left(\frac{a\,A}{b\,B}\right)^{\frac{a}{b-a}} + B\left(\frac{a\,A}{b\,B}\right)^{\frac{b}{b-a}}\ , \tag{14.18}$$

the potential of the field σ has a supersymmetric minimum with $W_{\rm KL}(\sigma_0) = 0$, $D_\rho W_{\rm KL}(\sigma_0) = 0$, and $V(\sigma_0) = 0$.

One may add an additional constant Δ (either positive or negative) to the superpotential (14.17). This will shift the minimum of the potential down to the AdS minimum with $V_{\rm AdS} = -3m_{3/2}^2 = -\frac{3\Delta^2}{8\sigma_0^3}$ [326, 330], after which one may use uplifting (as in KKLT) to make the cosmological constant as small as $\sim 10^{-120}$. Thus one has $m_{3/2}^2 = \frac{\Delta^2}{8\sigma_0^3} \ll 1$, which is the only weak-scale fine-tuning required in the KL model. Interestingly, exactly the same level of fine-tuning of the parameter W_0 is required in the simplest version of the KKLT scenario. This is the standard price for the desire to protect the Higgs mass by the smallness of supersymmetry breaking.

Finally, we turn to uplifting in the theory. Before we make the minuscule addition Δ to the KL superpotential, supersymmetry is unbroken, the gravitino mass vanishes, but the volume modulus mass is arbitrarily large, depending on the choice of the parameters A, a, B and b. This mass is virtually unchanged after adding Δ and uplifting. Thus, one achieves the desired strong vacuum stabilization and removes the cosmological constraint $H \lesssim m_{3/2}$. But this strong vacuum stabilization has an interesting implication for the resulting low-energy phenomenology: Just as in the KKLT scenario, $D_\rho W(\rho) = 0$ in the supersymmetric AdS minimum prior to uplifting in the KL model. However, in the simplest version of the KKLT scenario the value of the field ρ does shift slightly during uplifting, and $D_\rho W(\rho)$ becomes (approximately) as large as W, as seen in Eq. (14.16). In contrast, strong vacuum stabilization keeps moduli practically unchanged during the uplifting. As a result, after the uplifting in the KL scenario, *and in any other version of the KKLT scenario with strong vacuum stabilization*, one has $W = \Delta$ and $|D_\rho W(\rho)| \ll |\Delta|$, $m_{3/2}$.

In particular, in the KL model with an uplifting term $\sim \sigma^{-2}$, which appears in the models with uplifting due to anti-$D3$ branes in warped space, one has [326]

$$D_\rho W = 6\sqrt{2\sigma_0}\,\frac{m_{3/2}}{m_\sigma}\; m_{3/2}\;. \tag{14.19}$$

On the other hand, one can show that in the F-term uplifting models to be studied below, the uplifting term is proportional to σ^{-3}, and the result slightly changes,

$$D_\rho W = 9\sqrt{2\sigma_0}\,\frac{m_{3/2}}{m_\sigma}\; m_{3/2}\;. \tag{14.20}$$

What is most important for us is that in both cases one has $D_\rho W(\rho) \ll W$, $D_\rho W(\rho) \ll m_{3/2}$, under the condition of strong vacuum stabilization $m_\sigma \gg m_{3/2}$, which ensures vacuum stability during high energy inflation. Thus, independently of the particular choice of the stabilizing superpotential $W(\rho)$, one can simply take $D_\rho W(\rho) = 0$ at the minimum of the potential, before and after the uplifting. And this means, as one can easily check, that in all models of such type one has, at the minimum of the potential,

$$|W| = |\Delta| = (2\sigma_0)^{3/2}\, m_{3/2} \tag{14.21}$$

and

$$W_\rho = \frac{3\Delta}{2\sigma_0}\;. \tag{14.22}$$

14.3.2 F-term uplifting examples

In the discussion above, we assumed uplifting as an effect arising solely from string theory, for example, through the energy of anti-$D3$ branes placed in a highly warped throat. In the KL model when coupled to matter, the suppressed F-term given in equation (14.19) leads to extremely small values for gaugino masses and tri-linear supersymmetry breaking A-terms which are proportional to $D_\rho W$. In [326], the coupling of matter to the uplifting term was neglected and it was assumed that soft scalar masses remained equal to the gravitino mass. However, as can be seen from the analysis in Ref. [342], there is a cancelation which leaves only a tiny scalar mass also of order $m_{3/2}^2/M_P$. The resulting spectrum would then be dominated purely by anomaly mediation, and suffer from known phenomenological problems [343, 344].

However, it is in fact relatively simple to recover the result given in [326], by using F-term uplifting [345–348] instead of antibranes. In fact this possibility is quite generic and can be done in various ways, as we now describe in two explicit examples. The main idea is to use a SUSY breaking sector for uplifting, preferably with a dynamical scale leading to a small mass parameter $M << 1$, which breaks SUSY in the rigid limit $M_P \to \infty$. All masses in this sector will be determined, at the tree and one-loop level, by the dynamical scale and are much larger than the gravitino mass. Whereas in the rigid limit the uplift sector is decoupled from the KL sector, supergravity interactions couple the two sectors. However, in strong moduli stabilization models like KL, provided the KL modulus mass and masses of uplifted fields are much larger than gravitino mass, supergravity interactions only change the original KL and uplift sector minima in a very tiny way. As a result, the vacuum

structure is essentially unchanged and the modulus sector contribution to SUSY breaking is completely negligible, as will be seen in what follows.

This implies that in the models with strong stabilization, there is a certain decoupling of string theory moduli from the standard model phenomenology. While this could seem almost obvious on general grounds, it is not the case in the simplest versions of the KKLT scenario. Optimistically, this means that investigation of the low energy phenomenology may provide us with a possibility to test various mechanisms of moduli stabilization in string theory.

Furthermore, in what follows we will be considering models of strong stabilization for *all* moduli, including the F-term uplifting fields S_i. Just as strong stabilization of the volume modulus ρ was advantageous for cosmology and inflation, strongly stabilized uplifting fields, provides a simple mechanism to avoid cosmological problems associated with these moduli. While we do not enter into the details of the cosmology of the these moduli, we take the premise that all moduli are strongly stabilized.

F-term uplifting with a non-minimal Polonyi field

We begin with a very simple example based on a non-minimal version of the Polonyi model, known as O'KKLT [347, 349]. The O'KKLT model for F-term uplifting is realized with the following definitions of $K(S,\bar{S})$ and $W_F(S)$ used in eqs. (14.12) and (14.13) for a single Polonyi-like field S. We take

$$K(S,\bar{S}) = S\bar{S} - \frac{(S\bar{S})^2}{\Lambda^2} \ , \tag{14.23}$$

where we assume that $\Lambda \ll 1$ (in Planck units). As we will see, the second term in (14.23) provides strong stabilization for the field S. For the superpotential, we can take simply,

$$W_F(S) = M^2 S \ , \tag{14.24}$$

as in the Polonyi model, but without an additional constant which is necessary for the fine-tuning of the vanishingly small value of the cosmological constant. This constant is already provided by the KKLT/KL superpotential.

In the original O'KKLT model, it was assumed that the term $-\frac{(S\bar{S})^2}{\Lambda^2}$ appears after integrating out some heavy degrees of freedom in the O'Raifeartaigh model. A consistency of this assumption required careful investigation [347]. However, assuming that this interpretation of the term $-\frac{(S\bar{S})^2}{\Lambda^2}$ is available, one can simply consider this term as a part of a modified Polonyi model (14.23), (14.24) without further discussion of its origin [347, 349].

The simplest way to understand the main idea of this scenario is to consider it in the context of the KL model, or any other strongly stabilized model of that type. In this case, the position of the AdS minimum of the potential is strongly fixed. Therefore it is not affected by adding the Polonyi field to the theory. In order to find the value of the Polonyi field and its superpotential, it is sufficient to calculate the values of the superpotential $W(\rho)$ of the KL model and its derivative $W_\rho(\rho)$ at the minimum of the KL potential ignoring the Polonyi fields. The results of these calculations are given in (14.21) and (14.22). These results are then used in the calculation of the F-term potential of the field S.

Alternatively, one may wish to abandon any connection to string theory and simply consider the supersymmetry breaking sector of the Polonyi field S with strong stabilization

provided by the Kähler potential and superpotential given by Eqs. (14.23) and (superpol). For the superpotential, however, we must add back the constant term. For small Λ, we have strong stabilization and the mass of S can be large, as discussed below and its expectation value close to 0. So long as we continue to assume that the gauge kinetic function does not linearly depend on S, the phenomenological results discussed below will be unchanged.

These calculations, for strongly stabilized theories, show that the field S uplifts the AdS minimum to the nearly Minkowski minimum for

$$M^4 = 3\Delta^2 = 24\sigma_0^3 m_{3/2}^2 \ , \tag{14.25}$$

which determines the choice of the parameter M in (14.24).

The field S at the minimum of its potential is real, and its value is given by

$$\langle S \rangle = \frac{\sqrt{3}\Lambda^2}{6} \ . \tag{14.26}$$

The mass squared of the field S in both directions (real and imaginary) is given by

$$m_S^2 = \frac{3\Delta^2}{2\sigma_0^3\Lambda^2} = \frac{12 m_{3/2}^2}{\Lambda^2} \gg m_{3/2}^2 \ , \tag{14.27}$$

so it too is strongly stabilized. This is quite important. Indeed, the cosmological moduli problem appears because in the minimal Polonyi field model, the mass of the Polonyi field S is of the same order as the gravitino mass, which was supposed to be in the range of 1 TeV or below. In our model, $m_S^2 \gg m_{3/2}^2$ and the field is constrained to lie close to its minimum near $S = 0$ (for small Λ). Moreover, as we will soon see, in the models of this class one typically has $m_{3/2} \gg 1$ TeV. Therefore for sufficiently large $m_{3/2}$ and sufficiently small $\Lambda \ll 1$, the cosmological moduli and gravitino problems will disappear.

Strong stabilization of the field S is important in another respect as well. Since the field S is strongly stabilized, we can repeat the same procedure that we used before, and calculate the soft breaking terms in the standard model. The only additional parameters that we need for these calculations are the values of W_F and W_F' at the minimum of the potential for the field S, ignoring the standard model fields:

$$W_F = \frac{|\Delta|\Lambda^2}{2} \tag{14.28}$$

and

$$\partial_S W_F = \sqrt{3}\,|\Delta| \ . \tag{14.29}$$

As a result,

$$\begin{aligned} W &= W(\rho) + W_F = \Delta + \frac{\Delta\Lambda^2}{2} \simeq \Delta \ , \\ D_S W &= \partial_S W_F + K_S\,(W(\rho) + W_F) \\ &= \sqrt{3}|\Delta| + \frac{\sqrt{3}}{6}\Lambda^2(1 + \frac{1}{2}\Lambda^2)|\Delta| \simeq \sqrt{3}|\Delta|. \end{aligned} \tag{14.30}$$

Finally, we should note that in the context of this model, one can attribute the supersymmetry breaking parameter Δ either to the KL model, as we did earlier, or to the Polonyi

model, by adding it to the superpotential (14.24). Alternatively, one may add $\Delta_{\rm KL}$ to the KL superpotential (14.18) and $\Delta_{\rm P}$ to the Polonyi superpotential (14.24). The final results and the standard model phenomenology will depend only on the sum of these two parameters, $\Delta = \Delta_{\rm KL} + \Delta_{\rm P}$. This is another way to see that the degree of fine-tuning required in the KL model is exactly the same as in the Polonyi model.

F-term uplift with a dynamical ISS sector

As a second example, we display here another F-term uplifting [346,350] via the ISS mechanism [351], which leads to a qualitatively similar result to the one in the previous section. The difference is that in the present example the corresponding mass scale M has a dynamical origin, that naturally explain its smallness. The model is defined by

$$\begin{aligned} W &= W_{\rm KL}(\rho) \ + \ W_F(\chi^i) \ , \\ K &= -3 \ \ln(\rho + \bar\rho) \ + \ |q|^2 \ + \ |\tilde q|^2 \ + \ |S|^2 \ . \end{aligned} \tag{14.31}$$

In (14.31), χ^i denotes collectively the fields q_i^a, $\tilde q_a^{\bar j}$, $S_{\bar j}^i$ of the ISS model [351], where $i, \bar j = 1 \cdots N_f$ are flavor indices and $a, b = 1 \cdots N$ are color indices. Moreover, in (14.31)

$$W_F(\chi^i) \ = \ h \ Tr \ \tilde q \ S \ q \ - \ h \ M^2 \ TrS \ , \tag{14.32}$$

and $W_{\rm KL}$ is given in Eq. (14.17). As explained in [351], the sector q_i^a, $\tilde q_a^{\bar j}$ has a perturbative description in the free magnetic range $N_f > 3N$. SUSY is broken in (14.32) by the "rank condition", i.e. F-terms of meson fields $(F_S)_i^j = h \tilde q_a^j q_i^a - M^2 \delta_i^j$ cannot be set simultaneously to zero.

As is transparent in (14.31), the KL and the ISS sectors are only coupled through gravitational interactions. In the type II orientifold setup, if the ISS gauge group comes from D3 branes, the dynamical scale in the electric theory and therefore also the mass parameter M in the magnetic theory superpotential (14.32) depend on the dilaton S, which we assume is already stabilized by NS-NS and RR three-form fluxes [340]. As in the O'KKLT model, this decoupling between the uplift field(s) and modulus ρ is instrumental in getting the uplift of the vacuum energy.

At the global supersymmetry level and before gauging the color symmetry, the ISS model has a global symmetry $G = SU(N) \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)' \times U(1)_R$, broken explicitly to $SU(N) \times SU(N_f)_V \times U(1)_B \times U(1)_R$ by the mass parameter M. In the supergravity embedding (14.32), the R-symmetry $U(1)_R$ is explicitly broken. We consider here only the ungauged theory for simplicity, in which the $SU(N)$ is part of the global symmetry group. For the effects of the gauging, see e.g. [346] in the related context of the KKLT uplift. At the global supersymmetry level, the metastable ISS vacuum is

$$S_0 \ = \ 0 \ , \qquad q_0 \ = \ \tilde q_0^T \ = \begin{pmatrix} M I_N \\ 0 \end{pmatrix} \ , \tag{14.33}$$

where I_N is the $N \times N$ identity matrix and $M \ll \Lambda_m$, where $\Lambda_m \leq M_P$ denotes the mass scale associated with the Landau pole for the gauge coupling in the magnetic theory. The first

question to address is the vacuum structure of the model. In order to answer this question, we start from the supergravity scalar potential (14.11). By using (14.31)-(14.32), we find

$$
\begin{aligned}
V \;=\; & \frac{e^{\bar{\chi}_{\bar{i}}\chi^i}}{(\rho+\bar{\rho})^3}\left\{\frac{(\rho+\bar{\rho})^2}{3}\left|D_\rho W\right|^2\right. \\
& \left. +\sum_i \left|\partial_i W \;+\; \bar{\chi}_{\bar{i}} W\right|^2 \;-\; 3|W|^2\right\} \;. \qquad (14.34)
\end{aligned}
$$

Since $M \ll M_P$, the vev's in the ISS model are well below the Planck scale. Then an illuminating way of rewriting the scalar potential (14.34) is to expand it in powers of the fields χ^i/M_P, in which case it reads[2]

$$
\begin{aligned}
V \;=\; & \frac{1}{(\rho+\bar{\rho})^3}\, V_{ISS}(\chi^i,\bar{\chi}_{\bar{i}}) \;+\; V_{KL}(\rho,\bar{\rho}) \;+\; \frac{\bar{\chi}_{\bar{i}}\chi^i}{M_P^2}\, V_1(\rho,\bar{\rho}) \\
+ \;& \frac{1}{M_P^3}\,\left[\, W_{\rm ISS}(\chi^i)\, V_2(\rho,\bar{\rho}) + \chi^i\partial_i W_{\rm ISS}\, V_3(\rho,\bar{\rho}) \;+\; h.c.\right] \\
+ \;& \cdots \;, \qquad (14.35)
\end{aligned}
$$

where by comparing (14.35) with (14.34) we can check that $V_1 \sim m_{3/2}^2 M_P^2$, and $V_2, V_3 \sim m_{3/2} M_P^3$. The contribution to the vacuum energy from the ISS sector, in the global limit, is $\langle V_{\rm ISS}\rangle \;=\; (N_f - N)\; h^2\; M^4$. Since we are interested in $30-1000$ TeV scale gravitino masses, it is clear that the first two terms in the rhs of (14.35), V_{ISS} and V_{KL}, are the leading terms. Consequently, there should be a vacuum very close to an uplift KL vacuum $\langle\rho\rangle = \rho_0$ and the ISS vacuum $\langle\chi^i\rangle = \chi_0^i$. The cosmological constant at the lowest order is given by

$$
\Lambda \;=\; V_{KL}(\rho_0,\bar{\rho}_0) \;+\; \frac{(N_f-N)h^2M^4}{(\rho_0+\bar{\rho}_0)^3} \;, \qquad (14.36)
$$

which shows that the ISS sector plays indeed the role of un uplifting sector of the KL model. In the zeroth order approximation and in the large volume limit $\sigma_0 \gg 1$, we find that the condition of zero cosmological constant $\Lambda = 0$ implies roughly

$$
3\;|W|^2 \;\sim\; h^2\;(N_f - N)\;M^4 \;. \qquad (14.37)
$$

If we want to have a gravitino mass in the $30-1000$ TeV range, we need small values of $M \sim (10^{-5}-10^{-6})M_P$. Since M in the ISS model has a dynamical origin, this is natural. Moreover, the metastable ISS vacuum has a significantly large lifetime exactly in this limit. Therefore, the claimed value of the gravitino mass is natural in our model and compatible with the uplift of the cosmological constant.
In the rigid $M_P \to \infty$ limit, the ISS fields have masses of order

$$
\begin{aligned}
&\text{tree}-\text{level} \qquad m_0 \sim |hM| \;, \\
&\text{one}-\text{loop} \qquad m_1 \sim \frac{|h^2M|}{4\pi} \;. \qquad (14.38)
\end{aligned}
$$

[2]In most of the formulae of this letter, $M_P = 1$. In some formulae, however, we keep explicitly M_P.

Of course the goldstone bosons of the broken global symmetries are massless for the time being. It is easy to remove these massless states by breaking the global symmetry from the very beginning by having several mass parameters, $M^2 TrS \to \sum_i M_i^2 S_i^i$. Notice that supergravity corrections give tree-level masses to the pseudo-moduli fields of the ISS model. As explained in more general terms in [351], these corrections are subleading with respect to masses arising from the one-loop Coleman-Weinberg effective potential in the global supersymmetric limit. This can be explicitly checked starting from the supergravity scalar potential (14.34) and expanding in small fluctuations around the vacuum (14.33) to the quadratic order.

Similarly to the previous O'KKLT example, there is no moduli problem in the present setup: both the ρ modulus and the ISS fields are much heavier than the gravitino mass.

14.4 Soft masses for matter fields

While the particular form of the KL superpotential was instrumental in our analysis, the relation $D_\rho W(\rho) \ll W$, which we use in this section for computing soft terms for matter fields has a much more general validity. It follows directly from our requirement of strong vacuum stabilization, which solves the problem of decompactification during inflation with $H \gtrsim m_{3/2}$, as well as the cosmological moduli problem.

An additional assumption we will make in what follows is that there is no direct coupling in the Kähler potential, superpotential and gauge kinetic function between matter fields and the uplift fields. The absence of linear couplings to the SUSY breaking uplift fields in the gauge kinetic function and superpotential for matter fields can be argued at various levels:
- At the level of symmetries in the second uplift example based on ISS model, the meson fields S_i^j there transform under chiral symmetries $SU(N_f)_L \times SU(N_f)_R$, broken only by mass terms. It is expected that couplings to MSSM fields respect chiral symmetries of the uplift sector, therefore linear couplings to S should be absent.
- The uplift Polonyi or ISS sector does break SUSY in the rigid limit in the absence of additional couplings to matter and moduli fields. When these additional couplings are present, supersymmetry tends to be restored, especially for those couplings which break the R-symmetry of the uplift sector. It is possible that the vacuum we are discussing will still be a local miminum with a very long lifetime, however the absence of new couplings helps in avoiding new supersymmetric minima (this argument, however, does not pertain to the gauge kinetic function).
- From a string theory viewpoint, the linear terms in superpotentials present in both of our examples do not arise at tree-level in string perturbation theory. They can arise nonperturbatively by D-brane instanton effects. In this case S is actually a field charged under an "anomalous" $U(1)_X$. This $U(1)_X$ is broken close to the string scale by field-dependent Fayet-Iliopoulos terms, depending on some modulus field called T in what follows. The axionic field in the T multiplet is shifted nonlinearly under $U(1)_X$, $T \to T + i\,\delta_{GS}\,\alpha$, where α is the gauge transformation parameter, and is eaten up by the $U(1)_X$ gauge field. At the perturbative level, couplings of S are very restricted by the $U(1)_X$ symmetry. The gauge kinetic function h_A must clearly be invariant, and therefore S cannot appear there perturbatively. Instantonic effects are proportional to the D-instanton action $S_{\rm inst} = e^{-2\pi T}$, which

has a specific $U(1)_X$ charge. Linear terms in S can arise nonperturbatively in h_A and W via the gauge invariant combination $e^{-2aT}S$, where the $U(1)_X$ charge of e^{-2aT} compensates that of S [352]. In our uplift examples, it would mean that S couplings in h_A and the S-dependence of Yukawas are suppressed by the mass parameter $M^2 \sim e^{-2aT}$. For example $h_A = h_A^0(1+\beta_A e^{-2aT}S)$ or $y_{ijk} = y_{ijk}^0(1+c_{ijk}e^{-2aT}S)$. However, the "anomalous" symmetry does not forbid couplings in the Kähler potential, which we have argued against earlier.

Under the assumptions above we now show that strong moduli stabilization with any F-term uplifts leads to small A-terms which are dominated by anomaly contributions. As we will see, this fact alone forces one to large scalar masses and hence a large gravitino mass. This is acceptable if the symmetry preventing a linear coupling of S to matter is operative, and hence we are restricted to small gaugino masses also dominated by anomaly contributions. Couplings in the Kähler potential of the type $S^\dagger S\phi^\dagger\phi$, on the other hand, are invariant under all symmetries and, if present, they can change scalar masses in what follows. We will comment on their possible effects below.

Soft terms for matter fields generated in supergravity in the limit $M_P \to \infty$ with fixed gravitino mass $m_{3/2}$ [353] have a nice geometrical structure. For F-term SUSY breaking, they are given by [354]

$$\begin{aligned}
&m_{i\bar{j}}^2 = m_{3/2}^2\ (G_{i\bar{j}} - R_{i\bar{j}\alpha\bar{\beta}}G^\alpha G^{\bar{\beta}}\ )\ ,\\
&(B\ \mu)_{ij} = m_{3/2}^2\ (2\nabla_i G_j + G^\alpha \nabla_i\nabla_j G_\alpha)\ ,\\
&(A\ y)_{ijk} = m_{3/2}^2\,(3\nabla_i\nabla_j G_k + G^\alpha\nabla_i\nabla_j\nabla_k G_\alpha)\ \ ,\\
&\mu_{ij} = m_{3/2}\ \nabla_i G_j\ ,\\
&m_{1/2}^a = \frac{1}{2}(Re\ h_A)^{-1} m_{3/2}\ \partial_\alpha h_A\ G^\alpha\ ,
\end{aligned} \tag{14.39}$$

where $G = K + \ln|W|^2$, y_{ijk} are Yukawa couplings, h_A are the gauge kinetic functions and ∇_i denotes Kähler covariant derivatives

$$\nabla_i G_j = \partial_i G_j - \Gamma_{ij}^k G_k\,, \tag{14.40}$$

where $\Gamma_{ij}^k = G^{k\bar{l}}\partial_i G_{j\bar{l}}$ is the Kähler connection. Greek indices α, β in (14.39) refer to SUSY breaking fields S and ρ, latin indices refer to matter fields, whereas $R_{i\bar{j}\alpha\bar{\beta}}$ is the Riemann tensor of the Kähler space spanned by chiral (super)fields. In our models with strong moduli stabilization and decoupling between uplift fields and matter fields, the curvature terms in the scalar masses of matter fields are negligible and we find to great accuracy

$$m_0^2 = m_{3/2}^2\ , \tag{14.41}$$

where the gravitino mass is given by

$$m_{3/2}^2 = e^G = \frac{1}{8\sigma_0^3}|W(\rho) + W_F(S)|^2\,, \tag{14.42}$$

and fixes the universal mass scale for scalars. For the O'KKLT model described above, so long as $\Lambda^2 \ll 1$, the dominant contribution to the gravitino mass comes from $W(\rho) = \Delta$ at the minimum (see eq. (14.21)). The trilinear terms are given by

$$(Ay)_{ijk} = e^K K^{\alpha\bar{\beta}}\overline{D_\beta W}(K_\alpha + \nabla_\alpha)W_{ijk}\ , \tag{14.43}$$

where $W_{ijk} = \partial_i \partial_j \partial_k g$, where $g(\phi^i, \rho)$ is the superpotential for matter fields (14.13). In our case, more explicitly they equal

$$(Ay)_{ijk} = e^K \left[K^{\rho\bar{\rho}} \overline{D_\rho W} (K_\rho + \nabla_\rho) + K^{S\bar{S}} \overline{D_S W} K_S \right] W_{ijk} \,, \tag{14.44}$$

where we used, according to the arguments given above, our hypothesis that Yukawas depend very weakly on S. For bilinears B-terms, keeping also Giudice-Masiero like terms, we find

$$\begin{aligned} (B\mu)_{ij} &= e^K K^{\alpha\bar{\beta}} \overline{D_\beta W} (K_\alpha + \nabla_\alpha) W_{ij} - m_{3/2} e^{K/2} W_{ij} + \\ & m_{3/2}^2 (2 + G^\alpha \nabla_\alpha) K_{ij} - m_{3/2}^2 \Gamma^\alpha_{ij} (2G_\alpha + G^\beta G_{\alpha\beta}) \,. \end{aligned} \tag{14.45}$$

Notice that in our case, since $D_\rho W$ and $K_S \sim \bar{S}$ are very small, we find negligibly small A-terms. More precisely, we find that the dominant contribution to A_0 is given by $\bar{S}\bar{D}_{\bar{S}}\bar{W}$ so that at the tree-level one finds that the A-terms are given by

$$A_0 \simeq -\frac{1}{(2\sigma_0)^{3/2}} \frac{|\Delta|\Lambda^2}{2} = \frac{1}{2} m_{3/2} \Lambda^2 \tag{14.46}$$

and are extremely small if $\Lambda \ll 1$. This expression for A_0 is valid so long as $m_{3/2}/m_\sigma \ll \Lambda^2 \ll 1$. For $\Lambda^2 \ll m_{3/2}/m_\sigma \ll 1$ the parameter A_0 is proportional to $m_{3/2}^2/m_\sigma$, so in both cases $A_0 \ll m_{3/2}$. Thus we are driven to small values of A_0 as a direct consequence of strong stabilization. On the other hand, the μ and $B\mu$ parameters in the Higgs sector are given by

$$\begin{aligned} \mu &= m_{3/2} G_{12} = e^{K/2} W_{12} + m_{3/2} K_{12} = \mu_0 + m_{3/2} K_{12} \,, \\ B\mu &= (A_0 - m_{3/2}) \mu_0 + 2 m_{3/2}^2 K_{12} \,. \end{aligned} \tag{14.47}$$

where $W_{12} = \partial_{H_1} \partial_{H_2} W$, $K_{12} = \partial_{H_1} \partial_{H_2} K$, and $\mu_0 = e^{K/2} W_{12}$ is the μ-term in the absence of Giudice-Masiero terms. By combining eqs. (14.47), we find

$$B \;= (A_0 - m_{3/2}) \frac{\mu_0}{\mu} + \; \frac{2 m_{3/2}^2}{\mu} K_{12} \,, \tag{14.48}$$

that will be used in the next section for phenomenology. If $K_{12} = 0$, we get $\mu = \mu_0$ and $B = A_0 - m_{3/2}$, which is just the familiar mSUGRA relation $B_0 = A_0 - m_0$.

For a suitable choice of gauge kinetic functions $h_{\alpha\beta} = h(\rho)\delta_{\alpha\beta}$, one generates universal gaugino masses

$$m_{1/2} \;=\; \frac{\sqrt{2\sigma_0}}{6} \, \overline{D_\rho W(\rho)} \; \partial_\rho \ln \mathrm{Re}\, h \,, \tag{14.49}$$

where, according to our decoupling hypothesis, we have assumed that h does not explicitly depend on S [3]. In contrast to the universal scalar masses which are equal to the gravitino mass, $m_{1/2}$ is proportional to $D_\rho W$ and is suppressed by $m_{3/2}/m_\sigma$.

As a result, we obtain models resembling those mediated by anomalies [336], where the dominant contributions to gaugino masses and A-terms arise from loop corrections and give [342]

$$m_{1/2}^a = \frac{b_a g_a^2}{16\pi^2} \frac{F^C}{C_0} \tag{14.50}$$

[3] If we allow a coupling of the form $h_A = h_A^0 (1 + \beta_A e^{-2aT} S)$, we would find a suppression $m_{1/2} \propto m_{3/2}^2/M_P$.

and

$$A_{ijk} = -\frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2} \frac{F^C}{C_0} \ . \tag{14.51}$$

Here $b_a = 11, 1, -3$ for $a = 1, 2, 3$ are the one-loop beta function coefficients, γ_i are the anomalous dimensions of the matter fields y_i and

$$\frac{F^C}{C_0} = -\frac{1}{3} e^{K/2} K^{\alpha\bar{\beta}} K_\alpha \bar{D}_{\bar{\beta}} \bar{W} + m_{3/2} \ \simeq \ m_{3/2} \tag{14.52}$$

is related to the conformal compensator and equals to very high accuracy $m_{3/2}$ in the models we consider.

Because of the loop suppression factor in Eq. (14.50), we are forced to relatively large ($\mathcal{O}$(10-1000) TeV) gravitino masses in order to have acceptably large gaugino masses[4] Thus, the sparticle spectrum consists of relatively light gauginos whose masses are determined from anomaly mediation and large soft scalar masses fixed by the gravitino mass yielding a spectrum reminiscent of split supersymmetry [335]. The problem of tachyonic scalars normally associated with anomaly mediated models is absent here.

In what follows, we will examine the phenomenological consequences of the above model. In particular, we will see that it is difficult to construct consistent models if one wants to maintain the possibility of radiative electroweak symmetry breaking. If the input supersymmetry breaking scale is chosen to be the GUT scale (i.e. the scale at which gauge coupling unification occurs), one can not choose arbitrarily large universal scalar masses and insist on a well defined electroweak symmetry breaking vacuum (i.e., $\mu^2 > 0$). This difficulty can be alleviated in at least two ways:
- increasing the supersymmetry breaking scale, $M_{in} > M_{GUT}$. This is the case that we study in detail in the next section.
- Allow for direct couplings between the uplift field(s) S and the Higgs in the Kahler potential, by terms of the type $S^\dagger S H_i^\dagger H_i$. In this case, Higgs soft scalar masses acquire additional corrections proportional to $|F_S|^2$, where $F_S = e^{K/2} K^{S\bar{S}} D_S W$. They are no longer equal to the other scalar masses and are not necessarily degenerate anymore. These boundary conditions for scalar masses then resemble those assumed in non-universal Higgs mass models. This problem can be traced directly back to our assumption of strong moduli stabilization and small A-terms. With large A-terms, there is no difficulty in obtaining electroweak vacuum solutions with large m_0 and $\mu^2 > 0$.

Another challenge presented in these types of models stems from the mSUGRA relation between B_0 and A_0. Unlike CMSSM models [355, 356], this relation forces one to solve for $\tan\beta$ for a given choice of $m_{1/2}, m_0$, and A_0. In the present context, we expect no solutions as there is in effect only a single free parameter, namely $m_{3/2}$. However, an interesting extension of minimal supergravity is one where terms proportional $H_1 H_2$ are added to the Kähler potential as in the Giudice-Masiero mechanism [341]. By introducing a non-minimal coupling to the Kähler potential, one can effectively fix $\tan\beta$. If in addition, we take $M_{in} > M_{GUT}$, we can in fact formulate a consistent phenomenological model.

[4] A posteriori, we know that for very small A_0/m_0 the requirement for relatively large Higgs masses would lead us to the same conclusion regarding large scalar masses, which to control the relic density would also require anomaly mediation for gaugino masses.

In the next section we briefly review the GM extension to mSUGRA and the consequences of taking $M_{in} > M_{GUT}$. As a result, we are forced to consider a specific GUT and here for simplicity, we take minimal SU(5) as a concrete example. In section 5, we present the main results of the paper which include the low energy spectrum as a function of the gravitino mass. In particular, this amounts to the gaugino and Higgs masses as all of the other supersymmetric scalars are very heavy. Other phenomenological aspects of the models such as gluino production at the LHC and the direct and indirect detection of dark matter are discussed in section 6.

14.5 GM Supergravity and Super-GUT phenomenology

As described above, the KL phenomenological model has one free parameter, $m_{3/2}$, which when extended to include a Giudice-Masiero term, has two free parameters, which we take to be $m_{3/2}$ and $\tan\beta$. This is to be compared with mSUGRA models which have 3 free parameters or CMSSM models with 4 free parameters. In the present context, the gaugino masses, scalar masses, and A-terms are all determined by the gravitino mass. The alternative of coupling the uplift fields S directly to the Higgs sector, in order to obtain non-universal Higgs masses which are different from the gravitino mass, mentioned in the previous section, will not be pursued here for simplicity. On the other hand, direct couplings of uplift fields to squarks and sleptons $\lambda_{ij} S^\dagger S \phi^i \phi^\dagger_j$ in the Kahler potential would generically lead to flavor dependence and therefore to FCNC effects. Even for $30 - 50$ TeV scalar masses, which will be our typical values in what follows, FCNC effects require some degree of degeneracy. This is actually the main phenomenological reason we are imposing no direct couplings between uplift fields and matter fields in our paper.

From Eq. (14.41), we expect scalar mass universality at some renormalization scale, M_{in}. In the CMSSM, this scale is usually associated with the GUT scale[5]. If so, these masses are run down to low energy using standard renormalization group evolution. In contrast to the CMSSM, the gravity mediated part of gaugino masses and A-terms in the KL model are extremely small and their dominant contributions are determined by anomalies at any scale using Eqs. (14.50) and (14.51). In the CMSSM, μ and B are solved for in terms of M_Z and $\tan\beta$:

$$\begin{aligned} \mu^2 &= \frac{m_1^2 - m_2^2 \tan^2\beta + \frac{1}{2} m_Z^2 (1 - \tan^2\beta) + \Delta_\mu^{(1)}}{\tan^2\beta - 1 + \Delta_\mu^{(2)}} \,, \\ B\mu &= -\frac{1}{2}(m_1^2 + m_2^2 + 2\mu^2)\sin 2\beta + \Delta_B \,, \end{aligned} \tag{14.53}$$

where Δ_B and $\Delta_\mu^{(1,2)}$ are loop corrections [357–359], and $m_{1,2}$ are the Higgs soft masses (here evaluated at the weak scale). In mSUGRA models, however, B can not be determined independently as it must respect its boundary condition $B_0 = A_0 - m_0$ at M_{in}. Instead, one must solve for $\tan\beta$ (and μ) using the electroweak symmetry breaking conditions [360, 361].

[5]The GUT scale, M_{GUT}, is defined as the scale where SU(2) and U(1) gauge couplings unify and is approximately 1.5×10^{16} GeV.

In this sense, the KL models we are describing are more reminiscent of mSUGRA than the CMSSM.

There are, however, two immediate potential problems with the framework as described: 1) There is no guarantee that reasonable solutions for $\tan\beta$ exist while requiring $B_0 = A_0 - m_0$ at M_{in}. Indeed it is known [360,361] that only a limited portion of parameter space (defined by $m_0, m_{1/2}$ and A_0) possesses solutions for $\tan\beta$. 2) There is no guarantee that solutions with $\mu^2 > 0$ exist when m_0 is very large. This of course is a well known issue present in the CMSSM. For fixed $m_{1/2}$ and A_0, there is an upper limit to m_0 for which there are solutions to (14.53) with $\mu^2 > 0$ known as the focus point or hyperbolic branch [362]. This upper limit is also present in mSUGRA models as well, particularly when A_0/m_0 is small (as it is the case under consideration). As we now describe, neither problem is critical and there are known and relatively simple solutions to both.

To tackle the problem of $\tan\beta$, consider a Giudice-Masiero (GM) -like contribution to K of the form [341],

$$\Delta K = c_H H_1 H_2 + h.c.\,, \tag{14.54}$$

where c_H (equal to K_{12} in the previous section) is a constant, and $H_{1,2}$ are the usual MSSM Higgs doublets. The presence of ΔK affects the boundary conditions for both μ and the B term at the supersymmetry breaking input scale, M_{in}. The μ term is shifted as seen in Eq. (14.47) to

$$\mu_0 + c_H m_0\,. \tag{14.55}$$

However, since we solve for μ at the weak scale, its UV value is fixed by the low energy boundary condition. In contrast, the boundary condition on μB shifts from $\mu_0 B_0$ to

$$\mu_0 B_0 + 2c_H m_0^2\,. \tag{14.56}$$

We can add the GM term to better connect the solution of the minimization conditions to a supergravity boundary condition at M_{in}. Indeed, by allowing $c_H \neq 0$, we can fix $\tan\beta$ and derive μ and $B\mu$ at the weak scale. By running our derived values of $B(M_W)$ and $\mu(M_W)$ up to the GUT scale, we can write

$$B\mu(M_{GUT}) = (A_0 - m_0)\mu_0(M_{GUT}) + 2c_H m_0^2\,, \tag{14.57}$$

which is precisely eq. (14.47) of the previous section. Strictly speaking, (14.57) is valid at tree-level in SUGRA and does not include anomaly contributions. However, the latter are small compared to tree-level values of m_0, B and μ, so (14.57) is an excellent approximation. In what follows, we use Eq. (14.57) to derive the necessary value of c_H.

Of course, one must still check, whether the solution for c_H is reasonable (i.e., perturbative). In [361], it was indeed shown that over much of the mSUGRA parameter space $c_H \lesssim O(1)$. For fixed $\tan\beta$ and A_0/m_0, c_H is reasonably small for most choices of $m_{1/2}$ and m_0. Exceptions lying in the region where $m_{1/2} \gg m_0$ and the lightest supersymmetric particle (LSP) is the gravitino. When A_0 is large, these offending regions are further compressed to small m_0. Thus by allowing non-zero c_H, we can always satisfy the mSUGRA boundary condition for B_0 and check a posteriori that c_H is small.

As noted above, in the CMSSM and mSUGRA, there is generally an upper limit to m_0 for fixed $m_{1/2}, A_0$, and $\tan\beta$ determined by $\mu^2 = 0$ in Eq. (14.53). While it is common to assume

that the input supersymmetry breaking scale is equal to the GUT scale, it is quite plausible that M_{in} may be either below [363] (as in models with mirage mediation [342, 350, 364]) or above [361, 365–369] the GUT scale. Increasing M_{in} increases the renormalization of the soft masses which tends in turn to increase the splittings between the physical sparticle masses [367]. As a consequence, the focus-point solution for $\mu^2 = 0$ often moves out to very large values of m_0. This feature of super-GUT models is essential for KL model described here. Note that while the introduction of M_{in} adds a free parameter to the model, as we will see, our results are very insensitive to the choice of M_{in}. For consistency with the KL paradigm, we should also only consider values of $M_{in} < m_\sigma$.

To realize $M_{in} > M_{GUT}$, we need to work in the context of a specific GUT. Here, we use the particle content and the renormalization-group equations (RGEs) of minimal SU(5) [367, 370], primarily for simplicity: for a recent review of this sample model and its compatibility with experiment, see [371]. As this specific super-GUT extension of the CMSSM was studied extensively in Refs. [365, 372], we refer the reader there for details of the model.

The model is defined by the superpotential

$$\begin{aligned} W_5 &= \mu_\Sigma \operatorname{Tr} \hat{\Sigma}^2 + \frac{1}{6}\lambda' \operatorname{Tr} \hat{\Sigma}^3 + \mu_H \hat{\mathcal{H}}_1 \hat{\mathcal{H}}_2 + \lambda \hat{\mathcal{H}}_1 \hat{\Sigma} \hat{\mathcal{H}}_2 \\ &\quad + (\mathbf{h_{10}})_{ij} \hat{\psi}_i \hat{\psi}_j \hat{\mathcal{H}}_2 + (\mathbf{h_{\overline{5}}})_{ij} \hat{\psi}_i \hat{\phi}_j \hat{\mathcal{H}}_1 \,, \end{aligned} \tag{14.58}$$

where $\hat{\phi}_i$ ($\hat{\psi}_i$) correspond to the $\overline{\mathbf{5}}$ ($\mathbf{10}$) representations of superfields, $\hat{\Sigma}(\mathbf{24})$, $\hat{\mathcal{H}}_1(\overline{\mathbf{5}})$ and $\hat{\mathcal{H}}_2(\mathbf{5})$ represent the Higgs adjoint and five-plets. Here $i, j = 1..3$ are generation indices and we suppress the SU(5) index structure for brevity. There are now two μ-parameters, μ_H and μ_Σ, as well as two new couplings, λ and λ'. Results are mainly sensitive to λ and the ratio of the two couplings. In what follows, we will fix $\lambda' = 0.1$.

To generalize the GM solution for the B_0 boundary condition, we write

$$\Delta K = c_H \mathcal{H}_1 \mathcal{H}_2 + \frac{1}{2} c_\Sigma \operatorname{Tr} \Sigma^2 + h.c. \,, \tag{14.59}$$

where $\mathcal{H}_{1,2}$ are scalar components of the Higgs five-plets and Σ is the scalar component of the adjoint Higgs. Thus in principle, we have two extra parameters which can be adjusted to relate the CMSSM and supergravity boundary conditions for $M_{in} > M_{GUT}$. Nevertheless, these parameters have virtually no effect on the sparticle mass spectrum other than allowing us to fix $\tan\beta$ in a consistent manner.

For $M_{in} > M_{GUT}$, scalar mass universality is defined in terms of the scalar components of the Higgses and matter fields in the $\overline{\mathbf{5}}$ and $\mathbf{10}$ representations. At the GUT scale, these must be matched to their Standard Model counterparts. More importantly is the matching of the μ and B-terms from SU(5) to Standard Model parameters. These have been discussed extensively in Ref. [361, 373] and we do not repeat that analysis here.

There is one aspect of the matching of soft terms at M_{GUT} that is specific to the present model. Dominant contributions to gaugino masses and A-terms are provided by the conformal anomaly (14.50,14.51), with beta functions and anomalous dimensions computed with the spectrum at the given energy scale E. For the MSSM for example, gaugino masses at scale E are given by

$$m^a_{1/2}(E) = \frac{b_a g_a^2(E)}{16\pi^2} \frac{F^C}{C_0} \,. \tag{14.60}$$

Above M_{GUT}, on the other hand, we have a unified ($SU(5)$ in our case) theory, with a unified gauge coupling g_{GUT} and a unified beta function b_{GUT}. The unified gaugino mass is then given by

$$m_{1/2}^{GUT}(E) \;=\; \frac{b_{GUT} g_{GUT}^2(E)}{16\pi^2} \frac{F^C}{C_0} \tag{14.61}$$

and its value has to be taken into account for the running of soft terms between M_{in} and M_{GUT}. However, there is no matching at M_{GUT} between (14.60) and (14.61). The mismatch is to be interpreted as a threshold effect, due to the decoupling of heavy GUT particles at M_{GUT}. The argument is completely similar for the A-terms.

The additional running between M_{in} and M_{GUT} in CMSSM-like models is very efficient at raising the upper limit on m_0 [361,365,366,368] provided the Higgs coupling λ is sufficiently large. In mSUGRA-like models, however, we are still faced with the difficulty of satisfying the B_0 boundary condition and the GM parameters must be added. As shown in [361], for $M_{in} \gtrsim 10^{17}$ GeV, and fixed $\tan\beta$, A_0, m_0, and $m_{1/2}$, values of c_H are only small when λ is small[6]. At large λ (needed to raise the upper limit on m_0), values of c_H are of order 10 or larger. However, as also shown in [361], it is often possible to regulate c_H, by choosing $c_\Sigma \neq 0$ (yet still reasonably small). Thus, it is possible in the context of a superGUT version of the KL framework, to obtain a consistent sparticle spectrum.

Let us now summarize the ingredients of the phenomenological model we are considering. We begin with a no-scale type Kähler potential for the moduli with a racetrack superpotential as in the KL model. Uplifting is accomplished with an extra heavy Polonyi-like field. Unlike the KKLT model, here the moduli are superheavy, while the gravitino remains relatively light. Scalar mass universality is input at a renormalization scale M_{in}, with $m_0 = m_{3/2}$ at that scale. Gravity mediation supplies vanishingly small gaugino masses of order $m_{3/2}^2/m_\sigma$ and A-terms of order $\Lambda^2 m_{3/2} \ll m_{3/2}$, and thus these quantities receive their dominant contribution from anomalies. As a result, we have $m_{1/2}/m_0 \ll 1$ as well as $A_0/m_0 \ll 1$, and choose $M_{in} > M_{GUT}$ to allow solutions to the EWSB equations with $\mu^2 > 0$. In addition, we introduce GM parameters, c_H, and c_Σ so that $\tan\beta$ can be held fixed and still satisfy the boundary condition for B_0. We will see that for $c_\Sigma \gtrsim -1$, we can obtain c_H close to 0. We expect that the alternative road we mentioned, of imposing boundary conditions at $M_{\rm in} = M_{\rm GUT}$, and changing Higgs scalar masses via direct couplings of uplift fields to the Higgs sector, will lead to a similar phenomenology.

14.6 Low-energy spectra

We next describe the resulting sparticle spectrum and the predicted low energy phenomenology we expect from the KL motivated class of models. As should be clear from the preceding discussion, below the superheavy scale (GUT fields and moduli), there are sparticles with masses of order $m_{3/2}$, and those with much lighter masses as determined from anomalies.

Because of the boundary condition $m_0 = m_{3/2}$, all of the scalar partners of the matter fields have mass close to $m_{3/2}$. For example, let us take $m_{3/2} = 32$ TeV, with $\tan\beta = 25$, $M_{in} = 5 \times 10^{17}$ GeV, and $\lambda = 1.35$. By choosing $c_\Sigma \simeq -0.85$, we can obtain $c_H = 0$ for

[6]Note that the upper limit on m_0 can be raised significantly for small λ and large A_0/m_0. Of course in the KL models discussed here, A_0/m_0 is very small.

this point. Not surprisingly, all of the first and second generation sfermions have masses close to $m_{3/2}$. The lighter stau lepton has a mass of 29.6 TeV, and the lighter stop and sbottom quarks have masses of 24.2 and 26.9 TeV. All of which are past the current (and future) reach of the LHC. The Higgs soft masses run considerably and both mass-squared are negative with $|m^2|^{1/2} = 10.9$ and 25.1 TeV. The μ parameter from Eq. (14.53) is also very large (20.4 TeV) unlike the assumption made in many models of split supersymmetry. As a result, the pair of Higgsinos and the Higgs-like chargino have large masses (22.0 TeV). Similarly, the heavy Higgs scalar and pseudo-scalar have a mass of 20 TeV. These results are not very sensitive to any of the assumed input parameters and to a first approximation are all close to $m_{3/2}$. The full spectrum for this point and other test points are given in Table 14.2.

In contrast, the gauginos and one (wino-like) chargino remain relatively light. Their masses are dominated by the anomaly contributions given in Eq. 14.50. For the specific choice of parameters adopted above, we have a nearly degenerate neutral and charged pair of winos at 107 GeV (the neutral wino is the LSP) and a 314 GeV bino. The gluino mass is 1.0 TeV. Note that as we use two-loop expressions for the running of gaugino masses, our results differ slightly from the approximate values given in Eq. (14.60).

As we saw, the supersymmetry breaking scalar masses at the weak scale are $M_{SUSY} \simeq m_{3/2} \sim 10-1000$ TeV, while the gauginos are significantly lighter. Such heavy scalars lead to important quantum corrections in the Higgs sector. These are enhanced by large logarithms of the type $\log(M_{weak}/M_{SUSY})$. In fact, most of the public codes assume M_{SUSY} less than a few TeV, otherwise the computations become unreliable. Thus in the computation of the light CP-even Higgs boson mass, m_h, we follow the procedure described in Refs. [374, 375] and we add in the contributions due to sbottoms-induced corrections, though these are small in the models considered. The Higgs mass is effectively given by its quartic coupling

$$m_h^2 = 2v^2\lambda_H(Q)\,, \tag{14.62}$$

where v is the Higgs vacuum expectation value and we evaluate the quartic coupling at $Q = m_t$. The tree-level quartic coupling is simply

$$\lambda_H = \frac{1}{4}(\frac{3}{5}g_1^2 + g_2^2)\cos^2 2\beta \tag{14.63}$$

at the scale where heavier higgs fields decouple. The masses of the heavy CP-even and CP-odd Higgs bosons are of order M_{SUSY} and thus can be reliably computed by conventional techniques.

The heavy scalars decouple at approximately the scale, which we take to be $M_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, and the low-energy theory contains only SM fermions, gauginos, and one SM-like higgs doublet. Thus, in the effective theory below M_{SUSY}, in addition to the regular SM gauge, Yukawa and Higgs quartic couplings, we have Yukawa-like Higgs-higgsino-gaugino couplings. Since SUSY is broken below M_{SUSY} those Yukawa-like couplings are no longer equal to corresponding gauge couplings and also renormalize differently. We perform RGE evolution of all effective-theory couplings at the 2-loop level and take into account 1-loop threshold effects, thus obtaining the Higgs mass, m_h at full next-to-leading order accuracy.

The sparticle spectra for a few values of $m_{3/2}$ are given in Table 14.2. The (light) fields have masses which scale with the gravitino mass. In Fig. 14.1, we show the resulting masses

Figure 14.1: *The gaugino and chargino masses and the μ-term as a function of the gravitino mass, $m_{3/2}$. Here we have chosen,* $\tan\beta = 25$, $M_{in} = 5 \times 10^{17}$ *GeV,* $\lambda = 1.35$.

of the three gauginos, chargino (degenerate with the wino-like gaugino) and Higgs mass as a function of $m_{3/2}$. For reference, we also plot μ. For this example, we have chosen the same input parameters discussed above, namely $\tan\beta = 25$, $M_{in} = 5 \times 10^{17}$ GeV, and $\lambda = 1.35$. In addition, here and in all subsequent figures, we have taken $\lambda' = 0.1$ and $c_\Sigma = -0.85$. For the range of $m_{3/2}$ shown, c_H can be made to vanish for $c_\Sigma = -0.85$ - -0.91. We emphasize that these parameters do not affect the mass spectrum, but allow us to take fixed $\tan\beta$. As one can see the ratio between the gaugino masses is relatively fixed as one might expect from the anomaly conditions. Any deviations from this are a result of two loop effects. Notice that on this vertical scale, the Higgs mass is nearly constant.

Note that the LEP bound on the chargino mass of 104 GeV, provides a direct lower limit to $m_{3/2}$ in these models. As can be seen from Fig. 14.1, this chargino mass limit implies $m_{3/2} \gtrsim 31$ TeV. The Higgs mass at this value of $m_{3/2}$ is 125.3 GeV (at $\tan\beta = 25$). Thus, the limit on the chargino mass directly implies a lower limit on the Higgs mass of roughly 125 GeV in amazing agreement with the recent discovery claimed at the LHC [376]. Furthermore, at the lower bound on $m_{3/2}$, the gluino mass is 970 GeV close to the current lower bound from the LHC [377].

Because, the gaugino and chargino spectrum is determined by anomalies, it is almost completely independent of the choice of the parameters chosen in Fig 14.1. We have explicitly verified that for several choices of $\tan\beta$ and λ, and a range of input scales, M_{in}, up to the Planck scale, differences in the mass spectra are negligible.

The exception is the Higgs mass which we show separately in Figs. 14.2 and 14.3. In Fig. 14.2, we show the Higgs mass as a function of $m_{3/2}$ for several sets of input parameters. In addition to our base model with $\tan\beta = 25$, $M_{in} = 5 \times 10^{17}$ GeV, $\lambda = 1.35$, we vary each of these to show the behavior of the Higgs mass as a function of $m_{3/2}$. For the most part, we see that they are largely insensitive to the choice of input parameters and all yield masses

parameter	1	2	3	4	5
$m_{3/2}$ [TeV]	32	50	100	500	1000
$m_{\tilde{g}}$ [TeV]	1.0	1.5	2.7	11.1	20.8
$m_{\tilde{\chi}_1}$ [GeV]	107	168	338	1705	3423
$m_{\tilde{\chi}_2}$ [GeV]	314	495	1000	5130	10400
$m_{\tilde{\chi}_3}$ [TeV]	22.0	34.9	70.7	367	745
$m_{\tilde{\chi}_4}$ [TeV]	22.0	34.9	70.7	367	745
$m_{\chi_1^+}$ [GeV]	107	168	338	1705	3420
$m_{\chi_2^+}$ [TeV]	22.0	34.9	70.7	367	745
$m_{\tilde{t}_1}$ [TeV]	24.2	38.0	77.2	397	803
$m_{\tilde{t}_2}$ [TeV]	26.8	42.1	84.6	428	860
$m_{\tilde{b}_1}$ [TeV]	26.9	42.1	84.7	428	860
$m_{\tilde{b}_2}$ [TeV]	30.6	47.9	96.0	483	969
$m_{\tilde{q}_L}$ [TeV]	31.4	49.2	98.5	494	990
$m_{\tilde{u}_R}$ [TeV]	31.5	49.3	98.7	495	990
$m_{\tilde{d}_R}$ [TeV]	31.6	49.4	98.9	496	992
$m_{\tilde{\tau}_1}$ [TeV]	29.6	46.2	92.3	459	917
$m_{\tilde{\tau}_2}$ [TeV]	31.2	48.7	97.5	488	978
$m_{\tilde{\nu}_\tau}$ [TeV]	31.2	48.7	97.5	488	978
$m_{\tilde{e}_L}$ [TeV]	31.9	49.8	99.6	498	996
$m_{\tilde{e}_R}$ [TeV]	32.0	50.0	100	500	1000
$m_{\tilde{\nu}_L}$ [TeV]	31.9	49.8	99.6	498	996
m_h [GeV]	125	127	128	132	133
μ [TeV]	20.4	32.3	65.0	333	673
m_A [TeV]	19.5	30.6	58.4	262	494
$\Omega_{\tilde{\chi}} h^2$	0.0003	0.0008	0.0030	0.067	0.26
$\sigma^{SI}(\chi_1 p) \times 10^{14}$ [pb]	4.74	1.81	0.44	0.02	0.003
$\sigma^{SD}(\chi_1 p) \times 10^{12}$ [pb]	6.78	0.94	0.04	0.0008	0.001

Table 14.2: Input parameters and resulting masses and rates for benchmark points with $M_{in} = 5 \times 10^{17}$ GeV, $\lambda = 1.35$, $\lambda' = 0.1$, $c_\Sigma = -0.85$, $\tan\beta = 25$, $\mu > 0$ and $m_t = 173.1$ GeV.

Figure 14.2: *The Higgs mass as a function of the gravitino mass,* $m_{3/2}$*. Here we have chosen, several combinations of* $\tan\beta$*,* M_{in}*, and* λ *as indicated on the figure.*

between 125 GeV (when $m_{3/2} > 30$ TeV so that $m_{\chi^+} > 104$ GeV) and ~ 130 GeV [7] Indeed the Higgs mass determination allows one to set an upper limit on the split susy scale [**?**,375] or $m_{3/2}$ in the present context.

In Fig. 14.3, we show the Higgs mass as a function of $\tan\beta$. For $\tan\beta \gtrsim 10$, the Higgs mass is relatively insensitive to $\tan\beta$ and decreases for smaller values of $\tan\beta$. Several of the curves are cut off at low $\tan\beta$ when the chargino mass falls below the LEP limit. We see again that the results are also insensitive to the particular choice of model for $m_{3/2} =$ 32 TeV, but increases with $m_{3/2}$ as we saw in Fig. 14.2.

Note that the curves in Fig. 14.3 terminate at large $\tan\beta$ when solutions to the electroweak symmetry breaking conditions can no longer be obtained. This is due to the effect of the bottom (and to a lesser degree, tau) Yukawa coupling on the RGE evolution of Higgs mass parameters. At these values of $\tan\beta$, the bottom Yukawa coupling is large and comparable in magnitude to the top Yukawa coupling. Since Yukawa terms tend to decrease scalar masses, m_1^2 at the weak scale is driven to smaller and even to negative values. At some point it becomes impossible to find solutions to Eqs. (14.53). A similar decrease in m_1^2 is found when both M_{in} and $m_{3/2}$ become sufficiently large [365], as can be seen from the behaviour of the orange curve in Fig. 14.2 that terminates at $m_{3/2} \simeq 60$ TeV.

[7] We note that there is at least a 2-3 GeV uncertainty in the Higgs mass calculation, so that we can not out of hand exclude results with Higgs masses at 130 GeV.

Figure 14.3: *The Higgs mass as a function of* $\tan\beta$*. We have chosen, several combinations of* $m_{3/2}$*,* M_{in}*, and* λ *as indicated on the figure.*

14.7 Other phenomenological aspects

14.7.1 Gluinos

Multi-TeV scalars have severe consequences for the low energy spectra and LHC discovery prospects. It seems obvious that except for a hypothetical SLHC extension, the scalar sector of constructions such as the one described above is unreachable at the LHC. Nevertheless, the gaugino spectrum is relatively light (reduced by a loop factor generated by anomaly mediation) and a 1 TeV gluino can still be detectable. However, the classical 2-body decay mode $\tilde{g} \to \tilde{q}q$ is kinematically forbidden. The dominant decay modes become:
1) The three-body decay $\tilde{g} \to q\tilde{q}^* \to qq\widetilde{\chi}_1^0$ (through the exchange of a virtual squark $\tilde{q}^*$) with a 2 jet plus missing E_T signature.
2) The 2-body decay modes $\tilde{g} \to g\widetilde{\chi}_1^0$ generated by squark-quark loop diagrams.
There have been several analyses of gluino decay in split supersymmetry [379, 380]. These showed that as long as the scalar masses are below $\sim 10^4$ TeV, the lifetime of the gluino is too short to be detected. Since the two body decays occur primarily via the Higgsino component of the neutralino [381], these will be highly suppressed in the models under consideration with $\mu \gg m_{1/2}^a$ and a negligible Higgsino component in the either of the two light neutralinos. While there is a gaugino component to the 2-body decay, it cancels in the limit of degenerate squark masses due to the tracelessness of the diagonal generators of the electroweak gauge group. Since in our scenario, squarks are of order 10-1000TeV and close in mass, the 2-body loop decay is always negligible.

The three body gluino decay rate to winos can be written as [380]

$$\Gamma(\tilde{g} \to q\bar{q}\widetilde{\chi}) \;\approx\; \frac{9}{4}\frac{g_3^2 g_2^2}{768\pi^3}\frac{m_{\tilde{g}}^5}{m_{\tilde{q}}^4}\,. \tag{14.64}$$

Assuming that we are sufficiently far from thresholds (which we are in the cases under consideration), the above expression is a reasonable approximation. Numerically, however, the decay rates for $m_{3/2} < 1000$ TeV are sufficiently short so as to make the displaced vertices in gluino decays unobservable at the LHC. For the benchmark points above, the decay rates range from $5.3 \times 10^{-11} - 2.1 \times 10^{-10}$ TeV for $m_{3/2} = 32 - 1000$ TeV. This corresponds to a decay length of 3.7 – 0.9 nm, respectively. These lengths are far below the LHC resolution of order 10 μm.

Nevertheless, gluinos can be observed at the LHC since their pair production cross section is above the fb level for $m_{\tilde{g}} \lesssim 1.8$ TeV [382]. Gluinos can be searched for using the usual multi-jet plus missing energy analysis. It was shown that the gluino-wino mass difference can be determined from the endpoint of the di-jet invariant mass distribution with the accuracy of 5% [383]. Additional information about the gluino mass can be obtained from the measurements of the total gluino production cross section [382] or from the effective mass M_{eff} distribution [384].

14.7.2 Charginos

A related possibility for accelerator detection is through the observation of charged tracks of winos, as was suggested for AMSB-type gauginos [385]. When μ is much larger than M_2, the lighter chargino $\widetilde{\chi}_1^{\pm}$ is mostly wino and is only slightly heavier than the wino-like LSP. The chargino decay rate depends strongly on the $\widetilde{\chi}_1^{\pm} - \widetilde{\chi}_1^0$ mass splitting: if the splitting is larger than the mass of the charged pion, the dominant decay is $\widetilde{\chi}_1^{\pm} \to \widetilde{\chi}^0 + \pi^{\pm}$. The rate of this decay is quite slow and the chargino lifetime is of order 10^{-10}s. The resultant pion is very soft and one will only see the charged track stub, i.e. an abruptly terminating charged track, from $\widetilde{\chi}_1^{+}$. That track stub is only a few centimeters long for the roughly 200 MeV mass gap we have, so it will be difficult to observe at LHC. However, it has been argued that the track stubs could be observed in a specific mono-jet searches, provided that wino pairs are produced in the association with a hard jet that will serve as a trigger [385, 386].

14.7.3 Dark Matter

Since the only 'light' sparticles in the spectrum are the neutral gauginos and charged wino, and since these sparticle masses are fixed by anomalies, it is easy to see that the LSP is a wino. As a consequence, the relic density of LSPs is in general relatively small. The neutral and charged winos are nearly degenerate and the annihilations and co-annihilation cross sections to W^+W^- are large. Thus for our reference point at $m_{3/2} = 32$ TeV and $\tan\beta = 25$, the relic density of winos is $\Omega_\chi h^2 = 2.8 \times 10^{-4}$, far below the density ascertained by the cosmic microwave background anisotropy spectrum [13]. In Fig. 14.4, we show the wino relic density as a function of $m_{3/2}$. As one can see, for values of $m_{3/2}$ near the lower bound of 30 TeV, the relic density is far too small to account for the dark matter density inferred from the CMB. There are two noticeable dips at $m_{3/2} = 12 - 13$ TeV, which occur in all cases considered. These are due to the strong co-annihilation between the nearly degenerate neutral and charged winos. The first dip occurs because of the s-channel W-exchange in neutral and charged winos when $m_\chi \simeq m_{\widetilde{\chi}^{\pm}} \simeq m_W/2$, whilst the second dip occurs at slightly higher $m_{3/2}$ because of s-channel Z-exchange in chargino annihilation when

Figure 14.4: *The LSP (neutral wino) relic density, $\Omega_\chi h^2$, as a function of the gravitino mass, $m_{3/2}$. Here we have chosen, several combinations of $\tan\beta$, M_{in}, and λ as indicated on the figure.*

$m_\chi \simeq m_{\widetilde{\chi}^\pm} \simeq M_Z/2$. Of course, these chargino masses are well below the LEP bound and the value of $m_{3/2}$ where the dips occur is excluded.

We must therefore, address the question of dark matter in the context of this class of phenomenological models. Three possibilities are relatively straightforward:

1) The dark matter is something else other than a wino (e.g. an axion). This is a rather obvious possibility. While supersymmetric dark matter is an attractive possibility, it is not unique and axions present us with another well motivated candidate. Originally it was assumed that axions give a proper contribution to dark matter only if their mass is $O(10^{-5})$ eV [388]. However, later it was found that if the PQ symmetry was broken during inflation, and the Hubble constant during the last stage of inflation is sufficiently small, which is necessary to suppress isocurvature axion perturbations, then anthropic considerations lead to a desirable axionic contribution to dark energy for a broad range of axion masses $m_a \ll 10^{-5}$ eV [389]. One may also consider models with many light axion fields, the axiverse [390].

2) LSPs are created non-thermally via gravitino or moduli decay. In principle, moduli decay to gravitinos which subsequently decay to neutralinos can be quite dangerous, as their late decays may often overproduce LSPs leaving one with a relic density in excess of the WMAP determination. In our case, the modulus of concern is the Polonyi-like field S. However, because of the structure of the Kähler potential in Eq. (14.23), we expect the field S, to be pinned to the origin during inflation, when its mass is large (H^2/Λ [330, 332, 333]) and subsequent oscillations to the final minimum of S will have an amplitude no larger than Λ^2, thus greatly suppressing the energy density in its oscillations. For $\Lambda < 10^{-3} - 10^{-4}$, these oscillations and decays become harmless. Furthermore, if the adiabatic relaxation mechanism in the models with small Λ is operative [392], then the amplitude of oscillations is exponentially suppressed, and therefore the contribution of the decay products of the Polonyi field to the LSP relic density is exponentially small. Thus, strong moduli stabilization may play an additional important role in this context. While the complete evolution of these

moduli in the context of inflation is interesting for further study, here we will assume that because of the suppression in moduli oscillations, they will not contribute significantly to the relic density of dark matter. On the other hand, since suppression of the LSP production as a result of the Polonyi field decay is sensitive to the choice of Λ, one might find such values of this parameter where this effect may lead to a significant increase of the dark matter density.

Instead of moduli decay, we focus on the well known fact that gravitinos are produced during reheating after inflation [393–396]. The calculation of Ref. [396] yields

$$\frac{n_{\tilde{G}}}{n_\gamma} = 1.2 \times 10^{-11} \left(1 + \frac{m_{\tilde{g}}^2}{12 m_{3/2}^2}\right) \times \left(\frac{T_R}{10^{10}\ \mathrm{GeV}}\right), \tag{14.65}$$

where T_R is the reheating temperature achieved at the end of the inflationary epoch.

Since $m_{\tilde{g}} \ll m_{3/2}$ in the models considered here, we can ignore the middle term in Eq. (14.65), which can be rewritten as

$$\Omega_{3/2} h^2 \simeq 0.4 (\frac{m_{3/2}}{\mathrm{TeV}})(\frac{T_R}{10^{10}\mathrm{GeV}}). \tag{14.66}$$

Then the relic density of winos is given simply by

$$\Omega_\chi h^2 = \frac{m_\chi}{m_{3/2}} \Omega_{3/2} h^2 = 0.4 (\frac{m_\chi}{\mathrm{TeV}})(\frac{T_R}{10^{10}\mathrm{GeV}}). \tag{14.67}$$

Thus for a wino mass of about 100 GeV, and a reheat temperature of $T_R = 3 \times 10^{10}$ GeV, we obtain the correct dark matter relic density.

Typically, reheating temperatures as large $O(10^{10})$ GeV are excluded from big bang nucleosynthesis (BBN) [329, 393, 395–398]. These limits are obtained from the effects of gravitino decay on the background of light elements produced during or after BBN. The gravitino decay rate can be written as

$$\Gamma_{3/2} = \frac{m_{3/2}^3}{4\pi}, \tag{14.68}$$

for example, for a decay to a gluon and gluino, corresponding to a lifetime

$$\tau_{3/2} \simeq 3 \times 10^4 \mathrm{s} \left(\frac{1\mathrm{TeV}}{m_{3/2}}\right)^3. \tag{14.69}$$

Clearly, for gravitino masses of order 100 GeV, the lifetime is of order 10^8 s and can affect the light element abundances. However, for gravitino masses in excess of 30 TeV, the lifetime is of order 1 s and thus gravitinos decay before BBN and there is practically no limit on the reheat temperature. A problem appears only if gravitinos may decay to LSPs, which may give an additional contribution to dark matter. But this is exactly what we want.

While the reheat temperature may not be constrained by BBN, reheating after inflation in KL-type models requires some attention. In Ref. [330], an examination of possible inflaton decay channels in KL supergravity showed that reheating is in general suppressed as in the cases of no-scale supergravity [399] or racetrack inflation [400]. However a decay channel

to gauge bosons is possible if the gauge kinetic term, $h_{\alpha\beta}$ has a non-trivial coupling to the inflaton ϕ. Expanding $h_{\alpha\beta}$ in terms of ϕ, we can write

$$h_{\alpha\beta} = \big(h(\rho) + d_\phi \phi\big)\delta_{\alpha\beta}. \tag{14.70}$$

As written, h is a function of the moduli (normally included for the generation of gaugino masses - though these are greatly suppressed in the present context) and the inflaton, ϕ. The coupling d_ϕ is a constant. This coupling was shown to lead to a reheating temperature of order

$$T_R \sim d_\phi \times 10^9 \text{ GeV} . \tag{14.71}$$

for $m_\phi = 6 \times 10^{-6}$ in Planck units. Thus barring other possible enhancements, we would require $d_\phi \gtrsim 10$ to achieve reheating large enough to obtain the correct relic density of neutralinos after gravitino decay.

A similar situation appears if the inflaton field is a pseudo-scalar coupled to vectors as $d_\phi \phi F\tilde{F}$. Such models, with $d_\phi \sim 10^2$ and large reheating temperature, were recently considered in [401] as a potential source of non-Gaussian inflationary perturbations.

3) We increase $m_{3/2}$ sufficiently (to $\gtrsim$ 500 TeV) to get the right relic density. As is clearly seen in Fig. 14.4, for suitable $m_{3/2} \simeq 650$ TeV, we can obtain the WMAP density of $\Omega_\chi h^2 \simeq 0.11$. Of course at this value of $m_{3/2}$, we obtain $m_h \simeq 128.5$ GeV slightly in excess of the recent LHC measurement. However, as we noted earlier, there is certainly some uncertainty in the calculated value of m_h and so we can not entirely exclude this possibility.

14.7.4 Direct and indirect detection of dark matter

A wino–like dark matter candidate has several features which makes its direct detection modes, through the measurement of its scattering off a nucleus, difficult to observe. Indeed the two main scattering modes are the t-channel Higgs exchange (h or H) and the s- and t-channel squark exchange (see Fig.14.5). The t–channel SM Higgs exchange is strongly suppressed due to the coupling of the LSP to the Higgs boson. Indeed, this coupling is proportional to the product of the Higgsino and gaugino components of the neutralino. With such a heavy Higgsino ($\simeq 20-30$ TeV), the lightest neutralino (wino) has a very small higgsino component (much less than 1 %), thus reducing the effective $\widetilde{\chi}_1^0\ \widetilde{\chi}_1^0\ q\bar{q}$ coupling. Processes with heavy Higgs exchange or squark exchange (Fig.14.5), are both also strongly suppressed with such a heavy scalar spectrum ($\simeq$ 30 TeV), giving a reduction by a factor 10^3 to 10^4 compared to typical WIMP interactions on nucleon. This is clearly illustrated in Fig. 14.7 where a typical cross section for a 100 GeV neutralino ($m_{3/2} \simeq 30$ TeV) is 10^{-14} pb whereas classical WIMP interaction should lie between 10^{-8} to 10^{-12} pb. This result is relatively independent of $\tan\beta$ or M_{in} as the arguments developed above are quite general. For reference, the anticipated reach of a XENON 1 ton detector is about a few $\times 10^{-11}$ pb [402] and is shown by the black curve.

On the other hand, it is well known that a neutralino with a dominant wino component has a large s-wave annihilation cross section, which implies possibilities for indirect signals. The main annihilation channel is the t-channel exchange of the chargino (see Fig. 14.6). In the anomaly mediation scenario, the mass degeneracy between $\widetilde{\chi}_1^0$ and $\widetilde{\chi}_1^+$ together with the relatively strong $SU(2)$ coupling generates a high rate of W^+W^- final states (around

Figure 14.5: *Direct detection processes for the neutralino-nucleon elastic scattering.*

Figure 14.6: *Main neutralino annihilation channel for indirect detection constraint imposed by FERMI.*

Figure 14.7: *The spin independent elastic cross section, $\sigma_{\chi p}$, as a function of the gravitino mass, $m_{3/2}$. Also shown is the projected limit for a XENON-1 ton detector [402].*

80 %). However, such final states are strongly constrained by the recent analysis of dwarf galaxies by the FERMI telescope [403]. Due to the lower limit on the chargino mass from LEP constraint, this is also the lower limit for the lightest neutralino as they are nearly degenerate. For the benchmark point $m_{3/2} = 32$ TeV giving $m_{\tilde{\chi}_1^0} = 107$ GeV, we obtained $\langle \sigma v \rangle = 3.5 \times 10^{-7} \mathrm{GeV}^{-2} \simeq 4 \times 10^{-24} \mathrm{cm}^{-2} \mathrm{s}^{-1}$ which corresponds to the derived 95% CL upper limit of FERMI [403] in the case of W^+W^- final state. Moreover, as the dependence of the fluxes on the mass is proportional to $1/m_\chi^2$, and the limit of FERMI is even less constraining for heavier DM masses, we can safely conclude that the combined LEP/FERMI data do not affect the parameter space considered.

14.8 Conclusions

The Polonyi, or more generally, the moduli problem [327] has been a persistent problem for supergravity models put in a cosmological context. These problems were compounded in KKLT models of moduli stabilization as they place a severe requirement on the Hubble parameter during inflation, $H \lesssim m_{3/2}$, in order to avoid destabilization of the volume modulus. The mass of the volume modulus is relatively small, so even if one manages to avoid the volume modulus destabilization and the resulting decompactification, one may encounter a version of the cosmological moduli problem related to this field. Furthermore, if F-term supersymmetry breaking is used to uplift the AdS supersymmetric vacuum in KKLT, one may have a classic Polonyi problem in addition to the cosmological problems associated with the string theory moduli. Depending on its mass, the gravitino may also present a plethora of problems.

On the other hand, a relatively simple modification of the KKLT superpotential allows for strong moduli stabilization. In the particular example of a racetrack superpotential, as in the KL model, the volume modulus may be very heavy (*i.e.*, GUT scale or above), thus removing not only the cosmological problem involving the density of moduli, but also the problem of vacuum destabilization for $H \gtrsim m_{3/2}$. In the KL model, the constraint $H \leq m_{3/2}$ does not apply, inflation is possible even in the simplest models of chaotic inflation where the Hubble constant may be extremely large, exceeding 10^{13} GeV [330, 331]. However, as we already noted, uplifting in the KL model based only on anti-D3 branes in warped space leads to pure anomaly mediation, which is problematic from the point of view of phenomenology. Therefore, we were led to extended versions of the KL model involving F-term uplifting.

In section 2, we have given two explicit examples of F-term uplifting which generate scalar masses equal to the gravitino mass. Relying once more on strong moduli stabilization, the fields associated with uplifting do not suffer from the Polonyi problem. In the first example, we used a non-minimal form for the Kähler potential for the Polonyi-like field S [347]. The mass of S was found to be much larger than the gravitino mass which is sufficient for avoiding the Polonyi problem. A second example based on the ISS mechanism [351] was shown to give qualitatively similar results.

The main result of strong moduli stabilization with a decoupled uplift sector is the generation of scalar masses equal to the gravitino mass, while gaugino masses and supersymmetry breaking trilinear terms are heavily suppressed by either the F-term of the modulus proportional to $D_\rho W \propto m_{3/2}^2/m_\sigma \ll m_{3/2}$ or the vev of the Polonyi like field $\langle S \rangle \simeq \Lambda^2 \ll 1$ [347].

The resulting smallness of the A-terms necessitates a large gravitino mass to obtain a reasonably large Higgs mass as is now required. Thus we find a low energy spectrum built from very few parameters, reminiscent of split supersymmetry [335]. Arguments for the decoupling of the uplift sector, which is important in obtaining these results, were discussed in Sections 2 and 3.

However, given the theoretical framework for moduli stabilization described above, producing a viable low energy spectrum consistent with radiative electroweak symmetry breaking requires sensible supersymmetry breaking input parameters at some UV scale. Indeed, split supersymmetry models with universal scalar masses at the UV input scale put severe restrictions on the universal scalar mass, if these are input at the GUT scale as is commonly assumed in the CMSSM or mSUGRA models. While one could impose non-universality in the Higgs sector with non-minimal couplings between S and the Higgs multiplets, we have assumed that this non-universality is not present in the simple models based on KL supergravity with O'KKLT uplifting. Instead, we have shown [361] that large scalar masses are in fact consistent with radiative electroweak symmetry breaking if the UV input scale for supersymmetry breaking is input above the GUT scale. In this case, the extra running between M_{in} and M_{GUT} generates sufficient non-universality for the low energy minimization conditions to be satisfied. Finally, to satisfy the boundary condition given by Eq. (14.48), we include a Giudice-Masiero like contribution to the Kähler potential.

On the phenomenological side, the Higgs mass determination of $\simeq 125$ GeV may indicate significant splitting in the stop sector (through a large A-term) or a high mass scale of supersymmetry. Supergravity models with strong modulus stabilization choose the latter. As we have seen the only "light" supersymmetric particles in the theory are the charged and neutral gauginos. Indeed the LEP lower limit on the chargino mass coincides with a Higgs mass $\gtrsim 125$ GeV for $m_{3/2} \gtrsim 30$ TeV. While gaugino masses and A-terms are generated predominantly through anomaly mediation, the scalar sector of the theory resembles that of split supersymmetry. However, unlike most split-SUSY models in the literature, the μ-term derived here through radiative EWSB is of order the gravitino mass rather than the gaugino mass. Furthermore, the requirement of radiative EWSB in split SUSY models is indicative of a supersymmetry breaking input scale above the GUT scale or of direct couplings of uplift fields to the Higgs sector. The model described above, which followed the first option, demonstrates that a UV completion of split SUSY models is not only possible but is phenomenologically viable.

We have seen that despite the added complexity involved even in choosing minimal SU(5) as a GUT, the low energy spectrum and in particular, the Higgs mass is relatively insensitive to the choice of SU(5) couplings or the supersymmetry breaking input scale. While there is some sensitivity to $\tan\beta$, the predominant sensitivity of the low energy spectrum lies with $m_{3/2}$. The large value of $m_{3/2}$ required now guarantees not only the absence of cosmological moduli problems (due to strong stabilization), but also the absence of any gravitino problem as well, as the massive gravitino decays well before nucleosynthesis.

