Design of Toy Proteins Capable to Rearrange Conformations in a Mechanical Fashion

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We design toy protein mimicking a machine-like function of an enzyme. Using an insight gained by the study of conformation space of compact lattice polymers, we demonstrate the possibility of a large scale conformational rearrangement which occurs (i) without opening a compact state, and (ii) along a linear (one-dimensional) path. We also demonstrate the possibility to extend sequence design method such that it yields a "collective funnel" landscape in which the toy protein (computationally) folds into the valley with rearrangement path at its bottom. Energies of the states along the path can be designed to be about equal, allowing for diffusion along the path. They can also be designed to provide for a significant bias in one certain direction. Together with a toy ligand molecule, our "enzimatic" machine can perform the entire cycle, including conformational relaxation in one direction upon ligand binding and conformational relaxation in the opposite direction upon ligand release. This model, however schematic, should be useful as a test ground for phenomenological theories of machine-like properties of enzymes.

I. INTRODUCTION

A. The problem and the context

Theoretical progress in understanding proteins in the recent years was concentrated on folding, along with connected questions of sequence design and evolution (see book¹ and references therein for the recent overview). Folding attracts theorists not only because it is so important for fundamental biology and for pharmaceutical industry, but also because it is a robust universal phenomenon. Vast number of proteins exhibit the ability to fold, and there is a widely recognized necessity to understand the physical principle behind the selection of sequences capable of fast and reliable folding.

In our opinion, there is one more aspect of proteins which is equally robust and appealing for theoretical analysis in terms of some minimal model. We mean here the ability of many proteins to function in a machinelike fashion through ordered conformational rearrangement. This is most obvious for motor proteins whose function is directly related to certain mechanical (conformational) movements². This is also clear for ion channels whose function is to mechanically move molecules (or ions) from one place to the other (e.g., across the membrane). Although less obvious, conformational motions appear to be also very much at play in proteins whose function is purely electronic, such as, e.g., electron transfer in bioenergetics or catalysis of a chemical reaction. This latter point was first formulated by McClare³ and independently by Blumenfeld⁴. More recently, it was extensively discussed in the book⁶. Somewhat different viewpoint on this subject was also recently presented in the book 5 .

The new experimental data by H. Gruler et al⁷ support the idea of slow conformational relaxation being important for the operation of enzimatic molecular machines. More detailed phenomenological models of enzyme operation based on the concept of conformational relaxation as a biased diffusion process have been successfully implemented to interpret the experimental results⁸. There is now the Data Base of conformational movements in proteins⁹.

We emphasize two general properties of functionrelated conformational movements in proteins. First, they occur without significant opening of the dense globular structure. Viewed in the context of contemporary folding theories, this property seems quite exciting. Indeed, as native globule is pretty dense, frequently modeled as a maximally compact self-avoiding polymer¹⁰, the inside movements may be expected to be strongly suppressed. Counterargument to this suggests that in fact real protein globule does have certain voids and is not absolutely dense¹¹. Nevertheless, the density of a typical protein globule is similar to that of a polymer melt, for which reason it may be expected to be extremely viscous if not altogether glassy. The observation of significant conformational movements inside such a dense polymeric conglomerate challenges theory to offer an explanation.

Second general property we would like to mention is the presence of some preferred collective degree of freedom - which is almost a synonym to a functioning device. For instance, enzymes work in cycles, and each cycle means a turn around some loop in the conformational space. For channel-forming protein, a part of this loop corresponds to a transported ion moving from one place to the other, the rest of the loop corresponds to the protein coming back. According to the arguments developed a long time ago (see the book⁶ and references therein), it is important that there is only one collective degree of freedom along the loop of function (which, of course, does not rule out "transverse" fluctuations¹²).

Importantly, machine-like function is realized well away from equilibrium conditions. That means, there is no detailed balance, and the system moves along the loop in one direction and not in the other - there is no, and should not be, detailed balance. This, however, does not rule out the possibility that some parts of the cycle

may present themselves as being the motions along the same path in opposite directions, like, e.g., a piston moving up and down in a steam engine. We shall return to this point later.

The notion of preferred function-related degree of freedom may be compared in some respects to the concept of reaction coordinate much discussed in folding studies^{13,14}. In both cases, the presence of transverse fluctuations is important. In case of folding, this gives rise to the understanding that, e.g., transition "state" is not a microstate, determined to atomic details, but rather an ensemble of (micro)states¹⁵. The preferred functional degree of freedom must be considered in pretty much the same way.

It is an exciting question whether these two collective degrees of freedom, relevant for folding and function, are connected to one another. One could even speculate that they may be the same, or similar to some approximation. As of today, this question remains open. However, we note in passing that turn-around times reported in modern single molecule experiments on enzymes such as ¹⁶, on the order of a fraction of a millisecond, are not drastically different from typical folding times.

In the present paper, our goal is modest, but two-fold. First, we want to see, at least for the simplest model, how one can imagine a collective degree of freedom allowing orderly motion without opening the compact globule. Second, we want to design such a sequence that the globule energy changes in some desirable fashion while moving along the preferred coordinate. This way, we want to mimic a molecular spring.

Thus, our work is organized as follows. We first describe the model and formulate our problem in a more explicit way for that model. We then discuss the possibility to design the native state conformation, or, better to say, the ensemble of nearly-native state conformations, in such a way that they can realize the one-dimensional motion within an almost compact globule. After that, we design the sequence capable to fold into such a functioning conformation. At the end we study some properties of thus designed toy protein.

