PROBLEMS IN PHYSICS

PART I:
REPRESENTATIVE PROBLEMS

PART II:
TOPIC WISE PROBLEMS
[IIT - JEE 1972 ONWARDS]

ANAND K. MISHRA

G. K. PUBLISHERS (P) LTD.

PROBLEMS IN PHYSICS

(EXCLUSIVELY FOR IIT-JEE MAINS)

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PREFACE...

The book is in two parts. The first part of the book has new and representative problems. Answers have been given to all the problems but solutions are given for odd numbered problems to help the students in developing their problem solving aptitutde.

However I have given the solution but students are advised to refer these solutions only after attempting the questions.

Part II is having previous years problems from IIT- JEE (1972 onwards). In part II of the book students will get an idea about the patterns and level of the questions being asked in IIT- JEE for different topics. Part II is specially designed for last hour preparation.

Last but not the least, I wish to convey my thanks to the publisher of this book Mr. Rakesh Mittal, Mrs. Poonam Mittal & Mr. Nitin Bhargava for bringing out this book in an excellent form.

All efforts have been made to keep the book free from errors. Inspite of my best possible efforts, some printing errors might have occured. I shall be grateful to the readers if the same are brought to my notice.

Suggestions for the improvement of this book are welcome.

April 2005
New Delhi

Anand K. Mishra
B. Tech

PRACTICE PROBLEMS

PART-I

MECHANICS

1

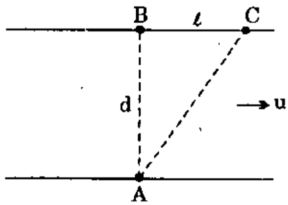
On a cricket field, a rabbit is at the origin of coordinate system and a dog at the position (46 m, 28 m). The rabbit runs on the ground with a constant velocity $\left(7.5\hat{i}+10\hat{j}\right)$ m/s. The dog can run with a speed 5 m/s. If the dog starts to run immediately the rabbit starts, what is the minimum time in which the dog could catch the rabbit?

2

A particle A has speed of 5 m/s. At time t = 0 its position relative to origin of coordinate system is (–11 m, 16 m) with the aim getting as close as possible to another particle B. At time $t = 0$, the particle B is at a point (4 m, 36 m) and is moving with constant velocity $\left(10\hat{i}-5\hat{j}\right)$ m/s. Find the velocity vector of particle A and the minimum distance of approach.

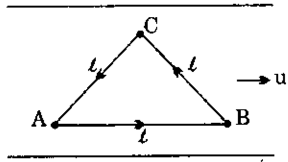
3

A swimmer can swim relative to water with a velocity v and the river flow velocity is u. The swimmer starts from a point A on one bank to reach a point C on the other bank which is at a distance l from the point B directly opposite to A on the other bank, as shown in the figure. The width of the river is d. Find the time taken by the swimmer to go from A to C.



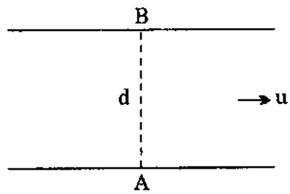
4

A boat is moving in a triangular course in a river. The flow velocity of the river is u and the velocity of boat relative to water is $u\sqrt{3}$. The length of the each side of the triangle ABC shown in the figure is l. Find the time taken to complete the full course.



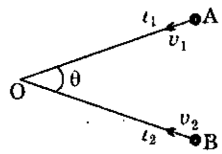
5

A man can swim with a velocity v relative to water and can run on the ground with a velocity w. The flow velocity of the river is u. The width of the river is d. The man is at a point A on one bank and wants to reach point B lying right across on the other bank of the river as shown in the figure. Find the minimum time in which the man can reach the destination.



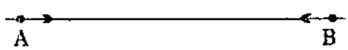
6

Two particles A and B move with constant velocities v_1 and v_2 along two straight lines inclined at angle θ towards the intersection point O. At the moment $t = 0$ the particles were located at the distance l_1 and l_2 from the point O as shown in the figure. Find the shortest distance between the particles.



7

In a still water lake a steamer is observed by two persons travelling at speed v into two boats in opposite directions on a straight track AB. To an observer in one boat the steamer appears to cross the track AB at right angles while to the observer in the other boat the angle appears to be 45°. At what angle does the boat actually cross the track and what is its speed?



8

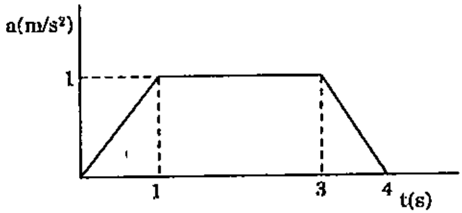
A man running on a horizontal road finds the rain falling at angle α with the vertical. If he doubles his speed angle becomes β. What will be the angle when he triples his speed?

9

A particle starts from rest with zero initial acceleration. The acceleration increases uniformly with time. Find the time average and distance average of velocity upto a certain instant when the velocity becomes v.

10

A particle starts from rest and moves on a straight line. The acceleration of the particle varies with time as shown in the figure. Find the velocity and displacement of the particle at $t = 4$s.



11

A particle moves along a straight line such that its displacement x from a fixed point on the line at time t is given by

$$x^2 = at^2 + 2bt + c.$$

Find acceleration as a function of displacement x.

12

A particle moves in a straight line under an attraction towards a fixed point O on the line from rest at a distance 'a' from O. If its acceleration when at a distance x from O varies as $\frac{\mu}{x^2}$, find the time period of the oscillatory motion about O.

13

A particles moves along the x-axis. The velocity of the particle as a function of x is given by

$$v^2 = 12x - 3x^3.$$

If f be the instantaneous acceleration, express velocity as a function of f.

14

A body of mass m is thrown straight up with initial velocity v_0. The air drag equals kv^2, where k is a constant and v is the velocity of the body. The acceleration due to gravity is g. Find the velocity with which the body comes down.

15

A particle moves along the x-axis. The accleration of the particle is given by

$$a = (kt - w^2x), \text{ where } k \text{ and } w \text{ are positive constants.}$$

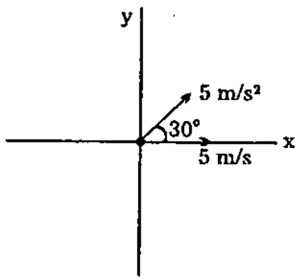
At $t = 0, x = 0$ and $\frac{dx}{dt} = 0$. Find the displacement x as a function of time t.

16

A particle moves along the x-axis. The acceleration of the particle is given by, $f = \left(\frac{\lambda}{x^3} - \frac{\mu}{x^2}\right)$, where λ and μ are positive constants. The particle starts from rest from $x = a$. Find the time period of the oscillatory motion performed by the particle.

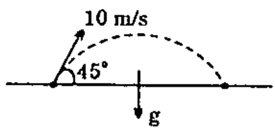
17

A particle is projected in X–Y plane from origin at $t = 0$ with initial velocity 5 m/s. A constant acceleration of 5 m/s^2 making an angle 30° with x-axis, as shown in figure, acts on the particle. Find the displacement and distance travelled by the particle at $t = 5$ seconds.



18

A particle is projected from the ground at an angle 45° with a initial velocity 10 m/s, as shown in the figure. The acceleration due to gravity is $g = 10$ m/s^2. Find the average velocity and average speed upto the instant when particle comes back to the ground.

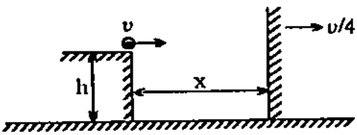


19

Two particles are projected from the same point with velocities v and $2v$ making equal angle $\theta = 30°$ with the horizontal in opposite directions as shown in the figure. Find the separation between them when their velocity vectors become mutually perpendicular. The acceleration due to gravity is g.

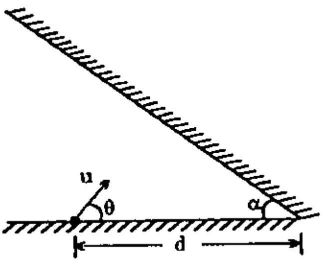
20

A particle is thrown from a height h horizontally towards a vertical wall moving away with a speed v as shown in the figure. If the particle returns to the point of projection after suffering two elastic collisions, one with the wall and another with the ground, find the initial separation x between the particle and the wall.



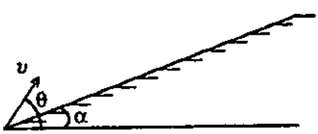
21

A projectile is fired with velocity u at an angle θ so as to strike a point on the inclined plane inclined at an angle α with the horizontal. The point of projection is at a distance d from the inclined plane on the ground as shown in the figure. The angle θ is adjusted in such a way that the projectile can strike the inclined plane in minimum time, find that minimum time.



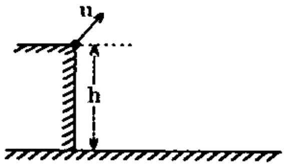
22

A particle is projected with initial velocity v at angle θ with the horizontal. There is an inclined plane at an angle α through the point of projection as shown in the figure. The angle θ is adjusted so that the particle strikes that plane at an angle 45°. Find the height h of the point struck above the point of projection.



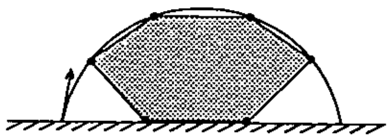
23

A particle is projected with an initial speed u from a point at height h above the horizontal plane as shown in the figure. Find the maximum range on the horizontal plane.



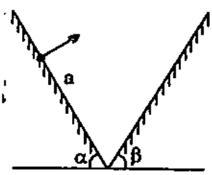
24

A particle is projected from the ground such that it touches the four corners of a regular hexagon of side length a, as shown in the figure. Find the corresponding horizontal range on the ground and the time of flight.



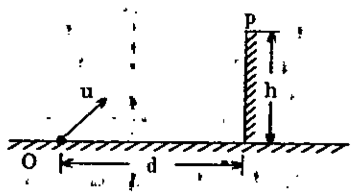
25

Two planes are inclined at angles α and β with the horizontal and a particle is projected at right angle to the one plane from a point at a distance 'a' from the point of intersection of the planes as shown in the figure. If the particle strikes to the other plane at right angle, find the time of flight.



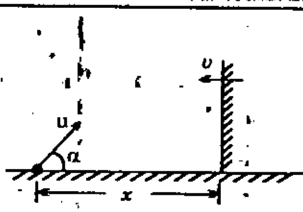
26

A particle is projected from the point O on the ground to hit a target P at a height h above the ground as shown in the figure. Find the least value of the velocity of projection u and the time taken by the particle to reach from O to P.



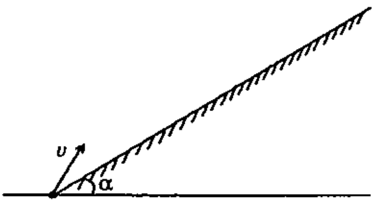
27

A particle is projected from the ground with initial velocity u at angle α to the horizontal and strikes elastically a vertical wall moving towards it with a horizontal velocity v as shown in the figure. The particle is bounded back to the point from where it was projected. Find the initial distance x of the wall from the point of projection.



28

A particle is projected with initial velocity v as shown in the figure. After elastic collision with the inclined plane the particle rebounds vertically. Then the particle retraces its path and comes back to the point of projection. Find the time it takes to return to the point of projection.



29

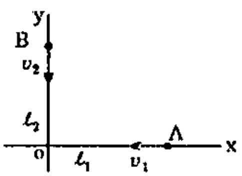
A stone is projected with initial velocity u at angle α with the horizontal. Find the angular velocity of the stone with respect to the point of projection, when it is at its maximum height.

30

A particle is projected from the ground at $t = 0$ with initial velocity u at an angle α with the horizontal. Find the radius of curvature of the path at time t.

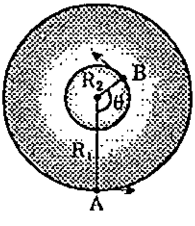
31

Two particles A and B are moving with uniform velocities v_1 and v_2 along two perpendicular lines as shown in the figure. At $t = 0$ their distances from O are l_1 and l_2 respectively. Find angular velocity of the line joining them at time t and its maximum value.



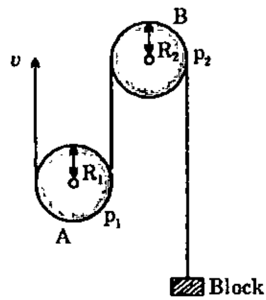
32

Two particles A and B describe coplanar concentric circles of radii R_1 and R_2 with angular velocities ω_1 and ω_2 as shown in the figure. Find the angular velocity of A with respect to B.



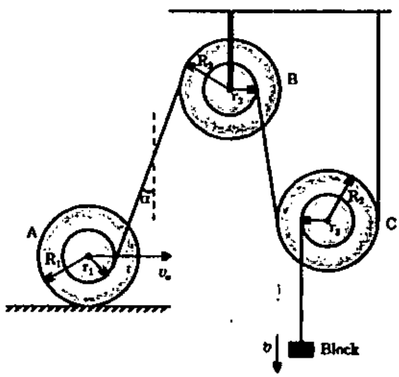
33

A string, attached with a block passing over two fixed pulleys A and B, is being pulled as shown in the figure. The radii of pulley A and B are $R_1 = 10$ cm and $R_2 = 5$ cm respectively. If the normal component of acceleration of a point P_1 on the pulley A is 40 m/s^2 and the tangential component of acceleration of a point P_2 on the pulley B is 30 m/s^2 at the instant, calculate the corresponding speed v of the tape, the magnitude of total acceleration of P_1, and the magnitude of the total acceleration of P_2.



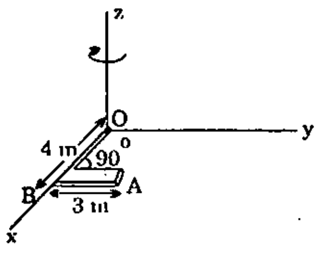
34

Consider an arrangement shown in the figure. 'A' is a bobbin, which can roll on the horizontal plane, B is a fixed pulley and C is a movable pulley. The string connecting the bobbin 'A' to the pulley 'B' makes an angle 'α' with the vertical at any instant and the block attached with the pulley C is being pulled down with an instantaneous speed v. Find the velocity of the centre of the bobbin at this instant if it rolls without slipping.



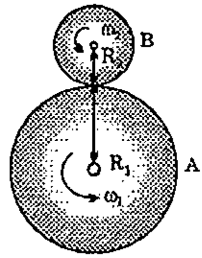
35

A rod is bent into the "L" shape and hinged at O so that it can be rotated about z-axis in x-y plane as shown in the figure. At the position shown in the figure, the angular velocity is 2 rad/sec and angular velocity is decreasing at the rate of 4 rad/s^2. Find the acceleration (in vector form) of the end A of the rod?



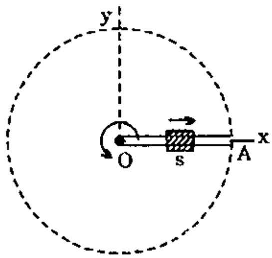
36

A sphere 'A' of radius R_1 is rotating about its centre O with a constant angular velocity ω_1. Another sphere B of radius R_2 rolls without slipping with angular velocity ω_2 over the sphere A as shown in the figure. Find the acceleration of the point of contact of the rolling sphere.



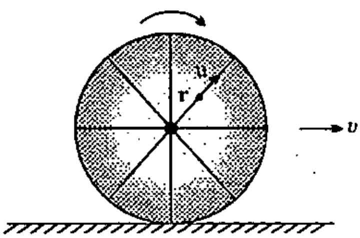
37

A rod OA is rotating about O with counter clockwise angular velocity 4 rad/s about z-axis which is decreasing at the rate 10 rad/sec^2. The motion of a slider 's' on the rod is separately controlled. At a particular instant, as shown in the figure, the slider 's' is at a distance 6 cm from O and it is moving away from O with an instantaneous velocity 5 cm/s and acceleration 81 cm/s^2 with respect to the rod. Determine the absolute velocity and acceleration of 's' for this position in vector form.



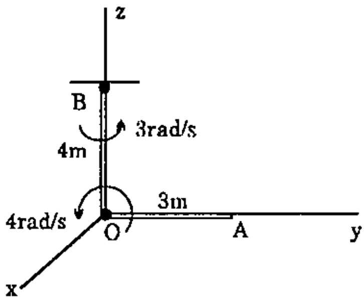
38

A wheel of radius R rolls on a horizontal ground with a speed v. An insect crawls at a constant speed u along a spoke of the wheel as shown in the figure. Find the acceleration of the insect with respect to the ground in terms of the distance r of insect from the centre of the wheel?



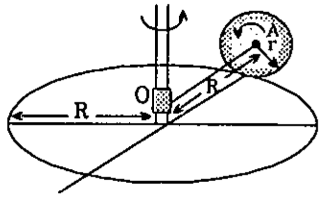
39

A rigid rod OA is freely attached with another rigid rod OB at O as shown in the figure. The lengths of the rods OA and OB are 3 m and 4 m respectively. The entire assembly rotates about Z-axis with a constant angular velocity 3 rad/s. Simultaneously, the rod OA is being rotated (raised) about an axis through O at the constant angular velocity 4 rad/s. At a particular instant the rod OA is along Y-axis as shown in the figure. Find the velocity of the end A at this instant in vector form.



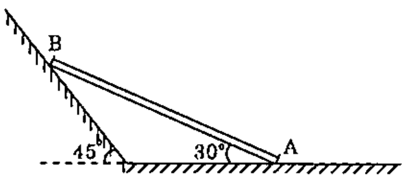
40

A disc of radius r, attached with a horizontal axle OA, rolls without sliding on a circular track of radius R as shown in the figure. In the process, the centre of the disc moves with a constant speed v. Find the angular acceleration of the disc.



41

A rod AB of length 4 m slides with its ends in contact with the floor and the inclined plane. At a particular instant the position of the rod is shown in the figure. At his instant the end A is moving towards right with velocity 4 m/s and acceleration 5 m/s^2. Find :



(a) angular velocity of the rod

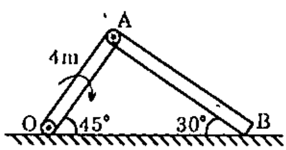
(b) velocity of the end B

(c) angular acceleration of the rod

(d) acceleration of the end B.

42

A rod OA of length 4 m is hinged at O. Another rod AB is attached freely at A with the rod OA. At a particular instant the positions of the rods are shown in the figure and the rod OA is rotating clockwise with angular velocity 1 radian/s. Find :

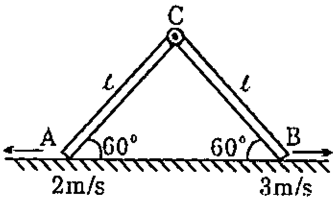


(a) angular velocity and angular acceleration of the rod AB

(b) velocity and acceleration of the end B.

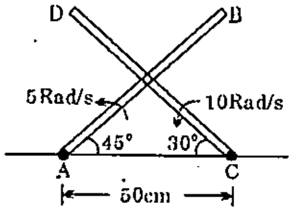
43

Two rods of equal length AC and BC are freely joined at C. Two ends A and B are pulled with speeds 2 m/s and 3 m/s respectively, as shown in the figure, at a particular instant. Find the speed of the point C at this instant.



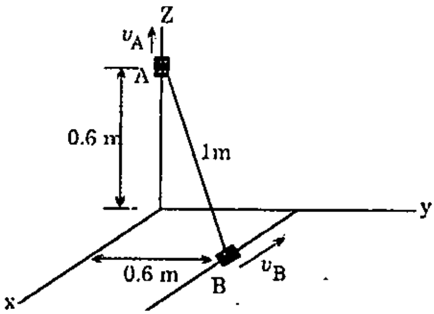
44

Two rods AB and CD rotate about ends A and C respectively in plane are immediately above the other, as shown in the figure. Find the velocity and acceleration of the point of crossing for the orientation given in the figure.



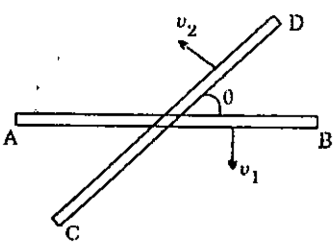
45

A rod AB of length 1 m is attached to two sliders at A and B moving on the guide bars as shown in the figure. Find the velocity of the slider B for the situation described in the figure when $v_A = 0.2$ m/s.



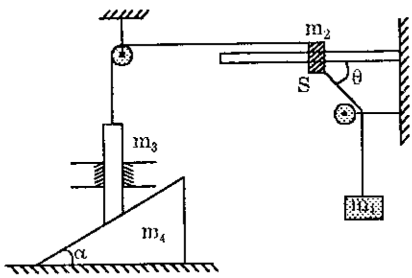
46

Two rods AB and CD are placed in a plane intersecting each other at an angle θ. The rods move perpendicular to themselves in the same plane with velocities v_1 and v_2 as shown in the figure. Find the velocity of the point of the intersection of two rods.



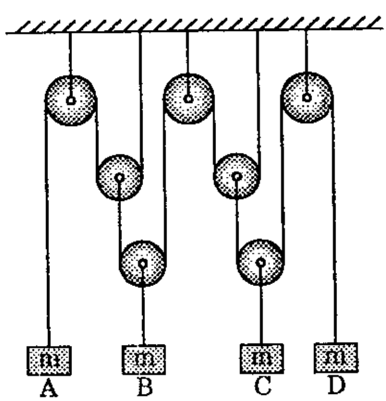
47

Consider an arrangement shown in the figure. The block of mass m_3 is constrained to more in the vertical direction only. The wedge of mass m_4 moves in the horizontal direction. The slider 's' of mass m_2 moves on a fixed horizontal rod. The friction between all the contact surfaces is negligible. At a particular instant the string connecting the slider 's' to the block of mass m_1 is making angle θ with the rod. Find the acceleration of the block of mass m_1 at this instant? The acceleration due to gravity is g.



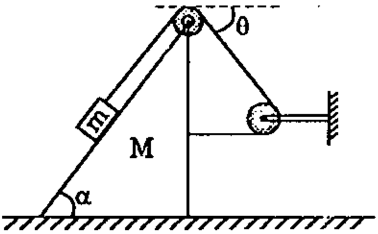
48

In the arrangement shown in the figure, the masses of all the four blocks A, B, C and D is equal to m. The masses of pulley and the thread are negligible. The friction in the pulleys is absent. If the system is released only one block moves up and the other three blocks come down. Which block starts moving up and with what acceleration? The acceleration due to gravity is g.



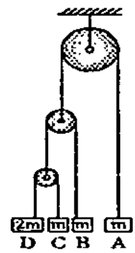
49

In the arrangement shown in the figure, friction at all the contact surfaces is absent. The mass of pulleys and thread is negligible. If the system is released from rest in the position shown in the figure, find the acceleration of the wedge?



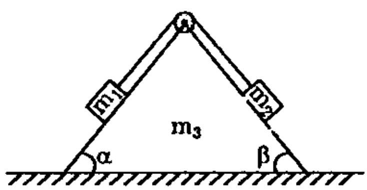
50

In the arrangement shown in the figure pulleys are massless and frictionless. The masses of the blocks A, B, C is equal to m and the mass of the block D is $2m$. If the system is released from rest, find the acceleration of the block A.



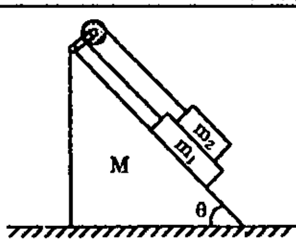
51

Two blocks of masses m_1 and m_2 are connected with a string and are placed on a wedge of mass m_3 as shown in the figure. The friction between all contact surfaces is negligible and the masses of the pulley and thread are also negligible. If the system is released find the acceleration of the wedge?



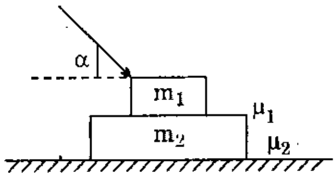
52

In an arrangement shown in the figure, there is no friction at any contact surface. The masses of pulley and thread are negligible. If the system is released, find the acceleration of the wedge.



53

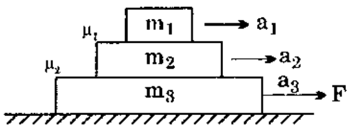
A block of mass m_1 is placed over a plank of mass m_2 as shown in the figure. The coefficient of friction between plank and the ground is μ_2 and between block and plank is μ_1. A force of magnitude F is applied on the block at an angle α with the horizontal. It is found that the plank moves on the ground but the block does not move relative to the plank, find the range of F (Assume $\cot \alpha > \mu_1 > \mu_2$).



54

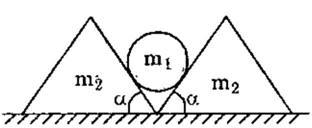
In the arrangement of the blocks shown in the figure, $m_1 = 2$ kg, $m_2 = 3$ kg and $m_3 = 7$ kg. The coefficient of friction between m_1 and m_2 is $\mu_1 = 0.2$ and between m_2 and m_3 is $\mu_2 = 0.3$. There is no friction between m_3 and ground. Find the acceleration a_1, a_2 and a_3 of the blocks m_1, m_2 and m_3 if the magnitude of the force F applied on m_3 equals to

(a) 12 N, *(b)* 30 N, *(c)* 48 N.



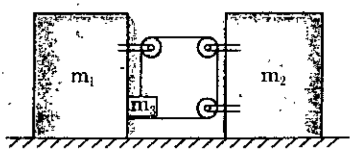
55

In the arrangement shown in the figure, a cylinder of mass m_1 is placed in the groove formed by two identical prisms each of mass m_1. The coefficient of friction for all the contact surfaces is μ. Find the initial acceleration of the cylinder, just after the release of the system from rest.



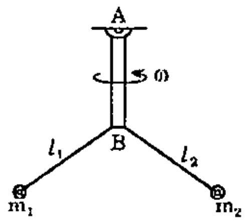
56

In the arrangement shown in the figure, the coefficient of friction for all the contact surfaces is μ. The masses of pulleys and threads and negligible. The block of mass m_3 always remains in contact with the block of mass m_1. Find the acceleration of the block of mass m_1 just after the release of the system from rest.



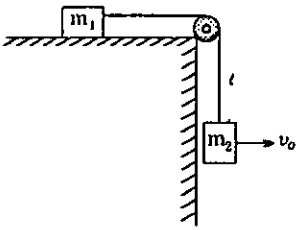
57

Two particles of masses m_1 and m_2 are connected with the two strings of lengths l_1 and l_2 with the end B of a rod AB as shown in the figure. The rod AB is rotated about the vertical axis so that the rod AB remains vertical. Find the angular velocity ω of rotation? The acceleration due to gravity is g.



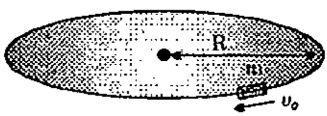
58

In the arrangement shown in the figure, initially the block of mass m_1 is in limiting equilibrium. The coefficient of friction between the block of mass m_1 and floor is μ. Now the particle of mass m_2 is given a horizontal velocity v_0. The initial distance of the block of mass m_2 from the pulley is l. The masses of pulley and threads and negligible. Find the initial acceleration a of the block of mass m_1 and initial radius 'R' followed by the block of mass m_2.



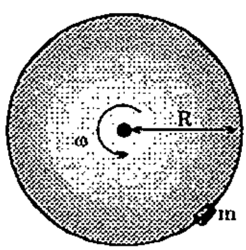
59

A thin rod is bent to form a horizontal circular track of radius R as shown in the figure. A ring of mass m is given an initial velocity of magnitude v_0. If the coefficient of kinetic friction is μ, determine the distance travelled before the ring comes to rest? The acceleration due to gravity is g.



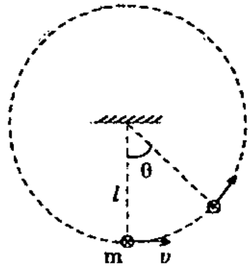
60

A rough vertical circle of radius R, carrying a bead of mass m, rotates in its own plane about its centre with uniform angular velocity ω. The coefficient of friction is μ. Find the minimum value of ω so that the bead will never slip? The acceleration due to gravity is g.



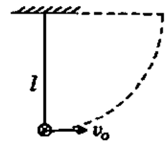
61

A particle of mass m is tied to one end of a light inextensible string of length l the other end of which is fixed as shown in the figure. The particle is projected with a velocity v from the lowest position. Find the net force acting on the particle, when the string makes an angle θ with the vertical? The acceleration due to gravity is g.



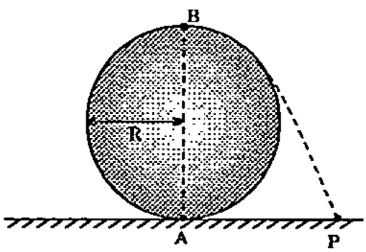
62

A particle suspended by a thread is projected with a velocity $v = 2\sqrt{gl}$ from the lowest position as shown in the figure. Find the time taken by the particle to reach the horizontal position.



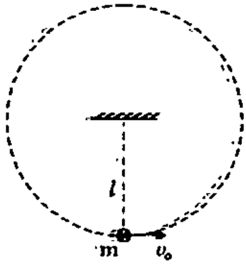
63

A particle slides down the surface of a smooth fixed sphere of radius R starting from rest at the highest point B. The particle leaves the sphere at some point and then strikes the horizontal plane through the lowest point A of the sphere, at a point P as shown in the figure. The acceleration due to gravity is g. Find the distance AP.



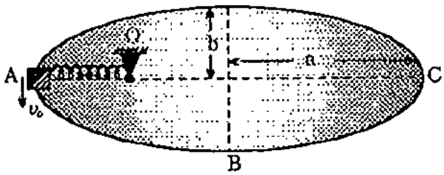
64

A particle is attached to a fixed point by a fine string of length l and is projected horizontally from the lowest point with velocity $v_0 = \sqrt{lg\left(2+\frac{3\sqrt{3}}{2}\right)}$ as shown in the figure. In the subsequent motion the string first becomes slack and again becomes tight and then passes through lowest point along circular path. Find the speed v with which the string will pass through the lowest point first time.



65

A slider can slide on a smooth vertical elliptical guide as shown in the figure. The fixed point O is located at one focus of the elliptical guide. One end of spring is connected at O and the other end is connected to the slider. The spring is unstretched when the slider is at A. The speed v_0 given to the slider at A is such that the speed approaches to zero at C. The semi-major and semi-minor axes are 'a' and 'b' respectively. The acceleration due to gravity is g. Determine the speed of the slider at B.

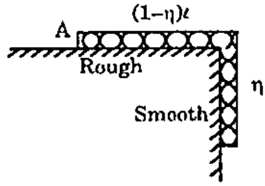


66

In the arrangement shown in the figure $m_1 = 2$kg, $m_2 = 1$ kg, $K_1 = 15$ N/cm and $K_2 = 5$ N/cm. The acceleration due to gravity is 10 m/s^2. Initially the system is in equilibrium. Find the work done by external agent in slowly pulling down m_2 by a distance of 8 cm.

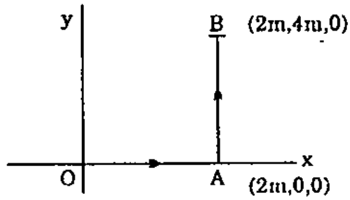
67

A uniform chain of length l rests on a rough table so that one end hangs over the edge. The chain slides off the table all by itself when η fraction of the length of the chain hangs, as shown in the figure. The chain starts sliding from rest. Find the speed of the end A of the chain when it completely slides off the table.



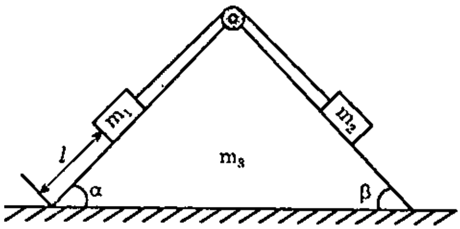
68

Find the work done by a force, $\vec{F} = [(y^2 - x^2 + Z^2)\,\hat{i} + (3xy - 5Z)\,\hat{j} + (4Z)\,\hat{k}]$ in taking particle from origin O to the point B $(2m, 4m, 0)$ along the path OAB as shown in the figure.



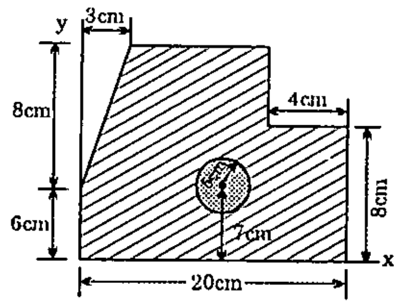
69

In the arrangement shown in the figure, all the contact surfaces are frictionless. Find the distance covered by the wedge of mass 'm' on the horizontal plane till the mass m_1 is lowered by a distance l along the surface of wedge.



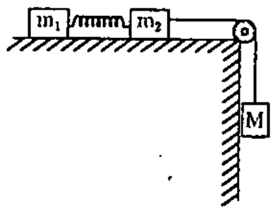
70

A uniform plate having a circular hole is shown in the figure. Find the co-ordinates of the centre of mass.



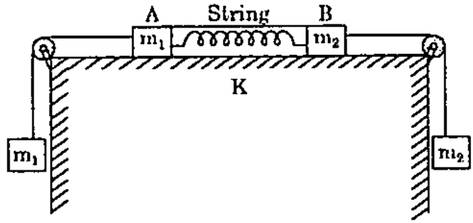
71

In the arrangement shown in the figure, m_1 = 3kg, m_2 = 2 kg, M = 5 kg and there is no friction at any contact surface. The acceleration due to gravity is 10 m/s^2. The system is released from rest. After 1 second, the velocity of m_1 is 3 m/s. Find the kinetic energy of the system at this instant.



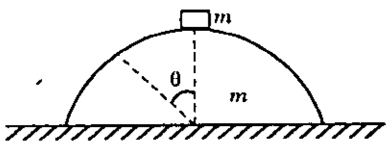
72

Consider an arrangement shown in the figure. There is no friction at any contact surface. The system is released from rest and the string connecting the blocks A and B is also cut. Find the maximum extension in the spring connecting the blocks A and B in the subsequent motion. The acceleration due to gravity is g.



73

A heavy particle of mass m is placed on the top of a smooth hemisphere also of mass m which is placed on a smooth horizontal plane as shown in the figure. The system is released from rest. Find the angle θ with the vertical where the particle will loose contact with the hemisphere.

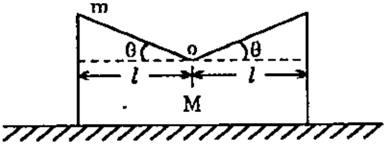


74

A particle of mass m is placed on the wedge shaped block of mass M as shown in the figure. The system is released from rest. Neglect the friction at all the contact surfaces and also neglect the impact at the centre O. Find

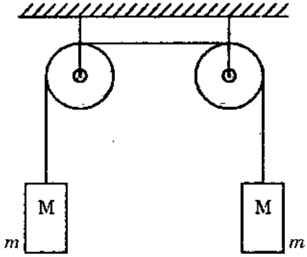
(a) maximum velocity acquired by the block

(b) average speed of the block over one period of oscillation.



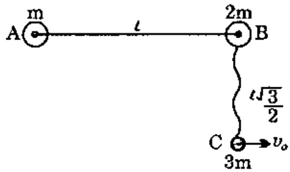
75

Two men each of mass M stand on two platforms each of mass m hanging as shown in the figure. One of the man leaps vertically upward with a velocity v_0 relative to the platform. Find the distance through which that platform will descend.



76

Three particles A, B and C of mass m, $2m$ and $3m$ respectively lie on a smooth horizontal table. A and B as well as B and C are connected by light inextensible strings each of equal length l. The string connecting A and B is tight. The initial distance between B and C is $\frac{l\sqrt{3}}{2}$ and particle C is given a velocity v_0 parallel to AB as shown in the figure. Find the velocity of the particle A, just after its starts moving.

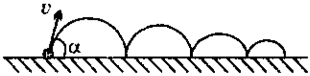


77

A particle is projected from a point on a smooth horizontal ground with velocity v at angle α to the horizontal. The coefficient of restitution for the collision of the particle with the ground is e. The acceleration due to gravity is g. Find

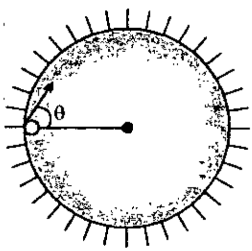
(a) the total time for which the particle rebounds on the ground.

(b) the distance on the ground from the starting point when the particle ceases to rebound.



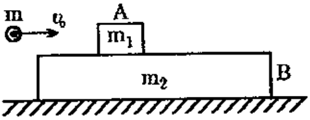
78

A smooth circular table is surrounded by a smooth rim whose interior surface is vertical. A particle whose coefficient of restitution is e is projected along table from a point on the rim in a direction making angle θ with the radius through the point and returns to the point of projection after three impacts on the rim. Find the angle θ ?



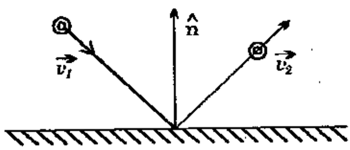
79

A block A of mass m_1 is resting on a long plank B of mass m_2 as shown in the figure. There is no friction between plank B and the ground. The coefficient of friction between block A and plank is μ. A bullet of mass m is fired with velocity v_0 into the block A and gets embedded in it. Find the displacement of block A on the plank B.



80

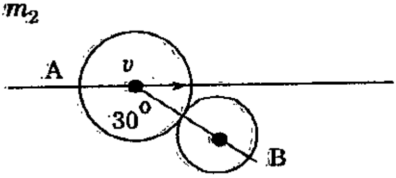
A particle strikes the floor with a velocity $\vec{v}_1$. The coefficient of restitution between the particle and floor is e. The unit vector normal to the floor is $\hat{n}$. Find the velocity (vector) $\vec{v}_2$ of the particle with which it rebounds.



81

A moving sphere A of mass m_1 experience a perfectly elastic collision with a stationary sphere B of mass m_2 as shown in the figure. At the instant of collision the velocity vectors of A makes an angle of 30° with the line joining the centres of A and B. After collision the spheres fly apart symmetrically relative to the initial motion direction of the sphere A with the angle of divergence 60°.

Find the ratio $\frac{m_1}{m_2}$.

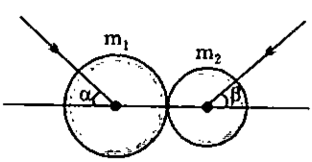


82

A sphere of mass m is moving with a velocity $\vec{v}_1 = (\hat{i} + 2\hat{j})$ m/s and another sphere of mass $2m$ is moving with a velocity $\vec{v}_2 = (-\hat{i} + 3\hat{j})$ m/s. At the instant of thier collision the line joining their centres is parallel to a vector $\vec{r} = (\hat{i} - \hat{j})$ m. The coefficient of restitution, $e = \frac{1}{2}$. Find the velocities $\vec{v}_1'$ and $\vec{v}_2'$ of the spheres just after collision.

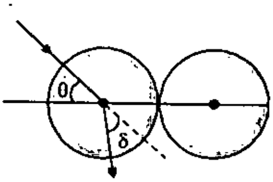
83

Two discs whose masses are m_1 and m_2 are moving in the directions, making angles α and β with the line joining their centres, collide as shown in the figure. The directions of motion after the impact is found to be perpendicular to their direction before impact. Find the coefficient of restitution.



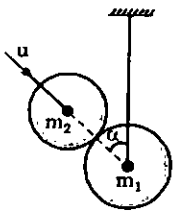
84

A smooth sphere impinges obliquely on an identical sphere at rest. Before impact the first sphere was moving in a direction making an angle θ with line joining centres at the moment of impact as shown in the figure. The direction of motion of the first sphere is turned through an anlge δ by the impact. The angle θ can be changed to get the maximum angle of deviation δ. The coefficient of restitution is e. Find the maximum value of angle of deviation δ and the corresponding value of the angle θ.



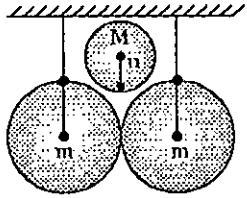
85

A smooth sphere of mass m_1 is tied to a fixed point by an extensible string and another sphere mass m_2 impinges directly on it with velocity u in a direction making an acute angle α with the thread as shown in the figure. The coefficient of restitution is e. Find the velocity of the suspended sphere of mass m_2 just after the impact.



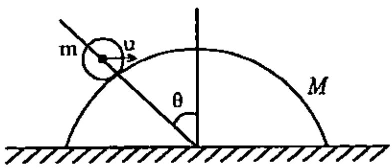
86

Two identical spheres, of mass m are suspended by vertical string so that they are in contact at the same level. A third sphere of same radius but of different mass M falls vertically and strikes other two simultaneously so that their centres at the instant of impact form an equilateral triangle in a vertical plane, as shown in the figure. If u be the speed of centre of the sphere M just before impact, find the velocities of the suspended spheres just after impact. The coefficient of restitution is e.



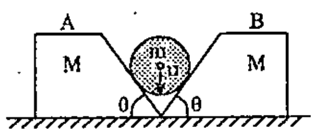
87

A hemisphere of mass M rests on a smooth horizontal floor and may move freely in the horizontal direction. A sphere of mass m moving horizontally with a velocity v strikes the hemisphere at a point where the common normal makes an angle θ with the vertical, as shown in the figure. The coefficient of restitution is e. Find the velocity of the hemisphere just after the impact.



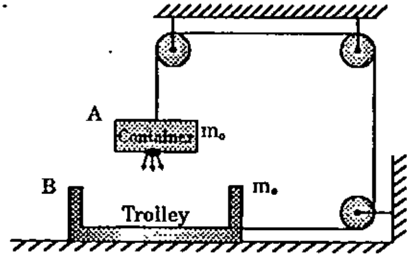
88

Two wedge shaped blocks A and B each of equal mass M are placed on the frictionless horizontal floor. A smooth sphere of mass m moving vertically down with the velocity v strikes the wedges symmetrically as shown in the figure. The coefficient of restitution is e. Find the velocities of wedges just after the collision?



89

Consider an arrangement shown in the figure in which all the contact surfaces are frictionless. The container 'A' contains water and its initial mass is m_0. A trolly 'B' which can store water has also mass m_0 when it is empty. There is a hole at the bottom in the container, which is when opened water leaks out at a constant rate of μ kg/s. The water falls on the trolly and comes to rest relative to the trolly. Initially the system is at rest. The system is released and hole is opened at $t = 0$. Find the velocity of trolly after time t.

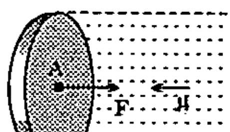


90

A spherical rain drop of radius r_0 starts falling freely receives water particles during its fall, so that its radius increases at a constant rate $\frac{dr}{dt} = \mu$. The acceleration due to gravity is g Find the distance that the drop has fallen in time t?

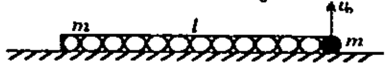
91

A disc of mass m_0 and cross-sectional area A is being pulled by a constant force F in a region where uniform cloud of dust particles are moving towards the disc with a velocity u as shown in the figure. All the particles meeting the disc attach to it. The density of the dust cloud is ρ. The disc starts moving from rest at $t = 0$. Find the velocity of the disc as a function of time.



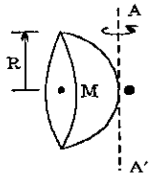
92

A heavy particle of mass m is attached to one end of a chain of length l and same mass m. The chain is placed on a horizontal floor at rest. The particle is projected vertically upward with an initial velocity v_0, as shown in the figure. The acceleration due to gravity is g. Find the least value of v_0 so that the whole chain may leave the floor.



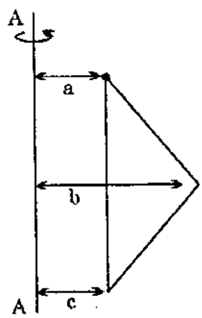
93

Find the moment of inertia of a hemisphere of mass M and radius R shown in the figure, about an axis AA′ tangential to the hemisphere.



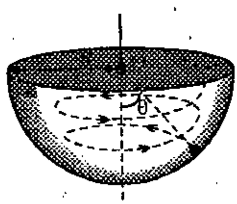
94

The distances of the vertices of triangle of mass M, from an axis AA′ in its plane are a, b and c as shown in the figure. Find the moment of inertia of the triangle about this axis.



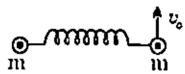
95

A particle is projected horizontally along the interior of a smooth hemispherical bowl of radius R. The particle moves on the helical path of increasing radius as shown in the figure. The initial position of the particle is at an angle θ with the vertical. Find the initial velocity v_0 required for the particle to just reach to the top of the bowl.



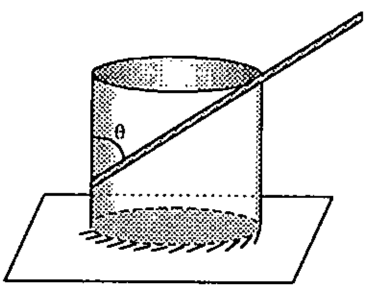
96

Two particles each of mass m, rest on a smooth horizontal floor and are connected by an ideal non deformed spring of natural length l and spring constant k. If one of the particle is given a velocity v_0 perpendicular to the line joining the particles as shown in the figure, find maximum elnogation x of the spring in process of motion assuming $x<<l$.



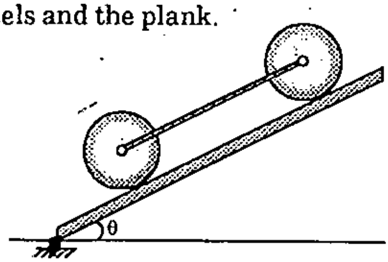
97

A vertical hollow cylinder is fixed on the ground. A uniform rod can be balanced partly in and partly out of the cylinder with the lower end of the rod resting against the vertical wall of the cylinder, as shown in the figure. The angle made by rod with the vertical in equilibrium is θ. If the maximum and minimum values of θ are α and β respectively, then find the coefficient of friction between rod and cylinder.



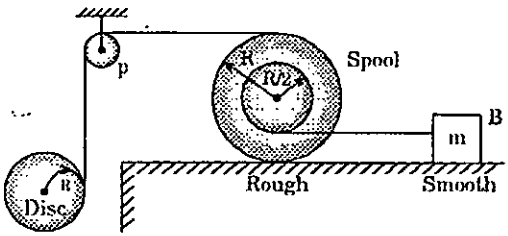
98

Two wheels are connected by an axle and are placed over a plank as shown in the figure. The angle of inclination θ of the plank can be varied. It is found that the system just slips down when upper wheel is locked at angle $\theta = \alpha$ and when lower wheel alone is locked slips down at angle $\theta = \beta$. Find the coefficient of friction between wheels and the plank.



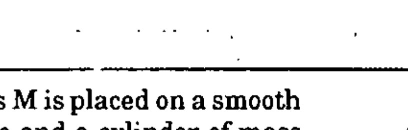
99

Consider an arrangement shown in the figure. The pulley P is frictionless and the threads are massless. The mass of the spool is m and moment of inertia of the spool is $\frac{1}{2}mR^2$. The mass of the disc of radius R is also m. The surface below the spool is rough to ensure pure rolling of spool. The mass of the block is m and the surface below the block is smooth. Find the initial acceleration of the block when the system is released from rest.



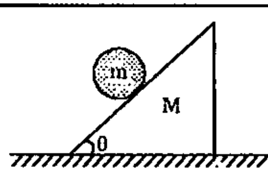
100

Consider an arrangement shown in the figure. The surface below the plank and block are frictionless. The pulleys attached with the block are frictionless and the thread are massless. The contact surface between cylinder and plank is rough enough to ensure pure rolling. A horizontal force F is applied at the centre of mass of the cylinder towards left. Find the initial acceleration of the plank.



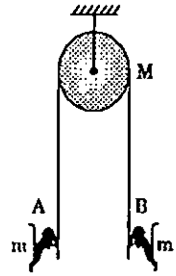
101

A wedge of mass M is placed on a smooth horizontal plane and a cylinder of mass m is placed on the wedge as shown in the figure. The contact surface between cylinder and wedge is rough. Find the acceleration of the wedge when the cylinder begins to roll down.



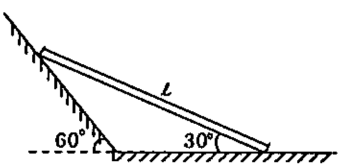
102

A light rope of length l passes over a pulley of mass M and radius R which can rotate about a horizontal axis. The moment of inertia of the pulley is $\frac{1}{2}MR^2$. Two monkeys A and B of equal mass m are initially at rest at the lower ends of the string as shown in the figure. The monkey A starts moving up with speed v and simultaneously monkey B starts moving up with $\frac{v}{2}$, both relative to the rope. Which monkey will reach the top first and how much time it will take? Assume $R << l$.



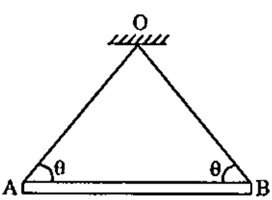
103

A uniform rod of length l is released from rest from the position shown in the figure. The acceleration due to gravity is g. There is no friction at any surface. Find the initial angular acceleration of the rod.



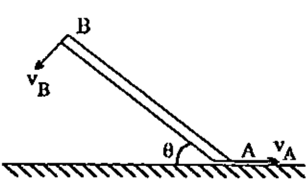
104

A uniform rod AB of length l and mass m is suspended by two identical strings OA and OB from a fixed point O as shown in the figure. The rod is in the horizontal position and each string makes an angle θ with the rod. If the string OB is cut what will be the tension in the string OA just after the cut?



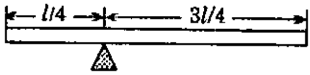
105

A uniform rod of mass m and length l is standing vertically on a smooth horizontal floor. A slight disturbance causes the lower end A slip on the floor. Find the velocities of the ends of the rod at the instant it makes angle θ with the horizontal as shown in the figure.



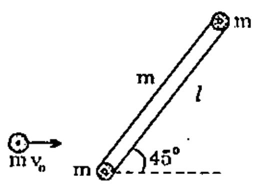
106

A uniform rod of length l is placed in horizontal position on a nail as shown in the figure. The rod is released from rest from this position. Find the coefficient of friction between the rod and the nail, if the rod starts sliding after turning through an angle θ.



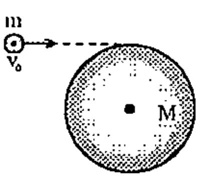
107

A rigid rod of length l and mass m joins two particles each of mass m. The rod lies on a frictionless table and is struck at one end by a particle of mass m moving with velocity v_0 as shown in the figure. After the collision, the particle moves straight back. Assume that the collision is perfectly elastic. Find the angular velocity of the rod after the collision.



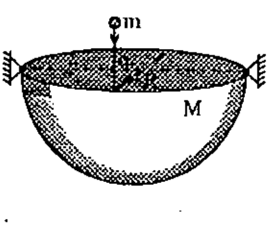
108

A disc of mass M is lying on a frictionless horizontal surface. A small particle of mass m strikes the edge of the disc with a velocity v_0 in a tangential direction of disc and gets embedded in it, as shown in the figure. Find the angular velocity of the disc just after the collision?



109

A solid hemisphere of mass M = 8 kg and radius R = 5 m is supported at the ends of the diameter of the circular cross-section. The supports are frictionless. A particle of mass m = 1 kg is dropped freely from a height h and strikes the end of the diameter of the circular section perpendicular to the diameter passing through the supports and sticks, as shown in the figure. The hemisphere just reaches a position where the flat face of it is vertical. Find h. Also calculate the reactive impulse 'J' of the supports during collision.

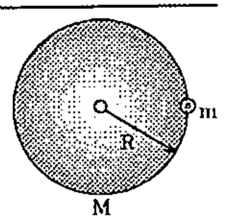


110

A solid sphere of mass m is rolling on a rough horizontal surface with velocity v_0. It collides elastically with a cubical block of same mass m at rest. The height of centre of mass of both the bodies is same. Assume that there is no friction between the sphere and the block. The coefficient of friction at all the surfaces is μ. Find at what time and at what distance from first collision the second collision will take place.

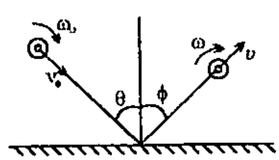
111

A circular ring of mass M and radius R lies on a smooth horizontal surface. An insect of mass m starts moving round the ring with uniform velocity v relative to the ring. Find angular velocity of insect with respect to the ground.



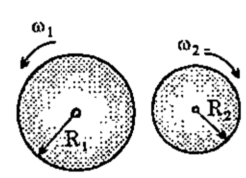
112

A solid spherical ball spinning with an angular velocity ω_0 strikes a horizontal floor with a velocity v_0 at angle θ with the normal to the floor and rebounds as shown in the figure. The coefficient of restitution is e and the coefficient of friction is μ. Find after the rebound the angle ϕ, the angular velocity ω and the velocity v.



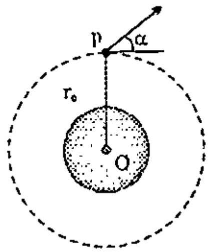
113

Two cylinders of radii R_1 and R_2 are of equal length and are made up of same material. Initially cylinders are rotating about their axes with angular velocities ω_1 and ω_2 as shown in the figure. The cylinders are moved closer to each other keeping their axes paralle. The cylinders first slip over each other at the contact but the slipping finally ceases due to the friction between them. Find the angular velocties ω_1' and ω_2' of the cylinders after the slipping ceases.



114

A satellite is projected into space from a point P at a distance r_0 from the centre of the earth at an angle α with the horizontal as shown in the figure. The velocity of projection equals to the velocity of the satellite in the circular orbit of radius r_0 around the earth. It is found that the satellite is propelled into an elliptic orbit. Find the length of semi-major axis 'a' and eccentricity e of the elliptic orbit.



115

A body is projected vertically upward from the surface of earth with a velocity equal to the escape velocity. The radius of earth is R and the acceleration due to gravity on the surface of the earth is g. Find the time taken by the body to reach to the height h.

116

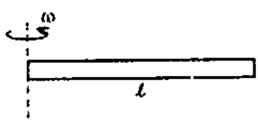
The density ρ inside a solid sphere of radius R and mass M varies as

$$\rho \propto \frac{1}{r}, \text{ where } r \text{ is the distance from the centre.}$$

Find the gravitational potential V at the centre of the sphere and also find the internal potential energy (self energy) u of the matter forming sphere.

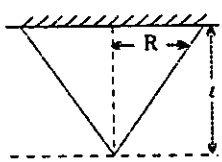
117

A uniform rod of length 'l', mass 'm' cross sectional area 'A' and made up of a material of young's modulus Y is rotated about one end with an angular velocity ω as shown in the figure. Find the elongation Δl in the length of the rod, assuming $\Delta l << l$.



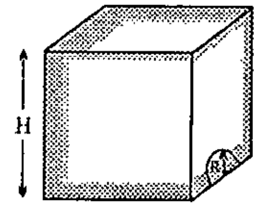
118

A cone of circular cross-section having base radius R, height l and mass m is suspended from its base as shown in the figure. The material of cone has Young's modulus Y. The acceleration due to gravity is g. Find elastic potential energy stored in the cone.



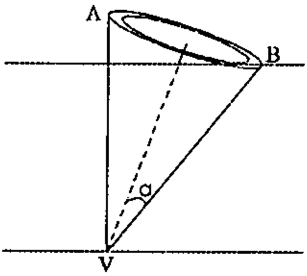
119

A tank of rectangular cross-section is filled with a liquid of density ρ upto height H. There is a semicircular gate of radius R hinged along the base line of the container as shown in the figure. Find the torque acting on the gate about base line. The acceleration due to gravity is g.



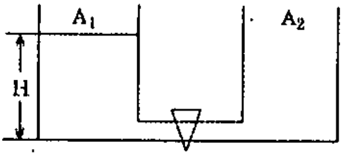
120

A right circular cone of semi-vertical angle α can float vertex down in water with generator vertical and the base just clear of the water as shown in the figure. Find the relative density of the material of the cone.



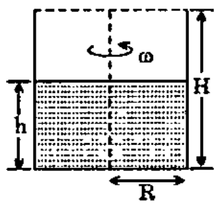
121

Two cylindrical containers of cross-sectional area A_1 and A_2 are connected together at their bottoms by a thin pipe of uniform cross-sectional area a as shown in the figure. Initially the container of cross sectional area A_1 is filled upto height H with a non-viscous incompressible liquid. Now the tap is opened and liquid flows from one container to the other container. Find the time taken to equalize the level of liquid in both the container. The acceleration due to gravity is g.



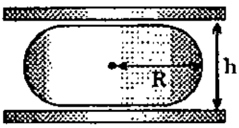
122

A cylindrical container of radius R and height H is filled with a non-viscous incompressible liquid upto a height h as shown in the figure. Now the cylinder is rotated about its axis with a constant angular velocity ω. Find the maximum value of ω for which the water will not spill over.



123

A mercury drop shaped as a round tablet of radius R and thickness h is placed between two horizontal glass plates. Now a mass m is placed on the upper plate and due to its weight the distance between the plates is diminished n times. The surface tension is T and acceleration due to gravity is g. Find the angle of contact θ. (Assume is h <<R)

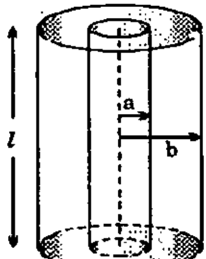


124

The surface tension of the soap water solution is S and the atmospheric pressure is P_0. Find the work to be done in order to blow a soap bubble of radius a under the isothermal condition.

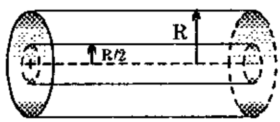
125

The space between two coaxial cylinders of radii a and b is filled with a liquid of coefficient of viscosity η. The length of cylinder is l. The outer cylinder can be rotated about its axis. Find the torque required to rotate the outer cylinder with a constant angular velocity ω.



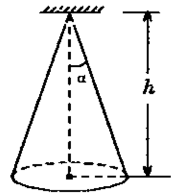
126

A viscous liquid is flowing through a pipe of cross-sectional radius R. The volume of liquid flowing per unit time is V_0. Find the volume of the liquid flowing per unit time through the section upto radius $\frac{R}{2}$ from the axis of the pipe.



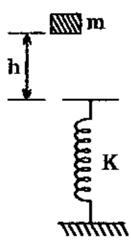
127

A solid cone of semi-vertical angle α and height h is suspended from its vertex as shown in the figure. The acceleration due to gravity is g. Find the time period of small oscillation if the cone is slightly displaced from its vertical position.



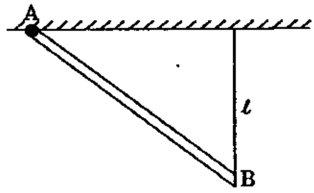
128

A block of mass m is dropped from a height h on the free end of a vertical spring of force constant K as shown in the figure. The block first compresses the spring and it leaves the spring when spring stretches to its natural length. Then the block rises to its original height. Find the time period of oscillations. The acceleration due to gravity is g.



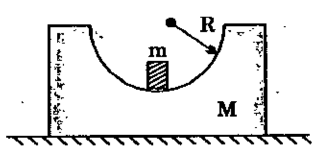
129

A thin uniform rod is hinged at one end A and the other end B is supported by a vertical massless inextensible thread of length l as shown in the figure. Now the end B is slightly displaced. Find the time period of small angular oscillations of this system? The acceleration due to gravity is g.



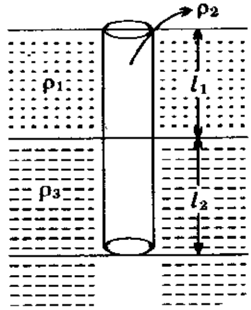
130

A symmetrical block of mass M having a notch of hemispherical shape of radius R is placed on a horizontal surface. A small particle of mass m is placed inside the hemispherical notch and can slide without friction. Find the time period of small oscillations of the particle and the block. The acceleration due to gravity is g.



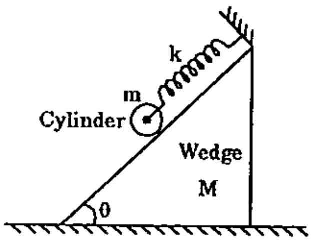
131

A cylindrical rod of length $(l_1 + l_2)$ and made up of a material of density ρ_2 is in equilibrium with the upper end at the surface of the upper liquid. The densities of upper and lower liquids are ρ_1 and ρ_3 respectively. The densities ρ_1, ρ_2 and ρ_3 are in geometric progression. The acceleration due to gravity is g. Find the time period of small vertical oscillations of the rod.



132

Consider an arrangement shown in the figure. There is no friction between wedge and the horizontal surface but there is sufficient friction between cylinder and the wedge so that the cylinder can roll without slipping. Find the time period of small oscillations if the cylinder is slightly displaced downward from equilibrium position.

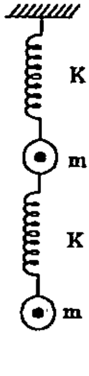


133

Two SHMS $x = a \cos 3\omega t$ and $y = b \sin \omega t$ along x and y-axis respectively are supposed on a particle. Find the equation of path traced by the particle in x–y plane and also plot the graph?

134

Consider a system shown in the figure. Find the angular frequencies of possible oscillations along the vertical direction.



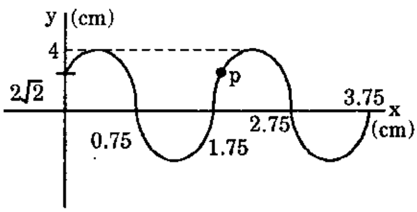
WAVES

135

A simple harmonic transverse wave of amplitude 8 mm travels in the direction of positive x-axis. At $t = 0.1$ second, for a particle at a distance of 10 cm from the origin the displacement is 6 mm, and for a particle at a distance of 25 cm from the origin the displacement is 4 mm. Find the equation of the wave. (Given $\sin 49° \sim 0.75$)

136

A snap shot, of a vibrating string at $t = 0$, is shown in the figure. A particle at point P is observed moving up with velocity $\left(20\sqrt{2}\,\pi\right)$ cm/s and the tangent at the point P makes an angle $\theta = \tan^{-1}(2\sqrt{2}\pi)$ with the positive x-axis. Find the equation of the wave.



137

The equation of a transverse wave is given by

$\Psi = (10^{-3}) \sin \pi (10t - \sqrt{3}x - y)$

where x, y and Ψ are expressed in metre and t in second.
Find *(a)* the direction of propagation of the wave
(b) wavelength and
(c) phase difference between two points A ($\sqrt{3}\,m$, $1m$) and B ($2\sqrt{3}\,m$, $2m$).

138

The equation of a plane longitudinal wave travelling through a homogeneous gaseous medium of density 1 kg/m³ is given by
$\Psi = (10^{-9})(1000t - 5x)$
where x and Ψ are in metre and t in second. Find *(a)* intensity of the wave and
(b) the amplitude of pressure variation.

139

The shape of pulse of a plane progressive wave is given by an equation $y = \dfrac{b^3}{b^2 + (x - ct)^2}$

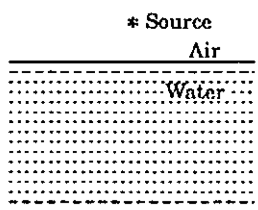
Find :

(a) the maximum value of particle velocity v_p at $t = 0$ and also plot the graph showing variation of v_p with respect to x.

(b) the equation of this wave in a reference frame moving in positive x-direction with a constant velocity v.

140

A source of sound is placed in air at some height above the air and water interface. The temperature of air and water is 20° C. The mean molar mass of air is 28.8 and $\dfrac{c_p}{c_v} = 1.4$. The bulk modulus of water is 2.088×10^9 Pa and density of water is 1 gm/c.c. Consider only refraction of sound at the interface. Find the percentage of power transmitted from air to water?

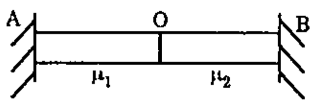


141

A string of mass 0.8 kg/m is stretched to a tension 500 N. Find the mean power required to maintain a travelling wave of amplitude of 10 mm and wavelength 0.5 m.

142

Two strings of mass per unit length $\mu_1 = 0.2$ kg/m and $\mu_2 = 0.8$ kg/m are joined together and stretched between two supports as shown in the figure. A sinosudal wave of amplitude A is incident from the side AO and transmission and reflection takes place at the interface O. Find



(a) the amplitude of the transmitted wave.

(b) the percentage of incident power transmitted to the second string.

143

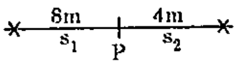
A stretched string of length l clamped at both the ends has a tension T. Now the string is pulled aside a distance h ($<< l$) at its centre and released, as shown in the figure. Assume that the tension remains unchanged by small transverse displacement. Find the energy of subsequent oscillations.

144

Two sources separated by 12 m vibrate according to the equations

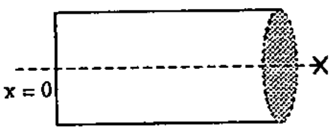
$y_1 = (3 \sin 4\pi t)$ cm and $y_2 = (4 \sin 4\pi t)$ cm.

They send out the waves with speed 32 m/s. What is equation of motion of a particle P, 8 m from first source and 4m from second source as shown in the figure ?



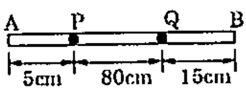
145

Consider a closed pipe of length l located along x-axis with closed end at the origin as shown in the figure. The density of air is ρ and the velocity of sound is v. The pipe resonates in n^{th} overtone with displacement amplitude A. Find the expression showing pressure variation at point $(x, 0)$.



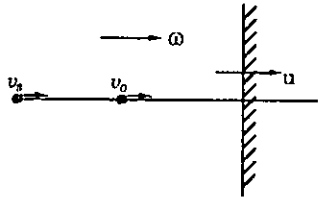
146

A metal rod AB of length 1m is clamped at two points P and Q as shown in the figure. Find the maximum possible wavelength for the natural longitudinal oscillations of the rod.



147

Consider a situation shown in the figure. Source and observer both are moving towards the wall with velocities v_s and v_0 respectively. The wall is moving away with velocity u. The wind is blowing with a velocity ω towards the wall. The frequency of the sound emitted by the source is n_0 and the velocity of sound with respect to air is v. Find the frequency and wavelength of the reflected waves received by the observer.

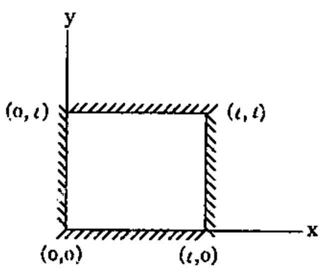


148

At a particular instant a source of sound of frequency 1000 Hz is at $(2m, 1m)$ and an observer is at $(5m, 5m)$. The velocity of source and observer are $30\left(\hat{i}+\hat{j}\right)$ m/s and $15\left(\hat{i}+\sqrt{3}\hat{j}\right)$ m/s respectively at this instant. The velocity of sound in air is 330 m/s. Find the frequency and wavelength of the sound received by the observer.

