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1 NUMBER SYSTEM AND NUMBER SERIES

1.1 NUMBERS AND THEIR CLASSIFICATION

A number p may be,

(i) a **natural** number (N)
(ii) a **whole** number (W)
(iii) an **integer** (Z)
(iv) a **rational** number (Q)
(v) a **real** number (R)
(vi) an **irrational** number

For example,

set of **natural** numbers is {1, 2, 3, ...}
set of **whole** numbers is {0, 1, 2, 3, ...}
set of **integers** is { ... –3, –2, –1, 0, 1, 2, 3, ...}
set of **rational** numbers is $\frac{1}{2}, \frac{3}{5}, -\frac{8}{5}, 0, +3, -150, \ldots$
set of **irrational** numbers is $\pi, \sqrt{2}, \sqrt{5}, \sqrt{7}, \ldots$

Besides the above cited number, we often come across numbers like $\sqrt{-8}, \sqrt{-7}, \sqrt{-5}, \sqrt{-1}, 3+\sqrt{-7}$ and so on. These are undefined numbers, called **complex** numbers.

A positive integer, except 1, is called a **prime** number, if its factors are 1 and the number itself 2, 3, 5, 7, ... are prime numbers.

1.2 TEST FOR DIVISIBILITY OF NUMBERS

There are certain tests for divisibility of numbers by any of the numbers 2, 3, 4, 5, 6, 8, 9, 10 and 11 such that by simply examining the digits in the given number, one can easily determine whether or not a given number is divisible by any of these numbers. Such tests are detailed as follows:

i. Divisibility by 2

If the last digit is an even number or it has zero (0) at the end.

Example: 74, 148, 1210 are all divisible by 2.

ii. Divisibility by 3

If the sum of the digits of the given number is divisible by 3.

Example: The sum of the digits of number 3705 is 3 + 7 + 0 + 5 = 15
Since 15 (the sum of digits) is divisible by 3, the number 3705 is also divisible by 3.

iii. Divisibility by 4

If the number formed by the last two digits of the given number is divisible by 4, or if the last two digits are '00'.

Example: 216560 is a number whose last two digits are 60. Since 60 is divisible by 4, the given number 216560 is also divisible by 4.

iv. Divisibility by 5

If the last digit of the given number is 0 or 5.

Example: 865, 1705, 4270 are all divisible by 5.

v. Divisibility by 6

If the given number is divisible by both 2 and 3.

Example: Let us consider the number 629130. It has 0 as the last digit, so it is divisible by 2.
Sum of the digits = 6 + 2 + 9 + 1 + 3 + 0 = 21
This sum 21 is divisible by 3, so the number is divisible by 3.
Since, 629130 is divisible by both 2 and 3, the number is also divisible by 6.

vi. Divisibility by 8

If the number formed by the last three digits of the given number is divisible by 8 or if the last three digits are '000'.

Example: The number 81976 has 976 as the last three digits. Since 976 is divisible by 8, 81976 is also divisible by 8. The number 6145000 ends with '000' and so, it is divisible by 8.

vii. Divisibility by 9

If the sum of the digits of the given number is divisible by 9.

Example: 870111 is a number the sum of whose digits = 8 + 7 + 0 + 1 + 1 + 1 = 18.
Since 18 (sum of digits) is divisible by 9, the number 870111 is also divisible by 9.

viii. Divisibility by 10

If the last digit of the number is zero (0).

Example: 730 has 0 at the end, so it is divisible by 10.

ix. Divisibility by 11

If the difference of the sum of its digits in odd places (i.e. first, third, fifth . . .) and the sum of its digits in even places (i.e. second, fourth, sixth . . .) is either zero (0) or a multiple of 11.

Example: Let us consider the number 647053.
Sum of digits at odd places = 6 + 7 + 5 = 18
Sum of digits at even places = 4 + 0 + 3 = 7
Difference of the sums = 18 – 7 = 11
Since the difference (= 11) is a multiple of 11, 647053 is also divisible by 11
Let us consider another number 9610260.
Sum of digits at odd places = 9 + 1 + 2 + 0 = 12
Sum of digits at even places = 6 + 0 + 6 = 12

Difference of the sums = 12 – 12 = 0
Since the difference is 0, 9610260 is divisible by 11.

1.3 GENERAL PROPERTIES OF DIVISIBILITY

There are some general properties of divisibility that help in determining the divisibility of a natural number by other natural numbers (other than detailed in 1.2)

Property 1

If a number x is divisible by another number y, then any number divisible by x, will also be divisible by y and by all the factors of y.

Example: The number 84 is divisible by 6. Thus any number that is divisible by 84, will also be divisible by 6 and also by the factors of 6, i.e. by 2 and by 3.

Property 2

If a number x is divisible by two or more than two co-prime numbers then x is also divisible by the product of those numbers.

Example: The number 2520 is divisible by 5, 4, 13 that are prime to each other (i.e. co-prime), so, 2520 will also be divisible by 20 (= 5 × 4), 65 (= 5 × 13), 52 (= 4 × 13).

Property 3

If two numbers x and y are divisible by a number 'p', then their sum $x + y$ is also divisible by p.

Example: The numbers 225 and 375 are both divisible by 5. Thus their sum = 225 + 375 = 600 will also be divisible by 5.

Note: It is also true for more than two numbers.

Property 4

If two numbers x and y are divisible by a number 'p', then their difference $x \sim y$ is also divisible by p.

Example: The numbers 126 and 507 are both divisible by 3. Thus their difference = 507 – 126 = 381 will also be divisible by 3.

1.4 TEST OF A PRIME NUMBER

A prime number is only divisible by 1 and by the number itself. The first prime number is 2. Every prime number other than 2 is odd, but every odd number is not necessarily a prime number. Again any even number (other than 2) cannot be a prime number. To test whether any given number (if odd) is a prime number or not, following steps are to be considered:

Step 1 Find an integer (x) which is greater than the approximate square root of the given number.

Step 2 Test the divisibility of the given number by every prime number less than x.

Step 3
- If the given number is divisible by any of them in Step 2, then the given number is NOT a prime number.
- If the given number is not divisible by any of them in Step 2, then the given number IS a prime number.

Example: Consider a number 203. Test if it is a prime number or not.

Step 1 The approximate square root of 203 is 14 plus. Take $x = 15$.

Step 2 Check the divisibility of 203 by the prime numbers less than 15 i.e. by 2, 3, 5, 7, 11, 13.

Step 3 203 is divisible by 7. Thus, it is not a prime number.

1.5 DIVISION AND REMAINDER

When a given number is not exactly divisible by any number, then there is a remainder number at the end of such division.

Suppose we divide 25 by 7 as,

$$\begin{array}{r} 7)\ 25\ (3 \\ \underline{21} \\ 4 \end{array}$$

then, divisor = 7, dividend = 25

quotient = 3, and $\boxed{\text{remainder} = 4}$

So, we can represent it as

$$\begin{array}{l} \text{divisor })\ \text{dividend } (\ \text{quotient} \\ \qquad\qquad \underline{} \\ \qquad\quad \text{remainder} \end{array}$$

Thus $\boxed{\text{dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}}$ **(i)**

So, if a number x is divided by k, leaving remainder 'r' and giving quotient 'q' then the number can be found by using (i)

$$x = kq + r$$

Hence, if the number x is exactly divisible by k, then remainder = $r = 0$

$\therefore$ $x = kq$

and so $\dfrac{x}{k} = q$, implying that x is divisible exactly by k and q is an integer.

1.5.1 Methods to Find a Number Completely Divisible by Another

Consider a given number x. When divided by d, it gives a quotient q and remainder 'r'.

It implies that the given number 'x' is not exactly divisible by 'd'

$$\begin{array}{l} d\ )\ x\ (\ q \\ \quad\ \overline{\ r\ } \end{array}$$

Now, to find a number exactly divisible by 'd', we can use either of the following two methods to reduce the remainder to zero. (If a number is exactly divisible, then remainder is zero).

Method 1

- By subtracting the remainder from the given number (dividend).

  $\therefore$ the required number that is exactly divisible by 'd' = $x - r$

  Hence 'remainder' is the **least number that can be subtracted** from any number to make it exactly divisible.

Method 2

- By adding the (divisor – remainder) to the given number.

  $\therefore$ the required number that is exactly divisible by $d = x +$ (divisor – remainder)

  Therefore, (divisor – remainder) is the **least number that can be added** to any given number to make it exactly divisible.

Example: Find the least number, that must be
(a) subtracted from
or (b) added to a given number 5029, to make it exactly divisible by 17.

Solution: On dividing 5029 by 17, we find that

```
17) 5029 (295
    34
    ---
    162
    153
    ---
     99
     85
    ---
     14
```

∴ remainder = 14.

(a) The least number to be subtracted to make it exactly divisible = remainder = 14. (By method 1)

(b) The least number to be added to make it exactly divisible = divisor − remainder = 17 − 14 = 3. (By method 2)

1.5.2 Greatest *n*-digit and Least *n*-digit Number Exactly Divisible by a Number

(a) To find out the **greatest *n*-digit** number exactly divisible by a divisor '*d*', we use Method 1 (1.5.1)

∴ the required number = Greatest *n*-digit number − remainder.

(b) To find out the **least *n*-digit** number exactly divisible by a divisor '*d*', we use Method 2 (1.5.1), because if we use method 1, then subtracting any number from the *n*-digit least number will reduce it to $(n-1)$ digit number.

∴ the required number = Least *n*-digit number + (divisor − remainder)

Example: Find the (a) greatest 3-digit number divisible by 13.
(b) the least 3-digit number divisible by 13.

Solution: (a) 13) 999 (
11

∴ the required 3-digit greatest number
= 999 − 11 = 988

(b) 13) 100 (
9

∴ the required 3-digit least number
= 100 + (13 − 9)
= 104.

1.6 REMAINDER RULES

Rule 1

This rule is applied to find the **remainder for the smaller divison,** when the same number is divided by the two different divisors such that one divisor is a multiple of the other divisor and also the remainder for the greater divisor is known.

If the remainder for the greater divisor = r,
and the smaller divisor = d, then

Rule-1 states, that **when $r > d$**, the required remainder for the smaller divisor will be the remainder found out by dividing the 'r' by 'd'. [Case I]
and when $r < d$, then the required remainder is 'r' it self. [Case II]

Example: If a number is divided by 527, the remainder is 42.
What will be the remainder if it is divided by 17?

Solution: Here the same number is divided by two divisors: 527 and 17.

Now, $\frac{527}{17} = 31$, so, 527 is a multiple of 17

Hence Rule 1 can be applied.
Remainder for the greater divisor (i.e., for 527) = 42
Smaller divisor = 17.

$$\begin{array}{r|l} 17) & 42 \; (\\ & \underline{34} \\ & 8 \end{array}$$

8 = required remainder for smaller divisor (i.e. 17)
Hence, if 527 is divided by 17, the remainder will be 8.

Rule 2

If two different numbers a and b, on being divided by the same divisor leave remainders r_1 and r_2 respectively, then their sum $(a + b)$ if divided by same divisor will leave remainder R, given by

$$R = (r_1 + r_2) - \text{divisor}$$

⇒ The required remainder R = sum of remainders − divisor
(When sum is divided)

Note: If R becomes negative in the above equation, then the required remainder will be the sum of the remainders.
∴ the required remainder = sum of remainders

Example: Two different numbers, when divided by the same divisor, leave remainders 15 and 39 respectively, and when their sum was divided by the same divisor, the remainder was 7. What is the divisor?

Solution: Using the Rule 2

$$7 = (15 + 39) - \text{divisor}$$

$$\Rightarrow \quad \text{divisor} = 47$$

Example: Two different numbers, when divided by 47, leave remainders 13 and 23 respectively. If their sum is divided by the same number 47, what will be the remainder?

Solution: Using Rule 2,

$$\text{The required remainder} = (13 + 23) - 47 = -11$$

Since the remainder is (−) ve, so, the actual remainder will be 23 + 13 = 36 (refer to NOTE under Rule 2)

Rule 3

When two numbers, after being divided by the same divisor leave the same remainder, then the difference of those two numbers must be exactly divisible by the same divisor.

Example: Two numbers 147 and 225, after being divided by a 2-digit number, leave the same remainder. Find the divisor.

Solution: By Rule 3, the difference of 225 and 147 must be perfectly divisible by the divisor.

The difference = 225 − 147 = 78

Now, 78 = 13 × 2 × 3.

Thus, 1-digit divisors = 2, 3 and 2 × 3

2-digit divisors = 13 × 2, 13 × 3, 13, 13 × 2 × 3

∴ the possible divisors are 26, 39, 13, 78.

Rule 4

If a given number is divided successively by the different factors of the divisor leaving remainders r_1, r_2 and r_3 respectively, then the true remainder (i.e. remainder when the number is divided by the divisor) can be obtained by using the following formula:

True remainder = (first remainder) + (second remainder × first divisor)
+ (third remainder × first divisor × second divisor)

Example: A number, being successively divided by 5, 7 and 11 leaves 3, 1, 2 as remainders respectively. Find the remainder if the same number is divided by 385.

Solution: Here, the divisor is 385, whose factors are 5, 7 and 11.

∴ by Rule 3,

True remainder (i.e. remainder when divided by 385) = 3 + (1 × 5) + (2 × 5 × 7)
= 3 + 5 + 70
= 78

Rule 5

When $(x + 1)^n$ is divided by x, the remainder is always 1, where x and n are natural numbers.

Example: What will be the remainder when $(17)^{21}$ is divided by 16?

Solution: $(17)^{21} = (16 + 1)^{21}$,

∴ when $(16 + 1)^{21}$ is divided by 16, the remainder = 1.

Rule 6

When $(x - 1)^n$ is divided by x, then

the remainder = 1, when n is an even natural number

but the remainder= $x - 1$, when n is an odd natural number.

Example: What will be the remainder when $(29)^{75}$ is divided by 30?

Solution: $(29)^{75} = (30 - 1)^{75}$, here index = 75 (which is odd) so, when $(30 - 1)^{75}$ is divided by 30, the remainder will be $x - 1 = 30 - 1 = 29$

1.7 NUMBER SERIES

In the number series, some numbers are arranged in a particular sequence. All the numbers form a series and change in a certain order. Sometimes, one or more numbers are wrongly put in the number series. One is required to observe the trend in which the numbers change in the series and find out which number/numbers misfit into the series. That number/numbers is the ODD NUMBER of the series.

1.7.1 Important Number Series

Following are some of the important rules or order on which the number series can be made.

I. Pure Series

In this type of number series, the number itself obeys certain order so that the character of the series can be found out.

The number itself may be:

— perfect square
— perfect cube
— prime
— combination of above

II. Difference Series

Under this category, the **change in order** for the differences between each consecutive number of the series is found out as shown in Table 1.1.

III. Ratio Series

Under this category, the change in order for the ratios between each consecutive number of the series is found out as shown in Table 1.2.

IV. Mixed Series

Here, the numbers obeying various orders of two or more different type of series are arranged alternately in a single number series.

V. Geometric Series

Under this category, each successive number is obtained by multiplying (or dividing) the previous number with a fixed number. (See Table 1.2).

Example: 5, 35, 245, 1715, 12005, . . .
43923, 3993, 363, 33, 3, . . .

VI. Two-tier Arithmetic Series

Under this category, the differences of successive numbers form an arithmetic series. (See Table 1.1)

Example: 4, 5, 9, 16, 26, 39, 55, 74.

+1 +4 +7 +10 +13 +16 +19 → Arithmetic Series

3 3 3 3 3 3

VII. Three-tier Arithmetic Series

The differences of successive numbers form a two-tier arithmetic series. The successive term difference in this, in turn form an arithmetic series.

Table 1.1 Change in Order for the Difference Series

Series Code	Nature/Order of the Number Series	Examples for the Series	
D1	Difference between consecutive numbers is same.	2 7 12 17 +5 +5 +5	35 27 19 11 –8 –8 –8
D2	Differences between consecutive numbers are in arithmetic progression (A.P.)	15 26 42 63 +11 +16 +21 Here 11, 16, 21 are in A.P.	34 25 18 13 –9 –7 –5 Here –9, –7, –5 are in A.P.
D3	Difference between consecutive numbers is a perfect square number.	7 16 41 90 $+3^2$ $+5^2$ $+7^2$	70 54 45 41 -4^2 -3^2 -2^2
D4	Differences between consecutive numbers are multiples of a number.	11 23 47 95 +12 +24 +48 Here 12, 24 and 48 are multiples of 12.	204 160 127 105 –44 –33 –22 Here –44, –33 and –22 are multiples of 11.
D5	Differences between consecutive numbers are prime numbers.	2 3 5 8 13 +1 +2 +3 +5 Here, 1, 2, 3 and 5 are prime numbers.	29 22 15 4 –5 –7 –11 Here –5, –7 and –11 are all prime numbers.
D6	Difference between consecutive numbers is a perfect cube number.	7 15 42 167 $+2^3$ $+3^3$ $+4^3$	121 57 30 22 -4^3 -3^3 -2^3
D7	Difference between consecutive numbers are in geometric progression (G.P.).	1 2 5 14 41 124 +1 +3 +9 +27 +81 Here 1, 3, 9, 27 and 81 are in G.P. 33 17 9 5 3 2 –16 –8 –4 –2 –1 Here –16, –8, –4, –2 and –1 are in G.P.	

Table 1.2 Change in Order for the Ratio Series

Series Code	Nature or Order of the Number Series	Examples for the Series
R1	Ratio between each consecutive numbers is the same.	$32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4$ $3 \xrightarrow{\times 5} 15 \xrightarrow{\times 5} 75 \xrightarrow{\times 5} 375$
R2	Ratio between each consecutive numbers is in arithmetic progression (A.P.)	$2 \xrightarrow{\times 2} 4 \xrightarrow{\times 4} 16 \xrightarrow{\times 6} 96$ $420 \xrightarrow{\div 5} 84 \xrightarrow{\div 4} 21 \xrightarrow{\div 3} 7$
R3	Ratio between consecutive number is a perfect square number.	$3 \xrightarrow{\times 2^2} 12 \xrightarrow{\times 4^2} 192 \xrightarrow{\times 6^2} 6912$ $44100 \xrightarrow{\div 7^2} 900 \xrightarrow{\div 5^2} 36 \xrightarrow{\div 3^2} 4$
R4	Ratio between consecutive number is the multiple of a number.	$5 \xrightarrow{\times 2} 10 \xrightarrow{\times 4} 40 \xrightarrow{\times 8} 320$ Here 2, 4 and 8 are multiples of 2. $972 \xrightarrow{\div 9} 108 \xrightarrow{\div 6} 18 \xrightarrow{\div 3} 6$ Here 9, 6 and 3 are multiples of 3.
R5	Ratio between consecutive numbers is a prime number.	$5 \xrightarrow{\times 1} 15 \xrightarrow{\times 2} 30 \xrightarrow{\times 3} 90 \xrightarrow{\times 5} 450$ Here, 2, 3 and 5 are prime numbers. $770 \xrightarrow{\div 11} 70 \xrightarrow{\div 7} 10 \xrightarrow{\div 5} 2$ Here, 11, 7 and 5 are prime numbers.

(Contd.)

Table 1.2 (*Contd.*)

Series Code	*Nature or Order of the Number Series*	*Examples for the Series*
R6	Ratio between consecutive numbers is a perfect cube number.	2 2 54 6750 $\times 1^3$ $\times 3^3$ $\times 5^3$ 13824 216 8 1 $\div 4^3$ $\div 3^3$ $\div 2^3$
R7	Ratios between consecutive numbers are in geometric progression (G.P.).	1 1 2 8 64 1024 $\times 1$ $\times 2$ $\times 4$ $\times 8$ $\times 16$ Here 1, 2, 4, 8 and 16 are in G.P. 729 27 3 1 $\times 27$ $\times 9$ $\times 3$ Here 27, 9 and 3 are in G.P.

Example: 392, 245, 140, 70, 28, 7, 0 . . .

147 105 70 42 21 7

42 35 28 21 14 → Arithmetic Series

7 7 7 7

VII. Twin Series

Under this category, two series are alternatively placed in one.

Example: 1, 2, 3, –2, 5, –8, 7, –16, 9 . . . .

2 2 2 2

–4 –6 –8 (first, third, fifth, seventh terms are in A.P.)

–2 –2 (second, fourth, sixth, terms form two-tier Arithmetic Series).

1.8 THREE STEPS TO SOLVE A PROBLEM ON SERIES

Step 1 Determine whether the series is increasing, decreasing or alternating.

Step 2 If the series is increasing or decreasing, then check:

(a) if change is slow or gradual, then it is a difference series.

(b) if the change is equally sharp, throughout, then it is a ratio series.

(c) if the rise is very sharp initially, but slows down later, then the series may be formed by adding squared, or cubed numbers.

If the series is alternating or irregular, there may be either a mix of two series or two different kinds of operations going on alternately.

Step 3 Complete the series accordingly.

1.9 TWO-LINE NUMBER SERIES

A two-line numbe. series, as the name suggests, consists of number series in two lines. If one complete series is given in first line, with an incomplete series in second line, and it is given that the series in both the lines have the same definite rule, we need to work it out as follows:

Applying the very definite rule of the series in the first line, the series in second line can be completed. The pattern/type of series in the first line may be any of the types described in 1.7.

Example: 15 28 51 84 127 . . .

22 a b c d e . . .

In the first line, the differences of two successive terms of the series are 13, 23, 33, 43.

Hence following the pattern of first line series, the number series in second line are:

$a = 22 + 13 = 35$, $b = 35 + 23 = 58$, $c = 58 + 33 = 91$

$d = 91 + 43 = 134$, $e = 134 + 53 = 187$.

1.10 SUM RULES ON NATURAL NUMBERS

Rule 1

Sum of all the first n natural numbers $= \dfrac{n(n+1)}{2}$
(starting from 1)

Example: Sum of 1 to 74 $= \dfrac{74 \times 75}{2} = 2775$

Rule 2

Sum of first n odd numbers $= n^2$
(starting from 1)

Example: Sum of first 7 odd numbers (1 + 3 + 5 + 7 + 9 + 11 + 13) = 72 = 49

Rule 3

Sum of first n even numbers $= n(n+1)$
(starting from 1)

Example: Sum of first 9 even numbers (2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18) = 9 (9 + 1) = 90.

Rule 4

Sum of squares of first n natural numbers $= \dfrac{n(n+1)(2n+1)}{6}$
(starting from 1)

Example: Sum of squares of first 8 natural numbers

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = \frac{8(8+1)(2\times 8+1)}{6}$$
$$= \frac{8\times 9\times 17}{6} = 204$$

Rule 5

Sum of cubes of first n natural numbers $= \left[\dfrac{n(n+1)}{2}\right]^2$

Example: Sum of cubes of first 6 natural numbers $= 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = \left[\dfrac{6(6+1)}{2}\right]^2 = 441$

Note: For applying Rule 2 and Rule 3, it is required to find how many odd numbers or even numbers are there in the given series.

In the first 'n' natural numbers,

if n is even, then there are $\dfrac{n}{2}$ odd numbers and $\dfrac{n}{2}$ even numbers

if n is odd, then there are $\dfrac{n+1}{2}$ odd numbers and $\dfrac{n-1}{2}$ even numbers

Example: From 1 to 30, as 30 is even, there are 15 odd numbers and 15 even numbers.

From 1 to 29, as 29 is odd, there are $\dfrac{29+1}{2} = 15$ odd numbers and $\dfrac{29-1}{2} = 14$ even numbers

1.11 BASE AND INDEX

If a number 'b' is multiplied by itself 'n' times, then the product is called the nth power of b, i.e.

$b \times b \times b$ . . . . up to n times = b^n

Here, b is called the base and n is called the index.

1.11.1 Last Digit (Digit at Unit's Place) in $(xyz)^n$

Here the given number is $\underbrace{(xyz)}_{\text{base}}^{n} \longrightarrow$ index

z is the last digit of the base.

To find out the last digit in $(xyz)^n$, following steps are to be followed.

Divide the index (n) by 4, then

Case I

if remainder = 0

then check if z is **odd** (except 5), then last digit = 1.

and if z is **even**, then last digit = 6.

Case II

if remainder = 1, then required last digit = last digit of base (i.e. z)

if remainder = **2**, then required last digit = last digit of $(z)^2$

if remainder = **3**, then required last digit = last digit of $(z)^3$

Note: If z is 5, then last digit in the product = 5

Example: Find the last digit in $(295073)^{\overset{130}{n}}$

Solution: Dividing 130 by 4, the remainder = 2

∴ refering to Case II, the required last digit is the last digit of $(z)^2$, i.e. $(3)^2 = 9$, (because $z = 3$)

Example: Find the last digit in $(81678)^{199}$

Solution: Dividing 199 by 4, the remainder = 3

∴ the required last digit is the last digit of $(z)^3$, i.e. $(8)^3 = 512$ (as $z = 8$)

Hence the last digit is 2

1.11.2 Number of Zeroes at the End of a Product

On multiplying two or more given numbers, the zeroes are produced at the end of the resulting product due to the following reasons:

(a) If there is any zero at the end of any of the factors (or numbers being multiplied)

Example: 7 × 20 = 140

This zero is produced at the end of the product also

(b) If 5 or a multiple of 5 is multiplied by any even number.

Example: $45 \times 12 = 540$

$\underset{\text{multiple of 5}}{45} \times \underset{\text{even}}{12} = 540$

Combining the above two reasons, we may say that:

(i) Resolve all the given numbers into their factors.

(ii) Count the number of 2s and 5s.

i.e. $(5)^x \times (2)^y$, say.

(iii) No. of zeroes at the end of product = No. of 2s or no. of 5s, whichever is less

Example: Find the number of zeroes at the end of the product of :

$$15 \times 32 \times 25 \times 22 \times 40 \times 75 \times 98 \times 112 \times 125$$

Solution: We need not multiply these numbers to find the number of zeroes at the end of the resultant product.

All we need to do is to find the numbers of 5s and 2s in the given numbers by resolving the numbers into their factors:

$$(5 \times 3) \times (2^5 \times 5^2) \times (2 \times 11) \times (2^3 \times 5) \times (5^2 \times 3) \times (2 \times 7 \times 7) \times (2^4 \times 7) \times 5^3$$

Only no. of 2s and 5s are relevant here, so, we have:

(i) No. of 5s in $(5^{1+2+1+2+3})$ i.e. $5^9 = 9$

(ii) No. of 2s in $(2^{5+1+3+1+4})$ i.e. $2^{14} = 14$.

Since $9 < 14$.

the no. of zeroes = 9 at the end of the product.

1.12 BINARY NUMBER SYSTEM

This system has a base 2 and uses only 0 and 1; whereas the conventional decimal system having a base 10, uses 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

For any number system it is common that

— the position of each digit within a number affects the total value of the number. In fact, the position of each digit carries a specific weight according to the base of the system.

— each digit has a distinct value and it cannot equal or exceed the base of the system.

In the binary number system, the base = 2 and the number of digits used in the binary system is 2 (0 and 1). So the digits can have value, 0 or 1.

The positional weights of a binary number have been indicated in Table 1.3.

Table 1.3 Positional Weights of a Binary Number

Positions from Right to Left ←	*8th* 7	*7th* 6	*6th* 5	*5th* 4	*4th* 3	*3rd* 2	*2nd* 1	*1st* 0 ← *value*
Positional weight of a	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Binary number	128	64	32	16	8	4	2	1

Using Table 1.3, the decimal equivalents of any binary number can be found out.

$$(10101)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 16 + 0 + 4 + 0 + 1 = (21)_{10}$$

Alternatively,

1	0	1	0	1
16	$\cancel{8}$	4	$\cancel{2}$	1

Hence, $(10101)_2 = 16 + 4 + 1 = (21)_{10}$

1.12.1 Conversion of a Decimal Number to a Binary Number

To convert a decimal number to a binary number, the following steps are to be considered.

Step 1 Divide the decimal number by 2.

Step 2 Go on dividing the quotients (obtained at each stage) by 2 till the quotient is 0.

Step 3 Write down the remainders on the right side after each of the above divisions.

Step 4 Arrange the remainders (as obtained in Step 3) in the reverse order to get the equivalent binary number.

For example, conversion of 25 to a binary number is given as:

		Remainder
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0 ⇒ last quotient	

Arrange the remainders

The equivalent binary number = $(11001)_2$

1.13 CALCULATION IN THE BINARY SYSTEM

Mathematical calculations (i.e. addition, subtraction and multiplication) in the binary system follow their own rules and are similar to those in the decimal system.

1.13.1 Binary Addition

It is easy to add two binary numbers. The rules for binary addition are as follows:

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 10$ (Put 0 in the same column and carry 1 to the **next left column**)

What is carry 1?
In the decimal system, when two numbers are added and the sum of two digits exceeds the highest digit (i.e. 9), then 1 is carried to the next higher digit position (left column).

Similarly, in binary system, when the sum of two digits (bits) exceeds the highest digit (i.e. 1), then 1 is carried to the next higher digit (bit) position (next left column).

In binary addition, the numbers are written one below the other with their rightmost digits aligned. In case of the numbers having fractional parts, the binary points are aligned. Adding is started from right to left, as done in the decimal system.

For example, 1011 + 111

carry	→	1	1	1	1	
(Augend) upper row	→		1	0	1	1
(Addend) lower row	→	+	0	1	1	1
		1 ←	0 ←	0 ←	1 ←	0

Carry 1 Carry 1 Carry 1 Carry 1

Explanation of Addition (column-wise)

Column 1 (rightmost column) $\quad 1 + 1 = \underbrace{1}_{\text{carry}} + \underbrace{\boxed{0}}_{\text{result}}$

Column 2 (from rightmost) $\quad$ 1 (carry) + 1 (upper row) = 1 0 $\quad$ carry

0 + 1 (lower row) = $\boxed{1}$ result

Column 3 (from rightmost) $\quad$ 1 (carry) + 0 (upper row) = 1

1 + 1 (lower row) = 1 $\boxed{0}$ result

← carry

Column 4 (from rightmost) $\quad$ 1 (carry) + 1 (upper row) = 1 0

0 + 0 (lower row) = $\boxed{0}$ result $\quad$ carry

and put in the left-most column

1.13.2 Binary Subtraction

It is easy to subtract a binary number from another binary number. The rules for binary subtraction are as follows:

$0 - 0 = 0$
$1 - 1 = 0$
$1 - 0 = 1$

But to find 0 – 1, we write 1 in the result and also we borrow 1 from the next left column.

What is Borrow 1?

In decimal system, when one number is subtracted from another and it happens that a greater digit is to be subtracted from a smaller one, then 10 is borrowed from the next digit position (next left column).

Similarly, in the binary system if it is required to subtract 1 from 0, then 1 is borrowed from the next digit position (next left column) and the result is 1.

For example, to find $(1000)_2 - (11)_2$, we proceed as follows.

Upper Row	1	0	0	0
Lower Row	0	0	1	1
Borrow	1	1	1	
		1	0	1

Explanation of Subtraction (Column-wise)

Column 1 (rightmost) — 0 – 1 yields a result $\boxed{1}$ and a borrow 1 which is placed below column 2.

Column 2 (from rightmost) — 0 – 1 yields a **temporary result 1** and a borrow 1 (placed below column 3).
From this temporary result 1, subtract borrow 1 which has already been placed below column 2. So, 1 – 1 yields a result $\boxed{0}$.

Column 3 (from rightmost) — 0 – 0 yields a **temporary result 0**. From this temporary result, **subtract borrow 1** (which has already been placed below column 3).
So, 0 – 1 yields a result $\boxed{1}$ and a borrow 1 (placed below column 4).

Column 4 (from rightmost) — 1 – 0 yields a temporary result 1. From this temporary result 1, subtract borrow 1 (which has already been placed below column 4).
So, 1 – 1 yields a result $\boxed{0}$.

1.13.3 Binary Multiplication

Binary multiplication is as simple as multiplication in decimal system. The four rules that are followed in multiplication of two binary numbers are summarised below.

0×0	$= 0$
0×1	$= 0$
1×0	$= 0$
1×1	$= 1$

For example, $(11011)_2 \times (101)_2$, we write as

			1	1	0	1	1
					1	0	1
			1	1	0	1	1
		0	0	0	0	0	
	1	1	0	1	1		
1	0	0	0	0	1	1	1

carry (1) to the next higher digit position (next left column) and add, because

1 + 1 = 1 0

result

carry

1.13.4 Binary Division

In binary division, the method that is applied is similar to that in decimal system. The two rules which are followed here are,

$$\frac{0}{1} = 0 \quad \text{and} \quad \frac{1}{1} = 1$$

Here also, the value of $\frac{1}{0}$ is undefined.

For example, to divide 100111 by 111

```
111 ) 1 0 0 1 1 1 ( 1 0 1
        0 1 1 1
      ---------
          1 0 1 1
            1 1 1
          -------
            1 0 0
```

∴ Quotient = 101
Remainder = 100

Note: In the problems on binary system it has been found in the examinations that
0 is written as *
1 is written as •

Hence it is to be remembered that all the rules of binary system are applicable, but only type of coding may vary. Write * for 0 and • for 1.

Solved Examples

E-1 On dividing 15625 by 41, what is the quotient and the remainder?

S-1 Divisor = 41, dividend = 15625, quotient = 381 and remainder = **4.**

E-2 On dividing 397246 by a certain number, the quotient is 865 and the remainder is 211. Find the divisor.

S-2 Divisor $= \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$= \frac{397246 - 211}{865} = 459$ ∴ The divisor is **459.**

E-3 What is the number which on dividing $(x + ak)$ gives 'a' as the quotient and x as the remainder?

S-3 Divisor $= \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$= \frac{x + ak - x}{a} = k$ ∴ the divisor is $\boldsymbol{k}$.

E-4 Find the least number, that must be subtracted from 87375, to get a number exactly divisible by 698.

S-4 On dividing 87375 by 698, the remainder is 125

By Method 1 (Refer 1.5.1)

The least number to be subtracted from the dividend is the remainder

∴ the least number to be subtracted = **125.**

E-5 What least number must be added to 49123 to get a number exactly divisible by 263

S-5 On dividing 49123 by 263, the remainder is 205.

By Method 2 (Refer 1.5.1)

the least number to be added to the dividend = divisor – remainder = 263 – 205 = 58

∴ the least number to be added = **58.**

E-6 Find the greatest number of 3 digits, which is exactly divisible by 35.

S-6 The greatest 3 digit number = 999.

On dividing 999 by 35, remainder = 19 Now by applying *Method* 2, we obtain

the required number = (dividend) – (remainder) = 999 – 19 = **980.**

E-7 Find the least number of 3 digits, which is exactly divisible by 14.

S-7 The least number of 3 digits = 100

On dividing 100 by 14, remainder = 2

To determine exactly divisible least number, we follow the method 2,

∴ The required number = Dividend + (Divisor – Remainder) = 100 + (14 – 2) = **112.**

E-8 A number when divided by 602 leaves a remainder 36. What remainder would be obtained by dividing the same number by 14?

S-8 Here divisor is a multiple of the other divisor, we find that one

i.e. $\frac{602}{14}$ = an integer. Rule 1 of Remainder Rules is to be used. (Refer 1.6) and so.

$$\begin{array}{r} 14)\ 36\ \ (2 \\ \underline{28} \\ 8 \end{array}$$

∴ the required remainder = 8

E-9 A number, when divided by 357, leaves a remainder 5. What remainder would be obtained by dividing the same number by 17?

S-9 Here, $\frac{357}{17}$ = an integer. i.e. one divisor is multiple of the other divisor.

∴ But the remainder by the greater divisor = 5, which is even less than the smaller divisor (=17), so using Case II of Rule-1, we find as

the required remainder by smaller divisor = the remainder by greater divisor = 5.

E-10 In each of the following number series, one number is wrong. Find out the odd number.

(i) 2, 8, 20, 44, 92, 184, 380

(ii) 60, 48, 38, 28, 24, 20, 18

(iii) 380, 188, 92, 48, 20, 8, 2

(iv) 3, 4.5, 9, 22.5, 67.5, 270, 945

(v) 7, 9, 17, 42, 91, 172, 293

(vi) 5, 15, 30, 135, 405, 1215, 3645

(vii) 2, 9, 28, 65, 126, 216, 344

(viii) 1, 2, 6, 21, 86, 445, 2676

(ix) 3, 5, 12, 38, 154, 914, 4634

(x) 696, 340, 168, 80, 36, 14, 3

(xi) 445, 221, 109, 46, 25, 11, 4

(xii) 1, 2, 4, 12, 36, 71, 144, 432

(xiii) 7, 10, 12, 14, 17, 19, 22, 24

(xiv) 1, 3, 10, 21, 64, 129, 356, 777

(xv) 200, 165, 148, 118, 104, 77, 68

(xvi) 2, 6, 24, 96, 285, 568, 567

(xvii) 6072, 1008, 200, 48, 14, 5, 3

(xviii) 3, 10, 36, 180, 1080, 7560

(xix) 318, 368, 345, 395, 372, 422, 400, 449

(xx) 54, 9, 15, 6, 24, 4, 16

(xxi) 444, 153, 156, 52, 60, 20, 28

(xxii) 2, 6, 12, 27, 58, 121, 248

(xxiii) 3, 9, 18, 54, 110, 324, 648

(xxiv) 5, 6, 15, 41, 89, 170, 291

(xxv) 8544, 1420, 280, 44, 18, 5

(xxvi) 1, 1, 4, 36, 586, 14400

(xxvii) 812, 398, 190, 90, 40, 16

(xxviii) 7, 8, 12, 24, 37, 62, 98

(xxix) 2, 2, 4, 12, 66, 420, 4620

(xxx) 7, 8, 10, 18, 17, 22, 28

(xxxi) 1, 3, 4, 8, 16, 36, 64

(xxxii) 12, 20, 19, 26, 24, 31, 30

(xxxiii) 4, 8, 11, 22, 18, 33, 25, 50

(xxxiv) –1, 2, 7, 14, 28, 34, 47

(xxxv) 100, 50, $33\frac{1}{3}$, 22, 20

(xxxvi) 1, 7, 16, 2, 8, 15, 3, 9, 18

(xxxvii) $\frac{1}{8}, \frac{1}{4}, \frac{3}{4}$, 4, 15, 90

(xxxviii) 6, 15, 19, 30, 32, 43, 45

(xxxix) 2348, 3437, 4346, 5436, 6344, 7433

(xxxx) 87, 86, 82, 75, 57, 32, –4

(xxxxi) 0, 3, 9, 15, 24, 35, 48

(xxxxii) $-\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{2}, 2$

(xxxxiii) 2, 4, 6, 3, 9, 13, 4, 16, 20

(xxxxiv) 15, 1, 14, 2, 12, 4, 9, 11, 5

(xxxxv) 21, 28, 29, 36, 38, 46, 48, 55

(xxxxvi) 9, 7, 64, 6, 3, 18, 1, 8, 9

(xxxxvii) 27, 54, 58, 116, 232, 240, 244

(xxxxviii) –3, 9, 41, 113, 262, 577

S-10 (i)

2 8 20 44 92 $\boxed{184}$ 380

+6 +12 +24 +48 +96 +192

Wrong number = **184**, correct number = **188** (Refer D7, Table 1.1)

(ii)

60 48 38 $\boxed{28}$ 24 20 18

–12, –10, –8 –6 –4 –2

Wrong number = **28**, correct number = **30** (Refer D2, Table 1.1)

(iii)

380 188 92 $\boxed{48}$ 20 8 2

–192 –96 –48 –24 –12 –6

Wrong number = **48**, correct number = **44** (Refer D7, Table 1.1)

(iv)

3 4.5 9 22.5 67.5 $\boxed{270}$ 945

× 1.5 × 2 × 2.5 × 3 × 3.5 × 4

Wrong number = **270**, correct number = **236.25** (Refer R2, Table 1.2)

(v)

7 $\boxed{9}$ 17 42 91 172 293

$+ 1^2$ $+ 3^2$ $+ 5^2$ $+ 7^2$ $+ 9^2$ $+ 11^2$

(1) (9) (25) (49) (81) (121)

Wrong number = **9**, correct number = **8** (Refer D3, Table 1.1)

(vi)

5 15 $\boxed{30}$ 135 405 1215 3645

× 3 × 3 × 3 × 3 × 3 × 3

Wrong number = **30**, correct number = **45** (Refer R1, Table 2.1)

(vii)

2   9   28   65   126 $\boxed{216}$ 344

$1^3 + 1$ $2^3 + 1$ $3^3 + 1$ $4^3 + 1$ $5^3 + 1$ $6^3 + 1$ $7^3 + 1$

Wrong number = **216**, correct number = **217** (It is a pure number series).

