Intersecting domain walls in MQCD

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ABSTRACT

We argue that MQCD admits intersecting domain walls that are realized as Cayley calibrations of the MQCD M5-brane. We discuss various dual realizations and comment on how branes can realise domain walls in N=1 supersymmetric theories in D=3.

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1 Introduction

Supersymmetric quantum chromodynamics (SQCD) exhibits many of the features of QCD in that it is believed to be a confining theory with a mass gap and to exhibit spontaneous breaking of a discrete chiral symmetry. The chiral symmetry of SQCD is a residual Z_n R-symmetry, which is broken by the non-vanishing expectation value of a gluino bilinear. For gauge group $SU(n)$ there are n isolated supersymmetric vacua, which are permuted by the action of the Z_n R-symmetry.

Domain walls can interpolate between the discrete vacua in SQCD and in [1] it was conjectured that they are BPS saturated. The mechanism for domain walls to be BPS saturated is the appearance of a topological central charge in the supertranslation algebra [2], and they were first found as BPS solutions of the Wess-Zumino (WZ) model [3, 4]. Whether SQCD domain walls are in fact BPS saturated is a subtle dynamical issue. Building on earlier work on the low-energy dynamics of SQCD [5, 6], it was argued in [7, 8] that in the large n limit the BPS equations for SQCD domain walls reduce to those of a WZ model with a Z_n invariant superpotential admitting n isolated vacua permuted by the Z_n symmetry. In a recent development it has been shown that, for $n \geq 3$, such WZ models also admit 1/4 supersymmetric configurations in which domain walls intersect on junctions [9, 10] (see [11, 12, 13] for some subsequent developments). This suggests the existence of 1/4 supersymmetric domain wall junctions in $SU(n)$ SQCD for $n \geq 3$, for which evidence has been presented in [14, 15].

Given these features of SQCD it would be natural to expect something similar for the closely related MQCD [16]. This theory describes the fluctuations of a single M5-brane wrapped on a particular holomorphic 2-cycle in an S^1-compactified D=11 Minkowski spacetime. It was argued in [16] that MQCD admits 1/2 supersymmetric domain walls, which were identified as the MQCD M5-brane wrapped on an associative 3-cycle, and this was partially confirmed in subsequent work [17, 18]. An interesting feature of these walls is that they are D-branes for the MQCD string (which suggests that the SQCD domain walls are also D-branes for the SQCD string that is expected to govern the large n dynamics). Here we shall argue that MQCD admits 1/4 supersymmetric intersecting domain wall configurations, which we identify as the MQCD M5-brane wrapped on Cayley-calibrated 4-cycles (although, as for the associative 3-cycles, the explicit construction of appropriate Cayley 4-folds will be left to future work). Intersecting domain walls thus provide a physical realization of the Cayley calibrations discussed (along with other cases) in various recent works [19, 20, 21, 22, 23, 24].

The plan of this letter is as follows. We argue that MQCD domain wall junctions are realised via Cayley 4-folds in section 2. Section 3 discuss a number of dual formulations of the domain wall junctions. Section 4 comments on the relevance of associative and Cayley calibrations of M5-branes to D=3 supersymmetric field theories with N=1 supersymmetry.

2 Cayley calibrations and MQCD

Since $SU(n)$ MQCD has n distinct vacua one would expect it to have domain walls for $n \geq 2$ and domain wall junctions for $n \geq 3$. We begin our investigation with a brief review of the essential features of MQCD. The vacua are identified as an M5-brane in an $\mathbb{E}^{(1,9)} \times S^1$ spacetime with an $\mathbb{E}^{(3,1)} \times \Sigma_2$ worldvolume, where Σ_2 is a holomorphic curve in a complex 3-dimensional subspace of $\mathbb{E}^9 \times S^1$ of the form $\mathbb{C}^2 \times (\mathbb{E}^1 \times S^1)$. Such surfaces can be described by the zero locus of two holomorphic functions $f_1(v, w, t)$ and $f_2(v, w, t)$, where v, w are complex coordinates for $\mathbb{C}^2$ and t is a complex coordinate for the cylinder $\mathbb{E}^1 \times S^1$. The choice of holomorphic functions that yields MQCD with gauge group $SU(n)$ is [16]

$$f_1 = vw - \zeta \,, \qquad f_2 = v^n - t \tag{1}$$

where ζ is a complex constant. Note that spatial infinity of the fivebrane has two components, $v \to \infty$ or $w \to \infty$ and that only ζ^n appears in the description there. The n values of ζ for a given ζ^n correspond to the n vacua of MQCD.