149

The edges of a square plate of side length l are clamped and transverse waves are set up in it. The coordinates of the corners of the square plate are shown in the figure. Take z-axis along the direction of displacement. Take $t = 0$ at the instant the centre of the plate passes through its mean position. The maximum amplitude of oscillation is A and the plate is vibrating in its fundamental mode. Write the equation describing the standing wave in the plate ?



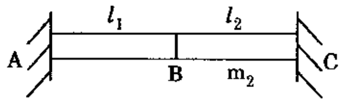
150

A uniform rectangular membrane of length 20 cm and bredth 10 cm is clamped at its edges. The mass of the membrane is 20 gm. The surface tension in the membrane is 5 kg/cm. Find the fundamental frequency of oscillation.

HEAT AND THERMODYNAMICS

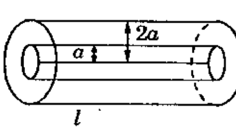
151

Two rods AB and BC of equal cross-sectional area are joined together and clamped between two fixed supports as shown in the figure. For the rod AB length is l_1, coefficient of thermal linear expansion is α_1, Young's modulus is Y_1 and density is ρ_1. For the rod BC, length is l_2, coefficient of thermal linear expansion is α_2, Young's modulus of elasticity is Y_2 and density is ρ_2. Now the temperature of the compound rod is increased by θ. Find the time taken by a transverse wave pulse to travel from end A to other end C of the compound rod. Assume that there is no significant change in the lengths of the individual rods due to heating.



152

Consider a composite rod shown in the figure. The radius of inner rod is a and outer most radius is $2a$. The Young's modulus and coefficient of thermal linear expansion for inner rod are Y_1 and α_1 and that for outer rod are Y_2 and α_2 respectively. The length of rod is l. Find the final length of the rod, if the temperature of the composite rod is increased by θ.

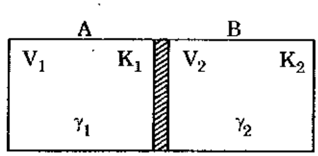


153

A long horizontal glass capillary tube open at both ends contains a mercury thread 1m long at 20° C. A scale is etched on the glass tube. The scale is correct at 20° C. Find the length of mercury thread as shown by this scale at 120° C (It is given $\alpha_{glass} = 9 \times 10^{-6}$/K and $\gamma_{mercury} = 18.5 \times 10^{-5}$/K)

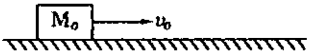
154

A tube of uniform cross-section is divided into two parts by a movable conducting separator. The two parts of the tube are filled with two different liquids A and B. For the liquid A, volume, coefficient of thermal cubical expansion and bulk modulus are V_1, γ_1 and K_1 respectively and those for liquid B are V_2, γ_2 and K_2 respectively. Find the final volumes of the liquids A and B if the temperature of the system is increased by θ. Neglect the expansion of tube.



155

An ice cube of mass M_0 is given a velocity v_0 on a rough horizontal surface with coefficient of friction μ. The block is at its melting point and latent heat of fusion of ice is L. The block receives heat only due to the friction forces and all work is converted into heat. Find the mass of the remaining ice block after time t.

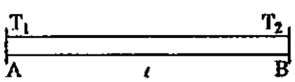


156

A container of mass 0.5 kg contains 1 kg ice at – 23° C. The specific heat of container varies with temperature T according to the relation S = A + BT, where A = 100 cal/kg-K and B = 2×10^{-2} cal/kg–K^2. The specific heat of ice is 0.5 cal/gm–K and the latent heat of fusion of ice is 80 cal/gm. Find the mass of steam at 100° C required to convert the ice in the container into water at 27° C. The latent heat of vaporisation of water is 540 cal/gm and specific heat of water is 1 cal/gm–K.

157

A rod AB of length l with thermally insulated lateral surface has uniform cross-section. The coefficient of thermal conductivity of the material of the rod varies with temperature as $K \propto \frac{1}{T}$. The ends of the rod are maintained at temperatures T_1 and T_2. Find the temperature T at a distance x from the end whose temperature is T_1.

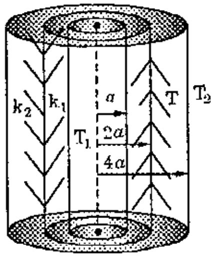


158

Two blocks are connected by a metal rod of length l, area of cross-section A and Thermal conductivity K. For the left block mass is m_1 and specific heat is S_1 and those for right block are m_2 and S_2 respectively. At $t = 0$ the temperature of left block is T_1 and the temperature of right block is T_2. Find the temperatures of the blocks after time t ?

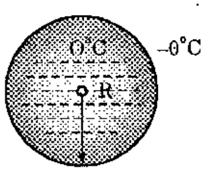
159

Consider a composite cylinder shown in the figure. The conductivity of the material the cylinder from radius a to $2a$ is K_1 and that for the cylinder from radius $2a$ to $4a$ is K_2. The temperature of inner most wall is maintained at temperature T_1 and the outermost wall is maintained at temperature T_2. Find the temperature T of the intermediate cylindrical wall of radius $2a$.



160

A thin spherical shell of radius R made up of highly conducting material contains water at 0° C. The temperature of the atmosphere is $-\theta°$ C. The thermal conductivity of ice is K and the latent heat of fusion of ice is L. The density of water is ρ. Find the time required to freeze the whole mass of water. Neglect the expansion during freezing.

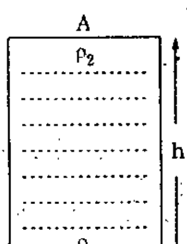


161

A body cools according to Newton's Law of cooling. The temperature of the body cools from θ_1 to θ_2 in time t. The temperature of surrounding is θ_0. Find the time taken for the temperature of the body to decrease from θ_2 to θ_3.

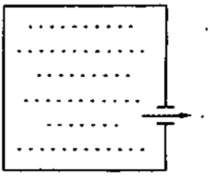
162

A cylindrical container of cross-sectional area A and height h contains a gas at a constant temperature. The density ρ of the gas varies due to acceleration due to gravity. The density of the gas at the bottom is ρ_1 and at the top becomes ρ_2. Find the mass of the gas in the container.



163

A vessel of volume V_0 contains an ideal gas at a pressure P_0. Gas is continuously pumped out of this vessel at a constant rate $\frac{dV}{dt} = r$, keeping the temperature constant. The pressure of the gas taken out equals the pressure inside the vessel. Find the pressure of the gas inside the vessel as a function of time.

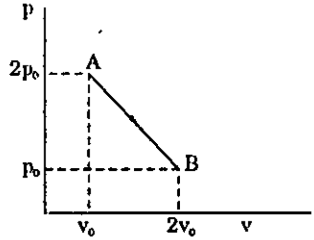


164

Find the speed of sound in a mixture of gases containing 3 moles of He, 2 moles of H_2 and 1 mole of CO_2 at temperature 27° C.

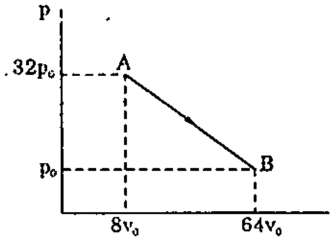
165

An ideal gas is taken along the process AB shown in the P-V diagram. Find the volume of the gas where the temperature becomes maximum.



166

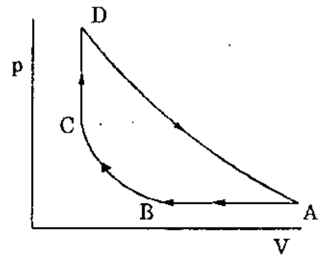
An ideal monoatomic gas is taken along the process AB shown in the P-V diagram. Find the volume of the gas where the heat flow reverses its direction.



167

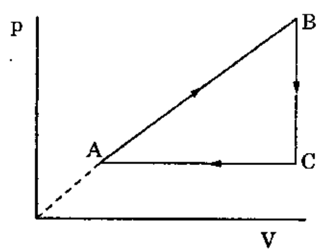
One mole of an ideal mono atomic gas is taken around a cyclic process shown in the figure. The processes AB, BC, CD and DA are isochoric, adiabatic, isobaric and isothermal respectively. It is given

$T_A = 4T_B$ and $V_A = \left(8\sqrt{2}\right) V_D$. Find the efficiency of the cyclic process.



168

One mole of a mono atomic ideal gas is taken around a cyclic process shown in the figure. It is given $T_B = 4T_A$. Find the efficiency of the cyclic process.



169

An amount Q of heat is aded to a mono atomic ideal gas in a process in which the gas performs a work $\frac{Q}{2}$ on its surrounding. Find

(a) equation of the process, and
(b) molar heat capacity for the process.

170

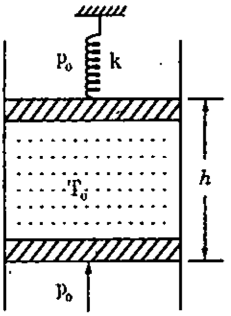
In an adiabatic container the molar specific heat at a constant volume of an ideal gas varies as

$C_V = (a + bT)$, where a and b are constant and T is temperature in Kelvin.

Find the equation of the process . The universal gas constant is R.

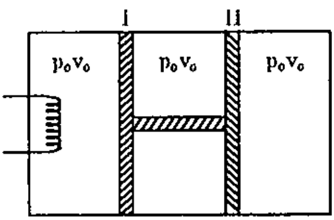
171

A cylindrical container of cross-sectional area A contains an ideal gas at atmospheric pressure P_0 and temperature T_0. The pistons used are massless. Initially the spring is unstretched and the separation between the pistons is h as shown in the figure. The force constant of the spring is k. Find the work done in displacing the lower piston upward by a distance $h/2$, keeping the temperature of the gas constant.



172

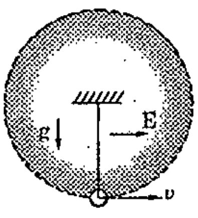
An insulated container divided into three parts contains an ideal gas ($\gamma = 1.5$) at the same pressure P_0 and temperature. Initial volume of each part is V_0. The pistons I and II are connected by a non-conducting rigid rod and can move without friction in side the cylinder. Piston I is conducting and the piston II is non-conducting. The leftmost part is slowly given heat through a heater such that the final volume of the rightmost part becomes $\frac{4}{9}V_0$. Find the amount of heat supplied by the heater ?



ELECTRO MAGNETICS

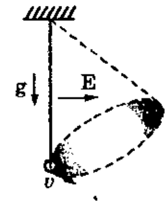
173

A simple pendulum with a bob of mass m, charge q and string length l is given a horizontal velocity v at its bottom most point. A horizontal electric field of strength E exists in space. The acceleration due to gravity is g. Find the minimum value of velocity v so that the particle may complete the vertical circle.



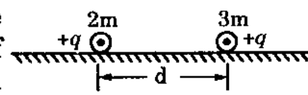
174

A positively charged particle of charge q and mass m is suspended from a point by a string of length l. In the space a uniform horizontal electric field E exists. The particle is drawn aside so that the string becomes vertical and then it is projected horizontally with velocity v such that the particle starts to move along a circle with same constant speed v. Find the speed v.



175

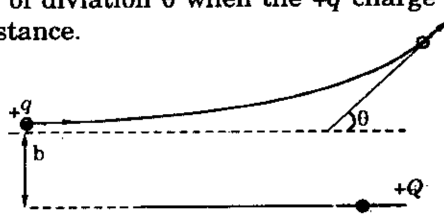
Two charged particles of masses $2m$ and $3m$ and having equal charges $+q$ are released from rest at distance of separation d on a frictionless horizontal surface as shown in the figure. Find the time after which the distance between the charges will become $3d$.



176

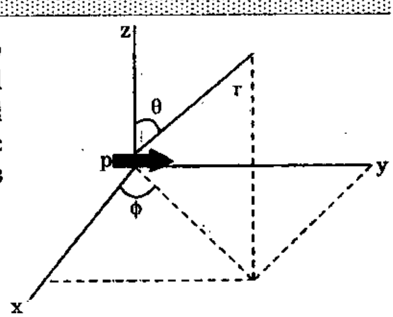
A charged particle of charge $+q$ and mass m is projected towards another fixed charge + Q from a very large distance with initial speed v_0 with an impact parameter b as shown in the figure. Find
(a) distance of closest approach of the two charges, and

(b) the angle of diviation θ when the $+q$ charge finally goes to a very large distance.



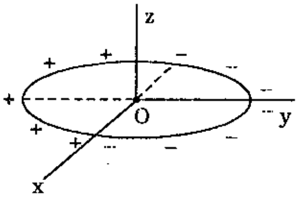
177

A small dipole of dipolemoment p is placed at the origin oriented along the y-axis. Find the potential V and the magnitude of electric field E at the point P (r, θ, ϕ) as shown in the figure.



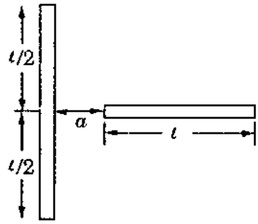
178

A thin non-conducting ring of radius a is placed in x-y plane with its center at the origin O as shown in the figure. A charge +q is distributed uniformly over one half of the ring and a charge $-q$ is distributed uniformly over other half of the ring. Find the electric field intersity vector $\vec{E}$ on the axis of the ring as a function of the distance Z from its centre.



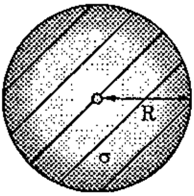
179

Two thin rods each of length l and uniform linear change density λ are arranged as shown in the figure. Find the force of interaction F between the rods.



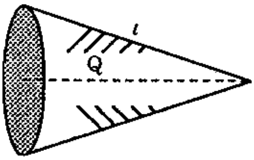
180

A thin disc of radius R carries a uniform surface change density σ. Find the potential energy of the disc.



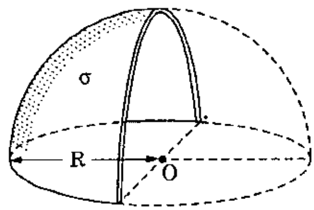
181

The surface of a cone of slant length l as shown in the figure is charged uniformly with the charge Q. Find the potential at the vertex of the cone.



182

The half part of a hemisphere of radius R, as shown in the figure is uniformly charged with a surface charge density σ. Find the magnitude of electric field E at the centre O.



183

The electric field in a region is radially outward and is given by $E = \alpha r$, where α is a constant and r is the distance from origin. Find the charge contained in a sphere of radius R centred at the origin.

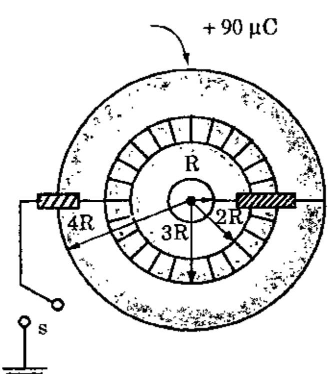
184

The electric field in a region is given by $\vec{E} = \dfrac{a\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{\left(x^2 + y^2 + z^2\right)^{3/2}}$.

Find the flux through the surface of a cube of side length l centred at the origin.

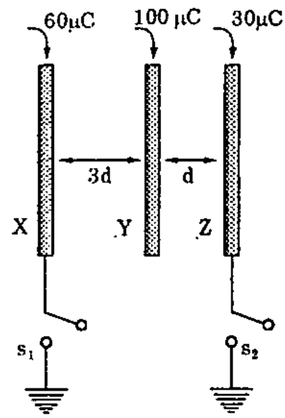
185

A solid conducting sphere of radius R is surrounded by a hollow conducting sphere of inner radius 2R and outer radius 3R which in turn enclosed by a thin conducting spherical shell of radius 4R as shown in the figure. The innermost sphere and outermost spherical shell are connected together by a conducting thin wire. The outer shell is given a charge equal to 90μC. If the middle sphere is grounded by closing a switch S, then find the charge on the outer surface of the outermost shell.



186

Three identical parallel conducting plates X, Y and Z are placed as shown in the figure. Switches S_1 and S_2 are open and canconnect X and Z to earth when closed. The charges 60 μc, 100 μc and 30 μc are given to the plates X, Y and Z respectively.



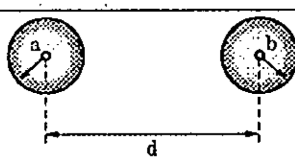
(a) If S_1 is closed with S_2 open, how much charge will flow from plate X to ground ?

(b) If S_2 is closed with S_1 open, how much charge will flow from plate Z to ground ?

(c) If S_1 and S_2 are closed together, how much charges will flow from plates X and Z to the ground through S_1 and S_2 respectively ?

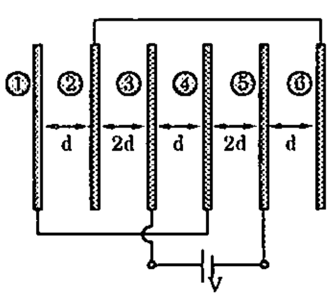
187

Two conducting spheres of radii a and b are separated by a large distance d. Find the capacitance of the system under the condition $d >> a$ and $d >> b$.



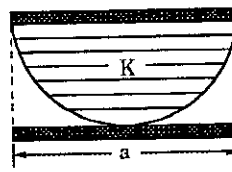
188

Consider an arrangement of six large parallel plates as shown in the figure. The emf of the battery is V. Find the electric field between the plates numbered (1) and (2).



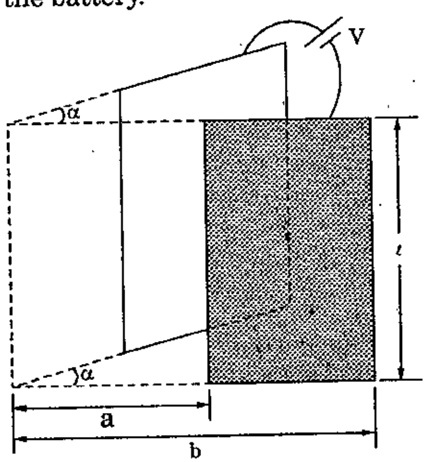
189

A parallel plate capacitor consists of two square plates of side length a. There is dielectric slab in the shape of a cylinder of parabolic cross-section between the plates, as shown in figure. The dielectric constant of the material is K. The length of latus rectum of the parabola is equal to the side length of the plate. Find the capacitance.



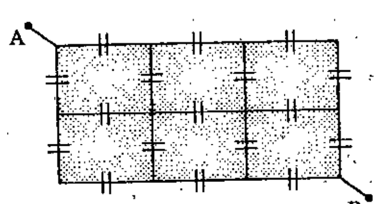
190

Two identical rectangular conducting large plates of length l and breadth $(b-a)$ are arrangeed as shown in the figure. The inclination between two plates is α. The emf of battery connected across the plates is V. Find the charge on the plate connected to the positive terminal of the battery.



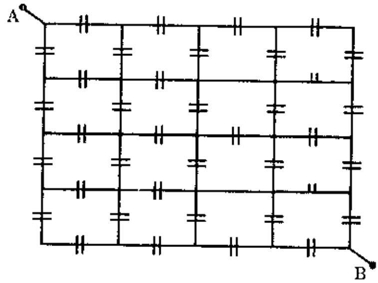
191

Find the equivalent capacitance of the circuit shown in the figure between points A and B. The capacitance of each capacitor is 121 μf.



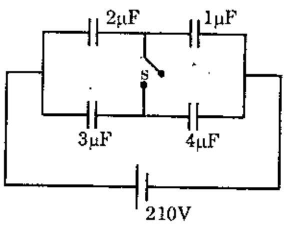
192

Find the equivalent capacitance of the circuit shown in the figure between points A and B. The capacitance of each capacitor is 47 μf.



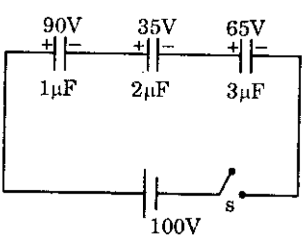
193

Consider a network shown in the figure. Initially the switch S was open. If the switch S is closed now, then find *(a)* the amount of charge flowing through the switch. *(b)* the amount of heat loss.



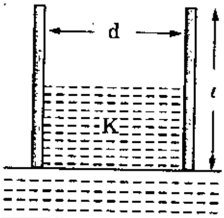
194

Initially charged three capacitors and a battery of 100 V are arranged as shown in the figure. Now the switch S is closed. Find *(a)* the final charges on the capacitors and *(b)* the amount of heat loss.



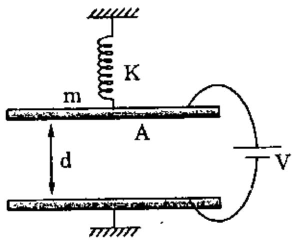
195

A parallel plate capacitor made up of two rectangular plates of length l and separated by a distance d is charged to a potential difference V and then disconnected from the battery. Now the plates of the capacitor are held vertically and the lower ends of the plates are brought in contact with the surface of a dielectric liquid of density ρ and dielectric constant K. It is found that the level of liquid rises inside the plates as shown in the figure. The acceleration due to gravity is g. Find the height of liquid level inside the plates of the capacitor.



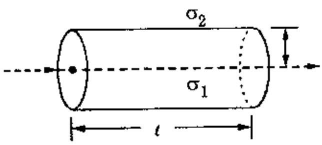
196

A parallel plate capacitor with air as dielectric is arranged horizontally. The lower plate is fixed and the upper plate is connected with a spring of force constant K as shown in the figure. The area of each plate is A and the mass of the upper plate is m. The capacitor is connected with an electric source of voltage V. In steady position, the distance betwen the plates is d. Now the upper plate is slightly displaced in the vertical direction. Find the angular frequency of small oscillations.



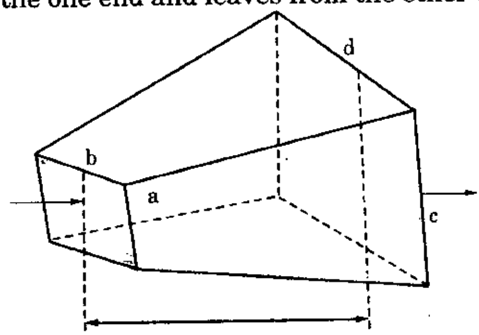
197

A long conductor of circular cross-section has radius r and length l as shown in the figure. The conductivity of the material near the axis is σ_1 and increases linearly with the distance from axis and becomes σ_2 near the surface. Find the resistance of the conductor if the current enters from the one end and leaves from the other end.



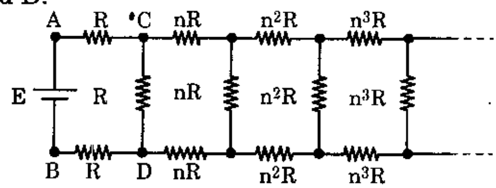
198

A long prismatic conductor or of varying rectangular cross-section and of length l is shown in the figure. The dimension of rectangular cross-section on the one end is $a \times b$ and on the other end it becomes $c \times d$. The side length of rectangular cross-section varies linearly with the distance from one end. The resistivity of the material of the conductor is ρ. Find the resistance of the conductor, if the current enters from the one end and leaves from the other end.



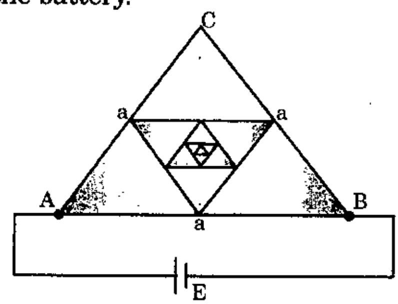
199

Consider an infinite ladder of net work shown in the figure. Find :
(*a*) the equivalent resistance between the terminals A and B, and
(*b*) the current through the resistance R connected between points C and D.



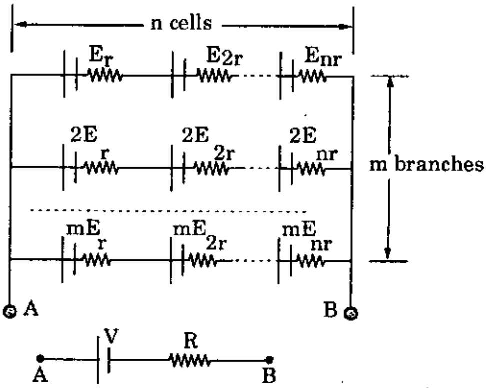
200

A frame made of thin homogeneous wire of resistance per unit length r is connected with a battery of emf E between the terminals A and B as shown in the figure. The frame ABC is in the form of equilateral triangle and the number of successively embeded equilateral triangles (with sides decreasing by half) tends to infinity. The side length of equilateral triangle ABC is a. Find the current flowing through the battery.



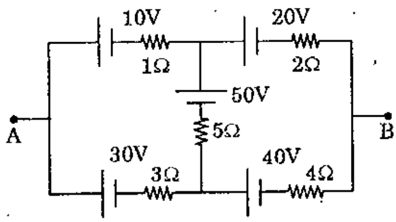
201

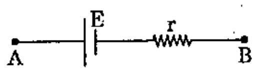
Consider an arrangement of cells shown in the figure. There are m-branches and in each branch there are n-cells. Suppose this combination of cells can be replaced by a single battery of emf V and internal resistance R, then find V and R.



202

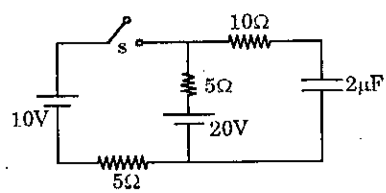
Consider an arrangement of cells shown in he figure. Suppose this combination of cells can be replaced by a single battery of emf E and internal resistance r, then find E and r.





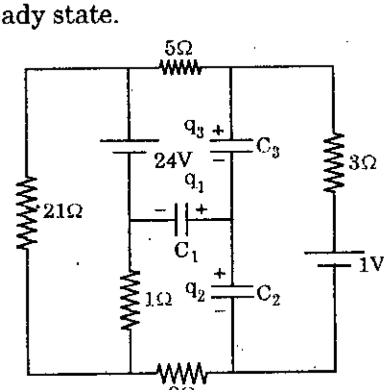
203

In the circuit shown in the figure, the switch S was initially open. Now the switch S is closed ($t = 0$). Find the current through the switch as a function of time.



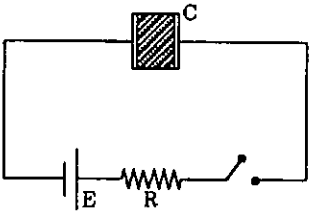
204

In the circuit shown in the figure $c_1 = 3\ \mu F$, $c_2 = 6\ \mu F$ and $c_3 = 9\ \mu F$. Find the charges q_1, q_2 and q_3 on the capacitors c_1, c_2 and c_3 respectively in steady state.



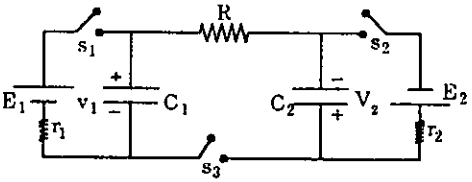
205

A capacitor of capacitance C filled with a resistive material of effective resistance r is connected with a battery as shown in the figure. Find the charge on the capacitor and current through the battery as a function of time. Assume that the switch is closed at $t = 0$.



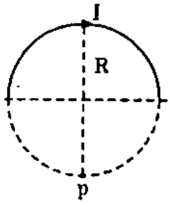
206

Initially switches S_1 and S_2 were closed and S_3 was open. Now the switches S_1 and S_2 are opened and S_3 is closed (at $t = 0$). Find the potential differences V_1 and V_2 across the capacitors c_1 and c_2 respectively as a function of time t.



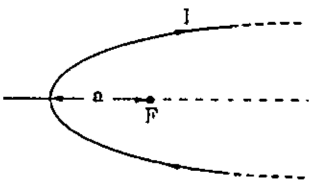
207

A thin wire bent into the form of a semicircle of radius r carries a current I. Find the magnetic field of induction at the point P as shown in the figure.



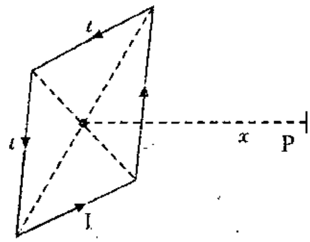
208

An infinitely long wire is bent into the form of a parabola of focal length a. The wire carries a steady current I. Find the magnetic field of induction at the focus F as shown in the figure.



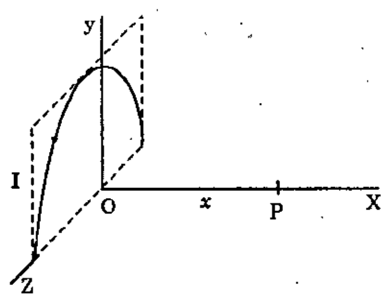
209

A square frame made of a thin wire carries a current I. The side length of the square is l. Find the magnetic field of induction at the point P on the axis at a distance x from the centre of the square.



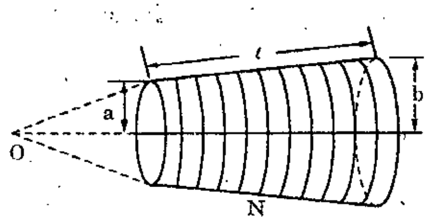
210

A wire carrying current I and in the form of a semicircle of radius a lies in the y-z plane with its centre at the origin O as shown in the figure. There is a point P on the x-axis at a distance x from O. Find the magnetic field induction vector $\vec{B}$ at the point P.



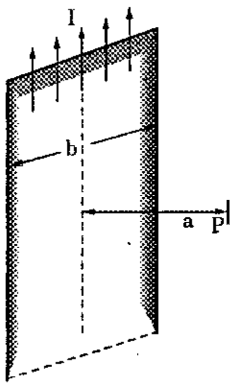
211

A frustum of cone having end radii a and b and slant length l is wound with N turns of wire carrying current I, as shown in the figure. Assume continuous distribution of wires over the surface. Find the magnetic field of induction B at the vertex O of the cone.



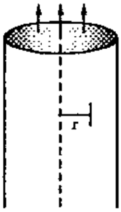
212

An infinitely long tape of width b carries a current I which is distributed uniformly over its width as shown in the figure. Find the magnetic field of induction B due to this tape at a point P which is on the bisector position of the tape. The distance of the point P is 'a' from the tape.



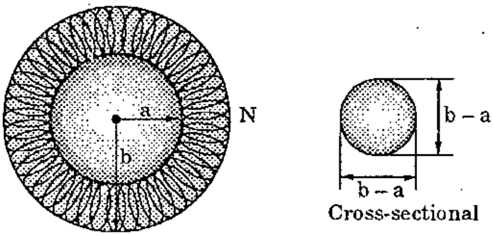
213

An infinitely long wire of uniform circular cross-section carries a current whose density varies as $j = br^{\alpha}$, where b and α are constants and r is the distance from axis of the wire. Find the magnetic field of induction as a function of distance r inside the wire.



214

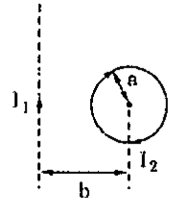
Consider a toroid of inner radius a and outer radius b as shown in the figure. The total number of turns of wire is N and the current is I. The cross-section of toroid is circular and the diameter of the circular cross-section is $(b - a)$. Find the magnetic flux through the cross-section of the toroid.



Cross-sectional

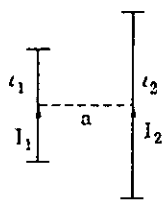
215

A circular loop of radius a and carrying a current I_2 is placed near an infinitely long straight wire carrying a current I_1 as shown in the figure. The straight wire and circular loop are coplanar. The distance of the centre of the loop from the wire is b. Find the force of interaction between the straight wire and the loop.



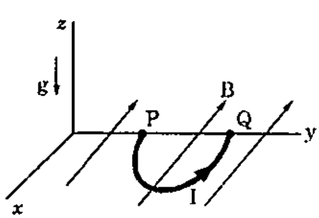
216

Two straight wires of length l_1 and l_2 are placed in a plane parallel to each other of a separation of distance a, as shown in the figure. The currents are I_1 and I_2 in the wires of lengths l_1 and l_2 respectively. The mid-points of the wires are at the same level. Find the force of interaction between the wires.



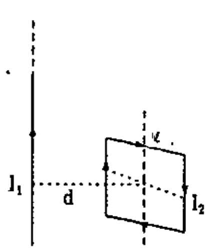
217

A semicircular wire of radius 'a' carries a current I and lies in a horizontal plane (x-y plane) with its ends P and Q hinged on the y-axis as shown in the figure. A uniform magnetic field $\vec{B} = -B\hat{i}$ exists in the space. The acceleration due to gravity is g. If the loop is released it is found to stay in horizontal position in equilibrium. Find the mass of the loop.



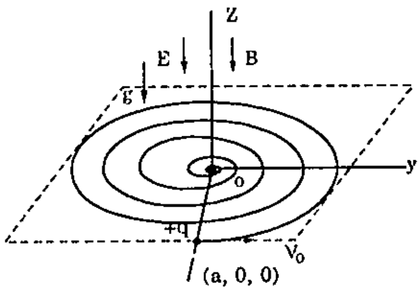
218

A square loop of side length l and carrying a current I_2 placed near an infinitely long straight wire, as shown in the figure. The straight wire carries a current I_1. The plane of the square loop is perpendicular, to the plane containing straight wire. Find the magnitude of force F and torque τ acting on the loop.



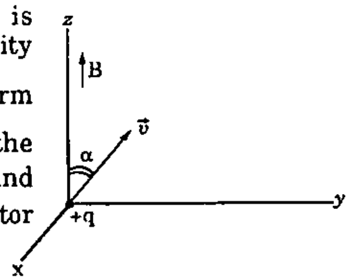
219

A particle of charge q and mass m is projected on a rough horizontal plane (x-y plane) from a point $(a, 0, 0,)$ with initial velocity $\vec{v} = v_0\hat{j}$. In the space uniform electric field $\vec{E} = -E\hat{k}$ and uniform magnetic field $\vec{B} = -B\hat{k}$ exists. The acceleration due to gravity is g. The coefficient of friction between the particle and the plane is μ. The particle moves on a spiral path and finally reaches to the origin O. Find the time taken by the particle to reach to the origin.



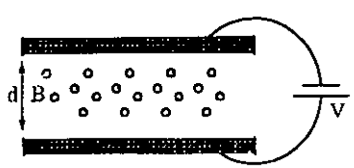
220

A particle of charge q and mass m is projected at $t = 0$ from origin with velocity $\vec{v} = v_0(\sin\alpha\,\hat{j} + \cos\alpha\,\hat{k})$. A uniform magnetic field $\vec{B} = B\hat{k}$ exists in the space. Neglect the effect of gravity. Find the velocity vector $\vec{v}$ and position vector $\vec{r}$ as a function of time.



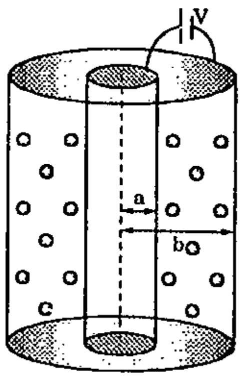
221

The seperation between the plates of a parallel plate capacitor is d. A battery of emf V is connected across the plates. A uniform magnetic field B exists in the region between the plates as shown in the figure. A positively charged particle of charge q and mass m starts from the positive plate of capacitor with zero initial velocity. Find the minimum value of V for which the particle can reach to the negative plate. Assume that the space is gravity free.



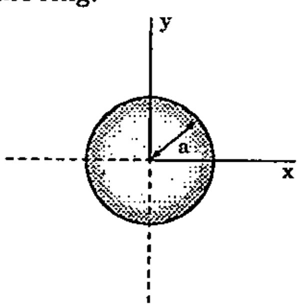
222

A cylindrical capacitor consists of two coaxial cylinders of radii a and b as shown in the figure. A battery of emf V is connected across the cylinders. A uniform magnetic field B exists in the region between the cylinders. A positively charged particle of charge q and mass m leaves the inner cylinder with zero initial velocity. Find the minimum value of V for which the particle can reach to the outer cylinder. Assume that the space is gravity free.



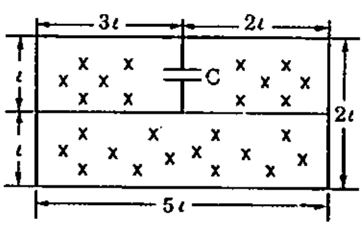
223

A circular ring of radius a and resistance R is placed (fixed) in x-y plane with its centre at origin as shown in the figure. A magnetic field $\vec{B} = (\alpha x)\hat{i} + (\beta y)\hat{j} + (\gamma t)\hat{k}$ is switched on at $t = 0$. Find the force acting on the ring.



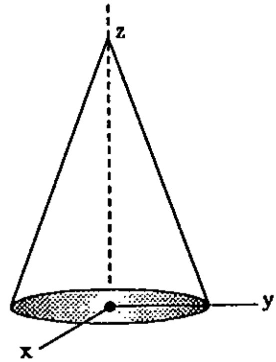
224

Consider a frame made up of uniform wire shown in the figure. A time varying magnetic field $B = b\,t$ is switched on. Find the charge on the capacitor in steady state.



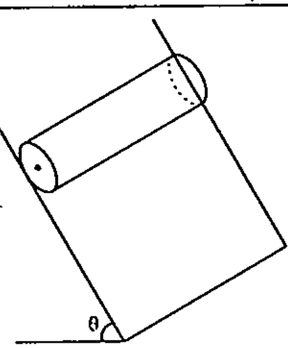
225

A right circular solid cone is made up of non-conducting material. The axis of the cone is along z-axis as shown in the figure. The mass of the cone is m. A charge q is distributed uniformly over its surface. A time varying magnetic field $\vec{B} = bt\,\hat{k}$ is switched as at $t = 0$. If the cone is free to rotate about its axis, find angular velocity $\vec{w}$ of the cone in terms of magnetic field vector $\vec{B}$.



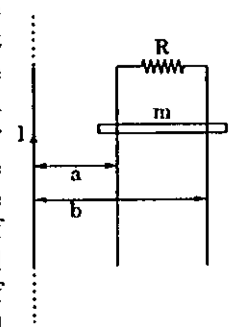
226

A long hollow cylinder of length l and radius R has total charge Q distributed uniformlly over its surface. The mass of the cylinder is m. The cylinder rolls down on an inclined plane of inclination θ with the horizontal as shown in the figure. The acceleration due to gravity is g. There is sufficient friction between the cylinder and the inclined plane to ensure pure rolling. Find the acceleration of the centre of mass of the cylinder.



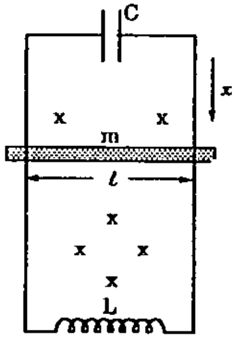
227

A long straight wire lying in a vertical plane carries a current I. At distances a and b from it there are two other wires, lying in the same vertical plane, which are inerconnected by a resistance R as shown in the figure. A conductor slides down without friction along the wires. The mass of the conductor is m. The acceleration due to gravity is g. Assume that the resistances of the wires, the conductor the sliding contacts, and the self-inductance of the frame is negligible. If the conductor is released from zero initial velocity at $t = 0$. Find the velocity of the conductor as a function of time.



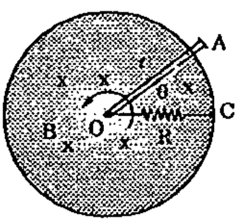
228

A conductor of mass m and length l can slide freely on a pair of smooth, vertical rails. A magnetic field B exists in the region in direction perpendicular to the plane of rails. The rails are connected at the top end by a capacitor of capacitance C and at the bottom end by an inductor of inductance L as shown in the figure. The acceleration due to gravity is g. The conductor is released from the top from rest at $t = 0$. Find the displacement x of the conductor from the top as a function of time.



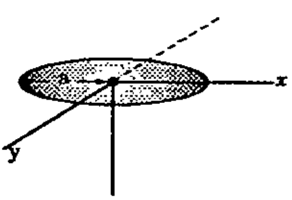
229

A conducting circular loop of radius l is placed in uniform magnetic field B as shown in the figure. The magnetic field is perpendicular to the plane of the circular loop. The centre O and a fixed point C on the loop are connected by a wire of resistance R. A metal rod OA of length l and mass m is pivoted at the centre O and the other end of the rod touches the loop at A. At $t = 0$ the rod is given an angular velocity ω_0 in the anticlock wise direction from the position OC. Find the angular displacement θ as a function of time.



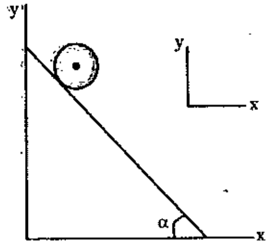
230

A conducting ring of mass m, radius a, inductance L and negligible resistance is located in the space as shown in the figure. The magnetic field $\vec{B} = -b(x\hat{i} + y\hat{j} + z\hat{k})$ exists in the space. The acceleration due to gravity is g. The z-axis is vertically down ward. The ring is released from rest at $t = 0$ from the position shown in the figure. Find the displacement z of the centre of the ring as a function of time.



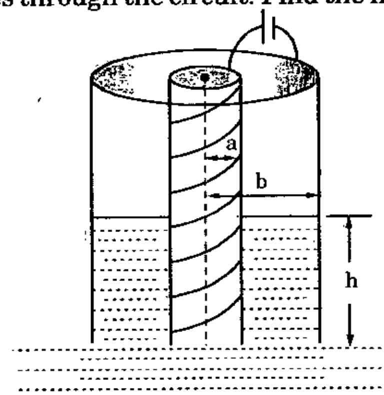
231

A small conducting ring of mass m, radius a and resistance R is released from rest along a smooth inclined plane of inination θ with the horizontal as shown in the figure. The plane of the ring is perpendicular to the magnetic field existing in the space. The magnetic field is given by $\vec{B} = -B_0(1 + \alpha x)\hat{k}$. If the ring is released from rest at $t = 0$, find the speed of the ring as a function of time.



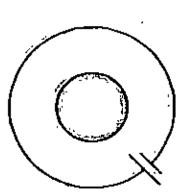
232

Two coaxial cylinders of radi a and b are made up of high conducting material. The inner cylinder is solid. This system is immersed in a magnetic liquid of permeability μ and mass density ρ. The constant current I passes through the circuit. Find the height h to which the liquid rises.



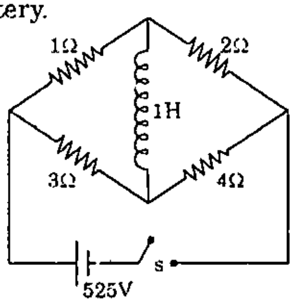
233

Two planar coils have mutual inductance M. By introducing a voltage source a current $I = I_0 \left(1 - e^{-\alpha t}\right)$ is maintained in the bigger coil. The inner coil has the resistance R and inductance L. If the switch in bigger coil is closed at $t = 0$, find the current in the smaller coil as a function of time.



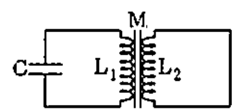
234

Consider a circuit shown in the figure. The switch S is closed at $t = 0$. Find the currents as a function of time through *(a)* the inductor and *(b)* the battery.



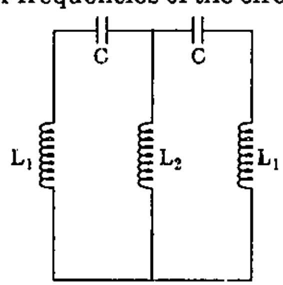
235

An oscillating circuit consists of a capacitor of capacitance C and an inductor of inductance L_1. The inductor is inductively coupled with a short circuited coil having inductance L_2 and negligible resistance. Their mutual inductance is M. Find the natural angular frequency of the given oscillating circuit.



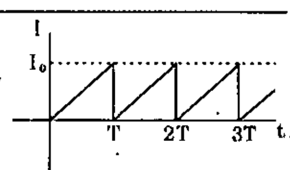
236

Consider an oscillating circuit shown in the figure. Find the possible natural angular frequencies of the circuit.



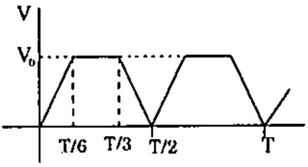
237

The time dependence of current is shown in the figure. The peak value of the current is I_0. Find *(a)* the average value of the current and *(b)* the root mean square value of the current.



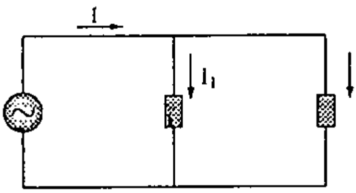
238

The time dependence of voltage is shown in the figure. The peak value of voltage is V_0. Find *(a)* the average value of the voltage and *(b)* the root mean square value of the voltage.



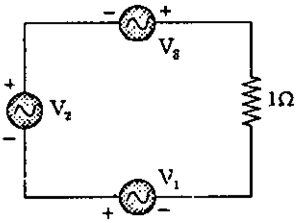
239

The current through the two branches of the network are $I_1 = \left(10\sqrt{2}\sin\omega t\right)$ Amp. and $I_2 = \left[20\sqrt{2}\sin(\omega t + 60°)\right]$ Find the current supplied by the voltage source ?



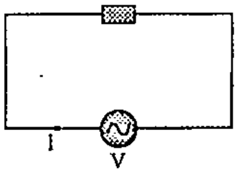
240

Three alternating voltage sources $v_1 = (3\sin\omega t)$ volt, $v_2 = 5\sin(\omega t + 30°)$ volt and $v_3 = 6\sin(\omega t - 120°)$ volt are connected across a resistance 1Ω as shown in the figure. Find the current through the resistor.



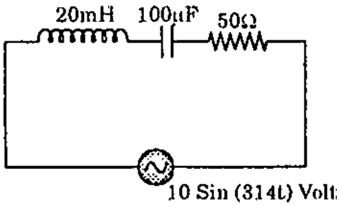
241

An alternating voltage source $V = (170)\sin(377t + 10°)$ volt supplies a current $I = (14.4)\sin(377t - 20°)$ Amp to the network connected to it. Find the average power supplied by the voltage source.



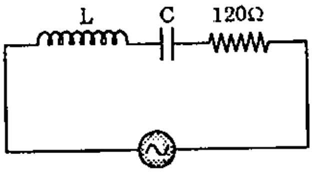
242

Consider a circuit shown in the figure. Find the energy dissipated in the circuit in 20 minutes.



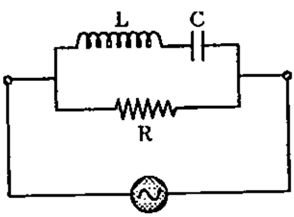
243

The resonance frequency of the circuit shown in the figure is (4×10^5) rad/s. At resonance the potential difference across resistor is 60V and across inductor is 40V. Find the values of inductance L and capacitance C.



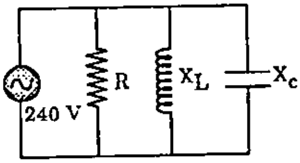
244

Consider a network shown in the figure. When an ac 250 V, 2250 rad/s is applied a current 1.25 A flows from the voltage source. When the angular frequency of the voltage source is changed to 4500 rad/s the current through the voltage source becomes maximum. Now if the ac voltage source is replaced by a dc voltage source of 250 V, then a current of 1 Amp. flows from the voltage source. Find the values of inductance L capacitance C and resistance R.



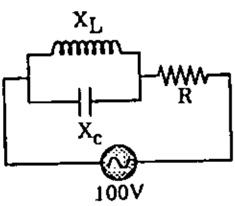
245

In the circuit shown in the figure $R = 8\Omega$, $X_L = 6\Omega$ and $X_C = 3\Omega$. Find the current through the voltage source.



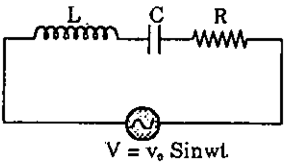
246

In the circuit shown in the figure, $X_L = 6\Omega$, $X_C = 3\Omega$ and $R = 8\Omega$. Find the current through the voltage source.



247

Consider a circuit shown in the figure. The frequency of the ac voltage source can be varied without changing the voltage amplitude. Find the angular frequencies at which the power in the circuit becomes half of the maximum power.

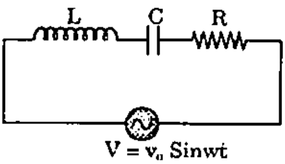


248

Consider a circuit shown in the figure. The frequency of the ac voltage source can be varied without changing the voltage amplitude. Find the angular frequency ω at which the voltage amplitude is maximum

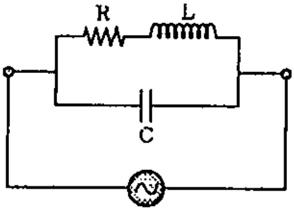
(a) across the capacitor

(b) across the inductor.



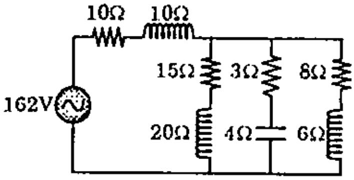
249

Consider the circuit shown in the figure. Find the resonance frequency for the circuit.



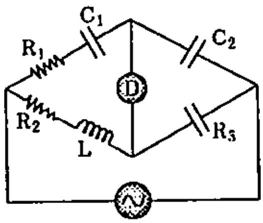
250

Consider a network shown in the figure. Find the current supplied by the ac voltage source.



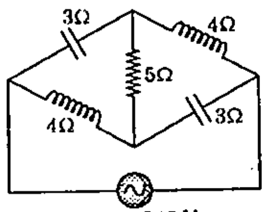
251

Consider a network shown in the figure. Find the condition that the detector D detects no current.



252

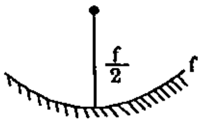
Consider a network shown in the figure. Find the current supplied by the ac voltage source connectd in the circuit.



OPTICS

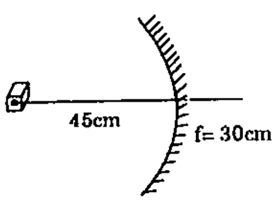
253

A particle is dropped along the axis from a height $f/2$ on a concave mirror of focal length f as shown in the figure. The acceleration due to gravity is g. Find the maximum speed of image.



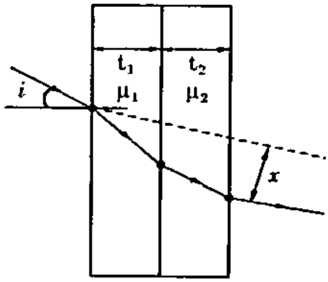
254

A cube of side length 1mm is placed on the axis of a concave mirror at a distance of 45 cm from the pole as shown in the figure. One edge of the cube is parallel to the axis. The focal length of the mirror is 30 cm. Find the volume of the image.



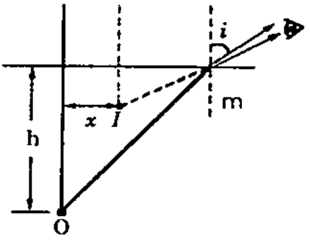
255

A ray of light is incident on a composite slab at an angle of incidence i as shown in the figure. Find the lateral shift x of the ray when it comes out from the otherside.



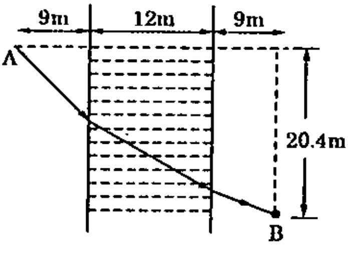
256

An object O in side water at a depth h from the surface is seen by an observer out side the water. The line of vision makes an angle i with the normal to the surface as shown in sthe figure. Find the horizontal distance x of the image I from the vertical line through object O. The refractive index of water is μ.



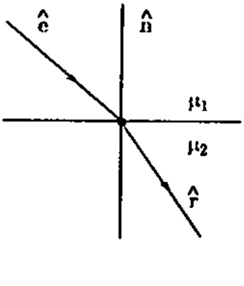
257

A person at A wants to reach to the point B after crossing a lake of still water of width 12 m as shown in the figure. The speed of walking of the person is 2.5 m/s and the speed of swimming is 1.875 m/s. Find the minimum time taken by the person to go from A to B.



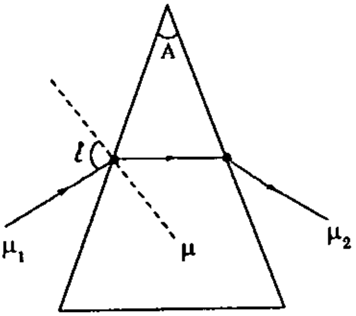
258

A ray of light is incident along the unit vector $\hat{e}$ on the interface of two mediums of refractive indices μ_1 and μ_2 as shown in the figure. The unit vector along the normal to the interface is $\hat{n}$. The ray is incident from the side of medium of refractive index μ_1 and gets transmitted to the medium of refractive index μ_2. Find the unit vector $\hat{r}$ along the refracted ray.



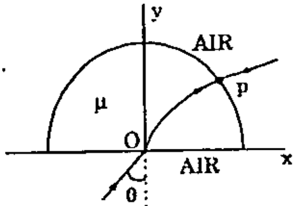
259

A prism of apex angle A is made up of a material of refractive index μ. The refractive indices of the mediums on the left and right sides are μ_1 and μ_2 respectively. A ray of light is incident from the side of medium of refractive index μ_1 at an angle i and comes out from the other side as shown in the figure. Find the angle of deviation.



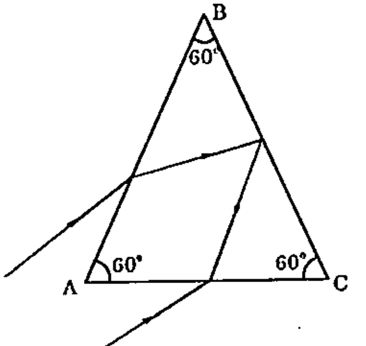
260

A hemisphere of radius $a/2$ and made up of a material of variable refractive index is placed with its base centre O at the origin as shown in the figure. The refrative index of the material of the hemisphere varies as $\mu = \dfrac{a}{a-x}$. A ray of light is incident at the point O at an angle θ with the normal in the xy plane and comes out through a point P on its curved surface. Find the coordinate of the point P if $\theta \to 0$.



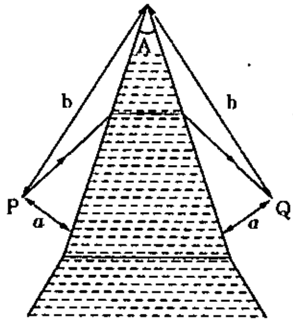
261

Consider an equilateral prism ABC as shown in the figure. A ray of light is incident on the face AB and gets transmitted into the prism. Then total internal reflection takes place at the face BC and the ray comes out of prism through the face AC. The total angle of deviation is 120°. Find the refractive index μ of the material of the prism.



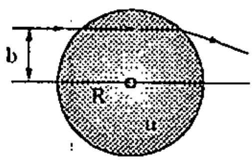
262

A person at point P wants to go to the point Q after crossing a triangular lake of still water, as shown in the figure. The speed of walking of the person is v and the speed of swimming is v/n. Find the minimum time required for the person to go from the point P to the point Q.



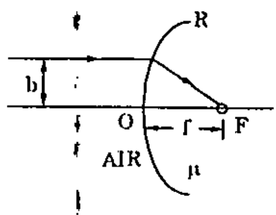
263

A ray of light is incident on the sphere of radius R and refractive index μ as shown in the figure. The incident ray is parallel to a horizontal diameter and the distance between the incident ray and the horizontal diameter is b. Find the angle of deviation δ suffered by the ray.



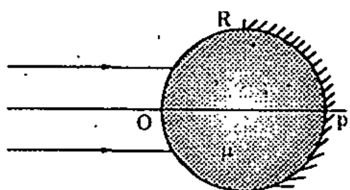
264

A ray of light is incident on the sphrerical surface of radius of curvature R as shown in the figure. The refractive index on the right side of spherical surface is μ. The medium on the left side of the spherical surface is air. The distance of the incident ray from the axis of the spherical surface is b. After refraction the ray intersects the axis at a point F. Find the distance f of the point F from the pole O.



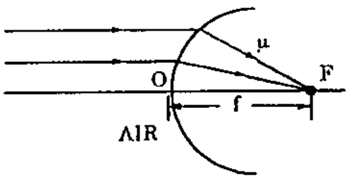
265

A sphere of radius R and refractive inde $\mu = 3/2$ is polished on its half portion as shown in the figure. A paraxial beam of light is incident along the axis (OP) of the hemisphere. Find the position of the final focus.



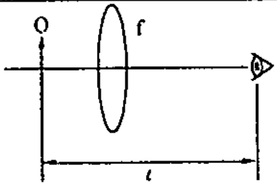
266

A curved surface separates a medium of refractive index μ from air. All the rays incident parallel to axis after refraction pass through a point 'F' at a distance f from the pole O as shown in the figure. Show that the curved surface is elliptic. Find the semi-major axis a and eccentricity e of the ellipse.



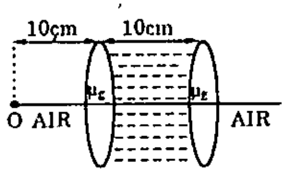
267

The distance between object and eye is fixed equal to l. A convex lens of focal length f is to be placed in between object and eye. The distance between object and lens is less than focal length f. Find the maximum possible angular magnification.



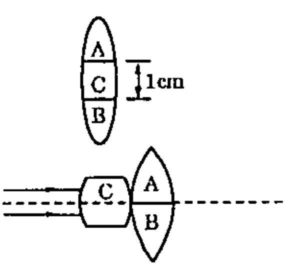
268

Consider an arrangement of two equibi convex lenses of focal length in air 10 cm. The refractie index of the glass of which the lenses are made is $\mu_g = 3/2$ and the refractive index of water filling the space between the two lenses is $\mu_w = 4/3$. A small object O is placed on the axis at a distance of 10 cm from the first lens in air as shown in the figure. The distance of separation between the two lenses is 10 cm. Find the position and magnification of the final image.



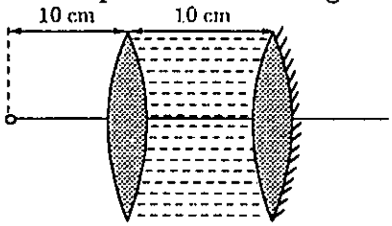
269

A thin convex lens of focal length 1m is cut into three parts A, B and C along the diameter. The thickness of the middle layer C is 1 cm. The middle layer is now removed and the two parts A and B are put together to form a composite lens. Then the part C is also placed infront of this composite lens symmetrically as shown in the figure. A paraxial beam of light is incident along the axis of the part C. Find the distance between the two images formed.



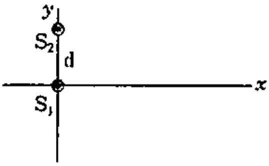
270

An equibiconvex lens of focal length 10 cm in AIR and made up of material of refractive index 3/2 is polished on one side. Another identical lens (not polished) is placed infront of the polished lens at a distance of 10 cm as shown in the figure. The space between the two lenses is filled with a liquid of refractive index 4/3. An object O is placed infront of the unpolished lens at a distance of 10 cm. Find the final position of the image.



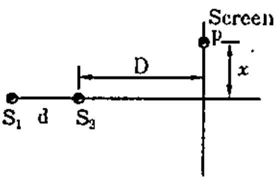
271

There are two coherent point sources S_1 and S_2 pleced on the y-axis as shown in the figure. The wavelength of the light emitted from the sources is λ. The distance between two sources is $d = 2\lambda$. Find the locations of all the minima on the positive x-axis.



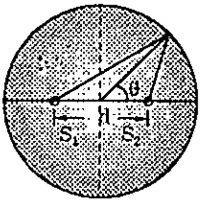
272

Two coherent point sources S_1 and S_2 are placed on a line perpendicular to the screen as shown in the figure. The wavelength of the light emitted by the sources is λ. The distance between two sources is $d = 2\lambda$. The distance of S_2 from the screen is D ($>> \lambda$). Find the minimum (non zero) distance x of a point P on the screen at which maxima is obtained.



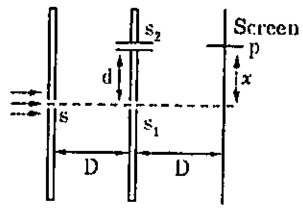
273

Two choerent point sources S_1 and S_2 emitting light of wavelength λ are placed symmetrically about the centre of a circle of large radius, as shown in the figure. The distance between the two sources is $d = 2\lambda$. Find the positions of the maxima on the circle in terms of the angle θ.



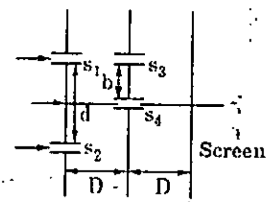
274

Consider an arrangement of slits shown in the figure. It is given that $d = \sqrt{\frac{\lambda D}{2}}$ and $D >> d$. Here λ is the wavelength of the light used. Find the minimum distance x of the point P on the screen where maximum is obtained.



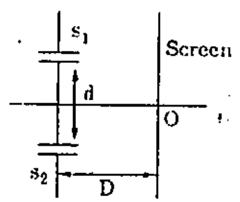
275

Consider an arrangement of slits and screen shown in the figure. The wavelength of the light used is λ. The distance between S_1 and S_2 is d (<< D). The distance between S_3 and S_4 is $b = \frac{\lambda D}{4d}$. Find the ratio of the maximum to minimum intensity observed on the screen.



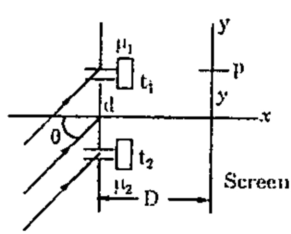
276

Consider an arrangement of slits and screen as shown in the figure. The wavelength of the light used is λ. The distance between slits S_1 and S_2 is $d = 2\lambda$. The distance of screen from the slit plane is D (>>d). Find the distance between two minima on the either side of the mid-point O of the screen.



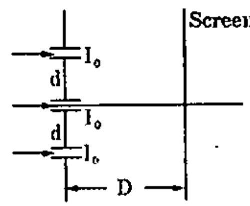
277

In young's double slit experiment two thin films are placed in front of two slits and the parallel beam of coherent light is incident at angle θ with the normal to the slit plane, as shown in the figure. It is given $\mu_1 = 1.7$, $\mu_2 = 1.5$, $t_1 = 3$mm, $t_2 = 2$mm, $\theta = 30°$, $l = 2\ mm$ and $D = 1m$. find the y-coordinate of the central maxima P.



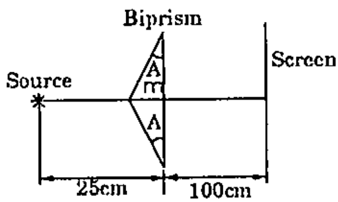
278

Consider an arrangement of three slits and screen as shown in the figure. The wavelength of the light used is λ. It is given $D >> d$. It is observed that there are two types of maxima formed on the screen called principal and secondary maxima. Show that the intensity of principal maxima is $9I_0$ and the intensity of secondary maxima is I_0. Also find the width of principal maxima β_1 and the width of secondary maxima β_2.



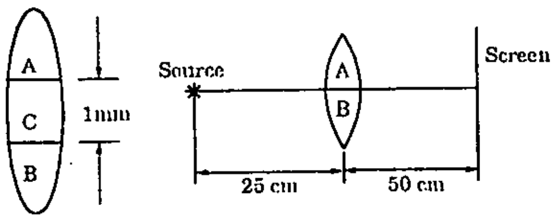
279

Consider an arrangement of a biprism and screen as shown in the figure. The refracting angle of the biprism is $A = 20'$ and the refractive index of its material is $\mu = 1.5$. The wavelength of the monochromatric light emitted from the source is 6000 Å. Find the fringe width of the interference pattern obtained on the screen.



280

A convex lens of focal length 50 cm is cut along the diameter into two identical halves A and B and in the process a layer C of the lens of thickness 1 mm is lost. Then the two halves A and B are put together to form a composite lens. Now infront of this composite lens a source of light emitting wavelength $\lambda = 6000$ Å is placed at a distance of 25 cm as shown in the figure. Behind the lens there is a screen at a distance 50 cm from it. Find the fringe width of the interference pattern obtained on the screen.



281

A parallel beam of light of wavelength 6000 Å is incident on a thin film of infractive index 1.33. The angle of incidence is 60°. Find the minimum thickness of the film for maximum reflection.

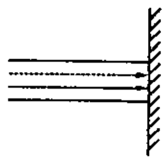
282

A parallel beam of light of wavelength 5500 Å is incident normally on the surface of a wedge made up of glass of refractive index $\mu = 1.5$. The angle between wedge faces is 3′. Find the distance between the two neighbouring maxima on the surface of the wedge.

MODERN PHYSICS

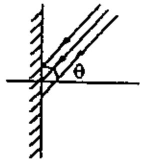
283

A parallel beam of electromagnetic radiation of wavelength 200 nm is incident on a perfectly reflecting surface. The power incident on 1 mm² area is (5×10^{-3}) watts. Find the pressure exerted by the radiation.



284

A plane electromagnetic wave of intensity 5×10^{-3} W/m² is incident on a plane surface with reflection coefficient 50%. The angle of incidence is $\theta = 45°$. Find the normal pressure exerted by the electromagnetic wave on that surface.

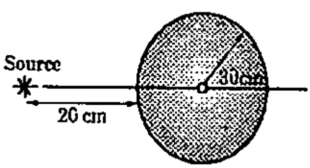


285

A metallic surface is illuminated alternatively with electromagnetic radiations of wavelengths 3000 Å and 6000 Å. It is observed that the maximum speeds of the photo electrons under these illuminations are in the ratio 3 : 1. Calculate the work function of the metal.

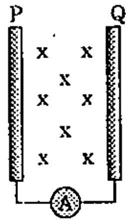
286

A source of power 0.1W and emitting light of wavelength 5000 Å is placed at a distance 20 cm from a sphere of radius 30 cm as shown in the figure. The work function of the sphere is 1.5 eV. The efficiency of photoelectric emission of the sphere is 10^{-6}. Find the time from the beginning of the experiment after which the photoelectric current stops.



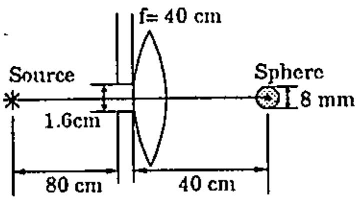
287

Two metallic plates P and Q are separated by a distance of 0.1 M. These are connected through an ideal ammeter as shown in the figure. A magnetic field B exists parallel to the plates. A light of wavelengths between 4000 Å and 6000 Å fall on the plate Q whose work function is 2.39 eV. Find the minimum value of magnetic field B for which the current registered by the ammeter is zero.



288

Consider an arrangement shown in the figure. The width of the slit is 1.6 cm. The convex lens placed in contact with the slit transmits 80% of the light incident on it. The radius of the sphere is 8 mm and the efficiency of photoelectric emission of the sphere is 10^{-4}. The power of the source is 3.2W and the energy of the photons of the radiation emitted by the squrce is 5 eV. Find the photoelectric current from the sphere.



289

An electron of kinetic energy 100 eV collides with a stationary helium ion He^+ in its ground state and excites it to a higher level. After collision He^+ ion emits two photons in succession with wavelengths 1085 Å and 304 Å. Find the kinetic energy of electron after collision. Neglect the recoil speed of He^+ ion.