(viii)

1   2   6   21 $\boxed{86}$ 445   2676

$\times 1 + 1$ $\times 2 + 2$ $\times 3 + 3$ $\times 4 + 4$ $\times 5 + 5$ $\times 6 + 6$

Wrong number = **86**, correct number = **88** (It is a mixed series).

(ix)

3   5   12   38   154 $\boxed{914}$ 4634

$\times 1 + 2$ $\times 2 + 2$ $\times 3 + 2$ $\times 4 + 2$ $\times 5 + 2$ $\times 6 + 2$

Wrong number = **914**, correct number = **772.**

(x)

696   $\boxed{340}$ 168   80   36   14   3

-352 -176 -88 -44 -22 -11

Wrong number = **340**, correct number = **344** (Refer D7, Table 1.1)

(xi)

445   221   109 $\boxed{46}$ 25   11   4

-224 -112 -56 -28 -14 -7

Wrong number = **46**, correct number = **53** (Refer D7, Table 1.2)

(xii)

1   2   4   12   36 $\boxed{71}$ 144   432

$\times 2$ $\times 2$ $\times 3$ $\times 3$ $\times 2$ $\times 2$ $\times 3$

Here, one series is multiple of 2 and other is multiple of 3 and both are repeated in a batch of two numbers.

Wrong number = **71**, correct number = **72**. It is a mixed series (Refer R4, Table 1.2)

(xiii)

7   10   12   14   17 $\boxed{19}$ 22   24

$+3$ $+2$ $+2$ $+3$ $+3$ $+2$ $+2$

Wrong number = **19**, correct number = **20**. It is a mixed series.

(xiv)

1 3 10 21 64 129 $\boxed{356}$ 777

$\times 2 + 1$ $\times 3 + 1$ $\times 2 + 1$ $\times 3 + 1$ $\times 2 + 1$ $\times 3 + 1$ $\times 2 + 1$

Wrong number = **356**, correct number = **388**. It is a mixed series.

(xv)

200 165 148 $\boxed{118}$ 104 77 68

$14^2 + 4$ $13^2 - 4$ $12^2 + 4$ $11^2 - 4$ $10^2 + 4$ $9^2 - 4$ $8^2 + 4$

Wrong number = **118**, correct number = **117**. It is a pure series (Refer 1.3.1 (I)).

(xvi)

2 6 $\boxed{24}$ 96 285 568 567

$\times 6 - 6$ $\times 5 - 5$ $\times 4 - 4$ $\times 3 - 3$ $\times 2 - 2$ $\times 1 - 1$

Wrong number = **24**, correct number = **25**.

(xvii)

6072 $\boxed{1008}$ 200 48 14 5 3

$- 12 \div 6$ $- 10 \div 5$ $- 8 \div 4$ $- 6 \div 3$ $- 4 \div 2$ $- 2 \div 1$

Wrong number = **1008**, correct number = **1010**.

(xviii)

3 $\boxed{10}$ 36 180 1080 7560

$\times 3$ $\times 4$ $\times 5$ $\times 6$ $\times 7$

Wrong number = **10**, correct number = **9** (Refer R2, Table 1.1).

(xix)

318 368 345 395 372 422 $\boxed{400}$ 449

+ 27 + 27 + 27

+ 27 + 27 + 27

Wrong number = **400**, correct number = **399**.

(xx)

54   9 $\boxed{15}$ 6   24   4   16

$54 \xrightarrow{\div 6} 9$, $15 \xrightarrow{\div 6} 6$, $24 \xrightarrow{\div 6} 4$

Wrong number = **15**, correct number = **36**.

(xxi)

444   $\boxed{153}$ 156   52   60   20   28

$444 \xrightarrow{\div 3} 153$, $156 \xrightarrow{\div 3} 52$, $60 \xrightarrow{\div 3} 20$

Wrong number = **153**, correct number = **148**.

(xxii)

2   $\boxed{6}$ 12   27   58   121   248

$\times 2 + 1$ $\times 2 + 2$ $\times 2 + 3$ $\times 2 + 4$ $\times 2 + 5$ $\times 2 + 6$

Wrong number = **6**, correct number = **5**.

(xxiii)

3   9   18   54 $\boxed{110}$ 324   648

$\times 3$ $\times 2$ $\times 3$ $\times 2$ $\times 3$ $\times 2$

Wrong number = **110**, correct number = **108**.

(xxiv)

5   6   15 $\boxed{41}$ 89   170   291

$+ 1^2$ $+ 3^2$ $+ 5^2$ $+ 7^2$ $+ 9^2$ $+ 11^2$

Wrong number = **41**, correct number = **40** (Refer D3, Table 1.1).

(xxv)

8544   1420   280 $\boxed{44}$ 18   5

$\div 6 - 4$ $\div 5 - 4$ $\div 4 - 4$ $\div 3 - 4$ $\div 2 - 4$

Wrong number = **44**, correct number = **66**.

(xxvi)

1   1   4   36 $\boxed{586}$ 14400

$\times 1^2$ $\times 2^2$ $\times 3^2$ $\times 4^2$ $\times 5^2$

Wrong number = **586**, correct number = **576** (Refer R3, Table 1.2).

(xxvii)

812   398 $\boxed{190}$ 90   40   16

$\div 2 - 8$ $\div 2 - 7$ $\div 2 - 6$ $\div 2 - 5$ $\div 2 - 4$

Wrong number = **190**, correct number = **192**.

(xxviii)

7   8   12 $\boxed{24}$ 37   62   98

$+1$ $+2^2$ $+3^2$ $+4^2$ $+5^2$ $+6^2$

Wrong number = **24**, correct number = **21** (Refer D3, Table 1.1).

(xxix)

2   2   4   12 $\boxed{66}$ 420   4620

$\times 1$ $\times 2$ $\times 3$ $\times 5$ $\times 7$ $\times 11$

Wrong number = **66**, correct number = **60** (Refer R5, Table 1.2).

(xxx)

7   8   10 $\boxed{18}$ 17   22   28

$+1$ $+2$ $+3$ $+4$ $+5$ $+6$

Wrong number = **18**, correct number = **13** (Refer D2, Table 1.1).

(xxxi)

1   3   4   8   16 $\boxed{36}$ 64

$4 \downarrow 1 + 3$

$8 \downarrow 1 + 3 + 4$

$16 \downarrow 1 + 3 + 4 + 8$

$36 \downarrow 1 + 3 + 4 + 8 + 16$

$64 \downarrow 1 + 3 + 4 + 8 + 16 + 32$

Wrong number = **36**, correct number = **32**.

(xxxii)

12   20   19   26 $\boxed{24}$ 31   30

$+8$ -1 $+7$ -1 $+6$ -1

Wrong number = **24**, correct number = **25**. It is a mixed series.

(xxxiii)

+ 7 + 7 + 7

4 8 11 22 18 $\boxed{33}$ 25 50

+ 14 + 14 + 14

Wrong number = **33**, correct number = **36**.

(xxxiv)

–1 2 7 14 $\boxed{28}$ 34 47

+ 3 + 5 + 7 + 9 + 11 + 13

Wrong number = **28**, correct number = **23**.

(xxxv)

100 50 $33\frac{1}{3}$ $\boxed{22}$ 20

100 ÷ 1 100 ÷ 2 100 ÷ 3 100 ÷ 4 100 ÷ 5

Wrong number = **22**, correct number = **25**.

(xxxvi)

1 7 16 2 8 $\boxed{15}$ 3 9 18

x *y* *z* *x* *y* *z* *x* *y* *z*

There are three series *x*, *y* and *z*. mixed

x series contains 1, 2, 3

y series contains 7, 8, 9

z series contains 16, 15, 18. (Wrong number = **15**, correct number = **17**)

(xxxvii)

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{4}$ $\boxed{4}$ 15 90

× 2 × 3 × 4 × 5 × 6

Wrong number = **4**, correct number = **3** (Refer R2, Table 1.2).

(xxxviii)

+ 13 + 13 + 13

6 15 19 30 32 $\boxed{43}$ 45

+ 15 + 15

Wrong number = **43**, correct number = **45**. It is a mixed series.

(xxxix)

2348   3437   4346 $\boxed{5436}$ 6344   7433

+ 1089   + 909   + 1089   + 909   + 1089

Here, 1089 and 909 are added alternately to continue the series.

Wrong number = **5436**, correct number = **5435**

(xxxx)

87   86   82 $\boxed{75}$ 57   32   – 4

-1^2 -2^2 -3^2 -4^2 -5^2 -6^2

Wrong number = **75**, correct number = **73** (Refer D3, Table 1.1).

(xxxxi)

0   3 $\boxed{9}$ 15   24   35   48

+ 3   + 5   + 7   + 9   + 11   + 13

Wrong number = **9**, correct number = **8**.

(xxxxii)

$-\frac{1}{2}$ 0 $\frac{1}{2}$ $\boxed{\frac{1}{4}}$ $\frac{3}{2}$ 2

$+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$

Wrong number = $\mathbf{\frac{1}{4}}$, correct number = **1**.

(xxxxiii)

2   4   6   3   9 $\boxed{13}$ 4   16   20

2 + 4   3 + 9   4 + 16

Wrong number = **13**, correct number = **12**. It is a mixed series.

(xxxxiv)

+ 1   + 2   + 3

15   1   14   2   12   4   9 $\boxed{11}$ 5

–1   –2   –3   –4

There are two series arranged alternately.

Series 1 contains 1, 2, 4, 11

Seies 2 contains 15, 14, 12, 9, 5

Wrong number = **11**, correct number = **7** (Refer D2, Table 1.1).

(xxxxv)

21 28 29 36 38 $\boxed{46}$ 48 55

+ 7 + 1 + 7 + 2 + 7 + 3 + 7

Wrong number = **46**, correct number = **45**.

(xxxxvi)

9 7 64 6 3 $\boxed{18}$ 1 8 9

$9 \times 7 + 1$ $6 \times 3 + 1$ $1 \times 8 + 1$

Wrong number = **18**, correct number = **19**.

(xxxxvii)

27 54 58 116 $\boxed{232}$ 240 244

× 2 + 4 × 2 + 4 × 2 + 4

Wrong number = **232**, correct number = **120**.

(xxxxviii)

– 3 9 41 113 $\boxed{262}$ 577

+ 12 + 32 + 72 + 152 + 312

+ 20 + 40 + 80 + 160

Here, 20, 40, 80 and 160 are in G.P.

Wrong number = **262**, correct number = **265**.

E-11 7 13 78 83 415

3 *a* *b* *c* *d* *e*

What should replace *c*?

S-11 The pattern of the first line series is obtained as:

7, 7 + 6, 13 × 6, 78 + 5, 83 × 5

Therefore, the second line series can be completed on the basis of the same pattern, as:

3, $a = 3 + 6$, $b = 9 \times 6$, $c = 54 + 5$, . . .

∴ The value of *c* is 59.

E-12 3 6 24 72 144 576

1 *a* *b* *c* *d* *e*

What should replace *d*?

S-12 The pattern of the first line series is obtained as:

3, 3 × 2, 6 × 4, 24 × 3, 72 × 2, 144 × 4

Therefore, the second line series can be completed on the basis of the same pattern as:

1, $a = 1 \times 2$, $b = 2 \times 4$, $c = 8 \times 3$, $d = 24 \times 2$

∴ The value of *d* is 48

E-13 4 6 15 49 201 1011

15 *a* *b* *c* *d* *e*

What should replace *e*?

S-13 The pattern of the first line series is obtained as:

4, 4 × 1 + 2, 6 × 2 + 3, 15 × 3 + 4, 49 × 4 + 5, 201 × 5 + 6

Therefore, the second line series can be completed on the basis of the same pattern as:
15, $a = 15 \times 1 + 2$, $b = 17 \times 2 + 3$, $c = 37 \times 3 + 4$, $d = 115 \times 4 + 5$, $e = 465 \times 5 + 6$
$\therefore$ The value of e is 2331

E-14 –1 0 10 65 345 1750
–2 a b c d e
What should come in place of d?

S-14 The pattern of the first line series is obtained as:
$-1, (-1 + 1) \times 5, (0 + 2) \times 5, (10 + 3) \times 5, (65 + 4) \times 5, (345 + 5) \times 5$
Therefore, the second line series can be completed on the basis of the same pattern as:
-2, $a = (-2 + 1) \times 5$, $b = (-5 + 2) \times 5$, $c = (-15 + 3) \times 5$, $d = (-60 + 4) \times 5$
$\therefore$ the value of d is –270

E-15 –1 0 –8 3 –52 –135
21 a b c d e
What should come in place of c?

S-15 The pattern of the first line series is obtained as:
$-1, (-1 \times 1) + 1^3, (0 \times 2) - 2^3, (-8 \times 3) + 3^3, (3 \times 4) - 4^3, -52 \times 5 - 5^3.$
Therefore, the second line series can be completed on the basis of the same pattern as :
21, $a = 21 \times 1 + 1^3$, $b = 22 \times 2 - 2^3$, $c = 36 \times 3 + 3^3$
$\therefore$ the value of c is 135.

E-16 3000 191 2216 847 1688 959
3435 a b c d e
What should come in place of b?

S-16 The pattern of the first line series is obtained as:
$3000, 3000 - 53^2, 191 + 45^2, 2216 - 37^2, 847 + 29^2, 1688 - 21^2$
Therefore, the second line series can be completed on the basis of the same pattern as:
3435, $a = 3435 - 53^2$, $b = 626 + 45^2$
$\therefore$ the value of b is 2651.

Directions (17–21): In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer.

For example, if an interchange of number marked 1 and the number marked 4 is required to establish the series, your answer is 1. If it is not necessary to interchange the position of the numbers to establish the series, give 5 as your answer. Remember that when the series is established, the numbers change from left to right (i.e. from the unmarked number to the last marked number) in a specific order.

E-17 1200 40 1000 50 750 75
1) 2) 3) 4) 5)

S-17 The pattern of the series is:
$1200, 1200 \div 30, 40 \times 25, 1000 \div 20, 50 \times 15, 750 \div 10$
Here, it is not necessary to interchange the position of any number to establish the series.
Hence, 5) is the answer.

E-18 2 5 26545 177 4424 44
1) 2) 3) 4) 5)

S-18 The pattern of the series is:
$2, 2 \times 2 + 1, \underline{5 \times 3^2 - 1}, 44 \times 4 + 1, 177 \times 5^2 - 1, \underline{4424 \times 6 + 1}$
5) ——— to interchange ——— 2)

Here, the numbers marked 2) and marked 5) are to be interchanged in the position to establish the series.
Hence 2) is the answer.

E-19 1 1 1 2 8 4

1	1	1	2	8	4
	1)	2)	3)	4)	5)

S-19 The pattern of the series should go as

$1, \quad 1^2, \quad 1^3, \quad 2, \quad \underline{2^2}, \quad \underline{2^3}$

5) 4)

to interchange

Here, the numbers marked 4) and marked 5) are to be interchanged in their position to establish the series.
Hence 4) is the answer

E-20

48	16	13	12	17	25.25
	1)	2)	3)	4)	5).

S-20 The pattern of the series should go as

$48, \quad 48 \times 0.25 + 4, \quad \underbrace{16 \times 0.5 + 4}_{3)}, \quad \underbrace{12 \times 0.75 + 4}_{2)}, \quad 13 \times 1 + 4, \quad 17 \times 1.25 + 4$

to interchange

Here, the numbers marked 2) and marked 3) are to be interchanged in their position to establish the series.
Hence 2) in the answer.

E-21

82	83	165	9916	1983	496
	1)	2)	3)	4)	5).

S-21 The pattern of the series should go as

$82, \quad 82 \times 1 + 1, \quad 83 \times 2 - 1, \quad \underbrace{165 \times 3 + 1}_{5)}, \quad 496 \times 4 - 1, \quad \underbrace{1983 \times 5 + 1}_{3)}$

to interchange

Here, the numbers marked 5) and marked 3) are to be interchanged to establish the series.
Hence 3) is the answer.

E-22 Find the binary equivalents of
(i) 30; (ii) 27; (iii) 41

S-22 (i) Remainders

$$\begin{array}{r|l}
2 & 30 \longrightarrow 0 \\ \hline
2 & 15 \longrightarrow 1 \\ \hline
2 & 7 \longrightarrow 1 \\ \hline
2 & 3 \longrightarrow 1 \\ \hline
2 & 1 \longrightarrow 1 \\ \hline
 & 0
\end{array}$$

So, the binary equivalent is $\mathbf{(11110)_2}$.

(ii) Remainders

$$\begin{array}{r|c l}
2 & 27 & \longrightarrow 1 \\ \hline
2 & 13 & \longrightarrow 1 \uparrow \\ \hline
2 & 6 & \longrightarrow 0 \\ \hline
2 & 3 & \longrightarrow 1 \\ \hline
2 & 1 & \longrightarrow 1 \\ \hline
 & 0 &
\end{array}$$

So, the binary equivalent is $\mathbf{(11011)_2}$.

(iii) Remainders

$$\begin{array}{r|c l}
2 & 41 & \quad 1 \\ \hline
2 & 20 & \quad 0 \uparrow \\ \hline
2 & 10 & \quad 0 \\ \hline
2 & 5 & \quad 1 \\ \hline
2 & 2 & \quad 0 \\ \hline
2 & 1 & \quad 1 \\ \hline
 & 0 &
\end{array}$$

So, the binary equivalent is $\mathbf{(101001)_2}$.

E-23 Simplify the following for question mark (?)

(i) $10110_2 - 1011_2 = (?)_2$

(ii) $1001_2 + 1010_2 = (?)_2$

(iii) $101_2 \times 100_2 = (?)_2$

(iv) $1100_2 \div 11_2 = (?)_2$

(v) $101_2 + 1100_2 \div 10_2 = (?)_2$

S-23 (i)

1	0	1	1	0	Upper row
	1	0	1	1	Lower row
1		1	1		Borrow
0	1	0	1	1	

Hence the required result is $\mathbf{(1011)_2}$.

(ii)

$$\begin{array}{ccccc}
 & 1 & 0 & 0 & 1 \\
+ & 1 & 0 & 1 & 0 \\ \hline
1 & 0 & 0 & 1 & 1
\end{array}$$

Hence the required result is $\mathbf{(10011)_2}$.

(iii)

```
            1   0   1
        ×   1   0   0
    ---------------------
            0   0   0
        0   0   0
    1   0   1
    ---------------------
    1   0   1   0   0
```

Hence the required result is $(\mathbf{10100})_2$.

(iv)

```
11 ) 1100 ( 100
     11
     ---
     000
```

Hence the required result is $(\mathbf{100})_2$.

(v) $101_2 + 1100_2 \div 10_2$

Firstly,

```
10 ) 1100 ( 110
     10
     ---
      10
      10
      ---
       00
```

Now $101_2 + 110_2$ is

```
      101
      110
     ----
     1011
```

Hence the required result is $(\mathbf{1011})_2$.

E-24 In a certain code, the symbol for 0 (zero) is * and for 1, it is ●. There are no other symbols for other numbers and all numbers greater than 1 are written using these two symbols only. The value of symbol 1 doubles itself everytime it shifts one place to the left.

Thus, 0 is written as *

1 is written as ●

2 is written as ●*

3 is written as ●●

On the above coding system, answer the following questions.

(i) What is the ratio of ●●* and ●**●?

(ii) Find the value of 10 ÷ 5 + 2 in the above code.

(iii) Evaluate: ●●* + ●*● – ●●●.

(iv) If 50% of ●*●* is added to ●*●, then what will be the result?

(v) Find the product of ●●● and ●*.

S-24 (i) ●●* = $(110)_2 = 4 + 2 = 6_{10}$

●**● = $(1001)_2 = 8 + 1 = 9_{10}$

$\therefore$ Required ratio = $\frac{6}{9} = \frac{2}{3}$.

(ii) $10 \div 5 + 2 = 4_{10}$

2	4	0
2	2	0
2	1	1
	0	

So, the required value is $(\mathbf{100})_2$ = ●**.

(iii) ●●* + ●*● – ●●●.

= 110 + 101 – 111

So, firstly

	1	1	0
+	1	0	1
1	0	1	1
–	1	1	1
	1	0	0

So, the required value is $(\mathbf{100})_2$, = ●**

(iv) ●*●* = $(1010)_2$

$= 8 + 2 = 10_{10}$

●*● = $(101)_2$

$= 4 + 1 = 5_{10}$.

So, 50% of ●*●* + ●*●

$= 50\%$ of $10_{10} + 5_{10}$

$= 5_{10} + 5_{10} = 10_{10}$

Now,

2	10	0
2	5	1
2	2	0
2	1	1
	0	

So the required result is ●*●*.

(v) ●●● × ●*

$$
\begin{array}{cccc}
 & 1 & 1 & 1 \\
 & & 1 & 0 \quad \times \\
\hline
 & 0 & 0 & 0 \\
1 & 1 & 1 & \\
\hline
1 & 1 & 1 & 0
\end{array}
$$

i.e. ●●●*

REGULAR PROBLEMS

Section A: Number System

(1) In a division, find the divisor if dividend = 27541, quotient = 233 and remainder = 47
(a) 172 (b) 238 (c) 126 (d) 194 (e) 118

(2) The least number that must be subtracted from 104075 to make it exactly divisible by 437 is:
(a) 31 (b) 69 (c) 50 (d) 44 (e) 38

(3) The greatest 5-digit number that is exactly divisible by 100 is:
(a) 99899 (b) 99800 (c) 99900 (d) 99889 (e) 98990

(4) Which of the following numbers is a prime number?
(a) 541 (b) 323 (c) 217 (d) 551 (e) None

(5) Which is the least 7-digit number, that leaves a remainder of 3 when divided by 7?
(a) 1000003 (b) 1000010 (c) 1000005 (d) 1000002 (e) 1000007

(6) When a certain number is multiplied by 21, the product consists of only fours. The smallest such number is :
(a) 21164 (b) 4444 (c) 444444 (d) 444 (e) 3126

Hint: Assuming the product, you can find the other multiplicand

(7) A number when divided by 627 leaves a remainder 43. By dividing the same number by 19, the remainder will be
(a) 19 (b) 24 (c) 43 (d) 5 (e) 13

(8) What will be the remainder when $(29)^{36}$ is divided by 28?
(a) 0 (b) 1 (c) 29 (d) 5 (e) Cannot be determined

(9) The sum of all odd numbers from 1 to 41 is:
(a) 372 (b) 505 (c) 441 (d) 398 (e) 516

Hint: Refer 1.10

(10) What is the total number of numbers up to 9999?
(a) 98900 (b) 10000 (c) 9999 (d) 98100 (e) None of these

(11) The digit in the unit place in $(1038)^{67}$ is:
(a) 2 (b) 4 (c) 1 (d) 6 (e) 8

Hint: Refer 1.11.1

(12) The number of prime numbers in $(25)^{13} \times (10)^{7} \times (27)^{5}$ is:
(a) 25 (b) 32 (c) 55 (b) 50 (e) 42

Hint: Resolve each number into its prime factors, then count the number of such factors by adding the index.

(13) The number of zeros at the end of the product **(SSC, '97)**
$16 \times 22 \times 15 \times 50 \times 65 \times 115 \times 18 \times 90$ is:
(a) 5 (b) 6 (c) 12 (d) 7 (10)

Hint: Refer 1.11.2 in the text

(14) What is the difference in intrinsic value and local value of 6 in 8631?
(a) 625 (b) 594 (c) 600 (d) 496 (e) 0

(15) How many such numbers are there between 1 and 100 such that each of which is not only divisible by 4, but also has one digit as 4 in the number?
(a) 5 (b) 12 (c) 6 (d) 15 (e) 7

(16) A number is greater than 3 but less than 8. Also, the number is greater than 6 but less than 10. What is the number?
(a) 5 (b) 4 (c) 9 (d) 6 (e) 7

(17) In a division, a student took 63 as divisor instead of 36. His answer was 24. The correct answer is:
(a) 42 (b) 32 (c) 48 (d) 28 (e) 38

(18) $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by:
(a) 3 (b) 11 (c) 13 (d) 17 (e) None of these

(19) How many numbers are there between 500 and 600 in which 9 occurs only once?
(a) 18 (b) 19 (c) 20 (d) 21 (e) 22

(20) Replace the * in the number 6* 106 by a suitable digit so that the number formed is exactly divisible by 11.
(a) 3 (b) 4 (c) 2 (d) 1 (e) 1

(21) The value of $101 + 102 + 103 + \ldots + 200$ is:
(a) 15050 (b) 20200 (c) 10909 (d) 16500 (e) None of these

(22) What are the values of '*a*' and '*b*', if 4266 *ab* is divisible by 45?
(a) 4 and 5 (b) 1 and 7 (c) 9 and 0 (d) 3 and 6 (e) either (a) or (c)

Hint: Refer 1.2

(23) The digit in the unit's place of the number
$17^{1999} + 11^{1999} - 7^{1999}$ is:
(a) 7 (b) 4 (c) 1 (d) 3 (c) None of these

Hint: Use of the concept (in ref 1.11.1) can be avoided here if we use a little common sense

(24) The number, one less than 7^{19} is divisible by:
(a) 49 (b) 7 (c) 16 (d) 18 (e) 6

Answers

1. (e)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (c)
10. (c)	**11.** (a)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (e)	**16.** (e)	**17.** (a)	**18.** (d)
19. (a)	**20.** (c)	**21.** (a)	**22.** (e)	**23.** (c)	**24.** (e)			

Section B: Number Series

(1) Which of the following does not fit in the series
(a) 3, 7, 12, 27, 51, 105, 204. **(RRB Kolkata '02)**

Hint: $3 \times 2 + 1, 7 \times 2 - 2, 12 \times 2 + 3\ \ldots$

(b) 4443, 2433, 4322, 4511, 6221 **(RRB Kolkata '02)**

Hint: Digits add to 11

(c) 0, 6, 24, 90, 120, 210 **(RRB Kolkata '02)**
(d) 318, 368, 345, 395, 372, 422, 400, 449 **(AAO '98)**
(e) 196, 168, 143, 120, 99, 80 63
(f) 3, 4, 20, 38, 87, 168, 289 **(BSRB, Bang, '97)**

Hint: $+1^2, +3^2, +5^2 \ldots$

(g) 698, 343, 170, 82, 38, 16, 5 **(BSRB, Bang, '97)**
(h) 698, 554, 454, 390, 347, 338, 334 **(BSRB, Bang, '97)**

Hint: $-12^2, -10^2, -8^2 \ldots$

(i) 376, 188, 88, 40, 16, 4, –2 **(BSRB, Delhi, '93)**

Hint: $\div 2 - 4$

(j) 10, 15, 24, 35, 54, 75, 100 **(IBPS Bank PO, '02)**

Hint: $+5, +9, +13 \ldots.$

(k) 1, 3, 4, 7, 11, 18, 27, 47 **(IBPS Bank PO, '02)**

Hint: Middle number = difference of succeeding and proceeding number

(l) 2, 8, 32, 148, 765, 4626, 32431 **(IBPS, '02)**

Hint: $\times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2 \ldots$

(m) 2, 3, 11, 38, 102, 229, 443 **(IBPS, '02)**

Hint: Differences are $1^3, 2^3, 3^3 \ldots$

(n) 0, 6, 23, 56, 108, 184, 279 **(RRB, Bhubaneswar, 2000)**

Hint: $1^3 - 2^0, 2^3 - 2^1, 3^3 - 2^2, 4^3 - 2^3 \ldots$

(o) 49, 56, 64, 71, 81, 90, 100, 110 **(RRB, Bhubaneswar, 2000)**
(p) 13700, 1957, 326, 65, 16, 6, 2 **(Andhra Bank PO, '99)**

Hint: $-1 \div 7, -1 \div 6, -1 \div 5 \ldots$

(q) 5, 7, 13, 25, 45, 87, 117 **(Andhra Bank PO, '99)**

(2) What will come in place of the question mark (?) in the following number series? (Andhra Bank Specialist Officer's Exam, 2002)
(a) 2, 9, 30, 105, ?, 2195
(b) 3, 4, 12, 45, ?, 1005
(c) 1, 3, 9, 31, ?, 651
(d) 5, ?, 4, 7.5, 17, 45
(e) 15, 30, ?, 40, 8, 48

Direction (3–23): In each of the following questions, a number series is given. After the series, below it, a number is given followed by (A), (B), (C), (D) and (E). You have to complete the series starting with the number given, following the sequence for the given series on top. Then answer the question below it

(3) 616 496 397 317 254 **(SBI ASSC Bank PO, '97)**
838 A B C D E
What will come for E?

(4) 6 9 27 121.5 729 **(SBI ASS. Bank PO, '97)**
8 A B C D E
What will come for C?

(5) 6 14 35 111 149 **(SBI ASS. Bank PO, '97)**
3 A B C D E
What will come for E?

(6) 434 353 417 368 404 379 **(SBI ASS Bank PO, '97)**
108 A B C D E
What will come for D?

(7) 5 6 16 57 244 1245 **(RBI Grade-B, '02)**
2 A B C D E
What will come for D?

(8) 3 5 9 17 33 65 **(RBI Grade-B, '02)**
7 A B C D E
What will come for C?

(9) 7 4 5 9 20 52.5 **(RBI Grade-B, '02)**
3 A B C D E
What will come for C?

Hint: $\times 0.5 + 0.5$, $\times 1 + 1$, $\times 1.5 + 1.5 \ldots$

(10) 3 10 32 111 460 2315 **(RBI Grade-B, '02)**
2 A B C D E
What will come for B?

Hint: $10 = 3 \times 1 + 7 \times 1$, $32 = 10 \times 2 + 6 \times 2$, $111 = 32 \times 3 + 5 \times 3$, $460 = 111 \times 4 + 4 \times 4 \ldots$

(11) 5 8 6 10 7 12 **(RBI Grade-B, '02)**
7 A B C D E
What will come for C?

Hint: $\times 2 - 2$, $\div 2 + 2$, $\times 2 - 2, \ldots$

(12) – 4 2 10 96 6150 **(RBI Grade-B, '02)**
–10 A B C D E
What will come for D?

Hint: $\times 1^0 + 6$, $\times 2^1 + 6$, $\times 3^2 + 6 \ldots$

(13) 4 6 15 79 704 8480 **(NABARD, '01)**
12 A B C D E
What will come for B?

(14) 200 184 193 157 182 118 167 **(NABARD '01)**
150 A B C D E
What will come for E?

Hint: -4^2, $+3^2$, $-6^2, \ldots$

(15) 60 121 131 264 284 571 601 **(NABARD, '01)**
120 A B C D E
What will come for D?

(16) 0.25 1.25 –3 0 –64 **(NABARD, '01)**
45 A B C D E
What will come for C?

(17) 5 7 10 36 136 690 **(SBI PO, '01)**
2 A B C D E
What will come for D?

Hint: $\times 1 + 2$, $\times 2 - 4$, $\times 3 + 6, \ldots$

(18) 8 9 13 12 8 9
12 A B C D E
What will come for E?

(19) 3 20 118 587 2344 **(SBI PO, '01)**
12 A B C D E
What will come for C?

(20) 0 16 48 112 240 **(SBI PO, '01)**
120 A B C D E
What will come for D?

Hint: $(+ 2^1 \times 8), (+ 2^2 \times 8) (+ 2^3 \times 8) \ldots$

(21) 108 52 24 10 3
64 A B C D E
What will come for D?

(22) 5 12 60 340 **(BSRB, Mumbai, '98)**
7 A B C D E
What will come for D?

Hint: $\times 8 - 28, \times 7 - 24, \times 6 - 20 \ldots$

(23) 4 7 24 93 **(BSRB, Chennai, '98)**
2 A B C D E
What will come for D?

Answers

1. (a) 51, (b) 4443, (c) 90, (d) 400, (e) 196, (f) 20, (g) 343, (h) 347, (i) 188, (j) 35, (k) 27, (l) 32, (m) 229, (n) 108, (o) 71, (p) 6, (q) .87.
2. (a) 436, (b) 196, (c) 129, (d) 3, (e) 10.
3. 428 **4.** 162 **5.** 1889 **6.** 78 **7.** 172 **8.** 49 **9.** 6 **10.** 30 **11.** 14
12. –762 **13.** 23 **14.** 68 **15.** 524 **16.** 1611 **17.** 64 **18.** 13 **19.** 2477 **20.** 360
21. 0.25 **22.** 5044 **23.** 360

REAL PROBLEMS

Section A: Number System

(1) The greatest 5-digit number, that leaves a remainder of 19 if divided by 23 is:
(a) not possible (b) 99980 (c) 99982 (d) 99977 (e) 99962

Hint: Here the actual remainder is less than the desired remainder (= 19). So, it can not be found out by the conventional method of simply adding the desired remainder to the exactly divisible 5-digit greatest number. This is because conventional method is applicable only when actual remainder (= 18) is greater than the desired remainder.

(2) The four integers consecutively lower than 81, and the four consecutively higher than 81, are added together. This sum is divisible by:
(a) 7 (b) 9 (c) 11 (d) 13 (e) None of these

(3) In a question, divisor is $\frac{2}{3}$ of the dividend and twice the remainder. If the remainder is 5, then the dividend is **(SSC, '94)**
(a) 85 (b) 145 (c) 225 (d) 65 (e) None of these

(4) The number that is nearest to 2160 and exactly divisible by 52 is:
(a) 2132 (b) 2148 (c) 2184 (d) 2177 (e) None of these

Tips: Check if the remainder obtained is more or less than half the divisor, because nearest number is to be found out.

(5) What will be the remainder when $(16^{27} + 37)$ is divided by 17?
(a) 4 (b) 19 (c) 13 (d) 2 (e) 14

Hint: Refer the Remainder Rules

(6) What is the number of digits of the smallest number which when multiplied by 7 gives a result consisting entirely of nines?
(a) 3 (b) 6 (c) 5 (d) 4 (e) 7

(7) What will be the digit in the unit place in the product $(3807)^{194} \times (932)^{84}$?
(a) 9 (b) 1 (c) 2 (d) 4 (e) 7

Hint: Refer 1.11.1

(8) The numbers 1, 3, 5 . . . 25 are multiplied together. The number of zeroes at the right end of the product is:
(a) 22 (b) 8 (c) 13 (d) 6 (e) 0

(9) A number when divided by a divisor, leaves a remainder of 63. If the remainder is 55 when twice the number is divided by the same divisor, then the divisor is:
(a) 21 (b) 37 (c) 16 (d) 49 (e) None of these

Hint: Divisor must be always greater than the remainder. Use this concept. Do not go for calculation.

(10) The sum of two numbers is 's' and their quotient is $\frac{p}{q}$. The numbers are:

(a) $\frac{ps}{q}, \frac{qs}{p}$ (b) $\frac{s}{p}, \frac{s}{q}$ (c) $\frac{s-p}{q}, \frac{s-q}{p}$ (d) $\frac{ps}{p+q}, \frac{qs}{p+q}$ (e) None

(11) How many digits are required to number a book containing 200 pages?
(a) 200 (b) 600 (c) 492 (d) 372 (e) 250

(12) How many numbers between 101 and 300 are divisible by both 3 and 5?
(a) 107 (b) 20 (c) 127 (d) 14 (e) 34

(13) A number is multiplied by 5 and 25 is added to it. The result is divided by 5 and the original number is subtracted from the same. The remainder will be:
(a) 0 (b) 1 (c) 2 (d) 3 (e) 5

Tips: Do not waste your time by forming a linear equation. Rather use the concept of remainder

(14) $\left(\frac{1}{3} - \frac{1}{4}\right)$ is added to a number. From the sum so obtained, $\frac{1}{3}$ of $\frac{1}{4}$ is subtracted and the remainder is $\left(\frac{1}{3} + \frac{1}{4}\right)$. The number is:

(a) $\frac{1}{12}$ (b) $\frac{7}{12}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{3}{4}$

(15) A number when divided by 5 leaves a remainder 3. What is the remainder when the square of the same number is divided by 5?
(a) 9 (b) 3 (c) 0 (d) 4 (e) 1

(16) There is a number 8 * 20 which if multiplied by 6, the product is divisible by 8. The digit replacing * mark is:
(a) 4 (b) 1 (c) any digit in between 0 and 9 (d) 7 (e) 0

(17) If $B = 2 \times 4 \times 6 \ldots 98 \times 100$, then the number of zeroes at the end of B will be:
(a) 330 (b) 11 (c) 10 (d) 101 (e) 12

(18) In the product of $24 * \times 981 \times 79 \times 104$ if the digit in the unit place is 2, then what will come in place of the asterisk?
(a) 2 (b) 3 (c) 6 (d) 7 (e) either (a) or (d)

(19) A number consists of four digits having 8 in the unit's place. If the digit in the extreme left is shifted to the immediate right to the unit's place, keeping all other numbers as they are, the new number formed exceeds the original number by 1305. Find the original number. **(ASSL. Grade, '96)**
(a) 4358 (b) 2731 (c) 3478 (d) 3316 (e) 4387

(20) A 4-digit number divisible by 7 becomes divisible by 3 when 10 is added to it. The largest such number is:
(a) 9999 (b) 9996 (c) 9989 (d) 9987 (e) 9993

Answers

1. d) **2.** (b) **3.** (e) **4.** (c) **5.** (d) **6.** (b) **7.** (d) **8.** (e) **9.** (e)
10. (d) **11.** (c) **12.** (d) **13.** (a) **14.** (b) **15.** (d) **16.** (c) **17.** (e) **18.** (e)
19. (c) **20.** (c)

Section B: Number Series

(1) How many 7s are there in the following series that are not immediately followed by 3 but immediately preceeded by 8 **(ASM, '02)**
8987622632697328727787737794
(a) 2 (b) 3 (c) 4 (d) 6 (e) 7

(2) If the given numbers are arranged in the descending order based on the sum of the digits of each number, which number will be in the middle?
842 641 961 479 715 216 523
(a) 961 (b) 216 (c) 479 (d) 715 (e) 523

(3) If by beginning with 1, consecutive numbers are continuously written to its right, then which digit will be written on thirty-first position?
(a) 1 (b) 2 (c) 3 (d) 0 (e) 4

(4) If the given numbers are arranged in such a way that each group of three assending numbers is followed by their LCM and the beginning number is 1, then 11th number is how many times of the fifth number?
1, 2, 3, 4, 5, 6, 5, 6, 6, 7, 60
(a) $\frac{3}{2}$ (b) 1.75 (c) 1.4 (d) $\frac{1}{3}$ (e) None of those

Directions (5–6): Study the following number series to answer these questions
2 6 7 5 4 3 7 4 8 9 4 3 2 5 4 7 9 8 6 8 7 1 2 5 3 7 6 8 9 3 6

(5) How many such numbers are there in the series which are not immediately followed by its multiple?
(a) 25 (b) 4 (c) 27 (d) 21 (e) 20

(6) If the order of last 15 numbers is reversed, which number will be eighth to the right of thirteenth number from left?
(a) 5 (b) 6 (c) 7 (d) 3 (e) 2

Directions (7–11): One number is wrong in the number series given in each of the following questions. You have to identify that number. Assuming that a new series starts with that number, and following the same logic as in the given series, which of the numbers given in (1), (2), (3), (4) and (5) given below the series will be the third number in the new series? **(SBI PO, '99)**

(7) 3 5 12 38 154 914 4634

(1) 1636 (2) 1222 (3) 1834 (4) 3312 (5) 1488

Hint: × 1 + 2, × 2 + 2, × 3 + 2 . . . So, 914 is incorrect. The new series begins with 914 and third number in the new series will be 1834

(8) 3 4 10 34 136 685 1446

(1) 22 (2) 276 (3) 1374 (4) 72 (5) 12

(9) 214 18 162 62 143 90 106

(1) –34 (2) 110 (3) 38 (4) 10 (5) 91

Hint: $-(14)^2, +(12)^2, -(10)^2, +(8)^2, \ldots$ So, 143 is incorrect.