While our starting point was the strong stabilization of moduli with appropriate F-term uplifting, our resulting spectrum and phenomenological consequences resemble the recent models of pure gravity mediation [404]. For a related analysis based on M-theory compactifications, see Ref. [405]. Here, however, we have followed the full renormalization group evolution of masses and couplings from the supersymmetry breaking input scale, M_{in},

to the electroweak scale insuring consistent electroweak symmetry breaking. An important difference between our approach and that of [405] is our emphasis on the strongly stabilized versions of string theory/supergravity, which provides a simple solution of the problem of moduli destabilization during inflation, as well as of the cosmological moduli problem after inflation. This approach led us directly to the phenomenological models described above.

There are several consequences of the derived sparticle spectrum. Because squarks are heavy, gluino decay is suppressed. However, given the ranges of gravitino masses considered here, the gluino decay length still lies below the LHC resolution. Because gaugino masses are determined by anomaly contributions, the LSP is the wino and at the minimal value $m_{3/2} \sim 30$ TeV the relic density of winos is far too small to make up the dark matter. This may indicate that the gravitino mass is larger ($m_{3/2} \simeq 650$ TeV) in which case, the wino mass is above 2 TeV and annihilations produce the desired WMAP relic density (in this case, there is a one-to-one correlation between the relic density and the gravitino mass); the reheat temperature after inflation is sufficiently large so as to produce a sufficient density of gravitinos which decay to winos and supply the correct relic density (in this case, there is a one-to-one correlation between the relic density and the inflationary reheat temperature); or simply, something else makes up the dark matter (e.g., an axion).

In the case that the wino is the dark matter, we have seen that the elastic scattering cross section on nucleons is far below the projected limit of a XENON 1T detector [402]. In contrast, the expected signal for gamma-rays from wino annihilation in the halo is close to the current FERMI limit when the gravitino mass is close to its lower limit.

In summary, we have argued that models of strong moduli stabilization lead to a very specific low energy spectrum and resulting phenomenology. The spectrum resembles that of split supersymmetry with scalars in the mass range $30 - 1000$ TeV. Smaller masses result in an excessively light chargino, while higher masses result both in an excessive wino relic density and excessive Higgs masses. Thus from the assumptions of strong moduli stabilization and decoupling of the uplift sector, upon applying the LEP limit on chargino masses, we are immediately led to a lower bound of roughly 125 GeV for the Higgs mass and a lower bound of roughly 1 TeV for the gluino mass. Furthermore, the upper limit on the gravitino mass from the limit on the wino relic density also corresponds to roughly the current upper limit on the Higgs mass at the LHC. Thus, there exists a narrow window where all current experimental results are satisfied. While direct detection of wino dark matter will be difficult in this model, detecting gamma rays from wino annihilations may indicate the realization of models of this type.

Chapter 15

The Gravitino : an example of dark matter candidate in Gauge Mediated scenarios**

15.1 Constraints on the reheating temperature T_R from BBN

Another natural candidate for any supersymmetric extension of the SM is the gravitino $\tilde{G}$. For a given reheating temperature T_R, its *thermal* abundance is given by

$$\Omega_{\tilde{G}}^{\rm TP} h^2 = \left(\frac{m_{3/2}}{1{\rm GeV}}\right)\left(\frac{T_R}{10^{10}{\rm GeV}}\right)\sum_r y'_r g_r^2(T_R)(1+\delta_r)\left(1+\frac{M_r^2(T_R)}{3m_{3/2}^2}\right)\ln\left(\frac{k_r}{g_r(T_R)}\right) \quad (15.1)$$

If the gravitino is the LSP, one should look carefully to the NLSP lifetime ($\tau_{\rm NLSP}$) that should not violate BBN through its hadronic decay ($\tau_{\rm NLSP} \lesssim 100$ s) or its bound state formation with Lithium ($\tau_{\tilde{\tau}_1} \lesssim 5000$ seconds for a $\tilde{\tau}_1$ NLSP). The NLSP lifetime can be written

$$\tau_{\rm NLSP} = \frac{48\pi m_{3/2}^2 M_P^2}{m_{\rm NLSP}^5}\left(1-\frac{m_{3/2}}{m_{\rm NLSP}}\right)^{-4} \simeq 6.1\times 10^6\left(\frac{m_{3/2}}{100\,{\rm GeV}}\right)^2\left(\frac{100\,{\rm GeV}}{m_{\rm NLSP}}\right)^5\left(1-\frac{m_{3/2}^2}{m_{NLSP}^2}\right)^{-4}{\rm s}\,. \quad (15.2)$$

Why is it so difficult to have high reheating temperature with a stable or unstable gravitino? With *unstable* gravitinos, we face the so–called gravitino problem, as for example in generic gravity mediation scenarios of supersymmetry breaking with a neutralino LSP. If gravitinos are overproduced, their decay products can destroy the otherwise successful prediction of big Bang Nucleosynthesis (BBN). This in turn, leads to strong upper limits on the reheating temperature T_R (Eq.15.1)which may be in conflict with the temperature required for the leptogenesis. For instance, a gravitino with mass $m_{3/2} \lesssim 5$ TeV have long lifetimes $\tau_{\tilde{G}} \gtrsim 100$ s and decay during or after BBN. Successful BBN predictions imply a bound on the reheating temperature after inflation T_R which governs the abundance before their decay [432, 444] : $T_R \lesssim 10^8$ GeV for $m_{3/2} \lesssim 5$ TeV [434]. An alternative to avoid this problem is to have very

Figure 15.1: Left: Cosmological constraints on the masses of the gravitino LSP and a purely right handed $\tilde{\tau}_1$ NLSP from [440]; Right: Allowed regions of the $(m_{\tilde{G}}, \delta m = m_{NLSP} - m_{\tilde{G}} - M_Z)$ parameter space in the gravitino LSP scenario assuming a bino NLSP from [439]

heavy gravitinos ($m_{3/2} \gtrsim 10$ TeV) as in anomaly-mediated scenarios, or mirage mediation models: such heavy gravitinos decay before the onset of nucleosynthesis.

In scenarios with *stable* gravitinos, such as gauge mediated supersymmetry breaking, it is also difficult to reach high reheating temperature while maintaining consistency with the BBN bounds. From Eq.15.1 we can see that for a given relic density $\Omega^{\rm TP}_{\tilde{G}}h^2 \sim 0.11$ (if we neglect at the first approximation the non-thermal contribution), one maximize the reheating temperature T_R by minimizing the gravitino mass $m_{3/2}$. However this low value of $m_{3/2}$ may imply an NLSP lifetime (Eq.15.2) for which its relic thermal abundance (before its decays) violates the BBN bound.

In the case of stau NLSP, as its decay products are manly leptonic, we can evade the classical lifetime limit $\tau_{\tilde{\tau}_1} \lesssim 100$ s, which is for a pure hadronic decaying particle, but to evade the Li abundance constraints coming from bound state formation, one need $\tau_{\tilde{\tau}_1} \lesssim 5000$ s. This constraint for the stau is more stringent that the BBN bounds coming from late hadronic energy released by the 3–bodies decaying channel[1]. This imply an upper bound on the gravitino mass, *ie* $m_{\tilde{\tau}_1}/m_{3/2} \ll 1$.

For the neutralino NLSP, the hadronic BBN bounds are much more stringent. This is expected as neutralino decays contribute to hadronic energy at the two-body level through decays $\chi^0_1 \to Z\tilde{G}$. The branching fractions for electromagnetic and hadronic decays are therefore not too different, and the extreme stringency of the hadronic constraints makes them the dominant bound. A sum up of the constraints are given in Fig. 15.1 where we see that the majority of the parameter space is already excluded by the hadronic BBN constraint.

[1]The 2-bodies decay $\tilde{\tau}_1 \to \tilde{G}\tau$ could in principle induce hadronic cascades. However, as shown in Refs. [437, 438] for decay times $\tau > 10^3 - 10^4$ s, the hadronic interaction time of all pions and kaons is much longer than their decay time. The decays of the staus therefore only contribute to elecrtromagnetic and not hadronic cascades.

Figure 15.2: Left: Hadronic branching fractions for $\tilde{\nu}$ NLSP as functions of the $\tilde{\nu}$ lifetime from [437]; Right: Allowed regions of the $(m_{\tilde{G}}, \delta m = m_{NLSP} - m_{\tilde{G}} - M_Z)$ parameter space in the gravitino LSP scenario assuming a Sneutrino NLSP from [439]

For a sneutrino $\tilde{\nu}$ NLSP, the BBN constraints are quite weak. Indeed, the two–body decays $\tilde{\nu} \to \nu\tilde{G}$ are essentially invisible and do not contribute to either electromagnetic or hadronic cascades. The three body decay are thus important. We show in the left panel of Fig. 15.2 the evolution of the hadronic branching fraction as function of the lifetime for a sneutrino NLSP. It appears clearly that for a lifetime longer than $\sim 10^7$ s, the hadronic decay of the sneutrino is very small and thus, the BBN constraints disappear. This means that *contrarly to the stau case*, where short lifetime (high $\Delta m = m_{\tilde{\tau}_1} - m_{\tilde{G}}$) were necessary to avoid bound state formation with Lithium, in the case of a neutral sneutrino, very long lifetime (small $\Delta m = m_{\tilde{\nu}} - m_{\tilde{G}}$) is needed to avoid BBN constraints. One can see in the right panel of Fig.15.2 that the more reliable D and ^4He constraints disfavor $\Delta m \gtrsim 300$ GeV while ^{6}Li is sensitive to $\Delta m \gtrsim 200$ GeV. Of course, very heavy sneutrino ($\gtrsim 2$ Tev) are still possible as they decay much before the BBN era.

From Eq.15.1 we can clearly see that the reheating temperature would be maximized for low and degenerate values of the gaugino masses at SUSY scale ($M_i \sim m_{\rm NLSP}$). It was shown in [441] that for a stau NLSP, we can reach at most, $T_R = 5.10^8$ GeV, and $T_R = 7.10^9$ GeV for a sneutrino NLSP with degenerate gaugino masses at SUSY scale ($M_i \sim m_{\rm NLSP}$). For a GUT like relation (unification of gaugino masses at GUT scale), the maximum reheating temperature are respectively $T_R = 5.10^7$ GeV, and $T_R = 2.10^9$ GeV. This range can become interesting for thermal leptogenesis.

One should include in the calculation of the relic abundance the gravitinos produced non thermally in the NLSP decays:

$$\Omega_{\tilde{G}} h^2 = \Omega_{\tilde{G}}^{\rm TP} h^2 + \Omega_{\tilde{G}}^{\rm NTP} h^2 = \Omega_{\tilde{G}}^{\rm TP} h^2 + \frac{m_{3/2}}{m_{\rm NLSP}} \Omega_{\rm NLSP} h^2 \tag{15.3}$$

which traduce the fact that for each NLSP decaying, one new gravitino contributes to the

dark matter relic: $\Omega_{\rm NLSP}$ represents the thermal relic abundance that the NLSP would have had, had it not decayed into gravitinos and Standard Model particles. One can also write $\Omega_{\tilde{G}}^{\rm NTP} = m_{\tilde{G}}\ n_{\rm NLSP}/\rho_c = m_{\tilde{G}}\ Y_{\rm NLSP}\ n_\gamma^{\rm BG}/\rho_c$, with $Y_{\rm NLSP} = n_{\rm NLSP}/n_\gamma^{\rm BG}$ where $n_\gamma^{\rm BG}$ is the background photon number density $n_\gamma^{\rm BG} = 2\xi(3)T^3/\pi^2$. [2] The nonthermal contribution, for maximized values of T_R, does not exceed 20 % of the total relic abundance. However, for low values of T_R, it could be the dominant contribution as one can see below.

15.1.1 Constraints of warm/hot Dark Matter

Decay produced particle dark matter, like a non-thermal gravitino, is often warm/hot, i.e. is endowed with primordial free–streaming velocities leading to the early erasure of small–scale perturbations, due to the kinetic energy imparted on the decay product during the decay process itself. A clear analysis in different framework can be found in [435]. For a decay NLSP $\to \gamma\ \tilde{G}$, one may compute the velocity of the DM particle $\tilde{G}$:

$$v_{DM}^0 \simeq 4.57 \times 10^{-5} \frac{\rm km}{\rm s} \left(\frac{m_{\rm NLSP}^2 - m_{3/2}^2}{2 m_{\rm NLSP} m_{3/2}} \right) g_d^{-1/12} \left(\frac{\tau_{\rm NLSP}}{1{\rm s}} \right)^{1/2} \tag{15.4}$$

with g_d the statistical weight of the plasma at the decay epoch and $\tau_{\rm NLSP}$ the lifetime of the NLSP. This velocity is often called free–streaming velocity and may become appreciable (and dangerous for structure formation) for either light DM particle or late decay of the NLSP : if the NLSP decays early (small τ_{NLSP}) the nowadays velocity is small because it has decreased with the temperature of the plasma. If we suppose that the relic density is totally composed by relativistically freezing out gravitino produced by the NSLP decay (as opposed to thermally produced gravitino), we find

$$v^0 \simeq 0.044{\rm kms}^{-1} \left(\frac{\Omega_{\tilde{G}} h^2}{0.15} \right)^{0.34} \left(\frac{m_{3/2}}{1keV} \right)^{-1.34} \tag{15.5}$$

One can also deduce the minimal wavelength reached by such velocity [435] :

$$R^0 \simeq 0.226{\rm Mpc} \left(\frac{\Omega_m h^2}{0.15} \right)^{-0.14} \left(\frac{v^0}{0.05{\rm km/s}} \right)^{0.86} \tag{15.6}$$

Warm dark matter should respect 3 fundamental cosmological constraints :

- Observation of the Lyman-α forrest by the WMAP mission has yielded a limit $v^0 \lesssim 0.1$km s^{-1}.
- To avoid late reonization, which would give too late formation of massive star, we need $v^0 \lesssim 0.002$km s^{-1}, but this measure is entached with large uncertainties.
- To permit the formation of Galaxy clusters on the scale of 20 Mpc, we need $v^0 \lesssim 1$km s^{-1}. One can avoid this constraint in the mixed scenario where only 10-20% of the DM is non-thermally produced.

[2] Another common definition is $Y_{\rm NLSP} = n_{\rm NLSP}/s$, where $s = (2\pi^2/45) g_* T^3$ is the entropy density. In the era of NLSP decays to gravitinos, $s \simeq 7.0 n_\gamma^{\rm BG}$

Figure 15.3: Contour plot of present-day free-streaming velocities where the gravitino dark matter is generated by the late decay of a bino NLSP. v^0 exceeds the limit in the parameter space below the lines [435]. The figure is quite similar for a stau or sneutrino NLSP.

Combining Eq.(15.2) and Eq.(15.4), we find

$$v^0_{DM} \propto \frac{(m_{\rm NLSP} + m_{3/2})}{m^{3/2}_{\rm NLSP}(m_{\rm NLSP} - m_{3/2})} = \frac{\Delta m + 2m_{3/2}}{(m_{3/2} + \Delta m)^{3/2}\Delta m} \tag{15.7}$$

with $\Delta m = m_{\rm NLSP} - m_{3/2}$. One can see that for a given $m_{3/2}$, the DM velocity decreases with increasing Δm (even if the gravitino produced in the decay will acquire more center of mass energy, the decrease of $\tau_{\rm NLSP}$ makes the gravitino to be produced earlier and its velocity at the structure formation epoch will be much smaller through expansion) and decreases with increasing $m_{3/2}$ for a given Δm (for a given center of mass energy, the velocity decrease if the mass of the corresponding particle increases). Any cosmological constraints on the free–streaming velocity v^0_{DM} will thus restrict the ($m_{3/2}$, Δm) plane and is shown in Fig. 15.3.

If you compare Fig. 15.3 and Fig. 15.2 one observe that in the case of a sneutrino NLSP, a large part (if not all) of the parameter space allowed by BBN constraint is exculded by structure formation constraints. This is not the case for a stau NLSP which is cleraly the most favored candidate respecting both constraints if sufficiently heavy ($\gtrsim$ 3 TeV for a 100 GeV gravitino).

15.2 Gravitino Dark Matter in GMSB

Gauge-mediated SUSY breaking (GMSB) [443] is usually implemented by adding a term

$$W = S\Phi_M\overline{\Phi}_M + \Delta W(S, Z_i) \tag{15.8}$$

to the superpotential where Φ_M and $\overline{\Phi}_M$ are messenger left chiral superfields with $SU(3) \times SU(2) \times U(1)$ quantum numbers, whereas the spurion left chiral superfield S and the secluded

sector Z_i fields are electroweak- and strong- interactions singlets. Upon the development of a non-vanishing *vev* $\langle S\rangle$ of the scalar component of the spurion superfield and a SUSY-breaking expectation value of the spurion auxiliary field F_S, due to unspecified dynamics in the secluded sector $\Delta W(S,Z)$, fermionic messengers combine into Dirac fermions of masses $M_{X,1/2} = M_X = \langle S\rangle$, whereas their bosonic partners mix in a mass matrix of eigenstates ϕ and $\overline{\phi}$ with masses $M_\phi = M_X(1 - F_S/M_X^2)^{1/2}$ and $M_{\overline{\phi}} = M_X(1 + F_S/M_X^2)^{1/2}$. In terms of the messengers bosonic components, $\phi = (-\Phi_M^* + \overline{\Phi}_M)/\sqrt{2}$ and $\overline{\phi} = (\Phi_M + \overline{\Phi}_M^*)/\sqrt{2}$. Note that ϕ denotes a set of scalar fields transforming under some representation of the grand unified gauge group, and that the mass degeneracy of this multiplet is lifted by $D-$terms and radiative corrections.

Since the messengers share the standard model gauge interactions, the gaugino and scalar spartners acquire mass at the one- and two- loop levels respectively:

$$\tilde{m}_{1/2} \sim \left(\frac{g^2}{16\pi^2}\right)\frac{F_S}{M_X}, \qquad \tilde{m}_0^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2\left(\frac{F_S}{M_X}\right)^2 \tag{15.9}$$

g being the gauge coupling under which is charge the messenger field, dhence the quantity $\Lambda = F_S/M_X$ is the supersymmetry breaking scale in the visible sector. Provided $F_S/M_X \sim$ 100 TeV, this generates the required order of magnitude for the soft parameters. Note that $F_S < M_X^2$ is mandatory otherwise one of the messengers bosons acquires a negative mass squared. This also implies $M_X \gtrsim 100$ TeV, and for $M_X \gg 100$ TeV, $F_S \ll M_X^2$, hence M_X sets the mass scale for the messenger sector. In particular, $M_\phi \simeq M_X$.

The gravitino mass is related to the fundamental SUSY breaking scale, $m_{3/2} = F/\sqrt{3}M_P$, with $F = F_S + \sum_i F_{Z_i}$ the sum of $F-$terms in the secluded sector. We can define the parameter $k = F_S/F \leq 1$, so that $m_{3/2} = F_S/(k\sqrt{3}_P)$. In pure direct gauge mediation, $k \simeq 1$ whereas in scenarios in which the transmission of supersymmetry breaking to the messenger sector is loop suppressed, one may find $k \ll 1$. One can easily see that for a pure gauge mediation, if the messenger masses are around 100 TeV, we find $F_S \sim 10^{10}$ GeV for a TeV SUSY mass spectrum, imply $m_{3/2} \sim 10^{-9}$ GeV. On another hand, to obtain a gravitino of the order of 100 GeV, we need $M_X \sim 10^{16}$ GeV which is GUT scale. In this case, we have to look at gravity mediated effects on the soft breaking term, leading to hybrid scenarios [446]. This specific class of models is discussed in a following section.

15.3 Different solutions to increase the reheating temperature

From the cosmological perspective, there exists a big difficulty in the GMSB models. That is the so–called cosmological gravitino problem. In these models, we have no natural explanation for the dark matter in the present universe. Thermal relics of the gravitino, the LSP in the GMSB models, overclose the Universe once they are thermalized in the early Universe. In order to avoid the overproduction of the gravitinos, there is a severe upper bound on the reheating temperature of inflation T_R which is about $T_R \lesssim 10^6$ GeV for $m_{3/2} = 10$ MeV for instance, and it even reaches $T_R \lesssim 10^3$ GeV in the case of the lighter gravitino $m_{3/2} \lesssim 100$ keV. This is a big drawback of the GMSB models with respect to the standard mSUGRA

scenario [444]. We can have reproduce roughly these values of T_R needed for the gravitino to account for the relic abundance with Eq.(15.1) which can be approximated for light gravitino by

$$\Omega_{\tilde{G}}^{\rm TP} h^2 \sim \left(\frac{m_{3/2}}{1{\rm GeV}}\right)\left(\frac{T_R}{10^{10}{\rm GeV}}\right)\frac{M_r^2}{m_{3/2}^2} \lesssim 0.1 \tag{15.10}$$

One solution to increase the reheating temperature would be to look for models with gravitino masses around the GeV scale. However, in GMSB this means very heavy messengers as we shows in the previous section and leads to other issues that we develop in this section.

15.3.1 Dilution of the gravitino relic by increasing the entropy

Even if the gravitino is light, GMSB scenarios generically lead to another cosmological catastrophe, as they predict that the lightest messenger should overclose the Universe. In effect, messenger gauge interactions as well as those derived from Eq.(15.8) conserve messenger number so that the lightest messenger (a boson) is stable in this minimal version of the theory. Through explicit computation one can show that the lightest messenger generically overcloses the Universe unless its mass $M_X \lesssim 10$ TeV [445] [3]. This is in contradiction with the preceding section where we show that one needed a messenger mass of the order of $\gtrsim 100$ TeV to obtain a reasonable SUSY spectrum around the TeV scale. The authors of [445] shown that the relic abundance of the lightest messenger would be of the right order of magnitude provided its mass $\sim 10-30$ TeV. For such values of M_X, F_S would be of the order of $\lesssim (100\ {\rm TeV})^2$ to avoid tachyon, giving a gravitino mass of ~ 4 eV. The only possibility to account with gravitino dark matter with a mass around the keV or GeV scale, is to allow the lightest messenger to decay, violating the messenger number. It has been shown in [442] that the late decay of the lightest messenger to visible sector particles can induce a substantial amount of entropy production which would result in the dilution of the predicted gravitino abundance. As a result, the light gravitino problem could be turned into a light gravitino blessing, i.e. one would obtain suitable gravitino dark matter for arbitrarily high reheating temperature, and avoiding the overclosure of the Universe by the messengers by their late time decays.

One can evaluate the dilution factor of the late–time entropy production generated by the decay of the lightest messengers of mass M_X after the freeze out time of the gravitino:

$$\Delta_X = \frac{s_{\rm after}}{s_{\rm before}} \simeq \frac{4}{3}\frac{MY_X}{T_d} \simeq 4.9\times 10^2 \left(\frac{M_X}{10^8\ {\rm GeV}}\right)^2 \left(\frac{10\ {\rm MeV}}{T_d}\right) \tag{15.11}$$

which can be written

$$\Delta_X \simeq 28 \left(\frac{M_X}{10^8\ {\rm GeV}}\right)\left(\frac{Y_X}{10^{-10}}\right)\left(\frac{\Gamma_X}{10^{-25}\ {\rm GeV}}\right)^{-1/2}\left(\frac{g_*}{10}\right)^{1/4} \tag{15.12}$$

where $s_{\rm before}$ ($s_{\rm after}$) denotes the entropy density of the universe before (after) the decays of the lightest messengers, T_d their decay temperature, Y_X their relic density and Γ_X their decay rate. Γ_X can be estimated if we assume a mixing between the SM and messenger

[3] We will neglect from now the off-diagonal terms in the messenger mass matrix and consider $M_\phi \sim M_X$

multiplets through the small R-symmetry-breaking effects caused by a term added in the superpotential :

$$\delta W = f\frac{\langle W\rangle}{M_P^2}\mathbf{5_X}\overline{\mathbf{5}}_\mathbf{X} \tag{15.13}$$

where f denotes some unknown coefficient of order 1. By virtue of the above small mixing term, the lightest messenger can decay into a SM lepton and a gaugino. The decay rate is estimated to be

$$\Gamma_X \simeq \frac{g_2^2}{16\pi}\left(\frac{fm_{3/2}}{M_X}\right)^2 M_X. \tag{15.14}$$

Assuming the stability of the lightest messenger, one can also estimate their relic abundance [445]:

$$Y_X \simeq 3.65\times 10^{-10}\left(\frac{M_X}{10^6\ \text{GeV}}\right) \tag{15.15}$$

Combining Eqs. (15.12), (15.14) and (15.15) we obtain

$$\Delta_X \simeq \frac{23}{g_2 f}\left(\frac{g_*}{10}\right)\left(\frac{10^{-5}\ \text{GeV}}{m_{3/2}}\right)\left(\frac{M_X}{10^8\ \text{GeV}}\right)^{5/2} \tag{15.16}$$

Increasing the entropy increases the total energy of the universe ρ_{tot}

$$\rho_{tot} = \frac{\pi^2}{30}g_*(T)T^4 + \frac{2\pi^2}{45}g_*(T)T^3 M_X Y_X \tag{15.17}$$

where the first and second terms represent the contributions from the radiation and from the lightest messenger, respectively, diluting in the mean time the relic abundance (ρ_i/ρ_{tot}) of all species after their freeze–out time (out of thermal equilibrium). We can hence derive easily the final gravitino abundance after the dilution. The number-to entropy ratio $Y_{\tilde{G}} = n_{\tilde{G}}/s$ is diluted to $Y_{\tilde{G}}/\Delta_X$, giving the same dilution factor for

$$\Omega_{\tilde{G}}h^2 = 2.81\times 10^8\left(\frac{m_{3/2}}{1\text{GeV}}\right)Y_{\tilde{G}}. \tag{15.18}$$

After the decay of the lightest messenger[4] we thus obtain $\Omega_{\tilde{G}}h^2 = \Omega_{\tilde{G}}h^2|_{initial}/\Delta_X$.

15.3.2 Playing with non-universal gaugino masses

Other types of models can increase the allowed reheating temperature T_R. The authors of [441] shows that non-universal gaugino breaking terms induce natural larger reheating temperatures than in the minimal universal version of SUGRA. Indeed, a brief look at Eq. (15.1) shows that the reheating temperature is maximized for $M_1 = M_2 = M_3 = m_{3/2}$. *at electroweak scale*. This corresponds to a non-universal scenario at GUT scale which is however difficult to embed in a coherent generalized gauge mediation framework. Their

[4]We can check that this decay which occurs in the matter dominated universe, starts well after the freeze–out time of the gravitinos (occuring in the radiation dominated Universe)

Figure 15.4: The maximal reheating temperature for four mass patterns of the gauginos at the low scale for a stau NLSP (left) and a sneutrino NLSP (right) from [441].

results are sum up in Fig.(15.4) where we clearly see that the reheating temperature can reach several 10^8 GeV for a stau NLSP and even several 10^9 GeV for a sneutrino NLSP.

15.4 An example in hybrid gauge-gravity models

An interesting option was presented in [446] where we proposed a mix scenario called "hybrid scenario" where the SUSY breaking is communicated to the visible sector both from gravitational *and* gauge interaction. These type of models can keep the interesting features of GMSB while avoiding the so called "$\mu-$problem inherent to GMSB (see the appendix for details). The breaking terms can be written

$$\begin{aligned} M_a &= M_a^{\rm Grav} + M_a^{\rm GMSB} = m_{3/2}\left(\tilde{\epsilon} + g_a^2 S_Q\ \alpha\right)\ , \\ m_i^2 &= (m_i^{\rm Grav})^2 + (m_i^{\rm GMSG})^2 = m_{3/2}^2\left(1 + C_i S_Q(-\delta + \frac{\alpha^2}{N})\right)\ , \end{aligned} \tag{15.19}$$

where S_Q is the Dynkin index of the messenger representation (1/2 for the fundamental representation of $SU(\mathcal{N})$), g_a^2 are the gauge couplings and $C_i = \sum_a g_a^4 C_i^a$, C_i^a being the Casimir of the MSSM scalar fields representations (in our normalization the Casimir of the fundamental representation of $SU(\mathcal{N})$ is $(\mathcal{N}^2-1)/(2\mathcal{N})$, that of $U(1)_Y$ is simply Y^2)[5].

For the phenomenological analysis we studied in detail the flavor universal case, where the gravity-mediated contributions are dominated by the term

$$(m_{ij}^{\rm Grav.})^2 \simeq m_{3/2}^2\,\delta_{ij}. \tag{15.20}$$

keeping in mind that in principle, there could be some flavor-mixing effects from the gravity side. However this assumption is largely justified in generic SUGRA constructions as was shown in [447]. The extra parameter $\tilde{\epsilon}$ includes the effects of gravity mediation for gauginos.

[5]See the appendix for the detailes formulae

In this case the gravitational contributions are there only if the gauge kinetic functions depend on the modulus field T. Moreover, since the correct values of these gaugino mass terms come from the ratio F^T/T, in the cases in exam the universal coefficient of proportionality with the gravitino mass is naturally of order $\mathcal{O}(10^{-1})$ [447]. Therefore it is slightly suppressed with respect to the correspondent contribution for the scalar fields. Different values should be taken into account in the cases where extra (for instance, secluded) sectors are included in the model. Finally, N is the "effective" number of messenger fields contributing to GMSB.

Unlike the classical GMSB at low energy, the mediation in the hybrid models occurs around the GUT scale where the gauge contributions to the gaugino masses M_a (proportional to their gauge couplings g_a) are approximately gauge universal. Thus, the gauge non-universality only affects the scalars masses. Concerning the trilinear couplings $A_{i=t,b,\tau}$, there is no 1-loop messenger contribution to the SUSY-breaking trilinear terms. However, A_i terms are generated in the leading log approximation by the RG evolution and are proportional to gaugino masses. At the GUT scale we will make no assumption on the value of the trilinear couplings and consider them universal and encoded in the free parameter A.

15.5 A model independant approach

In 2008, F. Steffen proposed a way of testing the maximum reheating temperature using collider measurements of the slepton mass and lifetime [436].

Part IV

Dark Matter in accelerators*

Chapter 16

Higgs and dark matter

In this chapter , we review the latest results concerning the Higgs searches at the LHC combined with dark matter searches from LUX and XENON. We place ourself in the context of the Higgs portal as it is the simplest possible extension of the SM, and treated scalar, fermonic and vectorial dark matter. In the first section, we show that when applying the direct detection constraint and the WMAP/PLANCK relic abundance data to the model, the Higgs should have coupled strongly to the dark sector, and thus not being observed at ATLAS or CMS. In a second section, we relax the relic abundance constraint, just keeping the hypothesis of a local presence of dark matter. In this condition, we show that monojet searches on the Higgs coupling to dark matter can be competitive with direct detection technology, especially in the low mass regim ($m_\chi \lesssim 30$ GeV) where the experiments dedicated to direct searches are much less sensible.

16.1 Combining fermionic/scalar/vectorial dark matter with a 126 GeV Higgs

16.1.1 Introduction

The ATLAS and CMS collaborations have recently reported on the search of the Standard Model (SM) Higgs boson with 5 fb^{-1} data [704]. Higgs bosons have been excluded in a significant mass range and, ignoring the unlikely possibility of a very heavy particle, only the very narrow window $m_h \approx 115$–130 GeV is now left over. There is even a slight excess of events in the data which could correspond to a SM like Higgs boson with a mass of 125 ± 1 GeV. Although the statistics are not sufficient for the experiments to claim discovery, one is tempted to take this piece of evidence seriously and analyze its consequences.

In this Letter, we study the implications of these LHC results for Higgs-portal models of dark matter (DM). The Higgs sector of the SM enjoys a special status since it allows for a direct coupling to the hidden sector that is renormalizable. Hence, determination of the properties of the Higgs boson would allow us to gain information about the hidden world. The latter is particularly important in the context of dark matter since hidden sector particles can be stable and couple very weakly to the SM sector, thereby offering a viable dark matter candidate [705]. In principle, the Higgs boson could decay into light DM

particles which escape detection [706]. However, given the fact that the ATLAS and CMS signal is close to what one expects for a Standard Model–like Higgs particle, there is little room for invisible decays. In what follows, we will assume that 10% is the upper bound on the invisible Higgs decay branching ratio, although values up to 20% will not significantly change our conclusions.

We adopt a model independent approach and study generic scenarios in which the Higgs-portal DM is a scalar, a vector or a Majorana fermion. We first discuss the available constraints on the thermal DM from WMAP and current direct detection experiments, and show that the fermionic DM case is excluded while in the scalar and vector cases, one needs DM particles that are heavier than about 60 GeV. We then derive the direct DM detection rates to be probed by the XENON100–upgrade and XENON1T experiments. Finally, we discuss the possibility of observing directly or indirectly these DM particles in collider experiments and, in particular, we determine the rate for the pair production of scalar particles at the LHC and a high-energy e^+e^- collider.

16.1.2 The models

Following the model independent approach of Ref. [742], we consider the three possibilities that dark matter consists of real scalars S, vectors V or Majorana fermions χ which interact with the SM fields only through the Higgs-portal. The stability of the DM particle is ensured by a Z_2 parity, whose origin is model–dependent. For example, in the vector case it stems from a natural parity symmetry of abelian gauge sectors with minimal field content [708]. The relevant terms in the Lagrangians are

$$\begin{aligned}
\Delta\mathcal{L}_S &= -\frac{1}{2}m_S^2 S^2 - \frac{1}{4}\lambda_S S^4 - \frac{1}{4}\lambda_{hSS} H^\dagger H S^2 \ , \\
\Delta\mathcal{L}_V &= \frac{1}{2}m_V^2 V_\mu V^\mu + \frac{1}{4}\lambda_V (V_\mu V^\mu)^2 + \frac{1}{4}\lambda_{hVV} H^\dagger H V_\mu V^\mu, \\
\Delta\mathcal{L}_f &= -\frac{1}{2}m_f \bar{\chi}\chi - \frac{1}{4}\frac{\lambda_{hff}}{\Lambda} H^\dagger H \bar{\chi}\chi \ .
\end{aligned} \tag{16.1}$$

Although in the fermionic case above the Higgs–DM coupling is not renormalizable, we still include it for completeness. The self–interaction terms S^4 in the scalar case and the $(V_\mu V^\mu)^2$ term in the vector case are not essential for our discussion and we will ignore them. After electroweak symmetry breaking, the neutral component of the doublet field H is shifted to $H^0 \to v + h/\sqrt{2}$ with $v = 174$ GeV and the physical masses of the DM particles will be given by

$$\begin{aligned}
M_S^2 &= m_S^2 + \frac{1}{2}\lambda_{hSS} v^2 \ , \\
M_V^2 &= m_V^2 + \frac{1}{2}\lambda_{hVV} v^2 \ , \\
M_f &= m_f + \frac{1}{2}\frac{\lambda_{hff}}{\Lambda} v^2 \ .
\end{aligned} \tag{16.2}$$

In what follows, we summarize the most important formulas relevant to our study. Related ideas and analyses can be found in [709, 711, 712, 731] and more recent studies of Higgs-portal scenarios have appeared in [713, 714].

The relic abundance of the DM particles is obtained through the s–channel annihilation via the exchange of the Higgs boson. For instance, the annihilation cross section into light fermions of mass $m_{\rm ferm}$ is given by

$$\begin{aligned}
\langle \sigma^S_{\rm ferm} v_r \rangle &= \frac{\lambda^2_{hSS} m^2_{\rm ferm}}{16\pi} \, \frac{1}{(4M_S^2 - m_h^2)^2} \, , \\
\langle \sigma^V_{\rm ferm} v_r \rangle &= \frac{\lambda^2_{hVV} m^2_{\rm ferm}}{48\pi} \, \frac{1}{(4M_V^2 - m_h^2)^2} \, , \\
\langle \sigma^f_{\rm ferm} v_r \rangle &= \frac{\lambda^2_{hff} m^2_{\rm ferm}}{32\pi} \, \frac{M_f^2}{\Lambda^2} \, \frac{v_r^2}{(4M_f^2 - m_h^2)^2} \, ,
\end{aligned} \tag{16.3}$$

where v_r is the DM relative velocity. (The cross section for Majorana fermion annihilation was computed in [715] in a similar framework.) We should note that in our numerical analysis, we take into account the full set of relevant diagrams and channels, and we have adapted the program micrOMEGAs [716] to calculate the relic DM density.

The properties of the dark matter particles can be studied in direct detection experiments. The DM interacts elastically with nuclei through the Higgs boson exchange. The resulting nuclear recoil is then interpreted in terms of the DM mass and DM–nucleon cross section. The spin–independent DM–nucleon interaction can be expressed as [742]

$$\begin{aligned}
\sigma^{SI}_{S-N} &= \frac{\lambda^2_{hSS}}{16\pi m_h^4} \frac{m_N^4 f_N^2}{(M_S + m_N)^2} \, , \\
\sigma^{SI}_{V-N} &= \frac{\lambda^2_{hVV}}{16\pi m_h^4} \frac{m_N^4 f_N^2}{(M_V + m_N)^2} \, , \\
\sigma^{SI}_{f-N} &= \frac{\lambda^2_{hff}}{4\pi \Lambda^2 m_h^4} \frac{m_N^4 M_f^2 f_N^2}{(M_f + m_N)^2} \, ,
\end{aligned} \tag{16.4}$$

where m_N is the nucleon mass and f_N parameterizes the Higgs–nucleon coupling. The latter subsumes contributions of the light quarks (f_L) and heavy quarks (f_H), $f_N = \sum f_L + 3 \times \frac{2}{27} f_H$. There exist different estimations of this factor and in what follows we will use the lattice result $f_N = 0.326$ [717] as well as the MILC results [718] which provide the minimal value $f_N = 0.260$ and the maximal value $f_N = 0.629$. We note that the most recent lattice evaluation of the strangeness content of the nucleon [719] favors f_N values closer to the lower end of the above range. In our numerical analysis, we have taken into account these lattice results, which appear more reliable than those extracted from the pion–nucleon cross section.

If the DM particles are light enough, $M_{\rm DM} \leq \frac{1}{2} m_h$, they will appear as invisible decay products of the Higgs boson. For the various cases, the Higgs partial decay widths into invisible DM particles are given by

$$\begin{aligned}
\Gamma^{\rm inv}_{h \to SS} &= \frac{\lambda^2_{hSS} v^2 \beta_S}{64\pi m_h} \, , \\
\Gamma^{\rm inv}_{h \to VV} &= \frac{\lambda^2_{hVV} v^2 m_h^3 \beta_V}{256\pi M_V^4} \left(1 - 4\frac{M_V^2}{m_h^2} + 12 \frac{M_V^4}{m_h^4} \right) , \\
\Gamma^{\rm inv}_{h \to \chi\chi} &= \frac{\lambda^2_{hff} v^2 m_h \beta_f^3}{32\pi \Lambda^2} \, ,
\end{aligned} \tag{16.5}$$

where $\beta_X = \sqrt{1 - 4M_X^2/m_h^2}$. We have adapted the program HDECAY [720] which calculates all Higgs decay widths and branching ratios to include invisible decays.

16.1.3 Astrophysical consequences

The first aim of our study is to derive constraints on the various DM particles from the WMAP satellite [13] and from the current direct detection experiment XENON100 [727], and to make predictions for future upgrades of the latter experiment, assuming that the Higgs boson has a mass $m_h = 125$ GeV and is approximately SM–like such that its invisible decay branching ratio is smaller than 10%; we have checked that increasing this fraction to 20% does not change our results significantly.

In Fig. 16.1, we delineate the viable parameter space for the Higgs-portal scalar DM particle. The area between the two solid (red) curves satisfies the WMAP constraint, with the dip corresponding to resonant DM annihilation mediated by the Higgs exchange. We display three versions of the XENON100 direct DM detection bound corresponding to the three values of f_N discussed above. The dash–dotted (brown) curve around the Higgs pole region represents $\mathrm{BR}^{\mathrm{inv}} = 10\%$ such that the area to the left of this line is excluded by our constraint $\mathrm{BR}^{\mathrm{inv}} < 10\%$. The prospects for the upgrade of XENON100 (with a projected sensitivity corresponding to 60,000 kg-d, 5-30 keV and 45% efficiency) and XENON1T are shown by the dotted lines.

Figure 16.1: Scalar Higgs-portal parameter space allowed by WMAP (between the solid red curves), XENON100 and $\mathrm{BR}^{\mathrm{inv}} = 10\%$ for $m_h = 125$ GeV. Shown also are the prospects for XENON upgrades.

We find that light dark matter, $M_{\mathrm{DM}} \lesssim 60$ GeV, violates the bound on the invisible Higgs decay branching ratio and thus is excluded. This applies in particular to the case of scalar DM with a mass of 5–10 GeV considered, for instance, in Ref. [711]. On the other hand, heavier dark matter, particularly for $M_{\mathrm{DM}} \gtrsim 80$ GeV, is allowed by both $\mathrm{BR}^{\mathrm{inv}}$ and XENON100. We note that almost the entire available parameter space will be probed by the XENON100 upgrade. The exception is a small resonant region around 62 GeV, where the Higgs–DM coupling is extremely small.

In the case of vector Higgs-portal DM, the results are shown in Fig. 16.2 and are quite similar to the scalar case. WMAP requires the Higgs–DM coupling to be almost twice as large as that in the scalar case. This is because only opposite polarization states can annihilate through the Higgs channel, which reduces the annihilation cross section by a factor of 3. The resulting direct detection rates are therefore somewhat higher in the vector case. Note that for DM masses below $m_h/2$, only very small values $\lambda_{hVV} < \mathcal{O}(10^{-2})$ are allowed if $\mathrm{BR}^{\mathrm{inv}} < 10\%$.

Figure 16.2: Same as Fig. 1 for vector DM particles.

Similarly, the fermion Higgs-portal results are shown in Fig. 16.3. We find no parameter regions satisfying the constraints, most notably the XENON100 bound, and this scenario is thus ruled out for $\lambda_{hff}/\Lambda \gtrsim 10^{-3}$.

This can also be seen from Fig. 16.4, which displays predictions for the spin–independent DM–nucleon cross section σ_{SI} (based on the lattice f_N) subject to the WMAP and $\mathrm{BR}^{\mathrm{inv}} < 10\%$ bounds. The upper band corresponds to the fermion Higgs-portal DM and is excluded by XENON100. On the other hand, scalar and vector DM are both allowed for a wide range of masses. Apart from a very small region around $\frac{1}{2}m_h$, this parameter space will be probed by XENON100–upgrade and XENON1T. The typical value for the scalar σ_{SI} is a few times 10^{-9} pb, whereas σ_{SI} for vectors is larger by a factor of 3 which accounts for the number of degrees of freedom.

16.1.4 Monojet constraints on the invisible width

In this section we estimate the sensitivity of current monojet searches at the LHC to a Higgs particles that decays invisibly. We rely on the search for monojets performed by the CMS collaboration which makes use of 4.7 fb^{-1} of data at 7 TeV center of mass energy [722]. The basic selection requirements used by the CMS experiment for such a topology are as follows:

- at least 1 jet with $p_T^j > 110$ GeV and $|\eta^j| < 2.4$;
- at most 2 jets with $p_T^j > 30$ GeV;
- no isolated leptons;
- missing transverse momentum $p_T^{\mathrm{miss}} \geq 200$ GeV.

Figure 16.3: Same as in Fig.1 for fermion DM; λ_{hff}/Λ is in GeV^{-1}.

Figure 16.4: Spin independent DM–nucleon cross section versus DM mass. The upper band (3) corresponds to fermion DM, the middle one (2) to vector DM and the lower one (1) to scalar DM. The solid, dashed and dotted lines represent XENON100, XENON100 upgrade and XENON1T sensitivities, respectively.

A second jet with p_T^j above 30 GeV is allowed provided it is not back-to-back with the leading one, $\Delta\phi(j_1, j_2) < 2.5$. Incidentally, this is advantageous from the point of view of invisible Higgs searches, as Higgs production at the LHC is often accompanied by more than one jet; vetoing the second jet would reduce the signal acceptance by a factor of ~ 2. The CMS collaboration quotes the event yields for 5 different cuts on the missing transverse momentum $p_T^{\rm miss}$ between 200 and 400 GeV. These are largely dominated by the SM backgrounds, namely Z+jets, where the Z boson decays invisibly, and W+jets, where the W boson decays leptonically and the charged lepton is not reconstructed. In particular, with 4.7 fb^{-1} data, the CMS collaboration estimates the background to be 24535 (1224) events for $p_T^{\rm miss} > 200$ (350) GeV, with an uncertainty that is below the 10% level.

A Higgs boson produced with a significant transverse momentum and decaying to invisible particles can also lead to the topology that is targeted by monojet searches. In Fig.16.5,

we show the fraction of Higgs events produced at the parton level in the ggF and VBF processes with p_T^j above a given threshold, assuming $M_H = 125$ GeV. One observes that about 1% (0.1%) of ggF events are produced with $p_T^j > 200$ (350) GeV which, assuming the SM production cross section, corresponds to about 750 (75) events in 5 fb^{-1} data at the $\sqrt{s} = 7$ TeV LHC. For the VBF and VH production processes, that fraction is larger by a factor of ~ 3. These numbers are comparable to the uncertainty on the SM monojet background quoted by the CMS collaboration. This suggests that if an invisible Higgs boson is produced with rates that are comparable or larger than the total rate of a SM–like Higgs boson, the monojet searches may already provide interesting constraints.

Figure 16.5: The fraction of events with Higgs transverse momentum above a given threshold for the ggF (red circles) and VBF (blue squares) production modes. The distributions were obtained at NLO using the program `POWHEG` [734]. In the case of ggF, the simulations included the finite quark mass effects [735], and we find good agreement with the NNLO distribution obtained using the program `HRes` [733] (black line).

In order to estimate the sensitivity of the CMS monojet search to the invisible Higgs signal, we generated the $pp \to H + \text{jets} \to \text{invisible} + \text{jets}$ process. We used the program `POWHEG` [734, 735] for the ggF and VBF channels at the parton level, and `Madgraph 5` [736] for the VH channels. Showering and hadronisation was performed using `Pythia 6` [744] and `Delphes 1.9` [738] was employed to simulate the CMS detector response. We imposed the analysis cuts listed above on the simulated events so as to find the signal efficiency. As a cross-check, we passed $(Z \to \nu\nu)$ + jets background events through the same simulation chain, obtaining efficiencies consistent within 15% with the data–driven estimates of that background provided by CMS.

The signal event yield depends on the cross section in each Higgs production channel and on the Higgs branching fraction into invisible final states. Thus, strictly speaking, the quantities that are being constrained by the CMS search are[1] $R_{\rm inv}^{\rm ggF}$ and $R_{\rm inv}^{\rm VBF}$ that are given in Eq. (**??**). Currently available data do not allow us to independently constrain $R_{\rm inv}^{\rm ggF}$ and $R_{\rm inv}^{\rm VBF}$. Thus, for the sake of setting limits, we assume that the proportions of ggF, VBF and VH rates are the same as in the SM, and we take the inclusive cross sections to be $\sigma(gg \to H)_{\rm SM} = 15.3$ pb, $\sigma(qq \to Hqq)_{\rm SM} = 1.2$ pb and $\sigma(q\bar{q} \to HV)_{\rm SM} = 0.9$ pb [739]. With this assumption, after the analysis cuts, the signal receives about 30% contribution

[1] Assuming custodial symmetry, $R_{\rm inv}^{\rm VH} = R_{\rm inv}^{\rm VBF}$.

$p_T^{\rm miss}$	$N_{\rm inv}^{\rm ggF}$	$N_{\rm inv}^{\rm VBF}$	$\Delta N_{\rm Bkg}$	$R_{\rm inv}^{\rm exp}$	$R_{\rm inv}^{\rm obs}$
200	630	260	∼1200	2.6	1.8
250	250	110	∼380	2.0	1.3
300	110	50	∼170	2.1	2.2
350	46	25	101	2.8	1.6
400	22	13	∼70	3.8	2.3

Table 16.1: Predicted event yields $N_{\rm inv}$, the 1σ background uncertainty $\Delta N_{\rm Bkg}$, and the expected and observed 95% CL limits on the invisible Higgs rate $R_{\rm inv}$ for each reported missing energy cut of the CMS monojet search. The event yields are given separately for the ggF and VBF+VH production modes, assuming the SM Higgs production cross sections in these channels.

from the VBF and VH production modes, and the rest from ggF; thus one constrains the combination $R_{\rm inv}^{pp} \approx \frac{2}{3} R_{\rm inv}^{\rm ggF} + \frac{1}{3} R_{\rm inv}^{\rm VBF}$.

Our results are presented in Table 16.1. We display the predicted event yield $N_{\rm inv}$ in the ggF and VBF channels for each cut on $p_T^{\rm miss}$ reported by the CMS collaboration. Comparing it to the 1σ background uncertainty[2] listed by CMS, we derive the expected 95% confidence level (CL) limit on $R_{\rm inv}$. We find the best expected limit is at the level of $R_{\rm inv} \leq 2$ for the $p_T^j \geq 250$ and 300 GeV cuts. The observed limit is better than the expected one in most cases, thanks to a $\sim 1\sigma$ downward fluctuation of the SM background. We find that the best limit corresponds to the $p_T^j \geq 250$ GeV cut, which yields the constraint $R_{\rm inv} \leq 1.3$ (1.0) at 95% (90%) CL. Finally, we can separately constrain $R_{\rm inv}^{\rm ggF}$ and $R_{\rm inv}^{\rm VBF}$ assuming only *one* Higgs production mode is present. Again the best limit is found using the CMS results for $p_T^j \geq 250$ GeV, and we find $R_{\rm inv}^{\rm ggF} \leq 1.9$ (when VBF is absent) or $R_{\rm inv}^{\rm VBF} \leq 4.3$ (when ggF is absent) at 95% CL.

Assuming Higgs produced with the SM cross section, the direct bounds from monojet searches do not yet constrain the invisible branching fraction. However, as discussed previously, in many models beyond the SM the Higgs production rate can be significantly enhanced, especially in the gluon fusion channel, in which case the current monojets bounds become non-trivial. In any case we should note that the sensitivity of monojet searches to invisible Higgs turns out to be much better than expected. Indeed, early studies [740], focusing mainly on the VBF production channel, concluded that observation of invisible Higgs decays was only possible at the highest LHC energy, $\sqrt{s} = 14$ TeV, and with more than 10 fb^{-1} data. Bounds on invisible Higgs based on the 1 fb^{-1} monojet search in ATLAS [244] were studied in Ref. [**?**], where a weaker limit of $R_{\rm inv}^{\rm ggh} \lesssim 4$ was obtained for $M_h \sim 125$ GeV. In contrast, our analysis shows that with 5 fb^{-1} data at 7 TeV, the LHC is already sensitive to $R_{\rm inv}$ of order 1. Note also that monojet searches are sensitive mostly to the ggF mode, thus they can also probe invisible Higgs in models where the Higgs couplings to the W, Z bosons are reduced, providing complementary information to invisible Higgs searches targeting the

[2]The CMS collaboration quotes the estimate of each important background for every $p_T^{\rm miss}$ cut but the errors on those estimates are given only for the $p_T^j \geq 350$ GeV cut. In the remaining cases, we estimated the uncertainty assuming that the error on the $Z \to \nu\nu$+jets background is dominated by statistics of the $Z \to ll$+jets control sample, while the error on the $W \to l\nu$+jets background is systematic and equals 11.3%. The remaining (subleading) backgrounds are assigned 100% uncertainty. Note that we did not consider the theoretical uncertainties on the cross sections [739] and the efficiencies of the p_T cuts which, although significant, are currently smaller than the experimental ones.

VBF mode.

We expect the bounds on the invisible Higgs width to be improved in the ongoing 8 TeV run. To make an estimate of the future sensitivity, we assume the error on the $Z \to \nu\nu$+jets background contribution will be dominated, as in the current run, by the statistics of the $Z \to \mu\mu$+jets control sample. We also (arbitrarily) assume the systematic error on the $W \to \ell\nu$+jets background contribution will be brought down to 5%. Taking into account the increased cross section[3] and selection efficiency for the signal and the V+jets backgrounds, we estimate the expected bound to be $R_{\rm inv}^{\rm exp} < 0.9$ at 95%CL with 15 fb^{-1} at $\sqrt{s} = 8$ TeV. Obviously, this is just a crude estimate, as it crucially depends on experiments' ability to control systematic errors on the V+jets backgrounds.

16.1.5 Invisible branching fraction and direct detection

If the invisible particle into which the Higgs boson decays is a constituent of dark matter in the universe, the Higgs coupling to dark matter can be probed not only at the LHC but also in direct detection experiments. In this section, we discuss the complementarity of these two direct detection methods. We consider generic Higgs-portal scenarios in which the dark matter particle is a real scalar, a real vector, or a Majorana fermion, $\chi = S, V, f$ [728, 742]. The relevant terms in the effective Lagrangian in each of these cases are

$$\begin{aligned}
\Delta\mathcal{L}_S &= -\frac{1}{2}m_S^2 S^2 - \frac{1}{4}\lambda_S S^4 - \frac{1}{4}\lambda_{hSS} H^\dagger H S^2 \ , \\
\Delta\mathcal{L}_V &= \frac{1}{2}m_V^2 V_\mu V^\mu + \frac{1}{4}\lambda_V (V_\mu V^\mu)^2 + \frac{1}{4}\lambda_{hVV} H^\dagger H V_\mu V^\mu, \\
\Delta\mathcal{L}_f &= -\frac{1}{2}m_f f f - \frac{1}{4}\frac{\lambda_{hff}}{\Lambda} H^\dagger H f f + \text{h.c.} \ .
\end{aligned} \tag{16.6}$$

The partial Higgs decay width into dark matter $\Gamma(H \to \chi\chi)$ and the spin–independent χ–proton elastic cross section $\sigma_{\chi p}^{\rm SI}$ can be easily calculated in terms of the parameters of the Lagrangian, and we refer to Ref. [728] for complete expressions. For the present purpose, it is important that both $\Gamma(H \to \chi\chi)$ and $\sigma_{\chi p}^{\rm SI}$ are proportional to $\lambda_{H\chi\chi}^2$; therefore, the ratio $r_\chi = \Gamma(H \to \chi\chi)/\sigma_{\chi p}^{\rm SI}$ depends only on the dark matter mass M_χ and known masses and couplings (throughout, we assume the Higgs mass be $M_H = 125$ GeV). This allows us to relate the invisible Higgs branching fraction to the direct detection cross section:

$$\mathrm{BR}_\chi^{\rm inv} \equiv \frac{\Gamma(H \to \chi\chi)}{\Gamma_H^{\rm SM} + \Gamma(H \to \chi\chi)} = \frac{\sigma_{\chi p}^{\rm SI}}{\Gamma_H^{\rm SM}/r_\chi + \sigma_{\chi p}^{\rm SI}} \tag{16.7}$$

with $\Gamma_H^{\rm SM}$ the total decay width into all particles in the SM. For a given M_χ, the above formula connects the invisible branching fraction probed at the LHC to the dark matter-nucleon scattering cross section probed by XENON100. For $m_p \ll M_\chi \ll \frac{1}{2}M_H$, and assuming the visible decay width equals to the SM total width $\Gamma_H^{\rm SM} = 4.0$ MeV [743], one can write down

[3]The inclusive cross sections at $\sqrt{s} = 8$ TeV are found to be 20.1, 1.6 and 1.1 pb for, respectively, ggF, VBF and VH.

the approximate relations in the three cases that we are considering,

$$
\begin{aligned}
\mathrm{BR}_S^{\mathrm{inv}} &\simeq \frac{\left(\frac{\sigma_{Sp}^{\mathrm{SI}}}{10^{-9}\mathrm{pb}}\right)}{400\left(\frac{10\ \mathrm{GeV}}{M_S}\right)^2+\left(\frac{\sigma_{Sp}^{\mathrm{SI}}}{10^{-9}\mathrm{pb}}\right)} \\
\mathrm{BR}_V^{\mathrm{inv}} &\simeq \frac{\left(\frac{\sigma_{Vp}^{\mathrm{SI}}}{10^{-9}\mathrm{pb}}\right)}{4\times 10^{-2}\left(\frac{M_V}{10\ \mathrm{GeV}}\right)^2+\left(\frac{\sigma_{Vp}^{\mathrm{SI}}}{10^{-9}\mathrm{pb}}\right)} \\
\mathrm{BR}_f^{\mathrm{inv}} &\simeq \frac{\left(\frac{\sigma_{fp}^{\mathrm{SI}}}{10^{-9}\mathrm{pb}}\right)}{3.47+\left(\frac{\sigma_{fp}^{\mathrm{SI}}}{10^{-9}\mathrm{pb}}\right)}
\end{aligned}
\tag{16.8}
$$

Thus, for a given mass of dark matter, an upper bound on the Higgs invisible branching fraction implies an upper bound on the dark matter scattering cross section on nucleons. In Fig. 16.6 we show the maximum allowed values of the scattering cross section, assuming the 40% bound on $\mathrm{BR}_\chi^{\mathrm{inv}}$, as advocated in Ref. [**?**]. Clearly, the relation between the invisible branching fraction and the direct detection cross section strongly depends on the spinorial nature of the dark matter particle, in particular, the strongest (weakest) bound is derived in the vectorial (scalar) case.

In all cases, the derived bounds on $\sigma_{\chi p}^{\mathrm{SI}}$ are stronger than the direct one from XENON100 in the entire range where $M_\chi \ll \frac{1}{2}M_H$. In other words, the LHC is currently the most sensitive dark matter detection apparatus, at least in the context of simple Higgs-portal models. This conclusion does not rely on the assumption that the present abundance of χ is a thermal relic fulfilling the WMAP constraint of $\Omega_{DM} = 0.226$ [13], and would only be stronger if χ constitutes only a fraction of dark matter in the universe. We also compared the bounds to the projected future sensitivity of the XENON100 experiment (corresponding to 60,000 kg-d, 5-30 keV and 45% efficiency).

Of course, for $M_\chi > \frac{1}{2}M_H$, the Higgs boson cannot decay into dark matter[4], in which case the LHC cannot compete with the XENON bounds.

16.1.6 Conclusions

We have shown that monojet searches at the LHC already provide interesting limits on invisible Higgs decays, constraining the invisible rate to be less than the total SM Higgs production rate at the 90% CL. This constrains the invisible branching fraction in models where the Higgs production cross section is enhanced, for example by the presence of additional heavy chiral quarks. Monojets searches are sensitive mostly to the gluon–gluon fusion production mode and, thus, they can also probe invisible Higgs decays in models where the Higgs coupling to the electroweak gauge bosons is suppressed. The limits could be significantly improved when more data at higher center of mass energies are collected, provided systematic errors on the Standard Model contribution to the monojet background can be reduced. We expect that the monojet data which will be collected in the ongoing 8 TeV

[4]In this case, one should consider the pair production of dark matter particles through virtual Higgs boson exchange, $pp \to H^* X \to \chi\chi X$. The rates are expected to be rather small [**?**, 728].

Figure 16.6: Bounds on the spin-independent direct detection cross section $\sigma^{\rm SI}_{\chi p}$ in Higgs portal models derived for $M_H = 125$ GeV and the invisible branching fraction of 40 % (colored lines). The curves take into account the full M_χ dependence, without using the approximation in Eq. 16.8. For comparison, we plot the current and future direct bounds from the XENON experiment (black lines).

LHC run will place non-trivial constraints on the Higgs invisible branching fraction, even if the Higgs production rate is SM–like.

We also analyzed in a model–independent way the interplay between the invisible Higgs branching fraction and the dark matter scattering cross section on nucleons, in the context of effective Higgs portal models. The limit $\mathrm{BR}_{\rm inv} < 0.4$, suggested by the combination of Higgs data in the visible channels, implies a limit on the direct detection cross section that is stronger than the current bounds from XENON100, for scalar, fermionic, and vectorial dark matter alike. Hence, in the context of Higgs-portal models, the LHC is currently the most sensitive dark matter detection apparatus.

Part V

Historical perspective: how the mechanics have been made so far

In this section, that I consider myself as fundamental, I want to give the reader the fundamental insights of the development of our field, analyzing how pieces of puzzles have been put together, year after year, to finally frame the picture we already have of our Universe (frame that is constantly changing). I will concentrate on the first publications, and give the mathematical tools, sufficient for any student to recover the main results obtained by the authors at that time. Indeed, I always truly believed (maybe by egocentrism as it is my case) that one understand better a formula, or a concept, once we know its source, the human ways and thoughts that brought it to life. This is my attempt in this part of the book that I consider honestly as the more important of the whole book.

Chapter 17

History of Dark Matter

17.1 The early years (1930-1960)

17.1.1 Introduction

It can be argued that the first success of the dark matter hypothesis, in the context of Newtonian gravity, is due to the French mathematician Urbain Le Verrier who, puzzled by small systematics peculiarities in the orbital motion of Uranus in the context of Newtonian gravity, pointed out that the existence of an unseen planet beyond the orbit of Uranus would resolve the anomalies. He did more than that: he actually determined the orbit of the undiscovered planet and, in 1846, predicted its position in the sky. Neptune was then quickly discovered by Johann Galle at the Berlin observatory (Le Verrier was independently followed in his prediction a few days later by the Englishman, John Adams).

These developments must actually be viewed as an outstanding success of Newtonian theory, which we now know works extremely well on the scale of the Solar System apart from tiny corrections due to the more complete theory of general relativity. But the success was realized only with the independent (i.e. visual) detection of Neptune: if no planet had been seen, Le Verrier's prediction would have remained an unconfirmed hypothesis. In that case, the anomalous motion of Uranus would have pointed to a breakdown of Newtonian gravity in the Solar System. This has relevance to the modern concept of dark matter as we shall see.

17.1.2 The virial theorem

Before entering into the details of the historical article of Zwicky published in 1933, I would like to remind the demonstration of the viral theorem, which is the powerful tool he used to compute the mass of the Coma cluster.

The virial theorem is an important theorem in mechanics. There are various corollaries of the theorem which are sometimes labelled *the Virial Theorem*. Those corollaries will be dealt in due course. Consider a system of point particles indexed by i. Let $\vec{r}_i$, $\vec{v}_i$ and $\vec{p}_i$ be the position, velocity and momentum vectors respectively, for the ith particle. Its mass

is denoted as m_i. Let $\vec{F}_i$ be the net force, internal and external, impinging upon the *ith* particle.

Statement of the virial theorem:

For the n point particles, bound together into a system, the *time average* of the kinetic energy of the particles, $\sum \frac{1}{2} m_i v_i^2$, plus one half of the *time average* of $\sum \vec{F}_i.\vec{r}_i$ is equal to zero.

Proof: for each particle

$$\begin{aligned} \vec{v}_i &= \frac{d\vec{r}_i}{dt} \\ \vec{p}_i &= m\vec{v}_i \\ \frac{d\vec{p}_i}{dt} &= \vec{F}_i \end{aligned} \tag{17.1}$$

Defining the virial $H = \sum \vec{p}_i.\vec{r}_i$ then $dH/dt = \sum (d\vec{p}_i/dt).\vec{r}_i + \sum \vec{p}_i.(d\vec{r}_i/dt)$. Because $(d\vec{p}_i/dt) = \vec{F}_i$, the first term can be written $\sum \vec{F}_i.\vec{r}_i$. Because $p_i = m_i\vec{v}_i = m_i(d\vec{r}_i/dt)$ the second term reduces to $\sum \vec{p}_i.(d\vec{r}_i/dt) = \sum m_i(d\vec{r}_i/dt)^2 = \sum m_i(\vec{v}_i)^2$. This last expression is just twice the kinetic energy K of the system, i.e. $2\sum \frac{1}{2} m_i v_i^2$. Thus

$$\frac{dH}{dt} = \sum \vec{F}_i.\vec{r}_i + 2K \tag{17.2}$$

One needs then to time average the expression above. The time average of a variable $y(t)$ over the interval 0 to τ is defined as $\overline{y} = (1/\tau)\int_0^\tau y(t)dt$. Time averaging the equation of dH/dt gives

$$\overline{\left(\frac{dH}{dt}\right)} = \sum \overline{\vec{F}_i}.\overline{\vec{r}_i} + 2\overline{K}. \tag{17.3}$$

The time average of dH/dt is just $\overline{(dH/dt)} = (1/\tau)[H(\tau) - H(0)]$. If the system is cyclical such that it returns to its initial state after an interval than τ can be chosen equal to the cycle period and $\overline{(dH/dt)}$ reduces to 0. If the system is not cyclical, then for the system being bounded the limit of $\overline{(dH/dt)}$ as τ increases without bounds is zero. Thus

$$\begin{aligned} &\sum \overline{\vec{F}_i}.\overline{\vec{r}_i} + 2\overline{K} = 0 \quad \text{or, equivalently} \\ &\overline{K} + \frac{1}{2}\sum \overline{\vec{F}_i}.\overline{\vec{r}_i} = 0 \end{aligned}$$

which is the virial theorem. If the forces are generated as the gradient of a potential $V(r_1, ..., r_n)$ then $\vec{F}_i = -\partial V/\partial r_i$ and hence

$$\overline{K} - \frac{1}{2}(\frac{\partial V}{\partial r_i}.r_i) = 0 \tag{17.4}$$

For forces which obey and inverse distance squared law, the potential is just inversely proportional to the distance. For systems held together by mutual gravitation or electrostatic attraction the virial theorem reduces to

$$\overline{K} = -\frac{1}{2}\overline{V} \tag{17.5}$$

For systems held together by mutual gravitational attraction the potential energy is negative so the kinetic energy is positive. The average total energy of the system $T = K + V$ is given by

$$\overline{T} = \frac{1}{2}\overline{V} \tag{17.6}$$

Astronomy has made great use of the virial theorem as a way to measure gravitational mass. Consider a set of n galaxies, each of mass m. Let $\overline{v^2}$ be the measured time averaged squared velocity of a galaxy and $\underline{v^2}$ the average of this quantity over the n galaxies. Then the time averaged kinetic energy of the system is $n[\frac{1}{2}m\underline{v^2}]$.
The gravitational potential for two galaxies separated by a distance R is then $-Gm^2/R$, where G is the gravitational constant[1]. Let $1/\underline{R}$ be the cluster average of the time average of $(1/R)$. There are $n(n-1)/2$ pairs of galaxies, so the time averaged potential of the system is then $-[n(n-1)/2][Gm^2/R]$. Then, according to the virial theorem,

$$\frac{1}{2}nm\underline{v^2} = \frac{1}{2}\frac{n(n-1)}{2}Gm^2\left(\frac{1}{\underline{R}}\right) \tag{17.7}$$

giving for the total mass of the cluster nm

$$nm = \frac{2\underline{v^2}}{G}\frac{n}{n-1}\,\underline{R} \tag{17.8}$$

where $\underline{R}$ is the reciprocal of $(1/\underline{R})$. In some clusters n is of the order of 1000 so $n/(n-1)$ is essentially unity.
Considering a general form of the potential $V = -\alpha\frac{GM^2}{R}$, α being a structure constant dependant on the profile of the distribution of the galaxies in the system ($\alpha = 3/5$ for an homogenous spherical system for instance) we can write

$$\frac{M}{2}\underline{v^2} = \frac{1}{2}\alpha\frac{GM^2}{R} \tag{17.9}$$

17.1.3 "The Redshift of Extragalactic Nebulae" by F. Zwicky (1933)

Fritz Zwicky (see chapter G.4 for more details) was one of those rare unorthodox geniuses who occasionaly emerge in astronomy or, for that matter, in any field. A swiss citizen who lived and worked in United State (Caltech) for many years, Zwicky made profound contributions to modern astronomy and astrophysics (from the observations and theory of

[1]To be more precise, the gravitational potential should depend on the exact form of the halo and should be generalized as $V = -\alpha GM^2/R$ where α is a structure parameter of the order of unity ($\alpha = 3/5$ for a homogenous spherical system) and R some characteristic size of the system (usually the core of half–mass radius)

Die Rotverschiebung von extragalaktischen Nebeln

von **F. Zwicky.**

(16. II. 33.)

Inhaltsangabe. Diese Arbeit gibt eine Darstellung der wesentlichsten Merkmale extragalaktischer Nebel, sowie der Methoden, welche zur Erforschung derselben gedient haben. Insbesondere wird die sog. Rotverschiebung extragalaktischer Nebel eingehend diskutiert. Verschiedene Theorien, welche zur Erklärung dieses wichtigen Phänomens aufgestellt worden sind, werden kurz besprochen. Schliesslich wird angedeutet, inwiefern die Rotverschiebung für das Studium der durchdringenden Strahlung von Wichtigkeit zu werden verspricht.

Figure 17.1: Left: original version of the Zwicky paper published in Helv. Phys. Acta 6, 110-127 (1933); Right: The Coma Cluster of Galaxies. This is a highly regular gravitationally bound system of thousands of galaxies at a distance of about 100 Mpc (NASA, SDSS)

exploding stars like supernovae and their remnants - the neutron stars- to the study and classification of galaxies and clusters of galaxies). He was decade ahead of his time, and the fact that much of his work was ignored by contemporaries no doubt contributed to his famously irascible behavior, he was, by any criterion, a difficult person.

Even if its work is widely considered as being the first one introducing the "dark matter" concept, it is only half true. In fact, in 1932, one year before Zwicky's paper on the Coma cluster, the Dutch astronomer Jan Oort, first used the term "dark matter" in discussing the mass density in the disk of the Milky Way as deduced from the motion of stars above the galactic plane. From the kinematic data, he calculated that only about one third of the dynamically inferred mass was present in bright visible stars. It is clear from the context that, in characterizing the reminder as *dark*, Oort was describing all matter not in the form of visible stars with luminosity comparable to or larger than that of the Sun. Oort included both low–mass stars and the undetected component of the interstellar medium (the gas and dust between the stars) in the category of dark matter, and concluded that when a proper accounting of all forms of conventional undetected matter is included, the resulting mass is not likely to be inconsistent with the dynamical mass. The first suggestion of a significant dynamical discrepancy on the scale of galaxies emerged a few years later from observation of rotation curves.

Measuring the distances in astrophysics: the *parsec*

During years, astrophysicist measured distance in angles, having no notion of absolute distances in space. It is then not surprising that the unit of distance in astrophysics is the parsec, which corresponds to the distance D at which a celestial object (a star) has an apparent movement of one second of arc (1/3600 degree) during the phase of rotation of the earth around the sun. Considering that the sun to earth distance is about 150 million kilometers, we deduce $D = 1.5 \times 10^8 \times 3600 \times 360/2\pi \simeq 3 \times 10^{13}$ kms $\simeq 3$ light-years. The stars we see with our unaided eyes are within 200 parsec (abreviated pc) distance, whereas a size of galaxy like the Milky-Way is of the order of the kilo-parsec (kpc, thousand of parsec). The Sun is at a distance of 8 kpc from the Galactic Center whereas the first nearby galaxy, Andromeda (M31) is at 700 kpc from the Sun. Others galaxies are at a distance of Megaparsec (Mpc) whereas the nearest cluster of galaxies, the Coma cluster is at a distance of 100 Mpc. The Horizon line (distance at which a particle travelling at the speed of light reached the earth from the primordial era, c/H_0, H_0 being the Hubble constant) is 4 Gigaparsec (Gpc).

We show in Figs.(18.2-17.3) the excerpt of the article where Zwicky propose his dark matter hypothesis (the rest of the article concerns mainly summary of the state of the art nebulae's searches in the last 10 years). He concetrate his work on the coma cluster (Fig.18.2) which is the largest nearby cluster, at a distance of about 100 Mpc from the Sun. However, as we can see from the table on Fig.(17.2), Zwicky took 45 light-years (around 15 Mpc) for the distance, in large part due to the error in the measurement of the Hubble constant at this epoch which was believed to be $H_0 = 558$ kms/s/Mpc compare to the measured value $H_0 = 74$ km/s/Mpc measured now.

Apparent velocities in the Coma cluster

$v =$	8500 km/s	6900 km/s
	7900	6700
	7600	6600
	7000	5100 (?)

Table II.[3]

Nebular cluster	Number of nebulae in the cluster	Apparent diameter	Distance in 10^6 light-years	Average velocity km/s
Virgo	(500)	12°	6	890
Pegasus	100	1°	23.6	3810
Pisces	20	0.5	22.8	4630
Cancer	150	1.5	29.3	4820
Perseus.	500	2.0	36	5230
Coma	800	1.7	45	7500
Ursa Major I	300	0.7	72	11800
Leo	400	0.6	104	19600
Gemini	(300)	—	135	23500

These results are shown graphically in Fig. 2.

Figure 17.2: Left: Values of the measured velocity of different galaxies in the Coma cluster; Right: table of the data used by F. Zwicky concerning the distance and size of different Cluster nearby the Milky Way, known in 1933. Both are excerpt from the original article of Zwicky Helv. Phys. Acta 6, 110-127 (1933).

But the more interesting idea of Zwicky was while he looked more in detail the dispersion in the velocities of the nebulae (name of galaxies at this time) measured by other groups (Fig.17.2 left). He noticed a large discrepancy, of around 1000 km/s that he attributed in the local individual of each member of the Coma. He then considered that the mean velocity of the galaxies in the Coma cluster is of the order of 1000 km/s. From that data, he was able to apply cleverly the virial theorem (Eq.17.9, his Eqs.4 and 6 in Fig.17.3) to measure the total mass of the Coma cluster. He applied the theorem supposing an homogenous sphere, hence the factor 3/5 in his Eq.(6) Fig.(17.3) compared to our Eq.(17.7).

This factor 3/5 for the potential can be recover by computing the gravitational potential V (Ω in the Zwicky's convention) for a continuous homogenous sphere of density ρ: $V = G\int_0^R(\frac{4}{3}\pi\rho r^3)(4\pi r^2 dr)/r = \frac{3}{5}\frac{GM^2}{R}$, M being the total mass of the sphere.

In fact, to be more precise, the reasoning of Zwicky was a little bit different. From the mass and radius of the cluster he inferred, he deduced the relative mean velocities of the galaxies. From the angular size of the Coma cluster (1.7 degree, Fig.17.2) one can compute its radius R because its distance D is known by the mean velocity of the galaxies (7500 km/s Fig.17.2) and the application of Hubble law (with the value $H_0 = 558$ km/s/Mpc used at this epoch) gives $D = 45$ light-years. This gives $R = 1.7 \times 45\pi/180 \simeq 1$ million light-year. He also observed that the cluster contains around 800 galaxies (nebulae) with each of the nebulae having a mass of approximatively 10^9 solar mass. The Sun mass being 2×10^{33} g he obtained $M_{\text{Coma}} = 800 \times 10^9 \times 2 \times 10^{33}$ g $=1.6 \times 10^{42}$ kg. as one can see on his Eq.(5). Then, applying the virial theorem he computed

$$\underline{v^2} = \frac{3}{5}\frac{GM}{R} = \frac{3}{5} \times \frac{6.67 \times 10^{-11} \times 1.6 \times 10^{42}}{10^{22}} \Rightarrow \sqrt{\underline{v^2}} \simeq 80 \text{ km/s}. \tag{17.10}$$

As Zwicky himself concludes in his paper *In order to obtain the observed value of an average Doppler effect of 1000 km/s or more, the average density in the Coma system would have to be at least 400 times larger than that derived on the grounds of observations of luminous matter. If this would be confirmed we would get the surprising result that dark matter is present in much greater amount than luminous matter.*. This is technically the very first proposition of missing mass in our Universe. Of course, due of the numerous approximations and errors (like the Hubble parameter) it is easy to understand that this work, in 1933, has been completely overlooked. Nobody really was interested in this conclusion, as a lot of observators considered that the lack of information in the interstellar and/or inter-nebulae gas, or even the absorption lines can explain such a huge discrepancy. It is only while working at the galactic scale, many years after, that the evidence of presence of dark matter in the Universe begins to appear.

§5. Remarks concerning the dispersion of velocities in the Coma nebular cluster.

As the data in §3 show, there are in the Coma cluster differences in velocity of at least 1500 to 2000 km/sec. In the context of this enormous variation of velocities the following considerations can be made:

1. Under the supposition that the Coma system has reached, mechanically, a stationary state, the Virial Theorem implies

$$\overline{\epsilon}_k = -\tfrac{1}{2}\overline{\epsilon}_p, \tag{4}$$

where $\overline{\epsilon}_k$ and $\overline{\epsilon}_p$ denote average kinetic and potential energies, e.g. of the unit of mass in the system. For the purpose of estimation we assume that the matter in the cluster is distributed uniformly in space. The cluster has a radius R of about one million light-years (equal to 10^{24} cm) and contains 800 individual nebulae with a mass of each corresponding to 10^9 solar masses. The mass M of the whole system is therefore

$$M \sim 800 \times 10^9 \times 2 \times 10^{33} = 1.6 \times 10^{45} \text{ g}. \tag{5}$$

This implies for the total potential energy Ω:

$$\Omega = -\frac{3}{5}\Gamma\frac{M^2}{R} \tag{6}$$

$$\Gamma = \text{Gravitational constant}$$

or

$$\overline{\varepsilon}_p = \Omega/M \sim -64 \times 10^{12} \text{ cm}^2\text{s}^{-2} \tag{7}$$

and then

$$\overline{\varepsilon}_k = \overline{v^2}/2 \sim -\overline{\varepsilon}_p/2 = 32 \times 10^{12} \text{ cm}^2\text{s}^{-2}$$

$$\left(\overline{v^2}\right)^{1/2} = 80 \text{ km/s}. \tag{8}$$

In order to obtain the observed value of an average Doppler effect of 1000 km/s or more, the average density in the Coma system would have to be at least 400 times larger than that derived on the grounds of observations of luminous matter.[8] If this would be confirmed we would get the surprising result that dark matter is present in much greater amount than luminous matter.

