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# 15 Practice Sets for JEE Main 2019

## Joint Engineering Entrance Main

**PHYSICS CHEMISTRY MATHEMATICS**

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# Contents



# Practice Sets

*for*

# JEE Main (1-15)

# JEE Main

## Joint Entrance Examination

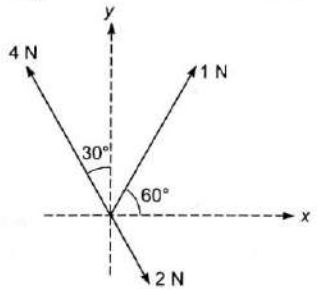
# Practice Set 1

## Instructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

1. Three forces acting on a body are shown in the figure. To have the resultant force only along the $y$-direction, the magnitude of the minimum additional force needed is




(a) $\frac{\sqrt{3}}{4}$ N (b) $\sqrt{3}$ N (c) 0.5 N (d) 1.5 N

2. A uniform metal disc of radius $R$ is taken and out of it a disc of diameter $R/2$ is cut-off from the end. The centre of mass of the remaining part will be

(a) $\frac{R}{4}$ from the centre (b) $\frac{R}{3}$ from the centre (c) $\frac{R}{5}$ from the centre (d) $\frac{R}{6}$ from the centre

3. A tank is filled with water of density 1 g/cm$^3$ and oil of density 0.9 g/cm$^3$. The height of water layer is 100 cm and of the oil layer is 400 cm. If $g = 980$ cm/s$^2$, then the velocity of efflux from an opening in the bottom of the tank is

(a) $\sqrt{900 \times 980}$ cm/s (b) $\sqrt{100 \times 980}$ cm/s (c) $\sqrt{920 \times 980}$ cm/s (d) $\sqrt{950 \times 980}$ cm/s

4. A charge $Q$ is divided into two parts $q$ and $q'$ and separated by a distance $R$. The force of repulsion between them will be maximum, when

(a) $q = \frac{Q}{4}$ (b) $q = \frac{Q}{2}$ (c) $q = Q$ (d) None of these

5. A parallel plate capacitor has plates with area $A$ and separation between it is $d$. A battery charges the plates to a potential difference $V_0$. The battery is then disconnected and a dielectric slab of thickness $d$ is introduced. The ratio of energy stored in the capacitor before and after the slab is introduced is

(a) $K$ (b) $\frac{1}{K}$ (c) $\frac{A}{d^2K}$ (d) $\frac{d^2K}{A}$

6. You are given $n$ resistors, each of resistance $r$. They are first combined to get minimum possible resistance, then they are connected to get maximum possible resistance. The ratio between minimum to maximum resistances is

(a) $\frac{1}{n}$ (b) $n$ (c) $n^2$ (d) $\frac{1}{n^2}$

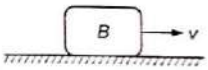
7. A rectangular coil of 100 turns and size 0.1 m × 0.05 m is placed perpendicular to a magnetic field of 0.1 T. The induced emf when the field drops to 0.05 T is 0.05 s is

(a) 0.5 V (b) 1.0 V (c) 1.5 V (d) 2.0 V

8. A bomb is dropped from an aeroplane when it is at a height $h$ directly above the target. If the aeroplane is moving horizontally at a speed $v$, the distance by which bomb will miss the target is given by

(a) $2v\sqrt{\frac{h}{g}}$ (b) $v\sqrt{\frac{h}{g}}$ (c) $v\sqrt{\frac{2h}{g}}$ (d) $v\sqrt{\frac{h}{2g}}$

9. A block $B$ is pushed momentarily along a horizontal surface with an initial velocity $v$. If $\mu$ is the coefficient of sliding friction between $B$ and the surface, block $B$ will come to rest after a time




(a) $\frac{g\mu}{v}$ (b) $\frac{g}{v}$ (c) $\frac{v}{g}$ (d) $\frac{v}{g\mu}$

10. A machine which is 80% efficient, uses 20 J of energy in lifting up a 2 kg mass through a certain distance. The mass is then allowed to fall through that distance. The velocity at the end of its fall is

(a) $2\sqrt{10}$ m/s (b) $4\sqrt{2}$ m/s (c) 4 m/s (d) $4\sqrt{5}$ m/s

11. A cylinder of height $h$ is placed on an inclined plane, the angle of inclination of which is slowly increased. It begins to slip when the angle of inclination is 45°. What is the radius of the cylinder?

(a) $h$ (b) $\frac{3}{4}h$ (c) $\frac{1}{2}h$ (d) $\frac{1}{4}h$

12. A person feels 2.5% difference of frequency of a motorcar horn. If the motorcar is moving to the person and the velocity of sound is 320 m/s, then the velocity of car will be

(a) 8 m/s (approx) (b) 800 m/s (c) 7 m/s (d) 6 m/s (approx)

13. Two concentric spheres of radii $R$ and $r$ have similar charges with same surface charge densities ($\sigma$). What is the electric potential at their common centre?

(a) $\frac{\sigma}{\varepsilon_0}$ (b) $\frac{\sigma}{\varepsilon_0}(R - r)$

(c) $\frac{\sigma}{\varepsilon_0}(R + r)$ (d) None of these

14. A transformer is used to step-up an transformation emf of 220 V to 4.4 kV in order to transmit 6.6 kV of power. If the primary coil has 100 turns, what is the number of turns in the secondary?

(a) 20 (b) 200 (c) 2000 (d) 20000

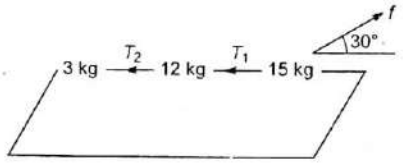
15. A body is projected vertically upwards. If $t_1$ and $t_2$ be the instants at which it is at a height $h$ above the point of projection, while ascending and descending respectively, then

(a) $h = gt_1t_2$ (b) $h = 2gt_1t_2$ (c) $h = \frac{1}{2}gt_1t_2$ (d) $h = \frac{1}{4}gt_1t_2$

16. A particle of mass $4m$ at rest explodes into three fragments. Two of the fragments each of mass $m$ move with speed $v$ at right angles to each other. The kinetic energy released in the process is

(a) $2mv^2$ (b) $\frac{3}{2}mv^2$ (c) $\frac{1}{2}mv^2$ (d) $3mv^2$

17. In figure, if the surfaces are frictionless, the ratio $T_1 : T_2$ is

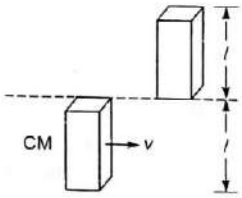



(a) $\sqrt{3} : 2$ (b) $1 : \sqrt{3}$ (c) $1 : 5$ (d) $5 : 1$

18. A chord is used to lower vertically a block of mass $M$ a distance $d$ at a constant downward acceleration of $\frac{g}{4}$. Then, the work done by the chord on the block is

(a) $\frac{Mgd}{4}$ (b) $\frac{-Mgd}{4}$ (c) $\frac{3Mgd}{4}$ (d) $\frac{-Mgd}{4}$

19. A bar of mass $m$ length $l$ is in pure translatory motion with its centre of mass velocity $v$. It collides with and sticks to another identical bar at rest as shown in figure. Assuming that after collision it becomes one composite bar of length $2l$, the angular velocity of the composite bar will be




(a) $\frac{3v}{4l}$, anti-clockwise (b) $\frac{4v}{3l}$, anti-clockwise

(c) $\frac{3v}{4l}$, clockwise (d) $\frac{4v}{3l}$, clockwise

20. An U tube of uniform bore of cross-sectional area $A$ is set up vertically with open ends up. A liquid of mass $M$ and density $d$ is poured into it. The liquid column will oscillate with a period

(a) $2\pi\sqrt{\frac{M}{g}}$ (b) $2\pi\sqrt{\frac{MA}{dg}}$ (c) $2\pi\sqrt{\frac{M}{Adg}}$ (d) $2\pi\sqrt{\frac{M}{2Adg}}$

21. In a ball, a person receives direct sound waves from a source 120 m away. He also receives wave from the same source which reach him after being reflected from one 25 m high ceiling at a point half-way between them. The two waves interfere constructively for wavelengths (in metre) of

(a) 10, 5, $\frac{5}{2}$, ... (b) 20, $\frac{20}{3}$, $\frac{20}{5}$, ... (c) 30, 20, 10, ... (d) 35, 25, 15, ...

22. A dip circle lying initially in the magnetic meridian is rotated through angle $\theta$ is the horizontal plane. The tangent of angle of dip is increased in the ratio
(a) $\cos\theta : 1$ (b) $\sin\theta : 1$ (c) $1 : \cos\theta$ (d) $1 : \sin\theta$

23. A rectangular coil of single turn, having area $A$, rotates in a uniform magnetic field $B$ with an angular velocity $\omega$ about an axis perpendicular to the field. If initially the plane of the coil is perpendicular to the field, then the average induced emf when it has rotated through 90°, is
(a) $\frac{\omega BA}{\pi}$ (b) $\frac{\omega BA}{2\pi}$ (c) $\frac{\omega BA}{4\pi}$ (d) $\frac{2\omega BA}{\pi}$

24. A 60 W bulb is placed at a distance of 4 m from you. The bulb is emitting light of wavelength 600 nm uniformly in all directions. In 0.1 s, how many photons enter your eye if the pupil of the eye is having a diameter of 2 mm? [Take, $hc = 1240$ eV-nm]
(a) $2.84 \times 10^{12}$ (b) $2.84 \times 10^{11}$ (c) $9.37 \times 10^{11}$ (d) $6.48 \times 10^{11}$

25. Two identical non-relativistic particles $A$ and $B$ move at right angles to each other, possessing de-Broglie wavelengths $\lambda_1$ and $\lambda_2$ respectively. The de-Broglie wavelength of each particle in their $C$ frame of reference is
(a) $\lambda_1 + \lambda_2$ (b) $\frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$ (c) $\frac{\lambda_1\lambda_2}{\sqrt{\lvert\lambda_1^2 - \lambda_2^2\rvert}}$ (d) $\frac{\lambda_1 + \lambda_2}{2}$

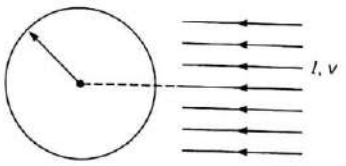
26. When the voltage applied to an X-rays tube increases from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength interval between $K_\alpha$ line and cut-off wavelength of continuous spectrum increases by a factor of 3. Atomic number of the metallic target is
(a) 28 (b) 29 (c) 65 (d) 66

27. The binding energy of an electron in the ground state of He-atom is equal to $E_0 = 24.6$ eV. The energy required to remove both the electrons from the atom is
(a) 24.6 eV (b) 79.0 eV (c) 54.4 eV (d) None of these

28. A photosensitive material is at 9 m to the left of the origin and the source of light is at 7 m to the right of the origin along $x$-axis. The photosensitive material and the source of light start from rest and moves with $8\mathbf{i}$ m/s and $4\mathbf{i}$ m/s respectively. The ratio of intensity at $t = 0$ to $t = 3$ s as received by the photosensitive material is
(a) 16 : 1 (b) 1 : 16 (c) 2 : 7 (d) 7 : 2

29. Hydrogen atoms in a sample are excited to $n = 5$ states and it is found that photons of all possible wavelengths are present in emission spectra. The minimum number of hydrogen atoms in the sample would be
(a) 5 (b) 6 (c) 10 (d) infinite

30. A parallel beam of light of intensity $I$ and frequency $\nu$ is incident on a solid sphere of radius $R$ as shown in the figure




For this situation mark out the correct statement.
(a) The force exerted by light on the sphere is the greatest when surface of sphere is perfectly reflecting and is equal to $\frac{2I \times \pi R^2}{c}$.
(b) The force exerted by light on the sphere is independent of the nature of surface, *i.e.*, it is same for perfect reflector, perfect absorber, and is partially reflecting and for all it is equal to $\frac{I \times \pi R^2}{c}$.
(c) The force exerted by light on sphere is least when surface of the sphere a is perfect absorber, and is equal to $\frac{I \times \pi R^2}{c}$.
(d) Both (a) and (c) are correct.

# Chemistry

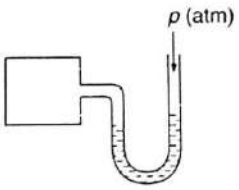
31. A gaseous substance $AB_2(g)$ converts to $AB(g)$ in the presence of solid $A(s)$ as

$$AB_2(g) + A(s) \rightleftharpoons AB(g)$$

The initial pressure and equilibrium pressure are 0.7 and 0.95 bar. Now the equilibrium mixture is expanded reversibly and isothermally till the gas pressure falls to 0.4 bar. The volume percentage of $AB(g)$ at the final equilibrium is

(a) 67.5 (b) 32.5 (c) 45.3 (d) 63.1

32. A open manometer attached to a flask containing ammonia gas have no difference in mercury level initially as shown in fig. After sparking into the flask, ammonia is partially dissociated as $2NH_3(g) \longrightarrow N_2(g) + 3H_2(g)$. Now it have difference of 18 cm in mercury level into columns, what is partial pressure of $H_2(g)$ at equilibrium?

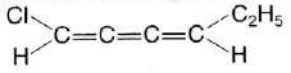


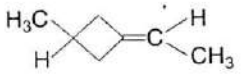

(a) 9 cm Hg (b) 18 cm Hg (c) 27 cm Hg (d) None of these

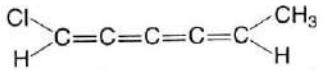
33. An inorganic red coloured compound ($A$) on heating gives a compound ($B$) and a gas ($C$). ($A$) on treatment with conc. $HNO_3$ gives compound ($D$), brown colour substance ($E$) and a neutral oxide ($F$). Compound ($D$) on warming gives off again gas ($C$). Then ($E$) will be

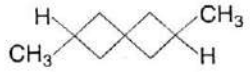
(a) $Mn_3O_4$ (b) $PbO_2$ (c) $Pb_3O_4$ (d) $Fe_2O_3$

34. Which of the following are optically active?

(I) 

(II) 

(III) 

(IV) 

(a) I and II (b) II and III (c) III and IV (d) II, III and IV

35. In which of the following pairs, both the species have same hybridisation and shape?

(a) $SF_4$, $XeF_4$ (b) $I_3^-$, $XeF_2$ (c) $ICl_4^+$, $SiCl_4$ (d) $ClO_3^-$, $PO_4^{3-}$

36. At what temperature will the translational kinetic energy of H-atom equal to that for H-atom of first line Lyman transition? (Given $N_A = 6 \times 10^{23}$)

(a) 780 K (b) $1.32 \times 10^5$ K (c) $7.84 \times 10^4$ K (d) 1900 K

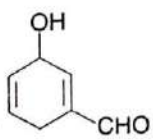
37. The correct order of bond angles in $NH_3$, $NF_3$ and $NCl_3$ are

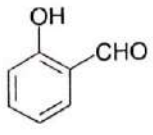
(a) $NH_3 > NF_3 > NCl_3$ (b) $NH_3 > NCl_3 > NF_3$ (c) $NCl_3 > NH_3 > NF_3$ (d) $NF_3 > NCl_3 > NH_3$

38. At 25°C, the vapour pressure of pure liquid $A$ (mol. wt. = 40) is 100 torr and that of pure liquid $B$ (mol. wt. = 80) is 60 torr. Find the vapour pressure of solution at 25°C if the mole ratio of $A$ and $B$ in vapour phase is 2 : 3.

(a) 60.3 torr (b) 65.7 torr (c) 71.42 torr (d) None of these

39. The tautomerism involving (1, 5) migration of hydrogen atom can be observed in


(I)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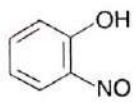
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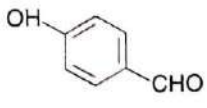
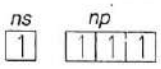
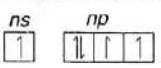
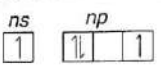

(IV)

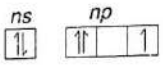
(a) I and II (b) II and III (c) II, III and IV (d) I, II and III

40. In which of the following Aufbau principle and Hund's rule is violated?

(a) 

(b) 

(c) 

(d) 


41. Which of the following order are correct?

(i) Solubility in water LiOH < NaOH < KOH < RbOH < CsOH

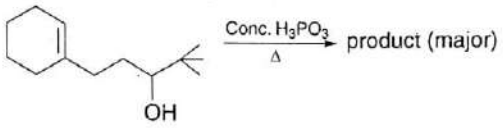
(ii) Thermal stability $BeSO_4 < MgSO_4 < CaSO_4 < SrSO_4 < BaSO_4$

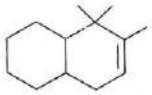
(iii) Melting point NaCl > KCl > RbCl > CsCl > LiCl

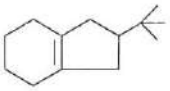
(iv) Solubility in water $Li_2CO_3 < Na_2CO_3 < K_2CO_3 < Rb_2CO_3$

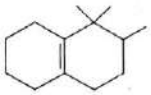
(a) I, II and III (b) I, II and IV (c) II, III and IV (d) All of these

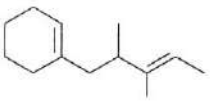
42. The product obtained from the following reaction is




(a) 

(b) 

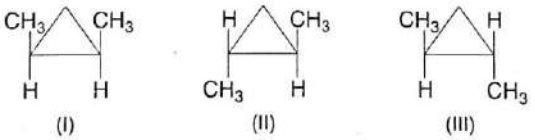
(c) 

(d) 

43. A metal complex of coordination number six having three different types of ligands *a*, *b* and *c* of composition $Ma_2b_2c_2$ can exist in several geometrical isomeric forms; the total number of such isomer is

(a) 3 (b) 5 (c) 7 (d) 9

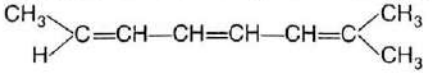
44. Which of the following is incorrect about the following?




(a) I and II are geometrical isomers (b) I and III are geometrical isomers

(c) II and III are optically active (d) II and III are same

45. The number of geometrical isomers shown by the following compound is




(a) 4 (b) 6 (c) 8 (d) None of these

46. A reaction $A \rightarrow B$ involves the following mechanism

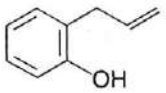
*Step* 1 $2A \xrightarrow{k_1} C$ (fast) *Step* 2 $C \xrightarrow{k_2} D$ (slow)

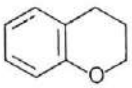
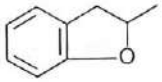
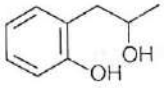
*Step* 3 $3D \xrightarrow{k_3} B$ (fast)

The rate law of the reaction may be given as

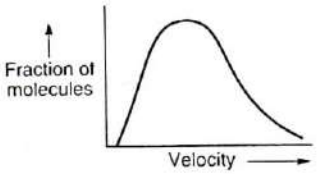
(a) rate = $k_1[A]^2$ (b) rate = $k_1k_2[A]^2$ (c) rate = $k_2[C][D]$ (d) rate = $k_1k_2k_3[C][D]$

47. Identify the correct statement.
(a) $P_4O_{10}$ contains peroxide linkages
(b) $P_4O_{10}$ contains $p\pi$-$d\pi$ back bonding
(c) In $P_4O_{10}$ each P atom is bonded to three oxygen atoms
(d) $P_4O_{10}$ hydrolyse in water forming phosphorous acid

48.  Product (major)

(a)  (b)  (c)  (d) None of these

49. The following graph is obtained when the fraction of molecules is plotted against velocity. The profile curve will be same for the gases.




(I) Oxygen gas at $T$ temperature
(II) Hydrogen gas at $2T$ temperature
(III) Helium gas at $2T$ temperature
(IV) Sulphur dioxide gas at $2T$ temperature

(a) I and II (b) II and III (c) I and IV (d) III and IV

50. A reactant ($A$) forms two products

$$A \xrightarrow{k_1} B \text{ Activation energy } E_{a_1}$$

$$A \xrightarrow{k_2} C \text{ Activation energy } E_{a_2}$$

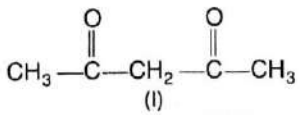
If $E_{a_2} = 2E_{a_1}$, then $k_1$ and $k_2$ will be related as

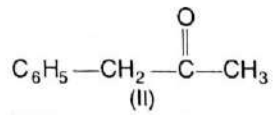
(a) $k_2 = k_1 e^{-E_{a_1}/RT}$ (b) $k_2 = k_1 e^{-E_{a_2}/RT}$
(c) $k_1 = k_2 e^{-E_{a_1}/RT}$ (d) $k_1 = 2k_2 e^{E_{a_2}/RT}$

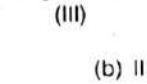
51. The halogen oxide used for the estimation of carbon monoxide in automobile exhaust gases is
(a) $Cl_2O_7$ (b) $I_2O_5$ (c) $ClO_2$ (d) $BrO_3$

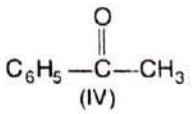
52. During the electrolysis of concentrated aqueous solution of sodium chloride, the pH of solution (electrolyte)
(a) increases (b) decreases (c) remains same (d) None of these

53. Which of the following can give iodoform test?


(I)

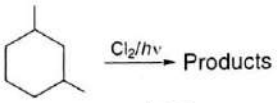

(II)


(III)


(IV)

(a) Only IV (b) II and IV (c) III and IV (d) All of these

54. The monochlorinated products obtained from the reaction is

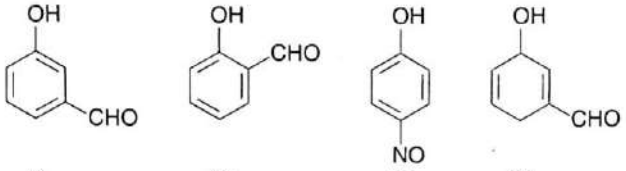



(a) 4 (b) 5 (c) 6 (d) 7

55. Oxygen atoms forms fcc unit cell with '*A*' atoms occupying all tetrahedral voids and '*B*' atoms occupying all octahedral voids. If atoms are removed from two of the body diagonals then determine the formula of resultant compound formed.

(a) $A_4B_4O_7$ (b) $A_8B_6O_7$ (c) $A_8B_8O_7$ (d) $A_6B_8O_6$

56. Which of the following compounds will not undergo tautomerism?



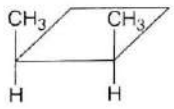

(I) (II) (III) (IV)

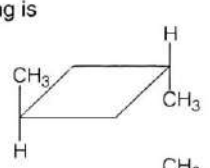
(a) (II) and (III) (b) (I), (II) and (III) (c) Only (I) (d) All of these

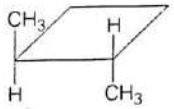
57. To 100 mL of 0.1 M solution of sodium dihydrogen phosphate 75 mL of 0.1 M sodium phosphate is added. Calculate the pH when 25 mL of 0.1 M HCl is added to the above solution (stepwise acid dissociation constant for phosphoric acid are $10^{-3}, 10^{-6}$ and $10^{-13}$)

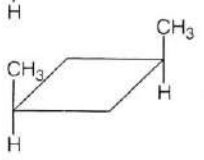
(a) 6 (b) 4 (c) 8.5 (d) None of these

58. The optically active compound among the following is

(a) 


(b) 


(c) 


(d) 

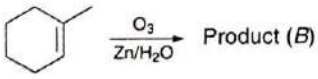

59. Flocculating value of ion depends on

(a) the shape of flocculating ion

(b) the amount of flocculating ion

(c) nature of the charge on the flocculating ion

(d) both, the nature and magnitude of the charge of the flocculating ion

60. 


The product '*B*' on reaction with base $OH^{\ominus}$ can undergo

(a) cross aldol condensation (b) intramolecular aldol condensation

(c) cross Cannizzaro's reaction (d) intramolecular Cannizzaro's reaction

# Mathematics

61. If $k\mathbf{r}+\mathbf{r}\times\mathbf{a}=\mathbf{b}$, where $k$ is non-zero scalar and $\mathbf{a},\mathbf{b}$ are two given vectors, then $\mathbf{r}$ will be

(a) $\frac{1}{k^2+\mathbf{a}^2}\left(k\mathbf{b}+\frac{\mathbf{a}\cdot\mathbf{b}}{k}\mathbf{a}+\mathbf{a}\times\mathbf{b}\right)$ (b) $\frac{1}{k^2+\mathbf{a}^2}\left(k\mathbf{b}-\frac{\mathbf{a}\cdot\mathbf{b}}{k}\mathbf{a}+\mathbf{a}\times\mathbf{b}\right)$

(c) $\frac{1}{k^2+\mathbf{a}^2}\left(k\mathbf{b}-\frac{\mathbf{a}\cdot\mathbf{b}}{k}\mathbf{a}-\mathbf{a}\times\mathbf{b}\right)$ (d) $\frac{1}{k^2-\mathbf{a}^2}\left(k\mathbf{b}-\frac{\mathbf{a}\cdot\mathbf{b}}{k}\mathbf{a}-\mathbf{a}\times\mathbf{b}\right)$

62. The value of $\tan^{-1}(e^{i\theta})$ is equal to

(a) $\frac{n\pi}{2}+\frac{\pi}{4}+\frac{i}{2}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ (b) $\frac{n\pi}{2}-\frac{\pi}{4}-\frac{i}{2}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

(c) $\frac{n\pi}{2}+\frac{\pi}{4}+\frac{i}{2}\log\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ (d) $\frac{n\pi}{2}+\frac{\pi}{4}-\frac{i}{2}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

63. If $\sum_{i=1}^{n}\mathbf{a}_i=\mathbf{0}$, where $|\mathbf{a}_i|=1,\forall i$, then the value of $\sum_{1\le i\le j<n}\sum\mathbf{a}_i\cdot\mathbf{a}_j$ is

(a) $-\frac{n}{2}$ (b) $-n$ (c) $\frac{n}{2}$ (d) $n$

64. If $\sin^2 x+\cos^2 y=2\sec^2 z$, then values of $x$, $y$ and $z$ are respectively (where, $m,n,r\in I$)

(a) $\left(\left(n+\frac{1}{2}\right)\pi, m\pi, n\pi\right)$ (b) $\left((n-1)\frac{\pi}{2}, m\pi, n\pi\right)$

(c) $\left((2n-1)\frac{\pi}{3}, m\pi, \frac{n\pi}{2}\right)$ (d) $\left((n+1)\frac{\pi}{2}, m\pi, n\pi\right)$

65. If $\alpha,\beta$ and $\gamma$ are the altitudes of the $\Delta ABC$ from the vertices $A,B$ and $C$ respectively, then value of $4\Delta^2\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)$ is

(a) $a^2+b^2+c^2$ (b) $\frac{1}{2}(a^2+b^2+c^2)$ (c) $a^2-b^2+c^2$ (d) $\frac{3}{\Delta}$

66. The number of real solutions of $(x, y)$, where $y=|\sin x|$, $y=\cos^{-1}(\cos x)$, $-2\pi\le x\le 2\pi$, is

(a) 2 (b) 1 (c) 3 (d) 4

67. If $p,q$ and $r$ are simple propositions, then $(p\wedge q)\wedge(q\wedge r)$ is true, then

(a) $p, q$ and $r$ are all false (b) $p, q$ and $r$ are all true

(c) $p$ and $q$ are true and $r$ is false (d) $p$ is true and $q$ and $r$ are false

68. Probability of product of a perfect square when two dice are thrown together

(a) $\frac{1}{9}$ (b) $\frac{2}{13}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$

69. The minimum value of $|z_1-z_2|$ and $z_1$ and $z_2$ vary over the curve $|\sqrt{3}(1-2z)+2i|=2\sqrt{7}$ and $|\sqrt{3}(-1-z)-2i|=|\sqrt{3}(9-z)+18i|$, respectively.

(a) $\frac{7\sqrt{7}}{2\sqrt{3}}$ (b) $\frac{5\sqrt{7}}{2\sqrt{3}}$ (c) $\frac{14\sqrt{7}}{\sqrt{3}}$ (d) $\frac{7\sqrt{7}}{5\sqrt{3}}$

70. There are 10 girls and 8 boys in a classroom including Mr Ravi, Ms Rani and Ms Radha. A list of speakers consisting of 8 girls and 6 boys has to be prepared Mr Ravi refuses to speak. If Mr Ravi refuses to speak, if Ms Rani is a speaker. The number of ways the list can be prepared is

(a) 202 (b) 308 (c) 567 (d) 952

71. The value of the integral $\int_0^2 \frac{\log(x^2+2)}{(x+2)^2}dx$ is

(a) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\frac{5}{12}\log 2-\frac{1}{4}\log 3$ (b) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\frac{5}{12}\log 2-\frac{1}{12}\log 3$

(c) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}+\frac{5}{12}\log 2+\frac{1}{4}\log 3$ (d) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}-\frac{5}{12}\log 2+\frac{1}{\sqrt{12}}\log 3$

72. Let $f: R \to R$ be defined by $f(x)=2x+\sin x$ for $x \in R$. Then, $f$ is

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

73. Let $M$ be a $3\times 3$ non-singular matrix with $|M|=\alpha$. If $M^{-1}\,\text{adj}\,(\text{adj}\,M)=kI$, then the value of $k$ is

(a) 1 (b) $\alpha$ (c) $\alpha^2$ (d) $\alpha^3$

74. If $y^x - x^y = 1$, then the value of $\frac{dy}{dx}$ at $x=1$ is

(a) $2(1-\log 2)$ (b) $2(1+\log 2)$ (c) $2-\log 2$ (d) $2+\log 2$

75. In the interval $\left[0,\frac{\pi}{2}\right]$, the equation $\cos^2 x-\cos x-x=0$ has

(a) no solution (b) exactly one solution

(c) exactly two solutions (d) more than two solutions

76. Let $f:(0,1)\to(0,1)$ be a differential function such that $f'(x)\neq 0$ for all $x\in(0,1)$ and $f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$.

Suppose for all $x$,

$$\lim_{t\to x}\frac{\int_0^t \sqrt{1-\{f(S)\}^2}\,dS-\int_0^x \sqrt{1-\{f(S)\}^2}\,dx}{f(t)-f(x)}=f(x).$$

Then, the value of $f\left(\frac{1}{4}\right)$ belongs to

(a) $\{\sqrt{7},\sqrt{15}\}$ (b) $\left\{\frac{\sqrt{7}}{2},\frac{\sqrt{15}}{2}\right\}$ (c) $\left\{\frac{\sqrt{7}}{3},\frac{\sqrt{15}}{3}\right\}$ (d) $\left\{\frac{\sqrt{7}}{4},\frac{\sqrt{15}}{4}\right\}$

77. The set of all $2\times 2$ matrices which commute with the matrix $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ with respect to matrix multiplication is

(a) $\left\{\begin{bmatrix}a & b\\ c & a-b\end{bmatrix};a,b,c\in R\right\}$ (b) $\left\{\begin{bmatrix}a & b\\ b & c\end{bmatrix};a,b,c\in R\right\}$

(c) $\left\{\begin{bmatrix}a-b & a\\ b & c\end{bmatrix};a,b,c\in R\right\}$ (d) $\left\{\begin{bmatrix}a & b\\ b & a-b\end{bmatrix};a,b\in R\right\}$

78. Let, $a$ be a non-zero real number and $\alpha,\beta$ be the roots of the equation $ax^2+5x+2=0$. Then, the absolute value of the difference of the roots of the equation

$$a^3(x+5)^2-25a(x+5)+50=0 \text{ is}$$

(a) $|\alpha^2-\beta^2|$ (b) $|\alpha\beta(\alpha^2-\beta^2)|$ (c) $\left|\frac{\alpha^2-\beta^2}{\alpha\beta}\right|$ (d) $\left|\frac{\alpha^2-\beta^2}{\alpha^2\beta^2}\right|$

79. Suppose an ellipse and a hyperbola have the same pair of foci on the $x$-axis with centres at the origin and that they intersect at (2, 2). If the eccentricity of the ellipse is $\frac{1}{2}$, then the eccentricity of the hyperbola is

(a) $\sqrt{\frac{7}{4}}$ (b) $\sqrt{\frac{7}{3}}$ (c) $\sqrt{\frac{5}{4}}$ (d) $\sqrt{\frac{5}{3}}$

80. The equation of the circle which cuts each of the three circles $x^2 + y^2 = 4, (x-1)^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ orthogonally is

(a) $x^2 + y^2 + x + 2y + 4 = 0$ (b) $x^2 + y^2 + x - 2y + 4 = 0$
(c) $x^2 + y^2 - x - 2y + 4 = 0$ (d) $x^2 + y^2 - x + 2y + 4 = 0$

81. The number of solutions of the equation $\cos^2\left(x+\frac{\pi}{6}\right)+\cos^2 x - 2\cos\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{6} = \sin^2\frac{\pi}{6}$ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(a) 0 (b) 1 (c) 2 (d) 3

82. Let $f(x) = \begin{cases} |x|, & \text{for } 0 < |x| \le 2 \\ 1, & \text{for } x = 0 \end{cases}$. Then, at $x = 0$, $f$ has

(a) a local maximum (b) no local minimum (c) a local minimum (d) no extremum

83. The value of the sum of the series $3 \cdot {}^nC_0 - 8 \cdot {}^nC_1 + 13 \cdot {}^nC_2 - 18\,{}^nC_3 + \ldots$

(a) 0 (b) $3^n$ (c) $5^n$ (d) None of these

84. Let $f(0) = \frac{1}{\tan^9 0}\{(1+\tan\theta)^{10} + (2+\tan\theta)^{10} + \ldots + (20+\tan\theta)^{10}\} - 20\tan\theta$. The left hand limit of $f(0)$ as $0 \to \frac{\pi}{2}$ is

(a) 1900 (b) 2000 (c) 2100 (d) 2200

**Directions** (Q. Nos. 85 and 86) *Let $ABC$ be a triangle, $R$ be the circumradius of the triangle. Also, given $R^2 = \frac{1}{8}(a^2 + b^2 + c^2)$, then*

85. Which of the following is true?

(a) $\sum \cos 2A = -1$ (b) $\sum \cos 2A = 1$ (c) $\sum \sin 2A = 1$ (d) $\sum \sin 2A = -1$

86. Hence, the $\Delta ABC$ can be

(a) equilateral (b) isosceles and scalene
(c) cannot say (d) right angled

**Directions** (Q. Nos. 87 and 88) *Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three non-coplanar vectors, i.e., $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \neq 0$. The three new vectors $\mathbf{a}'$, $\mathbf{b}'$ and $\mathbf{c}'$ defined by the equation*

$\mathbf{a}' = \frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$, $\mathbf{b}' = \frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$ *and* $\mathbf{c}' = \frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$ *are called reciprocal system to the vectors* $\mathbf{a}, \mathbf{b}$ *and* $\mathbf{c}$.

87. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ are reciprocal system of vectors, then the value of $\mathbf{a}\times\mathbf{a}' + \mathbf{b}\times\mathbf{b}' + \mathbf{c}\times\mathbf{c}'$ is

(a) $2(\mathbf{a}\times\mathbf{b}\times\mathbf{c})$ (b) $2(\mathbf{a}'\times\mathbf{b}'\times\mathbf{c}')$
(c) $\frac{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}{2}$ (d) $\mathbf{0}$

88. The reciprocal set of the vetors $2\mathbf{i}+3\mathbf{j}-\mathbf{k}, \mathbf{i}-\mathbf{j}-2\mathbf{k}$ and $-\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ are

(a) $\frac{2\mathbf{i}+\mathbf{k}}{3}, \frac{-8\mathbf{i}+3\mathbf{j}-7\mathbf{k}}{3}, \frac{7\mathbf{i}+3\mathbf{j}+5\mathbf{k}}{3}$ (b) $\frac{2\mathbf{i}+\mathbf{k}}{3}, \frac{-8\mathbf{i}+3\mathbf{j}+7\mathbf{k}}{3}, \frac{7\mathbf{i}+3\mathbf{j}+5\mathbf{k}}{3}$

(c) $\frac{2\mathbf{i}+\mathbf{k}}{3}, \frac{-8\mathbf{i}+3\mathbf{j}+7\mathbf{k}}{3}, \frac{7\mathbf{i}-3\mathbf{j}+5\mathbf{k}}{3}$ (d) $\frac{2\mathbf{i}+\mathbf{k}}{3}, \frac{-8\mathbf{i}+3\mathbf{j}-7\mathbf{k}}{3}, \frac{-7\mathbf{i}+3\mathbf{j}-5\mathbf{k}}{3}$

**Directions** (Q. Nos. 89 and 90) *For the following questions, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

89. Let us define the function as $\cos^{-1}(\cos\theta)=\theta$ and $2\tan^{-1}x=\frac{2x}{1-x^2}$.

**Statement I** If $\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}]=0$, then $x=\pm1,\pm(1\pm\sqrt{2})$

**Statement II** $\cot(2\tan^{-1}x)=\frac{1-x^2}{2x}$

90. Consider the identity $\frac{\sin\frac{\theta}{2}-\sin\frac{\phi}{2}}{\cos\frac{\theta}{2}+\cos\frac{\phi}{2}}=\tan\frac{\theta-\phi}{4}$.

**Statement I** $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n=\begin{cases}2\cot^n\frac{A-B}{2}, & \text{if } n \text{ is odd}\\ 0, & \text{if } n \text{ is even}\end{cases}$.

**Statement II** $\frac{\cos A+\cos B}{\sin A-\sin B}=\cot\frac{A-B}{2}$.

# Answer *with* Explanations

## Physics

**1.** (c) Taking $x$-components, the total should be zero.

$1 \times \cos 60° + 2 \cos 60° + x - 4 \cos 60° = 0$

$\therefore \quad x = 0.5\,\text{N}$

This is the required force in $x$-direction.

**2.** (d) Mass of the disc removed

$$= \frac{M}{\pi R^2} \times \pi \left\{\frac{R}{2}\right\}^2 = \frac{M}{4}$$

Remaining mass $= M - \frac{M}{4} = \frac{3M}{4}$

Let, the origin of the coordinate system coincide with centre of mass of whole disc. Now we know that,

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$X_{CM}$ will be zero, when

$$m_2 x_2 = -m_1 x_1$$

$$\therefore \quad x_2 = \frac{m_1}{m_2} x_1$$

Here, $\quad m_1 = \frac{M}{4}, x_1 = \frac{R}{2}$

and $\quad m_2 = \frac{3M}{4}$ (for remaining mass)

Hence, $\quad x_2 = -\frac{M/4}{3M/4} \cdot \frac{R}{2} = \frac{-R}{6}$

*i.e.*, $\frac{R}{6}$ from the centre (on LHS).

**3.** (c) Let, $d_w$ and $d_o$ be the densities of water and oil, then the pressure at the bottom of the tank

$$= h_w d_w g + h_o d_o g$$

Let, this pressure be equivalent to pressure due to water of height $h$. Then,

$$h d_w g = h_w d_w g + h_o d_o g$$

$$\therefore \quad h = h_w + \frac{h_o d_o}{d_w}$$

$$= 100 + \frac{400 \times 0.9}{1}$$

$$= 100 + 360 = 460$$

According to Toricelli's theorem,

$$v = \sqrt{2gh} = \sqrt{2 \times 980 \times 460}\ \text{cm/s}$$

$$= \sqrt{920 \times 980}\ \text{cm/s}$$

**4.** (b) As total charge $= Q = q + q'$

$$\therefore \quad q' = (Q - q)$$

$\therefore$ Force between the charges $q$ and $q'$

$$F = \frac{1}{\pi \varepsilon_0} \frac{Q(Q-q)}{R^2}$$

For maximum value of $F$

$$\frac{dF}{dq} = 0 \text{ or } \frac{d}{dq}\left[\frac{Kq(Q-q)}{R^2}\right] = 0$$

$$\Rightarrow \quad [Q - 2q] = 0$$

or $\quad q = \frac{Q}{2}$

**5.** (a) $\quad U = \frac{Q^2}{2C}$

Now, $\quad C' = KC$

As battery is disconnected, $Q$ remains unaltered. So,

$$U' = \frac{Q^2}{2C'} = \frac{1}{2}\frac{Q^2}{KC}$$

$$\therefore \quad \frac{U}{U'} = \frac{Q^2/2C}{Q^2/2KC} = K$$

**6.** (d) For minimum resultant, the $n$ resistors are connected in parallel

$$R_{min} = \frac{r}{n}$$

For maximum resultant, the $n$ resistors are connected in series,

$$R_{max} = nr$$

$$\therefore \quad \frac{R_{min}}{R_{max}} = \frac{r}{n} \times \frac{1}{nr} = \frac{1}{n^2}$$

**7.** (a) Area of the coil $= 0.1 \times 0.05 = 5 \times 10^{-3}\ \text{m}^2$,
Change in magnetic field $= 0.05 - 0.1 = -0.05\ \text{T}$
Therefore, change in flux is

$$\Delta\phi = -0.05 \times 5 \times 10^{-3}$$

$$= -0.25 \times 10^{-3}\ \text{Wb}$$

Induced emf for one turn $e = -\frac{\Delta\phi}{\Delta t}$

$$= \frac{0.25 \times 10^{-3}\ \text{Wb}}{0.05\ \text{s}} = 5 \times 10^{-3}\ \text{V}$$

Induced emf for 100 turns (in magnitude)

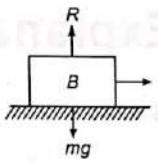
$$= 100\left(\frac{\Delta\phi}{\Delta t}\right)$$

$$= 5 \times 10^{-3} \times 100 = 0.5\ \text{V}$$

**8.** (c) As the bomb is released directly above the target, the distance by which the bomb will miss the target must be equal to its horizontal range, *i.e.*,

$$R = v\sqrt{\frac{2h}{g}}$$

**9.** (d) Given, $u = v$, final velocity = 0




Using, $v = u + at$

$0 = v - at$

$\therefore \quad -a = \frac{0 - v}{t} = -\frac{v}{t}$

$f = \mu R = \mu mg$

($f$ is the force of friction)

Retardation, $a = \mu g$

$\therefore \quad t = \frac{v}{a} = \frac{v}{g\mu}$

**10.** (c) We have, $\eta = \frac{P_0}{P_i}$

$\therefore$ Output energy = 16 J

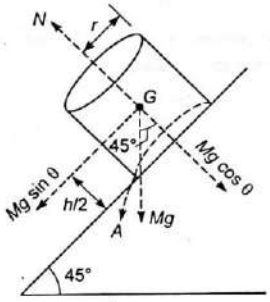
By conservation of energy,

$\frac{1}{2}mv^2 = 16$ J

$v = 4\ \text{ms}^{-1}$

**11.** (c) The cylinder will just begin to slip and will not topple, if moment of force $Mg \sin\theta$ about $A$

= Moment of $N$ about $A$




$Mg \sin\theta \times \frac{h}{2} = Mg \cos\theta \times r$

$\tan\theta = \frac{r}{h/2}$

As $\theta = 45°$, hence, $r = \frac{h}{2}$

*i.e.*, Vertical line through centre of gravity makes angle $\theta = 45°$ with the normal to plane.

**12.** (a) By Doppler's formula

$n' = \frac{v}{v - v_s} \times n$

$\Rightarrow \quad \frac{n'}{n} = \frac{v}{v - v_s}$

Now adding 1 to both the sides.

$\frac{\Delta n}{n} = \frac{v_s}{v - v_s}$

$\frac{2.5}{100} = \frac{v_s}{320 - v_s} = \frac{1}{40}$

or $\quad 40v_s = 320 - v_s$

or $\quad v_s = \frac{320}{41} \simeq 8$ m/s

**13.** (c) Let $Q$ and $q$ be the charges on the spheres. The potential at the common centre will be

$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R}\right] + \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}\right]$

$= \frac{1}{\varepsilon_0}\left[\frac{Q}{4\pi R^2} \times R + \frac{q}{4\pi r^2} \times r\right]$

But $\quad \frac{Q}{4\pi R^2} = \frac{q}{4\pi r^2} = \sigma$

$\therefore \quad V = \frac{1}{\varepsilon_0}[\sigma R + \sigma r] = \frac{\sigma}{\varepsilon_0}[R + r]$

**14.** (c) We have,

$N_S = \left(\frac{\rho_S}{\rho_P}\right) \times N_P$

$= \frac{(4.4 \times 10^3) \times 100}{220} = 2000$

Here, $\rho_S$ and $\rho_P$ are see secondary and primary voltages of the transformer.

**15.** (c) Taking downward direction as positive, we get

$-h = -ut_1 + \frac{1}{2}gt_1^2 \quad \ldots(i)$

$-h = -ut_2 + \frac{1}{2}gt_2^2 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

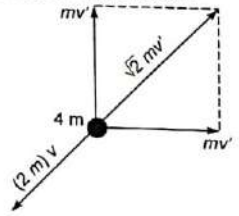
$0 = u(t_2 - t_1) + \frac{1}{2}g(t_1^2 - t_2^2)$

$u = \frac{1}{2}g(t_1 + t_2) \quad \ldots(iii)$

Substituting value of $u$ from Eq. (iii) in Eq. (i), we get

$h = \frac{gt_1t_2}{2}$

**16.** (b) The resultant momentum of two particles which fly-off at right angles is $\sqrt{2}\,mv$. By law of conservation of momentum




$\sqrt{2}\,mv' = (2m)\,v$

$\Rightarrow \quad v' = \frac{v}{\sqrt{2}}$

Total energy released $E$ is the sum of kinetic energies of all the three particles

$E = \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}(2m)v^2$

$= mv'^2 + mv^2$

$\Rightarrow \quad m\left(\frac{v}{\sqrt{2}}\right)^2 + mv^2 = \frac{3}{2}mv^2$

**17.** (d) Let the common acceleration of the system of masses is a

Then, for mass 12 kg

$$T_1 - T_2 = 12a \quad \ldots(i)$$

And for the mass 3 kg

$$T_2 = 3a \quad \ldots(ii)$$

Put the value of $T_2$ from Eq. (ii), in Eq (i)

$$T_1 = 15a$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{15a}{3a} = \frac{5}{1}$$

$$\Rightarrow \quad T_1 : T_2 = 5 : 1$$

**18.** (c) Tension in the chord is

$T = M\left(g - \frac{g}{4}\right) = \frac{3Mg}{4}$ in upward direction, since Chord is displaced in the downward direction, so

$$W = \mathbf{T} \cdot \mathbf{d} \Rightarrow W = Td$$

$$\Rightarrow \quad W = -Td = -\frac{3Mgd}{4}$$

Here, negative sign shows that $T$ and $d$ are in the opposite directions.

**19.** (c) By law of conservation of angular momentum

$$\Sigma mvr = (I_{system})\,\omega$$

$$\Rightarrow \quad mv\frac{l}{2} = \frac{(2m)(2l)^2}{12}\omega = \frac{2m(4l^2)}{12}\omega$$

$$\Rightarrow \quad \omega = \frac{3v}{4l} \text{ (anti-clockwise)}$$

Not that clockwise or anti-clockwise rotation can only be determined here by the given figure.

**20.** (d) $T = 2\pi\sqrt{\frac{L}{g}}$

where $L$ is the length of the liquid is one of the limbs. However, if $L$ is taken to be the length of the liquid column, then length of liquid in each limb is $\frac{L}{2}$.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{L}{2g}}$$

$$\Rightarrow \quad M = (AL)\,d \Rightarrow L = \frac{M}{Ad}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{M}{2Adg}}$$

**21.** (b) As, in $\Delta DBS$, $SD = \sqrt{60^2 + 25^2}$

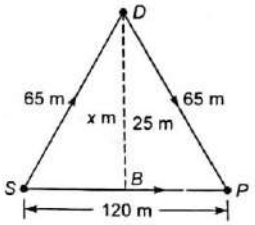
$$= \sqrt{4225} = 65 = DP$$

Path difference $\Delta x = (SA + AP) - SP$

$$\Rightarrow \quad \Delta x = (65 + 65) - 120$$

$$\Rightarrow \quad \Delta x = 10\,\text{m}$$

But at $A$, the wave suffers reflection at the surface of rigid/fixed end or denser medium, hence the wave must suffer an additional path change of $\frac{\lambda}{2}$ or a phase change of $\pi$.




Net path difference $= \left(10 - \frac{\lambda}{2}\right)$

For maximum, net path difference $= (2n)\frac{\lambda}{2}$; $n = 0, 1, 2, \ldots$

$$10 - \frac{\lambda}{2} = (2n)\frac{\lambda}{2},\ n = 0, 1, 2, \ldots$$

or $\quad 10 = (2n+1)\frac{\lambda}{2},\ n = 0, 1, 2, \ldots$

$$\Rightarrow \quad \lambda = \frac{20}{2n+1},\ n = 0, 1, 2, \ldots$$

$$\Rightarrow \quad \lambda = 20, \frac{20}{3}, \frac{20}{5}, \frac{20}{7}, \ldots$$

**22.** (c) If $\phi$ is true angle of dip, then

$$\tan\phi = \frac{V}{H} \quad \ldots(i)$$

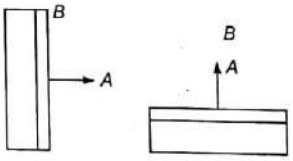
When the dip circle is rotated in the horizontal plane through an angle $\theta$ from the magnetic meredian, the effective horizontal component in the new plane becomes $H' = H\cos\theta$, while the vertical component remains the same. If $\phi'$ is apparent dip, then

$$\tan\phi = \frac{V}{H'} = \frac{V}{H\cos\theta} \quad \ldots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\tan\phi'}{\tan\phi} = \frac{V/H\cos\theta}{\frac{V}{H}} = \frac{1}{\cos\theta}$$

**23.** (d)




$\phi_1$ = Flux passing through the area of the coil when it is far to magnetic field = $BA$

and $\phi_2$ = Flux passing through the area of the coil when it is || to the magnetic field = 0

$$\therefore \quad |\xi| = \left|\frac{-\Delta\phi}{\Delta t}\right| = \left|-\left(\frac{0 - BA}{\frac{T}{4} - 0}\right)\right| = \frac{4BA}{T} \qquad (\Delta\phi = \phi_2 - \phi_1)$$

But $\quad T = \frac{2\pi}{\omega}$

$$\Rightarrow \quad \xi = \frac{4BA\omega}{2\pi} = \frac{2BA\omega}{\pi}$$

**24.** (a) The intensity of light of the location of your eye is,

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi \times 4^2}\ \text{W/m}^2$$

The energy entering into your eye per second is

$$P_1 = I \times \frac{\pi d^2}{4}$$

where $d$ is diameter of pupil

$$P_1 = \frac{60}{4\pi \times 4^2} \times \frac{\pi \times (2\times 10^{-3})^2}{4}$$

$$= 9.375 \times 10^{-7}\ \text{J/s}$$

Let $n$ be the number of photons entering into the eye per sec, then

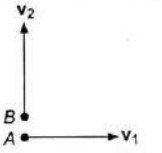
$$P_1 = n \times \frac{hc}{\lambda}$$

$$\Rightarrow \quad 9.375 \times 10^{-7} = n \times \frac{1240 \times 1.6 \times 10^{-19}}{600}$$

$$\Rightarrow \quad n = 2.84 \times 10^{12}\ \text{photons/s}$$

So, the number of photons entering the eye in 0.1 s, is $0.1\,n = 2.84 \times 10^{11}$.

**25.** (b) Let, $m$ is the mass of each particle, then $\lambda_1 = \frac{h}{mv_1}$ at $\lambda_2 = \frac{h}{mv_2}$, where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of two particles as shown in figure.




$$\mathbf{v}_{CM} = \frac{m\mathbf{v}_1 + m\mathbf{v}_2}{2m} = \frac{\mathbf{v}_1 + \mathbf{v}_2}{2}$$

Velocity of $A$ w.r.t. $C$ frame is,

$$\mathbf{v}_{1c} = \mathbf{v}_1 - \mathbf{v}_{CM} = \frac{\mathbf{v}_1 - \mathbf{v}_2}{2}$$

$$|\mathbf{v}_{1c}| = \frac{\sqrt{v_1^2 + v_2^2}}{2} = |\mathbf{v}_{2c}|$$

(as angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is 90°)

So, required wavelength is

$$\lambda = \frac{h}{m|\mathbf{v}_{1c}|} = \frac{h}{m} \times \frac{2}{\frac{h}{m}\sqrt{\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}}}$$

$$\left(\text{as } \lambda = \frac{h}{mv} \therefore v^2 = \frac{h^2}{m^2\lambda^2}\right)$$

$$\Rightarrow \quad \lambda = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

**26.** (b) The cut-off wavelength when $V = V_1 = 10$ kV is,

$$\lambda_1 = \frac{hc}{eV_1} = 1243.125 \times 10^{-13}\ \text{m} \qquad \ldots(\text{i})$$

The cut-off wavelength when $V = V_2 = 20$ kV is,

$$\lambda_2 = \frac{hc}{eV_2} = 621.56 \times 10^{-13}\ \text{m} \qquad \ldots(\text{ii})$$

The wavelength corresponding to $K_\alpha$ line is,

$$\frac{1}{\lambda} = \frac{3R}{4}(Z-1)^2 \qquad \ldots(\text{iii})$$

From given information, $(\lambda - \lambda_2) = 3(\lambda - \lambda_1)$

Solving above equation, we get $Z = 29$.

with the help of Eqs. (i), (ii) and (iii).

**27.** (b) The total energy required to make the electron free from nucleus is the sum of the energy required to separate the electrons from the influence of each other and the energy required to separate the electrons from the influence of nucleus, *i.e.*,

Total energy required

= BE of electron in He atom + Ionization energy of He-atom

$$= (24.6 + 13.6 \times 2^2)\ \text{eV}$$

$$= (24.6 + 54.4)\ \text{eV} = 79\ \text{eV}$$

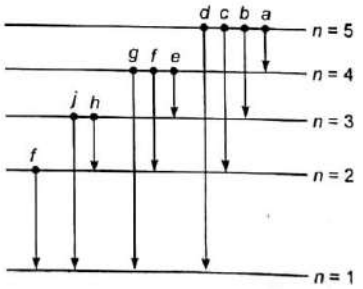
**28.** (b) The separation between source and photosensitive material at $t = 0$ is 16 m, therefore, intensity received by photosensitive material at $t = 0$ is $I_1 = \frac{P}{4\pi \times 16^2}$, where $P$ is the power of source of light.

At $t = 3$ s, the source is at (15, 0) and detector is at (19, 0), so the separation between them is 4 m.

$$I_2 = \frac{P}{4\pi \times 4^2}$$

So, $\quad \frac{I_1}{I_2} = 1:16$

**29.** (b) The wavelengths present in emission spectra are shown in the figure below

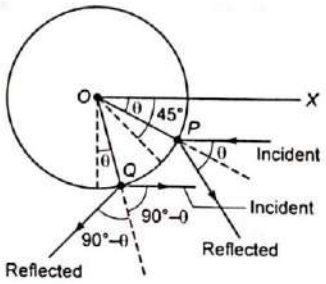



$a, e, h, f \rightarrow$ one atom

$b, j \rightarrow$ one atom

Remaining all by individual atoms.

**30.** (b) Here, due to the change in momentum imparted to the sphere, the sphere experiences a force due to incident light. Here, first of all, we see that what is the momentum after reflection in three different cases.




Consider two elements of small area $\Delta A$ at $P$ and $Q$ as shown in the figure. Let, reflection coefficient for the shown case be $r$.

At $P$, $\frac{\Delta P_f}{\Delta t} = r \times \frac{I}{c} \times \Delta A \cos 20$ along $OX$

At $Q$, $\frac{\Delta P_f}{\Delta t} = r \times \frac{I}{c} \times \Delta A \cos 20$ along $XO$

So, in horizontal direction, rate of change in final momentum is zero and hence due to reflected light force experienced by sphere is zero. In case of perfectly absorbing surface no reflection is there and in this case also, the force experienced by sphere in horizontal direction is only due to incident light.

In vertical direction also, the momentum imparted to sphere due to reflected light is zero due to cancellation of lower and upper haves.

Thus, we can say that force experienced by sphere is only due to incident light and hence is same in all the three cases, *i.e.*, *a* and *c* are wrong.

To compute the value of force experienced by the sphere due to incident light we can use the concept of projection.

$$F = \frac{I}{c} \times \pi R^2$$

# Chemistry

**31.** (a) $AB_2(g) + A(s) \rightleftharpoons 2AB(g)$

$0.7 - x \qquad 2x$

$\therefore \quad 0.7 - x + 2x = 0.95$

$x = 0.25$

$$K_p = \frac{(p_{AB})^2}{(p_{AB_2})} = \frac{(0.25 \times 2)^2}{(0.7 - 0.25)} = 0.555$$

Now, $AB_2(g) + A(s) \rightleftharpoons 2AB$

$1 - x \qquad 2x$

$$K_p = \frac{(p_{AB})^2}{p_{AB_2}} = \frac{\left(\frac{2x}{1+x} p_T\right)^2}{\left(\frac{1-x}{1+x}\right) p_T}$$

or $$0.555 = \frac{(2x)^2}{1-x^2} p_T$$

$$= \frac{(2x)^2 \times 0.4}{1-x^2}$$

$x = 0.507$

$x \approx 0.51$

$\therefore$ Volume % of $AB = \frac{2x}{1+x} \times 100 = 67.54\%$

**32.** (c) $2NH_3(g) \longrightarrow N_2(g) + 3H_2$

$76 - 2x \qquad x \qquad 3x$

$76 + 2x = 76 + 18$

$2x = 18$

$\therefore \quad x = 9$

$\therefore \quad p_{H_2} = 3 \times 9$

$= 27$ cm of Hg

**33.** (b) $\underset{(A)}{2Pb_3O_4} \xrightarrow{\Delta} \underset{(B)}{6PbO} + \underset{(C)}{O_2(g)}$

$\underset{(A)}{Pb_3O_4} + 4HNO_3 \longrightarrow \underset{(D)}{2Pb(NO_3)_2}$

$+ \underset{(E)}{PbO_2} + \underset{(F)}{2H_2O}$

Brown

$2Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 4NO_2(g) + \underset{(C)}{O_2(g)}$

**34.** (d) I contains plane of symmetry. Therefore, achiral (optically inactive).

**35.** (b) Species having the same number of bond pair and lone pair of electrons, have same shape and hybridisation.

$I_3^- = 2bp + 3lp = 5$ hybrid orbitals $\Rightarrow sp^3d$ hybridisation and planar geometry

$XeF_2 = 2bp + 3lp = 5$ hybrid orbitals $\Rightarrow sp^3d$ hybridisation and planar geometry.

**36.** (c) $E = h\nu$

where, $\bar{\nu} = R_H(1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$= \frac{3}{4} R_H$

$= h \frac{c}{\lambda}$

$= hc\bar{\nu} = 6.626 \times 10^{-34} \times 3 \times 10^8 \times \frac{3}{4} \times 1.097 \times 10^7$

$= 1.63 \times 10^{-18}$ J

KE of H atom $= \frac{3}{2} \frac{R}{N_A} T$

$$\frac{3}{2}\times\frac{8.314}{6\times10^{23}}\times T = 1.63\times10^{-18}$$

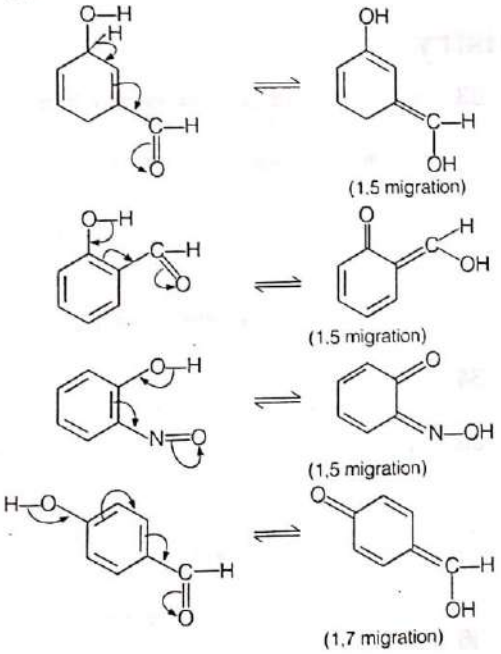
$$T = 7.84\times10^{4}\ \text{K}$$

**37.** (c) In $NCl_3$ back bonding is possible whereas in $NH_3$ and $NF_3$ back bonding is not possible due to the absence of vacant *d*-orbital.

Both $NH_3$ and $NF_3$ contain 1 lone pair but since F is more electronegative, the electron density about N-atom in $NF_3$ is less as compared to in $NH_3$ (107°). Hence, bond angle decreases in $NF_3$ (102°).

**38.** (c)
$$\frac{1}{p_T} = \frac{x_A}{p_A^0} + \frac{x_B}{p_B^0}$$
$$\frac{1}{p_T} = \frac{2/5}{100} + \frac{3/5}{60}$$
$$p_T = 71.42\ \text{torr}$$

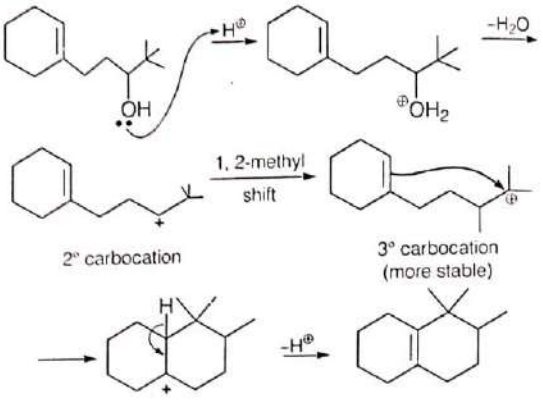
**39.** (d)



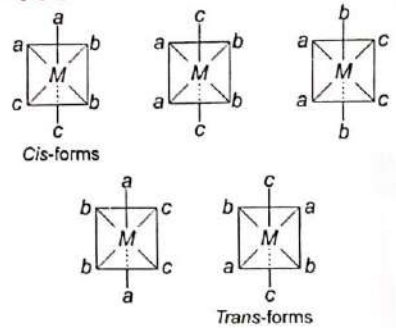

**40.** (c) In (a) only Aufbau principle is violated.
In (b) Aufbau principle and Pauli's principle is violated.
In (c) Aufbau principle and Hund's rule is violated.
In (d) Pauli's principle and Hund's rule is violated.

**41.** (d) (i) Solubility of hydroxides of alkali metals increases with increase in the size of alkali metal ion.
(ii) Thermal stability of alkaline earth metal sulphate increases on moving downwards due to their similar lattice energy and decrease in hydration energy.
(iii) Melting point ∝ ionic character.
(iv) Solubility of carbonates of alkali metal increases on moving downwards.

**42.** (c)




**43.** (b) The possible geometrical isomers of the formula $Ma_2b_2c_2$ are.




**44.** (d) Both II and III are the *trans*-forms of I but not the same.

**45.** (a) Number of geometrical isomers (as in the molecule ends groups about doubly bonded carbon atom are same. Hence, this bond is not considered in geometrical isomerism.)

$$= 2^n = 2^2 = 4$$

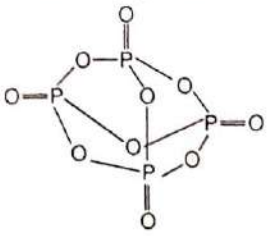
**46.** (b) $\text{Rate} = k_2[C]$

Now, $k_1 = \dfrac{[C]}{[A]^2}$

$\therefore\quad [C] = k_1[A]^2$

$\therefore\quad \text{Rate} = k_2 \cdot k_1[A]^2$

**47.** (b) The structure of $P_4O_{10}$ is as



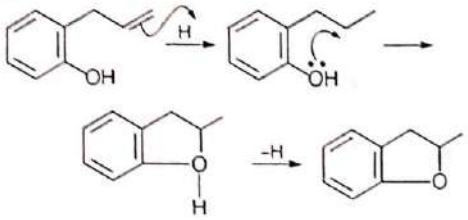
Therefore each P atom is bonded with four oxygen atom.

$p\pi$-$d\pi$ back bonding is found in the P=O of $P_4O_{10}$.

It forms orthophosphoric acid when hydrolysed

$$P_4O_{10} + 6H_2O \longrightarrow 4H_3PO_4$$

**48.** (b)




**49.** (c) Graph having same $M/T$ value will have same profile curve

$$\therefore \quad \frac{(u_{rms})_{O_2}}{(u_{rms})_{SO_2}} = \sqrt{\frac{T_{O_2} \cdot M_{SO_2}}{T_{SO_2} \cdot M_{O_2}}}$$

$$= \sqrt{\frac{T \times 64}{2T \times 32}}$$

$$= \sqrt{1} = 1$$

$$\therefore \quad (u_{rms})_{O_2} = (u_{rms})_{SO_2}$$

Hence, they have the same profile curve.

**50.** (a)

$$k_2 = Ae^{-E_{a_2}/RT}$$

$$k_1 = Ae^{-E_{a_1}/RT}$$

$$\frac{k_2}{k_1} = e^{(-E_{a_2} + E_{a_1})/RT}$$

(Since, $E_{a_2} = 2E_{a_1}$)

$$\frac{k_2}{k_1} = e^{-E_{a_1}/RT}$$

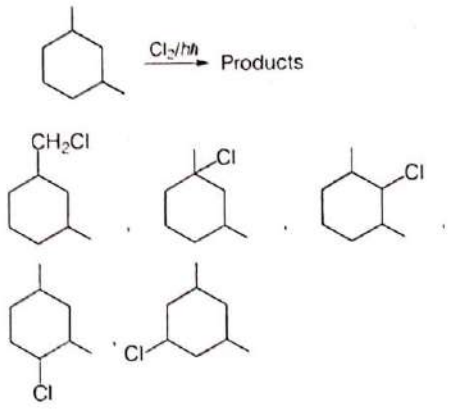
or $$k_2 = k_1 e^{-E_{a_1}/RT}$$

**51.** (b) $I_2O_5$ is used to estimate CO in automobile exhaust gases as it oxidises CO to $CO_2$ quantitatively liberating iodine, which can be titrated with hypo.

**52.** (a) During electrolysis of concentrated aqueous solution of NaOH, chlorine gas is released at anode and $H_2$ gas is liberated at cathode leaving $Na^+$ and $OH^\ominus$ ions in the solution, *i.e.*, $[OH^-]$ concentration increases. Therefore, pH increases.

**53.** (c) In (I) and (II) the hydrogen atom of $-CH_3$ group is not attacked by the base rather more acidic H-atom about two carbonyl group in (I) and methylene H-atom about phenyl group in (II) is attacked.

**54.** (b) Number of monochlorinated products depends upon the type of H-atoms present in the molecule.




**55.** (b) Since O-atom form fcc unit cell, the number of O atoms per unit cell = $6 \times \frac{1}{2}$ (face atoms) + $8 \times \frac{1}{8}$ (corner atoms) $= 4$

Number of *B* atoms at octahedral voids = Number of O-atoms = 4

Number of *A* atoms at tetrahedral voids = 2 × Number of O-atoms = 2 × 4 = 8

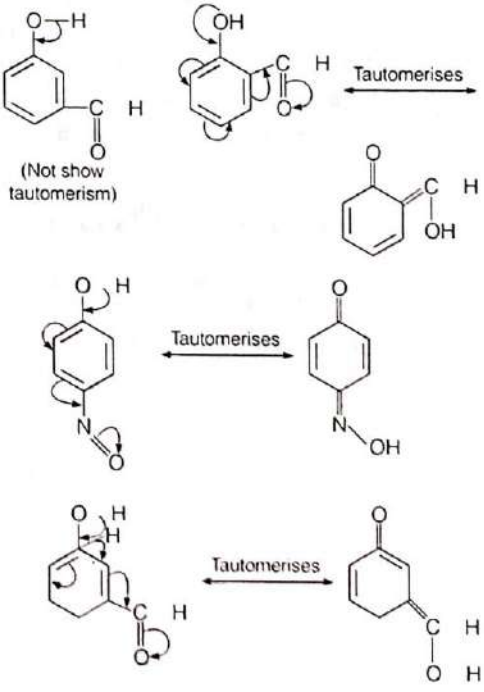
Initial formula of compound is $A_8B_4O_4$

After removal about two body diagonal

$$A_{(8-1\times4)}B_{(4-1\times1)}O_{\left(4-\frac{1}{8}\times4\right)}$$

$A_4B_3O_{7/2}$ or $A_8B_6O_7$

**56.** (c)



**57.** (d) $NaH_2PO_4$ (100 mL of 0.1 M)

↓

75 mL of 0.1 M $Na_3PO_4$ added

↓

25 ml of 0.1 M HCl added

| | $Na_3PO_4$ | + HCl → | $Na_2HPO_4$ | + NaCl |
|---|---|---|---|---|
| M moles | 75 × 0.1 | 25 × 0.1 | 0 | 0 |
| added | 7.5 | 2.5 | | |
| After reaction | 7.5 – 2.5 = 5 | 0 | 2.5 | |

It forms buffer solution of

| | $NaH_2PO_4$ | and | $Na_2HPO_4$ |
|---|---|---|---|
| M moles | 100 × 0.1 = 10 | | 2.5 |

$$pH = pK_{a_2} + \log \frac{[salt]}{[acid]}$$

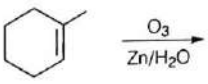
$$= -\log(10^{-6}) + \log \frac{[Na_2HPO_4]}{[NaH_2PO_4]}$$

$$= 6 + \log \frac{2.5}{10}$$

$$= 6 - 2\log 2 = 5.398$$

**58.** (c) (a) and (d) contain plane of symmetry. Hence, achiral. (b) contains centre of symmetry as well as plane of symmetry. Hence, achiral. (c) because of the absence of symmetry elements, is chiral so optically active.

**59.** (d) Higher the magnitude of opposite charge, higher is the coagulating power and lower is the flocculating value. Thus, flocculating value depends upon both the nature and magnitude of charge of flocculating ion.

**60.** (b) 


$$CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-CH_2-CH_2-CHO$$

The product contains 'α' hydrogen as well as CHO group. Hence, can undergo intramolecular aldol condensation.

# Mathematics

**61.** (a) $k\mathbf{r} + \mathbf{r} \times \mathbf{a} = \mathbf{b}$ ...(i)

Taking dot product with $\mathbf{a}$ in Eq. (i)

$$k(\mathbf{a}\cdot\mathbf{r}) + \mathbf{a}\cdot(\mathbf{r}\times\mathbf{a}) = \mathbf{a}\cdot\mathbf{b}$$

$$\Rightarrow \quad k(\mathbf{a}\cdot\mathbf{r}) + 0 = \mathbf{a}\cdot\mathbf{b} \quad \text{...(ii)}$$

$[\because [\mathbf{a}\,\mathbf{r}\,\mathbf{a}] = 0]$

Taking cross product with $\mathbf{a}$ in Eq. (i), we get

$$k(\mathbf{a}\times\mathbf{r}) + (\mathbf{a}\cdot\mathbf{a})\mathbf{r} - (\mathbf{a}\cdot\mathbf{r})\mathbf{a} = \mathbf{a}\times\mathbf{b}$$

$$\Rightarrow \quad k(k\mathbf{r} - \mathbf{b}) + \mathbf{a}^2\mathbf{r} - \frac{\mathbf{a}\cdot\mathbf{b}}{k}\mathbf{a} = \mathbf{a}\times\mathbf{b}$$

[from Eqs. (i) and (ii)]

$$\Rightarrow \quad (k^2 + \mathbf{a}^2)\mathbf{r} = k\mathbf{b} + \frac{\mathbf{a}\cdot\mathbf{b}}{k}\mathbf{a} + \mathbf{a}\times\mathbf{b}$$

$$\Rightarrow \quad \mathbf{r} = \frac{1}{k^2 + \mathbf{a}^2}\left(k\mathbf{b} + \frac{\mathbf{a}\cdot\mathbf{b}}{k}\mathbf{a} + \mathbf{a}\times\mathbf{b}\right)$$

**62.** (a) Let $\tan^{-1} e^{\theta} = A + iB$

$$\Rightarrow \quad e^{\theta} = \tan(A + iB)$$

$$\therefore \quad \tan^{-1} e^{-\theta} = A - iB$$

$$\Rightarrow \quad e^{-\theta} = \tan(A - iB)$$

Now, $\cos 2A = \cos[(A + iB) + (A - iB)]$

$$= \cos(A + iB)\cos(A - iB) - \sin(A + iB)\sin(A - iB)$$

$$= \cos(A + iB)\cos(A - iB)[1 - \tan(A + iB)\tan(A - iB)]$$

$$= \cos(A + iB)\cos(A - iB)[1 - e^{\theta}e^{-\theta}] = 0$$

$$\Rightarrow \quad 2A = (2n + 1)\frac{\pi}{2}$$

$$\Rightarrow \quad A = \frac{n\pi}{2} + \frac{\pi}{4}$$

Also, $\tan 2iB = \tan[(A + iB) - (A - iB)]$

$$= \frac{\tan(A + iB) - \tan(A - iB)}{1 + \tan(A + iB)\tan(A - iB)}$$

$$= \frac{e^{\theta} - e^{-\theta}}{1 + e^{\theta}e^{-\theta}} = i\sin\theta$$

$$\Rightarrow \quad \frac{e^{-2B} - e^{2B}}{i\cdot e^{-2B} + e^{2B}} = i\sin\theta$$

$$\Rightarrow \quad \frac{e^{2B} - e^{-2B}}{e^{2B} + e^{-2B}} = \sin\theta \qquad (\because i^2 = 1)$$

$$\Rightarrow \quad \frac{e^{2B}}{e^{-2B}} = \frac{1 + \sin\theta}{1 - \sin\theta}$$

(by using componendo and dividendo)

$$= \frac{1 + \cos\left(\frac{\pi}{2} - \theta\right)}{1 - \cos\left(\frac{\pi}{2} - \theta\right)}$$

$$\Rightarrow \quad e^{4B} = \frac{\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}$$

$$\Rightarrow \quad e^{2B} = \cot\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$= \cot\left\{\frac{\pi}{2} - \left(\frac{X}{4} + \frac{\theta}{2}\right)\right\}$$

$$= \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

$$\Rightarrow \quad B = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

$$\therefore \quad \tan^{-1} e^{i\theta} = \frac{n\pi}{2} + \frac{\pi}{4} + \frac{i}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

**63.** (a) $\sum_{i=1}^{n} \mathbf{a}_i = 0$

$$\left(\sum_{i=1}^{n} \mathbf{a}_i\right)\cdot\left(\sum_{i=1}^{n} \mathbf{a}_i\right) = \sum_{i=1}^{n} |\mathbf{a}_i^2| + 2\sum_{1\le i\le j\le n}\sum \mathbf{a}_i\,\mathbf{a}_j$$

$$\Rightarrow \quad 0 = n + 2\sum_{1\le i\le j\le n}\sum \mathbf{a}_i\,\mathbf{a}_j$$

$$\Rightarrow \quad \sum_{1\le i\le j\le n}\sum \mathbf{a}_i\,\mathbf{a}_j = -\frac{n}{2}$$

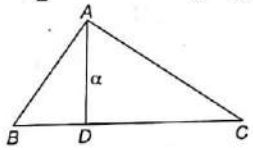
**64.** (a) $\because$ LHS $\le 2$ and RHS $\ge 2$

So, $\sin^2 x = 1, \cos^2 y = 1$ and $\sec^2 z = 1$

or $\cos^2 x = 0, \sin^2 y = 0$ and $\sin^2 z = 0$

$\therefore x = \left(n + \frac{1}{2}\right)\pi, y = m\pi$ and $z = r\pi$

**65.** (a) Clearly, $\frac{1}{2}\alpha\cdot a = \Delta \Rightarrow \frac{1}{\alpha} = \frac{a}{2\Delta}$




$$\therefore \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{a^2 + b^2 + c^2}{4\Delta^2}$$

$$\Rightarrow \quad 4\Delta^2\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right) = (a^2 + b^2 + c^2)$$

**66.** (c) For $x \in [0, \pi]$, $y = |\sin x|$, $y = \cos^{-1}(\cos x) = x$

For $x \in [\pi, 2\pi]$, $y = |\sin x|$,

$y = \cos^{-1}[\cos(2\pi - x)] = 2\pi - x$

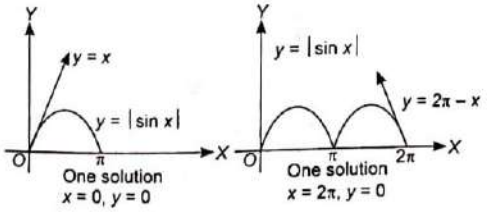
For $x \in [-\pi, 0]$, $y = |\sin x|$,

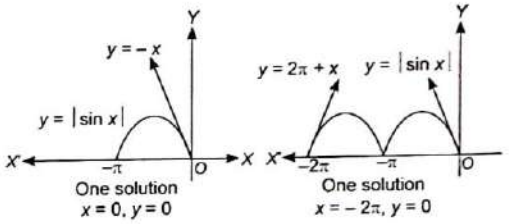
$y = \cos^{-1}[\cos(-x)] = -x$

For $x \in [-2\pi, -\pi]$, $y = |\sin x|$,

$y = \cos^{-1}[\cos(2\pi + x)] = 2\pi + x$

On plotting the graphs, we have




**67.** (b) $(p \wedge q) \wedge (q \wedge r)$ is true means $p \wedge q$ and $q \wedge r$ are both true.

Hence, $p, q$ and $r$ are all true.

**68.** (c) Total outcomes = 36

Favourable outcomes

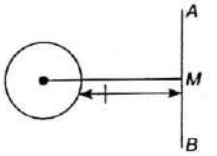
$E = \{(1,1), (1,4), (2,2), (3,3), (4,4), (4,1), (5,5), (6,6)\}$

$$n(E) = 8$$

$$\therefore \quad P(E) = \frac{8}{36} = \frac{2}{9}$$

**69.** (b) $|\sqrt{3}(1 - 2z) + 2i| = 2\sqrt{7}$

$$\Rightarrow \quad \left|z - \left(\frac{1}{2} + \frac{1}{\sqrt{3}}i\right)\right| = \frac{\sqrt{7}}{\sqrt{3}}$$




Represents a circle with centre $\left(\frac{1}{2}, \frac{1}{\sqrt{3}}\right)$ and radius $\frac{\sqrt{7}}{\sqrt{3}}$ Now,

$$|\sqrt{3}(-1 - z) - 2i| = |\sqrt{3}(9 - z) + 18i|$$

$$\Rightarrow \quad \left|z + 1 + \frac{2}{\sqrt{3}}i\right| = |z - 9 - 6\sqrt{3}\,i|$$

Represents a perpendicular bisector of the line segment joining $\left(-1, -\frac{2}{\sqrt{3}}\right)$ and $(9, 6\sqrt{3})$ is $M\left(4, \frac{8}{\sqrt{3}}\right)$

Equation of perpendicular bisector of $AB$ is

$$\frac{-10}{\frac{20}{\sqrt{3}}}(x - 4) = y - \frac{8}{\sqrt{3}}$$

$$\Rightarrow \quad y - \frac{8}{\sqrt{3}} = -\frac{\sqrt{3}}{2}(x - 4)$$

$$\Rightarrow \quad 2y - \frac{16}{\sqrt{3}} = -\sqrt{3}\,x + 4\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}\,x + 2y = \frac{28}{\sqrt{3}}$$

$\therefore$ Minimum value of $|z_1 - z_2|$

= Perpendicular distance of centre of circle to the line − Radius of circle

$$= \frac{\left|\sqrt{3}\cdot\frac{1}{2} + 2\cdot\frac{1}{\sqrt{3}} - \frac{28}{\sqrt{3}}\right|}{\sqrt{3}+4} - \frac{\sqrt{7}}{\sqrt{3}}$$

$$= \frac{\left|\frac{\sqrt{3}}{2} - \frac{26}{\sqrt{3}}\right|}{\sqrt{7}} - \frac{\sqrt{7}}{\sqrt{3}}$$

$$= \frac{49}{2\sqrt{7}\sqrt{3}} - \frac{\sqrt{7}}{\sqrt{3}} = \frac{7\sqrt{7}}{2\sqrt{3}} - \frac{2\sqrt{7}}{2\sqrt{3}} = \frac{5\sqrt{7}}{2\sqrt{3}}$$

**70.** (b) 10 girls and 8 boys (Select 8 girls and 6 boys)

Mr Ravi refuses to speak, if Ms Rani is a speaker.

