INSTANTONS AND SEVEN-BRANES IN TYPE IIB SUPERSTRING THEORY.

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Instanton and seven-brane solutions of type IIB supergravity carrying charges in the Ramond-Ramond sector are constructed. The singular seven-brane has a quantized $R \otimes R$ 'magnetic' charge whereas its dual is the instanton, which is non-singular in the string frame and has an associated *global* 'electric' charge. The product of these charges is constrained by a Dirac quantization condition. The instanton has the form of a space-time wormhole in the string frame, and is responsible for the non-conservation of the Noether current.

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1. **Introduction**

Recent developments in our understanding of superstring theory suggest that all known theories may be viewed as different perturbative approximations of a single underlying theory. Viewed from the standpoint of any given string theory the fundamental strings of other theories are solitonic states that are not apparent in string perturbation theory. Other p-branes also arise as solitons, suggesting an important rôle for objects with all possible values of p in the theory. In type II theories the fundamental strings carry the charges of the Neveu–Schwarz-Neveu–Schwarz (NS $\otimes$ NS) sector but do not carry Ramond–Ramond (R $\otimes$ R) charges. These latter charges are associated with $(p+1)$-form potentials or $(p+2)$-form field strengths, F_{p+2}, that are carried by some of the known p-branes of the these theories. The solitons that carry the R $\otimes$ R charge that have been constructed so far are the zero-brane (black hole), two-brane, four-brane and six-brane solutions of the type IIA theory and the one-brane (string), self-dual three-brane and five-brane solitons of type IIB string theory (for a review see [1]). In fact, there is an infinite-dimensional $SL(2,\mathbb{Z})$ multiplet of both 'dyonic' one-branes [2] and five-branes in the type IIB theory.

Recently, some p-brane solitons have been associated with superstring configurations known generically as D-branes (or D-instantons in the $p = -1$ case) [3] which are sources of the R$\otimes$R charge. This association suggests that there should be solitons for *all* values of p from $p = -1$ to $p = 9$. The case $p = 9$ is very special since the accompanying field strength vanishes identically, and is connected with the presence of chiral anomalies in type I theories with any gauge group other than $SO(32)$. The $p = 8$ soliton constructed in [45] couples to a cosmological constant in the type IIA theory [3, 6] and is a solution of 'massive' type IIA supergravity [7] . This paper considers the $p = -1$ (instanton) and $p = 7$ (seven-brane) solutions of the type IIB theory which have R $\otimes$ R charges that are related by a Dirac-like quantization condition. D-instantons were previously considered in the bosonic theory in [8,9] and the BPS boundary condition for the type IIB theory D-instanton was obtained in [10].

Although the construction to be described bears a resemblance to the construction of the previously discovered p-branes, there are fascinating new features. For example, the instanton solution is non-singular in the string frame – in fact it is a wormhole. Whereas other p-brane solutions can be thought of as wormholes with infinitely long throats the instanton is genuinely an Einstein-Rosen wormhole [11] which connects two asymptotically euclidean regions of space-time. The 'electric' charge carried by the instanton is an R $\otimes$ R charge that flows through the wormhole throat, and is interpreted as a violation of the conservation of a global charge in physical processes. The seven-brane solution, carrying the dual 'magnetic' charge, is related to the stringy cosmic string solution of [12].

The solutions of type IIB supergravity we will consider are ones in which the two scalar fields (the dilaton, ϕ, and the R $\otimes$ R scalar, a) and the metric have non-trivial behaviour while the other bosonic fields (the two third-rank field strengths and the self-dual fifth-rank field strength) vanish. The ten-dimensional lagrangian for the non-vanishing fields is

$$\mathcal{L} = R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}e^{2\phi}(\partial a)^2 \tag{1}$$

in the Einstein frame (where the signature is $(-+++++++++)$). Defining a nine-form field strength, $F_{(9)} = e^{2\phi} * da$, i.e.,

$$F_{\mu_1...\mu_9} = e^{2\phi}\epsilon^{\mu}{}_{\mu_1...\mu_9}\partial_\mu a, \tag{2}$$

the lagrangian can be written in the equivalent form,

$$\hat{\mathcal{L}} = R - \tfrac{1}{2}(\partial\phi)^2 - \frac{1}{2(9!)}e^{-2\phi}F_{\mu_1...\mu_9}F^{\mu_1...\mu_9}. \tag{3}$$

The passage from (1) to (3) is a standard duality transformation. The field equation for $F_{(9)}$ coming from (3) is

$$\nabla_\mu\left(e^{-2\phi}F^{\mu}{}_{\mu_1...\mu_8}\right) = 0, \tag{4}$$

which is equivalent to the Bianchi identity for a.