Figure 17.3: Excerpt of the article "The Redshift of Extragalactic Nebulae" by F. Zwicky published in 1933 (English translation)

Figure 17.4: Example of a spiral galaxy (Andromeda, M31) on the left and an elliptical galaxy (the giant elliptical M87) on the right

17.1.4 At the galactic scale, the early attempts (1939-1957)

Galaxies

There exists several types of galaxies in the Universe. Two of the more important ones are the spiral ones and the elliptical ones, with completely different characteristics, and more precisely their mass-to-light ratio M/L. This ratio is equal by one for the Sun by definition corresponding to $M_\odot/L_\odot = 5133$ kg/W . Typical old stars have a value $M/L > 1$ because they are less luminous due to the hydrogen they already consumed in their lifetime. $M/L = 5$ is the maximum values obtained by models of stars, corresponding with observations, which means a measurement of $M/L \gg 5$ means the presence of dark masses around the star. On the opposite, very bright ones can easily have M/L of the order of 10^{-4}.
The Mily Way galaxy contains about 100 billion (10^{11}) stars. Its mass in stars would be a few times 10^{10} solar masses. The mass of the Sun, in a standard unit of mass (2×10^{30} kilograms) in astronomy. The mass of the Galaxy, therefore would be of the order of 10^{41} kilograms. Galaxies in general range between 10^7 to 10^{12} solar masses (abreviated $M_\odot$), at least in visible stars.

- The **spiral galaxies** like the Milky Way, are composed of a flat spiral disk, with a population of young stars, and hot gas in large quantity, where stars are still in formation. It then emit a "blue" light and its stability against gravitational collapse is mainly due to its rotation, the centrifugal force. Its population of stars respect in the vast majority a mass-to-light ratio $M/L = 1$. The nearest galaxy to the Milky Way is also a spiral galaxy, Andromeda or M31 (Fig.17.4) which is at a distance of 700 kpc from the Sun.
- The **elliptical galaxies** are composed of old cold stars, almost not gas and is then much more situated on the red part of the spectrum. They respect an almost elliptically flatten shape, due to the age of its stars, the typical mass-to-light ratio in such galaxies are $M/L \simeq 5$. Their stability against the gravitational collapse is due to the random movement of the stars inside the galaxy more than a rotational movemement like in the case of spiral galaxies. It is a pressure support system.

Chapter 18

History of nucleosynthesis

In 1948, 11 papers have been published within a team of 3 authors: Gamow, Alpher (his PhD student) and Hermann (his postdoc). All these papers were concentrated on one issue: the formation of the first nuclei in the primordial Universe. The first paper, is the most famous and most cited one (usually cited for wrong reasons, even in several Nobel lectures!), the $\alpha\beta\gamma$ paper, written by Alpher and Gamow. The "β" is for Bethe who appears as an authors whereas he did not collaborate. Gamow just added it for the sounding of "Alpher, Bethe, Gamow", which seemed to be a quite good strategy as the $\alpha\beta\gamma$ paper is the more known on the subject. The article is based on the thesis work of Alpher, but Gamow wanted a short version of it in 1948 before the thesis defense. The article is "wrong in almost every detail" as said Arno Penzias in his Nobel lecture in 1978. Even if the treatment has a lot of mistakes (and some fundamentally false, like the fact they supposed a matter dominated Universe whereas at such high temperatures, radiation dominates the expansion rate) the bases of nucleosynthesis, leading also to the Cosmic Microwave Background (CMB) prediction 20 years before its discovery, are settled.

18.1 Historical premices

Without any doubt, Gamow was one of the genius of our field, and opened many new roads toward the understanding of the primordial Universe (see section G.5 for more details). After studying with Friedmann and then with Bohr, he had all the necessary tools when he arrived at Washington University to develop a new discipline: the study of nuclear reactions during the very first moments of the Universe. The concept was proposed by Alpher in his thesis under the supervision of Gamow from 1946 till 1949. They compiled their result in the $\alpha\beta\gamma$ paper (see summary frames) in April 1948. Their conclusion was that the Universe has formed the first light elements in about 20 seconds. What is intriguing is that they used a completely wrong model and context, supposing a matter-dominated Universe with a density 9 times larger that the real one (!!). Gamow corrected it in a next paper in June the same year "The Origin of Elements and the Separation of Galaxies" affirming "ρ_0.. was given incorrectly in the previous paper because of a numerical error in the calculations". Which is not true, as the basic hypothesis (matter-dominated Universe) renders all the arguments of the $\alpha\beta\gamma$ paper completely wrong. More probably, Alpher and Gamow noticed the mistake during the completion of the Thesis of Alpher and corrected the reasoning afterwards.

The Origin of Chemical Elements

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and

H. Bethe
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and

G. Gamow
The George Washington University, Washington, D. C.
February 18, 1948

On the other hand, according to the relativistic theory of the expanding universe[4] the density dependence on time is given by $\rho \cong 10^6/t^2$. Since the integral of this expression diverges at $t=0$, it is necessary to assume that the building-up process began at a certain time t_0, satisfying the relation:

$$\int_{t_0}^{\infty} (10^6/t^2)dt \cong 5\times 10^4, \qquad (2)$$

which gives us $t_0 \cong 20$ sec. and $\rho_0 \cong 2.5\times 10^5$ g sec./cm³. This

Figure 18.1: Original version of the $\alpha\beta\gamma$ paper by Alpher, Bethe and Gamow in 1948, where they predicted for the first time that the first light elements have been built up after 20 seconds. Very strangely, as Gamow will correct it 3 months after, the value of density ρ_0 they use is 9 orders of magnitude (2.5×10^5 g/cm^3 instead of $\sim 10^{-4}$ g/cm^3). This was coming from the fact that Gamow considered by mistake at this time a matter dominated universe at this energy scale, whereas it was still radiation dominated, with a density $\rho_0 = \rho_0^{rad} \simeq 1$g/cm^3 which is more or less the density of water.

Then Gamow proposed that *all* the observed heavy elements were built up from the lighter ones (which we know now is not true as the primordial BBN stopped to the lightest elements, the rest being produced by the stellar nucleosynthesis proposed by Hoyle et al. some years later). This is an original idea at this time, where the community was still attached to the decaying process of a very heavy primeval atom of long-lived Uranium type developed by Lemaitre. Even if this hypothesis was not supported by the astrophysical measurements of nuclei abundance the building up process was far to be obvious at this time.

The $\alpha\beta\gamma$ paper controversy

Physicist George Gamow was born in Odessa (now in Ukraine), in 1904. He grew dissatisfied with the Soviet Union, and after one failed attempt, he fled and immigrated to the United States in 1934. He took a position at George Washington University in Washington, DC. The research needed knowledge of nuclear physics, but most nuclear physicists in the US at the time had been recruited to the Manhattan project, so Gamow was essentially alone in working on the problem of nucleosynthesis. He started making calculations, beginning by looking at the density of matter in the universe and essentially running the expansion of the universe backwards to get an estimate of what the early universe might have looked like. He then began trying to figure out the probabilities of nuclear reactions in early universe. As the universe expands, conditions constantly change, so the calculations were complicated. Not particularly adept at mathematical calculations himself, Gamow recruited PhD student Ralph Alpher to help. They started by imagining the early stage of the universe as an extremely hot dense gas of neutrons, (which they called "ylem," after a medieval word for matter).
Alpher's calculations of nuclear processes used some of the first electronic digital computers, which had been developed during World War II. He was also able to use new data on nuclear reaction cross sections that had become available after the war ended. The calculations agreed with the known abundance of helium. Pleased with their result, Alpher and Gamow submitted a brief communication to the Physical Review, titled "The Origin of Chemical Elements." They celebrated with a bottle of liqueur, which Gamow relabeled "ylem." Gamow, who was known for his sense of humor, saw that the paper they had submitted to Phys. Rev. was to appear on April 1, 1948. He added the name of his friend Hans Bethe, who was known for work on nuclear reactions in stars, among other things, to the paper, so the authors would be Alpher, Bethe, and Gamow, a pun on the first three letters of the Greek alphabet. Alpher, as a PhD student struggling to make a name for himself, objected to the addition, fearing that the name of the famous Bethe would overshadow his own, reducing the credit he received for his crucial contribution to an important piece of research. But Gamow published it with Bethe's name, despite Alpher?s objections. The paper, still known as the alpha-beta-gamma paper, not only explained the origin of the most abundant elements in the universe, but also provided the first support for the big bang model since Hubble's discovery in 1929 that distant galaxies are redshifted in proportion to their distance from us.
It is probable that this little conflict was a source of tensions between Alpher and Gamow, explaining why the Cosmic Microwave Background prediction is proposed in separate papers, one by Alpher and Hermann (post-doc of Gamow) in 1948 and the other one in an article of Gamow in 1950.

18.2 How to predict a CMB?

The prediction of the CMB was in fact just a remark in the 1948 paper of Alpher and Hermann who quoted ".. the temperature in the Universe at the present time is found to be 5 K". The second genius idea of Gamow (after the first one which was the hypothesis that the elements were building up and not generated by decaying massive particles) is that, if the Universe was dense and hot, there was a time where the temperature dropped below the binding energy of the deuterium (2.2 MeV $\simeq 2.2 \times 10^{10}$ K). Even more cleverly, Gamow applied the Saha equation[1] (see section 2.3.2) and concluded that the temperature T_D below which the photons of the thermal bath will not be able to dissociate anymore the deuterium ($\gamma + d \to p + n$) is not the binding energy but much lower, $T_D \simeq 0.1$ MeV $= 10^9$ K. Below T_D, only the reverse production process $p + n \to d + \gamma$ will take place until there will not be sufficiently decaying neutrons left for the fusion to occur. The unbounded protons left will form the hydrogen, whereas the deuterium will continue to react on other deuterium nucleus in a serial of reactions to produce Helium and Lithium.

[1] Saha equation states that even if the mean energy of the primordial plasma is below the binding energy, there are still statistically sufficiently high energy photons which are able to destroy the deuterium.

Then Gamow applied the Friedmann equation, which links temperature, space and time, combined with the conservation of entropy which links space and temperature, to compute how many seconds after the big bang corresponds to a temperature $T_D = 10^9$ K. More precisely, from

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\rm rad}(T) = \frac{8\pi G}{3}\frac{\pi^2}{15}T^4 \tag{18.1}$$

and using entropy conservation

$$aT = \text{constant} \quad \Rightarrow \quad \frac{da}{a} = -\frac{dT}{T} \tag{18.2}$$

one deduces

$$\frac{dT}{T^3} = -\sqrt{\frac{8\pi^3 G}{45}}dt \quad \Rightarrow \quad t = \frac{M_{PL}}{T^2}\sqrt{\frac{45}{32\pi^3}} \simeq 0.2\frac{M_{PL}}{T^2} \tag{18.3}$$

which gives for $T = T_D = 10^9\text{K} = 10^{-4}$ GeV

$$t \simeq 3 \times 10^{27}\ \text{GeV}^{-1} \sim 200 \text{ seconds.} \tag{18.4}$$

Noticing that at the time of dissociation t_D, the Universe is radiation dominated, one can compute the density of mass at this time, $\rho_m(10\text{K}) = n(10^9\text{K})m_p$. $n(10^9\text{K})$ is obtained remarking that at the time t_D at least one reaction should have occurred[2] one can write

$$n(t_D)\sigma v t_D \simeq 1 \quad \Rightarrow \quad n(t_D) \simeq \frac{1}{\sigma v t_D} \tag{18.5}$$

Having measured the formation rate $\sigma \sim 10^{-29}\text{cm}^2$, and computing that at 10^9 K, the kinetic energy of the nucleons is $v = \sqrt{\frac{3T_D}{m_p}} \times c \simeq 5 \times 10^8$ cm/s, one deduces

$$n(t_D) \sim 10^{18}\ \text{cm}^{-3} \quad \Rightarrow \quad \rho_m(10^9\ \text{K}) \sim 10^{18}\ \text{GeV/cm}^3 = 1.78 \times 10^{-6}\ \text{g/cm}^3. \tag{18.6}$$

Then, remarking that mass should be conserved in an expanding Universe, $\rho_m a^3 \propto \rho_m/T^3 =$ constant, one obtains

$$T^{now} = \left(\frac{\rho_m^{now}}{\rho_m(10^9\ \text{K})}\right)^{1/3} 10^9\ \text{K} = \left(\frac{10^{-30}}{1.78 \times 10^{-6}\text{g/cm}^3}\right)^{1/3} 10^9\ \text{K} \simeq 8\ \text{K}. \tag{18.7}$$

where we have supposed that the density of mass *today*, measured by experimentalists like *Oort* is around 10^{-30}g/cm^3 (the critical density ρ_c is $2 \times 10^{-29}\text{g/cm}^3$).
A more complete treatment, taking into account the chains of nuclear fusions to obtain Helium, Lithium and Boron primordial abundances was initiated by Gamow in 1948, but really solved by Peebles et al. in 1965, the year of the Cosmic Microwave Background discovery by Penzias and Wilson. The aim of a more precise treatment is to obtain a value

[2]This statement simplifies strongly the problematic, which should take into account a chain of reactions, solving a complex system of differential equations, but it gives an accurate estimate of $n(t_D)$.

light-years. The temperature of the gas at the time of condensation was 600° K., and the temperature in the universe at the present time is found to be about 5° K.

We hope to publish the details of these calculations in the near future.

Figure 18.2: Excerpt of the paper "Evolution of the Universe" by Alpher and Hermann in 1948 where they predicted a 5 K relic temperature from emitted from the dissociation period.

of the initial condition $\rho_m(t_D)$ more solid that the back-to-the-enveloppe argument from Eq.(18.5). From his solution, Peebles predicted the amount of light elements, corresponding to the values measured experimentally.

Chapter 19

History of neutrino

Chapter 20

History of unification

Chapter 21

History of inflation

Chapter 22

History of black-body radiation

Part VI

Appendix

Appendix A

Astrophysics

A.1 General relativity feelings

A.1.1 Deflection angle

As it is well known, the general theory of relativity has been confirmed in 1919 by Eddington when he measured a deviation in the light emitted by stars and passing near the sun during the 1919 eclipse (see section H.15 for historical details). It is obviously beyond the scope of this book (and this appendix) to give a complete mathematical description of the theory of general relativity (I let the reader jump to [746] for a nice introduction in the subject).
The basic of deflected light, or even the computation of the perihelion of Mercury is based on the fact that light as particles move on a timeline geodesic, and so we classically study some of the geodesics of the Schwarzschild solution:

$$2K = (1-2m/r)\dot{t}^2 - (1-2m/r)^{-1}\dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\phi}^2 \tag{A.1}$$

with $2K = 0, +1$ or -1 depending on wether the tangent vector is null, or has positive or negative length respectively. In other words, when computing the Mercury perihelion, $2K = 1$ whereas the light travels on a null geodesic and thus $2K_\gamma = 0$. For the light, the equation (A.1) then becomes (you can check it as an exercise)

$$\frac{d^2u}{d\phi^2} + u = 3mu^2 \tag{A.2}$$

with $u = 1/r$. In the limit of special relativity, m vanishes and the equation becomes

$$\frac{d^2u}{d\phi^2} + u = 0, \tag{A.3}$$

the general solution of which can be written in the form

$$u = \frac{1}{b}\sin(\phi - \phi_0) \tag{A.4}$$

where b is the impact parameter, the distance of closest approach to the origin (see Fig.(A.1) for illustration). This is the equation of a straight line as ϕ goes from ϕ_0 to $\phi_0 + \pi$. The straight line motion is the same as is predicted by Newtonian theory.

The equation of a light ray in Schwarzschild space-time can be thought of as a perturbation of the classical equation, treating m/r as small. We therefore look for a solution of the form

$$u = u_0 + 3mu_1 \tag{A.5}$$

where u_0 is the solution of (A.4). Taking $\phi_0 = 0$ for convenience, we obtain

$$u_1'' + u_1 = u_0^2 = \frac{\sin^2\phi}{b^2} \quad \Rightarrow \quad u \simeq \frac{\sin\phi}{b} + \frac{m(1 + C\cos\phi + \cos^2\phi)}{b^2} \tag{A.6}$$

where C is an arbitrary constant of integration for u_1. Since m/b is small, this is clearly a perturbation from straight line motion. We are interested in determining the angle of deflection δ for a light ray in the presence of a spherically symmetric source, such as the sun. A long way from the source $r \to \infty$ and hence $u \to 0$, which requires the right–hand side of the equation (A.6) to vanish. Let us take the values of ϕ for which $r \to \infty$, that is the angles of the asymptote, to be $-\epsilon_1$ and $\pi + \epsilon_2$ respectively, as shown in Fig.(**??**). Using the small angle approximation for ϵ_1 and ϵ_2, we get

$$-\frac{\epsilon_1}{b} + \frac{m}{b^2}(2 + C) = 0, \qquad -\frac{\epsilon_2}{b} + \frac{m}{b^2}(2 - C) = 0. \tag{A.7}$$

adding, we find

$$\delta = \epsilon_1 + \epsilon_2 = \frac{4m}{b} \tag{A.8}$$

or, in a non-relativistic units,

$$\delta = \frac{4GM}{c^2 b} \tag{A.9}$$

Figure A.1: Illustration of the deflected angle due to gravitational field in general relativity.

Appendix B

Cosmology

B.1 Useful cosmology

B.1.1 Friedmann equation

The scale factor of the Universe (his size) is governed by Friedmann's equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} = \frac{8\pi}{3M_{\rm Pl}^2}\rho - \frac{K}{a^2}, \tag{B.1}$$

where we have ignored the cosmological constant for simplicity. Negelecting in the first step the curvature term (as one can do at early time), and approximating $\dot{a} \sim a/t$, Eq.(B.1) implies the familiar result that the expansion time scale $t \propto \rho^{-1/2}$. We choose the boundary condition $a(t_0) = 1$ at present time and $a(0) = 0$ at the early stage of the Universe. The Hubble constant H_0 is the present time value of the Hubble parameter $H_0 = (\dot{a}/a)_{t_0}$. K is the value of hypersurfaces of space–time, and ρ is the total matter density. We define the quantity

$$\rho_{\rm cr} = \frac{3H_0^2}{8\pi G} \approx 2 \times 10^{-29}\ h^2 {\rm g\ cm}^{-3} = 1.1 \times 10^{-5}\ {\rm GeV\ cm}^{-3} \tag{B.2}$$

is called the critical density for reasons that will become clear below. The density Ω_0 is the current total matter density $\rho(t_0) = \rho_0$ in unit of $\rho_{\rm cr}$,

$$\Omega_0 = \frac{\rho_0}{\rho_{\rm cr}}. \tag{B.3}$$

The Hubble constant is commonly written

$$H_0 = 100\ h\ {\rm km\ s}^{-1}\ {\rm Mpc}^{-1} \approx 3.2 \times 10^{-18} h\ {\rm s}^{-1}, \tag{B.4}$$

where $0.5 < h < 1$ express our ignorance of H_0. We can then extract from the Friedmann's equation (B.1) remembering that $a(t_0) = 1$,

$$K = H_0^2(\Omega_0 - 1) \tag{B.5}$$

In other words, a universe which contains the critical matter density is flat.

The first law of thermodynamics with conservation of entropy, $dS = dU + pdV$ can be written as

$$\frac{d(\rho a^3)}{dt} + p\frac{da^3}{dt} = 0 \tag{B.6}$$

For ordinary matter, $\rho \gg p \sim 0$, hence $\rho \propto a^{-3}$. For relativistic matter, $p = \rho/3$, hence $\rho \propto a^{-4}$. Therefore, the matter density changes with a as

$$\rho(a) = \rho_0 a^{-n} = \rho_{\rm cr}\Omega_0 a^{-n}, \tag{B.7}$$

where $n = 3$ for ordinary matter ("dust"), and $n = 4$ for relativistic matter ("radiation").

Summarizing, we can rewrite Friedmann's equation into the form

$$H^2(a) = H_0^2[\Omega_0 a^{-n} - (\Omega_0 - 1)a^{-2}] \tag{B.8}$$

Since the universe expands, $a < 1$ for $t < t_0$, and so the expansion rate $H(a)$ was larger in the past. At very early times, $a \ll 1$, the first term in Eq.(B.8) dominates because $n \geq 3$ and we can write

$$H(a) = H_0\Omega_0^{1/2}a^{-n/2} \tag{B.9}$$

This is called the Einstein-de Sitter limit of Friedmann's equation.

B.1.2 The Horizon

The size of casually connected regions of the Universe is called the *horizon size*. It is given by the distance a photon can travel in the time since the Big Bang. Since the appropriate time scale is provided by the inverse Hubble parameter, the horizon size is $d'_H = cH^{-1}(a)$, and the *comoving* horizon size is (reminding that the velocity of light $c = 1$)

$$d_H(a) = \frac{c}{aH(a)} = \frac{1}{aH(a)} = \frac{1}{H_0}\Omega_0^{-1/2}a^{n/2-1}, \tag{B.10}$$

where we have inserted the Einstein-de Sitter limit of Friedmann's equation. $cH_0^{-1} = 3\ h^{-1}$ Gpc is called the *Hubble radius*.

The previous calculations show that the matter density today is completely dominated by ordinary rather than relativistic matter. But since the relativistic matter density grows faster with decreasing scale factor a than the ordinary matter density, there had to be a time $a_{eq} \ll 1$ before which relativistic matter dominated. Using Eq.(2.95), the condition

$$\Omega_0^R a_{eq}^{-4} = \Omega_0^M a_{eq}^{-3} \tag{B.11}$$

yields[1]

$$a_{eq} = \frac{3.7 \times 10^{-5}}{0.15} \simeq 2.7 \times 10^{-4} \tag{B.12}$$

[1]For another way of computing the equilibrium time or temperature, see section (2.3.7)

which means that the Universe was tens thousands time smaller at the epoch when radiation and matter were in equilibrium. We have seen in section 2.8 that a_{eq} plays a big role in structure formation constraints. If we suppose that matter dominated completely for all $a > a_{eq}$ (i.e. ignoring the contribution from radiation to the matter density), Eq.(B.10) yields

$$d_H(a_{eq}) = \frac{\Omega_0^{-1/2}}{H_0} a_{eq}^{1/2} \approx 90 \text{ Mpc}. \tag{B.13}$$

As we noticed in Eq.(B.7) the photon temperature evolves as a^{-4}. We also have demonstrated in Eq.(2.37) that the energy density of radiative particles evolve as T_γ^4 which implies that $T_\gamma \propto a^{-1}$. We can then compute tha temperature of the photon a the time when $a = a_{eq}$:

$$T_\gamma(a_{eq}) = T_\gamma^0 \times a_{eq}^{-1} = \frac{2.4 \times 10^{-4}}{2.7 \times 10^{-4}} \simeq 0.9 \text{ eV}. \tag{B.14}$$

We recover the results obtained in the section 2.3.7

We can also compute the time dependance of the evolution of the scale factor. Indeed, from $\dot{a}/a = H \propto \rho_0^{1/2} a^{-n/2}$ which gives after integration

$$t \propto a^{(n-2)/2} \tag{B.15}$$

which means

$$\begin{aligned} &a \propto t^{1/2} \text{ [radiation dominated]} \qquad a \propto t^{2/3} \text{ [matter dominated]} \\ &\Rightarrow H = \frac{\dot{a}}{a} = \frac{1}{2t} \text{ [radiation dominated]} \qquad \frac{2}{3t} \text{ [matter dominated]} \end{aligned} \tag{B.16}$$

From the previous section, we computed the energy density of relativistic particle $\rho_{\gamma\nu} = \frac{3g}{\pi^2} e^{\mu/T} T^4$, which means $\rho_{\gamma\nu} \propto T^4$ for a null chemical potential μ. Moreover, if we write $p = adr/dt$, and knowking that t scales as a^{-2} in radiation time, one can deduce that p scales as a^{-1} in radiation dominated epoch.

$$p \propto a^{-1}, \tag{B.17}$$

which means that for a non-relativistic particle $p = mv$ implies $v \propto a^{-1}$.
Reminding that the TOTAL entropy of the Universe $S = sa^3 = g_{*S} T^3 a^3$ is constant, one can deduce

$$a(T) \propto g_{*S}^{1/3} T^{-1} \quad \text{[in matter AND radiation dominated epoch]} \tag{B.18}$$

implying

$$a(T) = a_0 \frac{2.4 \times 10^{-4} \text{ eV}}{T} = a_{DEC} \frac{0.3 \text{ eV}}{T} \tag{B.19}$$

Combining with the results from the thermodynamics equilibrium of the previous section and supposing that during the early radiation-dominated epoch $\rho \sim \rho_R$, Eq.(2.37) combined with Eq.(B.1), one obtains

$$H = \sqrt{\frac{8\pi G}{3}}\rho_R^{1/2} = 1.66 g_*^{1/2}\frac{T^2}{M_P} \tag{B.20}$$

Moreover, if $a \propto t^n$, one observe that $H = \dot{a}/a = nt^{-1}$ implying

$$t = 0.301 g_*^{-1/2}\frac{M_P}{T^2} \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \text{ [radiation dominated : n = 1/2]} \tag{B.21}$$

$$t \propto T^{-3/2} \text{ [matter dominated : n = 2/3]} \tag{B.22}$$

where $g_* = g_{boson} + \frac{7}{8} g_{fermion}$ counts the total number of massless degrees of freedom ($m \ll T$).

B.2 Matter/radiation domination

As we have already seen , the radiation[2] ($\gamma\nu$) and matter (M) densities are directly linked to the scale R of the Universe by (see the preceding paragraph)

$$\rho_\gamma \propto a^{-4} \qquad \rho_\nu \propto a^{-4} \qquad \rho_M \propto a^{-3} \tag{B.23}$$

From this relation, one can understand that with time, and the expansion of the Universe, the radiation density decreases faster than the matter density. Nowadays, the last measurements of the present densities give $\rho^0_{\gamma\nu} \simeq 7.8 \times 10^{-34}(T_0/2.725)^4$ g/cm^3 and $\rho^0_M \simeq 1.88 \times 10^{-29}\Omega_M h^2$ g/cm^3. From Eq.(B.23) we obtain $\rho_{\gamma\nu}(t)/\rho_M(t) \propto a_0/a(t) = 1 + z$ one can deduce the radius/redshift/temperature of equal matter and radiation densities (a_{EQ}, z_{EQ}, T_{EQ}) with $T_0 = 2.725$ K and $\Omega_M h^2 \simeq 0.146$:

$$1 + z_{EQ} = a_0/a_{EQ} = 2.3 \times 10^3 \tag{B.24}$$

$$T_{EQ} = T_0(1 + z_{EQ}) = 0.77 \text{ eV} = 9000K \tag{B.25}$$

$$t_{EQ} = 1.4 \times 10^5 \text{ years} \tag{B.26}$$

One can also calculate directly T_{EQ} from $\frac{T_{EQ}}{T_0} = \frac{\Omega_m}{\Omega_{\gamma\nu}}$ with $T_0 = 2.75$ K, $\Omega_{\gamma\nu}h^2 = 4.15 \times 10^{-5}$ and $\Omega_m h^2 = 0.146$ gives $T_{EQ} = 9000$ K/ 0.81 eV/ 80 000 years.

B.3 Thermodynamical fundamental relations

$$\begin{aligned} g_*(today) &= 3.36; \quad g_{*s}(today) = 3.91 \\ \rho_R &= \frac{\pi^2}{30} g_* T^4 = 8.09 \times 10^{-34} \text{g cm}^{-3} \text{ today} \\ s &= \frac{2\pi^2}{45} g_{*s} T^3 \simeq 2970 \text{ cm}^{-3} \text{ today} \\ n_\gamma &= \frac{2\zeta(3)}{\pi^2} T^3 = 422 \text{ cm}^{-3} \text{ today} \end{aligned} \tag{B.27}$$

[2]We will note $\rho_{\gamma\nu}$ the energy density of relativistic particle (CMB + ν) and ρ_γ the energy density of the CMB

Average energy per particles:

$$\langle E\rangle_{E\gg m} \equiv \frac{\rho}{n} = \frac{\pi^2}{30\zeta(3)}T \simeq 2.701\ T \quad \text{(BOSE)} \tag{B.28}$$
$$\langle E\rangle_{E\gg m} = \frac{7\pi^4}{180\zeta(3)}T \simeq 3.151\ T \quad \text{(FERMI)}$$
$$\langle E\rangle_{E\ll m} = m + \frac{3}{2}\ T$$

with $\zeta(3) \simeq 1.20206$.

$$\frac{n_B}{n_\gamma} = \eta = 2.68 \times 10^{-8}\ \Omega_B h^2 \tag{B.29}$$

B.4 An exemple : computing n_{eq}

In the Boltzmann approximation ($e^{\frac{E}{T}} \gg 1$), on can have an analytical solution of n_{eq} of a population of particles with g internal degrees of freedom

$$n_{eq} \sim g\int \frac{d^3p}{(2\pi)^3}e^{-\frac{E}{T}} = \frac{g}{2\pi^2}\int |p|EdEe^{-\frac{E}{T}} = \frac{g\ m^3}{2\pi^2}\int \sqrt{(E/m)^2-1}\left(\frac{E}{m}\right)d\left(\frac{E}{m}\right)e^{-\frac{E}{m}\frac{m}{T}}$$
$$= \frac{g\ m^3}{2\pi^2}\int \sqrt{z^2-1}\ zdze^{-zx} = \frac{g\ m^2\ T}{2\pi^2}K_2\left(\frac{m}{T}\right) \tag{B.30}$$

with $x = m/T$

B.4.1 Classical thermodynamic : the Laplace law

The law that is used in thermodynamics of the early Universe, are the ones corresponding to a gas of particle transforming adiabatically. We demonstrate in this section the Laplace law $PV^\gamma =$ constant, valid in such transformations.
The first principle of thermodynamics affirms that " the change in the internal energy dU of a closed system is equal to the amount of heat δQ supplied to the system, plus the amount of work δW received by the system from its surroundings (or equivalently minus the amount of work done by the system on its surroundings)". In the case of a thermodynamical system, only the internal energy varies

$$dU = \delta W + \delta Q$$

The mechanical work is the product of the external pressure P multiplied by the volume V, or, if we consider the work *received* by the system, one should write

$$\delta W = -PdV$$

Moreover, an adiabatic process is generated without exchange of heat, so $\delta Q = 0$. One then obtains

$$dU = -PdV$$

If one considers the enthalpy of the system, $H = U + PV$, one can write for an adiabatic transformation

$$dH = VdP$$

Supposing that the gas is perfect, the enthalpy and internal energy depends only on the temperature

$$dU = C_v dT, \qquad dH = C_p dT$$

where C_v and C_p (in units of Joule per Kelvin) are respectively the specific heat at constant volume and pressure and T the temperature[3]. We can then deduce

$$\begin{aligned} C_p dT &= VdP \\ C_v dT &= -PdV \end{aligned}$$

which gives after combining both equations

$$\gamma \frac{dV}{V} + \frac{dP}{P} = 0, \qquad \Rightarrow \qquad PV^\gamma = \text{constant} \tag{B.31}$$

with $\gamma = \frac{C_p}{C_v}$. This is the Laplace's law. Another interesting relation is between internal energy and pressure. If one define α by $P = \alpha u_k$, u_k being the density of internal energy (kinetic one in the absence of other sources and in adiabatic processes), one can write $PV = \alpha U$. Combining with Eq.(B.31) and remembering the $dU = -PdV$ one obtains

$$\alpha dU = PdV + VdP = PdV - \gamma PdV = PdV(1-\gamma) = -\alpha PdV \quad \Rightarrow \quad \alpha = \gamma - 1 \tag{B.32}$$

B.5 Tooling with Math

B.5.1 Function $\Gamma(z)$

Definition, propriety

$$\begin{aligned} \Gamma(z) &= \int_0^\infty t^{z-1} e^{-t} dt \\ \Gamma(z+1) &= z\Gamma(z) \\ \Gamma(n) &= (n-1)! \end{aligned} \tag{B.33}$$

[3]A specific heat, or heat capacity is the heat required to raise a unit mass of the material by one degree. This can be done at constant volume or at constant pressure and the corresponding symbols are C_v and C_p.

Some values

$$
\begin{aligned}
&\Gamma(-3/2) = \frac{4}{3}\sqrt{\pi} \approx 2.36 \\
&\Gamma(-1/2) = -2\sqrt{\pi} \approx -3.54 \\
&\Gamma(1/2) = \sqrt{\pi} \approx 1.77 \\
&\Gamma(1) = 0! = 1 \\
&\Gamma(3/2) = \frac{1}{2}\sqrt{\pi} \approx 0.89 \\
&\Gamma(2) = 1! = 1 \\
&\Gamma(5/2) = \frac{3}{4}\sqrt{\pi} \approx 1.33 \\
&\Gamma(3) = 2! = 2 \\
&\Gamma(7/2) = \frac{15}{8}\sqrt{\pi} \approx 3.32 \\
&\Gamma(4) = 3! = 6
\end{aligned}
\tag{B.34}
$$

B.5.2 The Riemann zeta function $\zeta(z)$

Definition, propriety

$$
\zeta(z) = \sum_{1}^{\infty} n^{-z} = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + \frac{1}{4^z} + \frac{1}{5^z} + ... \tag{B.35}
$$

Some values

$$
\begin{aligned}
&\zeta(0) = -\frac{1}{2} \\
&\zeta(1/2) \approx -1.46 \\
&\zeta(1) = \infty \\
&\zeta(3/2) \approx 2.61 \\
&\zeta(2) \approx 1.64 \\
&\zeta(3) \approx 1.2 \\
&\zeta(4) \approx 1.08
\end{aligned}
\tag{B.36}
$$

B.5.3 Modified Bessel Function of the 2nd kind $K_n(z)$

Definition, propriety

$$
\begin{aligned}
& z^2 K_n''(z) + z K_n'(z) - (z^2 + n^2) K_n(z) = 0 \\
& K_n(z) = \frac{\sqrt{\pi} z^n}{2^n \Gamma(n + \frac{1}{2})} \int_1^\infty e^{-zt} (t^2 - 1)^{n - \frac{1}{2}} dt = \frac{\Gamma(n + \frac{1}{2})(2z)^n}{\sqrt{\pi}} \int_0^\infty \frac{\cos t dt}{(t^2 + z^2)^{n + \frac{1}{2}}} \\
& \frac{n}{z} K_n(z) - K_n'(z) = K_{n+1}(z) \\
& K_0(z) \sim -\log z \\
& K_1(z) = z \int_1^\infty e^{-zt} \sqrt{t^2 - 1} dt \\
& K_2(z) = \frac{z^2}{3} \int_1^\infty e^{-zt} (t^2 - 1)^{3/2} dt = K_1'(z) - \frac{1}{z} K_1(z) = z \int_1^\infty t e^{-zt} \sqrt{t^2 - 1} dt \\
& z \to \infty, \quad K_n(z) \simeq \sqrt{\frac{\pi}{2z}} e^{-z}
\end{aligned}
\tag{B.37}
$$

Some values

$$
\begin{aligned}
& K_1(0.01) = 99.97; \quad K_1(0.1) = 9.84; \quad K_1(1) = 0.60; \quad K_1(10) = 1.9 \times 10^{-5}; \quad K_1(100) = 4.7 \times 10^{-45} \\
& K_2(0.01) = 20000; \quad K_2(0.1) = 200; \quad K_2(1) = 1.63; \quad K_2(10) = 2. \times 10^{-5}; \quad K_2(100) = 4.7 \times 10^{-45} \\
& \int_0^\infty z^2 K_1(z) dz = 2; \quad \int_0^\infty z^4 K_1(z) dz = 16; \quad \int_0^\infty z^3 K_2(z) dz = 8; \quad \int_0^\infty z^5 K_2(z) dz = 96;
\end{aligned}
$$

Some approximations

For $x \gtrsim 10$, we can write

$$
K_1(x) \simeq \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-x} \left[1 + \frac{3}{8x}\left(1 - \frac{5}{16x}\left(1 - \frac{21}{24x}\right)\right)\right] \tag{B.38}
$$

B.5.4 Useful integrals

$$\int e^{-cx^2} = \sqrt{\frac{\pi}{4c}}\mathrm{erf}(\sqrt{c}x) \quad \text{erf being the error function}$$
$$\int xe^{-cx^2} = -\frac{1}{2c}e^{-cx^2}$$
$$\int x^2e^{-cx^2} = \sqrt{\frac{\pi}{16c^3}}\mathrm{erf}(\sqrt{c}x) - \frac{1}{2c}xe^{-cx^2}$$
$$\int x^3e^{-cx^2} = -\frac{x^2}{2c}e^{-cx^2} - \frac{1}{2c^2}e^{-cx^2}$$
$$\int x^4e^{-cx^2} = \frac{3}{8}\sqrt{\frac{\pi}{c^5}}\mathrm{erf}(\sqrt{c}x) - \frac{e^{-cx^2}}{4c^2}[3x + 2cx^3]$$
$$\int x^5e^{-cx^2} = -\frac{e^{-cx^2}}{c^3}\left[1 + cx^2 + \frac{c^2}{2}x^4\right]$$
$$\int x^6e^{-cx^2} = \frac{15}{16}\sqrt{\frac{\pi}{c^7}}\mathrm{erf}(\sqrt{c}x) - e^{-cx^2}\left[\frac{15}{8c^3}x + \frac{5}{4c^2}x^3 + \frac{1}{2c}x^5\right]$$
$$\int \frac{e^{cx}}{x}dx = \ln|x| + \sum_{n=1}^{\infty}\frac{(cx)^n}{n.n!}$$
$$\int \frac{e^{cx}}{x^n} = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}} + c\int\frac{e^{cx}}{x^{n-1}}\right) \quad (\text{for n} \neq 1)$$
$$\int_0^{+\infty} e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$
$$\int_0^{+\infty} x^2e^{-ax^2}dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$
$$\int_0^{+\infty} x^4e^{-ax^2}dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}$$
$$\int_0^{+\infty} x^ne^{-ax^2}dx = \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}} \quad (\text{n} = 2\text{k}, \text{k integer})$$
$$\int_1^{\infty} t(t^2-1)^{1/2}e^{-zt}dt = \frac{1}{z}K_2(z)$$
$$\int_0^{\infty} dx\left(\frac{x^n}{e^x - \delta}\right) = \Gamma(n+1)\zeta(n+1)y(\delta) \tag{B.39}$$

with $y(\delta) = 1$ if $\delta = 1$ and $1 - \frac{1}{2^n}$ if $\delta = -1$. $n!!$ is the double factorial defined as $n!! = n \times (n-2) \times (n-4) \times ..$
and finally,

$$\int_0^{\infty}\frac{dx}{e^x+1} = \log(2)\ ; \qquad \int_0^{\infty}\frac{dx}{e^x-1} = \infty \tag{B.40}$$

$$\int\frac{1}{ae^{\lambda x}+b}dx = \frac{x}{b} - \frac{1}{b\lambda}\ln\left(ae^{\lambda x}+b\right) \tag{B.41}$$

Euler-Masheroni constant γ

$$\gamma = 0.577 \tag{B.42}$$

Gauss error function erf

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{B.43}$$

B.5.5 Change of variables in multidimensional integrations

Appendix C

Particle Physics

C.1 Feynman Rules

In this section we collect together all of the rules for the calculation of relativistic cross sections and decay rates. The rules fall into two parts. There are rules for the calculation of the cross section and decay rates from the relativistic scattering matrix, $\mathcal{M}$, and then there are rules for calculating $\mathcal{M}$ in a given theory. The former are quite general, but the latter, referred to as the Feynman rules, depend on the specific theory.

C.1.1 Decay rates and cross sections

The differential n-body decay rate, dW_n, for a particle with energy E is obtained from the following factors:

- a factor of $(2\pi)^4\delta^4(p_f - p_i)$where p_f is the total four-momentum of the n decay products and p_i is the four-momentum of the decaying particle,
- a factor
$$\frac{d^3k_i}{(2\pi)^3 E_{ki}}$$
for each particle in the final state where k_i and E_{ki} are the momentum and energy of the ith particle
- a factor $\frac{1}{2E}$ for the initial partcle which is decaying,
- and the absolute square of the $\mathcal{M}$–matrix

The differential rate is then (we let the reader go through the section (C.7) for a more detailed analysis of the cross section computation, inspired by the Fermi's golden rule.

$$dW_n = (2\pi)^4\delta^4(p_f - p_i)\frac{1}{2E}\prod_{i=1}^{i=n}\frac{d^3k_i}{(2\pi)^3 2E_{ki}} \tag{C.1}$$

the total decay rate is obtained by integrating Eq.(C.1) over all outgoing momenta and summing over all outgoing spins

$$W = \int \sum_{\text{spins}} dW_n. \tag{C.2}$$

The differential cross section for the production of n particles (elastic scattering occurs when $n = 2$ *and* the final particles are identical to the initial ones) is obtained from the following factors:

- a factor of $(2\pi)^4\delta^4(p_f - p_i)$where p_f is the total four-momentum of the n final states and p_i is the four-momentum of the two initial particles, particle,
- a factor
$$\frac{d^3k_i}{(2\pi)^3 E_{ki}}$$
for each particle in the final state where k_i and E_{ki} are the momentum and energy of the ith particle
- a factor $\frac{1}{4EE'}$ where E and E' are the energies of the two particles in the initial states,
- a factor of $1/v$ where v is the flux, or relative velocity of the two (colinear) colliding particles, equal to
$$v = \frac{p}{E} + \frac{p'}{E'} \tag{C.3}$$
where p and p' are the magnitudes of their momenta, and
- the absolute square of the $\mathcal{M}$–matrix

The differential cross section is therefore

$$d\sigma = (2\pi)^4\delta^4(p_f - p_i)\frac{1}{4EE'v}\prod_{i=1}^{i=n}\frac{d^3k_i}{(2\pi)^3 2E_{ki}}|\mathcal{M}|^2 \quad . \tag{C.4}$$

The unpolarized cross section for scattering into some final state in the phase volume $\Delta\Omega$ is therefore obtained by integrating Eq.(C.4) over all outgoing momenta in $\Delta\Omega$, summing over all final spins and *averaging* over initial spins:

$$\Delta\sigma = \int_{\Delta\Omega}\frac{1}{(2s+1)(2s'+1)}\sum_{\text{spins}} d\sigma \quad , \tag{C.5}$$

where s and s' are the spins of the initial particles.

Finally, in calculating both decay rates and cross sections, for *each* set of m identical particles in the final states, the integral over momenta must be*either* be divided by $m!$ or limited to the restricted cone $\theta_1 < \theta_2 < ... < \theta_m$.

C.2 Feynamn rules

C.2.1 General rules

The Feynman rules for calculation of the $\mathcal{M}$–matrix depend of the theory used to do the calculation. The basic rules are given first, and then the forms required for specific theories. Any diagram will either be a *tree* diagram (with no loops) or will have one or more closed *loops*.

- The diagrams consist of *lines* and *vertices*.
- Each *internal* line represents the propagation of a particular particle from one space-time point to another, and the vertices are the points in spacetime where particles are created or destroyed, as described by the interaction Lagrangian of the theory.
- Label the momenta of each external particle, and use energy-momentum conservation to determine the four-momentum of each internal line. *Tree* diagrams have no closed loops, and each internal momentum can be fixed in terms of the external momenta. *Loop* diagrams have momenta which cannot be uniquely specified, and these must be integrated over. There will be one undetermined four-momentum for *each* loop.

The Feynman rules tell how to associate a number with each Feynman diagram. There are several basic rules from which the number is constructed :

Rule 0

A factor of i.

Rule 1

An operator for each vertex, the precise form of which depends on the theory and the particular particles involved.

Rule 2

A propagator for each internal line with four-momentum k , the precise form of which depends on the particle propagating. For spin zero bosons with isospin indices i, j , for fermions with Dirac indices α, β, and for photons or massive vector bosons with polarization indices μ , ν, the forms are:

Spin 0 : $\Delta_{ij}(k) = \frac{i\delta_{ij}}{m^2-k^2-i\epsilon}$

Spin 1/2 : $S_{\alpha\beta}(k) = \frac{i(m+\not{k})_{\alpha\beta}}{m^2-k^2-i\epsilon}$

photon or gluon : $\Delta_{\mu\nu}(k) = \frac{-i}{-k^2 - i\epsilon}\left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}(1-\alpha)\right]$

vector boson : $\Delta_{\mu\nu}(k) = \frac{-i[\eta_{\mu\nu} - k_\mu k_\nu/m^2]}{m^2 - k^2 - i\epsilon}$

In all cases, k is constrained by momentum conservation, and for the photon or gluon, α is the gauge parameter (which could be unity).

Rule 3

For fermions, assemble the incoming fermion spinors, vertex operators, propagators, and outgoing fermion spinors in *order* along each fermion line to make a well-formed matrix element. In particular:

- mutliply from the $left$ by $\bar{u}(p_-, s_-)$ for each *outgoing* $fermion$ with momentum p_- and spin s_-.
- multiply from the $right$ by $u(k_-, s_-)$ for each *incoming* $fermion$ with momentum k_- and spin s_-.
- multiply from the $right$ by $v(p_+, s_+)$ for each *outgoing* $antifermions$ with momentum p_+ and spin s_+.
- multiply from the $left$ by $\bar{v}(k_+, s_+)$ for each *incoming* $antifermion$ with momentum k_+ and spin s_+.

for photons and vector bosons, construct well–formed vector products by saturating any free vector polarization indices μ on current operators γ^μ by :

- multiplying by ϵ^*_μ for each *outgoing particle* with polarization index μ.
- multiplying by ϵ_μ for each *incoming particle* with polarization index μ.

Rule 4

Symmetrize between identical bosons in the initial or final state, antisymmetrize between identical fermions in the initial or final state.

Rule 5

Integrate over each internal four–momentum k not fixed by energy–momentum conservation with a weight

$$\int \frac{d^4k}{(2\pi)^4}$$

Rule 6

For each closed fermion loop, a minus sign

Rule 7

Multiply the proper symmetry factor, which is $\frac{1}{2}$ for bubbles with *two identical neutral bosons* and $\frac{1}{3!}$ for bubbles with *three identical neutral bosons*.

Rule 8

Renormalize with a factor of $\sqrt{Z_2}$ for each external fermion and $\sqrt{Z_3}$ for each external photon (this factors are unity in all lowest order processes).

Rule 9

For each particle with a mass which could be shifted by self–interactions, a mass counter-term $i\delta m$ is added to remove the mass shift (this is zero to lowest order).

C.2.2 Majorana rules

In the case of Majorana fermions, the Feynman rules has to be adapted. Indeed, Majorana flows violates the fermion number, and we cannot compute the amplitude exactly the same way. We compute explicit exemples in section (C.7.8). Please, read also the section (C.3.6) before this one to recall the clear definition of a Majorana fermion and its Lagrangian (which differs from a Dirac one by a factor 1/2). There are so many factors 2 in the calculation, that it is easy to be lost, and computation should be made carefully to avoid double counting.
Our point of departure will be the Lagrangian (C.47). We will take the specific exemple of the Majorana coupling to a Higgs–like field ϕ. The Majorana part is

$$\frac{1}{2}y_\lambda \phi \bar{\lambda}\lambda \tag{C.6}$$

where we took $\Gamma = \mathbb{1}$ but the reasoning is valid for any kind of coupling structure. The Feynman rules, in Majorana case, is like for identical particles in the final states: one should multiply by 2 the coupling which appears in the lagrangian (so a factor 4 in the amplitude square) and divide by 2 the physical process (antisymmetrisation because 2 identical fermions in the final state) We will explain the former factor "2" in detail in this section.
Indeed, to compute an amplitude, one needs to develop λ into the creation and destruction operators[1] $b^\dagger$ and b.

$$\lambda = \int \frac{d^3k}{(2\pi)^3 2E}(b(k)ue^{-ikx} + b(k)^\dagger ve^{ikx}) \tag{C.7}$$

[1]In the case of a Dirac fermions, $b^\dagger$ should be replaced by $d^\dagger$, because a Dirac fermion ψ do not follows $\psi^C = \psi$

with u and v the dirac spinors studied in section (C.3.2). If one look for any process, the fact of having two "b's" in the development of λ will generate a factor 2 of symmetry. This funademtally comes from the fact that one cannot trace a fermion flow with Majorana fermion and "entering" or "exciting" flows can combine together.

Let's take a concrete example. A particle ϕ decaying into two λ's

$$\langle\lambda(p_1)\lambda(p_2)|\mathcal{L}|\phi\rangle = \langle 0|b(p_1)b(p_2)\ \ \frac{1}{2}y_\lambda\phi\lambda\lambda\ |0\rangle = \frac{y_\lambda}{2}[\bar{u}(p_1)v(p_2)+\bar{u}(p_2)v(p_1)] = y_\lambda\bar{u}(p_1)v(p_2)$$

where we used the identity for Majorana particles in the last equality, $u = C\bar{v}^T$ and $v = C\bar{u}^T$. As one notices, the factor $\frac{1}{2}$ in front of y_λ disappear, and we recover *the same* Feynman rule than in the case of the Dirac Fermion. That the fundamental point, and main result of this section. For a concrete exemple, have a look at section (C.7.8) where we computed the decay of a scalar into Majorana and Dirac particles.

Another interpretation

One can find another interpretation in the literature in [747]. This interpretation released on the fermionic fluxes. Indeed, whereas for general fermion fields χ, a Lagrangian can be written

$$\bar{\chi}\Gamma\chi = \bar{\tilde{\chi}}\Gamma'\tilde{\chi} \tag{C.8}$$

where Γ is the coupling matrix, $\tilde{\chi} = C\bar{\chi}^T$ (C being the charge conjugate operator (see section C.3.5) and $\Gamma' = C\Gamma^T C^{-1}$. This means that the *same term in the Lagrangian* will generate a process with $particle(p_1) - antiparticle(p_2)$ in the final state, and the symmetric one, $antiparticle(p_1) - particle(p_2)$ p_1 and p_2 being their respective momentum. Both final states are of course *different*. For a Majorana final state $\tilde{\chi} = \chi$. Therefore, if both $\chi's$ are Majorana fermions, $\Gamma = \Gamma'$. In other way, there is no flow of fermions, and one should compute matrix elements with Majorana states without distinguishing the possible flows of particles/antiparticles. This is illustrated more clearly in Fig.(C.2.2) where we show he Feynman rules for fermionic vertices with orientation. As we can see, the two figures on top are the same, and one should not add them in a computation as it will be double counting. On the other hand, the two last figures represent the Dirac case, and two different states of particle/antiparticle with exchanged momentum. Only one of this diagram should also contribute to the process. In conclusion, for the computation of the matrix element, in the Majorana as in the Dirac case, one should count *only* one flow, as in one case (Dirac) the other flow correspond to another final state with exchanged momentum, whereas in Majorana case, that would be double-counting.

Let's be more concrete. In the computation in a rate, where two Majorana fermions are in the final state, that is important to add a symmetry factor in the final result. Let's consider for instance the simplest example of the decay of a scalar ϕ into two Majorana fermions λ compared to a (Dirac) pair of electron positron final state of momentum p_1 and p_2. The Lagrangian

Figure C.1: The Feynman rules for fermionic vertices with orientation (thin arrow

$$\mathcal{L} = y_\phi^e \ \phi\bar{e}e + y_\phi^\lambda \phi\bar{\lambda}\lambda \tag{C.9}$$

gives respectively the width (see Eq.C.114) $\Gamma_\phi^i = \frac{|\mathcal{M}|^2}{16\pi \times sym \ m_\phi}$

$$\Gamma_\phi^{e^+e^-} = \frac{(y_\phi^e)^2 \ m_\phi}{8\pi}, \quad \Gamma_\phi^{\lambda\lambda} = \frac{(y_\phi^\lambda)^2 \ m_\phi}{16\pi} \tag{C.10}$$

where we neglected the final state masses. Indeed, in the Dirac (electron) case, the amplitude is $\mathcal{M} = -iy_\phi \bar{u}(p_1)v(p_2)$ wich implies $|\mathcal{M}|^2 = 4p_1.p_2 = 2m_\phi^2$, and no symmetry factor sym is present for Dirac final state because both particles are different. In the Majorana case λ, as we explained above, only one flow should be considered to avoid double-counting. Then, $|\mathcal{M}|^2 = 2p_1.p_2 = 2m_\phi$, exactly as in the Dirac case.

The $\left(\frac{1}{2}\right)$ symmetry factor sym between these be understood in Fig.(**??**). For a giving final state angle, the e^+e^- system is definite, whereas there are two possibilities for the $\lambda\lambda$ final state : both can have p_1 and/or p_1. There is then a double counting once we integrate on the total solid angle, and one should symmetrize it to take it into account, multiplying the final result by $\left(\frac{1}{2}\right)$

C.2.3 Standard Model couplings

We show in the following diagrams the Feynman rules in the Standard Model context and its simplest extension (Feynman rules = $i\mathcal{L}\times$ symmetry factors). In the case of the anomalies mediated trivectorial couplings, we are based on the Lagrangian Eq.(11.1)

$$\mathcal{L}_{CS} = \alpha_1 \ \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu F^Y_{\rho\sigma} + \alpha_2 \ \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu F'_{\rho\sigma} \ , \tag{C.11}$$

which gives the interactions Eq.(11.2) :

$$\begin{aligned}
&\Gamma^{\mu\nu\sigma}_{Z'ZZ}(p_3;p_1,p_2) = 2\alpha_1 s_W \ \epsilon^{\mu\nu\rho\sigma}(p_1-p_2)_\rho \ , \\
&\Gamma^{\mu\nu\sigma}_{Z'Z\gamma}(p_3;p_1,p_2) = 2\alpha_1 c_W \ \epsilon^{\mu\nu\rho\sigma}(p_2)_\rho \ , \\
&\Gamma^{\mu\nu\sigma}_{Z'ZZ'}(p_3;p_1,p_2) = 2\alpha_2 \ \epsilon^{\mu\nu\rho\sigma}(p_2-p_3)_\rho \ ,
\end{aligned} \tag{C.12}$$

with the obvious notation $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$. Notice the decomposition $Y_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$ and the symmetrization for the identical final state $Z'_\mu Z_\nu(p_1) Z_\sigma(p_2)\epsilon^{\mu\nu\rho\sigma}(p_2)_\rho = \frac{1}{2} Z'_\mu Z_\nu(p_1) Z_\sigma(p_2)\epsilon^{\mu\nu\rho\sigma}(p_2-p_1)_\rho$multiply by the symmetry factor (2)

C.3 Diracology

C.3.1 Matrices

Pauli Matrices

$iq\ e\ \gamma_\mu$

$iq\ e(p+p')_\mu$

$-2ie^2q^2\eta_{\mu\nu}$

$$\frac{ig}{4\cos\theta_W}\gamma_\mu\Big[(-4q\sin^2\theta_W \pm 1) \mp \gamma^5\Big] \quad (T_3 = +\frac{1}{2}/-\frac{1}{2})$$

$$\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

$$-ie\Big[\eta_{\mu\nu}(p_1-p_2)_\sigma + \eta_{\nu\sigma}(p_2-p_3)_\mu + \eta_{\sigma\mu}(p_3-p_1)_\nu\Big]$$

$$-ig\cos\theta_W\Big[\eta_{\mu\nu}(p_1-p_2)_\sigma + \eta_{\nu\sigma}(p_2-p_3)_\mu + \eta_{\sigma\mu}(p_3-p_1)_\nu\Big]$$

$$-i(g\cos\theta_W)^2\Big[2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\Big] \qquad -ig^2\sin\theta_W\cos\theta_W\Big[2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\Big]$$

$$-ie^2\Big[2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\Big] \qquad -ig^2\Big[2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\Big]$$

$$ig\ M_W\ \eta_{\mu\nu}$$

$$\frac{ig}{\cos^2\theta_W} M_W\ \eta_{\mu\nu}$$

$$\frac{i}{2}\left(\frac{g}{\cos\theta_W}\right)^2 \eta_{\mu\nu}$$

$$\frac{i}{2} g^2 \eta_{\mu\nu}$$

$$-\frac{3i}{2}\left(\frac{gM_H}{M_W}\right)^2$$

$$-\frac{3i}{4}\left(\frac{gM_H}{M_W}\right)^2$$

$$-i\frac{Y_f}{\sqrt{2}} = -i\frac{m_f}{v} = -i\frac{gm_f}{2M_W}$$

$$m_f = Y_f\langle H\rangle = Y_f\frac{v}{\sqrt{2}}$$

$$2i\epsilon^{\mu\nu\rho\sigma}(p_2)_\rho = 2i\cos\theta_W(p_2^A)_\rho - 2i\sin\theta_W(p_2^Z - p_1^Z)_\rho$$

$$2i\epsilon^{\mu\nu\rho\sigma}(p_2 - p_3)_\rho$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{C.13}$$

Dirac matrices

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} I_4; \quad \gamma_\mu = \eta_{\mu\nu}\gamma^\nu \quad \{\gamma^5, \gamma^\mu\} = 0, \quad \gamma^0\gamma^\dagger_\mu\gamma^0 = \gamma_\mu \tag{C.14}$$

we name the operator $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$.
with $\eta^{\mu\nu}$ the Minkowski metric with the signature (+,-,-,-) and I_4 the 4×4 identity matrix and

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{C.15}$$

which gives in explicit form,

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$
$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0 \end{pmatrix} \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{C.16}$$

C.3.2 Dirac equation

The Dirac equation for a spinor ψ of mass m and energy $E^2 = p^2 + m^2$ should be written

$$(i\gamma^\mu\partial_\mu \ - \ m)|\Psi\rangle \ = (i\gamma^0\partial_t - i\gamma^i\partial_{x_i} \ - \ m) \ |\Psi\rangle \ = \ 0 \tag{C.17}$$

The solution of the equation are then :

$$|\Psi_1\rangle = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ 0 \\ \frac{p_3}{E+m} \\ \frac{p_1+ip_2}{E+m} \end{pmatrix} e^{-ipx} = u_1 e^{-ipx}; \qquad |\Psi_2\rangle = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 0 \\ 1 \\ \frac{p_1-ip_2}{E+m} \\ \frac{-p_3}{E+m} \end{pmatrix} e^{-ipx} = u_2 e^{-ipx};$$
$$|\Psi_3\rangle = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \frac{p_3}{E+m} \\ \frac{p_1+ip_2}{E+m} \\ 1 \\ 0 \end{pmatrix} e^{ipx} = v_2 e^{ipx}; \qquad |\Psi_4\rangle = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \frac{p_1-ip_2}{E+m} \\ \frac{-p_3}{E+m} \\ 0 \\ 1 \end{pmatrix} e^{ipx} = v_1 e^{ipx}; \tag{C.18}$$

C.3.3 The spin matrix

The spin matrix Σ_i can be written

$$\Sigma_i = \frac{\hbar}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \rightarrow \Sigma_1 = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}; \quad \Sigma_2 = \frac{\hbar}{2}\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix};$$
$$\Sigma_3 = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \tag{C.19}$$

we let the reader check that, in the case of a particle at rest, $\Sigma_3|\Psi_{1,3}\rangle = \frac{\hbar}{2}|\Psi_{1,3}\rangle$ and $\Sigma_3|\Psi_{2,4}\rangle = -\frac{\hbar}{2}|\Psi_{2,4}\rangle$. For $p_3 \neq 0$ the result is still valid as a translation in z axis does not change the projection of the spin momentum on the z axis. However, after a translation on the x or y axis, a p_1 and p_2 component appears (relativistic boost). One can also give another useful expression for the spin matrice:

$$\langle\Psi^\dagger|\begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}|\Psi\rangle = \langle\Psi^\dagger|\gamma^0\gamma^i\gamma^5|\Psi\rangle = \langle\bar{\Psi}|\gamma^i\gamma^5|\Psi\rangle \tag{C.20}$$

where we have used

$$\bar{\psi} = \psi^\dagger\gamma^0. \tag{C.21}$$

Some useful relations are sometimes used to simplify the calculation of amplitudes. From the Dirac equation we can write

$$\not{p}_1 u(p_1) = mu(p_1); \quad \not{p}_1 v(p_1) = -mv(p_1); \quad \bar{u}(p_2)\not{p}_2 = m\bar{u}(p_2); \quad \bar{v}(p_2)\not{p}_2 = -m\bar{v}(p_2); \tag{C.22}$$

The helicity operators $P_L = \frac{1-\gamma^5}{2}$ and $P_R = \frac{1+\gamma^5}{2}$ can then be written

$$P_R = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad P_L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{C.23}$$

We can then decompose a dirac spinor into its helicity (eigenvalues) component,

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \tag{C.24}$$

C.3.4 Parity operator

The parity operator, $\mathcal{P}$, inverts all space coordinates used in the description of a physical process. Consider for instance a scalar wave function $\psi(x, y, z, t)$. Performing the parity operation on this wave function will transform it to $\psi(-x, -y, -z, t)$, or

$$\mathcal{P}\psi(x, y, z, t) = \psi(-x, -y, -z, t).$$

The parity transformation can be viewed as a mirroring with respect to a plane (for instance $z \to -z$) followed by a rotation around an axis perpendicular to the plane (the $z-$axis). As angular momentum is conserved, physics will be invariant under the rotation and so the parity operation tests for invariance to mirroring with respect to a plane of arbitrary orientation. Parity conservation or $\mathcal{P}-$symmetry implies that any physical process will proceed identically, when viewed in mirror image. this sounds rather natural. After all, we would not expect a dice for instance to produce a different distribution of numbers if one swaps the position of the one and the six of the dice.

It is important to underline that we discuss in this section the *intrinsic* parity associated with each particle, and not the one associated with the orbital wave function of the particle. In other words, **the parity of the wave function of a particle or a system of particles is the product of the orbital parity times the product of the intrinsic parities of the particles involved.**
In interactions where parity is conserved (electromagnetic and strong) we can use this to compute selection rules for the various reactions. We do this aided by the following two ideas :

- Under parity, the orbital wave function will change by $(-1)^L$ where L is the angular momentum quantum number. This is because the parity operator $x_i \to -x_i$, changes $\theta \to \theta - \pi$ and $\phi \to \phi + \pi$. Due to this change in the angle, the orbital wave function, which always involves spherical harmonics (central potential) changes as $(-1)^L$. This is actually the reason that the photon has negative parity, and is referred to as an 1^- state. Photons are emitted via atomic dipole transitions where $\Delta L = \pm 1$. Hence, the atomic parity changes by (-1) during these transitions, and for the overall parity of the system (atom + photon) to be conserved (electromagnetic interactions) we must have that the photon has negative parity.

- The overall wave function must be symmetric for systems of bosons and antisymmetric for systems of fermions.

In summary, the overall parity of the wave function of a set of particles is given by

$$P = (-1)^L \times P_1 \times P_2 \times ... \times P_s \tag{C.25}$$

where L is the total angular momentum and $P_1, P_2, .., P_s$ are the intrinsic parities of the particles involved.

Fermion case

For fermions fields respecting the Dirac equation, imposing that the laws of physics are invariant under the change of space coordinates : $x_i \to \tilde{x}_i = -x_i$ Eq.(C.17) transforms as

$$\begin{aligned}&(i\gamma^0\partial_t + i\gamma^i\partial_{x_i} - m)|\Psi(-x)\rangle = 0 \quad \Rightarrow \quad \gamma^0(i\gamma^0\partial_t + i\gamma^i\partial_{x_i} - m)|\Psi(-x)\rangle = 0 \\ &\Rightarrow (i\gamma^0\partial_t - i\gamma^i\partial_{x_i} - m)\gamma^0|\Psi(-x)\rangle = 0 \end{aligned} \tag{C.26}$$

which means that under parity $|\Psi(x)\rangle \to |\Psi(-x)\rangle = e^{i\phi}\ \gamma^0|\Psi\rangle = \mathcal{P}|\Psi\rangle$, $e^{i\phi}$ being present as it is a general unitarity transformation:

$$|\Psi(x)\rangle \quad \to_{\mathcal{P}} \quad |\Psi(-x)\rangle = \mathcal{P}|\ \Psi(x)\rangle \ = \ e^{i\phi}\ \gamma^0|\Psi(x)\rangle; \qquad \mathcal{P} = e^{i\phi}\ \gamma^0 \tag{C.27}$$

Note that under the parity $\mathcal{P} = \gamma^0$, the spinor $\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}$ transforms into $\Psi = \begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix}$. This is easily understandable as the helicity (projection of the spin on the velocity vector) changes orientation in space under the parity operation: a left-handed particle becomes a right-handed particle. This was in fact the way taken by Wu for her experiment in 1956 [748] to show the violation of parity. Indeed, finding a process which produces a particle with a preferred helicity also proves that $\mathcal{P}-$symmetry is violated. Wu did it measuring the products of ^{60}Co decay.
We then can consider the parity of different bilinear forms that are useful to compute some process and evaluate there existence or not:

$$\begin{aligned}
&\text{Scalar :} \quad \bar{\Psi}\Psi \ \to \Psi(\gamma^0)^\dagger\gamma^0\Psi = +\bar{\Psi}\Psi, \quad \mathcal{P}_{\bar{\Psi}\Psi} = +1 \\
&\text{Pseudoscalar :} \quad \bar{\Psi}\gamma^5\Psi \ \to \Psi(\gamma^0)^\dagger\gamma^5\gamma^0\Psi = -\bar{\Psi}\gamma^5\Psi, \quad \mathcal{P}_{\bar{\Psi}\gamma^5\Psi} = -1 \\
&\text{Vector :} \quad \bar{\Psi}\gamma^i\Psi \ \to \Psi(\gamma^0)^\dagger\gamma^i\gamma^0\Psi = -\bar{\Psi}\gamma^i\Psi, \quad \mathcal{P}_{\bar{\Psi}\gamma^i\Psi} = -1; \ \ \mathcal{P}_{\bar{\Psi}\gamma^0\Psi} = +1 \\
&\text{Axial :} \quad \bar{\Psi}\gamma^i\gamma^5\Psi \ \to \Psi(\gamma^0)^\dagger\gamma^i\gamma^5\gamma^0\Psi = +\bar{\Psi}\gamma^i\gamma^5\Psi, \quad \mathcal{P}_{\bar{\Psi}\gamma^i\gamma^5\Psi} = +1; \ \ \mathcal{P}_{\bar{\Psi}\gamma^0\gamma^5\Psi} = -1 \\
&\text{Tensor :} \quad \bar{\Psi}\sigma_{ij}\Psi \ \to \Psi(\gamma^0)^\dagger\sigma_{ij}\gamma^0\Psi = +\bar{\Psi}\sigma_{ij}\Psi, \quad \mathcal{P}_{\bar{\Psi}\sigma_{ij}\Psi} = +1; \ \ \mathcal{P}_{\bar{\Psi}\sigma_{0j}\Psi} = -1
\end{aligned} \tag{C.28}$$

with $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu] = \gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu$. The equation (C.28) summarize all the nature of couplings one can find in any specific microscopic models.
Moreover, in view of the form of the matrix γ^0 we can notice that $\mathcal{P}|\Psi_{1,2}\rangle = +|\Psi_{1,2}\rangle$ and $\mathcal{P}|\Psi_{3,4}\rangle = -|\Psi_{3,4}\rangle$. In other words, the antiparticles states have opposite parity than the particle states.

Boson case

In the case of a bosonic particle, one names *scalar* the particle ϕ with an even parity, $[\phi(-x_i, t) = \phi(x_i, t)]$ and *pseudoscalar* the particle $\tilde{\phi}$ with an odd parity $[\tilde{\phi}(-x_i, t) = -\tilde{\phi}(x_i, t)]$. In the standard model, the two kinds of bosons exists: the Higgs h, which is a scalar particle, and the neutral pion π^0, a pseudo-scalar one. But how we can measure the parity of a particle?

C.3.5 The charge conjugate operator

The charge conjugaison operator C transform a particle ψ into its anti-particle ψ^c. We can find his expression as we did for the parity operator, directly from the Dirac equation. Indeed, in the presence of an electromagnetic field, one should add the interaction of the photon field A_μ with the fermion of charge e. the Dirac equation for the field ψ then becomes

$$i\gamma^\mu(\partial_\mu - ieA_\mu)\psi - m\psi = 0 \tag{C.29}$$

which imply after taking the complex conjugate of the equation and multiplying by γ^2

$$-i\gamma^2(\gamma^\mu)^*(\partial_\mu + ieA_\mu)\psi^* - \gamma^2 m\psi^* = 0 \quad \Rightarrow \quad i\gamma^\mu(\partial_\mu + ieA_\mu)(\gamma^2\psi^*) - m(\gamma^2\psi^*) = 0 \tag{C.30}$$

where we used $(\gamma^{0,1,3})^* = \gamma^{0,1,3}$ and $(\gamma^2)^* = -\gamma^2$. Defining ψ^c as $\psi^c = i\gamma^2\psi^* = C\bar{\psi}^T = i\gamma^2\gamma^0\bar{\psi}^T$ (the factor i is added to integrate a phase and to keep C as a real operator), we notice that the Dirac equation for ψ^c is exactly the same than for ψ, but with an opposite charge: ψ^c is the antiparticle of ψ, and C the charge conjugate operator. We can then find the expression of C in the space of 4-dimensional spinors $\psi = (\chi, \eta)^T$. The operation can be written

$$\psi^c = C\bar{\psi}^T = i\gamma^2\gamma^0\bar{\psi}^T = C\gamma_0^T\psi^* \tag{C.31}$$

where

$$C = i\gamma^2\gamma^0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \Rightarrow \quad \psi = \begin{pmatrix} \chi \\ \eta \end{pmatrix} \rightarrow_C \psi^c = C\bar{\psi}^T = \begin{pmatrix} i\sigma_2\eta^* \\ -i\sigma_2\chi^* \end{pmatrix} \tag{C.32}$$

we can then prove easily the following useful relations

$$C^{-1} = -C^*; \quad C^\top = -C; \quad C^{-1}\gamma_\mu C = -\gamma_\mu^\top; \quad C^\dagger = C^{-1} \tag{C.33}$$

A Majorana particle is a particle which charge conjugate is equal to itself, $\psi^c = \psi$. We can then write

$$\psi = \begin{pmatrix} \chi \\ \eta \end{pmatrix} \quad \Rightarrow \quad \psi^c = i\gamma^2\gamma^0\bar{\psi}^T \begin{pmatrix} i\sigma_2\eta^* \\ -i\sigma_2\chi^* \end{pmatrix}, \quad \psi = \psi^c \quad \Rightarrow \quad \psi = \begin{pmatrix} \chi \\ -i\sigma_2\chi^* \end{pmatrix} \tag{C.34}$$

which is the generic form for a Majorana particle. Have a look at section (C.3.6) for a more detailed definition of a Majorana fermion in the context of Dirac equation and section (C.2.2) for the Feynmann rules with Majorana particles. Some operators are suppressed for Majorana particles: For instance, if $\psi^c = \psi$ we can write

$$\begin{aligned} \bar{\psi}^c\gamma^\mu\psi^c &= \psi^\top C^\dagger\gamma^\mu\gamma^0 C\psi^* = \psi^\top C^{-1}\gamma^\mu\gamma^0 C\psi^* = \psi^\top(\gamma^\mu)^\top(\gamma^0)^\top\psi^* \\ &= (\psi^\top(\gamma^\mu)^\top(\gamma^0)^\top\psi^*)^\top = -\bar{\psi}\gamma^\mu\psi \quad \Rightarrow \quad \bar{\psi}\gamma^\mu\psi = \frac{1}{2}\left(\bar{\psi}\gamma^\mu\psi + \bar{\psi}^c\gamma^\mu\psi^c\right) = 0 \end{aligned} \tag{C.35}$$

the negative sign is coming for the exchange of two fermions. As a consequence, we can deduce that a Majorana dark matter does not couple to a Z_μ (as the dark matter has no T^3 charges, no coupling to γ^5) or Z'_μ boson without axial coupling as the sum of both contributions will cancel. On the other hand if a majorana dark matter has an axial coupling to Z or Z' the annihilation rate is not null. We can also easily show with the same method that

$$\bar{\psi}^c\psi^c = \bar{\psi}\psi \tag{C.36}$$

showing that a Majorana dark matter can have a coupling to the Higgs boson.
Another useful relation (used in the section (5.6) concerning the neutrino is (we let the reader to prove it)

$$\bar{\psi}_L\psi_R = \bar{\psi}^c_R\psi^c_L \tag{C.37}$$

C.3.6 The Majorana case

Definition

The Feynman rules for a Majorana fermion differs from Dirac Fermion, the same manner Feynman rules for real scalar differs from imaginary scalars. All can be, at the end, summarize by factors 2 or 1/2, but it is important (and interesting) to understand how to treat the rules accurately. First of all, one should remember the basic facts of a Dirac equation. If one decompose a (4-dimension) Dirac spinor ψ into its (2-dimension) Weyl spinors ζ and η, we can write $\psi = \begin{pmatrix} \chi \\ \eta \end{pmatrix}$ which generates the Dirac equation (C.17) if one writes the Lagrangian

$$\mathcal{L}_{Dirac} = i\chi^\dagger\sigma^\mu\partial_\mu\chi + i\eta^\dagger\sigma^\mu\partial_\nu\eta - M\eta^\dagger\chi - M^*\chi^\dagger\eta = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi. \tag{C.38}$$

It is important to keep in mind that the 4-component Dirac spinor Ψ has been built from Eq.(C.38) which is the Lagrangian leading to the set of 2 Dirac equations for the 2-component spinors χ and η. We will try now, to build the same Lagrangian for a Majorana spinor. Before going into the details, one can have a guess of the result. Indeed, ψ is a 4-component spinors, but have only 2 degrees of freedom in a Majorana case. A Majorana spinor to a Dirac one has the same correspondance than a real field to a scalar one. One understand then that there should be a factor 1/2 entering into the game, $\mathcal{L}_{Majorana} = \frac{1}{2}\mathcal{L}_{Dirac}$. This is what we will demonstrate more rigorously.

Dirac–like Majorana equation

As we saw in the previous section, a (4-component) Majorana spinor λ is defined as $\lambda^C = C\bar{\lambda}^T = \lambda$. If one write, by analogy with the Dirac spinor $\lambda = \begin{pmatrix} \chi \\ \eta \end{pmatrix}$, and writing (see Eq. C.31)

$$\lambda^C = C\bar{\lambda}^T = i\gamma^2\gamma^0\bar{\lambda}^T = \begin{pmatrix} i\sigma_2\eta^* \\ -i\sigma_2\chi^* \end{pmatrix} = \begin{pmatrix} \epsilon^{ab}\eta^*_b \\ -\epsilon^{ab}\chi^*_b \end{pmatrix} \tag{C.39}$$

with

$$\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{C.40}$$

is the Kronecker symbol. The Kronecker symbol is in fact fundamental when dealing with Grassmann (anti-commuting) variables. One can consider it as a metric in this case, allowing to write a scalar product for anti-commuting variables. Indeed, if one defines $\eta.\chi = \eta_a \epsilon^{ab} \chi_b$, it is straightforward to check that $\eta.\chi = \chi.\eta$ using the property[2] of ϵ^{ab} and $\chi_a \eta_b = -\eta_b \chi_a$.

The condition $\lambda^C = \lambda$ leads to

$$\lambda = \begin{pmatrix} \chi \\ -\epsilon\, \chi^* \end{pmatrix} \quad \Rightarrow \quad \bar{\lambda} = \lambda^\dagger \gamma^0 = \left(-\chi^T \epsilon^T \ ; \ \chi^\dagger\right) \tag{C.41}$$

which implies

$$\bar{\lambda}\lambda = \chi^T \epsilon \chi - \chi^\dagger \epsilon \chi^* \tag{C.42}$$

In the "Left-Right" representation

The Weyl representation in which we made all the calculation (defining our $\gamma's$ matrices) it is easy to rewrite the mass term using the "left" and "right" eigenvectors of the projection operators P_L and P_R (Eq.C.23). If one defines

$$\lambda_L = P_L \lambda = \begin{pmatrix} \chi \\ 0 \end{pmatrix} \quad \text{and} \quad \lambda_R = (\lambda_L)^C = \begin{pmatrix} 0 \\ -\epsilon \chi^* \end{pmatrix} \ , \quad \lambda = \lambda_L + \lambda_R \tag{C.43}$$

we can then write

$$\bar{\lambda}\lambda = \bar{\lambda}_L \lambda_R + \bar{\lambda}_R \lambda_L = \chi^T \epsilon \chi - \chi^\dagger \epsilon \chi^* \tag{C.44}$$

(notice the change of sign comparing Eq.(C.42) and Eq.(C.44). This reminds of course the definition of a Majorana mass term $M_M \overline{(\lambda_L)^C} \lambda_L$ which is the mass term we can write from a 4-component Majorana spinor. For more details on the Majorana mass, and its application to see-saw mechanism, see section (5.7).

However, the Dirac equation[3] $i\sigma_\mu \partial_\mu \chi = M\chi$ is derived from the Lagrangian

$$\mathcal{L} = i\chi^\dagger \sigma_\mu \partial_\mu \chi - \frac{M}{2} \chi.\chi - \frac{M^*}{2} \chi^*.\chi^* \tag{C.45}$$

where we used the definition of the scalar product defined above. We notice the factor 1/2 in front of the mass term that correspond to the same factor when one compare mass term for complex scalar and real scalar, and is naturraly present when one derive the Dirac equation from the equation of motion $\frac{\partial \mathcal{L}}{\partial \chi} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)}$. In 4-component notation, the Lagrangian (C.45) is then given by (for real mass term)

[2]In the case of the complex conjugate, the conventional notation is $\eta^*.\chi^* = (\eta^*)^a \epsilon_{ab} (\chi^*)^b$ with $\epsilon_{ab} = -\epsilon^{ab}$. This convention makes the scalar product easier to manipulate as one can see from Eq.(C.45) to Eq.(C.46).

