PEARSON Education

The Pearson Guide to

Quantitative Aptitude

for the CAT

Second Edition

- Short-cut methods and problem solving tricks
- A section on Vedic mathematics
- Solved original CAT papers
- Numerous examples and fully solved problems

Nishit K. Sinha

THORPES

PEARSON Education

To
my parents
Shri Kumar Kalyan Prasad Sinha
and
Smt Sanjila Sinha

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Preface to the Second Edition

I am thankful to everyone—my friends in the CAT coaching industry, students and most importantly the users of this book—who have shared their feelings about the first edition of this book. Their suggestions inspired me to bring out the second edition.

This edition contains the following changes:

1. Detailed solutions to more questions.
2. The CAT 2006 and CAT 2007 question papers with solutions.
3. Two new chapters:
   - Elements of Algebra, which deals with the constituents of algebra like algebraic calculations and factorization.
   - Managerial Decision Making, which includes the basic concepts used by managers in decision making. This is one of the new areas where CAT may venture in future.

I hope this will facilitate the users of this book in clearing the CAT and getting admission in their desired B-Schools.

Any feedback, positive or negative, is welcome. I can be reached at **nishitkumarsinha@rediffmail.com**

NISHIT K. SINHA

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Preface to the First Edition

In the last five years that I have been training students for the Common Admission Test (CAT) and other major B-school entrance examinations, I have not come across a single book for quantitative aptitude (QA) that puts adequate emphasis on all the topics covered in these exams with a dedicated focus on the CAT. Hence, I decided to write a book that would cater exclusively to candidates taking the CAT and at the same time reflect the changing patterns of the quantitative aptitude exam. I have, therefore, included some not-so-conventional topics like graphs and maxima minima, calendars, inequalities, and height and distance. The current trend of the QA paper reflects that more emphasis is being laid on these topics.

I have tried to make this book useful for even those students who are not comfortable with math. This book has ample worked-out examples to enable the students to understand the problem-solving techniques before moving on to the practice problems. In my perception, students can be categorized into the following four types: novice, apprentice, practitioner and expert. The table below summarizes their abilities and approach towards this exam. This book is designed to take care of the needs of students of all these four types.

Students' response	*Novice*	*Apprentice*	*Practitioner*	*Expert*
Interpretation	Do not understand the basic concepts	Do not understand all the concepts underlying the problems. So, unable to attempt all the qustions	Understand the concepts and attempt most of the main problems	Can solve the problems correctly
Approach	Do not know the methods of solving the problems	Unable to pick the correct method; mostly rely on luck to get the right answer	Know and pick the correct method; have a sound strategy and solve the problems with skill, not luck	Use two separate strategies for a good number of questions
Accuracy level	Unable to get correct answers most of the time	Manage to get correct answers, through there are some mistakes	Correct answers with hardly any mistakes. Use appropriate units	Good level of accuracy as well as speed

This book is divided into 19 chapters, each chapter containing four different levels of problems: warm up, foundation, moderate and advanced. These levels are hierarchical and sequential, and students are expected to progress through them in the same order. Movement from one level to another is largely a function of appropriate experience with the specific content. A student is unlikely to perform well at a particular level without the experience of the preceding level, which allows an intuitive functioning later. If one level is not mastered before moving on to the next,

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though good students would appear to perform well at that higher level and may get the right answers, it would be without any sound reasoning most of the times.

A benchmarking test has been given at the end of every section with comparative scores. There are four review test papers too at the end of the book to enable you to benchmark your performance. Apart from these, there are CAT papers of the last five years. Solving these will serve as a revision of important questions as well as give you a feel of the exam.

While writing this book, the key on my laptop which I used the maximum number of times was ← (the back-space). I had to use this key whenever I found something inappropriate or not relevant; also when the language was not simple or when I wanted to simply improve. This is exactly what I would advise all who aspire to qualify the CAT; it is always a good idea to go back to what has been done earlier. Revision and practice are important stepping stones to success. It has been rightly said, "The more I sweat, the more I shine."

Although I have tried to be meticulous in preparing this text, some errors may have crept in. I invite all of you to contribute in enhancing the value of this book with your suggestions and by bringing to our notice the errors, if any, so that they can be corrected in the subsequent editions. I can be reached at nishitkumarsinha@rediffmail.com

NISHIT K. SINHA

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This book bears the imprint of many people— my colleagues, my students and my teachers who have had a significant impact on my thought process and have generously extended help whenever I needed.

I am thankful to my present organization for giving me enough time to work on this project.

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My special thanks to my brothers Ravi Shankar Prasad, Sharat Chandra Mayank, Amit Kumar and Vinit Kumar.

I extend heartfelt thanks to all my students, past and present, who have helped me improve the content and the presentation of the book.

I would like to thank Mrs Preeta Priyamvada for her painstaking efforts in going through the manuscript. A special mention of thanks to Mr Raza Khan, Sr Managing Editor, Pearson, who played a pivotal role in getting this book printed in such a short span of time.

I may have forgotten some names here. I wish to express my gratitude towards all who have contributed in the making of this book.

NISHIT K. SINHA

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Everything You Need to Know About CAT

What is CAT

CAT stands for Common Admission Test. It is a test conducted by the IIMs for admission to several programmes offered by them, especially MBA. Besides the IIMs, there are a good number of colleges that accept the CAT score in their first round of selection process. A detailed list of the same can be seen at http://www.catiim.in.

Myths and Realities About CAT

Myth 1 The CAT is difficult.

Reality 1 There is a common adage, 'nothing is impossible'. Hence, there is no difficulty that cannot be overcome. What is required is a strong desire to achieve your goal. If you really wish to study at any of the IIMs, you can make your dream come true with a well-planned strategy and a focussed approach.

Myth 2 I am not made for IIMs/CAT.

Reality 2 Believe for a moment that you can do well in the CAT. How does it feel? Did this belief harm you in any way? I think this belief has only instilled confidence in you and infused you with a desire to work towards this goal. The point that I want to drive at here is that all your actions or strategies germinate as thoughts. So, why assume from the very outset that you are incapable of cracking the CAT? Have a positive attitude and the right action will naturally follow. At the very worst, if you have prepared well for the CAT, other entrance exams will become a cakewalk for you.

Myth 3 I need to be a math-wizard to crack CAT.

Reality 3 Many students who have cleared the CAT have studied math only till Class 10. A look at the problems comprising the QA paper of the CAT shows that they can be solved by the application of simple logic and basic mathematical concepts. Every year the CAT features questions of about 9-10 marks that can be solved through sheer logic, and without the application of any mathematical concept. I would advise you to go to Part 7 of this book, which contains previous years' CAT papers and verify this. Apart from these, about 10 marks are allocated to questions relating to the number system, and another 10 marks to geometry questions. Thus, a total of 30 marks, which is roughly 60% of the whole QA paper, can be scored without using any other mathematical concept (Please keep in mind that this does not imply that you should limit your preparation to these 2 topics). The range of marks needed to clear the cut-off is 12 – 13.

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Eligibility for CAT

Any student who has completed his Bachelor's degree—including engineering, medicine or their equivalent — and has scored at least 50 per cent or more (or its equivalent in the Cumulative Grade Point Average/ Grade Point Average system) is eligible to sit for this exam. Students who are in the final year of their Bachelor's degree are also eligible, provided they have more than 50 per cent aggregate marks in each of the preceding years of their Bachelor's study. Candidates belonging to the category of Scheduled Caste/Scheduled Tribe/Persons With Disabilities require at least 45 per cent marks.

What is the Duration of the CAT 2006 exam?

The duration of this year's paper will be two and a half hours, instead of the two hours as it was earlier. This is one surprise nobody anticipated. However, it might actually be a good thing; you now have about 30 minutes more to write the exam.

What is the Right Time to Start the Preparation for CAT?

This will vary from person to person and there cannot be any tailor-made suggestion for all. I know a few people who do not prepare for CAT and still qualify the CAT every year. How good or bad you are at English or math or reasoning is known only to you, so you alone can judge what will be the right time for you to start preparation. Ideally, if you are fairly good at English or maths or reasoning, you should start preparing some 6 to 8 months before the exam. If you think you need to put in extra effort, you should begin earlier and give yourself some more time.

How Many Hours Every Day Must You Study?

It all depends on *your* ability and the state of *your* preparedness. If you start your preparation early, two to three hours a day would give you a good start. However, if you start, say, in May/June, then about four to five hours per day may be required. And if you start in September, then even six hours a day will not be enough.

Is There Any Sectional Cut-off?

Yes, there is a requirement of minimum marks in every section to qualify for that section and finally for the CAT. Sectional cut-offs vary mostly on the basis of the performance of the students who appear for the exam in that year. A new trend has been observed in recent years; this is the announcement of the minimum cut-off by IIM Ahemadabad for its courses before the CAT exam. More information regarding the same can be accessed at www.iimahd.ernet.in

Parameter	*CAT 2007*	*CAT 2006*	*CAT 2005*	*CAT 2004*	*CAT 2003*
Number of sections	3	3	3 × 2	3 × 2	3
Number of questions	75	75	90	123	150 (50 × 3)
Difficulty level (Quant)	Difficult	Easy	Average	Average	Average
Difficulty level (EU + RC)	Difficult	Difficult	Average	Average	Average
Difficulty level (DI + DS + AR)	Moderate	Moderate	Difficult	Difficult	Difficult
Expected cut-off (Overall)	88-92	105-110	48	54	56
Expected cut-off (Quant)	20-22	38-42	12-13	16	14
Expected cut-off (EU + RC)	18-20	28-32	17-18	21	22
Expected cut-off (DI + DS +AR)	32-34	30-33	10-11	14	16

* This comparison is only indicative. IIMs do not provide any such list.

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Comparison of Recent Years'CAT Papers

What is the Pattern of the CAT Exam

Simply put, there is no pattern. One of the reasons why the CAT is considered tough is the change in the pattern every year. In the 1990s, it was a four-section test with English taking up a major part. However, since 1999, it has become a three-section exam with English taking up just one-third of the total paper. So, I would suggest that the best you can do is to arm yourself with skill-sets and develop tools in advance so that you have an edge over the others on the final day.

The pattern of the Quantitative Aptitude section for the last five years is given below.

Section	*CAT 2007*	*CAT 2006*	*CAT 2005*	*CAT 2004*	*CAT 2003*
Quantitative Ability	25 (100)	25 (100)	30 (50)	35 (50)	50 (50)
Data Interpretation and Logic	25 (100)	25 (100)	30 (50)	38 (50)	50 (50)
Verbal and Reading Comprehension	25 (100)	25 (100)	30 (50)	50 (50)	50 (50)
Total Questions (Marks)	**75 (300)**	**75 (300)**	**90 (150)**	**123 (150)**	**150 (150)**

Do All Questions Carry Equal Marks

Till 2003, all questions carried equal marks. But as can be seen from the table above, in 2004 and 2005, there was differential marking. Last year, for example, all sections had 30 questions out of which 10 were allotted one mark each and 20 were allotted two marks each, and 50 marks per section.

Is There Any Negative Marking

Yes, a wrong answer costs you one-fourth of the total marks for that particular question. So, if a question carries one mark, then you get –1/4 (note the negative sign) for the wrong answer. And if the question carries 2 marks, then you will get –1/2.

The XAT exam 2006/07 (Xavier Labour Relations Institute Admission Test, the entrance test for XLRI, Jamshedpur) had progressive negative marking. For the first five questions that were wrong, the negative mark was 1/3. For any additional mistake, the negative mark was 1/2.

Can We Predict the CAT

The pattern of questions in the CAT varies every year. Hence, it is not possible to predict the style and level of questions that may be asked. However, it has been noticed that questions from number systems and geometry comprise close to 20% of the paper. These topics seem to be popular both in terms of coverage and representation. Besides, there are several concepts from algebra (e.g., sequence and series, functions, quadratic equations) and from arithmetic (e.g., time, speed and distance) from which questions are taken regularly.

Logistics Related to CAT

The application forms for this exam are out in the month of July (on the 2nd or 3rd Sunday in major daily national newspapers). The forms can be obtained from the specified bank till around 3rd Saturday of August. They can also be obtained from several IIMs till around the 4th Saturday of August (i.e., till a week after the date on which the bank closes its date for Application Forms). The exam is held normally on the 3rd Sunday of November. This has been the trend for the last three years. The results are announced around the 4th week of December or the 1st week of January. Group Discussion (GD)/PI starts in the second week of February and continues till the 3rd week of April.

The fees of different IIMs range from Rs. 1.4 lac per annum to Rs. 1.7 lac per annum. Bank loans for the same are very easily available from a number of banks. Besides, some IIMs offer scholarships based on the need and merit of the students. More information can be obtained from any of the IIMs Web site:

IIM Ahmedabad - www.iimahd.ernet.in
IIM Calcutta - www.iimcal.ac.in
IIM Indore - www.iimidr.ac.in
IIM Bangalore - www.iimb.ernet.in
IIM Lucknow - www.iiml.ac.in
IIM Kozhikode - www.iimk.ac.in

For any information related to MBA/PGDM from any of the institutes, you can also mail to me at: nishitkumarsinha@rediffmail.com

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How to Use This Book

Let me share with you what my feelings were before I started working on this book. I thought of the kind of book I would prefer if I had to learn the theories of public administration when I do not have any background of this subject. I realized I would want a book which is strong in fundamental concepts but lucid in language; I would need a book which is self-explanatory but not verbose; I would need a book which is all-encompassing but not irrelevant; I would need a book with a good number of practice questions but no repetitive questions (if I cannot learn a concept with 200 relevant questions, chances are low that I will be able to learn that concept with 250 questions having the same level of relevance); I would need a book which can help me judge my progress from time to time. And this is what I have sincerely tried to provide in this book.

How to Go Ahead with This Book

Step 1 Go through the learning objectives and remain focused on them. After completing the chapter, check if these objectives have been fulfilled.

Step 2 Start with the concepts. Before moving on to the next concept, go through the worked-out examples related to that concept. Move ahead only when you have internalized them. Sometimes this might appear to be drudgery, but you must do it.

Step 3 Do the warm-up exercise. It is a precursor to the problems. Most of the questions in the warm-up exercise will help you check only your understanding of the concept and not the application of it. Ideally, you should not give more than 2 minutes to any question in the warm-up exercise. If you get less than 75 per cent questions correct, go over the concepts for which you got the answers wrong.

Step 4 Do the foundation exercise. Solve the problems in this set without any time constraint. This level tests your comprehension of the concept, and is a precursor to the application-based questions. Try to solve all the questions in one sitting, whatever time it takes (one or two breaks of 10 minutes can be taken). If you don't get the correct answers, attempt the same question once again in the next sitting (preferably the next day). This will help you develop:

- A thorough understanding of the concepts
- An experience of the problems being asked at the basic level
- Confidence building

Step 5 Next is the moderate exercise. This level tests your ability to apply a particular concept, or a combination of concepts, to a problem. You might find that concepts of geometry are being used in problems on time, speed and distance. This will help you develop:

- The ability to identify easy and difficult questions
- Mental images of the problems
- A logical connection between concepts and their application.

Step 6 Do not go for the advanced level immediately. Relax. Do not solve any QA question for one day. The following day, take the benchmarking test. Your performance in this test can tell you a lot about your progress. Ideally, you should aim for 85 percentile or more.

Step 7 After you have done all the moderate exercises and benchmarking tests, go over the topics again. Only then attempt the advanced questions. Most of the questions here are above the regular CAT level. The idea is to prepare you for a higher level. If you excel at this level, CAT will be a cakewalk for you.

Step 8 Take the Review Tests. Target anything above 98 percentile. If you get it consistently in all the RTs, you are prepared for the CAT. If not, repeat the process from Step 5. If in any one of the RTs, you get less than 85 percentile, repeat the process from Step 4.

All the best.

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Smart table

Number of questions asked from various topics in CAT (QA) during 2005–2007.

Chapter	*CAT 05*	*CAT 06*	*CAT 07*
Part 1—Numbers and their properties			
Vedic Mathemetics	0	0	0
Number System	6	3	1
Part 2—Arithmetic			
Percentage	0	0	0
Profit Loss and Discount	0	0	0
Averages and their applications	0	0	3
Ratio proportion and variation	0	2	0
Time and work	1	0	0
Time Speed and Distance	2	1	2
Part 3—Higher Mathematics			
Basics of Algebra	0	6	2
Equations	2	0	3
Sequence and series	0	2	1
Functions	1	0	1
Maxima minima and graph	2	2	1
Logarithm	1	1	0
Permutations and Combinations	1	1	3
Probability	0	0	0
Part 4—Measurement			
Geometry	7	4	2
Mensuration	2	1	1
Part 5—Miscellaneous			
Set Theory	0	1	0
Miscellaneous	2	1	1
Managerial Decision Making	0	0	4

Good news

It may be heartening to know that the cut-off for QA section in the CAT has not gone beyond 30 per cent marks of QA till now. There have been times when the cut-off for QA was as low as 17%–18%. Besides, the fact that you have close to 50 minutes for QA, under CAT 06/07 situation solving 10-12 questions out of 25 questions will get you a high enough score.

Happy learning!

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How to Prepare for QA

Purpose of QA Being Included in the CAT

Have you ever thought why quantitative-aptitude questions are asked in the CAT? Is it to check your math ability with questions from algebra, or permutation and combination? If that were the case, they would have asked questions also from group theory and applied mathematics, which they don't. The questions asked in quantitative aptitude help test the student's reasoning ability based on mathematical skills. They also test the student's comprehension of basic mathematical concepts. And even if somebody has not studied mathematics after class 10, they can solve these problems with a little practice. What is required is an understanding of the basic concepts. It is, therefore, clear that through the test of QA, the examiners intend to check functional ability pertaining to basic mathematical operations.

Five-point Action Plan for QA

1. Address your math-phobia first You will not be in the best state of mind to crack the CAT if past failures or inabilities in mathematics haunt you. As it is said, it's okay to lose a couple of battles. Keep your eyes set on the war!

2. Make a roadmap 'Divide and win' is the key. Your preparation should be divided into topics and every topic should be further divided into sub-topics. Once you have a topic-centric roadmap of the entire QA syllabus, carefully distribute the time you want to devote to each section on a daily basis.

3. Strengthen your fundamentals Till you have understood the concepts and their various applications thoroughly, do not start solving the problems. Let this process take some time. Now, before you start solving questions, go over the fundamental concepts once again and see which concept should be applied to get the right answer.

4. Adopt a systematic practice technique Don't jump to solving problems immediately. Make an effort to understand the basic theories behind the mathematical concepts, however trivial they may seem. And only then solve the problems.

5. Focus on weaknesses If students are weak in a particular area (say, geometry or permutations and combinations), they tend to neglect these questions and leave the solutions to choice. This is not only unproductive but also damaging. Identify the areas with which you are not comfortable. Numbers, geometry and algebra account for a major portion of the questions in the CAT; hence, they deserve due attention.

Consistency with Perseverance Is the Key

If you do not find ways to solve a problem on the first day, sleep on it and tackle it the next day. Sometimes even the simplest of answers elude you and lead to frustration. Even the best of mathematicians face this problem; therefore, leave it for another day. And if you are unable to solve this problem even on the second day, try it on the third day or the third time. If you are still not able to solve it, check the solution provided at the end of the book or take help. The bottom line is that only consistent efforts and practice can bring in positive results. If you want to go through this book in 10 days' time, you can do that also. And you will be able to learn the fundamentals, but remember it will be only short-term. You will be required to go through the same chore again after a few days and repeat this exercise.

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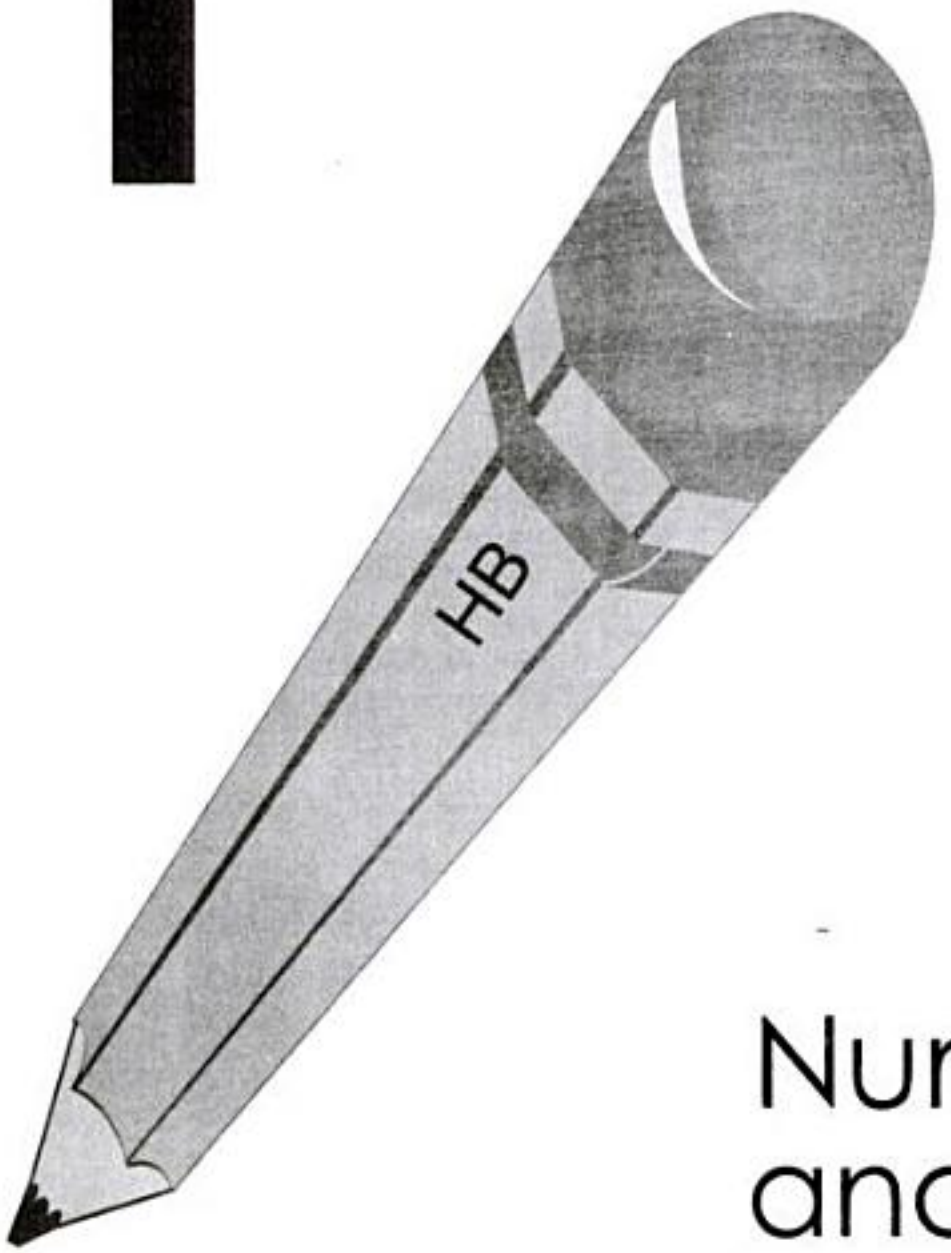
PART 1



Numbers and their Properties

- Vedic Mathematics
- Number System
- Benchmarking Test

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1

VEDIC MATHEMATICS

Introduction to the topic

Vedic mathematics is the ancient system of mathematics drawn from the Vedas. The Vedas are ancient texts that encompass a broad spectrum of knowledge, covering all aspects of life. These include the sutras (verses) pertaining to mathematics. In the early 20th century, Swami Shri Bharati Krishna Tirthaji Maharaja claimed to have rediscovered a collection of 16 ancient mathematical sutras from the Vedas and published it in a book titled *Vedic Mathematics*. Historians, however, do not agree on whether or not these were truly a part of the Vedic tradition. If these sutras dated back to the Vedic era, they would be a part of an oral rather than a written tradition. Despite controversies, they are a novel and useful approach to computation; they are flexible in application and easy to remember. They can often be applied in the algebraic contexts and in simple arithmetic as well.

Vedic mathematics is a relevant and an important topic for students preparing for the CAT as this will help them carry out lengthy calculations in an incredibly short time, which undoubtedly is a pre-requisite for this examination. Having a good calculation speed helps in saving time that can be used to garner a higher score in any test. It can be a moral booster too. So, we can consider these techniques as important tools that we should acquire to qualify the CAT the quicker way.

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TYPES OF CALCULATIONS

The different types of calculations that form the basis of mathematics are:

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Ratio comparison
6. Percentage calculations

When we talk about the techniques of calculations, addition and subtraction can simply not have any short-cuts. Since addition and subtraction are the basic units, we can at best only approximate the values.

In case of multiplication, the techniques of vedic maths can be used.

Ratio comparison techniques are discussed in the chapter on ratio, proportion and variation and the percentage calculations in the chapter on percentage.

VEDIC MATH TECHNIQUES IN MULTIPLICATION

There are several techniques of multiplication. We will discuss them one by one.

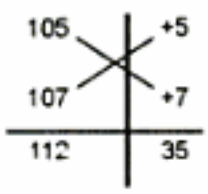
Method 1: Base Method

In this method, one number is used as a base; for example, 10, 50, 100 etc. The number that is closer to both the numbers should be taken as the base.

Example 1 105 × 107

In this case, both the numbers are close to 100, so 100 is taken as the base. We will now find the deficit/surplus from the base.

Base = 100, Surplus = 5 and 7



The right part (after slash) ⇒ this is the product of the surplus. Since the base = 100 and the surpluses are 5 and 7, the product would be 5 × 7 = 35.

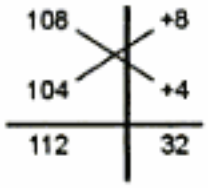
The left part (before slash) ⇒ It could be either of the numbers plus the surplus of the other multiplicand. Hence, the left part would be either (105 + 7) or (107 + 5) = 112 (both will always be the same) i.e., 112.

The left part would be equivalent to the number × 100. In this case, 112 × 100 = 11200

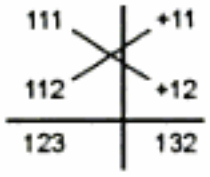
Now, we add both the right part and the left part = 11200 + 35 = 11235

Hence, the result of the multiplication would be 11235.

Example 2 108 × 104



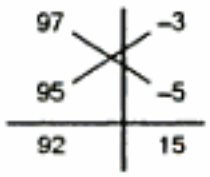
Example 3 111 × 112



Here, it is 11 × 12 = 132. But it can have only two digits. Thus, 1 will be carried over to the left part and the right part will be only 32. Left part will be either 111 + 12 + 1 (1 for the carry over) or (112 + 11 + 1), i.e., 124. So, the result will be 12432.

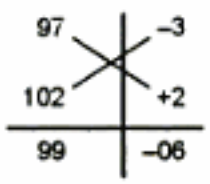
For 102 × 104, the answer will be 10608. Please note that the right part will be 08 and not simply 8.

Example 4 97 × 95



Base = 100, Deficit=97 – 100 = –3 and 95 – 100=–5

Example 5 97 × 102



96 × 108

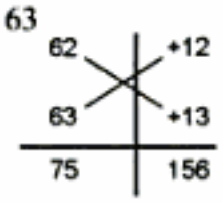
Base = 100, Deficit = 97 – 100 = –3,

Surplus = 102 – 100 = 2

The right part will now be (–3) × 2, i.e., –06. To take care of the negative, we will borrow 1 from the left part, which is equivalent to borrowing 100 (because we are borrowing from the hundred digits of the left part). Thus, this part will be 100 – 06 = 94.

So, the answer = 98 94

Example 6 62 × 63



To know more about Vedic maths, go to *www.pims.math.ca/pi/issue6/page15-16.pdf*

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We will assume here the base as 50 owing to the fact that the numbers are close to 50.

Base = 50, Surplus = 62 – 50 = 12, Surplus = 63 – 50 = 13

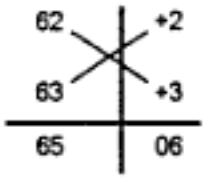
The left hand side = 156 and the right hand side = 75. Since the base is assumed to be equal to 50, so the value of the right hand side = 75 × 50 = 3750. Besides only two digits can be there on the right hand side, so 1(100) is transfered to the left hand side leaving 56 only on the left hand side.

So, the value on the right hand side = 3750 + 100 = 3850

Value on the left hand side = 56

Net value = 3850 + 56 = 3906

Let us do the same multiplication by assuming 60 as the base.



Base = 60, Surplus = 62 – 60 = 2, Surplus = 63 – 60 = 3

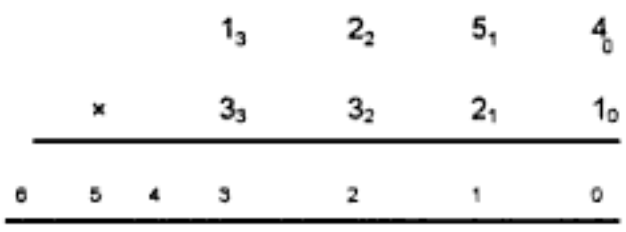
Since the base is assumed to be equal to 60, the value of the right hand side = 65 × 60 = 130 × 30 = 3900

So, net value = 3906

Method 2: Place Value Method

In this method of multiplication, every digit is assigned a place value and we do the multiplication is done by equating the place values of multiplicands with the place value of the product.

Let us see this with some examples:

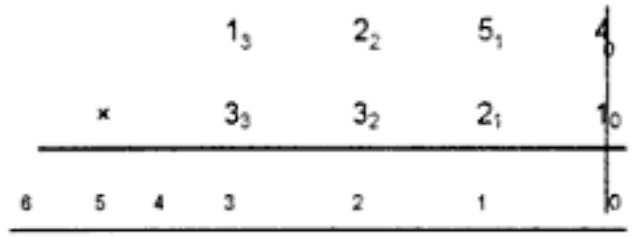


Conventionally, the unit digit is assigned a place value 0, the tens place digit is assigned a place value 1, the hundreds place digit is assigned a place value 2, the thousands place digits is assigned a place value 4 and so on.

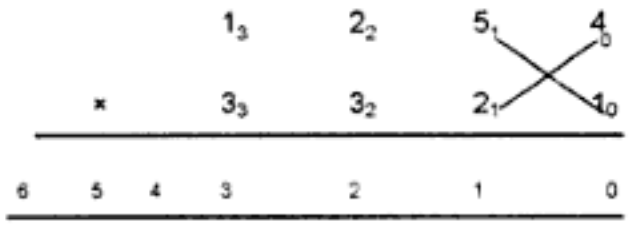
This multiplication is a two-step process.

Step 1 Add the place values of the digits of the numbers given (1254 × 3321) to obtain the place value of the digits of the product.

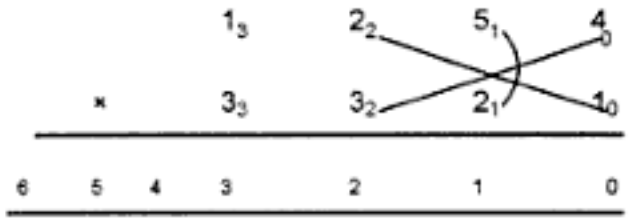
For example, using the place values of the multiplicands, i.e., using 0, 1, 2 and 3 of the number 1254 and the same place values 0, 1, 2 and 3 of the another multiplicand 3321, we can get 0 place value in the product in just one way, i.e., adding 0 and 0.



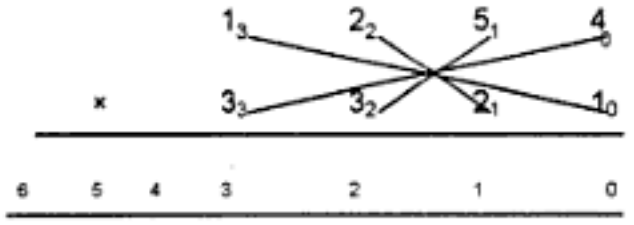
Place value 1 in the product can be obtained in two ways.



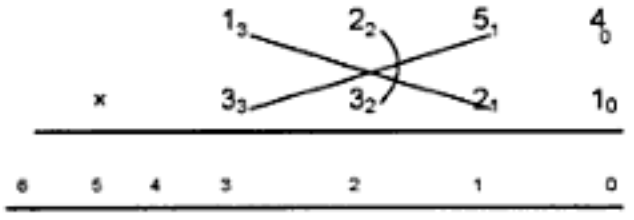
Place value 2 can be obtained in three ways.



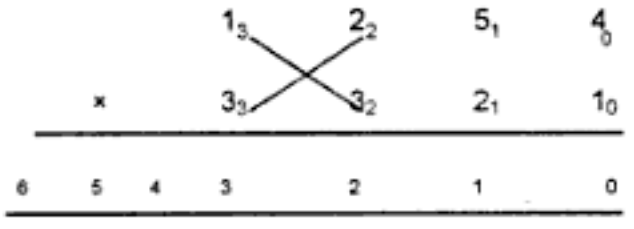
Place value 3 can be obtained in four ways.



Place value 4 can be obtained in three ways.



Place value 5 can be obtained in two ways.



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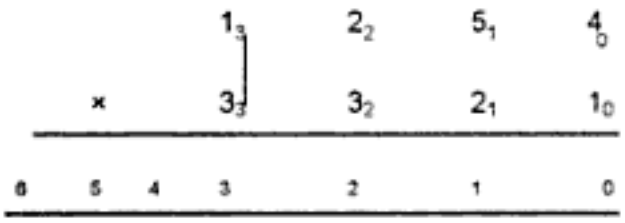
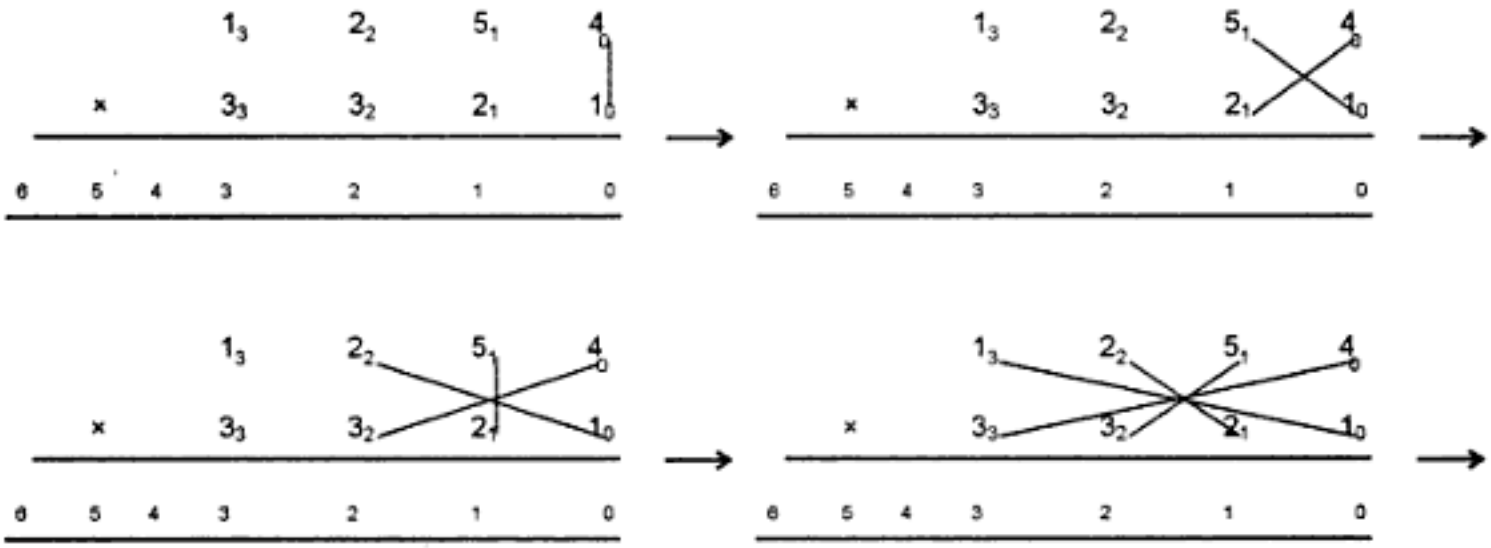
Place value 6 can be obtained in one way.



And this is the maximum place value that can be obtained.

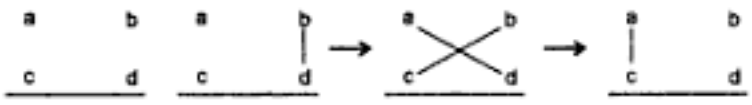
Step 2 Multiply the corresponding numbers one by one.



In this manner, we can find the product = 4164534

This method is most useful in case of the multiplications of 2 digits × 2 digits or 2 digits × 3 digits or 3 digits × 3 digits multiplication.

Example $ab \times cd$



Similarly, we can have a proper mechanism of multiplication of 2 digits × 3 digits or 3 digits × 3 digits also using the place value method.

Method 3: Units Digit Method

This method of multiplication uses the sum of the units digit, provided all the other digits on the left hand side of the unit digit are the same.

Example 1 75 × 75

1.0+7	5
7	5
56	25

The sum of the units digit = 10, so we add 1.0 in one of the digits on the left hand side.

Example 2 62 × 63

0.5+6	3
6	2
39	06

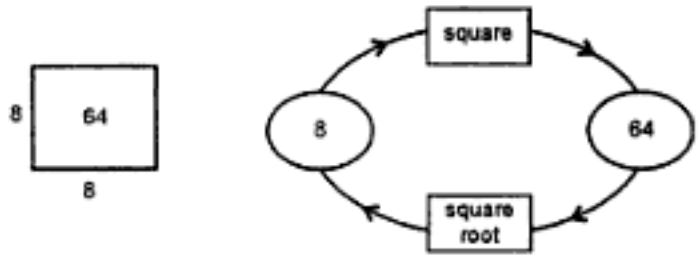
The sum of the units digit = 5, so we add 0.5 in one of the digits on the left hand side.

SQUARING

A **square number**, also called a **perfect square**, is an integer that can be written as the square of some other integer. In other words, a number whose square root is an integer is known as the square number of a perfect square.

For example, 9 is a square number since it can be written as 3 × 3.

This can be seen through the following flow-chart also.



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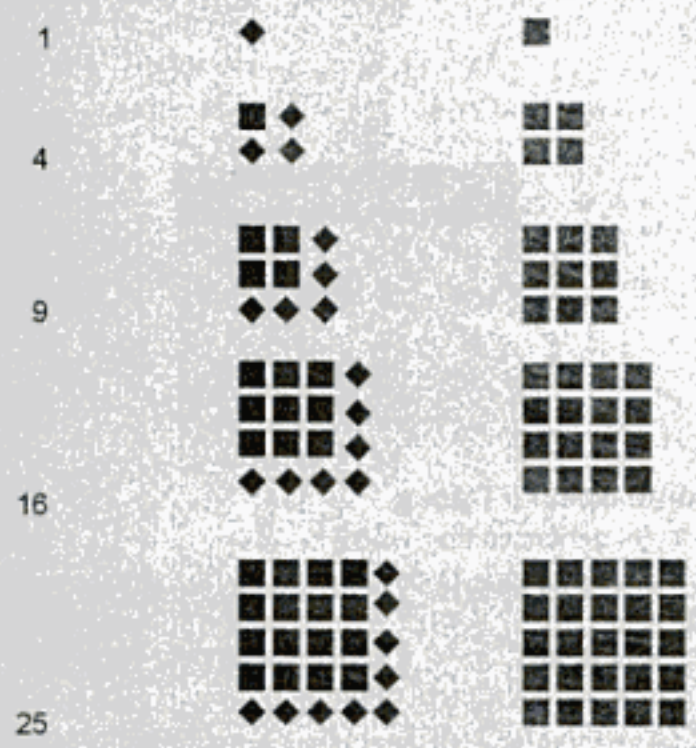
Properties of a Square Number

1. The number N is a square number if it can be arranged as N points in a square:



Thus, it can be deduced that the formula for the nth square number is n^2. This is also equal to the sum of the first n odd numbers $n^2 = \sum_{k=1}^{n}(2k-1)$, as can be seen in the above figure, where a square results from the previous one by adding an odd number of points (marked as '◆'). For example, $5^2 = 25 = 1+3+5+7+9$.

It should be noted that the square of any number can be represented as the sum $1+1+2+2+\ldots+n-1+n-1+n$. For instance, the square of 4 or 4^2 is equal to $1+1+2+2+3+3+4 = 16$. This is the result of adding a column and row of thickness 1 to the square graph of three. This can also be useful for finding the square of a big number quickly. For instance, the square of $52 = 50^2 + 50 + 51 + 51 + 52 = 2500 + 204 = 2704$.

2. A square number can only end with digits 00, 1, 4, 6, 9, or 25 in base 10, as follows:
3. If the last digit of a number is 0, its square ends in 00 and the preceding digits must also form a square.
4. If the last digit of a number is 1 or 9, its square ends in 1 and the number formed by its preceding digits must be divisible by four.
5. If the last digit of a number is 2 or 8, its square ends in 4 and the preceding digit must be even.
6. If the last digit of a number is 3 or 7, its square ends in 9 and the number formed by its preceding digits must be divisible by four.
7. If the last digit of a number is 4 or 6, its square ends in 6 and the preceding digit must be odd.
8. If the last digit of a number is 5, its square ends in 25 and the preceding digits (other than 25) must be 0, 2, 06, or 56.
9. A square number cannot be a perfect number. (If the sum of all the factors of a number excluidng the number itself is equal to the number, then the number is known to be a perfect number.)
10. The digital sum of any perfect square can be only 0, 1, 4, 9, 7. (Digital sum of any number is obtained by adding the digits of the number until we get a single digit. Digital sum of $385 = 3 + 8 + 5 = 1 + 6 = 7$).

An easy way to find the squares is to find two numbers which have a mean of it. This can be seen through the following example:

To find the square of 21, take 20 and 22, then multiply the two numbers together and add the square of the distance from the mean: $22 \times 20 = 440 + 1^2 = 441$. Here, we have used the following formula

$$(x - y)(x + y) = x^2 - y^2$$

known as the difference of two squares. Thus, $(21-1)(21+1) = 21^2 - 1^2 = 440$.

Odd and Even Square Numbers

Squares of even numbers are even, since $(2n)^2 = 4n^2$.

Squares of odd numbers are odd, since $(2n+1)^2 = 4(n^2+n)+1$.

Hence, we can infer that the square roots of even square numbers are even, and square roots of odd square numbers are odd.

Methods of Squaring

As we have seen in the case of multiplication, there are several methods for squaring also. Let us see the methods one by one.

Method 1: Base 10 Method

Understand it by taking few examples:

- Let us find out the square of 9. Since 9 is 1 less than 10, decrease it still further to 8. This is the left side of our answer.
- On the right hand side put the square of the deficiency that is 1^2. Hence, the answer is 81.
- Similarly, $8^2 = 64$, $7^2 = 49$.
- For numbers above 10, instead of looking at the deficit we look at the surplus. For example,
  $11^2 = (11 + 1).\ 10 + 1^2 = 121$
  $12^2 = (12 + 2).\ 10 + 2^2 = 144$

$14^2 = (14 + 4).\ 10 + 4^2 = 18\ .\ 10 + 16 = 196$

and so on.

This is based on the identities $(a + b)(a - b) = a^2 - b^2$ and $(a + b)^2 = a^2 + 2ab + b^2$.

We can be use this method to find the squares of any number, but after a certain stage, this method loses its efficiency.

Method 2: Base 50*n* Method here, (*n* is any natural number)

This method is nothing but the application of $(a + b)^2 = a^2 + 2ab + b^2$.

This can be seen in the following example:

Example Find the square of 62.

Because this number is close to 50, we will assume 50 as the base.

$(62)^2 = (50 + 12)^2 = (50)^2 + 2 \times 50 \times 12 + (12)^2 = 2500 + 1200 + 144$

To make it self explanatory a special method of writing is used.

$(62)^2 = [100\text{'s in (Base)}]^2 + \text{Surplus} \mid \text{Surplus}^2$

$= 25 + 12 \mid 144 = 38 \mid 44$ [Number before the bar on its left hand side is number of hundreds and on its right hand side are last two digits of the number.]

$(68)^2 = 25 + 18 \mid 324 = 46 \mid 24$

$(76)^2 = 25 + 26 \mid 676 = 57 \mid 76$

$(42)^2 = 25 - 8 \mid 64 = 17 \mid 24$ [$(a - b)^2 = a^2 - 2ab + b^2$]

Example Find the square of 113.

Since this number is closer to 100, we will take 100 as the base.

$(112)^2 = (100 + 12)^2 = (100)^2 + 2 \times 100 \times 12 + (12)^2 = 10000 + 2 \times 1200 + 144$

$(112)^2 = [100\text{'s in (Base)}]^2 + 2 \times \text{Surplus} \mid \text{Surplus}^2$

$= 100 + 2 \times 12 \mid 12^2 = 125 \mid 44$

Alternatively, we can multiply it directly using base value method.

Had this been 162, we would have multiplied 3 in surplus before adding it into $[100\text{'s in (Base)}]^2$ because assumed base here is 150.

$(162)^2 = [100\text{'s in (Base)}]^2 + 3 \times \text{Surplus} \mid \text{Surplus}^2$

$= 225 + 3 \times 12 \mid 12^2 = 262 \mid 44$

Method 3 : 10^n Method

This method is applied when the number is close to 10^n.

With base as 10^n, find the surplus or deficit (x)

Again answer can be arrived at in two parts

$(B + 2x) \mid x^2$

The right-hand part will consist of n digits. Add leading zeros or carry forward the extra to satisfy this condition.

$108^2 = (100 + 2 \times 8) \mid 8^2 = 116 \mid 64 = 11664$

$102^2 = (100 + 12 \times 2) \mid 2^2 = 104 \mid 04 = 10404$

$93^2 = (100 - 2 \times 7) \mid (-7)^2 = 86 \mid 49 \Rightarrow 8649$

$1006^2 = (1000 + 2 \times 6) \mid 6^2 = 10|12 \mid 036 = 1012036$

The right-hand part will consist of 2 digits. Add leading zeros or carry forward the extra to satisfy this condition.

$63^2 = (25 + 13) \mid 13^2 = 38 \mid 169 = 3969$

$38^2 = (25 - 12) + (-12)^2 = 13 \mid 144 = 1444$

Square Mirrors

$14^2 + 87^2 = 41^2 + 78^2$
$15^2 + 75^2 = 51^2 + 57^2$
$17^2 + 84^2 = 71^2 + 48^2$
$26^2 + 97^2 = 62^2 + 79^2$
$27^2 + 96^2 = 72^2 + 69^2$

Some Special Cases

1. Numbers ending with 5

If a number is in the form of n5, the square of it is $n(n+1) \mid 25$

Examples $45^2 = 4 \times 5 \mid 25 = 2025$

$135^2 = 13 \times 14 \mid 25 = 18225$

This is nothing but the application of the multiplication method using the sum of units digits.

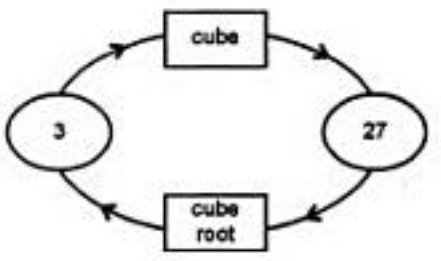
We can use this method to find out the squares fractions like $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2} \ldots$ also.

Process: Multiply the integral portion by the next higher integer and add $\frac{1}{4}$.