B. A digression: from spherical cow to lattice protein

We shall work with the toy lattice model of protein. We understand that these words will run the emotions high and negative with many readers, and so we want to answer that from the very beginning. Everyone understands that there are no lattices in biological world, but this argument itself, however obvious, is too cheap to prove the lattice models useless. Indeed, for example, everyone laughs at the famous anecdote about a spherical cow, but at the same time everyone tacitly agrees that the model of a spherical cow is useful, e.g., to understand the scaling laws relating animal body mass and the rate of oxygen consumption¹⁷. Thus, the dispute about useful-

ness, or the lack of one, for the lattice models in protein studies cannot be resolved on the level of philosophy, this is the question of specific purpose of certain studies. Not entering the details, there are questions for which lattice models are totally inappropriate (and may deserve laugh), there are some other questions for which using lattice models is legitimate. As we hope to prove by the results, our present paper belongs to the latter category.

Thus, we use standard toy lattice model in which protein is represented as a self-avoiding walk of the desirable length. The protein changes its conformation by means of elementary moves, including corner flips, end flips, crankshafts, and null moves¹⁸. The advantage of this move set is that the resulting system is known to be ergodic¹⁹. For this model, polymer moves by making discrete succession of steps from one conformation to the next. Accordingly, the preferred degree of freedom must be associated with certain a linear (one-dimensional) succession of conformations, in which every conformation may only move into either previous or next conformation of the same group.

II. DESIGN OF CONFORMATION

A. Space of conformations

We stated above why the protein needs to have a selected degree of freedom for functional work. Here, we design lattice toy protein in such a way that its conformations, while remain compact, can rearrange along one-dimensional linear path in conformational space.

The concept of linear path can be easily explained if the conformational space of lattice protein is visualized as a graph^{20,21,22}. In this representation, every conformation is denoted as a node of the graph, and two nodes are connected if and only if the transition between corresponding conformations is possible via single elementary move (see Figure 1).

We are looking for linear paths in the conformational space graph (CSG). Obviously, linear path is the succession of such nodes each of which is connected to exactly two other nodes. For swollen, non-compact polymer, a multitude of conformational motions is possible, the corresponding nodes of the graph have very many connections, and so none of the swollen conformations belong to the linear path. By constrast, in compact conformations conformational freedom of the polymer is very limited, and there is a hope to find linear paths.

In order to find one-dimensionally-connected paths of compact conformations, we examined from this point of view the properties of CSG of the short lattice polymer.



FIG. 1: The conformational space of the lattice polymer can be visualized as a graph. The nodes represent conformations. Two nodes are connected if and only if the transition between them is possible via single elementary move. The linear rearrangement pathway is shown in bold.

B. Properties of the space of the compact conformations

We build on the findings of the work²³. In that work, it was shown that placing the polymer inside a restricted size box makes the conformational space graph disconnected, consisting of several disjoint pieces, or chambers. This finding was based on computer simulation of lattice polymers of various lengths N, each confined in the $3\times3\times3$ box on the lattice. For our present purposes, it is important to address another physical situation, in which N is fixed, while the degree of compactness of the chain may change. We achieve this by restricting the gyration radius of the chain R_g and then looking at various specific values of R_g . More specifically, this was done in the following manner.

We consider lattice polymer of the length N=18. We start from maximally compact conformation and allow it to make all elementary moves consistent with the chain self-exclusion, but possibly (and necessarily!) violating the compactness. We accept the conformation and place it as a node on the graph if and only if the chain gyration radius in this conformation is less than the chosen threshold R_q . All the accepted conformations are pictured on the graph as nodes and their connections with all other accepted conformations are established through exhaustive search. Then these new conformations again allowed to make elementary moves, new conformations are accepted if they do not exceed the same threshold R_q , and the process repeats. As regards the limiting value of R_q , we choose it experimentally, and it regulates both the compactness of the conformations and the number of conformations in the graph. The graph constructed in such a manner consists of several disjoint regions²³, or chambers. That means, for two conformations, which

belong to different chambers, there is no sequence of elementary moves, which transforms one of them into another without breaking the restriction on R_q . Thus, the procedure must be repeated starting from different maximally compact conformations to list all the chambers of the graph. The number of conformations in different chambers varies, reflecting the distribution of the clusters in the bond percolation problem (in this case, we deal with percolation in conformation space 23). Figure 2 shows the dependence of the number of the small chambers in the conformational space graph on the number of conformations locked in chamber. In terms of the underlying physical idea, this figure is similar to the result reported earlier in the work²³ for a different model. In the present model, we vary R_g at the fixed number of monomers N. In the work²³, the similar alleged percolation in the conformational space was controlled by changing N while locking the polymer inside the $3 \times 3 \times 3$ cube on the lattice, which of course implies fixed R_q .

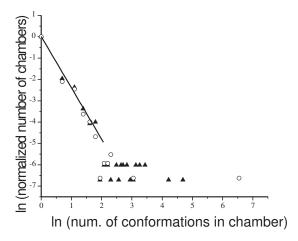
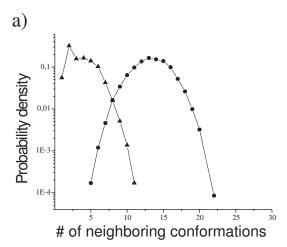


FIG. 2: The dependence of the number of the small chambers in the CSG on the number of conformations locked in chamber. Two different CSGs were built limited by $R_g=1.304$ (triangles) and $R_g=1.305$ (circles). For a comparison, maximally compact conformations of lattice 18-mer have $R_g=1.2583$. The conformational space of lattice 18-mer restricted by $R_g=1.304$ consist of 1094 small chambers. The "infinite" cluster incorporates 23536 compact conformations.