[3]Remember that the fundamental Dirac equation should be written in 2-component notation. The 4-component one, especially in the Weyl ("Left-Right") representation is mainly practical in the Standard Model because the helicities have different quantum number under the SU(2) gauge group.

$$\mathcal{L}_{Majorana} = \frac{1}{2} i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - \frac{M}{2}\bar{\lambda}\lambda \tag{C.46}$$

which corresponds indeed to our first guess we made at the beginning of the section $\mathcal{L}_{Majorana} = \frac{1}{2}\mathcal{L}_{Dirac}$. If moreover, one supposes that the mass term are coming from yukawa–like couplings with a field ϕ, as in the Standard Model, one should write for generic couplings Γ (which can contains $\gamma' s$ and combinations of $\gamma' s$ matrices),

$$\mathcal{L} = \mathcal{L}_{Dirac} + \mathcal{L}_{Majorana} = \bar{\psi}(i\gamma^\mu\partial_\mu - M_\psi)\psi + \frac{1}{2}\bar{\lambda}(i\gamma^\mu\partial_\mu - M_\lambda)\lambda + y_\psi\phi\,\bar{\psi}\Gamma\psi + \frac{1}{2}y_\lambda\phi\,\bar{\lambda}\Gamma\lambda \tag{C.47}$$

The factor $\frac{1}{2}$ in front of the Yukawa coupling y_λ is fundamental when one needs to compute processes through the Feynman rules. Indeed, if ϕ is a scalar field developing a vev $\phi = \frac{1}{\sqrt{2}}(v + a + ib)$, the mass is then defined with the same convention, $M_\psi = \frac{y_\psi}{\sqrt{2}}v$, $M_\lambda = \frac{y_\lambda}{\sqrt{2}}v$. That point is very important once we made any computation with Majorana spinors: to check that the convention of the couplings fits with the mass term in the Lagrangian.

Furry's theorem

C.3.7 Traces

$$
\begin{aligned}
&\eta^{\mu\nu}\eta_{\mu\nu} = 4\\
&\mathrm{Tr}[\gamma^\mu] = \mathrm{Tr}[\gamma^5] = 0\\
&\mathrm{Tr}[\gamma^5\gamma^\mu] = \mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu] = \mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho] = 0\\
&\mathrm{Tr}[1] = 4\\
&\mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4\eta^{\mu\nu}\\
&\mathrm{Tr}[\not{p}_1.\not{p}_2] = 4p_1.p_2\\
&\mathrm{Tr}[(\not{p}_1 - m_1)(\not{p}_2 - m_2)] = 4p_1.p_2 + 4m_1m_2\\
&\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho] = 0\\
&\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4\left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right)\\
&\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5] = -4i\epsilon^{\mu\nu\rho\sigma}\\
&\mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu] = 4\left(p_1^\mu p_2^\nu - \eta^{\mu\nu}p_1.p_2 + p_1^\nu p_2^\mu\right)\\
&\mathrm{Tr}[\not{p}_1\not{p}_2\not{p}_3\not{p}_4] = 4\left[(p_1.p_2)(p_3.p_4) - (p_2.p_4)(p_1.p_3) + (p_2.p_3)(p_1.p_4)\right]\\
&\mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]\,\mathrm{Tr}[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu] = 32\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)\right]\\
&\mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]\,\mathrm{Tr}[\gamma_\mu\gamma_\nu] = 32p_1.p_2\\
&\mathrm{Tr}[(\not{p}_1 - M_\chi)\gamma^\mu(\not{p}_2 + M_\chi)\gamma^\nu]\,\mathrm{Tr}[(\not{p}_3 - m_f)\gamma_\mu(\not{p}_4 + m_f)\gamma_\nu]\\
&= 32\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3) + 2M_\chi^2m_f^2 + (p_1.p_2)m_f^2 + (p_3.p_4)M_\chi^2\right]\\
&\mathrm{Tr}[(\not{p}_1 + M_\chi)\gamma^\mu(\not{p}_2 + M_\chi)\gamma^\nu]\,\mathrm{Tr}[(\not{p}_3 + m_f)\gamma_\mu(\not{p}_4 + m_f)\gamma_\nu]\\
&= 32\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3) + 2M_\chi^2m_f^2 - (p_1.p_2)m_f^2 - (p_3.p_4)M_\chi^2\right]\\
&\mathrm{Tr}[\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4\eta^{\alpha\beta}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) - 4\eta^{\alpha\mu}(\eta^{\beta\nu}\eta^{\rho\sigma} - \eta^{\beta\rho}\eta^{\nu\sigma} + \eta^{\beta\sigma}\eta^{\nu\rho})\\
&+4\eta^{\sigma\nu}(\eta^{\beta\mu}\eta\rho\sigma - \eta^{\beta\rho}\eta^{\mu\sigma} + \eta^{\beta\sigma}\eta^{\mu\rho}) - 4\eta^{\alpha\rho}(\eta^{\beta\mu}\eta^{\nu\sigma} - \eta^{\beta\nu}\eta^{\mu\sigma} + \eta^{\sigma\beta}\eta^{\mu\nu})\\
&+4\eta^{\alpha\sigma}(\eta^{\beta\mu}\eta^{\nu\rho} - \eta^{\beta\nu}\eta^{\mu\rho} + \eta^{\rho\beta}\eta^{\mu\nu})
\end{aligned}
\tag{C.48}
$$

Through the book, we will use the notation P_μ for a quadrivector and p_i vor a 3-dimensional vector.

C.3.8 The generators T_i^a

The generators of the non-abelian gauge groups $SU(2)$ and $SU(3)$ are given by :

For $SU(2)$, $T_2^i = \frac{\sigma_i}{2}$, σ_i being the Pauli matrices.

$$
T_2^1 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \qquad T_2^2 = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \qquad T_2^3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}
$$

For $SU(3)$

$$
T_1^2 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \qquad T_2^2 = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \qquad T_3^2 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}
$$

C.4 Weyl, Dirac or Majorana spinor?

C.5 The SM conventions

C.6 Useful kinematic formulae

C.7 Lorentz invariant scattering cross section and phase space

C.7.1 The FERMI's golden rule

Normalisation

In classical quantum mechanics, the wave function ϕ is normalized such as

$$\int |\phi|^2 d^3x = \int |\phi|^2 dV = 1 \tag{C.49}$$

This mean that the particle has a probability of presence of 1 in the entire universe. However, in field theory, there exist the possibility of create or annihilate particles from the void. The normalisation is then a free parameter and a question of taste, as long as one stays coherent. Moreover, it is clear that the expression (C.49) is not Lorentz invariant due to the presence of the volume element of $dV = d^3x$. To compensate it, it is natural to normalise with a parameter with a normalisation $\sqrt{\gamma}$. The usual convention is to $2E$ particle per unit of volume, E being the energy of the particle:

$$\int |\psi|^2 dV = 2E, \qquad \phi = \frac{1}{\sqrt{2E}}\psi$$

Once developed in the momentum phase space, as function of its Fourier transformation,

$$\psi(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E}} \left[\mathbf{a}(p)e^{-ipx} + \mathbf{a}^\dagger e^{ipx}\right] \tag{C.50}$$

$\mathbf{a}$ and $\mathbf{a}^\dagger$ being expansion coefficients. The $\frac{1}{\sqrt{(2\pi)^3}}$ is the classical coefficient to normalize the wave function. Indeed, the plane wave function $\phi(x) = \frac{1}{\sqrt{2\pi}} \int e^{ipx} dp$ is normalized such as $|\phi(x)|^2 = 1$. We let the reader check it, knowing that the delta function $\delta(x)$ can be written $\delta(x) = \frac{1}{2\pi} \int e^{ipx} dp$.

The S-operator

Let's denote $|i\rangle$ and $|f\rangle$ the initial and final state respectively in a Fock space. We can define the transition between $|i\rangle$ and $|f\rangle$ by a $\mathcal{S}$ matrix which represents the dynamics of the process and whose amplitude squared is the probability of transition:

$$\mathcal{S}_{fi} = \langle f|\mathcal{S}|i\rangle.$$

Hence, the probability for the process $|i\rangle \to |f\rangle$ is;

$$P(|i\rangle \to |f\rangle) = |\mathcal{S}_{fi}|^2. \tag{C.51}$$

In general, we can write

$$\mathcal{S}_{fi} = \delta_{fi} + i(2\pi)^4\delta^{(4)}(p_f - p_i)\ .\ \ \mathcal{T}_{fi}$$

or

$$\mathcal{S} = \mathbb{I} + i\mathcal{T},$$

the first part representing a non–interacting particle, while the second part is the dynamical part with the condition of conservation of energy-momentum. Moreover, put this way, it is a reminiscent of an exponential development, $\mathcal{S} = e^{i\mathcal{T}}$. What is physically of interest for us is the transition probability per unit time

$$w_{fi} = \frac{|\mathcal{S}_{fi}|^2}{T}$$

From Eq.(C.51), we see that we must address the issue of defining the value of a squared Dirac $\delta-$function. To do this we use a rather pragmatic approach due to Fermi himself:

$$\begin{aligned}
&[2\pi\delta(p_f^0 - p_i^0)]^2 = \int dt e^{i(p_f^0 - p_i^0)t} 2\pi\delta(p_f^0 - p_i^0) = T2\pi\delta(p_f^0 - p_i^0)\\
&[(2\pi)^3\delta^{(3)}(\vec{p}_f - \vec{p}_i)]^2 = \int d^3x e^{i(\vec{p}_f - \vec{p}_i).\vec{x}}(2\pi)^3\delta^{(3)}(\vec{p}_f - \vec{p}_i) = V(2\pi)^3\delta^{(3)}(\vec{p}_f - \vec{p}_i)\\
&\Rightarrow w_{fi} = \frac{|\mathcal{S}_{fi}|^2}{T} = V(2\pi)^4\delta^{(4)}(p_f - p_i).|\mathcal{T}_{fi}|^2
\end{aligned} \tag{C.52}$$

To talk about the transition rate, we look at Fock–space with a fixed number of particles. Experimentally, the angle and energy–momentum is only accessible up to a given accuracy. We therefore use differential cross–section in angle $d\Omega$ and energy–momentum dp near Ω, p respectively.

Computing the rate

In a cubic box of volume $V = L^3$ with infinitely high potential wells, the authorized momentum–values are discretely distributed.

$$p = \frac{2\pi}{L}n \quad \Rightarrow \quad dn = \frac{L}{2\pi}dp \quad \Rightarrow \quad d^3n = \left(\frac{L}{2\pi}\right)^3 d^3p$$

and hence

$$dw_{fi} = V\ (2\pi)^4\delta^{(4)}(p_f - p_i)\ |\mathcal{T}_{fi}|^2 \prod_{f=1}^{n_f} \frac{V}{(2\pi)^3} d^3p_f \tag{C.53}$$

where n_f stands for the number of particles in the final state. In order to get rid of the normalisation factors, we define a new matrix element $\mathcal{M}_{fi}$ by,

$$\mathcal{T}_{fi} = \left(\prod_{i=1}^{n_i} \frac{1}{\sqrt{2E_i V}}\right)\left(\prod_{f=1}^{n_f} \frac{1}{\sqrt{2E_f V}}\right)\mathcal{M}_{fi} \tag{C.54}$$

At first sight, the apparition of the energies of both the initial and final states might be surprising. It is however needed in order to compensate the non–invariance of the volume, so that EV is a Lorentz invariant quantity as we saw in the previous section. This corresponds to the $2E$ normalisation. Substituting the definition (C.54) in Eq.(C.53), we get the fundamentally important expression for the rate $R = dw_{fi}$

$$\boxed{R = dw_{fi} = \frac{V^{1-n_i}}{(2\pi)^{3n_f}}(2\pi)^4\delta^{(4)}(p_f - p_i)|\mathcal{M}_{fi}|^2 \prod_{i=1}^{n_i}\frac{1}{2E_i}\prod_{f=1}^{n_f}\frac{d^3p_f}{2E_f} \quad \text{(C.55)}}$$

Application

We can apply the expression (C.55) to decay rate and scattering. For the decay of a particle a into n_f particle

$$a \to 1 + 2 + ... + n_f.$$

We have for the **total decay width**,

$$\boxed{\Gamma_a = \frac{1}{2E_a}\frac{1}{(2\pi)^{3n_f}}\int \frac{d^3p_1}{2E_1}..\frac{d^3p_{n_f}}{2E_{n_f}}(2\pi)^4\delta^{(4)}(p_f - p_i)|\mathcal{M}_{fi}|^2 \quad \text{(C.56)}}$$

We remark that since E_a is not a Lorentz invariant quantity, Γ_a also depends on the reference frame.

For the scattering cross section, the case of two particles interacting via the reaction

$$a + b \to 1 + 2 + ... + n_f \tag{C.57}$$

thus getting the **scattering cross section** $\sigma(a + b \to 1 + 2 + ... + n_f)$ defined by

$$\sigma = \frac{\#\text{ of transition } \ \mathrm{a + b \to 1 + 2 + ... + n_f} \ \text{ per unit time}}{\#\text{ of incoming particles per unit surface and time}} = \frac{w_{fi}}{\text{incoming flux}} \tag{C.58}$$

The denominator can also be stated as

$$\text{incoming flux} = (\text{number density}) \ . \ (\text{relative velocity}) = \tfrac{v_{ab}}{V}$$

we then find

$$\boxed{\sigma_{i\to n_f} = \frac{1}{4F}\frac{1}{(2\pi)^{3n_f}}\int\left(\prod_{f=1}^{n_f}\frac{d^3p_f}{2E_f}\right)(2\pi)^4\delta^{(4)}(\sum_{f=1}^{n_f}p_f - p_a - p_b)|\mathcal{M}_{fi}|^2 \qquad (C.59)}$$

in which we see once more the Lorentz invariant Møller flux factor

$$\begin{aligned} F &= E_aE_bv_{ab} = \sqrt{(p_a.p_b)^2 - m_a^2m_b^2} \\ &= \sqrt{(s-(m_a+m_b)^2)(s-(m_a-m_b)^2)} \end{aligned} \tag{C.60}$$

We see that the total cross section is manifestly a Lorentz invariant quantity, since it only depends on Lorentz invariants.

C.7.2 Special case

In the early Universe, the initial momentum of the scattering particles 1 and 2 are not uniquely determined as they live in a thermal plasma, with a statistical distribution of their momentum and energies f_1 and f_2. Moreover, it is not possible to define the relative velocity between the particles as they are all relativistic. One then need to use the fundamental expression for the rate $R = n_1n_2\langle\sigma v\rangle$:

$$\begin{aligned} R &= \int f_1f_2d\Pi_1d\Pi_2(2\pi)^4\delta^4(P_1+P_2-P_3-P_4)d\Pi_3d\Pi_4|M_{1,2\to3,4}|^2 \\ &= \int f_1f_2\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}(2\pi)^4|M_{1,2\to3,4}|^2\frac{d\Phi_2}{2E_12E_2} \end{aligned} \tag{C.61}$$

with $d\Pi_i = \frac{d^3p_i}{(2\pi)^32E_i}$ and where $d\Phi_2$ is the two body phase space, computed in the next section and $d\Omega$ the solid angle between particle 1 and 3 in the center of mass frame of (3,4). In the massless case ($m_3 = m_4 = 0$), one obtains $d\Phi_2 = \frac{d\Omega}{512\pi^6}$, see Eq.(C.68), which gives

$$R = \int f_1f_2\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{|M_{1,2\to3,4}|^2}{128E_1E_2}d\Omega \tag{C.62}$$

I let the reader go to the section 2.6.2 to have a more detailed study of this specific case.

C.7.3 Computing the phase space

We can show that $d\Pi = \frac{d^3p}{(2\pi)^32E}$ is a Lorentz invariant quantity. Indeed, if one considers a Lorentz boost of parameter[4] β in the z direction,

$$p'_x = p_x, \quad p'_y = p_y, \quad p'_z = \gamma(p_z - \beta E), \quad E' = \gamma(E - \beta p_z). \tag{C.63}$$

[4]Where, as usual, $\gamma = 1/\sqrt{1-\beta^2}$

One then need to check that $d\Pi = \frac{d^3p}{(2\pi)^3 2E} = \frac{dp'}{(2\pi)^3 2E'}$, or more simply $\frac{dp_z}{E} = \frac{dp'_z}{E'}$ For that, one can write

$$\frac{dp'_z}{E'} = \frac{\gamma dp_z - \beta\gamma dE}{\gamma(E - \beta p_z)} = \frac{dp_z(1 - \beta\frac{p_z}{E})}{E(1 - \beta\frac{p_z}{E})} = \frac{dp_z}{E} \tag{C.64}$$

where we have used the fact that $E = \sqrt{p^2 + m^2} \Rightarrow \frac{dE}{dp_z} = \frac{p_z}{E}$. That proves the Lorentz invariant character of $d\Pi$.

2–body phase space

One can then compute the two–body phase space $d\Phi_2$ in a reaction with center of mass energy $\sqrt{s}$

$$\begin{aligned} d\Phi_2 &= d\Pi_1 d\Pi_2\ \delta^{(4)}(\sqrt{s} - P_1 - P_2) = \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\ \delta^{(4)}(\sqrt{s} - P_1 - P_2) \\ &= \frac{p_2^2 dp_2 d\Omega}{4(2\pi)^6 E_1 E_2}\delta(\sqrt{s} - E_1 - E_2) \end{aligned} \tag{C.65}$$

with $E_1 = \sqrt{p_2^2 + m_1^2}$ and $E_2 = \sqrt{p_2^2 + m_2^2}$ in the center of mass frame. We can indeed compute the phase space in the center of mass frame as we showed in the previous section that it is Lorentz invariant. Using the relation $\delta(g[x]) = \frac{\delta(x)}{g'(x)}$ on can write

$$\delta(\sqrt{s} - E_1 - E_2) = \delta\sqrt{s} - \sqrt{p_2^2 + m_1^2} - \sqrt{p_2^2 + m_2^2} = \frac{\delta(p_2)}{\frac{p_2}{\sqrt{p_2^2+m_1^2}} + \frac{p_2}{\sqrt{p_2^2+m_2^2}}} \tag{C.66}$$

which gives

$$d\Phi_2 = \frac{p_2\ d\Omega}{4(2\pi)^6\sqrt{s}} = \frac{[s - (m_1 - m_2)^2]^{1/2}[s - (m_1 + m_2)^2]^{1/2}}{512\pi^6 s}d\Omega \tag{C.67}$$

here we used $p_2 = \frac{[s-(m_1-m_2)^2]^{1/2}[s-(m_1+m_2)^2]^{1/2}}{2\sqrt{s}}$. In the massless states, one obtains

$$d\Phi_2(m_i = 0) = \frac{d\Omega}{512\pi^6} \tag{C.68}$$

N–body phase space

One can generalize the previous computation in the case of N–body particles system. We can rewrite n–body phase space in the centre of mass frame

$$\begin{aligned} d\Phi_n(\sqrt{s}; p_1, .., p_n) &= \prod_{i=1}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i} \times \delta(\sum_{i=1}^{n} p_i)\ \delta(\sum_{i=1}^{n} E_i - \sqrt{s}) \\ &= \frac{d^3p_n}{(2\pi)^3 2E_n}\prod_{i=1}^{n-1}\frac{d^3p_i}{(2\pi)^3 2E_i}\delta(\sum_{i=1}^{n-1} p_i - (-p_n))\delta(\sum_{i=1}^{n-1} E_i - \sqrt{s} - (-E_n)) = \frac{d^3p_n}{(2\pi)^3 2E_n}d\Phi_{n-1}(\epsilon; p_1, .., p_{n-1}) \end{aligned} \tag{C.69}$$

with $\epsilon = (\sqrt{s}-E_n)^2-p_n^2$, where we used the Lorentz invariance of the phase space to compute $d\Phi_{n-1}$ in the center of mass frame of the system $(p_1,..,p_{n-1})$ passing from $(\sqrt{s}-E_n;-p_n)$ to $(\sqrt{(\sqrt{s}-E_n)^2-p_n^2};0)$ after a boost of momentum $+p_n$.

As an example, one can compute the three–body phase space, from the two–body one we computed already in Eq.(C.67).

$$d\Phi_3 = \frac{d^3p_3}{(2\pi)^3 2E_3} \times \frac{p_2(\epsilon_3)d\Omega}{4(2\pi)^6\epsilon_3}$$

with $\epsilon_3^2 = (\sqrt{s}-E_3)^2 - p_3^2$, which gives

$$d\Phi_3 = \frac{d^3p_3 d\Omega}{8\times(2\pi)^9 E_3}\frac{[(\sqrt{s}-E_3)^2-p_3^2-(m_1-m_2)^2]^{1/2}[(\sqrt{s}-E_3)^2-p_3^2-(m_1+m_2)^2]^{1/2}}{2\sqrt{(\sqrt{s}-E_3)^2-p_3^2}}$$

which gives in the massless case :

$$d\Phi_3^{m_i=0} = \frac{p_3 dp_3 d\Omega_{13} d\Omega_{12}}{16(2\pi)^9} \tag{C.70}$$

Summary in the massless case

We can then summarize our result for 2-, 3- and 4-body final states in massless cases:

$$d\Phi_2^{m_i=0} = \frac{d\Omega_{12}}{8(2\pi)^6}$$
$$d\Phi_3^{m_i=0} = \frac{p_3 dp_3 d\Omega_{13} d\Omega_{12}}{16(2\pi)^9}$$

Examples

We will compute in the next section, specific examples of decay or annihilation rates. They are given by

$$\Gamma = \frac{|\mathcal{M}|^2}{2M}(2\pi)^4 d\Phi_n(P;p_1,...,p_n) \tag{C.71}$$

with

$$d\Phi_n(P;p_1...,p_n) = \delta^4(P-\sum_{i=1}^n p_i)\prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3 2E_i}$$

for the width of a particle of mass M and amplitude of decay $\mathcal{M}$ decaying into n particles. In the case of annihilation between particles 1 and 2 into n final particles, the cross section $d\sigma$ can be written

$$d\sigma = \frac{(2\pi)^4|\mathcal{M}|^2}{4\sqrt{(P_1.P_2)^2-m_1^2m_2^2}} \times d\Phi_n(p_1+p_2;p_3,...,p_{n+2}) \tag{C.72}$$

where we used Eq.(C.60) for the Lorentz invariant flux.

C.7.4 Annihilation

<u>**General formulae**</u>

As we just saw in the previous section, the annihilation cross section between two–body states $(1,2) \to (3,4)$ can be written

$$d\sigma = \frac{(2\pi)^4 |\mathcal{M}|^2}{4\sqrt{(P_1.P_2)^2 - m_1^2 m_2^2}} \delta^4(P_1 + P_2 - P_3 - P_4) \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4}. \tag{C.73}$$

Using $s = (P_1+P_2)^2 = m_1^2 + m_2^2 - 2P_1.P_2$ and $m_1 = m_2 = M_\chi$, one has $\sqrt{(P_1.P_2)^2 - m_1^2 m_2^2} = \sqrt{(\frac{s}{2} - M_\chi^2)^2 - M_\chi^4} = \sqrt{(\frac{s}{2} - 2M_\chi^2)\frac{s}{2}}$. After integration on $d^3p_3 \delta^3(p_1 + p_2 - p_3 - p_4)$ one can then rewrite Eq.(C.73)

$$d\sigma = \frac{|\mathcal{M}|^2}{64\pi^2 \sqrt{(\frac{s}{2} - 2M_\chi^2)\frac{s}{2}}} \frac{|p_4|^2 d\phi d\cos\theta d|p_4|}{\sqrt{|p_4|^2 + m_3^2}\sqrt{|p_4|^2 + m_4^2}} \delta\left(\sqrt{s} - \sqrt{|p_4|^2 + m_3^2} - \sqrt{|p_4|^2 + m_4^2}\right) \tag{C.74}$$

where we place ourself in the center of mass frame ($\mathbf{p_3} = -\mathbf{p_4}$, $|p_3| = |p_4|$) and $E_4 = \sqrt{|p_4|^2 + m_4^2}$. Reminding that the function delta respects $\delta[f(x)] = \frac{\delta(x)}{|f'(x)|}$,

$$\delta\left(\sqrt{s} - \sqrt{|p_4|^2 + m_3^2} - \sqrt{|p_4|^2 + m_4^2}\right) = \frac{\delta(|p_4|)}{\frac{|p_4|}{\sqrt{|p_4|^2+m_3^2}} + \frac{|p_4|}{\sqrt{|p_4|^2+m_4^2}}} \tag{C.75}$$

one can write Eq.(C.74)

$$d\sigma = \frac{|\mathcal{M}|^2}{64\pi^2 \sqrt{(\frac{s}{2} - 2M_\chi^2)\frac{s}{2}}} \frac{|p_4| d\phi d\cos\theta}{\sqrt{|p_4|^2 + m_3^2} + \sqrt{|p_4|^2 + m_4^2}} = \frac{|\mathcal{M}|^2}{64\pi^2 \sqrt{(\frac{s}{2} - 2M_\chi^2)\frac{s}{2}}} \frac{|p_4| d\phi d\cos\theta}{\sqrt{s}} \tag{C.76}$$

where we used $\sqrt{|p_4|^2 + m_3^2} + \sqrt{|p_4|^2 + m_4^2} = E_3 + E_4 = E_1 + E_2 = \sqrt{s}$. Moreover, $|p_4|^2 = E_4^2 - m_4^2$ and $E_4^2 = \frac{(s+m_4^2-m_3^2)^2}{4s}$ implies $|p_4|^2 = \frac{1}{4s}(s^2 + m_4^4 + m_3^4 - 2sm_4^2 - 2sm_3^2 - 2m_3^2 m_4^2)$. Replacing this expression for $|p_4|$ in Eq.(C.76) we obtain, with $d\Omega = d\phi d\cos\theta$

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{|\mathcal{M}|^2}{128\pi^2 s} \frac{\sqrt{s^2 - 2m_3^2 s - 2m_4^2 s + (m_3^2 - m_4^2)^2}}{\sqrt{(P_1.P_2)^2 - m_1^2 m_2^2}} \\ &= \frac{|\mathcal{M}|^2}{64\pi^2 s} \sqrt{\frac{[s - (m_3 - m_4)^2][s - (m_3 + m_4)^2]}{[s - (m_1 - m_2)^2][s - (m_1 + m_2)^2]}} = \frac{|\mathcal{M}|^2}{64\pi^2 s} \frac{\sqrt{s - 2m_3^2 - 2m_4^2 + \frac{(m_3^2 - m_4^2)^2}{s}}}{\sqrt{s - 4M_\chi^2}} \end{aligned} \tag{C.77}$$

But it is important to include the symmetric/statistic factors : 1/4 (1/(2S+1)*1/(2S+1)) for a fermionic dark matter and 1/Sym for a symmetry factor if the particles 3 and 4 are identical ($Z^0 Z^0$ for instance but not W^+W^- or $f\overline{f}$). One then obtain:

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2S_1+1)(2S_2+1)Sym}\frac{|\mathcal{M}|^2}{64\pi^2 s}\frac{\sqrt{s-2m_3^2-2m_4^2+\frac{(m_3^2-m_4^2)^2}{s}}}{\sqrt{s-4M_\chi^2}}$$
$$\simeq \frac{1}{(2S_1+1)(2S_2+1)Sym}\frac{|\mathcal{M}|^2}{32\pi^2 vs}\sqrt{1-\frac{2m_3^2+2m_4^2}{4M_\chi^2}+\frac{(m_3^2-m_4^2)^2}{16M_\chi^4}} \tag{C.78}$$

where we have developed $s \simeq 4M_\chi^2 + M_\chi^2 v^2$, v being the relative velocity between the two colliding particles (also called Moller velocity). The simplified formulae for massless final state and amplitude which does not depend on the diffusion angle (scalar–like interactions) after integration on the solid angle, is

$$\sigma v \simeq \frac{1}{(2S_1+1)(2S_2+1)Sym}\frac{\int |\mathcal{M}|^2 d\cos\theta}{16\pi s} \tag{C.79}$$

A shorter formulation

In the literature, especially when discussing dark matter processes one sometimes prefers to write another way Eq.(C.73) :

$$d\sigma = \frac{1}{2E_1 2E_2|v_2-v_1|}\left(\prod_{final}\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)(2\pi)^4\delta^{(4)}\left(P_1+P_2-\sum_{final}P_f\right)|\bar{\mathcal{M}}|^2 \tag{C.80}$$

where $|\bar{\mathcal{M}}|$ should be understood to mean the spin averaged squared amplitude. The two–body phase space is

$$dPS_2(p_3,p_4) = \left(\prod_{final}\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)\delta^{(4)}\left(P_1+P_2-\sum_{final}P_f\right) = \frac{d\Omega_{CM}}{256\pi^6}\left(\frac{|p_3|}{E_{CM}}\right). \tag{C.81}$$

I want to add an important note here. One must be careful when using "v". In Eq.(C.78) v represents the *relative* velocity of particle 1 with respect to particle 2, also noted $|v_2 - v_1|$, and *not* the absolute velocity or the velocity in the center of mass (CoM) frame. It is usual in dark matter literature to name v the *relative* velocity (in expressions of $\langle\sigma v\rangle$ for instance). However, when one compute cross section in quantum field theory, we usually do it in the center of mass frame, and $v_{CoM} = v/2$ In other words, $v_{dark\ matter} = v = |v_2 - v_1| = 2\ v_{CoM}$. This $v_{dark\ matter} = |v_2 - v_1|$ is the one used in Eq.(2.128) for instance. In other words, one can make all the computation in CoM frame, then when implementing velocities dependance replacing v_{CoM} by $v/2$, v being the relative velocity, of interest for us as it is the one "measured" by the rotation of the dark matter halo.
Let illustrate it concretely. When one has to compute $s = (P_1 + P_2)^2$, supposing $m_1 = m_2 = m$, one needs to develop

$$s = (P_1+P_2)^2 = m_1^2 + m_2^2 + 2E_1E_1 - 2\vec{p}_1.\vec{p}_2 = 4m^2 + 4|p_{CoM}|^2. \tag{C.82}$$

remembering that $\vec{p}_{CoM} = \gamma m v_{CoM}$, one deduce

$$s = 4m^2(1+\gamma^2 v_{CoM}^2) = \frac{4m^2}{1-v_{CoM}^2} = \frac{4m^2}{1-v^2/4} \simeq 4m^2 + m^2 v^2 \tag{C.83}$$

justifying the approximation used in Eq.(C.78).

A note on the symmetry factor

We want to add a little remark concerning the symmetry factor $1/Sym$ in Eq.(C.79). If there are n_f identical particles in the final states, the symmetry factor is $Sym = \Pi_f(n_f!)$ and $\sigma = \frac{1}{Sym}\int d\sigma$. We need the symmetry factor because merely integrating over all the outgoing momenta in the phase space treats the final state as being labeled by an *ordered* list of momentum. But if some outgoing particles are identical, this is not correct the momentum of the *identical particles* should be specified by an *unordered* list, because for example the state $a_1^\dagger a_2^\dagger|0\rangle$ is identical to the state $a_2^\dagger a_1^\dagger|0\rangle$. The symmetry factor provides the appropriate correction.

The specific case of Majorana or identical initial particle

The counting of states in annihilation processes for Majorana fermions is non-trivial, and has led to a factor-of-two ambiguity in the literature which also propagated into different codes present in the market. In short, the $\langle\sigma v\rangle$ used when computing a relic abundance should not be confounded by the σv used when one needs to deal with indirect detection rates. In the dirac case, or complex scalar dark matter, there are no differences, whereas a factor 1/2 appears when dealing with Majorana/identical particles, the halo annihilation being twice less than the Boltzmann one.

The clearest way to see the origin of the factor of 1/2 is probably to go back to the Boltzmann equation. In essence, one can view $\langle\sigma v\rangle$ as the thermal average (averaged over momentum and angles) of the cross section times velocity in the zero momentum limit; in this average one integrates over all possible angles. For identical particles in the initial state, one includes each possible initial state twice, therefore one needs to compensate by dividing by a factor of 2; the prefactor in the zero-momentum limit becomes then $\langle\sigma v\rangle/2$. In the Boltzmann equation describing the time evolution of the dark matter candidate number density, the 1/2 does not appear as it is compensated by the factor of 2 one has to include because 2 dark matter particles are depleted per annihilation, but we need to include the factor of 1/2 explicitly in other cases where we need the annihilation rate (like for annihilation in the halo). Indeed, when integrating on phase space, we count twice $\chi(p_1)\chi(p_2)$ annihilation as they will exchange impulsion in the process of integrating on θ and ϕ.

Scalar Fermi–like interaction

To illustrate this result, we can compute the annihilation cross section for a scalar-scalar type interaction of the form :

$$\mathcal{L}_S = \frac{G_S}{\sqrt{2}}\overline{\chi}\chi\overline{f}f \tag{C.84}$$

The amplitude may be written

$$\begin{aligned}\mathcal{A} &= \frac{G_S}{\sqrt{2}}\overline{u}(p_\chi)u(p_\chi)\overline{u}(p_f)u(p_f) \Rightarrow |\mathcal{A}|^2 = \frac{G_S^2}{2}\,\mathrm{Tr}(\not{p}_\chi - M_\chi)(\not{p}_\chi + M_\chi) \times \mathrm{Tr}(\not{p}_f - m_f)(\not{p}_f + m_f) \\ &= \frac{G_S^2}{2}(4P_\chi.P_\chi - 4M_\chi^2)(4P_{\bar{f}}.P_f - 4m_f^2) = \frac{G_S^2}{2}(2s - 8M_\chi^2)(2s - 8m_f^2) \end{aligned} \tag{C.85}$$

which imply

$$\sigma_S = \frac{G_S^2}{32\pi}c_f\frac{\sqrt{s-4m_f^2}}{\sqrt{s-4M_\chi^2}}\left(\frac{(s-4M_\chi^2)(s-4m_f^2)}{s}\right) \tag{C.86}$$

with c_f the color factor, equal to 3 for quarks and 1 for leptons. The annihilation cross section becomes, after developing around v

$$\sigma_S v \simeq \frac{G_S^2 M_\chi^2}{16\pi}c_f\left(1-\left(\frac{m_f}{M_\chi}\right)^2\right)^{\frac{3}{2}}v^2 \tag{C.87}$$

Vector Fermi–like interaction

Another example is a vector-vector-like interaction :

$$\mathcal{L}_V = \frac{G_V}{\sqrt{2}}\overline{\chi}\gamma^\mu\chi\overline{f}\gamma_\mu f \tag{C.88}$$

One can compute directly $|\mathcal{M}|^2$:

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{G_V^2}{2}\,\mathrm{Tr}[(\not{p}_1 - M_\chi)\gamma^\mu(\not{p}_2 + M_\chi)\gamma^\nu]\,\mathrm{Tr}[(\not{p}_3 - m_f)\gamma_\mu(\not{p}_4 + m_f)\gamma_\nu] \\
&= \frac{G_V^2}{2}32\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3) + 2M_\chi^2 m_f^2 + (p_1.p_2)m_f^2 + (p_3.p_4)M_\chi^2\right] \\
&= \frac{G_V^2}{2}32\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3) + \frac{s}{2}\left(M_\chi^2 + m_f^2\right)\right]
\end{aligned} \tag{C.89}
$$

Developing in the center of mass frame, where $\mathbf{p_1} = \mathbf{p_2} \Rightarrow E_1 = E_2 = \sqrt{s}/2 = E_3 = E_4$, one deduces $|p_1| = |p_2| = \sqrt{s/4 - M_\chi^2}$, $|p_3| = |p_4| = \sqrt{s/4 - m_f^2}$ and $p_1.p_3 = p_2.p_4 = E_1E_3 - |p_1||p_3|\cos\theta \Rightarrow (p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3) = (p_1.p_3)^2 + (p_1.p_4)^2 = (E^2 - |p_1||p_3|\cos\theta)^2 + (E^2 + |p_1||p_3|\cos\theta)^2 = 2E^4 + 2|p_1|^2|p_2|^2\cos^2\theta$.
After integrating on the solid angle $d\cos\theta$, one obtain

$$
\int d\cos\theta\left[(p_1.p_3)^2 + (p_2.p_4)^2\right] = 4E^4 + \frac{4}{3}|p_1|^2|p_3|^2 = \frac{s^2}{4} + \frac{4}{3}(\frac{s}{4} - M_\chi^2)(\frac{s}{4} - m_f^2) \tag{C.90}
$$

which gives finally

$$
\begin{aligned}
\sigma_V &= \frac{G_V^2}{2}\frac{2\pi}{4}\frac{1}{64\pi^2}c_f\frac{\sqrt{s-4m_f^2}}{\sqrt{s-4M_\chi^2}}32\left(\frac{s}{4} + \frac{4}{3s}(\frac{s}{4} - M_\chi^2)(\frac{s}{4} - m_f^2) + M_\chi^2 + m_f^2\right) \\
&= \frac{G_V^2}{32\pi}c_f\frac{\sqrt{s-4m_f^2}}{\sqrt{s-4M_\chi^2}}\left(s + \frac{1}{3s}(s-4M_\chi^2)(s-4m_f^2) + 4(M_\chi^2 + m_f^2)\right)
\end{aligned} \tag{C.91}
$$

Usually, when you will compute a cross section, you will use the easiest units of masses and energy which will be the GeV. However, fluxes are usually measured in more common units for experimentalists *ie* cm^{-2} or seconds. The conversion factor will be : $1\ \mathrm{GeV}^{-2} = 1.2\times10^{-17}\mathrm{cm}^3\mathrm{s}^{-1}$

<u>Neutrino interaction</u>

$$
\mathcal{L} = \frac{G_F}{\sqrt{2}}\bar{\nu}\nu\bar{e}e \Rightarrow |\mathcal{M}|^2 = \frac{G_F^2}{2}2s(2s - 8m_e^2) \tag{C.92}
$$

with $s = (p_\nu + p_{\bar{\nu}})^2 = 4E_\nu^2$ in the rest-frame. Applying directly Eq.(C.78) to our case (being careful to use a massless neutrino approximation) one obtains

$$\sigma_{\nu\nu} = \frac{G_F^2}{32\pi}\frac{(s-4m_e^2)^{3/2}}{\sqrt{s}} = \frac{G_F^2}{8\pi}\frac{(E_\nu^2-m_e^2)^{3/2}}{E_\nu} \tag{C.93}$$

which finally gives, using the thermal average of relativistic species Eq.(B.28) :

$$\sigma_{\nu\nu} \simeq \frac{G_F^2}{8\pi}\left(\frac{7\pi^4}{180\zeta(3)}\right)^2 T^2 \simeq \frac{9.93G_F^2}{8\pi}\times T^2 \tag{C.94}$$

Annihilation into monochromatic photons

For a complex scalar dark matter, this process is effectively generated through the Lagrangian[5]

$$\mathcal{L} = \frac{1}{\Lambda^2}SS^*F_{\mu\nu}F^{\mu\nu} \tag{C.95}$$

with Λ a typical mass scale of particles running in the loop generating the effective interaction. These particles being charged under the electromagnetic forces they should be above $\simeq$ 500 GeV as no such particles has been found at LHC yet. This type of process is useful when searching for monochromatic lines in the sky, as it is still considered as a smoking gun signal for dark matter detection (see section 11 for more details). Following the Feynman rule, the coupling can then be written

$$\mathcal{C}_{SS\gamma\gamma} = \frac{2}{\Lambda^2}(\partial_\mu A_\nu^1 - \partial_\nu A_\mu^1)(\partial^\mu A_2^\nu - \partial^\nu A_2^\mu) \tag{C.96}$$

A_1 and A_2 being the photons fields 1 and 2 respectively, the factor 2 arising from the symmetry in the exchange of identical particles $\gamma_{(1)} \leftrightarrow \gamma_{(2)}$. We then need to develop the coupling, contracting it with the polarization of the photons to compute the amplitude $\mathcal{M}$:

$$\mathcal{M} = \frac{2}{\Lambda^2}(p_\mu^1\epsilon_\nu^1 - p_\nu^1\epsilon_\mu^1)(p_2^\mu\epsilon_\nu^2 - p_2^\nu\epsilon_2^\mu) = \frac{4}{\Lambda^2}\Big[(p_1.p_2)(\epsilon_1.\epsilon_2) - (p_2.\epsilon_1)(p_1.\epsilon_2)\Big] \tag{C.97}$$

From $\mathcal{M}$, one can deduce $|\mathcal{M}|^2$

$$|\mathcal{M}|^2 = \frac{16}{\Lambda^4}\sum_{\mu,\nu,\mu',\nu'}\Big[(p_1.p_2)\epsilon_\mu^1\epsilon_2^\mu - (p_\mu^2\epsilon_1^\mu)(p_\nu^1\epsilon_2^\nu)\Big]\Big[(p_1.p_2)\epsilon_{\mu'}^1\epsilon_2^{\mu'} - (p_{\mu'}^2\epsilon_1^{\mu'})(p_{\nu'}^1\epsilon_2^{\nu'})\Big] \tag{C.98}$$

[5]Some other dimension 6 point like interactions of the form $SS^*F_{\mu\nu}\tilde{F}^{\mu\nu}$ can also be generated, see section (C.3.5) for some other examples.

after developing and using $\sum_{\mu\mu'} \epsilon^i_\mu \epsilon^i_{\mu'} = -\eta_{\mu\mu'}$ one obtains

$$|\mathcal{M}|^2 = \frac{32}{\Lambda^4}(p_1.p_2)^2 = \frac{8}{\Lambda^4}s \quad \Rightarrow \quad \sigma v = \frac{|\mathcal{M}|^2}{2 \times 8\pi s} = \frac{s}{2\pi\Lambda^4} \simeq \frac{2m_s^2}{\pi\Lambda^4} \tag{C.99}$$

where we have developed around $s \simeq 4m_s^2$ for low velocity[6] and divided by the symmetry factor 2 for the photons in the final state. It is possible to see other similar results in [749]

Annihilation in the case of real scalar Dark Matter to pairs of fermions

The two parts of the Lagrangian one needs to compute the scalar annihilation of Dark Matter $SS \to H \to \bar{f}f$ are (see appendix)[7]

$$\mathcal{L}_{HSS} = -\lambda_{HS}\frac{M_W}{2g}HSS \quad \to \mathcal{C}_{HSS} = -i\frac{\lambda_{HS}M_W}{g} \quad \text{and} \quad \mathcal{L}_{Hff} = -\frac{gm_f}{2M_W}\bar{f}f \tag{C.100}$$

which gives

$$|\mathcal{M}|^2 = \frac{\lambda_{HS}^2 m_f^2(s/2 - 2m_f^2)}{(s - M_H^2)^2 + \Gamma_H^2 M_H^2} \tag{C.101}$$

Γ_H being the width of the Higgs boson (including its own decay into SS, see next section). When ones implement this value of $|\mathcal{M}|^2$ into Eq.(C.78) one obtains after simplification

$$\langle\sigma v\rangle^S_{f\bar{f}} = \frac{|\mathcal{M}|^2}{8\pi s}\sqrt{1 - \frac{m_f^2}{M_S^2}} = \frac{\lambda_{HS}^2(M_S^2 - m_f^2)}{16\pi M_S^2(4M_S^2 - M_H^2)^2}\sqrt{1 - \frac{m_f^2}{M_S^2}} \tag{C.102}$$

the factor 2 in the denominator being the factor of symmetry.

Annihilation in the case of vectorial Dark Matter to pairs of fermions

[6] Do not forget that v is the relative velocity of the colliding particles.

[7] Notice the factor 2 between $\mathcal{L}_{HSS}$ and $\mathcal{C}_{HSS}$ coming from the fact that S is real: $S = S^*$ (it corresponds to the 2 possible contractions)

One can compute this annihilation cross section by the normal procedure or noticing that a neutral vectorial dark matter of spin 1 corresponds to 3 degrees of freedom. After averaging on the spin one can then write $\langle\sigma v\rangle^V = \frac{3}{3\times 3}\langle\sigma v\rangle^S = \langle\sigma v\rangle^S/3$. The academical computation for $V_\mu(p_1)V_\mu(p_2) \to f\overline{f}$ gives

$$\begin{aligned}
&\mathcal{L} \supset -\frac{\lambda_{hV}}{4}\eta_{\mu\nu}H^\dagger H X^\mu(p_1)X^\nu(p_2) \Rightarrow \mathcal{C}_{hXX} = -\lambda_{hV}M_W\eta_{\mu\nu}\epsilon_1^\mu\epsilon_2^\nu \qquad \text{(C.103)}\\
&\text{and} \quad \mathcal{C}_{hf\overline{f}} = -\frac{gm_f}{2M_W}\overline{u}(p_f)v(p_{\overline{f}}) \quad \text{implying}\\
&\mathcal{M} = \epsilon_{1\lambda}^\mu\epsilon_{2\rho}^\nu\eta_{\mu\nu}\frac{\lambda_{hV}m_f}{2}\overline{u}(p_f)v(p_{\overline{f}}) \qquad \mathcal{M}^* = \epsilon_{1\lambda}^{\alpha *}\epsilon_{2\rho}^{\beta *}\eta_{\alpha\beta}\frac{\lambda_{hV}m_f}{2}\overline{v}(p_{\overline{f}})u(p_f).
\end{aligned}$$

Using $\sum_\lambda \epsilon_{1\lambda}^\mu\epsilon_{1\lambda}^{\alpha *} = -\eta^{\mu\alpha} + \frac{p_1^\mu p_1^\alpha}{m_X^2}$ one obtains

$$|\mathcal{M}|^2 = \frac{2\lambda_{hV}^2 m_f^2(m_X^2 - m_f^2)}{(s-m_h^2)^2 + \Gamma_h^2 m_h^2}\left(4 + \frac{(p_1.p_2)^2}{m_X^4} - \frac{p_1.p_1}{m_X^2} - \frac{p_2.p_2}{m_X^2}\right) = \frac{6\lambda_{hV}^2 m_f^2(m_X^2 - m_f^2)}{(s-m_H^2)^2 + \Gamma_h^2 m_h^2} \qquad \text{(C.104)}$$

implying, after averaging on the spin 1 initial state particles

$$\langle\sigma v\rangle_{f\overline{f}}^V = \frac{\lambda_{hV}^2(m_X^2 - m_f^2)}{48\pi m_X^2[(4m_X^2 - m_h^2)^2 + \Gamma_h^2 m_h^2]}\sqrt{1 - \frac{m_f^2}{m_X^2}} \qquad \text{(C.105)}$$

Exchange of a vector: the case of the vectorial coupling

We compute in this section the process $\chi\bar{\chi} \to Z' \to f\bar{f}$.
from the Lagrangian $i\bar{\chi}\gamma^\mu D_\mu\chi + i\bar{f}\gamma^\mu D_\mu f$ we extract the interaction $g_D\left[q_\chi\bar{\chi}\gamma^\mu Z'_\mu\chi + q_f\bar{f}\gamma^\mu Z'_\mu f\right]$ which gives

$$\begin{aligned}
&\mathcal{M} = -i\frac{g_D^2 q_\chi q_f}{s - M_{Z'}^2}\bar{u}(p_f)\gamma^\mu v(p_{\bar{f}})\left[\eta_{\mu\nu} - \frac{p_\mu^{Z'}p_\nu^{Z'}}{M_{Z'}^2}\right]\bar{v}(p_{\bar{\chi}})\gamma^\nu u(p_\chi) \qquad \text{(C.106)}\\
&\Rightarrow |\mathcal{M}|^2 = 32\frac{g_D^4 q_\chi^2 q_f^2}{(s - M_{Z'}^2)^2}[(p_f.p_{\bar{\chi}})(p_{\bar{f}}.p_\chi) + (p_f.p_\chi)(p_{\bar{f}}.p_{\bar{\chi}}) + m_f^2(p_\chi p_{\bar{\chi}}) + m_\chi^2(p_f.p_{\bar{f}}) + 2m_\chi^2 m_f^2]\\
&\Rightarrow |\mathcal{M}|^2 = 64\frac{g_D^4 q_\chi^2 q_f^2}{(s - M_{Z'}^2)^2}[E^4 + (E^2 - m_f^2)(E^2 - m_\chi^2)\cos^2\theta + E^2(m_f^2 + m_\chi^2)]
\end{aligned}$$

After the average on the entering spins (1/4) we obtain

$$
\begin{aligned}
d\sigma &= C_f \frac{|\bar{\mathcal{M}}|^2}{64\pi^2 s} \frac{\sqrt{s-4m_f^2}}{\sqrt{s-4m_\chi^2}} \, 2\pi d\cos\theta \\
\Rightarrow \sigma &= \frac{4\pi C_f \ \alpha_D^2 \ q_\chi^2 q_f^2}{3E^2(4E^2-M_{Z'}^2)^2} \frac{\sqrt{4E^2-4m_f^2}}{\sqrt{4E^2-4m_\chi^2}} \, (2E^2+m_\chi^2)(2E^2+m_f^2) \\
&= \frac{4\pi C_f \ \alpha_D^2 \ q_\chi^2 q_f^2}{3s(s-M_{Z'}^2)^2} \frac{\sqrt{s-4m_f^2}}{\sqrt{s-4m_\chi^2}} \, (s+2m_\chi^2)(s+2m_f^2)
\end{aligned} \tag{C.107}
$$

C_f being the color index of the fermions in the final state and $\alpha_D = \frac{g_D^2}{4\pi}$.

C.7.5 Spin Independent diffusion, elastic scattering

If one want to compute the diffusion of a scalar particle of mass m_S and impulsion p_S on a proton of impulsion p_p and with impulsion p'_p after the hit, following Eq.(C.77) and noticing $\sqrt{(P_1.P_2)^2 - m_1^2 m_2^2} = \frac{1}{2}\sqrt{s^2 - 2m_3^2 s - 2m_4^2 s + (m_3^2 - m_4^2)^2}$ we can express

$$
\frac{d\sigma_{S-p}^{SI}}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 s} \quad \Rightarrow \quad \sigma_{S-p}^{SI} = \frac{|\mathcal{M}|^2}{16\pi s} \simeq \frac{|\mathcal{M}|^2}{16\pi(m_p+m_S)^2} \tag{C.108}
$$

Applying Eq.(C.100) one can write

$$
\mathcal{M} = -\frac{1}{2(p_H^2 - M_H^2)} (\sum_q \lambda_{HS} m_q \langle p|\bar{q}q|p\rangle) \bar{u}(p_p)u(p_p)\bar{u}(p'_p)u(p'_p) \tag{C.109}
$$

If one define $f_q = m_q/m_p \langle p|\bar{q}q|p\rangle$, $f = \sum_q f_q$, and notice $P_H^2 - M_H^2 \simeq -M_H^2$ Eq.(C.109) becomes

$$
|\mathcal{M}|^2 = \frac{1}{2}\frac{m_p^2 \lambda_{HS}^2 f^2}{M_H^4}(P'_p.P_p + m_p^2) \simeq \frac{m_p^4 \lambda_{HS}^2 f^2}{M_H^4} \quad \Rightarrow \quad \sigma_{S-p}^{SI} = \frac{m_p^4 \lambda_{HS}^2 f^2}{16\pi(m_p+m_S)^2 M_H^4} \tag{C.110}
$$

because the proton is mainly at rest[8] $P'_p.P_p + m_p^2 \simeq 2m_p^2$.

[8]The first $\frac{1}{2}$ is coming from the mean on the spin of one fermion, the initial proton.

C.7.6 Bhabha scattering (1926)

C.7.7 Higgs lifetime

General formulae

One can compute the width of a particle of mass M and quadri–impulsion P to two particles of masses m_1 and m_2 of impulsion $\mathbf{p_1}$ and $\mathbf{p_2}$ respectively through the general formula:

$$d\Gamma = \frac{(2\pi)^4}{2M}|\mathcal{M}|^2\delta^4(P - p_1 - p_2)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2} \tag{C.111}$$

which gives after integration on the final impulsions

$$d\Gamma = \frac{1}{32\pi^2}|\mathcal{M}|^2\frac{|\mathbf{p_1}|}{M^2}d\phi_1 d\cos\theta_1 \tag{C.112}$$

ϕ_1 and θ_1 being the spherical coordinates of the particle 1, with

$$|\mathbf{p_1}| = |\mathbf{p_2}| = \frac{[(M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2)]^{1/2}}{2M}. \tag{C.113}$$

In the case where $m_1 = m_2$, the formula is greatly simplified, after integration on the phase space to

$$\Gamma = \left(\frac{1}{2}\right)\frac{|\mathcal{M}|^2}{16\pi M^2}\sqrt{M^2 - 4m_1^2} \tag{C.114}$$

the factor $\left(\frac{1}{2}\right)$ coming form the (anti)symmetrization of identical (fermions) bosons in the exchange $1 \leftrightarrow 2$

Higgs lifetime from $H \to f(\mathbf{p_1})\bar{f}(\mathbf{p_2})$

Combining Eq.(C.114), and the SM Lagrangian (Section **??**) one obtains

$$\mathcal{C}_{Hf\bar{f}} = -i\frac{gm_f}{2M_W} \quad \Rightarrow \quad \mathcal{M} = \bar{u}(p_1)\frac{gm_f}{2M_W}u(p_2) \quad \Rightarrow \quad |\mathcal{M}|^2 = \frac{g^2m_f^2}{4M_W^2}\mathrm{Tr}(\not{p}_1 + m_f)(\not{p}_2 - m_f) \tag{C.115}$$

Using Eq.C.48 on can write

$$\Gamma_{H\to f\bar{f}} = \frac{g^2m_f^2}{32\pi M_W^2 M_H^2}(M_H^2 - 4m_f^2)^{3/2} \tag{C.116}$$

Higgs lifetime from $H \to Z(p_1)Z(p_2)$

In this case, one use the HZZ coupling from the Lagrangian given in Eq.(D.6), we obtain

$$\mathcal{M}_{\mu\nu} = igM_Z\eta_{\mu\nu}\epsilon^{\mu}_{1\lambda}\epsilon^{\nu}_{2\rho} \quad \mathcal{M}^*_{\mu\nu} = -igM_Z\eta_{\alpha\beta}\epsilon^{\alpha *}_{1\lambda}\epsilon^{\beta *}_{2\rho} \tag{C.117}$$

λ and ρ being the polarization of the $Z(p_1)$ and $Z(p_2)$ respectively. After the summation on the polarization, $\sum_\lambda \epsilon^{\mu}_{1\lambda}\epsilon^{\alpha *}_{1\lambda} = -\eta^{\mu\alpha} + p_1^\mu p_1^\alpha/M_Z^2$ one obtains

$$|\mathcal{M}|^2 = g^2 M_Z^2 \eta_{\mu\nu}\left(-\eta^{\mu\alpha} + \frac{p_1^\mu p_1^\alpha}{M_Z^2}\right)\eta_{\alpha\beta}\left(-\eta^{\nu\beta} + \frac{p_1^\nu p_1^\beta}{M_Z^2}\right) = \frac{g^2 M_Z^2 M_H^4}{4M_Z^4}\left(1 - 4\frac{M_Z^2}{M_H^2} + 12\frac{M_Z^4}{M_H^4}\right) \tag{C.118}$$

which gives

$$\Gamma_{H\to ZZ} = \frac{g^2 M_H^3}{128\pi M_W^2}\sqrt{1 - \frac{4M_Z^2}{M_H^2}}\left(1 - 4\frac{M_Z^2}{M_H^2} + 12\frac{M_Z^4}{M_H^4}\right) \tag{C.119}$$

Higgs lifetime from $h \to W^+W^-$

Similarily, on can compute

$$\Gamma_{H\to WW} = \frac{g^2 M_H^3}{64\pi M_W^2}\sqrt{1 - \frac{4M_W^2}{M_H^2}}\left(1 - 4\frac{M_W^2}{M_H^2} + 12\frac{M_W^4}{M_H^4}\right) \tag{C.120}$$

The difference between the WW and ZZ pre-factor comes from the symmetry ($Z = Z^* \Rightarrow$ 1/2 of WW final state.)

Higgs lifetime from $H \to SS$ with singlet scalar Dark Matter

From the simplest singlet extension of the Standard Model, one can extract (see appendix) the HSS interaction term after the $SU(2) \times U(1)_Y$ breaking[9],

$$\mathcal{L}_{HSS} = -\lambda_{HS}\frac{M_W}{2g}HSS \quad \rightarrow \mathcal{C}_{HSS} = -i\frac{\lambda_{HS}M_W}{g} \quad \Rightarrow \quad |\mathcal{M}|^2 = \lambda_{HS}^2\frac{M_W^2}{g^2} \tag{C.121}$$

From Eq.(C.114) one can compute

$$\Gamma_{H\rightarrow SS} = \frac{\lambda_{HS}^2 M_W^2}{32\pi g^2 M_H^2}\sqrt{M_H^2 - 4M_S^2} \tag{C.122}$$

Higgs lifetime from $h \rightarrow X_\mu X_\mu$

There are different ways of computing this process. The easiest way would be to see the vector as a 3-degree of freedom scalar. In this case, the sum on the 3 degrees of freedom convoluated with the spin averages gives

General scalar width : $T \rightarrow \chi\ \bar{\chi}$

Generically, a scalar coupling to a fermion can be written $y\ T\bar{\chi}\chi$. T can be a Higgs field or a moduli field decaying into 2 dark matter candidates for instance. In this case,

$$\begin{aligned}
&\mathcal{M} = -iy\ \bar{u}(p_\chi)v(p_{\bar{\chi}}), \quad \Rightarrow |\mathcal{M}|^2 = 4y^2(p_\chi.p_{\bar{\chi}} - m_\chi^2) \\
&\Rightarrow \Gamma_T = \frac{|\mathcal{M}|^2}{16\pi M_T^2}\sqrt{M_T^2 - 4m_\chi^2} = y^2\frac{(M_T^2 - 4m_\chi^2)^{3/2}}{8\pi M_T^2}
\end{aligned} \tag{C.123}$$

T can also be the inflaton field to compute the reheating temperature.

C.7.8 Majorana case

To compute the decay of the scalar into a pair of Majorana particle, we let the reader have a look at sections (C.2.2) and (C.3.6) to understand the origin of the factors $\frac{1}{2}$ appearing in the Lagrangian and 2 in the Feynman rules, canceling each other at the end. Writting the Lagrangian $\mathcal{L} = y_\chi T\bar{\chi}\chi + \frac{1}{2}y_\lambda T\bar{\lambda}\lambda$, one obtain

[9]Notice the factor 2 between $\mathcal{L}_{HSS}$ and $\mathcal{C}_{HSS}$ coming from the fact that there are 2 identical particles in final states (it corresponds to the 2 possible contractions)

$$\mathcal{M}_\lambda = -iy_\lambda \bar{u}(p_\lambda)v(p_\lambda) \quad \Rightarrow |\mathcal{M}_\lambda|^2 = 4y_\lambda^2(p_\lambda.p_{\bar{\lambda}} - m_\lambda^2)$$
$$\Rightarrow \Gamma_T^\lambda = \left(\frac{1}{2}\right)\frac{|\mathcal{M}_\lambda|^2}{16\pi M_T^2}\sqrt{M_T^2 - 4m_\lambda^2} = y_\lambda^2\frac{(M_T^2 - 4m_\lambda^2)^{3/2}}{16\pi M_T^2} \tag{C.124}$$

The last factor $\left(\frac{1}{2}\right)$ is the symmetry factor (antisymetrization because identical final state). We can also apply it to the decay of the scalar into a spin 3/2 Majorana (like the gravitino for instance[10])

C.7.9 Vector lifetime

Generalities

We can write a general vectorial Lagrangian of a vector V_μ of mass M, impulsion P and polarization λ coupling to fermions ψ_i of masses $m_1,\ m_2$ and impulsions p_1 and p_2,

$$\mathcal{L}_V = i\bar{\psi}_i\gamma^\mu(\partial_\mu - ig_V V_\mu)(\frac{V_i - A_i\gamma^5}{2})\psi_i \tag{C.125}$$

which gives the amplitude

$$\mathcal{M} = \frac{i}{2}g_V\bar{u}(p_1)\gamma^\mu(V_i - A_i\gamma^5)v(p_2)\epsilon_\mu^\lambda, \qquad \mathcal{M}^* = \frac{i}{2}g_V\bar{v}(p_2)\gamma^\nu(V_i - A_i\gamma^5)u(p_2)\epsilon_\nu^{\lambda *}, \tag{C.126}$$

After the summation on the polarization, $\sum_\lambda \epsilon_{1\lambda}^\mu \epsilon_{1\lambda}^{\nu *} = -\eta^{\mu\nu} + P^\mu P^\nu/M_V^2$ one obtains

$$|\mathcal{M}|^2 = C_i\frac{g_V^2}{4}Tr\left[(\not{p}_1 + m_1)\gamma^\mu(V_i - A_i\gamma^5)(\not{p}_2 - m_2)\gamma^\nu(V_i - A_i\gamma^5)\right]\left[-\eta_{\mu\nu} + \frac{P_\mu P_\nu}{M^2}\right]$$
$$= \frac{g_V^2}{2}\left[(|V_i|^2 + |A_i|^2)\left(M^2 - m_1^2 - m_2^2 + \frac{M^4 - (m_2^2 - m_1^2)^2}{M^2}\right) + 6(|V_i|^2 - |A_i|^2)m_1 m_2\right]$$

C_i being the color number of the fermion ψ_i. After a mean on the spin, $|\bar{\mathcal{M}}|^2 = \frac{1}{2J+1}|\mathcal{M}|^2 = \frac{1}{3}|\mathcal{M}|^2$ and using Eq.(C.112) we can write

$$\Gamma_V = \frac{g_V^2 M}{96\pi}\left[(|V_i|^2 + |A_i|^2)\left(2 - x_1^2 - x_2^2 - (x_2^2 - x_1^2)^2\right) + 6(|V_i|^2 - |A_i|^2)x_1 x_2\right]$$
$$\times\sqrt{[1 - (x_1 + x_2)^2][1 - (x_2 - x_1)^2]}, \qquad \text{with} \quad x_i = \frac{m_i}{M} \tag{C.127}$$

[10] Be aware that the Lagrangian of supergravity exhibit more complex structures that the simplest one we propose to study here.

$W^\pm$ **and** Z **widths**

In the Standard Model, the lagrangian involving $W^\pm$ is written following Eq.(C.125) with $g_V = g/\sqrt{2}$, $V_i = A_i = 1$ and neglecting the masses of the final states ($x_1 = x_2 = 0$) we obtain

$$\Gamma(W^+ \to e^+\nu_e) = \frac{g^2 M_W}{48\pi} \tag{C.128}$$

In the case of the Z boson $g_V = g/\cos\theta_W$the couplings A_i and V_i depends on the fermion ψ_i, we have

$$\Gamma(Z \to \psi_i\bar{\psi}_i) = C_i \frac{g^2 M_Z}{48\pi\cos^2\theta_W}\left(|V_i^Z|^2 + |A_i^Z|^2\right) \tag{C.129}$$

with $A_i^Z = T_i^3$ and $V_i^Z = -2q_i\sin^2\theta_W + T_i^3$, q_i being the electric charge of the particle i and $T_i^3 = \pm 1/2$ its isospin.

C.7.10 n-body decay: the exemple of the muon

The phase-space

Application to the muon decay

C.8 s-wave, p-wave, helicity suppression and all that

C.8.1 Velocity suppression

We used frequently in the analysis of the annihilation cross section the development

$$\sigma v = a + bv^2 \tag{C.130}$$

This comes from the fact that, on a general ground, a cross section can be decomposed on the basis of the orbital momentum l : $\sigma v = \sum_l (\sigma v)_l \propto |\sum_l (2l+1) a_l P_l(\cos\theta)|^2$, P_l being the Legendre polynomial function. The lth partial wave contribution to the annihilation rate is suppressed as v^{2l}, where v is the relative velocity between the annihilating particles. Indeed, the lth partial wave includes a factor $P_l(\cos\theta)$. Since the amplitude is an analytic function of the Lorentz invariants s, $t(\cos\theta)$ and $u(\cos\theta)$, the true argument of P_l is proportional to $\bar{p}_i\bar{p}_f\cos\theta$ for an initial particle of impulsion p_i and final particle of impulsion p_f, the latter scaling as $\bar{v}_i\bar{v}_f$. Integration over singularities picks out the highest power of P_l, and we get an amplitude scaling as $(\bar{v}_i\bar{v}_f)^l$. If annihilating particles have a non–zero spin, l is replaced by j and $P_l(\cos\theta)$ by the Wigner function $d^j_{\mu\lambda(\theta)}$. In the expression $a + bv^2$, a is then call $s-$wave ($l = 0$) term and bv^2 $p-$wave ($l = 1$) term.

The virial velocity in our Galactic halo is about ~ 300km s^{-1} $\sim 10^{-3}c$, so even for $l = 1$ the suppression is considerable. Thus only the $l = 0$ partial wave gives an unsuppressed annihilation rate in today's Universe. The $l > 1$ states are too suppressed to give any observable rate. When computing the relic abundance the velocity is less suppressed. Indeed,

the velocity of a dark matter particle χ of mass m_χ can be expressed as function of the temperature of the Plasma from Eq.(2.118), $\langle v^2 \rangle = 6\frac{T}{m}$. As we computed in Eq.(2.125), at the freeze out time T_{FO}, $\frac{m}{T_{FO}} \approx 20$ implying

$$\langle v^2 \rangle_{FO} = 6\frac{T_{FO}}{m_\chi} \approx 0.3. \tag{C.131}$$

C.8.2 Spin selection

For any kind of process occurring at low velocity (like annihilation of dark matter in the halo for instance), one can have indications concerning the form of the annihilation cross section from selection rules. Indeed, fermion bilinears of momentum L have parity $\mathcal{P} = (-1)^{L+1}$. The $(-1)^L$ factor coming from the parity of the Legendre polynomials in the wave function decomposition (the polynomials are odd in in $\cos\theta$ for odd values of L and even for even values of L, parity transforming $\cos\theta$ into $-\cos\theta$. The extra -1 factor exist for Majorana as for Dirac fermions (but not for scalar particles of course) but for different reasons. In the Dirac case, the u and v spinors (equivalently the positive and negative energy states) are independent and have opposite parity corresponding to the ± 1 eigenvalues of the parity operator γ^0 (see section C.3.4). Reinterpreting the two spinor types, or positive and negative energy states, as particle and antiparticle, then leads automatically to opposite intrinsic parity for the particle–antiparticle pair. In the Majorana case, the fermion has intrinsic parity $\pm i$, and so the two–particle state has intrinsic parity $(\pm i)^2 = -1$.

As we discussed in the previous section, the Lth partial wave contribution to the annihilation rate is suppressed as v^{2L}. A Majorana pair is even under charge–conjugation (particle–antiparticle exchange), and so from the relation $C = (-1)^{L+S} = +1$ one infers that L and S must be either both even, or both odd for the pair. The origin of the $C = (-1)^{L+S}$ rule is as follows: under particle–antiparticle exchange, the spatial wave function contributes $(-1)^L$, and the spin wave function contributes (+1) if in symmetric triplet $S = 1$ state, and (-1) if in the antisymmetric $S = 0$ singlet state, i.e. $(-1)^{S+1}$. In addition, there is an overall (-1) for anticommutation of the two particle–creation operators $b^\dagger d^\dagger$ for the Dirac case, and $b^\dagger b^\dagger$ fro the Majorana case. In the case of a bosonic initial state, the formula is the same $C = (-1)^{L+S}$ because the singlet/triplet spin cancel the anti commutation relation.

C.8.3 Application to specific models

We can apply such reasoning to specific models to understand the behavior of the cross section we obtained. For instance, the results we obtained in the section 16.1 where scalar dark matter as well as vectorial dark matter had no velocity dependance (and thus small couplings to the Higgs) whereas fermionic dark matter is excluded as the annihilation is dominated by the p–wave and so is excluded by the direct detection experiments due to the large value of the dark matter coupling to the Higgs to still realize relic abundance constraint (see Fig.16.4). Indeed, the scalar bilinear of the form $\bar{\Psi}\Psi$ is even under $\mathcal{P}$ (see Eq. C.28) which means for a fermionic dark matter, from $\mathcal{P} = (-1)^{L+1}$, that $L = 1$, and the wave function is velocity suppressed. On the contrary, terms of the form $\bar{\Psi}\gamma^5\Psi$ of $\bar{\Psi}\gamma^\mu\Psi$ are $\mathcal{P}$-odd and thus $L = 0$ is authorized. This is valid for Majorana as for Dirac cases. If the dark matter is a boson, the parity is $\mathcal{P} = (-1)^L = +1$ the S^*S or V^*V are thus not velocity

suppressed contrarily to the fermion bilinear $\bar{\Psi}\Psi$. We then conclude that scalar operators for bosonic dark matter and pseudo-scalar ones for fermionic dark mater are the only ones not velocity suppressed.
At the more microscopic level, an amplitude from a Lagrangian containing a $\bar{\Psi}\Psi$ gives an amplitude proportional to $\mathcal{M} \propto \bar{v}u$, applied to the Eq.(C.18) we obtain $\mathcal{M} \propto |p|/m_\Psi$ and is thus velocity suppressed. On the other hand, since γ^5 is antidiagonal, annihilation through $\bar{\Psi}\gamma^5\Psi$ is not velocity suppressed. Concerning the direct detection however, the difference is that we have now an initial state particle and final state particle, the the amplitude is proportional to $\mathcal{M} \propto \bar{u}u$ which is unsuppressed. On the other hand, the $\bar{\Psi}\gamma^5\Psi$ is velocity suppressed due to the antidiagonal γ^5. So the operator $\bar{\Psi}\gamma^5\Psi$ is very efficient: it induces unsuppressed annihilation and suppressed direct detection[11].
We observe this effect in supersymmetry where the Majorana neutralino annihilates mainly through the exchange of the pseudo–scalar A as the charge invariant term is $\bar{\chi}_1^0\gamma^5\chi_1^0 A$ (A being a pseudoscalar, $A^* = -A$, ensuring that the Lagrangian is neutral with respect to electromagnetic charge). Terms of the type $\bar{\chi}_1^0\chi_1^0 H/h$ exist but are velocity suppressed as we explained above.

C.8.4 Helicity suppression

We just saw that only some configurations of the dark matter two–particle wave function survive from the velocity suppression effect. However, for some of them, there is another source of suppression called helicity suppression which acts on the s-wave function. Indeed, in the Majorana case, the operator $\bar{\Psi}\gamma^5\Psi$ is even under charge conjugate ($\Psi \to \Psi$ under $\mathcal{C}$ for a Majorana). As we discussed in the previous section, $\mathcal{C} = (-1)^{L+S}$ which means that both L and S should be even/odd. In the s-wave part of the function, $L = 0 \Rightarrow S = 0 \Rightarrow J = L + S = 0$ (another way to see it is to observe that at a null velocity, for a spinless particle, no direction is space is preferred so $J = 0$). The helicity of the initial state is thus null. If the final state is made of standard model fermions, the bilinear operator should be of the form $\bar{f}_R f_L$, particles and anti–particles with opposite helicity, which imply $J_{\text{final}} \neq 0$, see Fig.(C.2). The only way to conserve the total momentum J in the annihilation is to flip by mass insertion a flipping operator transforming the left handed fermion f_L into a right handed fermion f_R. The amplitude has thus to be proportional to m_f/m_Ψ. This is what we call the helicity suppression mechanism.
It is important to notice that this argument is valid for a Majorana fermion (argument of $J = 0$) but is also valid for a spin 0 particle. However, we cannot apply the conclusion to a Dirac fermion or fermions with vector–like coupling of the style $\bar{\Psi}\gamma^\mu\Psi$, because the vectorial exchanged particle will discuss only to the left handed fermions of the Standard Model.

C.8.5 Summary

We saw that for the annihilation process, it is the C and P quantum numbers which determines the nature of a final state, given an initial state. If one consider that the initial state and final state consist of a pair particle/antiparticle, the operators are given (for fermions)

[11] We have to be careful as the operator $\bar{\Psi}\gamma^5\Psi$ is CP-odd and contribute to several processes, limiting the value of its coupling.

Figure C.2: Illustration of dark matter annihilating into a pair of Standard Model Dirac fermions.

$$C = (-1)^{L+S} \qquad P = (-1)^{L+1} \tag{C.132}$$

whereas for a bosonic pair

$$C = (-1)^{L+S} \qquad P = (-1)^{L} \tag{C.133}$$

The only allowed s-wave states are then $L = 0$, $S = 0$ ($J = 0$); $L = 0$, $S = 1$ ($J = 1$) and $L = 0$, $S = 2$ ($J = 2$). We list in the Tables (C.1) below the value of J, C and P for sermonic and bosonic final state. For any bosonic or sermonic bilinear, the transformation of the bilinear under rotations determines the total angular momentum of the state that this bilinear either creates or annihilates. This information along with the C and P quantum numbers of the bilinear are thus sufficient to determine the spin and orbital angular momentum of the initial and final state. The S and L quantum numbers of the states created (annihilated) by every lowest–dimension bilinear are listed in Table (C.2).

One can recover all the results presented in table (**??**) noticing that for the annihilation, $|\bar{v}(p_1)u(p_2)|^2 \propto s - 4m_\chi^2 \propto v^2$ and for the direct detection, $|\bar{u}(p_1)u(p_2)|^2 \propto s + 4m_\chi^2$; whereas the role are opposite in the case of pseudo-scalar couplings $|\bar{v}(p_1)\gamma^5 u(p_2)|^2$ as the introduction of a γ^5 matrix inverses the impulsion $p_2 \to -p_2$. Similar arguments are valid for $\bar{u}(p_1)\gamma^\mu u(p_2) = \frac{p_2^\mu}{m}\bar{u}(p_1)u(p_2)$ after applying the Dirac equation.

