String Fluid from Unstable D-branes

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ABSTRACT

We consider Sen's effective action for unstable D-branes, and study its classical dynamics exactly. In the true vacuum, the Hamiltonian dynamics remains well-defined despite a vanishing action, and is that of massive relativistic string fluid of freely moving electric flux lines. The energy(tension) density equals the flux density in the local co-moving frame. Furthermore, a finite dual Lagrangian exists and is related to the Nielsen-Olesen field theory of "dual" strings, supplemented by a crucial constraint. We conclude with discussion on the endpoint of tachyon condensation.

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1 Introduction

When an unstable D-brane or a pair of D-brane and anti-D-brane decays, all open string states should disappear. Around the new true vacuum of the theory where brane tension is all dissipated away, it follows, the theory must consist of closed string states only. Exactly how this is achieved from the open string point of view, however, is far from clear. In particular, very little is known about whether any open string degrees of freedom survive, say, in the form of light coherent or solitonic states, and if so how they are related to the closed string degrees of freedom. One possibility is that such states correspond to usual closed string modes repopulating the worldvolume. It is the purpose of this note to shed some light one some of these issues.

There has been incremental progress in understanding how open string state might be removed from the spectrum. The pioneering work by A. Sen addressed the fate of some massless open string modes [1]. In particular, in a D-brane and anti-D-brane system, one linear combination of massless gauge fields was shown to acquire a massgap via an Abelian Higgs mechanism. While this finite mass seemed to fall short of removing the state altogether, it is a step toward the right direction.

This naturally leads to the other linear combination of gauge fields that has no charged fields in perturbative open string sector, which posed extra difficulty in understanding how it was removed from the spectrum [2, 3]. On the other hand, while no perturbative state is charged under U(1), there are nonperturbative states that carry magnetic charge. This gives one a possibility that a dual Higgs mechanism exists, induces a massgap, and confines the U(1) gauge field [4]. For instance, one finds precisely this nonperturbative behavior from S-dualizing the perturbative sector of D3-anti-D3 system.

One drawback of the picture is that details of the scenario have been checked largely in the strong coupling picture. The practical matter of how one may study the mechanism in the weak coupling regime, beyond the formal manipulation such as introducing gauge-invariant mass term for dual gauge field, still remains. However, regardless of such details, it seems clear that the confined flux strings must emerge in any regime, if one is to believe in the charge conservation; for the electric flux of this U(1) carries the fundamental string charges [5].

Understanding of this gauge field at weak coupling were considerably improved with the introduction of an effective action by Sen [6, 7]. The tachyon potential is identified with the vanishing tension of the decaying branes which appears in front

of the Born-Infeld action,

$$-V(T)\sqrt{-Det(\eta+\mathcal{F})},\tag{1}$$

and sets the inverse coupling of the gauge field in question. The subsequent large coupling would certainly confine charged particles, if there were any. However, this by itself does not quite explain how the gauge field might be removed. The charge confinement here is more like that in the classical QED_{2+1} , where the confinement is simply due to the behavior of the Green's function. For instance, no massgap is to be found.¹ It is here where the dual or magnetic objects can play their most important role. They can generate a mass gap, correctly quantize the electric flux and confine it into a thin tube. In particular, it was shown [5] in the 2+1 dimensional case, where the dual "objects" generate the potential term for the dual photon à la Polyakov, that the electric flux tube tends to be completely squeezed and the tension approaches the exact value of the fundamental strings.

While this U(1) gauge field is only one of infinitely many open string modes, proper understanding of its fate may lead to something far more interesting than how open string modes disappear. To see this, it suffices to recall that \mathcal{F} is the open string mode that carries the fundamental string charge, and its dynamics may as well play a role in the true vacuum which is supposed to be a closed string theory.

In this note, we will take the classical Lagrangian of the general form (1), and study the dynamics in some detail. First we reformulate the theory in the Hamiltonian formalism. We must emphasize that this is not a sterile mathematical exercise, the crucial point being that, in the limit of V=0, the correct physical degrees of freedoms appear naturally in the Hamiltonian approach [5]. From general form of Hamiltonian, we learn that the theory reduced to that of stringy fluid of electric flux lines. A finite and fully relativistic, dual formulation is shown to exist, and used to study the stringy behavior of the theory. Finally we discuss on possible interpretations.