Mrs Rani refuses to speak, if Ms Radha is a speaker.

Let $A \rightarrow$ Mr Ravi is a speaker

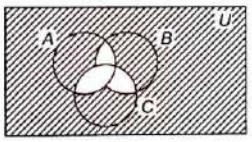
$B \rightarrow$ Ms Rani is a speaker

$C \rightarrow$ Ms Radha is a speaker

Set satisfying the given condition is represented in the Venn diagram

$= n(U) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

Number of ways of preparing the list




$= {}^{10}C_8 \cdot {}^8C_6 - {}^9C_7 \cdot {}^7C_5 - {}^8C_6 \cdot {}^8C_6 + {}^7C_5 \cdot {}^8C_6$

$= 45 \cdot 28 - 36 \cdot 21 - 28 \cdot 28 + 28 \cdot 21 = 308$

**71.** (d) Let $I = \int_0^2 \log(x^2+2)\cdot\frac{1}{(x+2)^2}\,dx$

$$= \left[\log(x^2+2)\cdot\frac{-1}{x+2}\right]_0^2 - \int_0^2 \left(\frac{-1}{x+2}\right)\cdot\left(\frac{2x}{x^2+2}\right)dx$$

$$= -\frac{\log 6}{4} + \frac{2\log 2}{4} + \int_0^2 \frac{2x}{(x+2)(x^2+2)}\,dx$$

$$= -\frac{\log 6}{4} + \frac{2\log 2}{4} + \int_0^2 \left\{\frac{-2}{3(x+2)} + \frac{2}{3}\frac{(x+1)}{(x^2+2)}\right\}dx$$

(resolved in partially)

$$= \frac{-\log 6}{4} + \frac{2\log 2}{4} + \left[-\frac{2}{3}\log(x+2)\right]_0^2 + \frac{2}{3}\int_0^2 \frac{x\,dx}{x^2+2} + \frac{2}{3}\int_0^2 \frac{1}{x^2+2}\,dx$$

$$= \frac{1}{4}\log\left(\frac{4}{6}\right) + \left[-\frac{2}{3}\log 4 + \frac{2}{3}\log 2\right] + \frac{1}{3}[\log(x^2+2)]_0^2 + \frac{2}{3}\times\frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{x}{\sqrt{2}}\right]_0^2$$

$$= \frac{1}{4}\log\left(\frac{2}{3}\right) - \frac{2}{3}\log 2 + \frac{1}{3}[\log 6 - \log 2] + \frac{\sqrt{2}}{3}[\tan^{-1}\sqrt{2}]$$

$$= \frac{1}{4}\log 2 - \frac{1}{4}\log 3 - \frac{2}{3}\log 2 + \frac{1}{3}\log 3 + \frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2}$$

$$= \frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2} - \frac{5}{12}\log 2 + \frac{1}{12}\log 3$$

**72.** (a) We have, $f(x) = 2x + \sin x$

$f'(x) = 2 + \cos x > 0$ for all $x$

$\therefore$ $f(x)$ is strictly increasing.

$\therefore$ $f$ is one-one.

Also, $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = 0$

$\therefore$ $f$ is onto.

**73.** (d) Given, $|M| = \alpha$

and $M^{-1}\,\text{adj}\,(\text{adj}\,M) = kI$

$\Rightarrow$ $|M^{-1}||\text{adj}\,(\text{adj}\,M)| = |kI|$

$[\because |\text{adj}\,(\text{adj}\,A)| = |A|^{(n-1)^2}]$

$\Rightarrow$ $\frac{1}{|M|}|M|^4 = |k| \times 1$

$\Rightarrow$ $k = \alpha^3$

**74.** (a) $y^x - x^y = 1$

Put $x = 1$, we get $y = 2$

On differentiating both sides w.r.t. $x$, we get

$$y^x \log y + y^x \cdot \frac{x}{y}\frac{dy}{dx} - x^y \log x \frac{dy}{dx} - \frac{y}{x}\cdot x^y = 0$$

$$\Rightarrow 2^1 \log 2 + 2^1 \cdot \frac{1}{2}\frac{dy}{dx} - 1^2 \log 1 \frac{dy}{dx} - \frac{2}{1}\cdot 1^2 = 0$$

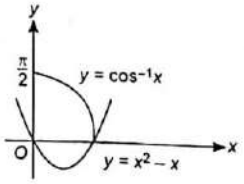
(put $x = 1$, $y = 2$)

$$\Rightarrow \frac{dy}{dx} = 2(1 - \log 2)$$

**75.** (b) $\cos^2 x - \cos x - x = 0$

$\cos^2 x - \cos x = \cos^{-1}(\cos x)$

From graph there is only one solution.



**76.** (d) Given, $f:(0,1)\to(0,1)$

$f'(x)\neq 0$ for all $x\in(0,1), f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$

$$\lim_{t\to x}\frac{\int_0^t\sqrt{1-|f(S)|^2}\,dS-\int_0^x\sqrt{1-|f(S)|^2}\,dS}{f(t)-f(x)}$$

$=f(x)$

By L'Hospital rule,

$$f(x)=\lim_{t\to x}\frac{\sqrt{1-(f(t))^2}}{f'(t)}$$

$$\Rightarrow\quad f(x)=\frac{\sqrt{1-(f(x))^2}}{f'(x)}$$

$$\Rightarrow\quad yy_1=\sqrt{1-y^2}\qquad[\because f(x)=y]$$

$$\Rightarrow\quad\int\frac{y}{\sqrt{1-y^2}}dy=\int dx$$

$$\Rightarrow\quad-\sqrt{1-y^2}=x+C\qquad\left[\because f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}\right]$$

$$\Rightarrow\quad C=-1$$

$$\therefore\quad x=1-\sqrt{1-y^2}$$

$$\Rightarrow\quad\frac{1}{4}=1-\sqrt{1-y^2}\qquad\left(\text{put } x=\frac{1}{4}\right)$$

$$\Rightarrow\quad y=\frac{\sqrt{7}}{4}$$

**77.** (d) Let the matrix be $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$\Rightarrow\quad\begin{bmatrix}a+b & a\\ c+d & c\end{bmatrix}=\begin{bmatrix}a+c & b+d\\ a & b\end{bmatrix}$$

$$\because\quad a=c+d, a=b+d$$

$$\Rightarrow\quad d=a-b, b=c$$

Set of all matrices that commute with $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ w.r.t. matrix multiplication

$$=\left\{\begin{bmatrix}a & b\\ b & a-b\end{bmatrix}:a,b\in R\right\}$$

**78.** (a) Since, $\alpha$ and $\beta$ are the roots of $ax^2+5x+2=0$

$\alpha-5$ and $\beta-5$ are roots of the transformed equation

$$a(x+5)^2+5(x+5)+2=0$$

Let $\alpha_1$ and $\beta_1$ be roots of

$$a^3(x+5)^2-25a(x+5)+50=0$$

$\Rightarrow(\alpha_1+5)$ and $(\beta_1+5)$ are roots of

$$a^3x^2-25ax+50=0$$

Difference between roots $=\alpha_1-\beta_1$

$$=\sqrt{(\alpha_1+5+\beta_1+5)^2-4(\alpha_1+5)(\beta_1+5)}$$

$$=\sqrt{\left(\frac{25a}{a^3}\right)^2-4\left(\frac{50}{a^3}\right)}$$

$$=5\sqrt{\frac{25}{a^4}-\frac{8}{a^3}}=\frac{5}{a^2}\sqrt{25-8a}$$

$$=\frac{5}{a}\sqrt{\left(\frac{5}{a}\right)^2-4\left(\frac{2}{a}\right)}$$

$$=(\alpha+\beta)\sqrt{(\alpha+\beta)^2-4\alpha\beta}$$

$$=|\alpha^2-\beta^2|$$

**79.** (b) Let equation of hyperbola and ellipse be

$$\frac{x^2}{a_1^2}-\frac{y^2}{a_1^2(e_1^2-1)}=1\qquad\ldots(i)$$

and $$\frac{x^2}{a_2^2}+\frac{y^2}{a_2^2(1-e_2^2)}=1\qquad\ldots(ii)$$

$\Rightarrow\quad a_1e_1=a_2e_2$ (since, same pair of foci)

$$e_2=\frac{1}{2}\qquad\text{(given)}$$

Both intersect at (2, 2),

$$\frac{4}{a_2^2}+\frac{4}{a_2^2\left(\frac{3}{4}\right)}=1\qquad[\text{from Eq. (ii)}]$$

$$\Rightarrow\quad 28=3a_2^2$$

$$\Rightarrow\quad a_2^2=\frac{28}{3}$$

Also, $$a_1^2e_1^2=a_2^2e_2^2$$

$$\Rightarrow\quad a_1^2e_1^2=\frac{28}{3}\times\frac{1}{4}=\frac{7}{3}\qquad\ldots(iii)$$

Now, $$\frac{4}{a_1^2}-\frac{4}{\frac{7}{3}-a_1^2}=1\qquad[\text{from Eq. (i)}]$$

$$\Rightarrow\quad\frac{28}{3}-4a_1^2-4a_1^2=\frac{7}{3}a_1^2-a_1^4$$

$$\Rightarrow\quad 3a_1^4-31a_1^2+28=0$$

$$\Rightarrow\quad 3a_1^4-3a_1^2-28a_1^2+28=0$$

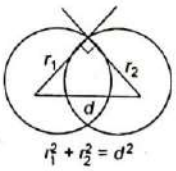
$$\Rightarrow\quad a_1^2=\frac{28}{3}\ \text{ or }\ a_1^2=1\qquad(\because e_1>1)$$

$$\therefore\quad a_1=1\ \Rightarrow\ e_1=\sqrt{\frac{7}{3}},\qquad[\text{from Eq. (iii)}]$$

**80.** (c) Given, $x^2+y^2=4$

$(x-1)^2+y^2=4$

and $x^2+(y-2)^2=4$




$r_1^2+r_2^2=d^2$

Let the required circle be

$(x-h)^2+(y-k)^2=r^2$

Now, by condition of orthoganality,

$h^2+k^2=r^2+4$ ...(i)

$\Rightarrow$ $(h-1)^2+k^2=r^2+4$ ...(ii)

and $h^2+(k-2)^2=r^2+4$ ...(iii)

$\Rightarrow$ $h=\frac{1}{2}$ and $k=1$

$\therefore$ $r^2+4=\frac{1}{4}+1$ [from Eq. (i)]

$\Rightarrow$ $r^2=-\frac{11}{4}$

$\therefore$ Required equation of circle

$$\left(x-\frac{1}{2}\right)^2+(y-1)^2=-\frac{11}{4}$$

$\Rightarrow$ $x^2+y^2-x-2y+4=0$

**81.** (c) Given, $\cos^2\left(x+\frac{\pi}{6}\right)+\cos^2 x-2\cos\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{6}=\sin^2\frac{\pi}{6}$

On adding $\cos^2\frac{\pi}{6}$ on both sides, we get

$\Rightarrow \cos^2\left(x+\frac{\pi}{6}\right)+\cos^2\frac{\pi}{6}+\cos^2 x-2\cos\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{6}=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{6}$

$\Rightarrow \left[\cos\left(x+\frac{\pi}{6}\right)-\cos\frac{\pi}{6}\right]^2+\cos^2 x=1$

$(\because \sin^2\theta+\cos^2\theta=1)$

$\Rightarrow \left[2\sin\frac{x}{2}\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)\right]^2=\sin^2 x$

$\Rightarrow 4\sin^2\frac{x}{2}\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right)=4\sin^2\frac{x}{2}\cos^2\frac{x}{2}$

$=4\sin^2\frac{x}{2}\cdot\sin^2\left(\frac{\pi}{2}-\frac{x}{2}\right)$

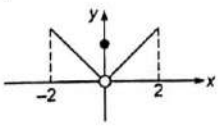
On comparing both sides, we get

$\Rightarrow$ $\sin\frac{x}{2}=0$ or $\frac{x}{2}+\frac{\pi}{6}=\frac{\pi}{2}-\frac{x}{2}$

$\Rightarrow$ $x=0$ or $x=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$

Hence, number of solutions are 2.

**82.** (a) $f(x)=\begin{cases}|x|, & 0<|x|\le 2\\ 1, & x=0\end{cases}$




Clearly, from the graph $f(x)$ has point of local maxima at $x=0$.

**83.** (a) The general term of the series is

$T_r=(-1)^r(3+5r)\,{}^nC_r, \quad r=0,1,2,\ldots,n$

$\therefore$ Sum of the series is given by

$$S=\sum_{r=0}^{n}(-1)^r(3+5r)\,{}^nC_r$$

$$=3\left(\sum_{r=0}^{n}(-1)^r\,{}^nC_r\right)+5\left(\sum_{r=1}^{n}(-1)^r n\cdot{}^{n-1}C_{r-1}\right)$$

$$=3\left(\sum_{r=0}^{n}(-1)^r\,{}^nC_r\right)-5n\left(\sum_{r=1}^{n}(-1)^{r-1}\,{}^{n-1}C_{r-1}\right)$$

$=3(1-1)^n-5n(1-1)^{n-1}=0$

**84.** (c) $f(\theta)=\frac{1}{\tan^9\theta}[(1+\tan\theta)^{10}+(2+\tan\theta)^{10}+\ldots+(20+\tan\theta)^{10}]-20\tan\theta$

By binomial theorem,

$\because (1+\tan\theta)^{10}={}^{10}C_0+{}^{10}C_1\tan\theta+\ldots+{}^{10}C_8\tan^8\theta+{}^{10}C_9\tan^9\theta+{}^{10}C_{10}\tan^{10}\theta$

$\Rightarrow \frac{(1+\tan\theta)^{10}}{\tan^9\theta}=\frac{1}{\tan^9\theta}({}^{10}C_0+{}^{10}C_1\tan\theta+\ldots+{}^{10}C_8\tan^8\theta)+{}^{10}C_9+{}^{10}C_{10}\tan\theta$

$$\Rightarrow \lim_{\theta\to\frac{\pi}{2}^-}\frac{(1+\tan\theta)^{10}}{\tan^9\theta}$$

$$=\lim_{\theta\to\frac{\pi}{2}^-}\left[\frac{1}{\tan^9\theta}({}^{10}C_0+{}^{10}C_1\tan\theta+\ldots+{}^{10}C_8\tan^8\theta+{}^{10}C_9+{}^{10}C_{10}\tan\theta\right]$$

$$=0+{}^{10}C_9+\lim_{\theta\to\frac{\pi}{2}^-}\tan\theta$$

$$=10+\lim_{\theta\to\frac{\pi}{2}^-}\tan\theta$$

Similarly;

$$\lim_{\theta\to\frac{\pi}{2}^-}\frac{(1+\tan\theta)^{10}}{\tan^9\theta}=20+\lim_{\theta\to\frac{\pi}{2}^-}\tan\theta,$$

$$\lim_{\theta\to\frac{\pi}{2}^-}\frac{(3+\tan\theta)^{10}}{\tan^9\theta}=30+\lim_{\theta\to\frac{\pi}{2}^-}\tan\theta$$

.....

.....

$$\lim_{\theta\to\frac{\pi}{2}^-}\frac{(20+\tan\theta)^{10}}{\tan^9\theta}=200+\lim_{\theta\to\frac{\pi}{2}^-}\tan\theta$$

On adding all these terms; we get

$$\lim_{\theta\to\frac{\pi}{2}^-} f(\theta) = (10 + 20 + \ldots + 200) + 20 \lim_{\theta\to\frac{\pi}{2}^-} \tan\theta - \lim_{\theta\to\frac{\pi}{2}^-} 20d \tan\theta$$

$$\Rightarrow \lim_{\theta\to\frac{\pi}{2}^-} f(\theta) = 10(1 + 2 + 3 + \ldots + 20) = \frac{10 \times 20 \times 21}{2} = 2100$$

**85.** (a) By Sine rule,

$$\sum a^2 = 4R^2 \sum \sin^2 A$$

$$\Rightarrow 4R^2 \sum \sin^2 A = 8R^2 \quad (\because a^2 + b^2 + c^2 = 8R^2)$$

$$\Rightarrow 2\sum \sin^2 A = 4$$

$$\Rightarrow \sum(1 - \cos 2A) = 4$$

$$\Rightarrow \sum \cos 2A = -1$$

**86.** (d) By above solution $\sum \cos 2A = -1$

$$\Rightarrow \cos 2A + \cos 2B + (\cos 2C + 1) = 0$$

$$\Rightarrow 2\cos(A + B)\cos(A - B) + 2\cos^2 C = 0$$

but $A + B = \pi - C$

$$\Rightarrow 2\cos(\pi - C)\cdot\cos(A + B) + 2\cos^2 C = 0$$

$$\Rightarrow -2\cos C\cdot\cos(A - C) + 2\cos^2 C = 0$$

$$\Rightarrow -2\cos C[\cos(A - C) - \cos C] = 0$$

$\Rightarrow$ Either $\cos C = 0$, *i.e.*, $C = \frac{\pi}{2}$

$$\Rightarrow \cos(A - B) - \cos C = 0$$

$$\Rightarrow 2\sin\left(\frac{C + A - B}{2}\right)\sin\left(\frac{C - A + B}{2}\right) = 0$$

*i.e.*, $C + A - B = 0$ or $C - A + B = 0$

*i.e.*, $B = \frac{\pi}{2}$ or $A = \frac{\pi}{2}$

**87.** (d) $\therefore \mathbf{a}\times\mathbf{a}' + \mathbf{b}\times\mathbf{b}' + \mathbf{c}\times\mathbf{c}'$

$$= \frac{1}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}[\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) + \mathbf{b}\times(\mathbf{c}\times\mathbf{a}) + \mathbf{c}\times(\mathbf{a}\times\mathbf{b})]$$

(for cyclic order)

$$= \frac{1}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}[0] = 0$$

**88.** (d) Let $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \mathbf{b} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}$

and $\mathbf{c} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

Here, $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = 3$

$$\left[\because \begin{vmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ -1 & 2 & 2 \end{vmatrix} = 2(2) - 3(0) - 1(1) = 3\right]$$

and $\mathbf{b}\times\mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -2 \\ -1 & 2 & 2 \end{vmatrix} = 2\mathbf{i} + \mathbf{k}$

$$\mathbf{a}' = \frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]} = \frac{2\mathbf{i} + \mathbf{k}}{3}$$

Similarly, $\mathbf{b}' = \frac{-8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}}{3}$

and $\mathbf{c}' = \frac{-7\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}}{3}$

**89.** (a) $\sin\theta = 0 \Rightarrow \theta = n\pi$

$$\Rightarrow 2\cos^{-1}[\cot(2\tan^{-1}x)] = n\pi$$

$$\Rightarrow \cos^{-1}[\cot(2\tan^{-1}x)] = \frac{n\pi}{2}$$

$$\Rightarrow \cot(2\tan^{-1}x) = \cos\frac{n\pi}{2}$$

$$\Rightarrow \frac{1}{\tan\left(\tan^{-1}\frac{2x}{1 - x^2}\right)} = \cos\frac{n\pi}{2}$$

$$\Rightarrow \frac{1 - x^2}{2x} = \cos\frac{n\pi}{2}$$

Now, $\cos\frac{n\pi}{2} = 0$, if $n = 1, 3, 5, \ldots$

$$\therefore \frac{1 - x^2}{2x} = 0 \Rightarrow x = \pm 1$$

Now, $\cos\frac{n\pi}{2} = 1$, if $n = 0, 4, \ldots$

$$\therefore \frac{1 - x^2}{2x} = 1 \Rightarrow x = -1 \pm \sqrt{2}$$

Now, $\cos\frac{n\pi}{2} = -1$, if $n = 2, 6, \ldots$

$$\therefore \frac{1 - x^2}{2x} = -1 \Rightarrow x = 1 \pm \sqrt{2}$$

Hence, $x = \pm 1, \pm 1 \pm \sqrt{2}$

**90.** (d) $\because \left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$$= \left(\frac{2\cos\frac{A + B}{2}\cdot\cos\frac{A - B}{2}}{2\cos\frac{A + B}{2}\cdot\sin\frac{A - B}{2}}\right)^n + \left(\frac{2\sin\frac{A + B}{2}\cdot\cos\frac{A - B}{2}}{2\sin\frac{B - A}{2}\cdot\sin\frac{A + B}{2}}\right)^n$$

$$= \cot^n\frac{A - B}{2} + \cot^n\frac{B - A}{2}$$

Now, $\cot^n\frac{A - B}{2} + \cot^n\frac{B - A}{2}$

$$= \begin{cases} 0, & \text{if } n \text{ is odd} \\ 2\cot^n\frac{A - B}{2}, & \text{if } n \text{ is even} \end{cases}$$

# JEE Main

## Joint Entrance Examination

# Practice Set 2

## Instructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

**1.** The equation of the stationary wave is

$$y = 2A \sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

Which of the following statement(s) is wrong?

(a) The unit of $ct$ is same as that of $\lambda$.

(b) The unit of $x$ is same as that of $\lambda$.

(c) The unit of $2\pi c / \lambda$ is same as that of $2\pi x / \lambda t$

(d) The unit of $c / \lambda$ is same as that of $x / \lambda$.

**2.** A particle is projected from ground making an angle $\theta$ with the horizontal. The value of $\theta$ for which, at the highest point of its trajectory, kinetic energy of particle will be equal to its potential energy, is

(a) 30° (b) 45° (c) 60° (d) 75°

**3.** A spherical ball rolls on a table without slipping. Then, the fraction of its total energy associated with rotation is

(a) 2/5 (b) 2/7 (c) 3/5 (d) 3/7

**4.** If $B$ is the bulk modulus of a metal and a pressure $p$ is applied uniformly on all sides of the metal with density $D$, then the fractional increase in density is given by

(a) $\frac{B}{p}$ (b) $\frac{p}{B}$ (c) $\frac{pD}{B}$ (d) $\frac{BD}{p}$

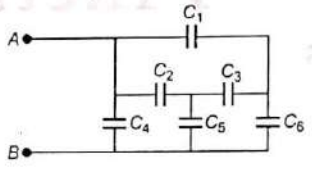
5. A water barrel having water upto a depth $d$ is placed on a table of height $h$. A small hole is made on the wall of the barrel at its bottom. If the stream of water coming out of the hole falls on the ground at a horizontal distance $R$ from the barrel, then the value of $d$ is

(a) $\frac{4h}{R^2}$ (b) $4hR^2$ (c) $\frac{R^2}{4h}$ (d) $\frac{h}{4R^2}$

6. If two like charges of magnitude $1\times10^{-9}$ C and $9\times10^{-9}$ C are separated by a distance of 1 m, then the point on the line joining the charges, where the force experienced by a charge placed at that point is zero, is

(a) 0.25 m from the charge $1\times10^{-9}$ C (b) 0.75 m from the charge $9\times10^{-9}$ C
(c) Both (a) and (b) (d) at all points on the line joining the charges

7. Six equal capacitors, each of capacitance $C$ are connected as shown in the figure. Then, the equivalent capacitance between $A$ and $B$ is




(a) $6C$ (b) $C$
(c) $2C$ (d) $\frac{C}{2}$

8. A conducting wire is stretched to increase its length by 2%. What is the percentage change in its resistance?

(a) 2% (b) 4% (c) 8% (d) 1%

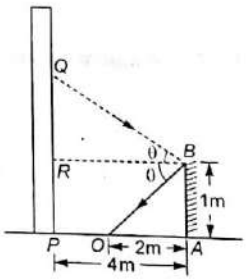
9. Flux $\phi$ (in weber) in a closed circuit of resistance $10\,\Omega$ varies with time $t$ (in seconds) according to the equation

$$\phi = 6t^2 - 5t + 1$$

The magnitude of the induced current in the circuit at $t = 0.25$ s is

(a) 0.2 A (b) 0.6 A (c) 0.8 A (d) 1.2 A

10. A 1 m long mirror is placed at a distance 4 m from a tall building as shown in figure. What height of the building can be seen in the mirror from a point $O$ at a distance 2 m from the mirror?




(a) 1 m
(b) 2 m
(c) 3 m
(d) 4 m

11. A block of pure silicon at 300 K has a length of 10 cm and an area of $1.0\times10^{-4}$ m$^2$. If a battery of emf 2 V is connected across it, what is the electron-current? The mobility of electrons is $0.14\,m^2V^{-1}s^{-1}$ and their number density is $1.5\times10^{16}$ m$^{-3}$.

(a) $6.72\times10^{-4}$ A (b) $6.72\times10^{-5}$ A
(c) $6.72\times10^{-6}$ A (d) $6.72\times10^{-7}$ A

12. A freely falling body, falling from a tower of height $h$ covers a distance $h/2$ in the last second of its motion. The height of tower is nearly

(a) 58 m (b) 50 m (c) 60 m (d) 65 m

13. A ball collides elastically with another ball of the same mass. The collision is oblique and initially one of the balls was at rest. After the collision, the two balls move with same speeds. What will be the angle between the velocity of the balls after the collision?

(a) 30° (b) 45° (c) 60° (d) 90°

14. A cubical block of mass $m$ and edge $a$ slides down a rough inclined plane of inclination $\theta$ with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude

(a) zero (b) $mga$ (c) $mga \sin\theta$ (d) $\frac{mga \sin\theta}{2}$

15. We have two (narrow) capillary tubes $T_1$ and $T_2$. Then lengths are $l_1$ and $l_2$ and radii of cross-sections are $r_1$ and $r_2$ respectively. The rate of flow of water under a pressure difference $p$ through the tube $T_1$ is 8 $cm^3/s$. If $l_1 = 2l_2$ and $r_1 = r_2$. What will be the rate of flow when the two tubes are connected in series and pressure difference across the combination is same as before ($= p$)?

(a) 4 $cm^3/s$ (b) 16/3 $cm^3/s$ (c) 8/17 $cm^3/s$ (d) None of these

16. The elastic flux $\phi$ through a hemisphere surface of radius $R$, placed in a uniform electric field of intensity $E$ parallel to the axis of its plane is

(a) $2\pi RE$ (b) $2\pi R^2E$ (c) $\pi R^2E$ (d) $\frac{4}{3}\pi R^2E$

17. If the velocity $v$ of a particle moving along a straight line decreases linearly with its displacement $S$ from 20 $ms^{-1}$ to a value approaching zero at $S = 30$ m, then acceleration of the particle at $S = 15$ m, is

(a) $\frac{2}{3} ms^{-2}$ (b) $-\frac{2}{3} ms^{-2}$ (c) $\frac{20}{3} ms^{-2}$ (d) $-\frac{20}{3} ms^{-2}$

18. A disc of mass 10 kg is kept floating horizontally by throwing 10 marbles per second against it from below. The marbles strike the disc normally and rebound downwards with the same speed. If the mass of each marble is 5 g, the velocity with which the marbles are strike the disc is ($g = 9.8\ ms^{-2}$)

(a) 0.98 $ms^{-1}$ (b) 9.8 $ms^{-1}$ (c) 1.96 $ms^{-1}$ (d) 19.6 $ms^{-1}$

19. The displacement $x$ of a particle of mass $m$ kg moving in one dimension, under the action of a force is related to the time $t$ by the equation $t = \sqrt{x} + 3$, where $x$ is in metre and $t$ is in second. The work done by the force in the first six seconds in joule is

(a) zero (b) 3 m (c) 6 m (d) 9 m

20. Two cones $A$ and $B$ are made of two different materials, the density of $A$ being greater than that of $B$. The height of $B$ is greater than that of $A$ but their base areas and masses are the same. The correct statement about the moment of inertia of the two cones about their axes, is

(a) $A$ will have larger moment of inertia than $B$
(b) $B$ will have larger moment of inertia than $A$
(c) in such a situation, it is dependent upon the height of the cone, the mass of the cone and radius of the base
(d) the moment of inertia of the two will be the same as it is not dependent upon height of the cone but depends only upon the mass and the base area

21. There are some passengers inside a stationary railway compartment. The centre of mass of the compartment itself is $c_1$, while the centre of mass of the 'compartment plus passengers' system is $c_2$. If the passengers move about inside the compartment

(a) both $c_1$ and $c_2$ will move w.r.t. ground
(b) neither $c_1$ and $c_2$ will move w.r.t. ground
(c) $c_1$ will move but $c_2$ will be stationary w.r.t. ground
(d) $c_2$ will move but $c_1$ will be stationary w.r.t. ground

22. If $g$ is the acceleration due to gravity at the surface of the earth, then the energy required to launch a satellite of mass $m$ from the surface of the earth into a circular orbit at an altitude of $2R$ where $R$ being the radius of the earth, is

(a) $\frac{mgR}{6}$ (b) $\frac{mgR}{3}$ (c) $\frac{2mgR}{3}$ (d) $\frac{5}{6}mgR$

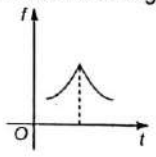
23. A cubical wooden block of side 10 cm floats at the interface between oil and water with its lower face 2 cm below the interface. If the density of oil is 0.6 g cm$^{-3}$, the mass of the block is

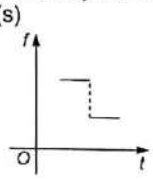
(a) 200 g (b) 340 g (c) 480 g (d) 680 g

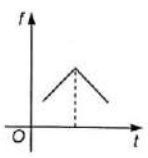
24. Springs of constants $k, 2k, 4k, 8k, \ldots 2048k$ are connected in series. A mass $m$ is attached to one end and the system is allowed to oscillate. The time period is approximately

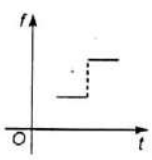
(a) $2\pi\sqrt{\frac{m}{2k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$ (c) $2\pi\sqrt{\frac{m}{4k}}$ (d) $2\pi\sqrt{\frac{4m}{k}}$

25. A source of sound emitting a note of constant frequency is moving towards a stationary listener, and then recedes from the listener with constant velocity maintained throughout the motion. The frequency head by the listener ($f$) when plotted against time ($t$) will give one of the following curve(s)

(a) 

(b) 

(c) 

(d) 


26. A system is taken from state $A$ to state $B$ along two different paths 1 and 2. The work done on the system along these two paths are $W_1$ and $W_2$, respectively. The heat absorbed by the system along these two paths are $Q_1$ and $Q_2$, respectively. The internal energies at $A$ and $B$ are $U_A$ and $U_B$ respectively. Then,

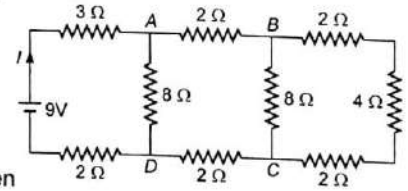
(a) $W_1 = W_2 = U_B - U_A$ (b) $Q_1 = Q_2 = U_A - U_B$
(c) $Q_1 + W_1 = Q_2 + W_2 = U_A + U_B$ (d) $Q_1 + (-W_1) = Q_2 + (-W_2) = U_B - U_A$

27. Equal charges $q$ are placed at the four corners $A, B, C, D$ of a square of side $a$. The magnitude of the force on the charge at $C$ will be

(a) $\frac{3q^2}{4\pi\varepsilon_0 a^2}$ (b) $\frac{4q^2}{4\pi\varepsilon_0 a}$ (c) $\left(\frac{1+2\sqrt{2}}{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$ (d) $\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$

28. In the circuit shown in figure, the current through

(a) the 3 Ω resistor is 0.50 A
(b) the 3 Ω resistor is 0.25 A
(c) the 4 Ω resistor is 0.50 A
(d) the 4 Ω resistor is 0.25 A




29. A blue object on a white background when seen through a blue filter will appear

(a) blue on a white background (b) black on a blue background
(c) blue on a red background (d) invisible

30. A solid rubber ball of density $\rho$ and radius $R$ falls vertically through air. Assume the air resistance on the ball is $F = kRv$, where $k$ is a constant and $v$ is the velocity. Because of this air resistance, the ball attains a constant velocity called terminal velocity $v_T$ after some time, then $v_T$ is

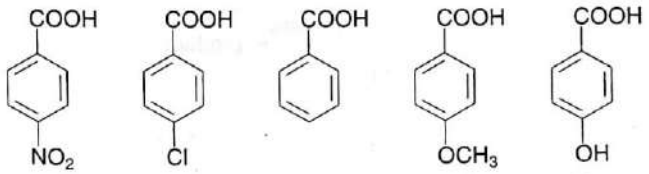
(a) $\frac{4\pi\rho g^2}{3k}$ (b) $\frac{4\pi R\rho g}{k}$ (c) $\frac{4}{3}\frac{\rho g}{kR^2}$ (d) $\frac{4\rho g R^2}{3k}$

# Chemistry

31. The number of structural isomers possible with the compound of molecular formula $C_4H_{10}O$ is
(a) 4 (b) 5 (c) 7 (d) 8

32. In the reaction, $I_2 \longrightarrow IO_3^- + I^-$ (Acidic medium)
The equivalent weight of $I_2$ is
(a) 120.8 (b) 127 (c) 152.4 (d) 254

33. The acidic order of the following acids is




(I) (II) (III) (IV) (V)

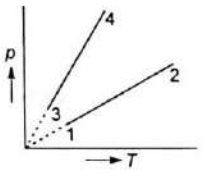
(a) I > II > III > IV > V (b) I > II > III > V > IV
(c) I > III > II > IV > V (d) II > I > III > IV > V

34. $CH_3C{\equiv}CH \xrightarrow{NaNH_2} A \xrightarrow{CH_3Br} B \xrightarrow[H_2SO_4]{HgSO_4} C$
The number of hydrogen atoms present in one molecule of *C*
(a) 5 (b) 6 (c) 7 (d) 8

35. The ratio of the radii of the first three Bohr orbits is
(a) 1 : 0.5 : 0.33 (b) 1 : 2 : 3 (c) 1 : 4 : 9 (d) 1 : 8 : 27

36. The hybridisation of the central atom in $ICl_2^+$ is
(a) $dsp^2$ (b) $sp$ (c) $sp^2$ (d) $sp^3$

37. Pressure *versus* temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternative.




(a) $V_1 = V_2 = V_3 = V_4$
(b) $V_4 > V_3 > V_2 > V_1$
(c) $V_1 = V_2, V_3 = V_4$ and $V_2 > V_3$
(d) $V_1 = V_2, V_3 = V_4$ and $V_2 < V_3$

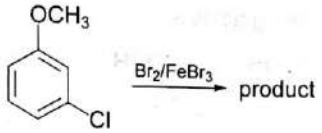
38. For the given reaction,
$$2A(s) + B(g) \rightleftharpoons C(g) + 2D(s) + E(s)$$
the degree of dissociation of *B* was found to be 20% at 300 K and 24% at 500 K. The rate of backward reaction
(a) increases with increase in pressure and temperature
(b) increases with increase in pressure and decrease in temperature
(c) depends on temperature only and decreases with increase in temperature
(d) increases with increasing the concentration of *B* and increasing the temperature

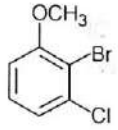
39. The concentration of $OH^-$ ion in a solution left after mixing 100 mL of 0.1 M $MgCl_2$ and 100 mL of 0.2 M NaOH ($K_{sp}$ $[Mg(OH)_2] = 1.2 \times 10^{-11}$) is
(a) $2.8 \times 10^{-3}$ (b) $2.8 \times 10^{-4}$ (c) $2 \times 10^{-7}$ (d) $10^{-7}$

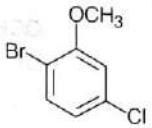
40. Two reactions one of first order and other of second order have same values of rate constants ($k_1$ and $k_2$) when concentrations are expressed in mol/dm$^3$. If the concentrations are expressed in mol/mL, the relationship between their rate constants $k_1$ and $k_2$ will be
(a) $k_1 = k_2$ (b) $k_1 \times 10^3 = k_2$ (c) $k_2 \times 10^3 = k_1$ (d) $k_1 = 10k_2$

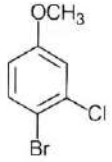
41. Which of the following leaves a black residue on addition of $NH_3$?
(a) AgCl (b) $PbCl_2$ (c) $Hg_2Cl_2$ (d) $HgCl_2$

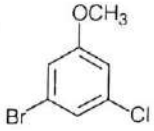
42. The major product formed on monobromination in the following reaction is




(a) 

(b) 

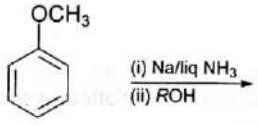
(c) 

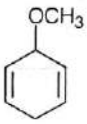
(d) 

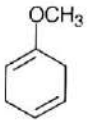

43. Pure $H_2S$ can be obtained by the action of water on
(a) FeS (b) $Al_2S_3$ (c) Flower of S (d) $FeS_2$

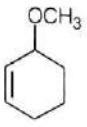
44. In normal spinel structures $MgAl_2O_4$, the percentage of tetrahedral void occupied is
(a) 12.5% (b) 25% (c) 50% (d) None of these

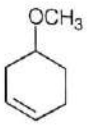
45. In the reaction, the product *P* is




(a) 

(b) 

(c) 

(d) 


46. Anhydrous $AlCl_3$ fumes in air because of
(a) dehydration (b) hydration (c) hydrolysis (d) reduction

47. In which of the following unit cells all sides and all angles are equal?
Cubic (I), Tetragonal (II), Orthorhombic (III), Rhombohedral (IV)
(a) I and II (b) I (c) I and III (d) I and IV

48. The change in entropy of 2 moles of an ideal gas upon isothermal expansion at 243.6 K from 20 L until the pressure becomes 1 atm is
(a) 1.385 cal/K (b) −1.2 cal/K (c) 1.2 cal/K (d) 2.77 cal/K

49. When $FeCl_3$ solution is added to NaOH, a negatively charged sol is obtained. It is due to the
(a) presence of basic group
(b) preferential adsorption of $OH^-$ ion
(c) self dissociation
(d) electron capture by sol particles

50. A 0.1 M solution of a certain cation will form a precipitate with 0.1 M solution of all these anions $OH^{\ominus}$, $CO_3^{2-}$, $Cl^-$, $SO_4^{2-}$. Which cation fits in this description?
(a) $Ba^{2+}$ (b) $Fe^{2+}$ (c) $Mg^{2+}$ (d) $Pb^{2+}$

51. Calculate the per cent dissociation of 0.15 M formic acid solution which is 0.1 M in HCl ($pK_a$ of HCOOH = 3.7).
(a) 0.1662 (b) 0.1993 (c) 0.3412 (d) 0.5126

52. The stop cock connecting the two bulbs of volume 5 L and 10 L containing an ideal gas at 9 atm and 6 atm respectively, is opened. What is the final pressure, if the temperature remains same?
(a) 2 atm (b) 7 atm (c) 10 atm (d) 12 atm

**Directions** (Q. Nos. 53-55) *A solution of sucrose [M (mass) = 342] has been prepared by dissolving 68.4 g of sucrose in 1 kg of water. $K_f$ for water is 1.86 kg $mol^{-1}$ and vapour pressure of water at 298 K is 0.024 atm. Assume density of the solution is 1 g/mL.*

53. The vapour pressure of the solution at 298 K will be
(a) 0.230 atm (b) 0.233 atm (c) 0.236 atm (d) 0.0239 atm

54. The freezing point of the solution will be
(a) –0.684°C (b) –0.342°C (c) –0.372°C (d) –0.186°C

55. Osmotic pressure of the solution is
(a) 4.89 atm (b) 0.34 atm (c) 0.24 atm (d) 0.14 atm

**Directions** (Q. Nos. 56-58) *Electrolysis involves electronation and de-electronation at the respective electrodes. Anode of electrolytic cell is the electrode at which de-electronation takes place whereas at cathode electronation is noticed.*

*If two or more ions of same charge are to be electronated or de-electronated, the ion having lesser discharge potential is discharged. Discharge potential of an ion refers for $E°_{OP}$ or $E°_{RP}$ as the case may be. The products formed at either electrode is given in terms of Faraday's law of electrolysis, i.e., $W = \frac{Eit}{96500}$.*

56. During electrolysis of $CH_3COONa(aq)$, the mole ratio of gases formed at cathode and anode is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

57. During electrolysis of $HCOONa(aq)$, the gas liberated at anode and cathode are respectively
(a) $H_2$, $CO_2$ and $H_2$ (b) $H_2$, $CO_2$ and $O_2$ (c) $H_2$ and $O_2$ (d) $O_2$ and $H_2$

58. During electrolysis of $CuSO_4(aq)$ using platinum electrodes, the pH of solution (electrolyte)
(a) increases
(b) decreases
(c) remains the same
(d) first increases then decreases

**Directions** (Q. Nos. 59 and 60) *For the following questions, choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

59. **Statement I** Basic radicals have been divided into groups based on $K_{sp}$ values.

**Statement II** Most alkali salts are water soluble.

60. **Statement I** $CH_3OH$ and $CH_3CH_2OH$ can be distinguished by iodoform test.

**Statement I** Iodoform test is given by 2° alcohol always.

# Mathematics

61. If the planes $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ pass through one line, then $a^2 + b^2 + c^2$ $2abc$ is equal to

(a) $ab$ (b) 1 (c) $bc$ (d) 0

62. The distance of the point, where the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4}$ meets the plane $x + 2y + 3z = 14$ from the origin, is

(a) $\sqrt{15}$ (b) $\sqrt{14}$ (c) 7 (d) $\sqrt{7}$

63. The area of the region enclosed by the curves $y = x \log x$ and $y = 2x - 2x^2$ is

(a) $\frac{7}{12}$ (b) $\frac{1}{2}$ (c) $\frac{5}{12}$ (d) None of these

64. If a circle of radius '$r$' is concentric with ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the common tangent is inclined to major axis at an angle

(a) $\tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$ (b) $\tan^{-1}\sqrt{\frac{r^2 - a^2}{b^2 - r^2}}$

(c) $\tan^{-1}\sqrt{\frac{r^2 - b^2}{r^2 - a^2}}$ (d) $\tan^{-1}\sqrt{\frac{r^2 - a^2}{r^2 - b^2}}$

65. If the inclination of the diameter $PP'$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the major axis is $\theta$ and $PP'^2$ is the AM of squares of major and minor axis, then $\tan\theta$ is equal to

(a) $\frac{b}{a}$ (b) $\frac{a}{b}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

66. A ray of light travels along the line $2x - 3y + 5 = 0$ and strikes a plane mirror lying along the line $x + y = 2$. The equation of the straight line containing the refracted ray is

(a) $2x - 3y + 3 = 0$ (b) $3x - 2y + 3 = 0$ (c) $21x - 7y + 1 = 0$ (d) $21x + 7y - 1 = 0$

67. Given, two points $A \equiv (-2, 0)$ and $B \equiv (0, 4)$. The coordinates of a point $M$ lying on the line $y = x$, so that the perimeter of the $\Delta AMB$ is least, is

(a) (1, 1) (b) (0, 0) (c) (9, 2) (d) (3, 3)

68. If $x+ky=1$ and $x=a$ are the equations of the hypotenuse and a side of a right angled isosceles triangle, then

(a) $k=\pm 1$ (b) $k=\pm a$ (c) $k=\pm\frac{1}{a}$ (d) $k=\pm 2$

69. The shortest distance between $z$-axis and the line $x+y+2z-3=0$ and $2x+3y+4z-4=0$ is

(a) 2 (b) 3 (c) 4 (d) 5

70. Consider any set of observations $x_1, x_2, ...., x_{10}$, it being given that $x_1 < x_2 < x_3 < ... < x_{100} < x_{101}$, then the mean deviation of this set of observations about a point $k$ is minimum when $k$ is equal to

(a) $x_1$ (b) $x_{51}$

(c) $\frac{x_1+x_2+...+x_{101}}{101}$ (d) $x_{50}$

71. The greatest distance of the point (10, 7) from the circle $x^2+y^2-4x-2y-20=0$ is $5\alpha$, then $\alpha$ is equal to

(a) 3 (b) 4 (c) 5 (d) 6

72. The locus of the mid-points of the chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which are tangent to the ellipse $\frac{x^2}{p^2}+\frac{y^2}{q^2}=1$ is

(a) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{p^2x^2}{a^4}+\frac{q^2y^2}{b^4}$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x^2}{p^2}+\frac{y^2}{q^2}$

(c) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{p^2}{a^2}+\frac{q^2}{b^2}$ (d) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{p^2}{q^2}+\frac{a^2}{b^2}$

73. If $f(x)=(2^x-1)(2^x-2)^2$, then $f(x)$ decreases in the interval

(a) $x\in\left(-\infty, \log_2\frac{4}{3}\right)\cup(1,\infty)$ (b) $x\in\left(\log_2\left(\frac{4}{3}\right), 0\right)$

(c) $x\in\left(\log_2\frac{4}{3}, 1\right)$ (d) Cannot say

74. Read the following mathematical statements carefully.

I. If $x, y$ and $z$ are all different real numbers, then

$$\frac{1}{(x-y)^2}+\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}=\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2.$$

II. $\log_3 x\cdot\log_4 x\cdot\log_5 x=(\log_3 x\cdot\log_4 x)+(\log_4 x\cdot\log_5 x)+(\log_5 x\cdot\log_3 x)$ is true for exactly for one real value of $x$.

III. A matrix has 12 elements. Number of possible orders it can have a six.

Now, indicate the correct choice

(a) exactly one statement is correct (b) exactly two statements are correct

(c) all statements are incorrect (d) all statements are correct

75. $\sum_{r=1}^{20}\sum_{p=1}^{r}p^2$ is equal to

(a) 16170 (b) 13300 (c) 16100 (d) None of these

76. If $n$th root of unity be $1, a_1, a_2, \ldots, a_{n-1}$, then $\sum_{r=1}^{n-1} \frac{1}{2+a_r}$ is equal to

(a) $\frac{n \cdot 2^{n-1}}{2^n - 1} - 1$ (b) $\frac{n(-2)^{n-1}}{(-2)^n - 1} - 1$ (c) $\frac{n(-2)^{n-1}}{1+(-2)^{n+1}} - \frac{1}{3}$ (d) None of these

77. Let $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ be two distinct infinite GP's. The sum of each one is 1. If $a_2 = b_2$ and $a_3 = \frac{1}{8}$, then $b_3$ is equal to

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}-2}{4}$ (c) $\frac{\sqrt{5}-2}{8}$ (d) $\frac{\sqrt{5}-1}{8}$

78. The sum of areas of all triangles whose vertices are also vertices of a cube of unit edge is $m + \sqrt{n} + \sqrt{p}$, where $m+n+p$ is

(a) 320 (b) 332 (c) 342 (d) 348

79. If four dice are rolled once, the probability of getting the sum atleast 10 is

(a) $\frac{61}{72}$ (b) $\frac{65}{72}$ (c) $\frac{121}{144}$ (d) $\frac{121}{216}$

80. If $z = a + ib, a, b \in R, b \neq 0$ and $|z| = 1$, then $z = \frac{c+i}{c-i}$, where $C$ is equal to

(a) $\frac{a}{b}$ (b) $\frac{a-1}{b}$ (c) $\frac{a+1}{b}$ (d) $\frac{a+1}{b+1}$

81. $\tan^3\theta + \cot^3\theta = 12 + 8\,\text{cosec}^2 2\theta$, if $\theta$ is equal to

(a) $\frac{7\pi}{12}$ (b) $\frac{7\pi}{6}$ (c) $\frac{7\pi}{3}$ (d) $\frac{7\pi}{2}$

82. In $\Delta ABC$, the slope of the median through $A$ is $-2$, $B = (-1,3)$ and $C = (3,5)$. If its area is 5, then the distance of the vertex $A$ from the origin is

(a) 2 (b) 4 (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

83. From the point (6, 2) three normals are drawn to the parabola $y^2 = 8x$. If a circle is drawn through the feet of normals, then the length of its intercept on the $Y$-axis is

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

**Directions** (Q. Nos. 84 and 85) *Let* $\frac{dy}{dx} = \frac{1}{x+y}$ *and* $y(0) = 0$. *Then,*

84. At $y = \log 3$, $\frac{dy}{dx}$ is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $-2$

85. $\int_0^1 x\,dx$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{e}{2}$ (c) $e - \frac{1}{2}$ (d) $e - \frac{5}{2}$

**Directions** (Q. Nos. 86 and 87) *For* $x \in R$, $f(x)$ *is defined as*

$$f(x) = \begin{cases} x+1, & 0 \le x < 2 \\ x-4, & x \ge 2 \end{cases}. \quad \text{For } x \in R, |x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$

86. For $0 \le x \le 1$, the solution set of $|x| f(x) > 2$ is

(a) $\phi$ (b) (0, 1) (c) $\left[\frac{1}{2}, \frac{1}{2}\right]$ (d) None of these

87. For $x \geq 3$, the solution set of $\{f(x) + |x-2|\} f(x) \leq 0$ lies in

(a) $(4, \infty)$ (b) $(-\infty, 3)$ (c) $[3, 4]$ (d) None of these

**Directions** (Q. Nos. 88–90) *For the following questions. Choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

88. We have, $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

**Statement I** If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit coplanar vectors, then

$$[2\mathbf{a} - \mathbf{b}\ \ 2\mathbf{b} - \mathbf{c}\ \ 2\mathbf{c} - \mathbf{a}] = 0$$

**Statement II** $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0$

89. We have, $\Sigma n = \frac{n(n+1)}{2}, \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$ and $\Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2$, $n \in N$

**Statement I** The sum of the series

$1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \ldots + (361 + 380 + 400)$ is 8000.

**Statement II** $\sum_{k=1}^{n} [k^3 - (k-1)^3] = n^3$ for any natural number $n$.

90. In onto functions, each image must be assigned atleast one preimage.

**Statement I** Let $E = \{1, 2, 3, 4\}$ and $F = \{a, b\}$, then the number of onto function from $E$ to $F$ is 14.

**Statement II** Number of ways in which 4 distinct object can be distribution into two different boxes is 14, if no box remain empty.

# Answer *with* Explanations

## Physics

**1.** (c) Here, $(2\pi ct/\lambda)$ as well as $(2\pi x/\lambda)$ are dimensionless. So, unit of $ct$ is same as that of $\lambda$. Unit of $x$ is same as that of $\lambda$.

Since, $\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right] = [M^0L^0T^0]$

Hence, $\frac{2\pi c}{\lambda} = \frac{2\pi x}{\lambda t}$

In the option (d), $\frac{x}{\lambda}$ is unitless. It is not the case with $\frac{c}{\lambda}$.

**2.** (b) Let, $E$ be the KE at the instant of projection, then

$$E = \frac{1}{2}mu^2$$

As KE at topmost point = PE at topmost point

$\therefore$ KE at topmost point, $K = \frac{E}{2}$

But $K = \frac{1}{2}mu^2\cos^2\theta$

$\therefore \quad \frac{1}{2}mu^2\cos^2\theta = \frac{1}{2}\left(\frac{1}{2}mu^2\right)$

$\Rightarrow \quad \theta = 45°$

**3.** (b) Total energy

$$K = K_R + K_T = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 + \frac{1}{2}mr^2\omega^2$$

$$= \frac{1}{5}mr^2\omega^2 + \frac{1}{2}mr^2\omega^2$$

$$= \frac{7}{10}mr^2\omega^2$$

Now, rotational kinetic energy

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{5}mr^2\omega^2$$

$$\frac{K_R}{K} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$$

**4.** (b) $\therefore \quad \frac{D'}{D} = \frac{V}{V-\Delta V} = \left[1 - \frac{\Delta V}{V}\right]^{-1} = 1 + \frac{\Delta V}{V}$

$$\frac{D'}{D} - 1 = \frac{\Delta V}{V}$$

$$\frac{D'-D}{D} = \frac{\Delta V}{V}$$

and we know that

$$\frac{\Delta V}{V} = \frac{p}{B}$$

$\therefore \quad \frac{D'-D}{D} = \frac{p}{B}$

or Fractional increase in density $= \frac{p}{B}$

**5.** (c) Velocity of efflux, $v = \sqrt{2gd}$

Time taken for the range, $R = \sqrt{\frac{2h}{g}}$

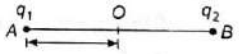
Range = velocity × time

$$R = \sqrt{2gd} \times \sqrt{\frac{2h}{g}}$$

or $\quad R^2 = 2dg \times \frac{2h}{g} = 4dh$

$\therefore \quad d = \frac{R^2}{4h}$

**6.** (c) $q_1 : q_2 = 1 : 9, L = 1\text{ m}$




Suppose at a distance $x$ from the charge of having minimum magnitude, the force on the test charge is zero

$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{9}{(1-x)^2}$

$\Rightarrow \quad (1-x)^2 = 9x^2$

$\Rightarrow \quad 1^2 + x^2 - 2x = 9x^2$

$\Rightarrow \quad 8x^2 + 2x - 1 = 0$

$\Rightarrow \quad 8x^2 + 4x - 2x - 1 = 0$

$\Rightarrow \quad (2x+1)(4x-1) = 0$

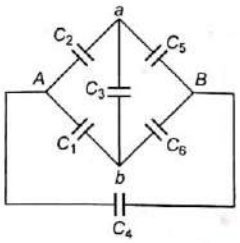
This gives $\quad x = -\frac{1}{2}$ or $x = \frac{1}{4}$

As $x$ is length

So, $\quad x = \frac{1}{4} = 0.25$

So, the required point is a distance of 0.25 m from the charge $1 \times 10^{-9}$ C or 0.75 m from the charge $9 \times 10^{-9}$ C

**7.** (c) The given network of capacitors can be redrawn as given ahead $C_4$ is in parallel to a balanced Wheatstone bridge made from the rest five capacitors as shown in the figure. Therefore, equivalent capacitance $= C + C = 2C$.




**8.** (b) Volume of wire remains constant when the wire is stretched. Thus,

$$V = ls = l \times \text{area of cross-section}$$

$$R = \rho \frac{l}{s} = \frac{\rho}{V} l^2$$

where, $V$ = Volume of wire,

$$\log R = \log \rho + 2 \log l - \log V$$

Differentiating,

$$\frac{dR}{R} = 0 + \frac{2dl}{l} - 0 = 2\frac{dl}{l}$$

Taking $\frac{dR}{R}$ and $\frac{dl}{l}$ as fractional errors (or % cross)

$$\frac{dR}{R} = 2 \times 2\% = 4\%$$

**9.** (a) The rate of change of flux gives the included emf. Thus,

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(6t^2 - 5t + 1)$$

$$= -12t + 5$$

At $t = 0.25$ s, $E = -12 \times 0.25 + 5 = -3 + 5 = 2$ V

$\therefore$ Induced current $I = \frac{E}{R} = \frac{2}{10} = 0.2$ A

**10.** (c) Triangles $QBR$ and $BOA$ are similar, hence,

$$\frac{AB}{OA} = \frac{QR}{RB}$$

or $$QR = RB\left(\frac{AB}{OA}\right) = 4 \times \frac{1}{2} = 2 \text{ m}$$

$\therefore$ Height of the building which can be observed in the mirror from point $O$,

$$PQ = PR + RQ = 1 + 2 = 3 \text{ m}$$

**11.** (d) Given, $\mu_e = 0.14 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$, $n_e = 1.5 \times 10^{16} \text{ m}^{-3}$, $l = 10$ cm $= 0.1$ m, $A = 1.0 \times 10^{-4} \text{ m}^2$ and $V = 2$ volts. The electric field in the block is

$$E = \frac{V}{l} = \frac{2}{0.1} = 20 \text{ Vm}^{-1}$$

The drift speed of electrons is

$$v_e = \mu_e E = 0.14 \times 20 = 2.8 \text{ ms}^{-1}$$

$\therefore$ Electron-current $I_e = n_e A \rho v_e$

$= (1.5 \times 10^{16}) \times (1.0 \times 10^{-4}) \times (1.6 \times 10^{-19}) \times 2.8$

$= 6.72 \times 10^{-7}$ A

**12.** (d) Let the total time of journey is $n$ seconds, then by using $s = ut + \frac{1}{2}at^2$, we get

$$h = \frac{1}{2}gn^2 \qquad [\because u = 0] \quad ...(i)$$

and by using $D_n = u + \frac{a}{2}(2n - 1)$, we get

$$\frac{h}{2} = \frac{1}{2}g(2n - 1) \qquad ...(ii)$$

On solving Eqs. (i) and (ii), we get

$$n = 3.73 \text{ s}$$

$\therefore$ $$h = \frac{1}{2}gn^2 = 65 \text{ m}$$

**13.** (b) Applying law of conservation of momentum along two axes of plane of motion, we get;

$$mv \sin \theta_1 - mv \sin \theta_2 = 0$$

*i.e.*, $$\theta_1 = \theta_2 \qquad ...(i)$$

Also, $$mu = mv(\cos \theta_1 + \cos \theta_2)$$

$$= 2mv \cos \theta \qquad [\theta_1 = \theta_2 = \theta \text{ (say)}]$$

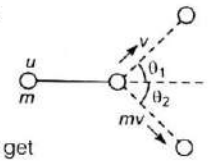
$$\cos \theta = \frac{u}{2v} \qquad ...(ii)$$

According to law of conservation of KE

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$u^2 = 2v^2$$

or $$u = \sqrt{2}\, v \quad ...(iii)$$




From Eqs. (ii) and (iii), we get

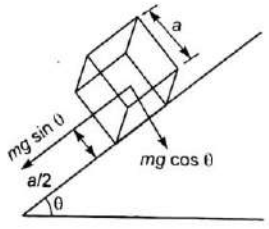
$$\cos \theta = \frac{\sqrt{2}\, v}{2v} = \frac{1}{\sqrt{2}} = \cos 45°$$

$\therefore$ $$\theta = 45°$$

**14.** (d) Because the cubical block slides with a uniform velocity and does not topple.

Hence, torque produced by weight

= torque due to normal force on the block




$\therefore$ Torque due to normal face

= torque due to weight

= component of weight parallel to plane

× perpendicular distance from lower face

$= (mg \sin \theta)\frac{a}{2}$

**15.** (b) For composite tube,

As we know that, $\frac{V}{t} = \frac{\pi pr^4}{8\eta l}$

$$V_1 = \frac{pr^4}{8\eta\left(l + \frac{l}{2}\right)} = \frac{2}{3} \times \frac{\pi pr^4}{8\eta l}$$

$[\because l_1 = l = 2l_2 \text{ or } l_2 = l/2]$

$$\therefore \quad V_1 = \frac{2}{3} \times 8 = \frac{16}{3} \text{ cm}^3/\text{s}$$

**16.** (c) The electric flux through any surface is equal to the product of electric field intensity at the surface and component of the area vector perpendicular to electric field

$$= E \times \pi R^2 = \pi R^2 E$$

**17.** (d) Slope of line $= -\frac{2}{3}$

Equation of line is $(v - 20) = -\frac{2}{3}(S - 0)$

$$\Rightarrow \quad v = 20 - \frac{2}{3}S \qquad \ldots(i)$$

Velocity of $S = 15$ m, *i.e.*,

$$v = \left.\frac{ds}{dt}\right|_{S=15\text{m}} = 20 - \frac{2}{3}(15) = 10 \text{ ms}^{-1}$$

Differentiating Eq. (i) w.r.t. time,

$$\text{Acceleration} = \frac{dv}{dt} = -\frac{2}{3}\frac{dS}{dt}$$

$$\therefore \quad \left.\frac{dV}{dt}\right|_{S=15\text{m}} = -\frac{2}{3}\left.\frac{dS}{dt}\right|_{S=15\text{m}} = -\frac{20}{3} \text{ ms}^{-2}$$

**18.** (a) Let $m$ be mass of each marble and $m_d$ be the mass of disc change in momentum of each marble $= 2mv$. Change in momentum of $nt$ marbles $= (2mv)(nt)$.

Let, the disc be balanced in mid air for a time $t$. Then, by impulse, momentum theorem

Impulse = Change in momentum.

$$(m_d g)\, t = 2mv \times nt$$

$$\Rightarrow \quad 10 \times 9.8 = 2 \times 5 \times v \times 10$$

$$v = \frac{98}{100} = 0.98 \text{ m/s}$$

**19.** (a) $x = (t - 3)^2 = t^2 - 6t + 9$

$$\Rightarrow \quad v = \frac{dx}{dt} = 2t - 6$$

Velocity at $t = 0$ is $u = -6 \text{ ms}^{-1}$

Velocity at $t = 6$ is $v = 6 \text{ ms}^{-1}$

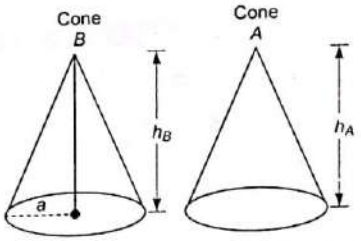
According to work-energy theorem,

Work done by all the forces = Change in KE

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \Rightarrow W = 0$$

**20.** (b) $h_A < h_B$

$d_A > d_B$




$B$ has larger shape and smaller density so, the same volume of $B$ as volume of $A$ has smaller value of its mass due to increase density.

**21.** (c) ∵ The compartment is stationary, so the combined centre of mass of the compartment and passengers is fixed, *i.e.*, $c_2$ is fixed. When the passengers move here and there in the compartment, then in an attempt to keep $c_2$ fixed, $c_1$ has to move.

**22.** (d) Total initial energy $= E_i$

Total final energy $= E_f$

So, $\quad E_i = \frac{-GMm}{R}$;

$$E_f = -\frac{GMm}{2(2R + R)} \qquad (\because \text{height} = 2R)$$

Energy required $= E_f - E_i$

$$\Rightarrow \quad \Delta E = \frac{-GMm}{6R} + \frac{GMm}{R} = \frac{5}{6}\frac{GMm}{R}$$

$$\Rightarrow \quad \Delta E = \frac{5}{6}mgR$$

**23.** (d) As, from the consideration of Archimedes' principle.

$$mg = V_w \rho_w g + V_{oil} \rho_{oil} g$$

where, $V_w$ and $V_{oil}$ are volumes of block inside water and oil, and $\rho_w$ and $\rho_{oil}$ are their respective densities.

$$\Rightarrow \quad m = (2 \times 10 \times 10 \times 1) + (8 \times 10 \times 10 \times 0.6)$$

$$m = 200 + 480 = 680 \text{ g}$$

**24.** (b) Let $x_1, x_2, x_3$ are the deformations in springs having stiffness $k_1, k_2, k_3, k_4 \ldots$, respectively then

Suppose, $x_s = x_1 + x_2 + x_3 + \ldots$

$$\Rightarrow \quad \frac{mg}{k_s} = mg\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots\right)$$

$$\therefore \quad \frac{1}{k_s} \quad \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots$$

If $k_1 = k, k_2 = 2k, k_3 = 3k, \ldots$ then

$$\frac{1}{k_s} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \ldots$$

$$\Rightarrow \quad \frac{1}{k_s} = \frac{1}{k}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right)$$

$$\Rightarrow \quad \frac{1}{k_s} = \frac{1}{k}\left(\frac{1}{1 - 1/2}\right)$$

$$\Rightarrow \quad \frac{1}{k_s} = \frac{2}{k} \Rightarrow k_s = \frac{k}{2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k_s}} \Rightarrow T = 2\pi\sqrt{\frac{2m}{k}}$$

**25.** (b) From the Doppler effect

We know that change in frequencies are as—

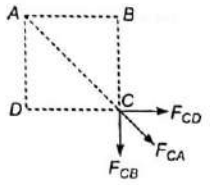
$$f_{\text{approach}} = \left(\frac{v}{v - v_s}\right) f_0 = f_1$$

and $$f_{\text{recede}} = \left(\frac{v}{v + v_s}\right) f_0 = f_2 (< f_1)$$

**26.** (d) By first law of thermodynamics,

$$Q_1 + (-W_1) = Q_2 + (-W_2) = U_B - U_A$$

**27.** (c) $$F_{\text{net}} = \frac{q^2\sqrt{2}}{4\pi\varepsilon_0 a^2} + \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}\,a)^2}$$




$$F_{\text{net}} = \left(\frac{1 + 2\sqrt{2}}{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$$

**28.** (d) $$I = \frac{V}{R} = \frac{9}{9} = 1\text{ A}$$

(Here, equivalent resistance can be found as $R = 9\,\Omega$)

At $A$, the current of 1 A divides into 0.5 A and 0.5 A.

At $B$, the current of 0.5 A divides into 0.25 A and 0.25 A.

**29.** (d) The blue filter will allow only blue light to pass through. So, light from the object passes through filter. Similarly, the white background will also look blue through the filter. Thus, we have a blue object under a blue background, which makes it indistinguishable.

**30.** (d) $$mg = kRv = \frac{4}{3}\pi R^3 \rho g = kRv_T$$

(At the condition of balance)

$$v_T = \frac{4}{3}\pi\frac{R^2\rho g}{k}$$

# Chemistry

**31.** (c) The structural isomers possible for the formula $C_4H_{10}O$ are

$CH_3—CH_2—CH_2—CH_2OH$ (butanol)

$CH_3—CH_2—\underset{\underset{OH}{|}}{CH}—CH_3$ (butane-2-ol)

$CH_3—\underset{\underset{CH_3}{|}}{CH}—CH_2—OH$ (2-methyl propane-1ol)

$CH_3—\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}—OH$ (2-methyl propane-2-ol)

$CH_3—CH_2—CH_2—O—CH_3$ (methoxy propane)

$CH_3—CH_2—O—CH_2—CH_3$ (ethoxy ethane)

$CH_3\diagdown\!\!\diagup CH—O—CH_3$ (2-methoxy propane)

**32.** (c) $$3I_2 \longrightarrow IO_3^- + 5I^-$$

$n = 5/3 \quad n = 5 \quad n = 1$

$$\therefore \quad \text{Eq. wt. of } I_2 = \frac{254}{5/3} = 152.4$$

**33.** (a) Acidic order can be determined by considering $-I$ effect of groups. Order of $-I$ effect is

$$—NO_2 > —Cl > —OCH_3$$

**34.** (d) $CH_3—C\equiv CH \xrightarrow{NaNH_2} \underset{(A)}{CH_3—C\equiv C^{\ominus}Na^{\oplus}}$

$\underset{(B)}{CH_3—C\equiv C—CH_3}$ ($A \xrightarrow{CH_3Br}$ $B$; $B \xrightarrow[H_2SO_4]{HgSO_4}$ …)

$CH_3—\underset{\underset{OH}{|}}{C}=CH—CH_3 \rightleftharpoons \underset{(C)}{CH_3—\overset{\overset{O}{\|}}{C}—CH_2—CH_3}$

Thus, $C$ contains 8 H atoms.

**35.** (c) $r_n = r_0 \times \frac{n^2}{z}$ are same

$z = 1$

$\therefore \quad r_1 = r_0$

$r_2 = 4r_0$

$r_3 = 9r_0$

and $r_1 : r_2 : r_3 = 1 : 4 : 9$

**36.** (d) $\left[\overset{\overset{\cdot\cdot}{(\cdot\cdot)}}{Cl—I—Cl}\right]^+$

I is central atom with two bonded pair and two lone pair. Hence, $sp^3$ hybridised.

**37.** (c) Straight line curve of $p$ *vs* $T$ shows that $V_1 = V_2$ and $V_3 = V_4$, *i.e.*, volume is constant.

Moreover, higher the $T$ and lesser the $p$, higher the value of $V$. Thus

$$V_2 > V_4 \text{ or } V_3$$

**38.** (c) Since, the number of moles in gaseous state in reactant and product side is same hence, no effect of pressure and rate depends only on temperature. Further the degree of dissociation increases with increase in temperature Hence, high temperature favours forward reaction or the rate of backward reaction decrease with rise in temperature.

**39.** (b) $MgCl_2 + 2NaOH \rightarrow Mg(OH)_2\downarrow + 2NaCl$

| | | | | |
|---|---|---|---|---|
| M moles | 10 | 20 | 0 | 0 |
| | 0 | 0 | 20 | 0 |

*i.e.*, almost all $Mg(OH)_2$ gets precipitated.

Hence, the remaining $Mg^{2+}$ and $OH^-$ concentration can be determined from $K_{sp}$ value of $Mg(OH)_2$.

$$Mg(OH)_2 \rightleftharpoons \underset{s}{Mg^{2+}} + \underset{2s}{2OH^-}$$

$$K_{sp} = 4s^3$$

$$s = \sqrt[3]{\frac{12\times10^{-12}}{4}} = 1.44\times10^{-4}$$

$$\therefore \quad [OH^-] = 2s = 2\times1.44\times10^{-4} = 2.88\times10^{-4}$$

**40.** (c) Since, $1\,dm^3 = 1\,L = 1000\,mL$

$$\therefore \quad k_2\times10^3 = k_1$$

**41.** (c) $Hg_2Cl_2 + 2NH_4OH \rightarrow \underset{\text{black ppt.}}{Hg} + Hg(NH_2)Cl + 2H_2O + NH_4Cl$

**42.** (b) $-OCH_3$ is electron donating in nature that is why strongly activating whereas $Cl^-$ is all over electron withdrawing by $-I$ effect so weakly deactivating.

Thus, the incoming group achieve a position *ortho* to $-OCH_3$ group as it is *o*/*p* directing.

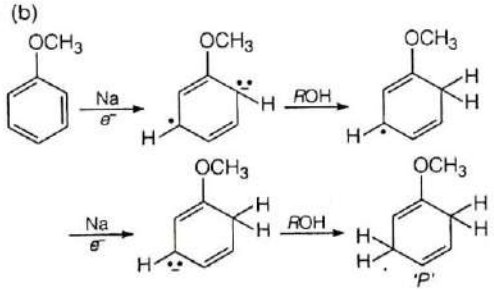
**43.** (b) $Al_2S_3 + 6H_2O \longrightarrow 2Al(OH)_3 + 3H_2S$

**44.** (a) Oxygen forms fcc structure in which $Mg^{2+}$ occupies tetrahedral voids and $Al^{3+}$ ions occupies octahedral void per unit cell. Total number of tetrahedral voids $= 2\times4 = 8$

$$\therefore \%\text{ tetrahedral void occupied} = \frac{1}{8}\times100$$

$$= 12.5\%$$

**45.** (b)




It is Birch reduction.

**46.** (c) Hydrolysis of $AlCl_3$ gives HCl which fumes in air.

$$AlCl_3 + 3H_2O \longrightarrow Al(OH)_3 + 3HCl$$

**47.** (d) Cubic ($a = b = c$ and $\alpha = \beta = \gamma = 90°$) and Rhombohedral ($a = b = c$ and $\alpha = \beta = \gamma \neq 90°$) have all sides and all angles equal.

**48.** (d) $$p_1 = \frac{nRT}{V_1} = \frac{2\times0.0821\times243.6}{20} = 2\,atm$$

$$\Delta S = nR\ln\left(\frac{p_1}{p_2}\right)$$

$$= 2\times2\ln2$$

$$= 2\times2\times0.693 \quad [\because \ln^2 \text{ or } \log_e 2 = 0.693]$$

$$= 2.77\,cal/K$$

**49.** (b) $FeCl_3 + 3NaOH \longrightarrow Fe(OH)_3 + 3NaCl$

$$NaOH \longrightarrow Na^{\oplus} + \dot{O}H^{\ominus}$$

$OH^{\ominus}$ ions having similarity with $Fe(OH)_3$ will be adsorbed on the surface of $Fe(OH)_3$ to form an electrical double layer.

**50.** (d) $Pb^{2+} + 2OH^{\ominus} \longrightarrow Pb(OH)_2\downarrow$ (white ppt.)

$Pb^{2+} + CO_3^{2-} \longrightarrow PbCO_3\downarrow$ (white ppt.)

$Pb^{2+} + 2Cl^- \longrightarrow PbCl_2\downarrow$ (white ppt.)

$Pb^{2+} + SO_4^{2-} \longrightarrow PbSO_4\downarrow$ (white ppt.)

**51.** (b)

$$HCOOH \rightleftharpoons HCOO^{\ominus} + H^{\oplus}$$

At equi $0.15 - x \sim 0.15 \quad x \quad 0.1$

$$\therefore \quad K_a = \frac{[HCOO^-][H^+]}{[HCOOH]}$$

$$[K_a = \text{Antilog}(-3.7) = 1.995\times10^{-4}]$$

$$1.995\times10^{-4} = \frac{x(0.1)}{0.15}$$

$$x = \frac{1.995\times10^{-4}\times0.15}{0.1}$$

$$[HCOO^-] = 2.99\times10^{-4}$$

$$\therefore \%\text{ dissociation} = \frac{2.99\times10^{-4}}{0.15}\times100$$

$$= 0.1995$$

**52.** (b) No. of moles of gas in bulb I $= \frac{pV}{RT} = \frac{5\times9}{RT}$

No. of moles of gas in bulb II $= \frac{10\times6}{RT}$

$\therefore$ Total moles when connected

$$= \frac{(45+60)}{RT} = \frac{105}{RT}$$

$\therefore$ Pressure of gas when connected

$$= \frac{nRT}{V_{Total}}$$

$$= \frac{105}{RT}\frac{RT}{15}$$

$$= 7\,atm$$

**53.** (d) $\frac{p_0 - p_S}{p_S} = \frac{W_B}{M_B} \times \frac{M_A}{W_A}$

$W_B$ = Mass of solute

$M_B$ = Molecular mass of solute

$W_A$ = Wt. of solvent

$M_B$ = Molecular mass of solvent

$$\frac{0.024 - p_S}{p_S} = \frac{68.4}{342} \times \frac{18}{1000}$$

$\therefore \quad p_S = 0.0239$ atm

**54.** (c) $\Delta T_f = K_f m$

$$= 1.86 \times \frac{68.4}{342} \times \frac{1}{1}$$

$$= 0.372$$

$\therefore$ Freezing point of aqueous solution

$= 0 - (0.372)^\circ C = -0.372^\circ C$

**55.** (a) Osmotic pressure,

$$\pi = \frac{n}{V} RT$$

$$= \frac{W_B}{M_B} \times \frac{RT}{V} \qquad (V = 1\,L)$$

$$= \frac{68.4}{342} \times \frac{0.0821}{1} \times 298 = 4.89 \text{ atm}$$

**56.** (c) **At anode**

$2CH_3COO^{\ominus} \longrightarrow 2CH_3COO^{\bullet} + 2e^-$

$2CH_3COO^{\bullet} \longrightarrow 2\dot{C}H_3 + 2CO_2$ (2 moles)

$CH_3^{\bullet} + \dot{C}H_3 \longrightarrow CH_3—CH_3(g)$ (1 mole)

Thus, at anode, 2 moles $CO_2$ and 1 mole $CH_3—CH_3$ is formed.

**At cathode**

$2H_2O + 2e^- \longrightarrow H_2 + 2OH^-$

*i.e.*, 1 mole of $H_2$ is liberated.

Thus, ratio of gases at cathode and anode is 1 : 3.

**57.** (a) **At anode**

$2HCOO^{\ominus} \longrightarrow 2HCOO^{\bullet} + 2e^-$

$2HCOO^{\bullet} \longrightarrow H_2 + 2CO_2$

**At cathode**

$2H_2O + 2e^- \longrightarrow H_2 + 2OH^{\ominus}$

$\therefore H_2$ and $CO_2$ are liberated at anode and $H_2$ is liberated at cathode.

**58.** (b) When $CuSO_4(aq)$ is electrolysed, Cu is deposited at cathode due to higher reduction potential but at anode instead of $SO_4^{2-}$ ions, $H_2O$ is oxidised due to higher oxidation potential.

*i.e.*, $2H_2O \longrightarrow O_2 + 4H^{\oplus} + 4e^-$

*i.e.*, $H^-$ ion is released in electrolytic solution, resulting in decrease of pH.

**59.** (b) Both the statements are true as basic radicals are analysed on the basis of $K_{sp}$ value only. Most alkali metal salts are soluble in water.

**60.** (c) $CH_3CH_2OH$ gives iodoform test, *i.e.*, it forms yellow crystals of iodoform when treated with $I_2$ and NaOH but $CH_3OH$ does not give this reaction.

$CH_3CH_2OH + 4I_2 + 6NaOH \longrightarrow HCOONa + \underset{\text{iodoform}}{CHI_3} + 5H_2O + 5NaI$

$CH_3OH + I_2 + NaOH \longrightarrow$ No reaction.

Only those 2° alcohols which have $CH_3\underset{\displaystyle OH}{\underset{|}{C}}H—$ unit give positive iodoform test. Thus, statement II is false.

# Mathematics

**61.** (b) The three planes can be written as

$x - cy - bz = 0$ ...(i)

$cx - y + az = 0$ ...(ii)

$bx + ay - z = 0$ ...(iii)

Let $l, m$ and $n$ be the DC's of the line of intersection of Eqs. (i) and (ii), then

$l - cm - bn = 0$

$cl - m + an = 0$

Now, by cross multiplication method,

$$\frac{l}{ac + b} = \frac{m}{bc + a} = \frac{n}{1 - c^2}$$

Since, planes (i) and (ii) passes through origin. Hence, there line of intersection will also passes through the point (0, 0, 0).

So, the equation of the line of intersection of Eqs. (i) and (ii) is

$$\frac{x}{ac + b} = \frac{y}{bc + a} = \frac{z}{1 - c^2} \qquad \text{...(iv)}$$

Now, the three planes will intersect in a line of Eq. (iv) satisfies Eq. (iii). Hence, the required condition is

$b(ac + b) + a(bc + a) - (1 - c^2) = 0$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2abc = 1$

**62.** (b) Let $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4} = r$ (say)

$\therefore \quad (x, y, z) = (2r - 1, 3r - 1, 4r - 1)$

$\therefore (2r - 1) + 2(3r - 1) + 3(4r - 1) = 14$

$\Rightarrow \quad 2r - 1 + 6r - 2 + 12r - 3 = 14$

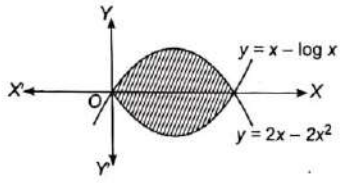
$\Rightarrow \quad r = 1$

So, the point on the plane is (1, 2, 3).

$\therefore$ The distance of $P(1, 2, 3)$ from origin is

$$\sqrt{1+4+9} = \sqrt{14}$$

**63.** (a) Given curves are $y = x \log x$ and $y = 2x - 2x^2$.




Required area

$$= \int_0^1 (2x' - 2x^2)dx - \int_0^1 x \log x \, dx$$

$$= \left[x^2 - \frac{2x^3}{3}\right]_0^1 - \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_0^1$$

$$= \left(1 - \frac{2}{3}\right) - \left[0 - \frac{1}{4} - \frac{1}{2}\lim_{x\to 0} x^2 \log x\right]$$

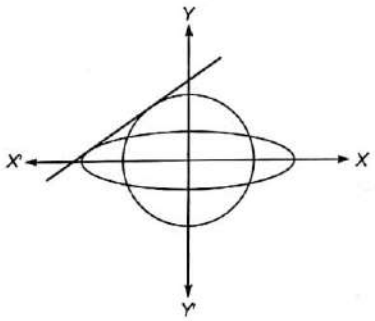
$$\left[\because \lim_{x\to 0} x^2 \log x = \lim_{x\to 0} \frac{\log x}{1/x^2} \ (0 \times \infty \text{ form}) \right.$$

$$= \lim_{x\to 0} \frac{1/x}{-2/x^3}$$

$$\left. = \lim_{x\to 0}\left(-\frac{x^2}{2}\right) = 0\right]$$

$$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

**64.** (a) Let equation of circle is $x^2 + y^2 = r^2$.




Tangent to ellipse is

$$y = mx + \sqrt{a^2m^2 + b^2}$$

If it is a tangent to the circle, then it is perpendicular from (0, 0) is equal to radius.

$$\therefore \quad \frac{\sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}} = r$$

$$\Rightarrow \qquad m = \frac{\sqrt{r^2 - b^2}}{\sqrt{a^2 - r^2}}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$$

**65.** (a) Let any two points on the ellipse be

$$P = (a\cos\phi, b\sin\phi).$$

and $\quad P' = (-a\cos\phi, -b\sin\phi)$

$$\therefore \quad PP'^2 = 4a^2\cos^2\phi + 4b^2\sin^2\phi$$

$$\therefore \ 4a^2 + 4b^2 = 2[4a^2\cos^2\phi + 4b^2\sin^2\phi]$$

$$\left[\because PP'^2 = \frac{(2a)^2 + (2b)^2}{2}\right]$$

$$\therefore \quad a^2 + b^2 = 2a^2\cos^2\phi + 2b^2\sin^2\phi$$

$$\therefore \quad a^2 + b^2 = 2a^2 - 2a^2\sin^2\phi + 2b^2\sin^2\phi$$

$$\therefore \quad b^2 - a^2 = 2(b^2 - a^2)\sin^2\phi$$

$$\Rightarrow \quad \sin^2\phi = \frac{1}{2} \text{ and } \cos^2\phi = \frac{1}{2} \quad [\because \tan\phi = 1]$$

$$\tan\theta = \text{Slope of } (PP') = \frac{-b\sin\phi - b\sin\phi}{a\cos\phi - a\cos\phi}$$

$$= \frac{b}{a}\tan\phi = \frac{b}{a}\cdot 1 = \frac{b}{a}$$

**66.** (b) The point of intersecton of $2x - 3y + 5 = 0$ and $x + y = 2$ is $\left(\frac{1}{5}, \frac{9}{5}\right)$

$\therefore$ Point $\left(\frac{1}{5}, \frac{9}{5}\right)$ is the point of incidence.

