# From Statistical Relational to Neuro-Symbolic Artificial Intelligence

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#### **Abstract**

Neuro-symbolic and statistical relational artificial intelligence both integrate frameworks for learning with logical reasoning. This survey identifies several parallels across seven different dimensions between these two fields. These cannot only be used to characterize and position neuro-symbolic artificial intelligence approaches but also to identify a number of directions for further research.

## 1 Introduction

The integration of learning and reasoning is one of the key challenges in artificial intelligence and machine learning today, and various communities have been addressing it. That is especially true for the field of neuro-symbolic computation (NeSy) [Besold et al., 2017; Garcez et al., 2019], where the goal is to incorporate symbolic reasoning into neural networks. NeSy already has a long tradition, and it has recently attracted a lot of attention from various communities (cf. e.g., the keynotes of Yoshua Bengio and Henry Kautz on this topic at AAAI 2020). Many approaches to NeSy aim at extending neural networks with logical reasoning.

Another domain that has a rich tradition in integrating learning and reasoning is that of statistical relational learning and artificial intelligence (StarAI) [Getoor and Taskar, 2007; De Raedt *et al.*, 2016]. But rather than focusing on how to integrate logic and neural networks, it is centred around the question of how to integrate logic with probabilistic graphical models. Despite the common interest in combining logic or symbolic reasoning with a basic paradigm for learning, i.e., probabilistic graphical models or neural networks, it is surprising that there are not more interactions between these two fields.

This discrepancy is the key motivation behind this survey: it aims at pointing out the similarities between these two endeavours and in this way stimulate more crossfertilization. In doing so, we start from the literature on StarAI because, arguably, there is more consensus on what the key concepts, challenges and issues are in StarAI than in NeSy (cf. the number of tutorials and textbooks on related topics such as [Russell, 2015; De Raedt *et al.*, 2016]

but see also [Besold et al., 2017; Garcez et al., 2019]). It turns out that essentially the same issues and techniques that arise in StarAI have to be addressed in NeSy as well. The key contribution of this survey is that we identify a set of seven dimensions that these fields have in common and that can be used to categorize both StarAI and NeSy approaches. These seven dimensions are concerned with (1) directed vs undirected models, (2) grounding vs proof based inference, (3) integrating logic with probability and/or neural computation, (4) logical semantics, (5) learning parameters or structure, (6) representing entities as symbols or sub-symbols, and, (7) the type of logic used. We provide evidence for our claim by positioning a wide variety of StarAI and NeSy systems along these dimensions and pointing out analogies between them. This, in turn, allows us to identify interesting opportunities for further research, by looking at areas across the dimensions that have not seen much work vet. Of course, there are also important differences between StarAI and NeSy, the most important one being that the former operates more at the symbolic level, lending itself naturally to explainable AI, while the latter operates more at the sub-symbolic level, lending itself more naturally for computer vision and natural language processing.

Unlike some other recent surveys or perspectives on neuro-symbolic computation [Besold *et al.*, 2017; Garcez *et al.*, 2019], the present survey limits itself to a logical and probabilistic perspective, which it inherits from StarAI, and to developments in neuro-symbolic computation that are consistent with this perspective. Furthermore, it focuses on representative and prototypical systems rather than aiming at completeness (which would not be possible given the page limitations). At the same time, unlike many early approaches to neuro-symbolic computation (see [Bader and Hitzler, 2005] for an overview), which focused more on modeling issues and principles, we focus on approaches that are also used for learning.

The following sections of the paper each describe a dimension. We summarize various neuro-symbolic approaches along these dimensions in Table 1. Furthermore, for ease of writing, the table mentions for each system the key reference (so that we do not always have to repeat these references).

### 2 Directed vs undirected

Within the graphical model community there is a distinction between the *directed* and *undirected* graphical models [Koller and Friedman, 2009], which has led to two distinct types of StarAI systems. The first generalizes directed models, and resembles Bayesian networks; the second generalizes undirected models like Markov networks or random fields. The key difference between the two is that the first class of models indicates a natural direction (sometimes the term "causal" is used) between the different random variables, while the second one does not.

StarAI, the first category includes known representations such plate notation [Koller and Friedman, 2009], probabilistic relational models (PRMs) [Friedman et al., 1999], probabilistic logic programs (PLPs) [De Raedt and Kimmig, 2015], and Bayesian logic programs (BLPs) [Kersting and De Raedt, 2007]. Today the most typical and popular representatives of this category are the probabilistic (logic) programs. The second category includes Markov Logic Networks (MLNs) [Richardson and Domingos, 2006] and Probabilistic Soft Logic (PSL) [Bach et al., 2017]. They essentially specify a set of weighted constraints, clauses or formulae.