2 Canonical Formulation of Born-Infeld Theory

The Lagrangian is

$$\mathcal{L} = -V(T)\sqrt{-Det(\eta + \mathcal{F} + \partial X^I \partial X_I)},$$
(2)

¹Recent computations [8, 9] suggest that an explicit mass term might be present in the tree-level open string field theory, as opposed to (1). See the last section of [8] for a discussion on this issue. See also [10] for a related computation. For a more recent discussion see [11].

where X^I , $I=1,\ldots,D-d$ are transverse scalars, with the spacetime dimension D, and $\mathcal{F}_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}$ is the curvature tensor of the gauge field. We will adopt the mostly positive signature for the metric. While we put the tachyonic potential V(T) in front, the analysis in this section is applicable when V is replaced by the constant tension. A more recent proposal [12] places tachyon kinetic term inside the determinant on equal footing as X^I . This possibility is also covered by the analysis in this section.

The determinant can be expressed more conveniently in terms of $E_i \equiv \mathcal{F}_{0i}$ and $F_{ij} = \mathcal{F}_{ij}$ as

$$- Det(\eta + F + \nabla X^I \nabla X_I) = Det(h)(1 - \dot{X}^I \dot{X}_I) - E_i^{(+)} D_{ik} E_k^{(-)}, \tag{3}$$

where we further defined

$$h_{ik} \equiv \delta_{ik} + F_{ik} + \partial_i X^I \partial_k X_I, \tag{4}$$

and the matrix D as

$$D_{ik} = (-1)^{i+k} \Delta_{ki}(h), \tag{5}$$

where Δ_{ki} denotes the determinant of the matrix with the k^{th} row and the i^{th} column omitted. When h is invertible, this is equivalent to

$$D = Det(h)h^{-1}. (6)$$

We also introduced a shorthand notation

$$E_i^{(\pm)} \equiv E_i \pm \dot{X}^I \partial_i X_I. \tag{7}$$

Now that we isolated the *E*-dependence of the Lagrangian, it is a straightforward exercise to find the canonical variables and find the Hamiltonian. Conjugate momenta are,

$$\pi^{i} = \frac{\delta \mathcal{L}}{\delta \dot{A}_{i}} = \frac{V}{\sqrt{-Det(\eta + \mathcal{F} + \partial X^{I} \partial X_{I})}} \frac{E_{k}^{(+)} D_{ki} + D_{ik} E_{k}^{(-)}}{2}, \tag{8}$$

which satisfy the Gauss constraint $\partial_i \pi^i = 0$, and

$$p_{I} = \frac{\delta \mathcal{L}}{\delta \dot{X}^{I}} = \frac{V}{\sqrt{-Det(\eta + \mathcal{F} + \partial X^{I} \partial X_{I})}} \left(Det(h) \dot{X}_{I} - \frac{E_{k}^{(+)} D_{ki} - D_{ik} E_{k}^{(-)}}{2} \partial_{i} X_{I} \right), \tag{9}$$

which are conjugate to X^{I} . The Hamiltonian consistent with the Gauss constraint is obtained by the following Legendre transformation

$$\mathcal{H} = \pi^i E_i + p_I \dot{X}^I - \mathcal{L},\tag{10}$$

which yields

$$\mathcal{H} = \frac{VDet(h)}{\sqrt{-Det(\eta + \mathcal{F} + \partial X^I \partial X_I)}}.$$
 (11)

This Hamiltonian, when expressed in terms of the canonical variables, becomes

$$\mathcal{H} = \sqrt{\pi^i \pi^i + p_I p^I + (\pi^i \partial_i X^I)^2 + (F_{ij} \pi^j + \partial_i X^I p_I)^2 + V^2 Det(h)}$$
(12)

As mentioned at the top of the section, this computation is carried over straightforwardly when we include a tachyon kinetic term inside the square-root, à la Bergshoeff et.al. All we need to do is to introduce another scalar T on equal footing as X^I .

An important set of physical quantities are the conserved Noether momenta. The time translation invariance leads to the conserved energy, \mathcal{H} , while the spatial translation leads to conserved momentum,

$$\mathcal{P}_i = -F_{ik}\pi^k - \partial_i X^I p_I, \tag{13}$$

where we covariantized the Noether momentum by adding a total derivative.

