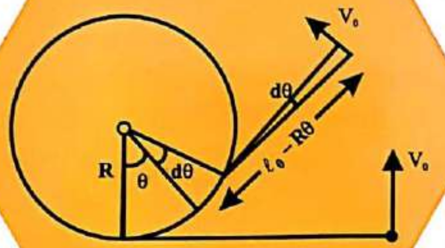
Solution of

IE IRODOV'S

Problems in General

PHYSICS



Vol. 1

Raj Kumar Sharma
B.Tech IIT(ISM Dhanbad)

RK Publications

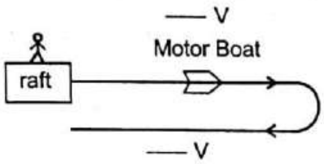
CONTENTS

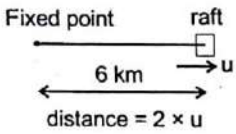
Part One
Physical Fundamentals of Mechanics

1.1 KINEMATICS.

1.1

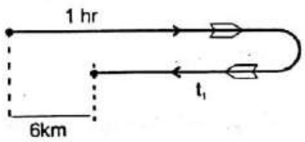
Method : 1 (Relative approach)





V = Relative speed of motor boat w.r.t. river which is constant
Observer on raft see that speed of motor boat w.r.t. river is constant because duty of motot boat w.r.t. river is constant. Hence if motor boat take 1 hrs in down stream journey then to reach again at raft motor boat will take 1 hrs in upstream journey because river is in rest w.r.t. motorboat.
Hence total time in complete journey = 2 hrs.
Motion of raft :
u = speed of river=speed of raft.
Then $2u = 6 \quad \Rightarrow u = 3$ km/hr. **Ans**

Method : 2 (With frame of ground)



Motion of raft : $u(1 + t_1) = 6$ ---------(i)
Motion of motor boat :
$(v + u) \times 1 - (v - u)t_1 = 6$
$v - vt_1 + ut_1 + u = 6$
$v - vt_1 + u(1 + t_1) = 6$
from (i)
$v - vt_1 + 6 = 6$
$t_1 = 1$ hrs.
put $t_1 = 1$ hr in (i) :
$u(1 + 1) = 6 \Rightarrow u = 3$ km/hr

Ans

1.2 :

Total distance travel by point is S.

Then time taken in first journey is t_1 :

$$t_1 = \frac{S/2}{V_0}$$

Time taken in second journey is t_2 :

$$\frac{S}{2} = V_1\frac{t_2}{2} + V_2\frac{t_2}{2}$$

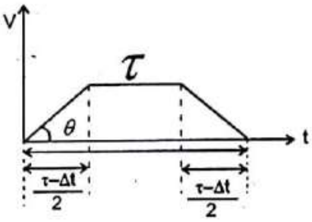
$$t_2 = \frac{S}{V_1 + V_2}$$

$$\text{Mean velocity} = \frac{S}{t_1 + t_2} = \frac{S}{\frac{S/2}{V_0} + \frac{S}{V_1 + V_2}}$$

$$\text{Mean velocity} = \frac{2V_0(V_1 + V_2)}{V_1 + V_2 + 2V_0}$$ **Ans**

1.3 **Method : 1 (Graphical Approach)**

$\tan\theta = \omega$



Average Velocity

$$= \frac{\text{Displacement}}{\text{Time}} = \frac{\text{Area of trapzium}}{\text{Total time}}$$

$$V = \frac{\frac{1}{2}(\Delta t + \tau)\left(\frac{\tau - \Delta t}{2}\right)\tan\theta}{\tau}$$

$$V = \frac{\frac{1}{2}(\Delta t + \tau)\left(\frac{\tau - \Delta t}{2}\right)\omega}{\tau}$$

$$\Delta t = \tau\sqrt{1 - \frac{4v}{\omega\tau}}$$ **Ans**

Method : 2 (Analytical)
Total displacement

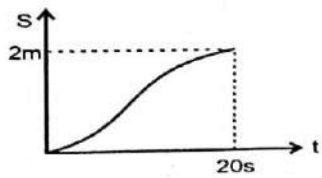
$$S = \frac{1}{2}\omega\left(\frac{\tau - \Delta t}{2}\right)^2 + \omega\Delta t\left(\frac{\tau - \Delta t}{2}\right) + \frac{1}{2}\omega\left(\frac{\tau - \Delta t}{2}\right)^2$$

Time taken = Δt

$$v = \frac{S}{\tau} = \frac{\frac{1}{2}\omega\left(\frac{\tau - \Delta t}{2}\right)^2 + \omega\Delta t\left(\frac{\tau - \Delta t}{2}\right) + \frac{1}{2}\omega\left(\frac{\tau - \Delta t}{2}\right)^2}{\tau}$$

$$\Delta t = \tau\sqrt{1 - \frac{4v}{\omega\tau}}$$ **Ans**

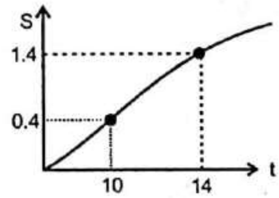
1.4 **(a)**



Average velocity = $\frac{2}{20} = 0.1 m/s = 10 cm/s$

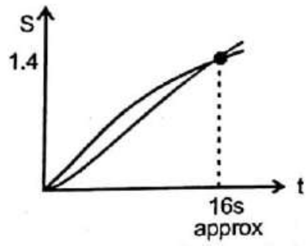
Ans.

(b) Velocity will be maximum when slope of S (t) curve will be maximum and it is seen in straight part approximately.



$V = \frac{1.4 - 0.4}{14 - 10}$ m/s = 0.25m/s= 25 cm/s **Ans.**

(c) Instanteneous velocity may be equal to mean velocity when slope of line joining final and intial point will be same as slope at point on curve. From curve t_0 = 16 s **Ans.**



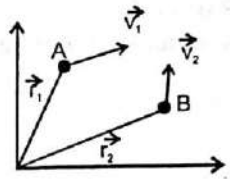
1.5

Velocity of B with respect to A :

$V_{B-A} = V_2 - V_1$

Position of B with respect to A :

$r_{B-A} = r_B - r_A = r_2 - r_1$



Particle B will be collide with A if velocity of B with respect A is directed toward observer A hence relative velocity should be antiparrallel to relative position.

Then $V_{B-A} \parallel r_{A-B}$

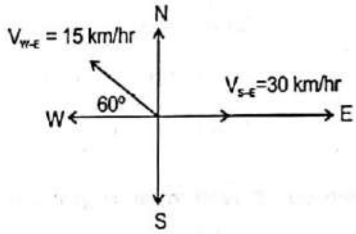
$\hat{V}_{B-A} = -\hat{r}_{B-A}$

Then $\frac{V_2 - V_1}{|V_2 - V_1|} = \frac{r_2 - r_1}{|r_2 - r_1|}$ **Ans.**

1.6

Method - 1(Coordinate approach)

In Irodov, by mistake at place of north-east, south-east is written. Suppose east direction is x axis and north direction is y axis.



$V_{S-E} = 30i$;

$V_{W-E} = -15\cos 60\hat{i} + 15\sin 60\hat{j}$

$V_{W-S} = V_{W-E} - V_{S-E}$

$= (-15\cos 60 - 30)\hat{i} + 15\sin 60\hat{j}$

$|V_{W-E}| = \sqrt{(-15\cos 60 - 30)^2 + (15\sin 60)^2}$

$\cong 40 km/hr$

$\tan\varphi' = \frac{15\sin 60}{|-15\cos 60 - 30|}$

$\phi' = 19°$ **Ans.**

Method - 2(Vector addition Method)

We know $V_{W-S} = V_{W-E} - V_{S-E}$

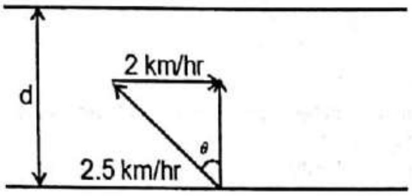
$|V_{W-S}| = \sqrt{V_{W-E}^2 + V_{S-E}^2 - 2V_{W-E}V_{S-E}\cos(180-60)}$

$\cong 40 km/hr$

$$\tan\varphi' = \frac{15\sin 60}{30+15\cos 60} \Rightarrow \phi' = 19^\circ \quad \textbf{Ans.}$$

1.7

Swimmer : 1



Time to cross the river is t then

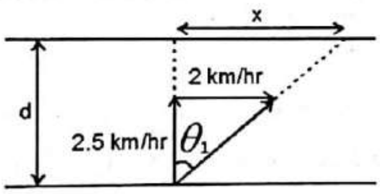
$$t = \frac{d}{2.5\cos\theta} \qquad \text{..........(i)}$$

From figure : $\sin\theta = \frac{2}{2.5} = \frac{4}{5}$

$$\cos\theta = \frac{3}{5}$$

Put in (i) : $t = \frac{d}{1.5}$ ....(ii)

Swimmer :2



Using trigonometry : $x = d\ \tan\theta_1$,

Time to reach at destination point:

$t = t_1 + t_2$.........(iii)

t_1 = Time to cross the river = $\frac{d}{2.5}$

t_2 = Time to walk on bank = $\frac{x}{u} = \frac{d\tan\theta_1}{u}$

Also $\tan\theta_1 = \frac{2}{2.5} = \frac{4}{5}$ ....(iv)

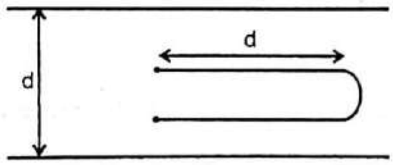
Put values of times and $\tan\theta_1$ in (iii)

$$t = \frac{d}{2.5} + \frac{d}{u}\times\frac{4}{5} \qquad \text{.......(v)}$$

from (ii) and (v) : $\frac{d}{1.5} = \frac{d}{2.5} + \frac{4d}{5u}$

u = 3 km/hr **Ans.**

1.8 **Boat A :**



Time to reach again at same point by boat A.

$$t_A = \frac{d}{V+W} + \frac{d}{V-W}$$

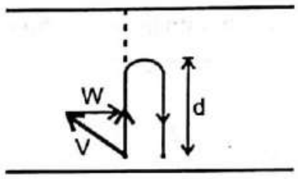
Also given $V = \eta W$

$$t_A = \frac{d}{\eta W + W} + \frac{d}{\eta W - W}$$

Where V = Speed of boat w.r.t. river

W = Speed of water w.r.t. earth.

Boat B :



Time to reach again at same point by boat B.

$$t_B = \frac{2d}{\sqrt{V^2 - W^2}}$$

Also given $V = \eta W$

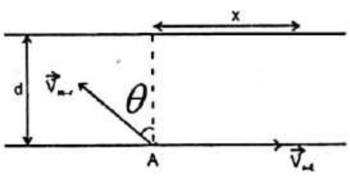
$$t_B = \frac{2d}{\sqrt{\eta W^2 - W^2}}$$

$$\text{Now } \frac{t_A}{t_B} = \frac{\frac{d}{\eta W + W} + \frac{d}{\eta W - W}}{\frac{2d}{\sqrt{\eta^2 W^2 - W^2}}} = \frac{\eta}{\sqrt{\eta^2 - 1}}$$

$$\frac{t_A}{t_B} = \frac{\eta}{\sqrt{\eta^2 - 1}} = 1.8\text{s}$$

1.9 :

Method : 1(Analytical Approach)



Time to cross the river : $t = \frac{d}{|V_{mr}|\cos\theta}$

Then drift $(x) = (V_{m-r} \sin\theta - V_{r-E}) \dfrac{d}{V_{mr}\cos\theta}$

Since $V_{m-r} < V_{r-E}$. Hence this in not possible that drift will be zero. We should have to minimize drift as because drift is function of x.

At maximum value of drift $\dfrac{dx}{d\theta} = 0$

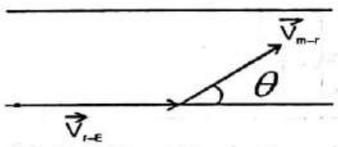
$$d\sec^2\theta - \frac{(V_{r-E})d}{V_{mr}}\sec\theta\tan\theta = 0$$

$$\sin\theta = \frac{V_{m-r}}{V_{r-E}} = \frac{1}{2}$$

$\theta = 30°$

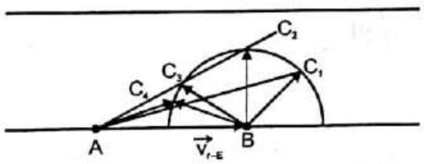
Angle made by boat with flow velocity of water $= 30° + 90° = 120°$ **Ans.**

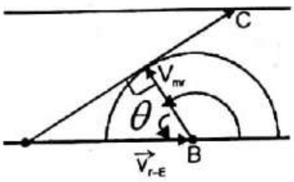
Method : 2 (Vector addition graph method)



$V_{m-E} = V_{m-r} + V_{r-E}$ Hence θ will taken any value between (0 – 180°) hence we can draw a semi circle of radius of length $|V_{m-r}|$.

Then there are some resulants as shown





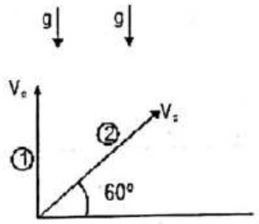
And resultant is given by C_1, C_2, C_3 and C_4, ... C_n. But for minimum drift resultant must be tangent at semicrirle. Then

$$\cos\alpha = \frac{V_{mr}}{V_{r-E}} = \frac{1}{2}$$

$\alpha = 60°$

Then $\theta = 180° - 60° = 120°$ **Ans.**

1.10



Relative acceleration of particle (1) and w.r.t. (2) = g – g = 0.

Relative velocity of particle (1) w.r.t. (2)

$$v_{1-2} = \sqrt{V_0^2 + V_0^2 - 2V_0\cos(90-\theta)}$$

$$= V_0\sqrt{2(1-\sin\theta)}$$

Where $90 - \theta$ is angle between two velocity does not change w.r.t. time because relative acceleration is zero.

Then

Distance between two particle at time t is

$= V_0 t\sqrt{2(1-\sin\theta)}$ = 22m **Ans.**

1.11

Method : 1 (Vector application)

Initial velocity in y direction of both particle are zero.

Hence vertical velocity of both particles at time t will be same then:

$V_y = u + at$

$V_y = gt$

Velocity of particle (1) at time t: $V_1 = -3\hat{i} + gt\hat{j}$

Velocity of particle (2) at time t: $V_2 = 4\hat{i} + gt\,\hat{j}$

Since $V_1 \perp V_2 \Rightarrow V_1 \cdot V_2 = 0$

$-12 + g^2t^2 = 0$

$t = \sqrt{0.12}$ s

Hence initial relative velocity=(4 – (–3)) =7

Distance between particles

$V_r \times t = 7 \times \sqrt{0.12} \cong 2.5\,m$ **Ans.**

Method:2 (Graphical application)

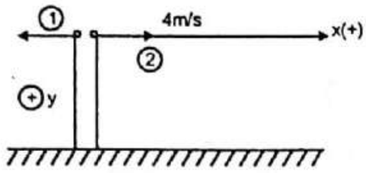
Since $V_1 \perp V_2 \Rightarrow \alpha + \beta = 90$

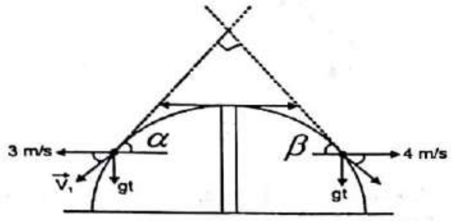
$\alpha = 90 - \beta$

$\tan\alpha = \tan(90-\beta)$

$\tan\alpha = \cot\beta$

$\tan\alpha\tan\beta = 1$ ......(i)





$$\tan\alpha = \frac{gt}{3}$$

$$\tan\beta = \frac{gt}{4}$$

Put in (i)

$$\left(\frac{gt}{3}\right)\left(\frac{gt}{4}\right) = 1$$

$t = \sqrt{0.12}$

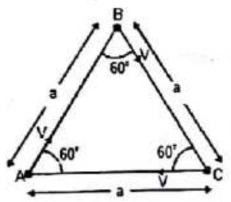
Initial relative velocity=(4 – (–3)) =7

Distance between particles

$$V_r \times t = 7\times\sqrt{0.12} \cong 2.5\,m \qquad \textbf{Ans.}$$

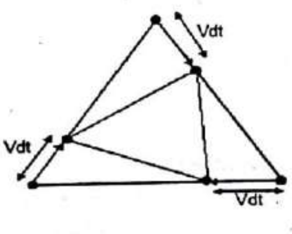
1.12:

Method : 1 (Vector application)

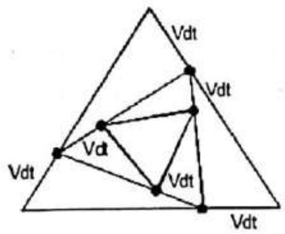


Since particle A heading to particle B and B to C and C to A.

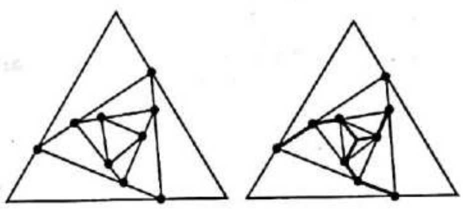
Now position of all particle at t = dt is as figure.



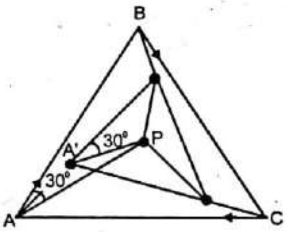
Again position of all particle at t = 2 dt is as figure.



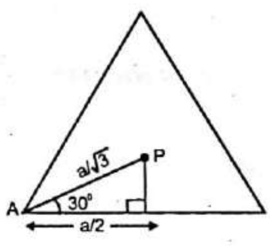
Again position at t = 3 dt and so on ....



Since at any time all particle travel same distance then at each moment of time,all particle will be at equilateral tringle. Then by symmetry you can say that all particle will be met at centriod of tringle then path of each particle will as :



Suppose at any instant of time, distance b/w particle A and centriod P is r.



Then line joing particle A and P make 30° angle with side of equilateral tringe then $\frac{dr}{dt}$ will always constant and equal to v cos 30° then

$$-\frac{dr}{dt} = V\cos 30^\circ$$

(–)ive sin because r is dicreasing function.

Finally r = 0 while initial $r = a/\sqrt{3}$ then

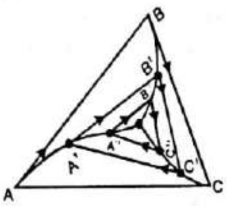
$$\int_{\frac{a}{\sqrt{3}}}^{0} -\frac{dr}{V\cos 30} = \int_0^t dt$$

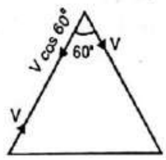
$$t = \frac{2a}{3V}$$

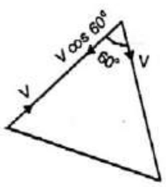
$$AP = \frac{a}{2}\sec 30^\circ = \frac{a}{\sqrt{3}}$$ **Ans.**

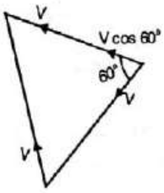
Method : 2 (Relative approach)

Let distance between A and B at time t is r then. At any instant of time, rate of decreasement of distance b/w two particle A and B will be constant as shown in figure.







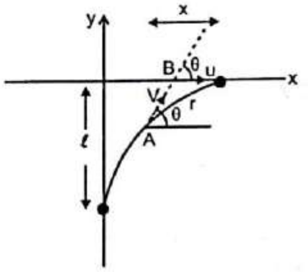


Then $\frac{dr}{dt} = -(V + V\cos 60^\circ) = -\frac{3V}{2}$

$$\int_a^0 \frac{2dr}{-3V} = \int_0^{dt} dt$$

$$t = \frac{3a}{V}$$ **Ans.**

1.13:



Suppose at time t, distance b/w A and B is r. Then rate of decrement of r is :

$$\frac{-dr}{dt} = -v + u\cos\theta$$

$$\int_r^0 -dr = \int_0^t (-v + u\cos\theta)dt$$

$$-l = -vt + u\int_0^t \cos\theta \, dt \text{ ....(i)}$$

Since $\int_0^t \cos\theta dt$ is not known then to find this integration,we use rate of decrement of distance between both in direction of x. Suppose it is x then

$$\frac{dx}{dt} = -V\cos\theta + u$$

$$\int_0^0 dx = \int_0^t (-V\cos\theta + u)dt$$

$$0 = ut - V\int_0^t \cos\theta \, dt$$

$$\int_0^t \cos\theta \, dt = \frac{ut}{V}$$

Put this value in (i) :

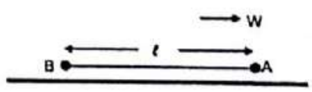
$$-l = -vt + u\left(\frac{ut}{V}\right)$$

$$t = \frac{V\ell}{V^2 - u^2}$$ **Ans.**

1.14 :

With frame of train :

With frame of train, train appear in rest then distance between two event is equal to l.



With frame of earth :

When event (1) will happen.

Velocity of train is $u_1 = u + at = wt$.

Since event (2) will be happen after time τ then.

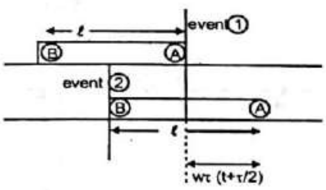
Distance travelled by headlight (A) in time τ=

Distance travelled by headlight (B) in time τ=

$$=u_1 t_1 + \frac{1}{2}at_1^2$$

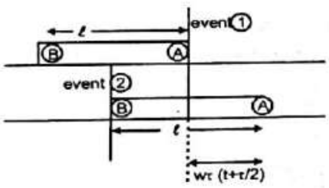
$$= wt(\tau)+\frac{1}{2}w\tau^2$$

$$= w\tau\left(t+\frac{\tau}{2}\right)$$



Then distance between two events is

$$= \ell - w\tau\left(t+\frac{\tau}{2}\right) = 0.24 \text{ km.}$$ **Ans**



It we want that both event will be happen at same point then event 2 should be at position of head light B.

Assume velocity of reference frame is v.

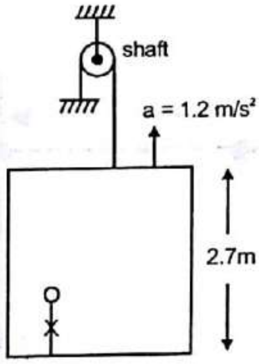
Distance travelled by headlight (B) in time τ

$$(u+wt)\tau+\frac{1}{2}w\tau^2 = l$$

$$u = \frac{\ell - w\tau\left(t+\frac{\tau}{2}\right)}{\tau}$$

$$u = \frac{0.24\text{km}}{60} = 4 \text{ m/s}$$ **Ans.**

1.15



(a) For observer inside lift at t = 2s. velocity of bolt = 0

Acceleration of bolt w.r.t. lift = 10 + 1.2 = 11.2

Then assume t time is taken by bolt to reach at floor.

$$s = \frac{1}{2}at^2$$

$$2.7 = \frac{1}{2} \times 11.2\, t^2 \qquad t = 0.7 \text{ s}$$ **Ans**

(b) Velocity of bolt w.r.t. ground at t = 2 sec.

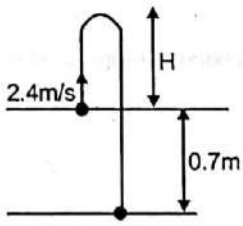
$V = 1.2 \times 2 = 2.4$ m/s.

Displacement(S) of bolt in next 0.7 sec

$$S = ut + \frac{1}{2}at^2$$

$$S = 2.4\,(0.7) - \frac{1}{2} \times 10\,(.7)^2 \cong -0.7 \text{ m}$$ **Ans**

Distance travelled by bolt:



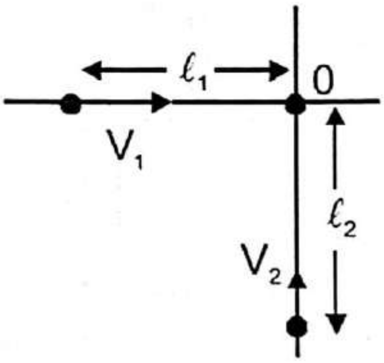
$$H = \frac{2.4 \times 2.4}{2g} = 0.288.$$

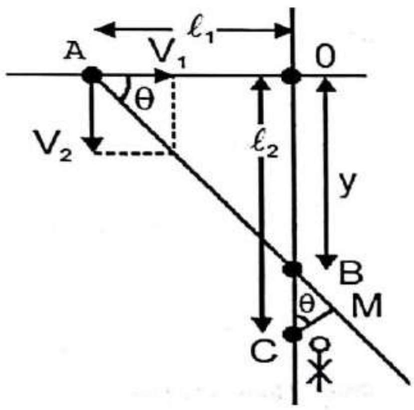
Distance travelled = 2 ×0.288 + 0.7 ≅ 1.3 m

Ans.

1.16

Method : 1 (Relative velocity)





Second figure is graph of relative velocity of 1 w.r.t. 2

From figure

$\tan\theta = \frac{V_2}{V_1}$ $\cos\theta = \frac{V_1}{\sqrt{V_1^2+V_2^2}}$

$\tan\theta = \frac{y}{\ell_1} = \frac{V_2}{V_1}$ $y = \ell_1\frac{V_2}{V_1}$

$BC = \ell_2 - \frac{\ell_1 V_2}{V_1}$

Shortest distance between two

$CM = BC\cos\theta = \left(\ell_2 - \frac{\ell_1 V_2}{V_1}\right)\frac{V_1}{\sqrt{V_1^2+V_2^2}}$

Shortest distance between two

$CM = \frac{|\ell_2 V_1 - \ell_1 V_2|}{\sqrt{V_1^2+V_2^2}}$ **Ans**

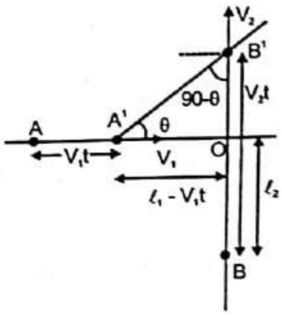
Time to reach at minimum distance

$t = \frac{AM}{\sqrt{V_1^2+V_2^2}} = \frac{AB+BM}{\sqrt{V_1^2+V_2^2}} =$

$\frac{\ell_1\sec\theta + CM\tan\theta}{\sqrt{V_1^2+V_2^2}} = \frac{V_1\ell_1+V_2\ell_2}{V_1^2+V_2^2}$

$t = \frac{V_1\ell_1+V_2\ell_2}{V_1^2+V_2^2}$ **Ans.**

Method : 2 (Velocity of approach)



At shortest distance velocity of approach will be zero.

$V_1\cos\theta - V_2\sin\theta = 0$

$V_1\cos\theta = V_2\sin\theta$

$\tan\theta = \frac{V_1}{V_2}$

In triangle A'B' O :

$\tan\theta = \frac{V_2 t - \ell_2}{\ell_1 - V_1 t} = \frac{V_1}{V_2}$

$V_2^2 t - \ell_2 V_2 = \ell_1 V_1 - V_1^2 t$

$t = \frac{\ell_1 V_1 + \ell_2 V_2}{V_1^2+V_2^2}$ **Ans**

Shortest distance between two

$\sqrt{(\ell_1 - V_1 t)^2 + (V_2 t - \ell_2)^2}$

Put the value of t then

Shortest distance $= \frac{|\ell_2 V_1 - \ell_1 V_2|}{\sqrt{V_1^2+V_2^2}}$ **Ans**

Method : 3 (Using formulas)

Shortest distance between two= $|\hat{V}_r \times \vec{r}|$**...(1)**

Relative velocity of 1 w.r.t. 2 : $\vec{V}_r = V_1\hat{i} - V_2\hat{j}$

Unit relative velocity: $\hat{V}_r = \dfrac{V_1\hat{i} - V_2\hat{j}}{\sqrt{V_1^2 + V_2^2}}$

Relative position of 1 w.r.t. 2 : $\vec{r} = -l_1\hat{i} + l_2\hat{j}$

Put in (1)

Shortest distance between two=

$$\left|\left[\frac{V_1\hat{i} - V_2\hat{j}}{\sqrt{V_1^2 + V_2^2}}\right] \times \left[-l_1\hat{i} + l_2\hat{j}\right]\right| = \frac{|\ell_2 V_1 - \ell_1 V_2|}{\sqrt{V_1^2 + V_2^2}}$$

Ans.

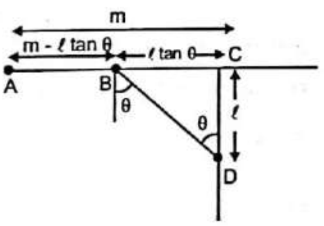
Time to reach at minimum distance= $\left|\dfrac{\hat{V}_r \bullet \vec{r}}{V_r}\right|$

$$= \left|\frac{\dfrac{\left[V_1\hat{i} - V_2\hat{j}\right]}{\sqrt{V_1^2 + V_2^2}} \bullet \left[-l_1\hat{i} + l_2\hat{j}\right]}{\sqrt{V_1^2 + V_2^2}}\right|$$

$$t = \frac{V_1\ell_1 + V_2\ell_2}{V_1^2 + V_2^2}$$ **Ans.**

1.17

Method : 1(Analytical Approach)



Suppose intial distance of person from point C is m and at ponit B, particle turn in his way

Time to reach at point D :

$$T = \frac{m - \ell\tan\theta}{V} + \frac{\ell\sec\theta}{V/n}.$$

Since T is function of θ then for minimum time T:

$$\frac{dT}{d\theta} = 0$$

$$-\frac{\ell}{V}\sec^2\theta + \frac{n\ell}{V}\sec\theta \times \tan\theta = 0$$

$\sec\theta = n\tan\theta$

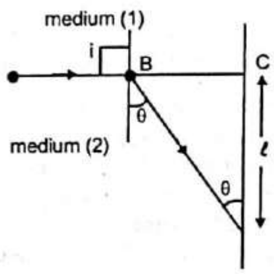
$\sin\theta = \dfrac{1}{n}.$

Then distance BC = $l\tan\theta$

$$BC = \frac{\ell}{\sqrt{n^2-1}}$$ **Ans.**

Method: 2 (Refration of light method)

We know that light travel via that path in which time will be less.



Using law of refration

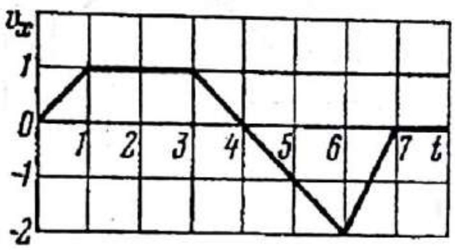
$$\frac{\sin i}{\sin\theta} = \frac{\text{speed in medium (1)}}{\text{speed in medium (2)}}$$

$$\frac{\sin 90}{\sin\theta} = \frac{V}{V/n}$$

$$\sin\theta = \frac{1}{\eta}$$

length BC $= l\tan\theta = \dfrac{\ell}{\sqrt{n^2-1}}$ **Ans.**

1.18

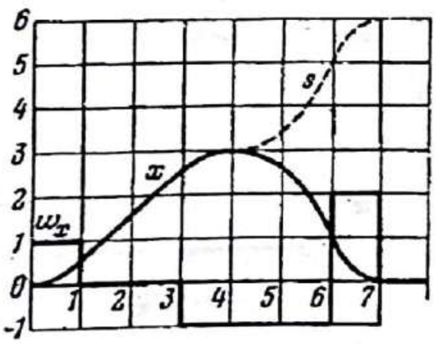


Acceleration graph:

In time interval 0-1 sec ....Slope is constant hence acceleration will be constant and to slope of curve 1 m/s^2.

In time interval 1-3 sec ....Slope is zero hence acceleration will be 0 m/s^2.
In time interval 3-6 sec ....Slope is constant hence acceleration will be constant and to slope of curve -1 m/s^2.
In time interval 7 sec onward ....Slope is zero hence acceleration will be 0 m/s^2.

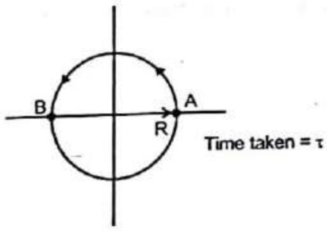
Displacement(x) graph:
In time interval 0-1 sec ....Acceleration is 1 m/s^2 displacement curve has concavity upward.
In time interval 1-3 sec ....Acceleration is 0 m/s^2 displacement curve has straightline.
In time interval 3-6 sec ....Acceleration is -1 m/s^2 hence displacement curve has concavity downward.
In time interval 7 sec onward ....velocity is zero hence curve be straightline.
Distanc(S) graph:
In time interval 0-1 sec ....Acceleration is 1 m/s^2 distance curve has concavity upward.
In time interval 1-3 sec ....Acceleration is 0 m/s^2 distance curve has straightline.
In time interval 3-6 sec ....Acceleration is -1 m/s^2 hence distance curve has concavity upward becuase it is increasing function.
In time interval 7 sec onward ....velocity is zero hence curve be straightline.



1.19
(a) Mean velocity in irodov is misprint and it is mean speed then mean speed = $\frac{\pi R}{\tau}$ **Ans.**

(b) Mean velocity = $\frac{2R}{\tau}$ **Ans.**

(c)



We know $\omega^2 = \omega_0^2 + 2\alpha\theta$

$\omega^2 = 2\alpha\pi$ ....(i)

$\theta = \frac{1}{2}\alpha t^2$

$\pi = \frac{1}{2}\alpha\tau^2$

$\alpha = \frac{2\pi}{\tau^2}$ ....(ii)

from (i) and (ii)

$\omega^2 = 2\left(\frac{2\pi}{\tau^2}\right)\pi$

$\omega = \frac{2\pi}{\tau}$

$V = RW = R\left(\frac{2\pi}{\tau}\right) = \frac{2\pi R}{\tau}$

Average acceleration

$= \frac{V_f - V_c}{\tau} = \frac{\frac{2\pi R}{\tau} - 0}{\tau} = \frac{2\pi R}{\tau^2}$ **Ans.**

1.20

It is one dimension motion because direction of position vector $\vec{r}$ is same as constant vector $\vec{a}$.

$\vec{r} = \vec{a}t(1-\alpha t) = \vec{a}\,t - \alpha t^2 \vec{a}$

(a) $\vec{V} = \frac{d\vec{r}}{dt} = \vec{a} - 2\alpha t\,\vec{a}$

$\vec{a}_{ac} = \frac{d\vec{v}}{dt} = -2\alpha\vec{a}$ **Ans.**

(b) At initial position time t = 0

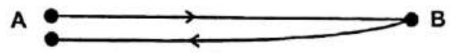
$\vec{r} = 0$

At final position at time t

$V = 0$

$\vec{a}t(1-\alpha t) = 0$

$t = \frac{1}{\alpha}$ **Ans.**



Initial at returning point B velocity will be equal to zero so that particle will be turn back at time t.

$$0 = a(1-2\alpha t) \Rightarrow t = \frac{1}{2\alpha}$$

Then position B is at distance r then

$$r = a\left(\frac{1}{2\alpha}\right)\left(1-\alpha\frac{1}{2\alpha}\right) = \frac{a}{4\alpha}.$$

Distance travelled in up and down journey is:

$$\frac{a}{4\alpha}+\frac{a}{4\alpha}=\frac{a}{2\alpha}$$ **Ans.**

1.21

(a)

$V=V_0(1-t/\tau)$

$$\int_0^x dx = \int_0^t V_0\left(1-\frac{t}{\tau}\right)dt$$

$$x = V_0\left(t-\frac{t^2}{2\tau}\right)$$

Afer putting valuesof time $x = 0.24, 0$ and -4.0 m

Ans.

(b)

Put $x = \pm 10$, $V_0 = 10$ and $\tau = 5$ in eqaution

$$x = V_0\left(t-\frac{t^2}{2\tau}\right)$$

$$\pm 10 = 10\left(t-\frac{t^2}{2\times 5}\right)$$

t= 1.1, 9 and 11 s **Ans.**

(c)

Here we see that velocity will change direction when it velocity will be zero.

$V=V_0(1-t/\tau)=0$

$t = \tau = 5$sec

Distance travelled in 4 sec:

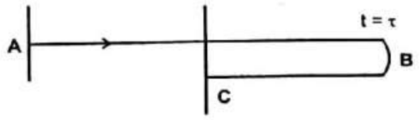
Since $t=4 < \tau = 5$

Velocity will be one direction and then displacement and distance will be equal.

$$x = V_0\left(t-\frac{t^2}{2\tau}\right) = \left(1-\frac{t}{2\tau}\right)V_0 t$$ **Ans.**

$$=10\left(4-\frac{4^2}{2\times 5}\right) = 24\text{cm}$$ **Ans.**

Distance travelled in 9 sec:



When $t < \tau$ $\quad V = \oplus v = \ominus$ ive

When $t > \tau$ $\quad v = \ominus$ ive

Since particle changes its direction hence we will calculate both up and down journey for distance calculation.

Up journey distance travelled to time $t = \tau$

$$AB = x = V_0\tau\left(1-\frac{\tau}{2\tau}\right)$$

Down journey distance travelled after time $t = \tau$ will be BC

$$AC = V_0 t\left(1-\frac{t}{2\tau}\right)$$

$$BC = AB - AC = \left[V_0\tau\left(1-\frac{\tau}{2\tau}\right) - V_0 t\left(1-\frac{t}{2\tau}\right)\right]$$

Total distance travelled = AB + BC

$$= V_0\tau\left(1-\frac{\tau}{2\tau}\right) + \left[V_0\tau\left(1-\frac{\tau}{2\tau}\right) - V_0 t\left(1-\frac{t}{2\tau}\right)\right]$$

$$= \frac{V_0\tau}{2}+\frac{V_0\tau}{2} - V_0 t\left(1-\frac{t}{2\tau}\right)$$

$$\text{Distance travelled} = V_0\tau - V_0 t\left(1-\frac{t}{2\tau}\right)$$

(This irodov ans given is not right)

Ans.

1.22

(a)

$v = \alpha\sqrt{x} = ax^{1/2}$ ....(1)

Differentiate w.r.t. time for acceleration:

$$a = \frac{dv}{dt} = \frac{1}{2}\alpha x^{-1/2}\frac{dx}{dt} = \frac{1}{2}\alpha x^{-1/2} v$$

Put value of v from(1)

$$a = \frac{\alpha}{2}x^{-1/2}\alpha x^{1/2} = \frac{1}{2}\alpha^2$$ **Ans.**

Since acceleration is constant; velocity will be :

V = u + at

$$V = \frac{1}{2}\alpha^2 t$$ **Ans.**

(b)

$$S = ut + \frac{1}{2}at^2$$

$$S = \frac{1}{2}\left(\frac{\alpha^2}{2}\right)t^2$$

$$t = \frac{2\sqrt{s}}{\alpha}$$

Mean velocity $<V> = \frac{S}{t} = \frac{S\alpha}{2\sqrt{S}}$

$$< V > = \frac{1}{2}\alpha\sqrt{s}$$ **Ans.**

1.23

Calculation of time

$w = -a\sqrt{v}$

($\ominus$ i ve sign is used to show deacceleration)

$$\frac{dv}{dt} = -a\sqrt{v}$$

$$\int_{v_0}^{0}\frac{dv}{\sqrt{v}} = -\int_0^{t_0} a\,dt$$

$$t_0 = \frac{2V_0^{1/2}}{a}$$ **Ans.**

Calculation of distance

$\omega = -a\sqrt{v}$

$$v\frac{dv}{dx} = -a\sqrt{v}$$

$$\int_{v_0}^{0} v^{1/2}dv = -a\int_0^{x_0} dx$$

$$x_0 = \frac{2V_0^{3/2}}{3a}$$ **Ans.**

1.24

(a)

$\vec{r} = at\,\hat{i} - bt^2\,\hat{j}$

Compare this equation with

$\vec{r} = x\,\hat{i} - y\,\hat{j}$

$x = at$

$y = -bt^2$

Eliminating t from above two coordinate

$$y = -b\left[\frac{x}{a}\right]^2$$ **Ans.**

(b)

$$\vec{v} = \frac{d\vec{r}}{dt} = a\,\hat{i} - 2bt\,\hat{j} \;.........(1)$$

$\vec{v} = a\,\hat{i} - 2bt\,\hat{j}$

$|\vec{v}| = \sqrt{a^2 + 2b^2t^2}$ **Ans.**

$$\vec{a} = \frac{d\vec{v}}{dt} = -2b\,\hat{j}$$

$\vec{a} = -2b\,\hat{j}$

$|\vec{a}| = 2b$ **Ans.**

(c)

Since direction of acceleration is toward y direction then angle made by velocity vector with y axis is known as angle b/w two vectors then.

$$\tan\alpha = \frac{a}{2bt}$$ **Ans.**

(d) $$\vec{V}_{mean} = \frac{\vec{s}}{t} = \frac{at\,\hat{i} - bt^2\,\hat{j}}{t}$$

$$= a\,\hat{i} - bt\,\hat{j}$$

$$|\vec{V}_{mean}| = \sqrt{a^2 + b^2t^2}$$ **Ans**

1.25

(a)

$x = a\,t$

$\frac{x}{a} = t$ -------(1)

$y = at(1-\alpha t)$.......(2)

Put value of t in equation(2)

$$y = a\frac{x}{a}\left(1 - \frac{\alpha x}{a}\right)$$

$$= x - \frac{\alpha x^2}{a}$$ **Ans.**

(b)

Position vector is

$\vec{r} = at\hat{i} + at(1-\alpha t)\hat{j}$

For velocity

$$\frac{d\vec{r}}{dt} = a\hat{i} + a(1-2\alpha t)\hat{j}$$

$$\left|\frac{d\vec{r}}{dt}\right| = a\sqrt{1 + (1-2\alpha t)^2}$$ **Ans.**

For acceleration

$$\frac{d\bar{v}}{dt} = -2a\alpha\ \hat{j}$$

$$\left|\frac{d\bar{r}}{dt}\right| = a\sqrt{1+(1-2\alpha t)^2}$$

(c)

$$\cos\theta = \hat{v}.\hat{a}$$

$$\cos\left(\frac{\pi}{4}\right) = \hat{v}.\hat{a}$$

$t = 1/\alpha$ **Ans.**

OR

$$-\tan\pi/4 = \frac{a}{a-2a\ \alpha t}$$

$- a - 2a\ \alpha t$

$t = 1/\alpha$ **Ans.**

1.26

(a)

$x = a \sin wt$

$v_x = \frac{dx}{dt} = a\omega\cos\omega t$ -------(i)

$y = a\ (1 - \cos wt)$

$v_y = \frac{dy}{dt} = a\omega\sin\omega t$ --------(i)

Squaring of both equations then add

$$v = \sqrt{v_x^2 + v_y^2} = a\omega$$

Since velocity is constant in magnitude

Distance travelled in time τ is = $(aw)\tau$ **Ans.**

Equation of circle of radius a. $\bar{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

(b)

Since motion is uniform circular motion.

Hence only radial acceleration is present

then angle b/w velocity vector and acceleration

will be $\frac{\pi}{2}$ **Ans.**

1.27

Method:(Comparing trajectory equation)

$y = ax - bx^2$

Compare with equation of trajectory

$$y = x\tan\theta + \frac{wx^2}{2u^2\cos^2\theta}$$

$\tan\theta = a$ then $\cos\theta = \frac{1}{\sqrt{1+a^2}}$

$\frac{w}{2u^2\cos^2\theta} = b$ then $\frac{w}{2b\cos^2\theta} = u^2$

$$u = \sqrt{\frac{w}{2b\cos^2\theta}}$$

Put value of $\cos\theta = \frac{1}{\sqrt{1+a^2}}$ in above eq.

$u = \sqrt{\frac{w}{2b}(1+a^2)}$ **Ans.**

Method:2(Diffrentiation method)

$y = ax - bx^2$

Differentiate w.r.t. time

$V_y = \frac{dy}{dt} = a\frac{dx}{dt} - b2x\frac{dx}{dt}$

$V_y = aV_x - b2xV_x$ ....(i)

At $x = 0$

$V_y = a\,V_x$

Differentiate equation (i) w.r.t. time:

$$a_y = \frac{dV_y}{dt} = a\frac{dV_x}{dx} - 2bx\frac{dV_x}{dt} - 2bV_x^2$$

$a_y = a\,a_x - 2bx\,a_x - 2b\,V_x^2$

At $x = 0$

Given $a_x = 0$ and $a_y = w$

$w = |-2b\,V_x^2|$

$w = 2b\,V_x^2$ ....(ii)

Speed at origin will be :

$V_x^2 = \frac{w}{2b}$

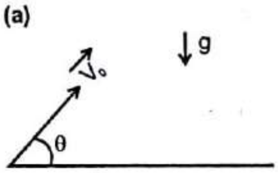
$V_y = a\,V_x$

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{\frac{w}{2b} + \frac{a^2w}{2b}}$$

$V = \sqrt{\frac{w}{2b}(1+a^2)}$ **Ans.**

1.28

(a)



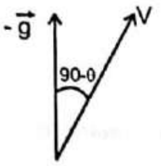
$$\bar{s} = \bar{u}t + \frac{1}{2}\bar{a}t^2$$

$\vec{s} = \vec{V}_0 t + \frac{1}{2}\vec{g}t^2$ **Ans.**

(b)

$< \vec{v} > = \frac{\vec{s}}{t}$

$< \vec{v} > = \vec{v}_0 + \frac{1}{2}\vec{g}t$ ....(1) **Ans.**



we know

$T = \frac{2u\sin\theta}{g}$

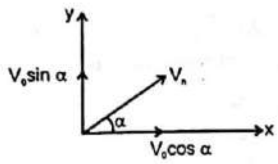
$u\sin\theta = \frac{-\vec{v}_0 \cdot \vec{g}}{g}$

From (1)

$< \vec{v} > = \vec{v}_0 + \frac{\vec{g}}{2}\left(\frac{-\vec{v}_0 \cdot \vec{g}}{g}\right)$

$< \vec{v} > = \vec{v}_0 - \vec{g}\frac{+\vec{v}_0 \cdot \vec{g}}{g^2}$ **Ans.**

1.29



(a) S = 0 in y direction in time period T

$0 = (V_0 \sin\alpha)T - \frac{1}{2}gT^2$

$T = \frac{2V_0 \sin\alpha}{g}$ **Ans.**

(c) At maximum height final velocity in y direction will be zero

$V_y^2 = u_y^2 + 2aH$

$0^2 = (V_0 \sin\alpha)^2 - 2gH$

$H = \frac{V_0^2 \sin^2\alpha}{2g}$ **Ans.**

Range(R) = $V_0 \cos\alpha \times T$

$= V_0 \cos\alpha \left(\frac{2V_0 \sin\alpha}{g}\right)$

$R = \frac{V_0^2 \sin 2\alpha}{g}$

When H = R

$\frac{V_0^2 \sin^2\alpha}{2g} = \frac{V_0^2 \sin 2\alpha}{g}$

$\frac{V_0^2 \sin^2\alpha}{2g} = \frac{2V_0^2 \sin\alpha \cos\alpha}{g}$

$\alpha = \tan^{-1}(4) = 76^0$ **Ans.**

(c)

$x = (v_0 \cos\alpha)t$

$t = \frac{x}{V_0 \cos\theta}$

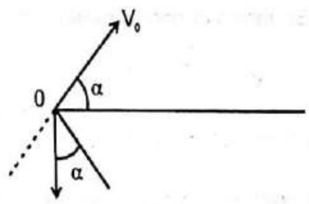
$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$ ....(1)

Put value of t in equation (1)

$y = v_0 \sin\alpha \left(\frac{x}{v_0 \cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos\alpha}\right)^2$

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ **Ans.**

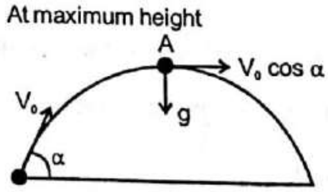
(d)



Radius of curvature = $\frac{V^2}{\text{normal acceleration}}$

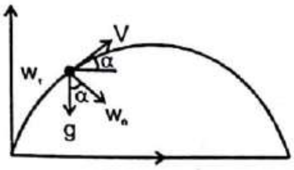
At projection point $R_0 = \frac{V_0^2}{g\cos\alpha}$

At maximum height



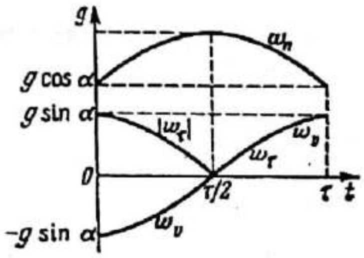
$R_A = \frac{V_0^2 \cos^2\alpha}{g}$ **Ans.**

1.30

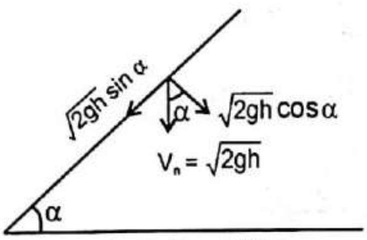


$w_t = g \sin \alpha$ and $w_n = g \cos \alpha$

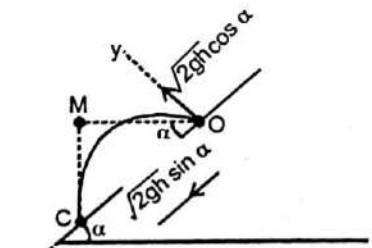
Here α is first dicreasing and then increasing. Projection of total acceleration on velocity vector will be (-)ive then $W_y = \vec{w}_1 . \vec{v} = -g \sin\alpha$



1.31 **Method:1(Analytical)**



Just before collision



Just after collision

Time to collide with incline

$$T = \frac{2u_y}{g\cos\alpha} = \frac{2\sqrt{2gh}\cos\alpha}{g\cos\alpha} = \frac{2\sqrt{2h}}{\sqrt{g}}$$

Length MO = (velocity in MO direction) x T

$$= \left(\sqrt{2gh}\cos\alpha\sin\alpha + \sqrt{2gh}\sin\alpha\cos\alpha\right) \times T$$

$$= \left(2\sqrt{2gh}\cos\alpha\sin\alpha\right) \times \frac{2\sqrt{2h}}{\sqrt{g}}$$

$$= 8h\cos\alpha\sin\alpha$$

Range(OC)=(MO)/cosα

$$= \frac{MO}{\cos\alpha} = \left(\frac{8h\ \cos\alpha\sin\alpha}{\cos\alpha}\right)$$

$R = 8h \sin \alpha$ **Ans.**

Method:2(Equation of kinematics)

Time to collide with incline

$$T = \frac{2u_y}{g\cos\alpha} = \frac{2\sqrt{2gh}\cos\alpha}{g\cos\alpha} = \frac{2\sqrt{2h}}{\sqrt{g}}$$

Write displacement eqaution in direction of incline.

$$S = ut + \frac{1}{2}at^2$$

$$S = (\sqrt{2gh}\sin\alpha)\frac{2\sqrt{2h}}{\sqrt{g}} + \frac{1}{2}(g\sin\alpha)\left[\frac{2\sqrt{2h}}{\sqrt{g}}\right]^2$$

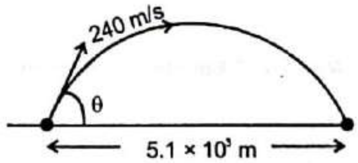
$R = 8h \sin \alpha$ **Ans.**

1.32.

We know $R = \dfrac{u^2 \sin 2\theta}{g}$

$$5.1 \times 10^3 = \frac{(240)^2 \sin^2\theta}{g}$$

$\theta_1 = 32.5°$.



Also we know that range will be same for $\theta_2 = 90 - 31.5 = 59.5°$

Then time of flight will be

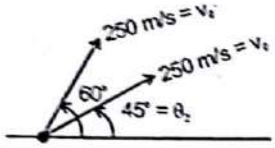
$$T_1 = \frac{2u\sin\theta}{g}$$

$$T_1 = \frac{2\times 240\sin 31.5}{10} = 24.3s = 0.41 \text{ min } \textbf{Ans.}$$

$$T_2 = \frac{2\times 240\sin 59.5}{10} = 42.3s = 0.69 \text{ min } \textbf{Ans.}$$

1.33

Method : 1(Using trajectory equation)



$\theta_1 = 60°$; $\theta_2 = 45°$

For both particles collision, x and y co-ordinate must be same then

Particle (i)

$$y = x\tan\theta_1 - \frac{gx^2}{2V_0^2\cos^2\theta_1} \quad ....(i)$$

Particle (ii)

$$y = x\tan\theta_2 - \frac{gx^2}{2V_0^2\cos^2\theta_2} \quad ....(ii)$$

Equating both (i) and (ii)

$$x\tan\theta_1 - \frac{gx^2}{2V_0^2\cos^2\theta_1} = x\tan\theta_2 - \frac{gt}{2V_0^2\cos^2\theta_2}$$

$$x = \frac{2V_0^2}{g}\frac{\sin(\theta_1-\theta_2)\cos\theta_1\cos\theta_2}{\cos^2\theta_1-\cos^2\theta_2} \quad ....(ii)$$

Timefor particle (i) : $t_1 = \dfrac{x}{V_0\cos\theta_1}$

Time for particle (ii) : $t_2 = \dfrac{x}{V_0\cos\theta_2}$

$$\Delta t = t_1 - t_2 = \frac{x}{V_0}\left[\frac{1}{\cos\theta_1} - \frac{1}{\cos\theta_2}\right] \quad ....(iv)$$

Put value of x on (iv) :

$$\Delta t = \frac{2V_0}{g}\left[\frac{\sin(\theta_1-\theta_2)}{\cos\theta_1+\cos\theta_2}\right] = 11s \quad \textbf{Ans.}$$

Method : 2(Equation of motion)

Particle (1) :

$x = V_0\cos\theta_1(t+\Delta t)$ ....(i)

$y = V_0\sin\theta_1(t+\Delta t) - \dfrac{1}{2}g(t+\Delta t)^2$ ......(ii)

Particle (2) :

$x = V_0\cos\theta_2(t)$ ....(iii)

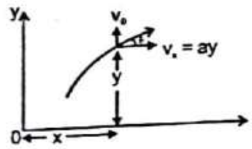
$y = V_0\sin\theta_2(t) - \dfrac{1}{2}gt^2$ ....(iv)

From (i), (ii), (iii) and (iv) :

$$\Delta t = \frac{2V_0}{g}\left[\frac{\sin(\theta_1-\theta_2)}{\cos\theta_1+\cos\theta_2}\right] = 11s \quad \textbf{Ans.}$$

1.34

(a)



When particle will be at height y, angle made by particle with horizontal will be θ

$$\tan\theta = \frac{V_0}{ay}$$

Since velocity in y direction is constant,time to reach at hight y :

$t = \dfrac{y}{V_0}$ ......(1)

$y = V_0 t$ ....(2)

Also $\dfrac{dx}{dt} = ay$

From equation(2)

$$\frac{dx}{dt} = a[V_0 t]$$

$$\int_0^x dx = aV_0\int_0^t t\,dt$$

$$x = \frac{aV_0t^2}{2}$$

From equation(1)

$$x = \frac{aV_0}{2}\left[\frac{y}{V_0}\right]^2$$

$$x = \left(\frac{a}{2V_0}\right)y^2 \quad \textbf{Ans.}$$

(b)

$a_y = 0$

$a_x = \dfrac{dV_x}{dt}$ ....(3)

Also we know $V_x = \dfrac{dx}{dt} = ay$

Put in (3)

$a_x = \frac{a\,dy}{dt} = aV_y = aV_0$

$a_{net} = aV_0$ **Ans.**

Tangential acceleration = a_τ
$= aV_0\cos\theta$

$$a_\tau = \frac{aV_0(ay)}{\sqrt{V_0^2 + (ay)^2}} = \frac{a^2y}{\sqrt{1+(ay/V_0)^2}}$$ **Ans.**

Radial acceleration or normal acceleration :

$$a_n = a\,V_0 \sin\theta = \frac{aV_0}{\sqrt{1+\left(\frac{ay}{V_0}\right)^2}}$$ **Ans.**

1.35

Since horizantal velocity is constant and equal to a then

$x = at$ ....(1)

$V_y = xb$

$V_y = (a\,t)\,b$

$$\frac{dy}{dt} = (a\,t)\,b$$

$$\int_0^y dy = ab\int_0^t t\,dt$$

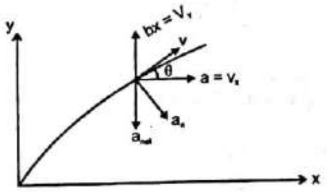
$$y = \frac{abt^2}{2}$$

Put value of t from equation(1)

$$y = \frac{ab}{2}\left(\frac{x}{a}\right)^2 \qquad y = \left(\frac{b}{2a}\right)x^2$$

Ans

(b)



$a_x = 0$

$$a_y = \frac{dV_y}{dt} = b\frac{dx}{dt} = ab$$

$a_{net} = ab$

Then normal acceleration

$a_n = a_{net}\cos\theta$

$a_{net} = ab$

Also we know

$$\tan\theta = \frac{A}{bx}$$

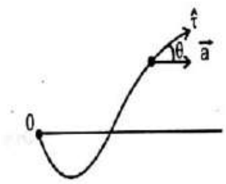
$$a_n = ab\,\frac{a}{\sqrt{a^2 + (bx)^2}}$$

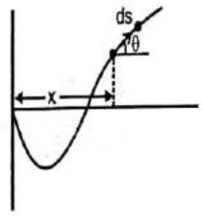
$$R = \frac{V^2}{a_n} = \frac{(a^2 + b^2x^2)^{3/2}}{a^2b}$$

$$R = \frac{a}{b}\left[1+\left(\frac{xb}{a}\right)^2\right]^{3/2}$$ **Ans**

1.36

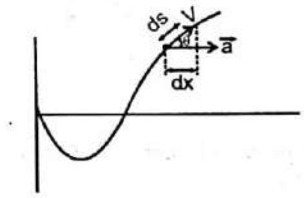
Method : 1 (Work - Energy Theorem)





Tangential acceleration is given by:

$|\vec{w}_\tau| = \vec{a}.\hat{\tau} = a\cos\theta$



Suppose at time t particle at posetion A and in small time dt particle displace by ds.

Work done by force

$dw = F_\tau ds = = m|w_\tau|\,ds = (ma\cos\theta)\,ds$

$= ma\,(ds\cos\theta)$

$$\int dw = \int ma\,(ds\cos\theta)$$

$$\Delta W = ma\int(ds\cos\theta) = ma\,(\Sigma ds\cos\theta) \;.....(1)$$

$(\Sigma ds\cos\theta) = x$ because it is summation of displacement in x direction.

$\Delta W = max$

Using work energy theorem

$$\Delta W = \frac{1}{2}mV^2$$

$$\max = \frac{1}{2} mV^2$$

$$V = \sqrt{2ax}$$ **Ans.**

Method : 2 (Kinematics)

Tangential acceleration is given by:

$|\vec{w}_\tau| = \vec{a}.\vec{\tau} = a\cos\theta$

We know that tangential acceleration is rate of change of speed then.

$$\frac{d|v|}{dt} = a \cos\theta$$

$$\frac{vdv}{ds} = a\cos\theta \Rightarrow$$

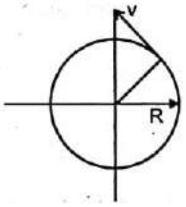
$$v\,dv = a\,(ds)\cos\theta$$

$$\int_0^v v\,dv = \int_0^x a\,(ds)\cos\theta = \int_0^x a dx$$

$$\frac{v^2}{2} = ax$$

$$v = \sqrt{2ax}$$ **Ans.**

1.37



Angle travel by particle is θ then

$\theta = 2\pi n$

Speed of particle

$v = Rw = at$ ....(1)

$$w = \frac{at}{R} = \frac{d\theta}{dt}$$

$$\int_0^{2\pi n} d\theta = \int_0^t \frac{at}{R} dt$$

$$2\pi n = \frac{at^2}{2R}$$

$$t^2 = \frac{4\pi nR}{a} \text{ ....(2)}$$

Radial acceleration

$$a_r = \frac{V^2}{R}$$

From (1) and(2)

$$\frac{V^2}{R} = \frac{a^2t^2}{R} = \frac{a^2}{R}\frac{4\pi nR}{a} = 4\pi na$$

$a_r = 4\pi na$

Tangential acceleration

$$a_t = \frac{dv}{dt} = a$$

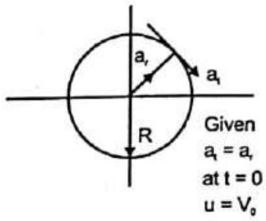
Total acceleration= $\sqrt{a_r^2 + a_t^2} = \sqrt{a^2 + (4\pi na)^2}$

Total acceleration = $a\sqrt{1+(4\pi n)^2} = 0.8 \text{ m/s}^2$

Ans.

1.38

(a)



At any time t

$$a_t = a_r = \frac{V^2}{R}$$

Since a is rate of change of speed then.

$$-\frac{dv}{dt} = \frac{V^2}{R}$$

$$\int_{V_0}^{V} -V^{-2}dV = \int_0^t \frac{dt}{R}$$

(–) sign because V is decreasing

$$V^{-1}\Big|_{V_0}^{V} = \frac{t}{R}$$

$$V = \frac{V_0}{1+\frac{V_0 t}{R}}$$ **Ans.**

(b)

$$a_{net} = \sqrt{a_t^2 + a_r^2} = a_t\sqrt{2}$$

$$a_t = \frac{v^2}{R} = -v\frac{dv}{ds}$$

$$-v\,dv = \frac{v^2}{R}dt$$

Where ds = distance travel by particle in time dt.

$$\int_{V_0}^{V} -\frac{dv}{v} = \int_0^s \frac{ds}{R}$$

$\ell n\frac{V}{V_0} = -\frac{S}{R}$ Then $V = V_0\, e^{-S/R}$

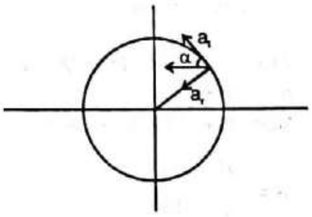
$$a_t = \frac{v^2}{R} = \frac{V_0^2 e^{-2S/R}}{R}$$

$$a_{net} = \frac{V^2}{R}\sqrt{2} = \frac{V_0^2\sqrt{2}e^{-2S/R}}{R}$$

Ans.

1.39

Method : 1



$v = a\sqrt{s}$

Squaring both side

$v^2 = a^2 s$

Compare this equation with $v^2 = u^2 + 2a_{ce}S$

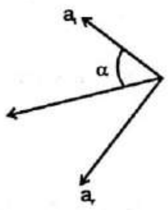
$u = 0$, $a_{ce} = \frac{1}{2}a^2$

Hence motion is constant magnitude tangential acceleration.

At any time t

$a_t = \frac{1}{2}a^2$ $a_r = \frac{V^2}{R} = \frac{a^2 s}{R}$

Since we know that velocity vector and tangential acceleration is parallel then.



$$\tan\alpha = \frac{a_r}{a_t} = \frac{\frac{a^2 s}{R}}{\frac{a^2}{2}} = \frac{2S}{R}$$ **Ans.**

Method : 2

$v^2 = a^2 s$

Differentiate this equation w.r.t time :

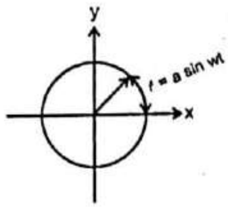
$$2v\frac{dv}{dt} = a^2\frac{ds}{dt} = a^2 v$$

$$\frac{dv}{dt} = a^2/2 = a_t$$

$$a_r = \frac{V^2}{R} = \frac{a^2 s}{R}$$

$$\tan\alpha = \frac{a_r}{a_t} = \frac{2S}{R}$$ **Ans.**

1.40



(a) $\ell = a \sin wt$

$$V = \frac{d\ell}{dt} = a\,w\cos wt$$

Tangential acceleration = $-a\,w^2 \sin wt$

$a_t = -aw^2 \sin wt$

$$a_r = \frac{V^2}{R} = \frac{a^2 w^2 \cos^2 wt}{R}$$

$$a_{net} = \sqrt{a_t^2 + a_r^2}$$

$$a_{net} = aw^2\sqrt{\sin^2 wt + \frac{a^2}{R^2}\cos^4 wt} \quad .....(i)$$

Now at $\ell = 0$ $t = 0$

$$a_{net} = \frac{a^2 w^2}{R}$$ **Ans.**

At $\ell = \pm a$ $t = \frac{\pi}{2w}, \frac{3\pi}{2w}$

$a_{net} = aw^2$ **Ans.**

(b)

For minimum value of acceleration, differentiate equation(i)

$$\frac{da_{net}}{dt} = 0$$

$$2w\sin wt\cos wt-\frac{a^2}{R^2}4w\cos^3 wt\sin wt=0$$

$$\cos wt=\frac{R}{\sqrt{2}\,a}$$

Put in equation ...... (i)

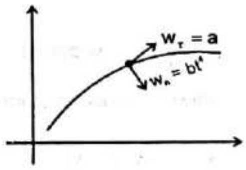
$$a_{min}=aw^2\sqrt{1-\left(\frac{R}{\sqrt{2}a}\right)^2+\frac{a^2}{R^2}\frac{R^4}{4a^4}}$$

$$=aw^2\sqrt{1-\left(\frac{R}{2a}\right)^2}$$ **Ans.**

$$\ell_m=a\sqrt{1-\left(\frac{R}{\sqrt{2}a}\right)^2}$$

$$\ell=a\sqrt{1-\frac{R^2}{2a^2}}$$ **Ans.**

1.41



Speed of particle when distance is S
$V^2 = 2aS$ ....(i)
Since $w_\tau = a_\tau = a$=const.

$$S=\frac{1}{2}at^2$$

$$t^2=\frac{2S}{a}$$

Then $$w_n=b\left[\frac{2S}{a}\right]^2=\frac{4bS^2}{a^2}$$

Radius of curvature:

$$R=\frac{V^2}{w_n}=\frac{2aSa^2}{4bS^2}\Rightarrow R=\frac{a^3}{2bS}$$ **Ans.**

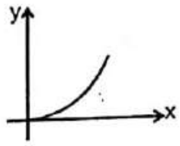
Net acceleration:

$$w=\sqrt{a_T^2+w_n^2}$$

$$w=\sqrt{a^2+\left(\frac{4bS^2}{a^2}\right)^2}$$ **Ans.**

1.42

(a) **Method :1(Conceptual)**



$y = ax^2$
Differentiate equation w.r.t. time

$$\frac{dy}{dt}=2ax\frac{dx}{dt}$$

$V_y = 2ax\, V_x$
Again differentiate w.r.t. time
$a_y = 2ax\, a_x + 2aV_x^2$

At x = 0, $V_y = 0$ Then $V_x = v$
$a_y = 2aV^2$
Normal acceleration=$2aV^2$

$$R=\frac{V^2}{a_n}=\frac{V^2}{2av^2}$$

$$R=\frac{1}{2a}$$ **Ans.**

Method :1(Formula based)

$$R=\left|\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}\right|$$

$y = ax^2$
Differentiate equation w.r.t. x

$$\frac{dy}{dx}=2ax$$

Again differentiate equation w.r.t. x

$$\frac{d^2y}{dx^2}=2a$$

Put values in radius of curvature formula

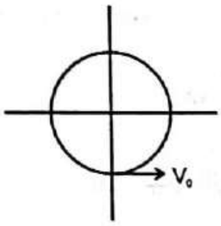
$$R=\left|\frac{\left(1+(2ax)^2\right)^{3/2}}{2a}\right|$$

Put x=0

$$R=\frac{1}{2a}$$ **Ans.**

(b)

Method :1(Conceptual)



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Differentiate equation w.r.t. time

$$\frac{2xV_x}{a^2}+\frac{2yV_y}{b^2}=0$$

Again differentiate w.r.t. time :

$$\frac{2xa_x}{a^2}+\frac{2V_x^2}{a^2}+\frac{2ya_y}{b^2}+\frac{2V_y^2}{b^2}=0$$

At $x = 0$ and $y = b$

$$\frac{2V_x^2}{a^2}+\frac{2ba_y}{b^2}+\frac{2V_y^2}{b^2}=0$$

As shown in figure

$V_y = 0$ and $V_x = V_0$

$$\frac{2V_0^2}{a^2}+\frac{2a_y}{b}=0$$

$$a_y=\frac{-b}{a^2}V_0^2$$

Normal acceleration= $\frac{-b}{a^2}V_0^2$

$$a_n=\frac{b}{a^2}V_0^2$$

Radius of curvature : $R=\frac{V_0^2}{a_n}$

$$R=\frac{V_0^2}{\frac{b}{a^2}V_0^2}$$

$$R=\frac{a^2}{b}$$ **Ans.**

Method :1(Formula based)

$$R=\left|\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}\right|$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Differentiate equation w.r.t. x

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0 \quad\text{.....................(1)}$$

Again differentiate w.r.t. x

$$\frac{2}{a^2}+\frac{2y}{b^2}\frac{d^2y}{dx^2}+\frac{2}{b^2}\left(\frac{dy}{dx}\right)^2=0 \quad\text{.................(2)}$$

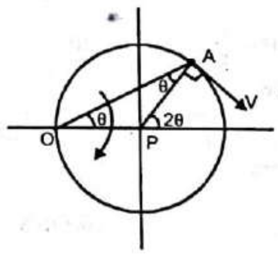
At x=0 , y=b from (1) and (2)

$$\frac{dy}{dx}=0\ ,\ \frac{d^2y}{dx^2}=\frac{-b}{a^2}$$

Put values in radius of curvature formula

$$R=\left|\frac{\left(1+(0)^2\right)^{3/2}}{\frac{-b}{a^2}}\right|=\frac{a^2}{b}$$ **Ans.**

1.43



Given that

Angular velocity of point A w.r.t. point O

$$\frac{d\theta}{dt}=w=\text{constant}$$

Angular velocity of point A w.r.t. point P

$$\frac{d(2\theta)}{dt}=2\frac{d\theta}{dt}=2w=\text{constant}$$

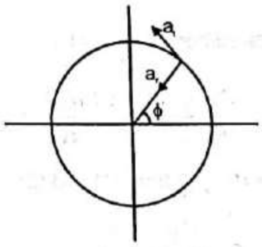
Since angular velocity of point A w.r.t. point P is constnat then.
$W_p = 2W$ and velocity will be perpendicular to position of A w.r.t. P
$V = R W_P = 2RW$ **Ans.**

W_P = constant

$$\frac{dW_P}{dt} = \alpha = 0$$

Tangential acceleration= 0
Radial acceleration = $R W_P^2 = 4 RW^2$
$a_{net} = 4 RW^2$
Direction is toward the centre. **Ans.**

1.44



$\phi = at^2$

$$\frac{d\phi}{dt} = w = 2at$$

$$\frac{dw}{dt} = \alpha = 2a$$

Tangential acceleration
$a_t = R\alpha = 2aR$
Radial acceleration
$a_r = RW^2 = R(2at)^2 = 4a^2t^2R$

$v = RW = R\,2at$

$$2aR = \frac{v}{t} \text{ ....(i)}$$

Now

$$a_{net} = \sqrt{a_t^2 + a_r^2} = 2aR\sqrt{1+(2at^2)^2}$$

From (i)

$$a_{net} = \frac{v}{t}\sqrt{1+(2at^2)^2} = 0.7 \text{ m/s}$$ **Ans.**

1.45

Since acceleration is constant.

$$\ell = \frac{1}{2}at^2$$

$V = at$

$$\ell = \frac{Vt}{2} \text{ .... (i)}$$

Angular acceleration is constant then

$$\theta = \frac{1}{2}\alpha t^2$$

$w = at$

$$\theta = \frac{1}{2}wt$$

Since particle taken n turn in its journey.
$\theta = 2\pi n$

$$2\pi\eta = \frac{1}{2}wt \text{ ....(ii)}$$

$$\text{From } {}^{(I)}\!/_{(II)} : \frac{\ell}{2\pi\eta} = \frac{V}{W}$$

$$W = \frac{2\pi\eta v}{\ell}$$ **Ans.**

1.46

$\phi = at - bt^3$

$$w = \frac{d\phi}{dt} = a - 3bt^2$$

when body is in rest then
$w = 0$
$0 = a - 3bt^2$

$$t = \sqrt{\frac{a}{3b}}$$

$\phi_{in} = 0$

$$\phi_{final} = t(a - bt^2) = \sqrt{\frac{a}{3b}}\left(a - b\frac{a}{3b}\right) = \sqrt{\frac{a}{3b}}\left(\frac{2a}{3}\right)$$

$$W_{avg} = \frac{\phi_{final} - \phi_{in}}{\Delta t} = \frac{\sqrt{\frac{a}{3b}}\left(\frac{2a}{3}\right)}{\sqrt{\frac{a}{3b}}} = \frac{2a}{3}$$ **Ans.**

Also
$W_{in} = a$

$$W_{final} = a - 3b\left(\frac{a}{3b}\right) = 0$$

$$\alpha_{avg} = \frac{W_{final} - W_{in}}{t} = \frac{a}{\sqrt{\frac{a}{3b}}} = \sqrt{3ab}$$ **Ans.**

$$\alpha = \frac{dw}{dt} = -6bt$$

$$= -6b\sqrt{\frac{a}{3b}}$$

$$= 2\sqrt{3ab}$$ **Ans.**

1.47

Given that
$\alpha = \beta = at$
To find tangential acceleration : (a_t)
$a_t = R\alpha = Rat$
To find radial acceleration : (a_r)

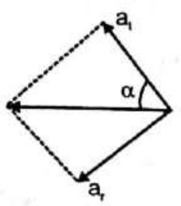
$a_r = Rw^2$

$\alpha = \dfrac{dw}{dt}$

$\int_0^w dw = \int_0^t \alpha\, dt$

$w = \int_0^t at\, dt = \dfrac{at^2}{2}$

$a_r = \dfrac{Ra^2t^4}{4}$.



$\tan\alpha = \dfrac{a_r}{a_t}$

$\tan\alpha = \dfrac{Ra^2t^4}{4Rat}$

$t = \sqrt[3]{\dfrac{4}{a}\tan\alpha} = 7s$ **Ans.**

1.48

Given that

$\beta \propto \sqrt{W}$

β = Angular acceleration

$\beta = k\sqrt{W}$

Where k = constant

$-W\dfrac{dW}{d\theta} = k\sqrt{W}$

(–) ive because W is decreasing.

$\int_{W_0}^{0} W^{1/2} dw = -\int_0^{\theta} k\, d\theta$

$\dfrac{2W_0^{3/2}}{3k} = \theta$ ....(i)

θ = Angular displacement.

Also $\beta = k\sqrt{W}$

$-dW = k\sqrt{W}\ dt$

$\int_{w_0}^{0} W^{-1/2} dw = -\int_0^t k\, dt$

$\dfrac{2W_0^{1/2}}{k} = t$ ....(ii)

Average angular velocity :

$W_{avg} = \langle W\rangle = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2W_0^{3/2}}{3k\left[\dfrac{2W_0^{1/2}}{k}\right]} = \dfrac{W_0}{3}$

Ans.

1.49

(a)

Given that

$W = W_0 - a\phi$ .... (i)

At t = 0

$\phi = 0$

$W = W_0$

Also $\dfrac{d\phi}{dt} = W_0 - a\phi$

$\int_0^{\phi} \dfrac{d\phi}{W_0 - a\phi} = \int_0^t dt$

$\dfrac{-1}{a}\ell n\ (W_0 - a\phi)\Big|_0^{\phi} = t$

$\ell n\dfrac{W_0 - a\phi}{W_0} = -at$

$\phi = \dfrac{W_0}{a}(1 - e^{-at})$ **Ans.**

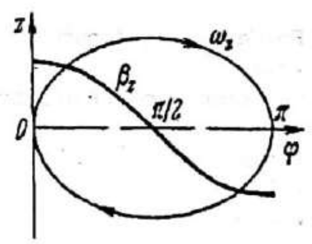
(b)

Put value of ϕ in equation (i) :

$W = W_0 - a\left[\dfrac{W_0}{a}(1 - e^{-at})\right]$

$W = W_0 e^{at}$ **Ans.**

1.50



Given that

$\beta = \beta_0 \cos\phi$

$\beta = W\dfrac{dW}{d\phi} = \beta_0 \cos\phi$

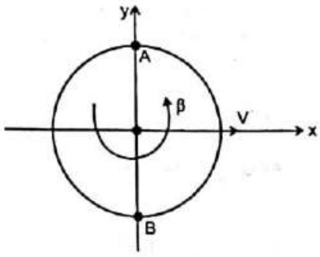
$\int_0^w W\, dw = \beta_0 \int_0^{\phi} \cos\phi\ d\phi$

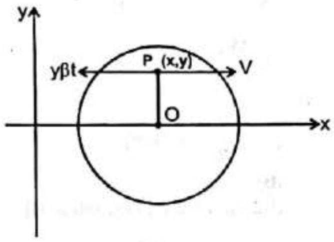
$$\frac{W^2}{2} = \beta_0 \sin\phi$$

$$W = \pm\sqrt{2\beta_0 \sin\phi}$$ **Ans.**

1.51

(a)





Instanteneous axis of rotation is passing through that point which velocity is zero always. If we observe carefully it point must be at line joining AB.

Then. at time t :

$V_{P-0} = RW = R\beta t = y\beta t$

Position of cente at time t:

$x = vt$ ....(i)

Net velocity of point P as instantaneous centre of rotation will be zero:

$V_P = 0$

$v - \beta\, ty = 0$

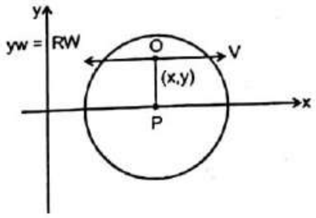
$v = \beta\, ty$

$t = \dfrac{v}{\beta y}$ put in (i)

$x = v\dfrac{v}{\beta y}$

$y = \dfrac{v^2}{\beta x}$ **Ans.**

(b)



$V_{O-P} = Rw = yw$.

$V_P = u + at = Wt$

Velocity of point O = 0

$yw - Wt = 0$

Then $yw = Wt$

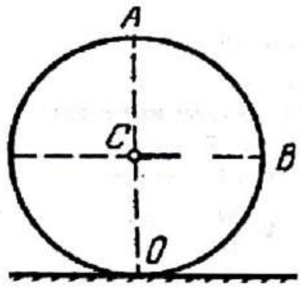
$t = \dfrac{wy}{W}$

Now $x = \dfrac{1}{2}at^2 = \dfrac{1}{2}W\left(\dfrac{wy}{W}\right)^2$

$x = \dfrac{1}{2}\dfrac{w^2y^2}{W}$ **Ans.**

1.52

(a)



Since there is no slipping at ground.

$V_o = 0$

$V_c = RW$ where V_o = velocity of contact point.

Acceleration of point A :

$a_{A-E} = a_{A-C} + a_{C-E}$ ....(1)

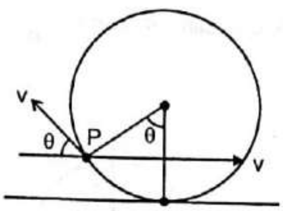
$a_{A-C} = R\omega^2 = \dfrac{V^2}{R}$

$a_{C-E} = 0$

Put in equation(1)

$a_{A-E} = R\omega^2 = \dfrac{V^2}{R}$ **Ans.**

(b)



To find distance, we have to find speed of a particle of rim at a tme t
$\theta = wt$

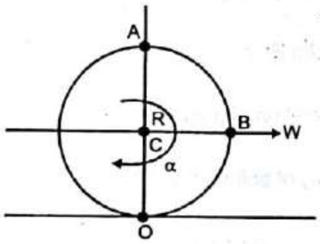
$V_P = \sqrt{v^2 + v^2 + 2vv\cos(\pi - \theta)}$
$= 2\,v\,\sin\theta/2 \;=\; 2V\sin(wt)/2$
$V_P = 2V\sin(wt)/2$ ....(2)
Time to one complets journey $= 2\pi/w$
Again from (2)
$V_P = 2\,v\,\sin wt/2$

$\frac{ds}{dt} = 2\,v\,\sin wt/2$

$\int_0^s ds = 2v\int_0^{2\pi/w} \sin\frac{wt}{2}\,dt$

$S = \frac{8v}{w} = 8R$ **Ans.**

1.53

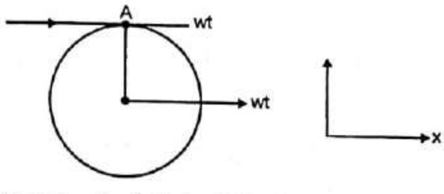


Acceleration of point C :
$a_c = w$
Velocity of point C at time t :
$V_c = u + at = wt$
Angular acceleration of ball:

$\alpha = {}^{a_c}\!/_R = \frac{w}{R}$

Angular velocity of ball at time t :
$w = w_0 + \alpha t = \alpha t$
(a)

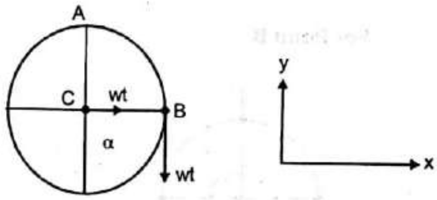
Acceleration Calculations
For Point A :



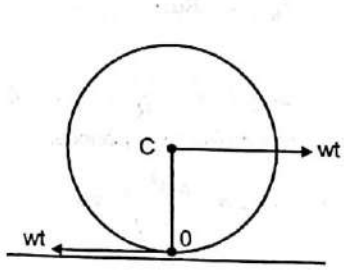
Velocity of point A w.r.t. centre C :

$V_{A-C} = R\omega = R\alpha\,t = Rt\frac{w}{R} = w\,t$
$V_{A-E} = V_{A-C} + V_A = 2\,w\,t$ **Ans.**

For Point B :



$V_{B-C} = R\omega = wt\,(-j)$
$V_C = wt\,i$
$V_{B-E} = V_{B-C} + V_C = wt\,(i - j)$
$V_{B-B} = wt\sqrt{2}$ **Ans.**
For Point O :

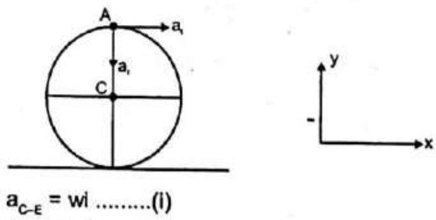


$V_{O-C} = -\,wt\,i$
$V_{C-E} = wt\,i$
$V_{O-E} = V_{O-C} + V_{C-E} = 0$ **Ans.**

(b)

Acceleration Calculations :

For Point A:



$a_{C-E} = wi$ ....(i)
$a_{A-C} = a_t\,i - a_r\,j$

Where a_t = tangential acceleration

a_r = radial acceleration. $= \dfrac{v^2}{R} = \dfrac{(wt)^2}{R} = \dfrac{w^2t^2}{R}$

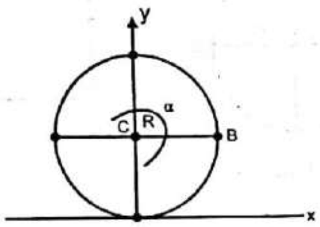
$$a_{A-C} = wi - \frac{w^2t^2}{R}\,j \text{ ....(ii)}$$

Adding (i) + (ii) equations:

$$a_{A-E} = 2wi - \frac{w^2t^2}{R}\,j$$

$$a_{A-E} = 2w\sqrt{1+\left(\frac{wt^2}{2R}\right)^2}$$ **Ans.**

For Point B:



$$\bar{a}_{C-E} = wi \quad \text{........(i)}$$

$$\bar{a}_{B-C} = (-R\alpha)\hat{j} - \left(\frac{v^2}{R}\right)\hat{i}$$

$$\bar{a}_{B-C} = -w\hat{j} - \frac{w^2t^2}{R}\hat{i} \text{ .... (ii)}$$

Adding (i) + (ii) equations:

$$\bar{a}_B = \left(w - \frac{w^2t^2}{R}\right)\hat{i} + w\hat{j}$$

$$a_B = w\sqrt{1+\left(1-\frac{wt^2}{2R}\right)^2}$$ **Ans.**

For Point O:

$$\bar{a}_{C-E} = wi \quad \text{........(i)}$$

$$\bar{a}_{O-C} = -w\hat{i} + \left(\frac{v^2}{R}\right)\hat{j} \text{ ....(ii)}$$

$$\frac{v^2}{R} = \frac{(wt)^2}{R} = \frac{w^2t^2}{R}$$

Adding (i) + (ii) equations:

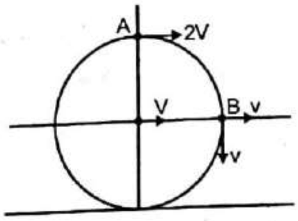
$$\bar{a}_{O-E} = \frac{w^2t^2}{R}\hat{j}$$

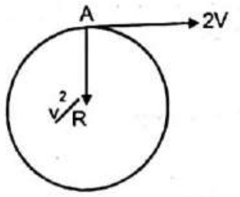
$$a_{O-E} = \frac{w^2t^2}{R}$$ **Ans.**

1.54

Velocity of point A = 2v

Acceleration of point A = $\dfrac{v^2}{R}$





Radius of curvature $= \dfrac{(\text{speed})^2}{\text{normal acceleration}}$

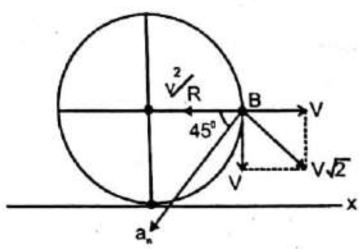
For point A:

$$R_A = \frac{(2V)^2}{V^2/R} = 4R$$ **Ans.**

For point B:

Acceleration of point B = $\dfrac{v^2}{R}$

Velocity of point B = $v\sqrt{2}$



Normal acceleration of point B:

$$a_n = \frac{v^2}{R}\cos 45^\circ = \frac{v^2}{\sqrt{2}R}$$

$$R_B = \frac{(V\sqrt{2})^2}{\dfrac{v^2}{\sqrt{2}R}} = 2\sqrt{2}R$$ **Ans.**

1.55*

Method :1 (axis is rotating)

Relative angular velocity calculation

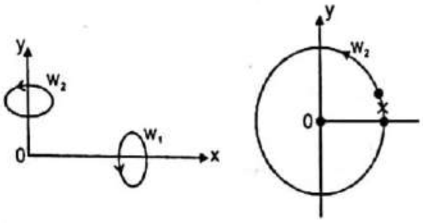
$\bar{w}_{1-0} = w_1\hat{i}$

$\bar{w}_{2-0} = w_2\hat{j}$

$\bar{w}_{1-2} = w_1\hat{i} - w_2\hat{j}$

$w_{1-2} = \sqrt{w_1^2 + w_2^2}$ **Ans.**

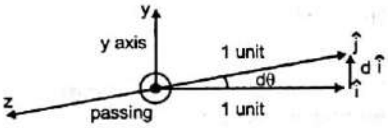
Relative angular acceleration calculation



$$\bar{\alpha}_{1-2} = \frac{d\bar{w}_{1-2}}{dt} = w_1\frac{d\hat{i}}{dt} - w_2\frac{dj}{dt} \quad ....(1)$$

Direction of $\frac{d\hat{i}}{dt}$ is toward Z axis and then this rate of change of direction of x axis.

Here x axis is directly attached with obsever then



$|d\hat{i}| = 1 \times d\theta$

$\frac{|d\hat{i}|}{dt} = \frac{d\theta}{dt} = w_2$

$\frac{d\hat{i}}{dt} = -w_2\hat{k}$

But because with frame of this it is observed that direction of axis does not rotating then

$\frac{d\hat{j}}{dt} = 0$

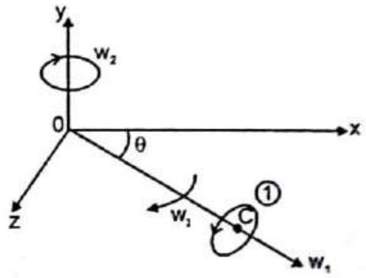
From(1)

$\bar{\alpha}_{1-2} = -w_1 w_2\ \hat{k}$

$\alpha_{1-2} = w_1 w_2$ **Ans.**

Method : 2 (axis is not rotating)

Relative angular velocity calculation



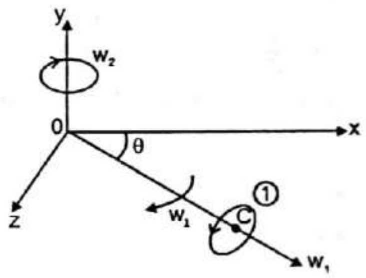
$\bar{w}_{1-C} = w_1\cos\theta\hat{i} + w_1\sin\theta\ \hat{k}$

$\bar{w}_{C-0} = w_2\hat{j}$

$\bar{w}_{1-0} = w_1\cos\theta\ \hat{i} + w_1\sin\theta\hat{k} + w_2\ \hat{j}$ -----(2)

$|\bar{w}_{1-0}| = \sqrt{w_1^2 + w_2^2}$

Relative angular acceleration calculation



$$\bar{\alpha} = \frac{d\bar{w}_{1-0}}{dt} = -w_1\sin\theta\left(\frac{d\theta}{dt}\right)\hat{i} + w_1\cos\theta\left(\frac{d\theta}{dt}\right)\hat{k}$$

$\frac{d\theta}{dt} = w_2$

$\bar{\alpha} = -w_1 w_2 \sin\theta\ \hat{i} + w_1 w_2 \cos\theta\hat{k}$

$|\bar{\alpha}| = w_1 w_2$ **Ans.**

1.56

$\bar{w} = at\ \hat{i} + b\,t^2\hat{j}$

$\bar{\beta} = \frac{d\bar{w}}{dt} = a\hat{i} + 2bt\,\hat{j}$

(a)

$|\bar{w}| = at\sqrt{1+\left(\frac{b}{a}t\right)^2}$ **Ans.**

(b)

$|\bar{\beta}| = a\sqrt{1+\left(\frac{2bt}{a}\right)^2}$

$\vec{w}.\vec{\beta} = w\beta \cos\theta$

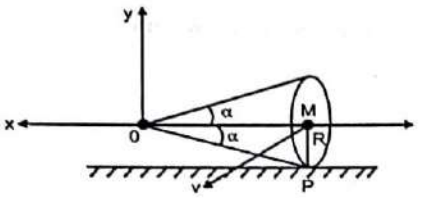
$$\cos\theta = \frac{\vec{w}.\vec{\beta}}{w\beta}$$

$$\cos\theta = \frac{a^2t + 2b^2t^3}{\sqrt{a^2t^2 + b^2t^4}\sqrt{a^2 + 4b^2t^2}}$$

$\theta = 17^\circ$ **Ans.**

1.57*

Method :1 (axis is rotating)
Angular velocity of cone calculation



We see carefully then x axis is rotating while y direction is not rotating.
Angular velocity of disc with respect to centre M is :

$$\vec{W}_{D-M} = \frac{V}{R}\hat{i} \text{ ....(i)}$$

Angular velocity of centre M w.r.t. origin.

$$\vec{W}_{M-O} = \frac{V}{OM}\hat{j}$$

$OM = R \cot\alpha$

$$\vec{W}_{M-O} = \frac{V}{R}\tan\alpha\,\hat{j}$$

Since angular velocity is vector quantity it follow vector addition law

$$\vec{W}_{D-O} = \vec{W}_{D-M} + \vec{W}_{M-O}$$

$$\vec{W}_{D-O} = \frac{V}{R}[\hat{i} + \tan\alpha\,\hat{j}] \text{..........(1)}$$

$$|\vec{W}_{D-O}| = \frac{V}{R}\sqrt{1+\tan^2\alpha} = \frac{V}{R\cos\alpha}$$ **Ans.**

Angular acceleration calculation
Angular acceleration (β) is

$$\vec{\beta} = \frac{d\vec{w}}{dt}$$

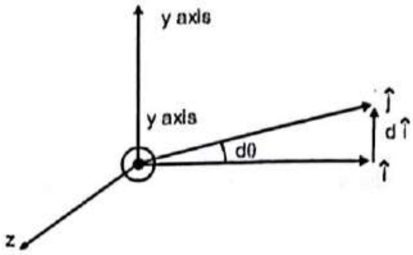
From equation(1)

$$\vec{\beta} = \frac{V}{R}\left[\frac{d\hat{i}}{dt} + \tan\alpha\frac{d\hat{j}}{dt}\right]$$

Since y direction is constant

$$\frac{d\hat{j}}{dt} = 0$$

But for $\frac{d\hat{i}}{dt}$:



$|d\hat{i}| = d\theta$

$$\left|\frac{d\hat{i}}{dt}\right| = \frac{d\theta}{dt} = w_{MO}$$

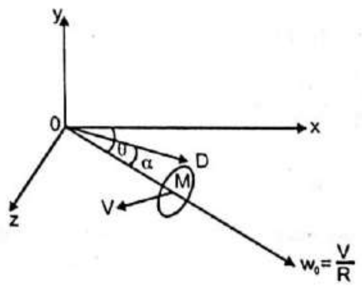
$$\frac{d\hat{i}}{dt} = -\frac{V}{R}\tan\alpha\,\hat{k}$$

$$\vec{\beta} = -\left(\frac{V}{R}\right)\left(\frac{V}{R}\right)\tan\alpha\,\hat{k}$$

$$\vec{\beta} = -\frac{V^2}{R^2}\tan\alpha\,\hat{k}$$

$$|\vec{\beta}| = \frac{V^2}{R^2}\tan\alpha$$ **Ans.**

Method : 2 (axis is not rotating)
Angular velocity calculation



Angular velocity of disc w.r.t. M :

$$W_{D-M} = \frac{V}{R}\cos\theta\,\hat{i} + \frac{V}{R}\sin\theta\,\hat{j}$$

Angular velocity of M w.r.t. O :

$$\vec{W}_{M-O} = \frac{V}{R\cot\alpha}\hat{j} = \frac{V}{R}\tan\alpha\,\hat{j}$$

Angular velocity of disc w.r.t. O :

$$\vec{W}_{D-O} = \vec{W}_{D-M} + \vec{W}_{M-O}$$

$$\vec{W}_{D-O} = \frac{V}{R}\cos\theta\,\hat{i} + \frac{V}{R}\sin\theta\,\hat{j} + \frac{V}{R}\tan\alpha\,\hat{j} \text{ ......(1)}$$

$$|\vec{W}_{M-O}| = \sqrt{\left(\frac{V}{R}\cos\theta\right)^2 + \left(\frac{V}{R}\sin\theta\right)^2 + \left(\frac{V}{R}\tan\theta\right)^2}$$

$$|\vec{W}_{M-O}| = \frac{V}{R\cos\alpha} \qquad \textbf{Ans.}$$

Angular acceleration calculation

$$\vec{\alpha}_{D-O} = \frac{d\vec{w}_{D-O}}{dt}$$

$$\vec{\alpha}_{D-O} = \frac{-V}{R}\sin\theta\left(\frac{d\theta}{dt}\right)\hat{i} + \frac{V}{R}\cos\theta\left(\frac{d\theta}{dt}\right)\hat{j} + 0 \quad .....(2)$$

$\frac{d\theta}{dt}$ is angular velocity of M w.r.t. 0 :

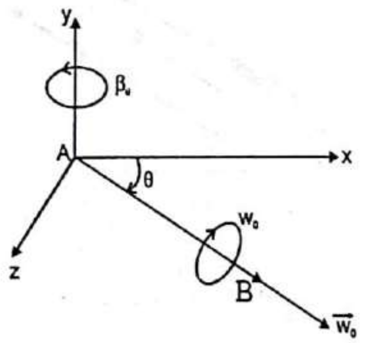
$$\frac{d\theta}{dt} = \frac{V}{R}\tan\alpha$$

$$\vec{a}_{D-O} = \frac{-V^2}{R^2}\sin\theta\tan\alpha\ \hat{i} + \frac{V^2}{R^2}\cos\theta\ \tan\alpha\ \hat{j}$$

$$|\vec{\alpha}_{D-O}| = \frac{V^2}{R^2}\tan\alpha \qquad \textbf{Ans.}$$

1.58 *

Method : 1 (axis is not rotating)
Angular velocity calculation



At time t line AB rotate by θ angle then

$$\vec{W}_P = w_0\cos\theta\,\hat{i} + w_0\sin\theta\,\hat{k} + \beta_0 t\,\hat{j} \quad ....(1)$$

$$\left|\vec{W}_P\right| = \sqrt{w_0^2\cos^2\theta + w_0^2\sin^2\theta + \beta_0^2 t^2}$$

$$\left|\vec{W}_P\right| = w_0\sqrt{1 + \left(\frac{\beta_0 t}{w_0}\right)^2} \qquad \textbf{Ans.}$$

Angular acceleration calculation

$$\frac{d\theta}{dt} = \beta_0 t$$

$$\vec{\alpha} = \frac{d\vec{W}_{P-A}}{dt}$$

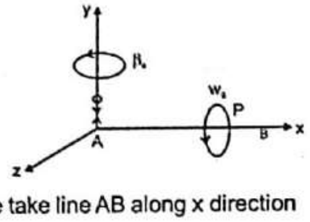
$$\vec{\alpha} = -W_0\sin\theta(\hat{i})\ \frac{d\theta}{dt} + W_0\cos\theta\left(\frac{d\theta}{dt}\right)\hat{k} + \beta_0\hat{j}$$

$$= -W_0\beta_0 t\sin\theta\,\hat{i} + W_0\beta_0\,t\cos\theta\,\hat{k} + \beta_0\,\hat{j}$$

$$|\vec{\alpha}| = \sqrt{w_0^2\beta_0^2 t^2 + \beta_0^2}$$

$$|\vec{\alpha}| = \beta_0\sqrt{1 + w_0^2 t^2} \qquad \textbf{Ans.}$$

Method :2 (axis is rotating)
Angular velocity of cone calculation



Here we take line AB along x direction

$$\vec{W}_{P-A} = w_0\,\hat{i} + \beta_0 t\,\hat{j} \quad ....(1)$$

$$|\vec{W}_{P-A}| = w_0\sqrt{1 + \frac{\beta_0^2 t^2}{w_0^2}} \qquad \textbf{Ans.}$$

$$\vec{\alpha} = \frac{d\vec{w}_{P-A}}{dt} = w_0\frac{d\hat{i}}{dt} + \beta_0\,t\frac{d\hat{j}}{dt} + \beta_0\,\hat{j}$$

Here $\frac{d\hat{j}}{dt} = 0$

But $\frac{d\hat{i}}{dt} = \beta_0 t\hat{k}$

Then $\frac{d\hat{i}}{dt} = \frac{d\theta}{dt}\hat{k}$

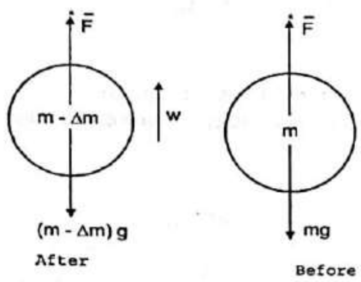
$$\frac{d\hat{i}}{dt} = \beta_0 t\hat{k}$$

$$\vec{\alpha} = w_0\beta_0\,t\hat{k} + \beta_0\hat{j}$$

$$|\vec{\alpha}| = \beta_0\sqrt{(1 + w_0^2 t^2)} \qquad \textbf{Ans.}$$

1.2 The Fundamentals Equation of Dynamcis

1.59

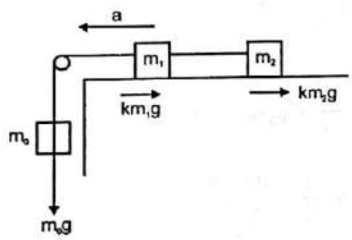


Where F is force due to air which will be constant then before mass blast
$mg - F = mw$ .... (1)
When Δm mass is taken then

$F - (m - \Delta m)\,g = (m - \Delta m)\,w$ ....(i)
Adding equations (i) and (ii)

$$\Delta m = \frac{2mw}{g + w}$$ **Ans.**

1.60



Net pulling force on system
$F = m_0 g - k\,m_1\,g - k\,m_2 g$
$F = ma$
$m_0 g - k\,m_1\,g - k\,m_2 g = (m + m_2 + m_0)\,a$

$$a = \left(\frac{m_0 - k(m_1 + m_2}{m_1 + m_2 + m_0}\right) g$$ **Ans.**

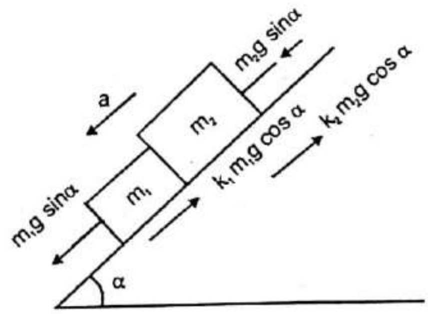
Tension in string attached with m_2.
F.B.D. of m_2 :

$T - k\,m_2\,g = m_2\,a$.......................(1)
Put value of a in (1) then

$$T = \frac{(1+k)m_0}{m_0 + m_1 + m_2} m_2\,g$$ **Ans.**

1.61

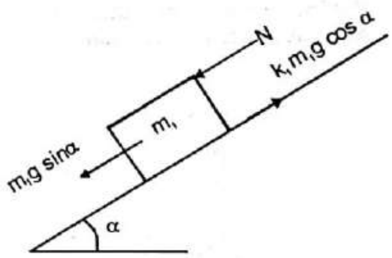
(a)



Net Pulling force on system
$F = m_1\,g \sin\alpha + m_2\,g \sin\alpha - k_1$ in$_1$ $g \cos\alpha - k_2\, m_2\, g \cos\alpha$.
Then common acceleration a is :

$$a = \frac{g\sin\alpha\ (m_1 + m_2) - g\cos\alpha\ (k_1 m_1 + k_2 m_2)}{m_1 + m_2}$$ ....(i)

F. B. D of m_1 :



N = Normal force between two blocks.
$m_2\,g \sin\alpha + N - k_1\,m_1\,g \cos\alpha\ = m_1\,a$ ....(ii)
Put value of (a) in (ii) :

$$N = \frac{(k_1 - k_2)m_1 m_2 g\cos\alpha}{m_1 + m_2}$$ **Ans.**

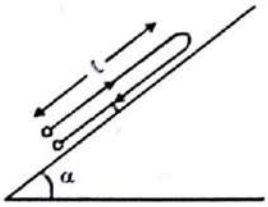
(b)
For minimum value of α acceleration will be zero and friction force will at maximum value then

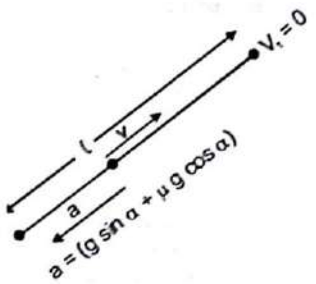
$$0 = \frac{[g\sin\alpha\,(m_1 + m_2 - g\cos\alpha(k_1 m_1 + k_2 m_2)]}{m_1 + m_2}$$

$$\tan\alpha = \frac{k_1 m_1 + k_2 m_2}{m_1 + m_2}$$ **Ans.**

1.62

Upward Journey :





We know displacement equation as

$$S = ut + \frac{1}{2}at^2$$

Also we know V=u+at then u=V–at

Put in above equation then

$$S = Vt - \frac{1}{2}at^2$$

Where V = final velocity in above equation

Suppose time for upward journey is t then

$V_f = 0$, $S = \ell$, $a = g \sin\alpha + \mu g \cos\alpha$

$$\ell = \frac{1}{2}(g \sin\alpha + \mu g \cos\alpha)\, t^2 \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Downward journey :

We know displacement equation as

$$S = ut + \frac{1}{2}at^2$$

u = initial velocity=0 m/s

$S = \ell$, $a = g \sin\alpha - \mu g \cos\alpha$

$$\ell = \frac{1}{2}(g \sin\alpha - \mu g \cos\alpha)\, (\eta t)^2 \ldots\ldots\ldots\ldots\ldots (ii)$$

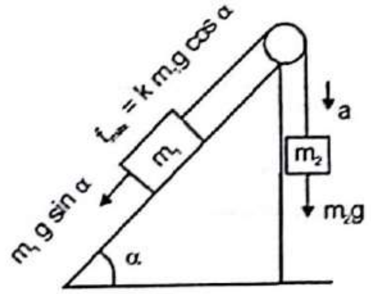
Divide both equations as $^{(i)}/_{(ii)}$

$$1 = \left(\frac{g\sin\alpha + \mu g\cos\alpha}{g\sin\alpha - \mu g\cos\alpha}\right)\frac{1}{\eta^2}$$

$$\mu = \left(\frac{\eta^2 - 1}{\eta^2 + 1}\right)\tan\alpha$$ **Ans.**

1.63

(a) Starts coming down



$m_2 g > m_1 g \sin\alpha + f_{max}$

$m_2 g > m_1 g \sin\alpha + k\, m_1 g \cos\alpha$

$$\frac{m_2}{m_1} > \sin\alpha + k\cos\alpha$$ **Ans.**

(b) Starts going up

$m_1 g \sin\alpha > m_2 g + k\, m_1 g \cos\alpha$

$$\sin\alpha > \frac{m_2}{m_1} + k\cos\alpha$$

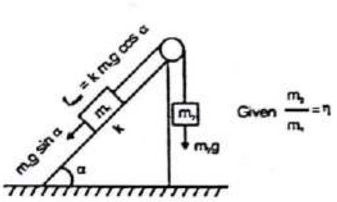
$$\frac{m_2}{m_1} < \sin\alpha - k\cos\alpha$$ **Ans.**

(c) At rest

Friction will be static hence ratio should be between calculated above value:

$$\sin\alpha - k\cos\alpha < \frac{m_2}{m_1} < \sin\alpha + k\cos\alpha$$ **Ans.**

1.64



To find tendency of sliding of block m_2

Pulling force in clockwise sense

$F_1 = m_2 g = m_1 \eta g = 2/3\, m_1 g \cong 0.66\, m_1 g$

Pulling force in anti clockwise sense

$F_2 = m_1 g \sin\alpha = m_1 g \sin 30^\circ = m_1 g/2 = 0.5\, m_1 g$

Since $m_2 g = 0.66\, m_1 g > m_1 g \sin\alpha = 0.5\, m_1 g$

Block m_2 has tendency to move down ward

Net pulling force

$F = m_2 g - m_1 g \sin\alpha - k\, m_1 g \cos\alpha$

Acceleration of system

$$a = \frac{m_2 g - m_1 g \sin\alpha - k m_1 g \cos\alpha}{m_1 + m_2}$$

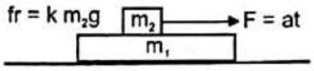
$$a = \frac{g[\eta - \sin\alpha - k\cos\alpha]}{\eta + 1}$$ **Ans.**

1.65

(a)

Before no sliding between m_1 and m_2:

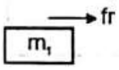
From F.B.D. of System :



Acceleration of both block will be same

$$w_1 = w_2 = w = \frac{F}{m_1 + m_2} = \frac{at}{m_1 + m_2}$$

Friction between m_1 and m_2 will be static then



$$fr = m_1 [w_1] = \frac{m_1 at}{m_1 + m_2}$$

For no sliding this required friction should be less than maximum value of static friction

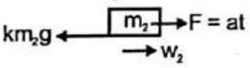
$$fr = \frac{m_1 at}{m_1 + m_2} < k\, m_2\, g$$

$$t < \frac{k m_2 g\,(m_1 + m_2)}{a m_1}$$

$$w_1 = w_2 = \frac{at}{m_1 + m_2}$$ **Ans.**

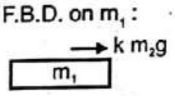
If $t > k\, m_2\, g\, \dfrac{(m_1 + m_2)}{a\, m_1}$

Sleeping between two block will be start then F.B.D. of m_2 :



$$w_2 = \frac{at - km_2 g}{m_2}$$ **Ans.**

F.B.D. on m_1 :



$$w_1 = \frac{km_2 g}{m_1}$$ **Ans.**

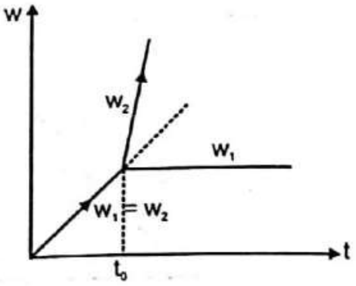
Assume $t_0 = \dfrac{k m_2 g (m_1 + m_2)}{a\, m_1}$

$t < t_0$:

Slope of w_2 = Slope of w_1

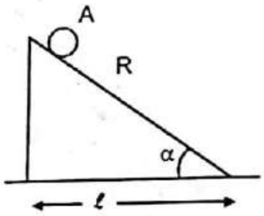
$t > t_0$:

Slope of w_2 > Slope of w_1 because acceleration of block (2) will be more than (1)

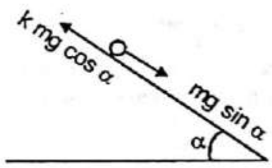


1.66

F.B.D of block :



k = coefficient of friction



$ma = mg \sin\alpha - k\, mg \cos\alpha$

$a = g \sin\alpha - kg \cos\alpha$

Time to reach at bottom after starting with rest:

$$s = ut + \frac{1}{2} at^2$$

$$\ell \sec\alpha = \frac{1}{2}(g\sin\alpha - kg\cos\alpha)t^2$$

$$t^2 = \frac{2\ell \sec\alpha}{g(\sin\alpha - k\cos\alpha)}$$

$$t^2 = \frac{2\ell}{g[\sin\alpha\cos\alpha - k\cos^2\alpha]}$$

$$t = \sqrt{\frac{2\ell}{g[\sin\alpha\cos\alpha - k\cos^2\alpha]}} \quad \text{.......(i)}$$

To minimise t

$(\sin\alpha\cos\alpha - k\cos^2\alpha)$ will be maximum

Assume

$x = \sin\alpha\cos\alpha - k\cos^2\alpha)$.