S	L	J	C	P
0	0	0	+	-
0	1	1	-	+
1	0	1	-	-
1	1	0,1,2	+	+
1	2	1,2,3	-	-
1	3	2,3,4	+	+

S	L	J	C	P
0	0	0	+	+
0	1	1	-	-
1	0	1	-	+
1	1	0,1,2	+	-
1	2	1,2,3	-	+
2	0	2	+	+
2	1	1,2,3	-	-
2	2	0,1,2,3,4	+	+
2	3	1,2,3,4,5	-	-
2	4	2,3,4,5,6	+	+

Table C.1: The C and P transformation properties of a fermion/anti–fermion (left) or boson/anti–boson (right) state for a given quantum number.

bilinear	C	P	J	state
$\bar{\psi}\psi$	+	+	0	S=1, L=1
$i\bar{\psi}\gamma^5\psi$	+	-	0	S=0, L=0
$\bar{\psi}\gamma^0\psi$	-	+	0	none
$\bar{\psi}\gamma^i\psi$	-	-	1	S=1, L=0, 2
$\bar{\psi}\gamma^0\gamma^5\psi$	+	-	0	S=0, L=0
$\bar{\psi}\gamma^i\gamma^5\psi$	+	+	1	S=1, L=1
$\bar{\psi}\sigma^{0i}\psi$	-	-	1	S=1, L=0, 2
$\bar{\psi}\sigma^{ij}\psi$	-	+	1	S=0, L=1
$\phi^\dagger\phi$	+	+	0	S=0, L=0
$iIm(\phi^\dagger\partial^0\phi)$	-	+	0	none
$iIm(\phi^\dagger\partial^i\phi)$	-	-	1	S=0, L=1
$B^\dagger_\mu B^\mu$	+	+	0	S=0, L=0; S=2, L=2
$iIm(B^\dagger_\nu\partial^0 B^\nu)$	-	+	0	none
$iIm(B^\dagger_\nu\partial^i B^\nu)$	-	-	1	S=0, L=1; S=2, L=1, 3
$i(B^\dagger_i B_j - B^\dagger_j B_i)$	-	+	1	S=1, L=0, 2
$i(B^\dagger_i B_0 - B^\dagger_0 B_i)$	-	-	1	S=0, L=1; S=2, L=1,3
$\epsilon^{0ijk}B_i\partial_j B_k$	+	-	0	S=1, L=1
$-\epsilon^{0ijk}B_0\partial_j B_k$	+	+	1	S=2, L=2
$B^\nu\partial_\nu B_0$	+	+	0	S=0, L=0; S=2, L=2
$B^\nu\partial_\nu B_i$	+	-	1	S=1, L=1

Table C.2: The C and P and J quantum numbers of any state that can be either created or annihilated by the bilinear. For each possible state, the S and L quantum numbers are also given.

operator	annihilation $\langle\sigma v\rangle$	direct detection [A,v_0]
$g\ \bar{\chi}\chi\bar{q}q$	$\propto g^2v^2$ [$\bar{\chi}\chi$: S=1, L=1]	SI $\propto g^2A^2 \times f^N(q)$
$g\ \bar{\chi}\gamma^5\chi\bar{q}q$	$\propto g^2m_q^2$ [$\bar{\chi}\gamma^5\chi$: S=0, L=0 $\Rightarrow J=0$:h.f.]	0 [$\propto (v_0^\chi)^2$]
$g\ \bar{\chi}\chi\bar{q}\gamma^5q$	$\propto g^2v^2$ [$\bar{\chi}\chi$: S=1, L=1]	0 [$\propto (v_0^q)^2$]
$g\ \bar{\chi}\gamma^5\chi\bar{q}\gamma^5q$	$\propto g^2m_q^2$ [$\bar{\chi}\gamma^5\chi$: S=0, L=0 $\Rightarrow J=0$:h.f.]	0 [$\propto (v_0^q)^2(v_0^\chi)^2$]
$g\ \bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$\propto g^2$	SI $\propto g^2 \times (2u+d) \times p_\chi.p_q$
$g\ \bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5q$	$\propto g^2$	0 [$\propto (v_0^q)^2$]
$g\ \bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$\propto m_q^2$ [L=0,S=0(C=1) $\Rightarrow$ J=0:h.f.]	0 [$\propto (v_0^\chi)^2$]
$g\ \bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5q$	$\propto m_q^2$ [L=0,S=0(C=1) $\Rightarrow$ J=0:h.f.]	SD $\propto g^2 \times (A-2Z)^2$

C.9 Schrodinger equation

C.9.1 Generalities

The Schrodinger equation for a particle of mass M with an impulsion k described by a wave function ψ in a potential V is usually written

$$\frac{1}{2M}\Delta\psi + V\ \psi = E\ \psi = \frac{k^2}{2M}\psi \tag{C.134}$$

with Δ the laplacian which can be expressed in spherical coordinates (r,θ,ϕ)

$$\Delta\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}\right] \tag{C.135}$$

For a radial potential, it is possible to decompose the wave function

$$\psi = \psi_{kl}^m(r) = R_{kl}(r)\ \Theta_{lm}(\theta)\ \Phi_m(\phi). \tag{C.136}$$

In the above equation, we have defined ψ_{kl}^m as the wave function eigenvector of the momentum k, kinetic moment l and its projection on z-axis m. The operator $\hat{l}_z\psi = m\psi = -i\frac{\partial}{\partial\phi}$, the solution for $\Phi(\phi)$ can then be solved directly after normalization $\int_0^{2\pi}\Phi_m^*(\phi)\ \Phi_{m'}(\phi)d\phi = \delta_{mm'}$

$$-iR_{kl}(r)\ \Theta_{lm}\ \frac{\partial\Phi_m(\phi)}{\partial\phi} = m\ R_{kl}(r)\ \Theta_{lm}(\theta)\ \Phi_m(\phi) \Rightarrow \Phi_m(\phi) = \frac{1}{\sqrt{2\pi}}e^{im\phi} \tag{C.137}$$

whereas the l dependance is included in the Θ function from the equation $\hat{l}^2\psi = l(l+1)\psi$ with $\hat{l}^2 = -\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$. Implementing Eq.(C.136) in the preceding equation we obtain

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta_{lm}}{d\theta}\right) - \frac{m^2}{\sin^2\theta}\Theta_{lm} + l(l+1)\Theta_{lm} = 0. \tag{C.138}$$

this equation is well known in the theory of spherical functions. Corresponding solutions Are called *associated Legendre polynomials* $P_l^m(\cos\theta)$. After normalization, the eigenfunctions solutions of the Eq.(C.138) can be written for $m > 0$,

$$\Theta_{lm} = (-1)^m i^l \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}} P_l^m(\cos\theta). \tag{C.139}$$

For any m, we combined $\Theta(\theta)$ and $\Phi(\phi)$ in $Y_{lm}(\theta,\phi)$

$$Y_{lm}(\theta,\phi) = (-1)^{\frac{m+|m|}{2}} i^l \left[\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}\right]^{1/2} P_l^{|m|}(\cos\theta)\ e^{im\phi} \tag{C.140}$$

The $m = 0$ solution is often useful for problems with a z-axis symmetry (independence on the angle ϕ)

$$Y_{l0} = i^l \sqrt{\frac{2l+1}{4\pi}} P_l(\cos\theta) \tag{C.141}$$

with

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right] \tag{C.142}$$

We then can summarize that the general solution of Schrodinger equation can be written

$$\psi_k = \sum_l A_l\ P_l(\cos\theta)\ R_{kl}(r) \tag{C.143}$$

where $R_{kl}(r)$ are the continuum radial functions associated with angular momentum l satisfying

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}R_{kl}\right) + \left(\frac{l(l+1)}{r^2} + 2M\ V(r)\right) R_{kl} = k^2\ R_{kl} \tag{C.144}$$

C.9.2 Solutions

Let's concentrate first in the case of the radial solution $Rkl^0(r)$ for a free particle ($V(r) = 0$) Defining $\chi_{kl}(r) = rR^0_{kl}(r)$, equation (C.144) yields

$$\frac{d^2\chi_{kl}(r)}{dr^2} + \left(k^2 - \frac{l(l+1)}{r^2}\right)\chi_{kl}(r) = 0. \tag{C.145}$$

For the simplest case $l = 0$, the two solutions are $\chi_{k0}(r) = \sin(kr)/\cos(kr)$ giving for $R^0_{k0}(r)$ the zeroth-order *Bessel* and *Neumann* functions respectively :

$$R^0_{k0}(r) = \frac{\sin(kr)}{kr} \quad \text{[Bessel]} \quad \text{or} \quad R^0_{k0}(r) = -\frac{\cos(kr)}{kr} \quad \text{[Neumann]} \tag{C.146}$$

Note that the Bessel function is the one well–behaved at the origin. To obtain the general solution for any value of the momentum l the idea is to write $R^0_{kl}(r) = (kr)^l\chi_{kl}(r)$ and replacing in Eq.(C.144) we can show that the function $\chi = \frac{1}{r}\frac{d}{dr}\chi_{kl}$ follows the equation for the momentum $l+1$. We can then write a recursion formula for generating all the χ_{kl} from χ_{k0}:

$$\chi_{k(l+1)}(r) = \frac{1}{k^2 r}\frac{d\chi_{kl}(r)}{dr} \quad \Rightarrow \quad R^0_{kl}(r) = (-kr)^l \left(\frac{1}{k^2 r}\frac{d}{dr}\right)^l (\frac{\sin kr}{kr}) \tag{C.147}$$

Developing[12] $\frac{\sin kr}{kr} = \sum_0^\infty (-1)^n \frac{(kr)^{2n}}{(2n+1)!}$, we can show that

$$R^0_{kl}(r) \simeq \frac{(kr)^l}{(2l+1)!!} \quad \text{as} \quad r \to 0 \tag{C.148}$$

This r^l behavior near the origin is the usual well-behaved solution to Schrodinger's equation in the region where the centrifugal term dominates. Now, for large values or r, it is obvious that the dominant term is generated by differentiating *only* the trigonometric function at each step which gives

$$R^0_{kl}(r) \simeq \frac{1}{kr} \sin\left(kr - l\frac{\pi}{2}\right) \quad \text{as} \quad r \to \infty \tag{C.149}$$

In fact, the real wave function should be a combination of the Bessel *and* Neumann wave function. One should thus apply the recursive procedure also to the function $\frac{\cos kr}{kr}$ which is equivalent to inroduce an extra phase factor called the *phase shift*, giving at the end the general solution

$$R^0_{kl}(r) \simeq \frac{1}{kr} \sin\left(kr - l\frac{\pi}{2} + \delta_l\right) \quad \text{as} \quad r \to \infty \tag{C.150}$$

In the case of a generic potential, the Schrodinger's equation can then be written

$$\frac{d^2\chi_{kl}(r)}{dr^2} + \left(k^2 - \frac{l(l+1)}{r^2} - 2MV(r)\right)\chi_{kl}(r) = 0. \tag{C.151}$$

C.10 Useful spectrum

C.10.1 Gamma spectrum

In order to determine these spectral functions, we generated 300000 events of standard model particles decaying (directly or through secondary decays) into γ-rays using the PYTHIA [744] package, taking care in order to include all possible decay channels. Following the method of Ref. [745], we fitted the resulting spectra through functions of the form:

$$\frac{dN^i_\gamma}{dx} = \exp\left[F_i(\ln(x))\right], \tag{C.152}$$

where i represents the i-th WIMP annihilation channel, $i = WW, ZZ, \text{etc}$; $x = E_\gamma/m_\chi$ with m_χ being the WIMP mass and F are seventh-order polynomial functions which were found

[12] The minus sign appearing in $(-kr)^l$ in Eq.(C.147) comes from the standard normalization of the function R^0_{kl}

to be the following:

$$
\begin{aligned}
WW(x) &= -7.72088528 - 8.30185509\,x - 3.28835893\,x^2 - 1.12793422\,x^3 \\
&- 0.266923457\,x^4 - 0.0393805951\,x^5 - 0.00324965152\,x^6 - 0.000113626003\,x^7, \\
ZZ(x) &= -7.67132139 - 7.22257853\,x - 2.0053556\,x^2 - 0.446706623\,x^3 \\
&- 0.0674006343\,x^4 - 0.00639245566\,x^5 - 0.000372241746\,x^6 - 1.08050617 \cdot 10^{-5}\,x^7, \\
b\bar{b}(x) &= -11.4735403 - 17.4537277\,x - 11.5219269\,x^2 - 5.1085887\,x^3 \\
&- 1.36697042\,x^4 - 0.211365134\,x^5 - 0.0174275134\,x^6 - 0.000594830839\,x^7, \\
u\bar{u}(x) &= -4.56073856 - 8.13061428\,x - 4.98080492\,x^2 - 2.23044157\,x^3 \\
&- 0.619205713\,x^4 - 0.100954451\,x^5 - 0.00879980996\,x^6 - 0.00031573695\,x^7, \\
d\bar{d}(x) &= -4.77311611 - 10.6317139\,x - 8.33119583\,x^2 - 4.35085535\,x^3 \\
&- 1.33376908\,x^4 - 0.232659817\,x^5 - 0.0213230457\,x^6 - 0.000796017819\,x^7, \\
\tau^-\tau^+(x) &= -5.64725113 - 10.8949451\,x - 7.84473181\,x^2 - 3.50611639\,x^3 \\
&- 0.942047119\,x^4 - 0.14691925\,x^5 - 0.0122521566\,x^6 - 0.000422848301\,x^7.
\end{aligned}
$$

The case of WIMP annihilation into $\mu^+\mu^-$ pairs has a relatively small decay contribution, to the photon spectrum, coming from the $\mu \to e^-\bar{\nu}_e\nu_\mu\gamma$ channel, which has a small branching ratio. e^+e^- pair production contributes to the gamma–ray spectrum through different (not decay) processes, mainly inverse Compton scattering and synchrotron radiation. These contributions depend crucially on the assumptions made concerning the intergalactic medium and will not be analysed here. This means, practically, that the e^+e^- and $\mu^+\mu^-$ spectral functions are set equal to zero. A graphical representation of these functions can be seen in Fig.C.3. These functions can afterward be used in order to generate any gamma-ray flux according to eq. (11.14)

As we can see, all contributions are quite similar, apart from the $\tau^-\tau^+$ channel which has a characteristic hard form. Nevertheless, at high energies, the form of all contributions becomes almost identical.

C.10.2 Positron spectrum

For the positron spectrum at the source, the same procedure gives us

Figure C.3: Separate contributions from standard model particles decaying into γ-rays for $m_\chi = 100$ and 500 GeV. The PYTHIA result points have been suppressed for the sake of clarity.

$$
\begin{aligned}
WW(x) &= -1.470768 - 1.865343\,x - 1.865343\,x^2 - 1.940513\,x^3 \\
&- 0.7538001\,x^4 - 01.432966\,x^5 - 0.01350088\,x^6 - 0.0005048965\,x^7, \\
ZZ(x) &= -2.909439 - 1.091164\,x - 1.184799\,x^2 - 1.621001\,x^3 \\
&- 0.6838962\,x^4 - 0.1359553\,x^5 - 0.01320953\,x^6 - 5.058684 \cdot 10^{-4}\,x^7, \\
b\bar{b}(x) &= -12.33180 - 27.10877\,x - 29.62288\,x^2 - 18.59724\,x^3 \\
&- 6.705565\,x^4 - 1.436652\,x^5 - 0.1813250\,x^6 - 0.01246600\,x^7, \\
c\bar{c}(x) &= -12.07344 - 17.57427\,x - 11.85548\,x^2 - 5.235151\,x^3 \\
&- 1.391299\,x^4 - 0.2141368\,x^5 - 0.01755747\,x^6 - 0.0005935255\,x^7, \\
t\bar{t}(x) &= -8.90339 - 21.5246\,x - 27.6548\,x^2 - 18.8567\,x^3 \\
&- 6.80256\,x^4 - 1.32199\,x^5 - 0.129489\,x^6 - 0.00494683\,x^7, \\
\tau^-\tau^+(x) &= -4.004877 - 6.630655\,x - 5.779419\,x^2 - 3.504474\,x^3 \\
&- 1.200937\,x^4 - 0.2254182\,x^5 - 0.02172664\,x^6 - 0.0008412294\,x^7. \\
HH(x) &= -8.731713 - 10.64431\,x - 6.924815\,x^2 - 3.131443\,x^3 \\
&- 0.7984188\,x^4 - 0.1135447\,x^5 - 0.0084599666\,x^6 - 0.00025748991\,x^7.((\text{fora200GeVHiggs})
\end{aligned}
$$

C.10.3 Antiproton spectrum

For the antiproton, we can parameterize the spectrum following :

$$
\frac{dN^i_{\bar{p}}}{dE} = \frac{1}{m_\chi \left(p^i_1 x^{p_3} + p^i_2 |\log_{10}(x)|^{p^i_4}\right)}, \tag{C.153}
$$

with

i	$1/p^i_1$	$1/p^i_2$	$1/p^i_3$	$1/p^i_4$
W	$306\ m_\chi^{0.28} + 7.4 \times 10^{-4} m_\chi^{2.25}$	$2.32\ m_\chi^{0.05}$	$-8.5\ m_\chi^{-0.31}$	$-0.39\ m_\chi^{-0.17} - 2 \times 10^{-2} m_\chi^{0.23}$
Z	$480\ m_\chi^{0.26} + 9.6 \times 10^{-4} m_\chi^{2.27}$	$2.17\ m_\chi^{0.05}$	$-8.5\ m_\chi^{-0.31}$	$-0.33\ m_\chi^{-0.075} - 1.5 \times 10^{-4} m_\chi^{0.71}$
t	$1.35\ m_\chi^{1.45}$	$1.18\ m_\chi^{0.15}$	-2.22	-0.21
b	$1.75\ m_\chi^{1.4}$	$1.54\ m_\chi^{0.11}$	-2.22	$-0.31\ m_\chi^{-0.052}$
c	$1.7\ m_\chi^{1.4}$	$3.12\ m_\chi^{0.04}$	-2.22	$-0.39\ m_\chi^{-0.076}$

Appendix D

Lagrangians

D.1 Standard Model

The SM Lagrangian can be written

$$\begin{aligned}\mathcal{L}_{SM} &= -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + |D_\mu H|^2 && \text{(D.1)}\\ &+ i\bar{\Psi}\gamma^\mu D_\mu \Psi && \text{(D.2)}\end{aligned}$$

with $T^a = \sigma^a/2$ (σ^a the Pauli matrices) and where D_μ represents the covariant derivative,

$$D_\mu = (\partial_\mu - ig_3 G^a T^a_3 - igW^a_\mu T^a_2 - ig'\frac{Y_H}{2}B_\mu) \tag{D.3}$$

Ψ are fermonic multiplet $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} \frac{1-\gamma^5}{2}\nu \\ \frac{1-\gamma^5}{2}e \end{pmatrix}$ in the case of the $SU(2)_L$ generators $W^a_\mu T^a$ for instance) and

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \qquad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{abc}W^b_\mu W^c_\nu \tag{D.4}$$

where under the $SU(2)\times U(1)_Y$ of parameter α^a and β, W_μ, B_μ and H transforms

$$\begin{aligned}&\Psi_L/H \to e^{-i\alpha_a T^a - i\beta\frac{Y}{2}}\Psi_L/H \qquad && \Psi_R \to e^{-i\beta\frac{Y}{2}}\Psi_R\\ &B_\mu \to B_\mu - \frac{1}{g'}\partial_\mu\beta \qquad && W^a_\mu \to W^a_\mu - \frac{1}{g}\partial_\mu\alpha^a - f^{abc}\alpha^b W^c_\mu\end{aligned}$$

D.1.1 Higgs couplings

The minimum of the Higgs potential is obtained for $v = \mu_H/\sqrt{\lambda_H}$

$$D_\mu H = (\partial_\mu - igW^a_\mu T^a - ig'\frac{Y_H}{2}B_\mu)H = \begin{pmatrix} \partial_\mu - ig\frac{W^3_\mu}{2} - ig'\frac{Y_H}{2}B_\mu & -i\frac{g}{2}(W^1_\mu - iW^2_\mu) \\ -i\frac{g}{2}(W^1_\mu + iW^2_\mu) & \partial_\mu + ig\frac{W^3_\mu}{2} - ig'\frac{Y_H}{2}B_\mu \end{pmatrix} H$$

If one develops around $H = \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix}$, $|D_\mu H|^2 = \begin{pmatrix} -i\frac{g}{2\sqrt{2}}(W^1_\mu - iW^2_\mu)(v+h) \\ \frac{\partial_\mu h}{\sqrt{2}} + i(\frac{gW^3_\mu}{2\sqrt{2}} - g'\frac{Y_H B_\mu}{2\sqrt{2}})(v+h) \end{pmatrix}^2$ if one defines $W^\pm_\mu = (W^1_\mu - iW^2_\mu)/\sqrt{2}$, one can identify $M^2_W|W^\pm|^2 + \frac{M^2_Z}{2}|Z_\mu|^2$ which gives

$$M_W = \frac{gv}{2} \qquad M_Z = \frac{\sqrt{g^2+g'^2}}{2}v = \frac{g}{2\cos\theta_W}v \tag{D.5}$$

For the Higgs coupling, the development of $|D_\mu H|^2$ gives $\mathcal{L} \supset \frac{g^2 v}{2} H W^+_\mu W^-_\mu + \frac{g^2 v}{4\cos^2\theta_W} H Z_\mu Z_\mu = gM_W H W^+_\mu W^-_\mu + \frac{gM_Z}{2} H Z_\mu Z_\mu$. The couplings are then

$$\mathcal{C}_{HWW} = igM_W, \quad \mathcal{C}_{HZZ} = \frac{igM^2_Z}{2M_W} \times 2 \ \text{(symmetry factor, Z = Z*)} = ig\frac{M^2_Z}{M_W}. \tag{D.6}$$

D.1.2 Vectorial couplings

Developing the Lagrangian (D.2) using (D.3) on can write

$$g\bar{\Psi}_L\gamma^\mu W^a_\mu T^a \Psi_L = \frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu W^+_\mu(1-\gamma^5)\nu + \frac{g}{2\sqrt{2}}\bar{\nu}\gamma^\mu W^+_\mu(1-\gamma^5)e \tag{D.7}$$

D.2 Singlet scalar

The simplest extension of the SM is the addition of a real singlet scalar field.Although it is possible to generalize to scenarios with more than one singlet, the simplest case of a single additional singlet scalar provides a useful framework to analyze the generic implications of an augmented scalar sector to the SM. The most general renormalizable potential involving the SM Higgs doublet H and the singlet S is

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{SM} + (D_\mu H)^\dagger(D^\mu H) + \frac{1}{2}\mu^2_H H^\dagger H - \frac{1}{4}\lambda_H H^4 \\ &+ \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{\lambda_S}{4}S^4 - \frac{\mu^2_S}{2}S^2 - \frac{\lambda_{HS}}{4}S^2 H^\dagger H \\ &- \frac{\kappa_1}{2}H^\dagger H S - \frac{\kappa_3}{3}S^3 - V_0 \end{aligned} \tag{D.8}$$

where D_μ represents the covariant derivative.

$$D_\mu H = (\partial_\mu - igW^a_\mu T^a - ig'\frac{Y_H}{2}B_\mu)H = \tag{D.9}$$

$$D_\mu H = (\partial_\mu - igW^a_\mu T^a - ig'\frac{Y_H}{2}B_\mu)H = \begin{pmatrix} \partial_\mu - ig\frac{W^3_\mu}{2} - ig'\frac{Y_H}{2}B_\mu & -i\frac{g}{2}(W^1_\mu - iW^2_\mu) \\ -i\frac{g}{2}(W^1_\mu + iW^2_\mu) & \partial_\mu + ig\frac{W^3_\mu}{2} - ig'\frac{Y_H}{2}B_\mu \end{pmatrix} H$$

and we have eliminated a possible linear term in S by a constant shift, absorbing the resulting S-independent term in the vacuum energy V_0. We require that the minimum of the potential

occur at $v = 246$ GeV . Fluctuations around this vacuum expectation value are the SM Higgs boson. For the case of interest here for which S is stable and may be a dark matter candidate, we impose a Z_2 symmetry on the model, thereby eliminating the κ_1 and κ_3 terms. We also require that the true vacuum of the theory satisfies $\langle S \rangle = 0$, thereby precluding mixing of S and the SM Higgs boson and the existence of cosmologically problematic domain walls. After the electroweak breaking, the scalar lagrangian can be written

$$\mathcal{L}_S = -\frac{M_S^2}{2} S^2 - \frac{1}{8}\lambda_{HS} S^2 h^2 - \lambda_{HS}\frac{M_W}{2g} h S^2 \tag{D.10}$$

In this case, the masses of the scalars are

$$v = \frac{\mu_H}{\sqrt{\lambda_H}}, \qquad M_H = \sqrt{2}\mu_H, \qquad m_S = \sqrt{\mu_S^2 + \frac{\lambda_{HS}}{\lambda_H}\frac{\mu_H^2}{2}} \tag{D.11}$$

and the HSS coupling generated is

$$\mathcal{C}_{HSS}: \quad -\frac{\lambda_{HS} M_W}{2g} \times 2 \ \text{(symmetry factor, S = S*)} \ = -\frac{\lambda_{HS} M_W}{g}. \tag{D.12}$$

D.3 Extra U(1) and kinetic mixing

The matter content of any *dark* $U(1)_D$ extension of the SM can be decomposed into three families of particles:

- The *Visible sector* is made of particles which are charged under the SM gauge group $SU(3) \times SU(2) \times U_Y(1)$ but not charged under $U_D(1)$ (hence the *dark* denomination for this gauge group)
- the *Dark sector* is composed by the particles charged under $U_D(1)$ but neutral with respect of the SM gauge symmetries. The dark matter (ψ_0) candidate is the lightest particle of the *dark sector*
- The *Hybrid sector* contains states with SM *and* $U_D(1)$ quantum numbers. These states are fundamental because they act as a portal between the two previous sector through the kinetic mixing they induce at loop order.

From these considerations, it is easy to build the effective Lagrangian generated at one loop :

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{SM}} - \frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{\delta}{2}\tilde{B}_{\mu\nu}\tilde{X}^{\mu\nu} \\ &+ i\sum_i \bar{\psi}_i \gamma^\mu D_\mu \psi_i + i\sum_j \bar{\Psi}_j \gamma^\mu D_\mu \Psi_j \end{aligned} \tag{D.13}$$

$\tilde{B}_\mu$ being the gauge field for the hypercharge, $\tilde{X}_\mu$ the gauge field of $U_D(1)$ and ψ_i the particles from the hidden sector, Ψ_j the particles from the hybrid sector, $D_\mu = \partial_\mu - i(q_Y \tilde{g}_Y \tilde{B}_\mu + q_D \tilde{g}_D \tilde{X}_\mu + g T^a W_\mu^a)$, T^a being the $SU(2)$ generators, and

$$\delta = \frac{\tilde{g}_Y \tilde{g}_D}{16\pi^2} \sum_j q_Y^j q_D^j \log\left(\frac{m_j^2}{M_j^2}\right) \tag{D.14}$$

with m_j and M_j being hybrid mass states [513] .

Notice that the sum is on all the hybrid states, as they are the only ones which can contribute to the $\tilde{B}_\mu \tilde{X}_\mu$ propagator. After diagonalization of the current eigenstates that makes the gauge kinetic terms of Eq.(D.13) diagonal and canonical, we can write after the $SU(2)_L \times U(1)_Y$ breaking[1] :

$$\begin{aligned} A_\mu &= \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu \\ Z_\mu &= \cos\phi(\cos\theta_W W_\mu^3 - \sin\theta_W B_\mu) - \sin\phi X_\mu \\ (Z_D)_\mu &= \sin\phi(\cos\theta_W W_\mu^3 - \sin\theta_W B_\mu) + \cos\phi X_\mu \end{aligned} \tag{D.15}$$

with, at the first order in δ:

$$\begin{aligned} \cos\phi &= \frac{\alpha}{\sqrt{\alpha^2 + 4\delta^2 \sin^2\theta_W}} \quad \sin\phi = \frac{2\delta \sin\theta_W}{\sqrt{\alpha^2 + 4\delta^2 \sin^2\theta_W}} \\ \alpha &= 1 - M_{Z_D}^2/M_Z^2 - \delta^2 \sin^2\theta_W \\ &\pm \sqrt{(1 - M_{Z_D}^2/M_Z^2 - \delta^2 \sin^2\theta_W)^2 + 4\delta^2 \sin^2\theta_W} \end{aligned} \tag{D.16}$$

and + (-) sign if $M_{Z_D} < (>) M_Z$. The kinetic mixing parameter δ generates an effective coupling of SM states $\psi_{\rm SM}$ to Z_D, and a coupling of ψ_0 to the SM Z boson which induces an interaction on nucleons. Developing the covariant derivative on SM and ψ_0 fermions state, we computed the effective $\psi_{\rm SM}\psi_{\rm SM} Z_D$ and $\psi_0 \psi_0 Z$ couplings at first order[2] in δ and obtained

$$\mathcal{L} = q_D \tilde{g}_D (\cos\phi \; Z'_\mu \bar{\psi}_0 \gamma^\mu \psi_0 + \sin\phi \; Z_\mu \bar{\psi}_0 \gamma^\mu \psi_0). \tag{D.17}$$

D.4 Other typical Lagrangians

In different field of physics, from inflation to symmetry breaking to dark matter one have sometimes to deal with different types of Lagrangians. Here are some exemples. This is mainly to check the signs and symmetries factors.

D.4.1 Real scalar lagrangian

For a real scalar, we can write

$$\mathcal{L}_{Rscal} = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{6}\mu\phi^3 - \frac{1}{24}\lambda\phi^4 \tag{D.18}$$

[1] Our notation for the gauge fields are $(\tilde{B}^\mu, \tilde{X}^\mu)$ before the diagonalization, (B^μ, X^μ) after diagonalization and (Z^μ, Z_D^μ) after the electroweak breaking.

[2] One can find a detailed analysis of the spectrum and couplings of the model in the appendix of [494].

whereas for a complex scalar one obtain

$$\mathcal{L}_{Cscal} = -\partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^2 \tag{D.19}$$

Appendix E

Useful statistics

E.1 5σ and p-value

E.1.1 5σ

An important example of constructing a confidence interval is when a single variable x follows a Gaussian distribution. This happens frequently when one needs to work with a great number ($\gtrsim 100$) of events. We then define the probability that a variable appear within an interval $\pm\delta$

$$1-\alpha = \frac{1}{\sqrt{2\pi}\sigma}\int_{\mu-\delta}^{\mu+\delta} e^{-(x-\mu)^2/2\sigma^2}dx = \mathrm{erf}\left(\frac{\delta}{\sqrt{2}\sigma}\right) \tag{E.1}$$

where μ is the mean of the variable x, and σ its variance. If one have to deal with a few number of events (dozens) it is more precise to use the poisson distribution, $f(n;\nu) = \frac{\nu^n e^{-\nu}}{n!}$ with $n = 1, 2, 3..$ and $\nu > 0$. We illustrate the distribution behavior and its $\sigma limits$ in the figure(E.1) and table (E.1). As a remark, the air below the Gaussian at 5σ correspond to a "null-hypothesis" (see definition of the p-value below) of $\sim 6\times10^{-7}$ (see table) so a probability $\pm3\times10^{-7}$. In other word, an event observed at 5 σ above a predicted background would have one chance on 3 millions to be part of the background (and thus 99.99997 % of chance to be a "something else" beyond the standard model). We will discuss this case more in detail when examining the look-elsewhere effect.

Indeed, as we will see later, a 5σ confidence level - expressed in terms of Higgs search for instance- discovery corresponds to the condition that the probability of finding an excess

Confidence level δ	α	Probability
1σ	0.3173	68.27 %
2σ	4.55×10^{-2}	95.45 %
3σ	2.7×10^{-3}	99.73 %
4σ	6.3×10^{-5}	99.9937 %
5σ	5.7×10^{-7}	99.99994 %
6σ	2.0×10^{-9}	99.9999998 %

Table E.1: Area of the tails α outside $\pm\delta$ from the mean of a Gaussian distribution.

Figure E.1: Gaussian distribution as function of standard σ deviations.

rate of the measured size that is caused by standard model particles without Higgs particle is lower than 3×10^{-7}. Setting the limit for calling the collected data a discovery is, of course, a matter of convention. It is based on a trade–off between the advantage of calling a viable scientific claim empirically well-established and the potential damage of endorsing a false scientific claim. In many scientific fields, a 3σ effect, that is a probability of less than 0.15 % that the null hypothesis hi olds, is considered sufficient for establishing a phenomenon. In particle physics, the discovery of a new particle is taken to be of very high importance and is used in the analysis of the background in all future high energy scattering experiments. Therefore, the risk of erroneously acknowledging a discovery of a particle should be kept particularly low and a stronger criterion seems advisable. On should also take account the look-elsewhere effect that can reduce considerably the 5σ effect (see below). Indeed, historically the 5σ limit was established based on largely pragmatic considerations. While statistical fluctuations at a 4σ level did and do occur from time to time in high energy physics experiments, no 5σ signal in a particle experiment has up to this point ever turned out to be a fluctuation. A 5σ limit therefore seemed plausible simply based on historical record. The fact that a 4σ fluctuations do occur can be statistically explained based on the number of experiments that are carried out in conjonction with the size of the look elsewhere effects which usually applies in those contexts.

E.1.2 p-value

The p-value is defined as the probability of obtaining a statistic at least as extreme as the one actually observed assuming that the null hypothesis is true. The smaller is the p-value, the less probable the "null hypothesis" is and so the results is "probably" a signal above a background. In other words, if the p-value computed by statistical tools (given by

experimentalists) is below 0.05 or 0.01 (depending the significance one is asking for) the researcher will reject the "null hypothesis": the observed result would be highly unlikely under the null hypothesis. The p-value can be seen as the probability that "something" observed happen assuming random coincidence.

E.2 Systematics vs statistics

Any experimental analysis has to deal with two kind of uncertainties. The statistical uncertainties and systematics ones. In many case the systematics uncertainties are comparable to the statistical ones. However, consistent definition and practice is elusive which leads to confusion and in some cases incorrect interference. We will try to give some practical definition in this section to help understanding the importance of each of them.

Examples of statistical uncertainties include the finite resolution of an instrument, the Poisson fluctuations associated with measurements involving finite sample sizes and random variations in the system one is examining. Systematic uncertainties, on the other hand, arise from uncertainties associated with the nature of the measurement apparatus, assumptions made by the experimenter, or the model used to make inferences based on observed data. Common examples of systematic uncertainty include uncertainties that arise from the calibration of the measurement device, the probability of detection of a given type of interaction (often called the "acceptance" of the detector), and parameters of the model used to make inferences that themselves are not precisely known.

Let's illustrate this correlations in a typical example, the measurement of the W boson cross section in hadronic colliders. The production rate of the W boson σ_W in $p\bar{p}$ collisions is theoretically known in the Standard Model. This cross section is also measured by a number of observed counts N_c (given some topologies and specific cuts depending on the channels analyzed by the experimenters), estimating the number of "background" events in this sample from other process N_b, estimating the acceptance of the apparatus including all selection requirements used to define the sample of events ϵ, and counting the number of $p\bar{p}$ annihilation, L. The cross section for W boson production is then

$$\sigma_W = \frac{N_c - N_b}{\epsilon L} \tag{E.2}$$

We present in Fig.(E.2-left) this measurement performed by CDF collaboration in 2003, where the transverse mass of a sample of candidate $W \to \epsilon\nu_e$ decays is illustrated. The measurement is quoted as

$$\sigma_W = 2.64 \pm 0.01(\text{stat}) \pm 0.18(\text{syst}) \text{ nb}, \tag{E.3}$$

where the first uncertainty reflects the statistical uncertainty arising from the size of the candidate sample (approximately 38 000 candidates) and the second uncertainty arises from the background subtraction in Eq.(E.2). We can estimate these uncertainties as

Figure E.2: Left: The transverse mass distribution for the W boson candidate as observed by CDF in 2003. The peak reflects the Jacobian distribution typical of W boson decays. The points are the measured distribution and the various histograms are the predicted distribution from W decays and the other background processes. Right: The variation of the background estimate as the isolation cut value is varied from 0.3 (the default value) up to 0.8. The isolation variable is a measure of the fraction of energy observed in a cone near the electron candidate normalized to the energy of the electron candidate.

$$\sigma_{stat} = \sigma_0 \sqrt{\frac{1}{N_c}}$$
$$\sigma_{syst} = \sigma_0 \sqrt{\left(\frac{\delta N_b}{N_b}\right)^2 + \left(\frac{\delta \epsilon}{\epsilon}\right)^2 + \left(\frac{\delta L}{L}\right)^2} \tag{E.4}$$

where the three terms in σ_{syst} are the uncertainties arising from the background estimate δN_b, the acceptance $\delta\epsilon$ and the integrated luminosity δL. The parameter σ_0 is the measured value.

The "paradox" that makes the definition of statistics and systematics so incertain is that in the *same* sample of events, experimenter also measured the acceptance ϵ through the Z boson observation from its decay into electron. The level of accuracy on ϵ becomes also a stochastic variable depending on the size of the sample. One can then also consider $\delta\epsilon$ as a statistical uncertainty and redefined

$$\sigma_{stat} = \sigma_0 \sqrt{\frac{1}{N_c} + \left(\frac{\delta\epsilon}{\epsilon}\right)^2}$$
$$\sigma_{syst} = \sigma_0 \sqrt{\left(\frac{\delta N_b}{N_b}\right)^2 + \left(\frac{\delta L}{L}\right)^2} \tag{E.5}$$

resulting in a different assignment of statistical and systematics uncertainties. This matter in a sense that as a definition we took for "systematic" uncertainty, it should *not* depend on the size of the sample used to make the observation. It is supposed to come from an external measurement like a calibration for instance, and does not scale with the sample size. In the specific case of the measurement of the W cross section, ϵ does not fill this requirement. We can include in this family of systematics any degree of freedom which depends stochastically on an external measurement (which means any observable which is dominate by statistics uncertainties). If a threshold or an energy calibration (systematic) is dominated by a measurement from another experiment dominated by statistics, these uncertainties could be considered in fact as statistical uncertainties.

The background systematics however has a different nature. Indeed, the backgrounds events are gluon or quarks, produced in the collision but which fake an electron or a muon. A reliable estimate of this background is difficult to make from first principle, as the rate of such QCD events is many orders of magnitude larger than the W boson cross section. The way to reject background event is to discriminate between events having high missing transverse energy (taken by the neutrino in the W decay) and what experimenters called the "isolation" value: events from QCD background will have more particles produced at proximity of the electron or muon candidate.

The dominant uncertainty on the background measurement is coming from the cut the experimenter will impose on the "isolation" parameter. It corresponds more or less of how you will define an isolated electron. The more isolated it will be, the less chance it corresponds to a background event. We show in Fig.(E.2-right) the number of background events as function of the isolation parameter. The default value used for the measurement of σ_W is 0.3. We clearly see that the influence on this parameter is luck stronger than the statistical one (82 events for statistics uncertainty while 672 for systematics one). In this case, one cas consider the global uncertainty coming from the background as a "pure" systematic uncertainty in the measurement of σ_W.

We remember of course also the "faster than light" neutrino results of OPERA that was claimed in 2011. this "faster-than-light" neutrino experiment suffered from a systematic error that affected all of the data; faulty cables consistently gave the researchers bad readings. No matter how many times physicists repeated the experiments, they would get the same yet inaccurate results.

E.3 Look-Elsewhere Effect (LEE)

E.3.1 Generality

The look-elsewhere effect is common in scientific analysis of experiments studying models with a lot of degrees of freedom. Indeed, the more degrees of freedom posses the theory the more probable to observe a statistically significant observation can occur. This was very useful for the hunting of the Higgs boson in 2011.

What is the look-elsewhere-effect in short? It is the main reason why the strong 5σ approach is applied in particle physics. Indeed, "5σ" corresponds to 3 events in 10 millions. But having such a signal which can be seen for instance as a bump at a given energy E in a diet spectrum corresponding to and invariant mass $M = E$. Naively speaking, suppose that such a bump is such away form that background that one has 1 chance on ten thousands for it to be a statistical fluctuation. It seems natural to claim fo a potential discovery, corresponding to the production of a new particle of mass M. Now, suppose the study has been made in an energy range between 1 to 100 GeV, and the bump has a width of 2 GeV (see Fig.(E.3). To know what is the real chance to observe such a bump at an energy E, say 60 GeV, on has to convolute the probability of observing such event in the 60 GeV bin to the general probability of having such a bump in another bin. In other words, if we do to "look elsewhere" the probability of such event to be the background is only 1/10000 (and so we can consider it as an "event"), however, if one consider that this can statistically happens in any bin, we have 50 chances (100 GeV/2 GeV) that this event can occur in another bin which mean a total probability of $50 \times 1/10000 = 1/200$ which cannot be considered anymore as a "discovery" signal. Put in another way, the probability of finding a signal *exactly* at 60 ± 2 GeV is one chance on 10000, whereas the probability of finding a signal "somewhere between 1 and 100 GeV" is one chance on 200. In general, a good rule of thumb is the following: if the signal has a width W, and if one examine a spectrum spanning a mass range from M_1 to M_2, then the "boost factor" due to the LEE is $(M_2 - M_1)/W$. This may easily reach a factor 10 or 100 depending on the detail of the searches. Of course, this is a simplified version of the LEE effect as I have supposed that the 50 possibilities have the same chance of occurrence (a flat distribution), which is never the case in any complex studies and explain the concrete difficulties to take into account this effect in modern analysis (especially at LHC).

One can also found effects of such LEE effect in the common life: a Swedish study in 1992 tried to determine whether or not power lines caused some kind of poor health effects. The researchers surveyed everyone living within 300 meters of high-voltage power lines over a 25-year period and looked for statistically significant increases in rates of over 800 ailments. The study found that the incidence of childhood leukemia was four times higher among those that lived closest to the power lines, and it spurred calls to action by the Swedish government. The problem with the conclusion, however, was that they failed to compensate for the look-elsewhere effect; in any collection of 800 random samples, it is likely that at least one will be at least 3 standard deviations above the expected value, by chance alone. Subsequent studies failed to show any links between power lines and childhood leukemia, neither in causation nor even in correlation.

Figure E.3: Illustration of the look elsewhere effect: a 5σ signal observed above the background in a specific energy range (left figure) can be reduced to a 4σ signal (or even lower) if one take into consideration that, looking elsewhere, this signal has also a probability of appearing somewhere else (right figure). If one know exactly the energy bin where the signal should appear (from other considerations or other experiments) this signal is effectively at 5σ of confidence level.

E.3.2 Applying the LEE effect to the Higgs discovery

Let us now look specifically at the Higgs detection at the LHC. The two main channels analyzed by the ATLAS and CMS collaborations are the $H \to \gamma\gamma$ and $H \to 4$ leptons final states, mainly because of the low background level of these two final states. In december 2011, the significance of the entire excess rate of both event types were close to 4σ. This corresponds to a probability close to 3×10^{-5} that the observed "signal" would arise as a fluctuation of standard model physics without a Higgs particle. This new boson was looked over a range of about 80 energy bins at the LHC. This corresponded to a probability of about 2.5×10^{-3} that an oscillation of the size of the December data could occur at the LHC experiment. This probability does not even amount a 3σ effect and therefore was clearly insufficient for establishing the existence of a new particle. The data of July 2012 then had a significance above the 5σ level, which corresponded to a probability of the null hypothesis of less than 3×10^{-5} after taking into account the LEE effect. This was sufficient for declaring the data a discovery.

What happened in December 2011 however, is that a lot of theoretician (the author of the present book included) considered that a 4σ limit can be considered seriously as the look elsewhere effect can be reduced by "theoretical" or "philosophical" argument. Indeed, there was strong evidence that 1) the Higgs field should exist and 2) that his mass range due to perturbative effects should lie below 1 TeV. Indeed, in this sense one can see the limit of the look elsewhere effect, and its part of arbitrariness. Up to which scale one should take the possibilities of statistical fluctuations? Which can of data from other experiments, or

Figure E.4: Count spectrum showing the 130 GeV line for the region of 3 degrees around the Galactic center [606].

theoretical arguments should we take into account in the analysis? Or should we play the role of completely blind physicists, unaware of other experiments or theoretical constructions and history? For instance, before the Higgs signal of December 2011, there was other measurements from LEP or Tevatron, limiting the Higgs mass in the window 115 to 141 GeV, reducing thus the analysis to 26 bins, increasing at the same time the significance by a factor 3, the probability of a statistical fluctuation being then below 10^{-3}. But of course, that one should be taken into account if one believe that the Higgs exists.

E.3.3 Applying the LEE effect to the dark matter searches

In April 2012 Christoph Weniger (not a member of the Fermi-LAT collaboration) reported "A tentative gamma-ray line from Dark Matter annihilation at the Fermi Large Area Telescope" [606] with a statistical significance of 4.6σ at 130 GeV. When taking into account the look elsewhere effect, the signal dropped to 3.2σ i.e. taking into the count the fact that he did search for a line in multiple regions of the sky and at multiple energies. We represent the data analyzed by Weniger in Fig.(E.4) In May 2013 the Fermi LAT collaboration has reported their "Search for Gamma-ray Spectral Lines with the Fermi Large Area Telescope and Dark Matter Implications" [608], computing the statistical significance of this line feature at 130 GeV at a lower statistical significance of 3.3σ, and only 1.6σ after taking the look-elsewhere effect into account. As we can see, the LEE effect was important in this case, as it seemed finally that such signal was not due to dark matter annihilation.

E.4 Bayesian vs frequentist approach

Since several decades, the fight exists between people sustaining the frequentist or bayesian approach. In the supersymmetric communities several authors used to claim in every conferences that the bayesian approach was *the one* to use in any supersymmetric study [751]. We will remind you in this section the fundamental of bayesian approach.

To understand the bayesian approach, we will take a very concrete example. Suppose a woman believes she may be pregnant after a single sexual encounter, but she is unsure. So she takes a pregnancy test that is known to be 90% accurate (meaning it gives positive result to positive cases 90% of the time) ands the test produces a positive result. Ultimately she would like to know the probability she is pregnant, given a positive test (p(preg—test +)); however, what she knows is the probability of obtaining a positive test result if she is pregnant (p(test + — preg)), and she knows the result of the test. Bayes' theorem offers a way to reverse conditional probabilities, and, hence, provides a way to answer this question.

Bayes' original theorem applied to point probabilities. The basic theorem states simply:

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)}, \tag{E.6}$$

which can be translated by the (conditional) probability of having B given A is equal to the (conditional) probability of having A given B, multiply by the (marginal) probability of having B divided by the (marginal) probability of having A[1].

Proof: we understand easily that the probability of having A and B is $p(A, B) = p(A|B)p(B)$. With the same reasoning, we also can write that $p(B, A) = p(B|A)p(A)$. $p(A, B) = p(B, A)$ one then can write

$$p(B, A) = p(A, B) \quad \Rightarrow \quad p(B|A)p(A) = p(A|B)p(B) \tag{E.7}$$

which gives the Bayes' result.

Now, back to our exemple, suppose that, in addition to the 90% accuracy rate, we also know that the test gives false positive results 50% of the time. In other words, in cases in which a woman is *not* pregnant, she will test positive 50% of the time. Thus $p(test+|not\ prey) = 0.5$. With this information, combined with some "prior" information concerning the probability of becoming pregnant from a single sexual encounter, Bayes' theorem provides a prescription for determining the probability of interest.

The prior information we need, $p(B) = p(prey)$, is the marginal probability of being pregnant, not knowing anything beyond the fact that the woman has had a single sexual encounter. This information is considered prior information, because it is relevant information that exists priory to the test. We may know from previous research that, without any additional information (e.g., concerning the date of last menstrual cycle), the probability of conception for any single sexual encounter is approximately 15%. With this information, we can determine $p(B|A) \equiv p(preg|test+)$ as

[1] If anyone still believes that $p(A|B) = p(B|A)$ [probability of A given B = probability of B given A], remind them that the probability of being pregnant, given that the person is female is ~ 3 %, while the probability of being female, given that they are pregnant, is considerably larger.

$$p(preg|test+) = \frac{p(test+|preg)p(preg)}{p(test+|preg)p(preg) + p(test+|not\ preg)p(not\ preg)} \tag{E.8}$$

which gives

$$p(preg|test+) = \frac{.90 \times 0.15}{0.90 \times 0.15 + 0.50 \times 0.85} = 0.241. \tag{E.9}$$

Thus, the probability the woman is pregnant, given the positive test is only 0.241. Using Bayesian terminology, this probability is called "posterior probability" because it is the estimated probability of being pregnant, obtained $after$ observing the data (the positive test). We can be surprised of such a low value given the "90%" accuracy of the test. This comes mainly from the 50% of probability (high) to obtain a positive test when the person is *not* pregnant.

If the woman is aware of these limitations, she can redo the test. But now, she can use the "updated" probability of being pregnant (p=0.241) as the new $p(B)$; that is the prior probability for being pregnant has now been updated to reflect the results of the first test. If she repeats the test again and again observes a positive result, her new "posterior probability" of being pregnant is

$$p(preg|test+) = \frac{0.90 \times 0.241}{0.90 \times 0.241 + 0.50 \times 0.759} = 0.364. \tag{E.10}$$

This result is still not very convincing evidence that she is pregnant, but if she repeats the test again and finds positive result, her probability increases to 0.507 up to 0.984 at the tenth positive attempt. From a Bayesian perspective, we begin with some prior probability for some event, and we update this prior probability with new information to obtain a posterior probability. The posterior probability can then be used as a prior probability in a subsequent analysis. From a Bayesian point of view, this is an appropriate strategy for conducting scientific research.

We can then understand how one should apply such statistical analysis in supersymmetric models. Indeed, before that tyne LHC gave any limits, the aim was to know how a point in supersymmetric parameter space was "probable" or not. Usually, frequentist (the author of the present book included) make a scan on the par mater space, and compute how far, or near (from a frequentist approach, *ie* 2 or 3σ), this point is from the data measured. Whereas the bayesian researchers would take the complementary approach: *given* experimental data, what is the probability that a point in the parameter space is around the data.

Appendix F

Numbers

I have included in this appendix the main tables, conversion factors and formulae I concretely regularly needed in my cosmological/astrophysical/particle computation. I hope you will enjoy using them as much as we enjoyed (sic!) writing them...

F.1 Useful formulae

F.1.1 Cosmology

$$T_0 = 2.34 \times 10^{-4} \text{ eV } = 2.725\ ^0\text{K}; \quad G = 1/M_P^2\ ; \quad M_P = 1.22 \times 10^{19} \text{ GeV}$$

$$H(T) = \sqrt{\frac{8\pi G}{3}\rho(T)} = \sqrt{g_\rho}\sqrt{\frac{4\pi^3}{45}}\frac{T^2}{M_P} \simeq 1.66 g_\rho^{1/2}\frac{T^2}{M_P}$$

$$\rho_c^0 = \frac{3H_0^2}{8\pi G} = 10^{-5}\ h^2 \text{ GeV cm}^{-3} \ = 2 \times 10^{-29} h^2 \ \text{ g/cm}^3; \quad \Omega_A = \frac{\rho_A}{\rho_c^0}$$

$$n_\gamma^0 = 394 \text{ cm}^{-3}; \rho_\gamma^0 = 2.4 \times 10^{-10}\text{GeV/cm}^3; \langle E_\gamma^0 \rangle = \rho_\gamma^0/n_\gamma^0 = 6 \times 10^{-4}\text{eV};$$

$$T_\nu^0 = 1.95\ ^0\text{K};\ n_\nu(T_\nu^0) = 109 \text{ cm}^{-3};$$

$$s(T) = \frac{2\pi^2}{45} g_s(T) T^3; \quad g_s^0 = 3.91\ ; \quad s(T_0) = 2.1 \times 10^{-38} \text{ GeV}^3 = 2787 \text{ cm}^{-3}$$

$$Y(T_0) = \frac{\Omega \rho_c^0}{m s_0} = \left(\frac{\Omega}{0.1}\right)\left(\frac{100 \text{ GeV}}{m}\right)\frac{135 \times 10^{-3} H_0^2 M_P^2}{16\pi^3 g_s^0 T_0^3}$$

$$Y(T_0) \simeq 5 \times 10^{-10}\left(\frac{1 \text{ GeV}}{m}\right)$$

$$f_A = \frac{g_A}{e^{(E-\mu_A)/kT} \pm 1}; \qquad n_A(T) = \frac{g_A}{(2\pi)^3}\int f_A(\vec{p}) d^3p$$

$$n_{EQ}^A(T \gg m)\left(\frac{3}{4}\right)\frac{\zeta(3)}{\pi^2} g_A T^3 \quad \text{(fermion)} \ \ \text{boson}$$

$$\rho_{EQ}^A(T \gg m)\left(\frac{7}{8}\right)\frac{\pi^2}{30} g_A T^4 \quad \text{(fermion)} \ \ \text{boson}$$

$$n_{EQ}^A(T \ll m) = g_A\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}; \quad \rho_{EQ}^A(T \ll m) = m \times n_{EQ}^A(T \ll m)$$

F.1.2 Particle physics

$$\langle H \rangle = v/\sqrt{2} \;\Rightarrow\; v = 246 \text{ GeV} \quad (174 \text{ GeV if } \langle H \rangle = v)$$

$$M_W = \frac{gv}{2} \;; \quad M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v = \frac{g}{2\cos\theta_W} v \;; \quad M_Z = \frac{M_W}{\cos\theta_W}$$

$$\sigma_{EW} \simeq |\mathcal{M}|^2/64\pi^2 s \simeq 10^{-9} \text{ GeV}^{-2}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \;; \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \;; \quad e = g\sin\theta_W$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu \;; \quad Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu$$

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2} \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

$$\sigma_T = 6.65 \times 10^{-29} \text{ m}^2$$

Perturbativity: $\frac{\alpha}{4\pi} = \frac{g^2}{(4\pi)^2} < 1$ giving $g < 4\pi$

F.2 Tables

Quantity	Symbol	Value
Speed of light in vacuum	c	299 792 458 m/s
Planck constant	h	6.626×10^{-34} J s $= 4.14 \times 10^{-24}$ GeV
Planck constant reduced	$\hbar \equiv h/2\pi$	1.055×10^{-34} J s $= 6.58 \times 10^{-22}$MeV s
Planck Mass	M_{Pl}	1.2211×10^{19} GeV $= 2.1768 \times 10^{-5}$ g
Reduced Planck Mass	$M_P = M_{Pl}/\sqrt{8\pi}$	2.43×10^{18} GeV
Conversion factor	$\hbar c$	197.3 MeV fm
		1 GeV $= 1.6 \times 10^{-10}$ J
Boltzmann constant	k_B	1.38×10^{-23}J K^{-1} $= 8.617 \times 10^{-5}$ eV K^{-1}
Weinberg angle	$\sin\theta_W / \sin^2\theta_W$	0.481/0.23122
Hypercharge coupling	g'	0.356
Weak coupling	g	0.65
Strong coupling	g_3	1.126
Electromagentic coupling	$e = g\sin\theta_W$	0.323
$\alpha_1^{-1}(M_Z)$	$(g_1^2/4\pi)^{-1}$	58.98 ± 0.04
$\alpha_2^{-1}(M_Z)$	$(g_2^2/4\pi)^{-1}$	29.57 ± 0.03
$\alpha_3^{-1}(M_Z)$	$(g_3^2/4\pi)^{-1}$	8.40 ± 0.14
Fermi constant	$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2}$	1.166×10^{-5} GeV^{-2}
Typical EW cross section	σ_{EW}	10^{-9} GeV^{-2}
Electron charge	e	1.6×10^{-19} C
Electron mass	m_e	0.511 MeV/c^2 $= 9.11 \times 10^{-31}$ kg
Proton mass	m_p	938 MeV/c^2 $= 1.672 \times 10^{-27}$ kg
Neutron mass	m_n	939.5 MeV/c^2 $= 1.675 \times 10^{-27}$ kg
Electron charge	e	1.6×10^{-19} C
Electron radius	$r_0 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e c^2}$	2.8×10^{-15} m
Vacuum permititivity	ϵ_0	8.854×10^{-12} F m^{-1} [A^{2}s^{4}kg^{-1}m^{-3}]
Vacuum permeability	$\mu_0 = 1/\epsilon_0 c^2$	$4\pi \times 10^{-7} = 12.566 \times 10^{-7}$ N A^{-2}
Gauss	G	1 Gauss $= 10^{-4}$ Tesla
Muon mass	m_μ	105.7 MeV/c^2
Pion mass	m_{π^0}	135 MeV/c^2
Tau mass	m_τ	1777 MeV/c^2
Deuteron mass	m_d	1876 MeV/c^2
Higgs *vev*	$\langle H \rangle = \frac{v}{\sqrt{2}}$	$\langle H \rangle = 174$ GeV; $v = 246$ GeV
Higgs mass / width	m_h / Γ_h	125 GeV/c^2 / 4.07 MeV/c^2
W mass / width	$M_W = \frac{g}{2}v$ / Γ_W	80.385 ± 0.015 GeV/c^2 / 2.085 ± 0.042 GeV/c^2
Z mass / width	$M_Z = \frac{M_W}{\cos\theta_W}$ / Γ_Z	91.1876 ± 0.0021 GeV/c^2 / 2.4952 ± 0.0023 GeV/c^2
up quark mass	m_u	1.5-4.5 MeV/c^2
down quark mass	m_d	5-8.5 MeV/c^2
strange quark mass	m_s	80-155 MeV/c^2
charm quark mass	m_c	1-1.4 GeV/c^2
bottom quark mass	m_b	4-4.5 GeV/c^2
top quark mass	m_t	175 GeV/c^2

Quantity	Symbol	Value
Conversion K/GeV		1K $=0.85 \times 10^{-4}$ eV
Conversion $\mathrm{GeV}^{-2}/\mathrm{cm}^3\mathrm{s}^{-1}$	σv	1 GeV^{-2} $=1.2 \times 10^{-17}\mathrm{cm}^3\mathrm{s}^{-1}$
Conversion GeV^{-1}/L		1 GeV^{-1}= 1.9733×10^{-14} cm
Conversion GeV^{-1}/t		1 GeV^{-1}= 6.67×10^{-25} s
Conversion GeV^{-2}/barn		1 GeV^{-2}= 4×10^{-28} cm^2 = 4×10^{-4} barn
Conversion cm^2/barn		$10^{-24}\mathrm{cm}^2$ = 1 barn
Conversion GHz/GeV		1GHz $=4 \times 10^{-6}$ eV
Conversion GeV/kg		1GeV $=1.78 \times 10^{-27}$ kg
Conversion barn/cm^2		1 barn = 10^{-24} cm^2 / 1 pbarn = 10^{-36} cm^2
Conversion cm^2 $\mathrm{g}^{-1}/\mathrm{GeV}^{-3}$		1 cm^2 g^{-1} = 4.4×10^3 GeV^{-3}
gravitational constant	G	$6.7 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$ $6.7 \times 10^{-39}\hbar c(\mathrm{GeV}/\mathrm{c}^2)^{-2}$
Present Hubble expansion rate	H_0	100 h km/s/Mpc $=2.13 \times 10^{-42}$ h GeV
Hubble time	H_0^{-1}	$3.08 \times 10^{17}h^{-1}$ sec = 9.77×10^9 h^{-1} yrs
Hubble distance	cH_0^{-1}	2998 h^{-1} Mpc = $9.25 \times 10^{27}h^{-1}$ cm
Reduced Hubble expansion rate	h	H_0 /(100 km/s/Mpc)
Normalized Hubble expansion rate	h	0.71 ± 0.07
Horizon size	L_H	14 Gpc
critical density	ρ_c	$10^{-5}h^2$ GeV cm^{-3} = $1.8 \times 10^{-32}h^2$ kg cm^{-3}
Age of the Universe	t_0	13 Gyr = 4×10^{17} s. = 6.1×10^{41} GeV
Mean free path of photon	λ_0	10^{26} m
CMB temperature	T_0	2.725 $\pm$0.001 K = 2.4×10^{-4} eV
Zeta Riemann	$\zeta(3)$	1.20206
Parsec	pc	1 pc = 3×10^{16} m $\simeq$ 3 light years
Solar velocity with respect to CMB		371 km/s
Jansky	Jy	10^{-26} W m^{-2} Hz^{-1}
Watt	W	J.s^{-1}=kg m^2 s^{-3}
Solar mass	$M_\odot$	1.98×10^{30} kg = 1.1×10^{57} GeV
Mass-to-light ratio	$M_\odot/L_\odot$	5133 kg/W
Binding energy of the Hydrogen	B_H	13.6 eV
Binding energy of the deuterium	B_D	2.2 MeV
Deuterium mass	m_D	1877.62 MeV

Quantity	z	Energy	Temperature	time
		0.85×10^{-4} eV	1 K	10^{17} seconds
Nucleosynthesis		0.1 MeV	10^9 K	100 seconds
Matter domination	3400	0.81 eV	9000 K	100 000 years
decoupling (atoms formation)	1100	0.3 eV	3000K	380 000 years
Actual	0	2.35×10^{-4} eV	2.75K	15×10^9 years

cross section	value (in pb)
$\sigma_{e^+e^-\to\nu\nu\gamma}$	2.7×10^{-3}

Field	$q = T^3 + Y/2$	T^3	Y
u_L	$+2/3$	$+1/2$	$+1/3$
d_L	$-1/3$	$-1/2$	$+1/3$
u_R	$+2/3$	0	$+4/3$
d_R	$-1/3$	0	$-2/3$
ν_{eL}	0	$+1/2$	-1
e_L	-1	$-1/2$	-1
e_R	-1	0	-2
N_R	0	0	0
H	0	$-1/2$	$+1$

Quantity	Symbol	Value
Total density	Ω_t	1.02 ± 0.02
Dark Energy density	Ω_Λ	0.73 ± 0.04
Total matter density	Ω_m	0.27 ± 0.04
Total matter energy density	ρ_0^M	$1.17\ \frac{\text{keV}}{\text{cm}^3}$
Baryonic density	Ω_b	0.044 ± 0.004
Hubble Constant	H_0	71 ± 4 km/s/Mpc
Age of the Universe	t_0	13.7 ± 0.2 Gyr
Size of the Universe	R_0	28 Gpc

Figure F.1: Periodic Table of the Elements

Appendix G

The actors

In this long-short story of the Universe, the actors we have crossed and shared ideas are numerous and original. On shoulder of giants, they pushed our art to the limits of the mind.

Solvay congress, 1927. From back to front and from left to right :

Auguste Piccard, Emile Henriot, Paul Ehrenfest, Edouard Herzen, Theophile de Donder, Erwin Schrodinger, Jules-Emile Verschaffelt, Wolfgang Pauli, Werner Heisenberg, Ralph Howard Fowler, Leon Brillouin,

Peter Debye, Martin Knudsen, William Lawrence Bragg, Hendrik Anthony Kramers, Paul Dirac, Arthur Compton, Louis de Broglie, Max Born, Niels Bohr,

Irving Langmuir, Max Planck, Marie Skodowska Curie, Hendrik Lorentz, Albert Einstein, Paul Langevin, Charles Eugene Guye, Charles Thomson Rees Wilson, Owen Willans Richardson

G.1 Max Planck (1858-1947)

Mac Planck was born at Kiel in 1858 and is considered as the father of Quantum Mechanics, especially from the proceedings of conferences he made in Berlin in 1900 (see section H.2 for details). Since his first years in University he was fascinated by thermodynamics and especially second principle on which was based his PhD dissertation. His professors in Berlin were Helmholtz and Kirchhhoff. His souvenirs that he recalls in his scientifical autobiography is that Helmhotz did not prepare for lectures, stammered and constantly made mistakes at the blackboard whereas Kirchhoff's lectures were "dry and monotonous". However, he was very impressed by their research works, and especially the activities of Clausius. thermodynamics and formulation of the second principle (Planck was the first to give the modern formulation of the second law: "in any natural (irreversible) process the sum of the entropies of all bodies involved in the process increases"). This fact is important to understand how Planck achieved to obtain the black body formulation and the quantization of energies. It is also a reason why he fought all his life to try to reconciliate classical (continuum) theory with quantum mechanics. Unsuccessfully. If in 1900 he doubted about the fundamental achievement of his model of black body, concluding his talk by "despite its evident achievements, the theory of black–body emission presented in this talk cannot be regarded as satisfactory in any case". Even in 1911 he wrote "it seems to me that the hypothesis of quantum emission, along with the original hypothesis of quanta of energy, must be regarded more as a figure of speech than as an expression of physical reality" (!!).

In 1880 he received a position in University of Munich as lecturer and was invited as professor at Kiel university (his native city) in 1885. He then wrote "of course, I did not guess that this good luck was not so much due to my own scientific successes as to the fact that

the professor of physics at Kiel was a close friend of my father. Nevertheless my joy was indescribable". In the spring 1889, after the death of Kirchhoff, Planck was invited to the university of Berlin. He kept his position until1928 when he gave up his professorship at the age of 70. His successor was Schrodinger. Between 1901 and 1910, Planck did not publish any original work on the quantum theory. He thought about his surprising result and tried to reconcile the quantization of energy with the classical theory. But it cannot be said as do some authors that in those years Planck supposedly drew no far–reaching conclusions from his theory nor even recognized the significance of the theory he had derived. He firmly believed in the validity of his theory, otherwise he would have to renounce to the second law of thermodynamics and this would have been harder for Planck. However, he did not dare introduce the concept of light quanta. This was first done by Einstein in 1905.
In the latter years of Planck's life, during the time of Hitler's fascism, Planck continued to serve in the Kaiser Wilhelm Institute (now called Max Planck Institute) in hopes of saving German science from total destruction (Heisenberg also stayed in Germany for the same reasons). "I think", subsequently wrote Max Born who left Germany in April, 1933 after the dismissal of the parliament, "that Max Planck hoped that the intolerance and oppression would subside with time. He did not see that an irreversible process goes further." Max Planck's son was a participant in the July plot against Hitler in 1944 and was executed by the Nazis. At this time Planck was 86 years old.
In conclusion, the beginning of the XXth century has taken a glorious path, revolutionary turn opened first by Planck, without any doubt one of the most eminent physicists of all time. His ideas were not accepted immediately and it took time and perseverance to impose them.The words of Planck are still valid in the modern science

"A great scientific idea rarely is introduced by the gradual convincing and conversion of its opponents, it rarely happens that "Saul" becomes Paul". In reality, the opponents gradually die out and the rising generation is familiar with the new idea from the beginning – an example of the fact that the future belongs to the young"

Max Planck

G.2 JJ. and G.P Thomson (1858-1947)

The story of the Thomson's family is one of the greatest in History of Science. The academic careers of J.J. (the father) and G.P. (the son) Thomson are linked to an entity very popular among historians and philosophers of science: the electron. Indeed, G.P. (for Georges Payet) was still a teenager when his father made the discovery of the electron in 1897 through the observation of the cathods rays. More precisely, from the observation of the deviation of the cathods ray induced by the presence of an electric field, J.J. (stands for Joseph John) proved that these puzzling rays could in fact be interpreted as corpuscles (names "electron") in opposition to the German movement of physicists led by prof. Hertz in Hamburg which interpretations were that these rays were waves propagating in aether. The scientific fight between these two interpretation reflects generic oppositions between the two european schools since Newton discovery of gravity. J.J. made his discovery i the Cavendish laboratory of Cambridge. He succeed a measure of the mass to charge ratio incredibly precise and proves this new particle was 2000 times lighter that the only "fundamental" substance known at this time, the "positive ions of Hydrogen) (which is known as the proton). In 1903 J.J receive the Nobel prize for his discovery and his son G.P. who attended to the ceremony was 14th years old. J.J. Thomson was often portrayed as a very "absent-minded" person, and his office at home was forbidden to any family member, like a sanctuary.

After brilant studies in Trinity College, G.P. went to Aberdeen in 1922 to take the chair of professor of Natural Philosophy (the same chair that James Clark Maxwell occupied). 2 years after, in 1924, the French physicist De Broglie published his thesis where he made the revolutionary hypothesis that, as the light can be considered as corpuscle or wave depending on the framework of study, we can apply the same principle for material particles. 2 years later, E. Schrodinger formalized the idea by expressing the evolution of particles in time describing it by a wave function following the "Schrodinger" equation. All these new ideas crossed the channel from the continent to England thanks to C.G. Darwin, young physicist from Manchester and the Edingburgh and one of the best friend of G.P. He convinced G.P.

to intend to observe diffraction figures that is observed in the case of X-rays, but in the case of a material object like the electron. After projecting electrons on different targets (lead, gold..) he effectively observed diffraction figures in 1927, leading G.P. to the Nobel prize in 1937 for the discovery of the electron diffraction.
The conflict between the father and the son becomes more obvious after this discovery. Indeed, J.J. was always reluctant to the quantum interpretation of atomic and nuclear physics. He never accepted the Bohr model of atomic structures, and even in the late 30's he was still building very complex models trying to satisfy the observations made by his son. Finally, G.P. also was influenced by his father point of view. He never really understood/accepted the Heisenberg interpretation of quantum mechanics in therms of matrices. He could accept the Schrodinger equation, as wave physics was similar to Maxwell propagation of light. In a sense, Schrodinger equation could be seen as classical physics in the sense "continuum" physics. The discretization of space time discovered by Heisenberg and then Pauli and Dirac was just very difficult for him to admit. In any case, 30 years after his father, G.P. closes the story of the first fundamental particles observed in humanity, the electron.

"Matter is still supposed made of discrete units, but instead of these units moving according to laws which concern them alone, as did the laws of Newtonian dynamics, we have had to introduce laws based on waves. Now a wave is essentially a continuous thing, evenif the continuity is only mathematical. It is spread through space, not divided into little lumps. So although the older belief in the discontinuity of matter still holds, it has lost some of his rigidity; continuity has crept in by the back door."

G.P. Thomson

G.3 Niels Bohr (1885-1962)

Niels Bohr can be considered as the first physicist which decided to implement quantum mechanics not as a "tool" to understand phenomena, as Planck and Einstein did some years before him, but really as inner part of the Nature. Bohr has 25 years old when he meets for the first time Rutherford. He just arrived to the Cavendish Laboratory of Cambridge, invited by J.J. Thomson on a Carlsberg fellowship (the brewery used to finance at these time some fundamental theoretical works, and later, will also finance the post-doc position of Gamow at the Bohr Institute). Just graduated, Bohr went to Thomson's, Nobel laureate in 1906 for his discovery of the electron in 1897. However, it is quite difficult to discuss with Sir Thomson for different reasons. First of all, Thomson is the head of the Cavendish Laboratory since 1884 (after Maxwell and Rayleigh, he was only 28 years old then!) and this activity is almost a full time job that occupies main of his days. On should also add a little absent mind and an obsession for very well done experimental work. In the meantime, Bohr cross in November Rutherford (in Manchester after his 8 years at Mac Gill) who passes by to visit his ex supervisor Thomson. Rutherford just comes back from the Solvay meeting and is very excited about is new planetary model of atoms he proposed there (but that nobody really cared as it was unstable), where a nucleus is surrounded by several electrons. Bohr is impressed by this tall New-zeland guy who speaks very loud with large movements of the hands, especially because Bohr always had troubles in English communication.

From this meeting, Bohr will ask the permission to Thomson to Join Rutherford for the 6 last months of his post-doc in Manchester. Thomson accepts, and then a real friendship will begin, such a rare friendship as Rutherford used to hate theoreticians : "Theoreticians just like to play with symbols, we, experimentalists, try to deal with real fact of Nature". During these 6 months, Bohr will try to solve the main issue of the model of Rutherford which is

the fact that by radiation, the electron turning around a nucleus are loosing energies and should fall down inside the nucleus in less than 10^{-6} seconds. Thus Bohr propose to fusion the idea of Planck with the model of Rutherford in a paper published in July 1913 in the review *Philosophical Magazine*. As used to say Heisenberg, Bohr was before everything a philosopher more than a physicist, in this sense, the article he sends to the editor of the review (who was Rutherford himself) is much too long and Rutherford answers:

I think in your endeavour to be clear, you have a tendency to make your papers much too long and a tendency to repeat your statements in different parts of the paper. The paper could be cut down without sacrificing anything to clarity. Long papers have a way of frightening readers. I suppose I may use my judgement in cutting out any matter I consider unnecessary? As you know it is the custom in England to put things very shortly and tersely, in contrast to the German method, where it appears to be a virtue to be as long-winded as possible

Bohr, who was then in Copenhagen when he received the answer, directly took a boat to Manchester to persuade Rutherford to keep any words in the paper. Indeed, Bohr used to write dozens of draft before submitting a right one. Each words had his place in the text, and as treating with Quantum Mechanics, he wanted to make it as clear as possible. Rutherford gave up the fight, and the paper was finally published in 1913 exactly on its original form.

If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet.

Niels Bohr.

G.4 Fritz Zwicky (1898-1974)

Fritz Zwicky was born in Bulgaria, was then citizen of switzerland before doing his main research works at Caltech. Fritz was famous for having a continuum of bunches of ideas that nobody really took seriously. He was famous to call his detractors "spherical bastard" because no matter what direction you look at him from, hes still a bastard. Millikan, Nobel prize of physics invited him at Mount Wilson, Caltech in 1925. To acknowledge him, Fritz had no other words that: "Millikan never had any great ideas during all his career". Discovering neutron stars, and even proposing that clusters of galaxies can act as gravitational lens 40 years before the discovery of the effects was some of his contribution to science. Of course, the observation of the Coma cluster of galaxies. He measured how fast these galaxies were moving, and then measured how far they are from the clusters center, and he thus figured out how much mass is in the cluster. Sounds like a good plan. Well, thats exactly what Fritz did. Know what he found? They are all moving way too fast! Not only did he find that they move so quickly, but that they move, according to his numbers, 160 times more quickly than youd expect from all the visible matter in this cluster. So, in 1933, Fritz published a radical idea that well over 90% of the matter in these clusters was dark. And the theory of dark matter was born. The only thing thats changed about this observation since 1933 is that we can see dimmer things. It turns out that it isnt a factor of 160, its only about a factor of 50.

He was also famous as a rude opponent to the Big Bang theory and invented the theory of the "tired light". In two words, to explain the redshift, Zwicky based his thinking about the fact that in any movement off a massive object (like a light escaping from the center of a galaxy for instance), the object is loosing energy through the gravity potential: an apple you throw in the air will stop and return back as its kinetic energy is transformed into potential energy. He applied this idea to the light. However, as the speed of light is constant, the effect of a photon escaping a galaxy would not have been to slow down but to redshift his frequency,

explaining the Hubble observation: the photon becomes tired. However, the computations shows that such an effect, if existing is really negligible, and can not account for the redshift observed by Hubble. Fritz then began a crazy battle, accusing Hubble to modified his data, with the help of the students to fit with the theory of the expansion of the Universe. In any case, Fritz Zwicky had several great ideas in his career, and let's remind these ones.

G.5 George Gamow (1904-1968)

Gamow was born in the city of Odessa, in the ex-Russian Empire (now Ukraine). Between 1923-1926 he went to study under the supervision of Alexander Friedmann in Lenningrad (now Saint-Petersburg) where he met Lev Landau. He then went to work in 1928 in University of Copenhagen, inventing the famous nuclear "liquid drop" model. Gamow first two attempts to defect with his wife were in 1932, and involved trying to kayak: first a planned 250-kilometer paddle over the Black Sea to Turkey, and another attempt from Murmansk to Norway. Poor weather foiled both attempts, but they had not been noticed by the authorities. In 1933 Gamow was suddenly granted permission to attend the 7th Solvay Conference on physics, in Brussels. He insisted on having his wife accompany him, even saying that he would not go alone. Eventually the Soviet authorities relented and issued passports for the couple. The two attended, and arranged to extend their stay, with the help of Marie Curie and other physicists. Over the next year, Gamow obtained temporary work at the Curie Institute, University of London, and University of Michigan.

In 1934, Gamow moved to George Washington University but was not included in the Manhattan project, despite his knowledge on nuclear physics, radioactivity and fusion, due to its "communist" origin. His most famous work was the paper he published in 1948 (1st of April, fools day) with his student (of Ukranian origin too) Alpher, "The origin of chemical elements". Even if the idea was obviously from Gamow mind, his computing abilities was helped enormous by Alpher who was used to drive complicated equations and was known for his deep understanding of mathematics (Gamow was much more intuitional). The paper became very famous in the community as the "Alpher Bethe, Gamow theory" as Gamow added the famous physicist (and friend) Bethe at the last moment (without telling to his student Alpher) for the "α, β, γ" joke. It turned out that Alpher was very upset by this "joke" (as the less famous of the tree but the one who did the calculation) and this event

change the relationship between the two men. In this paper, Alpher and Gamow computed the Helium to Hydrogen ratio in the Universe and confirms it fits outstandingly with the Big Bang prediction.
Alpher had his "revenge" in december of the same year (1948) when he predicted with Herman a diffuse cosmological background of photons of 5 degrees. This 5 degrees radiation should fill the Universe and be observed. At first, Gamow did not believe it and he would have answered to Alpher when he came to present his computation : "spend your time on something more useful and observable". Gamow fought against this Cosmological Microwave Background (CMB) prediction during at least 3 years. Gamow being his supervisor, the work of Alpher was attributed to Gamow during decades! The Alpher-Herman hypothesis stayed hypothesis until 1964 when Penzias and Wilson discovered, completely accidentally the famous radiation at 2.7 degrees, just 2.3 degrees below the Alpher-Herman prediction!! This discovery was awarded by the Nobel prize to Penzias and Wilson. Surprisingly, neither Gamow nor Alpher had received the Nobel prize.

G.6 Edwin Powell Hubble (1889-1953)

Bornt in Missouri and student of University of Chicago, Edwin Hubble was offered a staff position by Hale at the Mount Wilson Observatory in 1919. Died in 1953, no funeral was held and his wife never revealed his site of burial. His two most famous work concern the discoveries of external galaxies and the so–called "Hubble law" concerning the redshift and movement of galaxies. Using the Hooker Telescope at Mt. Wilson, Hubble identified Cepheid variables (a kind of star; see also standard candle) in several spiral nebulae, including the Andromeda Nebula and Triangulum. His observations, made in 1923, proved conclusively that these nebulae were much too distant to be part of the Milky Way and were, in fact, entire galaxies outside our own. This big debate opposed him with Shapley, his superior in the Mount Wilson observatory, who was convinced that these nebula were in fact part of our galaxy, and by consequence that the Milky Way was the only galaxy in the space. In 1920, Shapley quitted the Mount Wilson Laboratory for a promotion in Harvard University and as a consequence, the observatory which will dominate the astronomy observation for the next 40 years. After 4 years of incredible discipline and hard observational work in what was call "the monastery" (the telescope was also forbidden to women at these times), on the 4th of October of 1923 after having taken a long exposition (40 minutes) photo of the Nebula M31 (Andromeda) he observed a Cepheid variable from which he can deduce the distance between the earth and M31 : He obtained 900 000 light years, much more than the radius of the Milky Way (100 000 light years). His observation cut the debate, and when Shapley received the letter from Hubble, he told "This is the letter which destroyed my Universe". Now we know that the Andromeda Nebula is more than 2 million light years far away, but this observation by Hubble at this time was considerable and very mediatic. He increases the size of our Universe which was believed to lay in a 100 000 light years bubble. He received from this discoveries a lot of prizes and awards and became very "en vogue" in the Los Angeles nightlife and Hollywood stars.

Despite his new "star" lifestyle, Hubble never forgot he was before everything an astronomer. After a first observation of a redshift from different galaxies by Slipher in the Lowell Observatory, Hubble decided to clarify the issue. With the help of his great assistant Humason (ex-groom in the Mount Wilson Hotel) they measured the redshift of 46 galaxies in 1929. In almost all the case, it was a redshift that was observed, indicating that the galaxies were escaping from the Milky Way, but, more intriguing was when Hubble decided to draw a line in the plane (distance ; velocity): he found that the velocity of the galaxies were proportional to their distance from the earth. The fundamental conclusion of such observation was that, at a time being all the galaxies were together merged in one point, first hint that the Universe *had a beginning*, now called "Big Bang". It is worth noticing that Hubble never made any hypothesis about any beginning of the Universe in his article "A Relation Between Distance and Radial Velocity Among Extra-Galactic Nebulae", and never speculate on any interpretation of his measurement which was given an age of the Universe of 1.8 billions years (13.7 nowadays). In a word, Hubble, surely one of the most gifted and framed astronomer of his time gave a new dimension to the Universe and even an age.

G.7 Fred Hoyle (1915-2001)

Fred Hoyle was born in West Yorshire in 1915. He was professor in Cambridge and wrote several science fiction book after his "retirement" from the physics community in the seventies. He was very famous from his rejection of the "Big Bang" theory (name given by himself to make a joke on a BBC radio broadcast in 1949). Instead, he proposed with his two long-term friends and colleagues Thomas Gold and Hermann Bondi the theory of "steady state universe". The legend (confirmed by Gold himself) tells that out of a movie they saw together *Dead of Night* in 1946 Gold proposed a model equivalent to the scenario of the movie. In this movie, a man woke up from a bad dream. Then, after several adventures, he finishes exactly in the same situation of the nightmare before he woke up, and then.. woke up again but with very slightly changes, the story continues the same way it began. Gold the same evening, in the apartment they shared as students, proposed a model for the Universe where the galaxies born ("wake up") regularly, then follow the expansion, and some time later, in the same place they were born, another galaxy is created again and then follows the expansion, etc.. This Universe is very similar to a flow of water going down a valley, then in the sea, then forming clouds, and back again in the valley. However, even if very seducing, this model predicts a distribution population of young galaxies homogenous in all the Universe whereas the Big Bang theory predicts that the older galaxies pushed by the expansions should be further away than the younger ones, which corresponds to the observations. Moreover, the discovery of the CMB by Penzias and Wilson in 1965, impossible to explain in the steady state Universe but *predicted* by Alpher in the framework of the Big Bang theory definitively killed the Hoyle et al. model of the Universe.

Paradoxically, Hoyle himself solved the major problem of the model of the Big Bang, without wanting it, when he proposed a new mechanism for nucleosynthesis: the stellar nucleosynthesis. Inded, it was well known that the Big Bang nucleosynthesis was able to form the atoms of hydrogen, helium, lithium and bore but did not possess in the thermal bath the energy sufficient to generate heavier atoms. in 1957, Hoyle, W.A. Fowler, M. and G. Burbidge published the cornerstone B2FH paper (known for the initials of the four authors) in which the field of nucleosynthesis was defined: in this paper, the authors shows that stellar

processes are able to produce all the chemical elements observed in the cosmos, ejected from the heart of the star by supernovae. His co-worker William Alfred Fowler eventually won the Nobel Prize for Physics in 1983 (with Subrahmanyan Chandrasekhar), but for some reason Hoyles original contribution was overlooked, and many were surprised that such a notable astronomer missed out.