$\therefore$ Slope of incident ray $= \frac{2}{3}$

and Slope of normal $= 1$

Let slope of refracted ray be $m$.

Then, $\quad \frac{m-1}{1+m} = \frac{1-\frac{2}{3}}{1+\frac{2}{3}} = \frac{1}{5}$

$$\Rightarrow \qquad m = \frac{3}{2}$$

$\therefore$ The equation of the straight line containing the refracted ray is

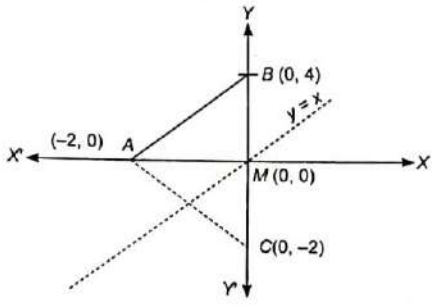
$$y - \frac{9}{5} = \frac{3}{2}\left(x - \frac{1}{5}\right)$$

$$\Rightarrow \ 3x - 2y + 3 = 0$$

**67.** (b) For perimeter to be minimum, $AM + BM$ should be minimum. ($\because$ $AB$ is fixed)

For $AM + BM$ to be minimum,

$M$ should be such that $AM$ is reflected along $MB$ from the line $x = y$.



$AM + BM = CM + BM = CB$

[$C$ is reflection of $A$ on line $x = y$ i.e., $C \equiv (0, -2)$]

So, the equation of $CB$ is $x = 0$.

$\therefore$ $M = (0, 0)$

**68.** (a) Since, given triangle is right angled isosceles triangle.

$\therefore$ Angle between two given lines = 45°

$\therefore$ $\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \tan 45°$ ...(i)

where, $m_1 = -\frac{1}{k}$ and $m_2 = \infty$

From Eq. (i), $\left|\frac{\frac{m_1}{m_2} - 1}{\frac{1}{m_2} + m_1}\right| = 1$

$\Rightarrow$ $\left|\frac{-1}{-\frac{1}{k}}\right| = 1$

$\Rightarrow$ $|k| = 1 \Rightarrow k = \pm 1$

**69.** (a) Given equation of the given line is $x + y + 2z - 3 = 0$ and $2x + 3y + 4z - 4 = 0$. The equation of the plane passing through the given line is

$(x + y + 2z - 3) + \lambda(2x + 3y + 4z - 4) = 0$ ...(i)

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (2 + 4\lambda)z - (3 + 4\lambda) = 0$

If this plane is parallel to $Z$-axis whose DC's are (0, 0, 1), then normal to the plane will be perpendicular to $Z$-axis.

$\therefore$ $(1 + 2\lambda)0 + (1 + 3\lambda)0 + (2 + 4\lambda)(1) = 0$

$\Rightarrow$ $\lambda = -\frac{1}{2}$

From Eq. (i)

$(x + y + 2z - 3) - \frac{1}{2}(2x + 3y + 4z - 4) = 0$

$\Rightarrow$ $y + 2 = 0$ ...(ii)

Now, the shortest distance is the distance of any point on the $Z$-axis for the plane (ii). Let us take (0, 0, 0) as a point on $Z$-axis.

$\therefore$ Shortest distance = Length of perpendicular from (0, 0, 0) on plane (ii)

$= \frac{2}{(\sqrt{1})^2} = 2$

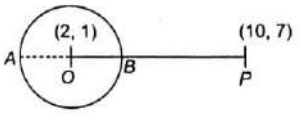
**70.** (b) Mean deviation is minimum when it is considered about the item, equidistant from the beginning and the end i.e., the median.

In this case, median is $\frac{101 + 1}{2}$th i.e., 51th term

$= x_{51}$

**71.** (a) Centre of circle = (2, 1)

and radius = $\sqrt{4 + 1 + 20} = 5$




$\therefore$ $OP = \sqrt{(10 - 2)^2 + (7 - 1)^2}$

$= \sqrt{64 + 36}$

$= 10$

$\because$ At point (10, 7),

$S_1 = 10^2 + 7^2 - 40 - 14 - 20 > 0$

$\therefore$ The point $P$ is outside the given circle.

Now, required largest distance = $PA$

$\therefore$ $PA = AB + BP$

$= 2$ radius + ($OP$ − radius)

$= 2 \times 5 + 10 - 5$

$= 10 + 5 = 15$

$\Rightarrow$ $5\alpha = 15$ (given)

$\therefore$ $\alpha = 3$

**72.** (a) Let the mid-point of any chord be $(h, k)$.

Then, $T = S_1$

$\Rightarrow$ $\left(\frac{xh}{a^2} + \frac{yk}{b^2}\right) = \frac{h^2}{a^2} + \frac{k^2}{b^2}$

$\Rightarrow$ $\frac{yk}{b^2} = -\frac{hx}{a^2} + \frac{h^2}{a^2} + \frac{k^2}{b^2}$

$\Rightarrow$ $y = \left(\frac{-b^2h}{a^2k}\right)x + \left(\frac{b^2h^2}{a^2k} + k\right)$

$\therefore$ Condition for tangency between the line $y = mx + c$ and ellipse is

$c^2 = a^2m^2 + b^2$

$\therefore$ $\left(\frac{b^2h^2 + a^2k^2}{a^2k}\right)^2 = p^2\left(\frac{-b^2h}{a^2k}\right)^2 + q^2$

Hence, locus of a point is

$(x^2b^2 + y^2a^2)^2 = p^2b^4x^2 + q^2a^4y^2$

$\Rightarrow$ $\left(\frac{b^2x^2 + y^2a^2}{a^2b^2}\right)^2 = \left(\frac{p^2x^2}{a^4} + \frac{q^2y^2}{b^4}\right)$

$\Rightarrow$ $\frac{p^2x^2}{a^4} + \frac{q^2y^2}{b^4} = \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2$

**73.** (c) Put $2^x = t$

$\therefore$ $f(t) = (t - 1)(t - 2)^2$

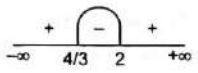
$\therefore$ $f'(t) = 2(t - 1)(t - 2) + (t - 2)^2$

$f'(t) = (t - 2)(3t - 4)$

For $f(t)$ to be decreasing,

$\therefore$ $f'(t) < 0$

$\Rightarrow$ $(t - 2)(3t - 4) < 0$




$\Rightarrow \quad \frac{4}{3} < t < 2$

$\Rightarrow \quad \frac{4}{3} < 2^x < 2$

$\Rightarrow \quad x \in \left(\log_2 \frac{4}{3}, 1\right)$

**74.** (b) I. $\text{RHS} = \frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} + \frac{2}{(x-y)(y-z)} + \frac{2}{(y-z)(z-x)} + \frac{2}{(x-y)(z-x)}$

$= \frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} + \frac{2(z-x+x-y+y-z)}{(x-y)(y-z)}$

$= \text{LHS}$

II. $\log_3 x \log_4 x \log_5 x$

$= \log_3 x \log_4 x + \log_4 x \log_5 x + \log_3 x \log_5 x$

$\frac{\log x}{\log 3} \cdot \frac{\log x}{\log 4} \cdot \frac{\log x}{\log 5}$

$= \frac{\log x}{\log 3} \cdot \frac{\log x}{\log 4} + \frac{\log x}{\log 4} \cdot \frac{\log x}{\log 5} + \frac{\log x}{\log 3} \cdot \frac{\log x}{\log 5}$

$\Rightarrow \frac{(\log x)^3}{\log 3 \cdot \log 4 \cdot \log 5}$

$= \frac{(\log x)^2 (\log 3 + \log 4 + \log 5)}{\log 3 \cdot \log 4 \cdot \log 5}$

$\Rightarrow \quad (\log x)^2 (\log x - \log 60) = 0$

$\Rightarrow \quad \log x = 0 \text{ or } \log x = \log 60$

$\Rightarrow \quad x = 1 \text{ or } x = 60$

So, $x = 1$ and 60 are the required solution.

III. Possible orders are

$(1 \times 12)/(12 \times 1)/(2 \times 6)/(6 \times 2)/(3 \times 4)/(4 \times 3)$.

**75.** (a) $\sum_{r=1}^{20} \sum_{p=1}^{r} p^2 = \sum_{r=1}^{20} \frac{r(r+1)(2r+1)}{6}$

$= \frac{1}{6} \sum_{r=1}^{20} (2r^3 + 3r^2 + r)$

$= \frac{1}{6}\left[2\left(\frac{20(20+1)}{2}\right)^2 + 3\left(\frac{20(20+1)(40+1)}{6}\right) + \frac{20(20+1)}{2}\right]$

$= \frac{1}{6}[88200 + 8610 + 210]$

$= \frac{97020}{6} = 16170$

**76.** (c) $\because 1, a_1, a_2, \ldots, a_{n-1}$ be $n$th root of unity.

$\therefore (x-1)(x-a_1)(x-a_2)\ldots(x-a_{n-1}) = x^n - 1$

On taking logarithm and then differentiation, we get

$\frac{1}{x-1} + \frac{1}{x-a_1} + \frac{1}{x-a_2} + \ldots + \frac{1}{x-a_{n-1}} = \frac{nx^{n-1}}{x^n - 1}$

On putting $x = -2$, we get

$\frac{1}{-2-1} + \frac{1}{-a_1} + \frac{1}{-2-a_2} + \ldots + \frac{1}{-2-a_{n-1}} = \frac{n(-2)^{n-1}}{(-2)^n - 1}$

$\Rightarrow \quad \sum_{r=1}^{n-1} \frac{1}{2+a_r} = \frac{n(-2)^{n-1}}{1-(-2)^n} - \frac{1}{3}$

$= \frac{n(-2)^{n-1}}{1+(-2)^{n+1}} - \frac{1}{3}$

**77.** (c) Take the two infinite GP is

$\frac{a}{r}, \frac{a}{1}, ar, \ldots \text{ and } \frac{a}{s}, a, as, as^2, \ldots$

Their sums are 1 each

$\therefore \quad a = r - r^2 = s - s^2 \qquad \left(\because \text{Sum} = \frac{A}{1-R}\right)$

$\Rightarrow \quad r + s = 1, \quad r \neq s$

$\therefore \quad ar = \frac{1}{8}, a = r - r^2$

$\Rightarrow \quad a = \frac{\sqrt{5}-1}{8}$

$r = \frac{\sqrt{5}-1}{4}$

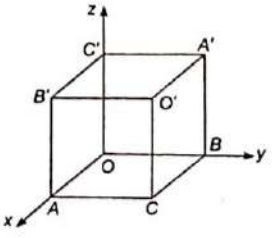
$s = \frac{3-\sqrt{5}}{4}$

Now, $b_3 = as = \frac{(\sqrt{5}-1)(3-\sqrt{5})}{32}$

$= \frac{\sqrt{5}-2}{8}$

**78.** (d) Here, vertices of a cube are $O(0, 0, 0)$, $O'(1, 1, 1)$, $A(1, 0, 0)$, $A'(0, 1, 1)$, $B(0, 1, 0)$, $B'(1, 0, 1)$, $C'(0, 0, 1)$ and $C(1, 1, 0)$.

Then; number of all triangles is ${}^8C_3 = 56$.



The number of right angled isosceles triangles like $\Delta OAB$ each of area $\frac{1}{2}$ is 24. The number of right angled scalar triangles like $\Delta OCO'$ each of area $\frac{1}{\sqrt{2}}$ is 24.

The number of equilateral triangles like $O'BC$, each of area $\frac{\sqrt{3}}{2}$ is 8.

$\therefore$ The total area $= 12 + 12\sqrt{2} + 4\sqrt{3}$

$= 12 + \sqrt{288} + \sqrt{48}$

$\Rightarrow \quad m = 12, n = 288$ and $n = 48$

$\therefore \quad m + n + p = 348$

**79.** (b) The number of ways of getting the sum 9 is the coefficient of $x^9$ in the expansion of

$(x + x^2 + x^3 + \ldots + x^6)^4$

*i.e.*, the coefficient of $x^5$ in $(1 + x + \ldots + x^5)^4$

$= (1 - x^6)^4 (1 - x)^{-5}$

$\therefore$ The number of ways of getting the sum $\le 9$ is the coefficient of $x^5$ in $(1 - x^6)^4 (1 - x)^{-5}$

$= (1 - 4x^6 - \ldots)(1 + {}^5C_1 x + {}^6C_2 x^2 + \ldots)$

$= {}^9C_5 = 126$

$\therefore$ The probability of getting the sum $\ge 10$ is

$$1 - \frac{126}{6^4} = 1 - \frac{7}{72} = \frac{65}{72}$$

**80.** (c) Let $a = \cos\theta$ and $b = \sin\theta$

$\therefore \quad z = \cos\theta + i\sin\theta \quad (\because |z| = 1)$

$\because \quad z = \frac{c+i}{c-i}$

$\therefore \quad c = \frac{i(z+1)}{z-1} = \frac{i(\cos\theta + i\sin\theta + 1)}{(\cos\theta + i\sin\theta - 1)}$

$$= \frac{i\left\{2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right\}}{\left\{-2\sin^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right\}}$$

$$= i\frac{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}{\left(-\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right)}\cdot\cot\frac{\theta}{2}$$

$$= \frac{i}{i}\cdot\frac{\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}\cdot\cot\frac{\theta}{2}$$

$(\because i^2 = 1)$

$$= \cot\frac{\theta}{2}$$

$$\Rightarrow \quad c = \cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta} = \frac{1+a}{b}$$

**81.** (a) $\frac{\sin^3\theta}{\cos^3\theta} + \frac{\cos^3\theta}{\sin^3\theta} = 12 + \frac{8}{\sin^3 2\theta}$

$\Rightarrow \sin^6\theta + \cos^6\theta = \frac{3}{2}\sin^3 2\theta + 1$

$\Rightarrow 1 - \frac{3}{4}\sin^2 2\theta = \frac{3}{2}\sin^3 2\theta + 1$

$\Rightarrow \sin 2\theta(\sin^2 2\theta + 2\sin 2\theta) = 0$

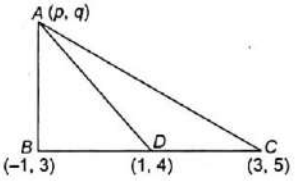
$\Rightarrow \sin 2\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right) \quad (\because \sin 2\theta \neq 0)$

$\Rightarrow 2\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\Rightarrow \theta = \frac{n\pi}{2} + \frac{\pi}{12}(-1)^{n+1}$

$\Rightarrow \theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

**82.** (c) Let $A = (p, q)$ and $D = (1, 4)$ is the mid-point of $BC$.




Slope of $AD$ is $-2$.

$\therefore \quad \frac{4-q}{1-p} = -2$

$\Rightarrow \quad 2p + q = 6 \quad \ldots(i)$

Area of $\Delta ABC = 5 \Rightarrow \begin{vmatrix} p & q & 1 \\ -1 & 3 & 1 \\ 3 & 5 & 1 \end{vmatrix} = \pm 10$

$\Rightarrow \quad -2p + 4q - 14 = \pm 10$

$\Rightarrow \quad -p + 2q - 7 = \pm 5$

$\Rightarrow \quad p - 2q = -2, -12 \quad \ldots(ii)$

$\therefore \quad (p, q) = (0, 6), (2, 2)$

$\therefore \quad OA = 6, 2\sqrt{2}$

**83.** (a) The normal at $P(t)$ to $y^2 = 8x$ is $xt + y = 2t^3 + 4t$.

The point (6, 2) lies on it.

$\therefore \quad t^3 - t - 1 = 0$

If $t_1, t_2$ and $t_3$ are roots.

$$\left.\begin{aligned} &t_1 + t_2 + t_3 = 0,\ t_1t_2 + t_2t_3 + t_3t_1 = -1 \\ \text{and} \quad &t_1t_2t_3 = 1 \end{aligned}\right\} \ldots(i)$$

Let The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meets the parabola at the point $t$.

$\therefore \quad 4t^4 + 16t^2 + 4gt^2 + 8ft + c = 0$

If $t_1, t_2, t_3$ and $t_4$ are the roots.

$t_1 + t_2 + t_3 + t_4 = 0,\ t_1t_2t_3t_4 = c$

$\sum t_1t_2t_3 = -2t$ and $\sum t_1t_2 = \frac{(16+36)}{4} = 4 + 9$

$\therefore \quad t_4 = 0 = c, g = -5, f = -\frac{1}{2}$ [using Eq. (i)]

$\Rightarrow \quad x^2 + y^2 - 10x - y = 0$

Its intercept on the $y$-axis is 1.

**84.** (c) $\frac{dx}{dy} - x = y$

$\therefore \quad IF = e^{-\int dy} = e^{-y}$

$\therefore \quad xe^{-y} = \int ye^{-y} dy$

$\therefore \quad xe^{-y} = -(y+1)e^{-y} + A$

$\because \quad x = 0, y = 0 \Rightarrow A = 1$

$xe^{-y} = -(y+1)e^{-y} + 1$

$\Rightarrow \quad x = e^{y} - (y+1)$

At $y = \log 3$,

$\Rightarrow \quad x = 2 - \log 3$

$\frac{dx}{dy} = 2 - \log 3 + \log 3$

$= 2$

$\therefore \quad \frac{dy}{dx} = \frac{1}{2}$

**85.** (d) $\int_0^1 x\,dy = \int_0^1 (e^y - y - 1)\,dy$

$= \left(e^y - \frac{y^2}{2} - y\right)_0^1$

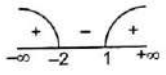
$= e - \frac{5}{2}$

**86.** (a) For $0 \le x \le 1$,

$|x|f(x) > 2 \Rightarrow x(x+1) > 2$

$\Rightarrow \quad x^2 + x - 2 > 0$

$\Rightarrow \quad (x+2)(x-1) > 0$



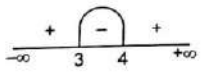

$\Rightarrow \quad x \in (-\infty, -2) \cup (1, \infty)$

Hence, there is no solution.

**87.** (c) $\{f(x) + |x-2|\} f(x) \le 0$

$\Rightarrow \quad (x - 4 + x - 2)(x - 4) \le 0$

$\Rightarrow \quad 2(x-3)(x-4) \le 0$




$\Rightarrow \quad 3 \le x \le 4$

**88.** (a) **Statement I**

$[2\mathbf{a} - \mathbf{b} \;\; 2\mathbf{b} - \mathbf{c} \;\; 2\mathbf{c} - \mathbf{a}]$

$= (2\mathbf{a} - \mathbf{b}) \cdot [(2\mathbf{b} - \mathbf{c}) \times (2\mathbf{c} - \mathbf{a})]$

$= (2\mathbf{a} - \mathbf{b}) \cdot [4\mathbf{b} \times \mathbf{c} - 2\mathbf{b} \times \mathbf{a} - 0 + \mathbf{c} \times \mathbf{a}]$

$= 8[\mathbf{abc}] - 4[\mathbf{aba}] + 2[\mathbf{aca}]$

$\quad - 4[\mathbf{bbc}] + 2[\mathbf{bba}] - [\mathbf{bca}]$

$= 8[\mathbf{aba}] - [\mathbf{bca}] = 7[\mathbf{abc}]$

$= 0$ ($\because \mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanars)

**89.** (a) $T_1 = 1, \quad T_2 = 7 = (8 - 1)$

$T_3 = 19 = (27 - 8)$

.....

.....

$T_n = n^3 - (n-1)^3$

$\therefore \quad S_n = \Sigma T_n = \Sigma(n^3 - n^3 + 1 - 3n + 3n^2)$

$= \Sigma(3n^2 - 3n + 1)$

$= \frac{3n(n+1)(2n+1)}{6} - \frac{3n(n+1)}{2} + n$

$= \frac{3n(n+1)}{6}(2n + 1 - 3) + n$

$= n(n+1)(n-1) + n$

$= n[n^2 - 1 + 1] = n^3$

$\therefore \quad S_{20} = (20)^3 = 8000$

**90.** (a) Now, if we consider the images $a$ and $b$ as two different boxes, then four distinct objects 1, 2, 3 and 4 (preimage) can be distributed in $2^4 - {}^2C_1(2-1)^4 = 16 - 2 = 14$.

Hence, both statements are true and statement II is the correct explanation of statement I.

# JEE Main

## Joint Entrance Examination

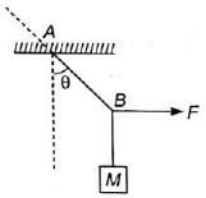
# Practice Set 3

## Instructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

1. A mass $M$ is suspended by a rope from a rigid support at $A$ as shown in figure. Another rope is tied at the end $B$ and it is pulled horizontally with a force $F$. If the rope $AB$ makes an angle $\theta$ with the vertical, then the tension in the string $AB$ is




(a) $F \sin\theta$ (b) $F/\sin\theta$
(c) $F \cos\theta$ (d) $F/\cos\theta$

2. The momentum of a body is increased by 100%. The percentage change in KE will be
(a) 480% (b) 100% (c) 200% (d) 300%

3. If the linear density of the rod of length $L$ varies at $\lambda_x = A + Bx$, then its centre of mass is given by

(a) $X_{CM} = \frac{L(2A + BL)}{3(3A + 2BL)}$ (b) $X_{CM} = \frac{L(3A + 2BL)}{3(2A + BL)}$ (c) $X_{CM} = \frac{L(3A + 2BL)}{3}$ (d) $X_{CM} = \frac{L(2A + 3BL)}{3}$

4. If a particle moves in the $x$-$y$ plane, the resultant angular momentum has
(a) only $x$-component (b) only $y$-component
(c) both $x$ and $y$-components (d) only $z$-component

5. The maximum vertical distance through which a fully dressed astronaut can jump on the earth is 0.5 m. If mean density of the moon is two-third that of the earth and radius is one quarter of the earth, the maximum vertical distance through which he can jump on the moon and the ratio of time of duration of the jump on the moon to that on the earth are
(a) 3 m, 6 : 1 (b) 6 m, 3 : 1 (c) 3 m, 1 : 6 (d) 6 m, 1 : 6

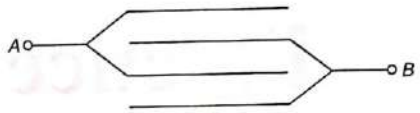
6. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square speed of gas molecules is $v$, then at 480 K it will be

(a) $4v$ (b) $2v$ (c) $\frac{v}{2}$ (d) $\frac{v}{4}$

7. Transverse waves of the same frequency are generated in two steel wires $A$ and $B$. The diameter of $A$ is twice that of $B$ and the tension in $A$ is half that in $B$. The ratio of the velocities of waves in $A$ and $B$ is

(a) 1 : 2 (b) $1:\sqrt{2}$ (c) $1:2\sqrt{2}$ (d) $3:2\sqrt{2}$

8. Four metallic plates, each with a surface area of one side $A$, are placed at a distance $d$ from each other. The alternating plates are connected to points $A$ and $B$ as shown in the figure. Then, the capacitance of the system is




(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$ (c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

9. The electric field part of an electromagnetic wave in a medium is represented by $E_x = 0$

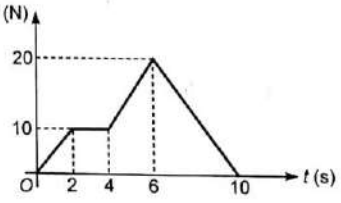
$E_y = 2.5\frac{\text{N}}{\text{C}}\cos\left[\left\{2\pi\times10^6\frac{\text{rad}}{\text{m}}\right\}t - \left\{\pi\times10^{-2}\frac{\text{rad}}{\text{s}}\right\}x\right]$; $E_z = 0$. The wave is

(a) moving along $x$-direction with frequency $10^6$ Hz and wavelength 100 m
(b) moving along $x$-direction with frequency $10^6$ Hz and wavelength 200 m
(c) moving along $y$-direction with frequency $10^6$ Hz and wavelength 200 m
(d) moving along $y$-direction with frequency $2\pi\times10^6$ Hz and wavelength 200 m

10. When $^{236}_{88}$Ra decays in a series by emission of 3$\alpha$-particles and 1$\beta$-particle isotope $X$ formed is

(a) $^{224}_{83}X$ (b) $^{218}_{84}X$ (c) $^{220}_{84}X$ (d) $^{223}_{87}X$

11. A particle of mass 2 kg is initially at rest. A force acts on it whose magnitude changes with time. The force time graph is shown in figure. The velocity of the particle after 10 s is




(a) $10\,\text{ms}^{-1}$ (b) $75\,\text{ms}^{-1}$
(c) $20\,\text{ms}^{-1}$ (d) $50\,\text{ms}^{-1}$

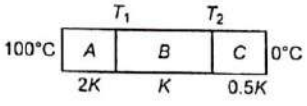
12. A mass $m$ moving horizontally with velocity $v_0$ strikes a pendulum of mass $m$. If the two masses stick together after the collision, then the maximum height reached by the pendulum is

(a) $v_0^2/8g$ (b) $v_0^2/2g$ (c) $\sqrt{2v_0 g}$ (d) $\sqrt{v_0 g}$

13. In a plant, a sucrose solution of coefficient of viscosity 0.0015 N-s/m$^2$ is driven at a velocity of $10^{-3}$ m/s through xylem vessels of radius 2 $\mu$m and length 5 $\mu$m. The hydrostatic pressure difference across the length of xylem vessels in N/m$^2$ is

(a) 5 (b) 8 (c) 10 (d) 15

14. Three identical rods $A, B$ and $C$ of equal lengths and equal diameters are joined in series as shown in figure. Their thermal conductivities are $2K, K$ and $K/2$ respectively. Calculate the temperature at two junction points

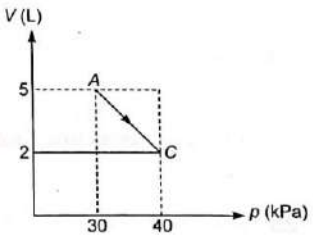



(a) 85.7, 57.1°C (b) 80.85, 50.3°C (c) 77.3, 48.3°C (d) 75.8, 49.3°C

15. Two sources $A$ and $B$ are sounding notes of frequency 680 Hz. A listener moves from $A$ to $B$ with a constant velocity $u$. If the speed of sound is 340 $ms^{-1}$, what must be the value of $u$ so that he hears 10 beats per second?
(a) 2.0 $ms^{-1}$ (b) 2.5 $ms^{-1}$ (c) 3.0 $ms^{-1}$ (d) 3.5 $ms^{-1}$

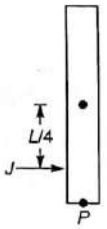
16. There is a temperature at which the reading on kelvin scale is numerically
(a) equal to that on the celsius scale (b) lower than that on the celsius scale
(c) equal to that on the fahrenheit scale (d) less than zero

17. A gas is taken along the path as shown. If heat supplied to the system is 20 J, then the change in the internal energy is




(a) 125 J
(b) – 10 J
(c) 10 J
(d) 55 J

18. A uniform rod of mass $M$ and length $L$ is placed on a smooth horizontal surface as shown in figure. The rod is given an impulse $J$ in a direction perpendicular to length of rod and at a distance $\frac{L}{4}$ from the centre of mass of road as shown in figure. Just after impact, the velocity of point $P$ is




(a) $\frac{5J}{2M}$ (b) $\frac{2J}{M}$ (c) $\frac{3}{2}\times\frac{J}{M}$ (d) $\frac{J}{M}$

19. Consider the transistor shown in the figure, its terminals are marked as 1, 2 and 3. Using multimeter a student tries to identify the base of transistor, he proceed as follows

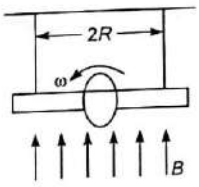

**Experiment I** He touches the common lead of the multimeter to 2, then on touching other lead of multimeter to 1, he has not get any beep (Indication of conduction) but when connected to 3, he gets a beep.
**Experiment II** He connects the common lead of multimeter to 1 and other lead to 2 and 3, turn by turn then, he gets a beep of both the connections. From this, we conclude that
(a) 1 is base (b) 2 is base (c) 3 is base (d) None of these

20. A particle is projected from a point (0, 1) on $y$-axis (assume $+y$ direction vertically upward) aiming towards a point (4, 9). It fell on ground along $x$-axis in 1 s. Take $g = 10\ m/s^2$ and all coordinates in metre. Find the $x$-coordinate where it fell
(a) (3, 0) (b) (4, 0) (c) (2, 0) (d) $(2\sqrt{5}, 0)$

21. The average speed of a moving object during a given interval of time is always
(a) the magnitude of its average velocity over the interval
(b) one-half its speed at the end of the interval
(c) distance covered during the interval divided by the time interval
(d) its acceleration multiplied by the time interval

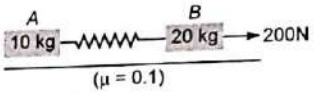
22. A solid conducting sphere having a charge $Q$ is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of solid sphere and that of the outer surface of the hollow shell be $V$. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces would be
(a) $V$ (b) $2V$ (c) $4V$ (d) $-2V$

23. A ring of radius $R$ having uniformly distributed charge $Q$ is mounted on a rod suspended by two identical light strings, as shown in the figure. The tension in the strings in equilibrium is $T_0$. Now a vertical magnetic field is switched on and the ring is rotated at constant angular velocity $\omega$. The maximum value $\omega$ that the ring can accquire if the strings can withstand a maximum tension of $\frac{3T_0}{2}$, is




(a) $\frac{2d}{3BQR\omega}$ (b) $\frac{d}{BQR\omega}$ (c) $\frac{3}{4}\times QB\omega R$ (d) $\frac{3}{2}\times QB\omega R$

24. Two blocks $A$ and $B$ connected to each other by a light spring are kept on a rough horizontal surface ($\mu = 0.1$) as shown in the figure. If at any instant acceleration of $A$ is $12\,m/s^2$, then at this instant acceleration of $B$ would be




(a) $12\,m/s^2$ (b) $16\,m/s^2$ (c) $24\,m/s^2$ (d) $2.5\,m/s^2$

25. A strong battery of emf 8 V and internal resistance 0.5 Ω is being charged by a 120 V DC supply using a series resistor of 15.5 Ω. The terminal potential difference of the battery during charging is

(a) 8 V (b) 4.5 V (c) 11.5 V (d) zero

26. A particle performs SHM about $x = 0$ such that at $t = 0$ it is at $x = 0$ and moving towards positive extreme. The ratio of time taken by it to go from $x = 0$ to $x = \frac{A}{2}, d$ and from here to $x = A$ is equal to

(a) 2 (b) 1/2 (c) 11/12 (d) 12/11

27. **Statement I** In verification of the Ohm's law using voltmeter-ammeter method, an experimenter noted down the reading of voltmeter as 2.15 V and ammeter reading as 1.60 A, then $R$ would be written upto 3 significant digits.

**Statement II** Reliability in the computed value cannot be greater than the reliability in the measured values. Mark your answer as

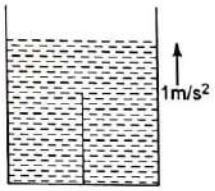
(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I.

(c) Statement I is true, Statement II is false.

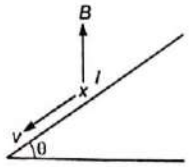
(d) Statement I is false, Statement II is true.

28. A tank accelerates uniformly in upward direction with acceleration $1\,m/s^2$ and contains water. A block of mass 1 kg and density 0.8 g/cc is held stationary inside the tank with the help of the string as shown in figure. The tension in the string is
[Take $\rho_{water} = 1000\,kg/m^3$ and $g = 10\,m/s^2$]



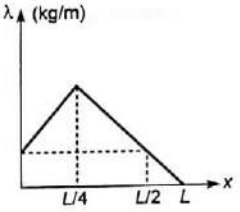

(a) 2 N (b) 1.5 N
(c) 10 N (d) 7 N

29. A conducting rod of length $L$ and mass $m$ is moving down a smooth inclined plane of inclination $\theta$ with constant speed $v$. A current $I$ is flowing in the conductor perpendicular to plane of paper (inwards). A vertically upward uniform magnetic field $B$ exists in space there. The magnitude of magnetic field will be




(a) $\frac{mg}{IL}\sin\theta$ (b) $\frac{mg}{IL}\cos\theta$ (c) $\frac{mg}{IL}\tan\theta$ (d) $\frac{mg}{IL}\sec\theta$

30. A rod of mass $M$ which is non-uniformly distributed over its length $L$ has been considered. Its linear mass density variation will distance $x$ from left end is shown in the figure.




From this information, we can conclude that the centre of mass of rod

(a) is on left of its mid-point
(b) is on right of its mid-point
(c) is coinciding with its mid-point
(d) Information insufficient

# Chemistry

31. The total number of stereoisomers shown by the compound $C_2FClBrI$ is
(a) 2 (b) 4 (c) 6 (d) 8

32. 20 mL of $H_2O_2$ solution liberates 560 mL of oxygen (measured at NTP). Normality of the given solution is
(a) 0.66 N (b) 1.25 N (c) 2.5 N (d) 5 N

33. In two hydrogen atoms $A$ and $B$, the electron move around the nucleus in circular orbits of radius $r$ and $4r$ respectively. The ratio of the time taken by them to complete one revolution is
(a) 1 : 4 (b) 1 : 2 (c) 1 : 8 (d) 2 : 1

34. Enthalpy of a reaction at 27°C is 15 kJ $mol^{-1}$. The reaction will be feasible if entropy is
(a) 15 J $mol^{-1}$ $K^{-1}$ (b) $-50$ J $mol^{-1}$ $K^{-1}$
(c) greater than 50 J $mol^{-1}$ $K^{-1}$ (d) less than 50 J $mol^{-1}$ $K^{-1}$

35. The metal which displaces the hydrogen from boiling caustic soda solution is
(a) Mg (b) Fe (c) As (d) Zn

36. Lead sulphate is soluble in
(a) conc. $HNO_3$ (b) conc. HCl
(c) solution of $CH_3COONH_4$ (d) water

37. Which of the following does not contain any peroxide linkage?
(a) $H_2S_2O_8$ (b) $H_2S_2O_7$ (c) $H_2SO_5$ (d) $CrO_5$

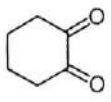
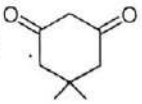
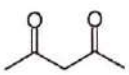
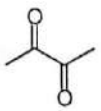
38. pH of $10^{-6}$ M $NH_4OH$ ($aq$) solution will be ($K_b = 2\times10^{-5}$)
(a) 6.02 (b) 7.0 (c) 7.98 (d) 8.65

39. 4.5 g of $PCl_5$ was completely vapourised at 523 K and the vapour occupied 1.7 L at 1 atm pressure. The $K_p$ for the reaction

$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g) \text{ is}$$

(a) 0.88 (b) 0.43 (c) 0.02 (d) 2.38

40. Which of the following compounds will have least percentage of enal content?

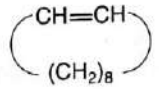
(a) 
(b) 
(c) 
(d) 

41. Which will show geometrical isomerism?


(I)

$CH_3-CH=NOH$

(II)

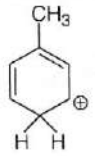
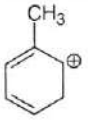
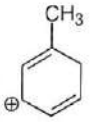

(III)

(a) I and II (b) II and III (c) I and III (d) I, II and III

42. An electron moving with velocity 'v' is found to have a certain value of de-Broglie wavelength. The velocity to be possessed by the neutron to have the same de-Broglie wavelength is

(a) $\frac{1840}{v}$ (b) $1840\,v$ (c) $\frac{v}{1840}$ (d) $v$

43. Which of the following carbocations is expected to be least stable?

(a) 
(b) 
(c) 
(d) 

44. ZnO is white when cold and yellow when heated. It is due to the development of

(a) Frenkel defect (b) Schottky defect
(c) Metal excess defect (d) Metal deficiency defect

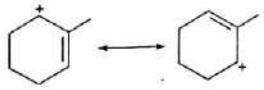
45. Fructose gives the silver mirror test because it

(a) contains an aldehyde group
(b) contains a keto group
(c) undergoes rearrangement under the alkaline conditions of the reagent to form a mixture of glucose and mannose
(d) None of the above

46. The correct order for the wavelength of absorption inthe visible region is

(a) $[Ni(NO_2)_6]^{4-} < [Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+}$ (b) $[Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+} < [Ni(NO_2)_6]^{4-}$
(c) $[Ni(H_2O)_6]^{2+} < [Ni(NH_3)_6]^{2+} > [Ni(NO_2)_6]^{4-}$ (d) $[Ni(NO_2)_6]^{4-} < [Ni(H_2O)_2]^{2+} < [Ni(NH_3)_6]^{2+}$

47. Which of the following is a pair of resonance structure?

(a) 
(b) 
(c) 
(d) 

48. One molal solution of a carboxylic acid in benzene shows the elevation of boiling point of 1.518 K. The degree of association for dimerisation of the acid in benzene is ($K_b$ for benzene = 2.53 K kg $mol^{-1}$)

(a) 60% (b) 70% (c) 75% (d) 80%

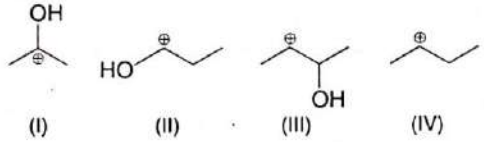
49. In fcc lattice, *A,B,C,D* atoms are arranged at corners, face centres, octahedral voids and tetrahedral voids respectively, then the body diagonal contains

(a) *2A,. C, 2D* (b) *2A, 2B, 2C* (c) *2A, 2B, D* (d) *2A, 2D*

50. Out of the following the one containing only nucleophiles is

(a) $AlCl_3, BF_3, NH_3$ (b) $NH_3, CN^{\ominus}, CH_3OH$ (c) $AlCl_3, NH_2^{\ominus}, H_2O$ (d) $RNH_2, :CX_2, H^{\oplus}$

51. The correct stability order for the following species is



(I) (II) (III) (IV)

(a) I > II > III > IV (b) I > II > IV > III (c) I > III > II > IV (d) IV > III > I > II

52. The standard electrode potential for the following reaction is + 1.33 V. What is the potential at pH = 2?

$$Cr_2O_7^{2-}(aq1M) + 14H^{\oplus}(aq) + 6e^- \longrightarrow 2Cr^{3+}(aq1M) + 7H_2O(l)$$

(a) +1.82 V (b) +1.99 V (c) +1.608 V (d) +1.054 V

**Directions** (Q. Nos. 53-55) *A white crystalline solid 'A' on boiling with caustic soda solution gives a gas 'B' which on passing through an alkaline solution of potassium tetraiodomercurate (II) solution gives a brown ppt. The substance 'A' on heating evolves a neutral gas 'C' which is inert at room temperature and reactive in the presence of catalyst and does not give brown fumes with nitric acid oxide.*

53. The gas '*B*' is

(a) $H_2S$ (b) $NH_3$ (c) HCl (d) $CO_2$

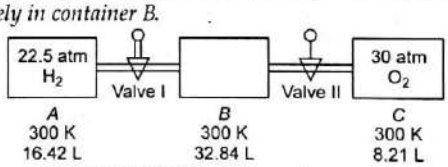
54. The gas '*C*' is

(a) $N_2O$ (b) $O_2$ (c) NO (d) $N_2$

55. The substance '*A*' is

(a) $NH_4Cl$ (b) $NH_4NO_3$ (c) $NH_4NO_2$ (d) $NaNO_3$

**Directions** (Q. Nos. 56-58) *First valve I is opened for a long period and then closed and after it valve II is opened for a long period and then closed. After closing the valve II,* $H_2$ *and* $O_2$ *are made to react completely in container B.*




***Given*** *Vapour pressure of* $H_2O$ *at* 300 K *is* 19 torr. *Assume temperature remains constant.*

56. Amount of $H_2O$ produced in the container *B* is

(a) 60 g (b) 72 g (c) 144 g (d) 212 g

57. Remaining moles of $H_2$ in container *A* is

(a) 5 (b) 10 (c) 75 (d) 15

58. Total pressure in container *B*, considering volume of $H_2O(l)$ formed is negligible is

(a) 2.75 atm (b) 3.0 atm (c) 2.057 atm (d) 3.025 atm

**Directions** (Q. Nos. 59 and 60) *For the following questions, choose the correct answers from the codes* (a), (b), (c) *and* (d) *defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

59. **Statement I** $PbI_4$ doesn't exist and converts into $PbI_2$ and $I_2$ spontaneously at room temperature but $PbCl_4$ needs heating to convert into $PbCl_2$ and $Cl_2$.
**Statement II** $Pb^{2+}$ is more stable than $Pb^{4+}$ due to inert pair effect.

60. **Statement I** For adsorption $\Delta G, \Delta H, \Delta S$ all have negative values.
**Statement II** Adsorption is an exothermic process in which randomness decreases due to force of attraction between adsorbent and adsorbate.

# Mathematics

61. If $L_r = \int \sin x\, d(i^r x)$, where $i = \sqrt{-1}$, then $\sum_{r=1}^{4n-1} L_r$ is $(n \in N)$

(a) $-\cos x + C$ (b) $\cos x + C$ (c) 0 (d) Not defined

62. The number of ordered 4-tuple $(x, y, z, w), \{x, y, z, w \in [0,10]\}$ which satisfies the inequality $2^{\sin^2 x}\, 3^{\cos^2 y}\, 4^{\sin^2 z}\, 5^{\cos^2 w} \geq 120$ is

(a) 0 (b) 144 (c) 81 (d) infinite

63. Let, $a_1, a_2, a_3, \ldots$ be in HP with $a_1 = 5$ and $a_{20} = 25$. The least positive integer '$n$' for which $a_n < 0$ is

(a) 22 (b) 23 (c) 24 (d) 25

64. Which statements represents the inverse of the statements. "If it is snowing, then Pinki wear a sweater"?

(a) If Pinki wears a sweater, then it is snowing.
(b) If Pinki does not wear a sweater, then it is not snowing.
(c) If it is snowing, then Pinki does not wear a sweater.
(d) If it is not snowing, then Pinki wears a sweater.

65. If $(1+x)^n = \sum_{r=0}^{n} a_r x^r$ and $b_r = 1 + \frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n} b_r = \frac{(101)^{100}}{(100)!}$, then $n$ is

(a) 99 (b) 100 (c) 101 (d) 102

66. If $f(x) = \begin{vmatrix} 1 & 2x & 3x^2 \\ 3 & a & 27 \\ 1 & 3 & 9 \end{vmatrix}$ and $\int_0^3 f(x)\,dx = 0$, then $a$ is equal to

(a) 3 (b) 6 (c) 9 (d) any real number

67. If the adjoint of a $3 \times 3$ matrix $P$ is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of $P$ are

(a) $\pm 2$ (b) $\pm 1$ (c) $\pm 3$ (d) $\pm 4$

68. The value of $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}}}} \ldots \infty\right)$ is

(a) 5 (b) 4 (c) 3 (d) 2

69. A boy is throwing stones at a target. The probability of hitting the target at any trial is $\frac{1}{2}$. The probability of hitting the target 5th time at the 10th throw is

(a) $\frac{5}{2^{10}}$ (b) $\frac{63}{2^9}$ (c) $\frac{{}^{10}C_5}{2^{10}}$ (d) $\frac{{}^{10}C_4}{2^{10}}$

70. The function $f:[0,3]\to[1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$ is

(a) one-one and onto (b) onto but not one-one
(c) one-one but not onto (d) neither one-one nor onto

71. If $K=\lim\limits_{x\to\infty}\left(\frac{\sum\limits_{k=1}^{1000}(x+K)^m}{x^m+10^{1000}}\right)$, then $K$ is $(m>101)$

(a) 10 (b) $10^2$ (c) $10^3$ (d) $10^4$

72. If $f(x)=\begin{cases} a+\frac{\sin[x]}{x}, & x>0 \\ 2, & x=0 \\ b+\left[\frac{\sin x-x}{x^3}\right], & x<0 \end{cases}$, (where [.] denotes the greatest integer function).

If $f(x)$ is continuous at $x=0$, then $b$ is equal to

(a) $a-2$ (b) $a-1$ (c) $a+1$ (d) $a+2$

73. If the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^3=16x$ intersect at right angles, then $[a^2]$ is equal to (where [.] denotes the greatest integer function)

(a) 1 (b) 2 (c) 3 (d) 0

74. If the greatest and least value of the function $f(x)=\begin{cases} 2x^2+\frac{2}{x^2}, & x\in[-2,2]-\{0\} \\ 1, & x=0 \end{cases}$ are $G$ and $L$ respectively, then

(a) $[G+L]=9$ (b) $[G+L]=8$ (c) $[G-L]=9$ (d) $[G-L]=8$

75. The integral $\int\frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals to (for some arbitrary constant $L$)

(a) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+L$

(b) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+L$

(c) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+L$

(d) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+L$

76. If $\frac{2x}{\pi}<\sin x<x$ for $0<x<\frac{\pi}{2}$, then the value of the integral $\int_0^{\pi/2}\frac{\sin x}{x}dx$ is

(a) $>1$ (b) $<1$ (c) $>\frac{\pi}{2}$ (d) $<\frac{\pi}{2}$

77. The area between the curve $y = 2x^4 - x^2$ the $x$-axis and the ordinates of two minima of the curve is

(a) $\frac{7}{120}$ sq unit (b) $\frac{9}{120}$ sq unit (c) $\frac{11}{120}$ sq unit (d) $\frac{13}{120}$ sq unit

78. The form of the differential equation of all central conics is

(a) $x = y\frac{dy}{dx}$ (b) $x + y\frac{dy}{dx} = 0$

(c) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$ (d) None of these

79. If $(\alpha, \beta)$ be the coordinates of the incentre of the triangle whose vertices are $(4, -2), (-2, 4)$ and $(5, 5)$. Then, the value of $(2\alpha + 10\beta)$ is

(a) 30 (b) 35 (c) 40 (d) 25

80. The set of values of $C$, so that the equations $y = |x| + C$ and $x^2 + y^2 - 8|x| - 9 = 0$ have no solution is

(a) $(-\infty, -3) \cup (3, \infty)$ (b) $(-3, 3)$

(c) $(-\infty, -5\sqrt{2}) \cup (5\sqrt{2}, \infty)$ (d) $(5\sqrt{2} - 4, \infty)$

81. Two perpendicular tangents $PA$ and $PB$ are drawn to $y^2 = 4ax$, minimum length of $AB$ is equal to

(a) $a$ (b) $4a$ (c) $8a$ (d) $2a$

82. If the normal at any point $P(\phi)$ to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ intersects it again at the point $Q(2\phi)$, then $\cos\phi$ is equal to

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

83. Pair of asymptotes of the hyperbola $xy - 3y - 2x = 0$ is

(a) $xy - 3y - 2x + 2 = 0$ (b) $xy - 3y - 2x + 4 = 0$

(c) $xy - 3y - 2x + 6 = 0$ (d) $xy - 3y - 2x + 12 = 0$

84. The point $P$ is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If $S$ is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to $QR$, then the length of the line segment $PS$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

**Directions** (Q. Nos. 85 and 86) *A system of vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ is said to be linearly dependent, if there exist a system of scalars $c_1, c_2, \ldots, c_n$ (not all zero) such that*

$$c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + \ldots + c_n\mathbf{a}_n = \mathbf{0}$$

*i.e., $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots, \mathbf{a}_n$ are linearly dependent iff one can be expressed as the linear combination of the others.*

*Again, $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n$ are linearly independent, if there exists scalars $c_1, c_2, \ldots, c_n$ such that*

$$c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + \ldots + c_n\mathbf{a}_n = \mathbf{0}$$

$$\Rightarrow \quad c_1 = c_2 = \ldots = c_n = \mathbf{0}$$

85. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are non-zero, non-coplanar vectors, then $\mathbf{r}_1 = 2\mathbf{a} - 3\mathbf{b} + \mathbf{c}$, $\mathbf{r}_2 = 3\mathbf{a} - 5\mathbf{b} + 2\mathbf{c}$ and $\mathbf{r}_3 = 4\mathbf{a} - 5\mathbf{b} + \mathbf{c}$ are

(a) linearly independent (b) linearly dependent

(c) $\mathbf{r}_3 = \alpha^2\mathbf{r}_1 + \beta^2\mathbf{r}_2, \alpha, \beta \in R$ (d) $\mathbf{r}_3^2 = \alpha^2\mathbf{r}_1 + \beta^2\mathbf{r}_2$

86. If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k}$ are linearly dependent vectors and $|\mathbf{c}|=\sqrt{3}$, then

(a) $\alpha=1,\beta=-1$ (b) $\alpha=1,\beta=\pm1$ (c) $\alpha=-1,\beta=\pm1$ (d) $\alpha=\pm1,\beta=1$

**Directions** (Q. Nos. 87 and 88) *For a given square matrix A, if there exists a matrix B such that $AB=BA=I$, then B is called inverse of A. Every non-singular square matrix possesses inverse and it exists, if $|A|\neq 0$.*

*If* $$A^{-1}=\frac{adj\,(A)}{det\,(A)}\Rightarrow adj\,(A)=|A|A^{-1}.$$

87. Let a matrix $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$, then it will satisfy the equation

(a) $A^2-4A+I=0$ (b) $A^2+4A+I=0$ (c) $A^2-4A+5I=0$ (d) $A^2-4A-5I=0$

88. Let matrix $A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$ satisfies the equation $A^2+aA+bI=0$, then the value of $\int_a^{4b} x^3\cos x\,dx$ is equal to

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-2b}{a-b}$ (c) $\frac{a+4b}{4a-b}$ (d) 0

**Directions** (Q. Nos. 89 and 90) *For the following questions. Choose the correct answers from the codes (a), (b), (c) and (d) defined as follows*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

89. Consider the two lines $L_1\equiv a_1x+b_1y+c_1=0$ and $L_2\equiv a_2x+b_2y+c_2=0$.

**Statement I** Equation of two bisectors of the lines $x=0$ and $y=0$ are given by $y=\pm x$.

**Statement II** The two bisectors of the line $L_1=0$ and $L_2=0$ is given by $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$.

90. Consider the circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$.

**Statement I** No transverse common tangent can be drawn to the two circles.

**Statement II** Two circles are not concentric circles.

# Answer *with* Explanations

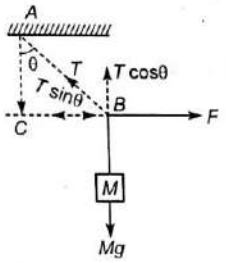
## Physics

**1.** (b) In the adjoining figure, the point $B$ is in equilibrium under the action of $T, F$ and $Mg$

Here, $T\cos\theta = Mg$ ...(i)

Also, $T\sin\theta = F$ ...(ii)

or $T = \dfrac{F}{\sin\theta}$




**2.** (d) KE and momentum are related as

$$K = \frac{p^2}{2m}$$

Suppose $p$ changes its value from $p_1$ and $p_2$

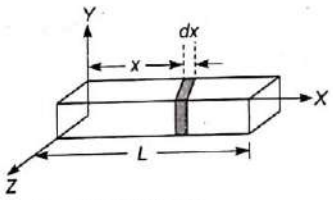
As $p_2 = 2p_1$

$\therefore$ $K_2 = 4K_1$

$\therefore$ Percentage change in KE

$$= \frac{K_2 - K_1}{K_1} \times 100\% = 300\%$$

**3.** (b) As the rod is along $X$-axis, for all points on it $Y$ and $Z$ will be zero, so, $Y_{CM} = 0$ and $Z_{CM} = 0$, *i.e.*, centre of mass will lie on the rod. Now consider an element of rod of length $dx$ at a distance $x$ from the origin, then




Mass of small part of rod

$$dm = \lambda dx = (A + Bx)\,dx$$

$$\text{So, } X_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A + Bx)\,dx}{\int_0^L (A + Bx)\,dx}$$

$$= \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}} = \frac{L(3A + 2BL)}{3(2A + BL)}$$

**4.** (d) By definition, angular momentum

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix}$$

Now for motion in $x$-$y$ plane $z = 0$ and $v_z = 0$

$$\text{So, } \mathbf{L} = m\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ v_x & v_y & 0 \end{vmatrix} = m\mathbf{k}(xv_y - yv_x)$$

Hence, the resultant angular momentum has only $Z$-components.

**5.** (a) (i) $g'$ = acceleration due to gravtiy on the moon

$$= G \cdot \frac{4}{3}\pi R' \rho'$$

$$= G \cdot \frac{4}{3}\pi \times \frac{R}{4} \times \frac{2}{3}\rho = \frac{g}{6}$$

Now, $h_{max} = \dfrac{u^2}{2g}$ or $h_m \propto \dfrac{1}{g}$

Hence, $h_m' g' = h_m g$

(mass of body is same everywhere)

or $h_m' = 6h_m = 6 \times 0.5 = 3\text{ m}$

(ii) Now, $t = \dfrac{2u}{g}$ or $t \propto \dfrac{1}{g}$

$t'g' = tg$

or $\dfrac{t'}{t} = \dfrac{g}{g'} = \dfrac{6}{1}$ $(\because g' = g/6)$

**6.** (b) Root mean square speed, $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

(According to kinetic theory of gases)

Since, $M$ remains the same

$\therefore$ $v_{rms} \propto \sqrt{T}$

$\therefore$ $\dfrac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\dfrac{T_1}{T_2}}$

$\therefore$ $\dfrac{v}{(v_{rms})_2} = \sqrt{\dfrac{120\text{ K}}{480\text{ K}}} = \dfrac{1}{2}$

$[\because (v_{rms})_1 = v$ (Given)]

or $(v_{rms})_2 = 2v$

**7.** (c) The velocity of transverse wave is given by $v = \sqrt{T/m}$, where $T$ = tension and $m$ = mass unit length of the wire. If $r$ is the radius of the wire and $\rho$ its density, then $m = \pi r^2\rho$. Therefore,

$$v = \frac{\sqrt{T}}{r\sqrt{\pi\rho}}$$

Thus, $v_A = \dfrac{\sqrt{T_A}}{r_A\sqrt{\pi\rho}}$

and $v_B = \dfrac{\sqrt{T_B}}{r_B\sqrt{\pi\rho}}$

Now, $\frac{v_A}{v_B} = \sqrt{\frac{T_A}{T_B}} \cdot \frac{r_B}{r_A}$

It is given that $r_A = 2r_B$ and $T_A = \frac{1}{2}T_B$.

Hence $\frac{v_A}{v_B} = \frac{1}{2\sqrt{2}}$

**8.** (c) The four plates are alternately connected and form three capacitors in parallel. The capacity of each capacitor is $(\varepsilon_0 A/d)$. Hence, the net capacitance between $A$ and $B$ is given by

$$C_{AB} = C + C + C = \frac{3\varepsilon_0 A}{d}$$

**9.** (b) As wave travelling along +ve $x$-direction is given by

$$E_y = E_0 \cos(\omega t - kx)$$
$$\omega = 2\pi f = 2\pi \times 10^6$$
$$\therefore \quad f = 10^6 \text{ Hz}$$
$$k = \frac{2\pi}{\lambda} = \pi \times 10^{-2} \text{ m}^{-1}$$
$$\lambda = 200 \text{ m}$$

**10.** (a) $^{236}_{88}\text{Ra} \xrightarrow{-3\alpha} {}^{22}_{82}S \xrightarrow{-\beta} {}^{224}_{83}X$

**11.** (d) Area under the $f$-$t$ curve = change in momentum

or $\frac{1}{2} \times 2 \times (10) + 2 \times 10 + \frac{1}{2}(10 + 20) \times 2 + \frac{1}{2} \times 4 \times 20 = m(v - 0)$

or $10 + 20 + 30 + 40 = 2(v - 0)$

or $100 = 2v$

or $v = 50 \text{ ms}^{-1}$

**12.** (a) Applying the law of conservation of momentum, we get;

$$mv_0 + 0 = 2m \times v$$

or $$v = \frac{v_0}{2}$$

$$\text{KE} = \frac{1}{2}(2m)v^2 = \frac{1}{2} \times 2m \times \left(\frac{v_0}{2}\right)^2 = \frac{mv_0^2}{4}$$

Let the system reach a height $h$.

Potential energy of the system = $2mgh$

$\because$ Loss in PE = Gain in KE change or magnitude of in KE = Magnitude of change in PE

Hence, $\frac{mv_0^2}{4} = 2mgh$

or $$h = \frac{v_0^2}{8g}$$

**13.** (d) According to Poiseull's formula

$$\eta = \frac{\pi pr^4}{8Vl}$$

Now, $V = \frac{\text{Volume}}{\text{Time}} = \frac{\pi r^2 l}{t} = \pi r^2 v$

$$\therefore \quad \eta = \frac{\pi pr^4}{8\pi r^2 vl} = \frac{pr^2}{8vl}$$

$$\therefore \quad p = \frac{\eta \times 8vl}{r^2} = \frac{0.0015 \times 8 \times 10^{-3} \times 5 \times 10^{-6}}{4 \times 10^{-12}} = 15 \text{ N/m}^2$$

**14.** (a) In the steady state,

$$\frac{Q_A}{t} = \frac{Q_B}{t} = \frac{Q_C}{t}$$
$$\frac{2KA(100 - T_1)}{d} = \frac{KA(T_1 - T_2)}{d} = \frac{KA(T_2 - 0)}{2d}$$
$$\Rightarrow \quad 2(100 - T_1) = T_1 - T_2 = \frac{T_2}{2}$$

From last two relation

$$T_1 = \frac{3}{2}T_2 \quad \text{or} \quad T_2 = \frac{2}{3}T_1$$

and from first-two relation

$$2(100 - T_1) = \frac{T_1}{3}$$

$\therefore$ $T_1 = 85.7°\text{C}$, $T_2 = 57.1°\text{C}$

**15.** (b) The listener moves away from $A$ and approaches $B$. Hence, the apparent frequencies are

$$\nu_1 = \nu\left(1 - \frac{u}{v}\right) \text{ and } \nu_2 = \nu\left(1 + \frac{u}{v}\right)$$

$$\therefore \quad \nu_2 - \nu_1 = \frac{2u}{v}$$

It is given that $\nu_2 - \nu_1 = 10$, $v = 340 \text{ ms}^{-1}$ and $\nu = 680$ Hz

Substituting these values, we get

$$10 = \frac{2 \times 680 \times u}{340}$$

or $$u = 2.5 \text{ ms}^{-1}$$

**16.** (c) From $\frac{C}{5} = -\frac{K - 273.15}{5} = \frac{F - 32}{9}$

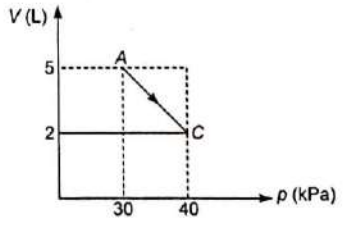
$C = K - 273.15$, for any value of $K$, $C \neq K$ and $K$ is always greater than $C$. Lower than zero reading on kelvin scale is not possible as 0 K itself cannot be realized.

From $\frac{K - 273.15}{5} = \frac{F - 32}{9}$,

for $$K \approx 319$$

Both values come out to be same.

**17.** (a)



From curve

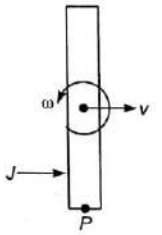
$$W = -\frac{1}{2}(5+2)\times10^{-3}\times10\times10^{3}$$

$$= -\frac{1}{2}\times7\times10 = -35\text{ J}$$

From $dU = dQ - W$

$$= 20 - (-35) = 55\text{ J}$$

**18.** (a) Let $v$ be the velocity of centre of mass of rod and $\omega$ be the angular velocity of rod about an axis passing through its centre of mass.




From linear impulse definition, $J = Mv$

(change in momentum of the rod)

From angular impulse definition

$$J\times\frac{L}{4} = \frac{ML^2}{12}\omega \Rightarrow \omega = \frac{3J}{ML}$$

Velocity of point $P$ is

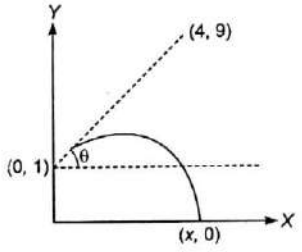
$$v + \frac{\omega L}{2} = \frac{J}{M} + \frac{3}{2}\frac{J}{M} = \frac{5J}{2M}$$

**19.** (a) To identify the base of transistor, the multimeter has to show conduction between emitter and base as well as between collector and base keeping one load of the multimeter common in both cases, then the terminal of the transistor to which the lead of multimeter is common of the base of transistor.

**20.** (c) $\tan\theta = \frac{8}{4} = 2$,

For vertical motion

$$-1 = u\times\sin\theta\times t - \frac{1}{2}\times gt^2$$




$$\Rightarrow \quad -1 = u\times\frac{2}{\sqrt{5}}\times1 - \frac{1}{2}\times10\times1^2$$

or $u = 2\sqrt{5}$ m/s

∴ Horizontal distance,

$$x = u\cos\theta\times t = 2\sqrt{5}\times\frac{1}{\sqrt{5}}\times1 = 2\text{ m}$$

So, coordinate will be (2, 0)

**21.** (c) From definition average speed may not be necessarily equal to magnitude of average velocity. If direction of motion is changing, then these two would have different values.

**22.** (a) Potential difference between two concentric conducting spheres is independent of the charge on outer sphere.

**23.** (c) Initially in equilibrium,

$$T_1 + T_2 = mg$$

and $T_1 = T_2 = T_0$

So, $T_0 = \frac{mg}{2}$

When magnetic field is switched on, let tensions in left and right strings be $T_1'$ and $T_2'$ respectively.

$$T_1' + T_2' = mg \quad \ldots(i)$$

and $(T_1' - T_2')R = \frac{Q}{2\pi/\omega}\times\pi R^2B$

$$\Rightarrow \quad (T_1' - T_2') = \frac{QB\omega R}{2} \quad \ldots(ii)$$

$$\Rightarrow \quad T_1' = \frac{mg}{2} + \frac{QB\omega R}{4} \quad \ldots(iii)$$

From Eqs. (i) and (ii)

So, $T_2' < T_1'$

It is clear by analyzing

Put $T_1' = \frac{3T_0}{2}$ in Eq. (iii)

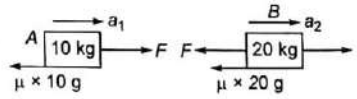
$$\frac{3R_0}{2} = T_0 + \frac{Q\omega RB}{4}$$

$$\Rightarrow \quad T_0 = \frac{Q\omega RB}{2}$$

$$\therefore \quad T_1' = \frac{3T_0}{2}$$

$$= \frac{3Q\omega RB}{4}$$

**24.** (d) Let at the instant considered in figure, the spring force in $F$. The FBD of both the blocks are as shown in the figure.




Here, $a_1 = 12\text{ m/s}^2$ and $\mu = 0.1$

Equation of motion of 10 kg block

$$F - 0.1\times10\text{ g} = 10\times12$$

$$\Rightarrow \quad F = 130$$

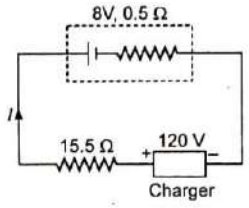
and for 20 kg block,

$$200 - 130 - 0.1\times20\times10 = 20a_2$$

$$\Rightarrow \quad a_2 = 2.5\text{ m/s}^2$$

**25.** (c) For charging process

$$V = E + Ir$$




$$I = \frac{112}{16} = 7 \text{ A}$$

So, $V = 8 + 7 \times \frac{1}{2} = 11.5$ volt

**26.** (b) From circular motion representation, $\theta_1 = \omega t_1$ where $t_1$ is time taken by particle to go from $x = 0$ to $x = \frac{A}{2}$.

$$\sin\theta_1 = \frac{\frac{A}{2}}{A} = \frac{1}{2}$$

$$\Rightarrow \quad \theta_1 = \frac{\pi}{6}$$

$$\Rightarrow \quad \omega t_1 = \frac{\pi}{6}$$

$$\Rightarrow \quad t_1 = \frac{\pi \times T}{6 \times 2\pi} = \frac{T}{12}$$

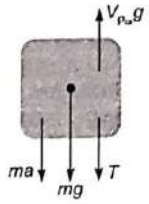
So, time taken to go from $x = \frac{A}{2}$ to $A$ is

$$t_2 = \frac{T}{4} - t_1 = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$$

Hence, $\frac{t_1}{t_2} = \frac{T/12}{T/6} = \frac{1}{2}$

**27.** (a) Here, if we see above computed value of $R$, it has 6 significant digits while measured values of $V$ and $I$ are reliable upto 3 digits, *i.e.*, in measured values, 3 significant digits are there. So, it is not possible that we have 6 significant digits in $\frac{V}{I}$, hence, correct way to compute $R$ from the measured values of $V$ and $I$ is to round off the computed answer upto 3 significant digits, *i.e.*, $R = 1.34\ \Omega$ (Rounded off 3 significant digits).

**28.** (b) Free body diagram of block is as shown in figure.




From equilibrium of block.

As net force on the block

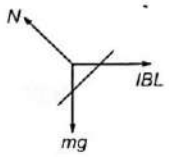
= mass × net acceleration of the block

$$V\rho_w g = T + mg + ma$$

$$\Rightarrow \frac{m}{800} \times 1000 \times 10 = T + m(10 + 1)$$

$$\Rightarrow \quad T = 1.5 \text{ N}$$

**29.** (c) As the conductor is moving with constant velocity, not force acting on it, must be zero. Forces acting on conductor are magnetic force, gravity force and contact force as shown.

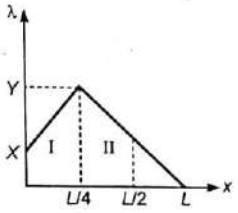



For dynamic equilibrium net force on the mass is equal to zero

$$\Rightarrow \quad mg \sin\theta = IBL \cos\theta$$

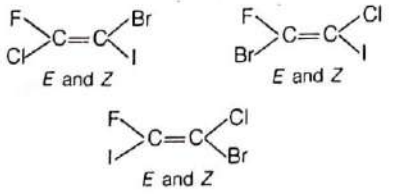
$$\therefore \quad B = mg \tan\theta / IL$$

**30.** (a) Area under $\lambda$ *versus* $x$ curve gives mass. For any $x$ and $y$ area of I > Area of II, it means more mass as on the left side, so centre of mass would be on left half.




# Chemistry

**31.** (c)




**32.** (d) 560 mL oxygen is liberated by 20 mL $H_2O_2$.

$\therefore$ 1 L of $H_2O_2$ will liberate $O_2$

$$= \frac{560 \times 1000}{20} = 28000 \text{ mL} = 28 \text{ L}$$

$$\therefore \quad N = \frac{V}{5.6} = \frac{28}{5.6} = 5 \text{ N}$$

**33.** (c) $v \propto \frac{1}{\sqrt{r}}$, $v_A \propto \frac{1}{\sqrt{r}}$, $v_B \propto \frac{1}{\sqrt{4r}}$

$$\therefore \quad \frac{v_A}{v_B} = 2$$

$$\frac{t_A}{t_B} = \frac{2\pi r/v_A}{2\pi(4r)/v_B} = \frac{1}{4} \times \frac{v_B}{v_A}$$

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = 1:8$$

**34.** (c) $\Delta G = \Delta H - T\Delta S$ for spontaneous reaction

$\Delta G$ = – ve

$T\Delta S > \Delta H$

*i.e.*, $300\Delta S > 15000$ J

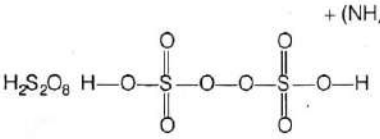
$\Delta S > 50$ J

**35.** (d) Zn metal gives reaction with boiling NaOH forming complex compound.

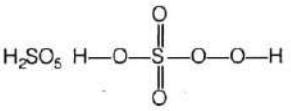
$Zn + 2NaOH + 2H_2O \longrightarrow Na_2[Zn(OH)_4] + H_2\uparrow$

**36.** (c) $PbSO_4 + 2CH_3COONH_4 \longrightarrow Pb(CH_3COO)_2 + (NH_4)_2SO_4$

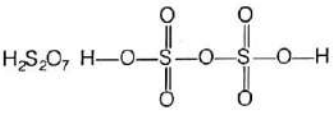
**37.** (b) $H_2S_2O_8$



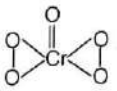
$H_2SO_5$



$H_2S_2O_7$



$CrO_5$



**38.** (c) $\underset{C(1-\alpha)}{NH_4OH} \rightleftharpoons \underset{C\alpha}{NH_4^+} + \underset{C\alpha}{OH^-}$

$$K_b = \frac{C\alpha \times C\alpha}{C(1-\alpha)}$$

$$K_b = \frac{C\alpha^2}{1-\alpha}$$

Here, $1 - \alpha \neq 1$ as concentration is very small.

$\therefore \quad \alpha = 0.95$

$$2 \times 10^{-5} = \frac{10^{-6}\,\alpha^2}{1-\alpha}$$

or $\quad a^2 + 20\alpha - 20 = 0$

$\therefore \quad [OH^-] = C\alpha$

$= 10^{-6} \times 0.95$

$pOH = -\log(OH^-)\,6.02$

$\therefore \quad pH = 14 - 6.02 = 7.977$

**39.** (b)

$$\underset{\text{At equi}}{PCl_5} \rightleftharpoons PCl_3 + Cl_2$$

$$\frac{4.5}{208.5} - x \qquad x \qquad x$$

$pV = nRT$

$$1 \times 1.7 = \left(\frac{4.5}{208.5} + x\right) 0.0821 \times 523$$

On solving, $x = 0.018$

$$\therefore \quad K_c = \frac{x \cdot x}{\left(\frac{4.5}{208.5} - x\right) \times V}$$

$$= \frac{0.018 \times 0.018}{\left(\frac{4.5}{208.5} - 0.018\right) \times 1.7} = 0.0533$$

$\therefore \quad K_p = 0.0533\,RT = 0.0533 \times 0.0821 \times 523$

$= 2.2886$

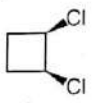
$\therefore K_p$ for the reaction

$PCl_3 + Cl_2 \rightleftharpoons PCl_5$

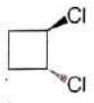
is $1/K_p = 1/2.2886 = 0.43$

**40.** (d) Enols of (a), (b) and (c) are stabilised by H-bonding whereas enol of (d) is not stabilised by H-bonding.

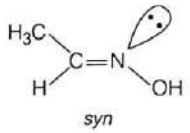
**41.** (d) I, II and III



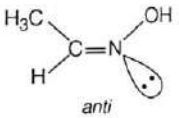
*cis*



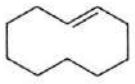
*trans*



*syn*



*anti*



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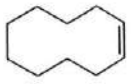


*cis*

**42.** (c) $\lambda_e = \frac{h}{m_e v_e}$ and $\lambda_n = \frac{h}{m_n v_n}$

$\therefore \quad \lambda_e = \lambda_n$

$$\frac{1}{m_e v_e} = \frac{1}{m_n v_n}$$

or $\quad \frac{u_n}{v_e} = \frac{m_e}{m_n} = \frac{1}{1840}$

or $\quad v_n = \frac{1}{1840} \times v_e = \frac{v}{1840}$

**43.** (c) (c) has no resonance structure in which positive charge moves to a carbon to which $CH_3$ (electron donating in nature) is attached.

**44.** (c) $ZnO \xrightarrow{\Delta} Zn^{2+} + \frac{1}{2}O_2 + 2e^-$

The remaining $Zn^{2+}$ ions occupy the interstitial voids of remaining crystal, thus causing metal excess defect.

**45.** (c) In alkaline medium, its some amount is converted into glucose due to this it gives silver mirror test.

**46.** (a) The absorption of energy by a complex compound depends on the charge of the metal ion and the nature of the ligands attached. The same metal ion with different ligands shows different absorption depending upon the type of ligand. The presence of weak field ligands make the central metal ion to absorb low energies, *e.g.*, of higher wavelength. The order of field strength of different ligands is

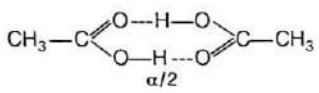
$$H_2O < NH_3 < NO_2$$

$\therefore$ the order of wavelength is

$$[Ni(NO_2)_6]^{4-} < [Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6)]^{2+}$$

**47.** (c) All other either involve movement of atom or are identical, so not a pair of resonance structure.

**48.** (d) $\underset{1-\alpha}{2CH_3COOH} \rightleftharpoons$



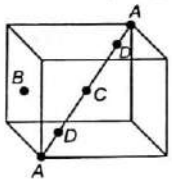

$\therefore$ van't Hoff factor

$$i = 1 - \alpha + \alpha/2$$
$$= 1 - \alpha/2$$

Boiling point elevation

$$\therefore \quad \Delta T_b = iK_b m$$
$$1.518 = (1 - \alpha/2) \times 2.53 \times 1$$
$$0.6 = 1 - \alpha/2$$
$$\alpha/2 = 0.4$$
$$\alpha = 0.8 \text{ or } 80\%$$

**49.** (a)




**50.** (b) $NH_3$, $CN^-$ and $CH_3OH$ all the three have at least one lone pair of electrons and hence, act as nucleophiles.

**51.** (b) In (I) and (II) the lone pair of oxygen atom are in resonance with +ve charge but (I) contains more number of α-H atoms. Hence, I is more stable than II. In (III), the —OH group withdraws electron by $-I$ effect hence decreases the stability of carbocation and in (IV) only hyperconjugation is operating.

$\therefore$ The order of stability is I > II > IV > III.

**52.** (d) $E_{Cr_2O_7^{2-}/Cr^{3+}}$

$$= E^\circ_{Cr_2O_7^{2-}/Cr^{3+}} - \frac{0.0591}{6} \log \frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}][H^+]^{14}}$$
$$= 1.33 - \frac{0.0591}{6} \log \frac{1}{(0.01)^{14}}$$
$$= 1.0542 \text{ V}$$

**Solutions** (Q. Nos. 53-55)

Since the gas B gives brown ppt with $K_2HgI_4$, it must be ammonia and hence *A* must be an ammonium salt Moreover, it gives a neutral, unreactive gas which is $N_2$. So *A* is ammonium nitrite and the reactions are as follows :

$$\underset{(A)}{NH_4NO_2} + NaOH \longrightarrow \underset{(B)}{NH_3}\uparrow + NaNO_2 + H_2O$$

Ammonium nitrite

$$NH_3 + 2K_2[HgI_4] + 3KOH \longrightarrow \underset{\text{Brown ppt}}{[HgO \cdot Hg(NH_2)_2]I}\downarrow + 3H_2O + 7KI$$

$$NH_4NO_2 \xrightarrow{\Delta} \underset{(C)}{N_2(g)} + 2H_2O$$

(neutral, inert at room temperature)

**53.** (b) **54.** (d) **55.** (c)

**Solutions** (Q. Nos. 56-58)

$$\text{Moles of } H_2 = \frac{22.5 \times 16.42}{0.0821 \times 300} = 15 \text{ mol}$$

$$\text{Moles of } O_2 = \frac{30 \times 8.21}{0.0821 \times 300} = 10 \text{ mol}$$

When valve I is opened $H_2$ diffuses from container *A* to container *B* till

$$(p_{H_2})_A = (p_{H_2})_B$$

$\therefore$ $(n_{H_2})_A = 5$ mol, $(n_{H_2})_B = 10$ mol

as volume of container *B* is twice of *A*

When value II is opened $H_2$ will flow from container *B* to container *C* and $O_2$ will flow from container *C* to *B* till

$$(p_{H_2})_B = (p_{H_2})_C \text{ and } (p_{O_2})_B = (p_{O_2})_C$$

$\therefore$ $(n_{H_2})_B = 8$ mol, $(n_{H_2})_C = 2$ mol

$(n_{O_2})_B = 8$ mol, $(n_{O_2})_C = 2$ mol

$$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$$

| | | | |
|---|---|---|---|
| $t = 0$ | 8 | 8 | |
| $t =$ end | 0 | 4 | 8 |

Amount of $H_2O$ produced $= 8 \times 18 = 144$ g

$$(n_{H_2})_A = 5 \text{ mol}$$

$$(p_{total})_B = \left(\frac{4 \times 0.082 \times 300}{32.84} + \frac{19}{760}\right) = 3.025 \text{ atm}$$

**56.** (c) **57.** (a) **58.** (d)

**59.** (b) $\underset{\text{Oxidising agent}}{Pb^{4+}} + \underset{\text{Strong reducing agent}}{4I^-} \longrightarrow PbI_2 + I_2$

Hence, $PbI_4$ does not exist.

$$\underset{\text{Oxidising agent}}{Pb^{4+}} + \underset{\text{Not so strong reducing agent like } I^-}{4Cl^-} \longrightarrow PbCl_4 \xrightarrow[\text{(above)}]{\Delta} PbCl_2 + Cl_2$$

**60.** (a) II is correct explanation of I as adsorption leads to arrangement *i.e.*, decreases randomness ($\therefore \Delta S$ = –ve) at the same time energy is released due to attraction between adsorbate and adsorbent ($\therefore \Delta H$ = –ve). Since, adsorption process is spontaneous.

$\therefore$ $\Delta G$ = –ve

# Mathematics

**61.** (b) $L_r = \int \sin x\, d(i^r x) = \int \sin x \frac{d(i^r x)}{dx} dx$

$= i^r \int \sin x\, dx$

$\therefore \sum_{i=1}^{4n-1} L_r = \left(\int \sin x\, dx\right) \sum_{i=1}^{4n-1} i^r$

$= (-\cos x)(-1) + C = \cos x + C$

**62.** (b) $\because x, y, z, w \in [0, 10]$

Given, $2^{\sin^2 x} \cdot 3^{\cos^2 y} \cdot 4^{\sin^2 z} \cdot 5^{\cos^2 w} \geq 2 \cdot 3 \cdot 4 \cdot 5$

$\Rightarrow 2^{1-\sin^2 x} 3^{1-\cos^2 y} 4^{1-\sin^2 z} 5^{1-\cos^2 w} \leq 1$

$\Rightarrow 2^{\cos^2 x}\, 3^{\sin^2 y}\, 4^{\cos^2 z}\, 5^{\sin^2 w} \leq 1$

$\Rightarrow \cos^2 x \log 2 + \sin^2 y \log 3 + \cos^2 z \log 4 + \sin^2 w \log 5 \leq 0$

It is possible only when

$\cos^2 x = 0 \Rightarrow x = m\pi + \frac{\pi}{2}, m \in I$

$\sin^2 y = 0 \Rightarrow y = m\pi, n \in I$

$\cos^2 z = 0 \Rightarrow z = r\pi + \frac{\pi}{2}, r \in I$

and $\sin^2 w = 0 \Rightarrow w = p\pi, p \in I$

but $x, y, z, w \in [0, 10]$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, y = 0, \pi, 2\pi, 3\pi$

and $z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, w = 0, \pi, 2\pi, 3\pi$

$\therefore$ Total number of orderd 4-tuples $(x, y, z, w)$

$= 3 \cdot 4 \cdot 3 \cdot 4 = 144$

**63.** (d) $\frac{1}{a_{20}} = \frac{1}{25} = \frac{1}{5} + 19d$

$\Rightarrow 19d = \frac{1}{25} - \frac{1}{5}$

$\Rightarrow d = -\frac{4}{25 \times 19}$

Also, $\frac{1}{a_n} = \frac{1}{5} - \frac{4}{25 \times 19}(n-1) < 0$

$\Rightarrow \frac{4(n-1)}{25 \times 19} > \frac{1}{5}$

$\Rightarrow n > \frac{99}{4}$

$\therefore$ The least integral value of $n$ is 25.