Now $\dfrac{dx}{d\alpha} = 0$

$\cos^2\alpha - \sin^2\alpha + 2k\cos\alpha\sin\alpha = 0$

$\cos 2\alpha = -k \sin 2\alpha \quad \Rightarrow \tan 2\alpha = -\frac{1}{k}$ **Ans.**

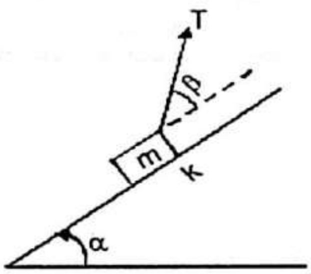
Put value of k : $\alpha = 49^\circ$

Put value of α in equation (i) :

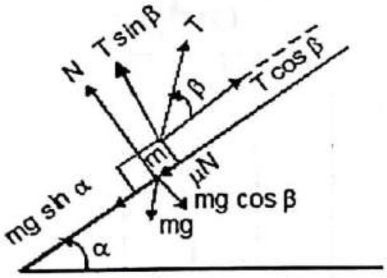
$$t_{min} = \sqrt{\frac{2\times 2.10}{10[\sin 49^\circ \cos 49^\circ - 0.14\cos^2 49]}} \cong 1.0s$$

Ans.

1.67



F.B.D. of block :



$N + T\sin\beta = mg\cos\beta$

$N = mg\cos\beta - T\sin\beta$

$fr = kN = k(mg\cos\beta - T\sin\beta)$

At just sliding condition:

$T\cos\beta = mg\sin\alpha + fr$

$T\cos\beta = mg\sin\alpha + k(mg\cos\beta - T\sin\beta)$

$$T = \frac{mg\sin\alpha + kmg\cos\beta}{\cos\beta + k\sin\phi}$$

For T_{min}, $y = \cos\beta + k\sin\beta$ should be maximum.

Maximum value of $\cos\beta + k\sin\beta = \sqrt{1+k^2}$

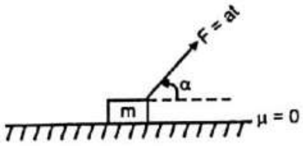
For this value $\quad \frac{dy}{dx} = 0$

$-\sin\beta + k\cos\beta = 0$

$\tan\beta = k$ **Ans.**

$$T_{min} = \frac{mg(\sin\alpha + k\cos\beta)}{\sqrt{1+k^2}}$$ **Ans.**

1.68



(a)

At time t_1 of breaking off the plane vertical component of $\vec{F}$ must be equal to weight mg.

$F\sin\alpha = mg$

$at_1\sin\alpha = mg$

$$t_1 = \frac{mg}{a\sin\alpha}$$

Motion equation of block :

a_1 = Accelration of block

$F\cos\alpha = m a_1$

$$a_1 = \frac{at\cos\alpha}{m} = \frac{dv}{dt}$$

$$\int_0^V \frac{mdV}{a\cos\alpha} = \int_0^{t_1} tdt$$

$$\frac{mV}{a\cos\alpha} = \frac{t_1^2}{2}$$

$$\frac{mV}{a\cos\alpha} = \frac{1}{2}\frac{m^2g^2}{a^2\sin^2\alpha}$$

$$V = \frac{mg^2\cos\alpha}{2a\sin^2\alpha}$$ **Ans.**

(b)

$$\int_0^V \frac{mdV}{a\cos\alpha} = \int_0^t tdt$$

$$\frac{mV}{a\cos\alpha} = \frac{t^2}{2}$$

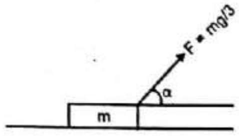
$$v = \frac{at^2\cos\alpha}{2m}$$

$$\frac{dx}{dt} = \frac{at^2\cos\alpha}{2m}$$

$$\int_0^x dx = \frac{a\cos\alpha}{2m}\int_0^{t_1 = \frac{mg}{a\sin\alpha}} t^2 dt$$

$$x = \frac{m^2 g^3 \cos\alpha}{6a^2 \sin^3\alpha}$$ **Ans.**

1.69



Angle made by force vector

$\alpha = aS$

Motion equation of block

$$\frac{mg}{3} \cos\alpha = m\, a_1$$

$$a_1 = \frac{g}{3} \cos\alpha$$

Where a is acceleration of block of mass m

$$a_1 = \frac{g}{3} \cos as$$

$$\int_0^v v\,dv = \frac{g}{3}\int_0^s \cos as\, ds$$

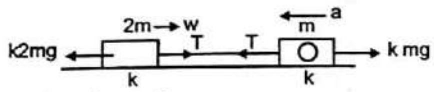
$$\left.\frac{v^2}{2}\right|_0^v = \frac{g}{3}\left.\frac{\sin as}{a}\right|_0^s$$

$$\frac{v^2}{2} = \frac{g}{3}\,\frac{\sin as}{a}$$

$$v = \sqrt{\frac{2g}{3}\,\frac{\sin as}{a}}$$ **Ans.**

1.70

Method :1(Equation of motion blocks)



Motion of mass 2 m

$T - k2mg = 2mw$

$T = 2\,mw + 2\,kmg$..................(1)

Motion of motor

$T - kmg = ma$ ....(2)

Adding both equations

$2\,mw + 2\,k\,mg - k\,mg = ma$

$a = 2\,w + kg$

Acceleration of 2 m w.r.t. m

$a_{2m-m} = a_{2m} - a_m$

$a_{2m-m} = w + a$

$= w + 2w + kg = 3w + kg$

Suppose in time t both blocks will be met then

$$S = \frac{1}{2}at^2$$

$$\ell = \frac{1}{2}\,(3w + kg)\,t^2$$

$$t = \sqrt{\frac{2\ell}{3w + kg}}$$ **Ans.**

Method : 2 (Equation on system)

On system,tension will be internal force then

$k2mg - k\,mg = -2\,mw + ma$

$a = 2w + kg$

Acceleration of 2 m w.r.t. m

$a_{2m-m} = a_{2m} - a_m$

$a_{2m-m} = w + a$

$= w + 2w + kg = 3w + kg$

Suppose in time t both blocks will be met then

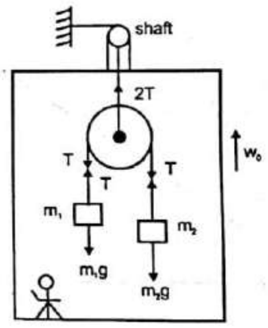
$$S = \frac{1}{2}at^2$$

$$\ell = \frac{1}{2}\,(3w + kg)\,t^2$$

$$t = \sqrt{\frac{2\ell}{3w + kg}}$$ **Ans.**

1.71

Method :1 (Observer inside lift)



For observer inside elevator

$$a_{1-car} = \frac{\text{Net pulling force along with pseudo force}}{\text{Total mass of system}}$$

$$a_{1-car} = \frac{m_2 g + m_2 w_0 - m_1 g - m_1 w_0}{m_1 + m_2}$$

$$a_{1-car} = \frac{(m_2 - m_1)(g + w_0)}{m_1 + m_2}$$

But ans in irodov is given in vector form

$$a_{1-car} = \frac{(m_2 - m_1)(-\vec{g} + \vec{w}_0)}{m_1 + m_2}$$

$$a_{1-car} = \frac{(m_1 - m_2)(\vec{g} - \vec{w}_0)}{m_1 + m_2}$$ **Ans.**

Acceleration of m_1 w.r.t. shaft or ground

$a_{1\text{-shaft}} = a_{1\text{-car}} + a_{car}$

$$= \frac{(m_1 - m_2)(\vec{g} - \vec{w}_0)}{m_1 + m_2} + \vec{w}_0$$

$$a_{1\text{-shaft}} = \frac{(m_1 - m_2)\vec{g} + 2m_2\vec{w}_0}{m_1 + m_2} \quad \textbf{Ans.}$$

Since tension is real force then from all frame it will be same

Force equation on block m_1

$T - m_1 g - m_1 w_0 = m_1 a_{1\text{-car}}$

$$T - m_1 g - m_1 w_0 = m_1 \frac{(m_2 - m_1)(g + w_0)}{m_1 + m_2}$$

$$T = \frac{2m_1 m_2 (g + w_0)}{m_1 + m_2}$$

Forced applied by pulley on ceiling

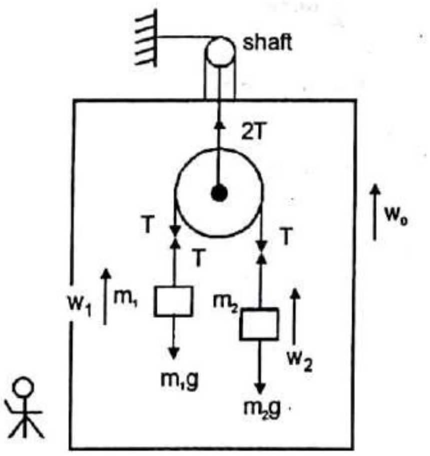
$$2T = \frac{4m_1 m_2 (g + w_0)}{m_1 + m_2}$$

Forced applied by pulley on ceiling as vector form :

$$-2T = \frac{4m_1 m_2}{m_1 + m_2}(-\vec{g} + \vec{w}_0) = \frac{4m_1 m_2}{m_1 + m_2}(\vec{g} - \vec{w}_0)$$

Ans.

Method :2 (Observer on ground)



Force equation on block m_1

$T - m_1 g = m_1 w_1$ ....(1)

Force equation on block m_2

$T - m_2 g = m_2 w_2$ ....(2)

Constraint relation between accelerations

$$w_0 = \frac{w_2 + w_1}{2} \text{ ....(3)}$$

From above three equations

$$w_1 = \frac{(m_2 - m_1)g + 2m_2 w_0}{m_1 + m_2}$$

$$a_{1\text{-shaft}} = \frac{(m_2 - m_1)g + 2m_2 w_0}{m_1 + m_2}$$

But ans in irodov is given in vector form

$$a_{1\text{-shaft}} = \frac{(m_1 - m_2)\vec{g} + 2m_2\vec{w}_0}{m_1 + m_2} \quad \textbf{Ans.}$$

Acceleration of block m_1 w.r.t. car

$a_{1\text{-car}} = w_1 - w_0$

$$= \frac{(m_2 - m_1)g + 2m_2 w_0}{m_1 + m_2} - w_0$$

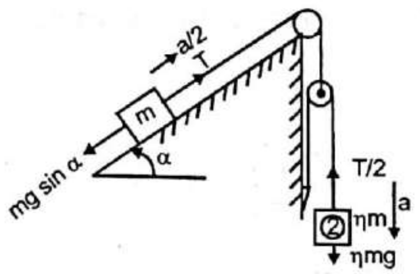
$$a_{1\text{-car}} = \frac{(m_2 - m_1)(g + w_0)}{m_1 + m_2}$$

But ans in irodov is given in vector form

$$a_{1\text{-car}} = \frac{(m_2 - m_1)(-\vec{g} + \vec{w}_0)}{m_1 + m_2}$$

$$a_{1\text{-car}} = \frac{(m_1 - m_2)(\vec{g} - \vec{w}_0)}{m_1 + m_2} \quad \textbf{Ans.}$$

1.72



Motion of body (2)

$\eta mg - T/2 = \eta ma$

$2\eta mg - T = 2\eta ma$ ....(i)

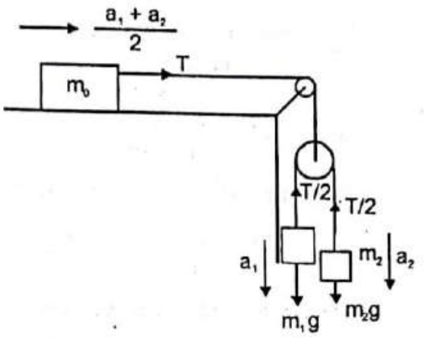
Motion of body on incline plane

$T - mg \sin\alpha = m\, a/2$ ....(ii)

From (1) and (ii)

$$a = \frac{2g\left[2\eta - \sin\alpha\right]}{\left[4\eta + 1\right]}$$ **Ans.**

1.73



Equation of motion of m_0

$$T = m_0\left(\frac{a_1 + a_2}{2}\right) \quad ....(i)$$

Equation of motion of m_1

$m_1g - T/2 = m_1 a_1$ ....(ii)

Equation of motion of m_2

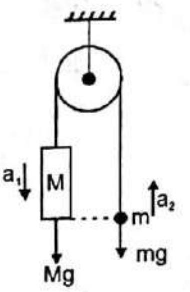
$m_2g - T/2 = m_2a_2$ ....(iii)

From (i), (ii) and (iii)

$$a_1 = \frac{4m_1m_2 + m_0(m_1 - m_2).g}{4m_1m_2 + m_0(m_1 + m_2)}$$ **Ans.**

1.74

Method :1 (Equation on system)



Friction will act as internal force then

Motion equation on system

$Mg - mg = M a_1 + m a_2$ .... (i)

Motion equation of m

$f_r - mg = ma_2$ ....(ii)

Relative acceleration of m w.r.t. M

$a_r = a_1 + a_2$

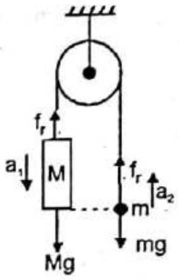
Since length of rod is ℓ then

$\ell = \frac{1}{2}(a_1 + a_2)t^2$.........(iii).

From (i), (ii) and (iii)

$$f_r = \frac{2mm\ell}{(M-m)t^2}$$ **Ans.**

Method :1 (Equation on each mass)



Motion equation M

$Mg - f_r = M a_1$ .... (i)

Motion equation of m

$f_r - mg = ma_2$ ....(ii)

Relative acceleration of m w.r.t. M

$a_r = a_1 + a_2$

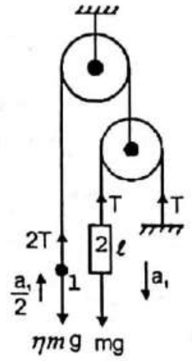
Since length of rod is ℓ then

$\ell = \frac{1}{2}(a_1 + a_2)t^2$.........(iii).

From (i), (ii) and (iii)

$$f_r = \frac{2mm\ell}{(M-m)t^2}$$ **Ans.**

1.75



Motion equation m

$mg - T = m\,a_1$ ....(i)

Motion equation η m

$$2T - \eta\, mg = \eta\, m\,\frac{a_1}{2} \text{ ....(ii)}$$

From (i) and (ii) $\quad a_1 = \dfrac{2g(2-\eta)}{\eta+4}$

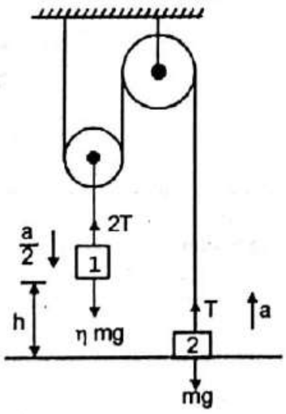
Relative acceleration of η m w.r.t. m

$$a_r = \frac{a_1}{2} + a_1 = \frac{3a_1}{2} = \frac{3}{2}\times\frac{2g(2-\eta)}{\eta+4} = \frac{3g(2-\eta)}{\eta+4}$$

$$l = \frac{1}{2}a_r t^2 = \frac{1}{2}\frac{3g(2-\eta)}{\eta+4}t^2$$

$$t = \sqrt{\frac{2(\eta+4)l}{3g(2-\eta)}}$$ **Ans.**

1.76



Motion equation of (2)

$T - mg = ma$ .... (i)

Motion equation of (1)

$\eta\, mg - 2T = \eta\, m\, a/2$...............(ii)

From (i) and (ii)

$$a = \frac{2g(\eta-2)}{\eta+4}$$

When body (1) travel h distance then in same time body (2) travel 2h distance in upward direction using constraint relation as acceleration.

String will be slack when body(1) travell h distance and strike on ground.

Now velocity of body (2) just before string slack.

$V^2 = 2a\,(2h)$..................(iii)

After that body(2) will be as projectile motion in air and continue moving in air in upward direction untill final velocity becomes zero.

Suppose body (2) travel x distance in projectile motion then

$V^2 = 2gx$ ....(iv)

From (iii) and (iv)

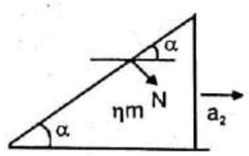
$$x = \frac{2ah}{g}.$$

Total hight travell from ground

$$H = \frac{2ah}{g} + h = \frac{6\eta h}{\eta+4}$$ **Ans.**

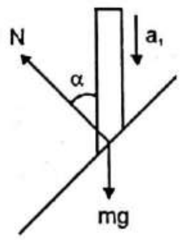
1.77

F.B.D. of wedge



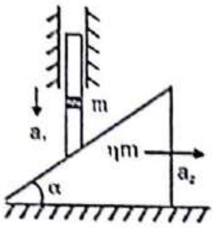
$N \sin\alpha = \eta\, m\, a_2$ ....(i)

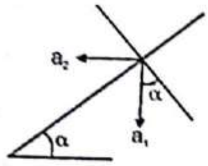
F.B.D. of rod



$mg - N\cos\alpha = m\,a_1$ ....(ii)

For constraint relation





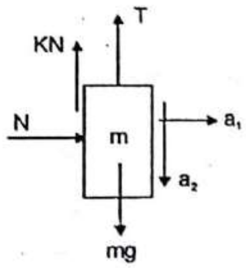
$a_2 \sin\alpha = a_1 \cos\alpha$ ....(iii)
From (i), (ii) and (iii):

$$a_1 = \frac{g}{1+\eta\cot^2\alpha}$$

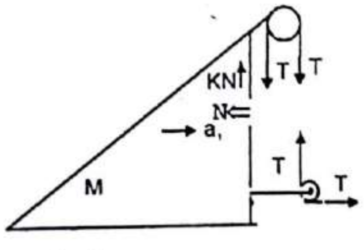
$$a_2 = \frac{g}{\tan\alpha + \eta\cot\alpha}$$ **Ans.**

1.78

F. B. D. of m



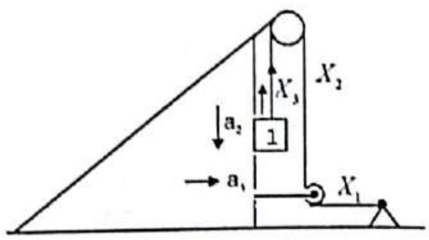
In y direction
$mg - KN - T = m\,a_2$ .... (i)
In x direction
$N = m\,a_1$ .... (ii)
F. B. D. of wedge



In x direction

$T - N = Ma_1$ ....(iii)

Using constraint relation



$$X_1 + X_2 + X_3 = const$$
$-a_1 + 0 + a_2 = 0$
$a_1 = a_2$......................(iv)
From (i), (ii), (iii) and (iv)

$$a_1 = \frac{mg}{2m+M+km} = \frac{g}{2+k+\frac{M}{m}}$$

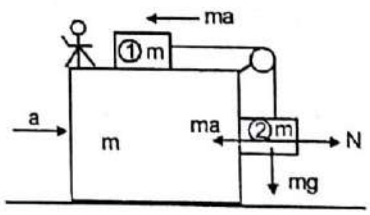
Net acceleration of block of mass m

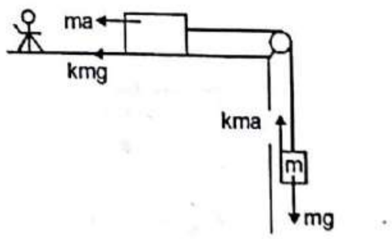
$$a = \sqrt{a_1^2 + a_2^2} = a_1\sqrt{2}$$

$$a = \frac{g\sqrt{2}}{2+k+\frac{M}{m}}$$ **Ans.**

1.79

F.B.D of bodies on frame of wedge:



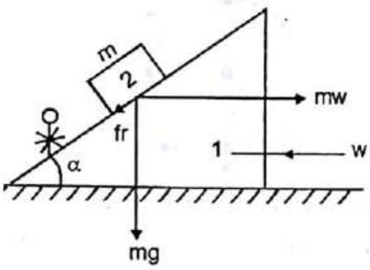


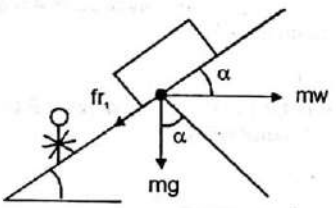
Since system is stationary on frame of wedge hence :
$mg = kma + ma + kmg$

$$a = \frac{g(1-k)}{1+k} \qquad \textbf{Ans.}$$

1.80

F.B.D. of block (2) with frame of wedge:





At maximum acceleration w block will have tendency to slip up the incline then

fr_1 will be maximum and direction on block will be down the incline.

$fr_1 = kN = k\,[mg\cos\alpha + mw\sin\alpha]$...............(1)

Since block is under rest with frame of wedge, then equilibrium equation of block along incline.

$fr_1 + mg\sin\alpha = mw\cos\alpha$

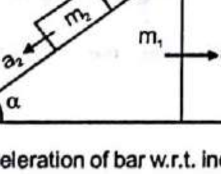
From equation (1)

$k\,(mg\cos\alpha + mw\sin\alpha) + mg\sin\alpha = mw\cos\alpha$

$$w = \frac{g(1+k\cot\alpha)}{\cot\alpha - k} \qquad \textbf{Ans.}$$

1.81.

Method : 1 (Equation on system w.r.t. ground)



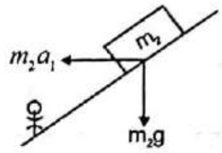
F.B.D. of System

a_2 = acceleration of bar w.r.t. incline.

Since no force on system in horizontal direction then

$0 = ma_1 + m\,[a_1 - a_2\cos\alpha]$...............(1)

F.B.D. of bar w.r.t. wedge



Equation along incline from frame of incline

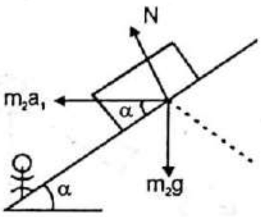
$m_2a_1\cos\alpha + m_2g\sin\alpha = m_2a_2$ ....(ii)

From (1) and (2)

$$a_1 = \frac{m_2g\sin\alpha\cos\alpha}{m_1 + m_2\sin^2\alpha} = \frac{g\sin\alpha\cos\alpha}{\frac{m_1}{m_2} + \sin^2\alpha} \qquad \textbf{Ans.}$$

Method : 2 (Equation on block w.r.t. wedge)

F.B.D. of bar

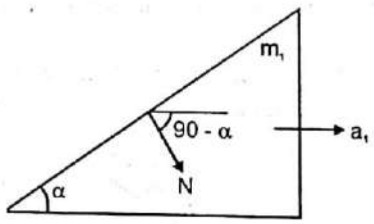


With frame of wedge, bar has zero acceleration perpendicular to incline then

Equation along incline

$N + m_2a_1\sin\alpha = m_2g\cos\alpha$ ....(i)

F.B.D. of wedge



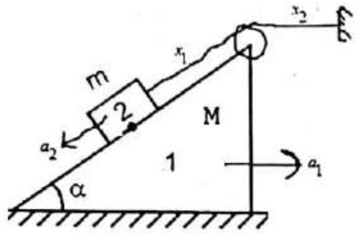
Equation along x direction

$N\sin\alpha = m_1a_1$ ....(ii)

from (i) and (ii)

$$a_1 = \frac{m_2g\sin\alpha\cos\alpha}{m_1 + m_2\sin^2\alpha} = \frac{g\sin\alpha\cos\alpha}{\frac{m_1}{m_2} + \sin^2\alpha} \qquad \textbf{Ans.}$$

1.82



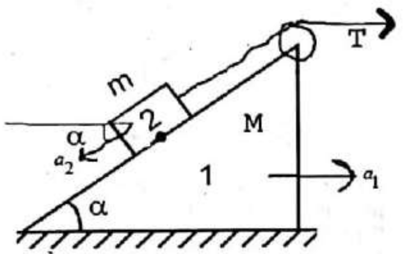
Using constraint relation

$x_1 + x_2 = const.$

$a_1 - a_2 = 0$

$a_1 = a_2$...........(1)

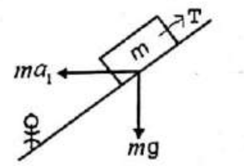
F.B.D. of System



a_2 = acceleration of block w.r.t. incline.
Since force T on system in horizontal direction then

$T = Ma_1 + m(a_1 - a_2 \cos\alpha)$..........(2)

F.B.D. of block w.r.t. wedge



Equation along incline from frame of incline

$ma_1 \cos\alpha + mg\sin\alpha - T = ma_2$ ....(3)

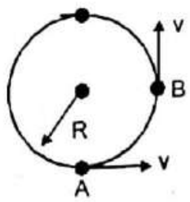
From (1),(2)and(3)

$$a_1 = a_2 = \frac{mg\sin\alpha}{2m(1-\cos\alpha)+M}$$

Ans.

1.83

(a) With constant velocity v



We know average force is equal to:

$$\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t}$$

Suppose particle is initially at point A then its intial momentum is

$$\vec{P}_i = mv\hat{i}$$

After quarter circle it will be at point B then its final momentum will be

$$\vec{P}_f = mv\hat{j}$$

Change in momentum

$$\Delta\vec{P} = \vec{P}_f - \vec{P} = mv\hat{j} - mv\hat{i}$$

Time taken in journey is t then

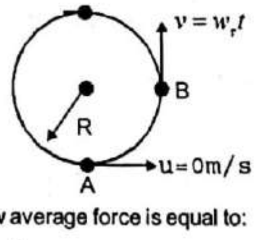
$$\Delta t = \frac{\frac{\pi}{2}R}{v}$$

Average force will be

$$\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t} = \frac{mv\hat{j} - mv\hat{i}}{\frac{\frac{\pi}{2}R}{v}} = \frac{2mv^2}{\pi R}\left(\hat{j}-\hat{i}\right)$$

$$\left|\vec{F}_{avg}\right| = \frac{2mv^2}{\pi R}\left|\left(\hat{j}-\hat{i}\right)\right| = \frac{2\sqrt{2}mv^2}{\pi R}$$ **Ans.**

(b) With constant tangential acceleration
Time taken in journey is t.



We know average force is equal to:

$$\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t}$$

Suppose particle start from point A then its

Intial momentum is

$\vec{P}_i = 0\hat{i}$

After quarter circle it will be at point B then its final momentum will be

$\vec{P}_f = mv\hat{j} = mw_r t\hat{j}$

Change in momentum

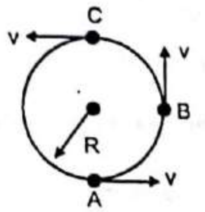
$\Delta\vec{P} = \vec{P}_f - \vec{P} = mw_r t\hat{j}$

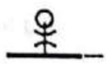
Average force will be

$$\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t} = \frac{mw_r t\hat{j}}{t} = mw_r\hat{j}$$

$$\left|\vec{F}_{avg}\right| = \left|mw_r\hat{j}\right| = mw_r$$ **Ans.**

1.84

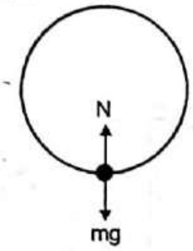




v = 360 km/hr = $\frac{360\times1000}{3600}$ = 100 m/s

Apparent weight is reading of normal reaction now

At point A



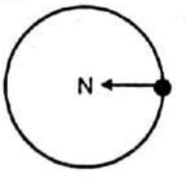
$N - mg = \frac{mv^2}{R}$

$N = mg + \frac{mv^2}{R}$

$= 70g + \frac{70[100]^2}{500} = 70g + \frac{70\times100\times100}{500}$

$= 70\text{ g} + 140\text{ g} = 210\text{ g} = 2100\text{ N} = 2.1\text{ kN}$ **Ans.**

At point B

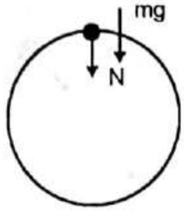


$N = \frac{mv^2}{R}$ ....(i)

Put value of m, V and R

N = 1.5 kN **Ans.**

At point C



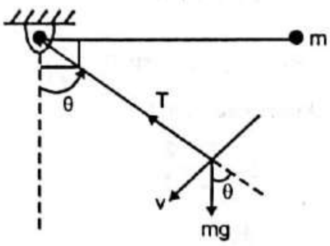
$N + mg = mv^2/R$

$N = mv^2/R - mg$ ....(ii)

Put value of V, m, R in (ii)

N = 0.7 kN **Ans.**

1.85



(a)

Tangential acceleration (a_t)

$mg\sin\theta = m\,a_t$

$a_t = g\sin\theta$ ....(1)

Radial acceleration (a_r)

$$a_r = \frac{v^2}{R} = \frac{v^2}{l}$$

Energy conservation

$mg\,\ell\cos\theta = \frac{1}{2}mv^2$

$v = \sqrt{2g\ell\cos\theta}$

$a_r = \frac{v^2}{\ell} = 2g\cos\theta$ ....(2)

$a_{net} = \sqrt{a_t^2 + a_r^2} = g\sqrt{\sin^2\theta + 4\cos^2\theta}$

$a_{net} = g\sqrt{1+3\cos^2\theta}$ **Ans.**

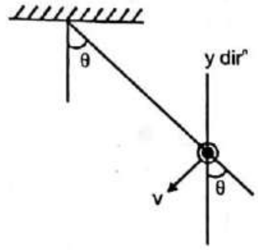
Force equation in radial direction at this instant

$T - mg\cos\theta = \frac{mv^2}{\ell}$

$T - mg\cos\theta = 2mg\cos\theta$

$T = 3\,mg\cos\theta$ **Ans.**

(b)



Component of velocity in y direction
$v_y = v \sin\theta$

$v_y = \sqrt{2g\ell\cos\theta}\ \sin\theta$

$v^2_y = 2g\ell\cos\theta\sin^2\theta$...............(1)
For v_y maximum v^2_y will be maximum
Then $x = 2g\ell\cos\theta\sin^2\theta$ will be maximum

$$\frac{dx}{d\theta} = 0 = 2g\ell[-\sin^3\theta + 2\cos^2\theta\sin\theta] = 0$$

$2\cos^2\theta = \sin^2\theta$

$\tan\theta = \sqrt{2}$, $\sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$, $\cos\theta = \frac{1}{\sqrt{3}}$

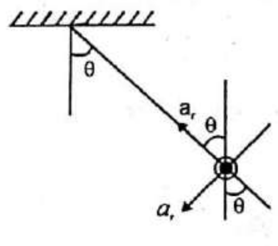
From equation(1)

$$v^2_y = 2g\ell\frac{1}{\sqrt{3}} \times \frac{2}{3}$$

$$v_y = \frac{4g\ell}{3\sqrt{3}}$$

$T = 3\,mg\cos\theta = 3\,mg\left(\frac{1}{\sqrt{3}}\right) = mg\sqrt{3}$ **Ans.**

(c)

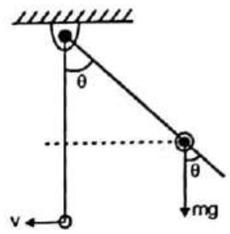


If no acceleration in y direction then
$a_r\cos\theta = a_t\sin\theta$
$2g\cos^2\theta = g\sin^2\theta$

$\tan\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{3}}$ **Ans.**

1.86



Since ball swings in vertical position then at extreme position
Only tangential acceleration is prerent
At extreme position
$a_t = g\sin\theta$
Lowest Position only radial acceleration is prerent
At lowest position
Using energy conservation

$$mg\,[\ell - \ell\cos\theta] = \frac{1}{2}mv^2$$

$v^2 = 2g\ell(1 - \cos\theta)$

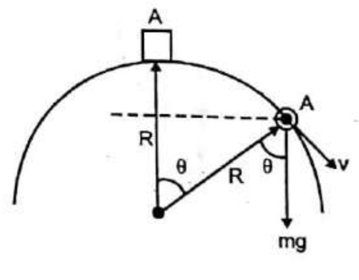
Radial acceleration$(a_r) = \frac{v^2}{\ell} = 2g(1 - \cos\theta)$

According to condition
$a_r = a_t$
$g\sin\theta = 2g(1 - \cos\theta)$
$\sin\theta + 2\cos\theta = 2$

$\cos\theta = \frac{3}{5}$

$\theta = 53°$ **Ans.**

1.87



Particle will break off sphere when normal reaction will be zero. At this instant force equation in radial direction

$$mg\cos\theta = \frac{mv^2}{R}$$

$V^2 = Rg\cos\theta$ ....(1)
Energy C

$$mg(R - R\cos\theta) = \frac{1}{2} mv^2$$

From equation(1)

$$mgR\,(1 - \cos\theta) = \frac{1}{2} mRg\cos\theta$$

$$1 - \cos\theta = \frac{\cos\theta}{2}$$

$$\frac{3\cos\theta}{2} = 1$$

$$\cos\theta = \frac{2}{3}$$

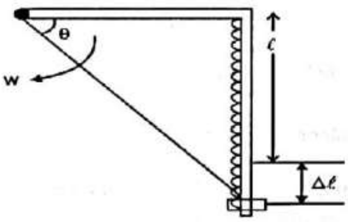
From equation(1)

$$v^2 = Rg\left[\frac{2}{3}\right]$$

$$v = \sqrt{\frac{2Rg}{3}}$$ **Ans.**

1.88

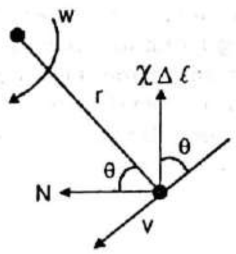
Top view



Spring force = F = $\chi \Delta \ell$

N = normal reaction on sleeve

Since no acceleration in tangential direction



$$N\sin\theta = \chi\Delta\ell\cos\theta$$

$$N = \frac{\chi\Delta\ell}{\sin\theta}\cos\theta \quad \text{.............(i)}$$

Equation in radial direction

$$N\cos\theta + \chi\Delta\ell\sin\theta = m\,r\,w^2$$

$$N\cos\theta + \chi\Delta\ell\sin\theta = m\,(\ell + \Delta\ell)\operatorname{cosec}\theta\ w^2$$

From (i)

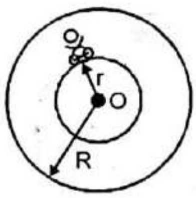
$$\frac{x\Delta\ell\cos\theta}{\sin\theta}(\cos\theta) + \chi\Delta\ell\sin\theta$$

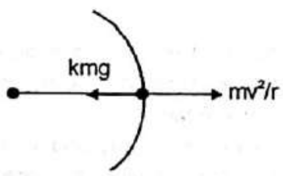
$$\frac{x\Delta\ell\cos\theta}{\sin\theta}(\cos\theta) + \chi\Delta\ell\sin\theta = m\,(\ell + \Delta\ell)\operatorname{cosec}\theta\ w^2$$

$$\chi\Delta\ell = m\ell + m\Delta\ell w^2$$

$$\Delta\ell = \frac{m\ell}{x - mw^2}$$ **Ans.**

1.89





$$k = k_0\left(1 - \frac{r}{R}\right)$$

Where k is friction coefficient

Suppose cyclist is at radius of r

Then friction provide centripital force to motion on circular path.

$$kmg = \frac{mv^2}{r}$$

$$k_0\left(1 - \frac{r}{R}\right) mg = \frac{mv^2}{r}$$

$$v^2 = k_0 g\left[r - \frac{r^2}{R}\right] \text{.............(1)}$$

For maximum value of v , v^2 should be maximum.

Assuming $v^2 = x = k_0 g\left[r - \frac{r^2}{R}\right]$

For x will be maximum

$$\frac{dx}{dr} = 0$$

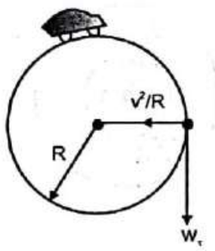
$$k_0 g\left[1 - \frac{2r}{R}\right] = 0 \qquad r = \frac{R}{2}$$ **Ans.**

From equation (1)

$$V_{max}^2 = k_0 g\left[\frac{R}{2} - \frac{R}{4}\right] = \frac{k_0 gR}{4}$$

$$V_{max} = \frac{1}{2}\sqrt{k_0 gR}$$ **Ans.**

1.90



Tangential and radial both acceleration is only provided by friction because friction is acting as external force.
Here maximum value of friction = kmg
Velocity of car after d distance travel
$v^2 = 2 w_\tau d$

$$a_r = \text{Radial acceleration} = \frac{V^2}{R} = \frac{2w_\tau d}{R}$$

$$a_t = \text{Tangential acceleration} = w_\tau$$

$$a_{net} = \sqrt{a_t^2 + a_r^2}$$

$$a_{net} = \sqrt{\left(\frac{2w_\tau d}{R}\right)^2 + w_\tau^2} = w_\tau\sqrt{1 + \frac{4d^2}{R^2}}$$

$$F_{net} = m\,a_{net} = m\,w_\tau\sqrt{1 + \frac{4d^2}{R^2}}$$

This force is provide by friction then

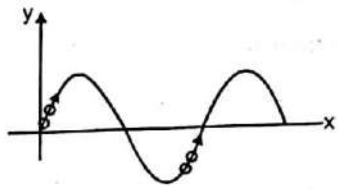
$$k\,mg = m\,w_\tau\sqrt{1 + \frac{4d^2}{R^2}}$$

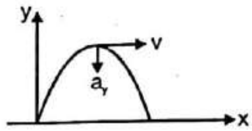
Squaring both side

$$w_\tau^2\left(1 + \frac{4d^2}{R^2}\right) = k^2 g^2$$

$$d = \frac{R}{2}\sqrt{\left(\frac{kg}{w_\tau}\right)^2 - 1}$$ **Ans.**

1.91





$$y = a\sin\left(\frac{x}{\alpha}\right)$$

k = friction coefficient

$$\text{Centripital force} = \frac{mv^2}{R}$$

Centripital force will be provided by friction
At limiting condition

$$\frac{mv^2}{R} \le k\,mg$$

$v^2 \le k\,Rg$........................(1)

Here all value of v even maximum value of v should be less than kRg
For v maximum R will be minimum
We know speed is constant. Also we are seeing that radius of curvature will be minimum at maximum point of curve because curvature is small at top.
Calculation of radius of curvature

$$a_y = \frac{d^2y}{dt^2} = \frac{v^2a}{\alpha^2}\sin\left(\frac{x}{\alpha}\right)$$

At minimum value of curve radius:

$$\sin\frac{x}{\alpha} = 1$$

$$a_y = \frac{v^2a}{\alpha^2}$$

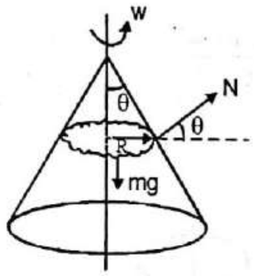
$$R = \frac{v^2}{a_y} = \frac{v^2\alpha^2}{v^2a}$$

$$R = \frac{\alpha^2}{a}$$

From equation(1)

$$v^2 \le k\frac{\alpha^2}{a}g \qquad v \le \alpha\sqrt{\frac{kg}{a}}$$ **Ans.**

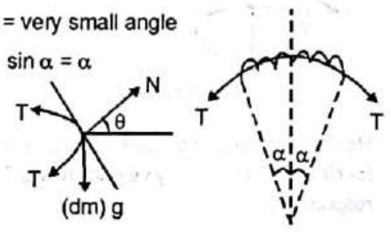
1.92



F.B.D. of differential element of length dl

α = very small angle

$\sin\alpha = \alpha$



Equation of motion :

$2T\sin\alpha - N\cos\theta = (dm)\,Rw^2$

$2T\alpha - N\cos\theta = (dm)\,Rw^2$ ....(i)

$N\sin\theta = (dm)\,g$

$$N = \frac{(dm)g}{\sin\theta}$$

Put value of N in (i)

$$2T\alpha - \frac{(dm)g}{\sin\theta}\cos\theta = (dm)\,Rw^2$$

$$2T\alpha - (dm)\,g\cot\theta = (dm)\,Rw^2$$

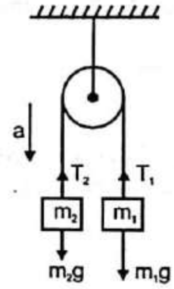
$$2T\alpha - \frac{m}{2\pi R}(R\,2\alpha)\,g\cot\theta = (R2\alpha)\frac{m}{2\pi R}Rw^2$$

$$T - \frac{mg\cot\theta}{2\pi} = \frac{mRw^2}{2\pi}$$

$$T = \frac{m}{2\pi}[Rw^2 + g\cot\theta]$$

$$T = \frac{mg}{2\pi}\left[\cot\theta + \frac{Rw^2}{g}\right]$$ **Ans.**

1.93



Since there is friction between pulley and string hence tension in both side of pulley will be different.

$$\frac{m_2}{m_1} = \eta_0$$

$T_1 - m_1g = m_1a$

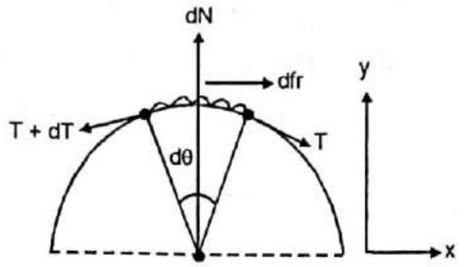
$T_1 = m_1g + m_1a$ ....(1)

$T_2 = m_2g - m_2a$...........(2)

Divide (1) by (2)

$$\frac{T_2}{T_1} = \frac{m_2(g-a)}{m_1(g+a)} \text{ ....(3)}$$

Relation between T_1 and T_2



Take a differential element that substend a very small angle $d\theta$

Force equation in y direction

$$(T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2} = dN$$

dT can be neglect w.r.t. T and $\sin d\theta \cong d\theta$

$$T\frac{d\theta}{2} + T\frac{d\theta}{2} = dN$$

$T d\theta = dN$ ....(4)

Since there is slipping then friction will be kinetic friction between pulley and string.

$dfr = \mu\, dN$

From(4)

$dfr = \mu\, T d\theta$...........................(5)

Force equation in x direction

$$(T + dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} = dfr$$

Since θ is very small

$\sin d\theta \cong d\theta$

$\cos d\theta \cong 1$

$dT = dfr$ ....(6)

From(5)

$dT = \mu\, T\, d\theta$

$$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\pi}\mu d\theta$$

$$\ln\left(\frac{T_2}{T_1}\right) = \mu\pi$$

$T_2 = T_1 e^{\mu\pi}$

$\frac{T_2}{T_1} = e^{\mu\pi}$ ....(7)

From (3) and (7)

$e^{\mu\pi} = \frac{m_2}{m_1}\left(\frac{g-a}{g+a}\right)$.................(8)

When string start slipping

$a = 0$ m/s and $\frac{m_2}{m_1} = \eta_0$

Put in (8)

$e^{\mu\pi} = \eta_0$

Taking ln of both side

$\mu = \frac{1}{\pi}\ell n\,\eta_0$ **Ans.**

(b)

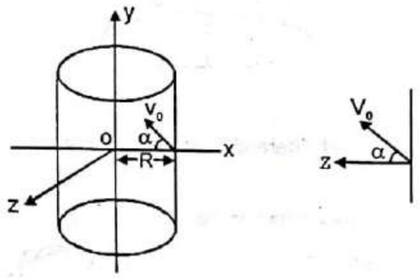
Put value $\mu = \frac{1}{\pi}\ell n\,\eta_0$ in (8) and $\frac{m_2}{m_1} = \eta$

$$e^{\ell n\,\eta_0} = \eta\left[\frac{g-a}{g+a}\right]$$

$$\frac{\eta_0}{\eta} = \frac{g-a}{g+a}$$

$a = g\left[\frac{n-\eta_0}{\eta+n_0}\right]$ **Ans.**

1.94



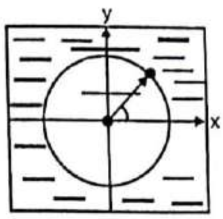
Hence velocity along y axis is not responsible for circular motion, only velocity along Z-axis is responsible.

$$N = \frac{mV_Z^2}{R}$$

$V_z = V_0 \cos\alpha$

$N = \frac{mV_0^2\cos^2\alpha}{R}$ **Ans.**

1.95



$x = a \sin wt$

$a_x = \frac{d^2x}{dt^2} = -\ aw^2 \sin wt$

$y = b \cos wt$

$a_y = \frac{d^2y}{dt^2} = -\,bw^2 \cos wt \Rightarrow$

$\vec{a}_{net} = a_x\hat{i} + a_y\hat{j}$

$= -\,a\,w^2 \sin wt\ \hat{i}\ -\ bw^2\ \cos wt\,\hat{j}$

$\vec{F} = m\vec{a}_{net}$

$\vec{F} = -mw^2[a\sin wt\hat{i} + b\cos wt\,\hat{j}]$ ....(1)

$\vec{r} = x\hat{i} + y\hat{j} = a\sin wt\,\hat{i} + b\cos wt\,\hat{j}$ ....(2)

From (1) and (2)

$\vec{F} = -m\vec{r}\,w^2$ **Ans.**

where $\vec{r} = x\hat{i} + y\hat{j}$ position vector of particle

$F = mrw^2$

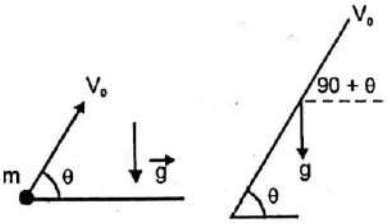
$r = \sqrt{x^2 + y^2}$

$F = mw^2\sqrt{x^2 + y^2}$ **Ans.**

1.96

Method : 1 (Impluse equation)

(a)



We know $\Delta\vec{P} =$ Impulse in time t

$$\Delta\vec{P} = \int_0^t F\,dt = \int_0^t -mg\,dt$$

$\Delta\vec{P} = -mgt$ **Ans.**

(b)

$\vec{V}_0 \cdot \vec{g} = V_0 g \cos(90 + \theta)$

$-\vec{V}_0 \cdot \vec{g} = V_0 g \sin\theta$

$$\frac{-\vec{V}_0 \cdot \vec{g}}{g} = V_0 \sin\theta$$

$$T = \frac{2V_0 \sin\theta}{g}$$

$$T = \frac{2\frac{-\vec{V}_0 \cdot \vec{g}}{g}}{g} = \frac{-2\vec{V}_0 \cdot \vec{g}}{g^2} =$$

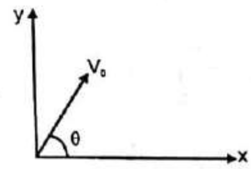
Momentum change in complete journey

$\Delta\vec{P} = -mgT$

$\Delta\vec{P} = +2m\,\frac{\vec{V}_0 \cdot \vec{g}}{g}$ **Ans.**

Method : 2 (Kinematic)

(a)



$\vec{V}_0 = V_0 \cos\theta\hat{i} + V_0 \sin\theta\hat{j}$

$\vec{V}_f = V_0 \cos\theta\hat{i} + (V_0 \sin\theta - gt)\hat{j}$

Where $\vec{V}_f$ = final velocity vector then

$\Delta\vec{P} = m\vec{V}_f - m\vec{V}_0$

$\Delta\vec{P} = -mgt\,\hat{j}$ **Ans.**

(b)

$\vec{V}_0 \cdot \vec{g} = V_0 g \cos(90 + \theta)$

$-\vec{V}_0 \cdot \vec{g} = V_0 g \sin\theta$

$$\frac{-\vec{V}_0\cdot\vec{g}}{g} = V_0\sin\theta$$

$$T = \frac{2V_0\sin\theta}{g}$$

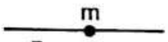
$$T = \frac{2\frac{-\vec{V}_0\cdot\vec{g}}{g}}{g} = \frac{-2\vec{V}_0\cdot\vec{g}}{g^2} =$$

Momentum change in complete journey

$$\Delta\vec{P} = -mgT$$

$$\Delta\vec{P} = +2m\frac{\vec{V}_0\cdot\vec{g}}{g} \qquad \textbf{Ans.}$$

1.97



$$\vec{F} = \vec{a}t(\tau - t)$$

(a)

$$\vec{F} = m\vec{w}$$

$$m\vec{w} = \vec{a}t(\tau - t)$$

$$\vec{w} = \frac{\vec{a}t(\tau - t)}{m}$$

$\vec{w}$ = linear acceleration

$$\vec{w} = \frac{d\vec{V}}{dt} = \frac{\vec{a}}{m}[\tau t - t^2]$$

$$\int_0^{\vec{V}} d\vec{V} = \frac{\vec{a}}{m}\left[\int_0^{t=\tau}\tau t\,dt - \int_0^{t=\tau}t^2\,dt\right] \quad ;$$

$$\vec{V} = \frac{\vec{a}}{m}\left[\frac{\tau t^2}{2} - \frac{\tau^3}{3}\right]_0^{\tau}$$

$$\vec{V} = \frac{\vec{a}\tau^3}{6m}$$

$$\vec{P}_f = m\vec{V} = \frac{\vec{a}\tau^3}{6} \qquad \textbf{Ans.}$$

(b) $w_\tau = \frac{\vec{a}}{m}\left[\tau t - t^2\right]$

$$\int_0^{\vec{V}} d\vec{V} = \frac{\vec{a}}{m}\left[\int_0^t \tau t\,dt - \int_0^t \tau t^2 dt\right]$$

$$\vec{V} = \frac{\vec{a}}{m}\left[\frac{t^2}{2}\tau - \frac{t^3}{3}\right]$$

$$\int_0^S dS = \frac{\vec{a}}{m}\left[\int_0^{\tau}\frac{t^2}{2}\tau\,dt - \int_0^{\tau}\frac{t^3}{3}dt\right]$$

$$\vec{S} = \frac{\vec{a}}{m}\left[\frac{\tau^3.\tau}{6} - \frac{\tau^4}{12}\right]$$

$$S = \frac{a}{m}\frac{\tau^4}{12}$$

$$S = \frac{a\tau^4}{12m} \qquad \textbf{Ans.}$$

1.98

$$F = F_0\sin wt$$

$$a = \frac{F_0}{m}\sin wt = \frac{dV}{dt}$$

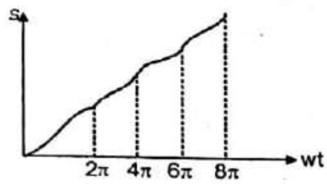
$$\int_0^V dV = \frac{F_0}{m}\int_0^t \sin wt\,dt$$

$$V = \frac{F_0}{mw}[1 - \cos wt]$$

$$\int_0^s ds = \frac{F_0}{mw}\int_0^t [1 - \cos wt]\,dt$$

$$S = \frac{F_0}{mw}\left[t - \frac{1}{w}\sin wt\right]$$

$$S = \frac{F_0}{mw^2}[tw - \sin wt] \qquad \textbf{Ans.}$$



Distance will be increasing function w.r.t. time.

1.99

$$F = F_0\cos wt$$

$$a = \frac{F_0}{m}\cos wt$$

$$\frac{dv}{dt} = \frac{F_0}{m}\cos wt$$

Calculation of time for first time stop

Intial and final both velocity will be zero

$$\int_0^0 dV = \frac{F_0}{m}\int_0^t \cos wt\, dt$$

$$0 = \frac{F_0}{mw} \sin wt$$

$\sin wt = 0 \qquad wt = \pi$

$$t = \frac{\pi}{w}$$ **Ans.**

Calculation of distance travell before first time stop

Intial and final both velocity will be zero

$$\int_0^V dV = \frac{F_0}{m}\int_0^t \cos wt$$

$$V = \frac{F_0}{mw} \sin wt \text{ ....(i)}$$

$$\int_0^s ds = \frac{F_0}{mw}\int_0^t \sin wt\, dt$$

$$S = \frac{F_0}{mw^2}(1-\cos wt)$$

Put $t = \frac{\pi}{w}$ in distance equation

$$S = \frac{2F_0}{mw^2}$$ **Ans.**

From (i), velocity will be maximum when when $\sin wt = 1$

$$V_{max} = \frac{F_0}{mw}$$ **Ans.**

1.100

(a)

$F = rV$

$$a = \frac{rV}{m}$$

$$\frac{dV}{dt} = -\frac{rV}{m}$$ (–) sign because velocity is decreasing

$$\int_{V_0}^{V} \frac{dV}{V} = -\frac{r}{m}\int_0^t dt$$

$$\ell n\left(\frac{V}{V_0}\right) = -\frac{rt}{m}$$

$$V = V_0 e^{-\frac{rt}{m}} \text{ ....(1)}$$

When V=0 m/s then $t \to \infty$ **Ans.**

(b)

$F = rV$

$$a = \frac{rV}{m}$$

$$V\frac{dV}{dx} = -\frac{rV}{m}$$ (–) sign because velocity is decreasing

$$\int_{V_0}^{V} dV = -\frac{r}{m}\int_0^s dx$$

$$V - V_0 = -\frac{r}{m}x$$

$$V = V_0 - \frac{r}{m}x \text{ ....(2)}$$

At final position V=0

Put this value in equation (2)

$$x = \frac{m}{r}V_0$$ **Ans.**

(b)

Put $V = \frac{V_0}{\eta}$ equation (1)

$$\frac{V_0}{\eta} = V_0 e^{-\frac{rt}{m}}$$

$$\ell n(\eta) = \frac{rt}{m}$$

$$t = \frac{m}{r}\ell n(\eta)$$

Put $V = \frac{V_0}{\eta}$ equation (2)

$$\frac{V}{\eta} = V_0 - \frac{r}{m}x$$

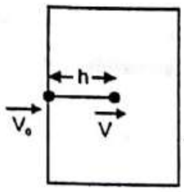
$$x = \left(V_0 - \frac{V_0}{\eta}\right)\frac{m}{r}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{x}{t}$$

$$\text{Average velocity} = \frac{\left(V_0 - \frac{V_0}{\eta}\right)\frac{m}{r}}{\frac{m}{r}\ell n(\eta)} = \frac{V_0(\eta - 1)}{\eta \ell n(\eta)}$$

Ans.

1.101



$F \propto v^2$

$F = -kv^2$ where k is any constant

$$a = \frac{-kv^2}{m}$$

$$\int_{V_0}^{V}\frac{dv}{v^2} = \frac{-k}{m}\int_0^t dt$$

$$\frac{-1}{V}\Big|_{V_0}^{V} = \frac{-kt}{m}$$

$$-\frac{1}{V_0} + \frac{1}{V} = \frac{kt}{m}$$

$$\left(\frac{V_0 - V}{V V_0}\right)\frac{m}{k} = t \text{ ....(i)}$$

Calculation of constant k

$$a = \frac{-kv^2}{m}$$

$$v\frac{dv}{dx} = \frac{-kv^2}{m}$$

$$v\,dv = \frac{-kv^2 dx}{m}$$

$$\int_{V_0}^{V}\frac{m\,dv}{V} = -k\int_0^h dx$$

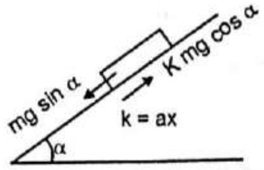
$$m\,\ell n\frac{V}{V_0} = -kh$$

$$k = \frac{m}{h}\,\ell n\frac{V}{V_0}$$

Put in (i)

$$t = \left(\frac{V_0 - V}{V V_0}\right)\frac{h}{\ell n\frac{V_0}{V_0}}$$ **Ans.**

1.102



Calculation of distance travell

Suppose at time t distance travel is x then

$F = mg\sin\alpha - kmg\cos\alpha$

$F = mg\sin\alpha - axmg\cos\alpha$

$mw = mg\sin\alpha - ax\,g\cos\alpha$

Where w = acceleration of block

$w = g\sin\alpha - ax\,g\cos\alpha$ ....(I)

$$\frac{v\,dv}{dx} = g\sin\alpha - ax\,g\cos\alpha \text{..............(2)}$$

$$\int_0^0 v\,dv = \int_0^{x_m}(g\sin\alpha - axg\cos\alpha)dx$$

$$0 = (g\sin\alpha)x_m - \frac{ax_m^2 g\cos\alpha}{2}$$

$$x_m = \frac{2g\sin\alpha}{a\,g\cos\alpha}$$

$$x_m = \frac{2}{a}\tan\alpha$$ **Ans.**

Calculation of maximum velocity

It will be maximum when

$$\frac{dv}{dt} = 0 = \text{w=acceleration}$$

From (1)

$0 = g\sin\alpha - ax\,g\cos\alpha$

$g\sin\alpha = axg\cos\alpha$

$$x = \frac{1}{a}\tan\alpha$$

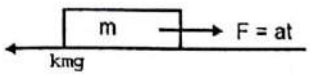
From(2)

$$\int_0^{V_{max}} v\,dv = \int_0^{\frac{1}{a}\tan\alpha}(g\sin\alpha - axg\cos\alpha)dx$$

$$\frac{V_{max}^2}{2} = g\sin\alpha\left(\frac{1}{a}\tan\alpha\right) - \frac{ag\cos\alpha}{2}\left(\frac{1}{a}\tan\alpha\right)^2$$

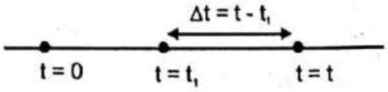
$$V_{max} = \sqrt{\frac{g\sin^2\alpha}{a\cos\alpha}} = \sqrt{\frac{g}{a}\sin\alpha\tan\alpha}$$ **Ans.**

1.103



K = friction coefficient
Block start sliding when
$F = at_1 = k\,m\,g$

$t_1 = \dfrac{kmg}{a}$ ....(1)



After this time , block start sliding then
Suppose acceleration of block after sliding start is w then
$mw = at - kmg$
Assuming $t = t_1 + \Delta t$.............(2)
$mw = a(t_1 + \Delta t) - kmg = at_1 + a\,\Delta t - kmg$
From (1)
$mw = a\,\Delta t$

$w = \dfrac{a}{m}\Delta t$

$$\int_0^v dV = \frac{a}{m}\int_0^{\Delta t} \Delta t\, d(\Delta t)$$

$$V = \frac{a}{2m}\Delta t^2$$

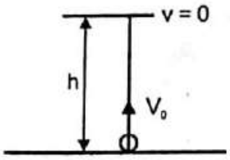
$$\int_0^s ds = \frac{a}{2m}\int_0^{\Delta t} \Delta t^2 d(\Delta t)$$

$$S = \frac{a}{6m}(\Delta t)^3$$

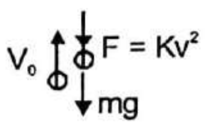
From (2) $\qquad S = \dfrac{a}{6m}(t - t_1)^3 \qquad$ **Ans.**

1.104

Upward journey of particle



Both mg and air drag will act in downward direction as below F.B.D.



$F_{net} = mg + kv^2$

$ma = mg + kv^2$

$a = g + \dfrac{kv^2}{m}$

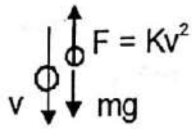
$-\dfrac{v\,dv}{ds} = \dfrac{mg + kv^2}{m}$

$-mv\,dv = (mg + kv^2)\,ds$

$$\int_{v_0}^{0} \frac{mv\,dv}{mg + kv^2} = -\int_0^h ds$$

$$h = \frac{m}{2k}\ell n\,\frac{KV_0^2 + mg}{mg} \qquad \text{............(i)}$$

Downward Journey of particle



$F_{net} = mg - kv^2$
$ma = mg - kv^2$

$a = \dfrac{mg - kv^2}{m} = v\dfrac{dv}{ds}$

$$\int_0^v \frac{mv\,dv}{mg - kv^2} = -\int_0^h ds$$

$$h = \frac{m}{2k}\;\ell n\,\frac{mg}{mg - kv^2} \qquad \text{..............(2)}$$

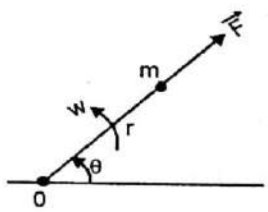
From(1) and (2)

$$\frac{m}{2k}\ell n\,\frac{KV_0^2 + mg}{mg} = \frac{m}{2k}\;\ell n\,\frac{mg}{mg - kv^2}$$

$$v = \frac{v_0}{\sqrt{1 + \dfrac{KV_0^2}{mg}}}$$ **Ans.**

1.105

(a)



Position vector of particle at time t when it rotate by θ angle

$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\theta = wt$

$\vec{F} = F\hat{r} = F(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$\vec{a} = \frac{F}{m}[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

$\int_0^{\vec{v}} d\vec{v} = \frac{F}{m}\int_0^t (\cos wt)dt\,\hat{i} + \frac{F}{m}\int_0^t (\sin wt)dt\,\hat{j}$

$\vec{V} = \frac{F}{mw}[\sin wt\;\hat{i} + (1-\cos wt)\,\hat{j}]$

$|\vec{V}| = \frac{F}{mw}\sqrt{(\sin wt)^2 + (1-\cos wt)^2}$

Speed$= |\vec{V}| = \frac{2F}{mw}\sin{}^{wt}\!/_{2}$ **Ans.**

(b)

Distance is calculated by speed

$\frac{ds}{dt} = V = \frac{2F}{mw}\sin{}^{wt}\!/_{2}$

$\int_0^s ds = \frac{2F}{mw}\int_0^t \sin{}^{wt}\!/_{2}\,dt$

$S = -\frac{2F}{mw^2}\times 2\cos\frac{wt}{2}\Big|_0^t$

$S = \frac{4F}{mw^2}\left(1-\cos\frac{wt}{2}\right)$ ....(1)

Velocity of particle will be zero for successive stops

$0 = \frac{2F}{mw}\sin\frac{wt}{2}$

$\sin\frac{wt}{2} = 0$

$t = \frac{2\pi}{w}$

put value of $t = \frac{2\pi}{w}$ in (1)

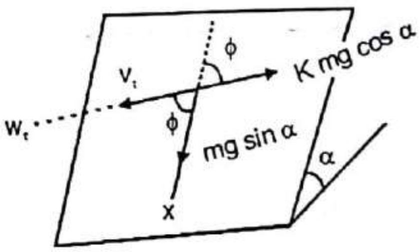
$S = \frac{4F}{mw^2}(1 - \cos\pi)$

$S = \frac{8F}{mw^2}$

Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{8F/mw^2}{2\pi/w}$

Average speed $= \frac{4F}{\pi mw}$ **Ans.**

1.106



$K = \tan\alpha$ = friction coefficient

When $= W_\tau$ = net tangential acceleration

W_x = Acceleration in x direction

Acceleration in velocity direction

$W_\tau = \frac{mg\sin\alpha\cos\phi - kmg\cos\alpha}{m}$

$W_\tau = g\,(\sin\alpha\cos\phi - K\cos\alpha)$

Put value of $K = \tan\alpha$

$W_\tau = g\sin\alpha\,[\cos\phi - 1]$.........(i)

Acceleration in x direction

$W_x = \frac{mg\sin\alpha - kmg\cos\alpha\cos\phi}{m}$

Put value of $K = \tan\alpha$

$W_x = g\sin\alpha\,[1-\cos\phi]$......(ii)

From (i) and (ii) : $W_\tau = -W_x$

$\int\frac{dV_\tau}{dt} = -\int\frac{dV_x}{dt}$

$V_\tau = -V_x + C$............(iii)

At $t = 0$

$V_\tau = V_0$ and $V_x = 0$

$V_0 = 0 + C$

$C = V_0$

From (iii)

$V_\tau = -V_x + V_0$ ....(iv)

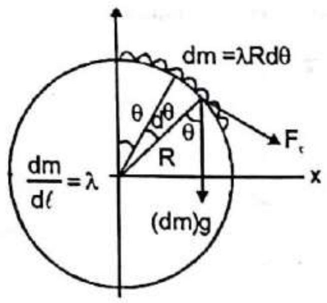
Also from figure

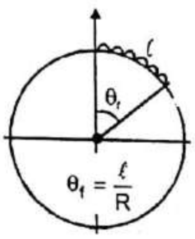
$V_x = V_\tau\cos\phi$ ....(v)

From (iv) and (v)

$V_\tau = \frac{V_0}{1+\cos\phi}$ **Ans.**

1.107





Tangential force on system is F_τ then

$$F_\tau = \int (dm)g\sin\theta = \int (\lambda R\, d\theta)g\sin\theta$$

$$F_\tau = \lambda Rg\int_0^{\theta_f}\sin\theta\, d\theta$$

$$F_\tau = -\lambda Rg\ \cos\theta\Big|_0^{\theta_f}$$

$$F_\tau = \lambda Rg\left[1-\cos\left(\frac{\ell}{R}\right)\right]$$

$$F_\tau = \lambda Rg\left[1-\cos\theta_f\right]$$

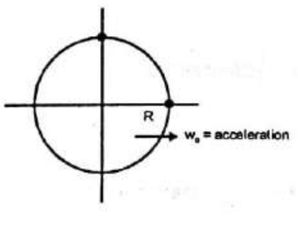
$$F_\tau = \lambda Rg\left[1-\cos\left(\frac{\ell}{R}\right)\right]$$

$$a_\tau = \frac{\lambda Rg}{m}\left[1-\cos\left(\frac{\ell}{R}\right)\right]$$

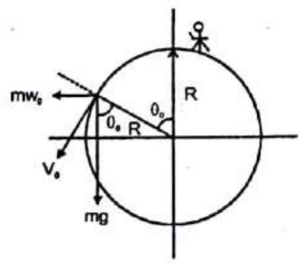
$$a_\tau = \frac{\lambda Rg}{\lambda\ell}\left[1-\cos\left(\frac{\ell}{R}\right)\right]$$

$$a_\tau = \frac{gR}{\ell}\left[1-\cos\left(\frac{\ell}{R}\right)\right]$$ **Ans.**

1.108



At time of break off, normal reaction will be zero.
F.B.D. of particle from frame of hemispher



Where mw_0= Psuedo force because observer at sphere

$$\frac{mV_0^2}{R} = mg\cos\theta_0 - m\, w_0\sin\theta_0$$

$V_0^2 = Rg\cos\theta_0 - R\, w_0\sin\theta_0$ ....(i)
Using work energy theorem equation
W_{all} forces $= K_f - K_i$
$W_{psuedo} + W_{mg} = K_f - K_i$

$$m\, w_0 R\sin\theta_0 + mg\,[R - R\cos\theta_0] = \frac{1}{2}mV_0^2$$

$V_0^2 = 2w_0 R\sin\theta_0 + 2gR\,[1-\cos\theta_0]$ ....(ii)
From (i) and (ii) :

$$V_0^2 = 2R\left[\frac{-V_0^2}{R}\right] + 2\,gR$$

$$V_0^2 = 2g\ \frac{R}{3}$$

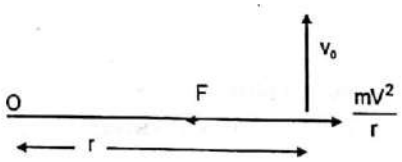
$$V_0 = \sqrt{\frac{2Rg}{3}}$$ **Ans.**

Put value of V_0 in equation (1)

$$\cos\theta_0 = \frac{2+\eta\sqrt{5+g[\eta]^3}}{3\left[1+(\eta)^2\right]}$$

Where $\eta = \dfrac{w_0}{g}$ **Ans.**

1.109



Given $F \propto \frac{1}{r^n} \Rightarrow F = \frac{K}{r^n}$
Where K = Constant.
A particle is said to steady if we displace particle away from origin, particle want to regain its position and also it we displaced particle toward origin, particle want to regain its original position. Then from rotating frame have same angular speed as particle using centrifugal force

$$F_{net} = \frac{mV^2}{r} - \frac{K}{r^n}$$

At equalibrium F_{net} =0 and $V = V_0$

$$\frac{K}{r^n} = \frac{mV_0^2}{r}$$ ....(i)

If we increases r then for stable equalibrium

$F_{net} < 0$

$$\frac{mV_0^2}{r+dr} - \frac{K}{(r+dr)^n} < 0$$

$$mV_0^2(r+dr)^{-1} - K(r+dr)^{-n} < 0$$

$$\frac{mV_0^2}{r}\left(1+\frac{dr}{r}\right)^{-1} - \frac{K}{r^n}\left(1+\frac{dr}{r}\right)^{-n} < 0$$

Using bionomial expression

$$\frac{mV_0^2}{r}\left(1-\frac{dr}{r}\right) - \frac{K}{r^n}\left(1-n\frac{dr}{r}\right) < 0$$

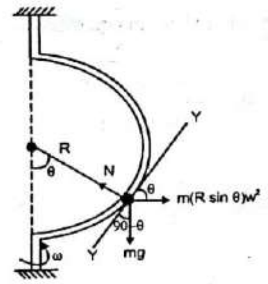
$$\frac{mV_0^2}{r} - \frac{mV_0^2}{r}\frac{dr}{r} - \frac{K}{r^n} + \frac{K}{r^n}n\frac{dr}{r} < 0$$

From(1)

$$-\frac{dr}{r} + n\frac{dr}{r} < 0$$

$n < 1$ **Ans.**

1.110



At steady state

$mg\sin\theta = m(R\sin\theta)w^2\cos\theta$

$\cos\theta = \dfrac{g}{Rw^2}$.

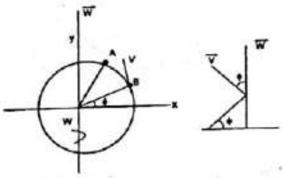
Case (i)

If $Rw^2 > g$ then $\cos\theta = \dfrac{g}{Rw^2}$ is defined and only one equilibrium position will be exist and will be steady.

Case (ii)

If $Rw^2 < g$ then $\cos\theta$ is not defined, here only $\cos\theta = 0°$ will be equilibrium position because tangential fore along arch of ring due to mg will be greater than that of centrifugal force and object will come at lower position of ring.

1.111*



Here point A is target and distance between A and B is $x_0 = 1$ km = S

Since horizonal velocity of bullet is v then time in which target kill : $t = \dfrac{x_0}{V} = \dfrac{S}{V}$ ....(1)

Because horizontal velocity is not afficted by gravity.

We know that earth is non inertial frame and it is rotating about y axis with angular velocity w. A frame attached with point B of earth, if we see the there is required corilious force from this frame, it provide corilious acceleration

$|\vec{a}_{cor}| = |2\vec{W}\times\vec{V}| = 2\omega V\sin\phi$ ....(2)

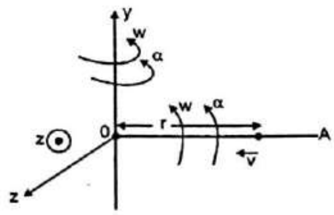
This direction is perpendicular to the plane of $\vec{W}$ and $\vec{V}$.

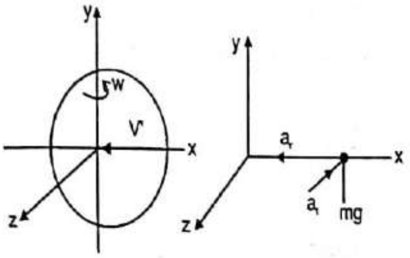
Now displacement (x) = $\frac{1}{2}|\vec{a}_{cor}|\ t^2 =$

$$x = \frac{1}{2}(2\omega V\sin\phi)\left(\frac{S}{V}\right)^2$$

$$x = \left(\frac{\omega S^2}{V}\right)\sin\phi = 7\,\text{cm}$$ **Ans.**

1.112*





If a particle on rod OA moving with velocity V toward centre and also rotating with angular velocity W.

Radial acceleration(a_r) = $\dfrac{d^2r}{dt^2} - rw^2$

Tangential acceleration(a_t) = $r\alpha + 2\vec{V}' \times \vec{W}$
Velocity of particle relative to rod OA
In questions radial velocity is constant and equal

to V' then $\dfrac{d^2r}{dt^2} = 0$.

Then radial acceleration $(a_r) = 0 - rw^2$
Since w is constant

$\dfrac{dw}{dt} = \alpha = 0$

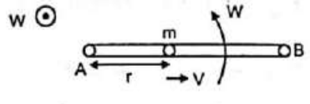
Tangential acceleration $(a_t) = 0 + 2V'w$

$\vec{F}_{net} = -m\,rw^2\hat{i} - 2m\,V'w\,\hat{k} - mg\,\hat{j}$.

$F_{net} = \sqrt{(mrw^2)^2 + (2mv'w)^2 + (mg)^2}$

$F_{net} = m\sqrt{g^2 + r^2w^4 + (2v'w)^2} = 8$ N. **Ans.**

1.113*



Coriolis acceleration(a_{co})= $2\vec{V} \times \vec{W} = 2VW$
Radial acceleration:

$a_r = \dfrac{d^2r}{dt^2} - rw^2$

Since there is no force along radial direction
$a_r = 0$

$\dfrac{d^2r}{dt^2} = rw^2$

$V\dfrac{dv}{dr} = rw^2$

$\int_{V_0}^{V} V\,dv = \int_0^r r w^2 dr$

$V^2 - V_0^2 = r^2w^2$
$V^2 = V_0^2 + r^2w^2$

$V = \sqrt{V_0^2 + r^2w^2}$

Coriolis force (F) =m [2V W] = $2m\sqrt{V_0^2 + r^2w^2}$

$F = 2mrw^2\sqrt{1 + \left(\dfrac{V_0}{wr}\right)^2} = 2.8$ N **Ans.**

1.114*

CONCEPT:
Velocity of particle w.r.t. with the help velocity of rotatory frame

$\vec{V}_{p-E} = \vec{V}_{rot} + \vec{\omega} \times \vec{R}$ ....**(1)**

Net virtual or fictitious force from non-inertial reference frame K' which rotates with a constant angular velocity $\vec{\omega}$ about an axis translating with an acceleration $\vec{W}_0$ will be

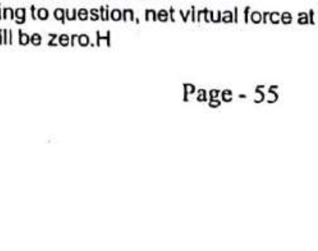
$\vec{F}_{inertial} = -m\vec{W}_0 + m\vec{R}\omega^2 + 2m\left(\vec{V} \times \vec{\omega}\right)$......**(2)**

where $\vec{R}$ is the radius vector of the point mass relative to the axis of rotation of the K' frame. Where V is relative velocity.

(a)
Suppose velocity of particle w.r.t disc is V then acceleration of particle with respect to disc is a then

$a = \dfrac{V^2}{R}$..................(3)

Calculation of V:



When particle will be at furtherest distance from axis then it will at point P.
According to question, net virtual force at this instant will be zero.H

$$\vec{F}_{inertial} = -m\vec{W}_0 + m\vec{R}\omega^2 + 2m\left(\vec{V}\times\vec{\omega}\right) = \vec{0}$$

Here $\vec{W}_0 = \vec{0}$ Then

$$0 = m2R\omega^2 + 2mV\omega$$

$$V = -R\omega$$

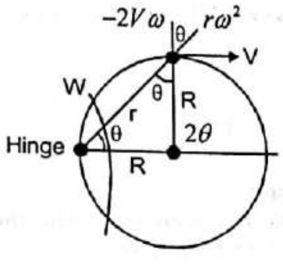
From(3)

$$a = R\omega^2$$ **Ans.**

(b)

When particle will be at r distance from axis

$$\vec{F}_{inertial} = -m\vec{W}_0 + m\vec{R}\omega^2 + 2m\left(\vec{V}\times\vec{\omega}\right)$$



$$\vec{F}_{inertial} = m\vec{R}\omega^2 + 2m\left(\vec{V}\times\vec{\omega}\right)$$

$$\left|\vec{F}_{inertial}\right| = m\left|\vec{R}\omega^2 + 2\left(\vec{V}\times\vec{\omega}\right)\right|$$

$$\left|\vec{F}_{inertial}\right| = m\sqrt{\left(r\omega^2\right)^2 + \left(2V\omega\right)^2 - 4rV\omega^3\cos\theta}$$ ....(4)

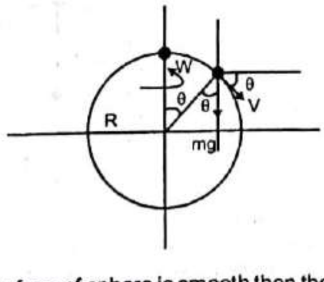
Here $2R\cos\theta = r$

$$\cos\theta = \frac{r}{2R}$$

Put value of $\cos\theta = \frac{r}{2R}$ in equation (4)

$$F_{in} = mrw^2\sqrt{\left(\frac{2R}{r}\right)^2 - 1}$$ **Ans.**

1.115



Since surface of sphere is smooth then there will be no effect of rotation of sphere on particle from inertial frame

At time of break off

$$mg\cos\theta = \frac{mv^2}{R} \text{ ....(i).}$$

Using energy conser vation

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta) \text{ ....(ii)}$$

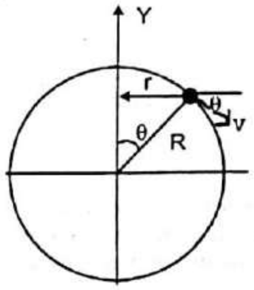
From (i) and (ii)

$$\cos\theta = \frac{2}{3}$$

From(1)

$$V^2 = \frac{2gR}{3} \text{ ....(3)}$$

(a)



Centrifugal force (F_c) = mrw^2

$$= mR\cos\theta\, w^2 = mRW^2\sqrt{1-\frac{4}{9}}$$

$$= mRW^2\frac{5}{9}$$ **Ans.**

(b*)

Coriolis force (F_{co})= $2m\left(\vec{V}_r\times\vec{\omega}\right)$

If reference frame fixed with sphere then velocity of particle from the frame of sphere will be

$$\vec{V}_r = rw\hat{k} + V\cos\theta\hat{i} - V\sin\theta\hat{j}$$

$$\vec{\omega} = w\hat{j}$$

$$\vec{F}_{CO} = 2m\left(rw\hat{k} + V\cos\theta\hat{i} - V\sin\theta\hat{j}\right)\times w\hat{j}$$

$$\vec{F}_{CO} = 2m\left(-w^2r\hat{i} + wV\cos\theta\hat{k}\right)$$

$$F_{CO} = 2mw\sqrt{(wr)^2 + (V\cos\theta)^2}$$

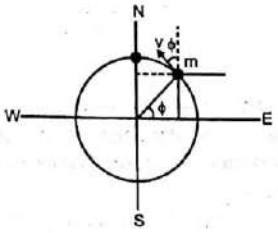
$$F_{CO} = 2mw\sqrt{(wR\sin\theta)^2 + \frac{2gR}{3}\cos^2\theta}$$

$$F_{CO} = \frac{2}{3} m w^2 R \sqrt{5 + \frac{8g}{3w^2 R}}$$ **Ans.**

1.116*

(a)

Magnitude of lateral force will be given by coriolis forces because coriolis force will be parallel to surface.



$F_{coriolis} = m\,|2\vec{V} \times \vec{W}| = 2\,mVW \sin\phi$ **Ans.**

(b)

$$\vec{F}_{inertial} = m\vec{R}\omega^2 + 2m\left(\vec{V} \times \vec{\omega}\right) = \vec{0}$$

$-2\vec{v} \times \vec{w} = \vec{r}w^2$

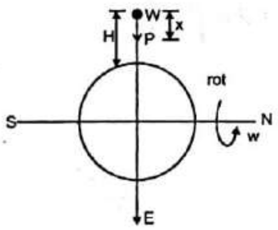
$2v = rw$

Angle between $\vec{w}$ and $\vec{v}$ must be parrallel to eachother sothat cross product become zero hence train must move from east to west

$2v = rw$

$v = \frac{1}{2} R w \cos\phi$ **Ans.**

1.117*



Suppose at time t particle at position P which is x unit below from starting point.

Let velocity is V then

$V = gt$

Coriolis acceleration

$a_{co} = 2\,v\,w = 2gtw$

$$\frac{dV_{CO}}{dt} = 2gtw$$

$$\int_0^{V_{CO}} dV_{CO} = 2g\int_0^t t\,w\,dt$$

$V_{CO} = gt^2 w$ ....(1)

$$h = \frac{1}{2} g t_1^2$$

$$t_1 = \sqrt{\frac{2h}{g}}$$

From (1)

$$\int_0^{x_{CO}} dx_{CO} = wg\int_0^{t_1} t^2\,dt$$

$$x_{CO} = \frac{wgt_1^3}{3}$$

Put $t_1 = \sqrt{\frac{2h}{g}}$ in (1)

$$X_{co} = \frac{2wh}{3}\sqrt{\frac{2h}{g}}$$ **Ans.**

1.3 Law of Conservation of Energy, Momentum and Angular Momentum

1.118

$d\bar{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$dw = \bar{F}.d\bar{r}$

$$w = \int_1^2 3\,dx + \int_2^{-3} 4\,dy = 3 + 4(-5) = -17 \text{ Joule}$$

Ans.

OR

Work done of constant force

$\Delta W = \vec{F}.\Delta\vec{R}$

$\vec{F} = 3\hat{i} + 4\hat{j}$

$\Delta\vec{R} = 1\hat{i} - 5\hat{j}$

$\Delta W = 3 - 20 = -17J$ **Ans.**

1.119

$v = a\sqrt{S}$ ....(i)

$dS = a\ S^{1/2}dt$

$$\int_0^s S^{-1/2}\,ds = a\int_0^t dt$$

$$2S^{1/2}\Big|_0^S = at$$

$S = \frac{a^2}{4}t^2$ ....(ii)

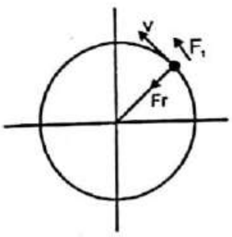
Put in (i)

$v = a\left(\frac{a}{2}t\right) = \frac{a^2t}{2}$

$$W_{\text{all forces}} = k_f - k_i = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{a^2t}{2}\right)^2$$

$\Delta W_{all} = \frac{ma^4t^2}{8}$ **Ans.**

1.120



$T = as^2 = k$ ————(i)

Calculation of tangential force (F_t)

Work done by force=Work done by tangential force (F_t)

$dw = F_t\,ds = dk$ ....——(iii)

Where dk = Change in K.E. in short time.

$F_t = \frac{dw}{ds} = \frac{dk}{ds}$

Differentiating equation(i)

$F_t = 2as$

Calculation of tangential force (F_r)

$\frac{1}{2}mv^2 = as^2$

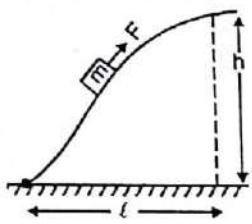
$mv^2 = 2as^2$

$F_r = \frac{mv^2}{R} = \frac{2as^2}{R}$

$f_{net} = \sqrt{F_t^2 + F_r^2}$

$F_{net} = 2as\sqrt{1+\left(\frac{S}{R}\right)^2}$ **Ans.**

1.121



Using work energy theory

$W_{all} = k_f - k_i$

$W_F + W_{mg} + W_{frictn} = k_f - k_i$

Since particle is solwly moving then

$k_i = 0$

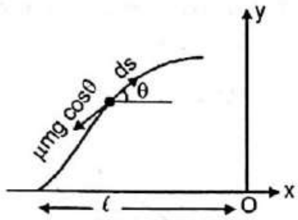
$k_f = 0$

$W_F + W_{mg} + W_{frictn} = 0$

$W_F = -(W_{mg} + W_{friction})$

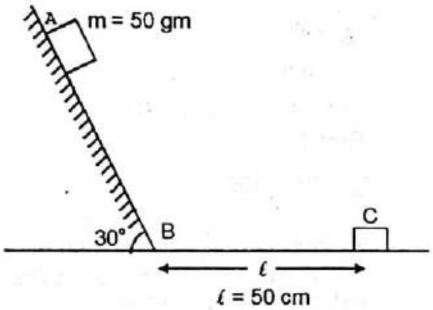
$= -(-mgh + W_{friction})$.......................(1)

Calculation of work done of friction



$W_{friction} = \Sigma(-\mu\, mg \cos\theta)\, ds$
$= -\Sigma\mu\, mg\, (ds \cos\theta)$
$= -\mu\, mg\, \Sigma\, (ds \cos\theta)$
$= -\mu\, mg\, \Sigma\, (dx)$
$= -\mu\, mg\ell$ In irodov $\mu = k$
$= -kmg\ell$
From(1)
$W_F = mgh + kmg\ell = mg\,(h + k\ell)$ **Ans.**

1.122



$K = \mu = 0.15$
We know
$W_{all\ forces} = k_f - k_i$
$W_{mg} + W_{friction} = 0 - 0 = 0$
$W_{friction} = -W_{mg}$ .... (i)
Soppose velocity at point B is V_β and distance between A and B is ℓ_1 then
$V_\beta^2 = 2[g \sin 30° - \mu g \cos 30°]\, \ell_1$ -------- (ii)
On horizontal plane
$V_\beta^2 = 2(\mu g)\, \ell$ ....(iii)
On incline plane
From (ii) and (iii)

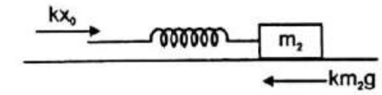
$$\mu g\ell = \frac{g\ell_1}{2} - \mu g\ell_1 \frac{\sqrt{3}}{2}$$

$$\ell_1 = \frac{2\mu\ell}{1-\sqrt{3}\mu}$$

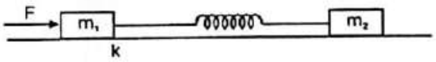
$$W_{mg} = mg\,\ell_1 \sin 30 = mg\,\frac{\mu\ell}{\left(1-\sqrt{3}\mu\right)} = 0.05\ \text{J}$$

From (i)
$W_{friction} = -0.05$ Joule **Ans.**

1.123



Condition for sliding of mass m_2
$kx_0 = km_2\, g$ — (i)



Now using work energy equation on m_1

$$Fx_0 - \frac{1}{2}kx_0^2 - km_1 g x_0 = \frac{1}{2}m_1 v^2 - 0$$

$$F - \frac{1}{2x_0}kx_0^2 - \frac{km_1 g x_0}{x_0} = \frac{m_1 v^2}{2x_0}$$

$$F = \frac{1}{2x_0}kx_0^2 + \frac{km_1 g x_0}{x_0} + \frac{1}{2x_0}m_1 v^2 \ \text{............(ii)}$$

From equation (i)

x_0 is constant , for minimum value of F , v should be minimum and equal to zero.

$$\frac{1}{2}m_1 v^2 \to 0$$

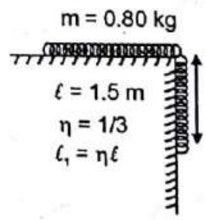
$$F_{min} = \frac{kx_0}{2} + km_1 g$$

From (i)

$$F_{min} = \frac{km_2 g}{2} + km_1 g$$

$$F_{min} = kg\left(m_1 + \frac{m_2}{2}\right)$$ **Ans.**

1.124

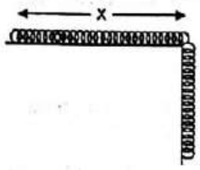


When $\eta\,\ell$ portion of chain is hanging, chain start sliding

$$(\eta\ell)\frac{m}{\ell}g = \mu\frac{m}{\ell}(\ell - \eta\ell)g$$

$\eta\ell = \mu\ell(1-\eta)$

$\mu = \frac{\eta}{1-\eta}$ ....(1)



Suppose at time t only x portion is at surface.
Work done by friction in next dx displacement
dw = fr (dx)

$$W = \int_{\ell-\eta\ell}^{0} \mu\frac{m}{\ell}xg\,dx$$

$$W = \frac{-\mu mg}{2\ell}[\ell-\eta\ell]^2$$

$$W = -\frac{\mu mg\ell}{2}[1-\eta]^2$$

Put the value of μ from equation (1)

$$W = \frac{-\eta}{2(1-\eta)}mg\ell(1-\eta)^2$$

$$W = -\frac{\eta(1-\eta)}{2}mg\ell$$ **Ans.**

1.125

(a)
Mean power of force

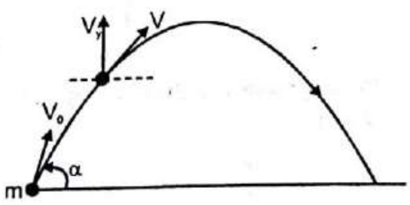
$$P_{avg} = \frac{\text{Net work done}}{\text{Time}} = \frac{\Delta W}{\Delta t}$$

Work done by force mg will be 0 because net displacement in y direction =0

$$\text{Mean power} = \frac{\text{Net work done}}{\text{Time}} = \frac{0}{\text{Time}} = 0$$

Ans.

(b)

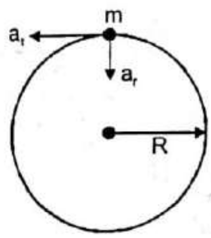


Instaneous power of force at time t

$P = \vec{F}.\vec{v} = -mgv_y$

$= -mg[V_0\sin\alpha - gt]$ **Ans.**

1.126



a_t = Tangential acceleration ;
a_r = Radial acceleration
(a)
$w_n = at^2$

$\frac{v^2}{R} = at^2$

$v^2 = aRt^2$

$v = t\sqrt{aR}$ ....(1)

From(1)

$a_t = \frac{dv}{dt} = \sqrt{aR}$

$F_t = ma_t = m\sqrt{aR}$ ....(2)

We know only tangential force provide work then
Instantenious power of force =

$P_{ins} = F_tV = m\sqrt{aR}\left(t\sqrt{aR}\right).$

$P_{inst} = m\,a\,R\,t$ **Ans.**

(b)

$$\int_0^s ds = \sqrt{aR}\int_0^t t\,dt$$

$$S = \frac{t^2}{2}\sqrt{aR}$$

Average power (P_{avg}) = $\frac{\Delta W}{t}$

$\Delta W = F_tS$

Where S = Distance travelled by particle

$$\Delta W = \left(m\sqrt{aR}\right)\left(\frac{t^2}{2}\sqrt{aR}\right) = \frac{maRt^2}{2}$$

$$P_{avg} = \frac{\Delta W}{t} = \frac{maRt^2}{2t} = \frac{maRt}{2}$$ **Ans.**

1.127

(a)

Time taken by particle to stop is t_1 then

$v = u + at$

$0 = v_0 - k g t_1$

$t_1 = \frac{V_0}{kg}$

Work done by f_r

Work done by (f_r)= $k_f - k_i = 0 - \frac{1}{2}mv_0^2$

$W_{friction} = -\frac{1}{2}mv_0^2$

Average power (P_{avg}) = $\frac{\Delta W}{t}$

$$<P> = \frac{W_{friction}}{t_1} = \frac{-\frac{1}{2}mv_0^2}{\frac{V_0}{kg}} = -\frac{1}{2}mV_0\ kg$$

Ans.

(b)

$a = -\mu g = -\alpha xg$..................(1)

$v\frac{dv}{dx} = -\alpha xg$

$$\int_{v_0}^{v} v\,dv = -\alpha g\int_0^x x\,dx$$

$v = \sqrt{v_0^2 - \alpha g x^2}$

Power of force(P) = Fv

F=ma=m α xg

$v = \sqrt{v_0^2 - \alpha g x^2}$

P=m α xg $\sqrt{v_0^2 - \alpha g x^2}$ ....(ii)

To maximum value of P

$\frac{dP}{dx} = 0$

$\alpha g x^2 - \frac{v_0^2}{2} = 0$

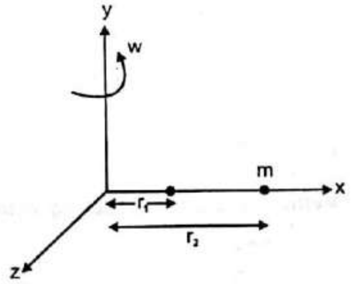
$x = \frac{v_0}{\sqrt{2\alpha g}}$.

Put (ii)

$P = -\frac{mv_0^2}{2}\sqrt{g\alpha}$.

Ans.

1.128



From frame of rotatory axis , centrifugal force on body when body is r distance from axis of rota tion, motion will be straight line.

$F = mrw^2$

$$\int dw = \int_{r_1}^{r_2} mrw^2 dr$$

$$w = \frac{mw^2}{2}\left[r_2^2 - r_1^2\right]$$ **Ans.**

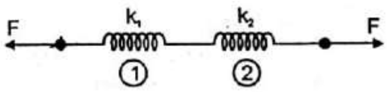
1.129

Method : 1 (Basic approach)

For minimum work done , straching is made very slowly then net foece on system is zero.

Then mass moment of system will be conserved.

Suppose x_1 elongation in spring (1) and x_2 in (2).



Using mass moment conservation

$k_1 x_1 = k_2 x_2$............(i)

$x_1 + x_2 = \ell$........ (ii)

From (i) and (ii)

$x_2 = \frac{k_1\Delta\ell}{k_2 + k_1}$

$x_1 = \frac{k_2\Delta\ell}{k_2 + k_1}$

Minimum work done by external agent

=Total Store Energy store in springs

$= \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$

$$E = \frac{1}{2}k_1\left[\frac{k_2\Delta\ell}{k_2+k_1}\right]^2 + \frac{1}{2}k_2\left[\frac{k_1\Delta\ell}{k_2+k_1}\right]^2 =$$

$$E = \frac{1}{2}\frac{\Delta\ell^2}{(k_2+k_1)^2}\left[k_1k_2^2 + k_2k_1^2\right]$$

$$E = \frac{1}{2}\frac{k_1k_2}{(k_1+k_2)}\Delta\ell^2 \qquad \textbf{Ans.}$$

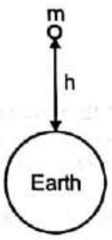
Method : 2 (Equivalent spring method)

$$k_{eq} = \frac{k_1k_2}{k_1+k_2}$$

$$E = \frac{1}{2}k_{eq}\,\Delta\ell^2$$

$$E = \frac{1}{2}\frac{k_1k_2}{(k_1+k_2)}\Delta\ell^2 \qquad \textbf{Ans.}$$

1.130



$\vec{F} = 2(ay-1)\,m\vec{g}$

$F_{up} = 2\,(1-ay)\,mg$.....................(1)

$F_{net}(up) = 2\,(1-ay)\,mg - mg$

$= mg\,(1-2\,ay)$

$a_{net} = g\,(1-2ay)$

$$a_{net} = \frac{vdv}{dy} = g\,(1-2ay)$$

$$\int_0^0 v dv = \int_0^y g\,(1\text{-}2ay)$$

$$0 = g\left[y_{max} - 2a\frac{y_{max}^2}{2}\right]$$

$$y_{max} = \frac{1}{a}$$

Half of ascent = $\frac{1}{2a}$

Work done by pulling force

$$W = \int_0^{1/2a} 2(1-ay)\,mg\,dy$$

$$W = \frac{3mg}{4}. \qquad \textbf{Ans.}$$

Increase in P. E. Due to gravity

$$= gm\left(\frac{y_{max}}{2}\right) = \Delta U = \frac{mg}{2a} \qquad \textbf{Ans.}$$

1.131 **(a)**

$$U = \frac{a}{r^2} - \frac{b}{r}$$

$U = ar^{-2} - br^{-1}$......................(1)

For equilibrium:

$$F = -\left.\frac{dU}{dr}\right|_{r=r_0} = 0$$

Differentiate equation(1)

$\frac{dU}{dr} = -2ar^{-3} + br^{-2}$ ....(2)

$$-2ar_0^{-3} + br_0^{-3} = 0$$

$$2ar_0^{-3} = br_0^{-2}$$

$$r_0 = \frac{2a}{b}$$

For stable equilibrium

$$\left.\frac{d^2U}{dr^2}\right|_{r=r_0} = \oplus \text{ ive}$$

Differentiate equation(2)

$$\frac{d^2U}{dr^2} = 6ar^{-4} - 2br^{-3} = 6ar^{-4} - 2br^{-3}$$

Put the value of $r_0 = \frac{2a}{b}$

$$\frac{d^2U}{dr^2} = 6a\left[\frac{2a}{b}\right]^{-4} - 2b\left[\frac{2a}{b}\right]^{-3}$$

$$= \left[\frac{2a}{b}\right]^{-3}\left[\frac{6ab}{2a} - 2b\right]$$

$$= \left[\frac{2a}{b}\right]^{-3} b$$

$= \oplus$ ive

Then equilibrium is stable **Ans.**

(b)

For attractive force(F)

$$F = \frac{dU}{dx} = -2ar^{-3} + br^{-2}$$

For maximum value of attractive force(F)

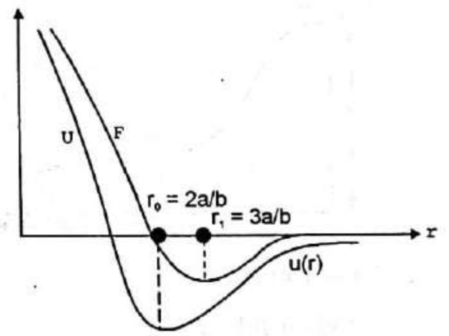
$$\frac{dF}{dr} = 0$$

$$6ar^{-4} - 2b\,r^{-3} = 0$$

$$\frac{6a}{r} - 2b = 0$$

$$r = \frac{6a}{2b} = \frac{3a}{b}$$

$$F_{max} = -2a\left(\frac{3a}{b}\right)^{-3} + b\left(\frac{3a}{b}\right)^{-2} = \frac{b^3}{27a^2}$$ **Ans.**



Ans.

1.132

(a) $U = \alpha x^2 + \beta y^2$ ....(1)

For central forced

$\vec{F} = k\vec{r}$ where K is any constant

$\vec{r}$ = Position vector of point of application of force

From (1)

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\right)\hat{i} + \left(\frac{\partial U}{\partial y}\right)\hat{j}$$

$$\vec{F} = -2\alpha x\hat{i} - 2\beta y\hat{j} = -2(\alpha x\hat{i} + \beta y\hat{j}) \text{...........(i)}$$

Suppose position vector is $\vec{r} = x\hat{i} + y\hat{j}$

This is not seen in equation (i) hence it is not central force.

(b)

Method : 1

We know equipotential surface are perpendicular to direction of force.

Hence equipotential surface will be in perpendicu -lar direction of slope on force.

From(i)

Slope of force $= \frac{dy_1}{dx_1} = \frac{\beta y}{\alpha x}$

Slope of equipotential surface

$m_1 m_2 = -1$

$$\frac{dy}{dx} \times \left(\frac{dx_1}{dy_1}\right) = -1$$

$$\frac{dy}{dx} = -\frac{\alpha x}{\beta y}$$

$$\int \beta y\, dy = -\int \alpha x\, dx$$

$$\beta \int y\, dy = -\alpha \int x\, dx$$

$$\frac{\beta y^2}{2} = -\frac{\alpha x^2}{2} + C$$

$$\frac{x^2}{2/\alpha} + \frac{y^2}{2/\beta} = C$$

This is ellipse.

Ratio of semi axis : $\frac{a}{b} = \frac{\sqrt{2/\alpha}}{\sqrt{2/\beta}} = \sqrt{\beta/\alpha}$ **Ans.**

Method : 2

Equipotential surface is that point at which po tential is constant then

$$\alpha x^2 + \beta y^2 = C_1$$

$$\frac{x^2}{\frac{1}{\alpha}} + \frac{y^2}{\frac{1}{\beta}} = C_1$$

This is ellipse.

Ratio of semi axis : $\frac{a}{b} = \frac{\sqrt{2/\alpha}}{\sqrt{2/\beta}} = \sqrt{\beta/\alpha}$ **Ans.**

If force is constant magnitude

$|\vec{F}|$ = constant

$\vec{F} = -2\,(\alpha x\hat{i} + \beta y\hat{j})$

$|\vec{F}| = 2\sqrt{\alpha^2x^2 + \beta^2y^2} = C_2$

$$\alpha^2x^2 + \beta^2y^2 = C$$

$$\frac{x^2}{\frac{1}{\alpha^2}} + \frac{y^2}{\frac{1}{\beta^2}} = C$$

This is surface at which magnitude of force is con stant which is an ellipse.

Ratio of semi maser axis $= \frac{\sqrt{1/\alpha^2}}{\sqrt{1/\beta^2}} = \beta/\alpha$ **Ans.**

1.133

If force fields are potential or conservative then

$$\vec{\nabla}\times\vec{F} = 0$$

For $\vec{F}_1 = ay\hat{i}$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ay & 0 & 0 \end{vmatrix}$$

$= \hat{i}(0)+\hat{j}(0-0)+\hat{k}(0-a) = -a\hat{k}$

Hence force is not potential.

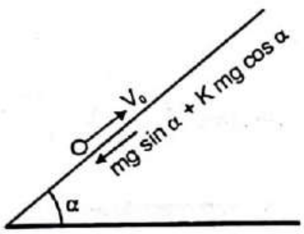
For $\vec{F}_2 = ax\hat{i} + by\hat{j}$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax & by & 0 \end{vmatrix}$$

$= \hat{i}[0-0]-\hat{j}[0-0]+\hat{k}[0-a] = \vec{0}$

Hence force is potential.

1.134



$a_{net} = g\sin\alpha + kg\cos\alpha$

This is retardation.

$V^2 = u^2 + 2as$

Final speed will be zero.

$0 = V_0^2 - 2(g\sin\alpha + kg\cos\alpha)S$

$$S = \frac{V_0^2}{2[g\sin\alpha + kg\cos\alpha]}$$

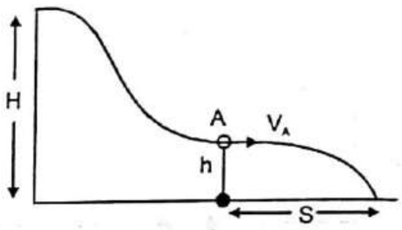
$W_{friction} = \Delta W = -f_r \times S = -(K mg\cos\alpha)S$

$$= -\left[\frac{V_0^2}{2[g\sin\alpha + kg\cos\alpha]}\right][Kmg\cos\alpha]$$

$$= \frac{-KmV_0^2\cos\alpha}{2[\sin\alpha + k\cos\alpha]}$$

$$W_{friction} = \frac{-mV_0^2K}{2(K+\tan\alpha)}$$ **Ans.**

1.135



Velocity at point A

$\frac{1}{2}mV_A^2 = mg(H-h)$

$V_A = \sqrt{2g(H-h)}$ ....(1)

Using equation of trajectory :

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ ....(2)

When particle is at point A ,

$\theta = 0$; $y = -h$; $u = V_A$; $x = S$

Put values in (2)

$-h = -\frac{gS^2}{2V_A^2}$

From (1)

$h = \frac{g}{2}\frac{S^2}{(2g)(H-h)}$

$S^2 = 4h(H-h) = 4hH - 4h^2$

$S^2 = 4hH - 4h^2$ ....(3)

For S maximum :

$\frac{dS}{dh} = 0$

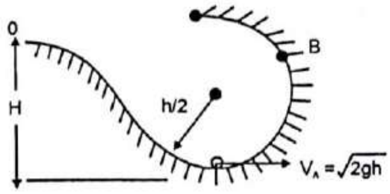
$[4H - 8h] = 0$

$h = \frac{H}{2}$ **Ans.**

Put value of h in (3)

$S_{max} = \sqrt{4\frac{H}{2}\left(H-\frac{H}{2}\right)} = H$ **Ans.**

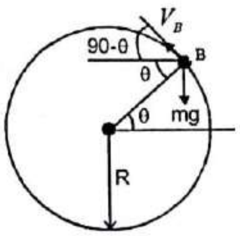
1.136



When particle are at bottom point, it velocity will be $V_A = \sqrt{2gh}$.

Suppose when particle reaches at point B, leave contact with surface.

Then at this point N = 0



Force equation in radial direction

$$mg \sin\theta = \frac{mV_B^2}{R}$$

$V_B^2 = Rg\sin\theta$ ....(i)

Energy conservation equation

$$\frac{1}{2}mV_A^2 = \frac{1}{2}m V_B^2 + mg(R + R\sin\theta)$$

$$V_A^2 = V_B^2 + 2gR(1+\sin\theta)$$

From (1)

$$2gh = V_B^2 + 2gR\left(1+\frac{V_B^2}{Rg}\right)$$

$$2gh = V_B^2 + 2gR + 2V_B^2$$

Given that R=h/2

$$2gh = 3V_B^2 + 2g\frac{h}{2}$$

$V_B = \sqrt{\frac{gh}{3}}$ ....(ii)

Put this value in equation (i)

$$\frac{1}{3}gh = \text{Rg}\sin\theta$$

$$\frac{1}{3}gh = \frac{h}{2}\text{g}\sin\theta$$

$\sin\theta = \frac{2}{3}$ ....(iii)

At maximum height after leave circular track, motion will be parabola and at maximum height only horizontal velocity present

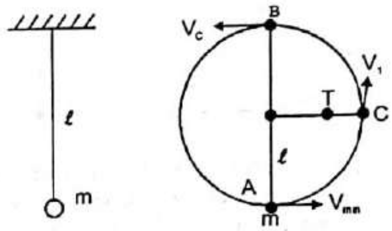
$$V_{Horizontal} = V_B \cos(90 - \theta)$$

$$= V_B \sin\theta$$

From (ii) and (iii)

$V_{Horizontal} = \frac{2}{3}\sqrt{\frac{gh}{3}}$ **Ans.**

1.137



V_{min} should be as at maximum height tension will be greater than equal to zero

Then force equation at that point B

$$\frac{mV_C^2}{\ell} = mg$$

$$V_C = \sqrt{g\ell}$$

Using energy equation

$$g\ell = V_{min}^2 - 4g\ell$$

$V_{min} = \sqrt{5g\ell}$ **Ans.**

Force equation at point C

$T = \frac{mV_1^2}{\ell}$ ....(1)

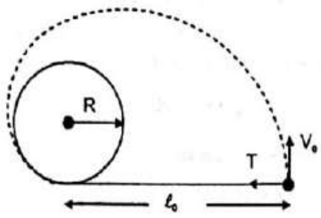
energy equation between A and C

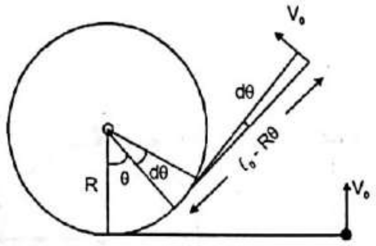
$$\frac{1}{2}mV_1^2 = \frac{1}{2}mV_{min}^2 - mgR$$

$$V_1^2 = 3gl$$

Put in (1) : T = 3 mg **Ans.**

1.138





Since initially tension will be perpendicular to velocity, it does not work and hence speed remain constant in magnitude hence we have to find total distance travel by particle before it strike again to cylinder.

At time = t

Suppose in next time dt particle traveled ds distance than

$$ds = (\ell_0 - R\theta)d\theta$$

$$\int_0^S ds = \int_0^{\theta_{max}} (\ell_0 - R\theta)d\theta \text{ ....(1)}$$

Where $\theta_{max} = \dfrac{\ell_0}{R}$

From (1)

$$S = \ell_0\theta_{max} - \frac{R\,\theta_{max}^2}{2}$$

$$S = \frac{\ell_0^2}{R} - \frac{R\ell_0^2}{2R^2}$$

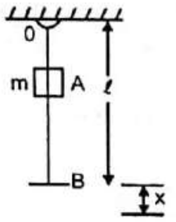
$$S = \frac{\ell_0^2}{R} - \frac{R\ell_0^2}{2R^2}$$

$$S = \frac{\ell_0^2}{2R}$$

Time to strike while speed is constant

$$t = \frac{S}{V_0} = \frac{\ell_0^2}{2RV_0}$$ **Ans.**

1.139



Total loss in P. E. of sleeve
= Gain in elastic potential energy of chord

$$mg(\ell + x) = \frac{1}{2}kx^2$$

$$2mg\ell + 2mgx = kx^2$$

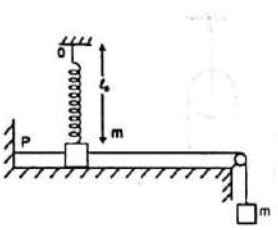
$$kx^2 - 2mgx - 2mg\ell = 0$$

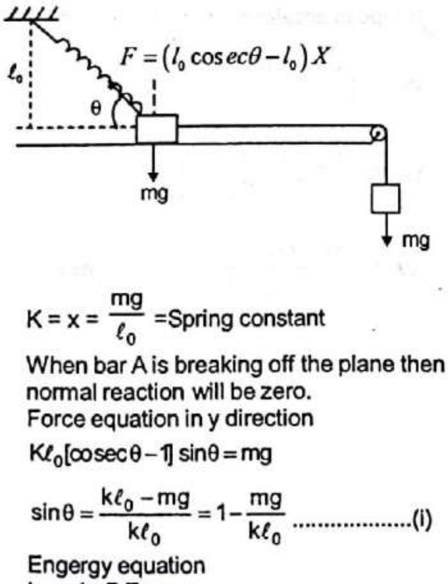
$$x = \frac{2mg + \sqrt{4m^2g^2 + 4k(2mg\ell)}}{2k}$$

$$x = \frac{mg}{k}\left(1 + \sqrt{1 + 2k\frac{\ell}{mg}}\right)$$

Ans.

1.140





$$K = x = \frac{mg}{\ell_0} = \text{Spring constant}$$

When bar A is breaking off the plane then normal reaction will be zero.
Force equation in y direction

$$K\ell_0[\operatorname{cosec}\theta - 1]\sin\theta = mg$$

$$\sin\theta = \frac{k\ell_0 - mg}{k\ell_0} = 1 - \frac{mg}{k\ell_0} \text{ ....(i)}$$

Engergy equation
Loss in P.E.
= Gain in K.E. of both block + spring energy

$$mg\ell_0\cot\theta = \left(\frac{1}{2}mV^2\right)\times 2 + \frac{1}{2}k\ell_0^2\,(\operatorname{cosec}\theta - 1)^2$$

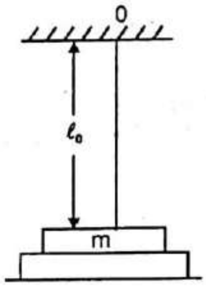
.........(ii)

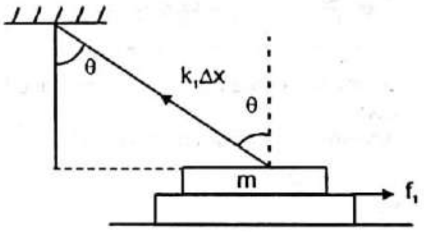
From (i) and (ii)

$$V = \sqrt{\frac{19g\ell_0}{32}}$$

Ans.

1.141





Elastic string behave as spring.
Let assume **elastic constant of string is k_1**

Elongation of spring $= \Delta x = l_0 \sec\theta - l_0$

Spring force $= k_1 \Delta x$

$= K_1(\ell_0\sec\theta - \ell_0) = K_1\ell_0(\sec\theta - 1)$...........(1)

For equation in horizontal direction on block of mass m at time of slipping

$k_1 \Delta x \sin\theta = f_1 = KN$

Where K=coefficient of friction

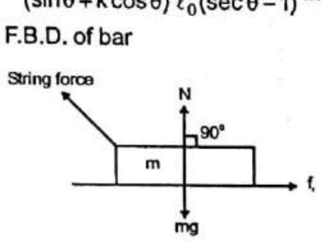
$k_1 \Delta x \sin\theta = K(mg - k_1\Delta x\cos\theta)$

$$k_1 = \frac{kmg}{\Delta x\sin\theta + k\Delta x\cos\theta}$$

$$k_1 = \frac{kmg}{(\sin\theta + k_1\cos\theta)\Delta x}$$

$$= \frac{kmg}{(\sin\theta + k\cos\theta)\,\ell_0(\sec\theta - 1)} \text{ ....(2)}$$

F.B.D. of bar



Since bar is moving slowly.