(10) 160 80 120 180 1050 4725 25987.5

(1) 60 (2) 135 (3) 3564 (4) 787.5 (5) 90

Hint: $\times\frac{1}{2}, \times\frac{3}{2}, \times\frac{5}{2}$ . . . Hence 180 is incorrect.

(11) 2 3 7 13 26 47 78

(1) 13 (2) 11 (3) 20 (4) 15 (5) 18

Directions (12–18): In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer. For example, if an interchange of the number marked '1' and the number marked '4' is required to establish the series, your answer is 1. If it is not necessary to interchange the positions of the numbers to establish the series give '5' as your answer.

(12) 8	4	12	6	4	30
	(1)	(2)	(3)	(4)	(5)

Hint: $\times\frac{1}{2}, \times 1, \times 1\frac{1}{2}, \times 2$ . . . So, the numbers at (2) and (4) are to be interchanged

(13) 829	436	661	300	557	508
	(1)	(2)	(3)	(4)	(5)

Hint: $-23^2, +19^2, -15^2, +11^2 + \ldots$

(14) 6	56	1	19	11	529
	(1)	(2)	(3)	(4)	(5)

Hint: $\times 1^2 + 5, \times 1^2 - 10, \times 2^2 + 15 \ldots$

(15) 21	29	23	21	41	61
	(1)	(2)	(3)	(4)	(5)
(16) 4	0	–7	–45	–20	–94
	(1)	(2)	(3)	(4)	(5)
(17) 0	6	184	56	109	23
	(1)	(2)	(3)	(4)	(5)
(18) 375	363	356	344	336	324
	(1)	(2)	(3)	(4)	(5)

Directions (19–24): In each of the following questions a number series is given. A number in the series is replaced by a letter 'A'. You have to find out the number that has been replaced by 'A' and use this number to find out the value that should be in the place of the question mark in the equation following the series.

(19) 36 216 64.8 388.8 A 699.84 209.952 **(BSRB Mumbai PO, '99)**

$A \div 36 = ?$

(1) 61.39 (2) 0.324 (3) 3.24 (4) 6.139 (5) 32.4

(20) $\frac{3}{8}$ $\frac{3}{4}$ $\frac{9}{16}$ $\frac{9}{8}$ $\frac{27}{32}$ $\frac{27}{16}$ A. **(BSRB Mumbai PO, '99)**

$\sqrt{A} = ?$

(1) $\frac{3}{2}$ (2) $\frac{6}{8}$ (3) $\frac{6}{4}$ (4) $\frac{3}{4}$ (5) $\frac{9}{8}$

(21) 99 163 A 248 273 289

$\sqrt{2A+17} = ?$

(1) 20.5 (2) 21 (3) 20 (4) 20.7 (5) 19

(22) A 12 9 $7\frac{1}{5}$ 6 $5\frac{1}{7}$ **(BSRB Bangalore, 2000)**

18% of A + 24% of A = ?

(1) 7.56 (2) 8.20 (3) 9.42 (4) 6.38 (5) 10.64

(23) 125 A 1127 1176 9408 9472

$A^2 - 2A = ?$

(1) 23799 (2) 28063 (3) 25599 (4) 27850 (5) 18749

Hint: Series is $+ 6^2, \times 7, + 7^2, \times 8 \ldots$

(24) 14.8 17.2 A 22 2.8 41.2 **(BSRB Bangalore, 2000)**

25% of $25A$ = ?

(1) 77.5 (2) 73.5 (3) 172.5 (4) 86.5 (5) 92.8

Answers

1. (a)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (3)	**8.** (4)	**9.** (5)
10. (2)	**11.** (2)	**12.** (2)	**13.** (1)	**14.** (1)	**15.** (1)	**16.** (3)	**17.** (2)	**18.** (5)
19. (3)	**20.** (5)	**21.** (2)	**22.** (1)	**23.** (3)	**24.** (1)			

2 HCF AND LCM OF NUMBERS

2.1 FACTORS

In a division, if a number f divides a number M completely (exactly) or in other words, if M is exactly divisible by f, then **'f' is the factor of M.**

Example: 5 divides 35 completely, so, 5 is a factor of 35.
Similarly, 2, 3, 4, 6 are all factors of 12, because each of the numbers 2, 3, 4, and 6 will divide 12 completely or, in other words 12 is divisible by 2, 3, 4 and 6.

2.2 MULTIPLES

From the above concept, if f is a factor of M, then M is a multiple of f.

Example: 63 is completely divisible by 7, 3, 9, 21. So, 63 is a multiple of 7 or 3 or 9 or 21.

2.3 PRINCIPLE OF PRIME FACTORISATION

Any natural number (>1) is either prime or non-prime (composite).

The principle of prime factorisation states:

Each non-prime (composite) number can be uniquely broken (reduced) into two or more prime numbers (prime factors). In other words, each non-prime number is divisible by any of the prime numbers.

With the use of this principle, a non-prime number is broken into its prime factor by dividing it with different prime numbers. This is known as division method of factorisation of a number. The same is explained in the following example.

Example: Resolve 20570 into its prime factors.

	Division by prime number		
	2	20570	
	5	10285	→ 1^{st} Quotient
Prime Factors	11	2057	→ 2^{nd} Quotient
	11	187	→ 3^{rd} Quotient
	17	17	→ 4^{th} Quotient
		1	

Thus, $20570 = 2 \times 5 \times 11 \times 11 \times 17$.
Hence, if the number is even, the division should start with 2; otherwise, rest of the prime numbers should be tried in succession.

2.4 HIGHEST COMMON FACTOR (HCF)

If two or more numbers are broken into their prime factors (as explained in 2.3), then the product of the maximum common prime factors in the given numbers is the H.C.F. of the numbers.

In other words, the HCF of two or more numbers is the greatest number (divisor) that divides all the given numbers exactly. So, HCF is also called the Greatest Common Divisor (GCD).

Example: Find the HCF of 72, 60, 96.
Here, we first find the prime factors of each given number.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

Here $72 = 2 \times 2 \times 2 \times 3 \times 3$
$60 = 2 \times 2 \times 3 \times 5$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
and so HCF = product of maximum common prime factors $= 2 \times 2 \times 3 = 12$

Note: The common factors in the given numbers have been encircled.

2.5 LCM (LOWEST COMMON MULTIPLE)

The LCM of two or more than two numbers is the product of the highest powers of all the prime factors that occur in these numbers.

Example: Find the LCM of 36, 48, 64 and 72

$$\begin{array}{r|rrrr} 2 & 36, & 48, & 64, & 72 \\ \hline 2 & 18, & 24, & 32, & 36 \\ \hline 2 & 9, & 12, & 16, & 18 \\ \hline 2 & 9, & 6, & 8, & 9 \\ \hline 3 & 9, & 3, & 4, & 9 \\ \hline 3 & 3, & 1, & 4, & 3 \\ \hline & 1, & 1, & 4, & 1 \end{array}$$

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 4 = 576$.

2.6 PRODUCT OF TWO NUMBERS

HCF of numbers × LCM of numbers = Product of numbers.

i.e., if the numbers are A and B, then

$$\text{HCF of } A \text{ and } B \times \text{LCM of } A \text{ and } B = A \times B$$

2.7 DIFFERENCE BETWEEN HCF AND LCM

HCF of x, y and z	LCM of x, y and z
is the **Highest Divisor** which can exactly divide x, y and z.	is the **Least Dividend** which is exactly divisible by x, y and z.

Table 2.1 Rapid Information List

Ref. No.	*Type of Problem*	*Approach to Problem*
2.3	Find the *GREATEST NUMBER* that will *exactly* divide x, y and z.	Required number = *HCF* of x, y and z (greatest divisor)
2.4	Find the *GREATEST NUMBER* that will divide x, y and z leaving remainders a, b and c respectively.	Required number (greatest divisor) = *HCF* of $(x - a)$, $(y - b)$ and $(z - c)$
2.5	Find the *LEAST NUMBER* which is *exactly* divisible by x, y and z.	Required number = *LCM* of x, y and z (least dividend)
2.6	Find the *LEAST NUMBER* which when divided by x, y and z leaves the remainders a, b and c respectively.	Then, it is always observed that $(x - a) = (y - b) = (z - c) = K$ (say). ∴ Required number = (*LCM* of x, y and z) − (K)
2.7	Find the *LEAST NUMBER* which when divided by x, y and z leaves the same remainder 'r' each case.	Required number = (*LCM* of x, y and z) + r.
2.8	Find the *GREATEST NUMBER* that will divide x, y and z leaving the same remainder in each case.	Required number = *HCF* of $(x - y)$, $(y - z)$ and $(z - x)$
2.9	Find the n-digit *GREATEST NUMBER* which when divided by x, y and z, (a) leaves no remainder (i.e., exactly divisible) (b) leaves remainder K in each case.	*LCM* of x, y and z = L (Step 1) L)*n-digit greatest number*((Step 2) *remainder* = R *By Rule I (Chapter 1)*, (a) Required number = n-digit greatest number − R (b) Required number = [n-digit greatest number − R] + K

(*Contd.*)

(Contd.)

Ref. No.	*Type of Problem*	*Approach to Problem*
2.10	Find the n-digit *SMALLEST NUMBER* which when divided by x, y and z *(a)* leaves no remainder (i.e., exactly divisible) *(b)* leaves remainder K in each case.	*LCM* of x, y and $z = L$ (Step 1) L) n-digit greatest number ((Step 2) *remainder* $= R$ *By Rule II (Chapter I)* (a) Required number $= n$-digit smallest number $+ (L - R)$ (b) Required number $= n$-digit smallest number $+ (L - R) + K$
2.11	Find the *HCF* of $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$	*HCF* of fractions $= \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$
2.12	Find the *LCM* of $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$	*LCM* of fractions $= \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$
2.13	Find the *HCF* of decimal numbers	**Step 1** Find the *HCF* of the given numbers **without** decimal. **Step 2** Put the decimal point (in the *HCF* of Step 1) from right to left according to the MAXIMUM decimal places among the given numbers.
2.14	Find the *LCM* of decimal numbers	**Step 1** Find the *LCM* of the given numbers **without** decimal. **Step 2** Put the decimal point (in the *LCM* of Step 1) from right to left at the place equal to the MINIMUM decimal places among the given numbers.

Solved Examples

E-1 Find the greatest number that will exactly divide 200 and 320.

S-1 Using the approach *2.3*,
required number = HCF of 200 and 320 = **40**.

E-2 Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively.

S-2 Using the approach *2.4*,
required number = HCF of (148 – 4),
(246 – 6) and (623 – 11) i.e. HCF of 144, 240, 612 is **12**.

E-3 Find the least number which when divided by 27, 35, 45 and 49 leaves the remainder 6 in each case.

S-3 Using the approach *2.7*,
required number = (LCM of 27, 35, 45 and 49) + 6 i.e. 6615 + 6 = **6621**.

E-4 Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively.

S-4 Using the approach *2.6*,

we get, $(36 - 25) = (48 - 37) = (64 - 53) = 11$

$\therefore$ required number = (LCM of 36, 48 and 64) – 11

= 576 – 11 = **565**.

E-5 Find the greatest possible length of a scale that can be used to measure exactly the following lengths of cloth: 3 m, 5 m, 10 cm and 12 m 90 cm.

S-5 The lengths of cloth to be measured are, 300 cm, 510 cm and 1290 cm.

$\therefore$ the required length of the scale is HCF of 300, 510 and 1290 i.e. 30

$\therefore$ the greatest possible length of the scale to be used = **30** cm.

E-6 Find the smallest number which when

(a) increased by 8 (or added by 8) ***(b)*** decreased by 8 (or subtracted by 8)

is exactly divisible by 15, 21, 30.

S-6 LCM of 15, 21, 30 = 210.

(a) the required number = LCM – (the number added) i.e. 210 – 8 = 202

(b) the required number = LCM + (the number subtracted) i.e. 210 + 8 = **218**.

E-7 Find

(a) the greatest number of 4 digits and ***(b)*** the smallest number of 4 digits

such that they are exactly divisible by 12, 15, 20 and 35.

S-7 ***(a)*** Using the approach *2.9(a)*,

Step 1 LCM of 12, 15, 20 and 35 = 420.

Step 2

$$\begin{array}{r} 420\ )\ 9999\ (\ 23 \\ \underline{9660} \\ 339 \end{array}$$

$\therefore$ required number = 9999 – 339 = **9663**.

(b) Using the approach *2.10(a)*,

Step 1 LCM of 12, 15, 20 and 35 = 420.

Step 2

$$\begin{array}{r} 420\ )\ 1000\ (\ 2 \\ \underline{840} \\ 160 \end{array}$$

$\therefore$ required number = 1000 + (420 – 160) = **1260**.

E-8 Four bells first begin to toll together and then at intervals of 6, 7, 8 and 9 seconds respectively. Find how many times the bells toll *together* in two hours and *at what interval they toll together*?

S-8 LCM of 6, 7, 8 and 9 = 504.

$\therefore$ All the bells toll together after each interval of 504 seconds.

$\therefore$ in two hours, no. of times they toll together = $\dfrac{2 \times 60 \times 60}{504}$ times $\approx$ **14 times**.

E-9 Find the

(a) the greatest 3-digit number, and ***(b)*** the smallest 3-digit number

such that when they are divided by 12, 18, 21 and 28, it leaves a remainder 3 in each case.

S-9 ***(a)*** Using the approach *2.9(b)*,

Step 1 LCM of 12, 18, 21 and 28 = 252

Step 2 252) 9999 (39
9828
———
171

$\therefore$ the required number = (9999 – 171) + 3 = **9931**.

(b) Using the approach *2.10(b)*,

Step 1 LCM of 12, 18, 21 and 28 = 252.

Step 2 252) 1000 (3
756
———
244

$\therefore$ the required number = 1000 + (252 – 244) + 3 = **1011**.

E-10 Find the numbers between 200 and 300 such that when they are divided by 6, 8 or 9,

(a) it leaves no remainder, i.e. exactly divisible.

(b) it leaves in each case a remainder 5

S-10 Here, the number to be found out are the DIVIDENDS. Now, the LCM of 6, 8, 9 = 72.

(a) Multiples of 72 which lie between 200 and 300 are $72 \times 3 = 216$ and $72 \times 4 = 288$

(b) Here, the remainder is 5 in each case.

$\therefore$ Required numbers are (216 + 5) and (288 + 5) i.e. **221** and **293**.

E-11 There are two electrical wires, one is a 9 m 60 cm long aluminium wire and the other is a 5 m 12 cm long copper wire. Find the

(a) maximum length that can be equally cut from each wire in such a way that the total length of each wire is exactly divisible by it.

(b) how many such largest possible pieces are available in each kind of wire?

S-11 9 metre 60 cm = 960 cm and 5 metre 12 cm = 512 cm.

(a) The required largest piece = HCF of 960 and 512 cm, i.e. **64 cm**.

(b) $\therefore$ Number of such aluminium wire pieces = $\frac{960}{64}$ **nos.**

and number of such copper wire pieces = $\frac{512}{64}$ **nos.**

E-12 HCF and LCM of two numbers are 16 and 240 respectively. If one of the numbers is 48, find the other number.

S-12 We know that, HCF × LCM = Product of two numbers

$\therefore$ second number = $\frac{16 \times 240}{48}$ i.e. **80**.

E-13 Among how many students, 175 bananas and 105 oranges can be equally divided?

S-13 HCF of 175 and 105 = 35

$\therefore$ The required number of students is **35**, or factors of 35, namely 5 or 7.

E-14 Find out the HCF of 11, 0.121 and 0.1331.

S-14 *Step 1* HCF of 11, 121 and 1331 is **11**.

Step 2 Resultant HCF = **0.0011** (Since maximum decimal places = 4 in 0.1331).

E-15 Find out the LCM of 2.2, 540 and 1.08.

S-15 *Step 1* LCM of 22, 540 and 108 is **5940**.

Step 2 Here minimum decimal place = 1 (in 2.2)

So, resultant LCM = **594**.

E-16 Find out the HCF of 3^5 , 3^9 and 3^{14}.

S-16 Here the base of each number is same (= 3) but indices are different.

So, the required HCF = number with the minimum index, i.e. $\mathbf{3^5}$.

E-17 Find out the LCM of 4^5, 4^{-81}, 4^{12} and 4^7.

S-17 Here the base of each number is the same (= 4) but indices (or powers) are different.

So, the required LCM = number with the maximum index, i.e. $\mathbf{4^{12}}$.

REGULAR PROBLEMS

(1) What is the greatest possible length of scale to measure exactly the following lengths, 20 feet, 13 feet 9 inches, 17 feet 6 inches, 21 feet 3 inches?

(a) 1 feet 6 inches (b) 1 feet 3 inches (c) 9 inches
(e) 2 feet 4 inches (e) 6 inches

(2) The greatest number that will divide 410,751 and 1030 leaving a remainder 7 in each case is:

(a) 29 (b) 13 (c) 17 (d) 37 (e) 31

Hint: Since the number to be found out is a GREATEST DIVISOR, so HCF is to be found out

(3) The ratio of two numbers is 15 : 11. If their HCF is 13, then the numbers are:

(a) 75, 55 (b) 45, 22 (c) 104, 44 (d) 195, 143 (e) None

Hint: Since HCF is 13, so, the numbers will be $13 \times \underbrace{15 \text{ and } 13 \times 11}_{\text{terms of ratio}}$ (Delhi Metro Rail, 2002)

(4) The LCM of two numbers is 1296 and HCF is 96. If one of the numbers is 864, then the other is:

(a) 72 (b) 64 (c) 144 (d) 11664 (e) 36

(5) Three men start together to walk along a road at the same rate. The length of their strides are 68 cm, 51 cm and 85 cm respectively. How far will they go before they will be 'in step' again:

(a) 102 m (b) 1020 m (c) 10.2 m (d) 150 m (e) 17 cm

(6) How many times is the HCF of 48, 36, 72 and 24 contained in their LCM?

(a) 10 (b) 12 (c) 120 (d) 2 (e) 15

(7) The greatest 4-digit number exactly divisible by 88 is:

(a) 8888 (b) 9944 (c) 9988 (d) 9999 (e) 8899

(8) Find the least number of soldiers in a regiment, such that they stand in rows of 18, 15 and 25 and form a perfect square?

(a) 900 (b) 1600 (c) 2500 (d) 450 (e) 400

Hint: Find LCM and then multiply by the factors to make it a perfect square

(9) Three strings of a musical instrument vibrate 6, 8, and 12 times a second respectively. If all the three begin to vibrate simultaneously, find the shortest time interval before all three vibrate together again?

(a) 2 sec (b) 48 sec (c) $\frac{1}{2}$ sec (d) $\frac{1}{24}$ sec (e) 24 sec.

Hint: Time to vibrate once is $\frac{1}{6}$ sec, $\frac{1}{8}$ sec and $\frac{1}{12}$ sec.

(10) Which is the smallest number that can be subtracted from 1936 so that on being divided by 9, 10, 15 the remainder is 7 everytime? **(RRB Ajmer, '97)**

(a) 93 (b) 46 (c) 76 (d) 39 (e) 53

(11) The smallest number from which if 4000 subtracted, is exactly divisible by 7, 11 and 13, is

(a) 5001 (b) 2999 (c) 1000 (d) 6303 (e) 5101

Hint: Using the concept of LCM, we find, LCM of 7, 11, 13 = Required number – 4000

(12) Find the least number which when divided by 20, 25, 30, 36 and 48 leaves the remainders 15, 20, 25, 31 and 43 respectively.
(a) 2165 (b) 144 (c) 3595 (d) 3600 (e) 2875

(13) Find the H.C.F. of 2.4, 0.36 and 7.2 (RRB, Patna, 2002)
(a) 12 (b) 120 (c) 1.2 (d) 0.12 (e) 0.012

(14) Traffic lights at three different points are changing respectively at 24, 48 and 72 seconds. If all the three are changed together at 9 : 10 : 24 hours, then when will the next change take place together? **(RRB, Guwhati, '97)**
(a) 9 : 12 : 25 hrs (b) 9 : 10 : 48 hrs. (c) 9 : 12 : 48 hrs.
(d) 9 : 10 : 50 hrs (e) None

(15) The HCF of two numbers is 12 and their difference is also 12. The numbers are
(a) 12, 84 (b) 100, 112 (c) 40, 52 (d) 84, 96 (e) 120, 124

Hint: Do not try to calculate. Only check which of the given choices satisfy the given condition

(16) The sum of two numbers is 45 and their difference is $\frac{1}{9}$ of their sum. Their LCM is
(a) 200 (b) 100 (c) 90 (d) 180 (e) 250

(17) John has a camera that takes film that allows 24 exposures, whereas Nancy has a camera that takes film that allows 36 exposures. Both of them want to be able to take the same number of photographs and complete their rolls of film. How many rolls should each buy?
(a) 12 (b) 72 (c) 3 and 2 (d) 6 (e) 144

Answers

1. (b)	**2.** (e)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (c)
10. (d)	**11.** (a)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (c)	

REAL PROBLEMS

(1) The least number by which 825 must be multiplied in order to produce a multiple of 715 is **(Bank PO, '90)**
(a) 11 (b) 5 (c) 13 (d) 17 (e) 19

(2) The LCM of two number is 630 and their HCF is 9. If the sum of the numbers is 153, then the ratio of the two numbers is
(a) 70 (b) 9 (c) 0.7 (d) $\frac{10}{7}$ (e) (c) or (d)

(3) The HCF of two numbers each consisting of four digits is 103 their LCM is 19261, then the numbers are
(a) 1133, 1751 (b) 1621, 2031 (c) 3031, 3523 (d) 2979, 2277 (e) 1833, 1651

(4) The least number which is a multiple of 31 and when divided by 15, 24 and 32 leaves the remainders 2, 11 and 19 respectively is, **(RRB, Secunderabad, '01)**
(a) 2356 (b) 2387 (c) 2325 (d) 2418 (e) 2722

Hint: Here, the remainder in each case is less than the divisor by 13. The number can be 480 k – 13, where, the minimum value of k will make (480 k – 13) divisible by 31. Put k = 1, 2 . . . and check with the given choices.

(5) Three men start together to travel the same way around a circular track of 11 kms. Their speeds are 4, $5\frac{1}{2}$ and 8 kms per hour respectively. When will they meet at the starting point?

(a) 22 hrs. (b) 12 hrs. (c) 11 hrs. (d) 44 hrs. (e) 36 hrs.

(6) In a seminar, the number of participants in Physics, Chemistry and Mathematics are 96, 36 and 180 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

(a) 12 (b) 21 (c) 36 (d) 26 (e) Not possible

(7) If the last divisor is 75 and the quotients are 3, 1, 1 and 3 respectively, in finding the HCF of two numbers by the method of division, then those two numbers are **(Mumbai Bank PO, '95)**

(a) 500, 1875 (b) 425, 1675 (c) 525, 1875 (d) 525, 1575 (e) 575, 1875

Hint: Assume two numbers as x and y. Follow the method of division to find HCF.

(8) Sum of two numbers is 56. If their LCM is 105, then the numbers are

(a) 7, 49 (b) 21, 34 (c) 24, 32 (d) 35, 21 (e) 27, 29

Tips: HCF of two numbers = HCF of (their sum and their LCM)

(9) The greatest number of three digits which when added to 45 is exactly divisible by 6, 8, 12 is **(RRB Kolkata Asst. Driver '01)**

(a) 963 (b) 987 (c) 984 (d) 980 (e) 1077

Hint: Find the greatest number which is less than 999 + 45 (= 1044) and divisible by the LCM of 6, 8 and 12 and then find the required number

(10) The sum of two numbers is pq and their difference is $\frac{1}{7}$ of their sum. Their HCF is

(a) $\frac{p+q}{pq}$ (b) $7\frac{(p-q)}{pq}$ (c) $\frac{12}{7}pq$ (d) $\frac{pq}{7}$ (e) $\frac{6}{7}pq$

(11) What is the smallest whole number that is exactly divisible by $1\frac{5}{28}$, $2\frac{2}{21}$ and $3\frac{1}{7}$?

(a) 132 (b) 130 (c) 138 (d) 124 (e) 112

Hint: Required number = numerator of the LCM.

(12) A boy running up a stair case finds that when he goes up two steps at a time there is one step over; when he goes up three at a time there are two over and when he goes up four at a time, there are three over. Find the number of stairs, which is somewhere between 40 and 50. **(BSRB, '99)**

(a) 47 (b) 45 (c) 42 (d) 49 (e) None

Hint: If there were one more stair, the no. of stairs would have been exactly divisible by each of the numbers 2, 3 and 4 (i.e. no. of steps at a time). So, the remainder in each case is less than divisor by 1. The LCM of 2, 3 and 4 = 12

(13) A number when divided by 10 leaves a remainder 9, when divided by 9 leaves a remainder of 8, when divided by 8 leaves a remainder of 7 . . . . . . when divided by 2 leaves a remainder of 1. Determine the number. **(SSC, '96)**

(a) 31 (b) 1029 (c) 2519 (d) 1679 (e) 189

(14) When 1388, 3309 and 7151 are divided by a certain number of three digits, the remainders are the same. Find the remainder.

(a) 17 (b) 32 (c) 113 (d) 11 (e) 1921

(15) Four prime numbers are written in ascending order of their magnitudes. The product of first three is 715 and that of last three is 2431. What is the largest given prime number?

(a) 5 (b) 19 (c) 17 (d) 23 (e) 31

Hint: Let a, b, c and d be the prime numbers in ascending order.

$$a\boxed{bc} = 715 \quad \therefore \text{ HCF of 715 and 2431} = bc.$$

$$\boxed{bc}d = 2431 \quad \therefore d = \frac{bcd}{bc}$$

(16) The LCM of two numbers is 2900% more than their HCF and the sum of LCM and HCF of two numbers is 310. If one of the numbers is 20, find the other number. **(SSC, '93)**

(a) 290 (b) 150 (c) 75 (d) 300 (e) 58

Hint: Firstly find the LCM and HCF, by using two given conditions & then, solve.

(17) The LCM of three numbers is 4752 and HCF is 6. If the two numbers are 48 and 66, find the third least number

(a) 72 (b) 99 (c) 48 (d) 528 (e) 54

Hint: Let the third least number be $6x$, then
$$6\,\underline{\big|\,48,\quad 66,\quad 6x}$$
$$\quad 8,\quad 11,\quad x$$

$\Rightarrow$ LCM $= 6 \times 8 \times 11 \times x = 4752$ (given)

$\therefore$ $x = 9 \Rightarrow 6x = 6 \times 9 = 54$

(18) A man has certain number of small boxes to pack into parcels. If he packs 3, 4, 5 or 6 boxes in parcel, he is left with one over; if he packs 7 in a parcel, none is left over. What is the number of boxes he may have to pack?

(a) 301 (b) 400 (c) 309 (d) 405 (e) 105

Answers

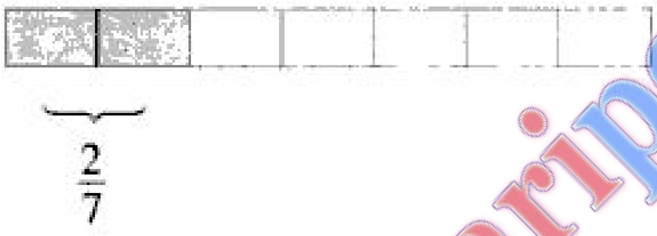
1. (c) **2.** (e) **3.** (a) **4.** (b) **5.** (a) **6.** (d) **7.** (c) **8.** (d) **9.** (b)
10. (d) **11.** (a) **12.** (a) **13.** (c) **14.** (b) **15.** (c) **16.** (b) **17.** (e) **18.** (a)

3 FRACTION

3.1 DEFINITION

A number of the type $\frac{x}{y}$ which represents x number of parts out of y number of equal parts of a thing is called a **fraction**.

$\therefore$ Fraction $\frac{2}{7}$ represents 2 equal parts out of 7 equal parts of a thing. In the figure, the shaded part represents $\frac{2}{7}$



$\therefore$ FRACTION = $\frac{\text{Numerator}}{\text{Denominator}}$. Such a fraction is known as common fraction or vulgar fraction

- A fraction, whose denominator is 10 or 100 or 1000 etc. is called a **decimal fraction**.
- Fractions whose denominators are same, are called **like fractions**, e.g. $\frac{3}{8}, \frac{5}{8}$ are like fractions.
- Fractions whose denominators are different, are called **unlike fractions**, e.g. $\frac{3}{4}, \frac{9}{11}$ are unlike fractions.

3.1.1 Comparison of Fractions

Two or more different fractions can be compared with the help of the following rules:

Rule 1

When two fractions have the same denominator, the greater fraction is that which has the greater numerator.

Example: Thus, $\frac{5}{11}$ is greater than $\frac{3}{11}$

Rule 2

When two fractions have the same numerator, the greater fraction is that which has the smaller denominator.

Example: Thus, $\frac{7}{13}$ is greater than $\frac{7}{19}$

Rule 3

When two or more fractions with different denominators and different numerators are to be compared, then the following simple technique is to be used:

Step 1 Among all the given fractions,
let the maximum number of digits in the numerator = n
the maximum number of digits in the denominator = d

Step 2 Find $(d - n)$.

Step 3 If $(d - n) = 0$ or 1, multiply each fraction by 10.
If $(d - n) = 2, 3, 4 \ldots$ multiply each given fraction by 10^2, 10^3, $10^4 \ldots$ respectively.

Step 4 After multiplication, find only the integer value of the resultant fraction.

Step 5 If in step 4, any of the two fractions have the same integer value, then find the next decimal place and so on.

Step 6 Compare the integer/decimal values obtained in step 4 or step 5. The fraction having the maximum value is the greatest fraction.

Note: In order to write the given fraction in ascending order the smallest fraction is written first, then the next greater one and so on. In order to write the given fraction in descending order, the greatest fraction is written first, then the next smaller and so on.

Example: Arrange $\frac{7}{13}, \frac{493}{971}, \frac{87}{165}, \frac{123}{235}$ in descending order

Solution: Here, maximum no. of digits in the numerator $n = 3$ (in 493 or in 123) and maximum no. of digits in the denominator = $d = 3$ (in 971 or in 235)
Now, $d - n = 3 - 3 = 0$
So, multiply the given fractions by 10.

$\frac{7}{13} \times 10,$	$\frac{493}{971} \times 10,$	$\frac{87}{165} \times 10,$	$\frac{123}{235} \times 10.$	
5	5	5	5	(integer values)

Since the integer values are same, so, find the next decimal digit for these fractions,

5.3	5.0	5.2	5.2

$\underbrace{5.2 \quad\quad 5.2}_{\text{same}}$

Since the value of two fractions are same, find the second decimal digit for these two fractions only, by dividing further

$$5.3 \quad 5.0 \quad 5.27 \quad 5.23.$$

Now, $5.3 > 5.27 > 5.23 > 5.0$

$\Rightarrow \frac{7}{13} > \frac{87}{165} > \frac{123}{235} > \frac{493}{971}$ (in descending order)

3.2 FRACTIONAL PART OF A NUMBER

Fractional part of a number (or quantity) is simply the product of the related fraction and the given number.

Example: Consider a given number as 60, then

Two-thirds of 60 = $\frac{2}{3} \times 60 = 40$

$\therefore$ $\frac{2}{3}$rd of 60 is 40 (fractional part)

$$\boxed{\text{Fractional part of any number = number} \times \text{its related fraction.}} \quad \textbf{(1)}$$

3.2.1 Different Fractional Parts of the Same Number

Consider any number, say, 36

then $\frac{3}{4}$th of 36 = 27 (fractional part of 36)

$\frac{1}{9}$th of 36 = 4 (fractional part of 36)

$\frac{2}{3}$rd of 36 = 24 (fractional part of 36)

From this, we find that as the fraction changes, the fractional part of the same number also changes.

In our earlier examples, we find that

$$\frac{27}{3/4} = \frac{4}{1/9} = \frac{24}{2/3} \ldots = \underbrace{36}_{\text{Original number}} \text{ (Fixed)}$$

In such cases, equation (1) can be re-written as

$$\frac{\text{Any fractional number}}{\text{its related fraction}} = \underbrace{\text{Original number}}_{\text{it is fixed.}} \quad \textbf{(2)}$$

Example: A man travels $\frac{1}{4}$th part by scooter, $\frac{3}{8}$th by car and rest 48 km by bus. Find the total distance covered.

Solution: Here, total distance (i.e. original quantity) is to be found out.

Fraction related to rest 48 km = $\underbrace{1}_{\text{related fraction to total distance}} - \left(\frac{1}{4} + \frac{3}{8}\right)$

Using the relation (2)

$$\frac{\text{Any fractional number}}{\text{its related fraction}} = \text{Original number,}$$

Here, we find,

$$\frac{\text{Rest distance}}{\text{its related fraction}} = \text{total distance}$$

$$\Rightarrow \qquad \frac{48}{1-\left(\frac{1}{4}+\frac{3}{8}\right)} = \text{total distance}$$

$$\Rightarrow \qquad \text{total distance} = \frac{48}{3/8} = 128 \text{ km}$$

Example: If $\frac{3}{17}$th of a number is 18, then find its two-third.

Solution: Let $\frac{2}{3}$rd of number = x.

Since both the fractions $\frac{3}{17}$ and $\frac{2}{3}$ are to be found out for the same number, the relation (2) can be used as*

$$\frac{\text{Fractional number}}{\text{its related fraction}} = \frac{\text{Another fractional number}}{\text{its related fraction}} = \text{original number (always)}$$

$$\Rightarrow \qquad \frac{18}{3/17} = \frac{x}{2/3}$$

$$x = 68$$

∴ Two-third of the number is 68.

**Note:* This relation can be used to find another fractional part directly without finding the original number.

3.3 TO FIND THE FRACTION RELATED TO BALANCE (REST) AMOUNT

Conventionally, we have learnt that
Fraction related to balance (rest) part = 1 – (sum of all other fractions)
It is used when **all fractions are independent.** Following example will illustrate the fact.

Example: A person spends $\frac{3}{8}$th part of his salary on food, $\frac{1}{12}$th part of his salary on education, $\frac{1}{4}$th part of his salary on clothing. He is now left with Rs. 550. Find his total salary.

Solution: Here, the spending on each item is independent, because each fraction has been indicated as out of total salary (original number).

$$\therefore \quad \text{fraction related to rest part} = 1-\left(\frac{3}{8}+\frac{1}{12}+\frac{1}{4}\right)$$

$$= \frac{5}{24}$$

$$\therefore \quad \text{total salary} = \frac{\text{Rest amount}}{\text{fraction related to rest part}}$$ *[Refer equation 2 of sec. 3.2.1]*

$$= \frac{550}{5/24} = \text{Rs } 2640.$$

Hence, for **independent fractions.**
Fraction for balance (rest) part = 1 – (sum of all independent fractions) **(3)**

Now, consider another example,

Example: A person spends $\frac{3}{8}$th part of his salary on food, $\frac{1}{12}$th of the rest part on education and $\frac{1}{4}$th of the remainder on clothing. He is now left with Rs 550. Find his total salary.

Solution: Here, spending on the second item (i.e. education) depends on the amount left after spending on the first item (i.e. food). Similarly, spending on the third item (i.e. clothing) depends on the amount left (remaining) after spending on the first item and the second item.

Here, spending on each item (except the first item) depends on the amount remaining, after spending on the previous item.

In such cases, all fractions (except the first one) are dependent on the previous fractions.

For **dependent fractions,**
Fraction for balance (rest) part = (1 – first fraction) × (1 – second fraction) **(4)**

So, in our example, using the relation 4

$$\text{fraction for balance (rest) part} = \left(1-\frac{3}{8}\right)\left(1-\frac{1}{12}\right)\left(1-\frac{1}{4}\right)$$

$$= \frac{5}{8}\times\frac{11}{12}\times\frac{3}{4}$$

$$= \frac{55}{128}$$

$$\therefore \quad \text{total salary} = \frac{\text{Rest amount}}{\text{Fraction related to rest part}}$$

$$= \frac{550}{55/128} = \text{Rs } 1280$$

Note: Observe the difference in the language of the two examples under 3.3

3.4 TO INSERT ANY NUMBER OF FRACTIONS IN BETWEEN TWO GIVEN FRACTIONS

Let two given fractions be $\frac{a}{b}$ and $\frac{x}{y}$. To insert a fraction lying between $\frac{a}{b}$ and $\frac{x}{y}$, the following steps are taken.

Step 1 The numerators of two given fractions are added to get the numerator of the **result** fraction, i.e. numerator of the result fraction = $a + x$

Step 2 The denominators are also added to get denominator of the **result** fraction. That is, denominator of the result fraction = $b + y$

Step 3 Result fraction = $\frac{a+x}{b+y}$

Hence, the result fraction so obtained has its magnitude (value) lying between the two given fractions. By this method, any number of fractions can be inserted between two given fractions.

Solved Examples

E-1 Arrange the following fractions in decreasing (descending) order:

(i) $\frac{5}{6}, \frac{3}{4}, \frac{5}{8}, \frac{6}{7}$ (ii) $\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{21}{50}$

(iii) $\frac{7}{12}, \frac{5}{16}, \frac{17}{36}, \frac{1}{3}$ (iv) $\frac{3}{5}, \frac{5}{7}, \frac{13}{16}, \frac{97}{104}$

(v) $\frac{2}{91}, \frac{5}{177}, \frac{22}{1091}, \frac{13}{558}$

S-1 Using the method 3.1.1

(i) Here $n = 1$ $d = 1$ $\therefore d - n = 0$

So, multiply the given fractions by 10.

