Again at any time t, p.d across the capacitor is

$$V_C = \frac{q}{c} = \left(\frac{Q}{C}\right)e^{\frac{-t}{RC}} \quad \Rightarrow \quad V_C = V_o \times e^{-t/RC} \tag{3}$$

And p.d across the resistor is, $\quad V_R = V_o \times e^{-t/RC}$ (4)

Again at any time t, energy stored in the capacitor is

$$U = \frac{q^2}{2C} = \left(\frac{Q^2}{2C}\right)e^{\frac{-2t}{RC}} = U_o\, e^{-2t/RC} \tag{5}$$

And heat produced in the resistor

$$dH = (i^2R)\ dt = \left(\frac{V_o}{R}\right)\ e^{-2t/RC}\, dt$$

$\therefore$ Heat produced for $t = 0$ to $t = t$ is

$$H' = \int_{t=0}^{t} dH = \left(\frac{V_0^2}{R}\right)\int_{t=0}^{t} e^{\frac{-2t}{RC}}dt = \left(\frac{V_0^2}{R}\right)\left(\frac{e^{\frac{-2t}{RC}}}{\frac{-2}{RC}}\right)_{t=0}^{t}$$

$$= \frac{1}{2}CV_0^2\left(1 - e^{\frac{-2t}{RC}}\right) = U_0\left(1 - e^{\frac{-2t}{RC}}\right) \tag{6}$$

And total heat produced is, $H = \int dH = \left(\frac{V_0^2}{R}\right)\int_{t=0}^{\infty} e^{\frac{-2t}{RC}}\ dt = U_o$ (7)

16.7.1 Power or Heat

Power supplied = ξi per sec

Power consumed = ξi

Energy supplied = $\xi it = q\xi$

Energy consumed = $q\xi$

Resistor (always consume)

Power consumed = $i^2R = \frac{V^2}{R} = Vi$

Energy (heat) = $i^2Rt = \frac{V^2}{R}t = Vit$

In parallel, $P = \frac{V^2}{R} \quad \therefore \quad P \propto \frac{1}{R}$

In series, $P = i^2R \quad \therefore \quad P \propto R$

In parallel, $P_{net} = P_1 + P_2$

In series, $\frac{1}{P_{net}} = \frac{1}{P_1} + \frac{1}{P_2} \Rightarrow P = \frac{P_1P_2}{P_1 + P_2}$

1. The conversion of electrical energy from thermal energy was discovered by Seebeck in 1826. According to him if the junctions of two different metals are kept at different temperatures, then there is an electric current in the circuit. This effect is called *Seebeck effect*.
2. The thermo-electric series is as: Animony, Fe, Cd, Zn, Ag, Au, Rb, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Pt, Ni, constantan, bismuth. If a thermocouple be madeup of copper and iron. The

current will be from iron to copper at the cold junction. The largest emf will generate in a thermocouple made of antimony and bismuth.

3. If t_C, t_n and t_i denote the temperature of the cold junction, the neutral temperature and the inversion temperature respectively, then

$$t_n - t_C = t_i - t_n \quad \text{or} \quad t_n = \frac{t_C + t_i}{2}$$

4. If the cold junction is kept in ice (0°C) and the hot junction at t(°C), then the thermo-emf depends on the temperature as

$$\xi_{AB} = a_{AB}t + \frac{b_{AB}}{2}t^2$$

where a_{AB} and b_{AB} are constants for a pair of metals A and B. It should be remembered that the parabolic variation is obtained in the case when either of a and b is negative.

$$\xi_{max} = -\frac{a^2}{b}$$

5. In 1834, Peltier discovered that if a current is allowed to pass through the junction of two different metals, heat is either evolved or absorbed at the junctions. That means the junction is either heated or cooled. This effect is known as Peltier effect and the heat evolved or absorbed is known as Peltier heat.

6. If a metal has a non-uniform temperature and a current is passed through it, heat is absorbed or evolved in different sections of the metal. This heat is over and above the Joule's heat. This effect is called Thomson effect. If a charge ΔQ is passed through a small section of the metal wire having temperature difference Δt between the ends, the Thomson coefficient

$$\sigma = \frac{\Delta H}{\Delta Q \Delta t}$$

16.8 FARADAY'S LAW OF ELECTROLYSIS

16.8.1 First Law

The mass of a substance liberated at an electrode is proportional to the amount of the charge passing through the electrolyte. Thus, $m = zQ$

If an electric current of constant magnitude i is passed through an electrolyte for a time t, then

$$Q = it \quad \therefore \quad m = zit$$

where z is a constant called electrochemical equivalent (ECE) of the substance. The SI unit of ECE is kg/C.

16.8.2 Second Law

The mass of a substance liberated at an electrolyte by a given amount of charge is proportional to the chemical equivalent of the substance. The chemical equivalent of a substance is

$$W = \frac{\text{Atomic mass}}{\text{Valency}} \quad \text{Also, } \frac{W_1}{z_1} = \frac{W_2}{z_2} = \text{Constant (F)}$$

or $$\frac{W}{z} = F$$

F is the proportionality constant called Faraday's constant. 1F = 96500 C/Kg-eq.

Table 16.3 E.M.F. of Cells: Volts

Cell	E.M.F.	Cell	E.M.F.
Daniell	1.08 – 1.09	Cadmium at 20°C	1.018 54
Grove	1.8 – 1.9		
Lechlanche	1.45	Lead accumulator	1.9 – 2.2
		Edison cell	1.45
Voltaic	1.01	Clarke	1.43
Bunsen	1.95	Ni-Fe	1.20

Table 16.4 Electro-chemical Equivalent of Elements

Element	Atomic Weight	Valency	E.C.E. (g/Coulomb)
Copper	63.57	2	0.000 329 5
Gold	197.2	3	0.006 812
Hydrogen	1.0080	1	0.000 010 45
Lead	207.21	2	0.001 073 6
Nickel	58.69	3	0.000 202 7
Oxygen	16.00	2	0.000 082 9
Silver	107.88	1	0.001 118 0

MAGNETIC EFFECT OF CURRENT AND MAGNETISM

17.1 MAGNETIC FIELD PRODUCED BY MOVING CHARGE OR CURRENT

SI unit $\rightarrow$ Tesla (T) = 1 weber m^{-2}, CGS unit $\rightarrow$ Gauss, $1T = 10^4 G$

Magnetic field at a point is said to be one tesla if a charge of 1 coulomb, when moving perpendicular to the direction of the magnetic field with a velocity of 1 m/s, experiences a force of 1 N.

17.2 MAGNETIC FORCE ON A MOVING CHARGE IN UNIFORM MAGNETIC FIELD

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \rightarrow \text{with sign or } F_m = Bqv \sin\theta \rightarrow \theta \text{ is the angle between } \vec{v} \text{ and } \vec{B}.$$

Direction of $\vec{F}_m$ can be decided with the help of right hand screw rule. A magnetic force can change path of a charged particle but cannot change its speed or kinetic energy.

17.3 PATH OF A CHARGED PARTICLE IN UNIFORM MAGNETIC FIELD

1. At rest $\vec{u} = 0$

   $\therefore\ F_m = 0 \quad \therefore\ \vec{a} = 0 \quad \therefore\ \vec{v} = 0$
2. Straight line $\theta = 0$ and $180°$, $\vec{v}$ = Constant
3. Uniform circular motion if $\theta = 90°$ when $\vec{v} \cdot \vec{B} = 0$

17.4 LIST OF FORMULAE IN UNIFORM CIRCULAR MOTION

1. $r = \dfrac{mv}{Bq} = \dfrac{p}{Bq} = \dfrac{\sqrt{2Km}}{Bq} = \dfrac{\sqrt{2qVm}}{Bq}$, where K is the kinetic energy of the charged particle and V is the potential difference. Here r is called the *gyroradius* or *cyclotron radius.*
2. $T = \dfrac{2\pi m}{Bq}, v = \dfrac{Bq}{2\pi m}$, here v is called the *gyrofrequency* or the *cyclotron frequency* and T is the time period of the uniform circular motion.
3. Angular frequency $\omega = \dfrac{Bq}{m}$

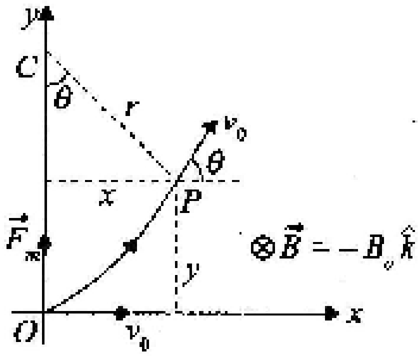
17.4.1 To Find Velocity and Position at Time t

1. $\theta = \omega t$
2. Plane of circle perpendicular to $\vec{B}$, make a plane of circle in your copy.
3. Assume particle starts from origin, final coordinates of particle

= Initial coordinate + Coordinate at time t

A particle of specific charge α enters a uniform magnetic field $\vec{B} = -B_o\hat{k}$ with velocity $\vec{v} = v_o\hat{i}$ from the origin.

The angle between $\vec{v}$ and $\vec{B}$ is 90°. Therefore, the path is a circle and its plane is x-y (perpendicular to the magnetic field). The sense of the rotation will be anticlockwise as shown in figure, because at origin the magnetic force is along positive y-direction (from right hand rule). Hence, the deviation and radius of the particle are,



$OC = CP$ = Radius of circle

$$\theta = \omega t = B_o\alpha t \text{ and } r = \frac{v_o}{B_o\alpha}$$

Velocity of the particle at any time t is,

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = v_o\cos\theta\,\hat{i} + v_o\sin\theta\,\hat{j} \quad \text{or} \quad \vec{v} = v_o\cos(B_o\alpha t)\hat{i} + v_o\sin(B_o\alpha t)\hat{j}$$

Position of particle at time t is, $\vec{r} = x\hat{i} + y\hat{j} = r\sin\theta\,\hat{i} + (r - r\cos\theta)\hat{j}$

Substituting the values of r and θ, we have

$$\vec{r} = \frac{v_o}{B_o\alpha}[\sin(B_o\alpha t)\hat{i} + \{1 - \cos(B_o\alpha t)\}\hat{j}]$$

17.4.2 Helical Path

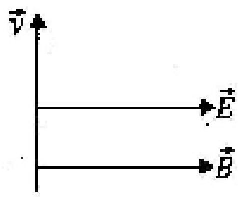
Angle between $\vec{v}$ and $\vec{B}$ neither 0° nor 180° nor 90°, path is helical.

Due to parallel component of $\vec{v}$, particle will travel in straight line, due to perpendicular component it will rotate in a circle.

Formulae in helical path.

1. $r = \dfrac{mv_\perp}{Bq} = \dfrac{mv\sin\theta}{Bq}$
2. $T = \dfrac{2\pi m}{Bq}$
3. $\omega = \dfrac{Bq}{m}$
4. Pitch $= v_{\parallel} \times T = (v\cos\theta) \times \left(\dfrac{2\pi m}{Bq}\right)$

Path of charged particle in both uniform electric and magnetic field $\vec{E}$ will be parallel to $\vec{B}$ and $\vec{v}$ is perpendicular to these two. Resultant path is helical with increasing pitch.

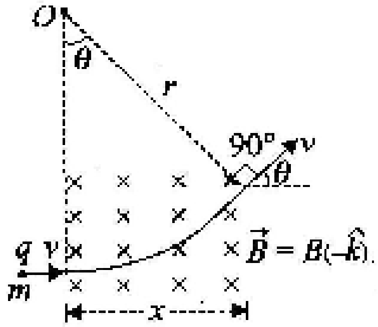


17.5 DEVIATION OF CHARGE PARTICLE IN MAGNETIC FIELD

Suppose a charged particle q enters perpendicularly in a uniform magnetic field $\vec{B}$. The magnetic field extends to a distance x, which is less than or equal to radius of the path, that is $x \le r$.

1. The radius of path $r = \dfrac{mv}{qB}$

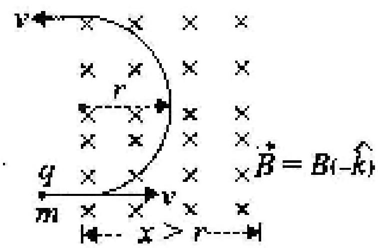
   and $\sin\theta = \dfrac{x}{r}$, when $x \le r$.



2. For $x > r$

   $r = \dfrac{mv}{qB}$

   and deviation, $\theta = 180°$ as clear from the diagram.

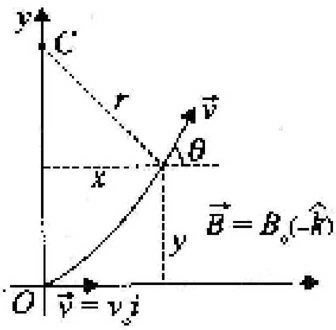


3. If particle moves for time t inside the field, then

   $\theta = \omega t = \left(\dfrac{Bq}{m}\right)t$

   As specific charge is $\alpha = q/m$

   $\therefore \quad \theta = B\alpha t$



4. Velocity of particle,

   We have, $\theta = B_o\alpha t$, $r = \dfrac{v_o}{\alpha B_o}$

   Velocity of particle at any time t, $\vec{v} = v_x\hat{i} + v_y\hat{j} = v_o\cos\theta\hat{i} + v_o\sin\theta\hat{j}$

   On substituting the value of θ, we have $\vec{v} = v_o\cos(B_o\alpha t)\hat{i} + v_o\sin(B_o\alpha t)\hat{j}$

5. Position of particle, $\vec{r} = x\hat{i} + y\hat{j} = r\sin\theta\hat{i} + (r - r\cos\theta)\hat{j}$

$$= r[\sin\theta\hat{i} + (1-\cos\theta)\hat{j}]$$

$$= \frac{v_o}{B_o\alpha}[\sin(B_o\alpha t)\hat{i} + \{1-\cos(B_o\alpha t)\}\hat{j}]$$

17.5.1 Lorentz Force

The force experienced by a charged particle moving in space where both electric and magnetic fields exist is called the Lorentz force.

$$\vec{F} = \vec{F}_e + \vec{F}_m \quad \text{or} \quad \vec{F} = q\vec{E} + q(\vec{v}\times\vec{B}) \quad \text{or} \quad \vec{F} = q[\vec{E} + (\vec{v}\times\vec{B})]$$

17.5.2 Cyclotron

It is a particle accelerator and is used to accelerate positive ions. Under the action of magnetic field, the positive ions move along spiral path and gain energy as they cross the alternating electric field again and again.

Cyclotron is based on the principle that the positive ions can be accelerated to high energies with a comparatively smaller alternating potential difference by making them to cross the electric field again and again, by making use of a strong magnetic field.

The positive ions of charge q and mass m in cyclotron attain maximum energy which is given by:

1. $E_{max} = \frac{1}{2} \cdot \frac{B^2 q^2 R^2}{m}$ where R is radius of the dees of the cyclotron.
2. $E_{max} = 2n(Vq)$ where n is number of revolutions completed by the positive ions before leaving the dees.

Limitations of the Cyclotron

1. Cyclotron cannot accelerate uncharged particle like neutron.
2. The positively charged particles having large mass i.e. ions cannot move at limitless speed in a cyclotron.

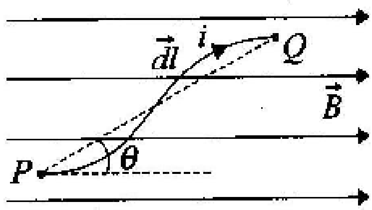
17.6 MAGNETIC FORCE ON A CURRENT CARRYING WIRE IN A UNIFORM MAGNETIC FIELD

$$\vec{F}_m = I(\vec{l} \times \vec{B}) \rightarrow \text{Straight line}$$

$$F_m = ilB \sin\theta \rightarrow \theta \text{ is the angle between } \vec{l} \text{ and } \vec{B} \text{ or current and } \vec{B}$$

Direction of F_m → $\vec{l} \times \vec{B}$; → RHR

17.7 MAGNETIC FORCE ON A CURVED WIRE IN UNIFORM $\vec{B}$

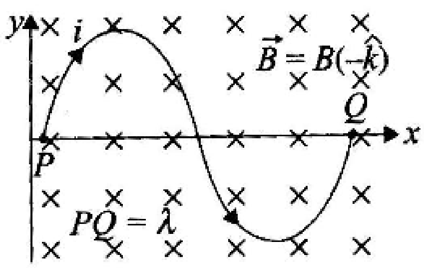


Let us consider a conducting wire of arbitrary shape and is placed in uniform magnitude field $\vec{B}$. The force on $\vec{dl}$ length of the conductor $\vec{dF} = i\,\vec{dl} \times \vec{B}$. To get force on the whole wire, we have to integrate $\vec{dF}$ over the length of the wire. Thus

$$\vec{F} = \int_P^Q i\vec{dl} \times \vec{B} = i\left[\int_P^Q \vec{dl}\right] \times \vec{B} = i\,\vec{PQ} \times \vec{B}$$

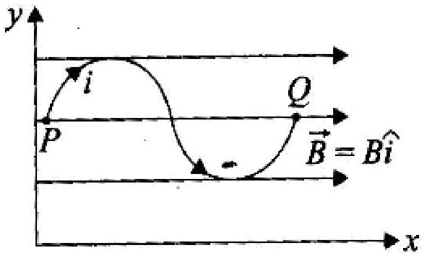
The other simpler way to get the force on current carrying wire is to draw straight line joining the ends of the conductor (here PQ), and then find its component perpendicular to $\vec{B}$, here it is $PQ \sin\theta$. Therefore $F = Bi(PQ \sin\theta)$.

(a)



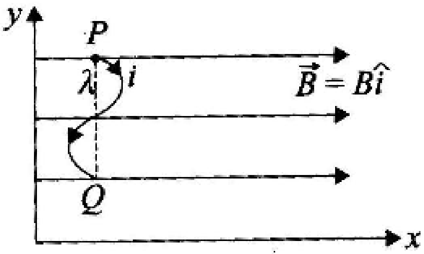
$F = Bi \times PQ = Bi\lambda$,
Direction of force along + y-axis

(b)



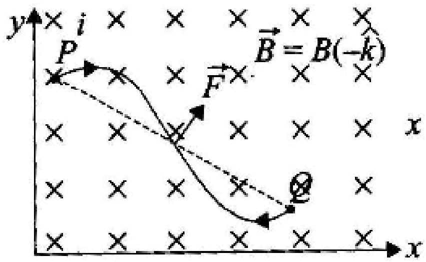
$\vec{F} = 0$

(c)



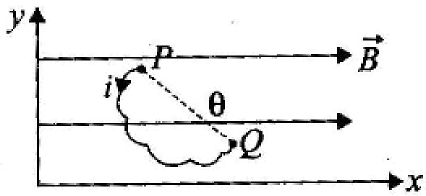
$\vec{F} = Bi\lambda\hat{k}$

(d)



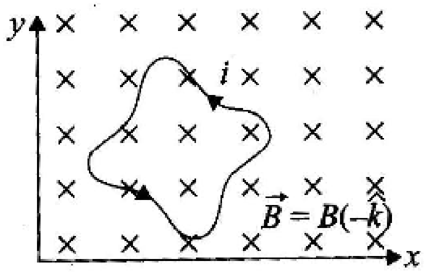
$F = Bi(PQ)$,
Direction of force perpendicular to line PQ

(e)

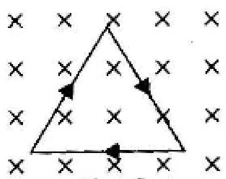


$F = Bi(PQ \sin\theta)$
Direction of force along + z-axiz

(f)



$F = 0$
i.e., Net magnetic force on a current carrying closed loop in field is zero



$F = 0$

17.8 TORQUE ON A CURRENT CARRYING COIL PLACED INSIDE A MAGNETIC FIELD

When a coil of area A having N turns and carrying current I is suspended inside a magnetic field of strength B, then torque on the coil is given by $\tau = NBIA\sin\theta$, when θ is angle between the direction of magnetic field and normal to the plane of the coil.

If the direction of magnetic field makes an angle α with the plane of the coil, then $\tau = NBIA\cos\alpha$.

The torque on the coil is maximum, when the plane of the coil is parallel to the magnetic field i.e., $\theta = 90°$ or $\alpha = 0°$.

A convenient vector notation for the above equation is $\vec{\tau} = I\vec{A} \times \vec{B}$. Here, $\vec{A}$ is area vector of the loop whose direction is determined by the right hand rule. $\vec{\tau}$ lies in the plane of the paper and is acting upwards.

Comparing equation with the equation for torque acting on a magnetic dipole of magnetic moment $\vec{m}$ in a uniform magnetic field $\vec{B}$, i.e.,

$$\vec{\tau} = \vec{m} \times \vec{B} \text{ we find that } \vec{m} = I\vec{A}.$$

Thus, a current carrying loop behaves as a bar magnet with its one face as south pole and the other face as north pole. The SI unit of magnetic moment is Am^2.

If magnetic dipole current carrying loop when placed in uniform magnetic field.

Similar to an electric dipole

1. $\vec{F}_m = 0$
2. $\vec{\tau} = \vec{M} \times \vec{B} = MB\sin\theta$
3. $U = -\vec{M} \cdot \vec{B} = -MB\cos\theta$
4. $W_{\theta_1 \to \theta_2} = (-\vec{M} \cdot \vec{B})_{\theta_1 \to \theta_2} = -MB(\cos\theta_1 - \cos\theta_2)$

17.9 MAGNETIC FIELD AT A POINT DUE TO A CURRENT OR SYSTEM OF CURRENT

Two methods:

1. Due to small current element → *M*-1 Bio-Savart Law + Integration; → *M*-2 Ampere Circuital Law

Biot-Savart law: According to Biot-Savart's law, the magnetic induction dB at a point P due to an infinitesimal element or current (length dl and current I) at a distance r is given by:

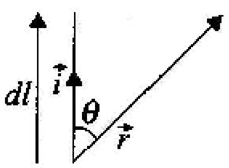
$dB = \frac{\mu_o}{4\pi} \times \frac{I\,dl\sin\theta}{r^2}$ μ_o is called permeability of free space. In SI unit, $\mu_o = 4\pi \times 10^{-7}$ henry/metre. Note that

$$1(\text{H/m}) = 1\frac{\text{Tm}}{\text{A}} = 1\frac{\text{Wb}}{\text{Am}} = 1\frac{\text{N}}{\text{A}^2} = 1\frac{\text{Ns}^2}{\text{C}^2}$$

The dimensions of $\mu_o = [M^1L^1T^{-2}A^{-2}]$

For vacuum: $\sqrt{1/\mu_o\varepsilon_o} = c = 3 \times 10^8$ m/s

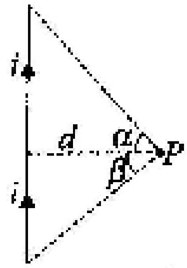
$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3}$$



2. Direction of $d\vec{B}$ → $\vec{dl} \times \vec{r}$; → Right hand rule; → Screw law

17.10 LIST OF FORMULAE

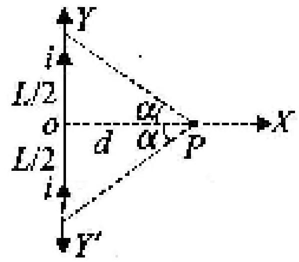
1. The magnetic field due to a current carrying straight conductor at a point is



$$B=\left(\frac{\mu_o i}{4\pi d}\right)(\sin\alpha+\sin\beta)\otimes$$

Where the conductor carries current i and its ends subtends angles α and β at the point P at which magnetic field is to be determined.

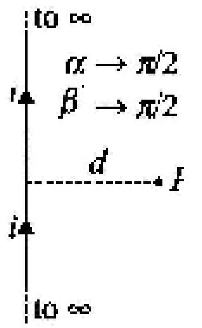
Case-I: If point P lies on the perpendicular bisector of the current carrying straight wire



$$B=\left(\frac{\mu_o i}{4\pi d}\right)2\sin\alpha$$

$$=\left(\frac{\mu_o i}{2\pi d}\right)\frac{L/2}{\sqrt{(L/2)^2+d^2}}\otimes$$

Case-II: If the current carrying wire becomes infinitely long, then

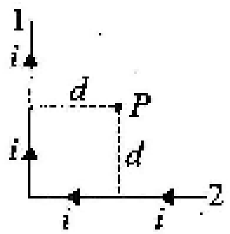


$$\alpha=\pi/2$$

$$\beta=\pi/2$$

$$\therefore \qquad B=\frac{\mu_o i}{2\pi d}\otimes$$

Case-III: If the point P lies on the intersection of perpendicular bisectors of the wires 1 and 2 as in figure, then



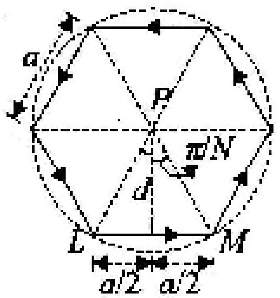
$$B_1=\left(\frac{\mu_o i}{2\pi d}\right)\left(\frac{L/2}{\sqrt{(L/2)^2+d^2}}\right)\otimes$$

$$B_2=\left(\frac{\mu_o i}{2\pi d}\right)\left(\frac{L/2}{\sqrt{(L/2)^2+d^2}}\right)\otimes$$

$$\therefore \qquad \vec{B}=\vec{B}_1+\vec{B}_2 \qquad \therefore \quad B=B_1+B_2$$

Two wires having parallel currents attract each other and having antiparallel current repel each other with the force per unit length $\left(\frac{\mu_o i_1 i_2}{2\pi d}\right)$. (There is no electrical interaction between the wires because they have no net charges.)

2. A current i flows along a thin wire shaped as a regular polygon with N sides which can be inscribed into a circle of radius R. Magnetic field due to a current carrying regular polygon shaped wire at its centre is



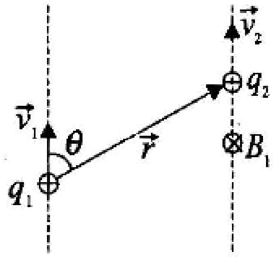
$$B=\left(\frac{2\mu_o i}{4r}\right)\frac{\tan(\pi/N)}{(\pi/N)}\odot$$

If $N \to \infty$ then the regular polygon is considered as a circle of radius r then magnetic field due to a current carrying circular loop at its centre is

$$B = \lim_{N\to\infty}\left(\frac{2\mu_o i}{4r}\right)\frac{\tan\pi/N}{\pi/N} = \frac{2\mu_o i}{4r}\lim_{N\to\infty}\frac{\tan\pi/N}{\pi/N} = \frac{\mu_o i}{2r}$$

$\therefore$ $$B = \left(\frac{\mu_o i}{2r}\right)\odot$$

3. The force of interaction between the two charges moving parallel to each other:



The magnetic field B set up by the moving charge q_1 at the location of charge q_2, at any instant is given by

$$\vec{B}_1 = \frac{\mu_o}{4\pi}\frac{q_1\vec{v}_1\times\vec{r}}{r^3}$$

Here $\vec{r}$ is the instantaneous position vector of the charge q_2 with respect to charge q_1.

The magnetic force $\vec{F}_m$ on the charge q_2 is given by

$$\vec{F}_m = q_2(\vec{v}_2\times\vec{B}_1) = q_2\vec{v}_2\times\frac{\mu_o}{4\pi}\frac{q_1(\vec{v}_1\times\vec{r})}{r^3}$$

$$= \frac{\mu_o}{4\pi}\frac{q_1q_2}{r^3}[\vec{v}_2\times(\vec{v}_1\times\vec{r})]$$

$$F_m = \frac{\mu_o}{4\pi}\frac{q_1q_2}{r^3}v_1v_2 r\sin\theta = \frac{\mu_o}{4\pi}\frac{q_1q_2}{r^2}v_1v_2\sin\theta$$

The electrical force at the same instant between the two charged particles is given by

$$F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$$

Comparing the magnetic and the electrical forces between the two charges,

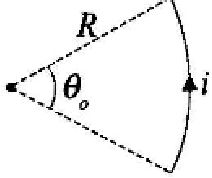
$$\frac{F_m}{F_e} = \mu_o\varepsilon_o v_1v_2\sin\theta = \frac{v_1v_2}{c^2}\sin\theta$$

where $c = \dfrac{1}{\sqrt{\mu_o\varepsilon_o}}$ is velocity of light in vacuum.

If $\theta = 90°$ then, $\dfrac{F_m}{F_e} = \dfrac{v_1v_2}{c^2}$

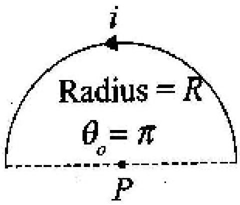
At any instant the magnetic force between two moving charges is much smaller than the electrical force between them.