$W_{all\ force} = k_f - k_i = 0$
$W_{string} + W_{friction} + W_{normal} + W_{mg} = 0$ ....(3)
$W_{normal} = 0$
$W_{mg} = 0$
From(3)
$W_{string} = -W_{friction}$..................(4)

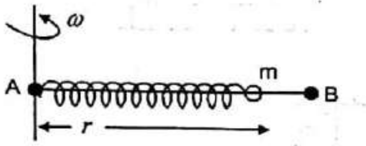
$$W_{friction} = -W_{string} = -\frac{1}{2}k_1\Delta x^2$$

$$W_{string} = -\frac{1}{2}\left[\frac{kmg}{(\sin\theta + k\cos\theta)\ell_0(\sec\theta - 1}\right]\ell_0^2(\sec\theta-1)^2$$

$$W_{friction} = \frac{kmg}{2}\ell_0\left[\frac{1-\cos\theta}{(\sin\theta + k\cos\theta)\cos\theta}\right]$$

Ans.

1.142



Suppose at time t sleeve at r distance from point A
Acceleration toward point A

$$a_{net} = -\left(\frac{d^2r}{dt^2} - r w^2\right)$$

$F_{net} = m\, a_{net}$

$$\chi(r-\ell_0) = m\left[\frac{d^2r}{dt^2} - r w^2\right] \text{...........(i)}$$

Since system start rotating slowly

$$v_r = \frac{dr}{dt} \to 0 \text{ and constant then } \frac{d^2r}{dt^2} = 0$$

Form (i) $\chi(r-\ell_0) = mrw^2$

$$r = \frac{\chi\ell_0}{\chi - mw^2}.$$

Work done by external agent
= Increse energy of block (sleeve) + P. E. of spring

Speed of sleeve = V = rw
Then using energy conservation equation
Work done by external agent

$$= \frac{1}{2}mV^2 + \frac{1}{2}\chi(r-\ell_0)^2$$

$$= \frac{1}{2}mr^2w^2 + \frac{1}{2}\chi\left[\frac{\chi\ell_0}{\chi - mw^2} - \ell_0\right]^2$$

$$= \frac{1}{2}mr^2w^2 + \frac{1}{2}\chi\left(\frac{m^2\ell_0^2w^4}{(\chi - mw^2)^2}\right)$$

$$= \frac{1}{2}w^2m\left(\frac{\chi\ell_0}{\chi - mw^2}\right)^2 + \frac{1}{2}\frac{m^2\ell_0^2w^4\chi}{(\chi - mw^2)^2}$$

$$= \frac{1}{2}\frac{mw^2\ell_0^2\chi}{(\chi - mw^2)^2}\left[\chi + mw^2\right]$$

Assume : $\eta = \dfrac{mw^2}{\chi}$

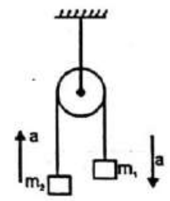
$$W = \frac{1}{2}\left(\frac{mw^2}{\chi^2}\right)\frac{\ell_0^2\chi}{\left(1-\frac{mw^2}{\chi}\right)^2}\left[\chi + mw^2\right]$$

$$W = \frac{1}{2}\left(\frac{mw^2}{\chi}\right)\frac{\ell_0^2\chi}{\left(1-\frac{mw^2}{\chi}\right)^2}\left(1+\frac{mw^2}{\chi}\right)$$

$$W = \frac{1}{2}\frac{\eta\ell_0^2\chi(1+\eta)}{(1+\eta)^2}$$

Ans.

1.143



$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

Suppose acceleration centre of mass is W_c

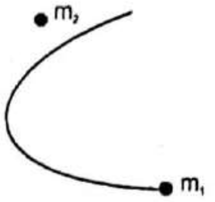
$$W_c = \frac{m_1a - m_2a}{m_1 + m_2}$$

$$W_c = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\frac{m_1 - m_2}{m_1 + m_2}g$$

$$W_c = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

Ans.

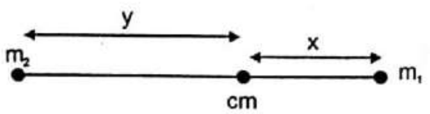
1.144



Given that $m_2 = \frac{m_1}{2}$

Since there is no more particle other than m_1 and m_2.

Then COM will be in rest then



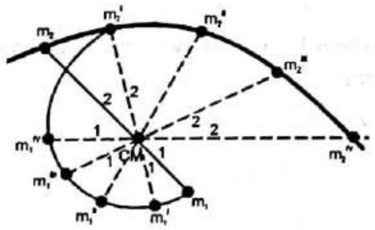
$m_2 y = m_1 x$

$\frac{m_1}{2} y = m_1 x$

$y = 2x$

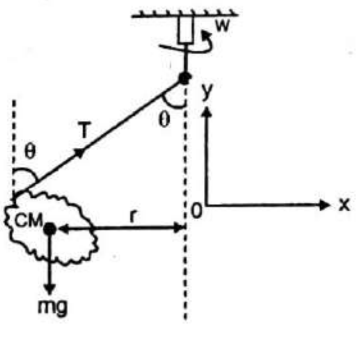
$\frac{x}{y} = \frac{1}{2}$

Distance of both particles frm their centre of will be constant and its ratio will be 2.



Ans.

1.145



Let distance between COM of ring and axis of rotation is r.

Then force equation in y direction

$T \cos\theta = mg$

$T = \frac{mg}{\cos\theta}$ ....(1)

Force equation in x direction

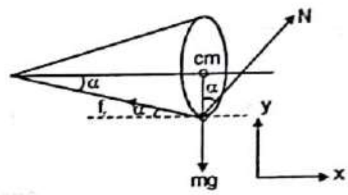
$T \sin\theta = mrw^2$

From(1)

$\left(\frac{mg}{\cos\theta}\right)\sin\theta = mrw^2$

$r = \frac{g\tan\theta}{w^2}$ **Ans.**

1.146



(a)

Acceleration of com $a^x_{cm} = \ell w^2$

Force equation in x direction

$f_r \cos\alpha - N\sin\alpha = m\,\ell\, w^2$...........(i)

Force equation in y direction

Since motion of com is horizantal circular motion

Acceleration in y direction $(a^y_{cm}) = 0$

$N\cos\alpha + f_r \sin\alpha = mg$ ....(ii)

From (i) and (ii)

$f_r = mg\left[\sin\alpha + \frac{w^2\ell}{g}\cos\alpha\right]$ **Ans.**

(b)

From (i) and (ii) For no slipping

$N = mg\cos\alpha - m\ell w^2 \sin\alpha$

For no slipping

$\mu N > f_r$

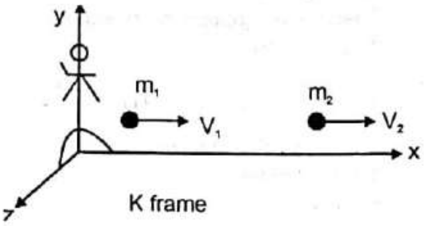
$\mu N > mg\left[\sin\alpha + \frac{w^2\ell}{g}\cos\alpha\right]$

$\mu(mg\cos\alpha - ml w^2 \sin\alpha) > mg\left[\sin\alpha + \frac{w^2\ell}{g}\cos\alpha\right]$

In irodov $\mu = k$

$W < \sqrt{\frac{g(K - \tan\alpha)}{\ell(1 + K\tan\alpha)}}$ **Ans.**

1.147



Let velocity of k' frame is V then
From this new frame k'
Velocity of $m_1 = V_1 - V$
Velocity of $m_2 = V_2 - V$
Kinetic energy of system from K' frame:

$$K_T = \frac{1}{2}m_1(V_1 - V)^2 + \frac{1}{2}m_2(V_2 - V)^2 \text{......(1)}$$

For K_T minimum $\frac{dK_T}{dV} = 0$

$$0 = \left[-\frac{1}{2}2m_1(V_1 - V)\right] + \left[-\frac{1}{2}(2)m_2(V_2 - V)\right]$$

$$V = \frac{m_1V_1 + m_2V_2}{m_1 + m_2}$$ **Ans.**

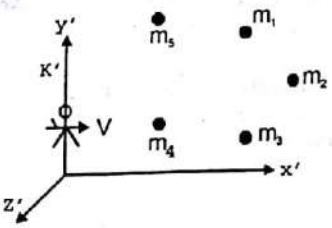
Kinetic energy from this frame:
Put value of V in equation (1)

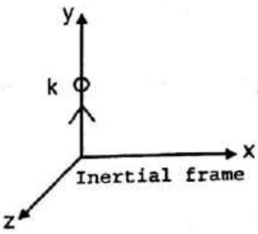
$$K_T = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(V_1 - V_2)^2$$

$$K_T = \frac{1}{2}\mu(V_1 - V_2)^2$$

$$\mu = \frac{m_1m_2}{m_1 + m_2}$$ **Ans.**

1.148



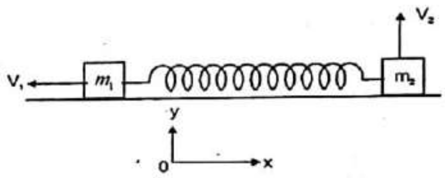


Let velocity of frame (k') is V w.r.t. inertial frame
Given that $V_{cm-k'} = 0$
It is possible when velocity of com is equal to velocity frame k'
$V_{cm-k} = V$
Assume $m_1 + m_2 + m_3 + m_4 = m$
Kinetic energy w.r.t COM = $\bar{E}$
Velocity of COM with frame (K) =V
We know that total kinetic of system =
Sum of energy of system w.r.t. com + K.E. of COM from K frame

$$K = \bar{E} + \frac{1}{2}m(V_{cm-k})^2$$

$$K = \bar{E} + \frac{1}{2}mV^2$$ **Ans.**

1.149



We know that kinetic energy of system in frame of COM

$$K.E. = \frac{1}{2}\mu V_{rel}^2$$

Where V_{rel} is velocity of one particle w.r.t. other

$$\bar{V}_{rel} = V_1\hat{i} - V_2\hat{i}$$

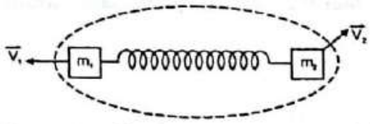
$$V_{rel} = \sqrt{V_1^2 + V_2^2}$$

$$\mu = \frac{m_1m_2}{m_1 + m_2}$$

$$K.E. = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(V_1^2 + V_2^2)$$

$$\bar{E} = \frac{1}{2}\mu(V_1^2 + V_2^2)$$ **Ans.**

1.150



On system force is only gravitation field.

$$\bar{V}_{cm} = \frac{m_1\bar{V}_1 + m_2\bar{V}_2}{m_1 + m_2}$$

$m_1 + m_2 = m$

Acceleration of COM

$$\bar{a}_{cm} = \frac{m_1\bar{g} + m_2\bar{g}}{m_1 + m_2} = \bar{g} \text{ = Constant}$$

Velocity at time t

V = u + at

$\bar{V} = \bar{V}_{cm} + \bar{g}t$ ....(1)

Momentum at time t

From (1)

$$\vec{P} = m\vec{V}$$

$\bar{P} = m\,\bar{V}_{cm} + m\bar{g}t \quad \bar{P} = \bar{P}_0 + m\bar{g}t$

Where $\bar{P}_0 = m\bar{V}_{cm}$

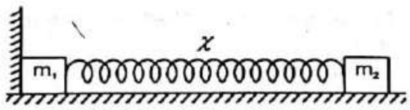
Ans.

Since acceleration of COM is constant then we can use

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{S} = \vec{V}_0 t + \frac{1}{2}\vec{g}t^2$$ **Ans.**

1.151



Since dispacement of bock m_1 is zero untill contact with wall break , wook done by Normal reaction of wall on m_1 will be zero Hence energy will be conserved

$$\frac{1}{2}\chi(x)^2 = \frac{1}{2}m_1(0)^2 + \frac{1}{2}m_2V_2^2$$

$$m_2V_2^2 = \chi(x)^2$$

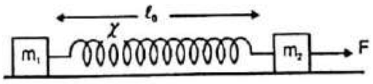
$$V_2 = x\sqrt{\chi/m_2}$$

$$V_{cm} = \frac{m_1 \times 0 + m_2 x\sqrt{\chi/m_2}}{m_1 + m_2}$$

$$V_{cm} = \frac{x\sqrt{m_2 X}}{m_1 + m_2}$$ **Ans.**

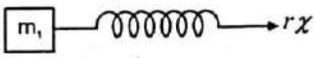
1.152

Method (1) (Non inertial frame)



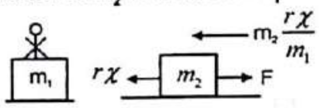
Suppose at time t,elongation in spring is r then

F.B.D. of m_1



$$a_1 = \frac{\chi r}{m_1}.$$

F.B.D. of m_2 in frame of m_1



Net force on m_2

$$F_{net} = F - \left(\chi r + \chi r\frac{m_2}{m_1}\right)$$

$$= -\chi r\left(\frac{m_1 + m_2}{m_1}\right) + F$$

$$a_2 = \frac{F_{net}}{m_2}$$

$$a_2 = \frac{F}{m_2} - \chi r\left(\frac{m_1 + m_2}{m_1 m_2}\right)$$

$$a_2 = V_2\frac{dV_2}{dt}$$

Because at maximum elongation or compres sion relative velocity will be zero

$V_r^{in} = 0$; $V_r^{fi} = 0$

$$\int_0^0 V_2\,dV_2 = \int_0^{r_{max}} \frac{F\,dr}{m_2} - \int_0^{r_{max}} \chi r\left(\frac{m_1 + m_2}{m_1 + m_2}\right)dr$$

$$0 = \frac{F}{m_2}r_{max} - \frac{\chi r_{max}^2}{2}\left(\frac{m_1 + m_2}{m_1\,m_2}\right)$$

$r_{max} = 0$ or $\quad r_{max} = \dfrac{2F}{m_2\chi}\dfrac{m_1m_2}{(m_1 + m_2)}$

(a)

Minimum distance = ℓ_0

Maximum distance = $\ell_0 + \dfrac{2m_1F}{(m_1 + m_2)x}$.

(b)

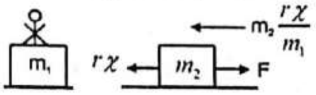
If $m_1 = m_2$

Minimum distance = ℓ_0

Maximum distance = $\ell_0 + \dfrac{F}{x}$

Ans.

Method - 2 (Work - energy theorem)
F.B.D. of m_2 w.r.f. m_1 :



$W_{all} = k_f - k_i$
Here $K_{in} = 0$ and $K_f = 0$
$W_{spring} + W_{pseudo} + W_F = 0$

$$-\frac{1}{2}\chi r_{max}^2 - m_2\chi \int_0^{r_{max}} \frac{r}{m_1} dr + Fr_{max} = 0$$

$r_{max} = 0$ or $r_{max} = \frac{2F}{m_2\chi}\frac{m_1 m_2}{(m_1+m_2)}$

(a)
Minimum distance = ℓ_0

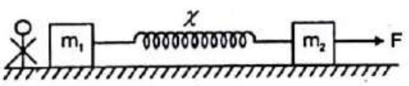
Maximum distance = $\ell_0 + \frac{2m_1 F}{(m_1+m_2)x}$.

(b)
If $m_1 = m_2$
Minimum distance = ℓ_0

Maximum distance = $\ell_0 + \frac{F}{x}$

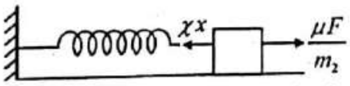
Ans.

Method - 3 [Reduce Mass Concept]



Reduce mass of system

$\mu = \frac{m_1 m_2}{m_1 + m_2}$



At maximum elongation final velocity will be zero. Using work energy theorm

$$\frac{\mu F}{m_2} r_{max} - \frac{1}{2}\chi r_{max}^2 = 0$$

$r_{max} = 0$ or $r_{max} = \frac{2\mu F}{m_2 \chi}$

$r_{max} = 0$ or $r_{max} = \frac{2F}{m_2\chi}\frac{m_1 m_2}{(m_1+m_2)}$

(a)
Minimum distance = ℓ_0

Maximum distance = $\ell_0 + \frac{2m_1 F}{(m_1+m_2)x}$.

(b)
If $m_1 = m_2$
Minimum distance = ℓ_0

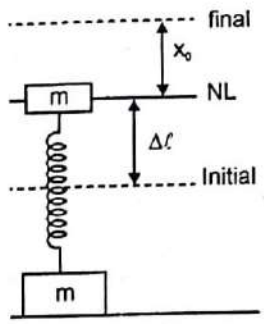
Maximum distance = $\ell_0 + \frac{F}{x}$

Ans.

1.153

Method : 1 (Energy conservation)

(a)



Suppose when elongation is x_0 lower block start moving upward then
Condition to leave ground of lower block
$\chi x_0 = mg$

$x_0 = \frac{mg}{\chi}$ .... (i)

Energy equation of upper block
Loss in P.E. of spring
= Gain in K.E. (ΔK)+ Gain in gravitational potential energy(ΔU)

$K_{in} = 0$, $K_f = 0$

$$\frac{1}{2}\chi\Delta\ell^2 = \frac{1}{2}mv^2 + mg(x_0 + \Delta\ell) + \frac{1}{2}\chi x_0^2 \text{ ....(ii)}$$

For $\Delta\ell$ is minimum , V will be minimum
V= 0
Put in (ii)

$$\frac{1}{2}\chi\Delta\ell^2 = mg\left(\frac{mg}{\chi} + \Delta\ell\right) + \frac{1}{2}\chi x_0^2$$

$$= mg\left(\frac{mg}{\chi} + \Delta\ell\right) + \frac{1}{2}\chi\left(\frac{mg}{\chi}\right)^2$$

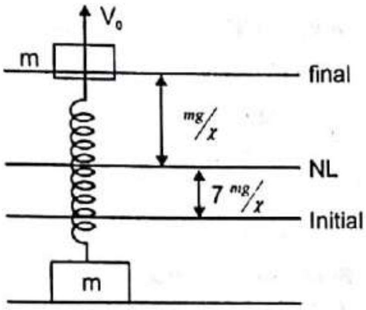
$$\frac{1}{2}\chi\Delta\ell^2 = \frac{m^2g^2}{\chi} + mg\Delta\ell + \frac{1}{2}\frac{m^2g^2}{\chi}$$

$\Delta\ell = \frac{3mg}{\chi}$ **Ans.**

(b)

When elongation in spring is $\frac{mg}{\chi}$ lower block leave the ground.

Energy equation between initial and final position.



$$\frac{1}{2}\chi\left(\frac{7\,mg}{\chi}\right)^2 = \frac{1}{2}mV_0^2 + mg\left(\frac{8mg}{\chi}\right) + \frac{1}{2}\chi\left(\frac{mg}{\chi}\right)^2$$

$$V_0 = \sqrt{\frac{32mg^2}{\chi}} \quad ....(1)$$

Velocity of centre of mass at this instant

$$V_{cm} = \frac{m\,V_0 + 0}{2m} = \frac{V_0}{2}$$

$a_{cm} = g$

At maximum height, final velocity of com = 0

$V^2 = u^2 + 2as$

$$0 = \frac{V_0^2}{2} - 2gh_1 \;.............(2)$$

From (1) and (2)

$$h_1 = \frac{4mg}{\chi}$$

This distance is travelled by com after leave the lower surface.

Before leave the lower surface

Displacement of COM :

$$h_2 = \frac{m\times 0 + m\left(\frac{8mg}{\chi}\right)}{2m}$$

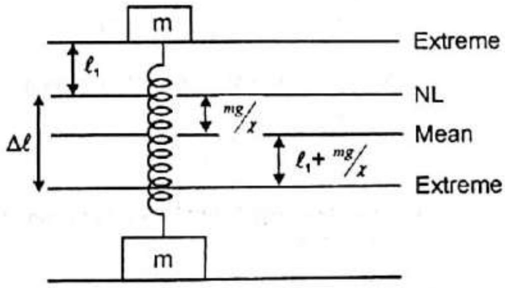
$$h_2 = \frac{4mg}{\chi}.$$

Net displacement of COM

$$h = h_1 + h_2 = \frac{8mg}{\chi} \qquad \textbf{Ans.}$$

Method : 2 (SHM APPLICATION)

(a)



For minimum value of $\Delta\ell$ final velocity of upper block will be zero and upper block perform S. H. M. and for ℓ_1

Condition to leave ground of lower block

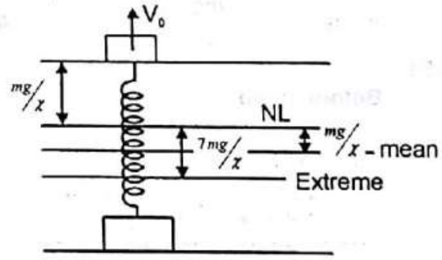
$\chi\, l_1 = mg$

$$l_1 = \frac{mg}{\chi} \;.........(i)$$

From figure

$$\Delta\ell = \frac{mg}{\chi} + \ell_1 + \frac{mg}{\chi} = \frac{3mg}{\chi} \qquad \textbf{Ans.}$$

(b)



$$\omega = \sqrt{\frac{K}{m}}$$

We know velocity equation in SHM

$V^2 = \omega^2 (A^2 - x^2)$

Amplitude of SHM= $^{6mg}/_{\chi}$

For calculation of velocity when block leave ground

$$V_0^2 = \frac{K}{m}\left[\left(\frac{6mg}{k}\right)^2 - \left(\frac{2mg}{k}\right)^2\right]$$

$$V_0^2 = \frac{K}{m}\left(\frac{32m^2g^2}{k^2}\right) = \frac{32mg^2}{k}$$

$$V_0 = \sqrt{\frac{32mg^2}{\chi}} \quad ....(1)$$

Velocity of centre of mass at this instant

$$V_{cm} = \frac{mV_0 + 0}{2m} = \frac{V_0}{2}$$

$a_{cm} = g$

At maximum height, final velocity of com = 0

$V^2 = u^2 + 2as$

$$0 = \frac{V_0^2}{2} - 2gh_1 \quad ....(2)$$

From (1) and (2)

$$h_1 = \frac{4mg}{\chi}$$

This distance is travelled by com after leave the lower surface.

Before leave the lower surface

Displacement of COM :

$$h_2 = \frac{m \times 0 + m\left(\frac{8mg}{\chi}\right)}{2m}$$

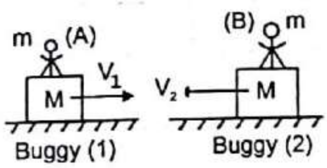
$$h_2 = \frac{4mg}{\chi}.$$

Net displacement of COM

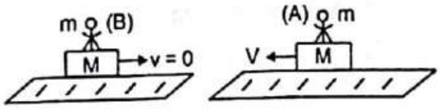
$$h = h_1 + h_2 = \frac{8mg}{\chi} \qquad \textbf{Ans.}$$

1.154

Before Jump



After Jump



During this exchange momentum will be con versed because there is no force in horizon tal direction on system.

Using LMC equation on sytem of all

$(M+m)V_1 - (M+m)V_2 = 0+(M+m)V$

$V_1 - V_2 = V$ ....(i)

When person jump perpendicular to track, velocity of buggies will not change.

Velocity of a buggy changes because per son landing on the buggy.

Using LMC on person (A) and buggy (B)

$$mV_2 - MV_1 = (m+M)0$$

$$mV_2 = MV_1 \quad ....(ii)$$

From (i) and (ii)

$$V_2 = \frac{MV}{M-m}$$

$$V_1 = \frac{mV}{M-m}$$

But in term of vector

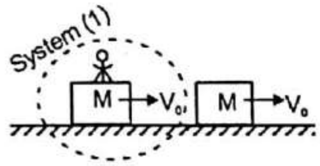
V_2 has opposite direction as V_1

$$V_2 = \frac{MV}{M-m} \qquad V_1 = \frac{-mV}{M-m}$$

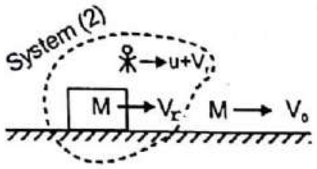
Ans.

1.155

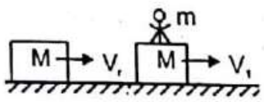
Before Jump



During Jump



After Jump



For Buggy (1)

Linear momentum of system (1), before jump and during jump will be conserved.

$(m+M)V_0 = m(V_r + u) + MV_r$

$$V_r = \frac{(m+M)V_0 - mu}{m+M}$$

$$V_r = V_0 - \left(\frac{m}{m+M}\right)u$$

For Buggy (2)

Momentum of man + buggy (2) will be con served during and after jump

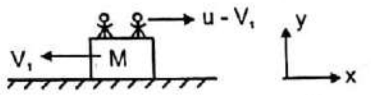
$$m(u+V_r)+MV_0=(m+M)V_1 \text{.......(1)}$$

Put the value of V_r in (1)

$$V_1 = V_0 + \frac{mMu}{(m+M)^2} \qquad \textbf{Ans.}$$

1.156

(a)



Using momentum conservation on system

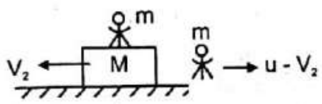
$2m\,(u-V_1) - MV_1 = 0$ .... (1)

$$V_1 = \frac{2Mu}{2m+M}$$

In term of vector form

$$\vec{V}_1 = \frac{-2M\vec{u}}{(2m+M)} \qquad \textbf{Ans.}$$

(b)

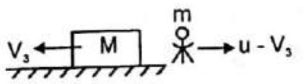


In first jump using LMC

$m(u-V_2)-(m+M)\,V_2 = 0$

$$V_2 = \frac{mu}{(2m+M)} \text{...........(i)}$$

After second jump using LMC



$-(m+M)\,V_2 = -MV_3 + m\,(u-V_3)$

$-(m+M)V_2 = -MV_3 + mu - mV_3$

$$V_3 = \frac{mu+(m+M)V_2}{m+M}$$

$$V_3 = V_2 + \frac{mu}{m+M} \text{.................(2)}$$

From (1) and (2)

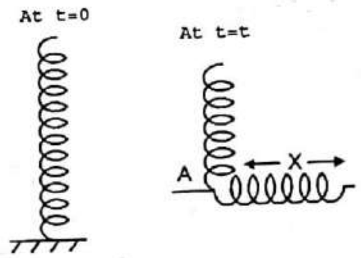
$$V_3 = \frac{mu}{(2m+M)} + \frac{mu}{(m+M)}$$

$$V_3 = \frac{mu(3m+2M)}{(m+M)(2m+M)}$$

In vector form

$$\vec{V}_3 = \frac{-m\vec{u}(3m+2M)}{(m+M)(2m+M)} \qquad \textbf{Ans.}$$

1.157



At t = 0 , l length in air.

Let us assume at time t, x part of chain is resting at floor.

Each particle of chain are in free fall then velocity of chain at this instant:

$v^2 = u^2 + 2as$

$v = \sqrt{2gx}$ ....(1)

Suppose in dt time, dm mass is collide with floor.

Then momentum transfer in dt time :

$dP = (dm)\,V$

$$\frac{dp}{dt} = \left(\frac{dm}{at}\right)V = \left(\frac{dm}{dt}\right)\sqrt{2gx}$$

$$F = \left(\frac{dm}{dt}\right)\sqrt{2gx} \text{ ....(i)}$$

If mass per unit length is λ then length travel in dt time

$dx = v\,dt$

Mass strike in dt time

$dm = \lambda\,dx = \lambda\,vdt$

$$\frac{dm}{dt} = \lambda V$$

Put in (i)

$$F = \lambda V\sqrt{2gx}$$

From(1) and $\lambda = \frac{m}{\ell}$

$$F = \frac{m}{\ell}\sqrt{2gx}\sqrt{2gx}$$

$$F = \frac{2mgx}{\ell}$$

Force applied by resting part of chain on ground

$F = m_1 g$

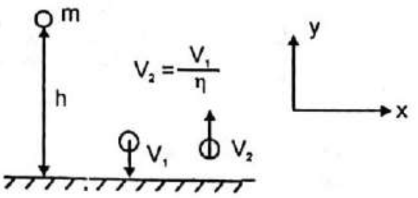
Where m_1 = mass of resting part on ground

$$F_r = \left(\frac{m}{L}\right) x g$$

Compare F are F_r

$F = 2Fr$ Proved.

1.158



Velocity before first impact

$$V_1 = \sqrt{2gh}$$

Just after first impact

$$V_2 = \frac{1}{\eta}\sqrt{2gh}$$

Momentum transfer in first impact

$$\Delta P = \left|m\bar{V}_2 - m\bar{V}_1\right| = m(V_2 + V_1)$$

$$\Delta P_1 = m\sqrt{2gh}\left(1 + \frac{1}{\eta}\right) \text{..........(1)}$$

Velocity before second impact

$$V_1 = \frac{1}{\eta}\sqrt{2gh}$$

Just after second impact

$$V_2 = \frac{1}{\eta^2}\sqrt{2gh}$$

$$\Delta P_2 = m\sqrt{2gh}\left(\frac{1}{\eta} + \frac{1}{\eta^2}\right) \text{.......(2)}$$

$$\Delta P_3 = m\sqrt{2gh}\left(\frac{1}{\eta^2} + \frac{1}{\eta^3}\right)$$

Now net mumentum transfer to ground

$$\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3 + \text{........} + \infty$$

$$= m\sqrt{2gh}\left(1 + \frac{1}{\eta}\right) + m\sqrt{2gh}\left(\frac{1}{\eta} + \frac{1}{\eta^2}\right) +$$

$$m\sqrt{2gh}\left(\frac{1}{\eta^2} + \frac{1}{\eta^3}\right) + \text{..........} + \infty$$

$$= m\sqrt{2gh}\left(1 + \frac{2}{\eta} + \frac{2}{\eta^2} + \text{.......} + \infty\right)$$

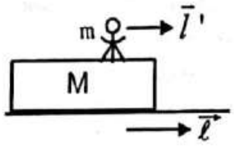
$$\Delta P = m\sqrt{2gh}\left(2 + \frac{2}{\eta} + \text{.......} + \infty\right) - m\sqrt{2gh}$$

$$= 2m\sqrt{2gh}\left[\frac{1}{1 - \frac{1}{\eta}}\right] - m\sqrt{2gh}$$

$$\Delta P = \frac{m\sqrt{2gh}\,(n+1)}{n-1}$$ **Ans.**

1.159

(a)

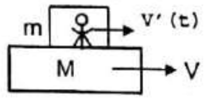


Since there is no external force in horizon tal direction ,also intial velocity of com will be zero always then change in mass moment of system will be zero always or mass momentum also be conserved then

$$M\vec{\ell} + m(\vec{\ell}' + \vec{\ell}) = 0$$

$$\vec{\ell} = \frac{-m\vec{\ell}'}{m+M}$$ **Ans.**

(b)



Since there is no external force in horizon tal direction then momentum of system also be conserved in horizontal direction.

$$MV + m[V + V'(t)] = 0$$

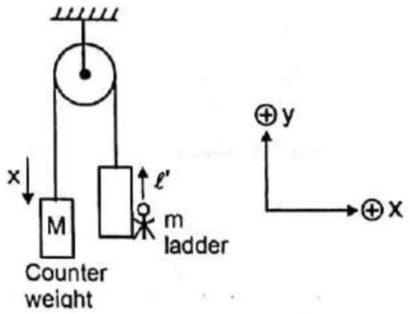
$$V = \frac{-mV'(t)}{m+M}$$

$$\bar{F} = M\frac{d\bar{V}}{dt}$$

$$\bar{F} = \frac{-Mm}{(m+M)}\frac{d\bar{V}(t)}{dt}$$ **Ans.**

1.160

Method(1)



If we open string assuming string is tight then since there is no horizontal force on system and initial velocity of COM = 0 then mass moment will be conserved. Assume ladder as will as counter weight move x distance then

$Mx + (M - m)\,x + m\,(x + \ell') = 0$

$x = -\dfrac{m\ell'}{2M}$.

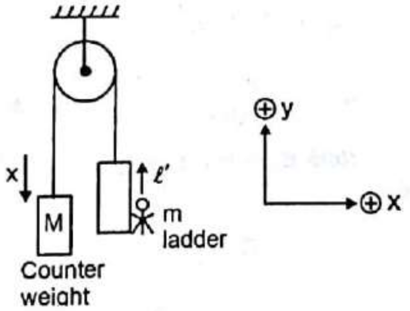
Displacement of COM

$$\Delta x_{COM} = \frac{-Mx + (M-m)x + m(x+\ell')}{M+M-m+m}$$

$$\Delta x_{COM} = \frac{m\ell'}{2M}$$ **Ans.**

Note. There is no need of calculation of x

Method(2)



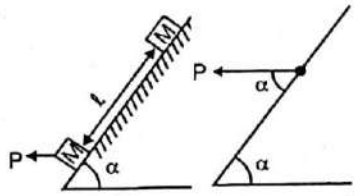
Assume ladder as will as counter weight move x distance then

Displacement of COM

$$\Delta x_{COM} = \frac{-Mx + (M-m)x + m(x+\ell')}{M+M-m+m}$$

$$\Delta x_{COM} = \frac{m\ell'}{2M}$$ **Ans.**

1.161



Velocity of cannon after travelling ℓ dis tance along incline

$V = \sqrt{2g\ell\sin\alpha}$

Momentum of cannon after travelling ℓ dis tance along incline before fire

$P_{intial} = m\sqrt{2gl\sin\theta}$

Momentum given by shell to cannon in up ward direction along in line due to fire

$= P\cos\alpha$.

Net momentum in upward direction of canon just after impact

$= P\cos\alpha - M\sqrt{2g\ell\sin\alpha}$.

Just after impact velocity in upward direction along the incline is V_0 then

$$MV_0 = \left[P\cos\alpha - M\sqrt{2g\ell\sin\alpha}\right]$$

$$V_0 = \frac{\left[P\cos\alpha - M\sqrt{2g\ell\sin\alpha}\right]}{M} \quad \text{......(1)}$$

Suppose cannon comes in rest in time τ

$0 = V_0 - (g\sin\alpha)\tau$

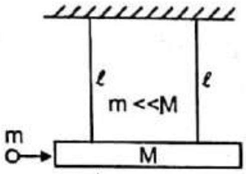
From (1)

$$\tau = \frac{P\cos\alpha - M\sqrt{2g\ell\sin\alpha}}{Mg\sin\alpha}$$ **Ans.**

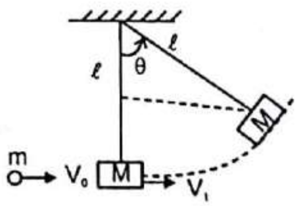
1.162

(a)

Front view



Side view



Since there is no force on system of (m + M) during collision in horizontal direction, momentum of system will be conserved

$mV_0 = (m + M)V_1$ ....(1)

Using energy consurvation in motion of body of M after impact made

At maximum deflection final velocity will be zero.

$$0^2 = V_1^2 - 2g[\ell - \ell\cos\theta]$$

$$V_1 = \sqrt{2g\ell(1-\cos\theta)}$$

$$V_1 = \left[2\sin\frac{\theta}{2}\right]\sqrt{g\ell} \quad \text{..................(2)}$$

From(1) and (2)

$$V_0 = \left(\frac{m+M}{m}\right)\left[2\sin\frac{\theta}{2}\right]\sqrt{g\ell}.$$

SinceM>>m

$$V_0 = \frac{M}{m}\left[2\sin\frac{\theta}{2}\right]\sqrt{g\ell}$$

$$V_0 = \left[\frac{2M}{m}\sin\frac{\theta}{2}\right]\sqrt{g\ell} \qquad \textbf{Ans.}$$

(b)

$$V_0 = \left[\frac{2M}{m}\sin\frac{\theta}{2}\right]\sqrt{g\ell} \quad \text{......(3)}$$

Dividing (1) by (2)

$$\frac{V_1}{V_0} = \frac{m}{M} \quad \text{......................(4)}$$

Heat developed in collsion= loss of kinetic energy of system during collision

$$\Delta H = \frac{1}{2}mV_0^2 - \frac{1}{2}(m+M)V_1^2$$

Fraction of energy loss (η)

$$\eta = \frac{\frac{1}{2}mV_0^2 - \frac{1}{2}(m+M)V_1^2}{\frac{1}{2}mV_0^2}$$

$$\eta = 1 - \left(\frac{m+M}{m}\right)\left(\frac{V_1^2}{V_0^2}\right)$$

since m<<M

$$\eta = 1 - \frac{M}{m}\left(\frac{V_1^2}{V_0^2}\right)$$

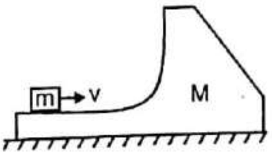
From (4)

$$\eta = 1 - \frac{M}{m}\left(\frac{m^2}{M^2}\right)$$

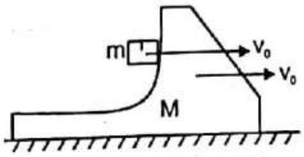
$$\eta = 1 - \frac{m}{M} \qquad \textbf{Ans.}$$

1.163

Intial Position



Final Position



At time of maximum height the body M has only horizontal velocity of both m and M will be same

Then using LMC in horizontal direction

$mv = (m + M)V_0$

$$V_0 = \frac{mv}{m+M} \text{ ....(1)}$$

Using energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}mV_0^2 + \frac{1}{2}MV_0^2 + mgh$$

From (1)

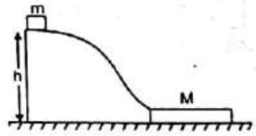
$$mv^2 = (m+M)\frac{m^2v^2}{(m+M)^2} + 2mgh$$

$$\frac{mMv^2}{m+M} = 2mgh$$

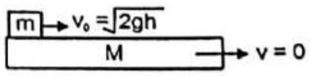
$$h = \frac{Mv^2}{2(m+M)g} \qquad \textbf{Ans.}$$

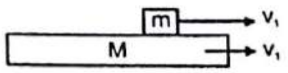
Note. Body leave wedge in return journey.

1.164



(a)





At final instant velocity of m and M will be same.
Using momentum conservation

$m v_0 + M \times 0 = (m + M) v_1$

$m\sqrt{2gh} = (m + M) V_1$

$$V_1 = \frac{m\sqrt{2gh}}{(m+M)} ....(i)$$

Work done by all forces

$W_{fr} = k_f - k_i$

$$= \frac{1}{2}mV_1^2 + \frac{1}{2}MV_1^2 - \frac{1}{2}mV_0^2$$

$$= \frac{1}{2}(m+M)\frac{m^2 2gh}{(m+M)^2} - \frac{1}{2}m2gh$$

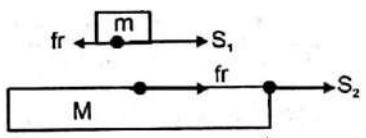
$$= \frac{2m^2gh}{2(m+M)} - \frac{2}{2}mgh$$

$$= mgh\left[\frac{m}{m+M} - 1\right]$$

$$= \frac{-mMgh}{m+M}$$

$$W_{fr} = \frac{-mMgh}{m+M}$$ **Ans.**

(b)



Displacement of m is S_1 and that of M is S_2 from a frame

$W_{fr} = -fr\ S_1 + fr\ S_2$

$W_{fr} = f_2 (S_2 - S_1)$

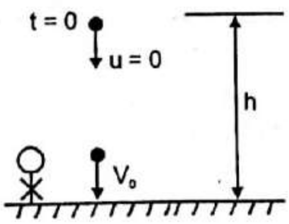
Here $S_2 - S_1$ is relative displacement of m w.r.t. M which does not depend on reference frame.

Work done by friction forcs on whole system is independent on reference frame.

Ans.

1.165

When observer or reference frame is in rest



$V_0^2 = 0^2 + 2gh$

$V_0 = \sqrt{2gh}$

When observer or reference frame is in moving with V_0

Intial velocity of ball from this frame $-V_0$

If velocity of collision is V then

$V^2 = V_0^2 - 2gh$

$V = \sqrt{V_0^2 - 2gh}$

Ans.

Note. Here displacement h is w.r.t. mov ing frame.

1.166

$\vec{V}_1 = 3\hat{i} - 2\hat{j}$

$\vec{V}_2 = 4\hat{i} - 6\hat{j}$

Since there is no net force , hence momen tum of system will be conserved.

Final velocity of both will be same and equal to V because collision is elastic.

$(m_1 + m)_2 \vec{V} = m_1\vec{V}_1 + m_2\vec{V}_2$

$$\vec{V} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1 + m_2}$$

$$\vec{V} = \frac{1\times(3\hat{i} - 2\hat{j}) + 2(4\hat{j} - 6\hat{k})}{3}$$

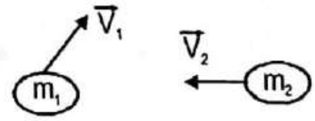
$$= \frac{3\hat{i} + 6\hat{j} - 12\hat{k}}{3}$$

$\vec{V}_{cm} = \hat{i} + 2\hat{j} - 4\hat{k}$

Ans.

1.167

Method: 1 (COM frame)
Before collision



After collision

If collision is perfictaly inelastic then final velocity of each particle will be same.
Because velocity of both particle have oppo site direction w. r. f. COM.
Where K is constant.
Initial momentum of system w. r. f. COM
= 0
Since collision is perfectly inelastic then final velocity will be same.

Using momentum conservation

$\bar{0} = (m_1 + m_2)\bar{V}$

$\bar{V} = \bar{0}$

Hence initially energy of system w. r.t. COM is completely converted into heat loss.

$$\Delta H = |\Delta K| = |K_f - K_i| = \left|0 - \frac{1}{2}\mu|\bar{V}_1 - \bar{V}_2|^2\right|.$$

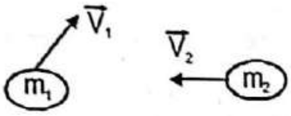
Where μ = reduce mass of system

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

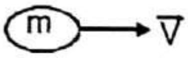
$$\Delta K = \frac{1}{2}\mu|\bar{V}_1 - \bar{V}_2|^2$$

$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)|\bar{V}_1 - \bar{V}_2|^2$$ **Ans.**

Method: 1 (Ground frame)
Before collision



After collision



Since collision is perfectly inelastic then final velocity will be same and equal to $\bar{V}$.

Using momentum conservation

$$(m_1 + m)_2\bar{V} = m_1\bar{V}_1 + m_2\bar{V}_2$$

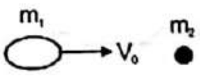
$$\bar{V} = \frac{m_1\bar{V}_1 + m_2\bar{V}_2}{m_1 + m_2}$$

$$\Delta H = |\Delta K| = |K_f - K_i|$$

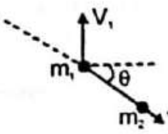
$$\Delta H = \left|\frac{1}{2}(m_1 + m_2)V^2 - \frac{1}{2}m_1V_1^2 - \frac{1}{2}m_2V_2^2\right|.$$

$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)|\bar{V}_1 - \bar{V}_2|^2$$ **Ans.**

1.168 **(a)**



Befor collision



After collision

L C M (linear momentum conservation) in x direction

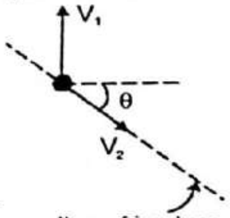
$m_1 V_0 = m_2 V_2 \cos\theta$.... (i)

L C M (linear momentum conservation) in y direction

$0 = m_1 V_1 - m_2V_2 \sin\theta$ ......(ii)

Since particle m_2 starts motion from rest then we can say that net impulse is along in direc tion of motion of m_2.

Then restitution equation (e) will be written along impulse direction



line of impluse

$$1 \times (V_0 \cos\theta) = V_2 + V_1 \sin\theta \quad \text{.......(iii)}$$

From (i), (ii) and (iii)

$$V_1 = \left(\sqrt{\frac{m_2 - m_1}{m_1 + m_2}}\right)V_0$$

Initial kinetic energy of particle (1)

$$k_i = \frac{1}{2}m_1V_0^2.$$

Final kinetic energy of particle (1)

$$k_f = \frac{1}{2}m_1V_1^2 = \frac{1}{2}\frac{m_1(m_2 - m_1)}{m_1 + m_2}V_0^2$$

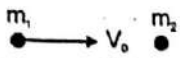
Loss of energy = $\Delta H = k_i - k_f$

Fraction of loss

$$\eta = \frac{\Delta H}{k_i} = \frac{k_i - k_f}{k_i} = 1 - \frac{k_f}{k_i}$$

$$\eta = 1 - \left(\frac{m_2 - m_1}{m_1 + m_2}\right) = \frac{2m_1}{m_1 + m_2}$$ **Ans.**

(b)



Before collision

After collision

LCM (linear momentum conservation)

$m_1 V_0 = m_1 V_1 + m_2 V_2$.... (i)

Equation of restitution

$V_0 = V_2 - V_1$ ....(ii)

From (1) and (2)

$$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_0$$

Fraction of loss

Initial kinetic energy of particle (1)

$$k_i = \frac{1}{2} m_1 V_0^2 .$$

Final kinetic energy of particle (1)

$$k_f = \frac{1}{2} m_1 V_1^2 = \frac{1}{2} m_1 \left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2 V_0^2$$

Loss of energy = $\Delta H = k_i - k_f$

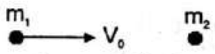
Fraction of loss

$$\eta = \frac{\Delta H}{k_i} = \frac{k_i - k_f}{k_i} = 1 - \frac{k_f}{k_i}$$

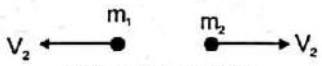
$$\eta = \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$ **Ans.**

1.169

(a)



Before collision



After collision

L C M (linear momentum conservation)

$m_1 V_0 = m_2 V_2 - m_1 V_2$ .... (i)

Equation of restitution

$1 \times V_0 = V_2 + V_2$

$V_2 = \frac{V_0}{2}$ ....(ii)

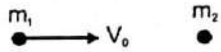
From (i) and (ii)

$m_1 V_0 = (m_2 - m_1) \frac{V_0}{2}$

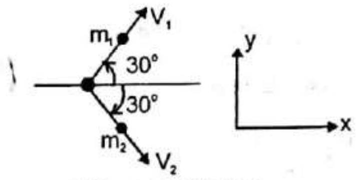
$2 m_1 = m_2 - m_1$

$\frac{m_1}{m_2} = \frac{1}{3}$ **Ans.**

(b)



Before collision



After collision

L M C in x direction

$m_1 V_0 = m_1 V_1 \cos 30° + m_2 V_2 \cos 30°$ .... (i)

L M C in y direction

$0 = m_1 V_1 \sin 30° - m_2 V_2 \sin 30°$ ...... (ii)

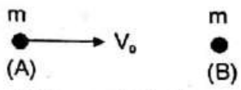
Since m_2 is initially rest hence e equation should be in direction of motion of m_2

$V_0 \cos 30 = V_2 - V_1 \cos 60$.... (iii)

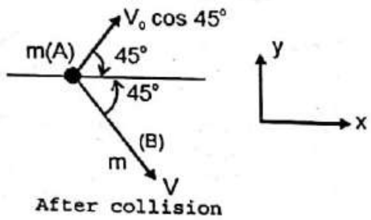
From (i), (ii) and (iii)

$\frac{m_1}{m_2} = 2$ **Ans.**

1.170



Before collision



After collision

Since particle (B) start motion from rest. Hence line of impact will be in direction of motion of B.

Then velocity of particle (A) perpendicular to direction of motion of (B) will be remain un changed because implusive force only in di rection of motion of (B).

Angle between motion of two ball will be 90 degree sothat momentum In y direction will be conserve.

At time of maxmimum potential energy velocity of both ball in direction of impact will be same.

L M C in direction of motion of B

$mV_0 \cos 45° = 2\, mV$

$$V = \frac{V_0}{2\sqrt{2}} \text{ ....(1)}$$

Final K.E. of B

$$K_B = \frac{1}{2}mV^2$$

Final K.E. of A

$$K_A = \frac{1}{2}m\left[V^2 + \left(V_0 \cos 45^0\right)^2\right] = \frac{5mV^2}{16}$$

Maximum P.E. store = Loss of kinetic energy

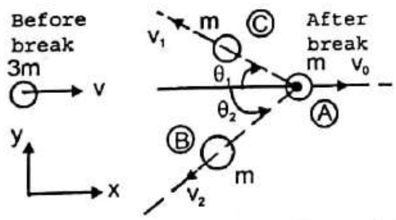
Now

Fraction loss of K.E.=Fraction store in P.E.

$$\eta = \frac{\frac{1}{2}mV_0^2 - \frac{1}{2}mV^2 - \frac{5mV_0^2}{16}}{\frac{1}{2}mV_0^2} = \frac{1}{4}$$

$\eta = 0.25$ **Ans.**

1.171 .



Suppose mass of ball is 3m. If speed of ball A is maximum then ball C and B must be in opposite motion direction as A. Becuase maximum momentum will be gain by particle A is only possible when B and C move opposite as A move sothat momentum conservation can hold even velocity of B and C are minimum other we have to take component of velocity of B and C in direction of intial velocity then velocity of A will be not maximum.

Hence θ_1 and θ_2 will be zero.

LMC (linear momentum conservation) in x direction

$3\,mv = mv_0 - mv_1 - mv_2$

$V_0 = 3v + v_1 + v_2$ ....(I)

For V minimum $v_1 + v_2$ should be minimum

Using

$AM \geq GM$

$\frac{V_1 + V_2}{2} \geq \sqrt{V_1V_2}$ because both are postive.

Equality is hold when

$V_1 = V_2$ ....(ii)

Using energy equation :

$$\eta\left(\frac{1}{2}(3m)v^2\right) = \frac{1}{2}mv_0^2 + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

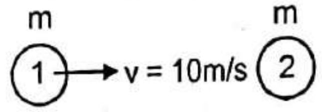
$3\eta V^2 = v_0^2 + v_1^2 + v_2^2$ ....(iii)

From (i), (ii) and (iii)

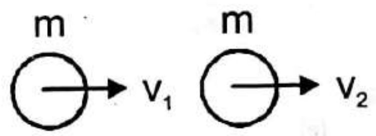
$v_0 = v\left[1 + \sqrt{2(\eta - 1)}\right]$ **Ans.**

1.172

Method: 1 (From earth frame)



Before collision



After collision

Using LCM

$mV = mv_1 + mv_2$

$v_1 + v_2 = V$ ....(i)

Using energy equation :

$$\frac{\frac{1}{2}mV^2 - \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2}{\frac{1}{2}mV^2} = \eta$$

$v_1^2 + v_2^2 = v^2(1-\eta)$ ....(ii)

Solving (1) and (ii) :

$$V_1 = \frac{V}{2}\left[1 \pm \sqrt{1 - \frac{\eta}{5}}\right]$$

Take negative sign because speed will be decrease

$$v_1 = \frac{V}{2}\left[1 - \sqrt{1 - \frac{\eta}{5}}\right]$$

Ans.

Method: 2 (COM frame)

Using LCM : $v_1 + v_2 = 10$ ......(i).

Energy equation from com frame :

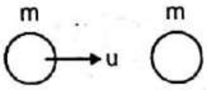
$$\frac{\frac{1}{2}\mu V^2 - \frac{1}{2}\mu(v_2 - v_1)^2}{\frac{1}{2}mV^2} = \eta \text{ ....(ii)}$$

$$\mu = \frac{mm}{m+m} = \frac{m}{2} \text{.................(iii)}$$

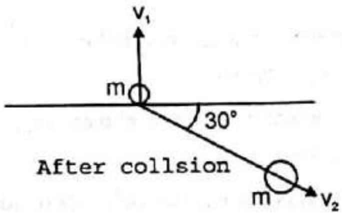
From (i), (iii) and (iv)

$$v_1 = \frac{v}{2}\left[1-\sqrt{1-\frac{\eta}{5}}\right]$$ **Ans.**

1.173



Before collsion



After collsion

Given that

$$\frac{M}{m} = 5$$

M=5m

LCM in x direction

$mu = M v_2 \cos 30^0$

$2u = 5v_2\sqrt{3}$ ....(i)

$$v_2 = \frac{2}{5\sqrt{3}}u$$

LCM in y direction

$0 = mv_1 - Mv_2 \sin 30^0$

$$v_1 = \frac{5}{2}v_2$$

$2v_1 = 5v_2$

$$v_1 = \frac{5}{2}\left[\frac{2}{5\sqrt{3}}u\right]$$

$$v_1 = \frac{u}{\sqrt{3}}$$

% Loss of K.E.

$$\frac{\Delta K}{K_{in}} \times 100 = \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv_1^2 - \frac{1}{2}Mv_2^2}{\frac{1}{2}mu^2} \times 100$$

$$= \left[1 - \left(\frac{v_1}{u}\right)^2 - \frac{M}{m}\left(\frac{v_2}{u}\right)^2\right] \times 100$$

$$= \left[1 - \frac{1}{3} - 5\left(\frac{4}{25\times 3}\right)\right] \times 100$$

$$= \left[1 - \frac{1\times 5}{3\times 5} - \frac{4}{15}\right] \times 100$$

$$= \left[1 - \frac{9}{15}\right] \times 100$$

$= 40\%$

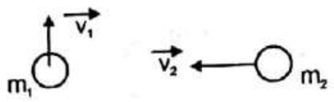
40% energy will be loss

Ans.

1.174

Method 1 :(Ground frame)

(a)



Liner momentum of system =

$$\vec{V}_{cm} = \frac{m_1 v_1\hat{j} + m_2 v_2\hat{i}}{m_1 + m_2}$$

Velocity of m_1 w.r.t. COM :

$$\vec{v}_{1-cm} = v_1\hat{j} - \frac{m_1 v_1\hat{j} + m_2 v_2\hat{i}}{m_1 + m_2}$$

$$\vec{v}_{1-cm} = \frac{m_2\left(v_1\hat{i} - v_2\hat{j}\right)}{m_1 + m_2}$$

$$\vec{P}_{1-cm} = m_1\vec{v}_{1-cm} = m_1\frac{m_2\left(v_1\hat{i} - v_2\hat{j}\right)}{m_1 + m_2}.$$

Similary

$$\vec{P}_{2-cm} = m_1\frac{m_2\left(v_1\hat{i} - v_2\hat{j}\right)}{m_1 + m_2}$$

$$\left|\vec{P}_{1-cm}\right| = \left|\vec{P}_{2-cm}\right| = \frac{m_1 m_2\sqrt{V_1^2 + V_2^2}}{m_1 + m_2}$$ **Ans.**

(b)

$$K_{system-cm} = \frac{1}{2}m_1(\vec{v}_{1-cm})^2 + \frac{1}{2}m_2(\vec{v}_{2-cm})^2$$

$$K_{system-cm} = \frac{1}{2}\frac{m_1\, m_2}{m_1 + m_2}\left|v_1\hat{i} - v_2\hat{j}\right|$$

$$= \frac{1}{2}\frac{m_1\, m_2}{m_1 + m_2}\sqrt{V_1^2 + V_2^2}$$

Ans.

Method (2) (COM Frame)

(a)

Momentum of particle (1) w.r.t com is :

$$\vec{P}_{1-cm} = \mu(\vec{v}_1 - \vec{v}_2)$$

$$\vec{P}_{1-cm} = \mu\left(v_1\hat{i} - v_2\hat{j}\right)$$

Where μ is reduce mass from COM frame

$$\mu = \frac{m_1\, m_2}{m_1 + m_2}$$

$$\vec{P}_{1-cm} = \frac{m_1\, m_2}{m_1 + m_2}\left(v_1\hat{i} - v_2\hat{j}\right)$$

momentum of particle (2) w.r.t. com

$$\vec{P}_{2-cm} = \mu(\vec{v}_2 - \vec{v}_1)$$

$$\vec{P}_{2-cm} = \frac{m_1\, m_2}{m_1 + m_2}(\vec{v}_2 - \vec{v}_1)$$

$$\left|\vec{P}_{1-cm}\right| = \left|\vec{P}_{2-cm}\right| = \frac{m_1 m_2\sqrt{V_1^2 + V_2^2}}{m_1 + m_2}$$ **Ans.**

(b)

Kinetic energy of system w.r.t. com :

$$K_{system-cm} = \frac{1}{2}\,\mu\,\left|\vec{v}_1 - \vec{v}_2\right|^2 \quad ;$$

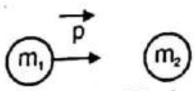
$$K_{system-cm} = \frac{1}{2}\left(\frac{m_1\, m_2}{m_1 + m_2}\right)\left|(\vec{v}_1 - \vec{v}_2)\right|^2$$

$$K_{system-cm} = \frac{1}{2}\frac{m_1\, m_2}{m_1 + m_2}\left|v_1\hat{i} - v_2\hat{j}\right|$$

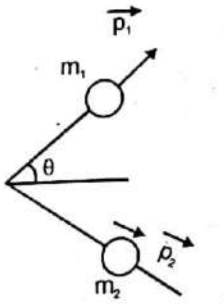
$$= \frac{1}{2}\frac{m_1\, m_2}{m_1 + m_2}\sqrt{V_1^2 + V_2^2}$$

Ans.

1.175 **Method : 1 (Vector Meethod)**



Before Collsion



After Collsion

Where $\vec{p}$ = linear momentum of block m_1 before collision.

$\vec{p}_1$ =linear momentum of block m_1 just after collision.

$\vec{p}_2$ = linear momentum of block m_2 just after collision.

Using LMC equation :

$$\vec{p} = \vec{p}_1 + \vec{p}_2$$

$$|\vec{p} - \vec{p}_1| = |\vec{p}_2|$$

$$|\vec{p}_2| = P^2 + P_1^2 - 2\,PP_1\cos\theta$$

$$P_2^2 = P^2 + P_1^2 - 2\,PP_1\cos\theta \quad ....(1)$$

Using energy equation

$$\frac{p^2}{2m_1} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} \quad ....(ii).$$

Put value of P_2 in equation (ii)

$$\frac{p^2}{2m_1} = \frac{p_1^2}{2m_1} + \frac{p^2 + p_1^2 - 2pp_1\cos\theta}{2m_2}$$

$$\frac{p^2}{2}\left(\frac{1}{m_1} - \frac{1}{m_2}\right) = \frac{p_1^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - \frac{2pp_1\cos\theta}{2m_2}$$

$$\frac{p^2}{2}\frac{(m_2 - m_1)}{m_1 m_2} = \frac{p_1^2(m_1 + m_2)}{2m_1\, m_2} - \frac{pp_1\cos\theta}{m_2}$$

$$p^2\left(\frac{m_2 - m_1}{m_1}\right) + \frac{2pp_1\cos\theta}{1} - \frac{p_1^2(m_1 + m_2)}{m_1} = 0$$

This is quatric equation in p and it has real root then

$D \geq 0$

$$(2p_1\cos\theta)^2 + 4\left[\frac{m_2-m_1}{m_1}\right]\left[\frac{p_1^2(m_1+m_2)}{m_1}\right] \geq 0$$

$$\cos^2\theta + \frac{m_2^2 - m_1^2}{m_1^2} \geq 0$$

$$\left(\frac{m_2}{m_1}\right)^2 \geq \sin^2\theta$$

$$\sin^2\theta \leq \frac{m_2}{m_1}.$$

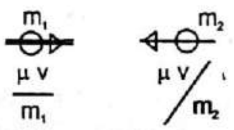
When θ_{max}

$$\sin\theta_{max} = \frac{m_2}{m_1}$$

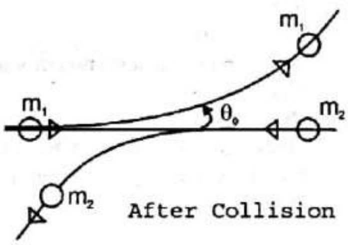
Only when $m_1 > m_2$

Ans.

Method (2) (COM Method)



Before Collsion



After Collision

In frame of com, in case of elastic collison, magnitude velocity of individual does not change only its direction is changed.

Calculation :

Momentum of individual particle = μv_r

Where μ = reduce mass = $\frac{m_1 m_2}{m_1 + m_2}$

v_r is relative velocity in ground frame.

Velocity of individual particle = $\frac{\mu v_r}{m}$

Where m = mass of particls.

In frame of com, θ_0 can take any possible value lie between (0——2π).

Also we know that

$\vec{v}_{1-\varepsilon} = \vec{v}_{1-com} + \vec{v}_{com-\varepsilon}$

Now using tringle method

Care (i) :

If $v_{1-cm} > v_{cm-e}$

$$\frac{\mu v}{m_1} > \frac{m_1 v}{m_1 + m_2}$$

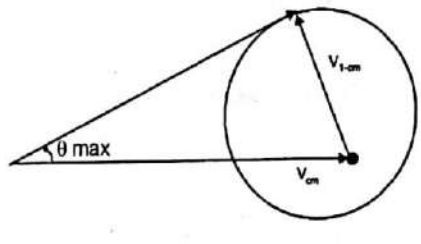
$$\frac{m_1 m_2}{m_1 + m_2} v > \frac{m_1 v}{m_1 m_2}$$

$$\frac{m_2}{m_1} > 1$$

$m_2 > m_1$

θ take any value.

Care (ii) :



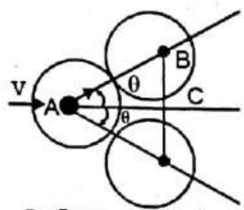
If $m_1 > m_2$ then $V_{1-cm} < V_{cm-6}$

$$\sin\theta_{max} = \frac{V_{1-cm}}{V_{cm-E}}$$

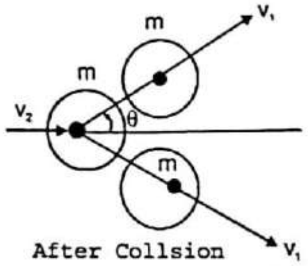
$$\sin\theta_{max} = \frac{\mu v / m_1}{m_1\, v / m_1 + m_2}$$

$$\sin\theta_{max} = \frac{m_2}{m_1}$$ **Ans.**

1.176



Before Collsion



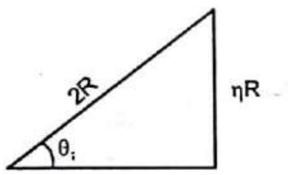
After Collsion

LCM in x direction
$mv = mv_2 + 2(mv_1 \cos\theta_1)$
$v = v_2 + 2v_1 \cos\theta_1$ ....(i)
Energy equation

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_2^2 + 2\left(\frac{1}{2}mv_1^2\right)$$

$v^2 = v_2^2 + 2v_1^2$ ....(ii)
From (i) and (ii):

$$v_2 = v\left(\frac{1-2\cos^2\theta_1}{1+2\cos^2\theta_1}\right) = v\left(\frac{2\sin^2\theta_1 - 1}{3-2\sin^2\theta_1}\right) \quad ..(1)$$



$\sin\theta_1 = \frac{\eta}{2}$
From (1)

$$v_2 = -v\left(\frac{2-\eta^2}{6-\eta^2}\right)$$

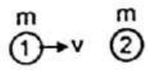
For stop
$v_2 = 0$
$\eta = \sqrt{2}$
For recoil in opposite direction
$V_2 \le 0$

$$-V\left(\frac{2-\eta^2}{6-\eta^2}\right) \le 0$$

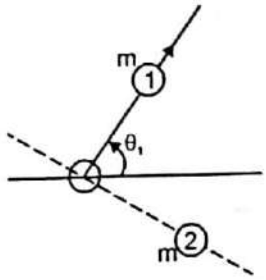
$$\frac{2-\eta^2}{6-\eta^2} \ge 0$$

Either
$\eta = \sqrt{2}$ or $\eta \ge \sqrt{6}$ **Ans.**

1.177
(a)



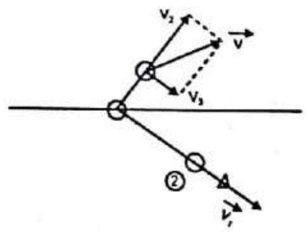
Before Collision



After Collsion

If collision is elastic then velocity of both molocules will be inter changed in direction of impact.
Also velocity of particle (1) perpendicular to directon of line of impact will not be changed.
Velocity of (1) after impact only along per pendicular to line of impact
$v_1 = v\cos\theta_1$
Velocity of (2) after impact only along line of impact
$v_2 = v\sin\theta_1$
Then angle between two molecules will be always 90^0. **Ans.**

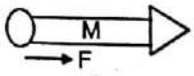
(b)
If collision is inelastic then velocity of particle (1) along line of impact will be not zero.



Angle between $\bar{v}$ and $\bar{v}_1$ is not equal to 90^0. Means whatever angle is present but not equal to 90^0. **Ans.**

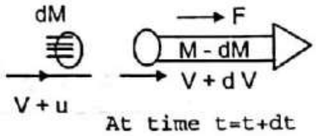
1.178

Method : 1(Ground frame)



At time $V_{t=t}$

Suppose at time t, mass of rocket M and its velocity is v and external force is acting on rocket as shown.
In next sort time dt, velocity of rocket is v + dv and its mass is M-dM then



At time t=t+dt

On System impluse equation

$F\,dt = P_F - P_i$

$F\,dt = (M - dM)(V + dV) + dM(V + u) - MV$
$= MV - V\,dM + M\,dV + dV\,dM + V\,dM + u\,dM - MV$

Since $(dM)(dV) \to 0$ is neglected

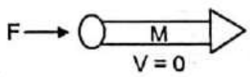
$F\,dt = u\,dM + M\,dV$

$$F = u\frac{dM}{dt} + M\frac{dv}{dt}$$

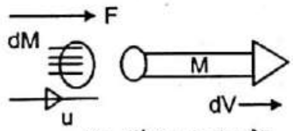
Here $\frac{dv}{dt} = W$ and $\frac{dM}{dt} = \mu$

$MW = F - \mu u$ **Proved**

Method : 2 (From reference frame)
At time t



At time t velocity of rocket is v and in dt time velocity of rocket is increase by dV.
Now we take a reference frame of velocity v
Impluse equation on rocket from this frame



At time t=t+dt

$F\,dt = P_f - P_i = M\,dV + u\,dM - 0$

$$F = u\frac{dM}{dt} + M\frac{dv}{dt}$$

Here $\frac{dv}{dt} = W$ and $\frac{dM}{dt} = \mu$

$MW = F - \mu u$ **Proved**

1.179

Force equation on rocket

$$\vec{F}_{net} = +\vec{v}_r \frac{dm}{dt}$$

$$m\,a = u\frac{dm}{dt}$$

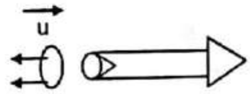
$m\,dv = u\,dm$

$$\int_0^v dv = u\int_{m_0}^{m} \frac{dm}{m}$$

$$v = u \ln\frac{m}{m_0}$$

$$v = -u \ln\frac{m_0}{m}$$ **Ans.**

1.180



Force equation

$$mw = -u\frac{dm}{dt}$$

$$\int_0^t w\,dt = -u\int_{m_0}^{m}\frac{dm}{m}$$

$$wt = -u\ell n\frac{m}{m_0}$$

$$wt = -u\,\ell n\frac{m}{m_0}$$

$m = m_0 e^{-wt/u}$ **Ans.**

1.181

Method (1): (Circular motion concept)

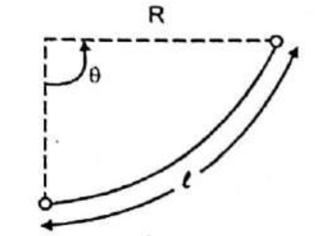
Sincs mass is ejicting parpendicular to direction with constant velocity at each instant then there is no change in magnitude of velocity only its direction will be changed.
Also since u is constant, F will constant in magnitude, hence motion will be circular motion.

$$F = -u\frac{dm}{dt} = \frac{mv_0^2}{R}$$

$$-uR\int_{m_0}^{m}\frac{dm}{m} = v_0^2\int_0^t dt$$

$$-uR\,\ell n\frac{m}{m_0} = v_0^2 t$$

$$R = \frac{V_0^2 t}{u\ln{}^{m_0}\!/_{m}} \quad \text{.................. (i)}$$



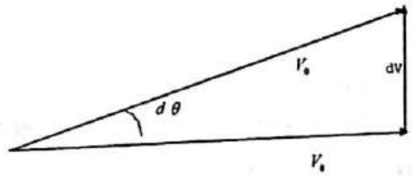
Since speed is constant,length of arch
$V_0 t = R\theta$
From (i)

$$V_0 t = \frac{V_0^2 t}{u \ln {m_0}/{m}}\theta$$

$$\theta = \frac{u}{v_0}\ell n \, {m_0}/{m} \qquad \textbf{Ans.}$$

Method (2): (Most General Approach)

Sincs mass is ejecting parpendicular to di rection with constant velocity at each instant then there is no change in magnitude of ve locity only its direction will be changed.



Velocity change in dt interval dV as shown in figure then

$$dV = \sqrt{V_0^2 + V_0^2 - 2V_0V_0\cos(d\theta)}$$

$$dV = 2V_0 \sin\frac{d\theta}{2}$$

$$dV = 2V_0\frac{d\theta}{2} = V_0 d\theta$$

$$\frac{dV}{dt} = V_0\frac{d\theta}{dt}$$

$$m\frac{dV}{dt} = mV_0\frac{d\theta}{dt}$$

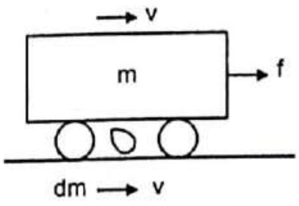
$$F = mV_0\frac{d\theta}{dt}$$

$$-u\frac{dm}{dt} = mV_0\frac{d\theta}{dt}$$

$$-u\int_{m_0}^{m}\frac{dm}{m} = V_0\int_0^{\theta} d\theta$$

$$\theta = \frac{u}{v_0}\ell n \, {m_0}/{m} \qquad \textbf{Ans}$$

1.182



Thrust force equation on van due sand out

$$\bar{F}_{thrust} = +\bar{v}_r\frac{dm}{dt}$$

$$V_r = 0$$

$$\bar{F}_{thrust} = \bar{0}$$

then
$F = ma$ ....(1)
At time t , mass of van is m then
$m = m_0 - \mu t$
From (1)
$(m_0 - \mu t)\, a = F$

$$a = \frac{F}{m_0 - \mu t} \qquad \textbf{Ans.}$$

Also

$$\frac{dv}{dt} = \frac{F}{m_0 - \mu t}$$

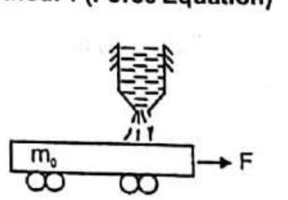
$$\frac{1}{F}\int_0^{v} dv = \int_0^{t}\frac{dt}{m_0 - \mu t}$$

$$v = \frac{F}{-\mu}\ell n\,(m_0 - \mu t)\Big|_0^t$$

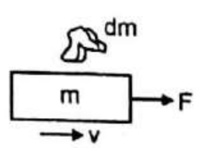
$$v = \frac{F}{\mu}\ell n\,\frac{mo}{m_0 - \mu t} \qquad \textbf{Ans.}$$

1.183

Method: 1 (Force Equation)



At time t



$$F_{net} = F - V_r\frac{dm}{dt} \text{ ....(1)}$$

Here relative velocity sand w.r.t. cart is V

and $\frac{dm}{dt} = \mu$

$$ma = F - V\frac{dm}{dt}$$

$ma = F - \mu V$ ....(2)

At time t , mass of van will be

$m = m_0 + \mu t$

From (2)

$(m_0 + \mu t)\, a = F - \mu V$

$$a = \frac{dv}{dt} = \frac{F - \mu V}{m_0 + \mu t} \text{ ......(i)}$$

$$\int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$$

$$\frac{1}{-\mu}\ell n\,(F - \mu v)\Big|_0^v = \frac{+1}{\mu}\ell n(m_0 + \mu t)\Big|_0^t$$

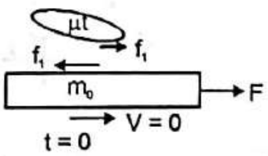
$$\ell n\frac{F}{F - \mu V} = \ell n\frac{m_0 + \mu t}{m_0}$$

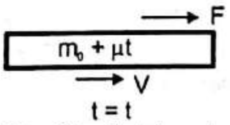
$$\frac{F}{F - \mu V} = \frac{m_0 + \mu t}{m_0}$$

$$V = \frac{Ft}{m_0 + \mu t} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \frac{m_0 F}{(m_0 + \mu t)^2} \qquad \textbf{Ans.}$$

Method : 2 (Impluse Equation on system)





At final time t=t

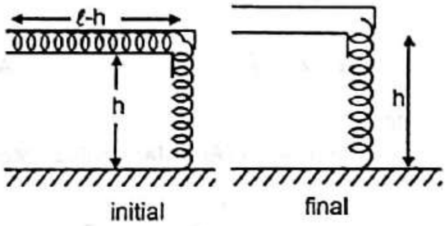
Impulse equation on system :

$Ft = (m_0 + \mu t)\, V - 0$

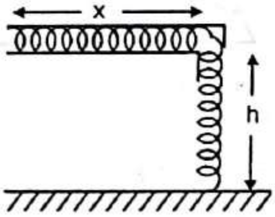
$$V = \frac{Ft}{m_0 + \mu t} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \frac{m_0 F}{(m_0 + \mu t)^2} \qquad \textbf{Ans.}$$

1.184



Assume at time t only x length on horizontal tube.



Pulling force = $\frac{M}{\ell}hg$

$$a = \frac{\frac{Mh}{\ell}g}{\frac{M}{\ell}(x+h)} = \frac{hg}{x+h}$$

$$-\frac{v\,dv}{dx} = \frac{h}{x+h}g$$

(–) ive because x is decreasing.

$$-\int_0^v v\,dv = h\int_{\ell-h}^{0} g\frac{dx}{x+h}$$

$$v^2 = 2\,hg\,\ln\left(\frac{\ell}{h}\right)$$

$$v = \sqrt{2hg\,\ell n\frac{\ell}{h}} \qquad \textbf{Ans.}$$

1.185

$\vec{M} = \vec{a} + \vec{b}t^2$ ....(i)

$\vec{N} = \frac{d\vec{M}}{dt} = 2\vec{b}t$ ....(ii)

$\vec{M} \cdot \vec{N} = MN \cos 45^\circ$

$2b^2t^3 = \sqrt{a^2 + b^2t^4}\; 2bt \cos 45^\circ$

$2\,b^2\,t^4 = a^2 + b^2t^4$

$t = \sqrt{\frac{a}{b}}$.

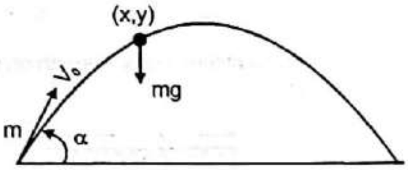
Then

$N = 2b\sqrt{\frac{a}{b}}$

$N = 2\sqrt{ab}$ **Ans.**

1.186

Method : 1 (Angular Impluse Method)



We know

$x = (V_0 \cos\alpha)\, t$

$\frac{dL}{dt} = \tau = mgx$

$\frac{dL}{dt} = mg(v_0 \cos\alpha)\, t$

$\int_0^L dL = mg\, V_0 \cos\alpha \int_0^t t\, dt$

$L = \frac{t^2}{2} mg\, V_0 \cos\alpha$

$L = \frac{1}{2} mg\, V_0 t^2 \cos\alpha$ **Ans.**

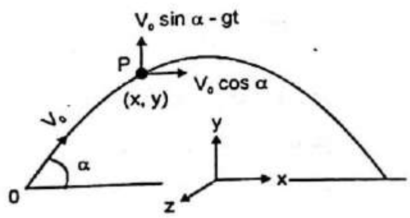
At highest point

$t = \frac{u_0 \sin\alpha}{g}$

$L = \frac{1}{2}\, mgV_0\left(\frac{V_0^2 \sin^2\alpha}{g^2}\right)\cos\alpha$

$L = \frac{1}{2} m\, \frac{V_0^3 \sin^2\alpha \cos\alpha}{g}$ **Ans.**

Method : 2 (Vector Method)



$\overline{op} = x\hat{i} + y\hat{j}$

$\overline{op} = (V_0 \cos\alpha)t\hat{i} + \left[(u_0 \sin\alpha)t - \frac{1}{2}gt^2\right]\hat{j}$ ..(1)

$\vec{V} = V_0 \cos\alpha\ \hat{i} + (V_0 \sin\alpha - gt)\hat{j}$ ....(2)

We know

$\vec{L}_0 = m\vec{r} \times \vec{V} = m\ \overline{op} \times \vec{V}$

From (1) and (2)

$\vec{L}_0 = \frac{1}{2} mg\, V_0\, t^2 \cos\alpha\, \hat{k}$ **Ans.**

At highest point

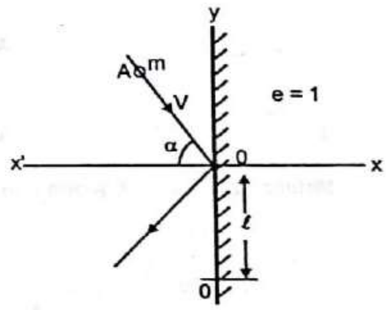
$t = \frac{u_0 \sin\alpha}{g}$

$L = \frac{1}{2}\, mgV_0\left(\frac{V_0^2 \sin^2\alpha}{g^2}\right)\cos\alpha$

$L = \frac{1}{2} m\, \frac{V_0^3 \sin^2\alpha \cos\alpha}{g}$ **Ans.**

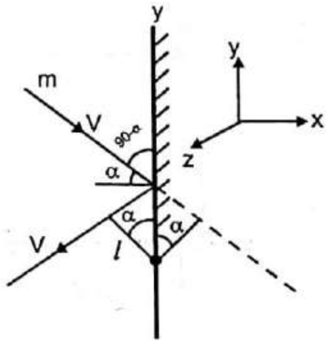
1.187

(a)



Since surface is smooth , there is no friction force. Here only normal reation will be present. Every points on x x' axis, torque will be zero, hence angular momentum will be conserved.

(b)



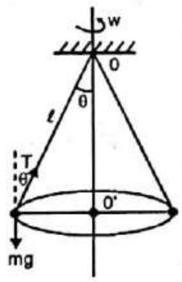
$\vec{L}_i = mV\ell\cos\alpha(-\hat{k})$

$\vec{L}_f = mV\ell\cos\alpha(\hat{k})$

$\Delta\vec{L} = \vec{L}_f - \vec{L}_i = 2mV\ \ell\cos\alpha(\hat{k})$

$\Delta L = 2m\ V\ \ell\cos\alpha$ **Ans.**

1.188



$T\cos\theta = mg$ ....(i)

$T\sin\theta = m\,Rw^2$

$T\sin\theta = m\,(\ell\sin\theta)\,w^2$ ....(ii)

Angular momentum about a point is conserved if torque about point is zero.

Torque about centre point (O')

= Torque due to mg + Torque due to T

= Torque due to mg + Torque due to T cos θ

+ Torque due to T sin θ ....(iii)

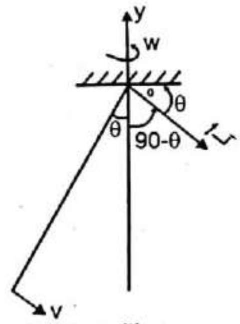
Torque due to mg = – (Torque due to T cos θ)

Torque due to T sin θ = 0

$\tau = 0+0+0=0$

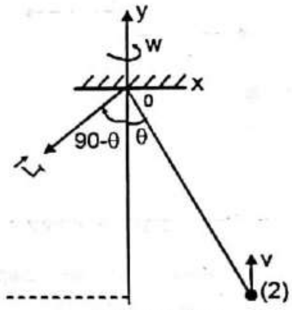
Hence angular momentum about O' will be conserved.

Change in Angular Momentum about O:



initial position

$\vec{L}_i = mv\ell\cos\theta - mv\ell\sin\theta\,\hat{j}$



final position

$\vec{L}_f = -mv\ell\cos\theta\,\hat{i} - mv\ell\sin\theta\,\hat{j}$

$\Delta\vec{L} = \vec{L}_f - \vec{L}_i = 2mv\ell\cos\theta\,\hat{i}$ ....(iii)

$v = (\ell\sin\theta)\ w$

$\Delta\vec{L} = 2m\ (\ell\sin\theta)\ w\,\ell\cos\theta\,\hat{i}$

$\Delta L = 2\,m\,\ell^2 w\sin\cos\theta$ ....(iv)

Devide equation (i) and (ii):

$$\frac{\cos\theta}{\sin\theta} = \frac{g}{(\ell\sin\theta)\,w^2}$$

$$\cos\theta = \frac{g}{\ell w^2}$$

$$\sin\theta = \frac{\sqrt{\ell^2 w^4 - g^2}}{\ell w^2}$$

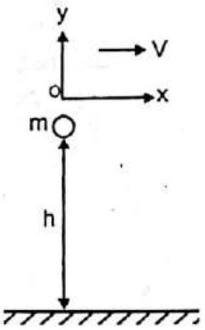
Put in (iv)

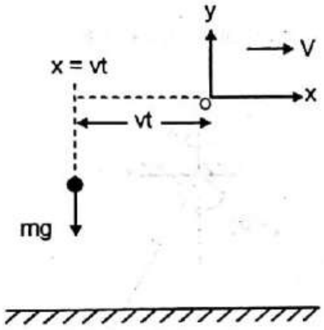
$$\Delta L = 2\,m\ell^2 w\frac{g}{\ell w^2}\ \frac{\sqrt{\ell^2 w^4 - g^2}}{\ell w^2}$$

$$\Delta L = \frac{2mg\ell}{w}\sqrt{1-\left(\frac{g}{\ell w^2}\right)^2}$$ **Ans.**

1.189

Method: 1 (Torque-Impluse Equation)





At time t, reference frame is displaced by vt displacement as shown in above diagram.
Torque mg about point O of refence frame = mg vt

Angular impluse = Change in angular momentum

$$\left|\Delta \vec{L}\right| = \int_0^{t_0} mg\ vt\, dt$$

Where t_o total time taken by particle before strite to floor.

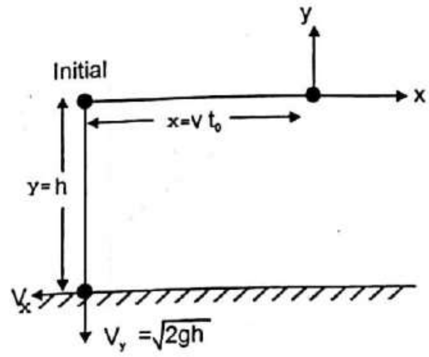
$$\left|\Delta \vec{L}\right| = mgv\frac{t_0^2}{2} = mv\left(\frac{1}{2}gt_0^2\right) \text{..........(i).}$$

Also we know $h = \frac{1}{2}gt_0^2$

Put value of h in (i)

$\left|\Delta \vec{L}\right| = mvh$ **Ans.**

Method : 2 (Using angular momentum formula)



$$h = \frac{1}{2}gt_0^2$$

$$t_0 = \sqrt{\frac{2h}{g}}$$

$v_y = \sqrt{2gh}$

Angular Momentum with respect to origin(O)

$L = mv_y x - mv_x y$ ....(1)

Where x is dispacement of reference frame and y is displacement of particle

Intial $v_y = 0$, $v_x = v$, $x = 0$

Velocity are w.r.t to reference frame.

From (1)

$L_i = 0$

Finally

$v_x = v$, $v_y = \sqrt{2gh}$, $x = vt_0$

From (1)

$L_f = m\sqrt{2gh}x - mhv$

$L_f = m\sqrt{2gh}V_0t_0 - mhv$

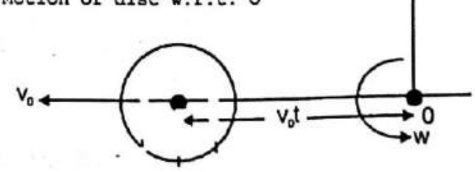
Put value of $t_0 = \sqrt{\frac{2h}{g}}$

$L_f = m\sqrt{2gh}V_0\sqrt{\frac{2h}{g}} - mhv = mhv$

Ans.

1.190 *

Motion of disc w.r.t. O



At time t

$F_{contious} = m\,2\left|\vec{V}\times\vec{w}\right| = 2mV_0w$

$\tau = \left|\vec{r}\times\vec{F}\right| = (2mV_0w)(V_0t)$

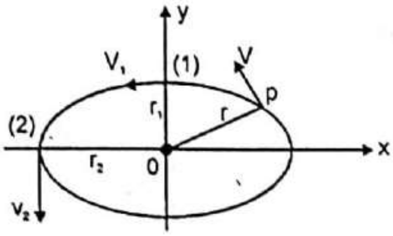
Angular implusein dt time :

$$\int dL = \int \tau dt$$

$dL = (2mV_0 w)(V_0 t)\, dt$

$$L = 2mV_0^2 w \int_0^t t\, dt = mV_0^2 wt^2$$ **Ans.**

1.191



Let assume, at time t, particle at point P and having velocity V and distance r.

$U = k r^2$

$$F = -\frac{dU}{dr} = -2kr.$$

This is central forcs hence angular momentum about O will be conserved.

$m V r$ = Constant ....(i)

Vr = constant

Differentiate w.r.t t

$$V\frac{dr}{dt} + r\frac{dV}{dt} = 0$$

$$\frac{dr}{dr} = -r\frac{dV}{dt} \text{ ....(ii)}$$

Also from energy conservation

$$\frac{1}{2}mv^2 + kr^2 = \text{constant}$$

Differentiate w.r.t. t

$$\frac{2}{2}mv\frac{dV}{dt} + 2kr\frac{dr}{dr} = 0$$

$$mV\frac{dV}{dt} = -2kr\frac{dr}{dr} \text{ ....(iii)}$$

Now divide (ii) by (iii)

$$\frac{-rdV/dt}{mvdv/dt} = \frac{V^{dr}/_{dt}}{-2kr^{dr}/_{dt}}$$

$$\frac{r}{mv} = \frac{v}{2kr}$$

$2kr^2 = mV^2$.............(iv)

At point (1)

$2k r_1^2 = mV_1^2$

$$V_1^2 = \frac{2kr_1^2}{m}$$

At point (2)

$$2kr_2^2 = mV_2^2$$

$$r_2^2 = \frac{mV_2^2}{2k}$$

Using energy conversation at (1) and (2) :

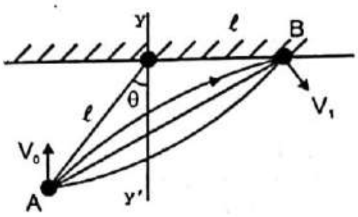
$$\frac{1}{2}mV_1^2 + kr_1^2 = \frac{1}{2}mV_2^2 + kr_2^2 :$$

$$\frac{1}{2}m\left[\frac{2kr_1^2}{m}\right] + kr_1^2 = \frac{1}{2}mV_2^2 + k\left(\frac{mV_2^2}{2k}\right)$$

$2kr_2^2 = mV_2^2$

$$m = \frac{2kr_1^2}{V_2^2}$$ **Ans.**

1.192



Using energy conservation between A and B.

$$\frac{1}{2}mV_0^2 - mg\ell\cos = \frac{1}{2}mV_1^2 \text{ ....(i).}$$

Angular momentum about axis yy' will be conserved.

Because at any instant torque of tension about axis y y' will be zero and mg is parallel to y y' then its torque also will be zero.

Hence angular momentum about y y' axis will be conserved.

$mV_0 \ell \sin\theta = mV_1 \ell$

$V_0 \sin\theta = V_1$ ....(ii)

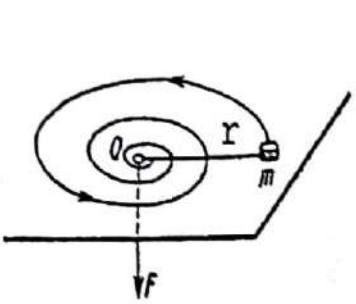
Put in (i)

$$\frac{1}{2}mV_0^2 - mg\ell\cos\theta = \frac{1}{2}mV_0^2 \sin^2\theta$$

$$\frac{V_0^2}{2}(1 - \sin^2\theta) = g\ell\cos\theta$$

$$V_0 = \sqrt{\frac{2g\ell}{\cos\theta}}$$ **Ans.**

1.193



Where r = distance between particle and origin.
Then using constraint equation

$$V = \frac{dr}{dt} = \text{constant}$$

$$\frac{d^2r}{dt^2} = 0$$

We know acceleration of particle = $rw^2 - \frac{d^2r}{dt^2}$

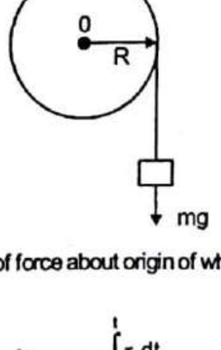
(toward centre)
Then
$a = rw^2$ ....(1)
$f = ma = mrw^2 = T$
$T = m\,r\,w^2$ ....(i)
Since tension is passing through O.
Torque of tension will the zero.
Angular momentom will be conserved
$m\,V\,r$ = constant
$mr^2 w = m\,r_0^2\,w_0$

$$w = \frac{r_0^2\,w_0}{r^2}$$

Put in (i)

$$T=F= mr\left(\frac{r_0^2\,w_0}{r^2}\right)^2 = \frac{mr_0^4\,w_0^2}{r^3} \quad \textbf{Ans.}$$

1.194
Method :1 (Angular Impulse Equation)



Torque of force about origin of whole system
$\tau = mgr$

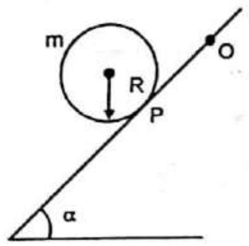
$$\text{Impulse of torque} = \int_0^t \tau\, dt$$

$L = mg\,Rt$ **Ans.**

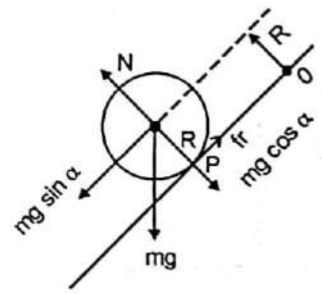
Method :2 (Formula based)

Angular momentum at time t
$L=m\,V\,R$.................(1)
Velocity after time t
$V = gt$
Put in (1)
$L = mgRt$ **Ans.**

1.195



Where O is point of initial moment and P is final moment. F.B.D. of sphere



Here $N = mg\cos\alpha$
Torque about O

$$\tau_{net} = \tau_{mg\sin\alpha} + \tau_{mg\cos\alpha} + \tau_{fr} + \tau_{normal} \text{..(1)}$$

$N = mg\cos\alpha$

$$\tau_{fr} = 0$$

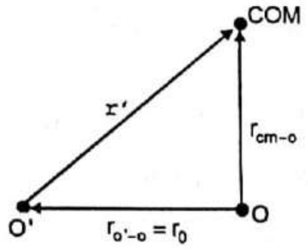
$$\tau_{mg\sin\alpha} = mgR\sin\alpha$$

$$\tau_{net} = mgR\sin\alpha$$

$$L = \int_0^t \tau_{net}\,dt = m\,R\,g\,t\sin\alpha \quad \textbf{Ans.}$$

If surface are smooth then also torque will be constant and equal to $R\,mg\sin\alpha$ hence angular momenta at time t again $Rmgt\sin\alpha$
Ans.

1.196



We know $\vec{L} = \vec{L}_{cm} + m\vec{r}_{cm} \times \vec{V}_{cm}$ .... (i)

Where $\vec{L}_{cm}$ = Angular momentum about com

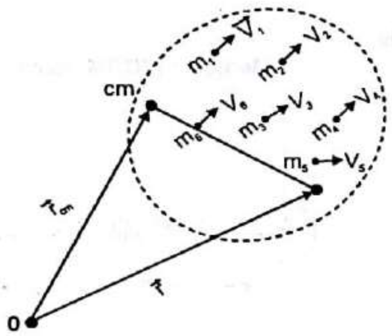
Angular momentom about O

$\vec{M} = \vec{L}_{cm} + m\,\vec{r}_{cm} \times \vec{V}_{cm}$ .....(ii)

Angular momentum about 0'

$\vec{M}' = \vec{L}_{cm} + mr' \times \vec{V}_{cm}$ ....(iii)

Ans.



Here we are seing that M' and M both are different and $\vec{p}$ is momentum of system. If we find a point in space from momentum of system is appear zero means

$$\vec{P} = \vec{0}$$

$$M\vec{V}_{cm} = \vec{0}$$

$$\vec{V}_{cm} = \vec{0}$$

Then from (ii) and (iii)

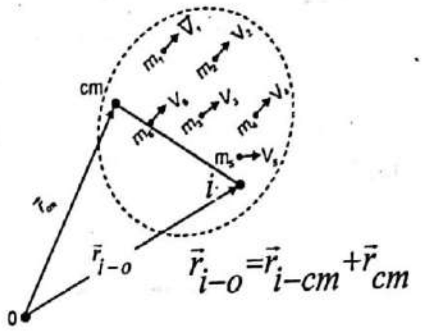
$$\vec{M} = \vec{L}_{cm}$$

$$\vec{M}' = \vec{L}_{cm}$$

then are can say that angular momentum does not depends on referencs point. e.g. this point can be COM because from COM frame linear momentum of system will be zero. Then $\vec{M}' = \vec{M}$.

Ans.

1.197



Let take a point i^{th}, having velocity V_i and position $\vec{r}_i$ then angular momentom of system about 0 :

$\vec{L} = m_i\,\vec{r}_i \times \vec{V}_i$........................(i)

$\vec{r}_i = \vec{r}_{i-cm} + \vec{r}_{cm-0} = \vec{r}_i = \vec{r}_{i-cm} + \vec{r}_{cm}$

Put in (i)

$\vec{L} = \sum m_i(\vec{r}_{i-cm} + \vec{r}_{cm}) \times \vec{V}_i$.

Also

$\vec{V}_i = \vec{V}_{i-cm} + \vec{V}_{cm}$

Then

$\vec{L} = \sum m_i(\vec{r}_{i-cm} + \vec{r}_{cm}) \times (\vec{V}_{i-cm} + \vec{V}_{cm})$

$= \sum m_i.\vec{r}_{i-cm} \times \vec{V}_{i-cm} + \sum m_i\,\vec{r}_{cm} \times \vec{V}_{i-cm} + \sum m_i\,\vec{r}_{i-cm} \times \vec{V}_{cm} + \sum m_i\,\vec{r}_{cm} \times \vec{V}_{cm}$

$\vec{L} = \sum m_i\vec{r}_{i-cm} \times \vec{V}_{cm} + \sum \vec{r}_{cm} \times m_i\vec{V}_{i-cm} + \sum m_i\vec{r}_{i-cm} \times \vec{V}_{cm} + \sum m_i\,\vec{r}_{cm} \times \vec{V}_{cm}$

................(ii)

Now

$$\frac{\sum m_i\vec{r}_{i-cm}}{M} = \vec{0}$$

Because it is position vector of CM w.r.t CM.

$\sum m_i\vec{r}_{i-cm} = \vec{0}\,; \sum m_i\vec{r}_{i-cm} \times \vec{V}_{cm} = \vec{0}$..........(a)

$\sum \vec{r}_{cm} \times m_i\vec{V}_{i-cm} = \vec{r}_{cm} \times \sum m_i\vec{V}_{i-cm} = \vec{0}$......(b)

Because it is momentum of system w.r.t. CM.

$\sum m_i \vec{r}_{i-cm} \times \vec{V}_{cm}$ is angular momentum of system w.r.t. CM.

$\sum m_i \vec{r}_{i-cm} \times \vec{V}_{cm} = \vec{L}_{cm}$ ....(c)

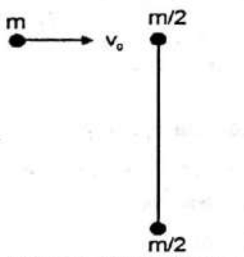
$\sum m_i \ \vec{r}_{cm} \times \vec{V}_{cm} = M\vec{r}_{cm} \times \vec{V}_{cm}$ ....(d)

Put all sigma (Σ) data in equation (ii) :

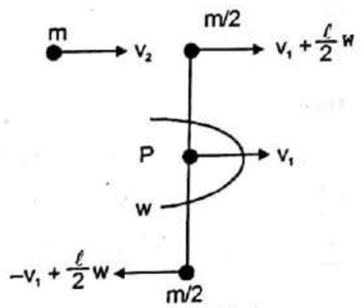
$\vec{L} = \vec{L}_{cm} + M\ \vec{r}_{cm} \times \vec{V}_{cm} = \vec{L}_{cm} + \vec{r}_{cm} \times \vec{P}_{cm}$

$\vec{M} = \vec{M} + [\vec{r}_c.\vec{P}]$ **Proved**

1.198



Before Collision



After Collision

Using LMC equation :

$$mV_0 = mV_2 + \frac{m}{2}\left(V_1 + \frac{\ell w}{2}\right) - \frac{m}{2}\left(\frac{\ell w}{2} - V_1\right)$$

$$V_0 = V_2 + \frac{1}{2}\left[V_1 + \frac{\ell w}{2} - \frac{\ell w}{2} + V_1\right]$$

$$V_0 = V_2 + V_1 \text{ ....(i)}$$

Equation of coefficient of restitution

$$V_0 = V_1 + \frac{\ell w}{2} - V_2 \text{ ....(ii)}$$

Using AMC (Angular Momentum Conservation) about inertial point P

$$mV_0\frac{\ell}{2} = mV_2\frac{\ell}{2} + 2 \times \frac{m}{2}\left(\frac{\ell}{2}\right)^2 w$$

$$V_0 = \frac{\ell w}{2} + V_2 \text{ ....(iii)}$$

Solving (i), (ii) and (iii)

$$w = \frac{4V_0}{3\ell}$$

$$V_1 = \frac{\ell w}{2}$$

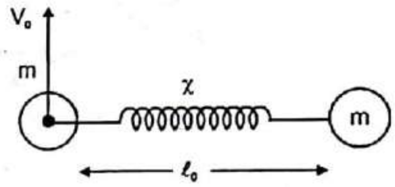
Angular momentum about COM

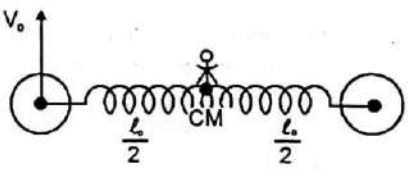
$$Lcm = \vec{M} = \left[\frac{m}{2}\left(\frac{\ell}{2}\right)^2 w\right] \times 2 = \frac{m\ell^2}{4}\left(\frac{4V_0}{3\ell}\right)$$

$$= \frac{mV_0\ell}{3}$$ **Ans.**

1.199

Method : 1 (COM frame)

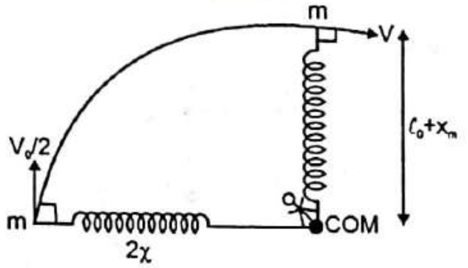




Since no force on COM, com will remain in rest then cut spring in two part as :
If we observe motion then in both minimum elongation or maximum elongation spring force will be perpendicular to velocity vector.

Velocity of COM will be $\frac{V_0}{2}$ in y direction.

From this frame velocity of both block will be $\frac{V_0}{2}$ but in opposite direction.



Then using energy conservation for one particle from COM frame

$$\frac{1}{2}m\left(\frac{V_0}{2}\right)^2 = \frac{1}{2}mV^2 + \frac{1}{2}2\chi\, x_m^2$$

$$m\frac{V_0^2}{4} = mV^2 + 2\chi\, x_m^2 \quad ....(1)$$

Using AMC (Angular Momentum Conservation)

$$\frac{mV_0}{2}\ell_0 = mV(\ell_0 + x_m)$$

$$V = \frac{V_0}{2}\left(\frac{\ell_0}{\ell_0 + x_m}\right)$$

Put the value in equation (1)

$$m\frac{V_0^2}{4} = m\left(\frac{V_0}{2}\left(\frac{\ell_0}{\ell_0 + x_m}\right)\right)^2 + 2\chi\, x_m^2$$

$$m\frac{V_0^2}{4}\left[1-\left(\frac{\ell_0}{\ell_0 + x_m}\right)^2\right] = 2\chi\, x_m^2$$

$$m\frac{V_0^2}{4}\left[1-\left(1+\frac{x_m}{\ell_0}\right)^{-2}\right] = 2\chi\, x_m^2$$

Using bionomial expression

$$m\frac{V_0^2}{4}\left[1-\left(1-\frac{2x_m}{\ell_0}\right)\right] = 2\chi\, x_m^2$$

$$x_m = \frac{mV_0^2}{2\ell_0 x}$$

In other half string also get same elongation.

Net elongation : $2x_m = \dfrac{mV_0^2}{\ell_0 \chi}$ **Ans.**

Method : 2

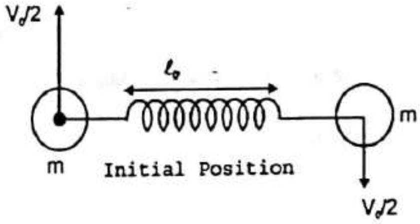
(Equation on combined system)

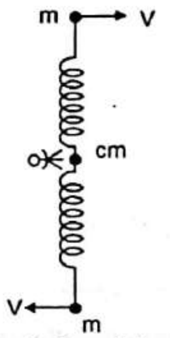
Since no force on COM, com will remain in rest.

If we observe motion then in both minimum elongation or maximum elongation spring force will be perpendicular to velocity vector.

Velocity of COM will be $\frac{V_0}{2}$ in y direction.

From this frame velocity of both block will be $\frac{V_0}{2}$ but in opposite direction.





Final Position

Using energy equation on system

$$\frac{1}{2}m\left(\frac{V_0}{2}\right)^2 \times 2 = \left(\frac{1}{2}mV^2\right)2 + \frac{1}{2}\chi x_m^2$$

$$m\left(\frac{V_0}{2}\right)^2 \times 2 = 2mV^2 + \chi x_m^2 \,.........(i)$$

Using AMC (Angular momentum Conserve w.r.t. COM)

$$\left(\frac{mV_0}{2}\frac{\ell_0}{2}\right)2 = 2(mV)\left[\frac{\ell_0 + x_m}{2}\right]$$

$$\left(\frac{mV_0}{2}\frac{\ell_0}{2}\right)2 = 2(mV)\left[\frac{\ell_0 + x_m}{2}\right]$$

$$V = \frac{V_0}{2(\ell_0 + x_m)}\ell_0$$

Put this value in (i)

$$m\left(\frac{V_0}{2}\right)^2 \times 2 = 2m\left(\frac{V_0}{2(\ell_0 + x_m)}\ell_0\right)^2 + \chi x_m^2$$

$$m\frac{V_0^2}{2} = m\frac{V_0^2}{2\left(1+\frac{x_m}{\ell_0}\right)^2} + \chi x_m^2$$

$$m\frac{V_0^2}{2} = m\frac{V_0^2}{2}\left(1+\frac{x_m}{\ell_0}\right)^{-2} + \chi x_m^2$$

Using bionomial expression

$$m\frac{V_0^2}{2} = m\frac{V_0^2}{2}\left(1-\frac{2x_m}{\ell_0}\right) + \chi x_m^2$$

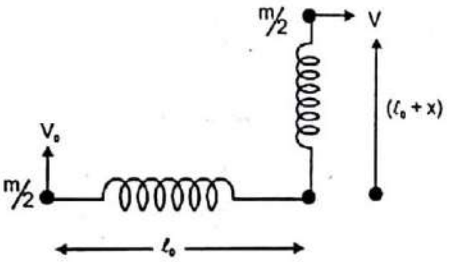
$$\frac{mV_0^2}{\chi\ell_0} = x_m \qquad \textbf{Ans.}$$

Method : 3 (Reduce Mass Concept)

Reduce Mass

$$\mu = \frac{m.m}{2m} = \frac{m}{2}$$

x = maximum elongation energy equation



Using energy equation on system

$$\frac{1}{2}\left(\frac{m}{2}\right)V_0^2 = \frac{1}{2}\left(\frac{m}{2}\right)V^2 + \frac{1}{2}(\ell_0 + x)^2 \text{ ....(i)}$$

Using AMC (Angular momentum Conserve w.r.t. COM)

$$\frac{m}{2}V_0\,\ell_0 = \frac{m}{2}V\,(\ell_0 + x)$$

$$V = V_0\left(1+\frac{x}{\ell_0}\right)^{-1}$$

Put (i)

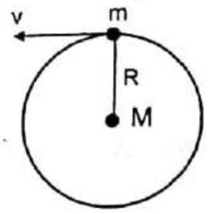
$$\frac{mV_0^2}{4}\left[1-\left(1-\frac{2x}{\ell_0}\right)\right] = \frac{1}{2}\chi x^2$$

$$\frac{mV_0^2}{4}\left(\frac{2x}{\ell_0}\right) = \frac{1}{2}\chi x^2$$

$$x = \frac{mV_0^2}{\ell_0\chi} \qquad \textbf{Ans.}$$

1.4 Universal Gravitation

1.200



Note : In irodov, M is mass of planet which is a mistake. Actually M is mass of sun. Here Mass of sun is M

$$\text{Time period (T)} = \frac{2\pi R}{V} \text{ ....(i)}$$

Also

$$\frac{GMm}{R^2} = \frac{mV^2}{R}$$

$$\frac{GM}{R} = V^2$$

$$R = \frac{GM}{V^2}.$$

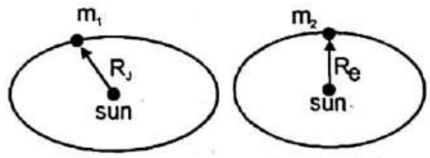
Put in (i)

$$T = \frac{2\pi GM}{V^3}.$$

In irodov G = Y

$$T = \frac{2\pi YM}{V^3} \qquad \textbf{Ans.}$$

1.201



Jupiter Motion — Earth Motion

We know $T \propto R^3$

$$T_J^2 = cR_J^3 \text{ ....(i)} \quad T = \pi\sqrt{\frac{\pi}{2\gamma M}(r+R)^3}$$

$$T_E^2 = cR_E^3 \text{ ....(ii)}$$

From (i) and (ii)

$$\frac{T_J^2}{T_E^2} = \left(\frac{R_J}{R_E}\right)^3 \text{ ....(iii)}$$

Given that $T_J = 12\,T_E$

Put in (iii)

$$\left(\frac{12T_E}{T_E}\right)^2 = \left(\frac{R_J}{R_E}\right)^3$$

$R_J = (144)^{1/3}$ Re

$R_J = 5.2$ Re **Ans.**

(b)

Here motion is taken as circular

Then

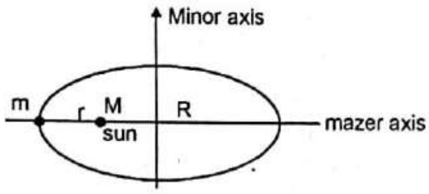
$$V = \sqrt{\frac{GM_J}{R_J}}$$

$$a = \frac{GM_J}{R_J^2}$$

Put values mass and distance to find numeri cal values.

1.202

(a)



Note : In irodov. M is mass of planet which is mistakes. M is mass of sun.

We know that

$$T^2 = \frac{4\pi^2}{GM}a^3 \text{.......................(1)}$$

Length of semi major axis (a) = $\frac{r+R}{2}$

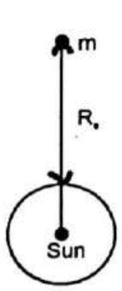
$$T^2 = \frac{4\pi^2}{GM}\left(\frac{r+R}{2}\right)^3$$

In irodov

$G = \gamma$

$$T = \pi\sqrt{\frac{\pi}{2GM}(r+R)^3}$$ **Ans.**

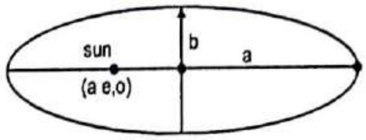
1.203

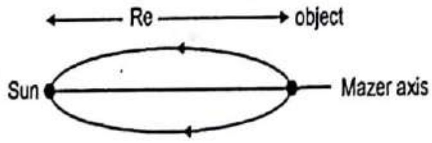


Where m is mass particle

Re = distance of particle and sun surface

= distance of earth to sun.





We can assume motion of object is flat ellipse whose length of semi minor axis is 0. e.g. If motion is staright line then we can as sume b = 0.

Also Co-ordinate of sun is on focus of ellipse = (ae, 0).

Using ellipse properties

$b^2 = a^2(1-e^2)$.

If b = 0 then e = 1.

Means co-ordinate of sun on mazor axis = (a, 0)

Hence in case of flat ellipse sun will be at end of mazor axis of flat ellipse

Also length of semi mazer axis = $\frac{Re}{2}$.

Time to collision with sun = $\frac{T}{2}$.

Where T = Time period of particle on flat ellipse.

Also using keppler's Law :

$T^2 \propto R^3$

For earth

$T_e^2 \propto R_e^3$..........(i)

Re = length of semi mazor axis of earth.

Te = Time period of earth

For object

$$T^2 \propto \left(\frac{Re}{2}\right)^3 \text{..........(ii)}$$

Now dividing (i) by (ii)

$$\frac{T_e^2}{T^2} = \frac{8}{1}$$

$$T = \frac{T_e}{2\sqrt{2}}$$

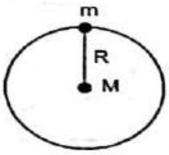
Then Time for collision $= \frac{T}{2} = \frac{T_e}{4\sqrt{2}}$ **Ans.**

We know time period of earth = 365 days.

Then collision time $\frac{365}{4\sqrt{2}} \cong 65$ days

Ans.