A holomorphic curve Σ_2 describing an MQCD vacuum is a 2-surface calibrated by an $SU(3)$-Kähler calibration. It is a general feature of such calibrated surfaces that they preserve $\nu = 1/8$ supersymmetry of the M-theory vacuum[1]. Such calibrations can be represented by the array of orthogonally intersecting fivebranes [21]

$$
\begin{array}{ccccccccccc}
M5: & 1 & 2 & 3 & | & 4 & 5 & - & - & - & - & - \\
M5: & 1 & 2 & 3 & | & - & - & 6 & 7 & - & - & - \\
M5: & 1 & 2 & 3 & | & - & - & - & - & 8 & 9 & - \\
\end{array}
\tag{2}
$$

Such arrays serve two purposes. Firstly, they easily allow one to recover associated string theory arrays of orthogonally intersecting branes upon dimensional reduction. Indeed if we assume that the 9 direction is a circle then we get the configuration corresponding to D4-branes suspended between the Neveu-Schwarz fivebranes. Secondly, they allow us to use some of the technology developed for orthogonal intersections as a guide to MQCD. The reason that this is possible is that one can consider arrays like the one above as a code for the constraints on the Killing spinors associated to the supersymmetries preserved by the fivebrane, since these constraints depend only on the type of calibration and not on its detailed form. In the present case, three M5-branes intersecting according to the above array defines an $SU(3)$-Kähler calibration, just as does the vacuum of MQCD[2]. With this in mind, the first three columns have

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[1] We will usually use the letter 'ν' to denote supersymmetry fractions relative to the M-theory vacuum. These fractions should not be confused with the fraction of supersymmetry preserved by objects in the vacuum of an N=1 D=4 theory. The fraction $\nu = 1/8$ corresponds to unbroken N=1 D=4 supersymmetry.

[2] As an aside we note that if the vacuum spacetime is taken to be $\mathbb{E}^{(1,10)}$ instead of $\mathbb{E}^{(1,9)} \times S^1$ then the three complex variables (v, w, t) can be taken to be coordinates on a $\mathbb{C}^3$ subspace of $\mathbb{E}^{10}$, and a choice of holomorphic functions corresponding to the intersecting fivebranes can be taken to be $f_1 = vwt$, $f_2 = vw + wt + tv$, which is invariant under the $U(1)$ symmetry group acting by simultaneous rotation in the v, w, t complex planes.

been separated from the rest to indicate that we interpret the first three directions as the spatial directions of an effective 3+1 dimensional field theory.

As shown in [16], the surface defined by (1) has a $Z_n \times Z_2 \times U(1)$ invariance group. The $U(1)$ symmetry group acts by phase rotation on the complex variables v, w and t, but this symmetry does not act on the fields of the effective D=4 N=1 super Yang-Mills (SYM) theory. The Z_2 symmetry flips v and w and corresponds to charge conjugation. The Z_n group is, as in SQCD, an R-symmetry group permuting the n supersymmetric vacua. By analogy with SQCD one therefore expects domain walls interpolating between adjacent vacua. Witten has argued persuasively that the M-theory realization of such a domain wall is an M5-brane wrapped on an associative 3-cycle in $\mathbb{E}^6 \times S^1$. Evidence for this interpretation is that associative 3-cycles preserve $\nu = 1/16$ of the supersymmetry of the M-theory vacuum and are thus naturally associated with 1/2 supersymmetric configurations of the effective D=4 N=1 field theory. Associative 3-cycles are also realized by four M5-branes intersecting according to the array [21]

$$
\begin{array}{llllllllllll}
M5: & 1 & 2 & 3 & | & 4 & 5 & - & - & - & - & - \\
M5: & 1 & 2 & 3 & | & - & - & 6 & 7 & - & - & - \\
M5: & 1 & 2 & 3 & | & - & - & - & - & 8 & 9 & - \\
- & & - & - & - & | & - & - & - & - & - & - & - \\
M5: & 1 & 2 & - & | & 4 & - & 6 & - & 8 & - & -
\end{array}
\tag{3}
$$