As an aside, we note that the Hamiltonian can be written formally as

$$\sqrt{\pi^M \pi^M + \mathcal{P}_M \mathcal{P}_M + V^2 Det(h)},\tag{14}$$

where π^M is conjugate to $(A_M) = (A_i, X_I)$, and the "conserved momenta" \mathcal{P}_M is

$$\mathcal{P}_N = -F_{NM}\pi^M,\tag{15}$$

with $F_{NM} = \partial_N A_M - \partial_M A_N$ with $\partial_I \equiv 0$. The potential piece Det(h) can also be written in a similar fashion [13];

$$Det(\delta_{ij} + F_{ij} + \partial_i X^I \partial_j X^I) = Det(\delta_{NM} + F_{NM}).$$
(16)

This reflects the underlying T-duality, and provides a consistency check of the Hamiltonian we derived.

3 Electric Flux after Tachyon Condensation

Suppose we started with an unstable brane with some electric flux on it, and let it decay via tachyon condensation. When the tachyon condenses to its true ground state where the potential V(T) vanishes [7, 14, 15], the Lagrangian vanishes. Despite this rather violent process, the electric flux must be preserved somehow. A crucial point we need to note is that the conserved electric flux is not given by E_i but rather by π^i .

The situation is analogous to that of massless relativistic particle. The Lagrangian $L = -m\sqrt{1-v^2}$ vanishes in the limit $m \to 0$ but the Hamiltonian $H = \sqrt{p^2 + m^2}$ survives the limit. (The analogy is more or less precise in the case of unstable D1-brane, as shown in [5] and later repeated in [16].) The necessity of use of π^i rather than E_i can also be seen in several ways. One way is to observe that π^i is the quantity that is constrained by the Gauss constraint. Perhaps a more intuitive way is to recall that π^i is the quantity that acts as the source for NS-NS spacetime two-form field B associated with the fundamental strings. This is because B appears in the Born-Infeld action in the combination $F + \hat{B}$ where \hat{B} denotes the pull-back of B to the worldvolume. In other words, π^i carries the spacetime fundamental string charge which has to be preserved no matter what.

At the end of decay process, when the D-brane tension is dissipated by emission of closed string states, the Hamiltonian is obtained by setting V = 0,

$$\mathcal{H} = \sqrt{\pi^i \pi^i + p_I p^I + (\pi^i \partial_i X^I)^2 + \mathcal{P}_i \mathcal{P}_i}.$$
 (17)

Given a total flux, the part of energy that cannot be dissipated away is the one associated with π . When all other excitations vanish, the energy of the system is simply

$$|\pi| = \sqrt{\pi^2}.\tag{18}$$

To minimize the energy, the flux line wants to be straight and directed along one direction. Once this is achieved, however, the energetics does not care how the flux lines are distributed. In the co-moving frame of straight and parallel flux lines, the energy per unit length equals the total flux [5].

For flux lines moving on the worldvolume, we can define a Lorentz-invariant quantity τ that reduces to $|\pi|$ in the local co-moving frame,

$$\tau = \pi^2 / \mathcal{H}. \tag{19}$$

The invariance property will be shown in section 5. Consider the effect of turning on momenta \mathcal{P} . The energy is,

$$\mathcal{H} = \sqrt{\pi^i \pi^i + \mathcal{P}_i \mathcal{P}_i} = \sqrt{\pi^2 + \mathcal{H}^2 v^2},\tag{20}$$

where $v_i \equiv \mathcal{P}_i/\mathcal{H}$ is the velocity field. The energy density of the moving flux lines is

$$\mathcal{H} = \tau \times \frac{1}{1 - v^2}.\tag{21}$$

The energy density of the moving flux lines are equal to boosted energy density. In other words, the flux lines behave as massive relativistic string fluid.

Turning on the transverse scalars as a perturbation, we find another interesting aspect of the flux line dynamics. When π is the dominant quantity, we may expand p_I to find

$$p_I \simeq -\mathcal{H}\dot{X}_I.$$
 (22)

When the flux lines move out of the worldvolume with a uniform velocity, then, the energy associated with a uniform motion becomes

$$\mathcal{H} \simeq \tau \times \frac{1}{1 - v^2 - (\dot{X}^I)^2}.$$
 (23)

This suggests that the transverse motion is also relativistic and on equal footing as motions within worldvolume.

When the flux string bends and stretches along the transverse directions, this turn on $(\pi^i \partial_i X^I)^2$ term, upon which an additional term in the Hamiltonian,

$$\mathcal{H} = \sqrt{\pi^2 (1 + (\partial_{\parallel} X^I)^2) + \cdots}.$$
 (24)

This precisely takes into account of the fact that the flux line has been lengthened. The effective tension density is again $|\pi|$ measured in the co-moving frame. Therefore, we conclude that the flux lines behave as if they are a continuum of massive relativistic strings that moves in all of spacetime.

