

9.1.1 Inertial Mass and Properties of Inertial Mass

1. It is defined as the ratio of the magnitude of external force applied on the body to the magnitude of acceleration produced in it,

$$\text{i.e., } m = (F/a)$$

2. (a) It is proportional to the quantity of matter present in the body.

- (b) It is independent of shape, size and state of the body.

- (c) It is conserved during chemical reaction.

- (d) It is not affected by the presence of other nearby bodies.

- (e) It increases as the speed of body increases.

- (f) If m_0 be the rest mass of the body and c be the speed of light, then, $m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}$.

9.2 GRAVITATIONAL MASS

Mass of the material of the body, which is determined by gravitational pull acting on it, is called as gravitational mass, i.e.,

$$m = \frac{FR^2}{GM}$$

Inertial and gravitational masses are equivalent. However, the definitions are independent of each other and two masses differ in the methods of their measurement.

9.3 ACCELERATION DUE TO GRAVITY

1. On the surface of earth $g = \frac{GM}{R^2} = 9.81 \text{ ms}^{-2}$

2. At height h from the surface of earth,

$$g_h = \frac{GM}{r^2} = \frac{GM}{(R+h)^2} = \frac{g}{(1+h/R)^2} \approx g \left(1 - \frac{2h}{R}\right) \text{ if } h \ll R$$

At a height of geostationary satellite $g_h = 0.225 \text{ m/s}^2$.

At a distance as moon from earth's centre $g_h = 0.0027 \text{ m/s}^2$.

3. At depth d from the surface of earth, $g_d = g[1 - (d/R)]$

At the centre of earth, $d = R$; $\therefore g_d = 0$

At the surface of earth, $d = 0$; $\therefore g_d = g$

4. Effect of rotation of earth at latitude θ , $g' = g - R\omega^2 \cos^2 \theta$

At equator $\theta = 0^\circ$, $g' = g - R\omega^2 = \text{Minimum value}$

At poles $\theta = 90^\circ$, $g' = g = \text{Maximum value}$

At equator, effect of rotation of earth is maximum and value of g is minimum.

At pole, effect of rotation of earth is zero and value of g is maximum.

If the earth stops rotating about its axis, the value of g at the equator will increase by about 0.35% but that at the poles will remain unchanged.

If the earth starts rotating at the angular speed of about 17 times its present value, there will be weightlessness on the equator, but g at the poles will remain unchanged. In such a case, the duration of the day will be about 84 minutes.

- Due to non-spherical shape of the earth, as polar radius $R_p >$ equatorial radius R_{eq} , and $g \propto (1/R)^2$, the value of g increases from equator to poles. Thus, due to the shape of the earth, g is maximum at poles and minimum at equator. Due to non-spherical shape of the earth.

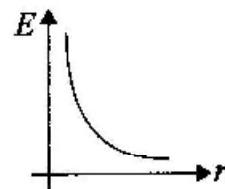
$$g_p - g_{eq} \approx 1.8 \text{ cm/sec}^2$$

9.4 GRAVITATIONAL FIELD STRENGTH

- Gravitational field strength at a point in gravitational field is defined as, $\vec{E} = \frac{\vec{F}}{m} =$ gravitational force per unit mass.

- Due to a point mass

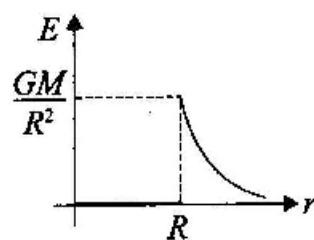
$$\vec{E} = \frac{GM}{r^2} \text{ (towards the mass)} \quad E \propto \frac{1}{r^2}$$



- Due to a uniform spherical shell

Inside points, $\vec{E} = 0$;

$$\text{Outside points, } \vec{E} = \frac{GM}{r^2}$$



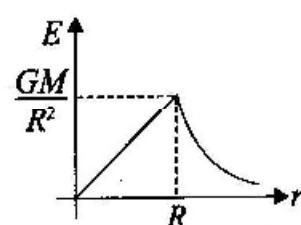
Just outside the surface, $E = \frac{GM}{R^2}$ (on the surface $E-r$ graph is discontinuous)

- Due to a uniform solid sphere

$$\text{Inside points, } \vec{E} = \frac{GM}{R^3}r \text{ (radially inwards)}$$

At $r = 0$ i.e., at centre, $E = 0$

$$\text{At } r = R \text{ i.e., on surface, } \vec{E} = \frac{GM}{R} \text{ (radially inwards)}$$



$$\text{Outside points, } \vec{E} = \frac{GM}{r^2} \text{ (radially inwards)} \quad E \propto \frac{1}{r^2}$$

As $r \rightarrow \infty$, $E \rightarrow 0$ (on the surface $E-r$ graph is continuous)

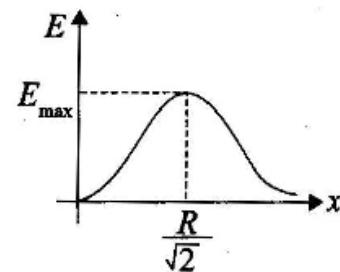
5. On the axis of a uniform circular ring

$$\vec{E} = \frac{GMx}{(R^2 + x^2)^{3/2}} \text{ (along the axis towards the centre)}$$

At $x = 0$ i.e., at centre, $E = 0$

If $x \gg R$, $E \approx \frac{GM}{x^2}$ i.e., ring behaves as a point mass

$$\text{As } x \rightarrow \infty, E \rightarrow 0 \quad E_{\max} = \frac{2GM}{3\sqrt{3}R^2} \text{ at } x = \frac{R}{\sqrt{2}}$$



9.5 GRAVITATIONAL POTENTIAL

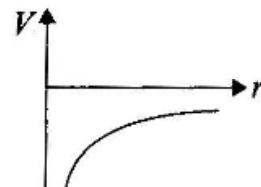
Gravitational potential at a point in a gravitational field is defined as the negative of work done by gravitational force in moving a unit mass from infinity to that point, Thus,

$$V_p = \frac{W_{R.L. \rightarrow p}}{m} \left(= \frac{W_{\infty \rightarrow p}}{m} \right)$$

1. Due to a point mass

$$V = -\frac{Gm}{r}$$

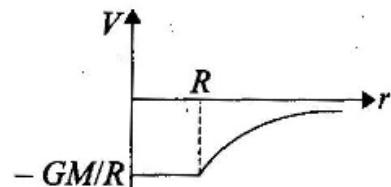
$V \rightarrow -\infty$ as $r \rightarrow 0$
and $V \rightarrow 0$ as $r \rightarrow \infty$



2. Due to a spherical shell

$$\text{Inside points} \quad V = -\frac{GM}{R} = \text{Constant}$$

$$\text{Outside points} \quad V = -\frac{GM}{r}$$

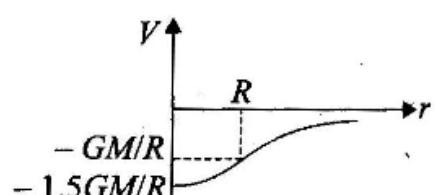


3. Due to a solid sphere

$$\text{Inside points } V = -\frac{GM}{R^3}(1.5R^2 - 0.5r^2)$$

$$\text{At } r = 0 \text{ i.e., at centre, } V = -1.5\frac{GM}{R}$$

$$\text{At } r = R \text{ i.e., on surface, } V = -\frac{GM}{R}$$



V - r graph is parabolic for inside points and potential at centre is 1.5 times the potential at surface.

$$\text{Outside points } V = -\frac{GM}{r}$$

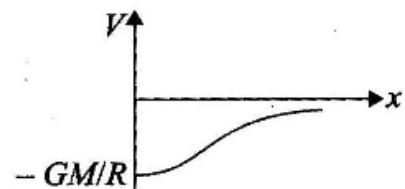
$$\text{At } r = R, V = -\frac{GM}{R} \text{ ie, on surface} \quad \text{As } r \rightarrow \infty, V \rightarrow 0$$

4. On the axis of a ring

$$V = -\frac{GM}{\sqrt{R^2 + x^2}}$$

$$\text{At } x = 0 \text{ i.e. at centre, } V = -\frac{GM}{R}$$

This is the minimum value as $x \rightarrow \infty, V \rightarrow 0$



9.6 GRAVITATIONAL POTENTIAL ENERGY

This is negative of work done by gravitational force in assembling the system in bringing from infinite separation to the present position.

1. Gravitational potential energy of two point masses is, $U = -\frac{Gm_1 m_2}{r}$
2. To find gravitational potential energy of more than two point masses i.e. the mutual potential energy we have to make unique pairs of masses.

$$U = -G \left[\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_1 m_4}{r_{14}} + \frac{m_2 m_3}{r_{23}} + \frac{m_2 m_4}{r_{24}} + \frac{m_3 m_4}{r_{34}} \right]$$

For n point masses, total number of unique pairs will be $\frac{n(n-1)}{2}$.

3. If a point mass m is placed on the surface of earth, the potential energy is $U = -\frac{GMm}{R}$
4. Potential energy of a point mass m at height h is $U_h = -\frac{GMm}{(R+h)}$
The difference in potential energy would be

$$\Delta U = U_h - U \text{ or } \Delta U = \frac{mgh}{1+h/R}$$

If $h \ll R, \Delta U = mgh$

9.6.1 Relation Between Field Strength E and Potential V

1. If V is a function of only one variable (say r) then

$$E = -\frac{dV}{dr} = -\text{Slope of } V-r \text{ graph}$$

2. If V is a function of three coordinate variables x, y and z then

$$\vec{E} = - \left[\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right]$$

Escape Velocity

The escape velocity is the minimum velocity with which a body be projected in order that it may just escape the gravitational field of the earth.

1. From the surface of earth, $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$ as $g = GM/R^2$, $v_e \approx 11.2 \text{ kms}^{-1}$
2. Escape velocity does not depend upon the angle at which particle is projected from the surface.
3. A planet will not have atmosphere if the root mean square velocity, $v_{rms} > v_e$.
4. If the velocity of a satellite orbiting near the surface of the earth is increased by 41.4 per cent, then it will escape away from the gravitational field of the earth.
5. If a body falls freely from infinite distance, then it will reach the surface of earth with a velocity of 11.2 km/sec.
6. The escape velocity on moon is low (as $g_m = \frac{g_E}{6}$) hence there is no atmosphere on moon, $v_e \approx 2.4 \text{ kms}^{-1}$.

Black Hole

A black hole is a body from whose surface nothing can be escaped, even light. From Einstein's theory of relativity, speed of any object cannot exceed the speed of light, $c = 3 \times 10^8 \text{ m/s}$. Thus, c is the upper limit to the projectile's escape velocity. Hence for a body to be a black hole

$$v_e \geq c \quad \text{or} \quad \sqrt{\frac{2GM}{R}} \geq c \quad \text{or} \quad R \leq \sqrt{\frac{2GM}{c^2}}$$

Motion of Satellites

A satellite is a body which continuously revolves around a much heavier body in a stable orbit. Moon is the natural satellite of the earth, which in turn is a satellite of Sun. A man made satellite is called artificial satellite.

- | | |
|---|--|
| 1. Orbital speed $v_o = \sqrt{\frac{GM}{r}}$ | 4. Potential energy $U = -\frac{GMm}{r}$ |
| 2. Time period $T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$ | 5. Total mechanical energy $E = -\frac{GMm}{2r}$ |
| 3. Kinetic energy $K = \frac{GMm}{2r}$ | 6. Binding energy $= \frac{GMm}{2r}$ |

Near the surface of earth, $r = R$ and $v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.9 \text{ kms}^{-1}$.

This is the maximum speed of earth's satellite.

Time period of such a satellite would be

$$T = \frac{2\pi}{\sqrt{GM}} R^{3/2} = 2\pi \sqrt{\frac{R}{g}} = 8.6 \text{ min.}$$

This is the minimum time period of any earth's satellite.

If the orbit of a satellite is elliptical

1. The energy $E = -\frac{GMm}{2a} = \text{Constant with } a \text{ as semi-major axis.}$
2. KE will be maximum when the satellite is closest to the central body (at perigee) and minimum when it is farthest from the central body (at apogee) (as for a given orbit $L = \text{Constant i.e., } mvr = \text{Constant i.e., } v \propto 1/r$).
3. PE = (E - K) will be minimum when KE = Max, i.e., the satellite is closest to the central body (at perigee) and maximum when KE = Min, i.e., the satellite is farthest from the central body (at apogee).

Some other important points concerning satellites:

1. Escape velocity and orbital velocity of a satellite are related as: $v_{es} = \sqrt{2v_o}$, i.e., if the speed of the satellite is increased by 41.4 per cent or its KE is increased by 100 per cent, it will escape out to infinite distance from the centre of the earth.
2. In an orbit, the magnitude of total energy of a body is equal to the magnitude of its KE.
3. As the radius of the orbit increases, i.e., the body moves away from the earth, its potential energy increases while KE decreases.
4. The total energy of a body in the orbit is always negative, i.e., the body is bound to the earth.
5. If we put a satellite in an orbit with a velocity v , then the velocity v_o for which the satellite revolves around the earth in a circular orbit of radius r , is given by $v_o = \sqrt{(GM/r)}$
 - (a) When $v < v_o$: the satellite fails to revolve around the earth and spirals inwards with a decreasing radius till it falls on the earth.
 - (b) When $v_{es} > v < v_o$: the body revolves in elliptical orbit.
 - (c) When $v > v_{es}$: the body escapes following a hyperbolic path.
6. If a body is thrown with a velocity v , then the sum of KE and PE is
 - (a) positive, when $v > v_{es}$
 - (b) zero, when $v = v_{es}$
 - (c) negative, when $v < v_{es}$

7. If the gravitational attraction of the sun on the planet varies as $(1/r^n)$, then the

(a) orbital velocity varies as $\frac{1}{\sqrt{r^{n-1}}}$;

(b) time period varies as $r^{(n+1)/2}$; where r is the distance of planet from the sun.

8. Geostationary satellite:

- (a) A satellite which appears to be stationary for a person on the surface of the earth is called geostationary satellite.
- (b) It revolves in the equatorial plane from west to east with a time period of 24 hours.
- (c) Its height from the surface of the earth is nearly 35600 km and radius of the circular orbit is nearly 42000 km.
- (d) The orbital velocity of this satellite is nearly 3.08 km/sec.
- (e) The relative velocity of geostationary satellite with respect to the earth is zero.
- (f) The orbit of a geostationary satellite is called as *parking orbit*.

9. Polar satellite orbit: Polar satellites travel around the earth in an orbit that travels around the earth over the poles. The earth rotates on its axis as the satellite goes around the earth. Thus over a period of many orbits it looks down on every part of the earth.

Kepler's Laws

Kepler's three empirical laws describe the motion of planets.

1. **First law:** Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.
2. **Second law:** The radius vector, drawn from the sun to a planet, sweeps out equal areas in equal time interval i.e., areal velocity is constant. This law is derived from law of conservation of angular momentum.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{Constant } (L \text{ is angular momentum and } m \text{ is mass of planet})$$

3. **Third law:** It states that, square of the time taken by the planet about the sun is proportional to the planet's mean distance from the sun.

If T be the time period of the planet and r be the mean distance of planet from the sun (average of maximum and minimum distances from the sun), the $r = \frac{r_{\min} + r_{\max}}{2} \Rightarrow \frac{T^2}{r^3}$ is same for all planets.

i.e., $T^2 \propto r^3$ where r is semi-major axis of elliptical path.

Circle is a special case of an ellipse. Therefore, second and third laws can also be applied for circular path. In third law, r is radius of circular path.

Perihelion distance is the shortest distance between the sun and the planet.

Aphelion distance is the largest distance between the sun and the planet.

$$\frac{V_{\text{aphelion}}}{V_{\text{perihelion}}} = \frac{r_{\text{aphelion}}}{r_{\text{perihelion}}}$$

If e is the eccentricity of the orbit, then $\frac{1+e}{1-e} = \frac{r_{\text{aphelion}}}{r_{\text{perihelion}}}$ and $r_{\text{aphelion}} + r_{\text{perihelion}} = 2r$

SOLIDS AND FLUIDS

10.1 INTERMOLECULAR FORCES

The force between two molecules is known as intermolecular forces.

These forces are of electrical origin. They are also known as van der Waal forces. These forces are very weak forces.

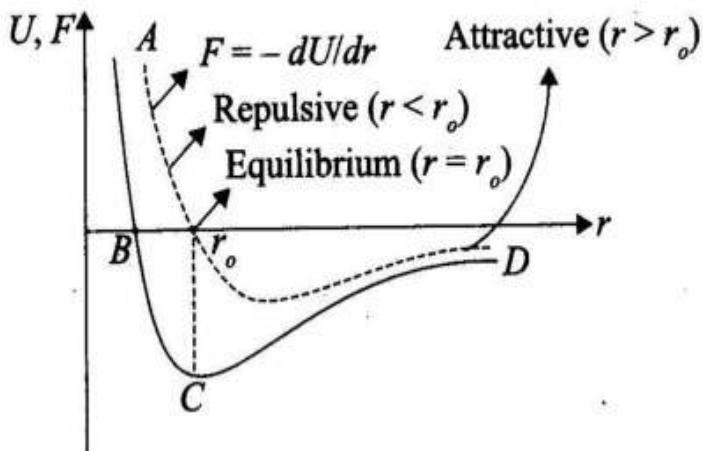
The potential energy is minimum and the kinetic energy is maximum when the two molecules are separated by $r = r_o$. This distance is known as normal distance.

The molecules are in the state of equilibrium. No net force acts between the molecules.

When $r > r_o$, the force is of attraction and potential energy is negative.

When $r < r_o$, the force is of repulsion and potential energy is positive.

At point A, the potential energy becomes minimum and the molecules cannot come closer. The distance OA denotes the minimum possible distance between molecules.



$F = -\frac{dU}{dr}$, where U = Potential energy and intermolecular force is

$$F(r) = \frac{A}{r^{13}} - \frac{A}{r^7}$$

- Force of cohesion and adhesion are intermolecular forces.

The potential energy is present in the molecular on account of intermolecular forces. The value of potential energy U can be zero, positive or negative. The potential energy due to forces of attraction is negative. The potential energy due to forces of repulsion is positive.

10.2 TYPES OF BONDING

There are mainly five types of bonding among the atoms or molecules.

1. **Ionic bonding:** Cohesive energy is high and electrical conductivity is very low, e.g., NaCl, LiF. It is the strongest bonding.
2. **Covalent bonding:** Cohesive energy is high. Semiconductors like Ge, Si, diamond have covalent bondings.
3. **Hydrogen bonding:** Cohesive energy is low. Insulators have this bonding.
4. Hydrogen bonding as in DNA and H₂O.
5. Metallic bonding as in metals.

10.3 FOUR STATES OF MATTER

1. **Solid state:** The intermolecular forces are strongest, the kinetic energy of molecules is less than their potential energy and intermolecular distance remains constant.
2. **Liquid state:** The intermolecular forces are less strong than solids and more strong than gases, kinetic energy is more than potential energy and the intermolecular distance does not remain constant.
3. **Gas state:** The intermolecular forces are weakest, the kinetic energy of molecules is much larger than their potential energy and intermolecular distance does not remain constant.
4. **Plasma state:** It is a state of matter in which the medium is in the form of positive and negative ions.

10.4 ELASTICITY

The property of a material body by virtue of which it regains its original configuration on the removal of the deforming force is called elasticity.

10.4.1 Strain

The ratio of the change in configuration of a body to its original configuration is called strain.

$$\text{Strain} = \frac{\Delta x}{x} = \text{Change per unit original dimension}$$

- **Longitudinal strain:** It is defined as the ratio of the change in length (ΔL) of the body to its original length (L).
- **Volume strain:** It is defined as the ratio of the change in volume (ΔV) of the body to its original volume (V).
- **Shearing strain:** It is defined as the ratio of lateral displacement (Δx) of a plane to the perpendicular, distance (L) of this plane from a fixed plane (ABCD).

10.4.2 Stress

The internal restoring force developed per unit area of a deformed body is called stress.

$$\text{Stress} = \frac{F}{A} = \text{Restoring force per unit area}$$

10.4.3 Elastic Moduli

- $Y = 2\eta(1 + \sigma)$
- $Y = 3K(1 - 2\sigma)$
- $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$ or $Y = \frac{9K\eta}{\eta + 3K}$
- $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$

Y = Young's modulus, η = Rigidity modulus

K = Bulk modulus, σ = Poisson's ratio (σ has no unit, no dimensions).

$$1. \text{ Workdone per unit volume} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$\text{or Energy density} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$2. \text{ Workdone} = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$3. \text{ Energy density} = \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \times \frac{\text{Stress}}{\text{Strain}} (\text{Strain})^2 \\ = \frac{1}{2} \times Y \times (\text{Strain})^2$$

$$4. \text{ Workdone per unit volume} = \frac{1}{2} \times \text{Stress} \times \frac{\text{Stress}}{Y} = \frac{(\text{Stress})^2}{2Y}$$

5. Solids have all three modulii of elasticities, Young's modulus, bulk modulus and shear modulus, whereas liquids and gases have only bulk modulus.

$$6. \text{ Modulus of elasticity: } E = \frac{\text{Stress}}{\text{Strain}}$$

7. Materials which offer more resistance to external deforming forces have higher value of modulus of elasticity.

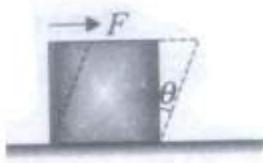
$$8. \text{ Young's modulus of elasticity: } Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\Delta l}$$

F = Force applied and A = Cross-sectional area

9. Bulk modulus of elasticity:

$$B = \frac{F/A}{\Delta V/V} = -\frac{\Delta P}{\Delta V/V} \quad \text{or} \quad -\frac{dP}{dV/V}$$

10. Shear modulus of elasticity or modulus of rigidity: $\gamma = \frac{F/A}{\theta}$



11. $\sigma = -1$ to 0.5 . When $\sigma = 0.5$, $K = \infty$ = Matter is incompressible and there shall be no change in volume of wire on loading. This is not possible in practice.

When $\sigma = -ve$, $\eta = \infty$ = Not possible. It means that if the length of a wire increases, its radius also increases. σ in practice cannot be negative.

$\sigma = 0.46$ for rubber, $\sigma = 0.25$ for steel, $\sigma = 0.20$ for glass.

12. Force required to prevent a rod from increasing in length when the rod is heated

$$(a) \text{Force} = YA\alpha\theta$$

where α = coefficient of linear expansion, θ = Change of temperature.

$$(b) \text{Thermal stress} = F/A = Y\alpha\theta.$$

13. Young's modulus Y and rigidity modulus η are possessed by solid materials only. For liquids, modulus of rigidity η is zero.

14. Every force produces extension along its own direction and simultaneous compression along perpendicular direction.

$$15. \text{Factor of safety} = \frac{\text{Breaking stress}}{\text{Normal working stress}} = \frac{\text{Breaking stress} \times \text{Area}}{\text{Force}}$$

Breaking stress is fixed for a material. It depends upon the material. Tensile strength is the breaking stress for a wire of unit cross-section.

$$16. \text{If real original length of wire} = L, \text{then} \quad L = \frac{F_2 L_1 - F_1 L_2}{F_2 - F_1}$$

where F_1 and F_2 are the stretching forces and L_1 and L_2 are the respective final length of a wire.

$$\begin{aligned} Y &= \frac{F_1 L}{A(L_1 - L)} = \frac{F_2 L}{A(L_2 - L)} \\ F_1(L_2 - L) &= F_2(L_1 - L) \\ \therefore L &= \frac{F_2 L_1 - F_1 L_2}{F_2 - F_1} \end{aligned}$$

(a) For a perfectly rigid body, strain produced by the applied force is zero.

$$\therefore Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Stress}}{0} = \infty$$

$Y = \infty = \text{Infinity}$

(b) For a perfectly rigid body, strain produced is zero.

$$\therefore K = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Pressure}}{\text{Zero}} = \infty = \text{Infinity}$$

(c) Quartz is the best example of a perfectly elastic body and Putty is the best example of a perfectly plastic body.

10.4.4 Interatomic Force Constant (k)

$$k = \frac{\text{Interatomic force}}{\text{Change in inter atomic distance}}$$

$k = \text{Young's modulus} \times \text{Interatomic distance}$

10.4.5 Cantilever and Beam

A beam clamped at one end and loaded at free end is known as cantilever.

Depression at the free end of cantilever = δ .

$\delta = \frac{Wl^3}{3YI_G}$, where l denotes length of cantilever, I_G = Geometrical moment of inertia of the cross-section of the beam.

For a beam with rectangular cross-section, $I_G = \frac{bd^3}{12}$ where b = Breadth, d = Thickness.

For a beam with circular cross-section having radius r , $I_G = \frac{\pi r^4}{4}$.

Depression produced at the centre of beam supported at two ends and loaded at the middle.

$$\delta = \frac{Wl^3}{48YI_G}, \text{ where } l \text{ denotes length of beam.}$$

1. $\delta = \frac{Wl^3}{4Ydb^3}$ for rectangular cross-section.

2. $\delta = \frac{Wl^3}{12\pi r^4 Y}$ for circular cross-section.

10.4.6 Torsion of a Cylinder and Workdone in Twisting

Couple per unit twist is $C = \frac{n\eta r^4}{2l}$ where l = Length of cylinder, r = Radius of cylinder and η = Modulus of rigidity of the material of cylinder.

Workdone in twisting the cylinder through an angle θ

$$= \int_0^\theta C \cdot \theta \cdot d\theta = \frac{1}{2} C \theta^2$$

1. Effect of temperature:

- (a) In general elasticity decreases as the temperature increases.
- (b) INVAR is an exception. There is no effect of temperature on elasticity of invar. Invar is infact a short form of invariable.

2. Effect of impurities:

- (a) If the impurity is more elastic, the elasticity of the material increases.
- (b) If the impurity is more plastic (less elastic) the elasticity of material decreases.

3. On hammering or rolling elasticity increases. On annealing i.e., on alternate heating and cooling elasticity decreases.

10.4.7 Compressibility

The reciprocal of bulk modulus of elasticity is defined as compressibility.

$$\text{Compressibility} = \frac{1}{B}$$

When pressure is applied on a substance, its volume decreases, while mass remains constant. Hence, its density will increase.

$$\rho' = \frac{\rho}{1 - \Delta P/B} \quad \text{or} \quad \rho' \approx \rho \left(1 + \frac{\Delta P}{B}\right) \text{ if } \frac{\Delta P}{B} \ll 1$$

10.4.8 Elastic Relaxation Time

The time delay in restoring the original form after removal of deforming force is called elastic relaxation time.

For phosphor, bronze, silver and gold, the time is negligibly small. For quartz, it is minimum. For glass, it is high.

10.4.9 Ductile Materials

The materials which exhibit large plastic range beyond the elastic limit such that they can be drawn into wire, springs and sheets are known as ductile materials. e.g., silver, steel, copper, aluminium.

10.4.10 Brittle Materials

These are those material which show a very small plastic range beyond the elastic limit. The breaking point lies very close to elastic limit e.g., rubber, glass.

10.5 FLUIDS

In fluid mechanics, the following properties of fluid would be considered:

1. When the fluid is at rest, *hydrostatics*.
2. When the fluid is in motion, *hydrodynamics*.

10.5.1 Pressure Due to a Liquid

The thrust exerted by a liquid at rest per unit area of the substance in contact with the liquid is called pressure. If F is the thrust exerted by a liquid on a surface of small area ΔA , then pressure is given by

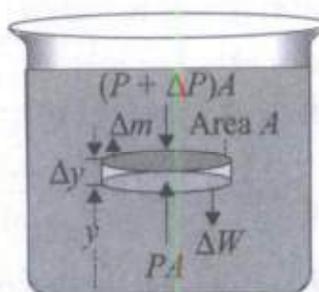
$$P = \lim_{\Delta A \rightarrow 0} (F/\Delta A)$$

The unit of pressure is dyne/cm² in CGS system and N/m² in SI. A pressure of one N/m² is also called pascal.

10.5.2 Variation of Pressure in a Fluid with the Height from the Bottom of the Fluid

For equilibrium, the resultant of the forces on mass Δm is zero. Hence

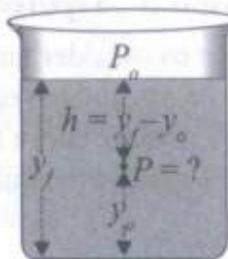
$$\begin{aligned} A \Delta P &= \rho (A \Delta y)g \\ \Rightarrow \frac{\Delta P}{\Delta y} &= -\rho g \\ \text{Also, } \lim_{\Delta y \rightarrow 0} \frac{\Delta P}{\Delta y} &= \frac{dP}{dy} = -\rho g \end{aligned}$$
(1)



It means that the pressure decreases with height y from the bottom of the fluid.

- If the fluid is incompressible, when the density of the fluid ρ is a constant, we can integrate the eqn. (1) to find the pressure P at any point in the fluid. Let us consider the pressure at height y from the bottom of the fluid as P and the pressure at the surface of the fluid as P_o , as shown in figure. Then equation (1) can be written as

$$\begin{aligned} dP &= -\rho g dy \quad \Rightarrow \quad \int_P^{P_o} dP = -\rho g \int_{y_o}^{y_f} dy \\ \Rightarrow (P_o - P) &= -\rho g(y_f - y_o) \quad \Rightarrow \quad P - P_o = \rho g(y_f - y_o) = \rho gh \\ \Rightarrow P &= P_o + \rho gh \end{aligned} \quad (2)$$

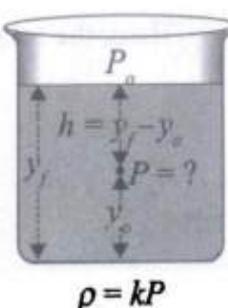


Where h is a positive distance measured downward from the surface. Equation (2) allows us to find pressure at any depth h within an incompressible fluid.

- If the fluid is compressible, the density is not constant, for example we can assume the density of the air is proportional to pressure to a good approximation (the assumption is not accurate due to temperature and other weather effects) and g is constant.

Let $\rho = kP$, where k is a constant. We assume a reference density ρ_o at a particular gas-pressure P_o . Take our reference at the earth's surface where $\rho = \rho_o$ and $P = P_o$.

$$\text{So, } \rho_o = kP_o \quad \Rightarrow \quad k = \frac{\rho_o}{P_o} \quad \text{Thus, } \rho = kP = \frac{\rho_o}{P_o} P$$

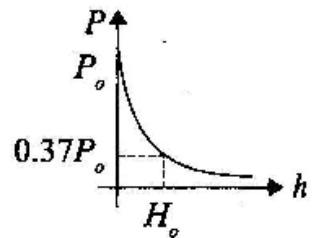


$$\text{As } dP = -\rho g dy \quad \Rightarrow \quad dP = -\frac{\rho_o}{P_o} Pg dy$$

$$\text{or } \frac{dP}{P} = -g \frac{\rho_o}{P_o} dy$$

The pressure is P_o when the altitude is zero and the pressure is P when the altitude is h , hence

$$\begin{aligned} \int_{P_o}^P \frac{dP}{P} &= -g \frac{\rho_o}{P_o} \int_0^h dy \Rightarrow \ln(P) - \ln(P_o) = -g \frac{\rho_o}{P_o} h \\ \Rightarrow \ln\left(\frac{P}{P_o}\right) &= -g \frac{\rho_o}{P_o} h \\ \therefore P &= P_o e^{\left\{-\frac{h}{[P_o/(g\rho_o)]}\right\}} \\ \Rightarrow P &= P_o e^{\left\{-\frac{h}{H_o}\right\}} \end{aligned} \quad (3)$$



The quantity $H_o = P_o/g\rho_o$ has the unit of length. The eqn. (3) shows exponential decrease in pressure with increasing altitude. When h increases by H_o , the pressure drops by $1/e$, i.e., to about 37 % of its original value.