From a logical perspective, the difference amounts to using a form of definite clauses (as in the programming language Prolog) versus the use of full clausal logic or even first order logic. On the one side, a definite clause is an expression of the form  $h \leftarrow b_1 \land ... \land b_n$  where h and the  $b_i$  are logical atoms of the form  $p(t_1,...,t_m)$ , with p being a predicate of arity m and the  $t_i$  being terms, that is, constants, variables, or structured terms of the form  $f(t_1,...,t_n)$ , where f is a functor and the  $t_i$  are again terms. On the other side, full clausal logic also allows for formulae of the form  $h_1 \lor ... \lor h_m \leftarrow b_1,...,b_n$ .

In the first type of rule the direction of the implication indicates, just like the direction of the arrows in a Bayesian network, what can be inferred from what. In the second type of rule, this relationship is blurred because of the disjunction in the head of the rule, which allows multiple conclusions for the same premises. This explains why the first type of rule is more directly used for inference, while the second more as a constraint. It also reflects the kind of knowledge that the user has about the problem. With directed models, one can express that a set of variables has a direct "causal" influence on another one, while with undirected ones one expresses a kind of (soft) constraints on a set of variables, that is, that the variables are related to one another.

Borrowing this view from StarAI, we can devise a first dimension for neuro-symbolic approaches, which relies entirely on the logical perspective outlined above.

The first category includes NeSy systems based on Prolog or Datalog, such as Neural Theorem Provers (NTPs) [Rocktäschel and Riedel, 2017], NLProlog [Weber et al., 2019], DeepProbLog [Manhaeve et al., 2018] and DiffLog [Si et al., 2019]. These systems retain the directed nature of logical inference as they exploit backward chaining. Lifted Relational

Neural Networks (LRNNs) [Šourek et al., 2018] and ∂ILP [Evans and Grefenstette, 2018] are other examples of non-probabilistic directed models, where definite clauses are compiled into a neural network architecture in a forward chaining fashion. The systems that imitate logical reasoning with tensor calculus, Neural Logic Programming (NeuralLP) [Yang et al., 2017] and Neural Logic Machines (NLM) [Dong et al., 2019], are likewise instances of directed logic.

The undirected NeSy approaches consider logic as a constraint on the behaviour of a predictive model. A large group of approaches, including Semantic Based regularization (SBR) [Diligenti *et al.*, 2017] and Semantic Loss (SL) [Xu *et al.*, 2018], exploits logical knowledge as a soft constraint over the hypothesis space in a way that favours solutions consistent with the encoded knowledge. SBR implements predicates as neural networks and translates the provided logical formulas into a real valued regularization by means of fuzzy logic, while SL uses marginal probabilities of the target atoms to define the regularization term and relies on arithmetic circuits [Darwiche, 2011] to evaluate it efficiently.

Another group of approaches, including Logic Tensor Networks (LTN) [Donadello et al., 2017], Neural Markov Logic Networks (NMLN) [Marra and Kuželka, 2019] and Relational Neural Machines (RNM) [Marra et al., 2020] extend MLNs, allowing either predicates (LTN) or factors (NMLN and RNM) to be implemented as neural architectures. Finally, [Rocktäschel et al., 2015; Demeester et al., 2016] compute ground atoms scores as dot products between relation and entities embeddings; implication rules are then translated into a logical loss by means of continuous relaxation of the implication operator.

## 3 Grounding vs proofs

From a logical perspective there is a model-theoretic and a proof-theoretic perspective to inference. This is clear when looking at the difference between Answer Set Programming and the programming language Prolog. In the model theoretic perspective, one first grounds out the clauses in the theory and then calls a SAT solver (possibly after breaking cycles), while in a proof theoretic perspective, one performs a sequence of inference steps in order to obtain a proof.

Grounding is the step whereby a clause c (or formula) containing variables  $\{V_1,...,V_k\}$  is replaced by all instances  $c\theta$  where  $\theta$  is a substitution  $\{V_1=c_1,...V_k=c_k\}$  and the  $c_i$  are constants (or other ground terms) appearing in the domain. The resulting clause  $c\theta$  is that obtained by simultaneously replacing all variables by the corresponding constants. Usually the grounding process is optimised in order to obtain only those ground clauses that are relevant for the considered inference task.