4 Dual Description and Equation of Motion

We saw that the dynamics is perfectly finite despite the vanishing action. All this says, on the other hand, is that the coordinate variables we started with, namely the U(1) gauge field, are not necessarily good variables. A natural followup question is whether there exists another set of variables which are good in this limit of V=0. For the remainder of the note, we will ignore the transverse scalars. Inclusion of them would proceed trivially along the same spirit as in (14), without extra work.

A standard trick [17] is to dualize the fields where one chooses to treat the magnetic fields, F_{ij} , to be the conjugate momenta of the new, dual variables. Performing Legendre transformation on F_{ij} , one finds,

$$\mathcal{L}' = \mathcal{H} - \frac{1}{2} F_{ij} K^{ij} = \sqrt{\pi^2 - K^2/2}, \tag{25}$$

with

$$K^{ij} = 2\frac{\delta \mathcal{H}}{\delta F_{ij}} = \frac{1}{\mathcal{H}} \left(F_{im} \pi^m \pi_j - \pi_i F_{jm} \pi_m \right). \tag{26}$$

The underlying, dual variables are (d-3)-form potential C, whose (d-2)-form field strength G=dC is related to π and K via,

$$\mathcal{K} \equiv \pi_i \, dt \wedge dx^i + \frac{1}{2} \, K_{ij} dx^i \wedge dx^j = *G, \tag{27}$$

where * is the Hodge-dual operator. The Legendre transformation gives

$$\mathcal{L}' = \sqrt{-\mathcal{K}^2/2} = \sqrt{G^2/2}.$$
 (28)

Note that $(G_{0\cdots})^2$ comes with the negative sign, so that stable field configurations are those with dominant magnetic G-fields, or equivalently those with dominant electric \mathcal{K} -field.

However, this \mathcal{L}' is not quite what we want. A subtlety arises from the fact that the Hamiltonian \mathcal{H} has flat directions in F_{ij} ; K^{ij} as a function of F_{ij} is rather restricted, and must be parallel to π_i . In particular \mathcal{K} obeys the constraint $\mathcal{K} \wedge \mathcal{K} = 0$. The subtlety then manifests itself in a pathology that the canonical analysis of \mathcal{L}' does not lead us back to the Hamiltonian \mathcal{H} . For this, there is a simple cure; we only need to impose a constraint via a Lagrange multiplier² (d-4)-form μ that imposes the constraint $\mathcal{K} \wedge \mathcal{K} = 0$;

$$\mathcal{L}_{\text{dual}} = \sqrt{G^2/2} + \langle *\mu, *G \wedge *G \rangle / 4. \tag{29}$$

The equation of motion is

$$0 = d\left(\frac{\mathcal{K}}{\sqrt{-\mathcal{K}^2/2}} + *(\mu \wedge \mathcal{K})\right),\tag{30}$$

which, in terms of the field strength $\mathcal{F}_{\mu\nu}$ of the original variable, is the Bianchi identity $d\mathcal{F} = 0$. The Bianchi identity for G, dG = 0, which reads as

$$\partial^{\mu} \mathcal{K}_{\mu\nu} = 0 \tag{31}$$

is equivalent to the equation of motion and the Gauss constraint of the original description. The derivation of these statements is rather technical and therefore is recorded in Appendix.

The constraint thus imposed $*G \wedge *G = \mathcal{K} \wedge \mathcal{K} = 0$ has a rather simple physical interpretation; it tells us that, at each point, the two-form \mathcal{K} is proportional to an

² All fields here must have an origin in string theory, yet the origin of μ appears a bit mysterious; No worldvolume field of such a type is apparent in the perturbative open string sector. One massless tensor that would be of rank (d-4) is the "would-be Goldstone boson" associated with the dual magnetic objects. In the present limit, it could indeed appear as a nondynamical field. Whether and why it should have an axion-like coupling is unclear, however.

area element of a two-dimensional surface. Furthermore, the stability requirement that the Lagrangian is real, tells us that the surfaces must be timelike ($\mathcal{K}^2 < 0$). From the definition of \mathcal{K} above, the family of these "surfaces" is clearly the foliation of the worldvolume by the flux fluid.