$\therefore \frac{5}{6} \times 10 \approx 8, \frac{3}{4} \times 10 \approx 7, \frac{5}{8} \times 10 \approx 6, \frac{6}{7} \times 10 \approx 8$ (integer value)

Since two fractions have the same integer value (= 8), find the next decimal digit for these two fractions only namely

$\approx$ 8.3 7 6 8.5

Now, $8.5 > 8.3 > 7 > 6 \Rightarrow \frac{6}{7} > \frac{5}{6} > \frac{3}{4} > \frac{5}{8}$ in descending order.

(ii) Here $n = 2$ $d = 2$ $\therefore d - n = 0$.

max^m no. of digits in numerator $= n = 2$ $\left(\text{in } \frac{21}{50}\right)$

max^m no. of digits in denominator $= d = 2$ $\left(\text{in } \frac{3}{10} \text{ or } \frac{21}{50}\right)$

So, multiply the given fraction by 10.

$\therefore \frac{1}{2} \times 10 \approx 5, \frac{3}{5} \times 10 \approx 6, \frac{3}{10} \times 10 \approx 3, \frac{21}{50} \times 10 \approx 4$ (integer value)

Now, $6 > 5 > 4 > 3 \Rightarrow \frac{3}{5} > \frac{1}{2} > \frac{21}{50} > \frac{3}{10}$

(iii) Here $n = 2$ $d = 2$ $\therefore d - n = 0$.

So, multiply the given fraction by 10.

$\therefore \frac{7}{12} \times 10 \approx 5, \frac{5}{16} \times 10 \approx 3, \frac{17}{36} \times 10 \approx 4, \frac{1}{3} \times 10 \approx 3$ (integer values)

Since the two fractions have the same integer value (= 3), find the next decimal place for these two fractions only.

i.e. $\approx$ 5 3.1 4 3.3

Now, $5 > 4 > 3.3 > 3.1 \Rightarrow \frac{7}{12} > \frac{17}{36} > \frac{1}{3} > \frac{5}{16}$

(iv) Using the method 3.1.1, here, maximum number of digits in numerator = 2 $\left(\text{in } \frac{13}{16}\right)$.

Maximum number of digits in denominator = 3 $\left(\text{in } \frac{97}{104}\right)$.

So, $n = 2$ $d = 3$ $\therefore$ $d - n = 1$

So, multiply the numerator of the given fraction by 10.

$\therefore \frac{3}{5} \times 10 \approx 6,\ \frac{5}{7} \times 10 \approx 7,\ \frac{13}{16} \times 10 \approx 8,\ \frac{97}{104} \times 10 \approx 9$ (integer value)

Now, $9 > 8 > 7 > 6 \Rightarrow \frac{97}{104} > \frac{13}{16} > \frac{5}{7} > \frac{3}{5}$

(v) Here $n = 2$ (in 22 or in 13) $d = 4$ (in 1091) $\therefore$ $d - n = 2$

So, multiply the numerator of given fractions by 10^2

$\therefore \frac{2}{91} \times 100 \approx 2,\ \frac{5}{177} \times 100 \approx 2,\ \frac{22}{1091} \times 100 \approx 2,\ \frac{13}{558} \times 100 \approx 2$ (integer value)

All the fractions have the same integer value, so, find the next decimal place, i.e.

$\approx$ 2.1 2.8 2.0 2.3

Now, $2.8 > 2.3 > 2.1 > 2.0 \Rightarrow \frac{5}{177} > \frac{13}{558} > \frac{2}{91} > \frac{22}{1091}$

E-2 $\frac{5}{12}$ part of what amount will be equal to $3\frac{3}{4}$ part of Rs 100.

S-2 Let the amount be Rs x

$\therefore \quad \frac{5}{12}x = 3\frac{3}{4} \times 100 \Rightarrow \frac{5}{12}x = \frac{15}{4} \times 100$

$\Rightarrow \quad x = \frac{12}{5} \times \frac{15}{4} \times 100 \Rightarrow x = \text{Rs } 900$

$\therefore$ Required amount is Rs 900.

E-3 What fraction is 6 bananas in 5 dozens?

S-3 Required fraction = 6 out of 5 dozen

$$= \frac{6}{5 \times 12} = \frac{1}{10}.$$

E-4 There are 40 students in a class. One day only $\frac{7}{10}$ students were present. Find the number of absentees on that day.

S-4 In solving the problem on fraction, the whole quantity is *always* considered as 1.

$\therefore$ Number of absentees = Fraction of absentees × Total number

$$= \left(1 - \frac{7}{10}\right) \times 40 = 12 \text{ students.}$$

E-5 A man spent $\frac{2}{7}$ of his savings and still has Rs 1,000 left with him. What were his savings?

S-5 In this type of problem, if balance amount is given, then this amount is to be related to the balance part (fraction). Using relation 2, for savings.

$$\text{Savings} = \frac{\text{balance amount}}{\text{fraction related to balance part}}$$

$$\Rightarrow \quad \text{savings} = \frac{1000}{\left(1 - \frac{2}{7}\right)} = \text{Rs } 1400.$$

E-6 A man reads $\frac{3}{8}$ of a book on a day and $\frac{4}{5}$ of the remainder, on the second day. If the number of pages still unread are 40, how many pages did the book contain?

S-6 It is a *dependent activity*, because on the second day he reads $\frac{4}{5}$ of the remaining pages.

Using the relation 4, the fraction related to balance part = $\left(1-\frac{3}{8}\right)\times\left(1-\frac{4}{5}\right)=\frac{5}{8}\times\frac{1}{5}=\frac{1}{8}$

using relation 2,

$$\text{Total pages} = \frac{\text{Pages unread}}{\text{fraction related to pages unread}}$$

$$= \frac{40}{1/8} = 320$$

$\therefore$ Total pages = 320

E-7 A man spends $\frac{2}{5}$ of his salary on food, $\frac{3}{10}$ of his salary on house rent and $\frac{1}{8}$ of the salary on clothes. He still has Rs 1,400 left with him. Find his salary.

S-7 The expenditure incurred on each item is expressed as part of the total amount (salary), so it consists of independent fractions.

Using the relation 3, we get

Fraction related to balance part = 1 − (sum of independent fractions)

$$= 1-\left(\frac{2}{5}+\frac{3}{10}+\frac{1}{8}\right)=1-\frac{33}{40}=\frac{7}{40}$$

Using relation 2, we get,

$$\text{Total salary} = \frac{\text{Balance amount}}{\text{fraction related to balance part}}$$

$$= \frac{1400}{7/40} = 8000$$

$\therefore$ Total salary = Rs 8000

E-8 A man spends $\frac{1}{3}$ of his income on food, of the rest $\frac{1}{4}$ on house rent and $\frac{1}{5}$ on cloth. He still has Rs 1760 left with him. Find his income.

S-8 Here, of the rest amount (after spending on food), $\frac{1}{4}$ is spent on house rent and $\frac{1}{5}$ is spent on clothes. So, spending on these last two items are independent of each other, but dependent on the expenditure incurred on the first item. It is a problem both on *dependent* and *independent* activities.

Using relation 3 and 4 together, we get,

$$\text{Fraction related to balance part} = \left(1-\frac{1}{3}\right)\times\left[1-\underbrace{\left(\frac{1}{4}+\frac{1}{5}\right)}_{\text{independent}}\right]$$

$$= \frac{2}{3}\times\frac{11}{20}$$

Using relation 2, we find, total income = $\frac{1760}{11/30} = 4800$

$\therefore$ total income = Rs 4800

E-9 $\frac{4}{7}$ of a pole is in the mud. When $\frac{1}{3}$ of it is pulled out, 250 cm. of the pole is still in the mud. Find the full length of the pole.

S-9 Using the relation 2, we get,

total length of pole = $\frac{\text{Length in mud}}{\text{Part in mud}} = \frac{250}{\frac{4}{7} - \frac{1}{3}} = 1050$

$\therefore$ Length of pole = 1,050 cm.

E-10 After covering five-eighth of my journey, I find that I have travelled 60 km. How much journey is left?

S-10 Using the relation 2, we get*

$$\frac{\text{Journey covered}}{\text{Fraction related to journey covered}} = \frac{\text{Journey left}}{\text{its related fraction}}$$

$$\Rightarrow \quad \frac{60}{5/8} = \frac{\text{Journey left}}{1 - \frac{5}{8}}$$

$\therefore$ Journey left = 36 km

**Note:* This relation is equal to total journey.

E-11 How much is to be added with 0.685 of 325 to get 300?

S-11 Let 'x' is to be added, then

$x + (0.685 \times 325) = 300 \Rightarrow x = 300 - 222.625 = 77.375.$

E-12 A man distributes 0.375 of his money to his wife and 0.4 to his son. He has still Rs 3,375 left with him. How much initial money did the man have? How much did his wife get?

S-12 Using the relation 3 and 2, we can write directly

[1 – (0.375 + 0.4)] × total money = balance money

$\Rightarrow$ [1 – 0.775] × Total money = 3375 $\Rightarrow$ Total money = $\frac{3375}{1 - 0.775}$ = Rs 15000

$\therefore$ Wife's share = 0.375 × total money = 0.375 × 15000 = Rs 5625.

E-13 Insert one fraction between

(i) $\frac{1}{3}$ and $\frac{4}{5}$ (ii) 2 and $3\frac{1}{2}$

S-13 (i) $\frac{1}{3}, \frac{4}{5}$

$= \frac{1}{3}, \frac{1+4}{3+5}, \frac{4}{5}$ (using the method 3.4)

$= \frac{1}{3}, \frac{5}{8}, \frac{4}{5}$

Thus, the resulting fraction $\frac{5}{8}$ is more than $\frac{1}{3}$ and less than $\frac{4}{5}$ in magnitude (value).

(ii) $2, 3\frac{1}{2}$

$= \frac{2}{1}, \frac{7}{2}$

$= \frac{2}{1}, \frac{2+7}{1+2}, \frac{7}{2}$ (using the method 3.4)

$= 2, 3, \frac{7}{2}$

Hence, the resulting fraction is 3.

E-14 Insert three fraction between $\frac{1}{3}$ and $\frac{4}{5}$.

S-14 $\frac{1}{3}, \frac{4}{5}$

$= \frac{1}{3}, \frac{1+4}{3+5}, \frac{4}{5} = \frac{1}{3}, \frac{5}{8}, \frac{4}{5}$ (inserting one fraction between $\frac{1}{3}$ and $\frac{4}{5}$)

$= \frac{1}{3}, \frac{1+5}{3+8}, \frac{5}{8}, \frac{5+4}{8+5}, \frac{4}{5}$ (inserting one fraction between $\frac{1}{3}$ and $\frac{5}{8}$, and one fraction between $\frac{5}{8}$ and $\frac{4}{5}$)

$= \frac{1}{3}, \frac{6}{11}, \frac{5}{8}, \frac{9}{13}, \frac{4}{5}$. (three fractions inserted between $\frac{1}{3}$ and $\frac{4}{5}$)

E-15 Which one of the following fractions is less than $\frac{1}{3}$? **(MBA '82)**

(a) $\frac{22}{63}$ (b) $\frac{4}{11}$ (c) $\frac{15}{46}$ (d) $\frac{33}{98}$

S-15 *Step 1* Reverse the test fraction, i.e. $\frac{1}{3}$ becomes $\frac{3}{1} = 3$.

Step 2 Reverse each alternative and find which alternative is greater than $\frac{3}{1}$.

$$\frac{63}{22} < \frac{3}{1}, \frac{11}{4} < \frac{3}{1}, \frac{46}{15} > \frac{3}{1}, \frac{98}{33} < \frac{3}{1}$$

$\therefore$ Required fraction is $\frac{15}{46}$.

Note: Reversing method is used because division process becomes easier when the numerator is greater than the denominator.

REGULAR PROBLEMS

(1) A badminton player, won 6 games and lost 4. The fraction of the games he won is:

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$ (e) $\frac{2}{5}$

(2) What fraction of 2 hours is 12 seconds?

(a) $\frac{1}{600}$ (b) $\frac{1}{12}$ (c) $\frac{1}{60}$ (d) $\frac{1}{5}$ (e) $\frac{3}{50}$

(3) A rope is $25\frac{1}{2}$ m long. How many pieces each of $1\frac{1}{2}$ m long can be cut from it?

(a) 16 (b) 21 (c) 13 (d) 11 (e) 17

(4) A lamp post has half of its length in mud, $\frac{1}{3}$ of its length in water and $3\frac{1}{3}$ m above the water. The total length of the post is:

(a) $4\frac{1}{6}$ m (b) $10\frac{1}{3}$ m (c) $16\frac{2}{3}$ m (d) 4 m (e) 20 m

(5) A man pays off $\frac{2}{5}$ of his debt and still has to pay Rs 240 to pay off the debt completely. The total amount of debt is:

(a) Rs 600 (b) Rs 400 (c) Rs 960 (d) Rs 480 (e) Rs 1200

(6) A drum of water is $\frac{3}{5}$ full. When 38 litres are drawn from it, it is just $\frac{1}{8}$ full. The half capacity of drum in litres is:

(a) 40 (b) 80 (c) 152 (d) 21.7 (e) 76

(7) The monthly salary of a man is Rs 480 and he spends $\frac{7}{8}$ of it. His income increases by $\frac{1}{6}$ of the present salary and his spending also increases by $\frac{2}{7}$ of the present expenditure. His savings will now

(a) increase by Rs 45 (b) decrease by Rs 40 (c) increase by Rs 40
(d) decrease by Rs 80 (e) decrease by Rs 60

(8) Which of the following fraction is the smallest?

(a) $\frac{7}{13}$ (b) $\frac{14}{33}$ (c) $\frac{11}{25}$ (d) $\frac{8}{15}$ (e) $\frac{9}{11}$

Hint: See 3.1.1

(9) Which of the following fraction is the greatest?

(a) $\frac{16}{21}$ (b) $\frac{11}{14}$ (c) $\frac{16}{19}$ (d) $\frac{16}{23}$ (e) $\frac{11}{17}$

(10) A man pays off $\frac{3}{20}$ of his debt every month. At the end of 6 months, his remaining debt is Rs A. How much amount has he cleared off in every month (in Rs)?

(a) $\frac{3A}{20}$ (b) $\frac{9A}{10}$ (c) $\frac{A}{10}$ (d) $\frac{3A}{10}$ (e) $\frac{3A}{2}$

(11) $\frac{3}{5}$ part of a kerosene tin is filled. If 6 bottles are taken out of it and 3 bottles are filled again, then half the tin is full. What is the capacity of the tin? (in bottles) **(RRB Guwahati, '97)**

(a) 20 (b) 30 (c) 45 (d) 50 (e) 40

(12) Reciprocal of sum of the reciprocals of $\frac{3}{5}$ and $\frac{7}{3}$ is:

(a) $\frac{1}{4}$ (b) $\frac{21}{44}$ (c) $\frac{4}{5}$ (d) $\frac{44}{21}$ (e) $\frac{15}{44}$

Hint: Start solving from backwards. First make reciprocals of $\frac{3}{5}$ and $\frac{7}{3}$ i.e. $\frac{5}{3}$ and $\frac{3}{7}$

Then sum it, as $\frac{5}{3}+\frac{3}{7}$, 8 then find reciprocal of the sum

(13) If the product of two numbers is 5 and one of the number is $\frac{3}{2}$, then what will be the sum of the numbers? **(RRB Trivendrum (Tech), '97)**

(a) $4\frac{1}{2}$ (b) $6\frac{1}{2}$ (c) $4\frac{5}{6}$ (d) 9 (e) $4\frac{2}{3}$

(14) In an examination, a student was asked to find $\frac{3}{14}$ of a certain number. By misake, he found $\frac{3}{4}$ of it. His answer was 150 more than the correct answer. The given number is:

(a) 450 (b) 300 (c) 270 (d) 180 (e) 280

(15) One of the rational numbers between $\frac{2}{7}$ and $\frac{3}{14}$ is:

(a) $\frac{5}{14}$ (b) $\frac{3}{49}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ (e) None

Hint: Refer

(16) Which of the following fractions is the greatest? **(RBI, '98)**

(a) $\frac{219}{337}$ (b) $\frac{221}{335}$ (c) $\frac{217}{339}$ (d) $\frac{215}{341}$ (e) $\frac{222}{339}$

Hint: Do not try to calculate. The greatest fraction can be found out by eliminating first the fractions with lower numerator and greater denominator, i.e, $\frac{215}{341}, \frac{217}{339}$, and $\frac{219}{337}$. Then compare $\frac{221}{335}$ and $\frac{222}{339}$

Answers

1. (c)	**2.** (a)	**3.** (e)	**4.** (e)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (c)
10. (e)	**11.** (b)	**12.** (b)	**13.** (c)	**14.** (e)	**15.** (c)	**16.** (b)		

REAL PROBLEMS

(1) One quarter of one-seventh of a land is sold for Rs 30000. What is the value of eight-thirty fifth of the land?

(a) Rs 192000 (b) Rs 212000 (c) Rs 27428 (d) Rs 36540 (e) Rs 150000

(2) It takes 40 days for a pond to get filled with rain water. If the level of water doubles each day, then how long would it take to fill $\frac{1}{4}$ of the pond?

(a) 10 days (b) 20 days (c) 30 days (d) 35 days (e) 38 days

(3) A post is divided into three parts, the first part is $\frac{1}{3}$ of the whole length, second $\frac{3}{8}$ of the first, and the third is 6 m 50 cm. The length of the post is:

(a) 15 m (b) 10 m (c) 12 m (d) $13\frac{1}{2}$ m (e) 18 m

(4) In a village, $\frac{5}{8}$ of the population are adults, $\frac{1}{2}$ of the adults are male, $\frac{4}{5}$ of adult females are illiterate. If 400 females are illiterate, then the population of the village is:

(a) 2000 (b) 1500 (c) 1800 (d) 1600 (e) 1200

Hint: Assume, population of village as x

(5) In a polling booth, total number of voters is 1575, of which 0.4 part are male voters. If a candidate gets 0.6 part of male voters and 0.4 part of female voters, then find how many votes did the candidate get?

(a) 189 (b) 756 (c) 378 (d) 630 (e) 945

(6) A man spends $\frac{1}{7}$ of his salary on food and $\frac{1}{2}$ of the remaining on clothing and $\frac{1}{3}$ of the remaining on entertainment. He is still left with Rs 600. How much does he spend on entertainment? **(BSRB, '95)**

(a) Rs 600 (b) Rs 450 (c) Rs 300 (d) Rs 700 (e) Rs 500

Hint: Refer E-6

(7) From a rope 30 metres long, a person cuts off as many pieces as possible, each $3\frac{1}{4}$ metres long. What fraction of the whole will be left?

(a) $\frac{1}{40}$ (b) $\frac{3}{4}$ (c) $\frac{8}{13}$ (d) $\frac{7}{13}$ (e) $\frac{13}{30}$

(8) A man left $\frac{1}{7}$ of his property to his daughter and the remaining to his sons to be equally divided among them. If the share of the each son be double of that of the daughter, find the number of sons. **(NABARD, '97)**

(a) 2 (b) 3 (c) 6 (d) 4 (e) 7

(9) A vessel, full of water, weighs 16.5 kg. When the vessel is $\frac{1}{4}$ full, it weighs 5.25 kg. The weight of the empty vessel (in kg) is:

(a) 1.125 (b) 4.5 (c) 1.5 (d) 3 (e) 2.5

(10) A scooter before overhauling requires $\frac{2}{3}$ hour service time every 45 days, while after overhauling it requires $\frac{2}{3}$ hour service time every 60 days. What fraction of pre-overhauling service time is saved in the latter case? **(MBA, '81)**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$ (e) $\frac{4}{9}$

(11) Sundari, Kusu and Jyoti took two tests each. Sundari secured $\frac{24}{60}$ marks in the first test and $\frac{32}{40}$ marks in the second test. Kusu secured $\frac{35}{70}$ marks in the first test and $\frac{54}{60}$ marks in the second test. Jyoti secured $\frac{27}{90}$ marks in the first test and $\frac{45}{50}$ marks in the second test. Who among them did register maximum progress? **(BSRB Bangalore, 2000)**

(a) Only Sundari (b) Only Kusu (c) Only Jyoti
(d) Both Sundari and Kusu (e) Both Kusu and Jyoti

Hint: Tabulate the score in each test with **common denominator** so that the progress for each person in second test over the first test can be found & compared.

		I		II			
Sundari	→	$\frac{4}{10}$	→	$\frac{8}{10}$	→	2 times	$\because \frac{8}{4} = 2$
Kusu	→	$\frac{5}{10}$	→	$\frac{9}{10}$	→	Less than 2 times	$\because \frac{9}{5} < 2$
Jyoti	→	$\frac{3}{10}$	→	$\frac{9}{10}$	→	Maximum progress, as in II test, score is 3 times the I test in terms of fraction	

(12) A boy on being asked $\frac{13}{14}$ of a certain fraction had made the mistake of dividing the fraction by $\frac{13}{14}$ and so got an answer that exceeded the correct answer by $\frac{3}{65}$. The correct is:

(a) $\frac{14}{45}$ (b) $\frac{12}{65}$ (c) $\frac{13}{45}$ (d) $\frac{2}{7}$ (e) $\frac{196}{585}$

(13) *A* has twice as much money as *B*. They play together, and at the end of the first game, *B* wins one third of *A*'s money from *A*; what fraction of the sum that *B* now has, must *A* win back in the second game so that they may have exactly equal money?

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{10}$ (e) $\frac{5}{18}$

Hint: Assume, before the start of first game, *B* has Rs 1 and *A* has Rs 2

(14) I bought a number of mangoes at 35 for 2. I divided the whole into two equal parts, one part of which I sold at 17, and the other at 18 mangoes per Rs 1. I spent and received an integral number of rupees, but bought the least possible number of mangoes. How many did I buy?

(a) 21420 (b) 24120 (c) 22014 (d) 1225 (e) 612

Hint: Assume that I buy 35 mangoes. Then on selling, I get $\left(\underbrace{\frac{35}{2}}_{\text{No. of mangoes}}\times\underbrace{\frac{1}{17}}_{\text{Unit price}}+\frac{35}{2}\times\underbrace{\frac{1}{18}}_{\text{Unit price}}\right)$ = Rs $\frac{1225}{612}$

But the number of rupees is an integer.

so, I must receive 612 times $\frac{1225}{612}$

Hence no. of mangoes I buy is 612 times 35

(15) Find out that minimum fraction which when added to $\frac{29}{12}+\frac{15}{16}$ will give a complete number.

(a) $\frac{21}{38}$ (b) $\frac{31}{38}$ (c) $\frac{31}{48}$ (d) $\frac{17}{48}$ (e) $\frac{23}{38}$

(16) In a class, 18 boys are there whose height is more than 160 cm. If they are three-fourth of the total number of boys and the total number of boys is two-third of the total number of students, then how many girls are there in the class?

(a) 18 (b) 6 (c) 12 (d) 24 (e) 8

(17) The fuel indicator in a car shows $\frac{1}{5}$th of the fuel tank as full. When 22 more litres of fuel are poured into the tank, the indicator rests at the three-fourth of the full mark. The capacity of the fuel tank (in litres) is:

(a) 30 (b) 40 (c) 36 (d) 28 (e) 45

Answers

1. (a) **2.** (e) **3.** (c) **4.** (d) **5.** (b) **6.** (c) **7.** (a) **8.** (b) **9.** (c)
10. (d) **11.** (c) **12.** (c) **13.** (d) **14.** (a) **15.** (c) **16.** (c) **17.** (b)

4

SIMPLIFICATION AND APPROXIMATION

4.1 OPERATION ORDER SEQUENCE

For simplifying an expression containing various types of fractions, the order of various operations involved should be strictly maintained. A simple technique for arranging the expression in the proper sequence, is by placing them in the order of the first letter appearing in **VBODMAS** where.

1. **V** Stands for vinculum or bar as ($\overline{}$)
2. **B** stands for bracket and operation of brackets in the order (), {} and then []
3. **O** stands for 'of'
4. **D** stands for division (÷)
5. **M** stands for multiplication (×)
6. **A** stands for addition (+)
7. **S** stands for subtraction (–)

4.2 APPLICATION FOR ALGEBRAIC FORMULA

Some algebraic formulae are used in solving the problems on simplification. Following important formula are to be memorised:

1. $(a + b)^2 = a^2 + b^2 + 2ab.$
2. $(a - b)^2 = a^2 + b^2 - 2ab.$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2).$
4. $(a + b)^2 - (a - b)^2 = 4ab.$
5. $(a + b) \times (a - b) = a^2 - b^2.$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   $= a^3 + b^3 + 3ab\,(a + b).$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   $= a^3 - b^3 - 3ab\,(a - b).$
8. $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$
10. $a^m \times a^n = a^{m+n}$
11. $a^m \div a^n = a^{m-n}$
12. $a^3 + b^3 + c^3 - 3abc = (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a + b + c = 0$, then the above identity reduces to $a^3 + b^3 + c^3 = 3abc$

4.2.1 Square Root and Square

When a number is multiplied by itself, the product obtained is called the square of the number since $6 \times 6 = 36$.

$\therefore$ 36 is the square of 6 or $6^2 = 36$

Also, $-3 \times -3 = 9 \Rightarrow (-3)^2 = 9$

$\therefore$ 9 is the square of -3

The square root of a given number is equal to the value whose square is the given number and the sign for square root is '$\sqrt{\ }$'

Since $6^2 = 36$ then

$$\sqrt{36} = 6, \text{ i.e. square root of 36 is 6}$$

$$\sqrt{25} = 5, \text{ i.e. square root of 25 is 5}$$

$$\sqrt{-25} = \text{an imaginary quantity}$$

$\therefore$ Square root of a negative number is an imaginary quantity

1. $\sqrt{?} = y$
   Then the required number $= y^2$
2. $\sqrt{a^2 \times b^2} = ab$
3. $\sqrt{a^3 \times b^3} = ab\sqrt{ab}$
4. $\sqrt{a^4 \times b^4 \times c^4} = a^2b^2c^2$
5. $\sqrt{a^n \times b^m} = a^{n/2} \times b^{m/2}$

4.2.2 Division Method for Finding the Square Root

$$\sqrt{64009} = ?$$

Step 1 Pairing the digits from right to left, we get

$$6\ \overline{40}\ \overline{09}$$

Step 2 Then take the first pair, here it is only '6' and find the largest whole number whose square is equal to 6 or less than 6. Such a whole number is 2.

Step 3 Hence, write 2 in the quotient and also in the divisor. (see next page)

Step 4 Subtract $2 \times 2 = 4$ from 6. The remainder is then 2.

Step 5 Bring down the second pair of digits (i.e. 40) double the quotient (i.e. $2 \times 2 = 4$) and write the result on the left of 240 and then repeat Step-2 till the remainder is zero. The whole process can be enumerated step-by-step as shown in the following table.

2	6 $\overline{40}$ $\overline{09}$	253 = Quatient
+ 2	4	
45	2 $\overline{40}$	
+ 5	2 25	(since 45 × 5 = 225)
503	15 $\overline{09}$	(since 503 × 3 = 1509)
	15 09	
	0	

$\therefore \quad \sqrt{64009} = 253$ (i.e. Quotient)

4.2.3 Properties of a Perfect Square Number

A number whose exact square root (which must be a whole number can be obtained, is called a *perfect square*:

(a) A number ending with 2, 3, 7 or 8 cannot be a perfect square.
(b) The last digit of a perfect square must be 0, 1, 4, 5, 6 or 9.
(c) A number ending with odd number of zeroes cannot be a perfect square, e.g. 9000, 25000, 16000, etc. are not perfect squares.
(d) A perfect square number is either exactly divisible by 3 or leaves a remainder of 1, when divided by 3.
e.g. 64 if divided by 3, will leave a remainder of 1
36 is exactly divisible by 3.
(e) A perfect square number is either exactly divisible by 4 or leaves a remainder of 1, when divided by 4
e.g. 81 if divided by 4, will leave a remainder of 1.
100 is exactly divisible by 4.

Note: Above properties are very useful to check if a given number is a perfect square or not.

4.2.4 Square Root of Vulgar Fraction

$$\sqrt{\frac{3}{7}} = ?$$

Step 1 Multiply the numerator and the denominator by the donominator.

Step 2 Find the square root of the new numerator and divide it by the new denominator.

$$\therefore \quad \sqrt{\frac{3}{7}} = \sqrt{\frac{3}{7} \times \frac{7}{7}} = \frac{\sqrt{21}}{7} = \frac{4.582}{7} = 0.654$$

4.2.5 Square Root of a Rational Decimal Fraction

$\sqrt{387.09126} = ?$

Step 1 Pair the integer part first

$$\overline{3}\,\overline{87}$$

Step 2 Check the number of decimal places.
If it is odd, then affix a zero on the extreme right of decimal part to make the even number of decimal places.
Here, no. of decimal places = 5, so, after placing a zero, it becomes .091260

Step 3 Pair the decimal part accordingly

$$.\overline{09}\,\overline{12}\,\overline{60}$$

Step 4 Start finding the square root by the division method as explained in 4.2.2 and put the decimal point in the square root as soon as the integer part is exhausted.

1	$3\,\overline{87}\cdot\overline{09}\,\overline{12}\,\overline{60}$	19.674
+ 1	1	
29	$2\overline{87}$	
+ 9	261 (integer part is over)	since 29 × 9 = 261
386	$26\overline{09}$	
6	2316	since 386 × 6 = 2316
3927	$293\overline{12}$	
7	27489	since 3927 × 7 = 27489
39344	$1823\overline{60}$	
	157376	since 39344 × 4 = 157376

The square root of 387. 09126 = $\sqrt{387.09126}$
= 19.674

4.3 SIMPLIFICATION OF DECIMAL FRACTION

The number of digits which are present on the RIGHT OF A DECIMAL POINT is called the number of decimal places.

That is, 32.0075 has four digits on the right of the decimal point. Therefore, the number is expressed to four decimal places.

A WHOLE NUMBER can also be written as a decimal fraction by putting a decimal after its LAST DIGIT and adding as many zeros as are required.
e.g. 12 = 12.0 = 12.000 and so on.

4.3.1 Addition

For addition of a decimal number with another decimal number or with another whole number write the given number in such a way that the number of decimal places are equal for all the numbers.

e.g. 1 + 0.59 + 0.008

Here maximum number of decimal places = 3 (three) in 0.008.
Convert all the number so that they have 3 decimal places.
∴ 1 + 0.59 + 0.008 = 1.000 + 0.590 + 0.008 = 1.598

4.3.2 Subtraction

In subtraction also, the given numbers are to be wirtten in such a way that the number of decimal places become equal for all the numbers (empty places are filled up with zeroes).

e.g. 2 – 0.283
In 0.283, number of decimal places = 3
In 2, number of decimal places = 0
So, make 2 as having 3 decimal places, i.e. 2.000
$\therefore$ 2 – 0.283 = 2.000 – 0.283 = 1.717.

4.3.3 Multiplication

0.005 × 0.08 × 0.4= ?

Step 1 Multiply the number only, i.e. 5 × 8 × 4 = 160

Step 2 Add the total number of decimal places in the given number, i.e. 3 + 2 + 1 = 6

Step 3 Write the result of Step 1 and convert it to a number with decimal places as obtained in Step 2 by shifting the decimal point to the left.

i.e. by six decimal places, we then get

0.005 × 0.08 × 0.4 = 0.000160 = 0.00016

Similarly $\underbrace{0.03 \times 0.7 \times 2}_{\text{Total of 3 decimal places}} = 0.042$

4.3.4 Division of Decimals

(a) When the Divisor (or Denominator) is a Whole Number

e.g. $\frac{3.0056}{7}$

Step 1 Simply divide the number without considering the decimal points given i.e. 7) 30056 (4293.7

Step 2 Count the no. of decimal places in the given number. Here it has 4 decimal places in 3.0056.

Step 3 Shift the decimal point in the quotient obtined to the same no. of decimal places as in Step-2

Hence the result becomes. $\overset{\text{Shift}}{42937}$ = 0.42937

(b) When the Divisor (Denominator) is also a Decimal Number

e.g. $\frac{12.598}{27.08 \times 1.417}$

Step 1 Shift the decimal point to the right of the numerator and of the denominator such that

— total decimal point shift in numerator = total decimal point shift in denominator.

— there is no decimal place left after the shift.

Here, no. of decimal place in numerator (in 12.598) = 3

no. of decimal place in denominator (in 27.08 and 1.417) = 2 + 3 = 5

since 5 > 3, so, total shift in decimal point to be made (in numerator and denominator) = 5

Now, 5 decimal points shifts are made,

$$\frac{12\overbrace{59800}^{5\text{ shift}}}{27\underbrace{08}_{2\text{shift}} \times 1\underbrace{417}_{3\text{shift}}} = \frac{1259800}{2708 \times 1417}$$

Step 2 Division process is continued with the resulting fraction obtained in step 1.

4.4 SIMPLIFICATION OF A MIXED FRACTION

A mixed fraction consists of two parts, the integer part and the fractional part.

e.g. $2\frac{7}{18}$ has 2 as an integer and $\frac{7}{18}$ as a fraction.

In fact $2\frac{7}{18} = 2 + \frac{7}{18}$

4.4.1 Addition

$$\boldsymbol{12\frac{5}{8} + 13\frac{7}{11} = ?}$$

Step 1 Add the integer part only i.e. 12 + 13 = 25

Step 2 Add the fractional part only i.e. $\frac{5}{8} + \frac{7}{11} = 1\frac{23}{88}$

Step 3 Add the results obtained in Step 1 and Step 2

$$\therefore \quad 12\frac{5}{8} + 13\frac{7}{11} = 25 + 1\frac{23}{88} = 26\frac{23}{88}$$

4.4.2 Subtration

$$\boldsymbol{10\frac{3}{7} - 18\frac{1}{6} = ?}$$

Step 1 Subtract the integer part only, i.e. 10 – 18 = – 8

Step 2 Subtract the fraction part only, i.e. $\frac{3}{7} - \frac{1}{6} = \frac{11}{42}$

Step 3 Add the result obtained in Step 1 and Step 2

Hence, $$10\frac{3}{7} - 18\frac{1}{6} = -8 + \frac{11}{42} = -8 + 1 - 1 + \frac{11}{42} = -(8-1) - \left(\frac{42-11}{42}\right) = -7 - \frac{31}{42}$$

$$= -\left(7 + \frac{31}{42}\right)$$ *[Please refer to Section 4.4.3]*

$$= -7\frac{31}{42}$$

Similarly $$12\frac{9}{11} - 15\frac{5}{8} = (12 - 15) + \left(\frac{9}{11} - \frac{5}{8}\right)$$

$$= -3 + \frac{17}{88} = -(3-1)\ \frac{88-17^{*}}{88} = -2\frac{71}{88}$$

*Explanation of in between steps of adding 1 & subtracting 1 have been explained in the previous problem, however, this step can be directly obtained after little practice.

(iii) $12\frac{5}{7} - 10\frac{2}{3} = (12 - 10) + \left(\frac{5}{7} - \frac{2}{3}\right) = 2 + \frac{1}{21} = 2\frac{1}{21}$

(iv) $5\frac{2}{3} - 2\frac{1}{7} + 6\frac{3}{8} = (5 - 2 + 6) + \left(\frac{2}{3} - \frac{1}{7} + \frac{3}{8}\right)$

$$= 9 + \left(\frac{112 - 24 + 63}{168}\right) = 9\frac{151}{168}$$

4.4.3 Subtraction of a Whole Number and Fraction

$5 - \frac{31}{48} = ?$

Step 1 Subtract 1 from the whold number i.e. + (5 – 1) = (4)

Step 2 Subtract the numerator from denominator and write in the numerator i.e. $\frac{31}{48} = \frac{48 - 31}{48} = \frac{17}{48}$

Step 3 Add the results obtained in Step 1 and Step 2

i.e. $5 - \frac{31}{48} = 4\frac{17}{48}$

$\therefore$ $12 - \frac{11}{52} = (12 - 1)\ \frac{52 - 11}{52} = 11\frac{41}{52}$

and $-6 + \frac{23}{36} = -(6 - 1)\ \frac{36 - 23}{36} = -5\frac{13}{36}$

4.4.4 Easy Method For Simplification

5542 + ? + 1369 = 4200

Step 1 First always put 'x' for (?)

Step 2 Proceed and follow the rules to find the value for 'x' (or finding the value of?)

4.4.5 Multiplication of a Whole Number and a Fraction

$4 \times 16\frac{2}{3} = ?$

Step 1 Multiply the integer part by the whole number

i.e. $4 \times 16 = 64$

Step 2 Multiply the fraction part by the whole number i.e. $4 \times \frac{2}{3} = 2\frac{2}{3}$

Step 3 Add the results obtained in Step 1 and Step 2

i.e. $64 + 2\frac{2}{3} = 66\frac{2}{3}$

$\therefore$ $4 \times 16\frac{2}{3} = 66\frac{2}{3}$

4.4.6 Division of Mixed Fraction by a Whole Number

$16\frac{2}{3} \div 4 = ?$

Step 1 Divide the integer part by the whole number,

i.e. $\frac{16}{4} = 4$

Step 2 Divide the fractional part by the whole number,

i.e. $\frac{2}{3} \div 4 = \frac{1}{6}$

Step 3 Add the results obtained Step 1 and Step 2,

i.e. $4 + \frac{1}{6} = 4\frac{1}{6}$

$\therefore \quad 16\frac{2}{3} \div 4 = 4\frac{1}{6}$

4.5 CONTINUED FRACTIONS AND ITS SIMPLIFICATION

Fractions of the form

(a) $7 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{3}}}$ or (b) $\cfrac{1}{2 - \cfrac{3}{8 + \cfrac{1}{4 - \cfrac{1}{5}}}}$

are called continued fractions

Simplification Rule

To simplify a continued fraction begin at the bottom and work upwards.

Example: Simplify $\cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{6}}}}$

$$= \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{\cfrac{7}{6}}}} \qquad = \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{6}{7}}}$$

$$= \cfrac{1}{3 + \cfrac{1}{\cfrac{41}{7}}} \qquad = \cfrac{1}{3 + \cfrac{7}{41}}$$

$$= \frac{1}{\frac{130}{41}}$$

$$= \frac{41}{131}$$

4.6 RECURRING DECIMALS

A decimal fraction in which a digit or set of digits is repeated continually is called a Recurring or Periodic decimal.

e.g. $\frac{1}{3} = 0.333...$

Here, on performing the division, it is found that the remainder is always 1 and in the quotient, the digit 3 is continually repeated. Hence it is written as $0.\dot{3}$, where the *dot* over 3 indicates that the 3 has to be continually repeated.

Similarly, $\frac{1}{7}$ is

7) 1.000000 (0.142857 142857...

So, if we continue the division, we shall get the same set of figures 142857 again and again and in the same order.