4. A current carrying arc shaped wire having radius R as in figure subtends an angle θ_o at its centre. The magnetic field due to this wire at its centre is



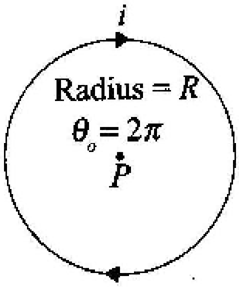
$$B = \left(\frac{\mu_o i}{4\pi R}\right)\theta_o\odot$$

Case-I:



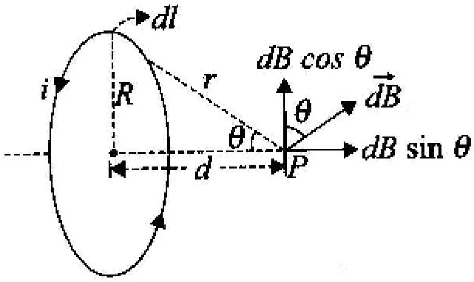
At P, $\vec{B} = \left(\dfrac{\mu_o i}{4\pi R}\right)\pi = \left(\dfrac{\mu_o i}{4R}\right) \odot$

Case-II:

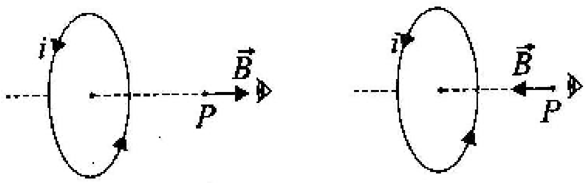


At P, $\vec{B} = \left(\dfrac{\mu_o i}{4\pi R}\right)2\pi \quad \Rightarrow \vec{B} = \left(\dfrac{\mu_o i}{2R}\right) \otimes$

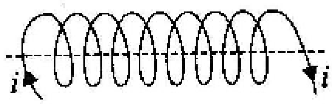
5. The magnetic field at a point P on the axis of the current carrying circular loop of radius R at a distance d from the centre of the loop is

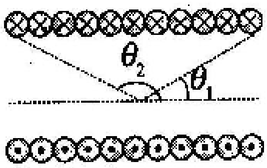


$$B = \left(\frac{\mu_o i}{2}\right)\frac{R^2}{(R^2 + d^2)^{3/2}}$$ (Along the axis away from the centre)



6. On the axis of solenoid,





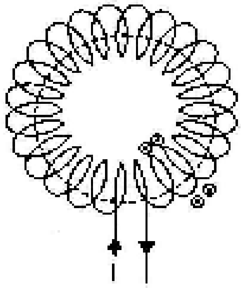
$$B = \frac{\mu_o n i}{2}(\cos\theta_1 - \cos\theta_2)$$

For very long solenoid $\theta_1 \to 0°$, $\theta_2 \to 180°$

$$B = \frac{\mu_o n i}{2}[1-(-1)] = \mu_o n i$$

At corner $\theta_1 \to 0°$, $\theta_2 \to 90°$ $\quad B = \dfrac{\mu_o n i}{2}$

7. Toroid:



Magnetic field at a point inside the core of the toroid

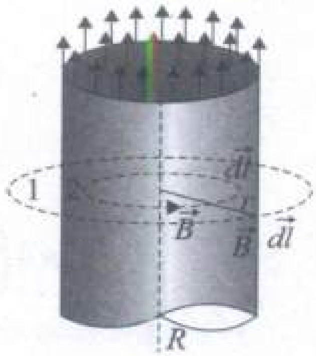
$$B = \frac{\mu_0 NI}{2\pi r} \qquad (1)$$

Thus, the field inside the toroid varies as $1/r$ and hence is non-uniform.

On the other hand, if the cross-sectional area of the toroid is very very small compared to r, we can neglect any variation in r. Considering $2\pi r$ to be the circumference of the toroid, $N/2\pi r$ will be a constant and equal to the number of turns per unit length (n). In this case, eqn. (1) takes the form,

$$B = \mu_0 nI$$

which is the same as for the long solenoid.



8. The magnetic field due to a current carrying long cylindrical wire of cross-sectional radius R and current i is

(a) At an outside point ($r \geq R$)

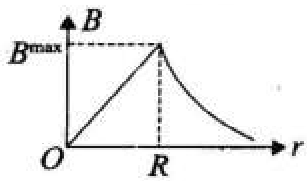
$$B = \frac{\mu_0}{2\pi}\frac{i}{r}$$

(b) At an inside point ($r < R$)

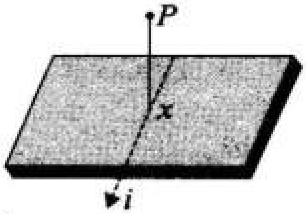
$$B = \frac{\mu_0}{2\pi}\frac{ir}{R^2}$$

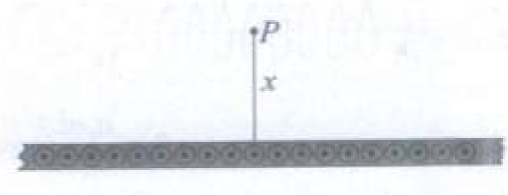
(c) On the surface point ($r = R$)

$$B = B^{max} = \frac{\mu_0}{2\pi}\frac{i}{R}$$



9. Figure shows a cross-section of a large metal sheet carrying an electric current along its axis.



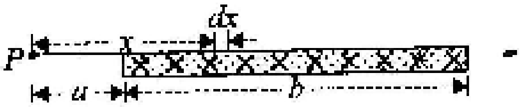


The current in a strip of width dl is kdl where k is constant. The magnetic field at a point P at a distance x from the metal sheet is

$$B = \frac{\mu_0 k}{2}$$

Clearly, magnetic field in this case is independent of x.

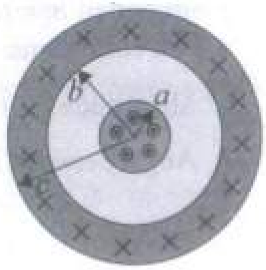
10. Figure shows a cross-section of a long thin ribbon of width b that is carrying a uniformly distributed total current i into the page.



The magnetic field $\bar{B}$ at a point P in the plane of the ribbon at a distance a from its edge is

$$B=\frac{\mu_o}{2\pi}\frac{i}{b}\ln\frac{(a+b)}{a}$$

11. Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_o and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance r from the axis where $r < a$, $a < r < b$, $b < r < c$ and $r > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.



According to the figure let the current be distributed uniformly over the cross-sections of outer and inner conductors. Current density in inner conductor $\frac{i}{\pi a^2}$.

Current density in outer conductor $= \frac{i}{\pi(c^2-b^2)}$

(a) **When $r < a$:** Consider a co-axial circular path (Ampereon loop) of radius r. Let B be the magnitude of magnetic field at this distance, then using Ampere's law

$$B\times 2\pi r=\mu_o\times\text{Current enclosed by path}=\mu_o\left(\frac{i}{\pi a^2}\times\pi r^2\right)=\frac{\mu_o i r^2}{a^2},$$

$$B=\frac{\mu_o}{4\pi}\times\frac{2ir}{a^2}\,\text{Wb/m}^2$$

(b) **When $a < r < b$:** In this case the circular path (Ampereon loop) of radius will enclose the current passing through inner conductor. Using Ampere's law

$$B\times 2\pi r=\mu_o i \qquad \text{or} \qquad B=\left(\frac{\mu_o i}{2\pi r}\right)\text{Wb/m}^2$$

(c) **When $b < r < c$:** Here, current enclosed by co-axial circular path (Ampereon loop) of radius r.

Current passing through inner conductor - current passing through portions of outer conductor lying between $r = b$ and $r = c$ (–ve sign is used because the currents in two conductors are in opposite directions).

By Ampere's law

$$B\times 2\pi r=\mu_o\left[i-\frac{i}{\pi(c^2-b^2)}\times\pi(r^2-b^2)\right]$$

$$\therefore \quad B=\frac{\mu_o i}{2\pi r}\left[1-\frac{(r^2-b^2)}{(c^2-b^2)}\right]$$

The net current enclosed by the circle (Ampereon loop) is zero and hence

$$B \times 2\pi r = 0 \quad \text{or} \quad B = 0$$

17.11 AMPERE'S CIRCUITAL LAW (ACL)

$$\oint \vec{B} \cdot \vec{dl} = \mu_o (i_{net})$$

Simplified $Bl = \mu_o (i_{net})$
Two conditions:

1. Magnitude of $\vec{B}$ at every point should be uniform in closed path.
2. Angle between $\vec{B}$ and $\vec{dl}$ should be 0° at every point.

17.12 MAGNET AND ITS CHARACTERISTICS

The term magnetism usually refers to the property by virtue of which a piece of iron or steel is attracted.

In the very beginning, it was established that pieces of the iron ore *magnetite* [Fe_3O_4] found in *magnesia* have the property of attracting certain other substances and pointing in north-south direction when suspended freely. These pieces are called *natural magnets* and the phenomenon *magnetism*. A natural magnet is an ore of iron (Fe_3O_4) which attracts small pieces of iron, cobalt and nickel towards it. Lode stone is a natural magnet.

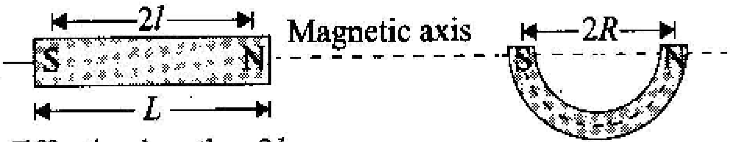
Due to their odd shapes and weak attracting power natural magnets are rarely used. Now a days, pieces of iron and other materials of suitable shapes and sizes are made magnets either by rubbing them with natural magnets or by passing direct current through a wire wound around them. The magnets which are prepared artificially are called *artificial magnet. e.g.* a bar magnet, a magnetic needle, electromagnet, a horse-shoe magnet etc.

17.13 PROPERTIES OF A MAGNET

1. It attracts iron or irons like substances towards it.
2. It aligns itself in geographical north-south direction when freely suspended. Clearly a magnet has both attractive as well as directive property.
3. The magnetic behaviour of a bar magnet is prominent near the ends and these points where the magnetic effect is prominent are called magnetic poles. (Poles of magnet are regions near the two ends of a magnet with maximum power of attraction. The strength of pole is called *pole strength* denoted by m or Q_m. S.I. unit of pole strength is *Am* (*Ampere metre*)
4. The pole strength of magnet depends on the material of the magnet, the state of magnetization and the cross-sectional area. At the centre of the magnet, the magnetism is almost zero and is considered as a neutral region.

5. Magnetic pole always exists in pairs. Magnetic monopole never exists (till today).
6. Magnetic poles are in a sense magnetic analogue of electric charges because the magnetic field of a magnetic dipole is identical to the electric field of an electric dipole. On this basis we could say that as electric charges exists in two forms, magnetic charges (called poles) also exists in two forms. North pole is magnetic analogue of +ve charge and south pole is magnetic analogue of –ve charge. So a north pole having a pole strength m experience a force $\vec{F} = m\vec{B}$ in a magnetic field $\vec{B}$.

   Similarly the magnetic field at a point at a distance of r from a north pole having pole strength m is given by $B = \frac{\mu_0}{4\pi}\frac{m}{r^2}$ along the line joining the point from the pole and away from it.
7. Poles are not exactly at the ends, they are a little inwards and the separation between the poles called *magnetic length.*



Effective length = $2l$
Actual length = L

Effective length = $2R$
Actual length = πR

   It should, however, be noted that no point as poles actually exists inside a magnet. It is due to the cummulative effect of the atomic magnets at the ends due to which it appears near the ends and the forces appear to be originating or terminating from or to that point.
8. As magnetic poles are magnetic analogues of electrical charges so the force between two magnetic poles is given by Coulomb's law.

   Coulomb's law: The force between any two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

   The force between two point poles of strength m_1 and m_2 at a distance d apart is given by $F = \frac{\mu m_1 m_2}{4\pi r^2}$, where μ is called the absolute permeability of the medium.

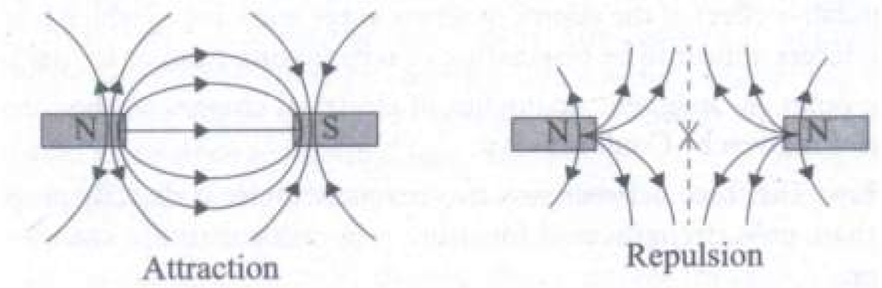
   Also, $\mu = \mu_o\mu_r$, where $\mu_o = 4\pi \times 10^{-7}$ henry/metre is the permeability of free space and μ_r is the relative permeability of the medium. μ_o is also expressed as TmA^{-1} (T = tesla).
9. Coulomb's law of magnetism is an outdated law because at present, we do not have magnetic monopoles (but the law gives the force between two monopoles). Now, we know that the magnetism and electricity are two facets of a single coin, called as electromagnetism.
10. Since like poles repel each other and unlike poles attract each other. (This is why it is said that repulsion is the surer test of magnetism).
11. When a magnet is suspended freely, it comes to rest along north-south direction. The end point towards geographic north is called north pole and the end point towards geographic south is called south pole.
12. *Unit pole* is defined as that pole which when placed in vacuum (or in air) at a distance of one meter from an equal and similar pole, repels it with a force equal to 10^{-7} newton.

17.14 MAGNETIC LINES OF FORCE AND THEIR CHARACTERISTICS

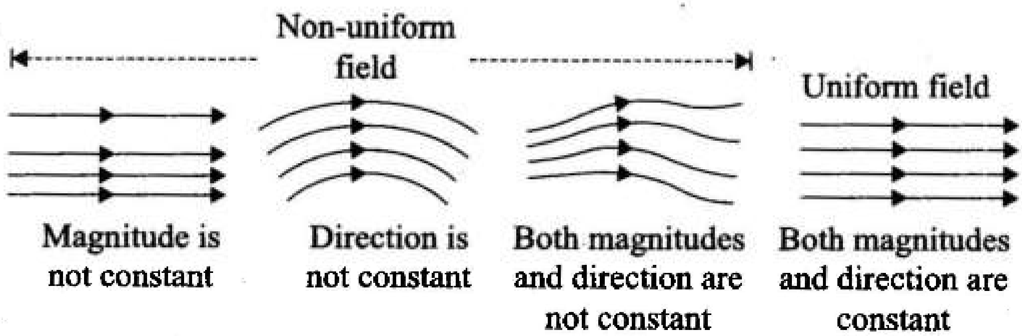
The space surrounding a magnet or magnetic configuration in which its effects are perceptible is called the magnetic field of the given magnet or magnetic configuration.

In order to visualize a magnetic field pictorially, **Michael Faraday** introduced the concept of lines of force. According to him a line of force is an imaginary curve the tangent to which at a point gives the direction of the field at that point.

1. Outside a magnet, lines of force are from north to south pole while inside, from south to north, i.e., magnetic lines of force are closed curves, i.e., they appear to converge or diverge at poles.
2. The number of magnetic lines of force originating or terminating on a pole is proportional to its strength, μ_o lines are assumed to be associated with a unit pole. So if a body encloses a pole of strength *m*, total lines of force linked with the body (called magnetic flux) will be $\mu_o(m_{encl})$.
3. Magnetic lines of force can never intersect each other because if they intersect at a point, intensity at that point will have two directions which is not possible according to definition of the field.
4. Magnetic lines of force are in longitudinal tension like a stretched elastic spring and repel each other laterally.



5.

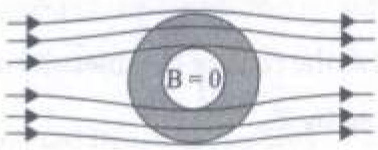


6. In a region of space where there is no magnetic field, there will be no lines of force. This is why, at a neutral point (where resultant field is zero) there cannot be any line of force.
7. Magnetic lines of force originates from or enter in the surface of a magnetic material at any angle.
8. Magnetic lines of force exist inside every magnetized material.

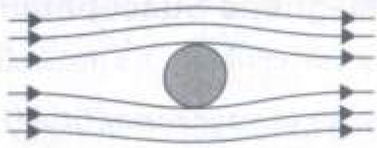
   (a) Gauss law in magnetism is $\oint \vec{B}\cdot d\vec{S} = \mu_o m_{in}$

As magnetic monopole never exists, the smallest unit of the source of magnetic field is a magnetic dipole, where the net magnetic pole is zero. Hence, net magnetic pole enclosed by any closed surface is always zero. Correspondingly, the flux of the magnetic field through any closed surface is zero. So, Gauss law for magnetism states $\oint \vec{B} \cdot d\vec{S} = 0$. Hence Gauss law for magnetism suggests that magnetic lines of forces are closed curves they neither originate from a given pole nor terminate on anyone. This is the only difference of magnetic lines of forces with that of electric lines of forces which originate from +ve charge and terminate on –ve charge and hence are open curves. As monopoles do not exist, the total magnetic flux linked with a closed surface is always zero.

(b) If a soft iron ring is placed in a magnetic field most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called *magnetic screening* or *shielding* and is used to protect costly wrist-watches and other instruments from external magnetic fields by enclosing them in a soft-iron case.



Iron ring in a field



Super conductor in a field

Magnetic Screening

(c) An arrangement of two unlike poles of equal strength and separated by a small distance is called *magnetic dipole.*

The distance $2l$ between the two magnetic poles is called the magnetic length of the magnetic dipole and is denoted by $(2\vec{l})$, a vector from south to north pole of the magnetic dipole.

S.No.	Physical Quantity	Magnetic Dipole
1.	Dipole moment	$M = m\,(2l)$
2.	Direction of dipole moment	From south to north pole
3.	Net force in uniform field	0
4.	Net torque in uniform field	$\vec{\tau} = \vec{M} \times \vec{B}$
5.	Field at far away point on the axis	$\frac{\mu_o}{4\pi} \cdot \frac{2M}{r^3}$ (along $\vec{M}$)
6.	Field at far away point on perpendicular bisector	$\frac{\mu_o}{4\pi} \cdot \frac{M}{r^3}$ (opposite $\vec{M}$)
7.	Potential energy	$U_\theta = -\vec{M} \cdot \vec{B} = -MB\cos\theta$
8.	Work done in rotating the dipole	$W_{\theta_1 - \theta_2} = MB(\cos\theta_1 - \cos\theta_2)$

17.14.1 Intensity of Magnetization

It is defined as the magnetic dipole moment developed per unit volume or the pole strength developed per unit area of cross-section of the specimen. It is given by $I = \frac{M}{V} = \frac{m}{a}$.

Here, V is volume and a is area of cross-section of the specimen. Magnetic induction, intensity of magnetization and magnetic intensity are related to each other as below:

$$B = \mu_o (H + I)$$

17.14.2 Magnetic Permeability

The magnetic permeability of a material is defined as the ratio of the magnetic induction (B) of the material to the strength of magnetizing field (H). It is given by $\mu = \frac{B}{H}$

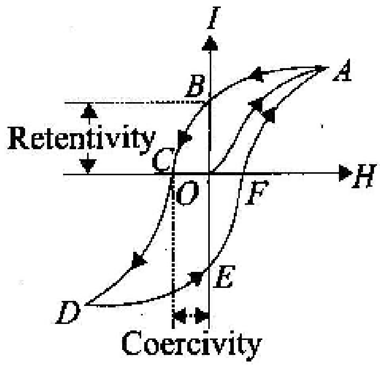
If μ_r is relative permeability of a medium, then $\mu_r = \frac{\mu}{\mu_o}$

17.14.3 Magnetic Susceptibility

The magnetic susceptibility of a material is defined as the ratio of the intensity of magnetization (I) and the strength of magnetizing field (H). It is given by $\chi_m = \frac{I}{H}$

Also $\mu = \mu_o (1 + \chi_m)$ so that $\mu_r = 1 + \chi_m$

- The resultant field produced inside a specimen placed in a magnetic field (along the field) is called magnetic induction B or magnetic flux density.
- **Hysteresis:** The lagging of intensity of magnetization (or magnetic induction) behind the magnetizing field, when a magnetic specimen is taken through a cycle of magnetization, is called *hysteresis*. The value of intensity of magnetization of the magnetic material, when the magnetizing field is reduced to zero, is called its *retentivity*. The value of the reverse magnetizing field, which has to be applied to the magnetic material so as to reduce the residual magnetization to zero, is called its *coercivity*.



- On the basis of magnetic properties, different materials have been classified into three categories; diamagnetic, paramagnetic and ferromagnetic substances.

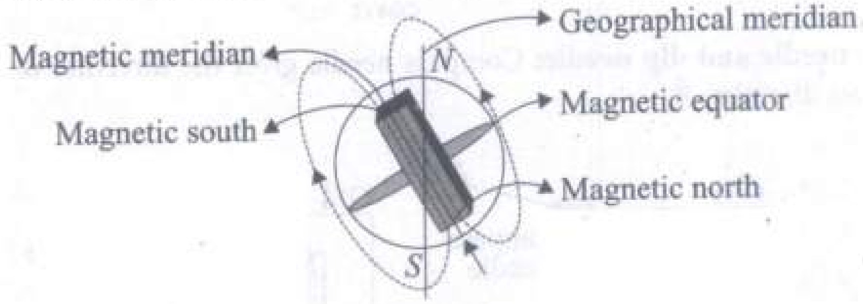
S. No.	Diamagnetism	Paramagnetism	Ferromagnetism
1.	Substances are feebly repelled by the magnet.	Substances are feebly attracted by the magnet.	Substances are strongly attracted by the magnet.
2.	Magngtization I is small, negative, and varies linearly with field.	I is small, positive and varies linearly with field.	I is very large, positive and varies non-linearly with field.
3.	Susceptibility χ is small, negative and temperature independent.	χ is small, positive and varies linearly with field.	χ is very large, positive and temperature dependent.

(*Continued*)

S. No.	Diamagnetism	Paramagnetism	Ferromagnetism
4.	Relative permeability μ_r is slightly less than unity, i.e. $\mu < \mu_0$	μ_r is slightly greater than unity, i.e. $\mu > \mu_0$	μ_r is much greater than unity, i.e. $\mu >> \mu_0$
5.	In it lines of force are expelled from the substance, i.e. $B < B_0$	In it lines of force are *pulled in* by the substance i.e. $B > B_0$	In it lines of force are *pulled in* strongly by the substance, i.e. $B >> B_0$
6.	It is practically independent of temperature.	It decreases with rise in temperature.	It decreases with rise in temperature and above Curie temperature becomes para magnet.
7.	Atoms do not have a permanent dipole moment	Atoms have permanent dipole moments which are randomly oriented.	Atoms have permanent dipole moments which are organized in domains.
8.	Exhibited by solids, liquids and gases.	Exhibited by solids, liquids and gases.	Exhibited by solids only, that too crystalline.
9.	*Bi, Cu, Ag, Hg, Pb*, water, hydrogen, *He, Ne* etc. are diamagnetic.	*Na, K, Mg, Mn, Al, Cr, Sn* and liquid oxygen are paramagnetic	*Fe, Co, Ni* and their alloys are ferromagnetic.

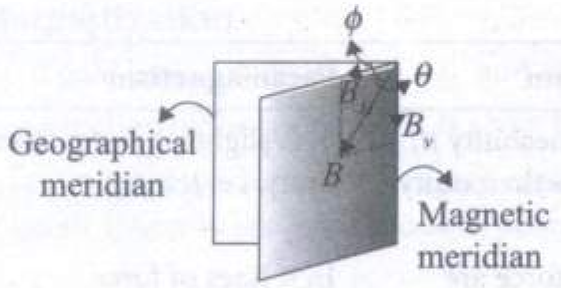
17.15 EARTH'S MAGNETISM

It is a well known fact that freely suspended magnet or current carrying solenoid rests in specific direction, called magnetic meridian. It shows that earth has its own magnetic field. The modern theory about earth magnetic field is that, the earth rotates about an axis and has the surrounding ionized region due to interaction of cosmic rays. Due to rotation of earth the surrounding ionized region gives rise to strong electric current which causes magnetic field. Its value on earth surface is 1 gauss.



To know about earth's magnetic field, we need three informations. They are:

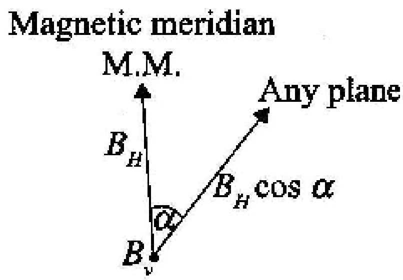
1. **Magnetic declination (ϕ):** Angle between geographical and magnetic meridian is known as angle of declination. It has an average value 17.5°.



2. **Angle of inclination or dip (θ):** It is the angle between the magnetic field of earth and the horizontal at that place. It is zero at magnetic equator and 90° at poles.

   In the magnetic northern hemisphere, the vertical component of earth's field points downward.

3. **Horizontal component of earth's magnetic field (BH):** At any place other than magnetic poles, there is horizontal component of field $B_H = B\cos\theta$ and vertical component $B_V = B\sin\theta$.



$$B = \sqrt{B_H^2 + B_V^2} \text{ and } \tan\theta = \frac{B_V}{B_H}$$

True dip and apparent dip: The angle of dip in magnetic meridian plane is called true dip (θ), and angle of dip in different plane from magnetic meridian plane is called apparent dip.

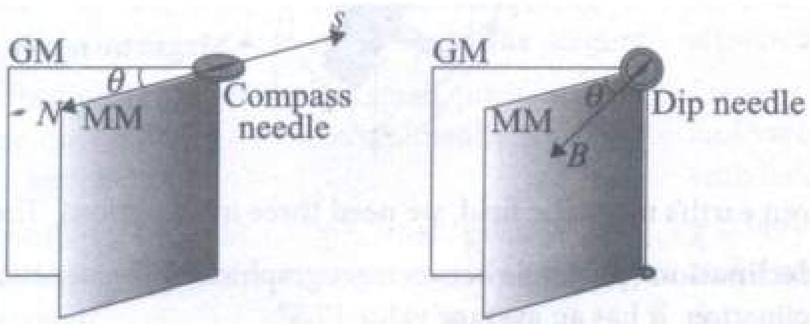
$$\tan\theta = \frac{B_V}{B_H} \quad \text{(i)}$$

On any other plane at an angle α from the meridian plane, the horizontal component of earth magnetic field will be $B_H\cos\alpha$ while vertical component remain as such. Thus apparent dip,

$$\tan\theta' = \frac{B_V}{B_H\cos\alpha} \quad \text{(ii)}$$

From equations (i) and (ii), we get $\tan\theta' = \dfrac{\tan\theta}{\cos\alpha}$ (iii)

Compass needle and dip needle: Compass needle gives the direction of $\vec{B}_H$ and the dip needle gives direction $\vec{B}$.



- **Magnetic map:** It is found that many places have the same value of magnetic elements. The lines drawn by joining all places on the earth having same value of magnetic element form magnetic map.
- **Isogonic line:** This is the line joining the places of equal angles of declination.
- **Agonic line:** This is the line which passes through places having zero declination. Magnetic meridian itself is a agonic line.
- **Isoclinic line:** This is the line joining the points of equal dip.
- **Aclinic line:** This is the line joining the places of zero dip. Magnetic equator is an aclinic line.
- **Isodynamic line:** This is the line joining the places of equal value of horizontal components of earth magnetic field.

18.1 MAGNETIC FLUX

$\phi = NBA\cos\theta$ or $\int \vec{B} \cdot d\vec{A}$

where B = strength of magnetic field, N = number of turns in the coil, A = area of surface and θ = angle between normal to area and field direction.

SI unit is wb m^{-2}

$$\left[\frac{\text{Electric flux}}{\text{Magnetic flux}}\right] \neq [\text{dimensionless}] \text{ i.e., } \left[\frac{E}{B}\right] = [v] = [LT^{-1}]$$

18.1.1 Faraday and Lenz Law (*I* from *B*)

Emf is induced due to the change in magnetic flux. The magnetic flux can be changed by

1. Keeping the magnetic field constant with respect to time and moving whole or part of the loop,
2. Keeping the loop at rest and changing the magnetic field,
3. Combination of the above (1) and (2).

$$\phi = \text{Constant (nothing will happen)} \quad \phi = \text{Vary} \begin{cases} \text{Induced emf} \\ \text{Induced current} \\ \text{Flow of charge} \end{cases}$$

According to Faraday's law of electromagnetic induction or Lenz's law,

(a) Whenever magnetic flux linked with a circuit (a loop of wire or a coil or an electric circuit in general) changes, induced emf is produced.

(b) The induced emf lasts as long as the change in the magnetic flux continues.

(c) The magnitude of induced emf is directly proportional to the rate of change of the magnetic flux linked with the circuit.

$$\xi_i = \frac{Nd\phi}{dt} \text{ where } N \text{ is turns in a coil.}$$

(d) By Faraday's second law of induction, $\xi_i = -d\phi/dt$

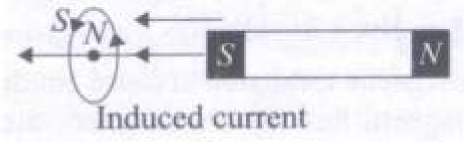
(e) Mathematical expression for Faraday's and Lenz law

$$\xi_i = \frac{d\phi}{dt} = \text{Induced emf},\ i = \frac{\xi_i}{R} = \frac{d\phi/dt}{R},\ \text{where } R \text{ is resistance}$$

$$dq = i\,dt = -\frac{d\phi}{R}$$

18.1.2 Lenz's Law

First law (attraction and repulsion law): Magnetic lines from right to left are increasing, hence induced current will produce magnetic lines from left to right.



In short, coming close → repulsion going far → attraction

Second law: If ⊗ magnetic field is increased then induced current will produce ⊙ if ⊗ magnetic field is decreased then induced current will produce ⊗.

Third law: When magnetic lines are tangential, flux is always zero.

In case of non-uniform magnetic field flux will be obtained by integration.

$i_{\text{induced}} \propto \langle x \rangle_{t_1 \text{ to } t_2}$ → main current

$I \propto t \Rightarrow i \to$ constant

If $I \propto t^2 \Rightarrow i \propto t \to$ linear

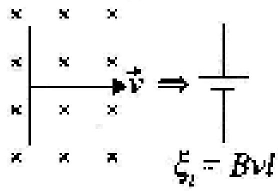
If $I \propto t^3 \Rightarrow i \propto t^2 \to$ quadratic

If $I \propto t^n \therefore i \propto t^{n-1}$

Motional emf: Potential difference or $\xi_i = Bvl$ if $\vec{B}, \vec{v}, \vec{l}$ are mutually perpendicular.

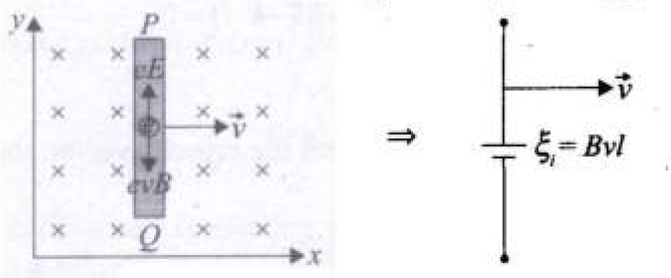
Direction → by RHR Upper side of palm → Higher potential

Lower → Lower potential



18.2 MECHANISM OF ELECTROMAGNETIC INDUCTION ACROSS A CONDUCTOR

Consider a conducting rod of length l moving with constant velocity $\vec{v}$ which is perpendicular to a uniform magnetic field $\vec{B}$ directed into the plane of paper. Let the rod is moving toward right as shown in figure. The free electrons also move to the right as they are trapped within the rod.