For example, $\left(6\frac{1}{2}\right) = 6 \times 7 + \frac{1}{4} = 42\frac{1}{4}$

CUBING

A number whose cube root is an integer is called a perfect cube.



Properties of a Cube

1. The sum of the cubes of any number of consecutive integers starting with 1 is the square of some integer. (For example, $1^3 + 2^3 = 9 = 3^2$, $1^3 + 2^3 + 3^3 = 36 = 6^2$, etc.)
2. Unit digit of any cube can be any digit from 0-9.

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Methods of Cubing

We can find the cube of any number close to a power of 10 say 10^n with base = 10^n by finding the surplus or the defict (x). The answer will be obtained in three parts. $B + 3x \mid 3 . x^2 \mid x^3$

The left two parts will have n digits.

104^3

Base B = 100 and surplus = $x = 4$

$(100 + 3 \times 4) \mid 3 \times 4^2 \mid 4^3 = 112 \mid 48 \mid 64 = 1124864$

109^3

Base B = 100 and $x = 9$

$(100 + 3 \times 9) \mid 3 \times 9^2 \mid 9^3 = 127 \mid 243 \mid 729 = 1295029$

98^3

Base B = 100 and $x = -2$

$(100 - 3 \times 2) \mid 3 \times (-2)^2 \mid (-2)^3 = 94 \mid 12 \mid -8 = 94 \mid 11 \mid 100 - 8 = 941192$

VEDIC MATHS TECHNIQUES IN ALGEBRA

1. If one is in ratio, the other one is zero

This formula is often used to solve simple simultaneous equations which may involve big numbers. But these equations in special cases can be visually solved because of a certain ratio between the coefficients. Consider the following example:

$6x + 7y = 8$

$19x + 14y = 16$

Here, the ratio of coefficients of y is the same as that of the constant terms. Therefore, the "other" is zero, i.e., $x = 0$. Hence, the solution of the equations is $x = 0$ and $y = 8/7$.

Alternatively,

$19x + 14y = 16$ is equivalent to. $(19/2)x + 7y = 8$.

Thus, x has to be zero and no ratio is needed, just divide by 2!

Note that it would not work if both had been "in ratio":

$6x + 7y = 8$

$12x + 14y = 16$

This formula is easily applicable to more general cases with any number of variables. For instance,

$ax + by + cz = a$

$bx + cy + az = b$

$cx + ay + bz = c$

which yields $x = 1, y = 0, z = 0$.

2. When samuccaya is the same, that samuccaya is zero

Consider the following symbols: N_1 – Numerator 1, N_2 – Numerator 2, D_1 – Denominator 1, D_2 – Denominator 2 and so on.

This formula is useful for solving equations that can be solved visually. The word "samuccaya" has various meanings in different applications. For instance, it may mean a term, which occurs as a common factor in all the terms concerned. For example, an equation "$12x + 3x = 4x + 5x$". Since "x" occurs as a common factor in all the terms, therefore, $x = 0$ is the solution. Alternatively, samuccaya is the product of independent terms. For instance, in $(x + 7)(x + 9) = (x + 3)(x + 21)$, the samuccaya is $7 \times 9 = 3 \times 21$, therefore, $x = 0$ is the solution. It is also the sum of the denominators of two fractions having the same numerical numerator, for example:

$1/(2x - 1) + 1/(3x - 1) = 0$ means $5x - 2 = 0$.

The more commonly used meaning is "combination" or total. For instance, if the sum of the numerators and the sum of denominators are the same then that sum is zero. Therefore,

$$\frac{2x+9}{2x+7} = \frac{2x+7}{2x+9}$$

Therefore, $4x + 16 = 0$ or $x = -4$.

This meaning ("total") can also be applied in solving the quadratic equations. The total meaning not only imply sum but also subtraction. For instance when given $N_1D_1 = N_2/D_2$, if $N_1 + N_2 = D_1 + D_2$ (as shown earlier) then this sum is zero. Mental cross multiplication reveals that the resulting equation is quadratic (the coefficients of x^2 are different on the two sides). So, if $N_1 - D_1 = N_2 - D_2$ then that samuccaya is also zero. This yields the other root of a quadratic equation.

The interpretation of "total" is also applied in multi-term RHS and LHS. For instance, consider

$$\frac{1}{x-7} + \frac{1}{x-9} = \frac{1}{x-6} + \frac{1}{x-10}$$

Here, $D_1 + D_2 = D_3 + D_4 = 2x - 16$. Thus $x = 8$.

There are several other cases where samuccaya can be applied with great versatility. For instance "apparently cubic" or "biquadratic" equations can be easily solved as shown below:

$$(x - 3)^2 + (x - 9)^3 = 2(x - 6)^3.$$

Note that $x - 3 + x - 9 = 2(x - 6)$. Therefore $(x - 6) = 0$ or $x = 6$.

Consider

$$\frac{(x+3)^3}{(x+5)^3} = \frac{x+1}{x+7}$$

Observe: $N_1 + D_1 = N_2 + D_2 = 2x + 8$.

Therefore, $x = -4$.

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2
NUMBER SYSTEM

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Numbers and their different types
- Definitions and properties of those numbers
- Concepts attached to these numbers
- Kinds of questions which are asked in the CAT
- Methods of solving

CAT 2007

Number of questions from this topic: 1 (4 Marks)

CAT 2006

Number of questions from this topic: 3 (12 Marks)

CAT 2005

Number of questions from this topic: 6 (11 Marks)

Introduction to the topic

With respect to preparation for the CAT, number system has been seen to be one of the most important topics in the quantitative appitude (QA) paper. During the last 15 years, it has been observed that the QA paper has had 20%–30% questions from the chapter on number system. Numbers in this chapter do not have that important a role to play as has logic. In other words, we can say that logical processes outweigh calculations in finding a solution to phenomenally lengthy mathematical problems in number system. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about numbers and their properties.

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TYPES OF QUESTIONS

Questions asked from this topic are of the following two kinds:

1. Based on definitions or properties of properties of numbers
2. Based on concepts.

1. Based on definitions Questions of this kind are based on definitions and properties of different kinds of numbers, involving formulae and some very basic calculations.

2. Based on concepts The other kind of questions are those that are based on the following concepts:

i. LCM and HCF
ii. Divisibility
iii. Divisibility rules (for base other than 10)
iv. Number of divisors
v. Number of exponents
vi. Remainders
vii. Base system
viii. Units digit
ix. Tens digit
x. Pigeon-hole principle

CLASSIFICATION OF NUMBERS/ INTEGERS

Natural Numbers

Natural numbers are counting numbers, i.e., the numbers which we use to count. For example, 1, 2, 3, ... The lowest natural number is 1.

Whole Numbers

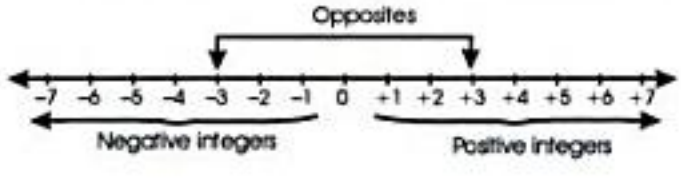
When zero is included in the list of natural numbers, the numbers are known as whole numbers. For example, 0, 1, 2, ... The lowest whole number is 0.

Integers

Integers are whole numbers, the negative of whole numbers and zero. For example, 43434235, 28, 2, 0, −28, and −3030 are integers, but numbers like 1/2, 4.00032, 2.5, Pi, and −9.90 are not whole numbers.

Number Line

The number line is used to represent the set of real numbers. A representation of the number line is given below:



Properties of Number Line

- The number line goes on till infinity in both directions. This is indicated by the arrows.
- Integers greater than zero are called positive integers. These numbers are to the right of zero on the number line.
- Integers less than zero are called negative integers. These numbers are to the left of zero on the number line.
- The integer zero is neutral. It is neither positive nor negative.
- The sign of an integer is either positive (+) or negative (−), except zero, which has no sign.
- Two integers are the opposites if each of the two are at the same distance from zero, but on the opposite side of the number line. One will have a positive sign while the other a negative sign. In the number line above, +3 and −3 are labelled as opposites. In other words, the whole negative number scale looks like a mirror image of the positive number scale, with a number like −15 being at the same distance away from 0 as the number 15 is.
- The number halfway between −1 and −2 is −1.5; just as the number halfway between 1 and 2 is 1.5.
- We represent positive numbers without using a +ve sign. For example, we would write 29.1 instead of +29.1. But when we talk of negative numbers, the sign must be there.

Prime Numbers and Composite Numbers

There are two types of natural numbers, prime and composite, except 1.

Prime numbers

The numbers which are divisible by 1 and itself only are known as prime numbers.

In other words, we can say that all the numbers which have only two factors are known as prime numbers.

Prime numbers can also be seen as the building blocks. We combine two or more than two prime numbers, same or distinct, to create numbers bigger than these prime numbers. For example, $3 \times 2 = 6$.

To know more about primes, go to *www.primes.utm.edu/largest.html*

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List of all the prime numbers less than 1000

2	3	5	7	11	13	17	19
23	29	31	37	41	43	47	53
59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131
137	139	149	151	157	163	167	173
179	181	193	197	199	211	223	227
229	233	239	241	251	257	263	269
271	277	281	283	293	307	311	313
317	331	337	347	349	353	359	367
373	379	383	389	397	401	409	419
421	431	433	439	443	449	457	461
463	467	479	487	491	499	503	509
521	523	541	547	557	563	569	571
577	587	593	599	601	607	613	617
619	631	641	643	647	653	659	661
673	677	683	691	701	709	719	727
733	739	743	751	757	761	769	773
787	797	809	811	821	823	827	829
839	853	857	859	863	877	881	883
887	907	911	919	929	937	941	947
953	967	971	977	983	991	997	

Number of prime numbers for every 100 numbers

Nos. from-to	1-100	101-200	201-300	301-400	401-500	501-600	601-700	701-800	801-900	901-1000
No. of primes	25	21	16	16	17	14	16	14	15	14

Largest prime number till date and the history of prime number The largest known prime number, today, is 7,816,230-digit prime number, $2^{25964951}-1$, found in early 2005. But how big have the "largest known primes" been historically and when can we see the first billion-digit prime number?

Table 1.1 Records before the advent of electronic computers

Number	*Digits*	*Year*	*Prover*	*Method*
$2^{17}-1$	6	1588	Cataldi	Trial division
$2^{19}-1$	6	1588	Cataldi	Trial division
$2^{31}-1$	10	1772	Euler	Trial division
$(2^{59}-1)/179951$	13	1867	Landry	Trial division
$2^{127}-1$	39	1876	Lucas	Lucas sequences
$(2^{148}+1)/17$	44	1951	Ferrier	Proth's theorem

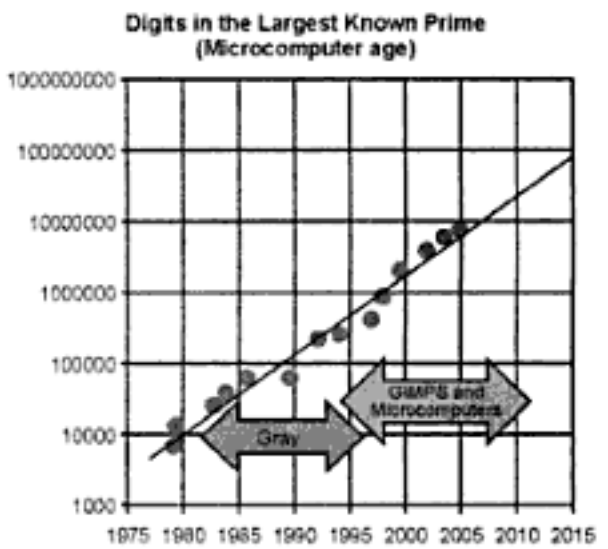
The prime number found by Lucas in 1876 was accepted as the largest prime number till 1951. In 1951, Ferrier used a mechanical desk calculator and techniques based on partial inverses of Fermat's little theorem (see the pages on remainder theorem) to slightly better this record by finding a 44-digit prime.

In 1951, Ferrier found the prime $(2^{148}+1)/17 = 20988936657440586486151264256610222593863921$. However, this record was very short-lived. In the same year, the advent of electronic computers helped in finding a bigger prime number. Miller and Wheeler began the electronic computing age by finding several primes as well as the new 79-digit record: $2^{127}-1$.

During this period everybody was working hard to find out the primes with the help of computers. Records were broken at a never-before pace. Sometimes, within a day.

When will we have a one-billion digit prime? Using the regression line given above, we can expect

- 10,000,000-digit prime by 2007,
- a 100,000,000-digit prime by 2016, and
- a 1,000,000,000-digit prime by early 2027.



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Formula for prime numbers

Till now, all attempts made to arrive at a single formula to represent all prime numbers have proved to be fruitless. It is all because there is no symmetricity between the differences among the prime numbers. Sometimes, two consecutive prime numbers differ by 2, sometimes by 4, and sometimes even by 10,000 or more.

However, there are some standard notations, which give us a limited number of prime numbers

$N^2 + N + 41 \rightarrow$ For all the values of N from –39 to +39, this expression gives us a prime number.

$N^2 + N + 17$ is again a similar example.

Properties of Prime Numbers

1. All prime numbers end in 1, 3, 7, or 9, except for 2 and 5. (Numbers ending in 0, 2, 4, 6 or 8 can be divided by 2 and numbers ending in 5 can be divided by 5, due to the use of a base 10 (2 × 5) system.)
2. If p is a prime number and p divides a product ab of integers, then p divides a or p divides b.
3. If p is prime and a is any integer, then $a^p - a$ is divisible by p. This is knows as **Fermat's Little theorem.**
4. If p is a prime number other than 2 and 5, 1/p is always a recurring decimal, with a period of p–1 or a divisor of p–1.
5. An integer $p > 1$ is prime if and only if the factorial $(p-1)! + 1$ is divisible by p. This is known as **Wilson's theorem.**
   Conversely, an integer $n > 4$ is composite if and only if $(n-1)!$ is divisible by n.
6. If n is a positive integer greater than 1, then there is always a prime number p with $n < p < 2n$.
7. All the prime numbers P > 3 give a remainder 1 or 5 when divided by 6; that is, all the prime numbers > 3 will be of the format $6N \pm 1$. But vice-versa is not always true.
8. There is only one set of three prime numbers with a gap of 2 between two prime numbers and that set is 3, 5 and 7.

Composite numbers

A number is composite if it is the product of two or more than two distinct or same prime numbers. For example, $\rightarrow$ 4, 6, 8,…

$4 = 2^2$

$6 = 2^1 \times 3^1$

Lowest composite number is 4.

Even and Odd Numbers

Suppose N is an integer. If there exists an integer P such that N = 2P + 1, then N is an *odd number*. If there exists an integer P such that N = 2P, then N is an *even number*.

Putting in a simple language, even numbers are those integers which are divisible by 2 and odd numbers are those integers which are not divisible by 2.

Even and odd numbers can be positive as well as negative also.

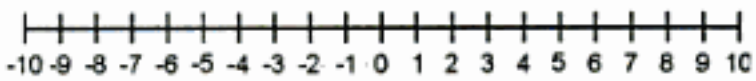
In other words, if x is any integer (even or not), then $2x$ will be an even integer, because it is a multiple of 2. Also x raised to any positive integer power will be an even number, so x^2, x^3, x^4 etc. will be even numbers.

Any integer that is not a multiple of 2 is called an *odd* number. For instance, –1, 3, 6883 and –8147 are all odd numbers. Any odd number raised to a positive integer power will also be an odd number, so if x is an odd number, then so will x^2, x^3, x^4 etc. be odd numbers.

The concept of even and odd numbers are most easily understood in the binary base. Above definition simply states that even numbers end with a 0, and odd numbers end with a 1.

Comparing Integers

We can compare two different integers by looking at their positions on the number line. For any two different places on the number line, the integer on the right is greater than the integer on the left. Note that every positive integer is greater than any negative integer.



Examples $9 > 4$, $6 > -9$, $-2 > -8$, and $0 > -5$
$-2 < 1$, $8 < 11$, $-7 < -5$, and $-10 < 0$

Points to Remember

- 1 is neither prime nor composite.
- 0 is neither positive nor negative.

Example 1 Two of a, b, c and d are even and two are odd, not necessarily in order. Which of the following is definitely even? (CAT 1997)

(a) $a + b + c - 2d$
(b) $a + 2b - c$
(c) $a + b - c + d$
(d) $2a + b + c - d$

Solution Since we don't know which two are even and which two are odd, we will have to do a bit of hit-and- trial to solve this problem, with the help of options.

In option (a), if a and b are even, and c and d are odd, then this will lead us to odd number.

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In option (b), if a and b are even, and c is odd, then this will lead us to odd number.

In option (c), whatever is the value of a, b, c and d, it is always going to be an even number.

In option (d), if a and b are odd, and c and d are even, then this will lead us to odd number.

Explanation Whatever kind of calculation we do with two even and two odd numbers, we will always get an even result.

So, the answer is option (c).

Example 2 If N, N + 2 and N + 4 are prime numbers, then the number of possible solutions for N is are:

(CAT 2003)

(a) 1 (b) 2 (c) 3 (d) None of these

Solution There is only one triplet of prime numbers where difference between any two prime number is 2, that is 3, 5 and 7. So, N = 3 is the only solution.

Hence, the answer is (a).

Proof of above example We know that prime numbers are of the form $6M \pm 1$ (except 2 and 3). Now if N is of the format $6M + 1$, then $N + 2$ will of $6M + 3$ format and $N + 4$ will be of $6M + 5$ format. Out of these three numbers, since $N + 2$ is of $6M + 3$ format, it will be divisible by 3.

Similarly, if N is of the format $6M - 1$, then $N + 2$ will of $6M + 1$ format and $N + 4$ will be of $6m + 3$ format. Out of these three numbers, since $N + 4$ is of $6M + 3$ format, it will be divisible by 3.

In both the cases, we find that one number out of given three number is divisible by 3. In the example given above (3, 5 and 7), one of the given three numbers is divisible by 3.

Example 3 Let x and y be positive integers such that x is prime and y is composite. Then which of the following is true? (CAT 2003)

(a) $y - x$ cannot be an even integer

(b) xy cannot be an even integer

(c) $\frac{x+y}{x}$ cannot be an even integer

(d) None of these

Solution Eliminating the options,

To eliminate option (a), If $y = 4$ and $x = 2$, then $y - x$ can be even.

To eliminate option (b), If $y = 4$ and $x = 2$, then yx can be even.

To eliminate option (c), If $y = 6$ and $x = 2$, then it can be even.

So, the answer is option (d).

QUESTIONS BASED ON CONCEPTS

In this section, we will discuss the concepts and questions based upon them.

LCM and HCF

Meaning of LCM

A common multiple is a number that is a multiple of two or more than two numbers. The common multiples of 3 and 4 are 12, 24,...

The least common multiple (LCM) of two numbers is the smallest positive number that is a multiple of both.

Multiples of 3 → 3, 6, 9, 12, 15, 18, 21, 24,...

Multiples of 4 → 4, 8, 12, 16, 20, 24, 28,...

So, the LCM of 3 and 4 will be 12, which is the lowest common multiple of 3 and 4.

First of all, the basic question is what kind of numbers we can use for LCM.

Let us see this through an example— LCM of 10, 20 and 25 is 100. It means that 100 is the lowest number. which is divisible by all these three numbers.

But cannot the LCM be (−100)? Since (−100) is lower than 100 and divisible by each of 10, 20 and 25. Or, it can be zero also.

Or, What will be the LCM of (−10) and 20?

Will it be (–20) or (–200) or (–2000) or the smallest of all the numbers, i.e., $-\infty$

Answer to all these questions is very simple. LCM is a concept defined only for positive numbers, be it an integer or a fraction i.e., LCM is not defined for negative numbers or zero.

Now, we will define a different method of finding out the LCM of two or more than two positive integers.

Process to find out LCM

Step 1 Factorize all the numbers into their prime factors.

Step 2 Collect all the distinct factors.

Step 3 Raise each factor to its maximum available power and multiply.

Example 4 Find LCM of 10, 20, 25.

Solution *Step 1* $10 = 2^1 \times 5^1$

$20 = 2^2 \times 5^1$

$25 = 5^2$

Step 2 2, 5

Step 3 $2^2 \times 5^2 = 100$

The biggest advantage of using this method lies in

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the fact that we can find out the LCM of any number of numbers in a straight line without using the conventional method. It can be understood in the following way with the previous example.

First of all, find out the LCM of 10, 20 = 20, and now the LCM of 20 and 25 = 100 (For this you will have to check which factor of 25 is not present in 20 and then multiply by this factor. Since 25 has 5^2 and 20 has 5^1 only, so we will multiply 20 by 5.)

Example 5 Find LCM of 35, 45, 55.

Solution First of all, find out the LCM of 35 and 45.

Now $35 = 5^1 \times 7^1$ and $45 = 3^2 \times 5^1$.

So, it can be observed here that 35 does not have 3^2 in it, so we will multiply 35 by 3^2.

So, LCM of 35 and 45 = 35×3^2. (You can start with 45 also to find out about the missing factors of 35 in 45.)

Now, we will find out the LCM of 35×3^2 and 55

$55 = 5^1 \times 11^1$

Now, 11^1 is not there with 35×3^2. So, we will multiply 35×3^2 with 11^1.

So, finally the LCM = $35 \times 3^2 \times 11^1 = 3465$.

Relationship between factors of a number and its LCM We can use the factors of a number in several ways to produce that under as the LCM.

Example N = 72. How many sets of two values (a, b) are there for which the LCM (a, b) is 72?

Solution $N = 72 = 2^3 \times 3^2$

Values of a and b have to be chosen in such a way that both 2^3 and 3^2 are contained in either a or b.

Possible sets are (72 ,1) (72, 2) (72, 3) (72, 4) (72, 6) ...(72, 36), (72, 72). These are a total of 12 sets.

The other possible sets are (9, 8), (9, 24), (8, 18), (8, 36),...

These are a total of six sets.

So, the total number of possible sets are 12 + 6 = 18 sets

In general, if $N = a^p \times b^q \times c^r \times \ldots$, then the set of values (x, y) such that LCM (x, y) is N can be summed up as:

Number of sets (x, y) =

$$\frac{(2p+1)(2q+1)(2r+1)\ldots-1}{2}+1$$

Meaning of HCF

Factors are those positive integral values of a number, which can divide that number. HCF, which is known as GCD (Greatest Common Divisor) also, is the highest value which can divide the given numbers.

Factors of 20: 1, 2, 4, 5, 10, 20.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30.

So, 10 will be the HCF of 20 and 30.

Process to find out HCF

Step 1 Factorize all the numbers into their prime factors.

Step 2 Collect all the common factors.

Step 3 Raise each factor to its minimum available power and multiply.

Example 6 Find HCF of 100, 200 and 250.

Solution

Step 1 $100 = 2^2 \times 5^2$
$200 = 2^3 \times 5^2$
$250 = 5^3 \times 2^1$

Step 2 2, 5

Step 3 $2^1 \times 5^2 = 50$

Alternatively, to find out HCF of numbers like 100, 200 and 250, it is required to observe the quantity which can be taken out common from these numbers. By writing these numbers as $(100x + 200y + 250z)$, it can be very easily observed that 50 is common out of these numbers.

Summarizing LCM and HCF

It is very essential to understand the mechanism of finding out LCM and HCF. We can simply understand the mechanism to find out lowest common multiple and highest common factor through this example.

Example Find out LCM and HCF of 16, 12, 24.

Solution

No.	Multiples	Factors
16	16, 32, 48, 64, 80, 96, 112, 128,...	1, 2, 4, 8, 16
12	12, 24, 36, 48, 60, 72, 84, 96, 108,...	1, 2, 3, 4, 6, 12
24	24, 48, 72, 96, 120, 144, 168, 192,...	1, 2, 3, 4, 6, 8, 12, 24
	Common Multiple	Common Factor
	48	1, 2, 3, 4
	Lowest Common Multiple	Highest Common Factor
	48	4

Standard Formulae

1. LCM × HCF = Product of two numbers.
   This formula can be applied only in case of two numbers. However, if the numbers are relatively prime to each other (i.e. HCF of numbers = 1), then this formula can be applied for any number of numbers.
2. LCM of fractions = LCM of numerator of all the fractions/HCF of denominator of fractions.
3. HCF of fractions = HCF of numerator of all the fractions/LCM of denominator of fractions.
4. HCF of (sum of two numbers and their LCM) = HCF of numbers.

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Example HCF of two natural numbers A and B is 120 and their product = 10,00 0. How many set of values of A and B is/are possible?

Solution HCF (A, B) = 120 ⇒ 120 is a common factor of both the numbers (120 being the HCF). Hence 120 is present in both the numbers. So the minimum product of A and B = 120 × 120 = 14400. Hence no set of A and B are possible satisfying the conditions.

QUESTIONS BASED ON STANDARD APPLICATION OF LCM AND HCF

Case 1 Time and work

Example 7 Tatto can do a work in 10 days and Tappo can do the same work in 12 days. How many days will it take if both of them start working together?

Solution Let us assume total work = LCM of (10, 12) units = 60 units. Now, since 60 units of work is being done by Tatto in 10 days, so Tatto is doing 6 units of work per day and similarly, Tappo is doing 5 units of work per day. Hence, together they are doing 11 units of work in one day. So, finally they will take $\frac{60}{11} = 5\frac{5}{11}$ days to complete the work.

Case 2 Time, speed and distance: Circular motion

Example 8 The speed of A is 15 m/s and the speed of B is 20 m/s. They are running around a circular track of length 1000 m in the same direction. After how much time will they meet at the starting point if they start running at same time?

Solution Time taken by A in taking one circle = 66.66

Time taken by B in taking one circle = 50

LCM (66.66, 50) = 200

Case 3 Number system: Tolling the bell

Example 9 There are two bells in a temple. Both the bells toll at a regular interval of 66.66 sec and 50 sec respectively. After how much time will they toll together for the first time?

Solution Time taken by the 1st bell to toll = 66.66

Time taken by the 2nd bell to toll = 50

LCM (66.66, 50) = 200

It can be observed here that mathematical interpretation of both the questions are same, just the language has been changed.

Case 4 Number system: Number of rows

Example 10 There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains the same number of fruits of one type. What is the minimum number of rows required for this to happen?

Solution We can put one fruit in one row, and in (24 + 36 + 60) 120 rows, we can arrange all the fruits. Or, we can put two fruits in one row and can arrange all the fruits in 60 rows. But for the rows to be minimum, the number of fruits should be maximum in one row.

HCF of 24, 36, 60 = 12, so 12 fruits should be there in one row.

Hence, the number of rows = 10

Case 5 Number system: Finding the remainder

Example 11 There is a number which when divided by 4 and 5 gives 3 as the remainder. What is the lowest three-digit number which satisfy this condition?

Solution Let us assume that there is no remainder. So, the number has to be a multiple of LCM of 4 and 5. Now, LCM (4, 5) = 20

But there is a remainder of 3 when divided by 4 and 5. So, the number will be of the form (20N + 3).

Hence, the numbers are 23, 43, 63, 83, 103 and so on…

So, 103 is the answer.

DIVISIBILITY RULES (FOR DECIMAL SYSTEM)

Divisibility rules are quite imperative, because with the help of these rules we can infer if a particular number is divisible by another number or not, without actually dividing it. Divisibility rules of numbers are specific to that particular number only. It simply means that divisibility rules of different numbers will be different. We shall now see a list of divisibility rules for some of the natural numbers.

Divisibility Rules

For 2 If unit digit of any number is 0, 2, 4, 6 or 8, then that number will be divisible by 2.

For 3 If sum total of all the digits of any number is divisible by 3, then the number will be divisible by 3. (e.g., 123, 456 etc.)

Example 12 How many values of A are possible if 3245684A is divisible by 3?

Solution Sum total of the number = 32 + A

For this number to be divisible by 3, A can take three values namely 1 or 4 or 7. (No other values are possible since A is the unit digit of the number.)

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For 4 If last two digits of a number are divisible by 4, then that number will be divisible by 4 (e.g., 3796, 248, 1256).

For 5 If last digit of the number is 5 or 0, then that number will be divisible by 5.

For 6 If last digit of the number is divisible by two and sum total of all the digits of the number is divisible by 3, then that number will be divisible by 6.

For 7 The integers are divisible by 7 if and only if, the difference of the number of its thousands and the remainder of its divisible by 1000, is divisible by 7.

For 7 If the difference between the numbers of tens in the number and twice the unit digits is divisible by 7, then the given number is divisible by 7.

Example Let us take the number 795. The units digit is 5 and when it is doubled it, we get 10. The remaining part of the number (i.e., the tens) is 79. If 10 is subtracted from 79 we get 69. Since this result is not divisible by 7, the original number 795 is not divisible by 7.

For 8 If last 3 digits of the number is divisible by 8, then the number itself will be divisible by 8 e.g., 128, 34568, 76232 etc.

For 9 If sum of the digits of the number is divisible by 9, then the number will be divisible by 9 e.g., 129835782.

$1+2+9+8+3+5+7+8+2=45$. Since 45 is divisible by 9, then the number will be divisible by 9.

Example 13 How many pairs of A and B are possible in number 89765A4B if the number is divisible by 9, given that last digit of the number is even?

Solution Sum of digits of number is $8+9+7+6+5+A+4+B=39+A+B$.

So, A + B should be 6 or 15. Next value should be 24 but since A and B are digits so it can't be more than 18. Possible pairs of A and B are

A	B
0	6
1	5
2	4
3	3
4	2
5	1
6	0
7	8
8	7
9	6
6	9

Since B is even, six possible set of values of A and B are there.

For 11 A number is divisible by 11, if the difference between the sum of the digits at the even places and the sum of the digits at the odd places is divisible by 11 (zero is divisible by 11) e.g., 6595149 is divisible by 11 as the difference of $6+9+1+9=25$ and $5+5+4=14$ is 11.

For 12 If the number is divisible by 3 and 4, then the number will be divisible by 12, e.g., 144, 348.

For 13 **(A + 4B)**, where B is the units place digit and A is all the remaining digits.

Example Checking the divisibility of 1404 by 13: Here, A =140 and B = 4, then $A+4B=140+4\times4=156$. 156 is divisible by 13, so 1404 will be divisible by 13.

For 14 If the number is divisible by both 2 and 7, then the number will be divisible by 14.

For 15 A number is divisible by 15, if the sum of the digits is divisible by 3 and unit digit of the number is 0 or 5, e.g., 225, 450, 375 etc.

For 16 A number is divisible by 16, if the number formed by the last 4 digits of the given number is divisible by 16 e.g., 12578320 is divisible by 16, since last 4 digits of the number, 8320 is divisible by 16.

For 17 **(A – 5B)**, where B is the units place digit and A is all the remaining digits.

For 18 Number should be divisible by both 9 and 2.

For 19 **(A + 2B)**, where B is the units place digit and A is all the remaining digits.

If the sum of the number of tens in the number and twice the unit digit is divisible by 19, then the number is divisible by 19.

For example, let us take the number 665. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 66. If 10 (which is the unit digit doubled) is added to 66 we get 76. Since, 76 is divisible by 19, it means the original number 665 is also divisible by 19.

For 20 The number should be divisible by 4 and 5.

Process to find out the divisibility rule for prime numbers

Process for the divisibility rule for P, a prime number is as follows:

Step 1 Find the multiple of P, closest to any multiple of 10. (This essentially will be of the form 10K + 1 or 10K –1.)

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Step 2 If it is 10 K – 1, then the divisibility rule will be A + KB, and if it is 10K + 1, then the divisibility rule will be A – KB, where B is the units place digit and A is all the remaining digits.

Example Finding out the divisibility rule of 23: Lowest Multiple of 23, which is closest to any multiple of 10 = 69 = 7 × 10 – 1

So, rule is A + 7 B.

Divisibility rule for any number of the format $10^n \pm 1$, where *n* is any natural number

In this method, we will be considering two factors, the first one being the value of n and the second one being the sign '+' or '–'. The value of '*n*' signifies how many digits will be taken one at a time and the sign signifies in what manner these digits will be taken. Let us see this with the help of some examples.

When *n* = 1

Case 1:$10^n + 1 = 11$

This method tells us that now we will be considering one digit at a time of the number which is to be divided by 11 and since the sign is '+' between 10^n and 1, we will alternately subtract and add starting from the right hand side.

Example To check if 123345 is divisible by 11, we will start from the right hand side, by subtracting the first number and then adding the third and then subtracting the fourth and so on…

Case 1:$10^n - 1 = 9$

Since the value of *n* is one, we will take one digit at a time and due to the '–' sign between 10^n and 1, we will keep on adding the digits from the right hand side (or, can be done from the left hand side also in this case).

When *n* = 2

Case 1:$10^n + 1 = 101$

Since the value of *n* here is two, we will make the pairs of digits from right hand side, and then we will alternately subtract and add each of the pairs.

Example Checking if 12345678 is divisible by 101: Make the pairs first – 12 34 56 78. Now we will do this calculation – (78) – (56) + (34) – (12) = 44, which is not a multiple of 101, so this number 12345678 is not divisible by 101.

Case 2:$10^n - 1 = 99$

Since the value of *n* is two, we will take two digit at a time after making the pairs and due to the '–' sign between 10^n and 1, we will keep on adding the pairs from right hand side (or, can be done from left hand side also in this case).

Example Checking if 122166 is divisible by 99: Let us make the pairs first: 12 21 66. Now add the pairs 12 + 21 + 66 = 99, so this number 122166 is divisible by 99.

And so on, we extend it for any natural number value of *n*.

NUMBER OF DIVISORS/FACTORS

If one integer can be divided by another integer an exact number of times, then the first number is said to be a *multiple* of the second, and the second number is said to be a *factor* of the first.

For example, 48 is a *multiple* of 6 because 6 goes into 48 an exact number of times (8 times in this case). Similarly, 6 is a *factor* of 48. On the other hand, 48 is not a multiple of 5, because 5 does not go into 48 an exact number of times. So, 5 is not a factor of 48.

When we talk about the number of divisors of any number, we are basically talking about positive integral divisor of that number. Likewise, it can be observed that 20 is having six divisors namely 1, 2, 4, 5, 10 and 20.

Its very essential here to understand the mechanism of formation of divisors.

Since $20 = 2^2 \times 5^1$

We will take three power of 2 viz., 2^0, 2^1 and 2^2 and two powers of 5 viz., 5^0 and 5^1.

Divisors will come from all the possible arrangements of powers of 2 and powers of 5.

$2^0 \times 5^0 = 1$
$2^0 \times 5^1 = 5$
$2^1 \times 5^0 = 2$
$2^1 \times 5^1 = 10$
$2^2 \times 5^0 = 4$
$2^2 \times 5^1 = 20$

Now, to find out the number of divisors of any number, we can use the following formula

If N is any number which can be factorised like $N = a^p \times b^q \times c^r \times \ldots$, where a, b and c are prime numbers.

Number of divisors = $(p + 1)(q + 1)(r + 1)\ldots$

Example 14 Find the number of divisors of $N = 420$.

Solution $N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

So, the number of divisors =
$(2 + 1)(1 + 1)(1 + 1)(1 + 1) = 24$

Example 15 Find the total number of even and prime divisors of $N = 420$.

Solution $N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

Odd divisors will come only if we take zero power of 2 (since, any number multiplied by any power (≥1) of 2 will give us an even number)

So, odd divisors will come if we take $N_1 = 2^0 \times 3^1 \times 7^1 \times 5^1$

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So, number of odd divisors = (0 + 1) (1 + 1) (1 + 1) (1 + 1) = 8

So, total number of even divisors = total number of divisors – number of odd divisors = 24 – 8 = 16

Alternatively, we can also find out the number of even divisors of $N = 420$ directly (or, in general for any number).

$420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

To obtain the factors of 420 which are even, we will not consider 2^0, since $2^0 = 1$

So, the number of even divisors of 420 = (2) (1 + 1) (1 + 1) (1 + 1) = 16

(We are not adding 1 in the power of 2, since we are not taking 2^0 here, i.e., we are not taking one power of 2.)

Prime divisor = 4 (namely 2, 3, 5 and 7 only)

Example 16 $N = 2^7 \times 3^5 \times 5^6 \times 7^8$. How many factors of N are divisible by 50 but not by 100?

Solution All the factors which are divisible by 50 but not divisible by 100 will have atleast two powers of 5, and one power of 2.

And its format will be $2^1 \times 5^{2+7}$.

So, number of divisors = $1 \times 6 \times 5 \times 9 = 270$

Alternatively, this is equal to (Number of factors divisible by 50) – (Number of factors divisible by 100).

Condition for two divisors of any number *n* to be co-prime to each other

Let us see it for $N = 12$

Total number of factors of 12 = 6 (namely 1, 2, 3, 4, 6, 12)

Now, if we have to find out the set of factors of this number which are co-prime to each other, we can start with 1.

Number of factors which are co-prime to 1 = 5 (namely 2, 3, 4, 6, 12)

Next in line is the number of factors which are co-prime to 2 = 1 (namely 3)

So, the total number of set of factors for 12 which are co-prime to each other = 6

So, we can induce that if we have to find out the set of factors which are co-prime to each other for $N = a^p \times b^q$, will be equal to $[(p + 1)(q + 1) - 1 + pq]$.

If there are three prime factors of the number, i.e., $N = a^p \times b^q \times c^r$, then set of co-prime factors can be given by $[(p + 1)(q + 1)(r + 1) - 1 + pq + qr + pr + 3\,pqr]$

Alternatively, we can find out the set of co-prime factors of this number by pairing it up first and then finding it out with the third factor.

Example 17 Find the set of co-prime factors of the number N = 720.

Solution $720 = 2^4 \times 3^2 \times 5^1$

Using the formula for three prime factors $[(p + 1)(q + 1)(r + 1) - 1 + pq + qr + pr + 3\,pqr]$

We get, $[(4 + 1)(2 + 1)(1 + 1) - 1 + 4.2 + 2.1 + 4.1 + 3.4.2.1] = 67$

Alternatively, let us find out first for $2^4 \times 3^2 = [(4 + 1)(2 + 1) - 1 + 4.2] = 22$

Now $p^{22} \times 5^1$ will give us $[(22 + 1)(1 + 1) - 1 + 22.1] = 67$

Sum of Divisors

Like the number of divisors of any number, we can find out the sum of divisors also.

If N is any number which can be factorised like $N = a^p \times b^q \times c^r \times \ldots$, where a, b and c are prime numbers. Then sum of the divisors

$$= \frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)}$$

REMAINDERS

Dividend = Quotient × Divisor + Remainder

Basics of Remainder

1. When any positive number A is divided by any other positive number B, and if B > A, then the remainder will be A itself.
   For example remainder of 5/12 = 5
   Remainder of 212/678 = 212
2. Remainder should always be calculated in its actual form i.e., you can not reduce the fraction to its lower ratio.
   For example, remainder of 1/2 = 1
   Remainder of 2/4 = 2
   Remainder of 3/6 = 3
   It can be observed that despite all the fractions being equal, remainders are different in each case.

Example 18 What is the remainder when 5×10^5 is divided by 6×10^6?

Solution As we know that we cannot reduce the fractions to its lower terms, the remainder obtained will be equal to 5×10^5.

3. Concept of negative remainder: As obvious from the name, remainder implies that something has been left out. So, remainder simply can never be negative. Its minimum value can be zero only and not negative.

Example 19 What is the remainder when 50 is divided by 7?

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Solution $-50/7 = (-56 + 6)/7$; which gives a remainder of 6.

Or, when we divide -50 by 7, we get -1 as the remainder. Now, since remainder has to be non-negative, so we add 7 (quotient) to it which makes final remainder as→ $-1 + 7 = 6$.

There are two methods to find out the remainder of any expression:

- Cyclicity Method
- Theorem Method

Cyclicity method

For every expression of a remainder, there comes attached a specific cyclicity of remainders.

Example 20 What is the remainder when 4^{1000} is divided by 7?

Solution To find the cyclicity, we keep finding the remainders until some remainder repeats itself. It can be understood with the following example

Number/7 →	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder →	4	2	1	4	2	1	4	2

Now, 4^4 gives us the same remainder as 4^1, so the cyclicity is of 3 (Because remainders start repeating themselves after 4^3.)

So, any power of 3 or a multiple of 3 will give a remainder of 1. So, 4^{999} will give 1 as the remainder.

Final remainder = 4

Example 21 What is the remainder when 4^{96} is divided by 6? **(CAT 2003)**

Solution Finding out the cyclicity,

Number/6 →	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder →	4	4	4	4	4	4	4	4

The remainder in all the cases is 4, so the final remainder will be 4. Actually, there is no need to find remainders upto 4^8 or even 4^3. 4^2 itself gives us a remainder of 4 when divided by 6, which is the same as the remainder obtained when 4^1 is divided by 6. So, we have got the cyclicity here, which is of 1.

Hence, final remainder = 4

It also can be observed here that if we write $4^{100}/6 = 2^{200}/6 = 2^{199}/3$, then remainder obtained will be 2, which is not the right answer (as given in CAT brochure of next year i.e., CAT 2004)

Remainder Theorem

The product of any two or more than two natural numbers has the same remainder when divided by any natural numbers, as the product of their remainders.

Let us understand this through an example.

Example 22 Remainder $\frac{12\times13}{7}$ = Remainder $\frac{156}{7} = 2$

Normal way of doing this is – Product →→→ Remainder

Theorem method – Remainder →→→ Product →→→ Remainder

So, first of all we will find the remainders of each number individually and then multiply these individual remainders to find out the final remainder.

Remainder 12/7 = 5

Remainder 13/7 = 6

Remainder $\frac{12\times13}{7}$ = Remainder $(5 \times 6)/7$

= Remainder $30/7 = 2$

Example 23 What is the remainder obtained when $(1421 \times 1423 \times 1425)$ is divided by 12? **(CAT 2000)**

Solution Remainder of $1421/12 = 5$

Remainder of $1423/12 = 7$

Remainder of $1425/12 = 9$

Remainder $(1421 \times 1423 \times 1425)/12$

= Remainder $(5 \times 7 \times 9)/12$

= Remainder $(5 \times 63)/12$

= Remainder $(5 \times 3)/12 = 3$

Concept of Successive Division

Suppose N is any number which is divided successively by 3 and 5, which means – first, we divide N by 3 and then the quotient obtained is divided by 5.

Example Let us see the case when 50 is divided by 5 and 3 successively.

50 divided by 5 gives 10 as the quotient. Now, we will divide 10 by 3. It gives finally a quotient of 3 and remainder of 1.

Example 24 When a number N is divided successively by 3 and 5, remainder obtained are 1 and 2 respectively. What is the remainder when N is divided by 15?

Solution It can be seen that we are required to calculate it from back-end.

Family of numbers which when divided by 5 gives remainder $2 = 5\,S + 2$

So, $N = 3\,(5\,S + 2) + 1 = 15\,S + 7$

Now, when N is divided by 15, remainder = 7

Fermat's remainder therorem

Let P be a prime number and N be a number not divisible by P. Then the remainder obtained when A^{P-1} is divided by P is 1.

Example 25 What is the remainder when 2^{100} is divided by 101?

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Solution Since it satisfies the Fermat's theorem format, remainder = 1

Euler's theorem of remainder

Let $f(n)$ be the number of integers less than n and co-prime with n, then the remainder obtained when $m^{f(n)}$ is divided by n is 1, where m and n are co-prime to each other.

As we can see:

$f(2) = 1, f(3) = 2, f(4) = 2, f(5) = 4$

So, the remainder obtained when $8^{f(5)=4}$ is divided by 5 is 1. It can also be seen that the remainder obtained when any power of 8 divisible by 4 such as 8^4 or 8^8 or 8^{12} and so on will give the same remainder when divided by 5.

Similarly, the remainder obtained when $12^{f(5)=4}$ is divided by 5 is 1;

or, the remainder obtained when $8^{f(7)=6}$ is divided by 7 is 1.

Derivations

1. $\frac{(A+1)^N}{A}$ will always give 1 as the remainder. (For all natural values of A and N)

Example 26 What is the remainder when 9^{100} is divided by 8?

Solution For A = 8, it satisfies the above condition. So, remainder = 1

Alternatively, we can apply either cyclicity or theorem method to find the remainder. (Do this yourself).

2. $\frac{A^N}{A+1}$ When N is even, remainder is 1 and when N is odd, remainder is A itself.

Example 27 What is the remainder when 2^{10} is divided by 3?

Solution Since here N is even, so remainder = 1

3. i. $(a^n + b^n)$ is divisible by $(a + b)$, if n is odd.
   Extension of the above formula – $(a^n + b^n + c^n)$ is divisible by $(a + b + c)$, if n is odd.
   Similarly, the above situation can be extended for any number of terms.
   ii. $(a^n - b^n)$ is divisible by $(a + b)$, if n is even.
   iii. $(a^n - b^n)$ is divisible by $(a - b)$, if n is either odd or even.

Example 28 What is the remainder when $(15^{23} + 23^{23})$ is divided by 19? **(CAT 2004, 2 marks)**

Solution It can be observed that $(15^{23} + 23^{23})$ is divisible by 38, so it will be divisible by 19 also. Hence, remainder = 0.

Alternatively, this problem can be done either by cyclicity method or theorem method.

Example 29 What is the remainder when $(16^3 + 17^3 + 18^3 + 19^3)$ is divided by 70? **(CAT 2005, 1 mark)**

Solution We know, this is a basic multiplication and division question. But using the above approach makes it a lot simple.

We know that $(a^n + b^n)$ is divisible by $(a + b)$, if n is odd. Taking cue from this, we can say that $(a^n + b^n + c^n)$ is divisible by $(a + b + c)$, if n is odd and similarly $(a^n + b^n + c^n + d^n)$ is divisible by $(a + b + c + d)$. Now 16 + 17 + 18 + 19 = 70, so remainder is zero.

UNIT DIGIT

As we have seen the cyclicity of remainders above, cyclicity exists for unit digit of the numbers also. (But there is no relation between the cyclicity of remainders and unit digit.)

Taking a very simple example– $2^5 = 32$, and so we know that unit digit of 2^5 is 2. But problem occurs when we start getting big numbers like 25678^{2345} etc. To find out the unit digit of these kinds of numbers, we have some standard results, which we use as formulae.

(Any even number)^{4n} = … 6

It means that any even number raised to any power, which is a multiple of 4, will give us 6 as the unit digit.

(Any odd number)^{4n} = … 1

It means that any odd number raised to any power, which is a multiple of 4, will give us 1 as the unit digit.

Exception 0, 1, 5, 6 [these are independent of power, and unit digit will be the same respectively]

Example 30 Find the unit digit of $25678^{2345} \times 3485^{4857}$.

Solution Unit digit of 25678^{2345} = Unit digit of 8^{45}

(To find out unit digit, we need to have unit digits only. And similarly, to find out tens digit we need to have the tens and units digit only. In the present case, we are considering only last two digits of the power because divisibility rule of 4 needs only the last two digits of the number.)

$8^{45} = 8^{44+1} = 8^{44} \times 8^1 = (\ldots 6) \times 8 = \ldots 8$

Example 31 What is the unit digit of ${}_{32}32^{32}$?

Solution 32 is an even number which is having a power of the form $4n$. So, it will give 6 as the unit digit.

Example 32 When 3^{32} is divided by 50, it gives a number of the format (asdf… xy) (xy being the last two digits after decimal). Find y.

Solution It can be observed that unit digit of $3^{32} = 1$. Now any number having 1 as the unit digit will always give 2 at the unit place when divided by 50.