290

A moving hydrogen atom makes a head on inelastic collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision both atoms move together. What is the minimum velocity of the moving hydrogen atom if one of the atom is to be given the minimum excitation energy after collision.

291

A hydrogen like ion in first excited state absorbs 10.2 eV photon and gets excited to higher energy level. Now the excited ion can emit six different photons, some of higher and same of lower energies than the absorbed photon. Find the quantum number n of the excited state and also find the atomic number Z of the ion.

292

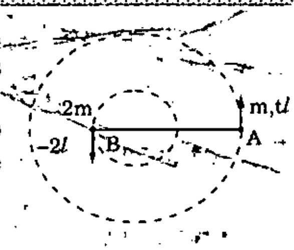
Two hydrogen like atoms A and B are of different masses and each atom contains equal number of protons and neutons. The energy difference between first Balmer lines emitted by A and B is 5.667 *e*V. When the atoms A and B moving with same velocity strike a heavy target they rebound back with the same velocity. In this process the atom B imparts twice momentum to the target that A imparts. Identify atoms A and B.

293

Suppose the potential energy between electron and proton at a distance r is given by $-\dfrac{e^2}{12\pi\varepsilon_0 r^3}$. Use Bohr's theory to obtain energy levels of such a hypothetical atom.

294

Two charged particles A and B move on circular paths about their centre of mass under their mutual electrostatic attraction. The particle A has mass m (electronic mass) and charge $+e$ (electronic charge). The particle B has mass $2m$ and charge $-2e$. Using Bohr's theory, obtain the energy level.



295

The wavelengths of K_α and K_β X-rays of a material are 21.3 pm and 18.5 pm respectively. Find the wavelength of L_α X-rays of the material.

296

In an X-rays tube the anti cathode material has atoms of atomic number Z = 28. Find the voltage applied to the X-ray tube if the wavelength difference between the K_α line and short-wave cut off of the the continuous X-ray spectrum is equal to 84 pm.

297

A (^{7}Li) target is bombarded with a proton beam currentof 10^{-4} Amp. for 1hr. to produce (^{7}Be) of activity 1.8×10^8 disc/sec. Assuming that one (^{7}Be) radioactive nucleus is produced by bombarding 1000 protons. Determine the half life of (^{7}Be).

298

A radionuclide A with decay constant λ transforms into a radionuclide B with same decay constant λ. Assume that at $t = 0$ the number of radionuclidie "A" is N_0 and number of radionuclide B is zero. Find the number of radionuclide B after time t.

$$A \xrightarrow[\lambda]{} B \xrightarrow[\lambda]{}$$

299

A nuclear explosion is designed with a nuclear fuel consisting of U-235 to run a reactor to deliver 1 MW of heat energy. The amount of energy released per fission is 200 MeV. Find the amount of U-235 needed to run this reactor for one year.

300

Consider a following nuclear reaction.

$$_1^2\mathrm{D} + _1^2\mathrm{D} \rightarrow _1^3\mathrm{T} + _1^1p + \Delta\mathrm{E}$$

It is given $m\left(_1^2\mathrm{D}\right) = 2.01458$ amu, $m\left(_1^3\mathrm{T}\right) = 3.01605$, amu, $m\left(_1^1p\right) = 1.00728$ amu. Calculate the mass of dueterium $\left(_1^2\mathrm{D}\right)$ required per day for a power output of 10^9W. Assume that the efficiency of process to be 50%.

MEASUREMENTS

301

In the relation $\mathrm{P} = \left(\frac{\alpha}{\beta}\right)e^{-\left(\frac{\alpha x}{k\theta}\right)}$, P is pressure, x is distance, k is Boltzman constant and θ is temperature. Find the dimensional formula of β.

302

A quantity X is given by $\mathrm{X} = \frac{\varepsilon_0 \mathrm{C}}{\mathrm{B}^2}$ where ε_0 is the permitivity of free space, C is capacitance and B is magnetic field of induction. Find the dimensional formula of X.

303

If the energy E, angular momentum J and universal gravitational constant G were chosen as base quantities, then find the dimensions of mass in terms of E, J and G.

304

Assuming that the classical radius r of electron depends upon the electronic mass m, electronic charge e, speed of light in vacuum C and the permitivity of free space ε_0, find the expression for radius r using dimensional analysis.

305

Assuming that the minimum possible measurable time t depends upon universal gravitational constant G, Planck's constant h and speed of light in vacuum C, find the expression for time t using dimensional analysis.

306

Check the dimensional accuracy of the relation $B = \dfrac{\mu_0 m^2 e^7}{256\pi^4 \varepsilon_0^3 h^3}$

where B is magnetic field of induction, μ_0 is permeability of free space, ε_0 is permitivity of free space, h is Planck's constant, m is electronic mass and e is electronic charge.

307

A cube has side of length 1.2×10^{-2} m and mass 3.10×10^{-3} kg. Calculate its density with due regard for significant figures.

308

Calculate the temperature of a gas in true significant figures from the Vander Walls equations $\left(P + \dfrac{a}{V^2}\right)(V - b) = nRT$ using following data.

$P = 90.00$ atm, $V = 0.250$ litre.

$n = 2.000$ moles, $R = 0.082 \dfrac{(\text{atm})(\text{litre})}{(\text{mole})k}$

$$a = 1.85\ \frac{(\text{atm})(\text{litre})^2}{(\text{mole})^2},\ b = 0.042\ (\text{litre})/(\text{mole})$$

309

The value of one main scale division is 1 mm and 9 main scale divisions are equal to 10 vernier scale divisions. There is no zero error in the instrument. When a rod is placed between the jaws of the calliperse the 39th division of the main scale coincides with the 7th division of the vernier scale. Find the length of the rod.

310

The value of one main scale division is 1 mm and 9 main scale divisions are equal to 10 vernier scale divisions. When nothing is put between the jaws of the calliperse, the zero of the vernier scale lies right side of the zero of the main scale and the 4th division of the vernier scale coincides with a main scale division. While measuring the inner diameter of a hollow cylinder the zero of the vernier scale lies between 2.4 cm and 2.5 cm of the main scale and 8th division of the vernier scale coincides with a main scale division. Find the inner diameter of the cylinder.

311

The value of 1 main scale division is 1 mm and there are 10 divisions on the vernier scale. The value of one vernier scale division is 0.9 mm. When the two jaws of the vernier calliperse touch each other the zero of the vernier scale lies to the left of the zero of the main scale and 3rd division of the vernier scale coincides with a main scale division. While measuring the depth of a hollow cylindrical can the 6th division of the vernier scale coincides with the 38th division of the main scale division. Find the depth of the cylindrical can.

312

In a screw gauge, the linear distance travelled by the head scale is 5 mm in 10 complete rotations. There are 100 divisions on the circular scale of the screw gauge. While measuring the diameter of the wire, the main scale reads 2.5 mm and the circular scale reads 33 divisions. Find the diameter of the wire. (There is no zero error in the screw gauge.)

313

The pitch of a screw gauge is 1 mm and the number of divisions on the circular scale is 100. When nothing is put in between its jaws, the zero of the circular scale lies 5 divisions below the reference line, *i.e.*, the zero error is +5 divisions on the circular scale. When a wire is held in between its jaws the reading of main scale is 8 divisions and on the circular scale the reading is 78 divisions. Find the diameter of the wire.

314

The pitch of a screw gauge is 1 mm and its cap is divided into 100 divisions. When nothing is placed between its studs, the zero of the circular scale is 6 divisions above the reference line, *i.e.*, the zero error is –6 divisions on the circular scale. When a wire is placed between its studs the main scale reading is 3 divisions and 37th division of circular scale coincides with reference line. Find the diameter of the wire.

315

A wire has (0.3 ± 0.03) gm, radius (0.5 ± 0.005) mm and length (6 ± 0.06) cm. Find the maximum percentage error in the measurement of its density.

316

The length and breadth of a rectangle are measured (33.2 ± 0.1) cm, and (16.8 ± 0.1) cm respectively. Find the percentage error in the measurement of its perimeter.

317

In an experiment the refractive index of the material of the prism are found to be 1.54, 1.53, 1.44, 1.54, 1.56 and 1.45 in successive measurements. Calculate the percentage error in finding the refractive index.

318

The diameter of a cylindrical wire is measured 0.247 cm using a screw gauge of least count 0.001 cm. The length of this wire is measured 5.63 cm using a vernier calliperse of least count 0.01 cm. Find the maximum permissible error in the measurement of its volume in cubic centimeter.

319

In an experiment to find the focal length of a concave mirror the readings of object distance and image distance are (40.0 ± 0.1) cm and (120.0 ± 0.1) cm. The image formed is real and inverted. Find the maximum percentage error in the measurement of focal length of the mirror.

320

The focal length of a thick biconvex lens is given by

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}-\frac{(M-1)t}{MR_1R_2}\right)$$ where R_1 and R_2 are radii of curvature, t is the thickness and μ is the refractive index of the glass. In an experiment to find the focal length of a thick lens following observations were taken.

$R_1 = (20.0 \pm 0.1)$ cm,
$R_2 = (60.0 \pm 0.1)$ cm,
$t = (15.0 \pm 0.1)$ cm, and
$\mu = (1.5 \pm 0.01)$ cm.

Find the focal length of the lens and express the result in terms of maximum absolute error.

ANSWERS

ANSWERS

1. 4 seconds

2. $\vec{v}_A = \left(4\hat{i} + 3\hat{j}\right) m/s$

   $r_{min} = 24$ m

3. $$\frac{d^2 + l^2}{ud + \sqrt{d^2 l^2 + l^2\left(v^2 - u^2\right)}}$$

4. $$\frac{\sqrt{3}\left(\sqrt{3} + 1\right)l}{2u}$$

5. $$t = \frac{d(u+w)\left[w\,(u+w) - v\sqrt{(u+w)^2 - v^2}\right]}{vw^2\sqrt{(u+w)^2 - v^2}}$$

6. $$d = \frac{(l_1 v_2 - l_2 v_1)\sin\theta}{\sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos\theta}}$$

7. $\alpha = \tan^{-1}(2)$

   $u = v\sqrt{5}$

8. $\theta = \tan^{-1}\left(2\tan\beta - \tan\alpha\right)$

9. $\langle v\rangle_{time} = \dfrac{v}{3}$, $\langle v\rangle_{space} = \dfrac{3v}{5}$

10. $v = 3$ m/s, S = 6 m

11. $$\left(\frac{ac - b^2}{x^3}\right)$$

12. $$T = \pi\sqrt{\frac{2a^3}{\mu}}$$

13. $v = \frac{1}{3}\left[8(6-f)(12+f)^2\right]^{\frac{1}{4}}$

14. $\frac{v_0}{\sqrt{1+\left(\frac{kv_0^2}{mg}\right)}}$

15. $x = \frac{k}{w^3}(wt - \sin wt)$

16. $T = \frac{2\pi\,\mu a^3}{(2\mu a - \lambda)^{\frac{3}{2}}}$

17. displacement = 85.1 m distance = 85.3 m

18. <velocity>= $5\sqrt{2}$ m/s, <speed> = 8.12 m/s

19. $d = \frac{(2\sqrt{7})v^2}{g}$

20. $x = 3v\sqrt{\frac{h}{2g}}$

21. $t = \frac{u \pm \sqrt{u^2 \pm gd\sin 2\alpha}}{g\cos\alpha}$

22. $h = \frac{v^2(1+\cot\alpha)}{g(2+2\cot\alpha+\cot^2\alpha)}$

23. $R_{max} = \frac{u\sqrt{u^2+2gh}}{g}$

24. $R = a\sqrt{7}$, $T = \sqrt{\frac{28a}{g\sqrt{3}}}$

25. $T = \sqrt{\frac{2a\sin^2(\alpha+\beta)}{g[\sin\alpha - \sin\beta\cos(\alpha+\beta)]}}$

26. $u = \left[g\left(h + \sqrt{h^2 + d^2}\right)\right]^{\frac{1}{2}}$, $T = \left[\frac{2}{g}\sqrt{h^2 + d^2}\right]^{\frac{1}{2}}$

27. $\dfrac{2u^2 \sin\alpha\cos\alpha(2v + u\cos\alpha)}{g(v + u\cos\alpha)}$

28. $T = \dfrac{6v}{g\sqrt{1 + 8\sin^2\alpha}}$

29. $w = \dfrac{2g\cos\alpha}{u\left(1 + 3\cos^2\alpha\right)}$

30. $\rho = \dfrac{\left[(u\sin\alpha - gt)^2 + (u\cos\alpha)^2\right]^{\frac{3}{2}}}{(ug\cos\alpha)}$

31. $w = \dfrac{(v_2 l_1 - v_1 l_2)}{(l_2 - v_2 t)^2 + (l_1 - v_1 t)^2}$, $w_{max} = \left(\dfrac{v_1^2 + v_2^2}{v_2 l_1 - v_1 l_2}\right)$

32. $w = \dfrac{w_1 R_1^2 + w_2 R_2^2 - R_1 R_2 (w_1 + w_2)\cos\theta}{R_1^2 + R_2^2 - 2R_1 R_2 \cos\theta}$

33. $v = 2\text{m/s}$, $a_1 = 50\ \text{m/s}^2$, $a_2 = 85.4\ \text{m/s}^2$

34. $v_0 = \dfrac{2vR_1R_2R_3}{r_2\,(r_3 + R_3)(R_1\sin\alpha - r_1)}$

35. $\left(-28\hat{i} + 4\hat{j}\right)\text{m/s}^2$

36. $\dfrac{R_1\left(R_1 w_1^2 - R_2 w_2^2 + 2R_2 w_1 w_2\right)}{R_1 + R_2}$

37. $\vec{v} = \left(5\hat{i} + 24\hat{j}\right)$ cm/s, $\vec{a} = -\left(15\hat{i} + 20\hat{j}\right)$ cm/s^2

38. $a = \dfrac{uv}{R}\sqrt{4 + \left(\dfrac{rv}{Ru}\right)^2}$

39. $\vec{v} = \left(-9\hat{i} + 16\hat{j} + 12\hat{k}\right)$

40. $\alpha = \dfrac{v^2}{rR}$

41. (*a*) $w = 0.732$ rad/s (*b*) $v_B = 3.6$ m/s
(*c*) $\alpha = 1.05$ rad/s^2 (*d*) $a_B = 6.65$ m/s^2

42. (*a*) $w = 0.816$ rad/s, $\alpha = 0.193$ rad/s^2
(*b*) $v_B = 1.115$ m/s, $a_B = 5.83$ m/s^2

43. $\sqrt{\dfrac{7}{3}}$ m/s

44. $v = 4.38$ m/s, $a = 56$ m/s^2

45. $v_B = 22.7$ m/s

46. $v = \left[\sqrt{\left(v_1^2 + v_2^2 + 2v_1v_2\cos\theta\right)}\right]\operatorname{cosec}\theta$

47. $a = \dfrac{g(m_3 - m_1\cos\theta)\cos\theta}{\left(m_1\cos^2\theta + m_2 + m_3 + m_4\cot^2\alpha\right)}$

48. Block C moves up with acceleration $a = \left(\dfrac{31}{97}\right)g$

49. $a = \dfrac{mg(1+\cos\theta)\sin\alpha}{M + m\left[(1+\cos\theta-\cos\alpha)^2 + \sin^2\alpha\right]}$

50. $a = \left(\dfrac{21}{43}\right)g$ upward

51.

$$a = \left[\frac{\left(\dfrac{1}{m_1}+\dfrac{1}{m_2}\right)(m_1\sin\alpha\cos\alpha - m_2\sin\beta\cos\beta) - (\sin\alpha+\sin\beta)(\cos\alpha-\cos\beta)}{\left(\dfrac{1}{m_1}+\dfrac{1}{m_2}\right)\left(m_3 + m_1\sin^2\alpha + m_2\sin^2\beta\right) + (\cos\alpha-\cos\beta)^2}\right]g$$

52. $a=\left[\dfrac{(m_1-m_2)^2 g\sin\theta\cos\theta}{M(m_1+m_2)+(m_1-m_2)^2\sin^2\theta+4m_1m_2}\right]$

53. $a=\dfrac{\mu_2(m_1+m_2)}{(\cos\alpha-\mu_2\sin\alpha)}$

$\leq F \leq \dfrac{m_1\,(m_1+m_2)(\mu_1-\mu_2)g}{\left(m_2\,(\cos\alpha-\mu_2\sin\alpha)-m_1\,(\mu_2+\mu_1)\sin\alpha\right)}$

54. (*a*) $a_1=a_2=a_3=1$ m/s^2
(*b*) $a_1=2$m/s^2 $a_2=a_3=2.6$ m/s^2

(*c*) $a_1=2$m/s^2, $a_2=3.67$ m/s^2, $a_3=4.71$ m/s^2

55. $a=\dfrac{g[m_1k-2\mu m_2(\cos\theta+\mu\sin\theta)]\tan\theta}{mk\tan\theta+2m_2(\cos\theta+\mu\sin\theta)}$

where $k=(\sin\theta-2\mu\cos\theta-\mu^2\sin\theta)$

56. $a=g\left[\dfrac{(2-\mu)(1+\mu)m_2m_3-\mu m_1\,(2m_3+m_2)}{\left(4-\mu^2\right)m_2m_3+\left(m_1+\left(\mu^2+1\right)m_3\right)(2m_3+m_2)}\right]$

57. $w=\left[\left(\dfrac{m_1^2-m_2^2}{m_1^2l_1^2-m_2^2l_2^2}\right)g^2\right]^{\frac{1}{4}}$

58. $a=\dfrac{\mu v_0^2}{(\mu+1)l}, R=(\mu+1)l$

59. $s=\dfrac{R}{2\mu}ln\left[\dfrac{v_0^2}{Rg}+\sqrt{1+\left(\dfrac{v_0^2}{Rg}\right)^2}\right]$

60. $w>\left[\dfrac{g}{R}\sqrt{1+\dfrac{1}{\mu^2}}\right]^{\frac{1}{2}}$

61. $F=mg\sqrt{\sin^2\theta+\left(\dfrac{v^2}{gl}-2(1-\cos\theta)\right)^2}$

62. $t = \left(\sqrt{\frac{l}{g}}\right) ln\left(\sqrt{2}+1\right)$

63. $\frac{5}{27}\left(\sqrt{5}+4\sqrt{2}\right)R$

64. $v = \sqrt{lg\left(2+\frac{9\sqrt{3}}{8}\right)}$

65. $v = \sqrt{\frac{3}{4}v_0^2 + 2gb}$

66. W = 1.2 Joule

67. $v = \sqrt{gl(1-\eta)}$

68. $\frac{136}{3}$ Joules

69. $\frac{l(m_1 \cos\alpha + m_2 \cos\beta)}{(m_1 + m_2 + m_3)}$

70. (9.49 cm, 6.11 cm)

71. 133.57 J

72. $x_{\max} = \frac{4m_1 m_2 g}{(m_1 + m_2)K}$

73. $\theta = \cos^{-1}\left(\sqrt{3}-1\right)$

74. (a) $v_{max} = \sqrt{\frac{2m^2 gl \sin\theta \cos\theta}{(m+M)\left(M + m\sin^2\theta\right)}}$

(b) $v_{average} = \sqrt{\frac{m^2 gl \sin\theta \cos\theta}{2(m+M)\left(M + m\sin^2\theta\right)}}$

75. $\dfrac{M(M+2m)v_0^2}{8(M+m)}$

76. $\dfrac{2v_0}{19}$

77. (a) $\dfrac{2v\sin\alpha}{g(1-e)}$, (b) $\dfrac{v^2\sin 2\alpha}{g(1-e)}$

78. $\theta = \tan^{-1}\left(e^{3/2}\right)$

79. $\dfrac{m^2 m_2 v_0^2}{2\mu g(m+m_1+m_2)(m+m_1)^2}$

80. $\vec{v}_2 = \vec{v}_1 - \left[(1+e)\vec{v}_1\cdot\hat{n}\right]\hat{n}$

81. $\dfrac{m_1}{m_2} = 2$

82. $\vec{v}_1' = \dfrac{1}{2}\left(-\hat{i}+7\hat{j}\right)\text{m/s}$, $\vec{v}_2' = \dfrac{1}{4}\left(-\hat{i}+9\hat{j}\right)\text{m/s}$

83. $e = \dfrac{m_1\sin^2\beta + m_2\sin^2\alpha}{m_1\cos^2\beta + m_2\cos^2\alpha}$

84. $\delta_{max} = \sin^{-1}\left(\dfrac{1+e}{3-e}\right), \theta = \tan^{-1}\sqrt{\dfrac{1-e}{2}}$

85. $\dfrac{m_2(1+e)u\sin\alpha}{m_1+m_2\sin^2\alpha}$

86. $\dfrac{\sqrt{3}(1+e)Mu}{M+6m}$

87. $\dfrac{(1+e)mu\sin^2\theta}{M+m\sin^2\theta}$

88. $\dfrac{mv_0(1+e)\sin\theta\cos\theta}{m\sin^2\theta+2M\cos^2\theta}$

89. $\left[\dfrac{3m_0g}{\mu}\left(1-e^{-\mu t/2m_0}\right)-gt\right]$

90. $\dfrac{1}{8}gt^2\left(\dfrac{2r_0+\mu t}{r_0+\mu t}\right)^2$

91. $\left[\left(\dfrac{Ft+m_0u}{\sqrt{m_0^2+\rho A\left(Ft^2+2m_0ut\right)}}\right)-u\right]$

92. $v=\sqrt{\dfrac{14}{3}gl}$

94. $\left(\dfrac{13}{20}\right)mR^2$

95. $\dfrac{M}{6}\left(a^2+b^2+c^2+ab+bc+ca\right)$

95. $\sqrt{2gR\sec\theta}$

96. $\dfrac{mv_0^2}{kl}$

97. $\mu=\tan\left[\dfrac{1}{2}\tan^{-1}\left(\dfrac{\sin^3\alpha-\sin^3\beta}{\sin^2\alpha\cos\alpha+\sin^2\beta\cos\beta}\right)\right]$

98. $\mu=\dfrac{4\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

99. $\dfrac{4}{37}g$

100. $\dfrac{F}{7m}$

101. $\dfrac{2mg\sin\theta\cos\theta}{3M + m(1+2\sin^2\theta)}$

102. Monkey A, $\dfrac{l}{2v}\left(\dfrac{M+4m}{M+3m}\right)$

103. $\dfrac{3\sqrt{3}g}{10l}$

104. $\dfrac{mg\sin\theta}{1+3\sin^2\theta}$

105. $v_A = \sqrt{\dfrac{3gl(1-\sin\theta)\sin^2\theta}{1+3\cos^2\theta}}$, $v_B = \sqrt{3gl(1-\sin\theta)}$

106. $\dfrac{13}{4}\tan\theta$

107. $\dfrac{36\sqrt{2}v_0}{65l}$

108. $\dfrac{2mv_0}{(M+3m)R}$

109. $h = 42\ m$, $J = 30$ kg m/s

110. time $= \dfrac{53v_0}{28\mu g}$, distance $= \dfrac{v_0^2}{2\mu g}$

111. $\dfrac{v}{R}\left(\dfrac{M+m}{M+2m}\right)$

112. $\phi = \tan^{-1}\left[\dfrac{\tan\theta}{e} + \left(1+\dfrac{1}{e}\right)\mu\right]$

$$\omega = \omega_0 - 5\mu\,(1+e)\left(\frac{v_0}{2R}\right)\cos\theta$$

$$v = v_0\sqrt{e^2\cos^2\theta + (\sin\theta + \mu(1+e)\cos\theta)^2}$$

113. $\omega'_1 = \dfrac{\left(\omega_1 R_1^3 + \omega_2 R_2^3\right)}{R_1\left(R_1^2 + R_2^2\right)}$, $\omega'_2 = \dfrac{\left(\omega_1 R_1^3 + \omega_2 R_2^3\right)}{R_2\left(R_1^2 + R_2^2\right)}$

114. $a = r_0$, $e = \sin\alpha$

115. $\dfrac{1}{3}\sqrt{\dfrac{2R}{g}}\left[\left(1 + \dfrac{h}{R}\right)^{\frac{3}{2}} - 1\right]$

116. $V = \dfrac{-2GM}{R}, U = -\dfrac{2GM^2}{3R}$

117. $\Delta l = \dfrac{m\omega^2 l^2}{3AY}$

118. $\dfrac{m^2 g^2 l}{10\pi R^2 Y}$

119. $2R^3 \rho g\left(\dfrac{H}{3} - \dfrac{\pi R}{16}\right)$

120. $(\cos 2\alpha)^{\frac{3}{2}}$

121. $\dfrac{A_1 A_2}{(A_1 + A_2)a}\sqrt{\dfrac{2H}{g}}$

122. $\dfrac{\sqrt{2g\,(H - h)}}{R}$

123. $\theta = \pi - \cos^{-1}\left(\dfrac{mgh}{2\pi R^2\left(n^2 - 1\right)}\right)$

124. $\dfrac{4}{3}\pi a^2\left[6S + (4S + P_0 a)\, ln\left(1 + \dfrac{4S}{P_0 a}\right)\right]$

125. $4\pi\eta l w\left(\dfrac{a^2b^2}{b^2-a^2}\right)$

126. $\left(\dfrac{7}{16}\right)V_0$

127. $2\pi\sqrt{\dfrac{h}{5g}\left(4+\tan^2\alpha\right)}$

128. $T=2\left[\sqrt{\dfrac{2h}{g}}+\sqrt{\dfrac{m}{k}}\left(\pi-\tan^{-1}\sqrt{\dfrac{2hk}{mg}}\right)\right]$

129. $2\pi\sqrt{\dfrac{2l}{3g}}$

130. $2\pi\sqrt{\dfrac{R}{g\left[1+\dfrac{m}{M}\right]}}$

131. $T=\pi\sqrt{\dfrac{l_1l_2\,(l_1+l_2)}{\left(l_1^2+l_2^2\right)g}}+\pi\sqrt{\dfrac{l_2\,(l_1+l_2)}{l_1g}}$

132. $2\pi\sqrt{\dfrac{3m\left(2M+3m\sin^2\theta\right)}{2K(2M+3m)}}$

133. $b^2x^2=a^2\left(b^2-y^2\right)\left(b^2-4y^2\right)$

134. $\omega=\sqrt{\left(3\pm\sqrt{5}\right)\dfrac{K}{2m}}$

135. $y=a\sin 2\pi\left(\dfrac{t}{T}-\dfrac{x}{\lambda}\right)$

where, $a=8$ mm, $T=0.58$ second, $\lambda=284.2$ cm.

136. $y = 4 \sin\left(10\pi t + \pi x + \frac{\pi}{4}\right)$

where x and y are in cm and t is in second.

137. *(a)* Along astraight line in xy plane through origin at 30° with x-axis.

(b) 1m

(c) 4π

138. *(a)* 10^{-10} W/m^2

(b) 2×10^{-4} N/m^2

139. *(a)* $(6\sqrt{3})$ C

(b) $y = \dfrac{b^3}{b^2 + (X - (c - v)t)^2}$

140. 1.44 %.

141. 98.7 W

142. *(a)* $\frac{2}{3}$ A

(b) 88.9 %

143. $\dfrac{2Th^2}{l}$

144. $y = [5 \sin \pi (4t - 0.7)]$ cm

145. $\Delta P = \left(n + \frac{1}{2}\right)\dfrac{\pi A \rho v^2}{l} \cos\left[\left(n + \frac{1}{2}\right)\dfrac{\pi x}{l}\right]$

146. 20 cm

147. $n = n_0 \left(\dfrac{v + w - u}{v + w - v_s}\right)\left(\dfrac{v - w + v_0}{v - w + u}\right)$

$$\lambda = \frac{(v - w + u)(v + w - v_s)}{n_0 (v + w - u)}$$

148. 1042 Hz, 30 cm

149. $z = A \sin \dfrac{\pi x}{l} \sin \dfrac{\pi y}{l} \sin \omega t$

150. 391 Hz.

151. $t = \left(l_1\sqrt{\rho_1} + l_2\sqrt{\rho_2}\right)\sqrt{\dfrac{(l_1 Y_2 + l_2 Y_1)}{\theta(l_1\alpha_1 + l_2\alpha_2) Y_1 Y_2}}$

152. $l\left[1 + \left(\dfrac{Y_1\alpha_1 + 3Y_2\alpha_2}{Y_1 + 3Y_2}\right)\theta\right]$

153. 1.0155 m

154. $V_A = V_1 + \left(\dfrac{K_1\gamma_1 - K_2\gamma_2}{V_1K_2 + V_2K_1}\right) V_1V_2\theta$

$V_B = V_2 + \left(\dfrac{K_2\gamma_2 - K_1\gamma_1}{V_1K_2 + V_2K_1}\right) V_1V_2\theta$

155. $-m_0 e^{-\frac{\mu g}{L}\left(v_0 t - \frac{1}{2}\mu g t^2\right)}$

156. 153.4 gm

157. $T = T_1\left(\dfrac{T_2}{T_1}\right)^{x/l}$

158. $T_1' = T_1 - \dfrac{m_2S_2(T_1 - T_2)}{m_1S_1 + m_2S_2}\left(1 - e^{-\alpha t}\right)$

$T_2' = T_2 + \dfrac{m_1S_1(T_1 - T_2)}{m_1S_1 + m_2S_2}\left(1 - e^{-\alpha t}\right)$

where $\alpha = \dfrac{KA}{l}\left(\dfrac{1}{m_1S_1} + \dfrac{1}{m_2S_2}\right)$

159. $T = \dfrac{T_1K_1 + T_2K_2}{K_1 + K_2}$

160. $\dfrac{\rho L R^2}{6K\theta}$.

161. $t\left[\dfrac{\ln\left(\dfrac{\theta_2 - \theta_0}{\theta_3 - \theta_0}\right)}{\ln\left(\dfrac{\theta_1 - \theta_0}{\theta_2 - \theta_0}\right)}\right]$

162. $\dfrac{Ah(\rho_1 - \rho_2)}{\left(\ln \dfrac{\rho_1}{\rho_2}\right)}$

163. $P = P_0\, e^{-rt/V_0}$

164. 607 m/s

165. $\dfrac{5}{4} V_0$

166. $(41.13)\, V_0$

167. 41 %

168. 8.33 %

169. *(a)* $PV^{\frac{1}{3}} = \text{const}$

(b) 3 R

170. $V^R T^a e^{bT} = \text{const}$

171. $W = nRT_0 \ln\left[\dfrac{1}{4} - \left(\dfrac{P_0A}{2kh}\right) + \sqrt{\dfrac{1}{16} + \dfrac{3P_0A}{4kh} + \left(\dfrac{P_0A}{2kh}\right)^2}\,\right]$

172. $18\, P_0V_0$

173. $v = \left[3gl\left(\dfrac{2}{3} + \sqrt{1 + \left(\dfrac{mg}{qE}\right)^2}\right)\right]^{\frac{1}{2}}$

174. $v = \dfrac{qE}{m}\sqrt{\dfrac{l}{g}}$

175. $t = \sqrt{2.4\,\pi\epsilon_0\, md}\ \left[\sqrt{6} + \ln\left(3 - \sqrt{2}\right)\right]\dfrac{d}{q}$

176. *(a)* $\left(\dfrac{qQ}{4\pi\varepsilon_0 m v_0^2}\right)^2 + \sqrt{b^2 + \left(\dfrac{qQ}{4\pi\varepsilon_0 m v_0^2}\right)^2}$

(b) $\theta = 2\tan^{-1}\left(\dfrac{qQ}{4\pi\varepsilon_0 m v_0^2 b}\right)$

177. $V = \dfrac{p\sin\theta\sin\phi}{4\pi\varepsilon_0 r^2}$

$$E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1+3\sin^2\theta\sin^2\phi}$$

178. $\vec{E} = \dfrac{qR\hat{j}}{\pi^2\varepsilon_0\left(z^2+R^2\right)^{3/2}}$

179. $F = \dfrac{\lambda^2}{2\pi\varepsilon_0}\ln\left[\dfrac{(a+l)\left(l+\sqrt{l^2+4a^2}\right)}{a\left(l+\sqrt{l^2+4(a+l)^2}\right)}\right]$

180. $\dfrac{2\sigma^2R^3}{3\varepsilon_0}$

181. $\dfrac{Q}{2\pi\varepsilon_0 l}$

182. $E = \dfrac{\sigma}{4\sqrt{2}\,\varepsilon_0}$

183. $4\pi\varepsilon_0\alpha R^3$

184. $4\pi a$

185. $+20\ \mu c$

186. *(a)* $190\ \mu c$

(b) $190\ \mu c$

(c) $85\ \mu c$ through S_1, and $105\ \mu c$ through S_2.

187. $\dfrac{4\pi\varepsilon_0}{\left(\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{2}{d}\right)}$

188. $\dfrac{V}{d}$

189. $\dfrac{4\varepsilon_0 aK}{\sqrt{K-1}}\tan^{-1}\left(\sqrt{K-1}\right)$

190. $\left(\dfrac{\varepsilon_0 lV}{\alpha}\right)\ln\dfrac{b}{a}$

191. $69\ \mu f$

192. $22\ \mu f$.

193. *(a)* 105 μc, *(b)* 2.625 mJ.

194. *(a)* 150 μc, – 10 μc and – 135 μc on 1 μf, 2 μf and 3 μf respectively.
(b) 3.3 mJ.

195. $\sqrt{\dfrac{l^2}{4(K-1)^2}+\dfrac{\varepsilon_0 l V^2}{\rho g d^2}}-\dfrac{l}{2(K-1)}$

196. $\sqrt{\left(\dfrac{K}{m}-\dfrac{\varepsilon_0 A V^2}{m d^3}\right)}$

197. $\dfrac{3l}{\pi a^2(\sigma_1+2\sigma_2)}$

198. $\left(\dfrac{\rho l}{ad-bc}\right)\ln\dfrac{ad}{bc}$

199. *(a)* $\dfrac{R}{2n}\left[(3n-1)+\sqrt{9n^2+2n+1}\right]$

(b) $\dfrac{E}{R}\left[\dfrac{\sqrt{9n^2+2n+1}-n-1}{\sqrt{9n^2+2n+1}+3n-1}\right]$

200. $\dfrac{E}{2ar}\left(\sqrt{7}+1\right)$

201. $V=\dfrac{n(n+1)E}{2}$

$R=\dfrac{n(n+1)r}{2m}$

202. E = 43.24 V, $r = 2.1\,\Omega$.

203. $\left(1+\dfrac{1}{5}e^{-t/\tau}\right)$Amp

where $\tau = 25\ \mu$ sec

204. $q_1 = 28\ \mu c$, $q_2 = 14\ \mu c$, $q_3 = 42\ \mu c$

205. Charge = $\left(\dfrac{CEr}{r+R}\right)\left(1-e^{-t/\tau}\right)$

Current = $\left(\dfrac{E}{r+R}\right)\left(1+\dfrac{r}{R}e^{-t/\tau}\right)$

where, $\tau=\dfrac{CRr}{R+r}$

206. $$V_1 = E_1 - \frac{c_2}{c_1 + c_2}(E_1 + E_2)\left(1 - e^{-t/\tau}\right)$$

and $$V_2 = E_2 - \frac{c_1}{c_1 + c_2}(E_1 + E_2)\left(1 - e^{-t/\tau}\right)$$

where, $\tau = R\left(\frac{c_1 c_2}{c_1 + c_2}\right)$

207. $\frac{\mu_0 I}{4\pi R} \ln\left(\sqrt{2} + 1\right)$

208. $\frac{\mu_0 I}{4a}$

209. $$\frac{\mu_0 I l^2}{2\pi\left(x^2 + \frac{l^2}{4}\right)\sqrt{\left(x^2 + \frac{l^2}{2}\right)}}$$

210. $$\frac{\mu_0 I a}{2\pi\left(x^2 + a^2\right)^{3/2}}\ (\pi a \hat{i} + x\hat{j})$$

211. $B = \frac{\mu_0 N I (b - a)}{l^2} \ln \frac{b}{a}$

212. $B = \frac{\mu_0 I}{\pi b} \tan^{-1}\left(\frac{b}{2a}\right)$

213. $\frac{\mu_0 b r^{\alpha + 1}}{\alpha + 2}$

214. $\mu_0 N I \left(\frac{a + b}{2} - \sqrt{ab}\right)$

215. $\mu_0 I_1 I_2 \left(\frac{b}{\sqrt{b^2 - a^2}} - 1\right)$

216. $$\frac{\mu_0}{2\pi}\ \frac{I_1 I_2}{a}\left[\sqrt{a^2 + \left(\frac{l_1 + l_2}{2}\right)^2} - \sqrt{a^2 + \left(\frac{l_2 - l_1}{2}\right)^2}\right].$$

217. $\frac{\pi^2 a B I}{4g}$

218. $F = \dfrac{\mu_0 I_1 I_2 l^2}{2\pi\left(d^2 + \dfrac{l^2}{4}\right)}$

$$\tau = \frac{\mu_0 I_1 I_2 l^2 d}{2\pi\left(d^2 + \dfrac{l^2}{4}\right)}$$

219. $\dfrac{aqB}{\mu(mg + qE)}$

220. $\vec{v} = (v_0 \sin\alpha \sin\omega t)\hat{i} + (v_0 \sin\alpha \cos\omega t)\hat{j} + (v_0 \cos\alpha)\hat{k}$

$\vec{r} = R(1 - \cos\omega t)\hat{i} + (R\sin\omega t)\hat{j} + (v_0 t\cos\alpha)\hat{k}$

where $\omega = \dfrac{qB}{m}$ and $R = \dfrac{mv}{qB}$.

221. $\dfrac{qB^2 d^2}{2m}$

222. $\dfrac{qB^2}{8m}\left(b - \dfrac{a^2}{b}\right)^2$

223. $F = \dfrac{\pi^2 a^4(\alpha + \beta)\gamma}{R}$

224. $\dfrac{l^2 bC}{7}$

225. $-\dfrac{5q\vec{B}}{6m}$

226. $\dfrac{g\sin\theta}{\left(2 + \dfrac{\mu_0 Q^2}{4\pi m l}\right)}$

227. $\dfrac{mgR\left(1 - e^{-\alpha t}\right)}{\left(\dfrac{\mu_0 I}{2\pi}\ln\dfrac{b}{a}\right)^2}$ where $\alpha = \dfrac{1}{mR}\left(\dfrac{\mu_0 I}{2\pi}\ln\dfrac{b}{a}\right)^2$

228. $x = \dfrac{mgL}{B^2 l^2}(1 - \cos\omega t)$

where, $\omega = \dfrac{Bl}{\sqrt{(m + cB^2 l^2)L}}$.

229. $\dfrac{4mR\omega_0}{3B^2l^2}\left(1-e^{-bt}\right)$, where $b=\dfrac{3B^2l^2}{4mR}$

230. $z=\dfrac{mgL}{2\pi^2a^4b^2}\,(1-\cos\omega t)$

where, $\omega=\pi a^2 b\sqrt{\dfrac{2}{mL}}$

231. $\dfrac{mgR\sec\theta\tan\theta}{\pi^2a^4B_0^2\alpha^2}\left(1-e^{-bt}\right)$

where $b=\dfrac{\pi^2a^4B_0^2\alpha^2\cos^2\theta}{mR}$

232. $h=\dfrac{(\mu-\mu_0)I^2\ln\dfrac{b}{a}}{4\pi^2\rho g\left(b^2-a^2\right)}$

233. $\left(\dfrac{MI_0\alpha}{R-\alpha L}\right)\left(e^{-\alpha t}-e^{-Rt/L}\right)$

234. *(a)* $21\left(1-e^{-21t/50}\right)$ Amp.

(b) $\left(252-2e^{-21t/50}\right)$ Amp.

235. $\sqrt{\dfrac{L_2}{\left(L_1L_2-M^2\right)C}}$

236. $\dfrac{1}{\sqrt{L_1C}},\ \dfrac{1}{\sqrt{(L_1+2L_2)C}}$

237. *(a)* $\dfrac{I_0}{2}$, *(b)* $\dfrac{I_0}{\sqrt{3}}$

238. *(a)* $\dfrac{2}{3}V_0$, *(b)* $\left(\dfrac{\sqrt{5}}{3}\right)V_0$

239. $[(37.4)\sin(\omega t+41°)]$ Amp.

240. $(5.1)\sin(\omega t-32°)$ Amp.

241. 1040 W.

242. 952 J.

243. $L = 0.2\ \text{mH},\ C = \left(\frac{1}{32}\right)\mu f$

244. $L = \frac{4}{81}\ \text{H},\ C = 1\ \mu f,\ R = 250\ \Omega$

245. 50 Amp.

246. 10 Amp.

247. $\omega = \left(\sqrt{\frac{1}{LC} + \frac{R^2}{4L^2}}\right) \pm \left(\frac{R}{2L}\right)$

248. *(a)* $\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$

(b) $\sqrt{\frac{2}{2LC - R^2C^2}}$

249. $\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

250. 10 Amp.

251. $L = C_2R_2R_1 = C_1R_3R_1$

252. 10 Amp

253. $\frac{3}{4}\sqrt{3fg}$

254. $16\ \text{mm}^3$

255. $x = \left[t_1\left(1 - \frac{\cos i}{\sqrt{\mu_1^2 - \sin^2 i}}\right) + t_2\left(1 - \frac{\cos i}{\sqrt{\mu_2^2 - \sin^2 i}}\right)\right]\sin i$

256. $\frac{h\left(\mu^2 - 1\right)\sin^3 i}{\left(\mu^2 - \sin^2 i\right)^{3/2}}$

257. 5 second.

$$\hat{r} = \frac{\mu_1}{\mu_2}\left[\hat{e} - \left\{(\hat{e}.\hat{n}) + \sqrt{(\hat{e}.\hat{n})^2 + \left(\frac{\mu_2}{\mu_1}\right)^2 - 1}\right\}\hat{n}\right]$$

259. $\delta = i - A + \sin^{-1}\left[\frac{\mu_1}{\mu_2}\left\{(\sin A)\sqrt{\left(\frac{\mu}{\mu_1}\right)^2 - \sin^2 i} - \cos A \sin i\right\}\right]$

260. $\left(\frac{a}{8}, \frac{a}{8}\sqrt{15}\right)$

261. $\mu = \sqrt{\frac{7}{3}}$

262. $\frac{2a}{v}\left[\sqrt{1 - n^2 \sin^2 \frac{A}{2}} + \left(n \sin \frac{A}{2}\right)\sqrt{\left(\frac{b^2}{a^2} - 1\right)}\right]$

263. $\delta = 2 \sin^{-1}\left[\frac{b}{\mu R}\sqrt{\mu^2 - \frac{b^2}{R}} - \sqrt{1 - \frac{b^2}{R^2}}\right]$

264. $f = R\left(1 + \frac{1}{\sqrt{\mu^2 - \frac{b^2}{R^2}} - \sqrt{1 - \frac{b^2}{R^2}}}\right)$

265. R/2 behind the polished surface.

266. $a = \frac{\mu f}{\mu + 1}, e = \frac{1}{\mu}$

267. $\frac{1}{1 + \left(\frac{l}{4f}\right)}$

268. 25 cm from the second lens on the right side, magnification $m =$

269. 0.5 cm

270. 6 cm back side of unpolished lens.

271. $x = \frac{7\lambda}{12}, \frac{15\lambda}{4}$

272. $D\sqrt{3}$

273. $\theta = 0, 30°, 90°, 150°, 180°, 210°, 270°, 330°$

274. d

275. 34

276. $\dfrac{2D}{\sqrt{15}}$

277. $y = 10.5$ cm

278. $\beta_1 = \dfrac{2\lambda D}{3d}$, $\beta_2 = \dfrac{\lambda D}{3d}$

279. 0.52 mm

280. 0.6 mm

281. 0.15 μm

282. 0.21 mm

283. 3.33×10^{-5} N/m^2

284. 1.25×10^{-5} N/m^2

285. 1.81 eV

286. 8.3 msec

287. 2.86×10^{-5} T

288. 5.12×10^{-8} Amp.

289. 47.7 eV

290. 6.24×10^4 m/s.

291. $n = 4$, Z = 2

292. A is $_2He^4$, B is $_1H^2$

293. $\dfrac{\epsilon_0^2 n^6 h^6}{24\pi^4 e^4 m^3}$

94. $\dfrac{-me^4}{3\varepsilon_0^2 n^2 h^2}$

5. 140.7 pm

. 15×10^3 V

40 min

$N_0 \lambda t e^{-\lambda t}$

382.4 gm

.3 kg

EXPLANATIONS

EXPLANATIONS

1. Suppose the velocity of the dog is $(v_1\hat{i} + v_2\hat{j})$.
If after time 't' the dog can catch the rabit, then

$$\left(7.5\hat{i} + 10\hat{j}\right)t = \left(46\hat{i} + 28\hat{j}\right) + \left(v_1\hat{i} + v_2\hat{j}\right)t$$

Comparing the coeficiants of $\hat{i}$ and $\hat{j}$ on both sides we get

$$7.5t - 46 = v_1 t \quad ...(i)$$

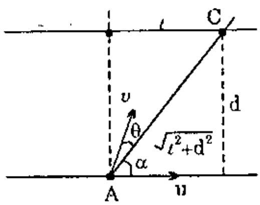
and $$10t - 28 = v_2 t \quad ...(ii)$$

also, we have $\sqrt{v_1^2 + v_2^2} = 5$

$\Rightarrow$ $$v_1^2 + v_2^2 = 25 \quad ...(iii)$$

Solving equations *(i)*, *(ii)* and *(iii)*, we get
$$t = 4s,\ 5.25s$$
Therefore $$t_{min} = 4s$$

3. Suppose swimmer's velocity v makes an angle θ with the line AC. The resultant velocity should be along AC. The components of velocity perpendicular to AC must cancel out.



$$v \sin\theta = u \sin\alpha \quad ...(i)$$

The resultant velocity along AC is ($u \cos\alpha + v \cos\theta$)

Therefore $$t = \frac{\sqrt{l^2 + d^2}}{u\cos\alpha + v\cos\theta} \quad ...(ii)$$

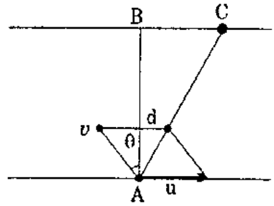
$\Rightarrow$ $$t = \frac{\sqrt{l^2 + d^2}}{u\cos\alpha + \sqrt{v^2 - u^2\sin^2\alpha}}$$

[using equn *(i)*]

$\Rightarrow$ $$t = \frac{l^2 + d^2}{ud + \sqrt{v^2d^2 + l^2\left(v^2 - u^2\right)}}$$

$$\left(\because \sin\alpha = \frac{d}{\sqrt{l^2 + d^2}}\right)$$

5. Suppose the man swims at an angle θ with the line A B and then walks the distance CB that he has been carried away by the stream.



Time taken to swim across the river

$$= \frac{d}{v\cos\theta}$$

The distance BC carried away by the stream is $\frac{d}{v\cos\theta}(u - v\sin\theta)$.

Time taken by the man to walk the distance BC is

$$\frac{d}{v\cos\theta}\left(\frac{u - v\cos\theta}{\omega}\right)$$

Therefore total time taken by the man to reach from A to B is

$$t = \frac{d}{v\cos\theta} + \frac{d}{v\cos\theta}\left(\frac{u - v\cos\theta}{\omega}\right)$$

For the minimum time, $\frac{dt}{d\theta} = o$

$$\Rightarrow \qquad \sin\theta = \left(\frac{v}{u+\omega}\right)$$

Substituting this value of θ, we get

$$\Rightarrow \qquad t = \frac{d(u+w)\left[w(w+u) - v\sqrt{(u+w)^2 - v^2}\right]}{vw^2\sqrt{(u+w)^2 - v^2}}$$

7. Suppose the steamer is moving at an angle α with AB with speed u as shown in the figures in two cases.

In the first case,

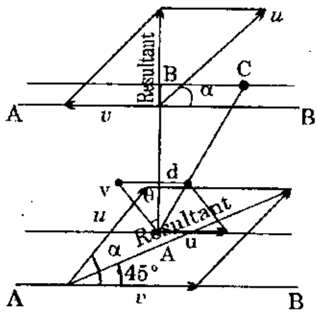
$$v = u\cos\alpha \qquad ...(i)$$

In the second case

$$\tan 45 = \frac{u\sin\alpha}{v + u\cos\alpha} \qquad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$\tan\alpha = 2$, and $u = v\sqrt{5}$



9. $$\frac{da}{dt} = \alpha$$

$\Rightarrow$ $$a = \alpha t$$

$\Rightarrow$ $$v = \frac{\alpha t^2}{2} \qquad ...(1)$$

$\Rightarrow$ $$s = \frac{\alpha t^3}{6} \qquad ...(2)$$

Now, $$< v >_{\text{time}} = \frac{s}{t} = \frac{1}{6}\alpha t^2 = \frac{v}{3} \qquad \text{[using (1) and (2)]}$$

Also, $$< v >_{\text{space}} = \frac{\int v ds}{s} = \frac{\int_0^s \left(\frac{9}{2}\alpha s^2\right)^{1/3} ds}{s} = \frac{3}{5}v$$

[using (1) and (2)]

11. $$x^2 = at^2 + 2bt + c \qquad ...(i)$$

Differentiating with respect to t, we get

$\Rightarrow$ $$x\frac{dx}{dt} = at + b$$

But $$\frac{dx}{dt} = v$$

$\therefore$ $$xv = at + b \qquad ...(ii)$$

Differentiating again with respect to t, we get

$$v^2 + x\frac{dv}{dt} = a$$

$\Rightarrow$ $$\frac{dv}{dt} = \frac{a - v^2}{x}$$

From equation (ii),

$\Rightarrow$ $$\frac{dv}{dt} = \frac{1}{x}\left[a - \left(\frac{at + b}{x}\right)^2\right] \qquad ...(iii)$$

From equation (i)

$$t = \frac{-b+\sqrt{b^2 - a\left(c-x^2\right)}}{a}$$

$$\Rightarrow \qquad (b + at) = \sqrt{b^2 - a\left(c - x^2\right)}$$

Substituting this value in equation *(iii)*, we get

$$\frac{dv}{dt} = \frac{ac - b^2}{x^3}$$

13. $$v^2 = 12x - 3x^3 \qquad ...(i)$$

$$\Rightarrow \qquad 2v\frac{dv}{dx} = \left(12 - 9x^2\right)v$$

$$\Rightarrow \qquad 2f = 12 - 9x^2$$

$$\Rightarrow \qquad x^2 = \frac{12 - 2f}{9} \qquad ...(ii)$$

Now, from equation *(i)*

$$v^4 = \left(12x - 3x^3\right)^2 = 9x^2\left(4 - x^2\right)$$

$$\Rightarrow \qquad v^4 = (12 - 2f)\left(4 - \left(\frac{12-2f}{9}\right)\right)$$

$$\Rightarrow \qquad v^4 = \frac{8(6-f)(12+f)^2}{81}$$

$$\Rightarrow \qquad v = \frac{1}{3}\left[8(6-f)(12+f)^2\right]^{1/4}$$

15. $$\frac{dv}{dt} = kt - \omega^2 x$$

$$\Rightarrow \qquad \frac{dv}{dt} + \omega^2 x = kt$$

$$\Rightarrow \qquad \frac{dv}{dt} + \omega^2 v = k \qquad \left(\because \frac{dx}{dt} = v\right)$$

Solution of this differential equation can be obtained as

$$v = \frac{k}{\omega^2}(1 - \cos\omega t)$$

Now $$\frac{dx}{dt} = \frac{k}{\omega^2}(1-\cos\omega t)$$

$\Rightarrow$ $$x = \frac{k}{\omega^3}(\omega t - \sin\omega t)$$

17. $$a_x = 5\cos 30^\circ = \frac{5\sqrt{3}}{2}$$

$$a_y = 5\sin 30 = \frac{5}{2}$$

$$x = (5\times 5)+\frac{1}{2}\left(\frac{5\sqrt{3}}{2}\right)(25) = 79.126 \text{ m}$$

$$y = \frac{1}{2}\left(\frac{5}{2}\right)(25) = 31.25$$

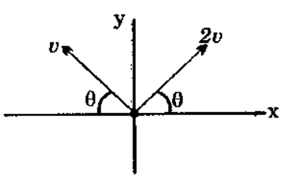
Displacement $$= \sqrt{x^2+y^2} = 85.1 \text{ m}$$

Distance, $$s = \int\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$s = \int_o^5\sqrt{\left(5+\frac{5\sqrt{3}}{2}t\right)^2+\left(\frac{5}{2}t\right)^2}\,dt$$

$\Rightarrow$ $$s = 85.3 \text{ m}$$

19. Suppose velocity of particle A is v and that of B is $2v$.



Then $\vec{v}_B = (2v\cos\theta)\hat{i} + (2v\sin\theta - gt)\hat{j}$

and $\vec{v}_A = (v\cos\theta)\left(-\hat{i}\right) + (v\sin\theta - gt)\hat{j}$

Now, $\vec{v}_B \cdot \vec{v}_A = 0$

$\Rightarrow$ $$-2v^2\cos^2\theta + (2v\sin\theta - gt)(v\sin\theta - gt) = 0$$

$\Rightarrow$ $$t = \frac{v}{2g}\left(3\sin\theta+\sqrt{1+7\cos^2\theta}\right) = \frac{2v}{g}\ (\because \theta = 30)$$

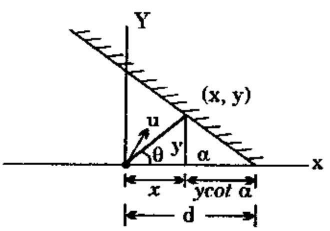
Relative velocity $v_{B/A} = (3v\cos\theta)\hat{i} + (v\sin\theta)\hat{j}$

$\therefore \quad |v_{B/A}| = v\sqrt{1+8\cos^2\theta} = v\sqrt{7}$

Separation between the particles

$$d = \left(v\sqrt{7}\right)\left(\frac{2v}{g}\right) = \left(2\sqrt{7}\right)\frac{v^2}{g}$$

21. Select a coordinate system with the origin at the point of projection. Suppose the coordinates of the point where projectile strikes the inclined plane is (x, y).



Now, $$y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad \text{...(i)}$$

and $$x = (u\cos\theta)t \quad \text{...(ii)}$$

From the geometry of the figure,

$$d = x + y\cot\alpha \quad \text{...(iii)}$$

From equations *(i)*, *(ii)* and *(iii)*

$$d = (u\cos\theta)t + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\cot\alpha \quad \text{...(iv)}$$

For $t_{\min}$, $$\frac{dt}{d\theta} = 0$$

$$\Rightarrow \quad \tan\theta = \cot\alpha$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} - \alpha$$

Now substituting this value of θ in equation *(iv)*

$$t_{min} = \left(\frac{u - \sqrt{u^2 - gd\sin 2\alpha}}{g\cos\alpha}\right)$$

23. Suppose the range is R corresponding to the angle of projection θ. Then

$$R = (u\cos\theta)t \qquad ...(i)$$

$$-h+\frac{1}{2}gt^2 = (u\sin\theta)t \qquad ...(ii)$$

From equations *(i)* and *(ii)*

$$R^2 + \left(\frac{1}{2}gt^2 - h\right)^2 = u^2t^2$$

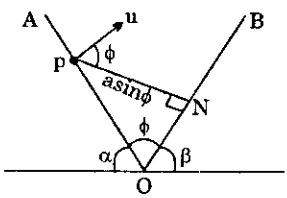
$$\Rightarrow \quad \frac{1}{4}g^2t^4 - (gh+u^2)t^2 + (R^2+h^2) = 0$$

For the maximum range, t^2 should be unique.

Therefore $(gh+u^2)^2 - g^2(R^2+h^2) = 0$

$$\Rightarrow \qquad R_{MAX} = \frac{u\sqrt{u^2+2gh}}{g}$$

25. PN is perpendicular to OB for motion parallel to OB.



$$O = (u\text{Sin}\ \phi) - (g\text{Sin}\ \beta)t \qquad ...(i)$$

For motion parallel to PN (perpendicular to OB)

$$a\sin\phi = (u\cos\phi)t - \frac{1}{2}(g\cos\beta)t^2$$

...*(ii)*

Also, $\phi = \pi - (\alpha+\beta)$ ...*(iii)*

From equations *(i)*, *(ii)*, and *(iii)*

$$u^2 = \frac{2ag\sin\beta}{\sin\alpha - \sin\beta(\cos(\alpha+\beta))} \qquad ...(iv)$$

Now from equations *(i)*, and *(iv)*

$$t = \sqrt{\frac{2a\sin^2(\alpha+\beta)}{g(\sin\alpha - \sin\beta\cos(\alpha+\beta))}}$$

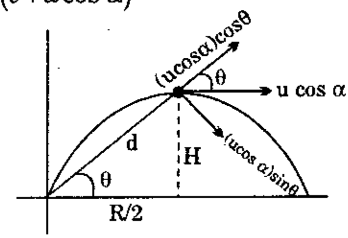
27. Time flight = (Time from beginning to impact with thewall)

+ (Time from wall to the floor)

$$\Rightarrow \quad \frac{2u\sin\alpha}{g} = \frac{x}{u\cos\alpha} + \frac{x}{2v+u\cos\alpha}$$

$$\Rightarrow \quad x = \frac{2u^2\sin\alpha\cos\alpha(2v+u\cos\alpha)}{g(v+u\cos\alpha)}$$

29. $d = \sqrt{\left(\frac{R}{2}\right)^2 + H^2}$



$$= \sqrt{\left(\frac{u^2\sin\alpha\cos\alpha}{g}\right)^2 + \left(\frac{u^2\sin^2\alpha}{2g}\right)^2}$$

$$= \frac{u^2\sin\alpha}{2g}\sqrt{(1+3\cos^2\alpha)}$$

Now, $\quad \sin\theta = \frac{H}{d}$

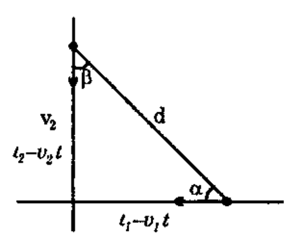
where $\quad H = \frac{u^2\sin^2\alpha}{2g}$

Now $\quad \omega = \frac{(u\cos\alpha)\sin\theta}{d} = \frac{H(u\cos\alpha)}{d}$

$$\Rightarrow \quad \omega = \frac{2g\cos\alpha}{u(1+3\cos^2\alpha)}$$

31. $\quad \omega = \frac{v_2\sin\beta - v_1\sin\alpha}{d}$

$$\Rightarrow \quad \omega = \frac{v_2(l_1 - v_1t) - v_1(l_2 - v_2t)}{d^2}$$



$$\omega = \frac{v_2l_1 - v_1l_2}{(l_2-v_2t)^2 + (l_1-v_1t)^2}$$

For ω_{max}; $\frac{d}{dt}\left((l_2-v_2t)^2 + (l_1-v_1t)^2\right) = 0$

$$t = \left(\frac{l_1v_1 + l_2v_2}{v_1^2 + v_2^2}\right)$$

Substituting, this value of t, we get

$$\omega_{MAX} = \left(\frac{v_1^2 + v_2^2}{l_1 v_2 - l_2 v_1}\right)$$

33. $\omega_1^2 R_1 = 40$

$\Rightarrow$ $\omega_1 = 20$ rad / s

$v = \omega_1 R_1 = 2$ m/s

$\omega_2 R_2 = v$

$\Rightarrow$ $\omega_2 = 40$ rad/s

Since $\alpha_2 R_2 = 30$

$\therefore$ $\alpha_1 R_1 = \alpha_2 R_2 = 30 \text{ m/s}^2$

Now $a_1 = \sqrt{(\omega_1^2 R_1)^2 + (\alpha_1 R_1)^2} = 50\,\text{m/s}$

$a_2 = \sqrt{(\omega_2^2 R_2)^2 + (\alpha_2 R_2)^2} = 85.4 \text{ m/s}^2$

Here, $\vec{\omega} = -2\hat{k}$, $\alpha = 4\hat{k}$

$\vec{r} = (4\hat{i} + 3\hat{j})$

Now, $\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) + (\vec{\alpha} \times \vec{r})$

$\Rightarrow$ $\vec{a} = (-28\hat{i} + 4\hat{j})$

37. It is given that $\vec{\omega} = 4\hat{k}$ rad / s, $\vec{\alpha} = -10\hat{k}$ rad / s^2, $\vec{r} = 6\hat{i}$ cm

$$\vec{v}_{rel} = \frac{d\vec{r}}{dt} = 5\hat{i} \text{ cm/s} \quad \vec{a}_{rel} = \frac{d^2 r}{dt^2} = 81\hat{i} \text{ cm/s}^2$$

Now, $\vec{v} = \vec{v}_{rel} + \vec{\omega} \times \vec{r}$

$\hat{v} = (5\hat{i} + 24\hat{j})$ cm / s^2

and $\vec{a} = \vec{a}_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r} + 2\vec{\omega} \times \vec{v}_{rel}$

$\Rightarrow$ $\vec{a} = -(15\hat{i} + 20\hat{j})$ cm / s^2

39. It is given that

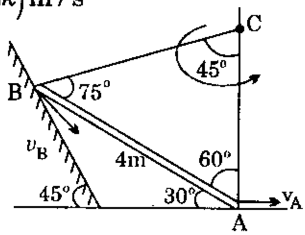
$$\vec{\omega} = \left(4\hat{i}+3\hat{k}\right)\text{rad/s}$$

and $$r = \left(3\hat{j}-4\hat{k}\right)\text{m}$$

therefore $$\vec{v} = \vec{\omega}\times\vec{r}$$

$\Rightarrow$ $$\vec{v} = \left(-9\hat{i}+16\hat{j}+12\hat{k}\right)\text{m/s}$$

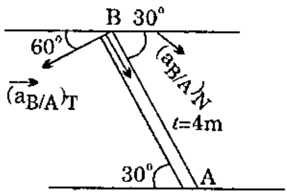
41. From geometry of the figure



$$CA = \left(\frac{\sin 75}{\sin 45}\right)4\text{ m}$$

$$= 5.46\text{ m}$$

Since, the instantaneous axis of rotation passes through point C.



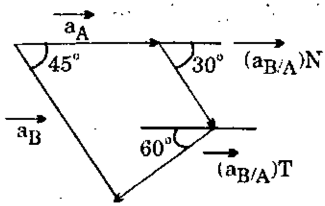
Therefore, $$\omega = \frac{v_A}{(CA)}$$

$\Rightarrow$ $$\omega = \frac{4}{5.46} = 0.732\text{ rad/s}$$

Also $$v_B = \omega\,(CB)$$

$$= \omega\left[\left(\frac{\sin 60}{\sin 45}\right)4\right]$$

$$= 3.6\text{ m/s}$$



Now $$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$= \vec{a}_A + \left(\vec{a}_{B/A}\right)_N + \left(\vec{a}_{B/A}\right)_T$$

Also, $$\left(a_{B/A}\right)_N = \omega^2 l$$

$$\left(a_{B/A}\right)_T = \alpha\, l$$

Using the vector diagram of accelerations, we have

$$(a_B)\cos 45 = (a_A) + \left(\omega^2 l\right)\cos 30 - (\alpha l)\cos 60 \qquad ...(i)$$

$$(a_B)\sin 45 = \left(\omega^2 l\right)\sin 30 + (\alpha l)\sin 60 \qquad ...(ii)$$

Solving equations *(i)* and *(ii)*, we get

$$\alpha = 1{\cdot}05\text{ rad/s}^2$$

and $$a_B = 6.65\text{ m/s}^2$$

43. The lengths of the rods remains same. For the rod BC the component of velocity of end C along CB must be equal to the component of velocity of end B along CB. Therefore,

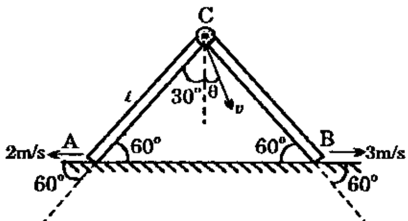
$$v \cos(30 - \theta) = 3 \cos 60 \qquad \text{...}(i)$$

Similarly for the rod AC

$$v \cos(30 + \theta) = 2 \cos 60 \qquad \text{...}(ii)$$

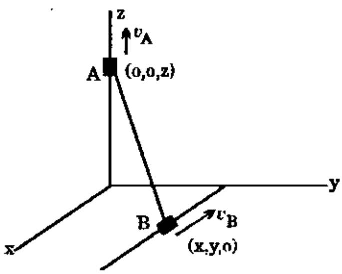
Solving equations *(i)* and *(ii)*, we get

$$v = \sqrt{\frac{7}{3}} \text{ M/s}$$



45. Suppose the coordinates of A and B are $(0, 0, z)$ and $(x, y, 0)$ respectively.

Then $\quad x^2 + y^2 + z^2 = l^2 \quad$ .....*(i)*



Now , it is given

$y = 0.6\ m, z = 0.6\ m, l = 1m$

Hence, from equation *(i)*, we get

$x = 0.53$ m

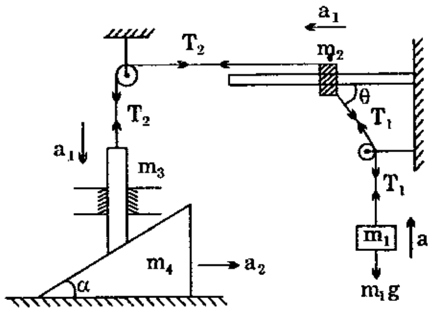
Since, the values of y and l are fixed, therefore,

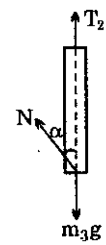
$$\frac{dx}{dt} = -\left(\frac{z}{x}\right)\frac{dz}{dt}$$

$\Rightarrow \qquad v_B = \left(\frac{z}{x}\right)v_A \qquad \left(\because \frac{dz}{dt} = v_A \text{ and } \frac{dx}{dt} = -v_B\right)$

Thus. $\qquad v_B = \left(\frac{0.6}{0.53}\right)(0.2) \text{ m/s} = 22.7 \text{ cm/s}$

47. From the geometry of the figure





$$a_1 = a\sec\theta \qquad ....(i)$$

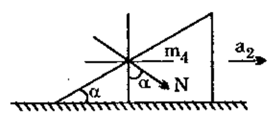
$$a_2 = a_1\cot\alpha \qquad ...(ii)$$

For the block of mass m_1

$$m_1a = T_1 - m_1g \qquad .....(iii)$$

For the slider 's'

$$m_2a_1 = T_2 - T_1\cos\theta \qquad ....(iv)$$



For the vertical motion of the block of mass m_3

$$m_3a_1 = m_3g - T_1 - N\cos\alpha \quad ....(v)$$

For the horizontal motion of the block of mass m_4

$$m_4a_2 = N\sin\alpha \qquad .....(vi)$$

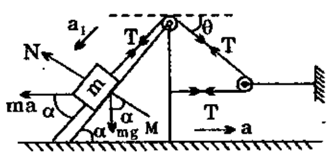
After solving these equations, we get

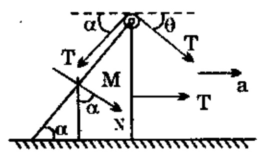
$$a = \frac{g(m_3 - m_1\cos\theta)\cos\theta}{\left(m_1\cos^2\theta + m_2 + m_3 + m_4\cot^2\alpha\right)}$$

49. Suppose acceleration of the wedge is 'a' and the acceleration of the block of mass m with respect to the wedge is a_1.