2. The instanton solution

The form of $\hat{\mathcal{L}}$ remains unchanged after Wick rotation to euclidean signature. The field equations that arise from the euclidean version of (3) have a form that could have been obtained from

$$\tilde{\mathcal{L}} = R - \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}e^{2\phi}(\partial\alpha)^2, \tag{5}$$

which is (1) with the substitution $a \to \alpha = ia$. The integrals of the Lagrangians (3) and (5) give actions that differ purely by surface terms that can arise in replacing da by $F_{(9)}$. The euclidean equations of motion are invariant under a euclidean version of $N = 2$ supersymmetry. The equations of motion that follow from (5) are

$$R_{\mu\nu} - \tfrac{1}{2}\left(\partial_\mu\phi\partial_\nu\phi - e^{2\phi}\partial_\mu\alpha\partial_\nu\alpha\right) = 0,$$
$$\nabla_\mu\left(e^{2\phi}\partial^\mu\alpha\right) = 0, \tag{6}$$
$$\nabla^2\phi - e^{2\phi}(\partial\alpha)^2 = 0.$$

We now turn to the conditions that need to be satisfied for a solution of the euclidean theory to preserve half the total supersymmetries which requires a brief discussion of the euclidean version of the $N = 2$ supersymmetry of the type IIB theory. This could be expressed in terms of symplectic spinors but it is more succinct to express the euclidean theory in a manner that parallels the usual discussion with lorentzian signature. Using the conventions in [13] the lorentzian signature supersymmetry transformations of the fields can be written in a complex notation. They can then be systematically adapted to the case of euclidean signature by replacing the usual algebra $\mathbb{C}$ of complex numbers,

generated over the reals $\mathbb{R}$ by 1 and i (where $i^2 = -1$) by the so-called 'double' or hyperbolic complex numbers $\mathbb{E}$, which are generated over the reals by 1 and e, where $e^2 = +1$. Conjugation of complex numbers is an automorphism taking 1 to 1 and i to $-i$. Similarly conjugation of double numbers takes 1 to 1 and e to $-e$. Thus a general double number $u = a + eb$ $(a, b \in \mathbb{R})$ is conjugate to $u^* = a - eb$ and $uu^* = a^2 - b^2$. As an algebra over $\mathbb{R}$ the double numbers are reducible $\mathbb{E} = \mathbb{R} \oplus \mathbb{R}$. The associated projectors $P_\pm = \frac{1}{2}(1 \pm e)$ are lightlike, $P_\pm P_\pm^* = 0$, with respect to the indefinite inner product uu^*.

The supersymmetry transformations of the $\mathbb{C}$-valued spin-1/2 and spin-3/2 fields, λ and ψ_μ are given in [13] (in the Einstein frame). In a bosonic background in which only $\tau = a + ie^{-\phi} = \tau_1 + i\tau_2$ and $g_{\mu\nu}$ are non-zero they are,

$$\delta\lambda = -\frac{1}{\tau_2}\left(\frac{\tau^* - i}{\tau + i}\right)\gamma^\mu(\partial_\mu\tau_1 + i\partial_\mu\tau_2)(\epsilon_1 - i\epsilon_2), \tag{7}$$

$$\delta\psi_\mu = \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_a\gamma_b - i\frac{1}{2}Q_\mu\right)(\epsilon_1 + i\epsilon_2), \tag{8}$$

where the composite $U(1)$ gauge potential is defined by

$$Q_\mu = \frac{-1}{4\tau_2}\left\{\left(\frac{\tau - i}{\tau^* - i}\right)(\partial_\mu\tau_1 - i\partial_\mu\tau_2) + \left(\frac{\tau^* + i}{\tau + i}\right)(\partial_\mu\tau_1 + i\partial_\mu\tau_2)\right\} \tag{9}$$

These expressions have been transformed from the $SU(1,1)$-invariant form in [13] to the parameterization with manifest $SL(2,\mathbb{R})$-invariance.