So, answer is 2.

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Example 33 What is the last non-zero digit of the number 30^{2720}? **(CAT 2005, 2 marks)**

Solution $30^{2720} = [30^4]^{680} = \ldots 1$

Unit digit can also be found out by cyclicity method as well.

It can be seen that

Unit digit of $2^1 = 2$

Unit digit of $2^2 = 4$

Unit digit of $2^3 = 8$

Unit digit of $2^4 = 6$

Unit digit of $2^5 = 2$

So, it can be inferred that unit digit of 2^1 = unit digit of 2^5 = unit digit of 2^9

Hence, cyclicity of 2 = 4, i.e., every fourth power of 2 will give the same unit digit.

Similarly,

Cyclicity of 3 = 4

Cyclicity of 4 = 2

Cyclicity of 7 = 4

Cyclicity of 8 = 4

Cyclicity of 9 = 2

Cyclicity of 0 or cyclicity of 1 or cyclicity of 5 or cyclicity of 6 = 1

TENS' DIGIT

There lies the cyclicity of tens' place digit of all the digits. This is given below:

Digits	*Cyclicity*
2, 3, 8–	20
4, 9 –	10
5–	1
6–	5
7–	4

Example 34 What is the tens' place digit of 12^{42}?

Solution For this, we need to break 12^{42} first by using binomial theorem as $(10+2)^{42}$. Obviously this expression will have 43 terms, and out of these 43 terms first 41 terms will have both of their tens and units place digit as 0.

Last two terms will be $\rightarrow {}^{42}C_{41} \times 10^1 \times 2^{41} + {}^{42}C_{42} \times 10^0 \times 2^{42}$

Now, we will find the tens' place digit of all these terms individually.

Tens digit of ${}^{42}C_{41} \times 10^1 \times 2^{41} = 42 \times 10 \times (02)$ [Cyclicity of 2 is 20, so 2^{41} will have same tens digits as 2^1] = 840, so 40 are the last two digits.

Similarly, ${}^{42}C_{42} \times 10^0 \times 2^{42} = 1 \times 1 \times 04 = 04$

So, finally last two digits are $\rightarrow 40 + 04 = 44$, so 4 is the tens' place digit.

Note: $(25)^n$ and $(76)^n$ will always give 25 and 76 as the units and tens digit for any natural number value of n.

NUMBER OF EXPONENTS

Let us take a simple number – 10^5

This is read as – 10 to the power 5, or we say that exponent of 10 is 5.

In simple terms, exponents are also known as Power.

Example 35 What is the maximum value of s if $N = (35 \times 45 \times 55 \times 60 \times 124 \times 75)$ is divisible by 5^s?

Solution If we factorize $N = (35 \times 45 \times 55 \times 60 \times 124 \times 75)$, then we can see that 5 appears 6 times, it means N is divisible by 5^6.

So, maximum value of $s = 6$

Exponent of any prime number P in $n!$

$= \left[\frac{n}{P}\right] + \left[\frac{n}{P^2}\right] + \left[\frac{n}{P^3}\right] + \ldots + \left[\frac{n}{P^x}\right]$, where $n \geq P^x$ and [.] denotes the greatest integer value i.e. we have to consider only the integral value.

Let us find the exponent of 5 in 1000! = $1000/5 + 1000/5^2 + 1000/5^3 + 1000/5^4 = 200 + 40 + 8 + 1 = 249$

Example 36 What is the highest power of 5 which can divide $N = (22! + 17894!)$?

Solution Number of times this number is divisible by 5 is same as the number of zeroes at the end of this number. Since 22! have 4 zeroes at its end, so N will also be having only four zeroes at its end. Hence, highest power of 5, which can divide N is 4.

Process to find out the exponent of any composite number in *n*!

There are three different kinds of composite numbers.

1. Product of two or more than two prime numbers with unit power of all the prime numbers
   For example, 15 (5×3), 30 $(2 \times 3 \times 5)$ etc.
2. (Any prime number)^n, where $n > 1$
   For example, 4 (2^2), 27 (3^3)
3. Product of two or more than two prime numbers with power of any one prime number more than 1.
   For example, 12 $(2^2 \times 3)$, 72 $(2^3 \times 3^2)$ etc.

Let us find out the exponents of the above-written composite numbers one by one:

- Find out the exponent of 15 in 100!

15 is the product of two distinct prime numbers 5 and 3. So, to find out the exponents of 15, we need to find out the exponents of 5 and 3 individually.

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$(0\text{-}9)_{10}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$(0\text{-}8)_9$	0	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18
$(0\text{-}7)_8$	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17	20	21
$(0\text{-}6)_7$	0	1	2	3	4	5	6	10	11	12	13	14	15	16	20	21	22	23
$(0\text{-}5)_6$	0	1	2	3	4	5	10	11	12	13	14	15	20	21	22	23	24	25
$(0\text{-}3)_4$	0	1	2	3	10	11	12	13	20	21	22	23	30	31	32	33	100	101
$(0\text{-}2)_3$	0	1	2	10	11	12	20	21	22	100	101	102	110	111	112	120	121	122

So, by applying the same formula of finding out the exponents for any prime number in both of these cases individually, minimum of those two will be the answer.

$100/5^x = [100/5] + [100/5^2] = 20 + 4 = 24$

$100/3^x = [100/3] + [100/3^2] + [100/3^3] + [100/3^4]$
$= 33 + 11 + 3 + 1 = 48$

Obviously, 24 is going to be the answer.

- Find out the exponent of 25 in 100
  $25 = 5^2$
  In this case, we first find out the exponents of 5 and then divide it by 2 (actually the power) to find the exponents of 25.
  $100/5^x = [100/5] + [100/5^2] = 20 + 4 = 24$
  So, $100/25^x = 24/2 = 12$
- Similarly, we can find out for the third category of numbers also.

BASE SYSTEM

In our decimal system of writing the numbers, we use 10 digits (0 – 9). In this system, largest number of single digit = 9, and the moment we have to form a number bigger than this number we need two-digit numbers starting from 10. Similarly, largest number of two digits = 99 and after this we have 100, a number of three digits.

Lets assume a system of writing where we use only 6 digits (0 – 5). Largest single digit number in this system will be 5 and next to this will be 10. Similarly, largest two digit number will be 55 and next to this is 100.

This whole procedure can be summed up in the following table:

Types of Questions

Questions from this concept are asked in three different ways:

1. $(\text{Base})_{10}$ to any other base and vice-versa
2. $(\text{Base})_x$ to $(\text{Base})_y$ and vice-versa; none of x and y being equal to 10 but x and y will be given.
3. $(\text{Base})_x$ to $(\text{Base})_y$, value of x and y will not be given.

1. $(\text{Base})_{10}$ to any other base and vice-versa

Method 1

Let us see in case of $(74)_{10}$.

$(74)_{10} = 7 \times 10^1 + 4 \times 10^0$, since the base is 10.

Now, if we have to convert this number in 9 base, then we will try to write it in terms of powers of 9.

$(74)_{10} = 8 \times 9^1 + 2 \times 9^0 = (82)_9$
$(74)_{10} = 1 \times 8^2 + 1 \times 8^1 + 2 \times 8^0 = (112)_8$
$(74)_{10} = 1 \times 7^2 + 3 \times 7^1 + 4 \times 7^0 = (134)_7$
$(74)_{10} = 2 \times 6^2 + 0 \times 6^1 + 2 \times 6^0 = (202)_6$

While converting the numbers from decimal system to any other system of writing the numbers, we should be concerned with the following two rules

i. Take maximum possible power of the base and then keep writing rest of the number with the help of lesser power of base (as illustrated in the above example).
ii. Once we have used $(\text{base})^n$; where n is the maximum power, we will be required to

Example Write the co-efficients of all the powers of base from 0 to $(n - 1)$ as in the case of $(74)_{10} = (202)_6$.

Now, suppose we have to convert $(356)_7$ in the base of 10.

$(356)_7 = 3 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 = (188)_{10}$

Method 2

Converting $(74)_{10}$ to the base of $(\ )_9$

Base	74	Remainder
9	8	2

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So, $(74)_{10} = (82)_9$

Converting $(74)_{10}$ to the base of $(\ )_8$

Base	74	Remainder
8	9	2
8	1	1

So, $(74)_{10} = (112)_8$

Converting $(74)_{10}$ to the base of $(\ )_7$

Base	74	Remainder
7	10	4
7	2	3
	1	

So, $(74)_{10} = (134)_7$

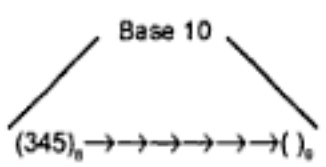
Converting $(74)_{10}$ to the base of $(\ )_6$

Base	74	Remainder
6	12	2
6	2	0
	Quotient	

So, $(74)_{10} = (202)_6$

2. $(\text{Base})_x$ to $(\text{Base})_y$ and vice-versa; none of x and y being equal to 10 but x and y will be given

Converting $(345)_8$ to the base of $(\ )_9$



We will do this problem with the help of creating a bridge of base 10 between base 8 and base 9.

Step 1 Convert $(345)_8$ into base 10.

$345 = 3 \times 8^2 + 4 \times 8^1 + 5 \times 8^0 = (229)_{10}$

Step 2 Now convert this number in base 10 into base 9.

$(229)_{10} = 2 \times 9^2 + 7 \times 9^1 + 4 \times 9^0 = (274)_9$

However, if new base is a power of old base and vice-versa, then it can be converted directly also in the new base, i.e., we are not needed to go to base 10 for these kind of conversions.

Example $(\text{Base})_2$ to $(\text{Base})_4$ or $(\text{Base})_2$ to $(\text{Base})_8$ conversion does not require a bridge of base 10.

Converting $(101110010)_2$ to Octal $(\ )_8$ system

At first, we will club three digits of binary number into a single block and then write the decimal equivalent of each group (left to right).

So, $(101110010)_2$ is now $(101)_2(110)_2(010)_2$

Now, $(101)_2 = 1 \times 2^2 + 0 + 1 \times 2^0 = 5$

$(110)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 6$

$(010)_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$

So, $(101110010)_2 = (562)_8$

Converting $(101110010)_2$ to Hexa-decimal $(\ )_{16}$ system

At first, we will club four digits of binary number into a single block and then write the decimal equivalent of each group (left to right).

So, $(101110010)_2$ is now $(0001)_2(0111)_2(0010)_2$

Now, decimal equivalent of $(0001)_2 = 1$

Decimal equivalent of $(0111)_2 = 7$

Decimal equivalent of $(0010)_2 = 2$

$(101110010)_2 = (172)_{16}$

3. $(\text{Base})_x$ to $(\text{Base})_y$, value of x and y will not be given

In these type of questions, normally some calculation is given in some unknown system of writing numbers and on the basis of that we will be required to solve the questions.

Example 37 In a system of writing of N digits, $4 \times 6 = 30$ and $5 \times 6 = 36$. What will be the value of $N = 3 \times 4 \times 5$ in the same system of writing?

Solution Let us assume that there are N digits in this system of writing.

So, $(30)_N = 3 \times N^1 + 0 \times N^0 = 24$

$\Rightarrow 3N = 24$

$\Rightarrow N = 8$

So, this system of writing has 8 digits.

In this system $3 \times 4 \times 5 = 60$ will be written as 74. $(60 = 7 \times 8^1 + 4 \times 8^0)$

Alternatively, since this system is having 6 as one of its digits, minimum value of N will be 7. Again, 24 is written as 30 in this system, so N is less than 10. Now use hit and trial for $N = 7$ or 8 or 9 to find out N in $24 = (30)_N$.

DECIMAL CALCULATION

So far we have seen the calculations involving natural numbers only. Let's work now with decimals.

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Converting decimal system numbers to any other system

Suppose (12.725) is a number in decimal system which is required to be converted into octal system (8 digits)

We will first convert 12 into octal system.

$(12)_{10} = (14)_8$

Now to convert $(0.725)_{10}$ into $(\ )_8$, we will apply the following method:

$0.725 \times 8 = 5.8$		Take out integral part from here.
$0.8 \times 8 = 6.4$		Take out integral part from here.
$0.4 \times 8 = 3.2$		Take out integral part from here.
$0.2 \times 8 = 1.6$		Take out integral part from here.

And keep doing this till we get decimal part as zero, i.e., the product should be an integer.

$(0.725)_{10} = (0.5632\ldots)_8$

So, $(12.725) = (14.5632\ldots)_8$

Converting any other system numbers to decimal system

Now suppose if $(15.453)_7$ is to be converted into decimal system, then the process is as follows:

We will first convert $(15)_7$ into decimal system.

$(15)_7 = 1 \times 7^1 + 5 \times 7^0 = (12)_{10}$

Now $(0.453)_7 = 4 \times 7^{-1} + 5 \times 7^{-2} + 3 \times 7^{-3} = 0.6821$

So, $(15.453)_7 = (12.6821)_{10}$

Basic Algebraic Calculations Involving Base Systems

Addition

$$\begin{array}{r} 325_7 \\ +\ 456_7 \\ \hline \end{array}$$

Start with the units place digit, 5 + 6 = 11 which is 14_7. So, unit digit is 4 and carry over is 1.

Next is tens place digit, 2 + 5 + 1 (carry over) = 8 which is 11_7. So, tens digit is 1 and carry over is again 1.

Next is 3 + 4 + 1 (carry over) = 8 which is 11_7.

$$\begin{array}{r} 325_7 \\ +\ 456_7 \\ \hline 1114_7 \\ \hline \end{array}$$

Subtraction

$456_8 - 367_8$

Starting with the units digit, since 6 is smaller than 7, we will borrow 1 from the tens place digit. So, now it is 14 (when the base is 10, we get 10 but here base is 8, so we get 8) and 7 subtracted from it = 14 – 7 = 7, which is the units digit.

Next, tens digit is now 4 and we have to subtract 5 from it. We again borrow 1 from hundred's place digit. So, now it is 12 and 12 – 6 = 6, which is the tens place digit.

Now, hundred's place digit is 3 (4 – 1), so 3 – 3 = 0

$$\begin{array}{r} 456_8 \\ -\ 367_8 \\ \hline 67_8 \end{array}$$

Note: Another method of doing these kinds of calculations is to convert these values (in whatever base) into decimal system, then do the actual calculation in decimal system itself and finally converting the numbers into the required or given system.

Some standard systems of writing

1. Decimal system
   Digits used: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
   Total digits used = 10 digits
2. Hexa-decimal system
   Digits used: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F.
   Total digits used = 16
3. Octal system
   Digits used: 0, 1, 2, 3, 4, 5, 6 and 7.
   Total digits used = 8
4. Binary system
   Digits used: 0, 1
   Total Digits used – 2

Comparison of Different Base Systems

It should be understood here that using different bases creates a difference only in the numbers (more than single digit) and not in the digit. This means in that $(5)_{10}$ is same as $(5)_9$ or any other system of writing the numbers.

Example How many different values of N are possible in the following calculation

$(2)_N \times (4)_N = (8)_N$?

Solution Any value of N which has 8 as a digit in that system will satisfy this calculation. So, any value of $N \geq 9$ is possible. Hence infinite values of N are possible.

This concept can be further understood through the mechanism of multiplication. Multiplication is basically the condensed form of addition. So, 2 added up four times will give 8, and hence all the base systems which have the following can be one of the values of N:

0 1 2 3 4 5 6 7 8.

Example How many different values of N are possible in the following calculation.

$(4)_N + (5)_N = (9)_N$?

Solution In this case also, the digits used can be of any system that has the digit 9. So, all the inegral values $N \geq 10$ are possible.

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Example How many different values of N are possible in the following calculation.

$(4)_N \times (5)_N = (24)_N$?

Solution Obviously, there can be either just one possible value of N or no possible value of N.

$(4)_{10} \times (5)_{10} = (20)_{10}$ [We know $(4)_N = (4)_{10}$ and $(5)_N = (5)_{10}$]

Now, $(24)_N = 2 \times N^1 + 4 \times N^0 = 20$

Hence, $N = 8$. And this will be the only value of N.

Divisibility Rules For Systems Other Than Decimal System

I would like to emphasize that different number systems are just different ways to write numbers. Thus the divisibility of one number by another does not depend on the particular system in which they are written.

At the same time, in each system there are some tricks to determine divisibility by certain specific numbers. These are the divisibility tests.

Let us investigate the tests for divisibility by 3 and 9. We will try to generalize these tests for any number base system.

Is 1234565642317 divisible by 6?

We know the divisibility rule for 9 – Sum of digits of the number should be divisible by 9.

Sum of the digits of this number is 42.

Now, we can answer this question easily, since the sum of the digits (which is 4210) is divisible by 6, the number itself is also divisible by 6.

In general, the sum of the digits of a number written in the base n system is divisible by $(n - 1)$ if and only if the number itself is divisible by $(n - 1)$.

So, divisibility rule for 4 in a base system of 5 in that the sum of the digits of the number should be divisible by 4. For example, 31_5 is divisible by 4.

Similarly, if we have to find out the divisibility rule of 12 in the base of 11, it will be nothing but same as the divisibility rule of 11 in the base of 10. Generalizing this whole concept, divisibility rule of any natural number N in the base of $(N - 1)$ will be same as divisibility rule of 11 on base 10.

Before you proceed, you should have a clear understanding of how to find the divisibility rule for any prime number in base 10.

The only difference is that in case of base 10, we find the multiples of prime numbers in the format $10n \pm 1$, and in case of base B, we find the multiples of prime numbers in the format $B\,n \pm 1$.

Let us find out the divisibility rule of $(5)_8$.

Multiples of 5 in base 10	5	10	15	20	25	30
Multiples of 5 in base 8	5	12	17	24	31	26

Obviously 17 is the first number in the format of $8n \pm 1$.

$17 = 8 \times 2 - 1$ [Yes, it is correct. 17_8 and RHS is in base 10]

So, divisibility rule of 5 on base 8 = $A + 2B_8$ where A and B have the same meaning as that in divisibility rules of 10.

We can check this by the following method:

First few numbers in base 8							
0	1	2	3	4	5	6	7
10	11	12	13	14	15	16	17
20	21	22	23	24	25	26	27
30	31	32	33	34	35	36	37

Every 5th number in this series will be divisible by 5. So, 5 is the first number divisible by 8. The next few numbers are 12, 17, 24, 31, 36.

We can use base to generate good concepts in remainders also.

Example What is the remainder when $(1234)_6$ is divided by 5?

Solution Divisibility rule of $(5)_6$ will be similar to that of the divisibility rule of $(9)_{10}$. So, if the sum of digits of the number in base 6 is divisible by 5, then it will be divisible by 5.

Adding the digits of $(1234)_6 = 10$, which is divisible by 5, the remainder obtained when $(1234)_6$ is divided by 5 is 0.

Example What is the remainder when $(12341)_6$ is divided by 5?

Solution The sum of the digits of $(12341)_6 = 11$, so the remainder obtained will be 1.

Pigeon-hole Principle

Although only a few questions have been asked from this concept so far in CAT, which is also true for some of the arithmetic topics like Profit and Loss; Calendar etc, the importance of this topic lies in the fact that this concept is purely logical and having a good practice over this

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concept gives a logical edge over others.

The concept is that if we put $(N+1)$ or more pigeons in N holes (nests), then at least one hole will be there which will have 2 or more pigeons.

Example 38 What is the minimum number of people in any group of five people who have an identical number of friends within the group, provided if A is a friend of B, then B is also a friend of A?

Solution Since there are five persons in the group, so possible number of friends is 0, 1, 2, 3, 4. It seems here that everybody is having a different number of friends, so the answer is zero. But anybody having four friends ensures that nobody is having 0 friends. So, at least two persons must have the same number of friends.

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PRACTICE EXERCISES

WARM UP

Q.1. Which of the following is the smallest?
(a) $5^{1/2}$ (b) $6^{1/3}$ (c) $8^{1/4}$ (d) $12^{1/5}$

Q.2. A number N is divisible by 6 but not divisible by 4. Which of the following will not be an integer?
(a) N/3 (b) N/2 (c) N/6 (d) N/12

Q.3. If a, b and c are consecutive positive integers, then the largest number which always divides $(a^2 + b^2 + c^2)$
(a) 14 (b) 55
(c) 3 (d) None of these

Q.4. $\dfrac{(3.134)^3 + (1.866)^3}{(3.134)^2 - 3.134 \times 1.868 + (1.866)^2} = ?$
(a) 25 (b) 2.68 (c) 1.038 (d) 5

Q.5. If n^2 is a perfect cube, then which of the following statement is always true?
(a) n is odd
(b) n is even
(c) n^3 is a perfect square
(d) n is a perfect cube

Q.6. If $(5x + 11y)$ is a prime number for natural number values of x and y, then what is the minimum value of $(x + y)$?
(a) 2 (b) 3 (c) 4 (d) 5

Q.7. For what values of x is $25^x + 1$ divisible by 13?
(a) All real values of x
(b) Odd natural values of x
(c) Even values of x
(d) All the integral values of x

Q.8. Which of the following numbers lies between 5/6 and 6/7?
(a) 71/84 (b) 31/42 (c) 129/168 (d) 157/339

Q.9. By multiplying with which of the following numbers does the product of $8 \times 9 \times 10 \times 11 \times 12$ become a perfect square?
(a) 55 (b) 11 (c) 165 (d) 310

Q.10. What is the difference between the sum of the cubes and that of squares of the first ten natural numbers?
(a) 5,280 (b) 2,640 (c) 3,820 (d) 4,130

Q.11. If $3 - 9 + 15 - 21 + \ldots$ up to 19 terms $= x$, then x is a/an
(a) odd number (b) even number
(c) prime number (d) irrational number

Q.12. What is the unit's digit of $21^3 \times 21^2 \times 34^7 \times 46^8 \times 77^8$?
(a) 4 (b) 8 (c) 6 (d) 2

Q.13. If the unit's digit in the product $(47n \times 729 \times 345 \times 343)$ is 5, what is the maximum number of values that n may take?
(a) 9 (b) 3 (c) 7 (d) 5

Q.14. In how many ways can 846 be resolved into two factors?
(a) 9 (b) 11
(c) 6 (d) None of these

Q.15. If a number is divided by 15, it leaves a remainder of 7. If thrice the number is divided by 5, then what is the remainder?
(a) 5 (b) 6 (c) 7 (d) 1

Q.16. A number when divided by 391 gives a remainder of 49. Find the remainder when it is divided by 39.
(a) 10 (b) 9
(c) 11 (d) Cannot be determined

Q.17. p and q are two prime numbers such that $p < q < 50$. In how many cases would $(q + p)$ be also a prime number?
(a) 5 (b) 6
(c) 7 (d) None of these

Q.18. How many distinct factors of 1,600 are perfect cubes?
(a) 3 (b) 4 (c) 6 (d) 2

Q.19. The LCM of 96,144 and N is 576. If their HCF is 48, then which of the following can be one of the values of N?
(a) 168 (b) 192 (c) 144 (d) 244

Q.20. If p and q are consecutive natural numbers (in increasing order), then which of the following is true?
(a) $q^2 < p$ (b) $2p > p$
(c) $(q + 1)^2 > p^2$ (d) $(p + 2)^3 < q^3$

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Q.21. $(17^{21} + 19^{21})$ is not divisible by
(a) 36 (b) 8 (c) 9 (d) 18

Q.22. Which of the following will divide $11^{12296} - 1$?
(a) 11 and 12 (b) 11 and 10
(c) 10 and 12 (d) 11 only

Q.23. If a, b, c and d are consecutive odd numbers, then $(a^2 + b^2 + c^2 + d^2)$ is always divisible by
(a) 5 (b) 7 (c) 3 (d) 4

Q.24. Four bells toll at intervals of 14, 21 and 42 min respectively. If they toll together at 11.22 a.m., when will they toll together for the first time after that?
(a) 11.56 a.m. (b) 12.04 p.m.
(c) 12.06 p.m. (d) 11.48 a.m.

Q.25. When x is divided by 6, remainder obtained is 3. Find the remainder when $x^4 + x^3 + x^2 + x + 1$ is divided by 6.
(a) 3 (b) 4 (c) 1 (d) 5

Q.26. I have 7^7 sweets and I want to distribute them equally among 2^4 students. After each of the student got maximum integral sweets, how many sweets are left with me?
(a) 8 (b) 5
(c) 1 (d) None of these

Q.27. When I distribute some chocolates to my 40 students, three chocolates will be left. If I distribute the same number of chocolates to my students and my colleague Manoj Dawrani, seven chocolates are left. Find the minimum number of chocolates I have.
(a) 1,443 (b) 1,476
(c) 1,480, (d) None of these

Q.28. The LCM of two numbers is 40 times their HCF. The sum of the LCM and HCF is 1,476. If one of the numbers is 288, find the other numbers?
(a) 169 (b) 180 (c) 240 (d) 260

Q.29. 1010101 ... 94 digits is a 94 digits number. What will be the remainder obtained when this number is divided by 375?
(a) 10 (b) 320
(c) 40 (d) None of these

Q.30. Chandrabhal adds first N natural numbers and finds the sum to be 1,850. But actually one numbers was added twice by mistake. Find the difference between N and that number.
(a) 40 (b) 33 (c) 60 (d) 17

Q.31. When I distribute a packet of chocolate to 7 students, I am left with 4 chocolates. When I distribute the same packet of chocolate to 11 students, I am left with 6 chocolates. How many chocolates will be left with me if I distribute the same packet of chocolate among 13 students (a packet of chocolate contains total number of chocolates N, $1000 < N < 1050$)
(a) 2 (b) 0 (c) 6 (d) 7

Q.32. How many prime numbers are there between 80 and 105?
(a) 3 (b) 4 (c) 5 (d) 8

Q.33. If x and y are consecutive natural numbers in an increasing order, then which of the following is always true?
(a) $x^y > y^x$ (b) $y^x > x^y$
(c) $x^x > y^y$ (d) $y^y > x^x$

Q.34. What is the remainder when 5^{79} is divided by 7?
(a) 1 (b) 0 (c) 5 (d) 4

FOUNDATION

Q.1. LCM of two natural numbers is 590 and their HCF is 59. How many sets of values are possible?
(a) 1 (b) 2 (c) 5 (d) 10

Q.2. MUL has a waiting list of 5005 applicants. The list shows that there are at least 5 males between any two females. The largest number of females in the list could be
(a) 920 (b) 835 (c) 721 (d) 1005

Q.3. HCF of two numbers A and B is 24. HCF of two other numbers C and D is 36. What will be the HCF of A, B, C and D?
(a) 12 (b) 24
(c) 36 (d) 6

Q.4. How many zeroes will be there at the end of $25 \times 35 \times 40 \times 50 \times 60 \times 65$?
(a) 6 (b) 8 (c) 5 (d) 7

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Q.5. What is the unit digit of $576847 \times 564068 \times 96467 \times 458576$?
(a) 2 (b) 4 (c) 6 (d) 8

Q.6. What is the unit digit of $1! + 2! + 3! + \ldots + 99! + 100!$?
(a) 3 (b) 1 (c) 5 (d) 6

Q.7. How many divisors will be there of the number 1020?
(a) 12 (b) 20 (c) 24 (d) 36

Q.8. In question 7, what is the difference between the number of even divisors and number of prime divisors?
(a) 13 (b) 12
(c) 11 (d) None of these

Q.9. $N = 7!^3$. How many factors of N are multiples of 10?
(a) 736 (b) 1008 (c) 1352 (d) 894

Q.10. A number N has odd number of divisors. Which of the following is true about N?
(a) All the divisors of this number will be odd.
(b) There will be atleast $(N - 11)$ prime divisors.
(c) N will be a perfect square.
(d) Atleast one divisor of the number should be odd.

Q.11. How many zeroes will be there at the end of the expression $N = 2 \times 4 \times 6 \times 8 \times \ldots \times 100$?
(a) 10 (b) 12
(c) 14 (d) None of these

Q.12. How many zeroes will be there at the end of the expression $N = 10 \times 20 \times 30 \ldots \times 1000$?
(a) 1280 (b) 1300
(c) 1320 (d) None of these

Q.13. How many zeroes will be there at the end of the expression $N = 7 \times 14 \times 21 \times \ldots \times 777$?
(a) 24 (b) 25
(c) 26 (d) None of these

Q.14. The number from 1 to 33 are written side by side as follows: 123456…33. What is the remainder when this number is divided by 9?
(a) 0 (b) 1 (c) 3 (d) 6

Q.15. The number 444444…(999 times) is definitely divisible by
(a) 22 (b) 44
(c) 222 (d) All of these

Q.16. Find the unit digit of $N = {}_{17}27!^{37!}$
(a) 1 (b) 3 (c) 7 (d) 9

Q.17. How many divisors of $N = 420$ will be of the form $4n + 1$, where n is a whole number?
(a) 3 (b) 4 (c) 5 (d) 8

Q.18. $N = 2^3 \times 5^3 \times 7^2$. How many sets of two factors of N are co-prime?
(a) 72 (b) 64
(c) 36 (d) None of these

Q.19. What is the unit digit of $2^{3^{4^5}}$?
(a) 2 (b) 4 (c) 8 (d) 6

Q.20. How many zeroes will be there at the end of $1003 \times 1001 \times 999 \times \ldots \times 123$?
(a) 224 (b) 217
(c) 0 (d) None of these

Q.21. How many zeroes will be there at the end of $36!^{36!}$?
(a) $7 \times 6!$ (b) $8 \times 6!$ (c) $7 \times 36!$ (d) $8 \times 36!$

Q.22. The number formed by writing any digit 6 times (e.g., 111111, 444444, etc) is always divisible by
(i) 7 (ii) 11 (iii)13
(a) i and ii (b) ii and iii
(c) i and iii (d) i, ii and iii

Q.23. What is the maximum value of HCF of $[n^2 + 17]$ and $(n + 1)^2 + 17]$?
(a) 69 (b) 85
(c) 170 (d) None of these

Q.24. What is the number of pairs of values of (x, y), which will satisfy $2x - 5y = 1$, where $x \le 200$, and x and y are positive integers?
(a) 38 (b) 39 (c) 40 (d) 41

Q.25. $N = 2^3 \times 5^3$. How many sets of two factors of N are co-prime to each other?
(a) 12 (b) 24 (c) 23 (d) 11

Q.26. What is the sum of digits of the least multiple of 13, which when divided by 6, 8 and 12 leave 5, 7 and 11 as the remainder?
(a) 5 (b) 6 (c) 7 (d) 8

Q.27. What is the unit digit of $7^{11^{22^{33}}}$?
(a) 1 (b) 3 (c) 7 (d) 9

Q.28. What is the remainder when $(1! + 2! + 3! + \ldots 1000!)$ is divided by 5?
(a) 1 (b) 2 (c) 3 (d) 4

Q.29. $A = 3^{150} \times 5^{76} \times 7^{140}$, $B = 3^{148} \times 5^{76} \times 7^{141}$, $C = 3^{148} \times 5^{80} \times 7^{139}$, $D = 3^{151} \times 5^{80} \times 7^{142}$, then the order of A, B, C and D from largest to smallest is
(a) DACB (b) CDBA
(c) CDAB (d) DCAB

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Q.30. The HCF of 0.3, 0.15, 0.225, 0.0003 is
(a) 0.0003 (b) 0.3 (c) 0.15 (d) 0.0015

Q.31. How many numbers between 1 to 550 are divisible by 5 but not by 9?
(a) 98 (b) 97
(c) 101 (d) None of these

Q.32. A and B are two distinct digits. If the sum of the two-digit numbers formed by using both the digits is a perfect square, what is the value of (A + B)?
(a) 9 (b) 11 (c) 13 (d) 17

Q.33. A number N = 897324P64Q is divisible by both 8 and 9. Which of the following is the value of P + Q?
i. 2 ii. 11 iii. 9
(a) either i or ii (b) either ii or iii
(c) either i or ii or iii (d) None of these

Direction for questions 34–35: Read the passage below and solve the questions based on it.

A = Set of first N positive numbers. There are 16 numbers in A which are divisible by both *X* and *Y*. There are 50 numbers in A which are divisible by *X* but not by *Y* and 34 numbers in A divisible Y but not by *X*.

Q.34. How many numbers in A are divisible by any of the two numbers?
(a) 100 (b) 50
(c) 200 (d) None of these

Q.35. How many numbers in *N* are divisible by *X*?
(a) 42 (b) 56
(c) 66 (d) None of these

Q.36. Nitin had forgotten his 6 digit bank account number but only remembered that it was of the form X515X0 and was divisible by 36. What was the value of X?
(a) 4 (b) 7 (c) 8 (d) 9

Q.37. Students from the Delhi Public School are writing their exams in Kendriya Vidyalaya. There are 60 students writing their Hindi exams, 72 students writing there French exam and 96 students writing their English exam. The authorities of the Kendriya Vidyalaya have to make arrangements such that each classroom contains equal number of students. What is the minimum number of classrooms required to accommodate all students of Delhi Pubic School?
(a) 19 (b) 38 (c) 13 (d) 6

Q.38. In the Jyotirmayi school all classes started at 9.00 a.m. The school has three sections: primary, middle and secondary. Each class for the primary section lasts for half an hour, for the middle section for forty five minutes and for the secondary section for half an hour. A lunch break has to be given for the entire school when each of three sections have just finished a respective class and are free. What is the earliest time for the lunch break?
(a) 11.00 a.m. (b) 10.30 a.m.
(c) 12.00 p.m. (d) 12.30 p.m.

Q.39. In the firing range 4 shooters are firing at their respective targets. The first, the second, the third and the fourth schooter hits the target once every 5s, 6s, 7s, 8s respectively. If all of them hits their target at 10.00 A.M., when will they hit their target together again?
(a) 10.14 a.m. (b) 10.28 a.m.
(c) 10.30 a.m. (d) 10.31 a.m.

Q.40. Two friends Harry and Jayesh were discussing about 2 numbers. They found the two numbers to be such that one was twice the other. However, both had the same number of prime factors while the larger one had 4 more factors than the smaller one. What are the numbers?
(a) 40,80 (b) 20,40
(c) 30,60 (d) 50,100

Q.41. To celebrate their victory in the World Cup, the Sri Lankans distributed sweets. If the sweets were distributed among 11 players, 2 sweets were left. When the sweets were distributed among 11 players, three extra's and 1 coach, even then 2 sweets were left. What is the minimum number of sweets in the box?
(a) 167 (b) 334 (c) 332 (d) 165

Q.42. The first 20 natural numbers from 1 to 20 are written next to each other to form a 31 digit number *N* = 1234567891011121314151617181920. What is the remainder when this number is divided by16?
(a) 0 (b) 4 (c) 7 (d) 9

Q.43. Two friends Kanti and Sridhar were trying to find the HCF of fifty distinct numbers. If they were finding the HCF of two numbers at a time, how many times this operation should be repeated to find the HCF of 50 numbers?
(a) 20 (b) 25 (c) 49 (d) 50

Q.44. How many zeroes will be there at the end of *N* = 18! + 19!?
(a) 3 (b) 4
(c) 5 (d) Cannot be determined

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Q.45. Manish was dividing 2 numbers by a certain divisor and obtained remainders of 437 and 298 respectively. When he divides the sum of the two numbers by the same divisor, the remainder is 236. What is the divisor?
(a) 499 (b) 735
(c) 971 (d) None of these

Q.46. I purchased a ticket for the football match between France and Italy in the World Cup. The number on the ticket was a 5 digit perfect square such that the first and the last digit were the same and the 2nd and 4th digit were the same. If the 3rd digit was 3, then what was the ticket number?
(a) 24,342 (b) 12,321
(c) 21,312 (d) None of these

Q.47. How many integers N in the set of integers $\{1, 2, 3, \ldots, 100\}$ are there such that $N^2 + N^3$ is a perfect square?
(a) 5 (b) 7 (c) 9 (d) 11

Q.48. In a birthday party, all the children were given candy bars. All the children got three candy bars each except the child sitting at the end who got only 2 candy bars. If each child had been given only 2 candy bars there would have been 8 candy bars remaining. How many children were there and how many candy bars were distributed?
(a) 9, 26 (b) 6, 26 (c) 9, 18 (d) 6, 15

Q.49. A natural number N satisfies following conditions.
(A) Number is having all the 9's
(B) It is divisible by 13.
How many digits are there in N?
(a) 5 (b) 6 (c) 7 (d) 8

Q.50. What is the minimum number of identical square tiles required to cover a floor of dimension 3.78 m × 4.8 m?
(a) 2,520 (b) 3,780 (c) 5,040 (d) 6,480

Q.51. What is the smallest five-digit number which when divided by 7, 11 and 21 leaves a remainder of 3 in each case?
(a) 10,019 (b) 10,001
(c) 10,111 (d) 10,167

Q.52. A milkman has 3 jars containing 57 litres, 129 litres and 177 litres of pure milk respectively. A measuring can leaves the same amount of milk unmeasured in each jar after a different number of exact measurements of milk in each jar. What is the volume of largest such can?
(a) 12 litres (b) 16 litres
(c) 24 litres (d) 48 litres

Q.53. A boy was carrying a basket of eggs. He fell down and some of the eggs were broken. The boy has 10 eggs left with him. When asked by his mother how many eggs were broken, the boy could not recall. However he recalled that when he counted the total number of eggs 3 at a time 1 egg was left. When counted 4 at a time, 1 egg was left and when counted 5 at time a no egg was left. How many eggs were broken?
(a) 15 (b) 25 (c) 30 (d) 35

Q.54. How many prime numbers are there which, when divided by another prime number, gives a quotient which is same as the remainder?
(a) 0 (b) 1
(c) 2 (d) More than 2

Q.55. The number of natural numbers n such that $\frac{(n+1)^2}{n+7}$ is an integer, is
(a) 4 (b) 5
(c) 6 (d) None of these

Q.56. Find the product of all the factors of 3^{16}.
(a) 3^{33} (b) 3^{68} (c) 3^{136} (d) 3^{128}

Q.57. What is the remainder when 90^{91} is divided by 13?
(a) 0 (b) 7 (c) 12 (d) 1

Q.58. Find the remainder when the product of 10 consecutive natural numbers starting from 8641 is divided by 8640.
(a) 1 (b) 55 (c) 10 (d) 0

Q.59. Palindromes are numbers which when read from both the ends will give us the same number, for example, 123321 etc. In $(P)_N$, P is a palindrome of S digits, where S is even, written in base N. Which of the following number will definitely divide P?
(a) N – 1 (b) N + 1
(c) 2 N + 1 (d) None of these

Q.60. Today, I bought a total of 615 chocolates and packed them into small packets, each packet containing an equal number of chocolates. I then distributed one packet each to all the boys of a class and one packet each to all the girls of the same class. The boys got a total of 120 chocolates and the girls got a total of 210 chocolates. If tomorrow, I want to distribute the remaining packets, one each to all the students of the class, then find the minimum number of packets that I will fall short of.
(a) 3 (b) 4 (c) 5 (d) 6

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Q.61. If $n^2 = 123.45654321$, which of the following is the exact value of n.
(a) 11.1001 (b) 11.1101
(c) 11.1111 (d) 11.1011

Q.62. A mule said to a horse, "If I take one sack off your back, my load will be double of yours and if you take one off my back our loads will be the same." How many sacks in all were they carrying?
(a) 5 (b) 7 (c) 12 (d) 14

Q.63. Divide 45 into 4 parts such that if the first is increased by 2, the second is decreased by 2, the third multiplied by 2 and the fourth divided by 2, the result is the same.
(a) 20, 8, 5, 12 (b) 12, 5, 20, 8
(c) 5, 8, 12, 20 (d) 8, 12, 5, 20

Q.64. Find the remainder when $3x^2-x^6+31x^4+21x+5$ is divided by $x+2$.
(a) 10 (b) 12
(c) 11 (d) None of these

Q.65. Four prime numbers are in ascending order of their magnitudes. The product of the first three is 385 and that of last three is 1001. The largest given prime number is
(a) 11 (b) 13 (c) 17 (d) 19

Q.66. What is the remainder when 4^{44} is divided by 15?
(a) 1 (b) 2 (c) 3 (d) 4

Q.67. LCM of two integers P and Q is 211. What is the HCF of P and Q?
(a) 37 (b) 1
(c) 3 (d) Cannot be determined

Q.68. How many times does the digit 6 appear when we count from 11 to 400?
(a) 34 (b) 74 (c) 39 (d) 79

Q.69. In Q.68, how many numbers will be having 8 as its digit?
(a) 74 (b) 75 (c) 76 (d) 77

Q.70. S is a number formed by writing 8 for 88 times. What will be the remainder of this number when divided by 7?
(a) 4 (b) 5 (c) 8 (d) 1

Q.71. We are writing all the multiples of 3 from 111 to 324. How many times will we write digit 3?
(a) 18 (b) 19 (c) 21 (d) 22

Q.72. What is the remainder when $7 + 77 + 777 + 7777 + \ldots$ (till 100 terms) is divided by 8?
(a) 0 (b) 2 (c) 4 (d) 6

Q.73. A number has exactly 15 composite factors. What can be the maximum number of prime factors of this number?
(a) 2 (b) 3 (c) 4 (d) 5

Q.74. $N = 204 \times 221 \times 238 \times 255 \times \ldots \times 850$. How many consecutive zeroes will be there at the end of this number N?
(a) 8 (b) 10 (c) 11 (d) 12

Q.75. 1st 126 natural numbers are put side by side in the ascending order to create a large number $N = 123456\ldots125126$. What will be the remainder when N is divided by 5625?
(a) 5126 (b) 26 (c) 126 (d) 156

Q.76. When a number S is divided by 3, 4 and 7 successively, remainders obtained are 2, 1 and 4 respectively. What will be the remainder when the same number is divided by 84?
(a) 43 (b) 53 (c) 63 (d) 73

Q.77. What is the remainder when $1714 \times 1715 \times 1717$ is divided by 12?
(a) 3 (b) 8 (c) 2 (d) 9

Q.78. $N^2 = 12345678987654321$. Find N.
(a) 101010101 (b) 11111
(c) 111111111 (d) 1000000001

Q.79. If a, b, c and d are distinct integers in the range 10 to 15 (both inclusive), the greatest value of $(a + b)(c + d)$ is—
(a) 750 (b) 731 (c) 700 (d) 729

Q.80. The smallest natural number which is a perfect square and is of the form abbb lies in between
(a) 1,000 to 2,000 (b) 2,000 to 3,000
(c) 3,000 to 4,000 (d) 4,000 to 5,000

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MODERATE

Q.1. How many number of zeroes will be there at the end of 12! expressed in base 6?
(a) 4 (b) 5 (c) 6 (d) 7

Q.2. Find the remainder when $2222^{5555} + 5555^{2222}$ is divided by 7.
(a) 1 (b) 3 (c) 0 (d) 5

Q.3 LCM of first 100 natural numbers is N. What is the LCM of first 105 natural numbers?
(a) $5! \times N$ (b) 10403 N
(c) 105N/103 (d) 4N

Q.4. How many divisors of 10^5 end with a zero?
(a) 1 (b) 3 (c) 9 (d) 16

Q.5. Following expression holds true if we replace some of '+' signs by '×' signs.
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 100$
How many '+' signs are needed to be replaced by '×'?
(a) 2 (b) 3 (c) 4 (d) 1

Q.6. In a particular country, all the numbers are expressed with the help of three alphabets a, b and c.
15 is written as abc.
6 is written as bc.
60 is written as bcbc.
How would one write 17 in that country?
(a) abb (b) bab (c) baa (d) aba

Q.7. When a certain two-digit number is added to another two digit number having the same digits in reverse order, the sum is a perfect square. How many such two-digit numbers are there?
(a) 4 (b) 6 (c) 8 (d) 10

Q.8. What is the remainder when ${}_{12}32^{32}$ is divided by 7?
(a) 2 (b) 3 (c) 4 (d) 6

Q.9. N is a 1001 digit number consisting of 1001 sevens. What is the remainder when N is divided by 1001?
(a) 7 (b) 700
(c) 777 (d) None of these

Q.10. Find four positive numbers such that the sum of the first, third and fourth exceeds the second by 8; the sum of the squares of the first and second exceeds the sum of the squares of the third and fourth by 36; the sum of the products of the first and second, and of the third and fourth is 42; the cube of the first is equal to the sum of the cubes of the second, third, and fourth.
(a) 2, 1, 9, 3 (b) 2, 4, 6, 8
(c) 6, 5, 4, 3 (d) None of these

Q.11. Digital sum of a number is obtained by adding all the digits of a number until a single digit is obtained. Find the digital sum of 19^{100}.
(a) 1 (b) 4 (c) 7 (d) 9

Q.12. Find the HCF of $(2^{100} - 1)$ and $(2^{120} - 1)$.
(a) $2^{10} - 1$ (b) $2^{20} - 1$
(c) 1 (d) None of these

Q.13. What is the remainder when $22^{33} + 10^{35}$ is divided by 45?
(a) 2 (b) 11
(c) 8 (d) None of these

Q.14. An intelligence agency forms a code of two distinct digits selected from 0, 1,....,9 such that the first digit of the code is non-zero. The code, handwritten on a slip, however, can potentially create confusion when read upside down, e.g. the code 91 may appear as 16. How many codes are there for which no such confusion can arise?
(a) 80 (b) 63
(c) 71 (d) None of these

Q.15. If p, p + 2, p + 4 are prime numbers, then the number of possible solutions for p is
(a) 0 (b) 1
(c) 2 (d) None of these

Q.16. Suppose N is an integer such that the sum of the digits of N is 2, and $10^9 < N < 10^{10}$. How many values of N are possible?
(a) 11 (b) 10 (c) 9 (d) 8

Q.17. Ten students solved a total of 35 questions in a maths Olympiad. Each question was solved by exactly one student. There is at least one student who solved exactly one problem, at least one student who solved exactly two problems and at least one student who solved exactly three problems. What is the minimum number of students who has/have solved at least five problems?
(a) 1 (b) 2
(c) 3 (d) None of these

Q.18. N! is having 37 zeroes at its end. How many values of N is/are possible?
(a) 0 (b) 1 (c) 5 (d) Infinite

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Q.19. In the above question, how many values of N will be even?
(a) 0 (b) 2 (c) 3 (d) Infinite

Q.20. N! is having 30 zeroes at its end. How many values of N is/are possible?
(a) 0 (b) 1 (c) 5 (d) Infinite

Q.21. What is the remainder when $(1^1 + 2^2 + 3^3 + \ldots + 100^{100})$ is divided by 4?
(a) 0 (b) 1 (c) 2 (d) 3

Q.22. A 3 digit number in which all the 3 digits are odd is such that if the cubes of the digit are added, the sum would be equal to the number itself. If one of the digit is 7, find the number.
(a) 171 (b) 371 (c) 575 (d) 775

Q.23. A teacher said that there were 100 students in his class, 24 of whom were boys and 32 were girls. Which base system did the teacher use in this statement?
(a) 9 (b) 5 (c) 6 (d) 8

Q.24. What is the remainder when 3^{450} is divided by 108?
(a) 3 (b) 1 (c) 27 (d) 81

Q.25. P is a natural number. 2P has 28 divisors and 3P has 30 divisors. How many divisors of 6P will be there?
(a) 35 (b) 40 (c) 45 (d) 48

Q.26. pqr is a three digit natural number such that $pqr = p^3 + q^3 + r^3$. What is the value of r?
(a) 0 (b) 1
(c) 3 (d) Cannot be determined

Q.27. There are two three-digit numbers. When one number is divided by another number, quotient obtained is 6 and remainder is 0. Sum total of both the numbers is a multiple of 504. What is the difference between the numbers.?
(a) 720 (b) 360 (c) 120 (d) 420

Q.28. LCM of 12^{24}, 16^{18} and N is 24^{24}. Number of all the possible values of N = S. What is the value of S?
(a) 25 (b) 1,800
(c) 1,825 (d) None of these

Q.29. Each of P, Q, R and S equals either 0 or 1. It is given that
If Q = 0, then R = 1
If R = 0, then P = S
If S = 0, then P = 1
Assume R = 0, find the value of (P + Q + R + S)?
(a) 0 (b) 1 (c) 2 (d) 3

Q.30. When asked about his date of birth in 1996, Mayank replied that "last two digits of my birth year stands for my age." When Siddharth was asked about his age, he also replied the same. But Siddharth is older to Mayank. What is the difference in their age?
(a) 46 (b) 50
(c) 0 (d) Cannot be determined

Directions for questions 31–35: Read the passage below and solve the questions based on it.