From the geometry of the figure.

$$a_1 = a\,(1 + \cos\theta) \qquad ....(i)$$





For the motion of the block of mass m,

$$ma_1 = mg\sin\alpha + ma\cos\alpha - T \qquad ...(ii)$$

$$N = mg\cos\alpha - ma\sin\alpha \qquad .....(iii)$$

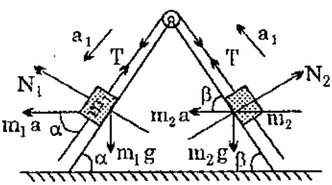
For the horizontal motion of the wedge,

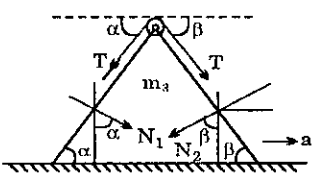
$$ma = T + T\cos\theta - T\cos\alpha + N\sin\alpha \quad ....(iv)$$

After solving these equations, we get

$$a = \frac{mg(1+\cos\theta)\sin\alpha}{M + m\left((1+\cos\theta-\cos\alpha)^2 + \sin^2\alpha\right)}$$

51. For the block m_1





$$m_1 a_1 = m_1 g\sin\alpha + m_1 a\cos\alpha - T \quad ...(i)$$

$$N_1 = m_1 g\cos\alpha - m_1 a\sin\alpha \quad ...(ii)$$

For the block m_2

$$m_2 a_1 = T - m_2 g\sin\beta + m_2 a\cos\beta \quad ...(iii)$$

$$N_2 = m_2 g\cos\beta + m_2 a\sin\beta \quad ...(iv)$$

For the horizontal motion of wedge,

$$m_3 a = T\cos\beta - T\cos\alpha + N_1\sin\alpha - N_2\sin\beta \quad ...(v)$$

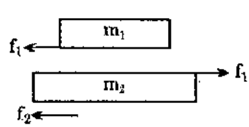
From these equation, we get

$$a = \left[\frac{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)(m_1\sin\alpha\cos\alpha - m_2\sin\beta\cos\beta) - (\sin\alpha+\sin\beta)(\cos\alpha-\cos\beta)}{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\left(m_{3+} m_1\sin^2\alpha + m_2\sin^2\beta\right) + (\cos\alpha-\cos\beta)^2}\right]g$$

53. The direction of friction forces are shown in the figure.

$$(f_1)_{MAX} = \mu_1(m_1 g + F\sin\alpha) \quad ...(i)$$

$$(f_2)_{MAX} = \mu_2(m_1 g + m_2 g + F\sin\alpha) \quad ...(ii)$$



The equations of motions are

$$F\cos a - f_1 = m_1 a \quad ...(iii)$$

$$f_1 - f_2 = m_2 a \quad ...(iv)$$

From equations *(iii)* and *(iv)*

$$F\cos\alpha - f_2 = (m_1 + m_2)\alpha \qquad ...(v)$$

If the plank moves on the ground, then

$$F\cos\alpha > (f_2)_{max}$$

$$\Rightarrow \qquad F > \frac{\mu_2(m_1+m_2)g}{(\cos\alpha - \mu_2\sin\alpha)}$$

From equation *(v)*

$$a = \frac{F\cos\alpha - (f_2)_{max}}{m_1+m_2} \qquad ...(vi)$$

Now from equation *(iii)*

$$f_1 = F\cos\alpha - m_1 a$$

$$\Rightarrow \qquad (F\cos\alpha - m_1 a) < (f_1)_{max}$$

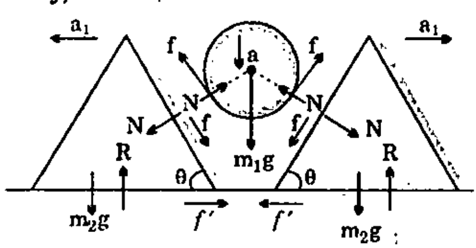
From equation *(iv)*

$$F\cos\alpha - m_1\left(\frac{F\cos\alpha - (f_2)_{max}}{m_1+m_2}\right) < (f_1)_{max}$$

Substituting $(f_1)_{max}$ and $(f_2)_{max}$, we get

$$f < \frac{m_1(m_1+m_2)(\mu_1 - \mu_2)g}{(m_2(\cos\alpha - \mu_2\sin\alpha) - m_1(\mu_1+\mu_2)\sin\alpha)}$$

55. From geometry,



$$a = a_1\tan\theta \qquad ...(i)$$

For cylinder

$$m_1 a = m_1 g - 2(N\cos\theta + f\sin\theta) \qquad ...(ii)$$

For prism

$$m_2 a_1 = N\sin\theta - f\cos\theta - f' \qquad ...(iii)$$

$$R = m_2 g + N\cos\theta + f\sin\theta \qquad ...(iv)$$

Also $f' = \mu R$ ...(v)

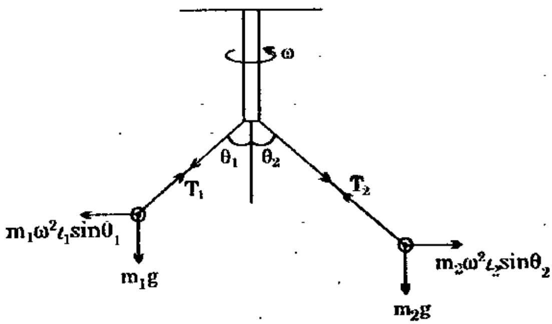
and $f = \mu N$ ...(vi)

From these equations, we get

$$a = \frac{g\left(m_1 k - 2\mu m_2 (\cos\theta + \mu \sin\theta)\right)\tan\theta}{mk \tan\theta + 2m_2(\cos\theta + \mu\sin\theta)}$$

where $k = \left(\sin\theta - 2\mu\cos\theta - \mu^2 \sin\theta\right)$

57.



$$T_1 \sin\theta_1 = T_2 \sin\theta_2 \qquad ...(i)$$

$$T_1 \cos\theta_1 = m_1 g \qquad ...(ii)$$

$$T_2 \cos\theta_2 = m_2 g \qquad ...(iii)$$

$$T_1 \sin\theta_1 = m_1 \omega^2 l_1 \sin\theta_1 \qquad ...(iv)$$

$$T_2 \sin\theta_2 = m_2 \omega^2 l_2 \sin\theta_2 \qquad ...(v)$$

From these equations

$$\omega = \left[\left(\frac{m_1^2 - m_2^2}{m_1^2 l_1^2 - m_2^2 l_2^2}\right) g^2\right]^{1/4}$$

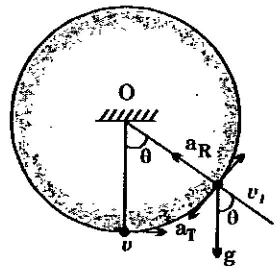
59.

$$mv\frac{dv}{ds} = -\mu\sqrt{\left(\frac{mv^2}{R}\right)^2 + (mg)^2}$$

$$\Rightarrow \quad \int_{v_0}^{0} \frac{v\,dv}{\sqrt{v^4 + R^2 g^2}} = -\int_{o}^{s} \frac{\mu\,ds}{R}$$

$$\Rightarrow \quad s = \frac{R\,ln}{2\mu}\left[\frac{v_o^2}{Rg} + \sqrt{1 + \left(\frac{v_o^2}{Rg}\right)^2}\right]$$

61. Suppose the velocity of the particle at the deflected position is v_1, From conservation of energy



$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 - mgl(1-\cos\theta)$$

$$\Rightarrow \quad v_1^2 = v^2 - 2gl(1-\cos\theta)$$

Radial acceleration at this position is

$$a_R = \frac{v_1^2}{l} = \frac{v^2}{l} - 2g(1-\cos\theta)$$

Tangential acceleration at the position is

$$a_T = g\sin\theta$$

Net acceleration

$$a = \sqrt{a_R^2 + a_T^2}$$

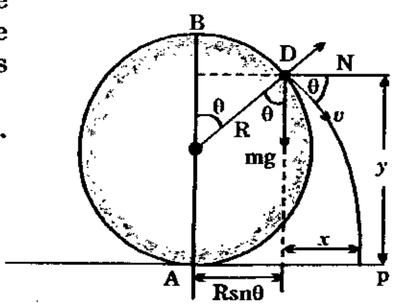
$$\Rightarrow \quad a = \sqrt{\left(\frac{v^2}{l} - 2g(1-\cos\theta)^2\right) + (g\sin\theta)^2}$$

Therefore, net force acting of the particle is

$$F = ma$$

$$\Rightarrow \quad F = mg\sqrt{\sin^2\theta + \left(\frac{v^2}{gl} - 2(1-\cos\theta)\right)^2}$$

63. Suppose the particle leaves the sphere at point D is shown in the figure. Then the particle follows a parabolic path.



If the speed of particle at D is v. then

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta) \quad ...(i)$$

and $$\frac{mv^2}{R} = mg\cos\theta - N$$

To leave the sphere $N = 0$,

$$\therefore \quad \frac{mv^2}{R} = mg\cos\theta \quad ...(ii)$$

From equation *(i)* and *(ii)*, we get

$$\cos\theta = \frac{2}{3}$$

$\therefore \qquad \sin\theta = \frac{\sqrt{5}}{3}$

Now for the parabolic path,

$$x = (v\cos\theta)t \qquad \text{...(iii)}$$

and $$y = (v\sin\theta)t + \frac{1}{2}gt^2 \qquad \text{...(iv)}$$

Also $$y = R(1+\cos\theta) \qquad \text{...(v)}$$

From equations (*iii*), (*iv*) and (*v*) , we get

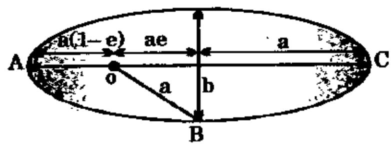
$$x = 4\left(\frac{5\sqrt{2}-\sqrt{5}}{27}\right)R$$

Now $$AP = R\sin\theta + x$$

$$\Rightarrow \qquad AP = \frac{5}{27}\left(\sqrt{5}+4\sqrt{2}\right)R$$

65. From geometry

$$OC - OA = 2ae$$



$$OB - OA = \left(\sqrt{b^2 + (ae)^2}\right) - a(1-e)$$

$$= a - a(1-e) = ae$$

$$\left(\because e = \sqrt{1 - \frac{b^2}{a^2}}\right)$$

From conservation of energy, for the points A and C

$$\frac{1}{2}mv_o^2 = \frac{1}{2}k(2ae)^2 \qquad \text{...(i)}$$

For the points A and B

$$\frac{1}{2}mv_o^2 + mgb = \frac{1}{2}mv^2 + \frac{1}{2}k(ae)^2 \qquad \text{...(ii)}$$

From equations *(i)* and *(ii)*, we get

$$v = \sqrt{\frac{3}{4}v_o^2 + 2gb}$$

67. Suppose λ is the mass perunit length of the chain. Then for equilibrium

$$\mu(1-\eta)l\lambda g = \eta l\lambda g$$

$$\mu = \frac{\eta}{1-\eta}$$

Work done against friction,

$$W = \int_o^{(1-\eta)l} \mu(\lambda x) g dx = \frac{1}{2}\mu\lambda g(1-\eta)^2 l^2 = \frac{\eta(1-\eta)\lambda g l^2}{2}$$

Decrease in P.E.,

$$\Delta U = U_{\text{inital}} - U_{\text{final}}$$

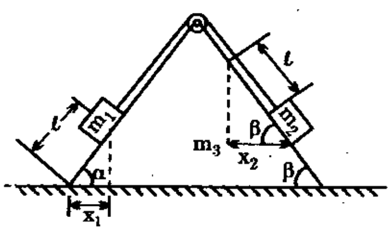
$$= \left[-(\eta l)\lambda g\left(\frac{\eta l}{2}\right)\right] - \left[-l\lambda g\left(\frac{l}{2}\right)\right] = \frac{\lambda l^2 g}{2}(1-\eta^2)$$

$$KE = \Delta U - W = \frac{\lambda g l^2(1-\eta)}{2}$$

$$\Rightarrow \qquad \frac{1}{2}(l\lambda)v^2 = \frac{\lambda g l^2}{2}(1-\eta)$$

$$\Rightarrow \qquad v = \sqrt{gl(1-\eta)}$$

69. Suppose the wedge gets displaced towards right by a distance x and the horizontal displacements of m_1 and m_2 are x_1 and x_2 respectively towards left with respect to the wedge. The centre of mass of the system remains undisplaced in the horizontal direction.



Therefore

$$m_1(x_1 - x) + m_2(x_2 - x) = m_3 x$$

$$\Rightarrow \qquad x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2 + m_3}$$

Now, from geometry

$$x_1 = l\cos\alpha \text{ and } x_2 = l\cos\beta$$

$$\therefore \qquad x = \frac{l(m_1\cos\alpha + m_2\cos\beta)}{m_1 + m_2 + m_3}$$

71. Change in momentum

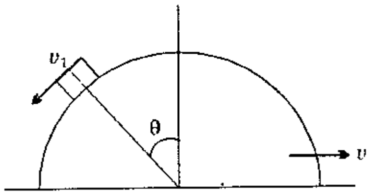
$$= (\text{force}) \times (\text{time})$$

$$\Rightarrow \qquad 3 \times 3 + (2+5)u = (5 \times 10) \times 1$$

$$\Rightarrow \qquad u = \frac{41}{7} \text{ m/s}$$

$$\text{KE} = \frac{1}{2}(3)(3)^2 + \frac{1}{2}(2+5)\left(\frac{41}{7}\right)^2 = \mathbf{133.57\ J}$$

73. Suppose the speed of the particle with respect to the hemisphere is v_1 and speed of hemisphere is v.



From conservation of momentum

$$mv = m(v_1 \cos\theta - v) \qquad ...(i)$$

From conservation of energy

$$\frac{1}{2}mv^2 + \frac{1}{2}m\left[(v_1\cos\theta - v)^2 + (v_1 \sin\theta)^2\right] = mgR(1-\cos\theta) \qquad ...(ii)$$

To leave the contact

Normal Reaction, N = O

$$\frac{mv_1^{\ 2}}{R} = mg\cos\theta \qquad ...(iii)$$

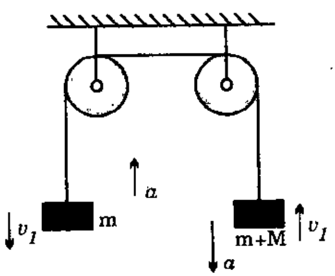
From equations *(i)*, *(ii)* and *(iii)*, we get

$$\cos^3\theta - 6\cos\theta + 4 = 0$$

$$\Rightarrow \qquad \cos\theta = \sqrt{3} - 1$$

$$\Rightarrow \qquad \theta = \cos^{-1}\left(\sqrt{3}-1\right) = 43°$$

75. Suppose the velocity of the platform just after jumping of the man is v_1.



Then $\quad m(v_0 - v_1) = [(M+m)+m]v_1 \qquad ...(i)$

Retardation of the plat form

$$a = \frac{(M+m)g-mg}{M+2m} = \frac{Mg}{M+2m} \quad ...(ii)$$

Now $$h = \frac{v_1^2}{2a}$$

$\Rightarrow$ $$h = \frac{M(M+2m)\,v_0^2}{8(M+m)}$$

77. Total time

$$T = 2\frac{(v\sin\alpha)g}{g} + \frac{2e(v\sin\alpha)}{g} + \frac{2e^2(v\sin\alpha)}{g} + ... = \frac{2v\sin\alpha}{g(1-e)}$$

Total distance,

$$d = \frac{2(v\sin\alpha)(v\cos\alpha)}{g} + \frac{2e(v\sin\alpha)(v\cos\alpha)}{g} + \frac{2e^2(v\sin\alpha)(v\cos\alpha)}{g} + ...$$

$$= \frac{v^2\sin 2\alpha}{g(1-e)}$$

79. Suppose the velocity of m_1, just after collision with bullet is v_1. Then

$$mv_0 = (m+m_1)v_1 \quad ...(i)$$

When the block m stop sliding with respect to the plank, then common velocity v_2 is given by

$$mv_0 = (m+m_1+m_2)v_2 \quad ...(ii)$$

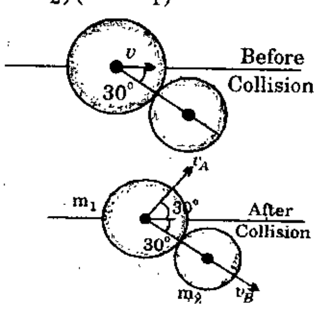
If the displacement of m_1 with respect to m_2 is x, then

$$\frac{1}{2}(m+m_1)v_1^2 - \frac{1}{2}(m+m_1+m_2)v_2^2 = \mu(m+m_1)gx \quad ...(iii)$$

From equations *(i)*, *(ii)* and *(iii)*, we get

$$x = \frac{m^2 m_2 v_o^2}{2\mu g(m+m_1+m_2)(m+m_1)^2}$$

81. Suppose along the line joining the centres of the sphere, the component of velocity of sphere A is v_1 and perpendicular to this line is v_2. The resultant of v_1 and v_2 is v_A as shown in the figure. The component of velocity perpendicular to the centre line remains unchanged.

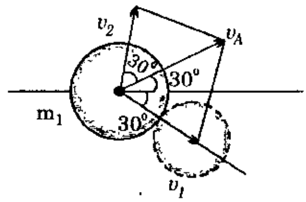


Therefore $\quad v_2 = v\sin 30°$ ...(i)

Along the centre line

$$m_1 v\cos 30° = m_1 v_1 + m_2 v_B \quad ...(ii)$$

and $\quad v\cos 30° = v_B - v_1$ ...(iii)



From equations (ii) and (iii), we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v\cos 30° \quad ...(iv)$$

Now, since v_A is resultant of v_1 and v_2, therefore

$$v_2 \sin 30° = v_1 \sin 60°$$

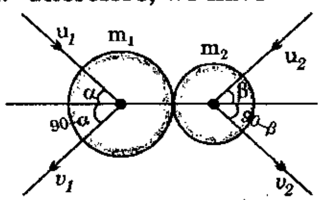
$$\Rightarrow \quad (v\sin 30°)\sin 30° = \left[\left(\frac{m_1 - m_2}{m_1 + m_2}\right) v\cos 30°\right]\sin 60°$$

Using equations (i) and (iv), we get

$$m_1 = 2m_2$$

$$\Rightarrow \quad \frac{m_1}{m_2} = 2$$

83. There is no impulse in the direction perpendicular to the common normal. Therefore, we have



$$u_1 \sin\alpha = v_1 \sin(90 - \alpha) \quad ...(i)$$

and $$u_2 \sin\beta = v_2 \sin(90 - \beta) \quad ...(ii)$$

From the conservation of momentum

$$m_2 v_2 \cos(90 - \beta) - m_1 v_1 \cos(90 - \alpha)$$
$$= m_1 u_1 \cos\alpha - m_2 u_2 \cos\beta \quad ...(iii)$$

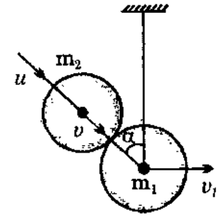
Form the definition of coefficient of restitution

$$v_2 \cos(90 - \beta) + v_1 \cos(90 - \alpha)$$
$$= e[u_1 \cos\alpha + u_2 \cos\beta] \quad ...(iv)$$

From equations (i), (ii), (iii) and (iv), we get

$$e = \frac{m_1 \sin^2\beta + m_2 \sin^2\alpha}{m_1 \cos^2\beta + m_2 \cos^2\alpha}$$

85. Suppose after the impact the velocity of sphere m_1 is v_1 in the horizontal direction and the velocity of sphere m_2 is v along the common normal.



Applying conservation of momentum in the horizontal direction

$$m_1 v_1 + m_2 v \sin\alpha = m_2 u \sin\alpha \qquad ...(i)$$

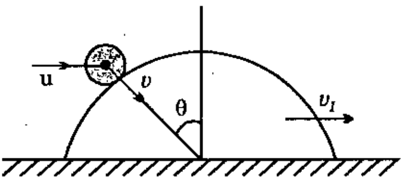
Along the common normal,

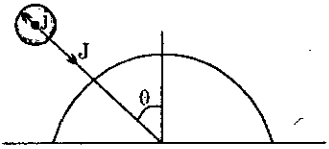
$$v_1 \sin\alpha - v = eu \qquad ...(ii)$$

Solving equations *(i)* and *(ii)*, we get

$$v_1 = \frac{m_2(1+e)u\sin\alpha}{m_1 + m_2\sin^2\alpha}$$

87. Suppose the velocity of the shpere after impact is v along the common normal and the velocity of the hemisphere is v_1 along the horizontal direction. If the impulse due to impact is J, then for the hemisphere





$$Mv_1 = J\sin\theta \qquad ...(i)$$

and for the sphere along the common normal

$$mv = mu\sin\theta - J \qquad ...(ii)$$

Also $\qquad v_1 \sin\theta - v = eu\sin\theta \qquad ...(iii)$

From equations *(i)*, and *(ii)* and *(iii)*, we get

$$v_1 \frac{(1+e)mu\sin^2\theta}{M + m\sin^2\theta}$$

89. Suppose the velocity of the trolly after time t is v towards right. The velocity of the container is also, v vertically downward.

If the tension in the string at this moment is T, then the equation of motion of the trolly will be

$$(m_0 + \mu t)\frac{dv}{dt} + \mu v = \mathrm{T} \qquad ...(i)$$

and equation of motion of the container will be

$$(m_0 - \mu t)\frac{dv}{dt} = (m_0 - \mu t)g - \mathrm{T} \qquad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$\frac{dv}{dt}+\left(\frac{\mu}{2m_0}\right)v=\left(\frac{1}{2}-\frac{\mu t}{2m_0}\right)g$$

Solving this differential equation, we get

$$v=\frac{3m_0 g}{\mu}\left(1-e^{-\mu t/2m_0}\right)-gt$$

91. The equation of motion of the disc is

$$\frac{d}{dt}(mv)=F-\frac{udm}{dt}$$

$\Rightarrow$ $$Fdt=d(mv)+udm$$

$\Rightarrow$ $$F.t+m_0u=m(v+u) \quad ...(i)$$

But at $t=0,\ v=0, m=m_0$

Now $$\frac{dm}{dt}=\rho A(v+u)$$

$\Rightarrow$ $$\frac{dm}{dt}=\rho A\left(\frac{Ft+m_0u}{m}\right)$$

$\Rightarrow$ $$\int_{m0}^{m} mdm = \int_0^t \rho A(Ft+m_0u)dt$$

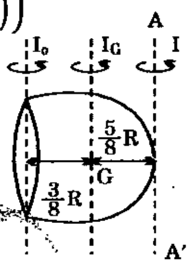
$\Rightarrow$ $\int_0^t \rho A(Ft+m_0u)dt$

$\Rightarrow$ $$m^2=m_0{}^2+\rho A\left(Ft^2+2m_0ut\right) \quad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$v=\left(\frac{Ft+m_0u}{m_0{}^2+\rho A\left(Ft^2+2m_0ut\right)}\right)-u$$

93. Moment of Inertia about an axis through the flat face of hemisphere



$$I_o=\frac{2}{5}mR^2.$$

From the parallel axis theorem, the moment of inertial through c.m. of the hemisphere

$$I_G=I_o-m\left(\frac{3}{8}R\right)^2=\left(\frac{83}{320}\right)mR^2$$

Using parallel axis theorem, Moment of inertia about the axis AA′,

$$I = I_G + m\left(\frac{5}{8}R\right)^2$$

$$\Rightarrow \qquad I = \left(\frac{13}{20}\right)mR^2$$

95. Suppose the velocity of the particle when it reaches to the topic top is v.

From conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgR\cos\theta \qquad ...(i)$$

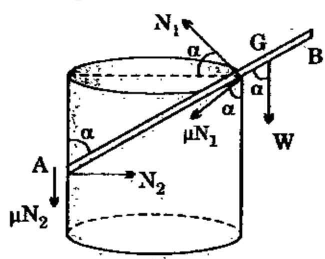
Now, applying conservation of angular momentum about the verticle axis through O, we have

$$mv_0(R\sin\theta) = mvR \qquad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$v_0 = \sqrt{2gR\sec\theta}$$

97. Suppose the radius of the cylinder is R and length of rod is $2l$. Consider the case when the end A slids up. Forces acting on the rod are shown in the figure.



Resolving forces horizontally and vertically, we have

$$N_2 = N_1\cos\alpha + \mu N_1\sin\alpha \qquad ...(i)$$

and $$N_1\sin\alpha = \mu N_2 + \mu N_1\cos\alpha + W \qquad ...(ii)$$

Taking moment about A,

$$N_1(2R\ \mathrm{cosec}\alpha) = W(l\sin\alpha) \qquad ...(iii)$$

From equations *(i)*, *(ii)* and *(iii)*, we get

$$2R = l\left[(1-\mu^2)\sin\alpha - 2\mu\cos\alpha\right]\sin^2\alpha \qquad ...(iv)$$

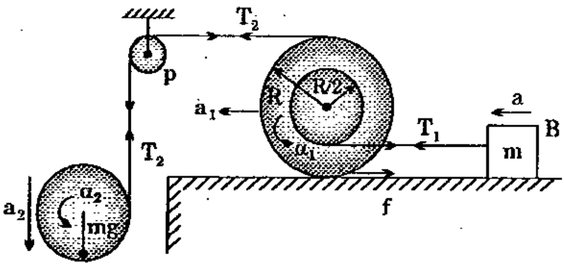
Similarly, when the rod makes least angle β, we get

$$2R = l\left[(1-\mu^2)\sin\beta + 2\mu\cos\beta\right]\sin^2\beta \qquad ...(v)$$

From equations *(iv)* and *(v)*, we get

$$\mu = \tan\left[\frac{1}{2}\tan^{-1}\left(\frac{\sin^3\alpha - \sin^3\beta}{\sin^2\alpha\cos\alpha + \sin^2\beta\cos\beta}\right)\right]$$

99. Suppose the acceleration of the block B is a, acceleration of disc is a_2 and the acceleration of centre of mass of spool is a_1, also suppose the angular accelerations of spool and disc are α_1 and α_2 respectively.



Now $$a = \alpha_1\left(R - \frac{R}{2}\right)$$

$\Rightarrow$ $$a = \frac{\alpha_1 R}{2} \qquad \text{...(i)}$$

$$a_1 = \alpha_1 R \qquad \text{...(ii)}$$

$$a_2 = \alpha_1(2R) + \alpha_2(R) \qquad \text{...(iii)}$$

For the block

$$ma = T_1 \qquad \text{...(iv)}$$

For the spool

$$ma_1 = T_2 - T_1 - f \qquad \text{...(v)}$$

$$\left(\frac{mR^2}{2}\right)\alpha_1 = T_2R + T_1R + fR \qquad \text{...(vi)}$$

For the disc

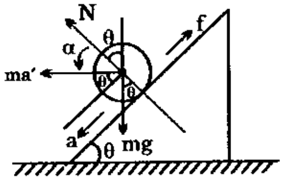
$$ma_2 = mg - T_2 \qquad \text{...(vii)}$$

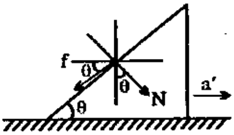
$$\left(\frac{1}{2}mR^2\right)\alpha^2 = T_2R \qquad \text{...(viii)}$$

From these equations, we get

$$a = \frac{4}{37}g$$

101. Suppose the acceleration of cylinder with respect to the wedge is a and the acceleration of the wedge is a'. If the radius of the cylinder is R, then the angular acceleration of the cylinder will be





$$\alpha = \frac{a}{R} \qquad ...(i)$$

For cylinder

$$\left(\frac{1}{2}mR^2\right)\alpha = fR \qquad ...(ii)$$

$$ma = mg\sin\theta + ma'\cos\theta - f \qquad ...(iii)$$

$$N = mg\cos\theta - ma'\sin\theta \qquad ...(iv)$$

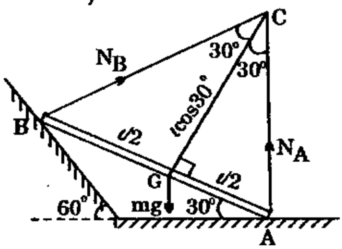
For wedge

$$Ma' = N\sin\theta - f\cos\theta \qquad ...(v)$$

From these equations, we get

$$a' = \frac{2mg\sin\theta\cos\theta}{3M + m\left(1 + 2\sin^2\theta\right)}$$

103. Suppose C is the point through which the instantaneous axis of rotation passes and G is the centre of mass of the rod. From the geometry of the figure



$$CG = l\cos 30°$$

The moment of inertia about C

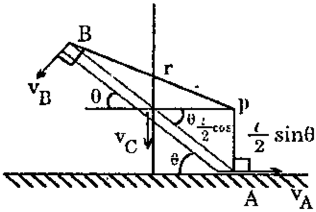
$$I = \frac{ml^2}{12} + m\,(l\cos 30°)^2 = \frac{5}{6}ml^2$$

If α is the angular acceleration, then

$$\left(\frac{5}{6}ml^2\right)\alpha = mg\left(\frac{l}{2}\cos 30^0\right)$$

$$\Rightarrow \qquad \alpha = \frac{3\sqrt{3}g}{10\,l}.$$

105. Suppose the centre of mass comes down by a distance h and the moment of inertia about the instantaneous axis of rotation (through point P) is I_p. Then, from conservation of energy



$$mgh = \frac{1}{2} I_p \omega^2$$

$$\Rightarrow \quad mg\frac{l}{2}(1-\sin\theta) = \frac{1}{2}\left[\frac{ml^2}{12} + m\left(\frac{l}{2}\cos\theta\right)^2\right]\omega^2$$

$$\Rightarrow \quad \omega = 2\sqrt{\frac{3g(1-\sin\theta)}{l\left(1+3\cos^2\theta\right)}}$$

From geometry, $\quad PA = \frac{l}{2}\sin\theta$

and $\quad PB = r = \frac{l}{2}\sqrt{1+3\cos^2\theta}$

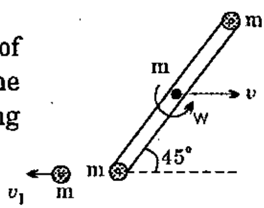
Now $\quad v_A = \omega\left(\frac{l}{2}\sin\theta\right)$

$$\Rightarrow \quad v_A = \sqrt{\frac{3gl(1-\sin\theta)\sin^2\theta}{1+3\cos^2\theta}}$$

Also $\quad v_B = \omega r = \omega\left(\frac{l}{2}\sqrt{1+3\cos^2\theta}\right)$

$$\Rightarrow \quad v_B = \sqrt{3gl(1-\sin\theta)}$$

107. Suppose after collision the angular velocity of rod is ω, the velocity of centre of mass of the rod is v and the velocity of the striking particle is v_1 in the backward direction.
From conservation of linear momentum



$$mv_0 = (3m)v - mv_1 \qquad ...(i)$$

From conservation of angular momentum about centre of mass

$$(mv_0)\frac{l}{2}(\sin 45^o) = \left(2\left(m\left(\frac{l}{2}\right)^2\right) + \frac{ml^2}{12}\right)\omega - (mv_1)\frac{l}{2}(\sin 45^o) \quad ...(ii)$$

From conservation of mechanical energy

$$\frac{1}{2}mv_0^{\ 2} = \frac{1}{2}mv_1^{\ 2} + \frac{1}{2}(3m)v^2 + \frac{1}{2}\left(2m\left(\frac{l}{2}\right)^2 + \frac{ml^2}{12}\right)\omega^2 \quad ...(iii)$$

From equations *(i)*, *(ii)* and *(iii)*

$$\omega = \frac{36\sqrt{2}\,v_o}{65l}$$

109. Suppose the particle strikes the hemisphere with a velocity v_0 and the angular velocity of hemisphere just after collision is ω. Then

$$v_0 = \sqrt{2gh} \quad ...(i)$$

From conservation of angular momentum about the line joining the supports

$$mv_0R = (mR^2)\omega + \left(\frac{2}{5}MR^2\right)\omega \quad ...(ii)$$

Applying conservation of energy after collision

$$\frac{1}{2}(mR^2)\omega^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = Mg\left(\frac{3}{8}R\right) - mgR \quad ...(iii)$$

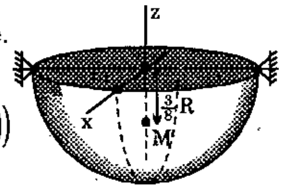
From equations *(i)*, *(ii)*, *(iii)* after substituting the given values we get

$$h = 42\ m$$

and $$\omega R = \frac{5}{21}v_0$$

Now select the axis as shown in the figure.

The reactive impulse



$$\vec{J} = \left[m\omega R(-\hat{k}) + M\left(\frac{3}{8}R\omega\right)(-\hat{i})\right] - (mv_0(-\hat{k}))$$

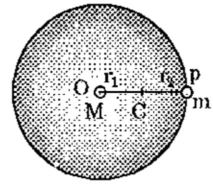
Substituting the numerical values, we get

$$\vec{J} = \frac{v_0}{21}(16\hat{k} - 15\hat{i})$$

when $$v_0 = \sqrt{2gh}$$

Then $$|\vec{J}| = 30\,\text{kg m/s}$$

111. Suppose the centre of mass of the system is at C as shown in the figure.



$$r_1 + r_2 = R \qquad ...(i)$$

and $$Mr_1 = mr_2 \qquad ...(ii)$$

From these equations $r_1 = \dfrac{mr}{m+M}$

and $$r_2 = \frac{MR}{m+M}$$

Suppose the angular velocity of backward spin of the ring is ω, then from conservation of angular momentum about the centre of mass of the system

$$mvr_2 - \left(mr_2^2\right)\omega = \left(MR^2 + Mr_1^2\right)\omega$$

Substituting the values of r_1 and r_2, we get

$$\omega = \frac{mv}{R(M+2m)}$$

Now suppose the angular velocity of the insect with respect to the ground is ω_1 then

$$\omega_1 = \frac{v}{R} - \omega$$

Substituting the value of ω, we get

$$\omega_1 = \frac{v}{R}\left(\frac{M+m}{M+2m}\right)$$

113. When the slipping ceases

$$\omega'_1 R_1 = \omega'_2 R_2 \qquad ...(i)$$

If the friction force is f between the cylinders, then

$$I_1(\omega'_1 - \omega_1) = -fR_1 t \qquad ...(ii)$$

and $$I_2(\omega'_2 - \omega_2) = fR_2 t \qquad ...(iii)$$

Also $$I_1 = \frac{1}{2}m_1R_1^2 = \frac{1}{2}\left(\pi R_1^2 l\rho\right)R_1^2 \qquad ..(iv)$$

and $$I_2 = \frac{1}{2}m_2R_2^2 = \frac{1}{2}\left(\pi R_2^2 l\rho\right)R_2^2 \qquad ..(5)$$

From these equations, we get

$$\omega'_1 = \left(\frac{\omega_1R_1^3 + \omega_2R_2^3}{R_1\left(R_1^2 + R_2^2\right)}\right)$$

and $$\omega'_2 = \left(\frac{\omega_1R_1^3 + \omega_2R_1^3}{R_2\left(R_1^2 + R_1^2\right)}\right)$$

115.
$$g = \frac{GM}{R^2} \quad \text{... (i)}$$

and
$$v_e = \sqrt{2gR} \quad \text{..(ii)}$$

Suppose the velocity of the body at a distance from the earth's center is v. Then from conservation of energy

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_e^{\,2} - \frac{GMm}{R}$$

$$\Rightarrow \quad v = R\sqrt{\frac{2g}{r}}$$

$$\Rightarrow \quad \frac{dr}{dt} = R\sqrt{\frac{2g}{r}}$$

$$\Rightarrow \quad \int_0^t dt = \frac{1}{R\sqrt{2g}}\int_R^{R+h}\sqrt{r}\,dr$$

$$\Rightarrow \quad t = \frac{1}{3}\sqrt{\frac{2R}{g}}\left[\left(l+\frac{h}{R}\right)^{3/2} - 1\right]$$

117. Suppose tension is F in the rod at a distance x from the end about which the rod is being rotated. Then

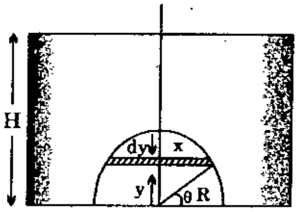
$$F = \int_x^l \left(\frac{m\,dx}{l}\right)\omega^2 x = \frac{m\omega^2 l^2}{2}\left(1 - \frac{x^2}{l^2}\right)$$

Now the elongation in the rod

$$\Delta l = \int_0^l \left(\frac{m\omega^2 l^2}{2}\right)\left(1 - \frac{x^2}{l^2}\right)\frac{dx}{YA} = \frac{m\omega^2 l^2}{3AY}$$

119. Consider a rectangular strip on the semi-circular gate as shown in the figure.

$$y = R\sin\theta \quad \text{...(i)}$$
$$x = R\cos\theta \quad \text{...(ii)}$$
$$d\tau = y(dF)$$



$$\Rightarrow \quad d\tau = y\ \rho g(H-y)(2x\,dy)$$

Now using equations *(i)* and *(ii)*, we get

$$T = 2R^3\rho g \int_0^{\pi/2}(H - R\sin\theta)\sin\theta\cos^2\theta\,d\theta$$

$$\Rightarrow \quad T = 2R^3\rho g\left(\frac{H}{3} - \frac{\pi R}{16}\right)$$

121. Suppose at any instant h is the difference in liquid levels, h_1 is height of level in left container and h_2 is height of level in second container.
Then

$$dh = dh_1 + dh_2$$

and $$A_1\, dh_1 = A_2\, dh_2$$

$\Rightarrow$ $$\frac{dh_1}{dt} = \left(\frac{A_2}{A_1 + A_2}\right)\frac{dh}{dt} \qquad ...(i)$$

Also $$A_1 \frac{dh_1}{dt} = -a\sqrt{2gh} \qquad ...(ii)$$

From equations *(i)* and *(ii)*

$$\left(\frac{A_1A_2}{A_1 + A_2}\right)\frac{dh}{dt} = -a\sqrt{2gh}$$

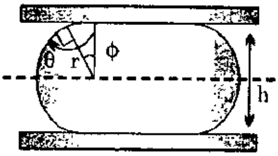
$\Rightarrow$ $$t = \frac{-A_1A_2}{(A_1 + A_2)a}\int_H^0 \frac{dh}{\sqrt{2gh}}$$

$\Rightarrow$ $$t = \frac{A_1A_2}{(A_1 + A_2)a}\sqrt{\frac{2H}{g}}$$

123. If the new radius of the mercury tablet is R′ , then

$$(\pi R^2)h = \pi(R')^2 \frac{h}{n} \Rightarrow R' = R\sqrt{n} \qquad ...(i)$$

From the geometry



$$r = \frac{h}{2\cos\phi} \qquad ...(ii)$$

$$\theta = 180 - \phi \qquad ...(iii)$$

Initial excess pressure inside drop

$$p = \frac{T}{r} = \frac{2T\cos\phi}{h} \qquad \text{[using equn } (ii)]$$

Initial force $$F = p(\pi R^2) = \frac{2\pi R^2 T\cos\phi}{h}$$

Similarly final force $$F' = \frac{2\pi(R')^2 T\cos\phi}{h/n}$$

Now $$F' - F = mg$$

$\Rightarrow$ $$\frac{2\pi(R')^2 T\cos\phi}{\left(\frac{h}{n}\right)} - \frac{2\pi R^2 T\cos\phi}{h} = mg$$

$$\cos\phi = \frac{mgh}{2\pi R^2 T(n^2-1)} \qquad \text{[using equn } (ii)]$$

Hence, from equation *(iii)*

$$\theta = \pi - \cos^{-1}\left(\frac{mgh}{2\pi R^2 + (n^2-1)}\right)$$

125. Consider an elementary cylindrical layer of radius r.
Then $\tau = fr$

$$\Rightarrow \qquad \tau = \left[h(2\pi rl)\left(\frac{rd\omega}{dr}\right)\right]r$$

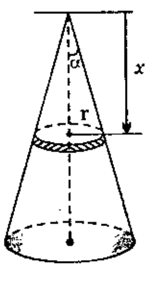
$$\Rightarrow \qquad \tau\int_a^b \frac{dr}{r^3} = 2\pi\eta l\int_0^{w} dw$$

$$\Rightarrow \qquad \tau = 4\pi\eta l\omega\left(\frac{a^2b^2}{b^2-a^2}\right)$$

127. Consider an elementary disc of a distance x from the vertex. Radius of this elementary disc

$r = x\tan\alpha$

Now moment of inertia about the line through suspension point



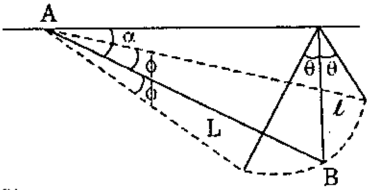
$$I = \int_0^h \frac{m}{\left(\frac{1}{3}\pi h^3\tan^2\alpha\right)}(\pi r^2 dx)\left(x^2 + \frac{r^2}{4}\right)$$

$$= \int_0^h \frac{m}{\left(\frac{1}{3}\pi h^3\tan^2\alpha\right)}(\pi x^2\tan^2\alpha\, dx)\left(x^2 + \frac{x^2\tan^2\alpha}{4}\right)$$

$$= \frac{3}{20}mh^2(4+\tan^2\alpha)$$

$$\text{Now} \qquad T = 2\pi\sqrt{\left(\frac{I}{mg\left(\frac{3}{4}h\right)}\right)}$$

$$\Rightarrow \qquad T = 2\pi\sqrt{\frac{h}{5g}(4+\tan^2\alpha)}$$



129. Suppose the length of the rod is L and the mass of the rod is m.

In equilibrium, tension in the thread

$$F = \frac{mg}{2} \qquad ...(i)$$

From geometry,

and $\qquad l\theta = L\phi \qquad ...(ii)$

Now, equation of motion is

$$\left(\frac{mL^2}{3}\right)\frac{d^2\phi}{dt^2} = -(F\sin\theta)L$$

$$= -FL\theta \qquad (\because \sin\theta \approx \theta)$$

$$= -\left(\frac{mg}{2}\right)L\left(\frac{L\phi}{l}\right) \qquad \text{[Using (i) \& (ii)]}$$

$$= -\left(\frac{mgL^2}{2l}\right)\phi$$

$$\Rightarrow \qquad \frac{d^2\phi}{dt^2} = -\left(\frac{3g}{2l}\right)\phi$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{3g}{2l}}$$

Thus, $\qquad T = \frac{2\pi}{\omega}$

$$\Rightarrow \qquad T = 2\pi\sqrt{\frac{2l}{3g}}$$

131. Suppose $\qquad \rho_1 = \rho$

Then $\qquad \rho_2 = \alpha\rho$

$$\rho_3 = \alpha^2\rho$$

For equilibrium

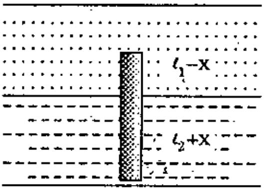
$$A(l_1 + l_2)\alpha\rho g = Al_1\rho g + Al_2\alpha^2\rho g$$

$$\Rightarrow \qquad l_1 = \alpha l_2$$

For downward motion

$$A(l_1 + l_2)\alpha\rho\frac{d^2x}{dt^2} = A(l_1 + l_2)\alpha\rho g$$

$$-A(l_2+x)\alpha^2\rho g$$
$$-A(l_1-x)\alpha\rho g$$



$$\Rightarrow \qquad \frac{d^2x}{dt^2} = -\omega_1^{\,2}x$$

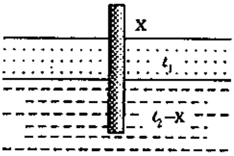
where, $\omega_1 = \sqrt{\left(\dfrac{l_1^{\,2}+l_2^{\,2}}{l_1\,l_2(l_1+l_2)}\right)g}$

For upward motion

$$A(l_1+l_2)\alpha\rho\frac{d^2x}{dt^2} = l_1\rho g+(l_2-x)\alpha^2\rho g-(l_1+l_2)\alpha\rho g$$

$$\Rightarrow \qquad \frac{d^2x}{dt^2} = -\omega_2^{\,2}x$$

where, $\omega_2 = \sqrt{\dfrac{l_1 g}{l_2(l_1+l_2)}}$



Now time period

$$T = \frac{\pi}{\omega_1}+\frac{\pi}{\omega_2}$$

$$\Rightarrow \qquad T = \pi\sqrt{\left(\frac{l_1 l_2(l_1+l_2)}{(l_1^{\,2}+l_2^{\,2})g}\right)}+\pi\sqrt{\frac{l_2(l_1+l_2)}{l_1 g}}$$

133. $x = a\cos 3\omega t$ ...(i)

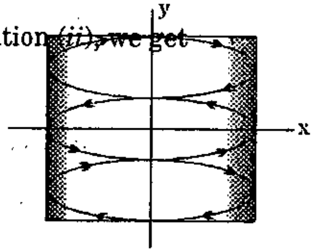
$y = b\sin\omega t$ ...(ii)

From equation *(i)*

$$x = a\cos\omega t\left(1-4\sin^2\omega t\right)$$

$$x = a\left(\sqrt{1-\sin^2\omega t}\right)\left(1-4\sin^2\omega t\right)$$

Now substituting $\sin\omega t = \dfrac{y}{b}$ from equation *(ii)*, we get



$$x = a\left(\sqrt{1-\frac{y^2}{b^2}}\right)x\left(1-\frac{4y^2}{b^2}\right)$$

$$x^2b^2 = a^2\left(b^2-y^2\right)\left(b^2-4y^2\right)$$

135. Suppose the equation of the wave is

$$y = a\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$$

In the first case

$$6 = 8\sin 2\pi\left(\frac{0.1}{T}-\frac{10}{\lambda}\right)$$

$$\Rightarrow \qquad 2\pi\left(\frac{1}{10T}-\frac{10}{\lambda}\right) = (49)\frac{\pi}{180}$$

$$\Rightarrow \qquad \frac{1}{10T}-\frac{10}{\lambda} = \frac{49}{360} \qquad \text{...}(i)$$

In the second case

$$4 = 8\sin 2\pi\left(\frac{0.1}{T}-\frac{25}{\lambda}\right)$$

$$\Rightarrow \qquad 2\pi\left(\frac{1}{10T}-\frac{25}{\lambda}\right) = \frac{\pi}{6}$$

$$\Rightarrow \qquad \frac{1}{10T}-\frac{25}{\lambda} = \frac{1}{12} \qquad \text{...}(ii)$$

From equations *(i)* and *(ii)*, we get
$\lambda = 284.2$ cm and $T = 0.58$ second

137. Comparing the given equation with

$$\Psi = a\sin\left(\omega t-\frac{2\pi}{\lambda}(x\cos\alpha+y\cos\beta)\right)$$

we get $\quad \cos\alpha = \frac{\sqrt{3}}{2} \quad$ and $\quad \cos\beta = \frac{1}{2}$

and $\quad \lambda = 1\text{m}$

Now, $\quad \cos\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \alpha = 30°$

and $\quad \cos\beta = \frac{1}{2}$

$\Rightarrow \quad \beta = 60°$

(a) The wave is moving along a straight line in x-y plane through origin making 30° with x-axis and 60° with y-axis.

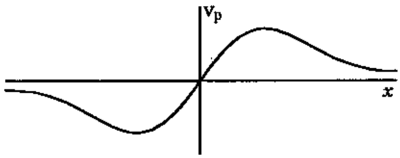
(b) The wavelength of the wave is 1m.

(c)
$$\Delta\phi = \frac{2\pi}{\lambda}\left[(x_2 - x_1)\cos\alpha + (y_2 - y_1)\cos\beta\right]$$
$$= \frac{2\pi}{1}\left[(\sqrt{3})\cos 30^\circ + (1)\cos 60^\circ\right]$$
$$= 4\pi$$

139. (a) Particle velocity

$$\frac{\partial y}{\partial t} = \frac{2b^3(x-ct)c}{\left[b^2 + (x-ct)^2\right]^2}$$

At $t = 0$
$$v_p = \frac{2b^3 c x}{(b^2 + x^2)^2}$$



Now
$$\frac{dv_p}{dx} = 0$$

$\Rightarrow$
$$x = \frac{b}{\sqrt{3}}$$

$\therefore$
$$(v_p)_{max} = \frac{2b^3 c\left(\frac{b}{\sqrt{3}}\right)}{\left(b^2 + \frac{b^2}{3}\right)^2} = (6\sqrt{3})c$$

(b) Let the displacement in moving reference frame is X then
$$x = X + vt$$
Therefore, the wave equation is

$$y = \frac{b^3}{b^2 + [(X + vt) - ct]^2}$$

$$y = \frac{b^3}{b^2 + [X - (c - v)t]^2}$$

141. $P = \frac{1}{2}\mu\omega^2 a^2 v$

Now $$\omega = \frac{2\pi v}{\lambda} \text{ and } v = \sqrt{\frac{T}{\mu}}$$

Therefore $$P = \frac{2\pi^2 a^2}{\lambda^2}\sqrt{\frac{T^3}{\mu}}$$

Substituting, $a = 10$ mm, $\lambda = 0.5$ m, T = 500 N and $\mu = 0.8$ kg/m we get

$$P = 98.7 \text{ W}.$$

143. Energy of oscillations equals to the work done against tension in giving the initial extension in the string. Therefore

$$U = T\left[2\sqrt{h^2 + \frac{l^2}{4}} - l\right] = Tl\left[\sqrt{1 + \frac{4h^2}{l^2}} - 1\right]$$

$$\cong \frac{2Th^2}{l}$$

145. In the n^{th} overtone

$$K = \frac{2\pi}{\lambda} = (2n + 1)\frac{\pi}{2l}$$

Amplitude of the pressure variation

$$\Delta P_m = \rho v^2 AK = \frac{\rho v^2 A\pi}{l}\left(n + \frac{1}{2}\right)$$

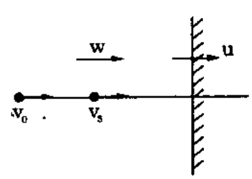
The equation for pressure variation

$$\Delta P = \Delta P_m \cos Kx$$

$$\Rightarrow \quad P = \left(n + \frac{1}{2}\right)\frac{\pi\rho v^2 A}{l}\cos\left[\left(n + \frac{1}{2}\right)\frac{\pi x}{l}\right]$$

147. Frequency of the sound received by the wall.

$$n_1 = n_0\left(\frac{v + \omega - u}{v + \omega - v_s}\right) \quad \ldots(i)$$



The waves reflected from the wall are received by the observer. The frequency received by the observer

$$n = n_1\left(\frac{v - \omega + v_0}{v - \omega + u}\right) \quad \ldots(ii)$$

Substituting n_1 from equation *(i)* in equation *(ii)*, we get

$$n = n_0 \left(\frac{v + \omega - u}{v + \omega - v_s}\right) \left(\frac{v - \omega + v_0}{v - \omega + u}\right)$$

Wavelength of the sound received by the observer

$$\lambda = \frac{v - w + u}{n_1}$$

Substituting n_1 from equation *(i)*, we get

$$\lambda = \frac{(v - \omega + u)(v + \omega - v_s)}{n_0 (v + \omega - u)}$$

149. The equation for standing wave will be in the form

$$z = A \sin K_1 x \sin K_2 y \sin (\omega t + \delta)$$

Since, at $t = 0$, the centre passes through mean position, therefore $\delta = 0$.

the edges of the plate are clamped.

Since,

$$K_1 l = n_1 \pi \quad \text{and} \quad K_2 l = n_2 \pi$$

For the fundamental model

$$n_1 = n_2 = 1$$

Thus $\quad K_1 = \frac{\pi}{l} \quad$ and also $K_2 = \frac{\pi}{l}$

Hence, the required equation is

$$z = A \sin \frac{\pi x}{l} \sin \frac{\pi y}{l} \sin \omega t.$$

151. Final length of the rod AB will be

$$l_1' = l_1 (1 + \alpha_1 \theta) - \frac{F l_1}{Y_1 A}$$

Final length of the rod BC will be

$$l_2' = l_2 (1 + \alpha_2 \theta) - \frac{F l_2}{Y_2 A}$$

Now $\quad l_1 + l_2 = l_1' + l_2'$

$$\Rightarrow \quad \frac{F}{A} = \frac{(l_1 \alpha_1 + l_2 \alpha_2) Y_1 Y_2 \theta}{l_1 Y_2 + l_2 Y_1}$$

Time taken by transverse wave pulse to travel from the end A to the end C will be

$$t = \frac{l_1'}{\sqrt{\dfrac{F}{A\rho_1}}} + \frac{l_2'}{\sqrt{\dfrac{F}{A\rho_2}}}$$

But it is given that $l_1' \simeq l_1$ and $l_2' \approx l_2$

$$\therefore \qquad t \simeq \frac{l_1}{\sqrt{\dfrac{F}{Al_1}}} + \frac{l_2}{\sqrt{\dfrac{F}{Al_2}}}$$

$$\Rightarrow \qquad t = \left(l_1\sqrt{\rho_1} + l_2\sqrt{\rho_2}\right)\sqrt{\frac{l_1 Y_2 + l_2 Y_1}{\theta(l_1\alpha_1 + l_2\alpha_2)Y_1Y_2}}$$

153.

$$V_2 = V_1\left(1 + \gamma_{Hg}(\theta_2 - \theta_1)\right)$$

$$\Rightarrow \qquad A_2 l_2 = A_1 l_1\left(1 + \gamma_{Hg}(\theta_2 - \theta_1)\right)$$

$$\Rightarrow \qquad l_2 = A_1 l_1 \frac{\left(1 + \gamma_{Hg}(\theta_2 - \theta_1)\right)}{A_1\left(1 + 2\alpha_g(\theta_2 - \theta_1)\right)}$$

$$= l_1\left[1 + \left(\gamma_{Hg} - 2\alpha_g\right)(\theta_2 - \theta_1)\right]$$

Now, Reading on scale

$$l = \frac{l_2}{1 + \alpha_g(\theta_2 - \theta_1)} = \frac{l_1\left[1 + \left(\gamma_{Hg} - 2\alpha_g\right)(\theta_2 - \theta_1)\right]}{1 + \alpha_g(\theta_2 - \theta_1)}$$

$$= l_1\left[1 + \left(\gamma_{Hg} - 3\alpha_g\right)(\theta_2 - \theta_1)\right]$$

Substituting the values, we get

$$l \doteq 1.0155 \text{ m}$$

155.

$$\mu\, mg\, v = -\left(\frac{dm}{dt}\right)L$$

$$\Rightarrow \qquad \int_{m_0}^{m}\frac{dm}{m} = -\frac{\mu g}{L}\int_0^t (v_0 - \mu g t)\,dt$$

$$\Rightarrow \qquad m = m_0\, e^{-\frac{\mu g}{L}\left(v_0 t - \frac{1}{2}\mu g t^2\right)}$$

157.
$$H = -KA\frac{dT}{dx}$$

Suppose $$K = \frac{\alpha}{T}$$

Then $$H = -\frac{\alpha A}{T}\frac{dT}{dx}$$

Now $$\int_0^l H\,dx = -\alpha A\int_{T_1}^{T_2}\frac{dT}{T}$$

$$\Rightarrow \qquad Hl = \alpha A \ln\frac{T_1}{T_2} \qquad \text{...(i)}$$

Also $$\int_0^x H\,dx = -\alpha A\int_{T_1}^{T}\frac{dT}{T}$$

$$\Rightarrow \qquad Hx = \alpha A\ln\frac{T_1}{T} \qquad \text{...(ii)}$$

From equation *(i)* and *(ii)*

$$T = T_1\left(\frac{T_2}{T_1}\right)^{x/l}$$

159. Since, the radial current from inner cylinder to the outer cylinder should be equal, Therefore

$$\frac{T_1 - T}{\left(\dfrac{\ln(2a/a)}{2\pi K_1 l}\right)} = \frac{T - T_2}{\left(\dfrac{\ln(4a/a)}{2\pi K_2 l}\right)}$$

$$\Rightarrow \qquad K_1(T_1 - T) = K_2(T - T_2)$$

$$\Rightarrow \qquad T = \frac{T_1K_1 + T_2K_2}{K_1 + K_2}$$

161. According to Newton's Law of cooling

$$\frac{d\theta}{dt} = -\alpha(\theta - \theta_0)$$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -\int_0^t \alpha\, dt$$

$$\Rightarrow \qquad \ln \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} = -\alpha t \qquad \text{...(i)}$$

$$\int_{\theta_2}^{\theta_3} \frac{d\theta}{\theta - \theta_0} = -\int_0^T \alpha\, dt$$

$$\Rightarrow \qquad \ln\left(\frac{\theta_3 - \theta_0}{\theta_2 - \theta_0}\right) = -\alpha T \qquad \text{...(ii)}$$

From equations *(i)* and *(ii)*, we get $T = t\left[\dfrac{\ln\left(\dfrac{\theta_2 - \theta_0}{\theta_3 - \theta_0}\right)}{\ln\left(\dfrac{\theta_1 - \theta_0}{\theta_2 - \theta_0}\right)}\right]$

163. The temperature of the gas is constant, therefore

$$PV = \text{constant}$$

$$\Rightarrow \qquad P\frac{dV}{dt} + V\frac{dP}{dt} = 0$$

$$\Rightarrow \qquad V\frac{dP}{dt} = -rP$$

$$\Rightarrow \qquad \int_{P_0}^{P} \frac{dP}{P} = -\frac{1}{V_0}\int_0^t r\, dt$$

$$\Rightarrow \qquad \ln\frac{P}{P_0} = \frac{-rt}{V_0}$$

$$\Rightarrow \qquad P = P_0\, e^{-rt/V_0}$$

165. The equation of the process AB is

$$P = 5P_0 - \frac{2P_0V}{V_0} \qquad \text{...(i)}$$

Now $\qquad PV = nRT$

Therefore, from equation *(i)*

$$nRT = 5P_0V - \frac{2P_0V^2}{V_0}$$

Now, $\quad$ for T_{max}, $\dfrac{dT}{dV} = 0 \Rightarrow V = \dfrac{5}{4}V_0$

167. Suppose $T_B = T_0$ and $V_C = V_D = V_O$

Then $\quad T_D = T_A = 4T_0$ and $V_A = \left(8\sqrt{2}\right)V_0$

Now $$V_B = \left(\frac{V_A}{T_A}\right) T_B = 2\sqrt{2}\, V_0$$

and $$T_C = T_B \left(\frac{V_B}{V_C}\right)^{\gamma-1} = 2\,T_0$$

$$\begin{aligned}
\Delta W &= \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA} \\
&= R\,(T_B - T_A) + nC_V\,(T_B - T_C) + 0 + RT_D \ln \frac{V_A}{V_D} \\
&= -\,3RT_0 - \frac{3}{2}RT_0 + 0 + 14\,RT_0 \ln 2 \\
&= (5.202)\,RT_0 \\
\Delta\,\theta_{absorbed} &= \Delta Q_{CD} + \Delta Q_{DA} \\
&= 3RT_0 + 14\,RT_0 \ln 2 \\
&= (12.702)\,T_0
\end{aligned}$$

$\therefore$ $\quad$ efficiency, $\eta = \dfrac{\Delta W}{\Delta Q_{absorbed}} = 0.41 = 41\,\%$

169. *(a)* Given $$\Delta W = \frac{Q}{2}$$

From first law of thermo dynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \quad \Delta U = \frac{Q}{2}$$

$$\therefore \quad dU = dW$$

$$\Rightarrow \quad C_V dT = PdV$$

$$\Rightarrow \quad \left(\frac{R}{\gamma - 1}\right) dT = PdV$$

$$\Rightarrow \quad \frac{3}{2}(PdV + VdP) = PdV \qquad \left(\because \gamma = \frac{5}{3}\right)$$

$$\Rightarrow \quad PdV + 3Vdp = 0$$

$$\Rightarrow \quad \frac{dV}{V} + \frac{3dP}{P} = 0$$

$$\Rightarrow \quad VP^3 = \text{constant}$$

$$\Rightarrow \quad PV^{1/3} = \text{constant}$$

(b)

$$C = C_V + P\frac{dV}{dT}$$

$$= \frac{3}{2}R + \frac{3}{2}R = 3R$$

171. Suppose the upper piston moves upward by a distance y when the lower piston is moved upward by a distance $\frac{h}{2}$. Then

$$P_0(Ah) = PA\left(h - \frac{h}{2} + y\right) = nRT_0 \qquad ...(i)$$

and

$$(P - P_0)A = Ky \qquad ...(ii)$$

From these equations, eliminating y, we get

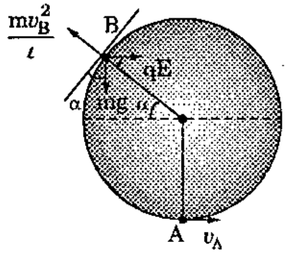
$$\frac{1}{P^2}(2P_0KAh) + \frac{1}{P}(2A^2P_0 - KAh) - 2A^2 = 0$$

$$\Rightarrow \quad \frac{P_0}{P} = \frac{1}{4} - \left(\frac{P_0A}{2Kh}\right) + \sqrt{\frac{1}{16} + \frac{3P_0A}{4Kh} + \left(\frac{P_0A}{2Kh}\right)^2}$$

Now, work done, $W = nRT_0 \ln \frac{P_0}{P}$

$$W = nRT_0 \ln\left[\frac{1}{4} - \left(\frac{P_0A}{2kh}\right) + \sqrt{\frac{1}{16} + \frac{3P_0A}{4kh} + \left(\frac{P_0A}{2kh}\right)^2}\right]$$

173. Suppose at the point B the tension in string just becomes zero. Therefore



$$qE \sin\alpha = mg \cos\alpha \quad ...(i)$$

and
$$\frac{mv_B^2}{l} = qE\cos\alpha + mg\sin\alpha \quad ...(ii)$$

Applying conservation of energy between points A and B, we get

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(l + l\sin\alpha) + qE(l\cos\alpha) \quad ...(iii)$$

From equations *(i)*, *(ii)* and *(iii)*, we get

$$v_A = \left[3gl\left(\frac{2}{3} + \sqrt{1 + \left(\frac{mg}{qE}\right)^2}\right)\right]^{1/2}$$

175. If the relative velocity of separtion of particles is v for a separation x. Then the relative acceleration.

$$v\frac{dv}{dx} = \frac{(kq^2/x^2)}{2m} + \frac{(kq^2/x^2)}{3m}$$

$$\left(\text{where } k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\Rightarrow \quad \int_0^v v\,dv = \frac{5kq^2}{6m}\int_d^x \frac{dx}{x^2}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{5kq^2}{6m}\left(\frac{1}{x} - \frac{1}{d}\right)$$

$$\Rightarrow \quad \frac{dx}{dt} = \sqrt{\frac{5kq^2}{3m}\left(\frac{1}{x} - \frac{1}{d}\right)}$$

$$\Rightarrow \quad \int_0^t dt = \sqrt{\frac{3md}{5kq^2}}\int_d^{3d}\sqrt{\frac{x}{d-x}}\,dx$$

$$\Rightarrow \qquad t = \sqrt{\frac{3md^2}{5kq^2}}\left[\sqrt{6} - \ln\left(3 - \sqrt{2}\right)\right]$$

$$\Rightarrow \qquad t = \sqrt{2.4\,\pi\,\varepsilon_0 md}\left[\sqrt{6} - \ln\left(3 - \sqrt{2}\right)\right]\frac{d}{q}$$

177. The angle between dipolemoment vector and radius vector γ is given by

$$\cos\gamma = \hat{j}\cdot\left[(\sin\theta\cos\phi)\,\hat{i} + (\sin\theta\sin\phi)\,\hat{j} + (\cos\theta)\,\hat{k}\right]$$

$$= \sin\theta\sin\phi$$

$\therefore$ Potential, $$V = \frac{k\,p\cos\gamma}{r^2} = \frac{p\sin\theta\sin\phi}{4\pi\varepsilon_0 r^2}$$

and magnitude of electric field

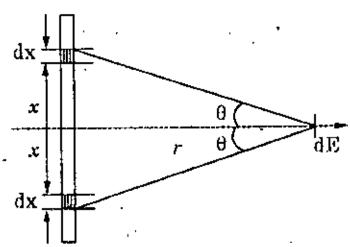
$$E = \frac{kp}{r^3}\sqrt{1 + 3\cos^2\gamma} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\sin^2\theta\sin^2\phi}$$

179. Consider a point on the perpendicular bisector of the rod at a distance r from the middle of the rod as shown in the figure. To find electric field at this point, consider two points on the rod at a distance x from the middle. Then

$$dE = 2\left[\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{\lambda\,dx}{r^2 + x^2}\right)\right]\cos\theta$$

$$\Rightarrow \qquad E = \frac{\lambda r}{2\pi\varepsilon_0}\int_0^{l/2}\frac{dx}{(r^2 + x^2)^{3/2}}$$

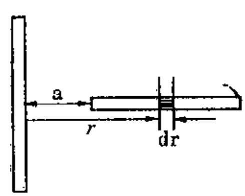
$$= \frac{\lambda l}{4\pi\varepsilon_0\sqrt{\dfrac{l^2}{4} + r^2}}$$



Now, consider an element on the second rod at a distance r of the length dr. Then

$$dF = E\,(\lambda\,dr)$$

$$F = \frac{\lambda^2 l}{4\pi\varepsilon_0}\int_a^{a+l}\frac{dr}{\sqrt{r^2 + \dfrac{l^2}{4}}}$$



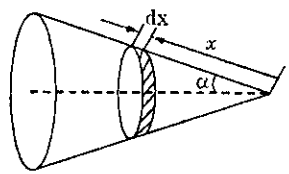
$$\Rightarrow \qquad F = \frac{\lambda^2}{2\pi\varepsilon_0} \ln\left[\frac{(a+l)\left(l+\sqrt{l^2+4a^2}\right)}{a\left(l+\sqrt{l^2+4(a+l)^2}\right)}\right]$$

181. Consider an elementary ring at a distance x from the vertex along the sloping length, as shown in the figure. Then

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{\left[\dfrac{Q}{\pi(l\sin\alpha)l} . 2\pi(x\sin\alpha)\,dx\right]}{x}$$

$$\Rightarrow \qquad V = \frac{Q}{2\pi\varepsilon_0 l^2}\int_0^l dx$$

$$\Rightarrow \qquad V = \frac{Q}{2\pi\varepsilon_0 l}.$$



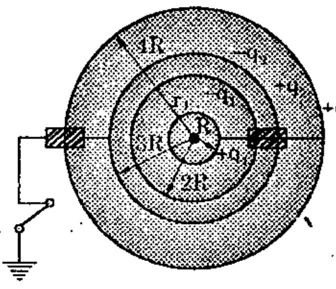
183. If a sphere is uniformly charged through out its volume with a total charge Q, then

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

$$\therefore \qquad \frac{Qr}{4\pi\varepsilon_0 R^3} = \alpha r$$

$$\Rightarrow \qquad Q = 4\pi\varepsilon_0\alpha R^3$$

185. Suppose the charges on all the surfaces are as shown in the figure.



Since, potential of the innermost sphere and outermost shell will be same, therefore,

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R} - \frac{q_1}{2R} - \frac{q_2}{3R} + \frac{q_2+q_3}{4R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{4R} - \frac{q_1}{4R} - \frac{q_2}{4R} + \frac{q_2 + q_3}{4R}\right)$$

$$\Rightarrow \qquad 6q_1 = q_2 \qquad \ldots(i)$$

Since, potential of the grounded sphere will be zero, therefore

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{2R} - \frac{q_1}{2R} - \frac{q_2}{3R} + \frac{q_2 + q_3}{4R}\right) = 0$$

$$3q_3 = q_2 \qquad \ldots(ii)$$

Since, the total charge of innermost sphere and outermost shell will remain same therefore

$$q_1 + q_2 + q_3 = 90\ \mu c \qquad \ldots(iii)$$

From equations *(i)*, *(ii)* and *(iii)*, we get

$$q_3 = 20\ \mu c$$

187. Suppose charge on the sphere of radius a is $+q$ and the charge on the sphere of radius b is $-q$.

Then potential difference between the spheres will be

$$V = \left(\frac{q}{4\pi\varepsilon_0 a} - \frac{q}{4\pi\varepsilon_0 d}\right) - \left(\frac{q}{4\pi\varepsilon_0 d} - \frac{q}{4\pi\varepsilon_0 b}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)$$

Now, $\qquad C = \dfrac{q}{V}$

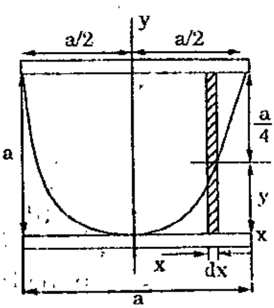
$$\Rightarrow \qquad C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)}$$

189. Consider the coordinate axes as shown in the figure.

Equation of the parabola is

$$x^2 = ay \qquad \ldots(i)$$

Now, consider an elementary capacitor at a distance x from the axis of parabola as shown. Then the capacitance of this elementary capacitor will be



$$dC = \frac{\varepsilon_0 (a\,dx)}{\frac{a}{4K} + \left(1 - \frac{1}{K}\right) y}$$

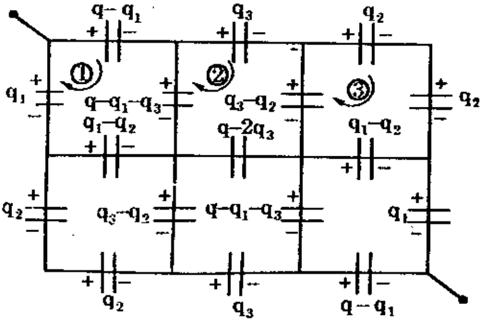
Now using equation *(i)*, we have

$$C = \int_{x=-a/2}^{x=a/2} \frac{\varepsilon_0 (a\,dx)}{\frac{a}{4K} + \left(1 - \frac{1}{K}\right) \frac{x^2}{a}}$$

$$\Rightarrow \qquad C = \frac{2\varepsilon_0 a^2 K}{K-1} \int_0^{a/2} \frac{dx}{x^2 + \left(\frac{a}{2\sqrt{K-1}}\right)^2}$$

$$\Rightarrow \qquad C = \frac{4\varepsilon_0 a K}{\sqrt{K-1}} \tan^{-1}\left(\sqrt{K-1}\right)$$

191. Using the symmetry of the network, the charges can be distributed as shown in the figure.



Now applying voltage law in the loops ①, ② and ③, we get

$$\frac{q-q_1}{C} + \frac{q-q_1-q_3}{C} - \frac{(q_1-q_2)}{C} - \frac{q_1}{C} = 0 \qquad \text{...(i)}$$

$$\frac{q_3}{C} + \frac{q_3-q_2}{C} - \frac{(q-2q_3)}{C} - \frac{(q-q_1-q_3)}{C} = 0 \qquad \text{...(ii)}$$

$$\frac{q_2}{C} + \frac{q_2}{C} - \frac{(q_1-q_2)}{C} - \frac{(q_3-q_2)}{C} = 0 \qquad \text{...(iii)}$$

Suppose the equivalent capacitance is C_{eq}. Then

$$\frac{q}{C_{eq}} = \frac{q-q_1}{C} + \frac{q_3}{C} + \frac{q_2}{C} + \frac{q_2}{C} + \frac{q_1}{C} \qquad ...(iv)$$

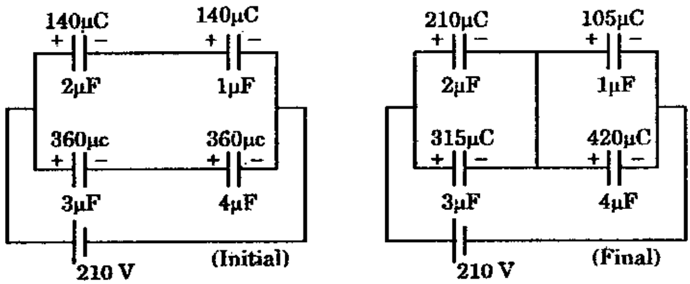
From equations *(i)*, *(ii)*, *(iii)* and *(iv)*, we get

$$C_{eq} = \left(\frac{69}{121}\right)C$$

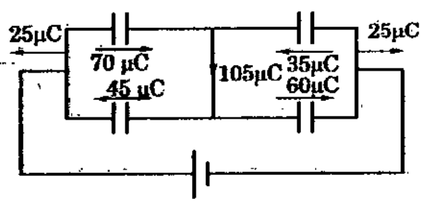
Now, it is given $\quad C = 121\,\mu F.$

$\therefore \quad C_{eq} = 69\,\mu F.$

193. *(a)* The charges on the capacitors in initial and final conditions can be calculated as shown in the figure.