Further, we established the connectivities of the nodes which belong to the largest chamber in the CSG. The distribution of the connectivities of the nodes is shown in Figure 3a, curve 1. It is compared with the distribution of the numbers of neighboring nodes for the same set of conformations, but not restricted with the R_g -condition (curve 2). Curve 2 is indistinguishable from the binomial distribution (which does not contradict the idea that non-restricted CSG is a small-world network²⁴). The CSG built under the restriction on the values of R_g of toy-protein conformations (curve 1) is significantly different, one can imagine this graph as a percolation cluster of the bond percolation problem on the lattice with the topology of the small-world network²⁵. For the small values

of R_q (weakly connected cluster) the peak of the distribution corresponds to the graph nodes connected to only two neighbors. The sharp peak on the curve 1 corresponding to the poorly connected conformations can be easily explained. The change of the geometry of the voids in the bulk of the protein globule is possible only via small number of local moves, because the excluded volume effect is very strong in compact conformations. Such subtle conformational moves do not affect significantly the value of the R_q of the protein chain, whereas opening of some loop on the surface of the globule leads to the increase of the R_q . On the other hand, the majority of the conformational moves accessible to the unrestricted protein chain occurs on the surface. Accordingly, the limiting of R_q from above forbids surface moves but does not restrict the changes in the bulk of the globule. That is why for small limiting R_g values the sharp peak of the distribution rises at the poorly connected conformations.



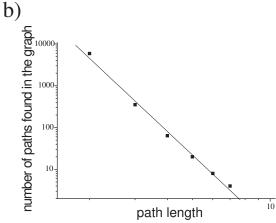


FIG. 3: (a) The distribution of the connectivities of the nodes within "infinite" cluster of the conformational space graph of the lattice 18-mer restricted by $R_g=1.304$ (triangles) compared with one for the non-restricted 18-mer (circles). The peak of the distribution corresponds to the nodes incorporated into the linear paths. (b) The distribution of the linear paths in the "infinite" cluster by length. The longest path found consist of 9 elementary moves.

Thus, the study of R_g -restricted conformations of the 18-mer provides an example in which there is the peak of connectivity distribution which corresponds to the graph nodes with connectivity 2. These are desired conformations forming linear paths. However, most of these linear paths are rather short and consist of only few elementary moves. The distribution of the lengths of the paths is shown in Figure 3b, it is exponential distribution.

Thus, our conclusion so far is this. Long linear paths exist, but they are not very common, they are rare. In this sense, the situation is similar to that of selection of sequences for toy proteins capable of folding¹. In sequence selection case, the "good" sequences are exponentially rare among the random ones. The goal of sequence design, or selection, is to fish them out. Similarly, our goal now is to identify the rare conformations which are connected by long linear paths in the conformation space. In order to do that, we need to understand better the local geometry of conformations belonging to the linear paths. It cannot be done for the chain length of N=18which is too small. It is also small compared with typical protein lengths, about one hundred or more in average. Thus, we repeated the same procedure of CSG mapping for the toy-protein of the length N=125. Of course, in this case no exhaustive enumeration of conformations is possible, and so we performed random sampling instead.

We started from the limiting value $R_g = 2.44949$, which is equal to the gyration radius of the maximally compact conformation. In this case, no conformational movement is possible, and, therefore, CSG consists of as many disconnected nodes as the number of compact conformations. We now increase R_q by a very small amount. By several attempts, we choose the step of R_g increment in such a way that after one step conformation space remains disintegrated, non-ergodic, consisting of disconnected chambers²³ most of which contain only few conformations. We then increase R_g step by step, every time mapping the newly obtained parts of conformation space on the graph, as described before. When R_q approaches 2.46, the percolation transition occurs and there appears an "infinite" cluster. We mapped into the graph 1000000 conformations from this cluster and searched for the long linear paths on the graph, as it was done in case of 18mer.

The longest paths we found for the 125-mer are organized as follows. The conformations along the path transform into one another in a set of subsequent corner flips in the bulk of compact conformation. When the first corner flip occurs, the vacant site opens producing the opportunity for another monomer to move. Then the similar process repeats many times, leading effectively to the vacancy traveling through the globule, finding the new corner flips on its way. These subsequent transformations are demonstrated in Figure 4 which for the simplicity of presentation shows a smaller polymer, namely 63-mer.

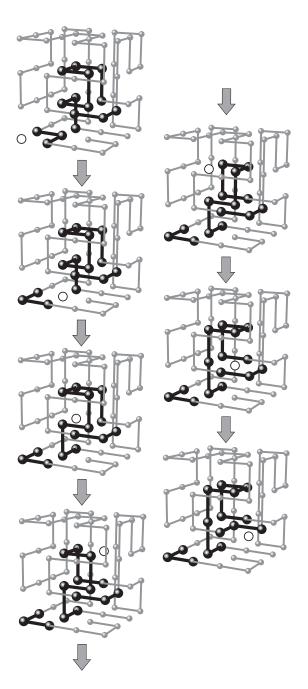


FIG. 4: A typical realization of the linear rearrangement path of the compact conformation of the lattice 63-mer. The vacancy travels through the bulk of the conformation in a set of subsequent corner flips. The bonds which participate in the conformation rearrangement are shown in bold color. The overlap between conformations on the opposite ends of the pathway comprises $Q^{1,7}=58$, whereas maximal number of contacts $Q^{max}=79$.

C. Design of the pathway in conformation space

Now, when the mechanism of rearrangements within the compact conformation for the toy protein is clarified, we can propose the effective and straightforward approach to design the linear path of compact conformations.

The algorithm includes two main stages. First, we arrange the switching elements along the path. Second, the rest of the polymer is computationally designed such that it is (almost) maximally compact and contains the necessary set of switching elements. As a result, we obtain one-dimensional path of compact conformations of the lattice toy protein.

1. Building the switching elements.

We build the linear path using flipping corners as switching elements.

In the beginning of the procedure we have an empty cubic lattice. We start from choosing initial position of the vacancy (Figure 5a, node (0,0,0)). The first flipping corner is drawn on the lattice in such a way that when it flips, the corner and the vacancy exchange their positions. The edges forming next switching element should be drawn in such a way, that when it will flip, vacancy will hop to the former corner position opening the way for the next switching element. In the Figure 5b two subsequent flipping corners are shown. After they both will move, the vacancy will take the position (1,2,1). Other flipping corners are drawn subsequently as many times as needed (Figure 5c).