This array suggests the domain wall interpretation, with the normal to the wall along the 3-axis; the gap between the first three rows and the fourth one is meant to facilitate this interpretation. More precisely [16], the 3 surface should degenerate to $R \times \Sigma_2$ and $R \times \Sigma_2'$ as $x_3 \to \pm\infty$ where Σ_2 and Σ_2' are the surfaces corresponding to a given ζ and $exp(2\pi i/n)\zeta$, respectively.

Consider now the following array of five intersecting M5-branes:

$$
\begin{array}{llllllllllll}
M5: & 1 & 2 & 3 & | & 4 & 5 & - & - & - & - & - \\
M5: & 1 & 2 & 3 & | & - & - & 6 & 7 & - & - & - \\
M5: & 1 & 2 & 3 & | & - & - & - & - & 8 & 9 & - \\
- & & - & - & - & | & - & - & - & - & - & - & - \\
M5: & 1 & 2 & - & | & 4 & - & 6 & - & 8 & - & - \\
M5: & 1 & - & 3 & | & 4 & - & 6 & - & - & 9 & -
\end{array}
\tag{4}
$$

This configuration is a $\nu = 1/32$ supersymmetric configuration, corresponding to 1/4 supersymmetry of an effective D=4 N=1 field theory. The format of the array has been chosen to remind us of this interpretation, and from it we see that it is naturally interpreted as the intersection along the 1-axis of two domain walls. As shown in [20, 21] this configuration defines a Cayley calibrated 4-cycle in $\mathbb{E}^7 \times S^1$ (the directions 2 to 9). Note that we could add an M-wave in the 1-direction while preserving supersymmetry, in accordance with the observation in [9] that one can add momentum along the intersection of domain walls in the WZ model.

The natural conclusion to be drawn from the above analysis is that 1/4 supersymmetric configurations of intersecting MQCD walls are to be identified with M5-branes

for which the worldvolume has the form $\mathbb{E}^{(1,1)} \times C_4$ with C_4 a Cayley calibrated 4-cycle in an $\mathbb{E}^7 \times S^1$ subspace of the $\mathbb{E}^{1,9} \times S^1$ M-theory vacuum. In order to have the MQCD interpretation we propose, this 4-cycle should degenerate in three or more directions in the 2,3 plane to a product of the form $\mathbb{R} \times A_3$ where A_3 is an associative 3-cycle in $\mathbb{E}^6 \times S^1$; in these directions we recover an MQCD domain wall. As discussed in [16], this should further degenerate, as we move away from the wall, into a product of the form $\mathbb{R} \times \Sigma_2$ where Σ_2 is a holomorphic 2-cycle. Tracing such a path in $\mathbb{E}^7 \times S^1$, the M5-brane worldvolume would be seen to pass through the sequence of 5-spaces

$$\mathbb{E}^{(1,1)} \times C_4 \to \mathbb{E}^{(1,2)} \times A_3 \to \mathbb{E}^{(1,3)} \times \Sigma_2 \,, \tag{5}$$

but there must be several such paths. In fact, there must be a minimum of three different endpoint 2-cycles Σ_2, corresponding to three distinct vacua because with only two distinct vacua we would have only one type of domain wall and no possibility of intersections. In other words, intersecting domain walls are possible, in principle, for $SU(n)$ MQCD with $n \geq 3$ but they are *not* possible for $SU(2)$ MQCD. This state of affairs is reminiscent of the 1/4 supersymmetric string-junction dyons in the N=4 D=4 SYM theory with gauge group $SU(n)$ on n parallel D3-branes; one again needs $n \geq 3$. We will briefly discuss a connection between the Cayley array and IIB string junctions in the next section.