1.204



We know time period is given by :

$$\frac{GMm}{R^2} = mRw^2$$

$$w = \sqrt{\frac{GM}{R^3}}$$

$$T = \frac{2\pi}{w} = 2\pi\sqrt{\frac{R^3}{GM}} \text{ ....(i)}$$

Assume radius of sun is r then

$$M = \rho\frac{4}{3}\pi r^3$$

Put in (i)

$$T = 2\pi\sqrt{\frac{R^3}{G\rho\frac{4}{3}\pi r^3}}$$

$$T = 2\pi\sqrt{\frac{3}{4G\rho\pi}\left(\frac{R}{r}\right)^3} \text{ ....(ii)}$$

Here ρ = Constant

If scale is down by η then both R and r is decresed by n

$$R_1 = \frac{R}{n} \quad ; \quad r_1 = \frac{r}{n}$$

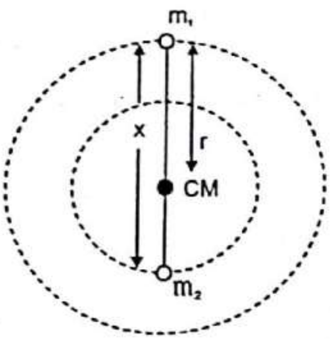
Put in (ii)

$$T = 2\pi\sqrt{\frac{3}{4G\rho\pi}\left(\frac{R/n}{r/n}\right)^3} = 2\pi\sqrt{\frac{3}{4G\rho\pi}\frac{R^3}{r^3}}$$

Not Change **Ans.**

1.205

Method : 1 (Basic calculation)



Distance between m_1 and COM

$$r = \frac{m_2 x}{m_1 + m_2} \text{ ....(1)}$$

Force equation on m_1

$$\frac{Gm_1m_2}{x^2} = m_1 r w^2$$

$$w = \sqrt{\frac{Gm_2}{x^2 r}} \text{ ....(2)}$$

Put the value of r from (1) in (2)

$$w = \sqrt{\frac{Gm_2(m_1 + m_2)}{x^2 m_2 x}}$$

Here $m_1 + m_2 = M$

$$w = \sqrt{\frac{GM}{x^3}} \text{ ....(3)}$$

$$w = \frac{2\pi}{T} \text{ ....(4)}$$

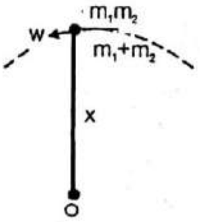
From (3) and (4)

$$\sqrt{\frac{GM}{x^3}} = \frac{2\pi}{T}$$

$$\frac{GM}{x^3} = \frac{4\pi^2}{T^2}$$

$$x = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$ **Ans.**

Method : 2 (Reduce Mass Concept)



Force equation

$$\frac{Gm_1m_2}{x^2} = \mu x w^2$$

Where μ is reduce mass of system.

$$\frac{Gm_1m_2}{x^2} = \left(\frac{m_1m_2}{m_1+m_2}\right) x\, w^2$$

$$\frac{G}{x^2} = \left(\frac{1}{m_1+m_2}\right) x w^2$$

Here $m_1 + m_2 = M$

$$\frac{GM}{w^2} = x^3$$

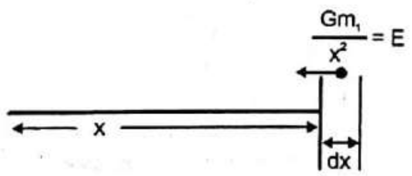
$$\frac{GM}{\left(\frac{2\pi}{T}\right)^2} = x^3$$

$$x = \sqrt[3]{GM\left(\frac{T}{2\pi}\right)^2}$$ **Ans.**

1.206

(a)

Gravitational Potential Calculation::



$$dV = -E.\,dr$$

$$\int_0^V dv = \int_\infty^r \frac{Gm_1}{x^2} dx$$

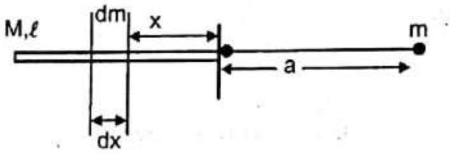
$$V = -\frac{Gm_1}{r}$$

Potential energy $= m_2.\, V = \frac{-Gm_1m_2}{r}$

Ans.

(b)

Potential Energy calculation



Potential energy between dm mass and m mass.

$$dU = -\frac{Gm\,dm}{a+x}$$

$$\int dU = \int_0^\ell -\frac{Gm}{a+x}\left(\frac{M}{\ell}dx\right)$$

$$U = -\frac{Gm}{\ell}\ell n\, a+x\Big|_0^\ell$$

$$= -\frac{GMm}{\ell}\ell n\left(\frac{a+\ell}{a}\right)$$ **Ans.**

Force calculation

$$dF = \frac{Gm\,dm}{(a+x)^2}$$

$$dF = \frac{Gm}{(a+x)^2}\left(\frac{M}{\ell}dx\right)$$

$$F = \frac{GmM}{\ell}\int_0^\ell \frac{dx}{(a+x)^2} = -\frac{GmM}{\ell}\left(\frac{1}{a+x}\right)\Big|_0^\ell$$

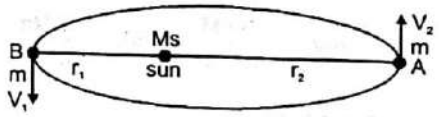
$$= \frac{GmM}{\ell}\left[\frac{1}{a} - \frac{1}{a+\ell}\right]$$

$$= \frac{GmM}{\ell}\frac{\ell}{a(a+\ell)}$$

$$F = \frac{GmM}{a(a+\ell)}$$ **Ans.**

Note: In Irodov G = γ

1.207



We know energy of particle of mass m is

$$E = -\frac{GmM_s}{2a}$$

Where a = length of semi mazor axis.
In above question

$a = \frac{r_1 + r_2}{2}$

$$E = -\frac{GmM_s}{2\left(\frac{r_1+r_2}{2}\right)} = -\frac{GmM_s}{(r_1+r_2)} \quad \text{.........(1)}$$

At point A

Using energy conservation

$$\frac{1}{2}mV_2^2 - \frac{GmM_s}{r_2} = -\frac{GmM_s}{r_1+r_2}$$

$$V_2 = \sqrt{\frac{2GM_Sr_1}{r_2(r_1+r_2)}}$$

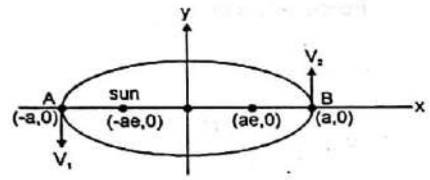
Angular momentom about sun

$$|\bar{L}| = mV_2r_2 = m\sqrt{\frac{2GM_Sr_1r_2}{r_1+r_2}}.$$

Inirodov G→Y

$$|\bar{L}| = m\sqrt{\frac{2GM_Sr_1r_2}{r_1+r_2}} \qquad \textbf{Ans.}$$

1.208



Where m = mass of planet

M = mass of sun

Where a = length of semimazor axis

Then distance of point A with sun

$= a(1-e)$

Distance of point B with sun

$= a(1+e)$

Using angular momentum conservation

$mV_1 a(1-e) = mV_2 a(1+e)$

$$V_1 = \frac{V_2(1+e)}{1-e} \quad \text{.................(i)}$$

Energy of planet

$$E = \frac{1}{2}mV_2^2 - \frac{GMm}{a(1+e)} \quad \text{................(ii)}$$

Using energy equation

$$\frac{1}{2}mV_1^2 - \frac{GmM}{a(1-e)} = \frac{1}{2}mV_2^2 - \frac{GMm}{a(1+e)} \quad \text{..........(iii)}$$

Put V_1 in (ii)

$$\frac{1}{2}mV_2^2\frac{(1+e)^2}{(1-e)^2} - \frac{GmM}{a(1-e)} = \frac{1}{2}mV_2^2 - \frac{GMm}{a(1+e)}$$

$$V_2 = \sqrt{\frac{GM(1-e)}{a(1+e)}}.$$

Put in (ii)

$$E = \frac{1}{2}mV_2^2 - \frac{GmM}{a(1+e)}$$

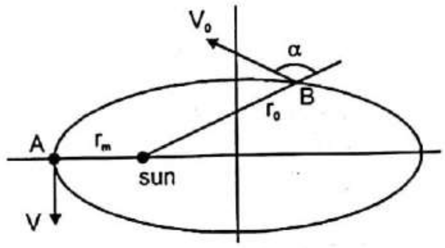
$$= \frac{1}{2}m\frac{GM(1-e)}{a(1+e)} - \frac{GmM}{a(1+e)}$$

$$= \frac{GmM}{a(1+e)}\left[\frac{1-e}{2} - 1\right]$$

$$= \frac{GmM}{a(1+e)}\left[\frac{-1-e}{2}\right]$$

$$E = -\frac{GmM}{2a} \qquad \textbf{Ans.}$$

1.209



m = mass of planet

Ms = mass of sun

Energy conservation at point A and B on points of trajectory

$$\frac{1}{2}mV^2 - \frac{GmM_S}{r_m} = \frac{1}{2}mV_0^2 - \frac{GmM_s}{r_0} \quad \text{............(i)}$$

Angular momentum conservation about sun because net torque about sun is zero

$mV_0 r_0 \sin\alpha = m V r_m$

$V_0 r_0 \sin\alpha = V r_m$ ....(ii)

From (i) and (ii)

$$r_m = \frac{r_0}{2-\eta}\left[1 - \pm\sqrt{1-(2-\eta)\eta\sin^2\alpha}\right].$$

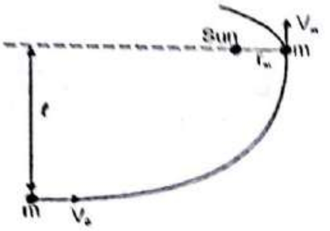
Where $\eta = \frac{r_0 v_0^2}{\gamma m_s}$

Here $\gamma = G$

⊕ sin for maximum distance

⊖ sign for minimum distance

1.210



Here M_s = mass of sun
m = mass of cosmicbody
r_m = minimum distance between sun and planet.
V_m = velocity at minimum distance
Using energy conservation

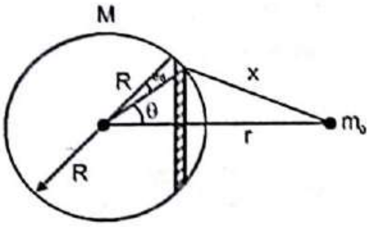
$$\frac{1}{2}mV_0^2 = \frac{1}{2}mV_m^2 - \frac{GmM_s}{r_m} \quad ......(i)$$

Using angular momentum conservation :
$mV_0 \ell = m V_m r_m$

$V_0 \ell = V_m r_m$ ....(ii)
From (i) and (ii)

$$r_m = \frac{GM_s}{V_0^2}\left[\sqrt{1+\left(\frac{\ell V_0^2}{GM_s}\right)^2} - 1\right] \qquad \textbf{Ans.}$$

1.211
(a)



Mass of differential element is dm.
This differential element is ring.
Then Potential energy

$$dU = -\frac{G(dm)m_0}{x}$$

$$U = \int_0^{\pi} \frac{Gm_0 \frac{M}{4\pi R^2}(2\pi R\sin\theta \, R \, d\theta)}{\sqrt{R^2 + r^2 - 2Rr\cos\theta}}$$

$$U = -\frac{Gm_0 M}{r}$$

In irodor
$G = \gamma$
$M_0 = m$

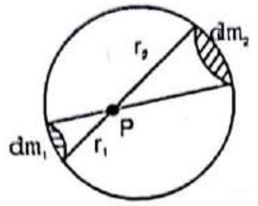
$$U = -\frac{GmM}{r} = -\frac{\gamma mM}{r} \qquad \textbf{Ans.}$$

(b)

$$F = -\frac{\partial U}{\partial r} = -\frac{\gamma mM}{r^2}$$

Ans.

1.212



Take cone form at point P as shown in figure solid angle made by dq_1 and dq_2 on P will be same.

$$\Omega = \frac{dA_1}{r_1^2} = \frac{dA_2}{r_2^2} \quad ....(1)$$

Where dA_1 is area at particle of dm_1 and dA_2 is area at particle of dm_2
Gravitational field at point P due to dm_1 mass

$$E_1 = G\frac{dm_1}{r_1^2}$$

Gravitational field at point P due to dm_2 mass

$$E_2 = G\frac{dm_2}{r_2^2}$$

Net gravitational field at point P
$E = E_1 - E_2$

$$E = G\frac{dm_1}{r_1^2} - G\frac{dm_2}{r_2^2} \quad ....(2)$$

Since mass density is uniform and equal to σ then

$dm_1 = \sigma dA_1$
$dm_2 = \sigma dA_2$
From (2)

$$E = G\frac{\sigma dA_1}{r_1^2} - G\frac{\sigma dA_2}{r_2^2} = G\sigma\left(\frac{dA_1}{r_1^2} - \frac{dA_2}{r_2^2}\right) ...(3)$$

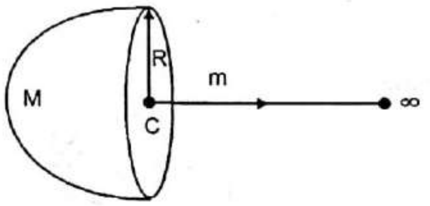
From (1)

$$\frac{dA_1}{r_1^2} = \frac{dA_2}{r_2^2}$$

Put in (3)

E=0
Then dm_1 and dm_2 is point like source then field at point P is zero due to dm_1 and dm_2 by symmetry. Net field at P due to all mass distribution will be zero.

1.213



Work performed by gravitation force

$W = -(U_\infty - U_0) = U_0 - U_\infty$

Assumption

$U_\infty = 0$

$W = U_0 = mV_0$.............(1)

Where

V_0 = Potential at origin

U_0 = Potential energy at origin of particle m

We know

Potential at centre

$$V_0 = \frac{V_1}{2} \text{ ....(2)}$$

Where V_1 is potential due complete solid sphere

$$V_1 = \frac{-3}{2}\frac{G(2M)}{R}$$

Put in (2)

$$V_0 = \frac{-3}{2}\frac{GM}{R}$$

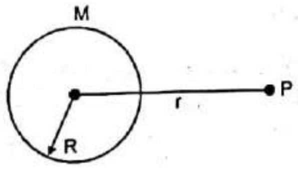
From (1)

$$W = -m\frac{3}{2}\frac{GM}{R} = \frac{-3GmM}{2R}$$ **Ans.**

1.214

At point P :

r > R :

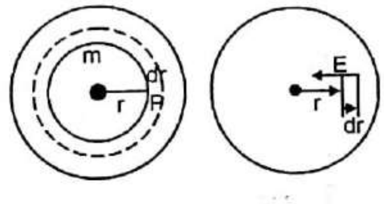


From Q. 1.211

$$E = -\frac{GM}{r^2}$$ **Ans.**

$$V = -\frac{GM}{r}.$$ **Ans.**

r < R :



Field is only due to mass inclosed in dotted sphere then let mass of dotted sphere is m then.

$$E_P = \frac{Gm}{r^2} \text{ ....(i)}$$

$$m = \frac{M}{\frac{4}{3}\pi R^3}\left(\frac{4}{3}\pi r^3\right) = \frac{Mr^3}{R^3}$$

Put in (i)

$$E_P = \frac{GMr^3}{r^2R^3} = \frac{GMr}{R^3}$$ **Ans.**

We know

$dV = -E\,dr$

$$\int_{V_R}^{V_r} dV = +\left(+\int_R^r \frac{GMr}{R^3}dr\right)$$

$$V_r - V_R = \left.\frac{GMr^2}{2R^3}\right|_R^r$$

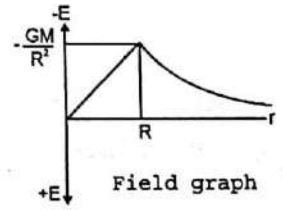
$$V_r - V_R = -\frac{GM}{2R} + \frac{GMr^2}{2R^3} \text{ ....(ii)}$$

Also we know

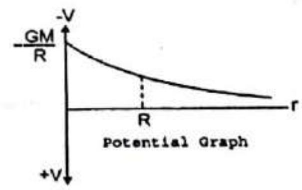
$$V_R = -\frac{GM}{R}$$

$$V_r = -\frac{GM}{2R} - \frac{GM}{R} + \frac{GMr^2}{2R^3}$$

$$V_r = -\frac{GM}{R}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right)$$ **Ans.**

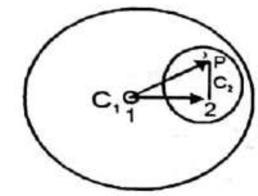


Field graph



Potential Graph

1.215



ρ = density of material

ℓ = distance between two centre of sphere

Using super position principle

$\vec{E}_P$ = Field at P

= field without cavity – field due to cavity

$$= -\frac{GM\vec{r}_{13}}{R_1^3} - \left(-\frac{Gm\vec{r}_{23}}{R_2^3}\right)$$

Where R_1 is radius of out sphere and R_2 radius of cavity

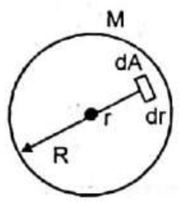
$$\vec{E}_P = -\frac{G\rho\frac{4}{3}\pi R_1^3}{R_1^3}\vec{r}_{13} + \frac{G\rho\frac{4}{3}\pi R_2^3}{R_2^3}\vec{r}_{23}$$

$$= -G\rho\frac{4}{3}\pi(\vec{r}_{13} - \vec{r}_{23}) = -G\rho\frac{4}{3}\pi\,\vec{r}_{12}$$

$$|\vec{E}_P| = \frac{4\pi}{3}G\rho\ell \qquad \textbf{Ans.}$$

Which is uniform.

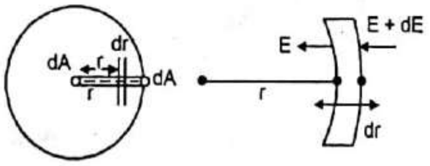
1.216 **Method : 1 (Force method)**



Take a differential area dA at r distance form centre.

Then force on this area

F.B.D. of differential area



Here we take cylindrical element of uniform cross section dA to calcution of pressure. dA must be uniform from centre to surface because at each point of force will be parrallel to each other sothat there is no need of to make component of force.

For calculation of pressure area will be same. Then

$$dF = \left(\frac{GMr}{R^3}\right)dm = -\frac{GMr}{R^3}\,(dA)\,(dr)\,\rho$$

$$= \left(\frac{GMr\,dr}{R^3}\right)\rho\,dA$$

$$dF = \rho\,dA\int_R^r \frac{GMr}{R^3}\,dr$$

Here dF is net force on surface at dA area

$$dF = \frac{GM}{2R^3}\rho\,dA\left[-r^2 + R^2\right]$$

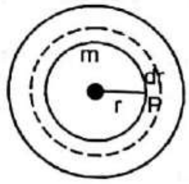
$$P_r = \frac{dF}{dA} = \frac{GM}{2R^3}\left(\frac{M}{\frac{4}{3}\pi R^3}\right)[R^2 - r^2]$$

$$P_r = \frac{3GM^2}{8\pi R^6}[R^2 - r^2]$$

$$P_r = \frac{3GM^2}{8\pi R^4}\left[1 - \frac{r^2}{R^2}\right]$$

Ans.

Method : 2 (Pressure method)



Pressure difference at r distance for thickness of dr

$$dF = \frac{GMr}{R^3}dm$$

$$dF = \frac{GMr}{R^3}\left(\frac{M}{\frac{4}{3}\pi R^3}4\pi r^2 dr\right)$$

$$dP = \frac{dF}{4\pi r^2} = \frac{GMr}{R^3}\left(\frac{M}{\frac{4}{3}\pi R^3}\right)dr$$

$$dP = \frac{3GM^2 r}{4\pi R^6}dr$$

$$P = \int_R^r \frac{3GM^2 r}{4\pi R^6}dr$$

$$P = \frac{3GM^2}{8\pi R^4}\left[1-\frac{r^2}{R^2}\right]$$

Ans.

1.217

(a)

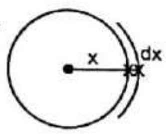
Let at time t only, m is prevent at surface and in nex dt time dm mass come then

$$dU = -\frac{Gmdm}{R}$$

$$U = -\int_0^M \frac{Gm}{R}dm$$

$$U = -\frac{GM^2}{2R}$$ **Ans.**

(b)



Let us only radius of x of sphere is formed, at time t and in next dt time dx redius again formed.
Now mass of sphere of x radius is m and that of dx is dm.
Self energy of m and dm mass

$$dU = \frac{-Gmdm}{x} \quad ....(1)$$

$$m = \rho\frac{4}{3}\pi x^3$$

$$dm = \rho 4\pi x^2\, dx$$

$$dU = \frac{-G}{x}\rho\frac{4}{3}\pi x^3\,\rho 4\pi x^2\,dx$$

$$U = -\frac{G}{3}\rho^2 4\pi\times 4\pi\int_0^R x^4\,dx$$

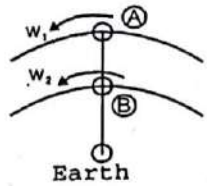
$$U = -\frac{G}{3}\rho^2 4\pi\times 4\pi\frac{R^5}{5}$$

Put $\rho = \dfrac{m}{\frac{4}{3}\pi R^3}$

$$U = -\frac{3}{5}\frac{GM^2}{R}$$ **Ans.**

1.218

Case (1)
Same sense of rotation



For satellites (A)

$$\frac{GMem}{r^2} = mrw^2$$

$$w = \sqrt{\frac{GMe}{r^3}}$$

$$w_1 = \sqrt{\frac{GMe}{r^3}}$$

$$w_2 = \sqrt{\frac{GMe}{(r+\Delta r)^3}}.$$

Relative angular velocity:
$w_r = w_1 - w_2$

$$= \sqrt{\frac{GMe}{r^3}}\left[1-\frac{1}{\sqrt{\left(1+\frac{\Delta r}{r}\right)^3}}\right]$$

$$w_r = \sqrt{\frac{GMe}{r^3}}\left(1-\left(1+\frac{\Delta r}{r}\right)^{-3/2}\right)$$

Using bionomial expansion

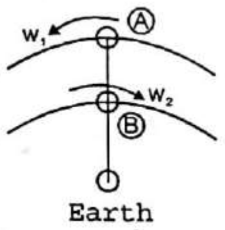
$$w_r = \sqrt{\frac{Gme}{r^3}}\left(\frac{3}{2}\frac{\Delta r}{r}\right).$$

Then time period

$$\Delta t = \frac{2\pi}{w_r} = \frac{2\pi}{\sqrt{\frac{GMe}{r^3}}\frac{3}{2}\frac{\Delta r}{r}}$$

$$\Delta t = \frac{2\pi r^{3/2}}{\sqrt{Gm_e}\left(\frac{3}{2}\frac{\Delta r}{r}\right)}$$ **Ans.**

Case (II)
Opposite sense of rotation



$$w_1 = \sqrt{\frac{GMe}{r^3}}$$

$$w_2 = \sqrt{\frac{GMe}{(r+\Delta r)^3}}$$

$$w_r = w_1 + w_2 = \sqrt{\frac{GMe}{r^3}}\left[1 + \frac{1}{\left(1+\frac{\Delta r}{r}\right)^{-3/2}}\right]$$

$$w_r = \sqrt{\frac{G\,Me}{r^3}}\left[2+\left(1+\frac{3\Delta r}{r}\right)\right]$$

$$= \sqrt{\frac{G\,Me}{r^3}}\left[2-\frac{3\Delta r}{r}\right]$$

$$T = \frac{2\pi}{w_r} = \frac{2\pi r^{3/2}}{\sqrt{GMe}\left(2-\frac{3\Delta r}{r}\right)}$$ **Ans.**

1.219

$$w_1 = \frac{GMe}{Re^2}$$

w_1 = Acceleration due to gravity
M_e = Mass of earth
R_e = Radius of earth

$$w_2 = R_e w^2$$

Where w_2 = centrifugal acceleration
w = Angular velocity

$$w_3 = \frac{GMs}{R_s^2}$$

R_s = Distance between sun and earth
M_s = Mass of sun

$w_1 \;:\; w_2 \;:\; w_3 \;:\; :$

$$\int \frac{GMs}{R_s^2} : R_e w^2 : \frac{GMs}{R_s^2}$$

$w_1 : w_2 : w_3 : : 1 : 0.0034 : 0.006$

Ans.

1.220

For 1% change in gravity:

We know $g = \dfrac{g_0}{\left(1+\frac{h}{R}\right)^2}$

If $h << R$

$$g = g_0\left(1-\frac{2h}{R}\right)$$

$$\frac{\Delta g}{g_0} = \frac{2h}{R}$$

For 1% change in gravity

$$\frac{\Delta g}{g_0}\times 100 = \frac{2h}{R}\times 100 = 1$$

$$h = \frac{R}{200}$$

We know R = 6400 km
$h \cong 32$ Km

Ans.

For $\frac{1}{2}$ change in gravity:

Then h is comparable to R

$$g_0 - g = g_0 - \frac{g_0}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_0-g}{g_0} = 1-\frac{1}{\left(1+\frac{h}{R}\right)^2}$$

Given that

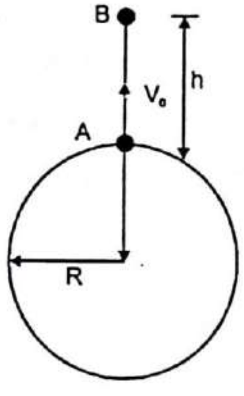
$$\frac{g_0-g}{g_0} = \frac{1}{2}$$

$$\left[1-\frac{1}{\left(1+\frac{h}{R}\right)^2}\right] = \frac{1}{2}$$

$$1+\frac{h}{R} = \sqrt{2}$$

$h = (\sqrt{2}-1)R \Rightarrow h \cong 2650$ km **Ans**

1.221



At maximum height fnal velocity will be zero.
Using energy conservation from A to B

$$\frac{1}{2}mV_0^2 - \frac{GMm}{R} = 0 - \frac{GMm}{R+h}$$

$$\frac{1}{2}mV_0^2 - \left(\frac{GM}{R^2}\right)mR = 0 - \left(\frac{GM}{R^2}\right)\frac{mR}{1+\frac{h}{R}}$$

We know $\frac{GM}{R^2} = g$

$$\frac{1}{2}mV_0^2 - mgR = -mg\frac{R}{1+\frac{h}{R}}$$

$$-\frac{V_0^2}{2gR} + 1 = \frac{1}{1+\frac{h}{R}}$$

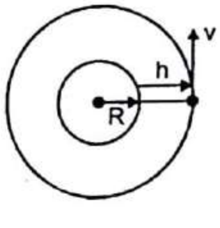
$$1+\frac{h}{R} = \frac{1}{1-\frac{V_0^2}{2gR}}$$

$$h = \left[\frac{R}{1-\frac{V_0^2}{2gR}} - R\right]$$

$$= \frac{R\frac{V_0^2}{2gR}}{1-\frac{V_0^2}{2gR}}$$

$$h = \frac{R}{\frac{2gR}{V_0^2} - 1}$$ **Ans.**

1.222



$$\frac{mv^2}{R+h} = \frac{GM_e m}{(R+h)^2}$$

$$\frac{mv^2}{R+h} = \frac{GM_e}{R^2}\frac{m}{(1+\frac{h}{R})^2}$$

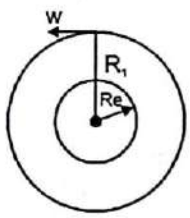
We Know

$$\frac{GM_e}{R^2} = g$$

$$h = \frac{gR^2}{V_0^2} - R$$

$$h = R\left(\frac{gR}{V_0^2} - 1\right)$$ **Ans.**

1.223



$$\frac{GM_e m}{R_1^2} = mR_1w^2$$

m = Mass of satillite

$$w^2 = \frac{GMe}{R_1^3}$$

Me = Mass of earth

$$w = \sqrt{\frac{GMe}{R_1^3}}$$

h = distance between satellite and earth centre

$$T = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{\frac{GMe}{R_1^3}}}.$$

Since satellite is motionless as seen from ground observer then time period of satellite must be same as earth about its own axis then

$$T = 24 \text{ hours} = 24 \times 60 \times 60 = \frac{2\pi}{\sqrt{\frac{GMe}{R_1^3}}}$$

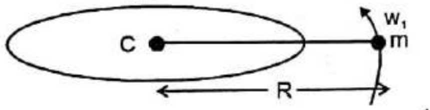
$R^1 = 4.2 \times 10^4$ km

$$V = R^1 w = 4.2 \times 10^4 \times \frac{2\pi}{24 \times 60 \times 60} = 3.1 \text{ km/s}$$

$$a = R^1 w^2 = 4.2 \times 10^4 \left(\frac{2\pi}{24 \times 60 \times 60}\right)^2$$

$$= 0.22 \text{ m/s}^2 \qquad \textbf{Ans.}$$

1.224



Since of rotation of earth is from west to east

$$w_2 = \frac{GmM_2}{R^2} = m R w_1^2$$

$$w_1 = \sqrt{\frac{G M_e}{R^3}}$$

Anglar velocity of earth is w_2

$$w_2 = \frac{2\pi}{T}$$

Relative angular velocity

$$w_r = -w_2 + w_1 = -\frac{2\pi}{T} + \sqrt{\frac{G M_e}{R^3}}$$

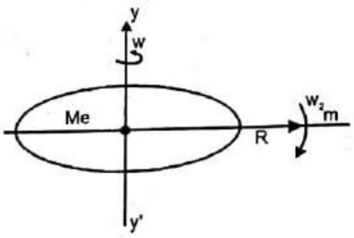
$$\frac{2\pi}{\tau} = -\frac{2\pi}{T} + \sqrt{\frac{G M_e}{R^3}}$$

$$2\pi\left(\frac{1}{\tau} + \frac{1}{T}\right) = \sqrt{\frac{G M_e}{R^3}}$$

$$\left(\frac{2\pi}{T}\right)^2 \left(\frac{T}{\tau} + 1\right)^2 = \frac{G M_e}{R^3}$$

$$Me = \frac{4\pi^2 R^3}{GT^2}\left(\frac{T}{\tau} + 1\right)^2 \qquad \textbf{Ans.}$$

1.225*



Where Me = Mass of earth
R = Distance between earth and satellite

w = Angular velocity of earth axis = $\frac{2\pi}{T}$

m = Mass of satellite.

Relative velocity (V_r)
On particle

$$\frac{G M_e m}{R^2} = m R w_0^2$$

$$w_0 = \sqrt{\frac{G M_e}{R^3}}.$$

Relative angular velocity
$w_r = w + w_0$
Relative velocity
$V_r = R (w + w_0)$

$$V_r = R\left[\frac{2\pi}{T} + \sqrt{\frac{G M_e}{R^3}}\right] = \frac{2\pi R}{T} + \sqrt{\frac{G M_e}{R^3}}$$

Ans.

Relative acceleration (a_r)
Force equation from y – axis

$$F = \frac{G M_e m}{R^2} = \text{Real force on particle}$$

$$F_{net} = F_{real} + \text{Centrifugal} + \text{Coriolis}$$

$$m\, a_r = F - mRw^2 + 2m\left[\frac{2\pi R}{T} + \sqrt{\frac{G M_e}{R^3}}\right] w$$

$$m\, a_r = F - mR\left[\frac{2\pi}{T}\right]^2 + 2m\left[\frac{2\pi R}{T} + \sqrt{\frac{G M_e}{R}}\right]\frac{2\pi}{T}$$

$$a_r = \frac{GM_e}{R^2} + \frac{2\pi}{T}\left[\frac{2\pi R}{T} + 2\sqrt{\frac{GM_e}{R}}\right]$$

Ans.

F = Gravitational force $= \frac{GM_e m}{R^2}$

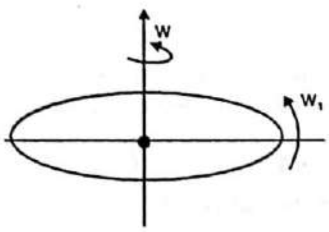
Centrifugal force = mRw^2

Coriolis force=2m(VW)

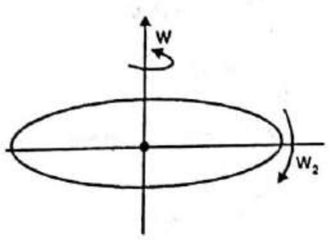
$$=2m\left[\frac{2\pi R}{T} + 2\sqrt{\frac{GM_e}{R}}\right]w$$

Ans.

1.226



Case (I)



Case (II)

Case (1)

$w_r' = w_1 + w$

Where $\frac{GM_2}{R_e^2} = R_e w_r^2$

$$w_1 = \sqrt{\frac{GM_e}{R_e^3}}$$

Case (2)

$w_r' = w_1 - w$

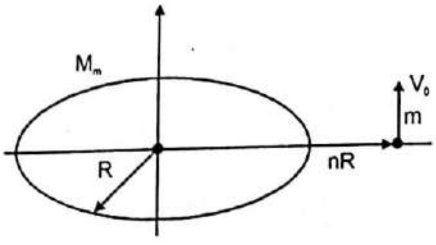
$$T = \frac{2\pi}{w}$$

$$\frac{T'}{T''} = \frac{w_r'}{w_r'} = \frac{w_1 + w}{w_1 - w}$$

$$\frac{T'}{T''} = \frac{\sqrt{\frac{GM_e}{R_e^3}} + \frac{2\pi}{T}}{\sqrt{\frac{GM_e}{R_e^3}} - \frac{2\pi}{T}} \cong 1.27$$

Ans.

1.227



$$V_0 = \sqrt{\frac{GM}{nR}} = \sqrt{\frac{gR}{n}}$$

Resistance force $F = \alpha v^2$

V_0 = Orbital velocity

M_m = mass of moon

$$m\frac{dV}{dt} = \alpha v^2$$

$$m\int_{v_0}^{v}\frac{dV}{V^2} = -\alpha\int_0^t dt$$

$$-\frac{1}{v}\Big|_{v_0}^{v} = -\frac{\alpha t}{m}$$

$$-\frac{1}{v} + \frac{1}{v_0} = -\frac{\alpha t}{m}$$

$$\frac{1}{v_0} + \frac{\alpha t}{m} = \frac{1}{v}$$

$$v = \frac{mv_0}{m + v_0\alpha t} \quad \text{.................(i)}$$

Let at time t distance of satellite from centre of earth is x then

$$\frac{GMm}{x^2} = \frac{mv^2}{x}$$

$$\frac{GM}{x^2} = \frac{v^2}{x} = \frac{1}{x}\left(\frac{mv_0}{m + v_0\alpha t}\right)^2$$

$$\frac{GM}{x^2} = \frac{1}{x}\left(\frac{m\,v_0}{m + v_0\,\alpha t}\right)^2$$

$$\frac{GM}{x^2} = \frac{1}{x}\left(\frac{mv_0}{m+v_0\,\alpha t}\right)^2$$

$$t = \frac{m}{v_0\alpha}\left[v_0\sqrt{\frac{x}{GM}} - 1\right] \text{.............(ii)}$$

Put $x = R$; $t = t_1$ in (ii)

$$t_1 = \frac{m}{v_0\alpha}\left[v_0\sqrt{\frac{R}{GM}} - 1\right]$$

Put $x = \eta R$; $t = t_2$ in (ii)

$$t_2 = \frac{m}{v_0\alpha}\left[v_0\sqrt{\frac{\eta R}{GM}} - 1\right]$$

Time to strike on surface

$$t = t_2 - t_1$$

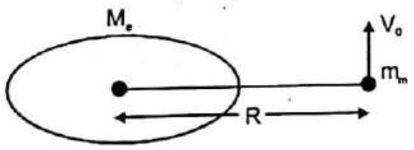
$$t = \frac{m}{v_0\alpha}\left[v_0\sqrt{\frac{\eta R}{GM}} - 1\right] - \frac{m}{v_0\alpha}\left[v_0\sqrt{\frac{R}{GM}} - 1\right]$$

$$t = \frac{m}{\alpha}\sqrt{\frac{R}{GM}}\left[\sqrt{\eta} - 1\right]$$

$$t = \frac{m}{\alpha}\sqrt{\frac{1}{\frac{GM}{R^2}R}}\left[\sqrt{\eta} - 1\right]$$

$$t = \frac{m(\sqrt{n} - 1)}{\alpha\sqrt{gR}}$$ **Ans.**

1.228



Orbital velocity :

$$\frac{GM_e m_m}{R^2} = \frac{m_m V_0^2}{R}$$

Where R = distance between moon and centre of earth.

$$V_o = \sqrt{\frac{GM_e m_m}{Rm_m}}$$

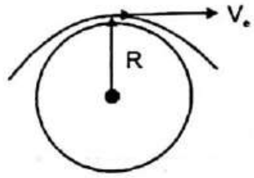
$$V_o = \sqrt{\frac{GM_e}{R}} = 1.67 \text{ m/s}$$ **Ans.**

Escape Velocity

$$0 = -\frac{GM_e m_m}{R} + \frac{1}{2}m_m V_e^2$$

$$V_e = \sqrt{\frac{2GM_e}{R}}$$ **Ans.**

1.229



Since path of spaceship is parabolic, means energy of space ship at moon surface is zero. This tell us velocity of space ship at moon surface will be

$$Ve = \sqrt{\frac{2\,GM}{R}}.$$

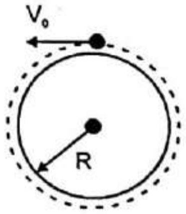
It we want to establish in R radius then

$$V_o = \sqrt{\frac{GM}{R}}$$

$$\text{Change in velocity} = \sqrt{\frac{GM}{R}}(1 - \sqrt{2})$$

Ans.

1.230



$$\frac{GM_e m}{R^2} = \frac{mV_0^2}{R}$$

$$V_0 = \sqrt{\frac{G M_e}{R}}$$

V_0 = orbital velocity satelite
Escape velocity(V_e)

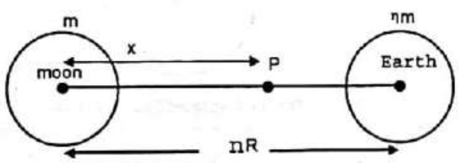
$$\frac{1}{2}mV_e^2 - \frac{GM_e m}{R} = 0$$

$$V_e = \sqrt{\frac{2G M_e}{R}}.$$

Additional velocity need:
$\Delta V = V_e - V_0$

$$\Delta V = (\sqrt{2}-1)\sqrt{\frac{G M_e}{R}}$$ **Ans.**

1.231



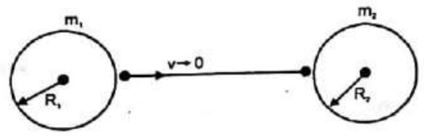
Assume at point P field is zero

$$\frac{Gm}{x} = \frac{G\eta m}{(nR-x)^2}$$

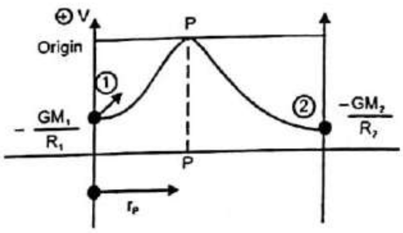
$nR - x = x\sqrt{\eta}$

$$x = \frac{nR}{1+\sqrt{\eta}}$$ **Ans.**

1.232



If particle move slowly from earth to moon then work done will be minimum.
Graph between potential and its position

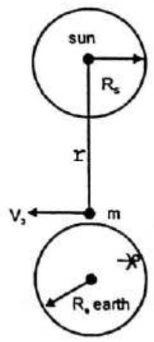


<u>In process 1 – P</u> :
Spaceship works against gravity to move slowly.
<u>In process P – 2</u> :
Spaceship works against gravity to move again slowly.
If we want to bring particle from (1) and (2) then we have to cross maximum potential barrier and at this point net gravitations pull on particle is zero. And this point P is far-far from earth centre and also $r_p >> R_1$ and $r_p >> R_2$. Hence potential of this point may be take to be zero because $r_p >> R_1$ & R_2.
Work done by spaceship
From (1) to (2)
$W_{1\text{-}P} + W_{P\text{-}2}$

$$= \frac{GM_1}{R_1}m + \frac{GM_2}{R_2}m = Gm\left(\frac{M_1}{R_1} + \frac{M_2}{R_2}\right)$$

Ans.

1.233*



Third cosmic velocity is the velocity of particle so that it leave solar system (Sun).
Energy of particle from earth observes

$$E = \frac{1}{2}mV_3^2 - \frac{GM_e m}{R_e} \text{ ....(1)}$$

V_3 = Relative velocity of particle from earth observer.
This energy shoud be more than binding energy of particle in sun gravitation.
Orbital velocity of particle in sun gravitation

$V_1 = \sqrt{\frac{GM_s}{r}}$

Escape velocity: $V_e = \sqrt{2}\,V_1$

Extra energy =

$$\frac{1}{2}m(V_e - V_1)^2 = \frac{1}{2}mV_1^2(\sqrt{2}-1)^2$$

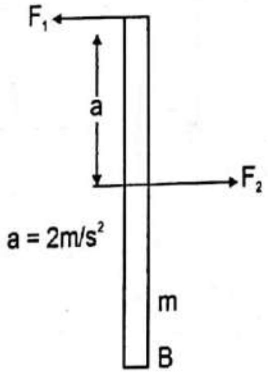
$$E = \frac{1}{2}mV_3^2 - \frac{GM_e m}{R_e} = \frac{1}{2}mV_1^2(\sqrt{2}-1)^2$$

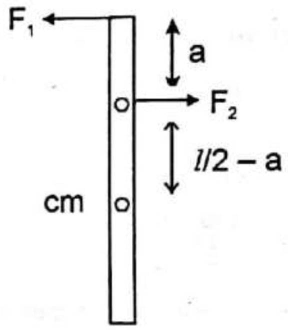
$$V_3 = \sqrt{2\frac{GM_e}{R_e} + (\sqrt{2}-1)^2 V_1^2}$$

$$V_3 = \sqrt{2v_1^2 + (\sqrt{2}-1)^2 V_1^2}$$ **Ans.**

1.7 Dynamics of solid body

1.234





Since rod is translatory then torque about com = 0

$$F_1\frac{\ell}{2} = F_2\left(\frac{\ell}{2} - a\right) \text{..........(i)}$$

From (1), it is seen that $F_2 > F_1$ because

$$\frac{\ell/2}{\ell/2 - a} > 1$$

$F_2 - F_1 = ma$ ....(ii)

$F_1 = F_2 - ma = 5\text{-}1\times 2 = 3N$

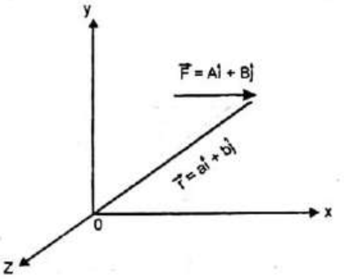
Put in (i):

$$3\left(\frac{\ell}{2}\right) = 5\left(\frac{\ell}{2} - a\right)$$

$$\frac{3\ell}{2} = \frac{5\ell}{2} - 5\times 0.2$$

$\ell = 1m$ **Ans.**

1.235



$\vec{N} = \vec{r} \times \vec{F} = (a\hat{i} + b\hat{i}) \times (A\hat{i} + B\hat{i})$

$\vec{N} = (aB - bA)\hat{k}$

$N = aB - bA$ ....(1) **Ans.**

We know

$N = r_\perp F$

Where $r_\perp$ is arm length of force.

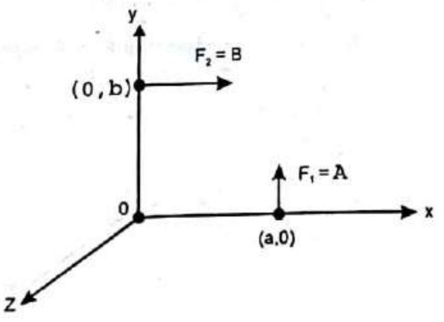
$r_\perp = \dfrac{N}{F}$ ....(2)

$F = \sqrt{A^2 + B^2}$

$r_\perp = \dfrac{(aB - bA)}{\sqrt{A^2 + B^2}}$ **Ans.**

$r_\perp$ = Arm length = Perpendicular distance between origin and line of force

1.236



$\vec{F}_{net} = A\hat{i} + B\hat{j}$.

$F = \sqrt{A^2 + B^2}$ ....(1)

Then torque of $\vec{F}_{net}$ about O

$N = -Bb\hat{k} + Aa\hat{k}$

$N = Aa - Bb$ ....(2)

We know

$N = r_\perp F$

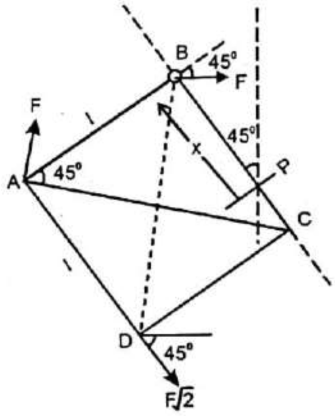
Where $r_\perp$ is arm length of force.

$r_\perp = \dfrac{N}{F}$

From (1) and (2)

$l = \dfrac{Aa - Bb}{\sqrt{A^2 + B^2}}$ **Ans.**

1.237



l = length of side of square

$\vec{F}_{net} = F_x i + F_y i$

$F_x = F + F\sqrt{2}\cos 45^\circ = 2F$

$F_y = F - F\sqrt{2}\,\text{sm}\, 45^\circ = 0$

$|\vec{F}_{net}| = 2F$

Direction of force is x axis.

Ans.

Suppose distance of point of application P is at x distance from B.

Then torque due to all force about point P =0

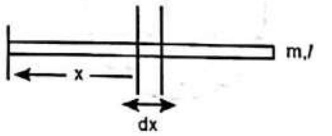
Torque about P

$$\left(F\cos 45^0\right)x-\left(F\sqrt{2}\right)l+$$
$$\left(F\cos 45^0\right)x+\left(F\cos 45^0\right)l=0$$

$$x=\frac{l}{2}$$ **Ans.**

1.238

(a)

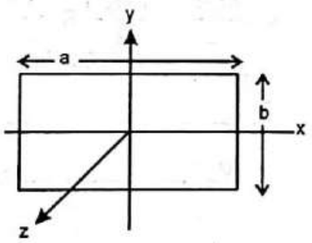


$$dI = (dm)\,x^2 = \left(\frac{m}{l}dx\right)x^2$$

$$\int dI = \frac{m}{l}\int_0^l x^2dx$$

$$I = \frac{ml^2}{3}$$ **Ans.**

(b)



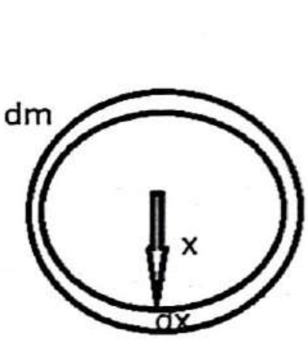
$$I_x=\frac{mb^2}{12}$$

$$I_y=\frac{ma^2}{12}$$

$$I_z = I_x + I_y = \frac{m}{12}\left(a^2+b^2\right)$$ **Ans.**

1.239

(a)



Moment of inertia of dx thickness disc

$dI = (dm)x^2$

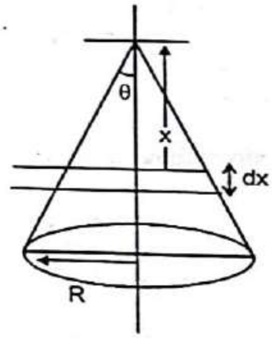
$= dm=\rho b2\pi x dx$

Where ρ = density of disc = $\frac{\text{mass}}{\text{volume}}$

$$I=\rho b2\pi\int_0^R x^3dx$$

$$I=\frac{\pi}{2}\rho bR^2$$ **Ans.**

(b)



Height of cone=h = Rcot θ

$$dI=\frac{(dm)}{2}r^2, =\frac{(dm)}{2}x^2\tan^2\theta$$

$$dI = \frac{x^2\tan^2\theta}{2}\left(\frac{m}{\frac{1}{3}\pi R^2 h}\right)\left(\pi x^2\tan^2\theta\right)dx$$

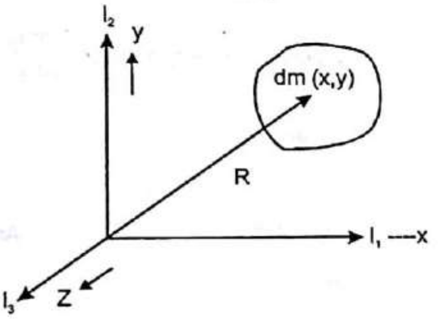
$$dI = \frac{3x^4\tan^4\theta\, m\, dx}{2R^2 R\cot\theta}$$

$$I = \frac{3}{2}\frac{m\tan^5\theta}{R^3}\int_0^h x^4 dx = \frac{3}{2}\times\frac{m\tan^5\theta}{5R^3}R^5\cot^5\theta$$

$$I = \frac{3}{10}mR^2$$ **Ans.**

1.240

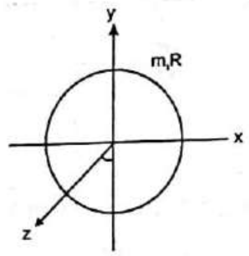
Perpendicular axis theorm



$I_3 = \Sigma\,(dm)R^2$
$I_3 = \Sigma\,(dm)\,(x^2+y^2) = \Sigma x^2 dm + \Sigma y^2\, dm$
$I_3 = I_y + I_x$
$I_3 = I_2 + I_1$

Proved

Uniform Disc

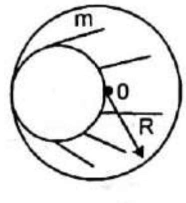


Using perpendicular axis theorm
$I_z = I_y + I_x$
By symmetry
$I_x = I_y$

$$I_x + I_y = \frac{mR^2}{2}$$

$$I_x = I_y = \frac{mR^2}{4}$$ **Ans.**

1.241



$$\text{Density of matrial } (\sigma) = \frac{m}{\pi R^2 - \pi\left(\frac{R}{2}\right)^2}$$

$$\sigma = \frac{4m}{3\pi R^2}$$

Moment of inertial about O

$I_0 = I_1 - I_2$
Where I_1 = moment of inertia due to complete disc (without cut)
I_2 = Moment of inertial of cutting disc

$$I_1 = \frac{M_2R^2}{2} = \frac{(\sigma\pi R^2)}{2}R^2 = \frac{4m}{3\pi R^2}\frac{\pi R^2R^2}{2}$$

$$= \frac{2mR^2}{3}$$

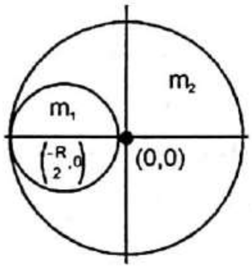
$$I_2 = \frac{M_1 R^2/4}{2} + M_1 R^2/4 = \frac{3M_1R^2}{8}$$

$$I_2 = \frac{3}{8}\left(\sigma\pi\left(\frac{R}{2}\right)^2\right)R^2 = \frac{3}{32}\pi R^2R^2\frac{4m}{3\pi R^2} = \frac{mR^2}{8}$$

$$I_0 = \frac{2mR^2}{3} - \frac{mR^2}{8} = \frac{13mR^2}{24}$$

$$I_0 = \frac{13mR^2}{24}$$

Calculation of com



$$x_{cm} = \frac{-\frac{m_2 R}{2}}{m_2 - m_1} = \frac{\frac{-m_2 R}{2}}{m} = -\frac{\sigma\pi\left(\frac{R}{2}\right)^2 \frac{R}{2}}{m}$$

$$= \frac{-4m}{3\pi R^2}\frac{\pi R^2 R}{8m} = \frac{-4R}{3\times 8} = \frac{-R}{6}$$

We know

$$I_0 = I_{cm} + m x_{cm}^2$$

$$\frac{13mR^2}{24} = I_{cm} + \frac{mR^2}{36}$$

$$I_{cm} = \frac{13mR^2}{24} - \frac{mR^2}{36}$$

$$I_{cm} = \frac{37}{72}mR^2$$

1.242

We know moment of Inertia of solid sphere

$$I = \frac{2}{5}mR^2$$

Then

$$I = \frac{2}{5}R^2\left(\rho\frac{4}{3}\pi R^3\right)$$

$$I = \frac{8}{15}\rho\pi R^5$$

Differentiate equation

$$\frac{dI}{dR} = \frac{8}{15}\rho\pi 5R^4$$

$$dI = \frac{8}{3}\rho\pi R^4\, dR \text{ ....(1)}$$

Mass of shell

$$dm = \left(4\pi R^2\, dR\right)\rho \text{ ....(2)}$$

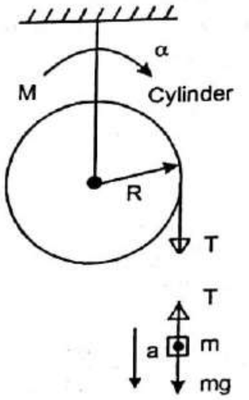
Divide (1) by (2)

$$dI = \frac{2}{3}(dm)R^2$$

If mass of shell is M then $I = \frac{2}{3}MR^2$

Ans.

1.143



Method : 1 (Force equation on each)

mg-T = ma ————— (i)

$$TR = \frac{MR^2}{2}\alpha \text{ ....(ii)}$$

$a = R\alpha$ ....(iii)

From (i), (ii) and (iii)

$$\alpha = \frac{mg}{\left(m + \frac{M}{2}\right)R}$$

$$W = W_0 + \alpha t = \frac{gt}{\left(1 + \frac{M}{2m}\right)R} \qquad \textbf{Ans.}$$

$$a = R\alpha = \frac{g}{\left(1 + \frac{M}{2m}\right)} \text{ ....(1)}$$

Work done of weight (mg) = Gain in K.E.

Work = mgs

$$\Delta W = mg\left(\frac{1}{2}at^2\right)$$

From (1)

$$\Delta W = m\frac{t^2}{2}\left(\frac{g^2}{\left(1+\frac{M}{2m}\right)}\right) \quad \textbf{Ans.}$$

Method : 2 (Force on system)

Torque equation about centre of cylinder on whole system

$$mgR = \left(\frac{MR^2}{2}+mR^2\right)\alpha$$

$$\alpha = \frac{g}{\left(1+\frac{M}{2m}\right)R}$$

$$W = W_0 + \alpha t = \frac{gt}{\left(1+\frac{M}{2m}\right)R} \quad \textbf{Ans.}$$

$$a = R\alpha = \frac{g}{\left(1+\frac{M}{2m}\right)} \text{......................(1)}$$

Work done of weight (mg) = Gain in K.E.

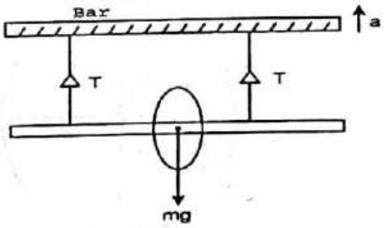
Work = mgs

$$\Delta W = mg\left(\frac{1}{2}at^2\right)$$

From (1)

$$\Delta W = m\frac{t^2}{2}\left(\frac{g^2}{\left(1+\frac{M}{2m}\right)}\right) \quad \textbf{Ans.}$$

1.244



In maxwell disc , bar is pull as acceleration of centre of disc is zero.

$F_{net} = 0$

$2T = mg$

$T = mg/2$

Torque about centre of disc

$2TR = I\alpha$

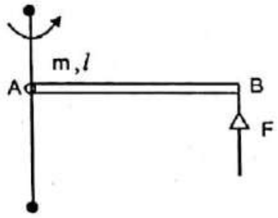
$mgR = I\alpha$

$$\alpha = \frac{mgR}{I}.$$

Acceleration of bar = $R\alpha$

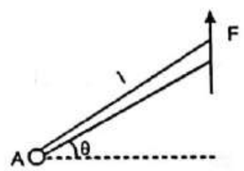
$$a = \frac{mgR^2}{I} \quad \textbf{Ans.}$$

1.245



Suppose a time t , rod make angle θ from its original position then.

Torque about A



$Fl\cos\theta = I\alpha$

$$F l \cos\theta = \frac{ml^2}{3}\alpha$$

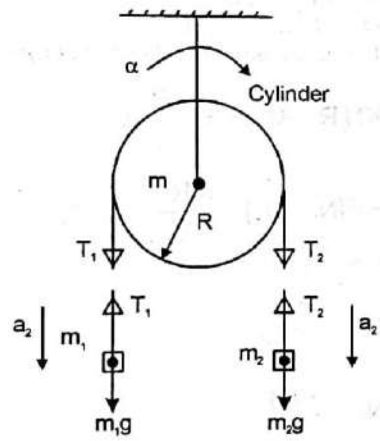
$$\alpha = \frac{3F\cos\theta}{ml}$$

$$\int_0^w w\,dw = \frac{3F}{ml}\int_0^\theta \cos\theta\, d\theta$$

$$\frac{w^2}{2} = \frac{3F}{ml}\sin\theta$$

$$w = \sqrt{\frac{6F}{ml}\sin\theta}$$ **Ans.**

1.246



Method :1 (Equation on each)

$m_2g - T_2 = m_2g_2$ ....(i)

$T_1 - m_1g = m_1g_2$ ....(ii)

$$(T_2 - T_1)R = \frac{mR^2}{2}\alpha \text{ ....(i)}$$

$a_2 = R\alpha$ ....(iv)

from (i), (ii) ,(iii) and (iv)

$$\alpha = \frac{(m_2 - m_1)g}{\left(m_2 + m_1 + \frac{m}{2}\right)R}$$

$T_2 = m_2g - m_2R\alpha$

$T_1 = m_1g + m_1R\alpha$

Dividing (2) by (1)

$$\frac{T_1}{T_2} = \frac{m_1g + m_1R\alpha}{m_2g - m_2R\alpha} = \frac{m_1(m + 4m_2)}{m_2(m + 4m_1)}$$

Ans.

Method 2: (Torque equation on system)

$$(m_2g - m_1g)R = \left(m_2R^2 + m_1R^2 + \frac{MR^2}{2}\right)\alpha$$

$$\alpha = \frac{(m_2 - m_1)g}{\left(m_2 + m_1 + \frac{m}{2}\right)R}$$

$m_2g - T_2 = m_2R\alpha$

$T_2 = m_2g - m_2R\alpha$ ....(1)

$T_1 - m_1g = m_1R\alpha$

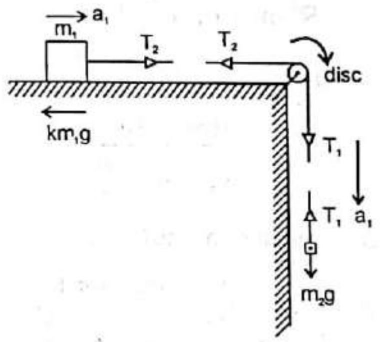
$T_1 = m_1g + m_1R\alpha$ ....(2)

Dividing (2) by (1)

$$\frac{T_1}{T_2} = \frac{m_1g + m_1R\alpha}{m_2g - m_2R\alpha} = \frac{m_1(m + 4m_2)}{m_2(m + 4m_1)}$$

Ans.

1.247



Method:1 (Force Equation on each)

$m_2g - T_1 = m_2a_1$ ....(i)

$T_2 - km_1g = m_1a_1$ ....(ii)

Torque equation on disc

$$(T_1 - T_2)R = \frac{mR^2}{2}\alpha \text{ ....(iii)}$$

$a_1 = R\alpha$ ....(iv)

From (i) , (ii) , (iii) and (iv) :

$$a_1 = \frac{g(m_2 - km_1)}{m_2 + m_1 + \frac{m}{2}}$$

Displacement in time t

$$S = \frac{1}{2}a_1t^2 = \frac{1}{2}\frac{g(m_2 - km_1)}{\left(m_2 + m_1 + \frac{m}{2}\right)}t^2$$

Workdone of friction =

$$km_1gS = \frac{1}{2}\frac{g^2km_1(m_2 - km_1)}{\left(m_2 + m_1 + \frac{m}{2}\right)}t^2$$

$$\Delta w = \left(\frac{km_1g^2(m_2 - km_1)t^2}{m + 2(m_1 + m_2)}\right) \qquad \textbf{Ans.}$$

Method 2: (Torque equation on system)

Torque equation on whole system about centre of pulley

$$(m_2g - km_1g)R = \left(m_2R^2 + m_1R^2 + \frac{mR^2}{2}\right)\alpha$$

$$\alpha = \frac{g(m_2 - km_1)}{R\left(m_2 + m_1 + \frac{m}{2}\right)}$$

Acceleration of mass m_1 :

$$a_1 = R\alpha = \frac{g(m_2 - km_1)}{m_2 + m_1 + \frac{m}{2}}$$

Displacement in time t

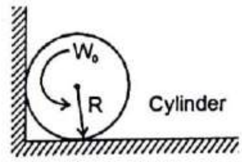
$$S = \frac{1}{2}a_1t^2 = \frac{1}{2}\frac{g(m_2 - km_1)}{\left(m_2 + m_1 + \frac{m}{2}\right)}t^2$$

Workdone of friction =

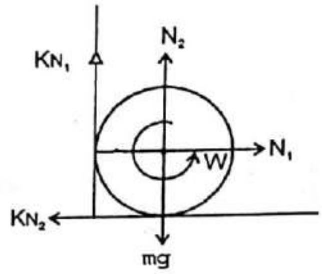
$$km_1gS = \frac{1}{2}\frac{g^2km_1(m_2 - km_1)}{\left(m_2 + m_1 + \frac{m}{2}\right)}t^2$$

$$\Delta w = \left(\frac{km_1g^2(m_2 - km_1)t^2}{m + 2(m_1 + m_2)}\right) \qquad \textbf{Ans.}$$

Q.1.248



F.B.D. of cylinder



Force equation :

$KN_1 + N_2 = mg$ ....(i)

$KN_2 = N_1$ ....(ii)

Torque equation about centre of cylinder

$$KN_1R + KN_2R = \frac{mR^2}{2}\alpha$$

$$KR[N_1 + N_2] = \frac{mR^2}{2}\alpha \quad ....(iii)$$

From (i) and (ii)

$$N_2 = \frac{mg}{1+k^2}$$

$$N_1 = \frac{kmg}{1+k^2}$$

Put in (iii)

$$\alpha = \frac{2kg(1+K)}{R(1+K^2)}.$$

We know

$W^2 = W_0^2 - 2\alpha\theta$

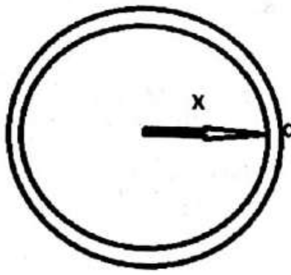
$0 = w_0^2 - 2\alpha\theta$

$$\theta = \frac{w_0^2R(1+K^2)}{4kg(1+K)}$$

No. of turns

$$N = \frac{\theta}{2\pi} = \frac{w_0^2R(1+K^2)}{8k\pi g(1+K)} \qquad \textbf{Ans.}$$

Q.1.249.



Calculation of torque

Where df = friction force on differential element

df = k [dN]

$$= k\left(\frac{m}{\pi R^2}2\pi x dx\right)g = \frac{2kmgxdx}{R^2}$$

The torque due to differential friction force

$$d\tau = xdf = \frac{2kmgx^2}{R^2}dx$$

$$\tau = \int d\tau = \int_0^R \frac{2kmgx^2dx}{R^2} = \frac{2}{3}kmgR$$

$$\tau = I\alpha$$

$$\frac{2}{3}kmgR = \frac{mR^2}{2}\alpha$$

$$\alpha = \frac{4}{3}\frac{kg}{R}$$

we know $W = W_0 + at$

$$0 = W - \frac{4}{3}\frac{kg}{R}t$$

$$t = \frac{3WR}{4kg}$$ **Ans.**

Q.1.250

$\tau\ \alpha - W^{1/2}$

$\tau = -cW^{1/2}$

$$I\alpha = -cW^{\frac{1}{2}}$$

$$\alpha = -\frac{c}{I}W^{\frac{1}{2}}$$

$$\alpha = -kW^{\frac{1}{2}}$$

Time calculation before comes in rest

$$\int_{w_0}^{0}\frac{dw}{w^{1/2}} = -\int_0^{t_1}kdt$$

$$t_1 = \frac{2w_0^{1/2}}{k}$$

Rotation angle calculation before comes in rest

$$wdw = -kw^{1/2}d\theta$$

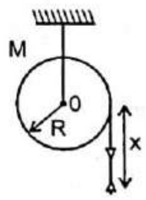
$$\int_{w_0}^{0} w^{1/2}dw = -\int_0^{0} kd\theta$$

$$\theta = \frac{2W_0^{1/2}}{3k}.$$

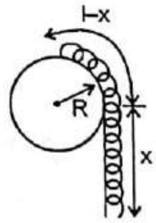
Mean angular velocity

$$\langle W\rangle = \frac{\theta}{t_1} = \frac{2W_0^{3/2}}{3k\left(\frac{2W_0^{1/2}}{k}\right)} = \frac{w_0}{3}$$ **Ans.**

Q.1.251



Moment of inertia of chord



Pulling force = $\frac{mgx}{l}$

$I = mR^2$

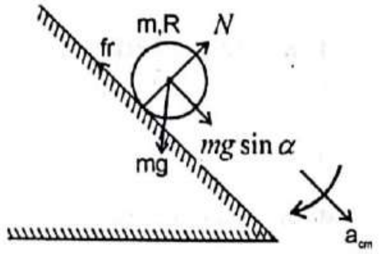
Toque about O

$\tau = I\,\alpha$

$$\frac{mgx}{l}R = \left(\frac{MR^2}{2} + mR\right)\alpha$$

$$\alpha = \frac{2mgx}{Rl(M+2m)}$$ **Ans.**

1.252



Method: 1 (General Approach)

Force equation along incline

$mg\sin\alpha - f_r = ma_{cm}$ ....(1)

Torque equation about COM

$Rf_r = I\alpha$ ....(2)

No slipping condition

$a_{cm} = R\alpha$ ....(3)

From (1), (2) and (3)

`$fr = \frac{10}{35}mg\sin\alpha$ ....(4)

At slipping

$fr = K\,mg\cos\alpha$ ....(5)

From (4) and (5)

$$kmg\cos\alpha = \frac{10}{35}mg\sin\alpha$$

$$K = \frac{10}{35}\tan\alpha$$

$$K = \frac{2}{7}\tan\alpha$$ **Ans.**

(b)

$$K.E. = \frac{1}{2}I_{IAR}w^2$$

$$K.E. = \frac{1}{2}\left(\frac{7}{5}MR^2\right)(\alpha t)^2$$

$$K.E. = \frac{7}{10}MR^2t^2\frac{25g^2\sin^2\alpha}{7\times7R^2}$$

$$= \frac{5mg^2\sin^2\alpha}{14}t^2$$ **Ans.**

Method: 2 (With the help IAR)

(a)

Torque equation about IAR

$$(mg\sin\alpha)R = \frac{7}{5}mR^2\alpha$$

$\alpha = \frac{5g\sin\alpha}{7R}$ ....(1)

Torque about com

$$Rfr = \frac{2}{5}mR^2\alpha$$

From (1)

$$fr = \frac{2}{5}mR\frac{5g\sin\alpha}{7R}$$

$fr = \frac{10}{35}mg\sin\alpha$ ....(2)

At slipping

$fr = K\,mg\cos\alpha$ ....(3)

From (2) and (3)

$$kmg\cos\alpha = \frac{10}{35}mg\sin\alpha$$

$$K = \frac{10}{35}\tan\alpha$$

$$K = \frac{2}{7}\tan\alpha$$ **Ans.**

(b)

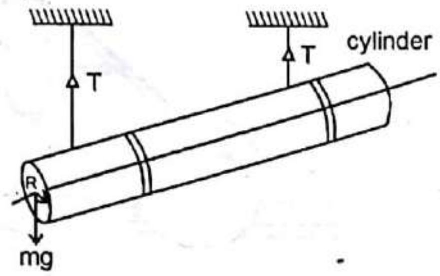
$$K.E. = \frac{1}{2}I_{IAR}w^2$$

$$K.E. = \frac{1}{2}\left(\frac{7}{5}MR^2\right)(\alpha t)^2$$

$$K.E. = \frac{7}{10}MR^2t^2\frac{25g^2\sin^2\alpha}{7\times 7R^2}$$

$$= \frac{5mg^2\sin^2\alpha}{14}t^2 \qquad \textbf{Ans.}$$

1.253



Method: 1 (General Approach)

Force equation

$mg - T = ma_{cm}$ ....(1)

Torque equation about COM

$2TR = I\alpha$ ....(2)

No slipping condition

$a_{cm} = R\alpha$ ....(3)

From (1), (2) and (3)

$$a_{cm} = \frac{2g}{3}$$

$$T = \frac{mg}{6} \qquad \textbf{Ans.}$$

(b)

Power of gravitational force

P = F.v

= mg[a_{cm}t]

$$= mg\left(\frac{2g}{3}t\right)$$

$$P = \frac{2}{3}mg^2t \qquad \textbf{Ans.}$$

Method: 2 (With the help IAR)

(a)

Torque equation of IAR which is attached with of string

$$mgR = \frac{3}{2}mR^2\alpha$$

$$\alpha = \frac{2g}{3R} \qquad \textbf{Ans.}$$

$$a_{cm} = R\alpha = \frac{2g}{3}$$

Force equation on cylinder

mg - 2T=ma

$$mg - 2T = \frac{m2g}{3}$$

$$T = \frac{mg}{6} \qquad \textbf{Ans.}$$

(b)

Power of gravitational force

P = F.v

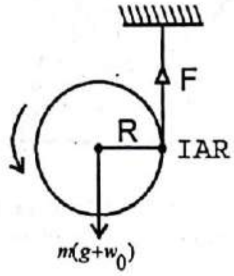
= mg[a_{cm}t]

$$= mg\left(\frac{2g}{3}t\right) = \frac{2}{3}mg^2t \qquad \textbf{Ans.}$$

1.254

With the help IAR

F.B.D. of cylinder in frame of lift



Torque about IAR

$$= m(g + w_0)R = \frac{3}{2}mR^2\alpha$$

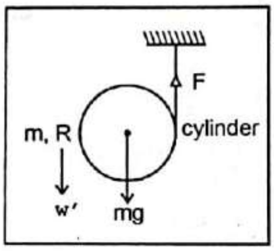
$$\alpha = \frac{2(g + w_0)}{3R}$$

$$W' = R\alpha = \frac{2}{3}(g + W_0)$$

In terms of vector

$$w' = \frac{2}{3}(\vec{g} - \vec{w}_0) \qquad \textbf{Ans.}$$

Torque about com



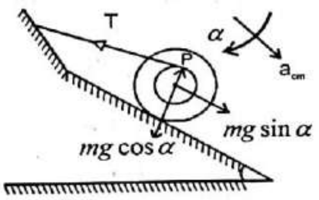
$$FR = I\alpha = \frac{mR^2}{2}\left(\frac{2}{3R}\right)(g + w_0)$$

$$F = \frac{m(g + w_0)}{3}$$

In vector form

$$F = \frac{m(\vec{g} - \vec{w}_0)}{3}$$ **Ans.**

Q.1.255



Method:1(General Approach)

Force equation along incline

$mg\sin\alpha - T = ma_{cm}$ ....(1)

Torque about CM

$Tr = I\alpha$ ....(2)

No slipping condition

$a_{cm} = r\alpha$ ....(3)

From (1) ,(2), (3)

$$a_{cm} = \frac{mgr^2\sin\alpha}{I + mr^2}$$ **Ans.**

Method:2 (With help of IAR)

Torque about IAR point P

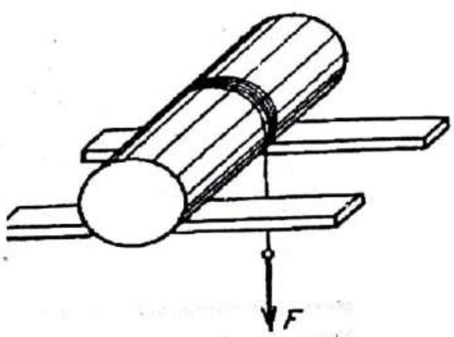
$mg\sin\alpha\ r = (I + mr^2)\alpha$

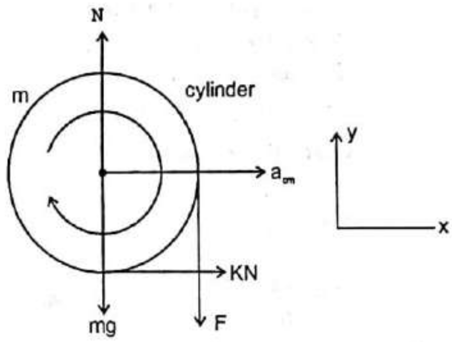
$$\alpha = \frac{mgr\sin\alpha}{I + mr^2}$$

$a_{cm} = r\alpha$

$$a_{cm} = \frac{mgr^2\sin\alpha}{I + mr^2}$$ **Ans.**

Q.1.256





At time of slipping friction will act at maximum value KN then

Force equation in y direction

N = F+mg ....(1)

Force equation in x direction

$KN = ma_{cm}$ ....(2)

Torque equation about COM

$$FR - KNR = \left(\frac{1}{2}mR^2\right)\alpha \text{ ....(3)}$$

No slipping condition

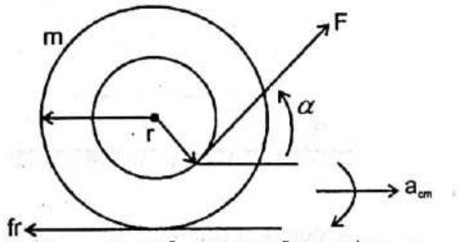
$a_{cm} = r\alpha$ ....(4)

From (1) ,(2), (3) ,(4)

$$a_{cm} = \frac{2kg}{2-3k}$$

$$F = \frac{3kmg}{2-3k}$$ **Ans.**

Q.1.257



Angular acceleration=a

(a)

Force equation along x- axis

$F \cos\alpha - fr = m\, a_{cm} =$ ....(i)

Torque equation equation about com

$$-Fr + Rfr = I a = \gamma m R^2 \frac{a_{cm}}{R}$$

$-Fr + Rfr = \gamma m R a_{cm}$ ....(ii)

From (i) and (ii)

$$a_{cm} = \frac{F(Cos\alpha - \frac{r}{R})}{m(\gamma+1)}$$ **Ans.**

(b)

Work done by force F is equal to change in kinetic energy:

Angular velocity at time t:

$$\omega = \omega_0 + \alpha t$$

$$\omega_0 = 0$$

$$\alpha = \frac{a_{cm}}{R}$$

$$\alpha = \frac{F(Cos\alpha - \frac{r}{R})}{mR(\gamma-1)}$$

$$\omega = \frac{F(Cos\alpha - \frac{r}{R})}{mR(\gamma+1)} t$$

$\Delta W = K_f - K_i = K_f - 0 = K_f$ ....(iii)

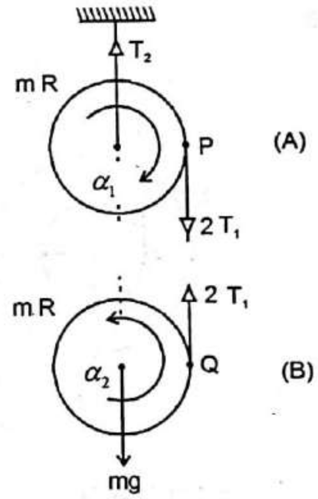
$$K_f = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} m V_{cm}^2$$

$$K_f = \frac{1}{2}\gamma m R^2 \omega^2 + \frac{1}{2} m R \omega^2 = \frac{1}{2} m R \omega^2 (1+\gamma)$$

$$K_f = \frac{1}{2} m(1+\gamma)\left(\frac{F(Cos\alpha - \frac{r}{R})}{mR(\gamma+1)} t\right)^2$$

$$\Delta W = \frac{F^2 t^2 \left(Cos\alpha - \frac{r}{R}\right)^2}{2m(1+\gamma)}$$ **Ans.**

1.258



Torque equation on sphere (A)

$$2T_1 R = \frac{mR^2}{2}\alpha_1$$

$$2T_1 = \frac{mR\alpha_1}{2} \quad ....(i)$$

Torque equation on sphere (B) :

$$2T_1R = \frac{mR^2}{2}\alpha_2$$

$$2T_1 = \frac{mR\alpha_2}{2} \quad ....(ii)$$

From (i) and (ii)

$\alpha_1 = \alpha_2$.

Force equation on (B)

$mg - 2T_1 = ma_B$ ....(iii)

Constraints relation

Acceleratin of point P

$R\alpha_1 = a_B - R\alpha_2$.

$a_B = R\alpha_1 + Ra_2 = 2Ra_1$

Put in (iii) :

$mg - 2T_1 = m2Ra_1 = 2mR\alpha_1$ .... (iv)

From (i)

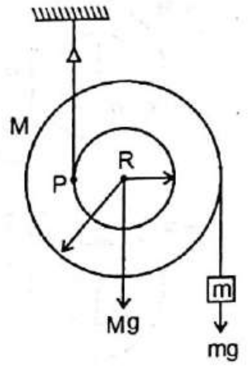
$mR\alpha_1 = 4T_1$

Put in (iv)

$mg - 2T_1 = 8T_1$

$$T_1 = \frac{mg}{10}$$ **Ans.**

1.259



Troque about point P for system

$$MgR + mg(2R + R) = \left[m(3R)^2 + (I + MR^2)\right]\alpha$$

$$\alpha = \frac{3mgR + MgR}{9mR^2 + I + MR^2} = \frac{gR(M + 3m)}{9mR^2 + I + MR^2}$$

Hence acceleation of particle

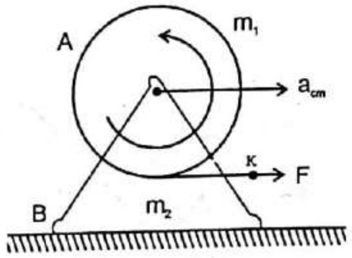
$$a_A = 3R\alpha$$

$$a_A = \frac{3R^2g(M + 3m)}{9mR^2 + I + MR^2}$$

$$a_A = \frac{3g(M + 3m)}{M + 9m + \frac{I}{R^2}}$$ **Ans.**

1.260

(a)



Soppose acceleration of mount is a_{cm}

Then

Force equation on system

$F = (m_1 + m_2)\, a_{cm}$

$$a_{cm} = \frac{F}{m_1 + m_2}$$

Torque equation on sphere about it centre

$$FR = \frac{m_1R^2}{2}\alpha$$

$$\alpha = \frac{2F}{m_1R}$$

Acculeration of point K or pulling agent

$$a_k = a_{cm} + R\alpha = \frac{F}{m_1 + m_2} + \frac{2F}{m_1}$$

$$a_k = \frac{F(3m_1 + 2m_2)}{m_1(m_1 + m_2)}$$ **Ans.**

(b)

W_F = Work done by F

= Grain in K.E.

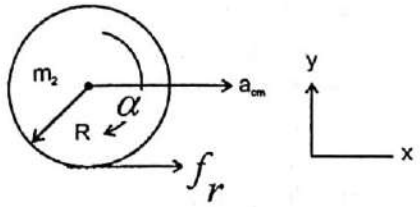
$$K.E. = F\left(\frac{1}{2}a_k t^2\right)$$

$$K.E. = \frac{1}{2}F^2\frac{(3m_1 + 2m_2)t^2}{m_1(m_1 + m_2)}$$

Ans.

1.261

Method: 1 (Basic Approach)



Suppose acceleration of sphere is a_{cm} and angular acceleration is α

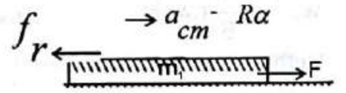
Force equation

$$f_r = m_2 a_{cm} \text{.............(1)}$$

Torque equation

$$-fR = \frac{2}{5} m_2 R^2 \alpha \text{.................(2)}$$

Negative sign because torque is opposite of angular acceleration in diagram.



Acceleration of plank (w)= $a_{cm} - R\alpha$

Force equation

$$F - f_r = m_1 \left(a_{cm} - R\alpha\right) \text{...........(3)}$$

From (1) ,(2) and (3)

$$a_{cm} = \frac{F}{\left(m_1 + \frac{2}{7} m_2\right)} \qquad \textbf{Ans.}$$

Acceleration of plank

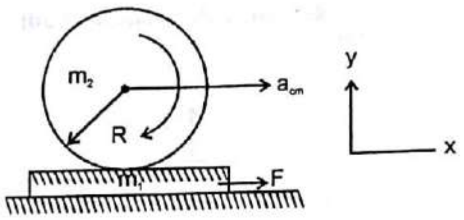
$w = a_{cm} - R\alpha$ ....(4)

From (1) ,(2) ,(3) and (4)

$$w = \frac{2}{7} \frac{F}{m_1 + \frac{2}{7} m_2}$$

$$w = \frac{2}{7} a_{cm} \qquad \textbf{Ans.}$$

Method: 2 (On system)



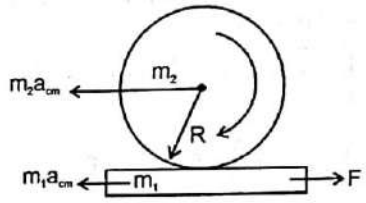
Suppose acceleration of sphere is a_{cm} and angular acceleration is α .

Acceleration of plank (w)= $a_{cm} - R\alpha$

force equation on system

(From ground frame)

$F = m_2 a_{cm} + m_1 (a_{cm} - R\alpha)$ ....(i)



Torque equation on system with frame attached with centre of sphere which is noninertial frame

$$m_1 a_{cm} R - FR = \left(m_1 R^2 + \frac{2}{5} m_1 R^2\right)\alpha$$

..........(ii)

From (i) and (ii)

$$a_{cm} = \frac{F}{\left(m_1 + \frac{2}{7} m_2\right)} \qquad \textbf{Ans.}$$

Acceleration of plank

$w = a_{cm} - R\alpha$ ....(iii)

Put a_{cm} in (i) and calculate α and put in ....(iii)

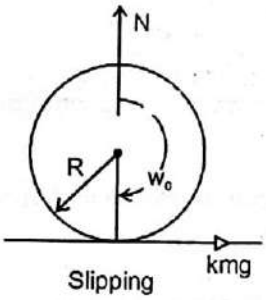
$$w = \frac{2}{7} \frac{F}{m_1 + \frac{2}{7} m_2}$$

$$w = \frac{2}{7} a_{cm} \qquad \textbf{Ans.}$$

1.262

Method :2 (Alternate Method)

(a)



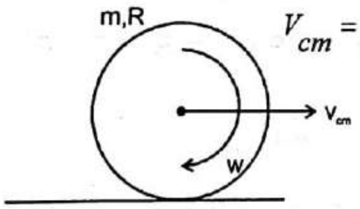
Torque about bottom point is zero. Hence angular momentom will be conserved about bottom point then

$\vec{L} = \vec{L}_{cm} + m\vec{R} \times \vec{V}_{cm}$ ....(1)

From (1)

$\left|\vec{L}_{initial}\right| = \left(\frac{1}{2}mR^2\right)w_0 + 0$ ....(2)

At time of pure rolling



From (2)

$\left|\vec{L}_{final}\right| = \frac{1}{2}mR^2 + mR(RW) = \frac{3}{2}mR^2 w$

............(ii)

Using angular momentum conservation

$L_{initial} = L_{final}$

From (i) and (ii)

$\frac{1}{2}mR^2 w_0 = \frac{3}{2}mR^2 w \Rightarrow w = \frac{w_0}{3}$

Torque equation about com :

$\text{kmgR} = \left(\frac{mR^2}{2}\right)\alpha$

$\alpha = \frac{2g}{kR}$ ....(iii)

We know that

$w = w_0 + \alpha t$ ....(iv)

Final angular velocity(w)

$w = \frac{w_0}{3}$

From (iii) and (iv)

$\frac{w_0}{3} = w_0 - \frac{2g}{kR}t$

$t = \frac{1}{3}\frac{w_0 R}{kg}$ **Ans.**

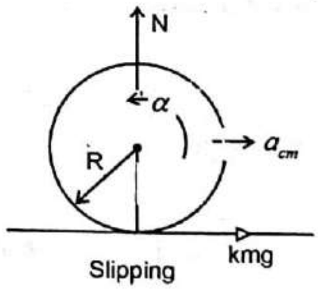
(b)

$w_{fr} = k_f - k_i$

$= \frac{1}{2}\left(\frac{3}{2}mR^2\right)w^2 - \frac{1}{2}m(RW_0)^2$

$W_{fr} = -\frac{1}{6}mw_0^2R^2$ **Ans.**

Method :1 (Alternate Method)



Force equation

$kmg = ma_{cm}$

$a_{cm} = kg$ ....(1)

Torque equation about centre

$kmgR = \frac{1}{2}mR^2\alpha$

$\alpha = \frac{2kg}{R}$ ....(2)

Using kinematics equations

$w = w_0 - \alpha t$

$$w = w_0 - \frac{2kg}{R}t \quad ....(3)$$

$V = a_{cm}t$

$V = kgt \quad ....(4)$

For pure rolling we know

$V = Rw$

From (3) and (4)

$$w = \frac{w_0}{3}$$

$$t = \frac{1}{3}\frac{w_0 R}{kg} \qquad \textbf{Ans.}$$

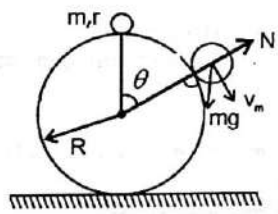
(b)

$w_{fr} = k_f - k_i$

$$= \frac{1}{2}\left(\frac{3}{2}mR^2\right)w^2 - \frac{1}{2}m(RW_0)^2$$

$$w_{fr} = -\frac{1}{6}mw_0^2R^2 \qquad \textbf{Ans.}$$

1.263



Suppose at angle θ ball leave the sphere then

Using energy conservation equation

Loss of P.E. = Gain of K.E

$$mg(R+r)(1-\cos\theta) = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}w^2$$

Put

$V_{cm} = Rw$

$$V_{cm}^2 = \frac{10}{7}g(R+r)(1-\cos\theta) \quad ......(i)$$

Force equation in direction of centre of bigger sphere

$$mg\cos\theta - N = \frac{mV_{cm}^2}{(R+r)}$$

For leave contact N = 0 then

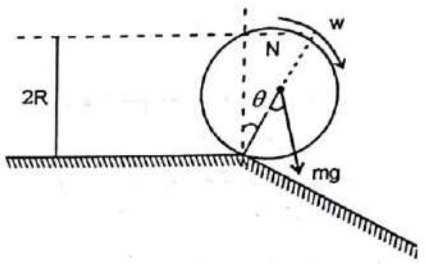
$$mg\cos\theta - 0 = \frac{mV_{cm}^2}{(R+r)}$$

$$\cos\theta = \frac{V_{cm}^2}{g(R+r)}$$

Put cosθ in (i)

$$V_{cm} = \sqrt{\frac{10}{17}g(R+r)} \qquad \textbf{Ans.}$$

1.264



Suppose at time t solid cylinder make θ angle with vertical

Using work energy equation

$$mg(R - R\cos\theta) = \frac{1}{2}\left(\frac{3}{2}mR^2\right)w^2 - \frac{1}{2}\left(\frac{3}{2}mR^2\right)\left(\frac{V_0}{R}\right)^2$$

$$\frac{3}{4}mV_0^2 = \frac{3}{4}mR^2w^2 - mgR + mgR\cos\theta \quad ......(i)$$

Force equation in direction of centre

$mg\cos\theta - N = mRw^2$

$mg\cos\theta = N + mRw^2 \quad ....(ii)$

Put mgcosθ value in equation (i)

$$\frac{3}{4}mV_0^2 = \frac{3}{4}mR^2w^2 - mgR + R(N + mRw^2)$$

$$N = \frac{3}{4R}mV_0^2 - \frac{7}{4}mRw^2 + mg \quad ....(iii)$$

For stopping bouncing , minimum value of N should be greater than 0 when W will be maximum , N will be minimum.

This condition will be come when θ = α.

Hence we have to avoid bouncing at θ = α then there will be no bouncing before θ = α.

Now for minimum V_0

Put N = 0 in equation (iii)

$$0 = \frac{3}{4R}mV_0^2 - \frac{7}{4}mRw^2 + mg$$

$$Rw^2 = \frac{3}{7R}V_0^2 + \frac{4}{7}g$$

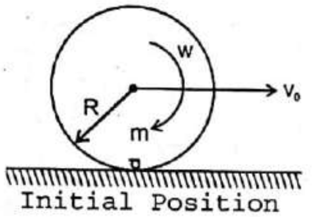
Put $\theta = \alpha$ and Rw^2 value in (i) :

$$V_0^2 = -\frac{4}{7}gR + \frac{7}{3}gR\cos\alpha$$

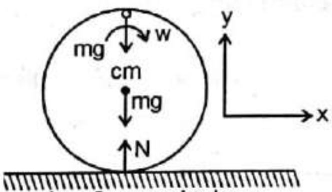
$$V_0 = \sqrt{\frac{gR}{3}(7\cos\alpha - 4)}$$

Ans.

1.265



Since angular velocity (w) is continuously decreasing then there is chance of minimum value of normal reaction at ground when particle is at maximum height or top of hoop.



Acceleration of centre in y direction will be zero.
Force equation in y direction on system w.r.t. centre of hoom.

$2mg - N = mRW^2$

Where w = final angular velocity
V=final velocity of centre

At leaving ground surface

$N = 0$

$2mg = mRW^2$

$$w^2 = \frac{2g}{R} \qquad ....(i)$$

Using energy equation

$$\frac{1}{2}mV_0^2 + \frac{1}{2}Iw_0^2$$

$$= mg(2R) + \frac{1}{2}mV^2 + \frac{1}{2}Iw^2 + \frac{1}{2}m(2V)^2$$

$$\frac{1}{2}mV_0^2 + \frac{1}{2}mR^2\left(\frac{V_0}{R}\right)^2$$

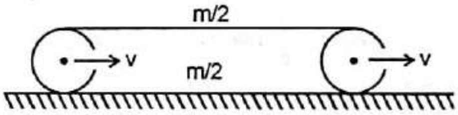
$$= mg(2R) + \frac{1}{2}m(Rw)^2 + \frac{1}{2}mR^2w^2 + \frac{1}{2}m(2Rw)^2$$

$$\frac{2mV_0^2}{2} = 2mgR + \frac{1}{2}(6mR^2)\frac{2g}{R}$$

$$V_0 = \sqrt{8gR} \qquad \textbf{Ans.}$$

1.266

Method:1 (Most general Approach)



Mass of crawler (m)
= Mass of curve part(m_1)+mass of straight part(m_2)

$$m = m_1 + m_2 \quad ....(1)$$

Where

m_1 = Mass of curve part

m_2 = Mass of straight part(Upper and lower)

Mass of upper straight part = $\frac{m_2}{2}$

Kinetic energy of upper straight part

$$K_1 = \frac{1}{2}\frac{m_2}{2}(2V)^2 = m_2V^2 \quad ....(2)$$

Kinetic energy of lower straight part = 0 ....(3)

Kinetic energy of curve part

On both crawler, curve becomes a circular ring

Then

$$K_2 = \frac{1}{2}m_1V^2 + \frac{1}{2}I\omega^2$$

$$K_2 = \frac{1}{2}m_1V^2 + \frac{1}{2}m_1R^2\left(\frac{V}{R}\right)^2$$

$$K_2 = m_1V^2 \quad ....(4)$$

From (2), (3), (4)
Total kinetic energy

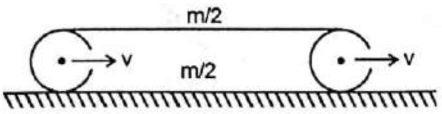
$$K = K_1 + K_2 + 0$$

$$K = (m_1 + m_2)V^2$$

From(1)

$$K = mV^2 \qquad \textbf{Ans.}$$

Method :2 (Assuming radius is very small compare to length of belt)



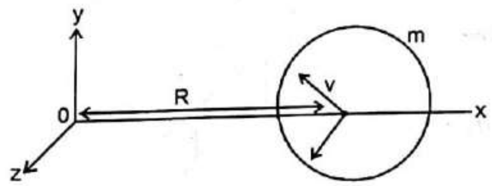
Kinetic energy of crawler is only in upper part and lower part has zero velocity hence kinetic energy of this part will be zero.
Velocity of top point
$V_p = 2V$

$$k = \frac{1}{2}\left(\frac{m}{2}\right)(2v)^2$$

$K = mV^2$ **Ans.**

Note. In case 2 , ans comes because this kinetic energy does not depend on radius of crawer. Hence second method is not appropite.

1.267



We know kinetic energy is given by

$$K = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}w^2 \text{ ....(1)}$$

If reference frame is not traslatory. And if reference frame is translatory then we know to use kinetic energy of COM.

$$V_{cm} = V \text{ ....(2)}$$

$$\vec{w} = w_x\hat{i} + w_y\hat{j}$$

$$w = \sqrt{w_x^2 + w_y^2} \text{ ....(3)}$$

$$w_x = \frac{V}{r}$$

$$w_y = \frac{V}{R}$$

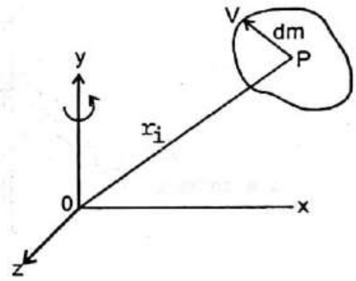
$$w = \sqrt{\left(\frac{V}{r}\right)^2 + \left(\frac{V}{R}\right)^2} \text{ .....(4)}$$

$$I_{cm} = \frac{2}{5}mR^2 \text{ ....(5)}$$

Put values from (2), (4) ,(5) in (1)

$$K = \frac{7}{10}mv^2\left(1 + \frac{2}{7}\frac{r^2}{R^2}\right) \qquad \textbf{Ans.}$$

1.268



(a)
From this rotatory reference frame , centrifugal force on particle of mass (dm) is :
$dF_{cf} = (dm)\, r_i w^2$

$$F_{cf} = \sum(dm)r_i w^2 = w^2\sum(dm)\, r_i$$

$$F_{cf} = w^2 m\left(\frac{\sum(dm)r_i}{m}\right)$$

$F_{cf} = m\, w^2 R_c$

Where $R_c = \dfrac{\sum(dm)r_i}{m}$ is position of COM.

Ans.

(b*)

Coriolis force

$$d\vec{F}_{cor} = 2dm[\vec{v} \times \vec{w}]$$

$$\vec{F}_{cor} = 2\sum (dm)\vec{v} \times \vec{w} = 2\left(\sum dm\,\vec{v}\right) \times \vec{w}$$

$$\vec{F}_{cor} = 2m\left(\frac{\sum dm \times \vec{v}}{m}\right) \times \vec{w}$$

$$\vec{F}_{cor} = 2m\vec{V}_C \times \vec{w}$$

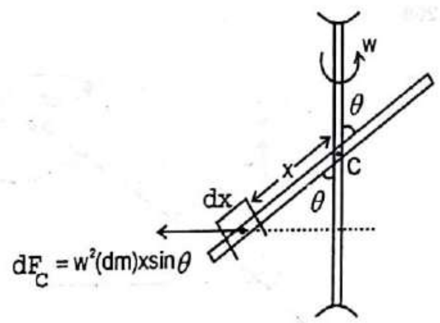
Where $\vec{V}_c = \dfrac{\sum(\text{dm}) \times \vec{v}}{\text{m}}$

$\vec{V}_c$ is velocity of COM.

Ans.

Q.1.269

Method :1 (Torque Method)



Moment of centrifugal force on differential element w.r.t. point C

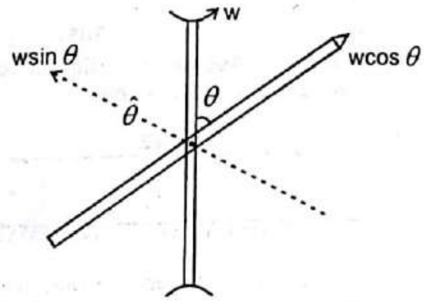
$$d\tau_c = (dF_c) \times x\cos\theta = dm\left(w^2x^2\sin\theta\cos\theta\right)$$

$$d\tau_c = \frac{m}{l}dx\left(w^2x^2\sin\theta\cos\theta\right)$$

$$\tau_c = w^2\frac{m}{l}\sin\theta\cos\theta\int_{-l/2}^{l/2} x^2dx$$

$$\tau_c = \frac{1}{24}mw^2l^2\sin 2\theta \qquad \textbf{Ans.}$$

Method : 2(Angular Momentum Method)



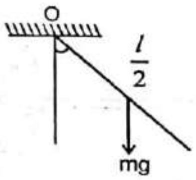
$$\vec{L} = \left(\frac{ml^2}{12}w\sin\theta\right)\hat{\theta}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{ml^2}{12}w\sin\theta\frac{d\hat{\theta}}{dt}$$

$$|\vec{\tau}| = \frac{ml^2}{12}(w\sin\theta)w\cos\theta$$

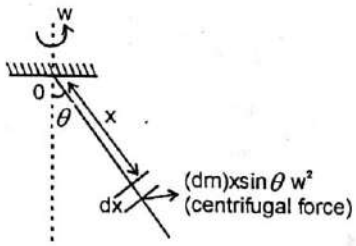
$$\tau = \frac{ml^2}{24}w^2\sin 2\theta \qquad \textbf{Ans.}$$

1.270



If we select a reference frame rotating with same angular velocity as rod , rod appears in rest. Because torque of centrifugal force will be balance by torque of gravitational force.

F.B.D. of rod:



Torque of centrifuglal force at differential elememt about O :

$$d\tau = \left[(dm)x\sin\theta w^2\right]x\cos\theta$$

$$\int d\tau = \int_0^l (dm)x\sin\theta w^2 x\cos\theta$$

$$\int d\tau = \int_0^l \frac{mdx}{l}x\sin\theta w^2 x\cos\theta$$

$$\int d\tau = \frac{m}{l}w^2\sin\theta\cos\theta\int_0^l x^2dx$$

$$\tau = \frac{l^2mw^2}{3}\sin\theta\cos\theta \quad ....(i)$$

Torque of mg about O

$$\tau_{mg} = mg\frac{l}{2}\sin\theta \quad ....(ii)$$

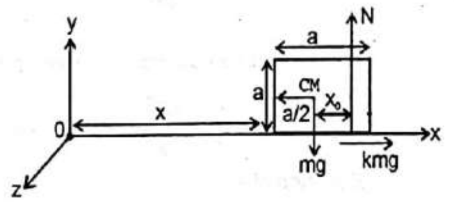
From (i) and (ii) for equilibrium

$\tau = \tau_{mg}$

$$\frac{l^2mw^2}{3}\sin\theta\cos\theta = mg\frac{l}{2}\sin\theta$$

$$\cos\theta = \frac{3g}{2w^2l}$$ **Ans.**

Q.1.271



Where k = coefficient of friction

x_0 = Perpendicular distance between line of mg and N

Force equation in y direction

N = mg

Since body is in rotational equilibrium then then torque about COM will be zero

$$Kmg\frac{a}{2} = Nx_0$$

$$Kmg\frac{a}{2} = mgx_0$$

$$x_0 = \frac{ka}{2}$$ **Ans.**

Suppose at time t cube is displaced by x distance then Torque in clockwise direction about origin is

$$\tau_0 = -N(x + x_0 + \frac{a}{2}) + mg(x + \frac{a}{2})$$

$$= -mg(x + x_0 + \frac{a}{2}) + mgx + mg\frac{a}{2}$$

$$= -mg(x_0 + \frac{a}{2}) - = mg\frac{a}{2}$$

$$= -mg\frac{ka}{2} - mg\frac{a}{2} + mg\frac{a}{2}$$

$$\tau_0 = -\frac{mgka}{2} \quad ....(1)$$

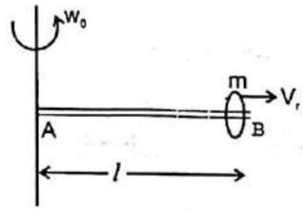
Initial angular momentum about origin in clockwise sense

$$L = mV_0\frac{l}{2} \quad ....(2)$$

Because of this torque and initial angular momentum about origin is opposite direction, angular momementum will be decreasing continuously and finall becomes zero.

Ans.

1.272



Energy conservation equation

$$\frac{1}{2}\left(\frac{Ml^2}{3}\right)w_0^2=\frac{1}{2}\frac{Ml^2}{3}w^2+\frac{1}{2}m\left[V_r^2+(lw)^2\right]$$

.............(1)

Using angular momentum conservation

$$\frac{Ml^2}{3}w_0=\frac{Ml^2}{3}w+m(lw)l$$

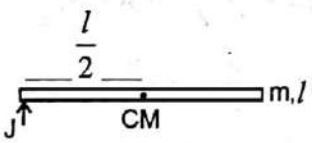
..........(2)

After eliminating ω from both equations

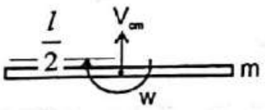
$$V_r=\frac{lw_0}{\sqrt{1+\frac{3m}{M}}}$$ **Ans.**

1.273

Before Impact:



After Impact:



Linear impluse equation

J = MV_{cm}

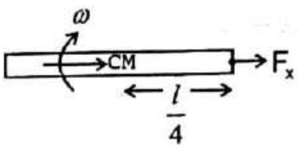
$$V_{cm}=\frac{J}{m}$$

Angular impluse equation about com

$$J\frac{l}{2}=\frac{ml^2}{12}w$$

$$w=\frac{6J}{ml}$$ ....(1)

F.B.D of half part of rod



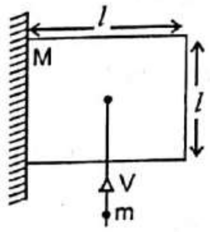
F_x = Centripital force

$$F_x=\frac{m}{2}\left(\frac{l}{4}\right)w^2=\frac{m}{2}\left(\frac{l}{4}\right)\frac{36J^2}{m^2l^2}$$

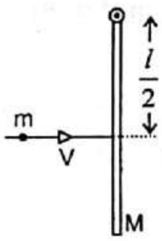
$$F_x=\frac{9J^2}{2ml^2}$$ **Ans.**

1.274

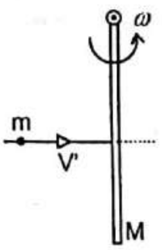
(a)



Before Impact:



After Impact:



Angular momentum conservation about hinge

$$mv\frac{l}{2}=mv'\frac{l}{2}+\frac{ml^2}{3}w$$ ....(i)

Equation of e

$$v=\frac{l}{2}w-v'$$ ....(ii)

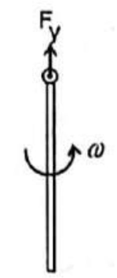
From (i) and (ii)

$$w = \frac{12mv}{l(3m+4m)}$$

$$v' = \left(\frac{3m-4M}{3m+4m}\right)V \qquad \textbf{Ans.}$$

(b)

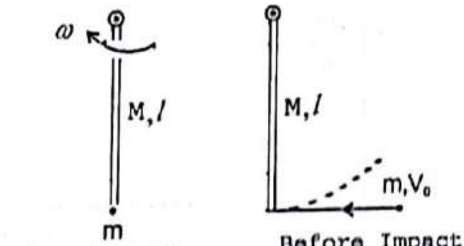
F.B.D. of rod



$$F_y = \left(\frac{Ml}{2}\right)w^2 = \frac{8MV^2}{l\left(1+\frac{4M}{3m}\right)^2} \qquad \textbf{Ans.}$$

1.275

(a)



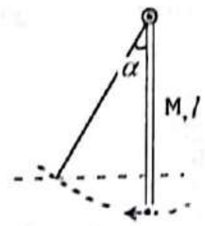
After Impact   Before Impact

Angular momentum conservation
We can neglect m compare to M:

$$mv_0 l = \frac{Ml^2}{3}w$$

$$w = \frac{3mv_0}{Ml} \quad \text{..........(i)}$$

Using energy conservation after collision
We can neglect m compare to M:



$$\frac{1}{2}\left[\frac{Ml^2}{3}w^2\right] = Mg\left(\frac{l}{2}-\frac{l}{2}\cos\alpha\right)$$

$$\frac{lw^2}{3} = g(1-\cos\alpha)$$

$$w = \sqrt{\frac{3g}{l}(1-\cos\alpha)} \quad \text{..........(ii)}$$

From (i) and (ii)

$$\frac{3mv_0}{Ml} = \sqrt{\frac{3g}{l}(1-\cos\alpha)}$$

$$v_0 = \frac{Ml}{3m}\sqrt{\frac{3g}{l}(1-\cos\alpha)}$$

$$v_0 = \frac{M}{m}\sqrt{\frac{2}{3}gl\sin^2\frac{\alpha}{2}}$$

$$v_0 = \frac{M}{m}\sqrt{\frac{2}{3}gl}\,\sin\left(\frac{\alpha}{2}\right) \qquad \textbf{Ans.}$$

(b)

$\Delta P = (m + M)\,V_{cm} - mV_0$

$= (m + M)\,\frac{l}{2}w - mV_0$

We can neglect m compare to M:

$\Delta P = M\frac{l}{2}w - mV_0$

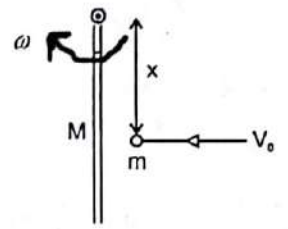
Put w and v_0 then

$$\Delta P = M\sqrt{\frac{1}{6}gl}\left(\sin\frac{\alpha}{2}\right)$$

Because hinge applied force during impact

Ans.

(c)
(Observation of motion of hinge)



For momentom of system constant, hinge force during collision will be zero .
For this purpose hinge velocity during collision will be zero.
Angular momentum conservation about hinge

$$mV_0x = \frac{Ml^2}{3}w \quad \text{..........(i)}$$

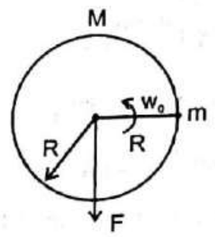
Using linear momentum conservation

$$mV_0 = m\frac{l}{2}w \quad \text{..........(ii)}$$

Divide both equations

$$x = \frac{2l}{3} \qquad \textbf{Ans.}$$

1.276
(a)



Using angular momentum conservation

$$\left(\frac{MR^2}{2} + mR^2\right)w_0 = \frac{MR^2}{2}w$$

$$w = \left(1 + \frac{2m}{M}\right)w_0 \qquad \textbf{Ans.}$$

(b)
Work done by force F = Change in K.E. of system

$$W_F = \frac{1}{2}\left(\frac{MR^2}{2}\right)w^2 - \frac{1}{2}\left(\frac{MR^2}{2} + mR^2\right)w_0^2$$

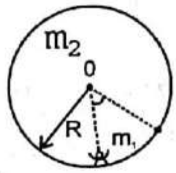
From ans (a) part

$$W_F = \frac{1}{2}\left(\frac{MR^2}{2}\right)\times\left(1 + \frac{2m}{M}\right)^2 w_0^2$$

$$-\frac{1}{2}\left(\frac{MR^2}{2} + mR^2\right)w_0^2$$

$$W_F = \frac{mR^2w_0^2}{2}\left(1 + \frac{2m}{M}\right) \qquad \textbf{Ans.}$$

1.277



(a)
Initially $\bar{L} = \bar{0}$ and no torque on system about centre of disc then $\bar{L}$ will be conseve always

$\Sigma I_1 W_1$ = constant

$$\Sigma I_1 \frac{d\theta}{dt} = 0$$

$$\Sigma I_1 d\theta = 0$$

$$I_1\theta_1 + I_2\theta_2 = 0$$

Where θ_1 and θ_2 is angular displacement of disc and person from ground frame. Then

$$\frac{m_2R^2}{2}(-\theta_1) + m_1R^2(\varphi' - \theta_1) = 0$$

$$\theta_1 = \frac{m_1\varphi'}{\frac{m_2}{2} + m_1} = \frac{2m_1\varphi'}{m_2 + 2m_1}$$

Ans.

(b)
Suppose angular velocity of disc is w then
Using angular momentum conservation

$$0 = -\frac{m_2R^2}{2}w + m_1R[v'(t) - Rw]$$

$$m_1Rv'(t) = w\left(\frac{m_2R^2}{2} + m_1R^2\right)$$

$$w = \frac{2m_1v'(t)}{(m_2 + 2m_1)R}$$

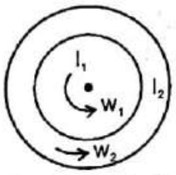
$$\alpha = \frac{dw}{dt}$$

$$\alpha = \frac{2m_1}{(m_2 + 2m_1)R}\frac{dV'(t)}{dt}$$

$$\tau = I\alpha = \frac{m_2R^2}{2}\left(\frac{2m_1}{R(2m_1 + m_2)}\right)\frac{dV'(t)}{dt}$$

$$\tau = \left(\frac{m_1m_2R}{2m_1 + m_2}\right)\frac{dV'(t)}{dt}$$ **Ans.**

1.278



(a)
Suppose final angular velocity is w. Since net torque on system is zero then
Using angular momentum conservation about centre of disc

$$I_1\vec{W}_1 + I_2\vec{W}_2 = (I_1 + I_2)\vec{w}$$

$$\vec{w} = \frac{I_1\vec{w}_1 + I_2\vec{w}_2}{I_1 + I_2}$$ **Ans.**

(b)
Work done by internal friction

$$\Delta W = \Delta K = K_f - K_i$$

$$\Delta W = \frac{1}{2}I_1W_1^2 + \frac{1}{2}I_2W_2^2 - \frac{1}{2}(I_1 + I_2)w^2$$

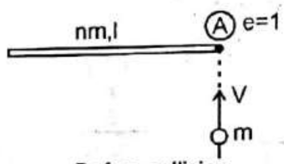
$$\Delta W = \frac{1}{2}I_1W_1^2 + \frac{1}{2}I_2W_2^2 - \frac{1}{2}(I_1 + I_2)\vec{w}.\vec{w}$$

From ans of part (a)

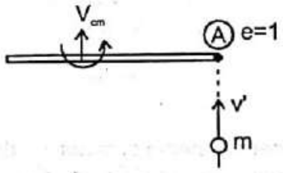
$$\Delta W = \frac{1}{2}I_1W_1^2 + \frac{1}{2}I_2W_2^2 - \frac{1}{2}(I_1 + I_2)\frac{I_1^2W_1^2 + I_2^2W_2^2 + 2I_1I_2\vec{w}_1.\vec{w}_2}{(I_1 + I_2)^2}$$

$$\Delta W = -\frac{1}{2}\frac{I_1I_2}{(I_1 + I_2)}|(\vec{w}_1.\vec{w}_2)|^2$$ **Ans.**

1.279



Before collision



After collision

Since there is no force on system , linear momentom will be conserve
Using linear momentom conservation

$mV = nm\,V_{cm} + mv'$

$V = nV_{cm} + v'$ ....(i)

Equation of restitution (e)

$$V = V_{cm} + \frac{l}{2}w - v' \quad \text{....... ....(ii)}$$

Using angular momentom conservation about point (A)

$0 + 0 = 0 + Lcm + m\vec{r} \times \vec{V}cm$

$$0 = 0 + \frac{ml^2}{12}w - \frac{ml}{2}V_{cm}$$

$$V_{cm} = \frac{lw}{6} \text{ ....(iii)}$$

From (i), (ii) and (iii)

$$v' = \frac{v(4-n)}{4+n}$$ **Ans.**

$$w = \frac{12v}{(4+n)l}$$ **Ans.**

For v' = 0

n = 4 **Ans.**

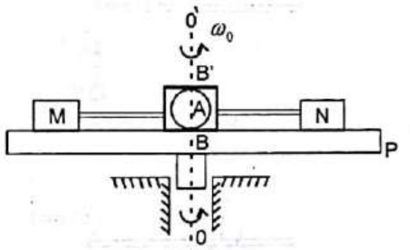
For reverse direction

v' = (–) ive

n > 4. **Ans.**

1.280

(a) 90° rotation



initial

Moment of inertia of motor + patform = I

Angular velocity of sphere = w_0

Moment of inertia of sphere = I_0

Angular momentum about 00' axis will be conserved because torque on system about 00' is zero.