Therefore $\frac{1}{7} = 0.\dot{1}4285\dot{7}$ or $0.\overline{142857}$

The repeated digits or repeated set of digits is called the period of the recurring decimal.
There are two types of Recurring Decimals,

(a) *Pure recurring decimal:* Such a decimal in which all the decimal digits recur, e.g. $0.\dot{1}4285\dot{7}$

(b) *Mixed recurring decimal:* Such a decimal in which all the decimal digits do not recur, e.g. $0.71\dot{6}\dot{7}$

4.6.1 Conversion of a Pure Recurring Decimal to the form $\frac{p}{q}$

Steps (a) Write the decimal part without the decimal point as the numerator.
(b) Write as many 9s as there are different repeating digits for the denominator.

e.g. $0.\overline{587} = \frac{587}{999}$, $\underset{\downarrow}{3}.\overline{17} = 3 + \frac{17}{99}$

(remains unchanged as it is integer part)

4.6.2 Conversion of a Mixed Recurring Decimal to the form $\frac{p}{q}$

Step (1) First, write the decimal part without the decimal point and subtract the non-repeating part from it and write the result in the numerator

Step (2) Write a number in the denominator with as many 9s as there are repeating digits in the decimal part and followed by as many zeroes as there are non-repeating digits in the decimal part.

Example: Express $7.00\dot{8}\dot{1}$ as vulgar fraction $\left(\frac{p}{q}\text{ form}\right)$

Solution $7.04\dot{8}\dot{1}$

$$= 7\frac{481-04}{9900}$$

$$= 7\frac{477}{9900}$$

4.7 IMPORTANT DERIVATIONS

(i) $\left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{+m/n}$

(ii) $\left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{+m/n}$

(iii) If $a \div b$, then $a = c \times b$ and $a \div c = b$

(iv) $N = \left[\frac{N+1}{2}\right]^2 - \left[\frac{N-1}{2}\right]^2$

4.8 APPROXIMATE VALUE

In this type of questions, candidates do not have to find out the exact value, but all they have to do is

Step 1 To round off the numbers given in the question

Step 2 To simplify

Step 3 To round off the result obtained in Step-2

Very Important: In some of the questions, the choices given are very close to each other. In such case, Step-1 is to be avoided, and we should go directly to Step-2.

4.8.1 Rounding Off Numbers

On some occasions for ease in simplification, we require only a rough estimation and not the exact value. In such cases we round off the values to the nearest tens, or hundreds or thousands.

Rounding off a number to the nearest ten, hundred or thousand means finding the multiple of 10, 100 or 1000 which is closest to (or approximate) the original number. It can be done by the following procedure.

(a) rounding off to the nearest 10: Replace the digit at unit's place by 0. If the replaced digit is 5 or more, then add 1 to the digit at tens place, otherwise digit at tens place remains unchanged.

e.g. $47 \xrightarrow[\text{off}]{\text{rounded}} 50$

$\downarrow$

>5, so 1 is added to digit at tens place i.e. 1 + 4 = 5 (digit at tens place after rounding off)

But, $92 \longrightarrow 90$

$\downarrow$

less than 5, so, 9 (digit at tens place) remains unchanged.

$75 \longrightarrow 80$

$\downarrow$

equal to 5, so 1 is added to 7 i.e. 1 + 7 = 8 becomes the ten's place digit

$295 \longrightarrow 300$

(b) rounding off to the nearest 100: Replace the digit at unit and tens places by 00. If the replaced digit at **tens place** is 5 or more, then add 1 to the digit at hundreds place, otherwise the digit at hundreds place remains unchanged.

e.g. $264 \xrightarrow{+1} 300$

$\downarrow$

>5, so 1 is added to digit at hundreds place i.e. 1 + 2 = 3

$5660 \longrightarrow 5700$, because 660 is rounded off as 700.

$841 \longrightarrow 800$

(c) rounding off to the nearest 1000: Replace the ones, tens and hundreds digits by 000. If the replaced digit at hundreds place is 5 or more, then add 1 to the digit at thousands place, otherwise not.

$1973 \longrightarrow 2000.$

NB: Whether a given number is to be rounded off to the nearest 10, or 100 or 1000, it depends on the other numbers involved in the simplification. It will be explained in the examples provided subsequently.

4.8.2 Rounding off a Number to a Decimal Place

To round off a number to the rth decimal place; following steps are to be checked.

Step 1 Check the digit immediately, next right to the rth place.

Step 2 If the next right digit is 5 or more, then add 1 to the digit in the rth place, otherwise the digit remains unchanged.

Step 3 Delete all the digits in places to the right of the rth place.

e.g. $5.792 \xrightarrow[\text{to 2nd place}]{\text{rounded off}} 5.79$

$5.795 \longrightarrow 5.8$

Example: What approximate value should come in place of (?) in the following equation

$9876 \div 24.96 + 215.005 - ? = 309.85$

Solution: $9876 \div 24.96 + 215.005 - ? = 309.85$

Put x in place of ?, then approximating the terms to the nearest values,

$9900 \div 25 + 215 - x = 310$

$$\Rightarrow \quad x = \frac{9900}{25} + 215 - 310$$

$\Rightarrow \quad x = 396 + 215 - 310$
$= 301 \approx 300$

Therefore the approximate value in place of (?) is 300.

Example: What approximate value should come in place of (?) in the following equation.

96895 + 589 + 22497 = ?

(a) 120000 (b) 125000 (c) 122000 (d) 99000 (e) 130000.

Solution: Here, out of the 5 choices, the values in three choices are very close to each other. Now, put (x) in place of (?) and after approximating, we get

$$x = 96895 + 589 + 22497$$
$$\approx 96900 + 600 + 22500$$
$$= 120000$$

Hence answer is (a)

Solved Examples

E-1 Simplify

$$10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - \overline{6-4})\}\right]$$

S-1
$$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - 2)\}\right] \qquad \overline{6-4} = 2\ (V)$$
$$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - 5\}\right] \qquad (7 - 2) = 5\ (B)$$
$$= 10\frac{1}{2} - \left[8\frac{1}{2} + 1\right] \qquad \{6 - 5\} = 1\ (B)$$
$$= 10\frac{1}{2} - 9\frac{1}{2} = \mathbf{1}. \qquad \left[8\frac{1}{2} + 1\right] = 9\frac{1}{2}\ (B)$$

E-2 $0.75 \times 0.75 + 0.25 \times 0.75 \times 2 + 0.25 \times 0.25$

S-2 Let $0.75 = a$ and $0.25 = b$

By 4.2 (1), we have $a^2 + 2ab + b^2 = (a + b)^2$
$= (0.75 + 0.25)^2 = \mathbf{1}.$

E-3 Simplify $\dfrac{(0.87)^3 + (0.13)^3}{(0.87)^2 + (0.13)^2 - (0.87 \times 0.13)}$

S-3 Let $0.87 = a$ and $0.13 = b$

$$\therefore \quad \frac{a^3 + b^3}{a^2 + b^2 - ab} = \frac{(a+b)(a^2 - ab + b^2)}{(a^2 + b^2 - ab)} = a + b \qquad \text{[Refer 4.2]}$$
$$= 0.87 + 0.13 = \mathbf{1}.$$

E-4 Find the missing number.

$$\frac{9840}{\sqrt{?}} = 410$$

S-4 $\therefore$ Required Number = $\left(\frac{9840}{410}\right)^2$ *[Refer 4.2.1]*

$= (24)^2 =$ **576.**

E-5 Simplify $(40^2 - 30^2) = 10 \times ?$

S-5 Let $40 = a$, $30 = b$, required number = x

$a^2 - b^2 = (a + b)(a - b)$

$\therefore$ Required Number $(x) = \frac{40^2 - 30^2}{10} = \frac{(40+30)(40-30)}{10} =$ **70.**

E-6 $3^4 \times 3^6 \div 3^9 = ?$

S-6 here $a = 3$ and the base is the same. Now, on multiplication, the exponents are added, and on division the difference of the exponents are taken. Now applying 4.2 (10) and (11), we get the required number $(x) = 3^{(4+6-9)} = 3$

E-7 $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2} = ?$ **(AGE '93)**

S-7 $2 + \sqrt{2} + \left[\frac{\sqrt{2}-2+2+\sqrt{2}}{(2+\sqrt{2})(\sqrt{2}-2)}\right]$ [Since $a = \sqrt{2}$ $b = 2$

$= 2 + \sqrt{2} + \frac{2\sqrt{2}}{2-4} \Rightarrow 2 + \sqrt{2} + \frac{2\sqrt{2}}{-2} =$ **2.** $\therefore (a + b)(a - b) = a^2 - b^2$]

E-8 If $x * y = (x + 2)^2 (y - 2)$ then $7 * 5 = ?$

S-8 Substituting $x = 7$ and $y = 2$, we get,

$7 * 5 = (7 + 2)^2 (5 - 2)$

$= (9)^2 \times 3 =$ **243.**

E-9 If m and n are whole numbers such that $m^n = 121$, then $(m - 1)^{n+1} = ?$

S-9 Given that $m^n = 121 \Rightarrow m^n = (11)^2$

Hence $m = 11$ and $n = 2$, and substituting these values,

$(m - 1)^{n+1} = (11 - 1)^{2+1} = 10^3 =$ **1000.**

E-10 If $\frac{x}{y} = \frac{3}{4}$, then $\frac{6}{7} + \frac{(y-x)}{(y+x)} = ?$

S-10 Substruting $x = 3k$

$y = 4k$, we get,

$\frac{6}{7} + \frac{(y-x)}{(y+x)} = \frac{6}{7} + \frac{4k-3k}{4k+3k}$

$= \frac{6}{7} + \frac{1}{7} =$ **1.**

E-11 If $\sqrt{x} - \sqrt{y} = 1$ and $\sqrt{x} + \sqrt{y} = 17$, then $\sqrt{xy} = ?$

S-11 $\sqrt{x} + \sqrt{y} = 17$ **(i)**

and, $\sqrt{x} - \sqrt{y} = 1$ **(ii)**

Adding equations (i) and (ii) $\sqrt{x} = 9$

Subtracting equation (ii) from (i), $\sqrt{y} = 8$

Substituting these values,

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y} = 9 \times 8 = \mathbf{72}.$$

E-12 $\dfrac{? \div 12}{0.2 \times 3.6} = 2$

S-12 Putting x in place of ?, we get

$$\frac{x \div 12}{0.2 \times 3.6} = 2$$

$$\Rightarrow \quad x \div 12 = 2 \times 0.2 \times 3.6 \Rightarrow \frac{x}{12} = 2 \times 0.2 \times 3.6$$

$$\Rightarrow \quad x = 12 \times 2 \times 0.2 \times 3.6 = \mathbf{17.28}.$$

E-13 $\sqrt{? \times 7} \times 18 = 84$

S-13 Subsituting x for ?, we get,

$$\sqrt{x \times 7} \times 18 = 84$$

$$\Rightarrow \quad \sqrt{x \times 7} = \frac{84}{18}$$

$$\Rightarrow \quad \left(\sqrt{x \times 7}\right)^2 = \left(\frac{84}{18}\right)^2 \qquad \text{(squaring both sides)}$$

$$\Rightarrow \quad x \times 7 = \left(\frac{84}{18}\right)^2 \quad \Rightarrow \quad x = \frac{84 \times 84}{18 \times 18 \times 7} = \mathbf{3.11}.$$

E-14 $\left(2\dfrac{3}{x}\right) \times \left(y\dfrac{1}{2}\right) = 7\dfrac{3}{4}$ **(MBA, '82)**

S-14 Taking the quotients 2, y and 7, we get

$2y = 7$, which gives the quotient as 3 $\left(\text{Since} \quad y = \dfrac{7}{2} = 3\dfrac{1}{2}\right)$

$\therefore$ $y = 3$. Substituting the value of y, we get,

$$2\frac{3}{x} \times 3\frac{1}{2} = 7\frac{3}{4}$$

Now, $\dfrac{7\frac{3}{4}}{3\frac{1}{2}} = 2\dfrac{3}{x} \Rightarrow 2\dfrac{3}{14} = 2\dfrac{3}{x}$

$\therefore$ $x =$ **14** and $y =$ **3**

E-15 $2^{2^?} = 256$ **(IA, '87)**

S-15 Putting x for ?, we get

$$2^{2^x} = 256 = 2^8$$

$\Rightarrow$ $2^{2^x} = 2^{2^3}$

$\therefore$ $x =$ **3**.

E-16 $2^{x+13} = 4^{x+2}$ then $x = ?$

S-16 $2^{x+13} = (2^2)^{x+2}$

$\Rightarrow$ $2^{x+13} = 2^{2x+4}$

$\Rightarrow$ $x + 13 = 2x + 4 \Rightarrow x =$ **9**.

E-17 $3\sqrt{27} - \sqrt{75} + \sqrt{12} = ?$

S-17 $3\sqrt{27} - \sqrt{75} + \sqrt{12}$

$$= 3\sqrt{3\times3\times3} - \sqrt{5\times5\times3} + \sqrt{2\times2\times3}$$

$$= 3 \times 3\sqrt{3} - 5\sqrt{3} + 2\sqrt{3}$$

$$= (9 - 5 + 2)\ \sqrt{3} = \mathbf{6\sqrt{3}}$$

E-18 If $5\dfrac{3}{x} \times y\dfrac{1}{2} = 19$, then the value of $(x, y) = ?$

S-18 Equating the quotients of both sides,

$5y = 19$, we get the quotient as 3 $\left(\text{Since } \dfrac{19}{5} = 3\dfrac{4}{5}\right)$

$\therefore y = 3$

Hence, $5\dfrac{3}{x} \times 3\dfrac{1}{2} = 19$

$\Rightarrow$ $5\dfrac{3}{x} = \dfrac{19}{\frac{7}{2}} \Rightarrow 5\dfrac{3}{x} = 5\dfrac{3}{7}$

$\therefore$ $x = 7$ so $x =$ **7** and $y =$ **3**.

E-19 $8\dfrac{1}{4} + 8\dfrac{1}{2} + ? = 20\dfrac{1}{8}$

S-19 Substituting x for ?, we get,

$$8\frac{1}{4} + 8\frac{1}{2} + x = 20\frac{1}{8}$$

$$\Rightarrow x = 20\frac{1}{8} - 8\frac{1}{4} - 8\frac{1}{2}$$

$$= (20 - 8 - 8) + \left(\frac{1}{8} - \frac{1}{4} - \frac{1}{2}\right)$$ *[Refer 4.4.2]*

$$= 4 + \left(\frac{1-2-4}{8}\right)$$

$$= 4 + \frac{-5}{8} = (4 - 1)\ \frac{8-5}{8}$$ *[Refer 4.4.3]*

$$= \mathbf{3\frac{3}{8}}.$$

E-20 $\dfrac{\sqrt{1296}}{?} = \dfrac{?}{2.25}$

S-20 Putting x for (?), we get *[Refer 4.4.4]*

$\sqrt{1296} \times 2.25 = x^2$

$36 \times 2.25 = x^2 \;\Rightarrow\; x = \sqrt{36 \times 2.25} \;\Rightarrow\; x = 6 \times 1.5$ (Since $\sqrt{1296} = 36$)

$\therefore\; x = \mathbf{9}.$

E-21 65% of ? = 124.90 – 63.15

S-21 Putting x for (?),

$\dfrac{65}{100}$ of $x = 61.75$

$\Rightarrow x = \dfrac{61.75}{65} \times 100 \Rightarrow x = \mathbf{95}.$

E-22 If $\dfrac{a}{a+b} = \dfrac{17}{23}$, what is $\dfrac{a+b}{a-b}$ equal to? **(IA, '79)**

S-22 Given that $\dfrac{a}{a+b} = \dfrac{17}{23}$

i.e. if $a = 17$, then $a + b = 23$

or $b = 6$

$\therefore\ a - b = 17 - 6 = 11$

hence $\dfrac{a+b}{a-b} = \mathbf{\dfrac{23}{11}}.$

E-23 $\dfrac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = ?$

S-23 Putting x for (?) and solving for x, we get

$$x = \frac{2\sqrt{6} + 6\sqrt{6}}{4\sqrt{6}} \;\Rightarrow\; x = \frac{8\sqrt{6}}{4\sqrt{6}}$$

$\therefore\; x = \mathbf{2}.$

E-24 Simplify $(x + y - z)^2 - (x - y + z)^2$

S-24 Using the formula *[Refer 4.2 v]*

$$a^2 - b^2 = (a + b)(a - b)$$

$$\Rightarrow \quad (x + y - z)^2 - (x - y + z)^2$$

$$= (x + y - z + x - y + z)(x + y - z - x + y - z) = 2x(2y - 2z)$$

$$= \mathbf{4xy - 4xz}.$$

E-25 If $x = 5$ and $y = -2$, then what is the value of $(x - 2y)^{1/y}$?

S-25 $(x - 2y)^{1/y}$

$$= (5 + 4)^{1/-2} = (9)^{1/-2} = \frac{1}{(9)^{1/2}}$$

$$= \frac{1}{\sqrt{9}} = \mathbf{\frac{1}{3}}.$$

E-26 $2\frac{2}{3} \div \frac{4}{5}$ of ? $= \frac{1}{6}$

S-26 Putting x for (?) and solving for x, we get,

$$\Rightarrow \quad \frac{8}{3} \div \frac{4}{5}x = \frac{1}{6}$$

$$\Rightarrow \quad \frac{8}{3} = \frac{1}{6} \times \frac{4}{5}x \quad \Rightarrow \quad x = \frac{8 \times 6 \times 5}{3 \times 4}$$

$$\Rightarrow \quad x = \mathbf{20}.$$

E-27 Find the value of

(i) $\frac{5}{6}$ of $\frac{1}{20}$ of 24 rupees. **(ii)** $\frac{4}{7}$ of 14 times of $2\frac{1}{4}$ kg.

S-27 **(i)** $\frac{5}{6}$ of $\frac{1}{20}$ of 24 rupees.

$= \frac{5}{6} \times \frac{1}{20} \times 24$ rupees $=$ **1 rupee**.

(ii) $\frac{4}{7}$ of 14 times of $2\frac{1}{4}$ kg.

$= \frac{4}{7} \times 14 \times \frac{9}{4}$ kg. $=$ **1.8 kg.**

E-28 $\left(11 \div 2\frac{1}{5}\right) \div \frac{11}{5}$ of $2\frac{1}{2} - 2$

S-28 Applying **VBODMAS** Rules, we get *[Refer 4.1]*

$$= \left(11 \times \frac{1}{2\frac{1}{5}}\right) \div \frac{11}{5} \text{ of } 2\frac{1}{2} - 2 \qquad [B]$$

$$= 5 \div \frac{11}{5} \times \frac{5}{2} - 2 \qquad [O]$$

$$= 5 \times \frac{5}{11} \times \frac{5}{2} - 2 = \frac{125}{22} - 2$$ [D][M]

$$= \mathbf{3\frac{15}{22}}.$$ [S]

E-29 $\dfrac{\frac{2}{3}+\frac{1}{5}-\frac{1}{10}}{\frac{4}{5}\times\frac{1}{8}+\frac{1}{2}}$

S-29 $= \dfrac{\frac{20+6-3}{30}}{\frac{1}{10}+\frac{1}{2}} = \dfrac{\frac{23}{30}}{\frac{1+5}{10}}$

$$= \frac{23}{30}\times\frac{10}{6} = \mathbf{1\frac{5}{18}}$$

E-30 $\left(\frac{?}{18}\right) \times \left(\frac{?}{162}\right) = 1$ **(BSRB, '92)**

S-30 Putting x for (?) and solving it for x, we get

$$\frac{x}{18}\times\frac{x}{162} = 1$$ *[Refer 4.4.2.]*

$\Rightarrow \quad x^2 = 18 \times 162$

$\Rightarrow \quad x^2 = 18 \times 18 \times 9 \Rightarrow x = 18 \times 3$

$\therefore \quad x = \mathbf{54}.$

E-31 $\dfrac{(0.55)^2+(0.07)^2+(0.027)^2}{(0.055)^2+(0.007)^2+(0.0027)^2} = ?$

S-31 Let $0.55 = a$, $0.07 = b$ and $0.027 = c$

Then, the given expression becomes

$$\frac{a^2+b^2+c^2}{(0.1\times a)^2+(0.1\times b)^2+(0.1\times c)^2} = \frac{[a^2+b^2+c^2]}{0.01[a^2+b^2+c^2]}$$

$$\frac{1}{0.01} = \mathbf{100}.$$

E-32 $\dfrac{137\times137\times137+133\times133\times133}{137\times137-137\times133+133\times133} = ?$

S-32 Let $137 = a$ and $133 = b$

Then, the given expression becomes

$$\frac{a^3+b^3}{a^2-ab+b^2} = \frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)} = a + b$$ *[Refer 4.2 (8)]*

Putting the value of a and b, we get

$= 137 + 133 = \mathbf{270}.$

E-33 $\dfrac{20.25 \times 2.80}{28.35}$

S-33 $\dfrac{20\frac{1}{4} \times 2.80}{28.35}$ $\left(\text{Since} \quad 20.25 = 20\frac{1}{4}\right)$

$$= \frac{\frac{81}{4} \times 2.8}{28.35} = \frac{81 \times 0.7}{28.35}$$

$$= \frac{56.70}{28.35} = \mathbf{2}.$$

E-34 $54 \div 66 \div 33 = ?$

S-34 $\dfrac{54}{66} \div 33$

$$= \frac{9}{11} \div 33 = \frac{9}{11} \times \frac{1}{33} = \mathbf{\frac{3}{121}}$$

E-35 $\dfrac{2?9}{4} = \dfrac{916}{16}$

S-35 Putting x for (?) we get

$$\frac{2x9}{4} = \frac{916}{16} \quad \Rightarrow \quad \frac{2x9}{4} = \frac{229}{4}$$

$\therefore \quad x = \mathbf{2}$

E-36 $\sqrt{\dfrac{?}{10}} = 0.011$

S-36 Putting x for (?) we get

$\sqrt{\dfrac{x}{10}} = 0.011$ (Since x is under square root)

$\dfrac{x}{10} = (0.011)^2$ (squaring both sides)

$\Rightarrow \quad x = 10 \times 0.000121$

$\therefore \quad x = \mathbf{0.00121}.$

E-37 $\sqrt{\dfrac{67.6}{?}} = 0.26$ **(SBI, '80)**

S-37 Putting x for (?) we get

$$\sqrt{\frac{67.6}{x}} = 0.26$$

$\Rightarrow \quad \frac{67.6}{x} = (0.26)^2$ (squaring both sides)

$\Rightarrow \quad x = \frac{67.6}{0.0676} \quad \Rightarrow \quad x = \mathbf{1000}.$

E-38 $\sqrt{\frac{0.324}{10}} = ?$

S-38 Putting x for (?), we get

$$\sqrt{\frac{0.324}{10}} = x$$

Here x is not under square root, hence squaring is not done.

$\Rightarrow \quad \sqrt{0.0324} = x$

$\Rightarrow \quad x = \sqrt{\frac{324}{10,000}} \quad \Rightarrow \quad x = \sqrt{\frac{18 \times 18}{100 \times 100}}$

$\Rightarrow \quad x = \frac{18}{10} \quad \Rightarrow \quad x = \mathbf{0.18}.$

E-39 $\frac{(63+36)^2 + (63-36)^2}{63^2 + 36^2} = ?$ **(ITI, '83)**

S-39 Putting 63 = a and 36 = b in the given expression, we get

$\Rightarrow \quad x = \frac{(a+b)^2 + (a-b)^2}{a^2 + b^2}$

$\Rightarrow \quad x = \frac{2(a^2 + b^2)}{a^2 + b^2}$ (Since $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$)

$x = \mathbf{2}.$ *[Refer 4.2 (iii)]*

E-40 $44.60 \times 2.50 = ?$

S-40 Putting x for (?), we get

$x = 44.60 \times 2.50 \Rightarrow x = 44.60 \times \frac{10}{4}$ $\left(\text{Since} \quad 2.50 = \frac{10}{4}\right)$

$\Rightarrow \quad x = 11.15 \times 10$

$\Rightarrow \quad x = \mathbf{111.5}.$

E-41 $\frac{14 \times 14 - 46}{11 \times 6 - (4)^2} = ?$

S-41 Putting x for (?) and applying **VBODMAS** Rule,
We get,

$$x = \frac{196-46}{66-16} \Rightarrow x = \frac{150}{50} = \mathbf{3}.$$

E-42 2002 – 2002 ÷ 10.10 = ? **(IA, '80)**

S-42 Putting x for (?) and applying **VBODMAS** Rule, we get

$$x = 2002 - \frac{2002}{10.10}$$

$$\Rightarrow x = 2002 - 200 \Rightarrow x = \mathbf{1802}.$$

E-43 $\dfrac{18-3\times 4+2}{6\times 5-3\times 8} = ?$

S-43 Putting x for (?) and applying **VBODMAS** Rule, we get

$$\Rightarrow x = \frac{18-12+2}{30-24} \Rightarrow x = \frac{18+2-12}{30-24} \Rightarrow x = \frac{20-12}{30-24} \quad \text{[M][A]}$$

$$\Rightarrow x = \frac{8}{6} \quad \text{[S]}$$

$$\therefore x = \mathbf{\frac{4}{3}}.$$

E-44 Express 5005 into its prime factors. **(SSC, '86)**

S-44 $5005 = 5 \times 1001$
$5 \times 7 \times 143 = \mathbf{5 \times 7 \times 11 \times 13}.$

E-45 $(4^3)^4 \div (4^2)^3 \times (4^5)^0 = ?$ **(ITI, '84)**

S-45 Put x for (?) , Since all base are equal to 4, hence, put $a = 4$.

$$\Rightarrow x = (a^3)^4 \div (a^2)^3 \times (a^5)^0$$

$$\Rightarrow x = a^{12} \div a^6 \times 1 \Rightarrow x = a^{12-6} \Rightarrow x = a^6 \quad (\text{Since } (a^5)^0 = 1)$$

(Since $a^m \div a^n = a^{m-n}$) *[Refer 4.2 (9)]*

$x = \mathbf{4^6}$.

E-46 Find the value of x in the equation. **(SSC, '86)**

$$\sqrt{1+\frac{25}{144}} = 1+\frac{x}{12}$$

S-46 $\sqrt{\dfrac{144+25}{144}} = 1+\dfrac{x}{12}$

$$\Rightarrow \sqrt{\frac{169}{144}} = 1+\frac{x}{12} \Rightarrow \frac{13}{12} = 1+\frac{x}{12}$$

$$\Rightarrow x = \mathbf{1}.$$

E-47 If $\dfrac{b}{a}=\dfrac{1}{2}$, what is the value of the expression $\dfrac{a-b}{a+b}+\dfrac{2}{3}$

S-47 Given that $\dfrac{b}{a}=\dfrac{1}{2}$

i.e. if $b = 1$, then $a = 2$ (assume it, for convenience)

So, $\dfrac{a-b}{a+b}+\dfrac{2}{3} = \dfrac{2-1}{2+1}+\dfrac{2}{3} = \mathbf{1}$.

E-48 If $\sqrt{18\times 14\times x} = 84$, then x is equal to ? **(Auditors, '86)**

S-48 $\sqrt{18\times 14\times x} = 84$

Since x is under square root, so, squaring both sides we get

$18 \times 14 \times x = 84 \times 84 \Rightarrow x = \dfrac{84\times 84}{18\times 14}$

$\therefore \quad x = \mathbf{28}.$

E-49 $(243)^{0.8} \div (243)^{0.4} = ?$ **(BSRB, '88)**

S-49 Putting x for (?) and $243 = a$, we get,

$x = (a)^{0.8} \div (a)^{0.4} = (a)^{0.8-0.4}$ *[Refer 4.2 (ix)]*

$\Rightarrow \quad x = a^{0.4}$

$x = (243)^{0.4}$

$x = (243)^{4/10} \Rightarrow x = (3^5)^{2/5} \Rightarrow x = 3^2$

$\Rightarrow \quad x = \mathbf{9}.$

E-50 $1.2 \times 1.2 + 0.8 \times 0.8 + 2.4 \times 0.8 = ?$

S-50 Putting x for (?) and $1.2 = a$, $0.8 = b$,

we get

$x = a^2 + b^2 + 2ab \Rightarrow x = (a + b)^2$

$\Rightarrow \quad x = (1.2 + 0.8)^2 \Rightarrow x = \mathbf{4}.$

E-51 $\dfrac{\dfrac{64}{121}-\dfrac{9}{64}}{\dfrac{8}{11}+\dfrac{3}{8}} = ?$ **(SBI, '86)**

S-51 $x = \dfrac{(64^2-9\times 121)}{121\times 64}\times\dfrac{8\times 11}{(8\times 8+3\times 11)}$

$\Rightarrow \quad x = \dfrac{(64^2-3\times 3\times 11\times 11)}{(11\times 11\times 8\times 8)}\times\dfrac{8\times 11}{(64+33)}$

$$\Rightarrow \quad x = \frac{(64+33)(64-33)}{11 \times 8} \times \frac{1}{(64+33)}$$

$$\Rightarrow \quad x = \mathbf{\frac{31}{88}}.$$

E-52 $\dfrac{0.1+0.75}{2.5+0.05} \div \left(0.125 + \dfrac{1}{4.8}\right) = ?$

S-52 Putting x for (?)

$$\Rightarrow \quad x = \frac{0.85}{2.55} \div \left(\frac{1}{8} + \frac{10}{48}\right) \quad \Rightarrow \quad x = \frac{1}{3} \div \left(\frac{16}{48}\right)$$

$$\Rightarrow \quad x = \mathbf{1}.$$

E-53 $\left(\dfrac{216}{1}\right)^{-2/3} \div \left(\dfrac{27}{1}\right)^{-4/3} = ?$ **(Bank PO, '79)**

S-53 Putting x for (?), we get

$$\Rightarrow \quad \left(\frac{216}{1}\right)^{-2/3} \div \left(\frac{27}{1}\right)^{-4/3} = x \qquad \left(\text{Since } \left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}\right)$$

$$\Rightarrow \quad (6^3)^{2/3} \div (3^3)^{4/3} = x \qquad \Rightarrow \quad x = \frac{6^2}{3^4} \quad \Rightarrow \quad x = \mathbf{\frac{4}{9}}$$

E-54 $8^{5/3} \div (125)^{-2/3} = ?$

S-54 Putting x for (?)

$$(2^3)^{5/3} \times (125)^{2/3} = x \qquad (\text{Since } a^m \div b^{-n} = a^m \times b^n)$$

$$\Rightarrow \quad x = 2^5 \times (5^3)^{2/3} \quad \Rightarrow \quad x = 32 \times 25$$

$$\therefore \quad x = \mathbf{800}.$$

E-55 $\left(\dfrac{1}{6^{-2}}\right) \times (81)^{-3/4} = ?$

S-55 Putting x for (?) and solving

$$\frac{6^2}{1} \times \left(\frac{1}{81}\right)^{3/4} = x \qquad \left(\text{Since } \left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}\right)$$

$$\Rightarrow \quad x = \frac{6^2}{(3^4)^{3/4}} \quad \Rightarrow \quad x = \frac{6 \times 6}{3 \times 3 \times 3}$$

$$x = \mathbf{\frac{4}{3}}.$$

E-56 $\sqrt{147}+\sqrt{27} = ? \times \sqrt{3}$

S-56 Putting x for (?) and solving it for x,

$$\sqrt{3\times7\times7}+\sqrt{3\times3\times3} = x\times\sqrt{3}$$

$$\Rightarrow \quad x\times\sqrt{3} = 7\sqrt{3}+3\sqrt{3} \quad \Rightarrow \quad x\sqrt{3} = 10\sqrt{3}$$

$\therefore \quad x =$ **10**.

E-57 $\sqrt{98}-\sqrt{50} = ? \times \sqrt{2}$ **(Bank PO, '80)**

S-57 Putting x for (?) and solving it for x,

$$\sqrt{7\times7\times2}-\sqrt{5\times5\times2} = x\times\sqrt{2}$$

$$\Rightarrow \quad 7\sqrt{2}-5\sqrt{2} = x\times\sqrt{2}$$

$\therefore \quad x =$ **2**.

E-58 $\sqrt{\frac{9}{25}}+\sqrt{3\frac{1}{16}} = ?$

S-58 $\frac{3}{5}+\sqrt{\frac{49}{16}} = x \quad \Rightarrow \quad x = \frac{3}{5}+\frac{7}{4} \quad \Rightarrow \quad x = \mathbf{2\frac{7}{20}}$

E-59 $\sqrt{0.01+\sqrt{0.0064}} = ?$

S-59 Putting x for (?) and solving it for x,

$$\sqrt{0.01+0.08} = x \quad \Rightarrow \quad \sqrt{0.09} = x$$

$\therefore \quad x =$ **0.3**

E-60 $\sqrt{\frac{25.6}{36.1}} \div \sqrt{\frac{12.1}{81\times0.1}} = ?$

S-60 Putting x for (?), we get

$$\sqrt{\frac{256}{361}} \div \sqrt{\frac{121}{81}} = x \quad \Rightarrow \quad x = \frac{16}{19}\div\frac{11}{9}$$

$$\therefore \quad x = \frac{16\times9}{11\times19} = \mathbf{\frac{144}{209}}$$

E-61 Express the number 51 as the difference of squares of two numbers. **(Bank PO, '82)**

S-61 Using the formula

$$N = \left[\frac{N+1}{2}\right]^2 - \left[\frac{N-1}{2}\right]^2, \text{ where } N = \text{ original number}$$

Put $N = 51$

$$\Rightarrow \quad 51 = \left[\frac{51+1}{2}\right]^2 - \left[\frac{51-1}{2}\right]^2$$

$$\Rightarrow \quad \mathbf{51 = (26)^2 - (25)^2}.$$

E-62 Find the number whose seventh part multiplied by its eleventh part gives 1,232.

S-62 Let x be the number such that

$$\frac{x}{7} \times \frac{x}{11} = 1232$$

$\Rightarrow \quad x^2 = 7 \times 11 \times 1232 \quad \Rightarrow \quad x^2 = 7 \times 11 \times 7 \times 11 \times 4 \times 4$

$\Rightarrow \quad x = 7 \times 11 \times 4 \quad \Rightarrow \quad 308$

$\therefore$ the number is **308**.

E-63 Find the square root of $\dfrac{\left(3\frac{1}{4}\right)^4 - \left(4\frac{1}{3}\right)^4}{\left(3\frac{1}{4}\right)^2 - \left(4\frac{1}{3}\right)^2}$

S-63 Let $\left(3\frac{1}{4}\right)^2 = a$ and $\left(4\frac{1}{3}\right)^2 = b$, then

$$\text{Given expression} = \frac{a^2 - b^2}{a - b} = a + b$$

$$= \left(3\frac{1}{4}\right)^2 + \left(4\frac{1}{3}\right)^2 = \left(\frac{13}{4}\right)^2 + \left(\frac{13}{3}\right)^2$$

$$= \frac{169}{16} + \frac{169}{9} = 169\left(\frac{1}{16} + \frac{1}{9}\right) = 169 \times \frac{25}{144}$$

$$\text{Required square root} = \sqrt{169 \times \frac{25}{144}} = \sqrt{13^2 \times \frac{5^2}{12^2}}$$

$$= \frac{65}{12} = \mathbf{5\frac{5}{12}}$$

E-64 The highest score in an innings was $\frac{2}{9}$ of the total score and the next highest was $\frac{2}{9}$ of the remainder. These scores differed by 8 runs. What was the total score in the innings? **(NDA, '88)**

S-64 Let the total score be x runs, such that

$$\frac{2}{9}x - \frac{2}{9} \times \left(x - \frac{2}{9}x\right) = 8 \quad \Rightarrow \quad \frac{2}{9}x - \frac{2}{9} \times \frac{7}{9}x = 8$$

$$\Rightarrow \quad \frac{2}{9}x \times \frac{2}{9} = 8 \quad \Rightarrow \quad x = 162$$

$\therefore$ The total score in the innings was **162**

E-65 Simplify

$\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$ **(SSC, '94)**

S-65 $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

$= 1 + (8^2)^{-1/2} + (-1 \times 32)^{4/5}$

$= 1 + 8^{-1} + [(-1)^{4/5} \times (32)^{4/5}$

$= 1 + \frac{1}{8} + [(-1^2)^{2/5} \times (2^5)^{4/5}] = 1 + \frac{1}{8} + [2^4] = \mathbf{17\frac{1}{8}}.$

E-66 Simplify

$(243)^{0.12} \times (243)^{0.08}$ **(SSC, '94)**

S-66 $(243)^{0.12} \times (243)^{0.08}$

$= (243)^{0.12 + 0.08} = (243)^{0.2}$ (Since $a^m \times a^n = a^{m+n}$)

$= (3^5)^{1/5} = \mathbf{3}.$

E-67 If $\sqrt{2^n} = 64$, then find the value of n **(AGE, '90)**

S-67 $\sqrt{2^n} = 64$

$\Rightarrow \quad (2^n)^{1/2} = 2^6 \Rightarrow \frac{n}{2} = 6$ (Since bases are same)

$\therefore \quad n = \mathbf{12}.$

E-68 $\left(\frac{-1}{216}\right)^{-2/3} = ?$ **(SSC, '94)**

S-68 Putting x for (?), we get

$x = \left(\frac{-1}{216}\right)^{-2/3}$

$\Rightarrow \quad x = (-216)^{2/3} \Rightarrow x = (-6^3)^{2/3}$ $\left(\text{Since } \left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{m/n}\right)$

$\Rightarrow \quad x = \mathbf{36}.$

E-69 $(-2)^{-(2)^{(-2)}} = ?$ **(SSC, '94)**

S-69 Putting x for (?)

$x = (-2)^{-(2)^{(-2)}}$

$\Rightarrow \quad x = \left(-\frac{1}{2}\right)^{(2)^{(-2)}}$ $\left(\text{Since } \left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{m/n}\right)$

$\Rightarrow \quad x = \left[\frac{1}{4}\right]^{-2}$ $\left(\text{Since } \left(\frac{-1}{2}\right)^2 = \frac{1}{4}\right)$

$\Rightarrow \quad x = (4)^2 \Rightarrow x = \mathbf{16}.$

E-70 $11\frac{1}{3} \times 4\frac{8}{10} \div ? = 22\frac{2}{3}$ **(NDA, '83)**

S-70 Putting x for (?), we get

$$11\frac{1}{3} \times 4\frac{8}{10} \div x = 22\frac{2}{3}$$

$$\Rightarrow \quad 11\frac{1}{3} \times 4\frac{8}{10} = 22\frac{2}{3} \times x$$ (If $a \div b = c$ then $a = b \times c$)

$$\Rightarrow \quad x = \frac{1}{2} \times 4\frac{8}{10}$$ $\left(\text{Since } \frac{11\frac{1}{3}}{22\frac{2}{3}} = \frac{1}{2}\right)$

$$\Rightarrow \quad x = \mathbf{2.4}.$$

E-71 $(1.06 + 0.04)^2 - ? = 4 \times 1.06 \times 0.04$ **(CDS, '80)**

S-71 Putting x for (?) and solving for it

$(1.06 + 0.04)^2 - x = 4 \times 1.06 \times 0.04$

Assume, $1.06 = a$ and $0.04 = b$

$\therefore \quad (a + b)^2 - x = 4ab$

$\therefore \quad x = (a - b)^2 = (1.06 - 0.04)^2 = \mathbf{1.0404}.$ $[(a + b)^2 - (a - b)^2 = 4ab]$

E-72 If $a^2 + b^2 = 45$ and $ab = 18$, find $\frac{1}{a} + \frac{1}{b}$ **(MBA, '87)**

S-72 $$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

$$= \frac{\sqrt{a^2 + b^2 + 2ab}}{ab}$$ (Since $a + b = \sqrt{(a+b)^2}$)

$$= \frac{\sqrt{45 + 2 \times 18}}{18} = \frac{\pm 9}{18} = \pm\frac{\mathbf{1}}{\mathbf{2}}$$

E-73 If $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd}$, then find the value of $\frac{a+b}{a-b}$ in terms of c and d only. **(MBA, '87)**

S-73 $$\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd} \quad \Rightarrow \quad \frac{a^2 + b^2}{c^2 + d^2} = \frac{2ab}{2cd}$$

$$\Rightarrow \quad \frac{a^2 + b^2 + 2ab}{c^2 + d^2 + 2cd} = \frac{a^2 + b^2 - 2ab}{c^2 + d^2 - 2cd}$$ (By componendo and dividendo)

$$\Rightarrow \quad \left(\frac{a+b}{a-b}\right)^2 = \left(\frac{c+d}{c-d}\right)^2$$

$$\therefore \quad \frac{\boldsymbol{a+b}}{\boldsymbol{a-b}} = \pm\frac{\boldsymbol{c+d}}{\boldsymbol{c-d}}$$

E-74 Simplify

$$\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$$ **(MBA, '87)**

S-74 $$\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$$

$$=\frac{a^{1/2}+a^{-1/2}}{(1+a^{1/2})(1-a^{1/2})}+\frac{1-a^{1/2}}{1+a^{1/2}}$$ $[\text{Since } 1-a=(1)^2-(a^{1/2})^2=(1+a^{1/2})(1-a^{1/2})]$

$$=\frac{a^{1/2}+a^{-1/2}+(1-a^{-1/2})(1-a^{1/2})}{(1+a^{1/2})(1-a^{1/2})}$$

$$=\frac{a^{1/2}+a^{-1/2}+1-a^{-1/2}-a^{1/2}+1}{(1-a)}$$

$$=\frac{\mathbf{2}}{\mathbf{1-a}}.$$

E-75 Solve $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$ **(MBA, '87)**

S-75 $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$

The given equation is of the form

$5^2+12^2=13^2$ (By the Pythogoras theorem of numbers)

Comparing the two equations, we find

$\sqrt{x}=2$

$\Rightarrow$ $x=\mathbf{4}$.