**64.** (c) The inverse of the given statement is "if it is snowing, then Pinki does not wear a sweater."

**65.** (b) $\because (1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r = \sum_{r=0}^{n} a_r x^r$ (given)

$\therefore a_r = {}^nC_r$

Also, $b_r = 1 + \frac{a_r}{a_{r-1}} = 1 + \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{{}^{n+1}C_r}{{}^nC_{r-1}}$

$(\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

$\therefore b_r = \frac{n+1}{r}$

$\because \prod_{r=1}^{n} b_r = \prod_{r=1}^{n} \left(\frac{n+1}{r}\right) = \frac{(n+1)^n}{n!}$

$= \frac{(101)^{100}}{(100)!} \Rightarrow n = 100$

**66.** (d) $\int_0^3 f(x)\, dx = \begin{vmatrix} \int_0^3 1\, dx & \int_0^3 2x\, dx & \int_0^3 3x^2 dx \\ 3 & a & 27 \\ 1 & 3 & 9 \end{vmatrix}$

$= \begin{vmatrix} 3 & 9 & 27 \\ 3 & a & 27 \\ 1 & 3 & 9 \end{vmatrix}$

Taking 3 common from $R_1$,

$3\begin{vmatrix} 1 & 3 & 9 \\ 3 & a & 27 \\ 1 & 3 & 9 \end{vmatrix} = 0$

$(\because R_1$ and $R_2$ are identical)

$\therefore a \in$ any real number.

**67.** (a) $\text{adj}(P) = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$|\text{adj}(P)| = 1(3-7) - 4(6-7) + 4(2-1)$

$= -4 + 4 + 4 = 4$

$\because |\text{adj}(P)| = |P|^2 \Rightarrow |P|^2 = 4$

$\Rightarrow |P| = \pm 2$

**68.** (b) Let $y = \frac{1}{3\sqrt{2}}\sqrt{4-y}$

On squaring both sides,

$18y^2 = 4 - y$

$\Rightarrow y = \frac{4}{9}$ $(\because y > 0)$

$\therefore 6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3/2}\left(\frac{3}{2}\right)^{-2}$

$= 6 - 2 = 4$

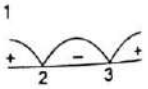
**69.** (b) The probability of hitting the target 5th time at the 10th throw = (the probability of hitting the target 4 time in the first 9 throws) × (the probability of hitting the target at the 10th throw)

$= {}^9C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^5 \times \frac{1}{2} = \frac{63}{2^9}$

**70.** (b) $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\therefore f'(x) = 6x^2 - 30x + 36$

$= 6(x-2)(x-3)$



So, $f'(x)$ changes sign in $[0, 3]$
Thus, $f(x)$ is increasing in $[0, 2]$ and decreasing in $[2, 3]$.
So, $f$ is not one-one.
$\Rightarrow$ $f(0) = 1, f(2) = 29, f(3) = 8$
Thus, range is $[1, 29]$ and so the function is onto.

**71.** (c) Divide numerator and denominator by $x^m$, then

$$K = \lim_{x\to\infty}\left(\frac{\sum_{k=1}^{1000}\left(1+\frac{K}{x}\right)^m}{1+\frac{10^{1000}}{x^m}}\right)$$

$$= \frac{1+1+1+\ldots\text{upto 1000 times}}{1+0}$$

$$= 1000 = 10^3$$

**72.** (c) $\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$$= \lim_{h\to 0}\left(a+\frac{\sin[h]}{h}\right)$$

$$= a + 0 = a$$

and $\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$$= \lim_{h\to 0} b + \left[\frac{-\sin h + h}{-h^3}\right]$$

$$= b + \left[-\left(\frac{h-\sin h}{h^3}\right)\right]$$

$$= b - 1 \qquad \left(\because \lim_{h\to 0}\frac{h-\sin h}{h^3} = \frac{1}{6}\right)$$

Hence, $f(x)$ is continuous at $x = 0$.

$\Rightarrow$ $\text{LHL} = \text{RHL}$

$\Rightarrow$ $b - 1 = a$

$\therefore$ $b = a + 1$

**73.** (a) $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ ...(i)

and $y^3 = 16x$ ...(ii)

$\Rightarrow$ $\frac{dy}{dx} = \frac{-4x}{a^2y}$

and $\frac{dy}{dx} = \frac{16}{3y^2}$

$\therefore$ $\left(-\frac{4x}{a^2y}\right)\times\left(\frac{16}{3y^2}\right) = -1 \quad (\because m_1m_2 = -1)$

$\Rightarrow$ $64x = 3a^2y^3$

$\Rightarrow$ $64x = 3a^2(16x)$ [from Eq. (ii)]

$\Rightarrow$ $a^2 = \frac{4}{3}$

$\Rightarrow$ $[a^2] = 1$

**74.** (a) The function $f(x)$ is differentiable except at $x = 0$.
$\therefore f(x)$ is not continuous at $x = 0$.

$\because$ $f(x) = 2x^2 + \frac{2}{x^2}$

$\Rightarrow$ $f'(x) = 4x - \frac{4}{x^3}$

So, the critical points are $x = 1, -1$.
Now, values of $f(x)$ at $x = -2, -1, 0, 1$ and $2$ are $f(-2), f(-1), f(0), f(1), f(2)$, *i.e.*, $\frac{17}{2}, 4, 1, 4$ and $\frac{17}{2}$.

$\Rightarrow$ $G = \frac{17}{2}$ and $L = 1$

$\Rightarrow$ $[G + L] = \left[\frac{17}{2}+1\right] = \left[\frac{19}{2}\right]$

$= (9.5) \approx 9$

**75.** (c) Let $\sec x + \tan x = t$

$\Rightarrow$ $\sec x(\sec x + \tan x)\,dx = dt$

$\Rightarrow$ $2(\sec^2 x + \sec x \tan x)\,dx = 2\,dt$

$\Rightarrow$ $[\sec^2 x + (1 + \tan^2 x) + 2\tan x \sec x]\,dx = 2\,dt$

$\Rightarrow$ $[1 + (\sec x + \tan x)^2]\,dx = 2\,dt$

$\Rightarrow$ $dx = \frac{2dt}{1+t^2}$

$\Rightarrow$ $I = \int \frac{\frac{1}{4}\left(t+\frac{1}{t}\right)^2\frac{2}{1+t^2}}{t^{9/2}}\,dt$

$$= \frac{1}{2}\int (t^{-\frac{9}{2}} + t^{-\frac{13}{2}})\,dt$$

$$= -t^{-\frac{11}{2}}\left[\frac{1}{7}t^2 + \frac{1}{11}\right] + L$$

$$= -\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left[\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right] + L$$

**76.** (d) For $0 < x < \frac{\pi}{2}$,

$\therefore$ $\frac{2}{\pi}x < \sin x < x$

$\Rightarrow$ $\frac{2}{\pi} < \frac{\sin x}{x} < 1$

$\Rightarrow$ $\int_0^{\pi/2}\frac{2}{\pi}dx < \int_0^{\pi/2}\frac{\sin x}{x}dx < \int_0^{\pi/2} 1\,dx$

$\Rightarrow$ $1 < I < \frac{\pi}{2}$

**77.** (a) Given curve is $y = 2x^4 - x^2$.

$\therefore \frac{dy}{dx} = 8x^3 - 2x$, for max or min $\frac{dy}{dx} = 0$

$\therefore$ $2x(4x^2 - 1) = 0$

$\Rightarrow$ $x = \frac{1}{2}, 0, -\frac{1}{2}$

Now, $\frac{d^2y}{dx^2} = 24x^2 - 2$

Then, $\left(\frac{d^2y}{dx^2}\right)_{x=-\frac{1}{2}} > 0, \left(\frac{d^2y}{dx^2}\right)_{x=0} < 0$

and $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} > 0$

$\therefore$ Required area $= \left|\int_{-1/2}^{1/2} (2x^4 - x^2)\,dx\right|$

$$= \left|\int_0^{1/2} (2x^4 - x^2)\right| dx$$

$$= \left|2\left[\frac{2x^5}{5} - \frac{x^3}{3}\right]_0^{1/2}\right|$$

$$= \left|2\left[2 \times \frac{1}{160} - \frac{1}{24}\right]\right|$$

$$= \frac{7}{120} \text{ sq units}$$

**78.** (c) Let central conic is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$\Rightarrow \quad \frac{2x}{a^2} + \frac{2y}{b^2} \cdot y_1 = 0$$

$$\Rightarrow \quad \frac{b^2}{a^2} + \frac{yy_1}{x} = 0$$

$$\Rightarrow \quad 0 + \frac{x \cdot \frac{d}{dx}(yy_1) - yy_1 \cdot 1}{x^2} = 0$$

$$\Rightarrow \quad x(yy_2 + y_1y_1) - yy_1 = 0$$

$$\Rightarrow \quad x(y_1)^2 + xyy_2 = yy_1$$

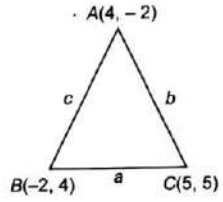
$$\Rightarrow \quad x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$$

**79.** (a) $a = \sqrt{(-2-5)^2 + (4-5)^2}$

$$= \sqrt{50} = 5\sqrt{2}$$

$$b = \sqrt{(5-4)^2 + (5+2)^2}$$

$$= \sqrt{50} = 5\sqrt{2}$$




$$c = \sqrt{(4+2)^2 + (-2-4)^2}$$

$$= \sqrt{72} = 6\sqrt{2}$$

Now, incentre of $\Delta ABC$ is

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$$\Rightarrow \quad \alpha = \frac{5\sqrt{2} \times 4 + 5\sqrt{2} \times (-2) + 6\sqrt{2} \times 5}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{5}{2}$$

and $\beta = \frac{5\sqrt{2}(-2) + 5\sqrt{2} \times 4 + 6\sqrt{2} \times 5}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}$

$$= \frac{40}{16} = \frac{5}{2}$$

$$\Rightarrow \quad \alpha = \frac{5}{2} \text{ and } \beta = \frac{5}{2}$$

$$\therefore \quad 2\alpha + 10\beta = 2 \times \frac{5}{2} + 10 \times \frac{5}{2} = 30$$

**80.** (d) Since, $y = |x| + C$ and $x^2 + y^2 - 8|x| - 9 = 0$ both are symmetrical about $y$-axis for $x > 0$, $y = x + C$ and equation of tangent to circle $x^2 + y^2 - 8x - 9 = 0$ parallel to $y = x + C$ is

$$y = x - 4 + 5\sqrt{1+1}$$

$$\Rightarrow \quad y = x + 5\sqrt{2} - 4$$

For no solution, $(C > 5\sqrt{2} - 4)$

$$\therefore \quad C \in (5\sqrt{2} - 4, \infty)$$

**81.** (b) Chord of contact of mutually perpendicular tangents is always a focal chord. Hence, min length of $AB$ is $4a$.

**82.** (b) Given equation of ellipse is $\frac{x^2}{14} + \frac{y^2}{5} = 1$.

$$\Rightarrow \quad P = (\sqrt{14}\cos\phi, \sqrt{5}\sin\phi)$$

and $\quad Q = (\sqrt{14}\cos 2\phi, \sqrt{5}\sin 2\phi)$

$\therefore$ Equation of normal at $P$ is

$$\sqrt{14}x \sec\phi - \sqrt{5}y \operatorname{cosec}\phi = 14 - 5$$

$$\Rightarrow \quad \frac{\sqrt{14}\,x}{\cos\phi} - \frac{\sqrt{5}\,y}{\sin\phi} = 9$$

Also, it passes through $Q$, then

$$\frac{14\cos 2\phi}{\cos\phi} - \frac{5\sin 2\phi}{\sin\phi} = 9$$

$$\Rightarrow \quad \frac{14(2\cos^2\phi - 1)}{\cos\phi} - 10\cos\phi = 9$$

$$\Rightarrow \quad 18\cos\phi - \frac{14}{\cos\phi} = 9$$

$$\Rightarrow \quad 18\cos^2\phi - 9\cos\phi - 14 = 0$$

$$\Rightarrow \quad (3\cos\phi + 2)(6\cos\phi - 7) = 0$$

$$\Rightarrow \quad \cos\phi = -\frac{2}{3} \qquad \left(\because \cos\phi \neq \frac{7}{6}\right)$$

**83.** (c) Pair of asymptotes is given by

$$xy - 3y - 2x + \lambda = 0 \qquad \ldots(i)$$

where, $\lambda$ is any constant such that it represents two straight lines.

$$\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow \quad 0 + 2 \times \left(-\frac{3}{2}\right) \times (-1) \times \left(\frac{1}{2}\right)$$

$$- 0 - 0 - \lambda \times \left(\frac{1}{2}\right)^2 = 0$$

$$\Rightarrow \quad \frac{3}{2} - \frac{\lambda}{4} = 0$$

$$\Rightarrow \quad \lambda = 6$$

$\therefore$ From Eq. (i), we get

$$xy - 3y - 2x + 6 = 0$$

**84.** (a) The equation of line $QR$ is

$$\frac{x-2}{-1} = \frac{y-3}{-4} = \frac{z-5}{-1} = k \quad \text{(say)}$$

So, any arbitrary point $P$ is

$$(-k + 2, -4k + 3, -k + 5)$$

Now, this will satisfy the equation of the given plane

$$5(-k + 2) - 4(-4k + 3) - (-k + 5) = 1$$

$$\Rightarrow \quad k = 2/3$$

So, the foot of the point $P$ is $\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$.

Now, let foot of the point $S$ is

$$(-\lambda + 2, -4\lambda + 3, -\lambda + 5)$$

So, DR's of $ST$ is $(-\lambda, -4\lambda + 2, -\lambda + 1)$.

$\therefore (-\lambda)(-1) - 4(-4\lambda + 2) - 1(-\lambda + 1) = 0$

$(\because a_1a_2 + b_1b_2 + c_1c_2 = 0)$

$$\Rightarrow \quad \lambda = \frac{1}{2}$$

So, the point $S$ is $\left(\frac{3}{2}, 1, \frac{9}{2}\right)$.

$\therefore$ Length,

$$PS = \sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + \left(\frac{1}{3} - 1\right)^2 + \left(\frac{13}{3} - \frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{1}{36} + \frac{4}{9} + \frac{1}{36}}$$

$$= \frac{1}{\sqrt{2}}$$

**85.** (b) Here, $\mathbf{r}_3 = x\mathbf{r}_1 + y\mathbf{r}_2$, where $x$ and $y$ are scalars. If the given vectors are linearly dependent, then $x$ and $y$ will exist uniquely.

Now, $\mathbf{r}_3 = x\mathbf{r}_1 + y\mathbf{r}_2$

$$\Rightarrow \quad (4\mathbf{a} - 5\mathbf{b} + \mathbf{c}) = x(2\mathbf{a} - 3\mathbf{b} + \mathbf{c}) + y(3\mathbf{a} - 5\mathbf{b} + 2\mathbf{c})$$

*i.e.*, $4\mathbf{a} - 5\mathbf{b} + \mathbf{c} = (2x + 3y)\mathbf{a} + (-3x - 5y)\mathbf{b} + (x + 2y)\mathbf{c}$

But $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are non-zero, non-coplanar vectors.

$\therefore$ $2x + 3y = 4$ ...(i)

$3x - 5y = 5$ ...(ii)

and $x + 2y = 1$ ...(iii)

On solving Eqs. (i) and (ii), we get $x = 5$ and $y = -2$ which satisfies Eq. (iii).

Since, $x$ and $y$ are unique.

$\therefore$ $\mathbf{r}_3 = 5\mathbf{r}_1 - 2\mathbf{r}_2$

So, $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{r}_3$ are linearly dependent vectors.

**86.** (d) Let $\mathbf{c} = x\mathbf{a} + y\mathbf{b}$

*i.e.*, $\mathbf{i} + \alpha\mathbf{j} + \beta\mathbf{k} = x(\mathbf{i} + \mathbf{j} + \mathbf{k}) + y(4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$

$$\Rightarrow \quad x + 4y = 1 \text{ and } x + 3y = \alpha$$

and $x + 4y = \beta$

From first and third, $\beta = 1$

Now, $|\mathbf{c}| = \sqrt{3}$ (given)

$$\Rightarrow \quad 1 + \alpha^2 + \beta^2 = 3$$

$$\Rightarrow \quad \alpha = \pm 1$$

**87.** (a) $A^2 - 4A + I = 0$

$$\Rightarrow \quad A^{-1}A^2 - 4A^{-1}A + A^{-1}I = 0$$

$$\Rightarrow \quad A^{-1} = 4I - A$$

$$\Rightarrow \quad \frac{1}{2}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \quad \text{(true)}$$

OR

Characteristic equation of $A$,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (2 - \lambda)^2 - 3 = 0$$

$$\Rightarrow \quad 4 + \lambda^2 - 4\lambda - 3 = 0$$

$$\Rightarrow \quad \lambda^2 - 4\lambda + 1 = 0$$

$$\Rightarrow \quad A^2 - 4A + I = 0$$

(by Caylay-Hamilton theorem)

**88.** (d) $\because \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$$\Rightarrow \quad \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 11 + 3a + b & 8 + 2a \\ 4 + a & 3 + a + b \end{bmatrix} = 0$$

$$\Rightarrow \quad 8 + 2a = 0, \quad 11 + 3a + b = 0$$

$$a = -4, b = 1$$

$$\therefore \quad \int_a^{4b} x^3 \cos x \, dx = \int_{-4}^{4} x^3 \cos x \, dx = 0$$

Therefore, $x^3 \cos x$ is an odd function.

**89.** (a) Equation of the two bisectors will be given by

$$\frac{y}{\sqrt{1^2 + 0^2}} = \pm \frac{x}{\sqrt{1^2 + 0^2}}$$

$$\Rightarrow \quad y = \pm x$$

**90.** (b) For two circles to have a transverse common tangent, they should either be seperated from each other or just touch each other externally. Here, two circles are intersecting in two distinct points.

# JEE Main

## Joint Entrance Examination

# Practice Set 4

## Instructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

1. A car starting from rest, accelerates at constant rate $\alpha$ for some time after which it decelerates at constant rate $\beta$ to come to rest. If the total time of journey is $t$, the maximum velocity attained by the car is given by

   (a) $\frac{\alpha\beta}{\alpha+\beta}t$ (b) $\frac{\alpha\beta}{\alpha-\beta}t$ (c) $\sqrt{\alpha\beta}\,t$ (d) $\frac{\alpha+\beta}{2}t$

2. Rain is falling vertically with a speed of 4 $ms^{-1}$. After some time, wind starts blowing with a speed of 3 $ms^{-1}$ in the north to south direction. In order to protect himself from rain, a man standing on ground should hold his umbrella at an angle $\theta$ given by

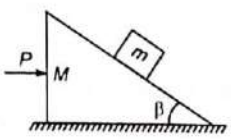
   (a) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards South

   (b) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards North

   (c) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards South

   (d) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards North

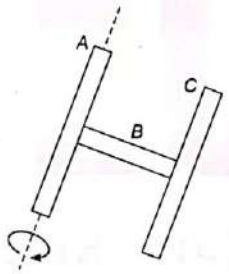
3. Two wooden blocks are moving on a smooth horizontal surface such that the mass $m$ remains stationary with respect to block of mass $M$ as shown in the figure. The magnitude of force $P$ is




   (a) $(M+m)g\tan\beta$ (b) $g\tan\beta$

   (c) $mg\cos\beta$ (d) $(M+m)g\,\text{cosec}\,\beta$

4. A rigid body is made of three identical thin rods, each of length $L$ fastened together in the form of letter $H$. The body is free to rotate about a horizontal axis that runs along the length of one of the legs of the $H$ shaped body. The body is allowed to fall from rest to a position in which the plane of $H$ is horizontal. What is the angular speed of the body when the plane of $H$ is vertical?



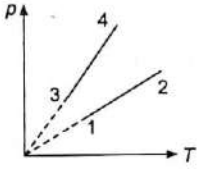

(a) $\sqrt{\frac{g}{L}}$ (b) $\frac{1}{2}\sqrt{\frac{g}{L}}$

(c) $\frac{3}{2}\sqrt{\frac{g}{L}}$ (d) $2\sqrt{\frac{g}{L}}$

5. A wooden ball of density $D$ is immersed in water of density $d$ to a depth $h$ below the uppermost surface of water and then released. Upto what height will the ball jump out of water?

(a) $\frac{d}{D}h$ (b) $\left(\frac{d}{D}-1\right)h$ (c) $h$ (d) Zero

6. Pressure *versus* temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternative.




(a) $V_1 = V_2, V_3 = V_4$ and $V_2 > V_3$

(b) $V_1 = V_2, V_3 = V_4$ and $V_2 < V_3$

(c) $V_1 = V_2 = V_3 = V_4$

(d) $V_4 > V_3 > V_2 > V_1$

7. A parallel plate capacitor of capacity 100 $\mu$F is charged by a battery of 50 volts. The battery remains connected and if the plates of the capacitor are separated, so that the distance between them becomes double the original distance, the additional energy given to the battery of the capacitor in joules is

(a) $\frac{125\times10^{-3}}{2}$ (b) $\frac{12.5\times10^{-3}}{2}$ (c) $\frac{1.25\times10^{-3}}{2}$ (d) $\frac{0.125\times10^{-3}}{2}$

8. The ratio of the resistance of conductor at temperature 15°C to its resistance at temperature 37.5°C is 4 : 5. The temperature coefficient of resistance of the conductor is

(a) $\frac{1}{25}$ °C$^{-1}$ (b) $\frac{1}{50}$ °C$^{-1}$ (c) $\frac{1}{80}$ °C$^{-1}$ (d) $\frac{1}{75}$ °C$^{-1}$

9. The work functions for metal $A, B$ and $C$ are 1.92 eV, 2.0 eV and 5 eV respectively. According to Einstein's equation, the metals which will emit photoelectrons for a radiation of wavelength 4000 Å is/are

(a) only $A$ (b) $A$ and $B$

(c) All the three metals (d) None of these

10. After 1$\alpha$ and 2$\beta$-emissions

(a) mass number reduces by 6 (b) mass number reduces by 4

(c) mass number reduces by 2 (d) mass number remains unchanged

11. If the input and output resistances in a common-base amplifier circuit are 400 $\Omega$ and 400 k $\Omega$, respectively. What is the voltage amplification when the emitter current is 2 mA and current gain, $\alpha = 0.98$?

(a) 9.8 (b) 98 (c) 980 (d) 9800

12. A ball is projected vertically upwards from the top of a tower of height 35 m with an initial velocity 30 ms$^{-1}$. How much time will it take to reach the ground? (take $g = 10$ ms$^{-2}$)

(a) 1 s (b) 7 s (c) 5 s (d) 3 s

13. A monkey climbs up and another monkey moving down a rope hanging from a tree with some uniform acceleration separately. If the respective masses of monkey are in the ratio 2 : 3, the common acceleration must be
(a) $g/5\,ms^{-2}$ (b) $6g\,ms^{-2}$ (c) $g/2\,ms^{-2}$ (d) $g\,ms^{-2}$

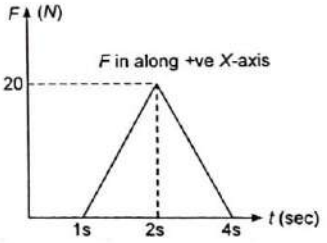
14. A simple pendulum of length $l$ is moved aside till the string makes an angle $\theta_1$, with the vertical. If the acceleration due to gravity is $g$, the kinetic energy of the bob when the string is inclined at $\theta_2$ to the vertical is
(a) $mgl\ \cos(\theta_1 - \theta_2)$ (b) $mgl\ (\cos\theta_2 - \cos\theta_1)$
(c) $mgl\ (\cos\theta_1 - \cos\theta_2)$ (d) $mgl\ \sin(\theta_1 - \theta_2)$

15. A rigid body rotates about a fixed axis with variable angular velocity equal to $(\alpha - \beta t)$ at time $t$ where, $\alpha$ and $\beta$ are constants. The angle through which it rotates before it comes to rest is
(a) $\frac{\alpha^2}{2\beta}$ (b) $\frac{\alpha^2 - \beta^2}{2\alpha}$ (c) $\frac{\alpha^2 - \beta^2}{2\beta}$ (d) $\frac{\alpha(\alpha - \beta)}{2}$

16. If a rocket is fixed with a speed, $v = 2\sqrt{gR}$ near the earth's surface and coasts upwards, its speed in the interstellar space is
(a) $4\sqrt{gR}$ (b) $\sqrt{2gR}$ (c) $\sqrt{gR}$ (d) $\sqrt{4gR}$

17. A beaker full of hot water is kept in a room. If it cools from 80°C to 75°C in $t_1$ min, from 75°C to 70°C in $t_2$ min and from 70°C to 65°C in $t_3$ min, then
(a) $t_1 = t_2 = t_3$ (b) $t_1 < t_2 = t_3$ (c) $t_1 < t_2 < t_3$ (d) $t_1 > t_2 > t_3$

18. A particle of mass 2 kg is moving along $X$-axis with a velocity of 5 m/s. It crosses the origin at $t = 0$. A time-varying force whose variation is as shown in the figure starts acting on particle at $t = 1$ s. For this situation, mark out the correct statement(s).
(a) The particle will come to rest instantaneously, at $t = 6.8$ s.
(b) The final velocity of the particle will be 18 m/s.
(c) Work done by the force on the particle is 375 J.
(d) All of the above



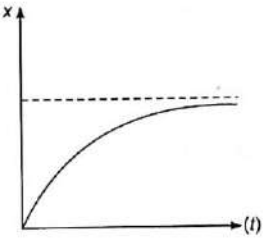

19. A solid sphere of radius $R$, made up of a material of bulk modulus $K$ is surrounded by a liquid in a cylindrical container. A massless piston of area $A$ floats on the surface of the liquid. When a mass $M$ is placed on the piston to compress the liquid, the fractional change in the radius of the sphere is
(a) $\frac{Mg}{AK}$ (b) $\frac{Mg}{3AK}$ (c) $\frac{3Mg}{AK}$ (d) $\frac{Mg}{2AK}$

20. If the gravitational force had varied as $r^{-5/2}$ instead of $r^{-2}$, the potential energy of a particle at a distance $r$ from the centre of the earth would be proportional to
(a) $r^{-1}$ (b) $r^{-2}$ (c) $r^{-3/2}$ (d) $r^{-5/2}$

21. The Zener diode normally operates under reverse bias condition, the major use of this fact is in the applications where, we require
(a) large value of current
(b) a constant voltage
(c) a current that is increasing without any change in applied voltage
(d) All of the above

22. For a hydrogen like atom, if electron moves from lower energy level to higher energy levels, then which of the following statement is correct?
(a) KE and PE both increases (b) KE and PE both decreases
(c) KE increases while PE decreases (d) KE decreases while PE increases

23. The displacement, $x$ of a particle as a function of time $t$ is shown in the figure. The figure indicates



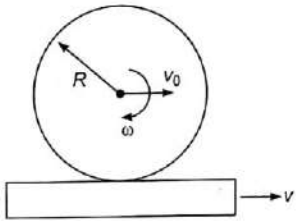

(a) the particle starts with a certain velocity, but the motion is related, finally the particle stops

(b) the velocity of the particle is constant throughout

(c) the acceleration of the particle is constant throughout

(d) the particle starts with certain velocity and then its motion is accelerated

24. A solid cylinder is performing pure rolling motion on a moving platform as shown. Which of the relation between, $v_0, \omega, R$ and $v$ is correct?



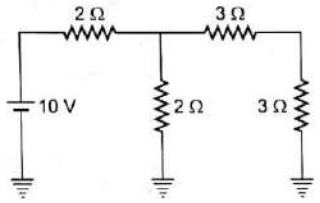

(a) $v_0 = R\omega$

(b) $v = v_0 + R\omega$

(c) $v = v_0 - R\omega$

(d) $v = v_0$

25. In the circuit shown, the current in 3 Ω resistor is



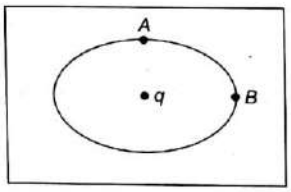

(a) 1 A

(b) $\frac{30}{7}$ A

(c) $\frac{5}{7}$ A

(d) Information insufficient

26. An elliptical cavity is curved within a perfect conductor. A positive charge $q$ is placed at the centre of the cavity. The points $A$ and $B$ are on the cavity surface as shown in the figure below.




Then,

(a) electirc field near $A$ in the cavity = electric field near $B$ in the cavity

(b) charge density at $A$ = charge density at $B$

(c) potential at $A$ = potential at $B$

(d) None of the above

27. In a hydroelectric installation, a turbine delivers 1500 HP to a generator, which in turn converts 80% of the mechanical energy into electrical energy. Under these conditions what current does the generator delivers at a terminal potential difference of 2000 V?

(a) 447.6 A (b) 559.5 A (c) 4 A (d) 800 A

28. A student in a hot air balloon ascends vertically at constant speed. Consider the following four forces that arise in this situation

$F_1$ : The weight of the hot air balloon

$F_2$ : The weight of the student

$F_3$ : The force of the student pulling on the earth

$F_4$ : The force of the hot air balloon pulling on the student

Which of the following relationships concerning the forces or their magnitudes is true?

(a) $F_4 > F_2$ (b) $F_3 = -F_4$ (c) $F_2 > F_1$ (d) $F_2 = -F_4$

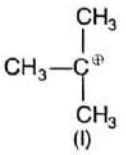
29. There are some passengers in a stationary railway compartment. The centre of mass of compartment itself (without passengers) is $c_1$, while centre of mass of compartment plus the passengers is $c_2$. If the passengers move inside the compartment [The surface on which compartment is at rest is smooth]
(a) both $c_1$ and $c_2$ will move w.r.t. ground
(b) neither $c_1$ nor $c_2$ will move w.r.t. ground
(c) $c_1$ will move but $c_2$ will remain stationary w.r.t. the ground
(d) $c_2$ will move but $c_1$ will remain stationary w.r.t. the ground

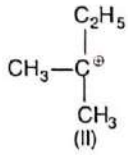
30. A chain of mass $m$ and length $l$ lies on the surface of a rough sphere of radius $R\,(>l)$ such that one end of chain is at the top most point of sphere. The chain is held at rest because of friction. The gravitational potential energy of the chain in this position (considering the horizontal diameter of sphere as reference level for gravitational potential energy), is

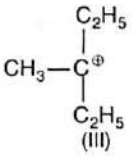
(a) $\frac{mgR^2}{l}$ (b) $\frac{mgR^2}{l}\sin\left(\frac{l}{R}\right)$ (c) $\frac{mgR^2}{l}\cos\left(\frac{l}{R}\right)$ (d) None of these

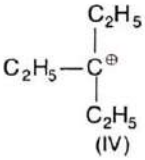
# Chemistry

31. Give the stability of following carbocations.


(I)

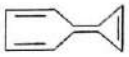
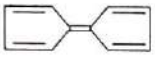
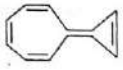

(II)


(III)


(IV)

(a) IV > III > II > I (b) IV > III > I > II (c) I > II > III > IV (d) I > III > II > IV

32. Which of the following will allow free rotation about double bond?

(a)  (b)  (c)  (d) 

33. The ratio of ionisation energy of H and $Be^{3+}$ is
(a) 1 : 1 (b) 1 : 3 (c) 1 : 9 (d) 1 : 16

34. The shape of the molecule $XeOF_2$ is
(a) tetrahedral (b) square pyramidal (c) T-shape (d) square planar

35. Equal masses of ethane and hydrogen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by hydrogen is
(a) 1 : 2 (b) 1 : 1 (c) 1 : 16 (d) 15 : 16

36. $S + \frac{3}{2}O_2 \longrightarrow SO_3 + 2x$ kcal

$SO_2 + \frac{1}{2}O_2 \longrightarrow SO_3 + y$ kcal

Find out the heat of formation of $SO_2$.
(a) $y - 2x$ (b) $2x - y$ (c) $x + y$ (d) $\frac{2x}{y}$

37. On adding 60 g of water to an oleum sample, 740 g of $H_2SO_4$ are obtained. The strength of given oleum is
(a) 103.8% (b) 105.9% (c) 108.8% (d) 110.5%

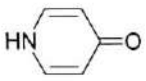
38. Argon gas taken in a closed vessel was heated from –13°C to 13°C, the percentage increase in pressure inside the bulb is
(a) 11% (b) 10% (c) 100% (d) 110%

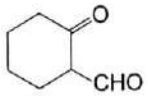
39. The number of peroxide linkages in $P_4O_{10}$ is
(a) 0 (b) 2 (c) 4 (d) 8

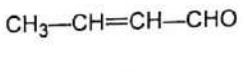
40. For the first order reaction, $2A(g) \longrightarrow 3B(g)$, $t_{1/2}$ =12 min. Initial pressure exerted by $A$ is 640 mm of Hg. The pressure of the reaction mixture after the time period of 36 min will be
(a) 560 mm Hg (b) 680 mm Hg (c) 920 mm Hg (d) 600 mm Hg

41. The compound which may exhibit tautomerism is


(I)

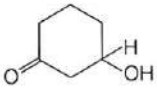

(II)


(III)


(IV)

(a) I, II and III (b) II, III and IV (c) I and III (d) I, II, III and IV

42. The cation which gives a yellow precipitate with potassium chromate is
(a) $NH_4^+$ (b) $Ba^{2+}$ (c) $Ca^{2+}$ (d) $Na^+$

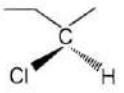
43. Which of the following compounds is achiral?
(a)  (b) $CH_3$—CHOHCN (c)  (d) 

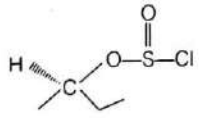
44. Which of the following complexes are inner-orbital complex and diamagnetic?
(a) $[Fe(CN)_6]^{4-}$ (b) $[Fe(H_2O)_6]^{2+}$ (c) $[Cu(NH_3)_6]^{2+}$ (d) $[Ni(CN)_6]^{4-}$

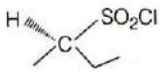
45. An ideal solution contains two volatile liquids $A$ ($p^\circ$ =100 torr) and $B$ ($p^\circ$ = 200 torr). If mixture contains 1 mole of $A$ and 4 moles of $B$, then total vapour pressure of the distillate is
(a) 150 torr (b) 180 torr (c) 188.88 torr (d) 198.88 torr

46. The product formed in the reaction,

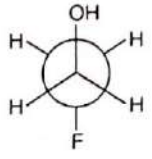
(a) 

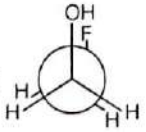
(b) 

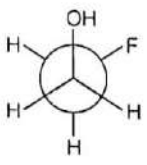
(c) 

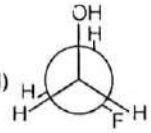
(d) 

47. The most stable form of the following conformers are

(a) 

(b) 

(c) 

(d) 

48. All the following substances react with water. The pair that gives the same gaseous product is
(a) K and $KO_2$ (b) Na and $Na_2O_2$ (c) Ca and $CaH_2$ (d) Ba and $BaO_2$

49. For the reaction, $2A(g)+B(g) \rightleftharpoons C(g)+D(g); K_c = 10^{12}$. If the initial moles of *A*, *B*, *C* and *D* are 2, 1, 7 and 3 moles respectively in 1 L vessel, what is the equilibrium concentration of *A* ?
(a) $4\times10^{-4}$ (b) $2\times10^{-4}$ (c) $10^{-4}$ (d) $8\times10^{-4}$

50. A galvanic cell is set up from a zinc bar weighing 100g and 1.0 L of 1.0 M $CuSO_4$ solution. How long would the cell run if it is assumed to deliver a steady current of 1.0 amp. (Atomic mass of Zn = 65)
(a) 1.1 hr (b) 46 hr (c) 53.6 hr (d) 24.00 hr

51. How many effective $Na^+$ and $Cl^-$ ions are present respectively in a unit cell of NaCl solid (Rock salt structure) if ions along line connecting opposite face centres are absent?
(a) 3, 3 (b) $\frac{7}{2}$, 4 (c) $\frac{7}{2}, \frac{7}{2}$ (d) 4, $\frac{7}{2}$

52. During the electro-osmosis of $Fe(OH)_3$ sol
(a) sol particles move towards anode
(b) sol particles move towards cathode
(c) the dispersion medium moves towards anode
(d) the dispersion medium moves towards cathode

**Directions** (Q. Nos. 53-55) *Solution of an acid and its anion (that is its conjugate base) or of a base and its common cation is buffer. On adding small amount of acid or base, the pH of solution changes very little (negligible change). The pH of buffer solution is determined as follows :*

$$pH \text{ of acidic in buffer} = pK_a + \log\frac{[\text{conjugate base}]}{[\text{acid}]}$$

$$pOH \text{ of basic buffer} = pK_b + \log\frac{[\text{conjugate acid}]}{[\text{base}]}$$

*A buffer solution can work effectively provided the value of* $\frac{[\text{conjugate base}]}{[\text{acid}]}$ *for acidic buffer or* $\frac{[\text{conjugate acid}]}{[\text{base}]}$ *for basic buffer lies within the range of* 1 : 10 *or* 10 : 1.

53. Calculate the pH of a solution made by adding 0.01 mole of HCl in 100 mL of a solution which is 0.2 M in $NH_3$ ($pK_b = 4.74$) and 0.3 M in $NH_4^+$. (Assuming no change in volume)
(a) 5.34 (b) 8.66 (c) 7.46 (d) None of these

54. 1 L of an aqueous solution contains 0.15 mole of $CH_3COOH$($pK_a = 4.8$) and 0.15 mole of $CH_3COONa$. After the addition of 0.05 mole of solid NaOH to this solution, the pH will be
(a) 4.5 (b) 4.8 (c) 5.1 (d) 5.4

55. When a 20 mL of 0.08 M weak base *B*OH is titrated with 0.08 M HCl, the pH of the solution at the end point is 5. What will be the pOH if 10 mL of 0.04 M NaOH is added to the resulting solution? (Given, log 2 = 0.30 and log 3 = 0.48)
(a) 5.40 (b) 5.88
(c) 4.92 (d) None of these

**Directions** (Q. Nos. 56-58) *Two liquids A and B have the same molecular weights and form an ideal solution. The solution of composition $X_A$ has the vapour pressure 700 mm Hg at 80°C. The above solution is distilled without reflux till 3/4 of the solution is collected as condensate. The composition of the condensate is $X'_A = 0.75$ and that of residue is $X_A = 0.3$. The vapour pressure of the residue at 80°C is 600 mm.*

56. Mole fraction of *A* in original sample is
(a) 0.6375 (b) 0.2375 (c) 0.8375 (d) 0.9375

57. Value of $p_A^\circ$ (*i.e.*, vapour pressure of pure *A*) is
(a) 807.41 mm of Hg (b) 511.11 mm of Hg (c) 707.41 mm of Hg (d) 207.41 mm of Hg

58. Value of $p_B^\circ$ (*i.e.*, vapour pressure of pure *B*) is
(a) 807.41 mm of Hg (b) 511.11 mm of Hg (c) 707.41 mm of Hg (d) 207.41 mm of Hg

**Directions** (Q. Nos. 59 and 60) *For the following questions choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*
(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.
(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

59. **Statement I** In the electrolytic cell, flow of electron is from anode to cathode through internal supply.
**Statement II** In an electrolytic cell, cathode is the electron rich electrode.

60. **Statement I** Internal energy of a system is an extensive property.
**Statement II** The internal energy of a system depends upon the amount and physical state of the substance.

# Mathematics

61. If $a,b,c,d,e$ are positive real numebrs such that $a+b+c+d+e=15$ and $ab^2c^3d^4e^5=(120)^3\cdot(50)$, then the value of $a^2+b^2+c^2+d^2+e^2$ is
(a) 40 (b) 45 (c) 50 (d) 55

62. Let $h(x)=\max\{-x,1,x^2\}$ for every real number $x$. Then,
(a) $h$ is continuous for all $x$ (b) $h$ is differentiable for all $x$
(c) $h'(a)=1, \forall\, x>1$ (d) $h$ is not differentiable at two values of $x$

63. The value of
$${}^{100}C_0\,{}^{200}C_{100} - {}^{100}C_1\,{}^{199}C_{100} + {}^{100}C_2\,{}^{198}C_{100} - {}^{100}C_3\,{}^{197}C_{100} + \ldots + {}^{100}C_{100}\,{}^{100}C_{100}$$
is equal to
(a) 1 (b) −1 (c) 0 (d) 2

64. The odd against a certain event is 5 : 2 and the odds in favour of another event is 6 : 5. If both the events are independent, then the probability that atleast one of the events will happens is
(a) $\frac{52}{77}$ (b) $\frac{50}{77}$ (c) $\frac{25}{88}$ (d) None of these

65. The number of solution(s) of the equation $2\tan^2 x + 2\tan^4 x - 2\sec^4 x \sin^2 x - 3 = 0$ in $\left(0, \frac{\pi}{2}\right)$ is

(a) 0 (b) 1 (c) 3 (d) 4

66. If the graph of $y = f(x)$ is symmetric about the curves $xy^2 - y^2 + x - 1 = 0$ and $xy^2 - 3y^2 + 2x - 6 = 0$, then fundamental period of $f(x)$ is

(a) 4 (b) 3 (c) 2 (d) 1

67. Let $A_1, A_2, A_3, \ldots, A_{2009}$ and $H_1, H_2, H_3, \ldots, H_{2009}$ are arithmetic and harmonic means inserted between two numbers '$a$' and '$b$', respectively. If $A_{1005}H_{1005}$ is 20011, then $\frac{A_{1006}\, H_{1004}}{A_{1004}\, H_{1006}}$ is

(a) 1 (b) 2 (c) 3 (d) 4

68. Let $x_1, x_2, \ldots, x_n$ be $n$ observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. Then, a possible value of $n$ among the following is

(a) 9 (b) 12 (c) 15 (d) 18

69. If vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ satisfy the condition $|\mathbf{a} - \mathbf{c}| = |\mathbf{b} - \mathbf{c}|$, then $(\mathbf{b} - \mathbf{a}) \cdot \left(\mathbf{c} - \frac{\mathbf{a} + \mathbf{b}}{2}\right)$ is

(a) 2 (b) 1 (c) –1 (d) 0

70. Consider the equation $3\sqrt{[32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1]^2} = \sin x$. Number of solutions of equation is

(a) 9, $x \in \left[0, \frac{\pi}{2}\right]$ (b) 10, $x \in [0, \pi]$ (c) 8, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) 12, $x \in [-\pi, \pi]$

71. If $\int \frac{12x^3 - 1}{x\sqrt{100x^2 - 36x^6 - 12x^3 - 1}}\, dx = a \sin^{-1}\{f(x)\} + C$, then

(a) $f(x) = \left(3x^2 - \frac{1}{2x}\right)$ (b) $f(x) = \frac{1}{5}\left(3x^2 - \frac{1}{2x}\right)$

(c) $a = 1$ (d) $a = 2$

72. The contrapositive of $(p \vee q) \to r$ is

(a) $\sim r \to (p \vee q)$ (b) $r \to (p \vee q)$

(c) $\sim r \to (\sim p \wedge \sim q)$ (d) $p \to (q \vee r)$

73. The equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the positive end of the latusrectum is

(a) $x + ey + e^3 a = 0$ (b) $x - ey - e^3 a = 0$

(c) $x + ey - e^2 a = 0$ (d) None of these

74. Range of the function $f(x) = \sin x + |\cos x|$ is

(a) $[-\sqrt{2}, \sqrt{2}]$ (b) $[0, \sqrt{2}]$ (c) $[1, \sqrt{2}]$ (d) $[-1, \sqrt{2}]$

75. The range of the function $f(x) = [4^x - 2^{x+1} + 1]$ is, where [.] denotes greatest integer function

(a) positive integers (b) whole numbers

(c) prime numbers (d) all integers

76. The value of the integral $\int_0^{\pi/2} \frac{a+b\cos x}{(b+a\cos x)^2}\,dx$.

(a) is independent of $a$
(b) is independent of $b$
(c) is independent of $a$ and $b$
(d) depends on both $a$ and $b$

77. The vertices of triangle are $A(1,2), B(3,2), C(2,\sqrt{3}+2)$ a point $P$ moves with a triangle such that it satisfies the relation $(d(P,AB) \le d(P,AC))$ and $(d(P,AB) \le d(P,BC))$, then the area of traced by point $P$, where $d(P,AB)$ denotes the distance of point $P$ from line $AB$ is

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

78. If the curve $C_1$ is $|z-1|=1$ and the point $z_1$ satisfies the relation $|z_1-8|+|z_1-6i|=10$, then the minimum value of $|z-z_1|$ is

(a) $\frac{16}{5}$
(b) $\frac{\sqrt{10025}}{25}-1$
(c) $\frac{\sqrt{10125}}{25}-1$
(d) 24

79. Let $M, N$ be feet of perpendicular from $P$ to $xy, yz$ plane, respectively. If $OP$ makes 30°, 45°, 60° angle with the planes $xy$, $yz$ and $zx$, respecitvely and angle $\theta$ with plane $OMN$, then the numerical value of $3\,\text{cosec}^2\,\theta$ is

(a) 17 (b) 23 (c) 22 (d) 19

80. If letters of the word 'CALCULUS' are arranged in such a way that all C's occur before all L's, then the total number of words formed is

(a) 600
(b) 820
(c) 840
(d) 620

81. Let $A$ be a matrix of order $3\times 3$ and matrices $B, C$ and $D$ are related such that $B=\text{adj}(A)$, $C=\text{adj}(\text{adj}\,A)$, $D=(\text{adj}(\text{adj}(\text{adj}\,A)))$ if $|\text{adj}(\text{adj}(\text{adj}(\text{adj}\,ABCD)))|$ is $|A|^k$, then $k$

(a) is less than 256
(b) has 21 divisors
(c) can't say
(d) is an odd number

82. If $Z$ is an idempotent matrix, then $(I+Z)^n$

(a) $I+2^nZ$
(b) $I+(2^n-1)Z$
(c) $I-(2^n-1)Z$
(d) None of these

83. If $(1+\tan 1°)(1+\tan 2°)(1+\tan 3°)\ldots(1+\tan 45°)=2^n$ and $a, b, c$ and $d$ are four numbers in the interval $[0,\pi]$ such that

$$\sin a+7\sin b=4(\sin c+2\sin d)$$
$$\cos a+7\cos b=4(\cos c+2\cos d)$$

and the numerical value of $\frac{7\cos(b-c)}{\cos(a-d)}$ is $m$. The correct statement is/are

(a) $m$ is a prime number
(b) $n$ is a composite number
(c) $m, n$ both are perfect square
(d) $m$ is divisible by $n$.

84. Number of identical terms in the sequence 2, 5, 8, 11,...,100 terms and 3, 5, 7, 9,...,100 terms are

(a) 17
(b) 33
(c) 50
(d) 47

**Directions** (Q. Nos. 85 and 86) *Sometimes use of graph is very important to find the number of solutions of an equation. To find the number of solutions of the equation* $f_1(x) = f_2(x)$. *We draw the graph of* $y = f_1(x)$, $y = f_2(x)$ *and the number of point of intersection of these graphs is equal to the number of solution. Let us consider the function*

$$f(x) = |x-1| + |x-3| + |x-7| + |x-13|$$

85. The minimum value of $f(x)$ is

(a) 10 (b) 16 (c) 20 (d) 28

86. The number of solutions of $f(x) = 16$ is

(a) zero (b) two (c) three (d) infinitely many

**Directions** (Q. Nos. 87 and 88) *The mean value of the continuous functions* $f(x)$ *in the interval* $[a, b]$ *is given by the formula*

$$\text{Mean value} = \frac{\int_a^b f(x)\,dx}{b-a}$$

87. Value of $\int_0^1 \frac{dx}{\sqrt{1-x^2}\,[(1-x^2)^{1/4} + x^{1/2}]^4}$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 0 (d) 1

88. The mean value of $f(x) = (x+1)(\tan x)^x \cdot \sec^2 x + \log_e(\tan x) \cdot (\tan x)^{x+1}$ in the interval $\left[0, \frac{\pi}{4}\right]$ is

(a) $\frac{2}{\pi}$ (b) $\frac{1}{\pi}$ (c) $\frac{4}{\pi}$ (d) $\frac{8}{\pi}$

**Directions** (Q. Nos. 89 and 90) *For the following questions. Choose the correct answer to the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

89. **Statement I** If $\sin x + \sin^2 x = 1$, then value of the expression

$$\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1 \text{ is } -1.$$

**Statement II** $\cos^2 x = \frac{\sqrt{5}-1}{2}$.

90. Let us consider the any $\Delta ABC$, whose sum of all angles is 180°.

**Statement I** If $\angle A$ is obtuse, then $\tan B \tan C < 1$.

**Statement II** $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

# Answer *with* Explanations

## Physics

**1.** (a) Let $v_0$ be the maximum velocity, then

**Case I** $u = 0, v = v_0, a = \alpha, t = t_1$ (say)

Using, $v = u + at \Rightarrow v_0 = \alpha t_1 \Rightarrow t_1 = \frac{v_0}{\alpha}$ ...(i)

**Case II** $u = v_0, v = 0, a = -\beta, t = t_2$ (say)

Using, $v = u + at \Rightarrow 0 = v_0 - \beta t_2$

$\Rightarrow$ $t_2 = \frac{v_0}{\beta}$ ...(ii)

But $t_1 + t_2 = t$

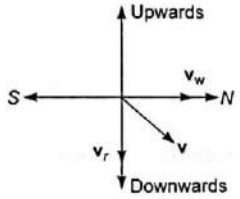
$$\frac{v_0}{\alpha} + \frac{v_0}{\beta} = t, \quad v_0 = \frac{\alpha\beta}{\alpha + \beta} t$$

**2.** (a) Resultant velocity of rain

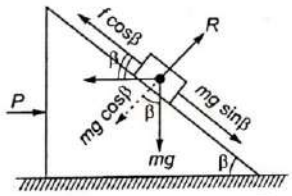
$$\mathbf{v} = \mathbf{v}_r + \mathbf{v}_w = 3\mathbf{i} + 4\mathbf{j}$$

$\therefore$ $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

As wind is blowing from south, umbrella should be hold towards south.




**3.** (a) The different forces acting on mass $m$ are shown in the adjoining figure.




Acceleration of the system $= \frac{P}{M + m}$

$\therefore$ Force on mass, $m = \frac{Pm}{M + m}$

Let the reaction of $m$ on $M$ be $f$. Then, $f = \frac{Pm}{M + m}$

Here, note that a pseudo force will act on the smaller mass $m$ as $f$ in backward to applied force.

According to figure, $m$ will be stationary when

$$f \cos\beta = mg \sin\beta$$

$$\frac{Pm}{M + m} \cos\beta = mg \sin\beta$$

$\therefore$ $P = (M + m) g \tan\beta$

**4.** (c) Moment of inertia of the system about the given axis,

$$I = I_A + I_B + I_C$$

Now, as rod is thin,

$$I_A = \Sigma m \times (0)^2 = 0$$

Rod $B$ is rotating about one end.

$\therefore$ $I_B = \frac{ML^2}{3}$

and for rod call points are always at distance $L$ from the axis of rotation, so

$$I_C = \Sigma mL^2 = ML^2$$

$\therefore$ $I = 0 + \frac{ML^2}{3} + ML^2 = \frac{4}{3} ML^2$

So, if $\omega$ is the desired angular speed, gain in kinetic energy due to rotation of $H$ from horizontal to vertical position,

$$K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left[\frac{4}{3} ML^2\right] \omega^2$$

$$= \frac{2}{3} ML^2\omega^2$$

and loss in potential energy of the system in doing so

$$= 0 + Mg \frac{L}{2} + MgL = \frac{3}{2} MgL$$

So, by conservation of mechanical energy,

$$\frac{2}{3} ML^2\omega^2 = \frac{3}{2} MgL$$

$$\omega = \frac{3}{2}\sqrt{\frac{g}{L}}$$

**5.** (b) Let the volume of the ball be $V$. Force on the ball due to upthrust $= Vdg$

Net upward force $= Vdg - VDg$

Consider the force of bouyancy

$\therefore$ Upward acceleration is given by

$$VDa = Vdg - VDg$$

$\therefore$ $a = \left(\frac{d - D}{D}\right) g$

Velocity on reaching the surface, $v = \sqrt{2ah}$

Further, $v = \sqrt{2gH}$

$\therefore$ $2ah = 2gH$

or $H = \frac{ah}{g} = \left(\frac{d - D}{D}\right) h = \left(\frac{d}{D} - 1\right) h$

**6.** (a) From ideal gas equation,

$$pV = \mu RT$$

Slope of $p$-$T$ curve,

$$\frac{p}{T} = \frac{\mu R}{V} \quad \text{or Slope} \propto \frac{1}{V}$$

It means lines of smaller slope represents greater volume of gas. For the figure given in problem, points 1 and 2 are on the same line, so they will represent same volume, *i.e.*, $V_1 = V_2$. Similarly, points 3 and 4 are on the same line, so they will represent same volume, *i.e.*, $V_3 = V_4$.

Also, slope of line 1-2 is less than that of the line 3-4, hence $(V_1 = V_2) > (V_3 = V_4)$.

**7.** (a) Initially energy stored in capacitor $E_1 = \frac{1}{2} CV^2$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times 50 \times 50$$

$$\Rightarrow \quad E_1 = 125 \times 10^{-3} \text{ J}$$

When the distance is doubled, the capacitance decreases to 1/2,

*i.e.*, $C' = 50\ \mu\text{F}$

∴ Final energy,

$$E_2 = \frac{1}{2} C'V^2 = \frac{1}{2} \times 50 \times 10^{-6} \times (50)^2$$

$$= \frac{125 \times 10^{-3}}{2}$$

Additional energy, $E = E_1 - E_2 = \frac{125 \times 10^{-3}}{2}$ J

**8.** (d) As, $R_t = R_0 (1 + \alpha t)$

At $T = 15°C$,

$$R_{15} = R_0 [1 + \alpha \times 15]$$

and $T = 37.5°$ C,

$$R_{37.5} = R_0 [1 + \alpha \times 37.5]$$

$$\therefore \quad \frac{R_{15}}{R_{37.5}} = \frac{1 + 15\alpha}{1 + 37.5\alpha}$$

or $$\frac{4}{5} = \frac{1 + 15\alpha}{1 + 37.5\alpha}$$

$$\therefore \quad \alpha = \left(\frac{1}{75}\right)^{\circ} \text{C}^{-1}$$

**9.** (b) $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{41 \times 10^{-8}}$ J

$$= \frac{19.89 \times 10^{-18}}{41 \times 1.6 \times 10^{-19}} \text{ eV} = 3.0 \text{ eV}$$

For metals *A* and *B*, the work function is less than 3.0 eV, so photoelectrons from metals *A* and *B* will be emitted with radiation of 4000 Å.

**10.** (b) After one α-emission, the mass number reduces by 4 and atomic number reduces by 2. Then, after two β-emissions the atomic number increases by 1 overall, the mass number reduces by 4 and atomic number remains unchanged.

**11.** (c) Emitter current is 2 mA, input resistance is 400 Ω, output resistance is $400\ \text{k}\Omega = 400 \times 10^3\ \Omega$, input voltage is $2 \times 400\ \text{mV} = 800\ \text{mV} = 0.800$ V and collector current, $I_C = 0.98 \times 2 = 1.96$ mA.

Output or collector voltage is

$$(1.96 \times 10^{-3}) \times (400 \times 10^{+3}) = 784 \text{ V}$$

Voltage amplification is,

$$\frac{784}{0.800} = 980$$

**12.** (b) Given,

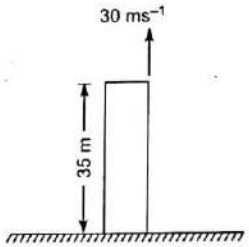
$$s = -35 \text{ m}, u = 30 \text{ ms}^{-1},$$

$$a = -g = -10 \text{ ms}^{-2}$$

Using, $s = ut + \frac{1}{2} at^2$, we get

$$-35 = 30t - 5t^2$$

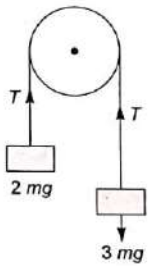
$$\Rightarrow \quad t^2 - 6t - 7 = 0$$




$t = -1, 7$ but $t$ cannot be –ve,

Hence, correct answer is $t = 7$ s.

**13.** (a)




For the monkey of mass $3m$

$$3mg - t = 3mf \qquad \ldots(i)$$

and for the monkey of mass $2m$

$$T - 2mg = 2mf \qquad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$mg = 5\,mf$$

∴ Acceleration, $f = \frac{g}{5}$ ms$^{-2}$

**14.** (b) PE at the point $B$ = TE

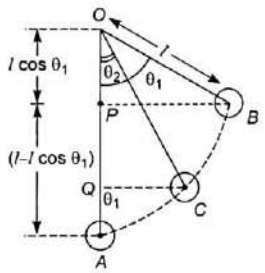
$= mg \times AP = mg(l - l\cos\theta_1)$ (∵ KE = 0 at *B*)

PE at the point $C = mg \times AQ = mg(l - l\cos\theta_2)$

If KE of the bob at the point *C* is *E*, then according to law of conservation of energy,

$E + mg(l - l\cos\theta_2) = mg(l - l\cos\theta_1)$

$E = mgl(\cos\theta_2 - \cos\theta_1)$ (Here $E$ = KE)




**15.** (a) $$\omega = \frac{d\theta}{dt} = \alpha - \beta t$$

or $$d\theta = (\alpha - \beta t)\, dt \quad \ldots(i)$$

When $\omega = 0, t = \frac{\alpha}{\beta}$

Now, integrating Eq. (i)

$$\int_0^{\theta} d\theta = \int_0^{t} (\alpha - \beta t)\, dt$$

$$\text{or } \theta = \alpha\,[t]_0^{\alpha/\beta} - \beta\left[\frac{t^2}{2}\right]_0^{\alpha/\beta} = \alpha \cdot \frac{\alpha}{\beta} - \beta\,\frac{\alpha^2}{2\beta^2}$$

$$= \frac{\alpha^2}{\beta} - \frac{\alpha^2}{2\beta} = \frac{\alpha^2}{2\beta}$$

**16.** (b) According to law of conservation of energy,

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mv_1^2 + 0$$

or $$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

or $$v_1^2 = v^2 - \frac{2GM}{R} = v^2 - v_e^2$$

$$\therefore \quad v_1 = \sqrt{v^2 - v_e^2} = \sqrt{4gR - 2gR} = \sqrt{2gR}$$

**17.** (c) According to Newton's law of cooling,
Rate of cooling ∝ Temperature difference of the body and its surroundings.
As the temperature of the body approaches the room temperature, rate of cooling decreases.

$$\therefore \quad t_1 < t_2 < t_3$$

**18.** (c) Initial velocity of the particle is along positive $X$-axis and force is also acting along positive $X$-axis for, $t = 1$ to $t = 4$ s, so particle will never come to rest upto $t = 4$ s, the velocity of the particle will change but then onwards, velocity becomes constant.
Now, from impulse-momentum theorem,

$$mv_f - mv_i = \int F\, dt$$

$$\Rightarrow \quad 2 \times v_f - 2 \times 5 = \frac{1}{2} \times 20 \times 3$$

$$\Rightarrow \quad v_f = 20 \text{ m/s}$$

and from work energy theorem,

$$\frac{mv_f^2}{2} - \frac{mv_i^2}{2} = \text{Work done by force } (W)$$

$$\Rightarrow \quad W = \frac{2 \times 20^2}{2} - \frac{2 \times 5^2}{2} = 375 \text{ J}$$

**19.** (b) Change in pressure due to placing of mass on piston is,

$$\Delta p = \frac{Mg}{A}$$

From bulk modulus definition,

$$K = \frac{-dp}{dV/V}$$

$$\Rightarrow \quad \left|\frac{dV}{V}\right| = \frac{\Delta p}{K} = \frac{Mg}{AK}$$

From $$\frac{dV}{V} = 3\frac{dR}{R} \qquad \left[\because V = \frac{4}{3}\pi R^3\right]$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{1}{3}\frac{dV}{V} = \frac{Mg}{3AK}$$

**20.** (c) Potential energy at a distance $r$ from the centre of earth is,

$$U = -\int_{\infty} -F\, dr$$

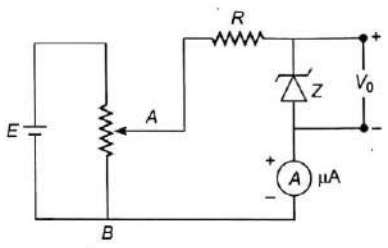
where, $F$ is the gravitational froce

$$F = \frac{GMm}{r^{5/2}}$$

$$U = \int_{\infty} \frac{GMm}{r^{5/2}}\, dr = -\frac{2}{3}\frac{GMm}{r^{3/2}}$$

*i.e.*, $$U \propto r^{-3/2}$$

**21.** (b) The circuit used for working Zener diode is as shown. Once the diode attains the breakdown voltage, there is no change in voltage across the diode even if we change the current in circuit by changing the position of rheostat and that's why the voltage across Zener diode is constant.




**22.** (d) For a hydrogen-like atom,

$$\text{TE} = -\frac{Ze^2}{4\pi\varepsilon_0 \times 2r}$$

$$\text{KE} = +\frac{Ze^2}{8\pi\varepsilon_0 r}$$

$$\text{PE} = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

As electron moves from lower energy state to higher energy state, its TE increases, and hence KE decrease and PE becomes less negative, *i.e.*, it increases. (Note that speed also decreases)

**23.** (a) The slope of $x$-$t$ graph at $t = 0$ is positive *i.e.*, initial velocity of the particle is +ve and $x$-$t$ graph

is concave down, so it means acceleration is negative but whether it is constant or not it can not be predicted from given information. As $v$ and acceleration are in opposite directions, the motion is the retarded one and finally the particle stops.

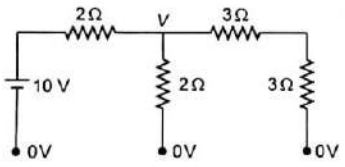
**24.** (c) This is a direct question based on concept of pure rolling. For no slipping relative velocity of point of contact has to be zero.
Here, velocity of the bottom-most point on cylinder is $v_0 - R\omega$, and velocity of platform is $v$. So, from definition of pure rolling $(v_0 - R\omega) - v = 0$.
Then, for the sphere

$$v = v_0 - R\omega.$$

**25.** (c) Apply Kirchhoff's current law at junction having potential $V$.

$$\frac{V-10}{2} + \frac{V-0}{6} + \frac{V-0}{2} = 0$$

$$\Rightarrow \quad V = \frac{30}{7} \text{ volt}$$




Both 3Ω resistors are connected in series, so the required current, $i = \frac{V-0}{6} = \frac{5}{7}$ A

**26.** (c) The charge distribution on the inner surface of cylinder (cavity) is non-uniform, so **E** near $A$ is not equal to **E** near $B$ in the cavity.
The conductor is an equipotential surface, so potentials at $A$ and $B$ are same.
Charge density depends upon radius of curvature.

**27.** (a) Power transferred to generator = 1500 × 746 W
Rate at which electrical energy is produced by generator is,

$$P = 0.8 \times 1500 \times 746 \text{ W}$$

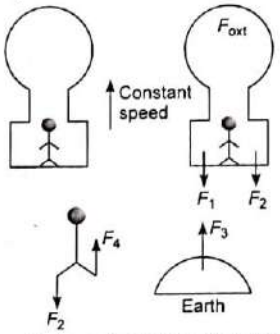
Let generator supplies current $I$, when potential difference is 2000 V, then

$$P = VI$$

$$\Rightarrow \quad I = \frac{0.8 \times 1500 \times 746}{2000}$$

$$= 447.6 \text{ A}$$

**28.** (d) Free body diagram of various components of the system is as shown in figure.




Since, balloon is moving at constant speed, net force on balloon and student must be zero. On balloon some other force must be acting which is not mentioned. For equilibrium of the student.

$$\mathbf{F}_2 + \mathbf{F}_4 = 0$$

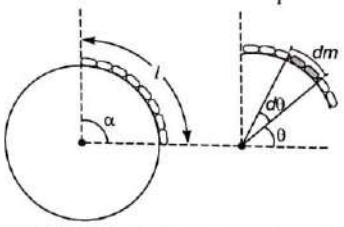
$$\Rightarrow \quad \mathbf{F}_2 = -\mathbf{F}_4$$

Relation between, $F_1$ and $F_2$ cannot be known.

**29.** (c) As no external force in acting on (compartment + passengers) system, so $c_2$ remains fixed. But as passengers move inside the compartment, friction force between compartment and passengers feet acting on compartment displaces its centre of mass.

**30.** (b) Here, $\alpha = \frac{l}{R}$

Consider the element of the chain as shown in the figure below, its mass is, $dm = \frac{m}{l} \times Rd\theta$.




Its PE w.r.t. horizontal diagram as the reference position is,

$$dU = dmg \times R \sin\theta$$

$$\Rightarrow \quad dU = \frac{mR^2g}{l} \sin\theta d\theta$$

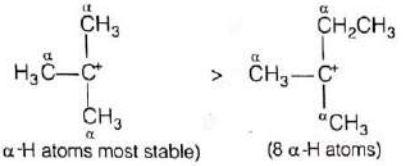
$$U = \int dU = \int_{\pi/2-\alpha}^{\pi/2} \frac{mR^2g}{l} \sin\theta d\theta$$

$$= \frac{mR^2g}{l} \sin\alpha$$

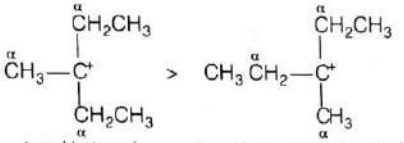
$$= \frac{mR^2g}{l} \sin\left(\frac{l}{R}\right)$$

# Chemistry

**31.** (c) The stability is determined by hyperconjugation. More the number of α-hydrogen atoms, more is the stability.



(9 α-H atoms most stable) (8 α-H atoms)



(7 α-H atoms) (6 α-H atoms least stable)

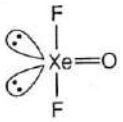
**32.** (a)

Both rings becomes aromatic after breaking of π-bond. So, it will allow free rotation because of the breaking of π-bond.

**33.** (d) $$E_n = E_0 \frac{Z^2}{n^2}$$

$$\frac{E_H}{E_{Be^{3+}}} = \frac{1}{(1)^2} \times \frac{1}{(4)^2} = \frac{1}{16}$$

**34.** (c) In $XeOF_2$ there are 3 bond pairs and 2 lone pair of electrons. So, the hybridisation is $sp^3d$ and structure should be trigonal bipyramidal. But its actual geometry is T-shaped because of the presence of two lone pairs which occupy equatorial position.



**35.** (d) Let the mass of ethane = hydrogen = $x$ g

$$\therefore \quad \text{Moles of ethane} = \frac{x}{30}$$

$$\text{and moles of hydrogen} = \frac{x}{2}$$

$$\therefore p_{H_2} = \frac{\frac{x}{2}}{\frac{x}{2} + \frac{x}{30}} \times p_{total} = \frac{1}{2} \times \frac{30}{16} p = \frac{15}{16} p$$

**36.** (b) Eq. (ii) – Eq. (i)

This gives $\Delta H = 2x - y$

**37.** (c) $H_2S_2O_7 + H_2O \longrightarrow 2H_2SO_4$

Wt. of oleum sample = 740 – 60 = 680 g

To 680 g of $H_2S_2O_7$ 60 g of water is added

$\therefore$ To 100 g of $H_2S_2O_7$ water acidic

$$= \frac{60}{680} \times 100 = 8.82 \text{ g}$$

$\therefore$ Strength of oleum = 100 + 8.82 = 108.82%

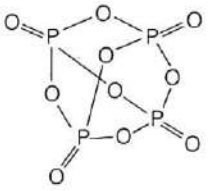
**38.** (b) $$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$\frac{p}{260} = \frac{p_2}{286}$$

$$1{\cdot}1p = p_2$$

$$\therefore \text{ \% increase } = \frac{1.1p - p}{p} = \frac{0.1p}{p} \times 100 = 10\%$$

**39.** (a) There is no peroxide linkage in $P_4O_{10}$.



**40.** (c) For first order reaction

$$2A(g) \longrightarrow 3B(g)$$

$$640-2x \quad 3x$$

$$\therefore \quad p_{Total} = 640 + x$$

$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{36} \log \frac{640}{640-2x}$$

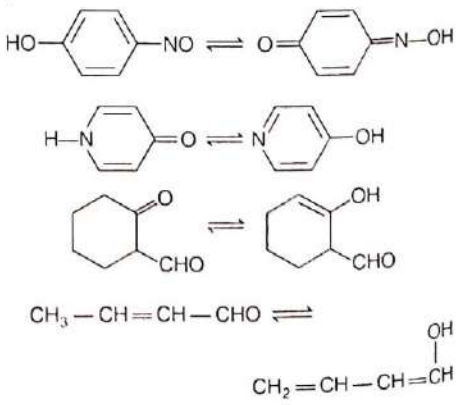
$$\frac{2.303 \log 2}{12} = \frac{2.303}{36} \log \frac{640}{640-2x}$$

$$\log 2^3 = \log \frac{640}{640-2x}$$

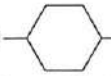
$$x = 280$$

$$\therefore \quad p_{Total} = 640 + 280 = 920 \text{ mm}$$

**41.** (d)



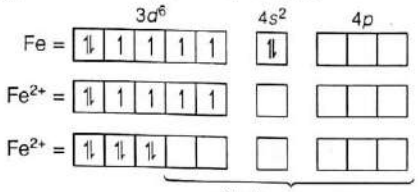
**42.** (b) $K_2CrO_4 + Ba^{2+} \longrightarrow \underset{\text{Yellow ppt.}}{BaCrO_4 \downarrow} + 2K^+$

**43.** (d)  contains plane of symmetry.

Hence, achiral.

**44.** (a) In $[Fe(CN)_6]^{4-}$, Fe is in +2 oxidation state.

$_{26}Fe - 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^6, 4s^2$




($CN^-$ is a strong field ligand and causes pairing of electrons.)

Therefore, $[Fe(CN)_6]^{4-}$ complex ion is inner-orbital and diamagnetic (as all electrons are paired).

**45.** (c) $p_{Total} = p_A^\circ X_A + p_B^\circ X_B$

$= 100 \times \frac{1}{5} + 200 \times \frac{4}{5}$

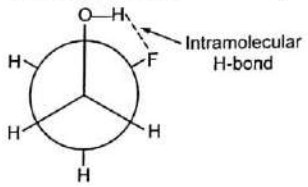
$= 20 + 160 = 180$

$\therefore \quad p_{distillate} = 100 \times \frac{20}{180} + 200 \times \frac{160}{180}$

$= 188.88$ torr

**46.** (a) The reaction takes place by $S_N2$ mechanism *i.e.*, involves inversion of configuration.

**47.** (c) Due to Intramolecular H-bonding.




**48.** (c) $Ca + 2H_2O \longrightarrow Ca(OH)_2 + H_2$

$CaH_2 + 2H_2O \longrightarrow Ca(OH)_2 + 2H_2$

whereas K, Na and Ba give $H_2$ gas and $Na_2O_2$ and $BaO_2$ give hydrogen peroxide, $KO_2$ gives $O_2$ and $H_2O_2$ on reaction with water.

**49.** (a)

| | $2A(g)$ | + | $B(g)$ | $\rightleftharpoons$ | $C(g)$ | + | $D(g)$ |
|---|---|---|---|---|---|---|---|
| Initial moles | 1 | | 1 | | 7 | | 3 |
| At equi. | $2-2x$ | | $1-x$ | | $7+x$ | | $3+x$ |

$\therefore$ Due to very high value of $K$, we can assume that reactant almost gets converted into products, so

$1 - x = y$

$2 - 2x = 2y \Rightarrow x \sim 1$

$\therefore \quad 10^{12} = \frac{[C][D]}{[A]^2[B]}$

$10^{12} = \frac{8 \times 4}{(2y)^2 (y)}$

$y^3 = 8 \times 10^{-12}$

$\Rightarrow \quad y = 2 \times 10^{-4}$

$\therefore$ Equilibrium concentration of $A = 2y$

$= 2 \times 2 \times 10^{-4} = 4 \times 10^{-4}$

**50.** (c) Moles of $Zn = \frac{100}{65}$

$= 1.53$ mol

1 L of 1M $CuSO_4$ solution contains 1 mol of $CuSO_4$.

$\therefore$ In $Zn + CuSO_4 \longrightarrow ZnSO_4 + Cu$

$CuSO_4$ is the limiting reagent.

$\therefore$ To deposit completely 1 mole of Cu,

Electricity required = $2 \times 96500$ C

$t = \frac{Q}{I} = \frac{2 \times 96500}{1}$ s

$= \frac{2 \times 96500}{3600}$ hr

$= 53.6$ hr

**51.** (a) Number of $Na^+ = 4 - 1 = 3$

Number of $Cl^- = 4 - 2 \times \frac{1}{2} = 3$

**52.** (c) In electro-osmosis, the sol. particles are prevented from migration whereas the dispersion medium migrates in the direction opposite to those of the particles. Here, the medium is negatively charged.

**53.** (b) $[HCl] = \frac{0.01}{0.1} = 0.1$ M

$pOH = 4.74 + \log\left(\frac{0.3 + 0.1}{0.2 - 0.1}\right) = 5.34$

$pH = 14 - 5.34 = 8.66$

**54.** (c) $pH = pK_a + \log \frac{[base]}{[acid]}$

$= 4.8 + \log \frac{(0.15 + 0.05)}{(0.15 - 0.05)}$

$= 4.8 + \log 2$

$= 4.8 + 0.30 = 5.1$

**55.** (b) $BOH + HCl \rightleftharpoons BCl + H_2O$

At equi. point $N_1V_1 = N_2V_2$

$[BCl] = \frac{20 \times 0.08}{20 + 20} = 0.04$

$pH = 7 - \frac{1}{2} pK_b - \frac{1}{2} \log C$

$\therefore \quad pK_b = 5.4$

$$B^+ + OH^- \longrightarrow BOH,$$

| | | | |
|---|---|---|---|
| Initial | 1.6 | 0.4 | |
| Final moles | 1.2 | – | 0.4 |

Basic buffer is formed.

$$pOH = pK_b + \log \frac{[B^+]}{[BOH]}$$

$$= 5.4 + \log\left(\frac{1.2}{0.4}\right)$$

$$= 5.4 + 0.48$$

$$pOH = 5.88$$

**Solutions** (Q. Nos. 56-58)

Let total moles of liquid taken initially = $x$

Condensate = $\frac{3}{4}x$ and Residue = $\frac{1}{4}x$

Moles of $A$ in condensate = $0.75 \times \frac{3}{4}x = \frac{9}{16}x$

Moles of $A$ in residue = $0.3 \times \frac{1}{4} = \frac{3}{40}x$

Total moles of $A = \left(\frac{9}{16} + \frac{3}{40}\right)x$

Mole fraction of $A$ in original sample

$$= \frac{51}{80} = 0.6375$$

(Neglecting moles of vapour as very small)

Total pressure over original solution

= 700 mm Hg

$$700 = p_A^\circ\left(\frac{51}{80}\right) + p_B^\circ\left(\frac{29}{80}\right) \quad \ldots(i)$$

Total pressure above residue = 600 mm Hg

$$600 = p_A^\circ\left(\frac{3}{10}\right) + p_B^\circ\left(\frac{7}{10}\right) \quad \ldots(ii)$$

From (i), $56000 = 51\,p_A^\circ + 29\,p_B^\circ$

From (ii), $[6000 = 3p^\circ + 7p_B^\circ \times 17$

$$102000 = 51\,p_A^\circ + 119\,p_B^\circ$$

$\therefore \quad 46000 = 90\,p_B^\circ$

$p_B^\circ = 511.11$ mm of Hg

Putting value of $p_B^\circ$ in (i)

$$700 = p_A^\circ\left(\frac{51}{80}\right) + \frac{4600}{9} \times \frac{29}{80}$$

$$p_A^\circ = \left[700 - \frac{4600}{9} \times \frac{29}{80}\right] \times \frac{80}{51}$$

= 807.41 mm of Hg

**56.** (a) **57.** (a) **58.** (b)

**59.** (d) In an electrolytic cell, flow of electrons is from cathode to anode through internal supply. In an electrolytic cell, cathode is the electron rich electrode.

**60.** (a) The internal energy of a system depends upon the amount and physical state of the substance, hence, it is an extensive property.

# Mathematics

**61.** (d) Given, $a + b + c + d + e = 15$

$$\frac{\left[a + \frac{b}{2} + \frac{b}{2} + \frac{c}{3} + \frac{c}{3} + \frac{c}{3} + \frac{d}{4} + \frac{d}{4} + \frac{d}{4} + \frac{d}{4} + \frac{e}{5} + \frac{e}{5} + \frac{e}{5} + \frac{e}{5} + \frac{e}{5}\right]}{15} = 1$$

$\Rightarrow \quad AM = 1$

Now, $GM = \left(a \times \frac{b^2}{2^2} \times \frac{c^3}{3^3} \times \frac{d^4}{4^4} \times \frac{e^5}{5^5}\right)^{1/15}$

$$= \left[\frac{(120)^3 \cdot 50}{2^2 \cdot 3^3 \cdot 4^4 \cdot 5^5}\right]^{1/15} = 1$$

$\therefore \quad AM = GM$

Hence, $a = \frac{b}{2} = \frac{c}{3} = \frac{d}{4} = \frac{e}{5}$

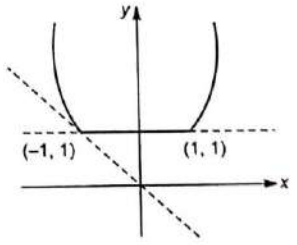
$\Rightarrow a = 1, b = 2, c = 3, d = 4, e = 5$

$\therefore \; a^2 + b^2 + c^2 + d^2 + e^2$

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 55$

**62.** (a) Now, we draw the graph of $y = -x$, $y = 1$ and $y = x^2$




From graph it is clear that $h(x)$ is continuous at all $x$ and it is not differentiable at $x = -1, 0, 1$.

**63.** (a) ${}^{100}C_0\,{}^{200}C_{100} - {}^{100}C_1\,{}^{199}C_{100} + {}^{100}C_2\,{}^{198}C_{100} - {}^{100}C_3\,{}^{197}C_{100} + \ldots + {}^{100}C_{100}\,{}^{100}C_{100}$

Coefficient of $x^{100}$ in

${}^{100}C_0(1+x)^{200} - {}^{100}C_1(1+x)^{199} + {}^{100}C_2(1+x)^{198} - \ldots + {}^{100}C_{100}(1+x)^{100}$

= Coefficient of $x^{100}$ in

$(1+x)^{100}\,[{}^{100}C_0\,(1+x)^{100} - {}^{100}C_1\,(1+x)^{99} + {}^{100}C_2\,(1+x)^{98} - \ldots + {}^{100}C_{100}]$

= Coefficient of $x^{100}$ in

$(1+x)^{100} \cdot x^{100} = 1$

**64.** (a) Let $A$ and $B$ are two given events. Then, the odds against $A$ are 5 : 2.

$\therefore \quad P(A) = \frac{2}{7}$

And the odds in favour of $B$ are 6 : 5.

$\therefore \quad P(B) = \frac{6}{11}$

$\therefore$ Required probability,

$$1 - P(\bar{A} \cap \bar{B}) = 1 - P(\bar{A})(\bar{B}) = 1 - \left(1 - \frac{2}{7}\right)\left(1 - \frac{6}{11}\right) = 1 - \left(\frac{5}{7}\right)\left(\frac{5}{11}\right) = \frac{77-25}{77} = \frac{52}{77}$$

**65.** (a) The given equation can be written as

$2\tan^2 x + \tan^4 x + \tan^4 x - 2\sec^2 x \tan^2 x - 3 = 0$

$\Rightarrow 2\tan^2 x + (\sec^2 x - 1)^2 + \tan^4 x - 2\sec^2 x \tan^2 x - 3 = 0$

$\Rightarrow 2\tan^2 x + \sec^4 x + 1 - 2\sec^2 x + \tan^4 x - 2\sec^2 x \tan^2 x - 3 = 0$

$\Rightarrow \quad (\sec^2 x - \tan^2 x)^2 = 4$

$\Rightarrow \quad 1 \neq 4$

Hence, no solution exists.