We will leave the explicit construction of the Cayley 4-folds to future work. It is interesting to note, however, that some properties of intersecting domain walls follow from the supersymmetry algebra alone [25]. The D=4 N=1 supersymmetry algebra allows 'electric' and 'magnetic' domain wall charges, which form a doublet of the $U(1)_R$ automorphism group of the supersymmetry algebra. A given wall may be 'electric', 'magnetic' or 'dyonic' but it has a definite electric-magnetic charge vector. It was shown in [25] that intersections of 1/2 supersymmetric domain walls preserve 1/4 supersymmetry if the angle at which they intersect equals the angle between them in 'electric-magnetic' charge space. We can see this behaviour mirrored in the array (4) where the last two fivebranes are rotated in the 2,3 and 8,9 planes by 90 degrees, although the projections on the supersymmetry parameters coded by this array are actually valid for arbitrary rotation angle.

Although the supersymmetry algebra allows for the 1/4 supersymmetric intersection of domain walls at arbitrary angles, only certain angles will be allowed in any given model. In models such as $SU(n)$ SQCD or MQCD in which $U(1)_R$ is broken by anomalies to Z_{2n} we can expect that all allowed configurations will be related by Z_n rotations (only a Z_n subgroup of Z_{2n} acts on the bosonic fields). It is instructive to first consider the action of Z_n for $n = 2$ and $n = 3$. In the $n = 2$ case there are two vacua and one wall interpolating between them. The Z_2 R-symmetry permutes the vacua but leaves invariant the wall. In the $n = 3$ case there are three vacua and three domain walls, which can meet at a Z_3-invariant domain-wall 'Y'-junction. Although the junction is Z_3-invariant the Z_3 group permutes not only the three vacua that meet at the junction but also the three domain walls. In this case, the orientation of the walls that meet at a junction is fixed. For $n > 3$ we can expect more than

one type of junction, in the sense that more than one set of angles is likely to be possible, but these will presumably be related by Z_n rotations of the walls meeting at the junction.

In the Wess-Zumino model it has been demonstrated that domain wall networks, or 'domain wallpaper', are only metastable [11], becoming longer lived as the widths of the domain wall decreases. Since the M5-brane action describes a structureless fivebrane, it seems possible that such networks could still be marginally stable in MQCD (whereas there is no obvious reason to suppose that they would be stable in SQCD). These configurations would be somewhat analogous to string networks in IIB string theory [26].

3 Dual formulations

The constraints on the supersymmetry spinor parameter ϵ that are associated with any particular brane configuration, the 'Killing spinors', generally have other interpretations. For example, the constraints associated with the $SU(3)$-Kähler calibrated configuration of three orthogonally intersecting M5-branes are also those obtained as the constraints on the Killing spinors in Ricci flat spacetimes of G_2 holonomy. This is reflected in the fact that spacetimes of G_2 holonomy are dual to intersecting brane configurations in the sense that there are standard M-theory dualities which transform the intersecting M5-brane configuration into one involving three 'intersecting' Kaluza-Klein M-theory monopoles, each of which in isolation is a fibre bundle with fibre S^1 and base $\mathbb{E}^3$.

The same duality chain takes the additional two M5-branes that we previously interpreted as intersecting domain walls in an effective D=4 theory to an M2-brane intersecting (or ending on) an M5-brane. Specifically, if we interchange the 8 and 10 directions in the Cayley array (4), reduce on the 10 direction, T-dualise on the 4 and 6 directions and then uplift back to eleven dimensions we obtain the following M-theory array:

$$
\begin{array}{llll|lllllll}
KK: & - & - & - & | & - & - & \times & o & o & o & - \\
KK: & - & - & - & | & \times & o & - & - & o & o & - \\
KK: & - & - & - & | & - & o & - & o & o & - & \times \\
- & & - & - & - & | & - & - & - & - & - & - & - \\
M2: & 1 & 2 & - & | & - & - & - & - & - & - & - \\
M5: & 1 & - & 3 & | & 4 & - & 6 & - & - & 9 & -
\end{array}
\tag{6}
$$

The notation is such that 'o' indicates a $\mathbb{E}^3$ direction and '$\times$' indicates the S^1 fibre. The spacetime metric for the intersecting KK monopoles has been given in [27]; it is a singular 7-dimensional space of G_2 holonomy but the singularities are presumably removable. In any case, the Killing spinor constraints are those of an arbitrary 7-manifold of G_2 holonomy so we may interpret this array as that corresponding to an M2-brane and an M5-brane intersecting in a spacetime of the form $\mathbb{E}^{(1,3)} \times J_7$ where J_7 is a 7-manifold of G_2 holonomy.