At the same time as the switching elements arranged on the lattice we can control the linearity of the path. Every time when the new flipping corner is set on the lattice there are several ways to locate it relative to the vacancy position. We choose only one location, but the problem is that the other flipping corners may appear later, on the stage of the design of the whole conformation (see Figure 5d). To prevent appearance of such parasitic switches we draw additional pieces of the conformation surrounding the pathway sites, as it is shown in Figure 5e.

2. Building the rest of conformation.

We now have the set of switching elements, which are supposed to be just the disconnected pieces of the polymer. We have to find now the rest of the polymer, such that it fills (almost) completely the whole volume of the cube, and connects all switching elements into a linear chain. For this purpose, we developed computational method which is the modification of the approach proposed by Ramakrishnan et al²⁶ to generate maximally compact conformations. Here, we describe our modified algorithm.

The conformation design starts with placing on the lattice the new edges connecting randomly chosen neighboring vertices (Figure 6 a,b). This process soon brings us to the state, where some vertex cannot be connected to its randomly chosen neighbor, because the neighbor already

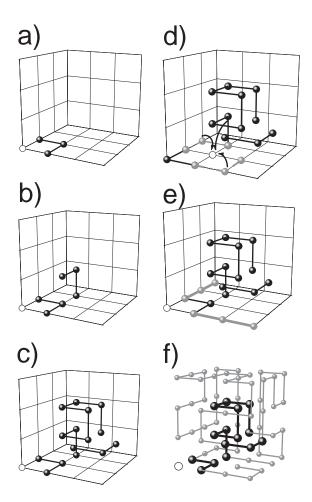


FIG. 5: The steps of the design of the linear rearrangement pathway. (a) The initial position of the vacancy and the first flipping corner are dawn on the lattice; (b,c) the other switching elements of the path are added; (d) possible parasitic flips, which may later appear on the stage of the design of the whole conformation, are shown in grey; (e) to prevent parasitic flips additional elements of the conformation surrounding the switching elements of the path are added (shown in yellow); (f) conformation design completed.

has two edges incident on it (Figure 6 c). In this case, special procedure called two-matching is applied²⁶. During this and the following steps of the algorithm, some randomly chosen edges can be removed from the lattice and changed by others. However, we impose the condition that the edges forming switching elements $cannot\ be\ removed$ at any step of conformation design.

Two-matching starts from picking up the vertex P, which is either not connected, or has only one incident edge. Then its neighbor Q is chosen randomly as an opposite end of the new edge. If Q belongs to the linear subchain, then the ends of this subchain are checked for the possibility to be connected with their neighbors. If it can be done, a new edge (the edge ST at the figure 6b) is added. One of the edges incident on Q is replaced by the edge PQ. Thus, in this procedure, the number

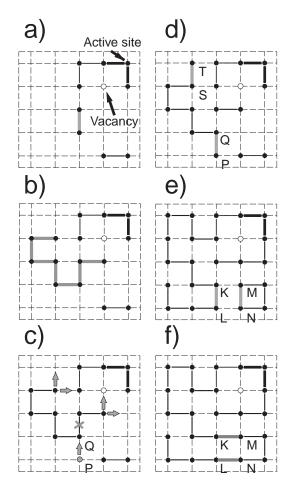


FIG. 6: The schematic representation of the application of the algorithm²⁶ for the purposes of the design of the conformation incorporating the rearrangement pathway. For the simplicity the steps of the algorithm are shown in two dimensions. (a) New edges randomly placed on the lattice; (b) Edges form subchains; (c) At some step edges can not be placed randomly; (d) Number of edges increases by one after applying two matching procedure; (e) Linear subchains an loops; (f) Subchains united.

of edges increases by one (Figure 6d). If, on the other hand, the vertex Q belongs to the looped subchain, one of its incident edges is removed and replaced by PQ. So, the loop is broken, and the total number of edges on the lattice remains unchanged. Typically, as a result of the work of this procedure, we obtain several looped and one linear subchain packed into the cubic lattice (Figure 6e).

Now, the subchains should be merged into one chain. This is achieved in the following way. Suppose four neighboring vertices (K, L, M, N) form a square. The connecting edges (K, L) and (M, N) belong to the different subchains. Excluding these edges and including instead (K, M) and (L, N), we would have merged subchains (see Figure 6f). Such an operation is known as patching. Each patching operation transforms a pair of subchains into one subchain. The edges to be involved in patching are chosen randomly. The process is stopped when

there is no more and no less than one linear chain on the lattice, which is the desired polymer conformation.

In the original work²⁶, this method was applied to generate maximally compact lattice conformations. It is worth repeating that for our purposes we use this method starting from a complicated lattice which is the cube minus elements chosen for switching elements.

Generally, it is possible to use also crankshafts and flipping ends to design the switching elements. One just needs to forbid all the states of these elementary moves but two. This should be done by placing on the lattice additional edges (as it was done to prevent parasitic corner flips), which restrict the extra states of these moves. However, the end flip can be used only twice as switching element, because there are two ends of the chain. As regards the crankshaft move, it needs two vacant sites to make switching possible, which means the conformation in question should be slightly less compact. For these reasons, we use only corner flips as switching elements in this work.

III. SEQUENCE DESIGN

After we have chosen conformations, we should find the sequence which fits the target conformations with low energies. For this purpose the sequence of the model protein is annealed and Monte Carlo optimization in the sequence space is performed. The details of the algorithm are previously published²⁷ (see also review article²⁸ and references therein for further details). It must be emphasized that we plan to work with the sufficiently large set of monomer species; in fact, we shall even use the so-called Independent Interaction Model, in which the number of distinct species is as large as the number of monomers in the chain. This allows us to avoid difficulties well known in the case of sequence design for the two-letter heteropolymers, such as the HP-model²⁹. In the context of the present work, sequence design method had to be modified in two respects. First of all, we need not only one target conformation to have a low energy, as it is typically assumed in protein folding simulations. We need the whole family of conformations - all conformations belonging to the rearrangement path - to have distinctly lower energies than all other states. Second, the more ambitious goal is to design the sequence in such a way that moving of the system along its pathway changes energy in an orderly fashion. Since our rearrangement path is one-dimensional, we can pretend energy to increase monotonically along this path, in this sense making our toy protein a model of a molecular spring.