**66.** (a) The given curve can be written as

$(y^2+1)(x-1) = 0 \Rightarrow x = 1$

$(y^2+2)(x-3) = 0 \Rightarrow x = 3$

Hence, $f(1+x) = f(1-x)$ ...(i)

$f(3+x) = f(3-x)$ ...(ii)

From Eqs. (i) and (ii), we get

$f(x) = f(2-x)$

and $f(x) = f(6-x)$

$\Rightarrow \quad f(2-x) = f(6-x)$

$\Rightarrow \quad f(x+4)$

Hence, period is 4.

**67.** (a) $A_1, A_2, A_3, \ldots, A_{2009}$ and $H_1, H_2, \ldots, H_{2009}$ are AM and HM between $a$ and $b$.

Hence,

$A_1H_{2009} = A_2H_{2008} = \ldots = A_{1005}\,H_{1005} = \ldots = A_{2009}H_1$

$\therefore \quad \frac{A_{1006}\,H_{1004}}{A_{1004}\,H_{1006}} = 1$

**68.** (d) Since, Root mean square $\geq$ Arithmetic mean

$\therefore \quad \sqrt{\frac{\Sigma x_i^2}{n}} \geq \frac{\Sigma x_i}{n}$

$\Rightarrow \quad \sqrt{\frac{400}{n}} \geq \frac{80}{n}$

$\Rightarrow \quad \frac{20}{\sqrt{n}} \geq \frac{80}{n}$

$\Rightarrow \quad \sqrt{n} \geq 4$

$\Rightarrow \quad n \geq 16$

**69.** (d) Given, $|\mathbf{a} - \mathbf{c}| = |\mathbf{b} - \mathbf{c}|$

$\Rightarrow \quad |\mathbf{a} - \mathbf{c}|^2 = |\mathbf{b} - \mathbf{c}|^2$

$|\mathbf{a}|^2 + |\mathbf{c}|^2 - 2\mathbf{a}\cdot\mathbf{c} = |\mathbf{b}|^2 + |\mathbf{c}|^2 - 2\mathbf{b}\cdot\mathbf{c}$

$\Rightarrow 2\mathbf{b}\cdot\mathbf{c} - (|\mathbf{b}|^2 - |\mathbf{c}|^2) - 2\mathbf{a}\cdot\mathbf{c} = 0$

$\Rightarrow \mathbf{b}\cdot\mathbf{c} - \frac{1}{2}(|\mathbf{b}|^2 - |\mathbf{c}|^2) - \mathbf{a}\cdot\mathbf{c} = 0$ ...(i)

$\therefore \quad (\mathbf{b}-\mathbf{a})\cdot\left(\mathbf{c} - \frac{\mathbf{a}+\mathbf{b}}{2}\right) = \mathbf{b}\cdot\mathbf{c} - \mathbf{b}\cdot\left(\frac{\mathbf{a}+\mathbf{b}}{2}\right) - \mathbf{a}\cdot\mathbf{c} + \frac{\mathbf{a}}{2}\cdot(\mathbf{a}+\mathbf{b})$

$= \mathbf{b}\cdot\mathbf{c} - \frac{1}{2}(|\mathbf{b}|^2 - |\mathbf{a}|^2) - \mathbf{a}\cdot\mathbf{c}$ [from Eq. (i)]

$= 0$

**70.** (d) $3\sqrt{[32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1]^2} = \sin x$

$= 3\sqrt{[2(16\cos^6 x - 24\cos^4 x + 9\cos^2 x) - 1]^2}$

$= 3\sqrt{2(4\cos^3 x - 3\cos x)^2 - 1}$

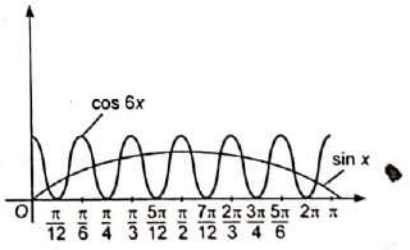
$= 3\sqrt{2\cos^2 3x - 1}$

$3\sqrt{\cos^2 6x} = \sin x$

$3|\cos 6x| = \sin x$

$6,\ x \in \left[0, \frac{\pi}{2}\right]$

$12,\ x \in [0, \pi]$




Clearly, number of solutions

$= 6,\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= 12,\ x \in [-\pi, \pi]$

**71.** (c) Given, $\int \frac{12x^3 - 1}{x\sqrt{100x^2 - 36x^6 - 12x^3 - 1}}\,dx$

$= a \sin^{-1}\{f(x)\} + C$

$\Rightarrow \int \frac{6x - \frac{1}{2x^2}}{\sqrt{25 - \left(3x^2 + \frac{1}{2x}\right)^2}}\,dx = a \sin^{-1}\{f(x)\} + C$

$= \int \frac{dt}{\sqrt{(5)^2 - t^2}} = \sin^{-1}\left(\frac{t}{5}\right) + C$

$\left(\text{let } t = 3x^2 + \frac{1}{2x}\right)$

$\Rightarrow \quad \sin^{-1}\left(\frac{3x^2 + \frac{1}{2x}}{5}\right) + C = a \sin^{-1}[f(x)] + C$

$\therefore \quad a = 1, f(x) = \frac{1}{5}\left(3x^2 + \frac{1}{2x}\right)$

**72.** (c) Contrapositive of $(p \vee q) \to r$ is

$\sim r \to \sim (p \vee q) \equiv \sim r \to (\sim p \wedge \sim q)$

**73.** (b) The equation of the normal at $(x_1, y_1)$ to the given ellipse is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

We know that coordinate of the latus rectum at positive end is $\left(ae, \frac{b^2}{a}\right)$.

Then, $x_1 = ae$ and $y_1 = \frac{b^2}{a}$

Therefore, the equation of the normal at positive end of the latusrectum is

$$\frac{a^2x}{ae} - \frac{b^2y}{b^2/a} = a^2e^2 \quad [\because b^2 = a^2(1 - e^2)]$$

$\Rightarrow \quad \frac{ax}{e} - ay = a^2e^2$

$\Rightarrow \quad x - ey - e^3a = 0$

**74.** (d) Use $-1 \le \sin x \le 1$ and $0 \le |\cos x| \le 1$

and $-\sqrt{a^2 + b^2} \le a \cos x + b \sin x \le \sqrt{a^2 + b^2}$

**75.** (b) $f(x) = [4^x - 2^{x+1} + 1]$, Let $2^x = t$

$= [t^2 - 2t + 1]$

$= [(t - 1)^2] = [(2^x - 1)^2]$

If $x = 0$, then $f(x) = 0$

Otherwise, $(2^x - 1)^2$ is always greater than zero, so range is $x \in \{0, 1, 2, 3, \ldots\}$ *i.e.*, the set of the whole numbers.

**76.** (a) Let $I = \int_0^{\pi/2} \frac{a + b \cos x}{(b + a \cos x)^2}\,dx$

$= \int_0^{\pi/2} \frac{a \operatorname{cosec}^2 x + b \cot x \operatorname{cosec} x}{(b \operatorname{cosec} x + a \cot x)^2}\,dx$

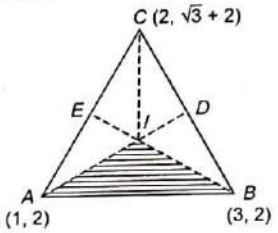
($\because$ divide Nr and Dr by $\sin^2 x$)

Let $b \operatorname{cosec} x + a \cot x = t$

$\Rightarrow \quad (-a \operatorname{cosec}^2 x - b \operatorname{cosec} x \cot x)\,dx = dt$

$\therefore \quad I = -\int_{\infty}^{b} \frac{1}{t^2}\,dt = \left(\frac{1}{t}\right)_{\infty}^{b} = \frac{1}{b}$

**77.** (c) $\Delta ABC$ is an equilateral triangle, $AB = BC = CA = 2$ units. $AD, BE$ are internal angle bisectors meet at a point I. Then,




Point $P$ will inside the triangle $IAB$

Area of $\Delta IAB = \frac{1}{3}\Delta ABC$

$= \frac{1}{3} \times 4 \times \frac{\sqrt{3}}{4} = \frac{1}{\sqrt{3}}$ unit

**78.** (a) $z_1$ lies on the line segment joining the points 8 and $6i$, equation of the line is $\frac{x}{8} + \frac{y}{6} = 1$. ...(i)

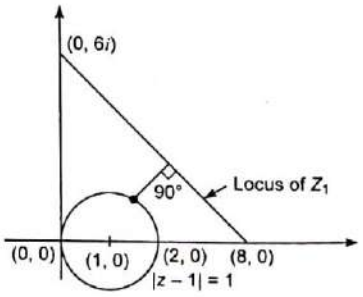
Slope is $-\frac{6}{8} = -\frac{3}{4}$.

A line is perpendicular to the line $\frac{x}{8} + \frac{y}{4} = 1$ and passing through the point (1, 0) is

$$y - 0 = \frac{4}{3}(x - 1)$$

$\Rightarrow \quad 4x - 3y = 4$ ...(ii)

Point of intersection of lines (i) and (ii) is $\left(\frac{88}{25}, \frac{84}{25}\right)$.




Min value of $|z - z_1|$ is

$= \sqrt{\left(1 - \frac{88}{25}\right)^2 + \left(0 - \frac{84}{25}\right)^2} - 1$

$= \sqrt{\left(\frac{63}{25}\right)^2 + \left(\frac{84}{25}\right)^2} - 1$

$$= \frac{\sqrt{11025}}{25} - 1 = \frac{105}{25} - 1$$

$$= \frac{80}{25} = \frac{16}{5}$$

**79.** (c) Let $P = (a, b, c)$

$M \to (a, b, 0), N \to (0, b, c)$

DR's of $OP = (a, b, c)$

DR's of perpendicular vector to $xy$-plane is $(0, 0, 1)$.

$$\sin 30° = \frac{a \times 0 + b \times 0 + c \times 1}{\sqrt{a^2 + b^2 + c^2}\sqrt{0^2 + 0^2 + 1^2}}$$

$$\sin 30° = \frac{c}{\sqrt{a^2 + b^2 + c^2}} \quad ...(i)$$

and similarly, $\sin 45° = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$ ...(ii)

and $\sin 60° = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$ ...(iii)

Plane $OMN$ passes through $(0, 0, 0), (a, b, 0)$ and $(0, b, c)$.

$A(x - 0) + B(y - 0) + C(z - 0) = 0$

$$Aa + Bb = 0$$

$$\Rightarrow \quad \frac{A}{B} = -\frac{b}{a}$$

and $Bb + Cc = 0$

$$\Rightarrow \quad \frac{B}{C} = -\frac{c}{b}$$

Let $A = -bk, B = ak,$

$$C = \frac{abk}{-c}$$

$$\Rightarrow -bx + ay - \frac{abz}{c} = 0$$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{b} + \frac{z}{c} = 0$$

DR's of perpendicular vector $= (-bc, ac, -ab)$

$$\therefore \quad \sin\theta = \frac{a(-bc) + b(ac) - a(bc)}{\sqrt{a^2 + b^2 + c^2}\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}$$

$$\Rightarrow \sin^2\theta = \frac{a^2b^2c^2}{(a^2b^2 + b^2c^2 + c^2a^2)(a^2 + b^2 + c^2)}$$

$$\Rightarrow \text{cosec}^2\theta = \frac{(a^2b^2 + b^2c^2 + c^2a^2)}{(a^2 + b^2 + c^2)a^2b^2c^2} \quad ...(iv)$$

Taking reciprocal of Eqs. (i), (ii) and (iii)

$$\frac{\sqrt{a^2 + b^2 + c^2}}{c} = \text{cosec } 30°$$

$$\frac{\sqrt{a^2 + b^2 + c^2}}{b} = \text{cosec } 45°$$

$$\frac{\sqrt{a^2 + b^2 + c^2}}{a} = \text{cosec } 60°$$

$$\Rightarrow \quad \text{cosec}^2 30° + \text{cosec}^2 45° + \text{cosec}^2 60°$$

$$= (a^2 + b^2 + c^2)\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)$$

$$\Rightarrow \quad 4 + 2 + \frac{4}{3} = \text{cosec}^2\theta \qquad \text{[from Eq. (iv)]}$$

$$\Rightarrow \quad 3\,\text{cosec}^2\theta = 22$$

**80.** (c) CALCULUS

□ □ □ □ □ □ □ □

All C's occur before all L's is

$${}^8C_4 = \frac{8!}{4!\,4!} = \frac{8 \times 7 \times 6 \times 5}{24} = 70$$

Now, there are four places remaining. These places are filled by the letters ($A, U, U, S$). So, number of ways of filling those four places

$$= \frac{4!}{2!} = 12$$

Total words $= 70 \times 12 = 840$

**81.** (a) Let $ABCD = E$

$$|\text{adj}(\text{adj}(\text{adj}(\text{adj}\,E)))|$$

$$= |E|^{(3-1)^4} = |E|^{16} = |ABCD|^{16}$$

$$= |A|^{16}|B|^{16}|C|^{16}|D|^{16}$$

$$= |A|^{16}(|A|^2)^{16}(|A|^4)^{16}(|A|^8)^{16}$$

$$= |A|^{16+32+64+128}$$

$$|A|^k = |A|^{240}$$

$$\Rightarrow \quad k = 240$$

**82.** (b) Since, $Z$ is idempotent, then

$$Z^2 = Z$$

$$\Rightarrow \quad Z^3, Z^4, \ldots, Z^n = Z \quad ...(i)$$

$$\therefore \quad (I + Z)^n = {}^nC_0 I^n + {}^nC_1 I^{n-1}Z + {}^nC_2 I^{n-2} \cdot Z^2 + \ldots + {}^nC_n Z^n$$

$$= I + ({}^nC_1 Z + {}^nC_2 Z + \ldots + {}^nC_n Z)$$

[from Eq. (i)]

$$= I + ({}^nC_1 + {}^nC_2 + \ldots {}^nC_n)Z$$

$$= I + (2^n - 1)Z$$

**83.** (a) $(1 + \tan 1°)(1 + \tan 2°)(1 + \tan 3°) \ldots (1 + \tan 45°) = 2^n$

$[(1 + \tan 1°)(1 + \tan 44°)][(1 + \tan 2°)(1 + \tan 43°)] \ldots [(1 + \tan 22°)(1 + \tan 23°)].(1 + \tan 45°)$

$$\left[\because (1 + \tan 1°)(1 + \tan 44°)\right.$$
$$= 1 + \tan 1° + \tan 44° + \tan 1° \tan 44°$$
$$= 1 + \tan 1° \tan 44° + (\tan 1° + \tan 44°)$$
$$\left.= 1 + \tan 45° = 2\right]$$

$$= \underbrace{2.2.2.\ldots.2.2}_{22 \text{ times}}.(2) = 2^{23}$$

$\Rightarrow \quad . \quad n = 23$

$\sin a + 7 \sin b = 4 \sin c + 8 \sin d$

$\sin a - 8 \sin d = 4 \sin c - 7 \sin b \quad \ldots(i)$

and second equation is

$\cos a - 8 \cos d = 4 \cos c - 7 \cos b \quad \ldots(ii)$

On Squaring and adding Eqs. (i) and (ii), we get

$1 + 8^2 - 16(\sin a \sin d + \cos a \cos d)$

$= 16 + 49 - 56(\sin b \sin c + \cos b \cos c)$

$\Rightarrow \quad -16 \cos(a - d) = -56 \cos(b - c)$

$\Rightarrow \quad \frac{\cos(b-c)}{\cos(a-d)} = \frac{2}{7}$

$\Rightarrow \quad \frac{7\cos(b-c)}{\cos(a-d)} = 2$

**84.** (b) The common terms of the given series are 5, 11, 17, ...

$\therefore$ $n$th term of common sequence

$T_n = 5 + (n-1)6 = 6n - 1$

Also, 100th term of the first sequence

$= 2 + (100 - 1)3 = 299$

And 100th term of the second sequence

$= 3 + (100 - 1)2 = 201$

$\Rightarrow \quad T_n \le 201$

$\Rightarrow \quad 6n - 1 \le 201$

$\Rightarrow \quad n \le 33\frac{2}{3}$

$\Rightarrow \quad n = 33$

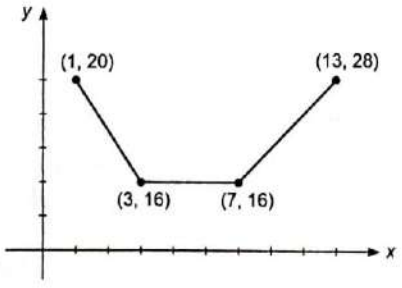
**85.** (b) $f(x) = |x-1| + |x-3| + |x-7| + |x-13|$

at $x = 1$, $f(x) = 20$

at $x = 3$, $f(x) = 16$

at $x = 7$, $f(x) = 16$

at $x = 13$, $f(x) = 28$




The min value of $f(x)$ is 16.

**86.** (d) $f(x) = 16$ has infinitely many solutions.

**87.** (a) Put $x = \sin\theta$

$$I = \int_0^{\pi/2} \frac{\cos\theta}{\cos\theta(\sqrt{\cos\theta} + \sqrt{\sin\theta})^4} d\theta$$

$$= \int_0^{\pi/2} \frac{1}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^4} d\theta$$

$$= \int_0^{\pi/2} \frac{1}{\cos^2\theta(1 + \sqrt{\tan\theta})^4} d\theta$$

$$= \int_0^{\pi/2} \frac{\sec^2\theta}{(1 + \sqrt{\tan\theta})^4} d\theta$$

Let $1 + \sqrt{\tan\theta} = z$

$\Rightarrow \quad \frac{1}{2\sqrt{\tan\theta}} \sec^2\theta \, d\theta = dz$

$$I = \int_1^{\infty} \frac{2(z-1)}{z^4} dz$$

$$= 2\int_1^{\infty} (z^{-3} - z^{-4}) dz$$

$$= 2\left[\frac{1}{-2z^2} + \frac{1}{3z^3}\right]_1^{\infty}$$

$$= 2\left[0 - \left(-\frac{1}{2} + \frac{1}{3}\right)\right]$$

$$= \frac{1}{3}$$

**88.** (c) $f(x) = (x+1)(\tan x)^x . \sec^2 x + \log_e(\tan x)(\tan x)^{x+1} \quad \ldots(i)$

Let $g(x) = (\tan x)^{x+1}$

$\log_e g(x) = (x+1)\log_e(\tan x)$

$\frac{1}{g(x)} g'(x) = (x+1)\frac{1}{\tan x}\sec^2 x + \log_e(\tan x).1$

$\Rightarrow \quad g'(x) = (\tan x)^{x+1} \left\{\frac{x+1}{\tan x} \cdot \sec^2 x + \log_e(\tan x)\right\}$

$\Rightarrow \quad g'(x) = (\tan x)^x (x+1)\sec^2 x + \log_e(\tan x)(\tan x)^{x+1}$

$\Rightarrow \quad g'(x) = f(x)$ [from Eq. (i)]

$\Rightarrow \quad \int f(x)\,dx = g(x)$

So, Mean value $= \frac{\int_0^{\pi/4} f(x)\,dx}{\frac{\pi}{4} - 0} = \frac{[g(x)]_0^{\pi/4}}{\frac{\pi}{4}}$

$$= \frac{[(\tan x)^{x+1}]_0^{\pi/4}}{\frac{\pi}{4}}$$

$$= \frac{(1)^{\frac{\pi}{4}+1} - (0)^{0+1}}{\frac{\pi}{4}} = \frac{4}{\pi}$$

**89.** (d) I. Given, $\sin^2 x + \sin x = 1$

$\Rightarrow \quad \sin x = 1 - \sin^2 x = \cos^2 x \quad ...(i)$

Now, the given expression is

$\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$

$= \cos^6 x\,[\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1] - 1$

$= \cos^6 x\,(\cos^2 x + 1)^3 - 1$

$= \sin^3 x\,(\sin x + 1)^3 - 1 \quad$ [from Eq. (i)]

$= (\sin^2 x + \sin x)^3 - 1$

$= (1)^3 - 1 = 1 - 1 = 0$

II. $\sin^2 x + \sin x - 1 = 0$

Now, $\sin x = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$= -\dfrac{-1 \pm \sqrt{5}}{2} = \dfrac{-1+\sqrt{5}}{2}$

$\therefore \quad \cos^2 x = 1 - \sin^2 x = 1 - \left(\dfrac{\sqrt{5}-1}{2}\right)^2$

$= 1 - \left(\dfrac{5+1-2\sqrt{5}}{4}\right)$

$= \dfrac{-2+2\sqrt{5}}{4}$

$= \dfrac{\sqrt{5}-1}{2}$

**90.** (a) I. $\tan(B+C) = \tan(\pi - A)$

$= \tan A = \dfrac{\tan B + \tan C}{\tan B \tan C - 1} < 0$

($\because A$ is obtuse)

$\therefore B$ and $C$ are acute, $\tan B + \tan C > 0$

$\because \quad \tan B \tan C - 1 < 0$

$\Rightarrow \quad \tan B \tan C < 1$

II. $\tan(A+B+C)$

$= \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$= \tan 180° = 0$

$\Rightarrow \quad \tan A + \tan B + \tan C = \tan A \tan B \tan C$

# JEE Main

## Joint Entrance Examination

# Practice Set 5

## nstructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

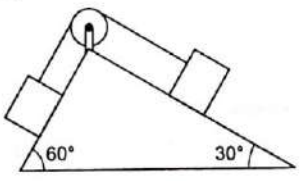
1. A cricketer can throw a ball to a maximum horizontal distance of 100 m with the same efforts, he throws the ball vertically upwards. What is the maximum height attained by the ball?

   (a) 50 m (b) 60 m
   (c) 70 m (d) 100 m

2. 80 railway wagons all of same mass $5 \times 10^3$ mg are pulled by an engine with a force of $4 \times 10^5$ N. The tension in the coupling between 30th and 31st wagon from the engine is

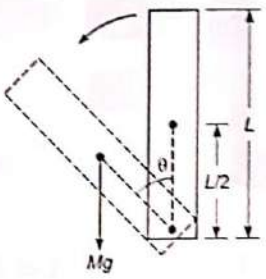
   (a) $25 \times 10^4$ N (b) $20 \times 10^4$ N
   (c) $32 \times 10^4$ N (d) $40 \times 10^4$ N

3. Two blocks of equal mass are tied with a light string which passes over a massless pulley as shown in the figure. The magnitude of acceleration of centre of mass of both the blocks is (neglect friction everywhere)




(a) $\frac{\sqrt{3}-1}{4\sqrt{2}}g$ (b) $(\sqrt{3}-1)g$ (c) $\frac{g}{2}$ (d) $\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)g$

4. A uniform rod of mass $M$ and length $L$ is pivoted at one end such that it can rotate in a vertical plane. There is negligible friction at the pivoting end. The free end of the rod is held vertically above the pivot and then released. The angular acceleration of the rod, when it makes an angle $\theta$ with the vertical is




(a) $g \sin\theta$ (b) $\frac{g}{L}\sin\theta$

(c) $\frac{3g}{2L}\sin\theta$ (d) $6gL\sin\theta$

5. The gravitational potential due to earth at infinite distance from it is zero. Let the gravitational potential at a point $P$ be $-5$ J/kg. Suppose, we arbitrarily assume the gravitational potential at infinity to be +10 J/kg, then the gravitational potential at $P$ will be

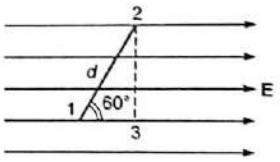
(a) −5 J/kg (b) +5 J/kg (c) −15 J/kg (d) +15 J/kg

6. A piece of ice is floating in a beaker containing water. When ice melts, the temperature falls from 20°C to 4°C and the level of water

(a) remains unchanged (b) falls

(c) rises (d) changes erratically

7. The electric field **E** between two points is constant in both magnitude and direction. Consider a path of length $d$ at an angle, $\theta = 60°$ with respect to field lines shown in figure. The potential difference between points 1 and 2 is




(a) $\frac{E}{d\cos 60°}$ (b) $Ed\cos 60°$

(c) $\frac{Ed}{\cos 60°}$ (d) $\frac{E}{d}\cos 60°$

8. Between the plates of a parallel plate capacitor of capacity $C$, two parallel plates, of the same material and area same as the plates of the original capacitor, are placed. If the thickness of these plates is equal to $\frac{1}{5}$ th of the distance between the plates of the original capacitor, then the capacity of the new capacitor is

(a) $\frac{5}{3}C$ (b) $\frac{3}{5}C$ (c) $\frac{3C}{10}$ (d) $\frac{10C}{3}$

9. Two identical cells whether connected in parallel or in series, gives the same current when connected to an external resistance 1.5 Ω. Find the value of internal resistance of each cell

(a) 1Ω (b) 0.5Ω (c) zero (d) 1.5Ω

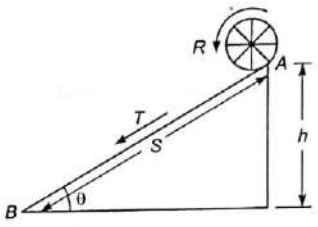
10. A ray of light travelling parallel to the principal axis of a concave mirror of radius of curvature $R$ strikes the mirror at angle of incidence $\theta$. At what distance from the pole will the reflected ray of light meet the principal axis?

(a) $\frac{R}{2}$ (b) $\frac{R}{2}\sec\theta$ (c) $\frac{R}{2}(1-\cos\theta)$ (d) $R\left(1-\frac{\sec\theta}{2}\right)$

11. Two weights $w_1$ and $w_2$ are suspended from the ends of a light string passing over a smooth fixed pulley. If the pulley is pulled up with acceleration $g$, the tension in the string will be

(a) $\frac{4w_1w_2}{w_1+w_2}$ (b) $\frac{2w_1w_2}{w_1+w_2}$ (c) $\frac{w_1-w_2}{w_1+w_2}$ (d) $\frac{w_1w_2}{2(w_1-w_2)}$

12. Suppose a body of mass $M$ and radius $R$ is allowed to roll on an inclined plane without slipping from its topmost point $A$. The velocity acquired by the body, as it reaches the bottom of the inclined plane, is given by




(a) $\sqrt{2gh}$ (b) $\sqrt{\beta \times 2gh}$

(c) $\sqrt{\frac{2gh}{\beta}}$ (d) $\frac{2gh}{\beta}$

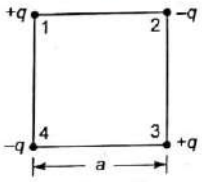
13. A projectile is fired vertically upwards from the surface of the earth with a velocity $kv_e$, where $v_e$ is the escape velocity and $k < 1$. If $R$ is the radius of the earth, the maximum height to which it will rise measured from the centre of earth will be (neglect air resistance)

(a) $\frac{1-k^2}{R}$ (b) $\frac{R}{1-k^2}$ (c) $R(1-k^2)$ (d) $\frac{R}{1+k^2}$

14. An organ pipe, open at both ends and another organ pipe, closed at one end, will resonate with each other, if their lengths are in the ratio of

(a) 1 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 2

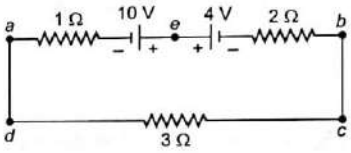
15. The work done required to put the four charges together at the corner of a square of side $a$, as shown in the figure is




(a) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$ (b) $-\frac{2.6}{4\pi\varepsilon_0}\frac{q^2}{a}$

(c) $+\frac{2.6}{4\pi\varepsilon_0}\frac{q^2}{a}$ (d) None of these

16. In the circuit diagram shown below, the magnitude and direction of the flow of current respectively, would be




(a) 7/3 A from $a$ to $b$ via $e$

(b) 7/3 A from $b$ to $a$ via $e$

(c) 1.0 A from $b$ to $a$ via $e$

(d) 1.0 A from $a$ to $b$ via $e$

17. A wire of length $l$ metre carrying a current $I$ ampere is bent in the form of a circle. The magnitude of the magnetic moment $i$

(a) $\frac{lI^2}{2\pi}$ (b) $\frac{lI^2}{4\pi}$ (c) $\frac{I^2 l}{2\pi}$ (d) $\frac{I^2 l}{4\pi}$

18. Steam at 1 atm and 100°C enters a radiator and leaves as water at 1 atm and 80°C, take $L_V = 540$ cal/g and $s_{water} = 1$ cal/g/°C. Of the total energy given off as heat, the percentage that arises from the cooling of water, is

(a) 100 (b) 54 (c) 3.6 (d) 8.2

19. Which statement about magnetism is correct?

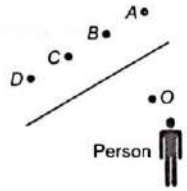
(a) A magnet attracts small pieces of aluminium.

(b) Steel makes a better permanent magnet than the iron does.

(c) There is no limit to the magnetic field strength of a magnet made from a steel bar.

(d) Two poles always attract each other.

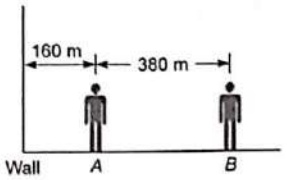
20. An object $O$ is placed in front of a plane mirror. A person looks into the mirror as shown in the figure. At what position is the image of $O$ is observed by the person?




(a) $A$ (b) $B$

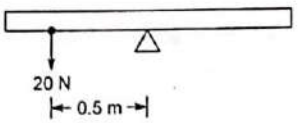
(c) $C$ (d) $D$

21. The persons $A$ and $B$ are standing in front of a wall as shown in figure. Person $A$ fires a pistol and $B$ hears two sounds at 1s interval. The speed of the sound in air is




(a) 330 m/s (b) 340 m/s
(c) 320 m/s (d) 360 m/s

22. A uniform beam is pivoted at its mid-point. A force of 20 N is acting on the rod in a direction perpendicular to length of rod. Which of the following force(s) can bring the system in rotational equilibrium. Assume that all forces are acting perpendicular to length of rod




(a) 20 N at 50 cm to right of mid-point (b) 10 N at 100 cm to right of mid-point
(c) 25 N at 40 cm to right of mid-point (d) All of these

23. Two helium nuclei move towards each other with same speed. Mark out the correct statement(s) related to above situation.
(a) The nuclei attract one another with a net gain in their energy
(b) The nuclei repel one another with a net loss in their energy
(c) The nuclei repel one another and their total energy remains constant
(d) The nuclei attract one another and their total energy remains constant

24. A tube of length $L$ is filled completely with an incompressible liquid of mass $M$ and the tube is closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with uniform angular velocity $\omega$. The force exerted by the liquid at the other end is

(a) $\frac{ML\omega^2}{2}$ (b) $ML\omega^2$ (c) $\frac{ML\omega^2}{4}$ (d) $\frac{ML^2\omega^2}{2}$

25. To identify whether the transistor is working or not, using multimeter, which statement serves the purpose?
(a) The common lead of multimeter is connected to base and other lead first to emitter and then to collector, only first connections shows the continuity
(b) The common lead of multimeter is connected to base and other lead first to emitter and then to collector, both the connections show the continuity
(c) The common lead of multimeter is connected to base and other lead to first emitter and then to collector, none of the connections shows the continuity
(d) All of the above

26. The length of an elastic string is $a$ metre when the tension in it is 4N and $b$ metre when the tension in it is 5N. The length of string in metre when the tension in the string is 9N, is
(a) $a + b$ (b) $5b - 4a$ (c) $4a - 5b$ (d) $9b - 5a$

27. In YDSE, how many maxima can be obtained on a screen *(including central maxima)* on both sides of the central maxima, if $d = \frac{5\lambda}{2}$ where $\lambda$ is the wavelength of light?
(a) 5 (b) 3 (c) 7 (d) 1

28. A solid sphere falls with a terminal velocity of 32 m/s in air. If it is allowed to fall in vacuum, then
(a) the terminal velocity will be 32 m/s
(b) the terminal velocity will be less than 32 m/s
(c) the terminal velocity will be greater than 32 m/s
(d) no terminal velocity is attained

29. For CE configuration of a transistor
(a) input resistance is very small while that of output resistance is very high
(b) input resistance is very large while that of output resistance is very small
(c) Both input and output resistances are very small
(d) Both input and output resistances are very high

30. An energetic α-particle striking a nitrogen nucleus at rest, causes the reaction $^{14}N + \alpha \rightarrow {}^{17}O + P$. If mass defect is 0.00129 u, the minimum energy required by the α-particle to trigger the reaction, is
(a) 1.54 MeV (b) 1.2 MeV (c) 3.08 MeV (d) 2.4 MeV

# Chemistry

31. The heat capacity of a bomb calorimeter is 500 J/°C. When 0.1 g of methane was burnt in this calorimeter, the temperature rose by 2°C. The value of $\Delta E$ per mole will be
(a) +10 kJ (b) −10 kJ (c) + 160 kJ (d) − 160 kJ

32. The number of possible isomers of an octahedral complex $[Co(C_2O_4)_2(NH_3)_2]^-$ is
(a) 3 (b) 4 (c) 6 (d) 8

33. $Cl_2$ gas is passed through Lassaigne's extract containing $CCl_4$. If the extract contains both NaBr and NaI then which colour will appear first in the $CCl_4$ layer?
(a) Violet (b) Brown (c) Green (d) Yellow

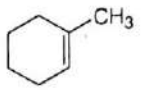
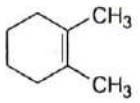
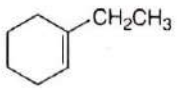
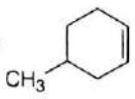
34. Which of the following reactions is likely to give the greatest yield of *n*-propyl bromide?
(a) $CH_3 \cdot CH{=}CH_2 + HBr \longrightarrow$ (b) $CH_3CH_2CH_3 + Br_2 \xrightarrow{h\nu}$
(c) $H_2C\text{—}CH_2 + HBr \longrightarrow$ (d) $CH_3CH_2CH_2Cl + NaBr \xrightarrow{\text{Acetone}}$

35. Which of the following does not exist?
(a) $KI + I_2 \longrightarrow KI_3$ (b) $KF + F_2 \longrightarrow KF_3$
(c) $KBr + ICl \longrightarrow K[BrICl]$ (d) $KF + BrF_3 \longrightarrow K[BrF_4]$

36. The decreasing order of the second ionisation enthalpy of K, Ca and Ba is
(a) K > Ca > Ba (b) Ca > Ba > K
(c) Ba > K > Ca (d) K > Ba > Ca

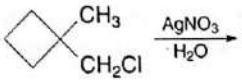
37. $H_2O_2$ on reacting with ethylene gives
(a) ethanal (b) ethane (c) ethanol (d) ethylene glycol

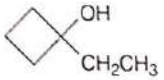
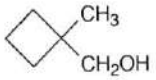
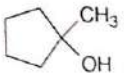
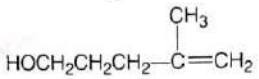
38. Which of the following is most stable?
(a) $Na_3N$ (b) $Li_3N$ (c) $K_3N$ (d) $Rb_3N$

39. A hydrocarbon (*A*) of molecular formula $C_7H_{12}$ on ozonolysis gives a compound (*B*) which undergoes aldol condensation to give 1-acetylcyclopentene. The hydrocarbon (*A*) is
(a)  (b)  (c)  (d) 

40. The quantity of electricity required to completely deposit copper from 200 mL of 0.5 M $CuSO_4$ solution will be
(a) 96500 C (b) 19300 C (c) 38600 C (d) 28950 C

41. Which of the following alcohols will be formed in major amount (besides the alkenes products) in the reaction shown below?



(a) 
(b) 
(c) 
(d) 

42. Maximum hydrogen bonding is observed in
(a) $H_2O$ (b) $H_2Se$ (c) $H_2S$ (d) HF

43. The van der Waals' constant '*a*' for the gases $O_2$, $N_2$, $NH_3$ and $CH_4$ are 1.3, 1.39, 4.17 and 2.253 $L^2$ atm-$mol^{-2}$ respectively. The gas which shows highest critical temperature is
(a) $O_2$ (b) $N_2$ (c) $NH_3$ (d) $CH_4$

44. Which one of the following will not scatter light?
(a) Rubber dissolved in benzene (b) Sodium chloride shaken with benzene
(c) Egg albumin shaken with water (d) Sulphur dissolved in $CS_2$

45. A synthetic rubber which is resistant to the action of oils, gasoline and other solvent is
(a) dacron (b) polyisoprene (c) neoprene (d) polystyrene

46. An organic compound does not reduce Tollen's reagent and Fehling's solution but gives red colour with ceric ammonium nitrate solution. The compound is
(a) a phenol (b) an aldehyde (c) a ketone (d) an alcohol

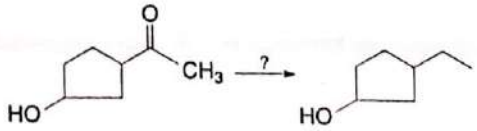
47. Which of the following pairs form the same osazone?
(a) Glucose and fructose (b) Glucose and galactose
(c) Glucose and arabinose (d) Lactose and maltose

48. Which of the following is not an electrophile?
(a) $\overset{+}{N}O_2$ (b) $Br^+$ (c) $R_4\overset{\oplus}{N}$ (d) $\overset{\oplus}{N}O$

49. In which of the following compounds, all the carbon atoms are $sp^2$ hybridised?
(a) Benzene (b) 1, 3-butadiene (c) 1, 3, 5-hexatriene (d) All of these

50. When borax is heated strongly it gives
(a) $B_2O_3$ (b) $Na_2B_4O_7$ (c) $NaBO_2$ (d) $NaBO_2 + B_2O_3$

51. For the reaction,



which reagent will be appropriate for the above conversion?
(a) Zn—Hg/HCl (b) $NH_2—NH_2/OH^-$ (c) $LiAlH_4/H^{\oplus}$ (d) HI/P

52. Equal volumes of two HCl solutions of pH = 3 and pH = 5 were mixed together. What is the pH of the resulting solution?
(a) 3.5 (b) 4.0 (c) 4.5 (d) 3.3

**Directions** (Q. Nos. 53-55) *Ammonium hydrogen sulphide dissociate as follows*

$$NH_4HS(s) \rightleftharpoons H_2S(g) + NH_3(g)$$

*If solid* $NH_4HS$ *is placed in an evacuated flask at a certain temperature, it will dissociate until the total gas pressure is* 500 *torr.*

53. Calculate the value of the equilibrium constant for the dissociation reaction.
(a) 0.108 $atm^2$ (b) 0.208 $atm^2$ (c) 0.308 $atm^2$ (d) 0.508 $atm^2$

54. Additional $NH_3$ is introduced into the equilibrium mixture without change in temperature until the partial pressure of ammonia is 700 torr. What is the partial pressure of $H_2S$ under these conditions?
(a) 0.107 atm (b) 0.117 atm (c) 0.237 atm (d) 0.327 atm

55. What is the total pressure in the flask?
(a) 0.038 atm (b) 2.038 atm (c) 3.038 atm (d) 1.038 atm

**Directions** (Q. Nos. 56-58) *Two moles of a perfect gas undergo the following processes*

I. *A reversible isobaric expansion from* (1.0 *atm*, 20 *L*) *to* (1.0 *atm*, 40 *L*)
II. *A reversible isochoric change of state from* (1.0 *atm*, 40 *L*) *to* (0.5 *atm*, 40 *L*)
III. *A reversible isothermal compression from* (0.5 *atm*, 40 *L*) *to* (1.0 *atm*, 20 *L*)

56. Calculate the total work done ($W$) involved in the above processes.
(a) – 620.77 J (b) 620.77 J (c) – 120.77 J (d) 120.77 J

57. The total heat change ($q$) involved in the above processes is
(a) – 620.77 J (b) 620.77 J (c) – 120.77 J (d) 120.77 J

58. The value of $\Delta E, \Delta H$ and $\Delta S$ for the overall process are
(a) 0, 0, 0 (b) 0, 0, – 620.77 (c) 0, –620.77, 120.77 (d) 0, –120.77, 0

**Directions** (Q. Nos. 59 and 60) *For the following questions, choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.
(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

59. **Statement I** $SO_2$ gas can be easily liquefied than $H_2$.
**Statement II** $SO_2$ has lower critical temperature than $H_2$.

60. **Statement I** Fluorine cannot be prepared from fluorides by chemical oxidation.
**Statement II** Fluorine is the strongest oxidising agent due to its highly positive potential.

# Mathematics

61. Locus of a point $z$ in Argand plane satisfying $|z^2 - (\bar{z})^2| = |z|^2$, $\text{Re}(z) \geq 0, \text{Im}(z) \geq 0$ is
(a) point (b) pair of straight lines
(c) hyperbola (d) ellipse

62. Let $I$ be the unit matrix of order $3 \times 3$ and $A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then $A^{-1}$ can be expressed as
(a) $A^2 - 5A + 8I$ (b) $A^2 + 5A - 8I$ (c) $3A^2 - 2A + 5I$ (d) $3A^2 - 2A - 5I$

63. If $x^3 + 3x^2 + 3x + 28 = 0$ has roots $\alpha, \beta$ and $\gamma$, then value of $\sum \frac{(\alpha+1)^2}{(\beta+1)(\gamma+1)}$ is equal to
(a) 0 (b) 1 (c) $-1$ (d) 3

64. The minimum value of $\{2\sin x(1 + 2\cos 2x)(\cos 3x - \sin 3x) + 1\}$, where $x \in R$, is
(a) $-\sqrt{3}$ (b) $-\sqrt{2}$ (c) $\sqrt{5}$ (d) $-\sqrt{6}$

65. The number of solutions of the equation
$$\tan^2\alpha - \text{cosec}^2\alpha + 4\cot^2 2\alpha + 4\tan\alpha\cot 2\alpha = 0 \text{ in } \left(0, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}\right\} \text{ is}$$
(a) 1 (b) 0 (c) 2 (d) 3

66. There are two bags each containing $n$ distinct balls. The number of ways in which a man can draw equal number of balls from the bags. If he selects atleast one ball from each bag, is
(a) ${}^{2n}C_n$ (b) ${}^{2n}C_n + 1$ (c) ${}^{2n}C_n - 1$ (d) $\frac{{}^{2n}C_n}{2}$

67. Number of solution(s) of the equation $x + \log_{10}(1 + 2^x) = x\log_{10} 5 + \log_{10} 6$ is
(a) 0 (b) 1 (c) 3 (d) infinitely many

68. If $\left(2\cos\frac{\theta}{2}\sec\theta \sin\frac{\theta}{2}\right)$ and $(\tan\theta\,\text{cosec}\,\theta)$, $\theta \neq n\pi$, $(2n+1)\frac{\pi}{2}$, $n \in Z$ be roots of $px^2 - qx + r = 0, p \neq 0$, then
(a) $p^4 + 4pq^2r + q^4 = 0$ (b) $p^4 - 4pqr + q^3 = 0$
(c) $(p^2 + q^2)(p^2 - q^2) + 4pq^2r = 0$ (d) $p^2 + 2pqr - q^2 = 0$

69. If $3(a^2 + b^2 + c^2) - 6(a + b + c) = 2abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) - 27$, then $a, b$ and $c$ may be in
(a) AP (b) HP
(c) GP (d) All of these

70. In $\Delta ABC$ with usual notations the value of
$$\frac{(b^2 + c^2)[c^2 - (a-b)(a+b)]}{2b^2c^2} + \frac{(c^2 + a^2)[a^2 - (b-c)(b+c)]}{2a^2c^2} + \frac{(a^2 + b^2)[b^2 - (c-a)(c+a)]}{2b^2a^2} \text{ is}$$
always equal to
(a) 1 (b) 20 (c) 10 (d) 3

71. If $x$ and $y$ be non-negative integers and $x > y$, then maximum number of order pairs $(x, y)$ satisfying $\frac{1}{x!(8-x)!} > \frac{1}{y!(8-y)!}$ is

(a) 17 (b) 18 (c) 19 (d) 16

72. $\int \frac{\cos^2 3x - \cos^2 6x}{\left(\frac{1}{2} + \cos 6x\right)} dx = g(x) + c$, then

(a) $g(x)$ is an even function
(b) $g(x)$ is a decreasing function
(c) $g(x)$ is an odd function
(d) $g(x)$ is a constant function

73. The number of solution(s) of the equation $|x-1| + |x-2| + |x-3| + |x-4| = 3$ is

(a) 2 (b) −1 (c) 0 (d) 4

74. If $x, y, z, s$ and $t \in R$ and $z \neq 0$, $t \neq 0$ and $f\left(\frac{x}{z} + \frac{y}{z}\right) = f\left(\frac{x}{z} + \frac{s}{t}\right) + f\left(\frac{y}{z} - \frac{s}{t}\right)$, then $\int_{(\ln 2010)}^{\left(\ln \frac{1}{2010}\right)} \frac{e^{\sin x}}{1 - e^{2 \sin x}} dx$ is equal to

(a) 0 (b) 1 (c) $2^{2010}$ (d) $3^{2009}$

75. Consider a function $f(x) =$ minimum $\{|\tan x|, |\cot x|\}$. The area bounded by $y = f(x)$, $x = \pm \frac{\pi}{4}$ and $y = 0$ is

(a) 2 log 2 (b) log 2 (c) $\frac{\pi}{2} + 1$ (d) $\frac{\pi}{2} - 1$

76. If $f(x) = \cos[\pi^2] x + \cos[-\pi^2] x$, where $[x]$ stands for greatest integer function, then

(a) $f\left(\frac{\pi}{2}\right) = 1$
(b) $f(\pi) = 1$
(c) $f(-\pi) = 0$
(d) $f\left(\frac{\pi}{4}\right) = 2$

77. Let $f$ be a differentiable function everywhere is defined as $f(xy) = f(x) f(y)$, $\forall\ x, y \in R$. If $f(1 + x) = 1 + g(\sin x)$, $g(0) = 0$, $\lim_{x \to 0} \frac{g(x)}{x} = 1$, then which of the following may be correct?

(a) $f(1 + 2 + 3 - \ldots - 100)^2 = f(1) + f(2) + f(3) + \ldots + f(100)^2$
(b) $f(1^2 + 2^2 + 3^2 + \ldots + 100^2) = f(1^2) + f(2^2) + f(3^2) + \ldots + f(100^2)$
(c) $f(\sin^{-1} x + \cos^{-1} x - \sec^{-1} x)$ have only two points in domain
(d) The function $\frac{f(x)^2}{x}$ is periodic

78. If $|\mathbf{a} + \mathbf{b}| = 1$, and $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, then the vector $\{\{\{(\mathbf{a} \times \mathbf{b}) \times \mathbf{a}\} \times \mathbf{a}\} \times \mathbf{a}\} \times \mathbf{b}$

(a) is parallel to $\mathbf{a} \times \mathbf{b}$
(b) has magnitude $\frac{\sqrt{3}}{4}$
(c) is parallel to $\mathbf{a}$
(d) None of these

79. The solution set of $\tan^2(\sin^{-1} x) > 1$ is

(a) $\left(-1, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$
(b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \sim \{0\}$
(c) $(-1, 1) \sim \{0\}$
(d) None of these

80. A square $S_1$ is inscribe in circle $C_1$ of radius 10, a circle $C_2$ is inscribed in $S_1$, then a square $S_2$ is inscribed in $C_2$ and so on. If the sum of area of $S_n$ and $C_n$ is greater than 1, then $n$ may be
(a) 6 (b) 14 (c) 12 (d) 15

81. $PBC$ is a triangle, a circle is drawn passing through $B$ and $C$ and intersect $PB$ and $PC$ at $A$ and $D$, respectively
(a) $\frac{1}{PA}+\frac{1}{PB}>\frac{4}{\sqrt{PD\times PT}}$ (b) $\frac{1}{PA}\times\frac{1}{PB}<\frac{2}{\sqrt{PD\times PC}}$
(c) $\frac{1}{PA}+\frac{1}{PB}+\frac{1}{PC}+\frac{1}{PD}>\frac{4}{PT}$ (d) $\frac{1}{PA}+\frac{1}{PB}+\frac{1}{PC}+\frac{1}{PD}<\frac{4}{PT}$

82. Consider the region formed by the lines $x=0$, $y=0$, $x=2$, $y=2$. Area enclosed by the curves $y=e^x$ and $y=\ln x$, within this region is being removed. The the area of remaining region is
(a) $(2\ln x+1)$ (b) $(4\ln 2-1)$
(c) $(\ln 2+1)$ (d) None of these

83. Maximum number of points of intersections of 4 circles and 4 parabolas is a multiple of
(a) 302 (b) 301 (c) 401 (d) 100

84. If ${}^{99}C_{51}+{}^{99}C_{50}=\frac{n!}{r!(n-r)!}$, then $\frac{n+2}{r}$ may be equal to
(a) 2 (b) 3 (c) 4 (d) 10

**Directions** (Q. Nos. 85 and 86) *Let us define a new differential operator is* $\frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{f(x+h)\,f'(x+h)-f(x)\,f'(x)}{h}$. *Then,*

85. $\frac{d}{dx}(x^2)$ is equal to
(a) $3x^2$ (b) $6x^2$ (c) $6x^3$ (d) $2x$

86. $\frac{d}{dx}(\sin x)$ is equal to
(a) $\cos 2x$ (b) $\cos x$ (c) $-\cos 2x$ (d) $-\cos x$

**Directions** (Q. Nos. 87 and 88) *To find the range of different functions. We can use the concept* $AM\geq GM\geq HM$. *The concept of quadratic equation, derivatives is also used to find the range. Also, we know that* $-\sqrt{a^2+b^2}\leq a\sin x+b\cos x\leq\sqrt{a^2+b^2}$, $\forall\, x\in R$. *Then,*

87. If $x\in\left(0,\frac{\pi}{2}\right)$ and $y\in\left(0,\frac{\pi}{2}\right)$, then minimum value of $\sin^3 x+\cos^3 y+3(\sin x+\cos y)$ $(\operatorname{cosec} x\sec y)$ is
(a) 8 (b) 7 (c) 6 (d) $\frac{13\sqrt{2}}{2}$

88. If $y=\frac{x^2y+x^2-xy-x+y+1}{x^2y+x^2+xy+x+y+1}$ and $y\neq -1$, then maximum value of $y$ is
(a) 5 (b) 4 (c) 3 (d) 2

**Directions** (Q. Nos. 89 and 90) *For the following questions. Choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

89. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be the equation of the circle, then

**Statement I** The least and the greatest distance of the point $P(10,7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ are 5 and 15 units, respectively.

**Statement II** A point $(x_1, y_1)$ lies outside a circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$, if $S_1 > 0$ where, $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$.

90. If $p$ =12 is an integer, $q$ =6 apple is sour, then

**Statement I** The symbolic form of 'if apple is not sour', then 12 is an even integer is $\sim q \Rightarrow p$.

**Statement II** It is a biconditional statement.

# Answer *with* Explanations

## Physics

**1.** (a) $H_{max} = \frac{u^2}{2g}$ and $R_{max} = \frac{u^2}{g}$

$\therefore \quad H_{max} = \frac{1}{2} R_{max} = 50\,m$

**2.** (a) Total force on train of 80 wagons is

$F = ma = mna$ (Here, $m = 5 \times 10^3$ kg and $n = 80$)

or $4 \times 10^5 = 80 \times 5 \times 10^3 \times a$

$\therefore \quad a = \frac{4 \times 10^5}{4 \times 10^5} = 1\,ms^{-2}$

Tension in the coupling between 30th and 31st wagon will be due to mass of remaining 50 wagons.

Now, mass of remaining 50 wagons

$= 50 \times 5 \times 10^3$ kg

$\therefore$ Required tension, $T = 50 \times 5 \times 10^3 \times a$

$= 50 \times 5 \times 10^3 \times 1 = 25 \times 10^4$ N

**3.** (a) Acceleration of system,

$$a = \frac{mg \sin 60° - mg \sin 30°}{2m}$$

or $$a = \left(\frac{\sqrt{3}-1}{4}\right) g$$

Now, $a_{common} = \frac{ma_1 + ma_2}{2m}$

Here, $a_1$ and $a_2$ are $\left(\frac{\sqrt{3}-1}{4}\right) g$ at right angles.

Hence, $|a_{common}| = \frac{\sqrt{2}}{2} a = \frac{a}{\sqrt{2}} = \left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right) g$

**4.** (c) The moment of inertia of the uniform rod about an axis through one end and perpendicular to length is

$$I = \frac{ML^2}{3}$$

Torque and acting on centre of gravity of rod is given by

$\tau = Mg\left[\frac{L}{2} \sin\theta\right]$ But $\tau = I\alpha$

or $$\frac{ML^2}{3} \alpha = Mg \frac{L}{2} \sin\theta$$

$\therefore \quad \alpha = \frac{3g}{2L} \sin\theta$

**5.** (b) According to the problem, as the potential at $\infty$ increases by + 10 J/kg, hence potential will increase by the same amount everywhere (potential gradient will remain constant). Hence, potential at point $P$

$= +10 - 5 = +5$ J/kg

**6.** (b) $pV = nRT$

$V$ = constant

For 32 g of $O_2$

$n = 1$

For 2 moles of $H_2$, temperature $2T$,

$$p' = \frac{2R\,2T}{V} = p' = 4p$$

Another way to analyse the answer is that, the density of ice is lesser than the water, therefore volume will be larger for same mass.

**7.** (b) Points 2 and 3 are equipotential points. Hence, potential difference between points 1 and 2 is the same as that between 1 and 3.

$\therefore \quad V = Ed \cos 60°$

Displacement vector **d** is the line joining from 1 to 2.

**8.** (a) Let $d$ be the distance between the plates of parallel plate capacitor. Then, total thickness of two plates placed between the plates of the original capacitor

$$t = \frac{d}{5} + \frac{d}{5} = \frac{2d}{5}$$

Original capacitance,

$$C = \frac{\varepsilon_0 A}{d}$$

New capacitance,

$$C' = \frac{\varepsilon_0 A}{d - t} = \frac{\varepsilon_0 A}{d - \frac{2d}{5}} = \frac{5}{3} \frac{\varepsilon_0 A}{d} = \frac{5}{3} C$$

**9.** (d) Let $E, r$ be the emf and internal resistance of each cell. In series combination of two cells, the current through external resistance 1.5 Ω will be

$$I_s = \frac{2E}{2r + 1.5}$$

In parallel combination of two cells, the current through the external resistance of 1.5 Ω will be

$$I_p = \frac{E}{\frac{r}{2} + 1.5} = \frac{2E}{r + 3}$$

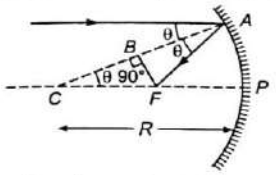
Given $I_s = I_p$

$$\frac{2E}{2r + 1.5} = \frac{2E}{r + 3}$$

or $r + 3 = 2r + 1.5$

or $r = 1.5\,\Omega$

**10.** (d) $P$ be the pole of concave mirror, $C$ its centre of curvature and $A$ be the point of incidence.




It is clear from figure, that $\Delta ACF$ is an isosceles, hence

$$CF = AF = CB \sec\theta$$

$$= \frac{1}{2} CA \sec\theta = \frac{R}{2} \sec\theta$$

$$\therefore PF = CP - CF = R - \frac{R}{2}\sec\theta = R\left(1 - \frac{\sec\theta}{2}\right)$$

**11.** (a) For system of pulley moving up with acceleration $a_P = g$. ...(i)

The apparent weights of masses $m_1$ and $m_2$ are

$$m_1 g' = m_1 (g + a_p) = 2m_1 g \quad ...(ii)$$

$$m_2 g' = m_2 (g + a_p) = 2m_2 g \quad ...(iii)$$

For equations of motion if $T$ is tension in string and $a$ is acceleration of masses, then

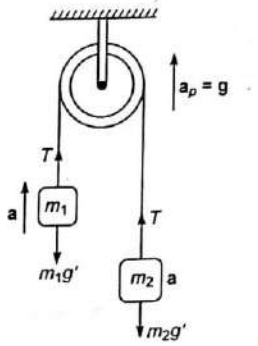
$$m_1 a = T - m_1 g' \quad ...(iv)$$

$$m_2 a = m_2 g' - T \quad ...(v)$$

Here, apparent value of gravitational acceleration is taken as $g'$ enstead of $g$ because a pseudo force (downward) acting on these bodies.

Adding, $a(m_1 + m_2) = (m_2 - m_1) g'$

or $$a = \frac{m_2 - m_1}{m_1 + m_2} g' \quad ...(vi)$$




Putting value of $a$ from Eq. (vi) in Eq. (iv)

$$T = m_1 (a + g') = \frac{2m_1 m_2}{m_1 + m_2} g'$$

or $$T = \frac{4m_1 m_2 g}{m_1 + m_2} = \frac{4m_1 g \times m_2 g}{(m_1 + m_2) g} = \frac{4w_1 w_2}{w_1 + w_2}$$

**12.** (c) As the body rolls the inclined plane, it losses potential energy. However, in rolling it acquires both linear and angular speeds and hence, gain in kinetic energy of translation and that of rotation. So, by conservation of mechanical energy.

$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

But as in rolling, $v = R\omega$

$$\therefore \quad Mgh = \frac{1}{2} Mv^2 \left[1 + \frac{I}{MR^2}\right]$$

Let $$1 + \frac{I}{MR^2} = \beta$$

$$\therefore \quad Mgh = \frac{1}{2} \beta M v^2$$

Hence, $$v = \sqrt{\frac{2gh}{\beta}}$$

**13.** (b) If a body is projected from the surface of the earth with a velocity $v$ and reaches a height $h$, then according to law of conservation of energy.

$$\frac{1}{2} mv^2 = \frac{mgh}{1 + \frac{h}{g}}$$

where $h = r - R$ and

$r$ = height from the earth's centre

Here, $$v = kv_e = k\sqrt{2gR}$$

$$\frac{1}{2} mk^2 . 2gR = \frac{mg(r - R)}{1 + \frac{(r - R)}{R}}$$

$$k^2 R\left[1 + \frac{r - R}{R}\right] = r - R$$

or $$k^2 r = r - R$$

or $$r = \frac{R}{1 - k^2}$$

**14.** (c) Fundamental frequency of open organ pipe,

$$\nu_1 = \frac{v}{2L_1}$$

and fundamental frequency of closed organ pipe,

$$\nu_2 = \frac{v}{4L_2}$$

But at the condition of resonance

$$\nu_1 = \nu_2$$

or $$\frac{v}{2L_1} = \frac{v}{4L_2}$$

or $$\frac{L_1}{L_2} = 2:1$$

**15.** (b) The work done required is equal to the potential energy of the system. Thus,

$W = U$

(here, number of pairs formed is $\frac{4 \times 3}{2} = 6$)

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_4}{r_{34}} + \frac{q_4 q_1}{r_{41}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_4}{r_{24}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{a}-\frac{q^2}{a}-\frac{q^2}{a}-\frac{q^2}{a}+\frac{q^2}{a\sqrt{2}}+\frac{q^2}{a\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{4q^2}{a}+\frac{2q^2}{a\sqrt{2}}\right] = -\frac{q^2}{4\pi\varepsilon_0 a}[4-\sqrt{2}]$$

$$= -\frac{2.6}{4\pi\varepsilon_0}\left(\frac{q^2}{a}\right)$$

**16.** (d) Total resistance of the circuit,

$$R = 1\Omega + 2\Omega + 3\Omega = 6\Omega$$

Current, $I = \frac{10-4}{6} = 1\,A$

The direction of the current would be from $a$ to $b$ *via. e.*

**17.** (d) Magnetic moment, $m = AI = \pi r^2 I$, where $r$ is the radius of the circular loop. Now, the circumference of the circle = length of the wire *i.e.*,

$$2\pi r = l$$

or $$r^2 = \frac{l^2}{4\pi^2}$$

Therefore, $$m = \pi r^2 I = \frac{\pi l^2 I}{4\pi^2} = \frac{l^2 I}{4\pi}$$

**18.** (c) Let $m$ be the amount of steam that passes off the radiator.

Heat released due to condensation of steam, $H_1 = mL_v = 540\,m$ cal

Heat released due to cooling of water,

$$H_2 = ms\,\Delta\theta = m \times 1 \times 20 = 20\,m \text{ cal}$$

Here, $m$ = mass of water given in the question.

Required percentage $= \frac{H_2}{H_1 + H_2} \times 100$

$$= \frac{20}{560} \times 100 = 3.6$$

**19.** (b) For $a$ — Aluminium is a non-magnetic substance and hence cannot be attracted by a magnet.

For $b$ — Steel retains induced magnetism and hence is a better choice to make a permanent magnet.

For $c$ — When a magnetic material is magnetised, then there is a limit to its strength.

For $d$ — Like poles repel each other.

**20.** (b) For image to be observed by the person, the reflected or refracted rays must reach to the person's eyes, which confirms that image of $O$ as observed by the person is at $B$.

**21.** (c) $B$ will hear two sounds, one directly and other after reflection from the wall. Let speed of sound in air be $v$.

The time taken by sound to reach directly to $B$ is $t_1 = \frac{380}{v}$ s.

The time taken by sound to reach $B$ after reflection is,

$$t_2 = \frac{2 \times 160 + 380}{v}$$

$\Rightarrow$ $t_2 - t_1 = 1$ s

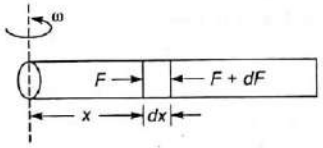
$\Rightarrow$ $v = 320$ m/s

**22.** (d) For the system to be in rotational equilibrium, the net torque about mid-point must be zero, which will come in all the 3 cases.

The torque force adjusts its value for vertical equilibrium.

**23.** (c) Helium nuclei are positively charged and like charges repel. The principle of conservation of energy ensures that total energy is always constant.

**24.** (a) Consider a small section of the liquid as shown in figure below




$$dF = dm\omega^2 x = \frac{M\omega^2}{L} x\,dx$$

$$\int_0^F dF = \int_0^L \frac{M}{L}\omega^2 x\,dx$$

$\Rightarrow$ $$F = \frac{M\omega^2 L}{2}$$

At $x = 0$, froce experienced by rod due to rotating fluid is zero.

**25.** (b) Option (a) tells that transistor is not working, as there is no connection between base and collector means there is some fault in this part of the transistor.

Option (b) tells that transistor is having no open circuit fault, *i.e.*, it is having the continuity.

Option (c) tells that transistor is faulty.

**26.** (b) From, $$Y = \frac{T/A}{\Delta l / l_0}$$

$\Rightarrow$ $$T = YA \times \frac{\Delta l}{l_0},$$

where $l_0$ is the initial length or natural length of string.

$$T = \frac{YA(l_1 - l_0)}{l_0} = k(l_1 - l_0)$$

where $l_1$ is the length of string when tension in string $T$.

$$T_1 = 4n = k(a - l_0)$$

$$T_2 = 5n = k(b - l_0)$$

$$T_3 = 9n = k(x - l_0)$$

Solving above equations, we get $x = 5b - 4a$.

**27.** (a) In YDSE, path difference, $\Delta x = d \sin\theta$

For maxima, $\Delta x = n\lambda$, where $n = 0, \pm 1, \pm 2, \ldots$

$$\frac{5\lambda}{2}\sin\theta = n\lambda \quad \left(\text{given that } d = \frac{5\lambda}{2}\right)$$

$$\sin\theta = \frac{2n}{5}$$

$$-1 \le \sin\theta \le 1$$

$$\Rightarrow \quad -1 \le \frac{2n}{5} \le 1$$

So, $$-\frac{5}{2} \le n \le \frac{5}{2}$$

So, possible values of $n$ are $-2, -1, 0, 1, 2$ thus, a total of 5 maximas will be formed.

**28.** (d) Terminal velocity is attained by an object due to some dragging force acting opposite to its motion. When sphere is allowed to fall in air, the air friction force causes the ball to acquire terminal velocity. But in vacuum, no dragging force is present and the ball falls down with acceleration $g$, so no terminal velocity is attained and velocity of the falling body regularly increases with increase in distance moved in downward direction.

**29.** (a) Input resistance $= \dfrac{\Delta V_{BE}}{\Delta I_B}$

Output resistance $= \dfrac{\Delta V_{CE}}{\Delta I_C}$

From input and output characteristic curve, we can find input and output resistances.

**30.** (a) Minimum energy required $= |Q| \times \left(\dfrac{M_\alpha}{M_N} + 1\right)$

But, $|Q| = \Delta mc^2$

$= 0.00129 \times 931$ MeV $= 1.2$ MeV

($\because$ 1 u = 931 MeV)

So, $K_{min} = 1.2 \times \left(\dfrac{4}{14} + 1\right)$ MeV $= 1.54$ MeV

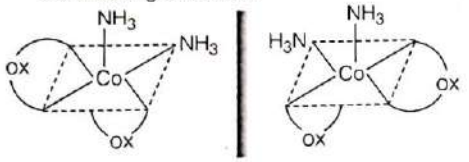
# Chemistry

**31.** (d) $\Delta E = -\dfrac{M}{w} \times C \times \Delta t$

$= -\dfrac{16}{0.1} \times 500 \times 2 = -160 \times 10^3$ J $= -160$ kJ

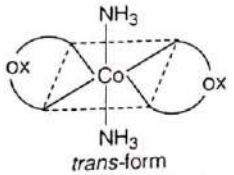
(minus sign is used as heat is evolved.)

**32.** (a) Octahedral complex $[Co(C_2O_4)_2(NH_3)_2]^-$ has the following 3 isomers.




*d* and *l* forms of *cis* $[Co(C_2O_4)_2(NH_3)_2]^-$

[Here $C_2O_4^{2-}$ = ox]




*trans*-form
(optically inactive)

**33.** (a) ($I^-$) ion is more easily oxidised than $Br^-$ ion and violet colour due to $I_2$ appears first.

**34.** (c) $H_2C(\text{—}CH_2\text{—})CH_2 + HBr \longrightarrow CH_3 - CH_2 - CH_2Br$

Other reactions may undergo rearrangement or give secondary alkyl halide.

**35.** (b) Due to the absence of *d*-orbitals, $F_2$ does not combine with $F^-$ to form $F_3^-$ ion.

**36.** (a) $_{19}K = 2, 8, 8, 1$

$_{20}Ca = 2, 8, 8, 2$

$_{56}Ba = 2, 8, 18, 18, 8, 2$

After loosing 1 electron, K gains inert gain configuration, due to which it is very difficult to withdraw electron from outermost orbital (completely filled). Therefore its second ionisation enthalpy is maximum. The size of Ba is larger as compared to Ca.

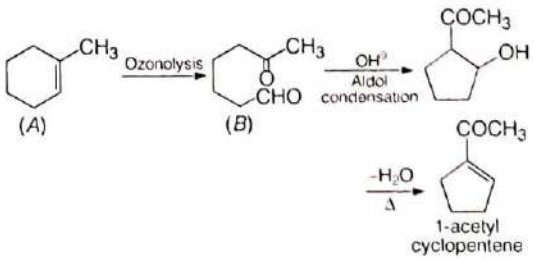
Therefore, it is easy to withdraw second electron from $Ba^+$as compared to $Ca^+$, due to which second ionisation enthalpy of Ba is minimum. Hence, the correct order of second ionisation enthalpy is

K > Ca > Ba

**37.** (d) $\underset{\text{Ethylene}}{CH_2\text{=}CH_2} + H_2O_2 \longrightarrow \underset{\text{Ethylene glycol}}{CH_2(OH)\text{—}CH_2(OH)}$

**38.** (b) Only $Li_3N$ is stable, others are not formed.

**39.** (a)



**40.** (b) $Cu^{2+} \xrightarrow{2e^-} Cu$

Number of moles of Cu

$$= \frac{200 \times 0.5}{1000} = 0.1$$

∵ 1 mole of Cu is deposited by electricity

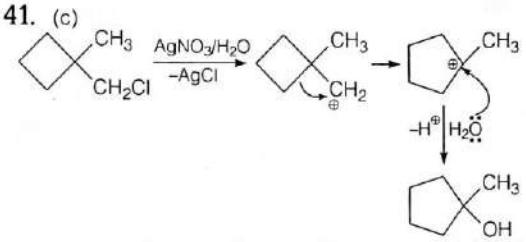
$= 2 \times 96500$ C

∴ 0.1 mole of Cu will be deposited by electricity

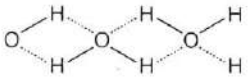
$$= \frac{2 \times 96500 \times 0.1}{1}$$

$= 19300$ C

**41.** (c)




**42.** (a)



In $H_2O$, four H-bonds per molecule are formed which is maximum among the other compounds.

**43.** (c) The gas which shows higher value of '*a*' will have higher value of critical temperature as van der Waals' constant '*a*' is directly proportional to critical temperature *i.e.*,

$T_C = \frac{8a}{27Rb}$.

Therefore, $NH_3$ having maximum value of *a*, will show highest critical temperature.

**44.** (d) Sulphur dissolved in $CS_2$ forms a true solution and hence, will not scatter light.

**45.** (c) Neoprene is an addition polymer of chloroprene. It is superior, resistant to aerial oxidation, oils, gasoline and other solvents.

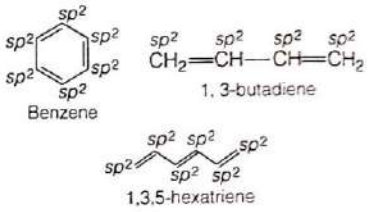
**46.** (d) Since, the compound does not reduce Tollen's reagent and Fehling's solution, it cannot be an aldehyde but gives red colour with ceric ammonium nitrate. It must be an alcohol.

**47.** (a) During osazone formation, the reaction occurs at $C_1$ and $C_2$ while rest of the molecule remains intact.

Glucose and fructose are different only at $C_1$ and $C_2$ atoms, so due to remaining C-atoms same, they form similar osazone.

**48.** (c) $R_4\overset{\oplus}{N}$ cannot act as electrophile as it cannot accept a pair of electron. Since, nitrogen already has 8 electrons.

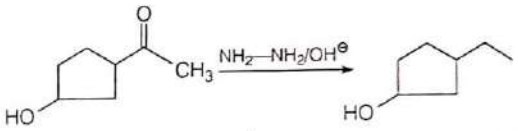
**49.** (d)




**Note** Double bonded carbon atoms are generally $sp^2$ hybridised.

**50.** (d) $\underset{\text{Borax}}{Na_2B_4O_7 \cdot 10H_2O} \xrightarrow{\Delta} 2NaBO_2 + B_2O_3 + 10H_2O$

**51.** (b)




In this reaction, $>C=O$ group is reduced to $>CH_2$ group which is carried by $NH_2—NH_2/OH^{\ominus}$ reagent.

Zn—Hg/HCl also reduces $>CO$ group to $>CH_2$ group but it also reacts with —OH group. That's why it is not a suitable reagent for this conversion. $LiAlH_4/H^{\oplus}$ converts $>CO$ group to $>CHOH$ group. HI/P affects both $>C=O$ and —OH groups.

**52.** (d) Let volume of solution $= V$

$[H^+]$ in I solution (pH = 3) $= 10^{-3}$

and, $[H^+]$ in II solution (pH = 5) $= 10^{-5}$

∴ $[H^+]$ in the mixture

$$= \frac{[H^+] \text{ of I solution} \times \text{Volume} + [H^+] \text{ of II solution} \times \text{Volume}}{\text{Total volume}}$$

$$= \frac{10^{-3}V + 10^{-5}V}{2V}$$

$$= \frac{10^{-5}(100+1)V}{2V} = \frac{101 \times 10^{-5}}{2}$$

pH of the mixture $= -\log[H^+]$

$$= -\log \frac{101 \times 10^{-5}}{2}$$

$$= \log 10^{-5} + \log 2 - \log 10$$

$$= 5 + 0.3010 - 2.0043$$

$$= 3.2967 \approx 3.3$$

**53.** (a) $NH_4HS(s) \rightleftharpoons \underset{p}{H_2S(g)} + \underset{p}{NH_3(g)}$

At equilibrium $2p = 500$ torr

∴ $p = 250$ torr

$$= \frac{250}{760} \text{ atm}$$

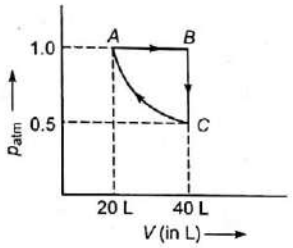
$$K_p = (p_{H_2S})(p_{NH_3})$$
$$= \left(\frac{250}{760}\right)^2$$
$$= 0.108 \text{ atm}^2$$

**54.** (b) If $p_{NH_3} = 700 \text{ torr} = \frac{700}{760} \text{ atm} = 0.921 \text{ atm}$

$$K_p = (p_{NH_3})(p_{H_2S})$$
$$0.108 = \frac{700}{760} \times p_{H_2S}$$
$$p_{H_2S} = 0.117 \text{ atm}$$

**55.** (d) $p_{total} = p_{H_2S} + p_{NH_3}$
$= 0.117 + 0.921$
$= 1.038$ atm

**56.** (a)




Total work, $W = W_{A\to B} + W_{B\to C} + W_{C\to A}$

$$W = -p(V_B - V_A) + 0 + 2.303\, nRT \log \frac{V_C}{V_A}$$
$$= -1(40 - 20) + 0 + 2.303\, p_A V_A \log \frac{V_C}{V_A}$$
$$= -20 + 0 + 2.303 \times 1 \times 20 \log \frac{40}{20}$$
$$= -20 + 13.87$$
$$= -6.13 \text{ L atm}$$
$$= -\frac{6.13 \times 8.314}{0.0821} \text{ J}$$
$$= -620.77 \text{ J}$$

**57.** (b) Total heat changed, $q = -W$
$= +620.77$ J

**58.** (a) Since, the overall process is cyclic. Therefore
$\Delta E = 0$, $\Delta H = 0$, $\Delta S = 0$ } as all are state functions.

**59.** (c) Critical temperature of $SO_2$ is quite higher than that of $H_2$, due to strong van der waals' forces of attraction. Hence, it can be easily liquefied in comparison to $H_2$.

**60.** (a) Fluorine is the strongest oxidising agent due to its highly positive standard potential. Therefore, fluorine has the highest tendency to get reduced to $F^-$. As a result, $F^-$ ion has the least tendency to get oxidised. That is why, fluorine cannot be prepared from fluorides by chemical oxidation.

# Mathematics

**61.** (b) Put $z = x + iy$ in given equation, we get
$|x^2 - y^2 + 2ixy - x^2 + y^2 + 2ixy| = x^2 + y^2$
$\Rightarrow \quad x^2 + y^2 = 4xy$
$\Rightarrow$ Pair of straight lines.

**62.** (a) The characteristic equation of $A$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 3 & 0 \\ 0 & 2-\lambda & -1 \\ 1 & 0 & 2-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)(2-\lambda)^2 - 3 = 0$
$\Rightarrow \quad (\lambda - 1)(\lambda^2 - 4\lambda + 4) + 3 = 0$
$\Rightarrow \quad \lambda^3 - 5\lambda^2 + 8\lambda - 1 = 0$
$\Rightarrow \quad A^3 - 5A^2 + 8A - 1 = 0$
$\Rightarrow \quad A^{-1} = A^2 - 5A + 8I$
(by Caley Hamilton theorem)

**63.** (d) Given equation can be rewritten as
$(x+1)^3 = -27$
$\Rightarrow \quad \alpha + 1 = -3, \beta + 1 = -3\omega$ and $\gamma + 1 = -3\omega^2$
$\therefore \quad \sum \frac{(\alpha+1)^2}{(\beta+1)(\gamma+1)}$

$$= \frac{9}{9\omega^3} + \frac{9\omega^2}{9\omega^2} + \frac{9\omega^4}{9\omega} = 3 \qquad (\because \omega^3 = 1)$$

**64.** (b) The given expression can be written as
$2\sin x\,[1 + 2(1 - 2\sin^2 x)(\cos 3x - \sin 3x)] + 1$
$= 2(3\sin x - 4\sin^3 x)(\cos 3x - \sin 3x) + 1$
$= 2\sin 3x\,(\cos 3x - \sin 3x) + 1$
$= \sin 6x + 1 - 2\sin^2 3x = \sin 6x + \cos 6x$
$\Rightarrow \quad -\sqrt{2} \le \sin 6x + \cos 6x \le \sqrt{2}$
Hence, minimum value $= -\sqrt{2}$

**65.** (b) The given equation can be written as
$(\tan\alpha + 2\cot 2\alpha)^2 - \text{cosec}^2\,\alpha = 0$

$$\Rightarrow \left[\tan\alpha + \frac{2(1 - \tan^2\alpha)}{2\tan\alpha}\right]^2 - \text{cosec}^2\,\alpha = 0$$

$\Rightarrow \quad \cot^2\alpha - \text{cosec}^2\,\alpha = 0$
$\Rightarrow \quad -1 = 0$
Hence, no solution exists.

**66.** (c) Total number of ways
$= ({}^nC_1)^2 + ({}^nC_2)^2 + ({}^nC_3)^2 + \ldots + ({}^nC_n)^2$
$= {}^{2n}C_n - 1$

**67.** (b) $\log_{10}(10^x) + \log_{10}(1 + 2^x) = \log_{10} 5^x + \log_{10} 6$
$\Rightarrow \quad 10^x(2^x + 1) = 6 \times 5^x$
$\Rightarrow \quad 2^x(2^x + 1) = 6$

$\Rightarrow \quad (2^x)^2 + 2^x - 6 = 0$

$\Rightarrow \quad (2^x + 3)(2^x - 2) = 0$

$\Rightarrow \quad 2^x = 2$

$(\because 2^x$ cannot be negative)

$\Rightarrow \quad x = 1$

**68.** (c) $\because 2\cos\frac{\theta}{2} \cdot \sec\theta \cdot \sin\frac{\theta}{2}$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

and $\tan\theta \operatorname{cosec}\theta = \sec\theta$

Clearly, the roots are $\tan\theta$ and $\sec\theta$.

$\therefore \quad \sec\theta + \tan\theta = \frac{q}{p}$ ...(i)

and $\sec\theta \tan\theta = \frac{r}{p}$

$\Rightarrow \sec\theta - \tan\theta = \frac{p}{q}$ $(\because \sec^2\theta - \tan^2\theta = 1)$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2\sec\theta = \frac{p}{q} + \frac{q}{p} \quad \text{...(iii)}$$

and $2\tan\theta = \frac{q}{p} - \frac{p}{q}$ ...(iv)

On multiplying Eqs. (iii) and (iv), we get

$$\frac{4r}{p} = \frac{q^2}{p^2} - \frac{p^2}{q^2}$$

$\Rightarrow \quad 4pq^2r = q^4 - p^4$

$\Rightarrow \quad p^4 + 4pq^2r - q^4 = 0$

$\Rightarrow \quad (p^2 + q^2)(p^2 - q^2) + 4pq^2r = 0$

**69.** (d) $3(a^2 + b^2 + c^2) - 6a - 6b - 6c - 2ab - 2bc - 2ac + 27 = 0$

$\Rightarrow \quad (a-3)^2 + (b-3)^2 + (c-3)^2 + (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Rightarrow \quad a = b = c = 3$

**70.** (d) Given expression can be written as

$$\left(\frac{b^2 + c^2}{bc}\right)\cos A + \left(\frac{c^2 + a^2}{ca}\right)\cos B + \left(\frac{a^2 + b^2}{ab}\right)\cos C \text{ (by Cosine law)}$$

$$= \left(\frac{b}{c}\cos A + \frac{c}{b}\cos A\right) + \left(\frac{c}{a}\cos B + \frac{a}{c}\cos B\right) + \left(\frac{a}{b}\cos C + \frac{b}{a}\cos C\right)$$

$$= \frac{1}{c}(b\cos A + a\cos B) + \frac{1}{b}(c\cos A + a\cos C) + \frac{1}{a}(b\cos C + c\cos B)$$

$= \frac{c}{c} + \frac{b}{b} + \frac{a}{a} = 3$ (by Projection formula)

**71.** (d) The given expression can be written as

$${}^8C_x > {}^8C_y$$

Order pairs satisfying the inequality is

(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3), (5, 0), (5, 1), (5, 2), (6, 0), (6, 1), (7, 0)

Total ordered pairs = 16

**72.** (c) $$\int \frac{\cos^2 3x - \cos^2 6x}{\frac{1}{2} + \cos 6x}\,dx$$

$$= \int \frac{\cos 6x - \cos 12x}{1 + 2\cos 6x}\,dx$$

$$= \int \frac{(2\sin 9x \sin 3x)\sin 3x}{\sin 3x + 2\cos 6x \sin 3x}\,dx$$

$$= \int \frac{2\sin 9x \sin 3x \sin 3x}{\sin 3x + \sin 9x - \sin 3x}\,dx$$

$$= \int 2\sin^2 3x\,dx = \int (1 - \cos 6x)\,dx$$

$$= x - \frac{\sin 6x}{6} + C$$

$\Rightarrow \quad g(x) = x - \frac{\sin 6x}{6} + C$

$\Rightarrow \quad g'(x) = 1 - \cos 6x \geq 0$

Hence, $g(x)$ is an increasing function, also $g(x) = -g(-x)$, hence $g(x)$ is odd function.