It has been shown that an interesting class of N=1 supersymmetric field theories in four dimensions, including SQCD, can be geometrically engineered using singular manifolds [28]. For field theories with discrete supersymmetric vacua we expect BPS domain walls and domain wall junctions. The above array makes it clear that, in general, there are electric and magnetic domain walls in accord with the discussion on the supersymmetry algebra in the last section. The details of which domain walls intersect and at what angles will depend on the details of which field theory model is being engineered.

Note that we can again add momentum along the intersection of the M2-brane with the M5-brane to the array without further reducing the fraction of supersymmetry preserved. Note also that M5 branes can wrap co-associative four-cycles in J_7. These would appear as strings in the effective 3+1-dimensional theory (they are dual to membranes in the original configuration, which were argued in [16] to decouple at low energy, but which are in the same homotopy class as the MQCD strings).

By reducing the above array (6) on the 6 direction and relabelling, we obtain the following IIA configuration:

$$
\begin{array}{llllllllll}
D6: & 1 & 2 & 3 & | & 4 & 5 & 6 & - & - & - \\
KK: & - & - & - & | & \times & o & - & - & o & o \\
KK: & - & - & - & | & - & o & \times & o & o & - \\
- & & - & - & - & | & - & - & - & - & - & - \\
D2: & 1 & 2 & - & | & - & - & - & - & - & - \\
D4: & 1 & - & 3 & | & 4 & - & - & - & - & 9
\end{array}
\tag{7}
$$

The first part of this array corresponds to the geometric engineering of N=1 supersymmetric theories obtained by wrapping D6-branes around special Lagrangian 3-cycles of Calabi-Yau 3-folds [29]. When the theories admit BPS domain walls and domain wall junctions they will correspond to D2-branes and D4-branes. It is interesting to note that in the context of this kind of geometric engineering there are in fact more possibilities for domain walls. For example domain walls in the 12 direction can also come from D6-branes wrapping 4-cycles of the Calabi-Yau: in the array language we could add D6: 124567, for example, to the array without breaking more supersymmetry. Similarly there are two sources of domain walls in the 13 directions: NS-5branes wrapping 3-cycles (eg NS5: 13579) as well as D8-branes wrapping the whole of the Calabi-Yau (D8:13456789). The D8-brane is probably not relevant in the field theory limit. Note that the lift of the D6 brane in the 124567 directions gives rise to a KK configuration in the M-theory array (6) in the 3689 directions which also does not seem relevant in the field theory limit. It would be very interesting to find more detailed evidence for domain wall junctions from the geometric engineering point of view.

Finally, it is instructive to consider also the following IIB dual of the Cayley

calibration array (4)

$$
\begin{array}{lccc|ccccccc}
KK: & - & - & | & o & - & - & o & \times & - & o \\
KK: & - & - & | & - & o & o & - & \times & - & o \\
KK: & - & - & | & - & o & - & o & - & \times & o \\
- & & - & - & | & - & - & - & - & - & - & - \\
D1: & 1 & - & | & - & - & - & - & - & - & - \\
F1: & - & 2 & | & - & - & - & - & - & - & - \\
\end{array}
\tag{8}
$$

(For example, up to a relabelling, this array can be obtained from (7) by T-dualising on the 1 direction, S-dualising and then T-dualising on the 4 and 9 directions). We again take the first three rows to represent a 7-space of G_2 holonomy. This type of IIB compactification on a possibly singular manifold allows one in principle to geometrically engineer N=2 supersymmetric field theories in D=3. The electric and magnetic domain walls have now become a D-string orthogonally intersecting a fundamental IIB string, although the constraints are those associated with any 1/4 supersymmetric intersection of (p,q) strings; in other words, a IIB string junction.