E-76 ***Directions (i)-(iv):*** What approximate value should come in place of the question mark (?) in the following questions:

(i) 139% of 459 + $5\frac{1}{2}$ of 384 = ? **(BSRB Bombay PO, '97)**

(ii) $\sqrt{2000}\times 0.7=(?)^2$ **(BSRB Bangalore PO, '97)**

(iii) ?% of 8999 + $\frac{599}{3}$ = 26300 **(BSRB Bangalore PO, '97)**

(iv) 3.9% 99 + $\frac{4}{9}$ of 700 = 40% of ? **(BSRB Bangalore PO, '97)**

S-76 (i) Assuming x for ? and approximating the terms to closest values, we get

$$x=140\%\text{ of }460+\frac{11}{2}\times 384$$

$\Rightarrow \quad x = \dfrac{140 \times 460}{100} + \dfrac{11}{2} \times 384$

$\Rightarrow \quad x = 644 + 2112 = 2756 \approx 2800$

$\therefore$ the required value is **2800**.

(ii) Assuming x for ? and approximating the terms to its closest values, we get

$$(x)^2 = \sqrt{2000} \times 0.7$$
$$= 20\sqrt{5} \times 0.7$$
$$= 20 \times 2.23 \times 0.7$$
$$= 31.22 \approx 31$$

$\therefore \quad x = \sqrt{31} = 5.56 \approx$ **5.6**.

(iii) Assuming x for ? and approximating the terms to its nearest values, we get

$$x\% \text{ of } 9000 = 26300 - \frac{600}{3}$$

$$\Rightarrow \quad x = \frac{26300 - 200}{9000} \times 100$$
$$= \mathbf{290}.$$

(iv) Assuming x for ? and approximating the terms to its nearest values, we get

$$40\% \text{ of } x = 4\% \text{ of } 100 + \frac{4}{9} \times 700$$

$$\Rightarrow \quad \frac{2}{5} \times x = 4 + 311$$

$$\Rightarrow \quad x = \frac{315 \times 5}{2} = \frac{1575}{2} = 787 \approx \mathbf{790}.$$

REGULAR PROBLEMS

(1) $\dfrac{2 \div (2 \times 2)}{(2 \div 2) \times 2} = ?$ **(UTI, '90)**

(a) 2 (b) 1 (c) 4 (d) $\frac{1}{4}$ (e) None of these

(2) Find the missing number: **(RRB Mumbai, '98)**

$\dfrac{8}{7} \times \dfrac{7}{12} \div ? = \dfrac{4}{9}$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{9}$ (d) $\frac{9}{2}$ (e) $\frac{1}{3}$

(3) Simplify: $2\frac{1}{2}$ of $\frac{3}{4}\times\frac{1}{2}\div\frac{3}{2}+\frac{1}{2}\div\frac{3}{2}\left(\frac{2}{3}-\frac{1}{2}\text{ of }\frac{2}{3}\right)$ to get

(a) 1 (b) $3\frac{3}{8}$ (c) $2\frac{7}{3}$ (d) $1\frac{5}{8}$ (e) $2\frac{3}{4}$

(4) If $\sqrt{13.69} = 3.7$ then **(ASM Exam, '01)**

$\sqrt{1369}+\sqrt{0.1369}+\sqrt{0.001369} = ?$

(a) 37.407 (b) 34.307 (c) 37.470 (d) 34.707 (e) 37.737

(5) If $46^2 - 44^2 = 45p$, then the value of 'p' will be:

(a) 10 (b) 15 (c) 4 (d) 12 (e) 6

(6) $\frac{196}{?} = \frac{?}{36}$ **(IA, '82)**

(a) 28 (b) 84 (c) 56 (d) 16.3 (e) 24

(7) If $a * b = a + b + \sqrt{ab}$, then $6 * 24$ is equal to **(RRB Bhopal, '98)**

(a) 41 (b) 42 (c) 43 (d) 44 (e) 45

(8) Find the value of $45^3 - 65^3 + 20^3$ **(RRB Ajmer, '98)**

(a) –175500 (b) 165500 (c) 0 (d) –174500 (e) –140055

Hint: Here, $a + b + c = 45 - 65 + 20 = 0$. Then use the identity no. 12 of 4.2

(9) $\frac{0.538\times0.538-0.462\times0.462}{1-0.924}=?$ **(Bank PO, '83)**

(a) 2 (b) 1.08 (c) 0.076 (d) 0.987 (e) 1

(10) What should come in place of the (?) mark? **(SBI PO, '99)**

$\frac{?}{24} = \frac{72}{\sqrt{?}}$

(a) 12 (b) 16 (c) 114 (d) 144 (e) None of these

(11) Find the value of $(512)^{-2/9}$

(a) 4 (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$ (e) $\frac{7}{4}$

(12) $8\times12\frac{1}{2} - 75 = 1\frac{2}{3}\times ?$

(a) 15 (b) 16 (c) 25 (d) $\frac{3}{5}$ (e) $2\frac{1}{3}$

Hint: $8\times12\frac{1}{2} = 100$

(13) $\sqrt{12\frac{3}{4}+13\frac{1}{4}+?} = 6$

(a) 10 (b) 16 (c) 12 (d) 36 (e) 18

(14) $7 - [? - \{4 - 7\} - \{5 - (4 - 5) + 2\}] = 16$

(a) 2 (b) 6 (c) 4 (d) 3 (e) – 4

(15) If $\sqrt{x}-\sqrt{y}=1$ and $\sqrt{x}+\sqrt{y}=17$ then $\sqrt{xy}=?$ **(BSRB, '93)**

(a) 51 (b) 16 (c) $\sqrt{72}$ (d) 72 (e) $\sqrt{17}$

(16) If $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{?}}=2$, then (?) is

(a) $2\sqrt{3}$ (b) 24 (c) 12 (d) 144 (e) $3\sqrt{2}$

(17) A man eats 'x' bananas in a week. In how many days will he eat 84 bananas?

(a) $\dfrac{84}{x}$ (b) $12x$ (c) $\dfrac{588}{x}$ (d) $\dfrac{12}{x}$ (e) $84x$

(18) 10 raised to the fifth power may be expressed as:

(a) 10×5 (b) 5^{10} (c) $\sqrt[5]{10}$
(d) $10 \times 10 \times 10 \times 10 \times 10$ (e) $(5)^{10/5}$

(19) Which of the following is the same as $50 \div 12$?
(a) $10\,(5 \div 3)$ (b) $(50 \div 6) + (50 \div 6)$ (c) $25 \div 12 \times 2$
(d) $50 \div 4 \times 3$ (e) $50 \div 4 \div 3$

(20) If $\left(\sqrt{3}\right)^{x}=1$, then the value of x is

(a) 1 (b) $\dfrac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 0 (e) 2

(21) $(16^{0.16} \times 2^{0.36})$ is equal to

(a) 64 (b) 16 (c) 2 (d) $\dfrac{1}{2}$ (e) 1

(22) If $0.5 \times A = 0.0003$, then the value of A will be
(a) 0.6 (b) 0.06 (c) 0.0006 (d) 0.006 (e) 0.175

Hint: Put $0.5=\dfrac{1}{2}$, for easy calculation

(23) Reciprocal of $\sqrt[5]{12\dfrac{209}{243}}$ is equal to

(a) $\dfrac{5}{3}$ (b) $\dfrac{3}{5}$ (c) $\dfrac{2}{5}$ (d) $\dfrac{5}{4}$ (e) $\dfrac{1}{3}$

(24) A decimal fraction is multiplied by itself. If the product is 477.4225, then the fraction is
(a) 19.325 (b) 23.715 (c) 22.75 (d) 21.85 (e) 18.65

(25) Simplify: $2.5-\dfrac{1}{3.25-\dfrac{2.5}{0.75+0.50}}$ **(Goods Guard Ex, '99)**

(a) 0.50 (b) 1.70 (c) 1.25 (d) 0.80 (e) 1.18

(26) Simplify: $\dfrac{0.\dot{3} \times 1.\dot{0}\dot{6}}{0.\dot{5} \times 0.\dot{4}}$

(a) $1.43\dot{1}\dot{8}$ (b) $0.2\dot{8}$ (c) $1.0\dot{3}\dot{2}$ (d) $1.6\dot{4}$ (e) $1.\dot{8}\dot{8}$

(27) If $\sqrt{\sqrt[3]{0.000001 \times x}} = 0.4$, then the value of x is

(a) $2^{1/6}$ (b) 4096 (c) $4^{1/6}$ (d) 64 (e) $4^{1/3}$

(28) $3\dfrac{1}{x} \times 3\dfrac{3}{4} = 12\dfrac{1}{2}$, then the value of '$x$' is

(a) 1 (b) $\dfrac{1}{3}$ (c) 2 (d) 3 (e) $\dfrac{1}{2}$

(29) If 3.2 ÷ 64 * 10 = 2.45 − 1.95, which of the following should replace the asterisk (*)? **(TC Clerk, '97)**

(a) + (b) − (c) × (d) ÷ (e) insufficient data

(30) The value of $(9.9)^3$ is

(a) 970.299 (b) 981.009 (c) 981.999 (d) 998.99 (e) 990.989

Hint: Write 9.9 = 10 − 0.1 and the use the identity $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

(31) Square of 21 of 64 is equal to

(a) square of 168 (b) square of 165 (c) square of 179
(d) square of 167 (e) square of 1344

(32) What is the value of x in $\dfrac{2}{x} - \dfrac{5}{3x} = \dfrac{1}{3}$ $(x \neq 0)$?

(a) cannot be determined (b) −2 (c) 3
(d) −1 (e) 1

(33) The value of $(16^{0.16} \times 2^{0.36})$ is equal to

(a) 0 (b) 1 (c) 2 (d) −1 (e) 4

(34) If '+' means '×', '−' means ÷, ÷ means + and '×' means '−', then what will be the value of 20 ÷ 40 − 4 × 5 + 6 = ?

(a) 0 (b) 13.5 (c) 60 (d) −15 (e) None of these

(Elec. Appr. Exam, '01)

(35) If the given interchanges are made in signs and numbers, which one of the four equations would be correct? **(RRB Secunderabad, '01)**

Given interchanges: signs × and ÷, numbers 4 and 9

(a) 94 × 7 ÷ 47 = 329 (b) 47 × 9 ÷ 94 = 18 (c) 49 × 7 ÷ 49 = 7
(d) 94 × 7 ÷ 97 = 324 (e) None

(36) Which of the following equations are equivalent? **(Mumbai PO, '99)**

(1) $\left(\dfrac{1}{2}M + \dfrac{2}{3}N\right)^2$ (2) $\dfrac{4}{9}N^2 + \dfrac{1}{4}M^2 + \dfrac{2}{3}MN$

(3) $\left(\dfrac{M}{2} + \dfrac{2}{3}N\right)\left(\dfrac{1}{2}M - \dfrac{2}{3}N\right)$ (4) $\dfrac{1}{4}\left(M + \dfrac{4}{3}N\right)^2$

(a) only (2) and (3) (b) only (1) and (4) (c) only (1) and (2)
(d) only (1) and (3) (e) only (1), (2) and (4)

Directions (37-41): What approximate value will come in place of the question mark (?) in the following equation?

(37) $\sqrt{625.04} \times 16.96 + 136.001 \div 17 = ?$ **(RBI, '02)**

(a) 4.18 (b) 4.41 (c) 425 (d) 433 (e) None

(38) $\left(115\frac{1}{24} + 234.92\right) \times 5\frac{3}{37} = ?$ **(Bank PO, '97)**

(a) 1400 (b) 1750 (c) 1350 (d) 1200 (e) 1650

(39) ? % of 6147 = $2\frac{1}{2} \times 245.76$ **(BSRB Mumbai Hindi Officer's, '97)**

(a) 16 (b) 10 (c) 18 (d) 20 (e) 15

(40) $5.6 \times 2569 + 2058 = 157\% \times 6529 + ?$

(a) 5800 (b) 6300 (c) 6200 (d) 6500 (e) 6000

(41) $0.\overline{91} + \sqrt{999} + \sqrt{111} + 0.\overline{11} = ?$

(a) 43 (b) 40 (c) 39 (d) 42 (e) 4.23

Answers:

1. (d)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (e)
10. (d)	**11.** (b)	**12.** (a)	**13.** (a)	**14.** (e)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (c)
19. (e)	**20.** (d)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (b)	**26.** (a)	**27.** (b)
28. (d)	**29.** (c)	**30.** (a)	**31.** (a)	**32.** (e)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (e)
37. (d)	**38.** (b)	**39.** (b)	**40.** (c)	**41.** (a)				

REAL PROBLEMS

(1) If $x = 0.5$ and $y = 0.2$, then $\sqrt{0.6} \times (3y)^x$ is equal to:

(a) 1.0 (b) 0.5 (c) 0.6 (d) 1.1 (e) $\sqrt{1.8}$

(2) $\sqrt[3]{1+\sqrt{2}} \cdot \sqrt[6]{3-2\sqrt{2}}$ is equal to:

(a) $2-\sqrt{2}$ (b) $\sqrt{2}-1$ (c) 1 (d) $3-2\sqrt{2}$ (e) $2+\sqrt{2}$

(3) The value of the expression $\frac{x-1}{x^{3/4}+x^{1/2}} \cdot \frac{x^{1/2}+x^{1/4}}{x^{1/2}+1} \cdot x^{1/4}$

when $x = 16$, is:

(a) 4 (b) 3 (c) 2 (d) 1 (e) 16

(4) $999\frac{998}{999} \times 13 = ?$

(a) $13888\frac{4}{999}$ (b) $12999\frac{986}{999}$ (c) $11988\frac{994}{999}$ (d) $12990\frac{1}{999}$ (e) $12907\frac{904}{999}$

Hint: $999\frac{998}{999} = 999 + \frac{998}{999} = 1000 - \frac{1}{999}$

(5) Arrange the following surds in ascending order of magnitude:

(i) $\sqrt[4]{3}$, (ii) $\sqrt[6]{7}$, (iii) $\sqrt[12]{48}$.

(a) (i), (ii), (iii) (b) (ii), (iii), (i) (c) (i), (iii), (ii) (d) (iii), (ii), (i) (e) (iii), (i), (ii)

Hint: The LCM of the orders of given surds, i.e. 4, 6, 12 are 12. Convert each one of the given surds into a surd of order 12

(6) Simplify: $\dfrac{2^{2^{2^2}} \div \left[\left(2^2\right)^3\right]^4}{\left(4^{4^4}\right) \div (4^4)^4}$

(a) 256 (b) 1024 (c) 64 (d) $\dfrac{1}{2^{488}}$ (e) 2^{4^2}

Hint: $a^{2^3} = a^{2\times2\times2} = a^8$, but $(a^2)^3 = a^{2\times3} = a^6$

(7) $38\frac{2}{3}$ divided by $\frac{2}{8}$ can be expressed as:

(a) $38\frac{2}{3}\times\frac{1}{4}$ (b) $\frac{2}{8}\times\frac{116}{3}$ (c) $\left(\frac{1}{4}\times 38\right)+\left(\frac{1}{4}\times\frac{2}{3}\right)$ (d) $\frac{116}{3}\times 4$

(e) $(38\times 4)+\left(\frac{1}{4}\times\frac{2}{3}\right)$

(8) If $2^n = \dfrac{4^5+4^5+4^5}{3^5+3^5}\times\dfrac{6^5+6^5+6^5+6^5}{2^5+2^5+2^5}$ and $n > 0$, then the value of n^2 is:

(a) 11 (b) 121 (c) 169 (d) 7 (e) 81

(9) If $4^b - 4^{b-1} = 24$, then $(2b)^b$ equals:

(a) 25 (b) $25\sqrt{5}$ (c) 125 (d) $5\sqrt{5}$ (e) $\sqrt{5}$

(10) If $a^3 + b^3 = 0$, then:

(a) $a + b = \sqrt{2ab}$ (b) $a + b = a^2 + b^2$ (c) $a + b = a^2 - b^2 + ab$

(d) $a + b = \sqrt{ab}$ (e) $a + b = \sqrt{3ab}$

(11) Replace the (*) mark in

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + (*)}} = \frac{8}{5}.$$

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) 1 (d) $\frac{1}{3}$ (e) $\frac{1}{4}$

(12) If $m\sqrt{m}\times m^3 \div m^{-3/2} = m^{(a+2)}$, then the value of $a^{(a+2)}$ is:

(a) 1876 (b) 2304 (c) 16 (d) 256 (e) 4096

(13) Evaluate: $1 + \frac{1}{1\times 2} + \frac{1}{1\times 2\times 4} + \frac{1}{1\times 2\times 4\times 8} + \frac{1}{1\times 2\times 4\times 8\times 16}$ up to four places of decimals.

(a) 1.6096 (b) 1.7062 (c) 1.3214 (d) 1.6416 (e) None of these

Hint: Do not waste your time by calculating individual term. Adopt any other convenient method

(14) Find the value of '*b*', if *b* is a natural number and

$$\sqrt{b\sqrt[3]{b}} - \sqrt[3]{b\sqrt{b}} = \sqrt{b}$$

(a) 4 (b) 16 (c) 256 (d) 64 (e) 128

(15) Evaluate: $\frac{(0.15)^2 + (0.28)^2}{(0.45)^2 + (0.84)^2} - \frac{(0.28)^3 + (0.47)^3 - (0.75)^3}{3(0.28)(0.47)(0.75)}$.

(a) 1 (b) 0.8 (c) $1\frac{1}{9}$ (d) 2.5 (e) $-\frac{8}{9}$

(16) If $\sqrt{\sqrt[3]{0.000001x}} = 0.2$, then $\frac{\sqrt{x}}{0.1}$ equals to:

(a) 14.14 (b) 640 (c) 80 (d) 20 (e) None of these

Hint: Remove the radicals one by one

(17) If $\left(\frac{p^2}{q^2}\right)^{5x+7} = \left(\frac{q^3}{p^3}\right)^{x-8}$, then the value of $(5x+7)$ is:

(a) 12 (b) $10\frac{11}{13}$ (c) 17 (d) $7\frac{2}{9}$ (e) $-20\frac{1}{7}$

Hint: Do not try to find the value of $5x + 7$ directly from the given relation

(18) What should come in place of the question mark (?) in the following equation?

(BSRB Patna PO, '01)

$$75^{7.5} \div 75^{3/2} \times 75^{-3} = \left(\sqrt{75}\right)^{?}$$

(a) 6 (b) $5\sqrt{3}$ (c) $\sqrt{3}$ (d) 3 (e) None of these

(19) What will replace the question mark (?) in the following equation?

$$27^{\sqrt{?-7}} = \frac{6^{19}\times 153}{2^{16}\times 136}$$

(a) 8 (b) 16 (c) 56 (d) 23 (e) 32

(20) If $5^x = 6^y = 30^7$, then the value of $\frac{xy}{x+y}$ is:

(a) $\frac{1}{7}$ (b) 3 (c) 6 (d) 7 (e) 1

(21) If $7x + 10 = x^2 - \frac{1}{N} = 13x - 2$, what is the value of N?

(a) $-\frac{1}{20}$ (b) 20 (c) 18 (d) $\frac{1}{18}$ (e) 22

(22) If $\frac{x(x+2)}{(x+4)4} = \frac{0.\dot{3} \times 1.0\dot{6}}{0.\dot{5} \times 0.\dot{4}}$, then the value of x is:

(a) 9 (b) 7 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$ (e) None of these

Hint: Do not waste your time by solving it like a quadratic equation

Directions (23-27): Following (A) to (H) are combinations of an operation and an operand:

(A) means ÷ 3 (B) means × 3 (C) means – 3 (D) means + 3
(E) means ÷ 2 (F) means × 2 (G) means – 2 (H) means + 2

You have been given one or more of these as answer choices for the following questions. Select the appropriate choice to replace the question mark in the equations. **(SBI PO, '99)**

(23) $42 \times 21 - 12? = 880$

(a) A (b) F (c) G (d) D (e) None of these

Hint: $42 \times 21 = 882$ ∴ $12? = 882 - 880 = 2$ Checking the given combination in place of ? By trial, (A) → $12 \div 3 = 4 \neq 2$, then
for combination (F) → $12 \times 2 = 24 \neq 2$,
(G) → $12 - 2 = 10 \neq 2$ and so on

(24) $36 + 12\ ? = 48$

(a) A followed by D (b) B followed by G (c) A followed by B
(d) F followed by H (e) None of these

Hint: Here the number on right hand side i.e. 48 is comparatively smaller than previous one (i.e. 880). So, we can directly test the combination of operation as per choices i.e. (a) → $36 + 12 \div 3 + 3 = 43 \neq 48$

(25) $48\ ? + 12 \times 4 = 80$

(a) E followed by B (b) D followed by A (c) B followed by F
(d) F followed by A (e) None of these

(26) $18 \times 3 \div 2 + 3 < 27\ ?$

(a) D followed by G (b) A followed by G (c) D followed by H
(d) D followed by A (e) None of these

(27) $(48 + 9) \div 19 \times 2 = 12\ ?$

(a) B followed by E (b) A followed by H (c) A followed by D
(d) C followed by A (e) None of these

(28) What would be the maximum value of Q in the following equation?

2P4 + 7R9 + 4Q7 = 1380

(a) cannot be determined (b) 9 (c) 6 (d) 7 (e) 8

Directions (29-36): What approximate value should replace the question mark (?) in the following equations:

(29) 208.78 of $7\frac{3}{5}\%$ + 423.547 of $24\frac{39}{50}\%$ = ?

(a) 120 (b) 117 (c) 123 (d) 114 (e) 130

Tips: Both the parts have % term, so take the % common outside a bracket. Hence calculation of % at each step should be avoided.

(30) 159% of 6531.8 + 5.5 × 1015.2 = ? + 5964.9 **(Mumbai PO, '98)**

(a) 11,000 (b) 11,500 (c) 10,000 (d) 10,800 (e) 12,000

(31) $152\sqrt{?} + 795 = 8226 - 3486$ **(Baroda PO, '99)**

(a) 675 (b) 550 (c) 860 (d) 925 (e) 500

(32) $\frac{5}{7}$ of 1596 + 3015 = ? – 2150

(a) 5500 (b) 6300 (c) 49000 (d) 7400 (e) 68000

(33) $16\sqrt{524} + 1492 - 250.0521 = ?$

(a) 1500 (b) 1350 (c) 2200 (d) 1800 (e) 1600

(34) 857 of 14% – 5.6 × 12.128 = ? **(Chennai PO, 2000)**

(a) 45 (b) 52 (c) 65 (d) 60 (e) 40

(35) 6.39 × 15.266 + 115.8 of $\frac{2}{5}$ = ?

(a) 160 (b) 150 (c) 145 (d) 170 (e) 130

(36) $33\frac{1}{3}$ % of 768.9 + 25% of 161.2 – 68.12 = ?

(a) 220 (b) 245 (c) 235 (d) 250 (e) 230

Hint: Before simplification, put the fractional equivalent of the percentage values as $33\frac{1}{3}\% = \frac{1}{3}$ and $25\% = \frac{1}{4}$

Directions (37-40): Find out the approximate value that should come in place of the question mark in the following questions. (You are not expected to find the exact value).

(37) $\sqrt{45689} = ?$

(a) 170 (b) 280 (c) 320 (d) 210 (e) 430

Hint: Do not try to find the square root by conventional method, but, in reverse way, check the square of which alternative gives closest value to 45689. This checking can be done mentally as square of 2, i.e. $(2)^2$ can give the first digit as 4. So either (b) or (d) can be the answer. Next you check $(28)^2 > 25^2$ (625) it is not the starting digit of the given number. So (d) 210 is the choice.

(38) $\frac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987 = ?$

(a) 30000 (b) 900000 (c) 300000 (d) 3000 (e) 60000

(39) $\frac{2}{5} + \frac{7}{8} \times \frac{17}{19} \div \frac{6}{5} = ?$

(a) $3\frac{1}{2}$ (b) $1\frac{1}{2}$ (c) 2 (d) 1 (e) $\frac{8}{11}$

(40) 399.89 + 206 × 11.009 = ?

(a) 2700 (b) 3100 (c) 6566 (d) 4336 (e) 2400

Directions (41-44): Four of the five parts numbered as (a), (b), (c), (d) and (e) in the following equations are exactly equal. You have to find out the part that is not equal to the other parts.

(41) $\sqrt[3]{729}+\sqrt[3]{625} = \sqrt{324}+\sqrt{256} = \sqrt[3]{216}\times\sqrt[3]{81} - \frac{1}{2}$ of 40

(a) (b) (c)

$\sqrt{441}+\sqrt[3]{2197} = \sqrt[3]{5832}+\sqrt[3]{2744}$

(d) (e)

(42) 8362.64 + 768.3 – 190.57 = 593.38 + 604.7 + 7742.29

(a) (b)

= 2235.925 × 4 = 9931.04 – 990.67 = 17880.74 ÷ 2

(c) (d) (e)

Hint: You have to do actual calculation here. Never try to round off

(43) 10.36 + 69.802 + 24.938 = 2207.1 ÷ 21 = $16\frac{2}{3}$% of 630.6

(a) (b) (c)

32.84375 × 3.2 = $\frac{1}{5}$ of $\frac{1}{9}$ of 4729.4

(d) (e)

(44) $x(x+5)+2(3x+2) = (x+2)^2+7x = (x+3)(x-2)+10(x+1)$

(a) (b) (c)

$= (x-2)^3-(x-3)^3-2(x^2-13x+8) = (x+7)(x+2)+2(x-5)$

(d) (e)

(45) If $\sqrt[3]{91125}$ = 45, then the value of

$\sqrt[3]{91.125}+\sqrt[3]{0.091125}+\sqrt[3]{0.000091125}$

(a) 49.95 (b) 5.495 (c) 4.995 (d) 5.405 (e) 4.545

Tips: On finding cube roots $\left(\sqrt[3]{\ }\right)$ the number of decimal places get reduced to $\frac{1}{3}$, as on finding square roots $\left(\sqrt[2]{\ }\right)$, the number of decimal places reduces to $\frac{1}{2}$

(46) If $x = \frac{1}{11} \div \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5}}}$, then find the value of $\left(x+\frac{5}{6}\right)$.

(a) 3 (b) $\frac{1}{5}$ (c) $5\frac{5}{6}$ (d) $\frac{1}{6}$ (e) 1

(47) If $(x+p+m) = p^3+m^3+x^3-3xpm$ and the sum of any two numbers is 12, then what is the ratio of that sum to the third number?

(a) –4 (b) –1 (c) 1 (d) –12 (e) 4

(48) If 8700 ÷ x = 300 and 4590 ÷ y = 170, then $(x - y) \times (x + y)$ = ?

(a) 29 (b) 56 (c) 112 (d) 27 (e) 81

Hint: Refer to 4.7

(49) If $\frac{8}{a} = \frac{3}{a} - 10$, then $\frac{a}{8} - \frac{3}{a}$ = ?

(a) –10 (b) $\frac{95}{16}$ (c) $\frac{16}{95}$ (d) $\frac{24}{a^2 - 8}$ (e) $\frac{a^2 - 24}{8a}$

Answers

1. (c)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (b)
10. (e)	**11.** (a)	**12.** (e)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (a)
19. (c)	**20.** d)	**21.** (a)	**22.** (b)	**23.** (e)	**24.** (c)	**25.** (d)	**26.** (c)	**27.** (b)
28. (c)	**29.** (a)	**30.** (c)	**31.** (a)	**32.** (b)	**33.** (e)	**34.** (b)	**35.** (c)	**36.** (e)
37. (d)	**38.** (c)	**39.** (d)	**40.** (a)	**41.** (e)	**42.** (c)	**43.** (e)	**44.** (d)	**45.** (c)
46. (e)	**47.** (b)	**48.** (c)	**49.** (b)					

5 PERCENTAGE

5.1 INTRODUCTION

The word 'per cent' means per hundred. Thus, 19 per cent means, 19 parts out of 100 parts. This can also be written as $\frac{19}{100}$.

Therefore, per cent is a fraction whose denominator is 100, and the numerator of this fraction is called the **Rate percent**. So, $\frac{19}{100}$ = 19 per cent. Here, 19 per cent is the rate. The sign for per cent is '%'.

Table 5.1 Fractional Equivalents of Important Percentages

$1\% = \frac{1}{100}$	$2\% = \frac{1}{50}$	$4\% = \frac{1}{25}$	$8\% = \frac{2}{25}$	$16\% = \frac{4}{25}$	$64\% = \frac{16}{25}$	$96\% = \frac{24}{25}$
$5\% = \frac{1}{20}$	$10\% = \frac{1}{10}$	$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$60\% = \frac{3}{5}$	$80\% = \frac{4}{5}$	$120\% = \frac{6}{5}$
$6\frac{1}{4}\% = \frac{1}{16}$	$12\frac{1}{2}\% = \frac{1}{8}$	$25\% = \frac{1}{4}$	$37\frac{1}{2}\% = \frac{3}{8}$	$50\% = \frac{1}{2}$	$87\frac{1}{2}\% = \frac{7}{8}$	$100\% = 1$
$8\frac{1}{3}\% = \frac{1}{12}$	$16\frac{2}{3}\% = \frac{1}{6}$	$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$	$83\frac{1}{3}\% = \frac{5}{6}$	$133\frac{1}{3}\% = \frac{4}{3}$	

Note: Similarity along the horizontal rows are to be observed for memorizing table 5.1.

5.2 FRACTION TO RATE PER CENT

To convert (or express) any fraction $\frac{a}{b}$ to rate per cent, multiply it by 100 and put $a(\%)$ sign.

$$\Rightarrow \underbrace{\boxed{\frac{a}{b}}}_{\text{a fraction}} = \underbrace{\boxed{\frac{a}{b} \times 100\%}}_{\text{rate per cent}}$$

Example: Express $\frac{3}{4}$ in rate per cent

Required rate per cent = $\frac{3}{4} \times 100\% = 75\%$

5.3 RATE PER CENT TO FRACTION

To convert a rate per cent to a fraction, divide it by 100 and delete the % sign.

Example: 8% can be converted to a fraction as $\frac{8}{100}$

5.4 RATE PER CENT OF A NUMBER

Rate per cent of a number is the product of equivalent fraction (of rate per cent) and the number.

$$\Rightarrow \quad p\% \text{ of } A = \left(\frac{p}{100}\right) \times A$$

Example: To find out 25% of 500

Solution: Required value $= 25\% \text{ of } 500$

$$= \underbrace{\left(\frac{25}{100}\right)}_{\text{equivalent fraction for } 25\%} \times 500.$$

$$= 125$$

5.4.1 Relation Among Rate Per cent, Number and Value

Let us consider a number, N.

Then N is considered as the base over which value of different rate per cents are found out,

$$10\% \text{ of } N = \frac{10}{100} \times N = \frac{N}{10} \text{ (value)}$$

$$25\% \text{ of } N = \frac{25}{100} \times N = \frac{N}{4} \text{ (value)}$$

and so on.

Therefore, it is found that as the rate per cent changes, its related value for the same number will also change.

Conversely, different values stand for different rate per cents of the same number. As in the above example, $\frac{N}{10}$ stands for 10% of N; $\frac{N}{4}$ stands for 25% of N and so on.

In the above context, a very useful relation is derived as:

$$\boxed{\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (base)}} \quad \textbf{(1)}$$

Example: 9% of what number is 36?

Solution: Using the relation 1,

the required number (base number) $= \frac{36}{9\%}$

$$= \frac{36}{9} \times 100$$
$$= 400$$

Note: Here, 36 is the value and its rate % of base number = 9%

Example: If 30% of a number is 48, then what is 70% of the number?

Solution: Here, unitary method can be used to save the time.

$$30\% \rightarrow 48$$

$$\Rightarrow \quad 1\% \rightarrow \frac{48}{30}$$

$$\Rightarrow \quad 70\% \rightarrow \frac{48}{30} \times 70 = 112$$

Hence, the required value is 112

Example: If 40% of the number exceeds the 25% of it by 54. Find the number.

Solution: Using the formula (1)

$$\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (i.e. base number)}$$

Here, 54 stands for the difference of (40% and 25% of number)

$$\Rightarrow \quad \frac{54}{(40-25)\%} = \text{number}$$

$$\Rightarrow \quad \text{required number} = \frac{54}{40-25} \times 100 = 360$$

5.5 EXPRESSING A GIVEN QUANTITY AS A PERCENTAGE OF ANOTHER GIVEN QUANTITY

Let one given quantity be x and another given quantity be y. It is often asked to find what percentage of y is x. Here both quantities (x and y) should be in same units. If not, they should be converted into the same unit.

Concept

The question requires us to express one given quantity 'x' as a percentage of another given quantity 'y'.

Since y is the basis of comparison, so, y will be in the denominator. But x is to be converted as percentage of y, hence x will be in the numerator of the fraction. Now to convert the fraction to percentage, we will multiply it by 100. So, we get

$$\text{the required percentage} = \frac{x}{y} \times 100\%.$$

Example: To find '30 is what per cent of 150' or 'what percentage of 150 is 30 ?

Solution: Using the earlier concept, we find here that 150 is the basis of comparison and hence 150 will be in the denominator.

$$\text{The required percentage} = \frac{30}{150} \times 100\% = 20\%.$$

Example: ?% of 320 = 86.4

Solution: Here, 320 is the basis of comparison and it will be in the denominator.

$$\therefore \quad \text{required percentage} = \frac{86.4}{320} \times 100\% = 27\%$$

5.6 CONVERTING A PERCENTAGE INTO DECIMALS

Case I

Let the percentage be a positive integer, then
place a decimal point after two places from the extreme right of the integer to convert it into a decimal. If the percentage is a single digit number, add one zero to the left of it and then place the decimal point for its conversion. % Sign is removed after conversion.

Example: 67% may be converted into decimals as 0.67, because 67% = $\frac{67}{100}$ = 0.67

8% may be written as 0.08

{Zero added to its left to make it a two digit number so that decimal point can be placed.}

253% is equivalent to 2.53

Case II

Let the percentage be a decimal fraction
The percentage being a decimal fraction, shift decimal by two places to the left. Add zero to the left of the fraction, if needed.

Two place left   Shifted

Example: 3.5% may be written as 0.035   (zero is added to its left so that decimal point can be shifted by two places to the left)

0.7% may be written as 0.007

Case III

Let the percentage be a fraction

If the percentage is a fraction of the form $\frac{a}{b}$, then convert it into a decimal fraction and then follow the rule detailed in case II

Example: $\frac{1}{4}$ % is equivalent to 0.25% which may be converted into decimals as 0.0025

5.7 CONVERTING A DECIMAL INTO A PERCENTAGE

In this case, the method of 5.6 is reversed, i.e. shift the decimal point two places to the right. Add zero to the extreme right if required. Then add % sign.

Example: 0.45 may be expressed as 45% (Shift)

0.032 is equivalent to 3.2%

1.7 is equivalent to 170% → zero is added so that decimal point can be shifted by two places.

5.8 EFFECT OF PERCENTAGE CHANGE ON ANY QUANTITY (NUMBER)

If any number (quantity) is increased by $x\%$, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100+x}{100}\right)$$

or

$$= \text{original number} \times (1 + \text{decimal equivalent of } x\%).$$

Similarly, if any number (quantity) is decreased by $x\%$, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100-x}{100}\right)$$

or

$$= \text{original number} \times (1 - \text{decimal equivalent of } x\%).$$

Note: In case of percentage decrease, a (–)ve sign is put before x, otherwise the formula is same.

Example: The present salary of A is Rs 3000. This will be increased by 15% in the next year. What will be the increased salary of A?

Solution: Here, the salary is to be increased by 15%.
15% is equivalent to 0.15

$$\therefore \quad \text{the increased salary} = 3000\,(1 + 0.15) \text{ or } 3000 \times \frac{100+15}{100}$$

$$= 3000 \times 1.15$$
$$= \text{Rs } 3450.$$

5.9 TWO STEP CHANGE OF PERCENTAGE FOR A NUMBER

In the first step, a number is changed (increased or decreased) by $x\%$, and in the second step, this changed number is again changed (increased or decreased) by $y\%$, then net percentage change on the original number can be conveniently found out by using the following formula,

$$\boxed{\text{net \% change} = x + y + \frac{xy}{100} \quad (+ \text{ or } -)} \qquad \textbf{(2)}$$

If x or y indicates decrease in percentage, then put a (–)ve sign before x or y, otherwise positive sign remains.

Example: If a number is increased by 12% and then decreased by 18%, then find the net percentage change in the number.

Solution: Using the formula (2)

$$\text{net \% change} = x + y + \frac{xy}{100}$$

where $x = 12 \qquad y = -18$

$$\Rightarrow \qquad \text{net \% change} = 12 - 18 + \frac{(12)\times(-18)}{100}$$

$$= -6 - 2.16$$

$$= -8.16$$

(–) sign signifies that there is percentage decrease in the result. Therefore –8.16 indicates net 8.16% decrease of the given number as a result of 12% increase and 18% decrease.
It also implies that 12% increase and 18% decrease are equivalent to 8.16% decrease.