The equation of motion (30) admits a scaling symmetry

$$\mathcal{K} \to f\mathcal{K},$$

$$\mu \to \mu/f, \tag{32}$$

for any reasonable function f. On the other hand, the Bianchi identity (31) for G restricts admissible form of λ to be those satisfying

$$0 = (\partial_{\mu} f) \mathcal{K}^{\mu\nu}. \tag{33}$$

In other words, f is allowed to vary only in the plane orthogonal to the flux fluid. For any static solution with electric flux π_i only, this generates other solutions of the form $f\pi_i$ as long as the function satisfies

$$\pi^i \partial_i f = 0. (34)$$

Nothing in the classical dynamics favors a particular distribution of flux lines along any orthogonal direction.

The above form of the action, without the constraint, was first discussed by Nielsen and Olesen [18], in 4-dimensional setting. They observed that the equation of motion admits magnetic G-flux strings as special solutions of the form,

$$\mathcal{K}^{\mu\nu}(x) = \int \delta^{(d)}(x - Z(\sigma)) dZ^{\mu} \wedge dZ^{\nu}, \tag{35}$$

where $Z(\sigma)$ is a map from the worldsheet coordinates $\sigma^{0,1}$ to the worldvolume, and the integral is over $\sigma^{0,1}$. These solutions satisfy the constraint by construction, so they are also solutions of (30) with $\mu = 0$. Nielsen and Olesen also showed that these solutions behave exactly like a Nambu-Goto string. However, it is quite clear that this confined form of the flux lines is an artifact of the ansatz, as the symmetry (32) shows.

5 Energy-Momentum and Fluid Motion

This dual action allows an easy computation of the energy momentum tensor, which comes out to be quite simple;

$$T_{\mu\nu} = 2 \frac{\delta}{\delta g^{\mu\nu}} \left(\sqrt{g} \sqrt{g^{\alpha_1 \beta_1} \cdots g^{\alpha_{d-2} \beta_{d-2}} G_{\alpha_1 \cdots \alpha_{d-2}} G_{\beta_1 \cdots \beta_{d-2}}} \right) = \frac{\mathcal{K}_{\mu\lambda} \mathcal{K}_{\nu}^{\lambda}}{\sqrt{-\mathcal{K}^2/2}}.$$
 (36)

The contribution from the Lagrange multiplier goes away once the constraint is used. It is straightforward to check the conservation of the energy-momentum, $\partial^{\mu}T_{\mu\nu} = 0$, using (30), (31), and the constraint.

The temporal part $T_{0\mu}$ reproduces the energy density and the momentum density faithfully,

$$T_{00} = \frac{\pi^2}{\sqrt{\pi^2 - K^2/2}} = \sqrt{\pi^2 + \mathcal{P}^2} = \mathcal{H},$$

$$T_{0i} = \frac{\pi^m K_{im}}{\sqrt{\pi^2 - K^2/2}} = F_{ij}\pi^j = -\mathcal{P}_i,$$
(37)

while the stress-tensor is equally simple;

$$T_{ij} = (-\pi_i \pi_j + \mathcal{P}_i \mathcal{P}_j) / \mathcal{H} = \frac{-\pi_i \pi_j + \mathcal{P}_i \mathcal{P}_j}{\sqrt{\pi^2 + \mathcal{P}^2}}.$$
 (38)

It is fairly easy to see how this form of energy-momentum would arise from a fluid of flux lines. Suppose we started with a bundle of straight flux lines, say pointing toward direction 1. Let the distribution in the co-moving frame be characterized by a Lorentz scalar function, which we call τ' , so that the rest frame energy-momentum is

$$\begin{pmatrix} \tau' & 0 & 0 & \cdots \\ 0 & -\tau' & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}. \tag{39}$$

Suppose we boosted the system along a direction orthogonal to the bundle, say direction 2, with speed v. The Lorentz transformation takes this energy-momentum tensor to

$$\begin{pmatrix}
\tau'\gamma^2 & 0 & -\tau'\gamma^2v & \cdots \\
0 & -\tau' & 0 & \cdots \\
-\tau'\gamma^2v & 0 & \tau'\gamma^2v^2 & \cdots \\
\vdots & \vdots & \vdots & \vdots
\end{pmatrix},$$
(40)

with the dilation factor $\gamma = 1/\sqrt{1-v^2}$. Identifying the energy density $\tau'\gamma^2$ with \mathcal{H} and the momentum density $\tau'\gamma^2v$ with \mathcal{P} , we see that the local form of the above energy-momentum tensor is reproduced precisely. It is a matter of identifying $\pi_i/|\pi|$ with the direction 1, and $\mathcal{P}_i/|\mathcal{P}|$ (which is necessarily orthogonal to π_i) with direction 2. As a final consistency check, one finds that τ' is correctly mapped to a Lorentz invariant quantity;