Comparing the initial and final conditions, the charges flowing through different branches can be obtained as shown in the figure.



Therefore, the charge flowing through the switch is 105μC.

(b) Increase in energy stored in capacitor

$$\Delta u_C = (\text{Final energy stored}) - (\text{Initial energy stored})$$

$$= \left[\frac{1}{2}\left(\frac{5}{2}\right)(210)^2 - \frac{1}{2}\left(\frac{50}{21}\right)(210)^2\right]\ \mu J$$

$$= 2.625 \text{ mJ}$$

Extra energy supplied by battery

$$\Delta u_B = 25 \times 210\ \mu J = 5.25 \text{ mJ}$$

Heat loss, $\quad \Delta H = \Delta u_B - \Delta u_C$

$$= (5.25 - 2.625) \text{ mJ} = \mathbf{2.625\ mJ.}$$

195. From conservation of energy

$$\frac{q^2}{2\left(\frac{\varepsilon_0 bl}{d}\right)} - \frac{q^2}{2\left[\frac{\varepsilon_0 b\{l+(K-1)x\}}{d}\right]} = (bd\rho x)\, g\frac{x}{2} \qquad ...(i)$$

Here $$q = \left(\frac{\varepsilon_0 bl}{d}\right)V \qquad ...(ii)$$

From equations *(i)* and *(ii)*

$$x = \sqrt{\frac{l^2}{4(K-1)^2} + \frac{\varepsilon_0 l V^2}{\rho g d^2}} - \frac{l}{2(K-1)}$$

197. Consider a coaxial cylindrical shell of radius x and thickness dx. All such type of cylindrical shells will be in parallel combination. Conductivity at the distance x from the axis

$$\sigma = \sigma_1 + \left(\frac{\sigma_2 - \sigma_1}{a}\right)x$$

Therefore, conductance of the cylindrical shell will be

$$dG = \frac{\sigma(2\pi x\, dx)}{l}$$

$$\Rightarrow \qquad G = \int_{x=0}^{x=a}\left[\sigma_1 + \left(\frac{\sigma_2 - \sigma_1}{a}\right)x\right]\frac{2\pi x}{l}\,dx$$

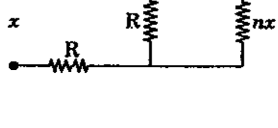
$$\Rightarrow \qquad G = \frac{\pi a^2(\sigma_1 + 2\sigma_2)}{3l}$$

Now, resistance of the conductor

$$R = \frac{1}{G}$$

$$\Rightarrow \qquad R = \frac{3l}{\pi a^2(\sigma_1 + 2\sigma_2)}$$

199. *(a)* Suppose the equivalent resistance between points A and B is x. Then the equivalent circuit can be drawn as shown in the figure. Therefore



$$x = 2R + \frac{R(nx)}{R + nx}$$

$\Rightarrow \quad nx^2 - (3n - 1)\,Rx - 2R^2 = 0$

$$\Rightarrow \quad x = \frac{R}{2n}\left[(3n-1)+\sqrt{9n^2+2n+1}\right]$$

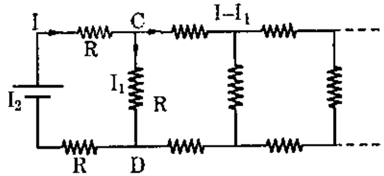
(b) Suppose the current through the section CD is I_1 and the current through the battery is I. Then

$$I\,x = E$$

and $\quad 2IR + I_1R = E$

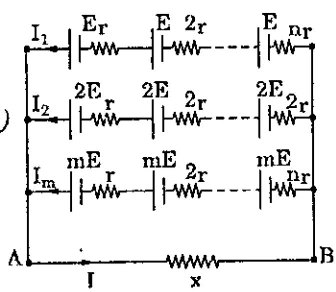
From these equations, we get

$$I_1 = \frac{E}{R}\left[\frac{\sqrt{9n^2+2n+1}-n-1}{\sqrt{9n^2+2n+1}+3n-1}\right]$$



201. Suppose an external resistance x is connected between terminals A and B. From Kirchoff's Current Law

$$I_1 + I_2 + ... + I_m = I \qquad ...(i)$$



Using Kirchoff's Voltage Law

$$I_1\left[\frac{n(n+1)}{2}r\right] + Ix = nE$$

$$I_2\left[\frac{n(n+1)}{2}r\right] + Ix = 2nE \qquad ...(ii)$$

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$$I_m\left[\frac{n(n+1)}{2}r\right] + Ix = mnE$$

From, these equations

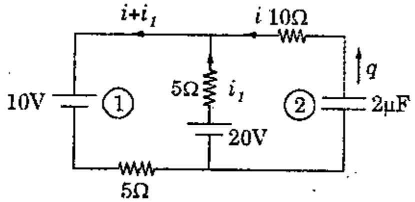
$$I = \frac{\left(\frac{n(m+1)}{2}\right)E}{x+\left(\frac{n(n+1)}{2m}\right)r}$$

This is of the form $\quad I = \dfrac{V}{x+R}$

Therefore $$V = \frac{n(m+1)}{2} E$$

and $$R = \frac{n(n+1)}{2m} r$$

203. Suppose charge q has flown from capacitor up to time 't' and at this instant the currents are distributed as shown in the figure.



Now, applying Voltage Law in loop(1) and(2), we get (currents are in amp. and charge is in μC)

$$5\,i_1 + 5\,(i + i_1) = 10 \qquad ...(i)$$

and $$\frac{q}{2} + 10i - 5i_1 = 0 \qquad ...(ii)$$

Also $$i = \frac{dq}{dt} \qquad ...(iii)$$

From these equations, we get

$$(25)\,\frac{dq}{dt} + q = 10$$

$\Rightarrow$ $$q = 10\left(1 - e^{-t/\tau}\right)\,\mu c \quad \text{where } \tau = 25\ \mu\text{sec}$$

Now, $$i = \frac{dq}{dt}$$

$\Rightarrow$ $$i = \frac{2}{5} e^{-t/\tau}\ \text{Amp.}$$

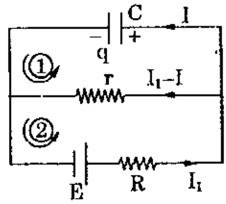
From equation *(i)*, $i_1 = 1 - \dfrac{i}{2}$

The current through the branch containing switch, will be

$$I = i + i_1 = 1 + \frac{i}{2}$$

$$I = \left(1 + \frac{1}{5} e^{-t/\tau}\right)\ \text{Amp.} \quad \text{where } \tau = 25\ \mu\text{sec}$$

205. The equivalent circuit diagram is shown in the figure. Suppose the charge on the capacitor at time t is q and the current through battery is I_1. Applying Voltage Law in loops ① and ②, we get



$$\frac{q}{C} - (I_1 - I)\,r = 0 \qquad ...(i)$$

and $$I_1R + (I_1 - I)\,r = E \qquad ...(ii)$$

Also $$I = \frac{dq}{dt} \qquad ...(iii)$$

From these equations

$$\left(\frac{rRC}{r+R}\right)\frac{dq}{dt} + q = \frac{CEr}{r+R}$$

$$\Rightarrow \qquad q = \left(\frac{CEr}{r+R}\right)\left(1 - e^{-t/\tau}\right)$$

where $$\tau = \frac{CRr}{r+R}$$

Now $$I = \frac{dq}{dt}$$

$$\Rightarrow \qquad I = \frac{E}{R}\,e^{-t/\tau}$$

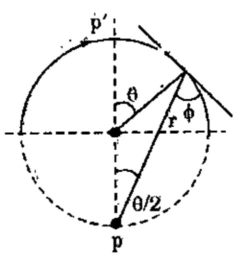
From equation *(ii)*

$$I_1 = \frac{E + Ir}{r+R}$$

$$\Rightarrow \qquad I_1 = \left(\frac{E}{r+R}\right)\left(1 + \frac{r}{R}e^{-t/\tau}\right)$$

207. Consider a current element at an angle θ with the radius OP′ as shown in the figure.

From geometry,



$$r = 2\,R\cos\frac{\theta}{2}$$

and $$\phi = 90° - \frac{\theta}{2}$$

Now, the magnetic field due to this current element at P is

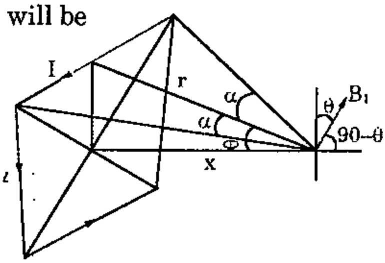
$$dB = \frac{\mu_0}{4\pi}\,\frac{I(Rd\theta)\sin\phi}{r}$$

$$\Rightarrow \qquad B = 2\left[\frac{\mu_0}{4\pi}\int_0^{\pi/2}\frac{I(Rd\theta)\sin\left(90°-\frac{\theta}{2}\right)}{2R\cos\frac{\theta}{2}}\right]$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{4\pi R}\ln\left(\sqrt{2}+1\right)$$

209. The magnetic field B_1 due to one side of square will be inclined at angle $(90-\theta)$ with the axis shown in the figure.

$\therefore$ Net magnetic field will be



$$B = 4\,B_1\sin\theta$$

$$B = 4\left(\frac{\mu_0}{2\pi}\frac{I}{r}\sin\alpha\right)\sin\theta$$

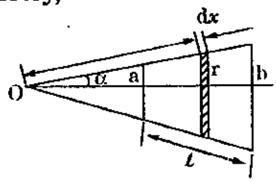
From geometry $\qquad r = \sqrt{x^2+\frac{l^2}{4}}$

$$\sin\alpha = \frac{l/2}{\sqrt{x^2+\frac{l^2}{2}}}$$

and $\qquad \sin\theta = \dfrac{l/2}{\sqrt{x^2+\frac{l^2}{4}}}$

Therefore $\qquad B = \dfrac{\mu_0 I l^2}{2\pi\left(x^2+\frac{l^2}{4}\right)\sqrt{\left(x^2+\frac{l^2}{2}\right)}}$

211. Consider an elementary ring of radius r at a distance x from the vertex along the sloping length. From geometry,



$$\sin\alpha = \frac{b-a}{l},\ r = x\sin\alpha$$

The magnetic field at O due to this ring is

$$dB = \frac{\mu_0 I r^2 (n\,dx)}{2x^3}$$

$$\therefore \qquad B = \int_{a\,\text{cosec}\,\alpha}^{b\,\text{cosec}\,\alpha} \frac{\mu_0 I (x\sin\alpha)^2 \left(\frac{N}{l}\,dx\right)}{2x^3}$$

$$\Rightarrow \qquad B = \frac{\mu_0 N I \sin^2\alpha}{l} \ln\frac{b}{a}$$

$$\Rightarrow \qquad B = \frac{\mu_0 N I (b-a)}{l^2} \ln\frac{b}{a}$$

213. The current enclosed within a loop of radius r,

$$I = \int j(2\pi x)\,dx$$

$$= \int_0^r b x^\alpha (2\pi x\,dx)$$

$$= \frac{2\pi b r^{\alpha+2}}{\alpha+2}$$

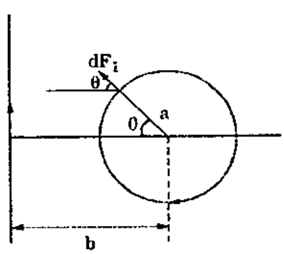
From Ampere's Law

$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

$$\Rightarrow \qquad B(2\pi r) = \mu_0\left(\frac{2\pi b r^{\alpha+2}}{\alpha+2}\right)$$

$$\Rightarrow \qquad B = \frac{\mu_0 b r^{\alpha+1}}{\alpha+2}$$

215. Consider an element on the circular loop at an angle θ with the horizontal. From the symmetry of the figure, only the horizontal component will contribute in net force. Therefore



$$dF = 2\,(dF_1)\cos\theta = 2\left(\frac{\mu_0 I_1}{2\pi\,(b - a\cos\theta)}\right)(I_2\, a\, d\theta)\cos\theta$$

$$F = \frac{\mu_0 I_1 I_2 a}{\pi}\int_0^{\pi}\frac{\cos\theta\, d\theta}{(b - a\cos\theta)}$$

$$\Rightarrow \qquad F = \mu_0 I_1 I_2\left(\frac{b}{\sqrt{b^2 - a^2}} - 1\right)$$

217. Consider an element on the semi-circle at an angle θ with the line PQ. The torque due to magnetic forces about line PQ,

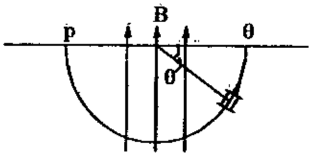
$$\tau = \int_0^{\pi}\left[BI\,(a\,d\theta)\sin\theta\right]a\sin\theta$$

$$\Rightarrow \qquad \tau = \frac{\pi B I a^2}{2}$$

The centre of mass is at a distance $\left(\frac{2a}{\pi}\right)$ from line PQ

For equilibrium

$$\frac{\pi B I a^2}{2} = (mg)\left(\frac{2a}{\pi}\right)$$



$$\Rightarrow \qquad m = \frac{\pi^2 a B I}{4g}$$

219. For the tangential direction

$$\frac{mdv}{dt} = -\mu\,(mg + qE) \qquad \text{...(i)}$$

For the radial direction

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow \qquad r = \frac{mv}{qB}$$

$$\Rightarrow \qquad dr = \frac{mdv}{qB}$$

$$\Rightarrow \qquad dr = -\frac{-\mu\,(mg + qE)}{qB}\,dt \qquad \text{[using equation (i)]}$$

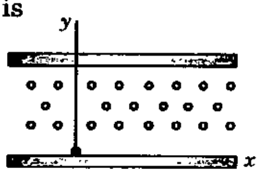
$$\Rightarrow \qquad \int_0^t dt = \frac{-1}{\mu}\left(\frac{qB}{mg + qE}\right)\int_a^0 dr$$

$$\Rightarrow \qquad t = \frac{aqB}{\mu(mg + qE)}$$

221. Select the axes as shown in the figure.

Then $\vec{B} = B\hat{k}$, and $\vec{E} = E\hat{j}$. Also, $E = \dfrac{V}{d}$.

Suppose the velocity at any instant of time t is



$$\vec{v} = (v_x)\,\hat{i} + (v_y)\,\hat{j}.$$

Now $\quad \vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

$\Rightarrow \quad \vec{F} = (qE - qv_xB)\,\hat{j} + (qBv_y)\,\hat{i}$

Therefore $\quad m\dfrac{dv_x}{dt} = qBv_y$ ...(i)

and $\quad m\dfrac{dv_y}{dt} = qE - qv_xB$ ...(ii)

From equation *(ii)*

$$\frac{d^2v_y}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_y \qquad \text{[using equation (i)]}$$

$$\therefore\ v_y = \left(\frac{E}{B}\right)\sin\left(\frac{qB}{m}\right)t, \qquad \because \text{ at } t = 0,\ v_y = 0, \frac{dv_y}{dt} = \frac{qE}{m}$$

$$\Rightarrow \qquad y = \int_0^t \frac{E}{B}\sin\frac{qB}{m}t = \frac{Em}{qB^2}\left(1 - \sin\frac{qBt}{m}\right)$$

Therefore $\quad y_{max} = \dfrac{2Em}{qB^2} = \dfrac{2Vm}{qB^2d} \qquad \left(\because E = \dfrac{V}{d}\right)$

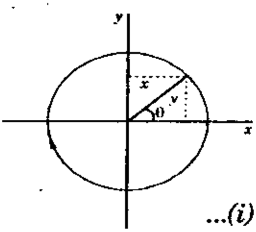
If the particle reaches to the other plate, then

$$y_{max} \geq d$$

$$\Rightarrow \qquad V \geq \frac{qB^2d^2}{2m}$$

223. Consider a current element on the circular ring at an angle θ with the positive x-axis as shown in the figure.

The current induced in the ring,



$$i = \frac{\pi a^2}{R}\left(\frac{dB_z}{dt}\right)$$

$$\Rightarrow \qquad i = \frac{\pi a^2 \gamma}{R} \qquad \text{...(i)}$$

Now, $\quad dF = (iad\theta)(\alpha x\cos\theta) + (iad\theta)(\beta y\sin\theta)$

$\therefore \quad F = 4i\alpha a^2 \int_0^{\pi/2} \cos^2\theta\, d\theta + 4i\beta a^2 \int_0^{\pi/2} \sin^2\theta\, d\theta$

$\Rightarrow \quad F = \pi i a^2 (\alpha + \beta)$

$\Rightarrow \quad F = \dfrac{\pi^2 a^4 (\alpha+\beta)\gamma}{R}$ [using equation *(i)*]

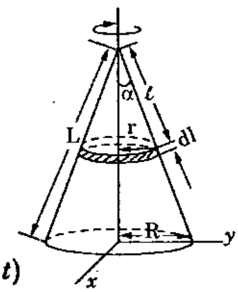
225. Suppose the base radius of the cone is R and slant height is L.

Then $\sin\alpha = \dfrac{R}{L}$.

Consider an elementary ring of radius r at distance l from the vertex along the slant height.

The induced electric field on the surface of the ring E is given by

$$\oint \vec{E}.d\vec{l} = \left|\frac{d\phi}{dt}\right|$$



$$\Rightarrow \quad E(2\pi r) = \pi r^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E = \frac{1}{2} r \frac{dB}{dt}$$

$$\Rightarrow \quad E = \frac{1}{2} rb \qquad (\because B = bt)$$

Now, torque acting on the ring due to induced electric field,

$$d\tau = \left[\left(\frac{rb}{2}\right)\left(2\pi r\, dl . \frac{Q}{\pi RL}\right)\right] r$$

$$\Rightarrow \quad \tau = \frac{bQ}{RL}\int r^3 dl = \frac{2bQ}{RL}\left(\sin^3\alpha\right)\int_0^L l^3\, dl \quad (\because r = l\sin\alpha)$$

$$\Rightarrow \quad \tau = \frac{bQR^2}{4} \qquad (\because L\sin\alpha = R)$$

Now $\quad \left(\dfrac{3}{10} mR^2\right)\dfrac{d\omega}{dt} = \dfrac{bQR^2}{4}$

$$\Rightarrow \quad \frac{d\omega}{dt} = \frac{5Qb}{6m}$$

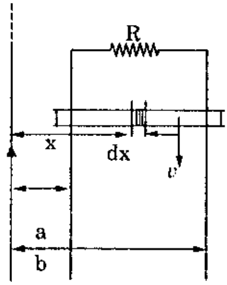
$$\Rightarrow \quad \omega = \frac{5Qbt}{6m}$$

$$\vec{\omega} = -\frac{5Q\vec{B}}{6m}$$

(Taking into account the direction of $\vec{w}$)

227. Suppose at any instant the conductor is moving down with velocity v. Consider an element on the conductor at a distance x from the current carrying wire.

The emf developed across this element,



$$dV = \left(\frac{\mu_0 I}{2\pi x}\right) v\, dx$$

$$\Rightarrow \quad V = \int_a^b \frac{\mu_0 I}{2\pi x} v\, dx$$

$$= \frac{\mu_0 I v}{2\pi} \ln \frac{b}{a}$$

The current through conductor will be

$$i = \frac{V}{R}$$

$$= \frac{\mu_0 I v}{2\pi R} \ln \frac{b}{a} \quad ...(i)$$

Now the force acting on the conductor

$$F = \int_a^b i \left(\frac{\mu_0 I}{2\pi x}\right) dx$$

$$= i \left(\frac{\mu_0 I}{2\pi}\right) \ln \frac{b}{a}$$

$$\Rightarrow \quad F = \left(\frac{\mu_0 I}{2\pi} \ln \frac{b}{a}\right)^2 \frac{v}{R}$$

[substituting i from equation *(i)*]

The equation of motion is

$$m \frac{dv}{dt} = mg - \left(\frac{\mu_0 I}{2\pi} \ln \frac{b}{a}\right)^2 \frac{v}{R}$$

$$\Rightarrow \quad \frac{dv}{dt} + \alpha v = g, \qquad \text{where } \alpha = \frac{1}{mR}\left(\frac{\mu_0 I}{2\pi} \ln \frac{b}{a}\right)^2$$

$$\Rightarrow \quad \int_0^v \frac{dv}{g - \alpha v} = \int_0^t dt$$

$$\Rightarrow \qquad v = \frac{g}{\alpha}\left(1 - e^{-\alpha t}\right)$$

$$\Rightarrow \qquad v = \frac{mgR\left(1 - e^{-\alpha t}\right)}{\left(\frac{\mu_0 I}{2\pi} \ln \frac{b}{a}\right)^2}$$

229. Suppose at time t the angular velocity of the rod is ω.

$\therefore$ Induced emf in the rod is $V = \frac{1}{2} B l^2 \omega$.

Therefore, torque acting on the rod

$$\tau = \left[\left(\frac{Bl^2\omega}{2R}\right)Bl\right]\frac{l}{2} = \frac{B^2 l^4 \omega}{4R}$$

The equation of motion is

$$\left(\frac{ml^2}{3}\right)\frac{dw}{dt} = -\frac{B^2 l^4 w}{4R}$$

$$\Rightarrow \qquad \frac{d\omega}{dt} = -b\omega, \qquad \text{where } b = \frac{3B^2 l^2}{4mR}$$

$$\Rightarrow \qquad \omega = \omega_0 e^{-bt}$$

$$\Rightarrow \qquad \frac{d\theta}{dt} = \omega_0 . e^{-bt}$$

$$\Rightarrow \qquad \theta = \int_0^t \omega_0 e^{-bt} dt$$

$$\Rightarrow \qquad \theta = \left(\frac{4mRw_0}{3B^2 l^2}\right)\left(1 - e^{-bt}\right)$$

231. (Work done by gravity) = (Increase in KE) + (heat dissipated)

$$\Rightarrow \qquad (mg)\, v \sin\theta\, dt = (mv)\, dv + (i^2 R\, dt) \qquad \ldots (i)$$

$$\frac{dx}{dt} = v \cos\theta$$

$$\frac{dy}{dt} = -v \sin\theta$$

Now, $i = \frac{(\pi a^2)}{R}\left|\frac{dB}{dt}\right| = \frac{\pi a^2 B_0 \alpha\, v \cos\theta}{R}$

Substituting this value of i in equation *(i)*, we get

$$\frac{dv}{dt} + bv = g \sin\theta$$

where, $$b = \frac{\pi^2 a^4 B_0^2 \alpha^2 \cos^2\theta}{mR}$$

$\Rightarrow$ $$v = \frac{mgR \sec\theta \tan\theta}{\pi^2 a^4 B_0^2 \alpha^2}\left(1 - e^{-bt}\right)$$

233. The induced emf in the inner coil is

$$V = M\frac{dI}{dt} = MI_0 \alpha e^{-\alpha t}$$

Applying Voltage Law for the inner coil

$$L\frac{di}{dt} + iR = MI_0 e^{-\alpha t}$$

Integrating, we get

$$i = \frac{MI_0\alpha}{(R - \alpha L)}\left(e^{-\alpha t} - e^{-Rt/L}\right)$$

235. Applying Voltage Law in the first and second circuits, we get

$$L_1\frac{di_1}{dt} + \frac{q}{C} + M\frac{di_2}{dt} = 0 \qquad \text{...(i)}$$

and $$L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = 0 \qquad \text{...(ii)}$$

From equations *(i)* and *(ii)*, we get

$$\left(L_1 - \frac{M^2}{L_2}\right)\frac{di_1}{dt} + \frac{q}{C} = 0 \qquad \text{...(iii)}$$

Now $$i_1 = \frac{dq}{dt}$$

Therefore, equation *(iii)* can be written as

$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

where $$\omega = \frac{1}{\sqrt{C\left(L_1 - \frac{M^2}{L_2}\right)}} = \sqrt{\frac{L_2}{C\left(L_1L_2 - M^2\right)}}$$

237. *(a)* The equation for the time dependence of the current is

$$I = \left(\frac{I_0}{T}\right)t$$

Now $$I_{average} = \frac{\int_0^T I\,dt}{\int_0^T dt} = \frac{\int_0^T \left(\frac{I_0}{T}\right) t\,dt}{T} = \frac{I_0}{2}$$

(b) $$I^2_{average} = \frac{\int_0^T I^2\,dt}{\int_0^T dt} = \frac{\int_0^T \left(\frac{I_0\,t}{T}\right)^2 dt}{T} = \frac{I_0^2}{3}$$

Now $$I_{rms} = \sqrt{I^2_{average}} = \frac{I_0}{\sqrt{3}}$$

239. $$I = I_1 + I_2$$

$$= \left(10\sqrt{2}\right) \sin\omega t + \left(20\sqrt{2}\right) \sin\,(\omega t + 60°)$$

$$= \left(10\sqrt{2}\right) \sin\omega t + \left(20\sqrt{2}\right)(\sin\omega t \cos 60° + \cos\omega t \sin 60°)$$

$$= \left(10\sqrt{2} + 20\sqrt{2}\cos 60°\right) \sin\omega t + \left(20\sqrt{2}\sin 60°\right) \cos\omega t$$

$$= \left(20\sqrt{2}\right) \sin\omega t + \left(10\sqrt{6}\right) \cos\omega t$$

$$= I_0 \sin\,(\omega t + \theta)$$

where, $I_0 = \sqrt{\left(20\sqrt{2}\right)^2 + \left(10\sqrt{6}\right)^2} = 37.4$

and $\tan\theta = \dfrac{10\sqrt{6}}{20\sqrt{2}} = \dfrac{\sqrt{3}}{2} \Rightarrow \theta = 41°$

$\therefore$ $I = [(37.4)\sin\,(\omega t + 41°)]$ Amp.

241. Given: $I_0 = 14.4$ Amp, $V_0 = 170$ Volt

Now $$P_{average} = \frac{1}{2} V_0 I_0 \cos\theta$$

$$= \frac{1}{2} \times 170 \times 14.4 \times \cos 30°$$

$$= 1040 \text{ Watt.}$$

243. For resistor

$$I = \frac{60}{120} = 0.5\text{ A}$$

For inductor $(\omega_0 L)\,I = 40$

$\Rightarrow$ $L = 2 \times 10^{-4}\text{H} = 0.2\,m\text{H}$

Now $\omega_0 = \dfrac{1}{\sqrt{LC}}$

$\Rightarrow$ $C = \dfrac{1}{\omega_0^2 L} \Rightarrow C = \left(\dfrac{1}{32}\right)\mu\text{F}$

245. The impedance of the circuit is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

$\therefore$ Current, $I = \dfrac{V}{Z} = V\sqrt{\dfrac{1}{R^2} + \left(\dfrac{1}{X_C} - \dfrac{1}{X_L}\right)^2}$

$$= 240\sqrt{\left(\frac{1}{8}\right)^2 + \left(\frac{1}{3} - \frac{1}{6}\right)^2}$$

$$= 50\text{ A}$$

247. $P_{max} = \dfrac{V^2}{R}$

and $P = \dfrac{V^2 R}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

Now $P = \dfrac{P_{max}}{2}$

$\Rightarrow$ $\dfrac{V^2 R}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} = \dfrac{V^2}{2R}$

$$\Rightarrow \quad \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\Rightarrow \quad \omega L - \frac{1}{\omega C} = \pm R$$

$$\Rightarrow \quad \omega = \left(\sqrt{\frac{1}{LC} + \frac{R^2}{4L^2}}\right) \pm \frac{R}{2L}$$

249. $\frac{1}{Z} = \frac{1}{R + jX_L} - \frac{1}{jX_C} = \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$

For the resonance, the imaginary part should be absent.

$$\therefore \quad \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = 0$$

$$\Rightarrow \quad R^2 + X_L^2 - X_C X_L = 0$$

$$\Rightarrow \quad R^2 + (\omega L)^2 - \left(\frac{1}{\omega C}\right)(\omega L) = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\Rightarrow \quad \nu = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

251. The detector D detects no current. Therefore the bridge is balanced.

$$\frac{R_1 + \frac{1}{j\omega C_1}}{\left(\frac{1}{j\omega C_2}\right)} = \frac{R_3 + j\omega L}{R_2}$$

$$\left(R_1 + \frac{1}{j\omega C_1}\right)(j\omega C_2) = \left(\frac{R_3 + j\omega L}{R_2}\right)$$

Equating real and imaginary parts, we get

$$\frac{C_2}{C_1} = \frac{R_3}{R_2} \quad \text{and} \quad = \frac{L}{R_2}$$

$$\therefore \qquad L = C_2R_2R_1 = C_1R_3R_1$$

253. If the distance of the virtual image from the pole is y, then

$$\frac{1}{y} - \frac{1}{x} = -\frac{1}{f}$$

$$\Rightarrow \qquad y = \frac{fx}{f-x}$$

Now $$\frac{dy}{dt} = \left(\frac{f}{f-x}\right)^2 \left(\frac{dx}{dt}\right)$$

$$\Rightarrow \qquad v_I = \left(\frac{f}{f-x}\right)^2 \sqrt{2g\left(\frac{f}{2}-x\right)}$$

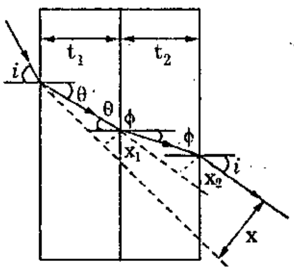
For v_I to be maximum

$$\frac{dv_I}{dx} = 0$$

$$\Rightarrow \qquad x = \frac{f}{3}$$

$$\therefore \qquad (v_I)_{max} = \frac{3}{4}\sqrt{3fg}$$

255. From the geometry of the figure



$$x = x_1 + x_2$$

$$\Rightarrow \qquad x = \frac{t_1}{\cos\theta}\sin(i-\theta) + \frac{t_2}{\cos\phi}\sin(i-\phi) \quad ...(i)$$

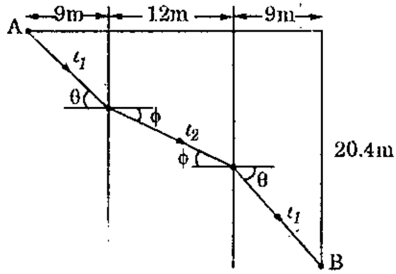
Now $$\sin\theta = \frac{\sin i}{\mu_1} \quad ...(ii)$$

and $$\sin\phi = \frac{\sin i}{\mu_2} \qquad ...(iii)$$

Substituting the values of sin θ and sin φ in equation *(i)*, we get

$$x = \left[t_1\left(1 - \frac{\cos i}{\sqrt{\mu_1^2 - \sin^2 i}}\right) + t_2\left(1 - \frac{\cos i}{\sqrt{\mu_2^2 - \sin^2 i}}\right)\right]\sin i$$

257. The person must move along the path of a "light ray" for the shortest time.



Refractive index $\mu = \frac{2.5}{1.875} = \frac{4}{3}$

From the geometry of the figure

$$2\,(9\tan\theta) + 12\tan\phi = 20.4 \qquad ...(i)$$

From Snell's Law,

$$\frac{4}{3}\sin\phi = \sin\theta \qquad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$\sin\theta = \frac{4}{5} \quad \text{and} \quad \sin\phi = \frac{3}{5}$$

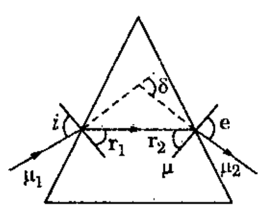
Now, $$l_1 = 9\sec\theta = 15 \text{ m and } l_2 = 12\sec\phi = 15 \text{ m}$$

$\therefore$ $$\text{Time} = \frac{2l_1}{2.5} + \frac{l_2}{1.875} = \mathbf{5\ sec.}$$

259. From Snell's Law

$$\sin r_1 = \left(\frac{\mu_1}{\mu}\right)\sin i \qquad ...(i)$$

$$\sin r_2 = \left(\frac{\mu_2}{\mu}\right)\sin e \qquad ...(ii)$$



From geometry

$$r_1 + r_2 = A \qquad ...(iii)$$

$$\delta = i + e - A \qquad ...(iv)$$

Now $$\sin e = \left(\frac{\mu}{\mu_2}\right) \sin r_2 \qquad \text{[using equation } (ii)]$$

$$\Rightarrow \quad \sin e = \frac{\mu}{\mu_2} [\sin (A - r_1)] \quad \text{[using equation } (iii)]$$

$$\Rightarrow \quad \sin e = \frac{\mu}{\mu_2} (\sin A \cos r_1 - \cos A \sin r_1)$$

$$\Rightarrow \sin e = \frac{\mu_1}{\mu_2} \left[(\sin A) \sqrt{\left(\frac{\mu}{\mu_1}\right)^2 - \sin^2 i} - \cos A \sin i \right]$$

[using equation (i)]

Therefore, from equation *(iv)*, we get

$$\delta = i - A + \sin^{-1}\left[\frac{\mu_1}{\mu_2}\left\{(\sin A)\sqrt{\left(\frac{\mu}{\mu_1}\right)^2 - \sin^2 i} - \cos A \sin i\right\}\right]$$

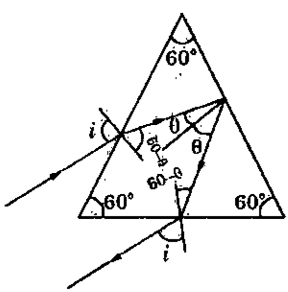
261. From Snell's Law

$$\mu \sin (60° - \theta) = \sin i \qquad ...(i)$$

For total internal reaction,

$$\mu \sin \theta \geq 1 \qquad ...(ii)$$

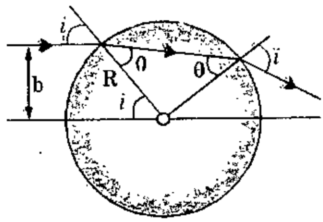
Since, total angle of deviation is 120°, hence, from geometry, we get, $\quad i = 30° \qquad ...(iii)$



Now, from equations *(i)*, *(ii)* and *(iii)*, we get

$$\mu = \sqrt{\frac{7}{3}}$$

263. From the geometry of the figure



$$\sin i = \frac{b}{R} \qquad ...(i)$$

From Snell's Law

$$\sin \theta = \frac{\sin i}{\mu} \qquad ...(ii)$$

Angle of deviation, $\delta = 2(i - \theta)$

$$\Rightarrow \qquad \sin\frac{\delta}{2} = \sin(i - \theta)$$

$$= (\sin i \cos\theta - \cos i \sin\theta)$$

$$= \frac{\sin i}{\mu}\left(\sqrt{\mu^2 - \sin^2 i} - \cos i\right)$$

[using equation *(ii)*]

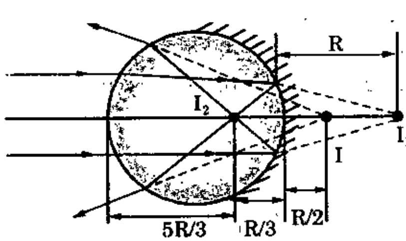
$$= \frac{b}{\mu R}\left(\sqrt{\mu^2 - \frac{b^2}{R^2}} - \sqrt{1 - \frac{b^2}{R^2}}\right)$$

[using equation *(i)*]

$$\Rightarrow \qquad \delta = 2\sin^{-1}\left[\frac{b}{\mu R}\left(\sqrt{\mu^2 - \frac{b^2}{R^2}} - \sqrt{1 - \frac{b^2}{R^2}}\right)\right]$$

265. For the refraction at the spherical surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{R} \qquad ...(i)$$



For the first refraction

$$\mu_1 = 1, \mu_2 = \frac{3}{2}, \; u = \infty, \mathrm{R} = \mathrm{R}$$

$\Rightarrow$ $$v = 3\mathrm{R}$$ [using equation *(i)*]

The image is formed at I_1

For the reflection at polished surface, $u = \mathrm{R}, f = -\dfrac{\mathrm{R}}{2}$.

$\therefore$ $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow$ $$v = -\frac{\mathrm{R}}{3}$$

The image due to reflection is formed at I_2.

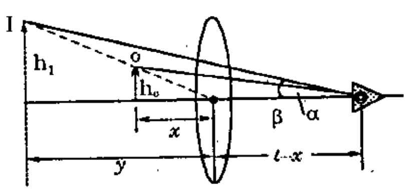
Now, for the refraction at the spherical surface

$$\mu_1 = \frac{3}{2}, \; \mu_2 = 1, \; u = -\frac{5}{3}\mathrm{R}, \mathrm{R} = -\mathrm{R}$$

$\Rightarrow$ $$v = -\frac{5}{2}\mathrm{R}$$ [using equation *(i)*]

Therefore, the final image is formed at I at a distance $\dfrac{\mathrm{R}}{2}$ behind the polished surface.

267. Suppose the distance of object from the lens is x and the distance of corresponding image from the lens is y.



Then $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow$ $$-\frac{1}{y} + \frac{1}{x} = \frac{1}{f}$$

$\Rightarrow$ $$y = \frac{fx}{f - x}$$

Now $$\frac{h_I}{h_o} = \frac{y}{x} = \frac{f}{f-x}$$

The angular magnification is

$$m = \frac{\beta}{\alpha} = \frac{h_I/(y+(l-x))}{h_0/l} = \left(\frac{h_I}{h_o}\right)\left(\frac{l}{l+(y-x)}\right)$$

$\Rightarrow$ $$m = \frac{fl}{x^2 + lf - lx}$$

Form m to be maximum

$$\frac{d}{dx}\left(x^2 + lf - lx\right) = 0$$

$\Rightarrow$ $$x = \frac{l}{2}$$

Therefore $$(m)_{max} = \frac{1}{1+\left(\frac{l}{4f}\right)}$$

269. The lens C will focus the parallel rays on its focus. Therefore, for the composite lens there will be a virtual object at a distance of 1 m on the right side.

From lens formula $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we have $$\frac{1}{v} - \frac{1}{1} = \frac{1}{1}$$

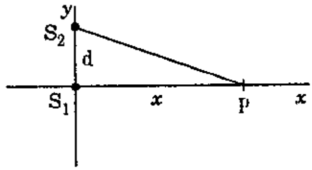
$\Rightarrow$ $$v = 0.5 \text{ m}$$

and magnification, $m = \frac{u}{v} = \frac{0.5}{1} = 0.5$

Hence, there will be two images at a distance of 0.5 m on the right side of the composite lens. The image due to part A will be below the axis at a distance of (0.5) (0.5 cm) = 0.25 cm from the axis. The image due to part B will be above the axis at a distance of 0.25 cm from the axis.

$\therefore$ Distance between the two images is (0.25 + 0.25) cm = 0.5 cm.

271. Consider a point P on the positive X-axis at a distance x from the origin, For minima



$$(S_2P - S_1P) = \left(n + \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad \sqrt{x^2 + (2\lambda)^2} - x = \left(n + \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad x = \frac{\left(4 - \left(n + \frac{1}{2}\right)^2\right)\lambda}{(2n+1)}, \text{ where } n = 1, 0$$

For $n = 1$, $\quad x = \dfrac{7}{12}\lambda$

and for $n = 0$, $\quad x = \dfrac{15}{4}\lambda$

273. For maxima $\quad d \sin\theta = \pm n\lambda$

$$\Rightarrow \quad \sin\theta = \pm \frac{n\lambda}{d}$$

$$\Rightarrow \quad \sin\theta = 0, \pm \frac{1}{2}, \pm 1$$

$$\Rightarrow \quad \theta = 0, 30°, 90°, 150°, 180°, 210°, 270° \; 330°$$

275. If the intensity of light through S_1 and S_2 is I_0. Then,
The intensity through S_3 will be $I_1 = 4I_0$.
The intensity through S_4 will be

$$I_2 = 4I_0 \cos^2 \frac{\phi}{2}$$

$$= 4I_0 \cos^2 \frac{1}{2}\left[\frac{2\pi}{\lambda}\left(\frac{\lambda D}{4d}\right)\frac{d}{D}\right]$$

$$= 4I_0 \cos^2 \frac{\pi}{4} = 2I_0$$

Now $\quad \dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$

$$= \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1} \right)^2 = \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right) \simeq 34$$

277. $$\frac{yd}{D} = (\mu_1 - 1)\, t_1 - (\mu_2 - 1)\, t_2 + d \sin\theta$$

$\Rightarrow$ $$y = \frac{D}{d}\, [(\mu_1 - 1)\, t_1 - (\mu_2 - 1)\, t_2 + d \sin\theta]$$

Substituting the given values, we get

$$y = 10.5 \text{ cm}$$

279. Angle of deviation due to prism

$$\delta = (\mu - 1)\, A$$

$$= (1.5 - 1) \left(\frac{20}{60} \right) \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{1080} \text{ Radian}$$

Distance between two coherent sources,

$$d = 2\,\delta\, (25 \text{ cm})$$

$$= 2 \times \frac{\pi}{1080} \times 25 = 0.145 \text{ cm}$$

Distance between coherent sources and screen

$$D = (25 + 100) \text{ cm} = 125 \text{ cm}$$

Now, fringe width $\beta = \dfrac{\lambda D}{d} = 0.52$ mm

281. If the angle of refraction is r, then

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 60°}{1.33} = 0.65$$

$\Rightarrow$ $$\cos r = 0.76$$

Now, for the maxima in the reflected rays

$$2\mu t \cos r = \left(n + \frac{1}{2} \right) \lambda$$

$\Rightarrow$ $$t = \left(n + \frac{1}{2} \right) \frac{\lambda}{2\mu \cos r}$$

$$\Rightarrow \quad t_{min} = \frac{\lambda}{4\mu\cos r}$$

$$\Rightarrow \quad t_{min} = \frac{6000}{4 \times 1.33 \times 0.76} \text{ Å}$$

$$= 0.15\ \mu m$$

283. $$\text{Pressure} = \frac{2I}{C} = \frac{2 \times \left(\dfrac{5 \times 10^{-3}}{1 \times 10^{-6}}\right)}{3 \times 10^{8}}$$

$$= 3.33 \times 10^{-5}\ \text{N/m}^2$$

285. $\lambda_1 = 3000$ Å,

$\lambda_2 = 6000$ Å

$$E_1 = \frac{hc}{\lambda_1} = 4.14\ eV,$$

$$E_2 = \frac{hc}{\lambda_2} = 2.07\ eV$$

Now $$(4.14)\ eV - W = \frac{1}{2}\ mv_1^2 \quad \text{...(i)}$$

$$(2.07)\ eV - W = \frac{1}{2}\ mv_2^2 \quad \text{...(ii)}$$

and $$\frac{v_1}{v_2} = 3 \quad \text{...(iii)}$$

From equations *(i)*, *(ii)* and *(iii)*, we get

$$W = 1.81\ eV.$$

287. $\lambda_{min} = 4000$ Å

$$\therefore \quad E_{max} = \frac{hc}{\lambda_{min}}$$

$$= 3.1\ eV.$$

KE of photoelectrons,

$$K = 3.1 - 2.39$$

$$= 0.7\ eV$$

Now, $$\frac{mv}{qB} < d$$

and $$\frac{1}{2}mv^2 = K$$

$\Rightarrow$ $$B > \sqrt{\frac{2mK}{qB}}$$

$\Rightarrow$ $$B_{min} = 2.86 \times 10^{-6}\ T$$

289. Here, $\lambda_1 = 1085$ Å,

$\lambda_2 = 304$ Å

$$E_1 = \frac{hC}{\lambda_1}$$

$$= 11.5\ eV,$$

$$E_2 = \frac{hC}{\lambda_2} = 40.8\ eV$$

$$E_1 + E_2 = 52.3\ eV$$

Kinetic energy of electrons after collision

$$E = 100 - 52.3$$
$$= 47.7\ eV.$$

291. Since, from the n^{th} energy level six photons are emitted, therefore

$$\frac{n(n-1)}{2} = 6$$

$\Rightarrow$ $$n = 4$$

It is given

$$E_4 - E_2 = 10.2$$

$\Rightarrow$ $$\left(-\frac{13.6}{(4)^2}z^2\right) - \left(-\frac{13.6}{(2)^2}z^2\right) = 10.2$$

$\Rightarrow$ $$2.55\,z^2 = 10.2$$

$\Rightarrow$ $$z = 2$$

293.
$$F = -\frac{du}{dr} = \frac{e^2}{4\pi\varepsilon_0 r^4}$$

Now
$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^4} \qquad ...(i)$$

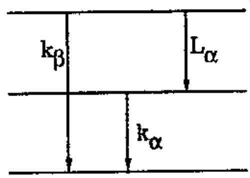
$$\Rightarrow \quad mvr = \frac{nh}{2\pi} \qquad ...(ii)$$

$$\text{Energy, E} = \frac{1}{2}mv^2 - \frac{e^2}{12\pi\varepsilon_0 r^3} \qquad ...(iii)$$

From equations *(i)*, *(ii)* and *(iii)*, we get

$$E = \frac{\epsilon_0^2 n^6 h^6}{24\pi^4 e^4 m^3}$$

295. From the energy level diagram



$$E_{K\beta} = E_{L\alpha} + E_{K\alpha}$$

$$\Rightarrow \quad \frac{hC}{\lambda_{K\beta}} = \frac{hC}{\lambda_{L\alpha}} + \frac{hC}{\lambda_{K\alpha}}$$

$$\Rightarrow \quad \frac{1}{\lambda_{L\alpha}} = \frac{1}{\lambda_{K\beta}} - \frac{1}{\lambda_{K\alpha}}$$

$$\Rightarrow \quad \lambda_{L\alpha} = 140.7 \text{ pm}$$

297. Number of protons incident per second

$$= \frac{10^{-4}}{1.6 \times 10^{-19}} = 6.25 \times 10^{14}$$

Number of (^{7}Be) atoms produced per second

$$= \frac{6.25 \times 10^{14}}{1000}$$
$$= 6.25 \times 10^{11}$$

Now, $$A = N\lambda$$

$$\Rightarrow \qquad \lambda = \frac{A}{N}$$

$$= \frac{1.8 \times 10^{8}}{6.25 \times 10^{11}}$$

$$= 2.885 \times 10^{11}/\text{sec}$$

$$\therefore \qquad \text{Half life, } \tau = \frac{0.693}{\lambda}$$

$$= \frac{0.693}{2.885 \times 10^{-4}} \text{ sec}$$

$$= 2.4 \times 10^{3} \text{ sec.} = 40 \text{ min.}$$

299. Number of fissions per second

$$= \frac{10^{6}}{\left(200 \times 10^{6}\right) \times \left(1.6 \times 10^{-19}\right)}$$

$$= 3.125 \times 10^{16}$$

Mass of U-235 required in one year

$$= \left[\frac{\left(3.125 \times 10^{16}\right)}{\left(6.023 \times 10^{23}\right)} \times 235\right] \times (365 \times 24 \times 60 \times 60)$$

$$= 382.4 \text{ gm.}$$

PART - II

Problems from Previous Years IIT-JEE (From 1972 onwards)

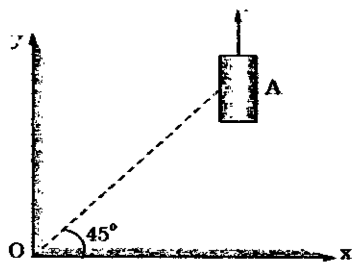
PRACTICE PROBLEMS

PART-II

KINEMATICS

1

On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $\left(\sqrt{3}-1\right)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle φ with the x-axis and it hits the trolley.



(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\varphi = \frac{4\theta}{3}$.

(IIT 2002)

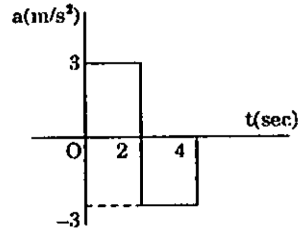
2

A mass A is released from the top of a frictionless inclined plane 18 m long and reaches the bottom 3 seconds later. At the instant when A is released, a second mass B is projected upward along the plane from the bottom with a certain initial velocity. The mass B travels a distance up the plane, stops and returns to the bottom so that it arrives simultaneously with A. The two masses do not collide with each other at any stage. Find the acceleration and initial velocity of B.

(IIT 1978)

3

A particle starts from rest at time $t = 0$ and undergoes acceleration a as shown in the figure.



(i) Draw a neat sketch showing the velocity of particle as a function of time during the interval 0 to 4 seconds, indicating each second on the abscissa.

(ii) Draw a neat sketch showing displacement of the particle as a function of time during the same interval. **(IIT 1977)**

4

A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time lapse is t seconds, evaluate :

(i) maximum velocity reached, and

(ii) the total distance travelled. **(IIT 1978)**

5

A metal ball is allowed to fall freely on a perfectly elastic plane from a height of 3 metres. At $t = 0$ the speed of the ball is zero. Diagramatically represent the variation of velocity with time specifying the proper units on the X and Y-axes. **(IIT 1975)**

6

Two cars are moving in the same direction with the same speed (= 30 km/h). They are separated by a distance 5 km. What is the speed of a car moving in the opposite direction if it met these two cars at an interval of 4 minutes ? **(IIT 1975)**

7

A rocket is fired vertically from the ground with a resultant vertical acceleration of 10 m/s^2. The fuel is finished in 1 minute and it continues to move up. What is the maximum height reached ? (g = 10 m/s^2) **(IIT 1975)**

8

A car covers the first half of the distance between two places at a speed of 40km/h and the second half at 60 km/h. What is the average speed of the car ? **(IIT 1974)**

9

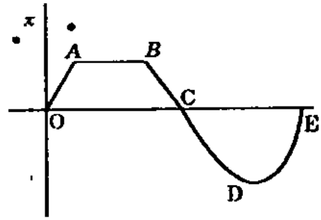
A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point. Which one of them will strike the ground
(*i*) earlier,
(*ii*) with greater velocity. **(IIT 1974)**

10

From the top of a building, a ball is dropped while another is thrown horizontally at the same time. Which ball will strike the ground first ? **(IIT 1974)**

11

Figure gives the displacement of a particle along the x-axis as a function of time. Find the direction of the velocity and acceleration of the particle between the following points.



(*i*) Between O and A
(*ii*) Between A and B
(*iii*) Between C and D **(IIT 1973)**

12

The acceleration due to gravity on a planet is 196 cm/s^2. If it is safe to jump from a height of 2 metres on the earth, what will be the corresponding safe height on the planet ?
(g_{earth} = 980 m/s^2) **(IIT 1972)**

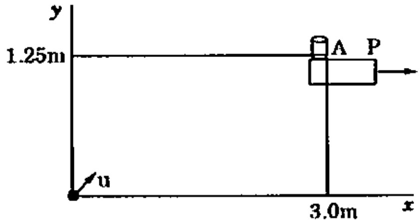
13

A block of ice starts sliding down from the top of the inclined roof of a house (angle of inclination of roof = 30° with the horizontal) along a line of maximum slope. The highest and lowest points of the roof are at heights of 8.1 metre and 5.6 metre respectively from the ground. At what horizontal distance from the starting point will the block hit the ground ? Neglect friction. (g = 9.8 m/s^2). **(IIT 1972)**

PROJECTILE

14

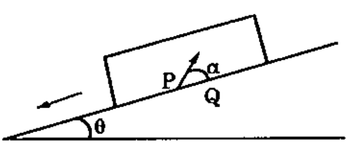
An object A is kept fixed at the point x = 3 and y = 1.25 m on a plank P raised above the ground. At time t = 0, the plank start moving alongthe $+x$ direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the x-y plane. Find u and the time after which the stone hits the object. Take $g = 10$ m/s^2.



(IIT 2000)

15

A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box, the initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom as shown in the figure.



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected. **(IIT 1998)**

16

Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ ms^{-1} at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find

(i) the time-interval between the firings, and

(ii) the coordinates of the point P.

Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. **(IIT 1996)**

17

A body falling freely from a given height 'H' hits an inclined plane in its path at a height 'h'. As a result of this impat the direction of the velocity of the body becomes horizontal. For what value of (h/H) the body will take maximum time to reach the ground ?

(IIT 1986)

18

A gun, kept on a straight horizontal road, is used to hit a car travelling along the same road away from the gun with a uniform speed of 72 km/h. The car is at a distance of 500 metres from the gun when the gun is fired at an angle 45° with the horizontal. Find *(i)* the distance of the car from the gun when the shell hits it *(ii)* the speed of projection of the shell from the gun. (g = 9.8 m/s^2)

(IIT 1974)

LAWS OF MOTION

19

A particle of mass 10^{-2} kg is moving along the positive x-axis under the influence of a force $F(x) = -\frac{K}{(2x^2)}$, where $K = 10^{-2}\ \mathrm{Nm}^2$.

At time $t = 0$ it is at $x = 1.0$ m and its velocity is $v = 0$.

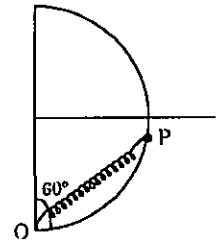
(a) Find its velocity when it reaches $x = 0.50$ m.

(b) Find the time at which it reaches $x = 0.25$ m **(IIT 1998)**

20

A smooth semicircular wire-track of radius R is fixed in a vertical plane

One end of a massless spring of natural length $\frac{3R}{4}$ is attached to the lowest point O of the wire-track. A small ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant



K = mg/R. Consider the instant when the ring is released, and

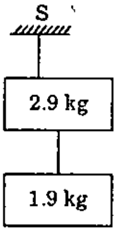
(i) draw the free body diagram of the ring,

(ii) determine the tangential acceleration of the ring and the normal reaction.

(IIT 1996)

21

Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid suport S by two inextensible wires each of length 1 metre see figure. The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg/m. The whole system of blocks wires and support have an upward acceleration of $0.2 m/s^2$. Acceleration due to gravity is $9.8 m/s^2$.



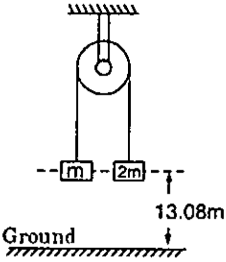
(i) Find the tension at the mid-point of the lower wire.

(ii) Find the tension at the mid point of the upper wire.

(IIT 1989)

22

Two masses m and 2m are connected by a massless string which passes over a light frictionless pulley as shown in the figure. The masses are initially held with equal lengths of strings on either side of the pulley. Find the velocity of the masses at the instant the lighter mass moves up a distance of 6.54 m. The string is suddenly cut at that instant. Calculate the time taken by each mass to reach the ground. ($g = 981 cm/s^2$)



(IIT 1977)

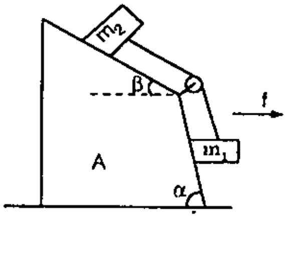
23

A horizontal uniform rope of length L, resting on a frictionless horizontal surface, is pulled at one end by a force F. What is the tension in the rope at a distance l from the end where the force is applied ?

(IIT 1978)

24

Two cubes of masses m_1 and m_2 be on two frictionless slopes of block A which rests on a horizontal table. The cubes are connected by a string which passes over a pulley as shown in the figure. To what horizontal acceleration f should the whole system (that is blocks and cubes) be subjected so that the cubes do not slide down the planes. What is the tension of the string in this situation ?



(IIT 1978)

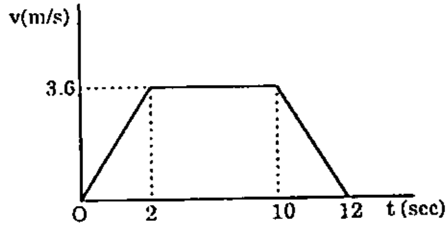
25

A spring of force constant K is cut into three equal parts. What is force constant of each part ? **(IIT 1978)**

26

A lift is going up. The total mass of the lift and the passengers is 1500 kg. The variation in speed of the lift is as given in the graph.

(a) What will be the tension in the rope pulling the lift at t equal to
(i) 1 sec
(ii) 6 sec
(iii) 11 sec.



(b) What is the height to which the lift takes the passengers ?

(c) What will be the average velocity and the average acceleration during the course of the entire motion ? ($g = 9.8\ m/s^2$).

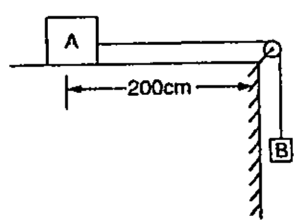
(IIT 1976)

27

Two balls A and B of masses 100 gm and 250 gm respectively and connected by a stretched string of negligible mass, and placed on a smooth table. When the balls are released simultaneously, the initial acceleration of ball B is 10 cm/s^2 westward. What is the magnitude and direction of the initial acceleration of the ball A ? **(IIT 1975)**

28

A mass A (500 gm) is placed on a smooth table with a string attached to it. The string goes over a frictionless pulley and is connected to another mass B (200 gm). At $t = 0$, the mass A is at a distance of 200 cm from the end and moving with a speed of 50 cm/sec towards left (see figure). What will be its position and its speed at $t = 1$ second ? ($g = 980\ cm/s^2$)



(IIT 1975)

29

A spring weighing machine kept inside a stationary elevator reads 50 kg when a man stands on it. What would happen to the scale reading if the elevator is moving upward with

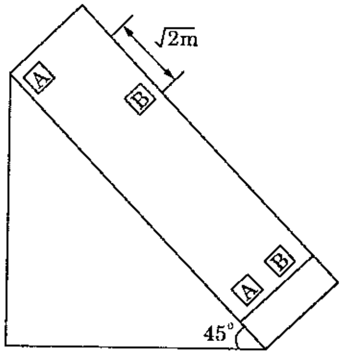
(i) constant velocity

(ii) constant acceleration ? **(IIT 1972)**

FRICTION

30

Two identical blocks A and B are placed on a rough inclined plane of inclination 45°. The coefficient of friction between block A and incline is 0.2 and that of between B and incline is 0.3. The initial separation between the two blocks is $\sqrt{2}$m. The two blocks are released from rest, then find



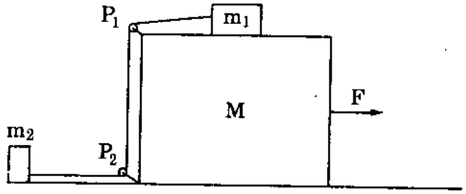
(a) the time after which front faces of both blocks come in same line and

(b) the distance moved by each block for attaining above position.

(IIT 2004)

31

In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the string are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applies to the mass M. Take g = 10 m/s^2.



(a) Draw a free body diagram for mass M, clearly showing the forces.

(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2.

For a particular F it is found that

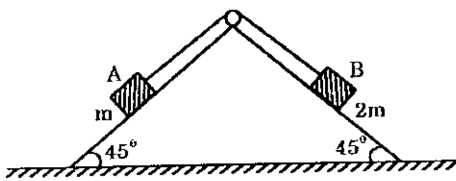
$$f_1 = 2f_2.$$

Find f_1 and f_2. Write down equations of motion of all the masses. Find F, tension in the string and accelerations of the masses.

(IIT 2000)

32

Block A of mass m block B of mass 2 m are placed on a fixed triangular wedge by means of a massless, in extensible string and a frictionless pulley as shown in Figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the system A and B is released from rest, find



(i) the acceleration of A,

(ii) tension in the string,

(iii) the magnitude and direction of friction acting on A.

(IIT 1997, May)

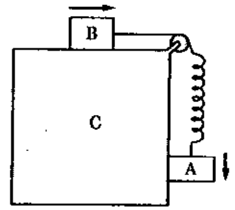
33

A particle of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

(IIT 1987)

34

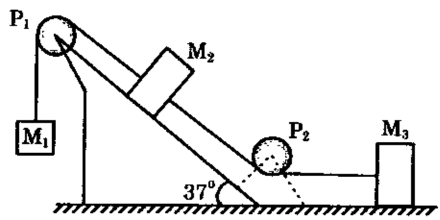
Two blocks A and B are connected to each other by a string and a spring; the string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of blocks is 0.2. Force constant of the spring is 1960 Newtons/m. If mass of block A is 2 kg, calculate the mass of block B and the energy stored in the spring.



(IIT 1982)

35

Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in the figure. The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal.



(IIT 1981)

If the mass M_1 moves downward with a uniform velocity, find :

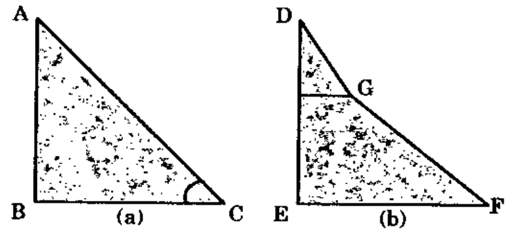
(i) the mass of M_1

(ii) tension in the horizontal portion of the string.