These two perspectives carry over to the StarAI perspective. Many StarAI systems use the logic as a kind of template to ground out the relational model in order to obtain a grounded model and perform inference. This grounded

model can be a graphical model, or alternatively, it can be a ground weighted logical theory on which traditional inference methods apply, such as belief propagation or weighted model counting. This is used in well known systems such as MLNs, PSL, BLPs, and PRMs. Some systems like PRMs and BLPs also use aggregates or combining rules in their knowledge base construction approach. The idea then is to combine multiple conditional probability distributions into one using, e.g., noisy-or.

Alternatively, one can follow a proof or trace based approach to define the probability distribution and perform inference. This is akin to what happens in probabilistic programming (cf. also [Russell, 2015]), in StarAI frameworks such as PLPs, probabilistic databases [Van den Broeck et al., 2017] and probabilistic unification based grammars such as Stochastic Logic Programs (SLPs) [Muggleton, 1996]. The idea is that a proof will form the basis for probabilistic inference. Just like pure logic supports the model-theoretic and proof-theoretic, both perspectives have been explored in parallel for some of the probabilistic logic programming languages such as ICL [Poole, 2008] and ProbLog [Fierens et al., 2015].

Again this carries over to neuro-symbolic methods. Approaches of NTPs, DeepProblog, ∂ILP and DiffLog are proof-based. The probabilities or certainties that these systems output are based on the enumerated proofs, and they are also able to learn how to combine them. In contrast, approaches of LRNN, LTNs, RNM, NMLN, NLM and NeuralLP are all based on grounding. Learning in these models is done through learning the (shared) parameters over the ground model and inference is based on possible groundings of the model.

# 4 Logic vs Probability vs Neural

When two paradigms are integrated, examining which of the base paradigms are preserved, and to which extent, tells us a lot about the strengths and weaknesses of the resulting paradigm. In StarAI, the traditional knowledge based model construction approach is to use the logic only to generate a probabilistic graphical model, implying that both the inference and the semantics are pushed inside the graphical model. The effect is that it is often harder to reason at a purely logical level with such systems. What is meant here is that it may become unclear how to apply logical inference rules such as resolution (or extensions that take into account the parameters) to such models or what the effect of applying such rules will be. This is what happens with systems such as PRMs, BLPs, PSL, and MLNs. For instance, in MLNs the addition of the resolvent of two weighted rules, makes it hard to predict the effect on the distribution. On the other hand, the opposite holds for PLPs and its variants. While it is clear what the effect of a logical operation is, it is often harder to directly identify and exploit properties such as conditional or contextual independencies, which are needed for efficient probabilistic inference.

This position on the spectrum between logic and probability has a profound influence on the properties of the

underlying model. For NeSy, the spectrum involves not only logic and neural networks, but often also probability. It has been argued that when combining different perspectives in one model or framework, such as neural, logic and probabilistic ones, it is desirable to have the originals or base paradigms as a special case, see also [De Raedt *et al.*, 2019].

The vast majority of current NeSy approaches focus on the neural aspect (i.e., they originated as a fully neural method to which logical components have been added). Some of these approaches like LTNs and Tensor-Log [Cohen et al., 2017] pursue a kind of knowledge-based model construction approach in which the logic is compiled away into the neural network architecture. A different family of NeSy approaches, which includes SL and SBR, turns the logic into a regularization function to provide a penalty whenever the desired logical theory or constraints are violated. This leads to the logic being compiled into the weights of the trained neural network.

A small number of NeSy methods, however, retain the focus on logic. Some of these methods start from existing logic (programming) frameworks and extend them with primitives that allow them to interface with neural networks and allow for differentiable operations. Examples include DeepProbLog and DiffLog. Other methods instead take an existing framework and turn it into a differentiable version. The key inference concepts are mapped onto an analogous concept that behaves identically for the edge cases, but is continuous and differentiable in non-deterministic cases. Such methods include  $\partial$  ILP,  $\partial$ 4 [Bošnjak et al., 2017] and NTPs.

Even for methods that focus on logic, it can be useful to map the problem onto an intermediate representation. One such idea concerns performing probabilistic inference by mapping it onto a weighted model counting (WMC) problem. This can then in turn be solved by compiling it into a structure (e.g. an arithmetic circuit) that allows for efficient inference. This has the added benefit that this structure is differentiable, which can facilitate the integration between logic based systems and neural networks. DeepProbLog, for example, uses this approach. In [Zuidberg Dos Martires et al., 2019], the authors argue that this intermediate representation can serve as an assembly language for AI.