4   D=3, N=1

Instead of interpreting an associative M5-brane calibration as a domain wall in MQCD, as in [16], we could interpret it as a vacuum of an N=1 D=3 SQFT. Cayley 4-folds with appropriate boundary conditions would then correspond to 1/2 supersymmetric domain walls in this effective theory (the possibility of such walls was demonstrated in [30, 9]).

Here we shall analyse this possibility in terms of orthogonally intersecting M5-branes. When reformatted to reflect the change of interpretation, the associative M5-brane array is

$$
\begin{array}{lcc|ccccccccc}
M5: & 1 & 2 & | & 3 & 4 & 5 & - & - & - & - & - \\
M5: & 1 & 2 & | & 3 & - & - & 6 & 7 & - & - & - \\
M5: & 1 & 2 & | & 3 & - & - & - & - & 8 & 9 & - \\
M5: & 1 & 2 & | & - & 4 & - & 6 & - & 8 & - & - \\
\end{array}
\tag{9}
$$

If we compactify in the 9-direction and relabel the 10th direction as the 9th, then we obtain the array

$$
\begin{array}{lcc|ccccccc}
NS5: & 1 & 2 & | & 3 & 4 & 5 & - & - & - & - \\
NS5': & 1 & 2 & | & 3 & - & - & 6 & 7 & - & - \\
D4: & 1 & 2 & | & 3 & - & - & - & - & 8 & - \\
NS5'': & 1 & 2 & | & - & 4 & - & 6 & - & 8 & - \\
\end{array}
\tag{10}
$$

The first three rows recall the IIA superstring interpretation of MQCD as k D4-branes suspended between a NS5 and a NS5'-brane separated in the 8-direction. The last row means that we can have, for example, two NS5''-branes in the $\{1, 2, 4, 6, 8\}$

planes separated in the 3-direction. The D4-brane world-volume is now cutoff in the 3 and 4 directions and the effective field theory is an $SU(k)$ D=3 N=1 gauge theory. Note that the above projections imply that we can also add NS5-branes in the $\{1, 2, 5, 7, 8\}$ plane and D4-branes in the $\{1, 2, 5, 6\}$ and $\{1, 2, 4, 7\}$ planes without breaking more supersymmetry, leading to further generalisations. The addition of D6-branes in the $\{1, 2, 3, 4, 5, 9\}$, $\{1, 2, 5, 7, 8, 9\}$, $\{1, 2, 3, 6, 7, 9\}$ or $\{1, 2, 4, 6, 8, 9\}$ planes can also be achieved without breaking more supersymmetry and will give rise to matter multiplets. It would be interesting to analyse these models in more detail along the lines of Refs. [31, 32]. The observations here indicate that these models can in principle be "solved" by determining how the specific brane configuration can be lifted to an associative 3-fold. In the cases where there are discrete quantum vacua, we expect BPS domain walls (strings) and they will be realised as Cayley 4-folds.

We previously saw how intersecting domain walls of MQCD could have various dual interpretations. Here too, reducing the Cayley array (4) on the 10 direction, T-dualising on the 5 direction, uplifting back to eleven dimensions yields

$$
\begin{array}{llllllllllll}
KK: & - & - & | & - & - & o & \times & - & - & o & o \\
KK: & - & - & | & - & - & o & \times & o & o & - & - \\
KK: & - & - & | & - & o & - & \times & - & o & - & o \\
KK: & - & - & | & o & - & - & \times & - & o & o & - \\
- & & - & - & | & - & - & - & - & - & - & - \\
M5: & 1 & - & | & 3 & 4 & 5 & 6 & - & - & - & -
\end{array}
\tag{11}
$$

where we have relabelled coordinates and reordered the array. The first four rows can now be interpreted as an eight-space with Spin(7) holonomy. Appropriate spaces will then allow one to engineer D=3 N=1 supersymmetric field theories with N=1 supersymmetry. To preserve 1/2 supersymmetry of this effective theory the M5-brane in the array must wrap a Cayley four cycle, giving rise to a supersymmetric domain wall (string). Again, momentum can be added in the 1 direction without breaking supersymmetry.

Acknowledgments

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