The magnetic field exerts force on the free electrons, $\vec{F}_m = -e(\vec{v} \times \vec{B})$ so they move towards the end Q within the rod. The end P of the rod becomes positively charged while end Q becomes negatively charged, hence an electric field $\vec{E}$ is set up within the rod which exerts force on the free electrons in opposite to magnetic force. At equilibrium

$$\vec{F}_e + \vec{F}_m = 0 \text{ or } -e\vec{E} + (-e)(\vec{v} \times \vec{B}) = 0 \text{ or } \vec{E} = -\vec{v} \times \vec{B}$$

The induced emf across the rod

$$\xi_i = \int \vec{E} \cdot d\vec{l} \text{ or } \xi_i = \int (\vec{v} \times \vec{B}) \cdot d\vec{l} \quad (1)$$

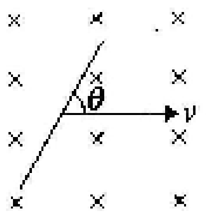
In the case under consideration

$$\xi_i = \int [v\hat{i} \times B(-\hat{k})] \cdot dl\hat{j} \quad \therefore \quad \xi_i = vBl \quad (2)$$

The mechanism of electromagnetic induction across a conductor or a conducting loop can also be explained in terms of magnetic flux (ϕ). The magnetic flux associated with an area is the dot product of magnetic field induction and the area vector. In the case of a moving conductor, the associated area is the sweeping area.

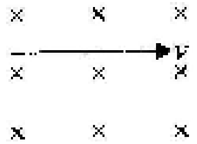
The polarity of the induced emf in the case of a moving conductor can be found with the help of the above equation (1). In the case of a conducting loop associated which magnetic flux is changing, the direction (clockwise or anticlockwise) of the induced current is such that the magnetic field produced due to the induced current compensates the change in the magnetic flux associated to the conductor.

Examples

1. 

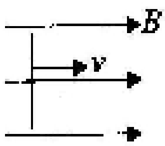
In this case we can make $\vec{v}$ perpendicular to length of the rod or l perpendicular to v. Thus,

$\xi_i = B(v \sin\theta)l$ or $Bv(l \sin\theta) = Bvl \sin\theta$

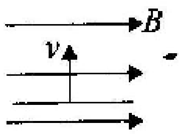
2. 

If we take the plane of motion of the rod as xy, then

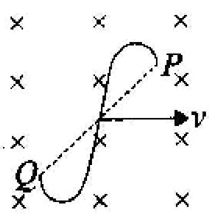
$$\xi_i = \int [v\hat{i} \times (-B\hat{j})] \cdot (l\hat{i})$$
$$= -vBl(\hat{k} \cdot \hat{i}) = 0$$

3. 

$$\xi_i = \int [v\hat{i} \times B\hat{i}] \cdot (l\hat{j}) = 0$$

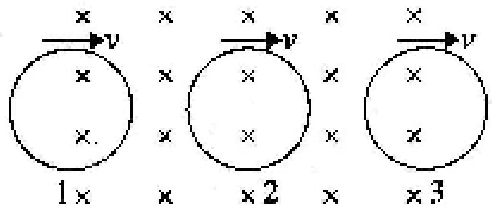
4. 

$$\xi_i = \int (v\hat{j} \times B\hat{i}) \cdot l\hat{i} = vBl(-\hat{k} \cdot \hat{i}) = 0$$

5. 

If PQ line makes θ with the velocity vector, then

$\xi_i = B(v \sin\theta)PQ$

6. 

If a closed conducting loop having constant area moves in uniform magnetic field as in figure (2), then there is no induced emf as well as induced current in the loop but at the time of entrance, figure (1) and at the time of emergence, figure (3), there occurs induced emf as well as induced current.

Figure (1): As the magnetic flux associated to the area is increasing directed into the plane of the paper hence the magnetic field produced due to the induced current must be coming out of plane of the paper. It means that the induced current is anticlockwise.

Figure (2): There is no change in magnetic flux hence induced current is zero.

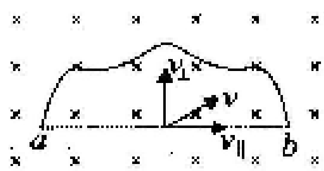
Figure (3): As the magnetic flux associated to the area is decreasing directed into the plane of the paper hence the magnetic field produced due to the induced current must be directed in to the plane of the paper. It means that the induced current is clockwise.

18.3 HOW TO SOLVE PROBLEMS RELATED TO MOTIONAL EMF

Assume a moving conductor as a battery of emf Bvl and polarity given by RHR, then solve the problem as a problem of current and electricity.

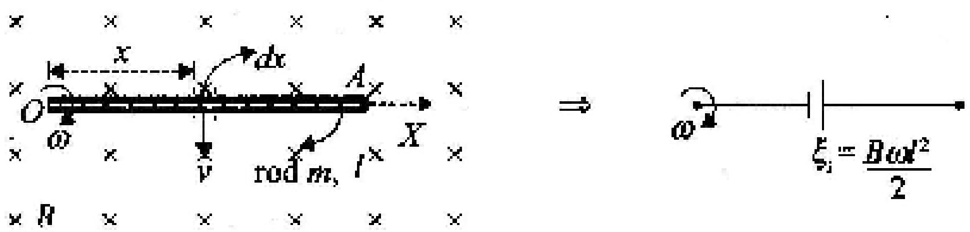
1. If conductor is not straight make it straight by joining initial and final points

   $V_a - V_b = B(v_\perp)(ab)$



2. The induced emf across a conductor if the conductor is rotating in uniform magnetic field

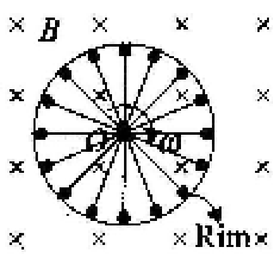
   (a) Let us consider a conducting rod of length l is rotating about an axis passing through one of its ends with constant angular velocity ω in an uniform magnetic field $\vec{B}$ as shown in figure.



Induced emf across the element is $d\xi_i = Bv(dx) = B(\omega x)dx$

Induced emf across the entire rod, $\xi_i = \int_0^l B\omega\, x dx$

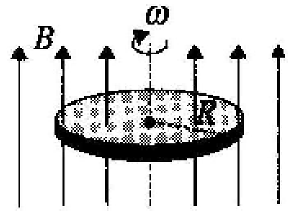
$$\Rightarrow \quad \xi_i = V_O - V_A = \frac{B\omega l^2}{2}$$



(b) Let us consider a cycle wheel is rotating about its own axis with constant angular velocity ω in uniform magnetic field. In this case each spoke becomes cell of emf $\xi_i = \dfrac{B\omega l^2}{2}$ because flux cutting by each metal spoke is same. All such cells are in parallel combination, therefore $\xi_{i,\text{ equivalent}} = \xi_i$. Each point on the periphery of wheel has same potential.

(c) Let us consider a metal circular disc of radius R is rotating about its axis with constant angular velocity in uniform magnetic field. The metal disc can be assumed to be made up of number of radial conductors. The emf induced across each conductor is $\xi_i = \dfrac{B\omega R^2}{2}$. All such conductors behave like a number of cells in parallel. Therefore

$$\xi_{i,\text{ equivalent}} = \xi_i = \frac{B\omega R^2}{2}$$



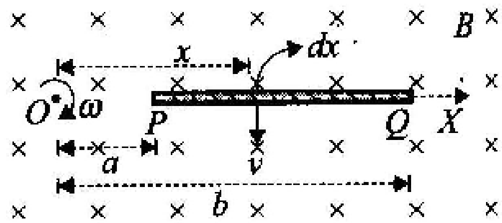
(d) A conducting rod PQ is rotated in a magnetic field about an axis passing through O. The one end of the rod is at a distance a and other end is at a distance b from O as shown in figure.

The induced emf across the element of length dx is

$$d\xi_i = Bvdx = B(\omega x)dx$$

The induced emf across the whole rod is

$$\xi_i = B\omega\int_a^b xdx = \frac{B\omega(b^2 - a^2)}{2}$$



3. Any problem of EMI can be solved by two methods
   (a) Faraday + Lenz
   (b) Motional emf

18.3.1 Self-inductance

The phenomenon, according to which on opposing induced emf is produced in a coil as a result of change in current or magnetic flux linked with the coil is called self-inductance.

As $\phi \propto I$, $\phi = LI$ where L is coefficient of self-induction or self-inductance

$$\Rightarrow \quad \xi_i = \frac{-d\phi}{dt} = -L\frac{dI}{dt}$$

1. The self-inductance L depends on geometry of coil or solenoid and the permeability of the core material of the coil or solenoid.
2. Unit of L is Henry.
3. For a small circular coil, $L = \dfrac{\mu_0 \mu_r N^2 \pi r}{2}$
4. For a solenoid, $L = \dfrac{\mu_0 \mu_r N^2 A}{l}$
5. For two coils connected in series

(a) when current flows in same direction in both,

$L_{eq} = L_1 + L_2 + 2M$

(b) when current flow in two coils in opposite directions

$L_{eq} = L_1 + L_2 - 2M$

If $M = 0$, $L_{eq} = L_1 + L_2$

6. For two coils connected in parallel

$$\frac{1}{L_{eq}} = \frac{1}{L_1 + M} + \frac{1}{L_2 + M} \Rightarrow L_{eq} = \frac{(L_1 + M)(L_2 + M)}{L_1 + L_2 + 2M}$$

If $M = 0$, $L_{eq} = \dfrac{L_1 L_2}{L_1 + L_2}$

7. Self-inductance of a toroid, $L = \dfrac{\mu_0 N^2 A}{2\pi r}$

18.3.2 Mutual Inductance (M)

1. Mutual inductance of two coils is numerically equal to magnetic flux linked with one coil, when a unit current flows through the neighbouring coil.

   As $\phi \propto I$, $\phi = MI$ where M is coefficient of mutual induction or mutual inductance

$$\Rightarrow \quad \xi_i = -M\frac{dI}{dt}$$

2. For two long co-axial solenoids, each of length l, common area of cross-section A wound on air core,

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

3. For two coupled coils, $M = K\sqrt{L_1 L_2}$ where K denotes the coefficient of coupling between the coils.
4. If $K = 1$, the coils are said to be tightly coupled such that magnetic flux produced in primary is fully linked with the secondary.

$$M = \sqrt{L_1 L_2} = \text{maximum value of } M.$$

18.3.3 Inductor (Solenoid and Toroid)

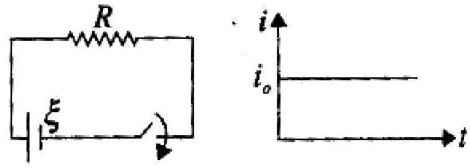
1. Potential energy stored in inductor $= \dfrac{1}{2}Li^2$
2. Energy density $u = \dfrac{B^2}{2\mu_0}$
3. Induced emf or potential difference across inductor is decided by logic. Current is increased this induced emf has tendency to decrease the current.

4. $\xrightarrow[\text{Kirchhoff along current}]{} -L\dfrac{dI}{dt}$

and $\xleftarrow[\text{Kirchhoff against current}]{} +L\dfrac{dI}{dt}$

5. If $\dfrac{di}{dt}$ = +ve, then $V_a - V_a$ = +ve and if constant current flows from an inductor it is just like a conducting wire $\rightarrow$ no potential difference.

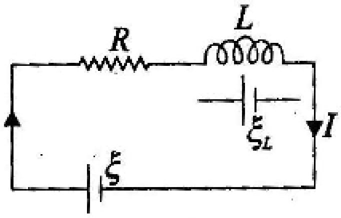
18.3.4 Current Growth in L–R Circuit



In a circuit having only resistor,

$i_o = \dfrac{\xi}{R}$ as soon as switch is closed,

but in L–R circuit inductor will oppose the increase in current from 0 to i_o. Steady state current is still i_o but will increase exponentially



The current in RL circuit at time t is $I = \dfrac{\xi}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$

1. **Charge:** $q = \int_o^t I\,dt = \dfrac{\xi}{R}t - \dfrac{\xi}{R^2}\left(1 - e^{\frac{-Rt}{L}}\right)$

2. **Voltage across resistor:** $V_R = IR = \xi(1 - e^{\frac{-Rt}{L}})$

3. **Voltage across inductor:** $\xi - V_R = \xi e^{\frac{-Rt}{L}}$

4. **Power from battery:** $P = \xi I = \dfrac{\xi^2}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$

5. **Energy in inductor:** $U_L = \dfrac{1}{2}LI^2 = \dfrac{L\xi^2}{2R^2}\left(1 - e^{\frac{-Rt}{L}}\right)^2$

6. **Heat developed in the resistor:**

$$H = \int_o^t I^2 R dt = \frac{\xi^2}{R}\int_o^t \left(1 - e^{\frac{-Rt}{L}}\right)^2 dt = \frac{\xi^2}{R}\int_o^t \left(1 + e^{\frac{2Rt}{L}} - 2e^{\frac{-Rt}{L}}\right) dt$$

$$= \frac{\xi^2}{R}\left(t - \frac{L}{2R}e^{\frac{-2Rt}{L}} + \frac{2L}{R}e^{\frac{-Rt}{L}}\right)_o^t$$

$$= \frac{\xi^2}{R}\left[t + \frac{L}{2R} - \frac{L}{2R}e^{\frac{-2Rt}{L}} - \frac{2L}{R} + \frac{2L}{R}e^{\frac{-Rt}{L}}\right]$$

$$= \frac{\xi^2}{R}\left[t + \frac{L}{2R}\left(1 - e^{\frac{-2Rt}{L}}\right) - \frac{2L}{R}\left(1 - e^{\frac{-Rt}{L}}\right)\right]$$

7. The steady state current $= \dfrac{\xi}{R}$

Magnetic field energy stored in the inductor in steady state

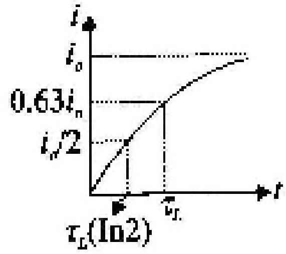
$$= \frac{1}{2}LI^2 = \frac{1}{2}\frac{\xi^2}{R^2}L$$

Energy at any time $t = \dfrac{1}{2}L\dfrac{\xi^2}{R^2}\left(1-e^{\frac{-Rt}{L}}\right)^2$

Let time taken for energy to be halved be t_1

$$\frac{1}{4}L\frac{\xi^2}{R^2} = \frac{1}{2}L\frac{\xi^2}{R^2}\left(1-e^{\frac{-Rt}{L}}\right)^2 \Rightarrow e^{\frac{-Rt_1}{L}} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

$$\Rightarrow \frac{-Rt_1}{L} = \ln\left(2-\sqrt{2}\right)-\ln 2 \Rightarrow t_1 = \frac{L}{R}\left[\ln 2 - \ln\left(\frac{1}{2-\sqrt{2}}\right)\right]$$



Let time taken for energy to be one-fourth be t_2.

$$\frac{1}{8}L\frac{\xi^2}{R^2} = \frac{1}{8}L\frac{\xi^2}{R^2}\left(1-e^{\frac{-Rt_2}{L}}\right)^2 \Rightarrow 1-e^{\frac{-Rt_2}{L}} = \frac{1}{2}$$

$$\Rightarrow e^{\frac{-Rt_2}{L}} = \frac{1}{2} \text{ or } t_2 = \frac{L}{R}\ln 2$$

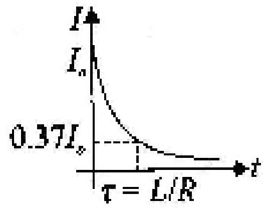
Hence $t_2 - t_1 = \dfrac{L}{R}\ln\left(\dfrac{1}{2-\sqrt{2}}\right)$ (It is the time taken for the magnetic energy stored in the circuit to change from one-fourth of the steady state value to half of the steady state value).

$$I = i_0\left(1-e^{\frac{-t}{\tau_L}}\right),$$

where $\tau_L = L/R \rightarrow$ Time constant

At $t = \tau_L$, $I = \left(1-\dfrac{1}{e}\right)i_o = 63\%$ of i_o (increasing graph)

18.3.5 Current Decay In L–R Circuit



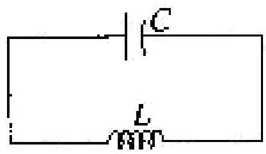
During decay, current at any instant of time is given by

$I = I_o.e^{-Rt/L}$

If $\tau = L/R = t =$ time constant of circuit

$I = I_o/e = 37\%\ I_o$

18.3.6 LC-oscillatory Circuit



Consider a LC-circuit shown in figure, a resistanceless inductor is connected between the terminals of a charged capacitor. At the instant when connections are made, the capacitor starts to discharge through the inductor. Let a capacitor C is given an initial charge Q and, at $t = 0$ is connected to the inductor of self-inductance L.

The sum of the energies of the system (the magnetic energy of the inductor $\left(\frac{1}{2}Li^2\right)$ at any time and the potential energy of the capacitor $\left(\frac{q^2}{2C}\right)$ remains constant. Therefore by conservation of energy

$$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$$

Differentiating above equation w.r.t. time, we get

$$\frac{1}{2}L \times 2i \times \frac{di}{dt} + \frac{1}{2C} \times 2q \times \frac{dq}{dt} = 0 \text{ or } L\frac{di}{dt} + \frac{q}{C} = 0$$

$$\text{As } i = \frac{dq}{dt}, \quad \therefore \quad \frac{d^2q}{dt^2} + \frac{q}{LC} = 0 \qquad (1)$$

Compare above equation with simple harmonical differential equation i.e., $\frac{d^2x}{dt} + \omega^2 x = 0$,

we get $\omega = \sqrt{\frac{1}{LC}}$ (2)

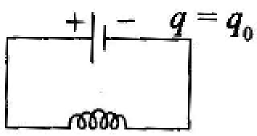
This is called natural frequency of the LC circuit.

Time period $T = 2\pi\sqrt{LC}$ Also, $q = Q\cos(\omega t + \phi)$

It means that the charging and discharging of the capacitor occur simple harmonically.

The capacitor has completely discharged and the potential difference between its terminals has decreased to zero. The current in the inductor has meanwhile establishes a magnetic field in the space around it. This magnetic field now decrease, inducing an emf in the inductor in the same direction as the current. The current therefore persists, although with decreasing magnitude, until the magnetic field has disappeared and the capacitor has been charged in the opposite sense to its initial polarity. The process now repeats itself in the reverse direction, and in the absence of any energy losses, the charges on the capacitor surge back and forth indefinitely. This process is called electrical oscillations. From the energy state point, the oscillations of an electrical circuit consist of a transfer of energy back and forth from electric field of capacitor to the magnetic field of the inductor, the total energy associated with the circuit remaining constant. This is analogous to the transfer of energy in an oscillating mechanical system from kinetic to potential and vice versa.

As oscillation are simple harmonic, q-t equation is sine or cosine equation.



At $t = 0$, $q = q_0$, applying KVL we get $q = q_0 \cos \omega t$

$$\Rightarrow \quad I = \left|\frac{dq}{dt}\right| \Rightarrow q_0\,\omega \sin \omega t.$$

1. $\omega = \frac{1}{\sqrt{LC}}$

2. $U_C = \frac{1}{2}\frac{q^2}{C}$

3. $U_B = \frac{1}{2}Li^2$

4. $\frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li_0^2 = \frac{1}{2}\frac{q_0^2}{C}$ = constant

5. $i_{max} = \omega q_0$

6. $\left(\frac{di}{dt}\right)_{max} = \omega^2 q = \frac{q}{LC}$

7. $i = \omega\sqrt{q_0^2 - q^2}$

8. $\frac{di}{dt} = -\omega^2 q$

Important points

1. $\left[\frac{1}{\sqrt{LC}}\right] = |\omega| = [T^{-1}]$

2. When $q = \pm q_0$, $i = 0$, $\frac{1}{2}Li^2 = 0$ $\left(\frac{di}{dt}\right) \to$ max

3. When $q = 0$, $i \to$ max., $\frac{1}{2}Li^2 \to$ max., $\frac{di}{dt} \to 0$, $\frac{1}{2}\frac{q^2}{C} \to 0$

Induced electric field can be produced by

1. change in system of charges
2. change in magnetic field

If charge particle is kept at rest in change magnetic field, it will experience electric force not magnetic

$\vec{F}_e = q\vec{E}_i \to$ induced electric field

How will you find value of E_i ?

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$$

18.3.7 Alternating Current (AC)

Average value of physical quantity x is

$$<x>_{t_1 \text{ to } t_2} = \frac{\int_{t_1}^{t_2} x\,dt}{t_2 - t_1}$$

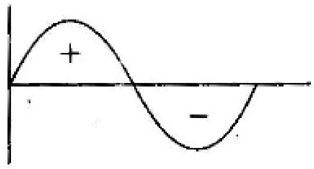
1. If x is linear function of time then average value can be

$$x_{av} = \frac{x_f + x_i}{2} \text{ or } \frac{x_{t_2} + x_{t_1}}{2}$$

2. In some cases there are fixed formula

e.g., average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

average speed $= \dfrac{\text{Total displacement}}{\text{Total time}}$

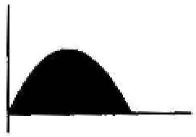


Average value of $\sin \omega t$ or $\cos \omega t$

$< \sin \omega t$ or $\cos \omega t >_{0-T} = 0$

or $\displaystyle\int_0^{2\pi/\omega} \frac{\sin \omega t dt}{2\pi/\omega} = 0$

$\Rightarrow \quad < \sin^2 \omega t$ or $\cos^2 \omega t >_{0-T} = \dfrac{1}{2}$



$\Rightarrow \quad < \sin \omega t >_{0-T/2} = \dfrac{2}{\pi} \approx 0.636$

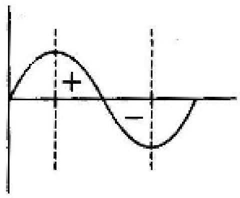
The average value of ac is defined for half the time period. The average value of ac is that steady current (i.e., dc) which sends the same amount of charge through a circuit, in a time equal to half the time period of ac, as is sent by ac through the same circuit in the same time.

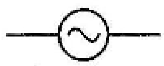
$$i_{av} = \frac{2}{\pi} i_o = 0.637 i_o$$

Hence, the average value of ac current over one half cycle is 0.637 times its peak value.
Similarly, it can be shown that
V_{av} (average alternating emf over one half cycle)

$$= \frac{2}{\pi} V_o = 0.637\ V_o$$

1. In half cycle also, average value may be zero.
2. In AC, value of emf changing, polarity changing and direction of current changing



Symbol 

Applied voltage

'+' and '−' not mentioned as they keep on changing.
In sine or cosine function, current or voltage have 4 values [$i = i_0 \sin \omega t$]

(a) Peak or maximum value $= i_0$

(b) Instantaneous value $= i_0 \sin \omega t$

(c) average value $\begin{cases} \text{whole cycle} = 0 \\ < i_0 \sin \omega t >_{0-\frac{T}{2}} = \dfrac{2}{\pi} i_0 \end{cases}$

(d) Rms value $i_{rms} = \dfrac{i_0}{\sqrt{2}} \approx 0.707 i_0$

Inductive reactance $X_L = \omega L \rightarrow$ unit $- \Omega$

Capacitive reactance $X_C = \dfrac{1}{\omega C} \rightarrow$ unit $- \Omega$

1. In DC voltage no oscillation take place.

$$\therefore\ \omega = 0, X_L = 0, X_C = \infty.$$

In steady state, resistance of inductor = 0 and resistance of capacitor = ∞.

2. Series C-R circuit: Total resistance of circuit is called impedence

$$Z = \sqrt{R^2 + (X_L \sim X_C)^2}$$

3. Phase difference between V and i

Only $R \rightarrow \phi = 0°$

Only $C \rightarrow \phi = 90°$, current leading

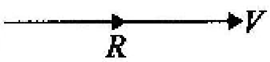
Only $L \rightarrow \phi = 90°$, voltage leading

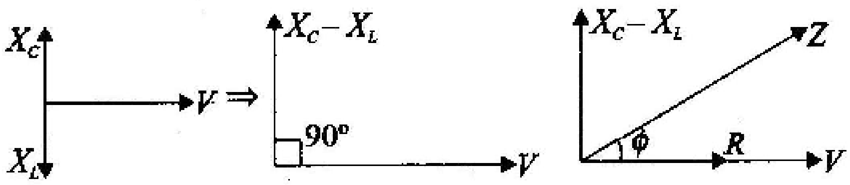
If $X_C > X_L \rightarrow$ current leading

If $X_L > X_C \rightarrow$ voltage leading

For LCR, ϕ is in between 0° and 90°

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \text{ or } \phi = \cos^{-1}\left(\frac{R}{Z}\right) \rightarrow \text{power factor}$$

Only R, $\phi = 0$ 



$$Z = \sqrt{R^2 + (X_C - X_L)^2} \quad \therefore\ \phi = \cos^{-1}\left(\frac{R}{Z}\right) \text{ or } \tan^{-1}\frac{(X_C - X_L)}{R}$$

$i_0 = \dfrac{V_0}{Z}$ $\quad i_{rms} = \dfrac{V_{rms}}{Z}$ but $V \neq iZ$

V can be iZ if $\phi = 0°$ *i.e.*, when only R

4. **Resonance frequency:** When $X_L = X_C \Rightarrow \omega = \dfrac{1}{\sqrt{LC}}$

If $\omega <$ resonance frequency, $X_C > X_L$ and circuit is capacitive in nature and current will lead.

5. **Voltage (only rms values):**

$V_R = I_{rms} R$, $V_C = I_{rms} X_C$, $V_L = I_{rms} X_L \Rightarrow V = \sqrt{V_R^2 + (V_C - V_L)^2}$

6. **Power:** There are two types of power (i) instantaneous power and (ii) average power.

(a) $P_i = V_i I_i$

(b) $P_{av} = I_{rms} V_{rms} \cos\phi = I^2_{rms} R \cos\phi$, $\cos\phi = \dfrac{R}{Z} \rightarrow$ Power factor

If $R = 0$, power factor $= 0$

$\therefore$ average power $= 0$ in one cycle but $i \neq 0$. This current is called wattless current.
P_{av} is also called the true power. $I_o \cos\phi$ is called the active or watt-full component and $I_o \sin\phi$ is called the wattless, idle or reactive component of the current.

18.3.8 Choke Coil

Sometimes, we have to reduce the value of the current in a circuit while keeping the supply voltage constant. If the current is drawn from a dc source, then its value can be reduced by using a rheostat. But in doing so, a power equal to I^2R will be wasted in the form of heat where I is the current flowing through the circuit whose resistance is R.

If instead of a dc source, we are using an ac source, then to change the value of the current in the circuit, inductance is used in place of resistance.

An inductance used in an ac circuit to control current is called a choke coil.

18.3.9 Transformer

A transformer is a device used for changing the form of electrical energy, e.g., for converting a low voltage alternating current into a high voltage alternating current or vice versa. When the voltage is raised, the transformer is called a step-up transformer and when the voltage is lowered, it is called a step-down transformer.

It is based on the phenomenon of mutual induction between two coils known as the primary coil and the secondary coil. Transformer does not amplify power. Law of conservation of energy holds good for a transformer. It does not operate on dc or direct voltage. It operates only on alternating voltages at input as well as at output. Frequency of output voltage across secondary coil is same as that of input voltage across primary coil.

It is used for transmission of ac over long distances at high voltages. The energy losses and cost of transmission are reduced by this device.

In Step-up transformer,

1. The output voltage V_s across secondary coil is greater than input voltage V_p in primary coil.
2. But $I_s < I_p$.
3. $N_s > N_p$ where N denotes the number of turns in the coils.
4. $\dfrac{V_s}{V_p} = \dfrac{I_p}{I_s} = \dfrac{N_s}{N_p} > 1$

In step-down transformer

1. The output voltage $V_s < V_p$
2. The output current $I_s > I_p$
3. The number of turns $N_s < N_p$
4. $\dfrac{V_s}{V_p} = \dfrac{I_p}{I_s} = \dfrac{N_s}{N_p} < 1$

Transformation ratio $K = \dfrac{V_s}{V_p} = \dfrac{I_p}{I_s} = \dfrac{N_s}{N_p}$

Efficiency of transformer $= \dfrac{\text{Output power}}{\text{Input power}}$ or $\eta = \dfrac{V_s I_s}{V_p I_p}$

Energy losses in transformer

For an ideal transformer, output power = input power

$$V_s I_s = V_p I_p \quad \Rightarrow \quad \frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

But in practice, there are losses and output power < input power.

Copper losses: Windings are made of copper wire. Energy is lost as heat in resistance of copper wire. It is reduced by use of suitably thin wires of copper.

Iron losses/eddy current losses: Energy is lost due to eddy currents in the core of transformer. It is reduced by using laminated soft iron core.

Flux leakage: Some magnetic flux leaks in air between primary and secondary coils. It is reduced by winding the secondary coil over a primary coil using insulator between them.

Hysteresis loss: The core is magnetized and demagnetized and energy is lost as heat. It is reduced by using soft iron core.

ELECTROMAGNETIC WAVES AND WAVE OPTICS

19.1 CONDUCTION CURRENT

It is the current, which arises due to the flow of electrons through the connecting wires in an electric circuit.

19.2 DISPLACEMENT CURRENT

It is the current, which arises due to time rate of change of electric flux (ϕ_E) in some part of the electric circuit. It is given by

$I_D = \varepsilon_o \frac{d\phi_E}{dt}$, where ε_o is absolute permitivity of free space.

When a capacitor is charged or a charged capacitor is allowed to discharge, the electric flux between the plates of the capacitor changes with time and it gives rise to displacement current between the plates. The current that flows through the connecting wires is called the conduction current.

19.3 MODIFIED AMPERE CIRCUITAL LAW

It states that the line integral of magnetic field $\vec{B}$ over a closed path is equal to μ_o times the sum of the conduction current (I_c) and the displacement current (I_D) threading the closed path.

Mathematically: $\oint \vec{B} \cdot d\vec{l} = \mu_o \left(I_C + \varepsilon_o \frac{d\phi_E}{dt} \right)$

It is also known as Ampere–Maxwell's circuital law.