For the application to the euclidean instanton solution, we change i to e everywhere in these transformations and make the identifications $\tau_1 = \alpha$ and $\tau_2 = e^{-\phi}$. The ansatz that leads to the preservation of half the supersymmetry is $d\tau_1 = \pm d\tau_2$ and the metric is flat, i.e.

$$d\alpha = \pm e^{-\phi}d\phi, \qquad g_{\mu\nu} = \delta_{\mu\nu}. \tag{10}$$

It follows from (7) that $\delta\lambda = 0$ if $\epsilon_1 = \pm\epsilon_2$. From hereon we will arbitrarily choose the plus sign. Furthermore, to obtain $\delta\psi_\mu = 0$ we note that the spin connection vanishes ($\omega_\mu^{ab} = 0$) since the metric is flat in the Einstein frame. Thus, after using the ansatz $\tau_1 = \tau_2 + k$ (with constant k) the transformation of the gravitino becomes,

$$\delta\psi_\mu = (1 + e)\left(\partial_\mu + \frac{k^2 - 1}{4\tau_2((\tau_2 + k)^2 - (1 + \tau_2)^2)}\right)\epsilon_1. \tag{11}$$

This can be made to vanish by setting $\epsilon_1 = f(\tau_2)\epsilon_1^0$, where ϵ_1^0 is an arbitrary constant real spinor, and choosing $f(\tau_2)$ appropriately.

The conditions (10) combined with the equations of motion (6) lead to $\partial^2 \phi = -(\partial \phi)^2$, so that

$$\partial^2(e^\phi) = 0. \tag{12}$$

This has a spherically symmetric solution describing a single instanton,

$$e^\phi = \left(e^{\phi_\infty} + \frac{c}{r^8} \right), \tag{13}$$

where ϕ_∞ is the value of the dilaton field at $r = \infty$ and c is a constant that is arbitrary at this stage, and will be shown below to be proportional to the instanton charge. From (10) α is given by

$$\alpha - \alpha_\infty = -e^{-\phi} + e^{-\phi_\infty} \tag{14}$$

where α_∞ is the constant value of α at $r = \infty$. The solution is specified by the value of the two constants ϕ_∞ and α_∞.

This single instanton solution is evidently singular at $r = 0$ in the Einstein frame. However, it is natural to transform from the Einstein frame (in which the metric is simply $ds_E^2 = dx^2$) to the string frame in which the metric is given by,

$$ds^2 = e^{\phi/2} ds_E^2 = \left(e^{\phi_\infty} + \frac{c}{r^8} \right)^{1/2} \left(dr^2 + r^2 d\Omega_9^2 \right), \tag{15}$$

where $d\Omega_n^2$ is the $SO(n)$-invariant line element on S^n. This metric is manifestly invariant under the inversion transformation,

$$r \to (ce^{-\phi_\infty})^{1/4} \frac{1}{r}, \tag{16}$$

which shows that the region $r \to 0$ is another asymptotically Euclidean region identical to that near $r = \infty$. In fact, the solution in this frame is a wormhole in which there are two asymptotically euclidean regions connected by a neck. The space-time is geodesically complete so in this sense it is non-singular.

The euclidean action of the instanton is given (in the Einstein frame), by

$$I_{inst} = \int_M \left(-R + \tfrac{1}{2}(\partial \phi)^2 + \frac{1}{2(9!)} e^{2\phi} F_{\mu_1 \ldots \mu_9} F^{\mu_1 \ldots \mu_9} \right) - 2 \int_{\partial M} [\mathrm{Tr} K], \tag{17}$$

where $[\mathrm{Tr} K]$ is the difference between the trace of the extrinsic curvature on the boundary and the value it would have if the boundary were filled in with flat space. For our instanton solution $R = 0$ and the boundary contributions also vanish. The contribution from infinity clearly vanishes. That from $r = 0$ also vanishes since $\mathrm{Tr} K = \frac{9}{r}$, but

when integrated over a small nine-sphere of radius r, it will vanish. Using the fact that $(\partial \phi)^2 = e^{-2\phi} F_{\mu_1 \ldots \mu_9} F^{\mu_1 \ldots \mu_9}/9!$, one obtains

$$I_{inst} = \int_{R^{10}} (\partial \phi)^2 = -\int_{R^{10}} \partial^2 \phi = -\int_{r=\infty} \partial_\mu \phi \, d\Sigma^\mu + \int_{r=0} \partial_\mu \phi \, d\Sigma^\mu. \qquad (18)$$