**73.** (c) As the minimum value of $|x-1| + |x-2| + |x-3| + |x-4|$ is 4 *i.e.*, it is above $x$-axis.

Hence, number of solution = 0.

**74.** (a) $f\left(\frac{x}{z} + \frac{y}{z}\right) = f\left(\frac{x}{z} + \frac{s}{t}\right) + f\left(\frac{y}{z} - \frac{s}{t}\right)$

Put $x = y = s = 0$

$f(0) = 0$

Now, put $x = y = 0, \frac{s}{t} = p$

$f(0) = f(p) + f(-p)$

$f(p) = -f(-p)$

$f(x)$ is an odd function.

$\therefore f\left(\log\frac{1}{2010}\right) = f(-\log 2010)$

$= -f(\log 2010)$

$$\int_{(\ln 2010)}^{-f(\ln 2010)} \frac{e^{\sin x}}{1 - e^{2\sin x}}\,dx = 0$$

$$\left[\because g(x) = \frac{e^{\sin x}}{1 - e^{2\sin x}},\ g(-x) = \frac{e^{-\sin x}}{1 - e^{-2\sin x}} = -g(x)\right]$$

**75.** (a) Area $= 2\int_0^{\pi/4} \tan x\,dx$

$= [2\log\sec x]_0^{\pi/4} = 2\log 2$

**76.** (c) $f(x) = \cos[\pi^2]x + \cos[-\pi]^2 x$

$= \cos 9x + \cos(-10)x$

$= \cos 9x + \cos 10x$

$\therefore \quad f\left(\frac{\pi}{2}\right) = 0 - 1 = -1$

$f(\pi) = -1 + 1 = 0$

$f(-\pi) = -1 + 1 = 0$

$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$

**77.** (b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{f(x) \cdot \left[f\left(1 + \frac{h}{x}\right) - 1\right]}{h}$

$= \lim_{h \to 0} \frac{f(x) \cdot g\left(\sin \frac{h}{x}\right)}{h}$

$f'(x) = \frac{f(x)}{x}$

$f(x) = x$

Hence, only option (b) is correct.

**78.** (b) Given, $|\mathbf{a} + \mathbf{b}| = 1$

$|\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}\cdot\mathbf{b} = 1 \quad (\because |\mathbf{a}| = |\mathbf{b}| = 1)$

$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

$\Rightarrow \quad \{(\mathbf{a}\times\mathbf{b})\times\mathbf{a} = \mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{a}\} \quad (\because \mathbf{a}\cdot\mathbf{a} = 1)$

$\Rightarrow \quad \{(\mathbf{a}\times\mathbf{b})\times\mathbf{a}\}\times\mathbf{a} = \mathbf{b}\times\mathbf{a} \quad (\because \mathbf{a}\times\mathbf{a} = 0)$

$\Rightarrow \quad \{\{(\mathbf{a}\times\mathbf{b})\times\mathbf{a}\}\times\mathbf{a}\}\times\mathbf{a} = (\mathbf{a}\cdot\mathbf{b})\mathbf{a} - \mathbf{b}$

$\{\{\{(\mathbf{a}\times\mathbf{b})\times\mathbf{a}\}\times\mathbf{a}\}\times\mathbf{a}\}\times\mathbf{b} = -\frac{1}{2}\mathbf{a}\times\mathbf{b}$

Now,

$|\{\{\{(\mathbf{a}\times\mathbf{b})\times\mathbf{a}\}\times\mathbf{a}\}\times\mathbf{a}\}\times\mathbf{b}| = \frac{1}{2}|\mathbf{a}\times\mathbf{b}|$

$= \frac{1}{2}|\mathbf{a}||\mathbf{b}|\sin\theta|$

$= \frac{1}{2}\times 1\times 1\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

**79.** (a) Given, $\tan^2(\sin^{-1}x) > 1$

$\Rightarrow \quad \frac{\pi}{4} < \sin^{-1}x < \frac{\pi}{4}$

or $\quad -\frac{\pi}{2} < \sin^{-1}x < -\frac{\pi}{4}$

$\Rightarrow \quad x \in \left(\frac{1}{\sqrt{2}}, 1\right)$

or $\quad x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad x \in \left(-1, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$

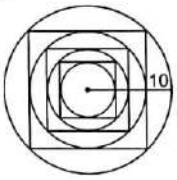
**80.** (a) Area of first circle,

$C_1 = \pi(10)^2 = 100\pi$

$\because$ Diameter of circle,

$d_1 = \sqrt{2}$ Side of square

$20 = \sqrt{2}(a) \Rightarrow a = 10\sqrt{2}$




Area of square

$S_1 = (10\sqrt{2})^2 = 200$

Area of $C_n = \frac{100\pi}{2^{n-1}}$

Area of $S_n = \frac{200}{2^{n-1}}$

$\frac{100\pi}{2^{n-1}} + \frac{200}{2^{n-1}} > 1$

$\Rightarrow \quad \frac{1}{2^n}\left[200\times\frac{22}{7} + \frac{400}{1}\right] > 1$

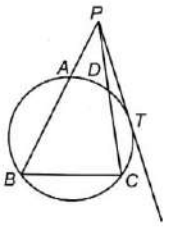
$2^n < 1028$

$n < 11$

**81.** (c) $PA \times PB = PD \times PC = PT^2$

Applying AM-GM

$\frac{1}{PA} + \frac{1}{PB} > \frac{2}{\sqrt{PA\times PB}} > \frac{2}{PD\times PC}$




$\Rightarrow \quad \frac{1}{PA} + \frac{1}{PB} > \frac{2}{\sqrt{PD\times PC}}$

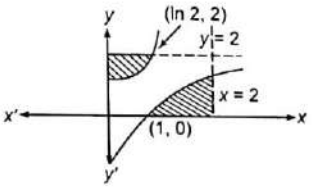
$\Rightarrow \quad \frac{1}{PA} + \frac{1}{PB} > \frac{2}{PT}$

$\Rightarrow$ Similarly, $\frac{1}{PD} + \frac{1}{PC} > \frac{2}{PT}$

$\Rightarrow \quad \frac{1}{PA} + \frac{1}{PB} + \frac{1}{PD} + \frac{1}{PC} > \frac{4}{PT}$

**82.** (d) $\therefore$ Required area = Shaded region

$= \int_0^{\ln 2} (2 - e^x)\,dx + \int_1^2 \ln x\,dx$




$= [2x - e^x]_0^{\ln 2} + [x\ln x - x]_1^2$

$= 2\ln 2 - 2 - (0 - 1) + [2\ln 2 - 2 - (0 - 1)]$

$= 4\ln 2 - 2$

**83.** (d) Maximum number of intersections

$$= 2 \times {}^4C_2 + 4 \times {}^4C_2 + 4 \times {}^4C_1\, {}^4C_1$$

$$= 12 + 24 + 64 = 100$$

**84.** (a) ${}^{99}C_{51} + {}^{99}C_{50} = {}^{100}C_{51} = {}^{n}C_r$

$(\because {}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1})$

$\Rightarrow \quad n = 100$ and $r = 51$

$\therefore \quad \dfrac{n+2}{r} = \dfrac{100+2}{51} = \dfrac{102}{51} = 2$

**85.** (b) We have,

$$\frac{d}{dx}(x^2) = \lim_{h \to 0} \frac{(x+h)^2\, 2(x+h) - x^2 \cdot 2x}{h}$$

$$= \lim_{h \to 0} \frac{2[(x+h)^3 - x^3]}{h}$$

$$= \lim_{h \to 0} \frac{2h[(x+h)^2 + x^2 + x(x+h)]}{h}$$

$$= 6x^2$$

**86.** (a) $\dfrac{d}{dx}(\sin x) = \lim\limits_{h \to 0} \dfrac{\left[\begin{matrix}\sin(x+h)\cos(x+h) \\ -\sin x \cos x\end{matrix}\right]}{h}$

$$= \lim_{h \to 0} \frac{\sin(2x+2h) - \sin 2x}{2h}$$

$$= \lim_{h \to 0} \frac{\cos(2x+h)\sin h}{h} = \cos 2x$$

**87.** (a) Using AM ≥ GM

$$\frac{\sin^3 x + \cos^3 y + \dfrac{3}{\sin x} + \dfrac{3}{\cos y}}{8}$$

$$\geq \sqrt[8]{\sin^3 x \times \cos^3 y \times \left(\frac{1}{\sin x}\right)^3 \left(\frac{1}{\cos y}\right)^3} = 1$$

$\therefore$ Minimum value = 8

**88.** (c) $y = \dfrac{(x^2 - x + 1)(y+1)}{(x^2 + x + 1)(y+1)} = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow \quad yx^2 + yx + y = x^2 - x + 1$

$\Rightarrow \quad x^2(y-1) + x(y+1) + (y-1) = 0$

For $x$ to be real

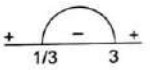
$$(y+1)^2 - 4(y-1)(y-1) \geq 0$$

$\Rightarrow \quad y^2 + 1 + 2y - (4y^2 - 8y + 4) \geq 0$

$\Rightarrow \quad 3y^2 - 10y + 3 \leq 0$

$\Rightarrow \quad 3y(y-3) + 1(y-3) \leq 0$

$\Rightarrow \quad (3y-1)(y-3) \leq 0$




$\Rightarrow \quad \dfrac{1}{3} \leq y \leq 3$

Hence, maximum value of $y$ is 3.

**89.** (b) I. Centre of the given circle is $C(2, 1)$ and radius is 5.

$\therefore$ Distance of $P(10, 7)$ from

$$C(2, 1) = \sqrt{(10-2)^2 + (7-1)^2}$$

$$= \sqrt{64 + 36} = 10$$

$\therefore$ Greatest distance = 5 + 10 = 15

and minimum distance = 10 − 5 = 5

II. It is also true but it is not a correct explanation of statement I.

**90.** (c) Statement I is true but statement II is not a true statement because for biconditional we use $\Leftrightarrow$ logical symbol.

# JEE Main

## Joint Entrance Examination

# Practice Set 6

## Instructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

1. Direction of magnetic field at equatorial point is
   (a) parallel to **M** (b) perpendicular to **M**
   (c) making an angle of 45° with **M** (d) anti-parallel to **M**

2. The magnetic needle lying parallel to the magnetic field requires $W$ units of work to rotate it through 60°. The torque needed to maintain the needle in this position is
   (a) $3W$ (b) $\sqrt{3}W$ (c) $\frac{W}{3}$ (d) $\frac{W}{\sqrt{3}}$

3. **Statement I** The lightning conductor at the top of high building has sharp pointed ends.
   **Statement II** The surface density of charge at sharp points is very high resulting in setting up of electric wind.
   (a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I.
   (b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement II.
   (c) Statement I is true but Statement II is false.
   (d) Both Statement I and Statement II are false.

4. A spring of spring constant $5 \times 10^3$ N-m$^{-1}$ is stretched initially by 5 cm from the unstretched position. Then, the work required to stretch it further by another 5 cm is
   (a) 12.50 N-m (b) 18.75 N-m (c) 25.00 N-m (d) 6.25 N-m

5. Two spheres of equal masses, one of which is a thin spherical shell and the other a solid, have the same moment of inertia about their respective diameters. The ratio of their radii will be
   (a) 5 : 7 (b) 3 : 5 (c) $\sqrt{3} : \sqrt{5}$ (d) $\sqrt{3} : \sqrt{7}$

6. **Statement I** $I_S$ and $I_H$ are the moments of inertia about the diameters of a solid and thin walled hollow sphere respectively. If the radii and the masses of the above spheres are equal, $I_H > I_S$.
**Statement II** In solid sphere, the mass is continuously and regularly distributed about the centre whereas the mass, to a large extent, is concentrated on the surface of hollow sphere.
(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I.
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement II.
(c) Statement I is true but Statement II is false.
(d) Both Statement I and Statement II are false.

7. Radius of orbit of satellite of earth is $R$. Its kinetic energy is proportional to
(a) $\frac{1}{R}$ (b) $\frac{1}{\sqrt{R}}$ (c) $R$ (d) $\frac{1}{R^{3/2}}$

8. **Statement I** Water in a U-tube executes SHM, the time period for mercury filled up to the same height in the U-tube be greater than that in case of water.
**Statement II** The amplitude of an oscillating pendulum goes on increasing.
(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I.
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement II.
(c) Statement I is true but Statement II is false.
(d) Both Statement I and Statement II are false.

9. The period of oscillation of a simple pendulum of constant length at surface of the earth is $T$. Its time period inside a mine will be
(a) cannot be compared (b) equal to $T$
(c) less than $T$ (d) more than $T$

10. A body is projected vertically upwards with a velocity $u$. It crosses a point in its journey at a height $h$ twice, just after 1 s and 7 s. The value of $u$ in $ms^{-1}$ is
(Take $g = 10\ ms^{-2}$)
(a) 50 (b) 40 (c) 30 (d) 20

11. A body moving along a circular path of radius $R$ with velocity $v$, has centripetal acceleration $a$. If its velocity is made equal to $2v$, then its centripetal acceleration is
(a) $4a$ (b) $2a$ (c) $\frac{a}{4}$ (d) $\frac{a}{2}$

12. If $a_r$ and $a_t$ represent radial and tangential accelerations, the motion of a particle will be uniformly circular if
(a) $a_r = 0$ and $a_t = 0$ (b) $a_r = 0$ but $a_t \neq 0$
(c) $a_r \neq 0$ but $a_t = 0$ (d) $a_r \neq 0$ and $a_t \neq 0$

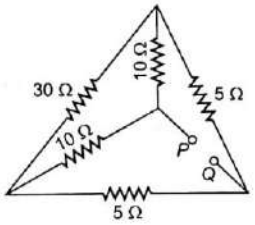
13. The voltage gain of an amplifier with 9% negative feedback is 10. The voltage gain without feedback will be
(a) 90 (b) 10 (c) 1.25 (d) 100

14. The energy that should be added to an electron to reduce its de-Broglie wavelength from 1 nm to 0.5 nm is
(a) four times the initial energy (b) equal to the initial energy
(c) twice the initial energy (d) thrice the initial energy

15. A bomb explodes on the moon. How long will it take for its sound to reach the earth?
(a) 10 s (b) 1000 s (c) 1 day (d) None of these

16. If the temperature difference on the two sides of a wall increases from 100°C to 200°C, its thermal conductivity
(a) remains unchanged (b) is doubled
(c) is halved (d) becomes four times

17. The equivalent resistance between the points $P$ and $Q$ in the network shown in the figure is given by

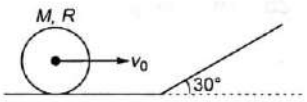



(a) 2.5 Ω
(b) 7.5 Ω
(c) 10 Ω
(d) 12.5 Ω

18. The internal resistance of a cell of emf 4 V is 0.1 Ω. It is connected to a resistance of 3.9 Ω. The voltage across the cell will be
(a) 3.9 V (b) 2 V (c) 0.1 V (d) 3.8 V

19. The deflection in a moving coil galvanometer falls from 50 divisions to 10 divisions, when a shunt of 12 Ω is connected with it. The resistance of galvanometer coil is
(a) 24 Ω (b) 12 Ω (c) 6 Ω (d) 48 Ω

**Directions** (Q. Nos. 20-22) *A solid ball of mass M and radius R is sliding on a smooth horizontal surface with velocity $v_0$ of shown in figure solely and smoothly it comes on the inclined plane of inclination* 30°.



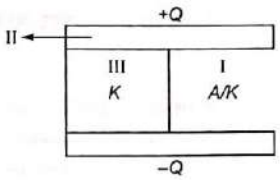

*Based on the above information answer the following questions :*

20. If incline is smooth, then
(a) ball will perform pure rolling motion
(b) ball will perform impure rolling motion on incline
(c) ball will slide down on incline
(d) ball will perform pure rolling motion on incline if it was initially in pure rolling motion on the horizontal surface

21. If incline is rough, then
(a) ball can perform pure rolling motion for some time
(b) ball can perform impure rolling motion for some time
(c) friction will aid the rotational motion
(d) All of the above

22. Mark out the correct statement(s).
(a) If incline is smooth, then the maximum height attained by the ball is $\frac{v_0^2}{2g}$.
(b) If initially ball is performing pure rolling motion on horizontal surface and incline is rough enough to prevent any slipping, then the maximum height attained by the ball is $\frac{7v_0^2}{10g}$.
(c) If initially ball is performing pure rolling motion on horizontal surface and the incline is smooth, then maximum height attained by the ball is $\frac{v_0^2}{2g}$.
(d) All of the above

**Directions** (Q. Nos. 23-25) *A parallel plate capacitor is as shown in figure. Half of the region in-between the plates of the capacitor is filled with a dielectric material of dielectric constant K and in the remaining half air is present. The capacitor is given a charge Q with the help of a battery. Some surfaces are marked on the figure.*




*Surface I is the right half inner surface of the upper plate of capacitor in the region in which air is present.*

*Surface II is the left half inner surface of the upper plate of capacitor.*

*Surface III is the surface of the dielectric slab which is near to the upper plate of capacitor.*

*Based on above information answer the following questions :*

23. Mark out the correct statement(s).
   (a) The electric field in region with dielectric is greater than that of electric field in the air-filled region.
   (b) The electric field in the region with dielectric is less than that of electric field in the air-filled region.
   (c) The electric field in the region with dielectric is equal to that of electric field in the air-filled region.
   (d) Nothing can be predicted about electric field from the given information.

24. The charge on surface I is
   (a) $\frac{Q}{2}$ (b) $\frac{Q}{1+K}$ (c) $\frac{KQ}{1+K}$ (d) $\frac{Q}{K-1}$

25. The charge on surface II is
   (a) $\frac{KQ}{1+K}$ (b) $\frac{K^2Q}{1+K}$ (c) $\frac{-K^2Q}{1+K}$ (d) $\frac{-K(K-1)Q}{1+K}$

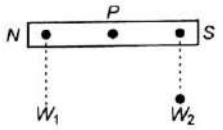
**Directions** (Q. Nos. 26-28) *For the following questions. Choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*
   (a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.
   (b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.
   (c) Statement I is true, Statement II is false.
   (d) Statement I is false, Statement II is true.

26. **Statement I** Charge is invariant in nature *i.e.*, in different inertial frames charge on an object is same.
   **Statement II** A gas consisting of hdyrogen molecules is neutral in nature.

27. **Statement I** If we consider an inertial frame $S$ in which two identical charges move towards each other with same speed, then it is impossible to find another inertial frame $S'$ in which only one of the fields either electric or magnetic would be observed.
   **Statement II** For above described situation, it is impossible to have an inertial frame $S'$ in which both the charges are at rest.

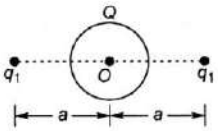
28. **Statement I** In Doppler effect, if the detector is stationary and the source is moving with constant velocity, then the apparent frequency as received by detector must be constant.
   **Statement II** In Doppler effect expression, $f_{AP} = f\left[\frac{v - v_d}{v - v_s}\right]$, where symbols have their usual meanings, $v_s$ represents the component of velocity of source along the line joining source and detector, at the instant when source emits the wave which is received by detector at some later instant $t_0$, and at this instant the detector receives the frequency $f_{AP}$.

29. Figure shows a bar magnet and two infinite long wires $W_1$ and $W_2$ carrying equal currents in opposite directions. The magnet is free to move and rotate. $P$ is the mid-point of magnet. For this situation mark out the correct statement(s).




(a) Magnet experiences a net torque in clockwise direction and zero net force.
(b) Magnet experiences a net force towards left and a net torque in anti-clockwise direction.
(c) Magnet experiences a net force towards right and a net torque in anti-clockwise direction.
(d) Magnet experiences zero net force and a net torque in anti-clockwise direction.

30. A conducting spherical shell having charge $Q$, is placed near two point charges as shown in figure. Assume all charges to be +ve. For this situation mark out the correct statement(s).

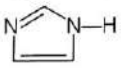
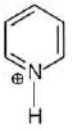
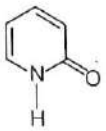



(a) The charge on outer surface of shell is uniformly distributed.
(b) The charge on outer surface of shell is non-uniformly distributed.
(c) The nature of distribution of charge on outer surface of shell cannot be predicted from the given information.
(d) None of the above

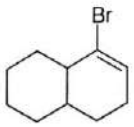
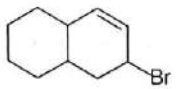
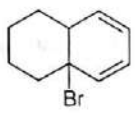
# Chemistry

31. What is the freezing point of a 10% (by weight solution of sodium chloride in water?
(a) $-7.06°C$ (b) $3.51°C$ (c) $-3.51°C$ (d) $10°C$

32. Photoelectric emission is observed from a surface for frequencies $\nu_1$ and $\nu_2$ of the incident radiation ($\nu_1 > \nu_2$). If the maximum kinetic energies of the photoelectrons in the two cases are in the ratio $1:k$ then the threshold frequency $\nu_0$ is given by
(a) $\frac{\nu_2 - \nu_1}{k-1}$ (b) $\frac{k\nu_1 - \nu_2}{k-1}$
(c) $\frac{k\nu_2 - \nu_1}{k-1}$ (d) $\frac{\nu_2 - \nu_1}{k}$

33. Identify the compound which is not aromatic.
(a) 
(b) 
(c) 
(d) 

34. The products formed when a solution of $CuSO_4$ is added to conc. solution of KI are
(a) $I_2 + Cu^{2+}$ (b) $CuI_2$ (c) $I_2 + Cu$ (d) $Cu_2I_2$

35. Which of the following bromide will undergo faster dehydrobromination?
(a) 
(b) 
(c) 
(d) 

36. An alloy of Cu, Ag and Au is found to have copper constituting the ccp lattice. If Ag atom occupy the edge centre and Au atom is present at body centre, the formula of this alloy is
(a) $Cu_4Ag_4Au$ (b) $Cu_4Ag_2Au$ (c) Au Ag Cu (d) $Cu_4Ag_3Au$

37. Which of the following cannot be used as an acylating agent?
(a) $CH_3COCl$ (b) $(CH_3CO)_2O$ (c) $CH_3COOH$ (d) $CH_3CH_2COCl$

38. The species having tetrahedral shape is
(a) $[PdCl_4]^{2-}$ (b) $[Ni(CN)_4]^{2-}$ (c) $[Pd(CN)_4]^{2-}$ (d) $[NiCl_4]^{2-}$

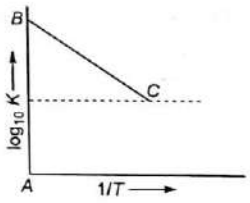
39. 16 g oxygen gas expands at STP to occupy double of its original volume. The work done during the process is
(a) 272.84 kcal (b) 260 kcal (c) 180 kcal (d) 130 kcal

40. An acid base reaction is observed when a compound '*X*' is treated with a base
Compound '*X*' + base $\longrightarrow$ salt
Compound '*X*' can be
(a)  (b)  (c)  (d) 

41. The following acids have been arranged in the order of decreasing acid strength. Identify the correct order
ClOH(I) BrOH(II) IOH (III)
(a) I > II > III (b) II > I > III (c) III > II > I (d) I > III > II

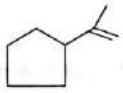
42. A gas can be liquefied most suitably at
(a) $T = T_c$ and $p < p_c$ (b) $T < T_c$ and $p = p_c$
(c) $T < T_c$ and $p > p_c$ (d) $T > T_c$ and $p > p_c$

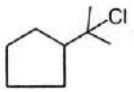
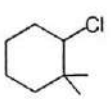
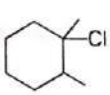
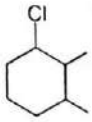
43. Figure shows a graph in $\log_{10} k$ *vs* $\frac{1}{T}$ where, $k$ is rate constant and $T$ is temperature. The straight line *BC* has slope, $\tan\theta = -\frac{1}{2.303}$ and an intercept of 5 on *y*-axis. Thus, $E_a$, the energy of activation, is




(a) 4.606 cal (b) $\frac{0.2}{2.303}$ cal
(c) 2 cal (d) None of these

44. The $[H^+]$ of a resulting solution that is 0.01 M acetic acid ($K_a = 1.8 \times 10^{-5}$) and 0.01 M in benzoic acid ($K_a = 6.3 \times 10^{-5}$) is
(a) $9 \times 10^{-4}$ (b) $81 \times 10^{-4}$ (c) $9 \times 10^{-5}$ (d) $2.8 \times 10^{-3}$

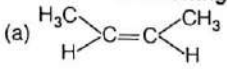
45.  $\xrightarrow[CCl_4]{HCl}$ Major product

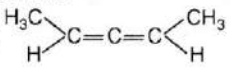
(a)  (b)  (c)  (d) 

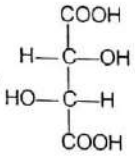
46. Which of the following trihalides of nitrogen behaves as the weakest base?
(a) $NF_3$ (b) $NCl_3$ (c) $NBr_3$ (d) $NI_3$

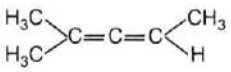
47. How much will the reduction potential of a hydrogen electrode change when its solution initially pH = 0 is neutralised to pH = 7?
(a) Increase by 0.059 V (b) Decrease by 0.059 V
(c) Increase by 0.41 V (d) Decrease by 0.41 V

48. Which of the following will not show stereoisomerism?

(a) 

(b) 

(c) 

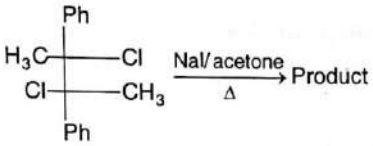
(d) 

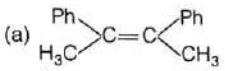
49. The compound insoluble in acetic acid is

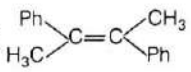
(a) calcium oxide (b) calcium carbonate (c) calcium oxalate (d) calcium hydroxide

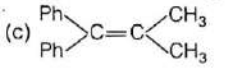
50. The arsenious sulphide sol has negative charge. The maximum coagulating power for precipitating it is of

(a) 0.1 N $Zn(NO_3)_2$ (b) 0.1 N $Na_3PO_4$ (c) 0.1 N $ZnSO_4$ (d) 0.1 N $AlCl_3$

51. 

(a) 

(b) 

(c) 

(d) No reaction

52. Which of the following cannot be oxidised by $H_2O_2$?

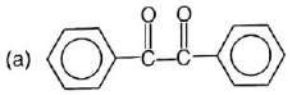
(a) KI + HCl (b) $O_3$ (c) PbS (d) $Na_2SO_3$

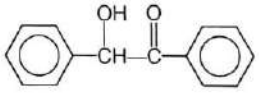
**Directions** (Q. Nos. 53-55) *An organic compound (A)* $C_7H_6O$ *gives positive test with Tollen's reagent. On treatment with alcoholic* $CN^{\ominus}$ *(A) gives the compound (B)* $C_{14}H_{12}O_2$. *Compound (B) on reduction with* Zn—Hg, HCl *and dehydration gives an unsaturated compound (C), which adds one mole of* $Br_2/CCl_4$. *The compound (B) can be oxidised with* $HNO_3$ *to a compound (D)* $C_{14}H_{10}O_2$. *Compound (D) on heating with* KOH *undergoes rearrangement and subsequent acidification of rearranged products yields an acidic compound (E)* $C_{14}H_{12}O_3$.

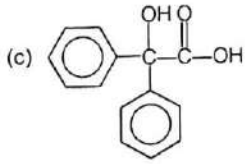
53. Compound (*A*) cannot undergo

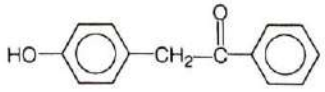
(a) benzoin condensation (b) aldol condensation
(c) Cannizzaro reaction (d) Perkin condensation

54. Structure of compound (*B*) is

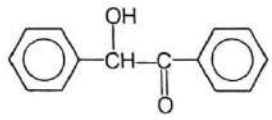
(a) 

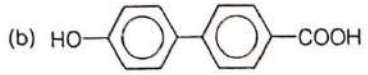
(b) 

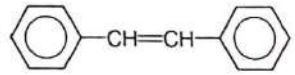
(c) 

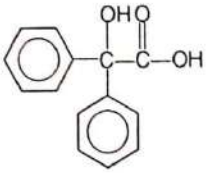
(d) 

55. Structure of compound (*E*) is

(a) 

(b) 

(c) 

(d) 

**Directions** (Q.Nos. 56-58) *A given sample of* $N_2O_4$ *in a closed vessel shows 20% dissociation in* $NO_2$ *at 27°C and 1 atm. The sample is now heated up to 127°C and the analysis of the mixture shows 60% dissociation at 127°C.*

56. The total pressure of equilibrium mixture in atm at 127°C is
(a) 1.78 atm (b) 2.01 atm (c) 3.18 atm (d) 4.33 atm

57. The molecular weight of mixture at 27°C is
(a) 76.66 (b) 78.69 (c) 66.52 (d) 80.24

58. The equilibrium constant ($K_p$) for the decomposition of $N_2O_4$ at 27°C is
(a) 0.165 (b) 0.29 (c) 0.523 (d) 0.625

**Directions** (Q. Nos. 59 and 60) *For the following questions choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.
(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

59. **Statement I** The dissociation constants of a polyprotic acid are in the order $K_1 > K_2 > K_3$.
**Statement II** The $[H^+]$ furnished in first step of dissociation exerts common ion effect to reduce the second dissociation and so on.

60. **Statement I** The vapour pressure of 0.1 M sugar solution is more that of 0.1 M KCl solution.
**Statement II** Lowering of vapour pressure is directly proportional to the number of species present in the solution.

# Mathematics

61. Let $n>3$. The expression $pq^nC_0-(p-1)(q-1)\,{}^nC_1+(p-2)(q-2)\,{}^nC_2-(p-3)(q-3)\,{}^nC_3+\ldots+(-1)^n(p-n)(q-n)\,{}^nC_n$, when simplified reduces to
(a) $npq$ (b) $pq$ (c) $2^npq$ (d) 0

62. Let $f\left(\frac{x}{2008}+2009y,\frac{x}{2008}-2009y\right)=xy$, then $f(p,q)+f(q,p)=0$ holds
(a) when only $p=q$ (b) when only $p\neq q$
(c) when only $p+q=0$ (d) for all $p,q\in R$

63. Area of the region enclosed between the curves $x=y^2-1$ and $x=|y|\sqrt{1-y^2}$ is
(a) 2 sq units (b) 2/3 sq unit (c) 1 sq unit (d) 4/3 sq units

64. An ellipse slides between two lines at right angle to one another. The locus of its centre is
(a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

65. Let normal at any point $P$ to the rectangular hyperbola meet the axes in $G$ and $g$ and $C$ be the centre of the hyperbola. Then, which of the following is true
(a) $PG=PG^2$ (b) $PG=PC$ (c) $PG=Gg$ (d) $PC=Cg$

66. Let $A$ be a $n\times n$ matrix such that $a_{ij}=\sin^{-1}(\sin(\mathbf{i}-\mathbf{j}))\ \forall\ \mathbf{i}$ and $\mathbf{j}$. Which of the following is true?
(a) If $n$ is odd, then $A$ is an invertible matrix. (b) If $n$ is even, then $A$ is not an invertible matrix.
(c) For all values of $n$, $A$ is not invertible matrix. (d) $A$ is a skew-symmetric matrix.

67. Let, $S$ be a relation on $R^+$ defined as $xSy\Leftrightarrow x^2-y^2=2(y-x)$. Then, $S$ is
(a) reflexive on $R^+$ (b) symmetric on $R^+$
(c) antisymmetric on $R^+$ (d) equivalence relation on $R^+$

68. Let $f:N\times N$ satisfying $f(0)=0, f(1)=1, f(n)=f(n-1)+f(n-2), n\geq 2$, then which of the following is true?
(a) $fof(2)=2$ (b) $fof(2)=1$
(c) $f(S_n)$ is divisible by 9 (d) $fof(5)=1$

69. If the mean and standard deviation of 20 observations $X_1, X_2, X_3, \ldots, X_{20}$ are 50 and 10 respectively, then $\sum_{i=1}^{20}X_i^2$ is equal to
(a) 2600 (b) 52000 (c) 2510 (d) None of these

70. Let $\int_0^1 e^{x^2}dx=k$, the sum of all possible values of $[k]$, where $[\cdot]$ denotes the greatest integer function
(a) 1 (b) 2 (c) 3 (d) 4

71. The distance between a point with position vector $5\mathbf{i}+\mathbf{j}+3\mathbf{k}$ and the line $\mathbf{r}=(3\mathbf{i}+7\mathbf{j}+\mathbf{k})+\lambda(\mathbf{j}+\mathbf{k})$ is
(a) 10 (b) 9 (c) 7 (d) 6

72. The minimum and maximum distance of a point (1, 2) from the ellipse $4x^2+9y^2+8x-36y+4=0$ are $k$ and $K$, then $(K-k)$ is equal to
(a) 4 units (b) 5 units (c) 6 units (d) 7 units

73. The slope of the tangent to the curve $y=f(x)$ at $(x,f(x))$ is $2x+1$. If the curve passes through the point $(1,2)$ and the area of the region bounded by the curve, the $x$-axis and the line $x=1$ is $k$ sq units, then $6k$ is equal to
(a) 4 (b) 5 (c) 7 (d) 8

74. If $p,q$ and $r$ are simple propositions with truth values T, F and T, then the truth value of $(\sim p \vee q) \wedge \sim p \Rightarrow p$ is
(a) F (b) T (c) T, if $r$ is F (d) None of these

75. The number of solutions of the system of equations $\{x\}+y+[z]=2.3$, $x+[y]+\{z\}=4.5$ and $[x]+\{y\}+z=6.2$, where $\{\cdot\}$ and $[\cdot]$ denote fractional part and greatest integer function respectively
(a) 0 (b) 1 (c) 2 (d) 3

76. Let $a=(\sin^{-1}x)^{\sin^{-1}x}$, $b=(\sin^{-1}x)^{\cos^{-1}x}$, $c=(\cos^{-1})^{\sin^{-1}x}$, $d=(\cos^{-1}x)^{\cos^{-1}x}$ and if $x\in(0,1)$, then
(a) $a>b>d>c$ (b) $d>c>a>b$
(c) $b>a>d>c$ (d) $a<b<d<c$

77. For the system of linear equation $kx+y+z=1$, $x+ky+z=k$ and $x+y+kz=k^2$. If $k=1$, then equation has
(a) unique solution (b) infinitely many solutions
(c) no solution (d) trivial solution

78. The value of $\lim\limits_{x\to\infty}(\sqrt{3x^2+\sqrt{3x^2+\sqrt{3x^2}}}-\sqrt{3x^2})$ is
(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{3}{2}$ (d) $\sqrt{3}$

79. The area of parallelogram formed by the tangents at the ends of conjugate diameters for the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$ is
(a) 16 sq units (b) 8 sq units (c) 32 sq units (d) $18\sqrt{3}$ sq units

80. If $|z|=5$, then the area of the triangle whose length of the sides are equal to $|z|$, $|\beta z|$ and $|z+\beta z+\beta^2 z+\beta^3 z|$ (where $\beta$ is fifth root of unit $y$ and $z$ be a complex number) is
(a) $\frac{25\sqrt{3}}{2}$ (b) $\frac{25\sqrt{3}}{4}$ (c) $\frac{75\sqrt{3}}{2}$ (d) $\frac{75\sqrt{3}}{4}$

81. The value of $\left(\frac{1}{\log_{\sqrt{3}}\frac{1}{5}}+\frac{1}{\log_{\sqrt{4}}\frac{1}{5}}\right)$ lies in the interval
(a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (−1, 0)

82. The number of solution(s) of the equation $\cos\frac{\pi x}{3\sqrt{3}}=x^2+4-2x\sqrt{3}$ will be
(a) 1 (b) 2 (c) infinitely many (d) 0

83. Let axis and latusrectum of a parabola intersect at (1, 1). If the equation of directrix is $5x+12y+22=0$, then the length of latus rectum is
(a) 3 (b) 6 (c) 9 (d) 12

84. The value of $\left[1+2\left(1+\frac{1}{\alpha}\right)+3\left(1+\frac{1}{\alpha}\right)^2+\ldots \text{ upto 50 terms}\right]$ is given by

(a) $\frac{(1+\alpha)^{50}(50-\alpha)+\alpha^{51}}{\alpha^{49}}$ (b) $\frac{(1+\alpha)^{51}(50-\alpha)+\alpha^2}{\alpha^{50}}$

(c) $\left(\frac{1+\alpha}{\alpha}\right)^{50}+\alpha^2$ (d) $\left(\frac{50-\alpha}{\alpha}\right)+\alpha^2$

**Directions** (Q. Nos. 85 and 86) *The geometrical meaning of* $|z_1-z_2|$, *where* $z_1$ *and* $z_2$ *are points in Argand plane is the distance between the points* $z_1$ *and* $z_2$ *based on this information, a class of problems about least value can be solved. The property that the sum of two sides of a triangle is greater than the third side is also very useful in solving these problems.*

85. The least value of $|z-2+2i|+|z-3|$, $z$ being a complex number, is

(a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) $2+\sqrt{5}$ (d) $1+\sqrt{5}$

86. The least value of $|z+i|+|z+3i|+|2-z|+|-z-7i|$, $z$ being a complex number, is

(a) $1+\sqrt{7}$ (b) $\sqrt{13}$ (c) $2+\sqrt{13}$ (d) $\sqrt{7}+\sqrt{13}$

**Directions** (Q. Nos. 87 and 88) *For the existence of limit at* $x=a$ *of* $y=f(x)$ *it must be true that* $\lim_{h\to 0} f(a-h)=\lim_{h\to 0} f(a+h)$. *Here,* $x=a$ *is not the end point of the interval,* $\lim_{h\to 0} f(a-h)$ *is called LHL and* $\lim_{h\to 0} f(a+h)$ *is called RHL.*

87. The value of limit $\lim_{x\to\frac{\pi}{2}}[(\sin x)]$, where $[\cdot]$ denotes hte greater integer function is

(a) 0 (b) Does not exist (c) $-1$ (d) 1

88. $\lim_{x\to 0}\left[\frac{\sin x}{\tan x}\right]$, where $[\cdot]$ denotes greatest integer function, is

(a) 0 (b) 1 (c) $-1$ (d) not in existence

**Directions** (Q. Nos. 89 and 90) *For the following questions. Choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

89. Let us define a function $f(x)=\sin\sqrt{2}x+\sin ax$

**Statement I** The maximum value of $f(x)$ cannot be 2, (where $a$ is positive rational number)

**Statement II** $\frac{\sqrt{2}}{a}$ is irrational.

90. Consider the equation of circle is $S\equiv x^2+y^2=a^2$

**Statement I** The chord of contact of tangent from these points $A, B, C$ to the circle $S$ are concurrent, then $A, B, C$ will be collinear.

**Statement II** $A, B, C$ always lies on the normal to the circle $S$.

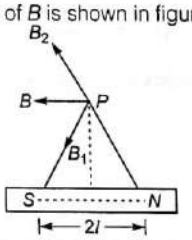
# Answer *with* Explanations

## Physics

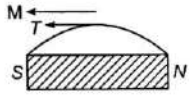
**1.** (a) Magnetic field due to a magnet at any point or equatorial line is given by

$$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

Direction of $B$ is shown in figure.




Hence, in equatorial position, the direction of magnetic field is parallel to magnetic axis direction is from north pole to south pole *i.e.*, parallel to **M**. We can also think about a tangent on magnetic induction curve from north pole of a magnet to the south pole of same magnet as shown in figure.




**2.** (b) The instantaneous moment of the deflecting couple or torque acting on the needle is

$\tau$ = force × perpendicular distance = work done

When axis of needle makes an angle $\theta$ with the magnetic field, then for magnetic moment $M$ and magnetic field $B$, we have

$$\tau = MB\sin\theta \quad ...(i)$$

$$W = MB\cos\theta \quad ...(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\tau}{W} = \frac{MB\sin\theta}{MB\cos\theta}$$

Given, $\theta = 60°$

$$\therefore \quad \tau = W\frac{\sin 60°}{\cos 60°} = W\sqrt{3}$$

**3.** (a) We know that surface density of charge is very large on the sharp ends of a conductor. This charge from pointed ends sets up a charged electric wind. This charged electric wind comes in contact with the charged clouds and then source of its charge is neutralised and so potential drops between building and clouds. Hence, chances of lightning on building is reduced.

Even if lightning strikes the building, charge is conducted by the lightning conductor to the earth and there is no harm to building.

**4.** (b) Magnitude of work done $|W_1| = \frac{1}{2}k \times x_1^2$

= Magnitude of elastic potential energy stored in spring block system.

$$= \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2$$

$$\Rightarrow \quad |U_1| = 6.25 \text{ J}$$

Now trually

$$|W_2| = \frac{1}{2}k(x_1 + x_2)^2$$

$$= \frac{1}{2} \times 5 \times 10^3 (5 \times 10^{-2} + 5 \times 10^{-2})^2$$

$$|U_2| = 25 \text{ J}$$

Net work done $= |W_2| - |W_1| = |U_2| - |U_1|$

$= 25 - 6.25$

$= 18.75 \text{ J} = 18.75 \text{ N-m}$

**5.** (c) Let the radii of the thin spherical shell and the solid sphere are $R_1$ and $R_2$ respectively.

Then, the moment of inertia of the spherical sheli about one of their diameters

$$I = \frac{2}{3}MR_1^2 \quad ...(i)$$

and the moment of inertia of the solid sphere about one of their diameters (respective to first one) is given by

$$I = \frac{2}{5}MR_2^2 \quad ...(ii)$$

It is given that the masses and moment of inertia for both the bodies are equal, then from Eqs. (i) and (ii)

$$\frac{2}{3}MR_1^2 = \frac{2}{5}MR_2^2$$

$$\Rightarrow \quad \frac{R_1^2}{R_2^2} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \sqrt{\frac{3}{5}}$$

$$\Rightarrow \quad R_1 : R_2 = \sqrt{3} : \sqrt{5}$$

**6.** (a) The moment of inertia of solid sphere about its any diameter

$$I_S = \frac{2}{5}MR^2$$

The moment of inertia of a thin walled hollow sphere about its diameter is

$I_H = \frac{2}{5}M\frac{(R_2^5 - R_1^5)}{(R_2^3 - R_1^3)}$, where $R_1$ and $R_2$ are its internal and external radii

$$\therefore \quad I_H > I_S$$

The reason is that in solid sphere the whole mass is uniformly and continuously distributed about its centre in the entire volume while in hollow sphere the mass is distributed on the surface of sphere.

**7.** (a) Kinetic energy of the satellite

$$KE = \frac{1}{2}mv_0^2$$

where, $v_0 = \sqrt{\left(\frac{GM}{R}\right)}$

Now, putting the value of $v_0$ in Eq. (i), we get

$$KE = \frac{1}{2}m\left(\sqrt{\left(\frac{GM}{R}\right)}\right)^2$$

$$= \frac{1}{2}\frac{mGM}{R}$$

Hence, $KE \propto \frac{1}{R}$

**8.** (d) The period of the liquid executing SHM in a U-tube does not depend upon the density of the liquid. Therefore, time period will be the same, when mercury is filled up to the same height as the water in the U-tube.

Now, as the pendulum oscillates, it drags air along with it. Therefore, its kinetic energy is dissipated in overcoming viscous drag due to air and hence, its amplitude goes on decreasing.

**9.** (d) The time-period ($T$) of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

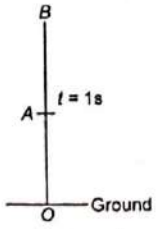
$$\Rightarrow \quad T \propto \frac{1}{\sqrt{g}}$$

Let mine be a depth $h$ below the surface of earth of radius $R$, then

At depth $h$, $g' = g\left(1 - \frac{h}{R}\right)$ hence, $g$ decreases.

$\left(\because \frac{h}{R} < 1\right)$

**10.** (b) The situation is shown in the figure. In the upward journey, the body is at $A$ at $t = 1$ s and in downward journey it is at $A$ at $t = 7$ s. Hence, we can conclude that during upward journey, the body will be at $B$ at $t = 4$ s.




Thus, $v = u - gt$

$$0 = u - 10 \times 4 \qquad (\because v = 0)$$

$$\therefore \quad u = 40\ ms^{-1}$$

**11.** (a) The centripetal acceleration

$$a = \frac{v^2}{R}$$

or $\quad a \propto v^2$

or $\quad \frac{a_1}{a_2} = \frac{v_1^2}{v_2^2}$

$$\therefore \quad \frac{a}{a_2} = \frac{v^2}{(2v)^2} = \frac{1}{4}$$

or $\quad a_2 = 4a$

**12.** (c) (a) If $a_r = 0$ and $a_t = 0$, then motion is uniform translatory

(b) If $a_r = 0$ but $a_t \neq 0$, then motion is accelerated translatory

(c) If $a_r \neq 0$ but $a_t = 0$, then motion is a uniform circular

(d) If $a_r \neq 0$ and $a_t \neq 0$, then motion is a non-uniform circular

**13.** (d) The process of injecting a fraction of output energy of some device back to the input is known as feedback. When the feedback energy (voltage or current) is out of phase with the input signal and thus opposes it, it is called negative feedback.

Voltage gain with feedback is

$$A_{Vf} = \frac{A_V}{1 + \beta A_V}$$

where, $A_V$ is voltage gain without feedback and $\beta$ is negative feedback.

Given, $\quad A_{Vf} = 10,\ \beta = 9\% = \frac{9}{100}$

$$\therefore \quad 10 = \frac{A_V}{1 + \frac{9}{100}A_V}$$

or $\quad 10 + \frac{9}{10}A_V = A_V$

or $\quad 0.1A_V = 10$ or $A_V = 100$

**14.** (d) de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \quad \frac{1 \times 10^{-9}}{0.5 \times 10^{-9}} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \quad 2 = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \quad \frac{E_2}{E_1} = 4$$

$$\therefore \quad E_2 = 4E_1$$

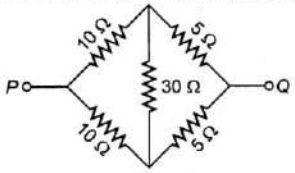
$\therefore$ Energy to be added $= E_2 - E_1$

$$= 4E_1 - E_1 = 3E_1$$

15. (d) Surface both are smaller compared to earth. Therefore the escape velocity at moon is 2.38 $kms^{-1}$. At the temperature of moon, the average velocity of gas molecules greater than this. Therefore, gas molecules cannot stay at moon and escaped out. Hence, there is no atmosphere around the moon.
Since, sound waves need a medium for propagation therefore, sound produced by the bomb explosion will never reach the earth.

16. (a) The coefficient of thermal conductivity of a material is the amount of heat flowing in 1 s through a rod of that material of 1 m length and 1 $m^2$ area of cross-section in the steady state; when the difference of temperatures between the two ends of the rod is 1°C and the flow of heat is normal to the end faces of the rod. Coefficient of thermal conductivity $K$ is a constant, whose value depends upon the material of the rod.
Hence, even if temperature increases or decreases, its thermal conductivity remains unchanged.

17. (b) We can show the network as below




From the circuit $\frac{10}{5} = \frac{10}{5}$ *i.e.*, $2 = 2$

So, it is balanced Wheatstone's bridge. Therefore, resistance of its middle arm will remain behaves as open circuited (inactive). So the net resistance in upper arms

$$R_U = 10 + 5 = 15\,\Omega \text{ (series)}$$

The net resistance in lower arms

$$R_L = 10 + 5 = 15\,\Omega \text{ (series)}$$

Hence, equivalent resistance of the network

$$R = \frac{R_U \times R_L}{R_U + R_L} \text{ (parallel)}$$

$$= \frac{15 \times 15}{15 + 15} = 7.5\,\Omega$$

18. (a) Emf of a cell ($E$) = 4 V
Internal resistance of a cell ($r$) = 0.1 Ω
External resistance ($R$) = 3.9 Ω
The potential drop across the cell

$$V = E - I \cdot r \qquad \ldots(i)$$

Now, the total resistance of the circuit

$$R' = r + R$$

$$R' = 0.1 + 3.9$$

$$\Rightarrow \quad R' = 4.0\,\Omega$$

Hence, current in the circuit is

$$I = \frac{E}{R'}$$

$$\Rightarrow \quad I = \frac{4}{4} = 1\text{ A}$$

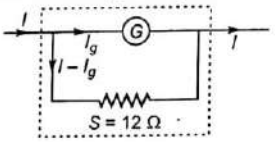
Now, from Eq. (i)

$$V = 4 - 1 \times 0.1$$

$$V = 4 - 0.1$$

$$V = 3.9 \text{ volt}$$

19. (d) Let the value of current for one division of galvanometer is $i$.
Then, the current through the ammeter ($I$) = $50i$ and current through the galvanometer ($I_g$) = $10i$




As we know that,

$$S = \frac{I_g}{(I - I_g)} G$$

$$\Rightarrow \quad G = S\left(\frac{I - I_g}{I_g}\right)$$

$$= 12 \times \left[\frac{50i - 10i}{10i}\right]$$

$$= 12 \times \left[\frac{40i}{10i}\right] = 48\,\Omega$$

20. (b) If incline is smooth, and the ball is initially not performing any rolling motion, then it continues to be in state of pure translational motion *i.e.*, sliding. From this we can also conclude that friction is necessary to cause the rolling motion.

21. (d) If incline is rough, then as the ball comes on incline the friction force comes into existence and the ball starts the rolling motion, thus we can say that friction aids the rotational motion component of rolling motion. And depending on the value of coefficient of friction, the ball can perform pure rolling or impure rolling motion.
Initially the ball performs impure rolling motion for some time and later on the ball can perform pure rolling motion if friction is enough to prevent slipping.

22. (d) If incline is smooth, then from energy conservation,

$$0 - \frac{mv_0^2}{2} = -mgh$$

$$\Rightarrow \quad h = \frac{v_0^2}{2g}$$

**For option (b)** $0 - \left(\frac{mv_0^2}{2} + \frac{I\omega_0^2}{2}\right) = -mgh$ where, $I = \frac{2}{5}mR^2$ and $\omega_0 = \frac{v_0}{R}$

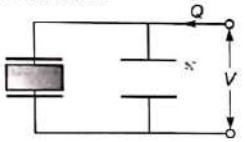
$$\Rightarrow \qquad h = \frac{7v_0^2}{10g}$$

**For option (c)** In this case the rotation kinetic energy of ball is not changing as ball moves up the incline as there is no friction and hence from work-energy theorem,

$$\frac{I\omega_0^2}{2} - \left(\frac{mv_0^2}{2} + \frac{I\omega_0^2}{2}\right) = -mgh$$

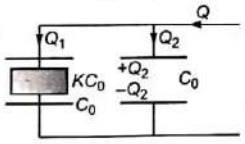
$$\Rightarrow \qquad h = \frac{v_0^2}{2g}$$

**23.** (c) The two halves of the capacitor can be considered as two capacitors in parallel having capacitances of $KC_0$ and $C_0$, where $C_0$ is the capacitance of air-filled part of capacitor. As the two capacitors are connected in parallel, so the potential difference across these two capacitors are same and as separation between the plates of both the capacitors are some so the electric field in between the plates of both capacitors would be the same




**24.** (b) Divide the charge $Q$ on two capacitors.

$$Q_2 = \frac{Q \times C_0}{KC_0 + C_0} = \frac{Q}{K+1}$$




**25.** (d) The charge on capacitor of capacitance $KC_0$ is $Q_1 = \frac{QKC_0}{KC_0 + C_0} = \left(\frac{QK}{1+K}\right)$. This charge on the capacitor is shared by surfaces II and III. Let, charge on surface II be $q_1$ and on III, it be $q_2$.

$$q_1 + q_2 = Q_1$$

And from polarisation concept,

$$q_1 = -q_2\left(1 - \frac{1}{K}\right)$$

Solving above equations, we get

$$q_1 = -\frac{-K(K-1)Q}{1+K}$$

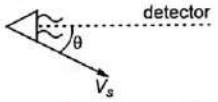
**26.** (b) Charges are always conversed for an isolated system. Electrons in hydrogen molecules move at much higher velocities than protons. Therefore, if the charge depends on the velocity, the charges of the electrons and protons would not compensate each other and the gas would be charged.

However, no charge has been observed in hydrogen gas upto a very good accuracy.

**27.** (b) The decomposition of electromagnetic field into its component electric and magnetic fields depends upon the frame of reference. If we are able to find a frame of reference in which both charges are at rest, then in this frame of reference the electromagnetic field is purely electrical in nature.

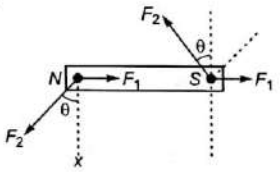
And in above case, there is no possibility that in any frame that both charges are all rest and hence, it is not possible that only one of the fields is existing.

**28.** (d) In shown case, detector is stationary while source is moving with constant velocity but the frequency as received by detector is changing as component of velocity of source along the line joining source and detector is changing.




$$f_{AP} = f\left[\frac{v - v_D}{v - v_s\cos\theta}\right]$$

**29.** (c) Both the poles of the magnet experience a force due to magnetic field produced by wires $W_1$ and $W_2$.

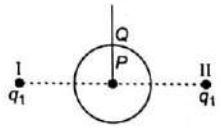



North pole experiences a force $F_1$ due to $W_1$, and $F_2$ due to $W_2$ as shown in figure. Similarly, south pole experiences a force $F_1$ due to $W_2$ and $F_2$ due to $W_1$.

From free body diagram of magnet, it is clear that magnet experiences a net force towards right and a torque in anti-clockwise direction.

**30.** (b) To solve this question we have to use the concept that "the resultant electric field at any point inside a conducting shell due to all outside charges is zero," and that "E at any internal point due to uniformly charged conducting spherical shell is zero."

Consider any point $P$ inside the shell, at $P$, the resultant electric field due to I, II and charge on outer surface of shell must be zero.




$$\mathbf{E}_P = \mathbf{E}_I + \mathbf{E}_{II} + \mathbf{E}_{\text{outer surface charge}} = 0$$

But $\mathbf{E}_I + \mathbf{E}_{II} \neq 0$, so $\mathbf{E}_{\text{outer surface charge}} \neq 0$, it means charge distribution is outer surface of shell is non-uniform.

# Chemistry

**31.** (a) 10% solution means, 1.00 kg water contains 0.100 kg NaCl and 0.900 kg $H_2O$)

$$\therefore (100 \text{ g NaCl}) \times \frac{1 \text{ mol}}{58.5 \text{ g}} = 1.709 \text{ mol}$$

$$= \frac{1.709 \text{ mol}}{0.900 \text{ kg}} = 1.89 \text{ m}$$

For NaCl, $i = 2$ ($\because$ NaCl $\rightarrow$ $Na^+ + Cl^-$)

$$\Delta T_f = i\, Kf \cdot m$$
$$= 2 \times (1.86^\circ \text{C/m}) \times (1.89 \text{ m})$$
$$= 7.06^\circ \text{C}$$

Freezing point of the solution,

$$T_b = T^\circ - \Delta T_f$$
$$= 0^\circ - 7.06^\circ \text{C} = -7.06^\circ \text{C}$$

**32.** (b) When frequency is $\nu_1$,

$$h\nu_1 = h\nu_0 + \frac{1}{2}mu_1^2 \quad \ldots(i)$$

When frequency is $\nu_2$,

$$h\nu_2 = h\nu_0 + \frac{1}{2}mu_2^2 \quad \ldots(ii)$$

$\because$ $\frac{1}{2}mu_1^2 = \frac{1}{k}\left(\frac{1}{2}mu_2^2\right)$

$\therefore$ From Eq. (i)

$$h\nu_1 = h\nu_0 + \frac{1}{2k}mu_2^2 \quad \ldots(iii)$$

or $\frac{1}{2}mu_2^2 = kh\nu_1 - kh\nu_0 \quad \ldots(iv)$

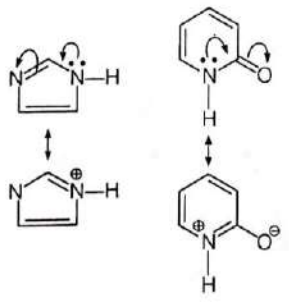
From Eqs. (ii) and (iv)

$$h\nu_2 = h\nu_0 + kh\nu_1 - kh\nu_0$$

or $\nu_0(1-k) = \nu_2 - k\nu_1$

or $\nu_0 = \frac{k\nu_1 - \nu_2}{k-1}$

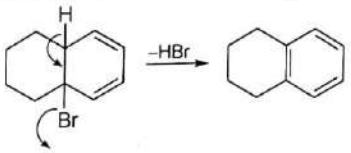
**33.** (d) In (a) and (c) the lone pair on N-atom is involved in resonance. Hence, these show aromaticity. (b) is already aromatic but structure (d) is not planar as one of the carbon is $sp^3$ hybridised.



**34.** (d) $I^-$ ions reduce $Cu^{2+}$ ions to $Cu^+$ ions.

$$2CuSO_4 + 4KI \longrightarrow Cu_2I_2 + 2K_2SO_4 + I_2$$

**35.** (d) Because on dehydrobromination it have one benzenoid ring *i.e.*, aromaticity is gained.




**36.** (d) Cu atoms forming ccp lattice

$\therefore$ No. of Cu atom $= \frac{1}{8} \times 8$ (at corner)

$$+ \frac{1}{2} \times 6 \text{ (at face)}$$

$$= 1 + 3 = 4$$

No. of Ag atom at edge of ccp $= \frac{1}{4} \times 12 = 3$

No. of Au atom at centre of ccp $= 1 \times 1 = 1$

$\therefore$ Formula of alloy $Cu_4Ag_3Au$.

**37.** (c) Acyl halide and acid anhydride can be used as acylating agent but an acid does not work as acylating agent.

**38.** (d) Only $[NiCl_4]^{2-}$ has tetrahedral structure while others show square planar structure. This is because of the involvement of $sp^3$ hybridised orbital as $Cl^-$ is weak field ligand (hence, $(n-1)d$ orbitals of Ni are not available for overlapping).

**39.** (a) At STP 16 g $O_2$ or 1/2 mol $O_2$ will occupy 11.2 L.

If volume is doubled *i.e.*, $(V_2 - V_1)$

$$= 22.4 - 11.2 = 11.2$$

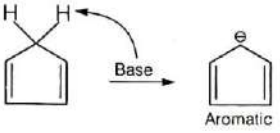
$\therefore$ $W = p(V_2 - V_1)$

$$= 1(11.2) = 11.2 \text{ L-atm}$$

$$= \frac{11.2 \times 2}{0.0821} \text{ cal}$$

$$= 272.84 \text{ cal}$$

**40.** (a)




Others become antiaromatic after giving $H^+$ ions.

**41.** (a) Acid strength decreases from HClO to HIO as the electronegativity of halogen decreases.

**42.** (c) For liquefication of gas, temperature should be lower than critical temperature. Also higher is the $p$ in comparison to critical pressure, easier is the liquefication.

**43.** (c) Rate constant, $k = Ae^{-E_a/RT}$

$$\ln k = \frac{-E_a}{RT} + \ln A$$

$$2.303 \log_{10} k = \frac{-E_a}{RT} + 2.303 \log_{10} A'$$

$$\log_{10} k = \frac{-E_a}{2.303R} \cdot \frac{1}{T} + \log_{10} A'$$

Now, $\frac{-E_a}{2.303R} = \tan\theta = -\frac{1}{2.303}$

$\therefore \quad E_a = R = 2$ cal

**44.** (a) $[H^+] = \sqrt{k_1 C_1 + k_2 C_2}$

(for mixture of two weak acids)

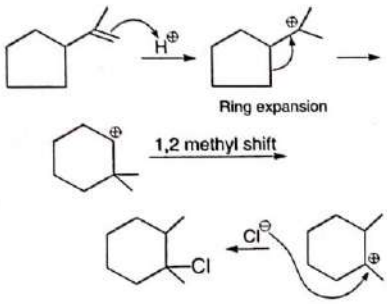
$$= \sqrt{1.8 \times 10^{-5} \times 0.01 + 6.3 \times 10^{-5} \times 0.01}$$

$$= \sqrt{18 \times 10^{-8} + 63 \times 10^{-8}}$$

$$= \sqrt{81 \times 10^{-8}}$$

$$= 9 \times 10^{-4}$$

**45.** (c)




**46.** (a) Due to high electronegativity of F-atom, the nitrogen atom attains possible charge and therefore, it has practically no electron donating properties.

**47.** (d) $2H^+ + 2e^- \longrightarrow H_2$

$$E_{(H^+/H_2)} = E^\circ_{(H^+/H_2)} - \frac{0.059}{2} \log \frac{1}{[H^+]^2}$$

$$= 0 + \frac{0.059 \times 2}{2} \log [H^+] = -0.059\,\text{pH}$$

when $\quad pH = 0$

$$E_{H^+/H_2} = 0$$

when $\quad pH = 7$

$$E_{H^+/H_2} = -0.059 \times 7$$

$$= -0.41\,\text{V}$$

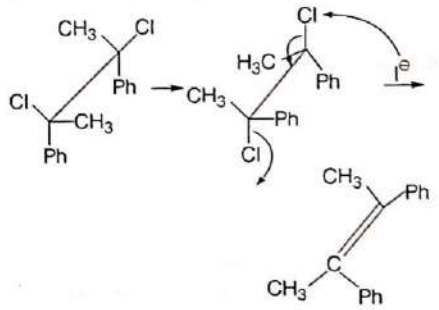
*i.e.*, reduction potential decreases by 0.41 V.

**48.** (d) Structure (a) shows geometrical isomerism because of the presence of different groups at double bonded carbon atoms.
Structure (b) and (c) show optical isomerism because of the absence of symmetry elements.
Structure (d) shows neither geometrical nor optical isomerism as it does not satisfy the conditions for optical and geometrical isomerism.

**49.** (c) Calcium oxalate is a salt which remains undissolved in acetic acid.

**50.** (d) More is the valence of effective ion, greater is its coagulating power (by Hardy Schulze rule). Since, the sol in negatively charged, positive ions are the effective ion. Thus, 0.1 N $AlCl_3$ because of the presence of $Al^{3+}$ ions, has the maximum coagulating power for this sol.

**51.** (a)



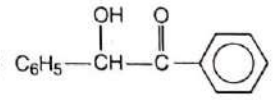

**52.** (b) $O_3$ is more powerful oxidising agent than $H_2O_2$.

So, $H_2O_2$ reduces $O_3$ to $O_2$

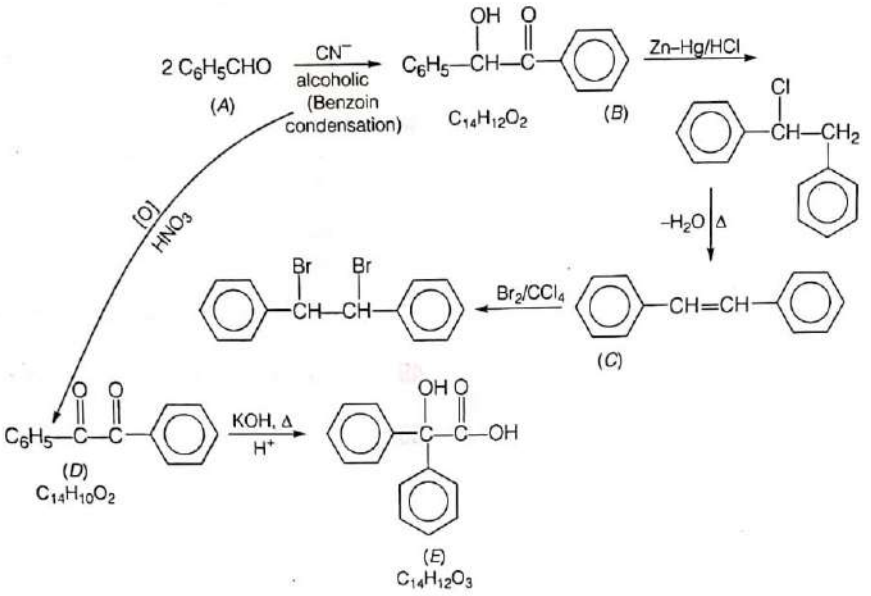
$$O_3 + H_2O_2 \longrightarrow H_2O + 2O_2$$

**53.** (b) Compound (*A*) gives positive test with Tollen's reagent $C_7H_6O$ and undergoes reaction with $CN^-$ giving condensation product. Hence, (*A*) is $C_6H_5CHO$. $C_6H_5CHO$. (Compound *A*) cannot under go aldol condensation as it does not contain α-H atom.

**54.** (b)



**55.** (d)




**56.** (a) At 300 K

| | $N_2O_4$ | $\rightleftharpoons$ | $2NO_2$ |
|---|---|---|---|
| Initial | $a$ | | 0 |
| at (equi.) | $a(1-0.2)$ $= 0.8a$ | | $2 \times 0.2a$ $= 0.4a$ |

At 400 K

| | $N_2O_4$ | $\rightleftharpoons$ | $2NO_2$ |
|---|---|---|---|
| Initial | $a$ | | 0 |
| At (equi.) | $a(1-0.6)$ $= 0.4a$ | | $2 \times 0.6a$ $= 1.2a$ |

$$pV = nRT$$

$$1 \times V = 1.2a \times R \times 300 \quad \ldots(i)$$

$$p \times V = 1.6a \times R \times 400 \quad \ldots(ii)$$

Solving Eqs. (i) and (ii)

$$p = 1.78 \text{ atm at } 400 \text{ K}$$

**57.** (a) $0.8a \times 92 + 0.4a \times 46$

$$= 1.2 \times a \times M \text{ at } 300 \text{ K}$$

$$M = 76.66 \text{ g mol}^{-1}$$

**58.** (a) At 300 K

$$p_{NO_2} = X_{NO_2} \cdot p_T$$

$$= \frac{0.4a}{1.2a} \times 1 = 0.33$$

$$p_{N_2O_4} = \frac{0.8a}{1.2a} \times 1 = 0.66$$

$$\therefore \quad K_p = \frac{(p_{NO_2})^2}{(p_{N_2O_4})} = \frac{(0.33)^2}{0.66} = 0.165$$

**59.** (a) Statement (II) is the correct explanation of (I)

$$\underset{\text{Acid}}{e.g.\,H_3A} \rightleftharpoons \underset{\substack{\text{Conjugate base} \\ \text{(weak acid)}}}{H_2A^{\ominus}} + H^{\oplus}$$

$$\underset{\text{Acid}}{H_2A^{\ominus}} \rightleftharpoons \underset{\substack{\text{Conjugate base} \\ \text{(weak acid)}}}{HA^{2-}} + H^{\oplus}$$

**60.** (c) Relative lowering in vapour pressure is a colligative properties, *i.e.*, depends upon the number of particles. More the number of particles lower is the vapour pressure. Thus, 0.1 M sugar solution has higher vapour pressure than the KCl solution of same concentration.

Relative lowering in vapour pressure is directly proportional to the mole fraction of the solute ($x_B$).

$$\frac{p^\circ - p}{p^\circ} \propto x_B$$

# Mathematics

**61.** (d) We have,

$$pq\,{}^nC_0-(p-1)(q-1)\,{}^nC_1+(p-2)(q-2)\,{}^nC_2-(p-3)(q-3)\,{}^nC_3+\ldots+(-1)^n(p-n)(q-n)\,{}^nC_n$$

$$=\sum_{k=0}^{n}(-1)^k(p-k)(q-k)\,{}^nC_k$$

$$=\sum_{k=0}^{n}(-1)^k\{pq-(p+q)k+k^2\}\cdot{}^nC_k$$

$$=pq\sum_{k=0}^{n}(-1)^k\,{}^nC_k-(p+q)\sum_{k=0}^{n}(-1)^k k\cdot{}^nC_k+\sum_{k=0}^{n}(-1)^k k^2\cdot{}^nC_k$$

$$=0$$

$$\left[\because \sum_{k=0}^{n}(-1)^k\,{}^nC_k=0=\sum_{k=0}^{n}(-1)^k k\,{}^nC_k=\sum_{k=0}^{n}(-1)^k k^2\,{}^nC_k\right]$$

**62.** (d) We have,

$$f\left(\frac{x}{2008}+2009y,\frac{x}{2008}-2009y\right)=xy$$

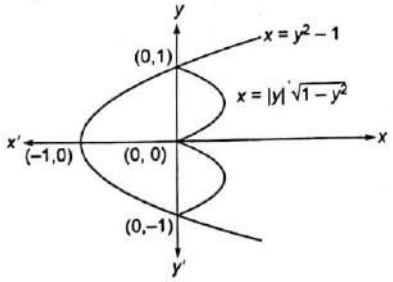
$$=1004\left(\frac{x}{2008}+2009y+\frac{x}{2008}-2009y\right)\times\frac{1}{2\times2009}\left(\frac{x}{2008}+2009y-\frac{x}{2008}+2009y\right)$$

$$\Rightarrow\ f(p,q)=\frac{502}{2009}(p+q)(p-q)=\frac{502}{2009}(p^2-q^2)$$

$$\therefore f(p,q)+f(q,p)=\frac{502}{2009}(p^2-q^2+q^2-p^2)=0$$

for all $p, q \in R$.

**63.** (a) Required area




$$A=2\int_0^1[y\sqrt{1-y^2}-(y^2-1)]\,dy$$

$$A=2\int_0^1 y\sqrt{1-y^2}\,dy-2\int_0^1(y^2-1)\,dy$$

Let $1-y^2=t^2$

$$-2y\,dy=2t\,dt$$

$$-y\,dy=t\,dt$$

$$A=-2\int_1^0 t^2\,dt-2\left[\frac{y^3}{3}-y\right]_0^1$$

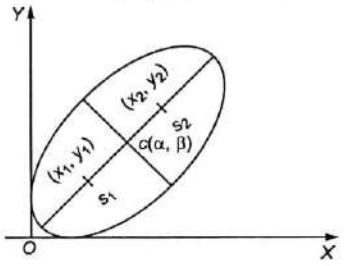
$$A=2\int_0^1 t^2\,dt-2\left(\frac{1}{3}-1\right)$$

$$A=2\left[\frac{t^3}{3}\right]_0^1-2\left(-\frac{2}{3}\right)$$

$$=2\left(\frac{1}{3}\right)+\frac{4}{3}=\frac{2+4}{3}$$

$$=2 \text{ sq units}$$

**64.** (a) Let the two lines be taken as $x$ and $y$ axes, respectively. Let $C(\alpha,\beta)$ be the centre of the ellipse of which $S_1(x_1, y_1)$ and $S_2(x_2, y_2)$ are foci.




Then, $x_1+x_2=2\alpha$

and $y_1+y_2=2\beta$

Since, $x$-axis and $y$-axis are the tangents to the ellipse.

$\therefore$ $x_1x_2=b^2$,

$y_1y_2=b^2$

Also, $S_1S_2^2=4a^2e^2$

$\Rightarrow (x_1-x_2)^2+(y_1-y_2)^2=4a^2e^2$

$\Rightarrow (x_1+x_2)^2+(y_1+y_2)^2-4x_1x_2-4y_1y_2=4a^2e^2$

$\Rightarrow 4\alpha^2+4\beta^2-4b^2-4b^2=4a^2e^2$

$\Rightarrow \alpha^2+\beta^2-a^2e^2=2b^2$

$\Rightarrow \alpha^2+\beta^2=2b^2+a^2e^2$

$\Rightarrow \alpha^2+\beta^2=2b^2+a^2-b^2$ $[\because b^2=a^2(1-e^2)]$

$\Rightarrow \alpha^2+\beta^2=a^2+b^2$

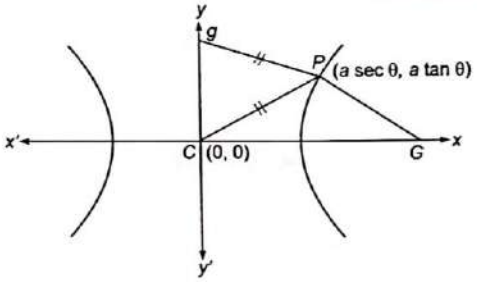
Locus of $(\alpha,\beta)$ is

$$x^2+y^2=a^2+b^2$$

which represents a circle.

**65.** (b) Let the equation of the rectangular hyperbola be $x^2-y^2=a^2$. Let $P$ be any point on the given rectangular hyperbola specified as $(a\sec\theta, a\tan\theta)$, then the equation of the normal

to the hyperbola at $P$ is $x \sin\theta + y = 2a \tan\theta$ which will intersect axes at




$G(2a \sec\theta, 0)$ and $g(0, 2a \tan\theta)$

Now, $PG^2 = (a \sec\theta - 2a \sec\theta)^2 + (a \tan\theta - 0)^2$

$= a^2(\sec^2\theta + \tan^2\theta)$

$Pg^2 = (a \sec\theta - 0)^2 + (a \tan\theta - 2a \tan\theta)^2$

$= a^2(\sec^2\theta + \tan^2\theta)$

and $PC^2 = (a \sec\theta - 0)^2 + (a \tan\theta - 0)^2$

$= a^2(\sec^2\theta + \tan^2\theta)$

Clearly, $PG = Pg = PC$

**66.** (d) Let $A = \begin{bmatrix} 0 & \sin^{-1}(-1) & \sin^{-1}(-2) \\ \sin^{-1}(1) & 0 & \sin^{-1}(-1) \\ \sin^{-1}(2) & \sin^{-1}(1) & 0 \end{bmatrix}$,

$= \begin{bmatrix} 0 & -\sin^{-1}(1) & -\sin^{-1}(2) \\ \sin^{-1}(1) & 0 & -\sin^{-1}(1) \\ \sin^{-1}(2) & \sin^{-1}(1) & 0 \end{bmatrix}$

As the matrix is skew-symmetric hence, $|A| = 0$, if $n$ is odd.

$\Rightarrow$ $A$ is not an invertible matrix, when $n$ is odd.

**67.** (d) Given, $x\,Sy \Leftrightarrow x^2 - y^2 = 2(y - x)$, $\forall\, x, y \in R^+$

$\because$ $x^2 - x^2 = 2(x - x)$, $\forall\, x \in R^+$

$0 = 0$

$\therefore$ $R$ is reflexive.

For symmetric of $S$, $xSy$

$\Rightarrow$ $x^2 - y^2 = 2(y - x)$

$\Rightarrow$ $-(y^2 - x^2) = -2(x - y)$

$\Rightarrow$ $y^2 - x^2 = 2(x - y)$

$\Rightarrow$ $y\,Sx$

$\therefore$ $S$ is symmetric.

Now, by definition of $S$,

$x^2 - y^2 = 2(y - x)$

and $y^2 - z^2 = 2(z - y)$

On adding, we get

$(x^2 - z^2) = 2(z - x)$

$\Rightarrow$ $xSz$

$\therefore$ $S$ is transitive.

Clearly, $S$ is an equivalence relation on $R^+$.

**68.** (b) Given relation

$f(n) = f(n-1) + f(n-2)$

$f(2) = f(1) + f(0) = 1 + 0 = 1$

$f(3) = f(2) + f(1) = 1 + 1 = 2$

$f(4) = f(3) + f(2) = 2 + 1 = 3$

$f(5) = f(4) + f(3) = 3 + 2 = 5$

$\therefore$ $fof(1) = f\{f(1)\} = f(1) = 1$

$fof(2) = f\{f(2)\} = f(1) = 1$

**69.** (b) $SD = \sqrt{\frac{\sum X_i^2}{n} - (\bar{X})^2}$

$\therefore$ $10 = \sqrt{\frac{\sum X_i^2}{20} - (50)^2}$

$\Rightarrow$ $100 = \frac{\sum X_i^2}{20} - 2500$

$\Rightarrow$ $\sum X_i^2 = 52000$

**70.** (a) We have,

$0 < x < 1$

$\Rightarrow$ $0 < x^2 < x < 1$

$\Rightarrow$ $e^{x^2} < e^x$

$\Rightarrow$ $\int_0^1 e^{x^2} dx < \int_0^1 e^x dx = e - 1$

$\Rightarrow$ $k < e - 1$

$\Rightarrow$ $[k] = 1$

**71.** (d) Given equation of the line is

$\frac{x-3}{0} = \frac{y-7}{1} = \frac{z-1}{1} = (k)$ (say)

Any point on the line is

$(3, 7 + k, 1 + k)$

The line joining $A(5, 1, 3)$ and $B(3, 7 + k, 1 + k)$ is perpendicular to the given line if

$0(-2) + (6 + k)\cdot(1) + (k - 2)\cdot(1) = 0$

$\Rightarrow$ $6 + k + k - 2 = 0$

$\Rightarrow$ $2k + 4 = 0$

$\Rightarrow$ $k = -2$

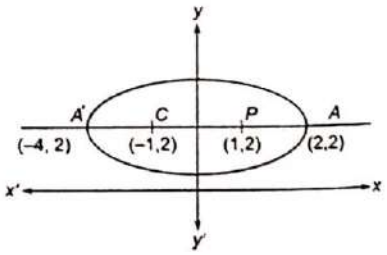
$\therefore$ Any point $B(3, 5, -1)$ lie on the line.

$\therefore$ Required distance $AB = \sqrt{2^2 + 4^2 + 4^2}$

$= \sqrt{36} = 6$

**72.** (a) Given equation can be rewritten as

$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$



can be rewritten as

whose centre is $(-1, 2)$ and the axis is parallel to $x$-axis. Vertices are $(2, 2)$ and $(-4, 2)$ and the point $(1, 2)$ lies inside the ellipse.

Minimum distance $= 2 - 1 = 1 = PA = k$

Maximum distance $= PA' = K = 5$

$\therefore \quad K - k = 5 - 1 = 4$ units

**73.** (b) We have,

$$\frac{dy}{dx} = 2x + 1$$

$\Rightarrow y = x^2 + x + C$, which will pass through $(1, 2)$

$\therefore \quad 2 = 1 + 1 + C \Rightarrow C = 0$

$\therefore y = x^2 + x$ is the required curve

$\therefore$ Required area $= \int_0^1 (x^2 + x)\, dx$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$$

$$\Rightarrow \quad \frac{5}{6} = k$$

$$\Rightarrow \quad 6k = 5$$

**74.** (b) Now, $(\sim p \vee q) \equiv (F \vee F) = F$

$(\sim p \vee q) \wedge \sim p \equiv F \wedge F = F$

$\therefore \quad (\sim p \vee q) \wedge \sim p \Rightarrow p$

$p \equiv F \Rightarrow T$

$\equiv T$

**75.** (b) On adding giving equations, we get

$x + \{x\} + [x] + y + [y] + \{y\} + z + [z] + \{z\} = 13$

$\Rightarrow \quad 2x + 2y + 2z = 13 \quad (\because x = [x] + \{x\})$

$\Rightarrow \quad x + y + z = 6.5$

Also,

$(x - \{x\}) + (z - [z]) = 6.5 - 2.3$

$\Rightarrow \quad [x] + \{z\} = 4.2$

$\Rightarrow \quad [x] = 4, \{z\} = 0.2$

Similarly,

$\{y\} + [z] = 2 \Rightarrow \{y\} = 0, [z] = 2$

Also, $\{x\} + [y] = 0.3$

$\Rightarrow \quad \{x\} = 0.3$ and $[y] = 0$

$\therefore \quad x = [x] + \{x\} = 4 + 0.3 = 4.3$

$y = [y] + \{y\} = 0 + 0 = 0$

$z = [z] + \{z\} = 2 + 0.2 = 2.2$

Hence, $(4.3, 0, 2.2)$ is the only solution.