### 5 Semantics

Traditionally, StarAI combines two semantics: a logical and a probabilistic one. In a *logical* semantics, atoms are assigned a truth value in the {*true*, *false*} set (i.e. {0,1}). In a *probabilistic* semantics, probability is defined as a measure over sets of possible worlds, where each possible world is an assignment of values to the random variables. This implies that a probabilistic logic semantics defines probability distributions over ground logical interpretations, that is, over sets of ground facts. Prominent examples in StarAI are ProbLog (from the directed side) and Markov Logic (from the undirected one). However, the complexity of inference in probabilistic logic has led to statistical relational

approaches (e.g. [Bach et al., 2017]), where the truth values are relaxed in the continuous interval [0,1] and logic operators are turned into real valued functions. This setting is described in terms of fuzzy logic (or soft logic) semantics, mathematically grounded in the t-norm theory. By exploiting the translation of Boolean formulas into real valued functions, the fuzzy semantics allows to exploit algebraic and geometric properties of t-norms (including especially their differentiability) to reduce complexity. The main issue of fuzzy semantics in the context of StarAI is that it is often not exploited to describe problems that are intrinsically vague [Fine, 1975], but, simplistically, as a continuous surrogate of Boolean logic. A side effect of this approximation is that many properties of the original logical theory can be realised in many different ways in their continuous translation. Indeed, the fuzzification procedure alters the logical properties of the original theory (such as satisfiability), depending on the particular connectives exploited in the conversion. For example, in the Łukasiewicz *t-norm*  $t_{\mathbf{L}}(x, y) = \max\{0, x + y - 1\}$ , the conjunction can be 0 (i.e. false) even without any of the elements being 0 (e.g. x = y = 0.5).

Neuro-symbolic approaches can easily be categorized in terms of the same logical, probabilistic or fuzzy semantics. Neural enhancements of the *logic* semantics either use neural networks to turn perceptive input to a logical atom or introduce a relaxed version of logical reasoning performed through tensor calculus. An instance of the former is ABL [Dai *et al.*, 2019], which use logical abduction to provide the feedback for a neural model processing the perceptive input. Tensor calculus approaches, such as NLM and NeuralLP, interpret predicates as tensors grounded over all constants in a domain and interpret clauses as a product of those matrices.

Neural enhancements of the *probabilistic* semantics usually reparameterize the underlying distribution in terms of neural components. In particular, DeepProbLog exploits neural predicates to compute the probabilities of probabilistic facts as the output of neural computations over vectorial representations of the constants, which is similar to SL in the propositional counterpart. NMLN and RNM use neural potentials in order to implement factors (or their weights) as neural networks. [Rocktäschel *et al.*, 2015] computes marginal probabilities as logistic functions over similarity measures between embeddings of entities and relations.

Neural enhancements of the *fuzzy* semantics are usually realised by allowing continuous truth values to be the outcome of a neural process and the differentiability of the corresponding t-norm allows for an easy integration with neural computation frameworks. In particular, SBR and LTN turn atoms into neural networks taking as inputs the feature representation of the constants and returning the corresponding truth value. Similarly, in LRNN and [Wang and Pan, 2019], the output of the neurons of the logical network can be interpreted as fuzzy truth values of the corresponding atoms.

Finally, there is a large class of methods [Minervini *et al.*, 2017; Demeester *et al.*, 2016; Cohen *et al.*, 2017; Weber *et al.*, 2019] realised by re-

laxing logical statements in a numeric way, without giving any other specific semantics, either probabilistic or fuzzy. Here, atoms are assigned scores in  $\mathbb R$  computed by a neural scoring function over embeddings. Numerical approximations are then applied either to combine these scores according to logical formulas or to aggregate proofs scores. The resulting neural architecture is usually differentiable and, thus, trained end-to-end.

### 6 Learning parameters or structure

StarAI distinguishes between two types of learning: structure learning, which corresponds to learning the logical clauses of the model [Kok and Domingos, 2005], and parameter learning in which the probabilities or weights of the clauses have to be estimated [Gutmann *et al.*, 2008; Lowd and Domingos, 2007].