19.3.1 Maxwell's Equations

Following four equations, which describe the laws of electromagnetism are called Maxwell's equation:

1. $\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_o}$ (Gauss's law in electrostatics)

2. $\oint \vec{B} \cdot d\vec{S} = 0$ (Gauss's law in magnetism)

3. $\oint \vec{B} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$ (Faraday's law of electromagnetic induction)

4. $\oint \vec{B} \cdot d\vec{l} = \mu_o \left(I_C + \varepsilon_o \frac{d\phi_B}{dt} \right)$ (Ampere–Maxwell's circuital law)

19.4 ELECTROMAGNETIC WAVES

The electric and magnetic fields varying in space and time and propagating through space, such that the two fields are perpendicular to each other and perpendicular to the direction of propagation, constitute electromagnetic waves.

1. The electromagnetic waves are transverse in nature.
2. The velocity of electromagnetic waves in free space is given by

$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 3 \times 10^8\ \mathrm{ms^{-1}}$. In a material medium, velocity of electromagnetic waves is given by

$v = \frac{1}{\sqrt{\mu \varepsilon}}$, where μ and ε are absolute permeability and absolute permitivity of that medium.

5. The refractive index (n) of a material medium is given by

$$n = \frac{c}{v} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \times \frac{\sqrt{\mu \varepsilon}}{1} = \sqrt{\frac{\mu \varepsilon}{\mu_o \varepsilon_o}}$$

Since $\mu/\mu_o = \mu_r$, relative permeability and $\varepsilon/\varepsilon_o = \varepsilon_r$, relative permitivity of the material medium, we have $n = \sqrt{\mu_r \varepsilon_r}$

6. The ratio of the amplitudes of electric and magnetic fields is constant and it is equal to velocity of the electromagnetic waves in free space. Mathematically, $\frac{E_o}{B_o} = c$.
7. The energy transported by electromagnetic waves per second per unit area is represented by a vector quantity $\vec{S}$, called poynting vector. It is given by $\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$.
8. The momentum transported by electromagnetic waves is given by $p = \frac{U}{c}$, where U is energy transported by electromagnetic waves in a given time and c is speed of electromagnetic waves in free space.
9. The intensity of electromagnetic waves *i.e.* energy crossing per second per unit area of a surface normally is given by

$$I = \frac{1}{2} \frac{E_o B_o}{\mu_o} = \frac{1}{2} \frac{B_o^2}{\mu_o} c = \frac{1}{2} \varepsilon_o E_o^2 c$$

19.5 ELECTROMAGNETIC SPECTRUM

The orderly distribution of electromagnetic waves (according to wavelength or frequency) in the form of distinct groups, having widely differing properties, is called electromagnetic spectrum.

The main parts of electromagnetic spectrum are namely: γ-rays, X-rays, ultra-violet rays, visible light, infra-red rays, heat radiation, microwaves and radio waves.

19.5.1 Radiowaves (Frequency Range: 500 kHz to About 1000 MHz)

Electromagnetic waves of wavelength longer than about 1 m are called radio waves. These waves are as a result of changes accelerating through conducting wires and are generated by such electronic devices as *LC* oscillators. Frequency bands used for different purposes in radio and TV communication are as follows.

AM band	:	530 kHz to 1.71 MHz
Short wave AM band	:	Upto 54 MHz
FM band	:	88 MHz to 108 MHz
TV	:	54 MHz to 890 MHz
Cellular phones	:	840 MHz to 935 MHz

Since these waves have long wavelengths, they spread round hills and buildings by diffraction. These waves are used (i) for radio, TV, telephone and satellite communications (ii) in radio astronomy and (iii) in nuclear magnetic resonance imaging (NMRI).

19.5.2 Microwaves (Frequency Range: 1 GHz to 100 GHz)

Electromagnetic waves with wavelengths in the approximate range of 1 mm to 0.3 mm (i.e., between infrared and radio waves) are called microwaves. These waves are produced by special electronic devices, called *klystrons* and *magnetrons*. Because to their short wavelength, they travel as a beam in a signal. Consequently, they are used (i) in radar systems for aircraft navigation (ii) for studying the atomic and molecular properties of matter (iii) in microwave ovens (microwaves of frequency 2.45 GHz and a wavelength of about 12 cm produce a heating like IR radiation) (iv) It has been suggested that solar energy could be harnessed by beaming microwaves down to Earth from a solar collector in space.

19.5.3 Infrared (IR) Waves (Frequency Range: 10^{11} Hz to 5×10^{14} Hz)

They have a wavelength range of 0.7 μm to about 1 cm. The sources of these waves are Sun, warm and hot objects such as fires. IR radiation is absorbed strongly by many molecules, including carbon dioxide and water. As it is absorbed, the wave energy is converted into thermal energy, warming the absorbing object. For this reason, infrared is often erroneously referred to as heat radiation. IR is also associated with maintaining the earth's temperature through the greenhouse effect. IR is not scattered as much as visible light. IR is used for (i) photography through haze and fog and in night and (ii) thermograph (a medical diagnostic technique) in which infrared film is used to detect tumours and other disorders.

19.5.4 Visible Light (Frequency Range: 4×10^{14} Hz to About 7×10^{14} Hz)

That small portion of the electromagnetic spectrum the human eye can detect is commonly referred to as light and it comprises an extremely small range of wavelengths between 400 nm to 700 nm. Within that range, we perceive what we call 'colours'.

19.5.5 Ultraviolet (UV) Radiation (Frequency Range: 10^{14} Hz to 10^{17} Hz)

The Sun, very hot objects, arcs and sparks, mercury vapour lamps are the sources of UV radiation. UV light is absorbed by glass, causes many chemical reactions, damages and kills living cells and causes sunburn.

19.5.6 X-rays (Frequency Range: 10^{17} Hz to 10^{19} Hz)

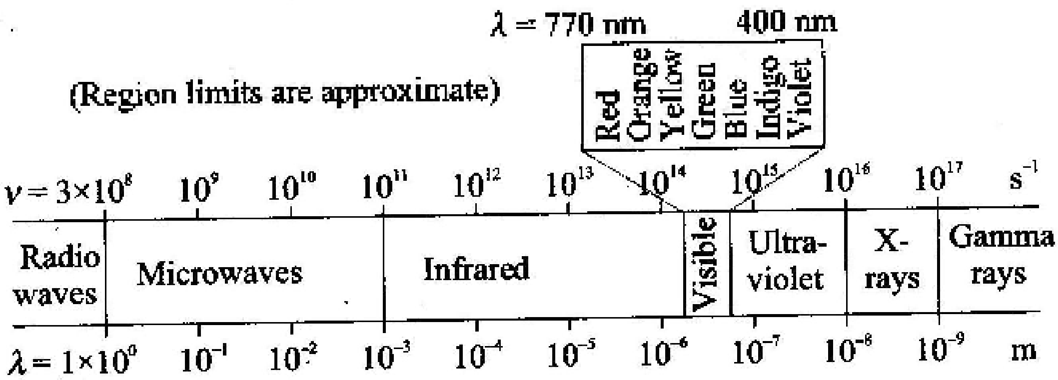
The most common source of X-rays is the deceleration of high-energy electrons bombarding a metal target. X-rays are very penetrating and dangerous. These are used (i) as a diagnostic tool in medicine—a combination of the computer with modern X-ray machines permits the formation of three-dimensional images by means of a technique called computerized tomograph (CT) (ii) to study crystall structure (X-ray crystallography) since their wavelength is comparable to atomic separation distances (= 0.1 nm) in solids and (iii) in the treatment of certain forms of cancer.

19.5.7 Gamma Rays (Frequency Range: 10^{18} Hz to 10^{22} Hz)

The electromagnetic waves of uppermost frequency range of the known electromagnetic spectrum are called gamma rays (γ-rays).

Gamma rays are produced in certain nuclear reactions, in particle accelerators and in certain types of nuclear decay. Cosmic rays from outer space is also a major source of γ-rays. Gamma rays have the same properties as the X-rays, the two differ only in the manner of their production.

These rays are used to kill cancerous growths, to find flaws in metals and, to sterilize equipment.



19.6 SOME USEFUL KEY POINTS

1. Whereas the conduction current is due to the flow of electrons through the connecting wires in an electric circuit, the displacement current arises due to the time rate of change of electric flux in some parts of the electric circuit.
2. The conduction and displacement currents are entirely different from each other. However, displacement current produces magnetic field in the same manner, as the conduction current does.
3. The displacement current is always equal to the conduction current.
4. Maxwell's equations are mathematical formulation of Gauss' law in electrostatics, Gauss' law in magnetism, Faraday's law of electromagnetic induction and Ampere's circuital law.
5. Electromagnetic waves are transverse in nature.
6. The frequency of electromagnetic waves is its inherent characteristic. When an electromagnetic wave travels from one medium to another, its wavelength changes but frequency remains unchanged.
7. All the types of electromagnetic waves travel with the same speed in free space.
8. The electric vector of an electromagnetic waves is responsible for its optical effect. For this reason, the electric vector is also called light vector.
9. When electromagnetic waves strike a surface, they exert pressure on the surface.

19.7 WAVE OPTICS

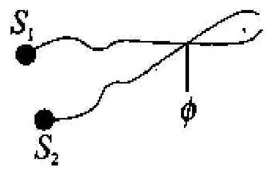
Light is the form of electromagnetic radiation to which the human eye is sensitive and on which our visual awareness of the universe.

The finite velocity of light was suspected by many early experimenters in optics, but it was not established until 1676 when *Ole Roemer* (1644–1710) measured it. *Sir Isaac Newton* investigated the optical spectrum and used existing knowledge to establish a primarily *corpuscular theory* of light, in which it was regarded as a stream of particles that set up disturbances in the *ether* of space. His successors adopted the corpuscles but ignored the wave-like disturbances until *Thomas Young* rediscovered the interference of light in 1801 and showed that a wave theory was essential to interpret this type of phenomenon. This view was accepted for most of the 19th century and it enabled *James Clerk Maxwell* to show that light forms part of the electromagnetic spectrum. He believed that waves of electromagnetic radiation required a special medium to travel through, and revived the name *luminiferous ether* for such a medium. The *Michelson–Morley* experiment in 1887 showed that, if the medium existed, it could not be detected; it is now generally accepted that the ether is an unnecessary hypothesis. In 1905 *Albert Einstein* showed that the photoelectric effect could only be explained on the assumption that light consists of a stream of discreate photons of electromagnetic energy. This renewed conflict between the corpuscular and wave theories has gradually been resolved by the evolution of the quantum theory and wave mechanics. While it is not easy to construct a model that has both wave and particle characteristics, it is accepted, according to the theory of complementarity proposed by *Neils Bohr*, that in some experiments light will appear wavelike, while in others it will appear to be corpuscular. During the course of the evolution of wave mechanics it has also become evident that electrons and other elementary particles have dual wave and particle properties.

The present stand point of physicist is to accept the fact that light is dualistic in nature. The phenomenon of light propagation may best be explained by electromagnetic wave theory (interference, diffraction and polarization) while the interaction of light with matter in the process of emission and absorption (photoelectric and compton effects) is a corpuscular phenomenon. The difference between a wave and a particle depends to a large extent on our point of view. Perhaps the best way to think of it that waves and particles are both simplified models of reality, and that light is a complicated phenomenon that does not quite fit either model alone. Both models are used extensively. Generally, we adopt whichever model provides an explanation of the optical behaviour under study.

19.7.1 Coherent Sources

1. In case the phase difference between the waves is either constant or zero, the waves are called coherent waves. Thus, waves having the same frequency and constant (or most often zero) phase difference are called coherent.

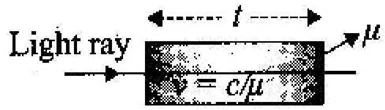


2. If the phase changes at random, the waves are called non-coherent.
3. Relation between Phase Difference ϕ and Path Difference (Δx)

$$\phi = \frac{2\pi}{\lambda}\Delta x \quad \text{If } \Delta x = 0, \lambda, 2\lambda \ldots, \text{ then } \phi = 0, 2\pi, 4\pi \ldots$$

3. Path difference and corresponding phase difference due to a glass slab

$$\Delta x = (\mu - 1)t \qquad \phi = \frac{2\pi}{\lambda}\Delta x$$



19.7.2 Interference

The redistribution of light energy due to superposition of two waves is called interference.

An increase in the intensity takes place at the points where the crest of one wave falls on the crest of the other or trough of one falls on the trough of the other. Such an interference is called constructive interference.

$$\left.\begin{array}{r} \Delta x = 0, \lambda, 2\lambda \ldots, \\ \phi = 0, 2\pi, 4\pi \ldots, \end{array}\right] \quad \text{Condition of constructive interference}$$

A decrease in the intensity takes place if the crest of one wave falls on the trough of the other or vice-versa. This type of interference is called destructive interference.

$$\left.\begin{array}{r} \Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots, \\ \phi = \pi, 3\pi, 5\pi \ldots, \end{array}\right] \quad \text{Condition of destructive interference}$$

It should be clearly understood that there is no loss of energy due to interference. Interference pattern is observed only when the light waves being superimposed are coherent. Two independent sources can never be coherent. They are produced from a single source of light.

19.7.3 Division of Wavefront

1. When light source is point source or line source, the wavefront is divided in two parts by reflection or refraction. In it, the coherent sources obtained are imaginary.
2. Young's double slit experiment, Fresnel's biprism and Lloyd's mirror use this technique for coherent sources.

19.7.4 Division of Amplitude

1. When light source is extended, the amplitude of wave is divided in two parts by partial reflection and partial refraction. In it, the coherent sources obtained are real.
2. Newton's rings, Michelson's interferometer and colour of thin films use this technique for obtaining two coherent sources.

19.8 YOUNG'S DOUBLE SLIT EXPERIMENT

1. In this experiment, two points of the same widths are used as two coherent sources.
2. The interference fringes are usually hyperbolic in shape. Locus of path difference between light waves from two slits is a hyperbola. Interference fringes obtained in Young's experiment consist of alternate bright and dark bands. When the screen is held at 90° to the line joining foci of the hyperbola, the fringes are circular. When distance of screen (D) is very large compared to the distance between the slits (d), the fringes are straight.
3. All bright fringes have same intensity and all dark fringes are perfectly dark.
4. Bright fringes are due to constructive interference. Dark fringes are due to destructive interference.
5. The central fringe is bright with monochromatic light. It is achromatic/white with white light.
6. If W_1 and W_2 represent width of two slits, I_1 and I_2 represents intensities of light from two slits, a and b represent amplitudes of waves from two slits, then $\frac{W_1}{W_2}=\frac{I_1}{I_2}=\frac{a^2}{b^2}$
7. As $I=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$ i.e., $I=\left(\sqrt{I_1}\right)^2+\left(\sqrt{I_2}\right)^2+2\sqrt{I_1I_2}\cos\phi$

   and $A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$ when $\cos\phi=-1$, i.e., destructive interference, then

$$A^{\min}=(A_1-A_2) \text{ and } I^{\min}=\left(\sqrt{I_1}-\sqrt{I_2}\right)^2$$

   when $\cos\phi=+1$, i.e. constructive interference, then

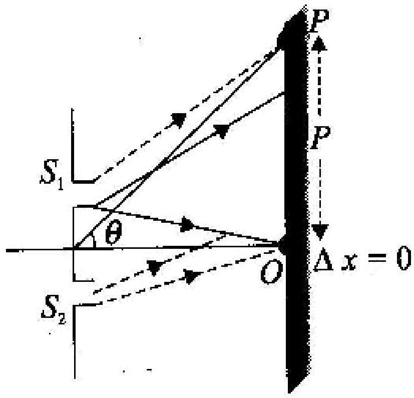
$$A^{\max}=(A_1+A_2) \text{ and } I^{\max}=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2$$

8. If amplitudes and intensities are equal then

$$A^{\max}=2A,\ I^{\max}=4I \quad A^{\min}=0;\ I^{\min}=0$$

$$\frac{I^{\max}}{I^{\min}}=\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{(I_2)^2}\right)}=\frac{\left(\sqrt{\frac{I_1+1}{I_2}}\right)^2}{\left(\sqrt{\frac{I_1}{I_2}}-1\right)^2}=\left(\frac{\frac{A_1+1}{A_2}}{\frac{A_2}{A_2}-1}\right)^2=\left(\frac{A_1+A_2}{A-A_2}\right)^2$$

9. Path difference $\Delta x = S_1P - S_2P$



$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

If amplitudes equal, then

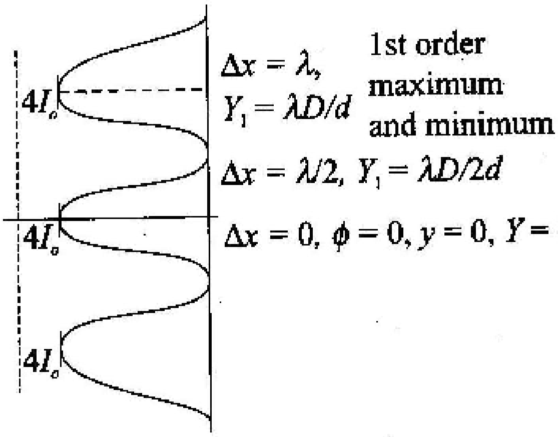
$$I_R = 4I\cos^2\frac{\phi}{2}$$

Minimum intensity or destructive interference

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots (2n-1)\frac{\lambda}{2}$$

$$\left[y_m = \frac{(2n-1)\lambda D}{2d}\right] y\text{-coordinate of } n\text{th order minima}$$

10. Intensity distribution on screen



11. Fringe width: $w = \frac{\lambda D}{d} \quad \Rightarrow \quad \{w \propto \lambda\}$

If *YDSE* apparatus is immersed in liquid then v and λ decreases μ times, therefore w also decreases μ time

$$\lambda_{\text{Red}} > \lambda_{\text{Voilet}} \quad \therefore \quad w_{\text{Red}} > w_{\text{Voilet}}$$

i.e., if whole set-up is in the water, fringe-width w decreases.

12. If the source S is covered by a black paper, no light is emitted by the source. S_1 and S_2 are not illuminated. There will be no interference pattern.

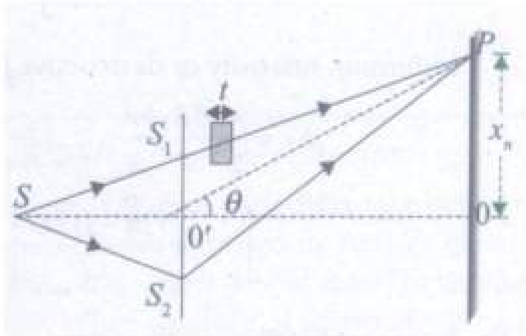
13. If any one of the slits S_1 or S_2 is covered with a black paper, light reaches the screen from the uncovered slit. There will be uniform illumination on the screen and no interference pattern.

14. The source S is moved closer to S_1 and S_2. The fringe-width remains the same, but the intensity of fringe increases.

15. If the source S is white light source, then the central fringe is white in colour (not just bright, but white). On either side, a few coloured fringes are seen followed by uniform illumination. The blue colour is nearer to the central fringe and red is far away.

16. In order to observe the fringes with a good contrast, the width of the slits must be considerably less than the fringe-width.

17. When a transparent thin film of mica or glass is inserted in the path of one of the beams, the whole of interference pattern gets shifted towards the side where the film is inserted.



Shift in fringe: $S = \dfrac{(\mu - 1)tD}{d}$

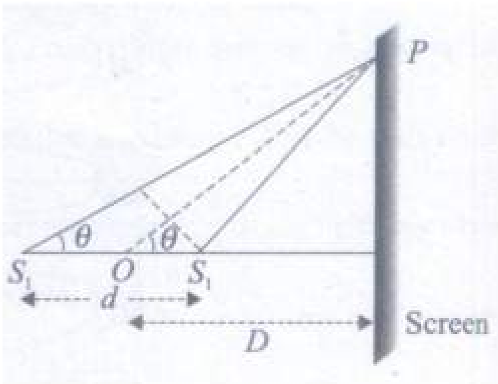
shift in 2nd order = shift in 3rd order

$\Rightarrow$ shift is independent of n and λ.

Number of fringes shifted $N = \dfrac{\text{Shift}}{w} = \dfrac{(\mu - 1)t}{\lambda}$

If size of one slit is increased then maximum intensity and minimum intensity both will increase.

18. Angular fringe width (θ):



$$\theta = \frac{w}{D} = \frac{\lambda \frac{D}{d}}{D} = \frac{\lambda}{d}$$

In $YDSE$, $\Delta x = d\cos\theta$

$\Delta x^{max} = d$ if $\theta = 0$

$\Delta x^{min} = 0$ if $\theta = 90°$ (Perpendicular bisector)

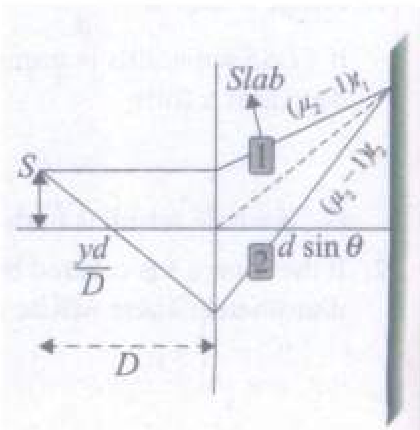
If $d << D$: $\Delta x^{max} = d$ for $\theta = 90°$

$\Delta x^{min} = 0$ for $\theta = 0°$ out of screen

$d\cos\theta << D$: $\Delta x^{max} = d$: $\theta = 0°$

$\Delta x^{min} = 0$, $\theta = 90°$ out of screen

19. Net path difference in mixed situation



$$\Delta x_{net} = \frac{yd}{D} + d\sin\theta + \{(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1\}$$
$$= n\lambda \text{ for } n\text{th maxima,}$$

where $n = 0, 1, 3,\ldots$

$= (2n-1)\frac{\lambda}{2}$ for nth minima,

where $n = 1, 2, 3\ldots$

$$\Delta x = \frac{yd}{D},\ \phi = \frac{2\pi}{\lambda}\Delta x,\ A_R = 2A\cos\frac{\phi}{2},\ I_R = 4I\cos^2\frac{\phi}{2}$$

Y	Δx	ϕ	$\frac{\phi}{2}$	I_R	A_R
$\frac{\lambda D}{d} = w$	λ	2π	π	$4I$	$2A$
$\frac{\lambda D}{6d} = \frac{w}{6}$	$\frac{\lambda}{6}$	$\frac{\pi}{3}$ or 60°	$\frac{\pi}{6}$	$3I$	$\sqrt{3}A$
$\frac{\lambda D}{4d} = \frac{w}{4}$	$\frac{\lambda}{4}$	$\frac{\pi}{2}$ or 90°	$\frac{\pi}{4}$	$2I$	$\sqrt{2}A$
$\frac{\lambda D}{3d} = \frac{w}{3}$	$\frac{\lambda}{3}$	$\frac{2\pi}{3}$ or 120°	$\frac{\pi}{3}$	I	A
$\frac{\lambda D}{2d} = \frac{w}{2}$	$\frac{\lambda}{2}$	π or 180°	$\frac{\pi}{2}$	0	0

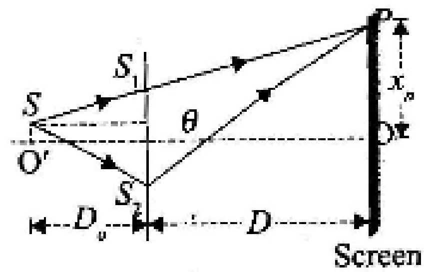
20. When the primary source S kept above the central axis

$S_1S_2 = (2d)$ and $O's = d_o$

The path-difference is

$$\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P)$$

$$\Delta x = \frac{(2d)d_o}{D} + \frac{x_n(2d)}{D}$$

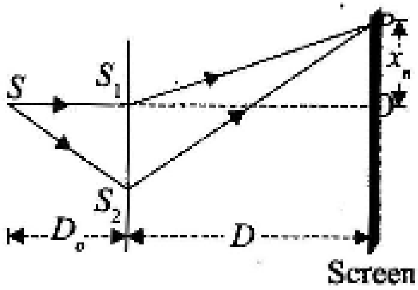


The value of position of nth bright fringe is $x_n = n\left(\frac{D\lambda}{2d}\right) - \left(\frac{Dd_o}{D_o}\right)$

The position of nth dark fringe is $x_n = \left(n + \frac{1}{2}\right)\frac{D\lambda}{2d} - \left(\frac{d_o D}{D_o}\right)$

i.e., the whole fringes shift in the downward direction.

21. When slit S_1 lies on the central line and S_2 is below the central line



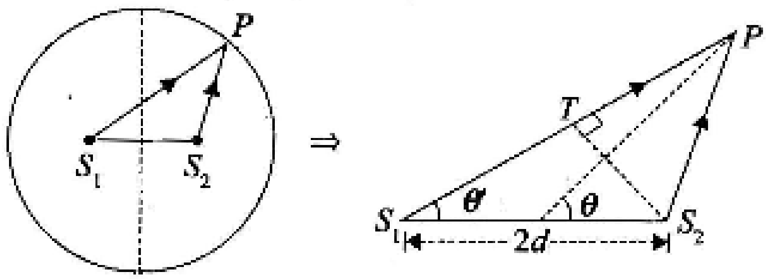
$$S_1S_2 = (2d) = d'(\text{say})$$

$$\Delta x = \frac{d'^2}{2D_o} + \frac{d'^2}{2D} + \frac{2x_n d'}{2D}$$

The value of position of nth bright fringe is given by $\Delta x = 2n\frac{\lambda}{2}$

The position of nth dark fringe is given by $\Delta x = (2n+1)\frac{\lambda}{2}$

22. Analysis of the positions of the fringes for spherical screen



$\Delta x = S_1P - S_2P = S_1T + TP - S_2P = S_1T = (2d\cos\theta)$ as $\theta' \approx \theta$ for very small θ.

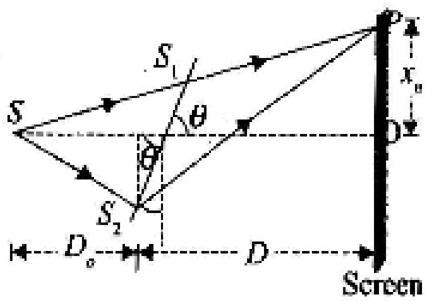
The value of position of nth bright fringe is given by $\Delta x = 2n\frac{\lambda}{2}$

For bright fringe, $(2d\cos\theta) = (2n)\left(\frac{\lambda}{2}\right) \Rightarrow \cos\theta = \left(\frac{n\lambda}{2d}\right)$

The position of nth dark fringe is given by $\Delta x = (2n+1)\frac{\lambda}{2}$

For dark fringe, $2d\cos\theta = (2n+1)\frac{\lambda}{2} \Rightarrow$
$\cos\theta = (2n+1)\left(\frac{\lambda}{4d}\right)$

23. When the plane of slits or sources is inclined at any angle θ



$$S_1S_2 = (2d)$$

The path-difference is

$$\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P)$$

$$= \left\{\sqrt{(D_o - d\cos\theta)^2 + (d\sin\theta)^2} + \sqrt{(D_o + d\cos\theta)^2 + (x_n + d\sin\theta)^2}\right\}$$
$$-\left\{\sqrt{(D_o + d\cos\theta)^2 + (d\sin\theta)^2} + \sqrt{(D - d\cos\theta)^2 + (x_n - d\sin\theta)^2}\right\}$$

The value of position of nth bright fringe is given by $\Delta x = 2n\frac{\lambda}{2}$

The position of nth dark fringe is given by $\Delta x = (2n+1)\frac{\lambda}{2}$

24. If wave is reflected from denser medium, $\phi = \pi$ and $\Delta x = \frac{\lambda}{2}$. A further $\Delta x = \frac{\lambda}{2}$ will be required to make a total λ or constructive interference.

    Hence, in this case maxima and minima are interchanged.

19.9 DIFFRACTION OF LIGHT

1. When light waves fall on a small aperture or a small-sized obstacle whose linear dimension e is comparable to the wavelength λ of the wave, then there is a deviation from straight line propagation and wave energy flares out into the region of geometrical shadow of the obstacle or aperture. The spreading of wave energy beyond the limits prescribed by the straight line propagation of the rays is called *diffraction*. Diffraction was discovered by *Grimaldi*. Diffraction effects become more prominent when (λ/e) increases.
2. As $\lambda_{sound} > \lambda_{light}$, diffraction is more easily observed in sound as compared to light.
3. Interference takes place when there is superposition of two separate wavefronts originating from two separate coherent sources. Diffraction takes place due to superposition of secondary wavelets starting from different points of the same wavefront.

19.9.1 Fraunhofer Diffraction Due to a Single Slit

When monochromatic light of wavelength λ is used to illuminate a single slit of width d, then condition of diffraction minima is given by

$$d \sin \theta = n\lambda; \text{ where } n = 1, 2, 3, 4\ldots$$

But the condition of secondary diffraction maxima is

$$d \sin \theta = (2n+1)\lambda/2; \text{ where } n = 1, 2, 3, 4\ldots$$

Angular position of nth secondary minima is given by $\sin\theta \approx \theta = n\frac{\lambda}{d}$

The angular width of central maxima is $2\theta = \frac{2\lambda}{d}$

The angular width of central maxima is double as compared to angular width of secondary diffraction maxima.

19.9.2 Fraunhofer Diffraction at a Circular Aperture

When monochromatic light of wavelength λ is used to illuminate a circular aperture of diameter d, then the angular radius of the first dark ring is given by $d \sin \theta = 1.22\lambda$ or $\sin\theta = (1.22\lambda/d)$ and θ also represents the radius of the central bright disc.

19.10 DIFFRACTION AT A PLANE GRATING

When polychromatic or monochromatic light of wavelength λ is incident normally on a plane transmission grating, the principal maxima are given by $(e + d) \sin \theta = n\lambda$, where n = order of maximum, θ = Angle of diffraction and $(e + d)$ = Grating element.