Let us consider these two aspects of sequence design one by one.

A. Sequence with multiple ground states

The model protein is determined by the set of the coordinates of the monomers $\mathcal{C} = \{\vec{r}_I\}$ and the sequence of monomers $seq = \{s_I\}$ (the species $s_I \in \{1, 2, ...q\}$ denote the identity of each monomer, q is the total number of species), index I counts monomers along the chain. The Hamiltonian is written as follows

$$\mathcal{H}(\text{seq}, \mathcal{C}) = \sum_{I < J}^{N} B_{s_I s_J} \Delta(\vec{r}_I - \vec{r}_J) , \qquad (1)$$

where the energy of the conformation is determined by the matrix of species-species energies $B_{i,j}$ for the contacting monomers and function $\Delta(\vec{r}_I - \vec{r}_J)$ is defined such that it is equal to 1 if monomers I, J are lattice neighbors and 0 otherwise. We use independent interactions model (IIM)³¹ and Miyazawa-Jernigan (MJ)³² matrices for Monte Carlo simulations in this article.

In our approach the goal is to design sequences for which the whole set of conformations $\{C_k\}$ have energies sufficiently below that of the rest of conformation space. Of course, our candidates for the target states $\{\mathcal{C}_k\}$ are the conformations which belong to the previously designed linear rearrangement path in conformation space. These conformations are supposed to form a deep valley in the energy landscape. For the set of two target conformations, sequence design was performed in the paper³⁰. In that work, the goal was to model proteins which can fold into two (or more) distinct "native" conformations, like prions³³. Accordingly, two target conformations were chosen to be totally dissimilar (nonoverlapping, or weakly overlapping). In more details, such design was examined in³⁴. In our case, the problem is almost the opposite. The conformations in question are very closely related, they can be mutually transformed into one another in just a few moves. Accordingly, the overlap between neighboring conformations along the path is very high. Of course, as the system walks along its pathway from one end to another, the overlap decreases, but still remains significant. For example, the overlap between the conformations at the opposite ends of the path shown in the Figure 4 is as high as about 75%.

The sequence optimization is governed by the following Hamiltonian:

$$\mathcal{H}_{\text{des}}(\text{seq}, \{\mathcal{C}_k\}) = \sum_{k=1}^{N_{\mathcal{C}}} \mathcal{H}(\text{seq}, \mathcal{C}_k) , \qquad (2)$$

where $N_{\mathcal{C}}$ is the total number of conformations along the rearrangement pathway.

The question which arises now is this. How efficient is the sequence optimization in the case of multiple closely related target states? Let us consider the simple situation when only energies of two target conformations are optimized. These conformations, namely \mathcal{C}_1 and \mathcal{C}_2 , could be, for example, the ends of the rearrangement

pathway. How deeply can their energies be lowered during the sequence design in comparison with an arbitrary other compact conformation C_{\star} ?

To make this estimate we can calculate the energy of the conformation \mathcal{C}_{\star} averaged over the sequence space

$$\langle E_{\star}(seq) \rangle = \frac{\sum_{seq} P_{seq}^{(0)} e^{-(\mathcal{H}_{des}(seq, \{\mathcal{C}_k\}))/T_{des}} \mathcal{H}(seq, \mathcal{C}_{\star})}{\sum_{seq} P_{seq}^{(0)} e^{-(\mathcal{H}_{des}(seq, \{\mathcal{C}_k\}))/T_{des}}},$$
(3)

where $P_{seq}^{(0)} = \prod_{I=1}^{N} p_{s_I}$ is the probability for the sequences made randomly from independent monomer species with occurrence probabilities p_i . The details of similar calculations are described in the review article²⁸. The result for the present case of two target conformations reads

$$\langle E_{\star}(seq)\rangle = \bar{B}Q^{\star} - \frac{\delta \hat{B}^2}{T_{\text{des}}}[Q^{1,\star} + Q^{2,\star}],$$
 (4)

where

$$\bar{B} = \sum_{ij} p_i B_{ij} p_j \quad \text{and} \tag{5}$$

$$\delta \hat{B}^2 = \sum_{ij} p_i (B_{ij} - \bar{B})^2 p_j \tag{6}$$

are the mean value and the variance of the interaction matrix,

$$Q^{\star} = \sum_{I < J}^{N} \Delta(\vec{r}_{I}^{\star} - \vec{r}_{J}^{\star}) \tag{7}$$

is the total number of contacts in the conformation C_{\star} and its overlap with arbitrary target conformation C_k is defined as

$$Q^{k,\star} = \sum_{I$$

As we can see from the expression (4), the energy of the designed sequence in the conformation C_{\star} depends on the similarity of this conformation to the target states. This similarity is measured by the overlap parameter $[Q^{1,\star} + Q^{2,\star}]$. It takes the maximal value for the a given pair C_1 , C_2 , when C_{\star} coincides with either C_1 or C_2 . Usually, if C_{\star} lies on the rearrangement pathway, the overlap parameter $[Q^{1,\star} + Q^{2,\star}]$ takes values close to the maximum. Therefore, not only energies of the target conformations are optimized, but conformations between them (e.g. along the designed linear path!) are optimized, too.

In principle, sequence design method may also lower the energies of other states, which are related to the target states, but do not belong to the rearrangement pathway, and thus are non desirable for us here. This is a well known problem, generally addressed through the "negative design" (see, for instance, 35). Luckily, in our specific case, conformation design, as discussed above,

helps to address this problem. Indeed, since there are no allowed conformations at all on the sides of the pathway, the only possible low energy decoys are structurally unrelated ones.