Fred Hoyle was also famous for being on regular basis very controversial and a serious conflict with the Nobel comitee in 1974 concerning the real discovery of the pulsars might be the origin of the reason for him having been forgotten by the same Nobel comitee in 1983. Clearly disappointed by this political issue, he resigned from his chair position in Cambridge and continued a life out of physics until his death in 2001.

G.8 Georges Lemaitre (1894-1966)

Georges Lemaitre was a Belgian priest and astronomer in the University of Louvain. He was the first to propose the theory of the expansion of the Universe and Hubble law in 1927, 2 years before the Hubble article. He also proposed its now famous "Big Bang" theory, which he called his "hypothesis of primeval atom". He obtained his PhD thesis in 1920 concerning "approximation of functions of several real variables" and ordained priest in 1923. Working in in University of Cambridge with sir Arthur Eddington as a graduate student, he then spent the following year in Cambridge, Massachusetts with Harlow Shapley who just got famous with his work on nebulae. Back to Belgium in 1925 he became famous after his publication "Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extragalactiques" ("A homogeneous Universe of constant mass and growing radius accounting for the radial velocity of extragalactic nebulae") where he presented his new idea of expanding Universe, derived Hubble's law and provided the first estimation of the Hubble constant, but not yet the idea of the primeval atom. Instead, the initial state was taken as Einstein's own finite-size static universe model. His paper was translated by Eddington in 1931 and Einstein refused his main idea of an expanding universe: he answers to his letter "Your math is correct but your physics is abominable". He proposed his theory of primeval atom in a conference organized by Eddington in 1931in London and published a note in Nature "The beginning of the World from the point of view of Quantum Theory" in 1931 too. He described himself his theory as "the Cosmic Egg exploding at the moment of the creation". It became better known as the "Big Bang theory", a term coined by Fred Hoyle.

His models was very unpopular and few physicists accepted his conception of expanding

Universe. In January 1933, he travelled with Einstein to California for a serial of conferences and which seemed to convince Einstein at least on the elegance of the construction. He then published a more complete and detailed version of his paper the same year in the *Annals of the Scientific Society of Brussels* and then became famous. In 1951, when pope Pius XII tried to use his theory as a justification for creationism, Lemaitre resented: he was reluctant to mis science and religion. He died on the 20th of June of 1966, shortly after having learned of the discovery of cosmic microwave background radiation, which provided further evidence for his intuitions about the birth of the Universe.

G.9 P.A.M. Dirac (1902-1984)

British physicist, Paul Adrien Dirac was born the 8th of August of 1902at Bristol, one of three children. His mother was English and his father Swiss and very strict, so intent on having his children learn French that he refused to speak to them unless they spoke in correct French. This may explain why Dirac was a man of few, carefully chose words. He once said : "I was taught at school to never start a sentence without knowing the end of it". His brother Reginald committed suicide in 1925, which brought Paul's already strained relationship with his father almost completely to an end. When Dirac was told he could invite his parents to his Nobel Prize ceremony, he invited only his mother. Dirac was awarded a grant to do research in theoretical physics at St. John's College, Cambridge, working in the famous Cavendish Laboratory. He had eleven papers in print before submitting his doctoral dissertation *Quantum mechanics* for which he was awarded a Ph.D. in 1926. In 1932 he was appointed Lucasian Professor of Mathematics at ST. John's College, holding the post for 37 years. He shared the Nobel prize of physics in 1933 with Shrodinger.

The revolutionary idea of Dirac was that the electron could be described by four wave functions, satisfying four simultaneous differential equations (see the section **??** for more details). He moved to Florida with his family in 1969 where he was appointed professor of physics in Florida State University. His wife Margit Wigner, whom he married in 1937, was the sister of noted Hungarian physicist Eugene Wigner. She had been married before and had two children. Both children adopted the name Dirac, and Paul and Margit had two daughters together. Dirac continued his research until his death in Tallahassee in 1984. In 1995 a plaque (see figure) to Dirac was unveiled in Westminster Abbey, London, making Dirac's equation the first to appear in the Abbey.

Like many scientists, Dirac was known to sleep during other people's lectures, and then wake up and suddenly make a penetrating remark. Once a speaker stopped, scratched his head and declared: "here is a minus where there should be a plus. I seem to have made an error of sign." Dirac opened one eye and said : "Or an odd number of sign". Dirac was known for saying exactly what he meant and no more. At a question period after a Dirac

lecture at the University of Toronto, someone in the audience remarked "Professor Dirac, I do not understand how you derived the formula on the top left side of the blackboard". "This is not a question," snapped Dirac, "It is a statement. Next question please."

The mathematician Mark Kac divided geniuses into two classes. There are the ordinary geniuses, whose achievements one imagines other people might emulate, with enormous hard work and a bit of luck. Then there are the magicians, whose inventions are so astounding, so counter to all the intuitions of their colleagues, that it is hard to see how any human could have imagined them. Dirac was a magician.

"The research worker, in his efforts to express the fundamental laws of Nature in mathematical form, should strive mainly for mathematical beauty. He should take simplicity into consideration in a subordinate way to beauty ... It often happens that the requirements of simplicity and beauty are the same, but where they clash, the latter must take precedence.

Paul A. M. Dirac

G.10 Wolfgang Pauli (1900-1958)

Pauli was born in Vienna to a chemist Wolfgang Joseph Pauli (n Wolf Pascheles, 18691955) and his wife Bertha Camilla Schtz. His middle name was given in honor of his godfather, physicist Ernst Mach. Pauli was known first by the "Pauli Effect" according to which Pauli could, by his mere presence, cause laboratory accidents and catastrophes of all kinds. He also managed to pass his driver's test only after taking one hundred driving lessons. there are indeed well–documented instances of Pauli's appearance in a laboratory causing machines to break down, vacuum systems to spring leaks, and glass apparatus to shatter. He followed the lectures of Sommerfeld at the University of Munich with Heisenberg with who they become relatively good friends. He then became assistant of Born in University of Gottingen in 1921 and then Hamburg before being appointed to the chair of physics at Zurich University in 1928. After being left by her wife (the young dancer Kathe Deppner) who cheated him with a chemist, a period of crisis ensued where he began an analysis with Carl Jung and a stable marriage in 1934 with Francisca Bertram.

He elaborated a set of key selection rules that dictated each electron's choice of quantum number in the atom. This principle was later called "exclusion principle" or "Pauli principle" : no two electrons in a given atom can occupy a state characterized by exactly the same set of values for the quantum numbers.

He also developed the concept of *spin* : electrons like planets, might have a spin motion around an internal axis, in addition to orbital motion around the nucleus of the atom. Two Dutch graduate students, George Uhlenbeck and Samuel Goudsmit arrived at the essentials of the electron spin concept in late 1925 at the University of Leiden. He then proposed the existence of a neutrino to keep valid the conservation of energy despite the measurement of continuous spectrulm in β–decay (see the original articles in the next appendice and his letter to Lise Meitner included below).

Pauli was known to be very critical, and even if his mind was supreme among his contemporaries, his full greatness did not equal that of Einstein, Bohr or Heisenberg. To some extent, Pauli was held in check by his own brillance. His critical sense became too refined and broad.

Original - Photocopie of PLC 0393

Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Nun handelt es sich weiter darum, welche Kräfte auf die Neutronen wirken. Das wahrscheinlichste Modell für das Neutron scheint mir aus wellenmechanischen Gründen (näheres weiss der Ueberbringer dieser Zeilen) dieses zu sein, dass das ruhende Neutron ein magnetischer Dipol von einem gewissen Moment μ ist. Die Experimente verlangen wohl, dass die ionisierende Wirkung eines solchen Neutrons nicht grösser sein kann, als die eines gamma-Strahls und darf dann μ wohl nicht grösser sein als $e \cdot (10^{-13}$ cm).

Ich traue mich vorläufig aber nicht, etwas über diese Idee zu publizieren und wende mich erst vertrauensvoll an Euch, liebe Radioaktive, mit der Frage, wie es um den experimentellen Nachweis eines solchen Neutrons stände, wenn dieses ein ebensolches oder etwa 10mal grösseres Durchdringungsvermögen besitzen würde, wie ein gamma-Strahl.

Ich gebe zu, dass mein Ausweg vielleicht von vornherein wenig wahrscheinlich erscheinen wird, weil man die Neutronen, wenn sie existieren, wohl schon längst gesehen hätte. Aber nur wer wagt, gewinnt und der Ernst der Situation beim kontinuierliche beta-Spektrum wird durch einen Ausspruch meines verehrten Vorgängers im Amte, Herrn Debye, beleuchtet, der mir kürzlich in Brüssel gesagt hat: "O, daran soll man am besten gar nicht denken, sowie an die neuen Steuern." Darum soll man jeden Weg zur Rettung ernstlich diskutieren.- Also, liebe Radioaktive, prüfet, und richtet.- Leider kann ich nicht persönlich in Tübingen erscheinen, da ich infolge eines in der Nacht vom 6. zum 7 Dez. in Zürich stattfindenden Balles hier unabkömmlich bin.- Mit vielen Grüssen an Euch, sowie an Herrn Back, Euer untertänigster Diener

gez. W. Pauli

Figure G.1: Original letter of Pauli to Lise Meitner for a convention in Tubingen, considering the problem of beta decay..

He was knwon to jump out of seminar room shouting "even not wrong" when the speaker was not at the taste of Pauli's exigency.

Letter to Lise Meitner, 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, because of the "wrong" statistics of the N- and Li-6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" (1) of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin 1/2 and obey the exclusion principle and that further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton mass. - The continuous beta spectrum would then make sense with the assumption that in beta decay, in addition to the electron, a neutron is emitted such that the sum of the energies of neutron and electron is constant.

Now it is also a question of which forces act upon neutrons. For me, the most likely model for the neutron seems to be, for wave-mechanical reasons (the bearer of these lines knows more), that the neutron at rest is a magnetic dipole with a certain moment ?. The experiments seem to require that the ionizing effect of such a neutron can not be bigger than the one of a gamma-ray, and then μ is probably not allowed to be larger than $e \times (10^{-13}\mathrm{cm})$.

But so far I do not dare to publish anything about this idea, and trustfully turn first to you, dear radioactive people, with the question of how likely it is to find experimental evidence for such a neutron if it would have the same or perhaps a 10 times larger ability to get through [material] than a gamma-ray.

I admit that my remedy may seem almost improbable because one probably would have seen those neutrons, if they exist, for a long time. But nothing ventured, nothing gained, and the seriousness of the situation, due to the continuous structure of the beta spectrum, is illuminated by a remark of my honored predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's better not to think about this at all, like new taxes." Therefore one should seriously discuss every way of rescue. Thus, dear radioactive people, scrutinize and judge. - Unfortunately, I cannot personally appear in Tubingen since I am indispensable here in Zurich because of a ball on the night from December 6 to 7. With my best regards to you, and also to Mr. Back, your humble servant

signed W. Pauli

G.11 Ettore Majorana (1906-1938)

Ettore Majorana was born in August 1906 in Catania, Sicilia, the year of the discovery of the atom by Jean Perrin. Ettore came from an intellectual and scientific family. His father, Fabio (1875-1934), was an engineer, and later administrator, in the telegraph and telephone industries. Ettore's uncle, Quirino Majorana (1871-1957), was an experimental physicist at the University of Bologna. He used to be a close child but attracted by arithmetic and strange life experience. Indeed, at 12 he decided to try the car of his father and crashed it directly in a wall. A scarce on his hand stayed as a souvenir of this event. Ettore Majorana was something of a prodigy. His first original work, published in 1928, dealt with atomic spectroscopy. He showed that the atom of Cesium has a strange property to possess a free "single" electron. He received his PhD thesis one year after, in 1929 in Roma. Unlike Fermi but like Pauli, he is not able to deal with experiments *but* stayed interested by them and was always aware of the more recent results.
Majorana's name is often associated with two distinguished elder Italian colleagues, Enrico Fermi and Emilio Segré. Both of whom eventually became Nobel laureates. Majorana served a professorship at the University of Naples. He later was appointed to a physics post at the University of Palermo, but died before actually assuming it. Had he lived longer, Majorana may have become a decisive force in the field of particle physics. Enrico Fermi himself used to consider different kind of physicists :

"*There are many categories of scientists, people of second and third rank, who do their best, but do not go very far. There are also people of first-class rank, who make great discoveries, fundamental to the development of science. But then there are the geniuses, like Galileo and Newton. Well, Ettore Majorana was one of them.*", E. Fermi

Contrarily to Pauli He published very rarely and used to fill his trash bin. 1932 is without

any doubt his "anus mirabilis". Indeed, he published a first paper[1] about the possibility for a spin of an atom to flip when passing through a magnetic field, effect called *spin flip*. This techniques is now used in medical imagery. His second impressive work dealt with the representation of the Dirac equation for any kind of spin structure[2] Even Pauli, known to give very few compliments, admitted in 1940 when he discover it that he was impressed by the beauty and impact of this work.
In 1932, when Irene Curie and Frederic Joliot misunderstood their own experiment, missing the discovery of the neutron, Ettore would have said : " *stronzi*, they did not understand it is a neutral proton". Chadwick wrote his article (cf. section H.3) only 3 weeks after! He then developed several models of the atoms, that Fermi urged to published. Ettore always refused , arguing that these models were not sufficiently mature to be published. A similar version of his nuclear construction was published by Heisenberg that he met in Leipzig in January 1933.
He was fascinated by the ascension of Hitler and turned to support antisemitist argument, even writting to Segrè (jewish) :

"*It is not conceivable that a population of 65 millions inhabitant can be conducted by 600 000 persons which even do not represent any kind of people. Jewish situation is not as dramatic as they want us to believe*", E. Majorana

His most impressive work[3] was published the same year, 1933, where he proposed the description of particles being their own antiparticle, nowadays called "Majorana particles" in opposition to the particles which are different from their anti-particle ("Dirac particles"). Neutrino could be one of them but we still don't know if it is a Majorana or Dirac particle. He was one of the first physicists to publish theories regarding neutrino masses. His most recent published paper (much has been published after his death), in 1937, detailed a symmetrical theory on the relationship and corresponding mass of protons and neutrons, and energies that gave stability to atoms of electrons and positrons.

Ettore Majorana disappeared from a ship during a routine trip from Palermo to Naples in March 1938 and was presumed dead. A theory of suicide has always been advanced. He seems that a loving affair could have been the reason of the suicide. This was still a mysteries (some even affirm that he quit Italy to make a new life in South America) especially because his trip to Palermo was just after he finally accepted his permanent position that he refused during 3 years, even after a strong pressure from Fermi.

[1] "Atomi orientati in campo magnetico variabile", *Nuovo Cimento*, 9, 1932 p. 43-50.

[2] "Teoria relativistica di particelle con momento intrinseco arbitrario", *Nuovo Cimento*, 9, 1932, p. 335-344.

[3] "Teoria simmetrica dell'elettrone e del positrone", *Nuovo Cimento*, 14, 1937, p.171-184

G.12 Sir Arthur Eddington (1882-1944)

Eddington was born in Cumbria in 1882, but moved with his mother to Somerset in 1884 when his father died. He was brought up as a devout Quaker, a fact that plays an important role in the story of the eclipse expedition. In 1912, aged only 30, he became the Plumian Professor of Astronomy and Experimental Philosophy at the University of Cambridge, the most prestigious astronomy chair in Britain, and two years later he became director of the Cambridge observatories. Sir Arthur Stanley Eddington was a slender, nervous–looking man who gave the impression of being totally ill–equipped for the rough and tumble of combative science. He was incredibly shy and quite incoherent in his way of expressing himself. As one of his Cambridge students put it : "his brain had no proper connection with his mouth". He was renowed as the worst lecturer in the world but was really talented by his writtings.

Eddington had led an expedition to Brazil in 1912 to observe an eclipse, so his credentials made him an ideal candidate to measure the predicted bending of light. Eddington was in England when Einstein presented the general theory of relativity to the Prussian Academy of Sciences in 1915. Since Britain and Germany were at war at that time, there was no direct communication of scientific results between the two countries. But Eddington was fortunate in his friendship with the astronomer Willem De Sitter, later to become one of the founders of modern cosmology, and who was in neutral Holland at the time. De Sitter received copies of Einsteins papers, and wasted no time in passing them onto Eddington in 1916. Eddington was impressed by the beauty of Einstein's work, and immediately began to promote it. In a report to the Royal Astronomical Society in early 1917, he particularly stressed the importance of testing the theory using measurements of light bending.

Although Britain and Germany had been at war since 1914, conscription into the armed forces was not introduced in England until 1917. At the age of 34, Eddington was eligible for the draft, but as a Quaker he let it be known that he would refuse to serve. The climate of

public opinion was heavily against conscientious objectors. Eddington might well have been sent with other Quaker friends to a detention camp and spent the rest of the war peeling potatoes. Dyson, and other prominent Cambridge academics, went to the Home Office to argue that it could not be in the nations interest to have such an eminent scientist killed in the trenches of the Somme. After much political wrangling, a compromise was reached. Eddington's draft was postponed, but only on condition that if the war ended by 29 May 1919, he must lead the expedition to Principe.

Eddington's work in astrophysics in the late 1920s and the 1930s continued his work in stellar structure, and precipitated further clashes with Jeans and Edward Arthur Milne. An important topic was the extension of his models to take advantage of developments in quantum physics, including the use of degeneracy physics in describing dwarf stars, and was one of the first to apply Bohr model of atoms to propose that the stellar energy is coming from fusion of hydrogen atoms. Gamow completed his model. The topic of extension of his models precipitated his famous dispute with Subrahmanyan Chandrasekhar, who was then a student at Cambridge. Chandrasekhar's work presaged the existence of black holes, which at the time seemed so absurdly non-physical that Eddington refused to believe that Chandrasekhar's purely mathematical derivation had consequences for the real world. Chandrasekhar's narrative of this incident, in which his work is harshly rejected, portrays Eddington as rather cruel, dogmatic and racial. This is at variance with Eddington's character as described by other contemporaries. Eddington's criticism seems to have been based on a suspicion that a purely mathematical derivation from quantum theory was not enough to explain the daunting physical paradoxes that were apparently part of degenerate stars.

Eddington is credited with devising a measure of a cyclist's long distance riding achievements. The Eddington Number in this context is defined as E, the number of days a cyclist has cycled more than E miles. For example an Eddington Number of 70 would imply that a cyclist has cycled more than 70 miles in a day on 70 occasions. Achieving a high Eddington number is difficult since moving from, say, 70 to 75 will probably require more than five new long distance rides since any rides shorter than 75 miles will no longer be included in the reckoning. Eddington's best E-number is a very impressive 84. The construct of the Eddington Number for cycling is analogous to the h-index that quantifies both the actual scientific productivity and the apparent scientific impact of a scientist. You can find a more detailed analysis of the general relativity effect of the deflection of the light in the section (A.1.1)

G.13 Hideki Yukawa (1907-1981)

Hideki Yukawa was born Hideki Ogawa in 1907 in Tokyo but made his study in Kyoto University. In 1933 he then became assistant professor in Osaka University at the age of 26. According to a Japanese tradition, Ogawa was adopted into his wife Sumi Yukawa's family after their marriage in 1932, and he took his wife's family name, becoming Hideki Yukawa. His main professor, Yoshio Nashina , considered as the "Bohr" of Japanese physics and who has studied in all the main European center, was the first researcher to teach nuclear physics between 1921 and 1928 after some visits by Heisenberg and Dirac in Riken. He earned his doctorate in physics in 1938 in University of Osaka. In 1935 he published his paper where he proposed a new particle the "meson". trying to explain strong interactions between neutrons and proton for the coherence of the nucleus that was at this time dominated by repulsive forces between protons positively charged. His prediction was a boson particle of mediate mass (so the name "meson) of 200 times the electron mass ($m \simeq 100$ GeV). When two years later Carl Anderson found signs of such a particle in cosmic-ray tracks physicists took this as supporting Yukawa's hypothesis and named the particle a μ-meson (now called a *muon*). However, although the muon had the appropriate mass it interacted with nucleons so infrequently that it could not possibly be the nuclear "glue". He then accepted a position at Columbia University and obtained the same year the Nobel prize of Physics following the discovery of the meson π or pion by by Cecil Frank Powell, Giuseppe Occhialini and Cesar Lattes in 1947, "for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces". He was the first Japanese to obtain a Nobel Prize, whereas Powell received the 1950 Nobel Prize in physics, in part for his discoveries concerning mesons.
His idea was to introduce a new for of potential, called potential of Yukawa $U = \frac{e^{-r/\lambda}}{r}$ with $\lambda \simeq 10^{-12}$ cm corresponds to the mean size of a nucleus. This potential, contrarily to the one representing the electromagnetism $V = \frac{1}{r}$, is a short range potential. Indeed, the exponential suppression ensures that the potential does not influence nucleons beyond the radius of the nucleus. In 1955 he was a signatory to the Russell-Einstein Manifesto, a letter from ten famed scientists calling for worldwide nuclear disarmament. Hideki Yukawa died at his home in Kyoto in 1981 from pneumonia and heart failure, aged 74.

Appendix H

Historical articles

In this chapter, I have compiled some of the more fundamental articles that could be considered as cornerstone in our field. Of course, the list included here is very far to be exhaustive but is my attempts to conciliate my (our) work with the famous sentence of Newton: "We are all standing on the shoulders of Giants". The articles are presented by thematic and (as far as possible) by chronological order. From the quantum basis (Planck, spin, Dirac equation..) to the cosmology passing through the discoveries of new particles and Einstein's contributions. I always extracted the original version of the article (when it is in German, I tried to find a translated version of it to be accessible at a larger community) to keep the original feelings of time passing by. It is striking how revolutionary articles are in fact usually very short ones (less than 3 pages in most of the case). However, when the article is too long, I just kept the important pages useful for the explanation of the ideas developed or results obtained. But to be as fair as possible, a companion book exist (by the same author) which includes the original articles in their complete and untouched version.
I also by this mean want to thank the participants of the "Magic Monday Journal Club" that is held in Orsay since 2010 (especially A. Falkowski) which gave me the opportunity to read/study/present the works of such giants, receiving illuminating remarks from interested colleagues.

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qui émettent des radiations fluorescentes de couleur jaune verdâtre p
impressionner la plaque photographique à travers les corps opaque

» Les résultats contradictoires ci-dessus s'expliquent donc très l
tenant compte des faits signalés par MM. Charles Henry, Niewenglo
surtout par notre confrère M. Henri Becquerel dans les dernières s
Les corps fluorescents émettent des radiations jouissant des pro
des rayons X conformément à l'hypothèse de notre confrère M. Po

» De tous ces faits il résulte que le rôle des rayons cathodiques d
expériences de Röntgen semble se borner à exciter la fluoresce
verre spécial composant l'ampoule de Crookes. »

PHYSIQUE. — *Sur les radiations invisibles émises par les corps phosphor*
Note de M. **Henri Becquerel**.

« Dans la dernière séance, j'ai indiqué sommairement les expé
que j'avais été conduit à faire pour mettre en évidence les radiatio
sibles émises par certains corps phosphorescents, radiations qui tra
divers corps opaques pour la lumière.

» J'ai pu étendre ces observations, et, bien que je me propose de
nuer et de développer l'étude de ces phénomènes, leur actual
conduit à exposer, dès aujourd'hui, les premiers résultats que j'ai ol

» Les expériences que je rapporterai ont été faites avec les rad
émises par des lamelles cristallines de sulfate double d'uranyle
potassium

$$[SO^4(UO)K + H^2O],$$

corps dont la phosphorescence est très vive et la durée de pers
lumineuse inférieure à $\frac{1}{100}$ de seconde. Les caractères des radiation
neuses émises par cette substance ont été étudiés autrefois par mo
et j'ai eu, depuis, l'occasion de signaler quelques particularités i
santes que présentent ces radiations lumineuses.

» On peut vérifier très simplement que les radiations émises pa
substance, quand elle est exposée au soleil ou à la lumière diffuse d
traversent, non seulement des feuilles de papier noir, mais encore
métaux, par exemple une plaque d'aluminium et une mince feu
cuivre. J'ai fait notamment l'expérience suivante :

» Une plaque Lumière, au gélatino-bromure d'argent, a été en
dans un châssis opaque en toile noire, fermé d'un côté par une

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d'aluminium; si l'on exposait le châssis en plein soleil, même pendant une journée entière, la plaque ne serait pas voilée; mais, si l'on vient à fixer sur la plaque d'aluminium, à l'extérieur, une lamelle du sel d'uranium, que l'on peut, par exemple, assujettir avec des bandes de papier, et si l'on expose le tout pendant plusieurs heures au soleil, on reconnaît, lorsqu'on développe ensuite la plaque par les procédés ordinaires, que la silhouette de la lamelle cristalline apparaît en noir sur la plaque sensible et que le sel d'argent a été réduit en face de la lamelle phosphorescente. Si la lame d'aluminium est un peu épaisse, l'intensité de l'action est moindre qu'au travers de deux feuilles de papier noir.

» Si, entre la lamelle du sel d'uranium et la lame d'aluminium ou le papier noir, on interpose un écran formé d'une lame de cuivre, de $0^{mm},10$ environ d'épaisseur, par exemple en forme de croix, on observe dans l'image la silhouette de cette croix, en plus clair, mais avec une teinte indiquant cependant que les radiations ont traversé la lame de cuivre. Dans une autre expérience, une lame de cuivre plus mince ($0^{mm},04$) a affaibli beaucoup moins les radiations actives.

» La phosphorescence provoquée, non plus par les rayons solaires directs, mais par les radiations solaires réfléchies sur le miroir métallique d'un héliostat, puis réfractées par un prisme et une lentille de quartz, a donné lieu aux mêmes phénomènes.

» J'insisterai particulièrement sur le fait suivant, qui me paraît tout à fait important et en dehors des phénomènes que l'on pouvait s'attendre à observer : Les mêmes lamelles cristallines, placées en regard de plaques photographiques, dans les mêmes conditions et au travers des mêmes écrans, mais à l'abri de l'excitation des radiations incidentes et maintenues à l'obscurité produisent encore les mêmes impressions photographiques. Voici comment j'ai été conduit à faire cette observation : Parmi les expériences qui précèdent, quelques-unes avaient été préparées le mercredi 26 et le jeudi 27 février et, comme ces jours-là, le soleil ne s'est montré que d'une manière intermittente, j'avais conservé les expériences toutes préparées et rentré les châssis à l'obscurité dans le tiroir d'un meuble, en laissant en place les lamelles du sel d'uranium. Le soleil ne s'étant pas montré de nouveau les jours suivants, j'ai développé les plaques photographiques le 1er mars, en m'attendant à trouver des images très faibles. Les silhouettes apparurent, au contraire, avec une grande intensité. Je pensai aussitôt que l'action avait dû continuer à l'obscurité et je disposai l'expérience suivante :

H.1 "Sur les radiations invisibles émises par les corps phosphorescents", by H. Becquerel, 1896

Becquerel was a man of obsession. Following his father and grandfather before him, both professor ar Ecole Polytechnique in Paris, he was intrigued by the peculiar ability of some minerals to glow, or 'fluoresce' after exposure to sunlight. He was studying the property of some salt of Uranium when in January 1896 the German Wilhelm Rontgen discovers the X-rays. After several attempts with different products, he succeeds on 24 February 1896 to produce penetrating X-rays by exposing a sample of potassium uranyl disulphate to the sun all the day, in front of a photographic plates recover by several layers of dark paper to avoid any direct effect from the sun. After repeating the experiment, introducing a copper cross between the sample and the photographic plate, he obtains a parfetc image of the cross, which proves that the radiation were coming from the sample and stopped by the copper cross. He wrote a note to the French science academy on the subject (p.420-421 of the report). However, on 26 February, much to Becquerel's frustration, the sky above Paris was completely overcast and he was unable to carry out his experiment. Instead, he took all his apparatus -uranium sample, copper cross and wrapped photographic plates- and placed it in the drawer of a cabinet :

(...) comme ces jours là, le soleil ne s'est montré que d'une manière intermittente, j'avais conservé les expériences toutes préparées et rentré les chassis à l'obscurité dans le tiroir

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« Au fond d'une boîte en carton opaque, j'ai placé une plaque photographique, puis, sur la face sensible, j'ai posé une lamelle du sel d'uranium, lamelle convexe qui ne touchait le gélatino-bromure que seulement en quelques points; puis, à côté, j'ai disposé sur la même plaque une autre lamelle du même sel, séparée de la surface du gélatino-bromure par une mince lame de verre; cette opération étant exécutée dans la chambre noire, la boîte a été refermée, puis enfermée dans une autre boîte en carton, puis dans un tiroir.

» J'ai opéré de même avec le châssis formé par une plaque d'aluminium, dans lequel j'ai mis une plaque photographique, puis à l'extérieur une lamelle du sel d'uranium. Le tout a été enfermé dans un carton opaque, puis dans un tiroir. Au bout de cinq heures, j'ai développé les plaques, et les silhouettes des lamelles cristallines ont apparu en noir, comme dans les expériences précédentes et comme si elles avaient été rendues phosphorescentes par la lumière. Pour la lamelle posée directement sur la gélatine, il y avait à peine une différence d'action entre les points de contact et les parties de la lamelle qui s'écartaient d'un millimètre environ de la gélatine; la différence peut être attribuée à la distance différente des sources des radiations actives. L'action de la lamelle placée sur une lame de verre a été très légèrement affaiblie, mais la forme de la lamelle a été très bien reproduite. Enfin, au travers de la feuille d'aluminium, l'action a été considérablement plus faible, mais cependant très nette.

» Il importe d'observer que ce phénomène ne paraît pas devoir être attribué à des radiations lumineuses émises par phosphorescence, puisque, au bout de $\frac{1}{100}$ de seconde, ces radiations sont devenues si faibles qu'elles ne sont presque plus perceptibles.

» Une hypothèse qui se présente assez naturellement à l'esprit serait de supposer que ces radiations, dont les effets ont une grande analogie avec les effets produits par les radiations étudiées par MM. Lenard et Röntgen, seraient des radiations invisibles émises par phosphorescence, et dont la durée de persistance serait infiniment plus grande que la durée de persistance des radiations lumineuses émises par ces corps. Toutefois, les expériences présentes, sans être contraires à cette hypothèse, n'autorisent pas à la formuler. Les expériences que je poursuis en ce moment pourront, je l'espère, apporter quelques éclaircissements sur ce nouvel ordre de phénomènes. »

d'un meuble, en laissant en place les lamelles du sel d'uranium

On Sunday March 1st however, he decided (who know why??) he decided to take of all the material and develop the plate beleiving he would find only fainted image of the cross on the plate as the exhibition to the sun was weak (do not forget he was looking for fluorescence, which means glowing of a material after its exposition to the sun). On the opposite, he observed the shadows of the cross with much brighter intensity. He then had to suppose that the radiation should continue even without the light of the sun. He then tried with different size and shape of uranium samples, at different distance of the plates. He then concludes

(...) supposer que ces radiations, dont les effets ont une grande analogie avec les effets produits par les radiations étudiées par MM. Lenard et Rontgen, seraient des radiations invisibles émises par phosphorescence, et dont la durée de persistance serait infiniment plus grande que la durée de persistance des radiations lumineuses émises par ces corps.

which means that he suppose that such radiation can be of a nature similar to the X-rays observed by Rontgen 3 months before and the time of radiation of such substance can be infinitely greater than normal fluorescence effect. What Becquerel had discovered was an entirely new phenomenon, one which, two years later, would be christened 'radioactivity'. The most remarkable characteristic of radioactivity was its persistence. As time passed, Becquerel could detect not the slightest weakening in the 'uranium rays'. Week after week, month after month, the rays flooded forth in an unending stream, drawing on an apparently bottomless well of energy. Becquerel obtained the Nobel Prize of Physics in 1905 with Pierre and Marie Curie (its students).

Ueber eine Verbesserung der Wien'schen Spectralgleichung; von M. Planck.

(Vorgetragen in der Sitzung vom 19. October 1900.)

(Vgl. oben S. 161.)

Die von Hrn. KURLBAUM in der heutigen Sitzung mitgeteilten interessanten Resultate der von ihm in Gemeinschaft mit Hrn. RUBENS auf dem Gebiete der längsten Spectralwellen ausgeführten Energiemessungen haben die zuerst von den Herren LUMMER und PRINGSHEIM auf Grund ihrer Beobachtungen aufgestellte Behauptung nachdrücklich bestätigt, dass das WIEN'sche Energieverteilungsgesetz nicht die allgemeine Bedeutung besitzt, welche ihm bisher von mancher Seite zugeschrieben worden war, sondern dass dies Gesetz vielmehr höchstens den Charakter eines Grenzgesetzes hat, dessen überaus einfache Form nur einer Beschränkung auf kurze Wellenlängen bez. tiefe Temperaturen ihren Ursprung verdankt.[1]) Da ich selber die Ansicht von der Notwendigkeit des WIEN'schen Gesetzes auch an dieser Stelle vertreten habe, so sei es mir gestattet, hier kurz darzulegen, wie sich die von mir entwickelte elektromagnetische Theorie der Strahlung zu den Beobachtungsthatsachen stellt.

Nach dieser Theorie ist das Energieverteilungsgesetz bestimmt, sobald die Entropie S eines auf Bestrahlung ansprechenden linearen Resonators als Function seiner Schwingungsenergie U bekannt ist. Ich habe indes schon in meiner letzten Arbeit über diesen Gegenstand hervorgehoben[2]), dass der Satz der Entropievermehrung an und für sich noch nicht hinreicht, um diese Function vollständig anzugeben; zur Ansicht von der Allgemeinheit des WIEN'schen Gesetzes wurde ich vielmehr durch eine besondere Betrachtung geführt, nämlich durch die Berechnung einer unendlich kleinen Entropievermehrung eines in einem stationären Strahlungsfelde befind-

1) Auch Hr. PASCHEN hat, wie er mir brieflich mitteilte, neuerdings merkliche Abweichungen vom WIEN'schen Gesetz festgestellt.

2) M. PLANCK, Ann. d. Phys. 1. p. 730. 1900.

lichen Systems von n gleichen Resonatoren auf zwei verschiedene Weisen, wodurch sich die Gleichung[1]) ergab:

$$d U_n \cdot \Delta U_n \cdot f(U_n) = n\, d U \cdot \Delta U \cdot f(U),$$

wobei

$$U_n = n U \quad \text{und} \quad f(U) = -\frac{3}{8}\frac{d^2 S}{d U^2},$$

aus welcher dann das WIEN'sche Gesetz in der Form hervorgeht:

$$\frac{d^2 S}{d U^2} = \frac{\text{const.}}{U}.$$

In jener Functionalgleichung stellt der Ausdruck auf der rechten Seite sicher die genannte Entropieänderung dar, weil sich n ganz gleiche Vorgänge unabhängig voneinander abspielen, deren Entropieänderungen sich daher einfach addiren müssen. Dagegen würde ich es wohl für möglich, wenn auch immer noch für nicht leicht begreiflich und jedenfalls schwer beweisbar ansehen, dass der Ausdruck links nicht allgemein die ihm früher von mir zugeschriebene Bedeutung besitzt, mit anderen Worten: dass die Werte von U_n, $d U_n$ und ΔU_n gar nicht hinreichen, um die fragliche Entropieänderung zu bestimmen, sondern dass dazu auch U selber bekannt sein muss. Im Verfolg dieses Gedankens bin ich schliesslich dahin gekommen, ganz willkürlich Ausdrücke für die Entropie zu construiren, welche, obwohl complicirter als der WIEN'sche Ausdruck, doch allen Anforderungen der thermodynamischen und elektromagnetischen Theorie ebenso vollkommen Genüge zu leisten scheinen wie dieser.

Unter den so aufgestellten Ausdrücken ist mir nun einer besonders aufgefallen, der dem WIEN'schen an Einfachheit am nächsten kommt, und der, da letzterer nicht hinreicht, um alle Beobachtungen darzustellen, wohl verdienen würde, daraufhin näher geprüft zu werden. Derselbe ergiebt sich, wenn man setzt[2]):

$$\frac{d^2 S}{d U^2} = \frac{\alpha}{U(\beta + U)}.$$

1) l. c. p. 732.

2) Ich gehe aus von dem zweiten Differentialquotienten von S nach U, weil diese Grösse eine einfache physikalische Bedeutung besitzt. (l. c. p. 731.)

H.2 "On an Improvement of Wien's Equation for the Spectrum", by M. Planck, 1900

"There is only one way to determine in what direction physical science is moving: to compare its current state with that it was previously", Max Planck. Indeed, the year 1900 is a turning point in theoretical physics. Two talks of Max Planck in Berlin and one experiments of Rubens and Kurlbaum will set the basis of the modern physics. On October 19, 1900, at a meeting of the German Physical Society Planck gave a brief report. It was modestly entitled "On an improvement of Wien's radiation law". It is fundamental to keep in mind, when reading any works made by Planck, that he was student of Helmholtz and Kirchhoff, was fan of Clausius and that his PhD thesis was about the second law of thermodynamics. Planck can abandon any kind of construction but would never be able to abandon the second principle. In other words, the entropy is the key of any revolution for him (contrarily to popular beliefs, the formula $S = k \log \Omega$ is not due to Boltzmann but to Planck).

First step: a thermodynamic for the black body.

At the beginning of 1900, the Wien law describing the density of energy in a black body at a temperature T $u_\nu(T)$ was valid at high frequencies and can be simplified $U(T) = Ae^{\frac{B\nu}{T}}$. From this, Planck deduce from the second principle $TdS = dU$ his second formula p.2: $\frac{d^2S}{dU^2} = -\frac{B}{U\nu}$. However, in October, Rubens and Kurlbaum in Berlin, just before the conference announced (in the paper also included below) that the radiation intensity at low frequencies was *proportional to the temperature*. Planck was shocked. That mean $U = kT$ at low frequencies ν, and so, still applying the second principle, $\frac{d^2S}{dU^2} = -\frac{A}{U^2}$. Intuitively Planck proposed in his third formula:

$$\frac{d^2S}{dU^2} = \frac{-\alpha}{U(\beta\nu + U)} \tag{H.1}$$

This formulation respect the Wien law at high frequencies and the new measurements at low frequencies. After one integration and combining with $\frac{dS}{dU} = \frac{1}{T}$ Planck obtained $U(T)$ in his last equation

$$U = \frac{\alpha'}{e^{\beta\nu/T} - 1}. \tag{H.2}$$

Sure of using the fundamental principles (thermodynamics) but not yet conscious of the revolution that he will provoke two months later, he concludes "I should therefore be permitted to draw your attention to this new formula which I consider to be the simplest possible, apart from Wien's expression, from the point of view of the electromagnetic theory of radiation".

Second step: experimental judges.

In October 1900 appeared the study of Rubens and Kurlbaum that we reproduce also hereafter. With a new apparatus, they study the radiation of several type of black bodies and compare 6 models. The first one is the classical Wien law, the 2-4 are phenomenological/empirical modifications to the Wien law and the last one (5) is the one proposed by Planck the same month. Their figure 2 summarize their results. Their conclusion in the last paragraph is clear : "We find therefore, that so far as the representation of Lummer and Pringsheim's results, as well as our own, is concerned, the formulae (4a) and (5) are excellently adapted, but that Planck's expression, in so far as it does the same thing, deserves the preference on account of its simplicity".

Third step: the quantization.

Then Planck concentrated in the entropy itself. Integrating Eq.(H.1) and obtained

$$S = k\left[(1 + \frac{U}{\beta\nu})\log\left(1 + \frac{U}{\beta\nu}\right) - \frac{U}{\beta\nu}\log\frac{U}{\beta\nu}\right] \tag{H.3}$$

With the fundamental hypothesis that the energy can be divided into finite quantum as he wrote on page 3 of the article "On the Theory of the Energy Distribution Law of the Normal Spectrum" he published in december: "*We consider, however - this is the most essential point of the whole calculation - E to be composed of a very definite number of equal parts and use thereto the constant of nature* $h = 6.55 \times 10^{-27}$ *erg.sec*". Planck remembered a computation made by Boltzmann himself (and published) about the entropy of a discrete system of N particles having P states (his last equation of page 3). Identifying the entropy as a measurement of the "disorder" $S = k \log W$ with $W \simeq \frac{(N+P)^{N+P}}{N^N P^P}$ for a set of N oscillators radiating energy with P energy elements. One obtain easily

$$S = kN\left[(1+\frac{P}{N})\log\left(1+\frac{P}{N}\right) - \frac{P}{N}\log\frac{P}{N}\right] \tag{H.4}$$

Identifying Eq.(H.3) with Eq.(H.4) one deduces $\frac{P}{N} = \frac{U}{\beta\nu} \Rightarrow NU = E = P\epsilon$, with $\epsilon = \beta\nu$ (nowadays $h\nu$, h being the Planck's constant) Planck noticed he will obviously recover by derivation the Eq.(H.2). The total energy radiated by the system of oscillator can thus only be done by a discrete amount of fundamental quanta ϵ: this is the act of birth of Quantum mechanics.

It is important to note that during long times, Planck believed in his method but doubted about the physical meaning of such quantization. Not only he not dare to introduce the concept of light quanta (this was first done by Einstein in 1905) but he even wrote in 1911 at his Solway's talk: "*it seems to me that the hypothesis of quantum emission, along with the original hypothesis of quanta of energy must be regarded more as a figure of speech than as an expression of physical reality*". He tried, unsuccessfully, all his life to reconciliate classical theory and quantum mechanics. The revolution he engaged began mature some 30 years later, when young scientists took the flame. As he used to say:

"A great scientific idea rarely is introduced by the gradual convincing and conversion of its opponents, it rarely happens that "Saul" becomes Paul". In reality, the opponents gradually die out and the rising generation is familiar with the new idea from the beginning – an example of the fact that the future belongs to the young"

Max Planck

welche S als logarithmische Function von U liefern (was anzunehmen die Wahrscheinlichkeitsrechnung nahe legt) und welche ausserdem für kleine Werte von U in den obigen WIEN'schen Ausdruck übergehen. Mit Benutzung der Beziehung

$$\frac{dS}{dU} = \frac{1}{T}$$

und des WIEN'schen „Verschiebungs"gesetzes [1]) erhält man hieraus die zweiconstantige Strahlungsformel:

$$E = \frac{C\lambda^{-5}}{e^{\frac{c}{\lambda T}} - 1},$$

welche, soweit ich augenblicklich sehen kann, den Gang der seither publicirten Beobachtungszahlen ebenso befriedigend wiedergiebt, wie die besten bisher aufgestellten Spectralgleichungen, nämlich die von THIESEN [2]), die von LUMMER-JAHNKE [3]) und die von LUMMER-PRINGSHEIM. [4]) (Wird an einigen Zahlen erläutert.) Ich möchte mir daher erlauben, Ihre Aufmerksamkeit auf diese neue Formel zu lenken, die ich vom Standpunkt der elektromagnetischen Strahlungstheorie aus nächst der WIEN'schen für die einfachste halte.

1) Der Ausdruck des WIEN'schen Verschiebungsgesetzes ist einfach:

$$S = f\left(\frac{U}{\nu}\right),$$

wo ν die Schwingungszahl des Resonators bedeutet. Ich werde dies bei einer anderen Gelegenheit darlegen.

2) M. THIESEN, Verhandl. d. Deutsch. Physikal. Gesellsch. 2. p. 67. 1900. Dort findet sich auch bemerkt, dass Hr. THIESEN seine Formel schon aufgestellt hatte, ehe die Herren LUMMER u. PRINGSHEIM ihre Messungen auf grössere Wellenlängen ausdehnten, was ich hier hervorhebe weil ich vor dem Erscheinen der citirten Publication eine etwas andere Darstellung gegeben hatte (M. PLANCK, Ann. d. Phys. 1. p. 719. 1900).

3) O. LUMMER u. E. JAHNKE, Ann. d. Phys. 3. p. 288. 1900.

4) O. LUMMER u. E. PRINGSHEIM, Verhandl. d. Deutsch. Physikal. Gesellsch. 2. p. 174. 1900.

M. Planck, Verhandl. Dtsch. phys. Ges., **2,** 202 **1900**

On an Improvement of Wien's Equation for the Spectrum

M. Planck
Berlin
(Received 1900)

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English translation from "The Old Quantum Theory," ed. by D. ter Haar, Pergamon Press, 1967, p. 79.

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The interesting result of long wave length spectral energy measurements which were communicated by Mr. Kurlbaum at today's meeting, and which were obtained by him and Mr. Rubens, confirm the statement by Mr. Lummer and Mr. Pringsheim, which was based on their observations that Wien's energy distribution law is not as generally valid, as many supposed up to now, but that this law at most has the character of a limiting case, the simple from of which was due only to a restriction to short wave lengths and low temperatures[1]. Since I myself even in this Society have expressed the opinion that Wien's law must be necessarily true, I may perhaps be permitted to explain briefly the relationship between the electromagnetic theory developed by me and the experimental data.

The energy distribution law is according to this theory determined as soon as the entropy S of a linear resonator which interacts with the radiation is known as function of the vibrational energy U. I have, however, already in my last paper on this subject[1] stated that the law of increase of by itself not yet sufficient to determine this function completely; my view that Wien's law would be of general validity, was brought about rather by special considerations, namely by the evaluation of an infinitesimal increase of the entropy of a system of n identical resonators in a stationary radiation field by two different methods which led to the equation

$$dU_n \cdot \Delta U_n \cdot f(U_n) = n dU \cdot \Delta U \cdot f(U),$$

[1]Mr. Paschen has written to me that he has also recently found appreciable deviations from Wien's law

where

$$U_n = nU \qquad \text{and} \qquad f(U) = -\frac{3}{5}\frac{d^2S}{dU^2}.$$

From this equation Wien's law follows in the form

$$\frac{d^2S}{dU^2} = \frac{\text{const}}{U}.$$

The expression on the right-hand side of this functional equation is certainly the above–mentioned change in entropy since n identical processes occur independently, the entropy changes of which must simply add up. However, I consider the possibility, even if it would not be easily understandable and in any case would be difficult to prove, that the expression on the left-hand side would not have the general meaning which I attributed to it earlier, in other words: that the values of U_n, dU_n and ΔU_n are not by themselves sufficient to determine the change of entropy under consideration, but that U itself must also be known for this. Following this suggestion I have finally started to construct completely arbitrary expressions for the entropy which although they are more complicated than Wien's expression still seem to satisfy just as completely all requirements of the thermodynamic and electromagnetic theory.

I was especially attracted by one of the expressions thus constructed which is nearly as simple as Wien's expression and which deserves to be investigated since Wien's expression is not sufficient to cover all observations. We get this expression by putting[2]

$$\frac{d^2S}{dU^2} = \frac{\alpha}{U(\beta + U)}.$$

It is by far simplest of all expressions which lead to S as a logarithmic function of U–which is suggested from probability considerations – and which moreover reduces to Wien's expression for small values of U. Using the relation

$$\frac{dS}{dU} = \frac{1}{T}$$

and Wien's "displacemen" law[3] one gets a radiation formula with two constants:

$$E = \frac{C\lambda^{-5}}{e^{c/\lambda T} - 1},$$

which, as far as I can see at the moment, fits the observational data, published up to now, as satisfactory as the best equations put forward for the spectrum, namely those of Thiesen[2][4] Lummer–Jahnke[4], and Lummer–Pringsheim[5]. (This was demonstrated by some numerical examples.) I should therefore be permitted to draw your attention to this new formula which I consider to be the simplest possible, apart from Wien's expression, from the point of view of the electromagnetic theory of radiation.

[2]I use the second derivative of S with respect to U since this quantity has a simple physical meaning.

[3]The expression of Wien's displacement law is simply

$$S = f(U/\nu),$$

where ν is the frequency of the resonator, as I shall show elsewhere.

References

[1] M. Planck Am. Physik. 1, 730 (1900);

[2] M. Thiesen, Verh. D. Phys. Ges. Berlin 2, 67 (1900);

[3] M. Planck, Ann. Physik 1, 719 (1900);

[4] O. Lummer and E. Jahnke, Ann. Physik Lpz. 3, 288 (1900);

[5] O. Lummer and E. Pringsheim, Verh. Dtsch. Phys. Ges. Berlin 2, 174 (1900)

[4]One can see there that Mr. Thiesen had put forward his formula before Mr. Lummer and Mr. Pringsheim had extended their measurements to longer wave lengths. I emphasise this point as I have made a statement to the contrary[3] before this paper was published.

Possible Existence of a Neutron

James Chadwick
Nature, p. 312 (Feb. 27, 1932)

It has been shown by Bothe and others that beryllium when bombarded by α-particles of polonium emits a radiation of great penetrating power, which has been an absorption coefficient in lead of about 0.3 $(cm)^{-1}$. Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionisation produced by this beryllium radiation in a vessel with a thin window, that the ionisation increased when matter containing hydrogen was placed in front of the window. The effect appeared to be due to the ejection of protons with velocities up to a maximum of nearly 3 x 10^9 cm. per sec. They suggested that the transference of energy to the proton was by a process similar to the Compton effect, and estimated that the beryllium radiation had a quantum energy of 50 x 10^6 electron volts.

I have made some experiments using the valve counter to examine the properties of this radiation excited in beryllium. The valve counter consists of a small ionisation chamber connected to an amplifier, and the sudden production of ions by the entry of a particle, such as a proton or α-particle, is recorded by the deflexion of an oscillograph. These experiments have shown that the radiation ejects particles from hydrogen, helium, lithium, beryllium, carbon, air, and argon. The particles ejected from hydrogen behave, as regards range and ionising power, like protons with speeds up to about 3.2 x 10^9 cm. per sec. The particles from the other elements have a large ionising power, and appear to be in each case recoil atoms of the elements.

If we ascribe the ejection of the proton to a Compton recoil from a quantum of 52 x 10^6 electron volts, then the nitrogen recoil atom arising by a similar process should have an energy not greater than about 400,000 volts, should produce not more than about 10,000 ions, and have a range in air at N.T.P. of about 1.3 mm. Actually, some of the recoil atoms in nitrogen produce at least 30,000 ions. In collaboration with Dr. Feather, I have observed the recoil atoms in an expansion chamber, and their range, estimated visually, was sometimes as much as 3 mm at N.T.P.

These results, and others I have obtained in the course of the work, are very difficult to explain on the assumption that the radiation from beryllium is a quantum radiation, if energy and momentum are to be conserved in the collisions. The difficulties disappear, however, if it be assumed that the radiation consists of particles of mass 1 and charge 0, or neutrons. The capture of the α-particle by the Be^9 nucleus may be supposed to result in the formation of a C^{12} nucleus and the emission of the neutron. From the energy relations of this process the velocity of the neutron emitted in the forward direction may well be about 3 x 10^9 cm. per sec. The collisions of the neutron with the atoms through which it passes give rise to the recoil atoms, and the observed energies of the recoil atoms are in fair agreement with this view. Moreover, I have observed that the protons ejected from hydrogen by the radiation emitted in the opposite direction to that of the exciting α-particle appear to have a much smaller range than those ejected by the forward radiation. This again receives a simple explanation of the neutron hypothesis.

If it be supposed that the radiation consists of quanta, then the capture of the α-particle by the Be^9 nucleus will form a C^{13} nucleus. The mass defect of C^{13} is known with sufficient accuracy to show that the energy of the quantum emitted in this process cannot be greater than about 14 x 10^6 volts. It is difficult to make such a quantum responsible for the effects observed.

It is to be expected that many of the effects of a neutron in passing through matter should resemble those of a quantum of high energy, and it is not easy to reach the final decision between the two hypotheses. Up to the present, all the evidence is in favour of the neutron, while the quantum hypothesis can only be upheld if the conservation of energy and momentum be relinquished at some point.

J. Chadwick.
Cavendish Laboratory,
Cambridge, Feb. 17.

H.3 "Possible Existence of a Neutron" by J. Chadwick, 1932

When W. Botrthe and H. Becker bombarded beryllium with polonium in 1931, they unknowingly observed the first neutron through the relation

$$He_2^4 \quad + \quad Be_4^9 \quad \rightarrow \quad C_6^{12} \quad + n_0^1 \tag{H.5}$$

The same year, I. Curie and her husband F. Joliot observed the same process, reproducing the experiments more carefully and deduce it was a *very* energetic X–ray even able to knock out protons out of paraffin. Chadwick understood that this interpretation of gamma–ray scattering on protons was a misinterpretation. He knew that Joliot and Curie has observed the neutral version of the proton.

Indeed, Chadwick noted that the velocity of the proton ejected through the collision was 1/10 (!!) the speed of light. The energy of the photon to produce such an energetic proton through a scattering should be around 50 MeV, unusually very high energy as the photon emitted in such processes never reach more than few MeV at the maximum. However, this issue is solved if one consider that the incident radiation due to the ejection of the proton consists of a neutral partner of a rest energy of about 1 GeV: a neutral partner of the proton. The discovery of the neutron was (they believed) the last piece of the puzzle. The modern atom was complete.

H.4 "The positive electron" by C. Anderson, 1932

In 1932, Chadwick just discovered a new neutral particle called the neutron. The same year, C.D. Anderson began his exploration of a new continent of particles through cosmic rays. Indeed at these times, the maximal energies that could be reached using X-rays and radioactive sources were limited to few MeV. The Austrian Viktor Hess some years before, observed some cosmic rays while embarked in a baloon at 5000 meters of altitude, whereas the soviet physicist Dimitry Skobeltzyn measured some tracks of cosmic particles in a cloud chamber.

Anderson was observing cosmic–rays particles in a cloud chamber he built in collaboration with R.A. Millikan at the California Institute of Technology. The chamber had a magnetic field of 15 000 Gauss, and a 6-mm plate of lead traversing it where the charged particles should lose part of their energy. Indeed, tha main difficulties in these chambers, measuring the curvature of the radius due to the magnetic field (for a taken photo), it was possible to determine the mass and the absolute value of the charge, but not its sign. In other word, how to distinguish a positive particle coming from above, from a negative particle coming from below. The passage through the lead solved this issue. Indeed, a greater curvature from one side of the lead plate indicates lower momentum, the result of the particle losing energy when it passed through the plate. From the photos he took, he concluded that he was dealing with a new positive particle with a charge less than twice that of the electron and a mass much less than that of the proton

Some year earlier, P.A.M. Dirac had proposed the existence of "antiparticles" through its equation (Dirac equation) where he conciliated special relativity with quantum mechanics. Originally Dirac proposed the proton to be the antiparticle of the electron. Even if the discovery of Anderson was fortuitous, he was aware of the article of Dirac. To be honest, as Anderson underline at the end of his articles, Blackett and Occhialini had taken some photos of similar events before but were not sufficiently sure of their results to publish them. These 2 researchers proposed the correct mechanism of production of the positron : an incident gamma–ray interacts withthe electromagnetic field surrounding a nucleus and an electron–positron pair is formed.

The Positive Electron

CARL D. ANDERSON, *California Institute of Technology, Pasadena, California*

(Received February 28, 1933)

Out of a group of 1300 photographs of cosmic ray tracks in a vertical Wilson chamber 15 tracks were of positive particles which could not have a mass as great as that of the proton. From an examination of the energy-loss and ionization produced it is concluded that the charge is less than, twice, and is probable exactly equal to, that of the proton. If these particles carry only positive charge the curvature and ionizations produced require the mass to be less than twenty times the electron mass. These particles will be called positrons. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.

Editor

ON August 2, 1932, during the course of photographing cosmic ray tracks produced in a vertical Wilson chamber (magnetic field of 15,000 gauss) designed in the summer of 1930 by Professor R. A. Millikan and the writer, the tracks shown in Fig. 1 were obtained, which seemed to be interpretable only on the basis of the existence in this case of a particle carrying a positive charge but having a mass of the same order of magnitude as that normally possessed by a free negative electron. Later study of the photograph by a whole group of men of the Norman Bridge Laboratory only tended to strengthen this view. The reason that this interpretation seemed so inevitable is that the track appearing in the upper half of the figure cannot possibly have a mass as large as that of a proton; for as soon as the mass is fixed the energy is at once fixed by the curvature. The energy of a proton of that curvature comes out 800,000 volts, but a proton of that energy according to well established and universally accepted determinations[1] has a total range of about 5 mm in air while that portion of the range actually visible in this case exceeds 5 cm without a noticeable change in curvature. The only escape from this conclusion would be to assume that at exactly the same instant (and the sharpness of the tracks determines that instant to within about a fifteenth of a second) two independent electrons happened to produce two tracks so placed as to give the impression of a single particle shooting through the lead plate. This assumption was dismissed on a probability basis, since a sharp track of this order of curvature under the experimental conditions prevailing occurred in the chamber only once in some 500 exposures, and since there was practically no chance at all that two such tracks should line up in this way. We also disregarded as completely untenable the assumption of an electron of 20 million volts entering the lead on one side and coming out with an energy of 60 million volts on the other side. A fourth possibility is that a photon, entering the lead from above, knocked out of the nucleus of a lead atom two particles, one of which shot upward and the other downward. But in this case the upward moving one would be a positive of small mass so that either of the two possibilities leads to the existence of the positive electron.

In the course of the now few weeks other photographs were obtained which could be interpreted logically only on the positive-electron basis, and a brief report was then published[2] with due reserve in interpretation in view of the importance and striking nature of the announcement.

MAGNITUDE OF CHARGE AND MASS

It is possible with the present experimental data only to assign rather wide limits to the magnitude of the charge and mass of the particle. The specific ionization was not in these cases measured, but it appears very probable, from a knowledge of the experimental conditions and by comparison with many other photographs of high- and low-speed electrons taken under the same conditions, that the charge cannot differ in magnitude from that of an electron by an amount as great as a factor of two. Furthermore, if the photograph is taken to represent a positive particle penetrating the 6 mm lead plate, then the energy lost, calculated for unit charge, is approximately 38 million electron volts, this value being practically independent of the proper mass of the particle as long as it is not too many times larger than that of a free negative electron. This value of 63 million volts per cm energy-loss for the positive particle it was considered legitimate to compare with the measured mean of approximately 35 million volts[3] for negative electrons of 200-300 million volts energy since the rate of energy loss for particles of small mass is expected to change only very slowly over an energy range extending from several million to several hundred million volts. Allowance being made for experimental uncertainties, an upper limit to the rate of energy loss for the positive particle can then be set at less than four times that for an electron, thus fixing, by the usual relation between rate of ionization and rate of energy loss, the charge of the particle as lying between the limits of twice and equal to that of the proton.

FIG. 1. A 63 million volt positive ($H\rho = 2.1 \times 10^3$ gauss-cm) passes through a 6 mm lead plate and emerging as a 23 million volt particle ($H\rho = 7.5 \times 10^2$ gauss-cm). The length of this bare path is at least ten times greater than the possible length of a proton path of this curvature.

[1] Rutherford, Chadwick and Ellis, *Radiations from Radioactive Substances*, p. 291. Assuming $R \propto v^3$ and using data there given the range of a 800,000 volt proton in air N.T.P. is about 5 mm.

[2] C. D. Anderson, Science 76, 238 (1932).

[3] C. D. Anderson, Phys. Rev. 43, 381A (1933).

charge, an upper limit to the charge less than twice that of the negative electron. It is concluded, therefore, that the magnitude of the charge of the positive electron which we shall henceforth contract to positron is very probably equal to that of a free negative electron which from symmetry considerations would naturally then be called a negatron.

FIG. 2. A positron of 20 million volts energy ($H\rho = 5.5 \times 10^2$ gauss-cm) and a negatron of 30 million volts energy ($H\rho = 10.2 \times 10^2$ gauss-cm) projected from a plate of lead. The curve of the positive particle provides no positive evidence of identity with the negative curvature.

It is pointed out that the effective depth of the chamber in the line of sight which is the same as the direction of the magnetic lines of force was 1 cm and its effective diameter at right angles to that line 14 cm, thus insuring that the particle crossed the chamber practically normal to the lines of force. The change in direction due to scattering in the lead[3] in this case about 8° measured in the plane of the chamber, is a probable value for a particle of this energy though less than the most probable value.

The magnitude of the proper mass cannot as yet be given further than to fix an upper limit to it about twenty times that of the electron mass. If Fig. 1 represents a particle of unit charge passing through the lead plate then the curvatures, on the basis of the information at hand on ionization, give but low a value for the energy-loss unless the mass is taken less than twenty times that of the negative electron mass. Further determinations of $H\rho$ for relatively low energy particles before and after they cross a known amount of matter, together with a study of ballistic effects such as those encountered with electrons, involving large energy transfers, will enable closer limits to be assigned to the mass.

To date, out of a group of 1300 photographs of cosmic ray tracks 15 of these show positive particles penetrating the lead, none of which can be ascribed to particles with a mass as large as that of a proton, thus establishing the existence of positive particles of unit charge and of mass small compared to that of a proton. In many other cases due either to the short section of track available for measurement or to the high energy of the particle it is not possible to differentiate with certainty between protons and positrons. A comparison of the six or seven hundred positive-ray tracks which we have taken is, however, still consistent with the view that the positive particle which is knocked out of the nucleus by the incoming primary cosmic ray is in many cases a proton.

FIG. 3. A group of six particles projected from a region in the wall of the chamber. The track at the left of the central group of four tracks is a negatron of about 18 million volts energy ($H\rho = 6.2 \times 10^2$ gauss-cm) and that at the right a positron of about 20 million volts energy ($H\rho = 7.0 \times 10^2$ gauss-cm). Identification of the two tracks in the center is not possible. A negatron of about 15 million volts is shown at the left. This group represents forty tracks which were broadened by the diffusion of the ions. The uniformity of this broadening for all the tracks shows that the particles entered the chamber at the same time.

From the fact that positrons occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from an atomic nucleus. If we retain the view that a nucleus consists of protons and neutrons (and, or particles and that a neutron represents a close combination of a proton and electron, then from the electromagnetic theory as to the origin of mass the simplest assumption would seem to be that an encounter between the incoming primary ray and a proton may take place in such a way as to expand the diameter of the proton to the same value as that possessed by the negatron. This process would release an energy of a billion electron-volts appearing as a secondary photon. As a second possibility the primary ray may disintegrate a neutron for more than once in the nucleus by the ejection either of a negatron or a positron with the result that a positive or a negative proton, as the case may be, remains in the nucleus in place of the neutron, the event occurring in this instance without the emission of a photon. This alternative, however, postulates the existence in the nucleus of a proton of negative charge, no evidence for which exists. The greater symmetry, however, between the positive and negative charges revealed by the discovery of the positron should prove a stimulus in search for evidence of the existence of negative protons. If the neutron should prove to be a fundamental particle of a new kind rather than a proton and negatron in close combination, the above hypotheses will have to be abandoned for the proton will then in all probability be represented as a complex particle consisting of a neutron and positron.

FIG. 4. A positron of about 300 million volts energy ($H\rho = 16.0 \times 10^2$ gauss-cm) penetrates the 12 mm lead plate and emerges with about 135 million volts energy ($H\rho = 4.2 \times 10^2$ gauss-cm). The distortion of the tracks represents a proton traversing the lead. It is inconsistent with the observed curvatures. The energies were then, respectively, about 20 million and 8 million volts above and below the lead, energies too low to permit the proton to have a range sufficient to penetrate a plate of lead 2.4 mm thick.

While this paper was in preparation press reports have announced that P. M. S. Blackett and G. Occhialini in an extensive study of cosmic-ray tracks have also obtained evidence for the existence of light positive particles confirming our earlier report.

I wish to express my great indebtedness to Professor R. A. Millikan for suggesting this research and for many helpful discussions during its progress. The able assistance of Mr. Seth H. Neddermeyer is also appreciated.

(*Reprinted from* NATURE, *February* 20, 1926.)

SPINNING ELECTRONS AND THE STRUCTURE OF SPECTRA

So far as we know, the idea of a quantised spinning of the electron was put forward for the first time by A. K. Compton (*Journ. Frankl. Inst.*, Aug. 1921, p. 145), who pointed out the possible bearing of this idea on the origin of the natural unit of magnetism. Without being aware of Compton's suggestion, we have directed attention in a recent note (*Naturwissenschaften*, Nov. 20, 1925) to the possibility of applying the spinning electron to interpret a number of features of the quantum theory of the Zeeman effect, which were brought to light by the work especially of van Lohuizen, Sommerfeld, Landé and Pauli, and also of the analysis of complex spectra in general. In this letter we shall try to show how our hypothesis enables us to overcome certain fundamental difficulties which have hitherto hindered the interpretation of the results arrived at by those authors.

To start with, we shall consider the effect of the spin on the manifold of stationary states which corresponds to motion of an electron round a nucleus. On account of its magnetic moment, the electron will be acted on by a couple just as if it were placed at rest in a magnetic field of magnitude equal to the vector product of the nuclear electric field and the velocity of the electron relative to the nucleus divided by the velocity of light. This couple will cause a slow precession of the spin axis, the conservation of the angular momentum of the atom being ensured by a compensating precession of the orbital plane of the electron.. This complexity of the motion requires that, corresponding to each stationary state of an imaginary atom, in which the electron has no spin, there shall in general exist a set of states which differ in the orientation of the spin axis relative to the orbital plane, the other characteristics of the motion remaining unchanged. If the spin corresponds to a one-quantum rotation there will be in general two such states. Further, the energy difference of these states will, as a simple calculation shows, be proportional to the fourth power of the nuclear charge. It will also depend on the quantum numbers which define the state of motion of the non-spinning electron in a way very similar to the energy differences connected with the rotation of the orbit in its own plane

[4]

arising from the relativity variation of the electronic mass. We are indebted to Dr. Heisenberg for a letter containing some calculations on the quantitative side of the problem.

This result suggests an essential modification of the explanation hitherto given of the fine structure of the hydrogen-like spectra. As an illustration we may consider the energy levels corresponding to electronic orbits for which the principal quantum number is equal to three. The scheme on the left side of the accompanying figure (Fig. 1) corresponds to the results to be expected from Sommerfeld's theory. The so-called azimuthal quantum number k is defined by the quantity of moment of momentum of the electron about the nucleus, $kh/2\pi$, where $k = 1, 2, 3$. According to the new theory, depicted in the scheme[1] on the right, this moment of momentum is given by $Kh/2\pi$, where $K = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. The total angular momentum of the atom is $Jh/2\pi$, where $J = 1, 2, 3$. The symbols K and J correspond to those used by Landé in his classification of the Zeeman effects of the optical multiplets. The letters S, P, D also relate to the analogy with the structure of optical spectra which we consider below. The dotted lines represent the position of the energy levels to be expected in the absence of the spin of the electron. As the arrows indicate, this spin now splits each level into two, with the exception of the level $K = \frac{1}{2}$, which is only displaced.

In order to account for the experimental facts, the resulting levels must fall in just the same places as the levels given by the older theory. Nevertheless, the two schemes differ fundamentally. In particular, the new theory explains at once the occurrence of certain components in the fine structure of the hydrogen spectrum and of the helium spark spectrum which according to the old scheme would correspond to transitions where K remains unchanged. Unless these transitions could be ascribed to the action of electric forces in the discharge which would perturb the electronic motion, their occurrence would be in disagreement with the correspondence principle, which only allows transitions in which the azimuthal quantum number changes by one unit. In the new scheme we see that, in the transitions in question, K will actually change by one unit and only J will

[1] Quite independently of the ideas discussed here, a scheme of levels corresponding to this figure has been previously proposed by the writers (*Physica*, 5, 266, 1925), on the ground of the formal analogy between spectral structures. From similar formal considerations, this scheme has recently also been arrived at by J. C. Slater (*Proc. Washington Acad.*, December 1925).

H.5 Introduction of the spin : "Spinning electrons and the structure of spectra" by G.E. Uhlenbeck and S. Goudsmit, 1926

The history of the discovery of the spin is a good summary of evolution of modern sciences. Goudsmit was, with Uhlenbeck, student of Ehrenfest. From a travel with his father, merchant in Leiden, he stopped a while to visit Pashen, a spectroscopist who was studying the ionized lines of Helium. One of the thing that stuck Goudsmit was the presence in Pashen experiment of "forbidden lines" in the fine structure of the Helium. When discussing with theoreticians about it, they usually answered to him "poor experimentalists".. Ehrenfest then sent Goudsmit to Amsterdam to work with Zeeman, confirming the presence of a fine structure. After a discussion with Uhlenbeck about the Pauli principle just discovered in 1925, Uhlenbeck answered "But don't you see what this implies? It means that there is a fourth degree of freedom for the electron. It means that the electron has a spin, that it rotates". Then the forbidden lines of Pashen were not forbidden anymore but real spectral lines. That is the first time the word "spin" appears on the page 6 of the article "the doublets in question may more properly be termed "spin" doublets, since the sole reason for their appearance is the difference in orientation of the spin axis relative to the orbital plane".

Figure H.1: Students of Ehrenfest in Leiden 1924. From left to right: Dieke, Goudsmit, Tinbergen, Ehrenfest, Kronig, Fermi. Note: Tinbergen later changed from physics to economy and became the first Nobel laureate in economy (1969).

In the page 5, the authors summarized clearly the new spectrum proposed adding the 2 spinorial degrees of freedom of the electron and, as a consequence, a split in 2 rays. However, the difference between the measurements and the quantum prediction was a factor 2. The author notice it at the end of the paper. It is Thomas who understood this factor 2 and convinced finally Pauli of the pertinence of the article. He then wrote a letter to Goudsmit reproduced here. 3 days before the publication of the article, after a discussion with Lorentz visiting Leiden, asking about the phenomena of "a charge that rotates", Lorentz answered "It is very difficult because it causes the self energy of the electron to be wrong". Uhlenbeck then got frightened and run to Ehrenfest to stop the process : "don't send it off, because it probably is wrong; it is impossible, one cannot have an electron that rotates at such high speed and has the right moment". And Ehrenfest replied : "It is too late, I have sent it off already. That is a nice idea, through it may be wrong. But you don't have a reputation, so you have nothing to lose". It is important to note that Kronig proposed the same interpretation of a spinning electron to explain the Pauli principle. However, he left Ehrenfest to join Pauli in 1926 and then was convinced by Pauli himself that this interpretation was not convincing, and he then stopped his research in this direction.

remain unchanged. Their occurrence is, therefor quite in conformity with the correspondence principl

The modification proposed is specially importar for explaining the structure of X-ray spectra. Thes spectra differ from the hydrogen-like spectra by th appearance of the so-called "screening" doublet which are ascribed to the interaction of the electror

FIG. 1.

within the atom, effective mainly through reducir the effect of the nuclear attraction. In our vie these screening doublets correspond to pairs of leve which have the same angular momentum J b different azimuthal quantum numbers K. Cons quently, the orbits will penetrate to different distanc from the nucleus, so that the screening of the nucle charge by the other electrons in the atom will ha different effects. This screening effect will, howeve be the same for a pair of levels which have the san K but different J's and correspond to the same orbital shape. Such pairs of levels were, on the older theory, labelled with values of k differing by one unit, and it was quite impossible to understand why these so-called "relativity" doublets should appear separately from the screening doublets. On our view, the doublets in question may more properly be termed "spin" doublets, since the sole reason for their appearance is the difference in orientation of the spin axis relative to the orbital plane. It should be emphasised that our interpretation is in complete accordance with the correspondence principle as regards the rules of combination of X-ray levels.

The assumption of the spinning electron leads to a new insight into the remarkable analogy between the multiplet structure of the optical spectra and the structure of X-ray spectra, which was emphasised especially by Landé and Millikan. While the attempt to refer this analogy to a relativity effect common to all the structures was most unsatisfactory, it obtains an immediate explanation on the hypothesis of the spin electron. If, for example, we consider the spectra of the alkaline type, we are led to recognise in the well-known doublets regular spin doublets of the character described above. In fact, this enables us to explain the dependence of the doublet width on the effective nuclear charge and the quantum numbers describing the orbit, as well as the rules of combination.

The simplicity of the alkaline spectra is due to the fact that the atom consists of an electron revolving round an atomic residue which contains only completed electronic groups, which are magnetically inert. When we pass to atoms in which several electrons revolve round a residue of this kind we meet with new features, since we have to take account of other directing influences on the spin axis of each electron besides the couple due to its own motion in the electric field. Not only does this enable us to account for the appearance of multiplets of higher complexity, but it also seems to throw light on the so-called "branching" of spectra, which usually accompanies the adding of a further electron to the atom, and for which hitherto no satisfactory explanation has been given. In fact, it seems that the introduction of the concept of the spinning electron makes it possible throughout to maintain the principle of the successive building up of atoms utilised by Bohr in his general discussion of the relations between spectra and the natural system of the elements. Above all, it may be possible to account for the important results arrived

at by Pauli, without having to assume an unmechanical "duality" in the binding of the electrons.

So far we have not mentioned the Zeeman effect, although the introduction of the spinning electron was primarily suggested by the analysis of the anomalous Zeeman effects shown by the components of multiplet structures. From the point of view of the correspondence principle, this effect shows that the influence of a magnetic field on the motion of the atom differs considerably from that to be expected if the electron had no spin. In fact, from the well-known theorem of Larmor we would expect the effect on any spectral line to be of the simple Lorentz type, quite independently of the character of the multiplet structure. Therefore the appearance of the anomalous Zeeman effects has hitherto presented very grave difficulties. However, these difficulties disappear at once when, as assumed, the electron has a spin and the ratio between magnetic moment and angular momentum of this spin is different from that corresponding to the revolution of the electron in an orbit large compared with its own size. On this assumption the spin axis of an electron not affected by other forces would precess with a frequency different from the Larmor rotation. It is easily shown that the resultant motion of the atom for magnetic fields of small intensity will be of just the type revealed by Landé's analysis. If the field is so strong that its influence on the precession of the spin axis is comparable with that due to the orbital motion in the atom, this motion will be changed in a way which directly explains the gradual transformation of the multiplet structure for increasing fields known as the Paschen-Back effect.

It seems possible on these lines to develop a quantitative theory of the Zeeman effect, if it is assumed that the ratio between magnetic moment and angular momentum due to the spin is twice the ratio corresponding to an orbital revolution. At present, however, it seems difficult to reconcile this assumption with a quantitative analysis of our explanation of the fine structure of levels. In fact it leads, in a preliminary calculation, to widths of the spin doublets just twice as large as those required by observation. It must be remembered, however, that we are here dealing with problems which for their final solution require a closer study of quantum mechanics and perhaps also of questions concerning the structure of the electron.

ness to Prof. Niels Bohr for an enlightening discussion, and for criticisms which helped us distinguish between the essential points and the more technical details of the new interpretation.

G. E. UHLENBECK.
S. GOUDSMIT.
Instituut voor Theoretische Natuurkunde,
Leyden, December 1925.

HAVING had the opportunity of reading this interesting letter by Mr. Goudsmit and Mr. Uhlenbeck, I am glad to add a few words which may be regarded as an addition to my article on atomic theory and mechanics, which was published as a supplement to NATURE of December 5, 1925. As stated there, the attempts which have been made to account for the properties of the elements by applying the quantum theory to the nuclear atom have met with serious difficulties in the finer structure of spectra and the related problems. In my article expression was given to the view that these difficulties were inherently connected with the limited possibility of representing the stationary states of the atom by a mechanical model. The situation seems, however, to be somewhat altered by the introduction of the hypothesis of the spinning electron which, in spite of the incompleteness of the conclusions that can be derived from models, promises to be a very welcome supplement to our ideas of atomic structure. In fact, as Mr. Goudsmit and Mr. Uhlenbeck have described in their letter, this hypothesis throws new light on many of the difficulties which have puzzled the workers in this field during the last few years. Indeed, it opens up a very hopeful prospect of our being able to account more extensively for the properties of elements by means of mechanical models, at least in the qualitative way characteristic of applications of the correspondence principle. This possibility must be the more welcomed at the present time, when the prospect is held out of a quantitative treatment of atomic problems by the new quantum mechanics initiated by the work of Heisenberg, which aims at a precise formulation of the correspondence between classical mechanics and the quantum theory.

N. BOHR.

I think you and Uhlenbeck
have been very lucky to get your
spinning electron published and talked
about before Pauli heard of it. It
appears that more than a year ago
Kronig believed in the spinning electron
and worked out something; the first
person he showed it to was Pauli.
Pauli ridiculed the whole thing so
much that the first person become also
the last and no one else heard anything
of it. Which all goes to show that
the infallibility of the Deity does not
extend to his self-styled vicar on earth.

Figure H.2: Letter by Thomas to Goudsmit, 25 March 1926.

H.6 Discovery of the meson : "On the Interaction of Elementary Particles" by H. Yukawa, 1935

In 1933, W. Heisenberg had given a model of nucleus made of neutrons and protons where he supposed that the neutron and proton were two states of one unique particle, the nucleon. He introduced a new quantum number to differentiate them: the *isotopic spin*, or *isospin* I_z, with $I_z = \pm 1/2$ for the neutron/proton. To try to understand the nuclear forces, one could be inspired by the two known forces : *the electromagnetism* and *the gravitation* described by the equations of Maxwell and Einstein. It was already supposed at that time that the photon was a bosons representative of the electromagnetism force and is exchanged between electrons. In 1934, the photon is the only known boson and was of null mass with long range force. However, if the nuclear force acts in nucleus to keep together stuck protons, it should be stronger than electromagnetism and at short distance, around 10^{-12} cm (typical size of a nucleus).