$(g = 9.8 \text{ m/s}^2, \sin 37° = \frac{3}{5})$

36

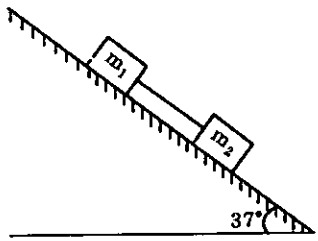
In the Figs. (a) and (b), AC, DG and GF are fixed inclined planes, BC = EF = x and AB = DE = y. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed V_c. The same



block is released from rest from the point D. It slides down DGF and reaches the point F with speed V_F. The coefficient of kinetic frictions between the block and both the surfaces AC and DGF are μ. Calculate V_C and V_F. **(IIT 1980)**

37

Two blocks connected by a massless string slide down an inclined plane having an inclination of 37°. The masses of two blocks are $m_1 = 4$ kg and $m_2 = 2$ kg respectively and the coefficient of friction of m_1 and m_2 with inclined plane are 0.75 and 0.25 respectively. Assuming the string to be taut, find

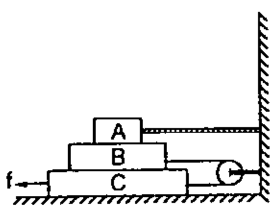


(i) the common acceleration of two masses and
(ii) the tension in the string ($\sin 37° = 0.6$, $g = 9.8$ m/s^2)

(IIT 1979)

38

In the diagram shown, the blocks A, B and C weight 3 kg, 4 kg and 5 kg respectively. The coefficient of sliding friction between any two surfaces is 0.25. A is held at rest by a massless rigid rod fixed to the wall while B and C are connected by a light flexible cord passing around a frictionless pulley.



Find the force F necessary to drag C along the horizontal surface to the left at constant speed. Assume that the arrangement shown in the diagram, B on C and A on B, is maintained all through. (g = 9.8 m/s^2) **(IIT 1978)**

39

An aeroplane requires for take off a speed of 80 km/h, the run on the ground being 100 metres. The mass of the plane is 10000 kg and the coefficient of friction between the plane and the ground is 0.2. Assume that the plane accelerates uniformly during the take off. What is the minimum force required by the engine of the plane for the take off ? ($g = 9.8$ m/s^2) **(IIT 1977)**

40

A block of mass 2 kg slides on an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\sqrt{\frac{3}{2}}$.

(i) What force should be applied to the block so that the block moves down without any acceleration ?

(ii) What force should be applied to the block so that it moves up without any acceleration ?

(iii) Calculate the ratio of the powers needed in the above two cases, if the block moves with the same speed in both cases. ($g = 9.8\ m/s^2$) **(IIT 1976)**

CIRCULAR MOTION

41

Two blocks of mass $m_1 = 10$ mg and $m_2 = 5$ kg, connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/s about a vertical axis passing through its centre O. The masses are placed along the diameter of the table on either side of the center O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1.

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position ?

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1? **(IIT 1997, July)**

42

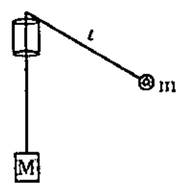
A hemispherical bowl of radius $R = 0.1$ m is rotating about its own axis (which is vertical), with an angular velocity ω. A particle of mass 10^{-2} kg on the frictionless inner surface of the bowl is also rotating with the same ω. The particle is at a height h from the bottom of the bowl.

(i) Obtain the relation between h and ω. What is the minimum value of ω needed, in order to have a non-zero value of h ?

(ii) It is desired to measure g (acceleration due to gravity) using this set-up, by measuring h accurately. Assuming that R and ω are known precisely, and that the least count in the measurement of h is 10^{-4} m, what is the minimum possible error Δg in the measured value of g ? **(IIT 1993)**

43

A large mass M and a small mass m hang at two ends of a string that passes over a smooth tube as shown in the figure. The mass m moves around a circular path which lies in a horizontal plane. The length of string from the mass m to the top of the tube is l and θ is the angle this length makes with the vertical. What should be the frequency of rotation of mass m, so that the mass M remains stationary ?



(IIT 1978)

44

The driver of a truck travelling with a velocity v suddenly notices a broad wall in front of him at a distance r. Is it better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall ? Why ? **(IIT 1977)**

45

A string of length 1 metre is fixed at one end and carries a mass of 100 gm at the other end. The string makes $\left(\frac{2}{\pi}\right)$ revolutions per second around a vertical axis passing through the fixed end. Calculate :

(i) the angle of inclination of the string with the vertical.

(ii) the tension in the string, and

(iii) the linear velocity of the mass.

($g = 9.8$ m/s^2) **(IIT 1976)**

46

A sphere of mass 200 gm is attached to an inextensible string of length 130 cm whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius 50 cm.

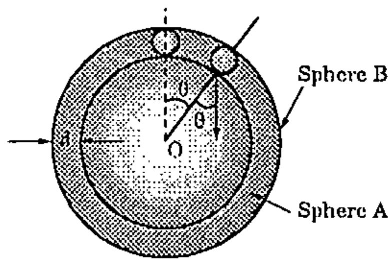
(i) Calculate the time period of one revolution.

(ii) What is the tension in the string.

($g = 980$ cm/s^2). **(IIT 1974)**

WORK, POWER & ENERGY

47

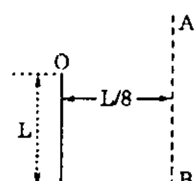


A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than *d*. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure).

(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axes. **(IIT 2002)**

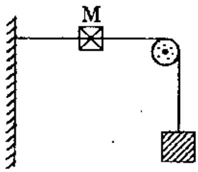
48



A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $\frac{L}{8}$ from O as shown. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find *u*. **(IIT 1999)**

49

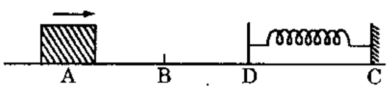
A string with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2 m from the wall, has a point mass M = 2 kg attached to it at a distance of 1 m from the wall. A mass *m* = 0.5 kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? ($g = 9.8$ m/s^2).



(IIT 1985)

50

A 0.5 kg block slides from the point A (see figure) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0.2 respectively. If the distances AB and BD are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely. (Take $g = 10$ m/s^2).



(IIT 1983)

51

Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ (rho). The height of the liquid in one vessel is h_1 and in other is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the two vessels are connected ? **(IIT 1981)**

52

The displacement x of a particle, moving in one dimension, under the action of a constant force is related to the time t by the equation

$t = \left(\sqrt{x}\right) + 3$, where x is in metres and t in seconds.

Find :

(i) the displacement of the particle when its velocity is zero, and

(ii) the work done by the force in first 6 seconds. **(IIT 1979)**

53

A ball falls under gravity from a height of 10 metres with an initial downward velocity v_0. It collides with ground loses 50% of its energy in collision and then rises back to the same height. Find :

(i) the initial velocity v_0 and

(ii) the height to which the ball would rise after collision, if the initial velocity v_0 was directed upward instead of downward. ($g = 9.8$ m/s^2). **(IIT 1979)**

54

A 40 kg mass, hanging at the end of a rope of length l, oscillates in a vertical plane with an angular amplitude θ_0. What is the tension in the rope when it makes an angle θ with the vertical? If the breaking strength of the rope is 80 kg, what is the maximum amplitude with which the mass can oscillate with the rope breaking? **(IIT 1978)**

55

A particle of mass m is moving in a horizontal circle of radius r under centripetal force $-\frac{k}{r^2}$, where k is a constant. What is the total energy of the particle?

(IIT 1977)

56

Two springs have their force constants as k_1 and k_2 ($k_1 > k_2$). In which spring is more work done :

(i) when their lengths are increased by the same amount

(ii) when they are stretched by the same force. **(IIT 1976)**

57

A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from a position where the string makes an angle of 60° with the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolutions with the nail as centre. Assume the length of the pendulum to be 1 metre. **(IIT 1975)**

58

A man weighing 60 kg climbs up a staircase carrying a 20 kg load on his head. The stair case has 20 steps and each step has a height 20 cm. If he takes 10 seconds to climb, calculate the power. ($g = 9.8$ m/s^2) **(IIT 1974)**

59

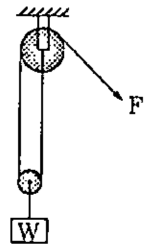
The length of a simple pendulum is 1 m. The bob of the pendulum of mass 10 gm is released when the string is horizontal. When it is at the lowest point of the path,

(i) what is its kinetic energy?

(ii) what is tension in the string?
($g = 980\ cm/s^2$). **(IIT 1973)**

60

The pulley system shown in the figure is used to lift a weight W with uniform velocity by applying a force F at the free end of the string.



(i) When the free end of the string is pulled through a distance x by what distance does W move up ?

(ii) What is corresponding change in the energy of W?

(iii) What is the magnitude of F and the work done by it?

(iv) What is mechanical advantage of the system?

(Neglect the friction and the mass of the pulley and strings). **(IIT 1973)**

SYSTEM OF PARTICLES

61

Two masses m_1 and m_2 connected by a light spring of natural length l_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along +ve x-axis pass through the origin at $t = 0$. At this position the string snaps. Position of mass m_1 at time t is given by the equation

$$x_1(t) = v_0 t - A(1 - \cos \omega t)$$

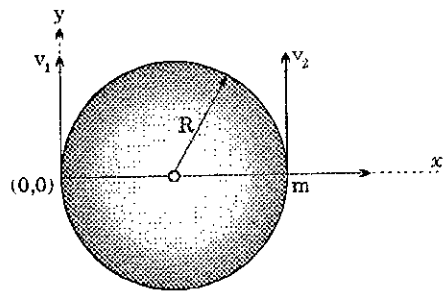
Calculate :

(a) position of the particle m_2 as a function of time.

(b) l_0 in terms of A. **(IIT 2003)**

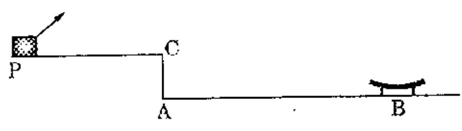
62

A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time $t = 0$ and a man starts moving with a velocity v_1 along the + ve y-axis from origin at time $t = 0$. Calculate the linear momentum of the particle with respect to the man as a function of time. **(IIT 2003)**



63

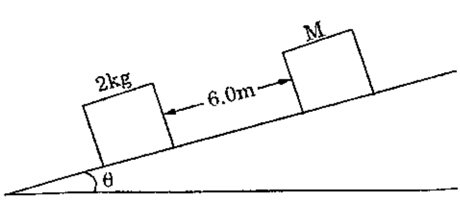
A car P is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30° with the horizozntal. The first cannon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0.



At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact ? **(IIT 2001)**

64

Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown. The coefficient of friction between each of the blocks and the inclined



plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collide with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M. [Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10$ m/s^2]

(IIT 1999)

65

A cart is moving along + x direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine :

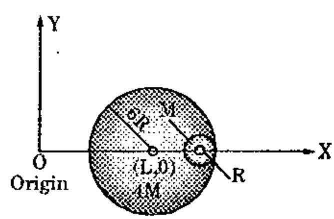
(i) the speed of the combined mass immediately after the collision with respect to an observer on the ground.

(ii) the length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

(IIT 1997, July)

66

A small sphere of radius R is held against the inner surface of a larger sphere of radius 6R (Fig.). The masses of large and small sphres are 4M and M, respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position.

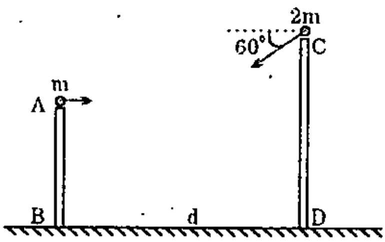


(IIT 1996)

67

Two towers AB and CD are situated a distance 'd' apart as shown in the figure. AB is 20 m high and CD is 30 m high from the ground.

An object of mass m is thrown from the top of AB horizontally with a velocity of 10 ms^{-1} towards CD. Simultaneously another object of mass 2m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other.



(i) Calculate the distance d between the towers and

(ii) Find the position where the objects hit the ground.

(IIT 1994)

68

A cylindrical solid of mass 10^{-2} kg and cross sectional ara 10^{-4} m^2 is moving paralle to its axis (the x-axis) with a uniform speed of 10^3 m/s in the positive direction. At $t = 0$, its front face passes the plane $x = 0$. The region to the right of this plane is filled with stationary dust particles of uniform density 10^{-3} kg/m^3. When a dust particle collides with the face of the cylinder, it sticks to its surface. Assuming that the dimensions of the cylinder remain practically unchanged, and that the dust sticks only to the front face of the cylinder, find the x-coordinate of the front of the cylinder at $t = 150$ sec. **(IIT 1993)**

69

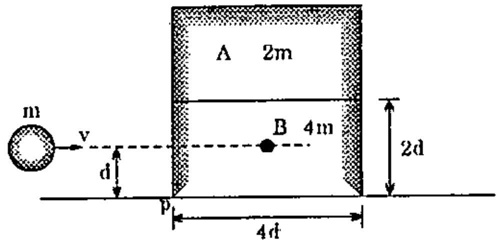
A uniform thin rod of mass M and length L is standing vertically along the Y-axis on a smooth horizontal surface with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on smooth surface along the positive X-axis, and the rod starts falling.

(i) What is the path followed by the centre of mass of the rod during its fall ?

(ii) Find the equation to the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point ? **(IIT 1993)**

70

A block 'A' of mass 2 m is placed on another block 'B' of mass 4 m which in turn is placed on a fixed table. The two blocks have a same length 4d and they are placed as shown in the figure. The coefficient of friction (both static and kinetic) between the block 'B' and table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass (cm) of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table.



(a) What is the minimum value of v (call it v_0) required to make the block A topple ?

(b) If $v = 2\,v_0$, find the distance (from the point P in the figure) at which the mass m falls on the table after collision.
(Ignore the role of friction during the collision.) **(IIT 1991)**

71

An object of mass 5 kg is projected with a velocity of 20 m/s at an angle of 60° to the horizontal. At the highest point of its path the projectile explodes and breaks up into two fragments of masses 1 kg and 4 kg. The fragments separate horizontally after the explosion. The explosion releases internal energy such that the kinetic energy of the system at the highest point is doubled. Calculate the separation between the two fragments when they reach the ground. **(IIT 1990)**

72

A bullet of mass M is fired with a velocity 50 m/s at an angle with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass 3 M suspended by a massless string of length 10/3 metres and gets embedded in the bob. After the collision, the string moves through an angle of 120°. Find :

(i) the angle θ ;

(ii) the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet.
(Take $g = 10\ m/s^2$). **(IIT 1988)**

73

A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degrees ?

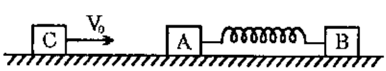
(IIT 1987)

74

A ball of mass 100 gm is projected vertically upwards from the ground with a velocity of 49 m/sec. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses.(g = 9.8 m/s^2) **(IIT 1985)**

75

Two bodies A and B of masses m and 2m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A as shown in Fig. At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further at this instant the compression of the spring is found to be x_0. Determine

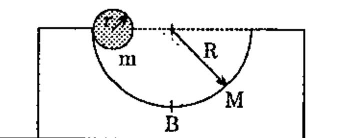


(i) the common velocity of A and B at time t_0; and

(ii) the spring constant. **(IIT 1984)**

76

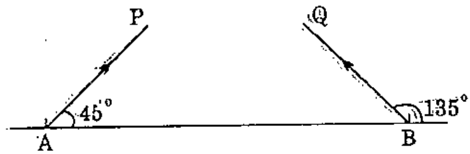
A block of mass M with a semi-circular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see figure). The cylinder slips on the semicircular frictionless track. How far has the



block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track ? **(IIT 1983)**

77

Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the figure. Each particle has an intial speed of 49 m/s. The separation AB is 245 m. Both particles travel in the same vertical plane and undergo a collision. After the collision P retraces its path.

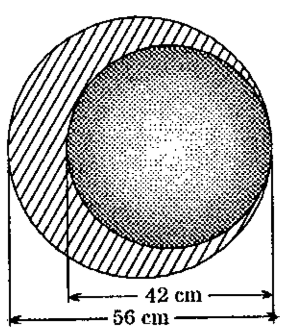


Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground ? ($g = 9.8\ m/s^2$)

(IIT 1982)

78

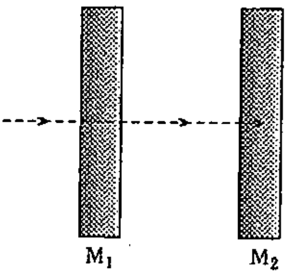
A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in the figure.



Find the position of the centre of mass of the remaining portion. **(IIT 1980)**

79

A 20 gm bullet pierces through a plate of mass $M_1 = 1$ kg and then comes to rest inside a second plate of mass $M_2 = 2.98$ kg, as shown in the figure. It is found that the two plates, initially at rest now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is between M_1 and M_2. Neglect any loss of material of the plates, due to action of the bullet.



(IIT 1979)

80

A body of mass m moving with velocity V in the X-direction collides with another body of mass M moving in Y-direction with velocity v. They coalesce into one body during collision. Calculate :

(i) the direction and magnitude of the momentum of the final body.

(ii) the fraction of initial kinetic energy transformed into heat during the collision in terms of the two masses. **(IIT 1978)**

81

A bullet is fired from a rifle. If the rifle recoils freely, determine whether the kinetic energy of the rifle is greater than, equal or less than that of the bullet.

(IIT 1978)

82

A plastic ball is dropped from a height of 1 M and rebounds several times from the floor. If 1.3 seconds elapse from the moment it is dropped to the second impact with the floor, what is the coefficient of restitution ? ($g = 9.8$ m/s^2) **(IIT 1977)**

83

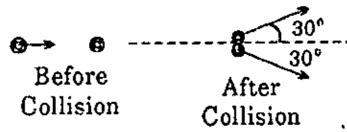
A mass of 2.9 kg is suspended from a string of length 50 cm, and is at rest. Another mass of 100 gm, which is moving horizontally with a velocity of 150 m/s strikes and sticks to it.

(i) What is the tension in the string when it makes 60° with the vertical ?

(ii) Will it complete a vertical circle ? ($g = 9.8$ m/s^2) **(IIT 1976)**

84

A ball moving with a speed of 9m/s strikes an identical ball such that after the collision the direction of each ball makes an angle 30° with the original line of motion (see figure).



Find speeds of the two balls after collision. Is the kinetic energy conserved in this collision process ? **(IIT 1975)**

85

A cricket ball of mass 150 grams is moving with a velocity of 12 m/s and is hitted by a bat so that the ball is turned back with a velocity of 20 m/s. The force of the blow acts on the ball for 0.01 seconds. Find the average force exerted on the ball by the bat. **(IIT 1974)**

86

A wooden block of mass 10 grams is dropped from the top of a cliff 100 metres high. Simultaneously a bullet of mass 10 grams is fired from the foot of the cliff vertically upwards with a velocity of 100 m/s.

(i) Where and after what time will they meet ?

(ii) If the bullet after striking the block gets embeded in it, how high will it rise above the cliff before it starts falling ? ($g = 9.8$ m/s^2). **(IIT 1973)**

87

A projectile of mass 50 kg is thrown vertically upward with an initial velocity of 100 m/s. After 5 seconds it explodes into two fragments, one of which having mass 20 kg travels vertically up with a velocity of 150 m/s.

(i) What is the velocity of the other fragment of this instant ?

(ii) Calculate the sum of momenta of the two fragments 3 seconds after the explosion. What would have been the momentum of the projectile at this instant if there had been no explosion ? ($g = 9.8$ m/s^2). **(IIT 1973)**

88

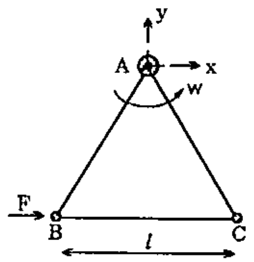
A ball moving on a horizontal frictionless plane hits an identical ball at rest with a velocity of 50 cm/s. If the collision is elastic, calculate the speed imparted to the target ball if the speed of the projectile ball after the collision is 30 cm/s. Show that two balls will move at right angles to each other after the collision.

(IIT 1972)

ROTATION

89

Three particles A, B and C each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.

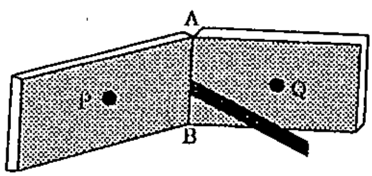


(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T. **(IIT 2002)**

90

Two heavy metallic plates are joined together at 90° to each other, a laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless.



The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 kgm^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB this distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacle with an angular velocity 1 rad/s and turns back. If the impulse of the sheet due to each obstacle is 6 N-s,

(a) find the location of the centre of mass of the laminar sheet from AB.

(b) at what angular velocity does the laminar sheet come back after the first impact ?

(c) after how many impacts, does the laminar sheets come to rest ? **(IIT 2001)**

91

A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in a direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

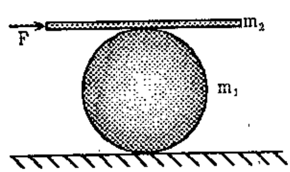
(a) Find the ratio m/M.

(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.

(c) Find the linear speed of the point P at a time $\left(\dfrac{\pi L}{3v_0}\right)$ after the collision. **(IIT 2000)**

92

A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown in the figure. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find :



(a) the accelerations of the plank and the centre of mass of the cylinder, and

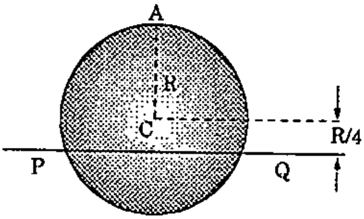
(b) the magnitudes and direction of frictional forces at contact points **(IIT 1999)**

93

A uniform circular disc has radius R and mass m. A particle also of mass m, is fixed at a point A on the edge of the disc as shown in the figure. The disc can rotate freely about a fixed horizontal chord

PQ that is at a distance $\frac{R}{4}$ from the centre C of the disc. The line AC is perpendicular to PQ.

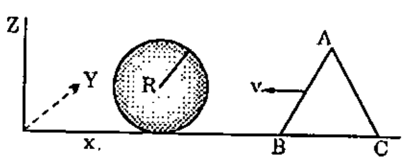
Initially, the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle as it reaches its lowest position.



(IIT 1998)

94

A wedge of triangular cross section (AB = BC = CA = 2R) is moving with a constant velocity $-v\hat{i}$ towards a sphere of radius R fixed on a smooth horizontal table as shown in the figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time Δt during which the sphere exerts a constant force F on the wedge :

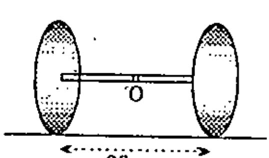


(a) Find the force $\vec{F}$ and also the normal force $\vec{N}$ exerted by the table on the wedge during the time Δt.

(b) Let h denote the perpendicular distance between the centre of mass of the wedge and the line of action of $\vec{F}$. Find the magnitude of the torque due to the normal force $\vec{N}$ about the centre of the wedge, during the interval Δt. **(IIT 1998)**

95

Two thin circular disks of mass 2 kg and raidus 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disk through their centres



(see fig.). This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of the motion of the truck.

Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x axis as the direction of motion of the truck and z axis as the vertically upwards direction. If the truck has an acceleration of 9 m/s^2, calculate

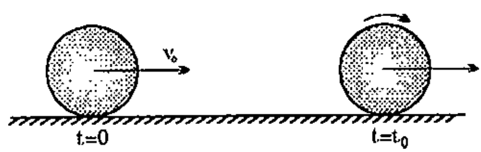
(i) the force of friction on each disk.

(ii) the magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object.
Express the torque in the vector form in terms of unit vectors in the $\hat{i}$, $\hat{j}$ and $\hat{k}$ in the x, y and z directions.

(IIT 1997, July)

96

A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at $t = 0$. After t_0 seconds, it acquires a purely rolling motion as shown in the figure.

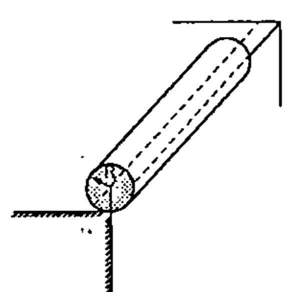


(i) Calculate the velocity of the centre of mass of the disc at t_0.

(ii) Assuming the coefficient of friction to be μ, calculate to. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0. **(IIT 1997, May)**

97

A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in the figure. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine :



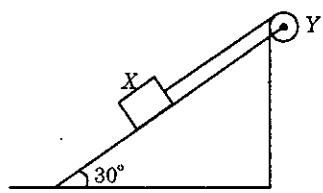
(a) the angle θ_c through which the cylinder rotates before it leaves contact with the edge.

(b) speed of the centre of mass of the cylinder before leaving contact with the edge, and

(c) ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge. **(IIT 1995)**

98

A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in the figure. The drum is given an initial angular velocity such that the block X starts moving up the plane.



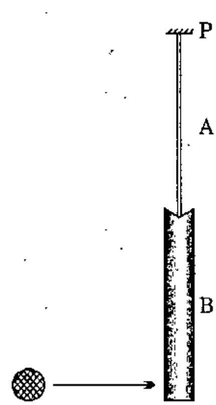
(i) Find the tension in the string during the motion.

(ii) At a certain instant of time, the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest.

(IIT 1994)

99

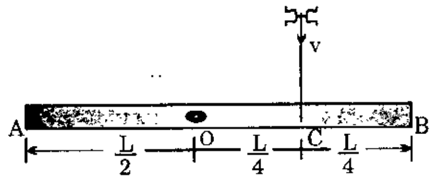
Two uniform this rods A and B of length 0.6 m each and of masses 0.01 kg. and 0.02 kg. respectively are rigidly joined, end to end. The combination is pivoted at the lihgter end P as shown in the figure, such that it can freely rotate about the point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position ?



(IIT 1994)

100

A homogeneous rod AB of length L = 1.8 m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (see figure). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω.



(a) Determine the angular velocity ω in terms of V and L.

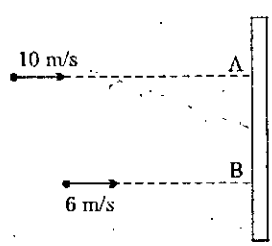
(b) If the insect reaches the end B when the rod has turned through an angle of 90°, determine V. **(IIT 1992)**

101

A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylinderical part of the carpet when its radius reduces to R/2. **(IIT 1990)**

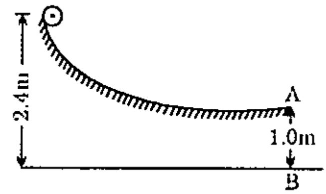
102

A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length metres. Two particles, each of mass 0.08 kg, are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity of 10 m/s and the other with 6 m/s, as shown in the figure. The first particle strikes the bar at point A and the other at point B. Points A and B are at a distance of 0.5 m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of kinetic energy of the system in the above collision process. **(IIT 1989)**



103

A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in the figure) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass ? Explain. **(IIT 1987)**

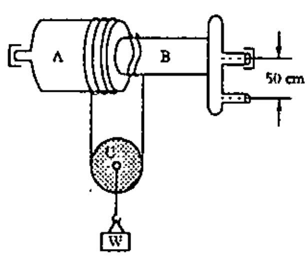


104

A particle is projected at time $t = 0$ from a point P on the ground with a speed v_0, at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t = \frac{v_0}{g}$. **(IIT 1984)**

105

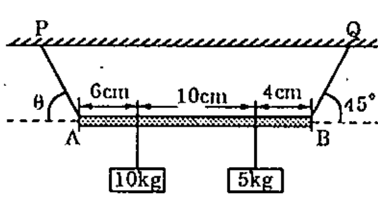
A pulley arrangement is shown in the figure. The cylinder A has a diameter of 30 cm and the cylinder B has a diameter of 20 cm. The working handle has an arm of 50 cm. The direction of winding of the rope on A is opposite to that on B.



Calculate the mechanicaladvantage of this arrangement. **(IIT 1980)**

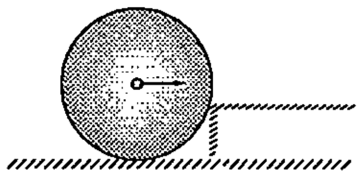
106

A thin bar AB, whose weight can be neglected, is suspended by strings from the two points A and B (see figure). The bar carries masses of 10 kg and 5 kg as shown. Find the tensions in the strings and the angle θ, if the system is the equilibrium. **(IIT 1977)**



107

A wheel of radius 40 cms rests against a step of height 20 cms as shown in the figure. What is the minimum horizontal force which, if applied perpendicular to the axle, will make the wheel climb the step ? The mass of the wheel is 2 kg. ($g = 9.8\ m/s^2$)



(IIT 1976)

108

A table has a heavy circular top of radius 1 metre and mass 20 kg. It has four light legs of length 1 metre fixed symmetrically on its circumference.

(i) What is the maximum mass that may be placed any where on this table without toppling the table ?

(ii) What is the area of the table top over which any weight may be placed without toppling it ? **(IIT 1974)**

GRAVITATION

109

There is a crater of depth $\frac{R}{100}$ on the surface of the moon (radius R). A projectile is fired vertically upward from the crater with a velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile.

(IIT 2003)

110

Distance between the centres of two stars is 10a. The masses of these stars are M and 16 M and their radii a and 2a, respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star ? Obtain the expression in terms of GM and a. **(IIT 1996)**

111

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth.

(i) Determine the height of the satellite above the earth's surface.

(ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth. **(IIT 1991)**

112

Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual separation a. Find the initial velocity that should be given to each particle and also the time period of the circular motion. **(IIT 1988)**

113

Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S_2 is closest to S_1, find

(i) the speed of S_2 relative to S_1,

(ii) the angular speed of S_2 as actually observed by an astronaut in S_1. **(IIT 1986)**

114

Consider an earth satellite so positioned that it appears stationary to an observer on the earth and serves the purpose of a fixed relay station for intercontinental transmission of television and other communication. What would be the height at which satellite be positioned and what would be the direction of its motion ?

(Radius earth = 6.4×10^8 cm, g = 980 cm/s^2) **(IIT 1973)**

115

The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum of the earth ?

(IIT 1973)

PROPERTIES OF MATTER

116

In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data. **(IIT 2004)**

117

A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses.

(For the steel wire : Young's Modulus = 2.1×10^{11} Pa; Density = 7860 kg/m^3; Specific heat = 420 J/kg–K). **(IIT 2001)**

118

A light rod of length 200 cm is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section 0.1 sq cm and the other is of brass of cross-section 0.2 sq cm. Find out the position along the rod at which a weight may be hung to produce

(i) equal stress in both wires

(ii) equal strains in bob wires.

($Y_{brass} = 1 \times 10^{12}$ dyne/cm^2, $Y_{steel} = 2 \times 10^{12}$ dyne/cm^2)

(IIT 1974)

119

A sphere of radius 10 cm and mass 25 kg is attached to the lower end of a steel wire which is suspended from the ceiling of a room, the point of support is 521 cm above the floor. When the sphere is set swinging as a simple pendulum, its lowest point just grates the floor. Calculate the velocity of the ball at its lowest position.

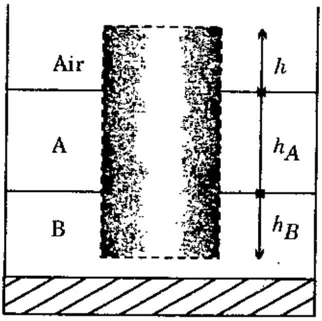
(Young's modulus of steel = 20×10^{11} dyne/cm^2. Unstretched length of wire = 500 cm, Radius of steel wire = 0.05 cm) g = 980 cm/s^2.

(IIT 1972)

HYDROSTATICS

120

A uniform solid cylinder of density $0.8\ g/cm^3$ floats in equilibrium in a combination of two non mixing liquids A and B with its axis vertical. The densities of the liquids A and B are $0.7\ g/cm^3$ and $1.2\ g/cm^3$, respectively. The height of liquid A is $h_A = 1.2$ cm the length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm.



(a) Find the total force exerted by liquid A on the cylinder.

(b) Find h, the length of the part of the cylinder in air.

(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released. **(IIT 2002)**

121

A wooden stick of length L, radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ ($> \rho$). **(IIT 1999)**

122

A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L.

(a) If $d < d_L$, obtain an expression (in terms of d, t_1, and d_L) for the time t_2 the ball takes to come back to the position from which it was released.

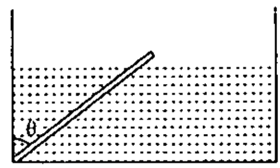
(b) Is the motion of the ball simple harmonic ?

(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid ? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

(IIT 1992)

123

A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in the figure. The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position.
(Exclude the case $\theta = 0°$)



(IIT 1984)

124

A cubical block of wood 10 cm along each side floats at the interface between an oil and water with its lower surface 2 cm below the interface. The heights of the oil and water columns are 10 cm each. The density of oil is 0.8 gm/c.c.

(i) What is the mass of the block ?

(ii) What is the pressure at the lower surface of the block ?

$(g = 10 \text{ m/s}^2)$ **(IIT 1977)**

125

A beaker containing water is placed on the pan of a balance which shows a reading of M gms. A lump of sugar of mass m gms and volume V cc. is now suspended by a thread in such a way that it is completely immersed in water without touching the beaker and without any overflow of water. What will be the reading of the balance just when the lump of sugar is immersed ? How will the reading change as the time passes on ? **(IIT 1978)**

126

A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube rises by 2 cm. What is the size of the cube ? **(IIT 1978)**

127

To what height a cylindrical vessel be filled with a homogeneous liquid to make the force with which the liquid presses on the sides of vessel equal to the force exerted by the liquid on the bottom of the vessel ? **(IIT 1976)**

128

A piece of cork is embeded inside an ice block which floats in water. What will happen to the level of water when all the ice melts ?

(IIT 1976)

129

A rod of length 6 metres has a mass of 12 kg. It is hinged at one end at a distance of 3 metres below a water surface.

(i) What weight must be attached to the other end of the rod so that 5 metres of the rod are submerged ?

(ii) Find the magnitude and direction of the force exerted by the hinge on the rod.

(specific gravity of the material of the rod is 0.5)

(IIT 1976)

130

A large block of ice 5 metre thick has a vertical hole drilled through and is floating in the middle of lake. What is the minimum length of a rope required to scoop up a bucket full of water through the hole ? (density of ice = 0.9 gm/c.c.) **(IIT 1975)**

131

A balloon filled with hydrogen has a volume of 10000 litres and its mass is 1 kg. What would be the volume of a block of a very light material which it can just lift ?

One litre of this material has a mass of 91.3 gm. (density of air = 1.3 gm/litre). **(IIT 1975)**

132

A block of ice is floating in a liquid of specific gravity 1.2 contained in a beaker. When the ice melts completely, will the liquid level in the beaker change ? **(IIT 1974)**

133

A cubical block of iron 5 cm on each side is floating on mercury in a vessel.

(i) What is the height of the block above the mercury level ?

(ii) Water is poured into the vessel until it just covers the iron block. What is the height of the water column ?

(density of mercury = 13.6 gm/c.c., density of iron = 7.2 gm/c.c.)

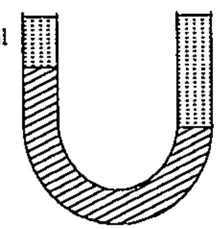
(IIT 1973)

134

A piece of ice with a stone frozen in it floats on water kept in a beaker. Will the level of water increase, decrease as remain the same when the ice completely melts ? **(IIT 1973)**

135

A vertical U tube of uniform inner cross-section contains mercury in both its arms. A glycerine (density 1.3 gm/c.c.) column of length 10 cm is introduced into one of its arms. Oil of density (0.8 gm/c.c.) is poured in the other arm until the upper surface of oil and glycerine are in the same horizontal level as shown in the figure. Find the length of the oil column. Density of mercury is 13.6 gm/c.c. **(IIT 1972)**



136

A piston of cross sectional area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dynes on the water. What is the cross sectional area of the other piston which supports a truck having a mass of 2000 kg ?
($g = 980$ cm/s^2). **(IIT 1972)**

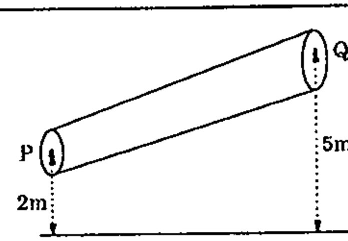
FLUID DYNAMICS

137

Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. Take $g = 10$ m/s^2. **(IIT 2004)**

138

A nonviscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of



2 meters and 5 meters are respectively 1×10^{-3} m^2 and 8×10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. **(IIT 1997, July)**

139

A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $\frac{A}{100}$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at t = 0, calculate

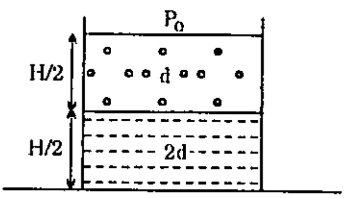
(i) the acceleration of the container, and

(ii) its velocity when 75% of the liquid has drained out

(IIT 1997, May)

140

A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and 2d, each of height $\frac{H}{2}$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0.



(a) A homogeneous solid cylinder of length L (L < H/2), cross-section area $\frac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $\frac{L}{4}$ in the denser liquid. Determine.

(i) the density D of the solid and,

(ii) the total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s (s << A) is punched on the vertical side of the container at a height h (h < H/2). Determine:

(i) the initial speed of efflux of the liquid at the hole,

(ii) the horizontal distance x travelled by the liquid initially, and

(iii) the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m.

(Neglect the air resistance in these calculations). **(IIT 1995)**

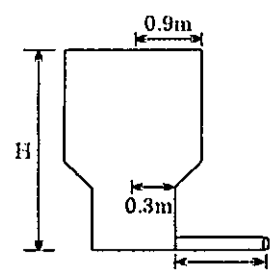
VISCOSITY

141

A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity. **(IIT 2004)**

142

A liquid of density 900 kg/m^3 is filled in a cylinderical tank of upper radius 0.9 m and lower radius 0.3 m. A capillary tube of length l is attached at the bottom of the tank as shown in the figure. The capillary has outer radius 0.002 m and inner radius a. When pressure P is applied at the top of the tank volume flow rate of the liquid is 8×10^{-6} m^3/s and if capillary tube is detached, the liquid comes out from the tank with a velocity 10 m/s. Determine the coefficient of viscosity of the liquid.

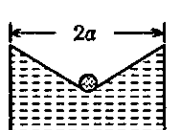


[Given : $\pi a^2 = 10^{-6}$ m^2 and $\dfrac{a^2}{l} = 2 \times 10^{-6}$ m] **(IIT 2003)**

SURFACE TENSION

143

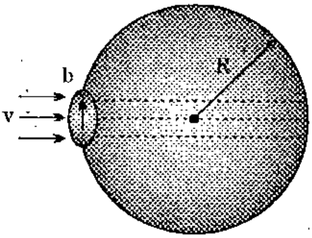
A container of width $2a$ is filled with a liquid. A thin wire of weight per unit length λ is gently placed over the liquid surface in the middle of the surface as shown in the figure. As a result, the liquid surface is depressed by a distance $y (y << a)$. Determine the surface tension of the liquid.



(IIT 2004)

144

A bubble having surface tension T and radius R is formed on a ring of radius b ($b \ll R$). Air is blown inside the tube with velocity v as shown in the figure.



The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring. **(IIT 2003)**

SHM

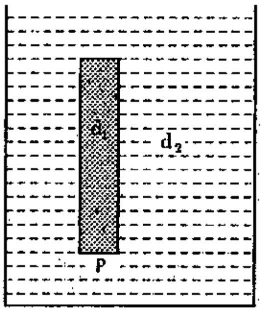
145

A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **(IIT 2004)**

146

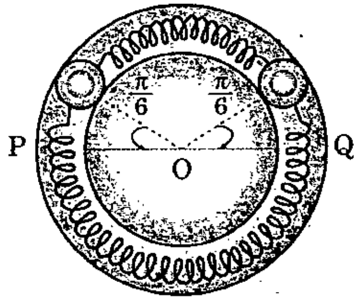
A thin rod of length L and area of cross section S is pivoted at its lowest point P inside a stationary, homogeneous and non viscous liquid (see figure).

The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **(IIT 1996)**



147

Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle, of radius 0.06 metre. Each spring has a natural length 0.06 π metre and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \frac{\pi}{6}$ radian with respect to the diameter PQ of the circle (as shown in the figure) and released from rest.



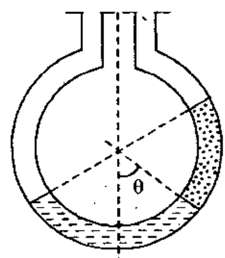
(i) Calculate the frequency of oscillation of ball B.

(ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

(iii) What is the total energy of the system ? **(IIT 1993)**

148

Two non-viscous, incompressible and immiscible liquids of densities ρ and 1.5 ρ are poured into the two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in the figure. Each liquid occupies one-forth the circumference of the tube.



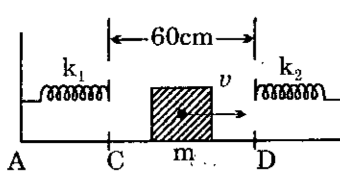
(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.

(b) If the whole is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

(IIT 1991)

149

Two light springs of force constants k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure.



The distance distance CD between the free ends of the springs is 60 cms. If the block moves along AB with a velocity 120 cm/sec in between the springs, calculate the period of oscillation of the block.(k_1 = 1.8 N/m, k_2 = 3.2 N/m, m = 200 gm) **(IIT 1985)**

150

A point mass m is suspended at the end of a massless wire of length l and cross section A. If Y is the Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line. **(IIT 1978)**

151

A particle of mass 10 gm is describing a simple harmonic motion along a straight line with a period of 2 seconds and amplitude of 10 cm. What is the kinetic energy when it is

(i) 2 cm from equilibrium position ?

(ii) 5 cm from equilibrium position ?

How would you account for the difference between these two values? **(IIT 1972)**

WAVES

152

A string tied between $x = 0$ and $x = l$ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **(IIT 2003)**

153

In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound in air at room temperature. **(IIT 2003)**

154

Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature.

(a) If the frequency to the second harmonic of the fundamental mode in pipe A is equal of the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of $\frac{M_A}{M_B}$.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B. **(IIT 2002)**

155

A 3.6 m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain heights in the pipe. Find the heights of water level (from the bottom of the pipe) at which resonances occur. Neglect end correction. Now, the pipe is filled to a height H ($\approx$ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurrence of first two resonances. Speed of sound in air is 340 m/s and g = 10 m/s^2.

(IIT 2000)

156

A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2 kg. The wire PQR is under a tension of 80 N. A sinusoidal wave-pulse of amplitude 3.5 cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave-pulse. Calculate

(a) the time taken by the wave-pulse to reach the other end R of the wire, and

(b) the amplitude of the reflected and transmitted wave-pulses after the incident wave-pulse crosses the joint Q. **(IIT 1999)**

157

The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation.

(a) Find the length L of the air column.

(b) What is the amplitude of pressure variation at the middle of the column ?

(c) What are the maximum and minimum pressures at the open end of the pipe ?

(d) What are the maximum and minimum pressures at the closed end of the pipe? **(IIT 1998)**

158

The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. **(IIT 1997, May)**

159

A metallic rod of length 1 m is rigidly clamped at its mid-point. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the mid-point. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the mid-point and those of the constituent waves in the rod. (Young's modulus = 2×10^{11} Nm^{-2}, density = 8000 kg m^{-3}) **(IIT 1994)**

160

Two radio stations broadcast their programs at the same amplitude A, and at slightly different frequencies ω_1 and ω_2 respectively. $\omega_2 - \omega_1 = 10^3$ Hz. A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq$ $2A^2$.

(i) Find the time interval between successive maxima of the intensity of the signal received by the detector.

(ii) Find the time for which the detector remains idle in each cycle of the intensity of the signal. **(IIT 1993)**

161

The displacement of the medium in a sound wave is given by the equation

$y_1 = A \cos(ax + bt)$ where A, a and b are positive constants.

The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 time that of the incident wave.

(a) What are the wavelength and frequency of incident wave ?

(b) Write the equation for the reflected wave.

(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

(d) Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave? What is the direction of propagation of travelling wave? **(IIT 1991)**

162

The following equations represent transverse waves :

$$z_1 = A\cos(kx - \omega t)$$
$$z_2 = A\cos(kx + \omega t)$$
$$z_3 = A\cos(ky - \omega t)$$

Identify the combination(s) of the waves which will produce

(i) standing wave(s),

(ii) a wave travelling in the direction making an angle of 45 degrees with the positive x and positive y axes. In each case, find the positions at which the resultant intensity is always zero. **(IIT 1987)**

163

The vibrations of a string of length 60 cm fixed at both ends are represented by the equation :

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

where x and y are in cm and t in seconds.

(i) What is the maximum displacement of a point at $x = 5$ cm ?

(ii) Where are the nodes located along the string ?

(iii) What is the velocity of the particle at x = 7.5 cm at t = 0.25 sec. ?

(iv) Write down the equations of the component waves whose superposition gives the above wave. **(IIT 1985)**

164

A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ? **(IIT 1984)**

165

A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of fundamental mode of vibration. **(IIT 1984)**

166

A sonometer wire under tension of 64 Newtons vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and a mass of 1 gm. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per second. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m/s. **(IIT 1983)**

167

A string 25 cm long and having a mass of 2.5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. **(IIT 1982)**

168

A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/sec. Estimate the diameter of the tube.

One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube. **(IIT 1980)**

169

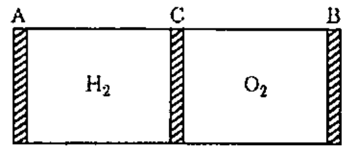
A metal wire of diameter 1 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire, vibrating with its fundamental frequency, and a vibrating tuning fork together produce 5 beats/sec. The tension in the wire is then reducted to 81 N. When the two are excited beats are heared again at the same rate, calculate.

(i) frequency of the fork.

(ii) density of the material of the wire. **(IIT 1980)**

170

AB is a cylinder of length 1 m fitted with a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which diaphragm C is a node ? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). **(IIT 1978)**



171

A column of air at 51°C and a tuning fork produces 4 beats per second when sounded together. As the temperature of the air column is decreased, the number of beats per second tends to decrease and when the temperature is 16°C, the two produce 1 beat per second. Find the frequency of the tuning fork. **(IIT 1977)**

172

Velocity of sound in a tube containing air at 20° C and a pressure of 76 cm of mercury is 330 metres/sec. What will be its velocity when the pressure is increased to 100 cm of mercury and the temperature is kept constant ? **(IIT 1976)**

173

Two wire of radii r and $2r$ respectively are welded together end to end. The combination is used as a sonometer wire and is kept under tension T. The welded point is midway between the two bridges. What would be the ratio of the number of loops formed in the wires such that the joint is a node when stationary vibrations are set up in the wires ? **(IIT 1976)**

174

A tuning fork having a frequency of 340 vibrations/sec is vibrated just above a cylindrical tube. The height of the tube is 120 cm.

Water is slowly poured in it. What is the minimum height of water required for resonance ?
(Velocity of sound in air = 340 m/s) **(IIT 1975)**

175

A wire of density 9 gm/c.c. is stretched between two clamps 100 cm apart while subjected to an extension of 0.05 cm. What is the lowest frequency of transverse vibrations in the wire assuming Young's modulus of the material to be 9×10^{11} dyne/cm^2 ? **(IIT 1975)**

176

A tuning fork of unknown frequency when sounded with another of frequency 256 Hz gives a 4 beats and when loaded with a certain amount of wax, it is again found to give 4 beats. Find the unknown frequency. **(IIT 1974)**

177

A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The druming rate is gradually increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 metres and finds that echo is not heard when the druming rate becomes 60 per minute. Calculate :

(i) distance between the mountain and the initial position of the man.

(ii) velocity of the sound. **(IIT 1974)**

178

A pipe of length 1.4 m closed at one end is filled with a gas and it resonates in its fundamental tone with a tuning fork. Another pipe of the same length but open at both ends is filled with air and is resonates in its fundamental tone with the same tuning fork. Calculate the velocity of sound at 0° C in the gas, given that the velocity of sound in air is 360 m/s at 30°C where the experiment is performed. **(IIT 1974)**

179

A sonometer wire fixed at one end has a solid mass M hanging from its other end to produce tension in it. It is found that 70 cm length of the wire produces a certain fundamental frequency when plucked. When the same mass M is hanging in water, completely submerged in it, it is found that the length of wire has to be changed by 5 cm in order that it will produce the same fundamental frequency. Calculate the density of the material of the mass M hanging from the wire. **(IIT 1972)**

DOPPLER'S EFFECT

180

A boat is travelling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm.

Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream ?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver. Given

Temperature of the air and water = 20°C

Density of river water = 10^3 kg/m^3

Bulk modulus of the water = 2.088×10^9 Pa

Gas constant R = 8.31 J/mol–K

Mean molecular mass of water = 28.8×10^{-3} kg/mol

$\frac{C_P}{C_V}$ for air = 1.4 **(IIT 2001)**

181

A band playing music at frequency f is moving towards a wall at a speed v_b. a motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist. **(IIT 1997, July)**

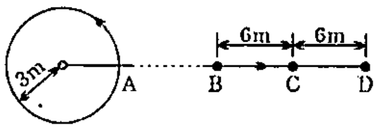
182

A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle.

(IIT 1996)

183

A source of sound is moving along a circular orbit of radius 3 metres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude BC = CD = 6 metres. The frequency of oscillation of the detector is $\frac{5}{\pi}$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. **(IIT 1990)**



184

A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find

(i) the frequency of the whistle as heard by an observer on the hill, and

(ii) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.

(Velocity of sound in air = 1,200 km/hr). **(IIT 1988)**

185

Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork.
(speed of sound = 340 m/s) **(IIT 1986)**

186

A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec ?

(IIT 1981)

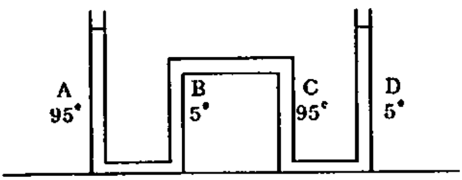
THERMAL EXPANSION

187

A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l, showing all the steps. **(IIT 2004)**

188

The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintaned at a temperature of 95°C while the column B and D are maintained at 5°C. The height of the liquid in A and D measured from the base line are 52.8 cm and 51 cm respectively. Determine the coefficient of thermal expansion of the liquid. **(IIT 1997, July)**



189

A thin rod of negligible mass and area of cross-section 4×10^{-6} m^2, suspended vertically from one end, has a length of 0.5 m at 100°C. The rod is cooled to 0°C, but prevented from contracting by attaching a mass at the lower end. Find

(i) this mass, and

(ii) the energy stored in the rod.

Given for the rod : Young's modulus = 10^{11} N/m^2, Coefficient of linear expansion 10^{-5} K^{-1} and $g = 10$ m/s^2. **(IIT 1997, May)**

190

A composite rod is made by joining a copper rod, end to end, with a second rod of a different material but of the same cross section. At 25°C, the composite rod is 1 meter in length, of which the length of copper rod is 30 cm. At 125°C the length of the composite rod increases by 1.91 mm. When the composite rod is allowed to expand by holding it between two rigid walls, it is found that the length of the two constituents do not change with the rise of temperature.

Find the Young's modulus and the coefficient of linear expansion of the second rod.

(For copper : $Y = 1.3 \times 10^{11}$ N/m^2, $\alpha = 1.7 \times 10^{-5}$/°C) **(IIT 1979)**

191

A copper wire is held at two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C.

(For copper : $\alpha = 1.7 \times 10^{-5}$/°C, $Y = 1.3 \times 10^{11}$ N/m^2, density $= 9 \times 10^3$ kgm^3) **(IIT 1979)**

192

A sinker of weight w_0 has an apparent weight w_1 when weighed in a liquid at a temperature t_1 and w_2 when weight in the same liquid at temperature t_2. The coefficient of cubical expansion of the material of sinker is β. What is the coefficient of volume expansion of the liquid. **(IIT 1978)**

193

A clock with an iron pendulum keeps correct time at 20°C. How much will it lose or gain if the temperature changes to 40°C? (Coefficient of cubical expansion of iron = 0.000036/°C).

(IIT 1977)

194

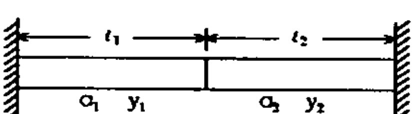
The brass scale of a barometer gives correct reading at 0°C. The coefficient of thermal expansion of brass is 0.00002/°C. The barometer reads 75 cm at 27°C. What is the atmospheric pressure at 0°C? **(IIT 1977)**

195

What should be the length of a steel and copper rod be at 0°C so that the length of the steel rod is 5 cm longer than the copper rod at any temperature? (Coefficient of linear expansion for copper = 1.7×10^{-5}/°C, coefficient of linear expansion for iron = 1.1×10^{-5}/°C **(IIT 1976)**

196

Two rods of different metals having the same area of cross-section A, are placed end to end between two massive walls as shown in figure. The first rod has length l_1, ceefficient of



linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for second rod are l_2, α_2 and Y_2. The temperature of both the rods is raised by T degrees.

(i) Find the force with which the rods act on each other (at the higher temperature) in terms of the given quantities.

(ii) Also find the lengths of the rods at higher temperature.

Assume that there is no change in the cross-sectional area of the rods and the rods do not bend. There is no deformation of the walls.

(IIT 1975)

197

A piece of metal weighs 46 gm in air. When it is immersed in a liquid of specific gravity 1.24 at 27°C it weighs 30 gm. When the temperature of liquid is raised to 42°C, the metal piece weighs 30.5 gm. Specific gravity of the liquid at 42°C is 12.0. Calculate the coefficient of linear expansion of the metal. **(IIT 1974)**

198

The difference between the length of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. Is this possible? **(IIT 1974)**

199

A one litre glass flask contains some mercury. It is found that at different temperatures the volume of air inside the flask remains same. What is the volume of mercury in this flask?
(coefficient of linear expansion of glass = 9×10^{-5}/°C, coefficient of cubical expansion of mercury = 180×10^{-6}/°C). **(IIT 1973)**

200

A metallic bob weighs 50 grams in air. It is immersed in a liquid at a temperature of 25°C it weighs 45 grams. When the temperature of the liquid is raised to 100°C, it weighs 45.1 grams. Calculate the coefficient of cubical expansion of the liquid. Assuming the coefficient of linear expansion of the metal to be 12×10^{-6}/°C.

(IIT 1973)

201

A steel wire of cross-sectional area 0.5 mm² is held between two fixed supports. If the tension in this wire is negligible and it is just taut at a temperature of 20°C, determine the tension when the

temperature falls to 0°C. Assume that the distance between the supports remains the same.
($\alpha = 12 \times 10^{-6}/°C$, $Y = 2.1 \times 10^{12}$ dyne/cm^2) **(IIT 1973)**

CALORIMETRY

202

An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation
$S = A + BT$,
where $A = 100$ cal/kg–K and $B = 2 \times 10^{-2}$ cal/kg–K^2.
If the final temperature of the container is 27°C, determine the mass of the container.
(Latent heat of fusion of water = 8×10^4 cal/kg–K,
Specific heat of water = 10^3 cal/kg–K) **(IIT 2001)**

203

The temperature of 100 gm of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose. **(IIT 1996)**

204

5 grams of water at 30°C and 5 grams of ice at – 20°C are mixed together in a calorimeter. Find the temperature of the mixture. Water equivalent of calorimeter is negligible.
(Specific heat of ice = 0.5 cal/gm-°C, Latent heat of ice = 80 cal/gm). **(IIT 1977)**

205

The temperatures of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C and when B and C are mixed it is 23°C. What will be temperature when A and C are mixed. **(IIT 1976)**

206

A vessel is filled completely with 500 gms of water and 1000 gms of mercury. When 21200 calories of heat are given to it water of mass 3.52 gms overflows. Calculate the coefficient of volume expansion of mercury. Expansion of vessel may be neglected.
(coefficient of volume expansion of water = $1.5 \times 10^{-4}/°C$,
Density of mercury = 13.6 gm/cc,
Density of water = 1 gm/cc,
Specific heat of mercury = 0.03 cal/gm-°C) **(IIT 1976)**

207

A mixture of 250 gm of water and 200 gm of ice at 0°C is kept in a calorimeter which has a water equivalent of 50 gm. If 200 gm of stream at 100°C is passed through this mixture, calculate the final temperature and weight of the contents of the calorimeter.

(IIT 1974)

208

A lead bullet strikes against a steel armour plate with a velocity of 300 m/s. If the bullet falls dead after the impact, find the rise in temperature of the bullet assuming that the heat produced is shared equally between the bullet and the target. (Specific heat of lead = 0.03 cal/gm-°C). **(IIT 1974)**

209

An aluminium container of mass 100 grams contains 200 grams of ice at – 20°C. Heat is added to the system at the rate of 100 calories per second. What is the temperature of the system after 4 minutes. Draw a rough sketch showing the variation of the temperature of the system as a function of time ?

(Specific heat of ice = 0.5 cal/gm-°C,
Specific heat of aluminium = 0.2 cal/gm-°C,
Latent heat of fusion of ice = 80 cal/gm) **(IIT 1973)**

210

In an industrial process 10 kg of water per hour is to be heated from 20°C to 80°C. To do this, steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. How many kg of steam are required per hour ? **(IIT 1972)**

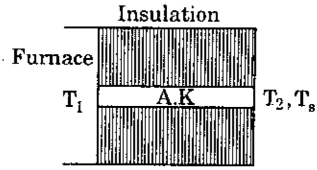
(Specific heat of steam = 1 cal/gm,
Latent heat of steam = 540 cal/gm)

HEAT TRANSFER

211

A uniform rod of length L, conductivity K is connected from one end to a furnace at temperature T_1. The other end of rod is at temperature T_2 and is exposed to atmosphere. The temperature of atmosphere is T_g. The lateral part of rod is insulated. If

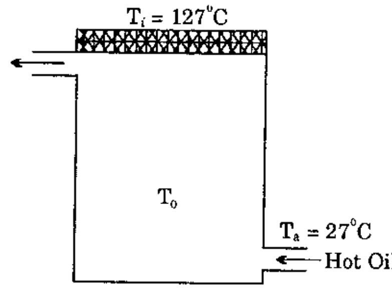
$T_2 - T_s << T_s$, $T_2 = T_s + \Delta T$ and $\Delta T \propto (T_1 - T_s)$, find proportionality constant of given equation. The heat loss to atmosphere is through radiation only and the emissivity of the rod is ε.



(IIT 2004)

212

Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity $K = 0.149$ J/(m–°C sec), thickness t = 5 mm, emissivity = 0.6. Temperature of the top of the lid is maintained at $T_l = 127°$. If the ambient temperature T_a = 27°C, calculate



(a) rate of heat loss per unit area due to radiation from the lid.

(b) temperature of the oil. (Given $\sigma = \frac{17}{3} \times 10^{-8}$) **(IIT 2003)**

213

A solid body X of heat capacity C is kept in an atmosphere whose temperature is $T_A = 300$ K. At time t = 0 the temperature of X is T_0 = 400 K. It cools according to Newton's law of cooling. At time t_1, its temperature is found to be 350 K.

At this time (t_1), the body X is connected to a large box Y at atmospheric temperature T_A, through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The cross sectional area A of the connecting rod is small compared to the surface area of X. The heat capacity of Y is so large that any variation in its temperature may be neglected. Find the temperature of X at time $t = 3t_1$. **(IIT 1998)**

214

A double-pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room-glass interface and the glass-outdoor interface are at constant temperatures of 27°C and 0°C respectively. Calculate the rate of heat flow through the window pane. Also find the temperatures of other interfaces. Given, thermal conductivities of glass and air as 0.8 and 0.08 Wm^{-1} K^{-1} respectively.

(IIT 1997, May)

215

A cylindrical block of length 0.4 m and area of cross-section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc is 600 J/kg.K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

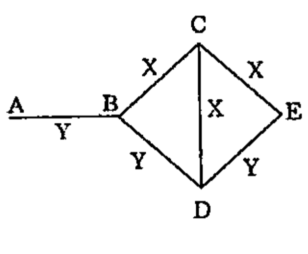
(IIT 1992)

216

An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of + 20°C inside it, when the outside temperature is – 10°C. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 watt/m/°C respectively. **(IIT 1986)**

217

Three rods of material X and three rods of material Y are connected as shown in the figure. All the rods are of identical length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C. Calculate the temperature of the junctionsB, C and D. The thermal conductivity of X is 0.92 cal/sec-cm-°C and that of Y is 0.46 cal/sec-cm-°C.



(IIT 1978)

218

A room is maintained at 20°C by a heater of resistance of 20 ohms connected to 200 volts mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal/m-sec-°C and mechanical equivalent of heat is 4.2 Joules/cal.

(IIT 1978)

219

A bar of copper of length 75 cm and a bar of steel of length 125 cm are joined together end to end. Both are of circular cross-section with diameter 2 cm. The free ends of copper and steel bars are maintained at 100°C and 0°C respectively. The surfaces of bars are thermally insulated. What is temperature of the copper-steel junction. What is the heat transmitted per unit time across the junction when the steady state has been reached. (Thermal conductivity of copper is 9.2×10^{-2} Kcal/m-sec-°C and that of steel is 1.1×10^{-2} Kcal/m-sec-°C) **(IIT 1977)**

220

Two rods A and B are of equal length. Each rod has its ends at temperature T_1 and T_2. What is the condition that will ensure equal rates of flow of heat through the rods A and B. **(IIT 1976)**

221

A 2 metre long wire of resistance 4Ω and diameter 0.64 mm is coated with plastic insulation of thickness 0.06 mm. When a current of 5 amps. flows through the wire find the temperature difference across the insulation in steady state. (Thermal conductivity of plastic = 1.6×10^{-3} cal/cm-s-°C). **(IIT 1974)**

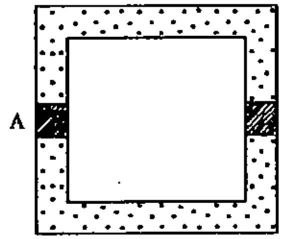
222

A slab of stone of area 3600 sq cm and thickness 10 cm is exposed on the lower surface to steam at 100°C. A block of ice at 0°C rests on the upper surface of the slab. In one hour 4800 gm of ice is melted. Calculate the thermal conductivity of the stone. (Latent heat of ice = 80 cal/gm) **(IIT 1972)**

223

A closed cubical box made of a perfectly insulating material has walls of thickness 8 cm and the only way for heat to enter or leave the box is through two solid, cylindrical, metallic plugs, each of cross-sectional area 12 cm^2 and length 8 cm fixed in opposite walls of the box (see figure). The outer surface of A of one plug is kept at 100°C while the outer surface of B of the other plug is maintained at 4°C. The thermal conductivity of the material of the plug is

0.5 cal/cm-sec-°C. A source of energy generecting 36 calories/sec is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface. **(IIT 1972)**



KINETIC THEORY OF GASES

224

A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root-mean-square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = \frac{25}{3}$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K.

(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box. **(IIT 2002)**

225

A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of 1×10^5 Nm^{-2}. The total mass of the mixture is 28 gm. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal. (Universal gas constant R = 8.314 J/mol. K) **(IIT 1994)**

226

A thin tube of uniform cross-section is sealed at both ends. It lies horizontally, the middle 5 cm containing mercury and the two equal ends containing air at the same pressure P. When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46 cm and 44.5 cm respectively. Calculate the pressure P in centimetres of mercury. (The temperature of the system is kept at 30°C). **(IIT 1986)**

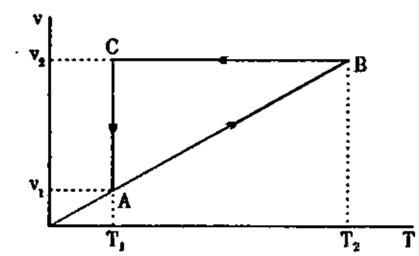
227

Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs ? The volume of the connecting tube is negligible. **(IIT 1985)**

228

A cyclic process ABCA shown in the V-T diagram (see figure) is performed with a constant mass of an ideal gas. Show the same process on a P-V diagram.

(In figure, CA is parallel to the V-axis and BC is parallel to the T-axis) **(IIT 1981)**



229

A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm of Hg. The temperatue of the jar is reduced by 1%. The saturated vapour pressures of water at the two temperatures are 30 and 25 mm of Hg. Calculate the new pressure in the jar. **(IIT 1980)**

230

A column of mercury of 10 cm length is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically? **(IIT 1978)**

231

Calculate the mass of 1 litre of moist air at 27°C when the barometer reads 753.6 mm of Hg and the dew point is 16.1°C.

(Saturation vapour pressure of water at 16.1°C is equal to 13.6 mm of Hg, density of air at NTP is equal to 0.001293 g/cc, density of saturated water vapour at NTP is equal to 0.000808 g/cc). **(IIT 1977)**

232

A glass capillary tube, sealed at both ends is 100 cm long. It lies horizontally with the middle 10 cm containing mercury. The two ends of the tube (which are equal in length) contain air at 27°C and at a pressure 76 cm of mercury. The tube is kept in a horizontal position such that the air column at one end is at 0°C and the other end is maintained at 127°C. Calculate the length of the air column which is at 0°C and its pressure. Neglect the change in volume of mercury and glass. **(IIT 1975)**

233

An electric bulb of volume 250 cc was sealed off during manufacture at pressure of 10^{-3} mm of Hg at 27°C. Compute the number of air molecules contained in the bulb. **(IIT 1974)**

234

At the top of a mountain a thermometer reads 7°C and a barometer reads 70 cm of Hg. At the bottom of the mountain they read 27°C and 76 cm of Hg. Compare the density of air at the top with that at the bottom. **(IIT 1974)**

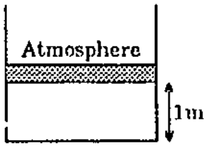
235

An ideal gas is trapped between a mercury column and the closed lower end of a narrow vertical tube of uniform bore. The upper end of the tube is open to the atmosphere (atmospheric pressure = 76 cm of mercury). The lengths of mercury and the trapped gas columns are 20 cm and 43 cm respectively. What will be the length of the gas column when the tube is tilted slowly in a vertical plane through an angle of 60°. Assume the temperature to be constant. **(IIT 1972)**

THERMODYNAMICS

236

An ideal diatomic gas is enclosed in an insulated chamber at temperature 300 K. The chamber is closed by a freely movable massless piston, whose initial height from the base is 1 m. Now the gas is heated such that its temperature becomes 400 K at constant pressure. Find the new height of the piston from the base. If the gas is compressed to initial position such that no exchange of heat takes place, find the final temperature of the gas. **(IIT 2004)**

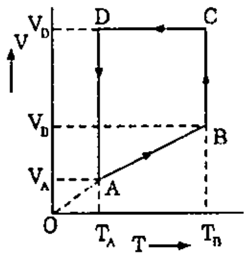


237

An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **(IIT 2003)**

238

A monatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The volume ratios are



$$\left(\frac{V_B}{V_A}\right) = 2 \text{ and } \left(\frac{V_D}{V_A}\right) = 4.$$

If the temperature T_A at A is 27°C, Calculate :

(a) temperature of the gas at point B.

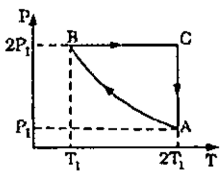
(b) heat absorbed or released by the gas in each process.

(c) the total work done by the gas during the complete cycle.