$$\tau' = \mathcal{H}/\gamma^2 = \mathcal{H}\left(1 - (\mathcal{P}/\mathcal{H})^2\right) = \frac{\pi^2}{\mathcal{H}} = \tau,\tag{41}$$

which we promised earlier to be Lorentz invariant. Indeed,

$$\tau = -\frac{1}{2} T_{\mu}^{\ \mu} = \sqrt{-\mathcal{K}^2/2},\tag{42}$$

is a Lorentz scalar. In the co-moving frame, therefore, the only force is the pull along the flux lines coming from this tension density τ . The theory reduced to that of flux lines with no mutual pressure.

For the sake of completeness, we record here the equations of motion for the flux lines. The conservation of the energy-momentum combined with the G-Bianchi identity produces,

$$\partial_0 n^i + v^k \partial_k n^i = n^k \partial_k v^i,$$

$$\partial_0 v^i + v^k \partial_k v^i = n^k \partial_k n^i,$$
(43)

where the vector fields n and v are defined as

$$\pi = \mathcal{H}n,$$

$$\mathcal{P} = \mathcal{H}v,$$
(44)

which satisfies the constraints

$$n^i n^i + v^i v^i = 1, \qquad n^i v^i = 0.$$
 (45)

The evolution of the energy density \mathcal{H} is then determined via,

$$\partial_0 \mathcal{H} + v^k \partial_k \mathcal{H} + \mathcal{H}(\partial_k v^k) = 0, \tag{46}$$

supplemented by an initial condition satisfying $0 = \partial_i \pi^i = \partial_i (\mathcal{H}n^i)$. The equations of motion (43) have exceedingly simple physical interpretations. The first is merely a continuity equation; the flux lines bend in response to the gradient in the velocity field. The second, dynamical equation says that the velocity fields adjust itself to straighten flux lines. The latter again confirms that in the comoving frame, the only acceleration arises from the tension.

The system of a 2-form field \mathcal{K} constrained by $\mathcal{K} \wedge \mathcal{K} = 0$ and $\mathcal{K}_{[\rho\lambda}\partial^{\mu}\mathcal{K}_{\nu]\mu} = 0$ with the energy momentum tensor (36) has been considered in [19, 20] and called string dust or string cloud model. (The second constraint follows from the G-Bianchi identity (31).)

6 Discussion

We analyzed Sen's effective action for unstable D-branes, and found that, classically, the gauge system reduces exactly to a theory of flux fluid which carries fundamental string charge density. Tension density, electric flux density, and fundamental string charge density all coincide. The theory contains Nambu-Goto strings as special solutions, but does not naturally form confined strings of unit flux. The symmetry (32) of the equation of motion pretty much guarantees this.

This brings us back to the original question of what is the endpoint of the tachyon condensation. In full string theories, there is little doubt that the true vacuum is some sort of closed string theory. For non-spacetime-filling branes, the unstable brane system can be thought of as a classical lump of energy in a closed string theory, and all that happen is the lump dissipates away to infinity, leaving behind a vacuum of the same closed string theory.

In order to obtain Sen's effective action, however, one must perform several truncations of the theory. First one considers the open string sector rather than the closed string theory. Here the original unstable branes cannot be thought of as classical lump anymore. One then truncates to the massless fields plus the tachyon, and also takes the classical limit of the theory. Note that the effective coupling of the truncated theory is not the string coupling but the string coupling divided by V. This somewhat violent limit must be responsible for the appearance of the string fluid.

The interpretation of this flux fluid remains to be found. The most cautious approach would be to view the classical theory as valid only in the macroscopic settings. If one cannot distinguish distribution of fundamental strings from continuum electric flux, and he may identify one with the other. Whatever the microscopic mechanism of forming the fundamental strings, macroscopic observers find the fundamental strings that moves in the entire spacetime, rather than just inside the D-brane worldvolume. This would be the correct interpretation if the classical dynamics is unable to resolve a length scale arbitrarily larger than $\sqrt{\alpha'}$, and if the unit flux of a fundamental string is viewed as infinitesimal.