19.11 POLARIZATION OF LIGHT

1. The ordinary light also called as *unpolarized light*. Unpolarized light is symmetrical about the direction of propagation. It consists of a very large number of vibrations in all planes with equal probability at right angles to the direction of propagation.
2. The light which has acquired the property of one sidedness is called polarized light or lack of symmetry of vibration around the direction of wave propagation is called polarization.
3. Transverse waves show polarization of light.
4. When the vibrations are confined only to a single direction in a plane perpendicular to the direction of propagation, it is called a plane polarized light. A plane passing through the direction of propagation and perpendicular to the plane of vibration is called as plane of polarization.
5. Plane polarized light can be produced by the following methods:
   (a) *By refraction:* According to Brewster's law $\tan \theta_p = \mu$ Moreover, in such an eventually the reflected and transmitted rays are mutually perpendicular. Thus, angle of refraction $r = (90 - \theta_p)^\circ$, where θ_p is the angle of polarization (Brewster's angle) and μ is the refractive index of the reflecting medium.
   (b) *By reflection:* When a beam of unpolarized light is reflected from the surface (unpolished) of a transparent medium of refractive index μ at the polarizing angle i_p, the reflected light is completely plane polarized.
   (c) *By dichroism:* Some doubly-refracting crystals have the property of absorbing strongly one of the two refracted rays and allowing the other to emerge with little loss. This selective absorption by the crystal is known as dichroism, e.g. tourmaline crystal.
   (d) *By double refraction (Nicol's prism):* When a ray of unpolarized light incident on a calcite (or quartz) crystals, splits up into two refracted rays, the phenomenon is called double refraction.
   (e) *By scattering*
6. **Polaroids:** These are artificially prepared polarizing materials in the form of sheets or plates capable of producing strong beam of plane polarized light. These are employed to observe stereoscopic motion picture showing the three dimensional effects. Polaroids are used in laboratories for producing and analysing plane polarized light.

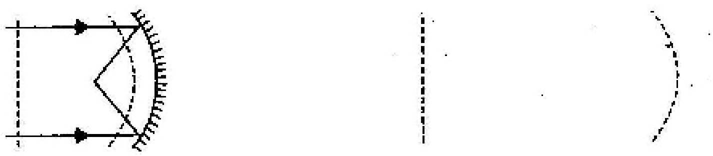
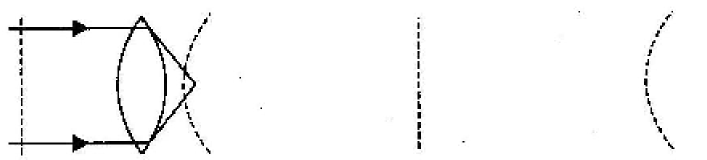
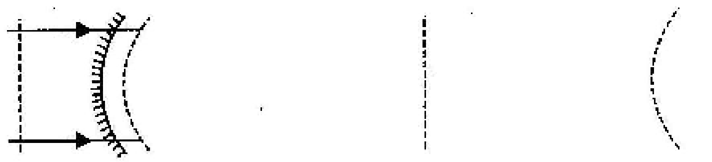
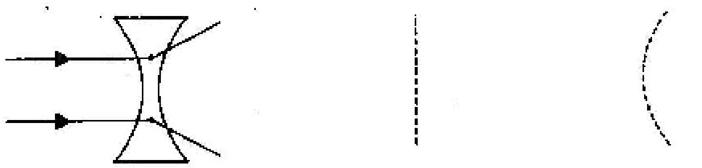
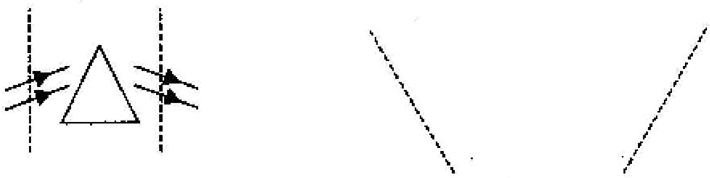
   Two crossed polaroids subjected to pre-determined stresses are used in optical stress analysis.
7. According to Malus law the intensity of emergent light out of analyser varies as $I \propto \cos^2\theta$ or $I = I_0 \cos^2\theta$, where θ is the angle between the planes of transmission of polariser and analyser.

8. Optical rotation is the phenomenon of rotating the plane of polarization of light about the direction of propagation of light, when passed through certain crystals or solutions. The substances which rotate the plane of polarization are called optically active substances. They are of two types: *dextro rotatory* (clockwise direction) and *laevo rotatory* (anti-clockwise direction). Quartz is dextro-rotatory as well as laevo rotatory.

9. It is found that quartz is available in both *laevo* and *dextro* varieties. Further, when in non-crystalline state, quartz is optically inactive. It is, therefore, obvious that the property of optical activity is closely associated with the asymmetric crystalline structure of substances. Many liquids like solutions of tartaric acid, sugar and turpentine oil are also optically active. These substances are found to be optically active even when dissolved in a solvent which itself is optically inactive and does not react with the substance chemically. Moreover, they remain optically active even in the vapour state. Thus, it follows that optical activity of liquids and their vapours is due to some asymmetry in their molecular structure.

19.11.1 Wave Front

Point are in same phase

	Incident Wave Front	Reflected/ Refracted	Type
			Concave mirror
			Convex lens
			Convex mirror
			Concave lens
			Prism

RAY OPTICS AND OPTICAL INSTRUMENTS

20.1 REFLECTION OF LIGHT

Reflection of light is the process of deflecting a beam of light. Experiments in reflection have yielded the following two laws:

1. The incident ray, the reflected ray, and the normal, all lie in the same plane, called the plane of incidence. The incident and reflected rays are on the opposite sides of the normal.
2. The angle of incidence is always equal to the angle of reflection.

   That is, $\angle i = \angle r$

   These angles are measured with the normal to the point of incident.
3. The vector form of the law of reflection is

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{n})\hat{n}$$

where $\hat{e}_1 =$ Unit vector along the incident ray

$\hat{e}_2 =$ Unit vector along the reflected ray

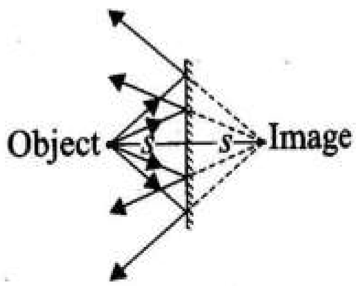
$\hat{t} =$ Unit vector along tangential direction

$\hat{n} =$ Unit vector along outside normal

20.2 CHARACTERISTICS OF IMAGE DUE TO REFLECTION BY A PLANE MIRROR

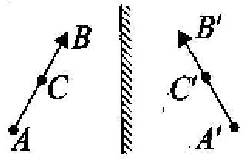
1. Distance of object from mirror = Distance of image from the mirror.

   All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called an image.

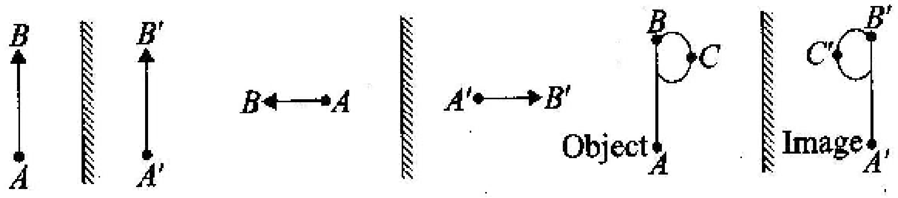


2. The line joining a point object and its image is normal to the reflecting surface.
3. The size of the image is the same as that of the object.
4. For a real object the image is virtual and for a virtual object the image is real.

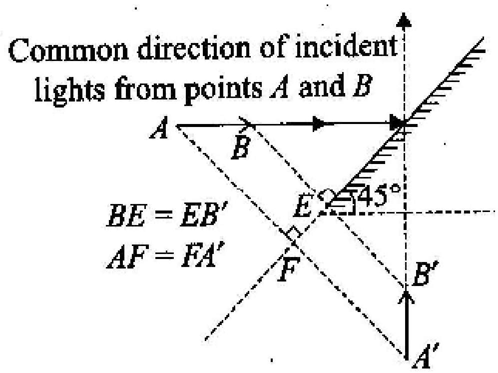
5. Size of extended object = Size of extended image.



6. The image is upright, if the extended object is placed parallel to the mirror.
7. The image is inverted if the extended object lies perpendicular to the plane mirror.



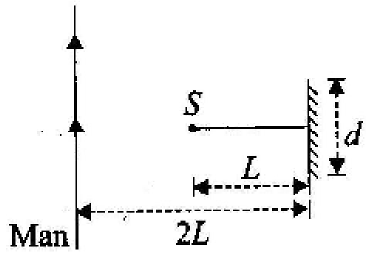
8. If an extended horizontal object is placed in front of a mirror inclined at 45° with the horizontal, the image formed will be vertical.
9. If an object in front of a plane mirror moves through a distance d away from the mirror, the image moves through the same distance but it should be noted that the displacement of the image is opposite to that of the object. (If v is the velocity between the object and the mirror, then velocity between the object and image is $2v$); whereas if the mirror moves parallel to itself through a distance d (the object remaining fixed), the image will move through a distance $2d$, but it is to be noted that the displacement of the mirror and the image are in the same direction.



(a) The minimum size of a plane mirror required to see the full image of an observer is half the size of that observer.

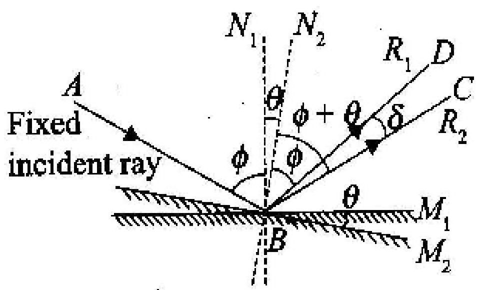
(b) A man is standing exactly at midway between a wall and a mirror and he wants to see the full height of the wall (behind him) in a plane mirror (in front of him). The minimum length of mirror in this case should be $H/3$, where H is the height of wall.

(c) A point source of light S, placed at a distance L in front of the centre of a mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown. The greatest distance over which he can see the image of the light source in the mirror is $3d$.



20.2.1 Effect of Rotation of Plane Mirror on the Image

1. When direction of incident ray is kept fixed. See figure M_1, N_1 and R_1 indicating the initial position of mirror, initial normal and initial direction of reflected light ray respectively. M_2, N_2 and R_2 indicate the final position of mirror, final normal and final direction of reflected light ray, respectively.



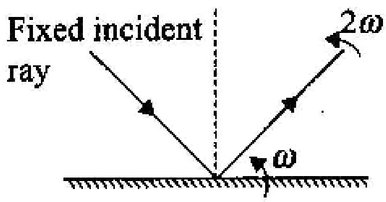
From figure it is clear that $\angle ABC = 2\phi + \delta = 2(\phi + \theta)$ or $\delta = 2\theta$.

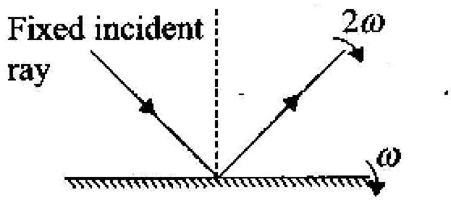
That means when incident ray is fixed, and mirror rotates through the angle θ:

(a) Then reflected ray rotates through the angle 2θ in the same sense as the mirror rotates.

(b) The angular velocity and angular acceleration of new reflected ray becomes twice as that of mirror.

$\omega_1 = 2\omega_2 \Rightarrow \omega_2 =$ angular velocity of mirror $\Rightarrow \omega_1 =$ angular velocity of reflected ray





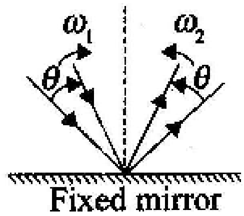
2. When mirror is fixed and incident ray rotates:

$\omega_1 =$ Angular velocity of incident ray

$\omega_2 =$ Angular velocity of reflected ray

$\omega_1 = -\omega_2$

(negative sign shows that the direction of angular velocity is opposite to each other)



20.2.2 Number of Images Formed by Two Inclined Plane Mirrors

Number of images formed by two inclined mirrors ($\theta =$ angle between mirrors)

1. If $\frac{360°}{\theta} =$ even number; number of image $= \frac{360°}{\theta} - 1$ because one image is common.

2. If $\frac{360°}{\theta} =$ odd number; number of image $= \frac{360°}{\theta} - 1$, if the object placed on the angle bisector.

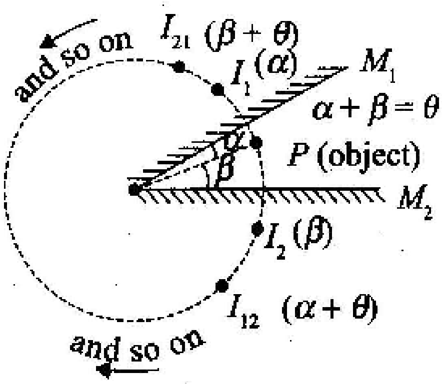
3. If $\frac{360°}{\theta} =$ odd number; number of image $\frac{360°}{\theta}$, if the object is not placed on the angle bisector.

4. If $\frac{360°}{\theta} \neq$ integer, then count the number of images as explained below.

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in figure.

$I_1 =$ Image of object P formed by the mirror M_1

$I_2 =$ Image of object P formed by the mirror M_2

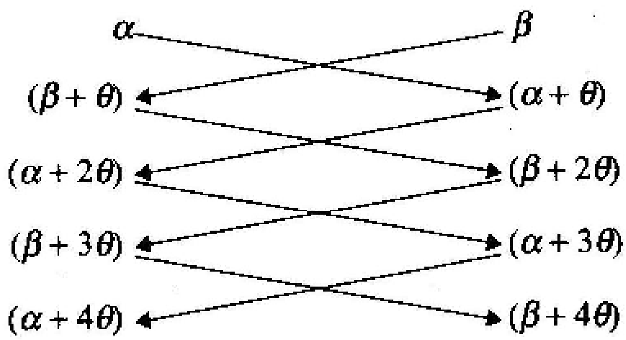


$I_{21} = I_2$ will act as an object for mirror M_1

$I_{12} = I_1$ will act as an object for mirror M_1 and so on

Image formed by Mirror M_1 (angles are measured from the mirror M_1)

Image formed by Mirror M_2 (angles are measured from the mirror M_2)



Stop if next angle will be more than 180° or equal

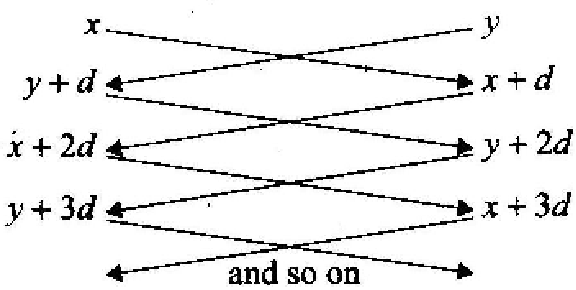
Stop if next angle will be more than 180° or equal

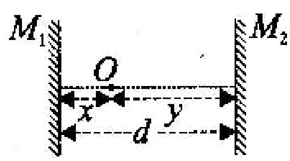
To check whether the final images made by the two mirrors coincide or not: add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Therefore in this case the last images coincide. Therefore the number of images = (number of images formed by mirror M_1 + number of images formed by mirror M_2) – 1(as the last images coincide)

- When $\theta = 90°$, three images are formed, but if two plane mirrors are placed parallel to each other, and facing each other then $n = \frac{360°}{\theta} = \infty$. This means infinite number of images must be formed, all lying on a straight line passing through the object and perpendicular to the mirrors. But in practice, only a limited number of images are seen because light is lost at each reflection. In this case, the distance of the image can be measured from the mirror with the help of the method given below.

Image formed by Mirror M_1 (distance of images measured from the mirror M_1)

Image formed by Mirror M_2 (distance of images measured from the mirror M_2)





20.2.3 Concept of Velocity of Image in the Plane Mirror

There are three components of velocity of image for a moving object.

1. Perpendicular to the plane mirror.
2. Other two are parallel to the plane mirror.

Concept of perpendicular component of velocity of image in the plane mirror:

$$(S_{IM})_\perp = -(S_{OM})_\perp$$

where S_{IM} = Distance of image w.r.t. to mirror

S_{OM} = Distance of object w.r.t. mirror

Differentiating both sides w.r.t. time, we get

$$(V_{IM})_\perp = -(V_{OM})_\perp \quad \Rightarrow \quad (V_{IG})_\perp - (V_{MG})_\perp = -[(V_{OG})_\perp - (V_{MG})_\perp]$$

$$(V_{MG})_\perp = \frac{(V_{OG})_\perp + (V_{IG})_\perp}{2}$$

If mirror is an $(x-y)$ plane and the perpendicular component is along z direction.

Concept of parallel components of velocity of image in the plane mirror:

$$(S_{IM})_\| = -(S_{OM})_\|$$

Differentiating both sides w.r.t. time, we get

$$(V_{IM})_\| = (V_{OM})_\| \Rightarrow (V_{IG})_\| - (V_{MG})_\| = (V_{OG})_\| - (V_{MG})_\|$$

$$\Rightarrow \quad (V_{IG})_\| = (V_{OG})_\|$$

It implies that parallel component of velocity of image w.r.t. ground or mirror will remain same as velocity of object w.r.t. ground or mirror. If mirror has $(x$-$y)$ plane then x and y component of velocity of image are the parallel components.

Notes

Again, differentiating the velocity equation w.r.t. time, we get the acceleration of image. It is concept for the acceleration of image

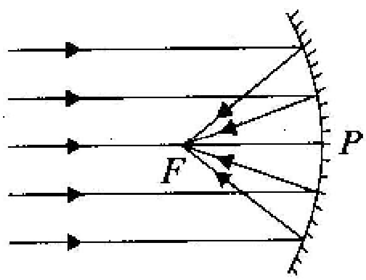
$$(a_{MG})_\perp = \frac{(a_{OG})_\perp + (a_{IG})_\perp}{2} \text{ and } (a_{IG})_\| = (a_{OG})_\|$$

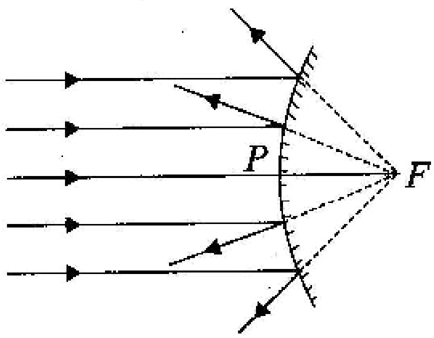
- If ray of light suffers successive reflections at two mirrors inclined at an angle θ, after falling on the first mirror at an angle of incidence α, its total deviation (δ) is independent of α. The deviation produced (δ) is the angle made by the emergent ray with the incident ray. When the mirrors are at right angles, a ray after successive reflections from both travels parallel to itself, but in a direction opposite to that before incidence, i.e., the direction of the ray is reversed in this case.

20.3 CURVED MIRRORS

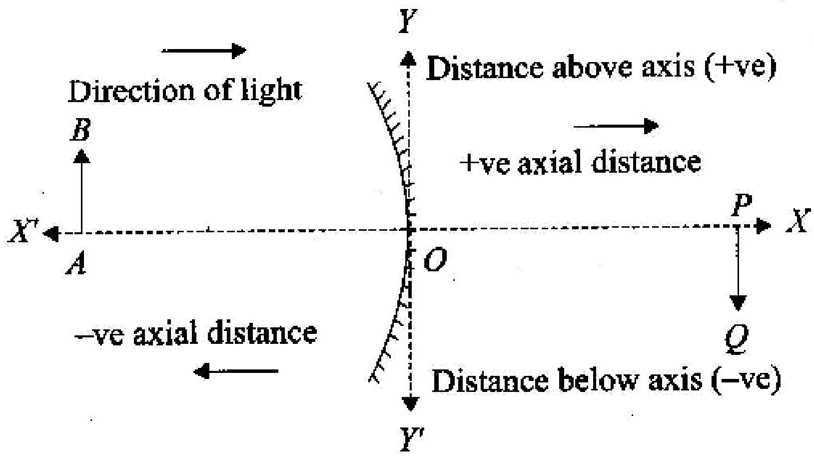
Mirrors whose reflecting surfaces are curved are called curved mirrors. The surfaces of curved mirrors are three dimensional and form part of a sphere or cylinder. A mirror which has parabolic section is called parabolic mirror. If the reflecting surface is a part of a sphere it is called a spherical mirror. If light gets reflected from inside the sphere it is a concave mirror and if it gets reflected from outside the sphere, it is called a convex mirror.

1. The centre of the sphere is called the centre of curvature.
2. The radius of the sphere is called the radius of curvature of the mirror.
3. The geometric centre of the mirror surface is called the pole of the mirror.
4. The straight line joining the pole and the centre of curvature of the mirror is called the principal axis of the mirror.
5. **Principal focus:** A narrow beam of rays, parallel and close to the principal axis, incident on a spherical mirror, after reflection, converges to a point on the principal axis, in the case of a concave mirror or appears to diverge from a point on the principal axis in the case of a convex mirror. This point is called the principal focus of a spherical mirror. F in figure.





6. Focal length (PF) is the distance between the pole and principal focus of the mirror. $f = \frac{R}{2}$ i.e., the focal length of a mirror is half of its radius of curvature.
7. **Sign convention:** This convention is in accordance with the conventions of coordinate geometry. The distance of object or the image from the reflecting surface are taken as vectors.



(a) The pole of the reflecting surface is assumed to be at the origin.

(b) The incident rays are drawn from left to right.

(c) The principal axis of the mirror is taken along the x-axis

(d) The distances measured to the left of the origin along the negative direction of x-axis, are taken as negative and the distances along the positive x-direction are taken as positive.

(e) Distances measured upward and perpendicular to the x-axis are taken as positive and downward distances are taken as negative.

While solving problems please remember this. Substitute the values of the given known quantities with +ve or –ve sign in the formula. Do not give any sign to the unknown quantity. Solve for the unknown quantity. The answer will contain the appropriate sign.

8. Mirror formula $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

9. Magnification:

(a) Lateral or transverse magnification $m_1 = \frac{\text{Lateral size of the image}}{\text{Lateral size of the object}} = \frac{h_2}{h_1}$ where h_2 is the height of the image and that of the object is h_1

(b) Axial or longitudinal magnification

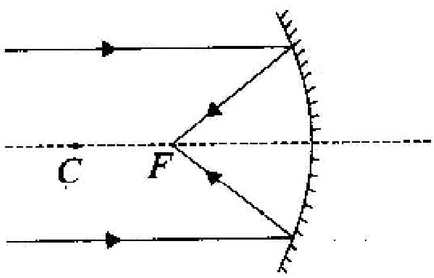
$$m_2 = \frac{\text{Axial distance of the conjugate image from the pole of the surface}}{\text{Axial distance of the object from the pole of the surface}} = \frac{v}{u}$$

(c) Angular magnification

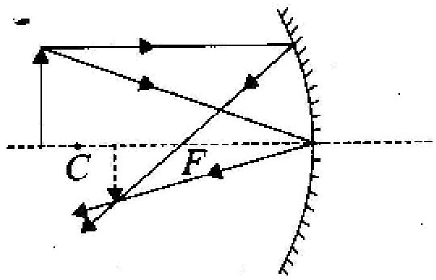
$$m_3 = \frac{\text{Sine of angle of conjugate image ray with the axis}}{\text{Sine of angle of object ray with the axis}} = \frac{\sin\theta_2}{\sin\theta_1}$$

10. Image formation:

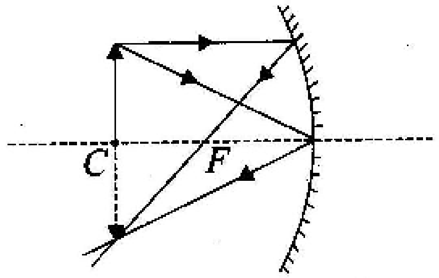
(a) Concave mirror



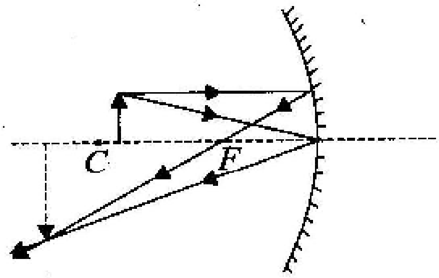
(a) Object at infinity; image at focus, small size and real.



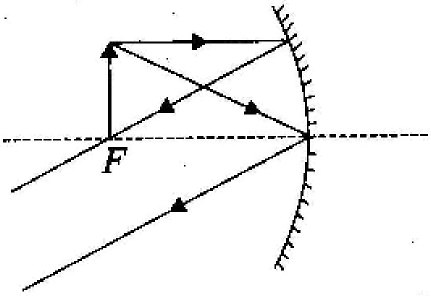
(b) Object placed beyond C; image is real, inverted, same size, formed between C and F



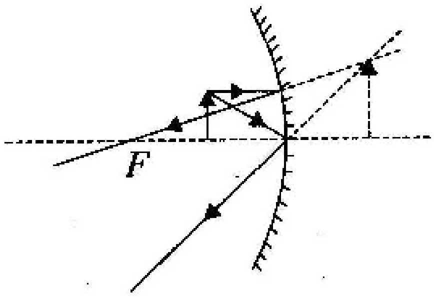
(c) Object at C (centre of curvature); image is real, inverted, same size, formed at C itself.



(d) Object between F and C; image is real, inverted, magnified, formed beyond C.

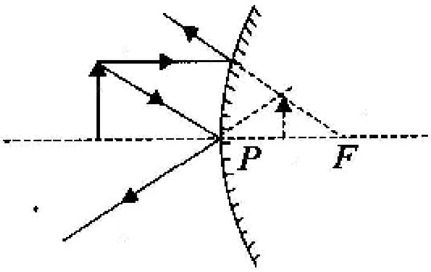


(e) Object at the focus F; image at infinity because the reflected rays being parallel cannot meet each other.



(f) Object between the pole and focus; image is virtual, magnified and erect.

(b) Convex Mirror



Object at any distance (provided it is real); image is always erect, diminished and virtual.

11. **Newton's formula:** Instead of measuring the distances from the pole of the spherical mirror suppose the distances of the object and image are measured from the focus as a and b respectively then, $ab = f^2$.

This is Newton's formula. The formula does not apply to convex mirror.

- In plane mirror, problems are solved by ray diagram but in spherical mirror, only formulae are used.

20.3.1 Concept of Velocity of Image in Spherical Mirrors

There are two components of velocity of an image:

1. Component along the *principal* axis that means perpendicular to the mirror
2. Component perpendicular to the *principal* axis that means parallel to the mirror

Velocity component along the axis:

Differentiating mirror formula on the both side w.r.t. time, we get

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0$$

(Since focal length of the mirror remains constant)

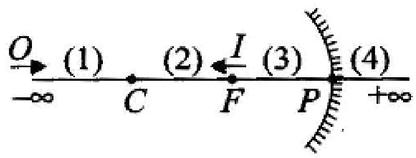
$$\Rightarrow \quad \frac{dv}{dt}=-\frac{v^2}{u^2}\frac{du}{dt} \Rightarrow (V_{IM})_{\parallel}=-\frac{v^2}{u^2}(V_{OM})_{\parallel}$$

$(V_{IM})_{\parallel}$ = Velocity of image w.r.t. mirror along the *principal* axis.

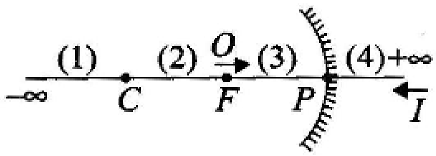
$(V_{OM})_{\parallel}$ = Velocity of an object w.r.t. mirror along the *principal* axis.

Direction of velocity of an object and mirror w.r.t. mirror is opposite to each other.

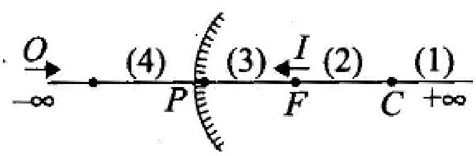
Any object in the region (1) will have its image in region (2) and vice versa and similarly to region (3) and (4).



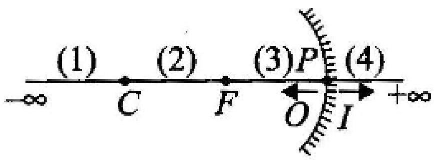
(a)



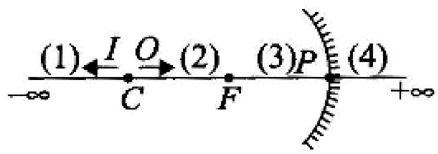
(b)



(c)



(d)



(e)

When an object moves from $-\infty$ to C, the image moves from F to C as shown in figure (a).

When object moves from F to P, the image moves from $+\infty$ to P as shown in figure (b).

When an object moves from $-\infty$ to P, image moves from F to P as shown in figure (c).

When an object moves from P to F, the image moves from P to $+\infty$ as shown in figure (d).

When an object moves from C to F, the image moves from C to $-\infty$ as shown in figure (e).

Velocity component perpendicular to axis

$$m = \frac{h_I}{h_O} \Rightarrow h_I = m h_O$$

Differentiating this equation on the both side w.r.t. time, we get

$$\frac{dh_I}{dt} = \frac{m dh_O}{dt} + h_O \frac{dm}{dt} \Rightarrow (V_{IM})_\perp = m(V_{OM})_\perp + h_O \frac{dm}{dt}$$

$(V_{IM})_\perp$ = Velocity of image w.r.t. mirror perpendicular to the principal axis.

$(V_{OM})_\perp$ = Velocity of object w.r.t. mirror perpendicular to the principal axis.

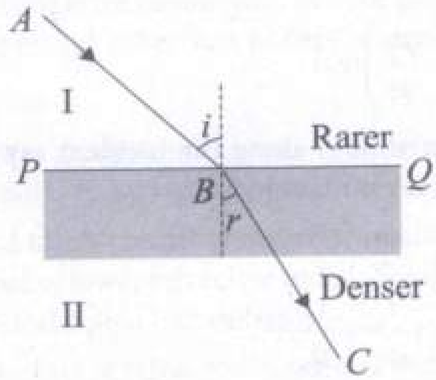
$\frac{dm}{dt}$ = Rate of change of magnification (its unit is per sec)

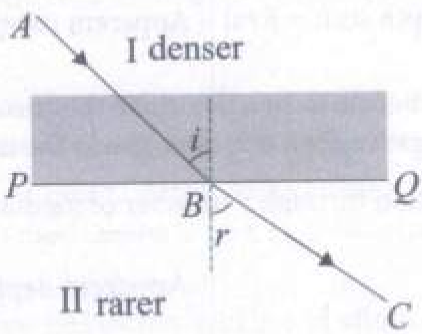
$m = -\frac{v}{u}$, by differentiating this equation on the both sides w.r.t. time, we get

$$\frac{dm}{dt} = \left[\frac{u(V_{IM})_{\parallel} - v(V_{OM})_{\parallel}}{u^2}\right]$$

20.4 REFRACTION OF LIGHT

A beam of light passing from one transparent medium to another obliquely undergoes an abrupt change in direction. This bending of light ray at the surface of separation of two media is called refraction.