10.5.3 Hydrostatic Force Due to Many Liquid Layers

Let us consider number of liquid layers; densities and heights of the liquids are, ρ_1 and h_1 , ρ_2 and h_2 ρ_n and h_n respectively.

Let us take a thin horizontal strip of area dA at a distance y from the top of n th liquid. The hydrostatic force dF on the thin strip is

$$dF = P dA \quad (3)$$

where P is the hydrostatic pressure on the thin strip.

We know that the pressure due to all liquid layers is

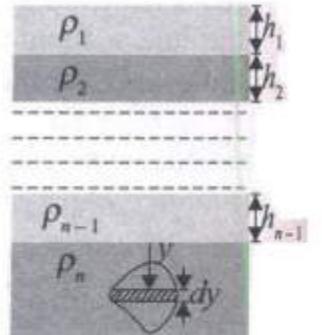
$$P = \rho_1 gh_1 + \rho_2 gh_2 + \dots + \rho_n gy \quad (4)$$

Substituting P from the equation (4) in equation (3), we have

$$dF = (\rho_1 gh_1 + \rho_2 gh_2 + \dots + \rho_n gy) dA$$

Then, the total hydrostatic force on the strip is

$$\begin{aligned} F &= \int dF = \int (\rho_1 gh_1 + \rho_2 gh_2 + \dots + \rho_n gy) dA \\ &= \sum_{i=1}^{i=n-1} (\rho_i gh_i) A + \rho_n g \int y dA \Rightarrow F = \left(\sum_{i=1}^{i=n-1} \rho_i h_i + \rho_n y_n \right) g A \end{aligned}$$



where y_n = Vertical distance of the centroid of the given area (patch) from the top of n th liquid.

10.5.4 Pascal's Law

Pressure applied to enclosed liquid is transmitted equally in all directions, to every position of liquid and wall of container. Brahma's hydraulic press is based upon Pascal's law of liquid pressure.

Unit of pressure is Pascal. Its symbol is Pa.

Bar = 105 Pa, torr = 1 mm of Hg column.

Density and Relative Density

1. Density = Mass/Volume

For water, density = 10^3 kg/m^3

2. One litre = $1000 \text{ cc} = 1000 \text{ cm}^3 = 1000 \times (10^{-2}\text{m})^3 = 10^{-3} \text{ m}^3$

3. Relative density = $\frac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}} = \frac{\text{Weight of substance in air}}{\text{Loss of weight in water}}$

Relative density, also known as specific gravity, has no unit, no dimension.

4. Density of mixture = $\frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right)} \frac{(m_1 + m_2)\rho_1\rho_2}{(m_1\rho_2 + m_2\rho_1)}$

where m denotes mass and ρ denotes density of liquid.

(a) If $m_1 = m_2 = m$, $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ or $\frac{2}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2}$

\therefore Density of mixture of two liquids is harmonic mean of the two densities.

(b) If $V_1 = V_2 = V$, $\rho = \rho_o \left[1 + \frac{\Delta P}{K} \right]$, where ΔP = Change in pressure,

K = Bulk modulus of elasticity of liquid.

10.5.5 Archimede's Principle

It states that when a solid body is immersed, partly or wholly, in a liquid at rest, it loses a weight which is equal to weight of the liquid displaced by the immersed portion of solid body.

1. Observed weight = True weight – Weight of liquid displaced

$$\text{or } T = Mg - mg = a\rho gh - a\sigma gh = ahg(\rho - \sigma)$$

$$\text{or } T = ahg\rho \left(1 - \frac{\sigma}{\rho} \right) = W \left(1 - \frac{\sigma}{\rho} \right)$$

where ρ = Density of solid, σ = Density of liquid

W = True weight of body

2. Weight of liquid displaced = Upthrust = Loss in weight of body.

Laws of Floatation

- The floating body will be in stable equilibrium when the metacentre lies above centre of gravity of body.
- The floating body will be in unstable equilibrium when the metacentre lies below centre of gravity of body.
- The floating body will be in neutral equilibrium when the meta centre coincides with centre of gravity of body.
- The upward force of upthrust acting on the body immersed in a liquid is known as Buoyant force of buoyancy.
- The centre of gravity of displaced liquid is known as centre of buoyancy.

If a person floats on his back on the surface of water, the apparent weight of the person is zero.

Law of floatation states that:

Weight of floating body = Weight of liquid displaced or
Volume of body immersed = Volume of liquid displaced.

1. If density of solid body is greater than density of liquid, the body will sink in the liquid. Here $d_s > d_L$ or $\rho > \sigma$.

Weight of body > Weight of liquid displaced.

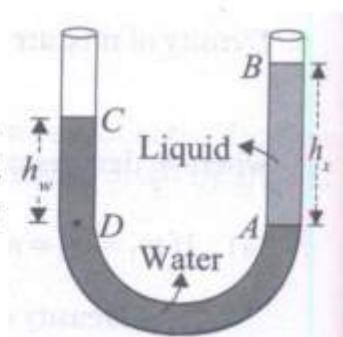
2. If $d_s < d_L$ or $\rho < \sigma$, the solid body floats on the liquid surface.

Here weight of body < Weight of liquid displaced.

3. If $d_s = d_L$ or $\rho = \sigma$, the body will stay at rest anywhere in the liquid.

Figure shows an open U-tube which contains some water and a less dense liquid poured in on the right side. If the density of the

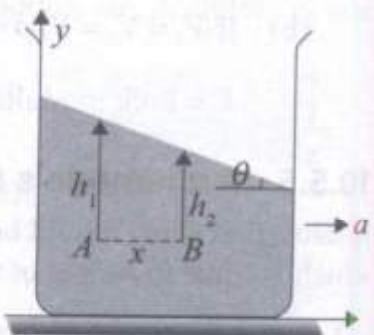
$$\text{unknown liquid is } \rho_x, \text{ then } \rho_x = \frac{\rho_w h_w}{h_x}$$



10.5.6 Variation of Pressure in a Liquid in a Container If the Container Is to Be Accelerated

Let A and B be two points in the same horizontal line at a separation x then, pressure difference is given by

$$P_A - P_B = \rho g(h_1 - h_2)$$



i.e., pressure along a horizontal line is not the same in case of horizontally accelerated liquid.

1. Initially the container is completely filled with a liquid of density ρ , now the container is given a uniform horizontal acceleration a . Pressures at different points will be as follows,

$$P_A = 0$$

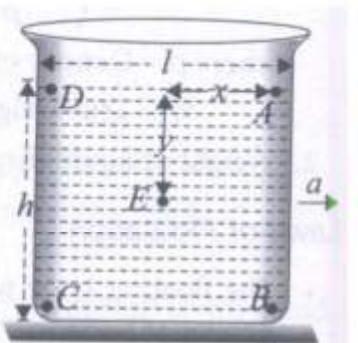
$$P_B = P_A + \rho gh = \rho gh$$

$$P_C = P_B + \rho al = \rho gh + \rho al$$

$$P_D = P_A + \rho al = \rho al$$

$$P_E = P_A + \rho gy + \rho ax = \rho gy = \rho ax$$

2. Initially the container is completely filled with a liquid of density ρ , now the container is given a uniform vertical acceleration a .



$$\begin{aligned} a &\uparrow \\ P &= \rho h (g + a) \\ \rho & \\ g_{\text{eff.}} &= (g + a) \end{aligned}$$

(a)

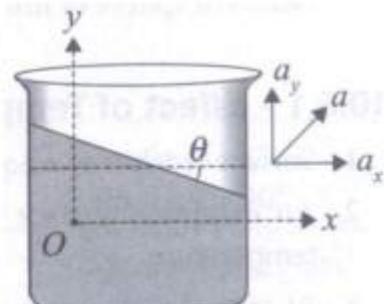
$$\begin{aligned} a &\downarrow \\ P &= \rho h (g - a) \\ \rho & \\ g_{\text{eff.}} &= (g - a) \end{aligned}$$

(b)

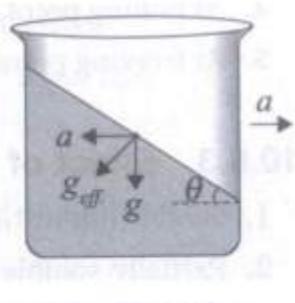
- (a) **Upward:** In this case the surface remains horizontal and pressure at every point increases.
- (b) **Downward:** If the container moves vertically downward then $g_{\text{eff}} = (g - a)$ depending on whether $g > a$ or $g < a$.
3. Initially the container is completely filled with a liquid of density ρ , now the container is given a uniform acceleration a in direction as in figure.

The pressure gradients along x , y and z direction are,

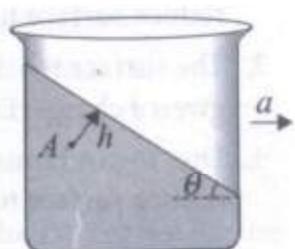
$$\frac{\partial P}{\partial x} = -\rho a_x; \frac{\partial P}{\partial y} = -\rho(a_y + g); \frac{\partial P}{\partial z} = 0$$



4. Due to acceleration of container, the free surface of liquid remains normal to the direction of effective gravity. The inclination angle of free surface of liquid from horizontal is $\theta = \tan^{-1}\left(\frac{a}{g}\right)$.
5. Pressure at every point in a liquid layer parallel to the free surface, remains same. e.g. if we find pressure at a point A in the accelerated container as shown in figure is given as



$$P_A = P_0 + h\rho\sqrt{a^2 + g^2}$$



where h is the depth of the point A below the free surface of liquid along effective gravity and P_0 is the atmospheric pressure acting on free surface of the liquid.

10.6 SURFACE TENSION

The free surface of every liquid has always a tendency to contract to a minimum possible surface area and thus behaves like a stretched membrane having a tension in all directions parallel to the surface. Thus, surface tension is the property of liquid by virtue of which its free surface behaves like a stretched membrane. The force which acts along the surface of a liquid, tending to contract its area to a minimum, is called the surface tension of the liquid.

The dimension of surface tension is $[MT^{-2}]$.

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}} = \frac{\text{Workdone}}{\text{Change in area}}$$

Unit of surface tension = Newton/Metre = Joule/(Metre) 2 .

- Surface tension is the molecular phenomenon. Cohesive force is the force of attraction between molecules of same substance. Adhesive force is the force of attraction between molecules of different substances.
Cohesive force (F_c) or adhesive force (F_a) is inversely proportional to eight power of the distance between the molecules.

i.e., $F_c \propto \frac{1}{r^8}$ or $F_a \propto \frac{1}{r^8}$, molecular range $\approx 10^{-9}$ m. It depends upon material. It is equal to the maximum distance upto which the molecules attract each other. Also it is equal to the radius of sphere of influence.

10.6.1 Effect of Temperature on Surface Tension

1. Surface tension of a liquid decreases with rise of temperature.
2. An exception-surface tension of molten cadmium or copper increases with increase in temperature.
3. At critical temperature, surface tension of a liquid becomes zero.
4. At boiling point, surface tension of a liquid becomes zero.
5. At freezing point, surface tension becomes maximum.

10.6.3 Effect of Impurity on Surface Tension

1. Soluble impurities cause increase in surface tension.
2. Partially soluble impurities cause decreases in surface tension. Soaps, detergents, phenol reduce surface tension of water.
3. The surface tension of the liquid decreases due to electrification. Soap bubble expands when given a charge. Due to charge, a normal force acts in the outward direction on liquid surface.
4. Due to contamination, surface tension decreases. Dust particles and lubricating materials reduce surface tension.

10.7 SURFACE ENERGY

To increase the surface area of liquid work has to be done against the force of surface tension. This additional potential energy stored per unit area of the surface is called surface energy. It can be shown that the surface energy per unit area is numerically equal to the surface tension of liquid.

10.7.1 Angle of Contact

A liquid is to be kept in a vessel and thus, liquid is in contact with some solid surface. For a pair of solid and liquid, the angle of contact θ is defined as the angle between tangent to the liquid surface drawn at the point of contact and the solid surface inside the liquid. The angle of contact does not depend upon the inclination of the solid in the liquid. The rise in temperature increases angle of contact. Addition of soluble impurities increases angle of contact.

The angle of contact lies between 0° and 180° .

If $\theta < 90^\circ$, the liquid will have a meniscus concave upwards.

If $\theta > 90^\circ$, the liquid surface will have a meniscus convex upwards.

If $\theta = 90^\circ$, the surface of liquid at the point of contact is plane.

The water proofing material increases the angle of contact. Acute angle is converted into obtuse angle.

- The angle of contact depends upon
 - The nature of solid and liquid in contact
 - The given pair of the solid and the liquid
 - The impurities and the temperature

Table 10.1 Angle of Contact, Meniscus, Shape of Liquid Surface

Property	Angle of Contact < 90°	Angle of Contact = 90°	Angle of Contact > 90°
Substances	Water and glass	Water and silver	Mercury and glass
Angle of contact	Almost zero or acute angle	Right angle = 90°	Obtuse angle = 130°
Meniscus shape	Concave	Plane	Convex
Capillary action	Liquid rises	No effect	Liquid falls
Sticking to solid	Sticks/Wets	Does not wet	Does not wet
Relation between F_a and F_c	$F_a > \frac{F_c}{\sqrt{2}}$ $F_a > F_c$	$F_a = \frac{F_c}{\sqrt{2}}$	$F_a < \frac{F_c}{\sqrt{2}}$ $F_a > F_a$
Shape of liquid surface	Almost round	Spreads on surface	Flat

10.7.2 Excess Pressure Due to Surface Tension

- The pressure on concave side of a curved liquid surface is greater than the pressure on convex side.
- Excess pressure inside a liquid drop $P = 2T/R$, where R is the radius of drop. A drop has only one surface of contact of liquid and air.
- Excess pressure inside a soap bubble: A bubble has two surfaces of contact of soap and air.
Excess pressure $P = 4T/R$.
- Excess pressure in air bubble inside a liquid $P = 2T/R$.

10.7.3 Radius of New Bubble When Two Bubbles Coalesce

Let r_1 and r_2 be the radii of two bubbles. Let them coalesce into a bubble of radius r , under isothermal conditions. Let T denote the surface tension,

$$P_1 = \frac{4T}{r_1}, \quad P_2 = \frac{4T}{r_2}, \quad P = \frac{4T}{r}$$

By Boyle's law,

$$P_1 V_1 + P_2 V_2 = PV$$

$$\left(\frac{4T}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(\frac{4T}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right) = \frac{4T}{r}\left(\frac{4}{3}\pi r^3\right)$$

$$r_1^2 + r_2^2 = r^2.$$

10.7.4 Radius of Interface

Consider two soap bubbles of radii r_1 and r_2 in contact with each other. Let r denote the radius of the common boundary/interface.

Let P_1 and P_2 denote the excess pressure on the two sides of the interface and let P denote the resultant excess pressure.

$$P = P_1 - P_2 \Rightarrow \frac{4T}{r} = \frac{4T}{r_1} - \frac{4T}{r_2} \Rightarrow \frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$$

Hence $r = \frac{r_1 r_2}{r_2 - r_1}$.

When two soap bubbles of equal radii coalesce, then the shape of resultant surface shall be plane.

$$r = \frac{r_1 r_2}{r_2 - r_1} = \frac{r_2}{\text{Zero}} = \text{Infinity} = \text{Plane surface}$$

10.7.5 Capillarity

A tube of very small radius is called a capillary. When such a clean tube of glass open at both ends is dipped vertically in water, the water rises in the tube upto a certain height above the water level outside the tube. On the other hand, if the tube is dipped in mercury, the mercury level falls below the outside level. The phenomenon of rise or fall of the liquids in a capillary tube is called capillarity.

Liquid rises (water in glass capillary) or falls (mercury in glass capillary) due to property of surface tension.

$$T = \frac{R\rho gh}{2\cos\theta}$$

where, R = Radius of capillary tube, h = Height of liquid.

ρ = Density of liquid, θ = Angle of contact.

T = Surface tension of liquid, g = Acceleration due to gravity.

10.7.6 Zurin's Law

$$Rh = \text{Constant} \Rightarrow R_1 h_1 = R_2 h_2$$

R = Radius of capillary tube, h = Height of liquid in capillary tube.

Tube of Insufficient Length

Liquid may rise to a height h but if the length of tube is less than this height, overflow of liquid does not occur. The liquid rises upto upper end and acquires a meniscus r' such that $r'h' = rh$.

If the tube kept in inclined position, then the vertical height to which liquid rises, remains the same.

$$\frac{h}{\cos\alpha} = l = \text{Inclined length}$$

When wax is coated on a glass capillary tube, it becomes water-proof. The angle of contact increases and becomes obtuse. Water does not rise in it. Rather it falls in the tube by virtue of obtuse angle of contact.

10.7.7 Poiseuille's Formula and Liquid Resistance

Volume of liquid flowing per second V through a horizontal capillary tube of length l , radius r ,

across a pressure difference P , under streamline motion, is given by $V = \frac{\pi P r^4}{8\eta l} = \frac{P}{R}$ and liquid resistance $R = \frac{8\eta l}{\pi r^4}$.

1. (a) If the two capillary tubes are joined in series, then the pressure $P = P_1 + P_2$ and V is same through the two tubes.
 (b) Equivalent liquid resistance, $R_s = R_1 + R_2$.
2. (a) If the two capillary tubes are joined in parallel, then $V = V_1 + V_2$ but pressure difference P is same across both tubes.
 (b) Equivalent liquid resistance, $R_p = \frac{R_1 R_2}{R_1 + R_2}$ or $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$.

Streamlined and Turbulent Flow

When a liquid flows in such a way that each liquid particle when passed through the same point follows exactly the same path as followed by the proceeding particles when passed through the same point then the flow is said to be streamlined and the path is called streamline.

The fluid flows in streamline only when its velocity is less than certain value called critical velocity. While the motion of the particles of the fluid are disorderly, if velocity is more than critical value, the disorder motion of fluid is called turbulent flow.

- Critical velocity, $v_c = \frac{K \cdot \eta}{r \cdot \rho}$

where K = Reynold's number, η = Coefficient of viscosity of liquid

r = Radius of capillary tube, ρ = Density of liquid

When $v \leq v_c$, the flow of liquid is streamlined. v_c denotes the maximum velocity of a liquid under streamline flow. Viscosity dominates the flow.

When $v > v_c$, the flow of liquid is turbulent. Such a flow is dominated by density of the fluid while viscosity has little effect on it. Lava, for example, is highly thick fluid emerging from a volcano with high speed.

Coefficient of Viscosity

When a layer of a liquid slips or tends to slip on another layer in contact, the two layers exert tangential force on each other. The directions are such that the relative motion between the layers is opposed. This property of a liquid to oppose relative motion between the layers is called viscosity.

Viscosity is due to transport of momentum. The value of viscosity (and compressibility) for ideal liquid is zero.

$$\text{Tangential force/Viscous drag } F = -\eta A \frac{dv}{dx}$$

where dv/dx denotes velocity gradient between two layers of liquid each of area A and η = Coefficient of viscosity of liquid.

- S.I. unit of η is decapoise = Nsm^{-2} or pascal-second.
c.g.s. unit η is poise = Dyne sec cm $^{-2}$.
Dimension of η = [ML $^{-1}$ T $^{-1}$].
- The cause of viscosity in liquids is the cohesive forces among molecules.
The cause of viscosity in gases is diffusion.
The viscosity of water is much higher than that of air. Hence it is more difficult to run through water than in air.
In heavy machines lubricating oils of high viscosity are used.
In light machines low viscosity oils are used for lubrication.

Factors Affecting Viscosity

1. Effect of temperature

- (a) If temperature increases, viscosity of liquid decreases. Viscosity of water, for example, at 80°C falls to one-third of its value at 10°C
- (b) Viscosity of gases remains constant at high pressure but in low pressure region the viscosity of gases is directly proportional to pressure.

2. Effect of pressure

- (a) If pressure increases, viscosity of liquid increases but viscosity of water decreases at few hundred atmosphere.
- (b) Viscosity of gases remains constant at high pressure but in low pressure region the viscosity of gases is directly proportional to pressure.

10.7.8 Stoke's Law and Terminal Velocity

Stoke's law relates the backward dragging force F acting on a small sphere of radius r moving through a viscous medium of viscosity η with the velocity v .

$$F = 6\pi\eta rv.$$

Terminal velocity is the maximum constant velocity of sphere of density ρ which falls freely in a viscous medium of density ρ_0 .

$$v = \frac{2}{9} \cdot \frac{r^2(\rho - \rho_0)g}{\eta}$$

1. If $\rho > \rho_0$, the body falls downwards.
2. If $\rho < \rho_0$ the body moves upwards with the constant velocity.
3. If $\rho_0 \ll \rho$, $v = \frac{2r^2 \rho g}{9\eta}$

10.7.9 Reynold's Number

$$K = \frac{v_c \cdot \rho \cdot r}{\eta}$$

1. $K = \frac{\text{Inertial force}}{\text{Viscous force}}$

2. For $K < 3000$, the flow of liquid is streamlined.
3. For $K > 5000$, the flow becomes turbulent.
4. K has no unit, no dimension. It is a pure number.
5. For narrow tubes and water, $K \approx 1000$.

Equation of Continuity

Equation of continuity is a special case of general law of conservation of matter stated as for a steady state flow of an ideal fluid (incompressible and non-viscous) in a pipe, the rate of mass flow across any cross section is constant. If two sections A_1 and A_2 at right angles to a tube of flow be considered at two different, then velocities of flow v_1 and v_2 respectively at these positions are related to the sections as $A_1 v_1 = A_2 v_2$.

Thus, when fluid flows through a smaller cross-section, its velocity increases. This is known as equation of continuity.

- A fluid in steady or streamline flow may possess any or all of the three types of energy:
 - **Kinetic energy:** Kinetic energy per unit volume = $\frac{1}{2} \rho v^2$
 - **Potential energy:** Potential energy per unit volume = ρgh
 - **Pressure energy:** Pressure energy of an incompressible fluid is because of its hydrostatic pressure P and pressure energy per unit volume = P .

10.7.10 Bernoulli's Theorem

It is the principle of conservation of energy for a flowing liquid. Under streamlined motion of a liquid, the sum total of pressure energy, kinetic energy and potential energy per unit volume at every point along its path remains constant.

$$\text{Mathematically, } P + \frac{1}{2} \rho r^2 + \rho gh = \text{Constant.}$$

Also, it is expressed as follows:

$$-\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{Constant where } \frac{P}{\rho g} = \text{Pressure head, } \frac{1}{2} \frac{v^2}{g} = \text{Velocity head, } h = \text{Potential head.}$$

The theorem is applicable to ideal liquid i.e., a liquid which is non-viscous, incompressible and irrotational.

Hence, at greater depth, P is large and so v is small. Liquid flows slow at greater depths. Deeper waters run slow accordingly.

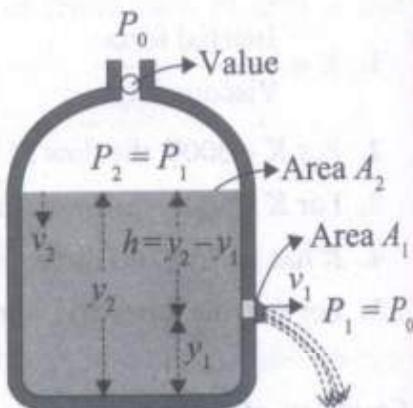
10.7.11 Torricelli's Theorem

A tank has a small hole in its side at a height y_1 . It is filled with a fluid of density ρ to a height y_2 . If the pressure at the top of the fluid is P_0 , assuming that the cross-sectional area of the tank is larger as compared to that of the hole, then

1. The velocity with which it leaves the tank:

From equation of continuity,

$$v_2 = \frac{A_1}{A_2} v_1$$



Since $A_1 \ll A_2$, v_2 must be very small compared to velocity of efflux at the hole, therefore we can take $v_2 \approx 0$. Fluid emerging from the hole is open to atmospheric pressure P_0 . We take two points A and B at the top of the fluid and at the hole respectively. From Bernoulli's principle,

$$\begin{aligned} P_0 + \frac{1}{2} \rho v_1^2 + \rho g y_1 &= P_t + \rho g y_2 \\ \Rightarrow \text{Speed of efflux } v_1 &= \sqrt{\frac{2(P_t - P_0)}{\rho} + 2gh} \\ \text{where } h &= y_2 - y_1 \end{aligned}$$

When the valve at the top of the tank is open to atmosphere,

$$P_t = P_0, \text{ the velocity is } v_1 = \sqrt{2gh}$$

Thus the speed of efflux of a liquid through a small hole in an open tank is equal to the velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called Torricelli's law.

If a pump is attached to the open valve so that pressure at the top of the tank is made large, then

$$\frac{2(P_t - P_0)}{\rho} \gg 2gh \quad \text{and} \quad v_1 = \sqrt{2 \left(\frac{P_t - P_0}{\rho} \right)}$$

It shows that the velocity of the emerging water/liquid depends on the magnitude of the applied pressure P_t .

2. The distance at which liquid strikes and also the maximum distance at which liquid strikes:

Let y_2 is the height of liquid in the open container. The time taken by liquid to hit the ground

$$(y_2 - h) = 0 + \frac{1}{2} g t^2 \quad \therefore \quad t = \sqrt{\frac{2(y_2 - h)}{g}}$$

The distance

$$x = v_1 t = \sqrt{2gh} \times \sqrt{\frac{2(y_2 - h)}{g}} = 2\sqrt{h(y_2 - h)}$$

For maximum x ,

$$\frac{dx}{dh} = 0 \quad \text{or} \quad \frac{d}{dx} [2\sqrt{h(y_2 - h)}] = 0$$

$$\Rightarrow h = \frac{y_2}{2} \quad \text{and} \quad x^{\max} = 2\sqrt{\frac{y_2}{2} \left(y_2 - \frac{y_2}{2} \right)} \quad \text{or} \quad x^{\max} = y_2$$

- (a) Velocity of efflux is the velocity acquired by a freely falling body in falling through a vertical distance h which is equal to depth of a hole, below free surface of liquid, from which liquid flows out.
- (b) Horizontal range when water issues out of a hole at depth h below surface of water in a tank filled with water upto a height H . Water issues out from A.

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- 1. JEE Advance, 2015
- 2. JEE (Mains), 2014
(Supported by Mathematician Dr. K. C. Sinha and Mr. Anand Kumar)
- 3. JEE Advance, 2013 4. IIT-JEE, 2012 5. IIT-JEE, 2011
- 6. AIEEE, 2012 7. AIEEE, 2011 8. J.A.C (XI), 2015
- 9. J.A.C (XII), 2014 (Admitted by J.A.C)
- 10. J.A.C (XII), 2015

And ALSO pointed out mistake in Official Solution released by IIT-JEE, 2011

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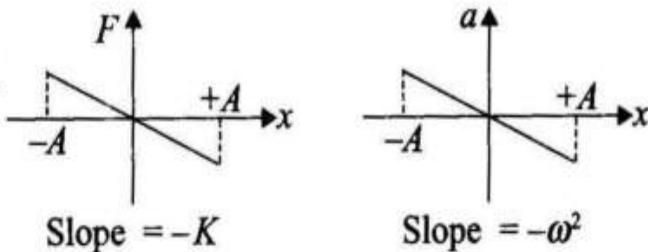
आईआईटी के एक सवाल को गलत साबित किया

ranchi@next.co.in

RANCHI (18 June): न्यूटन क्लासेस के हारिमेटा आर्क मॉलक ने एक बार फिर आईआईटी एडवांस के पेपर के एक सवाल को गलत साबित किया। उन्होंने 24 वां को शाम को ही वेबसर अलब्रेष्ट के एक सवाल को गलत बताया। आईआईटी ने भी जब 8 बजे को आगामी दिन डिक्लोर किया हो उस सवाल को सही बताया। आर्क मॉलक ने आईआईटी को पूछा: उस सवाल का ध्यान दिलाया तो 13 बजे को आईआईटी ने अपनी गतीय मात्री और साथ ही कॉलेजियम की 4 अधिकृत पार्किंग रेने ही दोष को आलोचित की गयी।

OSCILLATIONS AND WAVES (ACOUSTICS)

11.1 DIFFERENT EQUATIONS IN SHM



1. $F = -kx$, where k is force constant

$$2. a = \frac{F}{m} = -\left(\frac{k}{m}\right)x = -\omega^2 x$$

$$3. \omega = \sqrt{\frac{k}{m}} = \text{Angular frequency of SHM}$$

4. General solution of differential equation $\frac{d^2x}{dt^2} = -\omega^2 x$ is

$$x = A \sin(\omega t \pm \phi) \quad \text{or} \quad x = A \cos(\omega t \pm \phi)$$

Here, x is displacement from mean position (not x -coordinate), A is amplitude of oscillation or SHM, ϕ is phase angle at $t = 0$ (also called initial phase) and $(\omega t \pm \phi)$ is phase angle at a general time t , also called instantaneous phase.

$$5. \text{If } x = A \sin \omega t \text{ then, } v = \frac{dx}{dt} = \omega A \cos \omega t \text{ and } a = \frac{dv}{dt} = -\omega^2 A \sin \omega t$$

From these three equations we can see that $x-t$, $v-t$ and $a-t$ all three functions have simple harmonical variations with same angular frequency ω . Here, x oscillates between $+A$ and $-A$, v between $+\omega A$ and $-\omega A$ and a between $+\omega^2 A$ and $-\omega^2 A$.

6. Phase difference between $x-t$ and $v-t$ functions or between $v-t$ and $a-t$ functions is $\frac{\pi}{2}$. But phase difference between $x-t$ and $a-t$ function is π .

$$v = \omega \sqrt{A^2 - x^2} \quad \text{and} \quad a = -\omega^2 x$$

7. Potential energy $= U_0 + \frac{1}{2}kx^2$; Kinetic energy $= \frac{1}{2}k(A^2 - x^2)$ and total mechanical energy = $PE + KE = U_0 + \frac{1}{2}kA^2$

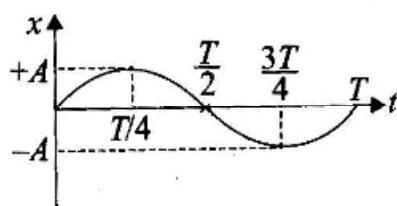
Here U_0 is minimum potential energy at mean position and $\frac{1}{2}kA^2$ or $\frac{1}{2}m\omega^2 A^2$ is called energy of oscillation. This much work is done on the system when displaced from mean position to extreme position. This much energy keeps on oscillating between potential and kinetic during oscillation.

Physical Quantity	At Mean Position	At Extreme Position	At General Point
Speed	ωA	zero	$\omega\sqrt{A^2 - x^2}$
Acceleration	zero	$\pm\omega^2 A$	$-\omega^2 x$
Force	zero	$\pm kA$	$-kx$
Kinetic energy	$\frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2$	zero	$\frac{1}{2}k(A^2 - x^2)$
Potential energy	U_0	$U_0 + \frac{1}{2}kA^2$	$U_0 + \frac{1}{2}kx^2$
Total mechanical energy	$U_0 + \frac{1}{2}kA^2$	$U_0 + \frac{1}{2}kA^2$	$U_0 + \frac{1}{2}kA^2$

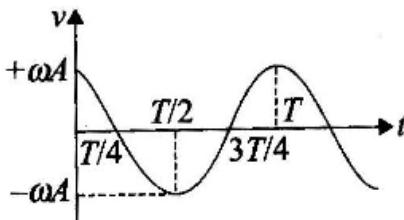
U_0 or minimum potential energy at mean position be zero also.