A better interpretation would be that we are allowed, within the classical dynamics, to distinguish between isolated unit flux tube and well-dispersed distribution. If this is the case, we have a natural framework to address the question of how fundamental string is recovered from the originally dispersed flux lines. As emphasized above, the classical system does not care how flux lines are distributed in a rest

frame.³ While the smallest of perturbation in the gauge sector could render the flux lines repulsive or attractive, such a purely energetical mechanism is unlikely to produce strings of precise unit flux. What one needs is *quantization of flux in local sense*, rather than in global sense, which points toward a mechanism involving a dual object.

A dual (i.e., magnetic) Higgs mechanism could do the trick and confine the flux into strings of unit quantum quite naturally [4]. In the language of section 4, this would generate a local, gauge-invariant, Higgs-like mass term of the dual field C [4, 21], and force the flux lines to come together. A candidate magnetic object has been identified for, say, the D-brane and anti-D-brane system: open D-brane of two less dimensions ending on the pair has the right charge. Due to the nature of the classical string fluid (that any profile in the transverse direction has the same total energy), a slight effect of the dual object can squeeze the electric flux into a very thin tube. As in the 2+1 dimensional case analyzed in [5], if that happens the tension of the flux string approaches the exact value of the fundamental string tension. (In [5], it was claimed for $p \geq 3$ cases that it was necessary to assume that the scale associated with the dual Higgs mechanism is of order 1 in the string coupling. However, it is now quite clear that one does not need such an assumption, given the lack of classical interactions between adjacent flux lines.)

Alternatively, one may think of the dual object as local deformation of world-volumes of the D-branes, accompanied by flux lines. When extended, the magnetic object is heavy and unsuitable for weak-coupling description, but they could reappear as light worldvolume fields once the decay process commences and fluctuations of D-brane become cheap.

An interesting generalization of this strong-coupling limit of Born-Infeld theory is for the M-theory branes. An unstable configuration of an M5-anti-M5 is expected to decay and produce co-dimension-three worldvolume solitons to be identified with M2-brane. It is not unreasonable to expect a membrane fluid dynamics in the limit where the five-brane tension is dissipated away. The worldvolume theory need not be chiral in this case, and we should find a simple dual description analgous to (29). This study may provide us more information on M-theory branes.

Finally we would like to comment that there have been several studies of Born-Infeld action in the limit of vanishing tension or (effectively) infinite α' . Ref. [22] studies the latter limit for D3, in effect, from which we benefited much. There were

³It was recently suggested [11] in different contexts that such a degeneracy might be a result of inappropriate choice of coordinate variables. However, the infinite degeneracy here occurs in the profile of *energy density*. Energy density does make sense in the effective field theory we are considering and should not depend on the choice of coordinates.

more recent studies of tensionless D-branes in Ref. [23], which also found string-like degrees of freedom. It is unclear to us at the moment how the latter work is related to ours.

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Appendix: Equations of Motion and Bianchi Identities

We record here some of the details in the algebraic manipulations used in section 4. In particular, we show here that the G-Bianchi identity is the \mathcal{F} -equation of motion (including the Gauss constraint), and the G-equation of motion is the \mathcal{F} -Bianchi identity. As in section 4, we set all $X^I = 0$ and the Lagrangian is

$$\mathcal{L} = -V\sqrt{Det(h) - E^T DE}.$$
 (A.1)

The conjugate momenta simplify to

$$\pi = \frac{V(D + D^T)E}{2\sqrt{Det(h) - E^T DE}},\tag{A.2}$$

and obey the Gauss law constraint $\partial_i \pi_i = 0$. The Hamiltonian is given by

$$\mathcal{H} = \sqrt{\pi^i \pi^i + (F_{ij} \pi^j)^2 + V^2 Det(h)} = \frac{V Det(h)}{\sqrt{Det(h) - E^T DE}}.$$
 (A.3)

Useful formulae to note are

$$(1 - F)\pi = \frac{VDE}{\sqrt{Det(h) - E^TDE}},$$

$$(1 + F)\pi = \frac{VD^TE}{\sqrt{Det(h) - E^TDE}},$$

$$(1 - F^2)\pi = \frac{VDet(h)E}{\sqrt{Det(h) - E^TDE}}.$$

$$(A.4)$$

Using these formulae it is straightforward to show that the variation of the Lagrangian is

$$\delta \mathcal{L} = \delta E^T \pi - \frac{V^2}{2\mathcal{H}} \text{Tr}(D\delta F) + \frac{1}{\mathcal{H}} \pi^T \delta F F \pi. \tag{A.5}$$