There is a prison with 100 cells inside it. Cells are numbered from 1 to 100 and every cell is occupied by one prisoner only. One day jailer decides to release some of the prisoners and for this he defines an algorithm of 100 steps which follows:

Step 1 Reverse the position of all the cells which are divisible by 1.

Step 2 Reverse the position of all the cells which are divisible by 2.

Step 3 Reverse the position of all the cells which are divisible by 3.

.....

.....

.....

Step 99 Reverse the position of all the cells which are divisible by 99.

Step 100 Reverse the position of all the cells which are divisible by 100.

Initially all the cells are closed. After executing all these steps, prisoners of all the cells which remain open are released.

Q.31. How many prisoners are released?
(a) 25 (b) 10
(c) 5 (d) None of these

Q. 32. Which of the following cell number will be open at the end?
(a) Cell number 56 (b) Cell number 64
(c) Cell number 72 (d) Cell number 84

Q.33. Which of the following is true about the family of cell numbers N that will be open at the end?
(a) All the elements of N will be having only two factors including the number itself.
(b) All the elements of N will be having odd number of factors excluding the number itself.

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(c) All the elements of N will be having odd number of factors including the number itself.
(d) None of these

Q.34. If number of prisoners in each cell is equal to the cell number, then how many prisoners will be released?
(a) 285 (b) 385
(c) 485 (d) None of these

Q.35. There are many islands on earth for which we do not have much information, or sometime we do not have any information about those islands. On such an island, which uses a system of S digits ($S \neq 10$) to write the numbers, selling price of a goat is Rs 1,143. Parul bought a goat and paid the shopkeeper Rs 1,150. Now shopkeeper returned Parul Rs 5. How many digits are used to write the numbers in that system?
(a) 9 (b) 8 (c) 7 (d) 6

Q.36. What is the value of N in the following expression $345_6 + 632_7 + 487_9 = (N)_5$?
(a) 11,412 (b) 11,214
(c) 10,412 (d) 21,412

Q.37. In a certain system of writing of numbers we write $(12\Phi)_4 = (124)_\Phi$, where Φ is the unit digit in the first place and number of digits in that system in the second place.
How many values of Φ are possible?
(a) 0 (b) 1 (c) 3 (d) Infinite

Q.38. Digital sum of a number is obtained by adding all the digits of a number until a single digit is obtained. What is the digital sum of 4444^{4444}?
(a) 2 (b) 6 (c) 7 (d) 3

Directions for questions 39–41: Read the passage below and solve the questions based on it.

$N = A^2 \times B^3 \times C^4$, where A, B and C are prime numbers.

Q.39. How many factors of N are perfect squares?
(a) 6 (b) 12
(c) 8 (d) None of these

Q.40. How many factors of N are cube of any natural number?
(a) 2 (b) 3 (c) 4 (d) 6

Q.41. How many factors of N will be having exactly 3 factors?
(a) 2 (b) 3
(c) 4 (d) None of these

Directions for questions 42–43: Read the passage below and solve the questions based on it.

M and N are two natural numbers such that M + N = 949. LCM of M and N is 2628.

Q.42. What is the HCF of M and N?
(a) 23 (b) 73
(c) 69 (d) None of these

Q.43. When M and N are divided by P, remainder obtained is same. How many values of P are possible?
(a) 0 (b) 1 (c) 2 (d) 3

Q.44. What is the difference between the largest four digit number and the smallest four digit number, both written in hexadecimal system?
(a) FFFF (b) EFFF
(c) EEEE (d) EFEF

Q.45. A ten digit number is said to be interesting if its digits are all distinct and it is a multiple of 11111. How many interesting integers are there?
(a) 1,001 (b) 101
(c) 10,001 (d) None of these

Q.46. Sum of a three-digit number and its mirror image is a multiple of 111. What is this sum (provided 6 is one of the digits of this three-digit number)?
(a) 999 (b) 666 (c) 888 (d) 777

Q.47. Consider the following statements:
(i) If $(2)_N \times (4)_N = (8)_N$, where $(X)_N$ is a number written in a system of writing having N digits, then N can have infinite values.
(ii) If $(4)_N \times (5)_N = (24)_N$, where $(X)_N$ is a number written in a system of writing having N digits, then N will have more than 1, but finite values.
(iii) If $(5)_N \times (6)_N = (3A)_N$, where $(X)_N$ is a number written in a system of writing having N digits and A is the unit digit of this number (3A) written in a system of writing the numbers having N digits, then A can have four values.
How many of the above written statements are true?
(a) 1 (b) 2
(c) 3 (d) None of these

Q.48. How many numbers between 1 to 250 can be represented in the form x^y, where $y > x > 1$?
(a) 6 (b) 7 (c) 8 (d) 9

Q.49. What will be the ten's place digit in the following expression:
$1! + 2! + 3! + \ldots + 70!$?
(a) 1 (b) 3 (c) 5 (d) 6

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Q.50. A two-digit number is K times the sum of its digits. When these numbers are interchanged, it equals N multiplied by the sum of digits. Which of the following is equal to N?
(a) K – 11 (b) 99 – K
(c) 11 – K (d) K – 99

Q.51. What is the remainder when $_{32}33^{34}$ is divided by 7?
(a) 4 (b) 2 (c) 3 (d) 1

Q.52. 1's are given 100 times, 2's are given 100 times and 3's are given 100 times. Now numbers are made by arranging these 300 digits in all possible ways. How many of these numbers will be perfect squares?
(a) 10101 (b) 1001001
(c) 100010001 (d) None of these

Q.53. Which of the following number is the greatest number dividing a family of numbers ($N^5 - N$), where N is any natural number?
(a) 0 (b) 15 (c) 30 (d) 120

Q.54. When Rahul Ghosh distributes Rs 5 to each of his employees, he is left with Re 1. And when he distributes Rs 6 to each of his employees, he is left with Rs 5. What is the sum of digits of the minimum possible sum with Rahul Ghosh if he is having more than Rs 100?
(a) 1 (b) 2 (c) 3 (d) 4

Q.55. The integers 34041 and 32506,when divided by a three digit integer N, leave the same remainder. What can be the value of N?
(a) 289 (b) 307 (c) 317 (d) 319

Q.56. In the above question, how many values of N is/are possible?
(a) 2 (b) 1
(c) 4 (d) None of these

Q.57. What is the remainder when 77777 ... upto 56 digits is divided by 19?
(a) 1 (b) 7 (c) 9 (d) 13

Q.58. There are 3 consecutives odd integers such that the product is a prime number. Find the largest of the prime number.
(a) 1 (b) 3 (c) 5 (d) 7

Q.59. A square floor of the dimensions 72 cm × 72 cm has to be laid with rectangular tiles whose length and breadth are in the ratio 3:2. What is the difference between the maximum number of tiles and minimum numbers of tiles, given that the length and the breath are integers?
(a) 858 (b) 864 (c) 868 (d) 872

Q.60. A man took a 5 digit number ending in 9 and raised it to an even power greater than 50. He then multiplied it with 17 raised to a multiple of 4. What is the last digit of the resulting number?
(a) 1 (b) 9 (c) 3 (d) 7

Q.61. Rahul took an odd number of prime numbers and added them. He found the result to be even. Which of the following could be concluded?
(a) The numbers include an even number.
(b) The numbers do not include an even number.
(c) There are even number of even numbers.
(d) None of the above.

Q.62. When asked as to how many guavas Lovely has, she replies, "if you add three quarters of the number of guavas that I have to three quarters of one, guava, you will get the numbers of guavas I have". How many guavas does she have?
(a) 2 (b) 5
(c) 7 (d) None of these

Directions for questions 63–65: Read the passage below and solve the questions based on it.

Rohan purchased some pens, pencils and erasers for his young brothers and sisters for the ensuing examinations. He had to buy at least 11 pieces of each items in a manner that the number of pens purchased be more than the number of pencils, which is more than the number of erasers. He purchased a total of 38 pieces.

Q.63. How many erasers did Rohan purchase?
(a) 11 (b) 10
(c) 8 (d) Cannot be determined

Q.64. If each eraser costs Rs 3, each pencil Rs 2 each pen Rs 10, what is the maximum amount that Rohan could have spent?
(a) 207 (b) 255 (c) 288 (d) 300

Q.65. If the number of pencils cannot be equally divided among his 4 brothers and sisters, how many pens did he purchase?
(a) 11 (b) 12 (c) 13 (d) 14

Q.66. What is the remainder when [100! + 1] is divided by 101?
(a) 100 (b) 1 (c) 0 (d) 2

Q.67. A is a set of first ten natural numbers. B is a subset of A such that B consists of exactly 2 numbers, both co-primes to each other. Find the maximum number of such possible subsets.
(a) 29 (b) 28 (c) 33 (d) 31

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Q.68. When written in the lowest term, rational numbers A and B, have denominators 60 and 70 respectively. Now A and B have been added. What is the smallest possible denominator of (A + B)?
(a) 72 (b) 84 (c) 105 (d) 148

Directions for questions 69–70: Read the passage below and solve the questions based on it.

The multiplication of two numbers is shown below.

```
 AD4
  xE
 ----
 A206
 ----
```

where A, D, and E are all distinct digits.

Q.69. The value of D is
(a) 3 (b) 9 (c) 2 (d) 7

Q.70. The value of A + E is
(a) 6 (b) 9 (c) 11 (d) 10

Q.71. P, Q, R, S and T are five prime numbers, where P < Q < R < S < T. It is also given that P + Q + R + S + T = 482. What is the value of P^5?
(a) 243 (b) 32
(c) 16807 (d) More than one value

Q.72. If n is a natural number greater than 1, then $1075^n - 1075^{n-1}$ is not divisible by
(a) 15 (b) 23 (c) 43 (d) 179

Directions for questions 73–74: Read the passage below and solve the questions based on it.

(a, b) is an ordered pair such that a and b are two-digit numbers a is the number obtained by interchanging the digits of b.

Q.73. How many ordered pairs are there such that $(a+b)$ is a perfect square?
(a) 4 (b) 8
(c) 3 (d) None of these

Q.74. How many ordered pairs are there such that $(a - b)$ is a perfect square greater than zero?
(a) 3 (b) 10
(c) 13 (d) None of these

Q.75. N is an even number when written in the decimal system. If S is the sum of the digits of N, when it is written in base 7, we can surely say that S would always be
(a) Even (b) Odd
(c) Composite (d) Both (a) and (c)

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Q.1. Find the last non zero digit of 96!.
(a) 2 (b) 4 (c) 6 (d) 8

Q.2. What will be remainder when 1212121212…300 times, is being divided by 99?
(a) 18 (b) 81 (c) 54 (d) 36

Q.3. Two players A and B are playing a game of putting '+' and '–' signs in between any two integers written from 1 to 100. A starts the game by putting a plus sign anywhere between any two integers. Once all the signs have been put, the result is calculated. If it is even then A wins and if it is odd then B wins, provided they are putting signs by taking turns one by one and either of them can put any sign anywhere between any two integers. Who will win at the end?
(a) A (b) B
(c) Either A or B (d) Cannot be determined

Q.4. What is the remainder when 2^{100} is divided by 101?
(a) 1 (b) 100
(c) 99 (d) None of these

Q.5. If $S = \{X_1, X_2, X_3 \ldots X_n\}$, where $n > 0$, $X_1 = 1$ is a set of positive numbers where $X_n = X_{n-1} + 2n - 1$. Which of the following sets of numbers does S resemble?
(a) Fibonacci (b) Cube
(c) Square (d) Prime

Q.6. A three-digit number ABC is a perfect square and the number of factors of this number is also a perfect square. If (A + B + C) is also a perfect square, then what is the number of factors of the 6-digit number ABCABC?
(a) 32 (b) 52
(c) 72 (d) Cannot be determined

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Q.7. How many divisors of 10^5 will have atleast one zero at its end?
(a) 9 (b) 12 (c) 15 (d) 25

Q.8. Let $V_1, V_2, V_3, \ldots V_{100}$ be hundred positive integers such that $V_i + V_{i+1} + V_{i+2} + V_{i+3} = K$, where K is a constant and i = 1, 2, 3, … 97. If $V_3 = 9$, then what is the value of V_{90}?
(a) 9 (b) K – 9
(c) (K/2 – 9) (d) Cannot be determined

Q.9. In the above question, if $V_3 = 7$, then what is the value of V_{90}?
(a) 7 (b) K – 7
(c) (K – 7)/2 (d) Cannot be determined

Q.10. *abcd* is a four-digit number in base 7 such that 2 (*abcd*) = *bcda*. ($a, b \neq 0$) Find the value of *a*.
(a) 1 (b) 2
(c) 3 (d) Cannot be determined

Q.11. If K is any natural number, such that $100 \leq K \leq 200$, how many values of K exist such that K! has 'z' zeroes at its end and (K + 2)! has 'z + 2' zeroes at its end?
(a) 2 (b) 4
(c) 6 (d) None of these

Q.12. Tatto bought a notebook containing 96 pages leaves and numbered them which came to 192 pages. Tappo tore out the latter 25 leaves of the notebook and added the 50 numbers she found on those pages. Which of the following is not true?
(a) She could have found the sum of pages as 1990
(b) She could have found sum of pages as 1275
(c) She could have got sum of pages as 1375
(d) None of these

Directions for questions 13–15: Read the passage below and solve the questions based on it.

There are 50 integers a1, a2, a3, …, a50; not all of them necessarily different. Let the greatest integer of these 50 integers be referred to as G and smallest integer be referred to as L. The integers a1 to a24 form a sequence S1 and the rest form a sequence S2. Each member of S1 is less than or equal to each member of S2.

Q.13. All values in S1 are changed in sign, while those in S2 remain unchanged. Which of the following statements is true?
(a) Every member of S1 is greater than or equal to every member of S2
(b) G is in S1
(c) If all the numbers originally in S1 and S2 had the same sign, then after the change of sign, the largest number of S1 and S2 is in S1
(d) None of these

Q.14. Elements of S1 are in ascending order and those of S2 are in descending order. a24 and a25 are interchanged then which of the following is true?
(a) S1 continues to be in ascending order
(b) S2 continues to be in descending order
(c) Both (a) and (b)
(d) Cannot be determined

Q.15. Every element of S1 is made greater than or equal to every element of S2 by adding to each element of S1 an integer *x*. Then, *x* cannot be less than
(a) 2^{10}
(b) The smallest value of S2
(c) The largest value of S2
(d) (G-L)

Q.16. Twenty-five boxes of sweets are delivered to Mr Roy's home. Mr Roy had ordered sweets of three different types. What is the minimum number of boxes of sweets which are having sweets of same type?
(a) 1 (b) 8
(c) 9 (d) Cannot be determined

Q.17. A warehouse contains 200 shoes of size 8, 200 shoes of size 9 and 200 shoes of size 10. Of these 600 shoes, there are 300 left shoes and 300 right shoes. What is the minimum number of usable shoes?
(a) 50 (b) 100
(c) 200 (d) None of these

Q.18. A teacher was doing some calculation exercise on the blackboard. When the teacher went out, a naughty student Chunmun erased some of the numbers written on the blackboard. Now it appeared like this

$$\begin{array}{r} 23_5_ \\ +1_642 \\ \hline 42423 \end{array}$$

When teacher entered the room, he realized that still this calculation was right, but in some other system of writing (i.e., not 10). How many digits are there in that system?
(a) 11 (b) 9 (c) 7 (d) 8

Q.19. Tatto, Tappo and Bubbly were solving problems from a problem book. Each solved exactly 60 problems, but they solved only 100 problems altogether. Any problem is known as "easy" if it was solved by all of them, and "difficult" if it was

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solved by only one of them. What is the difference between the number of "difficult" problems and number of "easy" problems?
(a) 10 (b) 20 (c) 30 (d) 40

Q.20. LCM of two numbers A and B = $P^x \times Q^y$, where P and Q are prime numbers and x and y are positive whole numbers. How many set of values are possible for A and B?
(a) $xy(x+y)$ (b) $xy(x-y)$
(c) $x^2y^2(x+y)$ (d) None of these

Q.21. When 7179 and 9699 are divided by another natural number N, remainder obtained is same. How many values of N will be ending with one or more than one zeroes?
(a) 24 (b) 124
(c) 46 (d) None of these

Q.22. There exists a 5 digit number N with distinct and non-zero digits such that it equals the sum of all distinct three digit numbers whose digits are all different and are all digits of N. Then the sum of the digits of N is a necessarily.
(a) Perfect square (b) Cube
(c) Even (d) None of these

Q.23. Starting with 1, positive integers are written one after the other. What is the 40,0000th digit that will be written?
(a) 3 (b) 6
(c) 8 (d) None of these

Q.24. Which of the following would always divide a six-digit number of the form ababab?
(a) 10,101 (b) 11,111
(c) 10,001 (d) None of these

Q.25. If in the number system of a particular country, 25 means 5 tens and 2 units, 467 means 7 hundreds, 6 tens and 4 units. Then find the value of 173 × 425?
(a) 4,04,491 (b) 7,35,255
(c) 6,22,744 (d) 5,25,376

Q.26. Let A be the set of integers N such that
i. $100 \le N \le 500$
ii. N is even
iii. N is divisible by either 2 or 3 or 4 but not by 7.
How many elements are there in set A?
(a) 171 (b) 172 (c) 170 (d) 173

Q.27. Three distinct prime numbers, less than 10 are taken and all the numbers that can be formed by arranging all the digits are taken. Now, difference between the largest and the smallest number formed is equal to 495. It is also given that sum of the digits is more than 13. What is the product of the numbers?
(a) 30 (b) 70 (c) 105 (d) 315

Q.28. What is the remainder when ${}_{55}56^{57}$ is divided by 17?
(a) 1 (b) 4 (c) 13 (d) 17

Q.29. What is the remainder when ${}_{20}51^{97}$ is divided by 17?
(a) 10 (b) 9 (c) 15 (d) 7

Q.30. [111 … 111(200 digits) – 222…22(100 digits)]$^{1/2}$ is equal to
(a) 1313 … 1313(100 digits)
(b) 2121…2121(100 digits)
(c) 1111…1111(100 digits)
(d) 3333…333(100 digits)

Q.31. N is a number which when divided by 10 gives 9 as the remainder,
when divided by 9 gives 8 as the remainder,
when divided by 8 gives 7 as the remainder,
when divided by 7 gives 6 as the remainder,
when divided by 6 gives 5 as the remainder,
when divided by 5 gives 4 as the remainder,
when divided by 4 gives 3 as the remainder,
when divided by 3 gives 2 as the remainder,
when divided by 2 gives 1 as the remainder.
What is N?
(a) 2519 (b) 841 (c) 839 (d) 2521

Q.32. What is the remainder when $(10^3 + 9^3)^{752}$ is divided by 12^3?
(a) 1 (b) 729 (c) 752 (d) 1000

Q.33. $f(x, x) = x^x$ and $f(f(x, x) = {}_{f}x^x$ and so on its keeps on going. What is the value of $f(f(f(f(7,7))))$ is divided by 5?
(a) 1 (b) 2 (c) 3 (d) 4

Q.34. The students of class 10th of Morgan High School took a test which had a maximum of 50 marks. The teacher misplaced the text notebooks of two of the students— Robin and Garry, but remembered that Garry had scored something between 10 and 15 and Robin something between 32 to 40. She also remembered that the product of the marks obtained by the two students was also equal to 10 times the marks obtained by two of them. How many marks did Garry scored?
(a) 11 (b) 12 (c) 13 (d) 14

Q.35. The History teacher was referring to a year in the 19th century. Rohan found an easy way to remember the year. He found that the number, when viewed in a mirror, increased 4.5 times. Which year was the teacher referring to?

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(a) 1,801 (b) 1,810
(c) 1,818 (d) More than one value

Q.36. Srini wrote his class 10th board examination this year. When the result came out he searched for his hall ticket to see his roll number but could not trace it. He could remember only the first three digits of the 6 digit number as 267. His father, however, remembered that the number was divisible by 11. His mother gave the information that the number was also divisible by 13. They tried to recollect the number when all of a sudden Srinivasa told that the number was a multiple of 7. What was the unit digits of the number?
(a) 5 (b) 7
(c) 2 (d) Cannot be determined

Q.37. Prof. Mathur and Prof. Singh attended the All India Historian's meet last week. Prof. Mathur told Prof. Singh, "I found out that your teaching experience is twice that of mine". Prof. Singh replied in the affirmative. Prof. Mathur continued, "But last time when both of us came for the same meet, I remember that your teaching experiencr was thrice that of mine". "That was 2 years ago." Prof. Singh said. How many years has Prof. Singh been working?
(a) 8 yrs (b) 10 yrs (c) 12 yrs (d) 16 yrs

Directions for questions 38–39: Read the passage below and solve the questions based on it.

ABCDEF is a 6-digit number with distinct digits. Further, the number is divisible by 11 and the sum of its digits is 24. Further A > C > E and B > D > F.

Q.38. The sum A + C + E is equal to
(a) 12 (b) 6
(c) 8 (d) Cannot be determined

Q.39. A + B is always
(a) 10 (b) 9
(c) 6 (d) Cannot be determined

Q.40. Raju had to divide 1080 by N, a two-digit number. Instead, he performed the division using M which is obtained by reversing the digits of N and ended up with a quotient which was 25 less than what he should have obtained otherwise. If 1080 is exactly divisible both by N and M, find the sum of the digits of N.
(a) 6 (b) 8
(c) 9 (d) None of these

Q.41. Let S = {1, 2, 3, ... n} be a set of N natural numbers. Let T be a subset of S such that the sum of any three elements of T is not less than N. Find the maximum number of elements in any such subset T for N = 40?
(a) 26 (b) 27
(c) 28 (d) None of these

Q.42. The last digit of the LCM of $(3^{2003} - 1)$ and $(3^{2003} + 1)$ is
(a) 8 (b) 2 (c) 4 (d) 6

Q.43. a, b and c are positive integers such that, $a + b + c = 2003$. Let $E = (-1)^a + (-1)^b + (-1)^c$. Find the number of possible values of E.
(a) 2004 (b) 3 (c) 1003 (d) 2

Q.44. Ajay took a 4-digit number in base 5 notation. He subtracted the sum of the digits of the numbers from the number. From the result, he struck off one of the digits. The remaining 3 digits were 1, 0 and 2. Then the digit struck off by Ajay was:
(a) 2 (b) 1
(c) 4 (d) Cannot be determined

Directions for questions 45–46: Read the passage below and solve the questions based on it.

N is a single digit integer satisfying the following two conditions.
i. N is non-zero.
ii. N is the right most digit of the number $(n!)^4$, where n is a natural number greater than 1.

Q.45. What is the number of possible values of N?
(a) 1 (b) 2
(c) 0 (d) None of these

Q.46. If condition (a) is relaxed, the number of possible values of N is.
(a) 1 (b) 2
(c) 0 (d) More than 2

Q.47. A teacher wrote a number on the blackboard and the following observations were made by the students
The number is a four-digit number.
The sum of the digits equals the product of the digits.
The number is divisible by the sum of the digits.
The sum of the digits of the number is
(a) 8 (b) 10 (c) 12 (d) 14

Q.48. The N of odd numbers are taken. Product of these odd numbers is of the form $(4n + 1)$, where n is any natural number Which of the following is true regarding the number of numbers?
(a) There must have been an odd number of numbers of the form $4n + 1$

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(b) There must have been an even number of numbers of the form $4n + 1$
(c) There must have been an even number of numbers of the form $4n + 3$
(d) None of these

Q.49. 16 students were writing a test in a class. Rahul made 14 mistakes in the paper, which was the highest number of mistakes made by any student. Which of the following statements is definitely true?
(a) At least two students made the same number of mistakes
(b) Exactly two students made the same number of mistakes
(c) At most two students made the same number of mistakes
(d) All students made different number of mistakes.

Q.50. The sum of the factorials of the three-digits of a 3-digit number is equal to the three-digit number formed by these three digits, taken in the same order. Which of the following is true of the number of such three-digit numbers, if no digit occurs more than once?
(a) No such number exists
(b) Exactly one such number exists
(c) There is more than one such number, but they are finite in number
(d) There are infinite such numbers

Directions for questions 51–52: Read the passage below and solve the questions based on it.

The inhabitants of planet Romeo, a remote planet in the milky way, use a number system which is similar to the decimal system used here on the earth, except that it has eleven distinct digits instead of the usual ten digits. The extra digit is an alien digit called x, which is inserted between the digits 7 and 8. Now $8 - 7 = 2$ and x is the digit, which is equidistance from 7 and 8, i.e., $x - 7 = 8 - x = 1$. All the algebric signs and operations carry the same meaning as in the usual sense.

Q.51. Romeo, an inhabitant of planet Romeo, evaluated an expression and arrived at an answer of '$6x$'. Then which of the following can be the expression that Romeo evaluated?
(a) $x^2 + x - 1$ (b) $x^2 + x + 1$
(c) $x^2 + x + 2$ (d) None of these

Q.52. What is the decimal equivalent of the two digit number '$9x$'?
(a) 98 (b) 107
(c) 118 (d) None of these

Q.53. If P is the product of four consecutive positive integers, then which of the following statements is not true?
(a) P is perfectly divisible by 24
(b) P is not a perfect square
(c) $(P + 1)$ is a perfect square
(d) $(P - 2)$ is a perfect square

Q.54. Let S be a two-digit number such that both S and S^2 end with the same digit and none of the digits in S equals zero. When the digits of S are written in the reverse order, the square of the new number so obtained has the last digit as 6 and is less than 3000. How many values of S are possible?
(a) 3 (b) 4 (c) 5 (d) 6

Q.55. How many different four digit numbers are there in the octal (Base 8) system, expressed in that system?
(a) 3584 (b) 2058 (c) 6000 (d) 7000

Q.56. How many numbers less than or equal to 500 are there, each of which is the product of more than three distinct prime numbers?
(a) 3 (b) 4 (c) 5 (d) 6

Q.57. How many numbers below 100 can be expressed as a difference of two perfect squares in only one way?
(a) 15 (b) 25 (c) 35 (d) 45

Q.58. The LCM of the two quotients, obtained when two numbers a and b, are divided by their HCF is 77. LCM of another pair of quotients, when other two numbers c and d have the same HCF as that of a and b, and when divided by it, is 221. Which of the following could be, ratio of the sum of a and b to that of the difference between c and d?
(a) 4 : 17 (b) 11:4 (c) 2:9 (d) 39:2

Q.59. Let p, q and s be integers such that $p^2 = sq^2$. Then it follows that
(a) p is an even number
(b) if s divides p, then s is a perfect square
(c) s divides p
(d) q^2 divides p

Q.60. Let N be a positive integer not equal to 1. Then none of the numbers 2, 3,...., N is a divisor of $(N! - 1)$. Thus, we can conclude that
(a) $(N! - 1)$ is a prime number
(b) At least one of the numbers $N + 1, N + 2, \ldots, N! - 2$ is a divisor of $(N! - 1)$
(c) The smallest number between N and N! which is a divisor of $(N! + 1)$, is a prime number
(d) None of the foregoing statement is necessarily correct

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TRUE/FALSE

Q.1. A natural number M is divided by another natural number N and the remainder obtained = R ≠ 0 and M > N. It is always possible to find out the exact remainder when M is divided by P (where P is a factor of N).

State whether True or False.

Q.2. A natural number M is divided by another natural number N and the remainder obtained = R '" 0 and M > N. It is always possible to find out the exact remainder when M is divided by P (where P is a multiple of N).

State whether True or False.

Q.3. A, B, C are three natural numbers. If we know the LCM of A, B and LCM of B, C separately, then it is always possible to find out the LCM of A, B, C.

State whether True or False.

Q.4. A, B, C are three natural numbers. If we know the HCF of A, B and HCF of B, C separately, then it is always possible to find out the HCF of A, B, C.

State whether True or False.

Q.5. A, B, C, D are four natural numbers. If we know the LCM of A, B and LCM of C, D separately, then it is always possible to find out the LCM of A, B, C, D.

State whether True or False.

Q.6. A, B, C, D are four natural numbers. If we know the HCF of A, B and HCF of C, D separately, then it is always possible to find out the HCF of A, B, C, D.

State whether True or False.

Q.7. If we know the total number of odd factors of a number, then we can always find out the total number of factors of that number.

State whether True or False.

Q.8. If we know the total number of even factors of a number, then we can always find out the total number of factors of that number.

State whether True or False.

Q.9. If a number is odd, then it cannot have total number of factors as an even number.

State whether True or False.

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ANSWERS

WARM UP

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Answer	(d)	(d)	(d)	(d)	(d)	(d)	(b)	(a)	(c)	(b)	(a)	(a)	(d)	(c)	(d)	(d)	(b)
Q. No.	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Answer	(a)	(b)	(c)	(b)	(c)	(d)	(b)	(c)	(d)	(d)	(b)	(d)	(a)	(b)	(c)	(d)	(c)

FOUNDATION

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(a)	(a)	(a)	(a)	(c)	(b)	(b)	(c)	(b)	(d)	(c)	(c)	(c)	(a)	(b)	(d)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(a)	(c)	(b)	(d)	(c)	(c)	(d)	(a)	(b)	(b)	(a)	(a)	(c)	(c)	(a)	(b)	(a)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(a)	(a)	(c)	(b)	(a)	(b)	(c)	(a)	(b)	(c)	(d)	(c)	(a)	(b)	(a)	(c)	(c)	(d)	(b)	(a)
Q. No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Answer	(c)	(c)	(d)	(d)	(b)	(a)	(b)	(d)	(b)	(b)	(d)	(d)	(a)	(b)	(c)	(b)	(c)	(c)	(d)	(a)

MODERATE

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(b)	(c)	(b)	(a)	(c)	(c)	(b)	(c)	(a)	(b)	(c)	(c)	(b)	(b)	(a)	(c)	(c)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(b)	(c)	(d)	(a)	(d)	(a)	(c)	(c)	(b)	(b)	(b)	(c)	(b)	(b)	(a)	(a)	(c)	(b)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(d)	(b)	(c)	(b)	(d)	(c)	(a)	(b)	(a)	(c)	(d)	(d)	(c)	(b)	(b)	(c)	(a)	(b)	(a)	(a)
Q. No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75					
Answer	(a)	(d)	(a)	(a)	(c)	(c)	(d)	(b)	(a)	(d)	(b)	(b)	(a)	(d)	(d)					

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Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(a)	(a)	(c)	(d)	(d)	(a)	(d)	(d)	(c)	(a)	(d)	(a)	(d)	(c)	(b)	(c)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(a)	(d)	(d)	(a)	(b)	(b)	(a)	(a)	(d)	(a)	(a)	(c)	(d)	(c)	(b)	(a)	(a)	(c)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(d)	(c)	(d)	(b)	(d)	(d)	(a)	(c)	(a)	(b)	(c)	(c)	(d)	(a)	(d)	(b)	(c)	(d)	(b)	(d)

TRUE / FALSE

Ans.1. True

To find out the remainder when M divided by P, we simply need to divide R by P.

Ans.2. False

If we divided M by P, we would get a range of remainders in terms of R and not the exact value of remainder in all the cases.

Ans.3. True

LCM, by its meaning, is the lowest number divisible by all the numbers constituting it. Final LCM will be the LCM of the pairs of numbers.

Ans.4. True

HCF, by its meaning, is the highest number that can divide the numbers constituting it. Final HCF will be the HCF of the pairs of numbers.

Ans.5. True

LCM, by its meaning, is the lowest number divisible by all the numbers constituting it. Final LCM will be the LCM of the pairs of numbers.

Ans.6. True

HCF, by its meaning, is the highest number that can divide the numbers constituting it. Final HCF will be the HCF of the pairs of numbers.

Ans.7. False

Ans.8. False

Ans.9. False

Total number of factors do not have any relationship with the number being odd or even. For example, all the perfect squares (irrespective of being odd or even) have total number of factors = Odd number.

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permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered that he had missed one number in the sequence during addition. The number he missed was (1 mark)

(a) Less than 10 (b) 10
(c) 15 (d) More than 15

Q.8. $1000^7 - 17$ is written in decimal notation. What is the sum of its digits? (1 mark)

(a) 181 (b) 182
(c) 183 (d) None of these

Q.9. N is an integer. For how many distinct values of N, $N^4 - 20N^2 + 4$ is a prime number? (1 mark)

(a) 1 (b) 3
(c) 3 (d) None of these

Q.10. Of 128 boxes of oranges, each box contains atleast 120 and at most 144 oranges. The number of boxes containing the same number of oranges is at least: (2 marks)

(a) 5 (b) 6
(c) 103 (d) None of these

Q.11. What is the value of $\sum_{N=1}^{99} \frac{1}{\sqrt{N}+\sqrt{N+1}}$? (1 mark)

(a) 9 (b) $\sqrt{99}$
(c) $\sqrt{100}-\sqrt{99}$ (d) None of these

Q.12. a, b, c, d and e are prime numbers, not necessarily consecutive. If $a+b+c+d+e=350$, and $a<b<c<d<e$, how many values of a^5 is/are possible? (1 mark)

(a) 1 (b) More than 1, but finite
(c) Infinite (d) Nothing can be said

Q.13. The ten-digit number $9793x\ 6160y$ is divisible by 11. If $0 < x < y$, find the sum of remainders when the number is divided by $x + y$ and x successively. (1 mark)

(a) 4 (b) 6 (c) 7 (d) 9

Q.14. What is the remainder when 128^{500} is divided by 153? (1 mark)

(a) 63 (b) 67
(c) 123 (d) None of these

Q.15. What is the minimum number of weights which enable us to weigh any integer number of grams of gold from 1 gm to 100 gms on a standard balance with two pans, weights can be put on either pan of balance? (2 marks)

(a) 4 (b) 5
(c) 6 (d) None of these

Q.16. $N = 2^8 \times 3^{10} \times 5^8 \times 7^2$. What is the number of factors of this number N which are factors of 360 but not the factors of 540? (2 marks)

(a) 120 (b) 210
(c) 114 (d) 144

Q.17. What is the remainder when 2^{104} is divided by 101?
(2 marks)

(a) 1 (b) 2 (c) 16 (d) 64

Q.18. Given that:
$X_1 = 5, X_2 = 25$ and $X_{n+2} = \text{GCD}\,(X_{n+1}, X_n) + X_n$.
What is the LCM of (X_{19} and X_{20})? (2 marks)

(a) 1840 (b) 2560
(c) 2160 (d) 2440

Q.19. $N_1 + N_2 + N_3 + \ldots + N_N = 100$, where $N_1, N_2, N_3 \ldots$ and N_N are $n(n>1)$ consecutive natural numbers such that $N_1 < N_2 < N_3 < N_4 \ldots < N_N$. If $N_1 > 8$, then how many values of n is/are possible? (2 marks)

(a) 4 (b) 8 (c) 2 (d) Infinite

Q.20. A teacher writes 101 numbers on the blackboard, of which 50 are zeros, and 51 are ones. A student is asked to perform the following operation 100 times on the board: strike out any two numbers. If they are equal, write another zero. If they are unequal, write a one. What are the numbers left on the board? (2 marks)

(a) Single One (b) Single Zero
(c) Two Ones (d) None of these

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3
PERCENTAGE

CAT 2007

Number of questions from this topic: 0

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- What is percentage
- Definitions and properties
- Percentage–ratio equivalence
- Usage of percentage as a calculation device
- Methods of solving questions

CAT 2006

Number of questions from this topic: 0

CAT 2005

Number of questions from this topic: 0

Introduction to the topic

Percentage is of immense importance when it comes to problem solving in quantitative aptitude (QA) and data interpretation (DI). Despite the fact that the number of questions asked from this concept is not very high in the CAT, a sound understanding and good practice of percentage calculations comes handy in almost all the topics of QA and DI. However, percentage is an important concept for all the other B-school and aptitude examinations.

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WHAT IS PERCENTAGE

Before we talk of the utility of the concept 'percentage', we should be clear regarding when should we read (%) as per cent and when as percentage. Whenever (%) symbol is attached to any value, known or unknown, the word which we use for symbol (%) is per cent, e.g., x% is read as x per cent or 20% is read as 20 per cent, not 20 percentage. The word percentage is used whenever (%) is not attached to any value.

This whole nomenclature can be understood with the help of the following example:

The salary of Abhishek is 20% more than the salary of Abhinav. By how much percentage is the salary of Abhinav less than the salary of Abhishek?

Let us first have some standard meaning of percentage:

- Percentage as a common platform
- Percentage as a rate measurer

Percentage As a Common Platform

The concept of percentage can be further understood with the help of the following table, which gives us the marks obtained by different students in their Class 10 exam:

Student in country	*Marks obtained*
America	100
India	25
China	45
France	50

Right now, by just having this piece of information, we cannot decide that students of which country have performed the best and the data required here is the total marks obtained in each country.

Now, suppose if we add the following data to this question, then the whole situation looks like:

Actually, here we are using percentage as a common platform to compare all the given values.

Student in country	*Marks obtained*	*Total marks*	*Marks obtained/ 100 marks*
America	100	1000	10%
India	25	25	100%
China	45	300	15%
France	50	100	50%

To know more about percentage, go to *www.gomath.com/exercises/percentage.php*

Percentage As a Rate Measurer

The concept of percentage is best-suited to find out the rate of change when two or more than two quantities are attached to each other by any common relationship. In this case, we get the respective change which should be brought in the value of quantities, when the value of other quantities are changing. We will see the application of percentage as a rate measurer while doing product stability ratio.

Before we move ahead, it is important to understand the basic statements used in percentage.

Basic statement 1

What is x% of y?

$$\Rightarrow \frac{x \times y}{100}$$

It can also be seen that x% of y = y% of x

For example 4.5% of 20,000 = 20,000% of 4.5

This one simple fact can be used to divide or multiply any number by 50 or 25 or so. Let us see this with the help of an example: We are trying to find out the value of 25×32, which is nothing but $32 \times 100/4 = 800$. Similarly if we have to divide any number by 50, we should be multiplying the number by 100 and dividing it by 2 finally.

Using this, we can see that if we have to calculate 24% of 25 (or any other calculation of similar nature), it is better to find out 25% of 24 ($= 24 \times 100/4$) = 600.

Example 1 What is 20% of 50% of 40% of 20?

Solution Percentage means 'per hundred'.

So, 20% of 50% of 40% of 20 =

$(20/100) \times (50/100) \times (40/100) \times 20 = 0.8$

What we can observe here is that even if we change the order of values here, the final result will be the same.

Example 2 Statistics show that 20% of smokers get lung cancer and 80% of lung cancer patients are smokers. If 30% of the population smokes, then the percentage of population having lung cancer is

(a) 4 (b) 3 (c) 8 (d) 7.5

Solution Assume that the total population is 100.

30 people smoke, and out of them 6 people will be having lung cancer. This 6 represents 80% of lung cancer patients (because they smoke). Hence, total percentage of population having lung cancer $= \frac{6}{80} \times 100 = 7.5\%$

Basic statement 2

What percentage of x is y?

$$\Rightarrow \frac{y \times 100}{x}$$

(This can be easily deduced by assuming Z% of x is y and then proceeding as given in statement 1.)

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Some more example of the same kind are given below:

1. What is the percentage growth in the sales value of the year 2002-03 over the sales value of 2003-04?
2. What is the percentage hike in the salary of Manoj this year compared to his salary hike last year?

Basic statement 3

Change in value

(a) Percentage change = $\frac{\text{Change}}{\text{Initial value}} \times 100$

(b) Percentage point change – It is the numerical difference between the values for which we have to calculate change.

Let us assume some values to understand the above written concept

Market share	*2002-03*	*2003-04*
Maruti	40%	48%
Honda	30%	26%

Percentage change in the market share of Maruti over the years $= \frac{48-40}{40} \times 100 = 20\%$

Percentage point change in the market share of Maruti over the years = 48% – 40% = 8%

Similarly, if we have to increase any quantity N by S%, then it is equal to $N\left(1+\frac{S}{100}\right)$ and when the same quantity N is to be decreased by S%, then final quantity $= N\left(1-\frac{S}{100}\right)$

It is worth mentioning here that

$100 \rightarrow 20\%\uparrow \rightarrow 100\times1.2=120$ $\quad$ $100 \rightarrow 20\%\downarrow \rightarrow 100\times0.8=80$

$150 \rightarrow 30\%\uparrow \rightarrow 150\times1.3=195$ $\quad$ $150 \rightarrow 30\%\downarrow \rightarrow 150\times0.7=105$

$210 \rightarrow 27\%\uparrow \rightarrow 210\times1.27=266.7$ $\quad$ $210 \rightarrow 27\%\downarrow \rightarrow 210\times0.73=153.3$

So, if the final value and percentage increase or percentage decrease is given and we have to find out the initial value, then it can be done in similar way.

Using $S \rightarrow 30\%\uparrow \rightarrow S \times 1.3 = 195$

So, if the final value 195 and 30%↑ is given, then initial value $S = \frac{195}{1.3} = 150$

Example 3 My Reliance India phone bill for the month of May is Rs B. Moreover, there is a service tax of S% which is to be levied upon this value. But since they are overcharging their customers they offer a discount of D% on it. So, now I have two options to make the payment

Rs B → S%↑→ D%↓→ Final bill

Rs B → D%↓→ S%↑→ Final bill

Which option is beneficial for me if S > D?

Solution Prima facie, it might appear that the 1st one is better than the 2nd one or the 2nd one is better than 1st one, but a close and deep inspection will reveal that final bill is same in both the cases.

It can be checked with the help of assuming values also.

B = Rs 100, S% = 20% and D% = 10%

Rs 100 – (20%↑) – Rs 120→ (10%↓) – Rs 108 (Final bill)

Rs 100 – (10%↓) – Rs 90→ (20%↑) – Rs 108 (Final bill)

So, both the values are same at the end.

Importance of base Whenever we are talking about percentage, it is important to specify what it is relative to, i.e., what is that total which corresponds to 100%. The following situation illustrates this point:

In a certain school 60% of all students are male, and 10% of all students are computer science majors. If 5% of males are computer science majors, what percentage of computer science majors are male?

Here, we are asked to calculate the ratio of male computer science majors to all computer science majors. We know that 60% of all students are male, and among hese 5% are computer science majors, so we conclude that .6 × .05 = .03 or 3% of all students are male computer science majors. Dividing this by the 10% of all students that are computer science majors, we arrive at the answer: 3%/10% = .3 or 30% of all computer science majors are male.

While in QA, choosing the right denominator is often not a problem but sometimes it becomes very problematic in Data Inerpretation (DI), because we are unsure as to which value should be taken as denominator or base. However, if we go by some keywords viz., of/than/over/to, the quantity, which is attached to these keywords will be the denominator.

It can be seen through the examples also.

Example 4 A student multiplies a number x by 5 instead of dividing it by 5. What is the percentage change in the result due to this mistake?

Solution Let us assume $x = 5$

So, actual result should have been 100 ÷ 5 = 20

But the result obtained = 100 × 5 = 500

So, percentage change = (500 – 20) × 100/20 = 2400%

(It can be very well observed that if we take base as 500, percentage change = 96%. What we should primarily take care of, with comparison to which number we are calculating percentage change?)

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SUCCESSIVE PERCENTAGE CHANGE

Suppose we have to increase a quantity successively by 20% and 30%. Ideally this should be done by taking 100 as the initial value and then changing this initial value first by 20% and then by 30%. It can be seen below:

$100 \rightarrow 20\%\uparrow \rightarrow 120 \rightarrow 30\%\uparrow \rightarrow 156$

So, net percentage increase = 56%

This is known as the **straight line method** of solving the problems.

Alternatively, we can use successive change formula also: $\left(a+b+\frac{ab}{100}\right)\%$

Using this formula, $20 + 30 + \frac{600}{100} = 56\%$

This kind of calculation is very much important in DI questions, especially in pie chart which can be seen with the help of following example

The following figure shows the sales figure of Due North Inc. for the year 2002-03 and 03-04. It is also given that the share of north India in total sales figure is 20% and 24% in respective years.

2002-03 2003-04

Total sales for the year 2002-03 is Rs 1202 crores and for the year 2003-04, it is Rs 1381 crores.

Now the question is: What is the percentage growth in the sales value of north India in 2003-04 over the sales value of 2002-03?

Normal way of doing this question is:

Sales value of north India in 2002-03 = Rs 240.4 crores

Sales value of north India in 2003-04 = Rs 331.4 crores

So, growth = Rs 91 crore

So, the percentage growth =

$$\frac{\text{Growth}}{\text{Initial value}} \times 100 = 37.93\%$$

Alternatively, percentage increases in two factors in the given pie-charts viz., share is increasing from 20% to 24% and then total sales value is increasing from Rs 1202 crores to Rs 1381 crores.

Rather than calculating the sales value of individual years, we can directly calculate the net percentage change by calculating percentage changes independently in the above-mentioned two factors.

Percentage growth in share = 20%

$\left(\frac{\text{Growth} \times 100}{\text{Initial value}}\text{; and remember that is not 4\%}\right)$

Percentage growth in total value ≅ Approx. 15%

So, $100 \rightarrow 20\%\uparrow \rightarrow 120 \rightarrow 15\%\uparrow \rightarrow 138$, or

$100 \rightarrow 15\%\uparrow \rightarrow 115 \rightarrow 20\%\uparrow \rightarrow 138$

So, net percentage growth = 38%

Example 5 The price of petrol is increased by 20%. But expenses increase only by 10%. What is the percentage increase or decrease in consumption?

Solution If consumption remains constant, then the expenses should have also increased by 20%. But since expenses increase by only 10%, so consumption has been reduced.

These kinds of problems can be done in three ways:

i. Straight line method

100 (Initial expenditure)→ 20%↑ in price → 120 → x%↓ in consumption → 110 (Final expenditure),

$$x\% = \frac{10}{120} \times 100 = 8.33\%$$

ii. 10 × 10 method

We know, Price × Consumption = Expenditure

Assume Initial price = Initial consumption = Rs 10

So, Initial expenditure = Rs 100

	Price		Consumption		Expenditure
Initially… →	10	×	10	=	100
Finally… →	12	×	C	=	110

So, C = 110/12 = Rs 9.16

So, percentage reduction = 8.33%

iii. Formula method

Here a = 20%, b is not known and final percentage change = 10%

$20 + b + 20b/100 = 10$

$\Rightarrow 20 + b + b/5 = 10$

$\Rightarrow b = -8.33\%$

So, there is a decrease of 8.33% in the consumption.