5.10 PERCENTAGE CHANGE AND ITS EFFECT ON PRODUCT

$$\text{Let} \quad \underset{\text{variable}}{\underset{\downarrow}{A}} \quad \times \quad \underset{\text{variable}}{\underset{\downarrow}{B}} \quad = \text{result (product)}$$

If A is changed (increased or decreased) by $x\%$, and also B is changed (increased or decreased) by $y\%$, then the net percentage change (increase/decrease) of the product of A and B can be found out easily by the formula

$$\boxed{\underset{(+\text{ or }-)}{\text{net \% change in product}} = x + y + \frac{xy}{100}} \qquad \textbf{(2A)}$$

If x or y indicates decrease in percentage, then put a (–)ve sign before x or y, otherwise positive sign remains.
This formula (2A) is same as formula (2)

Above formula can be used to find out the *net percentage change*, if it involves the *product of any two variable quantities which have also the % change.*

Application of the Formula (2A)

The formula (2A) can be used to find out

(a) % effect on expenditure, when rate and consumption are changed, since rate × consumption = expenditure [$A \times B$ = product]
(b) % effect on area of rectangle/square/triangle/circle, when its sides/radius are changed, since $\text{Side}_1 \times \text{Side}_2$ = area, or radius × radius = area [$A \times B$ = product]
(c) % effect on distance covered, when time and speed are changed, since time × speed = distance. [$A \times B$ = product]

Example: If the length of rectangle increases by 30% and the breadth decreases by 12%, then find the % change in the area of the rectangle.

Solution: Since, length × breadth = area, and both the length and breadth are changed, so, using the formula (2A), we get

$$\text{net \% change in product} = x + y + \frac{xy}{100}$$

where $x = 30,\quad y = -12$

$$\Rightarrow \quad \text{net \% change in area} = 30 - 12 + \frac{30 \times -12}{100}$$
$$= 18 - 3.6$$
$$= +14.4$$

It implies that there is 14.4% increase in the area of the rectangle.

5.10.1 To keep the product of two variable quantity as fixed

As we have seen in 5.10, $A \times B$ = product, where A and B are two quantities which are changing and product is also changing.

Now, we want to **keep the product fixed**, even if A and B are changed (increased/decreased). Then, if one quantity increases, the other quantity will decrease and vice-versa so that product remains unchanged.

Hence the net percentage effect on product is zero in the formula (2A).
Put net % change in product = 0 in formula (2A),

$$x + y + \frac{xy}{100} = 0$$

$$\Rightarrow \quad y = -\frac{x}{100 + x} \times 100, \text{ (–) ve sign shows decrease}$$

From the above derivation, we thus find that

if one quantity A increases by $x\%$, then other quantity B decreases by $\boxed{\frac{x}{100 + x} \times 100\ \%}$

and if one quantity A decreases by $x\%$, then putting (–) x, in place of x,

we find that the other quantity B increases by $\boxed{\frac{x}{100 - x} \times 100\ \%.}$

These formula are used to find out:

(a) % change in either rate or consumption, when expenditure is to be kept fixed, because, rate × consumption = expenditure [$A \times B$ = product = fixed]

(b) % change in either length or breadth, when area of rectangle is to be kept fixed, because, length × breadth = area [$A \times B$ = product = fixed]

(c) % change in either time or speed, when distance is to be kept fixed because, time × speed = distance [$A \times B$ = product = fixed]

Example: If the price of coffee is increased by 10%, then by how much percentage must a house wife reduce her consumption, to have no extra expenditure?

Solution: Since price × consumption = expenditure and expenditure has to be kept fixed (or unchanged), so, when the price increases by 10%, the % reduction in consumption $= \dfrac{10}{100+10} \times 100\%$

$$= 9\frac{1}{11}\%.$$

5.11 RATE CHANGE AND CHANGE IN QUANTITY AVAILABLE FOR FIXED EXPENDITURE

Let the original rate of an item = Rs x per unit quantity.
Expenditure is fixed.

$$\text{Quantity of the item available} = \frac{\text{Expenditure}}{x} \qquad \textbf{(a)}$$

Now, the original rate (x) changes (increase/decrease) to a new rate. Since the amount of expenditure is fixed, so, with the *change in rate*, it is obvious that the quantity of the item available in equation (a) will also change (decrease/increase) accordingly.
Hence, due to rate change,

$$\text{New quantity of the item available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$\Rightarrow$ Original quantity available ± change in quantity available $= \dfrac{\text{Expenditure}}{\text{New rate}}$, Using equation (a), we get

$$\Rightarrow \boxed{\frac{\text{Expenditure}}{x} \pm \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}} \qquad \textbf{(3)}$$

Put (–)ve when quantity available decreases due to rate change, (x = original rate per unit quantity.)

Example: A reduction of 25% in the price of sugar enables the person to get 10 kg more on a purchase for Rs 600. Find the reduced rate of sugar.

Solution: Let the original rate = Rs x per kg.
Since, there is a rate reduction of 25%, so,
New rate (or reduced rate) $= (1 - 0.25)\ x$

$$= 0.75\ x = \frac{3}{4}\ x$$

Expenditure = Rs 600.

Using the formula (3)

$$\frac{\text{Expenditure}}{x} + \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$$\Rightarrow \qquad \frac{600}{x} + 10 = \frac{600}{\frac{3}{4}x}$$ (+ 10, for quantity available increases after rate change)

$$\Rightarrow \qquad \frac{600}{x}\left(\frac{4}{3} - 1\right) = 10 \quad \Rightarrow \quad x = 20$$

Therefore, reduced rate = $\frac{3}{4}x = \frac{3}{4} \times 20$ = Rs 15/kg.

5.12 % EXCESS OR % SHORTNESS

When a number A exceeds the another number B by x %, then % shortness of $B = \frac{x}{100+x} \times 100$

It implies that B is less than A by $\frac{x}{100+x} \times 100\%$.

Similarly, if a number A is short of (or less than) B by x%, then % excess of $B = \frac{x}{100-x} \times 100$

i.e. B is more than A by $\frac{x}{100-x} \times 100\%$

Example: If the income of Ram is more than that of Shyam by 25%, then by how much percentage Shyam's income is less than that of Ram?

Solution: Required % shortness (less) income of Shyam $= \frac{25}{100+25} \times 100\%$

$= 20\%$.

Therefore, income of Shyam is 20% less than that of Ram.

Solved Examples

E-1 Express the following in terms of percentage:

(a) 0.4 *(b)* 1.0 *(c)* $\frac{5}{3}$ *(d)* $\frac{7x}{y}$ *(e)* 1.23

S-1 *(a)* Refer 5.2, multiply the decimal fraction by 100.

$\therefore$ 0.4 = (0.4 × 100) % = **40 %**.

(b) 1.0 = (1.0 × 100) % = **100 %**.

(c) $\frac{5}{3} = \left(\frac{5}{3} \times 100\right)\% = \mathbf{166\frac{2}{3}}\%$.

(d) $\frac{7x}{y} = \left(\frac{7x}{y} \times 100\right)\% = \frac{\mathbf{700x}}{\mathbf{y}}\%$.

(e) 1.23 = (1.23 × 100) % = **123 %**.

E-2 Express the following in terms of fractions:

(a) $22\frac{1}{2}$ % *(b)* 35% *(c)* $\frac{a^2}{b}$ % *(d)* 0.3% *(e)* $\frac{7}{2}$ %

S-2 ***(a)*** Divide the given percentage by 100 to convert it into a fraction. *[Refer 5.3]*

$$\therefore \quad 22\frac{1}{2}\ \% = \frac{22\frac{1}{2}}{100} = \frac{45}{2\times 100} = \mathbf{\frac{9}{40}}.$$

(b) $35\ \% = \dfrac{35}{100} = \mathbf{\dfrac{7}{20}}.$

(c) $\dfrac{a^2}{b}\ \% = \dfrac{a^2}{b\times 100} = \mathbf{\dfrac{a^2}{100b}}.$

(d) $0.3\ \% = \dfrac{0.3}{100} = \mathbf{\dfrac{3}{1000}}.$

(e) $\dfrac{2}{7}\ \% = \dfrac{2}{7\times 100} = \mathbf{\dfrac{1}{350}}.$

E-3 Find

(a) 9% of 27

(b) 0.02% of 6500

(c) $\dfrac{7}{2}$ % of 80

(d) 125% of 64

(e) 10% of 5% of 320

S-3 ***(a)*** Refer 5.4. Multiply the number by $\dfrac{p}{100}$, if p% of the number is to be calculated.

$$\therefore \quad 9\ \%\text{ of } 27 = \frac{9}{100}\times 27 = \mathbf{\frac{243}{100}}.$$

(b) $0.02\ \%\text{ of } 650 = \dfrac{0.02}{100}\times 6500 = \mathbf{\dfrac{13}{10}}.$

(c) $\dfrac{7}{2}\ \%\ 80 = \dfrac{7}{2\times 100}\times 80 = \mathbf{\dfrac{14}{5}}.$

(d) $125\%\text{ of } 64 = \dfrac{125}{100}\times 64 = \mathbf{80}.$

(e) $10\%\text{ of } 5\%\text{ of } 320 = \dfrac{10}{100}\times\dfrac{5}{100}\times 320 = 1.6 = \mathbf{\dfrac{8}{5}}.$

E-4 Find the following:

(a) 36 is what % of 144.

(b) $\dfrac{7}{8}$ is what % of $\dfrac{3}{4}$.

(c) What % of 80 is 16.

(d) 0.625 is equal to what % of $1\dfrac{7}{28}$.

(e) 36 × 14 is what % of 1400.
(f) R is what % of N.

S-4 Refer 5.5, the required percentage = $\frac{x}{y} \times 100$ %, where y is the base.

(a) Here, the base is 144. So, the denominator will be 144.

$\therefore$ required percentage = $\frac{36}{144} \times 100\% = 25\%$.

(b) Here, the base is $\frac{3}{4}$.

$\therefore$ required percentage = $\frac{7/8}{3/4} \times 100\ \% = 116\frac{2}{3}\%$.

(c) Here, the base is 80.

$\therefore$ required percentage = $\frac{16}{80} \times 100\% = 20\%$.

(d) Here, the base is $1\frac{7}{28}$

$\therefore$ required percentage = $\frac{0.625}{1\frac{7}{28}} \times 100 = 50\%$.

(e) Here, the base is 1400

$\therefore$ required percentage = $\frac{36 \times 14}{1400} \times 100 = 36\%$.

(f) Here, the base is N

$\therefore$ required percentage = $\frac{R}{N} \times 100 = \frac{100R}{N}$ %.

E-5 Find the following:
(a) 36 is 6 % of what?
(b) 2.5 is 5 % of what?
(c) 12 is 25 % of 20 % of what?
(d) $\frac{4}{7}$ is 24 % of what?

S-5 Refer 5.4.1, using equation (1),

$$\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (original)}$$

(a) required number = $\frac{36}{6\%} = \frac{36}{6} \times 100 = 600$.

(b) required number = $\frac{2.5}{5\%} = \frac{2.5}{5} \times 100 = 50$.

(c) required number = $\dfrac{12}{25\% \times 20\%} = \dfrac{12}{25 \times 20} \times 100 \times 100 = 240$

(d) required number = $\dfrac{4/7}{24\%} = \dfrac{4}{7 \times 24\%} = \dfrac{4}{7 \times 24} \times 100 = \dfrac{50}{21}$

E-6 25 % of a number is 20, what is 40 % of that number? Also find the number.

S-6 25 % $\rightarrow$ 20

$\Rightarrow$ 40 % $\rightarrow \dfrac{20}{25} \times 40 = 32$

Refer 5.4.1, using equation (1), the number = $\dfrac{\text{any value}}{\text{its rate \%}} = \dfrac{20}{25\%}$

$\Rightarrow$ required number = $\dfrac{20}{25} \times 100 = 80$

E-7 p_1 % of number N_1 is equal to p_2 % of number N_2. Find what per cent of N_1 is N_2?

S-7 It is required to find what per cent of N_1 is N_2, i.e. the base is N_1

$\therefore$ required percentage = $\dfrac{N_2}{N_1} \times 100\%$.

It is given that $\dfrac{p_1}{100} \times N_1 = \dfrac{p_2}{100} \times N_2 \quad \therefore \dfrac{N_2}{N_1} = \dfrac{p_1}{p_2}$

Putting the value of $\dfrac{N_2}{N_1}$, we find the required percentage = $\left(\dfrac{p_1}{p_2} \times 100\right)\%$

$\therefore$ N_2 is equal to $\left(\dfrac{p_1}{p_2} \times 100\right)\%$ of N_1

E-8 A number A exceeds B by 25%. By what per cent is B short of A?

S-8 Refer 5.12

$$\% \text{ Short} = \frac{\% \text{ excess}}{(100 + \% \text{ excess})} \times 100$$

$$= \frac{25}{(100 + 25)} \times 100 = \mathbf{20}$$

E-9 A number X is short of Y by 40%. By what per cent is Y in excess of X?

S-9 Refer 5.12. Using the formula,

$$\% \text{ Excess} = \frac{\% \text{ short}}{100 - \% \text{ short}} \times 100$$

$$= \frac{40}{100 - 40} \times 100 = \mathbf{66\frac{2}{3}}\ \%.$$

E-10 The ratio of salary of a worker in July to that in June was $2\frac{1}{2} : 2\frac{1}{4}$. By what % was the salary of July more than salary of June? Also find by what %, salary of June was less than of July.

S-10 Let Salary of July $= \frac{5}{2}x$

and Salary of June $= \frac{9}{4}x$

Refer 5.5, here the basis of comparison is either the salary of June or the salary of July.
Salary of July more than that of June by per cent

$$= \frac{\text{Difference}}{\text{Salary of June}} \times 100$$

$$= \frac{\left(\frac{5}{2} - \frac{9}{4}\right)x}{\frac{9}{4}x} \times 100 = \mathbf{11\frac{1}{9}\ \%}$$

Salary of June less than that of July by per cent

$$= \frac{\text{Difference}}{\text{Salary of July}} \times 100 = \frac{\left(\frac{5}{2} - \frac{9}{4}\right)x}{\frac{5}{2}x} \times 100 = \mathbf{10\%}.$$

E-11 In 1987, the enrolment in a school was 1,500. Next year it increased by 10 %. What was the enrolment in 1988?

S-11 Enrolment in 1988 = $1500 \times \frac{110}{100}$ = **1,650.**

E-12 The side of a square increases by p%, then find by what % does its area increase?

S-12 Suppose, side of a square = b
Original area of the square = b^2, i.e. result = $A \times B$
Here, both sides are increased by p%,
So, for finding out the % change in area, Refer formula (2A).

Net % change in area = $x + y + \frac{xy}{100}$, where, $x = y = +\ p$

$\Rightarrow$ Net % change in area $= p + p + \frac{p^2}{100}$

$$= 2p + \frac{p^2}{100}$$

Hence area increases by $\left(2\boldsymbol{p} + \frac{\boldsymbol{p}^2}{\mathbf{100}}\right)$ **%**.

Note: This formula is also applicable when the radius of a circle is increased by $p\%$.

Then its area increases by $\left[2p + \left(\frac{p}{10}\right)^2\right]\%$.

E-13 The daily wage is increased by 15 %, and a person now gets Rs 23 per day. What was his daily wage before the increase?

S-13 Original daily wage $= \dfrac{\text{Increased daily wage}}{100 + \%\text{ increase}} \times 100$ *[Refer 5.8]*

$$= \frac{23}{115} \times 100 = \mathbf{20}$$

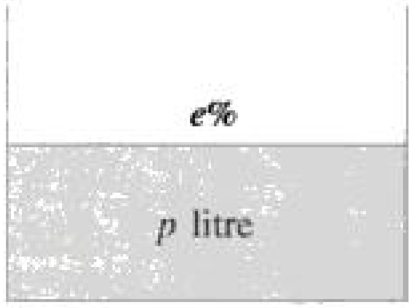
NB: → *In case of decrease use* (–) *ve sign*, before % value.

E-14 'p' litres of oil was poured into a tank and it was still e % empty. How much oil must be poured into the tank in order to fill it to the brim. Find the capacity of the tank.

S-14 Suppose, 'x' litres are to be poured to fill the tank completely, then, $x = e\%$ of capacity

Now, $p + e\%$ of capacity = full capacity $\Rightarrow p + \frac{e}{100} \times$ capacity = capacity

$\Rightarrow$ capacity $= \dfrac{p \times 100}{100 - e}$



$\therefore\ x = e\%$ of capacity $= \dfrac{e}{100} \times \dfrac{p \times 100}{100 - e}$ litres

$x = \dfrac{pe}{100 - e}$ litres = amount of oil to be poured to fill the tank completely.

E-15 A student X passes his examination with 515 marks, having scored 3% above the minimum. If Y had obtained 710 marks, what % would he have been above the minimum?

S-15 $\dfrac{\text{Marks of Y}}{\text{Marks of X}} = \dfrac{100 + \%\text{ above minimum of Y}}{100 + \%\text{ above minimum of X}}$

$$\frac{710}{515} = \frac{100 + Y}{100 + 3} \Rightarrow Y = +\ 42\ \%$$

Hence Y gets 42% above minimum.

Note: Similarly, if the % marks is below minimum, formula would have been,

$$\frac{\text{Marks of Y}}{\text{Marks of X}} = \frac{100 - \%\text{ below minimum of Y}}{100 - \%\text{ below minimum of X}}$$

Remember,

$$\frac{\text{Marks of Y}}{\text{Marks of X}} = \frac{100 \pm \%\text{ above/below minimum of Y}}{100 \pm \%\text{ above/below minimum of X}}$$

Here, (+) ve sign indicates above minimum and (–)ve sign indicates below minimum.

E-16 If a is x more then b and b is $y\%$ less than a, find the relation between x and y.

S-16 *[Refer 5.12]*

$$y\% = \frac{x}{100+x} \times 100\%$$

$$\Rightarrow \quad y = \frac{x \times 100}{100+x} \Rightarrow \frac{100+x}{100 \times x} = \frac{1}{y}$$

$$\Rightarrow \quad \frac{1}{y} - \frac{1}{x} = \frac{\mathbf{1}}{\mathbf{100}}.$$

NB: *y and x represent only the numerical value, i.e. if a is 3% more than b put only x = 3.*

E-17 The ratio of number of boys and girls in a school is 3 : 2 if 20% of the boys and 25% of the girls are holding scholarship, find the % of school students who

(a) hold scholarship

(b) do not hold scholarship

S-17 Percentage of scholarship holders

= (Boys × % boys who are scholarship holders) + (Girls × % girls who are scholarship holders)

$$= \left(\frac{3}{2+3} \times 20\right) + \left(\frac{2}{3+2} \times 25\right) = 22.$$

Similarly, percentage of non scholarship holders = $\left(\frac{3}{2+3} \times 80\right) + \left(\frac{2}{2+3} \times 75\right) = 78$

(Since 100 – 20 = 80, 100 – 25 = 75)

E-18 Groundnut oil is now being sold at Rs 27 per kg. During last month its cost was Rs 24 per kg. Find by how much % a family should reduce its consumption, so as to keep the expenditure the same.

S-18 New rate = Rs 27/kg

Original rate = Rs 24/kg.

∴ Change in rate = 27 – 24 = Rs 3

Here, the % change in consumption can be found out directly without finding the % change in rate by the following short-out method.

Short-cut: Since, % change in rate = $\frac{\text{rate change}}{\text{old rate}} \times 100$

So, % change in consumption = $\frac{-\text{rate change}}{\text{New rate}} \times 100$ (observe the difference in the denominator of these two formula)

$$= \frac{-3}{27} \times 100\ \% = -11\frac{1}{9}\ \%$$

Hence, family has to reduce its consumption by $\mathbf{11\frac{1}{9}}$ %.

E-19 A reduction of Rs 2 per kg enables a man to purchase 4 kg more sugar for Rs 16. Find the original price of sugar.

S-19 Here expenditure if fixed (= Rs 16), but as the rate **reduces** (by Rs 2/kg), so, the quantity of sugar available **increases** (by 4 kg). Let original price be Rs x/kg, using the formula 3. *[Refer 5.11]*

$$\frac{\text{Expenditure}}{x} + \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$$\Rightarrow \frac{16}{x} + 4 = \frac{16}{x-2} \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x = 4)(x - 2) = 0 \Rightarrow x = 4 \text{ or } -2$$

Considering the +ve value, original price = **Rs 4 per kg**.

E-20 If the price of coffee is increased by 3%, by how much % must a housewife reduce her consumption of coffee, so as to have no extra expenditure?

S-20 Refer 5.10.1

For fixed expenditure, % change in consumption = $\frac{\%\text{ change in rate}}{100 + \%\text{ change in rate}} \times 100$

$\therefore$ decrease in consumption = $\frac{100 \times 3}{100 + 3}\% = \frac{300}{103}\%$.

$\therefore$ % decrease in consumption = $2\frac{94}{103}\%$

The housewife must reduce her consumption of coffee by $\mathbf{2\frac{94}{103}}$ **%.**

E-21 If from a man's salary of Rs 'S', p% is deducted on house rent, k% of the rest on education and q% of the rest on food, and still there is balance Rs 'b' left, find the value of 'S'.

S-21 Here we know that this is a case of **dependent activity**, also refer the chapter on Fraction.

$\therefore$ $(1 - p\,\%) \times (1 - k\,\%) \times (1 - q\,\%)$ of $S = b$.

$$\Rightarrow S = \frac{b}{(1 - p\,\%) \times (1 - k\,\%) \times (1 - q\,\%)}.$$

E-22 From a man's salary, 10% is deducted on tax, 20% of the rest is spent on education, and 25% of the rest is spent on food. After all these expenditures, he is left with Rs 2,700. Find his salary.

S-22 Using the relation of E-21, we get,

$$\frac{(100 - 10)}{100} \times \frac{(100 - 20)}{100} \times \frac{100 - 25}{100} \times \text{salary} = 2700$$

$$\frac{90}{100} \times \frac{80}{100} \times \frac{75}{100} \times \text{salary} = 2700$$

$$\therefore \text{ Salary} = \frac{2700}{90 \times 80 \times 75} \times 100 \times 100 \times 100 = \mathbf{5000.}$$

E-23 When the price of sugar was increased by 32%, a family reduced its consumption in such a way that the expenditure on sugar was only 10% more than before. If 30 kg were consumed per month before, find the new monthly consumption.

S-23 We know that, expenditure = price × consumption.

So, new expenditure becomes 110% of 30

$$\Rightarrow \frac{110}{100} \times 30 = \frac{132}{100} \times \text{new consumption} \qquad \text{[Expenditure increases by 10\%]}$$

$\therefore$ New consumption = **25kg.**

E-24 A man's income is increased by Rs 1, 200 and at the same time, the rate of tax to be paid is reduced from 12% to 10%. He now pays the same amount of tax as before. What is his increased income if 20% of his income is exempted from tax in both cases?

S-24 Since, the same percentage of his income is exempted from tax in both cases, this data is not to be considered.

Now, let x be the increased income.

Amount of Tax_1 = Amount of Tax_2

$(x - 1200) \times 12\% = x \times 10\%.$

x = **Rs 7,200 = income after increase.**

E-25 When N is reduced by 4, it becomes 80% of itself. What is the value of N?

(Bank P.O. '91)

S-25 The problem implies that (100 – 80) % of N = 4

$$20\% \text{ of } N = 4 \quad N = \frac{4}{20} \times 100 \qquad (\text{Since } 100 - 80 = 20)$$

$\therefore$ N = **20.**

E-26 When N is increased by 6, it becomes 102% of itself. What is the value of N? **(IA '84)**

S-26 The problem implies that

(102 – 100) % of N = 6

$$\Rightarrow \quad 2\% \text{ of } N = 6 \Rightarrow N = \frac{6 \times 100}{2}$$

$\therefore$ N = **300.**

E-27 Increase

(i) 200 by 60 %

(ii) 11 by 100 %

(iii) 35 by 200 %

(iv) 48 by $12\frac{1}{2}$ %

(v) 1000 by 3.5 %

S-27 If a number (N) is to be increased by p% then multiply N by $\frac{100+p}{100}$ *[Refer 5.8]*

(i) So, 200 becomes $200 \times \frac{(100+60)}{100}$, i.e. **320.**

(ii) 11 increases to $11 \times \frac{100+100}{100}$, i.e. **22.**

(iii) Increased number $= N \times \frac{100+p}{100}$

$$= 35 \times \frac{100+200}{100} = \mathbf{105.}$$

(iv) Increased number $= N \times \dfrac{100+p}{100}$

$$= 48 \times \frac{100+12\frac{1}{2}}{100}$$

$$= 48 \times \frac{225}{200}$$

$$= 48 \times \frac{9}{8} = \mathbf{54}.$$

(v) Increased number $= N \times \dfrac{100+p}{100}$

$$= 1000 \times \frac{100+3.5}{100}$$

$$= 10 \times 103.5 = \mathbf{1{,}035}.$$

E-28 Decrease

(i) 200 by 40 %

(ii) 16 by 25 %

(iii) 216 by $37\frac{1}{2}$ %

(iv) 300 by 2.5 %

(v) 1000 by $\frac{1}{5}$ %

(vi) 50 by $12\frac{1}{2}$ %

S-28 In order to decrease a number N by p%, multiply N by $\dfrac{100-p}{100}$. *[Refer 5.11]*

(i) Decreased number $= N \times \dfrac{100-p}{100}$

$$= 200 \times \frac{100-40}{100} \quad \mathbf{120}.$$

(ii) Decreased number $= N \times \dfrac{100-p}{100}$

$$= 16 \times \frac{100-25}{100} = \mathbf{12}.$$

(iii) Decreased number $= N \times \dfrac{100-p}{100}$

$$= 216 \times \frac{100-37\frac{1}{2}}{100}$$

$$= 216 \times \frac{62\frac{1}{2}}{100}$$

$$= 216 \times \frac{125}{200}$$

$$= 216 \times \frac{5}{8} = \mathbf{135}.$$

(iv) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 300 \times \frac{100 - 2.5}{100}$$

$$= 3 \times 97.5 = \mathbf{292.5}.$$

(v) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 1000 \times \frac{100\frac{1}{5}}{100}$$

$$= 10 \times \frac{499}{5} = \mathbf{998}.$$

(vi) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 50 \times \frac{100 - 12\frac{1}{2}}{100}$$

$$= 50 \times \frac{175}{200}$$

$$= 50 \times \frac{7}{8} = \mathbf{43.75}.$$

E-29 If 10% of an electricity bill is deducted, Rs 45 is still to be paid. How much was the bill?

S-29 Here Rs 45 refers to (100 – 10) % of the bill.

Since 90% of bill = 45

$\Rightarrow$ 100% of bill $= \dfrac{45}{90} \times 100 = 50$

Hence the bill was 50.

E-30 The weight of a sand bag is 40 kg. In a hurry, it was weighed as 40.8 kg. Find the error percentage. **(Bank PO '89)**

S-30 $\%\ \text{Error} = \dfrac{\text{False weight} - \text{Actual weight}}{\text{Actual weight}} \times 100$

$$= \frac{40.8 - 40}{40} \times 100 = 2\%$$

$\therefore$ The error is **2%**.

E-31 If the price of one kg of cornflakes is increased by 25%, the increase is Rs 10. Find the new price of cornflakes per kg. **(C.D.S '86)**

S-31 Using the formula (1)

$$\text{Original price} = \frac{\text{Difference in price}}{\text{Difference in per cent}} \times 100$$

$$= \frac{10}{25} \times 100 = 40$$

$$\therefore \text{ New price} = 40 \times \frac{125}{100} = 50$$

$\therefore$ New price of cornflakes per kg is **Rs 50**.

E-32 The price of a book is reduced by 25%, what is the ratio of

(i) change in price to the old price?

(ii) old price to the new price?

Also find

(iii) the factor by which the old price should be multiplied to give the new price.

S-32 Consider the old price = Rs 100.

(i) $\dfrac{\text{Change in price}}{\text{Old price}} = \dfrac{25}{100} = \mathbf{\dfrac{1}{4}}$

(ii) $\dfrac{\text{Old price}}{\text{New price}} = \dfrac{100}{75} = \mathbf{\dfrac{4}{3}}$

(iii) Refer 5.8

$$\text{Decreased price (new price)} = \text{old price} \times \frac{100 - p}{100}, \text{ where } p = 25$$

$$\text{New price} = \text{old price} \times \left(\frac{75}{100}\right)$$

So the required factor is $\mathbf{\dfrac{3}{4}}$.

E-33 If A is more than B by 10%, then find

(i) $B = A \times ?$

(ii) $A = B \times ?$

(iii) $\dfrac{A}{B} = ?$

S-33 Since A is more than B by 10%,

(i) So $A = B \times \dfrac{100+10}{100}$

$$\Rightarrow \quad \mathbf{B = A \times \frac{10}{11}}.$$

(ii) $\mathbf{A = B \times \dfrac{11}{10}}$.

(iii) $\dfrac{A}{B} = \dfrac{100+10}{100}$

$$\therefore \quad \frac{A}{B} = \mathbf{\frac{11}{10}}.$$

E-34 If X is 20% less than Y, then find

(i) $\dfrac{X}{Y} = ?$

(ii) $\dfrac{Y-X}{Y} = ?$

(iii) $\dfrac{X}{Y-X} = ?$

S-34 Since X is 20% less than Y, therefore

(i) $\dfrac{X}{Y} = \dfrac{100-20}{100}$

$$\frac{X}{Y} = \mathbf{\frac{4}{5}}.$$

(ii) X is 20% less than Y, therefore, if Y is 100 then X is 80

$$\Rightarrow \frac{Y-X}{Y} = \frac{100-80}{100}$$

$$\therefore \frac{Y-X}{Y} = \mathbf{\frac{1}{5}}.$$

(iii) $\dfrac{X}{Y-X} = \dfrac{80}{100-80}$

$$\frac{X}{Y-X} = \mathbf{\frac{4}{1}}.$$

E-35 If $2\frac{1}{2}$ % of the weight of a table is 0.2 kg, then what will be 120% of it? **(C.D.S '83)**

S-35 $2\frac{1}{2}$ % of weight = 0.2 kg (use the unitary method)

$$\Rightarrow \quad 120\% = \frac{0.2}{2\frac{1}{2}} \times 120 = \frac{0.2 \times 120}{5} \times 2 = 9.6$$

$\therefore$ 120% of the weight of the table is 9.6 kg.

REGULAR PROBLEMS

(1) The number 0.05 is what percentage of 20?
(a) 1.5 (b) 0.025 (c) 0.25 (d) 2.5 (e) 25

(2) 12 is 0.2% of?
(a) 2400 (b) 600 (c) 240 (d) 6000 (e) 24

(3) In an election, one of the two candidates got 30% of the total votes polled, but he lost by 210 votes. What was the total number of votes polled?
(a) 525 (b) 700 (c) 610 (d) 300 (e) 520

(4) There are 850 students in a class. Out of these, 44% are Muslims, 28% Hindus, 10% Sikhs and remaining students belong to the other communities. How many students are there of other communities? **(RRB Secunderabad, '01)**
(a) 173 (b) 143 (c) 153 (d) 163 (e) 133

(5) The price of an article is cut by 10%, to restore to its original value, the new price must be increased by:
(a) 10% (b) $9\frac{1}{11}$% (c) 11% (d) $11\frac{1}{9}$% (e) 90%

Hint: Use the concept: if A is x% less than B, then B exceeds A by $\frac{x}{100-x}$%
(A ↓ new price; B ↓ original price)

(6) The difference between one-fifth of 1000 and one-fifth per cent of 1000 is:
(a) 800 (b) 80 (c) 198 (d) 998 (e) 0

(7) A man spends 80% of his income and saves the rest. What percentage of his expenditure does he save?
(a) 20 (b) 25 (c) 30 (d) 40 (e) Data insufficient

(8) 15% of x subtracted from x is equal to multiplying x by which number?
(a) 0.15 (b) $\frac{23}{20}$ (c) 115 (d) $\frac{17}{20}$ (e) $\frac{0.85}{100}$

(9) A pudding is made of 400 gm sugar, 200 gm of eggs, 800 gm of flour and 100 gm of dry fruit. The percentage of sugar present in the pudding is :
(a) $11\frac{1}{9}$% (b) 40% (c) 20% (d) $26\frac{2}{3}$% (e) $12\frac{1}{2}$%

(10) If 16% of 40% of a number is 8 then the number is
(a) 200 (b) 225 (c) 125 (d) 325 (e) 512

(11) The radius of a circle is so increased that its circumference increases by 5%. Then the area of the circle will increase by:
(a) 10% (b) 25% (c) 10.25% (d) 12.5% (e) 5%

Hint: Refer 5.10

(12) 40% of the population of a town are men and 35% are women. If the number of children are 20000, then the number of men will be:
(a) 3200 (b) 80000 (c) 32000 (d) 3,20,000. (e) 2,00,000.

(13) A medical student has to secure 40% marks to pass. He gets 80 and fails by 60 marks. Find the maximum marks.
(a) 150 (b) 250 (c) 350 (d) 450 (e) 500

Hint: Refer 5.4.1. Here, maximum marks is the (base) number, because % pass marks is related as 40% of maximum marks

(14) 40% of the students of a college are from West Bengal and out of this, 40% are from Kolkata. What % of the students are not from Kolkata?
(a) 60 (b) 16 (c) 40 (d) 84 (e) 20

(15) A school has a student population of 560. The number of girls is $14\frac{2}{7}$ % of the number of boys. How many girls are in the school?
(a) 100 (b) 70 (c) 80 (d) 140 (e) 240.

Hint: $14\frac{2}{7}\% = \frac{1}{7}$

(16) A man saves $3\frac{1}{3}\%$ from his salary of Rs 750 every month. In how many months will he be able to save an amount equal to his monthly salary?
(a) 40 months (b) 50 months (c) 30 months (d) 45 months (e) None of these

(17) In an examination, '*P*' scored 130 points, which are 10 points above 40%, and *Q* scored 75%. The points scored by *Q* are
(a) 225 (b) 250 (c) 200 (d) 275 (e) 300

(18) When 75% of a number is added to 75, the result is the number again. The number is
(a) 150 (b) 300 (c) 100 (d) 450 (e) 350

(19) The percentage of total quantity represented by a 12° sector in a circle graph (pie diagram) is,
(a) 12% (b) 24% (c) $33\frac{1}{3}\%$ (d) $3\frac{1}{3}\%$ (e) 36%

Hint: Total quantity in a circle graph (pie diagram) = 360°

(20) If the price of sugar is increased by 7%, then by how much per cent should a housewife reduce her consumption of sugar, to have no extra expenditure
(a) 7 over 107% (b) 107 over 100% (c) 100 over 107%
(d) 7% (e) 93 over 100%.

Hint: Refer 5.10.1

(21) What is the total number of customers in a shop on a particular day, when 31% of them purchases on credit, and the number of those who do cash purchase exceeds the number of credit purchases by 247?
(a) 605 (b) 560 (c) 650 (d) 1650 (e) 620

(22) 5 out of 2250 parts of the earth is sulphur. The percentage of sulphur in the earth is
(a) 11 over 50% (b) 2 over 9 (c) 1 over 45 (d) 2 over 45 (e) None of these

(23) In Mathematics examination, a student scored 30% in the first paper of 180 marks. How much % marks should he score in the second paper of 150 marks if he is to get an overall percentage of at least 50%
(a) 20 (b) 74 (c) 30 (d) 65 (e) 70

Tips: Do not try to calculate the marks obtained in each paper, because here the % marks are used in the problem and also % marks is to be foundout. Total marks = 180 + 150 = 330, As per question, 50% (330) = 30% of 180 + x% of 150.

(24) Nagamani had a car to sell. Loknayak offered him a sum of money for the car that he refused as it was 13%, below its value. Loknayak then offerred Rs 450 more and the second offer was 5% more than the estimated value. What was the value of the car?
(a) Rs 3000 (b) Rs 2500 (c) Rs 3800 (d) Rs 2800 (e) None

Answers

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (d)
10. (c)	**11.** (c)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (b)
19. (d)	**20.** (a)	**21.** (c)	**22.** (b)	**23.** (b)	**24.** (b)			

REAL PROBLEMS

(1) Which number is 40% less than 360?
(a) 90 (b) 144 (c) 216 (d) 270 (e) 288

(2) p is 6 times that q. By what per cent is q less than p?
(a) 500 (b) $83\frac{1}{3}$ (c) 5 (d) 20 $16\frac{2}{3}$

(3) If 90% of A = 30% of B and $B = x\%$ of A, then the value of x is
(a) 600 (b) 800 (c) 900 (d) 300 (e) None

(4) If $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of a number is 12 then 30 per cent of the number will be **(BSRB, Chennai, 2000)**
(a) 48 (b) 64 (c) 54 (d) 42 (e) None of these

(5) A piece of cotton cloth 20 m long, shrinks by 0.5% after washing. The length of the cloth after washing is
(a) 19.5 m (b) 19.9 m (c) 19.95 m (d) 19.92 m (e) 19.05 m.

(6) The percentage by which 120 is to be diminished to get 90 is: **(RRB Kolkata, '01)**
(a) 30 (b) 25 (c) 20 (d) $33\frac{1}{3}$ (e) None of these

(7) Two numbers are 20% and 25% less than the third number. By how much per cent is the second number to be enhanced to make it equal to the first number?
(a) $6\frac{2}{3}$ (b) $6\frac{1}{4}$ (c) 25 (d) $33\frac{1}{3}$ (e) 20

Tips: When any number 'x' is related to two or more than two numbers in terms of percentage, then it is convenient to assume $x = 100$

(8) 2 over 3 is? per cent of 5 over 7. The value of (?) is
(a) 90 (b) 93.33 (c) 94 (d) 9 (e) None of these

(9) There is an increase of 30% in the production of milk chocolates in Amul Dairy in one month. If now it is 9100 milk chocolates per month, what was it one month ago?
(a) 13000 (b) 10300 (c) 8400 (d) 7000 (e) 11700

(10) A positive number is by mistake divided by 6 instead of being multiplied by 6. What is the % error on the basis of correct answer?
(a) 3 (b) 97 (c) 17 (d) 83 (e) 100

(11) When 30 per cent of a number is added to another number the second number increases to its 140 per cent. The second number = x% of the first number. The value of x is **(Bank of Baroda PO, '99)**

(a) 130 (b) 75 (c) $133\frac{1}{3}$ (d) $33\frac{1}{3}$ (e) 110

(12) If the length of a rectangle is increased by 20% and the breadth is reduced by 20%, what will be the effect on its area? **(Guwahati PO, '99)**

(a) 4% increase (b) 6% increase (c) 4% decrease (d) No change (e) None of these

(13) If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area? **(SBI PO, '99)**

(a) No change (b) 16% increase (c) 8% decrease (d) 16% decrease (e) None of these

(14) If two numbers are respectively 20% and 50% of the third number, then what % is the first number of the second?

(a) 30 (b) 70 (c) 40 (d) 30 (e) 50.