When the angle of incidence $i = 0$ (i.e., normal incidence) or the ray strikes the surface of separation, normally the ray does not bend.

During refraction the frequency of the light ray remains constant. The velocity and wavelength change.

20.5 LAWS OF REFRACTION

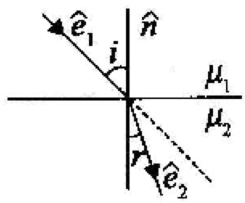
1. The incident ray, the refracted ray and the normal to the surface of separation, at the point of incidence, all lie in the same plane. The two rays lie on the opposite side of the normal.
2. For two particular media and for a given colour of light, the ratio of the sine of angle of incidence to the sine of angle of refraction is a constant. This is known as Snell's law.

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2$$

Snell's law can also be written as $\mu_1 \sin i = \mu_2 \sin r$.

3. Vector form of Snell's law,

$\mu_1(\hat{e}_1 \times \hat{n}) = \mu_2(\hat{e}_2 \times \hat{n})$



where $\hat{e}_1$ = Unit vector along the incident ray

$\hat{e}_2$ = Unit vector along the refracted ray

$\hat{n}$ = Unit vector along the normal

20.5.1 Refraction at Plane Surface

If the object and viewer both are in different media and eyes lie near the normal to the interface which passes through the object then vision is known as normal vision. For this, angle of incidence and angle of refraction are very small. If the object placed in medium-I of μ_1 and viewer is in medium-II of μ_2, then

$$\text{Apparent depth} = \text{Real depth} \times \left(\frac{\mu_2}{\mu_1}\right)$$

i.e., If $\mu_2 > \mu_1$; then Apparent depth > Real depth

If $\mu_1 > \mu_2$; then Real depth > Apparent depth

$$\therefore \quad \text{Depth shift} = \text{Real} - \text{Apparent depth} = \left(1 - \frac{\mu_2}{\mu_1}\right)\text{real}$$

If shift becomes positive then the direction of shift is along the incident ray and if shift becomes negative then it is opposite to the incident ray w.r.t. object.

- Refraction through a number of media:

$$\text{Apparent depth} = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \frac{t_3}{\mu_3} + \ldots$$

20.5.2 Total Internal Reflection

Total internal reflection is the phenomenon of reflection of light into the denser medium from the boundary of the denser medium and rarer medium.

The angle of incidence i in denser medium for which the angle of refraction in rarer medium is 90° is called the critical angle for the pair of media under consideration.

20.5.3 Refractive Index (R.I.) and Critical Angle

When the light ray passes from vacuum (in practice air) into a medium then the ratio of sine of angle of incidence to the sine of angle of refraction is called the absolute refractive index of the medium denoted by ${}_1\mu_2$ or μ.

$$\mu = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in the medium}} = \frac{c}{v}$$

for air is $\mu = 1.000292$, for water $\mu = 1.333$
for ice $\mu = 1.309$, for crown glass $\mu = 1.52$
for flint glass $\mu = 1.66$, for zircon $\mu = 1.923$
for carbon disulphide $\mu = 1.628$
(These values are measured with light of wavelength 589 nm at 20°C).

Critical angle is the angle of incidence in denser medium for which angle of refraction in rarer medium is 90°.

$$^{m}\mu_{a} = \frac{\sin i}{\sin r} \text{ by Snell's law}$$

Ray travels from the denser medium to air

$$^{m}\mu_{a} = \frac{\sin C}{\sin 90^\circ} = \sin C \text{ or, } ^{a}\mu_{m} = \frac{1}{\sin C} = \operatorname{cosec} C$$

Critical angle increases with temperature. Denser the medium, less the critical angle.
Regarding colour/wavelength or frequency of light, $\mu_V > \mu_R$. Hence $C_V < C_R$.
Critical angle for violet colour is lowest and for red colour is highest.
Critical angle for diamond = 24°, for glass = 42°, for water = 48°

Total internal reflection occurs if angle of incidence in denser medium exceeds critical angle.

1. Mirage is an optical illusion observed in deserts and roads on a hot day when the air near the ground is hotter and hence rarer than the air above.
2. Optical fibres consist of long fine quality glass or quartz fibres, coated with a thin layer of a material of lower refractive index. The device is used as a light pipe in medical diagnosis and for optical signal transmission.
3. The product of refractive index and distance travelled by the light in a medium (μd) is called optical path.

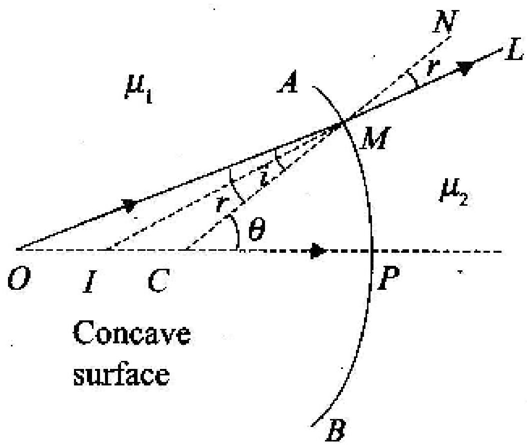
The optical path is the distance that the light travels in vacuum in the same time it travels a distance d in the medium.

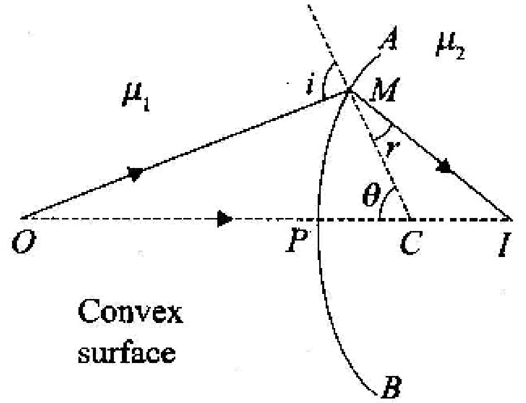
20.5.4 Spherical Refracting Surfaces

When a ray of light is incident upon a homogeneous transparent medium, it is refracted according to the laws of refraction. When the surface of the transparent refracting medium be spherical, it is called spherical refracting surface. It is of two kinds: (i) concave and (ii) convex.

In the concave refracting medium (through which the light passes), the contact surface is curved inward and in convex it is bulged outward.

20.5.5 Refraction from Spherical Surface





$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$ where, u = Object distance

v = Image distance

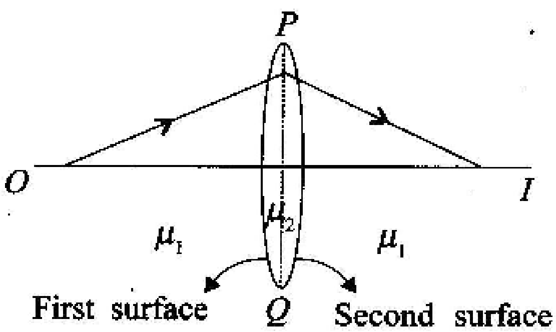
20.6 LENS

A lens is a piece of transparent refracting material which is bounded by two spherical surfaces or by one spherical surface and one plane surface.

When the lens is thicker in the middle than at the edges, it is called a convex lens or converging lens. When it is thinner in the middle, it is called a concave lens or diverging lens.

20.6.1 Lens Maker's Formula

PQ is a thin lens having two refracting surfaces of radii of curvature R_1 and R_2, respectively.



Using the formula for refraction at single spherical surface:
For first surface,

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad (1)$$

For second surface,

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \quad (2)$$

Adding eq. (1) and eq. (2),

$$\frac{\mu_1}{v}-\frac{\mu_1}{u}=(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{v}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$ It is lens maker's formula.

When $u=\infty$, $v=f$

$$\therefore \quad \frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Hence,

$$\therefore \quad \left(\frac{1}{v}-\frac{1}{u}\right)=\frac{1}{f}.$$ It is lens formula.

It is Gausion form of lens formula.

Limitations of this formula: (i) The lens must or should be thin. (ii) The medium on either side of the lens should be same.

1. If any limitation is violated, then we have to use the refraction at the curved surface formula for both the surfaces.
2. If mirror is immersed in a liquid then no change in focal length, but if lens is immersed, then it will change its nature.

20.6.2 Nature of Image Formation by Convex Lens and Concave Lens

For convex lens:

1. Object is at infinity

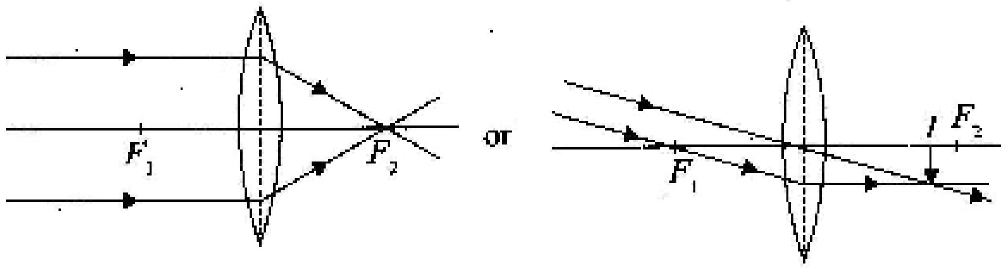


Image is real, inverted and diminished (i.e., $m << -1$) and is at focus.

2. Object is at in ∞ and $2F$:

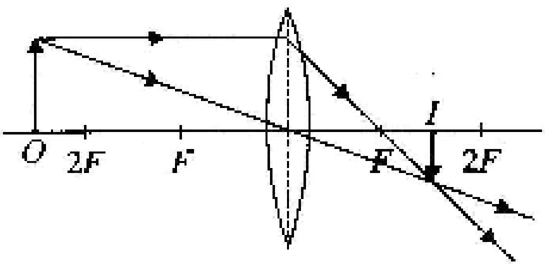


Image is real, inverted, diminished and in between F and $2F$.

3. Object is at $2F$:

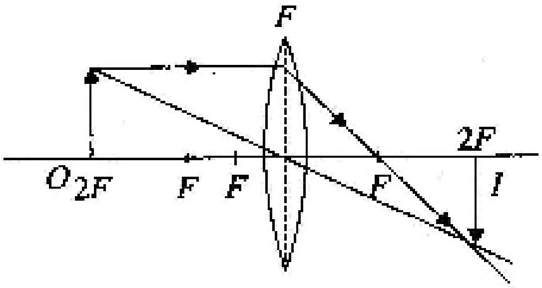


Image is real, inverted, equal (i.e., $m = -1$) and at $2F$.

4. Object is in between F and $2F$:

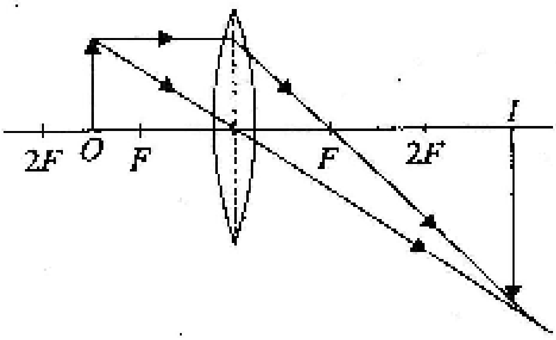


Image is real, inverted, and magnified (i.e., $m >$ 1) larger and at between $2F$ and ∞.

5. Object is at F:

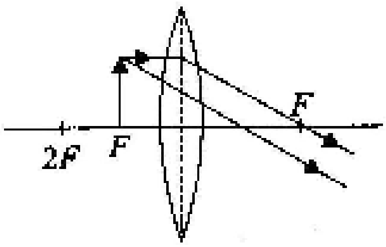


Image is real, inverted magnified (i.e., $m >> -1$) and at infinity.

6. Object is at F and optic centre.

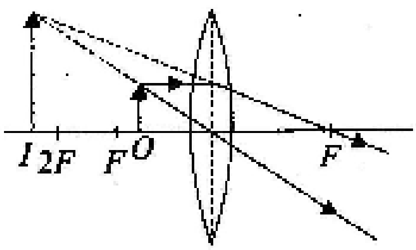


Image is virtual, erect, magnified (i.e., $m > +1$) and at between ∞ and $2F$ and on same side of object.

For Concave Lens

1. **Object is at infinity:**

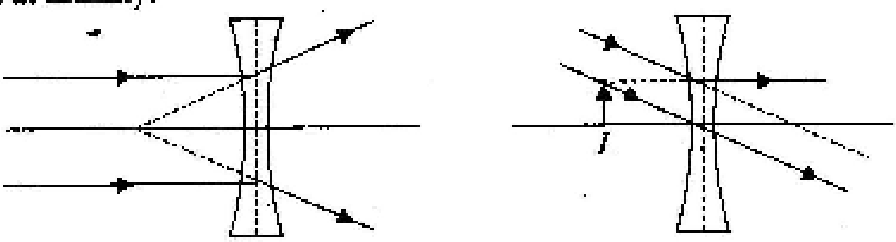


Image is virtual, erect, diminished ($m << +1$) and at F.

2.

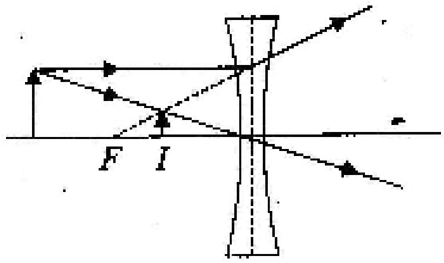


Image is virtual, erect diminished ($m < +1$) and between F and optical centre.

20.6.3 Concept of Velocity of Image in the Refraction Through Spherical Surface and Plane Surface

The formula of spherical surface is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

On differentiating the above equation w.r.t. time, we get

$$\mu_2\left(-\frac{1}{v^2}\right)\frac{dv}{dt} - \mu_1\left(-\frac{1}{u^2}\right)\frac{du}{dt} = 0 \quad \Rightarrow \quad \frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\left(\frac{\mu_1}{\mu_2}\right)\frac{du}{dt}$$

$$\Rightarrow \quad V_{IP} = \left(\frac{v^2}{u^2}\right)\left(\frac{\mu_1}{\mu_2}\right) = V_{OP} \qquad (1)$$

where V_{IP} = Velocity of image w.r.t. pole of refracting surface

V_{OP} = Velocity of object w.r.t. pole of refracting surface

For the plane surface, $R = \infty$

$$\therefore \quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = 0 \Rightarrow \frac{v}{u} = \frac{\mu_2}{\mu_1} \qquad (2)$$

From equation (1) and (2), we get

$$V_{IP} = \left(\frac{\mu_2}{\mu_1}\right)^2\left(\frac{\mu_1}{\mu_2}\right)(V_{OP}) = V_{IP} = \left(\frac{\mu_2}{\mu_1}\right)(V_{OP})$$

20.6.4 Concept of Velocity of Image in the Refraction Through Lens

Velocity of image of moving object (in lens)

There are two components of velocity of image

1. Component along the principal axis that means perpendicular to the lens.
2. Component perpendicular to the principal axis that means parallel to the lens.

Velocity component along the principal (Optical) axis:

Differentiating lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ on the both side w.r.t. time, we get

$$-\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0 \text{ (Since focal length of the lens remains constant)}$$

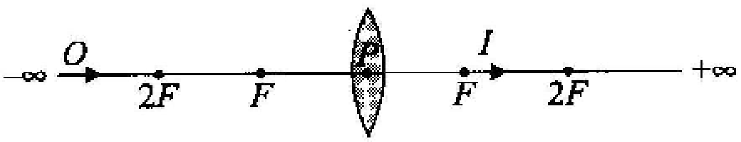
$$\Rightarrow \quad \frac{dv}{dt}=+\frac{v^2}{u^2}\frac{du}{dt} \Rightarrow (V_{IL})_{||}=(V_{OL})_{||}$$

where $(V_{IL})_{||}$ = Velocity of image w.r.t. lens along the principal axis

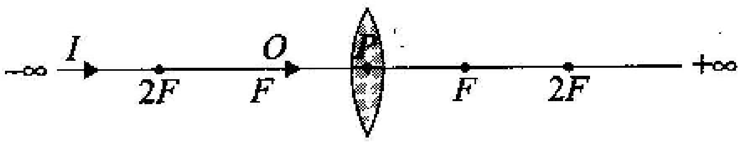
$(V_{OL})_{||}$ = Velocity of object w.r.t. lens along the principal axis

Direction of velocity of object and image w.r.t. lens is same to each other.

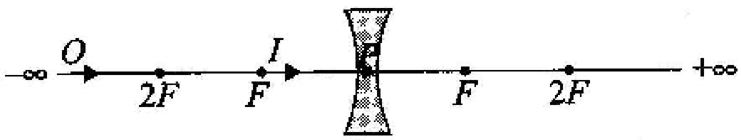
When object is moving from $-\infty$ to $2F$, the image is coming from F to $2F$ as shown in the figure.



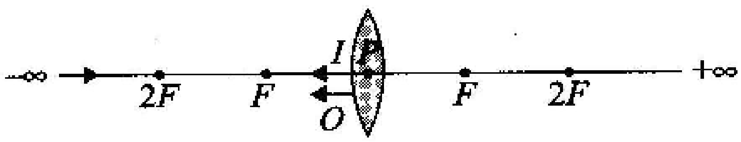
When the object shifts from F to P. The image formed moves from$(-\infty)$ to P as shown in figure.



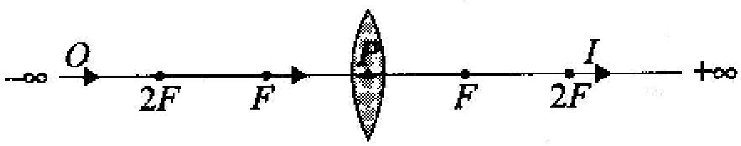
When the object shifts from $(-\infty)$ to P, the image moves from F to P, as shown in figure.



When the object shifts from P to F, the image moves from P to $(-\infty)$, as shown in figure.



When the object shifts from $2F$ to F, the image moves from $2F$ to $(+\infty)$, as shown in figure.



Velocity component perpendicular to axis:

$$m=\frac{h_i}{h_o} \Rightarrow h_i=mh_o$$

Differentiating this equation on the both side w.r.t. time, we get

$$\frac{dh_i}{dt}=\frac{m\,dh_o}{dt}+h_o\frac{dm}{dt};\ (V_{IL})_\perp=m(V_{OL})_\perp+h_o\frac{dm}{dt}$$

where $(V_{IL})_\perp$ = Velocity of image w.r.t. lens perpendicular to the principal axis

$(V_{OL})_\perp$ = Velocity of object w.r.t. lens perpendicular to the principal axis

$\frac{dm}{dt}$ = Rate of change of magnification (its unit is per sec)

As $m=+\frac{v}{u}$, by differentiating this equation on the both sides w.r.t. time, we get

$$\frac{dm}{dt}=\left[\frac{u(V_{IL})_{||}-v(V_{OL})_{||}}{u^2}\right]$$

20.7 POWER OF THE LENS

$P=\frac{1}{f}$ (dioptre) if f in metre

f is +ve for convex and –ve for concave

20.7.1 Combinations of the Lenses

1. For a system of lenses, the net power of the system will be

$$P_{equivalent}=P_1+P_2+P_3+\ldots$$

(provided all the thin lenses are in close contact)

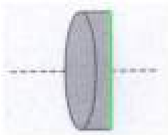
Hence, the focal length of the net system will be

$$\frac{1}{f_{equivalent}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots$$

($f_{equivalent}$ should be taken with proper sign).

When the image of an object formed by several lenses is made by a single lens at the some place and of the same magnification then this single lens is said to be an equivalent lens to the several lenses.

2. When a convex and a concave lens of equal focal length are placed in contact, the equivalent focal length is equal to infinite. Therefore the power becomes zero.



3. If the lenses are kept at a separation d, then the effective focal length f is given as

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$$

$$P=P_1+P_2-P_1P_2d$$

4. The overall magnification M of the system is given as the product of individual magnification m;

$$M = m_1\, m_2\, m_3 \ldots$$

5. Deviation of a ray due to a thin lens

$$\delta = \frac{h}{f}$$

As f is constant, δ depends on the height of the point from where the resultant deviation of a ray takes place.

6. When a lens (convex lens, say) is kept in any medium other than air of R.I. μ_m, the focal length f is

$$\frac{1}{f} = \left(\frac{\mu}{\mu_m} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(a) If $\mu_m < \mu$, then $f > f_o$ and $P < P_o$

where f_o = Focal length of lens in air

P_o = Power of lens in air

i.e., focal length increases, power decreases; but nature of the lens remains unchanged as f is still +ve. i.e., it remains converging.

(b) If $\mu_m = \mu$, then $f = \infty$ and $P = 0$ i.e., and lens behaves like a glass slab (plate).

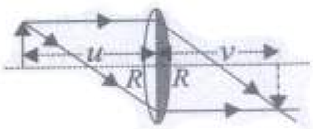
(c) If $\mu_m > \mu$, then $f < f_o$ and $P > P_o$ i.e., and focal length decreases and power increases numerically.

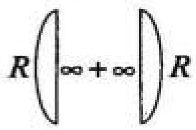
$\mu_m > \mu \quad \therefore (\mu - \mu_m) < 0$

$\Rightarrow \quad \frac{f}{f_0} < 0 \Rightarrow f$ is –ve i.e., nature of the lens changes from converging to diverging and vice-versa.

7. If a lens is dissected

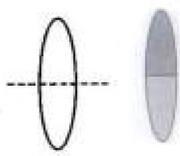
(a)



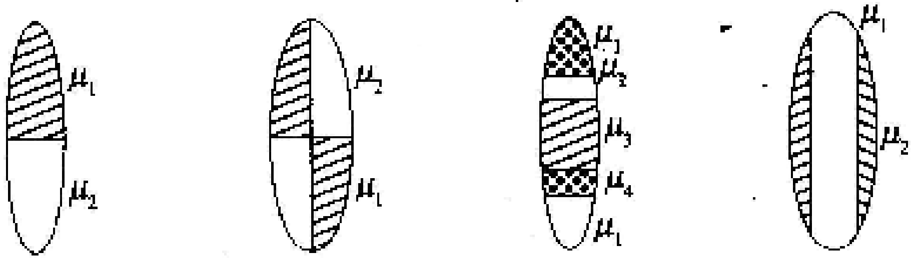


When a biconvex lens is cut transversely into two equal halves, the radius of curvature of the lens in the cutting side increases to ∞. Now the focal length f increases to twice the previous value and the size of image increases four folds. The brightness of images reduced (because size of image increases but the amount of light forming the image remains the same.)

(b) When a lens is cut into two equal halves parallel to principal axis, the focal length of each part remains constant and hence v and m remain unaffected (but less light gets refracted by lens and hence brightness of image is reduced.)



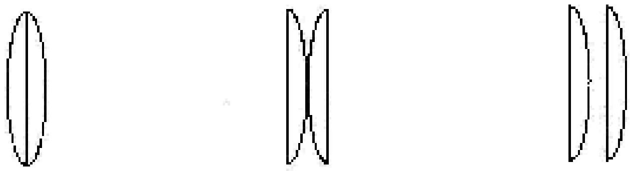
8. A convex lens is made of different material as shown in the figure below. An object is placed in front of it.



(a) (b) (c) (d)

No. of images = 2 No. of images = 1 No. of images = 4 No. of images = 1

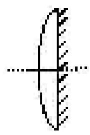
9. Two plano-convex lenses made of the same material and of the same curvature are arranged as shown in the figure.



(a) (b) (c)

The relative magnitude of the resultant focal lengths of the lenses as arranged are $f_a : f_b : f_c = 1 : 1 : 1$.

10. (a) Figure shows a plane-convex lens, with its plane surface silvered in figure



Let the effective focal length be F; $\dfrac{1}{F} = \dfrac{2}{f} + \dfrac{1}{f_m}$

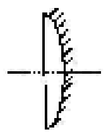
where f is the focal length of the convex surface.

A light ray, entering the lens undergoes refraction twice (so two is put) before coming out of the lens after reflection from the plane surface, f_m is the focal length of the plane mirror $f_m = \infty$.

$\dfrac{1}{F} = \dfrac{2}{f}$. But $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right) = \dfrac{n-1}{R}$

$$\therefore \quad F = \frac{f}{2} = \frac{R}{2(n-1)}$$

(b) When the curved surface is silvered



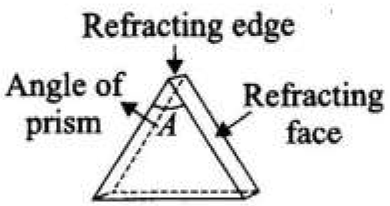
$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m}$$

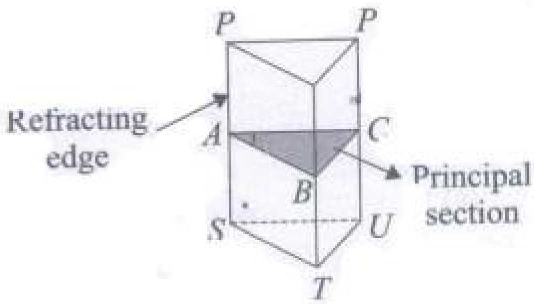
In this case $f_m = \dfrac{R}{2}; \dfrac{1}{f} = \dfrac{n-1}{R}$

$$\frac{1}{F} = \frac{2(n-1)}{R} + \frac{2}{R} = \frac{2n}{R}, \quad F = \frac{R}{2n}$$

20.8 PRISM

A transparent medium, bound by two or three plane surfaces, inclined to each other is called a prism.





1. Prism formula is given as $\mu = \dfrac{\sin[(A+\delta_m)/2]}{\sin(A/2)}$

   where δ_m is the minimum deviation and A is the angle of prism.
2. Dispersion of light is the phenomenon of splitting of white light into its constituent colours on passing through a prism. This is because different colours have different wavelengths.
3. Angular dispersion = $\delta_v - \delta_r = (\mu_v - \mu_r)A$, where μ_v and μ_r represents refractive index for violet and red lights.
4. Dispersive power, $\omega = \dfrac{\mu_v - \mu_r}{\mu - 1}$, where $\mu = \dfrac{\mu_v + \mu_r}{2}$ is the mean refractive index.

20.9 DEFECTS OF VISION OF HUMAN EYE

1. **Myopia or shortsightedness:** Myopia is that defect of the eye due to which it can see distinctly objects lying near it but cannot do so for those objects which are lying beyond a certain distance.

   Myopia my be due to (a) the elongation of the eye ball and (b) decrease in the focal length of the eye-lens.

   To correct myopia, a concave lens, whose focal length is equal to the distance of the far point from the eye, should be placed in front of the eye.

   For myopic eye, $f = -x$

   where f is the focal length of the concave lens used to correct a myopic eye whose far point lies at a distance x from it.
2. **Hypermetropia or hyperopia or longsightedness:** Hypermetropia is that defect of the eye due to which it can see far off objects distinctly but cannot do so for those objects which are lying nearer than a certain distance.

   Hypermetropia may be due to (a) the contraction of the eye-ball and (b) an increase in the focal length of the eye-lens.

To correct hypermetropia, a lens (convex) of such a focal length is placed in front of the eye that the rays starting from distance D (least distance of distinct vision) should appear to come from point N (near point).

- For hypermetropic eye, $\frac{1}{f} = \frac{1}{D} - \frac{1}{y}$

where f is the focal length of the convex lens used to correct a hypermetropic eye whose near point lies at a distance y instead of distance D (distinct vision distance), $y > D$.

3. **Presbyopia:** As age advances, the power of accommodation gradually decreases. It is due to this reason that an old person, while reading, is found holding the book farther away from the eyes. This defect is remedied by using low-power convex lenses. When accommodation becomes very much limited, bifocal lenses are used.
4. **Astigmatism:** When rays passing through an eye in mutually perpendicular planes (say the horizontal and the vertical) cannot be brought to a focus on the retina simultaneously, the defect is known as astigmatism.

   This defect is corrected by using plane-cylindrical, sphere-cylindrical or toric lenses.

- Angular magnification, $M = \frac{\beta}{\alpha}$

where β and α are the visual angles subtended by the final image formed by an optical instrument and the object (at the unaided eye) respectively.

20.9.1 Simple Microscope

A simple microscope consists of a convex lens of small focal length i.e., large power.

1. $M = 1 + D/f$

   where M is the magnifying power of a simple microscope (magnifying glass) of focal length f and D (about 25 cm.) is the distance of distinct vision where image is formed.
2. When the image is formed at infinity, $M = D/f$

20.9.2 Compound Microscope

A compound microscope consists of an objective and an eyepiece.

The objective is a convex lens of small focal length f_o.

The eye-piece is a convex lens of small focal length f_e but $f_o < f_e$

1. Distinct vision adjustment, $M = \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$

   where M is the magnifying power of a compound microscope, L is the length of the tube and f_o, f_e are the focal lengths of the objective and the eyepiece, respectively.
2. Normal adjustment, final image is at infinity, $M = \frac{LD}{f_o f_e}$

3. Distinct vision adjustment, $M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

4. Normal adjustment, $M = \frac{v_o D}{u_o f_e}$

5. The object is placed very near the objective but beyond the focus of objective. The objective forms a real, enlarged and inverted image of the object. This image serves as the virtual object for eyepiece of microscope. Final image is enlarged and inverted with respect to the object viewed by microscope.

   - Resolving power of a microscope is given by

$$\text{R.P.} = \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$$

   where d is the minimum distance between two point objects which can just be resolved, λ is the wavelength of light used, μ is the refractive index of the medium between the object and lens and θ is the half angle of the cone of light from the point object on the objective lens.

20.9.3 Astronomical Telescope

1. Distinct vision adjustment, $M = f_o / f_e + f_o / D$

   The object lies at infinity. The final image is formed at least distance of distinct vision (D).

2. Normal adjustment, $M = f_o/f_e$

   where M is the magnifying power of an astronomical telescope.

   The object lies at infinity. The final image is also formed at infinity.

   Tube length $= f_o + f_e$.

20.9.4 Terrestrial Telescope

Magnifying power of terrestrial telescope, $M = -f_o/f_e$

Resolving Power of Telescope

1. The ability of an optical instrument to produce separate diffraction patterns of two nearby objects is known as resolving power.