Final angular velocity along 00' axis will be same for motar and sphere.

Using angular momentum conservation about 00' axis

$$0 + I_0 w_0 = (I + I_0)w$$

$$w = \frac{I_0 w_0}{I + I_0} \text{ ....(1)}$$

Work done by motar= change in kinetic energy

$$\Delta W = \left[\left(\frac{1}{2}I_0 w_0^2 + \frac{1}{2}I_0 w^2\right) + \frac{1}{2}Iw^2\right] - \left(\frac{1}{2}I_0 w_0^2\right)$$

$$\Delta W = \frac{1}{2}(I + I_0)w^2$$

From (1) , Put the value of w

$$\Delta W = \frac{1}{2}(I + I_0)\frac{I_0^2 w_0^2}{(I + I_0)^2}$$

$$\Delta W = \frac{1}{2}\frac{I_0^2 w_0^2}{(I + I_0)}$$

Work done by motor $= \frac{1}{2}\frac{I_0^2 w_0^2}{I + I_0}$ **Ans.**

180° rotation

Angular momentum conservation about 00' axis.

$0 + I_0 w_0 = IW - I_0 w_0$

$$w = \frac{2I_0 w_0}{I} \text{ ....(1)}$$

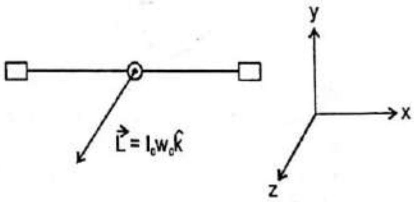
Work done by motar= change in kinetic energy

$$\Delta W = \left(\frac{1}{2}Iw^2 + \frac{1}{2}Iw_0^2\right) - \frac{1}{2}I_0 w_0^2$$

$$\Delta W = \frac{1}{2}Iw^2 = \frac{1}{2}I\left(\frac{2I_0 w_0}{I}\right)^2$$

$$\Delta W = \frac{2I_0^2 w_0^2}{I}$$ **Ans.**

(b)

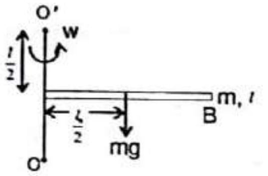


$$|\vec{\tau}| = \left|\frac{d\bar{L}}{dt}\right| = I_0 w_0 \left|\frac{d\hat{k}}{dt}\right| = I_0 w_0 w$$

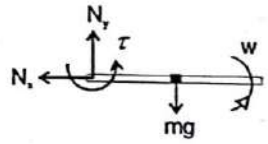
$$\tau = I_0 w_0 \frac{I_0 w_0}{I + I_0}$$

$$\tau = \frac{I_0^2 w_0^2}{I + I_0}$$ **Ans.**

1.281



F.B.D. of rod
A fixed end provide both force and torque



Force equation in x and y directions
$N_y = mg$ ....(1)

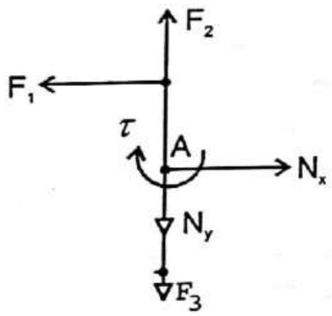
$$N_x = ma_{cm} = m\frac{l_0}{2}w^2 \text{ ....(2)}$$

Torque equation on rod about fixed point

$$\tau - mg\frac{l_0}{2} = 0$$

$$\tau = mg\frac{l_0}{2} \text{ ....(3)}$$

F.B.D of axis OO'



Force equation in x direction

$$F_1 = N_x = m\frac{l_0}{2}w^2 \text{ ....(4)}$$

Torque about point A

$$F_1\frac{l}{2} - \tau = 0$$

$$F_1 = \frac{2\tau}{l}$$

From (3)

$$F_1 = \frac{2}{l}mg\frac{l_0}{2} = mg\frac{l_0}{l} \qquad \textbf{Ans.}$$

From (4)

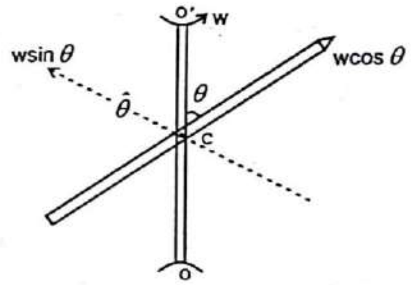
$$m\frac{l_0}{2}w^2 = mg\frac{l_0}{l}$$

$$w = \sqrt{\frac{2g}{l}} \qquad \textbf{Ans.}$$

1.282

(a)

Angular momentum w.r.t. C



$$\vec{L} = \vec{M} = \left(\frac{ml^2}{12}w\sin\theta\right)\hat{\theta}$$

$$|\vec{L}| = M = \frac{ml^2}{12}w\sin\theta \qquad \textbf{Ans.}$$

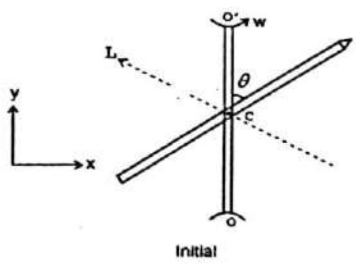
Angular momentum along axis OO'

Component of angular momentum w.r.t. point C along axis is angular momentum along axis

$$L_{oo'} = L\cos(90^0 - \theta) = L\sin\theta = M\sin\theta$$

Ans.

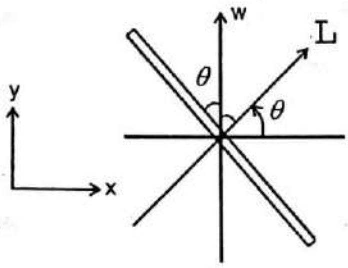
(b)



Initial

$$\vec{L}_i = L\sin\theta\hat{j} - L\cos\theta\hat{i}$$

$$\vec{L}_i = \frac{ml^2}{12}w\sin^2\theta\hat{j} - \frac{ml^2}{12}w\sin\theta\cos\theta\hat{i}$$



final

$$\vec{L}_f = L\sin\theta\hat{j} + L\cos\theta\hat{i}$$

$$\vec{L}_f = \frac{ml^2}{12}w\sin^2\theta\hat{j} + \frac{ml^2}{12}w\sin\theta\cos\theta\hat{i}$$

$$\Delta\vec{L} = \vec{L}_f - \vec{L}_i$$

$$\Delta\vec{L} = \frac{2ml^2}{12}w\sin\theta\cos\theta\hat{i}$$

$$\Delta L = \frac{ml^2}{12}w\sin 2\theta$$ **Ans.**

(c)

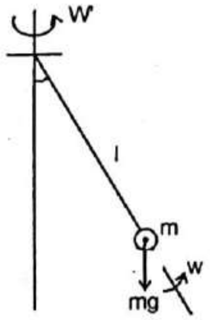
$$|\vec{\tau}| = \left|\frac{d\vec{L}}{dt}\right| = \left|\frac{dL\hat{L}}{dt}\right| = L\left|\frac{d\hat{L}}{dt}\right| = Lw\cos\theta$$

$$= \frac{ml^2}{12}w^2\sin\theta\cos\theta$$

$$\tau = \frac{ml^2w^2\sin 2\theta}{24}$$ **Ans.**

1.283*

(a)



Here w' = Angular presession velocity which is along axis.

Then gyroscopic torque $\vec{\tau} = \vec{w} \times \vec{L}$

$\tau = w'IW\sin\theta$ .....(i)

Torque of gravity about hinge

$\tau = mgl\sin\theta$ ......(ii)

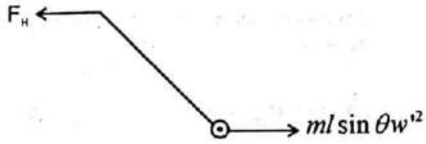
Sine we know that gyroscopic torque is provided by gravity

$mgl\sin\theta = w'Iw\sin\theta$

$$w' = \frac{mgl}{Iw}$$ **Ans.**

(b)

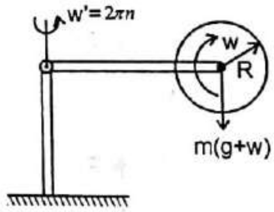
F.B.D. of rod



F_H = centrifugal force

$F_H = ml\sin\theta w'^2$ **Ans.**

1.284*



Moment of gyroscopic forces

$$\tau = \vec{w}' \times \vec{L}$$

$$\tau = 2\pi nIw' = 2\pi n\frac{mR^2}{2}w' = \pi mnR^2w'$$ ....(i)

Torque of pusedo force about origin

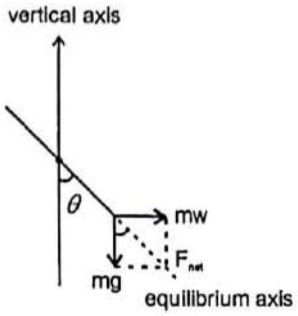
$\tau = m(g+w)\ l$ ....(ii)

From (i) and (ii)

$m(g+w)l = \pi nmR^2w'$

$$w' = \frac{(g+w)l}{\pi nR^2}$$ **Ans.**

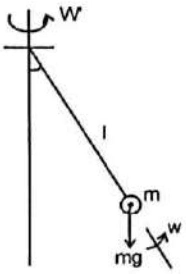
1.285*



$$\tan\theta = \frac{w}{g}$$

$$F_{net} = \sqrt{m^2g^2 + m^2w^2} = m\sqrt{g^2 + w^2}$$

Now gyroscope motion



Torque about fixed point C

Torque of gyroscope

$$\vec{\tau} = \vec{w}' \times \vec{L}$$

$$|\vec{\tau}| = w' I w \sin\theta_1 \text{ ....(i)}$$

Torque of net force

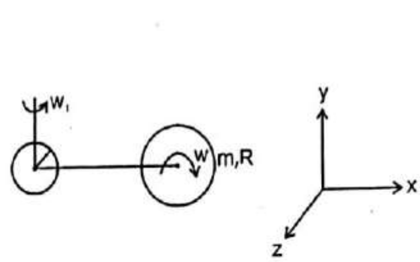
$$\tau = m\sqrt{g^2 + w^2}\, l \sin\theta_1 \text{ ....(ii)}$$

From (i) and (ii)

$$w' I w \sin\theta_1 = m\sqrt{g^2 + w^2}\, l \sin\theta_1$$

$$w' = \frac{ml\sqrt{g^2 + w^2}}{IW}$$ **Ans.**

1.286*



$$\vec{w}_1 = w_1\hat{j}$$

$$\vec{w} = w\hat{k}$$

Moment of inertia about own axis

$$I = \frac{2}{5}mR^2$$

Torque of gyroscope

$$\vec{\tau} = w' \times \vec{L}$$

$$\tau = w'\frac{2}{5}mR^2 w$$

Since bearing distance is l then

$$Fl = \frac{2}{5}mR^2 ww'$$

$$F = \frac{2mR^2 ww'}{5l}$$ **Ans.**

1.287

w' is angular preccession then

$$w' = \frac{2\pi}{T}$$

$$\phi = \phi_m \sin \omega t$$

$$\phi = \phi_m \sin\frac{2\pi}{T}t$$

$$w' = \frac{d\phi}{dt} = \phi_m \frac{2\pi}{T}\cos\frac{2\pi}{T}t \text{ ....(i)}$$

Torque of gyroscope

$$\vec{\tau} = \vec{w}' \times \vec{L}$$

$$\tau = w' \times Iw$$

From (i)

$$\tau = \left(\phi_m \frac{2\pi}{T}\cos\frac{2\pi}{T}t\right)\frac{mr^2 w}{2}$$

This is equal to bearing torque of F

$$Fl = \left(\phi_m \frac{2\pi}{T}\cos\frac{2\pi}{T}t\right)\frac{mr^2 w}{2}$$

$$F=\left(\phi_m\frac{2\pi}{T}\cos\frac{2\pi}{T}t\right)\frac{mr^2w}{2l}$$

For F maximum

$$\cos\frac{2\pi}{T}t=1$$

$$F=\frac{mr^2w\pi\phi_m}{lT}$$ **Ans.**

1.288

$$w=2\pi n$$

Angular preccession

$$w'=\frac{v}{R}$$

Torque of gyroscope

$$\vec{\tau}=\vec{w}'\times\vec{L}$$

$$\tau=\frac{v}{R}Iw$$

$$\tau=\frac{v}{R}I2\pi n$$

$$\tau=2\pi nI\frac{v}{R}$$

Ans.

1.289

$$w=2\pi n$$

Angular preccession

$$w'=\frac{v}{R}$$

Direction of w is in negative y direction while direction of w' is in x direction

Torque of gyroscope

$$\vec{\tau}=\vec{w}'\times\vec{L}$$

$$\tau=\frac{v}{R}Iw$$

$$\tau=\frac{v}{R}I2\pi n$$

$$\tau=2\pi nI\frac{v}{R}$$

Let gyroscopic force is F

$$Fl=2\pi nI\frac{v}{R}$$

$$F=2\pi nI\frac{v}{Rl}$$ **Ans.**

1.7 Elastic Deformations of Solid body

1.290

Initial length of rod = l_0

Thermal expansion cofficient of rod = α

Suppose there is no external pressure then due to tempreture increase , increase in length of rod

$$\Delta l = l_0 \alpha \Delta T \quad ....(1)$$

When extra pressure (ΔP) is applied on rod then decrease in length will be same as above sothat length become constant.

Using Hooks'law

$$E = \frac{\text{stress}}{\text{strain}}$$

Where E = young's modules

$$E = \frac{F/A}{\frac{\Delta l}{l}} = \frac{\Delta P}{\frac{\Delta l}{l}}$$

$$\Delta P = E\frac{\Delta l}{l}$$

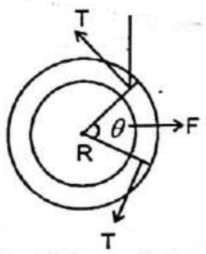
From (1)

$$\Delta P = E\alpha\Delta T$$ **Ans.**

1.291

(a)

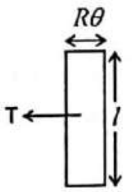
Top view



θ = is very very small

Suppose internal presure is P and length of cyl inder is l.

We select a differential element of very small angle θ.



Then force due to this pressure on this element

F = P(area) = P (Rθ) l ....(1)

This force is balanced by tensile force T

$$2T\sin\frac{\theta}{2} = F$$

Since θ is very very small

$$\sin\theta \cong \theta$$

$$T\theta = F \quad ....(2)$$

$$T\theta = RP\theta l$$

$$T = PRl$$

If breaking strength is σ_m then

$$\sigma_m = \frac{T}{l\Delta r}$$

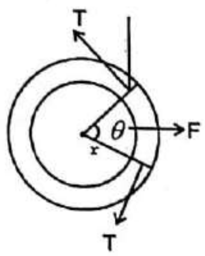
From (2)

$$\sigma_m = \frac{PRl}{l\Delta r}$$

$$\sigma_m = \frac{PR}{\Delta r}$$

$$P = \frac{\sigma_m \Delta r}{R} = \frac{\sigma_m \Delta r}{r}$$ **Ans.**

(b)



Take a cone of very small apex angle θ

Suppose internal presure is P.

Then force due to this pressure on this element

F = P(area)

$$F = P \times \pi\left[r\sin\frac{\theta}{2}\right]^2 \quad ....(1)$$

Since θ is very very small

$\sin\theta \cong \theta$

$$F = \frac{P\pi r^2\theta^2}{4} \quad \text{.............(1)}$$

This force is balanced by tensile force T

$$T\sin\frac{\theta}{2} = F$$

Since θ is very very small

$$T\frac{\theta}{2} = F \quad \text{................(2)}$$

From (1) and (2)

$$\frac{T\theta}{2} = \frac{P\pi r^2\theta^2}{4}$$

$$T = \frac{P\pi r^2\theta}{2} \quad \text{............(3)}$$

We know that

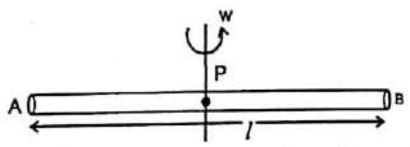
$$\sigma_m = \frac{T}{\left(2\pi r\sin\frac{\theta}{2}\right)\Delta r}$$

From (3)

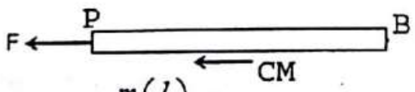
$$\sigma_m = \frac{\frac{P\pi r^2\theta}{2}}{\left(2\pi r\sin\frac{\theta}{2}\right)\Delta r} = \frac{Pr^2\theta}{2r\theta\Delta r}$$

$$P = \sigma_m\frac{2\Delta r}{r}$$ **Ans.**

1.292



Rod ruptures where pressure is maximum and it is a point on axis then this point is P
To find force at point P
F.B.D. of half of rod



$$\frac{m}{2}\left(\frac{l}{4}\right)w^2$$

Contripital force

$$\frac{m}{2}\left(\frac{l}{4}\right)w^2 = \left(\rho\frac{l}{2}A\right)\left(\frac{l}{4}\right)w^2 = \frac{\rho A l^2 w^2}{8}$$

Force equation on rod

$$F = \frac{\rho A l^2 w^2}{8}$$

$$Stress(\sigma_m) = \frac{F}{A} = \frac{\rho l^2 w^2}{8}$$

$$\frac{\rho l^2 w^2}{8} = \sigma_m$$

$$w = \frac{2\sqrt{\frac{2\sigma_m}{\rho}}}{l}$$

$$2\pi n = \frac{2\sqrt{\frac{2\sigma_m}{\rho}}}{l}$$

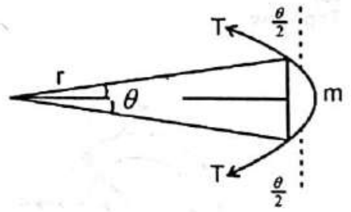
$$n = \frac{\sqrt{\frac{2\sigma_m}{\rho}}}{\pi l}$$

Where σ_m = Braking strength of rod
ρ = volume density of rod

Ans.

1.293

F.B.D of arch making θ angle at centre



Here θ is very-very small
dm = mass of differential element
= ρ volume = ρ(A) (rθ)
Where A = area of cross section of wire
Force equation toward centre

$$2T\sin\frac{\theta}{2} = (dm)rw^2$$

Since θ is very very small

$\sin\theta \cong \theta$

$$2T\frac{\theta}{2} = (\rho A r\theta)w^2 r$$

$T = \rho A r^2 w^2$

$$w = \sqrt{\frac{T}{A\rho r^2}}$$

$$w = \sqrt{\frac{1}{\rho r^2}\left(\frac{T}{A}\right)}$$

$$w = \sqrt{\frac{\sigma}{\rho r^2}}$$

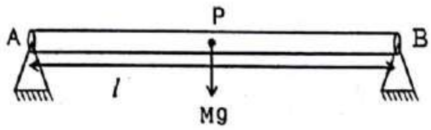
$$2\pi n = \sqrt{\frac{\sigma}{\rho r^2}}$$

$$n = \frac{1}{2\pi}\sqrt{\frac{\sigma}{\rho r^2}} = \frac{1}{2\pi r}\sqrt{\frac{\sigma}{\rho}}$$

Where σ = Breaking strength of wire

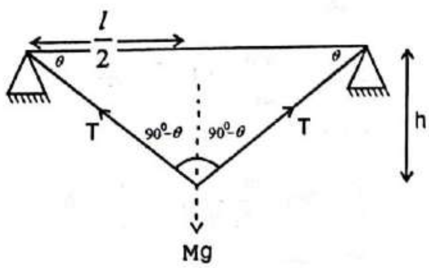
Ans.

1.294



$$R = \frac{d}{2}$$

Where R = radius of wire
d= Diameter of wire
Suppose mass of wire is M
F.B.D. of rod



2T sin θ = Mg ....(1)

Again we know
stress/strain = young moldules(E)
Elongation in rod

$$\Delta l = 2\left(\frac{l}{2}\sec\theta - \frac{l}{2}\right)$$

$$\Delta l = l\left(\sec\theta - 1\right) \text{.................(2)}$$

Again we know
stress/strain = young moldules(E)

$$\frac{\dfrac{T}{\dfrac{\pi d^2}{4}}}{\dfrac{\Delta l}{l}} = E$$

From (2)

$$\frac{\dfrac{T}{\dfrac{\pi d^2}{4}}}{\dfrac{l(\sec\theta - 1)}{l}} = E$$

$$T = \frac{E\pi d^2}{4}(\sec\theta - 1) \text{ .... (3)}$$

From (2) and (3)

$$\frac{Mg}{2\sin\theta} = \frac{E\pi d^2}{4}\left[\frac{1}{\cos\theta} - 1\right]$$

$$M = \frac{E\pi d^2}{2g}\left[\tan\theta - \sin\theta\right] \text{ ....(4)}$$

Here

$$\tan\theta = \frac{2h}{l}$$

$$\sin\theta = \frac{2h}{\sqrt{l^2 + 4h^2}} = \frac{2h}{l\sqrt{1 + \frac{4h^2}{l^2}}}$$

$$\sin\theta = \frac{2h}{l}\left(1 + \frac{4h^2}{l^2}\right)^{-\frac{1}{2}}$$

Using bionomial expression

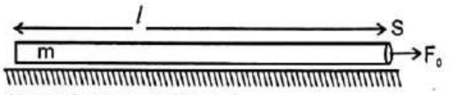
$$\sin\theta = \frac{2h}{l} - \frac{4h^3}{l^3}$$

Put value of $\tan\theta$ and $\sin\theta$ in (4)

$$M = \frac{E\pi d^2}{2g}\left[\frac{2h}{l} - \frac{2h}{l} + \frac{4h^3}{l^3}\right] = \frac{E\pi d^2}{2g}\left[\frac{4h^3}{l^3}\right]$$

$$h \cong l\sqrt[3]{\frac{mg}{2\pi d^2 E}}$$ **Ans.**

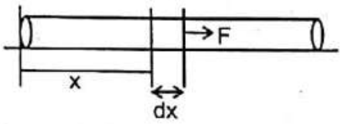
1.295



Area of cross sectin = S
Young modulus = E
Acceleration of plank

$$a = \frac{F_0}{m}$$

Elongation will be different at each element then Force on a cross section at x distance from one end



$$F = m_x a = \frac{m}{l} \times \left(\frac{F_0}{m}\right)$$

$$F = \frac{F_0 x}{l} \quad ....(1)$$

Again we know
stress/strain = young moldules(E)

$$E = \frac{\frac{F}{S}}{\frac{d(\Delta x)}{dx}}$$

$$d(\Delta x) = \frac{F}{ES} dx$$

Put value of F from (1)

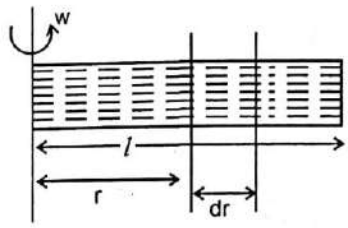
$$d(\Delta x) = \frac{F_0 x}{lES} dx$$

$$\int_0^{\Delta x} d(\Delta x) = \frac{F_0}{ElS} \int_0^l x\,dx$$

$$\Delta x = \frac{F_0 l}{2ES}$$ **Ans.**

1.296

Tension Calculation
Area of cross section = S
F.B.D. of dr part



$$-dF = (dm) r w^2$$

$$-\int_0^F dF = \int_l^r \left(\frac{m}{l} dr\right) r w^2$$

$$-\int_0^F dF = \frac{m w^2}{l} \int_l^r r\,dr$$

$$F = \frac{-m}{2l} w^2 \quad r^2 \Big|_l^r$$

$$F = \frac{m w^2}{2} l \left(1 - \frac{r^2}{l^2}\right) \quad ....(1)$$ **Ans.**

Elongation Calculation

$$\frac{Stress}{Strain} = E$$

$$\frac{F/S}{\frac{d(\Delta r)}{dr}} = E$$

$$d(\Delta r) = \frac{m w^2}{2ES} l \left(1 - \frac{r^2}{l^2}\right) dr$$

From (1)

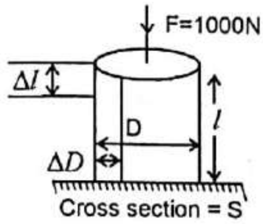
$$\int_0^{\Delta l} d(\Delta r) = \frac{m w^2}{2ES} l \int_0^l \left(1 - \frac{r^2}{l^2}\right) dr$$

After putting

$$m = \rho S l$$

$$\Delta l = \frac{1}{3E} \rho w^2 l^3$$ **Ans.**

1.297



$V_{old} = Sl$

We know
stress/strain = young moldules(E)

$$\frac{F/S}{\frac{\Delta l}{l}} = E$$

$$\Delta l = \frac{Fl}{SE}$$

$$\frac{\Delta l}{l} = \frac{F}{SE} \text{ ....(1)}$$

$$l_{new} = l - \Delta l = l - \frac{Fl}{SE} = l\left(1 - \frac{Fl}{SE}\right)$$

Also

$$\mu = \frac{\Delta D/D}{\frac{\Delta l}{l}}$$

$$\frac{\Delta D}{D} = \frac{\mu \Delta l}{l}$$

From (1)

$$\frac{\Delta D}{D} = \frac{\mu F}{SE} \text{ ....(2)}$$

$$V_{old} = \left(\frac{\pi D^2}{4}\right) l$$

$$V_{new} = \frac{\pi (D + \Delta D)^2}{4}(l - \Delta l)$$

$$V_{new} = \frac{\pi D^2 l}{4}\left(1 + \frac{\Delta D}{D}\right)^2 \left(1 - \frac{\Delta l}{l}\right)$$

$$V_{new} = \frac{\pi D^2 l}{4}\left(1 + 2\frac{\Delta D}{D}\right)\left(1 - \frac{\Delta l}{l}\right)$$

$$V_{new} = V_{old}\left(1 + \frac{2\Delta D}{D} - \frac{\Delta l}{l}\right)$$

$$\Delta V = V_{old} - V_{new} = V_{old} - V_{old}\left(1 + \frac{2\Delta D}{D} - \frac{\Delta l}{l}\right)$$

$$\Delta V = V_{old}\left(\frac{\Delta l}{l} - \frac{2\Delta D}{D}\right)$$

From (1) and (2)

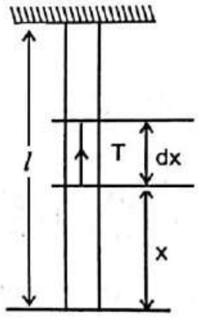
$$\Delta V = V_{old}(1 - 2\mu)\frac{F}{SE}$$

Initial volume $V_{old} = Sl$

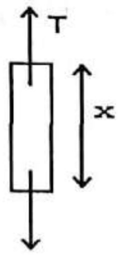
$$\Delta V = \frac{Fl}{E}(1 - 2\mu) \qquad \textbf{Ans.}$$

1.298

(a)



Area of corss section = S
F.B.D. of part x



$$\frac{m}{l}xg$$

$$T = \frac{m}{l}xg \text{ ....(1)}$$

$$\frac{Stress}{Strain} = E$$

$$\frac{T/S}{\frac{d(\Delta x)}{dx}} = E \quad \text{.................(2)}$$

Where d (Δx) is elongation in part dx
From (1) and (2)

$$\int_0^l \frac{mxgdx}{lS} = E\int_0^{\Delta l} d(\Delta x)$$

$$\Delta x = \frac{mgl^2}{2SEl} = \frac{mgl}{2SE}$$

Also

$$\rho = \frac{m}{Sl}$$

Then $\Delta x = \frac{1}{2}\frac{\rho g l^2}{E}$ **Ans.**

(b)
This part is same as Q : 1.297

$$V_{old} = \frac{\pi D^2}{4}$$

$$V_{new} = \frac{\pi(D-\Delta D)^2}{4}(l+\Delta l)$$

$$V_{new} = \frac{\pi D^2 l}{4}\left(1-\frac{\Delta D}{D}\right)^2\left(1+\frac{\Delta l}{l}\right)$$

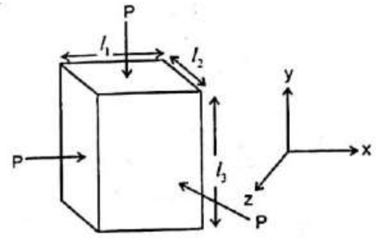
$$V_{new} = \frac{\pi D^2 l}{4}\left(1-2\frac{\Delta D}{D}\right)\left(1+\frac{\Delta l}{l}\right)$$

$$V_{new} = V_{old}\left(1-2\frac{\Delta D}{D}\right)\left(1+\frac{\Delta l}{l}\right)$$

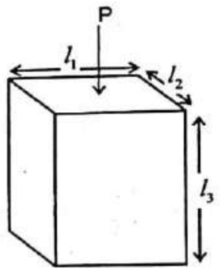
$$\frac{\Delta V}{V_{old}} = \frac{V_{now} - V_{old}}{V_{old}}$$

$$\frac{\Delta V}{V_{old}} = (1-2\mu)\frac{\Delta l}{l}$$ **Ans.**

1.299
(a)



Here we have to find decreasement of volume due to x direction, y direction and z direction pressure and add all three which will be answer. For this we select same question but under single force



$$\frac{P}{\frac{\Delta l_3}{l_3}} = E$$

$$\Delta l_3 = \frac{l_3 P}{E} \quad \text{......................(1)}$$

$$l_3^{new} = l_3 + \Delta l_3 = l_3\left(1-\frac{P}{E}\right) \quad \text{................(2)}$$

Also we know that

$$\frac{\frac{\Delta l_1}{l_1}}{\frac{\Delta l_3}{l_3}} = \mu$$

$$\frac{\Delta l_1}{l_1} = \mu\frac{\Delta l_3}{l_3}$$

From (1)

$$\frac{\Delta l_1}{l_1} = \mu \frac{P}{E}$$

$$l_1^{new} = l_1 + \Delta l_1 = l_1\left(1 + \frac{\mu P}{E}\right) \text{..........(3)}$$

Similary

$$l_2^{new} = l_2 + \Delta l_2 = l_2\left(1 + \frac{\mu P}{E}\right) \quad \text{......(4)}$$

$$V_{new} = l_1^{new} l_2^{new} l_3^{new}$$

$$V_{new} = l_1 l_2 l_3 \left(1 - \frac{P}{E}\right)\left(1 + \frac{\mu P}{E}\right)^2$$

$$V_{new} = l_1 l_2 l_3 \left(1 - \frac{P}{E}\right)\left(1 + \frac{2\mu P}{E}\right)$$

$$\frac{V_{new}}{V_{old}} = 1 - \frac{P}{E} + \frac{2\mu P}{E}$$

$$\frac{\Delta V}{V} = \frac{V_{old} - V_{new}}{V_{old}} = \frac{P}{E}(1 - 2\mu)$$

$$\frac{\Delta V}{V} = \frac{P}{E}(1 - 2\mu) \text{ ....(5)}$$

This is only due to force from y direction
Similarly
Due to force from y direction

$$\frac{\Delta V}{V} = \frac{P}{E}(1 - 2\mu)$$

Due to force from z direction

$$\frac{\Delta V}{V} = \frac{P}{E}(1 - 2\mu)$$

Total decrement due to forces are applied by all direction

$\left(\frac{\Delta V}{V}\right)_{net}$ = Sum of all deformaations

$$\left(\frac{\Delta V}{V}\right)_{net} = \frac{3P}{E}(1 - 2\mu)$$ **Ans.**

(b)
Compresibility

$$\beta = \frac{-1}{V}\frac{dV}{dP} \text{ ....(6)}$$

dP = Applied pressure

$$\frac{dV}{V} = Volume\, strain = \frac{3P}{E}(1 - 2\mu)$$

Here

$dP = P$
Put values in (6)

$$\beta = \frac{3P}{PE}(1 - 2\mu).$$

$$\beta = \frac{3}{E}(1 - 2\mu)$$ **Ans.**

(c)
Compressibility can not zero if we applied forces from all direction

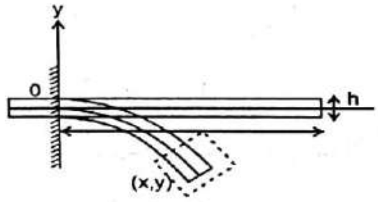
$$\beta > 0$$

$$\frac{3}{E}(1 - 2\mu) > 0$$

$$\mu < \frac{1}{2}$$

Ans.

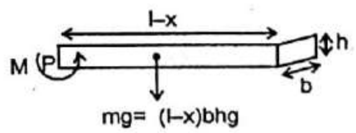
1.300*



This is a cantilever and bending moment of cantiliver is given by

$$M = E\frac{1}{R}$$

(At point (x,y) where radius is R)
F.B.D. of cantiliver in dotted part

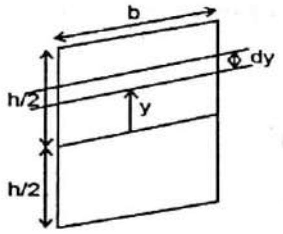


b = width of cantiliver
Torque about point P (x , y)

$$\tau = mg\frac{(l-x)}{2} = \frac{1}{2}\rho bhg(l-x)^2$$

I = Glometrical moment of inertia w.r.t. neurtal line

See F.B.D. of cross section of rod



$$\mathrm{I} = \int_{-h/2}^{h/2} y^2 b\,dy = \frac{bh^3}{12}$$

$$M = \frac{Ebh^3}{12R}$$

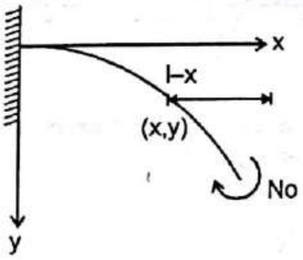
This bending moment must be equal to torque of mg for rotational equalibrium

$$\frac{Ebh^3}{12R} = \frac{1}{2}\rho bhg(l-x)^2 \quad \text{At } x = 0$$

$$R = \frac{Eh^2}{6\rho l^2 g} \qquad \textbf{Ans.}$$

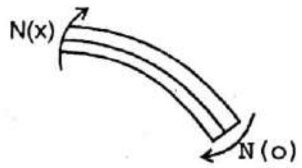
1.301*

(a)



$$N(x) = EI\frac{d^2h}{dx^2}$$

F.B.D. of length of $(l-x)$



Now

$N(x) = N(o)$

$$EI\frac{d^2y}{dx^2} = N(o)$$

$$\frac{d^2y}{dx^2} = \frac{N(o)}{EI}$$

$$\frac{dy}{dx} = \frac{N(o)}{EI}x + C_1$$

At $x = 0$; $\frac{dy}{dx} = 0$ as shown in figure

Then $C_1 = 0$

$$\frac{dy}{dx} = \frac{N(0)x}{EI}$$

$$y = \frac{N(o)x^2}{2EI} + C_2$$

At $x = 0$; $y = 0$

$C_2 = 0$

$$y = \frac{N(o)x^2}{2EI}$$

At $x = l$

$$y = \frac{N(o)x^2}{2EI}$$

From Q.1.300

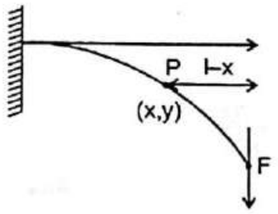
$$\mathrm{I} = \frac{bh^3}{12}$$

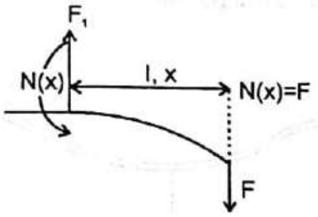
Here $b = h = a$

$$\mathrm{I} = \frac{a^4}{12} \qquad \textbf{Ans.}$$

(b)

F.B.D. of length $(l-x)$





Here $F_1 = F$
$N(x) = F(l - x)$

$$EI = \frac{d^2E}{dx^2} = E(l-x)$$

$$\frac{d^2y}{dx^2} = \frac{F}{EI}(l-x)$$

$$d\left(\frac{dy}{dx}\right) = \frac{F}{EI}\int (l-x)dx$$

$$\frac{dy}{dx} = \frac{Flx}{EI} - \frac{Fx^2}{2EI} + C_1$$

At x = 0

$$\frac{dy}{dx} = 0$$

$C_1 = 0$

$$\frac{dy}{dx} = \frac{Flx}{EI} - \frac{Fx^2}{2EI}$$

$$y = \frac{Flx^2}{2EI} - \frac{Fx^2}{6EI} + C_1$$

At x = 0
y = 0
$C_2 = 0$

$$y = \frac{Flx^2}{2EI} - \frac{Fx^2}{6EI}$$

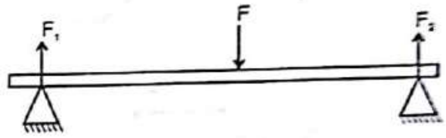
Put x = l

$$y = \frac{Flx^3}{2EI} - \frac{Fl^3}{6EI}$$

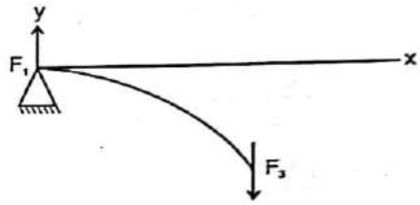
$$y = \frac{Flx^3}{3EI}$$

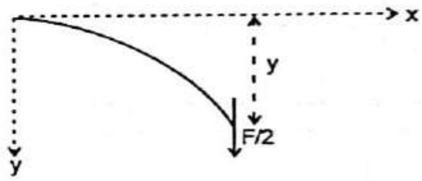
Ans.

1.302*



By symmetry
$F_1 = F_2 = F/2.$
F.B.D. of half of length





Then F_3 must be equal to F_1
$F_3 = F_1 = F/2.$
Now or question is as : Similar as Q : 1.301

$$y = \frac{F_0 l_0^3}{3EI}$$

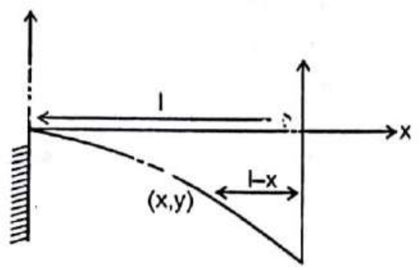
$$l_0 = \frac{l}{2}; F_0 = \frac{F}{2}$$

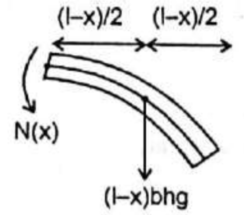
$$y = \frac{Fl^3}{3 \times 2EI \times 8} = \frac{Fl^3}{48EI}$$

Ans.

1.303*

(a)
F.B.D. of (1–x)





$$N(x) = \frac{\rho bhg(l-x)^2}{2}$$

$$EI\frac{d^2y}{dx^2} = \frac{\rho bhg}{2}(l-x)^2$$

$$\frac{d^2y}{dx^2} = \frac{\rho bhg}{2}(l-x)^2$$

$$\frac{dy}{dx} = -\frac{\rho bhg}{2}\frac{(l-x)^3}{12} + C_1x + C_2$$

$$x = 0$$

$$\frac{dy}{dx} = 0$$

$$C_1 = \frac{\rho bhg}{2}$$

Again

$$y = +\frac{\rho bhg}{2EI}\frac{(l-x)^4}{12} + C_1x + C_2$$

At $x = 0$
$y = 0$

$$C_2 = \frac{\rho bhgl^4}{24EI}$$

At $x = l$
$y = C_1 l + C_2$

$$= \frac{\rho bhgl^4}{6EI} - \frac{\rho bhgl^4}{24EI} = -\frac{\rho bhgl^4}{8EI}$$

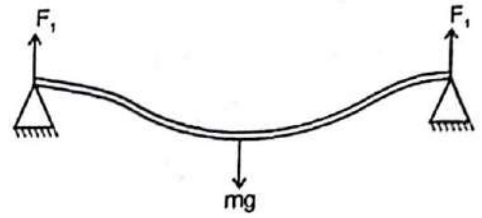
$$I = \frac{bh^3}{12}$$

$$y = \lambda = \frac{3}{2}\frac{\rho bhgl^4}{8EI}$$

Ans.

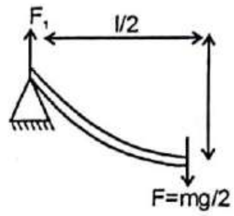
(b)

Here $F_1 = mg/2$
F.B.D. of half of portions



Similar as part (a)

$$\lambda = \frac{5}{2}\frac{\rho gl^4}{Eh^2}$$

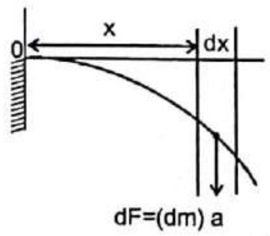


Because $F + F_1 = 0$

Ans.

1.304*

Torque about origin



$$N(x) = \rho lh\beta\int_x^l x^2dx = \frac{1}{3}\rho lh\beta(l^3 - x^3)$$

..................(i)

Now we know

$$N(x) = \frac{Elh^3}{12}\frac{d^2h}{dx^2} \qquad \text{.........(ii)}$$

From (i) and (ii)

$$E\frac{lh^3}{12}\frac{d^2h}{dx^2}=\frac{1}{3}\rho lh\beta(l^3-x^3)$$

$$\frac{dh}{dx}=\frac{h\rho\beta}{Eh^2}\left(l^3x-\frac{x^4}{4}\right)+C_1$$

At

$$x=0;\frac{dy}{dx}=0;C_1=0$$

$$y=\frac{4\rho\beta}{Eh^2}\left(\frac{l^3x^2}{2}-\frac{x^5}{20}\right)+C_2$$

at $x = 0$; $y = 0$; $C_2 = 0$

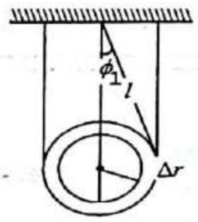
$$y=\frac{4\ell\beta}{Eh^2}\left(\frac{l^3x^2}{2}-\frac{x^5}{20}\right)$$

Put $x = l$

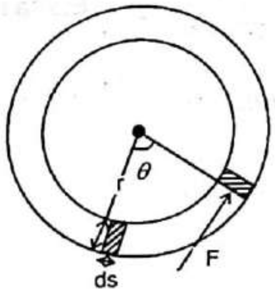
$$y=\frac{9}{5}\frac{\rho\beta l}{Eh^2}$$ **Ans.**

1.305

(a)



Here ϕ_1 = shear strain



Due to force F rod is rotated by angle then ϕ_1 will be shear strain like above figure. Then

$$\frac{\text{Shear stress}}{\varphi_1}=\eta$$

$$\frac{F}{(\Delta r)2\pi r}=\eta\varphi_1$$

$$F=\eta\varphi_1(\Delta r)2\pi r \quad \text{.....................(1)}$$

The torque τ = N
then
N = r F

$$N=2\pi\varphi_1\eta r^2\Delta r \quad \text{......................(2)}$$

Also we know

$$r\theta=l\varphi_1$$

$$\varphi_1=\frac{\theta r}{l}$$

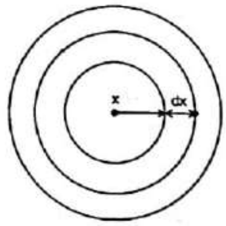
$$N=2\pi\eta r^2\Delta r\left(\frac{\theta r}{l}\right)$$

$$\varphi=\theta=N\left(\frac{l}{2\pi\eta r^3\Delta r}\right)$$

Ans.

$\theta = Torsion\,angle$

(b)



Take differential ring at radius x
From part (a)

$$\Delta r=dr;N=dN;r=x$$

$$dN=2\pi\eta x^2\theta\frac{x}{l}dx$$

$$\int dN=2\pi\eta\frac{\theta}{l}\int_0^r x^3dx$$

$$N = \frac{2\pi\eta\theta r^4}{4l}$$

$$\varphi = \theta = \left(\frac{2l}{\pi\eta r^4}\right)N$$

Ans.

1.306

From Q.No. 1.305

$$\int dN = \frac{2\pi\eta\theta}{l}\int_{d_1/2}^{d_2/2} x^3 dx$$

$$N = \frac{2\pi\eta\theta}{4l}\left[\frac{d_2^4 - d_1^4}{16}\right]$$

$$N = \frac{\pi\eta\theta}{32l}\left[d_2^4 - d_1^4\right]$$

Here $\eta = G$

$$N = \frac{\pi G\theta(d_2^4 - d_1^4)}{32l}$$ **Ans.**

1.307

From Q. No. 1.305

$$N = \frac{\pi\eta r^4\theta}{l}$$

Power of force

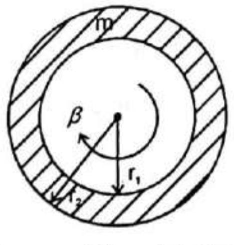
$$P = (\vec{\tau}).\vec{w} = \vec{N}.\vec{w}$$

$$P_{max} = Nw = \frac{\pi\eta r^4\theta w}{2l}$$

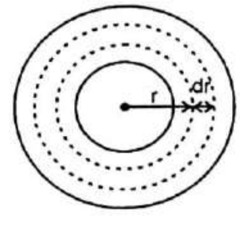
Here $\eta = G$

$$P_{max} = \frac{\pi G r^4\theta w}{2l}$$ **Ans.**

1.308



dF is force on differential element of length dr



$$dF = (dm)a_t = (dm)r\beta = \frac{m(2\pi r dr)r\beta}{(\pi r_2^2 - \pi r_1^2)}$$

$$\int_0^r d\tau = \int_{r_2}^r r dF = \frac{2m\beta}{r_2^2 - r_1^2}\int_{r_2}^r r^3 dr$$

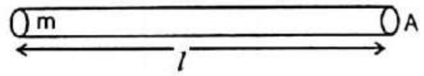
$$\tau = \frac{2m\beta}{4(r_2^2 - r_1^2)}(r^4 - r_2^2)$$

$$\tau = \frac{m\beta(r^4 - r_2^4)}{2(r_2^2 - r_1^2)}.$$

Ans.

Note in integratino range take from r_2 to r because elastic torque will be zero at boundary.

1.309



we know

$$\frac{\text{elastic energy}}{\text{volume}} = \frac{1}{2}\text{stress} \times \text{strain}.$$

Also we know

stress = E strain

$$\frac{\text{elastic energy}}{\text{volume}} = \frac{1}{2}\text{E(strain)}^2 = \frac{1}{2}\text{E}\varepsilon^2$$

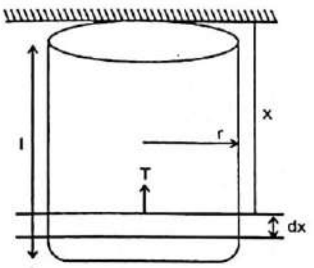
$$Elastic\ energy = \left(\frac{1}{2}E\varepsilon^2\right)Volume$$

$$\Delta U = \left(\frac{1}{2}E\varepsilon^2\right)\left(\frac{m}{\rho}\right)$$

Ans.

1.310

(a)



Tension is due to weight of lower part
$T = \rho\pi r^2 (l - x)g$

$$Stress = \frac{T}{\pi r^2} = \rho(l-x)g$$

we know

$$\frac{Elastic\,energy}{volume} = \frac{1}{2} stress \times strain$$

dU = elastic energy

$$dU = \frac{1}{2}\frac{(stress)^2}{E} \times \pi r^2 dx$$

$$dU = \frac{1}{2}\frac{\rho^2 (l-x)^2 g^2}{E} \times \pi r^2 dx$$

$$U = \frac{1}{2}\frac{\rho^2 \pi g^2 r^2}{E}\int_0^l (l-x)^2 dx$$

$$U = \frac{\rho^2 \pi r^2 g^2}{2E}\left(\frac{(l-x)^3}{3}\right)_0^l$$

$$U = \frac{1}{6E}\rho^2 \pi r^2 g^2 l^3$$

Ans.

(b)

$$\frac{Stress}{Strain} = E$$

$$\frac{\rho(l-x)g}{E} = Strain = \frac{d(dx)}{dx}$$

$$\int_0^{\Delta l} d(dx) = \int_l^0 \frac{\rho(l-x)g}{E} dx$$

$$\Delta l = \frac{\rho g}{E}\frac{(l-x)^2}{2}\Bigg|_l^0$$

$$\rho g l = 2E\frac{\Delta l}{l}$$

From (a)

$$U = \frac{\pi r^2 l}{6E}(\rho g l)^2$$

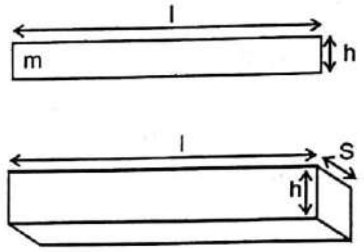
$$= \frac{\pi r^2 l}{6E}\left(2E\frac{\Delta l}{l}\right)^2$$

$$U = \frac{2\pi r^2}{3}\left(El\frac{\Delta l}{l}\right)^2$$

Ans.

1.311

(a)



If radius of hoop is R then

$$2\pi R = l$$

$$R = \frac{l}{2\pi}$$

We take an element at x distance from neutral line.
Increase in length of this element

$$dl = (R+x)2\pi - 2\pi R = 2\pi x$$

length of neutral surface

$$l = 2\pi R$$

Strain in this element $= \frac{2\pi x}{R 2\pi} = \frac{x}{R}$ ....(1)

Volume of this element $dV = 2\pi h R(dx)$

$$dU = \frac{1}{2}E(strain)^2 \times (2\pi R)h(dx)$$

From (1)

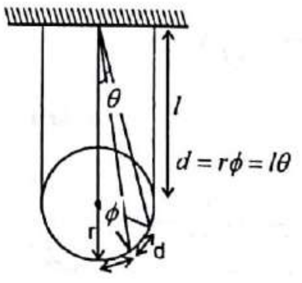
$$U = \int_{-\delta/2}^{\delta/2} E\left(\frac{x}{R}\right)^2 \times 2\pi h R(dx) = \frac{E\pi h\delta^3}{12R}$$

Put

$$R = \frac{l}{2\pi}$$

$$U = \frac{1}{6}\pi^2 h \delta^3 \frac{E}{l}$$ **Ans.**

1.312



$$\frac{r\phi}{l} = \theta \text{ ....(1)}$$

We know that

$$\frac{Stress}{\theta} = G$$

Stress = $G\theta = \dfrac{Gr\varphi}{l}$

This stress will be constant in next $d\phi$ rotation then

$$dE = \frac{1}{2}(stress \times strain)\pi r^2 l$$

$$dE = \frac{1}{2}\frac{Gr\varphi}{l} \times (d\theta)\pi r^2 l$$

From (1)

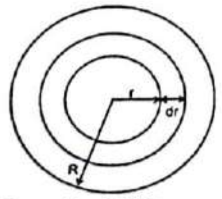
$$d\theta = \frac{r(d\phi)}{l}$$

$$dE = \frac{1}{2} Gr\varphi\pi r^2 \left[\frac{r}{l} d\varphi\right]$$

$$E = \frac{G\pi r^4}{2l}\int_{\varphi}^{\varphi} \varphi d\varphi$$

$$E = \frac{G\pi r^4 \varphi^2}{4l}$$ **Ans.**

1.313



From Q : 1.312

$$E = \frac{G\pi r^4 \varphi^2}{4l}$$

$$\frac{dE}{dr} = \frac{4G\pi r^3 \varphi^2}{4l}$$

$$dE = \frac{G\pi r^3 \varphi^2}{l} dr$$

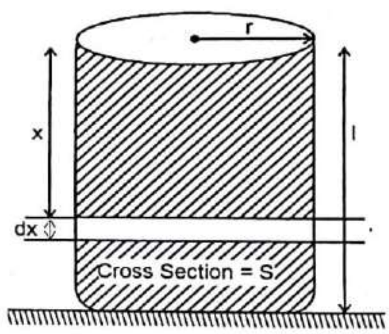
$$\frac{dE}{dV} = \frac{Energy}{Volume} = \frac{dE}{(2\pi r dr)l}$$

$$\frac{dE}{dV} = \frac{G\pi r^3 \varphi^2 dr}{l(2\pi r dr)l}$$

$$\frac{dE}{dV} = \frac{1}{2}\left(\frac{G\varphi^2 r^2}{l^2}\right)$$

Ans.

1.314



$$\beta = -\frac{1}{V}\frac{dV}{dP}$$

$$\beta(dP) = -\frac{dV}{V}$$

Here dP is pressure at depth x due to weight of water or dydrostatic pressure then

$$\frac{dV}{V} = -\beta[x\rho g] \text{ ....(1)}$$

Also we know

Energy density= $\frac{1}{2}$ (stress) × strain

$$= \frac{1}{2} B(strain)^2$$

Where B = Bulk modulus ; β =Compressibility

Energy density $= \frac{1}{2\beta}(strain)^2$

Energy Density $= \frac{1}{2}\times\frac{1}{\beta}(\beta x\rho g)^2$

$= \frac{1}{2}\times\beta(x\rho g)^2$

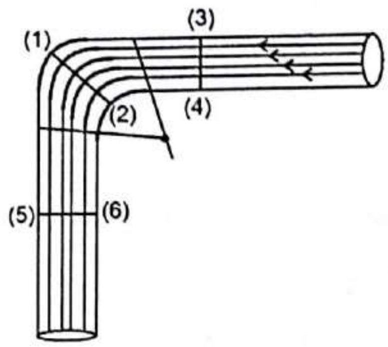
If x = h

Energy Density= $\frac{dE}{dV} = \frac{1}{2}\beta(h\rho g)^2$ **Ans.**

1.7 Hydrodynamics

1.315

F.B.D



By Observations

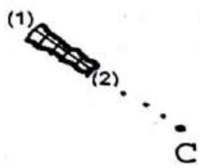
$P_3 = P_4$

$P_5 = P_6$

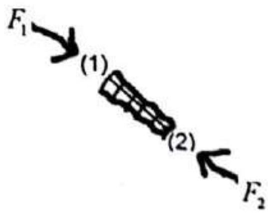
$V_3 = V_4$

$V_5 = V_6$

Pressure Comparision between P_1 and P_2



Take F.B.D. of differential element as shown in graph



Since this element is in circular motion toward centre C, ($F_1 - F_2$) will provide required centripital force toward centre .

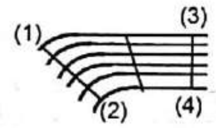
Hence

$F_1 > F_2$

$P_1 > P_2$ **Ans.**

Because F_1 and F_2 will only arised due to pressure difference.

Velocity Comparision between V_1 and V_2

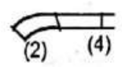


$$P_3 = P_4$$

$$V_3 = V_4$$

Also

Upper element



Lower element

Area of cross section (1-2) is more than that of (3-4)
Then using continuity equation

$$V_1 < V_3$$

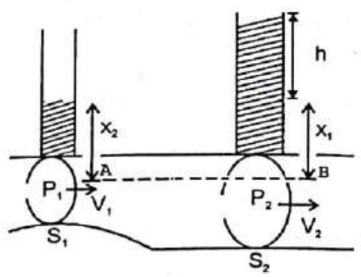
We can say velocity deceases from (3) to (1) or from (4) to (1)
Also from section (a)

$$P_1 > P_2$$

From above pressuse comparision, we can say that retardation of liquid particles from (3) to (1) will be more than retardation of liquid particles from (4) to (2).
Also displacement is more in path (3) to (1) than displacement in path (4) to (2).
Since both retardation and displacement are more in (3) to (1) than that of path (4) to (2).
Using kinematic we can say

$$V_1 < V_2$$ **Ans.**

1.316



Bernaullis equations at crossection S_1 and S_2

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \quad \text{..........(1)}$$

Using continuity equation
$S_1V_1 = S_2V_2$

$$V_1 = \frac{S_2V_2}{S_1}$$

Put in (1)

$$P_1 + \frac{1}{2}\rho\frac{S_2^2V_2^2}{S_1^2} = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 - P_1 = \frac{1}{2}\rho V_2^2\left(\frac{S_2^2}{S_1^2} - 1\right) \text{..................(2)}$$

$$P_1 = P_0 + x_2\rho g$$

$$P_2 = P_0 + (x_2 + \Delta h)\rho g$$

Then

$$P_2 - P_1 = \Delta h\rho g$$

Put in (2)

$$\Delta h\rho g = \frac{1}{2}\rho V_2^2\left(\frac{S_2^2}{S_1^2} - 1\right)$$

$$V_2 = \sqrt{\frac{2\rho g\Delta h}{\rho\left(\frac{S_2^2}{S_1^2} - 1\right)}} = \sqrt{\frac{2g\Delta h}{\left(\frac{S_2^2}{S_1^2} - 1\right)}} \quad \text{...............(3)}$$

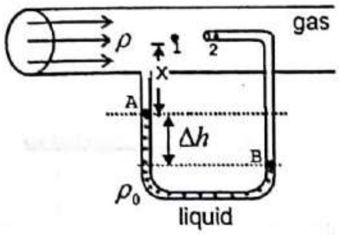
Volume flow rate

$$Q = \left(\frac{d\text{Volume}}{dt}\right) = S_2V_2$$

$$Q = S_2\sqrt{\frac{2g\Delta h}{\left(\frac{S_2^2}{S_1^2} - 1\right)}} = S_1S_2\sqrt{\frac{2g\Delta h}{S_2^2 - S_1^2}}$$

Ans.

1.317



Speed of liquid at point A and B will be zero.
Using Bernaullis equation at crossecton (A) and (B)

$P_A + \rho_0 g \Delta h = P_B$ ......(1)

Also

$P_A = P_1$ ....(2)

Because we are moving perpendicular to air flow

$P_B = P_2$ ....(3)

Because air is in rest inside liquid
At point (1) and point (2) pressure difference is arised due to air strike.
Using Bernaullis equation at crossecton (1) and (2)

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2$$

From (2) and (3)

$$P_A + \frac{1}{2}\rho V_1^2 = P_B \text{ ....(4)}$$

From (1) and (4)

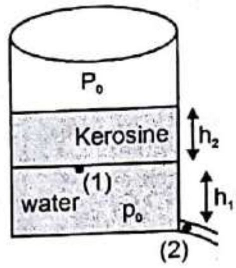
$$\rho_0 g \Delta h = \frac{1}{2}\rho V_1^2$$

$$V_1 = \sqrt{\frac{2\rho_0 g \Delta h}{\rho}}$$

Volume flow rate:

$$Q = SV_1 = S\sqrt{2g\left(\frac{\rho_0}{\rho}\right)\Delta h}$$ **Ans.**

1.318



P_0 = atmospheric pressure
Bernaullis equation at point(1) and (2)

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho_1 g h_1 = P_0 + \frac{1}{2}\rho V_1^2 + 0$$

Hence $V_1 << V$ because $A >> a$

$$P_1 + \rho_1 g h_1 = P_0 + \frac{1}{2}\rho_1 V^2 \text{ ....(1)}$$

Static pressure equation between (1) and atmosphere

$P_1 = P_0 + h_2\rho_2 g$

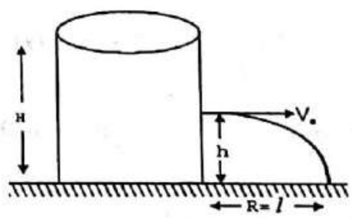
Put in (1)

$$P_0 + h_2\rho_2 g + \rho_1 g h_1 = P_0 + \frac{1}{2}\rho_1 V^2$$

$$V^2 = \frac{2g(h_2\rho_2 + h_1\rho_1)}{\rho_1}.$$

$$V = \sqrt{2g\left(h_1 + h_2\frac{\rho_2}{\rho_1}\right)}.$$ **Ans.**

1.319



Velocity of efflux

$V_e = \sqrt{2g(H-h)}$

Time to reach at bottom is t then

$$x = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$R = V_e\, t = \sqrt{2g(H-h)}\,\sqrt{\frac{2h}{g}}$$

$$R = \sqrt{4(Hh - h^2)} \text{ ....(1)}$$

$R^2 = 4\,(Hh - h^2)$

For R max , R^2 should be maximum

$$\frac{d(R^2)}{dt} = 0$$

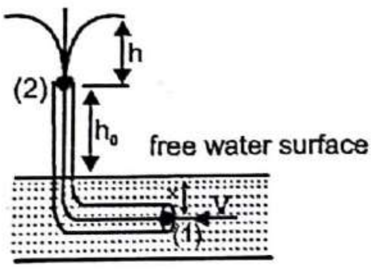
$H - 2h = 0$

$h = H/2$ **Ans.**

Put value of x in (1)

$R_{max} = H$ **Ans.**

1.320



Bernaullis equation at point (1) and (2) along stream line

$$P_1 + \frac{1}{2}\rho V^2 = P_0 + \frac{1}{2}\rho V_e^2 + \rho g(h_0+x)$$

$$P_0 + \rho gx + \frac{1}{2}\rho V^2 = P_0 + \frac{1}{2}\rho V_e^2 + \rho g(h_0+x)$$

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_e^2 + \rho g h_0$$

$$V_e = \sqrt{V^2 - 2gh_0} \text{ ....(1)}$$

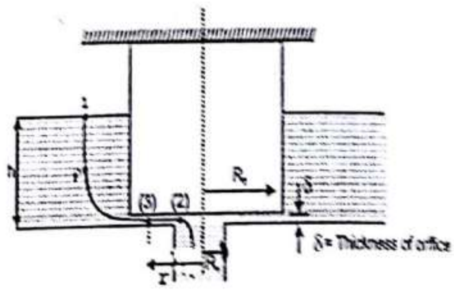
After that liquid will be in free fall

$$0^2 = V_e^2 - 2gh \text{ ....(2)}$$

From (1) and (2)

$$h = \frac{V^2}{2g} - h_0$$ **Ans.**

1.321

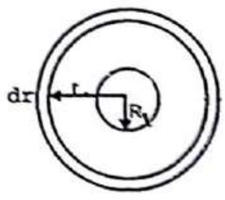


Bernaulli equation at points (1) and (2)

$$P_0 + \frac{1}{2}\rho(0)^2 + \rho gh = P_0 + \frac{1}{2}\rho V_2^2$$

$$\rho gh = \frac{1}{2}\rho V_2^2$$

$$V_2 = \sqrt{2gh} \text{ —(1)}$$



Continuety equation at (3) and (2)

$A_3V_3 = A_2V_2$

$$V_2(2\pi R_1)\delta = V_3(2\pi r)\delta$$

$$V_3 = \frac{R_1 V_2}{r} = \frac{R_1\sqrt{2gh}}{r} \text{ ....(2)}$$

Bernaulli equation at points (3) and (2)

$$P + \frac{1}{2}\rho V_3^2 = P_0 + \frac{1}{2}\rho V_2^2 \text{ ....(3)}$$

From (1) and (2) , put values in (3)

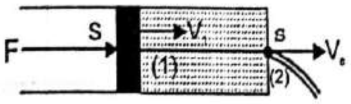
$$P = P_0 + \rho gh - \frac{1}{2}\rho\frac{R_1^2 2gh}{r^2}$$

$$P = P_0 + \rho g h\left(1-\frac{R_1^2}{r^2}\right)$$

Ans.

1.322

Using Work energy theorm



We know

$W_{\text{all forces}} = \Delta K.E.$

$W_F + W_{\text{atmosphere}} = \Delta K.E.$ ....(1)

$$\text{Power} = \frac{dw}{dt} = FV = PSV$$

Here work done by atmosphere = 0

Because power due to atomosphere

Power = $P_0SV_1 - P_0sV_e$

From continuity equations

$SV_1 = sVe$

Hence power of atmosphere = 0

$W_{\text{atmosphere}} = 0$

From (1)

$W_F = \Delta K.E.$

Calculation of change in kinetic energy

$$W_F = \Delta K = \frac{1}{2}mV_e^2 = \frac{1}{2}V\rho V_e^2 \text{ ....(1)}$$

Using volume conservation

Suppose time t is taken in eject out then

$$V = (sV_e)t$$

$$V_e = \frac{V}{st}$$

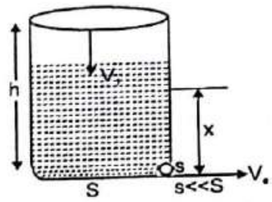
Put in (1)

$$W_F = \frac{1}{2}V\rho\left(\frac{V}{st}\right)^2$$

$$W_F = \frac{1}{2}\frac{V^3}{s^2t^2}\rho$$

Ans.

1.323



Suppose at time t, x length of water is inside tube

$Ve = \sqrt{2gx}$

Now using continuity equation

$SV_1 = s\,Ve$

$$V_1 = \frac{s}{S}\sqrt{2gx}$$

$$-\frac{dx}{dt} = \frac{s}{S}\sqrt{2gx}$$

$$-\int_h^0 x^{-1/2}dx = \frac{s}{S}\sqrt{2g}\int_0^\tau dt$$

$$-2x^{1/2}\Big|_h^0 = \frac{s}{S}\sqrt{2g}\,\tau$$

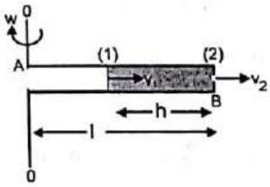
$$2\sqrt{h} = \frac{s}{S}\sqrt{2g}\,\tau$$

$$\tau = \frac{S}{s}\sqrt{\frac{2h}{g}}$$

Ans.

1.324

Method : 1 (Most Genaral Bernaulli equation)



Bernaulli equation w.r.t. rotatory axis

$$P + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho(rw)^2 + \rho gz = \text{constant}$$

Equation at points (1) and (2)

$$P_0 + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho(l-h)^2w^2 = P_0 + \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho l^2 w^2$$

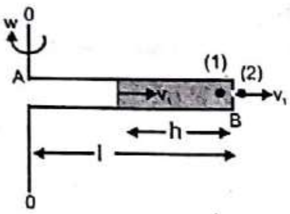
Here

$$\frac{1}{2}\rho V_1^2 \to 0$$

$$\frac{1}{2}\rho V_2^2 = \frac{1}{2}\rho l^2 w^2 - \frac{1}{2}\rho (l-h)^2 w^2$$

$$V_2 = wh\sqrt{\frac{2l}{h} - 1}$$ **Ans.**

Method : 2 (Basic Equation)

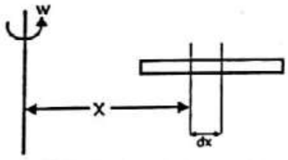


Bernaullis equation at (1) and (2) just inside and just outside

$$P_1 + \frac{1}{2}\rho(0)^2 + 0 = P_0 + \frac{1}{2}\rho V_2^2 + 0$$

$$P_1 - P_0 = \frac{1}{2}\rho V_2^2 \text{ ---------(i)}$$

Calculation $P_1 - P_0$



$dF = (dm)xw^2$
$(dP)S = (\rho S dx)\, xw^2$

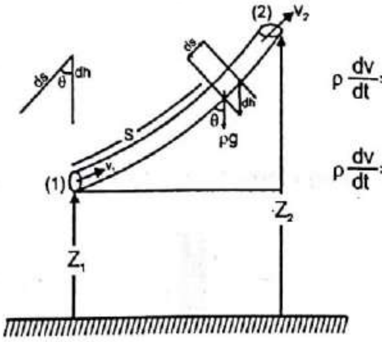
$$\int_{P_0}^{P_1} dP = \rho w^2 \int_{\ell-h}^{\ell} x dx$$

$$P_1 - P_0 = \frac{\rho w^2}{2}\left(\ell^2 - (\ell-h)^2\right)$$

Put in (1)

$$V_2 = wh\sqrt{\frac{2l}{h} - 1}$$ **Ans.**

1.325*



$$\rho\frac{dv}{dt} = f - \Delta P$$

$$\rho\frac{dv}{dt} = f - \frac{dp}{ds}$$

At differential element

$$\rho\frac{dV}{dt} = -\rho g \cos\theta - \frac{dP}{ds}$$

$$\rho V\frac{dV}{ds} = -\rho g \cos\theta - \frac{dP}{ds}$$

$$\int \rho v dv = -\int \rho g(\cos\theta) ds - \int dP$$

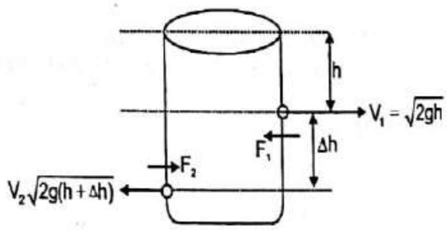
$$\rho\int_{V_1}^{V_2} v dv = -\rho g\int_{z_1}^{z_2} dz - \int_{P_1}^{P_2} dP$$

$$\rho\frac{V_2^2}{2} - \frac{\rho V_1^2}{2} = -\rho g z_2 + \rho g z_1 - P_2 + P_1$$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

Ans.

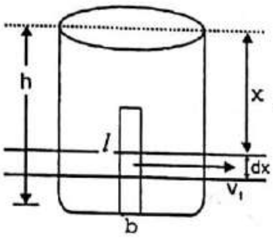
1.326



We know force F = ρAV^2

$F_1 = \rho S V_1^2 = 2\rho sgh$
$F_2 = \rho S(2g(h+\Delta h)) = 2\rho sg\,(h+\Delta h)$
$F_{net} = F_2 - F_1$
$F_{net} = 2\rho sg\Delta h$ **Ans.**

1.327



Velocity of efflux at x distance below the top

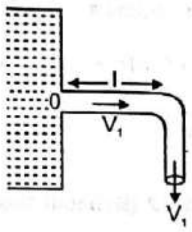
$V_1 = \sqrt{2gx}$

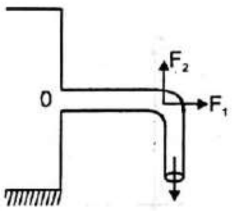
Then force $dF = \rho A V^2 = \rho (bdx) 2gx$

$$\int dF = 2\rho g b \int_{h-l}^{l} x\,dx$$

$F = \rho g b l\,(2h - l)$ **Ans.**

1.328





Using volume flow rate equation

$Q = AV_1$

$V_1 = \dfrac{Q}{A}$ ....(1)

F.B.D to tube

$A = \pi r^2$ = Area of cross-section

$F_1 = F_2 = \rho A V^2$

From (1)

$$F_1 = F_2 = \rho A \left[\frac{Q}{A}\right]^2 = \frac{\rho Q^2}{\pi r^2}$$

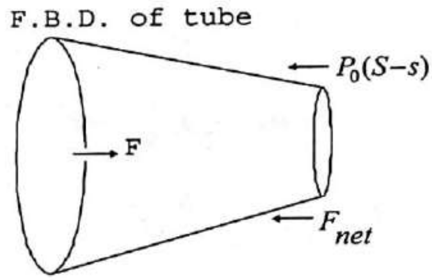
Torque about O

$\tau = F_2 l - F_1 \times 0$

$\tau = \dfrac{\rho Q^2 l}{\pi r^2}$ **Ans.**

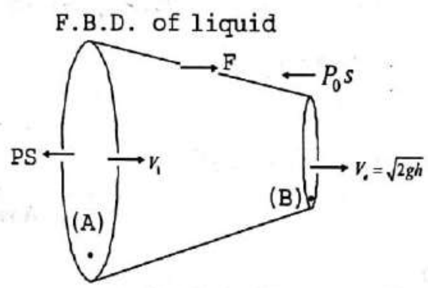
1.329

F.B.D. of tube



$F = F_{net} + P_0(S - s)$ ....(1)

F.B.D. of liquid



Using continuity equation

$SV_1 = s\sqrt{2gh}$

$V_1 = \dfrac{s\sqrt{2gh}}{S}$

Using force equation on liquid

$PS - F - P_0 s = \dfrac{dP}{dt}$ ....(2)

Calculation of $\frac{dP}{dt}$

$$\frac{dP}{dt}=\left(\frac{dm}{dt}\right)V_e-\left(\frac{dm}{dt}\right)V_1$$

$$=\rho sV_e\left(V_e-\frac{sV_e}{S}\right)=\rho sV_e^2\left(\frac{S-s}{S}\right)$$

Calculation of P

Using Bernaulli equation at point (A) and (B)

$$P+\frac{1}{2}\rho V_1^2=P_0+\frac{1}{2}\rho V_e^2$$

$$P=P_0+\rho gh-\frac{1}{2}\rho\left(\frac{s}{S}\right)^2 2gh$$

$$P=P_0+\rho gh\left[1+\frac{S^2-s^2}{S^2}\right]$$

Put value of P and $\frac{dm}{dt}$ in (2)

$$P_0S+\rho gh\frac{S^2-s^2}{S}-P_0s-F_{ext}-\rho_0(S-s)$$
$$=\frac{\rho s(S-s)2gh}{S}$$

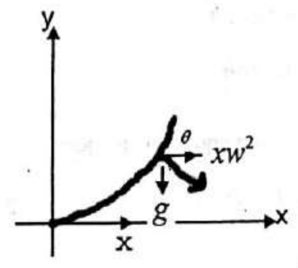
$$F_{ext}=\frac{\rho gh}{S}(S-s)^2$$

Ans.

1.330

Method : 1 (Basic Equation)

(a)



Effective acceleration is perpendicular to free surface of liquid.

$$\tan\theta=\frac{xw^2}{g}$$

$$\frac{dy}{dx}=\frac{xw^2}{g}$$

$$dy=\frac{xw^2}{g}dx$$

$$\int_0^y dy=\frac{w^2}{g}\int_0^x xdx$$

$$y=\frac{x^2w^2}{2g}$$

$$y=\left(\frac{w^2}{2g}\right)r^2$$ **Ans.**

(b)

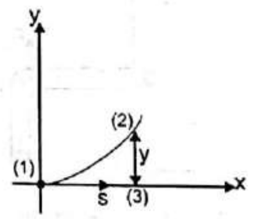
Bernalli's equation at (1) and (3)

$$P_0+0+0=P_0+0-\frac{1}{2}\rho(xw)^2$$

$$P=P_0+\frac{1}{2}\rho(xw)^2$$ **Ans.**

Method : 2 (Benaulli theorm from ground frame)

(a)



Pressure equation in horizantal direction from rotatory frame from 1 to 3

$$P_0+x\rho\frac{x}{2}w^2=P_3$$

$$P_0+\frac{1}{2}\rho(xw)^2=P_3 \text{ ....(1)}$$

Also pressure equation in vertical direction from 3 to 2

$P_0+\rho gy=P_3$ ....(2)

From (1) and (2)

$$y=\frac{x^2w^2}{2g}$$

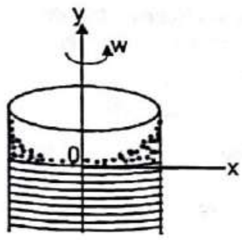
$$y = \left(\frac{w^2}{2g}\right) r^2 \qquad \textbf{Ans.}$$

(b)
Bernalli's equation at (1) and (3)

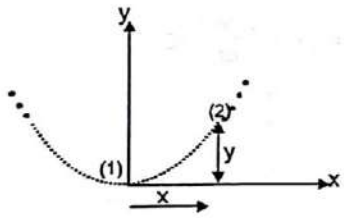
$$P_0 + 0 + 0 = P_0 + 0 - \frac{1}{2}\rho(xw)^2$$

$$P = P_0 + \frac{1}{2}\rho(xw)^2 \qquad \textbf{Ans.}$$

Method:3 (Benaulli theorm from rotatory frame)
(a)



Dotted line express isobaric surface.
Then F.B.D. of one of the dotted line.



Bernaullis equation between point (1) and (2)
From rotatory referece frame

$$P_0 + 0 + 0 = P_0 + 0 + \rho gh - \frac{1}{2}\rho(xw)^2$$

$$\rho gh = \frac{1}{2}\rho x^2 w^2$$

$$h = \frac{x^2 w^2}{2g}$$

Here x = r and h=y

$$y = \frac{x^2 w^2}{2g}$$

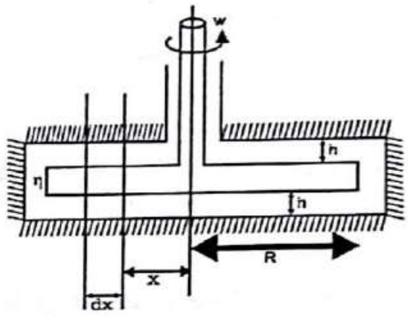
$$y = \left(\frac{w^2}{2g}\right) r^2 \qquad \textbf{Ans.}$$

(b)
Bernalli's equation at (1) and (3)

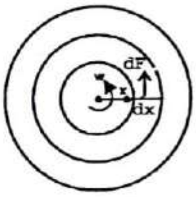
$$P_0 + 0 + 0 = P_0 + 0 - \frac{1}{2}\rho(xw)^2$$

$$P = P_0 + \frac{1}{2}\rho(xw)^2 \qquad \textbf{Ans.}$$

1.331



Top view



Viscous force at (dx) element at lower surface

$$dF = \eta A\frac{dV}{dz} = \eta(2\pi x\, dx)\ \frac{xw}{h}$$

Net viscous force at (dx) element from both lower and upper surface

$$dF_{net} = 2df = \frac{4\pi\eta w}{h} x^2 dx$$

Torque due to this force

$$d\tau = x dF_{net} = \frac{4\pi\eta w x^3}{h} dx$$

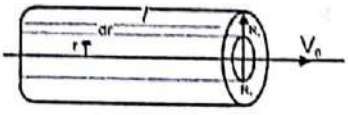
Power to this torque

$$dP = w d\tau = \frac{4\pi\eta w^2 x^3}{h} dx$$

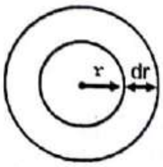
$$P_{net} = \frac{4\pi\eta w^2}{h}\int_0^R x^3 dx$$

$$P_{net} = \frac{\pi\eta w^2 R^4}{h} \qquad \textbf{Ans.}$$

1.332



Cross-sectional view



Viscous force on this differential element of liquid off radius r , thickness dr and length l

$$dF = \eta(2\pi rl)\ \frac{dv}{dr}$$

Where $2\pi rl$ = Curve cross-sectional area
In laminor flow

$$\frac{dF}{dx} = \text{constant} = C$$

$$\eta 2\pi rl \frac{dV}{dr} = C$$

$$2\eta\pi l\int dV = c\int \frac{dr}{r}$$

$2\,\eta\,\pi\, lV = C \ln r + C_1$ ....(1)
Using boundary condition
$r = R_1$; $V = V_0$
$2\eta\pi\, lV_0 = C \ln R_1 + C_1$ ....(2)
Also
$r = R_2$; $V = 0$
$C_1 = -C \ln R_2$ ....(3)
From (2) and (3)

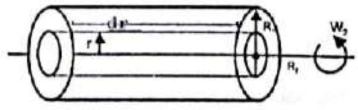
$$C = \frac{2\eta\pi lV_0}{\ln \frac{R_1}{R_2}}$$

$$C_1 = \frac{-2\eta\pi lV_0}{\ln\left(\frac{R_1}{R_2}\right)}\ln(R_2)$$

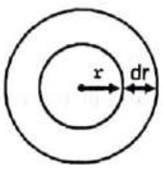
Put in (1)

$$V = \frac{V_0}{\ln R_1/R_2}\ln\frac{r}{R_2}$$ **Ans.**

1.333
(a)



Cross-sectional view



Viscous force on this differential element of liquid off radius r , thickness dr and length l

$$dF = \eta r\left(\frac{dw}{dr}\right)2\pi rl$$

Torque due to this force

$$d\tau = rdF = \eta r^2\left(\frac{dw}{dr}\right)2\pi rl$$

$$d\tau = 2\eta\pi lr^3\left(\frac{dw}{dr}\right)$$

For laminor flow this torque is constant then

$$d\tau = 2\eta\pi lr^3\left(\frac{dw}{dr}\right) = C$$

$$2\pi\eta lr^3\ \frac{dw}{dr} = C$$

$$C\int\frac{dr}{r^3} = 2\pi\eta l\int dw$$

$$2\pi\eta wl = -\frac{C}{2r^2} + C_1 \text{ ----(1)}$$

To find C and C_1
Use boundary condition
$r = R_1$; $w = w_2$

$$2\pi\eta l\, w_2 = -\frac{C}{R_1^2} + C_1 \text{ ---- (2)}$$

$r = R_2$; $w = 0$

$$2\pi l \times 0 = \frac{-C}{R_2^2} + C_1 \text{ ---- (3)}$$

From (2) and (3) find C_1 and C_2 and put in (1)

$$w = w_2\left(\frac{R_1^2R_2^2}{R_1^2R_1^2}\right)\left(\frac{1}{R_1^2} - \frac{1}{r^2}\right)$$ **Ans.**

(b)
From part (a)

$$\frac{d\tau}{dx} = 2\pi\eta r^3\left(\frac{dw}{dr}\right)$$

At $r = R_2$

$$\frac{d\tau}{dx} = 2\pi\eta R_2^{\,3}\left(\frac{dw}{dr}\right) \text{.............(4)}$$

$$w = w_2\frac{R_1^2R_2^2}{\left(R_2^2 - R_1^2\right)}\left(\frac{1}{R_1^2} - \frac{1}{r^2}\right)$$

$$\frac{dw}{dr} = \frac{w_2R_1^2R_2^2}{R_2^2 - R_1^2}\left(\frac{+2}{r^3}\right)$$

$$\left.\frac{dw}{dr}\right|_{r=R_2} = \left(\frac{w_2R_1^2R_2^2}{R_2^2 - R_1^2}\right)\left(\frac{2}{R_2^3}\right)$$

$$= \frac{2w_2R_1^2}{R_2\left(R_2^2 - R_1^2\right)}$$

Put in (4)

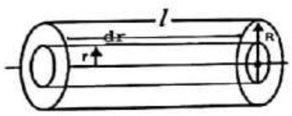
$$\frac{d\tau}{dx} = 2\pi\eta R_2^3\left(\frac{2w_2R_1^2}{R_2\left(R_2^2 - R_1^2\right)}\right)$$

$$\frac{d\tau}{dx} = \frac{4\pi\eta w_2R_1^2R_2^2}{R_2^2 - R_1^2}$$

Ans.

1.334

(a)



$$v = v_0\left(1 - \frac{r^2}{R^2}\right) \text{.................(1)}$$

Volume flow rate through length l, radius r and thickness dr

$$dQ = (2\pi r dr)v_0\left(1 - \frac{r^2}{R^2}\right)$$

$$dQ = (2\pi r dr)v_0\left(1 - \frac{r^2}{R^2}\right)$$

$$\int dQ = 2\pi v_0\int_0^R\left(r - \frac{r^3}{R^2}\right)dr$$

$$Q = 2\pi v_0\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$Q = \frac{\pi v_0R^2}{2}$$

Ans.

(b)

Kinetic energy of differential element of dr thickness

$$dK = \frac{1}{2}(dm)v^2$$

$$dK = \frac{1}{2}\rho\left(2\pi rl(dr)\right)v^2$$

$$dK = \frac{1}{2}\rho\left(2\pi rl(dr)\right)\left(v_0\left(1 - \frac{r^2}{R^2}\right)\right)^2$$

$$\int_0^K dK = \frac{1}{2}2\pi l\rho v_0\int_0^R\left(r\left(1 - \frac{r^2}{R^2}\right)\right)^2 dr$$

$$K = \frac{1}{6}\pi lR^2\rho v_0^2$$

Ans.

(c)

$$f_r = \eta A\frac{dV}{dr}$$

$$f_r = \eta\left(2\pi r\ell\right)\frac{dV}{dr} \text{ ....(2)}$$

Differentiate (1) w.r.t. r

$$\frac{dV}{dr} = v_0\left(\frac{2r}{R^2}\right) \quad \text{Put in (2)}$$

$$f_r = \eta\left(2\pi R\ell\right)v_0\left(\frac{2r}{R^2}\right) = 4\pi\eta lv_0$$

Ans.

(d)

From option (a)

$$Q = \frac{1}{2}\pi V_0R^2 \text{ —— (i)}$$

From poseuille 's law

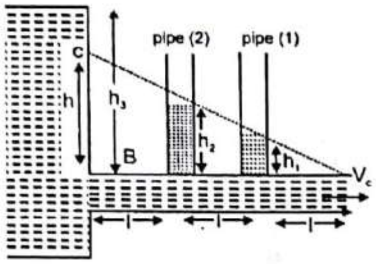
$$Q = \frac{\pi R^4\left(P_1 - R_2\right)}{8\eta l}$$

Comparing (i) and (ii)

$$\frac{1}{2}\pi V_0R^2 = \frac{\pi R^4\left(P_1 - R_2\right)}{8\eta l}$$

$$P_1 - P_2 = \frac{4\eta l V_0}{R^2}$$ **Ans.**

1.335*



If we see carefully, we get that height are different in pipe (1) and (2). While velocity of flow ar bottom of both pipes are same because cross-section are horizontal piple is same at each point.

Heights difference are due to friction loss and bent loss in pipes still we can say that dotted line show isobaric surface.

Height above isobaric surface provide, velocity at efflux.

$$V_e = \sqrt{2g(AC)} = \sqrt{2g(h_3 - h)}$$

Calculation of h

Using similar tringle properties

$$\frac{h}{h_2} = \frac{3l}{2l}$$

$$h = \frac{3h_2}{2} = \frac{3}{2} \times 20$$

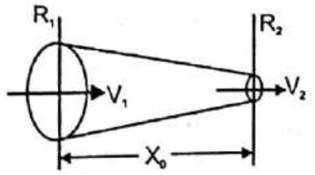
$h = 30$ cm

$h_3 - h = 35$ cm-30cm $= 5$ cm

$$V_e = \sqrt{2 \times g \times 5 \times 10^{-2}} = 1\text{m/s}$$

Ans.

1.336*



We know Reynold's no for circular cross-section is

$$Re = \frac{\rho v l}{\eta}$$

Where l = length of charactersitic and for circular tube with full of water

$D = 2R$ Then

$$\frac{Re_1}{Re_2} = \frac{\rho V_1 D_1 / \eta}{\rho V_2 D_2 / \eta} = \frac{v_1 R_1}{v_2 R_2} \quad \text{------ (i)}$$

Also we know

$$Q = V_1 \pi R_1^2 = V_1 \pi R_2^2$$

$$\frac{V_1}{V_2} = \frac{R_2^2}{R_1^2}$$

Put in (i)

$$\frac{Re_1}{Re_2} = \left(\frac{R_2^2}{R_1^2}\right)\left(\frac{R_1}{R_2}\right) = \frac{R_2}{R_1} = \frac{r_0 e^{-\alpha \Delta x}}{r_0^{-(0)}}$$

$$\frac{Re_1}{Re_2} = e^{-\alpha \Delta x}$$

Ans.

1.337*

Maximum value of Reynold 's no for glycerin for laminor flow

$$R^g = \frac{\rho V_{max} D}{\eta}$$

$$R^g = \frac{\rho_1 v_1 2r_1}{\eta_1}$$

Reynold no. for water

$$R^w = \frac{\rho_2 v_2 2r_2}{\eta_2}$$

We know

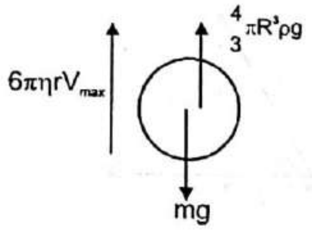
Reynold's no. for turbulent flow > that for laminor

$$\frac{\rho_2 v_2 2r_2}{\eta_2} > \frac{\rho_1 v_1 2r_1}{\eta_1}$$

$$v_2 > \frac{\rho_1 v_1 r_1 \eta_2}{\rho_2 v_2 r_2 \eta_1}$$ **Ans.**

1.338*

F.B.D. of sphere



ρ_0 = Density of gylcerin

ρ = Density of lead

At terminal velocity

$F_{net} = 0$

$$0 = 6\pi\eta r V + \frac{4}{3}\pi R^3 \rho g - mg$$

$$\left(\frac{4}{3}\pi R^3 \rho\right) g + 6\pi\eta r V = mg$$

$$\left(\frac{4}{3}\pi R^3 \rho\right) g + 6\pi\eta r V = \left(\frac{4}{3}\pi R^3 \rho_0\right) g$$

$$V = \frac{2r^2 g}{9\eta}(\rho_0 - \rho)$$

Rynold no

$$R_e = \frac{\rho V D}{\eta}$$

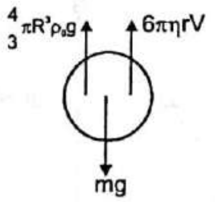
$$R_e = \frac{\rho x 2r}{\eta} \times \frac{2r^2 g}{9\eta}(\rho_0 - \rho)$$

$$r = \sqrt[3]{\frac{9R_e\eta^2}{4\rho g(\rho_0 - \rho)}}$$

$$\text{Diameter} = 2r = r = \sqrt[3]{\frac{18R_e\eta^2}{\rho g(\rho_0 - \rho)}}$$

Ans.

1.339



Where ρ_0 = density of olive oil.

Since radius of ball is $R = \frac{3}{2} \times 10^{-3}$ mm which is very-2 small.

Then

$$\frac{4}{3}\pi R^3 \rho_0 g \Rightarrow 0$$

Net force on object in downward direction

$F_{net} = mg - 6\pi\eta r v$

$$F_{net} = m\frac{dv}{dt} = mg - 6\pi\eta r v$$

$$m dV = (mg - 6\pi\eta r v)\, dt$$

$$\int_0^{\frac{V_{max}}{100}} \frac{m\, dV}{mg - 6\pi\eta r V} = \int_0^{t_1} dt$$

$$\frac{m}{-6\pi\eta r}\ln(mg - 6\pi\eta r V)\Big|_0^{\frac{V_{max}}{100}} = t_1$$

$$\frac{m}{-6\pi\eta r}\ln\left(\frac{mg - 6\pi\eta r\frac{V_{max}}{100}}{mg}\right) = t_1 \quad \text{................(1)}$$

Calculation of V_{max}

At maximum velocity

$F_{net} = 0$

$mg - 6\pi\eta r V_{max} = 0$

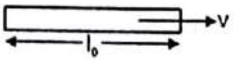
$$V_{max} = {}^{mg}\!/_{6\pi\eta r}$$

Put in (1)

$$t_1 = \frac{-\rho d^2}{18\eta}\ln n$$ **Ans.**

1.8 Relativistic Mechanics

1.340*



We know $l = l_0\sqrt{1-v^2/c^2}$

$$\Delta l = l_0 - l = l_0\left(1-\left(1-\frac{v^2}{c^2}\right)^{1/2}\right)$$

$$\frac{\Delta l}{l_0} = 1-\left(1-\frac{v^2}{c^2}\right)^{1/2}$$

$$1-\frac{v^2}{c^2} = \left(1-\frac{\Delta l}{l_0}\right)^2$$

Here $\Delta l << l_0$

$$1-\frac{v^2}{c^2} = 1-\frac{2\Delta l}{l_0},$$

$$V^2 = \frac{2\Delta l c^2}{l_0} \quad \text{----- (i)}$$

$$\frac{\Delta l}{l_0}\times 100 = 0.5$$

$$\frac{\Delta l}{l} = \frac{0.5}{100}$$

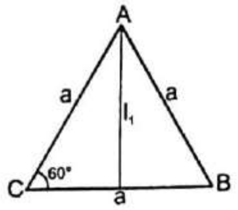
Put in (i)

$$V^2 = 2\left(\frac{0.5}{100}\right)c^2$$

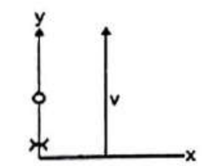
$V = 0.1C$ **Ans.**

1.341*

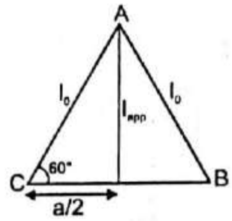
(a)



Here $l_1 = a \sin 60 = \frac{a\sqrt{3}}{2}$



length l_1 appeared from reference frame (x–y)



$$l_{app} = l_1\sqrt{1-v^2/c^2}$$

$$l_{app} = \frac{a\sqrt{3}}{2}\sqrt{1-v^2/c^2}$$

$$l_0 = \sqrt{\left(\frac{a}{2}\right)^2 + l_{app}^2}$$

$$= \sqrt{\frac{a^2}{4}+\frac{3a^2}{4}(1-v^2/c^2)} = \frac{a}{2}\sqrt{4-3\frac{v^2}{c^2}}$$

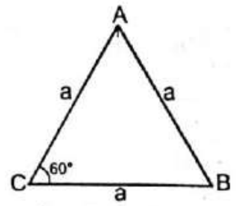
Perimeter (P) $= l_0 + l_0 + a = a + a\sqrt{4-3v^2/c^2}$

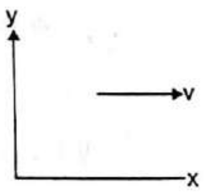
$$P = a\left(1+\sqrt{4-3v^2/c^2}\right)$$ **Ans.**

For $v << c$

$P = a(1+2) = 3a$ **Ans.**

(b)

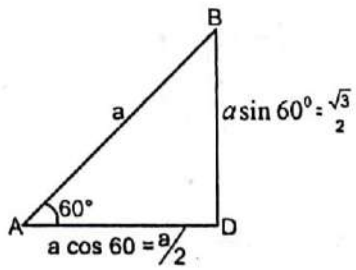




Length AC

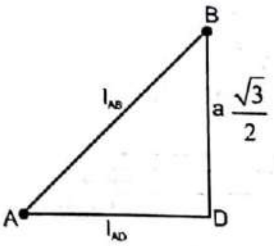
$$l_{AC} = a\left(\sqrt{1-v^2/c^2}\right)$$

Length AB



There will no change of length of BD while there is change in length of AD

$$l_{AD} = \frac{a}{2}\sqrt{1-{v^2}/{c^2}}$$



Now appeared length AB

$$l_{AD} = \sqrt{l_{AD}^2 + a^2\frac{3}{4}} = \frac{a}{2}\sqrt{4-{v^2}/{c^2}}$$

Now perimeler (P)

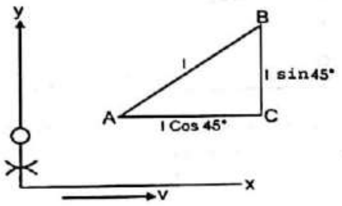
$$P = 2l_{AB} + AC = a\sqrt{4-{v^2}/{c^2}} + a\sqrt{4-{v^2}/{c^2}}$$

$$P = a\left(\sqrt{4-{v^2}/{c^2}} + \sqrt{1-{v^2}/{c^2}}\right) \qquad \textbf{Ans.}$$

$v << c$

$P = 3a$ **Ans.**

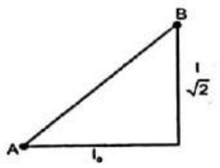
1.342*



Here only length in x direction is changed.
Actual length of AC = l_0
Then

$$l\cos 45° = l_0\sqrt{1-{v^2}/{c^2}}$$

$$l_0 = \frac{l}{\sqrt{2}}\sqrt{1-{v^2}/{c^2}}$$



Then original length AB is

$$l_{AB} = \sqrt{l_0^2 + \left(\frac{l}{\sqrt{2}}\right)^2} = \sqrt{\frac{l^2}{2\left(1-{v^2}/{c^2}\right)} + \frac{l^2}{2}}$$

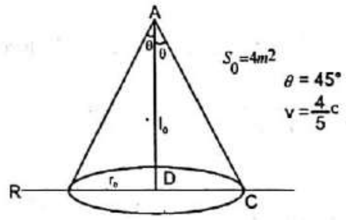
$$= \sqrt{\frac{l^2}{2}\left(1 + \frac{1}{1-{v^2}/{c^2}}\right)}$$

$$l_{AB} = l\sqrt{\frac{1}{2}\frac{\left(2-{v^2}/{c^2}\right)}{1-{v^2}/{c^2}}} = l\sqrt{\frac{1-{v^2}/{2c^2}}{1-{v^2}/{c^2}}}$$

$$l_{AB} = l\sqrt{\frac{1-{v^2}/{c^2}\,\sin^2 45°}{1-{v^2}/{c^2}}} \qquad \textbf{Ans.}$$

1.343*

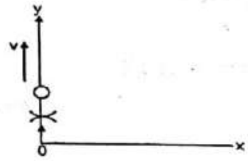
(a)



BD = radius of cone = r_0
length of AD = heigth of cone = l_0

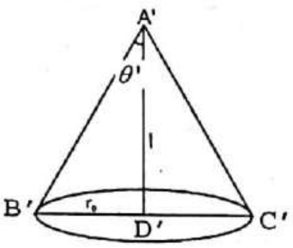
$$\tan\theta = \frac{r_0}{l_0}$$

Then lateral surface area
$S_0 = \pi r_0 l_0 \sec\theta$



From this reference frame length along x axis doest not appear to change .
Hence from this reference frame
BD = r_0
length l_0 will be appear as l

$$l = l_0\sqrt{1-{v^2}/{c^2}}$$



$$\tan\theta' = \frac{r_0}{l} = \frac{r_0}{l_0\sqrt{1-v^2/c^2}}.$$

$$\tan\theta' = \frac{\tan\theta}{\sqrt{1-v^2/c^2}}$$

Put value of θ and v in above function

$\theta' = 59°$ **Ans.**

(b)

$A'B' = l \sec\theta'$

Lateral surface area

$$\delta = \pi r_0\, l\sec\theta' = \pi r_0 l_0 \sqrt{1-v^2/c^2}\,\left[1+\tan^2\theta'\right]^{1/2}$$

$$= \pi r_0 l_0 \sqrt{1-v^2/c^2}\left[1+\frac{\tan^2\theta}{1-v^2/c^2}\right]^{1/2}$$

$$\delta = \frac{S_0}{\text{Sec}\theta}\sqrt{1-v^2/c^2}\left[\frac{\text{Sec}^2\theta - v^2/c^2}{1-v^2/c^2}\right]^{1/2}$$

$$= \frac{S_0}{\text{Sec}\theta}\sqrt{1-v^2/c^2}\,\frac{\left(\text{Sec}^2\theta\left(1-v^2/c^2\right)\right)^{\frac{1}{2}}}{\left(1-v^2/c^2\right)^{1/2}}$$

$$S = S_0\left[1-\frac{v^2\cos^2\theta}{c^2}\right]^{1/2}.$$

Put value of v and θ and S_0

$S = 3.3\ m^2$ **Ans.**

1.344*

Time measured by moving clock = t

Actual time of moving clock = t-Δt

We know

$$t = \frac{t-\Delta t}{\sqrt{1-v^2/c^2}}$$

$$t - \Delta t = t\sqrt{1-v^2/c^2}$$

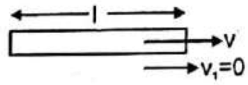
$$\left(\frac{t-\Delta t}{t}\right)^2 = 1 - v^2/c^2$$

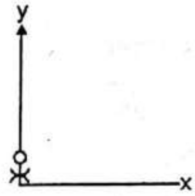
$$v^2/c^2 = \frac{t^2-(t-\Delta t)^2}{t^2} = \frac{(2t-\Delta t)\Delta t}{t^2}$$

$$v = c\sqrt{\left(2-\frac{\Delta t}{t}\right)\frac{\Delta t}{t}}$$

$v = 0.6 \times 10^8$ m/s **Ans.**

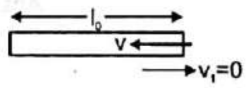
1.345*

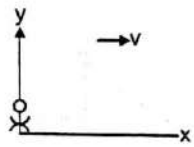




$$\Delta t = \frac{l}{v} \text{ —— (i)}$$

$$v = \frac{l}{\Delta t}$$





Proper length = l_0 = length of rod appeared by that frame from which rod will be appear to stationary.

$$l = l_0\sqrt{1-v^2/c^2}$$

$$l_0 = \frac{l}{\sqrt{1-\frac{v^2}{c^2}}}$$

Now

$$\Delta t' = \frac{l_0}{v} = \frac{1}{v\sqrt{1-v^2/c^2}}$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{l^2}{c^2\Delta t^2}}}$$

$$1-\frac{l^2}{c^2\Delta t^2}=\left(\frac{\Delta t}{\Delta t}\right)^2$$

$$l^2=c^2\Delta t^2\left(1-\left(\frac{\Delta t}{\Delta t'}\right)^2\right)$$

$$t=c\Delta t\sqrt{1-\left(\frac{\Delta t}{\Delta t'}\right)^2}$$ **Ans.**

1.346*

Asssume velocity of particle w.r.t laboratory frame is v then

$$\Delta t=\frac{\Delta t_0}{\sqrt{1-v^2/c^2}}$$

$$1-v^2/c^2=\left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\frac{v^2}{c^2}=1-\left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$v=c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}$$

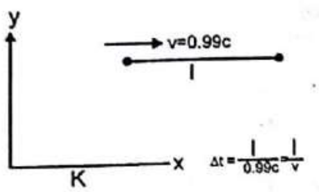
Distance travel in laboratory frame is

$$\text{Distance}=C\Delta t\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}$$

Ans.

1.347*

(a)



We know

$$\Delta t=\frac{\Delta t_0}{\sqrt{1-v^2/c^2}}$$

$$1-v^2/c^2=\left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\Delta t_0^2=\Delta t^2\left(1-v^2/c^2\right)$$

$$\Delta t_0=\Delta t\sqrt{1-v^2/c^2}$$

$$\Delta t_0=\frac{l}{v}\sqrt{1-v^2/c^2}$$ **Ans.**

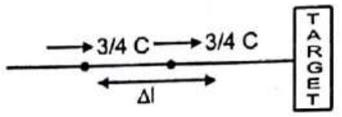
(b)

Actual distance travel is

Distance = Actual velocity x proper time

$$=v\Delta t_0=l\sqrt{1-v^2/c^2}$$ **Ans.**

1.348*



$$\Delta l=\frac{3}{4}C\Delta t \quad \text{----- (i)}$$

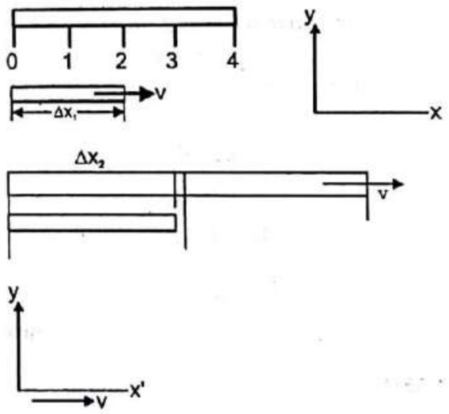
We know

$$\Delta l=\Delta l_0\sqrt{1-v^2/c^2}$$

$$\Delta l_0=\frac{\Delta l}{\sqrt{1-v^2/c^2}}=\frac{\frac{3}{4}C\Delta t}{\sqrt{1-v^2/c^2}}=\frac{v\Delta t}{\sqrt{1-v^2/c^2}}$$

$$\Delta l_0=\frac{v\Delta t}{\sqrt{1-v^2/c^2}}$$ **Ans.**

1.349*



We know proper length of rod does not change w.r.t. reference frame then

$$\Delta x_1=l_0\left(1-v^2/c^2\right)^{1/2} \text{ ---- (i)}$$

$$\Delta x_2 = l_0 \frac{1}{\left(1-v^2/c^2\right)^{1/2}} \text{ ------ (ii)}$$

Dividing both equations

$$\frac{\Delta x_1}{\Delta x_2} = 1 - v^2/c^2$$

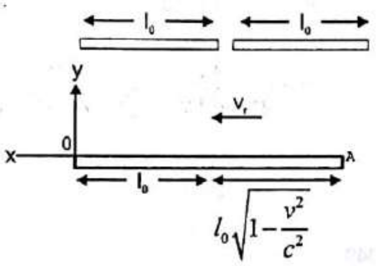
$$v^2/c^2 = 1 - \left(\frac{\Delta x_1}{\Delta x_2}\right)$$

$$v = c\sqrt{1-\left(\frac{\Delta x_1}{\Delta x_2}\right)}$$ **Ans.**

Put in (i)

$$l_0 = \sqrt{\Delta x_1 \Delta x_2}$$ **Ans.**

1.350*



Time to move from A to 0:

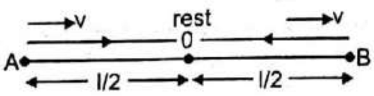
$$\Delta t = \frac{l_0 + l_0\sqrt{1-v_r^2/c^2}}{V_r}$$

$$(V_r\Delta t - l_0)^2 = l_0^2\left(1-\frac{Vr^2}{c^2}\right)$$

$$Vr = \frac{2l_0/\Delta t}{1+\frac{l_0^2}{c^2\Delta t^2}}$$

Ans.

1.351*



$$Ct_A + Vt_A = l/2 \Rightarrow t_A = \frac{l}{2(c+v)}$$

$$Ct_B - Vt_B = l/2 \Rightarrow t_B = \frac{l}{2(c-v)}$$

$t_A < t_B$ means later event occur first

$\Delta t = t_B - t_A$

$$= \frac{l}{2(c-v)} - \frac{l}{2(c+v)}$$

$$= \frac{1}{2}\left[\frac{c+v-c+v}{c^2-v^2}\right] = \frac{lV}{c^2-v^2}$$

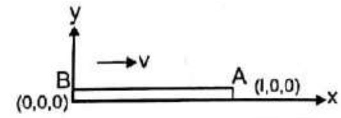
$$\Delta t = \frac{l\frac{v}{c}}{c(1-v^2/c^2)}$$

Assume $c/v = \beta$

$$\Delta t = \frac{l\beta}{C(1-\beta^2)}$$ **Ans.**

1.352*

(a)



Since rod is moving with speed V with respect to this frame, it length will be appear less than proper length

$X_A - l = Vt_A$

$x_B = Vt_B$

$(x_A - x_B) - l = V(t_A - t_B)$

$l = (x_A - x_B) - V(t_A - t_B)$

Now proper length

$$l_0 = l/\sqrt{1-v^2/c^2}$$

$$l_0 = \frac{(x_A - x_B) - V(t_A - t_B)}{\sqrt{1-v^2/c^2}}$$ **Ans.**

(b)

Here

$|x_A - x_B| = l_0$

$x_A - x_B = l_0$

or

$x_A - x_B = -l_0$

Then

$$l_0 = \frac{l_0 - V(t_A - t_B)}{\sqrt{1-v^2/c^2}}$$

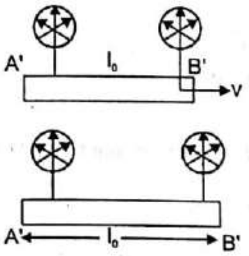
$$t_A - t_B = \frac{l_0}{V}\left(1-\sqrt{1-v^2/c^2}\right)$$

or

$$l_0 = \frac{l_0 - V(t_A - t_B)}{\sqrt{1-v^2/c^2}}$$

$$t_A - t_B = \frac{l_0}{V}\left(1-\sqrt{1-v^2/c^2}\right)$$ **Ans.**

1.353*



(a)
Reading of clock at B

$$t(B) = \frac{l_0}{V}$$ **Ans.**

Reading of clock at B'

$$l = l_0\sqrt{1-v^2/c^2}$$

$$t'(B) = \frac{l}{V} = \frac{l_0}{V}\sqrt{1-v^2/c^2}$$ **Ans.**

(b)
Appeanent length of AA' from A

$$l_{AA'} = l_0\sqrt{1-v^2/c^2}$$

$$t(A) = \frac{l_0}{V}\sqrt{1-v^2/c^2}$$ **Ans.**

When B' will be at B then distance travelled by point A' in frame or rod A'B' will be

$$t(A') = \frac{l_0}{V}$$ **Ans.**

1.354*

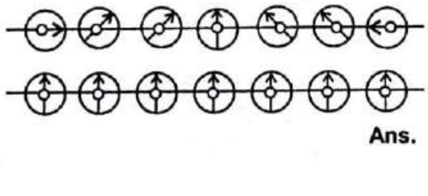
Lorentz transformation of time

$$t' = \frac{t - xV/c^2}{\sqrt{1-v^2/c^2}}.$$

If take t =0 for k frame then

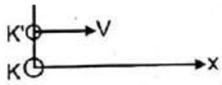
$$t' = \frac{-xV/c^2}{\sqrt{1-v^2/c^2}}$$

If $x > 0 \Rightarrow t < 0$
If $x < 0 \Rightarrow t' > 0$



Ans.