Using the explicit form of ϕ in (13) the contribution from $r = 0$ is seen to vanish whereas the contribution from $r = \infty$ gives the total action,

$$I_{inst} = \frac{8c}{g} \text{ Vol } (S^9) = \frac{2\pi^{5/2}}{3} c, \qquad (19)$$

where $\text{Vol}(S^9) = 2\pi^{5/2}/\Gamma(5)$ is the volume of the unit nine-sphere and where $g = e^{\phi_\infty}$ is the string coupling constant.

An electric charge $Q^{(-1)}$ is defined by the Noether charge for the translation symmetry $\alpha \to \alpha + \text{constant}$. Thus

$$Q^{(-1)} = \int_{S^9} e^{2\phi} F^\mu d\Sigma_\mu = \int_{S^9} J_\mu^{Noether} d\Sigma^\mu = \int_{S^9} F_{(9)} \qquad (20)$$

where $F_\mu = \partial_\mu \alpha$, $J_\mu^{Noether} = e^{2\phi} \partial_\mu \alpha$, and the integration is over the nine sphere at $r = \infty$. Conservation of the Noether current is equivalent to the field equation for $F_{(1)}$,

$$\nabla^\mu J_\mu^{Noether} = \nabla_\mu (e^{2\phi} F^\mu) = 0 \qquad (21)$$

For our solution

$$Q^{(-1)} = \int e^{2\phi} \partial_\mu \alpha d\Sigma^\mu = \int e^\phi \partial_\mu \phi d\Sigma^\mu = -8cg \text{ Vol}(S^9) \qquad (22)$$

so that

$$I_{inst} = \frac{|Q^{(-1)}|}{g}. \qquad (23)$$

This shows the expected dependence of the instanton action on the coupling constant. The energy of the p-brane solitons of the $R \otimes R$ sector have a similar $1/g$ dependence [14,15]. Such effects are expected from general considerations in string theory [16] based on estimates of the divergence of closed-string perturbation theory and matrix model calculations. Furthermore, we will show later that the constant c is determined by a quantization condition that fixes $Q^{(-1)}$ to be $2\pi n$, where n is an integer and the action (23) is that of a D-instanton [3]. The charge $Q^{(-1)}$ has a clear interpretation in the string frame as the charge that flows down the throat of the wormhole. It thus represents the violation of the charge in a physical spacetime process.

Multi-instanton solutions, with $k + 1$ asymptotically Euclidean regions located at x_i are obtained by taking the solution of (12),

$$e^\phi = e^{\phi_\infty} + \sum_{i=1}^{i=k} \frac{c_i}{(x - x_i)^8},$$

(24)

where c_i are positive constants that define the charges, $|Q_i^{(-1)}| = 8c_i g \, \mathrm{Vol}(S^9)$. All of these instantons are simply connected.

3. The seven-brane solution

The seven-brane is a soliton with an eight-dimensional world-volume characterized by non-trivial behaviour of the scalar fields and the metric in the two transverse euclidean dimensions. Defining $\tau_1 = a$ and $\tau_2 = e^{-\phi}$ (where $\tau = \tau_1 + i\tau_2$ spans the upper-half plane) the lagrangian (1) is identified with the $SL(2, \mathbb{R})$-invariant lagrangian considered in [12],

$$I = \int_M \left(-R + \frac{(\partial \tau_1)^2 + (\partial \tau_2)^2}{2\tau_2^2} \right) - \int_{\partial M} 2[\mathrm{Tr}K],$$

(25)

The global $SL(2, \mathbb{R})$ symmetry of this action is expected to be broken in string theory to a residual $SL(2, \mathbb{Z})$ that is interpreted as a local symmetry, so that τ should be restricted to the fundamental domain of the modular group, $|\tau| \geq 1$, $-1/2 \leq \tau_1 \leq 1/2$. Thus, the target space of the sigma model is a non-compact orbifold of finite volume with two orbifold points at $\tau = i$ and $\tau = e^{\frac{i\pi}{3}}$, and a cusp at $\tau_2 = \infty$. Many arguments have been advanced that point to this breaking of the continuous global symmetry. This restriction of the domain of the scalar fields is an essential ingredient in the following. Moreover it appears that unless one restricts the domain in this way, there are probably no finite energy solutions.