11.2 GRAPHS RELATED TO SHM

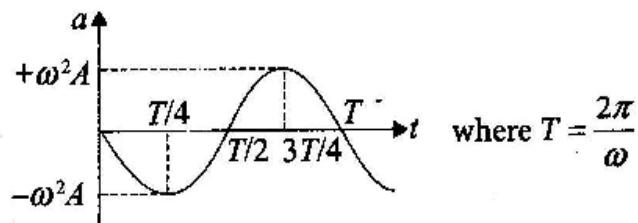
- If $x = A \sin \omega t$, then,



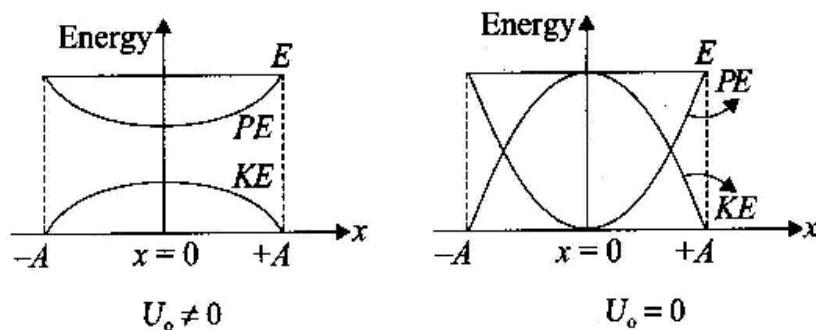
2. $v = \omega A \cos \omega t$,



3. $a = -\omega^2 A \sin \omega t$

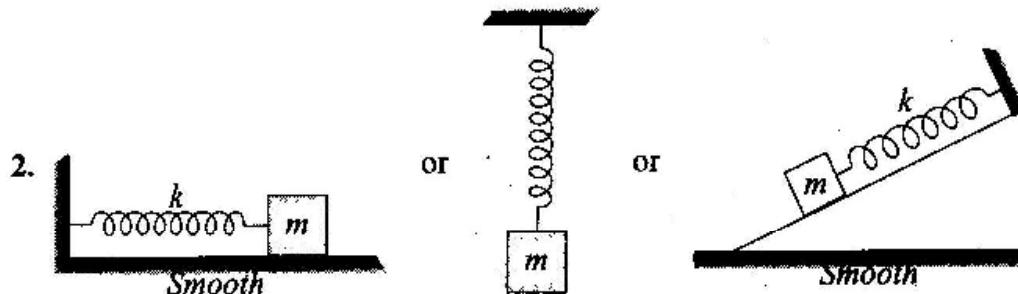


4. Potential energy versus x or kinetic energy versus x graph is parabola, while total energy versus x graph is a straight line as it remains constant.

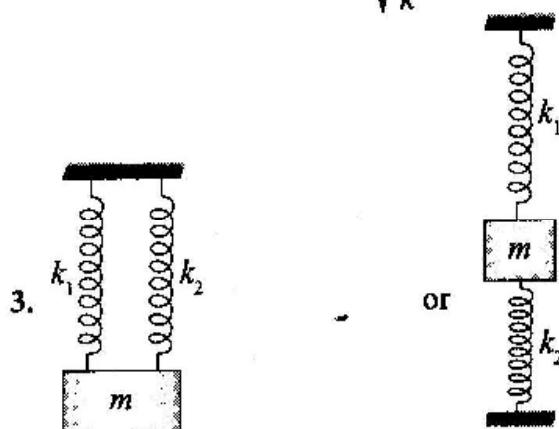


11.2.1 Spring Block System

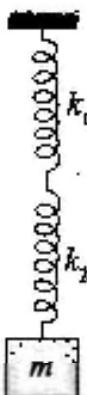
1. $\omega = \sqrt{\frac{k}{m}}$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ $v = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$



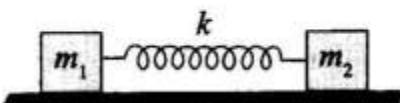
In all three cases, $T = 2\pi\sqrt{\frac{m}{k}}$



In both cases $k_{eq} = k_1 + k_2$ and $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$



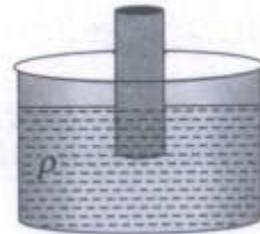
4. $k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$ or $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$



5. In case of two body oscillation,

$$T = 2\pi \sqrt{\frac{\mu}{k}} \text{ where, } \mu = \text{reduced mass of two blocks} = \frac{m_1 m_2}{m_1 + m_2}.$$

6. A plank of mass m and area of cross section A is floating in a liquid of density ρ . When depressed, it starts oscillating like a spring-block system.



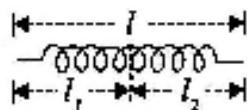
Effective value of k in this case is

$$k = \rho A g \quad \therefore \quad T = 2\pi \sqrt{\frac{m}{\rho A g}}$$

7. If mass of spring m_s is also given, then $T = 2\pi \sqrt{\frac{m + \frac{m_s}{3}}{k}}$

8. Every wire is also like a spring of force constant given by $k = \frac{YA}{l}$, where Y is the Young's modulus of the wire, A is cross-sectional area and l is the length of the wire.

9. If an ideal, uniform and massless spring of length l and spring constant k is cut into two pieces such that $l_1 / l_2 = n$, then the spring constant of the part having length l_1 and l_2 are respectively



$$k_1 = (1 + 1/n)k; \quad k_2 = (1 + n)k$$

Spring constant of a spring is inversely proportional to its length.

i.e., $k_1 l_1 = k_2 l_2 = k_3 l_3 = \dots = k l$

If length of spring is cut into N equal parts, then the spring constant of each part becomes (Nk) i.e., if length of spring is halved its spring constant will become two times.

11.2.2 Pendulum

- Only small oscillations of a pendulum are simple harmonic in nature. Time period of which is given by $T = 2\pi \sqrt{\frac{l}{g}}$
- Second's pendulum is one whose time period is 2s and length is 1 m.
- Time period of a pendulum of length of the order of radius of earth is

$$T = 2\pi \sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}} \quad \text{From here we can see that } T = 2\pi \sqrt{\frac{R}{g}} \text{ or } 84.6 \text{ min if } l \rightarrow \infty. \text{ Hence time}$$

period of a pendulum of infinite length is $2\pi \sqrt{\frac{R}{g}}$ or 84.6 min.

$$\text{Further, } T = 2\pi \sqrt{\frac{l}{g}} \quad \text{if } l \ll R \text{ or } \frac{1}{l} \gg \frac{1}{R}$$

- If point of suspension has an acceleration \vec{a} , then

$$T = 2\pi \sqrt{\frac{1}{|\vec{g}_e|}} \quad \text{Here } \vec{g}_e = \vec{g} - \vec{a} = \vec{g} + (-\vec{a})$$

For example if point of suspension has an upward acceleration \vec{a} , then $(-\vec{a})$ is downwards or parallel to \vec{g} . Hence,

$$|\vec{g}| = g + a \quad \text{or} \quad T = 2\pi \sqrt{\frac{1}{g+a}}$$

- If a constant force \vec{F} (in addition to weight and tension) acts on the bob then,

$$T = 2\pi \sqrt{\frac{l}{|\vec{g}_e|}} \quad \text{Here } \vec{g}_e = \vec{g} + \frac{\vec{F}}{m}$$

11.2.3 Physical Pendulum

$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

Here I = moment of inertia of the body about the point of suspension
 l = separation between the point of suspension and the centre of mass

11.3 SOME OTHER IMPORTANT POINTS CONCERNING SHM

- The simple harmonic oscillations may also be expressed as $y = A \sin \omega t + B \cos \omega t$, where A and B are constants related to the amplitude. We can write

$$y = A \sin \omega t + B \cos \omega t = A_R \sin(\omega t + \phi)$$

where $A_R = \sqrt{A^2 + B^2}$ and $\tan \phi = (B/A)$.

2. If a particle executes simple harmonic oscillations, then its velocity as well as acceleration also vary simple harmonically and velocity amplitude = $\omega \times$ displacement amplitude and acceleration amplitude = $\omega \times$ velocity amplitude = $\omega^2 \times$ displacement amplitude.
3. In SHM, the phase relationship between the displacement y , velocity (v) and acceleration (a) is as follows: (a) a leads y by $\pi/2$; (b) a lags behind the v by $\pi/2$; (c) a differs in phase with y by π .
5. When KE of SHM is 50 per cent of the total energy, the displacement is 71 per cent of the amplitude. At this stage, KE = PE.
6. Under weightlessness or in the freely falling lift $T = 2\pi\sqrt{L/0} = \infty$. This means, the pendulum does not oscillate at all.
7. The y , v , a of SHM vary simple harmonically with the same time period and frequency.
8. The KE as well as PE vary periodically but not simple harmonically in SHM. The frequency of KE or the PE is just two times that of y , v or a .
9. If a wire of length L , area of cross-section A , Young's modulus Y is stretched by suspending a mass m , then the mass can oscillate with time period,

$$T = 2\pi \sqrt{\frac{mL}{YA}}$$

10. If the lower surface of a cube of side L and modulus of rigidity η is fixed while fixing a particle of mass m on the upper face, a force parallel to upper face is applied to mass m and then withdrawn, the mass m can oscillate with a time period

$$T = 2\pi \sqrt{\frac{m}{\eta L}}$$

11. If a simple pendulum is suspended from the roof of a compartment of a train moving down an inclined plane of inclination θ , then the time period of oscillation is

$$T = 2\pi \sqrt{\frac{L}{g \cos \theta}}$$

12. If a ball of radius r oscillates in a bowl of radius R , then its time period of oscillation is

$$T = 2\pi \sqrt{\left(\frac{R-r}{g}\right)}$$

13. If a gas is enclosed in a cylinder of volume V_0 fitted with a piston of cross-section A and mass M and the piston is slightly depressed and released, the piston can oscillate with a frequency

$$f = \frac{1}{2\pi} \sqrt{\frac{BA^2}{MV_0}}$$

14. If a disc of radius r oscillates about a point at its rim, then its time period is given by

$$T = 2\pi \sqrt{\frac{r}{g}}$$

15. If a simple pendulum oscillates in a non-viscous liquid of density σ , then its time period is given by

$$T = 2\pi \sqrt{\frac{L}{\left(1 - \frac{\sigma}{\rho}\right)g}}$$

16. If the mass m attached to a spring oscillates in a non-viscous liquid of density σ , then its time period is given by

$$T = 2\pi \left[\frac{m}{k} \left(1 - \frac{\sigma}{\rho}\right) \right]^{1/2}$$

(where k = force constant and ρ = density of suspended mass).

11.3.1 Wave Equation

Wave motion is a form of disturbance (mode of momentum and energy transfer) which is caused due to the repeated periodic vibrations of the particles of the medium about their mean positions and the motion is handed over from one particle to the other without any net transport of the medium.

In any wave equation value of y is a function of position and time. In case of one dimensional wave position can be represented by one co-ordinate (say x) only. Hence, $y = f(x, t)$

Only those functions of x and t represent a wave equation which satisfy following condition

$$\frac{\partial^2 y}{\partial x^2} = (\text{constant}) \frac{\partial^2 y}{\partial t^2}; \quad \text{Here constant} = \frac{1}{v^2}$$

where v is the wave speed.

All functions of x and t of type, $y = f(ax \pm bt)$ satisfy above mentioned condition of wave equation, provided value of y should be finite for any value of t . If $y(x, t)$ function is of this type, then following two conclusions can be drawn.

1. Wave speed $v = \frac{\text{coefficient of } t}{\text{coefficient of } x} = \frac{b}{a}$

2. Wave travels along positive x -direction. If ax and bt have opposite signs and it travels along negative x -direction if they have same signs.

11.3.2 Plane Progressive Harmonic Wave

If oscillations of y are simple harmonic in nature then wave is called plane progressive harmonic wave. It is of two types (i) transverse wave, (ii) longitudinal wave

1. **Transverse wave:** A wave motion is said to be transverse if the particle of the medium through which the wave is travelling vibrate in a direction perpendicular to the direction of propagation of the wave.
2. **Longitudinal wave:** A wave motion is said to be longitudinal if the particle of the medium through which the wave is travelling vibrate in a direction parallel to the direction of propagation of the wave.

General equation of progressive harmonic wave is,

$$y = A \sin(\omega t \pm kx \pm \phi) \quad \text{or} \quad y = A \cos(\omega t \pm kx \pm \phi)$$

In these equations,

1. A is amplitude of oscillation,
2. ω is angular frequency and v is frequency of oscillation.

$$T = \frac{2\pi}{\omega} \quad \omega = 2\pi v \quad \text{and} \quad v = \frac{1}{T} = \frac{\omega}{2\pi}$$

3. k is wave number, $k = \frac{2\pi}{\lambda}$ ($\lambda \rightarrow$ wavelength)

(because its value is number of waves in 2π metres)

4. Wave speed $v = \frac{\omega}{k} = v\lambda$ (it is valid for waves of any kind)

5. ϕ is initial phase angle, and

6. $(\omega t \pm kx \pm \phi)$ is phase angle at time t at co-ordinate x .

(a) The *frequency* of a wave is always determined solely by the wave source. Thus, once a wave is formed, its frequency does not change even though the wave passes from one medium to another.

(b) The *speed of wave* is determined by the medium through which the wave travels.

(c) Wave speed in a given medium may depend on the frequency of the wave; that is waves of some frequencies travel faster than waves of other frequencies. This phenomenon is called *dispersion*.

(d) A *wavelength* depends on the source and the medium both.

(e) There are three categories of longitudinal mechanical waves which cover different ranges of frequencies: (i) Sound waves or audible waves (between 20 Hz to 20 kHz), (ii) Infrasonic waves (frequencies below 20 Hz) and (iii) Ultrasonic waves (frequency above 20 kHz).

Particle Speed (v_p) and Wave Speed (v)

In case of harmonic wave

1. $y = f(x, t)$ where x and t are two variables. So, $v_p = \frac{\partial y}{\partial t}$

2. In harmonic wave, particles are in SHM.

Therefore, all equations of SHM can be applied for particles also.

3. Relation between v_p and v is $v_p = -v \frac{\partial y}{\partial x}$

Phase Difference ($\Delta\phi$)**Case-I:** $\Delta\phi = \omega(t_1 - t_2)$

or $\Delta\phi = \frac{2\pi}{T} \Delta t$

= phase difference of one particle at a time interval of Δt .**Case-II:** $\Delta\phi = k(x_1 - x_2) = \frac{2\pi}{\lambda} \Delta x$ = phase difference at one time between two particles at a path difference of Δx .**Energy Density (u), Power (P) and Intensity (I)**

In harmonic wave

1. Energy density $u = \frac{1}{2} \rho \omega^2 A^2$ = energy of oscillation per unit volume.
2. Power $P = -\rho \omega^2 A^2 S v$ = energy transferred per unit time.
3. Intensity $I = \frac{1}{2} \rho \omega^2 A^2 v$ = energy transferred per unit time per unit area, here S is the cross sectional area.

11.3.3 Longitudinal Wave

1. There are three equations associated with any longitudinal wave

$$y(x, t), \Delta P(x, t) \text{ and } \Delta \rho(x, t)$$

2. y represents displacement of medium particles from their mean position parallel to direction of wave velocity.
3. From $y(x, t)$ equation, we can make $\Delta P(x, t)$ or $\Delta \rho(x, t)$ equations by using the fundamental relation between them,

$$\Delta P = -B \frac{\partial y}{\partial x} \quad \text{and} \quad \Delta \rho = -\rho \frac{\partial y}{\partial x}$$

4. ΔP_0 = pressure amplitude = $B A k$ and $\Delta \rho_0$ = density amplitude = $\rho A k$.
5. $\Delta P(x, t)$ and $\Delta \rho(x, t)$ are in same phase. But $y(x, t)$ equation has a phase difference of $\frac{\pi}{2}$ with rest two equations.

11.3.4 Wave Speed

1. Speed of transverse wave on a stretched wire, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho S}}$

- Sound is a form of energy which is emitted by a vibrating source and transmitted through a material medium producing in us the sensation of hearing. Sound waves are longitudinal.

2. Speed of longitudinal wave, $v = \sqrt{\frac{E}{\rho}}$

(a) In solids, $E = Y$ = Young's modulus of elasticity

$$\therefore v = \sqrt{\frac{Y}{\rho}}$$

(b) In liquids, $E = B$ = Bulk modulus of elasticity

$$\therefore v = \sqrt{\frac{B}{\rho}}$$

(c) In gases, according to Newton,

$E = B_T$ = Isothermal bulk modulus of elasticity = P

$$\therefore v = \sqrt{\frac{P}{\rho}}$$

But results did not match with this formula.

Laplace made correction in it. According to him,

$E = B_s$ = Adiabatic bulk modulus of elasticity = γP

$$\therefore v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma kT}{m}}$$

Factors Affecting Speed of Sound in Air or Gas

1. With temperature $v \propto \sqrt{\frac{\gamma kT}{m}} \Rightarrow v \propto \sqrt{T}$

with rise in temperature velocity of sound in a gas increases.

The velocity of sound in a given gas will become n times if temperature is changed from T to T' such that $T' = n^2 T$.

When change in temperature is small,

$$\frac{v'}{v_0} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{273+t}{273+0}} = \left[1 + \frac{t}{273}\right]^{1/2} \approx 1 + \frac{t^\circ C}{546}$$

i.e., $v' = v_0 \left(1 + \frac{t^\circ C}{546}\right) = [332 + 0.61(t^\circ C)] \text{ m/s}$

i.e., for small temperature variations at $0^\circ C$, the velocity of sound changes by 0.61 m/s when temperature changes by $1^\circ C$.

2. Pressure has no effect on speed of sound as long as temperature remains unchanged.
3. With increase in relative humidity in air, density decreases. Hence, speed of sound increases.

11.3.5 Echo

1. Sound persists on ear for 0.1 sec.

Velocity of sound in air = 330 m/s.

Distance covered by the sound = $330 \times 0.1 = 33$ m.

Distance between source of sound and reflector = 16.5 m

2. Let the first echo be heard after t_1 sec.

Let the second echo be heard after t_2 sec.

Then the third echo will be heard after $(t_1 + t_2)$.

3. *Articulate sound:* The sound produced by human beings is called articulate sound. A person can emit 5 syllables in one second. Each syllable is produced in 0.2 second.

Distance travelled by sound = $0.2 \times 330 = 66$ m.

Reflector should be at a distance = 33 m from person.

Sound Level (L)

$$L = 10 \log_{10} \frac{I}{I_o} \text{ (in dB)}$$

Here I_o = intensity of minimum audible sound = 10^{-12} watt/m²

While comparing loudness of two sounds we may write

$$L_2 - L_1 = 10 \log_{10} \frac{I_2}{I_1}$$

In case of point source, $I \propto \frac{1}{r^2}$ or, $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$

In case of line source, $I \propto \frac{1}{r}$ or, $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)$

However as human ear responds to sound intensities over a wide range, i.e., from 10^{-12} W/m² to 1 W/m², so instead of specifying intensity of sound in W/m², we use a logarithmic scale of intensity called *sound level* defined as, $L = 10 \log [I/I_o]$, where I_o is the threshold of human ear, i.e., 10^{-12} W/m². The sound level defined in this way is expressed in decibel (dB).

A sound of intensity I_o has an $L = 10 \log (I_o/I_o) = 0$ dB while sound at the upper range of human hearing called *threshold of pain* has an intensity of 1 W/m² or a $L = 10 \log (1/10^{-12}) = 120$ dB.

Doppler Effect in Sound

$v' = v \left(\frac{v \pm v_m \pm v_o}{v \pm v_m \pm v_s} \right)$ where v is the speed of sound, v_m is the speed of medium, v_o is the speed of observer and v_s is speed of source.

- For supersonic speeds of source or observer, v_s and v_o should be less than v i.e., $v_s < v$ and $v_o < v$.
- There should be a relative motion between source and observer for Doppler's effect to apply.
- This effect fails when source and observer move at right angles to each other. It is called Transverse Doppler's effect.
- Transverse Doppler's effect holds good for optical waves.

Beats

When two notes (or two sound waves) of nearly equal frequency travelling in same direction superpose each other at a given point, then the intensity of the resulting sound rises and falls periodically. This periodic rise and fall in the intensity of sound at a given point is called as *beats*.

- $v_b = v_1 - v_2$ ($v_1 > v_2$)
- Intensity of sound at a given point is not constant but varies periodically with time, i.e., interference is not sustained. As the persistence of ear is about $(1/10)$ sec., beats will be detected by the ear only when frequency $(v_1 - v_2) < 10$ Hz.
- If beat frequency is more than 10, beats produced will be more than 10 per sec but heard zero, as there will be continuous sound of intensity $I_{\text{average}} = [(I_{\max} + I_{\min})/2] = I_1 + I_2$ instead of waxing and waning of sound.
- Phenomenon of beats is used by musicians in tuning their instruments. They sound their instruments along with a standard source and tune them till beats are no more heard. In this situation, frequency of the instrument becomes equal to that of standard source.
- If two sound waves of wavelengths λ_A and λ_B produce v_b beats, then the velocity of sound is given by

$$\frac{v}{\lambda_A} \sim \frac{v}{\lambda_B} = v_b \text{ or } v = \frac{v_b \lambda_A \lambda_B}{\lambda_B - \lambda_A}$$

- If two open resonating air columns (closed) produce v_b beats per second, then velocity of sound is given by

$$\frac{v}{2l_1} \sim \frac{v}{2l_2} = v_b \text{ or } v = \left(\frac{2v_b l_1 l_2}{l_2 - l_1} \right) \text{ (for open pipe)}$$

and $v = \left(\frac{4v_b l_1 l_2}{l_2 - l_1} \right) \text{ (for closed pipe)}$

11.3.6 Stationary Waves

- Stationary waves are formed by the superposition of two identical waves travelling in opposite directions.
- Formation of stationary waves is really the interference of two waves in which coherent (same frequency) sources are required.
- By the word *identical waves* we mean that they must have same value of v , ω , and k . Amplitudes may be different, but same amplitudes are preferred.

4. In stationary waves, all particles oscillate with same value of ω but amplitudes varying from $(A_1 + A_2)$ to $(A_1 - A_2)$. Points where amplitude is maximum (or $A_1 + A_2$) are called *antinodes* (or points of constructive interference) and points where amplitude is minimum (or $A_1 - A_2$) are called *nodes* (or points of destructive interference).
5. If $A_1 = A_2 = A$, then amplitude at antinode is $2A$ and at node is zero. In this case, points at node do not oscillate.
6. Points at antinodes have maximum energy of oscillation and points at nodes have minimum energy of oscillation (zero when $A_1 = A_2$).
7. Points lying between two successive nodes are in same phase. They are out of phase with the points lying between two neighbouring successive nodes.
8. Equation of stationary wave is of type,

$$y = 2A \sin kx \cos \omega t \quad \text{or} \quad y = A \cos kx \sin \omega t \text{ etc.}$$

This equation can also be written as

$$y = A_x \sin \omega t \text{ or } y = A_x \cos \omega t$$

If $x = 0$ is an antinode then, $A_x = A_o \sin kx$

If $x = 0$ is an antinode then, $A_x = A_o \cos kx$

Here A_o is maximum amplitude at antinode.

9. Energy of oscillation in a given volume can be obtained either by adding energies due to two individual waves travelling in opposite directions or by integration. Because in standing wave amplitude and therefore energy of oscillation varies point to point.

Oscillations of Stretched Wire or Organ Pipes

1. Stretched wire:

Velocity of a transverse wave in stretched string = v

$$v = \sqrt{\frac{T}{\mu}} \quad \text{where } T \text{ denotes tension and } \mu \text{ denotes mass per unit length of string}$$

$$\mu = \frac{\text{Mass of string}}{\text{Length of string}} = \frac{\text{Volume} \times \text{density}}{l}$$

$$\mu = \frac{(\pi R^2 l) \rho}{l} = \pi R^2 \rho \quad \therefore \quad v = \sqrt{\frac{T}{\pi R^2 \rho}} = \frac{1}{R} \sqrt{\frac{T}{\pi \rho}}$$

It means that the frequency produced on the stretched wire is

$$v = n \times \frac{1}{2l} \sqrt{\frac{T}{\mu}}$$

$$v \propto \frac{1}{l}, v \propto \sqrt{T}, v \propto \frac{1}{\sqrt{\mu}}, v \propto \frac{1}{R}, v \propto \frac{1}{\sqrt{\rho}}$$

Law of loops (p) for a given length and given frequency is

$$p\sqrt{T} = \text{constant} \Rightarrow \frac{p_1^2}{p_2^2} = \frac{T_2}{T_1}. \text{ This is Melde's law.}$$

In longitudinal mode, vibrations of the prongs of tuning fork are along the length of the string. Frequency of vibration of string = $\frac{\text{frequency of tuning fork}}{2}$

$$v_L = \frac{p}{l} \sqrt{\frac{T}{\mu}}$$

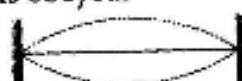
In transverse mode, vibrations of tuning fork are at 90° (right angles) to the length of string.

Frequency of vibration of string = frequency of tuning fork

$$v_T = \frac{p}{2l} \sqrt{\frac{T}{\mu}} = \frac{v_L}{2}$$

In both the modes, Melde's law ($p^2 T = \text{constant}$) is obeyed.

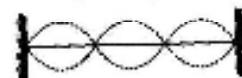
Fundamental tone or first harmonic ($n = 1$)



First overtone or second harmonic ($n = 2$)



Second overtone or third harmonic ($n = 3$)



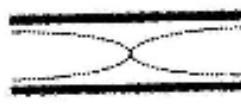
$$v = n \left(\frac{v}{2l} \right) \quad \text{Here, } n = 1, 2, 3, \dots$$

Even and odd both harmonics are obtained.

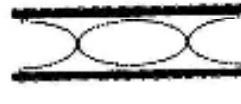
$$\text{Here, } v = \sqrt{\frac{T}{\mu}} \quad \text{or} \quad \sqrt{\rho S}$$

2. Open organ pipe:

Fundamental tone or first harmonic ($n = 1$)



First overtone or second harmonic ($n = 2$)



Second overtone or third harmonic ($n = 3$)



$$v = n \left(\frac{v}{2l} \right) \quad n = 1, 2, 3, \dots$$

Even and odd both harmonics are obtained.

Here, v = speed of sound in air.

v will be either given in the question, otherwise,

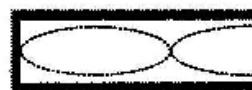
$$\text{Calculate from } v = \sqrt{\frac{\gamma RT}{M}}.$$

3. Closed organ pipe:

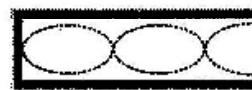
Fundamental tone or first harmonic ($n = 1$)



First overtone or third harmonic ($n = 3$)



Second overtone or fifth harmonic ($n = 5$)



$$v = n \left(\frac{v}{4l} \right) \quad n = 1, 3, 5, \dots$$

4. Stationary transverse waves are formed in stretched wire and longitudinal stationary waves are formed in organ pipes.
5. Open end of pipe is displacement antinode, but pressure and density nodes. Closed end of pipe is displacement node, but pressure and density antinodes.
6. Factors on which frequency of pipe depends

- (a) length of air column, $v \propto 1/l$
- (b) radius of air column, $v \propto 1/r$
- (c) velocity of sound in air column, $v \propto v$
- (d) temperature of air column, $v \propto \sqrt{T}$
- (e) pressure of air inside air column, $v \propto \sqrt{P}$
- (f) density of air, $v \propto 1/\sqrt{\rho}$

$$\text{Generally speaking, } v \propto \frac{v\sqrt{TP}}{lr\sqrt{\rho}} \propto \frac{v}{lr} \sqrt{\frac{TP}{\rho}}.$$

6. Laplace correction $e = 0.6 r$ (in closed organ pipe) and $2e = 1.2 r$ (in open organ pipe)

$$\text{Hence, } v = n \left[\frac{v}{2(l+1.2r)} \right] \text{ (in open organ pipe) and}$$

$$v = n \left[\frac{v}{4(l+0.6r)} \right] \text{ (in closed organ pipe).}$$

7. If an open organ pipe and a closed organ pipe are of same lengths then fundamental frequency of open organ pipe is two times the fundamental frequency of closed organ pipe.
8. *Resonance tube:* It is an example of a closed organ pipe. Water level acts as reflector and provides closed end.

$$\text{For first resonance, } \frac{\lambda}{4} = l_1 + x \quad (1)$$

$$\text{For second resonance, } \frac{3\lambda}{4} = l_2 + x \quad (2)$$

$$\therefore \lambda = 2(l_2 - l_1) \quad \text{Velocity of sound} = v\lambda$$

$$v = 2\lambda(l_2 - l_1)$$

$$\text{End-correction, eliminate } \lambda \text{ from (i) and (ii)} \quad x = \frac{l_2 - 3l_1}{2}$$

At resonance, frequency of tuning fork and the frequency of air column become equal. Amplitude of vibration of air column becomes large and a loud sound is produced at resonance.

- Seismic waves produced during earthquakes consist of transverse waves (*shear waves*) as well as longitudinal waves (*pressure waves*). Wave through an extended solid, e.g., crust of Earth is

$$\text{called Primary Earthquake waves, i.e., } P\text{-waves and its speed } v = \sqrt{\frac{B + (4/3)\eta}{\rho}}$$

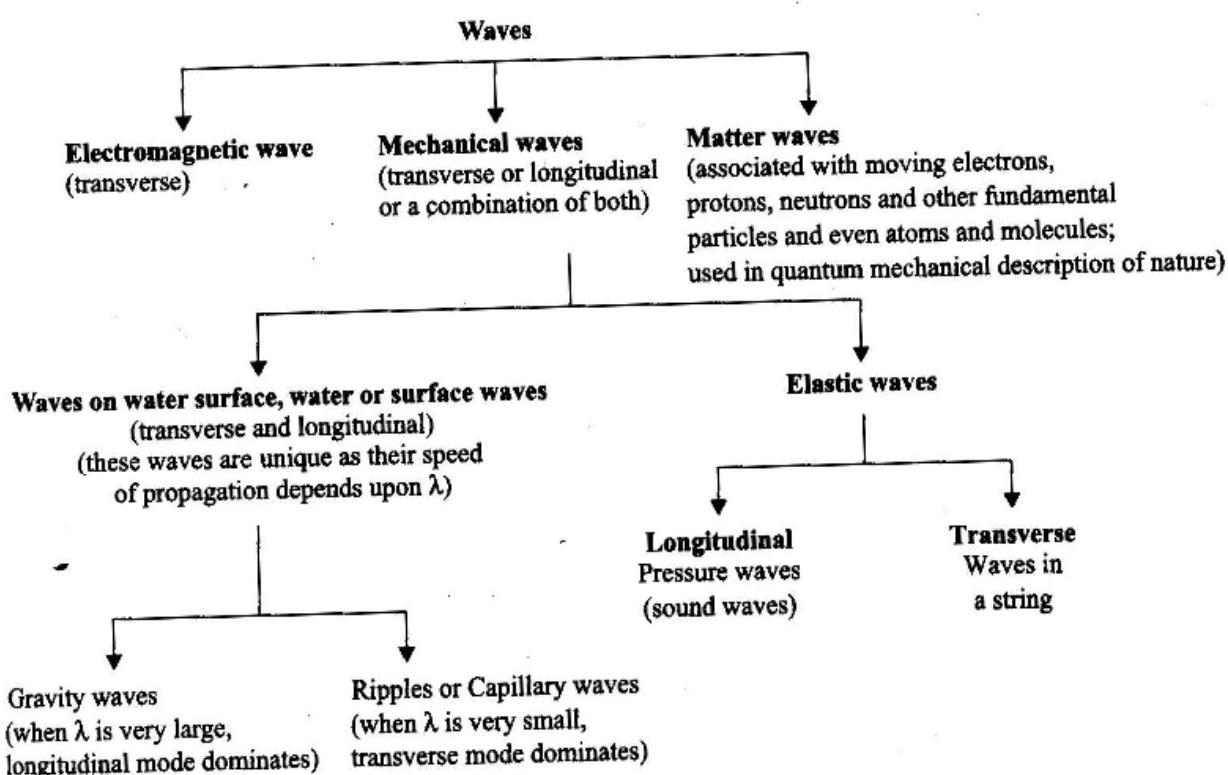
where B = bulk modulus of elasticity,

η = shear modulus of elasticity.