PERCENTAGE–RATIO EQUIVALENCE

The essence of percentage–ratio equivalence lies in the fact that most of the percentage calculations like 25%, 37.5%, 33.33% etc., hover around some particular ratios only. And having a good command over these ratios is definitely going to give a good percentage calculation speed which is very essential for DI calculations as well as QA calculations. (See more such calculation in ratio, proportion and variation chapter.)

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	100%	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
2	50	100	150	200	250											800
3	33.33	66.66	100													533.33
4	25	50	75	100												400
5	20	40	60	80	100											
6	16.66	33.33	50	66.66	83.33	100										
7	14.28	28.56	42.85	57.14	71.42	85.71	100									
8	12.5	25	37.5	50	62.5	75	87.5	100								
9	11.11	22.22	33.33	44.44	55.55	66.66	77.77	88.88	100							
10	10	20	30	40	50	60	70	80	90	100						
11	9.09	18.18	27.27	36.36	45.45	54.54	63.63	72.72	81.81	90.90	100					
12	8.33	16.66	25	33.33	41.66	50	58.33	66.66	75	83.33	91.66	100				
13	7.69												100			
14	7.14	14.28	21.42	28.57	35.71	42.85								100		
15	6.66	13.33	20	26.66	33.33	40	46.66	53.33	60	66.66	73.33	80	86.66	93.33	100	
16	6.25	12.5	18.75	25	31.25	37.5	43.75	50	56.25	62.5	68.75	75	81.25	87.5	93.75	100

PRODUCT STABILITY RATIO

In a number of topics/concepts, we encounter the relationship where the product of two quantities equals the third quantity.

For example, Speed × Time = Distance

Price × Consumption = Expenditure

Number of persons × Days = Work done

Length × Breadth = Area of rectangle

Apart form these cases, a number of times we see cases where one quantity is increased to get another quantity, e.g., if we increase cost price to obtain a certain profit, we obtain selling price or if we increase principal, we obtain amount.

If we generalize product stability ratio, it can be written like

$$A \times B = P$$

Now, if A is increased by a certain percentage, then B is required to be decreased by certain percentage so that the product (P) remains stable. For example, if we increase A by 25% and P has to be constant, then B is required to be decreased by 20%.

This procedure can be summed up in the following way:

Change in A	*Change in B*	*Change in P*
100%↑	50%↓	0%
50%↑	33.33%↓	0%
33.33%↑	25%↓	0%
25%↑	20%↓	0%
20%↑	16.66%↓	0%
16.66%↑	14.28%↓	0%
14.28%↑	12.5%↓	0%
12.5%↑	11.11%↓	0%
11.11%↑	10%↓	0%
10%↑	9.09%↓	0%
9.09%↑	8.33%↓	0%
8.33%↑	7.69%↓	0%
And so on.....		

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So, if A is increased by 25%, then we need to decrease B by 20% to maintain the product stable.

This one mathematical information can be used in so many forms:

1. **Percentage** If A is 25% more than B, then by how much percentage B is less than A?
2. **Profit and Loss** An article is sold for Rs 125 at a profit of 25%. What is the cost price of the article?
3. **Time, Speed and Distance (TSD)** When speed of a car is increased by 25%, time taken reduces by 20 minutes in covering a certain distance. What is the actual time taken to cover the same distance by actual speed?
4. **TSD** Mayank goes to his office from his home at a speed of 20 kmph and gets late by 10 minutes. However, when he increases his speed to 25 kmph, he is 20 minutes early. What is the distance from his office to his home?
5. **Time and Work** Efficiency of Amit is 25% more than Vinit. Vinit takes 20 days to complete a work. How many days will Amit take to do the same work?
6. **Time and Work** 20 men can do some job in 50 days. In how many days will 25 men do the same job?
7. **Simple Interest (SI)** Rate of interest is 12.5% per annum SI. What is the principal if amount obtained after two years is Rs 1250?
8. **Percentage** Due to a price hike of 25%, 5 kg less sugar is available for Rs 100. What is the original price per kg?
9. **Mensuration** Length of a rectangle is increased by 25%. By what percentage should the breadth be decreased so that area remains constant?

In all the above written situations, just one mathematical information has been used, i.e., if A is increased by 25%, then B decreases by 20%. Let us see the solution of all the questions given above.

Solution 1 Normal Method

Let us assume B = 100, then A = 125

Now, B is 25 less than A.

Percentage of B is less than A = 25/125 × 100 = 20%

Product Stability Ratio Method

Using product stability rule, since A is 25% more than B, so B is 20% less than A.

Solution 2 Normal Method

CP × 1.25 = SP

So, CP = SP/1.25 = 125/1.25 = Rs 100

Product Stability Ratio Method

If we increase CP by 25%, we will get SP.

So, if we decrease SP by 20%, we will get CP.

Hence CP = Rs 100

Solution 3 Normal Method

Since we know S = V × T (Distance = Speed × Time)

New speed = 1.25 V, so new time = T/1.25

So, reduction in time = T–T/1.25 = 0.25 T/1.25 = T/5

T/5 = 20 min ⇒ T = 100 min

Product Stability Ratio Method

Since speed has been increased by 25%, so time will reduce by 20%.

Now, 20% T (Time) = 20 min

So, Total time = 100 min

Solution 4 Normal Method

Let us assume that distance = D

So, D/20 – D/25 = 30/60 hr. = ½

So, D = 50 km

Product Stability Ratio Method

S = V × T

25%↑20%↓

So, 20% T = 30 min

⇒ T = 150 mins = 2½

So, total distance = 20 × 2½ = 50 km

Solution 5 Normal Method

Vinit is taking 20 days to complete the work i.e., Vinit is doing 100% work in 20 days. So, Vinit is doing 5% work in a day. Since efficiency of Amit is 25% more than Vinit, so Amit is doing 6.25% work per day.

So, number of days taken by Amit = 100/6.25 days = 16 days

Product Stability Ratio Method

Efficiency of Amit is 25% more than Vinit. So, Amit will take 20% less days than Vinit.

So, number of days taken by Amit = 16 days

Solution 6 Normal Method

Using Work = Number of persons × Number of days

Work = 20 × 50 = 1000

Now, 1000 = 25 × D

So, D = 40

Product Stability Ratio Method

Number of persons increases by 25%, so number of days will decrease by 20%.

So, number of days = 40 days

Solution 7 Normal Method

Using the formula for SI = PRT/100

P = (SI × 100)/RT

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Putting the values gives us P = Rs 1,000

Product Stability Ratio Method

Interest for two years = 25%

So, if we decrease the amount by 20%, then we will get the principal.

Hence, Principal = Rs 1,000

Solution 8 Normal Method

Let us assume that original price per kg = Rs P per kg

So, final price per kg = Rs 1.25 P

Hence, (120/P) – (120/1.25P) = 5

Solving this equation gives P = Rs 4 per kg

Product Stability Ratio Method

Since, the price hike is 25%, 20% less quantity of sugar will be available for Rs 100.

20% = 5 kg ⇒ 100% = 25 kg

So, 25 kgs were available for Rs 100 initially.

So, Price = Rs 4/kg

Solution 9 Normal Method

	Length		*Breadth*		*Area*
Initially…→	10	×	10	=	100
Finally… →	12.5	×	B	=	100

So, B = 8

Percentage decrease = 20%

Product Stability Ratio Method

Till now, it must have become very obvious that the breadth will decrease by 20% to keep the area constant.

Extension of Product Stability Ratio

This table is a two-way table, i.e., if we decrease A by 50%, then B is needed to be decreased by 100%.

If we express the percentage figures given in the product stability table in ratios, then it comes like the following:

Change in A	*Change in B*	*Change in P*
$\frac{1}{1}\uparrow$	$\frac{1}{2}\downarrow$	0 %
$\frac{1}{2}\uparrow$	$\frac{1}{3}\downarrow$	0 %
$\frac{1}{3}\uparrow$	$\frac{1}{4}\downarrow$	0 %

So, $\frac{1}{50}\uparrow$ corresponds to $\frac{1}{51}\downarrow$. It means that if we increase A by 2%, then B is needed to be decreased by 1.96% (approx.) so that P remains constant.

Similarly, it can be done with all the reciprocals.

But the problem which lies with this table is that it has values which are reciprocals only.

So, what are we required to do if we increase A by 15%?

Take this as

Change in A	*Change in B*	*Change in P*
$15\% = \frac{3}{20}\uparrow \Leftrightarrow$	$\frac{3}{23}\downarrow$	0%
$35\% = \frac{7}{20}\uparrow \Leftrightarrow$	$\frac{7}{27}\downarrow$	0%

APPLICATION OF PERCENTAGE

1. Simple Interest and Compound Interest

Interest is defined as the "Time value of money." As the time passes on, value of money keeps on changing. And while factors like inflation or depreciation in money decreases the value of money with the passage of time, interest the counters all these.

The basic difference between simple interest and compound interest lies in the fact that while in case of simple interest, the principal as well as the interest remain same for the entire period given, in case of compound interest, after a certain period, both the principal and the compound interest keep on changing.

Simple interest

In case of simple interest, interest is reckoned at a flat rate for the entire period of time for the amount borrowed.

Simple interest can be calculated by finding the percentage of principal for that period of time.

If the principal is P, rate of interest per annum = R% and time = T years, then Simple Interest will be equal to RT % of P.

$$\text{So, Simple Interest (SI)} = \frac{P\times(R\times T)}{100}$$

Example 6 A sum of money becomes 3 times in 5 years. In how many years will the same sum become 6 times at the same rate of SI?

Solution The sum of money gets 3 times means that 200% is being added to the original sum (principal) in 5 years.

So, 500% will be added up in $12\frac{1}{2}$ years.

Compound interest

In case of compound interest, interest is reckoned on the interest of previous years also apart from reckoning it on the current year.

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So, compound interest can be seen as the extension of simple interest in such a way that previous year interest also becomes principal for the next year.

If principal is P, rate of interest per annum = R% and time = n years, then

$$\text{Compound Interest} = P\left(1+\frac{R}{100}\right)^n - P$$

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

It can be seen that $P\left(1+\frac{R}{100}\right)^n$ is nothing but $P\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)\ldots n$ times, which is simply the application of successive percentage change.

Example 7 The difference between two years of compound interest and simple interest at 10% over Rs X is Rs 10. What is the value of x?

Solution

	SI	*CI*
At the end of 1st year	10%	10%
At the end of 2nd year	10%	10% + 10% of 10% =11%
	=20%	=21%

So, 1% = Rs 10

$\Rightarrow$ 100% = Rs 1000

It is pertinent to understand here that if the rate of Interest = R% per annum for both CI and SI, then the difference between CI and SI for 2 years will be equal to R% of R% = $R^2/100$%

In the above case, R = 10%, so the difference between CI and SI for 2 years = 1%

Comparison between SI and CI

Assume that two different sums are getting doubled at their respective rates of SI and CI in 5 years. The following table gives us the mechanism of getting money n times in the above situation.

	After 5 years	*After 10 years*	*After 15 years*	*After 20 years*
At SI	2times	3 times	4 times	5 times
At CI	2 times	4 times	8 times	16 times

2. Simple Annual Growth Rate (SAGR) and Compounded Annual Growth Rate (CAGR)

Consider the following table pertaining to the sales value of Due North Inc. in different years:

Year	*2004-05*	*2005-06*	*2006-07*	*2007-08*
Sales Value (Rs in crores)	100	110	124	131

If we find out the growth over the given period, then it is equal to 31%.

However, there are two more growth models, namely, Simple Annual Growth Rate (SAGR) and Compounded Annual Growth Rate (CAGR).

$$\text{SAGR} = \frac{\text{Overall growth\%}}{\text{No. of years for which growth can be calculated}}$$

$$= \frac{31\%}{3} = 10.33\%$$

To find out CAGR, we are needed to use the approach of CI and we will consider 131 as the amount and 100 as the principal.

$$\text{So, } 131 = 100\left(1+\frac{R}{100}\right)^3$$

To find out the value of R, we will apply the method of hit and trial, or otherwise find out the value of $\sqrt[3]{1.31}$.

Since we know, $(1.1)^3 = 1.331$, so value of R will be less than 10% and very close to 10%.

So, it will be around 9.7% – 9.8%.

Compound interest reckoned half-yearly or quarterly If the rate of interest is R% annually and CI is compounded half yearly for n years, then the expression for Compound

$$\text{Interest} = P\left(1+\frac{\frac{R}{2}}{100}\right)^{2n} - P$$

If the rate of interest is R% annually and CI is compounded quarterly, then the expression for Compound

$$\text{Interest} = P\left(1+\frac{\frac{R}{4}}{100}\right)^{4n} - P$$

3. Calculation

Percentage is mostly helpful in multiplication and division. Let us learn this with the help of examples.

Multiplication

To use this percentage method effectively for multiplying two values, we should be thorough with the equivalent

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ratio of the percentage figures. Besides, having a good addition speed will be an added advantage.

Example 8 63×72.

The moment we see any number, we should start mental scanning of the percentage–ratio equivalence. Here, $63 \times 72 = (62.5 + 0.5) \times 72$

$= 5/8 \times 100 \times 72 + 0.5 \times 72$ (5% of 72)

$= 4500 + 36$

$= 4536$

Example 9 $76 \times 24 = (75 + 1) \times 24 = ¾ \times 100 \times 24 + 1 \times 24 = 1800 + 24 = 1824$

Division

Example 10 Dividing 243 by 50.

$$\frac{243}{50} = \frac{243}{100} \times 2 = 2.43 \times 2 = 4.86$$

To divide any number by 50, we will divide it by 100 and multiply by 2.

Example 11 Dividing 312 by 25.

$$\frac{312}{25} = \frac{312}{100} \times 4 = 3.12 \times 4 = 12.48$$

To divide any number by 25, we will divide it by 100 and multiply by 4.

Similarly, while dividing any number by any such number for which we can find out a comparable value in terms of 100 $\left(\text{as } 25 = \frac{100}{4}\right)$ should be used.

PRACTICE EXERCISES

WARM UP

Q.1. What fraction is equal to 57.12% (approximately)?
(a) 4/8 (b) 5/8 (c) 5/7 (d) 4/7

Q.2. What is 50% of 40% of Rs 1,200?
(a) Rs 450 (b) Rs 100
(c) Rs 300 (d) Rs 240

Q.3. a% of a + b% of b = 2% of ab, then what percentage of a is b?
(a) 50% (b) 75%
(c) 100% (d) Cannot be determined

Q.4. Subtracting 34% of x from x is equivalent to multiplying x by N. Which of the following is the value of N?
(a) 0.66 (b) 66 (c) 0.34 (d) 6.6

Q.5. Subtracting 48% of N from N is equivalent to multiplying N by
(a) 0.48 (b) 1.48 (c) 0.52 (d) 0.42

Q.6. Tipu's salary has increased from Rs 7,200 to Rs 8,100. What is the percentage increase in Tipu's salary?
(a) $33\frac{1}{3}\%$ (b) 25% (c) $16\frac{2}{3}\%$ (d) $12\frac{1}{2}\%$

Q.7. 42% of a number is subtracted from 61% of itself and the resulting value is 38. What is the number?
(a) 100 (b) 250 (c) 200 (d) 300

Q.8. The length of rectangle is increased by 30% and the breadth is decreased by 25%. What is the percentage change in the area of the rectangle due to this?
(a) 2.5% increase (b) 25% decrease
(c) 25% increase (d) 2.5% decrease

Q.9. If a value is increased twice successively by 20%, then what is the net percentage increase in that value?
(a) 20% (b) 40% (c) 21% (d) 44%

Q.10. The numbers are respectively 30% and 40% of a third number. What percentage is the first number of the second number?
(a) 25% (b) 50% (c) 40% (d) 75%

Q.11. Two numbers are 15% and 20% less than a third number. What percentage is the first number as a percentage of the second number?
(a) $116\frac{2}{3}\%$ (b) $83\frac{1}{3}\%$
(c) $106\frac{1}{4}\%$ (d) $123\frac{2}{3}\%$

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Q.12. 10 litres of water is added to 50 litres of a solution containing 20% of alcohol in water. What is the concentration of alcohol in the solution now?

(a) 20% (b) $16\frac{2}{3}\%$

(c) $12\frac{1}{2}\%$ (d) $33\frac{1}{3}\%$

Q.13. A man spends 25% of his money on food. After spending 50% of the remaining, he is left with Rs 375. How much money was with that man initially?

(a) Rs 5,000 (b) Rs 4,000
(c) Rs 5,800 (d) Rs 1,000

Q.14. A man earns Rs 4,000 in a month and saves 30% of his income. What is his expenditure in a year?

(a) Rs 28,600 (b) Rs 18,400
(c) Rs 32,000 (d) None of these

Q.15. After spending 20% on clothes, 10% on books, 9% on purchasing gift for girl friend and 7% on others, Chandra has a balance of Rs 2,700. How much money was there with him initially?

(a) Rs 5,000 (b) Rs 6,000
(c) Rs 7,000 (d) None of these

Q.16. A man spends 60% of his income. His income increases by 40% and due to this, he increases his expenditure also by 40%. Find the percentage increase in his savings.

(a) 30% (b) 4%
(c) 4.44% (d) None of these

Q.17. If the numerator of a fraction is increased by 20% and its denominator decreased by 10% the fraction becomes 3/2. What is the value of the original fraction?

(a) $\frac{2}{3}$ (b) $\frac{7}{8}$ (c) $\frac{9}{8}$ (d) $\frac{6}{7}$

Q.18. In an exam of two papers maths and chemistry, 60% of the students pass in maths and 70% pass in chemistry. What is minimum percentage of students who could have failed in both the subjects?

(a) 0% (b) 30%
(c) 40% (d) None of these

Q.19. Everyday a fruit seller sells 50% of his stock. 10% of the stock gets spoiled during the night. If a total of 1983 fruits got spoiled in the three days period, how many fruits did he start with?

(a) 20,000 (b) 25,000
(c) 24,000 (d) None of these

Q.20. A certain sum becomes 7/3 times itself in ten years under simple interest. Find the rate of interest.

(a) $7\frac{1}{2}\%$ p.a (b) $13\frac{1}{3}\%$ p.a

(c) 20% p.a. (d) 10% p.a.

Q.21. An amount is lent at x% p.a. simple interest for two years. Instead, had it been lent at $2x$% p.a. simple interest for y more years, then the interest would have been five times the earlier interest. Find the value of y.

(a) 2 (b) 3 (c) 4 (d) 5

Q.22. According to a new plan rolled out by HISP Bank, the rate of simple interest on a sum of money is 8% p.a. for the first two years, 10% p.a. for the next three years and 6% p.a. for the period beyond the first five years. Simple interest accrued on a sum for a period of eight years is Rs 12,800. Find the sum.

(a) Rs 24,000 (b) Rs 16,000
(c) Rs 15,000 (d) None of these

Q.23. Find the least number of integral years in which a sum of money invested at 20% compound interest per annum will become more than double itself.

(a) 1 (b) 4 (c) 2 (d) 3

FOUNDATION

Q.1. In the country of Tattoland, a new term tercentage has been introduced in the place of percentage. While percentage is per 100, tercentage is per 125. What will be the tercentage marks obtained by a student in Tattoland if his percentage of marks is 60?

(a) 48 tercentage (b) 75 tercentage
(c) 60 tercentage (d) Cannot be determined

Q.2. In the country of Tappoland, a new term hercentage has been introduced in the place of percentage. While percentage is per 100, hercentage is per 1000. What will be the hercentage marks obtained by a student in Tappoland if his percentage is 60?

(a) 6 hercentage
(b) 600 hercentage

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(c) 166.66 hercentage
(d) Cannot be determined

Q.3. In a ΔABC measurement of $\angle ABC = 15^\circ$. What will be the value of $\angle ABC$ when seen through a magnifying glass which can magnify any object by 20%?
(a) Value reduce by 20%
(b) Same
(c) Value increased by 20%
(d) Value decreased by 16.66%

Q.4. In a maternity centre, 5% of all the childbirth cases result in twins. What is the approximate percentage of twins out of total children born?
(a) 5% (b) 7.6% (c) 9.5% (d) 10.4%

Q.5. Due to a price hike of 20%, 4 kg less sugar are available for Rs 120. What is the initial price per kg of sugar?
(a) Rs 5/kg (b) Rs 4/kg
(c) Rs 6/kg (d) Rs 5.5/kg

Q.6. A person has to pay 20% tax on first Rs 4,500 earned, for amount earned between Rs 4,500 and Rs 7,500 the tax is 25% and for the amount earned more than Rs 7,500, the tax is 30%. If he paid 23.33% of his income as tax, then what is his income?
(a) Rs 9,000 (b) Rs 8,500
(c) Rs 10,000 (d) Rs 10,500

Q.7. A clothes-shop offers a discount to all its customers on the basis of the value of the purchase. If the value of purchase is less than Rs 3,200, the customer gets a discount of 14%. The customer gets 16% discount on purchase between Rs 3,200 – Rs 6,300. The customer gets a discount of certain % on the purchase exceeding Rs 6,300. A customer gets a total discount of Rs 1,520 which is equal to 16% of the purchase value. How much discount does the customer get for amount exceeding Rs 6,300?
(a) 16% (b) 18% (b) 20% (b) 22%

Q.8. Three friends P, Q and R donate 8%, 7% and 9% of their salary to a charitable hospital in the given order. Salary of P and Q is same and the difference of their donations is Rs 74. The total donation by the P and Q is Rs 525 more than R's donation. What is the percentage of the total salary of the three donations?
(a) 7.95% (b) 8.3%
(c) 6.34% (d) None of these

Q.9. Fresh watermelons contain 90% water by weight whereas dry watermelons contain 20% water by weight. What is the weight of dry watermelon obtained from 20 kgs of fresh watermelon?
(a) 2.4 kg (b) 2 kg
(c) 2.5 kg (d) Cannot be determined

Q.10. In solution of sugar and water the ratio of sugar and water by weight is 1:4. This solution is heated and in the process it loses 50% weight. What is the ratio of weight of sugar and water in the final mixture?
(a) 4:1 (b) 2:1 (c) 2:3 (d) 3.5:2

Q.11. Rate of inflation is 1000% per annum. What is the value of an article two years from now if it costs Rs 6 today?
(a) Rs 66 (b) Rs 126
(c) Rs 660 (d) Rs 726

Q.12. Dataman Infosys Systems is a Lucknow-based software company which is growing tremendously. It doubled its turnover in 1998 from Rs 50 crores in 1997. Then it tripled its turnover in 1999 and grew by 50% in 2000. What is the turnover at the end of 2000?
(a) Rs 250 cr (b) Rs 450 cr
(c) Rs 300 cr (d) Rs 600 cr

Q.13. In the recently concluded CBSE board examination, a total of 6,00,000 students appeared. 40% of them were females while the rest were males. Pass percentage among males is 75% and the overall pass percentage is 70%. What is the pass percentage for females?
(a) 37.5% (b) 50% (c) 62.5% (d) 70%

Q.14. Because of the budget presented by Yashwant Sinha, the price of sugar increased by 40%. The Verma family reduced its consumption so that expenditure on sugar is up by 12%. If the total consumption of sugar before the rise in price was 50 kg. What is the present consumption of sugar (in kg)?
(a) 48 kg (b) 40 kg (c) 36 kg (d) 32 kg

Q.15. Tatto's working hours per day were increased by 25% and his wages per hour were increased by 20%. By how much per cent was his daily earnings increased?
(a) 20% (b) 25% (c) 50% (d) 45%

Q.16. A's income is reduced from Rs 75,000 to Rs 60,000, while B's income for the same period is increased from Rs 60,000 to Rs 75,000. What

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percentage of decrease in A's income is the increase in B's income?
(a) 125% (b) 75% (c) 133% (d) 100%

Q.17. 18% of A plus 15% of B plus 19% of C is equal to 17% of the sum of A, B, and C. If A – B = 500 and A – C = 3400, what is the value of A + B + C?
(a) 12,400 (b) 11,600
(c) 13,500 (d) None of these

Q.18. Out of 51,600 candidates who appeared in an examination, 35% failed, while 15% passed with honours. What is the number of candidates who passed the examination, but failed to obtain honours, assuming that no student who has failed can obtain honours?
(a) 25,000 (b) 30,000
(c) 27,309 (d) 28,509

Q.19. A and B have some guavas divided among themselves. A says to B "If I give you 25% of the guavas I have, I will still have 2 more guavas than you have." To this, B says "If you give me guavas equal to 70% of what I have now, I will have 4 more guavas than you have." What is the total number of guavas that they have?
(a) 80 (b) 64 (d) 36 (d) 88

Q.20. Raghupati Raghav was able to score a total of 600 in 12 tests. He scored less than or equal to 80% of his average score per test in four of these tests. If he did not score more than 60 in any of the tests, what is the minimum number of tests in which he should have scored more than 50?
(a) 8 (b) 4 (c) 3 (d) 2

Q.21. A locomotive engine runs at a speed of 50 km/h when no compartment is attached to it. For every new compartment that is attached to it, the speed of the train is reduced by 10% of the earlier speed. At most, how many compartments can be attached so that the train can cover a distance of 180 km in a maximum of 8 hours?
(a) 7 (b) 5 (d) 6 (d) 8

Q.22. Babloo organized a party by taking contributions from the participants. 60% of the participants contributed 80% of the funds. The average contribution of all the people who attended is Rs 50. What is the average contribution of the remaining 40% of the people?
(a) Rs 100 (b) Rs 150
(c) Rs 250 (d) Rs 25

Q.23. If the price of an article rose by 25% every odd year and fell by 20% every even year, what would be the percentage change after 180 years?
(a) 10% increase (b) 25% increase
(c) No change (d) 20% decrease

Q.24. Pranav saves 10% of his total salary. Next year he increases his expenses by 20% but his percentage savings remain the same. What is the percentage increase in his salary next year?
(a) 10% (b) 20% (c) 40% (d) 16.66%

Q.25. In an examination, 80% students passed in philosophy and 70% students passed in maths. At the same time 15% failed in both the subjects. If 390 students passed in both the subjects, how many students appeared in the examination?
(a) 500 (b) 400 (c) 80C (d) 600

Q.26. The salary of Anil and Vinit is 20% and 30% less than the salary of Dheeraj respectively. By what percentage is the salary of Anil more than the salary of Vinit?
(a) 33.33% (b) 50%
(c) 10% (d) 14.28%

Q.27. In the recent census report of Patna, it was found that 40% of total male adult population and 55% of total female adult population is married. What percentage of total adult population of Patna is married? (Polygamy is not in practice in Patna.)
(a) 46.32% (b) 43.6%
(c) 49.81% (d) 40%

Q.28. Sona and Ellaraputtu are very close friends. One day they were figuring out about their respective incomes and expenses. It was found that income of Sona and Ellaraputtu is 'S' and 'E' respectively. Sona spends 12% of her income and Ellaraputtu also spends the same amount. What percentage of her income is Ellaraputtu spending?
(a) E/(12S) (b) 12S/E
(c) 12 E/S (d) S/(12E)

Q.29. Sixty students appeared in the physics and math paper of an examination. Of these 60% students passed in physics and 70% passed in math. What is the maximum percentage of students who failed in both the subjects?
(a) 40% (b) 30% (c) 0% (d) 100%

Q.30. In an election, 10% of the voters on the voter's list did not cast their votes and 60 voters cast their ballot papers blank. There were only two candidates. The winner was supported by 47% of all voters in the list and he got 308 votes more than his rival. The number of voters on the list were.
(a) 3,600 (b) 6,200
(c) 4,575 (d) 6,028

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Q.31. A rainy day occurs once in every 10 days. Half of the rainy days produce rainbows. What in the percentage of all the days when we have no rainbows?
(a) 95% (b) 10% (d) 50% (d) 5%

Q.32. One-third of Mayank's marks in history equals his marks in geography. If he obtained 180 marks in two subjects together, how many marks did he get in geography?
(a) 40 (b) 45 (c) 60 (d) 36

Q.33. If the price of the sugar is increased by 20%, one can buy 2 kg less sugar in Rs 50. What is the quantity of sugar that could be bought before the price hike?
(a) 10 (b) 8 (c) 16 (d) 12

Q.34. The entry fee in an exhibition was Rs 10. Later this was reduced by 25%, which increased the sale of tickets by 20%. Find the percentage increase in the number of visitors.
(a) 54 (b) 57 (c) 60 (d) 66

Q.35. A car bought for Rs 13,350 depreciates each year by 12½%. In how many years will its value be reduced to Rs 6,000?
(a) 4 years (b) 6 years
(c) 8 years (d) 10 years

Q.36. An electrical contractor purchased a certain amount of wire, 10% of which was stolen. After using 85% of the remainder he had 47.25 m of wire left. How much wire did he purchase?
(a) 600 m (b) 103 m
(c) 325 m (d) 350 m

Q.37. Anoop got 273 marks in his graduation exam and got 5% more than the pass percentage. If Siddharth got 312 marks then by what percentage above the pass marks, did he pass the exam?
(a) 9% (b) 12.5%
(c) 20% (d) 25%

Q.38. There were only two contestants A and B in the recent assembly election in Bihar at Raghopur constituency. B got 50% of the votes that A got. Had A got 200 votes less, there would have been a tie. What is the total number of votes polled?
(a) 1,000 (b) 800
(c) 1,200 (d) Cannot be determined

Q.39. A city had a population of 30,00,000 in the beginning of 1999. Its average growth rate is 4% per year, but due to a massive earthquake in 2001, its population is reduced by 8% in that year. But it again maintained the same growth rate of 4% in following years. What will be the approx. population of the city at the end of 2003?
(a) 32,06,460 (b) 34,68,420
(c) 31,52,360 (d) 32,28,810

Q.40. A is an alloy of tin and copper and B is an alloy of copper and zinc. A has 30% copper while B has 50% copper. x gms of A is mixed with 30 gms of B to form another alloy which has 45% copper. What is the value of x?
(a) 5 (b) 10 (c) 15 (d) 20

Q.41. Rajesh Raj has Rs 90,000 with him. He purchases a Bike, a home theatre and a flat TV for Rs 15,000, Rs 13,000 and Rs 35,000 respectively and puts the remaining money in his bank account which pays 15 per cent per annum Compound Interest. After 2 years he sells off the three items at 80% of their original price and also withdraws his entire money from the bank by closing the account. What is the total change in his asset?
(a) −4.5% (b) +3.5%
(c) −4.32% (d) +5.5%

Q.42. After three successive raises, Aftab's salary became equal to $\frac{378}{125}$ of his initial salary. By what percentage was the salary raised the first time if the third rise was twice as high (in percentage) as the second rise was twice as high (in percentage) as the first rise?
(a) 10% (b) 15% (c) 20% (d) 25%

Q.43. One litre of water is evaporated from 6 litres of solution containing 4% of sugar. The percentage of sugar in the remaining solution is
(a) 1.4% (b) 5.2% (c) 4.8% (d) 3.4%

Q.44. A's income is 60% of B's income, and A's expenditure is 70% of B's expenditure. If A's income is 75% of B's expenditure, find the ratio of A's savings to B's savings.
(a) 5:1 (b) 1:5 (c) 3.5:1 (d) 2:7

Q.45. There are 1,10,000 books at Khuda Baksh Oriental Public Library, 40% of which are science books. It was decided to add 20,000 new books to the library. How many of the new books should be science books in order to bring the percentage of science books in the library equal to 45%?
(a) 15,000 (b) 1,500
(c) 1,450 (d) 14,500

Q.46. If 33 untrained labourers can do a piece of work in 15 days of 12 hours each, how many trained

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labourers can do 50% more work in 11 days of 9 hours each? (Assuming that it takes 2 trained labourer to do the work of 5 untrained labourers.)
(a) 42 (b) 36 (c) 90 (d) 100

Q.47. X's income is 75% of Y's income, and X's expenditure is 80% of Y's expenditure. If X's income is 90% of Y's expenditure, find the ratio of X's savings to Y's savings.
(a) 1:2 (b) 2:1 (c) 1:4 (d) 2:3

Q.48. Kajol went to a fruit market with a certain amount of money. With the money she has, she can buy either 40 apples or 70 mangoes. She retains 15% of her money for auto fare. If she buys 35 mangoes, how many more apples can she buy?
(a) 15 (b) 20
(c) 14 (d) None of these

Q.49. In an election for the post of class representative at Pitampura centre where two candidates were contesting, 10% of the students did not vote (because they were late, as usual) and 60 votes recorded were declared invalid (because stray marks were made on the ballot paper). Winning candidate won the election by 308 votes and it was found that he had been supported by 47% of the whole students of the class. Which of the following can be the number of valid votes recorded for each candidate?
(a) 2308, 2000 (b) 2904, 2596
(c) 2914, 2606 (d) 2464, 2772

Q.50. If the length and breadth of a rectangular plot are increased by 20% and 50% respectively, Then the new area is how many times the original area?
(a) 5/8 (b) 9/5 (c) 3/7 (d) 7/3

MODERATE

Q.1. There are fifty successive percentage discounts given in a series of 2%, 4%, 6%, 8%... and so on. What is the net discount?
(a) 98% (b) 2550%
(c) 100% (d) Infinite

Q.2. In Question 1, after how many such percentage discounts in succession will the effective discount be higher than 50%?
(a) 7th (b) 8th (c) 9th (d) 10th

Q.3. My Alliance India phone bill for the month of May is Rs B. On top of this, there is a service tax of S% which is to be levied upon this value. But since they are having a guilt feeling of overcharging their customers fraudulently, they offer a discount of D% also on top of this. So, my billing goes like this
Rs B → S%↑→ D% ↓→ Final Bill
But surprisingly, my final bill is equal to Rs B itself.
Now, which of the following is true regarding S and D(assume all the values to be positive)?
(a) S > D (b) S = D
(c) D > S (d) Cannot be determined

Q.4. Assuming percentage to be true for all the real values, which of the following can be true?
40% of A = 60% of B
i. A > B ii. B > A iii. B = A
(a) i and ii only (b) ii and iii only
(c) i and iii only (d) None of these

Directions for questions 5–6: Read the passage below and solve the questions based on it.

Alok has certain number of Oranges with him. He sells each orange for Rs 2. Shazmi bought 25% of the oranges and Sandeep bought one-third of what was left. Mohit bought 50% of what was left and Siddharth bought 4 oranges. Total amount paid by Shazmi and Mohit is Rs 24.

Q.5. How many oranges did Sandeep purchase?
(a) 3 (b) 4 (c) 6 (d) 8

Q.6. How many oranges were left with Alok?
(a) 3 (b) 4
(c) 6 (d) None of these

Directions for questions 7–8: Read the passage below and solve the questions based on it.

All students of class X took the pre-board test. One third of the class received B, one fourth received B+, one sixth received B−, and one eighth of the class failed (grade C). Everyone else got A.

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Q.7. What can be the minimum number of students in class X?
(a) 8 (b) 48
(c) 24 (d) Cannot be determined

Q.8. How many students in class X got A?
(a) 3 (b) 6
(c) 4 (d) Cannot be determined

Q.9. A manufacturer of detergent cakes increased the price of his product as a result of which there was a decrease of 20% in its sales. If the sales turnover increases by 20% due to this change what was the percentage increase in the price?
(a) 20% (b) 36% (c) 40% (d) 50%

Q.10. Ravi Shankar wishes to buy an air conditioner (AC) with the money in the bank, which currently is earning interest at the rate of 15 pcpa compounded annually. But Tanzar, his friend forecasts that the inflation rate applicable to AC is going to be 14%; 15% and 16% respectively for the next 3 consecutive years and advises Ravi Shankar to postpone the purchase by 3 years. Does Ravi Shankar gain monetarily, if he takes Tanzar's advice?
(a) Yes
(b) No
(c) He neither gains nor losses
(d) He gains only if the purchase is made in the second year.

Q.11. In the recently concluded referendum in New York, regarding calling back US troops from Iraq, 600 people voted on a referendum, but after some discussion the opponents increased by 150%. The motion was then rejected by a majority two times as great as that by, which it was formerly passed. How many people voted against the initial referendum?
(a) 400 (b) 300 (c) 200 (d) 500

Q.12. The number of students present in the class of Manoj Yadav is M and out of those M students, 22.1875% are stupids. If the number of stupids in the class is N, then what can be the minimum value of M + N?
(a) 1,955 (b) 391
(c) 16,200 (d) 12,21,785

Q.13. If 75% of a class of 40 students can sing and only 20% cannot dance, the maximum number of students who can neither sing nor dance is
(a) 0 (b) 6
(c) 18 (d) Cannot be determined

Q.14. There are five contestants A, B, C and D in the assembly election from the Bihar Sharif constituency. It is given that none of the contestants got less than 1% of the valid vote. Consider the following statements in isolation to each other
i. A got 49% of the total valid votes
ii. B got 55% of the total votes
iii. C got 46% of the total valid votes
iv. D got 48% of the total valid votes
With how many of the following statements when used independently, it is possible to find out the winner of the election?
(a) 0 (b) 1 (c) 2 (d) 3

Q.15. For admission in a graduation program of Delhi University, 90% of the candidates who appeared for the written test were males and the rest were females, 60% of the males and 80% of the females passed in the written test. What is the total number of students who appeared for the written test, if the total number of passed candidates was 1240?
(a) 2,500 (b) 2,000 (c) 2,750 (d) 3,500

Q.16. In a laboratory experiment, a sample of air, which is a mixture of oxygen and water vapour is taken. Water vapour, contains hydrogen and oxygen gases. If air contains a total of 70% oxygen (including that contained in the water vapour) by weight while water vapour contains 16.66% of oxygen by weight, how many kilograms of water vapour is present in 1 kilogram of air?
(a) 0.2 (b) 0.36 (c) 0.32 (d) 0.25

Q.17. In Patna University, out of every 7 students learning science, 3 take literature as well; for every student taking at least one of these two courses, there are 3 students who take up neither. If 10% of the students take only science, the percentages who have taken science is:
(a) $17^1/_2$% (b) $23^1/_3$%
(c) $7^1/_2$% (d) Cannot be determined

Q.18. 60% of the students of a class are boys and in an examination, 80% of the girls scored more than 40 marks (out of a maximum possible 150 marks). If 60% of the total students scored more than 40 marks in the same examination, find the fraction of the boys who scored 40 marks or less.
(a) 8/15 (b) 7/15 (c) 4/5 (d) 1/5

Q.19. A thousand people voted on a resolution with 10% of the votes being invalid. After some discussion 1,000 people voted again. This time there were 20% invalid votes. The opponents were increased by 50% while the motion was now rejected by a majority, which is 300% more than it was formerly

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passed by. How many people voted against the resolution after the discussion?
(a) 400 (b) 200 (c) 300 (d) 500

Q.20. Recently I had gone to a locality called Shadigarh for conducting a survey about the number of married persons in the locality. The population of the locality is 7,200 and $^{11}/_{18}$th of those are males and the rest females. If 40% of the males are married, find percentage of married females in the locality.
(a) $48^{1}/_{7}$% (b) $52^{4}/_{7}$%
(c) $62^{6}/_{7}$% (d) $71^{1}/_{7}$%

Q.21. Nitish was to get a 50% hike in his pay but the computer operator wrongly typed the figure as 80% and printed the new pay slip. He received this revised salary for three months before the organization realized the mistake. What percentage of his correct new salary will he get in the fourth month, if the excess paid to him in the previous three months is to be deducted from his fourth month?
(a) 20% (b) 25% (c) 40% (d) 33 1/3%

Q.22. In the US opinion polls held during August, 60% of the respondents favoured Walter Mondale while the rest favoured George W. Bush. It was found in September polls that 10% of Mondale supporters switched their preference to Bush, while the same percentage of Bush's supporters also switched their preference to Mondale. What percentage of the electorate should now switch their preference from Mondale to Bush so that they are at par?
(a) 6% (b) 8% (c) 10% (d) 14%

Q.23. An index of 12 shares contains, among others, the shares of Reliance, HLL and Infosys with weightage of 7%, 13% and 15% respectively. What is the increase in the prices of other shares, if these three rise by 9%, 10% and 4% respectively, while the index rises by 6%?
(a) 5.34% (b) 4.91%
(c) 4.58% (d) Cannot be determined

Q.24. A community service organisation has raised 80% of the donation that it needed for a new building by receiving an average donation of Rs 400 per person from a group of people. To raise the remaining amount, the organisation now intends approaching another group of people which has two-thirds the number of people as the first group. What should be the average donation per person from the new group?
(a) Rs 100 (b) Rs 150
(c) Rs 200 (d) Rs 250

Q.25. 60% of the employees of Due North Inc. are female, and 75% of the men earn more than Rs 25,000 per year. If 45% of the company's employees earn more than Rs 25,000 per year, what fraction of the women employed by the company earn Rs 25,000 per year or less?
(a) 2/11 (b) 1/4 (c) 1/3 (d) 3/4

Q.26. The pollution in normal atmosphere is less than 0.01%. Due to leakage of methyl isocyanide (MIC) from a factory the pollution is increased to 20%. If every day 80% of the pollution in the atmosphere is neutralized, in how many days will the atmosphere become normal?
(a) 5 days (b) 3 days
(c) 4 days (d) None of these

Q.27. The marks obtained by Arpit in English, maths, science and Hindi in Class 10 are as follows (Maximum marks per subject = 100)
(a) The marks obtained in maths is 1.5 times the marks obtained in English
(b) Arpit got 64% in these 4 subjects.
(c) Arpit got the maximum and minimum marks in science and Hindi, respectively, with a difference of 48 marks between them
(d) An addition of 50% of the marks obtained in English to the final score gives an overall percentage of 70%

What would be his percentage of marks if only maths and science marks are counted?
(a) 14 (b) 82 (c) 84 (d) 86

Q.28. Amit puts some amount in bank and obtained an interest of Rs 15 at the end of one year. He added Rs 85 to this amount and put the whole amount again in the bank for another year. At the end of the 2nd year, Amit got Rs 420 as total (interest + principal). What amount did he put in the beginning and what is the rate of interest offered by the bank if the minimum amount that can be deposited in the bank is Rs 75?
(a) 250, 5% (b) 300, 5%
(c) 200, 8% (d) 350, 3.5%

Q.29. Amul, Binit and Chand start a business by investing Rs 70,000 that earns them a profit of Rs 42,000 at the end of the year. Amul invests his share in the profit in a scheme that gives him 10% interest compounded annually and Binit invests his share in a scheme that gives him 20% interest compounded annually. Amul gets Rs 2,520 as interest at the end of 2 years and Binit gets an

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interest of Rs 4,200 at the end of one year. Find Chand's investment in the business.
(a) Rs 1,60,000 (b) Rs 15,000
(c) Rs 17,520 (d) Cannot be determined

Q.30. A food processing unit received fresh consignment of berries from Himachal Berry Corporation containing 'a' kilogram of berries. After analysis these berries were found to contain 99% water. After 15 days, sample of berries from the same lot was found to contain 98% water. What must be the weight of same lot at that time?
(a) $\frac{a}{2}$ kg (b) $\frac{2a}{3}$ kg
(c) $\frac{4a}{5}$ kg (d) $\frac{a}{3}$ kg

Q.31. Kishore is an employee of a MNC firm. He needs Rs 10 lakhs for buying a new flat in Patel Nagar. His company is offering housing loan at an interest rate of 15% for 5 years. The two banks ICICI and HFDC are offering loan for 5 year period at an interest rate of 18% and 13% respectively. How much should Kishore borrow from the two banks so that he can pay the interest rate same as that of his own MNC firm?
(a) 5 lakhs from HDFC and 5 from ICICI
(b) 3 lakhs from HDFC and 7 from ICICI
(c) 6 lakhs from HDFC and 4 from ICICI
(d) 8 lakhs from HDFC and 2 from ICICI

Q.32. The population of towns A and B is in the ratio of 1:4. For the next 2 years, the population of A would increase and that of B would decrease by the same percentage every year. After 2 years, their population became equal. What is the percentage change in the population?
(a) 33.33% (b) 66.66%
(c) 25% (d) Not possible

Directions for questions 33–35: Read the passage below and solve the questions based on it.

Petroleum product sales (in million tones)

Petroleum product	*2002-03*	*2003-04*
Diesel	36.0	36.6
Petrol	7.6	7.9
LNG	8.3	9.3
Crude oil	11.9	11.7
ATF	2.3	2.5
Total	**104.2**	**107.4**

Choose (a) if only A is true.
Choose (b) if only B is true.
Choose (c) if both A and B are true.
Choose (d) if neither A nor B is true.

Q.33. A. The percentage share of ATF in petroleum product sales has increased in 2003-2004 over the previous year.
B. Percentage share of diesel in petroleum product sales has decreased in 2003-2004 over the previous year.

Q.34. A. LNG has experienced the maximum percentage increase in petroleum product sales in 2000-2004 over the previous year.
B. Change in percentage shares of crude oil in petroleum product sales in 2003-2004 over the previous years is approximately 0.2%.

Q.35. A. Percentage decrease in crude oil sales in 2003-2004 over the previous year is more than percentage increase in diesel sales during the same period.
B. LNG sales in 2003-2004 is less than 25% of diesel sales in 2003-2004.

Directions for questions 36–38: Read the passage below and solve the questions based on it.

In each question, there are two statements—A and B, either of which can be right or wrong on the basis of the information given below

Direct taxes collected in different years (Rs in crores)

Taxes	*1998-99*	*1999-00*	*2000-01*	*2001-02*	*2002-03*
Corporation tax	22,750	23,460	25,190	27,080	28,880
Income tax	18,210	20,460	23,790	26,110	29,080
Service tax	220	240	200	230	260
Expenditure tax	300	320	340	350	345
Interest tax	1,280	1,190	1,350	1,410	1,490
Water tax	8	15	20	22	25

Mark your answer as
(a) if only A is right.

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(b) if only B is right.
(c) if both A and B are right.
(d) If both A and B are wrong.

Q.36. A. The growth rate for Service Tax from 1998-99 to 1999-2000 was higher than that for income tax for the same period.
B. The growth rate for Service Tax from 1998-99 to 1999-2000 was lower than that for the period 2001-02 to 2002-03.
(a) 4 (b) 2 (c) 3 (d) 1

Q.37. A. The ratio of income tax to corporation tax was higher in 1998-99 than in 1999-2000.
B. Service Tax, as a percentage of expenditure tax, was greater in 2000-01 than in 2001-02.
(a) 1 (b) 2 (c) 3 (d) 4

Q.38. A. The average annual growth rate for income tax during the period 1998-99 to 2002-03 was more than 1.5 times the same for corporation tax during the same period.
B. The average annual growth rate for interest tax was more than 5% during the given period.
(a) 1 (b) 3 (c) 2 (d) 4

Directions for questions 39–40: Read the passage below and solve the questions based on it.

Abhay, Bhaskar, Chitra and Danny work in the same office. Abhay's salary is 20% less than Bhaskar's salary. Chitra's salary is 25% more than Abhay's salary. Danny's salary is 20% more than Abhay's salary.

Q.39. If Abhay has a salary of Rs 10,000 per month, who has a salary of Rs 12,500 per month?
(a) Bhaskar (b) Chitra
(c) Danny (d) 1 and 2

Q.40. If Chitra received a raise and her new salary is 10% higher than Danny's, what is her new monthly salary? (use the data from the previous question).
(a) Rs 12,750 (b) Rs 13,200
(c) Rs 13,333 (d) Rs 14,225

Directions for questions 41–44: Read the passage below and solve the questions based on it.

There are two statements given in each question: A and B, either of which can be true or false on the basis of the information given below.

Following table gives us the detailed information regarding the admission process of a reputed B-School IIM-Jehanabad for the year 2003 and 2004.