The ansatz for the seven-brane solution is one in which the fields are trivial in the eight dimensions of the world-volume of the brane and non-trivial in the transverse space (r, θ) so that the metric in the Einstein frame takes the form,

$$ds_E^2 = -dt^2 + (dx^1)^2 + \dots (dx^7)^2 + \Omega^2(dr^2 + r^2 d\theta^2),$$

(26)

where $\Omega = \Omega(r, \theta)$. Introducing the complex co-ordinate $z = re^{i\theta}$, the complex scalar field τ will be taken to satisfy the holomorphic (or antiholomorphic) ansatz, $\bar\partial \tau = 0$ (or $\partial \tau = 0$). The Einstein equation for Ω is

$$\partial\bar\partial \ln \Omega = \frac{2\partial\tau\bar\partial\bar\tau}{(\tau - \bar\tau)^2} = 2\partial\bar\partial \ln \tau_2.$$

(27)

Just as in the case of the stringy cosmic string of [12] the single seven-brane solution is obtained by choosing $\tau(z)$ so that the pull-back of the elliptic modular function has a

single pole, for example a pole at infinity,

$$j(\tau(z)) = bz, \tag{28}$$

The constant b determines the value of the dilaton as $r \to \infty$. In that case $\Omega \sim r^{-\frac{1}{12}}$ as $r \to \infty$ and the space transverse to the seven-brane is asymptotically conical with deficit angle $\delta = \frac{\pi}{6}$. We can easily see that this is consistent with supersymmetry by considering (7) and (8). We will now use with complex numbers to describe supersymmetry, rather than double numbers, because the signature is Lorentzian. Demanding that $\gamma^1 \gamma^2 (\epsilon_1 + i\epsilon_2) = \pm i(\epsilon_1 + i\epsilon_2)$ enforces the condition $\delta\psi_\mu = 0$ since $\omega_\mu^{12} = Q_\mu$ for this ansatz. In other words the spin connection and the composite gauge connection cancel. Asymptotically conical spacetimes usually do not admit covariantly constant spinors, but in this case, the non-trivial gravitational holonomy is cancelled by that of the $U(1)$ gauge field. The fact that $\delta\lambda = 0$ follows from (7) making use of the holomorphicity of the field τ.

A solution of the equations of motion that follows from (25) has $R = \frac{1}{2}((\partial\tau_1)^2 + (\partial\tau_2)^2)/\tau_2^2$ so that the energy of the solution comes entirely from the boundary contribution. The energy per unit seven-volume of the seven-brane with these boundary conditions satisfies a Bogomol'nyi bound

$$E = 2\delta \geq \frac{\pi}{3}, \tag{29}$$

with equality if and only if τ is either holomorphic or antiholomorphic corresponding to the supersymmetric case [17].

The magnetic charge $P^{(7)}$ of the seven-brane is

$$P^{(7)} = \oint F_\mu d\Sigma^\mu = -\frac{1}{2\pi} \int_0^{2\pi} d\theta = -1 \tag{30}$$

where the line integral is taken around a closed loop at infinity, and we have used the fact that as $r \to \infty, a \sim -\frac{\theta}{2\pi}$. This charge is localized on the inverse images of i and $e^{\frac{i\pi}{3}}$, the two orbifold points of the target space. Thus this seven-brane behaves as one would expect of a singular source for the magnetic field.

One can straightforwardly extend the analysis of [12] for multi-strings to find multi-seven-brane solutions by replacing (28) by

$$j(\tau(z)) = \frac{P(z)}{Q(z)} \tag{31}$$

where $P(z)$ and $Q(z)$ are polynomials in z of order m and n respectively with no common factors. If $m > n$, we obtain a solution with $k = m$ seven-branes. If $m \leq n$, the

solution represents $k = n$ seven-branes. Provided that $k < 12$, the transverse space is asymptotically conical. If $k = 12$, the transverse space is asymptotically cylindrical, and if $k > 12$ the transverse space has finite volume; in general it is singular apart from the exceptional case of $k = 24$. There appears to be an interesting difference between multi-seven-branes and multi-instantons. Two seven-branes may continuously approach one another until they coincide, the resultant seven-brane having a charge equal to the sum of the individual charges. By contrast, for our instantons the analogous process does not seem to be possible.