- Ripples formed on the surface of a liquid are a combination of transverse and longitudinal waves although transverse waves are predominant. These waves are unique in the sense that their velocity of propagation depends upon their wavelength. Speed of ripples,

$$v = \sqrt{\frac{2\pi T}{g\lambda}} \quad (\text{when } \lambda \text{ is very small}).$$

(ρ = density of the liquid, T = surface tension of the liquid, g = acceleration due to gravity)



THERMOMETRY AND THERMODYNAMICS

12.1 TEMPERATURE

1. Temperature is a macroscopic physical quantity related to our sense of hot and cold.
2. Temperature is basically a measure of degree of hotness or coldness of a body.
3. Two bodies are said to be in thermal equilibrium with each other when no heat flows from one body to the other, i.e., when both the bodies are at the same temperature.
4. Temperature of a body cannot be lowered up to any extent while it can be raised up to any value. Theoretical lowest temperature is considered to be absolute zero. Highest possible temperature achieved in laboratory is about 10^8 K while lowest possible temperature attained is 10^{-8} K.
5. Branch of physics dealing with production and measurement of temperatures close to 0 K is known as *cryogenics* while that dealing with the measurement of very high temperatures is called as *pyrometry*.

Table 12.1 Temperature Scales

Name of the Scale	Symbol for Each Degree	Lower Fixed Point (LFP)	Upper Fixed Point (UFP)	Number of Divisions on the Scale
Celsius	°C	0°C	100°C	100
Fahrenheit	°F	32°F	212°F	180
Reaumur	°R	0°R	80°R	80
Rankine	°Ra	460°Ra	672°Ra	212
Kelvin	K	273.15 K	373.15 K	100

Temperature on one scale can be converted into other scale by using the following identity:

$$\frac{\text{Reading on any scale} - \text{Lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{Lower fixed point (LFP)}} = \text{Constant for all scales}$$

$$\text{Hence, } \frac{t^\circ C - 0^\circ}{100^\circ - 0^\circ} = \frac{t^\circ F - 32^\circ}{212^\circ - 32^\circ} = \frac{t^\circ R - 0^\circ}{80^\circ - 0^\circ}$$

$$= \frac{t^\circ Ra - 460^\circ}{672^\circ - 460^\circ} = \frac{tK - 273.15}{373.15 - 273.15}$$

Table 12.2 Different Types of Thermometers

Type of Thermometer	Thermometric Quantity	Formula	Range of Temperature
Liquid thermometer	Based on thermal expansion of a liquid in a narrow tube	$t_c = \frac{l_t - l_o}{l_{100} - l_o} \times 100^\circ C$	-50°C to 350°C
(a) Mercury			
(b) Alcohol	- do -	- do -	Lower range is extended to -80°C
Gas thermometer (Constant pressure or constant volume)	Pressure or volume	$t_c = \frac{P_t - P_o}{P_{100} - P_o} \times 100^\circ C$ $t_c = \frac{V_t - V_o}{V_{100} - V_o} \times 100^\circ C$	
(a) Hydrogen	- do -	- do -	-200°C to 500°C
(b) Nitrogen	- do -	- do -	-200°C to 1600°C
(c) Helium	- do -	- do -	-268°C to 500°C
Platinum resistance thermometer	Resistance	$t_c = \frac{R_t - R_o}{R_{100} - R_o} \times 100^\circ C$	-200°C to 1200°C
Thermoelectric thermometers	Thermo e.m.f.	$e = at + bt^2$	-200°C to 1600°C

12.1.1 Thermal Expansion

1. $L_t = L_o(1 + \alpha t)$, $\Delta L = L\alpha \Delta \theta$

(i) In differential form α can be written as; $\alpha = \frac{1}{L_o} \frac{dL}{dt}$

(ii) The above equation is applicable for small value of t ($t < 100^\circ C$), till the expansion of the material can be taken uniform. For higher range of temperature, L_t is given by

$$L_t = L_o(1 + \alpha_1 t + \alpha_2 t^2 + \dots), \text{ where } \alpha_1 > \alpha_2 > \dots$$

2. $A_t = A_o(1 + \beta t)$, $\Delta A = A\beta \Delta \theta$, and in differential form α can be written as $\beta = \frac{1}{A_o} \frac{dA}{dt}$

3. $V_t = V_o(1 + \gamma t)$, $\Delta V = V\gamma \Delta \theta$, and in differential form α can be written as $\gamma = \frac{1}{V_o} \frac{dV}{dt}$

4. $\beta = 2\alpha$ and $\gamma = 3\alpha$ for isotropic medium.

12.1.2 Effect of Temperature on Different Physical Quantities

1. With increase in temperature volume of any substance increases while mass remains constant, therefore density should decrease.

$$\rho' = \frac{\rho}{1 + \gamma \Delta \theta} \quad \text{or} \quad \rho' \approx \rho (1 - \gamma \Delta \theta) \text{ if } \gamma \Delta \theta \ll 1$$

2. When a solid whose density is less than the density of liquid is floating, then a fraction of it remains immersed. This fraction is $f = \frac{\rho_s}{\rho_l}$.

When temperature is increased, ρ_s and ρ_l both will decrease. Hence, fraction may increase, decrease or remain same. At higher temperature,

$$f' = f \left(\frac{1 + \gamma_l \Delta \theta}{1 + \gamma_s \Delta \theta} \right)$$

If $\gamma_l > \gamma_s$, $f' > f$ or immersed fraction will increase.

3. When a solid whose density is more than the density of liquid is immersed completely, then upthrust will act on 100% volume of solid and apparent weight appears less than the actual weight.

$$W_{\text{apparent}} = W - F$$

Here,

$$F = V_s \rho_l g$$

With increase in temperature V_s will increase and ρ_l will decrease, while g will remain unchanged. Therefore upthrust may increase, decrease or remain same. At some higher temperature,

$$F' = F \left(\frac{1 + \gamma_s \Delta \theta}{1 + \gamma_l \Delta \theta} \right)$$

If $\gamma_s > \gamma_l$, upthrust will increase. Therefore, apparent weight will decrease.

4. Time period of pendulum:

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \text{or} \quad T \propto \sqrt{l}$$

With increase in temperature, length of pendulum will increase. Therefore time period will increase. A pendulum clock will become slow and it loses the time. At some higher temperature,

$$T' = T(1 + \alpha \Delta \theta)^{\frac{1}{2}} \quad \text{or} \quad T' \approx T \left(1 + \frac{1}{2} \alpha \Delta \theta \right) \text{ if } \alpha \Delta \theta \ll 1$$

$$\Delta T = (T' - T) = \frac{1}{2} T \alpha \Delta \theta$$

$$\therefore \text{Time lost/gained } \Delta t = \frac{\Delta T}{T'} \times t$$

5. **Thermal stress:** If temperature of a rod fixed at both ends is increased, then thermal stresses are developed in the rod.

At some higher temperature we may assume that the rod has been compressed by a length,

$$\Delta l = l\alpha \Delta \theta \quad \text{or} \quad \text{Strain } \frac{\Delta l}{l} = \alpha \Delta \theta$$

$$\text{Stress} = Y \times \text{Strain} = Y \alpha \Delta \theta \quad (Y = \text{Young's modulus of elasticity})$$

$$F = A \times \text{stress} = YA\alpha\Delta\theta$$

Rod applies this much force on wall to expand. In turn, wall also exerts equal and opposite pair of encircled forces on rod. Due to this pair of forces only, we can say that rod is compressed.

12.2 KINETIC THEORY OF GASES

Claussius and Maxwell developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules.

The theory is based on the following assumptions:

1. The number of molecules in the gas is large and the average separation between them is large compared with their dimensions.
2. The molecules obey Newton's law of motion, but as a whole they move randomly.
3. The molecules interact only by short range forces during elastic collisions.
4. All the collisions between molecules among themselves or between molecules and the walls are elastic.
5. The gas under consideration is a pure substance, that is, all molecules are identical.
6. The duration of a collision is negligible compared to the time spent by the molecules between collision.
7. The molecules exert no force on each other or on the walls of the container except during collision.

Different equations used in kinetic theory of gases are listed below,

1. $PV = nRT = \frac{m}{M}RT$ (m = Mass of gas in gms)
2. Density $\rho = \frac{m}{V}$ (General), $\rho = \frac{PM}{RT}$ (for ideal gas)
3. Gas laws:

(a) Boyle's law is applied when $T = \text{Constant}$ or process is isothermal. In this condition,

$$PV = \text{Constant} \quad \text{or} \quad P_1 V_1 = P_2 V_2 \quad \text{or} \quad P \propto \frac{1}{V}$$

(b) Charles' law is applied when $P = \text{Constant}$ or process is isobaric. In this condition,

$$\frac{V}{T} = \text{Constant} \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \text{or} \quad V \propto T$$

(c) Pressure law or Gay Lussac's law is applied when $V = \text{Constant}$ or process is isochoric.

$$\text{In this condition, } \frac{P}{T} = \text{Constant} \quad \text{or} \quad \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \text{or} \quad P \propto T$$

4. Four speeds, $V = \sqrt{\frac{ART}{M}} = \sqrt{\frac{AkT}{m}} = \sqrt{\frac{AP}{\rho}}$

Here, m = Mass of one gas molecules

$A = 3$ for rms speed of gas molecules

$$= \frac{8}{\pi} \approx 2.5 \text{ for average speed of gas molecules}$$

= 2 for most probable speed of gas molecules

$$= \gamma = \frac{C_p}{C_v} \text{ for speed of sound in a gas}$$

5. $P = \frac{1}{3} \frac{mn}{V} v_{r.m.s}^2$

6. $P = \frac{2}{3} E$

Here, E = Total translational kinetic energy per unit volume

7. f = Degree of freedom

= 3 for monoatomic gas

= 5 for diatomic and linear polyatomic gas

= 6 for nonlinear polyatomic gas

(a) Vibrational degree of freedom is not taken into consideration.

(b) Translational degree of freedom for any type of gas is three.

8. Total internal energy of gas is, $U = \frac{nf}{2} RT$,

Here, n = Total number of gram moles

9. $C_v = \frac{dU}{dT}$ (where U = Internal energy of one mole of a gas = $\frac{f}{2} RT$)

$$C_v = \frac{f}{2} R = \frac{R}{\gamma - 1}$$

10. $C_p = C_v + R = \left(1 + \frac{f}{2}\right) R = \left(\frac{\gamma}{\gamma - 1}\right) R$

11. $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

12. Internal energy of 1 mole in one degree of freedom of any gas is $\frac{1}{2} RT$.

13. Translational kinetic energy of one mole of any type of gas is $\frac{3}{2} RT$.

14. Rotational kinetic energy of 1 mole of monoatomic gas is zero, of diatomic or linear polyatomic gas is $\frac{2}{2} RT$ or RT , of non-linear polyatomic gas is $\frac{3}{2} RT$.

15. Mixture of non-reactive gases:

$$(a) n = n_1 + n_2$$

$$(b) P = P_1 + P_2$$

$$(c) U = U_1 + U_2$$

$$(d) \Delta U = \Delta U_1 + \Delta U_2$$

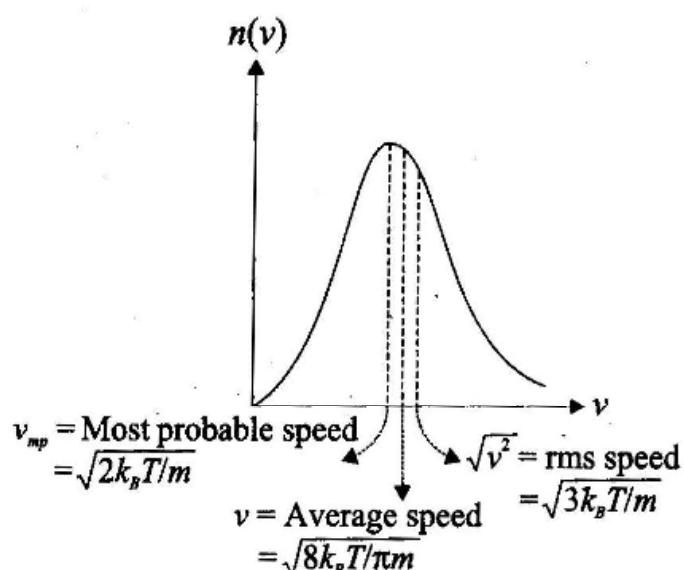
$$(e) C_v = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$$

$$(f) C_p = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 + n_2} = C_v + R$$

$$(g) \gamma = \frac{C_p}{C_v} \text{ or } \frac{n}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$(h) M = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

- Maxwell, on purely statistical considerations, showed that the distribution of molecular speeds in a gas takes place according to a definite law. This is known as Maxwell's law of distribution of molecular speeds.



Maxwell's law of speed distribution in a gas at temperature T is

$$n(v)dv = 4\pi n a^3 e^{-bv^2} v^2 dv$$

$$a = \sqrt{\frac{m}{2\pi k_B T}}, b = \frac{m}{2k_B T}$$

where N is the total number of molecules and $n(v)$ stands for the number of molecules between speed v and $v + dv$. The $n(v)$ versus v plot is the Maxwell's speed distribution.

12.2.1 Real Gases

Deviate at least slightly from ideal gas law because of two factors.

- Gas molecules attract one another.
- Gas molecules occupy a finite volume.

Both of these factors are neglected in the ideal gas law. Both increase in importance when molecules are close together (high P , low T).

The real gases obey the van der Waal's equation instead of ideal gas equation.

$$\left(p + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

where $\frac{n^2 a}{V^2}$ corrects for the attraction between molecules.

nb corrects for the volume of gas molecules.

a and b are called van der Waal's constants.

12.3 THERMODYNAMICS

1. **Molar heat capacity:** C = Heat required to raise the temperature of 1 mole of any substance by 1°C or 1 K .

$$C = \frac{Q}{n\Delta T} \quad \therefore Q = n C \Delta T$$

Molar heat capacity of solids and liquids is almost constant.

In case of gases C is process dependent. It varies from 0 to ∞ .

In isothermal process $C = \infty$ as $\Delta T = 0$

In adiabatic process $C = 0$ as $Q = 0$

C_p (molar heat capacity of isochoric process) are commonly used. In a general process $PV^x = \text{Constant}$, molar heat capacity is given by,

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-x}$$

2. **First law of thermodynamics:** It is a law of conservation of energy given by

$$Q = \Delta U + W$$

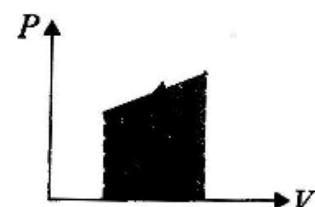
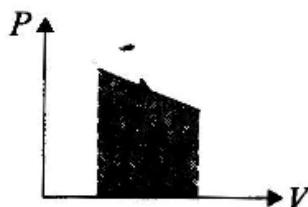
3. **Detailed discussion of three terms of first law of thermodynamics:**

- (a) **Work done:** Following methods are generally used to find the work done.

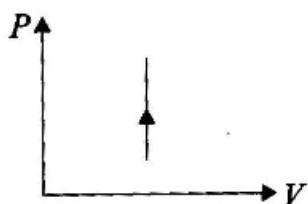
$$\text{Method 1: } W = \int_{V_i}^{V_f} P dV \text{ (because } dW = PdV\text{)}$$

Here, P should be either constant or function of V . If P is constant, it means process is isobaric, $W = P(V_f - V_i) = P \Delta V$

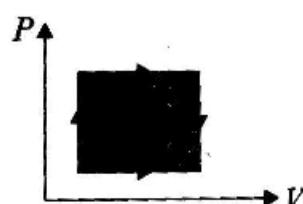
Method 2: Work done can also be obtained by area under $P-V$ diagram with projection on V -axis.



$W = +ve$ as volume is increasing; $W = -ve$ as volume is decreasing



$W = 0$ as volume is constant



$W = +ve$ as cyclic process is clockwise with P on y -axis

- (b) *Change in internal energy ΔU :* $\Delta U = n C_V \Delta T$ for all processes. For this C_V (or nature of gas), n and ΔT should be known. If either of the three terms is not known, we can calculate ΔU as $\Delta U = Q - W$.
- (c) *Heat exchange Q :* $Q = nC\Delta T$. For this n , ΔT and molar heat capacity C should be known. C is a process dependent. So, if either of the three terms (n , ΔT or C) is not known, we can calculate Q as $Q = \Delta U + W$.

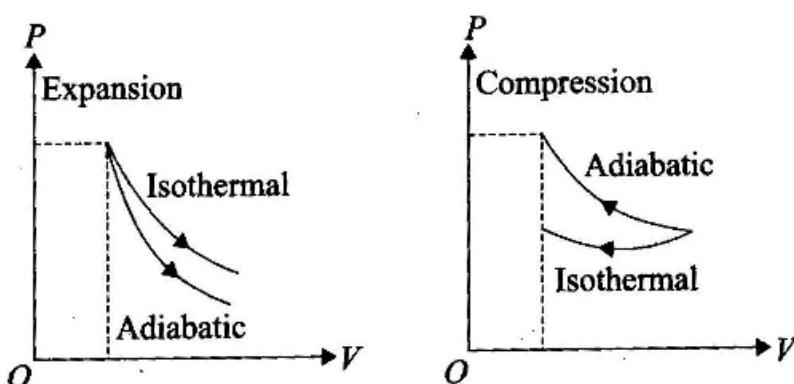
4. $\frac{dP}{dV} = -x \frac{P}{V}$ in process $PV^x = \text{Constant}$ or slope of P - V graph $= -x \frac{P}{V}$

In isobaric process $x = 0$, therefore slope = 0

In isothermal process $x = 1$, therefore slope $= -\frac{P}{V}$

In adiabatic process $x = \gamma$, therefore slope $= -\gamma \frac{P}{V}$

As $\gamma > 1$, therefore adiabatic curve at any point is steeper than the isothermal curve at that point.



5. **Heat engine:** It is a device which is used to convert heat energy into mechanical energy in a cyclic process.

Every heat engine basically consists of three parts:

- (a) A hot body called source
- (b) A working substance
- (c) A cold body called sink

In a heat engine working substance absorbs heat from the source at a higher temperature T_H , converts a part of it into useful work (motion of piston) and rejects the rest to the sink (usually atmosphere) at a lower temperature T_L and comes back to its initial state.

Efficiency of heat engine is given by

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done by engine}}{\text{Heat absorbed by engine from source}} = \frac{W}{Q}$$

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_L}{T_H}$$

A perfect heat engine is one which converts all heat into work, i.e., $W = Q_1$ so that $Q_2 = 0$ and hence, for it $\eta = 1$.

- 6. Efficiency in terms of adiabatic expansion ratio:** The ratio between the volume of the working substance in the cylinder at the end and at the start respectively of an adiabatic expansion stroke of the engine is called the adiabatic expansion ratio and is usually denoted by ρ .

$$\therefore \eta = 1 - \left(\frac{1}{\rho} \right)^{r-1}$$

12.3.1 Entropy

Entropy is defined as $\Delta S = \Delta Q/T$, where ΔQ represents the quantity of energy entering or leaving the body and T represents the absolute (or Kelvin or thermodynamic) temperature at which the energy transfer takes place.

At 0 K (absolute zero) the atoms of a substance are stationary. They form a well ordered arrangement. When energy flows into a body its atoms vibrate, they become a less well ordered arrangement.

Energy entering a body increases disorder and energy leaving a body decreases disorder.

Boltzmann showed that changes in entropy of a body can be considered as a direct measure of changes in the disorder of the arrangement of the particles.

When a hot body is brought into thermal contact with a cold body for a short time:

1. Each body will experience a change in the entropy of its particles.
2. The hot body experiences a decrease in entropy (a negative change) of magnitude $\Delta S_1 = \Delta Q/T_1$.
3. The cold body experiences an increase in entropy (a positive change) of magnitude $\Delta S_2 = \Delta Q/T_2$.
4. The net change in entropy $\Delta S = \Delta S_1 + \Delta S_2$.

12.3.2 Refrigerator or Heat Pump

1. A refrigerator or heat pump is basically a heat engine run in reverse direction. In it working substance takes heat Q_2 from a body at a lower temperature T_L , has a net amount of work done W on it by an external agent (usually compressor) and gives out a larger amount of heat $Q_1 (= Q_2 + W)$ to a hot body at temperature T_H (usually atmosphere).
2. A refrigerator or heat pump transfers heat from a cold to a hot body at the expense of mechanical energy supplied to it by an external agent. The working substance here is called refrigerant.

3. The coefficient of performance of a refrigerator is defined as:

$$\beta = \frac{\text{Heat extracted from the reservoir at low temperature } T_L}{\text{Work done to transfer the heat}}$$

$$= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

4. A perfect refrigerator is one which transfers heat from a cold to a hot body without doing any work, i.e., $W = 0$ so that $Q_1 = Q_2$ and hence for it $\beta = \infty$.

12.3.3 Second Law of Thermodynamics

1. **Kelvin's statement:** A perfect heat engine can never be constructed.
2. **Clausius statement:** Heat by itself cannot pass from a colder to hotter body.
3. **Boltzmann statement:** The effect of naturally occurring processes is always to increase the total entropy (or disorder) of the universe.

12.3.4 Third Law of Thermodynamics

We know that the efficiency of a Carnot engine is $\eta = 1 - \frac{T_2}{T_1}$.

As already said that η would be 100 per cent only if $T_2 = 0$. This is the absolute zero temperature. If T_2 were less than zero, that is, negative temperature, the efficiency would be greater than 100 per cent. This is impossible because it is incompatible with the law of conservation of energy. Hence, the lowest possible temperature is absolute zero. But is it possible to achieve absolute zero by any physical process? With the development of latest experimental techniques for liquefying and freezing of various gases, very low temperatures, such as 1 K, have been reached. By using the special techniques of adiabatic demagnetization, a temperature as low as 10^{-3} K has been reached. The extension of this method has made it possible to reach 10^{-6} K. There is enough evidence to make the following statement about the unattainability of absolute zero which is known as the third law of thermodynamics (or Nernst's law or Nernst's heat theorem). It states that *it is impossible to reach absolute zero of temperature in any physical process.*

CALORIMETRY AND HEAT TRANSFER

13.1 UNITS OF CALORIMETRY

When a hot body is placed in contact with a cold body, the former gets colder and the latter warmer i.e., a certain quantity of heat has passed from the hot body to the cold body. The measurement of the quantity of heat is called as calorimetry.

1. **Calorie** = The amount of heat required to raise the temperature of 1 gm of water by 1°C (from 14.5°C to 15.5°C) at a pressure of 1 atm.

(From experiments; it is found that from 0°C to 37.5°C the value of calorie decreases and from 37.5°C to 100°C , it increases, hence the range 14.5°C to 15.5°C is specified in the definition.)

2. **Kilo calorie** = The amount of heat required to raise the temperature of 1 kg of water through 1°C (from 14.5°C to 15.5°C). i.e., 1 K calorie = 1000 calorie.
3. **Joule (S.I) (J)** = The amount of heat produced by converting one joule of work into heat.

$$\therefore 1 \text{ calorie} = 4.2 \text{ J}$$

$$1 \text{ KJ} = 10^3 \text{ J} = \frac{1}{4.2} \times 10^3 \text{ calorie} = 240 \text{ calorie}$$

$$\therefore 1 \text{ KJ} = 240 \text{ calorie}$$

13.1.1 Specific Heat Capacity of a Substance

Substances differ from one another in the quantity of heat heated to produce a given rise of temperature in a given mass.

Specific heat (capacity) of a substance is the amount of heat required to raise the temperature of unit mass of the substance by 1°C .

(Capacity term is misleading). It is the characteristic of the material of which the body is composed. If $d\theta$ be the amount of heat required to raise the temperature of m mass of the substance by $d\theta$, then specific heat s of the substance is given by

$$s = \frac{1}{m} \left(\frac{dQ}{d\theta} \right) \Rightarrow dQ = ms d\theta$$

cgs Unit $\rightarrow \text{cal gm}^{-1} {}^{\circ}\text{C}^{-1}$ S.I. Unit $\rightarrow \text{J kg}^{-1} \text{K}^{-1}$ ($= 0.24 \times 10^{-3} \text{ cal gm}^{-1} {}^{\circ}\text{C}^{-1}$)

Heat lost or heat gained by way of change in temperature during the same state = $ms d\theta$
 = Mass × Specific heat capacity × Change in temperature
 = 1 cal gm⁻¹ °C⁻¹ = 4200 J kg⁻¹ K⁻¹

- Specific heat of the substance during adiabatic change is zero because

$$dQ = 0 \quad \therefore \quad s = 0$$

- Specific heat of the boiling water or melting ice (or isothermal change) is infinite because during change of state θ constant.

$$d\theta = 0 \quad \therefore \quad s = \infty$$

- Specific heat of water is 1 cal/g-°C or 4200 J/kg-K.

- Specific heat of ice is 0.5 cal/g-°C or 2100 J/kg-K.

- When s varies considerably with temperature, then for small change in temperature $d\theta$, we can write

$$dQ = ms d\theta \quad \therefore \quad Q = \int_{T_1}^{T_2} ms d\theta$$

Here T_1 and T_2 are the initial and final temperatures.

Molar Heat Capacity or Molar Thermal Capacity

It is the amount of heat required to raise the temperature of 1 mole of a substance through 1 kelvin.

S.I. Unit → J mol⁻¹ K⁻¹ and cgs Unit → cal mol⁻¹ K⁻¹

Heat Capacity of a Substance or Thermal Capacity

It is amount of heat required to raise the temperature of the given mass or the given amount of the substance by unity.

Let dQ be the amount of heat required to raise the temperature of given amount of a substance by $d\theta$.

$$\therefore \text{Thermal capacity } C = \left(\frac{dQ}{d\theta} \right)$$

$\therefore dQ = C d\theta$ and we know that $dQ = ms d\theta \therefore C = ms$

i.e., Heat capacity = Mass × Specific heat

S.I. unit → J K⁻¹ and cgs unit → cal °C⁻¹

13.1.2 Dulong and Petit's Law

According to Dulong and Petit's law, for a solid element the product of the molecular weight and the specific heat capacity is constant equal to about 6.4 cal mol⁻¹ °C⁻¹ or 25 J mol⁻¹ K⁻¹

In modern terms, the Dulong and Petit's law states the molar heat capacity of a solid element is approximately equal to $3R$ where R = Gas constant.

i.e., Heavier the element, lesser will be specific heat i.e., $S_{Hg} < S_{cu} < S_{Al}$

Water Equivalent of a Substance

It is the amount of water having the same heat capacity as the substance.

S.I. Unit → kg and cgs Unit → gm

∴ Water equivalent = C kg; C = Thermal capacity of the body.

Let m = Mass of the substance

s = Specific heat of the substance

W = Water equivalent of the substance

S_w = Specific heat of the water.

∴ Heat capacity of substance = ms

And heat capacity of W amount of water = WS_w

$$\text{By definition, } ms = WS_w \Rightarrow W = \left(\frac{ms}{S_w} \right)$$

$$\text{As } S_w = 4200 \text{ J Kg}^{-1} \text{ K}^{-1} \text{ (at } 15^\circ\text{C}) \quad \therefore \quad W = \left(\frac{ms}{4200} \right) \text{ Kg.}$$

13.1.3 Principle of Calorimetry

It states that the total heat given by the hot objects equals the total heat received by the cold objects. It is based on the principle of conservation of energy.

i.e., Heat lost = Heat gained or net change of heat = 0 (provided the two bodies are considered as a closed system).

The temperature of mixture can never be lower than the lowest temperature and never be greater than the highest temperature.

13.1.4 Change of State or Phase

When a certain amount of heat is supplied to any substance, its temperature rises. If the heat is continuously supplied, a time reaches when the temperature does not rise and in this case, the amount of heat supplied is being utilized in changing the molecular configuration which is turned as the change of phase. The amount of heat involved per unit mass is known as the (specific) latent heat L . The heat is *latent* in the sense that a thermometer fails to indicate its addition or subtraction.

If dQ be amount of heat to change the state of mass m of the substance at constant temperature.

$$\therefore L = \left(\frac{dQ}{m} \right) \Rightarrow dQ = mL \quad \text{S.I. Unit} \rightarrow \text{J/kg and cgs Unit} \rightarrow \text{cal/gm}$$

There are three types of specific latent heats corresponding to three types of changes of state from solid to liquid and vice-versa; liquid to vapour and vice-versa; and solid to vapour and vice-versa. i.e., specific latent heat of fusion; specific latent heat of vapourization and specific latent heat of sublimation.

1. Specific latent heat of fusion (of ice) = The amount of heat required for changing the phase of unit mass of the substance from solid to liquid or vice-versa.

$$\text{i.e., } L_f = \left(\frac{dq}{m} \right) \Rightarrow dq = m L_f$$

$$\text{For ice, } L_f = 80 \text{ cal/gm} = 80 \times 4.2 \times 10^3 \text{ J/Kg} = 3.36 \times 10^5 \text{ J/Kg}$$

2. Specific latent heat of vapourization (of water) = The amount of heat required to change the phase of any substance of unit mass from liquid to vapour or vice-versa.

i.e., $L_v = \left(\frac{dq}{m} \right) \Rightarrow dq = m L_v$

For water, $L_v = 540 \text{ cal/gm} = 540 \times 4.2 \times 10^3 \text{ J/kg} = 22.5 \times 10^5 \text{ J/kg}$, for mercury = 270 J/kg, for alcohol = 862 J/kg.

Note

Due to latent heat of steam, steam burns more serious than that of boiling water i.e., internal energy (internal K.E. remain the same but internal P.E. is increased) of steam at 100°C is more than that of water at 100°C

3. Specific latent heat of sublimation (L_s) = The amount of heat required to change the phase of any substance of unit mass directly from solid to vapour or vice-versa (i.e., Hoar Frost).

Specific Heat of a Gas

In the case of solid and liquid, if the heat is added, only temperature changes and only one specific heat capacity is defined. But in case of gaseous system, if heat is added, all the pressure, volume and temperature change and hence it is customary to speak of two specific heats of a gas:

1. Specific heat at constant volume (c_v) or isochoric specific heat = The quantity of heat required to raise the temperature of one gram of a gas through 1°C at constant volume.
2. Specific heat at constant pressure (c_p) or isobaric specific heat = The quantity of heat to raise the temperature of one gram of a gas through 1°C at constant pressure.
(For defining specific heat of a gas, it is essential to specify the condition otherwise it varies from $-\infty$ to $+\infty$)

Gram-molecular Specific Heat of Gas

The amount of heat required to raise the temperature of one gram-molecule of a gas through 1°C is called the gram-molecular specific heat. If M is the gram-molecular weight, then gram-molecular specific heat at constant volume is $C_v = M \times c_v$ and the gram-molecular specific heat at constant pressure is $C_p = M \times c_p$.