Express your answer in terms of the gas constant R. **(IIT 2001)**

239

Two moles of an ideal monatomic gas is taken through a cycle ABCA as shown in the P-T diagram. During the process AB, pressure and temperature of the gas vary such that PT = constant. If T_1 = 300 K, calculate



(a) the work done on the gas in the process AB and

(b) the heat absorbed or released by the gas in each of the process.

Give answers in terms of the gas constant R. **(IIT 2000)**

240

Two moles of an ideal monatomic gas, initially at pressure p_1 and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2.

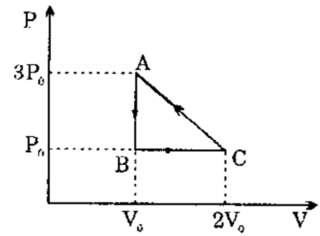
(a) Sketch the complete process on a p-V diagram.

(b) Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas.

[Give your answer in terms of p_1, V_1, V_2, Q and R] **(IIT 1999)**

241

One mole of an ideal monatomic gas is taken round the cyclic process ABCA as shown in the figure. Calculate



(a) the work done by the gas

(b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB.

(c) the net heat absorbed by the gas in the path BC.

(d) the maximum temperature attained by the gas during the cycle.

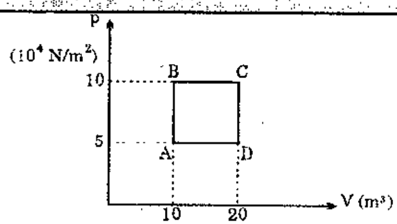
(IIT 1998)

242

One mole of a diatomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A. The process $A \to B$ is an adiabatic compression, $B \to C$ is isobaric expansion, $C \to D$ an adiabatic expansion and $D \to A$ is isochoric. The volume ratios are $\frac{V_A}{V_B} = 16$ and $\frac{V_C}{V_B} = 2$ and the temperature at A is $T_A = 300°$ K. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle. **(IIT 1997, July)**

243

A sample of 2 kg of monatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC (see figure). Given molecular mass of Helium = 4.



(i) What is the temperature of Helium in each of the states A, B, C and D ?

(ii) Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC ? Write Yes or No.

(iii) How much is heat involved in each of the processes ABC and ADC ? **(IIT 1997, May)**

244

At 27° two moles of an ideal mono atomic gas occupy a volume V. The gas expands adiabatically to a volume 2V. Calculate:

(i) the final temperature of the gas,

(ii) change in its internal energy, and

(iii) the work done by the gas during this process. **(IIT 1996)**

245

A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$ and another gas B with $\gamma = \frac{7}{5}$ at a certain temperature T. The gram molecular weights of the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic processes.

(a) Find the number of gram moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at T = 300 K.

(c) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

(d) The mixture is compressed adiabatically to $\frac{1}{5}$ of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities. **(IIT 1995)**

246

An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are: Q_1 = 5960 J, Q_2 = – 5585J, Q_3 = –2980 J and Q_4 = 3645 J respectively.

The corresponding works involved are:

W_1 = 2200 J, W_2 = – 825 J, W_3 = – 1100 J, and W_4 respectively.

(i) Find the value of W_4.

(ii) What is the efficiency of the cyclic ? **(IIT 1994)**

247

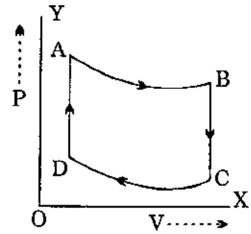
One mole of a monatomic ideal gas is taken through the cycle shown in the figure :

$A \longrightarrow B$: adiabatic expansion

$B \longrightarrow C$: cooling at constant volume

$C \longrightarrow D$: adiabatic compression

$D \longrightarrow A$: heating at constant volume



The pressure and temperature at A, B, etc. are denoted by P_A, P_B, T_A, T_B etc. respectively. Given that

$$T_A = 1000\text{ K},\ P_B = \left(\frac{2}{3}\right)P_A \text{ and } P_C = \left(\frac{1}{3}\right)P_A .$$

Calculate the following quantities :

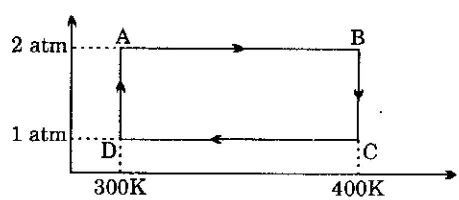
(i) The work done by the gas in process $A \longrightarrow B$

(ii) The heat lost by the gas in process $B \longrightarrow C$

(iii) the temperature T_D. [Given : $\left(\frac{2}{3}\right)^{2/5} = 0.85$] **(IIT 1993)**

248

Two moles of helium gas undergo a cyclic process as shown in the figure. Assume the gas to be ideal, calculate the following quantities in this process:



(a) The net change in the heat energy.

(b) The network done.

(c) The net change in internal energy. **(IIT 1992)**

249

Three moles of an ideal gas ($C_P = \frac{7}{2}R$) at pressure, P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally gas is compressed at constant volume to its original pressure P_A.

(a) Sketch P-V and P-T diagrams for the complete process.

(b) Calculate the net work done by the gas, and net heat supplied to the gas during the complete process. **(IIT 1991)**

250

An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $\frac{T}{2}$.

(i) How many degrees of freedom do the gas molecules have ?

(ii) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V. **(IIT 1990)**

251

Two moles of helium gas $\left(\gamma = \frac{5}{3}\right)$ are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.

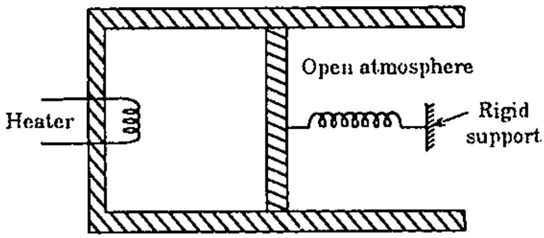
(i) Sketch the process on a p-V diagram,

(ii) What are the final volume and pressure of the gas ?

(iii) What is the work done by the gas ? **(IIT 1988)**

252

An ideal monatomic gas is confined in a cylinder by a spring-loaded piston of cross-section 8.0×10^{-3} m². Initially the gas is at 300 K and occupies a volume of 2.4×10^{-3} m³ and the spring is in its relaxed (unstretched, uncompressed) state, (see figure). The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat



supplied (in joules) by the heater. The force constant of the spring is 8000 N/m, atmospheric pressure is $1.0 \times 10^5\ Nm^{-2}$. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through the lead wires of the heater. The heat capacity of the heater coil is negligible. Assume the spring to be massless.

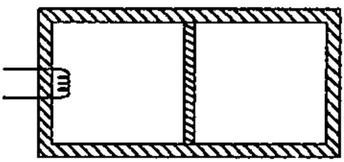
(IIT 1989)

253

An ideal gas has a specific heat at constant pressure $C_p = \frac{5R}{2}$. The gas is kept in a closed vessel of volume $0.0083\ m^3$, at a temperature of 300 K and a pressure of $1.6 \times 10^6\ N/m^2$. An amount of 2.49×10^4 Joules of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas. **(IIT 1987)**

254

The rectangular box shown in Fig. has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a monoatomic ideal gas $\left(\gamma = \frac{5}{3}\right)$ at a pressure P_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands pushing the partition until the final pressure in both chambers becomes $\frac{243 P_0}{32}$.



Determine

(i) the final temperature of the gas in each chamber

(ii) the work done by the gas in the right chamber. **(IIT 1984)**

255

One gram mole of oxygen at 27°C and one atmospheric pressure is enclosed in a vessel.

(i) Assuming the molecules to be moving with v_{rms}, find the number of collisions per second which the molecules make with one square metre area of the vessel wall.

(ii) The vessel is next thermally insulated and moved with a constant speed v_0. It is then suddenly stopped. The process results in a rise of the temperature of the gas by 1°C. Calculate the speed v_0. ($\gamma_{air} = 1.41$) **(IIT 1983)**

256

Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litres respectively. The final volume of the gas is 2 litres. Molar specific heat of the gas at constant volume is $\frac{3R}{2}$. **(IIT 1982)**

257

An ideal gas is enclosed in a vertical cylindrical container and supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. Atmospheric pressure is P_0, and when piston is in equilibrium, the volume of the gas is V_0. The piston is now displaced slightly from its equilibrium position. Assuming that system is completely isolated from its surrounding, show that the piston executes simple harmonic motion and find frequency of oscillations. **(IIT 1981)**

ELECTROSTATICS

258

Two infinitely large sheets having charge densities σ_1 and σ_2 respectively ($\sigma_1 > \sigma_2$) are placed near each other separated by distance 'd'. A charge 'Q' is placed in between two plates such that there is no effect on charge distribution on plates. Now this charge is moved at an angle of 45° with the horizontal towards plate having charge density σ_2 by distance '*a*' ($a < d$). Find the work done by electric field in the process. **(IIT 2004)**

259

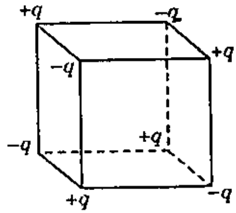
A positive point charge q is fixed at origin. A dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along positive x-axis. Find :

(a) the kinetic energy of the dipole when it reaches a distance d from the origin, and

(b) the force experienced by the charge q at this moment. **(IIT 2003)**

260

Eight point charges are placed at the corners of a cube of edge a as shown in the figure. Find the work done in disassembling this system of charges. **(IIT 2003)**



261

A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution. **(IIT 2001)**

262

The point charges + 8 μC, – 1 μC, – 1 μC, + 8 μC are fixed at the points $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and of charge + 0.1 μC moves along the $-x$ direction. Its speed at $x = +\infty$ is v_0.
Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin.
Assume that space is gravity free. Given $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9$ Nm²/C². **(IIT 2000)**

263

A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $q/m = \frac{4\varepsilon_0 g}{\sigma}$.

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position. **(IIT 1999)**

264

A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.

S_1 is given a charge Q, brought into contact with S_2, and removed. S_1 is recharged such that the charge on it is again Q; and it is again brought in to contact with S_2 and removed. This procedure is repeated *n* times.

(a) Find electrostatic energy of S_2 after n such contacts with S_1.

(b) What is limiting value of this energy as $n \to \infty$? **(IIT 1998)**

265

Two isolated metallic solid spheres of radii R and 2R are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere. **(IIT 1996)**

266

A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P $\left(R\sqrt{3}, 0, 0\right)$ on the positive x-axis directly towards O, with an initial speed v. Find the smallest (non zero) value of the speed v such that the particle does not return to P. **(IIT 1993)**

267

(a) A charge of Q coulomb is uniformly distributed over a spherical volume of radius R metres. Obtain an expression for the energy of the system.

(b) What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles ?

Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg-m.

(c) If the same charge of Q coul. as in part (a) above is given to a spherical conductor of the same radius R, what will be the energy of the system ? **(IIT 1992)**

268

Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the x-y plane.

(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.

(b) Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.

(c) If a particle of charge $+q$ starts from rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

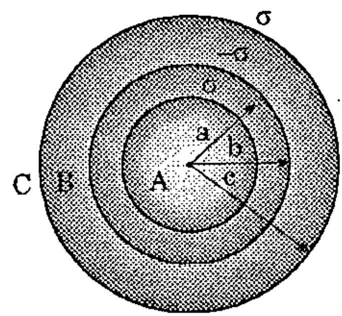
(IIT 1991)

269

Three concentric spherical metallic shells A, B and C of radii a, b and c $(a < b < c)$ have surface charge densities σ, $-\sigma$ and σ respectively.

(i) Find the potential of the three shells A, B and C.

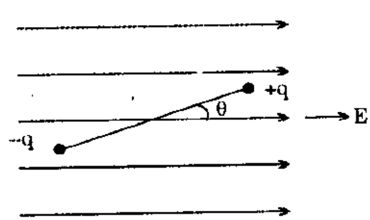
(ii) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c.



(IIT 1990)

270

A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say of about 5 degrees) with the field direction, (see figure). Find an expression for the minimum time needed for the rod to become parallel to the field after it is set free.



(IIT 1989)

271

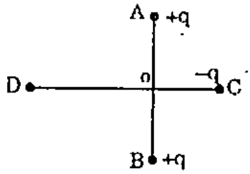
Three particles, each of mass 1 gm and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g = 10\ m/s^2$). **(IIT 1988)**

272

Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges ? **(IIT 1987)**

273

Two fixed, equal, positive charges, each of magnitude 5×10^{-5} coul. are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C. **(IIT 1985)**



274

A thin fixed ring of radius 1 metre has a positive charge 1×10^{-5} coulomb uniformly distributed over it. A particle of mass 0.9 gm and having a negative charge of 1×10^{-6} coulomb is placed on the axis at a distance of 1 cm from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillations.

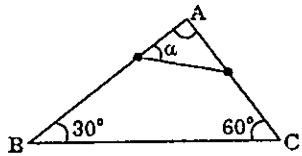
(IIT 1982)

275

A pendulum of mass 80 mg and carrying a charge of 2×10^{-8} coulomb is at rest in a horizontal uniform electric field of 20,000 V/m. Find the tension in the thread of the pendulum in the thread of the pendulum and the angle it makes with the vertical. ($g = 10\ m/s^2$) **(IIT 1979)**

276

A rigid insulated wire frame, in the form of right triangle ABC is set in a vertical plane. Two beads of equal masses m each carrying charges q_1 and q_2 are connected by a chord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine :



(i) the angle α,

(ii) the tension in the chord, and

(iii) the normal reactions on the beads.

If the chord is now cut, what are the values of the charges for which the beads continue to remain stationary? **(IIT 1978)**

277

Three charges each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle

(i) If $Q = -q$, will the charges at the corners move towards centre or fly away from it.

(ii) For what value of Q will the charges remain stationary ? In this situation how much work is done in removing the charges to infinity ? **(IIT 1978)**

278

A simple pendulum consists of a small sphere of mass m suspended by a thread of length l. The sphere carries a positive charge q. The pendulum is placed in an uniform electric field of strength E directed vertically upwards. With what periods will the pendulum oscillate if the electrostatic force acting on the sphere is less than the gravitational force ? (Assume that the oscillations are small). **(IIT 1977)**

279

Four charges $+q, +q, -q, -q$ are placed respectively at the corners A, B, C and D of a square with side a arranged in the given order. Calculate the electric potential and intensity at O, the centre of the square. If E and F are the mid points of the sides BC and CD, respectively, what will be work done in carrying a charge e from O to E and from O to F. **(IIT 1977)**

280

A particle of mass 9×10^{-31} kg and a negative charge of 1.6×10^{-19} coulomb is projected horizontally with a velocity of 10^6 m/sec into a region between two infinite horizontal plates of metal. The distance between plates is 0.3 cm and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected respectively to the positive and negative terminals of a 30 volt battery. Find the components of velocity of the particle just before it hits one of the plates. **(IIT 1977)**

281

A charge + Q is fixed at a distance d in front of an infinite metal plate. Draw the lines of force indicating the directions clearly. **(IIT 1976)**

282

Two hollow conductors are charged positively. The smaller is at 50 V and the bigger is at 100 V potential. How should they be arranged such that the charges flow from the smaller to the bigger conductor when connected by a wire? **(IIT 1976)**

283

Two identically charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 gm/cc the angle remains same, what is the dielectric constant of the liquid? The density of the material of the spheres is 1.6 gm/cc. **(IIT 1976)**

284

A particle having a charge of 1.6×10^{-19} coulombs enter midway between the plates of a parallel plate condenser. The initial velocity of the particle is parallel to the plane of the plates. A potential difference of 300 V is applied to the capacitor plates. If the length of the condenser plates be 10 cm and the condenser plates be separated by a distance of 2 cm, calculate the greatest initial velocity for which the particle will not be able to come out of the condenser plates. The mass of the charged particle is 12×10^{-24} kg. **(IIT 1976)**

285

Two identical charges + Q each, are at a distance r from each other. A third charge q is placed on the line joining the above two charges such that all the three charges are in equilibrium. What is the magnitude, sign and position of the charge q? **(IIT 1975)**

286

Two positive charges Q and 4 Q are fixed at a distance of 12 cm from each other. Sketch the lines of force and locate the neutral points if any. **(IIT 1975)**

287

A particle of mass 40 milligrams and carrying a charge of 5×10^{-9} coulombs is moving directly towards a fixed positive point charge of magnitude 10^{-8} coulombs. When it is at a distance of 10 cm from the fixed positive chrge it has a velocity of 50 cm/s. At what distance from the fixed charge will the particle come to momentarily to rest? Is the acceleration constant during motion? **(IIT 1975)**

288

A positive charge + Q is located at a point. What is work done if a unit positive charge is carried once completely around this charge along a circle of radius r about this point? **(IIT 1974)**

289

A spark is produced between two insulated surfaces maintained at a potential difference of 5×10^6 volts. If the energy output is 10^{-5} Joules, calculate the charge transferred during the spark. **(IIT 1974)**

290

Two point charges of value – 20 esu and + 20 esu are placed on the x-axis at $x = -10$ cm and $x = +10$ cm respectively. Calculate.

(i) potential, and

(ii) electric field at the point P ($x = 0, y = 10$ cm) and Q ($x = 20$ cm, $y = 0$).

(iii) Find the work done in carrying a unit positive charge from P to Q along a straight line.

(iv) Is there any path along which the work done is less than the above value? (3×10^9 esu = 1 coulomb) **(IIT 1974)**

291

A positively charged oil droplet remains stationary in the electric field between two horizontal parallel plates separated by a distance of 1 cm. If the charge on the drop is 96×10^{-10} esu and the mass of the droplet is 10^{-11} gm, what is the potential difference between the plates?
Now if the polarity of the plates is reversed, what is the instantaneous acceleration of the droplet?
(3×10^9 esu = 1 coulomb, g = 9.8 m/s^2) **(IIT 1974)**

292

Two copper spheres of the same radii, one hollow and the other solid are charged to the same potential. Which, if any, of the two will hold more charge? **(IIT 1974)**

293

An infinite number of charges each equal to q are placed along the x-axis at $x = a$, $x = 2a$, $x = 4a$, $x = 8a$, ... and so on.

(i) Find the potential and electric field at the point $x = 0$ due to this set of charges.

(ii) What will be potential and electric field if, in the above set up, the consecutive charges have opposite sign?

(IIT 1974)

294

A spherical liquid drop has a diameter 2 mm and is given a charge of 5×10^{-6} esu.

(i) What is the potential at the surface of the drop?

(ii) If two such drops coalesce to form a single drop, what is the potential at the surface of the drop so formed?

(3×10^9 esu = 1 coulomb) **(IIT 1973)**

295

A pith ball carrying a charge of 1 esu is suspended by an insulated thread of length 50 cm. When a uniform electric field is applied in a horizontal direction the ball is found to deflect by 2 cm from vertical. If the mass of the ball is 0.5 gram, what is the magnitude and direction of the electric field?
($g = 980$ cm/s^2, 3×10^9 esu = 1 coulomb) **(IIT 1973)**

296

Two point charges, one of + 100 esu and other of – 400 esu are kept 30 cm apart.

(a) Find the points of zero potential on the line joining the charges. (Assume the potential at infinity to be zero).

(b) Are there any other points of zero potential in the neighbourhood of the charges.

(c) Find the points where the electric field due to these two charges is zero. **(IIT 1972)**

297

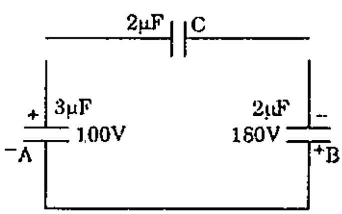
At the corner A of a square ABCD of side 10 cm is placed a charge + 200 esu. Another charge – 100 esu is located at the centre of the square. Find the work done in carrying a charge + 15 esu from the corner C to the corner B of the square.

(3×10^9 esu = 1 coulomb) **(IIT 1972)**

CAPACITANCE

298

Two capacitors A and B with capacities 3μF and 2μF are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate :



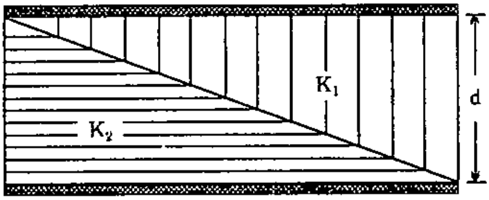
(i) the final charge on the three capacitors, and

(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit.

(IIT 1997, July)

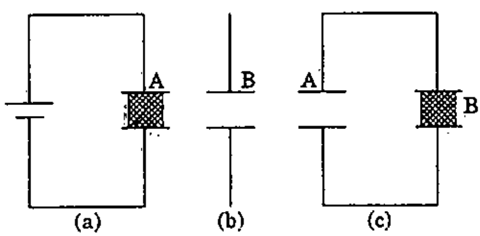
299

The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2, respectively (Figure). Find the capacitance of the resulting capacitor. **(IIT 1996)**



300

Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate area of A and B are 0.04 m^2 and 0.02 m^2 respectively. A slab of dielectric constant (relative permittivity) K = 9 has dimensions such that it can exactly fill the plates of capacitor B.



(i) The dielectric slab is placed inside A as shown in figure A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

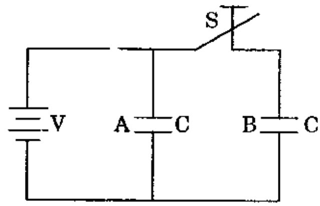
(ii) The battery is disconnected and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.

(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in the figure. Calculate the energy stored in the system.

(IIT 1993)

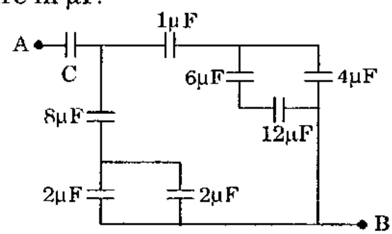
301

The Fig. shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **(IIT 1983)**



302

From the given figure, find the value of capacitance C if the equivalent capacitance between points A and B to be 1μF. All the capacitances are in μF. **(IIT 1977)**



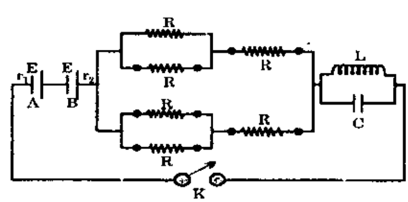
303

Two metal plates form a parallel plate condenser. The distance between the plates is d. A metal sheet of thickness $\frac{d}{2}$ and of the same area is inserted completely between the plates. What is the ratio of the capacitances in the two cases. **(IIT 1976)**

CURRENT

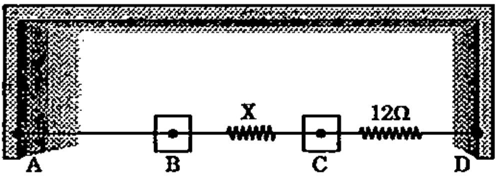
304

The two batteries A and B. Connected in given circuit, have equal e.m.f. E and internal resistance r_1 and r_2 respectively ($r_1 > r_2$), the switch K is closed at $t = 0$. After long time it was found that terminal potential difference across the battery A is zero. Find the value of R. **(IIT 2004)**



305

A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in the figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Whetstone bridge. Answer the following questions.



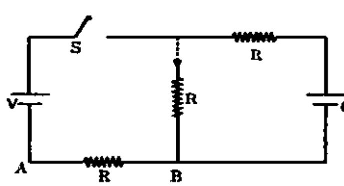
(a) Are there positive and negative terminals on the galvanometer ?

(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm. from A. Obtain the value of the resistance X. **(IIT 2002)**

306

In the circuit shown in the figure, the battery is an ideal one, with emf V, the capacitor is initially uncharged. The switch S is closed at time $t = 0$.

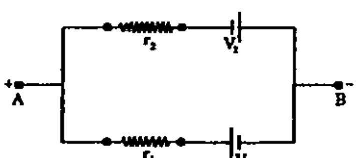


(a) Find the charge Q on the capacitor at time t.

(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$? **(IIT 1998)**

307

Find the emf (V) and internal resistance (r) of a single battery which is equivalent to a parallel combination of two batteries of emfs V_1 and V_2 and internal resistance r_1 and r_2 respectively, with polarities as shown in Fig.



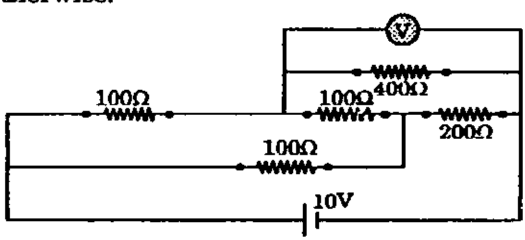
(IIT 1997, May)

308

A leaky parallel plate capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\ m^{-1}$. If the charge on the plate at the instant $t = 0$ is $q = 8.85\ \mu C$, then calculate the leakage current at the instant $t = 12$ s. **(IIT 1997, May)**

309

An electrical circuit is shown in the figure. Calculate the potential difference across the resistor of 400 ohm, as will be measured by the voltmeter V of resistance 400 ohm, either by applying Kirchoff's rules or ohtherwise. **(IIT 1996)**



310

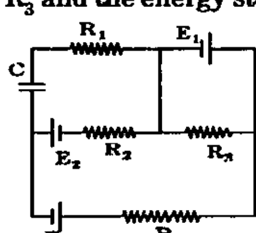
Two square metallic plates of 1 m side are kept 0.01 m apart, like a parallel plate capacitor, in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of $0.001\ ms^{-1}$. Calculate the current drawn from the battery during the process. (dielectric constant of oil = 11, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$) **(IIT 1994)**

311

In the given circuit,

$E_1 = 3E_2 = 2E_3 = 6$ volts, $R_1 = 2R_4 = 6\Omega$, $R_3 = 2R_2 = 4\Omega$, $C = 5\ \mu F$.

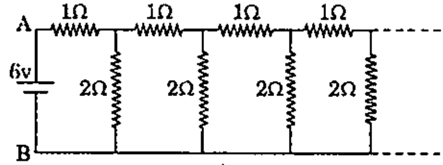
Find the current in R_3 and the energy stored in the capacitor. **(IIT 1988)**



312

An infinite ladder network of resistances is constructed with 1 ohm and 2 ohm resistance, as shown in the figure.

The 6 volt battery between A and B has negligible internal resistance.

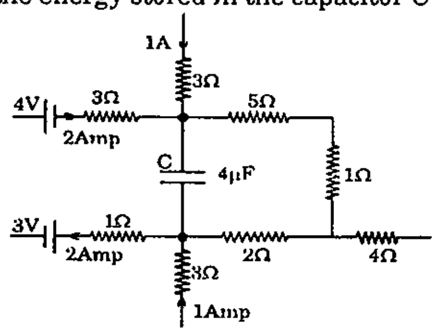


(i) Show that the effective resistance between A and B is 2 ohm.

(ii) What is the current that passes through the 2 ohm resistance nearest to the battery ? **(IIT 1987)**

313

A part of circuit in a steady state along with the currents flowing in the branches, the values of resistances etc. is shown in the figure. Calculate the energy stored in the capacitor C (4μF). **(IIT 1986)**



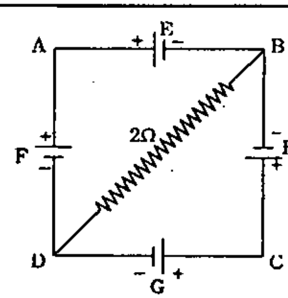
314

In the circuit shown in the figure, E, F, G, H, are cells of emf 2, 1, 3 and 1 volt respectively, and their internal resistances are 2, 1, 3 and 1 ohm respectively.

Calculate :

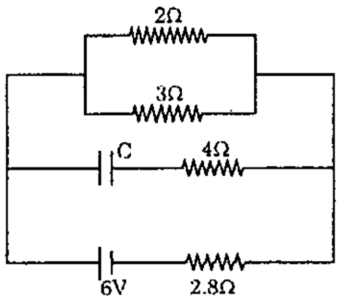
(i) the potential difference between B and D, and

(ii) the potential difference across the terminals of each cells G and H. **(IIT 1984)**



315

Calculate the steady state current in the 2-ohm resistor shown in the circuit in the figure. The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 microfarad.



(IIT 1982)

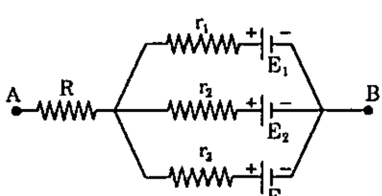
316

Two resistors, 400 ohms, and 800 ohms are connected in series with a 6-volt battery. It is desired to measure the current in the circuit. An ammeter of 10 ohms resistance is used for this purpose. What will be the reading in the ammeter ? Similarly, if a voltmeter of 10,000 ohms resistance is used to measure the potential difference across the 400 ohm resistor, what will be the reading in the voltmeter.

(IIT 1982)

317

In the circuit shown in Fig. E_1 = 3 volts, E_2 = 2 volts, E_3 = 1 volt and $R = r_1 = r_2 = r_3 = 1$ ohm.



(i) Find the potential difference between the points A and B and the currents through each branch.

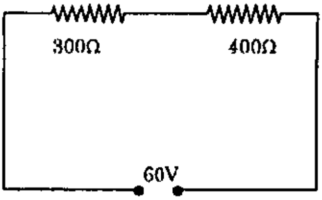
(ii) If r_2 is short circuited and the point A is connected to point B, find the currents through E_1, E_2, E_3 and the resistor R.

(IIT 1981)

318

(i) State Ohm's Law.

(ii) In the circuit shown in figure, a voltmeter reads 30 volts when it is connected across 400 ohm resistance. Calculate what the same voltmeter will read when it is connected across the 300 ohm resistance. **(IIT 1980)**



319

A battery of emf 2 volts and internal resistance 0.1 ohm is being charged with a current of 5 amps.

In what direction will the current flow inside the battery ?

What is the potential difference between the two terminals of the battery ? **(IIT 1980)**

320

A copper wire having cross-sectional area 0.5 mm^2 and a length of 0.1 m is initially at 25°C and is thermally insulated from surrounding. If a current of 10 amperes is set up in this wire.

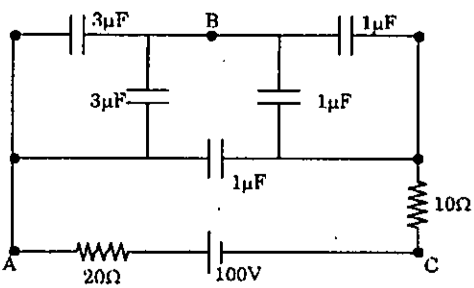
(i) Find the time in which the wire will start melting. The change of resistance with the temperature of the wire may be neglected.

(ii) What will this time be, if the length of wire is doubled.

(For copper : resistivity = 16×10^{-9} Ω-m, density = 8.9gm/cc,
specific heat = 390 J/kg-K)
Melting point = 1100°C **(IIT 1979)**

321

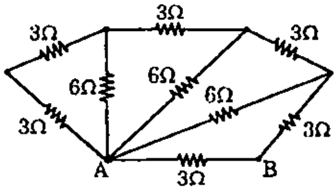
In the diagram shown below, find the potential difference between the points A and B and between the points B and C in steady state. **(IIT 1979)**



322

All the resistances in the diagram are in ohms.

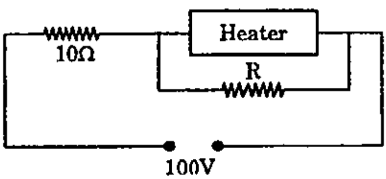
Find the effective resistance between the points A and B.



IIT 1979)

323

A heater is designed to operate with a power of 1000 watts in a 100 volt line. It is connected in a combination with a resistance of 10 ohms and a resistance R to a 100 volts mains as shown in the figure. What should be the value of R so that the heater operates with a power of 62.5 watts. **(IIT 1978)**



324

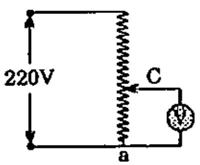
If a copper wire is stretched to make it 0.1% longer what is the percentage change in its resistance ? **(IIT 1978)**

325

Two electric bulbs, each designed to operate with a power of 500 watts in a 220 volt line are put in a series in a 110 volt line. What will be the power generated by each bulb. **(IIT 1977)**

326

A potential difference of 220 volts is maintained across a 12000 Ω-rheostat ab (see figure). The voltmeter V has a resistance of 6000 Ω and point C is at one fourth of the distance from *a* to *b*. What is the reading in the



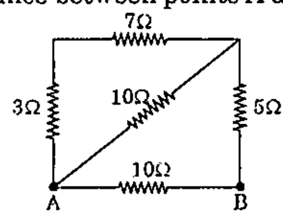
voltmeter. **(IIT 1977)**

327

A potentiometer wire of length 100 cms has a resistance of 10 ohms. It is connected in series with a resistance R and a cell of emf 2 volts and of negligible internal resistance. A source of emf 10 millivolts is balanced against a length of 40 cms of potentiometer wire. What is the value of external resistance R ? **(IIT 1976)**

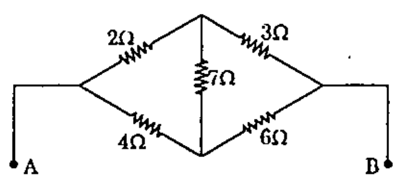
328

Five resistances are connected as shown in the diagram. Find the equivalent resistance between points A and B. **(IIT 1976)**



329

Five resistances are connected as shown in the diagram. What is effective resistance betwen the points A and B? **(IIT 1976)**



330

12 cells each having the same emf are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells identical with the others. The current is 3 amperes when the cells and battery aid each other and is 2 amperes when the cells and battery oppose each other. How many cells in battery are wrongly connected?

(IIT 1976)

331

A fuse made of lead wire has an area of cross-section 0.2 sq. mm. on short circuiting, the current in the fuse wire reaches 30 amps.

How long after short circuiting will the fuse begin to melt? Initial temperature of the wire is 20°C. Neglect the heat losses. (For lead : specific heat = 0.032 cal/gm-°C, melting point = 327 °C, density = 11.34 gm/cc, resistivity = 22×10^{-6} ohm-cm) **(IIT 1976)**

332

A battery of emf 1.4 volts and internal resistance 2 Ω is connected to a resistor of 100 Ω resistance through an ammeter. The resistance of ammeter is (4/3) ohm. A voltmeter has also been connected to find the potential difference across the resistance.

(i) Draw the circuit diagram.

(ii) The ammeter reads 0.02 amp. What is the resistance of voltmeter?

(iii) The voltmeter reads 1.10 volts. What is the error in this reading ? **(IIT 1975)**

333

An electric tea kettle has two heating coils. When one of the coils is switched on the kettle begins to boil in 6 minutes, and when the other is switched on the boiling begins in 8 minutes. In what time will the boiling begin if both coils are switched on simultaneously

(i) in series

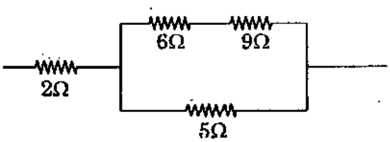
(ii) in parallel? **(IIT 1975)**

334

An electric current of 5 amps is divided into three branches forming a parallel combination. The lengths of the wire in the three branches are in the ratio 2, 3 and 4; their diameters are in the ratio 3, 4 and 5. Find the currents in each branch if the wires are of the same material. **(IIT 1975)**

335

In the circuit shown the 5Ω resistance develops 10.24 calories/second due to current flowing through it. Calculate
(i) the heat developed per second in 2 Ω resistor and
(ii) the potential difference across the 6 Ω resistance. **(IIT 1974)**



336

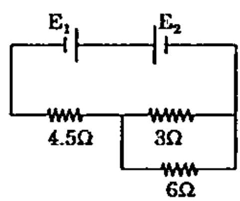
Two heater coils made of the same material are connected in parallel across the mains. The length and diameter of the wire of one of the coils is double that of the other. Which one of them will produce more heat ? **(IIT 1973)**

337

A standard 50 watt electric bulb in series with a room heater is connected across the mains. If 50 watt bulb is replaced by 100 watt bulb, will the heater output be larger, smaller or remain the same? **(IIT 1973)**

338

In the circuit shown in figure, the cells E_1 and E_2 have emfs 4V and 8V and internal resistances 0.5 Ω and 1 Ω respectively. Calculate the current in each resistor and the potential difference across each cell.



(IIT 1973)

339

A galvanometer together with an unknown resistance in series is connected across two identical batteries each of 1.5 volts. When the batteries are connected in series the galvanometer records a current of 1 ampere and when the battereies are in parallel the current is 0.6 ampere, what is the internal resistance of the battery? **(IIT 1973)**

340

An electric kettle 500 watts raises the temperature of one litre of water from 25°C to 100°C in 15 minutes. What percentage of electrical energy supplied is utilised in heating the water? **(IIT 1973)**

341

Three equal resistors connected in series across a source of emf together dissipate 10 watts of power. What would be the power dissipated if the same resistors are connected in parallel across the same source of emf? **(IIT 1972)**

342

A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 milliampere is passed through it.

(i) What is the value of the resistance which can convert this galvanometer into an ammeter giving a full scale deflection of 10 amperes?

(ii) A resistance of the required value is available but it will get burnt if the energy dissipated in it is greater than 1 watt. Can it be used for the above described conversion of the galvanometer?

(iii) When this modified galvanometer is connected across the terminals of a battery, it shows a current of 4 amperes. The current drops to 1 ampere, when a resistance of 1.5 Ω is connected in series with the modified galvanometer. Find the emf and the internal resistance of the battery. **(IIT 1972)**

MAGNETIC FIELD

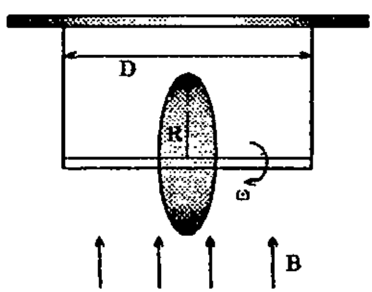
343

A proton and an alpha particle, after being accelerated through same potential difference, enter a uniform magnetic field the direction of which is perpendicular to their velocities. Find the ratio of radii of the circular paths of the two particles.

(IIT 2004)

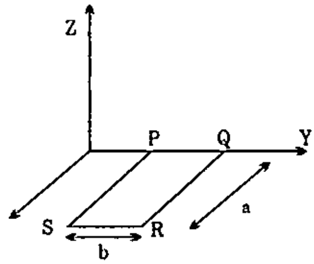
344

A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $\frac{3T_0}{2}$. **(IIT 2003)**



345

A rectangular loop PQRS made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the Z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})\, B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **(IIT 2002)**



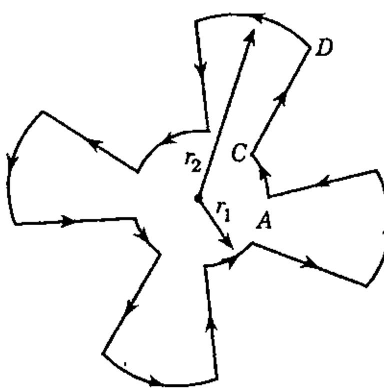
(a) What is the direction of the current I in PQ ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

346

A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii $r_1 = 0.08$ m and $r_2 = 0.12$ m. Each arc subtends the same angle at the centre.



(a) Find the magnetic field produced by this circuit at the centre.

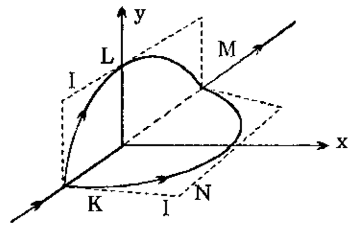
(b) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit.

What is the force acting on the wire at the centre due to the current in the circuit ? What is the force acting on the arc AC and the straight segment CD due to the current at the centre ?

(IIT 2001)

347

A circular loop of radius R is bent along a diameter and given a shape as shown in the figure. One the semicircle (KNM) lies in the x-z plane and the other one (KLM) in the y-z plane with their centres at the origin. Current I is flowing through each of the semicircles as shown in figure.



(a) A particle of charge q is released at the origin with a velocity $\vec{v} = -v_0\,\hat{i}$. Find the instantaneous force $\vec{f}$ on the particle. Assume that space is gravity free.

(b) If an external uniform magnetic field $B\hat{j}$ is applied, determine the forces $\vec{F}_1$ and $\vec{F}_2$ on the semicircles KLM and KNM due to this field and the net force $\vec{F}$ on the loop. **(IIT 2000)**

348

The region between $x = 0$ and $x = L$ is filled with uniform, steady magnetic field $B_0\,\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\,\hat{i}$ travels along x-axis and enters the region of the magnetic field. Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.

(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto 2.1L. **(IIT 1999)**

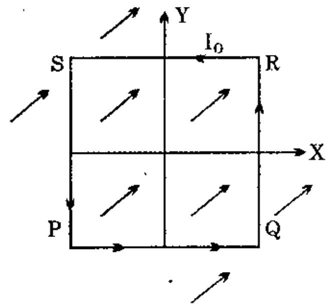
349

A particle of mass m and charge q is moving in a region where uniform, constant electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present. $\vec{E}$ and $\vec{B}$ are parallel to each other. At time $t = 0$ the velocity $\vec{v_0}$ of the particle is perpendicular to $\vec{E}$. (Assume that its speed is always << c, the speed of light in vacuum.) Find the velocity

$\vec{v}$ of the particle at time t. You must express your answer in terms of t, q, m. the vectors $\vec{v}_0$, $\vec{E}$ and $\vec{B}$, and their magnitudes v_0, E and B. **(IIT 1998)**

350

A uniform, constant magnetic field $\vec{B}$ is directed at an angle of 45° to the x-axis in the xy-plane. PQRS is a rigid, square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position shown in the figure, with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.



(a) What is the torque $\vec{\tau}$ about O acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δ t, and the axis about which this rotation occurs. (Δ t is so short that any variation in the torque during this interval may be neglected.)
Given : moment of inertia of the frame about an axis through its centre perpendicular to its plane is $\frac{4}{3}ML^2$? **(IIT 1998)**

351

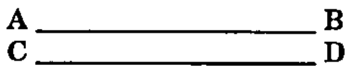
Three infinitely long thin wires, each carrying current I in the same direction are in x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$.

(i) Find the locus of the points for which the magnetic field B is zero.

(ii) If the central wire is displaced along the z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation. **(IIT 1997, July)**

352

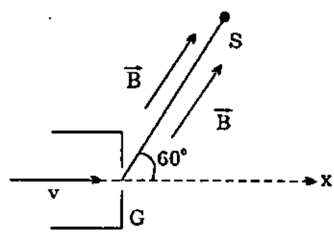
A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD, which is fixed in a horizontal plane and carries a steady current of 30A, as shown in the figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations.



(IIT 1994)

353

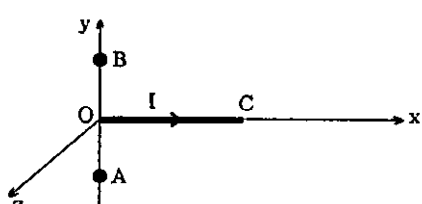
An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S where GS = 0.1 m, and the line GS makes and angle of 60° with the x-axis, as shown in the figure. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electrons hit S.



(IIT 1993)

354

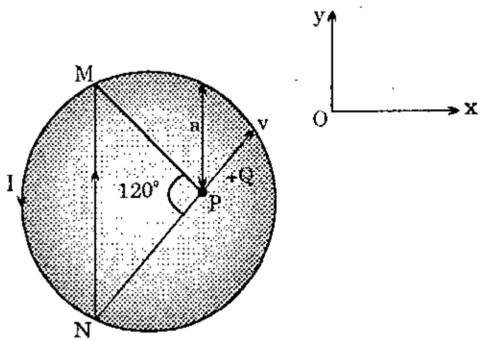
A straight segment OC (of length L metre) of a circuit carrying a current I amp is placed along the x-axis see figure. Two infinitely along straight wires A and B, each extending from z = – ∞ to + ∞, are fixed at y = – a metre and y = + a metre respectively, as shown in the figure. If the wires A and B each carry a current I amp into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed ?



(IIT 1992)

355

A wire loop carrying a current I is placed in the x-y plane as shown in the figure.



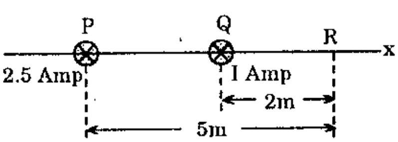
(a) If a particle with charge + Q and mass m is placed at the centre P and given a velocity $\vec{v}$ along NP (see figure), find its instantaneous acceleration.

(b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due to this field.

(IIT 1991)

356

Two long parallel wires carrying currents 2.5 amperes and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The point P and Q are located at a distance of 5 metres and 2 metres respectively from a collinear point R (see figure).



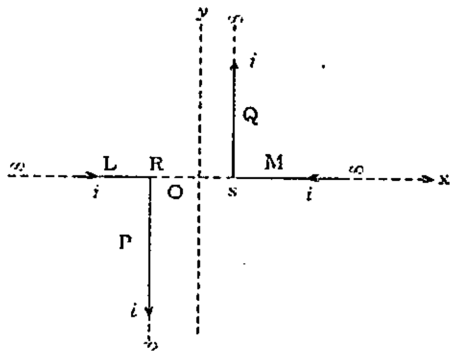
(i) An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(ii) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 amperes may be placed so that the magnetic induction at R is zero.

(IIT 1990)

357

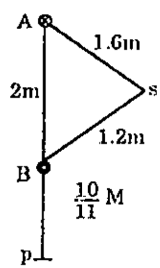
A pair of stationary and infinitely long bent wires are placed in the XY plane of as shown in the figure. The wires carry currents of $i = 10$ amperes each as shown. The segments L and M are along the X-axis. The segments P and Q are parallel to the Y-axis such that OS = OR = 0.02 m. Find the magnitude and direction of the magnetic induction at the origin O.



(IIT 1989)

358

Two long straight parallel wires are 2 metres a part, perpendicular to the plane of the paper see figure).



The wire A carries a current of 9.6 amps, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P, at a distance of $\frac{10}{11}$ m from the wire B is zero. Find

(i) the magnitude and direction of current in B.

(ii) the magnitude of the magnetic field of induction at the point S.

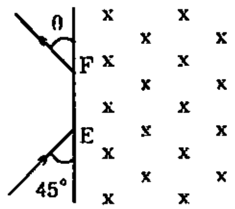
(iii) the force per unit length on the wire B. **(IIT 1987)**

359

A beam of protons with a velocity 4×10^5 m/sec. enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation). **(IIT 1986)**

360

A particle of mass $m = 1.6 \times 10^{-27}$ kg and charge $q = 1.6 \times 10^{-19}$ C enters a region of uniform magnetic field of strength 1 Tesla along the direction shown in the figure. The speed of the particle is 10^7 m/s



(i) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.

(ii) If the direction of the field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.

(IIT 1984)

361

A particle of mass 1×10^{-26} kg and charge $+ 1.6 \times 10^{-19}$ coulomb travelling with a velocity 1.28×10^6 m/s in the +X direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that

$E_x = E_y = 0$, $E_z = -102.4$ kV/m and $B_x = B_z = 0$, $B_y = 8 \times 10^{-2}$ weber/m^2. The particle enters this region at the origin at time t = 0. Determinethelocation (x, y and z coordinates) of the particle at $t = 5 \times 10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t = 7.45 \times 10^{-6}$ s? **(IIT 1982)**

362

A bar magnet with poles 25 cm apart and of strength 14.4 Amp-m rests with centre on a frictionless pivot. It is held in equilibrium at an angle of 60° with respect to a uniform magnetic field of induction 0.25 Wb/m^2, by applying a force F at right angle to its axis at a point 12 cm from pivot. Calculate F. What will happen if the force F is removed ?

(IIT 1978)

363

Two infinitely long parallel wires carry equal currents in the same direction.

(i) What is the direction of the magnetic field due to one of the wires at any point along the other wire?

(ii) What is the direction of force on one wire due to the other?

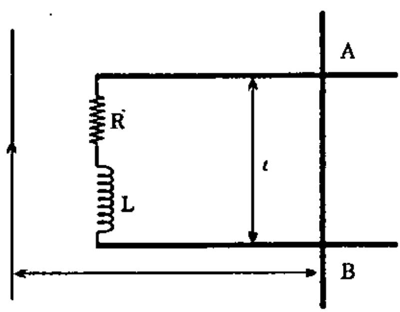
(iii) By what factor does this force change if the current in each wire is doubled?

(iv) What is the direction of magnetic field at a point midway between the two wires? **(IIT 1973)**

ELECTROMAGNETIC INDUCTION

364

A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At t = 0, it is made to slide on the rails away from the wire. Answer the following question.

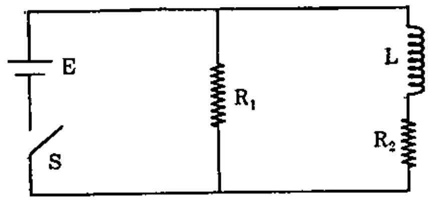


(a) Find a relation among i, $\frac{di}{dt}$ and $\frac{d\Phi}{dt}$, where i is the current in the circuit and Φ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time t = T, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from t = 0 to t = T.

(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time 2T. Find the value of $\frac{L}{R}$ in terms of the other given quantities. **(IIT 2002)**

365



An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2Ω and R_2 = 2Ω are connected to a battery of e.m.f. E = 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$.

What is the potential drop across L as a function of time ?

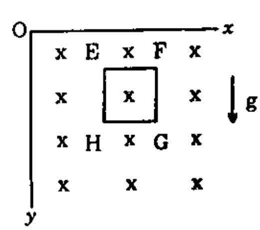
After the steady state is reached, the switch is opened. What is the direction and magnitude of current through R_1 as a function of time ? **(IIT 2001)**

366

A thermocole vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area 5×10^{-3} m², number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform, time varying magnetic field is set up to pass vertically through the coil at time t = 0. The field is first increased from zero to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s. This cycle is repeated 12000 time. Make sketches of the current through the coil and the power dissipated in the coil as functions of time for the first two cycles. Clearly indicate the magnitude of the quantities on the axes. Assume that no heat is lost to the vessel, or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of the metal = 500 $jkg^{-1} K^{-1}$ and the specific heat of water = 4200 $Jkg^{-1} K^{-1}$. Neglect the inductance of the coil. **(IIT 2000)**

367

A magnetic field $\vec{B} = \left(\frac{B_0 y}{a}\right)\hat{k}$ is into the paper in the + z direction. B_0 and a are positive constants. A square loop EFGH of side a, mass m and resistance R, in x-y plane, starts falling under the influence of gravity. Note the direction of x and y axes in the figure. Find :



(a) the induced current in the loop and indicate its direction.

(b) the total Lorentz force acting on the loop and indicate its direction, and

(c) an expression for the speed of the loop, v (t) and its terminal value. **(IIT 1999)**

368

An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF, and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor, and I the current in the circuit. It is found that the maximum value of Q is 200 μC

(a) When Q = 100 μ C, what is the value of $\left|\frac{dI}{dt}\right|$?

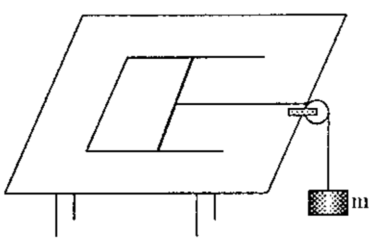
(b) When Q = 200 μ C, what is the value of I ?

(c) Find the maximum value of I.

(d) When I is equal to one half its maximum value, find the value of | Q | ? **(IIT 1998)**

369

A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string, hangs vertically. A constant magnetic field B exists perpendi-cular to the table. If the system is released from rest, calculate?



(i) the terminal velocity achieved by the rod, and

(ii) acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

(IIT 1997, July)

370

An infinitesimally small bar magnet of dipole moment M is pointing and moving with the speed v in the x-direction. A small closed circular conducting loop of radius a and negligible self inductance lies in the y-z plane with it centre at $x = 0$, and its axis coinciding with the x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.

(IIT 1997, May)

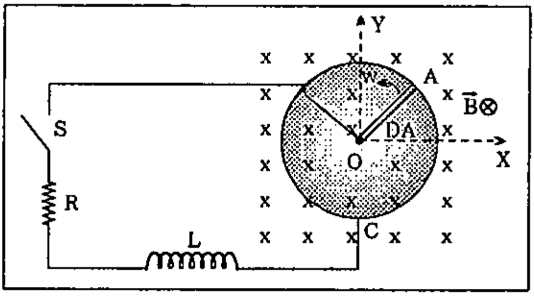
371

A solenoid has an inductance of 10 henry and a resistance of 2 ohm. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach $\frac{1}{4}$ of its maximum value ?

(IIT 1996)

372

A metal rod OA of mass m and length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



(a) What is the induced emf across the terminals of the switch ?

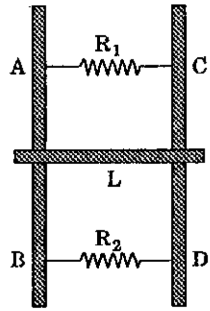
(b) The switch S is closed at time $t = 0$.

(i) Obtain an expression for the current as a function of time.

(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at $t = 0$. **(IIT 1995)**

373

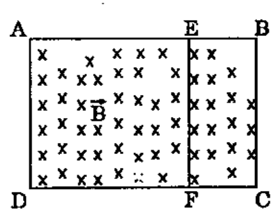
Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown in the figure. A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. **(IIT 1994)**



374

A rectangular frame ABCD made of uniform metal wire has a straight connection between E and F made of the same wire, as shown in the figure. AEFD is a square of side 1m, and EB = FC = 0.5 m. The entire circuit is placed in a steadily increasing, uniform magnetic field directed into the plane of the paper and normal to it. The rate of change of the magnetic field is 1 T/s. The resistance per unit length of the wire is 1 Ω/m. Find the magnitudes and directions of the currents in the segments AE, BE and EF.

(IIT 1993)

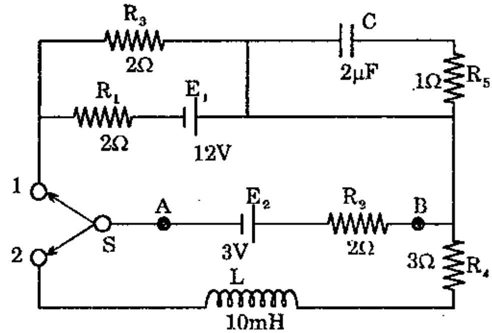


375

A circuit containing a two position switch S is shown in figure.

(a) The switch S is in position '1'. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.

(b) If now the switch S is put in position '2' at $t = 0$, find



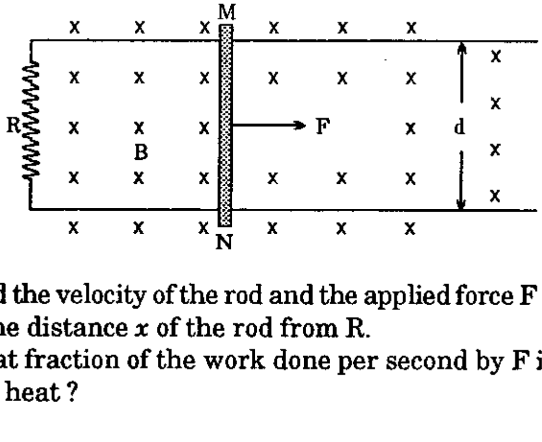
(i) steady current in R_4 and

(ii) the time when current in R_4 is half the steady value. Also calculate the energy stored in the inductor L at that time.

(IIT 1991)

376

Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current flows through R.



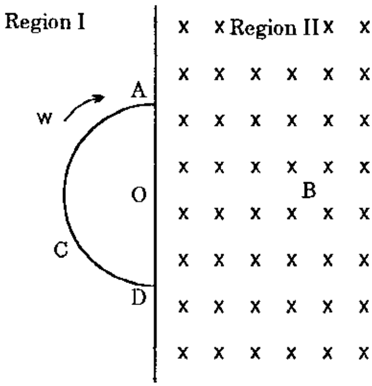
(i) Find the velocity of the rod and the applied force F as functions of the distance x of the rod from R.

(ii) What fraction of the work done per second by F is converted into heat ?

(IIT 1988)

377

Space is divided by the line AD into two regions. Region I is field free and the Region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semicircular conducting loop of radius r with center at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The effective resistance of the loop is R.



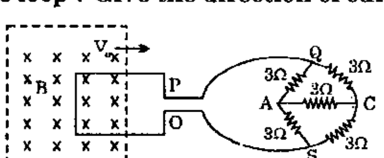
(i) Obtain an expression for the magnitude of the induced current in the loop.

(ii) Show the direction of the current when the loop is entering into the Region II.

(iii) Plot a graph between the induced e.m.f. and the time of rotation for two periods of rotation. **(IIT 1985)**

378

A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2\ Wb/m^2$ as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connectd to a network of resistors each of value 3 ohms. The resistances of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop ? Give the direction of current in the loop.



(IIT 1983)

379

The two rails of a railway track, insulated from each other and the ground, are connected to a millivolt-meter. What is the reading of the millivoltmeter when a train travels at a speed of 180 km/hour aong the track given that the horizontal components of earth's magnetic field is 0.2×10^{-4} webers/m^2 and the rails are separated by 1 metre ? **(IIT 1981)**

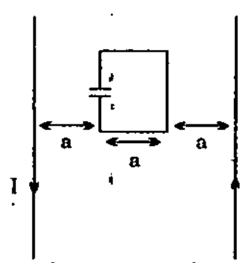
ALTERNATING CURRENT

380

In an LR series circuit, a sinusoidal voltage $V = V_0 \sin \omega t$ is applied. It is given that $L = 35$ mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\frac{\omega}{2\pi} = 50$ Hz and $\pi = \frac{22}{7}$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph. **(IIT 2004)**

381

A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the wires is given as $I = I_0 \sin \omega t$.



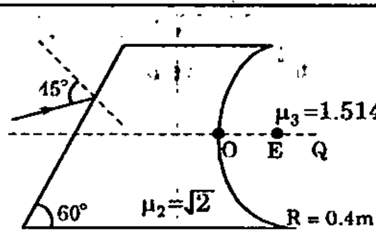
(a) Calculate maximum current in the square loop.

(b) Draw a graph between charges on the upper plate of the capacitor vs time. **(IIT 2003)**

RAY OPTICS

382

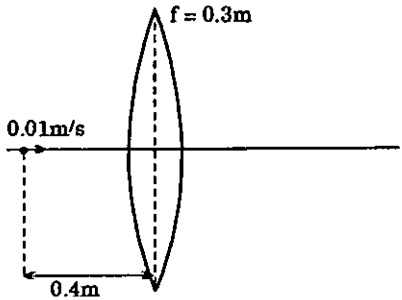
A light ray is incident on an irregular shaped slab of refractive index $\sqrt{2}$ at an angle of 45° with the normal on the incline face as shown in the figure. The ray finally emerges from the curved surface in the



medium of the refractive index $\mu = 1.514$ and passes through point E. If the radius of curved surface is equal to 0.4 m, find the distance OE correct up to two decimal places. **(IIT 2004)**

383

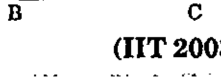
An object is approaching at thin convex lens of focal length 0.3 m with a speed of 0.01 m/s. Find the magnitudes of the rates of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens.



(IIT 2004)

384

A prism of refracting angle 30° is coated with a thin film of transparent material of refractive index 2.2 on face AC of the prism. A light of wavelength 5500 Å is incident on face AB such that angle of incidence is 60°, find



(a) the angle of emergence,

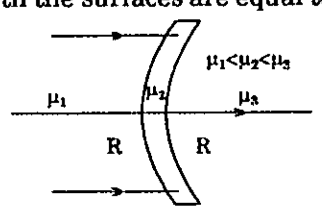
[Given refractive index of the material of the prism is $\sqrt{3}$].

(b) the minimum value of thickness of the coated film on the face AC for which the light emerging from the face has maximum intensity.

(IIT 2003)

385

Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to R.



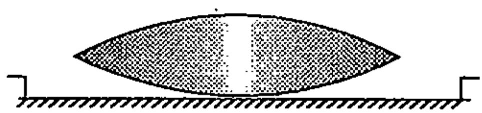
(IIT 2003)

386

The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle of this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system. **(IIT 2001)**

387

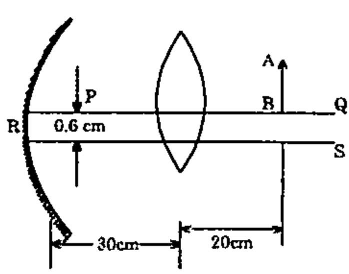
A thin biconvex lens of refractive index $\frac{3}{2}$ is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index $\frac{4}{3}$. It is found that when a point object is placed 15 cm above the lens on its principle axis, the object coincides with its own image.



On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid. **(IIT 2001)**

388

A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axes PQ and RS parallel but separated in vertical direction by 0.6 cm as shown. The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optic axis PQ of the lens at a distance of 20 cm from the lens. If A′B′ is the image after refraction from the lens and reflection from the mirror, find the distance of A′B′ from the pole of the mirror and obtain its magnification. Also locate position of A′ and B′ with respect to the optic axis RS. **(IIT 2000)**

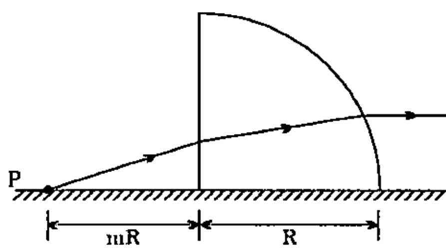


389

The x-y plane is the boundary between two transparent media. Medium – 1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium – 2 with $z \leq 0.0$, has a refractive index $\sqrt{3}$. A ray of light in medium – 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium-2. **(IIT 1999)**

390

A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in the figure.



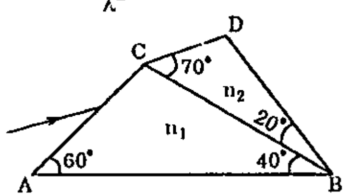
(IIT 1999)

391

A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together without a gap as shown in the figure. The angles of the prisms are as shown. n_1 and n_2 depend on λ, the wavelength of light, according to

$$n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2} \text{ and } n_2 = 1.45 + \frac{10.8 \times 10^4}{\lambda^2}$$

where, λ is in nm.

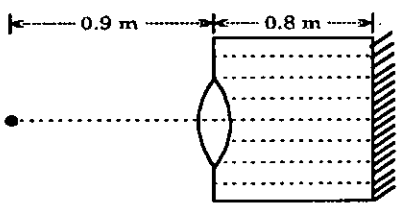


(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(b) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum. **(IIT 1998)**

392

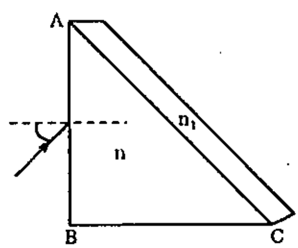
A thin equiconvex lens of glass of refractive index $\mu = \frac{3}{2}$ and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $\left(\mu = \frac{4}{3}\right)$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in figure. The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank in front of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to the lens) of the image of the object formed by the system.



(IIT 1997, May)

393

A right angle prism (45° – 90° – 45°) of refractive index n has a plate of refractive index n_1 ($n_1 < n$) cemented to its diagonal face. The assembly is in air. A ray is incident on AB (see figure),

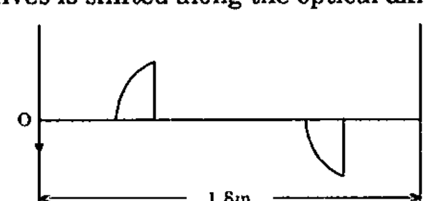


(i) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

(ii) Assuming n = 1.352, calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated. **(IIT 1996)**

394

A thin plano-convex lens of focal length f is split in to two halves : one of the halves is shifted along the optical axis (see figure).



The separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half-lenses is 2. Find the focal-length of the lens and separation between the two halves. Draw the ray diagram for image formation.

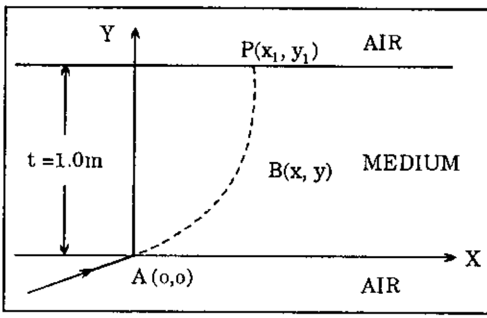
(IIT 1996)

395

A ray of light travelling in air is incident at grazing angle (incident angle = 90°) on a long rectangular slab of a transparent medium of thickness t = 1.0 m (see figure). The point of incidence is the origin A (0, 0). The medium has a variable index of refraction n (y) given by

$$n\,(y) = [Ky^{3/2} + 1]^{1/2}$$

where K = 1.0 (metre)$^{-3/2}$



The refractive index of air is 1.0.

(a) Obtain a relation between the slope of the trajectory of the ray at a point B (x, y) in the medium and the incident angle at that point.

(b) Obtain an equation for the trajectory y (x) of the ray in the medium.

(c) Determine the coordinates (x_1, y_1) of the point P, where the ray intersects the upper surface of the slab-air boundary.

(d) Indicate the path of the ray subsequently. **(IIT 1995)**

396

Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. Determine the least value of n so that the light entering the rod does not emerge from the curved surface of the rod irrespective of the value of α. **(IIT 1992)**

397

A parallel beam of light travelling in water (refractive index = $\frac{4}{3}$) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial,

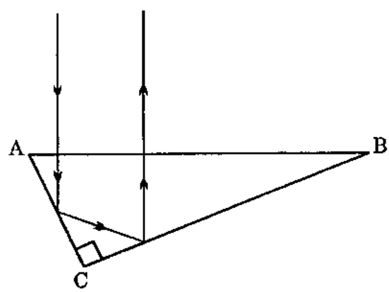
(i) find the position of the image due to refraction at the first surface and the position of the final image.

(ii) draw a ray diagram showing the positions of both the images.

(IIT 1988)

398

A right angled prism is to be made by selecting a proper material and the angles A and B (B ≤ A), as shown in Figure. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two internal reflections.



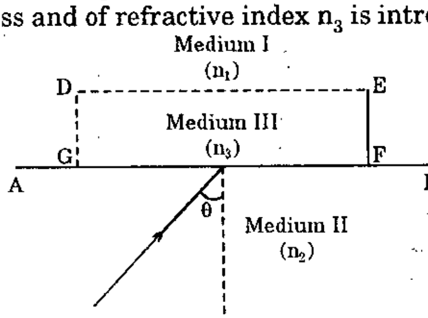
(i) What should be the minimum refractive index n for this to be possible ?

(ii) For $n = \frac{5}{3}$ is it possible to achieve this with the angle B equal to 30 degrees ?

(IIT 1987)

399

Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 ($n_2 > n_1$) at an angle of incidence θ as shown in the figure. The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab DEFG of uniform thickness and of refractive index n_3 is introduced on the



interface (as shown in the figure), show thatfor any value of n_3 all light will ultimately be reflected back again into medium II. Consider separately the cases.