**76.** (b) When $x \in (0, 1)$

$\Rightarrow \quad \cos^{-1} x > \sin^{-1} x$

Also, $\cos^{-1} x > 1$

and $\sin^{-1} x < 1$

$\therefore (\cos^{-1} x)^{\cos^{-1} x}$ is greatest and $(\sin^{-1} x)^{\cos^{-1} x}$ is least.

$\Rightarrow \quad (\sin^{-1} x)^{\sin^{-1} x} < (\cos^{-1} x)^{\sin^{-1} x}$

$\Rightarrow \quad d > c > a > b$

**77.** (b) If $k = 1$,

Then, $D = D_1 = D_2 = D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Clearly, for $k = 1$

$D = D_1 = D_2 = D_3 = 0$ has infinitely many solutions.

**78.** (a) $\lim\limits_{x \to \infty} \sqrt{3x^2 + \sqrt{3x^2 + \sqrt{3x^2}}} - \sqrt{3x^2}$

$$= \lim_{x \to \infty} \frac{3x^2 + \sqrt{3x^2 + \sqrt{3x^2}} - 3x^2}{\sqrt{3x^2 + \sqrt{3x^2 + \sqrt{3x^2}}} + \sqrt{3x^2}}$$

(by rationalization)

$$= \lim_{x \to \infty} \frac{\sqrt{3 + \sqrt{\frac{3}{x^2}}}}{\sqrt{3 + \sqrt{\frac{3}{x^2}} + \sqrt{\frac{3}{x^4}}} + \sqrt{3}}$$

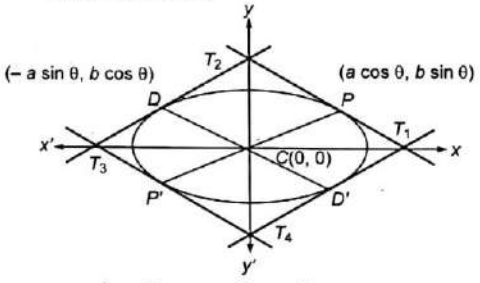
$$= \frac{\sqrt{3}}{\sqrt{3} + \sqrt{3}} = \frac{1}{2}$$

**79.** (c) Area of parallelogram $T_1T_2T_3T_4$

$= 4$ (Area of parallelogram $CP\,T_2\,D$)

$= 4$ (2 Area of $\Delta CPD$)

$= 8$ (Area of $\Delta CPD$)




$$= 8 \times \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \end{vmatrix}$$

$= 4(ab\cos^2\theta + ab\sin^2\theta)$

$= 4ab = 2a \times 2b$

= Product of the axes of the ellipse

$\therefore$ Area $= 4 \times 4 \times 2 = 32$ sq units

**80.** (b) Given, $|z| = 5, |\beta z| = 5 \quad (\because |\beta| = 1)$

and $|z + \beta z + \beta^2 z + \beta^3 z|$

$= |z||1 + \beta + \beta^2 + \beta^3|$

$= |z||-\beta^4| \quad (\because 1 + \beta + \beta^2 + \beta^3 + \beta^4 = 0)$

$= 5$

So, triangle will be an equilateral triangle.

$\therefore$ Area $= \frac{\sqrt{3}}{4}|z|^2 = \frac{\sqrt{3}}{4} \times 25 = \frac{25\sqrt{3}}{4}$

**81.** (b) $\frac{1}{\log_{1/3}\frac{1}{5}} + \frac{1}{\log_{1/4}\frac{1}{5}}$

$= \log_{1/5}\frac{1}{3} + \log_{1/5}\frac{1}{4}$

$= \log_5 3 + \log_5 4 = \log_5 12$

$\because \quad 5 < 12 < 5^2$

So, $\log_5 12 \in (1, 2)$

**82.** (d) $\cos\frac{\pi x}{3\sqrt{3}} = x^2 - 2\sqrt{3}x + 3 + 1$

$\cos\frac{\pi x}{3\sqrt{3}} = (x - \sqrt{3})^2 + 1$

$\because \quad (x - \sqrt{3})^2 + 1 \geq 1$

and $\cos\frac{\pi x}{3\sqrt{3}} \leq 1$

For $x = \sqrt{3}$, $\cos\frac{\pi x}{3\sqrt{3}} = \cos\frac{\pi}{3} = \frac{1}{2}$

and $(x - \sqrt{3})^2 + 1 = 1$

So, no value of $x$ will satisfy the equation.

**83.** (b) Clearly, focus $= (1, 1)$

Distance of focus from directrix

$= \frac{5 \times 1 + 12 \times 1 + 22}{13} = 3$

$\therefore$ Length of latusrectum $= 2 \times 3 = 6$

**84.** (a) Let $S = 1 + 2\left(1 + \frac{1}{\alpha}\right) + 3\left(1 + \frac{1}{\alpha}\right)^2$

$\ldots + 50\left(1 + \frac{1}{\alpha}\right)^{49} \quad \ldots(i)$

$S\left(1 + \frac{1}{\alpha}\right) = 1\left(1 + \frac{1}{\alpha}\right) + 2\left(1 + \frac{1}{\alpha}\right)^2$

$\ldots + 50\left(1 + \frac{1}{\alpha}\right)^{50} \quad \ldots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get

$-\frac{S}{\alpha} = 1 + \left(1 + \frac{1}{\alpha}\right) + \left(1 + \frac{1}{\alpha}\right)^2$

$+ \ldots + \left(1 + \frac{1}{\alpha}\right)^{49} - 50\left(1 + \frac{1}{\alpha}\right)^{50}$

$\Rightarrow \quad 50\left(1 + \frac{1}{\alpha}\right)^{50} - \frac{S}{\alpha} = \frac{\left(1 + \frac{1}{\alpha}\right)^{50} - 1}{1 + \frac{1}{\alpha} - 1}$

$\Rightarrow \quad 50\frac{(\alpha + 1)^{50}}{\alpha^{50}} - \frac{S}{\alpha} = \frac{(1 + \alpha)^{50} - \alpha^{50}}{\alpha^{49}}$

$\Rightarrow \quad 50(\alpha + 1)^{50} - S\alpha^{49} = \alpha(1 + \alpha)^{50} - \alpha^{51}$

$\Rightarrow \quad -S = \frac{(1 + \alpha)^{50}(-50 + \alpha) - \alpha^{51}}{\alpha^{49}}$

$\Rightarrow \quad S = \frac{(1 + \alpha)^{50}(50 - \alpha) + \alpha^{51}}{\alpha^{49}}$

**85.** (b) $|z - 2 + 2i| + |z - 3| \geq |z - 2 + 2i - z + 3|$

$= |1 + 2i| = \sqrt{5}$

$(\because |z_1| + |z_2| \geq |z_1 - z_2|)$

**86.** (b) $|z + i| + |z + 3i| + |-z + 2| + |-z - 7i|$

$\geq |z + i + z + 3i - z + 2 - z - 7i|$

$= |2 - 3i| = \sqrt{13}$

$(\because |z_1| + |z_2| + |z_3| + |z_4| \geq |z_1 + z_2 + z_3 + z_4|)$

**87.** (a) RHL $= \lim_{h \to 0}\left[\sin\left(\frac{\pi}{2} + h\right)\right] = 0$

LHL $= \lim_{h \to 0}\left[\sin\left(\frac{\pi}{2} - h\right)\right] = 0$

**88.** (a) $\lim_{x \to 0}\left[\frac{\sin x}{\tan x}\right] = 0$ as $\sin x < \tan x$

**89.** (b) The value of $\sin\sqrt{2}x + \sin ax$ can be equal to 2, if $\sin\sqrt{2}x$ and $\sin ax$ both are equal to one but both are not equal to one for any common value of $x$.

**90.** (c) Equation of chord of contact from $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$xx_1 + yy_1 - a^2 = 0$

$xx_2 + yy_2 - a^2 = 0$

and $xx_3 + yy_3 - a^2 = 0$

*i.e.*, $A, B$ and $C$ are collinear.

# JEE Main

## Joint Entrance Examination

# Practice Set 7

## Instructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

1. At what maximum angle, a ray of light must be incident on one of the refracting surfaces of a prism of angle 75° and refractive index $\sqrt{2}$, so that it does not come out of the other refracting surface?
   (a) 30° (b) 45° (c) 60° (d) None of these

2. A radioactive substance decays to 1/16th of initial activity in 40 days. The half-life of the radioactive substance expressed (in days) is
   (a) 2.5 (b) 5 (c) 10 (d) 20

3. **Statement I** The magnetic field produced by a current carrying solenoid is independent of its length and cross-sectional area.
   **Statement II** The magnetic field inside the solenoid is uniform.
   (a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I.
   (b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I.
   (c) Statement I is true but Statement II is false.
   (d) Both Statement I and Statement II are false.

4. The ground state energy of H-atom is –13.6 eV. The energy needed to ionize H-atom from its second excited state
   (a) 1.51 eV (b) 3.4 eV (c) 13.6 eV (d) None of these

5. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement $x$ is proportional to
   (a) $x^2$ (b) $e^x$ (c) $x$ (d) $\log_e x$

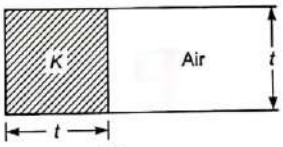
6. A charge $Q$ is placed at each corner of a cube of side $a$. The potential at the centre of the cube is

(a) $\frac{8Q}{\pi\varepsilon_0 a}$ (b) $\frac{4Q}{4\pi\varepsilon_0 a}$ (c) $\frac{4Q}{\sqrt{3}\,\pi\varepsilon_0 a}$ (d) $\frac{2Q}{\pi\varepsilon_0 a}$

7. If 20 J of work has to be done to move an electric charge of 4 C from a point, where potential is 10 V to another point, where potential is $V$ volt, find the value of $V$

(a) 2 V (b) 70 V (c) 5 V (d) 15 V

8. A parallel plate capacitor with air as the dielectric has capacitance $C$. A slab of dielectric constant $K$ and having the same thickness as the separation between the plates is introduced so as to fill one-fourth of the capacitor as shown in the figure. The new capacitance will be




(a) $(K+3)\frac{C}{4}$ (b) $(K+2)\frac{C}{4}$ (c) $(K+1)\frac{C}{4}$ (d) $\frac{KC}{4}$

9. The values of two resistors are $R_1 = (6 \pm 0.3)\,\text{k}\Omega$ and $R_2 = (10 \pm 0.2)\,\text{k}\Omega$. The percentage error in the equivalent resistance when they are connected in parallel is

(a) 5.125% (b) 2% (c) 3.875% (d) 7%

10. A car moves from $X$ to $Y$ with a uniform speed $v_u$ and returns to $Y$ with a uniform speed $v_d$. The average speed for this round trip is

(a) $\frac{2v_d v_u}{v_d + v_u}$ (b) $\sqrt{v_u v_d}$ (c) $\frac{v_d v_u}{v_d + v_u}$ (d) $\frac{v_u + v_d}{2}$

11. The coordinates of a moving particle at any time $t$ are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time $t$ is given by

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$ (c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

12. An alternating current is given by $I = I_1 \cos\omega t + I_2 \sin\omega t$. The root mean square current is given by

(a) $\frac{(I_1+I_2)}{\sqrt{2}}$ (b) $\frac{(I_1+I_2)^2}{2}$ (c) $\sqrt{\frac{I_1^2+I_2^2}{2}}$ (d) $\frac{\sqrt{I_1^2-I_2^2}}{2}$

13. A copper rod of length $l$ is rotated about one end, perpendicular to the uniform magnetic field $B$ with constant angular velocity $\omega$. The induced emf between two ends of the rod is

(a) $\frac{1}{2}B\omega l^2$ (b) $B\omega l^2$ (c) $\frac{3}{2}B\omega l^2$ (d) $2B\omega l^2$

14. The flux associated with coil changes from 1.35 Wb to 0.79 Wb within $\frac{1}{10}$ s. Then, the charge produced by the earth coil, if resistance of coil is 7 Ω is

(a) 0.08 C (b) 0.8 C (c) 0.008 C (d) 8 C

15. **Statement I** Taking into account the fact that any object which floats must have an average density less than that of water, during world war I, a number of cargo vessels were made of concrete.

**Statement II** Concrete cargo vessels were filled with air.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I.

(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I.

(c) Statement I is true but Statement II is false.

(d) Both Statement I and Statement II are false.

16. Consider the statements (A) and (B) and identify the correct answers.
   **A.** First law of thermodynamics specifies the conditions under which a body can use its heat energy to produce the work.
   **B.** Second law of thermodynamics states that heat always flows from hot body to cold body by itself.
   (a) Both A and B are true (b) Both A and B are false
   (c) A is true but B is false (d) A is false but B is true

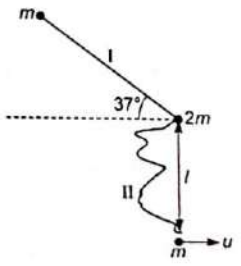
17. The amount of work, which can be obtained by supplying 200 cal of heat, is
   (a) 840 J (b) 840 erg (c) 840 W (d) 840 dyne

18. A man of mass $m$ is standing on a platform of mass $M$ kept on smooth ice. If the man starts moving on the platform with a speed $v$ relative to the platform, with what velocity relative to the ice does the platform recoil?
   (a) $V = \frac{Mv}{(M+m)}$ (b) $V = \frac{(M+m)v}{(m-M)}$
   (c) $V = \frac{(M-m)v}{(M+m)}$ (d) $V = \frac{mv}{M+m}$

19. In a series $L$-$R$ growth circuit, the maximum current and maximum induced emf in an inductor of 6 mH are 3 A and 8 V, respectively. In how much time the current in the circuit grows to 63.2% of its final value?
   (a) $\frac{9}{4}$ ms (b) $\frac{4}{9}$ ms (c) 16 ms (d) $\frac{1}{10}$ ms

20. A spherical shell of inner radius $R_1$ and outer radius $R_2$ is having variable thermal conductivity given by $K = a_0 T \times r$, where $a_0$ is constant, $T$ is temperature in kelvin and $r$ is the distance from the centre. Two surfaces of shell are maintained at temperature $T_1$ (inner surface) and $T_2$ (outer surface), respectively ($T_1 > T_2$). The heat current flowing through the shell would be
   (a) $\frac{4\pi a_0(T_1^2 - T_2^2)}{R_2 - R_1} \times R_1 R_2$ (b) $\frac{4\pi a_0 R_1^2 R_2^2 (T_1^2 - T_2^2)}{R_2^2 - R_1^2}$
   (c) $\frac{4\pi a_0 (T_1 - T_2) R_1 R_2}{R_2 - R_1}$ (d) $\frac{4\pi a_0 (T_1^2 - T_2^2)(R_1 + R_2)^2}{R_2 - R_1}$

**Directions** (Q. Nos. 21-23) *Two strings I and II of lengths l and 2l, respectively are arranged as shown in the figure. The ends of string* I *are connected to two pointd masses of the mass m and 2m while the ends of string* II *are connected to mass 2 m and m as shown in the figure. The entire system is placed on a smooth horizontal surface. Initially the string I is just on the verge of getting taut and* II *is slacked in the position shown. The mass m attached to string* II *is given a horizontal velocity u in a direction perpendicular to line joining 2m and m.*




*Answer the following questions based on the above information.*

21. The impulse experienced by point mass of $2m$ due to jerking of string II, is
   (a) $\frac{mu}{\sqrt{3}} + \frac{mu\cos^2 7°}{\sqrt{3}(6 + 3\sin^2 7° + 2\cos^2 7°)}$ (b) $\frac{\sqrt{3}\,mu\cos 7°}{6 + 3\sin^2 7° + 2\cos^2 7°}$
   (c) $\frac{\sqrt{3}\,mu}{4}\sin 7° \cos 7°$ (d) zero

22. The component of velocity of $2m$ perpendicular to the length of string II just after the strings get taut, is

(a) $\frac{\sqrt{3}\,u \sin 7^\circ \cos 7^\circ}{4}$ (b) $\frac{\sqrt{3}\,u \cos 7^\circ \sin 7^\circ}{2[6 + 3\sin^2 7^\circ + 2\cos^2 7^\circ]}$

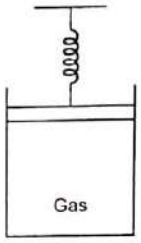
(c) $u + \frac{\sqrt{3}\,u \cos 7^\circ \sin 7^\circ}{2[6 + 3\sin^2 7^\circ + 2\cos^2 7^\circ]}$ (d) zero

23. The velocity of $m$ (connected to string I) just after the string becomes taut, is

(a) $\frac{\sqrt{3}\,u \sin 7^\circ \cos 7^\circ}{4}$ (b) $\frac{\sqrt{3}\,u \cos 7^\circ}{6 + 3\sin^2 7^\circ + 2\cos^2 7^\circ}$

(c) $u + \frac{\sqrt{3}\,u \cos 7^\circ \sin 7^\circ}{2[6 + 3\sin^2 7^\circ + 2\cos^2 7^\circ]}$ (d) zero

**Directions** (Q. Nos. 24-26) *One mole of an ideal monoatomic gas is contained in a piston-cylinder arrangement as shown in the figure. A spring is connected to piston as shown. The gas is initially at a pressure of* 2 atm *and temperature* 27°C. *The piston has a cross-section area of* $0.005\,m^2$ *and the spring is initially undeformed. The spring constant is* $10^4$ *N/m,* $g = 10\ m/s^2$, 1 atm $= 10^5\ N/m^2$, $R = 8.3$. *Heat is supplied to the system to increase its pressure to 5 atm. For this situation answer the following questions :*




24. The process according to which the state of gas is changing, is

(a) isobaric (b) adiabatic (c) isothermal (d) polytropic

25. How much heat must be supplied to increase the pressure to 5 atm?

(a) zero (b) 6352.5 J (c) 6165 J (d) 187.5 J

26. The work done by the system against the surrounding, is

(a) zero (b) −187.5 J (c) 187.5 J (d) 6165 J

**Directions** (Q. Nos. 27 and 28) *For the following questions. Choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.*

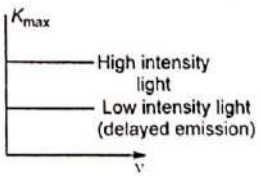
(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true and Statement II is false.

(d) Statement I is false but Statement II is true.

27. **Statement I** According to classical physics, the plot between maximum kinetic energy of the ejected photoelectrons *versus* frequency plot in photoelectric experiments would be as shown.

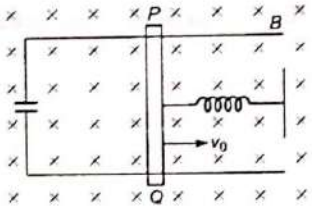



**Statement II** From classical theory, the photoelectric effect cannot be explained.

28. **Statement I** In an X-ray tube, if the energy with which an electron strikes the metal target increases, then the wavelength of the characteristic X-rays also changes.

**Statement II** Wavelength of characteristic X-rays depends only on the initial and final energy levels.

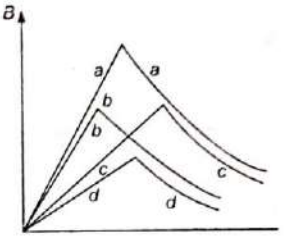
29. A conducting rod $PQ$ of mass $m$ and of length $l$ is placed on two long parallel (smooth and conducting) rails connected to a capacitor as shown below. The rod $PQ$ is connected to a non-conducting spring of spring constant $k$, which is initially in relaxed state. The entire arrangement is placed in a magnetic field perpendicular to the plane of figure.




Neglect the resistance of rails and rod. Now, the rod is imparted a velocity $v_0$ towards right, then acceleration of the rod as a function of its displacement $x$ is given by

(a) $\frac{kx}{m}$ (b) $\frac{kx}{m + B^2l^2C}$ (c) $\frac{kx}{m - B^2l^2C}$ (d) None of these

30. The curves in the figure shows the magnitude of magnetic field $B$ as a function of radial distance $r$ inside and outside the four current carrying wires (uniformly distributed across the cross-section) $a,b,c$ and $d$. The current density is maximum in wire




(a) $a$ and $b$
(b) $b$ and $c$
(c) $c$ and $d$
(d) $d$ and $a$

# Chemistry

31. How much amount of $CaCl_2$ ($i = 2.47$) is dissolved in 2.5 L of water so that its osmotic pressure become 0.75 atm at 27°C?
(a) 0.3 mol (b) 0.03 mol (c) 3 mol (d) 0.003 mol

32. Acidulated water is electrolysed by 1 A current for 16 min and 5 s using inert electrodes. The volume of gases liberated at STP will be
(a) 22.4 mL (b) 112 mL (c) 168 mL (d) 336 mL

33. The unit cell cube length of LiCl (NaCl structure) is 5.14 Å. Assuming anion-anion contact, the ionic radius of chloride is
(a) 1.82 Å (b) 3.63 Å (c) 0.91 Å (d) 2.57 Å

34. EAN (Effective Atomic Number) of the central metal in the complexes $K_2[Ni(CN)_4]$, $[Cu(NH_3)_4]SO_4$ and $K_2[PtCl_6]$ are respectively
(a) 36, 35, 86 (b) 34, 35, 85 (c) 34, 35, 86 (d) 34, 36, 86

35. If uncertainty in position and momentum are equal then uncertainty in velocity is
(a) $\frac{1}{m}\sqrt{\frac{h}{\pi}}$ (b) $\sqrt{\frac{h}{\pi}}$ (c) $\frac{1}{2m}\sqrt{\frac{h}{\pi}}$ (d) $\sqrt{\frac{h}{2\pi}}$

36. At 90°C, pure water has $[H^+] = 10^{-6}$ M. If 100 mL of 0.2 M HCl is added to 200 mL of 0.1 M KOH at 90°C, then pH of the resulting solution will be
(a) 5 (b) 6 (c) 7 (d) None of these

37. If one mole of a monoatomic gas ($\gamma = 5/3$) is mixed with one mole of a diatomic gas ($\gamma = 7/5$), the value of '$\gamma$' for the mixture is
(a) 1.4 (b) 1.5 (c) 1.53 (d) 3.07

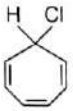
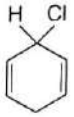
38. In a gaseous phase reaction,

$$A_2(g) \longrightarrow B(g) + \frac{1}{2}C(g),$$

the increase in pressure from 100 mm to 120 mm is noticed in 5 min. The rate of disappearance of $A_2$ in mm $min^{-1}$ is

(a) 4 (b) 8 (c) 16 (d) 2

39. Which of the following will give precipitate on treatment with $AgNO_3$ solution readily?

(a)  (b)  (c)  (d) 

40. The number of peroxide linkages in the compounds $H_2S_2O_7$ and $H_2S_2O_8$ respectively are

(a) 1, 0 (b) 1, 1 (c) 0, 1 (d) 1, 2

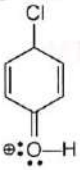
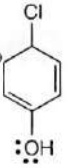
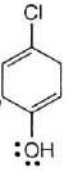
41. The work done in an open vessel at 300 K, when 112 g iron reacts with dil HCl is

(a) 1200 cal (b) 600 cal (c) 300 cal (d) 200 cal

42. The reaction of $XeF_6$ with silica gives

(a) $XeF_4$ and $SiF_4$ (b) $XeOF_2$ and $SiF_4$ (c) $XeOF_4$ and $SiF_4$ (d) None of these

43. Which one of the following is most stable?

(a)  (b)  (c)  (d) 

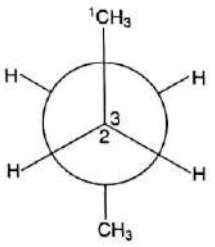
44. $C_2$ is rotated anti-clockwise 120° about $C_2 - C_3$ bond. The resulting conformer is

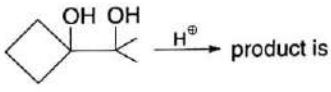
(a) staggered

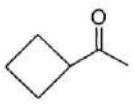
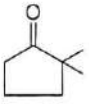
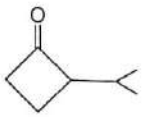
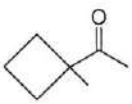
(b) *gauche*

(c) eclipsed

(d) partially eclipsed




45.  product is

(a)  (b)  (c)  (d) 

46. The property of colloidal suspension used to determine the nature of charge on the particles is

(a) dialysis (b) sedimentation (c) electrophoresis (d) ultrafiltration

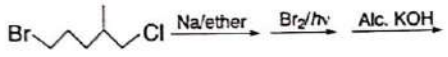
47. When orthoboric acid is heated strongly, it gives
(a) $B_2O_3$ (b) $H_2B_4O_7$ (c) $HBO_2$ (d) B

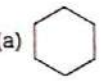
48. The number of alkynes possible with molecular formula $C_5H_8$ is
(a) 2 (b) 3 (c) 4 (d) 5

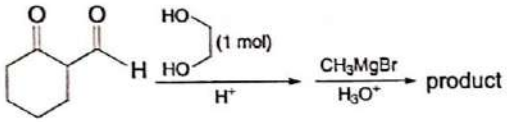
49. A mixture of nitrogen and hydrogen in the ratio of 1 : 3 reach equilibrium with ammonia when 50% of the mixture has reacted. If the total pressure is $p$, the partial pressure of ammonia in the equilibrium mixture was
(a) $\frac{p}{2}$ (b) $\frac{p}{3}$ (c) $\frac{p}{4}$ (d) $\frac{p}{6}$

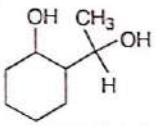
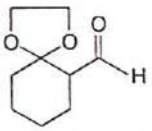
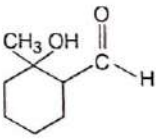
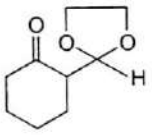
50. An aqueous solution of a substance gives a white precipitate on treatment with dilute HCl, which dissolved on heating. On passing $H_2S$ in hot acidic solution, a black precipitate is formed. The substance is
(a) $Hg_2^{2+}$ salt (b) $Cu^{2+}$ salt (c) $Ag^{+}$ salt (d) $Pb^{2+}$ salt

51. The major product of the following reaction is

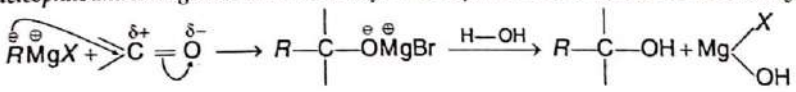



(a)  (b)  (c)  (d) 

52. 


(a)  (b)  (c)  (d) 

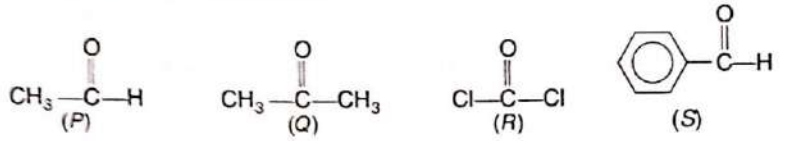
**Directions** (Q. Nos. 53-55) *Since, Grignard reagent resembles carbanion, so they are strong nucleophile and strong base. Their most useful nucleophilic reaction is addition to carbonyl group.*




53. Which of the following compound will not give acid-base reaction with $R$ Mg$X$?
(a) $CH_3—CH_2—Cl$ (b) $CH_3OH$ (c) $H_3C—N(=O)=O$ (d) $H_2S$

54. Which of the following compounds give racemic mixture on reaction with $CH_3$—MgBr?
(a) $H—C(=O)—H$ (b) $CH_3—C(=O)—CH_3$
(c) $Ph—C(=O)—H$ (d) $CH_3—C(=O)—Cl$

55. Arrange the following compounds in decreasing order of nucleophilic addition reaction.



(P) (Q) (R) (S)

(a) $R > P > S > Q$ (b) $P > Q > R > S$ (c) $Q > R > S > P$ (d) $R > S > P > Q$

**Directions** (Q. Nos. 56-58) *An unknown mixture contains one or two of the following compounds :* $CaCO_3$, $BaCl_2$, $AgNO_3$, $Na_2SO_4$, $ZnSO_4$ *and* NaOH. *The mixture is completely soluble in water and solution gives pink colour with phenolphthalein. When dilute hydrochloric acid is gradually added to the solution, a precipitate is formed which dissolves with further addition of the acid.*

56. Which of the following combinations of compounds is soluble in water?
(a) $BaCl_2$ and $AgNO_3$ (b) $AgNO_3$ and NaOH
(c) $BaCl_2$ and $Na_2SO_4$ (d) $ZnSO_4$ and excess NaOH

57. The aqueous solution of mixture gives white precipitate with dilute HCl which dissolves in excess of dilute HCl. It confirms
(a) $BaCl_2$ + NaOH (b) $Na_2SO_4$ + NaOH (c) $ZnSO_4$ + NaOH (d) $AgNO_3$ + NaOH

58. The white precipitate is
(a) $ZnSO_4$ (b) $Na_2ZnO_2$ (c) $Zn(OH)_2$ (d) $ZnCl_2$

**Directions** (Q. Nos. 59 and 60) *For the following questions, choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.
(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

59. **Statement I** 0.1 M $H_3PO_3(aq)$ solution has normality equal to 0.3 N when completely reacted with NaOH.
**Statement II** $H_3PO_3$ is a dibasic acid.

60. **Statement I** Liquid-liquid junction potential can be eliminated by putting a salt bridge of KCl in electrochemical cell.
**Statement II** The function of salt bridge is to remove liquid-liquid junction potential because the salt used has same speed of cations and anions.

# Mathematics

61. Two points $A$ and $B$ moves on the $x$-axis and the $y$-axis, respectively such that the distance between the two points is always the same. The locus of the middle point of $AB$ is
(a) a straight line (b) a circle (c) a parabola (d) an ellipse

62. There is a square of side 2 units. Another square is constructed in it having area half of that of original square. Inside the second square another square is constructed in same manner and the process of constructing squares in continued in the similar way infinitely many times. Sum of lengths of possible diagonals of all such squares is
(a) $8(\sqrt{2}-1)$ (b) $4(\sqrt{2}+1)$ (c) $8(\sqrt{2}+1)$ (d) $4(\sqrt{2}-1)$

63. Tangents drawn from $(6, \lambda)$ to the circle $2x^2 + 2y^2 - 12x + 8y + 1 = 0$ are perpendicular to each other. Then, $\lambda$ is equal to
(a) $-6$ (b) $-2$ (c) 6 (d) 0

64. Coefficient of $x^{18}$ in the expansion of $\sum_{r=0}^{100} (-1)^r {}^{100}C_r (2x+6)^{100-r} 4^r$ is
(a) $2^{82} \cdot {}^{100}C_{18}$ (b) $2^{100} \cdot {}^{100}C_{18}$ (c) $2^{18} \cdot {}^{100}C_{18}$ (d) $-2^{100} \cdot {}^{100}C_{18}$

65. If $x_1, x_2, x_3$ and $x_4$ are roots of equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos\beta - \sin\beta = 0$, then $\sum_{i=1}^{n} \tan^{-1} x_r$ is equal to
(a) $\beta$ (b) $\frac{\pi}{2} - \beta$ (c) $\pi - \beta$ (d) $-\beta$

66. Let two given curves $y^4 + x^2 - 2xy^2 = 0$ and $x^4 + 64y^2 - 16x^2 y = 0$ intersect each other at angle $\theta$. The value of $\tan\theta$ is
(a) $\frac{2}{5}$ (b) $\frac{-2}{5}$ (c) $\pm\frac{3}{5}$ (d) None of these

67. The value of $\frac{\sqrt{1+n^2}}{n^3} + \frac{2\sqrt{n^2+4}}{n^3} + \frac{3\sqrt{n^2+9}}{n^3} + \ldots + \frac{\sqrt{2}}{n} + \ldots$ is
(a) $\frac{2\sqrt{2}}{4}$ (b) $\frac{\pi}{2}$ (c) $\sqrt{2} + \frac{1}{3}$ (d) $\frac{1}{3}(2\sqrt{2} - 1)$

68. A cylindrical gas container is closed at the top and open at the bottom, if the iron plate of the top is $\frac{5}{4}$ times as well as the plate forming the cylindrical sides. The ratio of the radius to the height of the cylinder using minimum material for the same capacity is
(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$

69. A point is randomly chosen inside auxiliary circle of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, then
(a) probability that the point does not lie inside the ellipse is $\frac{1}{3}$
(b) probability that the point lies inside the ellipse may be $\frac{1}{3}$
(c) probability that the point lies outside the ellipse but lies in 1st or 3rd quadrant is $\frac{1}{5}$
(d) probability that the point lies inside the ellipse is $\frac{2}{4}$

70. If $\int \frac{\cos x \operatorname{cosec}^2 x}{(1-\operatorname{cosec} x)^{1/2}(1-2\operatorname{cosec} x)^{3/2}} dx = k\sqrt{\frac{\sin x - 1}{\sin x - 2}} + C$, where all the functions are well defined, then the value of $|k|$ is
(a) 4 (b) $-2$ (c) not possible (d) does not exist

71. A die is biased so that probability of getting a prime number is twice the probability of getting any other number. If getting a number greater than 4 is a success and probability of 2 success in 5 throws is $k$, then the value of $\frac{243k}{20}$ is
(a) 4 (b) 3 (c) 2 (d) 0

72. The length of diameter of circumcircle of $\Delta ABC$ with vertices $A(3, 0, -2)$, $B(3, 3, -2)$ and centroid at $(3, 1, -2/3)$ is
(a) 4 (b) 5 (c) 2 (d) 1

73. Tangent of the hyperbola $x^2 - 4y^2 = 4$ meets lines $x - 2y = 0$ and $x + 2y = 0$ at $A$ and $B$. Then, the value of $OA \times OB$ is

(a) 5 (b) 4 (c) 3 (d) $\frac{9}{20}$

74. In $\Delta ABC$ consider that $A \equiv (2,1)$ and the internal angle bisectors of $B$ and $C$ are respectively $y = x$ and $x + 2y + 1 = 0$. If the equation of $BC$ is $5x - y = k$, then the value of $k$ is

(a) $\frac{6}{7}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) 3

75. If the equation $x^3 - 6x + \lambda = 0$ has two distinct roots in $(-\sqrt{2}, \sqrt{2})$, then total number of existence of integral values of $\lambda$ is

(a) 1 (b) infinite (c) can't say (d) does not exist

76. Consider that $3x + 2y = 100$, $x, y \in R^+$. If the maximum value of $x^3y^2$ is $k$, then value of $(\log_{20} k)$ is

(a) 9 (b) $\frac{11}{5}$ (c) 5 (d) 35

77. If $f(x)$ is a periodic function with fundamental period 3, then the value of $f(45678) + f(123456789)$ is

(a) 0 (b) 1 (c) 45678 (d) 12345

78. If $f(x) = x^5 + e^{\frac{x}{3}}$ and $f^{-1}(x) = h(x)$ and $h(1) = 0$, then the value of $h'(1)$ is

(a) 3 (b) $-4$ (c) 5 (d) 6

79. If for curve $y = f(x)$ the lengths of subnormal and subtangent at any point are equal and $f(0) = 0$, then the number of solution(s) of the equation $(f(x))^2 = ||x| - 2|$ may be

(a) 5 (b) 4 (c) 3 (d) 2

80. If $\{\}$ represent the fractional function, then the value of $\int_1^3 ((x-1)(x-2)(x-3) + \{x\})\,dx$ is

(a) 3 (b) 2 (c) 1 (d) 0

81. If $f(x) = x^3 + x^2 + kx + 4$ is an injective function, the least positive integral value of $k$ is

(a) 1 (b) 2 (c) 3 (d) 4

82. The set of points in an argand diagram which satisfy both $|z| \le 4$ and $\arg z = \frac{\pi}{3}$ is

(a) a circle and a line (b) a raduis of a circle
(c) a sector of a circle (d) None of these

83. If $3^n$ is a factor of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^nC_1 & {}^{n+3}C_1 & {}^{n+6}C_1 \\ {}^nC_2 & {}^{n+3}C_2 & {}^{n+6}C_2 \end{vmatrix}$, then the maximum value of $n$ is

(a) 1 (b) 3 (c) 5 (d) 7

84. If the vector parallel to the line of intersection of the planes $x + y + z - 2 = 0$ and $x + 2y + z - 3 = 0$ is $(-\mathbf{i} + a\mathbf{j} + b\mathbf{k})$, then the value of $a + 3b$ is

(a) 3 (b) 4 (c) 5 (d) 6

**Directions** (Q. Nos. 85 and 86) *Read the following passage carefully and answer the questions given below it.*

*Let $y = f(x)$ satisfies the relations $xy = x + y + 1$, $x \in R - \{1\}$ and $g(x) = x f(x)$.*

85. $f(f(f(x)))$ is equal to

(a) $f(x)$ (b) $-f(x)$ (c) $\frac{1}{f(x)}$ (d) $\frac{-1}{f(x)}$

86. The function $g(x)$ has

(a) a maximum
(b) a minimum
(c) both maximum and minimum
(d) neither maximum nor minimum

**Directions** (Q. Nos. 87-90) *For the following questions. Choose the correct answers from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.
(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

87. If $p, q$ and $r$ be any three statements.

**Statement I** The statement $p \to (q \to r)$ is a tautology.

**Statement II** $(p \wedge q) \to r$ and $p \to (q \to r)$ are identical.

88. If $x_1$ and $x_2$ are the means of two groups having $n_1$ and $n_2$ observations.

**Statement I** The two distributions such that $\bar{x}_1 < \bar{x}_2$ and $\bar{x}$ is the mean of the combined distribution, then $x_2 < x < x_1$.

**Statement II** Combined mean $\bar{x} = \frac{\sum n_i \bar{x}_i}{\sum n_i}$.

89. The number of an arbitrary constant in an equation is called the order of the equation.

**Statement I** The differential equation of all circles in a plane must be of order 3.

**Statement II** There is only one circles passing through three non-collinear points.

90. Let $e$ and $e_1$ are the eccentricities of two hyperbolas.

**Statement I** If eccentricity of a hyperbola is 2, then eccentricity of its conjugate hyperbola is $\frac{2}{\sqrt{3}}$.

**Statement II** If $e$ and $e_1$, are the eccentricities of two conjugate hyperbolas, then $e\, e_1 > 1$.

# Answer *with* Explanations

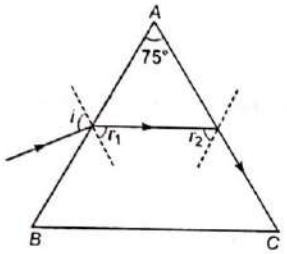
## Physics

**1.** (b) Critical angle for the material of prism,

$$\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

The ray of light will not come out the surface $AC$ if $r_2 \geq \theta_C$

$$\therefore \quad r_2 \min = \theta_c$$




or $r_2 \geq 45°$

But for prism

$$r_1 + r_2 = A$$

$$\therefore \quad r_1 \leq 30° \qquad (\because A = 75°)$$

By Snell's law,

$$\frac{\sin i}{\sin r} = \mu$$

or $\sin i \leq \sqrt{2} \sin 30°$

or $i \leq 45° \qquad (\because i_{max} = 45°)$

**2.** (c)

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{16}$$

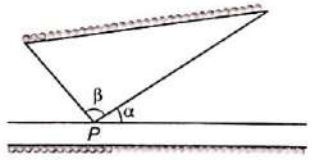
$$\Rightarrow \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n$$

or $2^4 = 2^n$

$$\therefore \quad n = 4$$

$$T_{1/2} = \frac{t}{n} = \frac{40 \text{ days}}{4} = 10 \text{ days}$$

**3.** (a) The magnetic field due to solenoid having $n$ number of turns/metre and carrying current $I$ is




$$B = \frac{\mu_0 nI}{2}(\cos\alpha - \cos\beta)$$

Here $(\cos\alpha - \cos\beta)$ is also always constant.

It is obvious that magnetic field is independent of length and area.

Also, magnetic field is uniform inside the solenoid.

**4.** (a) For second excited state, $n = 3$

$$\therefore \quad E_3 = -\frac{13.6}{3^2} = -1.51 \text{eV}$$

$$\text{IE} = E_\infty - E_3$$

$$= 0 - (-1.51 \text{eV}) = 1.51 \text{eV}$$

**5.** (a) From given information $a = -kx$, where $a$ is acceleration, $x$ is displacement and $k$ is a proportionality constant.

$$\frac{v\,dv}{dx} = -kx$$

$$\Rightarrow \quad v\,dv = -kx\,dx$$

Let, for any displacement from 0 to $x$, the velocity changes from $v_0$ to $v$.

$$\Rightarrow \quad \int_{v_0}^{v} v\,dv = -\int_0^x kx\,dx$$

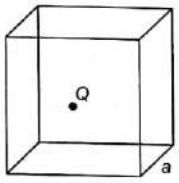
$$\Rightarrow \quad \frac{v^2 - v_0^2}{2} = -\frac{kx^2}{2}$$

$$\Rightarrow \quad m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mkx^2}{2}$$

$$\Rightarrow \quad \Delta K \propto x^2 \qquad (\Delta K \text{ is loss in KE})$$

**6.** (c) Let $V_1$ be the potential at the centre of the cube due to one charge.

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x} \quad \text{and} \quad x = \frac{a\sqrt{3}}{2}$$




Potential due to all eight corners of the cube

$$\Rightarrow \quad V = 8V_1$$

$$= 8\left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{\frac{\sqrt{3}}{2} \times a}\right]$$

$$= \frac{16Q}{4\pi\varepsilon_0 \sqrt{3} \times a} = \frac{4Q}{a\sqrt{3}\,\pi\varepsilon_0}$$

**7.** (d) If we move a charge $q$ from point $A$ to $B$, then potential difference

$$V_B - V_A = \frac{W_{AB}}{q} \qquad \ldots(i)$$

Here, $V_B = V, V_A = 10$ volt,

$$W_{AB} = 20 \text{ J}, q = 4 \text{ C}$$

On putting these values in Eq. (i),

$$V - 10 = \frac{20}{4}$$

$$V - 10 = 5$$

$$V = 15 \text{ volt}$$

**8.** (a) The condenser with air as the dielectric has capacitance

$$C_1 = \frac{\varepsilon_0}{d}\left(\frac{3A}{4}\right) = \frac{3\varepsilon_0 A}{4d}$$

Similarly, the condenser with $K$ as the dielectric constant has capacitance

$$C_2 = \frac{\varepsilon_0 K}{d}\left(\frac{A}{4}\right) = \frac{\varepsilon_0 AK}{4d}$$

Since, $C_1$ and $C_2$ are in parallel

$$C_{net} = C_1 + C_2$$

$$= \frac{3\varepsilon_0 A}{4d} + \frac{\varepsilon_0 AK}{4d}$$

$$= \frac{\varepsilon_0 A}{d}\left[\frac{3}{4} + \frac{K}{4}\right]$$

$$= \frac{C}{4}(K + 3)$$

**9.** (c) For parallel grouping,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Differentiating both sides,

$$-\frac{dR}{R^2} = -\frac{dR_1}{R_1^2} - \frac{dR_2}{R_2^2}$$

$$\Rightarrow \quad \frac{dR}{R^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}$$

$$\Rightarrow \quad \frac{dR}{R} = R\left(\frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}\right)$$

$$= \frac{6 \times 10}{(6 + 10)}\left(\frac{0.3}{(6)^2} + \frac{0.2}{(10)^2}\right)$$

$$= 3.875\%$$

**10.** (a) Average speed $= \dfrac{\text{Distance travelled}}{\text{Time taken}}$

Let $t_1$ and $t_2$ be times taken by the car to go from $X$ to $Y$ and then from $Y$ to $X$ respectively.

Then, $$t_1 + t_2 = \frac{XY}{v_u} + \frac{XY}{v_d}$$

$$= XY\left(\frac{v_u + v_d}{v_u v_d}\right)$$

Total distance travelled

$$= XY + XY = 2XY$$

Therefore, average speed of the car for this round trip is

$$v_{av} = \frac{2XY}{XY\left(\dfrac{v_u + v_d}{v_u v_d}\right)}$$

or $$v_{av} = \frac{2v_u v_d}{v_u + v_d}$$

**11.** (b) Given, $x = \alpha t^3$, $y = \beta t^3$

Then, $$v_x = \frac{dx}{dt} = 3\alpha t^2$$

and $$v_y = \frac{dy}{dt} = 3\beta t^2$$

Resultant velocity

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

**12.** (c) The equation of AC is

$$I = I_1 \cos \omega t + I_2 \sin \omega t$$

The resultant current is given by

$$I_0 = \sqrt{I_1^2 + I_2^2} \qquad \ldots(i)$$

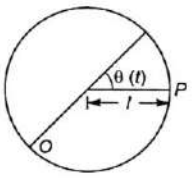
Hence, the rms current from relation is

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$= \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}} \qquad \text{[from Eq. (i)]}$$

$$= \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

**13.** (a) To calculate the emf we can imagine a closed loop by connecting the centre with any point on the circumference, say $P$ with a resistor. The potential difference across the resistor is then equal to the induced emf. It arises due to separation of charges.




$e = B \times$ (rate of change of area of loop)

If $\theta$ is the angle between the rod and the radius of circle at $P$ at time $t$, the area of the arc formed by the rod and radius at $P = \dfrac{1}{2} l^2\theta$.

(As $2\pi$ rad subtends $\pi l^2$ area)

where, $l$ is radius of the circle

$$\therefore \quad e = B \times \frac{d}{dt}\left(\frac{1}{2} l^2\theta\right)$$

$$= \frac{1}{2} B \cdot l^2 \frac{d\theta}{dt}$$

$$= \frac{1}{2} Bl^2\omega \qquad \left(\because \omega = \frac{d\theta}{dt}\right)$$

**14.** (a) As we know that

$$|e| = \left|\frac{\Delta\phi}{\Delta t}\right| = \left|\frac{1.35 - 0.79}{\frac{1}{10}}\right|$$

and $i = \frac{\varepsilon}{R} = \frac{10(1.35 - 0.79)}{R}$

Aslo $i = \frac{Q}{t} \Rightarrow Q = it$

$\Rightarrow \quad Q = \frac{10(1.35 - 0.79)}{R} \times \frac{1}{10}$

$\Rightarrow \quad Q = \frac{1.35 - 0.79}{7}$ (as $R = 7\,\Omega$, given)

$\therefore \quad Q = 0.08\ \text{C}$

**15.** (a) The density of concrete of course is more than that of water and a block of concrete will sink like a stone, if dropped into water. Concrete cargo were filled with air and as such, average density of cargo vessels

$$= \frac{\text{mass of concrete + mass of air}}{\text{volume of concrete + volume of air}}$$

It follows that the average density of cargo vessels must be less than that of water. As a result the concrete cargo vessels did not sink.

**16.** (a) According to first law of thermodynamics, amount of heat energy $Q$ taken or given to a thermodynamic system is equal to the sum of work done by or on the system $W$ and change in its internal energy $\Delta U$.

$$Q = W + \Delta U$$

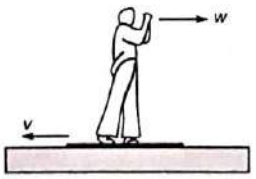
Second law of thermodynamics states that heat cannot be made to flow from cold body to a hot body without any external aid of machine but heat can flow from a hot body to a cold body by itself.

**17.** (a) Work done = heat supplied

We know that 1 cal = 4.2 J, therefore

$$W = 200\ \text{cal}$$
$$W = 200 \times 4.2$$
$$W = 840\ \text{J}$$

**18.** Consider the situation shown in figure. Suppose the man moves at a speed $w$ towards right and the plantform recoils at a speed $V$ towards left, both relative to the ice. Hence, the speed of the man relative to the platform is $V + w$. By the question,




$$V + w = v, \text{ or } w = v - V. \qquad \ldots(\text{i})$$

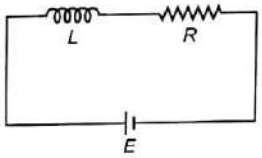
Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant. Initially, both the man and the platform were at rest. Thus,

$$0 = MV - mw$$

or $\quad MV = m(v - V)$

or $\quad V = \frac{mv}{M + m}$. [from Eq. (i)]

**19.** (a) In $L$-$R$ growth circuit current grows to 63.2% of its maximum value in one time constant, it means required time is $t = \tau = \frac{L}{R}$.




If the circuit is switched ON at $t = 0$, then

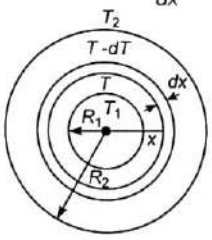
$$i(t) = \frac{E}{R}[1 - e^{-t/\tau}] \qquad \left[\tau = \frac{L}{R}\right]$$

$$i_{max} = \frac{E}{R}$$

$\Rightarrow \quad 3 = \frac{8}{R} \Rightarrow R = \frac{8}{3}\Omega$

So, $\quad t = \tau = \frac{L}{R} = \frac{6 \times 10^{-3}}{8/3}\,\text{s} = \frac{9}{4}\,\text{ms}$

**20.** (b) Temperature is decreasing as we are going out, let at a distance $x$ from the centre, the temperature gradient is $-\frac{dT}{dx}$.




At this location, $K = a_0 Tx$

From $\quad H = -KA\left(\frac{dT}{dx}\right)$

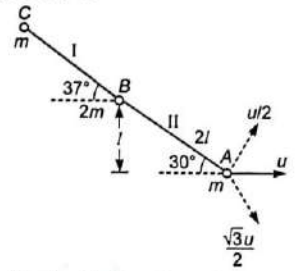
$\Rightarrow \quad H = -a_0 Tx \times 4\pi x^2 \times \frac{dT}{dx}$

$\Rightarrow \quad \int_{R_1}^{R_2} \frac{H\,dx}{x^3} = -\int_{T_1}^{T_2} 4\pi\alpha_0 T\,dT$

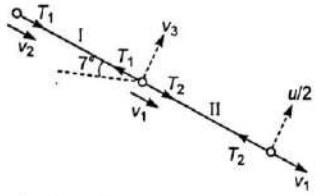
On solving we get,

$\Rightarrow \quad H = \frac{4\pi a_0 R_1^2 R_2^2 (T_1^2 - T_2^2)}{R_2^2 - R_1^2}$

**Solutions for Q. Nos. 21-23**




Position just before the string gets taut




Situation just after the string becomes taut.

The situations are shown just before the string gets taut and just after the string gets taut. The point masses are shown as $A$, $B$ and $C$.

Along the length of string, the two particles connected to two ends of a string have same velocity just after the string becomes taut. Let $T_1$ and $T_2$ be impulsive tensions in strings I and II, respectively.

Let $v_1$ be the velocity of $A$ along the length of string II which is same for $B$ also, but due to a component of $T_1$ perpendicular to spring II, $A$ also acquires a velocity $v_3$ as shown above.

Velocity of $C$ along the length of string I is $v_1$.

For $C$, $\int T_2\,dt = J_2 = -mv_1 - \left(-\dfrac{m\sqrt{3}\,u}{2}\right)$

$$= \frac{\sqrt{3}\,mu}{2} - mv_1 \qquad \ldots(i)$$

For $B$, $\int T_2\,dt - \int T_1 \cos 7°\,dt = 2mv_1$

$$\Rightarrow \quad J_2 - J_1 \cos 7° = 2mv_1 \qquad \ldots(ii)$$

For $B$, along a direction perpendicular to length of string II,

$$\int T_1 \sin 7°\,dt = J_1 \sin 7° = 2mv_3 \qquad \ldots(iii)$$

For $C$, $\int T_1\,dt = J_1 = mv_2 \qquad \ldots(iv)$

($v_2$ = Velocity of $C$ along string I)

Velocity of $B$ and $C$ along the string I would be the same, therefore,

$$v_2 = v_1 \cos 7° - v_3 \sin 7° \qquad \ldots(v)$$

Solving above Eq. (v), we get

$$v_1 = \frac{\sqrt{3}\,u}{6} - \frac{\sqrt{3}\,u\cos^2 7°}{3(6 + 3\sin^2 7° + 2\cos^2 7°)}$$

$$v_2 = \frac{\sqrt{3}\,u\cos 7°}{6 + 3\sin^2 7° + 2\cos^2 7°}$$

$$v_3 = \frac{\sqrt{3}\,u\sin 7°\cos 7°}{2[6 + 3\sin^2 7° + 2\cos^2 7°]}$$

$$J_1 = \frac{\sqrt{3}\,mu\cos 7°}{6 + 3\sin^2 7° + 2\cos^2 7°}$$

$$J_2 = \frac{mu}{\sqrt{3}} + \frac{mu\cos^2 7°}{\sqrt{3}(6 + 3\sin^2 7° + 2\cos^2 7°)}$$

In Q. No. 21, we have to find $J_2$.

In Q. No. 22, we have to find $v_3$.

In Q. No. 23, we have to find $v_2$.

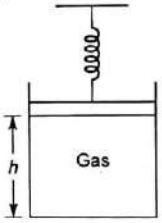
**21.** (a) **22.** (c) **23.** (b)

**Solution** (Q. Nos. 24-26)

Let mass of piston be $M$ and cross-section area be $A$, then from initial equilibrium situation of system,

$$Mg + p_0A = pA$$

where, $p_0$ is the atmospheric pressure = 1 atm




$l$ is the initial pressure of gas = 2 atm

$$\Rightarrow \quad M \times 10 = (p - p_0)\,A$$

$$M = 50\ \text{kg}$$

Let initially, the height of piston from base of cylinder be $h$, and when we supply heat to the system to increase pressure to 5 atm, the spring gets compressed by $x_0$.

As here are pressure is changing, volume is changing are temperature is changing. Heat exchange is taking place between the system and surroundings, so the process is neither adiabatic, isobaric, isochoric or isothermal.

Here, the process is polytropic.

Now, $dU = nC_V\,dT$

$$= 1 \times \frac{3R}{2}(T_f - T_i) = \frac{3R}{2}[T_f - 300]$$

where, $T_f$ is the final temperature of gas.

From $pV = nRT$ for initial and final situations,

$$\therefore\ 2 \times 10^5 \times 5 \times 10^{-3} \times h = 1 \times R \times 300$$

(Initial situation)

and $5 \times 10^5 \times 5 \times 10^{-3} \times (h + x_0) = 1 \times R \times T_f$

(Final situation)

From mechanical equilibrium of system,

$$Mg + Kx_0 + p_0A = p_fA, \text{ where } p_f = 5\ \text{atm}$$

$$\Rightarrow \quad 10^4 x_0 = 3 \times 10^5 \times 5 \times 10^{-3}$$

$\Rightarrow \quad x_0 = 0.15\,m$

From Ist law of thermodynamics,

$$\Delta Q = dU + p\,dV \qquad (i.e., \Delta W)$$

Here, work done by system against the surroundings is,

$$W = p\,dV = p_0 A x_0 + \frac{K x_0^2}{2}$$

$$= 10^5 \times 5 \times 10^{-3} \times 0.15 + \frac{10^4 \times (0.15)^2}{2}$$

$$= 187.5\ J$$

Solving the above equation, $dU = 6165$ J

and $\Delta Q = 6165 + 187.5 = 6352.5$ J

**24.** (d) **25.** (b) **26.** (c)

**27.** (b) From classical theory, light of sufficient intensity (energy per unit area in unit time) causes electrons to be emitted, independent of frequency and certainly without cut-off frequency. The maximum kinetic energy of the photoelectrons is more for light having high intensity and emission would be delayed for light having lower intensity.

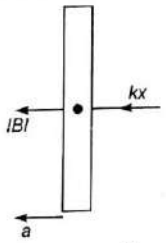
Classical physics is not able to explain photoelectric effect. Hence, here both the Statements are correct, but Statement II is not a correct explanation for Statement I.

**28.** (d) As the energy of striking electron is increased, the wavelength of the characteristic X-ray does not change as characteristic X-rays are emitted when electron are making transition from higher energy level to a lower energy level and energy of characteristic X-ray is given by

$$E = \frac{hc}{\lambda} = |E_f| - |E_i| = E_i - E_f$$

which does not depend at all on the energy of striking electron. The only dependence is that striking electron should possess enough energy to knock out an electron from the inner shell.

**29.** (c) Let the velocity of rod be $v$ when it has been displaced by $x$. Due to motion of rod an emf, will be induced in rod given by $e = Bvl$, due to this induced emf, charging of the capacitor takes place as a current flows in the circuit [for very small time] as a result of this current, the rod experiences a magnetic force given by $IBl$




From Newton's second law,

$$IBl + kx = ma$$

$$I = \frac{d}{dt}[Q] = \frac{d}{dt}[C \times Bvl] = CBl \times \frac{dv}{dt} \quad \left[\because a = \frac{dv}{dt}\right]$$

$$\Rightarrow a = \frac{kx}{m - B^2 l^2 C} = \omega^2 x, \text{ Here, } \frac{k}{m - B^2 l^2 C} = \omega^2$$

Which also shows that rod is performing SHM.

**30.** (a) Magnetic field at any inside point of the uniform distributed long current carrying wire, is

$B = \frac{\mu_0 J r}{2}$, where $J$ is the current density,

and at any outside point is,

$$B = \frac{\mu_0 I}{2\pi r}, \quad \text{where } I = JA = J \times \pi r^2$$

From $B = \frac{\mu_0 J r}{2}$

$B \propto J$ (current density)

*i.e.*, slope of $B$ *versus* $r$ curve is proportional to $J$.

So, current density is maximum for $a$ and $b$ (same for these two).

# Chemistry

**31.** (b) $CaCl_2 \longrightarrow Ca^{2+} + 2Cl^-$

$$\pi = i\,CRT$$

$$\pi = i\,\frac{n}{v}RT$$

$$0.75 = \frac{2.47 \times n \times 0.0821 \times 300}{2.5}$$

$$n = \frac{0.75 \times 2.5}{2.47 \times 0.0821 \times 300}$$

$$= 0.03\ mol$$

**32.** (c) **At Cathode**

$$H_2O + 2e^- \longrightarrow H_2 + 2OH^{\ominus}$$

**At Anode**

$$H_2O \longrightarrow \frac{1}{2}O_2 + 2H^{\oplus} + 2e^-$$

*i.e.*, on passing 2 mol of current, 1 mol $H_2(g)$ and $\frac{1}{2}$ mol $O_2(g)$ is liberated at cathode and anode respectively.

From, $Q = i\,t$

$$= 1 \times (16 \times 60 + 5) = 965\ C$$

The amount of gases liberated by passing 965 C of electricity $= \frac{1.5 \times 965}{2 \times 96500}$ mol

$$= 0.0075\ mol$$

and volume of gases liberated

$$= 0.0075 \times 22400 \text{ mL} = 168 \text{ mL}$$

**33.** (a) ∵ LiCl is a fcc crystal.

∴ Ionic radius of its chloride ion,

$$r = \frac{\sqrt{2}}{4}a$$

$$= \frac{\sqrt{2}}{4} \times 5.14$$

$$= 1.816 \text{ Å}$$

**34.** (c) EAN = Atomic number of central atom − Oxidation number + 2 × Coordination number

For Ni EAN = 28 − 2 + 2 × 4

= 34

For Cu EAN = 29 − 2 + 2 × 4

= 35

For Pt EAN = 78 − 4 + 2 × 6

= 86

**35.** (c) According to Heisenberg's uncertainty principle

$$\Delta p \cdot \Delta x \geq \frac{h}{4\pi}$$

or $m \cdot \Delta u \cdot \Delta x = \frac{h}{4\pi}$ $(\because \Delta p = m\,\Delta u)$

or $(m^2\,\Delta u^2) = \frac{h}{4\pi}$ $(\because \Delta x = \Delta p)$

or $\Delta u = \frac{1}{2m}\sqrt{\frac{h}{\pi}}$

**36.** (b) Meq of HCl $= M_1V_1$

$= 0.2 \times 100 = 20$

Meq of KOH $= M_2V_2$

$= 0.1 \times 200 = 20$

20 meq of HCl reacts completely with 20 meq of KOH. Hence, the final solution is neutral. Thus, at 90°C, pH = 6.

**37.** (b) For monoatomic gas

$$C_v = \frac{3}{2}RT$$

$$C_p = \frac{5}{2}RT$$

For diatomic gas

$$C_v = \frac{5}{2}RT$$

$$C_p = \frac{7}{2}RT$$

Thus, for mixture of 1 mol each

$$C_v = \frac{\frac{3}{2}RT + \frac{5}{2}RT}{2} = 2RT$$

$$C_p = \frac{\frac{5}{2}RT + \frac{7}{2}RT}{2} = 3RT$$

$$\therefore \quad \frac{C_p}{C_v} = \frac{3RT}{2RT} = 1.5$$

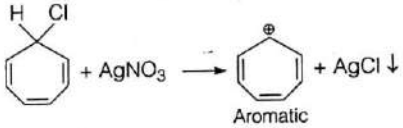
**38.** (b) $A_2(g) \longrightarrow B(g) + \frac{1}{2}C(g)$

$$-\frac{d[A_2]}{dt} = 2\frac{d[C]}{dt}$$

$$\frac{d[C]}{dt} = \frac{120 - 100}{5} = 4 \text{ mm min}^{-1}$$

$$\therefore \quad \frac{d[A_2]}{dt} = 2 \times 4$$

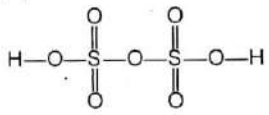
$$= 8 \text{ mm min}^{-1}$$

**39.** (b) 

Aromatic

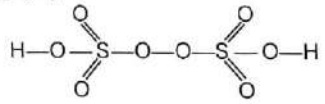
In other cases, the compound formed is either anti-aromatic or non-aromatic.

**40.** (c) $H_2S_2O_7$



(No peroxide linkage)

$H_2S_2O_8$



(One peroxide linakge)

**41.** (a) $Fe + 2HCl \longrightarrow FeCl_2 + H_2$

1 mol 1 mol

$\frac{112}{56} = 2$ mol 2 mol

Now, $V_2 = V_{H_2}$ and $V_1 = 0$

(For solid + liquid state)

Work done $= p(V_2 - V_1)$

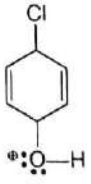
$= pV_{H_2} = n_{H_2}RT$

$= 2 \times 2 \times 300$

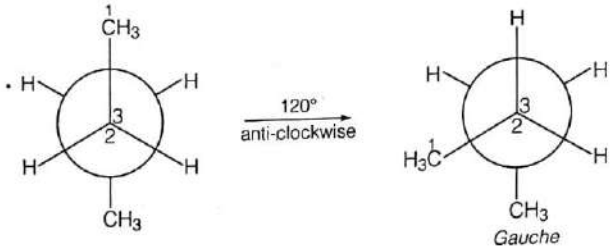
$= 1200$ cal

**42.** (c) $2XeF_6 + SiO_2 \longrightarrow 2XeOF_4 + SiF_4$

**43.** (b) In structure given in option (b), the octet of each element is complete and in other structures one carbon atom has only 6 electron in their valence shell.

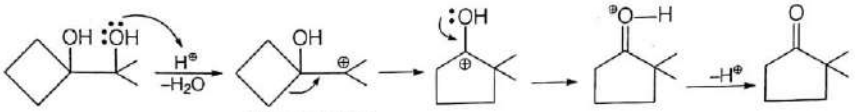


**44.** (b)




Gauche

**45.** (b)




Ring expansion

**46.** (c) Electrophoresis is used to determine the nature of charge on the colloidal particles. In this method, electric potential is applied across two platinum electrodes dipping in a colloidal solution. As a result colloidal particles move towards opositively charged electrodes.

**47.** (a) $2H_3BO_3 \xrightarrow[-H_2O]{\Delta} 2HBO_3 \xrightarrow[-H_2O]{\Delta} \underset{\text{Boric anydride}}{B_2O_3}$

**48.** (b) **Isomers of $C_5H_8$**

$CH_3—CH_2—CH_2—C\equiv CH$ $\qquad$ $CH_3—\underset{\displaystyle CH_3}{\underset{|}{CH}}—C\equiv CH$

$CH_3—CH_2—C\equiv C—CH_3$

**49.** (b)

| | $N_2$ | + $3H_2$ | $\rightleftharpoons$ $2NH_3$ | |
|---|---|---|---|---|
| Initially | 1 | 3 | 0 | |
| At equilibrium | $(1-x)$ | $(3-3x)$ | $2x$ | |
| | 0.5 | 1.5 | 1.0 | $(\because x = 0.5)$ |

$$\text{Mole fraction of } NH_3 = \frac{n_{NH_3}}{n_{Total}} = \frac{1}{0.5+1.5+1} = \frac{1}{3}$$

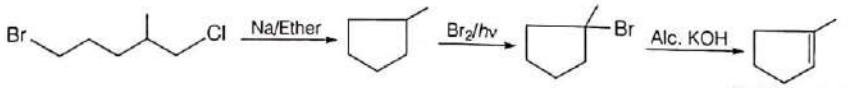
Thus, partial pressure of $NH_3$, $p_{NH_3} = p_{Total} \times$ mole fraction of $NH_3$

$$= p \times \frac{1}{3} = \frac{p}{3}$$

**50.** (d) Since aqueous solution of salt gives white precipitate with dil HCl, therefore its basic ion must be a group I radical. Again on passing $H_2S$ gas in hot acidic solution formation of black precipitate indicates that its basic ion must also be a group II radical. Since, only $Pb^{2+}$ is the only basic radical, present in both groups I and II. Therefore, substance is a $Pb^{2+}$ salt.

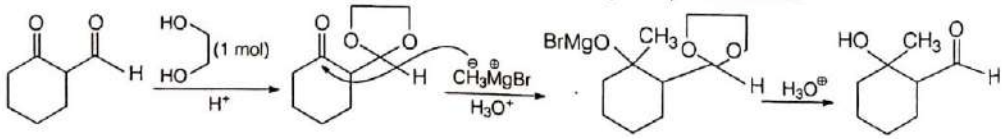
$$Pb^{2+} + HCl \xrightarrow{\Delta} \underset{\text{White}}{PbCl_2} \xrightarrow{H_2S} \underset{\text{Black}}{PbS}$$

**51.** (b)

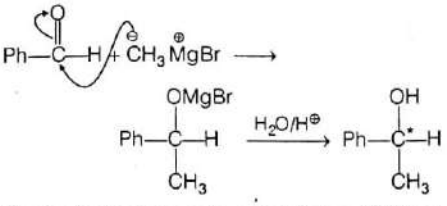



Major product

**52.** (c) Since, 1 mol of glycol is used. Hence, the carbonyl group of aldehyde is protected first.



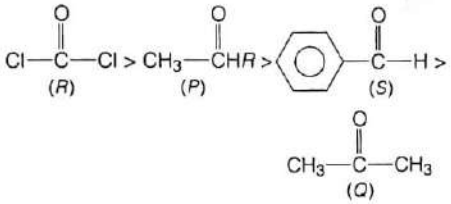

**53.** (a) Except $CH_3CH_2Cl$, all other compounds can readily give $H^{\oplus}$ ion to form $RH$, therefore, $CH_3CH_2Cl$ will not give acid-base reaction with $RMgX$.

54. (c) 

Due to the presence of a chiral C-atom (C*) final product formed is optically active. In this, attack on carbonyl group can take place from both the sides. Hence, racemic mixture is likely to be obtained. Other compounds give optically inactive compounds with Grignard reagent.

55. (a) Alkyl group being electron donating if present, increases the electron density at carbonyl carbon and thus, reduces its polarity. Hence, it becomes less reactive towards nucleophilic addition reaction.

Therefore, the order of nucleophilic reaction is



**Solution** (Q. Nos. 56-58)

The mixture contains excess of NaOH + $ZnSO_4$

$$4NaOH + ZnSO_4 \longrightarrow \underset{\text{(Soluble)}}{Na_2ZnO_2} + Na_2SO_4 + 2H_2O$$

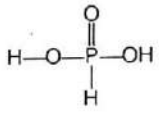
$$Na_2ZnO_2 + 2HCl \longrightarrow 2NaCl + \underset{\text{White precipitate}}{Zn(OH)_2}\downarrow$$

$$Zn(OH)_2 + 2HCl \longrightarrow \underset{\text{(Soluble)}}{ZnCl_2} + 2H_2O$$

56. (d) 57. (c) 58. (c)

59. (d) Statement I is not correct as $H_3PO_3$ is a dibasic acid. Hence, its normality will be 0.2 N.

Statement II is correct statement as structure of $H_3PO_3$ is as follows



60. (a) Both Statements are correct and Statement II is the correct explanation of Statement I.

# Mathematics

61. (b) Let us consider two points on coordinate axes are $A(a, 0)$ and $B(0, b)$.

Now, $AB = \sqrt{a^2 + b^2} = \lambda$ (say)

Let mid-point of $AB$ is $(x_1, y_1)$.

Then, $x_1 = \frac{a}{2}, y_1 = \frac{b}{2}$

$\Rightarrow$ $a = 2x_1,\ b = 2y_1$

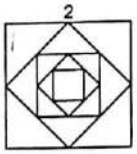
$\therefore$ $\sqrt{4x_1^2 + 4y_1^2} = \lambda \Rightarrow x_1^2 + y_1^2 = \frac{\lambda^2}{4}$

Hence, locus of a point is $x^2 + y^2 = \frac{\lambda^2}{4}$,

which is represents a equation fo circle.

62. (c) Sides of each squares are $2, \sqrt{2}, 1, \ldots$

Length of diagonal of each squares are $2\sqrt{2}, 2, \sqrt{2}, \ldots$




**Note** Each square consist of two diagonals

$$\therefore \text{ Sum of diagonals} = 2 \times \frac{2\sqrt{2}}{1 - \left[\frac{1}{\sqrt{2}}\right]}$$

$$\left(\because S_\infty \text{ of GP} = \frac{a}{1-r}\right)$$

$$= \frac{8}{\sqrt{2} - 1} = 8(\sqrt{2} + 1)$$

63. (a) $2(x^2 - 6x + 9) - 18 + 2(y^2 + 4y + 4) - 8 + 1 = 0$

$\Rightarrow$ $2(x - 3)^2 + 2(y + 2)^2 = 25$

$\because$ Angle is right angle, hence $(6, \lambda)$ lies on the director circle

$$(x - 3)^2 + (y + 2)^2 = 25$$

$\therefore$ $9 + (\lambda + 2)^2 = 25$

$\Rightarrow$ $\lambda + 2 = 4$ or $\lambda + 2 = -4$

$\Rightarrow$ $\lambda = 2$ or $\lambda = -6$

64. (b) ${}^{100}C_0(2x + 6)^{100} - {}^{100}C_1(2x + 6)^{99}4 + {}^{100}C_2(2x + 6)^{98}4^2 - \ldots + {}^{100}C_{100}4^{100}$

$= (2x + 6 - 4)^{100}$

$= 2^{100}(x + 1)^{100}$

$T_{r+1} = 2^{100} \cdot {}^{100}C_r x^{100-r}$

$\therefore$ Coefficient of $x^{18} = {}^{100}C_{82} \cdot 2^{100}$

$= 2^{100} \cdot {}^{100}C_{18}$

**65.** (b) Here, $\sum x_1 = \sin 2\beta$, $\sum x_1x_2 = \cos 2\beta$,

$\sum x_1x_2x_3 = \cos\beta$, $x_1x_2x_3x_4 = -\sin\beta$

$$\therefore \sum_{i=1}^{4} \tan^{-1} x_i = \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$$

$$= \tan^{-1}\left[\frac{\sum x_1 - \sum x_1x_2x_3}{1 - \sum x_1x_2 + x_1x_2x_3x_4}\right]$$

$$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$$

$$= \tan^{-1}\left(\frac{\cos\beta(2\sin\beta - 1)}{\sin\beta(2\sin\beta - 1)}\right)$$

$$= \tan^{-1}(\cot\beta) = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \beta\right)\right)$$

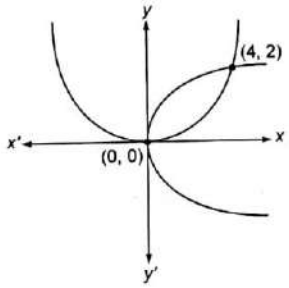
$$= \frac{\pi}{2} - \beta$$

**66.** (c) The given curves are

$$(y^2 - x)^2 = 0 \Rightarrow y^2 = x \quad \ldots(i)$$

and $(x^2 - 8y)^2 = 0 \Rightarrow x^2 = 8y \quad \ldots(ii)$

From Eqs. (i) and (ii), we find that point of intersections is (0, 0) and (4, 2).




Clearly, at (0, 0) the angle between tangent = 90°.

$\Rightarrow$ $\theta = 90°$ and hence $\sin\theta = 1$

From Eq. (i), we get

At point (4, 2),

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$\frac{dy}{dx} = \frac{1}{2\times 2} = \frac{1}{4}$$

From Eq. (ii), we get

$$\frac{dy}{dx} = \frac{x}{4}$$

At point (4, 2),

$$\frac{dy}{dx} = \frac{4}{4} = 1$$

$$\tan\theta = \pm\frac{\frac{1}{4} - 1}{1 + \frac{1}{4}} = \pm\frac{3}{5}$$

**67.** (d) Let $S_n = \frac{\sqrt{1+n^2}}{n^3} + \frac{2\sqrt{n^2+4}}{n^3} + \frac{3\sqrt{n^2+9}}{n^3} + \ldots + \frac{n\sqrt{n^2+n^2}}{n^3}$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r\sqrt{n^2+r^2}}{n^3}$$

$$= \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{r}{n}\sqrt{\left(\frac{r}{n}\right)^2 + 1}$$

$$= \int_0^1 x\sqrt{x^2+1}\,dx$$

$$= \int_1^{\sqrt{2}} t^2\,dt \qquad (\because \text{put } x^2 + 1 = t^2)$$

$$= \left[\frac{t^3}{3}\right]_1^{\sqrt{2}}$$

$$= \frac{1}{3}(2\sqrt{2} - 1)$$

**68.** (c) Let $k$ be the thickness of the sides, then the top will be $\frac{5}{4}k$.

Here, $S = (2\pi rh)k + (\pi r^2)\frac{5}{4}k$ (by condition)

$$\Rightarrow \quad S = 2\pi rk \cdot \frac{V}{\pi r^2} + \frac{5}{4}\pi r^2 k$$

$$= k\left(\frac{2V}{r} + \frac{5}{4}\pi r^2\right) \qquad (\because V = \pi r^2 h)$$

$$\Rightarrow \quad \frac{dS}{dr} = k\left(-\frac{2V}{r^2} + \frac{5}{2}\pi r\right)$$

For maxima or minima,

$$\frac{dS}{dr} = 0 \Rightarrow r^3 = \frac{4V}{5\pi}$$

$$\left[\frac{d^2S}{dr^2}\right]_{\text{at } r = \sqrt[3]{\frac{4V}{5\pi}}} = k\left(\frac{4V}{r^3} + \frac{5}{2}\pi\right)$$

$$= \pi k\left(5 + \frac{5}{2}\right) > 0,$$

where, $r^3 = \frac{4V}{5\pi}$

$$\Rightarrow \quad r^3 = 4 \times \frac{\pi r^2 h}{5\pi}$$

$$\Rightarrow \quad \frac{r}{h} = \frac{4}{5}$$

**69.** (a) Area of ellipse $= 6\pi$ $(\because A_e = \pi ab)$

$\because$ Equation of auxiliary circle of the given ellipse whose length of major axis is 3.

*i.e.*, $x^2 + y^2 = 9$

Auxiliary circle has radius 3.

Hence, area of auxiliary circle $= 9\pi$ $(\because A_c = \pi r^2)$

$$\therefore P(\text{point lies inside ellipse}) = \frac{6\pi}{9\pi} = \frac{2}{3}$$

$$P(\text{point lies outside ellipse}) = \frac{3\pi}{9\pi} = \frac{1}{3}$$

**70.** (b) The given integration can be written as

$$\int \frac{\cos x\, dx}{(\sin x-1)^{1/2}(\sin x-2)^{3/2}}$$

Put $t = \sin x$

$\Rightarrow$ $dt = \cos x\, dx$

$$= \int \frac{dt}{(t-1)^{1/2}(t-2)^{3/2}}$$

$$= \int \frac{dt}{(t-1)^2\left(\frac{t-2}{t-1}\right)^{3/2}}$$

Put $z = \frac{t-2}{t-1}$

$$dz = \frac{1}{(t-1)^2}dt$$

$$= \int \frac{dz}{z^{3/2}} = \left[\frac{z^{-1/2}}{-1/2}\right] + C$$

$$= -2\left(\frac{t-2}{t-1}\right)^{-1/2} + C$$

$$= -2\left(\sqrt{\frac{\sin x-1}{\sin x-2}}\right) + C$$

$\Rightarrow$ $|k| = -2$

**71.** (a) In a die, prime numbers are 2, 3, 5 and other numbers are 1, 4, 6.

Since, chance of other number is twice the chance of prime number. Therefore, total number of cases are 9.

$P(1) = \frac{1}{9}, P(2) = \frac{2}{9}, P(3) = \frac{2}{9}, P(4) = \frac{1}{9}, P(5) = \frac{2}{9}, P(6) = \frac{1}{9}$

Here, $p = P(5) + P(6) = \frac{1}{3}, q = \frac{2}{3}$

$P$(2 successes)

$$= {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3$$

$$= {}^5C_2 \times \frac{1}{9} \times \frac{8}{27} = \frac{80}{243} = k \quad \text{(given)}$$

$\therefore$ $\frac{243k}{20} = \frac{243}{20} \times \frac{80}{243} = 4$

**72.** (b) Let the vertices be $C(x, y, z)$.

$$\therefore \left(\frac{3+3+x}{3}, \frac{0+3+y}{3}, \frac{-2-2+z}{3}\right) = \left(3, 1, -\frac{2}{3}\right) \text{ (given)}$$

$\Rightarrow$ $x = 3, y = 0, z = 2$

$\therefore$ Coordinate of $C(3, 0, 2)$

$AB^2 = 0^2 + 3^2 + 0^2 = 0 + 9$

$BC^2 = 0 + 3^2 + 4^2 = 25$

and $AC^2 = 0 + 0 + 4^2 = 16$

$\therefore$ $BC$ is hypotenuse.

$\therefore$ Diameter $= 5$

**73.** (a) Equation of tangent at $(2\sec\theta, \tan\theta)$ is

$$x\sec\theta - 2\tan\theta = 2$$

Now, the intersection point of the above tangent from the given lines are

$\Rightarrow$ $A \equiv \left(\frac{2}{\sec\theta - \tan\theta}, \frac{1}{\sec\theta - \tan\theta}\right)$

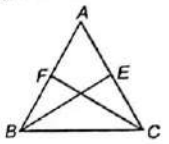
and $B \equiv \left(\frac{2}{\sec\theta + \tan\theta}, \frac{-1}{\sec\theta + \tan\theta}\right)$

$$\therefore OA \cdot OB = \sqrt{\frac{5}{(\sec\theta - \tan\theta)^2} \times \frac{5}{(\sec\theta + \tan\theta)^2}}$$

$= 5$ $(\because \sec^2\theta - \tan^2\theta = 1)$

**74.** (d) $BE : y = x$

$CF : x + 2y + 1 = 0$ (given)




The image of $A$ through the mirror $BE$ and $CF$ will lie on $BC$.

Clearly, $D \equiv (1, 2)$

Let $Q = (\alpha, \beta)$

$\Rightarrow$ $\frac{\alpha-2}{1} = \frac{\beta-1}{2} = \frac{-2(2+2+1)}{5}$

$\Rightarrow$ $\frac{\alpha-2}{1} = \frac{\beta-1}{2} = -2$

$\Rightarrow$ $\alpha = 0, \beta = -3$

$Q \equiv (0, -3)$

Hence, the equation of $BC$ is $5x - y = 3$.

**75.** (d) By Rolle's theorem, if $\alpha$ and $\beta$ $(\alpha < \beta)$ are two distinct roots, then $f'(y) = 0$ where, $y$ lies between $\alpha$ and $\beta$.

$\Rightarrow$ $3x^2 - 6 = 0$

$\Rightarrow$ $x^2 = 2$

$\Rightarrow$ $x = -\sqrt{2}$ or $\sqrt{2} \notin (-\sqrt{2}, \sqrt{2})$

Hence, Rolle's theorem fails.