Molar Specific Heat at Constant Volume

It is amount of heat required by one mole gas for unit rise in temperature at constant volume i.e.,

$$C_v = \frac{1}{n} \left(\frac{dQ}{d\theta} \right)_{V=\text{constant}} ; n = \text{Number of moles of gas}$$

dQ = Amount of heat for $d\theta$ rise in temperature

S.I. unit $\rightarrow \text{J mol}^{-1} \text{ K}^{-1}$

Molar Specific Heat at Constant Pressure

It is amount of heat required by one mole of gas for unit rise in temperature when pressure is constant.

$$C_p = \frac{1}{n} \left(\frac{dQ}{d\theta} \right)_{p=\text{constant}} \quad n = \text{Number of moles of gas}$$

dQ = Amount of heat for $d\theta$ rise in temperature

S.I. Unit $\rightarrow \text{J mol}^{-1} \text{K}^{-1}$

13.2 HEAT TRANSFER

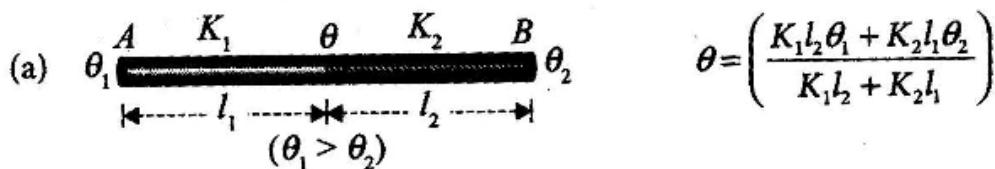
13.2.1 Heat Conduction Through a Rod

$$1. \text{ Heat flow in steady state } Q = \frac{kA(\theta_1 - \theta_2)}{l}$$

$$2. \text{ Rate of flow of heat} = \text{Heat current } i_{th} = \frac{dQ}{dt} = \frac{\text{T.D.}}{R}$$

Here T.D. = Temperature difference $= \theta_1 - \theta_2$ and R = Thermal resistance $= \frac{l}{KA}$

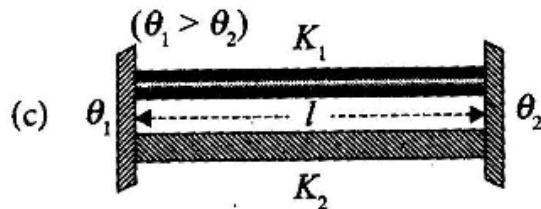
3. A composite system made of two rods of equal length and equal cross section as in figure. The thermal conductivities of the materials of the rods are K_1 and K_2 respectively. The end A and end B are at constant temperatures. All heat entering the face A goes out of the end B there being no loss of heat from the sides of the composite rod.



This is the temperature of the interface.

$$(b) \quad K = \frac{(l_1 + l_2)K_1 K_2}{l_1 K_2 + l_2 K_1} \quad \text{It gives equivalent thermal conductivity of composite rod.}$$

$$\text{If } l_1 = l_2 = l, \text{ then } K_{eq} = \left(\frac{2K_1 K_2}{K_1 + K_2} \right)$$



$$i_{th} = i_{th,1} + i_{th,2} = (\theta_1 - \theta_2) \left[\frac{1}{R_{th,1}} + \frac{1}{R_{th,2}} \right] = (\theta_1 - \theta_2) \times \frac{1}{R_{th}}$$

where R_{th} = The equivalent thermal resistance of the system

$$= \left(\frac{1}{R_{th,1}} + \frac{1}{R_{th,2}} \right)$$

- (d) Due to the similarity between electrical conductivity (or conduction) and thermal conduction, almost all the electrical laws e.g. Ohm's law, Kirchoff's voltage law and Kirchoff's current law etc. can be extended to combination of thermal conductors in the steady state.
- (e) When the temperature of the surrounding starts decreasing, the temperature of the surface of water, in contact with air, also decreases. When the temperature of surrounding becomes 0°C , the temperature of water surface also remain 0°C but if the temperature of the surrounding becomes negative, the heat will flow from water surface to surrounding and in this case, formation of ice takes place.

Time required for a thickness x of ice to grow from zero value

$$= \left(\frac{SL_f}{2K\theta} \right) x^2$$

where $-\theta^{\circ}\text{C}$ = Temperature of air above the lake.

x = Thickness of ice formed in time t .

S = Density of ice.

L_f = Latent heat of ice.

K = Thermal conductivity of ice.

13.3 WIEDEMANN-FRANZ LAW

This law is based on the fact that apart from thermal conductivity (k), the electrons are also responsible for electrical conductivity (σ) of metals. According to this law (given in 1853), for all metals, the ratio of thermal conductivity to the electric conductivity is directly proportional to absolute temperature T .

$$\text{Thus, } \frac{k}{\sigma} \propto T \Rightarrow \frac{k}{\sigma} = cT \text{ where } c \text{ is a constant.}$$

$$\text{At a given temperature, } \frac{k}{\sigma} = \text{Constant.}$$

In case, k is more, σ is also more. Obviously, a good conductor of heat is also a good conductor of electricity.

13.3.1 Thermal Convection

The process in which heat is transferred from one point to another by the actual movement of the heated material particles from a place of higher temperature to another place of lower temperature is called thermal convection.

When temperatures are not too high, liquids and gases are heated by this process. Examples are hot-air furnace, steam radiator and hot water heating system. If the medium is forced to move with the help of a fan or a pump, it is called *forced convection*. If the material moves because of the differences in density of the medium, the process is called *natural* or *free convection*.

The mathematical theory of thermal convection is much more complex and there is no simple equation that may be given as we did for the case of thermal conduction. The heat lost or gained depends upon such factors as:

1. The geometrical shape of the surface.
2. The characteristics of the fluid as its density, specific heat, conductivity and viscosity.
3. The type of flow, which may be streamline or turbulent.

The phenomena depending on the convection of air are ventilation, chimneys, winds, land and sea breezes, monsoons.

13.3.2 Radiation

Radiation is the process of transmission of heat in which heat travels directly from one place to another without the agency of any intervening medium.

In conduction and convection, heat is transmitted by means of material particles while in radiation heat can pass through vacuum.

Let Q be the radiant energy incident on the surface AB . If out of this incident energy, Q_1 is reflected, Q_2 is absorbed and Q_3 is transmitted, then

$$Q_1/Q = r \text{ (reflectance of the surface)}$$

$$Q_2/Q = a \text{ (absorptance of the surface)}$$

$$Q_3/Q = t \text{ (transmittance of the surface)}$$

None of these quantities (r , a and t) has a unit as these are pure ratios.

$$r + a + t = \frac{Q_1}{Q} + \frac{Q_2}{Q} + \frac{Q_3}{Q} = \frac{Q_1 + Q_2 + Q_3}{Q} = 1 \quad (\text{as } Q_1 + Q_2 + Q_3 = Q)$$

Thus, $r + a + t = 1$

In case, the surface does not transmit radiation, $t = 0$ and $r + a = 1$

Obviously, if a is more, r is less or vice-versa. Thus, good absorbers are bad reflectors and bad absorbers are good reflectors.

13.4 EMITTANCE, ABSORPTANCE AND ENERGY DENSITY

13.4.1 Monochromatic or Spectral Emittance (E_λ), Radiant Emittance or Radiance (E)

It is found that various bodies at the same temperature cool down at different rates depending upon the surface condition. Further, the energy emitted by a hot body is different at different wavelengths. To account for this, these are the following two terms.

- Monochromatic emittance (E_λ) corresponding to a wavelength λ is defined as the amount of energy emitted per unit time per unit area within a unit wavelength interval around λ , i.e., between $\left(\lambda - \frac{1}{2}\right)$ and $\left(\lambda + \frac{1}{2}\right)$. Here, unit wavelength interval stands for 1 Å.
- Radiant emittance (E) is defined as the total energy emitted per second per unit area of the body. Since the radiant energy can have wavelengths ranging from 0 to ∞ .

$$E = \int_0^\infty E_\lambda d\lambda$$

In CGS system, E is measured in erg/s cm² and in SI, it is measured in W/m².

13.4.2 Monochromatic Absorptance (a_λ)

Like emittance, the absorptance also depends upon the wavelength of incident radiations. The absorptance corresponding to a particular wavelength is called monochromatic absorptance.

Monochromatic absorptance corresponding to a wavelength λ is defined as the ratio of the energy absorbed per unit time per unit area within a unit wavelength interval around λ , i.e., between $\left(\lambda - \frac{1}{2}\right)$ and $\left(\lambda + \frac{1}{2}\right)$ to the incident energy.

Obviously, both E_λ and a_λ change with wavelength. The nature of this variation is dependent upon the physical conditions of the surface.

13.4.3 Monochromatic or Spectral Energy Density (U_λ) and Radiant Energy Density (U)

Since a body is emitting radiations in all directions, the space around it is filled with radiant energy of all possible wavelengths.

- Spectral energy density (U_λ) corresponding to a wavelength λ is defined as the radiant energy per unit volume within a unit wavelength interval around λ , i.e., between $\left(\lambda - \frac{1}{2}\right)$ and $\left(\lambda + \frac{1}{2}\right)$.
- Radiant energy density or total energy density (U) is the radiant energy per unit volume.

$$\text{Obviously, } U = \int_0^\infty U_\lambda d\lambda$$

The units of energy density are erg/cm³ and J/m³.

(a) Absorptive power $a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$; $a \leq 1$; $a = 1$ for perfectly black body.

(b) Spectral absorptive power $a_\lambda = \text{Absorptive power of wavelength } \lambda$.
 $a_\lambda \leq 1$; $a_\lambda = 1$ for perfectly black body.

- (c) Emissive power e = Energy radiated from a body per unit area per unit time.
 S.I. Units $\rightarrow \text{Js}^{-1} \text{m}^{-2}$ or Wm^{-2} .
- (d) Spectral emissive power e_λ = Emissive power of wavelength λ is known as spectral emissive power $e = \int_0^\infty e_\lambda d\lambda$.

Black-Body A perfect black-body is one which absorbs radiation of all wavelengths incident on it and its absorptance is unity as it neither reflects nor transmits any radiation.

The radiation given out by a perfect black-body is called black-body, full or total radiation. A black-body does not mean that its colour is black, although a black-coloured body may come close to being a black-body.

Prevost's Theory of Heat Exchanges According to this theory when we stand in front of fire, we feel hot due to the reason that we receive more radiation from the fire than we give to it. Reverse is the case when we stand in front of ice. Here, we lose more heat radiation than we gain from ice. Thus, we feel cold in front of ice.

The salient features of this theory are:

- All bodies at temperatures above 0 K emit thermal radiation irrespective of their surroundings.
- The amount of radiation emitted increases with temperature.
- There is a continuous exchange of heat between a body and its surroundings.
- The rise or fall in the temperature of the body is only due to this exchange.
- The exchange of heat between the body and the surroundings continues till a dynamic thermal equilibrium is established between them and their temperatures become equal.

3. Stefan's law: Emissive power of a body is given by, $e = e_r \sigma T^4$.

Here e_r = Emissivity, emittance, relative emissivity or relative emittance.

$e_r \leq 1$; $e_r = 1$ for a perfectly black body.

Note

Sometimes emissivity is also denoted by e . In that case differentiate them by their units. e_r is unitless while e has the units Wm^{-2} .

4. Total energy radiated by a body: $E = (e_r \sigma T^4 A) t$ Here, A = Surface area and t = Time.

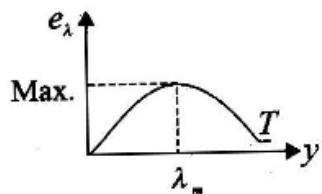
5. Kirchoff's law: If different bodies (including a perfectly black body) are kept at same temperatures, then $e_\lambda \propto a_\lambda$.

or
$$\left(\frac{e_\lambda}{a_\lambda} \right)_{\text{Body-1}} = \left(\frac{e_\lambda}{a_\lambda} \right)_{\text{Body-2}} = \left(\frac{e_\lambda}{a_\lambda} \right)_{\text{Perfectly black body}} = (e_\lambda)_{\text{Perfectly black body}}$$

From this law following two conclusions can be drawn.

- Good absorbers of a particular wavelength λ are also good emitters of same wavelength λ .
- At a given temperature, ratio of e_λ and a_λ for any body is constant. This ratio is equal to e_λ of perfectly black body at that temperature.

6. Wien's displacement law:



$$\lambda_m \propto 1/T \text{ or } \lambda_m T = \text{Constant} = \text{Wien's constant } b$$

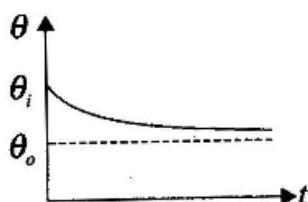
Here, $b = 2.89 \times 10^{-3} \text{ m-K}$, T = Absolute temperature emissive power which is proportional to T^4 .

7. Cooling of a body by radiation:

$$(a) \text{Rate of cooling } -\frac{d\theta}{dt} = \frac{e_r A \sigma}{ms} (\theta^4 - \theta_0^4) \text{ or } -\frac{d\theta}{dt} \propto (\theta^4 - \theta_0^4)$$

(b) Newton's law of cooling: rate of cooling \propto temperature difference.

(c) If body cools by radiation according to Newton, then temperature of body decreases exponentially.



θ_i = Initial temperature of body, θ_o = Temperature of atmosphere.

Temperature at any time t can be written as, $\theta = \theta_o + (\theta_i - \theta_o)e^{-\alpha t}$ which can be approximately written as $\left(\frac{\theta_i - \theta_o}{t}\right) = \alpha \left[\frac{\theta_i + \theta_o}{2} - \theta_o\right]$

13.5 FRAUNHOFER LINES

These are the dark lines present in the continuous spectrum of sun.

These are due to absorption of radiations. Photosphere, the central part of sun, is at a temperature of the order of 10^7 K . It emits continuous light of all wavelengths. These radiations pass through chromosphere which is at 6000 K . It contains certain elements in vapour form. These elements selectively absorb certain wavelengths. These missing wavelengths are dark Fraunhofer lines.

About 20000 such dark lines have been detected so far. These dark lines belong to hydrogen, helium, sodium, iron, calcium etc.

At the time of total solar eclipse, photosphere is covered by moon. The elements present in the chromosphere emit the characteristic wavelength they had absorbed. Therefore, Fraunhofer lines appear as bright lines at the time of total solar eclipse. These lines were named as A, B, C, D... etc.

Kirchoff's law explains this phenomenon.

13.5.1 Colours: Primary and Complementary

Primary colours are those which do not get dispersed when passed through a prism. The primary colours are red, blue and green.

Complementary colours are those two colours which when mixed produce white light.

The colour triangle indicate these colours as in figure.

Red + Green = Yellow

Green + Blue = Cyan (Turquoise)

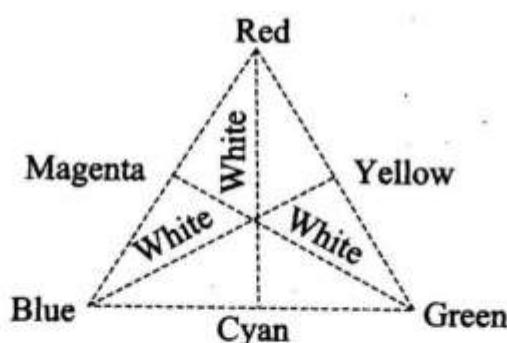
Blue + Red = Magenta (Purplish red)

Complementary Colours

Yellow + Blue = White

Magenta + Green = White

Cyan + Red = White



Red + Yellow + Blue = Black

When a green body is heated in a dark room then it appears red because it emits all colours except green and the emitted colours are dominated by red colour and vice-versa.

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the IIT-JEE Coaching Institutes
of India,
Including those of
Kota, Delhi, Mumbai,
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ELECTRIC FIELD, POTENTIAL AND GAUSS LAW

14.1 PROPERTIES OF ELECTRIC CHARGE

1. Charge like mass is a fundamental and intrinsic property of matter.
2. There are two types of charges: positive charge and the negative charge. Further, charge carried by a proton is positive and that carried by an electron is negative.
3. Unlike charges attract each other and like charges repel each other. The force is thought to result from the exchange of photons between the charged particles.
4. Electric charge is always conserved.
5. **Quantization of charge:** Charge is always in the form of an integral multiple of electronic charge and never its fraction.

$$q = \pm ne \text{ where } n \text{ is an integer and } e = 1.6 \times 10^{-19} \text{ coulomb}$$

$$= 1.6 \times 10^{-19} \text{ C}$$

Charge on an electron or proton is the minimum charge.

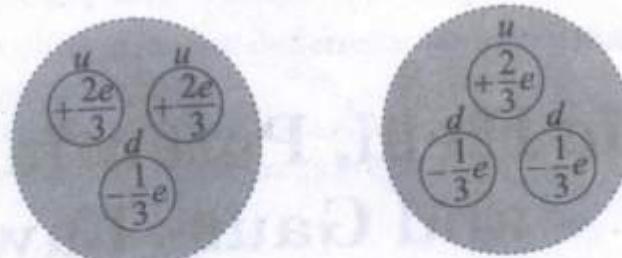
6. Electric charge is additive, i.e., total charge is the algebraic sum of the individual charges.
7. Electric charge is invariant as it does not depend upon the motion of the charged body or the observer. Mathematically $(q)_{\text{at rest}} = (q)_{\text{in motion}}$.

14.1.1 Quarks

Gell-Mann and Zweig who independently proposed these elementary fractionally charged particles in 1963, named them quarks after James Joyce's novel *Three quarks for Muster Mark* where a line exclaims. (The three *quarks* denote the three children of a character, Mister (Muster) Mark).

Quarks are truly elementary particles which carry charges that are fractions of electronic charge ($\pm \frac{2}{3}e$ and $\pm \frac{1}{3}e$). There are six types of quarks (referred to as six flavours of quarks) and these are: (i) up *u*, (ii) charm *c*, (iii) top or truth *t*, all having charge $+(2/3)e$, (iv) down *d*, (v) sideways or strange *s* and (vi) bottom or beauty *b*, all having charge $-(1/3)e$.

It is to be noted that quarks' names are arbitrary and therefore should not be taken literally. Antiquarks are designated by an overbar and have opposite charges compared with those of corresponding quarks, e.g. \bar{u} (antiquark of *u*, called anti-up) has a charge $-(2/3)e$.

Proton (uud), $q = +e$ Neutron (udd), $q = 0$

Protons, as well as neutrons (and other particles), are now known to be made up of quarks. There is firm experimental evidence of the existence of all six quarks and their six antiquarks within the nucleus, but free quarks have not been detected. Current theory implies that direct detection of quarks may, in principle, be impossible.

The quark composition of a proton and a neutron would be uud and udd respectively as shown in figure. Thus, a proton consists of two up quarks and one down quark and a neutron consists of one up quark and two down quarks.

1. Electric force on charge q_1 due to charge q_2 : $\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$

2. Field strength: $\vec{E} = \frac{\vec{F}}{q}$ N/C or $\vec{F} = q \vec{E}$

3. Field strength due to a point charge: $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

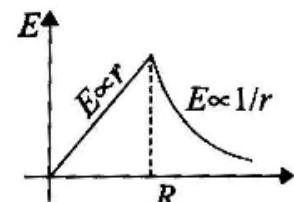
4. In terms of \hat{i}, \hat{j} and \hat{k} : $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{(\text{distance})^2} (\vec{r}_p - \vec{r}_q)$

5. Due to sphere of charge:

(a) Inside point ($r \leq R$), $E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} r; E \propto r$

(b) Outside point ($r \geq R$), $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}; E \propto \frac{1}{r^2}$

(c) On the surface ($r = R$), $E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$

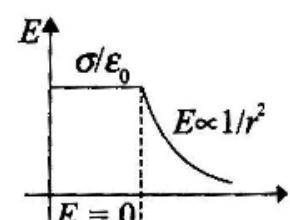


6. Due to hollow sphere of charge:

(a) Inside ($r \leq R$), $E = 0$

(b) Outside ($r \geq R$), $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

(c) On the surface ($r = R$), $E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} = \frac{\sigma}{\epsilon_0}$



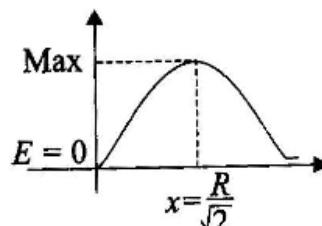
7. On the axis of ring:

(a) $E = \frac{1}{4\pi\epsilon_0} \frac{qx}{(R^2 + x^2)^{3/2}}$

(b) If $x = 0$ i.e. at centre $E = 0$

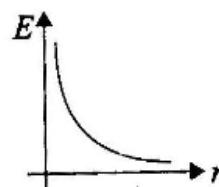
(c) If $x \gg R$ $E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$

If $x \rightarrow \infty, E \rightarrow 0$



8. Due to infinitely long line charge:

$$E = \frac{\lambda}{2\pi\epsilon_0 r}; E \propto \frac{1}{r}$$



9. Due to thin sheet of charge:

$$E = \frac{\sigma}{2\epsilon_0} \rightarrow \text{Constant}$$

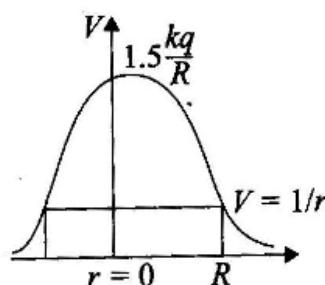
10. Electric potential: $V_p = -W_{\infty \rightarrow p}$; P is any point

11. Potential of point charge at some point: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

12. Due to solid sphere of charge:

(a) Inside ($r \leq R$), $V = \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} [1.5R^2 - 0.5r^2]$

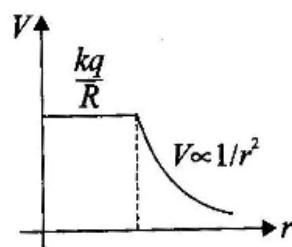
(b) Outside ($r > R$), $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$



13. Due to hollow sphere:

(a) Inside ($r \leq R$), $V_{\text{inside}} = V_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = \text{Constant}$

(b) Outside $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

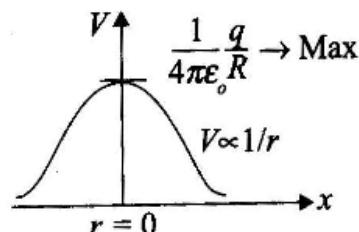


14. Axis of ring:

(a) $V = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{R^2 + r^2}}$

(b) $W_{A \rightarrow B} = q_o(V_B - V_A) \rightarrow \text{By external agent,}$

(c) $W_{A \rightarrow B} = q_o(V_A - V_B) \rightarrow \text{By field}$



15. Electrical potential energy: $U = qV$

(a) Two point charges $U = k \frac{q_1 q_2}{r}$

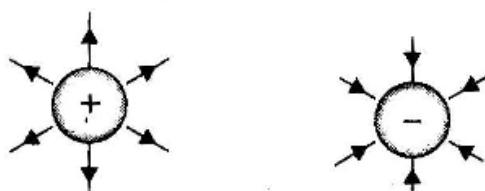
(b) More than two make pair $= \frac{n(n-1)}{2}$

14.1.2 Electric Lines of Force

An electric field line is an imaginary line or curve drawn through a region of space so that tangent to it at any point is in the direction of electric vector at that point.

An electric field line is also the path adopted independently by a positive test charge when placed in an electric field. A field line is, in fact, a space curve, that is, a curve in three dimensions.

1. Electric lines of force start from positive charge and terminate on negative charge.
2. From a positively charged conducting surface lines of force are normal to surface in outward direction.
3. Electric lines of force about a negative point charge are radial inwards and about a positive point charge are radial outwards.
4. Electric lines of force are always perpendicular to an equipotential surface.
5. These lines of force contract along the length but expand at right angles to their length. There is longitudinal tension and lateral pressure in a line of force. Contraction shows attraction between opposite charges while expansion indicates that similar charges repel.
6. The number of electric lines of force (flux) passing through unit normal area at any point indicate electric intensity at that point.
7. For a charged sphere these lines are straight and directed along radius.
8. Two lines of force never intersect or cut each other.
9. Lines of force are parallel and equally spaced in a uniform field.



Relation between \vec{E} and V :

$$\begin{aligned}\vec{E} &= - \left[\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right], \\ E &= - \frac{dV}{dr} = -(\text{Slope of } V-r \text{ graph}), \\ V_B - V_A &= - \int_A^B \vec{E} \cdot d\vec{r} = V_{BA}\end{aligned}$$

Potential difference between two point charges in uniform electric field:

$$V = Ed$$

14.1.3 Electric Flux

1. $\phi_E = EA$, where ϕ_E is the electric flux through a surface of area A , placed perpendicular to a uniform electric field E .
2. $\phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta$, where ϕ_E is the electric flux through a surface lying at an angle with the electric field and θ is the angle area which the vector \vec{A} makes with the electric field \vec{E} .
3. $\phi_E = \int_S \vec{E} \cdot d\vec{S}$ where ϕ_E is the electric flux over an entire surface (curved or plane) in a variable electric field \vec{E} .

There are two conditions to apply this formula as follows:

- At all points electric lines should be perpendicular to surface or $\theta = 0^\circ$.
- Magnitude of electric field should also be equal.

A hypothetical closed surface for which above two conditions can be applied for calculation of field intensity, is called as Gaussian surface.

Mathematical expression for Gauss theorem $\oint \vec{E} \cdot d\vec{s} = \frac{q_{net}}{\epsilon_0}$

14.1.4 Electric Dipole

A combination of two equal and opposite charges separated by a small distance and it behaves like a single entity, is called as electric dipole. Dipole moment is $\vec{p} = q(2l)$ and its direction is from negative charge to positive charge. All polar molecules are examples of dipoles.

	V	E
General point	Find $V(x, y, z)$	$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$
Axis	$\frac{kp}{r^2 - l^2} \approx \frac{kp}{r^2}$	$\vec{E} = \frac{2kpr}{(r^2 - l^2)^2} \approx \frac{2kp}{r^3}$
Perpendicular bisector	0	$E = \frac{kp}{(r^2 + l^2)^{3/2}} = \frac{kp}{r^3}$

Dipole when placed in uniform electric field:

- $F_{net} = 0$
- $\tau \neq 0, \vec{\tau} = \vec{p} \cdot \vec{E} = -pE \sin \theta$
- $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$

For equilibrium

$$U_{0^\circ} = -pE(\min) \rightarrow \text{Stable} \quad U_{180^\circ} = +pE(\max) \rightarrow \text{Unstable}$$

- When placed in non-uniform electric field, a dipole may undergo rotation as well as translational motion.

14.2 PRINCIPLE OF ELECTROSTATIC GENERATOR (VAN DE GRAFF GENERATOR)

Van de Graff designed this electrostatic machine in 1931 to build up high potential difference of the order of few million volt.

The generator is based on the following points:

- The action of sharp points i.e. the phenomenon of corona discharge.
- The property that the charge resides on the outer surface of a conductor. Charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it.

The high potential generated is used to accelerate charged particles like electrons, protons, ions etc. The particles hit the target with the huge energy acquired and carry out the artificial transmutation etc.

$$V_A - V_B = kq_1 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

where q_1 is the charge on the smaller sphere A, r_1 and r_2 are the radius of the smaller sphere A and bigger sphere B, respectively.

14.2.1 Behaviour of a Conductor in an Electrostatic Field

In the case of a charged conductor,

1. Charge resides only on the outer surface of conductor.
2. Electric field at any point inside the conductor is zero.
3. Electric potential at any point inside the conductor is constant and equal to potential on the surface of the conductor, whatever be the shape and size of the conductor.
4. Electric field at any point on the surface of charged conductor is directly proportional to the surface density of charge at that point, but electric potential does not depend upon the surface density of charge.
5. The electric field just outside a charged conductor has a magnitude σ/ϵ_0 , where σ is the charge per unit area at that point.
6. Electric field inside a cavity within a conductor is zero.
7. On an irregularly shaped conductor, charge tends to accumulate at locations where the radius of curvature of the surface is the smallest, that is, at sharp points. It is called as principle of corona discharge.

When charge leaks from sharp points on any conductor, we can sometimes see a faint glow of light and hear a hissing sound. This glow gives the process the name *corona discharge* because light surrounds the top of the pointed conductor like a crown. We can sometimes hear the hissing sound on overhead electricity power line when charge is leaking across the insulators.

14.2.2 Charged Soap Bubble

For equilibrium of a charged soap bubble, pressure due to surface tension = $\frac{4T}{r}$ acting inwards.

Electric pressure due to charging = $\frac{\sigma^2}{2\epsilon_0}$ acting outwards.

At equilibrium, $\frac{4T}{r} = \frac{\sigma^2}{2\epsilon_0}$ where σ = Surface density of charge

$$\Rightarrow \frac{4T}{r} = \frac{1}{2\epsilon_0} \left(\frac{q}{4\pi r^2} \right)^2 \quad \Rightarrow \quad q = 8\pi r \sqrt{2\epsilon_0 r T}$$

Here air pressures, inside and outside the bubble, are supposed to be same.

14.3 ATMOSPHERIC ELECTRICITY

The study of general electrical properties of the atmosphere, both under normal conditions and at the time of discharge (*i.e.*, a lightning), is called atmospheric electricity.

Atmospheric electricity is due to the following factors:

1. When evaporation takes place from the surface of water of the seas and the rivers, the water vapours which go up are positively charged thereby leaving the water behind as negatively charged.
2. Ultraviolet radiations, cosmic rays and radioactive rays (from radioactive sources) cause the ionization of the atmosphere.
3. The ions produced in the atmosphere bring about further ionization by collisions. The electric field region of the atmosphere (extending from 80 km to about 300 km above the earth's surface) is called ionosphere.

14.3.1 Main Features of Atmospheric Electricity

1. At the earth's surface, there exists an electric field of strength 100 V/m which is directed vertically downward all over the earth. The strength of this field decreases gradually with height and at about 50 km from the earth's surface, it becomes negligible. But the potential goes on increasing with height. The total potential difference between the earth's surface and the top of the atmosphere is 400 kV.
2. The total charge on the whole earth is -600 kC.
3. The downward electric field on the earth's surface produces a steady current density of about $3.5 \times 10^{-12} \text{ A/m}^2$. As a result of this, about +1800 C of charge is being deposited on the earth each second, *i.e.*, a discharging current of 1800 A flows to the earth.
4. The entire negative charge on the earth will be neutralized in about 5 minutes

$$\left(\frac{600 \text{ kC}}{1800 \text{ C/s}} \approx 300 \text{ s} \approx 5 \text{ min} \right)$$
But this does not happen. The atmosphere remains charged due to thunderstorm and lightning which occur all over the earth.
5. An average lightning flash has a potential of about $4 \times 10^9 \text{ V}$. It provides a charge of 15 C and possesses about $2 \times 10^{10} \text{ J}$ of energy. The average upward current is rather less than 1 A.

CAPACITORS

15.1 CAPACITANCE

Capacitance of a conductor is defined as the ratio of the charge on it to its potential.

The value of C depends upon:

1. The size and shape of the conductor,
2. The nature of the medium surrounding the conductor and
3. The position of the neighbouring charges.

It does not, however, depend upon the material of the conductor. Further, let $V = 1$, Therefore from equation (1),

$$Q = C \text{ or } C = Q$$

Thus, capacitance of a conductor is numerically equal to the amount of charge required to raise its potential through unity.