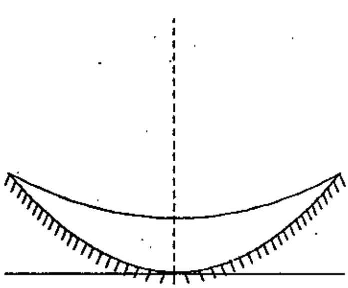
(i) $n_3 < n_1$ and *(ii)* $n_3 > n_1$. **(IIT 1986)**

400

A plano convex lens has a thickness of 4 cm. When placed on a horizontal table with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. Find the focal length of the lens. **(IIT 1984)**

401

The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface.



(i) Where should a pin be placed on the optic axis such that its image is formed at the same place ?

(ii) If the concave part is filled with water of refractive index $\frac{4}{3}$, find the distance through which the pin should be moved so that the image of the pin again coincide with the pin. **(IIT 1981)**

402

An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror.

Find the position of the final image formed.

(You may take the distance of the near surface of the slab from the mirrortobe 1 cm).

(IIT 1986)

403

A telescope has an objective of focal length 50 cm and an eye piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away from the objective. Calculate :

(i) the separation between the objective and the eye-piece.

(ii) the magnification produced. **(IIT 1980)**

404

A rectangular block of glass is placed on a printed page lying on a horizontal surface. Find the value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block. **(IIT 1979)**

405

The radius of curvature of the convex face of a plano convex lens is 12 cm and its refractive index is 1.5.

(i) Find the focal length of this lens.

(ii) The plane surface of the lens is now silvered. At what distance from the lens will parallel rays incident on the convex face converge.

(iii) Sketch the ray diagram to locate the image, when a point object is placed on the axis, 20 cm from the lens (polished).

(iv) Calculate the image distance when the object is placed as in *(iii)*. **(IIT 1979)**

406

A ray of light is incident at an angle of 60° on one face of prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism. **(IIT 1978)**

407

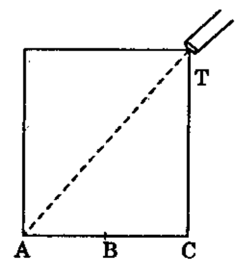
A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made of a material of refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature are 22 cm. Determine the position of the final image. Is the image real as virtual? **(IIT 1978)**

408

A glass lens has focal length 5 cm in air. What will be its focal length in water. (Refractive index of glass is 1.51 and that of water is 1.33). **(IIT 1977)**

409

A person looking through the telescope T just sees the point A on the rim at the bottom of a cylindrical vessel when the vessel is empty (see figure). When the vessel is completely filled with a liquid of refractive index 1.5, he observes a mark at the centre B of the bottom, without moving the telescope on the vessel. What is the height of the vessel if the diameter of the cross-section is 10 cm?



(IIT 1977)

410

A ray of light is travelling from diamond to glass. Calculate the minimum angle of incidence of the ray on the diamond glass interface such that no light is refracted into glass. What will happen if the angle of incidence exceeds the angle? (refractive index of glass is 1.51 and that of diamond is 2.47) **(IIT 1977)**

411

What is the velocity of light in glass of refractive index 1.5? (Velocity of light in air = 3×10^{10} cm/sec.) **(IIT 1976)**

412

The refractive index of the material of a prism of refracting angle 45° is 1.6 for a certain monochromatic ray. What should be minimum angle of incidence of this ray on the prism so that no total internal reflection takes place as the ray comes out of the prism. **(IIT 1976)**

413

Photographs of the ground are taken from an aircraft flying at an altitude of 2000 metres by a camera with a lens of focal length 50 cms. The size of the film in the camera is 18 cm × 18 cm. What area of the ground can be photographed by this camera at any one time?

(IIT 1976)

414

A point object O is placed at a distance of 12 cms on the axis of a convex lens of focal length 10 cms. On the other side of the lens, a convex mirror is placed at a distance of 10 cms from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of the convex mirror? **(IIT 1976)**

415

A rectangular glass block of thickness 10 cm and refractive index 1.5 is placed over a small coin. A beaker filled with water of refractive index $\frac{4}{3}$ to a height of 10 cm and is placed over the glass block.

(a) Find the apparent position of the object when it is viewed at near normal incidence.

(b) Draw a neat ray diagram.

(c) If the eye is slowly moved away from the normal at a certain position the object is found to disappear due to total internal reflection. At which surface does this happen and why ?

(IIT 1975)

416

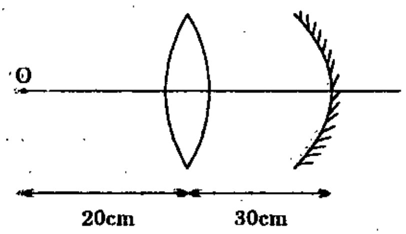
A projector lens has a focal length 10 cm. It throws an image of a 2 cm × 2 cm slide on a screen 5 metre from the lens. Find :

(i) the size of the picture on the screen and

(ii) ratio of illuminations of the slide and of the picture on the screen. **(IIT 1975)**

417

An object is placed at 20 cm left of the convex lens of focal length 10 cm. If a concave mirror of focal length 5 cm is placed at 30 cm to the right of the lens find the magnification and the nature of the final image. Draw the ray diagram and locate the position of the final image.



(IIT 1974)

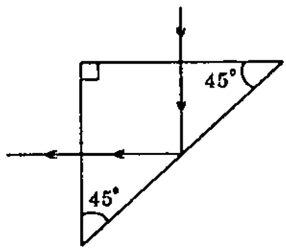
418

An object is placed in front of a convex mirror at a distance of 50 cms. A plane mirror is introduced covering lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cms, it is found that there is no parallux between the images formed by two mirrors. What is the radius of curvature of the convex mirror?

(IIT 1973)

419

A ray of light incident normally on one of the faces of a right angled isosceles prism is found to be totally reflected as shown in the figure. What is the minimum value of the refractive index of the material of the prism?

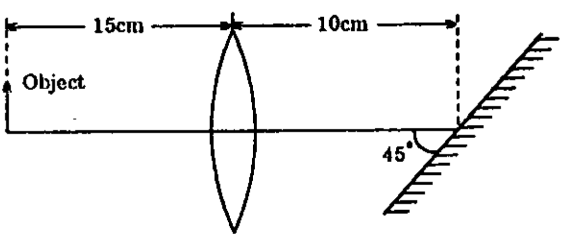


When the prism is immersed in water, trace the path of theemergent rays for the same incident ray, indicating the values of all the angles. $(\mu_\omega = \frac{4}{3})$

(IIT 1973)

420

An object of height 4 cm is kept to the left of and on the axis of a converging lens of focal length 10 cm as shown in figure. A plane mirror is placed inclined at 45° to the lens axis 10 cm to the right of the lens (see figure). Find the position and size of the image formed by the lens and mirror combination. Trace the rays forming the image.



(IIT 1972)

WAVE OPTICS

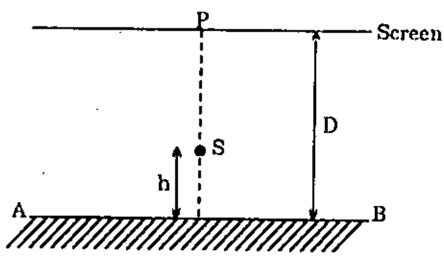
421

In a Young's double slit experiment light consisting of two wavelengths $\lambda_1 = 500$ nm and $\lambda_2 = 700$ nm is incident normally on the slits. Find the distance from the central maxima where the maximas due to two wavelengths coincide for the first time after central maxima. (given $\frac{D}{d} = 1000$) where D is the distance between the slits and the screen and d is the separation between the slits.

(IIT 2004)

422

A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it.



(a) What is the shape of the interference fringes on the screen ?

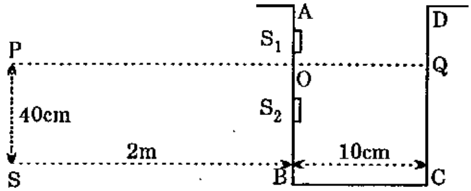
(b) Calculate the ratio of minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

(c) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

(IIT 2002)

423

A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the stlits is 0.8mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic ight source is kept at S, 40 cm below P and 2m from the vessel, to illuminate the slits as shown in the figure below. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled up to OQ.



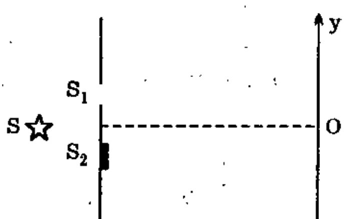
The central bright fringe is found to be at Q. Calculate the refractive index of the liquid. **(IIT 2001)**

424

A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain least value of t for which rays interfere constructively. **(IIT 2000)**

425

The Young's double slit experiment is done in a medium of refractive index $\frac{4}{3}$. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown.



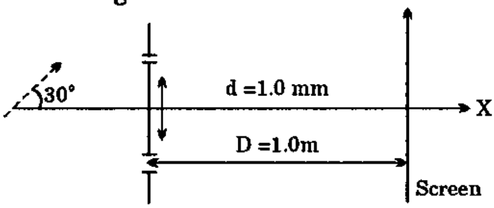
(a) Find the location of the central maximum (bright fringe with zero path difference) on the y-axis.

(b) Find the light intensity at point O relative to the maximum fringe intensity.

(c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O.
[All wavelengths in this problem are for the given medium of refractive index $\frac{4}{3}$. Ignore dispersion] **(IIT 1999)**

426

A coherent parallel beam of microwave of wavelength $\lambda = 0.5$ mm falls on a Young's double apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in the figure.



(a) If the incident beam falls normally on the double slit apparatus, find the coordinates of all the interference minima on the screen.

(b) If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in the figure), find the y-coordinates of the first minima on either side of the central maximum. **(IIT 1998)**

427

In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum (n = 0) fell before the glass plates were inserted now has $\frac{3}{4}$ the original intensity. It is further observed that what used to be the fifth maximum earlier, lies below the point P while the sixth minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected). **(IIT 1997, July)**

428

In Young's experiment, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength. **(IIT 1997, May)**

429

A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm, and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wave-length in air is 6300 Å.

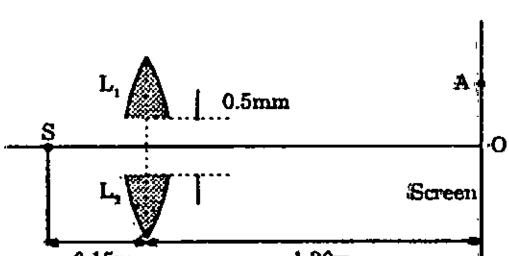
(i) Calculate the fringe-width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum to the axis.

(IIT 1996)

430

In the figure shown, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gass of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m, while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO.

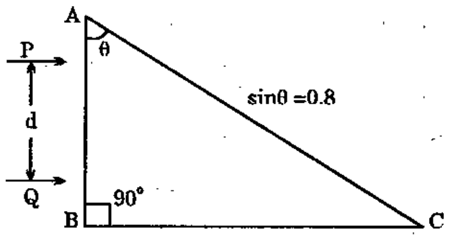


(i) If the third intensity maximum occurs at point A on the screen, find distance OA.

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease or remain the same ? **(IIT 1993)**

431

Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in Figure. The refractive index of the prism as a function of wavelength is given by the relation, $\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$ where λ is in Å and b is positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wave length and is not satisfied for the other.



(a) Find the value of b.

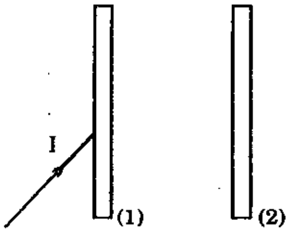
(b) Find the deviation of the beams transmitted through the face AC.

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and the lower beams immediately after transmission from the face AC, are 4I and I respectively, find the resultant intensity at the focus.

(IIT 1991)

432

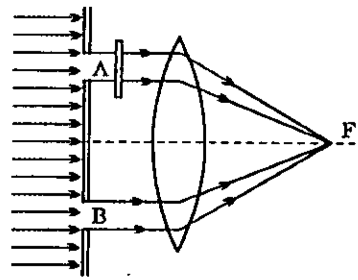
A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25 per cent of the light incident on it and transmits the remaining. Find the ratio



of the minimum and the maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate. **(IIT 1990)**

433

In a modified Young's double slit experiment, a mono-chromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $\left(\frac{10}{\pi}\right)$ Wm^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A, (see figure). Calculate the power (in watts) received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot. **(IIT 1989)**



434

A beam of light consisting of two wavelengths, 6500 Å and 5200 Å, is used to obtain interference fringe in a Young's double slit experiment :

(i) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 6500 Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide ?

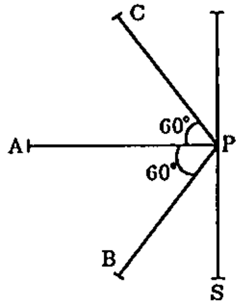
The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm. **(IIT 1985)**

435

In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wave length of the monomchromatic light used in the experiment. **(IIT 1983)**

436

Screen S is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards point P on the screen (see figure). Line AP is normal to the screen and the lines AP, BP and CP are in one plane The distances AP, BP and CP are 3 m, 1.5 m and 1.5 m respectively. The radiant powers of sources A and B are 90 watts and 180 watts respectively. The beam from C is of intensity 20 watts/m^2. Calculate the intensity at P on the screen. **(IIT 1982)**



PHOTOELECTRIC EFFECT

437

Two metallic plates A and B, each of area 5×10^{-4} m^2, are placed parallel to each other at a separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light with photons of energy 5 eV each, starts falling on plate A at t = 0 so that 10^{16} photons fall on it per square m/s. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV. Determine :

(a) the number of photoelectrons emitted up to t = 10 s.

(b) the magnitude of the electric field between the plates A and B at t = 10 s and

(c) the kinetic energy of the most energetic photoelectron emitted at t = 10s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B.

Take $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N-m^2 **(IIT 2002)**

438

Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form. **(IIT 1997, July)**

439

In a photoelectric effect set-up, a point source of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius 8.0×10^{-3} m. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission.

(a) Calculate the number of photoelectrons emitted per second.

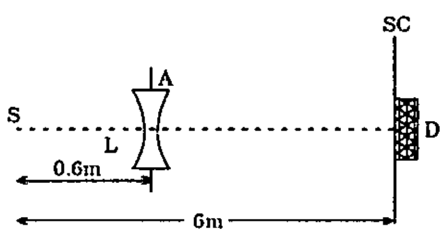
(b) Find the ratio of the wavelength of incident light to the de Broglie wavelength of the fastest photoelectrons emitted.

(c) It is observed that the photoelectron emission stops at a certain time t after the light source is switched on. Why ?

(d) Evaluate the time *t*. **(IIT 1995)**

440

A monochromatic point source S radiating wavelength 6000 Å, with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in figure. A photoemissive detector D of surface area 0.5 cm^2 is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9.



(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

(b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

(c) If the workfunction of the photoemissive surface is 1 eV, calculate the vlaues of the stopping potential in the two cases (without and with the lens in the aperture). **(IIT 1991)**

441

A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of 3.6×10^{-3} W m^{-2} equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds. **(IIT 1989)**

ATOMIC PHYSICS

442

In a photoelectric setup, the radiations from the Balmer seriesof hydrogen atom are incident on a metal surface of work function 2 eV. The wavelength of incident radiations lies between 450 nm to 700 nm. Find the maximum kinetic energy of photoelectron emitted. (Given hc/e = 1242 eV –nm) **(IIT 2004)**

443

Characteristic X-rays of frequency 4.2×10^{18} Hz are produced when transitions from L shell to K shell take place in a certain target material. Use Mosley's law to determine the atomic number of the target material. Given Rydberg constant R = 1.1×10^7 m^{-1}.

(IIT 2003)

444

A hydrogen-like atom (described by the Bohr model is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between –0.85 eV and –0.544 eV (including both these values).

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

Take hc = 1240 eV-nm, and ground state energy of hydrogen atom = – 13.6 eV. **(IIT 2002)**

445

(a) A hydrogen-like atom of atomic number Z is in an excited state of quantm number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also, calculate the minimum

energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is -13.6 eV.

(b) When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area $1.0 \times 10^{-4} m^2$ and work function 5.6 eV, 0.53% of incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV).
Take 1 eV = 1.6×10^{-19} J **(IIT 2000)**

446

Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoeclectrons pass through a region containing α- particles. A maximum energy electron combines with an α-particle to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination.
[Take, h = 4.14×10^{-15} eV –s] **(IIT 1999)**

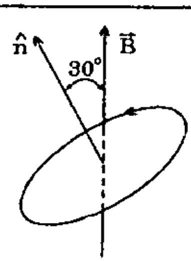
447

An electron, in a hydrogen-like atom, is in an excited state, It has a total energy of = -3.4 eV. Calculate :

(i) the kinetic energy, and

(ii) the de Broglie wavelength of the electron. **(IIT 1996)**

448



An electron in the ground state of hydrogen atom is revolving in anticlock-wise direction in a circular orbit of redius R.

(i) Obtain an expression for the orbital magnetic dipole momentum of the electron.

(ii) The atom is placed in a uniform magnetic induction B such that the plane-normal of the electron orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron. **(IIT 1996)**

449

A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV respectively. Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the values of n and Z. **(IIT 1994)**

((Ionization energy of hydrogen atom = 13.6 eV)

450

A neutron of kinetric energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of 90° with respect of its original direction.

(i) Find the allowed values of the energy of the neutron and that of the atom after the collision.

(ii) If the atom gets de-excited subsequently, by emitting radiation, find the frequencies of the emitted radiation.

[Given : (mass of the atom) = 4 (mass of neutron), (Ionisation energy of H atom) = 13.6 eV] **(IIT 1993)**

451

Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find

(a) the energy of the photons causing the photoelectric emission.

(b) the quantum numbers of the two levels involved in the emission of these photons,

(c) the change in the angular momentum of the electron in the hydrogen atom in the above transition, and

(d) the recoil speed of the emitting atom assuming it to be at rest before the transition.

(Ionization potential of hydrogen is 13.6 eV) **(IIT 1992)**

452

Electrons in hydrogen-like atoms (Z = 3) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 volts. Calculate the work function of the metal, and the stopping potential for the photoelectrons ejected by the longer wavelength.

(Rydberg constant = $1.094 \times 10^{7}\ m^{-1}$) **(IIT 1990)**

453

A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more and some have less than 2.7 eV.

(i) Find the principal quantum number of the initially excited level B.

(ii) Find the ionization energy for the gas atoms.

(iii) Find the maximum and the minimum energies of the emitted photons. **(IIT 1989)**

454

A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge $+3e$. (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system,

(i) derive an expression for the radius of the nth Bohr orbit.

(ii) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

(iii) find the wavelength of the radiation emitted when the mumeson jumps from the third orbit to the first orbit.

(Rydberg's constant = $1.097 \times 10^7\ m^{-1}$). **(IIT 1988)**

455

A doubly ionised Lithium atom is hydrogen-like with atomic number 3.

(i) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. (Ionisation energy cf the hydrogen atom equals 13.6 eV.)

(ii) How many spectral lines are observed in the emission spectrum of the above excited system ?

(IIT 1985)

456

The ionization energy of a hydrogen like Bohr atom is 4 rydbergs.

(i) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state?

(ii) What is the radius of the first orbit for this atom ?

(IIT 1984)

457

Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of Planck's constant. **(IIT 1983)**

458

Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum ? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV. **(IIT 1982)**

459

A single electron orbits around a stationary nucleus of charge + Ze, where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit.

Find :

(i) The value of Z.

(ii) The energy required to excite the electron from the third to the fourth Bohr orbit.

(iii) The wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.

(iv) The kinetic energy, potential energy and the angular momentum of the electron in the first Bohr orbit.

(v) The radius of the first Bohr orbit.

(The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} metre, velcoiy of light = 3×10^{8} m/sec. Planck's constant = 6.6×10^{-34} joule-sec).

(IIT 1981)

RADIO ACTIVITY

460

The age of a rock containing lead and uranium is equal to 1.5×10^9 yrs. The uranium is decaying into lead with half life equal to 4.5×10^9 yrs. Find the ratio of lead to uranium present in the rock. Assuming initially no lead was present in the rock

(Given $2^{1/3} = 1.259$) **(IIT 2004)**

461

A radioactive element decays by β emission. A detector records n beta particles in 2 seconds and in next 2 seconds it records 0.75 n beta particles. Find mean life correct to nearest whole number.

Given: $ln2 = 0.6931$, $ln3 = 1.0986$ **(IIT 2003)**

462

A radioactive nucleus X decays tc a nucleus Y with a decay constant $\lambda_X = 0.1 \text{ sec}^{-1}$. Y further decays to a stable nucleus Z with a decay constant $\lambda_Y = 1/30 \text{ sec}^{-1}$. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$. Set up the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function of time is given by

$$N_Y(t) = \left(\frac{N_0\lambda_x}{\lambda_x - \lambda_y}\right)\left(e^{-\lambda_y t} - e^{-\lambda_x t}\right)$$

Find the time at which N_Y is maximum and determine the population of X and Z at that instant. **(IIT 2001)**

463

Nuclei of a radicactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element.

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha = 2 N_0 \lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of N as $t \to \infty$.

(IIT 1998)

464

In an ore containing Uranium, the ratio of U^{238} to Pb^{206} nuclei is 3. Calculate the age of the ore, assuming that all the lead present in the ore is the final stable product of U^{238}. Take the half-life of U^{238} to be 4.5×10^9 years. **(IIT 1997 May)**

465

At a given instant there are 25% undecayed radio-active nuclei in a sample. After 20 seconds the number of undecayed nuclei reduces to 12.5%. Calculate (i) mean-life of the nuclei, and the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number.

(IIT 1996)

466

A small quantity of solution containing ^{24}Na radio-nuclide (half life 15 hours) of activity 1.0 micro-curie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hours shows an activity of 296 disintegrations per minute. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.

(1 Curie = 3.7×10^{10} disintegrations per second) **(IIT 1994)**

467

There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half life of neutrons is 700 seconds, what fraction of neutrons will decay before they travel a distance of 10 m ?

(IIT 1986)

NUCLEAR PHYSICS

468

In a nuclear reactcr ^{235}U undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required.

(IIT 2001)

469

A nucleus at rest undergoes a decay emitting an α particle of de-Broglie wavelength $\lambda = 5.76 \times 10^{-15}$m. If the mass of the daughter nucleus is 223.610 a.m.u. and that of the α particle is 4.002 a.m.u., determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in a.m.u.

(1 a.m.u. = 931.470 MeV/c^2) **(IIT 2001)**

470

The element Curium $^{248}_{96}\text{cm}$ had a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in α decay are as follows:

$$^{248}_{96}\text{Cm} = 248.072220 \text{ u},$$

$$^{244}_{94}\text{P}_\text{u} = 244.064100 \text{ u and } ^4_2\text{He} = 4.002603 \text{ u}.$$

Calculate the power output from a sample of 10^{20} cm atoms. (1u = 931 MeV/c^2). **(IIT 1997)**

471

A nucleus X, initially at rest, undergoes alpha-decay according to the equation,

$$^A_{92}\text{X} \rightarrow {}^{228}_{Z}\text{Y} + \text{a}$$

(a) Find the values of A and Z in the above process.

(b) The alpha particie produced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 Testla. Find the energy (in Mev) released during the process and the binding energy of the parent nucleus X.

Given that

$$m(Y) = 228.03 \text{ u}; \; m\left({}^1_1\text{H}\right) = 1.008 \text{ u};$$

$$m\left({}^1_0\text{n}\right) = 1.009 \text{ u}; m\left({}^4_2\text{He}\right) = 4.003 \text{ u};$$

(IIT 1991)

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It is proposed to use the nuclear fusion reaction

$$ {}_1^2H + {}_1^2H \rightarrow {}_2^4He $$

In a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day. (The masses of ${}_1^2H$ and ${}_2^4He$ are 2.0141 atomic mass units and 4.0026 atomic mass units respectively). **(IIT 1990)**

■ ■

ANSWERS

ANSWERS

1. (a) 45°
   (b) 2 m/s.

2. initial velocity = 6 m/s
   acceleration = 4 m/s^2 downward along the plane

4. (i) $\left(\frac{\alpha\beta}{\alpha+\beta}\right)t$,

   (ii) $\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$

6. 45 km/h.

7. 36 km.

8. 48 km/h.

9. (i) The block that falls freely will strike the ground earlier.
   (ii) Same speed.

10. Same time.

11. (i) Velocity = + X-axis, acceleration = 0
    (ii) Velocity = 0, acceleration = 0
    (iii) Velocity = – X-axis, acceleration = 0

12. 10 m.

13. 9 metres.

14. $u = 7.2$ m/s, $t = 1$ sec.

15. (a) $\frac{u^2 \sin 2\alpha}{g \cos\theta}$

    (b) $\frac{u \cos(\alpha+\theta)}{\cos\theta}$

16. (i) $\Delta t = 1$ second

    (ii) $\left(5\sqrt{3}\text{m}, 5\text{m}\right)$

17. $\frac{h}{H} = \frac{1}{2}$

18. *(i)* 747.1 metres

(ii) 85.55 m/s

19. *(a)* – 1 m/s

(b) 1.48 sec.

20. *(ii)* $a_T = \left(\frac{5\sqrt{3}}{8}\right) g$

$N = \left(\frac{3}{8}\right) mg$

21. *(i)* 20 N

(ii) 50 N

22. Velocity = 6.54 m/s

Time taken by m is 2.8 second.

Time taken by 2 m is $\frac{2}{3}$ second.

23. $F\left(1 - \frac{l}{L}\right)$.

24. $f = \left(\frac{m_1 \sin \alpha + m_2 \sin \beta}{m_1 \cos \alpha + m_2 \cos \beta}\right) g$

$T = \frac{m_1 m_2 g \sin (\alpha - \beta)}{(m_1 \cos \alpha + m_2 \cos \beta)}$

25. 3 K

26. *(a)* *(i)* 17400 N *(ii)* 14700 N *(iii)* 1200 N

(b) 36 m

(c) Average velocity = 3 m/s

Average acceleration = 0

27. 25 cm/s^2 Eastward.

28. 110 cm from the pulley.

230 cm/s towards right.

29. *(i)* remains same

(ii) increase

30. *(a)* $t = 2$ sec

(b) $S_B = 7\sqrt{2}$ m

$S_A = 8\sqrt{2}$ m

31. *(b)* $f_1 = 30$ N

$f_2 = 15$ N

F $= 60$ N

Tension = 18 N

(Acceleration of M) = (Acceleration of m_1) = 0.6 m/s^2

(m_1 is at rest w.r.t. M).

32. *(i)* zero

(ii) $\left(\dfrac{2\sqrt{2}}{3}\right)$ mg

(iii) $\dfrac{mg}{3\sqrt{2}}$

33. $F_{MIN} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

$\theta = \tan^{-1}\mu$ with horizontal

34. $M_B = 10$ kg, $\mu = 0.098$ J

35. *(i)* $M_1 = 4.2$ kg

(ii) 9.8 N

36. $V_C = V_F = \sqrt{2g\,(y - \mu x)}$

37. *(i)* 1.31 m/s^2

(ii) 5.2 N

38. 78.4 N

39. 4.43×10^4 N

40. *(i)* $9.8\left(\dfrac{3}{\sqrt{2}} - 1\right)$ N

(ii) $9.8\left(\dfrac{3}{\sqrt{2}} + 1\right)$ N

(iii) $\left(\dfrac{3-\sqrt{2}}{3+\sqrt{2}}\right)$

41. *(i)* 36 N

(ii) 11.67 rad/s

(iii) m_1 at a distance of 0.1 m from O and m_2 at a distance of 0.2 m from O.

42. *(i)* $h = R - \frac{g}{\omega^2}$

$$\omega_{min} = \sqrt{\frac{g}{R}} = 9.9 \text{ rad/s}$$

(ii) $|\Delta g|_{min} = 9.8 \times 10^{-3} \text{ m/s}^2$

43. $\frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$

44. To apply brakes.

45. *(i)* $\theta = \cos^{-1}\left(\frac{49}{80}\right) = 52.2°$

(ii) 1.6 N

(iii) 3.16 m/s

46. *(i)* 2.2 second

(ii) 2.123 N

47. $N_A = \begin{cases} mg\,(3\cos\theta - 2) & \theta \le \cos^{-1}\frac{2}{3} \\ 0 & \theta \ge \cos^{-1}\frac{2}{3} \end{cases}$

$N_B = \begin{cases} 0 & \theta \le \cos^{-1}\frac{2}{3} \\ mg\,(2 - 3\cos\theta) & \theta \ge \cos^{-1}\frac{2}{3} \end{cases}$

48. $u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$

49. 3.36 m/s.

50. 4.24 m

51. $\frac{A\rho g}{4}(h_1 - h_2)^2$.

52. *(i)* Zero

(ii) 18 m, where m is the mass of the body.

53. *(i)* 14 m/s

(ii) 10 m.

54. $40\,(3\cos\theta - 2\cos\theta_0)$ kg wt, 60°

55. $-\dfrac{k}{2r}$

56. *(i)* First

(ii) Second

57. 80 cm.

58. 313.6 Watt.

59. *(i)* K.E. = 0.098 J

(ii) T = 0.294 N

60. *(i)* $\dfrac{x}{2}$

(ii) $\dfrac{Wx}{2}$

(iii) $F = \dfrac{W}{2}$, work done $= \dfrac{Wx}{2}$

(iv) 2

61. *(a)* $x_2 = v_0 t + \dfrac{m_1}{m_2} A\,(1 - \cos\omega t)$

(b) $l_0 = \left(1 + \dfrac{m_1}{m_2}\right) A$

62. $m\left[\left(v_2 \cos\left(\dfrac{v_2 t}{R}\right) - v_1\right)\hat{j} - \left(v_2 \sin\left(\dfrac{v_2 t}{R}\right)\right)\hat{i}\right]$

63. $t_0 = 12$ seconds, $\dfrac{100\sqrt{3}}{11}$ m/s.

64. e = 0.84, $M = \dfrac{26}{\sqrt{3}}$ kg

65. *(a)* 2.5 m/s

(b) L = 0.32 m

66. (L + 2R, O).

67. *(i)* $d = 10\sqrt{3}$ m

(ii) $\dfrac{20}{\sqrt{3}}$ m from B.

68. 10^5 m.

69. *(i)* Vertical straight line.

(ii) $$\frac{x^2}{\left(\frac{L}{2}-r\right)^2}+\frac{y^2}{r^2}=1$$

(y-axis is a vertical line through C.M.)
The path of the point is a part of an ellipse.

70. *(a)* $5\sqrt{\frac{3}{2}\mu g d}$

(b) $6d\sqrt{3\mu}$

71. 44.18 m

72. *(i)* $\theta = \cos^{-1}\left(\frac{4}{5}\right) = 37°$

(ii) (120 m, 45 m)

73. $n = 4$.

74. 6.53 seconds

75. *(i)* $v = \frac{v_0}{3}$

(ii) $k = \frac{2mv_0^2}{3x_0^2}$

76. Distance $= \frac{m(R-r)}{(m+M)}$

Speed $= m\sqrt{\frac{2g(R-r)}{M(m+M)}}$

77. The particle Q hits the ground at the mid point of AB.

Time $= \frac{5}{\sqrt{2}}$ second.

78. 9 cm left from the centre of plate.

79. 25%.

80. *(i)* $\sqrt{(mV)^2+(Mv)^2}$, $\theta = \tan^{-1}\left(\frac{Mv}{mV}\right)$ with x-axis.

(ii) $\frac{Mm}{M+m}\left(\frac{V^2+v^2}{Mv^2+mV^2}\right)$

81. Less.

82. 0.94

83. *(i)* 135.3 N

(ii) Yes.

84. $3\sqrt{3}$ m/s, No.

85. 480 N.

86. *(i)* $t = 1$ second, 4.9 M below the top of the cliff.

(ii) 77.55 metres above the top of the cliff.

87. *(i)* 15 m/s downward

(ii) 1080 kgm/s, 1080 kgm/s

88. 40 cm/s.

89. *(a)* $\sqrt{3}\ m\omega^2 l$

(b) $F_y = \sqrt{3}\ m\omega^2 l$

$$F_x = -\frac{F}{4}$$

90. *(a)* 0.1 M

(b) 1 rad/s

(c) Infinite

91. *(a)* $\frac{m}{M} = \frac{1}{4}$

(b) $AP = \frac{2}{3}L$

(c) $V_P = \frac{v_0}{2\sqrt{2}}$

92. *(a)* $a_{\text{plank}} = \frac{8F}{3m_1 + 8m_2}$

$$a_{\text{cylinder}} = \frac{4F}{3m_1 + 8m_2}$$

(b) $f_{(\text{Plank \& Cylinder})} = \frac{3m_1 F}{3m_1 + 8m_2}$

$$f_{(\text{Cylinder \& ground})} = \frac{m_1 F}{3m_1 + 8m_2}$$

93. $\sqrt{5gR}$

94. *(a)* $\vec{F} = \frac{2mv}{\Delta t}\left(\hat{i} - \frac{\hat{k}}{\sqrt{3}}\right)$; $\vec{N} = \left(mg + \frac{2mv}{\sqrt{3}\,\Delta t}\right)\hat{k}$

(b) $T = \frac{4mvh}{\sqrt{3}\,\Delta t}$

95. *(i)* 6 N

(ii) $\vec{\tau}_1 = 0.6\left(\hat{k} - \hat{j}\right)$ Nm

$\vec{\tau}_2 = 0.6\left(-\hat{k} - \hat{j}\right)$ Nm

$\left|\tau_1\right| = \left|\vec{\tau}_2\right| = (0.6)\sqrt{2}$ Nm

96. *(i)* $\frac{2}{3}v_0$

(ii) $t_0 = \frac{v_0}{3\mu g}$

$W = -mv_0\mu gt + \frac{3}{2}m(\mu gt)^2$ for $t > t_0$, $W = -\frac{1}{6}mv_0^2$

97. *(a)* $\theta_c = \cos^{-1}\left(\frac{4}{7}\right)$

(b) $v = \sqrt{\frac{4}{7}gR}$

(c) $\frac{K_T}{K_R} = 6$

98. *(i)* T = 4.9 N

(ii) S = 1.224 m

99. 6.3 m/s

100. *(a)* $\frac{12V}{7L}$

(b) $\frac{7}{12}\sqrt{2gL} = 3.5$ m/s

101. $\sqrt{\frac{14}{3}gR}$

102. 2.72 J.

103. 2 m, sphere continues to rotate.

104. $\dfrac{mv_0^3}{2\sqrt{2}\,g}$

105. 20.

106. $T_{AP} = \sqrt{113}$ kg wt

$T_{BQ} = 7\sqrt{2}$ kg wt

$\theta = \tan^{-1}\left(\dfrac{8}{7}\right)$

107. 34 N.

108. *(i)* 48.3 kg

(ii) 2 m^2

109. 100 R

110. $v = \dfrac{3}{2}\sqrt{\dfrac{5\,GM}{a}}$

111. *(i)* $h = R = 6400$ km

(ii) $v = \sqrt{gR} = 7.92$ km/s

112. $v = \sqrt{\dfrac{Gm}{a}}$

$T = 2\pi\sqrt{\dfrac{a^3}{3Gm}}$

113. *(i)* $\pi \times 10^4$ km/hr

(ii) $\dfrac{\pi}{3}$ radians/hr

114. 35940 km, above the earth's surface.

115. $2\sqrt{2}$ second

116. 4.89%

117. 0.00457 K.

118. *(i)* 133.3 cm from steel wire

(ii) 100 cm from steel wire

119. 3.75 m/s

120. *(a)* zero

(b) 0.25 cm

(c) $\frac{g}{6}$

121. $(\pi R^2 L)(\sqrt{\rho\sigma} - \rho)$

122. *(a)* $\left(\frac{d_L}{d_L - d}\right) t_1$

(b) Periodic but not SHM.

(c) Ball will move with constant velocity within the liquid.

123. $\theta = 45°$

124. *(i)* 840 gm

(ii) $P_{atm} + 10^3$ N/m^2

125. In the beginning $(M + \rho_W V)$

where ρ_W = density of water

Then the reading increases and at the end it becomes $(M + m)$.

126. Side length = 10 cm.

127. $h = R$.

128. No change.

129. *(i)* $\frac{7}{3}$ kg

(ii) $\frac{17}{3}$ kgwt, vertically downward.

130. 0.5 metre.

131. 3.33 litres.

132. Liquid level will rise.

133. *(i)* 2.35 cm

(ii) 2.54 cm

134. Decrease.

135. 9.61 cm

136. $1.96\ m^2$

137. $2\ m$.

138. $W_{(gravity)} = -2.94 \times 10^4\ J/m^3$

$W_{(pressure)} = 2.9025 \times 10^4\ J/m^3$

139. *(i)* $\dfrac{g}{50}$

(ii) $\sqrt{\dfrac{2gm_0}{A\rho}}$

140. *(a)* *(i)* $\dfrac{5}{4}d$

(ii) $P_0 + \left(\dfrac{3H}{2} + \dfrac{L}{4}\right)dg$

(b) *(i)* $v = \sqrt{\dfrac{g}{2}(3H - 4h)}$

(ii) $x = \sqrt{h(3H - 4h)}$

(iii) $h_m = \dfrac{3}{8}H$

$x_m = \dfrac{3}{4}H$

141. $\dfrac{dQ}{dt} \propto r^5$

142. $1.41 \times 10^{-3}\ NS/m^2$

143. $\dfrac{\lambda a g}{2y}$.

144. $\dfrac{4T}{\rho v^2}$

145. $\dfrac{1}{2\pi}\sqrt{\dfrac{3g}{2R}}$

146. $\sqrt{\dfrac{3(d_2 - d_1)g}{2d_1 L}}$

147. *(i)* $\frac{1}{\pi}$ Hz

(ii) $\frac{\pi}{50}$ m/s

(iii) $4\pi^2 \times 10^{-5}$ J

148. *(a)* $\theta = \tan^{-1}\left(\frac{1}{5}\right)$

(b) $T = 2\pi\sqrt{\frac{\pi R}{2g\cos\theta}} \simeq 8\sqrt{\frac{R}{g}}$

149. $\left(1+\frac{7\pi}{12}\right)$ sec.

150. $\frac{1}{2\pi}\sqrt{\frac{YA}{ml}}$

151. *(i)* 4.8×10^{-4} J

(ii) 3.75×10^{-4} J

152. $\frac{A^2\pi^2 T}{4l}$

153. 336 m/s.

154. *(a)* $\frac{M_A}{M_B} = \frac{400}{189}$

(b) $\frac{n_A}{n_B} = \frac{3}{4}$

155. Heights of water level = 3.2 m, 2.4 m, 1.6 m, 0.8 m and 0; $\frac{dH}{dt} = \frac{-\sqrt{H}}{179}$

Time interval= 42.9 sec.

156. *(a)* 0.14 sec.

(b) $A_r = 2.1$ cm,

$A_t = 1.4$ cm

157. *(a)* $\frac{15}{16}$ m

(b) $\pm\frac{\Delta P_0}{\sqrt{2}}$

(c) $P_{min} = P_{max} = P_0$

(d) $P_{min} = P_0 - \Delta P_0$

$P_{max} = P_0 + \Delta P_0$

158. $L_{closed} = 0.75$ m
$L_{open} = 1.0067$ m or 0.9934 m

159. Equation of motion, $y = (2 \times 10^{-6}\text{ m}) \sin (0.1\pi) \cos (25000\pi t)$

Constituent waves, $y_1 = (1 \times 10^{-6}\text{ m}) \sin (5\pi x - 25000\pi t)$

$y_2 = (1 \times 10^{-6}\text{ m}) \sin (5\pi x + 25000\pi t)$

where x is in metre and t is in second.

160. *(i)* $2\pi \times 10^{-3}$ s.

(ii) $\frac{\pi}{2} \times 10^{-3}$ s.

161. *(a)* $\lambda = \frac{2\pi}{a}, \nu = \frac{b}{2\pi}$

(b) $y = -0.8\text{ A} \cos (ax - bt)$

(c) Maximum speed = 1.8Ab

Minimum speed = 0

(d) $y = y_{travelling} + y_{stationary}$

$= 0.2\text{A} \cos (ax + bt) - 1.6\text{ A} \sin ax \sin bt$

Antinodes of stationary wave,

$x = \left(n + \frac{1}{2}\right)\frac{\pi}{a}$ with $n = 0, 1, 2, \ldots\ldots$

The direction of propagation of travelling wave is negative X-axis.

162. *(i)* z_1 and z_2. Resultant intensity will be zero at $x = \left(m + \frac{1}{2}\right)\frac{\pi}{k}$

where $m = 0, 1, 2, \ldots\ldots$

(ii) z_1 and z_3. Resultant intensity will be zero at $(x - y) = (2n + 1)\frac{\pi}{k}$

where $n = 0, 1, 2, \ldots\ldots$

163. *(i)* 3.46 cm

(ii) $n = 0$, 15 cm, 30 cm, 45 cm, 60 cm

(iii) 0

(iv) $y_1 = 2\sin\left(96\pi t + \frac{\pi x}{15}\right)$, $y_2 = -2\sin\left(96\pi t - \frac{\pi x}{15}\right)$

164. 0.12 m

165. 11 Hz.

166. $\left(\frac{100}{133}\right)$ m/s

167. 27 N.

168. Diameter = 3.33 cm, $\nu = 163.3$ Hz.

169. *(i)* 95 Hz

(ii) 1.27×10^4 kg/m^3

170. 1650 Hz.

171. 50 Hz.

172. 230 m/s.

173. $\frac{\text{(Number of loops on wire of radius } r)}{\text{(Number of loops on wire of radius } 2r)} = \frac{1}{2}$

174. 45 cm.

175. $25\sqrt{2}$ Hz.

176. 260 Hz.

177. *(i)* 270 m

(ii) 360 m/s

178. 683.4 m/s

179. 7.27 gm/cc

180. *(a)* 100696 Hz

(b) 103039 Hz.

181. $f\left[\frac{2(v+v_m)v_b}{v^2 - v_b^2}\right]$

182. $\nu_{max} = 484$ Hz, $\nu_{min} = 403.3$ Hz.

183. $V_{max} = 442$ Hz, $V_{min} = 255$ Hz.

184. *(i)* 599 Hz

(ii) $\left(\frac{29}{31}\right)$ km, 620 Hz.

185. 1.5 m/s

186. 8 beats/sec.

187. $\gamma_l = 2\alpha_s$

188. Coefficient of volume expansion, $\gamma = 2 \times 10^{-4}/°C$

189. *(i)* 400 kg

(ii) 2 J

190. $Y = 1.1 \times 10^{11}$ N/m^2

$\alpha = 2 \times 10^{-5}/°C$

191. 70 m/s

192. $\left[\frac{(w_2 - w_1) + \beta(w_0 - w_1)(t_2 - t_1)}{(w_0 - w_2)(t_2 - t_1)}\right]$

193. Loses 10.37 seconds per day.

194. 75.04 cm.

195. Length of the copper rod = 9.16 cm
Length of the steel rod = 14.16 cm

196. *(i)* $F = \frac{AT(l_1\alpha_1 + l_2\alpha_2)}{\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right)}$

(ii) $l_1 + l_1\alpha_1 T - \frac{Fl_1}{Y_1 A}$, $l_2 + l_2\alpha_2 T - \frac{Fl_2}{Y_2 A}$

197. $2.31 \times 10^{-5}/°C$.

198. Yes, if $\frac{L_2}{L_1} = \frac{\alpha_1}{\alpha_2}$

199. 150 cc

200. $3 \times 10^{-4}/°C$.

201. 25.2 N.

202. 0.495 kg

203. 12 gm

204. 0°C

205. 20.25°C

206. $1.8 \times 10^{-4}/°C$

207. 100°C, (572.2 gm water + 77.5 gm steam)

208. 178.5°C

209. 25.5°C

210. 1 kg/hour.

211. $\dfrac{K}{\left[4\sigma\varepsilon T_s^3 L + K\right]}$

212. *(a)* 595 J/m^2-sec *(b)* 163°C

213. $\left(300 + (12.5)e^{-2KAt_1/CL}\right)K$

214. Rate of heat flow = 41.6 W
Temperature of Glass I and air =26.48°C
Temperature Glass II and air = 0.52°C

215. 166.4 seconds

216. 9000 W.

217. $T_B = 30°C$, and $T_C = T_D = 20°C$

218. 15.24°C.

219. 93.3°C, 0.257 cal/s

220. Ratio of radii, $\dfrac{r_A}{r_B} = \sqrt{\dfrac{K_B}{K_A}}$

221. 2°C

222. 0.3 cal/m-sec-°C

223. 76°C

224. *(a)* 160 K

(b) 3.312×10^{-21} J

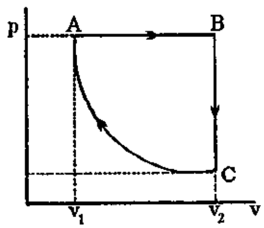
(c) 0.3012 gm

225. neon = 4.08 gm, argon = 23.92 gm.

226. 75.4 cm of mercury.

227. 83.75 cm of mercury.

228.



229. 817 mm of Hg

230. 3 cm.

231. 1.1588 g.

232. Length = 36.5 cm, pressure = 85.3 cm of Hg.

233. 8.02×10^{15}

234. $\dfrac{d_{TOP}}{d_{BOTTOM}} = \dfrac{75}{76}$

235. 48 cm.

236. $400\left(\dfrac{4}{3}\right)^{2/5}$ K

237. $\dfrac{mv_0^2}{3R}$

238. *(a)* 327°C

(b) $Q_{AB} = 1500$ R

$Q_{BC} = 831.8$ R

$Q_{CD} = -900$ R

$Q_{DA} = -831.8$ R

(c) 600 R

239. *(a)* + 1200 R

(b) $\Delta Q_{AB} = -2100\text{ R}$

$\Delta Q_{BC} = 1500\text{ R}$

$\Delta Q_{CA} = 831.77\text{ R}$

240. *(b)* $\Delta W = \frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$

$$\Delta U = Q - \frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

$$T = \frac{Q}{3R} + \frac{P_1V_1}{2R}\left(\frac{V_1}{V_2}\right)^{2/3}$$

241. *(a)* P_0V_0

(b) $Q_{CA}(\text{Rejected}) = \frac{5}{2}P_0V_0$

$Q_{AB}(\text{absorbed}) = 3P_0V_0$

(c) $Q_{BC}(\text{absorbed}) = \frac{1}{2}P_0V_0$

(d) $T_{max} = \frac{25P_0V_0}{8R}$

242. $T_B = 909\text{ K}$, $T_D = 791.4\text{ K}$, $\eta = 61.4\%$

243. *(i)* $T_A = 120.28\text{ K}$, $T_B = 240.56\text{ K}$, $T_C = 481.12\text{ K}$, $T_D = 240.56\text{ K}$

(ii) No.

(iii) $Q_{ABC} = 3.25 \times 10^6\text{ J}$

$Q_{ADC} = 2.75 \times 10^6\text{ J}$

244. *(i)* 189 K

(ii) – 2767 J

(iii) 2767 J

245. *(a)* 2 moles

(b) 401 m/s

(c) 0.167%

(d) $\Delta\kappa = \frac{V}{40T}$

246. *(i)* $W_4 = 765$ J

(ii) 10.82%

247. *(i)* 1870 J

(ii) 5300 J

(iii) 500 K

248. *(a)* 1153 J

(b) 1153 J

(c) zero

249. *(b)* $W = (0.58)\,RT_A$, $\Delta Q = (0.58)\,RT_A$

250. *(i)* 5

(ii) $\frac{5}{4}PV$

251. *(ii)* $V = 0.113$ m^3, $P = 4.41 \times 10^4$ N/m^2

(iii) 12450 J

252. 800 K, 720 J

253. $T = 675$ K, $P = 3.6 \times 10^6$ N/m^2

254. *(i)* $\left(\frac{207}{16}\right)T_0$ (Left chamber), $\left(\frac{9}{4}\right)T_0$ (Right chamber)

(ii) $15.58\,T_0$

255. *(i)* 1.94×10^{27}

(ii) $v_0 = 35.6$ m/s

256. 972 J.

257. $\frac{A}{2\pi}\sqrt{\frac{\gamma P_0}{MV_0}}$

258. $\frac{q(\sigma_1 - \sigma_2)a}{2\sqrt{2}\,\varepsilon_0}$

259. *(a)* $\frac{qp}{4\pi \epsilon_0 d^2}$

(b) $\frac{qp}{2\pi \epsilon_0 d^3}$

260. $\dfrac{5.824\, q^2}{(4\pi \epsilon_0) a}$

261. 5.86 m/s.

262. $v_0 = 3$ m/s, KE at origin $= 2.5 \times 10^{-4}$ J.

263. *(a)* $\dfrac{4}{3} a$,

(b) Height of equilibrium position $= \dfrac{a}{\sqrt{3}}$.

264. *(a)* $\dfrac{Q^2 R}{8\pi \epsilon_0 r^2}\left[1 - \left(\dfrac{R}{R + r}\right)^n\right]^2$

(b) $\dfrac{Q^2 R}{8\pi \epsilon_0 r^2}$

265. $\left(\dfrac{5}{6}\right)\sigma$

266. $\sqrt{\dfrac{\lambda q}{2 \epsilon_0 m}}$

267. *(a)* $\dfrac{3Q^2}{20\pi \epsilon_0 R}$

(b) 15×10^{31} J

(c) $\dfrac{Q^2}{8\pi \epsilon_0 R}$

268. *(a)* Equation of circle, $x^2 + y^2 - 10ax + 9a^2 = 0$
radius $= 4a$, centre at $(5a, 0)$

(b) $V = \dfrac{Q}{4\pi \epsilon_0}\left[\dfrac{1}{|x - 3a|} - \dfrac{2}{|x + 3a|}\right]$

(c) $\sqrt{\dfrac{qQ}{8\pi \epsilon_0 ma}}$

269. *(i)* $V_A = \frac{\sigma}{\epsilon_0}(a - b + c)$

$V_B = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b} - b + c\right)$

$V_C = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$

(ii) $c = a + b$

270. $t = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

271. 3.16×10^{-9} C

272. $2q$ and $8q$ at ends. q at a distance of 3 cm from $2q$ and 6 cm from $8q$.

E = 0.

273. $OD = 6\sqrt{2}\ m$

274. $\left(\frac{\pi}{5}\right)$ seconds.

275. 8.9×10^{-4} N, $\tan^{-1}\left(\frac{1}{2}\right)$

276. *(i)* 60°

(ii) $mg + \frac{q_1 q_2}{4\pi \epsilon_0 l^2}$

(iii) $mg\sqrt{3}$, mg when chord is cut $\frac{q_1 q_2}{4\pi \epsilon_0 l^2} = mg$

277. *(i)* move towards

(ii) $-\frac{q}{\sqrt{3}}$, zero.

278. $2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$

279. $V_0 = 0$, $E_0 = \frac{\sqrt{2}\,q}{4\pi \epsilon_0 a^2}$, $W_{0E} = 0$, $W_{0F} = \frac{(1-\sqrt{5})qe}{\sqrt{5}\,\pi \epsilon_0 a}$

280. Horizontal component = 10^6 m/s
Vertical component = 1.88×10^6 m/s (upward)

282. Smaller conductor is placed inside the bigger conductor.

283. K = 2.

284. 10^4 m/s.

285. $q = \frac{-Q}{4}$ on the middle point of the line joining two charges.

286. The neutral point is at a distance of 8 cm from + 4Q and 4 cm from + Q.

287. 4.73 cm, No

288. zero.

289. 2×10^{-12} coulomb.

290. *(i)* $V_P = 0, V_Q = 400$ volt,

(ii) $E_P = 4.242 \times 10^3$ N/C along –ve x-axis
$E_Q = 5.33 \times 10^3$ N/C along +ve x-axis

(iii) 400 J/C

(iv) No, independent of path

291. 3062.5 volt, 1.96 m/s^2

292. Same charges.

293. *(i)* $V = \frac{q}{2\pi \epsilon_0 a}, E = \frac{q}{3\pi \epsilon_0 a^2}$

(ii) $V = \frac{q}{6\pi \epsilon_0 a}, E = \frac{q}{5\pi \epsilon_0 a^2}$

294. *(i)* 1.5×10^{-2} volt

(ii) 2.382×10^{-2} volt

295. 5.88×10^5 N/C

296. *(a)* *(i)* 6 cm from + 100 esu and 24 cm from – 400 esu.

(ii) 10 cm from + 100 esu and 40 cm from – 400 esu.

(b) At many points in the space satisfying the condition $\frac{r_1}{r_2} = \frac{1}{4}$, where r_1 is distance from + 100 esu and r_2 is distance from – 400 esu.

(c) 30 cm from + 100 esu and 60 cm from – 400 esu.

297. 8.79×10^{-6} J.

298. *(i)* $Q_A = 90\ \mu C, Q_B = 150\ \mu C, Q_C = 210\ \mu C$

(ii) $U_{initial} = 4.74 \times 10^{-2}$ J

$U_{final} = 1.8 \times 10^{-2}$ J

299. $\frac{\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_2 - K_2}\right) ln \frac{K_2}{K_1}$

300. *(i)* 2×10^{-9} F, 1.21×10^{-5} J

(ii) 4.84×10^{-5} J

(iii) 1.1×10^{-5} J

301. $\frac{u_{Initial}}{u_{Final}} = \frac{3}{5}$

302. $\frac{32}{23}\ \mu F$

303. $\frac{C_{INITIAL}}{C_{FINAL}} = \frac{1}{2}$

304. $R = \frac{4}{3}(r_1 - r_2)$.

304. *(a)* No. *(c)* $8\,\Omega$.

306. *(a)* $\frac{CV}{2}\left(1 - e^{-2t/3RC}\right)$

(b) $I = \frac{V}{6R}\left(3 - e^{-2t/3RC}\right)$

At $t \to \infty, I = \frac{V}{2R}$.

307. $V = \left(\frac{V_1r_2 - V_2r_1}{r_1 + r_2}\right), r = \frac{r_1r_2}{r_1 + r_2}$

308. $0.2\ \mu A$.

309. $\frac{20}{3}$ V.

310. 4.425×10^{-9} A.

311. 1.5 A, 1.44×10^{-5} J.

312. *(i)* $2\,\Omega$ *(ii)* 1.5 A

313. 8×10^{-4} J.

314. *(i)* $V_B - V_D = \dfrac{2}{13}$ V

(ii) P.D. across G is $\dfrac{21}{13}$ V,

P.D. across H is $\dfrac{19}{13}$ V.

315. 0.9 A.

316. Reading of ammeter = 4.96×10^{-3} A

Reading of voltmeter = 1.95 V

317. *(i)* 2 V

(ii) Current through E_1 is 1 A

Current through E_2 is 2 A

Current through E_3 is 1 A

Current through R is 2 A

318. *(ii)* 22.5 V.

319. P.D. = 2.5 volts.

320. *(i)* 9 min 42 second.

(ii) Same.

321. $V_A - V_B = 25$ V, $V_B - V_C = 75$ V.

322. 2 Ω.

323. 5 Ω.

324. 0.2%.

325. 31.25 W.

326. 40 V.

327. 790 ohms

328. 5 Ω.

329. $\dfrac{10}{3}$ Ω.

330. 1 cell.

331. 0.095 sec.

332. *(i)* 200 Ω.

(ii) 0.23 volts.

333. *(i)* 14 minutes

(ii) $\frac{24}{7}$ minutes.

334. 1.40 amp, 1.66 amp, 1.94 amp.

335. *(i)* 7.28 calories

(ii) 5.86 volts.

336. Power in second coil is more.

337. Larger.

338. Current through 4.5 Ω, 3 Ω and 6 Ω are respectively 0.5 A, $\frac{1}{3}$A, $\frac{1}{6}$A. Potential difference across E_1 and E_2 are 4.25 V and 7.5 V respectively.

339. $\frac{1}{3}$ ohm.

340. 70%.

341. 90 watts.

342. *(i)* $\left(\frac{1}{100}\right)$ ohm.

(ii) Yes.

(iii) E = 2 volt, $r = 0.49\ \Omega$.

343. $\frac{r_p}{r_\alpha} = \frac{1}{\sqrt{2}}$.

344. $\frac{DT_0}{QR^2B}$.

345. *(a)* From P to Q

(b) $IbB_0\left(3\hat{k} - 4\hat{i}\right)$

(c) $\frac{mg}{6bB_0}$

346. *(a)* 6.54×10^{-5} T

(b) *(i)* Zero

(ii) Zero (on AC)

(iii) 8.11×10^{-6} N (on CD)

347. *(a)* $\left(\dfrac{\mu_0 v_0 q I}{4R}\right)(-\hat{k})$

(b) $\vec{F}_1 = (2IBR)\hat{i}$

$\vec{F}_2 = (2IBR)\hat{i}$

Force on the loop $\vec{F} = (4IBR)\hat{i}$

348. *(a)* $\dfrac{mv_0}{2qB_0}$

(b) $-v_0\,\hat{i}$

349. $\vec{v} = (\cos\omega t)\,\vec{v}_0 - (\sin\omega t)\dfrac{\vec{E}\times\vec{v}_0}{E} + \left(\dfrac{qt}{m}\right)\vec{E}$

where, $\omega = \dfrac{qB}{m}$

350. *(a)* $\vec{\tau} = \dfrac{I_0 L^2 B}{\sqrt{2}}(\hat{j} - \hat{i})$

(b) $\Delta\theta = \left(\dfrac{3I_0B}{4M}\right)(\Delta t)^2$

351. *(i)* $x = \pm\dfrac{d}{\sqrt{3}}$

(ii) $\dfrac{I}{2\pi d}\sqrt{\dfrac{\mu_0}{\lambda\pi}}$

352. T = 0.2 seconds.

353. 4.74×10^{-3} T.

354. $\left(\dfrac{\mu_0 I^2}{2\pi}\right)\ln\left(1 + \dfrac{a^2}{L^2}\right)$, zero.

355. *(a)* $\dfrac{\mu_0 I Q v}{6am}\left(\dfrac{3\sqrt{3}}{\pi} - 1\right)$

(b) $\vec{\tau} = BIa^2\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}\right)\hat{j}$

356. *(i)* I = 4 Amp.

(ii) 1 *m* from R on RX and 1 *m* from R on RQ.

357. 10^{-4} T upward.

358. *(i)* 3 Amp.

(ii) 1.3×10^{-6} T

(iii) 2.88×10^{-6} N/m

359. radius = 12 mm, pitch = 43.7 mm.

360. *(i)* EF = 0.1414 m, $\theta = 45°$

(ii) 4.71×10^{-8} S.

361. At $t = 5 \times 10^{-6}$ s, coordinates are (6.4 m, 0, 0)

At $t = 7.45 \times 10^{-6}$ s, coordinates are (6.4 m, 0, 2 m)

362. 6.5 N.

363. *(i)* Perpendicular to plane containing wires.

(ii) Attractive towards other.

(iii) 4 times.

(iv) Magnetic field is zero.

364. *(a)* $iR + \frac{L\,di}{dt} = \frac{-d\phi}{dt}$

(b) $\frac{1}{R}\left(\frac{\mu_0 I_0 l \ln 2}{2\pi} - L i_1\right)$

(c) $\frac{L}{R} = \frac{T}{2 \ln 2}$

365. $(12)\,e^{-5t}$ volt, $(6)\,e^{-10t}$ Amp.

366. Final temperature = 35.61°C

367. *(a)* $\left(\frac{B_0 a v}{R}\right)$ in the direction EHGFE (anticlock wise)

(b) $-\left(\frac{B_0^2 a^2 v}{R}\right)\hat{j}$

(c) $v = \frac{mgR}{B_0^2 a^2}\left(1 - e^{-\alpha t}\right)$

where $\alpha = \frac{B_0^2 a^2}{mR}$, $v_{\text{terminal}} = \frac{mgR}{B_0^2 a^2}$

368. *(a)* $\left|\dfrac{dI}{dt}\right| = 10^4$ A/s

(b) $I = 0$

(c) $I_{max} = 2A$

(d) $|Q| = 173\ \mu C$

369. *(i)* $\dfrac{mgR}{B^2L^2}$

(ii) $\dfrac{g}{2}$

370. $\left(\dfrac{21\mu_0^2 M^2 a^4 v}{4Rx^8}\right)$

371. 3.47 seconds.

372. *(a)* $\dfrac{1}{2}B\omega r^2$

(b) *(i)* $\dfrac{B\omega r^2}{2R}\left(1 - e^{-Rt/L}\right)$

(ii) $\dfrac{B^2 r^4 \omega}{4R} + \dfrac{mgr}{2}\cos\omega t$

373. $v = 1$ m/s, $R_1 = 0.474\ \Omega$, $R_2 = 0.3\ \Omega$.

374. $\dfrac{7}{22}$ Amp. from E to A,

$\dfrac{3}{11}$ Amp. from B to E,

$\dfrac{1}{22}$ Amp. from F to E.

375. *(a)* $V_A - V_B = -5$ V, Rate of production of joule heat in R_1 is 24.5 W.

(b) *(i)* 0.6 A

(ii) 1.386 ms, 4.5×10^{-4} J.

376. *(i)* $v = \dfrac{I}{Bd}(R + 2x\lambda)$, $F = IBd + \dfrac{2\lambda m I^2}{B^2 d^2}(R + 2x\lambda)$

(ii) $\dfrac{Q}{W} = \dfrac{1}{1 + \dfrac{2I\lambda m}{B^3 d^3}(R + 2x\lambda)}$

377. *(i)* $(-1)^n\left[\dfrac{Br^2\omega}{2R}\right]$.

where $n = 1, 2, 3, ...$ is the number of the half revolution.

378. $v_0 = 2$ cm/s, clockwise.

379. 1 millivolt.

380. $i_0 = 20$ Amp, $\Phi = \dfrac{\pi}{4}$.

381. *(a)* $\dfrac{\mu_0 C I_0 \omega^2 a \ln 2}{\pi}$

(b) $I = -\left(\dfrac{\mu_0 C I_0 \omega\, a \ln 2}{\pi}\right)\text{Cos}\,\omega t$

382. OE = 6.06 m.

383. 0.09 m/s, 0.3/s.

384. *(a)* 0

(b) 1250 Å.

385. $\dfrac{\mu_3 R}{\mu_3 - \mu_1}$

386. 4°, 0.04°.

387. 1.6

388. Distance of A'B' from pole of mirror 15 cm, magnification = – 1.5.
Distance of A' above RS is 0.3 cm,
Distance of B' below RS is 1.5 cm.

389. $\dfrac{\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)}{5\sqrt{2}}$.

390. $m = \dfrac{4}{3}$

391. *(a)* $\lambda_0 = 600$ nm,

(b) $i = \sin^{-1}\left(\dfrac{3}{4}\right)$

424. $2\mu t = \left(n + \frac{1}{2}\right)\lambda$, where $n = 0, 1, 2, 3, ...$

$t_{min} = 90$ nm.

425. (a) $y = 4.33$ mm

(b) $I = \frac{3}{4} I_m$

(c) 650 nm, 433.3 nm

426. (a) $y = \pm \frac{1}{\sqrt{15}} m, \pm \frac{3}{\sqrt{7}} m$;

(b) $y = \frac{1}{\sqrt{15}} m, \frac{3}{\sqrt{7}} m$

427. 9.3 μm

428. Thickness glass plate = 7 μm ;

Refractive index of glass for green light = 1.6

Change in fringe width = 57.1 μm

429. (i) 0.63 mm
(ii) 1.575 μm

430. (i) 1 mm
(ii) increase

431. (a) 8×10^{-15} m^2

(b) 27°

(c) 9I.

432. $\frac{I_{max}}{I_{min}} = 49$

433. 7×10^{-6} watts.

434. (i) 1.17 mm
(ii) 4.68 mm

435. 5892 Å

436. 14 W/m^2.

437. (a) 5×10^7
(b) 2000 N/C;
(c) 23 eV

438. 150.8 eV, 0.5 Å

439. *(a)* 10^5

(b) $\dfrac{\lambda_{\text{light}}}{\lambda_{\text{electron}}} = 286.2$

(c) Sphere gets positively charged *(d)* 111 seconds.

440. *(a)* $6 \times 10^{11}, 9.6 \times 10^{-8}$ A

(b) $1.478 \times 10^{11}, 2.13 \times 10^{-8}$ A

(c) 1.07 V in both the cases.

441. 1.1×10^{12}.

442. 0.55 ev.

443. $z = 42$.

444. *(a)* $z = 3$; *(b)* $\lambda_{min} = 4060$ nm

445. *(a)* $n = 2, z = 4, E_{min} = 10.58$ eV

(b) $N = 6.25 \times 10^{11}, E_{min} = 0, E_{max} = 5$ eV.

446. 3.3 eV, 3.94 eV, 2.64 eV.

447. *(i)* 3.4 eV

(ii) 6.663×10^{-10} M.

448. *(i)* $\left(\dfrac{eh}{4\pi m}\right)$; *(ii)* $\left(\dfrac{ehB}{8\pi m}\right)$

449. $z = 3, n = 6$.

450. *(i)* Neutron 6.36 eV, 0.32 eV; He atom 17.84 eV, 16.32 eV

(ii) 9.87×10^{15} Hz, 11.68×10^{15} Hz, 1.82×10^{15} Hz.

451. *(a)* 2.55 eV,

(b) $n = 2$ and $n = 4$,

(c) $\dfrac{h}{\pi}$

(d) 0.814 m/s

452. Work function = 2 eV

Stopping potential for longer wavelength = 0.756 V.

453. *(i)* 2 ; *(ii)* 14.4 eV ; *(iii)* 13.5 eV, 0.7 eV.

454. *(i)* $\dfrac{\varepsilon_0 h^2}{624\pi me^2}$

(ii) $n = 25$

(iii) 5.48×10^{-11} M.

455. *(i)* 114 Å,

(ii) Three.

456. *(i)* 300 Å

(ii) 0.26 Å

457. 6.6×10^{-34} Joule-sec.

458. Number of lines = 6

Longest wavelength = 1.88 μm

459. *(i)* $z = 5$,

(ii) 16.53 eV

(iii) 36.4 Å

(iv) KE = 340 eV, PE = – 680 eV, L = 1.05×10^{-34} J-sec.

(v) 1.06×10^{-11} m.

460. $\dfrac{N_{pb}}{N_u} = 0.259$

461. 0.145 /sec.

462. $t = [(15) \ln 3)$ seconds.

$N_X = 1.924 \times 10^{19}$.

$N_Z = 2.302 \times 10^{19}$.

463. *(a)* $N = \dfrac{d}{\lambda}\left(1 - e^{-\lambda t}\right) + N_0 e^{-\lambda t}$

(b) After one half life $N = \dfrac{3}{2} N_0$, At $\to \infty$ $N = 2N_0$

464. 1.8684×10^9 years.

465. Mean life = 14.43 seconds, $t = 40$ seconds.

466. 5954 cc.

467. 3.96×10^{-3}

468. 3.845×10^5 kg.

469. KE = 6.5 Mev; Mass = 227.62 a.m.u.

470. 3.32×10^{-5} W.

471. *(a)* A = 232, Z = 90

(b) Energy released = 5.34 MeV., binding energy = 1823 MeV.

472. 121 gm.

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