Year	*Gender*	*Number of applicants*	*Number of candidates appeared for written test*	*Number of candidates called for an interview*	*Number of candidates selected for the course*
2003	Male	6,100	5,900	60	12
	Female	1,900	1,500	15	5
2004	Male	6,300	6,000	64	13
	Female	4,000	3,500	30	6

Choose (a) if only A is true
Choose (b) if only B is true
Choose (c) if both A and B are true
Choose (d) if neither A nor B is true

Q.41. A. The success rate of moving from the written test to interview stage for females was worse than for males in 2003.
B. The success rate of moving for the written test to interview stage for males was better in 2003 than in 2004.

Q.42. A. The percentage of absentees in the written test among males decreased from 2003 to 2004.
B. The percentage of absentees in the written test in 2003 among females was larger than that among females in 2004.

Q.43. A. The percentage of the female candidates selected in 2004 from female applicants is more than the percentage of male candidates selected for year 2004 out of total male applicants of that year.
B. In 2003, from among those called for an interview, the males had a greater success rate than females.

Q.44. A. Difference in the total number of male and female applicants is more than 50% of the total candidates who appeared for the written test in 2004.
B. The total number selected for the job for both the years is more than 70% of the total female candidates called for an interview in both the years.

Q.45. In a laboratory, equal number of rats were injected with solution A. In a week, 55% of rats injected with solution A got infected with a disease and of

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that, equal number of rats were treated with vaccine C and D. Now, percentage of rats treated with vaccine C who recovered was 70% and that with vaccine D was 50%, while others died. Calculate the number of rats on whom the experiment was carried out in the laboratory, if the number of rats who died even after being treated with vaccines due to the disease is 77.

(a) 600 (b) 700
(c) 750 (d) 800

ADVANCED

Directions for questions 1–3: Read the passage given below and solve the questions based on it.

The table gives the production of five agricultural crops between 1999-2000 and 2000-2001.

Crops	*Target for 2001-2002 (in MT)*	*Output for 2000-2001 (in MT)*	*% increase in output over 1999-2000*
Rice	162.7	160.4	25
Wheat	18.9	16.2	18
Tea	20.5	22.5	35
Cotton	14.7	18.9	32
Coffee	12.3	14.8	16

Q.1. The production of rice in 1999-2000 (in MT) is approximately
(a) 124 (b) 128
(c) 132 (d) 136

Q.2. Output of which crop in 2000-01 is showing maximum percentage difference from the target set in 2001-02?
(a) Wheat (b) Coffee
(c) Cotton (d) Tea

Q.3. A new parameter is invented to judge the target: output ratio. That parameter is known as efficacy ratio and is defined as the ratio of target for 2001-02 and output for 2000-01. For how many given crops the efficacy ratio is more than 1?
(a) 1 (b) 3 (c) 2 (d) 4

Q.4. Some people form a joint account for one year with a condition that "each person will deposit an amount equal to the number of people in the account that month. Also the person who withdraws from the account in between will get their amount at the end of the year." After 6 months, 25% of the people withdrew and 33.33% of the remaining withdrew after 3 months. At the end of the year, there was an amount of Rs 4,860 in their account before paying off the withdrawn amounts. Find the number of people in the beginning of the year.
(a) 12 (b) 24
(c) 36 (d) None of these

Directions for questions 5–9: Read the passage below and solve the questions based on it.

The number of taxi drivers and auto drivers working in three shifts are given below:

Vehicle	*Shift*	*Number of drivers*	*Working hours*	*Total collection (in 1,000 Rs)*
Taxi	M1	425	10	132.1
	M2	672	8	240.2
	M3	135	6	42.3
Auto	M1	350	9	57.3
	M2	425	8	100.8
	M3	95	7	18.9

A taxi driver charges Rs 13 for every 1.6 kms, while an auto driver charges Rs 8 for every 1.3 kms. For the night shift, charges are 1.5 times the normal rate.

Q.5. By what percentage is the average collection per taxi driver more during the M2 shift than during the M1 shift?
(a) 4% (b) 15% (c) 34% (d) 82%

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Q.6. What is the average distance travelled by the auto drivers during the M3 shift?
(a) 1,356 km (b) 2,045 km
(c) 3,054 km (d) 2,672 km

Q.7. Who get the highest returns for their efforts with respect to work hours?
(a) Auto Drivers — M3 Shift
(b) Taxi drivers — M2 shift
(c) Taxi drivers — M3 shift
(d) Auto drivers — M1 shift

Q.8. What is the maximum difference between the average distances covered in an hour by drivers of any two shifts?
(a) 3,100 km (b) 3,300 km
(c) 3,500 km (d) 3,400 km

Q.9. Which of the following is false?
(a) Lowest contribution per driver is by auto drivers in the M1 shift
(b) Lowest returns for the efforts are for auto drivers in the M1 shift
(c) The percentage of drivers during the M3 shift is 21% of the total drivers
(d) None of these

Q.10. Consider a big cube formed of 64 smaller cubes of which 40 are coloured. The smaller cubes are arranged such that the exposure of the coloured cubes to the outside is minimized. What is the percentage of the exposed area that is coloured?
(a) 62.5% (b) 50%
(c) 41.6% (d) None of these

Q.11. On May 1, 2002, a person sold all his investments for Rs 10,8,000, incurring a loss of 10% with respect to the previous month. On April 1, 2002 he had invested all his money in shares, which represented a 20% increase over the sum he had on March 1, 2002 which represented a 20% decrease over the money he had on February 1, 2002 which represented a 25% increase over the money he had on January 1, 2002. The person had maximum money in
(a) February 2002 (b) January 2002
(c) April 2002 (d) None of these

Directions for questions 12–13: Read the passage below and solve the questions based on it.

Patna and Lucknow are the only cities of India which are having only adults. Female population in Patna grows by 50% to 60,000, but maintaining the same male to female ratio as previous year. Population in Lucknow increased by 12.5% to 90,000, but the ratio of male to female changed by 1 : 1 to 4:5. Male population in Patna was 10% more than the male population in Lucknow in the previous year.

Q.12. What is the female population in Lucknow in the second year?
(a) 55,000 (b) 45,000
(c) 4,00,000 (d) 50,000

Q.13. What is the total population in Patna after the increase (i.e., in the second year)?
(a) 80,000 (b) 1,00,000
(c) 20,000 (d) 26,000

14. The owner of a flower shop follows a particular pattern for his business. During a period of inflation, he raises his price by P% and during a slowdown he decreases his existing prices by P%. After a year in which there was inflation first, followed by a slowdown, the cost of a red-rose bouquet decreases by Rs 162. After another year, in which there was inflation once more followed by a slowdown, the cost of this bouquet reduced by a further Rs 147.42. What was the original price of the red-rose bouquet?
(a) Rs 1,500 (b) Rs 1,000
(c) Rs 1,800 (d) Rs 1,111

Directions for questions 15–17: Read the passage below and solve the questions based on it.

Three leading drug manufacturers, Arpit Pharma, Bittu Pharma and Chintu Pharma each launch an antiseptic drug. Each company uses all or some of the 5 constituents A, B, C, D and E. The table below gives the percentage composition of these constituents in the respective drugs manufactured by different companies.

	A	*B*	*C*	*D*	*E*
Arpit Pharma	12	15	20	28	25
Bittu Pharma	37	15	13	15	20
Chintu Pharma	26	15	10	34	15

The above values are in percentage of composition. All manufacturers produce their drug as a capsule of 300 mg.

Q.15. The effectiveness of constituents A, B, C and D are in the ratio 5:6:4:3, while E is just a sweetener and does not have any curing effect. Which of the following company's capsule is most effective?

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(a) Arpit Pharma (b) Bittu Pharma
(c) Chintu Pharma (d) Data Insufficient

Q.16. If the side effects caused by drugs are proportional to the ratio of C's composition to the sum of B's and E's composition, which drug has least side effect?
(a) Arpit's drug
(b) Bittu's drug
(c) Chintu's drug
(d) All of them have equal side effect

Q.17. Arpit Pharma finds that its sales are dipping because its drug tastes too sweet. So, it decides to decrease the composition of E and increase that of C, keeping other composition unchanged. It also wants to make sure that the side effect is not more than double its previous value. What could be the maximum percentage of C's composition in Arpit's Drug? (Use data from the previous questions)
(a) 25% (b) 30% (c) 34% (d) 37%

Q.18. If 3000 capsules are produced by these 3 companies per annum, the consumption of component C annually would be
(a) 60 kg (b) 90 kg
(c) 105 kg (d) Data Insufficient

Q.19. In Ranchi, only two newspapers *Dainik Jagran* and *Prabhat Khabar* are published. It is known that 25% of the city population reads *Dainik Jagran* and 20% reads *Prabhat Khabar* while 8% reads both *Dainik Jagran* and *Prabhat Khabar*. It is also known that 30% of those who read *Dainik Jagran* but not *Prabhat Khabar* look into advertisement and 40% of those who read *Prabhat Khabar* but not *Dainik Jagran* look into advertisements while 50% of those who read both *Dainik Jagran* and *Prabhat Khabar* look into advertisements. What is the percentage of the population who read an advertisement?
(a) 10.8% (b) 11.3%
(c) 13.9% (d) 12.7%

Directions for questions 20–22: Read the passage below and solve the questions based on it.

The following table gives the distribution of number of B-schools out of a sample of 100 B-Schools based on the scores obtained by them on five different parameters — Infrastructure, Placements, Research, Industry Interface and Intellectual Capitals. The maximum possible score in any of these parameters is 100 points.

Score/ Parameter	*80 points and above*	*60 points and above*	*40 points and above*	*20 points and above*
Infrastructure	16	30	50	65
Placements	12	28	37	50
Research	30	50	70	92
Industry Interface	24	42	57	73
Intellectual Capitals	27	43	47	70

For example, 16 out of 100 schools surveyed have scored 80 points or above in the parameter Infrastructure.

Q.20. What is the maximum number of business schools which could have scored 40 points or more but less than 60 points in at least three of the five parameters?
(a) 20 (b) 18
(c) 12 (d) None of these

Q.21. What is the minimum number of business schools which could have scored 40 points or more but less than 80 points in at least two of the five parameters?
(a) 10 (b) 5
(c) 12 (d) None of these

Q.22. Which of the following is a possible figure for number of schools which scored 20 points or more but less than 80 points in each of the five parameters?
(a) 38 (b) 36
(c) 42 (d) None of these

Q.23. In D-segment car market, only three competitors (Honda, Toyota and Hyundai) exist. Last year the sales of firm Toyota were 10% more than Honda. In a year, both the firms Honda and Toyota increased their respective sales by 20%. This year, the sales of the firm Toyota are five times that of Hyundai. How much were the sales of the firm Hyundai last year, if the total sales remained constant over the two years period?
(a) 46% (b) 25%
(c) 40.33% (d) 38.67%

Q.24. Chintu is given a quadratic equation $ax^2+bx+c=0$ and is asked to make another quadratic equation from this with $a = 1$. Also one root of the second quadratic equation is same as one of the roots of the first equation but opposite in sign and the other

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root of the second equation is two times the second root of the first equation. Find the percentage change in the constant term of the second equation as compared to the first equation?

(a) 200% increase (b) 300% decrease
(c) 400% increase (d) 100% decrease

Q.25. There are two investment schemes offered by two different financial institutions X and Y as follows

Scheme X: Rate of interest = 10% per annum and period of maturity = 2 years.
Amount is returned with compound interest compounded annually.

Scheme Y: Rate of interest = 11% per annum amount is returned with simple interest.
For what duration, should I invest Rs 25,000 in scheme Y, so as to get same amount in return if I had invested in scheme X?

(a) $\frac{21}{11}$ Years (b) $\frac{31}{11}$ Years
(c) $\frac{19}{7}$ Years (d) $\frac{19}{7}$ Years

Directions for questions 26–28: Read the passage below and solve the questions based on it.

Given below is the conversion rate of some of the currencies with respect to rupees.

	USD	*Pound*	*Euro*	*Canadian Dollar*	*Australian Dollar*
Rs Price/ unit	Rs 44	Rs 66	Rs 54	Rs 32	Rs 36

Q.26. Mr Kumar Kalyan Prasad Sinha is having two Pounds with him. How many Euros can he purchase for two Pounds?

(a) 2.33 (b) 2.44 (c) 2.55 (d) 2.66

Q.27. Value of USD is expected to rise by 10% against Rupee next year whereas value of Rupee is expected to decline by 10% against Canadian Dollar next year. How many USD can be purchased for 5 Canadian Dollars?

(a) 3.67 (b) 2.97
(c) 4.27 (d) None of these

Q.28. Mrs Sanjila Sinha has Rs 1,000 with her and her husband Mr Kumar Kalyan Prasad Sinha has Rs 2,000 with him. They want to purchase maximum number of currency notes given above but they are not ready to combine their money. What will be the maximum number of currency notes, as given above other than rupees which they can have?

(a) 62 (b) 94 (c) 93 (d) 32

Directions for questions 29–30: Read the passage below and solve the questions based on it.

The table given below shows the break-up of the percentage of people of different age groups frequenting bars in 4 different metro cities viz., Delhi, Hyderabad, Bangalore and Patna in the year 2002.

Q.29. Which city has the maximum number of people, upto 25 years of age, frequenting bars?

Cities	*Percentage break-up for age groups (Years) in 2002*						
	Up to 15	*15-20*	*20-25*	*25-30*	*30-35*	*35-40*	*Above 40*
Delhi	8	13	24	21	11	17	6
Hyderabad	3	8	35	23	10	16	5
Bangalore	4	21	27	11	8	14	15
Patna	1	7	43	32	9	5	3

(a) Bangalore (b) Patna
(c) Both 1 and 2 (d) Cannot be determined

Q.30. Which age group contributes to the maximum number of people frequenting bars in given cities?

(a) 20-25 (b) 25-30
(c) 30-35 (d) Cannot be determined

Directions for questions 31–34: Read the passage below and solve the questions based on it.

Five friends– Akshay, Binod, Chandu, Dev and Eeshwar– went on a shopping trip. Before shopping, the first person had Rs 400, the second person had 25% less than the first person, the third person and the fourth person had 50% less than the first person and the fifth person had 66.66% less than the second person. While shopping they did not lend or borrow from each other. After the shopping was over, it was observed that they were left with Rs 165, Rs 95, Rs 70, Rs 40 and Rs 10, not necessarily in that order. We don't know who is the first or the second or onwards. Following is known about the money they started with, they spent, or they were left with:

i. Akshay started with more money than Dev.
ii. Binod spent Rs 15 more than Chandu.

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iii. Eeshwar started with more money than just one another person of the group.
iv. Akshay spend the most but did not end with the least.
v. Chandu started with 66.66% of the money that Binod started with.
vi. Dev spend the least and ended with more than Akshay and Chandu.
vii. Eeshwar spent Rs 35.

Q.31. Who ended with the maximum amount of money?
(a) Akshay (b) Binod
(c) Chandu (d) Eeshwar

Q.32. How much money did Akshay spend?
(a) Rs 205 (b) Rs 190
(c) Rs 35 (d) Rs 360

Q.33. In ascending order of spending, Eeshwar would rank at which position?
(a) 1 (b) 2 (c) 4 (d) 5

Q.34. Who ended with Rs 40?
(a) Akshay (b) Binod
(c) Chandu (d) Dev

Q.35. In a certain town, at least 50% of the people read a newspaper. Among those who read a newspaper, at most 25% read more than one paper. Only one of the following statements follows from the statements given below. Which one is it?
(a) At the most 25% read exactly one newspaper
(b) At least 25% read all the newspaper
(c) At the most 37½% read exactly one newspaper
(d) At least 37½% read exactly one newspaper

Q.36. Two vessels contain equal quantities of 40% alcohol. Anil changed the concentration of the first vessels to 50% by adding extra quantity of pure alcohol. Balu changed the concentration of the second vessels to 50% replacing a certain quantity of the solution with pure alcohol. By what percentage is the quantity of alcohol added by Anil more than that replaced by Balu?
(a) 20% (b) 25%
(c) 40% (d) Cannot be determined

Directions for questions 37–40: Read the passage below and solve the questions based on it.

Six students Abhay, Bhupi, Chandi, Deb, Ellaraputtu and Fastinder appear in CAT 2006. Below is given the percentage marks obtained by them in different headings DS, DI, QA, EU, RC, GD and PI. (GD and PI are the part of CAT 06)

Its also known to us that each of DS, DI, QA, EU and RC carry same weightage. And the weightage given to PI is same as weightage given to GD.

Maximum marks in CAT 06 is 250, including GD and PI.

	Written Exam					*GD*	*PI*	*Total*
	DS	DI	QA	EU	RC			
A	45	50	40	90	80		75	62.4
B	55	60	65	70	75	80		66.4
C	75	72.5		65	82.5	88	84	
D	80	70	85	65	60	84	76	
E	90	60	80	60	50	88	72	70.4
F	85	80	75	45	70	72		

Q.37. What is the score obtained by Abhay in GD?
(a) 15 (b) 16 (c) 17 (d) 18

Q.38. If Deb has scored the maximum among the six students, what can be the maximum score that Chandi can get in QA?
(a) 20 (b) 21 (c) 22 (d) 23

Q.39. What is Bhupi's percentage score in PI?
(a) 42 (b) 56 (c) 64 (d) 72

Q.40. If Fastinder and Chandi end up with the same score, what is the difference between Chandi' QA score and Fastinder's PI score?
(a) 0 (b) 1 (c) 4 (d) 7

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TRUE / FALSE

Q.1. Length of a perimeter is increased by 30% and its breadth is increased by 10%. Perimeter of this triangle can increase by 25%.

State whether True or False.

Q.2. Length of a perimeter is increased by 30% and its breadth is increased by 10%. Perimeter of this triangle can increase by 32%.

State whether True or False.

Q.3. Length of a perimeter is increased by 40% and its breadth is increased by 20%. Percentage increase in the perimeter of the rectangle over the previous perimeter depends upon the dimension of the rectangle.

State whether True or False.

Q.4. Length of a perimeter is increased by 20% and its breadth is increased by 10%. Percentage increase in the area of the rectangle over the previous area depends upon the dimension of the rectangle.

State whether True or False.

ANSWERS

WARM UP

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12
Answer	(d)	(d)	(c)	(a)	(c)	(d)	(c)	(d)	(d)	(d)	(c)	(b)
Q. No.	13	14	15	16	17	18	19	20	21	22	23	
Answer	(d)	(d)	(a)	(d)	(c)	(a)	(c)	(b)	(b)	(d)	(b)	

FOUNDATION

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(c)	(a)	(a)	(b)	(a)	(c)	(c)	(d)	(b)	(c)	(b)	(c)	(d)	(c)	(d)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(d)	(c)	(b)	(d)	(d)	(a)	(b)	(b)	(b)	(a)	(c)	(d)	(c)	(b)	(d)	(c)	(d)	(d)	(b)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(d)	(c)	(c)	(b)	(d)	(b)	(a)	(c)	(c)	(b)										

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MODERATE

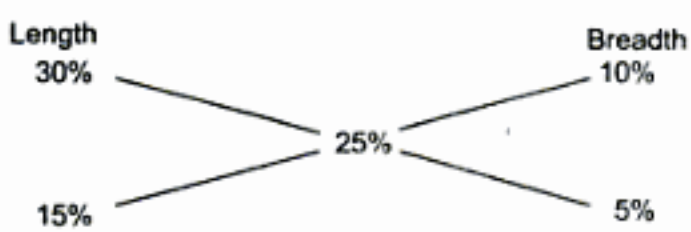
Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(b)	(a)	(d)	(c)	(d)	(c)	(d)	(d)	(a)	(c)	(b)	(c)	(b)	(b)	(d)	(a)	(b)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(d)	(a)	(b)	(b)	(a)	(b)	(b)	(b)	(a)	(c)	(a)	(c)	(a)	(a)	(c)	(b)	(b)	(d)	(b)
Q. No.	41	42	43	44	45															
Answer	(a)	(b)	(d)	(b)	(b)															

ADVANCED

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(c)	(b)	(b)	(b)	(b)	(d)	(c)	(c)	(a)	(d)	(d)	(c)	(b)	(c)	(b)	(d)	(c)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(b)	(b)	(a)	(b)	(a)	(c)	(d)	(a)	(d)	(d)	(b)	(a)	(d)	(a)	(b)	(c)	(c)	(b)

Q.1. True

Increase in perimeter has to be in between 10% and 30%. Hence, 25% increase in perimeter is possible. Actually it's a simple question of alligation. Ratio between length and breadth of rectangle should be 3:1.



Hence, the ratio between length and breadth = 15 : 5 = 3 : 1

Q.2. False

Taking a cue from the above problem, 32% increase in perimeter is not possible.

Q.3. True

Percentage increase in the perimeter of rectangle can be any value between 20% – 40%, and it depends upon the dimension of the rectangle (i.e., the length and breadth).

Q.4. False

Percentage increase in the area of rectangle does not depend upon the dimension of the rectangle and it remains constant for all the values of length and breadth. In this case, percentage increase in the area = 32%.

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4

PROFIT, LOSS AND DISCOUNT

HB

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Basic terms involved
- Mechanism of profit generation
- Honest and dishonest ways
- Kinds of questions which are asked in the CAT
- Methods of solving questions

CAT 2007

Number of questions from this topic: 0

CAT 2006

Number of questions from this topic: 0

CAT 2005

Number of questions from this topic: 0

Introduction to the topic

As far as the CAT is concerned, this chapter is important because the application of the concept of profit, loss and discount ranges from simple mathematical calculations in data interpretation (DI) to complex logical evaluations in QA. This chapter can be considered as an extension of percentage, the only difference being the terminology used in both the concepts. The terms we use in percentage are initial value, final value, percentage increase etc. But when we use the same terms in profit, loss and discount (PLD), the names get converted to cost price, selling price, profit percentage etc. Let us understand the concepts and learn problem solving methods for this concept.

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TERMINOLOGY

Cost Price

The price (amount) paid to purchase a product or the cost incurred in manufacturing a product is known as the cost price (CP) of that product.

Types of cost

1. **Fixed cost** As obvious from the name, it is that kind of cost which is fixed in all the cases.
2. **Variable cost** Variable costs are those costs which vary according to the number of units produced.
3. **Semi-variable cost** Semi variable costs are those costs which are fixed in one particular strata, but the costs varies among the different stratas.

One good example of fixed cost, variable cost and semi-variable cost is the bill we receive for the telephone connections at our home. A part of that bill, the rental, is fixed cost; and the rest of the part of the bill is calculated on the basis of the numbers of calls made.

Selling Price

The price at which a product is sold is called the selling price (SP) of the product.

Marked Price

The marked price or the mark-up price (MP) is the price which the shopkeeper/retailer fixes in anticipation of some discount they may be asked by a customer.

List Price

List price or the tag price, as the name suggests, is the price that is printed on the tag of the article.

For our calculations related to the concept of PLD, till the moment nothing is stated in the questions we would not see much difference between Marked Price and List price. This can be understood in the following example.

Suppose, the CP of an article is Rs 100 and the shopkeeper wants to earn a profit of 20%, i.e., he wants to have a profit of Rs 20. Now, he anticipates that a customer will ask for some discount and if he fixes the SP at Rs 120 he would not be able to give any discount to the customer. So, he now marks up his price over Rs 120 so that he can give some discount to his customer.

Remember that if nothing else is stated, then mark-up percentage is always calculated over CP.

$$MP \xrightarrow{\text{\% decrease}} SP \xrightarrow{\text{\% profit/\% loss}} CP$$

Example 1 After selling an article at a discount of 50%, profit percentage obtained is 20%. What is the mark-up over CP?

Solution Let us assume that CP = Rs 100.

So, SP = Rs 120.

Now, after giving a discount of 50% over MP, Rs 120 is the SP.

Using product stability ratio, 50%↓ = 100%↑

So, MP = 100%↑ over SP = Rs 240

Hence, percentage mark-up = 140%

Alternatively, 0.5 MP = 1.2 CP

MP/CP = 1.2/0.5 = 2.4

So, percentage mark-up $= \left(\frac{MP}{CP} - 1\right) \times 100$

$= (2.4 - 1) \times 100 = 140\%$

Further, as we know

If SP > CP, then there will be a profit and if SP < CP, then there will be a loss.

SP – CP = Profit

CP – SP = Loss

Profit % = Profit/CP × 100

Loss % = Loss/CP × 100

Profit% or Loss % is always calculated upon CP, unless something else is mentioned.

It can be seen that whenever, there is a profit, then MP > SP > CP.

However, when there is a loss, then we can not have some definite relationship between the above written three quantities.

Margin

When we calculate profit percentage as a percentage of SP, then it is known as margin.

Calculating CP/SP Profit % or Loss %

If we say that there is a profit of 20%, then

CP × 1.2 = SP

So, if CP = Rs 120, then SP = Rs 120 × 1.2 = Rs 144

Or, if SP = Rs 144, then CP = 144/1.2 = Rs 120

(Needless to say that if there is a profit of 30%, then we will take 1.3 in the place of 1.2, and if there is a loss of 27%, then we will take 0.73 at the place of 1.2)

So, if there is a profit of R%, and CP = C, then,

$$SP = \frac{C}{100+R} \times 100$$

And in case of loss of R%, $SP = \frac{C}{100-R} \times 100$

Alternatively, we can use product stability ratio also to find out SP if CP is given.

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Example 2 Two shopkeepers sell some article for Rs 4000 each. A shopkeeper calculates his profit per cent on his CP and another calculates his profit per cent wrongly on SP. What is the difference in their actual profit if both claim to have a profit of 20%?

Solution

In 1st case,	In 2nd case
SP = Rs 4,000	SP = Rs 4,000
Profit = 20% of CP	Profit = 20% of SP
CP = Rs 3,333.33	CP = Rs 3,200
Profit = Rs 666.66	Profit = Rs 800

So, the difference in profit = Rs 133.33

PROFIT GENERATION

The purpose of studying Profit, Loss and Discount is to understand the mechanism of profit generation under different situations. Broadly it can be seen in two ways:

Honest Ways

Questions which you will find here will be based either in terms of Rs or in terms of goods.

Type 1: Questions in terms of goods

In these types of questions, CP of a fixed number of goods are compared with the SP of another field number of goods. Let us see through an example:

Example 3 The CP of 30 articles is equal to the SP of 40 articles. What is the profit or loss percentage?

Solution To obtain the same amount of money, which was needed to purchase 30 articles, we need to sell 40 articles, which is more than what we have got for the same sum. It means we need to arrange 10 more articles apart from the articles which we have purchased. So, there will be a loss.

Now, CP of 30 articles = SP of 40 articles

Or, CP/SP = 30/40 = ¾

Or, 1 – CP/SP = 1 – ¾ = ¼

So, Loss percentage

= (1 – CP/SP) × 100 = ¼ × 100 = 25%

Alternatively,

CP of 30 articles = SP of 40 articles

= Rs 120 (Assume)

So, CP of one article = Rs 4

SP of one article = Rs 3

Obviously, there is a loss of Re 1

Loss percentage = ¼ × 100 = 25%

Or, Otherwise we can use Profit/loss Percentage

$$= \frac{(\text{Goods left/added})}{(\text{Goods sold})} \times 100$$

Type 2: Questions in terms of money

Basically questions in terms of money relate to the CP or SP. However, sometimes the results given below also come handy in solving problems.

Some Important Results

When SPs of two articles are same

i. First one is sold at a profit of x% and second one is sold at a profit of y%.
Ratio of $CP_1:CP_2 = (100 + y):(100 + x)$

ii. First one is sold at a profit of x% and second one is sold at a loss of y%.
Ratio of $CP_1:CP_2 = (100 - y):(100 + x)$

iii. First one is sold at a loss of x% and second one is sold at a loss of y%.
Ratio of $CP_1:CP_2 = (100 - y):(100 - x)$

iv. First one is sold at a loss of x% and second one is sold at a profit of y%.
Ratio of $CP_1:CP_2 = (100 + y):(100 - x)$

Example 4 SPs of two articles are same. One is sold at a loss of 20% and another one at a profit of 20%. What is the net loss/profit in the whole transaction?

Solution Assume that SPs of each of the article = Rs 100

So, CP_1 (For the article which is sold at a loss of 20%) = 100/0.8 = Rs 125

CP_2 (For the article which is sold at a profit of 20%) = 100/1.2 = Rs 83.33

So, net CP = Rs 125 + Rs 83.33 = Rs 208.33

As we can observe now that SP<CP, so, there is a loss.

Loss % = (8.33/208.33) × 100 = (1/25) × 100 = 4%

Alternatively, if SPs of two articles are same, and one is sold at a profit of x% and another is sold at a loss of x%, then in that case there will be a loss always. And

$$\text{loss percentage} = \frac{x^2}{100}\%$$

Example 5 Two articles are sold at Rs 12,000 each. One is sold at a profit of 20% and another one at a loss of 20%. What is the net loss?

Solution SP_1 = Rs 12,000

CP_1 = Rs 12,000/ 1.2 = Rs 10,000

SP_2 = Rs 12,000

CP_2 = Rs 12,000/ 0.8 = Rs 15,000

So, total CP = Rs 25,000 and total SP = Rs 24,000

So, loss = Rs 1,000.

Alternatively, since there is a loss of 4%. So, for every Rs 100 invested, Rs 96 is coming back and Rs 4 is lost. In our case, 96% = Rs 24,000; so, 4% = Rs 1,000.

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Example 6 Due North Inc. is the number 1 idea developer company worldwide. One day, they sold Idea 1 to CL and Idea 2 to AMS at a profit percentage of 20% and 30%, respectively. If the sum of developing Idea 1 and Idea 2 is Rs 25,000, what is the developing cost of Idea 1?

Solution The ratio of developing cost of Idea 1 and Idea 2 = 130:120

So, developing the cost of Idea 1 = $\frac{130}{250} \times 25{,}000$ = Rs 13,000

Dishonest Ways

Dishonest ways of earning profit includes adulterating or using faulty balance or both of these simultaneously.

Example 7 A shopkeeper sells his articles at his CP but uses a faulty balance which reads 1000 gm for 800 gm. What is his actual profit percentage?

Solution This question can be solved in several methods.

Method 1 Shopkeeper's net profit = 200 gm
CP of 1000 gm = SP of 800 gm
So, profit % = 200/800 × 100 = 25%

Method 2 Since while selling, 800 g = 1 kg
So, 200 gms = 1/4 kg
So, profit = 1/4 = 25%

Method 3 While selling, since the shopkeeper is branding 800 grams as 1 kg, so 1000 grams will be branded as 1000/800 = 1.25 kg and while purchasing, he has paid for just 1000 grams.
So, net profit = (1.25 − 1) = 0.25 kg
So, profit percentage = 0.25/1 × 100 = 25%

Example 8 A shopkeeper marks up his goods by 40% and gives a discount of 10%. Apart from this, he uses a faulty balance also, which reads 1000 gm for 800 gm. What is his net profit percentage?

Solution Let us assume his CP/1000 gm = Rs 100
So, his SP/kg (800 gm) = Rs 126
So, his CP/800 gm = Rs 80
So, profit = Rs 46
So, profit percentage = 46/80 × 100 = 57.5%

Some more examples

Example 9 A cloth store is offering "Buy 3, get 1 free." What is the net percentage discount being offered by the store?

Solution Suppose price of one article is Re 1. So, price of 4 articles = Rs 4. Now the whole scene can be understood as – We are paying only Rs 3 at the place of Rs 4. So, discount = Re 1. So, discount percentage = ¼ (discount/total price) = 25%

Example 10 A shopkeeper sells his goods at its CP only. But still he manages to gain a profit of 40% because he has manipulated his weights. Find how many grams he is actually selling at the place of 1000 gm.

Solution To earn a profit of 40%, shopkeeper needs to make 1.4 kg out of 1 kg. So, he will be selling $\frac{1000}{1.4}$ grams = 728 grams at the place of 1 kg.

Example 11 When an article is sold at 20% discount, SP is Rs 24. What will be SP when the discount is 30%?

Solution 20% discount = 0.8 of MP = Rs 24
So, MP = Rs 30
So, SP when discount is 30% = Rs 21

PRACTICE EXERCISES

WARM UP

Q.1. An article is bought for Rs 600 and sold for Rs 750. What is the profit percentage?
(a) 20% (b) 25%
(c) 30% (d) None of these

Q.2. By selling a VCD player for Rs 1,950, I got a profit of 30%. At what price should I have sold it in order to get a profit of 40%?
(a) Rs 2,000 (b) Rs 2,100
(c) Rs 2,500 (d) None of these

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Q.3. The profit obtained by selling a book for Rs 56 is the same as the loss obtained by selling this book for Rs 42. What is the cost price of the book?
(a) Rs 40 (b) Rs 50
(c) Rs 49 (d) None of these

Q.4. A man bought a shirt at 3/4th of its list price and sold it at 50% above its list price. What is his percentage gain in the transaction?
(a) 50% (b) 70% (c) 80% (d) 100%

Q.5. By selling a CD for Rs 150, a shop owner lost 1/16 of what it costs. What is the cost price of CD?
(a) Rs 120 (b) Rs 140
(c) Rs 150 (d) Rs 160

Q.6. Saurabh sold 250 cycles and had a gain equal to the selling price of 50 cycles. What is his profit percentage?
(a) 25% (b) 20%
(c) 10% (d) None of these

Q.7. If profit as a percentage of the selling price is 25%, then what is profit as a percentage of the cost price?
(a) 25% (b) $33\frac{1}{3}\%$
(c) 20% (d) 15%

Q.8. A man sold an article at a profit of 10%. If he had charged Rs 45 more his profit percentage would have been 25%. What is the cost price?
(a) Rs 200 (b) Rs 300
(c) Rs 250 (c) None of these

Q.9. Richa purchased a pen drive at 4/5th of its list price and sold it at 20% above the original list price. What is Richa's profit percentage?
(a) 25% (b) 40%
(c) 50% (d) None of these

Q.10. A man buys 200 oranges for Rs 10. How many oranges a rupee can he sell so that his profit percentage is 25%?
(a) 10 (b) 14
(c) 16 (d) 20

Q.11. The CP of 24 apples is the same as the SP of 18 apples. Find the percentage gain.
(a) $12\frac{1}{2}\%$ (b) $14\frac{2}{7}\%$
(c) $16\frac{2}{3}\%$ (d) $33\frac{1}{3}\%$

Q.12. A manufacturer earns Rs 20 on selling his product at 10% above his cost. What is the cost price?
(a) Rs 125 (b) Rs 150
(c) Rs 200 (d) None of these

Q.13. By selling an article for Rs 360, the loss incurred is 10%. At what minimum price should he sell that article to avoid loss?
(a) Rs 320 (b) Rs 324
(c) Rs 396 (d) Rs 400

Q.14. Nitika buys a kinetic for Rs 16,000 and sells it for Rs 18,500. If she wants to gain 40%, how much more should she charge for the kinetic?
(a) Rs 3,800 (b) Rs 4,000
(c) Rs 4,200 (d) None of these

Q.15. The owner of Book Point does not get either profit or loss by selling 15 books for Rs 225. How many books should he sell for the same amount to gain 25% profit?
(a) 10 (b) 11 (c) 12 (d) 13

Q.16. A reduction 20% in the price of the rice enables to get 6 kg more of rice for Rs 120. What is the initial price of rice (Rs/kg)?
(a) 4 (b) 5
(c) 6 (d) None of these

Q.17. CP of 12 apples is equal to the SP of 9 apples and the discount on 10 apples is equal to the profit on 5 apples. What is the percentage difference between the CP and SP of apples?
(a) 20% (b) 25%
(c) 16.66% (d) None of these

Q.18. A shopkeeper professes to sell his articles at CP but gives only 800 gm in the place of 1000 gm. What is his profit percentage?
(a) 20% (b) 25%
(c) 16.66% (d) 33.33%

Q.19. In winters, a metre scale shrinks and starts reading 900 cm for every 100 cm. If the shopkeeper is selling the cloth at his CP, what is his loss percentage?
(a) 11.11% (b) 10%
(c) 12.5% (d) None of these

Q.20. A shopkeeper professes to sell his articles at CP but gives only N grams in the place of 1000 gm. If his profit percentage is 100%, what is the value of N?
(a) 750 (b) 500
(c) 0 (d) Not possible

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FOUNDATION

Q.1. A car is sold for Rs 2,400 at a profit of 20%. What is the CP?
(a) Rs 2,100 (b) Rs 2,000
(c) Rs 1,800 (d) Rs 2,200

Q.2. A car is sold for Rs 2,400 at a profit of 20% over SP. What is the CP?
(a) Rs 2,000 (b) Rs 1,920
(c) Rs 1,980 (d) Rs 1,800

Q.3. A car is sold for Rs 2,400 at a profit of 20% over SP. What is the actual profit percentage?
(a) 16.66% (b) 25%
(c) 21.21% (d) 14.28%

Q.4. Apples are purchased @ 10 apples/Re. How many apples should be sold for one Rupee to obtain a profit of 25%?
(a) 6 (b) 8 (c) 12 (d) 4

Q.5. Sharat sells some articles to Chandra at a profit of 20%. Chandra now sells this article to Mayank at a loss of 30% and Mayank sells this article at a profit of 20%. If CP of Chandra is Rs 150, then what is the SP of Mayank?
(a) Rs 105 (b) Rs 87.5
(c) Rs 125 (d) Rs 126

Q.6. Petrol is purchased @ Rs 5/ litre and sold @ 5 litres/Re. What is the profit/loss %?
(a) Loss of 96% (b) No profit, no loss
(c) Profit of 2400% (d) None of these

Q.7. Despite giving a discount of 11.11% on the mark-up price, profit obtained is equal to 14.28%. What is the per cent mark up over CP?
(a) 14.28% (b) 28.56%
(c) 25% (d) 50%

Q.8. A shopkeeper wants to earn a profit of 20% and at the same time, the minimum discount which he wants to offer is 25%. What should be the minimum percentage mark-up over CP?
(a) 60% (b) 42.5%
(c) 62.5% (d) 35%

Q.9. In Q. No. 8, what should be the maximum percentage mark-up over CP?
(a) 50% (b) 100%
(c) 71.4% (d) Cannot be determined

Q.10. An article when sold at 10% discount on the marked price gave a profit of Rs 70. What is the CP?
(a) Rs 700 (b) Rs 350
(c) Rs 125 (d) Cannot be determined

Q.11. 100 kgs of gold are purchased for Rs 1,100. It is sold in such a way that after selling the whole quantity, the quantum of loss is equal to the amount obtained by selling 20 kgs of gold. What is the selling price?
(a) Rs 9.16 (b) Rs 18.32
(c) Rs 11.11 (d) Rs 25

Q.12. A milkman buys two cows for Rs 750. He sells first cow at a profit of 22% and the second cow at a loss of 8%. What is the SP of second cow if in the whole transaction there is no profit no loss?
(a) Rs 312 (b) Rs 506
(c) Rs 484 (d) Rs 532

Q.13. Two cars are sold for Rs 24,000 each. One is sold at a profit of 20% and another at a loss of 20%. What is the net profit percentage/loss percentage, and amount of profit/loss??
(a) 4% profit, Rs 2,000 profit
(b) [illegible] loss, Rs 2,000 loss
(c) [illegible] profit, Rs 500 profit
(d) 4% loss, Rs 500 loss

Q.14. A supplier sells 20 pencils at the marked price of 16 pens to a retailer. The retailer, in turn, sells them at the marked price. What is the percentage profit or percentage loss of the retailer?
(a) Loss 25% (b) Profit 25%
(c) Loss 20% (d) Profit 20%

Q.15. Sum of the CP's of two cars is Rs 1,00,000. 1st car is sold at a profit of 20% and 2nd car is sold at a loss of 20%. However, their SPs are same. What is the CP of 1st car?
(a) Rs 40,000 (b) Rs 60,000
(c) Rs 52,400 (d) Rs 47,600

Q.16. Sum of CP's of two cows is Rs 13,000. Both the cows are sold at a profit of 20% and 40% respectively with their SP's being the same. What is the difference of CPs of both the cows?
(a) Rs 1,000 (b) Rs 2,000
(c) Rs 1,500 (d) Rs 2,500

Q.17. Buy three, get one free. What is the percentage discount being offered here?
(a) 33.33% (b) 25%
(c) 20% (d) 28.56%

Q.18. CP of 40 articles is equal to the SP of 30 articles. What is the profit/loss percentage?
(a) 25% Profit (b) 33.33% profit
(c) 25% loss (d) 33.33% loss

Q.19. Due to a price hike of 20%, 4 kgs less tea is available for Rs 120. What is the original price of tea?
(a) Rs 4/kg (b) Rs 5/kg
(c) Rs 6/kg (d) Rs 4.5/kg

Q.20. A shopkeeper sells his goods at its CP only. But he uses 750 grams weight at the place of 1000 gms weight for a kg. What is his net profit percentage?
(a) 25% (b) 20%
(c) 16.66% (d) 33.33%

Q.21. A shopkeeper sells his goods at its CP only. But still he manages to gain a profit of 30% because he has manipulated his weights. Find out how many grams he is actually selling at the place of 1000 gms.
(a) 700 (b) 769 (c) 800 (d) 820

Q.22. Which of the following discount options is better for a customer?
i. Successive discounts of 20%, 30% and then pay a service tax of 10%.
ii. Successive discounts of 30%, 20% and then pay a service tax of 10%.
iii. Pay service tax of 10% first, then successive discounts of 20% and 30%.
(a) i or ii
(b) iii
(c) Either of these three
(d) i and iii or ii and iii

Q.23. A wholeseller supplies few chips to a retailer every year. Each chip costs Rs 10,000 to the wholeseller. 5% of the chips are defective and they are to be replaced without charging anything extra. If the wholeseller still makes a profit of 20%, at what price is he selling it to retailer?
(a) Rs 12,300 (b) Rs 12,600
(c) Rs 13,200 (d) None of these

Q.24. A seller calculated his intended selling price at 6% profit on the cost of a product. However owing to some mistake while selling, the units and tens digits of the selling price got interchanged. This reduced the profit by Rs 9 and profit percentage to 2.4%. What is the cost price of the product?
(a) Rs 240 (b) Rs 250
(c) Rs 400 (d) Rs 480

Q.25. A shopkeeper wrongly calculates his profit on SP and finds it to be 25%. What is the actual profit percentage?
(a) 20% (b) 33.33%
(c) 40% (d) 25%

Q.26. There are two shopkeepers selling the same article at the same price for same quantity. One day, first shopkeeper offers a price discount of 25% for the same quantity whereas second shopkeeper offers 25% more quantity for the same price. From a customer's point of view, which deal is better?
(a) 1st shopkeeper's deal
(b) 2nd shopkeeper's deal
(c) Both are equal
(d) Cannot be determined

Q.27. A shopkeeper sells two cows in such a way that CP of first cow is equal to the SP of second cow and SP of first cow is equal to the CP of second cow. Which of the following is final result after the whole transaction?
(a) There is a loss (b) There is a profit
(c) No profit, no loss (d) Cannot be determined

Q.28. When an article is sold for Rs 180, a profit of 20% is made. At what price should the article be sold so that profit percentage is double?
(a) Rs 200 (b) Rs 210
(c) Rs 192 (d) Rs 240

Q.29. Vinit sells a car to Amit at a profit of 20%. Now Amit sells this car to Vicky at a profit of 12%. And finally Vicky sells this car to Nishu at a loss of 21%. What is the sum of CPs of Amit and Nishu?
(a) Rs 250 (b) Rs 475
(c) Rs 540 (d) Cannot be determined

Q.30. Anoop sells a book to Mayank at a profit of 20% and Mayank sells this book to Siddharth at a profit of 25%. Now Siddharth sells this book at a loss of 10% to Shishir. At what percentage loss should Shishir sell this book now so that his SP becomes equal to Anoop's CP?
(a) 36.68% (b) 25.92%
(c) 48.66 (d) Cannot be determined

Q.31. When an article is sold for Rs X, loss percentage is equal to L%. However, when the same article is sold for Rs Y, profit percentage is equal to P%. What is the CP of that article?
(a) 100 (P+L)/(Y−X) (b) (P+L)/(Y−X)
(c) 100 (Y−X)/(P+L) (d) (Y−X)/(P+L)

Q.32. A milkman professes to sell milk at its CP only. But still he is making a profit of 20% since he has mixed some amount of water in the milk. What is the percentage of milk in the mixture?

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(a) 80% (b) 83.33%
(c) 75% (d) 66.66%

Q.33. There is some profit when an article is sold for Rs 720. However when the same article is sold for Rs 420, there is some loss. If the quantum of loss is two times the quantum of profit, find the cost price of the article.
(a) Rs 620 (b) Rs 700
(c) Rs 520 (d) None of these

Q.34. While I was shopping in Kamla Nagar market for my baby girl Tatto, I came across two shopkeepers who were giving me a discount of same amount for a top. While first shopkeeper was offering a discount of 14.28%, second shopkeeper was offering a discount of 12.5% over their respective mark-ups. Whose cost price is higher?
(a) 1st shopkeeper
(b) 2nd shopkeeper
(c) Either of them depending upon their respective MP
(d) Cannot be determined

Q.35. A seller offers discounts on the basis of the number of articles purchased. He gives a discount of 10% when 5 articles are bought and a discount of 12.5% when 8 articles are bought. If the profit he makes in each case is the same, then the ratio of the marked price to the cost price of the article is
(a) 4:3 (a) 3:4
(c) 6:5 (d) None of these

Q.36. A shopkeeper purchases his goods at a discount of 10% on the list price. He normally gives a discount of 6% to his customers. If during a special scheme, he gets an additional discount of 2% on his purchase price, find his approximate profit percentage.
(a) 8.8% (b) 7.8%
(c) 6.6% (d) None of these

Q.37. A shopkeeper purchases a packet of 50 pencils at Rs 10 per pencil. He sells a part of the packet at a profit of 30%. On the remaining part, he incurs a loss of 10%. If his overall profit on the whole packet is 10%, the number of pencils he sold at profit is
(a) 25 (b) 30 (c) 20 (d) 15

Q.38. The costs of a pen and a book are Rs 3 and Rs 4 respectively. A student has to spend a sum of exactly Rs 100 to buy pens and books, buying at least one pen and one book. How many combinations of books and pens can he buy?
(a) 6 (b) 8 (c) 12 (d) 10

Q.39. Anam bought a jute bag at 30% discount on the list price. He then sold it at a price which is 160% of the list price thereby making a profit of Rs 81. What is the list price of the bag?
(a) Rs 90 (b) Rs 100
(c) Rs 180 (d) Rs 200

Q.40. What is the profit percentage of a dishonest cloth merchant who professes to sell his articles at CP but uses a scale which weighs less by 16.67%?
(a) 20 (b) 15
(c) 16 (d) None of these

Q.41. A shopkeeper calculated his profit on the marked price and finds it to be 30%. He forgets the fact that he gave a discount of 20%. What is his actual profit percentage?
(a) $14^2/_7$% (b) 20% (c) 25% (d) $28^4/_7$%

Q.42. Santa Singh, the local fruit vendor, buys a certain number of oranges at Rs 7 a dozen and equal number at Rs 6 a dozen. He sells the oranges at Rs 7.50 a dozen and makes a profit of Rs 80. How many oranges does he buy?
(a) 20 dozens (b) 30 dozens
(c) 40 dozens (d) 80 dozens

Q.43. A milkman who wants to make a fast buck by using a false measure, measures only 900 ml instead of a litre. If his cost price and selling price per litre of milk are Rs 10 and Rs 12 respectively, what is his daily profit and profit percentage if he sells 20 litres in one day?
(a) Rs 60, 20% (b) Rs 60, $33^1/_3$%
(c) Rs 40,25% (d) Rs 40, $33^1/_3$ %

Q.44. Two lots of oranges with equal quantity, one costing Rs 20 per dozen and the other costing Rs 30 per dozen, are mixed together and the whole lot is sold at Rs 24 per dozen. Then what is the profit or loss?
(a) 4% profit (b) 5.6% profit
(c) 4% loss (d) 5.6% loss

Q.45. A shopkeeper marks up the price of his product by 40%. If he increases the discount from 5% to 10%, the profit would decrease by Rs 14. How much profit would he earn if he gives a discount of 20% on the marked price?
(a) Rs 56 (b) Rs 28
(c) Rs 32 (d) Rs 24

Q.46. Navneet is a bananawallah who buys a gross of bananas at Rs 10 a dozen. During transport and storage two dozen bananas get spoilt. If Navneet spends 10% of his outlay on transport costs and

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wants an overall profit of 20%, at what price must he sell a dozen bananas?
(a) Rs 14.64 (b) Rs 15.84
(c) Rs 16.24 (d) Rs 17.54

Q.47. Krishna sells his watch at a loss of 5%. If he had sold it for Rs 56.25 more he would have gained 10%. What will be his gain or loss percentage if he sells it for Rs 450?
(a) 10% (b) 15% (c) 20% (d) 30%

Q.48. A person sold his watch for Rs 24 and got a percentage of profit equal to the cost price; then the cost price is
(a) Rs 145 (b) Rs 18
(c) Rs 22 (d) Rs 20

Q.49. A merchant buying goods abroad at a discount of 33.33% on the catalogue price, has to pay import duty of 20% on the net cost of goods, realize a profit of 25% on his outlay, what must he charge for an article priced Rs 228 in the catalogue?
(a) 228
(b) 322
(c) Both of the above
(d) Cannot be determined

Q.50. A group of students prepared stuffed toys as part of their group activity. They spent Rs 100 on velvet, Rs 5 on thread and needle and Rs 27 on miscellaneous items. They made 30 toys, 50% of which were purchased by force by some senior students causing them a loss of 50%. At what % profit should they sell the remaining toys so as to gain 50% on their total cost?
(a) 100% (b) 150%
(c) 75% (d) None of these

MODERATE

Directions for questions 1–5: Read the passage below and solve the questions based on it.