2. The reciprocal of resolving power is defined as the limit of resolution.

3. For telescope, the limit of resolution $(d\theta) = (1.22\lambda)/a$, and

   resolving power $= 1/\theta$ or resolving power $= \frac{a}{1.22\lambda}$.

4. $d\theta \propto \lambda$ (wavelength of light used)

   $d\theta \propto \frac{1}{a}$ (a = diameter of aperture of objective).

ATOMS AND NUCLEI

21.1 ATOMS

Atoms were unseen hypothetical entities and were the dreams of philosophers for thousands of years.

Today, we can produce images of individual atoms by using a scanning tunnelling electron microscope. Thus, atoms which were once only a philosopher's dream, have become a physical reality. The fact that all matter is composed of atoms is perhaps the single most important piece of scientific knowledge we possess.

21.1.1 Dalton's Atomic Theory

John Dalton, in 1808, put forward his theory, according to which

1. All chemical elements are composed of tiny particles, called atoms. These particles cannot be subdivided further.
2. Atoms of a particular element resemble one another whereas atoms of different elements differ from one another.

21.1.2 Thomson's Atomic Model

Postulated in 1907 by J. J. Thomson.

1. Atoms consists of positively charged protons and negatively charged electrons.
2. An atom could be divided into its constituent elementary particles.
3. Atom is neutral.

Thomson's model could not explain:

1. The emission of spectral lines from the atoms.
2. The large angle scattering of α-particles by thin metal foils.

21.1.3 Rutherford's Atomic Model

Postulated in 1911

1. The whole positive charge of the atom and almost its entire mass is concentrated in small region of the atom. Rutherford named this region as the nucleus.

2. The nucleus is surrounded by electrons. As the atom on the whole is neutral, the total negative charge of the electrons is equal to the positive charge on the nucleus.
3. Using high speed α-particle, the nuclear diameter has been found to be of the order of 10^{-14} m.
4. For the stability of the atom, Rutherford assumed that the electrons are revolving at high speeds around the nucleus in closed circular orbits so that the force of attraction between the nucleus and the electrons is balanced by the centrifugal force acting on the electrons.

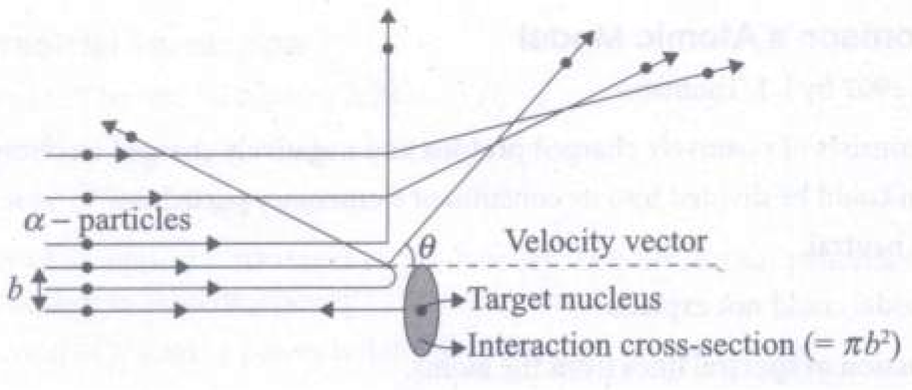
   (If the electrons in an atom were stationary, these would fall into the nucleus due to the electrostatic force of attraction between the electrons and the nucleus.)
5. The existence of sufficient empty space within the atom explains why most of the α-particles go undeflected. Small angle of scattering is accounted for by the fact that the nucleus occupies only a fraction of the total volume of the atom.

21.1.4 Impact Parameter and Angle of Scattering

When α-particle is scattered through 180° it reflected back along its initial path. It is under this condition that the particle has the closest approach to the nucleus. At the distance r_o of closest approach, the kinetic energy of the α-particle is converted into the potential energy of the system.

$$K = U \qquad \text{or} \qquad K = k_e\frac{Ze(2e)}{r_o} \qquad \text{or} \qquad r_o = k_e\frac{2Ze^2}{K}$$

where $k_e = \dfrac{1}{4\pi\varepsilon_o} = 9\times10^9\ \text{Nm}^2/\text{C}^2$.



The trajectory traced by an α-particle depends upon the impact parameter of collision.

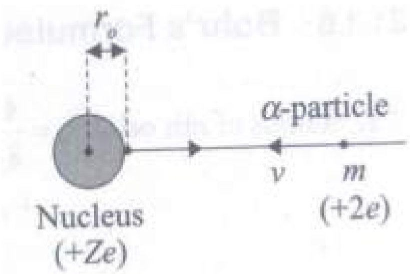
Impact parameter is defined as the perpendicular distance of the velocity vector of α-particle from the centre of the nucleus (when α-particle is far away from the atom).

It is denoted by b as shown in the figure.

Angle of scattering is defined as the angle between the direction of approach and the direction of recede of the α-particle.

It is denoted by θ. Rutherford calculated that

$$\cot\left(\frac{\theta}{2}\right)=\frac{2b}{r_o}$$



where $r_o = k_e \dfrac{2Ze^2}{K}$.

$$\text{Thus, } \cot\left(\frac{\theta}{2}\right)=\frac{2b}{k_e\dfrac{2Ze^2}{K}}=\frac{4\pi\varepsilon_o K}{Ze^2}b$$

For a given nucleus (constant Ze) and an α-particle of given energy K, $\cot(\theta/2) \propto b$. As such a graph between b and $\cot(\theta/2)$ is a straight line. Thus, it is clear that θ increases as b decreases. It implies that only an α-particle passing close to the nucleus, suffers large angle deflection.

- Number of α-particles scattered through angle θ is given by

$$N(\theta) \propto \frac{Z^2}{\sin^4(\theta/2)K^2}$$

where K is the kinetic energy of the α-particle and Z is the atomic number of the metal.

- The diameter of the nucleus is of the order of 10^{-14} m.

21.1.5 Bohr's Atomic Model

In 1913, Niels Bohr (1885–1962) explained the hydrogen atom spectrum by applying the quantum theory of radiation to Rutherford's atomic model. Bohr's theory is based on the following three postulates.

Postulate I

There is a positively charged nucleus at the centre of the atom around which the electron revolves in circular orbits. The necessary centripetal force is provided by the Coulomb's force of attraction exerted by the positively charged nucleus on the negatively charged electron.

Postulate II

The electron moves in certain discrete (non-radiating) orbits, called the stationary orbits, for which the total angular momentum of the moving electron is an integral multiple of $h/2\pi$, h being the Planck's constant.

Postulate III

When the electron jumps from one stationary orbit of higher energy E_i to another stationary orbit of lower energy E_f, it radiates energy as a single photon of frequency v.

- If energy of the electron in nth and mth orbits be K_n and E_m respectively, then, when the electron jumps from nth to mth orbit, the radiation frequency ν is emitted such that $E_n - E_m = h\nu$ is called Bohr's frequency equation.

21.1.6 Bohr's Formulae

1. Radius of nth orbit $r_n = \frac{4\pi\varepsilon_0 n^2 h^2}{4\pi^2 mZe^2}$, $r_n = \frac{0.529n^2}{Z}$ Å.

i.e.,
$$r \propto \frac{n^2}{Z} \propto \frac{1}{m}$$

2. Velocity of electron in the nth orbit $v_n = \frac{1}{4\pi\varepsilon_0}\frac{2\pi Ze^2}{nh} = \frac{2.2\times10^6 Z}{n}$ m/s

i.e.,
$$v \propto \frac{Z}{n} \text{ (Independent of } m\text{)}$$

3. The kinetic energy of the electron in the nth orbit

$$(K.E.)_n = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 me^4 Z^2}{n^2h^2} = \frac{13.6Z^2}{n^2} \text{ eV}$$

4. The potential energy of electron in nth orbit

$$U_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 me^4 Z^2}{n^2h^2} = \frac{-27.2Z^2}{n^2} \text{ eV}$$

5. Total energy of electron in nth orbit

$$E_n = U_n + (K.E.)_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 me^4 Z^2}{n^2h^2} = \frac{-13.6Z^2}{n^2} \text{ eV}$$

i.e.,
$$E_n \propto \frac{Z^2}{n^2} \propto m$$

$E_1 = -13.6$ eV and $v_1 = 2.2 \times 10^6$ ms^{-1}, and $r_1 = 0.529$ Å

6. If motion of nucleus is also to be considered, then their reduced mass $\frac{m_1 m_2}{m_1 + m_2}$ is taken.

7. Frequency of electron in nth orbit $\nu_n = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 Z^2 e^4 m}{n^3h^3}$

$$= \frac{6.62\times10^{15} Z^2}{n^3}$$

8. Wavelength of radiation in the transition from $n_2 \to n_1$ is given by

$$\frac{1}{\lambda} = Z^2 R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where R is called Rydberg's constant.

$$R = \left(\frac{1}{4\pi\varepsilon_o}\right)^2 \frac{2\pi^2 me^4}{ch^3} = 1.097 \times 10^7 \text{ m}^{-1}$$

9. Ionization energy $= \dfrac{13.6Z^2}{n^2}$ eV.

10. Ionization potential $= \dfrac{13.6Z^2}{n^2}$ volt.

11. When number of orbit n is increased, then potential energy, total energy, angular momentum and time are increased but speed, kinetic energy, angular frequency and frequency are decreased.

21.1.7 Hydrogen Spectrum

According to Bohr's postulate,

$$h\nu = E_i - E_f$$

It is called *Bohr's frequency condition.*

or $$h\nu = -\frac{me^4}{8\varepsilon_o^2 n_i^2 h^2} - \left(-\frac{me^4}{8\varepsilon_o^2 n_f^2 h^2}\right) = \frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \text{ as } E_n = -\frac{me^4}{8\varepsilon_o^2 n^2 h^2}$$

or $$\nu = \frac{me^4}{8\varepsilon_o^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

or $$\frac{c}{\lambda} = \frac{me^4}{8\varepsilon_o^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\left(\text{as } \nu = \frac{c}{\lambda}\right)$$

or $$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

or $$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{1}$$

where $R = \dfrac{me^4}{8\varepsilon_o^2 ch^3}$ is identified as the Rydberg constant.

From equation (1), it is clear that the radiations emitted by excited hydrogen atoms should consist of certain wavelengths only. Further, these wavelengths should fall into definite series depending upon the quantum number of the final energy level of the electron. As the initial quantum number n_i should always be greater than the final quantum number n_f, there will be an excess energy to be emitted as a photon. The first five series are as follows:

1. **Lyman series:** This series is emitted when $n_f = 1$ and $n_i = 2, 3,...$ etc.

   Thus, from equation (1), $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$.

   This series lies in the ultraviolet region and is invisible.

2. **Balmer series:** This series is obtained when $n_f = 2$ and $n_i = 3, 4,...$ etc.

   Thus, from equation (1), $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$

   This series lies in the visible region.

3. **Paschen series:** This series is obtained when $n_f = 3$ and $n_i = 4, 5,...$ etc.

   Thus, from equation (1), $\frac{1}{\lambda} = \left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$

   This series lies in the infrared region.

4. **Brackett series:** This series is obtained when $n_f = 4$ and $n_i = 5, 6,...$ etc.

   Thus, from equation (1), $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$

   This series lies in the far infrared region.

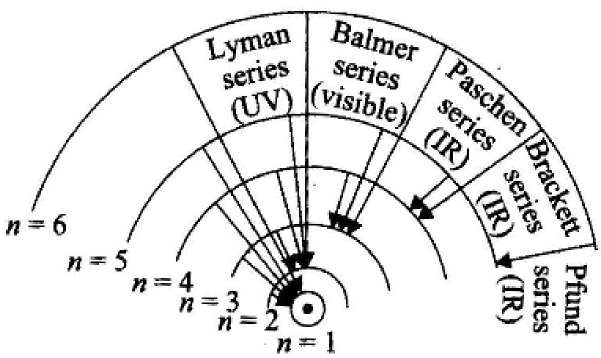
5. **Pfund series:** This series is emitted when $n_f = 5$ and $n_i = 6, 7, 8,...$ etc.

   Thus, from equation (1), $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$

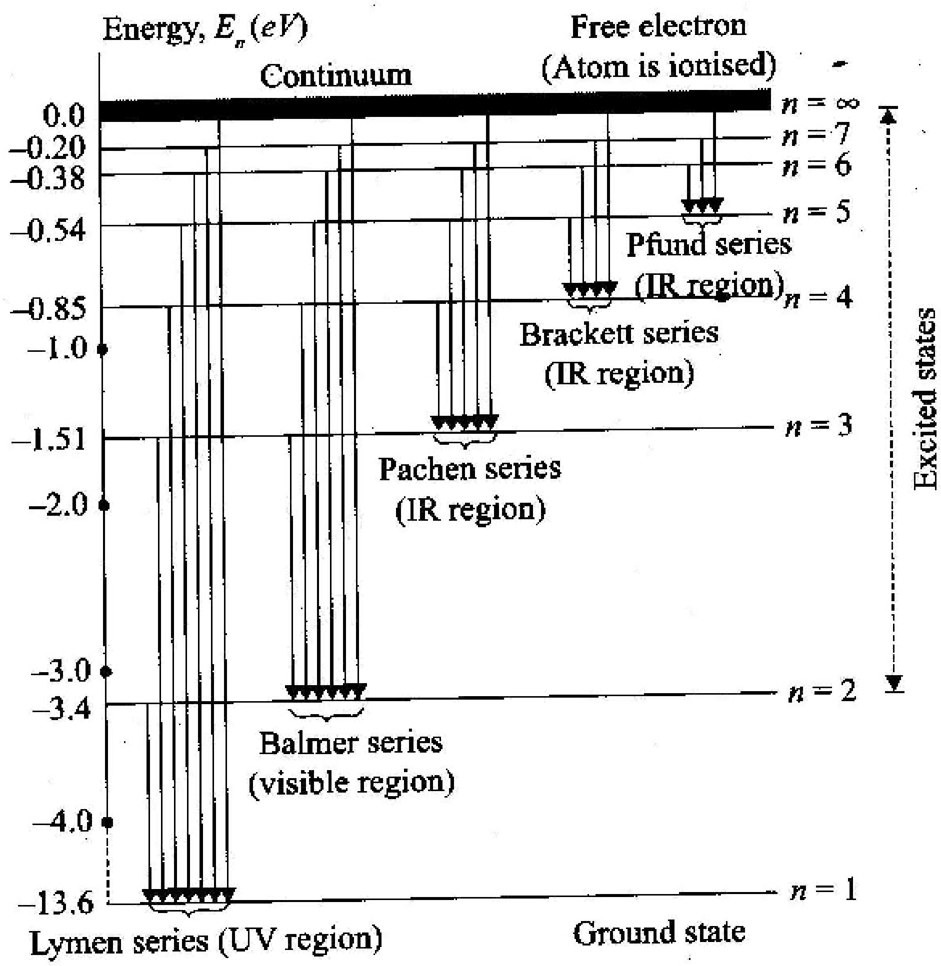
   This series also lies in the far infrared region.

   All these series have been shown in figure. This diagram is called the *Kossel diagram*.

21.1.8 Kossel Diagram



21.1.9 Energy Level Diagram of Hydrogen Atom



We know that $E = -\dfrac{me^4}{8\varepsilon_o^2 n^2 h^2} = -\left(\dfrac{me^4}{8\varepsilon_o^2 h^2}\right)\dfrac{1}{n^2}$.

Substituting, $m = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C,

$$\varepsilon_o = \frac{8.85 \times 10^{-12} C^2}{Nm^2}$$

$h = 6.626 \times 10^{-34}$ Js, we get

$$E = -\frac{2.17 \times 10^{-18}}{n^2} J = -\frac{\left(\dfrac{2.17 \times 10^{-18}}{1.60 \times 10^{-19}}\right)}{n^2} eV \text{ i.e., } E = -\frac{13.6}{n^2} eV \text{ (where } n = 1, 2, 3, \ldots)$$

In case $n = 1$, $E_1 = -13.6$ eV

E_1 is called the ground state of the atom.
In case $n = 2$, $n = 3$, $n = 4$, etc.,

$$E_2 = -\frac{13.6}{2^2} eV = -3.40 \text{ eV}$$

$$E_3 = -\frac{13.6}{3^2}\text{eV} = -1.51\,\text{eV}$$

$$E_4 = -\frac{13.6}{4^2}\text{eV} = -0.85\,\text{eV}$$

E_2, E_3, E_4, etc., are called the excited states of the atom.

When $n = \infty$, $E_\infty = 0$ and the electron is no longer bound to the nucleus.

E_1, E_2, E_3, E_4... etc. are also called the energy levels of the hydrogen atom. These energy levels have been plotted in figure and the various series have also been shown. Such a diagram is called the energy level diagram.

Vertical arrows drawn between the energy levels represent electronic transitions from one energy level to another. A downward transition corresponds to emission of light. The energy of the emitted photon is given by the energy difference between the levels.

Comparative Study of Spectral Series

1. As the order of spectral series increases, the wavelengths of lines increase.

$$\lambda_{\text{Pfund}} > \lambda_{\text{Brackett}} > \lambda_{\text{Paschen}} > \lambda_{\text{Balmer}} > \lambda_{\text{Lyman}}$$

2. The maximum number of spectral lines obtained due to transition of electrons present in nth orbit is

$$N = \frac{n(n-1)}{2}$$

21.1.10 Wave Model

1. It is based on wave mechanics.
2. It proposed that electrons do not move in a definite orbit and that the location of the electrons is based on how much energy each electron contains.
3. Quantum numbers are the numbers required to completely specify the state of the electrons.
4. In the presence of strong magnetic field, the four quantum numbers are
   (a) Principal quantum number (n) can have values 1, 2, ... ∞.
   (b) Orbital angular momentum quantum number (l) can have values 0, 1, 2, ... ($n-1$).
   (c) Magnetic quantum number (m_l) which can have values $-l$ to $+l$.
   (d) Magnetic spin angular momentum quantum number (m_s) which can have only two values $\pm 1/2$.

21.1.11 Work Function

The minimum energy that must be supplied to liberate the most weakly bound surface electrons from a metal without giving them any velocity is called the work function of the metal.

1. Work function is measured in electron volt (eV) where 1 eV = 1 $e \times 1$ V = $(1.6 \times 10^{-19}$ C) (1V) = 1.6×10^{-19} J.

2. The work function depends on the properties of the metal and the nature of its surface.
3. The work function is the highest for platinum and the lowest for caesium.

21.1.12 Electron Emission

Electron emission can take place by any of the following physical processes.

1. **Thermionic emission:** The release of electrons from a metal as a result of its temperature, *i.e.*, by heating is called thermionic emission.
2. **Field emission:** It is a kind of electron emission in which a very strong electric field pulls the electrons out of the metal surface.
3. **Photoelectric emission:** It is that kind of electron emission in which light of suitable frequency ejects the electrons from a metal surface.

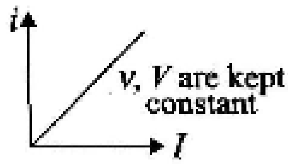
21.1.13 Photoelectric Effect

The phenomenon of emission of electrons by a good number of substances, mainly metals, when light of suitable wavelength falls on them is called the photoelectric effect.

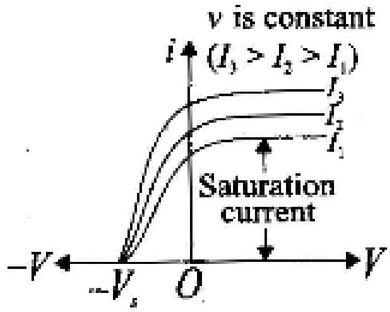
1. Alkali metals like lithium, sodium, potassium, rubidium and caesium show photoelectric effect with visible light whereas zinc and cadmium are sensitive only to UV radiation.
2. Non-metals also show photoelectric effect but only with short wavelengths.
3. Liquids and gases also show this phenomenon.

Laws of Photoelectric Effect

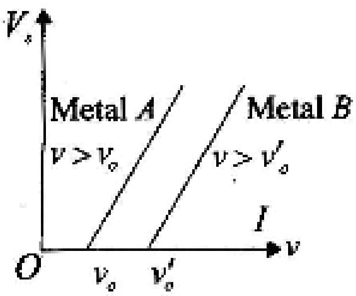
1. For a given photosensitive material and frequency of incident radiation (above the threshold frequency), the photoelectric current is directly proportional to the intensity of light.



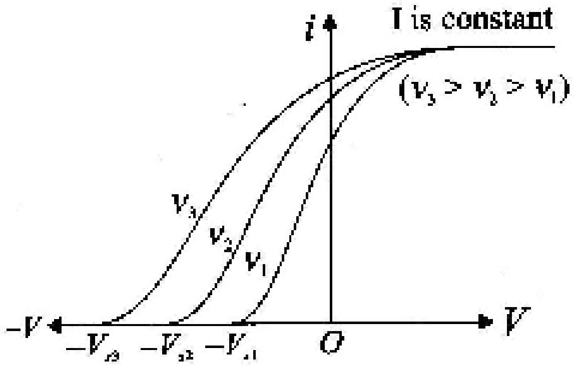
2. For a given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of radiation whereas the stopping potential is independent of intensity.

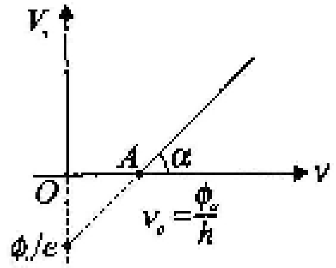


3. For a given photosensitive material, there exists a certain minimum cut-off frequency, called the threshold frequency, below which no emission of photoelectrons takes place, no matter how intense the light is. Threshold frequency is different for different metals. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity.



4. The maximum kinetic energy of the photoelectrons or stopping potential varies linearly with the frequency of incident radiation, but is independent of its intensity.





5. The photoelectric emission is an instantaneous process.

Einstein's Photoelectric Equation

A successful explanation of photoelectric effect was given by Einstein in 1905.

He assumed that light (or any em wave) of frequency ν propagating in vacuum can be considered a stream of photons. Each photon travels at speed c and has energy $h\nu$. When a photon interacts with an electron, it gives its entire energy to the electron and then exists no longer. The chance of two photons hitting the same electron simultaneously is practically zero as there is one-to-one interaction between the photons and the electrons. The energy of the photon is spent in two ways:

1. In liberating the electron from the metal surface by providing an energy ϕ_0 to it.
2. In imparting it kinetic energy, K_{max} being the maximum value of this kinetic energy.

The maximum kinetic energy of photoelectron is given by:

$$K_{max} = h\nu - \phi_0$$

where $h\nu$ is the energy of the incident photon, ϕ_0 is the work function.

The stopping potential is directly related to the maximum kinetic energy of the electrons emitted as

$$eV_s = \frac{1}{2}mv_{max}^2 = K_{max}$$

21.1.14 Properties of Photon

1. A photon has energy $h\nu$ and momentum $h\nu/c$ and it travels in vacuum with a speed of light c.
2. A photon has no rest mass.
3. Whatever the intensity of radiation may be, all photons of light of a particular frequency ν have the same energy $h\nu$ and momentum $h\nu/c$ $(= h/\lambda)$.
4. The energy of a photon is independent of intensity of radiation. With increase in intensity of light, only the number of photons crossing a given area per second increases.

5. Photons are electrically neutral and are not deflected by electric and magnetic fields.
6. In a photon-particle collision, the total energy and total momentum are conserved.
7. The number of photons in a collision may not be conserved. A photon may be absorbed (photoelectric effect) or a new photon may be created (Compton's effect).

Compton's Effect

When radiation of short wavelength (like X-rays) is incident on target of electron, the wavelength of scattered X-rays becomes longer than the wavelength of incident X-rays. This was first studied by Compton and therefore is known as Compton's effect.

According to him if λ and λ' are the wavelength of incident and scattered X-rays, then shift is wavelength $\Delta\lambda$ is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

where ϕ is the scattering angle.

$\Delta\lambda$ is known as Compton's shift and $\frac{h}{mc}$ is a constant, called the Compton wavelength. Compton shift depends only on the scattering angle. This phenomenon supported the view that both momentum and energy are transferred via photons.

21.2 MATTER WAVE OR DE BROGLIE WAVE OR WAVELENGTH

de Broglie introduced the idea that all moving material particles possess a wave character also.

The waves associated with moving material particles are called matter waves or de Broglie waves.

$$\text{de Broglie wavelength } \lambda = \frac{h}{P} = \frac{h}{mv} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}}$$

1. For electron, $m = 9.1 \times 10^{-31}$ kg, $q = 1.6 \times 10^{-19}$ C

$$\therefore \quad \lambda \text{ (in Å)} = \sqrt{\frac{150}{V\text{(in volt)}}}$$

2. For thermal neutron, $K = \frac{3}{2}kT$, $\quad \therefore \quad \lambda = \frac{h}{\sqrt{3mkT}}$

For neutron, $m = 1.67 \times 10^{-27}$ kg $\quad \therefore \quad \lambda = \frac{25.17}{\sqrt{T}}$ Å

21.3 X-RAYS

X-rays are produced when fast moving electrons accelerated by applying voltage strike a metal of high atomic number.

Molybdenum and tungsten provide suitable targets. These elements have large atomic number and high melting point for the purpose.

1. X-rays are electromagnetic waves of high energy, high frequency and low wavelength. Its wavelength is $\lambda \rightarrow 1\text{–}100$ Å and kinetic energy $= qV$ (J) $= V$(eV)
2. 98 per cent energy of electron is converted into heat, only 2 per cent is utilized in X-ray production.
3. Intensity of X-rays is proportional to filament current and penetration power of X-rays is proportional to potential difference between target and filament.
4. **Types:** (a) Continuous X-rays and characteristic X-rays.
   (b) Hard X-rays and soft X-rays
5. Hard X-rays have lower wavelength, high frequency, higher energy and greater penetration.

$$\text{Energy} \quad E = \frac{hc}{\lambda} = h\nu$$

6. Soft X-rays have greater wavelength (λ), lower frequency (ν), lower energy (E) and smaller penetration.

21.3.1 Mosley's Law

$\nu = a(Z - b)^2$ where a and b are constant and Z is atomic number of element. ν represents frequency of line.

Thus, $\nu \propto Z^2$ or $\sqrt{\nu} \propto Z$ for characteristic X-rays.

21.3.2 Isotopes

Isotopes are the atoms of the same element which have the same atomic number but different atomic masses.

1. Since chemical properties of an element are decided by its atomic number, therefore, all isotopes of an element possess identical chemical properties. However, their physical properties differ.
2. They occupy the same place in the periodic table of elements.
3. All the known elements have one or more isotopes. Mercury has as many as nine isotopes.
4. Examples of isotopes are:
   (a) Hydrogen (^1_1H, ^2_1H, ^3_1H): These are called protium or hydrogen (H), heavy hydrogen or deuterium (D) and tritium (T), respectively.
   (b) Helium (^3_2He, ^4_2He, ^5_2He).

(c) Carbon ($^{10}_{6}C$, $^{11}_{6}Ce$, $^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{6}C$)

(d) Oxygen ($^{15}_{8}O$, $^{16}_{8}O$, $^{17}_{8}O$, $^{18}_{8}O$, $^{19}_{8}O$)

21.3.3 Isobars

Isobars are the atoms of different elements which have the same atomic mass but different atomic numbers. They do not occupy the same place in periodic table of elements and differ widely in their chemical properties. (Isobar implies same weight).

For example, calcium ($^{40}_{20}Ca$) and argon ($^{40}_{18}Ar$) are isobars.

21.3.4 Isotones

Nuclei with the same number of neutrons but different number of protons are called isotones. For example, ($^{4}_{2}He$, $^{5}_{3}Li$) and ($^{13}_{6}C$, $^{14}_{7}N$).

21.3.5 Isomers

Isomers are the excited states of a stable nucleus. ($^{80}_{35}Br$) has two isomers, one with half-life of 18 minutes and the other with 4.4 hours.

21.3.6 Mass Defect (Δm)

Δm = Mass of (Neutron + Proton − Nucleus)

N = Number of neutrons in nucleus

Z = Number of protons in nucleus

M_n = Mass of one neutron

M_p = Mass of one proton

M_{ZA} = Mass of nucleus formed

Δm = Decrease in mass during the process of formation of nucleus

21.3.7 Binding Energy (ΔE)

1. ΔE = Energy obtained by converting Δm in energy.

   $$\Delta E = (\Delta m)c^2 = (NM_n + ZM_p - M_{ZA})c^2$$

2. Binding energy represents the stability of nucleus.
3. Energy equivalent of a nucleon = 931 MeV

   Energy due to 1 proton = Energy due to 1 neutron

   $$\Rightarrow \quad 1\text{ a.m.u} = (1.67 \times 10^{-27}) \times c^2 = \left(\frac{5}{3} \times 10^{-27}\right) \times (3 \times 10^8)^2 \text{ J}$$

$$\Rightarrow \qquad 1 \text{ a.m.u} = \frac{5\times 9\times 10^{-27}\times 10^{16}}{3} \text{ J}$$

$$\Rightarrow \qquad 1 \text{ a.m.u} = \frac{15\times 9\times 10^{11}}{1.6\times 10^{-19}} = \text{eV} = \frac{15}{16}\times 10^{9}\text{ eV} = 931 \text{ MeV}$$

$$\Rightarrow \qquad 1 \text{ a.m.u} = 931 \text{ MeV.}$$

4. Binding energy per nucleon $= \dfrac{\Delta E}{A}$

21.3.8 Packing Fraction (*P*)

Packing fraction is the difference between the exact nuclear mass M of a nucleus and its mass number A, divided by the mass number.

$$P = \frac{M-A}{A}$$

21.3.9 Radioactivity

Radioactivity is the property by virtue of which the nucleus of a heavy element disintegrates itself with the emission of radiation without being forced by any external agent to do so.