This distinction is less clear in the NeSy setting. Unlike what is common in StarAI, the NeSy approaches do not perform a search through the discrete space of possible clauses, but rather through the space of parameters of such clauses which are typically enumerated by following a template (often with a predefined complexity). Examples of such systems include NTPs,  $\partial$  ILP, DeepProbLog, NeuralLP and DiffLog. Alternatively, one can provide a *sketch* of the desired program – a program with certain decisions left blank – and learn a NeSy model to fill out the blanks, such as Deep-ProbLog and  $\partial$ 4.

A substantial number of approaches tries to leverage the best of both worlds. These ideas include using neural models to guide the symbolic search [Kalyan *et al.*, 2018; Ellis *et al.*, 2018a; Valkov *et al.*, 2018], or using a neural model to produce a program that is then executed symbolically [Ellis *et al.*, 2018b; Mao *et al.*, 2019].

#### 7 Symbols vs Sub-symbols

An important factor in both StarAI and NeSy systems is the representation of entities. StarAI generally represents entities by constants (symbols). But neural methods are numerical by nature and therefore symbols are replaced with sub-symbols, i.e., vectorized representations. If the entity has inherent numerical properties, these could be used as sub-symbols (e.g. the pixel data of an image). However, if this is not the case, a one-hot encoding or learned embedding can be used instead. This, of course, has an impact on the generalizability of the system towards unseen entities, as new embeddings have to be learned for new symbols. Naturally, among the neurosymbolic methods, there is a wide variety in how symbols and sub-symbols are used in representation and reasoning. The idea of mapping entities onto sub-symbols is made very explicit in LTNs, where in a first step, all symbols are replaced with sub-symbols. In DeepProbLog, entities are represented using symbols, but they sometimes have sub-symbolic representations that are only used inside the neural networks. Similarly, in [Lippi and Frasconi, 2009] and RNM, MLNs are conditioned on a feature representation of constants (e.g. images, audio signals, etc.). Finally, among those models exploiting learned embeddings,

we find [Rocktäschel et al., 2015; Minervini et al., 2017; Demeester et al., 2016].

Now that we discussed how entities can be represented by symbols and sub-symbols, let us discuss how they can be used for reasoning. Most methods either only work with logic reasoning on symbols, or perform algebraic operations on sub-symbols. However, some methods can use both simultaneously. A very powerful and elegant mechanism for reasoning about symbols in first order logic is *unification*. It is used to reason about equality at the symbolic level. For instance, the atomic expressions p(a, Y) and p(X, b) can be unified using the substitution  $\{X = a, Y = b\}$ . Unification not only works for constants but also for structured terms  $f(t_1, ..., t_n)$  where f is a structured term and the  $t_i$  are constants, variables or structured terms themselves.

While unification is not supported by standard neural networks, reasoning about equality corresponds closely to reasoning about similarity in embedding space. Entities are typically embedded in some metric space, and represented through their embeddings, that is, through sub-symbols. Reasoning typically proceeds by performing algebraic operations (such as vector addition) on these embeddings, and considering the similarity between two entities by using their distance in embedding space. It is quite interesting to see to what extent current neuro-symbolic approaches support unification on the one hand, and to what extent the use of embeddings has been integrated into the neuro-symbolic logics as a kind of *soft* equality or unification

This idea was implemented in NTPs and NLProlog as *soft* or *weak unification*. In these systems, two entities can be unified if they are similar, and not just if they are identical. As such, this system can interweave both symbols and subsymbols during inference. For each entity, an embedding is learned and their similarity is determined based on the distance between the embeddings using a radial basis function. However, this potentially adds a lot of different proof paths, which can result in computational issues for larger programs. This problem was solved in later iterations of the system [Minervini *et al.*, 2020].

## 8 Type of logic

There is a natural ordering of logical representations, starting with propositional logic (only arity 0 predicates), to relational logic (having no structured terms, so only constants and variables as terms, which is also the basis for the Datalog database language), to general first order logic (FOL), and then to logic programs (LP) as in the programming language Prolog. Logic programs are usually restricted to definite clauses, while the semantics of a definite clause program is given by its least Herbrand model, the set of all ground facts that are logically entailed by the program. This contrasts with the standard semantics of first order logic that would also allow for other models. This difference carries over to StarAI, where probabilistic logic programs and Markov Logic inherit their semantics from logic programming, respectively first order logic. This explains, for instance, why Markov Logic's semantics boils down to

a maximum entropy approach when a theory has multiple models (such as  $a \lor b$ ), cf. [De Raedt and Kimmig, 2015; De Raedt et~al., 2016] for more details. On the other hand, logic programs are also the basis for the programming language Prolog, which implies that they can be used to specify traditional programs such as sorting and data structures such as lists through structured terms. This is relevant especially for those approaches to neurosymbolic computation that are used to synthesize programs from examples.