Capacitance of a conductor is said to be one farad if its potential rises through one volt when a charge of one coulomb is given to it.

15.2 ISOLATED CONDUCTOR

$$1. V = \frac{Q}{C} \text{ or } C = \frac{Q}{V} \quad \text{S.I. unit} \rightarrow 1F \text{ (farad)} = 1 \text{ Coulomb/volt}$$

Capacitance of an isolated spherical conductor of radius r placed in a medium of dielectric constant K ,

$$C = 4\pi\epsilon_0 Kr \text{ farad}$$

For vacuum or air, $K = 1$, hence $C_0 = 4\pi\epsilon_0 r$ farad.

i.e., capacitance of a spherical conductor \propto radius.

2. Energy stored:

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 \quad \text{or} \quad U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV$$

3. Loss of energy during redistribution:

$$U_i = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}; U_f = \frac{1}{2} \frac{(Q_1 + Q_2)^2}{C_1 + C_2}; \Delta U = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$$

Clearly, during sharing of charge between two charged conductors there is an electric current in the connecting wire and hence, its thermal effect exists. The loss of energy of the system is converted into the heat produced in the connecting wire.

$$\therefore \text{Heat produced} = \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) (V_1 - V_2)^2$$

15.3 PARALLEL PLATE CAPACITOR

$$1. \text{ Capacitance } C = \frac{\epsilon_0 A}{d}$$

2. A parallel plate capacitor having plate area A is given a charge Q . The distance between the plates of the capacitor is d . One of the plates is fixed and the other plate is moved away from the other till the distance between them becomes d' , then

$$(a) \text{ Force of attraction per unit area} = \frac{F}{A} \quad (= \text{electrostatic stress})$$

$$(b) \text{ Electrostatic stress} = \frac{Q^2}{2A\epsilon_0} \times \frac{1}{A} = \frac{Q^2}{2\epsilon_0 A^2} = \frac{(Q/A)^2}{2\epsilon_0} = \frac{\sigma^2}{2\epsilon_0}$$

$$\therefore \text{Electrostatic stress} = \left(\frac{\sigma^2}{2\epsilon_0} \right) \text{ Unit} \rightarrow \text{N/m}^2$$

$$(c) \text{ The work done by the external agent } W = \frac{Q^2(d' - d)}{2\epsilon_0 A}$$

- (d) When voltage is kept constant, the force acting on each plate of capacitor will depend on the distance between the plates.

$$(e) \text{ The work done by the external agent } W = \frac{\epsilon_0 A \zeta^2}{2} \left[\frac{1}{d} - \frac{1}{d'} \right]$$

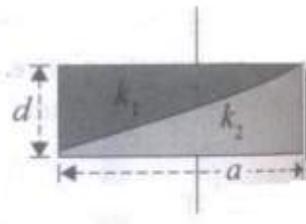
3. A parallel plate capacitor of plate area A and the separation between plates d is fully filled with a dielectric medium whose relative permittivity linearly increases from the value k_1 at the left plate to the value k_2 at the right plate, the capacitance of this capacitor is

$$C = \left(\frac{A\epsilon_0}{d} \right) (k_2 - k_1) \times \ln \left(\frac{k_1}{k_2} \right)$$

4. A parallel plate capacitor of plate area A and length L and the separation between plates d is fully filled with a dielectric medium whose relative permittivity linearly increases from the value k_1 at the upper layer to the value k_2 at the lower layer, the capacitance of this capacitor is

$$C = \frac{A\epsilon_0}{d} \frac{k_1 + k_2}{2}$$

5. A capacitor is formed by two square metal-plates of edge a , separated by a distance d . Dielectrics of dielectric constants k_1 and k_2 are filled in the gap as shown in figure, then the capacitance is



$$\frac{\epsilon_0 a^2}{d} \left(\frac{k_1 k_2}{k_1 - k_2} \right) \ln \frac{k_1}{k_2}$$

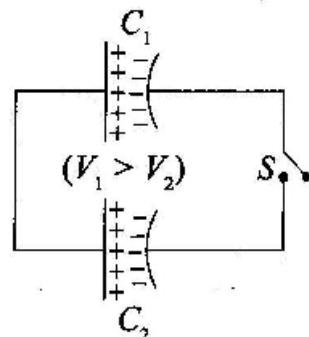
6. A capacitor of capacitance C_1 is charged to potential difference V_1 and another capacitor of capacitance C_2 is charged to potential difference V_2 . Both the capacitors are connected to each other. The sharing of charge between the two charged isolated capacitors occurs till the potential difference between the plates of capacitors becomes the same.

Case I: Common p.d = $V = \left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right)$

and $U'_1 = \frac{1}{2} C_1 V^2$ and $U'_2 = \frac{1}{2} C_2 V^2$

loss of electrostatic energy = $\frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) (V_1 - V_2)^2$

This loss of energy is converted into heat produced.

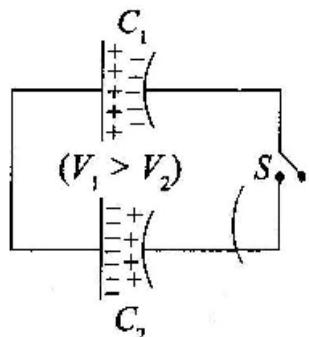


Case II: Common p.d = $V = \left(\frac{C_1 V_1 - C_2 V_2}{C_1 + C_2} \right)$ and $U'_1 = \frac{1}{2} C_1 V^2$ and

$U'_2 = \frac{1}{2} C_2 V^2$ and loss of electrostatic energy

$$= \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) (V_1 + V_2)^2$$

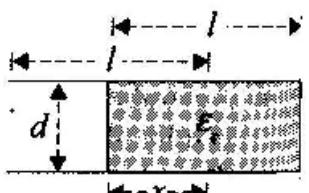
This loss of energy is converted into heat produced.



7. Two rectangular plates of length l and area A are arranged parallel to each other, a distance d apart. They are charged to a potential difference V . A dielectric of permittivity ϵ_r and thickness equal to the plate separation is drawn into the space between the plates.

- (a) The force acting on the dielectric in the direction of the field in terms of the length x of the plate between the plates if the capacitor remains isolated is

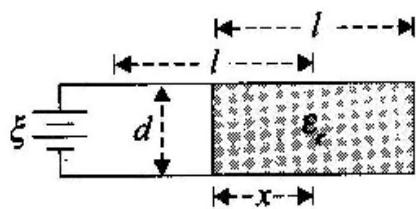
$$F = \frac{\epsilon_0 A \epsilon_r^2}{2ld} \frac{\epsilon_r - 1}{\left[1 + (\epsilon_r - 1) \frac{x}{l} \right]^2}$$



In this case the force acting on the dielectric slab is the function of x . The motion of slab is oscillatory but not an example of SHM.

- (b) The force when the dielectric plate is introduced with a charging battery ($\text{emf} = \xi$ volts) connected to the plates is

$$F = \frac{\epsilon_0 A \xi^2}{2ld} (\epsilon_r - 1)$$



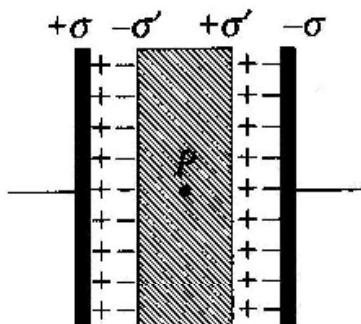
8. The capacitance of parallel plate capacitor is $C = \frac{\epsilon_0 A}{d}$ without dielectric and if a dielectric slab is inserted then the capacitance is

$$C' = \frac{\epsilon_0 A}{\left(d - t + \frac{t}{\epsilon_r} \right)}$$

9. A parallel plate capacitor has capacitance C in the absence of a dielectric. If a dielectric slab of relative permittivity ϵ_r is completely filled inside the air capacitor, the induced bound charge at the surface of the dielectric slab is

$$\sigma' = \sigma \left(1 - \frac{1}{\epsilon_r} \right). \text{ Then, putting } \sigma' = P \text{ and } \frac{\sigma}{\epsilon_0 \epsilon_r} = E_{\text{net}}, \text{ we have}$$

$$P = \epsilon_0 (\epsilon_r - 1) E_{\text{net}} \quad \text{or} \quad \vec{P} = \epsilon_0 \chi \vec{E},$$



where E = field inside the dielectric and $\chi = (\epsilon_r - 1)$, called susceptibility of the dielectric. The dielectrics obeying the above relation are called *linear dielectrics*.

10. If N dielectric slabs of different materials having dielectric constants k_1, k_2, \dots and of different thickness t_1, t_2, \dots are inserted in a charged isolated parallel plate capacitor then;

Charge = Q

$$\text{and p.d.} = \frac{Q}{A \epsilon_0} \left[d - t_1 \left(1 - \frac{1}{k_1} \right) - t_2 \left(1 - \frac{1}{k_2} \right) - t_3 \left(1 - \frac{1}{k_3} \right) \dots \right]$$

$$\text{Capacitance} = \frac{A \epsilon_0}{\left[d - t_1 \left(1 - \frac{1}{k_1} \right) - t_2 \left(1 - \frac{1}{k_2} \right) \dots \right]}$$

$$\text{and stored energy} = \frac{Q^2}{2 \times \text{Capacitance}}$$

11. If N dielectric slabs of different materials having dielectric constants k_1, k_2, \dots and of different thickness t_1, t_2, \dots are inserted in a charged isolated parallel plate capacitor then

$$\text{p.d.} = \xi \text{ and capacitance} = \frac{A\epsilon_0}{d - t_1 \left(1 - \frac{1}{k_1}\right) - t_2 \left(1 - \frac{1}{k_2}\right) \dots}$$

and charge = $(C'\xi)$ i.e., increased and stored energy = $\left(\frac{1}{2}C'\xi^2\right)$ i.e., increased.

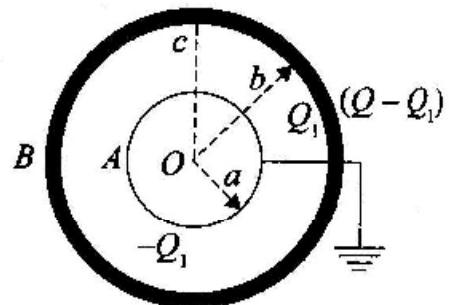
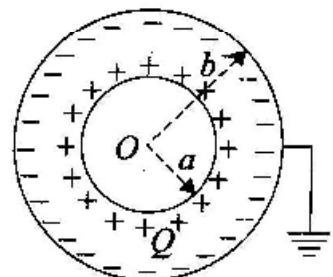
15.4 SPHERICAL CAPACITOR

A spherical capacitor consists of a spherical conductor surrounded by a concentric spherical conducting shell, the inner sphere is charged and the outer shell is earthed.

$$1. \text{ Capacitance } C = \frac{Q}{V} = \frac{4\pi\epsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

- If charge Q is given to the outer shell, then induced charge $-Q$ appears on the outer surface of the inner shell. The outer surface of the inner shell is earthed, then the capacity of the capacitor so formed is

$$C = 4\pi\epsilon_0 \left[\frac{ab}{b-a} + c \right]$$



15.5 CYLINDRICAL CAPACITOR

A cylindrical capacitor consists of a cylindrical conductor surrounded by a coaxial cylindrical conducting shell, the inner cylinder is charged and the outer cylindrical shell is earthed.

$$1. \text{ Capacitance is } C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

$$2. \text{ Capacity per unit length is } \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}$$

15.6 COMBINATION OF CAPACITORS

- A number of capacitors are said to be connected in series if each capacitor acquires the same charge and the sum of the voltage drops across all the capacitors is equal to the voltage of the battery charging the capacitors.

$$\text{In series, } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

$$\text{Two capacitors in series } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ and } \frac{V_1}{V_2} = \frac{C_2}{C_1}$$

2. A number of capacitors are said to be connected in parallel if potential difference across each capacitor is the same (and is equal to the voltage of the battery) and the total charge is equal to the sum of the charges on the individual capacitors.

$$\text{In parallel, } C_{eq} = C_1 + C_2 + C_3 + \dots$$

$$\text{Two capacitors in parallel, } C_{eq} = C_1 + C_2 \text{ and } \frac{q_1}{q_2} = \frac{C_1}{C_2}$$

- (a) Energy density of a charged capacitor is $u = \frac{1}{2} \epsilon_0 k E^2$ and the electrical energy stored in a charged capacitor is

$$U = u \times (\text{Total volume}) = \frac{1}{2} k \epsilon_0 E^2 V$$

- (b) If the dielectric breakdown occurs in a parallel plate capacitor, then the leakage current

$$\text{is } I = \frac{V}{R} = \frac{VA}{\rho d}$$

- (c) If n charged drops, each of capacity C , charged to potential V with charge q , surface density σ and potential energy U coalesce to form a single drop, then for such a drop, total charge = nq ,

$$\text{total capacity} = n^{1/3} C, \quad \text{potential} = n^{2/3} V$$

$$\text{Surface density of charge} = n^{1/3} \sigma,$$

$$\text{and total potential energy} = n^{2/3} U.$$

15.7 DIELECTRICS

Dielectrics are of two types: Non-polar and polar. The non-polar dielectrics (like N₂, O₂, benzene, methane) are made up of non-polar atoms or molecules, in which the centre of mass of negative coincides with the centre of mass of positive charge of the atom or molecule.

- The polar dielectrics (like H₂O, CO₂, NH₃, HC1) are made up of polar atoms or molecules, in which the centre of mass of positive charge does not coincide with the centre of mass of negative charge of the atom or molecule.
- A non-polar dielectric can be polarized by applying an external electric field on the dielectric.

15.7.1 Polarization of Dielectric Medium Placed in an Electric Field

A dielectric may be made up of polar or non-polar molecules. But the net effect of an external field is almost the same, i.e., the external field will compel the molecules to align their dipole moments along its own direction.

The alignment of the dipole moments of the permanent or induced dipoles with the direction of the applied electric field is called polarization.

Polarization of a dielectric is also defined as the transition into such a state when within a small volume of the dielectric, the vector sum of the dipole moment vectors of the molecules is not zero.

Polarization can also be thought of as a phenomenon in which an alignment of positive and negative charges takes place within the dielectric resulting in no net increase in the charge of the dielectric.

1. A non-conducting medium that can be polarized by an external field is called dielectric. A special class of substances (BaTiO_3 , KNbO_3 , LiTaO_3) which exhibit a permanent polarization even in the absence of an electric field, are called ferroelectrics.
2. The amount of polarization developed by nonpolar molecules is not as great as it is in polar molecules. However, for both polar and nonpolar dielectrics, the net effect of the external electric field is to give the dielectric positive and negative surface charges.

**An Institute
which challenged
ALL
the IIT-JEE Coaching Institutes
of India,
Including those of
Kota, Delhi, Mumbai,
Kolkata, Hyderabad,
Patna, Chennai etc., etc.....
(National or Local Level)
In a question
of JEE-Advanced 2014
and finally
winning through
decision of IIT
on 1st June, 2014.**

OHM'S LAW, THERMAL AND CHEMICAL EFFECT OF ELECTRICITY

16.1 ELECTRIC CURRENT

Electric current is defined as the rate of flow of electric charge through an area.

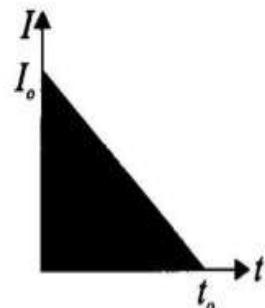
$$I_{av} = \left(\frac{\Delta q}{\Delta t} \right) \text{Cs}^{-1} \text{ or Ampere} \quad I_{ins} = \left(\frac{dq}{dt} \right) \text{Cs}^{-1} \text{ or Ampere}$$

If current is constant, then charge flown, $\Delta q = I \cdot \Delta t$

If current is function of time, then charge flown, $\Delta q = \int_{t_{initial}}^{t_{final}} I dt$

Charge = Area under the graph

$$= \frac{1}{2} \times t_o \times I_o$$



Since the electrons are negatively charged, they move from a point at negative potential (i.e., lower potential) to another point at positive (i.e., higher potential). This is contrary to the usual way of things, e.g., water flows from a higher level to a lower level and not from lower level to higher level. Therefore, conventionally the direction of current is defined as the direction in which a positive charge would move. This current is called the conventional current (I) and it flows from a point at a higher potential to another at a lower potential. The conventional current thus flows in a direction opposite to that of the electronic current.

1. How to find current in electrical circuit:

- (a) For simple circuit of single wire, $I = \frac{\text{Net emf}}{\text{Net resistance}}$
- (b) For complex circuit of more than one wire, with the help of Kirchoff's two laws
 - Law of conservation of charge applied at a junction, i.e., $\sum i = 0$ (KCL)
 - Law of conservation of energy applied in closed loop, i.e., $\sum \xi = \sum iR$ (KVL)

2. To find potential difference between two points in an electrical circuit:

$$V_{ab} = V_a - V_b = \text{Potential difference between } a \text{ and } b$$

- (a) Reach from a to b via any path and write $\pm \xi$ or $\pm IR$.
- (b) Take path with minimum number of batteries and resistor preferably.

16.1.1 Series Combination

1. $I \rightarrow$ Same
2. Potential difference distributed in direct ratio
3. $R = R_1 + R_2 + R_3 + \dots$

16.1.2 Parallel Combination

1. $V \rightarrow$ Same
2. Current distributed in inverse ratio
3. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$
4. Increase in number of parallel resistor, decrease in net resistance

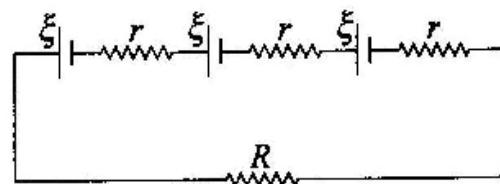
Grouping of Batteries

1. Series:

$$I = \frac{n\xi}{nr + R} \text{ where } n \text{ is the number of batteries.}$$

If polarity of m batteries is reversed then the current

$$I = \frac{(n-m)\xi}{nr + R}$$

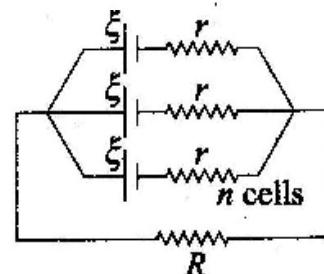


2. Parallel:

$$(a) \text{ Identical batteries, } I = \frac{\xi}{\frac{r}{n} + R}$$

(b) Unidentical batteries,

$$I = \frac{\sum \frac{\xi}{r}}{R + \frac{1}{\sum \left(\frac{1}{r} \right)}} \quad \xi_{net} = \frac{\sum \left(\frac{\xi}{r} \right)}{\sum \left(\frac{1}{r} \right)} \text{ (in parallel)}$$



$$(c) \text{ Special case: Identical batteries, } \xi_{net} = \frac{\frac{\xi}{1} - \frac{\xi}{1}}{\frac{1}{r} + \frac{1}{r}} = \frac{\xi}{2r}$$

If polarity of any battery reversed take $-\frac{\xi}{r}$ in numerator.

(d) Mixed grouping (having m rows and n calls)

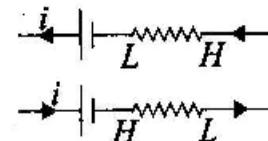
$$I = \frac{mn\xi}{(\sqrt{nr} - \sqrt{mR})^2 + 2\sqrt{mn} rR}$$

Current maximum when denominator is minimum.

$$(\sqrt{nr} - \sqrt{mR}) = 0 \quad \text{or} \quad R = \frac{nr}{m}$$

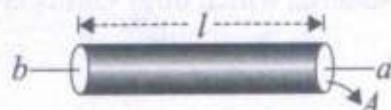
Potential Difference Across Terminals of a Battery

1. $V = \xi - ir, V < \xi$
2. $V = \xi + ir, V > \xi$
3. $V = \xi$ when $i = 0$



16.2 RESISTANCE OF A CONDUCTOR

$$R = \frac{l\rho}{A} \quad \text{or} \quad \frac{l}{\sigma A}$$



(σ) Units: ohm⁻¹ m⁻¹ = mho m⁻¹

1. In $R = \frac{l\rho}{A}$, length is parallel to ab and A (area of cross section) is perpendicular to A.
2. If area of cross-section is perpendicular to ab from a to b, direct formula can be used otherwise by integration.

On reshaping, Initial volume = Final volume

$A_i l_i = A_f l_f$, where l_i, A_i are initial length and area of cross-section of resistor and l_f, A_f are final length and area of cross-section of resistor.

$$\text{Now, } \frac{R_i}{R_f} = \frac{\frac{\rho l_i}{A_i}}{\frac{\rho l_f}{A_f}} = \frac{l_i}{l_f} \times \frac{A_f}{A_i} \Rightarrow \frac{R_i}{R_f} = \left(\frac{l_i}{l_f} \right)^2 \Rightarrow R \propto l^2$$

This means that resistance is proportional to the square of length during reshaping of resistor wire.

$$\text{Also, } \frac{R_i}{R_f} = \left(\frac{A_f}{A_i} \right)^2 \Rightarrow R \propto \frac{1}{A^2}$$

This means that resistance is inversely proportional to the square of the area of cross-section during reshaping of resistor wire.

$$\text{Since } A = \pi r^2 \text{ (for circular cross-section)} \quad \therefore \quad R \propto \frac{1}{r^4}$$

16.2.1 Variation of Resistivity

In conductors, increase in temperature, resistance is also increased but in semiconductors, resistance is decreased.

$$R_{t_0C} = R_{0^{\circ}C} (1 + \alpha t) \quad \text{or} \quad R_{t_2} = R_{t_1} [1 + \alpha(t_2 - t_1)]$$

where α is temperature coefficient.

$$\alpha_{\text{series}} = \frac{R_{01} \alpha_1 + R_{02} \alpha_2}{R_{01} + R_{02}} \quad \alpha_{\text{parallel}} = \frac{R_{01} \alpha_2 + R_{02} \alpha_1}{R_{01} + R_{02}}$$

Relation between current and drift velocity, $i = neAv_d$,

where n = Number of free electron per unit volume

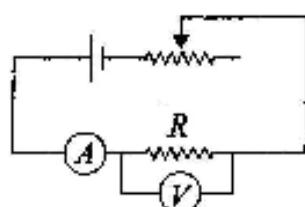
A = Area of cross section of conductor

16.3 OHM'S LAW: $V = IR$

Material which obey Ohm's law are called ohmic and has $\frac{V}{I} = \text{constant}$.

$V-I$ graph \rightarrow Straight line \rightarrow Ohmic

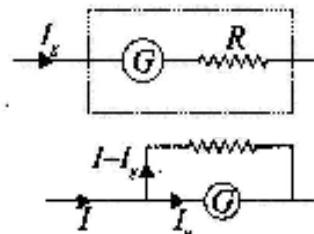
Circuit for Ohm's Law



Conversion of Galvanometer

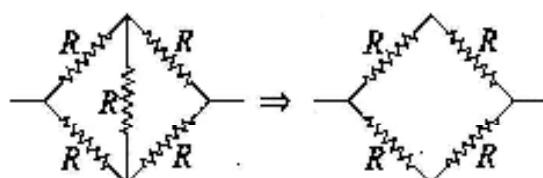
$$1. \text{ Into ammeter } \frac{I_g}{I_s} = \frac{S}{G} = \frac{I_g}{I - I_g}$$

$$2. \text{ Into voltmeter P.d. } = V = I_g(G + R)$$



Wheatstone Bridge

$$\frac{P}{Q} = \frac{R}{S} \rightarrow \text{Balanced bridge, then}$$



Meter Bridge

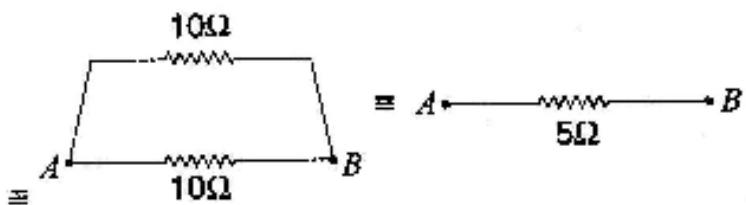
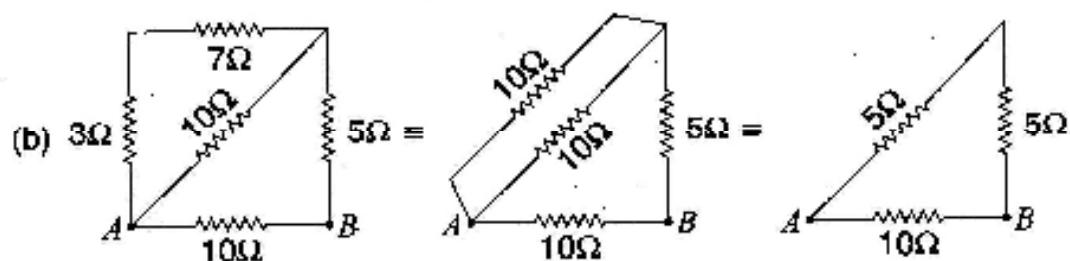
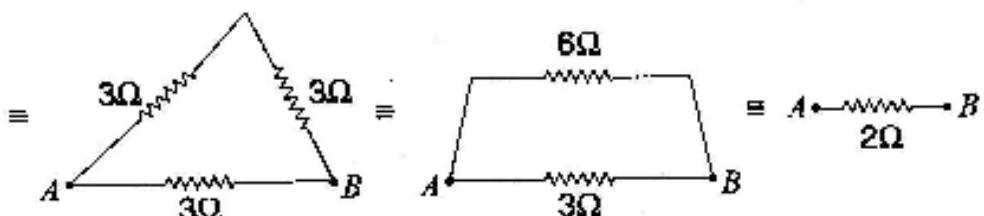
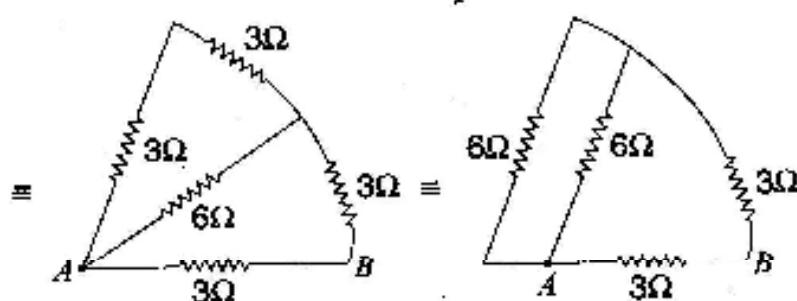
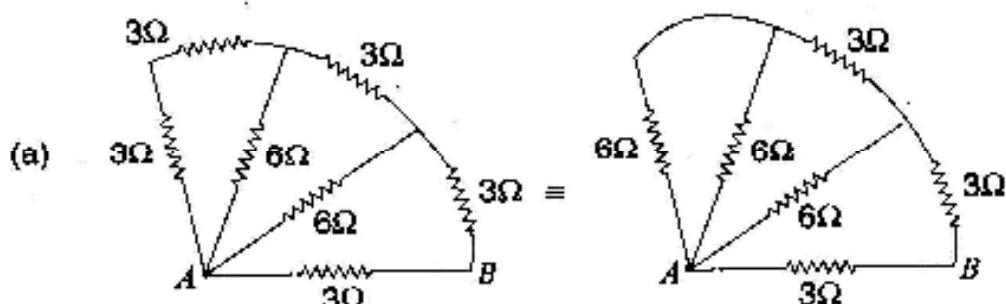
$$X = \frac{Rl}{100-l}$$

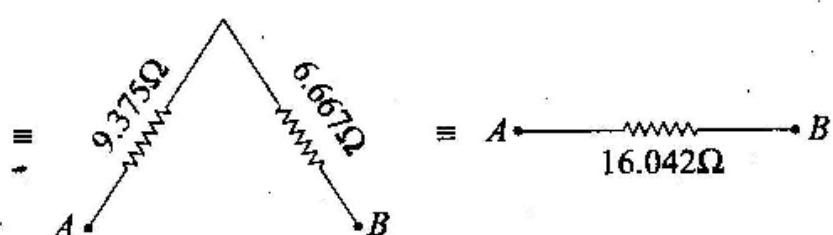
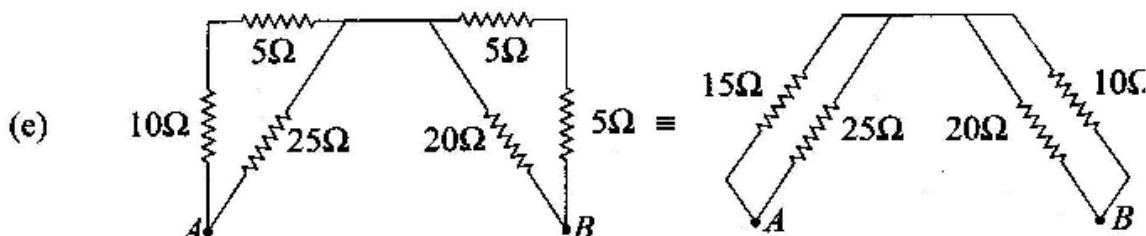
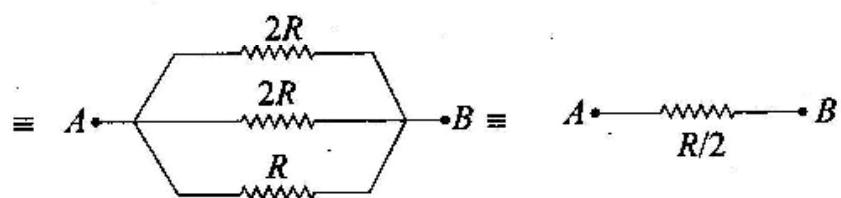
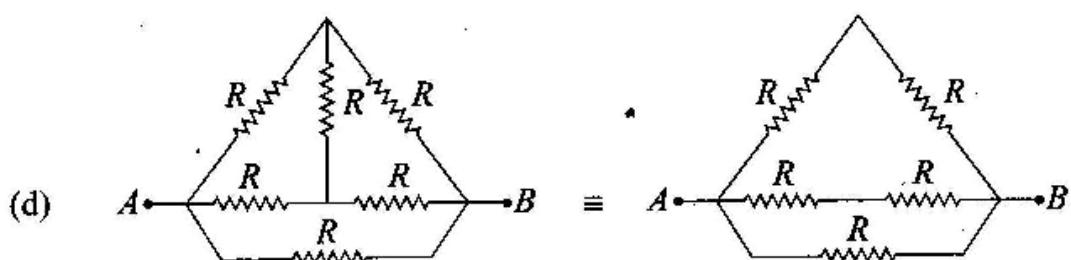
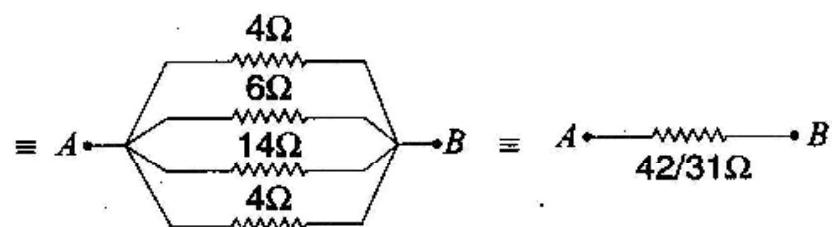
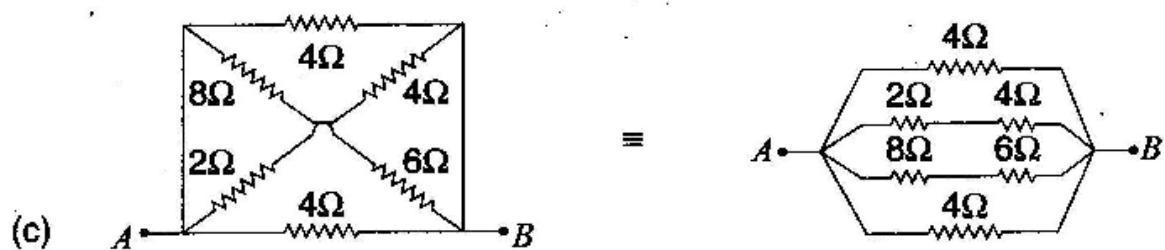
16.4 HOW TO FIND EQUIVALENT RESISTANCE

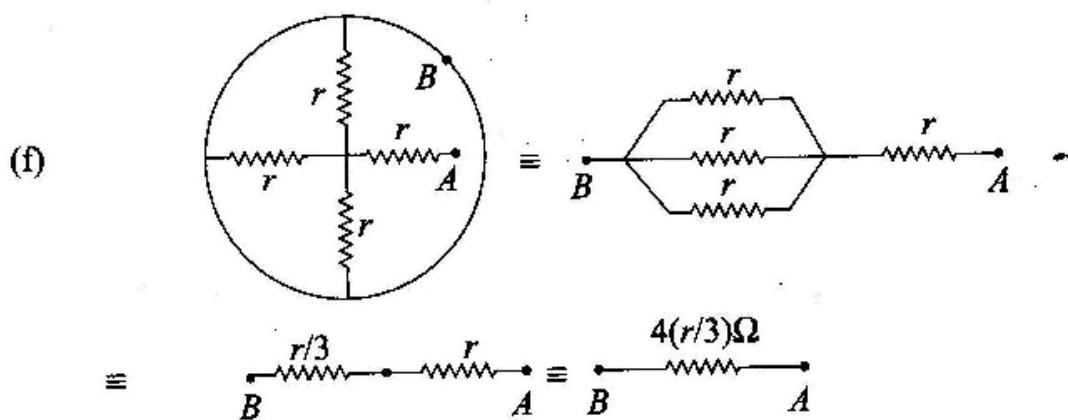
16.4.1 Successive Reduction Method

This method is applicable only when the resistors can be clearly identified as in series or parallel directly or by balanced wheatstone bridge.