4. **Quantization condition**

The $R \otimes R$ charges carried by the seven-brane and the instanton are related by the same quantization condition that applies to magnetic and electric charges of the other pairs of branes with p and $\tilde{p}$ [18,19,20] where in the case of superstrings $p + \tilde{p} = 6$,

$$P^{(\tilde{p})} Q^{(p)} = 2\pi n, \qquad n \in \mathbb{Z}. \tag{32}$$

In our case, $p = -1$ and $\tilde{p} = 7$, so that substituting the fact that the electric charge of the elementary seven-brane solution has a quantized charge, $P^{(7)} = m$ (where $|m| = 0, 1, \ldots, 11, 12, 24$), gives a quantization condition on the charge carried by the instanton,

$$Q^{(-1)} = 2\pi n. \tag{33}$$

Using (22) this determines c in the single instanton solution to be

$$c = \frac{3|n|}{\pi^{3/2}}. \tag{34}$$

With this value the action for the instanton in (23) agrees with that of the D-instanton [3], where it is determined in an altogether different procedure as a functional integral over a string world-sheet with the topology of a disk.

5. Discussion

In this paper we have argued that the D-Instanton of type IIB string theory may be identified with a BPS spacetime wormhole solution of the euclidean ten-dimensional type IIB supergravity theory carrying R⊗R electric charge. We have also argued that the dual magnetically charged seven-brane may be identified with a solution constructed from the stringy cosmic string of [12]. Consistent with our interpretation, the instanton solution has finite euclidean action $I_{inst} = \frac{2\pi}{g}|Q^{(-1)}|$, where $|Q^{(-1)}|$ is the electric charge violated by the instanton, (the charge flowing through the wormhole neck). The instanton is geodesically complete in the string frame and the dilaton field ϕ is everywhere finite though it diverges at infinity in one asymptotically flat region. It is also consistent that the seven-brane has finite energy per unit seven-volume, saturates a Bogomol'nyi bound and is geodesically complete in the Einstein frame, but not in string frame. The magnetic charge resides entirely at two point sources. The total magnetic charge $P^{(7)}$ is quantized and satisfies a Dirac-Teitelboim-Nepomechie quantization condition.

Our $R \otimes R$ electrically charged D-instanton or -1-brane has a number of features which distinguish it from other $R \otimes R$ $p-$branes. Firstly it has a finite throat and two asymptotically flat regions, while in other cases any throat is necessarily infinitely long. This may be partly understood using supersymmetry. The Lorentzian solutions have Killing spinors ϵ and the associated Killing vector $\bar{\epsilon}\gamma^{\mu}\epsilon$ can never become spacelike. Thus if these solutions have regular horizons they must be of extreme type which means that the surfaces of constant time resemble those in the extreme Reissner-Nordström solution and have the form of infinitely long throats with an internal infinity rather than the finite Einstein-Rosen throats encountered on the surfaces of constant time of non-extreme black holes. For our euclidean instanton one cannot construct a timelike Killing vector from the Killing spinor. In fact in the string frame the metric of our instanton coincides exactly with the constant time surfaces of the 11-dimensional vacuum black hole solution of the vacuum Einstein equations but we suspect that this a coincidence.