We drop the second term in the $V \to 0$ limit where the Hamiltonian is given by

$$\mathcal{H} = \sqrt{\pi^2 + (F\pi)^2}.\tag{A.6}$$

Then, the Euler-Lagrange equations read as (the first one is the Gauss constraint)

$$\partial_i \pi_i = 0, \tag{A.7}$$

$$\partial_0 \pi_i + \partial_j \left[\frac{1}{\mathcal{H}} \left(\pi_j (F\pi)_i - \pi_i (F\pi)_j \right) \right] = 0.$$
 (A.8)

The Bianchi identity for \mathcal{F} is expressed in terms of π^i and F_{ij} as

$$\partial_{[i}F_{jk]} = 0,$$

$$\partial_{0}F_{ij} = \partial_{i}\left(\frac{\pi_{j} - F_{jk}F_{kl}\pi_{l}}{\mathcal{H}}\right) - \partial_{j}\left(\frac{\pi_{i} - F_{ik}F_{kl}\pi_{l}}{\mathcal{H}}\right). \tag{A.9}$$

where we traded off E_i in favor of the physical field π_i .

Let us see whether the Bianchi identity and equation of motion for the dual side reproduces these equations. Let us consider the equation of motion first. First let us define

$$\mathcal{J}_{\mu\nu} := \frac{\mathcal{K}_{\mu\nu}}{\sqrt{-\mathcal{K}^2/2}} + \left[*(\mu \wedge \mathcal{K}) \right]_{\mu\nu}. \tag{A.10}$$

If μ is written in terms of a 4-form α as $\mu = *\alpha$ and if we denote $J_{ij} = \mathcal{J}_{ij}$, this decomposes into

$$\mathcal{J}_{0i} = \frac{\mathcal{K}_{0i}}{\sqrt{-\mathcal{K}^2/2}} - \frac{1}{2}\alpha_{0ijk}\mathcal{K}_{jk},\tag{A.11}$$

$$J_{ij} = \frac{\mathcal{K}_{ij}}{\sqrt{-\mathcal{K}^2/2}} + \alpha_{0ijk}\mathcal{K}_{0k} - \frac{1}{2}\alpha_{ijkl}\mathcal{K}_{kl}.$$
 (A.12)

From the constraint $\mathcal{K} \wedge \mathcal{K} = 0$ and the bound $-\mathcal{K}^2/2 \geq 0$, one can put

$$\mathcal{K}_{0i} = a_i, \tag{A.13}$$

$$\mathcal{K}_{ij} = -a_i b_j + a_j b_i, \tag{A.14}$$

with $a \cdot b = 0$ and $-\mathcal{K}^2/2 = a^2(1-b^2) \ge 0$. Taking the contractions of (A.12) with a_i and b_i and using (A.11), we obtain

$$b_i = \frac{J_{ij}a_j}{\sqrt{a^2 + (Ja)^2}},\tag{A.15}$$

$$\mathcal{J}_{0i} = \frac{a_i - (J^2 a)_i}{\sqrt{a^2 + (Ja)^2}}. (A.16)$$

Then, the equation of motion (30) reads as

$$\partial_{[i}J_{jk]} = 0, (A.17)$$

$$\partial_0 J_{ij} = \partial_i \left(\frac{a_j - (J^2 a)_j}{\sqrt{a^2 + (Ja)^2}} \right) - \partial_j \left(\frac{a_i - (J^2 a)_i}{\sqrt{a^2 + (Ja)^2}} \right),$$
 (A.18)

while the Bianchi identity (31) amounts to

$$\partial_i a_i = 0,$$
 (A.19)

$$\partial_0 a_i + \partial_j \left[\frac{a_j (Ja)_i - a_i (Ja)_j}{\sqrt{a^2 + (Ja)^2}} \right] = 0. \tag{A.20}$$

We see that these equations are equivalent to the \mathcal{F} -Bianchi identity (A.9) and \mathcal{F} equation of motion (A.8) under the identification

$$a_i = \pi_i, J_{ij} = F_{ij}. (A.21)$$

It is easy to see that this identification is consistent with the dualization of variables, (26) and (27).

This identification of the two descriptions determines the classical value of the Lagrange multiplier to be of the form,

$$\mu = * \left(\frac{dt \wedge \pi_i dx^i \wedge F}{\pi^2} + dt \wedge \lambda + \nu \right), \tag{A.22}$$

where 4-form ν is a purely spatial and otherwise arbitrary, while λ is appropriately constrained by 4-form ν , up to a 3-form orthogonal to π . Since the Lagrange multiplier is not part of the physical phase space, this ambiguity is harmless and may be ignored.

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