1. The radioactive radiations are of three types: α-particles, β-particles and γ-rays. An α-particle carries two units of positive charge and four units of mass. A β-particle carries unit negative charge and has negligible mass. A γ-rays carries no charge and has zero rest mass.
2. β-particles are the same as He nuclei. β^- particles are electrons and β^+ are positrons. γ^- rays are electromagnetic radiations emitted by excited states of nuclei.
3. (a) **α-emmission:** ${}_zX^A \xrightarrow{\alpha\text{-emmission}} {}_{z-2}X^{A-4}$

   (b) **β-emmission:** $n \to P + e^- + \bar{\nu}$

   ($\bar{\nu} \to$ Antineutrino and it is neutral)
4. **Soddy and Rutherford law (Radioactive decay law):**

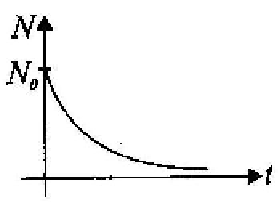
$$-\frac{dN}{dt} \propto N \Rightarrow -\frac{dN}{dt} = \lambda N,$$

where λ is decay constant or disintegration constant of radioactive element.

$\Rightarrow$ $N = N_o e^{-\lambda t}$, where N is remaining nucleus and N_o is number of radioactive nucleus initially.

Substituting $t = 1/\lambda$ in eqn.

$$N = N_o e^{-\lambda(1/\lambda)} = N_o e^{-1} = N_o/e$$



The disintegration constant is, therefore, the reciprocal of time during which the original number of radioactive nuclei (N_o) of a radioactive substance falls to N_o/e.

5. **Half-life period or half-life ($T_{1/2}$):** The time $T_{1/2}$ required for the disappearance of half of the amount of the radioactive substance originally present is called the half-life period or simply half-life.

The SI unit of $T_{1/2}$ is second (s).

The half-life period of radium is 1600 years. This means that 1600 years would elapse for 1/2 g of radium to disappear out of an original amount of 1 g.

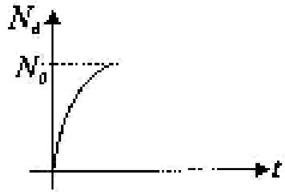
$$T_{1/2} = \frac{0.693}{\lambda}$$

6. **Average life or mean life τ:** The mean (average) life of radioactive nuclei is the average time for which the nuclei of a radioactive element exist.

Thus, average life, $\tau = \dfrac{\text{Total life time of all the nuclei}}{\text{Total number of nuclei}} = \dfrac{N_o/\lambda}{N_o} = \dfrac{1}{\lambda}$

The average life of an individual radioactive nucleus is the reciprocal of its disintegration constant *i.e.*, $\tau = 1.44\, T_{1/2}$.

7. **Number of nuclei decayed:**



$$N_d = N_o(1 - e^{-\lambda t})$$

where $\dfrac{1}{\lambda}$ = Time constant = Mean life or average life.

(a) At $t = \dfrac{1}{\lambda}$, $N = N_o e^{-\lambda t} = \dfrac{N_o}{e} =$ 36% of N_o

(b) At $t = \dfrac{1}{\lambda}$, $N_d = N_o(1 - e^{-\lambda t}) = \left(1 - \dfrac{1}{e}\right) N_o =$ 63% of N_o

8. **Decay rate or activity:** Rate of decay (R) of a radioactive substance is commonly known as its activity and it is determined by the number of nuclei that decay in a unit time or by the number of decays per second (decays/s) or disintegrations per second (dis/s or dps).

$$R = (\lambda N_o)e^{-\lambda t} \text{ or } R = R_o e^{-\lambda t}$$

where $R_o = \lambda N_o$ = Initial activity.

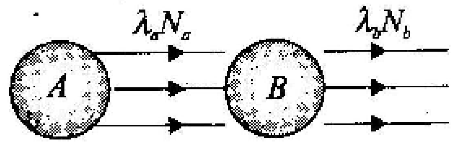
The units of activity

(a) 1 Curie = 1 Ci = 3.7×10^{10} dps

(b) 1 Rutherford = 1 rd = 10^6 dps

(c) 1 Becquerel = 1 Bq = 1 dps

9. **Probability:** (a) $P(\text{survived}) = e^{-\lambda t}$ (b) $P(\text{decay}) = (1 - e^{-\lambda t})$



10. Successive Radioactivity:

At $t = 0$, let N_o,

At $t = t$, $N_a = N_o e^{-\lambda_a t}$, $N_b = ?$

$$N_b = \frac{N_o \lambda_a}{\lambda_b - \lambda_a}(e^{-\lambda_a t} - e^{-\lambda_b t})$$

11. When heavy nucleus decays, then effective value of $\lambda = \lambda_1 + \lambda_2$.

If T be half life then $T = \frac{T_1 T_2}{T_1 + T_2}$

12. After every half life, fraction $= \left(\frac{1}{2}\right)^n$, % $\rightarrow \left(\frac{1}{2}\right)^n$, Nuclei $\rightarrow N_o\left(\frac{1}{2}\right)^n$

Decayed $= N_o - N_o\left(\frac{1}{2}\right)^n$ or decayed in ΔT is $\Delta N = \lambda N \Delta t$

21.4 NUCLEAR FISSION

1. In it, a heavy unstable nucleus breaks into two smaller parts. An energy of about 200 MeV is released and 3 neutrons are emitted when ${}_{92}U^{235}$ splits by the impact of a slow neutron.

$${}_{92}U^{235} + {}_0n^1 = {}_{92}U^{236} = {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,({}_0n^1) + 200\text{ MeV}$$

2. Atom bomb is based on nuclear fission.
3. About 99.9 per cent of energy is converted into heat. Rest is converted into kinetic energy of neutrons, γ-rays, light and product nuclei.
4. Nuclear reactor is the furnace in which energy is generated by controlled nuclear fission. Atomic reactors work on the basis of controlled chain reaction.
5. Controlled chain reaction is slow and needs only one neutron for further fission, on which control is possible. Uncontrolled chain reaction is fast and needs more than one neutron for further fission. Atom bomb is an example. It cannot be controlled once the fission starts.

21.5 NUCLEAR FUSION

1. In it, two or more than two lighter nuclei combined to form a heavy nucleus with liberation of energy constitute nuclear fusion.

$$4({}_1H^1) = {}_2He^4 + 2\,({}_{+1}e^0) + 2\nu + 27\text{ MeV}$$

Hydrogen bomb is based on fusion.

2. Temperature should be high about 10^7 K and pressure should be large about 10^6 atmosphere.

3. The kinetic energy of interacting nuclei must be greater than the Coulomb repulsive energy i.e. about 0.1 MeV.
4. Thermonuclear energy is the energy released during nuclear fusion.
5. Fusion reactors are better than fission reactors because harmful radiations are not produced in them.
6. Protons are needed for fusion while neutrons are needed for fission process.
7. Destruction caused by nuclear weapons on a mass scale is termed as nuclear holocaust.

22.1 INTRINSIC SEMICONDUCTOR

A pure semiconductor in which no impurity of any sort has been mixed, is called intrinsic semiconductor. Germanium (E_g = 0.72 eV) and silicon (E_g = 1.1 eV) are intrinsic semiconductors. In an intrinsic semiconductor the number of free electrons in conduction band n_e is exactly equal to the number of holes n_h in valence band. Thus, $n_e = n_h = n_i$ where n_i is called the number density of intrinsic carriers. At 0 K these behave as 100% insulators. But at any other temperature they have thermally generated charge carriers and thus behave as semiconductor. Conductivity of an intrinsic semiconductor is $\sigma = e(n_e\,\mu_e + n_h\,\mu_h)$, where n_e is free electron density, n_h is the hole density and μ_e and μ_h are their respective mobilities. Electrical conductivity of pure semiconductor is very small.

22.2 EXTRINSIC SEMICONDUCTOR

To prepare a n-type semiconductor a pentavalent impurity, eg., P, As, Sb is used as a dopant with Si or Ge. Such an impurity is called donor impurity because each dopant atom provides one free electron. In n-type semiconductor $n_e >> n_h$, i.e., electrons are majority charge carriers and the holes are minority charge carriers such that $n_e \times n_h = n_i^2$. A n-type semiconductor is electrically neutral and is not negatively charged.

To prepare a p-type semiconductor a trivalent impurity, eg., B, Al, In, Ga, etc. is used as a dopant with Si or Ge. Such an impurity is called acceptor impurity as each impurity atom wants to accept an electron from the crystal lattice. Thus, effectively each dopant atom provides a hole. In p-type semiconductor $n_h >> n_e$, ie, holes are majority charge carriers and electrons minority charge carriers such that

$$n_h \times n_e = n_i^2$$

A p-type semiconductor is electrically neutral and is not positively charged.

22.3 P-N JUNCTION

A p-n junction is obtained by joining a small p-type crystal with a n-type crystal without employing any other binding material in between them. Whenever a p-n junction is formed, electrons from n-region diffuse through the junction into p-region and the holes from p-region diffuse into n-region. As a result neutrality of both n and p-regions is disturbed and a thin layer of immobile negative charged ions appear near the junction in the p-crystal and a layer of positive ions appear near the junction in n-crystal. This layer containing immobile ions is called depletion layer. The thickness of depletion layer is approximately of the order of 10^{-6} m.

The potential difference created across the p-n junction due to diffusion of electrons and holes is called the potential barrier V_b (or emf of fictitious battery). For germanium diode barrier potential to 0.3 V but for Si diode its value is 0.7 V. The barrier electric field developed due to it is of the order of 10^5 Vm^{-1}.

22.4 HALF-WAVE RECTIFIER

In half-wave rectifier only one diode is used. In it no current flow takes place and no output signal is obtained.

Even during one half cycle the output obtained is a mixture of dc and ac.

$$\text{The ripple factor} = \frac{\text{Effective AC component of voltage}}{\text{Effective DC component of voltage}} = 1.21 \text{ or } 121\%$$

22.5 FULL-WAVE RECTIFIER

In full-wave rectifier two p-n junction diodes have been joined in complimentary modes. In this rectifier, we obtain a continuous unidirectional current through the load resistor R_L.

Ripple factor in full-wave rectifier $\dfrac{V_{AC}}{V_{DC}} = 0.48 = 48\%$

The average output in one cycle is

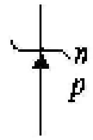
$$V_{dc} = \frac{2}{\pi}V_o \quad \Rightarrow \quad I_{dc} = \frac{2}{\pi}I_o$$

The ripple frequency for full-wave rectifier is twice that of ac input signal.

Table 22.1 Comparison Between Half- and Full-Wave Rectifier

Property	**Half-Wave Rectifier**	**Full-Wave Rectifier**
Average direct current, I_{DC}	I_o/π	$2I_o/\pi$
Average voltage, V_{AV}	V_o/π	$2V_o/\pi$
Ripple factor, r	1.21	0.48
	$\therefore I_{AC} > I_{DC}$	$\therefore I_{AC} < I_{DC}$
Efficiency, η	$\dfrac{0.406}{1+r_P/R_L} = 40\%$	$\dfrac{0.812}{1+r_P/R_L} = 80\%$
Form factor	1.57	1.11
Ripple frequency	ω	2ω
Pulse frequency	1/2 of input pulse frequency	input pulse frequency

22.6 DIODES



1. Zener diode is a highly doped *p-n* diode which is not damaged by high reverse current. It is always used in reverse bias in breakdown voltage region and is chiefly used as a voltage regulator.
2. Light emitting diode (LED) is a specially designed diode made of GaAsP, etc. When used in forward biased, it emits characteristic, almost monochromatic light.
3. Photo diode is a special diode used in reverse bias which conducts only when light of suitable wavelengths is incident on the junction of diode. The energy of incident light photon must be greater than the band gap of semiconductor.
4. Solar cell is a special *p-n* junction in which one of the semiconductors is made extremely thin so that solar radiation falling on it reaches junction of diode without any absorption. A solar cell directly converts solar energy into electrical energy.

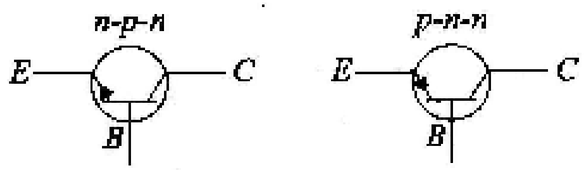
22.7 TRANSISTOR

A transistor is a combination of two *p-n* junctions joined in series. A junction transistor is known as bipolar junction transistor (BJT). Transistors are of two types: *n-p-n* and *p-n-p* transistor.

A transistor has three regions:

1. An emitter (*E*), which is most heavily doped and is of moderate size. It supplies large number of charge carriers, which are free electrons in a *n-p-n* transistor and holes in a *p-n-p* transistor.
2. A base (*B*), which is very lightly doped and is very thin (thickness $\approx 10^{-5}$ m).
3. A collector (*C*), which is moderately doped and is thickest.

A transistor is symbolically represented as shown in the figures.



(a) (b)

In a *n-p-n* transistor, electrons flow from emitter towards the base and constitute a current I_e. Due to larger reverse bias at base-collector junction, most of these electrons further pass into the collector, constituting a collector current I_c. But a small percentage of electrons (less than 5%) may combine with holes present in base. These electrons constitute a base current I_b. It is self evident that

$$I_e = I_c + I_b$$

22.8 TRANSISTOR CONFIGURATION

A transistor can be connected in either of the following three configurations:

1. Common emitter (CE) configuration
2. Common base (CB) configuration
3. Common collector (CC) configuration

1. In common emitter configuration, we obtain the values of the following parameters:

Input resistance $r_i = \left[\dfrac{\Delta V_{BE}}{\Delta I_B}\right]_{V_{CE}=\text{constant}}$

Input resistance $r_o = \left[\dfrac{\Delta V_{CE}}{\Delta I_C}\right]_{I_B=\text{constant}}$

AC current gain $\beta = \left[\dfrac{\Delta I_C}{\Delta I_B}\right]_{V_{CE}=\text{constant}}$

Transconductance $g_m = \dfrac{\Delta I_C}{\Delta V_{BE}} = \dfrac{\beta}{r_i}$

A transistor can be used as an amplifier. The voltage gain of an amplifier will be given by

$$A_V = \frac{V_o}{V_i} = \beta\frac{R_C}{R_B}$$

where R_C and R_B are net resistances in collector and base circuits respectively.

2. In common base configuration ac current gain is defined as

$$\alpha = \left[\frac{\Delta I_C}{\Delta I_E}\right]_{V_{CB}=\text{constant}}$$

Value of α is slightly less than 1. In fact $0.95 \le \alpha \le 1$.
Current gains α and β are correlated as

$$\beta = \frac{\alpha}{1-\alpha} \quad \text{or} \quad \alpha = \frac{\beta}{1+\beta}$$

Table 22.2

In CB	In CE	In CC
$i_B =$ Constant	$i_E =$ Constant	$i_C =$ Constant
As $i_E = i_B + i_C$	As $i_E = i_B + i_C$	As $i_E = i_B + i_C$
$\Rightarrow \Delta i_E = \Delta i_C$	$\Rightarrow \Delta i_B = -\Delta i_C$	$\Rightarrow \Delta i_E = \Delta i_B$
$\Rightarrow \alpha_{AC} = \dfrac{\Delta i_C}{\Delta i_E} = \alpha_{DC}$	$\Rightarrow A_R = \dfrac{R_o}{R_i} >> 1$	$\Rightarrow A_R = \dfrac{R_o}{R_i} >> 1$
$A_R = \dfrac{R_o}{R_i} = \dfrac{R_C}{R_E}$	$A_i = \dfrac{i_o}{i_i} = \dfrac{i_C}{i_B}$	$A_i = \dfrac{i_o}{i_i} = \dfrac{i_E}{i_B}$
$= \dfrac{R_{high}}{R_{low}} \approx 10^3$	$= \beta >> 1$	$= \gamma >> 1$
$A_i = \dfrac{i_o}{i_i} = \dfrac{i_C}{i_E} = \alpha = 1$	$A_V = A_iA_R = \beta A_R >> 1$	$A_V = \gamma A_R >> 1$
$A_V = A_iA_R = \alpha A_R >> 1$	$A_P = A_i^2A_R = \beta^2A_R >> 1$	$A_P = \gamma^2 A_R >> 1$
$A_P = A_i^2A_R \Rightarrow \alpha^2A_R >> 1$		

(Continued)

Table 22.2 (*Continued*)

In CB	In CE	In CC
i_E = Constant	i_B = Constant	i_C = Constant
Input and output signals are in phase ($\Delta\phi = 0$)	Input and output signals are out of phase ($\Delta\phi = \pi$)	Input and output signals are in phase
		$\gamma = \frac{i_E}{i_B} = \frac{i_E}{i_c} \times \frac{i_c}{i_A} = \frac{\beta}{\alpha}$ or $\gamma = 1 + \beta$

A logic gate is a digital electronic circuit which follows a logical relationship between its input and output. A logic gate may have one or more inputs but has only one output.

Logic gates follow Boolean algebra, which consists of three basic operations, namely AND($A \cdot B = Y$), OR ($A + B = Y$) and NOT ($\bar{A} = Y$).

22.9 LOGIC GATES

An electronic circuit which makes logic decision or binary decision between input and output signal is called *logic gate* i.e. a logic circuit or logic gate is a digital circuit that can implement Boolean algebraic equations.

There are 3 types of basic logic gates which are building blocks of logical circuit: OR gate, AND gate and NOT gate.

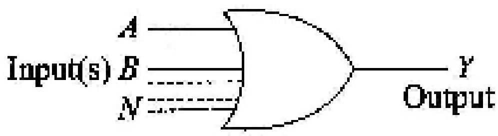
Each logic gate has its characteristic symbol and can be realised in practice using solid-state devices such as diode, transistors etc. provided the devices perform here in a nonlinear manner i.e. in a switching mode.

The working of a logic gate can be explained by either truth table or Boolean algebra. Truth table shows all possibilities of input and output.

22.9.1 OR Gate

An OR gate has two or more inputs but a single output.

1. **Symbol:**

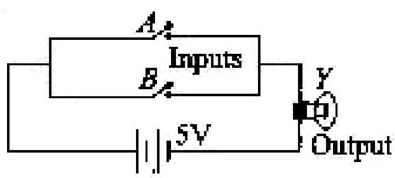


2. Truth Table for 2-input OR Gate

Inputs		Output
A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

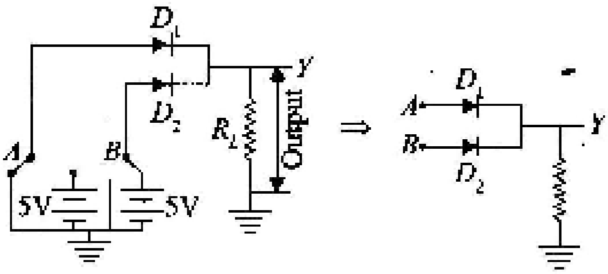
↓
Unique combination for Boolean algebra.

i.e. A + B = Y
i.e. A OR B = Y
" + " → OR symbol



OFF ⇒ 0 ON ⇒ 1

3. **Realization:** An OR gate can be realised by using diode, known as diode-logic (*DL*) system.



Case-I: When A - Low (0) and B - Low (0) i.e. D_1 and D_2 do not conduct. Hence, current through R_L is zero. i.e. output voltage is zero i.e. low i.e. $Y = 0$

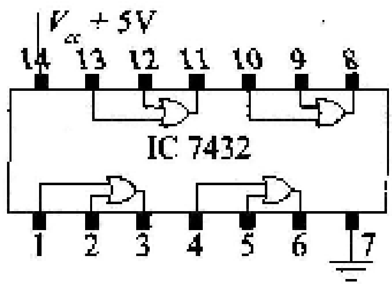
Case-II: When A - High (1) and B - Low (0) $Y = 0$ i.e. D_1 - Conducting and D_2 - Non-conducting Hence current flows through R_L i.e. $Y = 1$

Case-III: When A - Low (0) and B - High (1) i.e. D_1 - Non-conducting and D_2 - conducting; hence there is a current through R_L i.e. $Y = 1$.

Case-IV: When A - High (1) and B - High (1) i.e. D_1 and D_2 - Conducting; hence $Y = 1$.

4. The number of rows in a truth table equals 2^n, where n is the number of inputs.

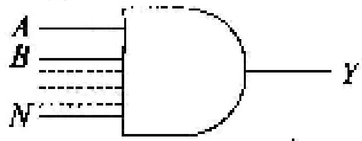
An OR gate can have as many inputs as desired; adding one diode for each additional inputs. No matter how many inputs, the action of any OR-gate is one or more high inputs produce a high output.



22.9.2 AND Gate

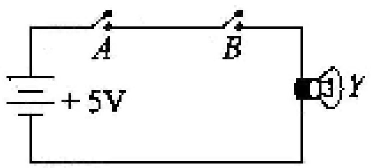
An AND gate possesses two or more inputs and a single output. An AND gate has a high output when all inputs are high.

1. **Symbol:**



2. Truth Table for 2-input AND Gate

Inputs		Output
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

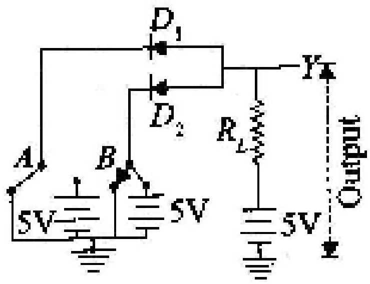


i.e. $Y = A \cdot B$

i.e. $Y = A$ AND B

"$\cdot$" – AND symbol

3. **Realisation:** Again, it has diode-logic (DL)



Case-I: A – Low and B – Low, i.e., the inputs are short-circuited to ground. The 5 V battery in the output side forward biases the D_1 and D_2. Hence, D_1 and D_2 – Conducting. The output is also shorted to ground through the diodes. Thus, output $Y = 0$.

Case-II: A – High; B – Low, i.e., D_1 – non-conducting but D_2 – Conducting and the output is short circuited to ground through this diode. Hence, $Y = 0$.

Case-III: A – Low; B – High, i.e., D_1 – Conducting and D_2 – non-conducting and the output is short-circuited to ground through the diode D_1. Hence, $Y = 0$.

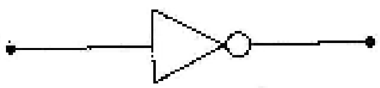
Case-IV: A – High; B – High, neither D_1 nor D_2 – Conducts. No current, therefore, flows through R_L and the output $Y = 1$.

Note

If in the truth table of a positive logic AND gate, 0' are replaced by 1' and vice-versa, we immediately get the truth table of a positive logic OR gate. Thus, a negative logic AND gate behaves as a positive logic OR gate, and vice-versa.

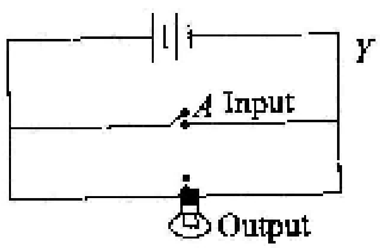
22.9.3 NOT Gate

The NOT circuit has a single input and a single output. The NOT gate inverts the sense of the output with regard to the input. Hence it is also called *inverter* and the NOT operation is also called as *Negation*.

1. **Symbol:** (Input) A  (B Output)

2. Truth Table

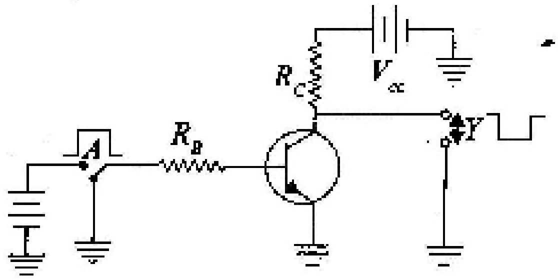
Inputs A	Output Y
0	1
1	0



i.e. $Y = \text{NOT A}$ if $A = 0$, $Y = \text{NOT } 0 = 1$

if $A = 1$, $Y = \text{NOT } 1 = 0$ i.e. $Y = \overline{A}$

3. **Realisation:** It can be used as a transistor i.e. transistor logic.



Case-I: When no signal is applied at the input i.e. $A = 0$ the transistor is cut-off, making the collector-current zero. Thus, potential drop across R_C is zero. The supply voltage of V_{CC} appears at the output terminal. Hence, output $Y = V_{CC} = 1$ (Yes).

Case-II: When a positive pulse is applied to A i.e. $A = 1$, the transistor conducts (fully ON) drawing maximum collector current. Hence, whole of V_{CC} drops across R_C and output $Y = 0$ (NO).

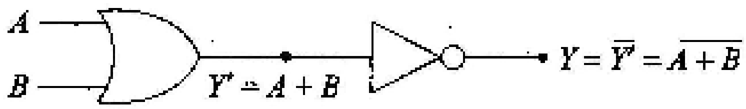
22.10 UNIVERSAL BUILDING BLOCKS

The three basic gates are naturally independent because by their repeated use, one gate cannot be obtained from the other. But by repeated use of 3 basic gates, two more fundamental gates can be obtained and hence these are called universal building blocks of digital electronics.

These fundamental gates are NAND and NOR gates.

All three gates can be obtained by any of the NAND or NOR gate.

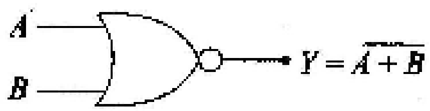
1. **NOR-gate:** OR gate + NOT gate ≡ NOR gate



i.e., Y equals NOT A OR B

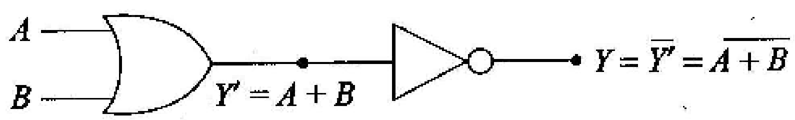
If the output of an OR gate is connected to input of a NOT gate, the resulting arrangement works as a NOR gate.

Symbol:

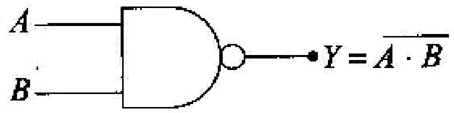


A	B	Y′ = A+B	Y = $\overline{Y'}$
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0

2. NAND-Gate: AND-gate + NOT-gate ≡ NAND-gate

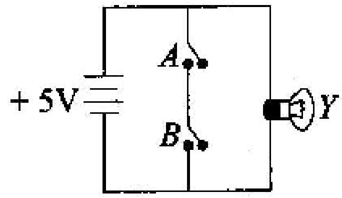


Logic symbol:

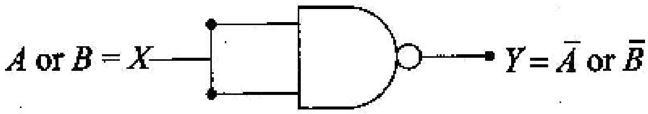


Truth Table

Input		Inter-output	Final output
A	B	$Y' = A \cdot B$	$Y = \overline{Y'} = \overline{A \cdot B}$
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

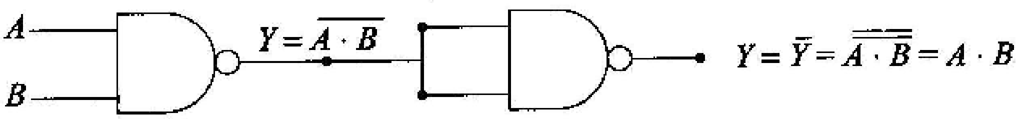


(a) *To obtain NOT-gate from NAND-gate*



NOT-gate can be obtained from NAND-gate by joining their both the inputs.

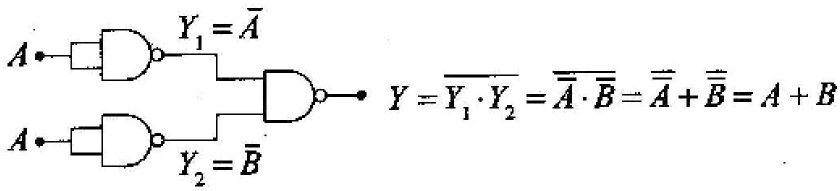
(b) *To obtain AND-gate from NAND-gate*



NAND gate　　　NOT gate obtained from NAND gate

If a output of NAND-gate is fed to the NOT-gate which is obtained from NAND-gate by joining their inputs. Resulting gate works as AND-gate.

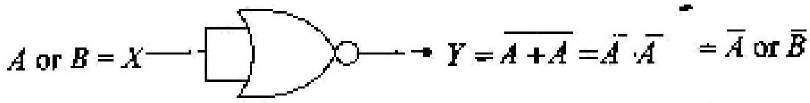
(c) *To obtain OR-gate from NAND-gate*

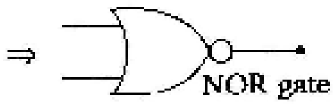


If the two outputs obtained from two NOT-gate (which is obtained from NAND-gate) are fed to input of a input of the NAND-gate. The resulting arrangement works as OR-gate.

Similarly:

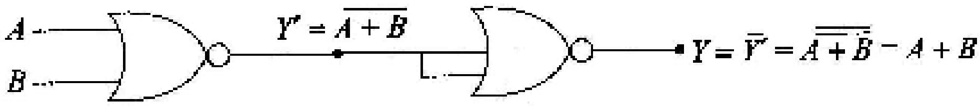
(a) *To obtain NOT-gate from NOR-gate*





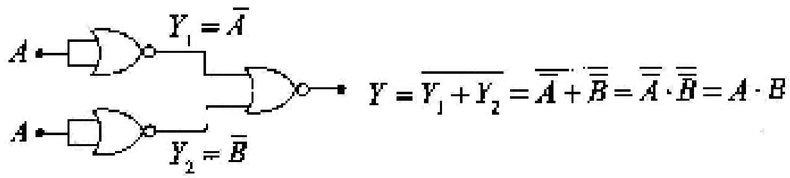
The NOT-gate is obtained from a one-input NOR-gate. Thus, single input NOR gate is yet another inverter circuit.

(b) *To obtain OR-gate from NOR-gate*



If a output of a NOR-gate is fed to the NOT-gate obtained from NOR-gate (i.e. joining the two inputs of NOR-gate). The resulting gate works as OR-gate.

(c) *To obtain AND-gate from NOR-gate*



If the two outputs obtained from two NOT-gates (which is obtained from NOR-gate) are fed to the inputs of a NOR-gate. The resulting arrangement works as AND-gate.

22.10.1 Arithmetic Circuits

Some of the arithmetic circuits, used in digital computers, are exclusive OR-gate (XOR-gate), exclusive NOR-gate (XNOR-gate), half adders and full adders.

Exclusive–OR-gate (Ex–OR-gate or XOR-gate)

The Boolean expression for XOR is given by

$Y = \bar{A}\cdot B + A\cdot\bar{B}$ or $Y = A \oplus B$ (XOR-binary operation is denoted by '$\oplus$')