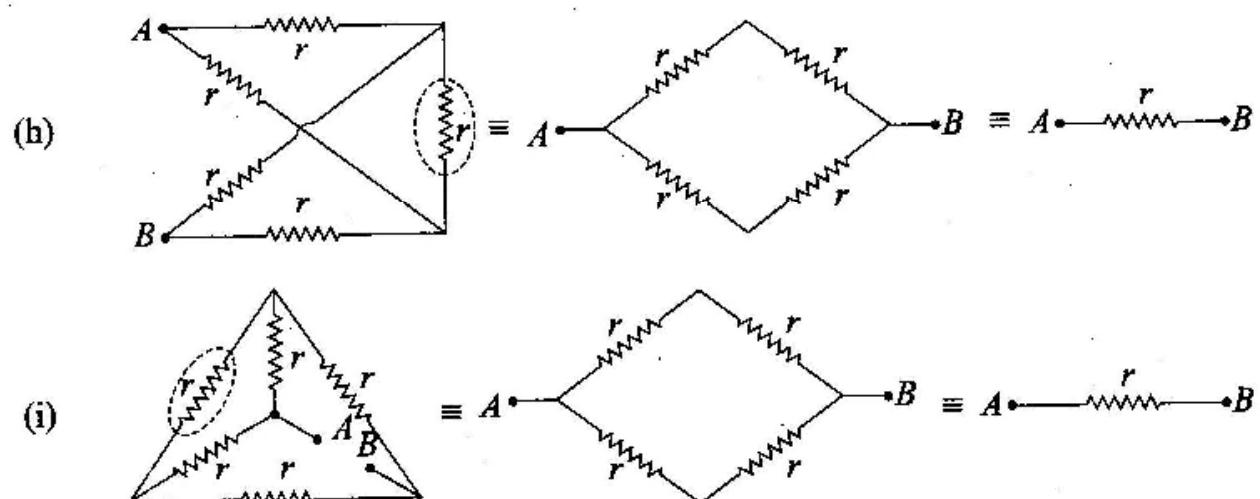
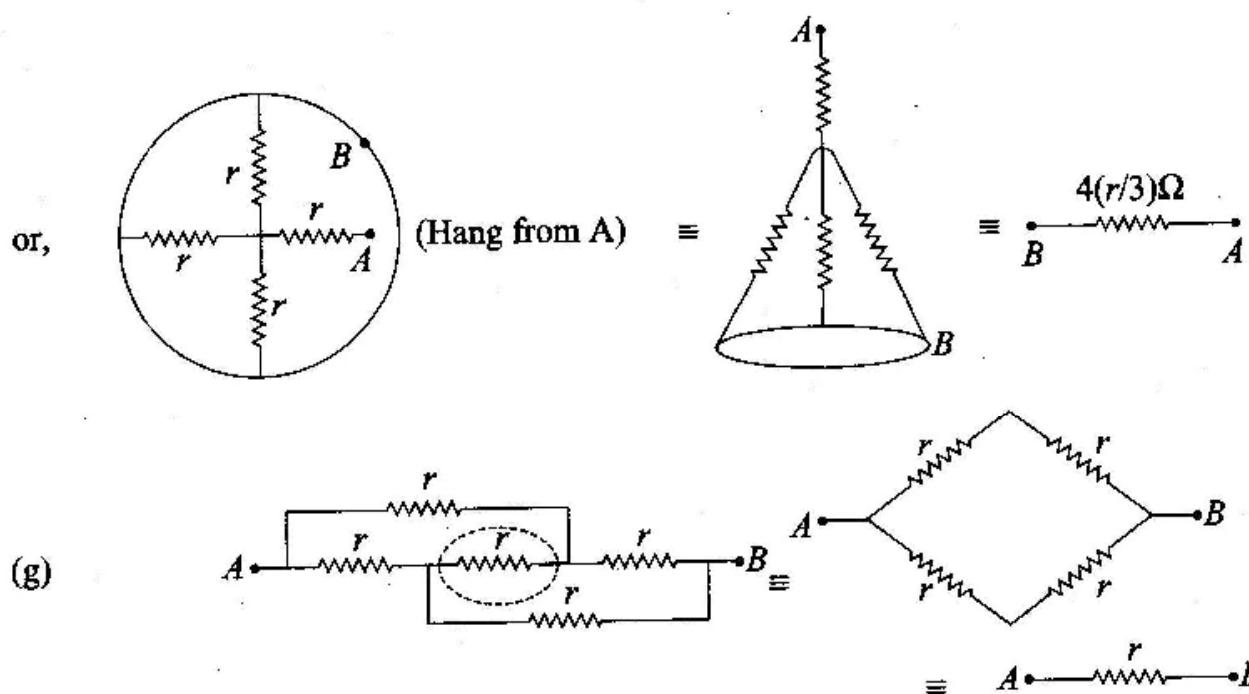
- Find the equivalent resistance between points A and B in the following circuits.

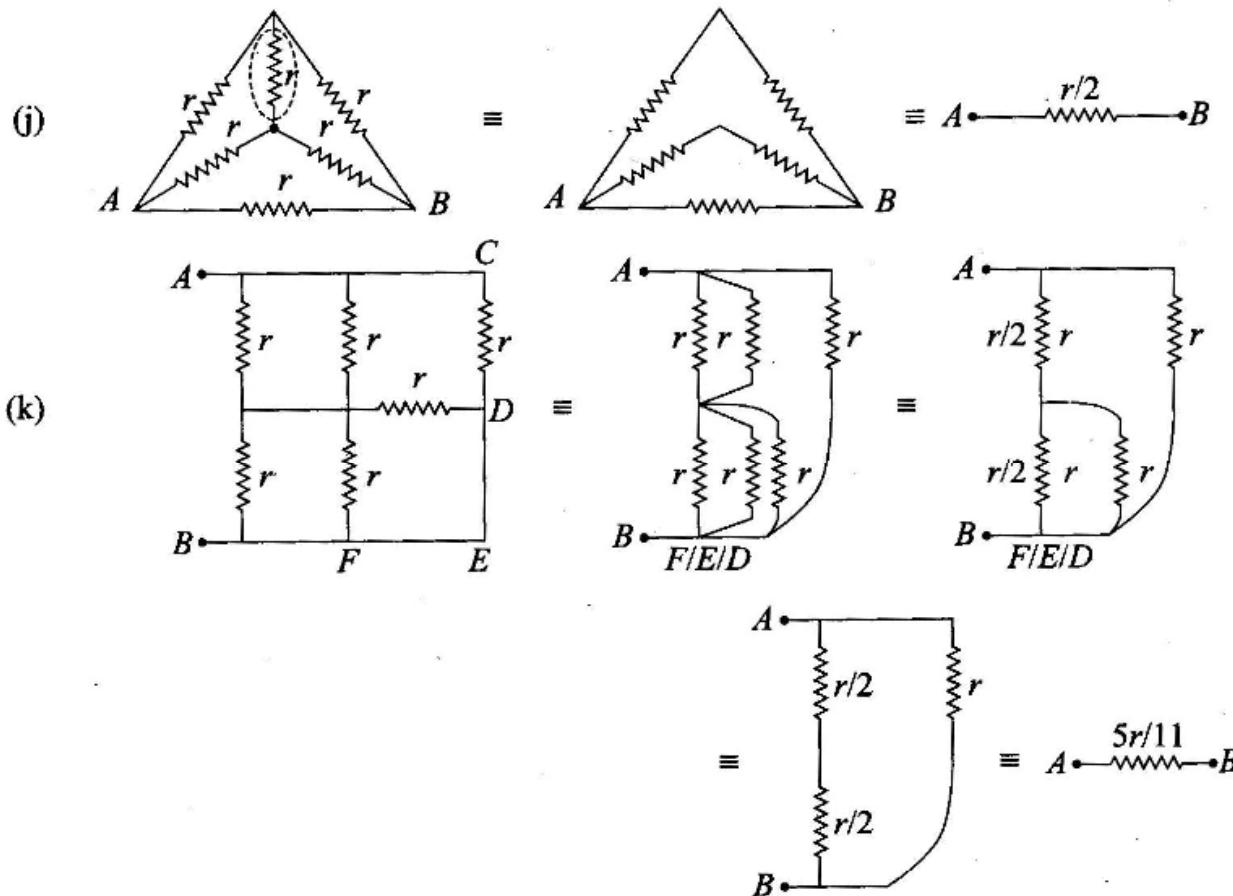






All points on the circumference are at same potential as there is no resistance on circumference.





16.4.2 Using Symmetry of the Circuit

- Find the equivalent resistance between points X and Y in the following circuits.

The adjacent circuit is symmetrical about XAEBY axis (**Axis symmetry**). This is because the upper part of the axis is the mirror image of lower part (resistors and current direction both)

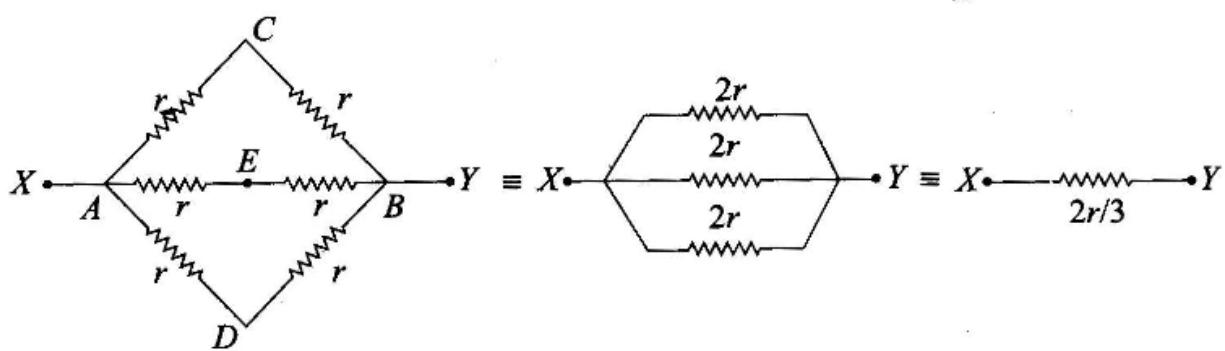
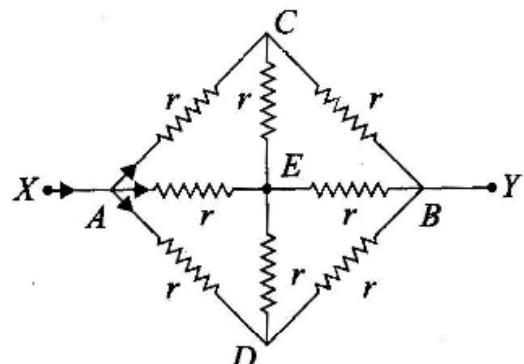
$$\therefore I_{AC} = I_{AD}; I_{CB} = I_{DB};$$

$$I_{AB} = I_{EB}$$

$$\because V_C = V_E = V_D$$

$$\Rightarrow I_{CE} = I_{ED} = 0$$

Therefore the circuit can be redrawn as it is now easier to find resistance between X and Y.



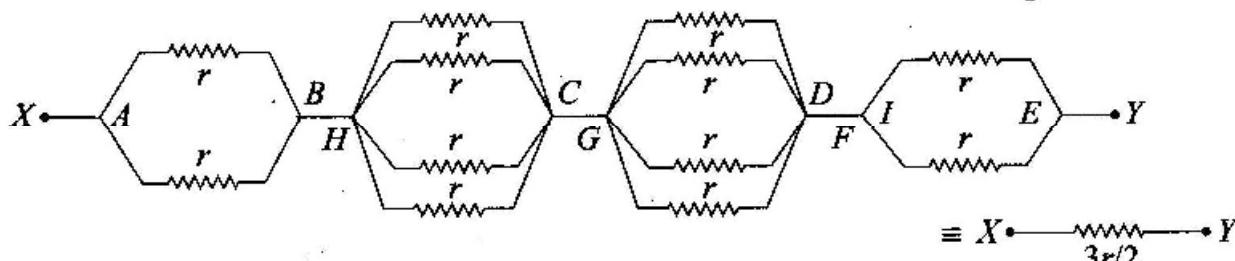
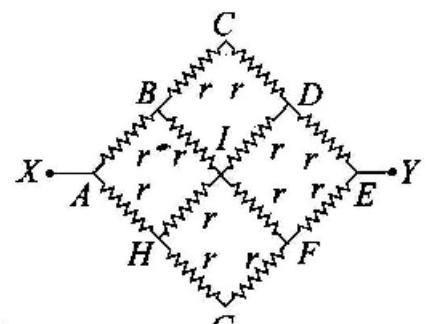
2. Find the equivalent resistance between points X and Y in the following circuits.

The adjacent circuit is symmetrical about axis XY.

Therefore,

$$V_B = V_H; V_C = V_I = V_G; V_D = V_F$$

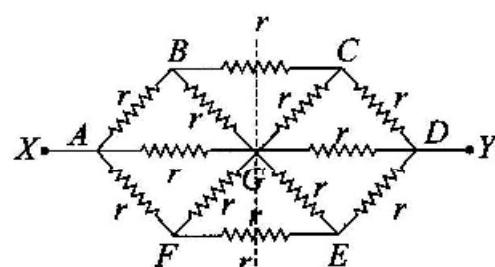
Therefore, the circuit can be recharged as



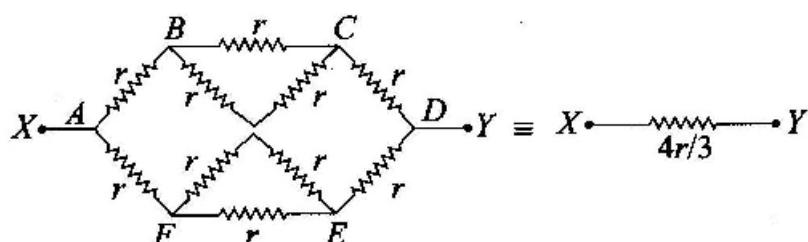
3. Find the equivalent resistance between points X and Y in the following circuits.

The circuit is symmetric about the dotted line

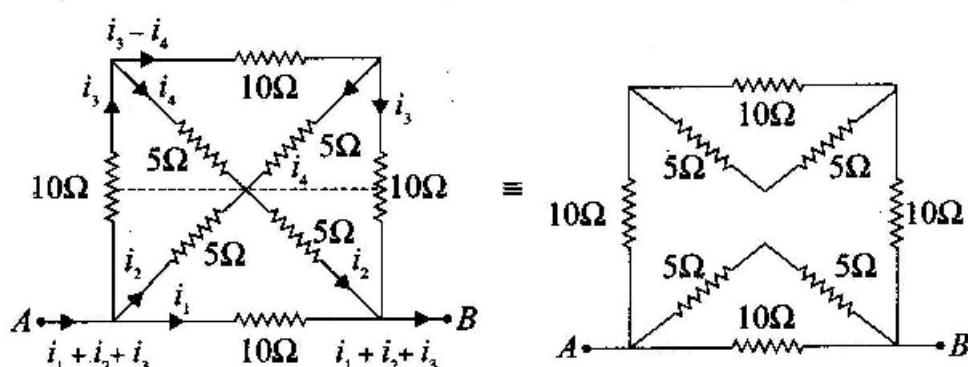
$$\therefore I_{BG} = I_{GC}; I_{FG} = I_{GE} \text{ and } I_{AG} = I_{GB}$$

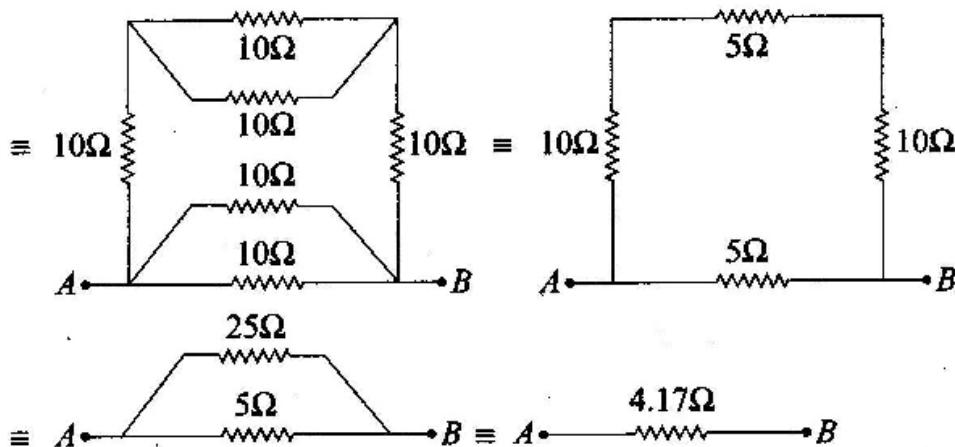


Therefore the circuit can be recharged as



4. Find the equivalent resistance between points A and B in the following circuits.





5. A five pointed regular star has been soldered together from a uniform wire. The resistance of the section CL is r :

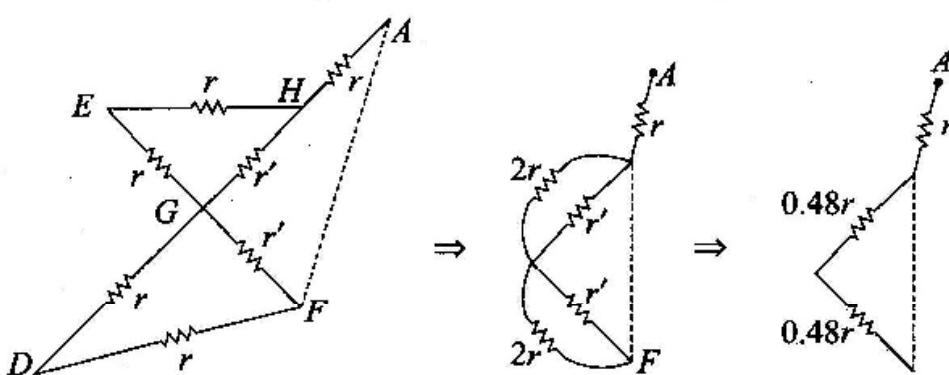
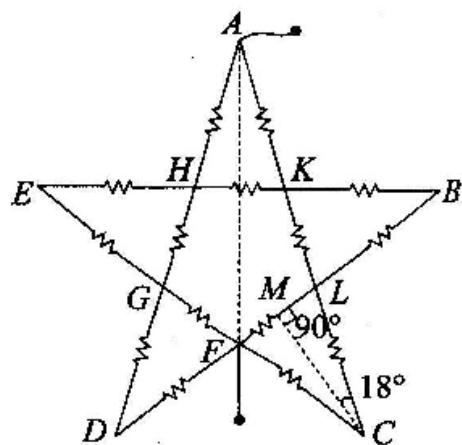
- Find the resistance of the section FL .
- What is the equivalent resistance across the terminals A and F ?

Suppose length of $LC = l$

The length of the section

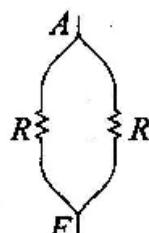
$$FL = 2ML = 2LC \sin 18^\circ = 2l \sin 18^\circ = 0.62l$$

- Since resistance of wire is proportional to its length, therefore resistance of section FL , $r' = 0.62r$.
- The given network of resistors is symmetrical about AF , so it can be broken into two identical parts, each with a resistance R . HK has no current because $V_H = V_K$. Therefore its resistance can be neglected. The resistance of left part R can be obtained as:

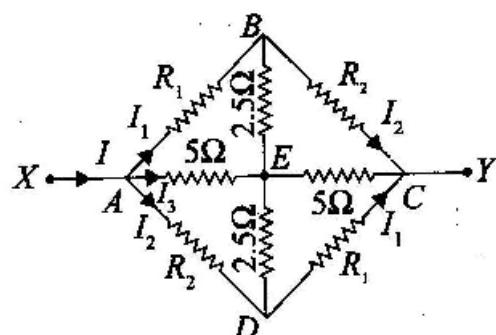


$$R = r + 0.48r + 0.48r = 1.96r$$

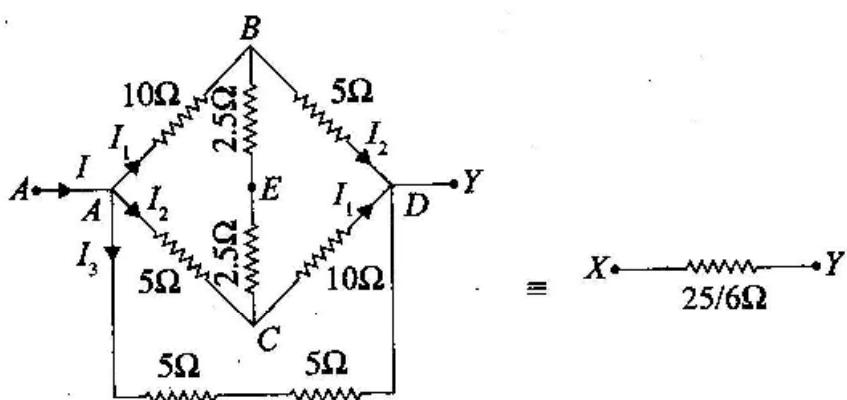
$$\therefore R_{AF} = \frac{R}{2} = \frac{1.96r}{2} = 0.98r$$



6. Find the equivalent resistance between points X and Y in the following circuits.

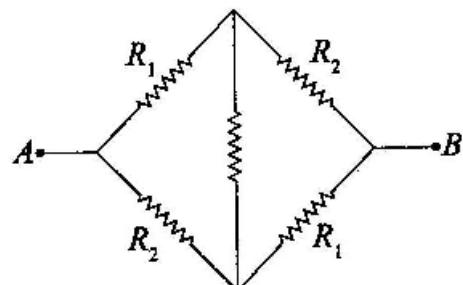


The above network of resistances is symmetrical but the positions of the resistance are shifted (shifted symmetry). Let I be the current in the circuit from A . The same leaves the circuit at C . Let current in AB , AD and AE be I_1 , I_2 and I_3 respectively. Since the same current flows in AE and EC , the detached equivalent circuit can be drawn as



Note: Short-cut

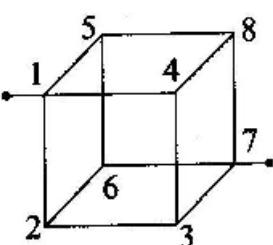
$$R_{AB} = \frac{R_1(R_1 + 3R_2)}{R_2 + 3R_1}$$



1. Twelve equal wires, each of resistance r ohm are connected so as to form a cube-frame. Find the equivalent resistance between the diagonally opposite points 1 and 7.

Connect a source between points 1 and 7.

The network is symmetrical (path symmetric) about the diagonal 1-7. Current in resistors are distributed symmetrically about the diagonal. All paths from one point to another which have the same

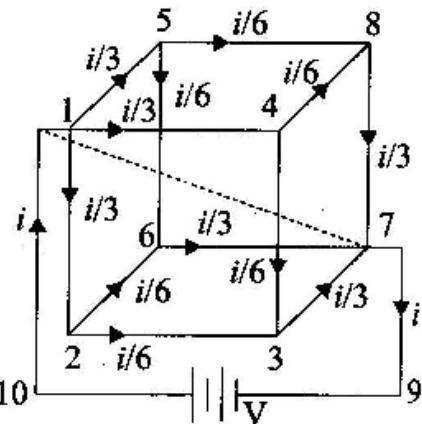


setting of resistances have the same amount of currents. The current distribution is shown in figure. Applying KVL for closed loop 1-2-3-7-9-10-1, we have

$$-r\frac{i}{3} - r\frac{i}{6} - r\frac{i}{3} + V = 0$$

or $\frac{V}{i} = \frac{5}{6}r$

or $R_{17} = \frac{V}{i} = \frac{5}{6}r$



2. Twelve equal wires, each of resistance r ohm are connected so as to form a cube-frame. Find the equivalent resistance between the diagonally opposite points A and C.

By path symmetry,

$$I_{AB} = I_{BC} = I_{AD} = I_{DC} = I$$

$$\therefore I_{AE} = I - 2I_1$$

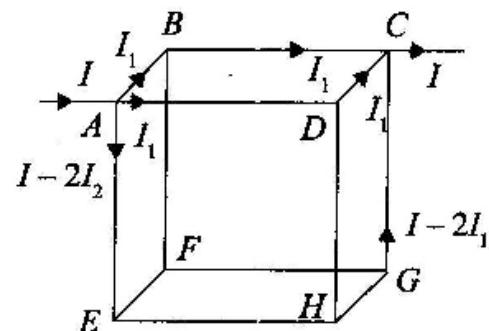
$$\Rightarrow I_{GC} = I - 2I_1$$

Since current in AB = Current in BC

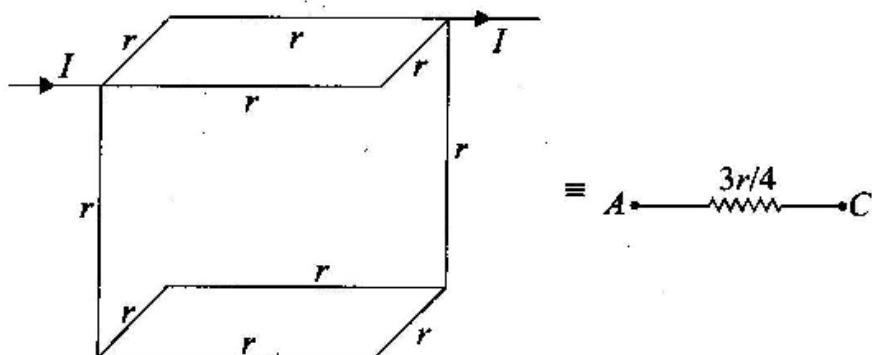
$$\Rightarrow I_{BF} = 0$$

$$\text{Also } I_{AD} = I_{DC}$$

$$\Rightarrow I_{DH} = 0$$

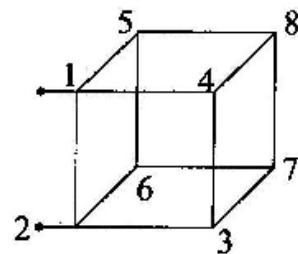


The equivalent circuit will be as shown. The resistance now clearly visible as in series and in parallel.



3. Twelve equal wires, each of resistance r ohm are connected so as to form a cube-frame. Find the equivalent resistance between the points 1 and 2.

Connect a source between points 1 and 2. Let current i enters through point 1 into the network. The network is symmetrical about dotted line (path symmetry). The currents above and below dotted line are symmetrically distributed as shown in figure.