It is also interesting to contrast our ten-dimensional $R \otimes R$ instanton with the closely related four dimensional axionic instanton of the $NS \otimes NS$ sector [21,22]. This may also be thought of as the four dimensional transverse space of the neutral five-brane [23]. In string frame, the solution is the product of flat 6-dimensional Minkowski spacetime $(t, y^1, ...y^5)$ with a curved four dimensional transverse space of the form $ds^2 = (e^{2\phi_{\infty}} + \frac{c}{r^2})(dr^2 + r^2 d\Omega_3^2)$. In the Einstein frame the transverse space is flat. If t is imaginary, the solution may be interpreted as an instanton with an infinite throat. In the flat four-dimensional Einstein frame the dilaton ϕ and $NS \otimes NS$ 3-form field strength $F_{(3)}$ satisfy the self-duality condition which guarantees supersymmetry: $d\phi = \pm e^{-2\phi} * F_{(3)}$ which is similar to the condition for our ten-dimensional instanton. The main difference is that due to the $NS \otimes NS$ three-form $F_{(3)}$ coupling with a different power of e^{ϕ} compared to the $R \otimes R$ fields, it is the square of e^{ϕ} which is harmonic. This is why the throat is infinite rather than finite. Moreover, for the same reason, the Euclidean action of this $NS \otimes NS$ instanton is proportional to $\frac{1}{g^2}$ not $\frac{1}{g}$ as it is for our ten-dimensional R⊗R instanton.

One of the most intriguing aspects of our work is the relation to the breaking of continuous global symmetries. On the one hand superstring theory is believed to have

no continuous symmetries. On the other hand, in low energy quantum gravity the thesis that black holes and possibly wormholes should lead to a violation of the conservation of charges associated with continuous and possibly discrete symmetries has been strenuously argued [24,25], and equally strenuously rebutted [26,27]. The connection between real black holes, virtual black holes and instantons in this context remains, however, obscure [28,29].

The relevant continuous symmetries are contained in $SL(2,\mathbb{R})$. From the point of view of string theory it is believed on quite general grounds that this must break down to the modular subgroup $SL(2,\mathbb{Z})$. Now $SL(2,\mathbb{R})$ is certainly a symmetry of the classical equations of motion of type IIB supergravity theory written in terms of the dilaton and pseudoscalar field a. In particular the equation of motion for a may be thought of as the conservation of a Noether current $J_\mu^{\text{Noether}} = e^{2\phi}\partial_\mu a$ arising from the translation subgroup: $a \to a + \text{constant}$. The associated charge is $Q^{(-1)}$. It may be argued that quantum mechanically spacetime wormholes will lead to the violation of the conservation of any Noether charge because some of the current $J_\mu^{\text{Noether}} = e^{2\phi}\partial_\mu a$ may flow down the throat. It may also be argued that black holes or black branes should lead to a violation of the conservation of Noether charges. We shall comment on this later.

The discussion so far has assumed that the global translational symmetry $a \to a+\text{constant}$ is well-defined. This would be true of our sigma model if the target space were the entire upper half plane $SL(2,\mathbb{R})/SO(2)$. But it is not. Rather it is the fundamental domain of the modular group $SL(2,\mathbb{Z})\backslash SL(2,\mathbb{R})/SO(2)$. Thus the translations do not act globally on the target space of our sigma model. In other words the continuous global symmetry is not well defined. There are two apparently unrelated reasons for quotienting out by the modular group. One is string inspired. The other is that unless we do so, we cannot obtain a seven-brane with finite total energy.

Thus both for stringy reasons and by virtue of wormhole effects we do not expect the electric charge to be conserved. What about the magnetic charge $P^{(7)}$ of the seven-brane and what about black holes? The usual physical arguments for the violation of global charges by black holes rely on two main planks. Firstly, there should be a no-hair theorem for the charge, and secondly there should be a lower bound to the mass of any state carrying the relevant charge. In our case, it is straightforward to show that if the target space is the entire upper half plane then there are no black hole solutions with regular event horizons and non-constant scalar fields. If the target space is the fundamental domain of the modular group, the argument is not quite so straightforward but as far as we can tell the result seems to hold. Moreover, the same no-hair results seem to apply to other p-branes. Secondly, the only states carrying $R \otimes R$ charge are p-branes, in particular the only states we know of that carry $P^{(7)}$ are the seven-branes of this paper, which satisfy a Bogomol'nyi bound on the energy per unit seven-volume. Therefore there are no light states in the theory that carry $R \otimes R$ charges. Thus it seems plausible that the conservation of $P^{(7)}$ could be violated by dropping seven-branes into a black hole or a black p-brane and letting it evaporate.

Finally we conclude with the observation that the construction of the D-instanton wormhole solutions in this paper opens up the prospect of studying a variety of non-perturbative effects of great interest in quantum gravity within a *controlled* computational scheme. These include the breaking of supersymmetry, the nature of a possible

non-perturbative dilaton potential and the problem of the cosmological constant.

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