Patna-mart, sister concern of Wal-mart, is planning to establish a mechanism to give discount. It is actually a Khul Ja Simsim kind of game where if you win any round, you will get some discount specific to that round, and it you lose, you will be losing all the discounts so far won.

Now, round 1 is having discount coupons of 2%.
Round 2 is having discount coupons of 4%.
Round 3 is having discount coupons of 6%.
And so on.....

There are infinite rounds and you can participate in all the rounds but you can participate in any round only once and before participating in round N (any particular round), you should have participated in round 1 – round N-1 (all the rounds before that particular round.)

All the discounts won will be successive, e.g., if you win Round 1 and then round 2 also, you will get a successive discount of 2% and then 4%. But again as the rule goes on, if you lose round 3, you will be losing all the discount coupons won so far. But despite losing round 3, you can participate in round 4 and hence.

Q.1. What is the minimum number of rounds to obtain maximum possible discount?
(a) 50 (b) 100
(c) Infinite (d) Cannot be determined

Q.2. What is the maximum possible discount?
(a) 1000% (b) 50%
(c) 100% (d) Infinite

Q.3. What is the minimum possible discount after 20 rounds provided that customer has won a minimum of 18 rounds?
(a) 100% (b) 50%
(c) 0% (d) None of these

Q.4. In the above question, what is the maximum possible discount?
(a) 100% (b) 50%
(c) 0% (d) None of these

Q.5. In a bid to attract more customers, Patna-mart is on a mass-compaigning move. With a new punch line "You can always check-out, but you can never leave", they are organizing several contact programs across the city. As a result, there is an increase in sale of 10% and 20% respectively on weekdays and weekends (Saturday and Sunday). If each of the articles sold by Patna-mart is sold at a profit of 40%, what is the maximum possible percentage growth in the profit of Patna-mart?
(a) 32% (b) 72%
(c) Minimum 50% (d) Cannot be determined

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Directions for questions 6–8: Read the passage below and solve the questions based on it.

Had the CP been 10% less and SP been 10% more, profit % would have been double than that of earlier case.

Q.6. What is the actual CP?
(a) Rs 70 (b) Rs 100
(c) Rs 140 (d) Cannot be determined

Q.7. What is the profit % in original case?
(a) 20% (b) 28.56%
(c) 14.28% (d) Cannot be determined

Q.8. How many values for actual SP is/are possible?
(a) 1 (b) 0
(c) 3 (d) Infinite

Directions for questions 9–10: Read the passage below and solve the questions based on it.

King Nandan is the king of Divyagarh. One day he decides to sell all the animals living in his big kingdom. Now, there are five states in his kingdom and the only animals which his kingdom is having are horses, cows and sheeps. King Nandan sells all the animals to eight neighboring states. Each of the neighboring states purchases same number of animals at the following rate.

Horse – Rs 1,700 each
Cow – Rs 200 each
Sheep – Rs 200 each
King Nandan receives Rs 28,500 in all?

Q.9. What can be the maximum number of animals?
(a) 80 (b) 130
(c) 160 (d) None of these

Q.10. What are the respective numbers of horses, cows and sheeps in any state?
(a) (3, 109, 8) (b) (1, 37, 3)
(c) (2, 74, 6) (d) Cannot be determined

Q.11. Sunny marks up his goods by 40% and gives a discount of 10%. Apart from this, he uses a faulty balance which reads 800 gms for 1000 gms. What is his net profit/loss percentage?
(a) 8% (b) 57.2%
(c) 37.6% (d) None of these

Q. 12. A shopkeeper marks up his goods by 20% and then gives a discount of 20%. Besides he cheats both his supplier and customer by 100 grams i.e., he takes 1100 gram from his supplier and sells only 900 grams to his customer. What is his net profit percentage?
(a) 24.5% (b) 17.33%
(c) 25% (d) 32.5%

Q.13. In question number 12, shopkeeper takes a discount of 20% from his supplier which he disregards while preparing his mark-up. What is his net profit percentage?
(a) 37.5% (b) 46.66%
(c) 52.33% (d) 58.33%

Q.14. In addition to all the data of question number 13, now he adulterates his goods in the ratio of 1:5 (adulterant: original). Assuming that adulterant comes free of cost, what is the net profit percentage?
(a) 62.5% (b) 66.66%
(c) 72.08% (d) None of these

Q.15. In question number 14, if the cost of adulterant is 20% the cost of original substance, what is the net profit percentage?
(a) 77.77% (b) 81.08%
(c) 86.75% (d) None of these

Q.16. Some mangoes are purchased at the rate of 8 mangoes/Re and some more mangoes at the rate of 6 mangoes/Re, investment being equal in both the cases. Now, the whole quantity is sold at the rate of 7 mangoes/Re What is the net percentage profit/loss?
(a) 0.6% profit (b) 0.6% loss
(c) 1.2% loss (d) No profit/no loss

Q.17. Some olives are purchased at the rate of 8 olives /Re and same number of olives at the rate of 6 olives/Re. Now, the whole quantity is sold at the rate of 7 olives/Re. What is the net percentage profit/ loss?
(a) 0.6% profit (b) 0.6% loss
(c) 1.2% loss (d) No profit/ no loss

Directions for questions 18–22: Read the passage below and solve the questions based on it.

A shop has the following discount layers
1st layer – Buy 1, get 2 free.
2nd layer – Buy 2, get 3 free.
3rd layer – Buy 3, get 4 free.
And so on there are infinite layers on the same pattern.

Q.18. For how many of these discount layers discount percentage offered is more than 50%?
(a) 4 (b) 27
(c) 99 (d) None of these

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Q.19. The shopkeeper wishes to lessen the percentage of discount by clubbing any two layers. Which two layers should be clubbed so that discount is minimized by maximum percentage?
(a) 1st + 2nd (b) 2nd + 3rd
(c) 3rd + 4th (d) None of these

Q.20. What is the net percentage discount if we club 1st three layers?
(a) 43.33% (b) 46.66%
(c) 53.33% (d) None of these

Q.21. A new condition has been introduced by the shop that anybody can avail one layer discount only once (i.e., if you are planning to purchase 6 articles, then you can not purchase 2 articles in 1st layer and get rest four free. Since you have availed 1st layer discount, you can now ask for only other layer discounts.) Sona is purchasing 8 articles from the store. What is the maximum possible discount that she can avail?
(a) 72% (b) 26%
(c) 38% (d) None of these

Q.22. In Q. 21, what is the minimum percentage discount that she can avail?
(a) 0% (b) 22%
(c) 44% (d) None of these

Directions for questions 23–24: Read the passage below and solve the questions based on it.

To produce a certain type of engine, two components are imported— Component P is imported from USA and component Q is imported from Japan. P and Q account for 30% and 40% of the production cost of the engine. A profit of 25% is to be made but discount of 10% is to be offered to customers for promotional reasons.

Q.23. If the price of US Dollar appreciates by 20% and the price of Japanese Yen appreciates by 30%, what percentage above the original CP should the product be marked?
(a) 33.3% (b) 48.5%
(c) 63.88% (d) 71.4%

Q.24. US Dollar appreciates by 4% and Japanese Yen by 32%. If marked price remains same, then what is the change in profit percentage?
(a) 4.38% (b) 3.33%
(c) 6.25% (d) 7.69%

Directions for questions 25–26: Read the passage below and solve the questions based on it.

A shopkeeper sells two types of custom confiscated goods, A and B, both at same price of Rs 120 whereas they cost him Rs 90 and Rs 160 respectively. Sale for A started from 1 for the first day and then went on increasing every day by 4 units, whereas sale of article B was 450 on the first day and went on decreasing every day by 6 units.

Q.25. Shopkeeper has planned the things in such a way that he starts earning profit exactly on diwali which is after N days. What is the value of N?
(a) 103 (b) 105 (c) 107 (d) 109

Q.26. On which day will he earn profit for the first time on day-to-day basis?
(a) 49 (b) 51 (c) 53 (d) 55

Q.27. In the year 2002, the cost price of an item was 90% of the selling price while in 2003, the cost price was 95% of the selling price. If the gross profit remains the same for both the years, What is the percentage increase in the cost price of the item from 2002 to 2003?
(a) 60% (a) 80%
(c) 100% (d) None of these

Directions for questions 28–33: Read the passage below and solve the questions based on it.

Mittal has recently acquired four companies viz., Bank of Bozoland (BOB), My Own Bank (MOB), Zany Obliterated Bank (ZOB), and Dogmatically Obscure Bank (DOB). He noticed that the sales of DOB are half that of BOB whereas the profits of DOB are double that of BOB. The expenses of ZOB are Rs 3 crores less than that of DOB whereas the profits of MOB is Rs 1 crore less than that of ZOB. The expenses of BOB are three times that of DOB. It is also known that the sales of ZOB are Rs 15 crore or one-fourth that of MOB. All figures are for 1992-93. An insider further informs Mittal that the sales of DOB are Rs 10 crores more than that of ZOB and the expenses of BOB are 90% of its own sales. Sales – Expenses = Profit

Q.28. The total sales of all the four companies is (Rs crores)
(a) 200 (b) 150 (c) 125 (d) 160

Q.29. The expenses of BOB exceed that of ZOB by (Rs crores)
(a) 31 (b) 43 (c) 33 (d) 62

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Q.30. Which bank had the maximum profit?
(a) BOB (b) MOB (c) ZOB (d) DOB

Q.31. Total profits of the four companies (Rs crores) is
(a) 51 (b) 52
(c) 53 (d) None of these

Q.32. What is the total expense (Rs crores) of all the four banks together?
(a) 110 (b) 130 (c) 150 (d) 160

Q.33. The profits of MOB form what percentage of the expenses of ZOB?
(a) 16% (b) 20% (c) 15% (d) 23%

Q.34. By selling a watch at a profit of 10 per cent, a man got Rs 15 more than half its price. What is the price of the watch?
(a) 10 (b) 15 (c) 25 (d) 5

Q.35. A man would gain 25% by selling a chair for Rs 47.5 and would gain 15% by selling a table for Rs 57.5. He sells the chair for Rs 36; what is the least price for which he must sell the table to avoid any loss on the two together?
(a) Rs 51.2 (b) Rs 58.5
(c) Rs 62.5 (d) None of these

Q.36. A bookseller marks his books at an advance of 69% on the actual cost of production. He allows a discount of 15% and also given a copy free for every dozen sold at a time. What rate per cent profit does the bookseller make, if books are sold in lots of 12?
(a) 32.6 (b) 47.5
(c) 24.9 (d) None of these

Q.37. While returning from Dubai festival, Mr. Sanjay Singh purchased a number of articles, all at a discount of $33\frac{1}{3}$% of the list price. However he is required to pay a duty of 20% on the his cost of goods. If he realizes a profit of 25% on his outlay, what must he charge for an article priced Rs 228 as the list price?
(a) 228 (b) 322
(c) Neither (a) nor (b) (d) Data insufficient

Q.38. A certain manufacturer sells a product to the distributor at 10% profit. Then the distributor sells it to the dealer and the dealer sells to the retailer at a mark up of 10% and 20% respectively. The retailer marks up his cost by 20% and then offers a 10% discount to the customer. If the customer had bought it from the distributor directly at the distributor's selling price, then how much reduction in price would he have got with respect to buying it from the retailer?
(a) 29.6% (b) 18% (c) 32% (d) 22.8%

Directions for questions 39–40: Read the passage below and solve the questions based on it.

Machine X produces articles at the rate of 50 units/h. SP of each article is Rs 100 and the cost of production is Rs 40 each. However, 20% of the articles produced are defective and hence cannot be sold. The rate of production of machine can be increased, but with every increase of N units/h production cost would increase by $2x$% and the number of defectives would become $(20 + 1.5x)$%.

Q.39. Find the maximum increase in production that can be undertaken without incurring losses?
(a) 14 unit (b) 15 unit
(c) 16 unit (d) 17 units

Q.40. What will be the profit in 1 hour if the machine produces 58 units per hour?
(a) Rs 1,839 (b) Rs 1,178
(c) Rs 1,253 (d) Rs 1,624

Directions for questions 41–42: Read the passage below and solve the questions based on it.

There is a growing demand of platinum in the Indian market. A firm dealing in platinum wants to earn a profit to the tune of Rs 5 lacs per annum. Platinum can be produced from a mine. It is required to employ 25 people whose average salary is Rs 2,000 per month. Cost of the raw material i.e., platinum ore is Rs 200 per ton of pure platinum produced. However, the fixed cost associated with the plant is Rs 1,00,000 annually.

Q.41. If the production rate is 20 ton/month, then what should be the price per ton of platinum to earn the required profit?
(a) Rs 4,300 (b) Rs 5,200
(c) Rs 6,300 (d) None of these

Q.42. If the price per ton is Rs 3,000, then how much ton of platinum is required annually to fulfill the profit target?
(a) 300 (b) 350
(c) 400 (d) None of these

Q. 43. Two pieces of the same cloth together cost Rs 910. On selling from the first piece as many metres as the second piece originally contained and from

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the second piece half as many metres as the first piece contained originally. The remaining of the first piece turned out to exceed the remaining of the second piece by 10 m. Each metre of the cloth costs Rs 14. How many metre of cloth did each piece originally contain?

(a) 45 m; 20 m (b) 45 m; 25 m
(c) 36 m; 29 m (d) 40 m; 25 m

Q.44. Rupesh marks up an article by p%, gives a discount of $\frac{p}{4}$% and gets a profit of $\frac{p}{4}$%. Had he marked up by $\frac{p}{2}$% and given a discount $\frac{p}{6}$%, what would be his profit percentage?

(a) 25% (b) $33\frac{1}{3}$%
(c) 50% (d) $66\frac{2}{3}$%

Q.45. Amit brought two cars. He then sold the first car at 10% profit and the second one at 25% profit. The selling price of the second car is 25% more than the selling price of the first car. What is the approximate profit per cent in both the cars together?

(a) 17.85% (b) 18.36%
(c) 16.19% (d) Cannot be determined

ADVANCED

Q.1. A publisher printed 3,000 copies of '*Future Shock*' at a cost of Rs 2,400. He gave 500 copies free to different philanthropic institutions. He allowed a discount of 25% on the published price and gave one copy free for every 25 copies bought at a time. He was able to sell all the copies in this manner. If the published price is Rs 3.25, then what is his overall gain or loss percentage in the whole transaction?

(a) 113% (b) 130% (c) 162% (d) 144%

Q.2. A car manufacturing company XYZ can make 300 cars per month at a cost of Rs 2 lacs per car and pay an excise duty of 30% per car. It has 500 cars in its storage room in a month and is able to sell 300 cars per month at a rate of Rs 3 lacs per car. In the budget in the last week of February, the excise duty was reduced by 10% and it will take effect from 1st April. But, the customers cannot wait till 1st April. So the company decides to give this rebate to every customer who buys a car in March as well, but for this particular month the company has to pay 30% duty. The company can reduce or increase manufacturing of car per month by a maximum of 25%. What is the profit or loss in the revenue to the company in the month of March?

(a) 15 lac (b) 60 lac
(c) 45 lac (d) 30 lac

Q.3. A shopkeeper uses a "point of sales" software which prepares the bill for customers based on the number of units per unit. Both of the above are 2-digit numbers. While entering the data, he erroneously swaps the digits of both quantity of units and price. As a result, the closing stock balance is lowered by 72 units, and the sales value shown is Rs 1,368. How many units have actually been sold?

(a) 27 (b) 91
(c) 83 (d) None of these

Directions for questions 4–5: Read the passage below and solve the questions based on it.

Given below is the production of fertilizers bags by ABC fertilizers during different days of the week.

Monday	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	*Saturday*
650	750	700	800	750	600

There are two modes of transporting these bags to warehouse. A truck has a capacity of 800 bags and cost Rs 1,100 per trip. A Matador has a capacity of 500 bags and cost Rs 700 per trip. The cost of storing the material in case it is not transported is Re 1 per bag per day. The cost of transportation includes the storing cost.

Q.4. In order to minimize transportation cost, how many trips should the Matador and the Truck respectively make to the warehouse during the week?

(a) 3, 4 (b) 2, 4 (c) 1, 5 (d) 4, 3

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Q.5. What should be the minimum cost of storing 1 bag/day, at which it is best to hire a truck everyday?
(a) Rs 0.4/day (b) Rs 1.4/day
(c) Rs 1.6/day (d) Rs 1.2/day

Q.6. Mohit goes to furniture shop to buy a sofa set and a center table. He bargains for a 10% discount on the center table and 25% discount on sofa set. However, the shopkeeper, by mistake, interchanged the discount percentage figures while making the bill and mohit paid accordingly. When compared to what he should pay for his purchases, what percentage did mohit pay extra given that the center table costs 40% as much as the sofa set.
(a) 12.3% (b) 7.2% (c) 8.1% (d) 6.3%

Q.7. In a laboratory experiment, a sample of air which is a mix of only oxygen and vapour is taken. Water contains hydrogen and oxygen gases. If air contains 70% oxygen (including that contained in the water vapour) by weight while water vapour contain 16.66% oxygen by weight, how many kilograms of water vapour is present in 1kilogram of water?
(a) 0.25 (b) 0.34 (c) 0.36 (d) 0.3

Q.8 A shopkeeper purchases a packet of 50 pens at Rs 10 per pen. He sells a part of the packet at a profit of 30%. On the remaining part, he incurs a loss of 10%. If his overall profit on the whole packet is 10%, the number of pens he sold at profit is
(a) 25 (b) 30 (c) 20 (d) 15

Q.9. Amar sold his moped to Bharat at 20% profit and Bharat sold it to Sridhar at 10% profit. Sridhar sold the same to a mechanic and received Rs 2,316. If Amar had sold the same moped to the mechanic and receive the same amount the mechanic paid to Sridhar, what profit percentage would Amar have made?
(a) 52% (b) 48%
(c) 33.3% (d) Cannot be determined

Directions for questions 10–13: Read the passage below and solve the questions based on it.

Newspaper major *"The Asian Times"* is trying to enter the city of "Paperabad", where already two newspapers *"The Deccan City"* and *"Good Morning Paperabad"* are in circulation. The population of the city is 35 lakhs of which, according to the recent survey, 60% read newspaper. The two evening newspapers sell at Rs 1.75 per copy on weekdays and Rs 3.00 per copy on Sundays. *"The Asian Times"* promised its readers newspapers at Rs 1.50 per copy per day. Since people were aware of the quality of the paper, the management thought that it can capture a major chunk of the newspaper reading population. On weekdays, the operational cost of the *"The Asian Times"* were Rs 62,500 per day and the cost for the material and printing charges was Rs 2.75 per newspaper. For the Sunday edition the cost of the newspaper per copy was Rs 3.00 and the operational cost were Rs 75,000. The number of the copies of the newspaper "The Asian Times" sold on Sunday was 16.66% less than the number of copies sold on any of the week days. The number of copies sold on any weekday being the same. The amount collected through the advertisement on any weekday was the same and equaled Rs 2,50,000 per day while for Sunday it was 4,35,000.

Q.10. If *"The Asian Times"* wanted to break even, how many copies of the newspaper are required to be sold every week?
(a) 21,25,800 (b) 16,97,143
(c) 47,50,500 (d) 50,50,500

Q.11. What percentage of the newspaper readers should read *"The Asian Times"* so that the newspaper makes a profit of Rs 12,500 every weekday?
(a) 7.61% (b) 12.4 %
(c) 12.8 % (d) None of these

Q.12. If 50,000 of the readers of *"The Deccan City"* now read *"The Asian Times"* on all weekdays, how much profit would *The Asian Times* expect to make every week?
(a) 2,55,000 (b) 3,85,000
(c) 4,57,000 (d) Cannot be determined

Q.13. How many copies of the newspaper should *"The Asian Times"* sell on Sunday so that there is a profit of Rs 60,000 every Sunday?
(a) 1,88,000 (b) 2,40,000
(c) 3,20,000 (d) 3,76,000

Directions for questions 14–16: Read the passage below and solve the questions based on it.

In the "Get-One" supermarket, a new software was installed in the computerized billing section which made the calculations of each of the item sold very easy on the first day of the operations. To verify the workability of the software only one of the item was billed on the computer.

Due to some technical problem in the software, it reversed both the numbers of the item sold and price per piece as a result of which the closing of the stock items showed 63 items less and the value on the items still left was shown Rs 936. It was known that both the stocks left and the price per piece are two digit numbers.

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this as the cost price) is Rs 20 per mango and Rs 10 per apple. In this transaction, the farmer lost Rs 15,000. What is the total number of trees in the garden?

(a) 75 (b) 80
(c) 25 (d) 60

Q.24. A spring balance reads 1 kg when actually 800 g is weighed on it. The shrewd grocer has already marked up the price of his goods by 20%. One day, a police inspector detects this and orders him to repair his spring balance and revert to the original list price of the goods. Moreover, he asked him to give a discount of 10% to all the customers for a month from then onwards as a punishment. A day after, Ramu paid Rs 16 for 1 kg rice. Before the raid, how much he would have paid for the same quantity?

(a) Rs 26.66 (b) Rs 27.5
(c) Rs 24 (d) Rs 28

Q.25. The evergreen shrubs at Ravi's nursery are planted in rows on a square plot of land measuring 2,401 ft^2. The shrubs are planted in such a manner that the centers of the adjacent shrub are 7 ft apart and the outer shrubs are planted along the edges of the plot, with a shrub at each corner. Ravi spent $ 896 to cover all the costs necessary for raising this crop of the evergreen shrubs. If Ravi succeeds in selling each shrub for $ 35, his profit will be what percentage of his total cost?

(a) 50% (b) 90% (c) 150% (d) 200%

Q.26. A retailer keeps Reebok shoes, which are listed at a mark-up of 10% above their factory price. However, on Monday he realized the shortage of demand and hence, decreased the list price by 10%. On the very next day, i.e., Tuesday, he realized he is making a loss so, he increased the list price by 10% again. He continues this trend indefinitely. On which day he will suffer a loss of more than 10% per shoes for the first time?

(a) Monday (b) Tuesday
(c) Thursday (d) Sunday

Directions for questions 27–28: Read the passage below and solve the questions based on it.

A person purchased 50,000 cups of glasses at a rate of Rs 10 per cup. For transporting these cups to his shop, he has four options as follows:

Mode of transportation	*Rent for one trip*	*Capacity of number of glass in one trip*	*Breakages during transportation*
Truck	Rs 2,000	20,000	8%
Small truck	Rs 1,000	12,000	10%
Jeep	Rs 750	10,000	15%
Rickshaw	Rs 500	5,000	10%

Even if a transport vehicle is not full to its capacity, rent for the trip will remain the same.

Q.27. Which mode of transportation is most profitable?

(a) Truck (b) Small truck
(c) Jeep (d) Rickshaw

Q.28. If the most profitable mode of transportation is chosen, then what should be his selling price per cup so as to get the profit of 10%?

(a) Rs 12.45 (b) Rs 14.67
(c) Rs 13.80 (d) Rs 12.10

Directions for questions 29–31: Read the passage below and solve the questions based on it.

The air conditioning market is one of the few segments that have been stable since the last three-four years. The total air-conditioning industry is estimated to be around Rs 3,000 crores with central air-conditioning including packaged and ducted air-conditioning systems estimated to be around Rs 1,200 crore. Windows and mini-split air-conditioners are estimated at Rs 1,800 crores, with window ACs cornering 80% of the market.

Prior to 1997, corporations, institutions and the government dominated the consumer segment. They did around 85–90 per cent of the buying. It was only in the year 1997 that the home segment picked up. The institutional segment accounted for 65 per cent while the rest went to the home segment. The average growth for the last five years in the residential segment has been around 24 per cent.

Q.29. LG has priced their split-ACs at Rs 35,000. If they have estimated a market share of 70% in the split-AC market, then how many split-ACs would they be selling approximately?

(a) 72,000 (b) 68,000
(c) 71,000 (d) 70,000

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Q.30. If prior to 1997, the institutional buyers comprised of Rs 2,700 crore worth of market, what is the approximate share of Samsung ACs in 1997 if they sold only to home buying segment? Their share prior to 1997 was 20% in the home AC segment, while it had increased by 15% in 1997 from its earlier levels. The overall market size for total AC market had remained the same.

(a) 300 crores (b) 69 crores
(c) 76 crores (d) 241.5 crores

Q.31. With respect to the previous question, if Samsung entered the institutional market in 1998 and had a share of 10% by 1999. What was the ratio of its share in the institutional market in 1998-1999 with respect to its share in the home market prior to 1997? The market size of institutional segment showed a 10% increase over its size in 1997. (Use the data from previous questions if required). (Ratio is required with respect to revenue).

(a) 3 : 1 (b) 2.75 : 1
(c) 3.57 : 1 (d) 2.8 : 1

Q.32. The cost price of a dozen mangoes is equal to the selling price of 10 mangoes. The selling price of apples per piece is the cost price of a mango (per piece). If the profit % made on selling an apple is half that of selling a mango, then what is the cost of an apple if the selling price of one mango is Rs X?

(a) Rs $\frac{X}{6}$ (b) Rs $\frac{5X}{6}$
(c) Rs $\frac{25X}{33}$ (d) Rs $\frac{33X}{25}$

Directions for questions 33–37: Read the passage below and solve the questions based on it.

Various costs involved in printing news papers
All the given cost are calculated in Rs
Now, the total cost of printing is calculated by the following formula:
Total cost = P × (Columns × Cost per column + Rows × Cost per row + Labour cost per print) + Plate cost + Setup cost
Where, P = The number of prints
Fixed cost = Plate Cost + Set up Cost
Total cost = Fixed cost + Variable Cost

Newspapers	*The Times of India*	*The Hindustan Times*	*The Pioneer*	*The Statesman*	*Dainik Jagran*
Columns	3	4	2	4	2
Cost/column	20	15	35	20	10
Rows	400	350	400	600	200
Cost/row	0.5	0.8	0.6	0.3	0.5
Plate	1000	800	750	800	600
Set-up cost	200	100	150	300	150
Labour cost per print	1.5	2	1.2	0.9	1

Q.33. If the number of prints of each newspaper (in thousands) is equal, which newspaper is more costly?

(a) *The Times of India*
(b) *Dainik Jagran*
(c) *The Hindustan Times*
(d) *The Pioneer*

Q.34. If one has to print hundred prints of *The Pioneer*, then what is the ratio of the variable cost to the fixed cost (approx)?

(a) 20 (b) 30
(c) 40 (d) 35

Q.35. Which is the costliest newspaper in terms of cost per ten prints?

(a) *The Times of India*
(b) *The Hindustan Times*
(c) *The Pioneer*
(d) None of these

Q.36. How many copies of the *Dainik Jagran* should be printed so that the total cost is equal to the cost of 100 prints in *The Times of India* (approx)?

(a) 150 (b) 175
(c) 225 (d) 200

Q.37. Which newspaper has the highest fixed cost?

(a) *The Pioneer*
(b) *The Statesman*
(c) *The Hindustan Times*
(d) None of these

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Directions for questions 38–40: Read the passage below and solve the questions based on it.

The following table shows the relationship between production volumes, unit price and unit cost of a particular product. Refer to the following table to answer the questions that follow:

Contribution = volume x (price – cost)

Production Volumes (No. of Units)	*Unit Price (Rs)*	*Unit Cost (Rs)*
1000	140	100
2000	125	95
3000	110	90
4000	100	80
5000	90	75
6000	75	70
7000	65	60

Q.38. At what volume will the contribution be maximum?
(a) 2,000 units
(b) 3,000 units
(c) 4,000 units
(d) 5,000 units

Q.39. If the unit cost goes down by Rs 10 at all levels of production, what will be the maximum possible contribution (Rs)?
(a) Rs 75,000 (b) Rs 1,25,000
(c) Rs 1,20,000 (d) Rs 80,000

Q.40. A breakeven point is the volume of production at which for the first time the contribution equals fixed costs. If the fixed costs are Rs 60,000, then the production level at which the break even is reached, is
(a) 2,000 units
(b) 4,000 units
(c) 5,000 units
(d) 6,000 units

ANSWERS

WARM UP

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(c)	(d)	(d)	(a)	(b)	(b)	(c)	(c)	(d)	(c)	(d)	(d)	(c)	(b)	(d)	(b)	(b)	(b)

FOUNDATION

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(b)	(d)	(a)	(b)	(a)	(d)	(d)	(a)	(b)	(b)	(b)	(a)	(a)	(b)	(b)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(c)	(b)	(b)	(b)	(a)	(c)	(b)	(d)	(b)	(c)	(b)	(c)	(d)	(c)	(d)	(a)	(a)	(a)	(a)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(a)	(d)	(b)	(c)	(d)	(b)	(c)	(d)	(a)	(b)										

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MODERATE

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(c)	(d)	(d)	(b)	(b)	(d)	(d)	(d)	(a)	(b)	(b)	(c)	(d)	(d)	(b)	(d)	(a)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(a)	(c)	(a)	(c)	(b)	(d)	(b)	(c)	(d)	(d)	(b)	(a)	(c)	(d)	(a)	(a)	(d)	(d)	(c)
Q. No.	41	42	43	44	45															
Answer	(b)	(d)	(d)	(c)	(a)															

ADVANCED

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(c)	(b)	(c)	(c)	(c)	(a)	(a)	(d)	(b)	(d)	(d)	(b)	(b)	(d)	(b)	(b)	(a)	(c)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(a)	(a)	(a)	(c)	(d)	(a)	(d)	(a)	(d)	(c)	(c)	(b)	(d)	(b)	(b)	(d)	(c)	(b)	(a)

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5

AVERAGES AND THEIR APPLICATIONS

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Average and its various interpretations
- Definitions and properties
- Applications of average in different cases such as alligation
- Different cases of mixtures and alligation
- Methods of solving questions

CAT 2007

Number of questions from this topic: 3 (12 marks)

CAT 2006

Number of questions from this topic: 0

CAT 2005

Number of questions from this topic: 0

Introduction to the topic

Averages and their applications was once an important topic for CAT, but the number of questions asked from this topic has been reduced in the last couple of years. However, this concept is used vigorously in other exams. As a concept, average should be seen more as a logical concept of group formation than a pure mathematical concept. And this is true about its applications also.

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AVERAGE

Traditionally, average is calculated by dividing the sum of all the numbers by the number of numbers.

$$\text{Average} = \frac{\text{Sum of numbers}}{\text{Number of numbers}}$$

For example, the average of the four numbers 214, 215, 219, 224 will be

$$\text{Average} = \frac{214+215+219+224}{4} = 218$$

Central Value Meaning of Average

Average can also be seen as the central value of all the given values.

Applying this definition for the above example let us assume the central value of all the given numbers = 214

Now, find the deviations of all the numbers from 214

214 215 219 224

When assumed central value is (214), the sum of the deviations

$= 0 + 1 + 5 + 10$

Now, finding the average of deviations gives us

$$\frac{0+1+5+10}{4} = \frac{16}{4} = 4$$

So, average = assumed central value + average of deviations = 214 + 4 = 218

Thus, we can assume any value to be the assumed average and then find the average of all the deviations; and when we add all the numbers and divide it by number of numbers, 0 is assumed to be the central value.

Example 1 Average age of A, B and C is 84 years. When D joins them the average age of A, B, C and D becomes 80 years. A new person E, whose age is 4 years more than D, replaces A and the average age of B, C, D and E becomes 78 years. What is the age of A?

Solution Since the average age of A, B and C is 84 years so, we can assume that age of A, B and C is 84 years.

A = 84 years

B = 84 years

C = 84 years

After D has joined them,

	Initially	*Finally*
A	84 years	80 years
B	84 years	80 years
C	84 years	80 years
D	.....	80 years

Decrease in the age of A, B and C can be attributed to the increase in the age of D. So, after getting 12 years in total (4 years each from A, B and C) D is at 80 years. The original age of D = 80 – 12 = 68 years.

So, age of E = 72 years

Now, the average age of A, B, C and D = 80 years; A + B + C + D = 320

And average of B, C, D and E = 78 years;

B + C + D + E = 312

(Since the average difference between the age of A and E is 2 years,)

Difference (A – E) = 2 × 4 = 8 years

Since E = 72 years, so A = 80 years

By using central value method of averages every question of average can be done by mental calculation only.

Example 2 Average of ten two digit numbers is S. However when we reverse one of the numbers AB as BA from the given 10 numbers, then the average becomes S + 1.8. What is the value of B – A?

Solution Average of 10 numbers is increasing by 1.8, so it can be assumed that 1.8 has been added to all the numbers.

So, BA is 1.8 × 10 = 18 more than AB.

There are so many two-digit numbers which satisfy above condition. Using hit and trial method, the number can be 13, 24, 35, 46, 57, 68, 79. In every case, difference between the digits = 2

Otherwise, we can use the formula

(BA – AB) = 9 × (B – A)

Where BA and AB are two-digit numbers.

Example 3 The average score of Rahul Dravid after 25 innings is 46 runs per innings. If after the 26th innings, his average runs increased by 2 runs, then what is his score in the 26th inning?

Solution Runs in 26th inning = Total runs after 26th innings – Total runs after 25th innings

$= 26 \times 48 - 25 \times 46 = 98$

Alternatively, this question can be done by the above given central value meaning of average. Since the average increases by 2 runs per innings, we can assume that 2 runs have been added to his score in each of the first 25 innings. Now, the total runs added in these innings have been contributed by the runs scored in the 26th inning, which must be equal to 25 × 2 = 50 runs.

And after contributing 50 runs, his score in the 26th inning is 48 runs.

Hence, runs scored in the 26th inning = new average + old innings × change in average

$= 48 + 25 \times 2 = 98.$

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To have a mental mapping, we can see the whole situation as:

Number of innings	*Average in the 1st 25 innings*	*Averate in the 1st 26 innings*	*Addition*
1	46	48	2
2	46	48	2
3	46	48	2
...	...	...	...
...	...	...	...
...	...	...	...
25	46	48	2
26		48	

Properties of Average

1. Average always lies in between the maximum and the minimum value. It can be equal to the maximum or minimum value if all the numbers are equal.
   For example—A1, A2, A3 and A4 are four numbers given where A1>A2>A3>A4.
   Average of these four numbers will always lie in between A1 and A4.
   However, if all the four numbers are equal (A1 = A2 = A3 = A4) then the average will be equal to each of these numbers.
   Average = A1 = A2 = A3 = A4.
2. Average is the resultant of net surplus and net deficit, as used in the central tendency method.
3. When weights of different quantities are same, then simple method is used to find the average. However, when different weights of different quantities are taken, then it is known as weighted average. Here the method of weighted average is used to find the average.
   For example, assume per capita income of India is USD 500 and per capita income of US is USD 200. Now if we merge India and US into one country then it is observed that per capita income of this new country will not be equal to $\frac{500+200}{2}$ = USD 350
4. If the value of each quantity is increased or decreased by the same value S, then the average will also increase or decrease respectively by S.
5. If the value of each quantity is multiplied by the same value S, then the average will also be multiplied by S.
6. If the value of each quantity is divided by the same value S ($S \neq 0$) then the average will also be divided by S.

Example 4 The average of 4 positive numbers is A and the average of all the possible triples formed out of these four positive numbers is B. Which of the following is true regarding A and B?

(a) A = B (b) A > B
(c) A < B (d) Cannot be determined

Solution Let us assume that the numbers are 1, 2, 3 and 4

Average of 1, 2, 3 and 4 : 1 + 2 + 3 + 4 = 10/4 = 2.5

The triplets are 1, 2 and 3; and the average = $\frac{6}{3}$

1, 2 and 4; and the average = $\frac{7}{3}$

1, 3 and 4; and the average = $\frac{8}{3}$

2, 3 and 4; and the average = $\frac{9}{3}$

Average of these four averages = $\frac{\frac{6}{3}+\frac{7}{3}+\frac{8}{3}+\frac{9}{3}}{4} = 2.5$

So, option a (A = B) is the answer.

Central value method

It should be observed here that when we find the average of all the possible triplets, all the numbers (1, 2, 3, 4) are added thrice. So effectively we are adding 12 numbers. Hence, the average should be equal to:

$$\frac{3(1+2+3+4)}{12} = 2.5$$

Thus, the average will be equal in all the cases.

Extension to this problem The average of four positive numbers is A and the average of all the possible pairs formed out of these four positive numbers is B. Which of the following is true?

(a) A = B (b) A > B
(c) A < B (d) Cannot be determined

Some Special Cases

1. Average involving time, speed and distance

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

However, while solving the questions involving time, speed and distance, we should assume some distance, preferably the LCM of all the given speeds.

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Example 5 Lovely goes to Patna from New Delhi at a speed of 40 km/h and returns with a speed of 60 km/h. What is her average speed during the whole journey?

Solution Assuming that the total distance between Patna and New Delhi is 120 km (LCM of 40 and 60) the total time taken (Patna – New Delhi and New Delhi – Patna) = 3 + 2 = 5 h

So, average Speed = $\frac{240}{5}$ = 48 km/h

2. Average involving age

Average of a group of n persons given at any point of time can be calculated in the following way

5 years ago	*10 years ago*	*Now*	*10 years later*	*5 years later*
N – 5	N – 10	N	N + 10	N + 5

Example 6 The average age of the 5 members of a family is 20 years. The youngest member of the family is 4 years old. At the time of his birth the average age of the rest of the family was N years. What is the average age of the family (in terms of N) excluding the youngest member?

Solution Sum of ages of all the members of the family = 100

Sum of ages of all the members of the family excluding the youngest number = 100 – 4 = 96

So, average age of all the members of the family excluding the youngest number = 96/4 = 24 = N

What is the average age of the family(in terms of N) excluding the youngest member = N + 4

3. Average involving number system

1. Average of 1st n consecutive natural numbers $= \frac{n+1}{2}$
   For example, the average of 1st five natural numbers = 3
2. The average of 1st n consecutive even natural numbers = $n + 1$
   Sum of 1st n consecutive even natural numbers $= n(n + 1)$
   For example, the average of 1st five even natural numbers = 6
3. The average of 1st n consecutive odd natural numbers = n
   Sum of 1st n consecutive odd natural numbers = n^2
   For example, the average of 1st five odd natural numbers = 5

Weighted Average

It is observed that the average can be calculated only if the weights of all the factors, is same. So, the weighted average is a more generalized form of average. This can be further understood with the following illustration

	Class A	*Class B*
No. of students	10	10
Average age	12 years	16 years

Now, if we combine both these classes, then the average age of all the students = $\frac{12+16}{2} = \frac{28}{2} = 14$ years.

This is one standard example of Average.

Let us see another example:

	Class A	*Class B*
No. of students	12	16
Average age	10 years	14 years

Now, if we combine these two classes, then the average can not be calculated by the above mentioned method, since the weights attached to different averages are different.

Some more cases of weighted average

1. As we have observed above in the case of average, if per capita income of India is USD 500 and per capita income of US is USD 200, and if we merge India and US into one country, then the per capita income of this new country (India + US) cannot be found by just adding the per capita income of both the countries and dividing it by 2.
   The weights, i.e., the population attached to the different averages, i.e., the per capita income would also have to the considered.
2. Average speed cannot be calculated by just adding the different speeds and then dividing it by 2. This can be understood by the following example:
   A person goes to A from B at a speed of 40 km/h and returns with a speed of 60 km/h, then the average speed for the whole journey can not be equal to 50 km/h.

   We know that average speed = $\frac{\text{Total distance}}{\text{Total time}}$

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Finding expression for weighted average

Let us assume there are N groups with the following structure:

Group no.	*No. of members*	*Avg. age of the group*
G_1	N_1	A_1
G_2	N_2	A_2
G_3	N_3	A_3
...	...	...
...	...	...
G_N	N_N	A_N

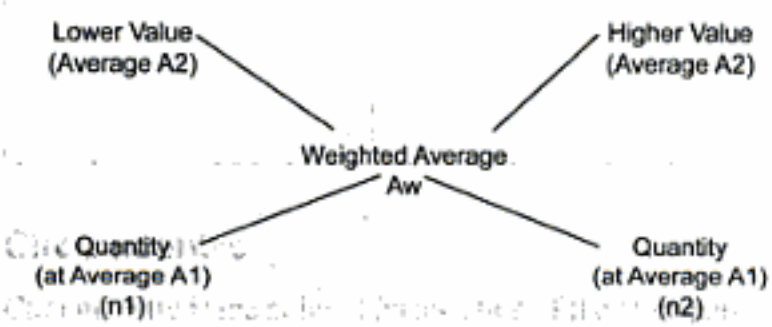
If we combine all these groups, then the average age of all the members = $(N_1 \times A_1 + N_2 \times A_2 \ldots + N_N \times A_N)/(N_1 + N_2 + N_3 + \ldots + N_N)$

$$= \frac{\sum AN}{\sum N}$$

Group no.	*No. of members*	*Avg. age of the group*
G_1	N_1	A_1
G_2	N_2	A_2

Considering that there are only two groups and both the groups are combined then the average age of all the members = $(N_1 \times A_1 + N_2 \times A_2)/(N_1 + N_2) = A_w$

Simplifying the above written expression, we get the conventional criss-cross method as given below



And we write this as: $\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$

i.e.

$$\frac{\text{Quantity (Lower Priced)}}{\text{Quantity (Higher Priced)}} = \frac{\text{Higher Price} - \text{Averge Price}}{\text{Average Price} - \text{Lower Price}}$$

It is quite obvious that the ratio of the number of persons/items in different groups is proportionate to the deviations of their average from the average of all the people combined.

This average of all the members combined is known as weighted average, and is denoted by A_w. This process of mixing the two groups is also referred as alligation.

Elements of weighted average

As we can see from the above derivation there are five quantities

i. Number of members in 1st group (n_1)
ii. Number of members in 2nd group (n_2)
iii. Average of 1st group (A_1)
iv. Average of 2nd group (A_2)
v. Weighted average (A_w)

Normally, in the case of weighted average, we get questions in which one of these five elements is missing, and with the help of the remaining four quantities, the value of that missing quantity is found. Different possibility (situations) are given below (Y represents – data given, N represents – data not given):

Situations	n_1	n_2	A_1	A_2	A_w
First	Y	Y	Y	Y	N
Second	Y	Y	Y	N	Y
Third	Y	Y	N	Y	Y
Fourth	Y	N	Y	Y	Y
Fifth	N	Y	Y	Y	Y
Sixth	N	N	Y	Y	Y

First Situation

Example 7 10 kg of rice priced at Rs 12 per kg is mixed with 6 kg of rice priced at Rs 16 per kg. What is the average price of the whole mixture?

Solution Lower priced value = Rs 12 per kg and its quantity = 10 kg

Higher priced value = Rs 16 per kg and its quantity = 6 kg

Using Alligation,

$$\frac{10}{6} = \frac{16 - A_w}{A_w - 12}, \text{ or, } A_w = \text{Rs } 13.5/\text{kg}$$

However, in my opinion, in this situation it is better to use the normal method rather than using the weighted average method of finding A_w

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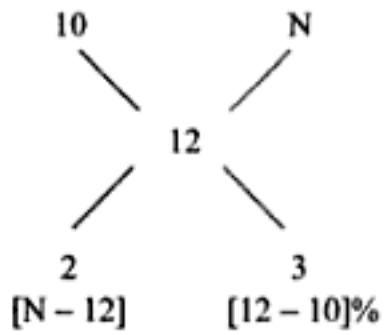
Normal method – Total value = 12 ×10 + 16 × 6 = 216

So, average price = $\frac{216}{16}$ = Rs 13.5/kg

Second/Third Situation

Example 8 Two varieties of rice are mixed in the ratio 2:3. The price of the mixture is Rs 12 per kg and the price of the variety having lower weight is Rs 10 per kg. Find the price of the other variety.

Solution

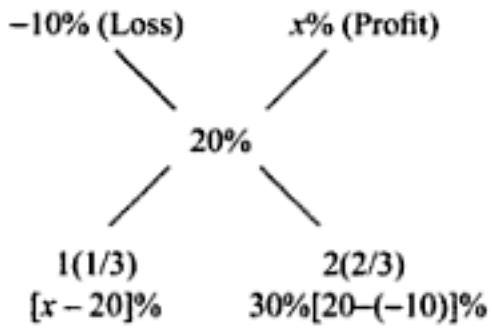


Now, 2:3 = [N – 12]/[12 – 10] = [N – 12]/2
So, N = Rs 13.33 per kg

Fourth/Fifth Situation

Example 9 Some articles are purchased for Rs 450. 1/3rd of the articles are sold at a loss of 10%. At what percentage profit should the remaining articles be sold to obtain a net profit of 20% on the whole transaction?

Solution



Now, 1:2 = [20–(–10)]/[x – 20]
So, x = 35%

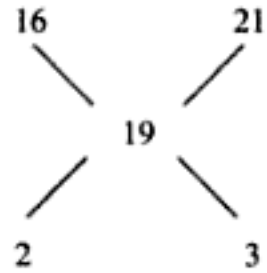
It is seen that the quantities are in the ratio of 1:2, so the deviation from mean percentage profit in the loss percentage and profit percentage will also be same.

2 corresponds to 30%, so 1 will correspond to 15%. So, x = 35%

Sixth Situation

Example 10 Two different qualities of sugar are mixed in some ratio. The price of one quality of sugar is Rs 16/kg and that of another quality is Rs 21/kg. In what ratio have the sugar of two qualities been mixed if the price of the mixture is Rs 19/kg?