1.355*



Since both show zero reading at origin. If clock (k) reads time t and clock (k') reads time t' then according to lorentz transformation.

$$t' = \frac{t - xV/c^2}{\sqrt{1-v^2/c^2}}$$

According to question : $t' = t$

Then

$$t' = \frac{t - xV/c^2}{\sqrt{1-v^2/c^2}} = t$$

Differentiate w.r.f. time

$$\frac{t - V_x\frac{V}{c^2}}{\sqrt{1-v^2/c^2}} = t$$

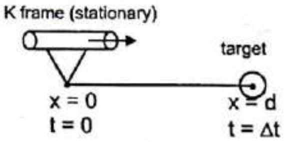
$$V_x = \frac{c^2}{V}\left(1-\sqrt{1-v^2/c^2}\right)$$

We know

$\beta = V/c$

$$V_x = \frac{c}{\beta}\left(1-\sqrt{1-\beta^2}\right)$$ **Ans.**

1.356*



Suppose a shot is made and it hit the target after time Δt then from k' frame (mouing with V velocity)

$$\Delta t' = \frac{\Delta t - x\,v/c^2}{\sqrt{1-v^2/c^2}}$$

$$\Delta t' = \frac{\Delta t - d\,V/c^2}{\sqrt{1-v^2/c^2}}.$$

If $\Delta t' < 0$ means target is hit before shot is fired then.
$\Delta t' - dV/c^2 < 0$
$dV > c^2\,\Delta t$

$$\left(\frac{d}{\Delta t}\right)V > c^2$$

If $\dfrac{d}{\Delta t} = V_x$

Then

$V_x V > c^2$

It is possible only if one of the $V_x > c$ or $V > c$ which is not possible .

It prove that target will be hit after shot made

1.357*

(a)

We know invariant formula

$$c^2t'^2_{12} - x'^2_{12} = c^2t^2_{12} - x^2_{12}$$

In frame K' both events occure at same point Then

$x'_{12} = 0$ Then

$$c^2t'_{12} = c^2t^2_{12} - (x_2 - x_1)^2$$

$$= c^2(t_2 - t_1)^2 - (x_2 - x_1)^2$$

$$c^2t'_{12} = (ct_2 - ct_1)^2 - (x_2 - x_1)^2$$

$$t'^2_{12} = \frac{16}{(3\times10^8)^2}$$

$$t'_{12} = \frac{4}{3\times10^8} \cong 1.3\times10^{-8}\,s \cong 13ns$$

Ans.

(b)

In frame k' if both occure simulaneously then

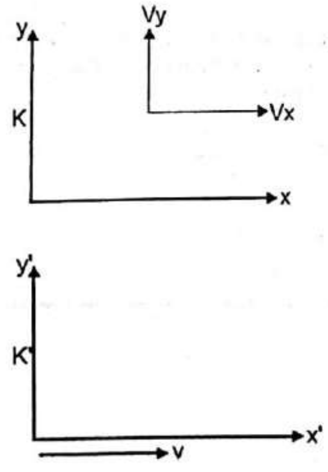
$$t'_{12} = 0$$

$$-x'^2_{12} = c^2t^2_{12} - x^2_{12} = 16$$

$$x'_{12} = 4m$$

Ans.

1.358*



We know

$$V'_x = \frac{V_x - V}{1 - \frac{V_x V}{c^2}}$$

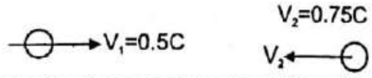
$$V'_y = \frac{V_y\sqrt{1 - v^2/c^2}}{1 - V_x\frac{V}{c^2}}$$

Then net velocity appear from k' frame

$$V_{net} = \sqrt{V'^2_x + V'^2_y}$$

$$V_{net} = \frac{\sqrt{(V_x - V)^2 + V_y^2(1 - V^2/c^2)}}{1 - V_x\ V/c^2}$$ **Ans.**

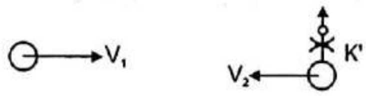
1.359*



Velocity of approach is taken from laboratory frame hence

Velocity of approach = $V_1 + V_2 = 0.5C + 0.75C$
$= 1.25C$ **Ans.**

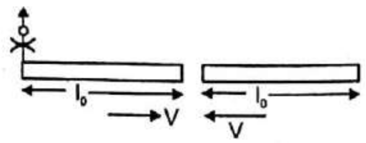
Relative velocity



$$V'_x = \frac{V_1 - (-V_2)}{1 - \frac{V_1(-V_2)}{c^2}}$$

$$V'_x = \frac{V_1 + V_2}{1 + V_1 V_2/c^2}$$ **Ans.**

1.360*



$$V_{rel} = \frac{V - (-V)}{1 - (V)(-V)/c^2} = \frac{2V}{1 + V^2/c^2} = \frac{2V}{1 + \beta^2}$$

Now apparent length is

$$l = l_0\sqrt{1 - \frac{V^2_{rel}}{c^2}}$$

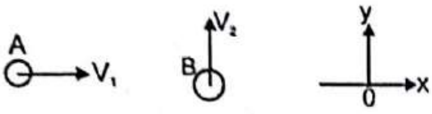
$$l = l_0\sqrt{1 - \frac{(2V)^2}{(1+\beta^2)^2c^2}}$$

$$l = l_0\sqrt{1-\frac{4\beta^2}{\left(1+\beta^2\right)^2}}$$

$$l = l_0\frac{\left(1-\beta^2\right)}{\left(1+\beta^2\right)}$$

Ans.

1.361*



$$\begin{array}{l|l} V_{Ax} = V_1 & V_{Bx} = 0 \\ V_{Ay} = 0 & V_{By} = V_2 \end{array}$$

From reference frame A : velocitycomponents are as shown

$$V'_{Bx} = \frac{0-V_1}{1-oV_1/c^2} = -V_1$$

$$V'_{By} = \frac{V_2\sqrt{1-V_1^2/c^2}}{1-0V_1/c^2} = V_2\sqrt{1-V_1^2/c^2}$$

Then

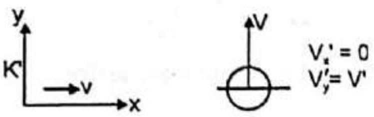
$$V_{net} = \sqrt{V'^2_{Bx} + V'^2_{By}}$$

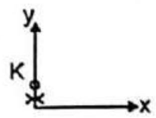
$$V_{net} = \sqrt{V_1^2 + V_2^2 + \left(1-V_1^2/c^2\right)}$$

$$V_{net} = \sqrt{V_1^2 + V_2^2 - \left(\frac{V_1V_2}{c}\right)^2}$$

Ans.

1.362*





To find velocity of particle in k frame assume components of velocity in k frame is Vx and Vy then using lorenz transformation :

$$V_x = \frac{V_x - V_y}{1-V_xV/c^2}$$

$$V_y = \frac{V_y\sqrt{1-V^2/c^2}}{1-V_xV/c^2}$$

Then

From frame K'

$$0 = \frac{V_x - V}{1-\frac{V_xV}{c^2}}$$

$$V_x = V$$

$$V' = \frac{V_y\sqrt{1-\left(\frac{V}{c}\right)^2}}{1-\left(\frac{V}{c}\right)^2}$$

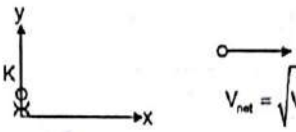
$$V_y = V'\sqrt{1-\left(\frac{V}{c}\right)^2}$$

Now

$$V_{net} = \sqrt{V^2 + V'^2\left(1-V^2/c^2\right)}$$

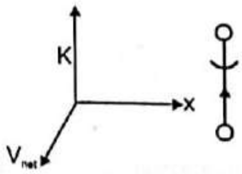
$$= \sqrt{V^2 + V'^2 - \frac{V^2V'^2}{c^2}}$$

Hence we say that V_{net} is velocity from k frame then



$$V_{net} = \sqrt{V^2 - V'^2 - \frac{V^2V'^2}{C^2}}$$

For proper time Δt_0,we have to choose another frame k" which is attached with particle. Then



Reference frame k will be appear to move with V_{net} speed then time interval will be appear to increase from this frame (k) then .

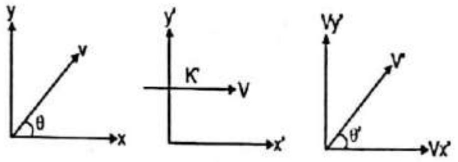
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-V^2_{net}/c^2}}$$

Distance travel in k frame

$$\text{Distance} = V_{net}\,\Delta t = V_{net}\frac{\Delta t_0}{\sqrt{1 - V^2_{net}/c^2}}$$

$$\text{Distance} = \frac{\Delta t_0\sqrt{V^2 + V'^2\left(1 - v^2/c^2\right)}}{\left(1 - v^2/c^2\right)\left(1 - v'^2/c^2\right)} \quad \textbf{Ans.}$$

1.363*

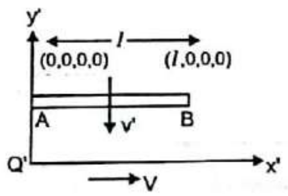


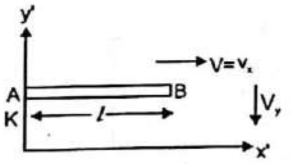
$$V'x = \frac{V_x - V}{1 - \frac{V_x V}{c^2}} = \frac{V\cos\theta - V}{1 - \frac{(v\cos\theta)V}{c^2}} \quad ---- \quad (i)$$

$$V_y' = \frac{V_y\sqrt{1 - v^2/c^2}}{1 - \frac{V_x V}{c^2}} = \frac{V\sin\theta}{1 - V\cos\theta\left(\frac{V}{c^2}\right)}\sqrt{1 - v^2/c^2}$$

$$\tan\theta' = \frac{Vy'}{Vx'} = \frac{V\sin\theta}{V\cos - V}\sqrt{1 - v^2/c^2} \quad \textbf{Ans.}$$

1.364*





Here in k frame length of rod will be contracted because it is moving with V velocity then

$l = l_0\sqrt{1 - v^2/c^2}$

Velocity of rod $V_x = V$

$$V_y = \frac{V_y'\sqrt{1 - (v/c)^2}}{1 - V_x' V/c^2} = \frac{v'\sqrt{1 - (v/c)^2}}{1 - ov/c^2}$$

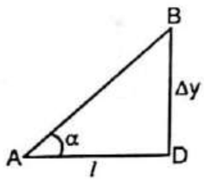
$$V_y = v'\sqrt{1 - v^2/c^2}$$

Time difference of measuring for A and B

$$t = \frac{t' - x'v/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t = \frac{l_0 v/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\Delta y = V_y\Delta t = l_0 V\, v'/c^2$$



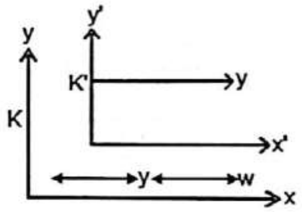
$$\tan\alpha = \frac{\Delta y}{l} = \frac{l_0 V\, v'/c^2}{l_0\sqrt{1 - v^2/c^2}}$$

$$\tan\alpha = \frac{V\,v'}{c^2\sqrt{1 - v^2/c^2}}$$

Ans.

1.365*

(a)



We know

$$Vx' = \frac{V_x - V}{1 - V_x V/c^2}$$

Now differentiate equation :

$$dVx' = \frac{\left(1 - Vx\,V/c^2\right)(dVx) - (Vx - V)\left(-(dVx)V/c^2\right)}{\left(1 - VxV/c^2\right)^2}$$

$$= dV_x\left[1 - Vx\,V/c^2 + Vx\,V/c^2 - V^2/c^2\right]/\left(1 - Vx\,V/c^2\right)^2$$

$$\frac{dVx'}{dt'} = \frac{(dv_x)}{dt}\left(1 - V^2/c^2\right)/\left(1 - V_x V/c^2\right)^2 \quad \text{—— (i)}$$

Also we know

$$t' = \frac{t - xV/c^2}{\sqrt{1-V^2/c^2}}$$

$$dt' = \frac{dt - (dx)V/c^2}{\sqrt{1-V^2/c^2}}$$

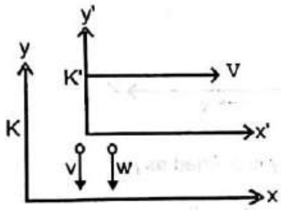
$$\frac{dVx'}{dt'} = \frac{(dVx)\sqrt{1-V^2/c^2}\left(1-V^2/c^2\right)}{\left(dt - dx\,V/c^2\right)\left(1-V_x\,V/c^2\right)^2}$$

$$= \frac{\left(\frac{dV_x}{dt}\right)\left(1-V^2/c^2\right)^{3/2}}{\left(1-\frac{V_xV}{c^2}\right)\left(1-\frac{V_xV}{c^2}\right)^2}$$

$$w' = \frac{w\left(1-V^2/c^2\right)^{3/2}}{\left(1-V_xV/c^2\right)^3}$$

$Vx = v$ **Ans.**

(b)



we know

$$Vx' = \frac{Vx - V}{1 - Vx\,V/c^2} \Rightarrow Vx' = \frac{0-V}{1-0} = -V$$

$a_{x'} = 0$

Also

$$V_y' = \frac{V_y\sqrt{1-V^2/c^2}}{1-V_x\,V/c^2}$$

$$V_y' = \frac{V_y\sqrt{1-V^2/c^2}}{1-0} = V_y\sqrt{1-V^2/c^2}$$

$$\frac{dV_y'}{dt'} = \frac{dV_y}{dt'}\sqrt{1-V^2/c^2}$$

From option (a) :

$$dt' = \frac{dt - (dx)V/c^2}{\sqrt{1-V^2/c^2}}$$

$$\frac{dV_y'}{dt'} = \frac{dV_y}{dt - (dx)V/c^2}\left(1-V^2/c^2\right)$$

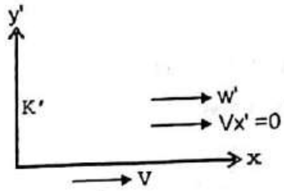
$$= \frac{\left(dV_y/dt\right)}{1-\left(\frac{dx}{dt}\right)V/c^2}\left(1-V^2/c^2\right)$$

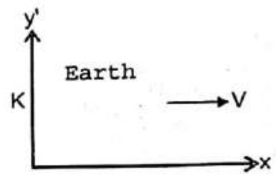
$$= \frac{w}{1-V_x\,V/c^2}\left(1-V^2/c^2\right)$$

$w' = w\left(1-V^2/c^2\right)$ **Ans.**

Here $V_x = 0$ **Ans.**

1.366*





Moving frame (k') is that frame in which particle is appear to instanteously rest. But its acceleration of particle in this frame may or may not zero.

Now we know:

$$w' = \frac{w\left(1-V^2/c^2\right)^{3/2}}{\left(1-V_xV/c^2\right)^3}$$

From Q : 1.365

Here Vx = V then

$$w' = \frac{w\left(1-V^2/c^2\right)^{3/2}}{\left(1-V^2/c^2\right)^3} = w\left(1-V^2/c^2\right)^{-3/2}$$

$$w = w'\left(1-V^2/c^2\right)^{3/2}$$

$$dv = w'\left(1-V^2/c^2\right)^{3/2}dt$$

$$\int_0^x \frac{dv}{\left(1-V^2/c^2\right)^{3/2}} = \int_0^t w'\,dt$$

$$V = \frac{w't}{\sqrt{1+\left(\frac{w't}{c}\right)^2}}$$ **Ans.**

$$\int_0^x dx = \int_0^t \frac{w't}{\left(1+(w't/c)^2\right)^{1/2}}dt$$

$$x = \frac{c^2}{w'}\left(\sqrt{1+\left(\frac{w't}{c}\right)^2}-1\right)$$

$$\frac{c-v}{c} = \frac{1}{2}\left(\frac{c}{w't}\right)^2$$

Ans.

1.367*

$$\tau_0 = \int_0^\tau \sqrt{1-(v/c)^2}\ dt \text{ —— (i)}$$

From Q : 1.366

$$V = \frac{w't}{\sqrt{1+\left(\frac{w't}{c}\right)^2}}$$

Put in (i)

$$\tau_0 = \int_0^\tau \sqrt{1-\frac{(w't)^2}{\left[1+\left(\frac{w't}{c}\right)^2\right]c^2}}dt$$

$$= \int_0^\tau \sqrt{1-\frac{(w't)^2}{c^2+(w't)^2}}\ dt = \int_0^\tau \sqrt{\frac{c^2}{c^2+(w't)^2}}\ dt$$

$$\tau_0 = \int_0^\tau \frac{dt}{1+\left(\frac{w't}{c}\right)^2} = \frac{c}{w'}\ln\left[\frac{w'\tau}{c}+\sqrt{1+\left(\frac{w'\tau}{c}\right)^2}\right]$$

$$\tau_0 = \frac{c}{w'}\ \ln\left[\frac{w'\tau}{c}+\sqrt{1+\left(\frac{w'\tau}{c}\right)^2}\right]$$ **Ans.**

1.368*

We know

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

Here $\beta = \frac{V}{c}$

$$m = \frac{m_0}{\sqrt{1-\beta^2}} = \frac{m_0}{\sqrt{(1-\beta)(1+\beta)}}$$

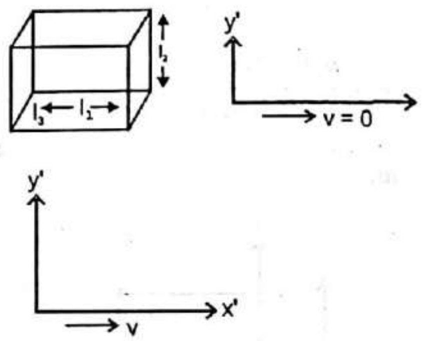
$$\beta = \frac{v}{c} = \frac{C-\frac{0.01}{100}C}{C} \cong 1-\frac{.01}{100}$$

$$\beta \cong 1$$

$$m = \frac{m_0}{\sqrt{2(1-\beta)}}$$

$$\frac{m}{m_0} = \frac{1}{\sqrt{2(1-\beta)}} = 70$$ **Ans.**

1.369*



Density is defined as ρ

$$\frac{\text{rest maes}}{\text{volume}} = \frac{m_0}{\text{volume}}$$

$$\rho_0 = \frac{m_0}{l_1l_2l_3}\ldots\ldots\ldots\ldots(1)$$

$$\rho = \frac{m_0}{l_1'l_2'l_3'}$$

Here $l_2 = l_2'$
$l_3 = l_3'$

$$\ell' = \ell_1\sqrt{1-v^2/c^2}$$

$$\rho = \frac{m_0}{l_1l_2l_3\sqrt{1-v^2/c^2}}\ldots\ldots\ldots\ldots(2)$$

From (1)/ (2) :

$$\frac{\rho_0}{\rho} = \sqrt{1-v^2/c^2}$$

$$\rho = (1+\eta)\rho_0 \Rightarrow \frac{\rho_0}{\rho} = \frac{1}{1+\eta}$$

Then $\frac{1}{1+\eta} = \sqrt{1-v^2/c^2}$

$$V = \frac{C}{(1+\eta)}\sqrt{\eta(2+\eta)}$$ **Ans.**

1.370*

Assume mass of proton is m_0 then
P=mV

$$P = \frac{mV_0}{\sqrt{1-\frac{V^2}{C^2}}}$$

Squaring both sides and value of v

$$V = \frac{P}{\sqrt{m_0^2 + \frac{P^2}{C^2}}} = \frac{C}{\sqrt{1+\frac{m_0^2C^2}{P^2}}}$$

$$\frac{C-V}{C} = 1 - \frac{V}{C} = 1 - \frac{1}{\sqrt{1+\frac{m_0^2C^2}{P^2}}}$$

$$\frac{C-V}{C} = 1 - \left(1+\frac{m_0^2C^2}{P^2}\right)^{-\frac{1}{2}}$$

Ans.

1.371*

We know Newtonium momentum
$P = m_0V$ —— (i)
Reletivistic momentum:

$$2P = \frac{m_0}{\sqrt{1-v^2/c^2}}V \text{ —— (ii)}$$

Using $^{(i)}/_{(ii)}$:

$$\frac{1}{2} = \sqrt{1-v^2/c^2}$$

$$\frac{1}{4} = 1 - v^2/c^2$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$v = \frac{\sqrt{3}}{2}c$$

Ans.

1.372*

Classical mechanics
$\Delta w = K_f - K_i$

$$= \frac{1}{2}m_0(0.8)^2c^2 - \frac{1}{2}m_0(0.6)^2c^2$$

$$= \frac{m_0}{2}(.64-.36)c^2$$

$\Delta w = 0.14\ m_0c^2$ **Ans.**

Relativistic mechanics

$$\Delta w = m_fc_f^2 - m_ic_i^2$$

$$= \frac{m_0}{\sqrt{1-0.8^2}}0.8^2c^2 - \frac{m_0}{\sqrt{1-0.6^2}}0.6^2c^2$$

$= m_0 \times 0.42\ c^2$

$\Delta w = 0.42\ c^2$ **Ans.**

1.373*

m_0c^2 = Rest mass energy
mc^2 = Total energy
Kinetic energy = $mc^2 - m_0c^2$
According to question
$m_0c^2 = mc^2 - m_0c^2$
$m = 2m_0$

$$\frac{m_0}{\sqrt{1-v^2/c^2}} = 2m_0$$

$$1 - v^2/c^2 = \frac{1}{4} \Rightarrow v^2/c^2 = \frac{3}{4}$$

$$v = \frac{\sqrt{3}}{2}c$$ **Ans.**

1.374*

Using classical mechanics :

$$T = \frac{1}{2}m_0v_c^2$$

where Vc = velocity calculated by classical mechanics

$$V_c = \sqrt{\frac{2T}{m_0}}$$

Relativistic mechanics :

$T = mc^2 - m_0c^2$

$$T = \frac{m_0c^2}{\sqrt{1-v^2/c^2}} - m_0c^2$$

$$\frac{T}{m_0c^2} = \left(1 - v^2/c^2\right)^{-1/2} - 1$$

$$= 1 + \frac{v^2}{2c^2} + \frac{(-1/2)(-1/2-1)}{2}\frac{v^4}{c^4} + \ldots - 1$$

$$\frac{T}{m_0c^2} = 1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^2}{c^4} - 1$$

$$= \frac{v^2}{2c^2}\left(1 + \frac{3}{4}v^2/c^2\right)$$

Approximately

$$\frac{v^2}{2c^2} \cong \frac{T}{m_0c^2}$$

Then

$$\frac{T}{m_0c^2} = \frac{v^2}{2c^2}\left(1 + \frac{3}{2}\frac{T}{m_0c^2}\right)$$

$$\frac{v^2}{c^2} = 2\frac{T}{m_0c^2}\left(1 + \frac{3}{2}\frac{T}{m_0c^2}\right)^{-1} = \frac{2T}{m_0c^2}\left(1 - \frac{3}{2}\frac{T}{m_0c^2}\right)$$

$$V = \sqrt{2T/m_0}\left(1 - \frac{3}{4}\frac{T}{m_0c^2}\right)$$

Now

$$\frac{V - V_c}{V_c} = \varepsilon$$

$$\frac{\sqrt{2T/m_0}\left(1 - 3/4\frac{T}{m_0c^2}\right) - \sqrt{\frac{2T}{m_0}}}{\sqrt{\frac{2T}{m_0}}} = \varepsilon$$

$$\frac{T}{m_0c^2} = \frac{4}{3}\varepsilon \text{ at max}$$ **Ans.**

1.375*

We know

$E^2 = m_0^2c^4 + p^2c^2$ —— (i)

Let us kinetic energy is T then

$E = m_0c^2 + T$

Put in (i)

$$\left(m_0c^2 + T\right)^2 = m_0c^4 + P^2c^2$$

$$m_0^2c^4 + T^2 + 2m_0c^2T = m_0^2c^4 + P^2c^2$$

$$P^2c^2 = T\left(T + 2m_0c^2\right)$$

$$P = \frac{1}{c}\sqrt{T\left(T + 2m_0c^2\right)}$$ **Ans.**

1.376*

From question no. 1.375

$$P = \frac{1}{c}\sqrt{T\left(T + 2m_0c^2\right)}$$

If no. of particle collide per second is n then

$ne = I$

$$n = \frac{I}{e}$$

Momentum transfer per second is :

$$F = \frac{dp}{dt} = np = \frac{I}{Ce}\sqrt{T\left(T + 2m_0c^2\right)}$$

Power = Energy radiate or absorb per second

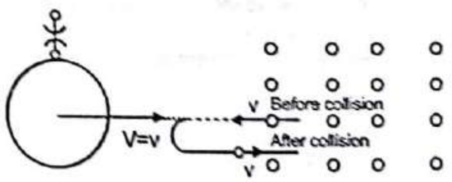
Power = n (kinetic energy of one particle)

n T

$$\text{Power} = \frac{I}{e}T$$ **Ans.**

1.377*

From sphere Reference frame



Velocity of gas particle in frame of sphere

$$V_x' = \frac{V_x - V}{1 - V_x V/c^2}$$

Here $V_x = 0$; $V = v$

$Vx' = -v$

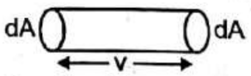
Momentum transfer in one collision

$$dP = \frac{m(v)}{\sqrt{1-v^2/c^2}} - \frac{(-mv)}{\sqrt{1-v^2/c^2}} = \frac{2mv}{\sqrt{1-v^2/c^2}}$$

Since volume will be decreased by a factor of $\sqrt{1-v^2/c^2}$

But actually no. of particles of gas does not change but its occupied volume is decreased and hence apparent no. of particles per unit volume is increased.

$$\eta_{app} = \frac{\eta}{\sqrt{1-v^2/c^2}}$$



No. of particles collides per second

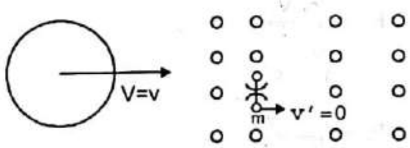
$$N_{app} = \left(\frac{\eta}{\sqrt{1-v^2/c^2}}\right)(dA)V$$

$$dF = \frac{dp}{dt}\left(\frac{2mv}{\sqrt{1-v^2/c^2}}\right)\left(\frac{\eta}{\sqrt{1-v^2/c^2}}\right)(dA)V$$

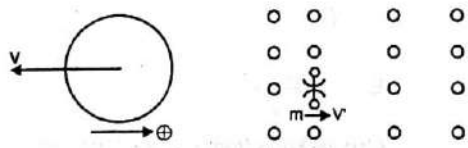
$$\text{Pressure} = \frac{dF}{dA} = \frac{2mv^2\eta}{1-v^2/c^2}$$

$$P = \frac{2\eta mv^2}{1-v^2/c^2}$$ **Ans.**

Gas Refrence frame



Before collision



After collision

We know

$$V_x' = \frac{V_x - v'}{1 - V_x V'/c^2}$$

$V_x' = -v$ = Velocity of sphere w.r.t. gas frame after collision

$V_x = v$ =Actual velocity of sphere after collision which does not change

$$-v = \frac{v - V'}{1 - v\dfrac{V'}{c^2}}$$

$$V^2\left(1 + \frac{v^2v'^2}{c^4} - \frac{2vv'}{c^2}\right) = V^2 + v'^2 - 2vv'$$

$$V'\left(\frac{v^4}{c^4} - 1\right) = \frac{2v^3}{c^2} - 2v$$

$$V' = \frac{2v}{1 + \dfrac{v^2}{c^2}}$$

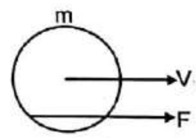
Momentum transfer in one collision$= \dfrac{m2V}{1 + v^2/c^2}$

$$\frac{dF}{dA} = \left(\frac{2mV}{1 + v^2/c^2}\right)V\eta$$

$$P = \frac{2\eta mv^2}{1 + v^2/c^2}$$ **Ans.**

Since in gas frame mass of gas particle and its density does not change

1.378*



At time t, letus velocity of phsere is V then

$$F = ma = \left(\frac{m_0}{\sqrt{1-v^2/c^2}}\right)\frac{dV}{dt}$$

$$\frac{F}{m_0}\int_0^t dt = \int_0^t \sqrt{1-v^2/c^2}\, dv$$

$$v = \frac{Fct}{\sqrt{m_0^2c^2 + F^2t^2}}$$

Also

$$v = \frac{ds}{dt} = \frac{Fct}{\sqrt{m_0^2c^2 + F^2t^2}}$$

$$\int_0^s ds = FC\int_0^t \frac{t\,dt}{\sqrt{m_0^2c^2 + F^2t^2}}$$

$$S = \sqrt{\left(m_0^2c^2/F\right)^2 + c^2t^2} - \frac{m_0c^2}{F}$$ **Ans.**

1.379*

Given

$$x = \sqrt{a^2 + c^2t^2}$$

$$v = \frac{dx}{dt} = \frac{c^2t}{a^2 + c^2t^2}$$

$$\frac{dV}{dt} = \left(a^2 + c^2t^2\right)c^2 - \frac{c^2t\left(2c^2t\right)}{a^2 + c^2t^2}$$

$$\frac{dV}{dt} = \frac{a^2 - c^2t^2}{a^2 + c^2t^2}$$

Momentum

$$P = mv = \frac{m_0}{\sqrt{1 - v^2/c^2}}v$$

$$\frac{dp}{dt} = F = \frac{\left(1 - v^2/c^2\right)m_0\left(\frac{dv}{dt}\right) - m_0 v\frac{d}{dt}\left(1 - v^2/c^2\right)}{\left(1 - v^2/c^2\right)}$$

Put value of dv/dt

$$F = \frac{m_0c^2}{a}$$ **Ans.**

1.380*

We know

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}\left(\frac{m_0\vec{V}}{\sqrt{1 - v^2/c^2}}\right) = m_0\frac{d}{dt}\left(\vec{v}\left(1 - v^2/c^2\right)^{-1/2}\right)$$

$$\vec{F} = m_0\left[\left(1 - v^2/c^2\right)^{-1/2}\frac{d\vec{V}}{dt} + \left(\frac{-1}{2}\right)\vec{V}\left(1 - v^2/c^2\right)^{-3/2}\left(\frac{-2V}{c^2}\right)\frac{dv}{dt}\right]$$

We know

$$\vec{a} = \frac{d\vec{v}}{dt} \text{ and } \frac{dv}{dt} = a_t = \vec{a}.\hat{v}$$

Then

$$\vec{F} = m_0\left[\frac{\vec{a}}{\sqrt{1 - v^2/c^2}} + \frac{V}{c^2}\left(1 - v^2/c^2\right)^{-\frac{3}{2}}\vec{V}.\vec{a}.\vec{v}\right]$$

If $\vec{a}\perp\vec{v} \Rightarrow \vec{a}.\vec{v} = 0 = av$ then

$$\vec{F} = \frac{m_0\vec{a}}{\sqrt{1 - v^2/c^2}}$$

If $\vec{a}\,||\,\vec{v} \Rightarrow \vec{a}.\vec{v} = av$ then

$$\vec{F} = \frac{m_0\vec{a}}{\sqrt{1 - v^2/c^2}} + \frac{m_0 a v\vec{v}}{c^2\left(1 - v^2/c^2\right)^{3/2}}$$

$$= \frac{m_0\vec{a}}{\sqrt{1 - v^2/c^2}} + \frac{m_0 v^2/c^2\vec{a}}{\left(1 - v^2/c^2\right)^{3/2}}$$

$$\vec{F} = \frac{m_0\vec{a}}{\sqrt{1 - v^2/c^2}}\left(1 + \frac{v^2/c^2}{\left(1 - v^2/c^2\right)}\right)$$

$$\vec{F} = \frac{m_0\vec{a}}{\left(1 - v^2/c^2\right)^{3/2}}$$

Ans.

1.381*

Assume velocity of particle in K frame is V_x then we know

$$P_x = \frac{m_0V_x}{\sqrt{1 - \frac{V_x^2}{c^2}}}$$

$$P_x = \frac{m_0c\,V_x}{\sqrt{c^2 - V_x^2}}$$

$$E = \frac{m_0c^2}{\sqrt{1 - \frac{V_x^2}{c^2}}}$$

$$E = \frac{m_0c^2}{\sqrt{c^2 - V_x^2}}$$

Also we know from invarient theorem

$(ds)^2 = c^2(dt)^2 - (dx)^2$ = constant

$= c^2(dt)^2 - (V_x dt)^2$

$(ds)^2 = (c^2 - V_x^2)(dt)^2$

$\sqrt{c^2 - Vx^2} = \frac{ds}{dt}$ = constant in any intertial frame of references

Then

$$P_x = \frac{m_0 c V_x}{(ds)} dt \text{ —— (i)}$$

$$E = m_0 c^3 \left(\frac{dt}{ds}\right) \text{ —— (ii)}$$

$V_x dt = dx$

Also

$$P_x{}' = \frac{m_0 c V_x{}' dt'}{(ds)}$$

$$= \frac{m_0 c}{ds}[dx'] = \frac{m_0 c}{ds}\left[\frac{dx - vdt}{\sqrt{1 - v^2/c^2}}\right]$$

$$P'_x = \frac{m_0 c}{\sqrt{1-\beta^2}}\left(\frac{dx}{ds}\right) - \frac{m_0 c V dt}{\sqrt{1-\beta^2} ds}$$

$$P'_x = \frac{P_x}{\sqrt{1-\beta^2}} - \frac{V}{\sqrt{1-\beta^2}}\left(\frac{m_0 c}{\sqrt{c^2 - V_x^2}}\right)$$

$$P_x{}' = \frac{P_x}{\sqrt{1-\beta^2}} - \frac{V/c^2}{\sqrt{1-\beta^2}}\left(\frac{m_0 c^3}{\sqrt{c^2 - V_x^2}}\right)$$

$$P_x{}' = \frac{P_x}{\sqrt{1-\beta^2}} - \frac{V/c^2 \, E}{\sqrt{1-\beta^2}}$$

$$P_x{}' = \frac{P_x - E\frac{V}{c^2}}{\sqrt{1-\beta^2}}$$ **Proved**

$$E' = m_0 c^3 \frac{dt'}{ds} = \frac{m_0 c^3}{ds}\left(\frac{dt - (dx)V/c^2}{\sqrt{1-\beta^2}}\right)$$

$$E' = \frac{m_0 c^3}{\sqrt{1-\beta^2}}\left(\frac{dt}{ds}\right) - \frac{m_0 c^3}{ds}\frac{dx \, V/c^2}{\sqrt{1-\beta^2}}$$

$$E' = \frac{E}{\sqrt{1-\beta^2}} - \frac{m_0 c V}{\sqrt{1-\beta^2}}\left(\frac{dx}{ds}\right)$$

$$= \frac{E}{\sqrt{1-\beta^2}} - \frac{m_0 c V}{\sqrt{1-\beta^2}}\left(\frac{V_x dt}{ds}\right)$$

$$= \frac{E}{\sqrt{1-\beta^2}} - \frac{m_0 c V_x dt}{ds}\left(\frac{V}{\sqrt{1-\beta^2}}\right)$$

$$E' = \frac{E}{\sqrt{1-\beta^2}} - \frac{P_x V}{\sqrt{1-\beta^2}}$$

$$E' = \frac{E - P_x V}{\sqrt{1-\beta^2}}$$

Ans.

1.382*

We know

$E^2 = P^2C^2 + m_0^2c^4$

Rest mass of proton (m_0)= 0

$E^2 = P^2C^2$

$E = PC$

From K frame

$$\varepsilon = p_x c$$

$$p_x = \frac{\varepsilon}{c}$$

Form k' frame

$$\varepsilon' = p'_x c$$

$$\varepsilon' = \left(\frac{p_x - E\frac{V}{c^2}}{\sqrt{1-\beta^2}}\right)c$$

$$\varepsilon' = \left(\frac{p_x - \frac{\varepsilon V}{c^2}}{\sqrt{1-\beta^2}}\right)c$$

$$\varepsilon' = \frac{\varepsilon - \frac{\varepsilon V}{c}}{\sqrt{1-\beta^2}} = \frac{\varepsilon(1-\beta^2)}{\sqrt{1-\beta^2}}$$

$$\varepsilon' = \varepsilon\sqrt{\frac{1-\beta}{1+\beta}}$$

Ans.

For

$$\varepsilon' = \frac{\varepsilon}{2} = \varepsilon\sqrt{\frac{1-\beta}{1+\beta}} \Rightarrow \beta = \frac{V}{c} = \frac{3}{5}$$ **Ans.**

1.383*

From Q1.381 We know

$$E = m_0c^3\left(\frac{dt}{ds}\right)$$

$$P_x = m_0c\frac{dx}{ds}$$

$$P_y = m_0c\frac{dy}{ds}$$

$$P_z = m_0c\frac{dz}{ds}$$

Then

$$P_x c = m_0c^2\frac{dx}{ds} \quad \text{..................... (i)}$$

$$P_y C = m_0c^2\frac{dy}{ds} \quad \text{..................... (ii)}$$

$$P_z C = m_0c^2\frac{dz}{ds} \quad \text{..................... (iii)}$$

Squaring and add ((i) + (ii) + (iii)):

$$\left(P_x^2 + P_y^2 + P_z^2\right)c^2 = \frac{m_0^2c^4}{(ds)^2}\left(dx^2 + dy^2 + dz^2\right)$$

$$p^2c^2 = \frac{m_0^2c^4}{(ds)^2}(dl)^2$$

$$E^2 - P^2C^2 = m_0^2c^6\left(\frac{dt}{ds}\right)^2 - m_0^2c^4\frac{(dl)^2}{(ds)^2}$$

$$E^2 - P^2C^2 = m_0^2c^4\left[\frac{c^2dt^2 - dl^2}{ds^2}\right] = m_0^2c^4$$

= const

$$E^2 - P^2C^2 = m_0^2c^4$$ **Ans.**

1.384*

(a)

Since maes of both particles are same hence magnitude of mometnum of both particles from com frame will be same.

Velocity of each particle from com frame
$= 0 - V_{cm} = -V_{cm}$

Energy in com frame

$$\tilde{E} = \frac{2m_0c^2}{\sqrt{1-\frac{V_{cm}^2}{c^2}}} = \sqrt{2m_0c^2(T+2m_0c^2)}$$

Using equation

$$\tilde{E} = \tilde{T} + 2m_0c^2$$

$$\tilde{T} = \sqrt{2m_0c^2(T+2m_0c^2)} - 2m_0c^2$$

$$\tilde{T} = 2m_0c^2\left(\sqrt{1+\frac{T}{2m_0c^2}} - 1\right)$$

Ans.

$$\tilde{P} = \left(\frac{m_0}{\sqrt{1-v^2/c^2}}\right)V_{cm} = \frac{m_0}{\sqrt{1-V_{cm}^2/c^2}}V_{cm}$$

Put value of V_{cm}

$$\tilde{P} = \sqrt{\frac{1}{2}m_0T}$$

Ans.

(b)

We know that

$$P = mV = \frac{mc^2}{c^2}V \quad \text{..............(1)}$$

Also we know

$$PC = \sqrt{T(T+2m_0c^2)}$$

From(1)

$$P = \frac{EV}{c^2}$$

Now

$$\frac{V}{C} = \frac{PC}{E}$$

For system

$$\frac{V_{cm}}{c} = \frac{P_{system}C}{E_{system}}$$

$$\frac{V_{cm}}{c} = \frac{\left(\sqrt{T(T+2m_0c^2)} + \sqrt{0(0+2m_0c^2)}\right)c}{(T+m_0c^2)+m_0c^2}$$

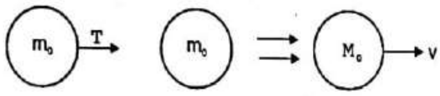
$$\frac{V_{cm}}{c} = \frac{C\sqrt{T(T+2m_0c^2)}}{T+m_0c^2}$$

$$= \sqrt{\frac{T}{T+2m_0c^2}}$$

$$V_{cm} = C\sqrt{\frac{T}{T+2m_0c^2}}$$

Ans.

1.385*



We know

$E^2 - P^2C^2 = \text{Invariant} = M_0^2C^4$

We assume energy does not losses

$$\left[\left(m_0c^2+T\right)+m_0c^2\right]^2 - T\left(T+2m_0c^2\right) = m_0^2c^4$$

$$m_0^2c^4 = c\sqrt{2m_0(2m_0c^2+T)}$$

$$m_0 = \frac{1}{c}\sqrt{2m_0(2m_0c^2+T)}$$

Also we know from Q : 1.384

$$P = \frac{EV}{c^2}$$

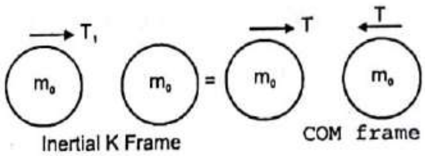
$$v = \frac{Pc^2}{E} = \frac{(Pc)c}{E}$$

$$v = \frac{c\sqrt{T(T+2m_0c^2)}}{T+2m_0c^2}$$

$$v = c\sqrt{\frac{T}{T+2m_0c^2}}$$

Ans.

1.386



Use invariant equation

$E^2 - P^2C^2 = \text{Invariant}$

$$\left(2m_0c^2+T_1\right)^2 - T_1\left(T_1+2m_0c^2\right) = \left[2\left(m_0c^2+T\right)\right]^2$$

Here momentum of system from com = 0

$$T_1^2 + 4T_1m_0c^2 + 4m_0^2c^4 - T_1^2 - 2T_1m_0c^2$$

$$= 4T^2 + 8Tm_0c^2 + 4m_0^2c^4$$

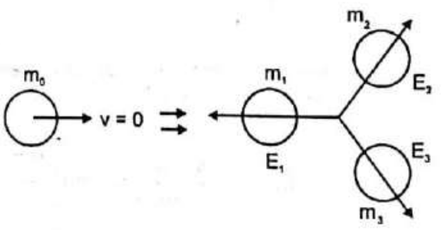
$$4T^2 + 8Tm_0c^2 = 2T_1m_0c^2$$

$$T_1 = \frac{4T(T+2m_0c^2)}{2m_0c^2}$$

$$T' = \frac{2T(T+2m_0c^2)}{2m_0c^2}$$

Ans.

1.387



$E_1 + E_2 + E_3 = m_0c^2$

$\bar{P}_1 + \bar{P}_2 + \bar{P}_3 = \bar{0}$

Take system of $(m_2 + m_3)$

$$(E_2+E_3)^2 - \left(\left|\bar{P}_1 - \bar{P}_2\right|\right)^2c^2$$

$$= \left(m_0c^2 - E_1\right)^2 - P_1^2c^2 = \text{In var ient}$$

$$\left(m_0c^2 + E_1\right)^2 - P_1^2c^2 = \left(m_2+m_3\right)^2c^4$$

Because this invariant is same in all frame of reference hence from com frame of $(m_2 + m_3)$

Invariant $= (m_2 + m_3)^2c^4$

Now

$$m_0^2c^4 + E_1^2 - 2m_0c^2E_1 - P_1^2c^2 = (m_2+m_3)^2c^4$$

$$m_0^2c^4 + m_1^2c^4 - 2m_0c^2E_1 = (m_2 + m_3)^2c^4$$

$$2m_0c^2E_1 = \left(m_0^2 + m_1^2\right)c^4 - \left(m_2 + m_3\right)^2 c^4$$

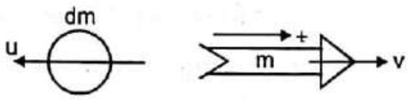
$$E_{1,max} = \left[\frac{\left(m_0^2 + m_1^2\right) - \left(m_2 + m_3\right)}{2m_0}\right]c^2$$

Ans.

$$\frac{v}{c} = \frac{1 - \left(\frac{m}{m_0}\right)^{2u/c}}{1 + \left(\frac{m}{m_0}\right)^{2u/c}}$$

Ans.

1.388



Suppose velocity of ejected mass is V_x from a frame having zero velocity then we know

$$V_x' = \frac{V_x - V}{1 - V_x \frac{v}{c^2}}$$

$$-u = \frac{V_x - V}{1 - V_x \frac{v}{c^2}}$$

$$V_x = \frac{V - u}{1 + \frac{uv}{c^2}}$$

Change in momentum of (dm) mass
dp = – (dm)V + dm V_x

$$= -dm\left[v - \frac{v - u}{1 - \frac{uv}{c^2}}\right] = -dm\left[\frac{u - u\frac{v}{c^2}}{1 - \frac{uv}{c^2}}\right]$$

Here $\frac{uv}{c^2} << u$ then

$$\frac{dP}{dt} = -\frac{(dm)}{dt}\frac{u}{1 - \frac{uv}{c^2}}$$ = Force on gas particle

Then force on rocket

$$F = -\left(\frac{dm}{dt}\right)\frac{u}{1 - \frac{uv}{c^2}} = m\frac{dv}{dt}$$

$$-u\int_{m_0}^{m}\frac{dm}{m} = \int_0^v\left(1 - \frac{uv}{c^2}\right)dv$$

PART: TWO

THERMODYNAMICS

AND

MOLECULAR

PHYSICS

2.1 Equation of gas state process

2.1. Suppose gas is at pressure P, volume V and tampreture T

Then we know

$PV = nRT$

If m = total mass of gas

M = molar mass of gas

$$PV = \frac{m}{M}RT$$

Here V = constant and T = constant

Then

$$(\Delta P)\,V = \frac{(\Delta m)RT}{M}$$

$$\Delta m = (\Delta P)V\frac{M}{RT} \quad ....(1)$$

At NTP (Normal temperature and pressure)

$T_0 = 0°C = 273\ K$

$P = 1\ atm = P_0 = 1 \times 10^5\ N/m^2$

Also we know

$$PM = \rho RT$$

$$\frac{M}{RT} = \frac{\rho}{P}$$

Then initially

$$\frac{M}{RT} = \frac{\rho_0}{P}$$

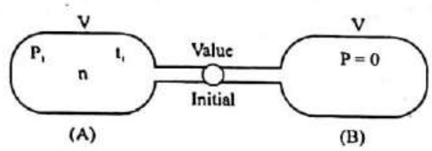
Put in (1)

$$\Delta m = (\Delta P)V\frac{\rho}{P_0}$$

$$\Delta m = \frac{\Delta P}{P_0}V\rho$$

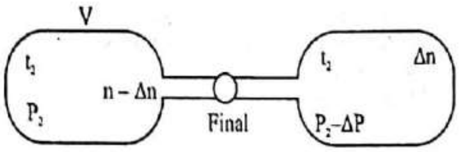
Ans.

2.2.



$$P_1V = nRT_1$$

$$n = \frac{P_1V}{RT_1}$$



Total no. of mole of gas is n and due to heating Δn mole of gas goes in other chamber.
Equation on first chamber:

$$P_2V = (n - \Delta n)RT_2 \Rightarrow n - \Delta n = \frac{P_2V}{RT_2} ......(i)$$

Equation on second chamber:

$$(P_2 - \Delta P)V = \Delta nRT_2$$

$$\Delta n = \frac{(P_2 - \Delta P)V}{RT_2}$$

Put value of n and Δn in (i) :

$$\frac{P_1V}{RT_1} - \frac{(P_0 - \Delta P)V}{RT_2} = \frac{P_2V}{RT_2}$$

$$\frac{P_1V}{RT_1} - \frac{(P_0 - \Delta P)V}{RT_2} = \frac{P_2V}{RT_2}$$

$$P_2\left[\frac{1+1}{T_2}\right] = \frac{P_1}{T_1} + \frac{\Delta P}{T_2}$$

$$P_2 = \frac{P_1T_2}{2T_1} + \frac{\Delta P}{2}$$

Increase of pressure of Vessel B

$$P_2 - \Delta P = \frac{P_1T_2}{2T_1} - \frac{\Delta P}{2} = \frac{1}{2}\left(\frac{P_1T_2}{T_1} - \Delta P\right)$$

Ans.

2.3

Let mass of H_2 gas is m_1 and that of He is m_2
Number of moles of H_2

$$n_1 = \frac{m_1}{2}$$

Number of moles He

$$n_2 = \frac{m_2}{2}$$

Also $m_1 + m_2 = m$ .... (i)
$PV = (n_1 + n_2)$ Rt

$$PV = \left(\frac{m_1}{2} + \frac{m_2}{4}\right)Rt$$

$$\frac{m_1}{2} + \frac{m_2}{4} = \frac{PV}{Rt}$$

$$m_1 + \frac{m_2}{2} = \frac{2PV}{Rt} ....(ii)$$

Now (i) - (ii)

$$\frac{m_2}{2} = m - \frac{2PV}{Rt}$$

$$m_2 = 2\left(m - 2\frac{PV}{Rt}\right)$$

Put in (i)
$m_1 = m - m_2$

$$= 4\frac{PV}{Rt} - m$$

$$\frac{m_1}{m_2} = \frac{4\frac{PV}{Rt} - m}{2\left(m - 2\frac{PV}{Rt}\right)}$$

$$= \frac{\frac{PV}{Rt} - \frac{m}{4}}{\frac{m}{2} - \frac{PV}{Rt}}$$

$$= \frac{\frac{PV}{Rt} - \frac{m}{m_2}}{\frac{m}{m_1} - \frac{PV}{Rt}}$$

Ans.

2.4

We know PM = ρRT .... (i)
Where M = Molecular weight of mixture
ρ = Density of mixture
Calculation of M
No. of mole of N_2
$n_1 = m_1/M_1$
No. of mole of CO_2
$n_2 = m_2/M_2$
Where M_1 and M_2 molecular weight of N_2 and CO_2

$$M = \frac{n_1M_1 + n_2M_2}{n_1 + n_2} = \frac{m_1 + m_2}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}$$

Put in (i)

$$P_0\left(\frac{m_1+m_2}{\dfrac{m_1}{M_1}+\dfrac{m_2}{M_2}}\right)=\rho RT$$

$$\rho_0=\frac{P_0(m_1+m_2)}{\left(\dfrac{m_1}{M_1}+\dfrac{m_2}{M_2}\right)}$$

Ans.

2.5

(a)

Suppose molar mass of O_2, N_2, CO_2 are M_1, M_2 and M_3.

$PV = nRT$

$PV = (\nu_1 + \nu_2 + \nu_3)\,RT$

$$P = (\nu_1 + \nu_2 + \nu_3)\frac{RT}{V}$$

Ans.

(b)

Total maes of mixture

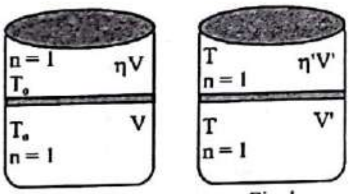
$M = \nu_1 M_1 + \nu_2 M_2 + \nu_3 M_3$

$$\text{Molar mass of mixuture} = \frac{\text{Total mass}}{\text{total no. of mole}}$$

$$M=\frac{\nu_1 M_1+\nu_2 M_2+\nu_3 M_3}{\nu_1+\nu_2+\nu_3}$$

Ans.

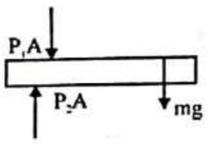
2.6



Initial Final

Here $(\eta' + 1)\,v' = (\eta + 1)v$ .... (i)

F.B.D. of piston



Net force will be zero in process

$$P_2A = P_1A + mg$$

$$P_2 - P_1 = \frac{mg}{A}$$

Similar $P_2' - P_1' = mg/A$

Now

$P_1\eta v = RT_0$

$$P_1 = \frac{RT_0}{\eta v}$$

$$P_2 = \frac{RT_0}{V}$$

$$P_2 - P_1 = \frac{RT_0}{V}\left(1-\frac{1}{\eta}\right)$$

Similar

$$P_2' - P_1' = \frac{RT}{v'}\left(1-\frac{1}{\eta}\right)$$

Since

$P_2 - P_1 = P_2' - P_1' = mg/A$

$$\frac{RT_0}{V}\left(1-\frac{1}{\eta}\right)=\frac{RT}{V'}\left(1-\frac{1}{\eta'}\right)$$

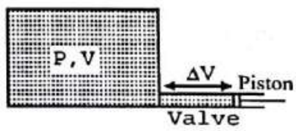
$$\frac{RT_0}{V}\left(\frac{\eta-1}{\eta}\right)=\frac{RT}{V'}\left(\frac{\eta'-1}{\eta'}\right)$$

$$=\frac{RT}{(\eta+1)}\,\frac{(\eta'+1)(\eta'-1)}{\eta'}$$

$$T=\frac{\eta'}{\eta}\frac{(\eta^2-1)}{(\eta'^2-1)}T_0$$

Ans.

2.7



This question is based on operation of an engine.

In this engine, first, piston pull right side and during pulling piston, value is opened and gas is filled in vacant space then value is closed. And gas between value and piston is removed. After that piston moves left then piston is again pull right way and value is opened and gas comes with piston. This process continues.

During each expansion, our system is remaining gas and we can assume no. of mole just afer start pull the piston and just before end of pull then we can use PV=constant

But in each expansion, no. of mole inside chamber will be constant.

<u>First stroke right</u>

$PV = P_1(V + \Delta V)$

$$P_1 = \frac{PV}{V+\Delta V} \quad \text{.............. (i)}$$

First stroke left

Now pressure becomes P_1 and volume will be V

<u>Second stroke right</u>

$P_1 V = P_2(V + \Delta V)$

$$\left(\frac{PV}{V+\Delta V}\right)V = P_2(V+\Delta V)$$

$$P_2 = \frac{PV^2}{(V+\Delta V)^2} \quad \text{.......... (ii)}$$

<u>In 3rd stroke</u>

$$P_3 = \frac{PV^3}{(V+\Delta V)^3}$$

Similarly in n^{th} stroke

$$P_n = \frac{PV^n}{(V+\Delta V)^n}$$

According to question

$$\frac{PV^n}{(V+\Delta V)^n} = \frac{P}{\eta}$$

$$n \ln \frac{V}{(V+\Delta V)} = -\ln \eta$$

$$n = \frac{\ln \eta}{\ln\left(\frac{V+\Delta V}{V}\right)} = \frac{\ln \eta}{\ln\left(1+\frac{\Delta V}{V}\right)}$$

Ans.

2.8

Suppose at time t, pressure is P and in next dt time dP pressre is increased.

But P + dP < P because P is decreasing function hence dP = (-) ive

Then

$PV = (P + dP)(V - Cdt)$

$PV = PV + VdP + CPdt + C(dP)(dt)$

$-VdP = (CP + CdP)\,dt$ .... (i)

$$CP + CdP \cong CP$$

Then

From (i)

$-VdP = CPdt$

$$-V\int_{P_0}^{P}\frac{dP}{P} = \int_0^t C\,dt$$

$$\ln P/P_0 = \frac{-C}{V}t$$

$$P = P_0 e^{\frac{-Ct}{V}}$$

Ans.

2.9

From Q : 2.8

$$P = P_0 e^{\frac{-Ct}{V}}$$

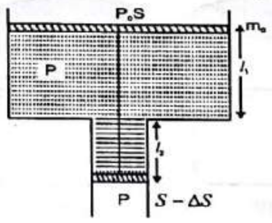
$$\frac{P_0}{\eta} = P_0 e^{\frac{-Ct}{V}}$$

$$\ln(\eta) = \frac{Ct}{V}$$

$$t = \left(\frac{V}{C}\right)\ln(\eta)$$

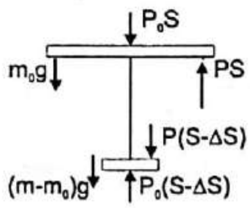
Ans.

2.10



Let assume pressure inside gas chember is P

F.B.D. of piston



Since process is very slow, piston always will be approximate equlibrium.

Force equation

$P_0S + P(S-\Delta S) + m_0g + (m-m_0)g = PS + P_0(S-\Delta S)$

$$P = P_0 + \frac{mg}{\Delta S} \quad ...... (i)$$

After increasing ΔT of tempreature, final tempreature is $T + \Delta T$

Since from (i) pressure is constant, process will be isobaric.

$$\frac{V}{T} = \text{constant}$$

$$\frac{V}{T} = \frac{V+\Delta V}{T+\Delta T}$$

$$\frac{T+\Delta T}{T} = 1 - \frac{\Delta V}{V}$$

$$\Delta T = T\left(\frac{\Delta V}{V}\right)$$

$$V_{initial} = S\ell_1 + (S-\Delta S)\ell_2$$

Change in volume

$\Delta V_{initial} = S\Delta l_1 + (S - \Delta S)\,\Delta l_2$

If l_2 increase, l_2 decrease in same amount.

$\Delta l_2 = l$

$\Delta l_2 = -l$

Then

$\Delta V_{initial} = \Delta S l = \Delta V$

Then

$$\Delta T = T\left(\frac{\Delta S\ell}{V}\right) \quad .... (ii)$$

Also

PV = nRT

Given n = 1

$$\left(P_0 + \frac{mg}{\Delta S}\right)V = RT$$

$$\frac{T}{V} = \frac{1}{R}\left(P_0 + \frac{mg}{\Delta S}\right)$$

Put in (i)

$$\Delta T = \frac{(\Delta S)\ell}{R}\left(P_0 + \frac{mg}{\Delta S}\right)$$

$$\Delta T = (mg + P_0\Delta S)\frac{\ell}{R}$$ **Ans.**

2.11

(a)

$P = P_0 - \alpha V^2$ .... (i)

No. of mole of gas = 1

We know PV = nRT

$$P = \frac{RT}{V}$$

Put in (i)

$$\frac{RT}{V} = P_0 - \alpha V^2$$

$$T = \frac{P_0V}{R} - \frac{\alpha V^3}{R} \quad .... (ii)$$

For T maximum

$$\frac{dT}{dV} = 0$$

$$0 = \frac{P_0}{R} - \frac{3\alpha V^2}{R}$$

$$V = \sqrt{\frac{P_0}{3\alpha}}$$

Put in (ii) :

$$T_{max} = \frac{P_0}{R}\sqrt{\frac{P_0}{3\alpha}} - \frac{\alpha}{R}\left(\frac{P_0}{3\alpha}\right)\sqrt{\frac{P_0}{3\alpha}}$$

$$T_{max} = \frac{2P_0}{3}\sqrt{\frac{P_0}{3\alpha}}$$

Ans.

(b)

$P = P_0e^{-\beta V}$

$$\frac{RT}{V} = P_0e^{-\beta V}$$

$$T = \frac{P_0}{R}Ve^{-\beta V} \quad ....(i)$$

For maximum Tempreature

$$\frac{dT}{dV} = 0$$

Calculate V and put in (i)

$$T_{max} = \frac{P_0}{e\beta R}$$

Ans.

2.12

$T = T_0 + \alpha V^2$ .... (i)

We know $PV = nRT = RT$

$$T = \frac{PV}{R}$$

Put in (i)

$$\frac{PV}{R} = T_0 + \alpha V^2 \text{ ....(ii)}$$

$$P = R\left[T_0 V^{-1} + \alpha V\right]$$

For

$$P_{min} : \frac{dP}{dV} = 0$$

Then from (ii)

$$0 = R\left[-T_0 V^{-2} + \alpha\right]$$

$$V = \sqrt{\frac{T_0}{\alpha}}$$

Put in (ii)

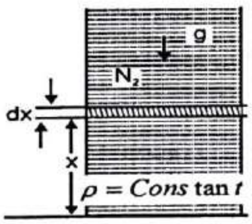
$$\frac{P}{R}\sqrt{\frac{T_0}{\alpha}} = T_0 + \alpha\frac{T_0}{\alpha} = 2T_0$$

$$\frac{P}{R\sqrt{\alpha}} = 2\sqrt{T_0}$$

$$P_{min} = 2R\sqrt{\alpha T_0}$$

Ans.

2.13



We know

$PM = \rho RT$

$(dP)\, M = \rho R(dT)$

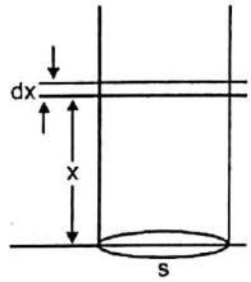
$-(\rho g dx)\, M = \rho R\, dT$

$$-\frac{dT}{dx} = \frac{\rho g M}{\rho R}$$

$$\frac{dT}{dx} = \frac{dT}{dh} = \frac{-Mg}{R}$$

Ans.

2.14



Here

$$\frac{P}{\rho^n} \text{ const} = C$$

$$P = C\rho^n$$

$$\rho = \left(\frac{P}{C}\right)^{\frac{1}{n}}$$

Also

$$PM = \rho RT$$

$$PM = \left(\frac{P}{C}\right)^{\frac{1}{n}} RT$$

$$P^{1-\frac{1}{n}} C^{\frac{1}{n}} = \frac{RT}{M}$$

Differentiate :

$$\left(1-\frac{1}{n}\right)P^{-\frac{1}{n}}(dP)C^{\frac{1}{n}} = \frac{R}{M}dT \text{ .... (i)}$$

Also we know

$dP = -\rho g dh$

Put in (i)

$$-\left(1-\frac{1}{n}\right)P^{\frac{-1}{n}}C^{\frac{1}{n}}(\rho g dh) = \frac{R}{M}dT$$

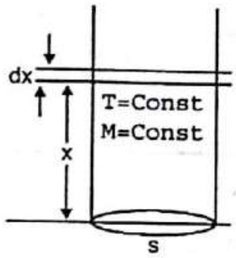
$$\frac{dT}{dh} = \frac{-(n-1)}{n}P^{\frac{-1}{n}}C^{\frac{-1}{n}}Mg\rho$$

$$\frac{dT}{dh} = -\left(\frac{n-1}{n}\right)P^{\frac{-1}{n}}Mg\rho\left(\frac{P}{\rho^n}\right)^{\frac{1}{n}}$$

$$\frac{dT}{dh} = -\frac{(n-1)}{n}Mg$$

Ans.

2.15



We know

$dP = -\rho g dz$

$$\rho = \frac{-dP}{gdz}$$

Also

$PM = \rho RT$

$$PM = RT\left(\frac{-dP}{gdz}\right)$$

In question dz = dh

$$-PM = \frac{RT}{gdh}dP$$

$$-PM = \frac{RT}{gdh}dP$$

$$\int_{P_0}^{P}\frac{dP}{P} = \frac{Mg}{RT}\int_0^h dh$$

$$P = P_0 e^{\frac{-Mgh}{RT}}$$

Above earth surface

$$\left.\begin{array}{l} h = 5\times10^3 m \\ g = 9.8 m/s^2 \\ R = 8.3 \end{array}\right| \begin{array}{l} T = 27°C = 300K \\ M = 78\times10^{-3} kg \end{array}$$

Put in above equation

$P = 0.5 \times 10^{+5}\ N/m^2 \cong 0.5$ Atm

Below earth surface

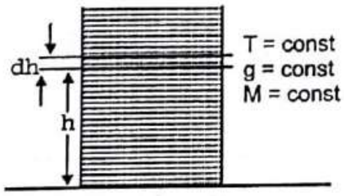
$$\left.\begin{array}{l} h = -5\times10^3 m \\ g = 9.8 m/s^2 \\ R = 8.3 \end{array}\right| \begin{array}{l} T = 27°C = 300K \\ M = 78\times10^{-3} kg \end{array}$$

Put

$P = 2 \times 10^{+5}\ N/m^2 = 2$ Atm

Ans

2.16



$PM = \rho RT$

$(dP) M = (d\rho) RT$ .... (i)

Also we know

$dP = -\rho g dz$ .... (ii)

Then from (i) and (ii)

$$\frac{(d\rho)RT}{M} = -\rho g dz$$

$$\int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \frac{-Mg}{RT}\int_0^h dz$$

$$\rho = \rho_0 e^{-\frac{Mgh}{RT}}$$

(a)

$$\rho = \frac{\rho_0}{e}$$

$$\frac{\rho_0}{e} = \rho_0 e^{-\frac{Mgh}{RT}}$$

$$e^{-1} = e^{-\frac{Mgh_1}{RT}}$$

$$Mgh_1 = RT$$

$$h_1 = \frac{RT}{Mg}$$

Ans.

(b)

$$\frac{-\rho + \rho_0}{\rho_0} = \eta$$

$$\rho = \rho_0(1-\eta)$$

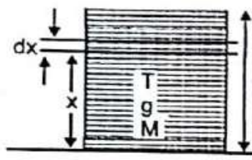
$$\rho_0(1-\eta) = \rho_0 e^{-\frac{Mgh_2}{RT}}$$

$$h_2 = \frac{RT}{Mg}\ell n(1-\eta)$$

$$h_2 = \frac{\eta RT}{Mg}$$

Ans.

2.17



Suppose pressure at height x is P

PV = nRT

Gas equation in differential volume

PSdx = (dN) RT ....(1)

We know

$$N = \frac{m}{M}$$

$$dN = \frac{dm}{M}$$

From(1)

$$PSdx = \left(\frac{dm}{M}\right)RT \quad ....(2)$$

Also we known

$$dP = -\rho g dx \text{ and } \rho = \frac{dm}{dV} = \frac{dm}{Sdx}$$

Then

$$dP = \left(\frac{-dm}{Sdx}\right)g\,dx$$

$$dm = \frac{SdP}{-g} \quad ......(3)$$

Put (1)

$$PS\,dx = \frac{RT}{M}\left(\frac{SdP}{-g}\right)$$

$$\frac{-Mg}{RT}\int_0^x dx = \int_{P_0}^{P}\frac{dP}{P}$$

$$P = P_0 e^{-\frac{Mgx}{RT}}$$

From (3) :

$$\int_0^m dm = -\int_{P_0}^{P}\frac{SdP}{g}$$

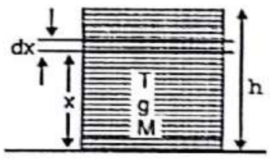
$$m = \frac{S}{g}(P - P_0) = \frac{P_0 S}{g}\left(1 - e^{-\frac{Mgx}{RT}}\right)$$

When x = h

$$m = \frac{P_0 S}{g}\left(1 - e^{-\frac{Mgh}{RT}}\right)$$

Ans.

2.18



We know

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

Also we know from Q : 1.215

$$P = P_0 e^{-Mgx/RT}$$

$$P(Sdx) = \frac{dm}{M}RT$$

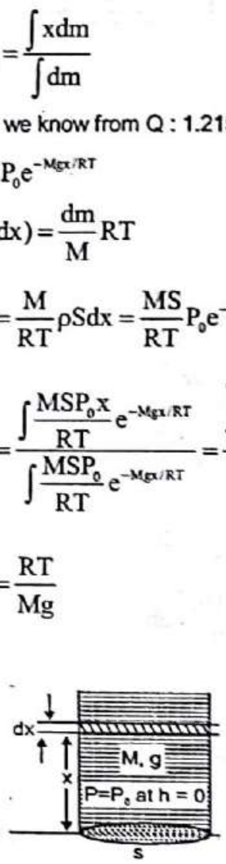
$$dm = \frac{M}{RT}\rho Sdx = \frac{MS}{RT}P_0 e^{-Mgx/RT}dx$$

$$x_{cm} = \frac{\int \frac{MSP_0 x}{RT} e^{-Mgx/RT}}{\int \frac{MSP_0}{RT} e^{-Mgx/RT}} = \frac{\int_0^\infty x e^{-Mgx/RT}dx}{\int_0^\infty e^{-Mgx/RT}dx}$$

$$x_{cm} = \frac{RT}{Mg}$$

Ans.

2.19



(a)

$T = T_0(1\text{-}ah)$

$$Psdx = \frac{dm}{M}RT \quad .... (i)$$

Also

$$dP = -\rho g dx = -\left(\frac{dm}{sdx}\right)gdx$$

$$dP = -\frac{g}{s}(dm)$$

$$dm = \frac{-s}{g}(dp)$$

Put in (i)

$$P s dx = \frac{RT}{M}\left(\frac{-s}{g}dP\right)..............(ii)$$

Also $T = T_0(1-ax)$
From (i)

$$\int_0^x \frac{-Mgdx}{RT_0(1-ax)} = \int_{P_0}^{P} \frac{dP}{P}$$

$$\frac{-Mg}{RT_0(-a)}\ell n(1-ax)\Big|_0^x = \ell n\left(\frac{P}{P_0}\right)$$

$$\frac{Mg}{aRT_0}\ell n(1-ax) = \ell n\left(\frac{P}{P_0}\right)$$

$$P = P_0(1-ax)^{\frac{-Mg}{aRT_0}}$$

Assume
$Mg/aRT_0 = n$ and $x = h$
$P = P_0(1-ah)^n$

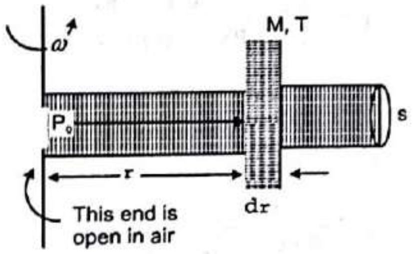
Ans.

(b)
Similarly as above

$$P = P_0/(1+ah)^n$$

Ans.

2.20



Force equation of dr element at r distance
$dF = (dm)rw^2$

$$dP = \frac{dF}{S} = \left(\frac{dm}{S}\right)rw^2$$

$$dP = \left(\frac{dm}{S}\right)rw^2$$

$$dm = \left(\frac{s}{rw^2}\right)dP$$

Also we know

$$P(Sdr) = \left(\frac{dm}{M}\right)RT$$

$$PS(dr) = \frac{RT}{M}\left(\frac{S}{rw^2}\right)dP$$

$$Mw^2\int_0^r rdr = RT\int_{P_0}^{P}\frac{dP}{P}$$

$$\frac{Mw^2r^2}{2} = RT\ell n\, P/P_0$$

$$P = P_0 e^{\frac{Mw^2r^2}{2RT}}$$

Ans.

2.21

Ideal gas equation
$PM = \rho RT$

$$P = \frac{\rho RT}{M}$$ **Ans.**

Vanderwal equation (for one mole gas)

$$\left(P+\frac{a}{V^2}\right)(V-b) = RT..............(1)$$

$$V = \frac{M}{\rho}$$

$$\left(P+\rho^2\frac{a}{M^2}\right)\left(\frac{M}{P}-b\right) = RT$$

$$P = \frac{RT}{\left(\frac{M}{\rho}-b\right)} - \frac{\rho^2 a}{M^2}$$

$$P = \frac{\rho RT}{M-\rho b} - \frac{\rho^2 a}{M^2}$$

Ans.

2.22

(a)
Using ideal gas equation
$PV = RT$

$$P_i = \frac{RT}{V}$$

Using vander wall equation

$$\left(P+\frac{a}{v^2}\right)(v-b) = RT$$

$$P_2 = \frac{RT}{V-b} - \frac{a}{V^2}$$

Here $\frac{P_1 - P_2}{P_2} = \eta$

$P_1 = P_2 = (1 + \eta)$

$$\frac{RT}{V} = \left(\frac{RT}{V-b} - \frac{a}{V^2}\right)(1+\eta)$$

$$T\left(\frac{R}{V} - \frac{(1+\eta)R}{V-b}\right) = -(1+\eta)\frac{a}{V^2}$$

$$T = \frac{(1+\eta)\frac{a}{V^2}}{\frac{(1+\eta)R}{V-b} - \frac{R}{V}} = \frac{(1+\eta)a(V-b)V}{V^2R(nV - V + b + V)}$$

$$T = \frac{a(V-b)(1+\eta)}{VR(nV-b)}$$

Ans.

(b)

Put value of this tempreture in vanderwall equation

$$P_2 = \frac{RT}{V-b} - \frac{a}{V^2}$$

Ans.

2.23

$$\left(P_1 + \frac{a}{V^2}\right)(V-b) = RT_1 \text{ ....(i)}$$

$$\left(P_2 + \frac{a}{V^2}\right)(V-b) = RT_2 \text{ .... (ii)}$$

From (i) and (ii)

$$a = V^2\left(\frac{T_1P_2 - T_2P_1}{T_2 - T_1}\right)$$

$$b = V - R\left(\frac{T_2 - T_1}{P_2 - P_1}\right)$$

Ans.

2.24

We know bulk modulus of a gas is given by

$$B = \frac{-dP}{dv/V}$$

While compressibility is given by :

$$\chi = -\frac{dv/V}{dP} = -\frac{1}{V}\left(\frac{dV}{dP}\right)$$

In vanderwall equation :

$$\left(P + \frac{a}{v^2}\right)(v-b) = RT \text{ ....(i)}$$

if process is isothermal

T = constant

Differentiate (i)

$$\left(dP + \frac{-a}{V^3}dV\right)(V-b) + \left(P + \frac{a}{V^2}\right)(dV) = 0$$

$$\left(\frac{dP}{dV} - \frac{a}{V^3}\right)(V-b) + \left(P + \frac{a}{V^2}\right) = 0$$

$$\frac{dP}{dV} = \frac{-\left(P + \frac{a}{V^2}\right)}{(V-b)} + \frac{a}{V^3}$$

$$\frac{dP}{dV} = \frac{-\left(PV^3 + av\right) + a(V-b)}{V^3(V-b)}$$

$$\chi = -\frac{1}{V}\left[\frac{V^3(V-b)}{(-PV^3 + av) + a(V-b)}\right]$$

$$\chi = \frac{V^2(V-b)}{(PV^3 - aV) + a(b-V)} \text{ ..... (ii)}$$

Put value of P from (i) in (ii)

$$\chi = \frac{V^2(v-b)^2}{\left[RTV^3 - 2a(v-b)^2\right]}$$

Ans.

2.25

Using ideal gas
PV = RT
$V = RT\,P^{-1}$

$$\frac{\partial V}{\partial P} = -RTP^{-2}$$

$$\chi_1 = \frac{RT}{VP^2} = \frac{V}{RT}$$

From Q : 2.24

$$\chi = \frac{V^2(V-b)^2}{RTV^3 - 2a(V-b)^2}$$

According to equation

$$\chi > \chi_1$$

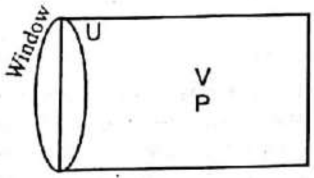
$$\frac{V^2(V-b)^2}{RTV^3 - 2a(V-b)^2} > \frac{V}{RT}$$

$$T > \frac{a}{b}R$$ **Ans.**

2.2

The first law of thermodynamics, Heat capacity

2.26



We know that room is an open thermodynamic system,in which no. of molecules may change suppose no. of mole of gas is n. Then
PV = nRT
Also we know Internal energy is given by :

$$U = nC_V T = \frac{nRT}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

$$\gamma = \frac{C_P}{C_V} = \text{constant}$$

And room pressure is also constant.
Then Internal energy inside room will constant.
U = constant
It is not depend on T.

Note:
When T increases no. of moles will be decreased in room and nT will be constant because PV = Constant according to question.

2.27

When vessel will stop suddenly , its directional kinetic energy is converted into random kinetic energy due to collision with walls of vessel hence tempreture is increased.
Decrease in directional kinetic energy = Increase in internal energy of gas

$$\frac{1}{2}MV^2 = \Delta U$$

$$\frac{1}{2}MV^2 = nC_v\Delta T$$

Here n=1 for one mole of gas

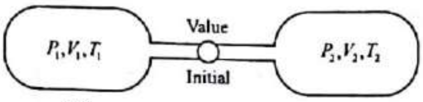
$$\frac{1}{2}MV^2 = C_v\Delta T$$

$$\frac{1}{2}MV^2 = \frac{R}{\gamma - 1}\Delta T$$

$$\frac{\gamma - 1}{2R}MV^2 = \Delta T$$

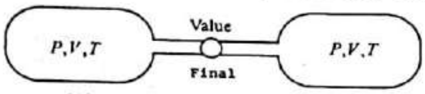
Ans.

2.28



(A) (B)

Due to valve open gases will transfer from high pressue to low pressure sotaht final pressure and final tempreture will be same everywhere. Let final pressure and tempreture are P and T.



(A) (B)

Since vessel is adiabatic and closed, internal energy of system will be conserve.

$$U_i = U_f$$

$$n_1C_vT_1 + n_2C_vT_2 = n_1C_vT + n_2C_vT$$

$$\frac{n_1T_1 + n_2T_2}{n_1 + n_2} = T \quad \text{..........(1)}$$

Also we know

$$P_1V_1 = n_1RT_1$$

$$n_1 = \frac{P_1V_1}{RT_1}$$

Simliarly

$$n_2 = \frac{P_2V_2}{RT_2}$$

Put values of n_1 and n_2 in eqution (1)

$$T = \frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$$

Ans.

Caculation of pressure

$$P(2V) = (n_1 + n_2)RT$$

Put value of T then

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$$

Ans.

2.29

Increase in internal energy

$$\Delta U = nC_v\Delta T = n\frac{R}{\gamma - 1}\Delta T \quad \text{..........(1)}$$

Also we know

PV = nRT

$$nR = \frac{P_0V}{T_0}$$

Put value of nR in (1)

$$\Delta U = -\frac{P_0V}{T_0}\frac{\Delta T}{\gamma - 1}$$

Ans.

$$\Delta Q = \Delta U + \Delta W$$

Since vessel is closed hence work done by gas will be zero.

$$\Delta W = 0$$

$$\Delta Q = \Delta U$$

$$\Delta Q = \frac{P_0V}{T_0}\frac{\Delta T}{\gamma - 1}$$

Ans.

2.30

We know

$$\Delta Q = \Delta U + \Delta W$$

Also we know

$$\Delta U = nC_v\Delta T = \frac{P\Delta V}{\gamma - 1}$$

In isobaric process

$$\Delta W = P\Delta V = A$$

Then

$$\Delta Q = \frac{P\Delta V}{\gamma - 1} + \Delta W$$

$$\Delta Q = \frac{A}{\gamma - 1} + A$$

$$\Delta Q = A\frac{\gamma}{\gamma - 1}$$

Ans.

2.31

We know

$$\Delta Q = \Delta U + \Delta W \quad \text{.........(1)}$$

Also we know

$$\Delta U = nC_v\Delta T = \frac{R}{\gamma - 1}\Delta T$$

Ans.

Also in question

$$\Delta Q = Q$$

Put values of ΔU and ΔQ in (1)

$$\Delta W = Q - \frac{R\Delta T}{\gamma - 1}$$

Ans.

2.32

Initial tempreture = T_0

Isochoric process

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\frac{P_0}{P_0/n} = \frac{T_0}{T_2}$$

$$T_2 = \frac{T_0}{n}$$

Heat given in isochoric process

$$Q_1 = vC_v\Delta T$$

$$Q_1 = vC_v\left(\frac{T_0}{n} - T_0\right) \quad \text{.................(1)}$$

Isobaric process

Heat given in isobaric process

$$Q_1 = vC_p\Delta T$$

$$Q_2 = vC_p\left(T_0 - \frac{T_0}{n}\right) \quad \text{..................(2)}$$

Total heat given

$$Q = Q_1 + Q_2$$

$$Q = -vC_v\left(\frac{T_0}{n} - T_0\right) + vC_p\left(T_0 - \frac{T_0}{n}\right)$$

$$Q = v\left(C_p - C_v\right)\left(T_0 - \frac{T_0}{n_0}\right)$$

$$Q = vR\left(T_0 - \frac{T_0}{n_0}\right)$$

$$Q = vRT_0\left(1 - \frac{1}{n_0}\right)$$

Ans.

2.33

$$C_v(mixture) = \frac{n_1C_1^v + n_2C_2^v}{n_1 + n_2}$$

$$C_p(mixture) = \frac{n_1C_1^p + n_2C_2^p}{n_1 + n_2}$$

$$\gamma(mixture) = \frac{C_p(mixture)}{C_v(mixture)}$$

$$\gamma(mixture) = \frac{n_1C_1^p + n_2C_2^p}{n_1C_1^v + n_2C_2^v} \quad \text{..........(1)}$$

We know

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p = \frac{R}{\gamma - 1} + R = \frac{\gamma R}{\gamma - 1}$$

For O_2 adiabatic exponent is γ_1 and number of mole is v_1 and that of CO_2 is γ_2 and number of mole is v_2

Then

$$C_1^v = \frac{R}{\gamma_1 - 1} \ ; \ C_2^v = \frac{R}{\gamma_2 - 1}$$

$$C_1^p = \frac{\gamma_1 R}{\gamma_1 - 1} \ ; \ C_2^p = \frac{\gamma_2 R}{\gamma_2 - 1}$$

Put all values in (1) and solve

$$\gamma(mixture) = \frac{v_1\gamma_1(\gamma_2 - 1) + v_2\gamma_2(\gamma_1 - 1)}{v_1(\gamma_2 - 1) + v_2(\gamma_1 - 1)}$$

Ans.

2.34

For N_2

$$n_1 = \frac{7}{28} = 0.25$$

$$C_1^v = \frac{5}{2}R$$

$$C_1^p = \frac{7}{2}R$$

For Ar

$$n_2 = \frac{20}{40} = 0.5$$

$$C_2^v = \frac{3}{2}R$$

$$C_2^p = \frac{5}{2}R$$

$$C_v(mixture) = \frac{n_1 C_1^v + n_2 C_2^v}{n_1 + n_2}$$

Put all above values

$$C_v(mixture) = \frac{0.25\frac{5}{2}R + 0.5\frac{3}{2}R}{0.25+0.5}$$

$$= \frac{1.375R}{0.75} = \frac{1.375 \times 8.3}{0.75} = 15.21\frac{J}{mol.K}$$

But in irodov ans is in per gram at place of per mole
Then
Molar mass of system(M)

$$M = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

$$M = \frac{0.25 \times 28 + 0.5 \times 40}{0.25+0.5} = 36gm$$

Then upper ans is for 36 gram then for 1 gram

$$C_v(mixture) = \frac{15.21}{36} = 0.42\frac{J}{g.K} \qquad \textbf{Ans.}$$

Similiar calculation is made of $C_p(mixture)$

$$C_p(mixture) = 0.65\frac{J}{g.K} \qquad \textbf{Ans.}$$

2.35

Suppose at time t , piston moves by x distance.

Since T = constant

$$P_1V_1 = P_2V_2$$
$$P_0V = (P_0 + \Delta P)(V + xS)$$

Here ΔP is change in pressure.

$$\Delta P = \frac{P_0 V}{V + xS} - P_0 = \frac{-xSP_0}{V + xS}$$

Force applied by person to move piston slowly

$$F = \Delta PS = \frac{-xS^2 P_0}{V + xS}$$

Work done by by this force

$$\Delta W = \int_0^x F dx = \int_0^x \frac{-xS^2 P_0}{V + xS} dx = \int_0^x \frac{-S^2 P_0}{V + xS} x dx$$

$$\Delta W = P_0 Sx - P_0 V \ell n\left(\frac{V + xS}{V}\right) \text{ ....(1)}$$

Where Sx is increase in volume then

$$Sx = nV - V = (n-1)V$$

Put in (1)

$$\Delta W = P_0(n-1)V - P_0 V \ell n(n)$$

Also $P_0 V = RT$

Then

$$\Delta W = RT\left[(n-1) - \ell \text{n}(n)\right] \qquad \textbf{Ans.}$$

2.36

Suppose at time t , piston moves by x distance in right side.
Since T = constant
In left part

$$P_1V_1 = P_2V_2$$
$$P_0V_0 = P_1(V + xS)$$
$$P_1 = \frac{P_0 V_0}{V + xS}$$

In right part

$$P_1V_1 = P_2V_2$$
$$P_0V_0 = P_1(V - xS)$$
$$P_1 = \frac{P_0 V_0}{V - xS}$$

Suppose ΔP is change in pressure

$$\Delta P = P_2 - P_1 = P_0 V_0\left(\frac{1}{V_0 - xS} - \frac{1}{V_0 + xS}\right)$$

$$\Delta P = \frac{2xSP_0V_0}{V_0^2 - x^2S^2}$$

Force applied by person to move piston slowly

$$F = \Delta PS = \frac{2xS^2 P_0 V_0}{V_0^2 - (xS)^2}$$

Work done by by this force

$$\Delta W = \int_0^x F dx = \int_0^x \frac{2xS^2 P_0 V_0}{V_0^2 - (xS)^2} dx \quad ....(1)$$

Suppose final volume of left part is V_1

$$V_1 = V_0 + xS = \frac{2V_0}{\eta + 1}$$

$$xS = \frac{2V_0}{\eta + 1} - V_0$$

$$x = \frac{1 - \eta_0}{\eta + 1} \frac{V_0}{S}$$

Put in (1)

$$\Delta W = P_0 V_0 \ell n \left[\frac{(\eta + 1)^2}{4\eta} \right]$$ **Ans.**

2.37

Heat given in isothermal process

$$\Delta Q_1 = \Delta W = \nu RT_0 \ell n \frac{V_2}{V_1} = \nu RT_0 \ell n(n)$$

$$\Delta Q_1 = \nu RT_0 \ell n(n) \quad ....(1)$$

Suppose initial tempreture is T_0 then due to volume increase , pressure will be reduced by n times in isothermal process.
In isochoric process
Due to pressure increase to initial value it should be raised by n time hence tempreture will be increase by n times and become nT_0
Heat given in isochoric process

$$\Delta Q_2 = \nu C_v \Delta T = \nu \frac{R}{\gamma - 1}(nT_0 - T_0)$$

$$\Delta Q_2 = \nu \frac{RT_0}{\gamma - 1}(n - 1) \quad ....(2)$$

Total heat given is Q then

$$Q = \Delta Q_1 + \Delta Q_2$$

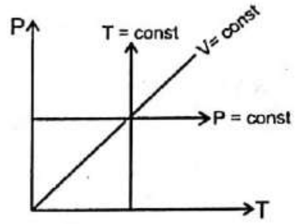
$$Q = \nu RT_0 \ell n(n)_1 + \nu \frac{RT_0}{\gamma - 1}(n - 1)$$

$$\gamma = 1 + \frac{n - 1}{\frac{Q}{\nu RT_0} - \ell n(n)}$$

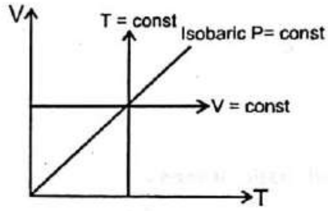
Ans.

2.38

(a)



(b)



2.39

(a)
We know

$$PV^\gamma = Const \quad \text{—(i)}$$

$$V = \frac{\nu RT}{P}$$

Put in (i)

$$P\left[\frac{\nu RT}{P}\right]^\gamma = \text{const}$$

$$P^{1-\gamma} T^\gamma = Const$$

$$P_0^{1-\gamma} T_0^\gamma = (\eta P_0)^{1-\gamma} T^\gamma$$

$$T = \left(\frac{1}{\eta}\right)^{1-\gamma/\gamma} T_0$$ **Ans.**

(b)
In adiabatic process

$$\Delta W = -\Delta U$$

work done b gas (A) = $\nu C_v \Delta T$

$$A = \frac{\nu R}{\gamma - 1}\left[T_0 \eta^{\gamma - 1/\gamma} - T_0\right]$$

Here $\nu = 1$

$$A = \frac{RT_0}{\gamma - 1}\left[\eta^{\gamma-1/\gamma} - 1\right]$$ **Ans.**

OR

We know work done by gas in adiabatic process

$$\Delta W = \frac{P_1V_1 - P_2V_2}{1-\gamma} = \frac{\nu R\Delta T}{1-\gamma}$$

$$A = \frac{\nu R\Delta T}{1-\gamma} = \frac{R\Delta T}{1-\gamma}$$

$$A = \frac{R\left(\left(\frac{1}{\eta}\right)^{1-\gamma/\gamma} T_0 - T_0\right)}{1-\gamma}$$

$$A = \frac{RT_0}{\gamma - 1}\left[\eta^{\gamma-1/\gamma} - 1\right]$$

Ans.

2.40

Adiabatic process

$$\Delta w_a = \nu C_v\Delta T = \frac{\nu R}{\gamma - 1}\Delta T \text{ ....(1)}$$

$$TV^{\gamma-1} = const$$

$$T_1\left(\frac{V_0}{\eta}\right)^{\gamma-1} = T_0V_0^{\gamma-1}$$

$$T_1 = T_0\eta^{\gamma-1}$$

$$\Delta T = T_0\left(\eta^{\gamma-1} - 1\right)$$

From (1)

$$\Delta w_a = \frac{\nu RT_0\left(n^{\gamma-1} - 1\right)}{\gamma - 1}$$

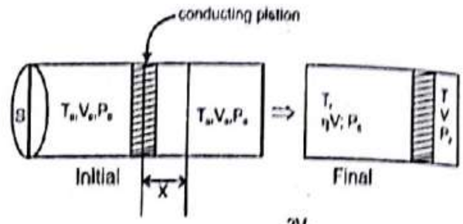
Isothermal Process

$$\Delta w_T = \nu RT_0 \ln {}^{V_2}\!/_{V_1} = \nu RT_0 \ln(\eta) \text{ ------- (2)}$$

$$\frac{\Delta w_a}{\Delta w_T} = \frac{\nu RT_0\left(n^{\gamma-1} - 1\right)}{(\gamma - 1)\nu RT_0 \ln\eta}$$

$$\frac{\Delta w_a}{\Delta w_T} = \frac{n^{\gamma-1} - 1}{(\gamma - 1)\ln\eta}$$ **Ans.**

2.41



$$\eta V + V = 2V_0 \Rightarrow V = \frac{2V_0}{1+\eta}$$

Work done by gas on system is zero here temprature is only increased because of work done by external agent.

$\Delta W_{ext} = \Delta U_{system}$

Calculation of ΔU_{system}

$$\Delta U_{system} = \left(\nu_1 + \nu_2\right)\frac{R}{\gamma - 1}\Delta T$$

Gas equation at time t

$$\frac{P_0V_0}{T_0} = \frac{P_1\left(V_0 + xs\right)}{T}$$

$$P_1 = \frac{P_0V_0T}{T_0\left(V_0 + xs\right)}$$

$$\frac{P_0V_0}{T_0} = P_2\frac{\left(V_0 - xs\right)}{T}$$

$$P_2 = \frac{P_0V_0T}{T_0\left(V_0 - xs\right)}$$

Force applied by extenal agent

$$F = \left(P_2 - P_1\right)S = \frac{P_0V_0T}{T_0}\left(\frac{1}{V_0 - xS} - \frac{1}{V_0 + xS}\right)S$$

$$F = \frac{P_0V_0TS}{T_0}\left(\frac{2xS}{V_0^2 - x^2S^2}\right)$$

Work done in displacement dx

$$dW = \frac{2P_0V_0TS^2x}{T_0(V_0^2 - x^2S^2)}dx$$

Increase in internal enegy

$$dU = \frac{\left(\nu_1 + \nu_2\right)R}{\gamma - 1}dT$$

On system

$W_{ext} = dU$

$$\frac{2P_0V_0TS^2x}{T_0\left(V_0^2 - x^2s^2\right)}\ dx = \frac{(v_1 + v_2)R\,dT}{\gamma - 1}$$

$$\frac{2P_0V_0TS^2x}{T_0\left(V_0^2 - x^2s^2\right)}\ dx = \left(\frac{P_0V_0}{RT_0} + \frac{P_0V_0}{RT_0}\right)\frac{RdT}{\gamma - 1}$$

$$s^2\int_0^x \frac{xdx}{V_0^2 - (xs)^2} = \frac{1}{\gamma-1}\int_{T_0}^{T}\frac{dT}{T}$$

$$T = T_0\left(\frac{(V_0 + xs)(V_0 - xs)}{V_0^2}\right)^{-\left(\frac{\gamma-1}{2}\right)} \text{ ------ (i)}$$

$$(V_0 + xs)\ (V_0 - xs) = \eta V.v = \eta v^2 = \frac{4\eta V_0^2}{(1+\eta)^2}$$

Put in (i)

$$T = T_0\left[\frac{(\eta+1)^2}{4\eta}\right]^{\frac{\gamma-1}{2}}$$

Ans.

2.42

Gas has internal energy which is due it random motion is converted into directional kinetic energy.

Using Barnauli's equation for gas particle at same level

$$P + \frac{1}{2}\rho V^2 + \frac{dU}{dVolume} = \text{const}$$

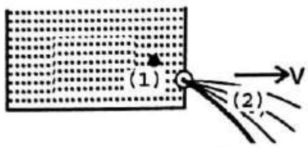
Where V= Directional velocity

$$\frac{dU}{dVolume} = \text{Internal energy per unit volume}$$

$$\text{For Gas } \frac{dU}{dVolume} \neq 0$$

While for liquid

$$\frac{dU}{dVolume} = 0$$



Now bernauli's equation between (1) and (2)

$$P_1 + 0 + \frac{nC_VT}{Volume} = 0 + \frac{1}{2}\rho V^2 + 0$$

$$\frac{\rho RT}{M} + \frac{m}{MV_{volume}}C_VT = \frac{1}{2}\rho V^2$$

$$\frac{\rho RT}{M} + \frac{\rho C_VT}{M} = \frac{1}{2}\rho V^2$$

$$\frac{2C_PT}{M} = v^2$$

$$V = \sqrt{\frac{2C_PT}{M}}$$

$$V = \sqrt{\frac{2\gamma TR}{(\gamma-1)M}}$$

Ans.

2.43

Method : 1(Basic Approach)

$$V = \frac{a}{T}$$

No. of mole of gas(n) = **1**

$$T = \frac{a}{V} \text{ ....(1)}$$

We know

$\Delta Q = \Delta U + \Delta W$

$$\Delta U = C_V\Delta T = \frac{R}{\gamma - 1}\Delta T \text{ ------- (2)}$$

$dw = PdV$

$$\Delta w = \int PdV \text{ -------- (3)}$$

$PV = nRT = RT$

$$P = \frac{RT}{V}$$

From **(1)**

$$P = \frac{RT}{V} = \frac{aR}{V^2}$$

Put in (2) :

$$\Delta w = \int_{V_1}^{V_2}\frac{aR}{V^2} = -aR\frac{1}{V}\Big|_{V_1}^{V_2} = aR\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

$$\Delta W = R\left[\frac{a}{V_1} - \frac{a}{V_2}\right] = R\left[T_1 - T_2\right] = -R\Delta T$$

$$\Delta Q = \Delta U + \Delta W = \frac{R}{\gamma-1}\Delta T - R\,\Delta T = R\Delta T\frac{(2-\gamma)}{\gamma - 1}$$

Ans.

Method : 2(Direct formula based)

We know molar heat capacity of gas

$$C = C_v + \frac{R}{1-x}$$

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x} \text{ ....(1)}$$

Here

$V = \frac{a}{T}$ then $T = \frac{a}{V}$

$PV = RT$ then $PV = \frac{Ra}{V}$

$PV^2 = aR$ = constant

Compare with PV^x = const

$x = 2$

Then from (1)

$$C = \frac{R}{1-\gamma} + \frac{R}{1-2}$$

$$C = \frac{R}{\gamma - 1} - R = \frac{R}{\gamma-1}(2-\gamma)$$

Also we know

$$\Delta Q = nC\Delta T = R\Delta T\frac{(2-\gamma)}{\gamma-1}$$ **Ans.**

2.44

ν = no. of mole of gas

$dW \propto dU$

$dW = KdU$

$PdV = K\nu C_V dT$ ....(1)

Where K = constant

$$PV = \nu RT$$

$$P = \frac{\nu RT}{V} \text{ ....(2)}$$

From (1)

$PdV = K\nu C_V dT$

From (2)

$$\frac{\nu RT}{V}dV = K\nu C_V dT$$

$$\frac{R}{KC_V}\int\frac{dV}{V} = \int\frac{dT}{T}$$

$$\frac{R}{KC_V}\ln V = \ln T + \ln K_1$$

Where K_1 = constant

$$\ell n\left(V^{R/KC_V}\right) = \ell n(K_1T)$$

$$V^{\frac{R}{KC_V}} = K_1T$$

$$K_1\frac{PV}{\nu R} = V^{R/KC_V}$$

PV^{1-R/KC_V} = constant

$1 - R/KC_V$ = n = const

PV^n = constant **Ans.**

2.45

PV^n = constant --------(i)

We know

n_1 = No. of mole of gas

$dQ = dU + dW$

$n_1CdT = n_1C_VdT + PdV$ ------- (ii)

Differentiate equation (i)

$nPV^{n-1}\, dV + V^n\, dP = 0$

$-nPdV = VdP$ ---------- (iii)

Also

$PV = n_1RT$

Also differentiate this equation

$PdV + VdP = n_1RdT$.

From (iii)

$PdV - nPdV = n_1RdT$

$$PdV = \frac{n_1RdT}{1-n}$$

Put in (ii)

$$n_1CdT = n_1C_VdT + \frac{n_1RdT}{1-n}$$

$$n_1C = n_1C_V + \frac{n_1R}{1-n}$$

$$C = C_V + \frac{R}{1-n}$$

$$C = \frac{R}{\gamma-1} - \frac{R}{n-1}$$ **Ans.**

2.46

There is propotionality relation between pressure and volume then it will be converted in Polytropic pricess then

PV^x = const

$$P_0V_0^x = \frac{P_0}{\beta}(\alpha V_0)^x$$

$$V_0^x = \frac{(\alpha V_0)^x}{\beta} = \frac{\alpha^x V_0^x}{\beta}$$

$\alpha^x = \beta$

$$x = \ln\beta / \ln\alpha$$

We know molar heat capacity of polytropic process is given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x}$$

$$C = \frac{R}{\gamma-1} + \frac{R}{1-\frac{\ln\beta}{\ln\alpha}}$$

Assume $= \frac{\ln\beta}{\ln\alpha} = n$

$$C = \frac{R}{\gamma-1} - \frac{R}{n-1}$$

$$C = \frac{R(n-\gamma)}{(n-1)(\gamma-1)}$$

Ans.

2.47

$$C = \frac{R}{\gamma-1} + \frac{R}{1-n}$$

(a)

$\Delta Q = \nu C \Delta T$

Here $\nu = 1$

$\Delta Q = C\Delta T$

$$\Delta Q = \frac{R(n-\gamma)}{(n-1)(\gamma-1)} \Delta T$$

Ans.

(b)

We know that work done in polytropic process is as

$$\Delta W = \frac{P_1V_1 - P_2V_2}{1-n} = \frac{\nu R \Delta T}{1-n}$$

Here $\nu = 1$

$$\Delta W = \frac{R}{1-n} \Delta T$$

Ans.

2.48

Method :1(Formula based)

$P = \alpha V$

Initially

$P_0 = \alpha V_0$ ....(1)

$PV^{-1} = \alpha$ ....(2)

Compare with polytropic gas equatin

PV^n = const

$n = -1$

(c)

Molar heat capacity of gas is given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-n} \text{ —— (i)}$$

$$C = \frac{R}{\gamma-1} - \frac{R}{-1-1} = R\left[\frac{1}{\gamma-1} + \frac{1}{2}\right]$$

$$C = \frac{R}{2}\left[\frac{1+\gamma}{\gamma-1}\right]$$

Ans.

(b) We know that work done in polytropic process is as

$$\Delta W = \frac{P_1V_1 - P_2V_2}{1-n} = \frac{\nu R \Delta T}{1-n}$$

$$\Delta W = \frac{\nu R}{1-n} \Delta T$$

$$\Delta W = \frac{\nu R}{1-(-1)} \Delta T$$

$$\Delta W = \frac{\nu R}{2} \Delta T \text{ ....(3)}$$

Calcultion of ΔT

From (2)

PV^{-1} =constant

$$\frac{\nu RT}{V}(V)^{-1} = const$$

$T = cV^2$

Where c = const

Initially

$T_0 = cV_0^{\,2}$ ....(4)

When volume increase η times

then

$T_f = c(\eta V)_0^{\,2}$ ....(5)

From (4) and (5)

$T_f = \eta^2 T_0$

then

$$\Delta T = T_f - T_i = \eta^2 T_0 - T_0 = (\eta^2 - 1)T_0$$

From (2)

$$\Delta W = \frac{\nu R}{2}(\eta^2 - 1)T_0 = \frac{1}{2}(\eta^2 - 1)P_0V_0$$

From (1)

$$\Delta W = \frac{\alpha(\eta^2 - 1)V_0^2}{2}$$

Ans.

(a)

$\Delta U = \nu C_V \Delta T$

$$\Delta U = \frac{\nu R}{r-1} \Delta T$$

Ans.

Method :1(Basic Approach)

(a)

$$T_0 = \frac{P_0V_0}{vR} = \frac{\alpha V_0^2}{vR}$$

Final tempreture

$$T_f = \frac{P_0V_f}{vR} = \frac{\alpha\eta^2 V_0^2}{vR}$$

$$\Delta T = T_f - T_0 = \frac{\alpha\eta^2 V_0^2}{vR} - \frac{\alpha V_0^2}{vR} = \frac{\alpha V_0^2}{vR}(\eta^2 - 1)$$

$$\Delta U = vC_v\Delta T = v\frac{R}{\gamma-1}\frac{\alpha V_0^2}{vR}(\eta^2-1)$$

$$\Delta U = \alpha V_0^2\left(\frac{\eta^2-1}{\gamma-1}\right)$$

Ans.

(b)

dw = PdV

$$\Delta W = \alpha\int_{V_0}^{\eta V_0} V dV = \frac{\alpha}{2}V^2\Big|_{V_0}^{\eta V_0}$$

$$\Delta W = \frac{\alpha(\eta^2-1)V_0^2}{2}$$

Ans.

(c)

Molar heat capacity of gas is given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-n} \text{ ——— (i)}$$

$$C = \frac{R}{\gamma-1} - \frac{R}{-1-1} = R\left[\frac{1}{\gamma-1} + \frac{1}{2}\right]$$

$$C = \frac{R}{2}\left[\frac{1+\gamma}{\gamma-1}\right]$$

Ans.

2.49

Method :1(Formula Based)

(a)

$dQ = -nC_v dT$

Here molar heat capacity is C then

$nCdT = -nC_v dT$

$$C = -C_v = \frac{-R}{\gamma-1}$$

Ans.

(b)

Also we know

$$C = \frac{R}{\gamma-1} - \frac{R}{x-1} = \frac{-R}{\gamma-1}$$

$$\frac{2}{\gamma-1} = \frac{1}{x-1}$$

$2x-2 = \gamma-1$

$$x = \frac{\gamma+1}{2} \text{ ....(1)}$$

We know that work done in polytropic process is as

$$\Delta W = \frac{P_1V_1 - P_2V_2}{1-x} = \frac{vR\Delta T}{1-x}$$

$$\Delta W = \frac{vR}{1-x}\Delta T$$

From (1)

$$\Delta W = 2\frac{vR}{\gamma-1}\Delta T \text{..............(2)}$$

Calculation of ΔT

$$T_0(V_0)^{\frac{\gamma-1}{2}} = T_f[\eta V_0]^{\frac{\gamma-1}{2}}$$

$$T_f = T_0\left[\frac{1}{\eta}\right]^{\frac{\gamma-1}{2}}$$

$$\Delta T = T_0\left[\left[\frac{1}{\eta}\right]^{\frac{\gamma-1}{2}} - 1\right]$$

From (2)

$$\Delta W = \frac{2vRT_0}{\gamma-1}\left[1 - \left[\frac{1}{\eta}\right]^{\frac{\gamma-1}{2}}\right]$$

$v = 1$

$$\Delta W = \frac{2RT_0}{\gamma-1}\left[1 - \left[\frac{1}{\eta}\right]^{\frac{\gamma-1}{2}}\right]$$

$$\Delta W = \frac{2RT_0}{\gamma-1}\left[1 - \eta^{\frac{1-\gamma}{2}}\right]$$

Ans.

Method : 2(Basic Approach)

(a)

$dQ = nC_v dT$

$nCdT = -nC_v dT$

$$C = \frac{-R}{\gamma-1}$$

Ans.

(b)

Also we know

$dQ = nC_v dT + PdV$

$-(nC_v dT) = nC_v dT + PdV$

$PdV = -2nC_v dT$

Also we know

$$P=\frac{nRT}{V}$$

$$\frac{nRT}{V}dV = -2n\frac{R}{\gamma-1}dT$$

$$\frac{-(\gamma-1)}{2}\int\frac{dV}{V} = \int\frac{dT}{T}$$

$$-\left(\frac{\gamma-1}{2}\right)\ln V = \ln KT$$

$$(KT)^{-1} = V^{\frac{\gamma-1}{2}}$$

$$TV^{\frac{\gamma-1}{2}} = \frac{1}{K} = \text{const}$$

$$TV^{\frac{\gamma-1}{2}} = \text{const}$$

Ans.

(c)

$$dW = -2nC_v dT = \frac{-2nR}{\gamma-1}dT$$

$$\Delta W = \frac{-2nR}{\gamma-1}\Delta T$$

Calculate ΔT as method : 1 and put

$$\Delta W=\frac{2RT_0}{\gamma-1}\left[1-(\eta)^{\frac{\gamma-1}{2}}\right]$$

Ans.

2.50

$P = \alpha T^{\alpha}$ ————(i)

$$T=\frac{PV}{vR}$$

Put in (i)

$$P = \alpha\left[\frac{PV}{vR}\right]^{\alpha}$$

$$P = \frac{\alpha}{v^{\alpha}R^{\alpha}}P^{\alpha}V^{\alpha}$$

$$P^{1-\alpha} = CV^{\alpha}$$

$$P = C_1 V^{\alpha/1-\alpha}$$

$$PV^{\alpha/\alpha-1} = \text{const}$$

Compare with PV^x = Const

$$x = \frac{\alpha}{\alpha-1}$$

(b)

$$C=\frac{R}{\gamma-1}-\frac{R}{x-1}$$

$$C = \frac{R}{\gamma-1}-\frac{R}{\frac{\alpha}{\alpha-1}-1} = \frac{R}{\gamma-1}+R(1-\alpha)$$

Ans.

(a)

$$dW = v\left[\frac{-R}{x-1}\right]dT$$

$$\Delta W = \frac{vR}{1-\frac{\alpha}{\alpha-1}}\Delta T = \nu R(1-\alpha)\Delta T$$

Here $\nu = 1$

$\Delta W = R(1-\alpha)\,\Delta T$

Ans.

2.51

$U = aV^{\alpha}$ ....(1)

We know that

$$U=\frac{PV}{\gamma-1}=\frac{vRT}{\gamma-1}$$

Then from (1)

$$\frac{PV}{\gamma-1} = aV^{\alpha}$$

$PV^{1-\alpha} = a(\gamma-1) = \text{const}$

Compare with PV^x = const

$x = 1-\alpha$

(b)

$$C=\frac{R}{\gamma-1}-\frac{R}{x-1}$$

$$C=\frac{R}{\gamma-1}-\frac{R}{1-\alpha-1}=\frac{R}{\gamma-1}+\frac{R}{\alpha}$$

Ans.

(a)

Given $\nu C_v\Delta T = \Delta U$

$$\frac{vR}{\gamma-1}\Delta T = \Delta U$$

$$\Delta T = \frac{(\gamma-1)\Delta U}{v\gamma}$$

$$\Delta Q = \nu C\Delta T = \nu\left[\frac{R}{\gamma-1}+\frac{R}{\alpha}\right]\frac{(\gamma-1)\Delta U}{v\gamma}$$

$$\Delta Q=\left[\frac{1}{\gamma-1}+\frac{1}{\alpha}\right](\gamma-1)\Delta U=\frac{(\alpha+\gamma-1)\Delta U}{\alpha}$$

$$\Delta Q=\Delta U\left[1+\frac{\gamma-1}{\alpha}\right]$$

Ans

$$\Delta W=\Delta Q-\Delta U=\Delta U\left[1+\frac{\gamma-1}{\alpha}\right]-\Delta U$$

$$\Delta W=\frac{\Delta U(\gamma-1)}{\alpha}$$

Ans

2.52

(a)

$$T=T_0e^{\alpha V}$$

$$dT=T_0\alpha e^{\alpha V}dV$$

$$dV=\frac{dT}{T_0\alpha e^{\alpha V}}\ \ldots\ldots\ldots\ldots(1)$$

We know

$dQ = dU + dW$ ——— (2)

$PV = \nu RT$

$PdV + VdP = \nu RdT$ ——— (3)

Also we know

$$\frac{PV}{\nu R}=T$$

Here $T = T_0e^{\alpha V}$

$$\frac{PV}{\nu R}=T_0e^{\alpha V}$$

$$P=\frac{\nu RT_0}{V}e^{\alpha V}$$

Now from (1)

$$PdV=\frac{\nu RT_0e^{\alpha V}}{V}\left[\frac{dT}{T_0\alpha e^{\alpha V}}\right]=\frac{\nu RdT}{V\alpha}$$

From (2)

$dQ = dU + dW$

$$\nu CdT=\nu C_v dT+PdV$$

$$\nu CdT=\nu C_v dt+\frac{\nu RdT}{V\alpha}$$

$$C=C_v+\frac{R}{\alpha V}$$ **Ans.**

2.53

$$P=P_0+\frac{\alpha}{V}$$

$$-\frac{\alpha}{V}=-P+P_0\ \ldots\ldots\ldots\ldots(1)$$

$$dP=-\frac{\alpha}{V^2}dV$$

(a)

We know

$dQ = dU = dU + PdV$

$nCdT = nC_vdT + pdV$ ——— (2)

We know $PV = nRT$

$PdV + VdP = nRdT$

$$PdV+V\left[\frac{-\alpha dV}{V^2}\right]=nRdT$$

$$PdV+\frac{-\alpha}{V}dV=nRdT$$

From (1)

$PdV + (P_0 - P)\,dV = nRdT$

$P_0dV = nRdT$

$$dV=\frac{nR}{P_0}dT$$

Put in (1)

$$nCdT=nC_vdT+\left(P_0+\frac{\alpha}{V}\right)\frac{nRdT}{P_0}$$

$$C=\frac{R}{\gamma-1}+\left(P_0+\frac{\alpha}{V}\right)\frac{R}{P_0}$$

$$C=\frac{R}{\gamma-1}+R\left(1+\frac{\alpha}{VP_0}\right)$$

$$C=\frac{\gamma R}{\gamma-1}+\frac{\alpha R}{VP_0}$$

Ans.

(b)

$PV = RT$

$$\left(P_0+\frac{\alpha}{V}\right)V=RT$$

$$T=\frac{P_0V+\alpha}{R}$$

$$\Delta T=\frac{P_0}{R}\Delta V=\frac{P_0}{R}[V_2-V_1]$$

$$\Delta U=C_v\Delta T=\frac{R}{\gamma-1}\Delta T$$

$$\Delta U = \frac{P_0}{\gamma - 1}[V_2 - V_2]$$

Ans.

$dW = PdV$

$$\Delta W = \int_{V_1}^{V_2}\left(P_0 + \frac{\alpha}{V}\right)dV$$

$$\Delta W = P_0(V_2 - V_1) + \alpha \ln {V_2}/{V_1}$$

Ans.

$\Delta Q = \Delta U + \Delta W$

$$= \frac{P_0}{\gamma-1}(V_2-V_1) + P_0(V_2-V_1) + \alpha \ln\frac{V_2}{V_1}$$

$$\Delta Q = \frac{P_0(V_2 - V_1)\gamma}{\gamma - 1} + \alpha \ln\frac{V_2}{V_1}$$

Ans.