Hence, it does not exist any value of $\lambda$.

**76.** (c) $\frac{x+x+x+y+y}{5} \geq (x^3y^2)^{1/5}$ $(\because$ AM $\geq$ GM$)$

$\Rightarrow$ $x^3y^2 \leq 20^5 = k$

$\Rightarrow$ $\log_{20} k = 5$

**77.** (a) $f(x) = -f(x) \Rightarrow f(0) = 0 = f(3) = f(6)$

$= \ldots = f(45678) \ldots = f(123456789)$

Hence, sum $= 0 + 0 = 0$

**78.** (a) $f(h(x)) = h(f(x)) = x$

$\Rightarrow f'(h(x)) \cdot h'(x) = h'(f(x))\, f'(x) = 1$

$$\Rightarrow \quad h'(x)=\frac{1}{f'(h(x))}$$

$$\Rightarrow \quad h'(1)=\frac{1}{f'(h(1))}$$

But $h(1)=0$

$$\Rightarrow \quad h'(1)=\frac{1}{f'(0)}=\frac{1}{1/3}=3$$

$$\left[\because f'(x)=5x^4+\frac{1}{3}e^{x/3} \Rightarrow f'(0)=\frac{1}{3}\right]$$

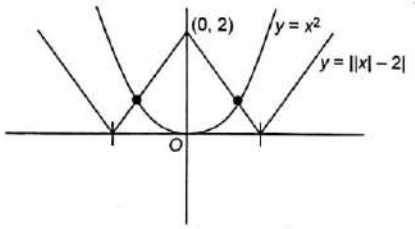
**79.** (d) $\left|y\frac{dy}{dx}\right|=\left|\frac{y}{\frac{dy}{dx}}\right|$ (by given condition)

$$\Rightarrow \quad \frac{dy}{dx}=\pm 1$$

$$\Rightarrow \quad y=\pm(x) \Rightarrow f(x)=\pm x$$

Given equation becomes $x^2=||x|-2|$

For number of solutions draw the graphs of $y=x^2$ and $y=||x|-2|$




Hence, number of solutions = 2

**80.** (c) Let $I=\int_1^3 \{x\}\,dx+\int_1^3 (x-1)(x-2)(x-3)\,dx$

Let $I_1=\int_1^3 \{x\}\,dx$

$$=\int_1^3 (x-[x])\,dx=\left[\frac{x^2}{2}\right]_1^3-\int_1^2[x]\,dx-\int_2^3[x]\,dx$$

$$=\frac{1}{2}(9-1)-1(2-1)-2(3-2)=4-1-2=1$$

$$I_2=\int_1^3 (x-1)(x-2)(x-3)\,dx$$

$$=\int_1^3 (3-x)(2-x)(1-x)\,dx=-I_2$$

$$\left(\because \int_a^b f(x)=\int_a^b f(a+b-x)\,dx\right)$$

$$\Rightarrow \quad I_2=-I_2 \Rightarrow I_2=0$$

$$\Rightarrow \quad I=I_1+I_2=1$$

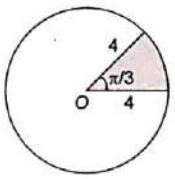
**81.** (a) $f'(x)=3x^2+2x+k\geq 0$

$$\Rightarrow \quad 4-4\times 3\times k\leq 0 \qquad (\because D\leq 0)$$

$$\Rightarrow \quad k\geq \frac{1}{3}$$

Hence, least positive integral value is 1.

**82.** (c) $|z|\leq 4$ and $\arg(z)=\frac{\pi}{3}$ represents a sector of a circle.




**83.** (b)

$$\Delta=\frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ n & (n+3) & (n+6)\\ n(n-1) & (n+3)(n+2) & (n+6)(n+5)\end{vmatrix}$$

Applying $C_2\to C_2-C_1$ and $C_3\to C_3-C_1$

$$=\frac{1}{2}\begin{vmatrix}1 & 0 & 0\\ n & 3 & 6\\ n^2-n & 6n+6 & 12n+30\end{vmatrix}$$

$$=27=3^3$$

$$\therefore \quad n=3$$

**84.** (a) Here, $\mathbf{n}_1=\mathbf{i}+\mathbf{j}+\mathbf{k}$

$$\mathbf{n}_2=\mathbf{i}+2\mathbf{j}+\mathbf{k}$$

Parallel vector $\mathbf{n}=\mathbf{n}_1\times\mathbf{n}_2$

$$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 1 & 1\\ 1 & 2 & 1\end{vmatrix}=-\mathbf{i}+\mathbf{k}$$

$$\Rightarrow \quad \mathbf{i}+\mathbf{k}=-\mathbf{i}+a\mathbf{j}+b\mathbf{k}$$

$$\therefore \quad a=0, b=1$$

$$\therefore \quad a+3b=3$$

**85.** (a) Given, $xy=x+y+1$

$$\Rightarrow \quad y=f(x)=\frac{x+1}{x-1}$$

Now, $f(f(x))=\frac{f(x)+1}{f(x)-1}$

$$\Rightarrow \quad f(f(x))=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}=x$$

$$\therefore \quad f(f(f(x)))=f(x)$$

**86.** (c) $g(x)=xf(x)=x\cdot\frac{(x+1)}{(x-1)}=\frac{x^2+x}{x-1}$

$$g'(x)=\frac{(x-1)(2x+1)-x^2-x}{(x-1)^2}$$

$$=\frac{x^2-2x-1}{(x-1)^2}$$

$g'(x)=0$ gives $x=1\pm\sqrt{2}$

$$g''(x)=\frac{(x-1)^2(2x-2)-2(x-1)(x^2-2x-1)}{(x-1)^4}$$

$$= \frac{4}{(x-1)^3}$$

$g''(x) < 0$, when $x = 1 - \sqrt{2}$

$g''(x) > 0$, when $x = 1 + \sqrt{2}$

Thus, $g(x)$ has maximum at $x = 1 - \sqrt{2}$ and $g(x)$ has minimum value at $x = 1 + \sqrt{2}$.

**87.** (d)

| $p$ | $q$ | $r$ | $q \to r$ | $p \wedge q$ | $p \to (q \to r)$ | $(p \wedge q) \to r$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | F | T | F | F |
| T | F | T | T | F | T | T |
| T | F | F | T | F | T | T |
| F | T | T | T | F | T | T |
| F | T | F | F | F | T | T |
| F | F | T | T | F | T | T |
| F | F | F | T | F | T | T |

From table columns of $p \to (q \to r)$ is not a tautology and column of $p \to (q \to r)$ and $(p \wedge q) \to r$ are identical.

**88.** (d) I. Now, $\bar{x} - \bar{x}_1 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} - \bar{x}_1$

$$= \frac{n_2(\bar{x}_2 - \bar{x}_1)}{n_1 + n_2} > 0$$

$\Rightarrow \quad \bar{x} - \bar{x}_1 > 0 \Rightarrow \bar{x} > \bar{x}_1$ ...(i)

and $\bar{x} - \bar{x}_2 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} - \bar{x}_2$

$$= \frac{n_1(\bar{x}_1 - \bar{x}_2)}{n_1 + n_2} < 0$$

$\Rightarrow \quad \bar{x} - \bar{x}_2 < 0 \Rightarrow \bar{x} < \bar{x}_2$ ...(ii)

$\therefore$ From Eqs. (i) and (ii) is

$$\bar{x}_1 < \bar{x} < \bar{x}_2$$

II. It is obviously true.

**89.** (b) I. Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Here, $g, f$ and $c$ are independent constant. Hence, order is 3.

II. It is also true but it is not a correct explanation of I.

**90.** (b) I. The conjugate hyperbolas are

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

Then, $e^2 = \frac{a^2 + b^2}{a^2}$

and $e_1^2 = \frac{a^2 + b^2}{b^2}$

$\because \quad \frac{1}{e^2} + \frac{1}{e_1^2} = 1$

$\therefore \quad \frac{1}{2^2} + \frac{1}{e_1^2} = 1$

$\Rightarrow \quad \frac{1}{e_1^2} = \frac{3}{4}$

$\Rightarrow \quad e_1 = \frac{2}{\sqrt{3}}$

II. Now, $e\,e_1 = 2 \times \frac{2}{\sqrt{3}} = \frac{4}{\sqrt{3}} > 1$

# JEE Main

## Joint Entrance Examination

# Practice Set 8

## Instructions

1. The test consists of 90 questions. The maximum marks are 360.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
3. Candidates will be awarded marks as stated in the above instruction no. 1 for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 3 above.
5. The test is of 3 hours duration.

## Physics

1. Two cells of emfs $E_1$ and $E_2$ and internal resistances $r_1$ and $r_2$ are connected in parallel. Then, the emf and internal resistance of the equivalent source is

   (a) $(E_1 + E_2)$ and $\frac{r_1 r_2}{(r_1 + r_2)}$

   (b) $(E_1 - E_2)$ and $(r_1 + r_2)$

   (c) $\frac{E_1 r_2 + E_2 r_1}{(r_1 + r_2)}$ and $\frac{r_1 r_2}{(r_1 + r_2)}$

   (d) $\frac{(E_1 r_2 + E_2 r_1)}{(r_1 + r_2)}$ and $(r_1 + r_2)$

2. An alternating voltage (in volts) varies with time $t$ (in seconds) as

   $$V = 100 \sin (50 \pi t)$$

   The peak value of voltage, the rms value of the voltage and the frequency respectively are

   (a) 100 V, $\frac{100}{\sqrt{2}}$ V, 50 Hz

   (b) $\sqrt{2}$ 100 V, 100 V, 25 Hz

   (c) $\frac{100}{\sqrt{2}}$ V, $\sqrt{2}$ 100V, 50 Hz

   (d) 100V, $\frac{100}{\sqrt{2}}$ V, 25 Hz

3. A prism of material of refractive index $\sqrt{3}$ can produce a minimum deviation 60°. The angle of prism is

   (a) 30° (b) 45° (c) 60° (d) 75°

4. **Statement I** Thermodynamic process in nature are irreversible.

   **Statement II** Dissipative effects cannot be eliminated.

   (a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I

   (b) Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I

   (c) Statement I is true but Statement II is false

   (d) Both Statement I and Statemet II are false

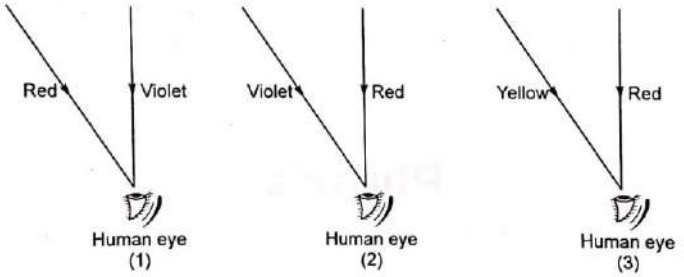
5. Efficiency of engine working at the temperatures 40°C and 20°C is
(a) 0.064% (b) 0.64% (c) 64% (d) 6.4%

6. Which one of the following statements is true for the speed $v$ and the acceleration $a$ of a particle executing simple harmonic motion?
(a) Where $v$ is maximum, $a$ is maximum
(b) Value of $a$ is zero, whatever may be the value of $v$
(c) When $v$ is zero, $a$ is zero
(d) When $v$ is maximum, $a$ is zero

7. The period of oscillation of a simple pendulum is $T$ in a stationary lift. If the lift moves upwards with acceleration of 8 $g$, the period will
(a) remain the same (b) decreases by $\frac{T}{2}$ (c) increases by $\frac{T}{3}$ (d) None of these

8. Blood is flowing at the rate of $200\,cm^3s^{-1}$ in a capillary of cross-sectional area $0.5\,m^2$. The velocity of flow (in $mms^{-1}$) is
(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4
(e) 0.5

9. The coming rays are forming rainbows




(a) Fig. (1) forms primary rainbow (b) Fig. (1) and (3) form primary rainbow
(c) Fig. (2) forms secondary rainbow (d) Fig. (1) forms secondary rainbow

10. If the refractive index of a glass prism is $\cot\left(\frac{A}{2}\right)$ and $A$ is angle of prism, then angle of minimum deviation is
(a) $\left(\frac{\pi}{2}-A\right)$ (b) $\left(2\pi-\frac{A}{2}\right)$ (c) $\left(\frac{\pi-A}{2}\right)$ (d) $(\pi-2A)$

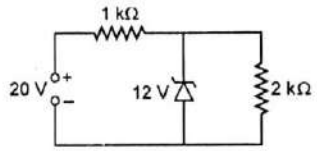
11. If the coefficient of static friction between the tyres and road is 0.5, what is the shortest distance in which an automobile can be stopped when travelling at 72 $kmh^{-1}$?
(a) 50 m (b) 60 m (c) 40.8 m (d) 80.16 m

12. Two tuning forks $P$ and $Q$ when set vibrating, give 4 beats $s^{-1}$. If a prong of the fork $P$ is filled, the beats are reduced to $2\,s^{-1}$. What is frequency of $P$, if that of $Q$ is 250 Hz?
(a) 246 Hz (b) 250 Hz (c) 254 Hz (d) 252 Hz

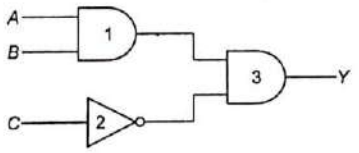
13. Two waves, represented by the following equations, are travelling in the same medium
$$y_1 = 5\sin 2\pi(75t - 0.25x)$$
$$y_2 = 10\sin 2\pi(150t - 0.50x)$$
The intensity $\frac{I_2}{I_1}$ of two waves is
(a) 8 : 1 (b) 2 : 1 (c) 4 : 1 (d) 16 : 1

14. In an unbiased $p$-$n$ junction
(a) potential at $p$ is more than that at $n$ (b) potential at $p$ is less than that at $n$
(c) potential at $p$ is equal to that at $n$ (d) potential at $p$ is +ve and that at $n$ is –ve

15. In the given circuit, the current through the resistor 2 k Ω is
(a) 2 mA (b) 4 mA
(c) 6 mA (d) 1 mA
(e) 10 mA




16. In the following circuit, the output $Y$ becomes zero for the inputs
(a) $A = 1, B = 0, C = 0$ (b) $A = 0, B = 1, C = 1$
(c) $A = 0, B = 0, C = 0$ (d) $A = 1, B = 1, C = 0$
(e) $A = 1, B = 1, C = 1$




17. **Statement I** A ball connected to a string is in circular motion on a frictionless horizontal table and is in equilibrium.
**Statement II** Magnitude of the centripetal force is equal to the magnitude of the tension in the string.
(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I
(b) Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false and Statement II is true

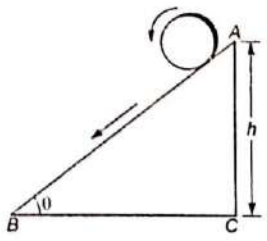
18. A bomber plane is moving horizontally with a speed of 500 $ms^{-1}$ and a bomb released from it, strikes the ground in 10 s. Angle at which the bomb strikes the ground is ($g = 10\ m\ s^{-2}$)
(a) $\tan^{-1}(1)$ (b) $\tan^{-1}(5)$ (c) $\tan^{-1}\left(\frac{1}{5}\right)$ (d) $\sin^{-1}\left(\frac{1}{5}\right)$

19. A body is projected vertically upwards. The times corresponding to height $h$ while ascending and while descending are $t_1$ and $t_2$ respectively. Then, the velocity of projection is ($g$ is acceleration due to gravity)
(a) $g\sqrt{t_1 t_2}$ (b) $\frac{g t_1 t_2}{t_1 + t_2}$ (c) $\frac{g\sqrt{t_1 t_2}}{2}$ (d) $\frac{g(t_1 + t_2)}{2}$

20. Velocity-time curve for a body, projected vertically upwards, is
(a) parabola (b) ellipse (c) hyperbola (d) straight line

21. The binding energy of a satellite of mass $m$ in a orbit of radius $r$ is ($R$ = radius of earth, $g$ = acceleration due to gravity)
(a) $\frac{mgR^2}{r}$ (b) $\frac{mgR^2}{2r}$ (c) $-\frac{mgR^2}{r}$ (d) $-\frac{mgR^2}{2r}$

22. If a sphere rolling on an inclined plane with velocity $v$ without slipping, the vertical height of the incline in terms of velocity will be
(a) $\frac{7v}{10g}$ (b) $\frac{7v^2}{10g}$
(c) $\frac{2v^2}{5g}$ (d) $\frac{3v}{5g}$



**Directions** (Q. Nos. 23 and 24) *For the following questions. Choose the correct answers from the codes (a), (b), (c) and (d) defined as follows*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I

(b) Statement I is ture, Statement II is also true and Statement II is not the correct explanation of Statement I

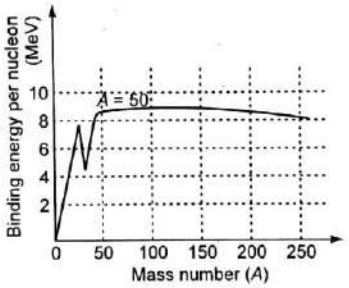
(c) Statement I is true but Statement II is false

(d) Statement I is false and Statement II is true

23. **Statement I** To determine the age of certain very old organic samples, dating of sample with radioactive isotopes having larger half-life is a better choice than with radioactive isotopes having smaller half-lives.

**Statement II** The activity of a radioactive sample having smaller half-life is negligibly small after a very long time and hence making it next to impossible to get detect.

24. **Statement I** From the plot of binding energy per nucleon *versus* mass number for various stable nuclei, as shown in figure. We can easily conclude that nuclear force are short range forces.




**Statement II** From the shown graph it is clear that binding energy per nucleon various slowly for nuclides having mass number greater than 40, which means nuclear forces are almost same in all nuclides having $A > 40$.

**Directions** (Q. Nos. 25-27) *A certain species of ionized atoms produces an emission line spectrum according to the Bohr model. A group of lines in the spectrum is forming a series in which the shortest wavelength is 22.79 nm and the longest wavelength is 41.02 nm. The atomic number of atom is Z.*

Based on above information, answer the following questions

25. The value of $Z$ is

(a) 2 (b) 3 (c) 4 (d) 5

26. The series belongs to

(a) Lyman (b) Balmer (c) Paschen (d) Brackett

27. The next to longest wavelength in the series of lines is

(a) 35.62 nm (b) 30.47 nm (c) 25.68 nm (d) 12.64 nm

**Directions** (Q. Nos. 28 and 29) *A conducting circular loop of face area* $2.5 \times 10^{-3}\ m^2$ *is placed perpendicular to a magnetic field which varies as* $B = (0.20\ T) \sin [(50\pi\ s^{-1})t]$.

28. The charge flowing through any cross-section during the time $t = 0$ to $t = 40$ ms is

(a) $\frac{i_0}{\omega}$ (b) $\frac{3i_0}{\omega}$ (c) 0 (d) $\frac{i_0}{\sqrt{2}\,\omega}$

29. If the resistance of the loop is 10 Ω, then, the thermal energy developed in the loop in this period is given by
(a) $1.25 \times 10^{-5}$ J (b) $2.25 \times 10^{-5}$ J (c) $1.50 \times 10^{-5}$ J (d) $2 \times 10^{-5}$ J

30. **Statement I** The amount of energy required to remove an average nucleon from different nuclei having different mass numbers is approximately the same, while to remove an average electron from atoms having different mass numbers is widely varying amounts of energy is required.
**Statement II** Nucleons in a nucleus are bounded by short range nuclear force while electrons in an atom are bounded by long range Coulomb's forces.
(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is also true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false and Statement II is true

# Chemistry

31. Alkaline earth metals produce bright blue colour when they dissolved in liquid ammonia. It is due to
(a) *d-d* transition in the solvated metal ion
(b) presence of unpaired electron
(c) charge-transfer transition
(d) absorption of light by the solvated electrons

32. Calculate relative rate of effusion of $O_2$ to $CH_4$ through a container containing $O_2$ and $CH_4$ in 3 : 2 mass ratio.
(a) $\frac{3\sqrt{2}}{4}$ (b) $\frac{3}{4\sqrt{2}}$ (c) $\frac{3}{2\sqrt{2}}$ (d) None of these

33. Which of the following methods gives rise to 1-bromo-2-phenyl ethane?
(a) Benzene + oxirane + $H^+$ followed by reaction with HBr
(b) Benzene + oxirane + $H^+$ followed by reaction with $P/Br_2$
(c) Propene + HBr followed by reaction with benzene in the presence of $AlCl_3$
(d) Styrene + $H_3O^+$ + followed by reaction with $P/Br_2$

34. Name the structure of silicate in which three oxygen atoms of $[SiO_4]^{4-}$ are shared.
(a) Pyrosilicate (b) Sheet silicate
(c) Linear chain silicate (d) Three dimensional sheet silicate

35. From the following reaction sequence

$$CaC_2 + H_2O \longrightarrow CaO + C_2H_2$$
$$C_2H_2 + H_2 \longrightarrow C_2H_4$$
$$nC_2H_4 \longrightarrow (C_2H_4)_n$$

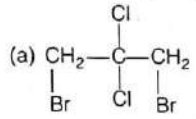
Determine the mass of polyethylene, which can be produced from 10 Kg of $CaC_2$.
(a) 4.375 g (b) 43.75 g (c) 4.375 kg (d) 4375 kg

36. A decapeptide (mol. wt. 796) on complete hydrolysis gives glycine (mol. wt.=75), alanine and phenylalanine. Glycine contributes 47% to the total weight of the hydrolysed products. The number of glycine units present in decapeptide is
(a) 3 (b) 4 (c) 5 (d) 6

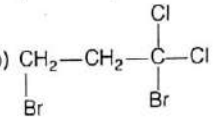
37. The solubility product of $Mg(OH)_2$ is $10^{-14}$. The solubility of $Mg(OH)_2$ in a buffer solution of pH = 8 is
(a) $10^{-8}$ (b) $10^{-6}$ (c) $10^{-2}$ (d) $10^{-4}$

38. Which of the following can be reduced by Zn?
(a) $Na[Ag(CN)_2]$ (b) $Na[Au(CN)_2]$ (c) Both (a) and (b) (d) None of these

39. 1, 1-dichloro cyclopropane reacts with $Br_2$. The possible product will be



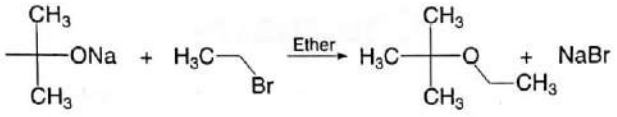
(b)

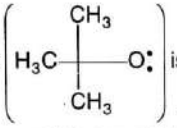


(c) Both (a) and (b) (d) None of these

40. When NaCl is doped with $1.0 \times 10^{-3}$ mole of $SrCl_2$, the number of cation vacancy is
(a) $6.023 \times 10^{18}$ (b) $6.023 \times 10^{20}$ (c) $2 \times 6.023 \times 10^{20}$ (d) $3.011 \times 10^{20}$

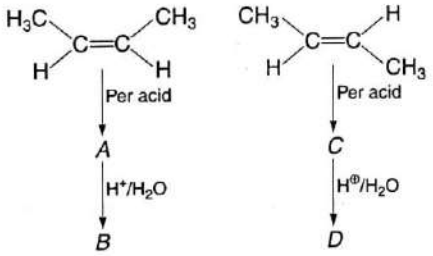
41. **Statement I** The reaction given below is not feasible.



**Statement II** *t*-butoxide  is a weak nucleophile but very strong base.

(a) Statement I is true, Statement II is true : Statement II is a correct explanation for Statement I.
(b) Statement I is true, Statement II is true : Statement II is not a correct explanation for statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

42.



Compounds in *B* and *D* are
(a) enantiomers (b) diastereomers
(c) constitutional isomers (d) geometrical isomers

**Directions** (Q. Nos. 43-45) *Dipole moment of a bond is a vector and physical quantity to calculate the percentage ionic character in a covalent bond. It is expressed as dipole moment,* $\mu = \delta \times d$

*Resultant dipole moment of two bond moments acting at an angle* $\theta$*, is given by*

$$\mu_{\text{Resultant}} = \sqrt{\mu_1^2 + \mu_2^2 + 2\mu_1 \cdot \mu_2 \cos\theta}$$

*If* $\mu \neq 0$ *molecule is polar.*

*Dipole moment plays an important role in deciding the stability order of alkanes. i.e., more stable alkane has less dipole moment.*

43. Dipole moment of HCl molecule is found to be 0.816 D. Assuming HCl bond length to be equal to 1Å, the % ionic character of HCl molecule is
(a) 10% (b) 15% (c) 17% (d) 40%

44. The correct increasing order of stability of the following alkenes,
I. *cis*-2-butene II. *trans*-2-butene III. isobutene
(a) II < III < I (b) III < I < II (c) I < II < III (d) III < II < I

45. Which one of the following species is non-polar?
(a) *trans*-pent-2-ene (b) *cis*-pent-2-ene
(c) *cis*-1-chloropropene (d) $SF_6$

46. Among the following, the correct statement is
(a) Order of Cl—O bond length is $ClO^- < ClO_2^- < ClO_3^- < ClO_4^-$
(b) Order of oxidising capacity is $HClO < HClO_2 < HClO_3 < HClO_4$
(c) Order of stability is $HClO < HClO_2 < HClO_3 < HClO_4$
(d) Order of reactivity is $IF_7 > BrF_5 > ClF_3$

47. 1 g magnesium atoms in vapour phase absorbs 50.0 kJ of energy to convert all Mg into Mg ions. The energy absorbed is needed for the following changes.

$$Mg(g) \longrightarrow Mg^+(g) + e^-; \quad \Delta H = 740 \text{ kJ mol}^{-1}$$

$$Mg^+(g) \longrightarrow Mg^{2+}(g) + e; \quad \Delta H = 1450 \text{ kJ mol}^{-1}$$

The % of $Mg^+$ and $Mg^{2+}$ in mixture (respectively) will be
(a) 31.72% and 68.28% (b) 68.28% and 31.27%
(c) 50% and 50% (d) 25% and 75%

48. The compressibility factor for a real gas at high pressure is
(a) 1 (b) $1 + \frac{pb}{RT}$ (c) $1 - \frac{pb}{RT}$ (d) $1 + \frac{RT}{pb}$

49. Which of the following compounds show geometrical isomerism?
(I) $[Co(NH_3)_4Cl_2]^+$ (II) $[Co(en)_2Cl_2]^+$ (III) $[Pt(gly)_2]$ (IV) $[Co(en)_3]^{3+}$
(a) I, II and III (b) II, III and IV (c) I, III and IV (d) I, II, III and IV

50. In brown ring Fe and NO exist as $Fe^+$ and $NO^+$ rather than $Fe^{2+}$ and NO. These forms can be differentiated by
(a) measuring the conductivity of an aqueous solution of the complex
(b) by careful crystallization of the complex under special conditions and measuring the magnetic moment in solid state
(c) estimating the concentration of iron
(d) determining elevation in boiling point using the complex as solute

51. Calculate the rate constant of first order reaction $CH_3CHO(g) \xrightarrow{\text{Heat}} CH_4(g) + CO(g)$, if the initial pressure of $CH_3CHO$ is 80 mm and the total pressure at the end of 20 min is 120 mm.
(a) $3.465 \times 10^{-2}$ min$^{-1}$ (b) 34.65 min$^{-1}$ (c) 3.465 min$^{-1}$ (d) 0.3465 min$^{-1}$

52. When the concentration of reactants is increased by 5 times, if the rate of the reaction increases by 15 times, the order of the reaction is
(a) 5 (b) 3 (c) $\log_{15}^{5}$ (d) $\log_{5}^{15}$

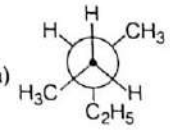
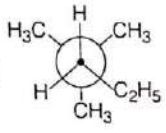
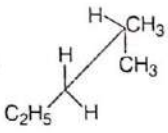
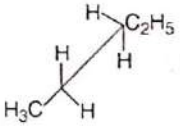
53. **Statement I** Metal hydroxides of Na and K cannot be stored in aluminium or zinc container.
**Statement II** A protective layer of oxide is formed on the surface of metal.

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I.
(b) Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

54. An acid-base indicator has a $K_a$ of $3 \times 10^{-5}$. The acid form of the indicator is red and the basic form is blue. The change in $H^+$ concentration (which is $9 \times 10^{-5}$) in order to change the indicator from 25% red to 75% blue is
(a) $2 \times 10^{-5}$ (b) $8 \times 10^{-5}$ (c) $9 \times 10^{-5}$ (d) $1 \times 10^{-5}$

55. Kinetic energy of an electron in second excited state of $He^+$ is
(a) 6.04 eV (b) 54.4 eV (c) 27.2 eV (d) – 54.4 eV

56. Which of the following conformational structures is of 2-methylpentane?

(a) 
(b) 
(c) 
(d) 

57. Two half cells, describd below

$$2H^+ + NO_3^- + e^- \longrightarrow NO_2 + H_2O;\ E^\circ = 0.80V$$
$$7H^+ + NO_3^- + 6e^- \longrightarrow NH_2OH + 2H_2O;\ E^\circ = 0.73\ V$$

were connected to each other. At a particular instance when activity of all other species except $[H^+]$ was unity, no current was flowing through the system. The uniform pH that must be maintained in order to achieve such a no current flowing situation is,
(a) at 1.0 m concentration of $H^+$ (b) pH = 1
(c) pH = 1.45 (d) pH = 6

58. The energy needed for $Li(g) \longrightarrow Li^{3+}(g) + 3e^-$ is $1.96 \times 10^4$ kJ mol$^{-1}$, if the first ionization enthalpy of Li is 520 kJ mol$^{-1}$, second ionization enthalpy of Li is approximately (ionization enthalpy of H is $2.18 \times 10^{-18}$ J/atom)
(a) 5950 kJ mol$^{-1}$ (b) 5950 J atom$^{-1}$ (c) 7270 kJ mol$^{-1}$ (d) 727.0 kJ mol$^{-1}$

59. Choose the incorrect statement.
(a) $BeCO_3$ is preserved in an atmosphere of $CO_2$ as it is thermally least stable.
(b) $BeF_2$ forms a complex compound with excess NaF, in which the complex entity containing Be, is a cation.
(c) Beryllium dissolves in an alkali to form $[Be(OH)_4]^{2-}$ ion.
(d) Beryllium exhibits no diagonal relationship with sodium.

60. The reaction $I^- + OCl^- \longrightarrow Cl^- + OI^-$ follows the rate law $\frac{d}{dt}[OI^-] = [I^-][OCl^-]$, but proves to be a function of hydroxide ion concentration. For hydroxide ion concentrations of 1M, 0.5M and 0.25M, the rate is equal to 61,120 and 230 L$^{-1}$ mol$^{-1}$s$^{-1}$ respectively and the mechanism of this reaction is

$$[OCl^- + H_2O \longrightarrow HOCl + OH^-\ \text{(rapid)}$$
$$HOCl + I^- \longrightarrow HOI + Cl^-\ \text{(slow)}$$
$$HOI + OH^- \longrightarrow H_2O + OI^-\ \text{(rapid)},$$

The order of reaction with respect to $OH^-$ is,
(a) 1 (b) 0 (c) –1 (d) 2

# Mathematics

61. The number of values of $\theta$ lying in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and satisfying the equation $\frac{(1-\tan\theta)(1+\tan\theta)}{\cos^2\theta} + 2^{\tan^2\theta} = 0$ is

(a) 0 (b) 2 (c) 3 (d) 4

62. The greatest value of $|3+4i+z|-|1+i+z|$, where $z$ being a complex number is

(a) $\sqrt{17}$ (b) $\sqrt{13}$ (c) $\sqrt{5}$ (d) 5

63. The value of the limit $\lim_{x\to 0}\left[\frac{|\sin x|}{\sqrt{x^2}}\right]$, where [.] denotes greatest integer functions

(a) 0 (b) 1 (c) –1 (d) doesn't exist

64. If $x>0$, then the minimum value of $\left(x+\frac{1}{x^3}\right)$ is

(a) $\frac{4}{3^{3/4}}$ (b) $\frac{3}{4^{4/3}}$ (c) $\sqrt{\frac{256}{27}}$ (d) $\sqrt{\frac{27}{256}}$

65. Number of 5-digit numbers without repetition formed using digits 0, 1, 2, 4, 5 which are divisible by 3.

(a) 96 (b) 24 (c) ${}^4C_1\,{}^4C_2\,2!$ (d) ${}^4C_1\,{}^4C_3\,3!$

66. The angle between two diagonals of a cube is

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\sin^{-1}\left(\frac{2\sqrt{2}}{\sqrt{3}}\right)$

(c) $\cos^{-1}\sqrt{\frac{2}{3}}$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

67. Perpendicular distance of point $P(1, 2, 3)$ from line joining points $A(1,0,5)$ and $B(7,3,3)$ is

(a) $\frac{7}{10}$ (b) $\sqrt{\frac{100}{49}}$ (c) $\frac{2}{7}\sqrt{73}$ (d) $\frac{\sqrt{291}}{7}$

68. Last digit of $(243)^{23}$ is

(a) 7 (b) 8 (c) 9 (d) 1

69. If parametric coordinates of a curve are given by $(1+4\cos\theta, -5+2\sin\theta)$, where $\theta$ is parameter, then distance between two vertices is

(a) 8 (b) 9 (c) 10 (d) $\frac{7}{2}$

70. For a series the value of mean deviation is 15. The most likely value of its quartile deviation is

(a) 12.5 (b) 11.6 (c) 13 (d) 9.7

71. The determinant of idempotent matrix is

(a) 0, 1 (b) 1, 2 (c) 1 (d) 0

72. The order of the differential equation of the ellipse having axes along coordinate axes is always greater than or equal to

(a) 2 (b) 3 (c) can't say (d) No possible

73. Non-zero number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y + k = 0$ for different values of $k$ cannot be
(a) 0 (b) 1 (c) 2 (d) 3

74. $f(x) = \sin^4 \pi x + \tan \pi x$, then period of $f(x)$ is
(a) 120 (b) 0
(c) non-periodic (d) None of these

75. In a $\Delta ABC$, with usual notations $\frac{r_1^2 r_3 + r_2^2 r_1 + r_3^2 r_2}{r_1 r_2 r_3}$ is always greater than
(a) 0 (b) 1 (c) 2 (d) 3

76. If ${}^8C_r$ is maximum, then $r$ is
(a) 4 (b) 5 (c) 1 (d) 2

77. The number of ways in which 5 different books placed in 3 different bags such that no bag remains empty is
(a) 150 (b) 149 (c) 148 (d) 147

78. The number 2006 is made up of exactly two zeroes and two other digits whose sum is 8. The number of 4-digit numbers with these properties (including 2006) is $m \times n$, where $m$ and $n$ are prime numbers. The absolute value of $m - n$ is
(a) 3 (b) 2 (c) 1 (d) 4

79. Let $PQRST$ be a regular pentagon with all sides equal to 4. The length of $PR$ is '$x$'. Then, the least integer greater than or equal to
(a) 7 (b) 6 (c) 5 (d) 4

80. The area of the region consisting the all points $(x, y)$ satisfying $x^2 + y^2 \leq 1 \leq |x| + |y|$ is $A$. Then, $\pi - A$ is equal to
(a) 0 (b) 1 (c) 2 (d) 3

81. The sum of the squares of the lengths of the chords intercepted by the line $x + y = n, n \in N$ on the circle $x^2 + y^2 = 4$ is $11k$, where $k$ is equal to
(a) 2 (b) 0
(c) can't say (d) None of these

82. The internal bisector of $\angle P$ for the $\Delta PQR$, the coordinates of vertices whose are $P(-8, 5)$, $Q(-15, -19)$ and $R(1, -7)$ has the equation $ax + 2y + c = 0$, then $\frac{c-1}{a}$ is equal to
(a) 1 (b) 9 (c) 7 (d) 8

83. Which of the following is not true any two statements $p$ and $q$ ?
(a) $\sim(p \vee (\sim q)) \equiv (\sim p) \sim \wedge q$ (b) $(p \vee q) \vee (\sim q)$ is a toutology
(c) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$ (d) None of these

84. $\lim_{x \to \infty} \int_0^x \left( \frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t} \right) dt$ is
(a) $\log 2$ (b) $\log \sqrt{2}$
(c) $\log 2^{1/4}$ (d) $\log 2^{1/5}$

**Directions** (Q. Nos. 85 and 86) *Consider three planes* $P_1, P_2, P_3$

$$P_1 : ax + by + cz = 0$$
$$P_2 : bx + cy + az = 0$$
$$P_3 : cx + ay + bz = 0. \text{ Then,}$$

85. If $a, b, c$ are the sides of a triangle and the planes meet in a common line, then triangle will be

(a) equilateral (b) right angled (c) obtuse angled (d) scalene

86. If $a, b$ and $c$ are the sides of an scalene triangle and plane meet at point $(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to

(a) 1 (b) 0 (c) 2 (d) 3

**Directions** (Q. Nos. 87 and 88) *Let* $f(x + y) = f(x) + f(y) + 4xy, \forall x, y \in R$ *is a differential function every where, then give the answer of the following questions.*

87. $f(2)$ is equal to

(a) 4 (b) 8 (c) 12 (d) 16

88. $f'(2009)$ is equal to

(a) 2009 (b) 6027 (c) 4018 (d) 8036

**Directions** (Q. Nos. 89 and 90) *For the following questions. Choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.*

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

89. **Statement I** The minimum distance of the fixed point $(0, y_0)$, where $0 \le y_0 \le \frac{1}{2}$, from the curve $y = x^2$ is $y_0$.

**Statement II** Maxima and minima of a function is always a root of the equation $f'(x) = 0$.

90. Let $I_n = \int_1^e (\log x)^n \, dx, n \in N$

**Statement I** $I_1, I_2, I_3, \ldots$ is an increasing sequence.

**Statement II** $\log x$ is an increasing function.

# Answer *with* Explanations

## Physics

**1.** (c) Let the equivalent resistance and emf of the parallel combination be $r$ and $E$

Then, $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$

or $r = \frac{r_1 r_2}{r_1 + r_2}$

However, maximum currents drawn from the batteries are $\left(\frac{E_1}{r_1}\right)$ and $\left(\frac{E_2}{r_2}\right)$. Hence, total current $I$ is given by

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2}$$

To obtain the same current $I$ from the equivalent source its emf $E$ is given by

$$E = I_r = \left[\frac{E_1 r_2 + E_2 r_1}{r_1 r_2}\right]\left[\frac{r_1 r_2}{r_1 + r_2}\right] = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

**2.** (d) An alternating voltage varies with time as

$$V = V_0 \sin \omega t \quad \ldots(i)$$

Given, $V = 100 \sin 50\pi t \quad \ldots(ii)$

Comparing Eqs. (i) and (ii), we have $V_0 = 100$ V and $\omega = 50\pi$ rad s$^{-1}$ or $2\pi\nu = 50\pi$, *i.e.*, $\nu = 25$ Hz

Also, $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}$ V

Hence, the correct choice is (d).

**3.** (c) If $A$ and $\delta_m$ be the angle of prism and minimum deviation respectively, then the refractive index

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

or $$\sqrt{3} = \frac{\sin\left(\frac{A}{2} + 30°\right)}{\sin\frac{A}{2}}$$

or $$\sqrt{3}\sin\frac{A}{2} = \sin\frac{A}{2}\cos 30° + \cos\frac{A}{2}\sin 30°$$

or $$\frac{\sqrt{3}}{2}\sin\frac{A}{2} = \frac{1}{2}\cos\frac{A}{2}$$

or $$\tan\frac{A}{2} = \frac{1}{\sqrt{3}}$$

or $$A = 60°$$

**4.** (c) The thermodynamic process is irreversible, as there always occurs a loss of energy due to energy spent in working against the dissipative force, which is not recovered back. Other irreversible process also occurs in nature such as friction where extra work is required to cancel out the effect of friction. (*i e*, dissipative effect is eliminated in that case)

**5.** (d) Efficiency of the engine is given by

$$n = \left(1 - \frac{T_2}{T_1}\right) \times 100 = \left(1 - \frac{273 + 20}{273 + 40}\right) \times 100$$

$$= \left(1 - \frac{293}{313}\right) \times 100$$

$$= 0.064 \times 100 = 6.4\%$$

**6.** (a) In simple harmonic motion, the displacement equation is,

$$y = A \sin \omega t$$

where, $A$ is amplitude of the motion,

Velocity $$v = \frac{dy}{dt} = A\omega \cos \omega t$$

$$v = A\omega\sqrt{1 - \sin^2 \omega t}$$

$$v = \omega\sqrt{A^2 - y^2} \quad \ldots(i)$$

Acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}(A\omega \cos \omega t)$$

$$a = -A\omega^2 \sin \omega t$$

$$a = -\omega^2 y \quad \ldots(ii)$$

When, $y = 0$; $v = A\omega = v_{max}$

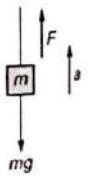
$a = 0 = a_{min}$

When, $y = A$; $v = 0 = v_{min}$

$a = -\omega^2 A = a_{max}$

Hence, it is clear that when $v$ is maximum, then $a$ is minimum (*i.e.*, zero) or *vice-versa*.

**7.** (d) When lift moves upwards then net force on it is in the upward direction. Therefore, from Newton's second law, the net force




$$F - mg = ma$$

$$\Rightarrow \quad F = m(g + a)$$

Given, $a = 8g$

$$\therefore \quad g' = g + 8g = 9g$$

Also time period

$$(T) = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{g}}$$

Since, $T^2 g = \text{constant}$

$$\therefore \quad T_1^2 g = T_2^2 \times 9g$$

$$\Rightarrow \quad T_2 = \frac{T_1}{3} = \frac{T}{3}$$

$\therefore$ Decrement in time-period

$$= T - \frac{T}{3} = \frac{2T}{3}$$

**8.** (d) By the equation of continuity, we have

$Av$ = rate of flow ($R$)

Given,

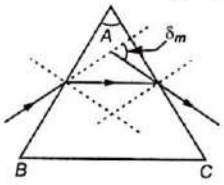
$$R = 200\ \text{cm}^3\text{s}^{-1},\ A = 0.5\ \text{m}^2$$
$$= 0.5 \times (10^{-3})^2\ \text{mm}$$

and $R = 200 \times (10)^3\ \text{mms}^{-1}$

$$\therefore \quad v = \frac{200 \times (10)^3}{0.5 \times 10^{-6}}$$
$$= 400 \times 10^{-6} \times 10^3 = 0.4\ \text{mms}^{-1}$$

**9.** (b) In secondary rainbow violet is above and red is below.

**10.** (d) The refractive index ($\mu$) of a prism of angle $A$, and minimum deviation $\delta_m$ is given by




$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

Given, $\mu = \cot\frac{A}{2}$

$$\therefore \quad \cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \cos\frac{A}{2} = \sin\left(\frac{A+\delta_m}{2}\right)$$

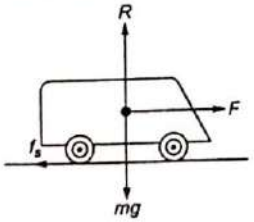
$$\Rightarrow \quad \sin\left(90° - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

$$\Rightarrow \quad 90° - \frac{A}{2} = \frac{A+\delta_m}{2}$$

$$\Rightarrow \quad 180° - A = A + \delta_m$$

$$\Rightarrow \quad \delta_m = 180° - 2A = \pi - 2A$$

**11.** (c) Free body diagram of automobile is shown




By Newton's third law

$$F = f_s = \mu R = \mu\, mg$$

where, $m$ is the mass of automobile.

Also, $F = ma$

or $ma = \mu mg$

$\therefore$ Retardation $a = \mu g = 0.5\, g$

Let automobile stops at a distance $x$, then from equation of motion

$$v^2 = u^2 - 2ax$$

Given, $v = 0,\ u = 72\ \text{km h}^{-1}$

$$= 72 \times \frac{5}{18}\ \text{ms}^{-1} = 20\ \text{ms}^{-1}$$

$$g = 9.8\ \text{ms}^{-2}.$$

$$\therefore \quad 0^2 = (20)^2 - 2 \times 0.5 \times 9.8\, x$$

$$\Rightarrow \quad x = \frac{20 \times 20}{2 \times 0.5 \times (9.78)} = 40.8\ \text{m}$$

**12.** (a) There are four beats between $P$ and $Q$, therefore the possible frequencies of $P$ are 246 or 254 (that is 250 ± 4) Hz.

When the prong of $P$ is filled, its frequency becomes greater than the original frequency.

If we assume that the original frequency of $P$ is 254, then on filing its frequency will be greater than 254. The beats between $P$ and $Q$ will be more than 4. But it is given that the beats are reduced to 2, therefore, 254 is not possible.

Therefore, the required frequency must be 246 Hz.

(This is true, because on filing the frequency may increase to 248, giving 2 beats with $Q$ of frequency 250 Hz).

**13.** (d) Here, $\omega_1 = 150\,\pi\text{rad s}^{-1}$

$$\omega_2 = 300\,\pi\ \text{rad s}^{-1}$$
$$v_1 = \frac{\omega_1}{K_1} = 300\ \text{ms}^{-1}$$
$$v_2 = \frac{\omega_2}{K_2} = 300\ \text{ms}^{-1}$$
$$\frac{P_1}{P_2} = \frac{5}{10}$$
$$\Rightarrow \quad \frac{P_2}{P_1} = \frac{10}{5}$$

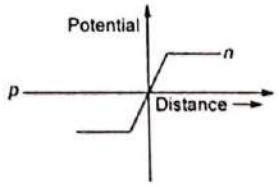
(as $P$ is always proportional to amplitude)

We have $I = \frac{1}{2} P\omega^2 A^2 v$

$$\therefore \quad \frac{I_2}{I_1} = \frac{\omega_2^2 P_2^2 v_2}{\omega_1^2 P_1^2 v_1} = 16$$

$$\Rightarrow \quad I_2 : I_1 = 16 : 1$$

**14.** (b) Graph between potential and distance in a $p$-$n$ junction diode is given by



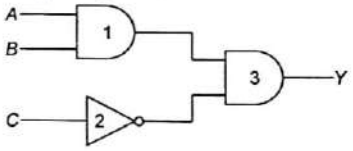

$\therefore$ Potential at $p$ is less than that at $n$.

**15.** (c) Voltage across 2 k$\Omega$ resistor = 12 V

Therefore, current passing through 2 k$\Omega$ resistor

$$= \frac{12}{2 \times 10^3} = 6 \times 10^{-3}\ \text{A} = 6\ \text{mA}$$

**16.** (e) Output of gate-1,




$$Y_1 = AB$$

Output of gate-2,

$$Y_2 = \overline{C}$$

Output of gate-3,

$$Y = Y_1 Y_2 = (AB)\overline{C}$$

Thus, output will be zero, if

$$A = 1, B = 1 \text{ and } C = 1 \qquad (\text{as } \overline{C} = 0)$$

**17.** (a) In a circular motion, the direction of resultant force is towards centre and its magnitude is

$$F = \frac{mv^2}{r} \text{ or } F = mr\omega^2$$

Thus, a centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle moving in a circle with constant speed.

**18.** (c) Let $h$ be height of plane from the ground, then from equation of motion, we have

$$h = ut + \frac{1}{2}gt^2$$

Since, initial velocity, $u = 0$

$$h = \frac{1}{2}gt^2$$

Given, $t = 10$ s, $g = 10\ \text{ms}^{-2}$

$$\therefore \qquad h = \frac{1}{2} \times 10 \times (10)^2 = 500\ \text{m}$$

Also, by equation

$$v^2 = u^2 + 2gh$$

We have for

$$u = 0,\ v = \sqrt{2gh}$$

$$\therefore\ v = \sqrt{2 \times 10 \times 500} = 100\ \text{ms}^{-1}$$

Now, as we know that

$$\tan\theta = \frac{\text{Vertical velocity}}{\text{Horizontal velocity}} = \frac{100}{500} = \frac{1}{5}$$

$$\Rightarrow\ \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

Here, $\theta$ is the angle between the velocity and one of the two axes.

**19.** (d) In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance.

Time of descent ($t_2$) = time of ascent ($t_1$) = $\frac{u}{g}$

$\therefore$ Total time of flight

$$T = t_1 + t_2 = \frac{2u}{g}$$

$$\Rightarrow \qquad u = \frac{g(t_1 + t_2)}{2}$$

**20.** (d) From equation of motion, we have

$$v = u - gt$$

When $v$ is final velocity, $u$ is initial velocity, $g$ is acceleration due to gravity and $t$ is time.

Since, body is projected vertically upwards final velocity $v = 0$.

$$\therefore \qquad u = gt$$

This equation is form of straight line equation

$$y = mx + c$$

Hence, graph between velocity and time is a straight line.

**21.** (b) The energy required to remove the satellite from its orbit around the earth to infinity is called binding energy of the satellite. It is equal to negative of total mechanical energy of satellite in its orbit.

Thus,

Binding energy earth-satellite system,

$$E = \frac{-GMm}{2r}$$

But, $$g = \frac{GM}{R^2}$$

$$\Rightarrow \qquad GM = gR^2$$

$\therefore$ But $$\text{BE} = \frac{gmR^2}{2r} \qquad \text{(in magnitude)}$$

**22.** (b) The velocity of solid sphere on the bottom of inclined plane is,

$$v = \sqrt{\left[\frac{2gh}{\left(1 + \frac{I}{MR^2}\right)}\right]}$$

where, $I$ = moment of inertia of sphere,

$M$ = mass of sphere,

and $R$ = radius of sphere.

The moment of inertia of solid sphere about one of its diameters,

$$I = \frac{2}{5}MR^2$$

$$v = \sqrt{\left[\frac{2gh}{\left(1 + \frac{2}{5}\right)}\right]} = \sqrt{\left(\frac{10}{7}gh\right)}$$

$$\therefore \qquad h = \frac{7v^2}{10g}$$

**23.** (a) If the half-life of a radioactive isotope is small as compared to the age of organic sample, then over the age of the sample the activity of radioactive isotope becomes very small and hence is impossible to detect. While this problem will not arise if we use radioactive isotope having larger half-life for dating with organic samples.

**24.** (a) From the figure, it is clear that $\frac{E_b}{A}$ for nuclides having $A > 40$ is very slowly varying *i.e.*, average energy per nucleon to be supplied to nucleus to break it into its constituents particles is almost the same for nuclides having $A > 40$. It means the energy required to overcome the nuclear forces per nucleon would be almost same and hence we can conclude that nuclear forces are short range forces.

**Solutions** (Q. Nos. 25-27)

Let $n$ be the lowest energy level of the given series of lines, then maximum wavelength is given by

$$\frac{hc}{\lambda_{max}} = 13.6 \times Z^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$\frac{hc}{\lambda_{min}} = 13.6 \times Z^2\left[\frac{1}{n^2} - \frac{1}{\infty^2}\right]$$

where, $\lambda_{max} = 41.02$ nm and $\lambda_{min} = 22.79$ nm

Solving above equations, we get $n = 2$, which corresponds to Balmer series.

$$Z = 4$$

For next to longest wavelength, transition takes place from $n_1 = 4$ to $n_2 = 2$

So, $$\frac{hc}{\lambda} = 13.6 \times 4^2\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\Rightarrow \quad \lambda = 30.47 \text{ nm}$$

Here, note that right side having unit eV which has been converted into joule.

**25.** (c) **26.** (b) **27.** (b)

**28.** (c) The face area of the loop is $A = 2.5 \times 10^{-3}$ m$^2$ and the magnetic field changes as $B = B_0 \sin\omega t$ where $B_0 = 0.20$ T and $\omega = 50\,\pi$ s$^{-1}$. The resistance of the loop is $R = 10\,\Omega$.

The flux through the loop at time $t$ is

$$\phi = B_0 A \sin\omega t$$

The emf induced is

$$\varepsilon = -\frac{d\phi}{dt} = -B_0 A\omega \cos\omega t$$

The current is

$$i = \frac{\varepsilon}{R} = \frac{B_0 A\omega}{R}\cos\omega t = -i_0 \cos\omega t$$

The current changes sinusoidally with the time period

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{50\pi s^{-1}} = 40 \text{ ms}$$

The charge flowing through any cross-section in 40 ms is

$$Q = \int_0^T i\,dt = -i_0 \int_0^T \cos\omega t\,dt$$

$$= -\frac{i_0}{\omega}[\sin\omega t]_0^T = 0$$

**29.** (a) The thermal energy produced in 40 ms is

$$H = \int_0^T i^2 R dt = i_0^2 R \int_0^T \cos^2\omega t\,dt$$

$$= \frac{i_0^2 R}{2}\int_0^T (1 + \cos 2\omega t)\,dt$$

$$= \frac{i_0^2 R}{2}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{i_0^2 RT}{2} = \frac{B_0^2 A^2\omega^2}{2R^2} R\left(\frac{2\pi}{\omega}\right) = \frac{\pi B_0^2 A^2 \omega}{R}$$

$$= \frac{\pi \times (0.20T)^2 \times (2.5 \times 10^{-3} m^2)^2 \times (50\pi s^{-1})}{10\,\Omega}$$

$$= 1.25 \times 10^{-5} \text{J}$$

**30.** (a) As in a nucleus, nucleons are bounded by short range nuclear force, so a given nucleon is in interaction only with neighbouring nucleons. So, to detach a nucleon from a nucleus is irrespective of the fact that how many nucleons are present in the nucleus, and moreover due to short range nuclear force only, the $\frac{E_b}{A}$ *versus* $A$ curve is slowly varying for $A > 40$.

While in atoms electrons are bound with nucleus by Coulomb's force which is not necessarily short ranged force and depends on the number of protons, in the nucleus and electron separation from nucleus. If we take the average of the energies required to detach all electrons from outermost shell to innermost $K$ shell, then this average increases rapidly with increase in atomic number.

# Chemistry

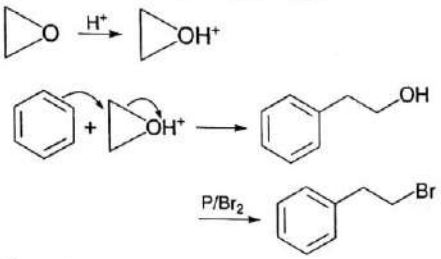
**31.** (d) $M + (x + 2y)\,NH_3 \longrightarrow [M(NH_3)_x]^{2+} + 2e^- (NH_3)_y$

(blue)

**32.** (b) $$\frac{r_{O_2}}{r_{CH_4}} = \frac{n_{O_2}}{n_{CH_4}} \times \sqrt{\frac{M_{CH_4}}{M_{O_2}}}$$

$$= \frac{3}{2} \times \frac{16}{32} \times \sqrt{\frac{16}{32}} = \frac{3}{4\sqrt{2}}$$

**33.** (b)




**34.** (b) Two dimensional sheet structures are formed when three oxygen atoms of each $[SiO_4]^{4-}$ tetrahedral are shared.

**35.** (c) $nCaC_2 + nH_2O + nH_2 \longrightarrow nCaO + (C_2H_4)_n$

$$\text{Moles of } CaC_2 = \frac{10 \times 1000}{64} \times \frac{1}{n}$$

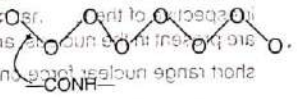
$$\text{Moles of polyethylene} = \frac{10 \times 1000}{64} \times \frac{1}{n}$$

Mass of polyethylene

$$= \frac{10 \times 1000}{64} \times \frac{1}{n} (12 \times 2 + 1 \times 4) \times n$$

$\therefore$ Mass = 4375 g or 4.37 kg

**36.** (d) A decapeptide has nine peptide (amide) linkage as




bond.

Therefore on hydrolysis it will absorb nine water molecules.

Hence, total mass of hydrolysis product

$$= 796 + 18 \times 9 = 958$$

Mass of glycine in hydrolysis product

$$= \frac{958 \times 47}{100} = 450.26$$

Number of glycine molecule in one molecule of decapeptide

$$= \frac{450.26}{75} = 6$$

**37.** (c) $Mg(OH)_2 \rightleftharpoons Mg^{2+} + 2OH^-$

$$K_{sp} = [Mg^{2+}][OH^-]^2$$

$$1 \times 10^{-14} = [Mg^{2+}][10^{-6}]^2$$

$\because$ pH = 8

$\therefore$ $[H^+] = 10^{-8}$ mol $L^{-1}$,

and $[OH^-] = 10^{-6}$ mol $L^{-1}$

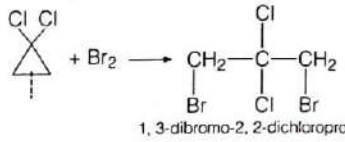
$$[Mg^{2+}] = \frac{10^{-14}}{10^{-12}} = 10^{-2} \text{ mol } L^{-1}$$

Solubility

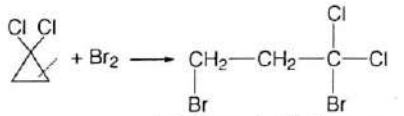
$[Mg^{2+}] = 10^{-2}$ mol $L^{-1}$

**38.** (c) 'Zn' is better reducing agent than Ag as well as Au due to lower reduction potential.

**39.** (c)




1, 3-dibromo-2, 2-dichloropropane




1, 3-dibromo-1, 1-dichloropropane

**40.** (b) Two $Na^+$ ions are replaced by one $Sr^{2+}$.

Hence, number of vacancies

= Number of $Sr^{2+}$ doped

$= 1 \times 10^{-3} \times 6 \times 10^{23} = 6 \times 10^{20}$

**41.** (d) The reaction is feasible. It is a $S_N2$ reaction. For better yield, alkyl halide should be primary and alkoxide should be secondary or tertiary.

**42.** (b) *B* is racemic butane-2, 3-diol while *D* is *meso* butane-2, 3-diol.

**43.** (c) Dipole moment,

$$\mu = \delta \times d$$

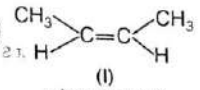
$$0.816 \times 10^{-18} = \delta \times 10^{-8}$$

$$\delta = 0.816 \times 10^{-10} \text{ esu}$$

% ionic character

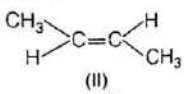
$$= \frac{0.816 \times 10^{-10} \times 100}{4.803 \times 10^{-10}} = 16.9\%$$

**44.** (b)



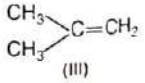

(I)

$\mu$ less than (III)




(II)

$\mu = 0$




(III)

$\mu$ = max.

**45.** (d) $SF_6$ has octahedral geometry so its dipole moment, $\mu = 0$

**46.** (c) The conjugate base of HClO, $HClO_2$, $HClO_3$ and $HClO_4$ are $ClO^-$, $ClO_2^-$, $ClO_3^-$ and $ClO_4^-$ respectively. As the number of oxygen atoms increases, the negative charge dispersal becomes more and more and thus lesser is the charge on Cl atom and more will be its stability.

**47.** (b) Total mole of $Mg = \frac{1}{24}$

Let (a) mole of $Mg^+$ and (b) mole of $Mg^{2+}$ are present, then

$$a + b = \frac{1}{24} \quad ...(i)$$

Also, $50 = a \times 740 + b \times 2190$ ...(ii)

From Eq. (i) and (ii)

$$a = 2.845 \times 10^{-2}$$

$$b = 1.322 \times 10^{-2}$$

$$\% \text{ of } Mg^+ = 68.28\%$$

$$\% \text{ of } Mg^{2+} = 31.72\%$$

**48.** (b) van der Waals' equation for one mole of a real gas is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

At high pressures, volume of gas being low and thus '$b$' should not be ignored. However, the term $\frac{a}{V^2}$ may be negligible in comparison to pressure. Thus, $p(V - b) = RT$

$pV = RT + pb$

or $\frac{pV}{RT} = 1 + \frac{pb}{RT}$

∴ Compressibility factor

$$Z = \frac{pV}{RT} = 1 + \frac{pb}{RT}$$

**49.** (a) IV complex cannot show geometrical isomerism due to presence of symmetrical ligand. Remaining three can exists in *cis* and *trans* forms.

**50.** (b) When the combination is $Fe^{2+}$ and NO, there will be 5 unpaired electrons in the complex (4 in $Fe^{2+}$ and 1 in NO) and when the combination is $Fe^+$ and $NO^+$, there will be only 3 unpaired electrons which can be differentiated by measuring the magnetic moments.

**51.** (a)

$$CH_3CHO(g) \longrightarrow CH_4(g) + CO(g)$$

| | | | |
|---|---|---|---|
| Initial | 80 mm Hg | 0 | 0 |
| Final | 80-$p$ | $p$ | $p$ |

After 20 min total pressure = $80 + p = 120$

$\Rightarrow$ $p = 40$ mm Hg

*i.e.*, 50% of reaction is complete,

*i.e.*, $t_{1/2} = 20$ min

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{20} = 3.465 \times 10^{-2} \text{ min}^{-1}$$

**52.** (d) Rate, $r = K(C)^m$

$$\left(\frac{r_1}{r_2}\right) = \left(\frac{C_1}{C_2}\right)^m$$

$$m = \log \frac{r_2}{r_1} \Big/ \left(\frac{C_2}{C_1}\right)$$

$$\frac{r_2}{r_1} = 15$$

$$\frac{C_2}{C_1} = 5$$

$$m = \log_5^{15}$$

**53.** (c) No protective layer is formed during the reaction with hydroxides.

**54.** (b)

$$\underset{\text{Acidic red}}{HIn} \longrightarrow H^+ + \underset{\text{Basic blue}}{In^-}$$

$$K_a = \frac{[H^+][In^-]}{[HIn]}$$

When 75% blue and 25% red, then

$$[H^+] = 3 \times 10^{-5} \times \frac{25}{75} = 1 \times 10^{-5}$$

∴ Change in $[H^+] = 9 \times 10^{-5} - 1 \times 10^{-5} = 8 \times 10^{-5}$

**55.** (a) $KE_3 = -E_3 = \frac{13.6 \times 4}{9} = 6.04$ eV

**56.** (c) 2-methyl pentane contains 5 carbon atoms which are also found in the conformation shown in (c).

Thus, it isthe conformation of 2-methyl pentane.

**57.** (c) $E_{NO_3^-/NO_2} = 0.80 - \frac{0.059}{1} \log \frac{a_{NO_2}}{[H^+]^2 [NO_3^-]}$

$$E^\circ_{NO_3^-/NH_2OH} = 0.73 - \frac{0.059}{6} \log \frac{[NH_2OH]}{[H^+]^7 [NO_3^-]}$$

When no current flows and concentration are unity,

$$0.8 - \frac{0.059}{1} \log [H^+]^{2-} = 0.73 - \frac{0.059}{6} \log [H^+]^{7-}$$

Solving $-\log [H^+] = pH = \frac{0.07}{0.048} = 1.45$

**58.** (c) $I_3$ of Li (*i.e.*,) ionization Enthalpy) of $Li^{2+} = 2.18 \times 10^{-18} \times 9$ J atom$^{-1}$

$= 2.18 \times 10^{-18} \times 9 \times 10^{-3} \times 6.023 \times 10^{23}$ kJ mol$^{-1}$

$= 11817$ kJ mol$^{-1}$

And also

$$I_1 + I_2 + I_3 = 19600 \text{ kJ mol}^{-1}$$

and $I_1 = 520$ kJ mol$^{-1}$

∴ $I_2 = 19600 - (11817 + 520) = 7263$ kJ mol$^{-1}$

**59.** (b) $BeF_2 + NaF \longrightarrow Na_2[BeF_4]$

Complex entity is an anion.

**60.** (c) For the slowest step,

$$\text{rate} = k [HOCl][I^-]$$

$$OCl^- + H_2O \xrightarrow{\text{Fast}} HOCl + OH^-$$

$$\therefore \quad K_c = \frac{[HOCl][OH^-]}{[OCl^-][H_2O]}$$

or $[HOCl] = \frac{K_c \cdot [OCl^-][H_2O]}{[OH^-]}$

$$\therefore \quad \text{Rate} = \frac{kK_c [OCl^-][H_2O][I^-]}{[OH^-]}$$

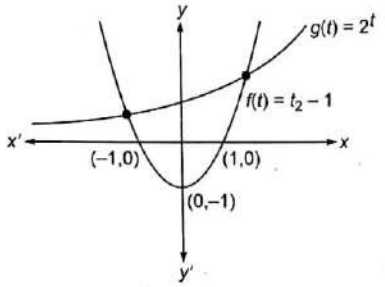
or $= kK_c [OCl^-][H_2O][I^-][OH^-]^{-1}$

Thus, the order of reaction with respect to $OH^-$ is $-1$.

# Mathematics

**61.** (b) Given equation, $(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

$\Rightarrow \quad (1-\tan^4\theta) + 2^{\tan^2\theta} = 0$




Let $\tan^2\theta = t$

$\Rightarrow \quad t^2 - 1 = 2^t$

So, two value of $t$ are possible but $\tan^2\theta \geq 0$, so negative values of $t$ will be neglected as

$t = 3 \Rightarrow t^2 - 1 = 2^t$

So, $\tan^2\theta = 3$

$\Rightarrow \quad \tan\theta = \pm\sqrt{3}$

$\Rightarrow \quad \theta = \left\{\frac{-\pi}{3}, \frac{\pi}{3}\right\}$

**62.** (b) $|3+4i+z| - |1+i+z|$

$\leq |3+4i+z-1-i-z|$ $(\because |z_1 - z_2| \geq |z_1| - |z_2|)$

$= |2+3i| = \sqrt{13}$

**63.** (a) $\lim_{x\to 0}\left|\frac{|\sin x|}{\sqrt{x^2}}\right| = \lim_{x\to 0}\left[\frac{|\sin x|}{|x|}\right]$

$= \lim_{x\to 0}\left[\left|\frac{\sin x}{x}\right|\right] = 0 \qquad \left(\because \frac{\sin x}{x} < 1\right)$

**64.** (a) Consider four numbers $\frac{x}{3}, \frac{x}{3}, \frac{x}{3}$ and $\frac{1}{x^3}$.

Apply AM ≥ GM

$$\frac{x + \frac{1}{x^3}}{4} \geq \left(\frac{x^3}{3^3} \times \frac{1}{x^3}\right)^{1/4}$$

$\Rightarrow \quad x + \frac{1}{x^3} \geq 4\left(\frac{1}{3^3}\right)^{1/4}$

$\Rightarrow \quad x + \frac{1}{x^3} \geq \frac{4}{3^{3/4}}$

**65.** (a) Since, sum of digits

$= 0 + 1 + 2 + 4 + 5 = 12$

So, all 5-digit numbers formed by 0, 1, 2, 4, 5 will be divisible by 3. Now,

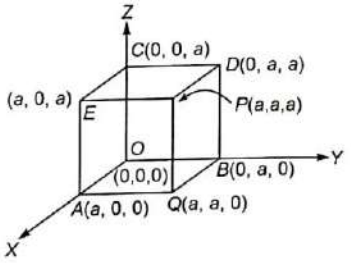
$\underset{4}{\downarrow}\ \underset{4}{\downarrow}\ \underset{3}{\downarrow}\ \underset{2}{\downarrow}\ \underset{1}{\downarrow}$

$= 4 \times 4! = 96$

**66.** (a) DR's of diagonal $OP = (a, a, a)$

DR's of diagonal $AD = (-a, a, a)$

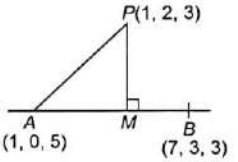
$\therefore \quad \cos\theta = \frac{-a^2 + a^2 + a^2}{\sqrt{3}\,a\,\sqrt{3}\,a} = \frac{1}{3}$




$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$

**67.** (c) $PM = \sqrt{AP^2 - AM^2}$

$\therefore$ $AM$ is projection of $AP$ on line $AB$.




DR's of line $AB = (6, 3, -2)$

DC's of line $AB = \left(\frac{6}{7}, \frac{3}{7}, \frac{-2}{7}\right)$ $(\because \sqrt{6^2 + 3^2 + 2^2} = 7)$

$\therefore \quad AM = \left|0\cdot\left(\frac{6}{7}\right) + (2-0)\left(\frac{3}{7}\right) + (3-5)\left(\frac{2}{7}\right)\right|$

$= \left|\frac{6}{7} + \frac{4}{7}\right| = \frac{10}{7}$

$\therefore \quad AP = \sqrt{0^2 + 4 + 4} = 2\sqrt{2}$

$PM = \sqrt{8 - \frac{100}{49}}$

$= \sqrt{\frac{292}{49}} = \frac{2\sqrt{73}}{7}$

**68.** (a) $(243)^{23} = (3^5)^{23} = (3)^{115}$

$= (3)^{112} \times 3^3 = (3^4)^{28} \times 3^3$

$= 1 \times 7 = 7$ [$\because$ unit digit of $(3^4)^{28} = 1$ and unit digit of $3^3 = 7$]

**69.** (a) Let $h = 1 + 4\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{h-1}{4}$ ...(i)

and $k = -5 + 2\sin\theta$

$$\Rightarrow \quad \sin\theta = \frac{k+5}{2} \qquad \ldots(ii)$$

$\because \sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \quad \left(\frac{k+5}{2}\right)^2 + \left(\frac{h-1}{4}\right)^2 = 1$$

$$\Rightarrow \quad \frac{(k+5)^2}{4} + \frac{(h-1)^2}{16} = 1$$

$$\Rightarrow \quad \frac{(h-1)^2}{4^2} + \frac{(k+5)^2}{2^2} = 1$$

which represent an ellipse.

Here, $a = 4$ and $b = 2$

$$\therefore \quad e = \sqrt{1 - \frac{2^2}{4^2}} = \frac{\sqrt{3}}{2}$$

$\therefore$ Distance between vertices $= 2a = 8$

**70.** (a) $\because MD = \frac{4}{5}\sigma$, $QD = \frac{2}{3}\sigma$

$$\therefore \quad \frac{MD}{QD} = \frac{6}{5}$$

$$\Rightarrow \quad QD = \frac{5}{6}MD$$

$$\Rightarrow \quad QD = \frac{5}{6}(15) = 12.5$$

**71.** (a) For idempotent matrix $A$,

$$A^2 = A$$

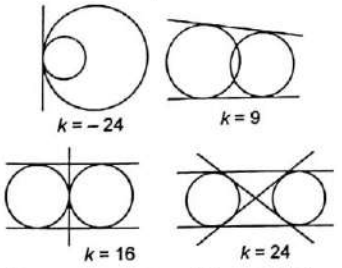
$$\Rightarrow \quad |A^2| = |A|$$

$$\Rightarrow \quad |A| = 0, 1$$

**72.** (a) Since, equation of Ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. There are two constants $a$ and $b$. Hence, order is 2.

**73.** (a) Distance between centre (0, 0) and $(-3, -4) = 5$

Radii are 2 and $\sqrt{25-k}$




If $k = -24$, number of tangent is 1.

$k = 9$, number of tangent is 2.

$k = 16$, number of tangent is 3.

$k = 24$, number of tangent is 4.

**74.** (a) $f(x) = \sin^4 \pi x + \tan \pi x$

$\because$ Fundamental period $= 1$

Hence, period $= 1, 2, 3, 4$ and $120$.

**75.** (d) Given, $\frac{r_1^2 r_3 + r_2^2 r_1 + r_3^2 r_2}{r_1 r_2 r_3}$

$$= \frac{r_1}{r_2} + \frac{r_2}{r_3} + \frac{r_3}{r_1}$$

$\because$ AM $\geq$ GM

$$\therefore \quad \frac{\frac{r_1}{r_2} + \frac{r_2}{r_3} + \frac{r_3}{r_1}}{3} \geq \left(\frac{r_1}{r_2}\cdot\frac{r_2}{r_3}\cdot\frac{r_3}{r_1}\right)^{1/3}$$

$$\Rightarrow \quad \frac{r_1}{r_2} + \frac{r_2}{r_3} + \frac{r_3}{r_1} \geq 3(1)^{1/3}$$

$$\Rightarrow \quad \frac{r_1^2 r_3 + r_2^2 r_1 + r_3^2 r_2}{r_1 r_2 r_3} \geq 3$$

**76.** (a) ${}^nC_r$ is maximum, when $r = \frac{n}{2}$

$\therefore {}^8C_r$ is maximum, when $r = \frac{8}{2} = 4$

**77.** (a) Total number of different ways

$$= 3^5 - {}^3C_1(3-1)^5 + {}^3C_2(3-2)^5$$

$$= 243 - 96 + 3$$

$$= 150$$

**78.** (d) Two digits with sum 8 can be 17, 26, 35, and 44.

Number of numbers with 17, 26, 35

$$= 3\left[\frac{4!}{2!} - 3!\right] = 18$$

Number of numbers with 44

$$= \frac{4!}{2!\,2!} - \frac{3!}{2!} = 6 - 3 = 3$$

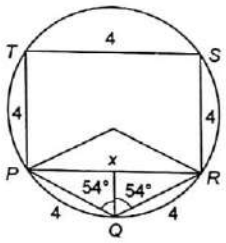
Total number of such numbers

$$= 18 + 3 = 21 = 7 \times 3$$

$$\Rightarrow \quad |m - n| = |7 - 3| = 4$$

**79.** (a) Apply cosine law in $\Delta PQR$, we have

$$x^2 = 16 + 16 - 2\cdot 4\cdot 4\cos 108°$$




$$= 32 - 32\cos 108°$$

$$= 32(1 - \cos 108°) = 64\sin^2 54°$$

$$\Rightarrow \quad x = 8\sin 54°$$

$$= 8\cos 36° = 2(\sqrt{5} + 1)$$

For the least integer greater than or equal to $x = 7$

**80.** (c) $\therefore$ Required area $= 4 \times$ Area of $APBA$

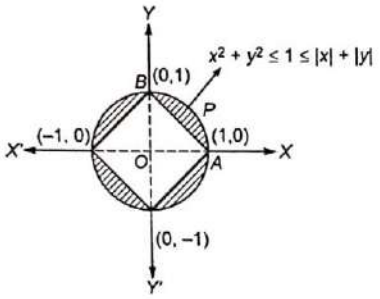
$A = 4$ (Area of $OAPBO$ − Area of $\Delta OAB$)

$$A = 4\left\{\int_0^1 \sqrt{1-x^2}\,dx - \frac{1}{2}(1)(1)\right\}$$

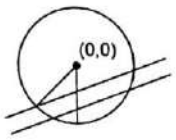
$$A = 4\left\{\left[\frac{x}{2}\log\left(x + \sqrt{1-x^2}\right) + \frac{1}{2}\sin^{-1}x\right]_0^1 - \frac{1}{2}\right\}$$

$$A = 4\left\{\frac{\pi}{4} - \frac{1}{2}\right\} = \pi - 2$$

$$\Rightarrow \quad \pi - A = 2$$




**81.** (a) For the intersection of the line $x + y = n$ and the circle $x^2 + y^2 = 4$. We must have,




$$x^2 + (n - x)^2 = 4$$

$$\Rightarrow 2x^2 - 2nx + n^2 - 4 = 0$$

For $D > 0$,

$$\Rightarrow \quad 4n^2 - 8(n^2 - 4) > 0$$

$$\Rightarrow \quad n^2 - 8 < 0$$

$$\Rightarrow \quad -2\sqrt{2} < n < 2\sqrt{2}$$

$$\Rightarrow \quad n = 1 \text{ or } 2$$

Hence, lines are $x + y = 1$ or $x + y = 2$

$\therefore$ Length of chord

$$l_1 = 2\left(\sqrt{4 - \frac{1}{2}}\right) = \sqrt{14}$$

and $\quad l_2 = 2\sqrt{4 - 2} = \sqrt{8}$

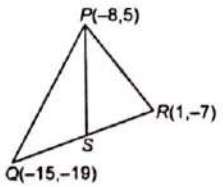
According to the given condition

$$11k = l_1^2 + l_2^2 = 22$$

$$\Rightarrow \quad k = 2$$

**82.** (c) We have, $PQ = \sqrt{7^2 + 24^2} = 25$

and $\quad PR = \sqrt{9^2 + 12^2} = 15$




$\therefore$ Line $PS$ divides the line $QR$ in 3 : 5.

Coordinate of the point $S$

$$= \left(\frac{-45 + 5}{8}, \frac{-57 - 35}{8}\right)$$

$$= \left(-5, \frac{-23}{2}\right)$$

Hence, the equation of $PS$ is

$$y - 5 = \frac{5 + 23/2}{-8 + 5}(x + 8)$$

$$\Rightarrow \quad -3(y - 5) = \frac{33}{2}(x + 8)$$

$$\Rightarrow \quad -6y + 30 = 33x + 264$$

$$\Rightarrow \quad 33x + 6y + 234 = 0$$

On comparing it with $ax + 2y + c = 0$, we get

$$\frac{a}{33} = \frac{2}{6} = \frac{c}{234}$$

$$\Rightarrow \quad a = 11 \text{ and } c = 78$$

$$\therefore \quad \frac{c - 1}{a} = \frac{77}{11} = 7$$

**83.** (c) $\therefore \sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

**84.** (a) Let $P = \lim\limits_{x \to \infty} \int_0^x \left(\frac{1}{\sqrt{1 + t^2}} - \frac{1}{1 + t}\right) dt$

$$= \lim_{x \to \infty} [\log(t + \sqrt{1 + t^2}) - \log(1 + t)]_0^x$$

$$= \lim_{x \to \infty} \{\log(x + \sqrt{1 + x^2}) - \log(1 + x)\}$$

$$= \lim_{x \to \infty} \log\left(\frac{x + \sqrt{1 + x^2}}{1 + x}\right)$$

$$= \log \lim_{x \to \infty}\left(\frac{x + \sqrt{1 + x^2}}{1 + x}\right)$$

$$= \log \lim_{x \to \infty}\left\{\frac{1 + \sqrt{\frac{1}{x^2} + 1}}{\frac{1}{x} + 1}\right\}$$

$$= \log\left(\frac{1+1}{1}\right) = \log 2$$

**85.** (a) For common line, $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$$\Rightarrow (a + b + c)(a^2 + b^2 + c^2 - abZ - bc - ca) = 0$$

$$\Rightarrow \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$$

$$\Rightarrow \quad a - b = 0, b - c = 0, c - a = 0$$

$$\Rightarrow \quad a = b = c$$

Hence, it is an equilateral triangle.

**86.** (b) Here, $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \neq 0$, hence point is (0, 0, 0).

$$\Rightarrow \quad \alpha + \beta + \gamma = 0$$

**87.** (b) $\because \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{f(h) + 4xh}{h}$$

$$[\because f(x+h) = f(x) + f(h) + 4xh]$$

$$= \lim_{h \to 0} \frac{f(h)}{h} + 4x$$

$$= f'(0) + 4x$$

$$= 0 + 4x = 4x$$

$\Rightarrow \quad f(x) = 2x^2 + c$

But at $x = 0, f(0) = 0$

$\Rightarrow \quad c = 0$

Hence, $f(x) = 2x^2$

$\therefore \quad f(2) = 8$

**88.** (d) $\because \quad f(x) = 2x^2$

$\therefore \quad f'(x) = 4x$

$\Rightarrow \quad f'(2009) = 4 \times 2009$

$\Rightarrow \quad f'(2009) = 8036$

**89.** (c) Let the point on the parabola be $(t, t^2)$.Let '$d$' be the distance between $(t, t^2)$ and $(0, y_0)$, then

$$d^2 = t^2 + (t^2 - y_0)^2 = t^4 + (1 - 2y_0)\, t^2 + y_0^2$$

$$= z^2 + (1 - 2y_0)\, z + y_0^2,\ z \geq 0 \quad (\because \text{ put } t^2 = z)$$

Its vertex is at $x = y_0 - \frac{1}{2} < 0$

$\therefore$ The minimum value of $d^2$ is at $z = 0$, *i.e.*, $t^2 = 0$

$\therefore \quad d = y_0$

$\therefore$ Statement I is true. Statement II is false because extremum can occur at a point where $f'(x)$ does not exist.

**90.** (d) Statement II is true, as if $f(x) = \log x$, then $f'(x) = \frac{1}{x} > 0$ (as $x > 0$, so that $f(x)$ is defined).

Statement I is not true as $0 < \log x < 1$,

$\forall\ x \in (1, e)$ and hence $(\log x)^n$ decreases as $n$ is increasing. So that it is a decreasing sequence.