Solution



So, the ratio of quantity of sugar of different qualities = 2:3

MIXTURES

When two or more than two pure substances/mixtures are mixed in a certain ratio, they create a mixture. Here we shall confine ourselves to mostly homogenous mixtures in view of the questions commonly asked in CAT.

Mixing without Replacement

In this particular type of mixing, two or more than two substances are mixed without any part of any mixture being replaced.

Example 11 In a mixture of 420 litres the ratio of milk and water is 6:1. Now, 120 litres of the water is added to the mixture. What is the ratio of milk and water in the final mixture?

Solution Volume of milk = 360 litres and volume of water = 60 litres.

When 120 litres of water is added, volume of water = 180 litres

So, the ratio of milk water = 2:1

Example 12 A milkman mixes 20 litres of water with 80 litres of milk. After selling one-fourth of this mixture, he adds water to replenish the quantity that he had sold. What is the current proportion of water to milk?

Solution Ratio of milk and water = 20:80

When 1/4th of this mixture is sold, total volume of mixture will be reduced by 25%, so 25% of milk and water both will reduce. So, volume of milk and water after selling out 1/4th of mixture = 60 litres and 15 litres respectively. Addition of 25 litres of water will finally give us the following: volume of milk = 60 litres and volume of water = 40 litres. Hence, the ratio of water and milk = 40:60 = 2:3.

Example 13 How many litres of fresh water should be mixed with 30 litres of 50% milk solution so that resultant solution is a 10% milk solution?

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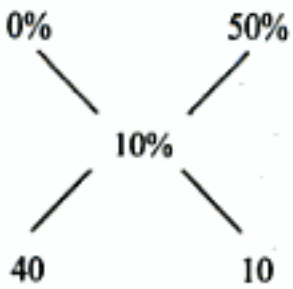
Solution

Method 1 *Using Alligation*



So, the ratio of fresh water added: milk solution = 4:1

Hence, 120 litres of fresh water should be added.

Method 2 *Principle of constant volume of one component*

Since we add fresh water, the volume of milk will be constant.

Now volume of milk = 15 litres = 10% of the new mixture.

So, 100% of the new mixture = 150 litres

So, volume of fresh water added = 150 – 30 = 120 litres.

Method 3 *Principle of inverse proportion*

We know that concentration is inversely proportional to the volume of solute added.

So, in this case $30 \times 50\% = 10\% \times (30 + x)$, where x is the volume of water added.

So, $x = 120$ litres

Method 4 *Using equation*

In the final mixture, $\frac{\text{Milk}}{\text{Total}} = 10\% = \frac{15}{30+x}$

So, $x = 120$ litres

Mixing with replacement In this particular type of mixing, two or more than two substances are mixed by replacing some part of a mixture. In these types of questions, total volume may or may not be the same and information regarding the same can be obtained from the question.

Case 1 When the quantity withdrawn and the quantity replaced are of the same volume.

Initially there are 40 litres of milk, and 4 litres of milk is replaced with 4 litres of water

Obviously, there will be 36 litres of milk and 4 litres of water.

Now, 4 litres of mixture is replaced with 4 litres of water

The quantity of milk and water being withdrawn here will be in the ratio of 9:1 (36:4). So, quantity of milk withdrawn = 9/10 × 4 = 3.6 l.

So, the volume of milk = 32.4

And the volume of water = 7.6

Now, again 4 litres of mixture is replaced with 4 litres of water

The quantity of milk and water being withdrawn here will be in the ratio of 81:19 (32.4:7.6). So, the quantity of milk withdrawn = (81/100) × 4 = 3.24 l

So, the volume of milk = 29.16

And the volume of water = 10.84

If we summarize the above values, then it looks like

	1st operation		*2nd operation*		*3rd operation*	
	Taken out	*Left*	*Taken out*	*Left*	*Taken out*	*Left*
Milk	4	36	3.6	32.4	3.24	29.16
Water	0	4	0.4	7.6	0.76	10.84

It can be seen that the quantity of water or milk withdrawn is 10% of the existing volume of milk or water because only 10% of the total volume of 40 litres taken out.

With this we can deduce a standard formula for these kinds of calculations.

If V is the initial volume of milk (or any liquid), and x litres of milk is always replaced by water, then quantity of milk left after n such operations $= V\left(1-\frac{x}{v}\right)^n$

This formula is very similar to the standard formula we have seen in the case of Compound Interest $\left[P(1+\frac{R}{100})n\right]$.

The only difference between the two formulae is that while the interest is being added every year (or for the given time-period), volume of milk gets reduced after every operation.

Using the values of the above example, quantity of milk left after 3 operations $= 40 \times \frac{36}{40} \times \frac{36}{40} \times \frac{36}{40} = 19.16$ litres

The same problem can be solved with straight-line approach of percentage also Since 10% of existing volume is taken out every time, the percentage of milk in the final mixture after the third operation = 72.9%

$$\left(100\% \xrightarrow{10\%\downarrow} 90\% \xrightarrow{10\%\downarrow} 81\% \xrightarrow{10\%\downarrow} 72.9\%\right)$$

Since 100% = 40, so 72.9% = 29.16 litres

Case 2 When the quantity withdrawn and the quantity replaced are of the same volume, but the total volume before replacement does not remain the same.

Initially there are 40 litres of milk, and 4 litres of milk is taken out and 4 litres of water is poured in

So, there will be 36 litres of milk and 4 litres of water.

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Now, 5 litres of mixture is taken out and 5 litres of water is poured in.

The quantity of milk and water being withdrawn here will be in the ratio of 36:4. So, the quantity of milk withdrawn $= \frac{36}{40} \times 5$

$$\text{Milk left} = 40 \times \frac{36}{40} \times \frac{35}{40}$$

Again, if now 6 litres of mixture is taken out and 6 litres of water is poured in

$$\text{Milk left} = 40 \times \frac{36}{40} \times \frac{35}{40} \times \frac{34}{40}$$

Case 3 When the quantity withdrawn and the quantity replaced are not of the same volume.

Initially there are 40 litres of milk, and 4 litres of milk is taken out and 5 litres of water is poured in

Obviously, there will be 36 litres of milk and 5 litres of water.

Now, 5 litres of mixture is taken out and 6 litres of water is poured in then the quantity of milk and water being withdrawn will be in the ratio of 36:5. So, the quantity of milk withdrawn $= \frac{36}{41} \times 5$

$$\text{Milk left} = 40 \times \frac{36}{40} \times \frac{36}{41}$$

Again 6 litres of mixture is taken out and 7 litres of water is poured in.

Thus, the volume of milk in the final mixture

$$= 40 \times \frac{36}{40} \times \frac{36}{41} \times \frac{36}{42}$$

Example 14 Two vessels A and B of equal capacities contain mixtures of milk and water in the ratio 4:1 and 3:1, respectively. 25% of the mixture from A is taken out and added to B. After mixing it thoroughly, an equal amount is taken out from B and added back to A. The ratio of milk to water in vessel A after the second operation is

(a) 79:21 (b) 83:17
(c) 77:23 (d) 81:19

Solution Assume there is 20 litres of the mixture in both the vessels.

In vessel A, milk = 16 litres and water = 4 litres

25% from A to B = milk in B = 15 + 4 = 19 litres

= water in B = 5 + 1 = 6 litres

ratio = 19:6

Equal amount from vessel B to vessel A

$$= \text{milk in A} = 12 + \frac{19}{5} = \frac{79}{5}$$

$$= \text{water in A} = 3 + \frac{6}{5} = \frac{21}{5}$$

Hence, the ratio is 79:21

PRACTICE EXERCISES

WARM UP

Q.1. Find the average age of a family of five members, whose ages are 42, 49, 56, 63 and 35 years respectively.

(a) 50 years (b) 49 years
(c) 45 years (d) 48 years

Q.2. Rahul Ghosh walks from A to B at 8 km/h and comes back from B to A at 12 km/h. What is his average speed for the entire journey?

(a) 8.8 km/h (b) 9.6 km/h
(c) 10.2 km/h (d) 11.4 km/h

Q.3. The average of five numbers is 39.20 and the average of three of these numbers is 41. Find the average of the remaining two numbers.

(a) 35.5 (b) 36.5 (c) 37.5 (d) 38.5

Q.4. In a class of 60 students, the average height of 30 students is x cm and that of the remaining students is y cm. Find the average height of the whole class.

(a) $(x + y)$ cm (b) 30 cm
(c) $\frac{x+y}{2}$ cm (d) None of these

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Q.5. What is the average of the first 15 multiples of 3?
(a) 21 (b) 27 (c) 24 (d) 15

Q.6. The average age of 7 members of a family is 25. If one of the members whose age is 28 years, is excluded, then others are in the ratio 1:2:3:4:5:6. Find the age of the eldest member of the family.
(a) 28 (b) 35 (c) 14 (d) 42

Q.7. The average of seven consecutive even numbers is 10. If the next three even numbers are included, then what is the new average?
(a) 10 (b) 12 (c) 13 (d) 14

Q.8. Average weight of three students is 40 kg. If the first student is twice the second and the second student is thrice the weight of the third student, then what is the weight of the student with lightest weight?
(a) 18 (b) 20
(c) 14 (d) None of these

Q.9. In a test, the average marks of five students was found to be 50. However during cross checking of the papers, it was discovered that the mark of one student was misread as 84 instead of 48. Find the correct average.
(a) 57.2 (b) 42.8 (c) 46.5 (d) 43.2

Q.10. The average weight of seven members of a family is 18 kg. If the head of the family is not considered, the average weight of the others would be 5 kg less. The weight of the head of the family is
(a) 42 kg (b) 48 kg
(c) 52 kg (d) 36 kg

Q.11. In a prep school, the average weight of 30 girls in a class of 50 students is 16 kg and that of the remaining students is 15.5 kg. What is the average weight of the all the students in the class?
(a) 15.2 kg (b) 15.8 kg
(c) 15.4 kg (d) None of these

Q.12. The average of the amount of money that 10 girls have is Rs 45. When two more girls join, the average increases by Rs 2. Find the average amount of the two new girls.
(a) Rs 57 (b) Rs 54
(c) Rs 48 (d) None of these

Q.13. In what ratio must 35% spirit solution be mixed with pure spirit to get a resultant solution of 56% spirit?
(a) 35:44 (b) 44:21 (c) 8:3 (d) 56:9

Q.14. A beggar has 50 coins with an amount of Rs 15 in the denominations of 25 paise and 50 paise. How many 25 paise coins are there with the beggar?
(a) 36 (b) 40 (c) 30 (d) 25

Q.15. Two litres of pure spirit is added to 6 litres of a spirit solution containing $16\frac{2}{3}\%$ spirit. What is the concentration of the spirit in the resultant solution?
(a) 12.5% (b) 25%
(c) 37.5% (d) None of these

Q.16. Three litres of pure milk is added to 10 litres of a milk solution containing 9% milk. Find the concentration of the resultant solution.
(a) $33\frac{1}{3}\%$ (b) 40%
(c) 50% (d) 30%

Q.17. In what ratio must a quality of sugar @ Rs 15.50/kg be mixed with another quality of sugar @ Rs 21.80/kg so that selling the mixture at Rs 20.02/kg results in 10% profit?
(a) 5:4 (b) 3:2 (c) 2:1 (d) 4:3

Q.18. Parul Jaiswal has Rs 11,200 in Rs 100 and Rs 500 denomination. What is the ratio of the number of notes, of the two types, if she has a total of 40 notes with her?
(a) 3:2 (b) 7:3 (c) 8:7 (d) 11:9

Q.19. Ordinarily sugar contains 32% impurities, while processed sugar has 18% impurities. How many kilograms of ordinary sugar must be processed to yield 102 kg of processed sugar?
(a) 185 (b) 246
(c) 123 (d) None of these

Q.20. A shrewd milkman mixes water and milk in the ratio of 2:3. What part of this mixture should be removed and replaced with water so that the solution contains water and milk in the ratio 1:1?
(a) 1/6 (b) 1/4 (c) 1/3 (d) 5/6

Q.21. A milkman sells his mixture of milk and water at the cost price of milk itself and thus he gains a profit of 20%. What is the ratio of milk and water in the mixture?
(a) 4:1 (b) 5:1
(c) 1:4 (d) None of these

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FOUNDATION

Q.1. Neo is going to NDLS from PNBE by his car at a speed of 40 km/h. While coming back, he returns with a speed of x km/h. What should be the value of x so that his average speed during the intre journey is 80 km/h?
(a) 160 km/h (b) 40 km/h
(c) 120 km/h (d) It is not possible

Q.2. There are 125 middle level employees in Due North Inc. The average monthly salary of the middle level employees is Rs 5,500 and that of the senior level employees is Rs 14,000. If the average monthly salary of all these employees is Rs 8,687.5, find the total number of employees in the company, if middle and senior level employees of Due North Inc. form 80% of their total employees.
(a) 175 (b) 200 (c) 220 (d) 250

Q.3. I went to a hotel along with 12 friends. I paid Rs 145 and all the others paid an equal amount. In the end, we did some calculations and found that the average sum paid by all of us was Rs 5 more than what was originally paid by each of my friends. How much money did each of my friends pay?
(a) Rs 120 (b) Rs 100
(c) Rs 80 (d) Rs 70

Q.4. Saketprakash married ten years ago at the age of 27 years. His wife was 23 years old then. Six years after their marriage, the average age of Saketprakash, his wife and their son was 22 years. After how many years of Saketprakash's marriage was his son born?
(a) 6 years (b) 3 years
(c) 2 years (d) 4 years

Q.5. Mr Anant Roy, the renowned author, recently got his new novel released. To his utter dismay he found that for the 1,007 pages on an average there were 2 mistakes every page. While, in the first 612 pages there were only 434 mistakes, they seemed to increase for the latter pages. Find the average number of mistakes per page for the remaining pages.
(a) 6 (b) 4
(c) 2 (d) None of these

Q.6. The class X of Nav Yug Vidhyalaya has four sections— A, B, C and D. The average weight of the students of A, B, C together and A, C, D together are 45 kg and 55 kg respectively, while the average weight of the students of A, B, D together and B, C, D together are 50 kg and 60 kg respectively. Which of the following could be the average weight of the students of all the four sections together?
(a) 47.6 kg (b) 52.5 kg
(c) 53.7 kg (d) 56.5 kg

Q.7. The average score of a cricketer in a certain number of innings is 44. He then played another eight innings in which he got out at 97, 3, 23, 0, 68, 40, 50 and 71 runs respectively. Then, which of the following statements is true regarding his new average score in all the innings together?
(a) The new average is more than the old average
(b) The new average is less than the old average
(c) The new average is the same as the old average
(d) Cannot conclude any of the above unless we know the number of innings played

Q.8. Average age of A, B and C is 84 years however when D joins them, then the average comes down to 80. Now, a new person E whose age is 3 years more than D replaces A and the new average is 79 years. What is the age of A?
(a) 75 (b) 79 (c) 81 (d) 77

Directions for questions 9–10: Read the passage below and solve the questions based on it.

The average age of the students in a class is 35 years.

Q.9. If a student whose age was 25 is absent from the class, the average of those present goes up by 1. How many students were there in the class originally?
(a) 9 (b) 10 (c) 11 (d) 12

Q.10. If 2 students whose ages were 30 years and 20 years left, by what value would the average of the remaining increase? (use the result from question 9.)
(a) $\frac{20}{9}$ (b) 2
(c) $\frac{20}{11}$ (d) None of these

Q.11. If the average of m numbers is a, and on adding x to the m numbers, the average of the $1 + m$ numbers is b. Then find the value of x.
(a) $m(b - a) + b$ (b) $m(b + a) + a$
(c) $m(a - b) + a$ (d) None of these

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Q.12. Brian Lara, the famous batsman, scored 6,000 runs in certain number of innings. In the next five innings he was out of form and hence, could make only a total of 90 runs, as a result of which his average fell by 2 runs. How many innings did he play in all, if he gets out in all the innings?
(a) 105 (b) 95 (c) 115 (d) 104

Q.13. The average weight of A, B and C is x kg. A and C lose y kg each after dieting and B gains $\frac{y}{2}$ kg. After this their average weight decreases by 1 kg. Find y.
(a) 1 kg (b) 2 kg
(c) 3 kg (d) Cannot be determined

Q.14. One out of the five weights A, B, C, D and E is of a different weight. A person makes a measurement and finds the following: A + B = C + D + E
Which of the following should be known to find out which one is of a different weight?
(a) A + C > B + D + E
(b) A + D + E < B + C
(c) A + B + C > D + E
(d) A + C < B + D + E

Q.15. In a class, with a certain number of students, if another student weighing 50 kg joins, then the average weight of the class increases by 1. If one more student weighing 50 kg joins then the average weight of the class increases by 1.5 over the original average. What is the original average weight of the class?
(a) 46 (b) 4 (c) 2 (d) 47

Q.16. The average weight of the books carried by Kuku to school is 1.5 kg and 2.5 kg is the average weight of the exercise books. If Kuku is carrying only text books and exercise books in the ratio of 3:4, what is the total weight of his school bag? (Ignore any other weights such as that of school bag, stationary etc.)
(a) 10.5 kg (b) 1 kg
(c) 6 kg (d) Data insufficient

Q.17. The average of n numbers is 32. If 3/4 of the numbers are increased by 4 each and the remaining are decreased by 6 each, then what is the new average?
(a) 32 (b) 32.5 (c) 33.5 (d) 34.5

Q.18. The average of eight numbers is 25, that of the first two is 20 and of the next three is 26. The sixth number is less than the seventh by 4, and less than the eighth by 6. The last number is
(a) 30 (b) 32 (c) 40 (d) 36

Q.19. The average amount of sales per day in a shop for 5 days starting from Monday is Rs 1,800. The average amount of sales per day for 5 days from Tuesday is Rs 1,890. By how much does the sale on Saturday exceed that on Monday?
(a) Rs 450 (b) Rs 500
(c) Rs 600 (d) Rs 650

Q.20. Three years ago, the average age of a family of 5 members was 17 years. Inspite of the birth of a child in the family, the present average age of the family remains the same. The present age of the child is
(a) 3 years (b) 1 year
(c) 2 years (d) 1.5 years

Q.21. A batsman has a certain average of runs for 16 innings. In the 17th inning, he makes a score of 85 runs, thereby increasing his average by 3 runs. What is the average after the 17th inning?
(a) 58 (b) 37
(c) 35 (d) None of these

Q.22. If the average marks of 'a' students in a class is 'c' and that of the remaining 'b' students is 'd', what is the average marks of the class?
(a) $\frac{ac+bd}{b+d}$ (b) $\frac{ab+cd}{a+d}$
(c) $\frac{ac+bd}{a+b}$ (d) $\frac{ad+cd}{b+d}$

Q.23. The average marks of a class of 48 students is 35. Of them, two score zero, of the rest, the first 30 scored an average of 40, the next fourteen scored an average of 20. If the remaining two scored equal marks, what are their individual marks?
(a) 80 (b) 75 (c) 90 (d) 100

Q.24. The total salary of A, B and C is Rs 444. If they spend 80%, 85%, 75% of their salaries respectively, their savings are as 7:6:9. Then salary of B is:
(a) Rs 140 (b) Rs 160
(c) Rs 144 (d) None of these

Q.25. The average age of 2/3rd of the class is 17. What should be the average the age of the remaining 1/3rd students so that the average age of the entire class is 20?
(a) 21 (b) 22 (c) 24 (d) 26

Q.26. Sidharth Tripathi travels from Delhi to Lucknow at a speed of 75 km/h and returns to Delhi at a speed of 50 km/h. He again leaves for Lucknow immediately at a speed of 30 km/h and returns back to Delhi at a speed of 60 km/h. What is his average speed for the entire journey?

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(a) 54 km/h (b) 48 km/h
(c) 56 km/h (d) Cannot be determined

Q.27. Average marks of a class of n students is 64. When eight new students with an average marks of 73 join the class, the new average of the entire class is a whole number. Find the number of students in the class given that n lies between 25 to 60.
(a) 44 (b) 32 (c) 36 (d) 72

Q.28. Average of 5 consecutive integers is N. What will be the new average when next two integers are also included?
(a) N + 2
(b) N + 2/7
(c) N + 1
(d) N + K / 7, where K depends upon the values taken.

Q.29. The average of 10 two-digit positive integers is Z. However, one number AB is taken as BA, then the average increases to Z + 2.7. What is the value of |B – A|?
(a) 1 (b) 2 (c) 3 (d) 4

Q.30. In the above question, how many set of values for A and B are possible?
(a) 5 (b) 6 (c) 7 (d) 8

Q.31. In a country, the average salary per month is calculated on the basis of 14 months and average expenses is calculated on the basis of 10 months. What will happen to the average savings per month with respect to actual savings (calculated on the basis of 12 months)?
(a) Decrease (b) Increase
(c) No change (d) Cannot be determined

Q.32. In the above question, what will be the net percentage change in the savings with respect to the actual savings?
(a) 11.11% (b) 22.22%
(c) 33.33% (d) Cannot be determined

Q.33. The average rainfall for the first 3 days out of five days was recorded to be 0.45 inches. The rainfall on the last two days was in the ratio 2:3. The average of five days was 0.40 inches. What was the rainfall on the last day?
(a) 0.45 inches (b) 0.39 inches
(c) 0.35 inches (d) 0.30 inches

Q.34. The average height of the girls of a class is 155 cm and the average height of the boys of the class is 4 cm more than the average of the class. If the number of girls is 205 less than the number of boys, what is the average height of the boys?
(a) 140 cm (b) 162 cm
(c) 152 cm (d) None of these

Q.35. The average price of a share is the average of 5 readings taken at regular intervals in a day. The index price is taken by a weighted arithmetic average price of a class A and class B stock. The respective weights are 1.1 and 0.9 for the two kinds of stocks. If the five readings of a class A stock were 19, 26, 31, 35, 39 and for a class B stock the readings were 7, 8, 17, 20, 23 then what was the index price that day?
(a) 46.5 (b) 25 (c) 23.25 (d) 45

Q.36. Ankit travels from New Delhi to Patna. He stops at equidistant places, i.e., Lucknow and Mirzapur during his journey. He covers the first 1/3rd distance at a speed of 50 km/h, the 2nd 1/3rd distance at a speed of 40 km/h and the last 1/3rd distance at a speed of 60 km/h. However, while returning, his uniform speed is 75 km/h and he stops only at Jaunpur, which is exactly at the mid-point between New Delhi and Patna. What is the average speed (in km/h) of Ankit during the whole journey, not considering his halt time?
(a) 59.8 (b) 59.01
(c) 60.8 (d) None of these

Q.37. The average age of 24 students and the class teacher is 16 years. If the age of the class teacher is excluded the average reduces by 1 year. What is the age of the class teacher?
(a) 38 (b) 39
(c) 40 (d) None of these

Q.38. First ten multiples of 1, 2,... 10 are taken. What is the average of all these 100 numbers?
(a) 25.5 (b) 50.5
(c) 75.5 (d) None of these

Q.39. There are 20 students in Mr. Rahul Ghosh's class. He conducts an examination out of 100 and then arranged the marks in an ascending order. He found Chandan, the topper of the class, had slipped to the tenth position. When he was adding the scores of the last 11 students the average was 64 and that of the top 10 was 67. If the average marks obtained by all the students of his class was 65, how many marks did Chandan score?
(a) 64 (b) 67
(c) 74 (d) Cannot be determined

Q.40. The average of 38, 84, 63, 45, 32 and n lies between 50 and 60. If n is an integer greater than the average of the above numbers, then n lies between
(a) $53 < n < 97$ (b) $50 < n < 100$
(c) $52 < n < 98$ (d) None of these

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Q.41. Milawati Sharma, the local grocer mixed three qualities of tea T_1, T_2 and T_3 priced at Rs 74 per kg, Rs 68 per kg and Rs 63 per kg in the ratio of 1:2:4. He added some more quantity of T_1 to 4 kg of this mixture. He sold this new mixture for Rs 84 per kg, thereby making a profit of 20%. How much of T_1 did he mix with the mixture?
(a) 4 kg (b) 6 kg
(c) 8 kg (d) None of these

Q.42. 60 litres of a 75% solution of wine is taken into a laboratory. 4 litres of the solution is flushed and 4 litres of water is added to compensate the loss. Then 5 litres of the solution is flushed again and 5 litres of water is added. Again 6 litres of the solution is flushed and 6 litres of water is added. The final concentration of wine in the solution is
(a) 255% (b) 57.75%
(c) 60.25% (d) 59%

Q.43. A and B are two alloys of gold and copper prepared by mixing the respective metals in the proportions of 7:2 and 7:11 respectively. If the alloys are melted and mixed to form a third alloy C, which has an equal proportion of gold and copper. What is the ratio of alloys A and B in the mixture?
(a) 2:5 (b) 5:2 (c) 11:2 (d) 1:2

Q.44. A vessel contains a mixture of two liquids A and B in the ratio of 4:1. When 10 litres of the mixture is replaced with liquid B, the ratio becomes 2:3. How many litres of liquid A was present in the jar initially?
(a) 12 litres (b) 14 litres
(c) 16 litres (d) 10 litres

Q.45. From a vessel containing pure milk, 9 litres is taken out. It is replaced with an equal quantity of water. This operation is repeated twice. The ratio of milk and water now in the vessel is 16:9. What was the volume of milk in the vessel initially?
(a) 54 litres (b) 36 litres
(c) 42 litres (d) 45 litres

Q.46. There are only deers and ducks in a zoo. When heads are counted, it is 204 and when the legs are counted, it is 512. How many deers are there in the zoo?
(a) 52 (b) 152 (c) 176 (d) 26

Q.47. Sharat invests a total of Rs 10,000 in two parts, some money @ 6% per annum and rest of the money @ 8% per annum of SI. After two years, he got a total of Rs 1,440 as interest. What is the amount put at 6% per annum interest?
(a) Rs 4,000 (b) Rs 6,000
(c) Rs 3,000 (d) Rs 7,000

Q.48. A vessel contains a mixture of two liquids A and B in the ratio of 7:5. When 9 litres of the mixture is taken out and the vessel is filled with B, the ratio of A and B is now 7:9. How many litres of liquid A was there in the vessel initially?
(a) 24 litres (b) 21 litres
(c) 18 litres (d) 27 litres

Q.49. A milkman dilutes 36 litres of pure milk with water. The percentage of milk in the solution is now 75%. How many litres of water did he add?
(a) 6 litres (b) 8 litres
(c) 12 litres (d) 16 litres

Q.50. Doodhimal, the local milkman was notorious for the quality of milk that he used to sell. Initially he had 60 liters of milk in his can. He removed one-third of it and replaced that with water. He then removed one-third of the mixture and again added water to it. How many times should this process be repeated if Doodhimal wants the concentration of milk to be just below 20%?
(a) 3 (b) 4 (c) 5 (d) 7

MODERATE

Q.1. The average marks of the girls in a class is equal to the number of boys and the average marks of boys is equal to the number of girls. If the class average is 4 less than the average of both the boys' and the girls' average marks, what will be the number of students in the class?
(a) 24 (b) 48
(c) 50 (d) 64

Q.2. In the above question, what can be the minimum number of students in the class?
(a) 0 (b) 2
(c) 8 (d) None of these

Q.3. A, B, C, D and E are five consecutive integers and the average of these five numbers is less than $\frac{1}{4}$th of A. Then A is

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(a) Positive (b) Negative
(c) Even (d) Data Sufficient

Q.4. Find the difference between the average of the first n even numbers and the average of all the even numbers up to n (n is an even number).

(a) $\frac{n-1}{2}$ (b) $\frac{n+1}{2}$
(c) $\frac{n}{2}$ (d) None of these

Q.5. I was born 30 years after my father was born. My sister was born 25 years after my mother was born. The average age of my family is 26.25 years right now. My sister will get married 4 years from now and will leave the family. Then the average age of the family will be $\frac{107}{3}$ years. What is the age of my father?
(a) 30 year (b) 35 year
(c) 40 year (d) 45 years

Directions for questions 6–9: Read the passage below and solve the questions based on it.

India is supposed to send its boxing team for Olympics in each of the following 10 weight group divisions.

A (48 kg - 52 kg) B (52 kg - 56 kg)
C (56 kg - 60 kg) D (60 kg - 64 kg)
E (64 kg - 68 kg) F (68 kg - 72 kg)
G (72 kg - 76 kg) H (76 kg - 80 kg)
I (80 kg - 84 kg) J (84 kg - 88 kg)

After selecting one player from each group, their average weight comes out to be 68 kg. If one of the players named X leaves the team, their average weight comes down to 66.5 kg.

Q.6. Player X is from group
(a) A (b) E
(c) I (d) None of these

Q.7. If X leaves the team, and two new players join the group, then their average weight increases to 68 kg. These players can be from group....
(a) A, C (b) D, J
(c) Both from G (d) Either 2 or 3

Q.8. What is the average weight (in kg) of all the players taken together?
(a) 56.7 (b) 58.8
(c) 61.4 (d) Cannot be determined

Q.9. In the average of all the groups together, which group contributes most in overall average?
(a) F (b) G
(c) H (d) Cannot be determined

Q.10. A shipping clerk has to weigh 6 distinct packets. He weighs them four at a time, weighing all the possible combinations of the packets from the six. The average weight of all the weighing combinations is found to be 500 gm. What is the combined weight of all the six packets?
(a) 375 gm (b) 1250 gm
(c) 750 gm (d) 1000/3 gm

Directions for questions 11–21: Read the passage below and solve the questions based on it.

There are three sections in class X at A. R. Academy. Below is the table indicating the number of students, and the range of marks obtained by the students of that particular section.

	Section A	*Section B*	*Section C*
No. of students	25	30	20
Minimum marks	22	24	20
Maximum marks	30	28	26

The minimum and the maximum marks provides us with only the range of marks which can be obtained by the students of that particular section.

Q.11. What is the average marks obtained by all the students of class X together?
(a) 24.25 (b) 26.33
(c) 26.75 (d) Cannot be determined

Q.12. If we shift 5 students from section A to section B and then 'x' students from section B to section C, then what should be the value of 'x' so that the minimum possible average of section B and the maximum possible average of section C are the same?
(a) 6 (b) 7
(c) 9 (d) Not possible

Q.13. If we combine section A and section C, then what will be the ratio of the maximum possible average of this combined section and the average of section B?
(a) 2:1 (b) 3:2
(c) 1:2 (d) Cannot be determined

Q.14. What is the maximum number of students that can be transferred from section A to section C, so

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that the maximum possible average of section A is still more than the maximum possible average of section C?
(a) 20 (b) 22
(c) 24 (d) None of these

Q.15. What can be the maximum possible average of class X when all the three sections are combined?
(a) 26.33 (b) 28.13 (c) 27.28 (d) 28.5

Q.16. We take a sample size of the following numbers from different sections

	Section A	*Section B*	*Section C*
Total no. of students selected	10	10	10
No. of students of minimum marks	5	6	4
No. of students of maximum marks	5	4	6

What is the average of this sample size?
(a) 25.06 (b) 26.16 (c) 25.83 (d) 26.71

Q.17. In the above question, which section has contributed the maximum in the average of the whole sample size?
(a) A
(b) B
(c) C
(d) Two sections contributed equal

Q.18. If we shift 5 students from section A to section B and then 10 students from section B to section C, what will be the maximum average of section C?
(a) 27 (b) 26.66 (c) 26 (d) 27.33

Q.19. In the above question, what can be the minimum average of section C?
(a) 20 (b) 21 (c) 21.33 (d) 22

Q.20. 10 students are shifted from each of the sections A and C to section B and then 10 students each are shifted from section B to each of the section A and section C. Now what is the ratio of maximum possible average of section A and section C?
(a) 1:1 (b) 15:14
(c) 14:15 (d) Cannot be determined

Q.21. In the above question, what is the ratio of the minimum possible average of section A and section C?
(a) 11:12 (b) 12:11
(c) 1:1 (d) Cannot be determined

Q.22. The average of 5 positive numbers is A. The average of the average of all the possible distinct triplets of these 5 numbers is B. Which of the following is true?
(a) A>B (b) A<B
(c) A=B (d) Cannot be determined

Q.23. The average of 5 positive numbers is A. The average of the average of all the possible distinct pairs of these 5 numbers is B. Which of the following is true?
(a) A>B (b) A<B
(c) A=B (d) Cannot be determined

Directions for questions 24–26: Read the passage below and solve the questions based on it.

The following operators have been defined for two numbers A and B.
@(A, B) = Average of A and B
*(A, B) = Product of A and B
/(A, B) = A divided by B

Q.24. If A = 2 and B = 4, then the value of @ (/(*(A, B), B), A) would be
(a) 2 (b) 4 (c) 6 (d) 16

Q.25. The sum of A and B is given by
(a) *(@ (A, B), 2) (b) /(@ (A, B), 2)
(c) @(*(A, B), 2) (d) @ (/(A, B) 2)

Q.26. The sum of A, B and C is given by
(a) * (@ (* (@ (B, A), 2), C), 3)
(b) * (@ (* (@ (B, A), 2), C), 2)
(c) /(* (@ (* (B, A), 2), C), 3)
(d) None of these

Q. 27. The average age of Mr. and Mrs. Sinha at the time of their marriage in 1972 was 23 years. On the occasion of their anniversary in 1976, they observed that the average age of their family had come down by 4 years compared to their average age at the time of their marriage. This was due to the fact that their son Vicky was born during that period. What was the age of Vicky in 1980?
(a) 6 (b) 7 (c) 8 (d) 5

Q.28. A man makes 60 articles in the 1st hour. His efficiency decreases by 25% in the 2nd hour, increases by 40% in the 3rd hour, decreases by 33% in the 4th hour and increases by 50% in the 5th hour. If he has to work for more than 1 hour, then in which hour the average number of articles produced per hour then would be minimum?
(a) 2nd hour (b) After 5th hour
(c) 3rd hour (d) None of these

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Q.29. Sachin Tendulkar has a certain batting average N (a whole number) in his career of 86 innings. In the 87th inning, he gets out after scoring 270 runs which increases his batting average by a whole number. The batting average is calculated by dividing the total number of runs scored by the total number of innings played by the player. How many values of his new average is/are possible?
(a) 0 (b) 1
(c) 2 (d) None of these

Q.30. In a school there are two sections for class X – section A and section B. The number of students in section A and section B are 60 and 70 respectively and their respective average weight is 35 kg and 38 kg. If one of the students from section A is shifted to section B then the average weight of both the sections decreases. What can be said about the weight of that student?
(a) His weight is less than 35 kg.
(b) His weight is more than 38 kg.
(c) His weight is more than 35 kg and less than 38 kg.
(d) Cannot be determined

Directions for questions 31–32: Read the passage below and solve the questions based on it.

The average score of a batsman for a certain number of innings was 21.75 per inning. He played 3 innings more and scored 28, 34 and 37 runs respectively, thus increasing his average by 1.25.

Q.31. How many innings did he play in all?
(a) 18 (b) 21
(c) 27 (d) None of these

Q.32. What was his total score?
(a) 520 (b) 601
(c) 621 (d) None of these

Q.33. A curious student of Statistics calculated the average height of all the students of his class as A. He also calculated the average of the average heights of all the possible pairs of students (two students taken at a time) as B. Further, he calculated the average of the average heights of all the possible triplets of students (three students taken at a time) as C. Which of the following is true of the relationship among A, B and C?
(a) A + 2B = C (b) A + B = 2C
(c) A = B = 3C (d) None of these

Q.34. A succession of numbers is said to be in the Arithmetic Progression (AP) if the difference between any given term and the previous term is constant throughout. In a AP $n_1, n_2, n_3, \ldots, n_{46}$, the sum of odd numbered terms $n_1, n_3, n_5, \ldots, n_{45}$, is 1272. What is the sum of all the 46 terms of this AP?
(a) 2,491 (b) 2,500
(c) 2,400 (d) Cannot be determined

Q.35. There are two vessels of milk of different prices with the volume of 220 litres and 180 litres respectively. Equal amounts of milk were poured off simultaneously from the two vessels and the milk poured off from the first vessel was poured into the second vessel and the milk poured off from the second vessel was poured into the first vessel. Then the price of milk in both the vessels becomes the same. How much milk was poured from one vessel into the other?
(a) 90 (b) 80
(c) 160 (d) 99

Q.36. There are two solutions of Sulphuric acid (acid + water) with concentration of 50% and 80% respectively. They are mixed in a certain ratio to get a 62% sulphuric acid solution. This solution is mixed with 6 litres of water to get back 50% solution. How much of the 80% solution has been used in the entire process?
(a) 15 litres (b) 12 litres
(c) 10 litres (d) None of these

Q.37. Two-fifths of the volume of the mixture of the milk and water is of the ratio of 4:3 is converted into a mixture of the ratio of 5:3 by the method of replacement with milk. The 4:3 mixture was prepared from a 4:1 mixture by the method of 'addition of the substances of the mixture'. If the replacement volume is 14 litres, what is the volume of the substance added?
(a) 12 litres (b) 60 litres
(c) 80 litres (d) 24 litres

Q.38. Rakesh has a stock of 1000 litres of pure milk. He decides to decrease the concentration to half, using the method of successive dilution, replacing 200 litres of contents by water, in each step of dilution. After the fourth dilution, he realizes that the concentration is less than the required value. Each of the contents removed during the dilution process is stored separately. He also has ample supply of water. Will he be able to make the mixture of desired concentration?

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(a) Not possible
(b) Possible
(c) Cannot be determined
(d) None of these

Q.39. An industrial solvent of 90% strength is prepared and stored in a 150 litres capacity container. The container is filled to 80% of its capacity. It is required to decrease the concentration of the solvent in the container to less than 60%. For doing this, either pure water can be added to it and/or solution can be removed. But removal can only be done if there is no place for adding water. At a given time, only 10 litres of the solution can be added or removed. How many times should water be added to the solution?
(a) 4 (b) 5 (c) 9 (d) 6

Q.40. Four gallons are drawn from a case full of wine. It is then filled with water. Four gallons of mixture are again drawn and the cask is re-filled with water. The ratio of the quantity of wine now left in the cask to that of the mixture in it is 36:49. How much does the cask hold?
(a) 30 gallons (b) 25 gallons
(c) 35 gallons (d) 28 gallons

Q.41. There is a vessel holding 40 litres of milk. 4 litres of milk is initially taken out from the vessel and 4 litres of water is then poured in. After this, 5 litres of mixture is replaced with 5 litres of water. And finally 6 litres of the mixture is replaced with 6 litres of water. How much milk (in litres) is there in the vessel?
(a) 26.775 (b) 29.16
(c) 24.72 (d) 27.42

Q.42. There is a vessel holding 40 litres of milk. 4 litres of milk is initially taken out from the vessel and 5 litres of water is poured in. After this, 5 litres of the mixture from this vessel is replaced with 6 litres of water. And finally 6 litres of mixture from this vessel is replaced with 7 litres of water. How much of the milk (in litres) is there in the vessel now?
(a) 22.42 (b) 27.09
(c) 24.72 (d) 29.42

Q.43. There is a vessel holding 40 litres of milk. 4 litres of milk initially is taken out from the vessel and 5 litres of water is poured in. After this, 6 litres of mixture from this vessel is replaced with 7 litres of water. And finally 8 litres of mixture from this vessel is replaced with 9 litres of water. How much of the milk (in litres) is there in the vessel now?
(a) 22.82 (b) 20.92
(c) 26.78 (d) 24.87

Q.44. There are three vessels A, B and C having a mixture of milk and water. Concentration of milk in the vessels is C_1, C_2 and C_3 respectively. When a part of vessel A is replaced with the mixture in vessel B, then the concentration of milk in the final mixture is less than C_1. However, when vessels B and C are mixed in the ratio of 1:1, then the quantity of milk in the final mixture is double than the quantity of milk in the vessel B. Which of the vessels is having the mixture of maximum concentration of milk?
(a) A (b) B
(c) C (d) Cannot be determined

Q.45. In the above question, which of the vessels is having the mixture of minimum concentration of milk?
(a) A (b) B
(c) C (d) Cannot be determined

Q.46. In question number 44, if we arrange the concentration of milk in different vessels in ascending order, then which of the following is true?
(a) ABC (b) BAC
(c) CAB (d) Cannot be determined

Q.47. In two alloys, the ratios of copper to zinc is 5:2 and 3:4. How many kilograms of the first alloy and the second alloy respectively should be melted together to obtain 28 kg of a new alloy with equal copper and zinc?
(a) 8 kg and 20 kg (b) 4 kg and 24 kg
(c) 3 kg and 25 kg (d) 7 kg and 21 kg

Q.48. From a 3:5 solution of milk and water, 20% is taken out and replaced by milk. How many times should this process be done to make the ratio of milk to water as 17:8?
(a) Once (b) Twice
(c) Thrice (d) Four times

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ADVANCED

Directions for questions 1–2: Read the passage below and solve the questions based on it.

In the National Baseball League (NBL), the batting average (AB) of a hitter is computed from the home runs scored, innings played, completed innings and incomplete innings.

R1 = Number of home runs scored in complete innings.

N1 = Number of complete innings.

R2 = Number of home runs scored in incomplete innings.

N2 = Number of incomplete innings.

$$AB = \frac{R_1 + R_2}{N_1}$$

To accurately assess a hitter's accomplishment, the NBL has devised two other statistical computations called the Quick Average (AQ) and the Comprehensive Average (AC), which are defined as follows:

$$AQ = \frac{R_1}{N_1} + \frac{N_2}{N_1}\max\left[0, \left(\frac{R_2}{N_2} - \frac{R_1}{N_1}\right)\right]$$

$$AC = \frac{R_1 + R_2}{N_1 + N_2}$$

Q.1. Based on the information, which of the following is true?

(a) $AQ \le AB \le AC$ (b) $AB \le AC \le AQ$
(c) $AC \le AB \le AQ$ (d) None of these

Q.2. An experienced hitter with no incomplete innings has AB of 50. The next time he bats, the inning is incomplete and he scores 45 runs. It can be inferred that

(a) AB and AQ will increase, while AC will decrease
(b) AB will increase, while AC will decrease
(c) AB will increase and we cannot comment on the changes in AQ and AC
(d) None of these

Q.3. If the average marks of 17 students in a class is A. The marks of the students when arranged in either an ascending or a descending order was found to be in arithmetic progression. The class teacher found that the students who were ranked 3rd, 7th, 9th, 11th, 15th had copied in the exam and hence got all of them rusticated. The average of the remainder of the class was B. Then

(a) A = B (b) A > B
(c) A < B (d) Data Insufficient

Q.4. The average weight of the students in four sections A, B, C and D is 60 kg. The average weights of the students of A, B, C and D individually are 45 kg, 50 kg, 72 kg and 80 kg respectively. If the average weight of the students of section A and B together is 48 kg and that of the students of B and C together is 60 kg, what is the ratio of the number of students in sections A and D?

(a) 4:3 (b) 3:2 (c) 8:5 (d) 12:7

Q.5. Illara, Puttu and Munchun had three bottles containing equal amounts of orange juice, mango juice and water respectively. Since Illara and Puttu wanted to dilute their drinks, Munchun gave one-fourth of the water that she had, to each of them. After mixing their bottles well, each of them gave to Munchun the same quantity of the solution that they had borrowed from her. Which of the following statements is true?

i. The concentration of water in Munchun's drink is more than the concentration of orange juice in Illara's drink
ii. The concentration of orange juice in Illara's drink is more than the concentration of mango juice in Puttu's drink
iii. The concentration of orange is more than the concentration of mango in Munchun's drink

(a) i and ii (b) i, ii and iii
(c) i, ii and iii (d) None of these

Q.6. We write down all the digits from 1-9 side by side. Now we put '+' between as many digits as we wish to, so that the sum of numbers become 666. It is explained below

1 2 3 4 5 6 7 8 9 = 666

Now suppose we put plus signs at following places:

12 + 345 + 67 + 89 = 513

Since there are four numbers, so the average can be calculated by dividing the sum by 4. What is the average if the sum is 666?

(a) 166.5 (b) 111
(c) 133.2 (d) Cannot be determined

Q.7. In the above question, what is the average if the sum is 261?

(a) 54 (b) 52.2
(c) 55 (d) None of these

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Directions for questions 8–11: Read the passage below and solve the questions based on it.

Go through the following definitions

A + B – All A are B.

A * B – Some A are B.

A @ B – No A are B.

A # B – Only A are B.

Following table gives the average age break-up of all the students at IIM-A (2005 batch)

Group of students	*Average age (years)*	*No. of persons in the group*
Males M	23	140
Females F	22	100
Engineers E	25	160
Fresher R	20	80
Working Professionals W	27	160

Assume that all the students mentioned above are either males or females. The minimum entry age in the IIM is 20 years (at the time of taking the entrance exam to qualify for IIM-A). No fresher can be a working professional and males/females/engineers can be either a fresher or a working professional. Everybody is either a fresher or a working professional.

Q.8. Following statement is definitely true: M + E. What is the average age (in years) of all the engineers who are not male?

(a) 39 (b) 40
(c) 38 (d) Cannot be determined

Q.9. Total number of working professionals is divided in the ratio of 3:2 among males and females. If A * B, then what is the range of the average age (x) of all the males who are engineers?

(a) $32 \le x \le 38$ (b) $39 \le x \le 43$
(c) $44 < x \le 49$ (d) Cannot be determined

Q.10. R + M and F + W are definitely true. What is the average age (in years) of all the males who are working professionals?

(a) 35 (b) 35.33
(c) 36 (d) None of these

Q.11. Consider the following statements – F @ E. What is the average age (in years) of all the working females?

(a) 23 (b) 24
(c) 24.5 (d) Data inconsistent

Directions for questions 12–14: Read the passage below and solve the questions based on it.

Ten different samples of milk solution— A, B, C, D, E, F, G, H, I and J are taken and the concentration of each sample (total quantity of milk as a percentage of total quantity of solution) was found out and the results were tabulated as follows

Sample	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Concentration (%)	78	57	82	84	98	66	34	87	79	71

Q.12. If two samples of milk are taken to form a sample whose concentration is more than 80%, then a maximum of how many distinct pairs of samples are there, which will never give the desired result for any ratio of their volumes?

(a) 30 (b) 48 (c) 12 (d) 15

Q.13. A triplet is a set of three distinct samples taken in some ratio of their volumes. How many such distinct triplets are there, which when mixed can give a sample with more than 80% concentration?

(a) 50 (b) 100 (c) 150 (d) 200

Q.14. There are how many distinct samples such that if two or mote than two such samples are mixed in any ratio, the resulting solution is always a sample of more than 80% concentration?

(a) 4 (b) 6 (c) 8 (d) 10

Q.15. There are three containers C_1, C_2 and C_3 having milk and water solutions in the ratio of a:b, where a ε (1, 2, 3) and b ε (4, 5, 6). Containers C_1, C_2 and C_3 can take any value of a and b. For how many different set of values of concentration of containers C_1, C_2 and C_3, the resultant mixture will have the ratio of milk and water as 1:1?

(a) 12 (b) 0
(c) Infinite (d) None of these

Q.16. In the above question, a ε (1, 2, 6) and b ε (4, 5, 3). Containers C_1, C_2 and C_3 can take any value of a and b. For how many different set of values of concentration of containers C_1, C_2 and C_3, the resultant mixture will have the ratio of milk and water as 1:1?

(a) 12 (b) 0
(c) Infinite (d) None of these

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