2.54

(a)

$T = T_0 + \alpha V$

$dT = \alpha dV$

$$dV = \frac{dT}{\alpha}$$

Also

$dQ = dU + dW$

$\nu CdT = \nu C_v dT + PdV$

$$\nu CdT = \nu C_v dT + \frac{PdT}{\alpha}$$

$$C = C_v + \frac{P}{\nu\alpha} \text{ ——— (i)}$$

$PV = \nu RT$

$$\frac{P}{\nu} = \frac{RT}{V}$$

$$C = C_v + \frac{RT}{V\alpha}$$

$$C = C_v + \frac{R}{V\alpha}[T_0 + \alpha V]$$

$$C = \frac{\gamma R}{\gamma - 1} + \frac{RT_0}{\alpha V}$$

$$C = C_P + \frac{RT_0}{\alpha V}$$

Ans.

(b)

$T = T_0 + \alpha V$

$dT = \alpha dV$

$dQ = \nu CdT$

Here $\nu = 1$

$$dQ = cdT = \left(C_P + \frac{RT_0}{\alpha V}\right)dT$$

$$= \left(C_P + \frac{RT_0}{\alpha V}\right)\alpha dV$$

$$\Delta Q = \alpha C_P \int_{V_1}^{V_2} dV + \int_{V_1}^{V_2} \frac{RT_0}{V} dV$$

$$\Delta Q = \alpha C_P(V_2 - V_1) + RT_0 \ln {V_2}/{V_1}$$

Ans.

2.55

(a)

$C = C_v + \alpha T$

$dQ = \nu CdT = \nu[C_v + \alpha T]dT$

$dQ = \nu C_v dT + \nu \alpha TdT$ ——— (i)

Also

$dQ = \nu C_v dT + PdV$ ——— (ii)

Compare (i) and (ii)

$PdV = \nu \alpha TdT$ ....(iii)

Also we know

$$P = \frac{\nu RT}{V}$$

Put in (iii)

$$\frac{\nu RT}{V} dV = \nu \alpha TdT$$

$$\int \frac{dV}{V} = \frac{\alpha}{R}\int dT$$

$$\ln V = \frac{\alpha}{R}T + \ln C_1$$

$$\frac{V}{C_1} = e^{\alpha T/R}$$

$$Ve^{-\alpha T/R} = C_1$$

Ans.

(b)

$C = C_v + \beta V$

$dQ = \nu CdT$

$dQ = \nu(C_v + \beta V)dT$

$dQ = \nu C_v dT + \nu \beta VdT$...................(1)

Also

$dQ = \nu C_v dT + PdV$ ....(2)

Compare with (1) and (2)

$PdV = \nu \beta VdT$

$$\frac{\nu RT}{V} dV = \nu \beta VdT$$

$$\int \frac{dV}{V^2} = \frac{\beta}{R}\int \frac{dT}{T}$$

$$-\frac{1}{V} = \frac{\beta}{R} \ln C_1T$$

$$C_1T = e^{\frac{-R}{\beta V}}$$

$$Te^{\frac{R}{\beta V}} = \text{const}$$

Ans.

(c)
$C = C_v + ap$
$PdV = \nu apdT$
$\int dV = \nu a \int dT$
$V = \nu aT + C_1$
For 1 mole gas
$\nu = 1$
$V = aT + C_1$
$V - aT = C_1 = \text{constant}$

Ans.

2.56

(a)

$$dQ = dU + dW$$

$$\nu CdT = \nu C_v dT + PdV$$

$$\nu \frac{\alpha}{T} dT = \nu C_v dT + PdV$$

Here $\nu = 1$ then

$$dW = \frac{\alpha}{T} dT - C_v dT$$

$$\Delta W = \int PdV = \int_{T_0}^{\eta T_0} \frac{\alpha}{T} dT - \int_{T_0}^{\eta T_0} C_V dT$$

$$\Delta W = \alpha \ln \eta - C_v T_0(\eta - 1)$$

$$\Delta W = \alpha \ln \eta - \frac{R}{\gamma - 1} T_0(\eta - 1)$$

Ans.

(b)

$$C = \frac{\alpha}{T}$$

$$dQ = \nu CdT = \nu \frac{\alpha}{T} dT$$

Also we know
$dQ = \nu C_v dT + pdV$
Then

$$\nu \frac{\alpha}{T} dT = \nu C_v dT + pdV$$

Here $\nu = 1$ then

$$C_v dT + PdV = \frac{\alpha}{T} dT \text{ ——— (i)}$$

Also we know

$$P = \frac{RT}{V}$$

Put in (i)

$$C_v dT + \frac{RT}{V} dV = \frac{\alpha}{T} dT$$

$$\left(C_v - \frac{\alpha}{T}\right) dT = \frac{-RT}{V} dV$$

$$\int (C_v - \alpha/T) dT = -R \int \frac{dV}{V}$$

$$\int \frac{C_V dT}{T} - \alpha \int \frac{dT}{T^2} = -R \int \frac{dV}{V}$$

$$C_v \ln T + \frac{\alpha}{T} = -R \ln KV$$

$$\ln \left(T^{C_V} \times (KV)^R\right) = \frac{-\alpha}{T}$$

$$T^{C_V} (KV)^R = e^{-\alpha/T}$$

$$T^{C_V} V^R e^{\alpha/T} = \text{const}$$

$$\left(\frac{PV}{R}\right)^{C_V} V^R e^{\frac{R\alpha}{PV}} = \text{const}$$

$$PV^{\frac{C_V + R}{C_V}} e^{\frac{R\alpha}{PVC_V}} = \text{const}$$

$$PV^{\gamma} e^{\frac{\alpha(\gamma - 1)}{PV}} = \text{const}$$

Ans.

2.57

We know vanderwall gas equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \text{ ——— (i)}$$

$$\Delta W = \int dW = \int PdV$$

From (1)

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

$$\Delta W = \int_{V_1}^{V_2} \left(\frac{RT}{V - b} - \frac{a}{V^2}\right) dV$$

$$\Delta W = RT \ln \frac{V_2 - b}{V_1 - b} + a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

Ans.

2.58*

(a)
Internal energy of one mole gas

$$U = C_v T - \frac{a}{V_M}$$

Here V_M = Volume of 1 mole gas at tempreture T

$$U_1 = C_V T - \frac{a}{V_1}$$

$$U_2 = C_V T - \frac{a}{V_2}$$

$$\Delta U = U_2 - U_1 = \frac{a}{V_1} - \frac{a}{V_2} = a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) \text{ ....(1)}$$

Ans.

(b)

We know

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

Here n = 1

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$P = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\Delta W = \int_{V_1}^{V_2} \frac{RT}{V-b} dV - \int_{V_1}^{V_2} \frac{a}{V^2} dV$$

$$= RT \ln \frac{V_2 - b}{V_1 - b} + a\left[\frac{1}{V_2} - \frac{1}{V_1}\right] \text{. ....(2)}$$

Now

$\Delta Q = dW + \Delta U$

From (1) and (2)

$$\Delta Q = RT \ln \frac{V_2 - b}{V_1 - b} + a\left[\frac{1}{V_2} - \frac{1}{V_1}\right] + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

$$\Delta Q = RT \ln \frac{V_2 - b}{V_1 - b}$$

Ans.

2.59*

(a)

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \text{ —— (i)}$$

$$U = C_V T - \frac{a}{V} \text{ ———— (ii)}$$

$$dU = C_v dT + \frac{a}{V^2} dV$$

For adiabatic process

$dQ = 0 = dU + pdV$

$$-dU = PdV$$

$$-\left(C_v dT + \frac{a}{V^2} dV\right) = \left[\frac{RT}{V-b} - \frac{a}{V^2}\right] dV$$

$$-\int \frac{C_V dT}{RT} = \int \frac{dV}{V-b}$$

$$-\frac{C_V}{R} \ln TK = \ln(V - b)$$

$$V - b = (TK)^{-C_V/R}$$

$$V - b = T^{-C_V/R} \times K^{-C_V/R}$$

$$(V - b)\, T^{C_V/R} = K^{-C_V/R}$$

$$T(V - b)^{R/C_V} = \text{const}$$

Ans.

(b)

$dQ = C_v dT + PdV$

$$C_p dT = C_v dT + \frac{a}{V^2} dV + PdV \text{ ——— (i)}$$

Also

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \text{ ——— (ii)}$$

$$P = \frac{RT}{V-b} - \frac{a}{V^2}$$

Given that P=constant(Isobaric process)

Differeniate(ii):

$$\left(P + \frac{a}{V^2}\right)(dV - 0) + (V - b)\left(0 + \frac{-a}{V^3} dV\right) = RdT$$

$$\frac{-(V-b)}{V^3} adV + PdV + \frac{a}{V^2} dV = RdT$$

$$\frac{-adV}{V^2} + \frac{abdV}{V^3} + PdV + \frac{a}{V^2} dV = RdT$$

$$PdV = RdT - \frac{abdV}{V^3} \text{ ....(iii)}$$

Put in (i)

$$C_p dT = C_V dT + \frac{a}{V^2} dV + RdT - \frac{ab}{V^3} dV$$

$$C_p - C_v = R + \frac{a}{V^2}\frac{dV}{dT} - \frac{ab}{V^3}\frac{dV}{dT}$$

$$C_p - C_v = R + \left(\frac{a}{V^2} - \frac{ab}{V^3}\right)\left(\frac{dV}{dT}\right) \text{ ———— (iv)}$$

From (iii) :

$$\frac{dV}{dT} = \frac{R}{P + \frac{ab}{V^3}}$$

From (iv)

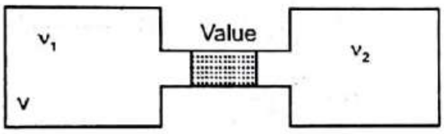
$$C_P - C_V = R + \left(\frac{a}{V^2} - \frac{ab}{V^3}\right)\left(\frac{R}{P + \frac{ab}{V^3}}\right)$$

$$C_P - C_V = \frac{R}{1 - 2a(V-b)^2 \big/ RTV^3}$$

Ans.

2.60*

We know
dQ = dU + dW
Here dW = 0
dQ = 0
Then dU = 0
$U_i = U_f$



$$\nu C_v T_1 - \frac{\nu^2 a}{V_1} = \nu C_v T_2 - \frac{\nu^2 a}{V_2 + V_1}$$

$$C_v (T_2 - T_1) = \nu a\left[\frac{1}{V_1 + V_2} - \frac{1}{V_1}\right]$$

$$C_v \Delta T = \nu a\left[\frac{-V_2}{V_1(V_1 + V_2)}\right]$$

$$\Delta T = \frac{\nu a V_2}{C_v V_1 (V_1 + V_2)}$$

$$\Delta T = \frac{\nu a V_2 (r-1)}{R V_1 (V_1 + V_2)}$$

Ans.

2.61*

We Know
$\Delta Q = \Delta U + \Delta W$
Calculation of ΔU

$$U_1 = \nu C_v T - \frac{\nu^2 a}{V_1}$$

$$U_2 = \nu C_v T - \frac{\nu^2 a}{V_2}$$

$$\Delta U = \nu^2 a\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

Calculation of ΔW

$\Delta w = \int PdV$ = zero

Because work done against Vaccum is zero

$\Delta Q = \Delta U + \Delta W$

$$\Delta Q = \nu^2 a\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

$$\Delta Q = \nu^2 a\left[\frac{V_2 - V_1}{V_1 V_2}\right]$$

Ans.

Kinetic theory of gases, Boltzmann's law and maxwell's distribution

2.62

We know : $P = nKT$

Where n = No. of gas particles/ volume

$$n = \frac{P}{KT}$$

Put values

$$n = \frac{P}{KT} = \frac{4\times10^{-15}\times1.01\times10^{5}}{8.3/6.023\times10^{23}\times300} / m^3 = 1\times10^{5} / cm^3$$

Ans.

Since in 1 cm^3 volume no. of molecules are 10^5

One molecules occupied volume is = 10^{-5} cm^3

This volume will be like cube then side of cube is

</>= $v^{1/3} = 10^{-5/3} = (10^{-5})^{1/3}$ cm

</>= $\left[10\times10^{-6}\right]^{1/3}$ cm $= \left(10^{1/3}\right)\times10^{-2}$ cm

$l = 2\times10^{-2}$ cm = 0.2 mm

Ans.

2.63

Suppose initial no. of molecules of gas is v_1 then

$PV = v_i RT$

$v_i = PV/RT$

No. of (molecules + atoms) in gas finally :

$$(1-\eta)\frac{PV}{RT} + 2\eta\frac{PV}{RT}$$

Due to dissociation. no. of particle is increase by 2 times

$$v_f = (\eta+1)\frac{PV}{RT},$$

$$P_f V = v_f RT = (\eta+1)\frac{PV}{RT}RT$$

$$P_f = (\eta+1)P$$

Calculation of initial pressure(P)

$$PV = \frac{m}{M}RT$$

$$P = \frac{mRT}{MV}$$

Put in (i)

$$P_f \frac{(\eta+1)mRT}{MV}$$

Ans.

2.64

Suppose no. of moles per unit volume of He is n_1 and that of N_2 is n_2

Also molar mass of H_1 and N_2 are M_1 and M_2.

We know

Density of mixture = Total mass/ Total volume

$$\rho = \frac{m_1+m_2}{v} = \rho_1 + \rho_2$$

$$\rho = \rho_1 + \rho_2$$

$$\rho = n_1 M_1 + n_2 M_2 \text{ ....(1)}$$

Also

$$P = nKT = (n_1+n_2)KT$$

$$\frac{P}{KT} - n_1 - = n_2 \text{ ....(2)}$$

From (i) and (ii)

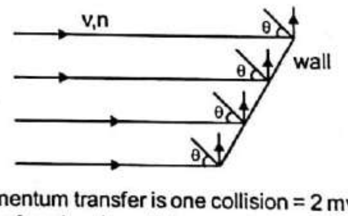
$$\rho = n_1 M_1 + \left(\frac{P}{KT} - n_1\right)M_2$$

$$n_1 = \frac{\left(\rho - \frac{PM_2}{KT}\right)}{M_1 - M_2} = \frac{\frac{\rho}{M_2} - \frac{\rho}{KT}}{\frac{M_1}{M_2} - 1}$$

$$n_1 = \frac{\frac{P}{KT} - \frac{\rho}{M_2}}{1 - \frac{M_1}{M_2}}$$

Ans.

2.65



Momentum transfer is one collision = 2 mvcosθ

No. of molcuels collides per second = η(VA)

$$\left(\frac{dP}{dt}\right) = \eta V(2mvCos\theta)A$$

$F = 2\eta m V^2 A \cos\theta$

$F/A = P = 2\eta m v^2 \cos\theta$

Ans.

2.66

We know

$$V_s = V_{sound} = \sqrt{\frac{\gamma P}{\rho}}$$

$$\gamma = \frac{V_s^2 P}{P} = 1 + \frac{2}{i}$$

Where i = modes of degree of freedom to contribute energy.

$$\frac{2}{i} = \frac{V_s^2 \rho}{P} - 1$$

$$i = \frac{2}{\frac{V_s^2 \rho}{P} - 1}$$

Put values

i = 5

Ans.

2.67

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{V}{V_{rms}} = \sqrt{\frac{\gamma}{3}} \text{ ....(1)}$$

Here

$$\gamma = 1 + \frac{2}{i} = 1 + \frac{2}{i} \text{ ....(2)}$$

$$\frac{v}{v_{rms}} = \sqrt{\frac{1 + 2/i}{3}} = \sqrt{\frac{(2+i)}{3i}}$$

Ans.

(a)

For Monoatomic gas

i = 3

$$\frac{v}{v_{rms}} = \sqrt{\frac{5}{9}}$$

Ans.

(b)

For Rigid diatomic gas

Vibrational mode will be not active

i = 5

$$\frac{v}{v_{rms}} = \sqrt{\frac{2+5}{3 \times 5}} = \sqrt{\frac{7}{15}}$$

Ans.

2.68*

Kinetic energy (K.E.) $= \frac{i}{2}KT$ ....(1)

i = no. of translational digree of freedom

+ no. of rotational digree of freedom

+2 (no. of vibration DOF)

For Linear N-atomic molecules

i = 3 + 2+ 2 (3N -5)

= 6N - 5

Form(1)

$$KE = \left(\frac{6N-5}{2}\right)KT = \left(3N - 5/2\right)KT$$

Network (non linear atomic)

i = 3 +3 +2 (3N – 6) = 6N +6 –12 = 6N – 6

$$K.E. = \frac{1}{2}(6N-6)\,KT = 3(N-1)\,KT$$

Ans.

2.69*

Molar heat capacity (C) at constant volume

$$C = C_v = \frac{R}{\gamma - 1} = \frac{i}{2}R$$

$$\gamma = 1 + \frac{2}{i}$$

(a)

Diatomic gas

i = 3 + 2 +2 (3 × 2 – 5) = 7

$$C = \frac{7}{2}R$$

Ans.

$$\gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

Ans.

(b)

Linear N atomic gas

i = 3 + 2+ 2 (3N –5) = 6N –5

$$C = \frac{6N-5}{2}R$$ **Ans.**

$$\gamma = 1 + \frac{2}{6N-5} = \frac{6N-3}{6N-5}$$

Ans.

(c)

Non linear -N-atomic

i = 3 + 3 + 2 (3N –6) = 6N –6

$$C = \frac{6N-6}{2}R = 3(N-1)$$ **Ans.**

$$\gamma = 1 + \frac{2}{6N-6} = \frac{6N-4}{6N-6} = \frac{3N-2}{3N-3} = \frac{N-2/3}{N-1}$$

Ans.

2.70*

For isobaric process

$dQ = nC_p dT$

$dW = nRdT$

$$\text{Ratio} = \frac{dW}{dQ} = \frac{R}{C_p}$$

$C_p = R + C_v$

From **Q.2.69**

$$C_v = \frac{6N-5}{2}R \text{ (For linear)}$$

$$C_p = R + \frac{6N-5}{2}R = \frac{(6N-3)R}{2} = \frac{3}{2}(2N-1)R$$

$$\text{Ratio} = \frac{Rx2}{3(2N-1)R} = \frac{2}{3(2N-1)} = \frac{1}{(3N-3/2)}$$

Ans.

For $C_v = 3(N-1)R$ (For nonlinear)

$C_p = R + 3(N-1)R = (3N-2)R$

$$\text{Ratio} = \frac{R}{(3N-2)} = \frac{1}{3N-2}$$

Ans.

2.71

$C_v = 0.65$

$C_p = 0.91$

$$\gamma = \frac{C_p}{C_v} = \frac{0.91}{0.65} = 1 + \frac{2}{i}$$

$$\frac{2}{i} = \frac{0.91}{.65} - 1 = \frac{0.26}{0.65}$$

$$i = 2 \times \frac{0.65}{0.26} = \frac{2 \times 65}{26} = 5$$

Calcalation of molar mass

We know

$R = 8.3$ joule/ mole K

$C_p - C_v = 0.91 - 0.65 = 0.26$ J/gK

For 1 gm, value of $C_p - C_v$ is 0.26

For 1 mole, value of $C_p - C_v$ is 0.26 M

where M is molar mass then

$0.26\,M = 8.3$

$M = 32$ **Ans.**

2.72

(a)

$C_p = R + C_v$

$$C_v = C_p - R = \frac{i}{2}R$$

$$i = 2\left(\frac{C_p}{R} - 1\right)$$

Ans.

(b)

$PT = \text{const}$

$$P\left[\frac{PV}{nR}\right] = \text{const}$$

$PV^{1/2} = \text{const}$

$PV^x = \text{const}$

$x = 1/2$

Also

$$C = \frac{R}{\gamma-1} - \frac{R}{x-1}$$

$$\frac{R}{\gamma-1} = C + \frac{R}{x-1}$$

$$\frac{1}{\gamma-1} = \frac{C}{R} + \frac{1}{x-1}$$

$$\frac{1}{1+\frac{2}{i}-1} = \frac{C}{R} + \frac{1}{x-1}$$

$$i = 2\left(\frac{C}{R} + \frac{1}{x-1}\right)$$

Ans.

2.73

$$C_v(\text{mix}) = \frac{v_1\frac{3}{2}R + v_2 \times \frac{5}{2}R}{v_1 + v_2}$$

$$C_v(\text{mix}) = \frac{3v_1R + 5v_2R}{2(v_1+v_2)} = \frac{R}{\gamma-1}$$

$$\gamma - 1 = \frac{2(v_1+v_2)}{3v_1+5v_2}$$

$$\gamma = \frac{2v_1+2v_2}{3v_1+5v_2} + 1$$

$$\gamma = \frac{5v_1+7v_2}{3v_1+5v_2}$$

Ans.

2.74

We know

$$P = \frac{\rho RT}{M}$$

$$\Delta P = \frac{\rho R}{M}\Delta T \text{ ---------- (i)}$$

Calculation of ΔT

$$nC_v\Delta T = \frac{1}{2}mv^2$$

$$\Delta T = \frac{1}{2}\frac{mv^2}{nC_V}$$

Put in (i)

$$\Delta P = \frac{\rho R}{M}\left(\frac{1}{2}\frac{mv^2}{nC_V}\right) = \frac{1}{2}\frac{\rho R v^2}{i/2R} = \frac{\rho v^2}{i}$$

$$\frac{\Delta P}{P} = \frac{\rho v^2 M}{i\rho RT}$$

$$\frac{\Delta P}{P} = \frac{Mv^2}{iRT}$$

Ans.

2.75

(a)

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Ans.

$$K.E. = \frac{3}{2}KT$$

(b)

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

Where m = mass of one drop

$$m = \rho\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4\pi\rho}{3}\times\frac{d^3}{8} = \frac{\pi\rho d^3}{6}$$

$$V_{rms} = \sqrt{\frac{3KT6}{\rho\pi d^3}}$$

$$V_{rms} = 3\sqrt{\frac{2KT}{\rho\pi d^3}}$$

Ans.

2.76

Adiabatic process

$$V_1 = \sqrt{\frac{3RT_1}{M}}$$

$$V_2 = \frac{V_1}{\eta} = \sqrt{\frac{3RT_2}{M}}$$

Divide both equaions :

$$\eta = \sqrt{\frac{T_1}{T_2}}$$

$T_1 = T_2\,\eta^2$ ------ (i)

Also we know

$$TV^{\gamma-1} = const$$

From (i) $T_2\eta^2 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \eta^2$$

$$\left(\frac{V_2}{V_1}\right)^{1+\frac{2}{i}-1} = \eta^2$$

$$\left(\frac{V_2}{V_1}\right)^{\frac{2}{i}} = \eta^2$$

$$\frac{V_2}{V_1} = n^i$$

Ans.

2.77

$$V_{rms} = \sqrt{\frac{3RT_1}{M}}$$

$$\eta V_{rms} = \sqrt{\frac{3RT_2}{M}}$$

Here $T_1 = T$

$$\frac{1}{\eta} = \sqrt{\frac{T_1}{T_2}}$$

$T_2 = \eta^2 T_1$ ---------- (i)

$\Delta Q = \left(\frac{m}{M}\right) C_V \Delta T$ ----------- (ii)

From (i)

$$T_1 - T_2 = \Delta T = \left(-1+\eta^2\right)T_1 = \left(\eta^2 - 1\right)T$$

Put in (ii)

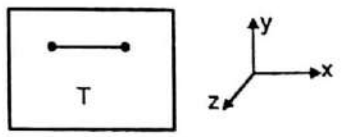
$$\Delta Q = \frac{m}{M}\,C_V\left(\eta^2 - 1\right)T$$

$$\Delta Q = \frac{m}{M}\frac{i}{2}R\left(\eta^2 - 1\right)T$$

Ans.

2.78

Suppose a diatomic gas atom is in x direction



$$w_{sq} = \sqrt{\left\langle w^2\right\rangle} = \sqrt{\left\langle w_{yy}^2 + w_{zz}^2\right\rangle} \;............(1)$$

By random motion

$$\left\langle w_{yy}\right\rangle = \left\langle w_{zz}\right\rangle$$

From (1)

$$w_{sq} = \sqrt{2\left\langle w_{yy}^2 \right\rangle} \text{ ....(2)}$$

Rotational kinetic energy of diatomic gas

$$K.E. = \frac{1}{2} I_{yy} W_{yy}^2 + \frac{1}{2} I_{zz} W_{zz}^2$$

According to equipartition law of energy
For each degree of freedom

$$K.E. = \frac{1}{2} KT$$

Then

$$\frac{1}{2} I_{yy} W_{yy}^2 = \frac{1}{2} kT$$

According to question

$$I_{yy} = I$$

$$\frac{1}{2} I W_{yy}^2 = \frac{1}{2} kT$$

$$W_{yy} = \sqrt{\frac{KT}{I}}$$

From (2)

$$w_{sq} = \sqrt{\frac{2KT}{I}}$$

Ans.

2.79

We know

$$TV^{\gamma-1} = const$$

Then

$$T_0 V^{\gamma-1} = T\left(\frac{V}{\eta}\right)^{\gamma-1}$$

$$T = T_0 \eta^{\gamma-1} \text{ ——— (i)}$$

Mean kinetic energy of rotational motion

$$K.E. = \frac{1}{2} (i) KT \text{ ——— (ii)}$$

Degree of freedom for rotational motion
i = 2
Then from (i) and (ii)

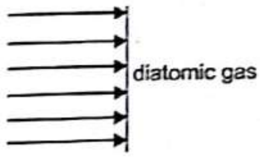
$$K.E. = \frac{1}{2} 2KT_0 \eta^{\gamma-1}$$

$$K.E. = KT_0 \eta^{1+\frac{2}{i}-1}$$

$$K.E. = KT_0 \eta^{\frac{2}{i}}$$

Ans.

2.80*



We know of collision per second per unt area of wall : $v = \frac{1}{4} n < v >$

Where n = no. of molecules per unit volume.
We know

$$TV^{\gamma-1} = const$$

$$T_i V^{\gamma-1} = T_f (\eta V)^{\gamma-1}$$

$$T_f = T_i \left(\frac{1}{\eta}\right)^{1+\frac{2}{i}-1}$$

$$T_f = T_i \left(\frac{1}{\eta}\right)^{\frac{2}{i}} \text{ —— (i)}$$

$$< v > = \sqrt{\frac{8RT}{\pi M}}$$

$$v_1 = \frac{1}{4}\left[\frac{N}{V}\right]\sqrt{\frac{8RT_i}{\pi M}} = \frac{N}{4V}\sqrt{\frac{8RTi}{\pi M}}$$

$$v_2 = \frac{1}{4}\left[\frac{N}{\eta V}\right]\sqrt{\frac{8RT_f}{\pi M}}$$

$$\frac{v_2}{v_1} = \frac{1}{\eta}\sqrt{\frac{T_f}{T_i}}$$

$$\frac{v_2}{v_1} = \frac{1}{\eta}\sqrt{\left(\frac{1}{\eta}\right)^{\frac{2}{i}}}$$

$$\frac{v_2}{v_1} = \frac{1}{\eta}\left(\frac{1}{\eta}\right)^{\frac{1}{i}}$$

$$\frac{v_2}{v_1} = \left(\frac{1}{\eta}\right)^{\frac{1+i}{i}} = \frac{1}{\eta^{\frac{1+i}{i}}}$$

Ans.

2.81*

We know

$$C = \frac{R}{\gamma - 1} - \frac{R}{x-1}$$

$$R = \frac{R}{2}i - \frac{R}{x-1}$$

For diatomic gas then
i = 5 then

$$1 = \frac{5}{2} - \frac{1}{x-1}$$

$$\frac{1}{x-1} = \frac{3}{2}$$

$$2 = 3x - 3$$

$$x = \frac{5}{3}$$

$PV^{5/3}$ = const

$$\frac{vRT}{V}V^{5/3} = \text{const}$$

$TV^{2/3}$ = const

$T_iV^{2/3} = T_f(\eta v)^{2/3}$

$$T_f = T_i\left(\frac{1}{\eta}\right)^{2/3}$$

Also we know

$$V_i = \frac{1}{4}\left(\frac{N}{V}\right)\sqrt{\frac{8RT_i}{\pi M}}$$

$$V_f = \frac{1}{4}\left(\frac{N}{\eta V}\right)\sqrt{\frac{8RT_f}{\pi M}}$$

Now

$$\frac{V_f}{V_i} = \frac{1}{\eta}\sqrt{\frac{T_f}{T_i}} = \frac{1}{\eta}\sqrt{\left(\frac{1}{\eta}\right)^{2/3}} = \frac{1}{\eta}\left(\frac{1}{\eta}\right)^{1/3} = \left(\frac{1}{\eta}\right)^{4/3}$$

$$\frac{V_f}{V_i} = \left(\frac{1}{\eta}\right)^{4/3} \quad \text{...........(1)}$$

Ans.

For diatomic gas

$$\left(\frac{1}{\eta}\right)^{(i-1)/i-2} = \left(\frac{1}{\eta}\right)^{5-1/5-2} = \left(\frac{1}{\eta}\right)^{4/3}$$

Then

$$\frac{V_f}{V_i} = \left(\frac{1}{\eta}\right)^{(i-1)/i-2}$$

Ans.

2.82*

No. of collision per second per unit area

$$\nu = \frac{1}{4}\left(\frac{N}{V}\right)\sqrt{\frac{8RT}{\pi M}}$$

$$\nu = \frac{N}{4}\sqrt{\frac{8RT}{\pi M}}\left(V^{-1}T^{1/2}\right)$$

Since ν = const
$V^{-1}T^{1/2}$ = const

$$V^{-1}\left(\frac{PV}{vR}\right)^{1/2} = \text{const}$$

$P^{1/2}V^{-1+1/2}$ = const
PV^{-1} = const
Compare with
PV^x = const
x = 1

$$C = \frac{R}{\gamma - 1} - \frac{R}{x-1} \quad \text{............(1)}$$

Also we know

$$\gamma = 1 + \frac{2}{i} \text{ then}$$

$$C = \frac{R}{2}i - \frac{R}{-1-1}$$

$$C = \frac{Ri}{2} + \frac{R}{2} = \frac{R}{2}(1+i)$$

Ans.

2.83

$$V_p = \sqrt{\frac{2RT}{M}}$$

We know $PM = \rho RT$

$$\frac{RT}{M} = \frac{P}{\rho}$$

$$V_p = \sqrt{\frac{2P}{\rho}}$$

$$\langle v \rangle = \sqrt{\frac{8RT}{\pi M}}$$

$$\langle v \rangle = \sqrt{\frac{8P}{\pi \rho}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} = \sqrt{\frac{3P}{\rho}}$$

Ans.

2.84*

$$d(f(u)) = \frac{dN}{N}(u) = \frac{4}{\sqrt{\pi}}e^{-u^2}u^2du$$

Here $u = \frac{v}{v_P}$

$\frac{dN(u)}{N}$ =sum of fraction of gas molecules having ratio $\left(\frac{v}{V_P}\right)$ between u to u + du

(a)

$$-\delta_\eta < \frac{V - V_P}{V_P} < \delta_\eta$$

$$1-\delta_\eta < \frac{V}{V_P} < 1+\delta_\eta$$

Then sum of fraction of molecules

$$f(u) = \int_{1-\delta n}^{1+\delta n} \frac{4}{\sqrt{\pi}}e^{-u^2}u^2du$$

$$f(u) = \frac{8}{\sqrt{\pi}}e^{\delta_\eta}$$ **Ans.**

(b)

$$-\delta_\eta < \frac{V - V_S}{V_S} < \delta_\eta$$

$$1-\delta_\eta < \frac{V}{V_s} < 1+\delta_\eta$$

Also

$$V_S = \sqrt{\frac{3RT}{M}}$$

$$V_P = \sqrt{\frac{2RT}{M}}$$

$$\frac{V_S}{V_P} = \sqrt{\frac{3}{2}}$$

$$V_s = V_P\sqrt{\frac{3}{2}}$$

$$1-\delta_\eta < \frac{V}{V_P}\sqrt{\frac{2}{3}} < 1+\delta_\eta$$

$$\sqrt{\frac{3}{2}}(1-\delta n) < \frac{V}{V_P} < \sqrt{\frac{3}{2}}((1+\delta n))$$

From (a) integration $f(u) = 12\sqrt{\frac{3}{2\pi}}e^{-3/2}\delta_\eta$ **Ans.**

2.85

(a)

$$V_P = \sqrt{\frac{2KT}{m}}$$

$$V_{rms} = \sqrt{\frac{3KT}{m}}$$

$$V_{rms} - V_P = \Delta V = \sqrt{\frac{3KT}{m}} - \sqrt{\frac{2KT}{m}}$$

$$\Delta V = \sqrt{\frac{3KT}{m}} - \sqrt{\frac{2KT}{m}}$$

$$\sqrt{T} = \frac{\Delta V}{\left(\sqrt{3}-\sqrt{2}\right)}\sqrt{\frac{m}{K}}$$

$$T = \frac{m\Delta V^2}{K\left(\sqrt{3}-\sqrt{2}\right)^2}$$

Ans.

(b)

F(u) will be maximum at maximum probable velocity.

$$V_P = V = \sqrt{\frac{2KT}{m}}$$

$$T = \frac{mV^2}{2K}$$

Ans.

2.86*

(a)

We know

$$F(u) = \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mv^2/2KT}\, 4\pi v^2$$

Let temp T at which for V_1 and V_2, F(u) are same

$$\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mv_1^2/2KT}\, 4\pi v_1^2 = \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mv_2^2/2KT}\, 4\pi v_2^2$$

Here range will be taken same for both

$$e^{\frac{-mv_1^2}{2KT}+\frac{mv_2^2}{2KT}} = v_2^2/v_1^2$$

$$T = \frac{m\left(v_2^2 - v_1^2\right)}{4K\ln v_2/v_1}$$

Ans.

(b)

$$\left(\frac{m}{2\pi KT_0}\right)^{3/2} e^{-mv^2/2KT_0}\, 4\pi v^2 = \left(\frac{m}{2\pi K\eta T_0}\right)^{3/2} e^{-mv^2/2\eta KT_0}\, 4\pi v^2$$

$$v = \sqrt{\frac{3kT_0}{m}\frac{\eta \ln \eta}{\eta - 1}}$$

Ans.

2.87

$$V_P = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2KT}{m}}$$

$$v_P^N = \sqrt{\frac{2KT}{mN}}$$

$$v_P^o = \sqrt{\frac{2KT}{m_0}}$$

$$v_P^N - v_P^o = \Delta v = \sqrt{\frac{2KT}{m_N}} - \sqrt{\frac{2KT}{m_0}}$$

$$\frac{\Delta v}{\sqrt{2K}} = \left(\frac{1}{\sqrt{m_N}} - \frac{1}{\sqrt{m_0}}\right)\sqrt{T}$$

$$\sqrt{T} = \frac{\Delta V}{\sqrt{2K}} \frac{\sqrt{m_N}\sqrt{m_0}}{\left(\sqrt{m_0} - \sqrt{m_N}\right)}$$

$$T = \frac{\Delta V^2}{2K} \frac{m_N m_0}{\left(\sqrt{m_0} - \sqrt{m_N}\right)^2}$$

$$T = \frac{\Delta V^2 m_N}{2K\left(1 - \sqrt{\frac{m_N}{m_0}}\right)^2}$$

Ans.

2.88*

$$\left(\frac{m_H}{2\pi KT}\right)^{3/2} e^{-\frac{m_H}{2KT}v^2}\ 4\pi v^2 = \left(\frac{m_{He}}{2\pi KT}\right)^{3/2} e^{-\frac{m_{He}}{2KT}v^2}\ 4\pi v^2$$

$$\left(\frac{m_H}{m_{He}}\right)^{3/2} e^{-\frac{m_H v^2}{2KT}} = e^{-\frac{m_{He} v^2}{2KT}}$$

$$\frac{3}{2}\ \ln\frac{m_H}{m_{He}} = v^2\left(\frac{m_H - m_{He}}{2KT}\right)$$

Here $m_H = m_{He}$

$m_{He} = M_1$

$$V = \sqrt{\frac{3KT\ln\left(m_2/m_1\right)}{m_2 - m_1}}$$

Ans.

2.89*

$$f(u) = \left(\frac{m}{2\pi KT}\right)^{3/2} e^{\frac{-m}{2KT}v^2}\ \ 4\pi v^2$$

Here v = constant

But T is varibale

Then for f(u) maximum

$$\frac{df(u)}{dT} = 0$$

$$f(u) = \left(\frac{m}{2\pi K}\right)^{3/2} \left(T^{-3/2} e^{-\frac{m}{2KT}v^2}\right) 4\pi v^2$$

$$\frac{df(u)}{dT} = 0$$

$$0 = T^{-3/2} e^{-\frac{mv^2}{2K}T^{-1}} \left(\frac{-mv^2}{2K}\right)(-1)T^{-2} + e^{-\frac{mv^2}{2K}T^{-1}}\left(\frac{-3}{2}\right)T^{-5/2}$$

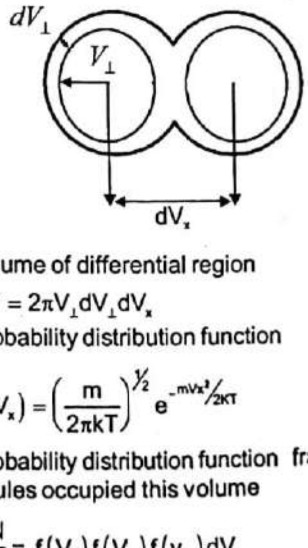
$$T^{-7/2}\frac{mv^2}{2K} = \frac{3}{2}T^{-5/2}$$

$$T^{-1}\frac{mv^2}{K} = 3$$

$$T = \frac{mv^2}{3K}$$

Ans.

2.90*



Volume of differential region

$dV = 2\pi V_\perp dV_\perp dV_x$

Probability distribution function

$$f(V_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-mV_x^2/2KT}$$

Probability distribution function fraction of molecules occupied this volume

$$\frac{dN}{N} = f(V_x)f(V_\perp)f(v_\perp)dV$$

$$\frac{dN}{N}=\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-(v_x^2+v_{\perp y}^2+v_{\perp z}^2)}dv$$

$$\frac{dN}{N}=\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-mv^2/2KT}2\pi V_\perp dV_\perp dV_x$$

Ans.

2.91*

We know Mean velocity

$$<Vx> = \frac{\int_{-\infty}^{\infty} V_x dN}{N} = \frac{\int_{-\infty}^{\infty} V_x N\left(\frac{m}{2\pi KT}\right)^{1/2} e^{-\frac{m}{2kT}V_x^2}dx}{N}$$

$$= \left(\frac{m}{2\pi kT}\right)^{1/2}\left(\frac{KT}{m}\right)e^{-\frac{m}{2kT}V_x^2}\Big|_{-\infty}^{\infty} = 0$$

Ans.

Mean speed

$$<|V_x|> = \frac{\int_0^{\infty}|V_x|N\left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m}{2kT}V_x^2}dVx}{N}$$

$$= 2\int_0^{\infty} V_x\left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m}{2kT}V_x^2}dVx$$

$$<|V_x|> = \frac{2\int_0^{\infty} V_x N\left(\frac{m}{2\pi KT}\right)^{1/2} e^{-\frac{m}{2kT}V_x^2}dVx}{N}$$

$$= 2\left(\frac{m}{2\pi kT}\right)^{1/2}\frac{KT}{m}\ e^{-mVx^2/2kT}\Big|_0^{\infty}$$

$$<|V_x|> = 2\sqrt{\frac{2}{2\pi kT}\times\frac{k^2T^2}{m^2}} = \sqrt{\frac{4kT}{2\pi m}}$$

$$<|V_x|> = \sqrt{\frac{2kT}{\pi m}}$$

Ans.

2.92

Method:1

$$<V_x^2>=\frac{\int_{-\infty}^{\infty} V^2{}_x dN}{N}$$

$$= 2\int_0^{\infty} V^2x\left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m}{2kT}V_x^2}dVx$$

$$<Vx^2>=\frac{KT}{m}$$ **Ans.**

Method :2

We know

$$\sqrt{<V^2>} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3KT}{m}}$$

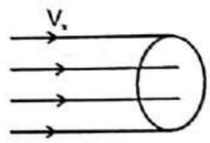
$$\langle V^2\rangle = \frac{3kT}{m}$$

Also

$$<V^2x> = <Vy^2> = <Vz^2> = \frac{\langle V^2\rangle}{3}$$

$$<Vx^2> = \frac{KT}{m}$$ **Ans.**

2.93



No. of molecules / Volume = n

Fraction of molecules having velocity Vx to Vx + dVx is

$$=\frac{dN}{N}$$

No. of molecules / volume having velocity Vx to Vx +dVx is

$$= n\left(\frac{dN}{N}\right)$$

No. of molecules approach per second toward wall with velocity Vx

$= n\left(\frac{dN}{N}\right)$ (volume traverese in one second with velocity Vx)

$$= n\left(\frac{dN}{N}\right)(Vx\, dA)$$

Total no. of molecules approach per second, per unit area of wall

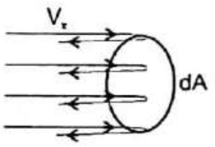
$$=\int nVx\left(\frac{dN}{N}\right)$$

$$\nu=\int nVx\left(\frac{m}{2\pi kT}\right)^{3/2} e^{\frac{-mVx^2}{2KT}}dVx$$

$$\nu = \frac{n}{4}\sqrt{\frac{8kT}{\pi m}} = \frac{n}{4}<V>$$

Ans.

2.94



No. of molecules per unit volume = n
Momentum transfer in one collision due to one molecule = 2 mVx

Fraction of molecules having velocity Vx is $\frac{dN}{N}$

No. of molecules per unit volume having velocity Vx to Vx + dVx

$$= n\left(\frac{dN}{N}\right)$$

No. of molecules collide per second with velocity Vx is

$$= n\left(\frac{dN}{N}\right) \text{ (volume travel in one second)}$$

$$= n\left(\frac{dN}{N}\right) (Vx\, dA)$$

Momentum transfer with Vx velocity per second

$$= n\left(\frac{dN}{N}\right) Vx(dA)\,(2m\,Vx)$$

Net moment transfer with wall per second

$$dF = \int_{-\infty}^{\infty} n\left(\frac{dN}{N}\right) Vx(dA)\,(2m\,Vx)$$

$$P = \frac{dF}{dA} = \int_{-\infty}^{\infty} 2nmVx^2\left(\frac{m}{2\pi kT}\right)^{1/2} e^{\frac{-m}{2kT}Vx^2}\,dVx$$

$P = nKT$ **Ans.**

2.95*

$$<\frac{1}{v}> = \frac{\int_0^{\infty}\frac{1}{v}(dN)}{N} = \int_0^{\infty}\frac{1}{v}\left(\frac{dN}{N}\right)$$

$$= \int_0^{\infty}\frac{1}{v}\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\frac{m}{2KT}v^2}\,4\pi v^2 dv$$

$<\frac{1}{v}> = \sqrt{\frac{2m}{\pi KT}} = 4\pi < V >$ **Ans.**

2.96*

We know

$$\frac{dN}{N} = \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\frac{mv^2}{2KT}}\; 4\pi v^2 dv$$

Now kinetic energy

$$\varepsilon = \frac{1}{2}mv^2 \;..............(1)$$

$$v = \sqrt{\frac{2\varepsilon}{m}}$$

Differentiate equation (1)

$$d\varepsilon = mvdv$$

$$vdv = \frac{d\varepsilon}{m}$$

$$\frac{dN}{N} = \left(\frac{m}{2\pi KT}\right)^{-3/2} e^{-\frac{\varepsilon}{KT}} 4\pi\sqrt{\frac{2\varepsilon}{m}}\frac{d\varepsilon}{m}$$

$\frac{dN}{N} = 2\pi(\pi kT)^{-3/2} e^{-\frac{\varepsilon}{KT}}\sqrt{\varepsilon}\; d\varepsilon$ **Ans.**

For maximum proble value of K.E. at which no. of molecules will be maximum

$$f = 2\pi(\pi kT)^{-3/2} e^{-\frac{\varepsilon}{KT}}\sqrt{\varepsilon}$$

$$\frac{df}{d\varepsilon} = 0$$

$\varepsilon_{Pr} = \frac{1}{2}KT$ **Ans.**

K.E. corresponding to most probable speed

$$\varepsilon = \frac{1}{2}m\left(\frac{2KT}{m}\right) = KT \neq \varepsilon_{pr}$$

2.97*

From Q .2.96

$$\frac{dN}{N} = 2\pi(\pi kT)^{-3/2} e^{-\frac{\varepsilon}{KT}}\sqrt{\varepsilon}\; d\varepsilon \quad \text{———— (i)}$$

$$\delta n = \frac{d\varepsilon}{\varepsilon}$$

$$d\delta = \varepsilon\delta n$$

$$\varepsilon = \frac{3}{2}KT$$

$$\frac{\varepsilon}{KT} = \frac{3}{2}$$

$$KT = \frac{2\varepsilon}{3}$$

Put in (1)

$$\frac{dN}{N} = 2\pi\left(\pi\frac{2\varepsilon}{3}\right)^{-3/2} e^{-3/2}\,\varepsilon\sqrt{\varepsilon}\,\delta n$$

$$\frac{dN}{N} = 2\pi\left(\frac{2\pi}{3}\right)^{-3/2} e^{-3/2}\,\delta n$$

$$\frac{dN}{N} = 3\sqrt{6\pi}\, e^{-3/2}\,\delta n$$

Ans.

2.98*

From Q, 2.96

$$\frac{dN}{N} = 2\pi(\pi kT)^{-3/2}\, e^{-\varepsilon/kT}\sqrt{\varepsilon}\, d\varepsilon$$

Where $\frac{dN}{N}$ = fraction of molecules which kinetic energy lies between ε to $\varepsilon + d\varepsilon$

Now we want sum of fraction of molecules whose kinetic energy $\varepsilon > \varepsilon_0$

Then

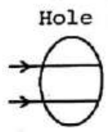
$$\sum\frac{dN}{N} = \int_{\varepsilon_0}^{\infty} 2\pi(\pi KT)^{-3/2}\, e^{-\varepsilon/KT}\sqrt{\varepsilon}\, d\varepsilon$$

$$\sum\frac{dN}{N} = \frac{2\pi}{(\pi KT)^{3/2}}\int_{\varepsilon_0}^{\infty} e^{-\varepsilon/KT}\sqrt{\varepsilon}\, d\varepsilon$$

Ans.

2.99*

(a)



$F = Av^3\, e^{-mv^2/2KT}$

Probable velocity is that velocity at which no. of molecules will be maximum.

For maximum no. of molecules F will be maximum and hence

$$\frac{dF}{dv} = 0$$

$$V = \sqrt{\frac{3KT}{m}}$$

(b)

$$\varepsilon = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2\varepsilon}{m}} \text{ ——— (i)}$$

$$d\varepsilon = mvdv \text{ ——— (ii)}$$

$$\frac{dN}{N} = Av^3 e^{-mv^2/2KT}\, dv$$

$$\frac{dN}{N} = A\left(\frac{2\varepsilon}{m}\right)^{3/2} e^{-\frac{\varepsilon}{kT}}\,\frac{d\varepsilon}{\sqrt{2m\varepsilon}}$$

$$F'(\varepsilon) = A\left(\frac{2\varepsilon}{m}\right)^{3/2}\left(\frac{1}{2me}\right)^{1/2} e^{-\frac{\varepsilon}{KT}}$$

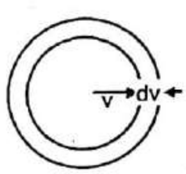
For $F'(\varepsilon)$ maximum

$$\frac{dF'(\varepsilon)}{d\varepsilon} = 0$$

$$\varepsilon = KT$$

Ans.

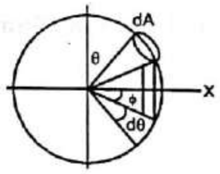
2.100*



No. of molecules making solid angle $d\Omega$ on centre

Using formula

$$dv' = (dN)\frac{d\Omega}{4\pi}$$



Using formula

$$d\Omega = \sin\theta d\theta(d\phi)$$

No. of collision persecond at angle θ on unit area

$dv = (dv')\,(\text{volume}) = (dv')\,(v\cos\theta) \times 1$

$$dv = \left(\frac{dN}{4\pi}d\Omega\right)(v\cos\theta)$$

$$= \left[N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2KT}\,4\pi v^2 dv\right]\frac{1}{4\pi}v\cos\theta\, d\Omega$$

$$= N\left(\frac{m}{2\pi kT}\right)^{3/2}\int_{v=0}^{\infty} e^{-\frac{mv^2}{KT}}\, v^3\cos\theta(dv)\sin\theta d\theta\int_0^{2\pi} d\phi$$

$$\frac{dv}{N} = \left(\frac{2kT}{m\pi}\right)^{1/2} \sin\theta \cos\theta d\theta$$ **Ans.**

2.101*

Similar like Q:2.100

$$dv = \int_{\theta=0}^{\pi/2} (dn)\frac{(d\Omega)}{4\pi} v\cos\theta$$

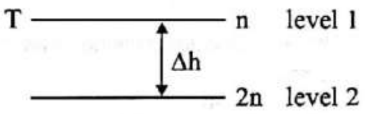
$dv =$

$$\left(N\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mv^2/2KT} 4\pi v^2 dv\right)\frac{1}{4\pi} v\cos\theta\sin\theta\int_0^{2\pi} d\varphi$$

$$\frac{dv}{N} = \pi\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mv^2/2KT} v^3 dv$$

Ans.

2.102*



Suppose potential energy of level (1) is zero.

$U_0 = 0$

$n_0 = n$

According to boltz man's formula

$n = n_0 e^{-(U-U_0)/KT}$

$2n = ne^{-U/KT}$

$$\ln 2 = \frac{U}{KT}$$

$U = KT \ln 2.$

Since filed is uniform hence force will be uniform then

Work done by force $= -(U_f - U_0)$

$F\Delta h = -U_f = -U$

$$F = \frac{-KT}{\Delta h}\ln 2$$

Magnitude of force

$$F = \frac{-KT}{\Delta h}\ln 2$$

Ans.

2.103* **Method:1**

On droplet, force is arised due to mass changed and hence

$$F = \Delta mg = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \Delta\rho = \frac{\pi d^3 \Delta\rho}{6} g$$

From Q.No.2.102

$$F = \frac{KT}{\Delta h}\ln n = \frac{\pi d^3 \Delta\rho g}{6}$$

$$K = \frac{(\pi d^3 \Delta\rho)hg}{6T\ln n} = \frac{R}{N_A}$$

$$N_A = \frac{6RT\ln n}{\pi d^3 \Delta\rho hg}$$

Ans.

Method:2

$n_f = n_0 e^{-U_f - U_i/KT}$

Hence $U_f = mgh$

$\Delta U_f = \Delta mgh$

Here $n_0 = n\, n_f$

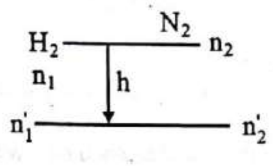
$n_f = nn_f\, e^{-\Delta mgh/KT}$

$$\ln n = \frac{\Delta mgh}{KT} = \left(\frac{\pi d^3 \Delta\rho gh}{6}\right)\left(\frac{N_A}{RT}\right)$$

$$N_A = \frac{6RT\ln n}{\pi d^3 \Delta\rho gh}$$

Ans.

2.104*



$$\frac{n_1}{n_2} = n_0$$

$$\frac{n'_1}{n'_2} = n$$

$n'_1 = n_1 e^{-M_1gh/KT}$

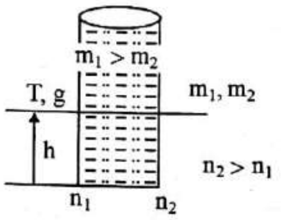
$n'_2 = n_2 e^{-M_2gh/KT}$

$$\frac{n'_1}{n'_2} = \frac{n_1}{n_2} e^{(M_2 - M_1)gh/KT}$$

$$\frac{n}{n_0} = e^{(M_2 - M_1)gh/KT}$$ **Ans.**

where M_1 is mass of H_2 molecule M_2 is mass of N_2 molecule

2.105*



$$n_1' = n_1 e^{-m_1 gh/KT}$$

$$n_2' = n_2 e^{-m_2 gh/KT}$$

Here

$$n_1' = n_2'$$

Then

$$n_1 e^{-m_1 gh/KT} = n_2 e^{-m_2 gh/KT}$$

$$\ln\frac{n_1}{n_2} = (m_1 - m_2)gH/kT$$

$$h = \frac{KT\ln\left[\frac{n_1}{n_2}\right]}{g(m_1 - m_2)}$$

Ans.

2.106*

We know P = nKT

n = density of molecules

$$n_f = n_0 e^{-mgh/KT}$$

$$\frac{P_f}{KT} = \frac{P_i}{KT} e^{-mgh/KT}$$

$$P_f = P_i e^{-mgh/KT}$$

At temp $T = T_0$

$$P_f' = P_i e^{-mgh/KT_0}$$

At temp $T = nT_0$

$$P_f'' = P_i e^{-mgh/K\eta T_0}$$

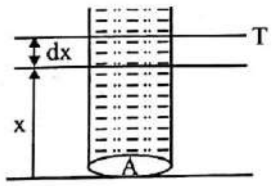
$$\frac{P_f''}{P_f'} = e^{-mgh/KT_0\left(\frac{1}{n}-1\right)}.$$

At bottom h = 0

$= P_f' =$ not change

Ans.

2.107*



$$<U> = \frac{\int (dm)gx}{N}$$

Where m_0 = mass of one molecule

n = number of molecules per unit volume.

Where N = Total number of molecules

$$dm = m_0(A\ dx)\ n = m_0 A\left(\frac{P}{KT}\right)dx$$

$$= \frac{m_0 A}{KT} P_0 e^{-\frac{mgh}{KT}} dx$$

$$N = \int nAdx = \int \frac{P_0 e^{-mgh/KT}}{KT} Adx$$

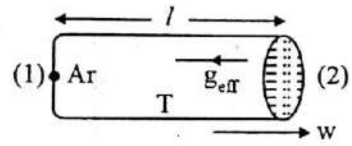
$$<U> = \frac{\int \frac{m_0 A}{KT} P_0 e^{-mgh/KT} gx\, dx}{\int \frac{P_0}{KT} e^{-mgh/KT} A\, dx}$$

$$= m_0 g \frac{\int_0^\infty x\, e^{-m_0 gh/KT} dx}{\int_0^\infty x\, e^{-m_0 gh/KT} dx}$$

$$<U> = KT$$

It is constant and not depends on type of molecule.

2.108*



Here effective accleration will be

$g_{eff} = w$

$n_f = n_0 e^{-mwl/KT}$

$$\frac{n_f}{n_0} - 1 = e^{-mwl/KT} - 1$$

$n + 1 = e^{-mwl/KT}$

$$\ln(n+1) = \frac{mw\ell}{KT}$$

$$W = \frac{-KT(\ln(1+n))}{ml}$$

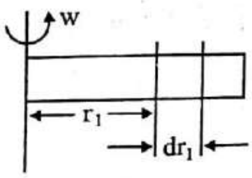
Magnitude of acceleration

$$W = -\frac{KT\ln(1+n)}{ml}$$

Also we know $\ln(1 + x) = x$

$$W = \frac{KTn}{ml}$$ **Ans.**

2.109*



Excess force on particle is arised due to change in density then

$F = (dm)rw^2$

Where $\frac{m}{\rho}$ = Volume density of particle

m = mass of one molecules

$$F = \left(\frac{m}{\rho}\right)(\rho - \rho_0)\, rw^2$$

Potential energy

$$U = \frac{m(\rho-\rho_0)w^2(r_2^2 - r_1^2)}{2\rho KT}$$

Now

$$n = n_0 e^{\frac{m(\rho-\rho_0)r^2w^2}{2\rho kT}}$$

$$\frac{n(r_2)}{n(r_1)} = n = e^{\frac{m(\rho-\rho_0)(r_2^2-r_1^2)w^2}{2\rho kT}}$$

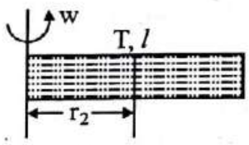
$$m = \frac{2\rho KT\ln n}{(\rho-\rho_0)w^2(r_2^2 - r_1^2)}$$

$$M = N_A m = \frac{N_A 2\rho KT\ln n}{(\rho-\rho_0)w^2(r_2^2 - r_1^2)}$$

$$= \frac{2\rho RT\ln n}{(\rho-\rho_0)w^2(r_2^2 - r_1^2)}$$

Ans.

2.110*



Centrigugal force on paritgcle of mass m. Where m is mass of one molecule of CO_2.

$F = mrw^2$

Potential energy

$$U = m\frac{r^2}{2}w^2$$

Then

$n_f = n_0 e^{mr^2/2w^2/KT}$

$$\frac{n_f}{n_0} = n = e^{ml^2/2w^2/KT}$$

$$\ln n = \frac{m^2w^2}{2KT}$$

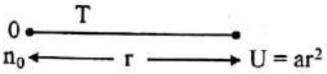
$$w = \sqrt{\frac{2(\ln n)KT}{ml^2}}$$

$$w = \sqrt{\frac{2KT\ln n}{ml^2}}$$

Ans.

2.111*

(a)



$n_r = n_0 e^{-ar^2/KT}$

Where n_0 = number of molecules / volume at centre

n_r = number of molecules / volume at distance r from centre

Then

$dN = n_r(4\pi r^2 dr)$

$$dN = n_0 e^{-ar^2/KT}\, 4\pi r^2 dr$$

Ans.

(b)

At most probable distance, tempreture = constant. then

For maximum dN

$f = e^{-ar^2/KT} r^2$ will be maximum then

$$\frac{df}{dr} = 0$$

$$r = \sqrt{\frac{KT}{a}}$$

Ans.

(c)

Fraction of molecules in region $r \to r+dr$.

$$\frac{dN}{N} = \frac{n_0 e^{-ar^2/KT} 4\pi r^2 dr}{\int_0^\infty n_0 4\pi r^2 e^{-ar^2/KT} dr}$$

$$\frac{dN}{N} = \left(\frac{a}{\pi KT}\right)^{3/2} 4\pi r^2 e^{-ar^2/KT}$$ **Ans.**

(d)

Compare Ans. (a) and Ans (c)

$$n_0 = N\left(\frac{a}{\pi KT}\right)^{3/2}$$

If Tempreture decreases n times

$$T_f = \frac{T_{in}}{n}$$

Then concentration increases by $n^{3/2}$

Ans.

2.112

(a)

$U = ar^2$ ....(1)

$$r = \sqrt{\frac{U}{a}}$$

Also from (1)

$dU = 2ardr$

$$dr = \frac{dU}{2ar}$$

Then

$$dN = n_0 4\pi\left(\frac{U}{a}\right)\frac{dU}{2\sqrt{aU}} e^{-U/KT}$$

$$dN = 2\pi n_0 a^{-3/2} U^{1/2} e^{-U/KT} dU$$

Ans.

(b)

For most probably value of U at constant temp.

$f = a^{-3/2} U^{1/2} e^{-U/KT}$ will be maximum then

$$\frac{df}{dU} = 0$$

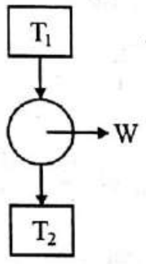
$$U = \frac{1}{2}KT$$

Ans.

2.4

The second law of thermodynamics, entropy

2.113



$$\eta = 1 - \frac{T_2}{T_1}$$

If T_1 increased by ΔT then new efficiency

$$n'_f = 1 - \frac{T_2}{T_1 + \Delta T} = 1 - \frac{T_2}{T_1}\left(1 + \frac{\Delta T}{T_1}\right)^{-1}$$

$$n'_f = 1 - \frac{T_2}{T_1} + \frac{T_2}{T_1^2}\Delta T \text{ ....(1)}$$

If T_2 by ΔT then

$$n''_f = 1 - \frac{T_2 - \Delta T}{T_1} = 1 - \frac{T_2}{T_1} + \frac{\Delta T}{T_1} \text{ ......(ii)}$$

From (i) and (ii)

$$\frac{T_2 \Delta T}{T_1} < \frac{\Delta T}{T_1}$$

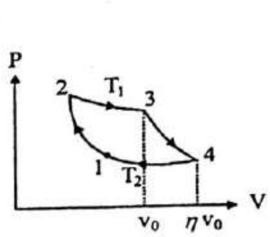
$$\frac{T_2}{T_1} < 1 \text{ because } T_2 < T_1$$

Then efficiency of second case will be more than that of fist $n''_f > n'_f$

Ans.

2.114

(a)



We know

$$TV^{\gamma} = const$$

$$T_1 V_0^{\gamma-1} = T_2 (\eta V_0)^{\gamma-1}$$

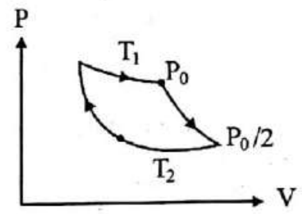
$$\frac{T_1}{T_2} = \eta^{\gamma-1}$$

Efficiency

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{\eta^{\gamma-1}} = 1 - \eta^{1-\gamma}$$

Ans.

(b)



$$PV^{\gamma} = const$$

$$P\left[\frac{\eta RT}{p}\right]^{\gamma} = \text{const}$$

$$P^{1-\gamma} T^{\gamma} = const$$

$$P_0^{1-\gamma} T_1^{\gamma} = \left(\frac{P_0}{2}\right)^{1-\gamma} T_2^{\gamma}$$

$$\left(\frac{T_2}{T_1}\right)^{\gamma} = 2^{1-\gamma}$$

$$\frac{T_2}{T_1} = 2^{\frac{1-\gamma}{\gamma}}$$

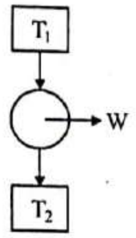
$$\text{Efficiency } (\eta) = 1 - \frac{T_2}{T_1} = 1 - 2^{\frac{1-\gamma}{\gamma}}$$

$$\eta = 1 - 2^{\frac{1}{\gamma} - 1}$$

Ans.

2.115

Carnot Engine

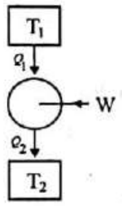


Efficiency of carnot engine

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 1 - \eta \text{ ....(1)}$$

Refrigerator



Efficiency of refrigerator

$$\eta = \frac{Q_2}{W}$$

$$\eta = \frac{Q_2}{Q_1 - Q_2}$$

$$\eta = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

Also we know

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

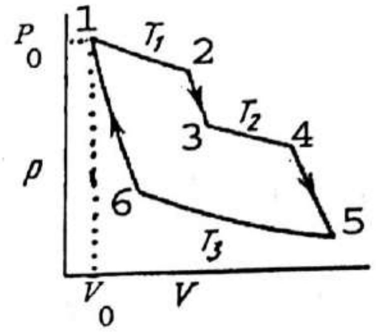
Then

$$\eta = \frac{1}{\frac{T_1}{T_2} - 1}$$

From(1)

$$\eta = \frac{1}{\frac{1}{1-\eta_2} - 1} = \frac{1-\eta}{\eta}$$ **Ans.**

2.216



Let initial pressure, volume and tempreture are P_0, V_0, T_1

Process 1-2(Isothermal):

$$\Delta Q = \Delta U + \Delta W \text{ ....(1)}$$

Here $\Delta U = 0$

From (1)

$$\Delta Q_{1-2} = \Delta W = \nu R T_1 \ln \frac{V_2}{V_1}$$

Suppose volume is increased by x times in each expansion

Then

$$\frac{V_2}{V_1} = x$$

$$\Delta Q_{1-2} = \Delta W = \nu R T_1 \ln x \text{ ....(2)}$$

This is positive value.

Process 2-3(Adiabatic):

$$\Delta Q_{2-3} = 0 \text{ ....(3)}$$

Process 3-4(Isothermal):

$$\Delta Q_{3-4} = \Delta W = \nu R T_2 \ln x \text{ ....(4)}$$

This is positive value.

Process 4-5(Adiabatic):

$$\Delta Q_{4-5} = 0 \text{ ....(5)}$$

Calculation of $\left(\frac{V_6}{V_5}\right)$

$$P_2 = \frac{P_0 V_0}{V_2} = \frac{P_0}{x}$$

$$T_1V_2^{\gamma-1}=T_2V_3^{\gamma-1}$$

$$V_3=V_2\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}=xV_0\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$$

Also

$$V_4=xV_3=x^2V_0\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$$

$$V_5=V_4\left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}}=x^2V_0\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}\left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}}$$

$$V_5=x^2V_0\left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}}$$

$$T_1V_0^{\gamma-1}=T_3V_6^{\gamma-1}$$

$$V_6=V_0\left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}}$$

Process 5-6(Isothermal):

$$\Delta Q_{5-6}=\Delta W=\nu RT_3\ln\left(\frac{V_6}{V_5}\right)$$

$$\Delta Q_{5-6}=\nu RT_3\ln\frac{V_0\left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}}}{x^2V_0\left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}}}$$

$$\Delta Q_{5-6}=-2\nu RT_3\ln x \qquad \text{............(6)}$$

This is negative value.
Heat given to system

$$\Delta Q_{given}=\Delta Q_{1-2}+\Delta Q_{3-4}$$

$$\Delta Q_{given}=\left(\nu RT\ln x\right)\left(T_1+T_2\right)$$

Work done

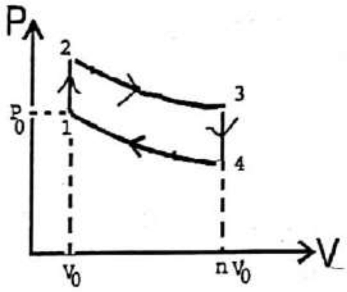
$$\Delta W=\Delta Q_{net}=\left(\nu RT\ln x\right)\left(T_1+T_2-2T_3\right)$$

Efficiency

$$\eta=\frac{\Delta W}{\Delta Q_{given}}=\frac{T_1+T_2-2T_3}{T_1+T_2}=1-\frac{2T}{T_1+T_2}$$

Ans.

2.117



Let initial pressure, volume and tempreture are P_0, V_0, T_0

Process 1-2(Isochoric):

$$\Delta Q_{1-2}=\nu C_v\left(T_2-T_0\right)$$

This is positive value.

Process 2-3(Adiabatic):

$$\Delta Q_{2-3}=0$$

Process 3-4(Isochoric):

$$\Delta Q_{3-4}=\nu C_v\left(T_4-T_3\right)$$

This is negative value.

$$\Delta W=\Delta Q_{1-2}+\Delta Q_{3-4}=\nu C_v\left(T_2-T_0+T_4-T_3\right)$$

Heat Given

$$\Delta Q_{1-2}=\Delta Q_{given}=\nu C_v\left(T_2-T_0\right)$$

Efficiency

$$\eta=\frac{\Delta W}{\Delta Q_{given}}=1+\frac{T_4-T_3}{T_2-T_0} \qquad \text{........(1)}$$

Process 4-1(Adiabatic):

$$T_0V_0^{\gamma-1}=T_4\left(nV_0\right)^{\gamma-1}$$

$$T_4=T_0n^{1-\gamma}$$

Process 2-3(Adiabatic):

$$T_2V_0^{\gamma-1} = T_3\left(nV_0\right)^{\gamma-1}$$

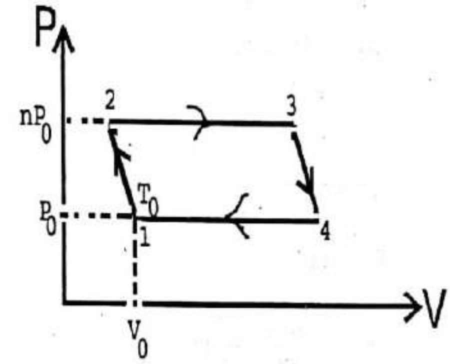
$$T_3 = T_2 n^{1-\gamma}$$

Put in (1)

$$\eta = 1+\frac{\left(T_0-T_2\right)n^{1-\gamma}}{T_2-T_0} = 1-n^{1-\gamma}$$

Ans.

2.118



Process 1-2(Adiabatic):

$\Delta Q = 0$

Process 2-3(Isobaric):

$$\Delta Q_{2-3} = \nu C_p\left(T_3-T_1\right) = \nu\frac{\gamma R}{\gamma-1}\left(T_3-T_1\right)$$

This is positive value.

Process 3-4(Adiabatic):

$\Delta Q = 0$

Process 4-1(Isobaric):

$$\Delta Q_{4-1} = \nu C_p\left(T_1-T_4\right) = \nu\frac{\gamma R}{\gamma-1}\left(T_0-T_4\right)$$

This is negative value.

$$\Delta W = \Delta Q_{4-1}+Q_{2-3} = \nu\frac{\gamma R}{\gamma-1}\left(T_3-T_2+T_0-T_4\right)$$

Heat given to system

$$\Delta Q_{given} = \nu\frac{\gamma R}{\gamma-1}\left(T_3-T_2\right)$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = \frac{T_3-T_2+T_0-T_4}{T_3-T_2} = 1+\frac{T_0-T_4}{T_3-T_2} \quad \text{...................(1)}$$

Calculation of T_3, T_2

$$T_3^{\gamma}\left(nP_0\right)^{1-\gamma} = T_4^{\gamma}\left(P_0\right)^{1-\gamma}$$

$$T_4 = T_3 n^{\frac{1-\gamma}{\gamma}}$$

Again

$$T_2^{\gamma}\left(nP_0\right)^{1-\gamma} = T_0^{\gamma}\left(P_0\right)^{1-\gamma}$$

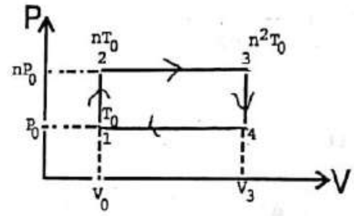
$$T_2 = T_0 n^{\frac{-1+\gamma}{\gamma}}$$

Put in (1)

$$\eta = 1+\frac{T_0-T_3 n^{\frac{1-\gamma}{\gamma}}}{T_3-T_0 n^{\frac{-1+\gamma}{\gamma}}} = 1-n^{\frac{1}{\gamma}-1}$$

Ans.

2.119



Process 1-2(Isochoric):

$$\Delta Q_{1-2} = \nu C_v\left(nT_0-T_0\right) = \nu C_v\left(n-1\right)T_0$$

This is positive value.

Process 2-3(Isobaric):

$$\Delta Q_{2-3} = \nu C_p\left(n^2T_0-nT_0\right) = \nu C_p n\left(n-1\right)T_0$$

This is positive value.

Process 3-4(Isochoric):

$$\Delta Q_{3-4} = \nu C_v\left(T_4-n^2T_0\right)$$

This is negative value.

Process 4-1(Isobaric):

$$\Delta Q_{4-1} = \nu C_p\left(T_0-T_4\right)$$

This is negative value.

Work done by gas

$$\Delta W = \Delta Q_{1-2}+Q_{2-3}+\Delta Q_{3-4}+\Delta Q_{4-1}$$

Heat given to system

$$\Delta Q = \Delta Q_{1-2}+\Delta Q_{2-3}$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}}$$

$$= \frac{\nu C_v(n-1)T_0 + \nu C_p n(n-1)T_0 + \nu C_v(T_4 - n^2 T_0) + \nu C_p(T_0 - T_4)}{\nu C_v(n-1)T_0 + \nu C_p n(n-1)T}$$

.................(1)

Relation between T_4, T

$$\frac{P_4}{P_3} = \frac{T_4}{n^2 T_0} \quad ....(2)$$

$$\frac{P_4}{P_3} = \frac{nT_0}{T_0} \quad ....(3)$$

From (2) and (3)

$$T_4 = n^3 T_0$$

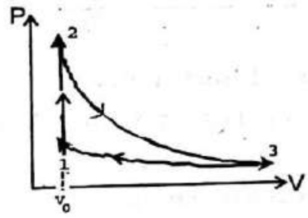
Put in (1)

$$\eta = 1 + \frac{n+\gamma}{1+n\gamma}$$

Ans.

2.120

(a)



$T_2 = nT_0$

$T_1 = T_3 = T_0$

Process 1-2(Isochoric):

$$\Delta Q_{1-2} = \nu C_v(nT_0 - T_0) = \nu C_v(n-1)T_0$$

This is positive value.

Process 3-1(Isothermal):

$$\Delta Q_{1-3} = \Delta W = \nu RT_0 \ln\left(\frac{V_0}{V_3}\right)$$

This is negative value.

Work done by gas

$$\Delta W = \Delta Q_{1-2} + Q_{1-3}$$

Heat given to system

$$\Delta Q = \Delta Q_{1-2}$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = \frac{Q_{12} + Q_{13}}{Q_{12}} = 1 + \frac{Q_{13}}{Q_{12}}$$

$$\eta = 1 + \frac{R \ln\left(\frac{V_0}{V_3}\right)}{C_v(n-1)} \quad ....(1)$$

Using adiabatic process

$$T_2 V_0^{\gamma-1} = T_3 V_3^{\gamma-1}$$

$$nT_0 V_0^{\gamma-1} = T_0 V_3^{\gamma-1}$$

$$n = \left(\frac{V_3}{V_0}\right)^{\gamma-1}$$

$$\frac{V_0}{V_3} = \frac{1}{n^{\frac{1}{\gamma-1}}}$$

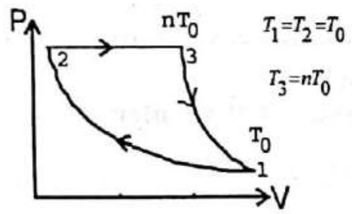
Put in (1)

$$\eta = 1 + \frac{R(\gamma-1)}{R(n-1)} \ln\left(\frac{1}{n^{\frac{1}{\gamma-1}}}\right)$$

$$\eta = 1 - \frac{1}{n-1}\ln(n)$$

Ans.

(b)



Process 1-2(Isothermal):

$$\Delta Q_{1-2} = \Delta W = \nu RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

This is negative value.

Process 2-3(Isobaric):

$$\Delta Q_{2-3} = \nu C_p(T_3 - T_2)$$

$$= \nu C_p(nT_0 - T_0)$$

$$= \nu C_p(n-1)T_0$$

This is positive value.

Work done by gas

$$\Delta W = \Delta Q_{1-2} + \Delta Q_{2-3}$$

Heat given to system

$$\Delta Q = \Delta Q_{2-3}$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = \frac{Q_{12}+Q_{23}}{Q_{23}} = 1+\frac{Q_{12}}{Q_{23}}$$

$$\eta = 1+\frac{R\ln\left(\frac{V_2}{V_1}\right)}{C_p(n-1)}$$

$$\eta = 1+\frac{(\gamma-1)\ln\left(\frac{V_2}{V_1}\right)}{\gamma(n-1)} \quad \text{............(1)}$$

Calculation of $\frac{V_2}{V_1}$

Using adiabatic process

$$TV^{\gamma-1} = const$$

$$(nT)(V_3)^{\gamma-1} = T_0(V_1)^{\gamma-1}$$

$$V_3 = V_1\left(\frac{1}{n}\right)^{\gamma-1}$$

Also

In isobaric process

$$\frac{T_3}{V_3} = \frac{T_2}{V_2}$$

$$\frac{n}{V_3} = \frac{1}{V_2}$$

$$V_3 = nV_2$$

Then

$$nV_2 = V_1\left(\frac{1}{n}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{V_2}{V_1} = \left(\frac{1}{n}\right)^{\frac{1}{\gamma-1}}$$

From (1)

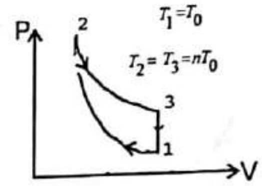
$$\eta = 1+\frac{\gamma-1}{\gamma(n-1)}\ln\left(\frac{1}{n}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 1-\frac{\ln(n)}{n-1}$$

Ans.

2.121

(a)



Process 2-3(Isothermal):

$$\Delta Q_{2-3} = \Delta W = \nu RT_0\ln\left(\frac{V_3}{V_2}\right)$$

This is positive value.

Process 3-1(Isochoric):

$$\Delta Q_{3-1} = \nu C_v(T_1 - T_3) = \nu C_v(1-n)T_0$$

This is negative value.

Work done by gas

$$\Delta W = \Delta Q_{2-3} + Q_{3-1}$$

Heat given to system

$$\Delta Q = \Delta Q_{2-3}$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = 1+\frac{Q_{31}}{Q_{23}}$$

$$\eta = 1+\frac{C_vT_0(1-n)}{RnT_0\ln\left(\frac{V_3}{V_2}\right)} = 1+\frac{1-n}{n(\gamma-1)\ln\left(\frac{V_3}{V_2}\right)} \quad \text{............(1)}$$

Calculation of $\frac{V_3}{V_2}$

Using adiabatic process

$$TV^{\gamma-1} = const$$

$$nT_0V_2^{\gamma-1} = T_0V_3^{\gamma-1}$$

$$V_3 = V_2(n)^{\frac{1}{\gamma-1}}$$

$$\frac{V_3}{V_2}=(n)^{\frac{1}{\gamma-1}}$$

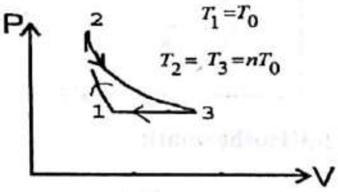
Put in (1)

$$\eta=1+\frac{1-n}{n(\gamma-1)\ln(n)^{\frac{1}{\gamma-1}}}$$

$$\eta=1-\frac{n-1}{n\ln(n)}$$

Ans.

(b)



Process 2-3(Isothermal):

$$\Delta Q_{2-3}=\Delta W=\nu RnT_0\ln\left(\frac{V_3}{V_2}\right)$$

This is positive value.

Process 3-1(Isobaric):

$$\Delta Q_{3-1}=\nu C_p(T_1-T_3)=\nu C_p(1-n)T_0$$

This is negative value.

Work done by gas

$$\Delta W=\Delta Q_{2-3}+Q_{3-1}$$

Heat given to system

$$\Delta Q=\Delta Q_{2-3}$$

Efficiency

$$\eta=\frac{\Delta W}{\Delta Q_{given}}=1+\frac{Q_{31}}{Q_{23}}$$

$$\eta=1+\frac{C_pT_0(1-n)}{RnT_0\ln\left(\frac{V_3}{V_2}\right)}=1+\frac{\gamma(1-n)}{n(\gamma-1)\ln\left(\frac{V_3}{V_2}\right)}$$

............(1)

Calculation of $\frac{V_3}{V_2}$

Using isobaric process

$$\frac{V_1}{V_3}=\frac{T_0}{nT_0}=\frac{1}{n}$$

$$V_1=\frac{V_3}{n}$$

Using adiabatic process

$$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$$

$$T_0\left(\frac{V_3}{n}\right)^{\gamma-1}=nT_0(V_2)^{\gamma-1}$$

$$\frac{V_3}{V_2}=(n)^{\frac{\gamma}{\gamma-1}}$$

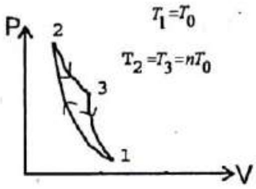
Put in (1)

$$\eta=1+\frac{(1-n)\gamma}{n(\gamma-1)\ln(n)^{\frac{\gamma}{\gamma-1}}}$$

$$\eta=1+\frac{1-n}{n\ln(n)}$$

Ans.

2.122



Process 2-3(Isothermal):

$$\Delta Q_{2-3}=\Delta W=\nu RnT_0\ln\left(\frac{V_3}{V_2}\right)$$

This is positive value.

Process 3-1:

$$\Delta Q_{3-1}=\nu C(T_0-nT_0)=\nu\left(C_v-\frac{R}{x-1}\right)(1-n)T_0$$

This is negative value.

Work done by gas

$$\Delta W=\Delta Q_{2-3}+Q_{3-1}$$

Heat given to system

$$\Delta Q=\Delta Q_{2-3}$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = 1 + \frac{Q_{31}}{Q_{23}}$$

$$\eta = \frac{Rn\ln\left(\frac{V_3}{V_2}\right) + \left(C_v - \frac{R}{x-1}\right)(1-n)}{Rn\ln\left(\frac{V_3}{V_2}\right)}$$

$$\eta = 1 + \frac{\left(C_v - \frac{R}{x-1}\right)(1-n)}{nR\ln\left(\frac{V_3}{V_2}\right)} \quad \text{............(1)}$$

Using adiabatic process

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_0(V_1)^{\gamma-1} = nT_0(V_2)^{\gamma-1}$$

$$\frac{V_3}{V_2} = (n)^{\frac{1}{\gamma-1} - \frac{1}{x-1}}$$

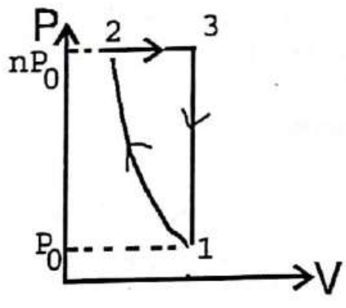
Put in (1)

$$\eta = 1 - \frac{n-1}{n\ln(n)}$$

Ans.

2.123

(a)



Process 2-3(Isobaric):

$$\Delta Q_{2\text{-}3} = \nu C_p(T_3 - T_2)$$

This is positive value.

Process 3-1(Isochoric):

$$\Delta Q_{3\text{-}1} = \nu C_v(T_1 - T_3)$$

This is negative value.

Work done by gas

$$\Delta W = \Delta Q_{2\text{-}3} + \Delta Q_{3\text{-}1}$$

Heat given to system

$$\Delta Q = \Delta Q_{2\text{-}3}$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = 1 + \frac{Q_{31}}{Q_{23}}$$

$$\eta = 1 + \frac{C_p(T_3 - T_2)}{C_v(T_1 - T_3)} \quad \text{................(1)}$$

Calculation of T_1, T_3

Using adiabatic process

$$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$$

$$P_0^{1-\gamma}T_1^{\gamma} = (nP_0)^{1-\gamma}T_2^{\gamma}$$

$$T_2 = T_1\left(\frac{1}{n}\right)^{\frac{1-\gamma}{\gamma}}$$

Using isochoric process

$$\frac{nP_0}{P_0} = \frac{T_3}{T_1} = n$$

$$T_3 = nT_1$$

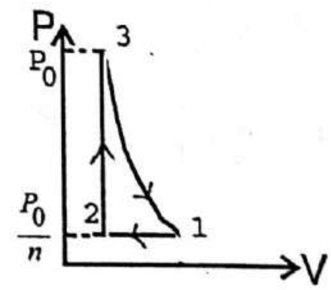
Put in (1)

$$\eta = 1 + \frac{\gamma\left(nT_1 - \left(\frac{1}{n}\right)^{\frac{1-\gamma}{\gamma}}T_1\right)}{T_1 - nT_1}$$

$$= 1 + \frac{(n-1)\gamma}{n^{\gamma} - 1}$$

Ans.

(b)



Process 1-2(Isobaric):

$\Delta Q_{1-2} = \nu C_p (T_2 - T_1)$

This is negative value.

Process 2-3(Isochoric):

$\Delta Q_{2-3} = \nu C_v (T_3 - T_2)$

This is positive value.

Work done by gas

$\Delta W = \Delta Q_{2-3} + \Delta Q_{1-2}$

Heat given to system

$\Delta Q = \Delta Q_{2-3}$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = 1 + \frac{Q_{1-2}}{Q_{2-3}}$$

$$\eta = 1 + \frac{C_p (T_2 - T_1)}{C_v (T_3 - T_2)} \quad \text{................(1)}$$

Calculation of T_2, T_3

Using adiabatic process

$$P_3^{1-\gamma} T_3^{\gamma} = P_1^{1-\gamma} T_1^{\gamma}$$

$$P_0^{1-\gamma} T_3^{\gamma} = \left(\frac{P_0}{n}\right)^{1-\gamma} T_1^{\gamma}$$

$$T_3 = T_1 \left(\frac{1}{n}\right)^{\frac{1-\gamma}{\gamma}}$$

Using isochoric process

$$\frac{P_3}{P_2} = \frac{T_3}{T_2} = n$$

$$T_2 = \frac{T_3}{n}$$

Put values in (1)

$$\eta = 1 - \frac{n-1}{n \ln(n)}$$

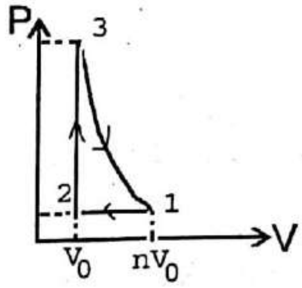
$$= 1 - \gamma \left(\frac{\frac{T_3}{n} - T_3 n^{\frac{1-\gamma}{\gamma}}}{T_3 - \frac{T_3}{n}} \right)$$

$$\eta = 1 - \frac{n^{\gamma} - 1}{\gamma (n-1) n^{\gamma-1}}$$

Ans.

2.124

(a)



Process 1-2(Isobaric):

$\Delta Q_{1-2} = \nu C_p (T_2 - T_1)$

This is negative value.

Process 2-3(Isochoric):

$\Delta Q_{2-3} = \nu C_v (T_3 - T_2)$

This is positive value.

Process 3-1(Isothermal):

$$\Delta Q_{3-1} = \Delta W = \nu R T_3 \ln\left(\frac{V_1}{V_2}\right)$$

This is positive value.

Work done by gas

$\Delta W = \Delta Q_{2-3} + \Delta Q_{1-2} + \Delta Q_{3-1}$

Heat given to system

$\Delta Q = \Delta Q_{2-3} + \Delta Q_{3-1}$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = 1 + \frac{\Delta Q_{1-2}}{\Delta Q_{2-3} + \Delta Q_{3-1}}$$

$$\eta = 1 + \frac{C_p (T_2 - T_1)}{C_v (T_3 - T_2) + R T_3 \ln\left(\frac{V_1}{V_2}\right)}$$

................(1)

Calculation of T_1, T_2

Process(1-2)

$$\frac{nV_0}{V_0} = \frac{T_1}{T_2}$$

$$T_2 = \frac{T_1}{2}$$

$$\frac{V_1}{V_2} = n$$

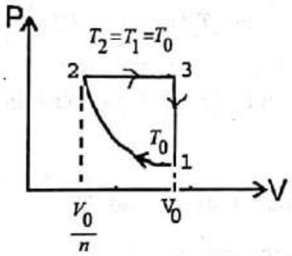
Put values in (1)

$$\eta = 1 + \frac{\frac{C_p}{C_v}\left(\frac{T_1}{n} - 1\right)}{\left(T_1 - \frac{T_1}{n}\right) + \frac{R}{C_v}\frac{T_1}{n}\ln(n)}$$

$$\eta = 1 - \frac{\gamma(n-1)}{(n-1) + (\gamma - 1)n\ln(n)}$$

Ans.

(b)



Process 1-2(Isothermal):

$$\Delta Q_{1-2} = \Delta W = \nu R T_0 \ln\left(\frac{V_2}{V_1}\right) = -\nu R T_0 \ln(n)$$

This is negative value.

Process 2-3(Isobaric):

$$\Delta Q_{2-3} = \nu C_p (T_3 - T_2) = \nu C_p (T_3 - T_0)$$

This is positive value.

Process 3-1(Isochoric):

$$\Delta Q_{3-1} = \nu C_v (T_1 - T_3) = \nu C_v (T_0 - T_3)$$

This is negative value.

Work done by gas

$$\Delta W = \Delta Q_{2-3} + \Delta Q_{1-2} + \Delta Q_{3-1}$$

Heat given to system

$$\Delta Q = \Delta Q_{2-3}$$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = 1 + \frac{\Delta Q_{1-2} + \Delta Q_{3-1}}{\Delta Q_{2-3}}$$

$$\eta = 1 + \frac{\nu C_v (T_0 - T_3) - \nu R T_0 \ln(n)}{\nu C_p (T_3 - T_0)} \quad \text{................(1)}$$

Calculation of T_0, T_3

Isochoric process

$$\frac{T_3}{T_0} = \frac{P_0}{P_1}$$

$$\frac{T_3}{T_0} = \frac{P_2}{P_1} \quad \text{...........(2)} \quad (\text{Because } P_2 = P_3)$$

Isothermal process

$$P_2\left(\frac{V_0}{n}\right) = P_1 V_0$$

$$\frac{P_2}{P_1} = n \quad \text{.............(3)}$$

From (2) and (3)

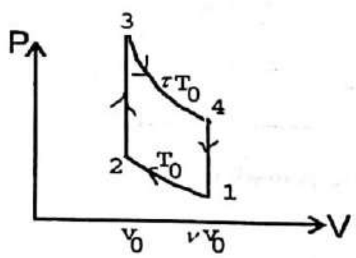
$$T_3 = nT_0$$

Put values in (1)

$$\eta = 1 + \frac{(n-1)(\gamma - 1)\ln(n)}{\gamma(1-n)}$$

Ans.

2.125



Process 1-2(Isothermal):

$$\Delta Q_{1-2} = \Delta W = nRT_0 \ln\left(\frac{V_0}{\nu V_0}\right) = -nRT_0 \ln(\nu)$$

This is negative value.

Process 2-3(Isochoric):

$\Delta Q_{2-3} = nC_v\left(T_3 - T_2\right)$

This is positive value.

Process 3-4(Isothermal):

$$\Delta Q_{3-4} = \Delta W = nR\tau T_0 \ln\left(\frac{\nu V_0}{V_0}\right) = nR\tau T_0 \ln(\nu)$$

This is positive value.

Process 4-1(Isochoric):

$\Delta Q_{4-1} = nC_v\left(T_0 - \tau T_0\right) = -nC_v T_0\left(\tau - 1\right)$

This is negative value.

Work done by gas

$\Delta W = \Delta Q_{1-2} + \Delta Q_{2-3} + \Delta Q_{3-4} + \Delta Q_{4-1}$

Heat given to system

$\Delta Q = \Delta Q_{2-3} + \Delta Q_{3-4}$

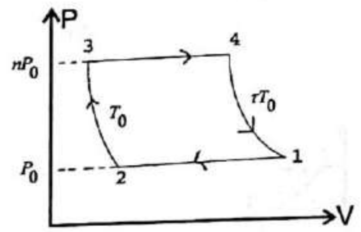
Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = \frac{\Delta Q_{1-2} + \Delta Q_{2-3} + \Delta Q_{3-4} + \Delta Q_{4-1}}{\Delta Q_{2-3} + \Delta Q_{3-4}}$$

$$\eta = \frac{(\tau - 1)\ln(\nu)}{\tau \ln(\nu) + \frac{(\tau - 1)}{\gamma - 1}}$$

Ans.

2.126



Process 1-2(Isobaric):

$\Delta Q_{1-2} = \nu C_p\left(T_0 - \tau T_0\right)$

This is negative value.

Process 2-3(Isothermal):

$$\Delta Q_{2-3} = \nu R T_0 \ln\left(\frac{V_3}{V_2}\right)$$

This is negative value.

Process 3-4(Isobaric):

$\Delta Q_{3-4} = \nu C_p\left(\tau T_0 - T_0\right)$

This is positive value.

Process 4-1(Isothermal):

$$\Delta Q_{4-1} = \nu R\tau T_0 \ln\left(\frac{V_1}{V_2}\right)$$

This is positive value.

Work done by gas

$\Delta W = \Delta Q_{1-2} + \Delta Q_{2-3} + \Delta Q_{3-4} + \Delta Q_{4-1}$

Heat given to system

$\Delta Q = \Delta Q_{4-1} + \Delta Q_{3-4}$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = \frac{\Delta Q_{1-2} + \Delta Q_{2-3} + \Delta Q_{3-4} + \Delta Q_{4-1}}{\Delta Q_{4-1} + \Delta Q_{3-4}}$$

$$\eta = 1 + \frac{\nu C_p T_0\left(1 - \tau\right) + \nu R T_0 \ln\left(\frac{V_3}{V_2}\right)}{\nu C_p\left(\tau T_0 - T_0\right) + \nu R\tau T_0 \ln\left(\frac{V_1}{V_4}\right)} \quad ...(1)$$

Calculation of $\frac{V_3}{V_2}$ and $\frac{V_1}{V_4}$

Isobaric process

$\frac{T_4}{T_3} = \frac{V_4}{V_3} = \tau$

$V_4 = V_3\tau$

Similiar

$V_1 = V_2\tau$

Isothermal process

$P_0V_2 = nP_0V_3$

$\frac{V_3}{V_2} = \frac{1}{n}$

$P_0V_1 = nP_0V_4$

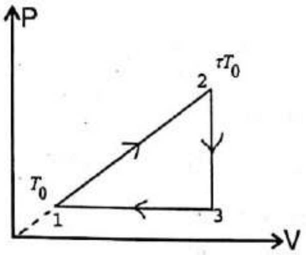
$\frac{V_1}{V_4} = n$

Put values in (1)

$$\eta = 1 + \frac{(\tau-1)\ln(n)}{(\tau-1)\frac{\gamma}{\gamma-1} + \tau\ln(\nu)}$$

Ans.

2.127



Process 1-2(Isotropic):
Here line(1-2) is passing through origin then

$P \propto V$

$PV^{-1} = const$

Compare with polytropic equation

$PV^{x} = const$

$x = -1$

$$\Delta Q_{1-2} = \nu C(\tau T_0 - T_0) = \nu\left(C_v - \frac{R}{x-1}\right)(\tau-1)T_0$$

$$= \nu\left(\frac{R}{\gamma-1} + \frac{R}{2}\right)(\tau-1)T_0$$

This is positive value.

Process 2-3(Isochoric):

$\Delta Q_{2-3} = nC_v(T_3 - \tau T_0)$

This is negative value.

Process 3-1(Isobaric):

$\Delta Q_{3-1} = \nu C_p(T_0 - T_3)$

This is negative value.
Work done by gas

$\Delta W = \Delta Q_{1-2} + \Delta Q_{2-3} + \Delta Q_{3-1}$.

Heat given to system

$\Delta Q = \Delta Q_{1-2}$

Efficiency

$$\eta = \frac{\Delta W}{\Delta Q_{given}} = \frac{\Delta Q_{1-2} + \Delta Q_{2-3} + \Delta Q_{3-1}}{\Delta Q_{1-2}}$$

$$\eta = 1 + \frac{C_v(T_3 - \tau T_0) + C_p(T_0 - T_3)}{\frac{(\gamma+1)R}{2(\gamma-1)}T_0(\tau-1)} \quad ......(1)$$

Calculation of T_3 and T_0

Isobaric process

$\frac{V_3}{V_1} = \frac{T_3}{T_1}$

Polytropic process

$P \propto V$

$PV^{-1} = const$

$TV^{-2} = const$

$T_1V_1^{-2} = \tau T_0 V_3^{-2}$

$\frac{V_3}{V_1} = \tau^{\frac{1}{2}}$

$\frac{T_3}{T_1} = \frac{T_3}{T_0}$

$T_3 = T_0\tau^{\frac{1}{2}}$

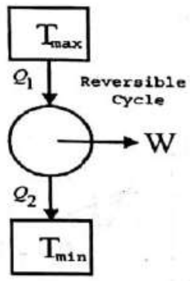
Put in (1)

$$\eta = 1 - 2\frac{\gamma + \sqrt{\tau}}{(\gamma+1)(1+\sqrt{\tau})}$$

Ans.

2.128*

Carnot Reversible Cycle



$$\eta = 1 - \frac{Q_2}{Q_1}$$

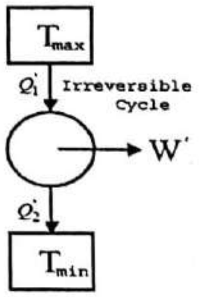
Using Calausis inequality

$\int \frac{dQ}{T} = 0$ for reversible cycle

$$\frac{Q_1}{T_{max}} = \frac{Q_2}{T_{min}}$$

$$\frac{Q_2}{Q_1} = \frac{T_{min}}{T_{max}} \quad ....(1)$$

Irreversible Cycle



$$\eta' = 1 - \frac{Q'_2}{Q'_1}$$

Using Calausis inequality

$$\int \frac{dQ}{T} < 0 \quad \text{for Irreversible cycle}$$

$$\frac{Q'_1}{T_{max}} - \frac{Q'_2}{T_{min}} < 0$$

$$\frac{Q'_2}{Q'_1} > \frac{T_{min}}{T_{max}} \quad ....(2)$$

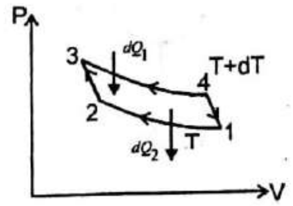
From (1) and (2)

$$\frac{Q'_2}{Q'_1} > \frac{Q_2}{Q_1}$$

$$n > n'$$

Ans.

2.129*



$$\text{Efficiency } (\eta) = \frac{dA}{dQ_1} = \frac{dQ_1 - dQ_2}{dQ_1}$$

$$\eta = 1 - \frac{\partial Q_2}{\partial Q_1} = 1 - \frac{T}{T+\partial T} = \frac{\partial T}{T+\partial T} = \frac{\partial T}{T} \quad .....(1)$$

$$\frac{\partial A}{\partial Q_1} = \eta$$

$$\partial Q_1 = \frac{\partial A}{\eta} = \left(\frac{\partial A}{\partial T}\right) T \quad ....(2)$$

From(1)

$$\partial A = (\partial P)(\partial V) = \left(\frac{\partial P}{\partial T}\right)_V (dT)\partial V \quad ....(3)$$

Also we know :

$$\partial \theta_1 = U_1 + PdV$$

$$\left(\frac{\partial A}{\partial T}\right) T = \partial U + PdV \quad ....(4)$$

From(2),(3) and (4)

$$\left(\frac{\partial U}{\partial V}\right)_V = \left(\frac{\partial P}{\partial T}\right)_V - P$$

Ans.

2.130

(a)
Isochoric Process
We know

$$ds = \frac{dQ}{T} \quad ....(1)$$

$$dQ = \nu C_v \, dT$$

Here $\nu = 1$
From(1)

$$\int ds = \int_{T_0}^{nT_0} C_V \frac{dT}{T}$$

$$\Delta S = C_V \, \ell n(n) = \frac{R}{\gamma - 1} \ell n(n)$$

Ans.

(b)
Isobaric Process

$$dQ = \nu C_p dt$$

$$\int ds = \int_{T_0}^{nT_0} C_P \frac{dT}{T}$$

$$\Delta S = C_P \, \ell n(n) = \frac{\gamma R}{\gamma - 1} \ell n(n)$$

Ans.

2.131

For isothermal process

$\Delta S = \frac{\Delta Q}{T}$ —(1)

Also know

$\Delta Q = \Delta U + \Delta w$

$\Delta Q = 0 + \nu RT\, \ell n \frac{V_f}{V_i}$ —(2)

Here $\Delta U = 0$ becuase T = constant

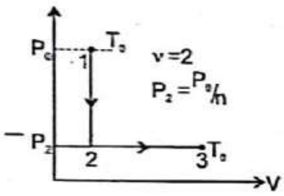
From (1) and (2)

$$\Delta S = \frac{\nu RT \ell n \frac{V_f}{V_i}}{T}$$

$$\frac{V_f}{V_i} = e^{\Delta S/\nu R}$$

Ans.

2.132



<u>Process 1-2 :</u>

$$\Delta S_{1-2} = \int \frac{dQ}{T} = \int_{T_1}^{T_2} \nu C_V \frac{dT}{T} = \nu C_V \ell n \, T_2/T_1$$

<u>Process 2–3 :</u>

$$\Delta S_{2-3} = \int \frac{dQ}{T} = \int_{T_2}^{T_1} \nu C_P \frac{dT}{T} = \nu C_P \ell n \, T_1/T_2$$

Net entropy change :

$$\Delta S = \nu C_V \ell n \, T_2/T_1 + \nu C_P \, \ell n \, T_1/T_2$$

$$= \nu C_V \ell n \, T_2/T_1 - \nu C_P \, \ell n \, T_2/T_1$$

$$\Delta S = \nu (C_V - C_P) \ell n \left(T_2/T_1\right) \text{ ———————— (i)}$$

Calculation of T_1 and T_2

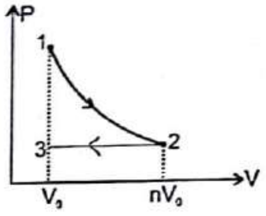
$$\frac{P_0}{P_0/n} = \frac{T_1}{T_2} \Rightarrow \frac{T_2}{T_1} = \frac{1}{n}$$

Put in (i)

$$\Delta S = (\nu \ell n n)(C_P - C_V)$$

Ans.

2.133



Process 1–2 (Adiabatic process)

$dQ = 0$

$$\Delta S_{1-2} = \int \frac{d\theta}{T} = 0$$

Process 2-3 (Isobaric process)

$$\Delta S_{2-3} = \int \frac{dQ}{T}$$

$$\Delta S_{2-3} = \int_{T_2}^{T_3} \frac{\nu C_P dT}{T} = \nu C_P \ln\left(\frac{T_3}{T_2}\right)$$

Calculation of T_3/T_2

Process 2–3

$$\frac{T_2}{nV_0} = \frac{T_3}{V_0} \Rightarrow \frac{T_3}{T_2} = \frac{1}{n}$$

$$\Delta S_{2-3} = -\nu C_P \ell n(n)$$

$$\Delta S_{net} = \Delta S_{1-2} + \Delta S_{2-3}$$

$$\Delta S_{net} = 0 + -\nu C_P \ell n(n)$$

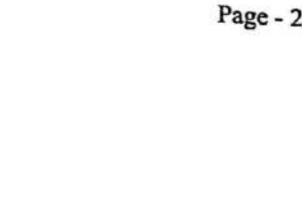
$$\Delta S_{net} = \frac{\gamma R}{\gamma - 1} \ln(n)$$

Ans.

2.134

Since entropy is state function, entropy change can be found by another process joining (1) and (2)

Now take a random process as diagram, first isochoric and then isobaric process



Process 1 – 3

$$\Delta S_{1-3} = \nu C_V \int_{T_1}^{T_3} \frac{dT}{T} = \nu C_V \ell n \frac{T_3}{T_1}$$

Process 3 – 2

$$\Delta S_{3-2} = \nu C_P \int_{T_3}^{T_2} \frac{dT}{T} = \nu C_P \ell n \frac{T_2}{T_3}$$

Calculation of $\frac{T_3}{T_1}$ **and** $\frac{T_2}{T_3}$

Process 1 – 3

$$\frac{P_0}{P_0/\beta} = \frac{T_1}{T_3} \Rightarrow \frac{T_3}{T_1} = \frac{1}{\beta}$$

Process 3 – 2

$$\frac{V_0}{\alpha V_0} = \frac{T_3}{T_2} \Rightarrow \frac{T_2}{T_3} = \alpha$$

Then

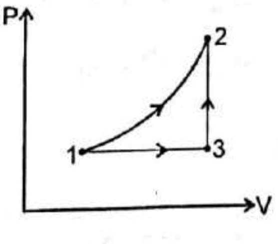
$$\Delta S_{net} = \nu C_V \ell n \frac{1}{\beta} + \nu C_P \ell n\alpha$$

$$\Delta S_{net} = -\nu \frac{R}{\gamma - 1} \ell n\beta + \nu \frac{\gamma R}{\gamma - 1} \ell n\alpha$$

$$= \frac{\nu R}{\gamma - 1} (\gamma \ell n\alpha - \ell n\beta)$$

Ans.

2.135



$$V_2 = V_3$$

$$\frac{T_1}{T_2} = \beta$$

$$\frac{V_2}{V_1} = \alpha$$

For entropy change $S_2 - S_1$ take a other process 1–3 –2

Process 1-3 :

$$\Delta S_{1-3} = \nu C_P \int_{T_1}^{T_3} \frac{dT}{T} = \nu C_P \ell n \frac{T_3}{T_1}$$

Process 3-2 :

$$\Delta S_{3-2} = \nu C_V \int_{T_3}^{T_2} \frac{dT}{T} = \nu C_V \ell n \frac{T_2}{T_3}$$

Calculation of $\frac{T_3}{T_1}$ **and** $\frac{T_2}{T_3}$

Process 1-3

$$\frac{V_1}{V_3} = \frac{T_1}{T_3}$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_3} = \frac{1}{\alpha}$$

$$\frac{T_3}{T_1} = \alpha \text{ ------------ (ii)}$$

Process 3-2

$T_1 = \beta T_2$

Put in (ii)

$$\frac{T_3}{\beta T_2} = \alpha \Rightarrow \frac{T_2}{T_3} = \frac{1}{\alpha\beta} \text{ ----(iii)}$$

Put in (i)

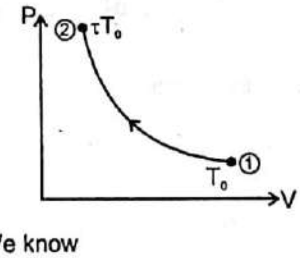
$$\Delta S_{net} = \nu C_P \ln\alpha - \nu C_V \ell n\alpha\beta$$

$$\Delta S_{net} = \frac{\nu\gamma R}{\gamma - 1} \ell n\alpha - \frac{\nu R}{\gamma - 1} (\ell n\alpha + \ell n\beta)$$

$$\Delta S_{net} = \nu R \left(\ell n\alpha - \frac{\ell n\beta}{\gamma - 1} \right)$$

Ans.

2.136



We know

$$\int dS = \int \frac{dQ}{T}$$

$$\Delta S = \int \frac{dQ}{T}$$

$$dQ = CdT$$

$$\Delta S = \int_{T_0}^{\tau T_0} C \frac{dT}{T}$$

$$\Delta S = C \ell n \tau$$

$$C=\left(\frac{R}{\gamma-1}-\frac{R}{n-1}\right)\ln(\tau)$$

$$\Delta S=\left(\frac{R}{\gamma-1}-\frac{R}{n-1}\right)\ell n\tau$$

$$\Delta S=\frac{R(n-\gamma)}{(\gamma-1)(n-1)}\ \ell n\tau$$

Ans.

2.137*

$P \propto V$

$PV^{-1}=\text{const}$

Compare with $PV^x=\text{Const}$

$x=-1$

$V_f=\alpha V_i$ (Given)

$$\Delta S=\int_{T_i}^{T_f}\frac{\nu CdT}{T}=\nu C\ell n\, T_f/T_i \quad \text{----- (i)}$$

Hence

$$C=\frac{R}{\gamma-1}-\frac{R}{-1-1}=R\left(\frac{1}{\gamma-1}+\frac{1}{2}\right)=\frac{R(\gamma+1)}{2(\gamma-1)}$$

Calculatoin of T_f/T_i

$PV^{-1}=\text{Const}$

$$\frac{\nu RTV^{-1}}{V}=\text{const}$$

$TV^{-2}=\text{const}$

$T_f(V_f)^{-2}=T_i(V_i)^{-2}$

$$\frac{T_f}{T_i}=\left(\frac{V_f}{V_i}\right)^2=\alpha^2$$

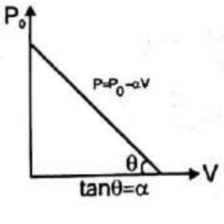
Put in (i)

$$\Delta S=\nu\left(\frac{R(\gamma+1)}{2(\gamma-1)}\right)\ell n\alpha^2$$

$$\Delta S=\frac{\nu R(\gamma+1)}{(\gamma-1)}\ell n\alpha$$

Ans.

2.138



We know $\Delta S=\int\frac{dQ}{T}$

For ΔS maximum, dQ must be (+) ive and when dQ will be negative ΔS will be decreasing.

The we know

$dQ=dU+dW$

$dQ=dU+PdV\geq 0$

$dU\geq -PdV$

$\nu C_v dT\geq -PdV$

$$\nu C_v\frac{dT}{dV}\geq -P$$

$$\frac{dT}{dV}\geq -\frac{P}{\nu C_v} \quad \text{------- (i)}$$

Also

$P=P_0-\alpha V$

$$\frac{\nu RT}{V}=P_0-\alpha V$$

$\nu RT=P_0V-\alpha V^2$

$$\nu R\frac{dT}{dV}=P_0-2\alpha V$$

$$\frac{dT}{dV}=\frac{P_0}{\nu R}-\frac{2\alpha V}{\nu R}$$

Put in(i)

$$\frac{P_0}{\nu R}-\frac{2\alpha V}{\nu R}\geq\frac{-P}{\nu C_v}$$

$$P_0-2\alpha V\geq\frac{-PR}{C_v}=\frac{-R}{C_v}(P_0-\alpha V)$$

$$P_0-2\alpha V\geq\frac{-RP_0}{C_v}+\frac{R\alpha V}{C_v}$$

$$P_0\left(1+\frac{R}{C_v}\right)\geq\left(2\alpha+\frac{R\alpha}{C_v}\right)V$$

$$V\leq\frac{P_0(R+C_v)}{C_v\left(\frac{2\alpha C_v+R\alpha}{C_v}\right)}$$

$$V\leq\frac{P_0\gamma}{\alpha(1+\gamma)}$$

Maximum Value of Volume

$$V_{max}=\frac{P_0\gamma}{\alpha(1+\gamma)}$$

Ans.

2.139

$$S = aT + C_V \ell nT$$

$$dS = adT + \frac{C_V}{T} dT$$

Also we know
dQ= dU + dW
Also
dQ = T ds
T ds = dU + P dV

$$T\left(adT + \frac{C_V}{T}dT\right) = C_V dT + \frac{2T}{V}dV$$

$$TadT = \frac{RT}{V}dV$$

$$a\int_{T_0}^{T} dT = R\int_{V_0}^{V}\frac{dV}{V}$$

$$a(T - T_0) = R\ell n \, V/V_0$$

$$T = T_0 + \frac{R}{a}\ell n \, V/V_0$$

Ans.

2.140*

$$\int dS = \int_0^{\Delta Q}\frac{dQ}{T}$$

$$\Delta S = \frac{1}{T}\int_0^{\Delta Q}\frac{dQ}{T}$$

(Because isothermal process)

$$\Delta S = \frac{1}{T}\,\Delta Q$$

$$\Delta Q = \int_{V_1}^{V_2} PdV + \Delta U$$

$$\Delta Q = \int_{V_1}^{V_2}\left(\frac{RT}{V_m - b} - \frac{a}{V_M^2}\right)dV_M + \Delta U$$

$$= RT\,\ell n\left(\frac{V_2 - b}{V_1 - b}\right) + \frac{a}{V_M}\Bigg|_{V_1}^{V_2} + \Delta U$$

$$\Delta Q = RT\ell n\left(\frac{V_2 - b}{V_1 - b}\right) + a\left(\frac{1}{V_2} - \frac{1}{V_1}\right) + \Delta U \;......(1)$$

Calculation of ΔU

$$U = C_v T - \frac{a}{V_m}$$

$$\Delta U = -\left(\frac{a}{V_2} - \frac{a}{V_1}\right)$$

Put in (i)

$$\Delta Q = RT\,\ell n\left(\frac{V_2 - b}{V_1 - b}\right)$$

$$\Delta S = \frac{\Delta Q}{T} = R\ell n\left(\frac{V_2 - b}{V_1 - b}\right)$$

Ans.