Sequence design method employed here may seem to contradict the results of the recent work³⁶. In this work, authors estimated the maximal possible number of "native" states which may be "memorized" by the sequence. They showed that this number is very limited, it is independent on protein length, and is fully determined by the alphabet - the number, q, of distinct monomer species (for instance, for the system with q=20 monomer species, there can be not more than four or five "native" states). In fact, there is no contradiction. The estimate of the work³⁶ determines the maximal number of unrelated conformations which can be designed into the sequence. In our case, all target states are very closely related, and they in fact belong to the same potential well, or funnel, in the free energy landscape. They, of course, cannot be considered as independent. As we show below, our sequence design process works successfully even when the number of states in the rearrangement path is as large as about ten.

B. Sequence design with energy gradient along the rearrangement path

So far, we have been discussing the sequence design procedure for which all the target states C_k were equal. Now, following Orwell³⁹, we shall consider some of the conformations more equal than others. Specifically, we should remember that conformations C_k form a onedimensional path. We did it on purpose, and we should use it now. To be specific, let us assume that conformations C_k are labeled with index k in a natural way, such that k changes orderly from 1 at the one end of the rearrangement pathway to the maximal value $N_{\mathcal{C}}$ at the other end. Then, we shall require that, say, C_1 has the lowest energy, C_2 has energy a little higher - preferably, by a certain amount W higher; \mathcal{C}_3 we want to be about W higher in energy than C_2 , ..., and this continues all the way up to C_{N_c} , which we want to be about $(N_c - 1) W$ above C_1 , but still much lower than all the other conformations.

Why do we want such energy landscape? First of all, being all connected in one valley, the conformations \mathcal{C}_k form together a basin of attraction for folding, or folding funnel. Second, when correctly folded and at the bottom of the funnel, the system can still travel back and forth along the one-dimensional rearrangement path. This travel may be either due to fluctuations or due to some externally applied force.

To achieve this aim we modified the design Hamiltonian (2) in the following way:

$$\mathcal{H}_{\mathrm{des}}(\mathrm{seq}, \{\mathcal{C}_k\}) = \sum_{k=1}^{N_{\mathcal{C}}} \mathcal{H}(\mathrm{seq}, \mathcal{C}_k) +$$

+
$$\lambda \sum_{k=2}^{N_C} \left[\mathcal{H}(\text{seq}, \mathcal{C}_k) - \mathcal{H}(\text{seq}, \mathcal{C}_1) - kW \right]^2$$
, (9)

where λ is the "experimentally" adjusted parameter, and W is the desired energy gap between neighboring conformations. The Monte-Carlo optimization ruled by the Hamiltonian (9) has a bias towards sequences with lower energy in conformation C_1 and subsequently higher energies in conformations C_2 , C_3 ,

Various regimes are possible here depending on the relation between the design temperature $T_{\rm des}$, real temperature T, and the value of W. Not entering the discussion of all these regimes, we mention that in what follows we have chosen the sequence optimization temperature to be $T_{\rm des} = 0.1 \delta B$.

IV. RESULTS

A. Design of the conformations and the sequence

To demonstrate the work of our design method, we generated two sets of conformations of lattice 63-mer as described above in the section II C. The linear rearrangement path of the conformation shown in Figure 4 includes 7 conformations. In another example, shown below in Figure 10, there are 6 conformations in the path. The location of the switching elements in these two cases is chosen differently. Switching elements in the former case are located in the bulk of the globule, whereas in the latter case the switching elements are all located near the surface.

For these two sets of target conformations, the sequence optimization procedure was applied. The values of parameters were as follows: $W = 0.3\delta B$, $T_{\rm des} = 0.1\delta B$, $\lambda = 15$, and the interaction matrix was chosen to be correspond to the independent interactions model (IIM).

B. Folding

First of all, we have to check that the chains can correctly fold into the conformation as designed. We used the set of conformations shown in the Figure 44 as target states. We compared folding rates for the sequence designed as proposed in this article and for the control sequence, designed in a more traditional way, with conformation C_1 as the only purported ground state. All the folding experiments were started from different random unfolded conformation. The Monte Carlo simulations were performed at temperatures in the range $T = 0.7 \div 0.9 T_m$, where $T_m \approx 1.3 \delta B$ was the midpoint temperature. Mean first passage time (MFPT) at every temperature was calculated by averaging over 30 folding runs. The results are shown in the Figure 7. The folding times for the sequence which has multiple ground states are approximately 3 times longer than for the sequence

with the unique native conformation. Further inspection suggests that this happens because the depth of the global minimum for the sequence with multiple ground states can not be as well optimized as that for the sequence with unique ground state. Nevertheless, the emphasis of our result here is on the good news, not the bad ones: it is not important that our sequences are slower, it is important that they are insignificantly slower, only by a factor of about 3 slower. That means, they do fold, and their folding time is of the same order of magnitude.

It is worth to emphasize that during this folding experiments, the chain was not confined in any restricted volume. We used volume restriction in the preliminary stage of this work, to elucidate the method of conformation design. Now, as we are done with the design, we let the polymer do whatever it wants, and the result is that it folds and spontaneously arrives into the valley where it has a linear chain of conformations at its disposal.

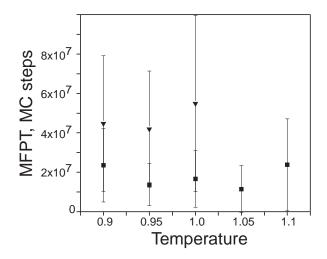


FIG. 7: The comparison of the folding rates of the two sequences. First sequence (squares) is optimized to have unique ground state , which is the conformation 1 in the Figure 4. Second sequence (triangles) is designed according to the approach described in this paper and has 7 low energy conformations (all are shown in the Figure 4. The same IIM matrix with $\langle B \rangle = -0.025, \ \delta B = 1.0$ used for the design and folding. Both sequences were designed at the temperature $T_{\rm des} = 0.1 \delta B$.