By KCL at 1, we have

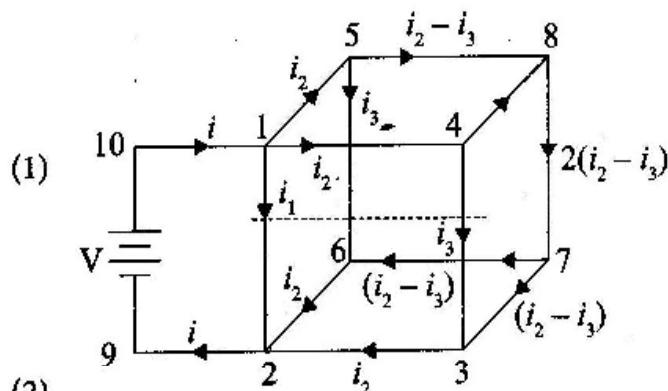
$$i = i_1 + 2i_2$$

$$\therefore R_{12} = \frac{V}{i} = \frac{V}{i_1 + 2i_2}$$

By KVL for closed loop

1-2-9-10-1, we have

$$-ri_1 + V = 0 \text{ or } i_1 = \frac{V}{r}$$



(1)

(2)

By KVL for closed loop 1-4-3-2-1

$$-ri_2 - ri_3 - ri_2 + ri_1 = 0 \quad \text{or} \quad i_1 - 2i_2 - i_3 = 0 \quad (3)$$

By KVL for closed loop 4-8-7-3-4

$$-r(i_2 - i_3) - r \times 2(i_2 - i_3) - r(i_2 - i_3) + ri_3 = 0$$

$$\text{or} \quad -4(i_2 - i_3) + i_3 = 0 \quad \text{or} \quad -4i_2 + 5i_3 = 0 \quad (4)$$

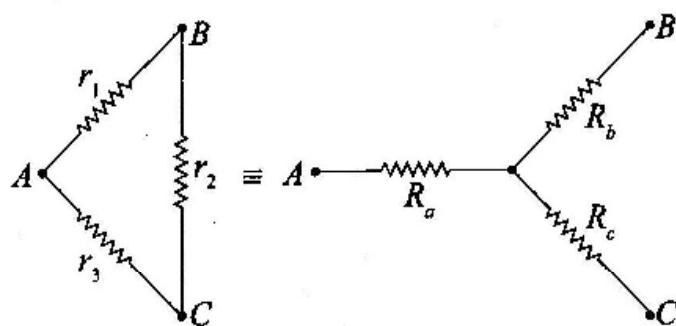
From equations (3) and (4), we get

$$-4i_2 + 5(i_1 - 2i_2) = 0 \quad \text{or} \quad 5i_1 = 14i_2$$

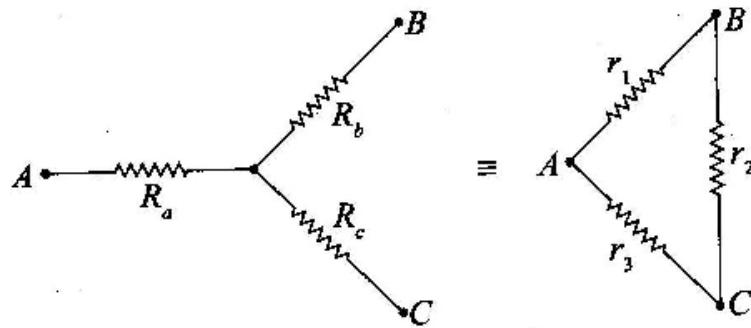
$$\text{Since} \quad i_1 = \frac{V}{r} \quad \therefore \quad i_2 = \frac{5}{14} \times \frac{V}{r}$$

$$\text{Now} \quad R_{12} = \frac{V}{i_1 + 2i_2} = \frac{V}{\frac{V}{r} + 2 \times \frac{5V}{14r}} = \frac{7}{12}r$$

16.4.3 Using Star-delta Conversion Method

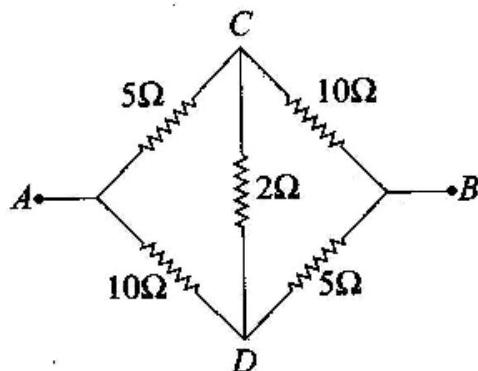


$$R_a = \frac{r_1 r_3}{r_1 + r_2 + r_3}; \quad R_b = \frac{r_1 r_2}{r_1 + r_2 + r_3}; \quad R_c = \frac{r_2 r_3}{r_1 + r_2 + r_3}$$

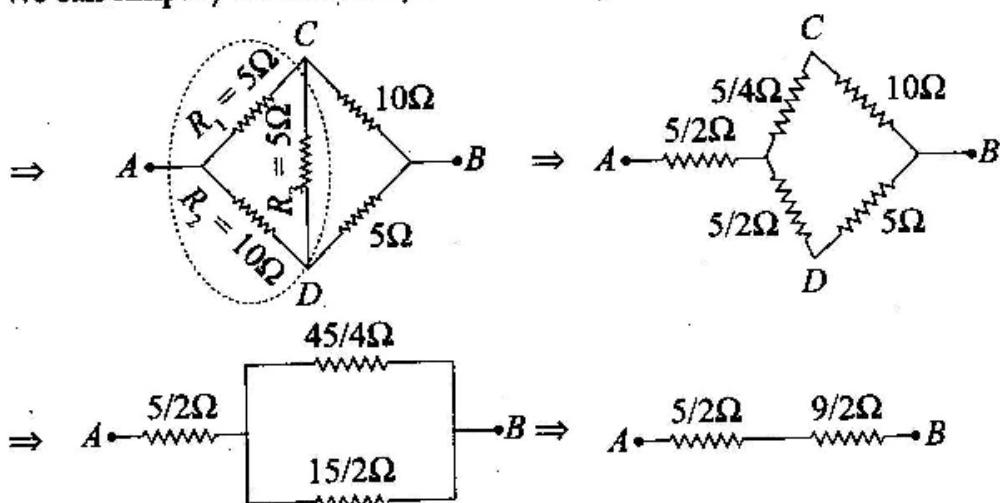


$$r_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}; \quad r_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}; \quad r_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

1. Find the equivalent resistance of the circuit shown in figure between the points A and B.



We can simplify the circuit by transforming delta ACD into star as follows.



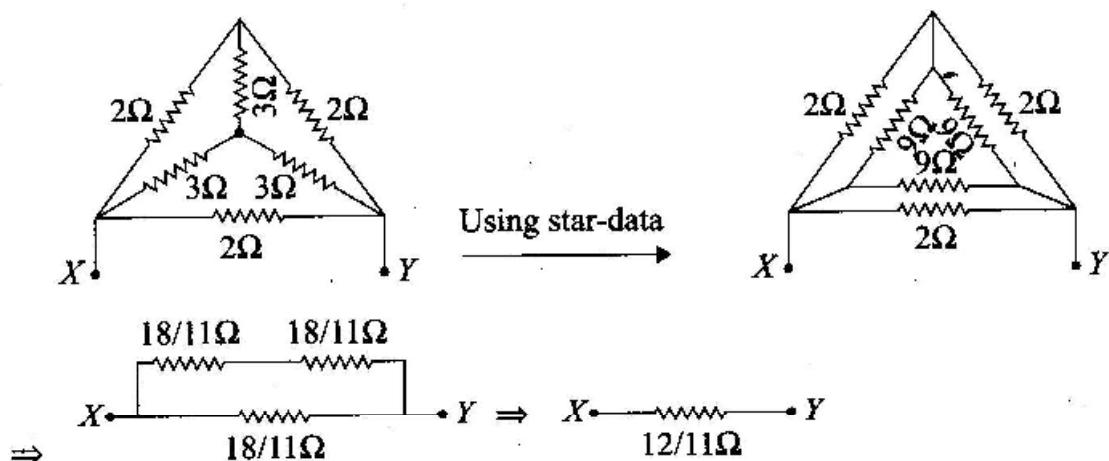
$$R_{12} = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{5 \times 10}{5 + 10 + 5} = \frac{5}{2} \Omega$$

$$R_{13} = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{5 \times 10}{5 + 10 + 5} = \frac{5}{4} \Omega$$

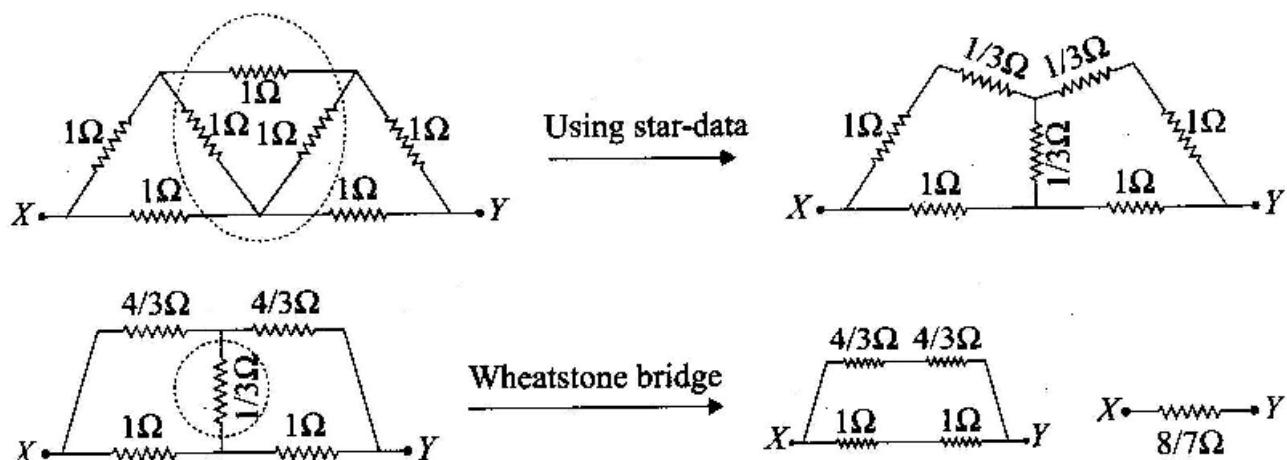
$$R_{23} = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{10 \times 5}{5 + 10 + 5} = \frac{5}{2} \Omega$$

$$\Rightarrow R_{AB} = \frac{5}{2} + \frac{9}{2} = 7 \Omega$$

2. Find the equivalent resistance between the points X and Y.

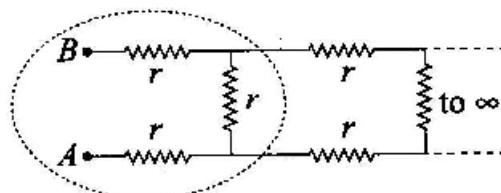


3. Find the equivalent resistance between the points X and Y.



16.4.4 Using Infinite Ladder Method

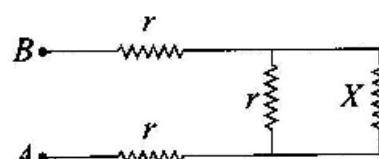
1. Find the equivalent resistance between the points A and B.



We observe that there is a repetitive unit extending to infinity. We assume that the equivalent resistance of all the units except one (shown dotted) is equal to X ohm.

The equivalent resistance across A and B is

$$R_{AB} = r + \frac{X \times r}{X + r} + r$$

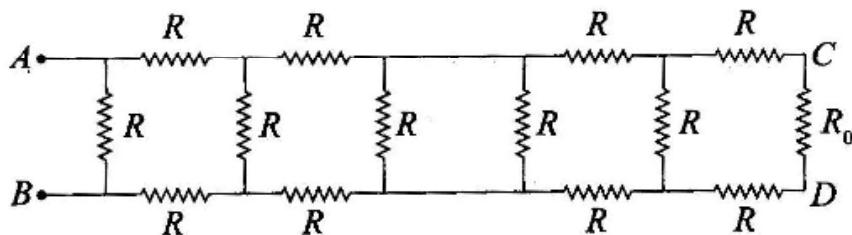


Please note that R_{AB} can be taken as X because if you add one unit to the sum of infinite units, then it will be approximately the same.

$$\therefore X = 2r + \frac{X \times r}{X + r}$$

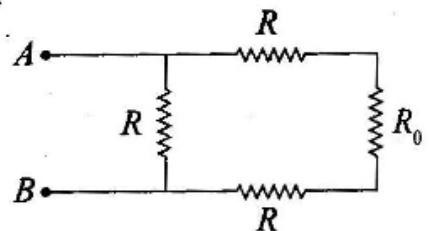
Solve the equation as a normal algebraic equation to find X .

2. For what value of R_o in the circuit shown the total resistance between point A and B will be independent of the number of links?



Here R_o should have such a value for which the resistance of the last link is also equal to R_o . If this condition is satisfied, then we can replace the last link by the resistance R_o . This process can be repeated from the next to the last chain. Hence the total resistance will not depend upon the number of links and it will be equal to R_o . Therefore, on the above assumption we can redraw a circuit as shown. Now from this circuit we have

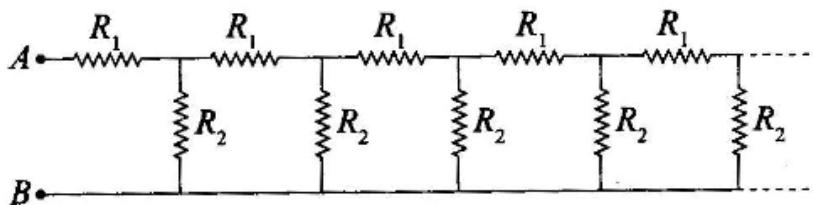
$$\frac{(R_o + 2R)R}{(R_o + 2R) + R} = R_o$$



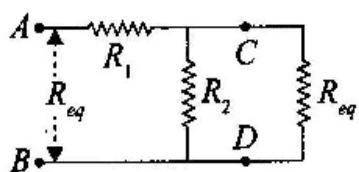
Solving the above equation, and on neglecting the -ve value of R_o , we get

$$R_o = R(\sqrt{3} - 1)$$

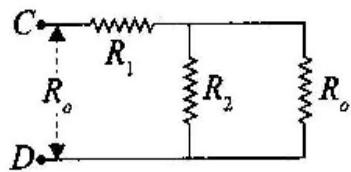
3. Consider an infinite ladder of network shown in figure. A voltage is applied between points A and B . If the voltage is halved after each section, find the ratio R_1/R_2 . Suggest a method to terminate it after a few sections without introducing much error in its attenuation.



The given infinite chain is equivalent to resistance R_1 in series with parallel combination of R_2 and R_{eq} as shown in figure. So,



(a)



(b)

$$R_{eq} = R_1 + [R_2 \text{ Parallel } R_{eq}] = R_1 + \frac{R_2 R_{eq}}{(R_2 + R_{eq})}$$

$$\text{or } R_{eq}^2 - R_1 R_{eq} - R_1 R_2 = 0$$

$$\text{i.e., } R_{eq} = \frac{1}{2} \left[R_1 \pm \sqrt{(R_1^2 + 4R_1 R_2)} \right]$$

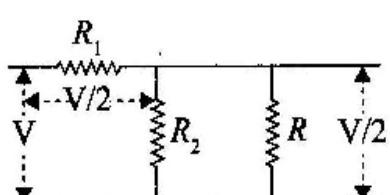
And as resistance cannot be negative

$$R_{eq} = \frac{1}{2} R_1 \left[1 + \sqrt{1 + 4 \left(\frac{R_2}{R_1} \right)} \right]$$

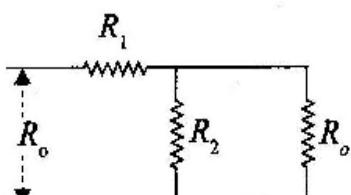
According to final result of above, we have

$$R \rightarrow R_{eq} = \frac{1}{2} R_1 \left[1 + \sqrt{1 + 4 \left(\frac{R_2}{R_1} \right)} \right] \quad (1)$$

Now as here voltage is halved after each section



(a)



(b)

$$R_1 = [R_2 \text{ Parallel } R] = \frac{R_2 R}{R_2 + R} \quad (2)$$

So, substituting the value of R from equation (1) in (2) and simplifying, we get

$$R_2 = 2R_1, \text{ i.e., } \frac{R_2}{R_1} = 2$$

And so with this condition equation (1) gives,

$$R = 2R_1 = R_2$$

Now the attenuation produced by the circuit on termination by a resistance will not be affected if R becomes independent of number of sections in the circuit. This is possible only if the terminating resistance R_o as shown in figure (b) is such that,

$$R_o = R_1 + \frac{R_2 R_o}{(R_2 + R_o)} = R_1 + \frac{2R_1 R_o}{(2R_1 + R_o)} \quad [\text{As } R_2 = 2R_1]$$

$$\text{or } R_o^2 - R_1 R_o - 2R_1^2 = 0, \text{ i.e., } R_o = 2R_1$$

i.e., attenuation produced by the circuit will not be affected if after a few sections it is terminated by a resistance

$$R_o = 2R_1 = R_2 = R$$

16.5 COLOUR CODE FOR CARBON RESISTORS

In electrical and electronic circuits, carbon resistors with a wide range of values are extensively used. To indicate the resistance value and its percentage reliability, a colour code is used. The resistor has a set of coloured concentric rings A, B, C and D on it as in Fig., with their significance indicated in Table.

A denotes the first digit (i.e., the first significant figure), B denotes the second digit (i.e., the second significant figure), C denotes the number of zeros (or power of ten) by which the above two significant figures are to be multiplied. D denotes the tolerance limits (i.e., the error in the value of the resistance).

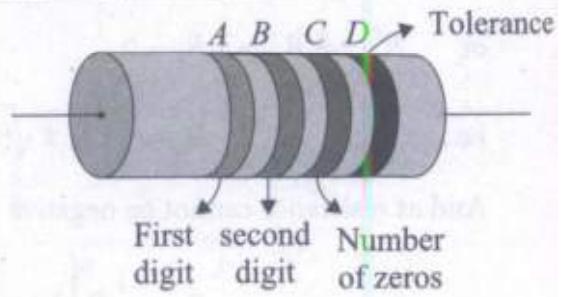


Table 16.1

Figure	Colour	Figure	Colour
0	Black	5	Green
1	Brown	6	Blue
2	Red	7	Violet
3	Orange	8	Grey
4	Yellow	9	White

Table 16.2

Colour	Tolerance
Gold	$\pm 5\%$
Silver	$\pm 10\%$
No colour (i.e., when the tolerance band is missing)	$\pm 20\%$

If any band is omitted, it is of the same colour as that of the body of the resistor.
Let us illustrate the colour code with a couple of examples.

Example 1

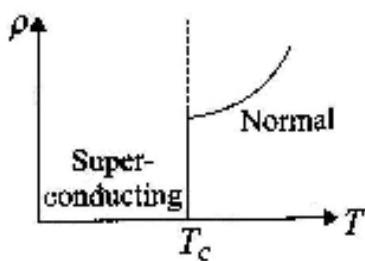
If A is green, B is violet, C is orange and D is silver, resistance is $(57000 \pm 10\%) \Omega$.

Example 2

If A is yellow, B is red, C is orange and D is gold, resistance is $(42000 \pm 5\%) \Omega$.

16.5.1 Superconductivity

There is a class of metals and compounds whose resistivity goes to zero below a certain temperature T_c , called the critical temperature. These materials are known as super-conductors and the phenomenon is called superconductivity. The resistivity-temperature graph for a superconductor follows that of a normal metal at temperatures above T_c as shown in Fig. When the temperature is at or below T_c , the resistivity drops suddenly to zero. Magnetic field is expelled from the interior of the superconductor during the normal ($T > T_c$) to super-conducting transition ($T < T_c$). This effect is called the Meissner' effect (exclusion efflux).

**16.5.2 Potentiometer**

An instrument used for measuring accurately the emf or potential difference is called a potentiometer. It was first designed by Poggendorff.

Since the wire has a uniform area of cross-section, its resistance per unit length at every point will be the same.

Let, I = Current flowing through the wire AB

r = Resistance per unit length of the wire AB

$e = pd$ across unit length of the wire

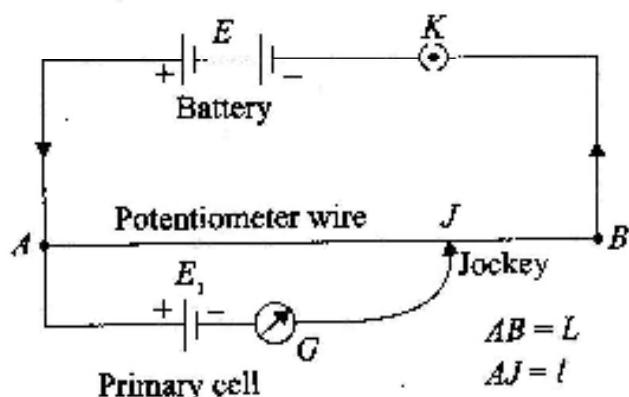
Clearly, $e = Ir$ (1)

If the current I through the wire does not change, from equation (1)

$$V = el$$

As e is a constant, $V \propto l$ (2)

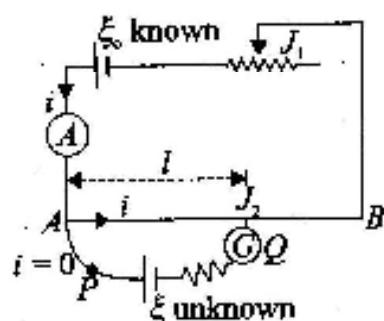
From equation (2), it is clear that the pd between any two points of a wire of uniform area of cross-section is directly proportional to the length of the wire between these points, provided the current through the wire remains constant. This is the principle of a potentiometer.

**1. To measure the emf of a cell (ξ):**

$$V_{AB} = V_{PQ}$$

$$iR_{AJ_2} = \xi$$

$$iel = \xi$$



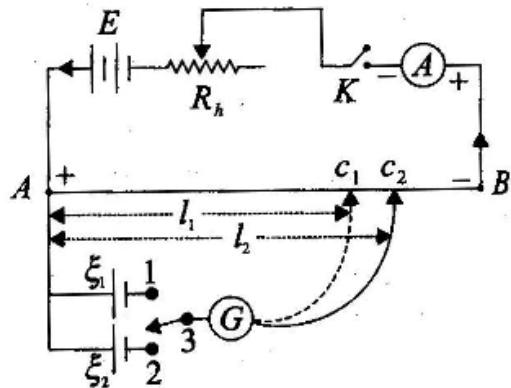
2. Comparison of emf's of two cells:

In case we want to compare the emf's (ξ_1 and ξ_2) of two cells, we find the balance points separately for each cell. If l_1 and l_2 are the distances of the respective balance points from A, then

$$\xi_1 \propto l_1 \text{ and } \xi_2 \propto l_2$$

$$\text{or } \frac{\xi_1}{\xi_2} = \frac{l_1}{l_2}$$

The circuit diagram is as shown in Fig. Here, C_1 and C_2 are the balance points for the cells ξ_1 and ξ_2 respectively.



3. Determination of internal resistance of a cell:

In order to determine the internal resistance (r) of a cell whose emf is ξ , the experimental set up is as shown in Fig. As discussed earlier, find the balance point C_1 when the key K' is open. In this case, the cell is in the open circuit and

$$\xi \propto pd \text{ across } AC_1$$

$$\text{or } \xi \propto l_1 \text{ or } \xi = kl_1$$

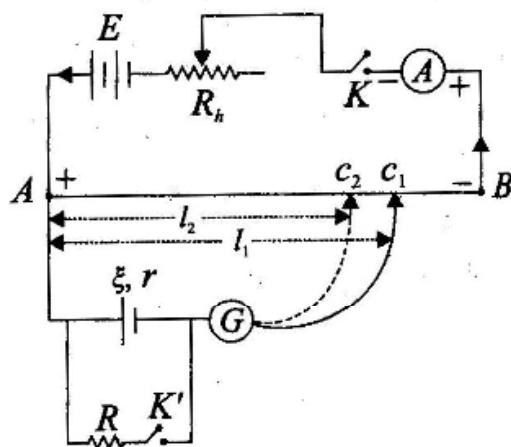
Again find the balance point C_2 when the key K' is closed and a resistance R is in the circuit. In this case the cell is in the closed circuit and

$$V = pd \text{ across } AC_2$$

$$\text{i.e., } V \propto l_2 \text{ or } \xi = kl_2$$

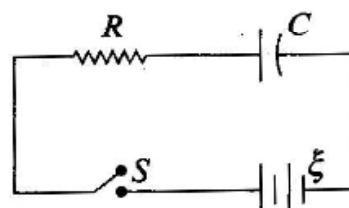
$$\text{We know that } r = \frac{\xi - V}{V} \times R$$

$$\text{Hence, } r = \frac{kl_1 - kl_2}{k_2} \times R \quad \text{or} \quad r = \left(\frac{l_1 - l_2}{l_2} \right) \times R$$



16.6 STUDY ABOUT R-C CIRCUIT

A capacitor in series with a resistor connected to a source of potential difference is called as R - C charging circuit. We have to study about transient state. Let at $t = 0$, the switch is closed and after t time the charge on capacitor be q and current through the resistor be i .



$$\text{Applying KVL, } -iR - \frac{q}{C} + \xi = 0$$

$$\Rightarrow -\left(\frac{dq}{dt} \right) R - \left(\frac{q}{C} \right) + \xi = 0 \Rightarrow -\left(\frac{dq}{dt} \right) R = \left(\frac{q}{C} - \xi \right)$$

$$\Rightarrow R \left(\frac{dq}{dt} \right) = \left(\xi - \frac{q}{C} \right) \Rightarrow \int \frac{dt}{R} = \int \frac{dq}{\left(\xi - \frac{q}{C} \right)}$$

$$\Rightarrow \frac{t}{R} = \frac{\ln \left(\xi - \frac{q}{C} \right)}{\left(\frac{-1}{C} \right)} + A \quad (1)$$

At $t = 0, q = 0$. Hence from equation (1),

$$0 = \frac{\ln \xi}{\left(\frac{-1}{C} \right)} + A \Rightarrow A = -\frac{\ln \xi}{\left(\frac{-1}{C} \right)}$$

$$\therefore \frac{t}{R} = \frac{\ln \left(\xi - \frac{q}{C} \right)}{\left(\frac{-1}{C} \right)} - \frac{\ln \xi}{\left(\frac{-1}{C} \right)} \Rightarrow \frac{t}{RC} = \ln \left(\frac{\xi - \frac{q}{C}}{\xi} \right)$$

$$\Rightarrow \frac{-t}{RC} = \ln \left(1 - \frac{q}{C\xi} \right) \Rightarrow 1 - \frac{q}{C\xi} = e^{-t/RC}$$

$$\Rightarrow \frac{q}{C\xi} = 1 - e^{-t/RC} \Rightarrow q = C\xi (1 - e^{-t/RC}) \quad (2)$$

When capacitor becomes fully charged ($q = q^{\max}$)

Then $i = 0 \Rightarrow V_R = 0$

i.e., $\xi = V_C \rightarrow \xi = \frac{q^{\max}}{C} \rightarrow q^{\max} = C\xi$

i.e., Equation (2) becomes

$$q = q^{\max} (1 - e^{-t/RC}) \quad (3)$$

On the basis of dimensional analysis, it is found that $R-C$ has the dimensions of time. It is called the time constant of the **R-C Charging Circuit**.

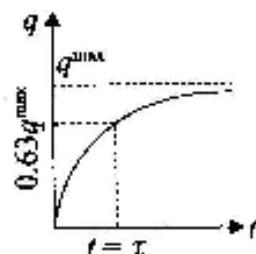
i.e., Time constant $\tau = R \times C$

If $t = \tau$,

$$\text{Then } q = q^{\max} [1 - e^{-\tau/RC}] = q^{\max} \left[1 - \frac{1}{e} \right] = q^{\max} [1 - 0.37] = (0.63) q^{\max}$$

And when $q = q^{\max}$ then $t \rightarrow \infty$

The time constant of **R-C charging circuit** is that time interval in which 63 per cent of charging of the capacitor has been done. A capacitor never becomes fully charged, but practically after 3 or 4 time constants the capacitor is considered as fully charged.



Again, at any time t , the energy stored in the capacitor is

$$U = \frac{q^2}{2C} = \frac{(q_{\max})^2}{2C} \left(1 - e^{-\frac{t}{RC}}\right)^2$$

$$U = \left(\frac{1}{2} C \xi^2\right) \left(1 - e^{-\frac{t}{RC}}\right)^2 = U_{\max} (1 - e^{-t/RC})^2 \quad (4)$$

After one time-constant, $U = U_{\max} (0.63)^2$

$\Rightarrow U \approx (0.40) U_{\max}$ and when $U \rightarrow U_{\max}$, $t \rightarrow \infty$

Again at any time t , current in the resistor is,

$$i = \frac{dq}{dt} = \frac{dt}{dt} \left[C \xi \left(1 - e^{-\frac{t}{RC}}\right) \right] \Rightarrow i = \left(\frac{\xi}{R}\right) e^{\frac{-t}{RC}} \quad (5)$$

At $t = 0$, $V_C = 0 \therefore \xi = V_R$ then current in resistor $= \frac{V_R}{R} = \left(\frac{\xi}{R}\right)$

and also $t \rightarrow \infty$, $i \rightarrow 0$

$$\text{i.e., } i = i_{\max} e^{-t/RC} \quad (\text{where } i_{\max} = \frac{\xi}{R} \text{ at } t = 0) \quad (5')$$

After one time-constant, $i = (0.37)i_{\max}$

Clearly in $R-C$ charging circuit the current through the resistor is not constant but for very short time interval dt it can be assumed as constant. The heat produced in the resistor for time-interval dt is

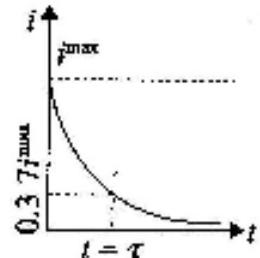
$$dH = (i^2 R) dt = \left(\frac{\xi^2}{R^2}\right) e^{-2t/RC} \times R dt \Rightarrow dH = \left(\frac{\xi^2}{R}\right) e^{-2t/RC} dt$$

\therefore Heat produced for $t = 0$ to $t = t$ is

$$H' = \int_{t=0}^t dH = \left(\frac{\xi^2}{R}\right) \int_{t=0}^{t=t} e^{-2t/RC} dt = \left(\frac{\xi^2}{R}\right) \left[\frac{e^{-2t/RC}}{(-2)} \right]_{t=0}^{t=t}$$

$$= -\left(\frac{\xi^2 RC}{2R}\right) \left(e^{-2t/RC}\right)_0^t = -\frac{C \xi^2}{2} \left(e^{-\frac{2t}{RC}} - 1\right)$$

$$H' = \left(\frac{1}{2} C \xi^2\right) \left(1 - e^{-\frac{2t}{RC}}\right) = U_{\max} \left(1 - e^{-\frac{2t}{RC}}\right) \quad (6)$$



(where U_{\max} = Total heat produced)

When a capacitor is charged by connecting it to a battery through a resistor, the energy dissipated (U_{\max}) as heat equals the energy stored (U_{\max}) in the capacitor.

Again, at any time t , p.d across the resistor is $V_R = iR = \left(\frac{\xi}{R}\right) e^{-t/RC} \times R$

$$\Rightarrow V_R = (\xi) e^{-t/RC} = V_R^{\max} \times e^{-t/RC} \quad (7)$$

(where $V_R^{\max} = \xi$ at $t = 0$)

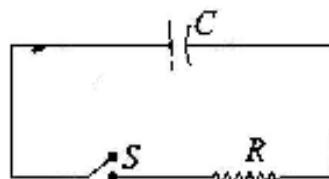
16.7 R-C DISCHARGING CIRCUIT

Initially at $t=0$,

Charge on capacitor = Q ,

P.d. of capacitor = $Q/C = V_0$ and

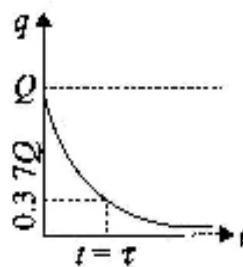
$$\text{Energy stored in capacitor} = \frac{Q^2}{2C} = U_0$$



The circuit in figure (the capacitor is initially charged) is called as R - C discharging circuit. During the discharging of the capacitor there is current in the resistor. Let us consider that at time t after the switch is closed, the current through resistor is i and the amount of the charge on the capacitor is q .

Applying KVL,

$$\begin{aligned} \frac{-q}{C} + iR &= 0 \Rightarrow \frac{-q}{C} + \left(\frac{-dq}{dt} \right) R = 0 \Rightarrow \frac{q}{C} + R \left(\frac{dq}{dt} \right) = 0 \\ \Rightarrow R \left(\frac{dq}{dt} \right) &= \frac{-q}{C} \Rightarrow \int \frac{dq}{q} = -\left(\frac{1}{RC} \right) \int dt \\ \Rightarrow \ln q &= -\left(\frac{t}{RC} \right) + A \\ \text{At } t=0, q &= Q \quad \therefore \ln q = 0 + A \\ \ln q &= \left(\frac{t}{RC} \right) + \ln Q \\ \Rightarrow \ln \left(\frac{q}{Q} \right) &= -t/RC \\ \Rightarrow \frac{q}{Q} &= e^{-t/RC} \Rightarrow q = Q e^{-t/RC} \end{aligned} \tag{1}$$



Clearly, the product RC has the dimension of time and it is called time-constant of R - C discharging circuit. i.e., $\tau = (RC)$

After one time-constant, $q = Q \times e^{-1} = (0.37)Q$

and $q \rightarrow 0$ i.e., $t \rightarrow \infty$

Again, at time t , the current in the resistor is

$$\begin{aligned} i &= \left(\frac{dq}{dt} \right) \Rightarrow i = -Q e^{-t/RC} \times -1/RC \Rightarrow i = \frac{Q}{RC} e^{-t/RC} \\ \Rightarrow i &= \left(\frac{Q/C}{R} \right) e^{-t/RC} \Rightarrow i = \frac{V_0}{R} e^{-t/RC} \\ \Rightarrow i &= i_0 e^{-t/RC} \end{aligned} \tag{2}$$

(where $i_0 = \frac{V_0}{R}$ at $t=0$)