2.141*

$$\Delta S = \int\frac{dQ}{T} = \int\frac{dU + pdV}{T} \;..... (i)$$

$$dU = C_V\,dT + \frac{a}{V^2}dV$$

__Vander wall equation__ :

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

$$Pd\,V = \frac{RT\,dV}{V - b} - \frac{a}{V^2}$$

Put in (i)

$$\Delta S = \int\frac{C_V dT + \frac{a}{V^2}dV + \frac{RTdV}{V - b} - \frac{a}{V^2}dV}{T}$$

$$\Delta S = \int_{T_1}^{T_2}\frac{C_V\,dT}{T} + \int_{V_1}^{V_2}\frac{R\,dV}{v - b}$$

$$\Delta S = C_V \ell n\, T_2/T_1 + R\ell n\left(\frac{V_2 - b}{V_1 - b}\right)$$

Ans.

2.142

$$dQ = C\,d\,T$$

$$ds = \frac{dQ}{T} = \frac{cdT}{T}$$

$$\int ds = \int C\frac{dT}{T} = \int\frac{aT^3 dT}{T}$$

$$\Delta S = \frac{aT^3}{3}$$

Ans.

2.143*

$$dQ = mC\,d\,T = m\,(a+bT)\,dT$$

$$ds = \frac{dQ}{T} = m\left(\frac{cdT}{T} + bdT\right)$$

$$\Delta S = m\left(\int_{T_1}^{T_2} \frac{ddT}{T} + \int_{T_1}^{T_2} \frac{bdT}{T}\right)$$

$$\Delta S = m\left(a\ell n\, {}^{T_2}\!/\!_{T_1} + b(T_2 + T_1)\right)$$

Ans.

2.144*

$$T = a\,s^n$$

$$s = \left(\frac{T}{a}\right)^{1/n}$$

$$ds = \frac{1}{na}\left(\frac{T}{a}\right)^{1/n-1} dT$$

Also

$$ds = \frac{dQ}{T} = \frac{CdT}{T}$$

Then

$$\frac{1}{na}\left(\frac{T}{a}\right)^{1/n-1} dT = \frac{CdT}{T}$$

$$C = \frac{T}{na}\left(\frac{T}{a}\right)^{1/n-1} = \frac{1}{n}\left(\frac{T}{a}\right)^{1/n}$$

$$C = \frac{1}{n}\left(\frac{aS^n}{a}\right)^{1/n}$$

$$C = \frac{S}{n}$$

$C < 0$

If $n < 0$

Ans.

2.145*

$$PV^x = \text{const}$$

$$C = \frac{R}{\gamma - 1} - \frac{R}{x - 1}$$

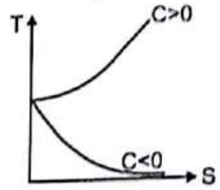
Then

$$\int_{S_0}^{S} ds = \int \frac{d\theta}{T} = \int_{T_0}^{T} \frac{CdT}{T}$$

$$S = S_0 + C\,\ell n \frac{T}{T_0}$$

$$\frac{T}{T_0} = e^{\left(\frac{S - S_0}{C}\right)}$$

$$T = T_0 e^{\left(\frac{S - S_0}{C}\right)}$$



2.146*

(a)

$$S = \frac{\alpha}{T}$$

$$dS = -\frac{\alpha}{T^2}dT$$

$$dQ = TdS = -\frac{T\alpha}{T^2}$$

$$dQ = -\frac{\alpha}{T}dT = CdT$$

$$C = -\frac{\alpha}{T}$$

Ans.

(b)

$$\int dQ = \int_{T_1}^{T_2} CdT = -\int_{T_1}^{T_2} \frac{\alpha}{T}dT = \alpha\ell n\, {}^{T_1}\!/\!_{T_2}$$

$$\Delta Q = \alpha\ell n\, {}^{T_1}\!/\!_{T_2}$$

Ans.

(c)

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U = \int_{T_1}^{T_2} C_V dT = C_V\left(T_2 - T_1\right)$$

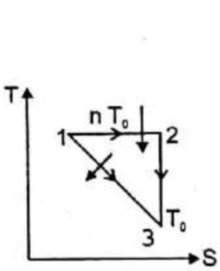
$$\Delta W = \Delta Q - \Delta U$$

$$\Delta W = \alpha\ell n\, {}^{T_1}\!/\!_{T_2} - C_V(T_2 - T_1)$$

Ans.

2.147

(a)



We know

$dQ = T\,ds$

$\Delta Q_{net} = \int d\theta = \int T\,ds$ = Area uner T VS S curve

$$\Delta Q_{net} = \frac{1}{2}(T_2 - T_3)(S_2 - S_1) = \frac{1}{2}(n-1)(S_2 - S_1)T_0$$

Also

$\Delta Q = \Delta U + \Delta w.$

One complete cycle

$\Delta U = 0$

$\Delta Q_{net} = \Delta W$

$\Delta Q_{given} = \int T\ ds$

if $ds = \oplus$ then heat given will be positive through process and this in $1 \rightarrow 2$ process

$\Delta\theta_{given} = \int T\ ds = nT_0(S_2 - S_1)$

Efficiency(η)

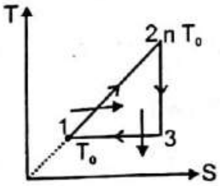
$$\eta = \frac{\Delta w}{\Delta Q_{given}} = \frac{\Delta Q_{net}}{\Delta Q_{given}}$$

$$\eta = \frac{n-1}{2n}$$

$$\eta = \frac{1}{2}\left(1 - \frac{1}{n}\right)$$

Ans.

(b)



$$\Delta Q_{net} = \frac{1}{2}(nT_0 - T_0)(S_3 - S_1)$$

$\Delta\theta_{given}$ = Area under process 1–2

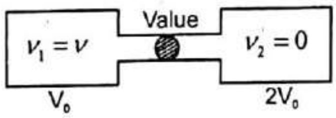
$$= \frac{1}{2}(T_0 + nT_0)(S_3 - S_1)$$

$$\text{Efficiency } (\eta) = \frac{\Delta\theta_{net}}{\Delta\theta_{given}} = \eta = \frac{n-1}{n+1}$$

Ans.

2.148*

Thermally insulated vessel



Initial volume = V_0

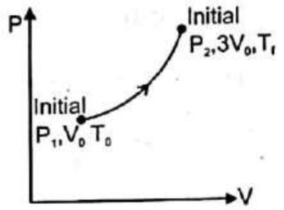
Final Volume= $3V_0$

Since valve open suddenly hence process is not slow.

Now we can not use PV = ν R T for intermediate process.

But we know entropy is state function and depends on intial and final state.

Hence we take another slow process in which change in volume occure very slowly.



Calculation of final temp (T_f)

Change in internal energy of system is 0.

$U_i = U_f$

$\nu\ C_V T_0 = \nu C_V T_f$

$T_f = T_0$

Take imaginary process in which heat may be exchange but final and initial statges are same

$$dS = \frac{d\theta}{T} = \nu C_V \frac{dT}{T} + \frac{pdV}{T}$$

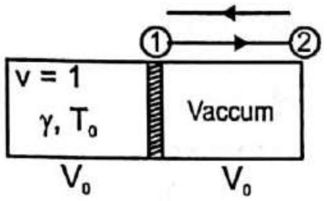
$$= \frac{\nu C_V dT}{T} + \frac{\nu RTdV}{VT}$$

$$\Delta S = \int ds = \int_{T_0}^{T_0} \nu C_V \frac{dT}{T} + \nu R \int_{V_0}^{nV_0} \frac{dV}{V}$$

$$\Delta S = \nu R \ell n(n)$$

Ans.

2.149



Internal energy calculation :

Process ① ⟶ ②

$U_i = U_f$

Because $\Delta Q = 0$ and $\Delta W = 0$

$0 = \Delta U + 0$

$T_i = T_f = T_2$

Process ② ⟶ ①

Adiabatice process

$V_2 = 2V_0$

$V_1 = V_0$

$\Delta U = U_f - U_i = \nu C_V T_f - \nu C_V T_i = \nu\ CV\ (T_f - T_i)$

$T_i = T_0 = T_2$

Since

$TV^{\gamma-1} = \text{const.}$

$T_0 (2V_0)^{\gamma-1} = T_f (V_0)^{\gamma-1}$

$T_f = 2^{\gamma-1}\ T_0$

Then

$\Delta U = \nu\ C_V\ (2^{\gamma-1} T_0 - T_0) = \nu\ C_V\ T_0\ (2^{\gamma-1} - 1)$

$\Delta U_{net} = 0 + \nu\ C_V T_0\ (2^{\gamma-1} - 1)$

$$\Delta U_{net} = \frac{\nu R T_0}{\gamma - 1}\left(2^{\gamma-1} - 1\right)$$

Put $\nu = 1$

$$\Delta U_{net} = \frac{R T_0}{\gamma - 1}\left(2^{\gamma-1} - 1\right)$$

Ans.

2.150

(a)

If process is made sudden in first part while slow in second part

then external force in part (a) will be more than part (b)

Hence $\Delta W_{(a)} < \Delta W_{(b)}$ (by gas)

Hence $\Delta U_{(a)} < \Delta U_{(b)}$ (of gas)

T_{final} of process (a) > T_{final} of process(b)

And $V_{final(a)} = V_{final(b)}$

Also we know

$$P = \frac{\nu R T_f}{V_f}$$

$P_{final(a)} > P_{final(b)}$

Ans.

2.151*

$C_v = 5/2\ R$ is for N_2 as well as O_2

At final equalibrium

$U_i = U_f$

$$\nu_1 C_V T_0 + \nu_2 C_V T_0 = (\nu_1 + \nu_2) C_V^{Mix} T_f \quad ......(1)$$

$$C_V^{Mix} = \frac{\nu_1 C_V + \nu_2 C_V}{\nu_1 + \nu_2} = C_v$$

Put in (1)

$T_f = T_0$

N_2 Gas :

$$\Delta S_{N_2} = \int ds = \int \frac{dQ}{T} = \int_{T_0}^{T_0} \frac{\nu_1 C_V dT}{T} + \int_{nV_0}^{nV_0+V_0} \frac{pdV}{T}$$

$$\Delta S_{N_2} = \int_{T_0}^{nV_0+V_0} \frac{\nu_1 R}{V} dV = \nu_1 R \ell n(n+1)$$

O_2 Gas :

$$\Delta S_{O_2} = \int_{T_0}^{T_0} \frac{\nu_2 C_V dT}{T} + \int_{nV_0}^{nV_0+V_0} \frac{pdV}{T}$$

$$= 0 + \int_{nV_0}^{(n+1)V_0} \frac{\nu_2 R}{V} dV = \nu_2 R \ell n\left(\frac{n+1}{n}\right)$$

$$\Delta S_{system} = \Delta S_{N_2} + \Delta S_{O_2}$$

$$\Delta S_{system} = \nu_1 R \ell n(n+1) + \nu_2 R \ell n\left(\frac{n+1}{n}\right)$$

$$\Delta S_{system} = \nu_1 R \ell n(n+1) + \nu_2 R \ell n\left(1 + \tfrac{1}{n}\right)$$

Ans.

2.152*

Suppose mass and tempreture of copper are $m_1 = 300$ gm , $t_1 = 97°$ C .

While mass and tempreture of water are $m_2 = 100$ gm , $t_2 = 7°C$.

Let us final temp is T_f

Then

Using calorimeter principle

Heat given by (m_1) = Heat taken by (m_2)

$m_1 C_1\ (t_1 - t_f) = m_2 C_2\ (t_f - t_2)$

$$t_f = \frac{m_1 C_1 t_1 + m_2 C_2 t_2}{m_1 C_1 + m_2 C_2} \quad \text{———(i)}$$

For m_1 :

$$\int ds = \int \frac{dQ}{T}$$

$$\Delta S_{m_1} = \int_{t_1}^{t_f} \frac{m_1 C_1 dT}{T} = m_1 C_1 \ell n\, t_f / t_1$$

For m_2 :

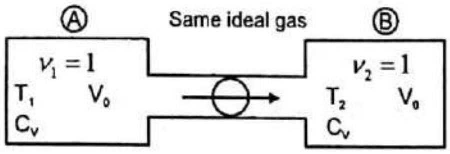
$$\Delta S_{m_2} = m_2 C_2 \ell n \frac{t_f}{t_2}$$

$$\Delta S_{system} = m_1 C_1 \ell n \frac{t_f}{t_1} + m_2 C_2 \ell n \frac{t_f}{t_2}$$

Where C_1 and C_2 are heat capacity is of coper and water.

2.153*

Case1 : If valve is open.



Let us final temp is T_f

$U_i = U_f$ (For system)

$$\nu_1 C_V T_1 + \nu_2 C_V T_2 = (\nu_1 + \nu_2) C_V T_f$$

$$T_f = \frac{T_1 + T_2}{2}$$

For chanber (A):

$$\int ds = \int \frac{dQ}{T} = \int_{T_1}^{T_f} \frac{C_V dT}{T} = \int_{T_1}^{2V_0} \frac{RdV}{V}$$

$$\Delta S_A = C_V \ell n \frac{T_f}{T_1} + R\ln 2$$

For chamber (B) :

Similiar

$$\Delta S_B = C_V \ell n \frac{T_f}{T_2} + R\ell n 2$$

$$\Delta S_{system} = \Delta S_A + \Delta S_B$$

$$\Delta S_{system} = C_v \left[\ell n \frac{T_f}{T_1} + \ell n \frac{T_f}{T_2} \right] + 2R\ell n 2$$

$$\Delta S_{system} = C_V \ell n \frac{(T_1 + T_2)^2}{4T_1 T_2} + 2R\ell n 2$$

Using AM $\geq$ GM

$$(T_1 + T_2)^2 > 4T_1 T_2$$

$$\Delta S \geq O$$

Case 2 :

If value is conducting

$$T_f = \frac{T_1 + T_2}{2}$$

For chamber A :

$$\Delta S = \int_{T_1}^{T_f} \frac{C_V dT}{T} + \int PdV$$

Here $\Delta V = 0$

$$\Delta S_A = C_V \ \ell n \left(\frac{T_f}{T_1} \right)$$

For chanber B

$$\Delta S_B = C_V \ \ell n \ \frac{T_f}{T_2}$$

$$\Delta S_{net} = \Delta S_A + \Delta S_B$$

$$\Delta S_{net} = Cv \ \ell n \left(\frac{(T_1 + T_2)^2}{4T_1 T_2} \right)$$

Using AM $\geq$ GM

$$(T_1 + T_2)^2 > 4T_1 T_2$$

$$\Delta S \geq O$$

From above long function.

Ans.

2.154*

(a)

Probability to find a molecule in one half part of vessel = 1/2

Probability of find N molecules on one half part

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \text{ N time}$$

$$P = \frac{1}{2} \ N$$

Ans.

(b)

Velocity of gass molecules is

$v = 10^5$ cm/s

Time to cross the vessel

$$\tau = \frac{1}{10^5} = 10^{-5} s$$

No. of time of cross the vessel = t/τ

Where $t = 10^{10}$ year

Then there will be a certainty to occopic this half will be 1

$$\frac{t}{\tau} \times \frac{1}{2^N} \cong 1$$

$$2^N \cong t/\tau$$

$$N = \frac{\ell n \, t/\tau}{\ell n 2}$$

Ans.

2.155*

We know statical wright on n molecules among N molecules

$$C_N^n = \frac{N!}{n!(N-n)!}$$

Since in half part N/2 molecules will be occupied then

Statical weight of n = N/2 molecules

$$C_N^{\frac{N}{2}} = \frac{N!}{\frac{N}{2}!\frac{N}{2}!}$$

Probability (P)

$$P = \left(C_N^{N_2}\right)\frac{1}{2^N}$$

$$P = \left(\frac{N!}{\frac{N}{2}!\frac{N}{2}!}\right)\frac{1}{2^N}$$

Ans.

2.156*

There is N molecules and all are free from each other. Suppose probality to one molecule to enter successfully in one part is P

Then probability of unsuccesful attempt is 1 – P

Then probality of n mole celes to enter in one part :

$$P_{net} = {}^nC_r P^n (1-P)^{N-n}$$

Here

$$P = \frac{1}{2}$$

$$P_{net} = \frac{N!}{n!(N-n)!2^N}$$

Ans.

Put N = 5 and n = 0,1,2,3 an get different result.

2.157*

Like Q : 2.156

$$P_{net} = \frac{N!}{n!(N-n)!}P^n(1-P)^{N-n}$$

Where P = Probability to find one molecules in certain part.

2.158*

No of molecules in sphere of diameter d

$$N = \frac{4}{3}\pi R^3 n_0$$

Where n_0 = Loschmidt's constant = no of molecules / volume

$$N = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 n_0$$

$$N = \frac{\pi d^3 n_0}{6}$$

We know relative fluctvation is

$$\frac{1}{\sqrt{N}} = n$$

$$N = \frac{1}{n^2}$$

$$\frac{1}{n^2} = \frac{\pi d^3 n_0}{6}$$

$$n = \sqrt{6/\pi d^3 n_0}$$

Ans.

Also are know that average no of molecules inside sphere

$$<n> = \frac{1}{n^2}$$ **Ans.**

2.159*

Gas is monoatomic gas and no. of mole , tempreture and volume are $\nu = 1, T_\sigma\, V_0$

Now $T_f = T_0 + T$; $V_f = V_0$

We know

$$\int dS = \int\frac{dQ}{T} = \int_{T_0}^{T_0+\Delta T}\frac{\nu C_V dT}{T}$$

$$\Delta S = C_V \ell n\frac{T_0+\Delta T}{T_0}$$

Also we know

$$\Delta S = K\ell n\Omega_2 - K\ell n\Omega_1 = K\ell n\,{}^{\Omega_2}\!/_{\Omega_1}$$

Then

$$C_V\ell n\frac{T_0+\Delta T}{T_0} = K\ell n\frac{\Omega_2}{\Omega_1}$$

Here $C_V = \frac{i}{2}R$ where i = Degree of freedom

$$\frac{iR}{2R}\ell n\left(1+\frac{\Delta T}{T_0}\right) = \ell n\frac{\Omega_2}{\Omega_1} \Rightarrow \frac{\Omega_2}{\Omega_1} = \left(1+\frac{\Delta T}{T_0}\right)^{\frac{iR}{2N_A}}$$

$$\frac{\Omega_2}{\Omega_1} = \left(1+\frac{\Delta T}{T_0}\right)^{\frac{i}{2}N_A}$$ N_A = Avogadro number

Ans.

2.5

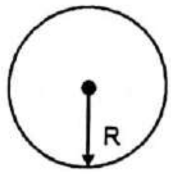
Liquids,

Capillary

Effects

2.160

(a)
Mercury drop :



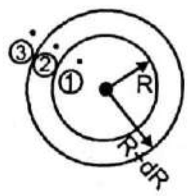
$$\Delta P = \frac{2T}{R}$$

Hence $T = \alpha$ = Surface tension
$R = d/2$

$$\Delta P = \frac{2\alpha}{d/2} \Rightarrow \Delta P = \frac{4\alpha}{d}$$

Ans.

(b)
Soap bubble :



$$P_1 - P_2 = \frac{2\alpha}{R} \text{ ....(1)}$$

$$P_2 - P_3 = \frac{2\alpha}{R + dR} \cong \frac{2\alpha}{R} \text{ ....(2)}$$

Adding (1) and (2)

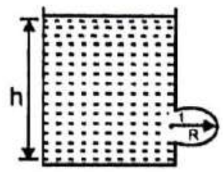
$$P_1 - P_3 = \frac{4\alpha}{R}$$

$$P_1 - P_3 = \frac{4\alpha}{R} = \frac{4\alpha}{d/2}$$

$$\Delta P = \frac{8\alpha}{d}$$

Ans.

2.161



Force applied by surface tension and due to hydrostatic pressure will be same.

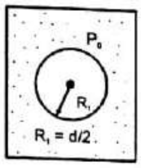
$$F=(\pi R^2)h\rho g=\left(\frac{2\alpha}{R}\right)\pi R^2$$

$$h=\frac{2\alpha}{\rho g\left(\frac{d}{2}\right)}$$

$$h=\frac{4\alpha}{\rho g d}$$

Ans.

2.162



Initial pressure inside bubble :

$$P_{in}=P_0+\frac{4\alpha}{R_1}=P_0+\frac{8\alpha}{d} \text{ ----(i)}$$

Final pressure inside bubble :

$$P_f=\frac{P_0}{n}+\frac{8\alpha}{d_f}$$

Given that

$$d_f=\eta d$$

$$P_f=\frac{P_0}{n}+\frac{8\alpha}{\eta d} \text{ ---- (ii)}$$

Since process is isothermal
PV = Constant (Inside soap bubble)

$$\left(P_0+\frac{8\alpha}{d}\right)\frac{4}{3}\pi R_1^3=\left(\frac{P_0}{n}+\frac{8\alpha}{\eta d}\right)\frac{4}{3}\pi(\eta R_1)^3$$

$$\alpha=\frac{1}{8}P_0 d\frac{\left(1-\frac{\eta^3}{n}\right)}{\eta^2-1}$$

Ans.

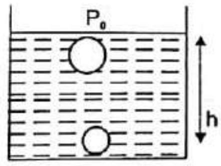
2.163

$$P_{inside}=P_0+h\rho g+\frac{2\alpha}{R}$$

$$P_{inside}=P_0+h\rho g+\frac{4\alpha}{d}$$

Ans.

2.164



$$P_{bottom}=P_0+h\rho g+\frac{2\alpha}{R_1}$$

$$P_{bottom}=P_0+h\rho g+\frac{4\alpha}{d}$$

$$P_{surface}=P_0+\frac{4\alpha}{nd}$$

Hence if process is isothermal then

$$P_{bottom}\frac{4}{3}\pi R_1^3=P_{surface}\frac{4}{3}\pi(nR_1)^3$$

$$\left(P_0+h\rho g+\frac{4\alpha}{d}\right)\times 1=\left(P_0+\frac{4\alpha}{nd}\right)n^3$$

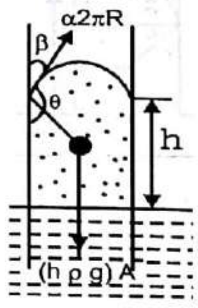
$$h=\frac{P_0(n^3-1)+4\alpha(n^2-1)}{\rho g}$$

Ans.

2.165

<u>Method : 1 (Force Method)</u>

F.B.D. of liquid coloum



At equilibrium

$\alpha 2\pi R\cos\beta = (h\rho g)A$

$\alpha 2\pi R\cos\beta = h\rho g\pi R^2$

$2\alpha\cos(\pi-\theta) = hR\rho g$

$$h = \frac{-2\alpha\cos\theta}{\rho g R}$$

Here
$d = 2R$

$$h = \frac{-4\alpha\cos\theta}{\rho g d}$$

First coloumn :

$$h_1 = \frac{-4\alpha\cos\theta}{\rho g d_1}$$

Second column

$$h_2 = \frac{-4\alpha\cos\theta}{\rho g d_2}$$

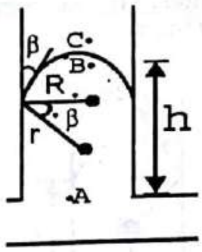
$\Delta h = h_1 - h_2$

$$\Delta h = \frac{4\alpha}{\rho g}\cos\theta\left(\frac{1}{d_2}-\frac{1}{d_1}\right)$$

$$\Delta h = \frac{4\alpha|\cos\theta|}{\rho g}\left[\frac{d_1-d_2}{d_1 d_2}\right]$$

Ans.

Method : 2 (Pressure Method)



P_0= Atmospheric pressure
Using pressure equation from point A to C

$$P_0 - h\rho g - \frac{2\alpha}{r} = P_0$$

$$h = \frac{2\alpha}{r\rho g}\text{.................}(1)$$

Also

$$r\cos\beta = R$$

$$r = \frac{R}{\cos\beta}$$

Then

From (1)

$$h = \frac{2T\cos\beta}{R\rho g}$$

$$\beta = \pi - \theta$$

$$h = \frac{-2\alpha\cos\theta}{\rho g R}$$

Here
$d = 2R$

$$h = \frac{-4\alpha\cos\theta}{\rho g d}$$

First coloumn :

$$h_1 = \frac{-4\alpha\cos\theta}{\rho g d_1}$$

Second column

$$h_2 = \frac{-4\alpha\cos\theta}{\rho g d_2}$$

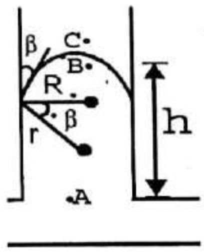
$\Delta h = h_1 - h_2$

$$\Delta h = \frac{4\alpha}{\rho g}\cos\theta\left(\frac{1}{d_2}-\frac{1}{d_1}\right)$$

$$\Delta h = \frac{4\alpha|\cos\theta|}{\rho g}\left[\frac{d_1-d_2}{d_1 d_2}\right]$$

Ans.

2.166



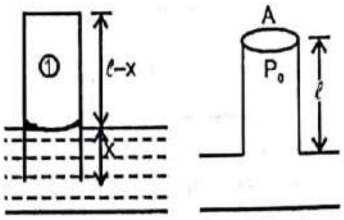
P_0= Atmospheric pressure
Using pressure equation from point A to C

$$P_0 - h\rho g - \frac{2\alpha}{r} = P_0$$

$$r = \frac{2\alpha}{h\rho g}$$

Ans.

2.167



Pressure at point (1)

$$P_1 = P_0 + \frac{2\alpha}{R} = P_0 + \frac{4\alpha}{d}$$

Using

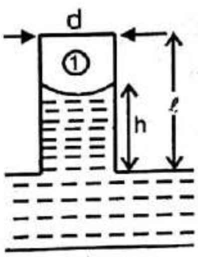
PV = constant (Because T is constant)

$$P_0 \ell A = \left(P_0 + \frac{4\alpha}{d}\right)(\ell - x)A$$

$$x = \frac{\ell}{1 + \frac{P_0 d}{4\alpha}}$$

Ans.

2.168



Pressure at point (1)

Calculated by Boyles law

$P_0 \ell A = P_1 (\ell - h) A$

$$P_1 = \frac{P_0 \ell}{\ell - h} \text{ —— (i)}$$

Also

$$P_1 = P_0 + h\rho g + \frac{4\alpha}{d}\cos\beta \text{ —— (ii)}$$

From (i) and (ii)

$$\frac{P_0 \ell}{\ell - h} = P_0 - h\rho g + \frac{4\alpha}{d}\cos\beta$$

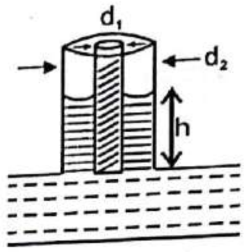
$$P_0\left[\frac{\ell}{\ell - h} - 1 + \frac{h\rho g}{P_0}\right] = \frac{4\alpha\cos\beta}{d}$$

$$P_0\left[\frac{h}{\ell - h} + \frac{h\rho g}{P_0}\right] = \frac{4\alpha}{d}\cos\beta$$

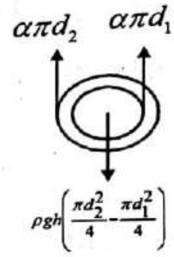
$$\alpha = \frac{\left[\rho g h + \frac{P_0 \ell}{\ell - h}\right]d}{4\cos\beta}$$

Ans.

2.169



F.B.D.



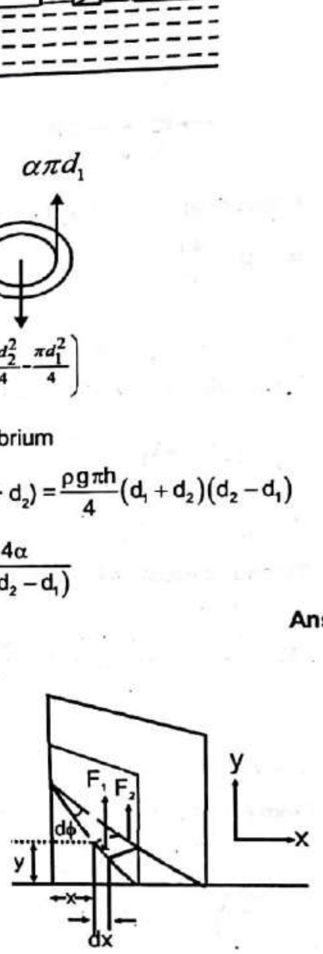
At equilibrium

$$\alpha\pi(d_1 + d_2) = \frac{\rho g \pi h}{4}(d_1 + d_2)(d_2 - d_1)$$

$$h = \frac{4\alpha}{\rho g(d_2 - d_1)}$$

Ans.

2.170



$F_y = (\alpha \cos\beta)\,dx$

$F_{net} = 2F_y = 2(\alpha\cos\beta)\,dx$

Since every differential elements are free from each other.

Then

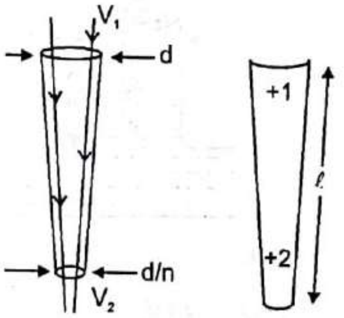
F_{net} = Weight of liquid column of differential length dx

$2\alpha \cos\beta \, dx = \rho g \, [y x \, d\phi dx]$

$$y = \frac{(2\alpha \cos\beta)}{\rho g \times d\phi}$$

Ans.

2.171



Pressure at point (1) and (2) are

$$P_1 = P_0 - \frac{4\alpha}{d}$$

$$P_2 = P_0 + \frac{4\pi}{d/n} = P_0 + \frac{4\alpha n}{d}$$

Continuity equation at (1) and (2) :

$$\frac{\pi d^2}{4} V_1 = \frac{\pi d^2}{4n^2} V_2$$

$$V_1 = \frac{V_2}{n^2} \text{..............(1)}$$

Bernaullis equation between point (1) and (2)

$$\left(P_0 - \frac{4\alpha}{d}\right) + \rho g \ell + \frac{1}{2}\rho V_1^2 = \left(P_0 - \frac{4\alpha n}{d}\right) + 0 + \frac{1}{2}\rho V_2^2$$

.... (2)

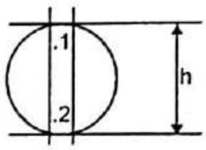
$$Q = V_1 \frac{d\pi^2}{4} \text{.............(3)}$$

From (1) and (2) calculate V_1 and put in (3)

$$Q = \frac{\pi d^2}{4}\sqrt{\frac{2gl - \frac{4\alpha(n-1)}{\rho d}}{n^4 - 1}}$$

Ans.

2.172



Here

$R_1 \cong R_2 \cong h/2$

Pressure point (1) and (2)

$$P_1 = P_0 + \frac{2\alpha}{R_1}$$

$$P_2 = P_0 + \frac{2\alpha}{R_2}$$

$$P_2 = P_1 + h\rho g$$

$$P_2 - P_1 = h\rho g$$

Then

$$\Delta P = 2\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = h\rho g$$

$$2\alpha\left(\frac{R_1 - R_2}{R_1 R_2}\right) = h\rho g$$

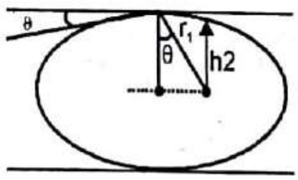
$$\frac{2\alpha \Delta R}{\frac{h}{2} \times \frac{h}{2}} = h\rho g$$

$$\Delta R = \frac{h^3 \rho g}{8\alpha}$$

Ans.

2.173

Method : 1 (Pressure Method)



$r_1 \cos\theta = h/2$

$$r_1 = \frac{h}{2\cos\theta}$$

$r_2 = R$

Here $R >> h$

We know

$$\Delta P = \alpha\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

Then

$$\Delta P = \alpha\left(\frac{1}{\frac{h}{2\cos\theta}}+\frac{1}{R}\right) = \alpha\left(\frac{2\cos\theta}{h}+\frac{1}{R}\right) \text{.........(1)}$$

Hence $\frac{1}{R} << \frac{1}{h}$

From(1)

$$\Delta P = \frac{2\alpha\cos\theta}{h}$$

Normal force due to surface tension

$$N = (\Delta P)\,\pi R^2 = \left(\frac{2\alpha\cos\theta}{h}\right)\pi R^2 \text{ ....(2)}$$

Also

$V = \pi R^2 h$

$\pi R^2 = V/h$

$$N = \left(\frac{2\alpha\cos\theta}{h^2}\right) V = m_1 g$$

Where m_1 = mass of plate

$$N = \left(\frac{2\alpha\cos\theta}{h^2}\right) V = m_1 g$$

$$N' = \left(\frac{2\alpha\cos\theta}{h'^2}\right) V = (m_1 + \Delta m)g$$

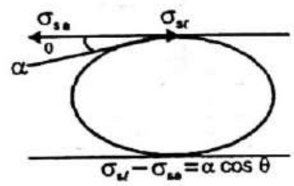
$$\Delta mg = (2\alpha\cos\theta)V\left(\frac{1}{h'^2}-\frac{1}{h^2}\right)$$

Also $h' = h^2/n^2$

$$\Delta m = \frac{(2\alpha\cos\theta)\pi R^2 (n^2-1)}{gh}$$

Ans.

Method : 2 (Surface energy)



Surface Energy

$$U = (-\sigma_{sa} + \sigma_{sl})\pi R^2$$

$$U = (\alpha\cos\theta)(2\pi R^2)$$

Also we know volume will be constant while R will be change if surface area increase.

Then

$V = \pi R^2 h$

$\pi R^2 = V/h$

$U = (\alpha\cos\theta)\, 2V/h$

Normal force

$$N = \frac{-dU}{dh} = \frac{2\alpha V\cos\theta}{h^2}$$

Then

$$\Delta N = 2\alpha V\cos\theta\left(\frac{1}{h'^2}-\frac{1}{h^2}\right) = \Delta mg$$

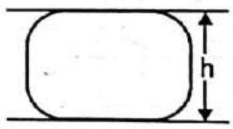
$$2\alpha V\cos\theta\left(\frac{1}{\frac{h^2}{n^2}}-\frac{1}{h^2}\right) = \Delta mg$$

$$2a(\pi R^2 h)\cos\theta\left(\frac{n^2-1}{h^2}\right) = \Delta mg$$

$$\Delta m = \frac{2\alpha\pi R^2\cos\theta}{h}(n^2-1)$$

Ans.

2.174



Normal reaction at surface from question no. Q.2.73

$$N = \frac{2\alpha V\cos\theta}{h^2}.$$

If wetting is complete $\cos\theta = 1$

$$N = \frac{2\alpha V}{h^2}$$

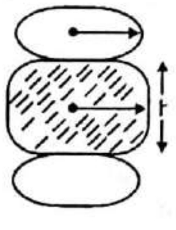
Here

$$V = \frac{m}{\rho}$$

$$N = \frac{2\alpha m}{\rho h^2}$$

Ans.

2.175



Normal force $N = \dfrac{2\alpha V \cos\theta}{h^2}$

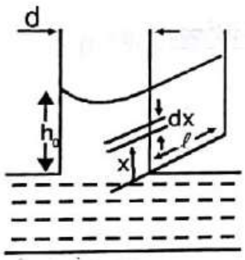
For complete wetting $\cos\theta = 1$

$$N = \frac{2\alpha V}{h^2} = \frac{2\alpha\left(\pi R^2 h\right)}{h^2}$$

$$N = \frac{2\alpha \pi R^2}{h}$$

Ans.

2.176



h_0 = height of liquid.

$2\alpha\ell = \rho(d\ell)\, h_0\, g$

$$h_0 = \frac{2\alpha}{\rho d g}$$

Pressure change at x height

$\Delta P = x\,\rho g$

Force on dx thickness :

$dF = (x\,\rho\, g)\,\ell\, dx$

$$F = \rho g \ell \int_0^{h_0} x\,dx$$

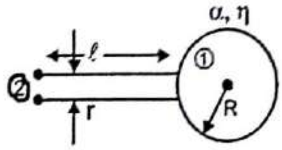
$$F = \rho g \ell \frac{h_0^2}{2}$$

$$F = \frac{\rho g \ell}{2}\left[\frac{2\alpha}{\rho d g}\right]^2$$

$$F = \frac{2\alpha^2 \ell}{\rho g d^2}$$

Ans.

2.177*



Suppose at time t, radius of bubble is x.
Then pressure at point (1)

$$P_1 = P_0 + \frac{4\alpha}{x}$$

Also we know

$$\frac{dQ}{dt} = \frac{(\Delta P)\pi r^4}{8\eta\ell}$$

$$\frac{dQ}{dt} = \left(\frac{4\alpha}{x}\right)\frac{\pi r^4}{8\eta\ell}$$

$$dQ = \frac{4\alpha\pi r^4}{8\eta\ell x}dt$$

Suppose in dt time dx radius is decreased then

$$dQ = (4\pi x^2)dx = \frac{4\alpha\pi r^4}{8\eta\ell x}dt$$

$$16/_2\, x^3 dx = \frac{\alpha r^4}{\eta\ell x}dt$$

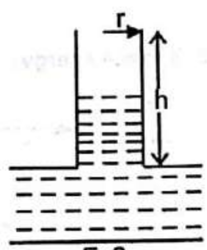
$$\frac{16}{2}\int_0^R x^3\,dx = \frac{\alpha r^4}{\eta\ell}\int_0^t dt$$

$$\frac{16R^4}{4\times 2} = \frac{\alpha r^4 t}{\eta\ell}$$

$$t = \frac{2R^4\eta\ell}{\alpha r^4}$$

Ans.

2.178



Calculation of energy due to rise of coloumn

$$E = (\rho\pi r^2 h g)\frac{h}{2} - (2\pi r h)\alpha$$

$$E = \frac{\rho\pi r^2 g}{2} h^2 - 2\pi r \alpha h$$

Also we know

$$h = \frac{2\alpha}{\rho g r}$$

$$E = \frac{\rho\pi r^2 g}{2}\frac{4\alpha^2}{\rho^2 g^2 r^2} - \frac{2\pi r\alpha}{\rho g r}2\alpha$$

$$E = \frac{4\pi g\alpha^2}{2\rho g^2} - \frac{4\pi r\alpha^2}{\rho g r}$$

$$E = \frac{2\pi\alpha^2}{\rho g} - \frac{4\pi\alpha^2}{\rho g} = -\frac{2\pi\alpha^2}{\rho g}$$

This show that energy is decreased and hence heat will be released

$$\Delta Q = \frac{2\pi\alpha^2}{\rho g}$$

Ans.

2.179

(a)

$$U = \alpha\left[4\pi\left(\frac{d}{2}\right)^2\right] = \pi\alpha d^2$$

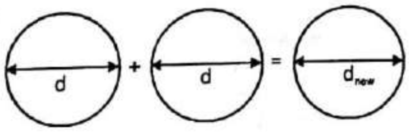
Ans.

(b)

$$U = \alpha\left[4\pi\left(\frac{d}{2}\right)^2 \times 2\right] = 2\pi\alpha d^2$$

Ans.

2.180



$$A = 2\left[4\pi\left(\frac{d}{2}\right)^2\right] = 2\pi d^2$$

Since volume is conserved then

$$\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times 2 = \frac{4}{3}\pi\left(\frac{d_{new}}{2}\right)^3$$

$$d_{new} = 2^{1/3} d$$

$$A_{new} = 4\pi\left(\frac{d_{new}}{2}\right)^2$$

$$A_{new} = 2^{2/3}\pi d^2$$

$$\Delta U = \alpha A_{new} - \alpha A$$

$$\Delta U = \alpha(A_{new} - A)$$

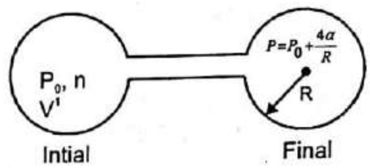
$$\Delta U = \alpha(2^{2/3}\pi d^2 - 2\pi d^2)$$

$$\Delta U = \alpha\,\pi\, d^2\,(2^{2/3} - 2)$$

$$\Delta U = 2\alpha\,\pi\, d^2\,(2^{-1/3} - 2)$$

Ans.

2.181



Volume of bubble

$$V = \frac{4}{3}\pi R^3 \Rightarrow PV = nRT.$$

Work done by external agent

ΔW = Work done against increase pressure from P_0 to $\left(P_0 + 4\alpha/R\right)$ + Work done against increase surface energy

Work done by gas = n RT $\ell n\, V_2/V_1$ = n RT $\ell n\, P_1/P_2$ = – (Work done against gas)

$$\Delta W = \left(nRT\,\ell n\left(\frac{P_0}{P_0 + \frac{4\alpha}{R}}\right)\right) + \left(\alpha 4\pi R^2\right)2$$

$$\Delta W = -PV\,\ell n\left(\frac{P_0}{P}\right) + 8\pi R^2\alpha$$

Ans.

2.182

Pressure inside bubble

$$P = P_0 + \frac{4\alpha}{r}$$

We know that no of molecules inside bubble is not change then

$$\left(P_0 + \frac{4\alpha}{r}\right)\frac{4}{3}\pi r^3 = \nu RT$$

Differentiate equation

$$\left(P_0+\frac{4\alpha}{r}\right)4\pi\, r^2dr+\frac{4}{3}\pi r^3\left(-\frac{4\alpha}{r^2}dr\right)=\nu RT$$

$$\left(P_0+\frac{8\alpha}{3r}\right)4r^2dr=\nu RdT$$

$$\left(P_0+\frac{8\alpha}{3r}\right)dV=\nu RdT$$

$$dV=\frac{\nu RdT}{P_0+\frac{8\alpha}{3r}}$$

Also we know

$dQ=\nu CdT=\nu C_VdT+pdV$

$$C=C_v+\left(P_0+\frac{4\alpha}{r}\right)\left(\frac{R}{P_0+\frac{8\alpha}{3r}}\right)$$

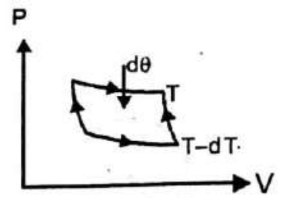
$$C=C_p-R+\left(P_0+\frac{4\alpha}{r}\right)\left(\frac{R}{P_0+\frac{8\alpha}{3r}}\right)$$

$$C-C_p=-R+\left(P_0+\frac{4\alpha}{r}\right)\left(\frac{R}{P_0+\frac{8\alpha}{3r}}\right)$$

$$C-C_p=\frac{1}{2}\frac{R}{\left(1+\frac{3}{8}\frac{P_0r}{\alpha}\right)}$$

Ans.

2.183



We know efficiency of cycle

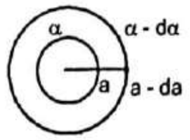
$$\frac{\partial A}{\partial\theta}=1-\frac{T-dT}{T}$$

Where ∂A = work done in cycle

$\partial\theta$ = Given energy in cycle

a = area of soap bubble

$$\frac{\partial A}{\partial\theta}=\frac{\partial T}{T}\ \text{--- (i)}$$



Also we know ∂A = work done due to given heat

Because of tempreture change,surface tension will be change

$\partial A=(\alpha-d\alpha)(a+da)-\alpha(\alpha)=-(da)\,(d\alpha)$

From (i)

$$\frac{(da)(d\alpha)}{d\theta}=\frac{\partial T}{T}$$

$$\left(\frac{d\theta}{da}\right)=-T\frac{d\alpha}{dT}$$

$$q=-T\frac{d\alpha}{dT}$$

Proved

2.184

We know

$$q=-T\frac{d\alpha}{dT}$$

There is two surface in bubble then heat given

$$\Delta Q=(q\Delta\sigma)2$$

$$\Delta Q=-2T\Delta\sigma\frac{d\alpha}{dT}$$

Entropy change

$$\Delta S=\frac{\Delta Q}{T}=-2(\Delta\sigma)\frac{d\alpha}{dT}$$

Ans.

Also we know

$\Delta Q=\Delta U+\Delta W$

Here

$\Delta W=-2\alpha\,\Delta\sigma$

$\Delta U=\Delta Q-\Delta W$

$$\Delta U=-2T\Delta\sigma\left(\frac{d\alpha}{dT}\right)+2\alpha\Delta\sigma$$

$$\Delta U=2\Delta\sigma\left(\alpha-T\frac{d\alpha}{dT}\right)$$

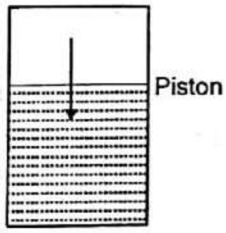
Ans.

2.6

Phase

Transformations

2.185



We know vapour will be condensed in liquid at constant temperature and pressure.
Then work done on gas
$\Delta W = \Delta A = P\Delta V$ ....(1)
Where ΔV = decrease in volume
Also
$PV = nRT$
$P\Delta V = \Delta nRT$ (Because P and T are constant)
From(1)
$\Delta W = \Delta nRT$

$$\Delta W = \frac{\Delta m}{M} RT$$

Ans.

Put values in S.I. Unit
$\Delta W = 1.2\ J$ **Ans.**

2.186*

Suppose volume of vapoar and liquid are V_v and V_ℓ
$V = V_v + V_\ell$
$m = m_v + m_\ell$
Also suppose specific volume of vapour and liquid are V_v^1 and V_ℓ^1

$$m_v = \frac{V_v}{V_v^1}$$

$$V_v + M_v V_v^1$$

$$m_\ell = \frac{V_\ell}{V_\ell^1}$$

$V_\ell = m_\ell V'_\ell$
Then

$$V = m_v V_v^1 + m_\ell V_v^1$$

$$V = m_v V_v^1 + m_\ell V'_\ell$$

$$V = m_v V_v^1 + (m - m_v) V_\ell^1$$

$$V = m_v V_v^1 + mV_\ell^1 - m_v V_\ell^1$$

$$m_v = \frac{V - mV_\ell^1}{V_v^1 - V_\ell^1}$$

Ans.

Put V = 6 lt : m = 5kg ; V_ℓ^1 = 1lit/kg ;

$V_v^1 = 50\,\ell/kg$

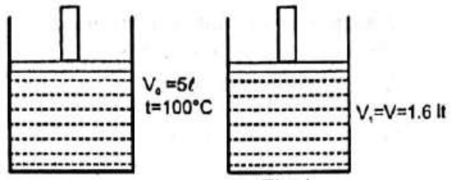
$m_v \cong 20gm.$

Ans.

Volume (V_v) = $20 \times 10^{-3} \times 50$ = 1 lt.

Ans.

2.187*



Initial Final

Since PV=Constant
Means if we compressed then pressure increased and according to phase diagram curve vapour will condensed to get saturation equalibrium.
Since temperature is 100°C and pressure will be equal to atomospheric pressure P_0.
Now volume of vapour freeze = $V_0 - V$ and equivalent mass will be equal to mass of liquid

$$P_0(V_0 - V) = \frac{\Delta m}{M}RT$$

$$\Delta m = \frac{P_0(V_0 - V)M}{RT}$$

Put $P_0 = 1.1 \times 10^5$ n/m²
T = 373° K
R = 8.3
$V_0 = 5 \times 10^{-3}$ m³
$V = 1.6 \times 10^{-3}$ m³
Δm = 20 gm

Ans.

2.188*

Given $\frac{V_V^1}{V_\ell} = N$

Suppose V = initial volume of vapour then initial

$$\text{Mass (m)} = \frac{V}{V_V^1}$$

Assume mass of liquid and vapour is m_l and m_y
Then

$$\frac{V}{V_V^1} = m_\ell + m_V$$

$$V = m_\ell N V_\ell^1 + m_V N V_\ell^1 \text{ .... (i)}$$

Also

$$\frac{V}{\eta} = V_\ell + V_V$$

$$\frac{V}{\eta} = m_\ell V_\ell^1 + m_V V_V^1$$

$$\frac{V}{\eta} = m_\ell V_\ell^1 + m_V N V_\ell^1$$

$$\frac{V}{\eta} = m_\ell V_\ell^1 + N m_V V_\ell^1 \text{ ..... (ii)}$$

Now(i) − ii)

$$V\left(1 - \frac{1}{\eta}\right) = m_\ell V_\ell^1 (N-1)$$

$$\frac{m_\ell V_\ell^1}{V/\eta} = \frac{n-1}{N-1}$$

$$\frac{V_\ell}{V/\eta} = \frac{n-1}{N-1} = \eta$$

Ans.

Here final volume

$$\frac{1}{\eta} = \frac{1+N}{2}$$

$$\eta = \frac{1}{N+1}$$

Ans.

2.189*

We know $\Delta Q = \Delta U + \Delta W$
At at atomospheric tempreture and pressure
$\Delta Q = mq$
Where q = latent heat of vapourisation
$\Delta W = P(\Delta V) = \Delta n\, RT$

$$\Delta W = \frac{m}{M}RT$$

$$\Delta U = mq - \frac{m}{M}RT$$

$$\Delta U = m\left(q - \frac{RT}{M}\right)$$

Ans.

$$\Delta S = \int \frac{dQ}{T}$$

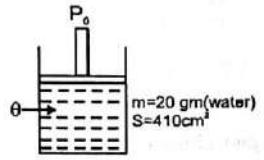
Since T = Constant

$$\Delta S = \frac{\Delta Q}{T}$$

$$\Delta S = \frac{mq}{T}$$

Ans.

2.190*



Heat required to increase temp by $\Delta T = 100°$ C, up to boiling point

$\theta_1 = m\,C\,\Delta T$

Where C = specific heat of water

Remaining heat

$\Delta\theta = \theta - m\,C\,\Delta T$

Also we know

$\Delta\theta = \Delta U + \Delta W$

Here

$\Delta W = P_0\,(Sh)$

Where h = height rise by pistion

$\Delta U = (\Delta m)q$

(increase internal energy of gas phase)

$$\Delta U = q\left[\frac{\Delta V\,M}{RT}\right]$$

$$\Delta U = q\frac{P_0 Sh\,M}{RT}$$

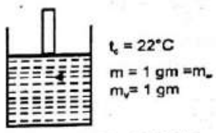
Then

$$\theta - mC\,\Delta T = P_0\,SH\left(1+\frac{qM}{RT}\right)$$

$$h = \frac{\theta - mC\,\Delta T}{P_0 S\left(1+\frac{qM}{RT}\right)}$$

Ans.

2.191*



Temp of vapour is T = 373° K

Heat required to increase temp t_0 to T

$\Delta\theta = m\,C\,(T - t_0)$

This heat will be transfer from gas to liquid

$\Delta\theta = m_v\,q$

Then

$$m_v = \frac{mC}{q}(T - t_0) = \text{mass of vapour}$$

Work done $P\,\Delta V = \Delta n\,RT$

$$\Delta w = \frac{m_v}{M}RT = \text{work done on gas}$$

$$\Delta w = \frac{mC}{q}\frac{(T - t_0)RT}{M}$$ **Ans.**

2.192*

We know

$$\Delta P = \left(\frac{\rho_v}{\rho_\ell}\right)\left(\frac{2\alpha}{r}\right)$$

$$\Delta P = \frac{\rho_v}{\rho_\ell}\left(\frac{4\alpha}{d}\right) = \eta P_{vapour}$$

$$\Delta P = \eta\left(\frac{\frac{m}{M}RT}{V_{vapour}}\right)$$

$$\Delta P = \frac{\eta\,R\,T}{M}\rho_v$$

$$d = \frac{4\alpha M}{\rho_\ell RT\eta}$$

Put values

$d \equiv 0.2\ \mu m$

Ans.

2.193*

No of molecules condense unit are a in unit second

$$\frac{dN}{dt} = \left(\frac{1}{4}n < v >\right)\eta$$

$$\frac{dN}{dt} = \frac{1}{4}n\eta\sqrt{\frac{8KT}{\pi m}}$$

Mass condense unit area in unit second

$$\mu = m\left[n\,\eta\sqrt{\frac{KT}{2\pi m}}\right]$$

m = mass of one molecule

$$\mu = P_0\,\eta\sqrt{\frac{M}{2\pi RT}}$$

Ans.

2.194*

From Q. 2193

$$m = P_0\sqrt{\frac{M}{2\pi RT}}$$

Here η =1 be cause other part is vacuum

Then

$$P_0 = \mu\sqrt{\frac{2\pi RT}{M}}$$

Ans.

Put values P_0 = 0.9 n pa

Ans.

2.195*

We know vander wall's equation

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb) = nRT$$

Assume n = 1

$$P=\frac{RT}{V-b}-\frac{a}{V^2}$$

Here a arised due to molecules attractions. Then change in pressure due to molecules attractions vanish.

$$\Delta P=\frac{a}{V^2}$$

Ans.

2.196*

We know internal pressure

$$P_i=\frac{a}{V^2} \text{ (For one mole liquid)}$$

Work done by internal pressure :

$$\int dw=\int_{V_\ell}^{V_v} P_i dV=\int_{V_\ell}^{V_v}\frac{a}{V^2}dV$$

$$\Delta w=\frac{a}{V_\ell}-\frac{a}{V_v}$$

But we know

$V_v >> V_\ell$

$$\Delta w=\frac{a}{V_\ell}=qM$$

(M - mass of 1 mole of liquid)

$$P_i=\frac{a}{V_\ell^2}=\frac{qM}{V_\ell}$$

$P_i = eq$

Ans.

2.197*

We know for one moles gas

$$P=\frac{RT}{V-b}-\frac{a}{V^2} \text{ — (ii)}$$

Also we know in P – V diagram critial point act as inflection point .

Then

$$\left(\frac{\partial P}{\partial V}\right)_T=0$$

as wall as

$$\left(\frac{\partial^2 P}{\partial V^2}\right)=0$$

Then

$$0=-\frac{RT}{(V-b)^2}+\frac{2a}{V^3}$$

$$\frac{RT}{(V-b)^2}=\frac{2a}{V^3} \text{ ....(i)}$$

Again differential w.r.t. V

$$\frac{2RT}{(V-b)^3}-\frac{6a}{V^4}=0$$

$$\frac{RT}{(V-b)^3}=\frac{3a}{V^4} \text{ .... (ii)}$$

Solving (i) and (ii)

$V = 3b = V_{MCr}$

Put V in(ii)

$$T_{cr}=\frac{8a}{27Rb}$$

Put in (iii)

$$P_{cr}=\frac{a}{27b^2}$$

Ans.

Also

$$\frac{P_{Cr}V_{Mcr}}{T_{Cr}}=3/8$$

Ans.

2.198*

We know

$$P_{cr}=\frac{a}{27b^2} \text{ — (i)}$$

$$T_{cr}=\frac{8a}{27bR} \text{ — (ii)}$$

Dividing (i) /(ii)

$$b=R\frac{T_{cr}}{8P_{cr}}$$

$$b=\frac{0.082\times 304}{73\times 8}$$

b = 0.043 lit/ mole

Put value of b in (ii)

$$T_{cr}=\frac{8a}{27R^2T_{cr}}\times 8P_{cr}$$

$$a=\frac{27}{64}\frac{R^2T_{cr}^2}{P_{cr}}$$

$$a=\frac{27}{64}\frac{R^2T_{cr}^2}{P_{cr}}$$

Ans.

2.199*

Specific volume

$$V_{cr}^{1} = \frac{V_{cr}}{M} = \frac{V_{cr}}{M} \text{ ---- (i)}$$

Also we know

$$P_{cr} V_{cr} = \frac{3}{8} R T_{cr}$$

$$V_{cr} = \frac{3RT_{cr}}{8 P_{cr}}$$

Put in (i)

$$V_{cr}^{1} = \frac{3RT_{cr}}{8MP_{cr}}$$

Ans.

2.7

TRANSPORT

PHENOMENA

2.200*

We know

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT \text{ — (i)}$$

$$P_{cr}V_{cr}=\tfrac{3}{8}T_{cr} \text{ — (ii)}$$

dividing (i) and (ii)

$$\left(\frac{P}{P_{cr}}+\frac{a}{V^2P_{cr}}\right)\left(\frac{V}{V_{cr}}+\frac{b}{V_{cr}}\right)=\frac{8}{3}\left(\frac{T}{T_{cr}}\right)$$

Here

$$\pi=P/P_{cr};\nu=\frac{V}{V_{cr}};\tau=\frac{T}{T_{cr}}$$

$$\left(\pi+\frac{27b^2}{9V^2}\right)\left(\nu-\frac{1}{30}\right)=\frac{8}{3}$$

Also

$b=V_{cr}/3$

$$\left(\pi+\frac{27}{g}\frac{V_{cr}^2}{V^2}\right)\left(\nu-\frac{1}{3}\right)=\tfrac{8}{3}\tau$$

$$\left(\pi+\frac{3}{\nu^2}\right)(\nu-\tfrac{1}{3})=\tfrac{8}{3}\tau$$

$$\left(\pi+\tfrac{3}{\nu^2}\right)(3\nu-1)=8\tau$$

Put $\pi=12$; $\nu=\tfrac{1}{2}$

$$\tau=\tfrac{8}{3}$$

Ans.

2.201*

(a)

We know at critical tempreture

$\rho_l=\rho_v$

After that only gas phase exist.

Then maxmimum volume of liquid phase is

$V_{Mcr}=3\,b\times n$

Where b = volume of 1 mole of liquid = 0.03 lt

$$n\,b=\frac{1000}{18}\times 0.03\text{ Lt.}$$

Here 0.03 lt is volume of one moles gas

$$V_{Mcr}=3\left[\frac{1000}{18}\times 0.03\right]\Rightarrow V_{mcr}=5\text{lit}$$

Ans.

Also P_{cr} is maximum vapour pressure in saturated water and vapour equilibrium

$$P_{cr}=\frac{a}{27b^2}$$

For n mole gas

$$P_{cr}=\frac{na}{27nb^2}$$

$$P_{cr}=\frac{5.47}{27\times 0.032}=225\text{ atom}$$

Ans.

2.202*

We know

$$T_{cr}=\frac{8a}{27Rb}=\frac{8\times 3.62}{27\times 0.082\times 0.043}\cong 304\text{K}$$

Also

$$\rho_{cr}V_{cr}=\frac{3}{8}R\,T_{cr}$$

$$\rho_{cr}=\frac{3}{8}\left(\frac{P_{cr}}{M}\right)RT_{cr}$$

$$\rho_{cr}=\frac{8}{3}\frac{P_{cr}M}{RT_{cr}}$$

$$\rho_{cr}=\frac{M}{R}\left(\frac{a}{27b^2}\times\frac{27Rb}{8a}\right)\times\frac{8}{3}=\frac{M}{8b}\times\frac{8}{3}$$

$$P_{cr}=\frac{M}{3b}=\frac{44}{3\times 43}=0.34\text{ gm/cc}$$

Ans.

2.203*

We know

$$P_{cr}V_{cr}=\frac{3}{8}R\,T_{cr}\text{ (for one mole gas)}$$

For n mole gas

$$P_{cr}V_{cr}=\frac{3}{8}\left(\frac{m}{M}\right)RT_{cr}$$

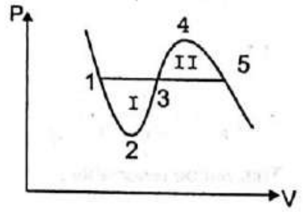
$$P_{cr}V_{cr}=\frac{3}{8}\left(\frac{V\rho}{M}\right)R\,T_{cr}$$

$$\eta=\frac{V}{V_{cr}}=\frac{8}{3}\frac{P_{cr}M}{R\,T_{cr}\rho}$$

Where V_{cr} is maximum volume of vapour at liquid vapour equilibrium.

Ans.

2.204*



Here 1–3–5 is a phase transition line at which pressure and temp are same while curve are isothermal.

For reversible process

$$T\oint ds = \oint du + \oint p\,dV$$

$$\oint ds = 0$$

Because P and T are same on line 1–3–5

$$\oint du = 0$$

Because curve is isothermal.

Hence

$$\oint P\,dV = 0$$

But in cycles

$$\oint P\,dV \neq 0$$

Which tell us this is irriversible.

Same 3–4–5 are irrieversible

But cycle 1 – 2 – 3 – 4 – 5

$$\oint P\,dV = 0$$

Which tell us at point 3 phase change from I and II phase

Ans.

2.205*

Fraction of water turn into ice

$$f = \frac{C\Delta t}{q}$$

Where

Δt = temp of liquid

C = specific heat of water

q = latent heat of fusion

$$f = 1 \times 20/80$$

$$f = 0.25$$

For

$$f = 1$$

$$1 = \frac{1 \times \Delta t}{80}$$

$$\Delta t = -80°C$$

Ans.

2.206*

We know clausius - clapeyron equation

$$\frac{dp}{dT} = \frac{q_{12}}{T\left(V_2^1 - V_1^1\right)}$$

$$\int dt = \frac{T\left(V_2^1 - V_1^1\right)}{q_{12}}\int dp$$

$$\Delta T = \frac{T\left(V_2^1 - V_1^1\right)\Delta P}{q_{12}}$$

Put value in S.I. Unit

$\Delta T = -0.0075$ K

Ans.

2.207*

From Q. 12.206

$$\Delta T = \frac{T\left(V_2^1 - V_1^1\right)\Delta P}{q_{12}}\ \Delta P$$

V_2^1 = Specific volume of vapour

V_1^1 = Specific volume of liquid

We know

$$V_1^1 << V_2^1$$

$$V_2^1 = \frac{q_{12}\Delta T}{T\,\Delta P}$$

$$V_2^1 = \frac{2250}{373} \times \frac{0.9}{3.2} \times 10^{-3}$$

$$= 1.7 m^3/kg$$

Ans.

2.208*

Here we know

$$\frac{dp}{dT} = \frac{q_{12}}{T\left(V_2^1 - V_1^2\right)}$$

Also

$$V_2^1 >> V_1^1$$

$$\frac{dp}{dT} = \frac{q_{12}}{T\,V_2^1}$$

$$dp = \frac{q_{12}}{T\,V_2^1}dT$$

$$\Delta P = \frac{q_{12}}{T\,V_2^1}\ \Delta T \text{ — (i)}$$

Also

$$P_0 V_2 = \frac{m}{M}RT$$

$$V_2^1 = \frac{V_2}{m} = \frac{RT}{P_0 M}$$

Put in (i)

$$\Delta P = \frac{q_{12}\Delta T P_0 M}{TRT} = \frac{P_0 M q_{12}\Delta T}{RT^2}$$

$$P = P_0 + \Delta P = P_0 + \frac{P_0 M q_{12}\Delta T}{RT^2}$$

$$P = P_0\left(1+\frac{Mq_{12}\Delta T}{RT^2}\right)$$

Ans.

2.209*

$$\frac{dP}{dT} = \frac{Mq}{RT^2}P \text{ (from Q ; 2.208)}$$

$$\frac{dP}{P} = \frac{Mq}{RT^2}dT \text{ --- (1)}$$

Also

$$PV = \frac{m}{M}RT$$

$$\ell nP + \ell nV = \ell nm + \ell nT + \ell n\frac{R}{M}$$

$$\frac{dP}{P} = \frac{dm}{m} + \frac{dT}{T}$$

$$\frac{dm}{m} = \frac{dp}{\rho} - \frac{dT}{T}$$

$$\frac{dm}{m} = \frac{MqdT}{RT^2} - \frac{dT}{R}$$

$$\frac{dm}{m} = \left(\frac{Mq}{RT} - 1\right)\frac{dT}{T}$$

Put value in S.I. Unit

$$\frac{dm}{m} = 4.85\%$$

Ans.

2.210*

$$\frac{dP}{dT} = \frac{MqP}{RT^2} \text{ (from Q. No. 2.209)}$$

$$\int_{P_0}^{P}\frac{dP}{P} = \frac{Mq}{R}\int_{T_0}^{T}\frac{dT}{T^2}$$

$$\ell n\, {}^{P}\!/_{P_0} = -\frac{Mq}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

$$P = P_0 e^{\frac{Mq}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right)}$$

$$P = P_0 e^{\frac{Mq}{R}\left(\frac{T-T_0}{TT_0}\right)}$$

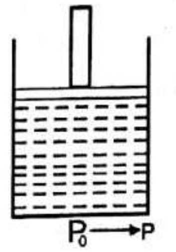
Ans.

If $T - T_0 << T_0$

$$P = P_0\left(1+\frac{Mq}{RT}\left(\frac{T-T_0}{T_0}\right)\right)$$

This will be resonable .

2.211*



Here $P \propto T$

We Know

$$\Delta P = \frac{q_{12}}{T\left(V_2^1 - V_1^1\right)}\,\Delta T$$

Then

$$\Delta T = \frac{T\Delta V^1}{q_{12}} = \text{lowering of melting point}$$

Also

$$m_1 C\,\Delta T = m_2\, q_{12}$$

$$\eta = \frac{m_2}{m_1} = \frac{C\Delta T}{q_{12}} = \frac{CT\Delta V^1}{q_{12}^2}$$

$$\eta = \frac{CT\Delta V^1}{q_{12}^2}$$

Ans.

2.212*

(a)

$$\log P = a - \frac{b}{T}$$

At triple point pressure and temperuture of solid CO_2 and vapour CO_2 will be same.

$$\log P = 9.05 - \frac{1.80\times10^3}{T} \text{ --- (i) (Sublimation)}$$

$$\log P = 6.78 - \frac{1.3\times10^3}{T} \text{ --- (ii) (Vaporisation)}$$

Solving (i) and (ii)

$T = 216$ K

$P = 5.1$ atm

(b)
Here

$$\ln P = a - \frac{b}{T}$$

$$P = e^{\left(a-\frac{b}{T}\right)} \text{ --- (iii)}$$

Compare with Q. 2.210

$$P = P_0 e^{\left[\frac{Mq}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)\right]} \text{ ---(iv)}$$

$$\frac{b}{T} = \frac{Mq}{R}$$

$$q_{sub} = \frac{Rb}{M}$$

$$= 8.31\times 2.303 \times \frac{1800}{44} = 783\ ^{j}/_{gm}$$

$$q_{liquid\ gas} = \frac{8.31\times 2.303\times 1310}{44} = 570$$

Ans.

2.213*

$$\Delta S = \int \frac{\Delta Q}{T} = \int_{t_1}^{t_2} \frac{mCdT}{T} + \frac{mq}{t_2}$$

$$\Delta S = mC\ell n\left(\frac{t_2}{t_1}\right) + \frac{mq}{t_2}$$

$$\text{Heat Given } (\Delta Q) = \int_{t_1}^{t_2} mCdT + mq$$

Ans.

2.214*

$$\Delta S = \int \frac{\Delta Q}{T} = \frac{mq_m}{T} + \int_{t_1}^{t_2} \frac{mCdT}{R} + \frac{mq_v}{T}$$

$$\Delta S = \frac{mq_m}{T} + mC\ell n\ ^{t_2}/_{t_1} + \frac{mq_v}{T}$$

Ans.

2.215*
Suppose final temp is T which is greater than 0° C
Then entropy change

$$\Delta S = \int \frac{dQ}{T} = \int_{t_1}^{T} \frac{mCdT}{T}$$

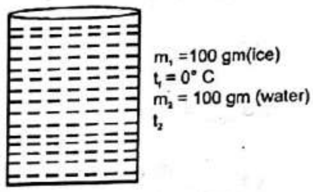
$$\Delta S = mC\ell n\ ^{T}/_{t_1}$$

Where C = specific heat of copper
Put values $\Delta S = -10$ J/K

Ans.

Calculation of T
C = 0.39 J/g K Þ Cw = 4.18 J/g K. Heat given by copper to make temp of mixture zero °C $\Rightarrow \theta$ = mc ΔT = 90×0.39 ×90. Heat taken by ice to convert into 0°C temp of liquid θ_2 , = 50×4.13×3+50×80×4.2 ; θ^1 =50 × 4.18 ×3 $\Rightarrow$ Here $\theta_2 > \theta_1$ and $\theta_1 > \theta^1$. Final temperature will be 0°C = 27.3° K.

2.216*



(a)
t_2 = 60C let us final temp is t then this
0°C < t < 60° C
Now
100×80+100×1×t=100×1×(60–t)
80= 60 –2 t
t < 0
Which tell us, some of ice is not melt. Then again assume, m mass of ice is melt
m × 30 = 100×1×60

$$m = \frac{600}{8} = \frac{300}{4} = 75 \text{ gm}$$

Finally mass of ice remain (m_1)
Mass of water (m_2) = (200 - 25) gm = 175 gm.
Now

$$\Delta S_{ice} = \int \frac{dQ}{T} = \frac{1}{t_1}\Delta Q = \frac{m_2 c_2 (t_2 - t_1)}{t_1}$$

$$\Delta S_{water} = \int \frac{dQ}{T} = \int_{t_2}^{t_1} \frac{m_2 c_2 dT}{T} = m_2 c_2 \ell n\ ^{t_1}/_{t_2}$$

$$\Delta S_{system} = m_2 c_2 \left(\frac{t_2}{t_1} - 1 + \ell n\ ^{t_1}/_{t_2}\right)$$

$$= m_2 c_2 \left(\frac{t_2}{t_1} - 1 - \ell n\ ^{t_2}/_{t_1}\right)$$

Ans.

(b)
t_2 = 94°C
Suppose all ice is melt and temp at equiiibium is T

0°C < T < 94°C
100 × 80 + 100 (1) T = 100 × 1 (t_2 –T)
80 + T = 94 – T

$-6 = -2T$
$T = 3°C$

$$\Delta S_{ice} = \int \frac{dQ}{T} = \frac{m_1 q}{t_1} + \int_{t_1}^{T} m_1 c_2 dt$$

$$\Delta S_{ice} = \frac{m_1 q}{t_1} + m_1 c_2 \ell n T / t_1$$

$$\Delta S_{water} = \int \frac{dQ}{T} = \int_{t_2}^{T} \frac{m_1 c_2 dT}{T}$$

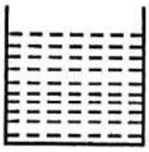
$$\Delta S_{water} = m_2 c_2 \ \ell n \ T/t_2$$

$$\Delta S_{system} = \frac{m_1 q}{t_1} + m_1 c_2 \ \ell n \frac{T}{t_1} + m_2 c_2 \ \ell n \frac{T}{t_2}$$

$$\Delta S_{system} = \frac{m_1 q}{t_1} + c_2 \left(m_1 \ell n \frac{T}{t_1} - m_2 \ell n \frac{t_2}{T} \right)$$

Ans.

2.217*



$$\Delta S_{lead} = \int \frac{dQ}{T} = \int_{t_2}^{t_1} \frac{mCdT}{T} - \frac{1}{t_2} \int dQ$$

Here t_2 = melting temp of lead

$$\Delta S_{lead} = mc \ \ell n(t_1/t_2) - \frac{1}{t_2} mq \text{ ---- (i)}$$

Since T is constant because amount of wate is very high

$$\Delta S_{lead} = \int \frac{dQ}{T} = \frac{1}{t_1} \int dQ$$

$$= \left(\frac{1}{t_1} \int_{t_1}^{t_2} mc\,dT + \frac{mq}{t_1} \right) = \frac{1}{t_1}(mc)(t_2 - t_1) + \frac{mq}{t_1}$$

$$\Delta S_{ystem} = mc\ell n \, {}^{t_1}\!/_{t_2} + mq\left(\frac{1}{t_1} - \frac{1}{t_2} \right) + mc\left(\frac{t_2}{t_1} - 1 \right)$$

$$\Delta S_{ystem} = mq\left(\frac{1}{t_1} - \frac{1}{t_2} \right) + mc\left(\frac{t_2}{t_1} - 1 - \ell n \, {}^{t_2}\!/_{t_1} \right)$$

Ans.

2.218*

We know

$$\frac{dp}{dT} = \frac{q}{TV^1}$$

Where V^1 = specific volume of gas = volume / kg
Also
$PV = RT$
Differentiate
$P\,dV + V\,dP = R\,dT$
Again
$dQ = dU + PdV$
$CdT = C_V dT + (R\,dT - Vd\,P)$

$$C = (C_V + R) - V \frac{dp}{dT}$$

$$C = C_P - V\left(\frac{q}{TV^1} \right)$$

$$C = C_P - m\frac{q}{T}$$

Where m = mass of vapour since amount of vapour taken is 1 mole
$m = M$

$$C = C_P - \frac{Mq}{T}$$

Ans.

2.219*

Here at T_2 tempretor, all water is in saturated vapour from which is boiling temp of vapour. if you decrease temp. from T_2, vapour will be converted into liquid
Then

$$\Delta S = \int \frac{dQ}{T} = \frac{\Delta Q}{T_2} + \int \frac{dQ_1}{T}$$

$$\Delta S = \frac{mq}{T_2} + \nu C_P \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\Delta S = \frac{mq}{T_2} + \nu C_P \ell n\left(\frac{T_2}{T_1} \right)$$

Here $\nu = 1$
$m = M$

$$\Delta S = \frac{mq}{T_2} + \nu C_P \ell n\left({}^{T_2}\!/_{T_1} \right)$$

$$\Delta S = \frac{mq}{T_2} + C_P \ell n\left({}^{T_2}\!/_{T_1} \right)$$

Ans.

2.7
Transport Phenomena

2.220*

(a)

We know fraction of molecules traversing distance S without collisions

$$\frac{N}{N_0} = e^{-S/\lambda}$$

Now we want to find sum of fraction of molecules having

$$\lambda \le S < \infty \Rightarrow 1 \le \frac{S}{\lambda} < \infty$$

Assume

$$x = \frac{S}{\lambda}$$

$$1 \le x < \infty$$

$$\frac{N}{N_0} = e^{-x}$$

Now

$$\sum_{x=1}^{\infty} \frac{N}{N_0} = \sum_{x=1}^{\infty} e^{-x}$$

$$\sum_{x=1}^{\infty} \frac{N}{N_0} = \int_1^{\infty} e^{x} dx \Rightarrow \sum_{x=1}^{\infty} N/N_0 = \left.\frac{e^{-x}}{-1}\right|_1^{\infty} = \frac{1}{e} \cong 0.37$$

Ans.

(b)

Similar

$$\sum_{x=1}^{2} \frac{N}{N_0} = \int_1^2 e^{-x} dx = \left(\frac{1}{e} - \frac{1}{e^{-2}}\right) \cong 0.23$$

Ans.

2.221*

Suppose intensity of molecules is N_0 at $S = 0$ while N at $S = \Delta\ell$ then

$$\frac{N}{N_0} = \frac{1}{\eta} = e^{-\Delta/\lambda}$$

$$\ell n\ \eta = \frac{\Delta\ell}{\lambda} \Rightarrow \lambda = \frac{\Delta\ell}{\ell n \eta}$$

Ans.

2.222*

Consider a molecule having velocity v

P(t) = Probability of molecule experiencing no collision in time interval (0,t)

α dt = Probaility of molecule suffers a collision between time t and t+ dt.

Here α is function of speed (v)

$1 - \alpha$ dt = Probability of a molecule suffer no collision between time t and t + dt.

Then

$$P(t + dt) = P(t)(1 - \alpha\, dt)$$

$$p(t) + \left(\frac{dP}{dt}\right) dt = P(t) - \alpha P(t)dt$$

$$\frac{dP}{\alpha P(t)} = -dT$$

$$\int \frac{dp}{P(t)} = -\int \alpha\, dt$$

$$\ell n\, P(t) = -\alpha t + C$$

$$p(t) = e^{-\alpha t + C}$$

At

$t \to 0$

$P(0) \to 1$

Because when time interval is very small, probability of no collision will be approach 1.

$$1 = e^C$$

$$C = 0$$

$$P(t) = e^{-\alpha t}$$

Ans.

(b)

Collision time or mean time or Relaxation teme (τ)

Probability that a gas molecenle survive for time t and suffer a collision in next dt time

$f(t) = P(t)(\alpha)(dt)$

$f(t) = \alpha\, e^{-t\alpha} dt$

$$\text{Mean time } (\tau) = \frac{\int_0^{\infty} f(t)t}{\int_0^{\infty} f(t)} = \frac{\int_0^{\infty} \alpha e^{-\alpha t} t\, dt}{\int_0^{\infty} \alpha e^{-\alpha t}\, dt} \Rightarrow \tau = \frac{1}{\alpha}$$

Ans.

2.223

(a)

Mean free path of a gas molecule

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Also we know

$P = nKT$

Then

$$\lambda = \frac{KT}{\sqrt{2}\pi d^2 P}$$

Put value

$$\lambda = \frac{1.38\times10^{-23}\times273}{\sqrt{2}\pi\left(0.37\times10^{-9}\right)^2\times10^5} = 6.2\times10^{-8}\text{m}$$

Also we know

Mean time $\left(\overline{\tau}\right) = \frac{\lambda}{|\overline{v}|}$

There

$\overline{v}$ = Average speed = $\sqrt{\frac{8KT}{\pi m}}$

$$\overline{\tau} = \frac{6.2\times10^{-8}}{\sqrt{\frac{8\times1.3\times10^{-23}\times273}{\pi\times28\times10^{-3}}}} = 0.13\text{ns}$$

Ans.

(b)

Similar like part (a)

$$\lambda = \frac{1.33\times10^{-23}\times273}{\sqrt{2}\pi\left(0.37\times10^{-9}\right)^2\times10^{-9}} = 6.2\times10^{6}\text{ m}$$

$$\overline{\tau} = \frac{6.2\times10^{6}}{\sqrt{\frac{8\times1.3\times10^{-23}\times2\times3}{\pi\times28\times10^{-3}}}} = 3.8\text{ hours}$$

Ans.

2.224*

We knwo at STP volume of 1 mole gas is 22.4 lt.

Volume occupied by one molecule is

$V_1\times6.023\times10^{23} = 22.4\times10^{-6}\text{ m}^3$

$$V_1 = \frac{22.4\times10^{-6}}{6.02\times10^{23}}$$

$$\frac{4}{3}\pi R^3 = \frac{22.4\times10^{-6}}{6.02\times10^{23}}$$

$R^3 \cong 10^{-29} \Rightarrow$

$R = \sqrt[3]{10}\times10^{-10}$

Distance b/w two molecules = $2R = 2\sqrt[3]{10}\times10^{-3}$

$\lambda = 6.2\times10^{-8}\text{m}$

$$\frac{\lambda}{2R} = \frac{10^{-8}}{10^{-10}} \cong 100\text{time}$$

Ans.

2.225*

We know vander wall constant b is four time of actual volume of gas molecule

$$b = 4N_A\frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$d^3 = \frac{3b}{4N_A\pi} \Rightarrow d = \left(\frac{3b}{4\pi N_A}\right)^{1/3}$$

And

$$\lambda = \frac{1}{\sqrt{2}\pi d^2\eta} = \frac{KT}{\sqrt{2}\pi d^2P}$$

$$\lambda = \frac{KT}{\sqrt{2}\pi P\left[\frac{3b}{4\pi N_A}\right]^{2/3}}$$

$$\lambda = \left(\frac{2\pi N_A}{32b}\right)^{23}\left(\frac{KT_0}{\sqrt{2}\pi P_0}\right)$$

Ans.

2.226*

We know acoustic wave speed

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Then

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{1}{f}\sqrt{\frac{\gamma RT}{M}}$$

$$f = \frac{1}{\lambda}\sqrt{\frac{\gamma RT}{M}}$$

Where

$$\lambda = \frac{1}{\sqrt{2}\pi d^2n}$$

$$f = \sqrt{2}\pi d^2n\sqrt{\frac{\gamma RT}{M}} = \frac{\sqrt{2}\pi\delta^2P_0N_A}{RT}\sqrt{\frac{\gamma RT}{M}}$$

$$f = \pi d^2P_0N_A\sqrt{\frac{2\gamma}{MRT_0}}$$

Ans.

2.227*

(a)

We know

$$\lambda = \frac{KT}{\sqrt{2}\pi d^2P} > \ell \Rightarrow P < \frac{KT}{\sqrt{2}\pi d^2\ell}$$

Put value P < 0.7 Pa

Ans.

(b)

P = n KT

$n = \frac{P}{KT}$

if P < 0.7 Pa

$n < \frac{0.7 \times 1.1 \times 10^5}{1.38 \times 10^{-23} \times 273} / m^3 \Rightarrow n < 2 \times 10^{14} / cm^3$

Ans.

And volume will be assume for one molecules is

$V_1 = \ell_0^3$

Then

$2 \times 10^{14} \times V_1 = 10^{-6}$

$2 \times 10^{14} \times \ell_0^3 = 10^{-6}$

$\ell_0 = 0.2\ \mu m$

Ans.

2.228*

(a)

We know

$\because \tau$ second, no of collision is 1

$\therefore$ 1 second no of collision is $f = \frac{1}{\tau}$

Also

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

$\tau = \frac{\lambda}{<v>} = \frac{1}{\sqrt{2}\pi d^2 n <v>}$

Then

$f = \sqrt{2}\pi d^2 n <v>$

Put values

$f = 0.74 \times 10^{10} / s$

Ans.

(b)

Suppose there are n molecules present then total no of collision persecond (N) = $\frac{1}{2} f(n)$

$N = \frac{1}{2}\sqrt{2}\,\pi d^2 n^2 <v>$

$\frac{1}{2}$ is taken because in one collision, two molecules are participated.

Ans.

2.229*

We know

d = effective diameter of molecule = const.

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

(a)

Isochoric process

$V = n\,[\ell^3]$

For isochoric process

$n = \frac{\text{Total no. of molecules}}{\text{Total Volume}} = \text{const}$

Since V and n are constant

ℓ will constant hence d = const

Then

λ= const

No of collision pur unit time (v)

$v = \frac{<v>}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{8RT}{M}}$

$v \propto \sqrt{T}$

Ans.

(b)

Isobaric process

$V = \frac{NRT}{P}$

$P = nKT$

Since

d = const

$\lambda = \frac{1}{\sqrt{2}\pi d^2}\frac{KT}{P}$

$\lambda \propto T$

$v = \frac{1}{\lambda}\sqrt{\frac{8RT}{M}} = \frac{1}{CT}\sqrt{\frac{8RT}{M}}$

$v \propto \frac{1}{\sqrt{T}}$

Ans.

2.231

(a)

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

d= const

$n = \frac{N}{V}$

Where N = Total no. of molecule

V = Volume of gas.

Hence $\lambda \propto V$

Ans.

Frequency (v) = $\frac{<v>}{\lambda} = \frac{1}{CV}\sqrt{\frac{8RT}{\pi M}}$

Where C = const

For adiabatic process

$TV^{\gamma-1} = const$

$$\nu = \frac{1}{CV}\sqrt{\frac{8RT}{\pi M}\frac{C_1}{V^{\gamma-1}}}$$

$$\nu \propto \frac{1}{V.V^{\gamma-1/2}} = \frac{1}{V^{\left(\frac{\gamma+1}{2}\right)}}$$

$$\nu \propto V^{-\left(\frac{\lambda-1}{2}\right)} = V^{-6/5}$$ **Ans.**

$\nu = K V^{-6/5}$ ---- (i)

Where K = const

(b)

$PV^{7/5} = K_1$

$V = (K_1P)^{-5/7}$

Put in (i)

$\nu = K (K_1P)^{6/7}$

$\nu \ \alpha \ P^{6/7}$

Ans.

(c)

Similiar

$\lambda \ \alpha \ T^{-5/2}$

$\lambda \ \alpha \ T^3$

Ans.

2.232*

Similar Q: 2.231

(a)

$\lambda \ \alpha \ V$

$\nu \ \alpha \ V^{-(n+1)/2}$

Ans.

(b)

$\lambda \ \alpha \ P^{-1/n}$

$\nu \ \alpha \ P^{(n+1)/n}$

Ans.

(c)

$\lambda \ \alpha \ T^{\ 1/1-n}$

$\nu \ \alpha \ T^{(n+1)\ 2(n-1)}$

Ans.

2.233*

(a)

$$\frac{\text{No of collision}}{time} = f = \frac{<v>}{\lambda}$$

$$f = \sqrt{2}\pi d^2 n\sqrt{\frac{8RT}{\pi M}}$$

$$f = \sqrt{2}\pi d^2 n\frac{N}{V}\sqrt{\frac{8RT}{\pi M}}$$

$$f = \sqrt{2}\pi\frac{d^2N}{V}\sqrt{\frac{8RT}{\pi M}\times\frac{PV}{NR}} = CP^{1/2}V^{-1/2}$$

Where C = const

No. of frequency per unit volume

$$f' = \frac{f}{V} = C\,P^{1/2}\ V^{1/2} = \text{const}$$

$PV^{-3} = \text{const.}$

Compare

$PV^x = \text{const}$

$x = -3$

$$C = \frac{R}{\gamma-1} - \frac{R}{x-1} = \frac{R}{\gamma-1} + \frac{R}{4}$$

$$C = R\left[\frac{4+\gamma-1}{4(\gamma-1)}\right] = \frac{R}{4}\left(\frac{3+\gamma}{\gamma-1}\right)$$

Also we know

$$\gamma = 1 + \frac{2}{i}$$

$$C = \frac{R}{4}\left(\frac{3+1+2/i}{1+2/i-1}\right) = \frac{R}{4}\left(\frac{4i+2}{2}\right)$$

$$C = \frac{R}{4}(2i+1)$$

Ans.

(b)

f= const

$PV^{-1} = \text{const}$

$x = -1$

Similiar as (a)

$$C = \frac{1}{2}R(i+2)$$

Ans.

2.234*

No of particle approach per second toward wall

$$\nu = \frac{1}{4}n<v>$$

No. of particles leaks out from hole in dt time :

$$dN = \frac{1}{4}\ n<v>sdt$$

Then decrease in concentration

$$-dn = \frac{dN}{V} = \frac{1}{4}<v>sdt$$

$$-\int_{n_0}^{n}\frac{dn}{n} = \frac{1}{4}\frac{<v>S}{V}\int_0^t dt$$

$$\ell n\frac{n}{n_0} = -\frac{1}{4}\frac{<v>S}{V}t$$

$$n = n_0\ e^{-\frac{<v>St}{4V}} = n_0\ e^{-t/\tau}$$

$$n = n_0 e^{-t/\tau}$$ **Ans.**

Where

$$\tau = \frac{4V}{S < V >}$$

$$< V > = \sqrt{\frac{8RT}{\pi M}}$$

Ans.

2.235*

(1)	(2)
T_0	ηT_0
P_0, V	P_0, V

$$\nu_2 = \frac{P_0 V}{nRT_0}$$

$$\nu_1 = \frac{P_0 V}{RT_0}$$

Also

$n_1 < V_1 > = n_2 < V_2 >$

$$\nu_1^1 \sqrt{T_0} = \nu_2^1 \sqrt{n T_0}$$

$$\nu_1^1 = \nu_2^1 \sqrt{n} \text{ ---- (i)}$$

Also

$$\nu_1^1 + \nu_2^1 = \nu_1 + \nu_2 = \frac{P_0 V}{RT_0}\left(\frac{1+\eta}{\eta}\right) \text{ — (ii)}$$

From (i) and (ii) :

$$\nu_2^1 = \frac{P_0 V}{nRT_0}\left(\frac{1+n}{1+\sqrt{n}}\right)$$

$$\frac{\nu_2^1}{\nu_2} = \frac{1+n}{1+\sqrt{n}}$$

Ans.

2.236

$$D = \frac{1}{3} < V > \lambda = \frac{1}{3}\sqrt{\frac{8KT}{\pi m}} \times \frac{1}{\sqrt{2}\pi d^2 n}$$

$$D = \frac{1}{3}\sqrt{\frac{8KT}{\pi m}} \times \frac{KT}{\sqrt{2}\pi d^2 P} = C\frac{T^{3/2}}{P}$$

$$T = \left(\frac{PD}{C}\right)^{2/3} \text{ —— (I)}$$

Now

$$n = \frac{1}{3} < v > \lambda P$$

$$n = \frac{1}{3}\sqrt{\frac{8KT}{\pi m}} \times \frac{1}{\sqrt{2}\pi d^2 P} \times \frac{KT}{P} \times \frac{PM}{RT}$$

$$n = C_1 T^{1/2}$$

$$T = \left(\frac{n}{C_1}\right)^2 \text{ ----- (II)}$$

From (I) and (II)

$$\left(\frac{n}{C_1}\right)^2 = \left(\frac{PD}{C}\right)^{2/3}$$

$$\left(\frac{n_f}{n_i}\right)^2 = \left(\frac{P_f}{P_i} \times \frac{D_f}{D_i}\right)^{2/3}$$

$$\alpha^2 = \left(\frac{P_f}{P_i}\right)^{2/3} \times \beta^{2/3}$$

$$\frac{\alpha^3}{\beta} = \frac{P_f}{P_i}$$

Ans.

2.237

Form Q :2.236

$$D = \frac{CT^{3/2}}{P}$$

$$n = C_1 T^{1/2}$$

(a)

Isothermal process

n = constant

Ans.

$$D = \frac{CT^{3/2}}{\gamma RT} V$$

$D \alpha n$

Ans.

(b)

Isobaric process :

$$D = C\,\frac{(PV/\nu R)^{3/2}}{P}$$

$D \propto V^{3/2}$

D increase $n^{3/2}$ times

Ans.

$$n = C_1\left[\frac{PV}{\nu R}\right]^{1/2}$$

$n \propto V^{1/2}$

n increase $n^{1/2}$ times

Ans.

2.238*

We know from Q: 2.236

$$D=\frac{CT^{3/2}}{P}=\frac{CT^{3/2}V}{NRT}=\frac{C}{NR}T^{1/2}V$$

For adiabatic process

$TV^{\gamma-1}$ =constant=C_1

$$D=C_2\frac{1}{V^{\gamma-1/2}}V=C_2V^{1-\left(\frac{\gamma-1}{2}\right)}=C_2V^{\frac{3-\gamma}{2}}$$

For diaatomic gas γ - 7/5

$$D=C_2\,V^{\frac{3-7/5}{2}}=C_2V^{4/5}$$

If V decrease n times then D will be decreased by $n^{4/5}$

Ans.

Also we know

$$\eta=C_3\,T^{1/2}=C_3\left[\frac{C_1}{V^{\gamma-1}}\right]^{1/2}\Rightarrow\eta=C_3\,C_1^{1/2}V^{\frac{1-\gamma}{2}}$$

$$\eta=C_3\,C_1^{1/2}\,V^{-1/5}$$

If V decrease n times then η will be increase $n^{1/5}$

Ans.

2.239*

From Q : 2.239

(a)

$$D=\frac{CT^{3/2}}{P}=\frac{C}{P}\left(\frac{PV}{\nu R}\right)^{3/2}$$

$P^{1/2}\,V^{3/2}$ = const

PV^3 = const

n = 3

Ans.

(b)

$$x=C_1\,T^{1/2}=C_1\left(\frac{PV}{\nu R}\right)^{3/2}$$

PV = const

n = 1

Ans.

(c)

$$x=\frac{\lambda}{3}<V>\rho C_V$$

$x=\eta\,C_V$

If x = const

η = const

Same as part (b)

η = 1

Ans.

2.240

We know

$$\eta=\frac{1}{3}<V>\lambda\rho$$

$$\eta=\frac{1}{3}\sqrt{\frac{8KT}{m}}\frac{\rho}{\sqrt{2\pi^2}d^2n}=\frac{2}{3}\sqrt{\frac{KT}{m}}\frac{\rho}{\pi d^2n}$$

$$\eta=\frac{2}{3}\sqrt{\frac{KT}{m}}\frac{PMkT}{RT\pi d^2\,P}$$

$$\eta=\frac{2}{3}\sqrt{\frac{KT}{m}}\frac{MK}{R\pi d^2}$$

$$d=\left(\frac{2}{3}\sqrt{\frac{KT}{m}}\frac{MK}{R\pi}\right)^{1/2}$$

Ans.

Put values

d = 0.18 nm

2.241*

Assume C = 8.7

$$\chi_{He}=\frac{1}{3}<V>\lambda\ \rho\ C_V=C\chi_{Ar}$$

$$\frac{1}{3}\sqrt{\frac{8RT}{\pi M_{He}}}\frac{\rho_{He}}{\sqrt{2}\pi d_{He}^2n_{He}}C_V^{He}=C\frac{1}{3}\sqrt{\frac{8RT}{\pi M_{Ar}}}\frac{\rho_{Ar}}{\sqrt{2}\pi d_{Ar}^2n_{Ar}}C_V^{Ar}$$

$$\sqrt{\frac{M_{Ar}}{M_{He}}}\left(\frac{\rho_{He}}{\rho_{Ar}}\right)\left(\frac{n_{Ar}}{n_{He}}\right)\left(\frac{C_V^{He}}{C_V^{Ar}}\right)=\frac{d_{He}^2}{d_{Ar}^2}$$

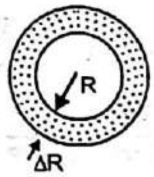
$$\frac{d_{He}}{d_{Ar}}=\left[\left(\sqrt{\frac{M_{Ar}}{M_{He}}}\right)\left(\frac{\rho_{He}}{\rho_{Ar}}\right)\left(\frac{n_{Ar}}{n_{He}}\right)\left(\frac{C_V^{He}}{C_V^{Ar}}\right)\right]^{1/2}$$

Put values

$$\frac{d_{Ar}}{d_{He}}=1.7$$

Ans.

2.242*



We know viscous force :

$$F\cong\eta A\frac{dV}{dz}$$

$$\frac{F}{\ell}=\eta(2\pi R\times1)\left(\frac{RW}{\Delta R}\right)$$

Moment of force per unit lemgth $\left(\frac{F}{\ell}\right)$:

$$N_A \cong RF = \frac{2\pi\eta W R^3}{\Delta R}$$

Ans.

Also we know viscous foce per unit area :

$$\frac{F}{A} = \frac{1}{6}\ \rho < V > (u_1 - u_2)$$

$$F = \frac{1}{6}\ \rho < V > RW \times 2\pi R \ell$$

$$N_1' = \left(F/\ell\right) R$$

$$N_1' = \frac{2\pi}{6} \rho R^3 W \langle V \rangle$$

Now

$$\frac{N_1}{N_1'} = \frac{2\pi\eta W R^3 \times 3}{\Delta R\ \pi\rho R^3 W < V >} = \frac{6\eta R}{\Delta R\ \rho < R > R}$$

$$\frac{N_1}{N_1'} = 6 \times \frac{1}{3} \frac{< V > \lambda\rho R}{\Delta R \rho < V > R} = \frac{2\lambda R}{\Delta R \times R} = \frac{2\lambda}{\Delta R}$$

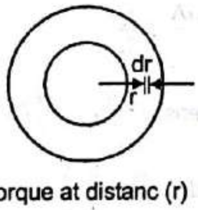
Since final moment is decreased by n time then

$$n = \frac{2}{\Delta R} \frac{1}{\sqrt{2}\pi d^2 n} = \frac{2KT}{\Delta R \sqrt{2}\pi d^2 P}$$

$$P = \frac{\sqrt{2} KT}{\pi d^2 \Delta R n}$$

Ans.

2.243



Viscous torque at distanc (r)

$$\tau = \eta r \left(\frac{\partial w}{\partial r}\right) (2\pi r \ell) r = N_1 \ell$$

Here it is assume Viscous torque (N_1) will be constant

$$N_1 \ell = 2\pi \eta r^3 \ell \frac{\partial w}{\partial r}$$

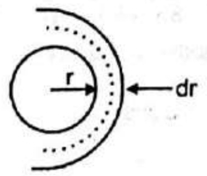
$$2\pi\eta\ell \int_0^w \partial w = \ell N_1 \int_{R_1}^{R_2} \frac{\partial r}{r^3}$$

$$2\pi\eta w = \frac{N_1}{2}\left(-\frac{1}{R_2^2} + \frac{1}{R_1^2}\right)$$

$$\eta = \frac{N_1}{4\pi w}\left(-\frac{1}{R_2^2} + \frac{1}{R_1^2}\right)$$

Ans.

2.244



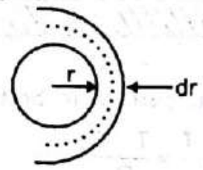
$$d\tau = \eta A \left(\frac{dV}{dz}\right) r = \eta\,(2\pi r\, dr) \left(\frac{r w}{h}\right) r$$

$$\tau = \frac{2\pi\eta w}{4} \int_0^a r^3 dr$$

$$\tau = \frac{\pi\eta w a^4}{2h}$$

Ans.

2.245



$$N = \left[\frac{1}{6} < V > \rho r w\right] [2\pi \rho\, dr] [r]$$

$$N = \frac{1}{3} \pi <V> \rho w \int_0^a r^3 dr$$

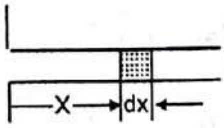
$$N = \frac{1}{12} \pi <V> \rho w a^4$$

$$N = \frac{1}{12} \pi \sqrt{\frac{8RT}{\pi M}} \frac{PM}{RT} w a^4$$

$$N = \frac{1}{3} w a^4 P \sqrt{\frac{\pi M}{2RT}}$$

Ans.

2.246



$$\frac{dQ}{dt} = \frac{\pi a^4 dP}{8\eta dx}$$

$$\frac{dm}{dt} = \frac{\rho\pi a^4 dP}{8\eta dx} \text{ —— (i)}$$

$$\frac{dm}{dt} = \frac{\pi a^4}{8\eta}\left(\frac{PM}{RT}\right)\frac{dP}{dx} = \frac{\pi a^4 M}{8\eta RT}\left(\frac{PdP}{dx}\right)$$

For isothermal process

$$\frac{PdP}{dx} = \text{Const} = C$$

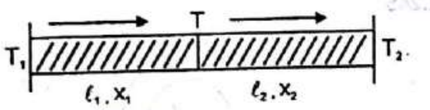
$$\int_{P_1}^{P_2} P\,dP = C\int_0^{\ell} dx$$

$$C = \frac{P_2^2 - P_1^2}{2\ell}$$

$$\frac{dm}{dt} = \frac{\pi a^4 M}{8\eta RT}\left(\frac{P_2^2 - P_1^2}{\ell}\right)$$

Ans.

2.247



Heat current in both rod will be same.

$$\frac{T_1 - T}{R_1} = \frac{T - T_2}{R_2}$$

$$\frac{T_1 - T}{\ell_1/\chi_1 A} = \frac{T - T_2}{\ell_2/\chi_2 A}$$

$$T = \frac{(\chi_1 T_1/\ell_1 + \chi_2 T_2/\ell_2)}{(\chi_1/\ell_1 + \chi_2/\ell_2)}$$

Ans.

2.248

For equivalent heat conductivity (χ) heat resistance of system will be equal to that of one rod.

Then

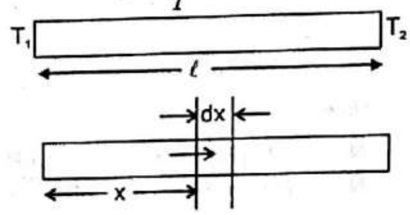
$$\frac{\ell_1 + \ell_2}{\chi A} = \frac{\ell_1}{\chi_1 A} + \frac{\ell_2}{\chi_2 A}$$

$$\chi = \frac{\ell_1 + \ell_2}{\frac{\ell_1}{\chi_1} + \frac{\ell_2}{\chi_2}}$$

Ans.

2.249

$$\chi = \frac{\alpha}{T}$$



Assume $T_1 > T_2$

$$\frac{dQ}{dt} = -\frac{\chi A\,dT}{dx} = \text{Const} = C$$

$$-\chi A dT = c dx$$

$$-\int_{T_1}^{T_2} \frac{\alpha A}{T} dT = C\int_0^{\ell} dx$$

$$-\alpha A \ell n \frac{T_2}{T_1} = C\ell$$

$$C = -\frac{\alpha A}{\ell}\ell n\frac{T_2}{T_1}$$

Heat density $\left(C/A\right) = \frac{\alpha}{\ell}\ell n\, T_1/T_2$

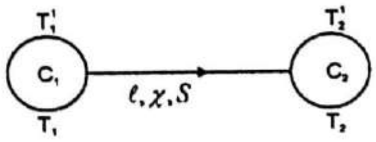
$$-\int_{T_1}^{T} \frac{\alpha A}{T} dT = \frac{\alpha}{\ell}\ell n\frac{T_2}{T_1}\int_0^x dx$$

$$T = T_1\left(T_2/T_1\right)^{x/\ell}$$

Ans.

2.250

Method: 1 (Basic Approach)



Assume tempreture of two chunks at t = 0 are T_1' and T_2' while at time t = t, tempreture are T_1 and T_2.

$$dQ = \frac{\chi S(T_1 - T_2)}{\ell}dt = -C_1 dT_1 = C_2\, dT_2 \text{ —— (i)}$$

Now

$$-C_1\int_{T_1'}^{T_1} dT_1 = C_2\int_{T_2'}^{T_2} dT_2$$

$$T_1 = \frac{C_2}{C_1}\left(T_2' - T_2\right) + T_1' \text{ —— (ii)}$$

$$\frac{\chi S(T_1 - T_2)}{\ell}\,dt = C_2\, d\,T_2$$

$$\frac{\chi S}{\ell}\int_0^t dt = C_2\int_{T_2'}^{T_2}\frac{dT_2}{T_1 - T_2}$$

$$\frac{\chi S}{\ell}\int_0^t dt = C_2\int_{T_2'}^{T_2}\frac{dT_2}{\frac{C_2}{C_1}\left(T_2' - T_2\right) + T_1' - T_2}$$

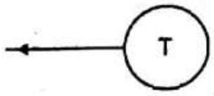
$$\Delta T = (\Delta T)_0\, e^{-\alpha t}$$

Where

$$\alpha = \left(\tfrac{1}{C_1} + \tfrac{1}{C_2}\right)\frac{S\chi}{\ell}$$

Ans.

Method :2 (Reduce heat capacity)



t = t

$$C = \frac{C_1C_2}{C_1 + C_2} = \text{Reduce heat capacity}$$

Let us difference of tempreture of object is T which provide tempreture difference at time t.

$$\frac{dQ}{dt} = \frac{\chi ST}{\ell}$$

$$dQ = \frac{\chi ST}{\ell}dt = -\,CdT$$

$$\int_{\Delta T_0}^{T}\frac{dT}{T} = \frac{-\chi S}{\ell C}\int_0^t dt$$

$$\ell n\frac{T}{\Delta T_0} = \frac{-\chi S}{\ell C}t$$

$$T = \Delta T_0\, e^{\frac{-\chi st}{\ell C}}$$

$$T = \Delta\, T_0\, e^{\frac{-\chi s}{\ell}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)t}$$

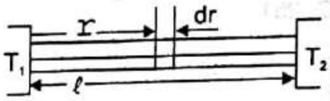
$$\text{Assume } \alpha = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{s\chi}{\ell}$$

$$T = \Delta T_0 e^{-\alpha t}$$

Ans.

2.251

$$\chi = C\sqrt{T}$$



Assume $T_1 > T_2$

$$\frac{dQ}{dt} = -\frac{\chi SdT}{dr} = \text{Constant} = C_1$$

$$-\chi\, S\, dT = C_1\, dr$$

$$-CS\int_{T_1}^{T} T^{1/2}dT = C_1\int_0^r dr$$

$$-\frac{2CS}{3}\left(T^{3/2} - T_1^{3/2}\right) = C_1 r$$

$$T^{\frac{3}{2}} = T_1^{\frac{3}{2}} - \frac{3\ell}{2CS}C_1$$

Put

$r = \ell\,; T = T_2$

$$\left(-T_2^{3/2} + T_1^{3/2}\right) = \frac{3\ell}{2CS}C_1$$

$$T = \left[T_1^{3/2} - \frac{3r}{2SC}\left(\frac{2CS}{3\ell}\right)\left(T_1^{3/2} - T_2^{3/2}\right)\right]^{2/3}$$

$$T = \left[T_1^{3/2} - \frac{r}{\ell}\left(T_1^{3/2} - T_2^{3/2}\right)\right]^{2/3}$$

Ans.

2.252*

We know

$$\chi = \frac{1}{3}\langle V\rangle\lambda\rho C_v$$

$$\chi = \frac{1}{3}\sqrt{\frac{8RT}{\pi m}}\left(\frac{KT}{\sqrt{2}\pi d^2 P}\right)\times\left(\frac{PM}{RT}\right)\left(\frac{R\mathring{i}}{2}\right)$$

$$\chi = \frac{1}{3}\left(\sqrt{\frac{8RT}{\pi M}}\right)\frac{KMR\mathring{i}}{2\sqrt{2}\pi d^2 R}$$

$$\chi = \frac{1}{6\sqrt{2}}\sqrt{\frac{8RT}{\pi M}}\ \frac{KM\mathring{i}}{\pi d^2}$$

$$\chi = \frac{1}{3}\sqrt{\frac{RT}{RM}}\ \frac{KM\mathring{i}}{\pi d^2}$$

From Q. 251

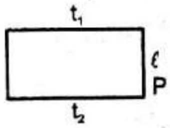
$$C_1 = \frac{2CS}{3\ell}\left(T_1^{3/2} - T_2^{3/2}\right)$$

$$q = C_1 = \frac{2S}{3\ell}\left(T_1^{3/2} - T_2^{3/2}\right)\ \frac{1}{3}\sqrt{\frac{R}{\pi M}}\frac{KM\mathring{i}}{\pi d^2}$$

$$q = \frac{2\mathring{i}R^{3/2}\left(-T_2^{3/2} + T_1^{3/2}\right)}{9\pi^{3/2}\ \ell d^2 N_A\sqrt{M}}$$

Ans.

2.253*



We know

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} = \frac{KT}{\sqrt{2}\pi d^2 P}$$

Since t_1 and t_2 not very large difference hence tempreture of mixture

$$T = \frac{t_1 + t_2}{2} \Rightarrow \lambda = \frac{KT}{\sqrt{2}\pi d^2 P}$$ **Ans.**

For rarefied gas

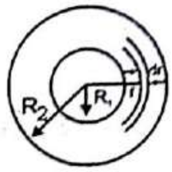
$$q = \frac{1}{6} <V> \rho\, C_V (t_1 - t_2)$$

$$q = \frac{1}{6} <V> \frac{PM}{RT}\left(\frac{R\mathring{i}}{2}\right)(t_1 - t_2)$$

$$q = \frac{1}{6}P <V> \frac{M}{R}\frac{\mathring{i}}{2}(t_1 - t_2)$$

Ans.

2.254



Suppose length of cylinder is ℓ

$$\frac{dQ}{dt} = -\chi\, 2\pi r\ell\left(\frac{dT}{dr}\right) = \text{constant} = C$$

$$\frac{-2\pi\ell\chi}{C}\int_{T_1}^{T} dT = \int_{R_1}^{r}\frac{dr}{r}$$

$$\frac{-2\pi\ell\chi}{C}\ (T - T_1) = \ell n\, r/R_1 \text{ —— (i)}$$

To find C :

Put $T = T_2$ and $r = R_2$

$$C = \frac{2\pi\ell\chi(T_1 - T_2)}{\ell n(R_2/R)_1}$$

Form (i)

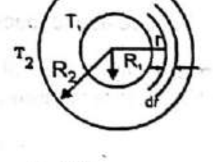
$$T = T_1 + \frac{C}{2\pi\ell x}\ \ell n(R_1/r)$$

$$T = T_1 + \frac{2\pi\ell\chi(T_1 - T_2)}{(\ell n(R_2/R)_1)2\pi\ell\chi}\ell n\, R_1/r$$

$$T = T_1 + (T_1 - T_2)\ \frac{\ell n^{R_1}/_{r}}{\ell n^{R_2}/_{R_1}}$$

Ans.

2.255



$$\frac{dQ}{dt} = \frac{-\chi 4\pi r^2 dT}{dr} = \text{Constant} = C$$

$$-4\pi\chi\int_{T_1}^{T} dT = C\int_{R_1}^{r}\frac{dr}{r^2}$$

$$-4\pi\chi\left(T - T_1\right) = -C\left(\frac{1}{r} - \frac{1}{R_1}\right)$$

For Calculition of C

Put $r = R_2$; $T = T_2$

$$4\pi\chi(T_2 - T_1) = C\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$C = \frac{4\pi\chi R_1 R_2 (T_2 - T_1)}{R_1 - R_2}$$

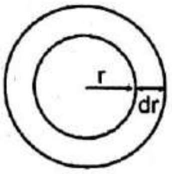
From (i)

$$T = T_1 + \frac{C}{4\pi\chi}\left(\frac{1}{r} - \frac{1}{R_1}\right)$$

$$T = T_1 + \left(\frac{R_1 R_2}{R_1 + R_2}\right)(T_2 - T_1)\left(\frac{1}{r} - \frac{1}{R_1}\right)$$

Ans.

2.256



$$\frac{dQ}{dt} = -\chi(2\pi r\ell)\frac{dT}{dr} = \omega\pi r^2\ell$$

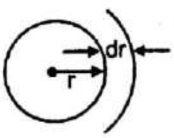
(At steady state)

$$2\chi\int_{T_0}^{T} dT = \omega\int_{R}^{r} r\,dr$$

$$T = T_0 + \frac{\omega}{4\chi}(R^2 - r^2)$$

Ans.

2.257



$$\frac{dQ}{dt} = -\chi\, 4\pi r^2\frac{dT}{dr} = \omega \times \frac{4}{3}\pi r^3$$

(At steady state)

$$-\chi\int_{T_0}^{T} dT = \frac{\omega}{3}\int_{R}^{r} r\,dr$$

$$T = T_0 + \frac{W}{6x}(r^2 - R^2)$$

Ans.

About Author ....

He has received his B.Tech Degree from IIT (ISM Dhanbad) in 2010.

While preparing for JEE, Physics got him intrigued the most and later on the decided to pursue this interest as his career.

Raj Kumar Sharma's career has centered entirely around giving coaching for IIT JEE and Physics Olympiad and has been associated with leading institutions of the country.

Through his teaching career, he has been active in promoting physics education and developing many short approaches to make physics interesting.

Since the beginning of teaching career, his Lectures has revolved around IE Irodov's problems.

Most importantly, he relishes spending time with his students, his wife and child.