C. Diffusion along the designed path

Now, since we have established that our model does fold, we have to check if it can move along the designed path. In reality, conformational relaxation of a functioning protein machine is triggered by the attachment or detachment of a substrate or other ligand; we shall consider this in the next section. Here, we want to perform the simpler test to see what happens without any stimuli. In this situation, we expect our toy protein to move randomly back and forth along the designed path. It should

be mostly in the lowest energy state C_1 , but since the energy difference between states along the path $W = 0.3\delta B$ is only a fraction of thermal energy, bias towards the C_1 end should be relatively weak, there should be plenty of fluctuations along the path. The point to be checked is that the system, while performing this random walk along the path, should not open up too frequently, the globule should stay compact.

To examine how this happens in the toy-protein shown in the Figure 4, we run a long Monte Carlo simulations at different temperatures starting from the conformation \mathcal{C}_1 . The events of passing the conformations $\{C_k\}$ of the predesigned path are recorded. Coordinate along the pathway takes the value x = k if the vacancy position along the path coincides with conformation C_k . A typical "trajectory" of conformational changes for the first designed toy protein is shown in Figure 8. The inset of Figure 8 displays the details of one particular passage along the path from conformation C_1 to C_7 and back. The events when conformation changes in a way other than walking along the path (say, some loop opens on the surface of the globule), are pictured as the change of the coordinate in the perpendicular plane. As one can see, below the midpoint temperature the toy-protein stays steadily on the rearrangement path and makes random walks back and forth along it, with very limited fluctuations in the multitude of transverse directions in the conformation space.

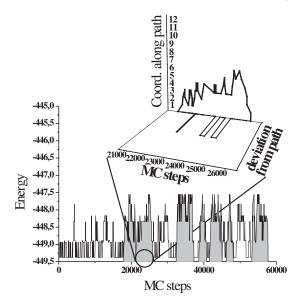


FIG. 8: A typical MC trajectory of the toy-protein shown in the Figure 4 at the ground state. The dependence of energy on the number of MC steps; (Inset) The dependence of the conformational coordinates on the number of MC steps. Coordinate along the linear rearrangement path is shown in the vertical plane. The deviations from the linear path conformations a shown schematically as the non-zero values in the horizontal plane. These deviations are related to the events of unfolding of some loops on the surface of the molecule.

Thus, what we observe confirms that our toy protein performs thermal fluctuations in the form of onedimensional movement along the designed path. This is important, because if fluctuations occur that way, that means, under some different circumstances, e.g., applied external force, the system can perform also a forced movement along that same pathway, as it is expected for the function. Thus, we need to test carefully that the observed fluctuations are indeed the random walks effectively in one dimension. To address this quantitatively we calculate the correlation function $\langle x(t)x(t+\tau)\rangle$ for the toy protein and compare it with that for the artificial auxiliary random walker which diffuses on the 1D discrete set of states with the same energy profile as for the protein. The good agreement between the two correlation functions is evident in Figure 9. Hence the designed toy protein does indeed move along the one-dimensional pathway and in this sense it remembers its "function."

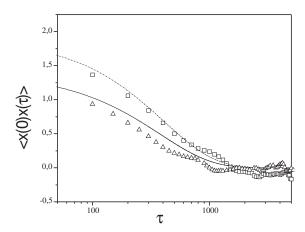


FIG. 9: The correlation function of the random walk along the linear path. The simulation results ($T=0.5\delta B$ - triangles; $T=0.7\delta B$ - squares) are compared with the curves calculated for the particle performing one-dimensional walk at the same potential and at the same temperatures ($T=0.5\delta B$ - solid line; $T=0.7\delta B$ - dots).

D. Simulation of the binding/dissociation with the small molecule

In the previous section we demonstrated that toyprotein can use the designed rearrangement path, performing a biased, but random motion. This biased walk can be triggered if the protein molecule is externally stimulated. As in the real biochemical world, this can be most efficiently done by attaching the ligand molecule to the protein globule. In this sense, the typical cycle of a protein work may be described as follows⁶.

We start from the protein molecule in its ground state. At some moment, a ligand molecule gets attached to the protein. With the ligand attached, protein conformation is no longer the ground state, some other conformation is now lower in energy, and, therefore, attachment of ligand initiates the relaxation process of the globule into the new conformational state, which is the ground state

for the protein-plus-ligand complex. When this relaxation is completed (or nearly completed), the new conformation turns out well suitable for certain chemical (or other small length scale) changes, the result of which for the protein is the desorbtion of a ligand initiating again the relaxation process, this time back into the original ground state. This type of conformational relaxation processes coupled with the ligand binding plays is well known for motor proteins³⁷, heme-containing proteins³⁸, etc. In this work, following our general strategy, we want to design a toy ligand with which our toy protein can perform the entire cycle of its "function."

We designed a toy protein which has binding site and is able to change its shape. The initial protein conformation and conformational changes activated by the adsorption of the toy ligand are shown in Figure 10. The toy protein was designed as explained above. The linear rearrangement path of the molecule includes 6 conformations. The conformation C_1 is the lowest energy state for the protein with no ligand. The energies of interaction of the ligand with the "active center" of the protein are chosen such that as soon as the complex forms, the energies of linear path conformations change in comparison with those in isolated protein molecule. The profile of the energy changes along the path of conformational rearrangements in the protein-ligand complex is shown in Figure 11. When the ligand is attached, the conformation \mathcal{C}_6 of the path has the lowest energy. Hence as the small molecule binds to the protein in the state C_1 , it induces the cascade of conformational changes which drives the protein to the conformation C_6 . The energy profile along the path of the conformational changes corresponds to the case of the suicide inhibitor due to the high barrier for the dissociation of the ligand in the conformation C_6 .

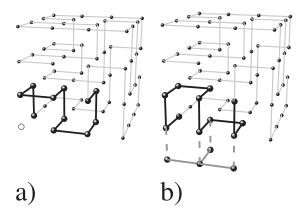


FIG. 10: Two conformations of the toy-protein representing the ends of the linear rearrangement path and the small molecule bound to the active center of the protein (a). The substrate binds when the protein stays in the conformation C_1 and unbinds at the conformation C_6 (b).