Neuro-symbolic representations typically extend one of these four types of logic: propositional, relational, first order logic, or logic programs. For instance, SL focuses only on the propositional setting. On the other hand, ∂ILP, NTPs and DiffLog are based on Datalog, which belongs to relational logic segment. LTNs and SBR use fuzzy logic to translate a general FOL theory into a training objective, either isolated or in conjunction with a supervised criterion. Just like Markov Logic, also RNM and NMLN use first order logic to generate a random field. Finally, DeepProbLog, NLProlog and LRNN are examples of neuro-symbolic logic programming frameworks.

# 9 Open challenges

To conclude, we now list a number of challenges for NeSy, which deserve, in our opinion, more attention.

**Probabilistic reasoning** Although relatively few methods explore the integration of logical and neural methods through probabilities perspective, we believe that a probabilistic approach is the best way to principally integrate the two [De Raedt *et al.*, 2019]. There should be further investigation into the applicability of probabilistic reasoning for neuro-symbolic computation.

Structure learning While significant progress has been made on learning the structure of purely relational models (without probabilities), learning StarAI models remains a major challenge due to the complexity of inference and the combinatorial nature of the problem. Incorporating neural aspects complicates the problem even more. NeSy methods have certainly shown potential for addressing this problem (Section 6), but the existing methods are still limited and mostly domain-specific which impedes their wide application.

**Scaling inference** Scalable inference is a major challenge for StarAI and therefore also for NeSy approaches with an explicit logical or probabilistic reasoning component. Investigating to which extent neural methods can help with this challenge by means of lifted (exploiting symmetries in models) or approximate inference, as well as reasoning from the intermediate representations [Abboud *et al.*, 2020], are promising future research directions.

**Data efficiency** A major advantage of StarAI methods, as compared to neural ones, is their data efficiency – StarAI methods can efficiently learn from small amount of data, whereas neural methods are data hungry. On the other hand, StarAI methods do not scale to big data sets, while neural methods can easily handle them. We believe that

	Dimension 1	Dimension 2	Dimension 3	Dimension 4	Dimension 5	Dimension 6	Dimension 7
	(D)irected (U)ndirected	(G)rounding (P)roofs	(L)ogic (P)robability (N)eural	(L)ogic (P)robability (F)uzzy	(P)arameter (S)tructure	(S)ymbols (Sub)symbols	(P)ropositional (R)elational (FOL) (LP)
∂ILP [Evans and Grefenstette, 2018]	D	P	L+N	L	P	S	R
DeepProbLog [Manhaeve et al., 2018]	D	P	L+P+N	P	P	S+Sub	LP
DiffLog [Si et al., 2019]	D	P	L+N	L	P+S	S	R
LRNN [Šourek <i>et al.</i> , 2018]	D	P	L+N	F	P+S	S+Sub	LP
LTN [Donadello et al., 2017]	U	G	L+N	F	P	Sub	FOL
NeuralLP [Yang et al., 2017]	D	G	L+N	L	P	S	R
NLM [Dong et al., 2019]	D	G	L+N	L	P+S	S	R
NLProlog [Weber et al., 2019]	D	P	L+P+N	P	P+S	S+Sub	LP
NMLN [Marra and Kuželka, 2019]	U	G	L+P+N	P	P+S	S+Sub	FOL
NTP [Rocktäschel and Riedel, 2017]	D	P	L+N	L	P+S	S+Sub	R
RNM [Marra et al., 2020]	U	G	L+P+N	P	P	S+Sub	FOL
SL [Xu et al., 2018]	U	G	L+P+N	P	P	S+Sub	P
SBR [Diligenti et al., 2017]	U	G	L+N	F	P	Sub	FOL
Tensorlog [Cohen et al., 2017]	D	P	L+N	P	P	S+Sub	R

Table 1: Taxonomy of a (non-exhaustive) list of NeSy models according to the 7 dimensions outlined in the paper.

understanding how these methods can help each other to overcome their complementary weaknesses, is a promising research direction.

**Symbolic representation learning** The effectiveness of deep learning comes from the ability to change the representation of the data so that the target task becomes easier to solve. The ability to change the representation on the symbolic level as well would significantly increase the capabilities of NeSy systems. This is a major open challenge for which neurally inspired methods could help achieve progress [Cropper, 2019; Dumančić *et al.*, 2019].

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