In the beginning of the 1930, Japan just began to develop research in atomic physics. Hideki Yukawa, student in Kyoto between 1926 and 1929 did not have the opportunity to meet Heisenberg or Dirac who made their conference in Riken. In 1931, the first official lectures about nuclear physics is given by Yoshio Nishina. Inspired by the nuclear model of W. Heisenberg, H. Yukawa tried to describe the β-decay by the creation of an electron associated by the transformation of a neutron to a proton in the nucleus. He first thought that the particle echanged for the interaction was the electron : $\bar{p} + n \to e^- \to n + \bar{p}$. Yukawa never published this work as it was obvious that there was several issue concerning spin-statistics and the force he obtained was long ranged. After a conference of Yukawa, Nishina noted that the exchange of a boson would solve these difficulties. Only the photon was known to be bosonic at these time, but an article of Klein and Gordon gave some years before an explicit wave function describing a massive boson of spin 0 and corresponding to a long range force. In october 1934, H. Yukawa colmpute the mass of a boson acting with a short range and obtained a mass around 200 times the mass of the electron ($\simeq$ 100 GeV, see first line of p.6 of the following article). His article reproduced here "On the Interaction of Elementary Particles" was published in the first numero of "Proceedings of the Physico Mathematical Society of Japan". Not any particle of this mass was known at this time as he wrote after the equation (12) even adding "the above theory seems to be on a wrong line". He called this particle "meson" which signify "intermediate mass" in greek[1]. Yukawa finished his article by a comment "The massive quanta may also have some bearing on the shower produced by cosmic ray". The meson π, or pion was discovered by Powell in 1948: Yukawa received the Nobel prize for the theory of the mesons in 1949, and Powell for the discovery of the pion in 1950.

[1]The first name proposed by Yukawa was "mesotron", bu Heisenberg, as a former professor of greek, made the remark to Yukawa that the letter "tr" was not existing in greek.

8×23 = 184
12×31 = 372

DEPARTMENT OF PHYSICS
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1

DATE Nov. 1, 1934
NO. 2

On the Interaction of Elementary Particles. I.

By Hideki Yukawa

§1. Introduction

At the present stage of the quantum theory little is known about the nature of interaction between elementary particles. For example, the interaction between a neutron and a proton whether is an ordinary attraction ~~force~~ or an "Platzwechsel" ~~exchange interaction~~ first proposed by Heisenberg. Recently Fermi[1] has treated the problem of β-ray disintegration on the hypothesis of ~~the existence of~~ "neutrino". According to this theory a neutron and a proton can interact by emitting and absorbing a neutrino and an electron. Unfortunately the energy of interaction calculated on ~~this~~ such assumption[2] is much too small to account for the binding of neutrons and protons in the nucleus. To remove this defect we may modify the theory of Heisenberg or Fermi in the following way.

The transition of a heavy particle from a neutron state to a proton state is not always accompanied

(1) E. Fermi, Zeits. f. Phys. 88, 161 (1934).
(2) Ig. Tamm, Nature, 133, 981 (1934); D. Iwanenko, ibid. 981 (1934).

On the Interaction of Elementary Particles.

By Hideki YUKAWA.

(Read Nov. 17, 1934)

§ 1. Introduction

At the present stage of the quantum theory little is kno the nature of interaction of elementary particles. Heisenberg c the interaction of "Platzwechsel" between the neutron and t to be of importance to the nuclear structure.[1]

Recently Fermi treated the problem of β-disintegratio hypothesis of "neutrino"[2]. According to this theory, the ne the proton can interact by emitting and absorbing a pair of and electron. Unfortunately the interaction energy calculated assumption is much too small to account for the binding e neutrons and protons in the nucleus.[3]

To remove this defect, it seems natural to modify the Heisenberg and Fermi in the following way. The transit heavy particle from neutron state to proton state is not alway panied by the emission of light particles, i. e., a neutrino and an but the energy liberated by the transition is taken up some another heavy particle, which in turn will be transformed fro state into neutron state. If the probability of occurrence of process is much larger than that of the former, the interactio the neutron and the proton will be much larger than in th Fermi, whereas the probability of emission of light particles i fected essentially.

Now such interaction between the elementary particles ca cribed by means of a field of force, just as the interaction bet charged particles is described by the electromagnetic field. T considerations show that the interaction of heavy particles v field is much larger than that of light particles with it.

(1) W. Heisenberg, Zeit. f. Phys. **77**, 1 (1932); **78**, 156 (1932); **80**, 587 (1933). denote the first of them by *I*.

(2) E. Fermi, ibid. **88**, 161 (1934).

(3) Ig. Tamm, Nature **133**, 981 (1934); D. Iwanenko, ibid. 981 (1934).

In the quantum theory this field should be accompanied by a new sort of quantum, just as the electromagnetic field is accompanied by the photon.

In this paper the possible nature of this field and the quantum accompanying it will be discussed briefly and also their bearing on the nuclear structure will be considered.

Besides such an exchange force and the oridinary electric and magnetic forces there may be other forces between the elementary particles, but we disregard the latter for the moment.

Fuller account will be made in the next paper.

§ 2. Field describing the interaction

In analogy with the scalar potential of the electromagnetic field, a function $U(x, y, z, t)$ is introduced to describe the field between the neutron and the proton. This function will satisfy an equation similar to the wave equation for the electromagnetic potential.

Now the equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\} U = 0 \tag{1}$$

has only static solution with central symmetry $\frac{1}{r}$, except the additive and the multiplicative constants. The potential of force between the neutron and the proton should, however, not be of Coulomb type, but decrease more rapidly with distance. It can be expressed, for example, by

$$+ \text{ or } -g^2\frac{e^{-\lambda r}}{r}, \tag{2}$$

where g is a constant with the dimension of electric charge, i. e., cm.$^{\frac{3}{2}}$ sec.$^{-1}$ gr.$^{\frac{1}{2}}$ and λ with the dimension cm.$^{-1}$

Since this function is a static solution with central symmetry of the wave equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = 0, \tag{3}$$

let this equation be assumed to be the correct equation for U in vacuum. In the presence of the heavy particles, the U-field interacts with them and causes the transition from neutron state to proton state.

Now, if we introduce the matrices[4]

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and denote the neutron state and the proton state by $\tau_3 = 1$ and $\tau_3 = -1$ respectively, the wave equation is given by

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = -4\pi g \bar{\Psi}\frac{\tau_1 - i\tau_2}{2}\Psi, \tag{4}$$

where Ψ denotes the wave function of the heavy particles, being a function of time, position, spin as well as τ_3', which takes the value either 1 or -1.

Next, the conjugate complex function $\tilde{U}(x, y, z, t)$, satisfying the equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} \tilde{U} = -4\pi g \bar{\Psi}\frac{\tau_1 + i\tau_2}{2}\Psi, \tag{5}$$

is introduced, corresponding to the inverse transition from proton to neutron state.

Similar equation will hold for the vector function, which is the analogue of the vector potential of the electromagnetic field. However, we disregard it for the moment, as there's no correct relativistic theory for the heavy particles. Hence simple non-relativistic wave equation neglecting spin will be used for the heavy particle in the following way

$$\left\{\frac{h^2}{4}\left(\frac{1+\tau_3}{M_N} + \frac{1-\tau_3}{M_P}\right)\Delta + ih\frac{\partial}{\partial t} - \frac{1+\tau_3}{2}M_N c^2 - \frac{1-\tau_3}{2}M_P c^2 - g\left(\tilde{U}\frac{\tau_1 - i\tau_2}{2} + U\frac{\tau_1 + i\tau_2}{2}\right)\right\}\Psi = 0, \tag{6}$$

where h is Planck's constant divided by 2π and M_N, M_P are the masses of the neutron and the proton respectively. The reason for taking the negative sign in front of g will be mentioned later.

The equation (6) corresponds to the Hamiltonian

$$H = \left(\frac{1+\tau_3}{4M_N} + \frac{1-\tau_3}{4M_P}\right)p^2 + \frac{1+\tau_3}{2}M_N c^2 + \frac{1-\tau_3}{2}M_P c^2 - g\left(\tilde{U}\frac{\tau_1 - i\tau_2}{2} + U\frac{\tau_1 + i\tau_2}{2}\right) \tag{7}$$

(4) Heisenberg, loc. cit. I.

where $\boldsymbol{p}$ is the momentum of the particle. If we put $M_N c^2 - M_P c^2 = D$ and $M_N + M_P = 2M$, the equation (7) becomes approximately

$$H = \frac{p^2}{2M} + \frac{g}{2}\{\tilde{U}(\tau_1 - i\tau_2) + U(\tau_1 + i\tau_2)\} + \frac{D}{2}\tau_3, \tag{8}$$

where the constant term Mc^2 is omitted.

Now consider two heavy particles at points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively and assume their relative velocity to be small. The fields at (x_1, y_1, z) due to the particle at (x_2, y_2, z_2) are, from (4) and (5),

$$\left.\begin{aligned} U(x_1, y_1, z_1) &= g\frac{e^{-\lambda r_{12}}}{r_{12}}\frac{(\tau_1^{(2)} - i\tau_2^{(2)})}{2} \\ \text{and} \qquad \tilde{U}(x_1, y_1, z_1) &= g\frac{e^{-\lambda r_{12}}}{r_{12}}\frac{(\tau_1^{(2)} + i\tau_2^{(2)})}{2}, \end{aligned}\right\} \tag{9}$$

where $(\tau_1^{(1)}, \tau_2^{(1)}, \tau_3^{(1)})$ and $(\tau_1^{(2)}, \tau_2^{(2)}, \tau_3^{(2)})$ are the matrices relating to the first and the second particles respectively, and r_{12} is the distance between them.

Hence the Hamiltonian for the system is given, in the absence of the external fields, by

$$\begin{aligned} H &= \frac{p_1^2}{2M} + \frac{p_2^2}{2M} + \frac{g^2}{4}\{(\tau_1^{(1)} - i\tau_2^{(1)})(\tau_1^{(2)} + i\tau_2^{(2)}) \\ &\quad + (\tau_1^{(1)} + i\tau_2^{(1)})(\tau_1^{(2)} - i\tau_2^{(2)})\}\frac{e^{-\lambda r_{12}}}{r_{12}} + (\tau_3^{(1)} + \tau_3^{(2)})D \\ &= \frac{p_1^2}{2M} + \frac{p_2^2}{2M} + \frac{g^2}{2}(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)})\frac{e^{-\lambda r_{12}}}{r_{12}} + (\tau_3^{(1)} + \tau_3^{(2)})D, \end{aligned} \tag{10}$$

where $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ are the momenta of the particles.

This Hamiltonian is equivalent to Heisenberg's Hamiltonian (1),[5] if we take for "Platzwechselintegral"

$$J(r) = -g^2\frac{e^{-\lambda r}}{r}, \tag{11}$$

except that the interaction between the neutrons and the electrostatic repulsion between the protons are not taken into account. Heisenberg took the positive sign for $J(r)$, so that the spin of the lowest energy state of H^2 was 0, whereas in our case, owing to the negative sign in front of g^2, the lowest energy state has the spin 1, which is required

(5) Heisenberg, I.

from the experiment.

Two constants g and λ appearing in the above equation be determined by comparison with experiment. For examp the Hamiltonian (10) for heavy particles, we can calculate t defect of H^0 and the probability of scattering of a neutron by provided that the relative velocity is small compared with velocity.[6]

Rough estimation shows that the calculated values agree experimental results, if we take for λ the value between 10^{12}c 10^{13}cm^{-1} and for g a few times of the elementary charge e, no direct relation between g and e was suggested in the above rations.

§ 3. Nature of the quanta accompanying the field

The U-field above considered should be quantized accordir general method of the quantum theory. Since the neutron proton both obey Fermi's statistics, the quanta accompanying field should obey Bose's statistics and the quantization can b out on the line similar to that of the electromagnetic field.

The law of conservation of the electric charge demands quantum should have the charge either $+e$ or $-e$. The field U corresponds to the operator which decreases the number of n charged quanta and increases the number of positively charge by one respectively. $\tilde{U}$ which is the complex conjugate of U, cor to the inverse operator.

Next, denoting

$$p_x = -ih\frac{\partial}{\partial x}, \quad \text{etc.}, \quad W = ih\frac{\partial}{\partial t},$$

$$m_U c = \lambda h,$$

the wave equation for U in free space can be written in the f

$$\left\{p_x^2 + p_y^2 + p_z^2 - \frac{W^2}{c^2} + m_U c^2\right\} U = 0,$$

so that the quantum accompanying the field has the proper mass

(6) These calculations were made previously, according to the theory of I by Mr. Tomonaga, to whom the writer owes much. A little modification is n our case. Detailed accounts will be made in the next paper.

Assuming $\lambda = 5\times10^{13}$cm^{-1}, we obtain for m_U a value 2×10^2 times as large as the electron mass. As such a quantum with large mass and positive or negative charge has never been found by the experiment, the above theory seems to be on a wrong line. We can show, however, that, in the ordinary nuclear transformation, such a quantum can not be emitted into outer space.

Let us consider, for example, the transition from a neutron state of energy W_N to a proton state of energy W_P, both of which include the proper energies. These states can be expressed by the wave functions

$$\Psi_N(x, y, z, t, 1) = u(x, y, z)e^{-iW_N t/h}, \quad \Psi_N(x, y, z, t, -1) = 0$$

and

$$\Psi_P(x, y, z, t, 1) = 0, \quad \Psi_P(x, y, z, t, -1) = v(x, y, z)e^{-iW_P t/h},$$

so that, on the right hand side of the equation (4), the term

$$-4\pi g\tilde{v}u e^{-i(W_N - W_P)t/h}$$

appears.

Putting $U = U'(x, y, z)e^{i\omega t}$, we have from (4)

$$\left\{\Delta - \left(\lambda^2 - \frac{\omega^2}{c^2}\right)\right\} U' = -4\pi g\tilde{v}u, \tag{13}$$

where $\omega = \frac{W_N - W_P}{h}$. Integrating this, we obtain a solution

$$U'(r) = g\iiint \frac{e^{-\mu|r-r'|}}{|r-r'|}\tilde{v}(r')u(r')dv', \tag{14}$$

where $\mu = \sqrt{\lambda^2 - \frac{\omega^2}{c^2}}$.

If $\lambda > \frac{|\omega|}{c}$ or $m_U c^2 > |W_N - W_P|$, μ is real and the function $J(r)$ of Heisenberg has the form $-g^2\frac{e^{-\mu r}}{r}$, in which μ, however, depends on $|W_N - W_P|$, becoming smaller and smaller as the latter approaches $m_U c^2$. This means that the range of interaction between a neutron and a proton increases as $|W_N - W_P|$ increases.

Now the scattering (elastic or inelastic) of a neutron by a nucleus can be considered as the result of the following double process: the neutron falls into a proton level in the nucleus and a proton in the latter jumps to a neutron state of positive kinetic energy, the total energy being conserved throughout the process. The above argument, then, shows that the probability of scattering may in some case increase

with the velocity of the neutron.

According to the experiment of Bonner[7], the collision cross section of the neutron increases, in fact, with the velocity in the case of lead whereas it decreases in the case of carbon and hydrogen, the rate of decrease being slower in the former than in the latter. The origin of this effect is not clear, but the above considerations do not, at least, contradict it. For, if the binding energy of the proton in the nucleus becomes comparable with $m_U c^2$, the range of interaction of the neutron with the former will increase considerably with the velocity of the neutron, so that the cross section will decrease slower in such case than in the case of hydrogen, i.e., free proton. Now the binding energy of the proton in C^{12}, which is estimated from the difference of masses of C^{12} and B^{11}, is

$$12{,}0036 - 11{,}0110 = 0{,}0926.$$

This corresponds to a binding energy 0,0152 in mass unit, being thirty times the electron mass. Thus in the case of carbon we can expect the effect observed by Bonner. The arguments are only tentative, other explanations being, of course, not excluded.

Next if $\lambda < \frac{\omega}{c}$ or $m_U c^2 < |W_N - W_P|$, μ becomes pure imaginary and U expresses a spherial undamped wave, implying that a quantum with energy greater than $m_U c^2$ can be emitted in outer space by the transition of the heavy particle from neutron state to proton state, provided that $W_N - W_P > m_U c^2$.

The velocity of U-wave is greater but the group velocity is smaller than the light velocity c, as in the case of the electron wave.

The reason why such massive quanta, if they ever exist, are not yet discovered may be ascribed to the fact that the mass m_U is so large that condition $|W_N - W_P| > m_U c^2$ is not fulfilled in ordinary nuclear transformation.

§ 4. Theory of β-disintegration

Hitherto we have considered only the interaction of U-quanta with heavy particles. Now, according to our theory, the quantum emitted when a heavy particle jumps from a neutron state to a proton state, can be absorbed by a light particle which will then in consequence of energy absorption rise from a neutrino state of negative energy to an

(7) T. W. Bonner, Phys. Rev. 45, 606 (1934).

§ 5. Summary

The interaction of elementary particles are described by considering a hypothetical quantum which has the elementary charge and the proper mass and which obeys Bose's statistics. The interaction of such a quantum with the heavy particle should be far greater than that with the light particle in order to account for the large interaction of the neutron and the proton as well as the small probability of β-disintegration.

Such quanta, if they ever exist and approach the matter close enough to be absorbed, will deliver their charge and energy to the latter. If, then, the quanta with negative charge come out in excess, the matter will be charged to a negative potential.

These arguments, of course, of merely speculative character, agree with the view that the high speed positive particles in the cosmic rays are generated by the electrostatic field of the earth, which is charged to a negative potential.[8]

The massive quanta may also have some bearing on the shower produced by cosmic rays.

In conclusion the writer wishes to express his cordial thanks to Dr. Y. Nishina and Prof. S. Kikuchi for the encouragement throughout the course of the work.

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(Received Nov. 30, 1934)

(8) G. H. Huxley, Nature 134, 418, 571 (1934); Johnson, Phys. Rev. 45, 563 (1934).

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The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charge electrons, does not give results in agreement with experiment. The discrepancies consist of "duplexity" phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton. This model for the electron has been fitted into the new mechanics by Pauli,* and Darwin,† working with an equivalent theory, has shown that it gives results in agreement with experiment for hydrogen-like spectra to the first order of accuracy.

The question remains as to why Nature should have chosen this particular model for the electron instead of being satisfied with the point-charge. One would like to find some incompleteness in the previous methods of applying quantum mechanics to the point-charge electron such that, when removed, the whole of the duplexity phenomena follow without arbitrary assumptions. In the present paper it is shown that this is the case, the incompleteness of the previous theories lying in their disagreement with relativity, or, alternatively, with the general transformation theory of quantum mechanics. It appears that the simplest Hamiltonian for a point-charge electron satisfying the requirements of both relativity and the general transformation theory leads to an explanation of all duplexity phenomena without further assumption. All the same there is a great deal of truth in the spinning electron model, at least as a first approximation. The most important failure of the model seems to be that the magnitude of the resultant orbital angular momentum of an electron moving in an orbit in a central field of force is not a constant, as the model leads one to expect.

* Pauli, 'Z. f. Physik,' vol. 43, p. 601 (1927).
† Darwin, 'Roy. Soc. Proc.,' A, vol. 116, p. 227 (1927).

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§ 1. *Previous Relativity Treatments.*

The relativity Hamiltonian according to the classical theory for a point electron moving in an arbitrary electro-magnetic field with scalar potential A_0 and vector potential $\mathbf{A}$ is

$$F \equiv \left(\frac{W}{c} + \frac{e}{c}A_0\right)^2 + \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 + m^2c^2,$$

where $\mathbf{p}$ is the momentum vector. It has been suggested by Gordon* that the operator of the wave equation of the quantum theory should be obtained from this F by the same procedure as in non-relativity theory, namely, by putting

$$W = ih\frac{\partial}{\partial t},$$

$$p_r = -ih\frac{\partial}{\partial x_r}, \qquad r = 1, 2, 3,$$

in it. This gives the wave equation

$$F\psi \equiv \left[\left(ih\frac{\partial}{c\,\partial t} + \frac{e}{c}A_0\right)^2 + \Sigma_r\left(-ih\frac{\partial}{\partial x_r} + \frac{e}{c}A_r\right)^2 - m^2c^2\right]\psi = 0, \qquad (1)$$

the wave function ψ being a function of x_1, x_2, x_3, t. This gives rise to two difficulties.

The first is in connection with the physical interpretation of ψ. Gordon, and also independently Klein,† from considerations of the conservation theorems, make the assumption that if ψ_m, ψ_n are two solutions

$$\rho_{mn} = -\frac{e}{2mc^2}\left\{ih\left(\psi_m\frac{\partial\bar{\psi}_n}{\partial t} - \bar{\psi}_n\frac{\partial\psi_m}{\partial t}\right) + 2eA_0\psi_m\bar{\psi}_n\right\}$$

and

$$\mathbf{I}_{mn} = -\frac{e}{2m}\left\{-ih(\psi_m \operatorname{grad}\bar{\psi}_n - \bar{\psi}_n \operatorname{grad}\psi_m) + 2\frac{e}{c}\mathbf{A}_m\psi_m\bar{\psi}_n\right\}$$

are to be interpreted as the charge and current associated with the transition $m \to n$. This appears to be satisfactory so far as emission and absorption of radiation are concerned, but is not so general as the interpretation of the non-relativity quantum mechanics, which has been developed‡ sufficiently to enable one to answer the question: What is the probability of any dynamical variable

* Gordon, 'Z. f. Physik,' vol. 40, p. 117 (1926).
† Klein, 'Z. f. Physik,' vol. 41, p. 407 (1927).
‡ Jordan, 'Z. f. Physik,' vol. 40, p. 809 (1927); Dirac, 'Roy. Soc. Proc.,' A, vol. 113, p. 621 (1927).

H.7 The Dirac equation : "Quantum Theory of Electron" by P.A.M. Dirac, 1926

As we noticed earlier, before quantum mechanics, there had been another revolution in physics, with Einstein's discovery in 1905 that Newton's mechanics fails for matter moving at speeds approaching that of light. The question arose: what is the physics of particles that are at the same time small (quantum) *and* moving fast (relativistic).This was a practical question: the electrons in atoms are small, and they move fast enough for the new quantum mechanics to be slightly inaccurate. Dirac's construction of his wave equation, trying to solve this issue was published in two papers in the *Proceedings of the Royal Society* in February and March 1928. We reproduce the first of them here. In this paper, he shows that the simplest wave satisfying the requirement is not a simple number but have four components called spinors. The name spinors is coming from the fact that 2 degrees of freedom of the wave

functions can be understood as a description of the Spin postulated by Pauli and Uhlenbeck and Goudsmit 2 years before.

Indeed, in the first page, he explain this problem of "*duplexity phenomena: the observed number of stationary states for an electron being twice the number given by the theory*". In the second paragraph of the first page, he then adds that "*the simplest Hamiltonian for a point-charge electron satisfying the requirements of both relativity and the general transformation theory leads to an explanation of all duplexity phenomena without further assumption*". The two other degrees of freedom introduced by the spinors are in fact the description of the $anti--electron$. Dirac is not completely aware of that, however is work is based on the Klein–Gordon description published in 1926-1927 (Eq.1). He notes after the equation of the third page that for negative energies "*One cannot do this on the quantum theory, since in general a perturbation will cause transitions from states with W positive to states with W negative. Such a transition would appear experimentally as the electron suddenly changing his charge from -e to +e, a phenomenon which has not been observed*". He then explain that in this paper, he will not solve this problem. Which is in fact perfect as these states correspond to the anti-matter that will be discover in 1932 by Anderson (this paper already discussed in this book).

Finally, after simple algebra, he derived his famous "Dirac Equation", its Eq.(11) in the case of free particle. After checking the invariance of his equation under the Lorentz transformation and adding the electromagnetic field to the game, he shows after the Eq.(18) that the real invariant of the motion is not the orbital angular momentum m but the sum of $m + \frac{1}{2}h\sigma = M$, σ being the Pauli matrices. he then wrote (first paragraph after Eq.(18)) "*We can interpret this result by saying that the electron has a spin angular momentum of $\frac{1}{2}h\sigma$, which, added to the orbital angular momentum m, gives the total angular momentum M which is a constant of the motion*". In other words, he has put in equation the idea of the existence of the spin, postulated by Uhlenbeck and Goudsmit to explain Zeeman effect and the fine structure of the hydrogen spectrum.

Having explained spin, it was natural for Dirac to try to explain electric charge, and in particular the mysterious fact that it is quantized: all charges found in nature are multiples of the charge of the electron. In classical electricity, there is no basis for this: charges can have any value. In 1931 Dirac gave a solution of this problem in an application of quantum mechanics so original that it still astounds us to read it today. He combined electricity with magnetism, in a return to the 18th century notion of a magnet being a combination of north and south magnetic poles. Dirac wondered if there was any way that magnetic monopoles could be brought into quantum physics without spoiling everything that had grown out of assuming that they did not exist. The existence of even one monopole anywhere in the Universe would suffice to ensure that electric charge must be quantized.

Niels Bohr: What are you working on Mr. Dirac?
Paul Dirac: Im trying to take the square root of something

now be a function of ζ as well as x_1, x_2, x_3, t. The result of α_μ multiplied into ψ will be a function $(\alpha_\mu\psi)$ of x_1, x_2, x_3, t, ζ defined by

$$(\alpha_\mu\psi)(x, t, \zeta) = \Sigma_{\zeta'}\, \alpha_\mu(\zeta\zeta')\, \psi(x, t, \zeta').$$

We must now find four matrices α_μ to satisfy the conditions (6). We make use of the matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which Pauli introduced* to describe the three components of spin angular momentum. These matrices have just the properties

$$\sigma_r^2 = 1 \qquad \sigma_r\sigma_s = -\sigma_s\sigma_r = 0, \qquad (r \neq s), \tag{7}$$

that we require for our α's. We cannot, however, just take the σ's to be three of our α's, because then it would not be possible to find the fourth. We must extend the σ's in a diagonal manner to bring in two more rows and columns, so that we can introduce three more matrices ρ_1, ρ_2, ρ_3 of the same form as $\sigma_1, \sigma_2, \sigma_3$, but referring to different rows and columns, thus :—

$$\sigma_1 = \begin{Bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{Bmatrix} \quad \sigma_2 = \begin{Bmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{Bmatrix} \quad \sigma_3 = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{Bmatrix},$$

$$\rho_1 = \begin{Bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{Bmatrix} \quad \rho_2 = \begin{Bmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{Bmatrix} \quad \rho_3 = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}.$$

The ρ's are obtained from the σ's by interchanging the second and third rows, and the second and third columns. We now have, in addition to equations (7)

$$\left.\begin{aligned} &\rho_r^2 = 1 \qquad \rho_r\rho_s + \rho_s\rho_r = 0 \qquad (r \neq s), \\ &\rho_r\sigma_s = \sigma_s\rho_r. \end{aligned}\right\}, \tag{7'}$$

and also

* Pauli, *loc. cit.*

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If we now take

$$\alpha_1 = \rho_1\sigma_1, \qquad \alpha_2 = \rho_1\sigma_2, \qquad \alpha_3 = \rho_1\sigma_3, \qquad \alpha_4 = \rho_3,$$

all the conditions (6) are satisfied, *e.g.*,

$$\alpha_1^2 = \rho_1\sigma_1\rho_1\sigma_1 = \rho_1^2\sigma_1^2 = 1$$
$$\alpha_1\alpha_2 = \rho_1\sigma_1\rho_1\sigma_2 = \rho_1^2\sigma_1\sigma_2 = -\rho_1^2\sigma_2\sigma_1 = -\alpha_2\alpha_1.$$

The following equations are to be noted for later reference

$$\left.\begin{aligned} \rho_1\rho_2 = i\rho_3 = -\rho_2\rho_1 \\ \sigma_1\sigma_2 = i\sigma_3 = -\sigma_2\sigma_1 \end{aligned}\right\}, \tag{8}$$

together with the equations obtained by cyclic permutation of the suffixes.

The wave equation (4) now takes the form

$$[p_0 + \rho_1(\boldsymbol{\sigma}, \mathbf{p}) + \rho_3 mc]\,\psi = 0, \tag{9}$$

where $\boldsymbol{\sigma}$ denotes the vector $(\sigma_1, \sigma_2, \sigma_3)$.

§ 3. *Proof of Invariance under a Lorentz Transformation.*

Multiply equation (9) by ρ_3 on the left-hand side. It becomes, with the help of (8),

$$[\rho_3 p_0 + i\rho_2(\sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 p_3) + mc]\,\psi = 0.$$

Putting

$$p_0 = ip_4,$$

we have

$$\rho_3 = \gamma_4, \qquad \rho_2\sigma_r = \gamma_r, \qquad r = 1, 2, 3, \tag{10}$$

$$[i\Sigma\gamma_\mu p_\mu + mc]\,\psi = 0, \qquad \mu = 1, 2, 3, 4. \tag{11}$$

The p_μ transform under a Lorentz transformation according to the law

$$p_\mu' = \Sigma_\nu a_{\mu\nu} p_\nu,$$

where the coefficients $a_{\mu\nu}$ are c-numbers satisfying

$$\Sigma_\mu a_{\mu\nu} a_{\mu\sigma} = \delta_{\nu\sigma}, \qquad \Sigma_\nu a_{\mu\nu} a_{\sigma\nu} = \delta_{\mu\sigma}.$$

The wave equation therefore transforms into

$$[i\Sigma\gamma_\mu' p_\mu' - mc]\,\psi = 0, \tag{12}$$

where

$$\gamma_\mu' = \Sigma_\nu a_{\mu\nu}\gamma_\nu.$$

Now the γ_μ, like the α_μ, satisfy

$$\gamma_\mu^2 = 1, \qquad \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 0, \qquad (\mu \neq \nu).$$

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Thus (15) becomes

$$0 = \Big[(p_0 + e'A_0)^2 + (\mathbf{p} + e'\mathbf{A})^2 + m^2c^2 + e'h(\boldsymbol{\sigma}, \operatorname{curl}\mathbf{A})$$
$$+ ie'h\rho_1\Big(\boldsymbol{\sigma}, \operatorname{grad} A_0 + \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\Big)\Big]\psi$$
$$= [-(p_0 - e'A_0)^2 + (\mathbf{p} + e'\mathbf{A})^2 + m^2c^2 + e'h(\boldsymbol{\sigma}, \mathbf{H}) - ie'h\rho_1(\boldsymbol{\sigma}, \mathbf{E})]\,\psi,$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic vectors of the field.

This differs from (1) by the two extra terms

$$\frac{eh}{c}(\boldsymbol{\sigma}, \mathbf{H}) + \frac{ieh}{c}\rho_1(\boldsymbol{\sigma}, \mathbf{E})$$

in F. These two terms, when divided by the factor $2m$, can be regarded as the additional potential energy of the electron due to its new degree of freedom. The electron will therefore behave as though it has a magnetic moment $eh/2mc.\,\boldsymbol{\sigma}$ and an electric moment $ieh/2mc.\,\rho_1\boldsymbol{\sigma}$. This magnetic moment is just that assumed in the spinning electron model. The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether the electric moment has any physical meaning, since the Hamiltonian in (14) that we started from is real, and the imaginary part only appeared when we multiplied it up in an artificial way in order to make it resemble the Hamiltonian of previous theories.

§ 5. *The Angular Momentum Integrals for Motion in a Central Field.*

We shall consider in greater detail the motion of an electron in a central field of force. We put $\mathbf{A} = 0$ and $e'A_0 = V(r)$, an arbitrary function of the radius r, so that the Hamiltonian in (14) becomes

$$F \equiv p_0 + V + \rho_1(\boldsymbol{\sigma}, \mathbf{p}) + \rho_3 mc.$$

We shall determine the periodic solutions of the wave equation $F\psi = 0$, which means that p_0 is to be counted as a parameter instead of an operator; it is, in fact, just $1/c$ times the energy level.

We shall first find the angular momentum integrals of the motion. The orbital angular momentum $\mathbf{m}$ is defined by

$$\mathbf{m} = \mathbf{x} \times \mathbf{p},$$

and satisfies the following "Vertauschungs" relations

$$\left.\begin{aligned} m_1x_1 - x_1m_1 = 0, \qquad m_1x_2 - x_2m_1 = ihx_3 \\ m_1p_1 - p_1m_1 = 0, \qquad m_1p_2 - p_2m_1 = ihp_3 \end{aligned}\right\}, \tag{17}$$

together with similar relations obtained by permuting the suffixes. Also $\mathbf{m}$ commutes with r, and with p_θ, the momentum canonically conjugate to θ.

We have

$$m_1F - Fm_1 = \rho_1\{m_1(\boldsymbol{\sigma}, \mathbf{p}) - (\boldsymbol{\sigma}, \mathbf{p})m_1\}$$
$$= \rho_1(\boldsymbol{\sigma}, m_1\mathbf{p} - \mathbf{p}m_1)$$
$$= ih\rho_1(\sigma_2\rho_3 - \sigma_3\rho_2),$$

and so

$$\mathbf{m}F - F\mathbf{m} = ih\rho_1\boldsymbol{\sigma} \times \mathbf{p}. \tag{18}$$

Thus $\mathbf{m}$ is not a constant of the motion. We have further

$$\sigma_2F - F\sigma_1 = \rho_1\{\sigma_1(\boldsymbol{\sigma}, \mathbf{p}) - (\boldsymbol{\sigma}, \mathbf{p})\sigma_1\}$$
$$= \rho_1(\sigma_1\boldsymbol{\sigma} - \boldsymbol{\sigma}\sigma_1, \mathbf{p})$$
$$= 2i\rho_1(\sigma_3p_2 - \sigma_2p_3),$$

with the help of (8), and so

$$\boldsymbol{\sigma}F - F\boldsymbol{\sigma} = -2i\rho_1\boldsymbol{\sigma} \times \mathbf{p}.$$

Hence

$$(\mathbf{m} + \tfrac{1}{2}h\boldsymbol{\sigma})F - F(\mathbf{m} + \tfrac{1}{2}h\boldsymbol{\sigma}) = 0.$$

Thus $\mathbf{m} + \frac{1}{2}h\boldsymbol{\sigma}$ ($= \mathbf{M}$ say) is a constant of the motion. We can interpret this result by saying that the electron has a spin angular momentum of $\frac{1}{2}h\boldsymbol{\sigma}$, which, added to the orbital angular momentum $\mathbf{m}$, gives the total angular momentum $\mathbf{M}$, which is a constant of the motion.

The Vertauschungs relations (17) all hold when M's are written for the m's. In particular

$$\mathbf{M} \times \mathbf{M} = ih\mathbf{M} \quad \text{and} \quad \mathbf{M}^2M_3 = M_3\mathbf{M}^2.$$

M_3 will be an action variable of the system. Since the characteristic values of m_3 must be integral multiples of h in order that the wave function may be single-valued, the characteristic values of M_3 must be half odd integral multiples of h. If we put

$$\mathbf{M}^2 = (j^2 - \tfrac{1}{4})h^2, \tag{19}$$

j will be another quantum number, and the characteristic values of M_3 will extend from $(j - \frac{1}{2})h$ to $(-j + \frac{1}{2})h$.* Thus j takes integral values.

One easily verifies from (18) that $\mathbf{m}^2$ does not commute with F, and is thus not a constant of the motion. This makes a difference between the present theory and the previous spinning electron theory, in which $\mathbf{m}^2$ is constant, and defines the azimuthal quantum number k by a relation similar to (19). We shall find that our j plays the same part as the k of the previous theory.

13. ***Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?***
von A. Einstein.

Die Resultate einer jüngst in diesen Annalen von mir publizierten elektrodynamischen Untersuchung[1]) führen zu einer sehr interessanten Folgerung, die hier abgeleitet werden soll.

Ich legte dort die Maxwell-Hertzschen Gleichungen für den leeren Raum nebst dem Maxwellschen Ausdruck für die elektromagnetische Energie des Raumes zugrunde und außerdem das Prinzip:

Die Gesetze, nach denen sich die Zustände der physikalischen Systeme ändern, sind unabhängig davon, auf welches von zwei relativ zueinander in gleichförmiger Parallel-Translationsbewegung befindlichen Koordinatensystemen diese Zustandsänderungen bezogen werden (Relativitätsprinzip).

Gestützt auf diese Grundlagen[2]) leitete ich unter anderem das nachfolgende Resultat ab (l. c. § 8):

Ein System von ebenen Lichtwellen besitze, auf das Koordinatensystem (x, y, z) bezogen, die Energie l; die Strahlrichtung (Wellennormale) bilde den Winkel φ mit der x-Achse des Systems. Führt man ein neues, gegen das System (x, y, z) in gleichförmiger Paralleltranslation begriffenes Koordinatensystem (ξ, η, ζ) ein, dessen Ursprung sich mit der Geschwindigkeit v längs der x-Achse bewegt, so besitzt die genannte Lichtmenge — im System (ξ, η, ζ) gemessen — die Energie:

$$l^* = l \frac{1 - \frac{v}{V} \cos \varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}},$$

wobei V die Lichtgeschwindigkeit bedeutet. Von diesem Resultat machen wir im folgenden Gebrauch.

1) A. Einstein, Ann. d. Phys. 17. p. 891. 1905.

2) Das dort benutzte Prinzip der Konstanz der Lichtgeschwindigkeit ist natürlich in den Maxwellschen Gleichungen enthalten.

Es befinde sich nun im System (x, y, z) ein ruhender Körper, dessen Energie — auf das System (x, y, z) bezogen — E_0 sei. Relativ zu dem wie oben mit der Geschwindigkeit v bewegten System (ξ, η, ζ) sei die Energie des Körpers H_0.

Dieser Körper sende in einer mit der x-Achse den Winkel φ bildenden Richtung ebene Lichtwellen von der Energie $L/2$ (relativ zu (x, y, z) gemessen) und gleichzeitig eine gleich große Lichtmenge nach der entgegengesetzten Richtung. Hierbei bleibt der Körper in Ruhe in bezug auf das System (x, y, z). Für diesen Vorgang muß das Energieprinzip gelten und zwar (nach dem Prinzip der Relativität) in bezug auf beide Koordinatensysteme. Nennen wir E_1 bez. H_1 die Energie des Körpers nach der Lichtaussendung relativ zum System (x, y, z) bez. (ξ, η, ζ) gemessen, so erhalten wir mit Benutzung der oben angegebenen Relation:

$$E_0 = E_1 + \left[\frac{L}{2} + \frac{L}{2}\right],$$

$$H_0 = H_1 + \left[\frac{L}{2} \frac{1 - \frac{v}{V} \cos \varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} + \frac{L}{2} \frac{1 + \frac{v}{V} \cos \varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}\right]$$

$$= H_1 + \frac{L}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}.$$

Durch Subtraktion erhält man aus diesen Gleichungen:

$$(H_0 - E_0) - (H_1 - E_1) = L \left\{\frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} - 1\right\}.$$

Die beiden in diesem Ausdruck auftretenden Differenzen von der Form $H - E$ haben einfache physikalische Bedeutungen. H und E sind Energiewerte desselben Körpers, bezogen auf zwei relativ zueinander bewegte Koordinatensysteme, wobei der Körper in dem einen System (System (x, y, z)) ruht. Es ist also klar, daß die Differenz $H - E$ sich von der kinetischen Energie K des Körpers in bezug auf das andere System (System (ξ, η, ζ)) nur durch eine additive Konstante C unterscheiden kann, welche von der Wahl der willkürlichen addi-

H.8 "Does the Inertia of a Body Depend Upon Its Energy Content?" by A. Einstein, 1905

This article was published late 1905 (september) and contain the famous consequence (the most popular equation of theoretical physics I would say) $E = mc^2$, a consequence of the most cited work which contains the basic of Lorentz transformation "A. Zur Elektrodynamik bewegter Krper" ("On the Electrodynamics of Moving Bodies"), Annalen der Physik 17 (1905): 891-921. This article in a sense, close the annus mirabilis of Einstein. The last formula of the paper $K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2$ and its comment : *if a body gives off the energy L in the form of radiation, its mass diminished by* L/c^2 that we would translate in a more modern formulation $\Delta m = \Delta E/c^2$ shows that he realized how important was his discovery. He even propose in the last paragraph a measurment of this effect through radium decay. He was aware of the recent work of the Curie on the subject some 3 years ago. Indeed, the emission of the radium was an issue at this time as it seemed to contradict the principle of energy conservation. The first *real* test of the mass-energy equivalence from nuclear reaction was made in 1932 by Cockroft and Walton with the reaction ${}^7_3Li + {}^1_1H \rightarrow {}^4_2He + {}^4_2He + Energy$: proton were accelerated and slammed into lithium atoms producing alpha–particles and energy. Thus the mass of proton and lithium was converted into the mass of two alpha–

also setzen:

$$H_0 - E_0 = K_0 + C,$$
$$H_1 - E_1 = K_1 + C,$$

da C sich während der Lichtaussendung nicht ändert. Wir erhalten also:

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1-\left(\frac{v}{V}\right)^2}} - 1\right\}.$$

Die kinetische Energie des Körpers in bezug auf (ξ, η, ζ) nimmt infolge der Lichtaussendung ab, und zwar um einen von den Qualitäten des Körpers unabhängigen Betrag. Die Differenz $K_0 - K_1$ hängt ferner von der Geschwindigkeit ebenso ab wie die kinetische Energie des Elektrons (l. c. § 10).

Unter Vernachlässigung von Größen vierter und höherer Ordnung können wir setzen:

$$K_0 - K_1 = \frac{L}{V^2}\frac{v^2}{2}.$$

Aus dieser Gleichung folgt unmittelbar:

Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um L/V^2. Hierbei ist es offenbar unwesentlich, daß die dem Körper entzogene Energie gerade in Energie der Strahlung übergeht, so daß wir zu der allgemeineren Folgerung geführt werden:

Die Masse eines Körpers ist ein Maß für dessen Energieinhalt; ändert sich die Energie um L, so ändert sich die Masse in demselben Sinne um $L/9 \cdot 10^{20}$, wenn die Energie in Erg und die Masse in Grammen gemessen wird.

Es ist nicht ausgeschlossen, daß bei Körpern, deren Energieinhalt in hohem Maße veränderlich ist (z. B. bei den Radiumsalzen), eine Prüfung der Theorie gelingen wird.

Wenn die Theorie den Tatsachen entspricht, so überträgt die Strahlung Trägheit zwischen den emittierenden und absorbierenden Körpern.

Bern, September 1905.

(Eingegangen 27. September 1905.)

DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY-CONTENT?

By A. EINSTEIN

September 27, 1905

The results of the previous investigation lead to a very interesting conclusion, which is here to be deduced.

I based that investigation on the Maxwell-Hertz equations for empty space, together with the Maxwellian expression for the electromagnetic energy of space, and in addition the principle that:—

The laws by which the states of physical systems alter are independent of the alternative, to which of two systems of coordinates, in uniform motion of parallel translation relatively to each other, these alterations of state are referred (principle of relativity).

With these principles* as my basis I deduced *inter alia* the following result (§ 8):—

Let a system of plane waves of light, referred to the system of co-ordinates (x, y, z), possess the energy l; let the direction of the ray (the wave-normal) make an angle ϕ with the axis of x of the system. If we introduce a new system of co-ordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the system (x, y, z), and having its origin of co-ordinates in motion along the axis of x with the velocity v, then this quantity of light—measured in the system (ξ, η, ζ)—possesses the energy

$$l^* = l\frac{1 - \frac{v}{c}\cos\phi}{\sqrt{1 - v^2/c^2}}$$

where c denotes the velocity of light. We shall make use of this result in what follows.

Let there be a stationary body in the system (x, y, z), and let its energy—referred to the system (x, y, z) be E_0. Let the energy of the body relative to the system (ξ, η, ζ) moving as above with the velocity v, be H_0.

Let this body send out, in a direction making an angle ϕ with the axis of x, plane waves of light, of energy $\frac{1}{2}L$ measured relatively to (x, y, z), and simultaneously an equal quantity of light in the opposite direction. Meanwhile the body remains at rest with respect to the system (x, y, z). The principle of

*The principle of the constancy of the velocity of light is of course contained in Maxwell's equations.

1

particles and kinetic energy. Cockroft and Walton won the Nobel Prize in physics for this experiment in 1951. To be honest, in 1905 the idea of equivalence between mass and energy was not completely new as Gustave Lebon (1841-1931) had several similar intuitions. More scientific works as the ones by Kaufmann in 1901 and Abraham in 1903 concluded that all the mass of the electron can be understood as part of the energy of the electromagnetic field.. Langevin in 1905 even extended this notion of "electromagnetic mass" to any massive body, representing any type of mass as a sum of positive and negative electrons. But Einstein was the first to really think the link between inertial mass and energy in a global and geometrical way, based on fundamental principles, with tremendous consequences. During the years 1906-1907, Einstein published 3 other demonstrations of the energy–mass equivalence principle, the more important one published in his famous review of 1907. To summarize, and to show that Einstein was very conscious of the revolution he achieved this year, we reproduce here the letter he sent to his professor and mentor at that time, Conrad Habicht, with its translation after the presentation of the original article of 1905.

energy must apply to this process, and in fact (by the principle of relativity) with respect to both systems of co-ordinates. If we call the energy of the body after the emission of light $\mathbf{E}_0$ or $\mathbf{H}_0$ respectively, measured relatively to the system (x, y, z) or (ξ, η, ζ) respectively, then by employing the relation given above we obtain

$$\begin{aligned}
\mathbf{E}_0 &= \mathbf{E}_1 + \frac{1}{2}\mathbf{L} + \frac{1}{2}\mathbf{L}, \\
\mathbf{H}_0 &= \mathbf{H}_1 + \frac{1}{2}\mathbf{L}\frac{1 - \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}} + \frac{1}{2}\mathbf{L}\frac{1 + \frac{v}{c}cos\phi}{\sqrt{1 - v^2/c^2}} \\
&= \mathbf{H}_1 + \frac{\mathbf{L}}{\sqrt{1 - v^2/c^2}}.
\end{aligned}$$

By subtraction we obtain from these equations

$$\mathbf{H}_0 - \mathbf{E}_0 - (\mathbf{H}_1 - \mathbf{E}_1) = \mathbf{L}\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The two differences of the form $\mathbf{H} - \mathbf{E}$ occurring in this expression have simple physical significations. $\mathbf{H}$ and $\mathbf{E}$ are energy values of the same body referred to two systems of co-ordinates which are in motion relatively to each other, the body being at rest in one of the two systems (system (x, y, z)). Thus it is clear that the difference $\mathbf{H} - \mathbf{E}$ can differ from the kinetic energy K of the body, with respect to the other system (ξ, η, ζ), only by an additive constant C, which depends on the choice of the arbitrary additive constants of the energies $\mathbf{H}$ and $\mathbf{E}$. Thus we may place

$$\begin{aligned}
\mathbf{H}_0 - \mathbf{E}_0 &= \mathrm{K}_0 + \mathrm{C}, \\
\mathbf{H}_1 - \mathbf{E}_1 &= \mathrm{K}_1 + \mathrm{C},
\end{aligned}$$

since C does not change during the emission of light. So we have

$$\mathrm{K}_0 - \mathrm{K}_1 = \mathbf{L}\left\{\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right\}.$$

The kinetic energy of the body with respect to (ξ, η, ζ) diminishes as a result of the emission of light, and the amount of diminution is independent of the properties of the body. Moreover, the difference $\mathrm{K}_0 - \mathrm{K}_1$, like the kinetic energy of the electron (§ 10), depends on the velocity.

Neglecting magnitudes of fourth and higher orders we may place

$$\mathrm{K}_0 - \mathrm{K}_1 = \frac{1}{2}\frac{\mathbf{L}}{c^2}v^2.$$

From this equation it directly follows that:—

If a body gives off the energy L in the form of radiation, its mass diminishes by $\mathbf{L}/c^2$. The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that

The mass of a body is a measure of its energy-content; if the energy changes by $\mathbf{L}$, the mass changes in the same sense by $\mathbf{L}/9 \times 10^{20}$, the energy being measured in ergs, and the mass in grammes.

It is not impossible that with bodies whose energy-content is variable to a high degree (e.g. with radium salts) the theory may be successfully put to the test.

If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.

About this Document

This edition of Einstein's *Does the Inertia of a Body Depend upon its Energy-Content* is based on the English translation of his original 1905 German-language paper (published as *Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?*, in *Annalen der Physik*. 18:639, 1905) which appeared in the book *The Principle of Relativity*, published in 1923 by Methuen and Company, Ltd. of London. Most of the papers in that collection are English translations by W. Perrett and G.B. Jeffery from the German *Das Relativitätsprinzip*, 4th ed., published by in 1922 by Tuebner. All of these sources are now in the public domain; this document, derived from them, remains in the public domain and may be reproduced in any manner or medium without permission, restriction, attribution, or compensation.

The footnote is as it appeared in the 1923 edition. The 1923 English translation modified the notation used in Einstein's 1905 paper to conform to that in use by the 1920's; for example, c denotes the speed of light, as opposed the V used by Einstein in 1905. In this paper Einstein uses L to denote energy; the italicised sentence in the conclusion may be written as the equation "$m = \mathbf{L}/c^2$" which, using the more modern E instead of L to denote energy, may be trivially rewritten as "$\mathrm{E} = mc^2$".

This edition was prepared by John Walker. The current version of this document is available in a variety of formats from the editor's Web site:

http://www.fourmilab.ch/

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Bern, 18 May 1905

Dear Habicht,

Such a solemn air of silence has descended between us that I almost feel as if I am committing a sacrilege when I break it now with some inconsequential babble. So, what are you up to, you frozen whale, you smoked, dried, canned piece of soul? Why have you still not sent me your dissertation? Don't you know that I am one of the 1.5 fellows who would read it with interest and pleasure, you wretched man? I promise you four papers in return. The first deals with radiation and the energy properties of light and is very revolutionary, as you will see if you send me your work first. The second paper is a determination of the true sizes of atoms. The third proves that bodies on the order of magnitude 1/1000 mm, suspended in liquids, must already perform an observable random motion that is produced by thermal motion. The fourth paper is only a rough draft at this point, and is an electrodynamics of moving bodies which employs a modification of the theory of space and time.

appearance the spectrum is very much like spectra of the Milky Way clouds in Sagittarius and Cygnus, and is also similar to spectra of binary stars of the W Ursae Majoris type, where the widening and depth of the lines are affected by the rapid rotation of the stars involved.

The wide shallow absorption lines observed in the spectrum of N. G. C. 7619 have been noticed in the spectra of other extra-galactic nebulae, and may be due to a dispersion in velocity and a blending of the spectral types of the many stars which presumably exist in the central parts of these nebulae. The lack of depth in the absorption lines seems to be more pronounced among the smaller and fainter nebulae, and in N. G. C. 7619 the absorption is very weak.

It is hoped that velocities of more of these interesting objects will soon be available.

A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY AMONG EXTRA-GALACTIC NEBULAE

BY EDWIN HUBBLE

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Communicated January 17, 1929

Determinations of the motion of the sun with respect to the extra-galactic nebulae have involved a K term of several hundred kilometers which appears to be variable. Explanations of this paradox have been sought in a correlation between apparent radial velocities and distances, but so far the results have not been convincing. The present paper is a re-examination of the question, based on only those nebular distances which are believed to be fairly reliable.

Distances of extra-galactic nebulae depend ultimately upon the application of absolute-luminosity criteria to involved stars whose types can be recognized. These include, among others, Cepheid variables, novae, and blue stars involved in emission nebulosity. Numerical values depend upon the zero point of the period-luminosity relation among Cepheids, the other criteria merely check the order of the distances. This method is restricted to the few nebulae which are well resolved by existing instruments. A study of these nebulae, together with those in which any stars at all can be recognized, indicates the probability of an approximately uniform upper limit to the absolute luminosity of stars, in the late-type spirals and irregular nebulae at least, of the order of M (photographic) $= -6.3$.[1] The apparent luminosities of the brightest stars in such nebulae are thus criteria which, although rough and to be applied with caution,

furnish reasonable estimates of the distances of all extra-galactic systems in which even a few stars can be detected.

TABLE 1

NEBULAE WHOSE DISTANCES HAVE BEEN ESTIMATED FROM STARS INVOLVED OR FROM MEAN LUMINOSITIES IN A CLUSTER

OBJECT	m_s	r	v	m_t	M_t
S. Mag.	..	0.032	+ 170	1.5	−16.0
L. Mag.	..	0.034	+ 290	0.5	17.2
N. G. C. 6822	..	0.214	− 130	9.0	12.7
598	..	0.263	− 70	7.0	15.1
221	..	0.275	− 185	8.8	13.4
224	..	0.275	− 220	5.0	17.2
5457	17.0	0.45	+ 200	9.9	13.3
4736	17.3	0.5	+ 290	8.4	15.1
5194	17.3	0.5	+ 270	7.4	16.1
4449	17.8	0.63	+ 200	9.5	14.5
4214	18.3	0.8	+ 300	11.3	13.2
3031	18.5	0.9	− 30	8.3	16.4
3627	18.5	0.9	+ 650	9.1	15.7
4826	18.5	0.9	+ 150	9.0	15.7
5236	18.5	0.9	+ 500	10.4	14.4
1068	18.7	1.0	+ 920	9.1	15.9
5055	19.0	1.1	+ 450	9.6	15.6
7331	19.0	1.1	+ 500	10.4	14.8
4258	19.5	1.4	+ 500	8.7	17.0
4151	20.0	1.7	+ 960	12.0	14.2
4382	..	2.0	+ 500	10.0	16.5
4472	..	2.0	+ 850	8.8	17.7
4486	..	2.0	+ 800	9.7	16.8
4649	..	2.0	+1090	9.5	17.0
Mean					−15.5

m_s = photographic magnitude of brightest stars involved.
r = distance in units of 10^6 parsecs. The first two are Shapley's values.
v = measured velocities in km./sec. N. G. C. 6822, 221, 224 and 5457 are recent determinations by Humason.
m_t = Holetschek's visual magnitude as corrected by Hopmann. The first three objects were not measured by Holetschek, and the values of m_t represent estimates by the author based upon such data as are available.
M_t = total visual absolute magnitude computed from m_t and r.

Finally, the nebulae themselves appear to be of a definite order of absolute luminosity, exhibiting a range of four or five magnitudes about an average value M (visual) $= -15.2$.[1] The application of this statistical average to individual cases can rarely be used to advantage, but where considerable numbers are involved, and especially in the various clusters of nebulae, mean apparent luminosities of the nebulae themselves offer reliable estimates of the mean distances.

Radial velocities of 46 extra-galactic nebulae are now available, but

H.9 "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae" by E. Hubble, 1929

In this article, Hubble following Shapley's measurements, "showed" that they could exist a relation between radial velocity and distance of nearby clusters. This article was very criticized when it was released in 1929, because of its large uncertainties in the measurements (just a glance at his Fig.1 can convince us why there was such debates). Moreover, the acceleration of the Universe computed with these data gives 745 km/s for a distance of 1.4 Mpc see first sentence of page 172), almost an order of magnitude of the actual value ($\simeq$ 74 km/s/Mpc). The age computed with such a value is 2 billions years whereas Jeans already computed that the age of the earth was more than 4 billions years. How to explain an Universe whose components are older than itself (see the article of G. Lemaitre for an attempt to solve this issue)?

individual distances are estimated for only 24. For one other, N. G. C. 3521, an estimate could probably be made, but no photographs are available at Mount Wilson. The data are given in table 1. The first seven distances are the most reliable, depending, except for M 32 the companion of M 31, upon extensive investigations of many stars involved. The next thirteen distances, depending upon the criterion of a uniform upper limit of stellar luminosity, are subject to considerable probable errors but are believed to be the most reasonable values at present available. The last four objects appear to be in the Virgo Cluster. The distance assigned to the cluster, 2×10^6 parsecs, is derived from the distribution of nebular luminosities, together with luminosities of stars in some of the later-type spirals, and differs somewhat from the Harvard estimate of ten million light years.[2]

The data in the table indicate a linear correlation between distances and velocities, whether the latter are used directly or corrected for solar motion, according to the older solutions. This suggests a new solution for the solar motion in which the distances are introduced as coefficients of the K term, i. e., the velocities are assumed to vary directly with the distances, and hence K represents the velocity at unit distance due to this effect. The equations of condition then take the form

$$rK + X \cos \alpha \cos \delta + Y \sin \alpha \cos \delta + Z \sin \delta = v.$$

Two solutions have been made, one using the 24 nebulae individually, the other combining them into 9 groups according to proximity in direction and in distance. The results are

	24 OBJECTS	9 GROUPS
X	-65 ± 50	$+3 \pm 70$
Y	$+228 \pm 95$	$+230 \pm 120$
Z	-195 ± 40	-133 ± 70
K	$+465 \pm 50$	$+513 \pm 60$ km./sec. per 10^6 parsecs.
A	288°	269°
D	+40°	+33°
V_0	306 km./sec.	247 km./sec.

For such scanty material, so poorly distributed, the results are fairly definite. Differences between the two solutions are due largely to the four Virgo nebulae, which, being the most distant objects and all sharing the peculiar motion of the cluster, unduly influence the value of K and hence of V_0. New data on more distant objects will be required to reduce the effect of such peculiar motion. Meanwhile round numbers, intermediate between the two solutions, will represent the probable order of the values. For instance, let $A = 277°$, $D = +36°$ (Gal. long. = 32°, lat. = +18°), $V_0 = 280$ km./sec., $K = +500$ km./sec. per million parsecs. Mr. Strömberg has very kindly checked the general order of these values by independent solutions for different groupings of the data.

A constant term, introduced into the equations, was found to be small and negative. This seems to dispose of the necessity for the old constant K term. Solutions of this sort have been published by Lundmark,[3] who replaced the old K by $k + lr + mr^2$. His favored solution gave $k = 513$, as against the former value of the order of 700, and hence offered little advantage.

TABLE 2

NEBULAE WHOSE DISTANCES ARE ESTIMATED FROM RADIAL VELOCITIES

OBJECT	v	v_0	r	m_r	M_r
N. G. C. 278	+650	−110	1.52	12.0	−13.9
404	−25	−65	..	11.1	..
584	+1800	+75	3.45	10.9	16.8
936	+1300	+115	2.37	11.1	15.7
1023	+300	−10	0.62	10.2	13.8
1700	+800	+220	1.16	12.5	12.8
2681	+700	−10	1.42	10.7	15.0
2683	+400	+65	0.67	9.9	14.3
2841	+600	−20	1.24	9.4	16.1
3024	+290	−105	0.79	9.0	15.5
3115	+600	+105	1.00	9.5	15.5
3368	+940	+70	1.74	10.0	16.2
3379	+810	+65	1.49	9.4	16.4
3489	+600	+50	1.10	11.2	14.0
3521	+730	+95	1.27	10.1	15.4
3623	+800	+35	1.53	9.9	16.0
4111	+800	−95	1.79	10.1	16.1
4526	+580	−20	1.20	11.1	14.3
4565	+1100	−75	2.35	11.0	15.9
4594	+1140	+25	2.23	9.1	17.6
5005	+900	−130	2.06	11.1	15.5
5866	+650	−215	1.73	11.7	−14.5
Mean				10.5	−15.3

The residuals for the two solutions given above average 150 and 110 km./sec. and should represent the average peculiar motions of the individual nebulae and of the groups, respectively. In order to exhibit the results in a graphical form, the solar motion has been eliminated from the observed velocities and the remainders, the distance terms plus the residuals, have been plotted against the distances. The run of the residuals is about as smooth as can be expected, and in general the form of the solutions appears to be adequate.

The 22 nebulae for which distances are not available can be treated in two ways. First, the mean distance of the group derived from the mean apparent magnitudes can be compared with the mean of the velocities

corrected for solar motion. The result, 745 km./sec. for a distance of 1.4×10^6 parsecs, falls between the two previous solutions and indicates a value for K of 530 as against the proposed value, 500 km./sec.

Secondly, the scatter of the individual nebulae can be examined by assuming the relation between distances and velocities as previously determined. Distances can then be calculated from the velocities corrected for solar motion, and absolute magnitudes can be derived from the apparent magnitudes. The results are given in table 2 and may be compared with the distribution of absolute magnitudes among the nebulae in table 1, whose distances are derived from other criteria. N. G. C. 404

FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

can be excluded, since the observed velocity is so small that the peculiar motion must be large in comparison with the distance effect. The object is not necessarily an exception, however, since a distance can be assigned for which the peculiar motion and the absolute magnitude are both within the range previously determined. The two mean magnitudes, −15.3 and −15.5, the ranges, 4.9 and 5.0 mag., and the frequency distributions are closely similar for these two entirely independent sets of data; and even the slight difference in mean magnitudes can be attributed to the selected, very bright, nebulae in the Virgo Cluster. This entirely unforced agreement supports the validity of the velocity-distance relation in a very

evident matter. Finally, it is worth recording that the frequency distribution of absolute magnitudes in the two tables combined is comparable with those found in the various clusters of nebulae.

The results establish a roughly linear relation between velocities and distances among nebulae for which velocities have been previously published, and the relation appears to dominate the distribution of velocities. In order to investigate the matter on a much larger scale, Mr. Humason at Mount Wilson has initiated a program of determining velocities of the most distant nebulae that can be observed with confidence. These, naturally, are the brightest nebulae in clusters of nebulae. The first definite result,[4] $v = +3779$ km./sec. for N. G. C. 7619, is thoroughly consistent with the present conclusions. Corrected for the solar motion, this velocity is +3910, which, with $K = 500$, corresponds to a distance of 7.8×10^6 parsecs. Since the apparent magnitude is 11.8, the absolute magnitude at such a distance is −17.65, which is of the right order for the brightest nebulae in a cluster. A preliminary distance, derived independently from the cluster of which this nebula appears to be a member, is of the order of 7×10^6 parsecs.

New data to be expected in the near future may modify the significance of the present investigation or, if confirmatory, will lead to a solution having many times the weight. For this reason it is thought premature to discuss in detail the obvious consequences of the present results. For example, if the solar motion with respect to the clusters represents the rotation of the galactic system, this motion could be subtracted from the results for the nebulae and the remainder would represent the motion of the galactic system with respect to the extra-galactic nebulae.

The outstanding feature, however, is the possibility that the velocity-distance relation may represent the de Sitter effect, and hence that numerical data may be introduced into discussions of the general curvature of space. In the de Sitter cosmology, displacements of the spectra arise from two sources, an apparent slowing down of atomic vibrations and a general tendency of material particles to scatter. The latter involves an acceleration and hence introduces the element of time. The relative importance of these two effects should determine the form of the relation between distances and observed velocities; and in this connection it may be emphasized that the linear relation found in the present discussion is a first approximation representing a restricted range in distance.

[1] *Mt. Wilson Contr.*, No. 324; *Astroph. J., Chicago, Ill.*, **64**, 1926 (321).

[2] *Harvard Coll. Obs. Circ.*, 294, 1926.

[3] *Mon. Not. R. Astr. Soc.*, **85**, 1925 (865–894).

[4] These PROCEEDINGS, **15**, 1929 (167).

H.10 "The Beginning of the World from the Point of View of Quantum Theory" by Georges Lemaitre; 1931

In this article, published in *Nature* in 1931 by Georges Lemaitre, the priest exposes his theory of "primeval atom" this theory was critized strongly by Einstein and the majority of the scientific community at this time as Lemaitre point out in his first sentence. At the first reading, the text can appear as a divulgation article for layman people. However, when combined with his more complete work "A homogeneous Universe of constant mass and growing radius accounting for the radial velocity of extragalactic nebulae" published in 1927, all becomes clear and very very modern. Indeed, its aim was to conciliate the recent measurements of Hubble which gives when applied the "Hubble law" proposed by Lemaitre himself, and age of the Universe of 2 billions years, much younger that the age of the earth which was dated in 1927 to range around 5 billions years. His solution was as elegant as modern: he proposed to combined the expansion due to a de Sitter vacuum (cosmological constant), responsible for the 2 billions years observed by Hubble, preceding by a time of stagnation which was following a huge expansion due to the decay of the "primeval atoms" (the text). This history of the Universe is summarized in the figure above. replacing the "primeval atom" by an inflaton and you have the present picture of the universe we have now.

Another interesting point is the draft he wrote before sending it to the review *Nature*. We reproduce it after the original text. We can notice that the last paragraph was erased. It was concerning the presence of God in the physical laws. As we know, this inflationary theory was strongly supported by the church as it was very compatible with a description of the Creation described in the holy texts.

The Beginning of the World from the Point of View of Quantum Theory.

SIR ARTHUR EDDINGTON [1] states that, philosophically, the notion of a beginning of the present order of Nature is repugnant to him. I would rather be inclined to think that the present state of quantum theory suggests a beginning of the world very different from the present order of Nature. Thermodynamical principles from the point of view of quantum theory may be stated as follows: (1) Energy of constant total amount is distributed in discrete quanta. (2) The number of distinct quanta is ever increasing. If we go back in the course of time we must find fewer and fewer quanta, until we find all the energy of the universe packed in a few or even in a unique quantum.

Now, in atomic processes, the notions of space and time are no more than statistical notions; they fade out when applied to individual phenomena involving but a small number of quanta. If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning; they would only begin to have a sensible meaning when the original quantum had been divided into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time. I think that such a beginning of the world is far enough from the present order of Nature to be not at all repugnant.

It may be difficult to follow up the idea in detail as we are not yet able to count the quantum packets in every case. For example, it may be that an atomic nucleus must be counted as a unique quantum, the atomic number acting as a kind of quantum number. If the future development of quantum theory happens to turn in that direction, we could conceive the beginning of the universe in the form of a unique atom, the atomic weight of which is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radioactive process. Some remnant of this process might, according to Sir James Jeans's idea, foster the heat of the stars until our low atomic number atoms allowed life to be possible.

Clearly the initial quantum could not conceal in itself the whole course of evolution; but, according to the principle of indeterminacy, that is not necessary. Our world is now understood to be a world where something really happens; the whole story of the world need not have been written down in the first quantum like a song on the disc of a phonograph. The whole matter of the world must have been present at the beginning, but the story it has to tell may be written step by step.

G. LEMAÎTRE.
40 rue de Namur,
Louvain.

[1] NATURE, Mar. 21, p. 447.

the universe in the form of a unique atom whose atomic weight is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radioactive process. Some rest of this process would, according to Sir Jeans idea, foster the heat of the stars until our low atomic number atoms may allow live to be possible.

Clearly the initial quantum could not conceile in itself the whole course of evolution; but, according to the indetermination principle, that is not necessary. Our world is now a world where something happens; the whole story of the world does not need to be written down in the first quantum as a song on the disc of a phonograp. The whole matter of the world must be present at the beginning, but the story it has to tell may be written step by step.

I think that every one who believes in a supreme being supporting every being and every acting, believes also that God is essentially hidden and may be glad to see how present physics provides a veil hiding the creation.

Figure 1: A copy of the original Lemaître typescript, with the last paragraph crossed by the author.

and theology dealt with two separate worlds.

Physik. Zeitschr. XII, 1911. v. Baeyer, Hahn u. Meitner, Magnet. Spektren der β-Strahlen. 1099

kurven des aktiven Niederschlages von Aktinium gelangt[1]). Indes war es mir nicht möglich, die erwähnten Unregelmäßigkeiten in der Abklingung des Aktinium *B* willkürlich zu reproduzieren.

Wurde die Lösung des aktiven Niederschlages unter den oben angegebenen Bedingungen mit Silber als Kathode elektrolysiert, so schied sich Aktinium $A+B+C$ mit einem Überschuß an Aktinium *A* ab. Da die Nachbildung von Aktinium *B* mit 2,1 Minuten, die von Aktinium *C* mit einer Periode von 5,1 Minuten erfolgt, so spielt hier die Geschwindigkeit, mit der das Präparat nach dem Entfernen aus der Lösung gemessen wird, eine ausschlaggebende Rolle für den gefundenen Überschuß an Aktinium *A*.

Die Abscheidung des Aktinium *B* auf Nickel ist wegen der Einfachheit des Verfahrens und der relativen Empfindlichkeit (es werden α-Strahlen gemessen) vielleicht zum Nachweis kleiner Aktiniummengen geeignet.

1) Mlle Blaauqis, C. R. 151, 57—60, 1910.

Berlin, Chemisches Institut der Universität.

(Eingegangen 15. September 1911.)

Magnetische Spektren der β-Strahlen des Radiums.

Von Otto v. Baeyer, Otto Hahn und Lise Meitner.

(Mit Tafel XIV.)

In zwei vorangehenden Arbeiten[1]) sind die magnetischen Spektren der β-Strahlen verschiedener radioaktiver Substanzen, besonders der Produkte der Thoriumreihe beschrieben worden. Dabei zeigte sich, daß in fast allen Fällen ein diskontinuierliches Spektrum erhalten wird. Besonders wohl definierte Streifen erhielt man in dem Gebiet der langsameren β-Strahlen (von 70 Proz. Lichtgeschwindigkeit an abwärts), während die schnelleren nur ziemlich verwaschene Bänder ergaben, was sich z. B. beim *RaE* besonders bemerkbar machte.

Die vorliegende Arbeit beschäftigt sich mit den β-Strahlen des Radiums selbst und seines schnell zerfallenden aktiven Niederschlags. Diese Untersuchung bot insofern ein besonderes Interesse, da bei früheren Versuchen, wie z. B. von Kaufmann und Bucherer, eine Diskontinuität des magnetischen Spektrums sich nicht bemerkbar gemacht hatte.

Die durchdringenden β-Strahlen der üblichen Radiumsalze rühren vor allem vom *RaC* her. Aus Absorptionsmessungen war für diese Substanz schon früher eine komplexe β-Strahlung angenommen worden. Außer dem Radium *C* emittiert das Radium *B* β-Strahlen, die im allgemeinen weniger durchdringend sind als die Strahlen des *RaC*, und nach Untersuchungen von H. W. Schmidt[1]) ebenfalls als komplex anzusehen sind. Schließlich ist das Radium selbst Träger einer typischen β-Strahlung, die von Hahn und Meitner[2]) aufgefunden und vor kurzem von Kolowrat[3]) bestätigt worden ist. Diese Strahlung beteiligt sich bei der gesamten β-Strahlung des Radiums plus aktivem Niederschlag, in der üblichen elektroskopischen Weise gemessen, nur zu etwa 1 Proz., läßt sich aber in einem entemanierten Radium mit Leichtigkeit nachweisen.

Bei der Aufnahme des magnetischen Spektrums muß man also 3 Strahlengruppen erwarten, die des *RaC*, des *RaB* und des eigentlichen Radiums.

Die β-Strahlen des aktiven Niederschlags des Radiums.

Zuerst wurde der aktive Niederschlag des Radiums untersucht und zwar $RaB+C$. Der an den Wänden eines geeigneten Emanationsgefäßes angesammelte aktive Niederschlag wurde nach den Angaben von L. Meitner[4]) in salzsaurer Lösung an einer Silberkathode elektrolytisch abgeschieden. Die verwendete Silberkathode bestand aus einem 1 cm langen, 0,2 mm dicken Silberdrähtchen. Die Aktivität eines solchen Präparats entsprach 2 bis 4 mg Radium und war hergestellt worden aus 12 mg Emanation.

Das Präparat wurde in dünne Zinnfolie (0,007 mm) eingehüllt, um ein Verstreuen der aktiven Substanz durch Rückstoß oder Erschütterungen zu vermeiden. Die experimentelle Anordnung zur photographischen Aufnahme des magnetischen Spektrums im Vakuum war dieselbe wie früher. Die Feldstärke betrug 215 Gauß, die Expositionsdauer 2 Stunden.

Die erhaltene Photographie ist in Fig. 1[5]) dreifach vergrößert wiedergegeben. Auf der

1) v. Baeyer u. Hahn, diese Zeitschr. 11, 488, 1910; v. Baeyer, Hahn u. Meitner, diese Zeitschr. 12, 273, 1911.

1) H. W. Schmidt, Ann. d. Phys. (4) 23, 609. 1906.
2) Hahn u. Meitner, diese Zeitschr. 10, 741. 1909.
3) Kolowrat, Le Radium 7, 269. 1910.
4) L. Meitner, diese Zeitschr. 12, 1094. 1911.
5) Die beiden Figuren befinden sich auf besonderer Tafel XIV.

1100 v. Baeyer, Hahn u. Meitner, Magnet. Spektren der β-Strahlen. Physik. Zeitschr. XII, 1911.

Originalplatte sind außer dem starken α-Strahlstreifen 7 Streifen zu erkennen, die sich mit genügender Genauigkeit ausmessen ließen. Auf der Reproduktion sind nicht alle Streifen zu erkennen, da, wie wir schon früher beobachteten, die schnelleren β-Strahlen ziemlich unscharfe Bilder ergaben. Die Ausmessung der Streifen erfolgte wie früher unter einem schwach vergrößernden Mikroskop.

Bei der Berechnung der Krümmungsradien aus der gemessenen Ablenkung und den Apparatdimensionen wurde nicht die früher verwandte einfache Formel benutzt, da diese bei der hier vorliegenden Größe der Ablenkung (bis zu 5 mm) nicht genügend genau ist[1]).

Die genauere Formel lautet

$$\varrho = \sqrt{R^2 + \frac{1}{2}A^2 + \frac{1}{2}B^2},$$

wo ϱ der gesuchte Krümmungsradius.

$$A = d_1 + \frac{d_2}{2}$$

$$B = \frac{d_2}{2}$$

$$R = \frac{d_1(d_1 + d_2)}{2d}.$$

d_1 Abstand des Spaltes von der photograph. Platte,
d_2 „ „ „ „ dem Präparat,
d die gemessene Aufspaltung.

Die Abweichungen der früheren Formel gegenüber dieser sind natürlich nur von Belang bei den langsameren β-Strahlen mit kleinem Radius und großer Ablenkung.

Bei der Bestimmung der magnetischen Ablenkbarkeit ist außerdem zu berücksichtigen, daß durch das Bedecken des Präparats mit Zinnfolie die langsameren β-Strahlen einen Geschwindigkeitsverlust erleiden, der immerhin schon nachweisbar ist. Deshalb wurde zur Bestimmung der Geschwindigkeit der langsameren β-Strahlen eine Aufnahme mit unbedecktem Präparat bei demselben Feld ausgeführt.

Außerdem wurde noch eine Aufnahme bei einem halb so starken Feld hergestellt ebenfalls mit einem unbedeckten Präparat. Die erwähnten 7 Streifen sind auf dieser Aufnahme zwar zu erkennen, aber nicht genügend getrennt, um eine Ausmessung zuzulassen. Dafür treten aber auf dieser Aufnahme noch 2 weitere schwächere, sehr stark abgelenkte Streifen auf, so daß nach den bisherigen Versuchen der aktive Niederschlag des Radiums 9 Streifen aufweist.

Wie schon erwähnt, gehören die entsprechenden β-Strahlen dem Radium *B* und Radium *C* an. Um nun zu erkennen, welche Streifen dem Radium *B* zuzuschreiben sind, wurde eine Aufnahme mit reinem Radium *C* gemacht. Das Radium *C* wurde auf einen 1 cm langen, 0,2 mm dicken Nickeldraht niedergeschlagen. Die Aktivität entsprach mehreren Milligrammen reinem Radiumbromid. Die Aufnahme ergab ein ganz anderes Bild wie Fig. 1, die fünf stark abgelenkten Streifen fehlen. Daraus folgt, daß diese fünf Streifen dem Radium *B* zuzuschreiben sind. Dagegen sind die übrigen vier Streifen zu erkennen. Leider ist die Aufnahme dieser β-Strahlen, die alle größere Geschwindigkeit besitzen, nicht so klar, daß sie sich zur Reproduktion eignen würde.

Die große Zerfallsgeschwindigkeit von Radium *C* ebenso wie Radium *B* macht die Aufnahme des magnetischen Spektrums dieser Substanzen ziemlich schwierig, da man bei nicht sehr großen zur Verfügung stehenden Radium- bezw. Emanationsmengen die elektrochemische Abscheidung mit sehr guter Ausbeute vornehmen muß, um überhaupt photographierbare Mengen zu erhalten.

Die Ergebnisse der Messungen sind in der folgenden Tabelle zusammengestellt. Zur Berechnung der Geschwindigkeit ist die Lorentz-Einsteinsche Formel verwendet unter Zugrundelegung des Wertes

$$\frac{e}{m_0} = 1{,}77 \cdot 10^7 \text{ elektrom. Einh.}$$

β-Strahlen des aktiven Niederschlags[1]) des Radiums.

Geschwindigkeit (% Lichtgeschwindigkeit)	
0,98	Radium *C*
0,94	
0,86	
0,80	
0,74	Radium *B*.
0,69	
0,63	
0,41	
0,36	

1) Bei den von uns früher angestellten Messungen der sehr langsamen β-Strahlen von Radium *D* und den zwei sehr langsamen und schwachen β-Strahlen des aktiven Niederschlages der Thoriumemanation hätte diese genauere Formel ebenfalls angewendet werden müssen, wurde aber wegen der durch die Unschärfe dieser Streifen bedingten Ungenauigkeit der Messung nicht benutzt. Die Berechnung nach der genaueren Formel ergibt für Radium *D* $\beta = 0{,}39$ und 0,33 statt 0,37 und 0,31, für Thorium $(A+D)$ $\beta = 0{,}36$ und 0,29 statt 0,34 und 0,28.

1) Vor kurzem sind von Herrn J. Danysz (C. R. 153,

H.11 "Magnetische Spektren der β–Strahlen des Radiums" by Baeyer, Hahn and Meitner; 1911

This article is in fact the first article where have been observed a continuum $\beta-$spectrum (electron emitted through the desintegration of a neutron into proton plus electron plus neutrino). Of course, in 1911 the hypothesis of neutrino was not at all in the mind of the authors of the articles. With a spectrometer of mass, they were believing to observe monochromatic spectrum (if we suppose only 2 bodies decay, forgetting the neutrino). In the german article, we can notice on page 2 that they have noticed something "wrong" or at least strange appeared in their results (see the photos they obtained included here too). Indeed, they remark that their spectrum is "unscharfe" which means "fuzzy", (due to the continuum and not monochromatic spectrum) but they conclude it should come from the preparation "preparats" in page 2) and the shield they used that can diffuse the spectrum. In 1914, Chadwick did the same experiment and confirmed the nature continuous of the spectrum.

Die β-Strahlen des Radiums selbst.