Thus, our model exhibits the conformational response to the ligand binding. When the ligand binds, it jump-starts the cascade of orderly occurring conformational transitions $\mathcal{C}_1 \to \mathcal{C}_2 \to \ldots \to \mathcal{C}_6$. In the computer exper-

iment, we have measured the relaxation times - the number of Monte Carlo steps in which the toy protein goes from \mathcal{C}_1 into \mathcal{C}_6 upon ligand binding. We repeated this measurement in 100000 independent MC runs, and Figure 12 shows the distribution of relaxation times. The tail of the distribution demonstrates exponential decay that is expected for the biased diffusion in one-dimensional system.

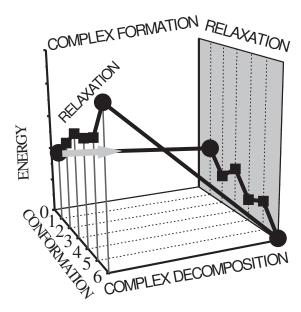


FIG. 11: The energies of the conformations along the path in the protein molecule and in the complex.

These simulation results are in good agreement with some experimental data. For example, Beece et. al. 38 studied the rebinding of carbon monoxide to myoglobin protein after dissociation induced by 1 μ s laser impulse. Such a short pulse synchronized conformational transformations of many protein molecules in solution. Under proper circumstances, the population of excited protein molecules exhibited exponential decay, quite similar to our data presented on Figure 12. To reproduce also the multi-exponential or power law decay on the medium time scale observed in some experiments 16 will require some modifications of the present model.

V. CONCLUDING REMARKS

In this study, we designed a toy model which exhibits quite a few protein-machine-like properties. First and foremost, it has the possibility to move like a mechanical system along the one-dimensional path, which we called rearrangement path. Like real protein, our model can fold into the valley of conformations the bottom of which is the rearrangement path. Thus, it folds, but not only it just folds, it does so to form a functional, movable state. While folded into its low energy valley, our toy protein can move along the path which is the valley bottom. Making only few transverse excursions, it performs

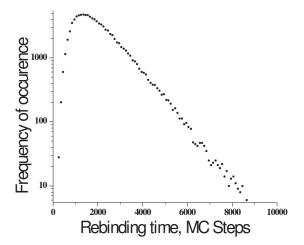


FIG. 12: The distribution of the relaxation times of the toy protein in response to the events of ligand binding. The MC simulations were performed for the sequence designed in MJ model, $\langle B \rangle = -1.78$, $\delta B = 1.0$, $T_{\rm des} = 0.1 \delta B$, $T = 0.4 \delta B$.

a biased random walk along this path. Attachment and detachment of the ligand can switch the direction of the bias, thus making toy protein to go orderly through the closed cycle mimicking protein function.

The usefulness of a toy is the possibility to play. Playing with our toy allows to gain some insights into the general concept of machine-like function of proteins. One question which we can discuss is that of storing energy in the deformed globule. Our model is designed in such a way that the energy of the globule at one end of the rearrangement path is higher than at the other - see, e.g., figure 11, left panel. Can we say this globule acts as a molecular spring? To begin with, this seems to contradict the formulation of the design Hamiltonian which purports to make energy linear in the coordinate k along the path (kW), while regular Hookean spring, of course, should have energy quadratic in deformation. In fact, this is unimportant, because design Hamiltonian can be easily modified to purport quadratic (or any other) energy profile; only discrete character of the lattice renders such fine tuning of the model useless. More importantly, all energies involved in our model are in fact free energies. It should be understood that proteins, although machines, are not heat engines: they function in essentially isothermic environment. Therefore, when we say, for instance, that contact of monomers i and j has energy δB_{ij} , we should have in mind that pre-averaging has been performed over a multitude of small scale, rapidly relaxing degrees of freedom (e.g., χ -angles of the residues side groups), and then δB_{ij} is the free energy of the corresponding contact. From that point of view, we can say

that at the upper-energy end of the rearrangement path our model protein stores free energy rather than energy. In other words, it acts not so much like a Hookean spring, but rather like a piece of rubber. This analogy gets even better if we remember that linearity of the strain-stress relation and reversibility of deformation are totally unrelated in the case of rubber. As in other mechanical devices, the question of reversibility in our toy machine is related to its speed: the slower is the rate of operation, the closer it can approach to the limit of being reversible. Of course, when our toy protein undergoes fluctuations along its designed path, it exchanges energy with the surrounding heat bath, because the process occurs at the constant temperature. When conformational relaxation takes place, because random walk along the path is strongly biased - then energy exchange with thermal bath is dominated by the energy transfer into the bath. However, when we imagine that ligand energy has driven the enzyme to the high energy end of its path, then this energy remains available and drives the subsequent conformational relaxation. In this delicate sense, our toy can be said to work like a molecular spring - or, once again, as a piece of rubber. It goes without saving that this is an overdamped spring (or rubber), no oscillations are in question in the course of conformational relaxation.

Our approach is, no doubt, very schematic. It uses to the full extent the discrete geometry of moves on the cubic lattice. This is the heavy price we have to pay for the tractability of the model. In our opinion, the wonderful properties which we found justify the study of this model. We hope also that the fundamental ideas behind our approach will be useful for the more realistic models, including off-lattice ones. These fundamental ideas are the search for a one-dimensional rearrangement pathways in the compact globule and the design of sequences capable of folding into a "collective funnel" formed together by all states belonging to this pathway. Design of the static protein backbone conformation⁴⁰, as well as sequence design for the static⁴¹ limited flexibility⁴² backbones are all familiar computational approaches. It is a challenge to incorporate the search for the one-dimensional rearrangement pathways into this already difficult area.

VI. ACKNOWLEDGEMENTS

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