Wie schon erwähnt, beteiligt sich die β-Strahlung des eigentlichen Radiums an der gesamten β-Strahlung nur zu einem sehr geringen Prozentsatz. Um diese Strahlung demnach deutlich wahrnehmbar zu machen und vor allem auch, um Störungen durch die α-strahlende Emanation zu vermeiden, wurde für die Aufnahme das Radium durch Auflösen und längeres Auskochen der Lösung von seiner Emanation befreit. Dann wurde es als Karbonat gefällt und das frisch getrocknete Pulver in einer kleinen Glasrinne zur Untersuchung gebracht. Mit großer Vorsicht wurde vermieden, daß Spuren von der Substanz außerhalb der Rinne verstreut wurden. Zur Sicherheit wurden alle Teile außer der Rinne mit Aluminiumblech von solcher Dicke bedeckt, daß es die α-Strahlen nicht hindurchließ.

Für die angestellten Versuche kam 1 mg Radium zur Verwendung. Das erhaltene Bild zeigt Fig. 2 (in zweifacher Vergrößerung). Man erkennt außer dem starken α-Strahl einen kräftigen β-Strahl und etwas weiter abgelenkt einen schwächeren β-Strahl.

Die Bestimmung der Geschwindigkeit dieser beiden β-Strahlen wurde leider nicht mit der wünschenswerten Genauigkeit ausgeführt, da das kostbare Präparat uns nur einmal zur Verfügung stand. Wir mußten uns damit begnügen, unter möglichst gleichen Umständen eine photographische Aufnahme der β-Strahlen des Thoriums $A+D$ zu machen. Da die Geschwindigkeiten dieser letzteren β-Strahlen aus unseren früheren Versuchen bekannt war, konnte man annähernd auch die Geschwindigkeiten der β-Strahlen des Radiums berechnen. Es ergab sich 0,65 und 0,52 Proz. Lichtgeschwindigkeit.

Zusammenfassung.

Es wurde konstatiert, daß auch der aktive Niederschlag des Radiums und das Radium selbst ein magnetisches Linienspektrum ergeben. Die Geschwindigkeit der einzelnen β-Strahlen wurde bestimmt aus der magnetischen Ablenkbarkeit.

Physikalisches Institut } der Universität Berlin.
Chemisches Institut }

(Eingegangen 27. September 1911.)

339, 1911) Messungen der Geschwindigkeit der β-Strahlen des aktiven Niederschlags der Radiumemanation mitgeteilt worden. Als Strahlungsquelle diente ein mit 120 mg Emanation gefülltes Glasröhrchen von 0,04 mm Wandstärke. Die erhaltenen Werte stimmen mit unseren Werten ziemlich gut überein. Es ist aber fraglich, ob ein direkter Vergleich gestattet ist, da Herr Danysz den der Rechnung zugrunde gelegten Wert von $\frac{e}{m_0}$ nicht angibt und die Geschwindigkeitsänderung, die durch die Glaswandung bedingt ist, nicht berücksichtigt.

Über die Temperaturabhängigkeit der Wärmeleitfähigkeit einiger Gase.

(Aus dem Physikalisch-chemischen Institut der Universität Berlin.)

Von A. Eucken.

1. Bei der Untersuchung der physikalischen Eigenschaften sehr verdünnter Gase, die neuerdings[1]) in den Vordergrund getreten ist, ist die mittlere freie Weglänge der Moleküle als so groß anzusehen, daß man die Zusammenstöße der Moleküle untereinander vernachlässigen kann. Der Wert der hierdurch erzielten Vereinfachung wird indessen durch den stark hervortretenden Einfluß des Materials der das Gas begrenzenden Wände herabgesetzt; so zeigte Knudsen[2]), daß der Wärmeaustausch zwischen einem Gas und einer platinierten Platinfläche bedeutend vollkommener ist, als zwischen dem Gas und einer glatten Platinfläche. Der Energieübergang zwischen den Molekülen eines festen Körpers und eines Gases ist daher nicht allein durch spezifische Eigenschaften des Gases, sondern auch des festen Körpers bedingt, eine Beobachtung, die qualitativ sehr einleuchtend, quantitativ aber schwer eindeutig zu analysieren ist[3]). Unter diesen Umständen scheint es günstiger zu sein, zunächst die Frage nach dem Austausch von Energie und Bewegungsgröße zwischen Gasmolekülen unter sich voranzustellen und daher noch einmal zur Untersuchung der inneren Reibung und Wärmeleitung bei höherem Druck zurückzukehren, wo nur die Zusammenstöße der Moleküle untereinander zu berücksichtigen sind, während der Einfluß der Beschaffenheit der Wandungen beseitigt ist. — Das bisherige experimentelle Material besitzt trotz seines Umfanges noch einige Lücken — insbesondere fehlen Wärmeleitfähigkeitsmessungen bei tiefen Temperaturen. Durch ihre Ausfüllung, zu der die im folgenden mitgeteilten Resultate beitragen sollen, darf man einen bessern Überblick über die Erscheinungen und daher vielleicht auch

1) F. Soddy u. A. J. Berry, Proc. Roy. Soc. London (A) **83**, 254 u. **84**, 576, 1910; M. Knudsen, Ann. d. Phys. **31**—**33**.
2) M. Knudsen, Ann. d. Phys. (4) **34**, 638, 1911.
3) Smoluchowski, Ann. d. Phys. (4) **35**, 983, 1911.

Aktiver Niederschlag des Radiums

Fig. 1.

Radium

Fig. 2.

O. v. Baeyer, O. Hahn u. L. Meitner. Magnetische Spektren der β-Strahlen des Radiums.

Verlag von S. Hirzel in Leipzig.

518 *Mr Sargent, The upper limits of energy in the*

To get the range the usual procedure is to plot the intensity as ordinate against the mass per cm.² of paper as abscissa a extrapolate the curve to $I=0$, where the abscissa is the requir value. Another method is to plot log I against x, the mass p cm.², and extrapolate this curve to the point where it cuts the a of abscissa practically at right angles, i.e. where $I \to 0$. Bo methods have been used, and the curves are shown in Fig. 3. T ranges obtained in this way from the three sets of data agree w with each other and give a final value 0·62 gm./cm.² with a probab

Fig. 3. (a) the absorption curve of β-rays from actinium (B + C) at position S_1, poi marked with circles; (b) the corresponding curve for actinium C″ at points marked with dots; (c) the absorption curve of β-rays from actini (B + C) at S_3, points marked with circles. These three curves are plot log I against x. In the case of (c), log I for each point is diminished by 0·8 to prevent confusion with (a) and (b); (d) represents the same results as (but I is now plotted against x.

error of 3 per cent. due to arbitrariness in extrapolation. Obvious this must be interpreted as the range of actinium C″ β-rays.

Towards the end of the range we are concerned with t intensity of the fastest rays, which form a very small fraction the total spectrum. Eventually, this ionization becomes a fe per cent. of the natural leak of the electroscope and measuremen are exceedingly difficult. Therefore, unless the initial activity the source is very large, the range may be underestimated.

Very little is known about the actinium B β-rays. Hahn a Meitner's work [7] gives an average mass absorption coefficie

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Fig. 3. Comparison of the counting rates for various values of the magnetic field without (solid curve) and with (dotted curve) the partition P in the box.

detailed study of this endpoint is shown in Fig. 4. This indicates an upper limit at H 3380±50 gauss. Readings were taken for values of the magnetic field as great as 4500 gauss and no increase in the counting rate above the residual value was found for any field strength above 3400 gauss. This upper limit represents an $H\rho = 6604 \pm 98$ corresponding to an energy of (15.34 ± 0.28) $\times 10^6$ electron volts. The value of ρ used in computing the upper limit was one-half the distance from the side of the wire nearest S_2 to the edge of S_2 nearest the wire, which was 1.954 cm. In computing the energy distribution curve, however, the value used was one-half the distance from the side of the wire nearest S_2 to the center of S_2, a distance of 1.975 cm.

This value of the upper limit is higher than that obtained by such investigators as Madgwick, Sargent, Feather and Champion who obtained values for the endpoint from $H\rho = 5000$ to $H\rho = 5500$. On the other hand Terroux found there was no definite upper limit, while Curie and d'Espine and Yovanovitch found a band extending from $H\rho = 6000$ to $H\rho = 12{,}000$.

The ordinates of the curve of Fig. 2 were divided by H to give the distribution with respect to $H\rho$ and then divided by $\sin\theta$ to give the distribution with respect to the energy which is shown in Fig. 5. This curve has a maximum at 3.87 $\times 10^6$ electron volts. The average value of the energy obtained from this curve by graphical

Fig. 4. Counting rates for various values of the magnetic field at the high energy end of the spectrum.

Fig. 5. Energy distribution curve of the beta rays.

means is 4.71 $\times 10^6$ electron volts. Fig. 6 shows the experimental curve and the curve obtained from the Fermi theory. Both curves have been reduced to unit area and the abscissae are proportional to the energy instead of being actual energies. Since all the energy distribution curves of the simple type of the different radioactive substances are of the same general form, Fermi's theory should apply to all. He, however, points out that the present theory which he has thus far developed does not apply to radium E. The curve of Fig. 6 shows this to be the case.

The endpoint of the β-ray spectrum of radium E has been estimated from the effective range of the particles in various substances such as paper and aluminum and has been found to be between an $H\rho$ of 5000 and 5500 corresponding to an energy of 10.3 to 12.1 $\times 10^6$ electron volts. This method was used by Schmidt, Gray, Douglas, Feather and Sargent. A kink always appears in the absorption curve which is taken to be the effective range of the fastest particles in the substance. From this it is possible to estimate the endpoint of the spectrum. It is quite often rather

One of the first measured spectrum (Hahn and Chadwick just "observed" the spectrum, but not measured it qualitatively) was made by Sargent in 1929 in the article "The Upper Limits of Energy in the β–ray Spectra of Actinium B and Actinium C" and then by Scott in "Energy Spectrum of the Beta-Rays of Radium E". We have included their spectrum below. This result remained a mystery until Pauli proposed the conservation of energy at the prize of introducing a new neutral particle, the neutrino, in a letter he sent to Lise Meitner in 1930 (see the section concerning Pauli for the original and translation of the letter). Then, in 1934, FERMI proposed a model of neutrino, including its physics in the Dirac Equation[2] where he proposed different theoretical spectrum with endpoints (if massive neutrino) or assymptotic (if non-massive one). The aim of the following experiences was then to discover (or not) an endpoint in the spectrum. Then in 1956, Cowan, Reines, Harrison, Kruse and Mc Guire from Rice University succeed to obtain a direct proof of the existence of a neutrino through its interaction on a proton : $\nu + p \to n + e^+$ and published it in "Detection of free neutrino : a confirmation". They obtained an incredible good value of the interaction cross section ($\sigma \simeq 6 \times 10^{-44}\text{cm}^2$). We also include this article in this section. Hahn (1944), Pauli (1945), Fermi (1938), Cowan (1995) and Reines (1995) received the Nobel prize of physics which shows that neutrino is a good source of promotions.

[2] "Versuch einer Theorie der p-Strahlen", Zeitschrift fur Physik 1934, Volume 88, Issue 3-4, pp 161-177

20 July 1956, Volume 124, Number 3212 SCIENCE

Detection of the Free Neutrino: a Confirmation

C. L. Cowan, Jr., F. Reines, F. B. Harrison, H. W. Kruse, A. D. McGuire

A tentative identification of the free neutrino was made in an experiment performed at Hanford (1) in 1953. In that work the reaction

$$\nu_e + p^+ \rightarrow B^+ + n^0 \qquad (1)$$

was employed wherein the intense neutrino flux from fission-fragment decay in a large reactor was incident on a detector containing many target protons in a hydrogenous liquid scintillator. The reaction products were detected as a delayed pulse pair; the first pulse being due to the slowing down and annihilation of the positron and the second to capture of the moderated neutron in cadmium dissolved in the scintillator. To identify the observed signal as neutrino-induced, the energies of the two pulses, their time-delay spectrum, the dependence of the signal rate on reactor power, and its magnitude as compared with the predicted rate were used. The calculated effectiveness of the shielding employed, together with neutron measurements made with emulsions external to the shield, seemed to rule out reactor neutrons and gamma radiation as the cause of the signal. Although a high background was experienced due to both the reactor and to cosmic radiation, it was felt that an identification of the free neutrino had probably been made.

Design of the Experiment

To carry this work to a more definitive conclusion, a second experiment was designed (2), and the equipment was taken to the Savannah River Plant of the U.S. Atomic Energy Commission, where the present work was done (3). This work confirms the results obtained at Hanford and so verifies the neutrino hypothesis suggested by Pauli (4) and incorporated in a quantitative theory of beta decay by Fermi (5).

In this experiment, a detailed check of each term of Eq. 1 was made using a detector consisting of a multiple-layer (club-sandwich) arrangement of scintillation counters and larger tanks. This arrangement permits the observation of prompt spatial coincidences characteristic of positron annihilation radiation and of the multiple gamma ray burst due to neutron capture in cadmium as well as the delayed coincidences described in the first paragraph.

The three "bread" layers of the sandwich are scintillation detectors consisting of rectangular steel tanks containing a purified triethylbenzene solution of terphenyl and POPOP (6) in a chamber 2 feet thick, 6 feet 3 inches long, and 4 feet 6 inches wide. The tops and bottoms of these chambers are thin to low-energy gamma radiation. The tank interiors are painted white, and the solutions in the chambers are viewed by 110 5-inch Dumont photomultiplier tubes connected in parallel in each tank. The energy resolution of the detectors for gamma rays of 0.5 Mev is about 15 percent half-width at half-height.

The two "meat" layers of the sandwich serve as targets and consist of polyethylene boxes 3 inches thick and 6 feet 3 inches by 4 feet 6 inches on edge containing a water solution of cadmium chloride. This provides two essentially independent "triad" detectors, the central scintillation detector being common to both triads. The detector was completely enclosed by a paraffin and lead shield and was located in an underground room of the reactor building which provides excellent shielding from both the reactor neutrons and gamma rays and from cosmic rays.

The signals from a bank of preamplifiers connected to the scintillation tanks were transmitted via coaxial lines to an electronic analyzing system in a trailer van parked outside the reactor building. Two independent sets of equipment were used to analyze and record the operation of the two triad detectors. Linear amplifiers fed the signals to pulse-height selection gates and coincidence circuits. When the required pulse amplitudes and coincidences (prompt and delayed) were satisfied, the sweeps of two triple-beam oscilloscopes were triggered, and the pulses from the complete event were recorded photographically. The three beams of both oscilloscopes recorded signals from their respective scintillation tanks independently. The oscilloscopes were thus operated in parallel but with different gains in order to cover the requisite pulse-amplitude range. All amplifier pulses were stored in long low-distortion delay lines awaiting electronic decision prior to this acceptance.

Manual analysis of the photographic record of an event then yielded the energy deposited in each tank of a triad by both the first and second pulses and the time-delay between the pulses. Using this system, various conditions could be placed on the pulses of the pair comprising an acceptable event. For example, acceptance of events with short time delays (over ranges up to 27 microseconds, depending on the cadmium concentration used) resulted in optimum signal-to-background ratios, while analysis of those events with longer time delays yielded relevant accidental background rates. Spectral analyses of pulses comprising events with short time delays were also made and compared with those with long delays.

This method of analysis was also employed to require various types of energy deposition in the two tanks of a triad. For instance, the second pulse of an event could be required to deposit at least a given energy in each tank, and in addition, maximum and minimum limits could be placed on the total energy of the pulse. Application of criteria such as these assisted in discriminating between events satisfying the physical aspects of a neutrino capture and the various backgrounds experienced. Simultaneous presentation of the three tank outputs on the three beams of the oscilloscopes also permitted rejection of pseudo events due to penetrating cosmic rays, thus utilizing the two triads as shields for one another.

The varying rates observed by changing the response of the system assisted in ascertaining that the gamma rays observed did indeed arise in the target tanks. The efficiency of the system was calibrated in each case by the use of a dissolved copper-64 positron source in the target tanks and by using a plutonium-beryllium neutron source. The neutron calibrations utilized the 9.2-Mev gamma ray emitted by the source as the first pulse of a delayed pair, the second being due to capture of the associated neutron in the cadmium. In addition, secondary calibrations were performed each week using the cosmic ray penetration pile-up peak (7) and standardized pulsers to check for drift in the apparatus. Standard pulses were recorded each day on the oscilloscope cameras to maintain a constant film calibration. Running controls were made of all single and prompt coincidence rates relevant during the experiment as checks for drift or changes in background. Long-term stability of the equipment was easily maintained, and the results of the two independent triad detectors agreed well throughout the experiment.

Experimental Results

Using this equipment near one of the reactors at the Savannah River Plant, the following results were obtained bearing on the reaction expressed by Eq. 1.

1) A reactor-power-dependent signal was observed which was (within 5 percent) in agreement with a cross section for reaction 1 of 6.5×10^{-44} cm^2. The predicted cross section (8) for the reaction, however, is uncertain by ± 25 percent. In one set of runs, the neutrino signal rate was 0.56 ± 0.08 count per hour, and with changed requirements it was 2.88 ± 0.22 counts per hour. The total running time, including reactor-down time, was 1371 hours. The signal-to-background ratio associated with the higher signal rate quoted was about 3 to 1. The neutrino signal was greater than 20 times the accidental background associated with the reactor.

2) A signal rate produced by reaction 1 must be a linear function of the number of protons provided as targets for the neutrinos. This was tested by diluting the light water solution in a target tank with a heavy water solution to yield a resultant proton density of one-half of normal. The neutron detection efficiency measured using the plutonium-beryllium source was essentially unchanged. The reactor signal fell to one-half of its former rate.

3) Reaction 1 states that the first pulse of a delayed pair observed must be due to the annihilation radiation of a positron in the target tank. This would produce one ½-Mev gamma ray entering each detector tank of the triad simultaneously after some degradation in the water target. Events were thus chosen which satisfied these time and spatial conditions. Analysis of the pulse-amplitude spectra of these gamma rays associated with short time-delay events yielded spectra which matched that produced by the dissolved copper-64 source, having a peak at about 0.5 Mev. Spectra obtained for the first pulse of events with long delays (accidental events) were, on the other hand, monotonically decreasing with energy, as was the background spectrum producing the accidental events.

A differential absorption measurement was made using first a ⅜-inch and then a ⅜-inch-thick lead sheet between the target tank and one scintillation tank of a triad. The measured neutron detection efficiency was changed to about 70 percent of its former value in the first case and to about 45 percent in the second. The reactor signal rate fell sharply, however, as required for events with first pulse gamma rays of 0.5 Mev originating in the target tank.

4) The second pulse of the delayed pair signal observed was identified as being due to the capture of a neutron by cadmium in the water target. In addition to the prompt spatial coincidence required and the total-energy limits of 3 to 11 Mev imposed on a pulse for acceptance, analysis of the time-delay spectrum yielded excellent agreement with that expected for the cadmium concentration used in the target water (7). Doubling of the cadmium concentration produced the expected shift in the time-delay spectrum without increasing the signal rate. Removal of the cadmium from the target water resulted in disappearance of the reactor signal.

5) As it is possible for a fast neutron or energetic gamma ray entering the detector from the outside to produce pseudo events with many of the characteristics of true neutrino captures, the observed reactor signal was tested for these effects. A strong americium-beryllium neutron source was used outside the detector shield to produce pseudo signals. Tests of the pseudo signal with the lead sheet described in paragraph 3 resulted in a negligible drop in rate beyond that accounted for by the lowered neutron detection efficiency mentioned in paragraph 3, in contrast with the strong response of the reactor signal. The spectrum of first pulse amplitude of the neutron-produced signal with short time delays fell monotonically with increasing energy, in contrast with the characteristic spectra obtained with both the reactor signal and the dissolved copper-64 positron source.

The results of the heavy water dilution measurement described in paragraph 2 also militates against reactor-produced neutron or gamma rays as the agent producing the signal observed.

Finally, a gross shielding experiment was performed in which the detector shield was augmented by bags of sawdust saturated with water. When stacked, the density of the added shield was 0.5 grams per cubic centimeter, its minimum thickness was 30 inches, and its average thickness was about 40 inches. This absorber would reduce the signal caused by neutrons to about one-tenth of its former rate, depending somewhat upon the direction of the incoming neutrons, and would produce a similar decrease in a signal caused by gamma rays. No decrease was observed in the reactor signal within the statistical fluctuations quoted in paragraph 1.

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References and Notes

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3. We wish to thank the many people at the Los Alamos Scientific Laboratory who assisted in the preparation of the experiment and to mention especially A. R. Rosche, C. W. Johnstone, and A. Bruneau for their help in the chemical and electronic problems. M. F. Warren and R. Jones were invaluable members of the group during both the preparation and field phase of the problem. We also wish to thank the E. I. du Pont de Nemours Company and their personnel at the Savannah River Plant for their constant cooperation and assistance during our stay at the reactor. This work was performed under the auspices of the U.S. Atomic Energy Commission.

4. W. Pauli, in *Rapp. Septième Conseil Phys. Solvay, Brussels 1933* (Gauthier-Villars, Paris, 1934).
5. E. Fermi, *Z. Physik* 88, 161 (1934).
6. Triethylbenzene scintillator, studied first in connection with the Hanford experiment in the search for higher proton densities, was purified by methods developed in collaboration with A. R. Rosche. POPOP, a scintillation spectrum shifter, was developed by F. N. Hayes, Hayes, Rogers, and Dos, *J. Am. Chem. Soc.* 77, 1850 (1955).
7. F. Reines, *et al.*, *Rev. Sci. Inst.* 25, 1061 (1954).
8. This value for the predicted cross section is calculated from the decay of the neutron as observed by J. M. Robson (*Phys. Rev.* 83, 349 (1951)) and the spectrum of beta radiation from fission fragments as measured by C. O. Muehlhause at Brookhaven National Laboratory. We are indebted to Muehlhause for communication of his results in advance of publication.

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also well suited for presentation to students who possess a limited knowledge of the theory of determinants and matrices.

II. PROOF OF THE THEOREM

Theorem. *If A and B are hermitian matrices of order n, A being positive definite, the rank of the matrix $B-\lambda_i A$ is exactly $n-k$, where k is the multiplicity of the root λ_i of the secular equation $|B-\lambda A|=0$*

Let the rank of $B-\lambda_i A$ be $n-r$. Then the equation

$$(B-\lambda_i A)\xi=0 \tag{1}$$

has r independent solutions, say $\xi_1, \cdots, \xi_r$. These solutions can be so chosen[1,2] that they also satisfy the orthonormality relations

$$\xi_i' A \xi_j=\delta_{ij}. \tag{2}$$

By selecting arbitrarily $n-r$ additional vectors,[6] say $\xi_{r+1}, \cdots, \xi_n$ so that the entire set of n vectors is orthonormal in the sense of (2), one obtains a non-singular matrix $X=[\xi_1, \cdots, \xi_n]$ such that $X'AX=I$. In view of this relation and the fact that the first r columns of X satisfy (1), the matrix $X'BX$ has the form

$$X'(B-\lambda A)X = \begin{pmatrix} \lambda_i-\lambda & & & & & \\ & \ddots & & & 0 & \\ & & \lambda_i-\lambda & & & \\ & & & \ddots & & \\ & 0 & & & B_1-\lambda I & \\ & & & & & \ddots \end{pmatrix} \tag{3}$$

where B_1 is hermitian and of order $n-r$. It follows that

Since the roots of the equation $|X'(B-\lambda A)X|=0$ are the same as the roots of the secular equation and, in view of (3), λ_i is a root of the equation $|X'(B-\lambda A)X|=0$ of multiplicity r at least. It follows that r cannot exceed the multiplicity k of the root λ_i for the secular equation. But if r is less than k then λ_i is necessarily a root of the equation $|B_1-\lambda I|=0$. This is impossible since the rank of $X'(B-\lambda_i A)X$ is equal to the rank of $B-\lambda_i A$, which is $n-r$ by assumption, and by (3) the rank of $X'(B-\lambda_i A)X$ is also equal to the rank of $B_1-\lambda_i I$, which is less than $n-r$ if $|B_1-\lambda_i I|=0$. It follows that $r=k$ and the rank of $B-\lambda_i A$ is $n-k$ as asserted.

[6] This device has been used by other authors to prove similar theorems. See, for example, P. R. Halmos, "Finite dimensional vector spaces," *Annals of Mathematics Studies* (Princeton University Press, Princeton, 1942), No. 7, pp. 125–136.

PHYSICAL REVIEW VOLUME 75, NUMBER 7 APRIL 1, 1949

Remarks on the Evolution of the Expanding Universe*,†

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The relativistic energy equation for an expanding universe of non-interconverting matter and radiation is integrated. The above result, together with a knowledge of the physical conditions that prevailed during the element forming process in the early stages of the expansion, is used to determine the time dependences of proper distance as well as of the densities of matter and radiation. These relationships are employed to determine the mean galactic diameter and mass when formed as 2.1×10^8 light years and 3.8×10^7 sun masses, respectively. Galactic separations are computed to be of the order of 10^8 light years at the present time.

I. INTRODUCTION

WITH the experimental and theoretical information now available it is possible to give a tentative description of the structure and evolution of the universe. Investigations of cosmological models of various types have been carried out which explain many of the features of the observed

* The work described in this paper was supported by the Bureau of Ordnance, U. S. Navy, under Contract NOrd-7386.
† A preliminary account of this work was given at the New York meeting of the American Physical Society, January, 1949.

universe.[3] It does not appear to have been possible to complete these speculations principally for lack of sufficient physical data. Recent studies of the origin and relative abundances of the elements have yielded new information concerning the physical state of the universe at the very early time during which the elements were apparently formed.[3,9] According to this theory the ylem (the primordial substance from which the elements were formed) consisted of neutrons at a high density and temperature. Protons were formed by neutron decay, and the successive capture of neutrons led to the formation of the elements. In order to predict the observed relative abundances of the elements, it is necessary to stipulate the magnitude and the time dependence of the temperature, and density of matter during the period of element formation.

On the basis of a simplified version of the neutron capture theory, namely, one which involves the building up of deuterons only, Gamow[9] has examined the state of the universe at early times and traced the evolution of the universe through the formation of galaxies. For reasons which will be discussed later, Gamow's formulation gives rise to certain difficulties.

We have reformulated this problem from a somewhat different point of view, following some of Gamow's basic ideas.[4,9] This reformulation, which is the main purpose of this paper, involves the use of the general non-static relativistic cosmological model together with knowledge of the physical conditions of matter and radiation which prevail now and also those which are required to predict the observed relative abundances of the nuclear species formed during the very early stages of the universe. As a consequence, it is possible to obtain the functional dependence of both the density of matter and radiation on time. On the basis of the foregoing, the formation of galaxies and other cosmological consequences are considered.

II. FORMULATION OF THE PROBLEM

The model of the expanding universe that we shall discuss is one in which there is a homogeneous and isotropic mixture of radiation and matter, assumed to be non-interconverting. This mixture is treated as a perfect fluid. If the pressure due to matter is neglected, one may write the relativistic energy equation for the non-static model in the following form:[3]

$$d[\exp(\tfrac{1}{2}g(t))]/dt=\pm[(8\pi/3)\rho\exp(g(t))-R_0^{-2}]^{\frac{1}{2}}, \tag{1}$$

which is in relativistic units. The cosmological constant Λ is taken equal to zero. In Eq. (1), ρ is the density of mass and the radius of curvature, R, is given by $R=R_0\exp(g(t))$, where $\exp(g(t))$ is the time-dependent factor in the spatial portion of the line element. Now,

$$\exp(\tfrac{1}{2}g(t))=l/l_0=R/R_0, \tag{2}$$

where l is any proper distance, and l_0, the unit of length, together with R_0, must be determined from the boundary conditions for Eq. (1). It should be pointed out that solutions of Eq. (1) involve l/l_0 and not l alone. The density of mass ρ, which determines the geometry of the space, is the sum of the density of matter, ρ_m, and the density of radiation, ρ_r. If matter is to be conserved we must have

$$\rho_m l^3=A=\text{constant}. \tag{3a}$$

Furthermore, if the universal expansion is adiabatic, the temperature, T, must vary[1] as l^{-1}. If one assumes that the universe contains blackbody radiation, then

$$\rho_r l^4=B=\text{constant}. \tag{3b}$$

It is to be noted that energy is not conserved in models of this type. Equations (3a) and (3b) obviously may be written as

$$\rho_r\rho_m^{-4/3}=\text{constant}. \tag{4}$$

It is clear that this relationship must hold throughout the universal expansion and that the density of mass at any time is

$$\rho=\rho_m+\rho_r=Al^{-3}+Bl^{-4}, \tag{5}$$

providing, as stated earlier, there is no interconversion of matter and radiation. If we substitute Eqs. (2) and (3) into Eq. (1), and convert to c.g.s. units, we obtain

$$dl/dt=+[(8\pi G/3)(Al^{-3}+Bl^{-4})l^2-c^2l_0^2/R_0^2]^{\frac{1}{2}}, \tag{6}$$

where the positive sign is taken to indicate expansion and c and G are the velocity of light and the gravitational constant, respectively. Equation (6) can be integrated and the result given in the form

$$t=K_1+K_2^{-1}[\gamma\rho_{r''}+\gamma\rho_{m''}L+K_2L^{2-1}(\gamma\rho_{m''}/2K_2^{\frac{1}{2}})\ln[(\gamma\rho_{r''}+\gamma\rho_{m''}L+K_2L^{2-1}+K_2^{\frac{1}{2}}L+(\gamma\rho_{m''}/2K_2^{\frac{1}{2}})]], \tag{7}$$

where

$$K_1=(\gamma\rho_{m''}/2K_2^{\frac{1}{2}})\ln[(\gamma\rho_{r''})^{\frac{1}{2}}+(\gamma\rho_{m''}/2K_2^{\frac{1}{2}})]-(\gamma\rho_{r''}/K_2^{\frac{1}{2}}), \tag{8}$$

[1] R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Clarendon Press, Oxford, 1934).
[2] G. Gamow, Phys. Rev. **70**, 572 (1946).
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[7] R. A. Alpher and R. C. Herman, Phys. Rev. **74**, 1737 (1948).
[8] G. Gamow, Nature, **162**, 680 (1948).
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H.12 "Remarks on the Evolution of the Expanding Universe" by R.A. Alpher and C. Herman, 1949

In this paper of 1949, Alpher and Herman compute for the first time a predicted temperature for the cosmological background of around 5 degrees kelvin. The confirmation came much later with the discovery of Penzias and Wilson (see following reference). Alpher was the ex-student of Gamow and begin his own work after the "controversy" of the "α, β, γ" paper ("On the Origin of the Chemical Elements that you can find in this appendix too) when Gamow added Bethe as the author for the consonance of the name (Alpher, Bethe, Gamow). He then computed by himself this result earlier, but Gamow seemed to warn him "to concentrate on useful and measurable things". Indeed, at that time, it was impossible to imagine a technology able to measure such a low temperature in the Universe. In the present paper, the authors write end of left column of the third page)

> *(the present density radiation, $\rho_{r''} \simeq 10^{-32} g/cm^3$) corresponds to a temperature now of the order of 5 K. This mean temperature for the universe is to be interpreted as the background temperature which would result from the universal expansion alone. However, the thermal energy resulting from the nuclear energy production in stars would increase this value*

So, according to them, the temperature characteristic to this radiation should be at least 5 degrees Kelvin. The calculation is mainly based on the hypothesis of two initial conditions (the temperature of the nucleosyntesis epoch and the matter density at this epoch) and

one measured one (the matter density today, measured by Hubble). All their hypothesis are clearly explained in the left column of their page 3. from that, using the relation $\rho_r \rho_m^{-4/3}$ =cst, their Eq.(4), Alpher and Herman deduced ρ_r(today) tand thus T_γ(today). But in fact, as you can notice, the existence, or discovery of residual temperature is just witten like a remark in the middle of the ext. Nor the title and even the abstract mention it. The man problem at these time was indeed the Galaxy formation time, as the age of the Universe computed using Hubble data gave a very young universe (around 4 billion years) in contradiction with the recent observation of older galaxies. In this work thay also give the time dependance of radius, matter and radiation density in the early Universe.
It is interesting to remark that in 1961 Gamow published a revised version of his popular book *The Creation of the Universe* (Gamow, 1961). This is the last work of Gamow where he discusses the temperature of interstellar space prior to the discovery of the cosmological radiation (CBR) by Penzias and Wilson in 1965. There is only one place in the book where he mentions the temperature of the CBR :

> *The relation previously stated between the value of Hubble's constant and the mean density of the universe permits us to derive a simple expression giving us the temperature during the early stages of expansion as the function of the time counted from the moment of maximum compression. Expressing that time in seconds and the temperature in degrees (see Appendix, pages 142-143), we have*
>
> $$temperature = 1.5 \times 10^{10}/[times]^{1/2}$$
>
> *Thus when the universe was 1 second old, 1 year old, and 1 million years old, its temperatures were 15 billion, 3 million, and 3 thousand degrees absolute respectively. Inserting the present age of the universe ($t = 10^{17}$ sec) into that formula, we find*
>
> $$T_{present} = 50 \text{ degrees absolute}$$
>
> *which is in reasonable agreement with the actual temperature f interstellar space.*

this temperature is obviously very different from that obtained by Penzias and Wilson in 1965, namely $T = 3.5 \pm 1.0$ K. Even if we take the actual age of the Universe, the approximation of Gamow would give $\simeq 24$ K. This was coming from approximation and over–simplification in the nucleosynthesis process. However it is impressive that from basic principles, Gamow, Alpher and Herman succeed to predict the existence of such a temperature. Their work will be forgotten until 1965.

In Eqs. (7) and (8), $L=l/l_0$, $\gamma=(8\pi G/3)$, $K_2=(c^2/R_0^2)$, and ρ_m and ρ_r are the densities of matter and radiation when $L=1$. In order to integrate Eq. (6) and evaluate the integration constant, it is necessary to specify the parameter R_0 and consequently l_0, which gives the units in which R_0 is measured. Examination of Eq. (6) indicates that R_0 can be determined only if it is possible to specify $[(dL/dt)/L]_{t=t_0}$, ρ_m, and ρ_r at any given time. Since $[(dL/dt)/L]_{t=t_0}$ is the expansion rate of space as determined by Hubble[10] and known, therefore, only at the present time, since ρ_m is also known now, and if we assume that $\rho_m \gg \rho_r$ now, one may evaluate R_0 and K_2. Introducing the value of the present expansion rate of the universe $[(dL/dt)/L]_{t=t_0}=1.8\times10^{-17}$ sec.$^{-1}$, taking $\rho_{m0}=10^{-30}$ g/cm^3 and $l=l_0=10^{28}$ cm, i.e., l_0 is the side of a cube containing one gram of matter now, one obtains $R_0=1.7\times10^{27}(-1)^{\frac{1}{2}}$ cm and $K_2=3.2\times10^{-18}$ sec.$^{-2}$. The constants appearing in Eqs. (7) and (8) involve the present densities of matter and radiation. Clearly, in utilizing Eqs. (7) or (8) one may introduce the density values at any other time providing one specifies a value of L at that time which leads to the present value of the density of matter. For convenience we have chosen l_0 to be the side of a cube containing one gram of matter at the present time, so that $L=1$ now. Furthermore, we have again for convenience assumed that $L=0$ at $t=0$. While Eq. (6) has a singularity at $t=0$ which is physically unreasonable, we have employed the solutions in such a manner that the singularity is of no consequence.

For purposes of computation it is convenient to employ an approximate form for Eq. (7) which is valid for early t, i.e., when

$$L[(\rho_{m0}/\rho_{r0})+(K_2/\gamma\rho_{r0})L^2]<1. \tag{9}$$

The expansion of Eq. (7) which satisfies the above inequality is

$$L=(4\gamma\rho_{r0})^{\frac{1}{2}}t^{\frac{1}{2}}-(\rho_{m0}/6\gamma^{\frac{1}{2}}\rho_{r0}^{\frac{1}{2}})t^{\frac{3}{2}}+(8\gamma/\rho_{r0})^{\frac{1}{2}}t^{2}$$
$$\times[(3\gamma\rho_{m0}^2/4\rho_{r0})-K_2]t^{5/2}+\cdots. \tag{10}$$

The validity of Eqs. (7) or (10) is questionable for very early times, i.e., in the vicinity of the singularity at $t=0$, when the energy of light quanta was comparable to the rest mass of elementary particles. In fact, Einstein[21] has pointed out that there is a difficulty at very early times because of the separate treatment of the metric field (gravitation) and electromagnetic fields and matter in the theory of

However, since we do not concern ourselves with the "beginning" this difficulty is obviated. In addition to the fact that the relativistic energy equation is not valid for very early times, there are the problems of angular momentum of matter in the universe, as well as certain physical factors involved in the formation of the elements, which we cannot handle satisfactorily at present.

In order to utilize the above equations, it is necessary to specify ρ_{m0}, ρ_{r0}, and K_2. While it may appear that one need specify the matter and radiation densities at the present time only, because of Eq. (4), specifying ρ_{m0} and ρ_{r0} is equivalent to specifying ρ_m and ρ_r, these being the densities at a time during the period of element formation. This time is to be specified later. (The primed quantities should not be confused with the running variables.) It must be remembered that the value of K_2 employed is that calculated from the present value of dL/dt.

III. PHYSICAL CONDITIONS DURING THE EXPANSION

Some information is available regarding the values of the matter and radiation densities at the present time and, recently, studies of the relative abundances of the elements have indicated values for these densities prevailing very early in the universe during the period of element formation. Because of Eq. (4) a knowledge of ρ_m and ρ_r during the element forming period together with ρ_{m0} fixes a value for ρ_{r0}, the present radiation density, which is perhaps the least well-known quantity.

In a recent paper Gamow,[6] by considerations which are different than those we have employed, found a set of physical conditions which prevailed during the early stages of the universe. He studied the formation of deuterons only, by the capture of neutrons by protons, taking into account the universal expansion. Equations for the formation of deuterons were integrated from $t=0$, subject to the condition that there were neutrons at the start (unit concentration by weight) and that the final concentration by weight of protons and deuterons was 0.5. This solution determined a parameter α which in turn defined the magnitude of the matter density,[12] $\rho_m=\rho_0 t^{-1}$.

[12] The expression for the parameter α, as given by Gamow in reference 8, has been found to be incorrect (see reference 9). We find that $\alpha t=\rho_m vtl/m$, where $\rho_m=\rho_0 t^{-2/3}$ is the density of

FIG. 1. The time dependence of the proper distance L, the densities of matter and radiation, ρ_m and ρ_r, as well as the temperature, T, are shown for the case where $\rho_{m0}\cong10^{-30}$ g/cm^3, $\rho_{r0}\cong10^{-36}$ g/cm^3, $\rho_{r0}\cong10^{-4}$ g/cm^3, and $\rho_{r0}\cong1$ g/cm^3. [See Eq. (12).]

We believe that a determination of the matter density on the basis of only the first few light elements is likely to be in error. Our experience with integrations required to determine the relative abundances of all elements[6,7] indicates that these computed abundances are critically dependent upon the choice of matter density. Furthermore, all formulations of the neutron capture process which have been made thus far neglect the thermal dissociation of nuclei, which is one of the important competing processes during the element forming period if elements are formed from a very early time.

In order to clarify the difficulties associated with the singularity at $t=0$, we digress here for an examination of the equations employed to describe the formation of the elements. These equations, recently given by the authors,[1] include neutron decay and universal expansion but do not take into account the effects of nuclear evaporation or any processes other than radiative capture of neutrons. In terms of concentrations by weight, $x_j=m_j n_j/\rho_m$, rather than particle concentrations, n_j, Eqs. (6)–(8) of reference 7 may be written as

$$dx_1/dt=\lambda x_2-(p_2\rho_m/m_1)x_2x_0, \tag{11b}$$

and

$$dx_j/dt=j(p_{j-1}\rho_m/m_{j-1})x_{j-1}x_0-j(p_j\rho_m/m_j)x_jx_0,$$
$$j=2,3,\cdots,J, \tag{11c}$$

where x_0, x_1, and x_j are the concentrations by weight of neutrons, protons, and nuclei of atomic weight $2\leq j\leq J$, respectively, m_j the nuclear mass, ρ_m the density of matter, λ the neutron decay constant, and p_j the effective neutron capture volume swept out per second by nuclei of species j. Gamow[4] has solved Eqs. (11a) and (11b) numerically, taking $J=1$, and thereby describing the building up of deuterons only. In general, Eqs. (11) have a singularity at the origin because when $t\to0$, $\rho_m\to\infty$ as t^{-1}. In the approximation used by Gamow this singularity is reduced because a relation for the capture cross section of protons for neutrons is employed which makes $p_1\rho_m(=\sigma_1v\rho_m)$ vary as t^{-1}.

It may be seen readily that Eq. (11c) can be written in the form

$$dx_j/d\tau=(p_{j-1}/\lambda m_{j-1})x_{j-1}-(p_j/\lambda m_j)x_j,$$
$$j=2,3,\cdots,J, \tag{11d}$$

where

$$\tau=\int_{\tau_0}^{\tau}j\rho_m(\tau)x_2(\tau)d\tau, \tag{11e}$$

and

$$\tau=\lambda t.$$

In general, the integrand in Eq. (11e) is singular at $\tau=0$, so that one must take $\tau_0>0$. This implies the choice of an initial time at which the element forming process started. Physically, one may not speak of an initial time because there were competing processes which became unimportant as the neutron capture process became important. Competing processes such as photo-disintegration and nuclear evaporation fall off approximately exponentially with time so that neutron capture would become significant rather rapidly, say in a time of the order of 10^3 seconds. The inclusion of this type of competing process in principle could be handled and would yield a better estimate of the relative abundances of the elements. However, without a better knowledge of cosmology at very early t it does not appear to be possible to avoid the above-mentioned difficulty. Finally, if Eqs. (11a), (11b), and (11c) are solved simultaneously for $J=4$, the remaining equations for $j>4$ are given by Eq.

time for the element forming process. Equations (11) have not yet been solved but are given to illustrate the singularity. So far as we know, any formulation of a theory of element building which includes the type of cosmology discussed will reflect these same difficulties.

In what follows we continue the discussion of the physical conditions employed in the solutions of the relativistic energy equation. The mean density of matter in the universe at the present time has been determined by Hubble[10] to be

$$\rho_{m0}\cong10^{-30}\text{ g/cm}^3. \tag{12a}$$

An estimate of the density of matter, ρ_{m0}, prevailing at the start of the period of element formation is obtained by integration of the equations for the neutron capture theory of the formation of the elements. Integrations in which neutron decay is explicitly included, but in which the expansion of the universe is not included, yield a matter density of 5×10^{-3} g/cm^3. Preliminary investigations of the equations, including the universal expansion, indicate that this density should be increased by a factor roughly of the order of 100 in order that one may correctly determine the relative abundance of the elements with the universal expansion taken into account. In fact, we have numerically integrated for the light elements the complete equations (see Eqs. (11)) with an "initial" density about 100 times the density used in obtaining solutions without the universal expansion.[7] We find that the above factor of ~100 is roughly what might be required. Accordingly, we have taken

$$\rho_{m0}\cong10^{-6}\text{ g/cm}^3. \tag{12b}$$

As discussed elsewhere,[6,7] the temperature during the element-forming process must have been of the order of 10^8–10^{9}°K. This temperature is limited, on the one hand, by photo-disintegration and thermal dissociation of nuclei and, on the other hand, by the lack of evidence in the relative abundance data for resonance capture of neutrons. For purposes of simplicity we have chosen

$$\rho_{r0}\cong1\text{ g/cm}^3, \tag{12c}$$

which corresponds to $T\cong0.6\times10^{9}$°K at the time when the neutron capture process became important.

In accordance with Eq. (4), the specification of ρ_{m0}, ρ_{m0}, and ρ_{r0} fixes the present density of radiation, ρ_{r0}. In fact, we find that the value of ρ_{r0} consistent with Eq. (4) is

$$\rho_{r0}\cong10^{-32}\text{ g/cm}^3, \tag{12d}$$

which corresponds to a temperature now of the order of 5°K. This mean temperature for the universe is to be interpreted as the background temperature which would result from the universal expansion alone. However, the thermal energy resulting from the nuclear energy production in stars would increase this value.

Since we have $\rho_r\gg\rho_m$ at early time the energy relation given in Eq. (6) may be integrated in a simpler form, with the result

$$T=[(32\pi G\rho_r)/(3c^4)]^{\frac{1}{4}}t^{-\frac{1}{2}}\text{°K}$$
$$=1.52\times10^{-2}t^{-\frac{1}{2}}\text{°K}. \tag{13a}$$

The density of radiation, ρ_r, may be found from $\rho_r=(a/c^2)T^4$, or

$$\rho_r=4.48\times10^{2}t^{-2}\text{ g/cm}^3. \tag{13b}$$

These expressions for T and ρ_r at early time are the consequence of the assumption of an adiabatic universe filled with blackbody radiation. It can also be shown that with the densities chosen in Eq. (12) we have for early time

$$\rho_m=1.70\times10^{-8}t^{-1}\text{ g/cm}^3. \tag{13c}$$

Using l and l_0 as already defined, we may determine the constants A and B in Eq. (3). With the densities discussed above we find $A=1$ g and $B=10^{8}$ g cm. These values of A and B fix the dependence of ρ_m and ρ_r on time through $L(=l/l_0)$. Using these values of A and B, we have computed L, ρ_m, ρ_r, and T. These quantities are plotted on a logarithmic scale in Fig. 1. It should be noted in Fig. 1 that all the quantities plotted bear simple relationships with the time to within several orders of magnitude

FIG. 2. The time dependence of the proper distance L, the densities of matter and radiation, ρ_m and ρ_r, as well as the temperature, T, are shown for the case where $\rho_{m0}\cong10^{-6}$ g/cm^3, $\rho_{r0}\cong10^{-36}$ g/cm^3, $\rho_{r0}\cong1.8\times10^{-11}$ g/cm^3, and $\rho_{r0}\cong1$ g/cm^3. [See Eq. (12).]

of the time when the universal expansion changes from one controlled by gravitation to one of free escape. This transition occurs in the region of about 10^{16}–10^{18} sec. Following this transition the quantities L, ρ_m, ρ_r, and T again are simple functions of the time. The relations for large t are as follows:

$$L=K_2^{\frac{1}{2}}t,$$
$$\rho_m=(\rho_{m0}/K_2^{\frac{3}{2}})t^{-3}, \tag{14}$$
$$\rho_r=(\rho_{r0}/K_2^2)t^{-4},$$

and,

$$T=(c^2\rho_{r0}/aK_2^{\frac{1}{2}})t^{-1}.$$

It is to be noted that in the region of transition to free escape the densities of matter and radiation become equal so that, in fact, prior to the transition the expansion is controlled chiefly by radiation and subsequent to the transition by matter. The universe is now in the freely expanding state, and, since the radius of curvature is imaginary, is of the open, hyperbolic type.

In order to study how sensitive this model is to the choice of densities, we have considered the following additional set of density values which satisfy Eq. (4):

$$\rho_m\cong1.78\times10^{-4}\text{ g/cm}^3,$$
$$\rho_r\cong1\text{ g/cm}^3, \tag{15}$$
$$\rho_{r0}\cong10^{-32}\text{ g/cm}^3,$$

and

$$\rho_{r0}\cong10^{-13}\text{ g/cm}^3.$$

The value obtained for ρ_{r0} in this case corresponds to a present mean temperature of about 1°K. The constants A and B are found to be 1 g and 10^{8} g cm, respectively. In Fig. 2 we have plotted the time dependence of the quantities of interest. One finds that the transition occurs at an earlier time than in the previous case, namely, at $\sim10^{9}$ sec., which implies that this universe would have been in a state of free expansion for a considerably longer time. Apparently the behavior of the model is extremely sensitive to the choice of density conditions. However, the simple type of relations for L, ρ_m, ρ_r, and T that were given previously still apply, but with different constants and different regions of validity.

The time at which $\rho_m=\rho_{m0}$ and $\rho_r=\rho_{r0}$ for both sets of densities given in Eqs. (12) and (15) are found from Eq. (13b) to be 6.7×10^{2} seconds, with a corresponding temperature of 0.59×10^{9}°K. We have chosen $\rho_{r0}\cong1$ g/cm^3 in both cases because the corresponding temperature is seen by independent considerations to be that required for the element forming process. As will be seen later, the densities given in Eq. (15) with $\rho_{m0}\cong1.78\times10^{-4}$ g/cm^3 do not yield a satisfactory description of the size and mass of galaxies. On the other hand, as already stated a density $\rho_{m0}\cong100(5\times10^{-6}$ g/cm$^3)$ is apparently enough to overcome the effect of the universal expansion and give the correct relative abundances of the elements. Thus, on the basis of these considerations one is led to the conclusion that when $t\cong6.70\times10^{2}$ sec., and $\rho_r\cong1$ g/cm^3 we have

$$5.0\times10^{-6}\text{ g/cm}^3\cong\rho_{m0}\cong1.8\times10^{-4}\text{ g/cm}^3.$$

While it is not particularly germane to the study reported in this paper, it is interesting to note that one may find the dependence of the universal expansion rate on the time in this type of model. This rate is the percentage change in proper distance per unit time determined by Hubble[10] from the red-shift in spectra of nebulae, and is given in $V=Hd$, where V is the velocity of recession of a nebula at a distance d. In our notation, we have, in general,

$$H=(dL/dt)/L=L^{-1}(c^2/L^2+K_2)^{\frac{1}{2}}. \tag{16}$$

For early time this reduces to

$$H\sim(2t)^{-1}, \tag{16a}$$

and, for late time, to

$$H\sim t^{-1}. \tag{16b}$$

For early and late t, the value of H does not depend upon the choice of densities. However, in the transition region where the functional form of H changes, the manner of change does depend on the existing density conditions. The universal expansion rate is the reciprocal of the age of the universe if measured during the period of free expansion.

IV. THE FORMATION OF GALAXIES

In his discussion of the evolution of the universe, Gamow[8] suggested that galactic formation occurred at the time when the densities of matter and radiation were equal. He assumes that the Jeans' criterion of gravitational instability may be applied at this time and as a consequence derives expressions for the galactic diameter and mass.[13] We have carried out calculations[9] based on Gamow's formulation using the corrected expressions for D and M given in footnote 13. We find that $\rho_m=\rho_r$ when $t_c\cong0.86\times10^{18}$ sec., which is greater than the age of the universe. This arises out of the fact that, in addition to the difficulties with density determinations mentioned earlier, there is involved an extra-

[13] Using the corrected form of α described in footnote 12, we find for the galactic diameter, D, and mass, M, the following corrected expressions according to Gamow's formulation:

$$D=\frac{10e^2t^2h}{3(mc)^{1/2}c^{1/2}}\{\mu_r[1+\alpha_r^{1/2}(e^{1/2}+\alpha^{1/2}e^{3/2})],$$

and

$$M\cong\rho_mD^3=\frac{5^3\pi^{1/2}e^{3/2}h^{3/2}}{2^{3/2}3^3G^{3/2}c^{3/2}m^{1/2}c^{1/2}}\{\mu_r+[\mu_r+\alpha_r^{1/2}(e^{1/2}+\alpha^{1/2}e^{3/2})]^3\},$$

where t_c is the time at which the densities of matter and radiation were equal.

1965ApJ...142..419D

high pressure, such as the zero-mass scalar, capable of speeding the universe through the period of helium formation. To have a closed space, an energy density of 2×10^{-29} gm/cm^3 is needed. Without a zero-mass scalar, or some other "hard" interaction, the energy could not be in the form of ordinary matter and may be presumed to be gravitational radiation (Wheeler 1958).

One other possibility for closing the universe, with matter providing the energy content of the universe, is the assumption that the universe contains a net electron-type neutrino abundance (in excess of antineutrinos) greatly larger than the nucleon abundance. In this case, if the neutrino abundance were so great that these neutrinos are degenerate, the degeneracy would have forced a negligible equilibrium neutron abundance in the early, highly contracted universe, thus removing the possibility of nuclear reactions leading to helium formation. However, the required ratio of lepton to baryon number must be $> 10^9$.

We deeply appreciate the helpfulness of Drs. Penzias and Wilson of the Bell Telephone Laboratories, Crawford Hill, Holmdel, New Jersey, in discussing with us the result of their measurements and in showing us their receiving system. We are also grateful for several helpful suggestions of Professor J. A. Wheeler.

R. H. Dicke
P. J. E. Peebles
P. G. Roll
D. T. Wilkinson

May 7, 1965
Palmer Physical Laboratory
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A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and

free from seasonal variations (July, 1964–April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.

The total antenna temperature measured at the zenith is 6.7° K of which 2.3° K is due to atmospheric absorption. The calculated contribution due to ohmic losses in the antenna and back-lobe response is 0.9° K.

The radiometer used in this investigation has been described elsewhere (Penzias and Wilson 1965). It employs a traveling-wave maser, a low-loss (0.027-db) comparison switch, and a liquid helium–cooled reference termination (Penzias 1965). Measurements were made by switching manually between the antenna input and the reference termination. The antenna, reference termination, and radiometer were well matched so that a round-trip return loss of more than 55 db existed throughout the measurement; thus errors in the measurement of the effective temperature due to impedance mismatch can be neglected. The estimated error in the measured value of the total antenna temperature is 0.3° K and comes largely from uncertainty in the absolute calibration of the reference termination.

The contribution to the antenna temperature due to atmospheric absorption was obtained by recording the variation in antenna temperature with elevation angle and employing the secant law. The result, 2.3° ± 0.3° K, is in good agreement with published values (Hogg 1959; DeGrasse, Hogg, Ohm, and Scovil 1959; Ohm 1961).

The contribution to the antenna temperature from ohmic losses is computed to be 0.8° ± 0.4° K. In this calculation we have divided the antenna into three parts: (1) two non-uniform tapers approximately 1 m in total length which transform between the $2\frac{1}{8}$-inch round output waveguide and the 6-inch-square antenna throat opening; (2) a double-choke rotary joint located between these two tapers; (3) the antenna itself. Care was taken to clean and align joints between these parts so that they would not significantly increase the loss in the structure. Appropriate tests were made for leakage and loss in the rotary joint with negative results.

The possibility of losses in the antenna horn due to imperfections in its seams was eliminated by means of a taping test. Taping all the seams in the section near the throat and most of the others with aluminum tape caused no observable change in antenna temperature.

The backlobe response to ground radiation is taken to be less than 0.1° K for two reasons: (1) Measurements of the response of the antenna to a small transmitter located on the ground in its vicinity indicate that the average back-lobe level is more than 30 db below isotropic response. The horn-reflector antenna was pointed to the zenith for these measurements, and complete rotations in azimuth were made with the transmitter in each of ten locations using horizontal and vertical transmitted polarization from each position. (2) Measurements on smaller horn-reflector antennas at these laboratories, using pulsed measuring sets on flat antenna ranges, have consistently shown a back-lobe level of 30 db below isotropic response. Our larger antenna would be expected to have an even lower back-lobe level.

From a combination of the above, we compute the remaining unaccounted-for antenna temperature to be 3.5° ± 1.0° K at 4080 Mc/s. In connection with this result it should be noted that DeGrasse *et al.* (1959) and Ohm (1961) give total system temperatures at 5650 Mc/s and 2390 Mc/s, respectively. From these it is possible to infer upper limits to the background temperatures at these frequencies. These limits are, in both cases, of the same general magnitude as our value.

We are grateful to R. H. Dicke and his associates for fruitful discussions of their results prior to publication. We also wish to acknowledge with thanks the useful comments and advice of A. B. Crawford, D. C. Hogg, and E. A. Ohm in connection with the problems associated with this measurement.

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H.13 "A Measurement of Excess Antenna Temperature at 4080 Mc/s" by A.A. Penzias and R.W. Wilson, 1965

Penzias and Wilson, two engineers at the Bell Telephone Company, passionated by astrophysical measurements obtained from their company the right to use for "personal purpose" an old radio–telescope in New Jersey. Their first aim was to clean it to be able to make refined radio–measurements of the sky. However, after 2 years of "cleaning duties" (even hunting pigeons to avoid them to nest inside it), they were still measuring a constant background of around 3.5 K (first sentence of their paper). Then, in a conference in Montreal end of 1964 where Penzias attended, he mention this "noise" problem to Bernard Burke from MIT. Some months later, Burke phoned to Penzias, over–excited as he just received a preliminary version of an article by Peebles and Dicke where they predicted a fossil radiation from the Big Bang (in fact, they mainly redo the Alpher, Herman and Gamow work without noticing it). They just add the work of Peebles and Dick to their article to be published in

Astrophysical Journal.
It is relevant to remember here a letter sent by Gamow to Arno Penzias in 1965 (which was curiously dated 1963). This letter was reproduced in Penzias' artcile entitled "Cosmology and microwave astronomy," (Penzias, 1972). We reproduce it here again

Thank you for sending me your paper on 3 K radiation. It is very nicely written except that "early history" is not "quite complete". The theory of, what is now known, as, "primeval fireball", was first developed by me in 1946 (Phys. Ver. 70, 572, 1946; 74, 505, 1948; Nature 162, 680, 1948). The prediction of the numerical value of the present (residual) temperature could be found in Alpher and Hermanns paper (Phys. Ver. 75, 1093, 1949) who estimate it as 5 K, and my paper (KongDansk. Ved. Sels 27 no. 10, 1953) with the estimate of 7 K. Even in my popular book Creation of the Universe (Viking 1952) you can find on p. 42) the formula $T = 1.5 \times 10^{10}/t^{1/2} K$*, and the upper limit of 50 K. Thus, you see the world did not start with almighty Dicke.*
Sincerely,
G. Gamow

Penzias and Wilson received the 1978 Nobel Prize for this work.

The Redshift of Extragalactic Nebulae

by F. Zwicky.

(16.II.33.)

Contents. This paper gives a representation of the main characteristics of extragalactic nebulae and of the methods which served their exploration. In particular, the so called redshift of extragalactic nebulae is discussed in detail. Different theories which have been worked out in order to explain this important phenomenon will be discussed briefly. Finally it will be indicated to what degree the redshift promises to be important for the study of penetrating radiation.

§1. Introduction.

It has been known for a long time that there exist in space certain objects which, when observed with small telescopes, appear to be quite fuzzy, self shining spots. These objects have different structures. Often they are spherical, often elliptical, and many of them have a spiral-like appearance, and are therefore occasionally called spiral nebulae. Thanks to the enormous resolving power of modern giant telescopes astronomers were able to establish that these nebulae lie beyond the limits of our own Milky Way. Photographs made with the Hundred-Inch-Telescope on Mount Wilson reveal that these nebulae are stellar systems, similar to our own Milky Way System. The extragalactic nebulae are on the whole homogeneously distributed over the sky and are, as could be shown, also homogeneous in space. They are seen as individuals or grouped in clusters. The following lines aim to give a short account of the most important characteristics and a description of the methods which made it possible to establish them.

play an essential role. Assuming effects which have their origin in direct spatial interaction between light and matter shows that this cannot explain the transparency of intergalactic space.

I proposed then another possible effect, which can however hardly be observed on earth, for the existence of which, nevertheless, some theoretical reasons can be given. According to the theory of relativity there corresponds to each photon or light quantum of frequency ν a gravitational as well as an inertial mass $h\nu/c^2$. Thus there is an interaction (attraction) between light and matter. If the photon is emitted resp. absorbed at two points P_1 and P_2 which have the *same* gravitational potential, it loses on the way from P_1 to P_2 a certain momentum and gives this to matter. The photon gets redder. This effect, which could be called gravitational friction, is caused mainly by the finite velocity of gravitational interaction. Its amount depends on the average density of matter and on its distribution. The redshift $\Delta\lambda/\lambda$ in this case depends not only on the distance but also on the distribution of matter. Explorations to test this second conclusion are being done now.

Finally it has to be said that none of the proposed theories is satisfying. All of them have been developed on a most hypothetical basis, and none of them has succeeded to uncover any new physical relationships.

§5. Remarks concerning the dispersion of velocities in the Coma nebular cluster.

As the data in §3 show, there are in the Coma cluster differences in velocity of at least 1500 to 2000 km/sec. In the context of this enormous variation of velocities the following considerations can be made:

1. Under the supposition that the Coma system has reached, mechanically, a stationary state, the Virial Theorem implies

$$\bar{\epsilon}_k = -\tfrac{1}{2}\bar{\epsilon}_p, \tag{4}$$

where $\bar{\epsilon}_k$ and $\bar{\epsilon}_p$ denote average kinetic and potential energies, e.g. of the unit of mass in the system. For the purpose of estimation we assume that the matter in the cluster is distributed uniformly in space. The cluster has a radius R of about one million light-years (equal to 10^{24} cm) and contains 800 individual nebulae with a mass of each corresponding to 10^9 solar masses. The mass M of the whole system is therefore

$$M \sim 800 \times 10^9 \times 2 \times 10^{33} = 1.6 \times 10^{45} \text{ g}. \tag{5}$$

H.14 "The redshift of extragalactic nebulae" by F. Zwicky, 1933 and "Rotation of the Andromeda nebula from a spectroscopic survey of emission regions" by V. Rubin and W. Ford, 1969.

I decided to present this article in the Magic Monday Journal CLub of Orsay because it was never clear to myself (and to several members of the audience as I noticed) who really made the first step toward the presence of dark matter in the Universe. After a quick pool in the room, 90% of the attendees answered Zwicky, and 10% Rubin. I include here the main pages of these two articles in this section.

Surprisingly, the article of Zwicky "The redshift of extragalactic nebulae" is not at all a paper concerning dark matter. It is a review of the recent discoveries concerning different clusters of galaxy (a good review by the way). It was published in German in *Helvetica*. The only remark concerning the dark matter is in the last section before conclusion entitled "Remarks concerning the dispersion of velocities in the Coma nebular cluster". He took the more recent data from the Coma cluster and compute, using the virial theorem, the mean

This implies for the total potential energy Ω:

$$\Omega = -\frac{3}{5}\Gamma\frac{M^2}{R} \tag{6}$$

Γ = Gravitational constant

or

$$\overline{\varepsilon}_p = \Omega/M \sim -64 \times 10^{12}\ \mathrm{cm^2 s^{-2}} \tag{7}$$

and then

$$\overline{\varepsilon}_k = \overline{v^2}/2 \sim -\overline{\varepsilon}_p/2 = 32 \times 10^{12}\ \mathrm{cm^2 s^{-2}}$$

$$\left(\overline{v^2}\right)^{1/2} = 80\ \mathrm{km/s}. \tag{8}$$

In order to obtain the observed value of an average Doppler effect of 1000 km/s or more, the average density in the Coma system would have to be at least 400 times larger than that derived on the grounds of observations of luminous matter.[8] If this would be confirmed we would get the surprising result that dark matter is present in much greater amount than luminous matter.

2. One could also assume that the Coma system is not in stationary equilibrium, but that all available energy has the form of kinetic energy. Then we would have

$$\overline{\varepsilon}_k = -\overline{\varepsilon}_p, \tag{9}$$

This assumption thus allows to get rid of a factor of only 2 compared to 1., and the necessity of an enormously large density of dark matter stays the same.

3. Let the average density in the Coma cluster be wholly determined by the presence of luminous matter (mass M above). Then the large velocities cannot be determined by considerations of type 1. or 2. If the observed velocities are indeed real ones, the Coma system should disperse in the course of time. The result of this expansion would be 800 individual nebulae (field nebulae), which, as follows from 2., would have eigenvelocities of the original order of magnitude (1000 to 2000 km/sec). From analogies it is to be expected that field nebulae with such large eigenvelocities would be observable also in the state of development the world is in today. This conclusion however

[8] In order of magnitude this would agree with the view of Einstein and de Sitter discussed in §4.

1970ApJ...159..379R

The Astrophysical Journal, Vol. 159, February 1970
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ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC SURVEY OF EMISSION REGIONS*

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Received 1969 July 7; revised 1969 August 21

ABSTRACT

Spectra of sixty-seven H II regions from 3 to 24 kpc from the nucleus of M31 have been obtained with the DTM image-tube spectrograph at a dispersion of 135 Å mm^{-1}. Radial velocities, principally from Hα, have been determined with an accuracy of ± 10 km sec^{-1} for most regions. Rotational velocities have been calculated under the assumption of circular motions only.

For the region interior to 3 kpc where no emission regions have been identified, a narrow [N II] λ6583 emission line is observed. Velocities from this line indicate a rapid rotation in the nucleus, rising to a maximum circular velocity of $V = 225$ km sec^{-1} at $R \approx 400$ pc, and falling to a deep minimum near $R = 2$ kpc.

From the rotation curve for $R \leq 24$ kpc, the following disk model of M31 results. There is a dense, rapidly rotating nucleus of mass $M = (6 \pm 1) \times 10^9\ M_\odot$. Near $R = 2$ kpc, the density is very low and the rotational motions are very small. In the region from 500 to 1.4 kpc (most notably on the southeast minor axis), gas is observed leaving the nucleus. Beyond $R = 4$ kpc the total mass of the galaxy increases approximately linearly to $R = 14$ kpc, and more slowly thereafter. The total mass to $R = 24$ kpc is $M = (1.85 \pm 0.1) \times 10^{11}\ M_\odot$; one-half of it is located in the disk interior to $R = 9$ kpc. In many respects this model resembles the model of the disk of our Galaxy. Outside the nuclear region, there is no evidence for noncircular motions.

The optical velocities, $R > 3$ kpc, agree with the 21 cm observations, although the maximum rotational velocity, $V = 270 \pm 10$ km sec^{-1}, is slightly higher than that obtained from 21-cm observations.

I. INTRODUCTION

The Andromeda Nebula offers a unique opportunity for the study of the dynamics of a galaxy. Because of its large angular extent on the sky, it is possible to observe individual emission regions spectroscopically and, in principle, to map the velocity field of a galaxy which resembles our Galaxy. In practice, however, the extreme faintness of the regions makes detailed optical studies very time-consuming, if not impossible. The very early observations of M31 were restricted to the bright central regions. In 1914, from observations made with the Lowell Observatory 24-inch refractor, V. M. Slipher (1914) reported inclined absorption lines across the nucleus of M31. From absorption lines, Pease (1918) measured the rotation out to 150 seconds of arc from the nucleus on a single plate of 79 hours' exposure, taken in 1917 during the months of August, September, and October. The Mount Wilson 60-inch reflector was used, the dispersion[1] was 192 Å mm^{-1}, and the resulting values are of sufficiently high accuracy to warrant serious scientific consideration today.

The first large-scale study of motions in M31 was made by Babcock (1939) with the 36-inch Crossley telescope at Lick Observatory. His observations included spectra of the unresolved central nucleus (exposure times 25 minutes to 22 hours) and a few spectra of

* *Contributions from the Kitt Peak National Observatory*, No. 492.

† Visiting Astronomer 1966, 1967, 1968, Lowell Observatory; and Visiting Astronomer 1967, 1968, Kitt Peak National Observatory.

‡ Operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

[1] Drs. N. U. Mayall and R. Minkowski have pointed out that there is a misprint in Pease's paper, and the observed wavelength range is actually 3930–4950 Å, not 4930–4950 Å as published.

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PLATE I

FIG. 1.—Identification chart for emission regions in M31 for which velocities have been obtained. Palomar 48-inch Schmidt ultraviolet photograph, 103aO plate + UG 1 filter, courtesy of Dr. S. van den Bergh.

RUBIN AND FORD (*see* page 380)

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24 kpc are 1.66×11^{11} $M_\odot$ (low minimum) and 1.67×11^{11} $M_\odot$ (higher minimum); those for the mass in the nucleus to $R = 1$ kpc are 5.2×10^9 $M_\odot$ and 6.2×10^9 $M_\odot$, respectively. When these values are increased 10 percent to compensate for the assumption of a flat disk (Brandt 1960), the total mass is 1.8×10^{11} $M_\odot$ out to 24 kpc. Also shown in Figure 10 are the variation in the mass surface density as a function of distance from the center, and the variation in the angular velocity, V/R, as a function of R. Note that the solution with the high inner minimum has a positive mass density everywhere. The solution with the low minimum has a negative surface density near $R = 1$ kpc. We take this to mean only that the density at this distance is vanishingly small for the low-minimum model.

FIG. 9.—Rotational velocities for OB associations in M31, as a function of distance from the center. *Solid curve*, adopted rotation curve based on the velocities shown in Fig. 4. For $R \leq 12'$, curve is fifth-order polynomial; for $R > 12'$, curve is fourth-order polynomial required to remain approximately flat near $R = 120'$. Dashed curve near $R = 10'$ is a second rotation curve with higher inner minimum.

Various other rotation curves for the data in Figures 3 and 4 have been formed, all from least-squares solutions, with polynomials of third, fourth, or sixth order. In Figure 11 we show, superimposed, the fourteen rotation curves from the polynomial representations. The various mass determinations from these rotation curves are listed in Table 4. Successive columns list the order of the polynomial, the resulting total mass, 1.1 times the mass, and the value of the maximum distance to which the mass has been determined. The final columns list the depth of the inner minimum, and notes concerning the solutions.

It is apparent from the calculations that there is only a small spread in total mass out to $R = 24$ kpc from all fourteen solutions. The shaded regions in Figure 12 indicate the range of masses which results from the fourteen rotation curves, as well as the range of surface densities. For the mass out to $R = 24$ kpc, a value of $M = (1.68 \pm 0.1) \times 11^{11}$ $M_\odot$ lies midway between all values. When this is increased 10 percent to compensate for the disk approximation, we obtain a mass $M = (1.85 \pm 0.1) \times 11^{11}$ $M_\odot$ out to $R = 24$ kpc; the error is estimated from the total range in values. For the entire galaxy,

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velocity of the coma. He obtained 80 km/s and concluded :

"If this would be confirmed we would get the surprising result that dark matter is present in much greater amount than luminous matter".

It was the first time a computation was made, based on measured data, giving the hint of the presence of dark matter in the Universe. However, the real measurements of rotation curves of galaxies was made by V. Rubin in 1969 and is also presented in this section. She first concentrated on the Andromeda nebula and, following the advice of Minkowski, published it before taking data on other galaxies. She never mentioned the word "dar matter" in the text. But, following the Figure.9 that I also included above, she wrote :

"Extrapolation beyond that point is clearly a matter of taste"

where "that point" means the last visible point that Rubin observed. She did not want to make any assumptions but of course believed strongly that the rotation curve should be extrapolate below this last visible point pointing to dark matter presence. A lot of people in the scientific community is still surprise that V. Rubin still never received the Nobel Prize.

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IX. *A Determination of the Deflection of Light by the Sun's Gravitational Field from Observations made at the Total Eclipse of May 29, 1919.*

By Sir F. W. Dyson, *F.R.S., Astronomer Royal, Prof.* A. S. Eddington, *F.R.S., and Mr.* C. Davidson.

(*Communicated by the Joint Permanent Eclipse Committee.*)

Received October 30, - Read November 6, 1919.

[Plate 1.]

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I. Purpose of the Expeditions.

1. The purpose of the expeditions was to determine what effect, if any, is produce by a gravitational field on the path of a ray of light traversing it. Apart from possib surprises, there appeared to be three alternatives, which it was especially desired t discriminate between—

(1) The path is uninfluenced by gravitation.

(2) The energy or mass of light is subject to gravitation in the same way as ordinar matter. If the law of gravitation is strictly the Newtonian law, this leads t an apparent displacement of a star close to the sun's limb amounting to 0″·8 outwards.

(3) The course of a ray of light is in accordance with Einstein's generalised relativit theory. This leads to an apparent displacement of a star at the limb amounti to 1″·75 outwards.

In either of the last two cases the displacement is inversely proportional to the distan of the star from the sun's centre, the displacement under (3) being just double th displacement under (2).

It may be noted that both (2) and (3) agree in supposing that light is subject to gravit tion in precisely the same way as ordinary matter. The difference is that, whereas (assumes the Newtonian law, (3) assumes Einstein's new law of gravitation. The sligh

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but having regard to possible systematic error, probably a quarter weight would more nearly represent the true value.

This determination is ignored in the subsequent discussion.

36. It is easy to calculate the effects of any errors in the adopted scale, orientation, &c., on the final result (deflection at the limb). We give some illustrations.

An error in the adopted scale of y of 10 units (in the fifth place of decimals) would lead to an error 0″·68 in the result from either plate. Thus the probable error ±2·1 in the determination of e' gives a probable error ±0″·14 in the final result; or, if we adopted the largest (rather discordant) value found for e' instead of the mean, we should reduce the result by 0″·52.

An error of 10 units in the orientation gives an error in the result of 0″·45 for plate X, and 0″·22 for Plate W. It is therefore of less importance, and further it is not likely to be systematic.

Errors in the measurement of x only affect the result through the orientation. For Plate X, a probable error of ±0″·20 in the x-measures would give an error ±4·0 in the orientation, leading to an error ±0″·18 in the result; whereas an error of the same magnitude in the y measures gives directly an error ±0″·35 in the result. For Plate W, the probable error of ±0″·20 in x gives an error ±3·5 in the orientation and ±0″·08 in the result, compared with ±0″·38 for similar inaccuracy in y. It is particularly fortunate that the x-measures are so unimportant for Plate W, because, as already mentioned, the images trailed on that plate.

Finally, it will be remembered that in order not to commit ourselves to the Einstein hypothesis prematurely we neglected the correction $\frac{1}{3}E$, in determining the orientation. This will make a difference of 0″·029 in the results from Plate W and 0″·092 from Plate X. The effect is that the deduced deflection needs to be decreased, and the mean correction −0″·06 should be applied to the mean result obtained, or rather, to make the adopted deflection for x consistent with the deduced value from y, the correction needed is −0″·04.

Discussion of the Results.

37. The four determinations from the two eclipse plates are

X − G	1″·94
X − H	1″·44
W − D	1″·53
W − I	1″·67
giving a mean of	1″·65.

They evidently agree with Einstein's predicted value 1″·75.

H.15 "A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May 29, 1919" by F.W. Dyson, A.S. Eddington and C. Davidson, 1919

In 1919 an expedition was sent to Principe Island and Sobral in northern Brazil to observe the eclipse of 29 May 1919 which path of totality ran across the Atlantic ocean from Brazil to West Africa. The position of the sun at that time was in front of a prominent grouping of 13 stars known as the Hyades. When totality occured, the sky behind the Sun has been glittering with bright stars whose position could be measured. Dyson began to investigate possible observing sites. One expedition was led by Eddington, and was to travel to the island of Principe off the coast of Spanish Guinea in West Africa, and the other led by Andrew Crommelin to Sobral. As precised in the introduction of the paper, there was 3 possibilities: or the light does not deviate, or deviates following a Newtonian law (angle $\theta_N = 2GM/rc^2 \simeq 0.87$ arc second for the sun) or deviates following the Einstein law of general relativity ($\theta_E = 4GM/rc^2 \simeq 1.74$ arc second). The factor 2 between Newtonian and Einstenian value is coming from a mistake Einstein made in 1911, forgetting the contribution

DETERMINATION OF DEFLECTION OF LIGHT BY THE SUN'S GRAVITATIONAL FIELD. 33

The result from declinations is about twice the weight of that from right ascension so that the mean result is

1″·98

with a probable error of about ±0″·12.

The Principe observations were generally interfered with by cloud. The unfavourabl circumstances were perhaps partly compensated by the advantage of the extremel uniform temperature of the island. The deflection obtained was

1″·61.

The probable error is about ±0″·30, so that the result has much less weight tha the preceding.

Both of these point to the full deflection 1″·75 of EINSTEIN's generalised relativit theory, the Sobral results definitely, and the Principe results perhaps with some u certainty. There remain the Sobral astrographic plates which gave the deflection

0″·93

discordant by an amount much beyond the limits of its accidental error. For t reasons already described at length not much weight is attached to this determinatio

It has been assumed that the displacement is inversely proportional to the distan from the sun's centre, since all theories agree on this, and indeed it seems clear fro considerations of dimensions that a displacement, if due to gravitation, must follow th law. From the results with the 4-inch lens, some kind of test of the law is possib though it is necessarily only rough. The evidence is summarised in the following tab and diagram, which show the radial displacement of the individual stars (mean from a the plates) plotted against the reciprocal of the distance from the centre. The displac ment according to EINSTEIN's theory is indicated by the heavy line, according to t Newtonian law by the dotted line, and from these observations by the thin line.

RADIAL Displacement of Individual Stars.

Star.	Calculation.	Observation.
	″	″
11	0·32	0·20
10	0·33	0·32
8	0·40	0·56
5	0·53	0·61
4	0·75	0·81
2	0·85	0·97
3	0·98	1·02

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of the space curvature in the weak field limit[3] :

$$ds^2_{GR} = \left(1 + \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right) dl^2. \tag{H.6}$$

Not only the measurement is easier than in the Newtonian case and also offered the possibility of a definitive test of the theory since it differs from the Newtonian value.In 1912 an Argentinian expedition and in 1914 a German one was sent without success. Eddington measured a deviation of 1.98 in Sorbal and 1.61 in Principe. As concluded Eddington: "both of these point to the full deflection 1.75 of Einstein generalised relativity theory". Einstein himself later down-played his excitement in a puckish remark about his friend and colleague, the physicist Max Planck:

He was one of the finest people I have ever known . . . but he didnt really understand physics, [because] during the eclipse of 1919 he stayed up all night to see if it would confirm the bending of light by the gravitational field. If he had really understood [the general theory of relativity], he would have gone to bed the way I did.

[3]You can find a more detailed analysis of the general relativity effect of the deflection of the light in the section (A.1.1)

Appendix I

Glossary

Parsec

The parsec (parallax of one arcsecond; symbol: pc) is a unit of length, equal to just under 31 trillion kilometres (about 19 trillion miles), 206265 AU, or about 3.26 light-years. The parsec measurement unit is used in astronomy. It is defined as the length of the adjacent side of an imaginary right triangle in space. The two dimensions that specify this triangle are the parallax angle (defined as 1 arcsecond) and the opposite side (defined as 1 astronomical unit (AU), the distance from the Earth to the Sun)

Virial radius

The virial radius is the radius of a sphere that encompasses around 200 times the critical density. The virial radius for the Milky Way is 428 kpc.

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