(15) In measuring the side of a square, an error of 5% in excess is made. The error percentage in the calculated area is

(a) $10\frac{1}{4}$ (b) $10\frac{3}{4}$ (c) $1\frac{3}{4}$ (d) 25 (e) 5

(16) A number exceeds by 40 when added by 20% of itself. The number is

(a) 200 (b) 60 (c) 80 (d) 320 (e) 120

(17) A rainy day occurs once in every 25 days. Half of the rainy days produce rainbows. The percentage of days having no rainbows is :

(a) 2 (b) $12\frac{1}{2}$ (c) 98 (d) $87\frac{1}{2}$ (e) 50

(18) In a class of 52 students, 25% are rich and others are poor. There are 20 females in the class, of whom 55% are poor. How many rich males are there in the class? **(NABARD, '96)**

(a) 13 (b) 4 (c) 39 (d) 2 (e) 28

(19) When any number is divided by 12, then dividend becomes $\frac{1}{4}$ of the other number. By how much per cent is first number greater than the second number? **(BSRB, Chennai, 2000)**

(a) 200 (b) 150 (c) 300 (d) data inadequate (e) None

(20) Naresh's monthly income is 30% more than that of Raghu. Raghu's monthly income is 20% less than that of Vishal. If the difference between the monthly incomes of Naresh and Vishal is Rs 800, what is the monthly income of Raghu? **(Baroda PO, '99)**

(a) Rs 16000 (b) Rs 20000 (c) Rs 12000 (d) Data inadequate (e) None

Hint: Since the difference of income has been given in respect of Naresh and Vishal. So, at first, find the relation of income for Naresh with Vishal and equate the difference.

(21) The rate for admission to an exhibition was Rs 5 and was later reduced by 20%. As a result, the sale proceeds increased by 44%.

The percentage increase in attendance was:

(a) 80 (b) 24 (c) 64 (d) 20 (e) None of these

Hint: Rate × attendance = sale proceeds

(22) An ore contains 40% mass impurity, while the metal extracted from this ore contains 4% impurity. How much metal will 24 tons of the ore yield?

(a) 10 tons (b) 20 tons (c) 9 tons (d) 12 tons (e) 15 tons

Tips: Since only pure part of the ore yields metal. So, equate the pure (quantity) part in the ore and pure part in the metal.

(23) At an examination in which maximum marks are 500, A got 10% less than B. B got 25% more than C. C got 20% less than D. If A got 360 marks, what percentage of marks was obtained by D?

(a) 60 (b) 72 (c) 55 (d) 80 (e) 40

Hint: Since D is the last in the % relation, so assume the marks obtained by $D = x$

Answers

1. (c)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (d)
10. (b)	**11.** (b)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (b)
19. (d)	**20.** (a)	**21.** (a)	**22.** (e)	**23.** (d)				

6

AVERAGE

6.1 INTRODUCTION

The idea of **average** is not new to us. We all are familiar with the following types of statements.

(i) The average runs scored by Sachin Tendulkar in a series is 72.

(ii) The average marks secured by Kana is 78%.

If a man earns Rs 40, Rs 50, Rs 56, Rs 46 and Rs 48 on five consecutive days of a week, then he earns a total of Rs (40 + 50 + 56 + 46 + 48) = Rs 240.

To find his average earning per day, his total earning is divided by the number of days, i.e.,

$$\text{Average} = \frac{240}{5} = \text{Rs } 48$$

Average earning does not mean that he earned Rs 48 everyday. But had he earned Rs 48 everyday, then his total earnings would have also been Rs 240 in 5 days.

Hence, to find the average of given quantities:

Step 1 The given quantities are added to get a **Sum**

Step 2 The **Sum** is divided by the **Number of items** to get the **Average.**

$$\therefore \quad \frac{\text{Sum of all the items}}{\text{Number of items}} = \text{Average} \qquad \textbf{(1)}$$

Note: The average is also called the **Mean.**

The quantities, whose average is to be determined, *should be in the same unit.*

Hence, $\boxed{\text{Sum of all the items} = \text{Average} \times \text{no. of items}}$ **(2)**

6.2 AVERAGE OF DIFFERENT GROUPS

Sometimes, the average of two different groups are known and the average of a third group (made by combining these two groups) is to be found out.

Let,

	Group 1	+	Group 2	makes	Combined Group (1 + 2)
No. of items =	m		n		$m + n$
Average =	a		b		A
Sum of all items =	ma		nb		$ma + nb$

Therefore, average of combined group = $\dfrac{\text{Sum of all items}}{\text{No. of items}}$

$$\boxed{A^* = \frac{ma + nb}{m + n}} \qquad \textbf{(3)}$$

*This formula is also applicable for more than two groups forming the combined group.

Example: The average weight of 17 girls is 20 kg and that of 23 boys is 22 kg. Find the average weight of the class.

Solution:

	Girls	Boys
No. in the class =	17	23
Average =	20	22

∴ average weight of the class = $\dfrac{17 \times 20 + 23 \times 22}{17 + 23}$

= 21.15 kg.

6.3 ADDITION OR REMOVAL OF ITEMS AND CHANGE IN AVERAGE

Since, Average = $\dfrac{\text{Sum of the items}}{\text{Number of items}}$

So, the original average may change (increase/decrease), if number of items change. The number of items may change in the following two cases,

Case I

When one or more than one NEW items are added

Let the average of N items = A

Now, 'n' New items are **added** and the average increases or decreases by x, then

$$\text{Average of New items added} = A \pm \left(1 + \frac{N}{n}\right)x \qquad \textbf{(4)}$$

Use (–), when average decreases
(+), when average increases.

When only one New item is added, put $n = 1$, then

Value* of the New item added = $A \pm (N + 1)x$

Case II

When one or more than one items are removed

In this case, items are removed, so on placing $-\dfrac{N}{n}$ in place of $+\dfrac{N}{n}$ in formula (4), it becomes,

* Here, it deals with ONE item only, so, average has got no meaning and thus, 'average' is replaced by 'value' of that ONE item.

$$\boxed{\text{Average of items removed} = A \pm \left(1 - \frac{N}{n}\right)x}$$

Use (–), when average decreases
(+), when average increases **(5)**

When only ONE item is removed, put $n = 1$, then

Value* of the item removed = $A \pm (1 - N)x$

Example: The average age of 40 students in a class is 15 years. When 10 new students are admitted, the average is increased by 0.2 year. Find the average age of the new students.

Solution: Here, 10 students are added, and average increases by 0.2 year. Therefore, using formula (4),

$$\text{Average age of new students} = A + \left(\frac{N}{n} + 1\right)x$$

$$= 15 + \left(\frac{40}{10} + 1\right) 0.2$$

$$= 15 + 1$$

$$= 16 \text{ years.}$$

Example: The average salary of 15 teachers is Rs 4500 per month. Three teachers left the school and the average salary of the remaining teachers dropped by Rs 175. Find the total salary of the teachers who left the school.

Solution: Here, 3 teachers have been removed from the group, and the average salary dropped by Rs 175. Therefore, using formula (5),

$$\text{Average salary of the teachers who left} = A - \left(1 - \frac{N}{n}\right)x$$

(–) sign, because average decreases

$$= 4{,}500 - \left(1 - \frac{15}{3}\right)(175)$$

$$= 4{,}500 - (-4)(175)$$

$$= \text{Rs } 5{,}200$$

Hence, total salary of 3 teachers who left the school = $3 \times 5{,}200$

= Rs 15,600

6.4 REPLACEMENT OF SOME OF THE ITEMS

Sometimes, when a number of items of a group are removed and these are replaced with equal number of different items, then the average of the group changes, (increases or decreases) by x.

Let there are N items in the group, then

$$\boxed{\text{Sum of New items added} - \text{Sum of removed items} = \pm Nx} \quad \textbf{(6)}$$

(–)ve, when average decreases
(+)ve, when average increases

Example: When a man weighing 80 kg is replaced by another man in a group of five persons, the average weight decreases by 3 kg. What is the weight of new man?

Solution: Using formula (6),

Weight of new man – Weight of removed man = $-Nx$ (–ve, average decrease)

$\Rightarrow$ Weight of new man – 80 = -5×3

$\Rightarrow$ Weight of new man = 80 – 15

= 65 kg.

6.5 SOME PROBLEM-SPECIFIC FORMULAE

(i) Before 't' years, the average age of 'N' members of a family was 'T' years. If the average remains same even after one more member joins the family, then present age of new member = $T - Nt$.

Example: Four years ago, the average age of six members of a family was 26 years. On the birth of a child in the family, the average remains the same. Find the present age of the child.

Solution: Present age of the child = $26 - 6 \times 4$

= 2 years.

(ii) Out of the given numbers, if the average of first 'n' numbers is 'x' and that of last 'n' numbers is 'y', then

First number – last number = $n(x - y)$.

Example: The average temperature of June, July and August was 31°C. The average temperature of July, August and September was 30 °C. If the temperature of June was 29 °C, find the temperature of September.

Solution: In the given problem, four months have been indicated, i.e.

June,	July,	August,	September
1 (first)	2	3	4 (last)

Out of these, the average temperature of first three (n = 3) months = 31°C = x

and the average temperature of last three (n = 3) months = 30 °C = y

Then, by the formula in (ii)

Temperature of first month – temperature of last month = $n(x - y)$

$\Rightarrow$ temperature of June – temperature of September = 3(31 – 30).

$\Rightarrow$ 29 – temp. of September = 3

$\therefore$ Temperature of September = 26 °C

Solved Examples

E-1 The average age of students in section A of 40 students is 10 years and the average age of students in section B of 30 students is 12 years. Find the average age of students in both sections taken together.

S-1 Here, average of 40 + 30 students is to be found out. Refer 6.2

$$\text{Average} = \frac{ma + nb}{m + n} = \frac{40 \times 10 + 30 \times 12}{(40 + 30)} = \frac{760}{70} = 10.855 \text{ years}$$

$\therefore$ Average age of all the students = 10.86 years.

E-2 The average of 5 quantities is 6. The average of three of them is 4. What is the average of remaining two quantities?

S-2 Let the average of two quantities be x,
then as per question,

$$6 = \frac{3 \times 4 + 2 \times x}{5}$$

$\Rightarrow$ $x = 9$

$\therefore$ required average = 9.

E-3 30 pens and 75 pencils were purchased for Rs 510. If the average price of a pencil was Rs 2.00, find the average price of a pen.

S-3 [Refer 6.1], using the formula (2),
Average of quantities × Number of quantities = Sum of quantities
Here quantity is the cost of pen.
Let average price of pen be Rs y

$\therefore$ $30 \times y + 75 \times 2 = 510$ = Sum of the cost.

$\Rightarrow$ $y = 12$,

$\therefore$ Average price of pen = **Rs 12**.

E-4 The average age of A and B is 20 years. If C were to replace A, the average would be 19 and if C were to replace B, the average would be 21. What are the ages of A, B and C ? **(MBA '82)**

S-4 Say, a, b, c are the ages of A, B, and C
Since the average age of A and B is 20 years, so,

$$\frac{a+b}{2} = 20$$

$\Rightarrow$ $a + b = 40$ **(i)**

As per question, when C replaces A, average drops by $19 - 20 = -1$,
So, using formula (6), we get, (refer 6.4)

$c - a = -2 \times 1$ **(ii)**

Similarly, when C replaces B, average increases by $21 - 20 = +1$,

$c - b = 2 \times 1$ **(iii)**

Adding (i), (iii) and (iii), we get

$c = 20$,

Then from (ii), $a = 22$ and from (iii), $b = 18$

$\therefore$ Age of A = 22 yrs, age of B = 18 yrs, age of C = 20 yrs.

E-5 The average monthly expenditure of a family was Rs 2,200 during first 3 months, Rs 2,550 during next 4 months and Rs 3,120 during last 5 months of the year. If the total saving during the year was Rs 1,260, find average monthly income.

S-5 Total yearly income = yearly expenditure + yearly saving
$= [2200 \times 3 + 2550 \times 4 + 3120 \times 5] + 1260 =$ Rs 33,660

Average monthly income = $\frac{33660}{12}$ = **Rs 2,805**.

E-6 The average temperature on Tuesday, Wednesday and Thursday was 37 °C. The average temperature on Wednesday, Thursday and Friday was 38 °C. If the temperature on Friday was 39 °C, find the temperature on Tuesday.

S-6

	Average Temperature
Tue + Wed + Thurs	37 °C
Fri + Wed + Thurs	38 °C

It is same.

Here, Tuesday is replaced by Friday. So, using the relation 6 for *replacement of* one quantity only.
Replacing quantity – replaced quantity = Change in average × Number of quantity

$\therefore$ Temperature of Friday – Temperature of Tuesday = (+) 1 × 3

$\Rightarrow$ 39 – Temperature of Tuesday = + 3

$\Rightarrow$ Temperature of Tuesday = 39 – 3 = **36 °C**.

E-7 The average weight of 29 students in a class is 48 kg. If the weight of the teacher is included, the average weight rises by 500 g. Find the weight of the teacher.

S-7 Here, weight of the teacher added and final average of the group increases by 0.5 kg. Since, here only one item (i.e. weight of one teacher) has been added, so using the formula *[Refer 6.3]*

$\therefore$ weight of teacher $= A + (N + 1)\ x$

$= 48 + (29 + 1) \times 0.5 = 63$ kg

E-8 There are 50 boys in a class. Their average weight is 45 kg. When one boy leaves the class, the average reduces by 100 g. Find the weight of the boy who left the class.

S-8 Since here only one item (i.e. weight of boy who leaves the class) has been removed, so, using formula (5),

Value of one item removed $= A - (1 - N)\ x$ (here, average drops)

$\Rightarrow \therefore$ weight of boy who left $= 45 - (1 - 50) \times 0.1$ [100 gm = 0.1 kg]

$= 45 + 4.9$

$= 49.9$ kg.

$\therefore$ weight of boy who left the class is **49.9 kg.**

E-9 The average of 11 results is 50. If the average of first six results is 49 and that of last six results is 52, find the sixth result.

S-9 Average of 11 results

1	2	3	4	5	6	7	8	9	10	11

Average of last 6 results = 52

Average of first 6 results = 49

From the above diagram, it is quite obvious that the 'sixth result' is included twice, once in the first six results and second in the last six results.

$\therefore$ Value of the sixth result = (Sum of first six results) + (Sum of last six results) – (Sum of 11 results)
$= 6 \times 49 + 6 \times 52 - 11 \times 50 =$ **56.**

E-10 A batsman's runs just before the last match of the season, adds up to 750. In his last two innings, he scores only 6 runs, and his average drops by 2. Find his final average of the season.

S-10 For a batsman, average $= \dfrac{\text{Total runs scored}}{\text{Total number of innings playing}}$

Suppose,
Just before the last match, the batsman played "N" number of innings.

$\therefore$ Original average $A = \dfrac{750}{N}$

Using formula (4),

average of two innings added $= A - \left(1 + \dfrac{N}{n}\right)x$ [(–)ve, average drops]

$\Rightarrow \quad \frac{6}{2} = \frac{750}{N} - \left(1 + \frac{N}{2}\right) 2$

$\Rightarrow \quad N^2 + 5N - 750 = 0$

$\Rightarrow \quad N = 25$

$\therefore$ Total no. of innings played before the last match = 25

$\therefore$ Original average = $\frac{750}{25} = 30$

$\therefore$ Final average = 30 – 2 = **28 runs.**

E-11 The average weight of 15 students in a class is increased by 1.5 kg when one of the students weighing 40 kg is replaced by a new student. Find the weight of the new student.

S-11 Using formula (6), for one item replacement,

weight of new student – weight of removed student = $+Nx$

$\Rightarrow$ weight of new student – 40 = 15 × 1.5

$\therefore$ weight of new student = 40 + 22.5 = **62.5 kg**.

E-12 Find the average of

(a) 9 consecutive odd numbers $a, b, c, d, e, f, g, h, i$

(b) 7 consecutive even numbers k, l, m, n, o, p, q

(c) 6 consecutive odd numbers a, b, c, d, e, f

(d) 4 consecutive numbers j, k, l, m

S-12 ***(a)*** There are 9 (odd) consecutive odd numbers.

Hence, the average of 9 consecutive odd numbers is the middle number (i.e. 5th number)

$\therefore$ the average is 'e'.

(b) There are 7 (odd) consecutive even numbers.

The average of 7 consecutive even numbers is the middle number (i.e. 4th number)

$\therefore$ the average is 'n' .

(c) There are 6 (even) consecutive odd numbers.

The average of 6 consecutive odd numbers is the average of two middle numbers (i.e. 3rd and 4th number)

$\therefore$ the average is $\frac{c+d}{2}$.

(d) There are 4 (four) consecutive numbers.

The average of 4 consecutive numbers is the average of two middle numbers (i.e. 2nd and 3rd numbers)

$\therefore$ the average is $\frac{k+l}{2}$.

REGULAR PROBLEMS

(1) The mean of marks secured by 25 students in section A of Class X is 47, that of 51 students of section B is 51 and that of 30 students of section C is 53. Find the mean of marks of the students of three sections of Class X.

(a) 50 (b) 50.6 (c) 52.5 (d) 54.5 (e) 51.5

(RBI, '96)

(2) There are Indians and Europeans in an army of 12,000. The average height of Europeans is 5′10″ and that of Indians is 5′9″ and the average height of the entire army is $5'\ 9\frac{3''}{4}$. Find out the number of Indians in the army.

(RRB Ahmedabad, '97)

(a) 2500 (b) 3000 (c) 2800 (d) 2200 (e) 2000

(3) The average age of a family of five persons is 20 years. If the youngest member is 8 years old. What was the average age of the family at the birth time of the youngest member?

(a) 12 years (b) 15 years (c) 18 years (d) 16 years (e) 23 years

Hint: Find the total age of the family now and 8 years earlier.

(4) The average of five consecutive numbers is 16. The highest of these numbers is:

(a) 21 (b) 20 (c) 18 (d) 19 (e) 22

Hint: Assume the first number as x, then consecutive numbers are $x + 1$, $x + 2$, $x + 3$ and $x + 4$.

(5) The average of six numbers is 12. If each number is increased by 2, the new average is:

(a) 8 (b) 4 (c) 14 (d) 24 (e) 12

Hint: Since each number is increased by same amount, so, average will also increase by same amount.

(6) If the average of three numbers a, b and c is A, then the average of a, b, c and A is:

(a) Data insufficient (b) $2A$ (c) $\frac{A}{2}$ (d) $4A$ (e) A

(7) The average temperature of the first three days of a week is 27 °C and of the next three days is 29 °C. If the weekly average is 28.5°C, then the temperature of the last day is:

(a) 31.5 °C (b) 28 °C (c) 42 °C (d) 21 °C (e) 26 °C

(8) A batsman has a certain average of runs for 9 innings. In the 10th innings, he makes a score of 100 runs thereby increasing his average by 8 runs. His new average is:

(a) 32 (b) 36 (c) 25 (d) 40 (e) 28

Hint: While finding the average, always use the basic formula, i.e. average = $\frac{\text{Sum of all quantities}}{\text{No. of quantities}}$

(9) The average age of 24 boys in a class is 16. If the teacher is included and one boy is excluded from the group, the average increases by one. The age of the teacher is:

(a) 41 (b) 32 (c) 45 (d) 24 (e) Data insufficient

(10) The average weight of 35 students in a class is 35 kg. If the teacher is also included, the average weight increases to 36 kg. The weight of the teacher is:

(a) 36 kg (b) 35 kg (c) 70 kg (d) 71 kg (e) 34 kg

(11) The average number of printing errors per page in a book of 512 pages is 4. If the total number of printing errors in the first 302 pages is 1208, the average number of printing errors per page in the remaining pages is:

(a) 0 (b) 4 (c) 840 (d) 90 (e) 9

(12) Of the three numbers, the first is four times the second and three times the third. If the average of all the three numbers is 95, then the third number is:

(a) 57 (b) 76 (c) 38 (d) 60 (e) 130

(13) The total production of 10 tea estates is 550 tonnes. By opening two new tea estates, the average increases by 3 tonnes. The average production of these two new tea estates (in tonnes) is:

(a) 70 (b) 64 (c) 67 (d) 73 (e) 68

(14) The average age of 10 members in a family is 21 years and due to death of one family member, the average age is reduced by 2 months. The age of the member who died is:

(a) 20 years 10 months (b) $20\frac{1}{3}$ years (c) 22 years

(d) $19\frac{1}{3}$ years (e) None

(15) If a, b, c, d and e are five consecutive odd numbers, their average is:

(a) $5(a + 4)$ (b) $\frac{abcde}{5}$ (c) $5(a + b + c + d + e)$

(d) $a + 4$ (e) None of these

(16) The average of a batsman for 40 innings is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded, his average drops by 2 runs. His highest score is:

(a) 86 (b) 92 (c) 174 (d) 170 (e) 88

(17) The average of three numbers is 45. The first is as much more than the average as the second number is less than the average. Find the third number.

(a) 40 (b) 41 (c) 20 (d) 45 (e) 43

(18) A mathematics teacher tabulated the marks secured by 35 students of 8th class. The average of their marks was 72. If the marks secured by Reema was written as 36 instead of 86 then find the correct average marks up to two decimal places.

(a) 73.41 (b) 74.31 (c) 72.43

(d) 73.43 (e) cannot be determined

Hint: Correct average = 72 + $\frac{(86-36)}{35}$, –36, because 36 is to be replaced

(19) Among three numbers, the first is twice the second and thrice the third. If the average of three numbers is 49.50, then what is the difference between the first and the third number?

(a) 54 (b) 28 (c) 39.50 (d) 41.50 (e) 33

Answers

1. (b) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (e) **7.** (a) **8.** (e) **9.** (e)
10. (d) **11.** (b) **12.** (d) **13.** (d) **14.** (c) **15.** (d) **16.** (c) **17.** (d) **18.** (d)
19. (a)

REAL PROBLEMS

(1) The average of two numbers is XY. If one number is X, then the other number is:

(a) Y (b) $\frac{Y}{2}$ (c) $2XY - X$ (d) $X(Y - 1)$ (e) $\frac{XY}{2}$

(2) Of three numbers whose average is 60, the first is $\frac{1}{4}$th of the sum of other two. The first number is:

(a) 30 (b) 60 (c) 36 (d) 24 (e) 20

(3) Out of four numbers, the average of the first three is 15 and that of the last three is 16. If the last number is 19, then first number is:
(a) 15 (b) 16 (c) 18 (d) 19 (e) 22

Tips: 1 2 3 4

↓ ↓

15 16

(4) A cricket player makes 200 runs in the 15th over. In doing so, his average at the end of the 14th over increases by 10. What was his average at the end of 15th over? **(RRB Guwahati, '97)**
(a) 60 (b) 50 (c) 40 (d) 45 (e) 35

(5) The average of Suresh's marks in English and History is 55. His average marks in English and Science is 65. What is the difference between the marks that he obtained in History and Science? **(Baroda Bank PO, '97)**
(a) 40 (b) 60 (c) 20 (d) data inadequate (e) None

(6) A body weighs 121 grams and 125.44 grams on two different pans of a faulty balance. Its true weight will be:
(a) 4.44 gms (b) 120 gms (c) 123.22 gms (d) 130 gms (e) 122 gms

(7) The average of three consecutive odd numbers is 14 more than one-third of the first of these numbers. What is the last of these numbers?
(a) 17 (b) 19 (c) 15
(d) data inadequate (e) None of these

(8) Ten years ago, the average age of a family of four members was 24 years. Three children having been born, the average age of the family is same today. What are the present ages of children, if two children are identical twins and differ by two years from the younger one?
(a) 12, 12, 10 (b) 8, 8, 6 (c) 13, 13, 11 (d) 14, 14, 12 (e) None

(9) The captain of a cricket team of 11 players is 25 years and the wicket keeper is 3 years older. If the age of these two are excluded, the average age of the remaining players is 1 year less than the average age of the whole team. The average age of the whole team is
(a) 24 years (b) 21 years (c) 26 years (d) 22 years (e) 25 years

(10) What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the two fractions?
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$ (e) None of these

(11) In a coconut groove, $(x + 2)$ trees yield 60 nuts per year, x trees yield 120 nuts per year and $(x - 2)$ trees yield 180 nuts per year. If the average yield per year is per tree be 100. Then the value of x is
(a) 8 (b) 4 (c) 12 (d) 10 (e) 14

(12) If the average weight of boys of Ram's age and height is 105 kg, and if Ram weighs 110% of the average, then the weight of Ram is
(a) 105 kg (b) data insufficient (c) 115.5 kg
(d) 110 kg (e) 107.5 kg

(13) After a certain number of matches, a bowler has had 200 runs knocked off him. In the next match, he takes 3 wickets for 52 runs and his average goes up by 1. The new average of the bowler is
(a) 9 (b) $8\frac{1}{3}$ (c) $9\frac{1}{3}$ (d) 8 (e) either (a) or (c)

Hint: Bowler's average = $\frac{\text{runs knocked off}}{\text{no. of wickets taken}}$, Refer formula no. 4

(14) A batsman has scored an average of 46 runs for a certain number of innings played in England. When he came back to India, he played another two test matches of two innings each and scored at an average of 55 runs. For the innings in England and in India taken together, he has improved his average by 2 runs over the matches played in England. The number of innings played in England was

(a) 7 (b) 9 (c) 14 (d) 18 (e) 11

(15) A batsman's scores in a particular innings is twice the average score in 4 previous innings. The percentage improvement in his average score now is:

(a) 6 (b) 20 (c) 5 (d) 10 (e) 12

(16) If the average of a, b, c be M and $ab + bc = -ca$, then the average of a^2, b^2, c^2 is:

(a) M^2 (b) $3M^2$ (c) $9M^2$ (d) $27M^2$ (e) None of these

Hint: $a^2 + b^2 + c^2 + 2(ab + bc + ca) = (a + b + c)^2$

(17) Among three numbers, the first number is thrice the second number and one-fourth of the third number. The average of all three numbers is 64. What is the average of first and third number?

(a) 30 (b) 24 (c) 90 (d) 78 (e) 48

Tips: Since the first number is related to both the second and the third number in the given condition, so, it is better to assume the first number = x]

(18) The average of 'n' numbers is A. If the number Y is replaced by the number Y^0, then the average becomes A^0. Which of the following equations is true?

(a) $A = A^0 \dfrac{(Y - Y^0)}{n}$ (b) $A = A^0 + n(Y - Y^0)$ (c) $A^0 = \dfrac{Y}{n} + \dfrac{Y^0}{A}$

(d) $A^0 + \dfrac{Y}{n} = A + \dfrac{Y^0}{n}$ (e) $\dfrac{A}{A^0} = n + \dfrac{Y}{Y^0}$

Answers

1. (c)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (e)	**9.** (d)
10. (c)	**11.** (b)	**12.** (c)	**13.** (e)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (d)

7 RATIO AND PROPORTION

7.1 INTRODUCTION

(a) Ratio A ratio is a comparison of two numbers (quantities) by division. The ratio of a to b is written as,

$a : b = \frac{a}{b} = a \div b$

In the ratio $a : b$, a and b are called the terms of the ratio; 'a' is the antecedent 'b' is the consequent. A ratio is a number, so to find out the ratio of two quantities, they must be expressed in the same units.

(b) Proportion A proportion is an expression which states that two ratios are equal.

e.g. $\frac{3}{12} = \frac{1}{4}$ is a proportion. It can also be expressed as $3 : 12 = 1 : 4$ or $3 : 12 :: 1 : 4$.

Each quantity in proportion is called **term** or **proportional**. The first and the last terms are called the **extremes,** whereas the second and the third terms are called the **middle terms**. When four quantities are in proportion, the last quantity is said to be **fourth propartional** to the other three and also we find, **product of middle terms = product of extremes**

2nd term × 3rd term = 1st term × 4th term

e.g. In $4 : 8 = 12 : 24$, we have

$8 \times 12 = 4 \times 24$

(Thus, if $a : b = x : y$, then

$bx = ay$).

7.2 PROPERTIES OF RATIO

(a) In a ratio, **two** quantities are **compared**. So, the quantities must be of the same kind, i.e. they must be expressed in the same units.
(b) The ratio of two quantities determines how many times one quantity is contained by the other.
(c) The order of the terms in a ratio $a : b$ is very important. Since $4 : 5$ is different from $5 : 4$.

7.3 DIVIDING A GIVEN NUMBER IN THE GIVEN RATIO

Let 'A' be the given number. The given ratio is $a_1 : a_2$

Here 'A' is to be divided in the ratio $a_1 : a_2$.

It implies that A is divided in two parts such that value of first part: value of second part = $a_1 : a_2$. Therefore,

$$\text{first part} = \frac{a_1}{a_1 + a_2} \times A = \text{first term of ratio} \times \left(\frac{\text{Sum of parts}}{\text{Sum of terms of ratio}}\right)$$

$$\text{second part} = \frac{a_2}{a_1 + a_2} \times A = \text{Second term of ratio} \times \left(\frac{\text{Sum of parts}}{\text{Sum of terms of ratio}}\right)$$

Since, A has been divided into two parts, so, first part + second part = A.

Example: Two numbers are in the ratio 8:9. If the sum of the numbers is 119, find the numbers.

Solution: Since the sum of two numbers is 119, so, the problem implies that 119 is divided in two parts in the ratio 8:9.
Therefore,

$$\text{first number} = \frac{8}{8+9} \times 119 = 56$$

$$\text{second number} = \frac{9}{8+9} \times 119 = 63 \quad \text{or} \quad (119 - 56 = 63)$$

Note: These relations are also true for dividing a given number into more than two ratios (i.e. more than two parts). When any number A is divided in more than one ratio such as $a : b : c : d : \ldots$
then,

$$\text{value of any part} = \frac{\text{its related ratio term}}{a+b+c+\ldots} \times A$$

e.g. $$\text{third part} = \frac{c}{a+b+c+\ldots} \times A.$$

Example: Dividing Rs 3,200 among P, Q, R in the ratio 5:2:9, find the amount received by Q.

Solution: Amount received by $Q = \dfrac{\text{its related ratio term}}{\text{sum of ratio terms}} \times \text{Total amount}$

$$= \frac{2}{5+2+9} \times 3200$$

$$= \text{Rs } 400$$

7.4 COMPARISON OF RATIOS

Let $a : b$ and $c : d$ be two ratios. Then

$a : b > c : d$ if $ad > bc$

i.e. $\dfrac{a}{b} > \dfrac{c}{d}$ if $ad > bc$

Similarly,

$$a : b < c : d \quad \text{if } ad < bc$$

and

$$a : b = c : d \quad \text{if } ad = bc$$

In brief, when we want to compare two ratios, i.e. whether

1st ratio > , = , or < 2nd ratio, then, it is necessary only to check,

1st term of 1st ratio	×	2nd term of 2nd ratio	>, = , or <	1st term of 2nd ratio	×	2nd term of 1st ratio

Example: Compare 5:7 and 2:3

Solution: Given ratios are

5 : 7 and 2 : 3

(7 × 2)

2nd multiplication

1st multiplication

(5 × 3)

Here, $5 \times 3 > 7 \times 2$ *(1st multiplication > 2nd multiplication)*

So, $5 : 7 > 2 : 3$ *(1st ratio > 2nd ratio)*

Note: This method is an easier one to use.

7.4.1 Two Important Results

If $\frac{a}{b} > 1$, it is implied that $a > b$, 1st term > 2nd term

and $\frac{a}{b} < 1$, then $a < b$

Result 1

Addition of same positive integer to both terms of the ratio

If $\frac{a}{b} = 1$, then $\frac{a+x}{b+x} = \frac{a}{b} = 1$ (adding of x does not affect the ratio equal to 1)

If $\frac{a}{b} > 1$, then $\frac{a+x}{b+x} < \frac{a}{b}$ (adding of x decreases the ratio greater than 1)

If $\frac{a}{b} < 1$, then $\frac{a+x}{b+x} > \frac{a}{b}$ (adding of x increases the ratio lesser than 1)

Result 2

Subtraction of same positive integer from both terms of the ratio.

If $\frac{a}{b} = 1$, then $\frac{a-x}{b-x} = \frac{a}{b} = 1$ (Subtraction of x does not affect the ratio equal to 1)

If $\frac{a}{b} > 1$, then $\frac{a-x}{b-x} > \frac{a}{b}$ (subtraction of x increases the ratio greater than 1)

If $\frac{a}{b} < 1$, then $\frac{a-x}{b-x} < \frac{a}{b}$ (subtraction of x decreases the ratio lesser than 1)

Memory Tips: These two results can be easily memorised through the following line diagram:

When $\frac{a}{b} > 1$, = $+x$ ↓ ratio decreases; ratio increases ↑ $-x$

When $\frac{a}{b} < 1$, = ratio increases ↑ $+x$; $-x$ ↓ ratio decreases

Example: Consider the ratio $\frac{7}{5}$ (> 1)

Then, addition of a positive quantity, say 3,

$$\frac{7+3}{5+3} < \frac{7}{5}$$

Similarly, subtraction of a positive quantity, say 2

$$\frac{7-2}{5-2} > \frac{7}{5}$$

Example: Consider the ratio $\frac{4}{9}$ (< 1)

Then, addition of a positive quantity, say 5,

$$\frac{4+5}{9+5} > \frac{4}{9}$$

and subtraction of a positive quantity, say 2,

$$\frac{4-2}{9-2} < \frac{4}{9}$$

7.5 USEFUL RESULTS ON PROPORTION

As we have learnt in 7.1, four quantities a, b, c and d are said to be in proportion if and only if

$$a : b = c : d$$

$\Rightarrow$ $$a \times d = b \times c$$

product of extremes = product of middles

Now, we will see some useful results, when $\boxed{a : b = c : d \quad \text{or,} \quad \frac{a}{b} = \frac{c}{d}}$

then,

(i) $\frac{b}{a} = \frac{d}{c}$ (ratio gets inverted)

(ii) $\frac{a}{c} = \frac{b}{d}$

(iii) $\frac{c}{a} = \frac{d}{b}$

(iv) $\frac{a+b}{b} = \frac{c+d}{d}$ $\left(\text{By componendo: } \frac{a}{b}+ = \frac{c}{d}+\right)$

(v) $\frac{a-b}{b} = \frac{c-d}{d}$ $\left(\text{By dividendo: } \frac{a}{b}- = \frac{c}{d}-\right)$

(vi) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (By componendo and dividendo)

(vii) $\frac{a}{a-b} = \frac{c}{c-d}$

Note: Important

If $a : b = c : d$

$a \rightarrow$ 1st proportion, $b \rightarrow$ 2nd proportion, $c \rightarrow$ 3rd proportion, $d \rightarrow$ 4th proportion

7.6 CONTINUED PROPORTION

Three quantities a, b, c of the same kind are said to be in continued proportion, when $a : b = b : c$.

In this case, the **last number** (c, here) is said to be **third proportional** to other two numbers (a and b, here). The middle number (b, here) is said to be a mean proportional to two extreme numbers (a and c). So, in such case of continued proportion, $b^2 = ac$

$$(\text{middle number})^2 = \text{first number} \times \text{last number}$$

Example: Find the mean proportional between 9 and 16.

Solution: Required mean proportional $= \sqrt{9 \times 16}$

$= 12$

Example: If 3, x, 27 are in continued proportion, then find the value of x.

Solution: Since 3, x, 27 are in continued proportion, therefore,

(middle number)^2 = First number × last number

$\Rightarrow \quad x^2 = 3 \times 27$

$\Rightarrow \quad x = \sqrt{18}$

$\Rightarrow \quad x = 9$

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$$\text{and, second part} = \frac{\text{Second ratio term}}{\text{Sum of ratio terms}} \times \text{Total amount}$$

$$= \frac{5}{(4+5)} \times 72 = \textbf{Rs 40}.$$

E-9 The ratio of the number of boys and girls in a school is 2 : 5. If there are 350 students in the school, find the number of girls in the school. **(Bank P.O. '80)**

S-9 $\text{Number of girls} = \frac{\text{Ratio term for girl}}{\text{Total sum of ratio}} \times \text{Number of students}$

$$= \frac{5}{2+5} \times 350 = \textbf{250 girls}.$$

E-10 The ratio between two numbers is 3 : 7. If their L.C.M. is 210, find the numbers.

S-10 Let the numbers be $3x$ and $7x$

L.C.M. is $3 \times 7 \times x = 21x$

$\therefore$ $21x = 210 \Rightarrow x = 10$

$\therefore$ Numbers are **30 and 70**.

E-11 In a ratio which is equal to 3 : 7, if the antecedent is 33, what is the consequent?

S-11 $\frac{3}{7} = \frac{33}{x} \Rightarrow x = \mathbf{77}$.

E-12 The ratio between two numbers is 3 : 5. If each number is increased by 4, the ratio becomes 2 : 3. Find the number.

S-12 Let the numbers be $3x$ and $5x$,

$$\Rightarrow \frac{3x+4}{5x+4} = \frac{2}{3}$$

$\Rightarrow$ $x = 4$ $\therefore$ Numbers are **12 and 20**.

E-13 The ratio between two numbers is 12 : 13. If each number is reduced by 20, the ratio becomes 2 : 3. Find the numbers.

S-13 Let the numbers be $12x$ an $13x$.

$$\therefore \frac{12x-20}{13x-20} = \frac{2}{3}$$

$\Rightarrow x = 2$, $\therefore$ Numbers are **24, 26.**

E-14 What must be subtracted from each term of the ratio 3 : 7 so that the ratio becomes 2 : 5?

S-14 Here number, say x, is subtracted from each term of the ratio. Hence, by the given problem,

$$\frac{3-x}{7-x} = \frac{2}{5}$$

$\Rightarrow$ $x = 1$ $\therefore$ 1 is subtracted from each term of the ratio **3 : 7**

E-15 The ratio of number of ladies to gents at a party was 1 : 2, but when 2 ladies and 2 gents left, the ratio became 1 : 3. How many people were originally persent at the party?

S-15 Let the number of ladies = x, then

the number of gents = $2x$

As per question, $\frac{x-2}{2x-2} = \frac{1}{3} = x = 4$

$\therefore$ Total number of people originally present = $x + 2x = 3x =$ **12.**

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(9) A lady has 25 paise and 50 paise coins in her bag. She has a total of 120 coins amounting to Rs 50. What is the respective number of 25 and 50 paise coins in her bag?

(a) 40, 80 (b) 70, 50 (c) 90, 30 (d) 60, 60 (e) 56, 64

Hint: Since total no. of coins and total amount (both) are given, so by mentally checking each pair of alternatives after multiplying them by $\frac{1}{4}$ (= 25 paise) and $\frac{1}{2}$ (= 50 paise), we can find the solution without actually solving it.

(10) The marks obtained by Suresh in English, Mathematics and Science are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{3}{5}$. If his total score is 860, his marks in English is:

(a) 160 (b) 300 (c) 228 (d) 312 (e) None of these

(11) It rains cx cm in t minutes on the first day and $\frac{c}{2}$ cm, in $\frac{t}{x}$ minutes on the next day. The ratio of rainfall on 1st day and 2nd day is: **(ITI, '89)**

(a) $2x : 1$ (b) $1 : 2t$ (c) $c : 2x$ (d) $2 : 1$ (e) $x : 1$

(12) Half of one number is equal to 0.07 of another. The ratio of the numbers is:

(a) 50 : 7 (b) 5 : 7 (c) 7 : 50 (d) 1 : 14 (e) 2 : 7

(13) Find the ratio whose terms differ by 40 and the measure of which is $\frac{2}{7}$.

(a) 15 : 55 (b) 2 : 42 (c) 16 : 56 (d) 14 : 49 (e) 18 : 63.

(14) In a bag, 25 paise, 10 paise and 5 paise coins are in the ratio of 1 : 2 : 3. If their total value is Rs 30, then the number of 5 paise coins is:

(a) 50 (b) 100 (c) 150 (d) 200 (e) None of these

Hint: Here total value is known, so, convert the each type of coin in equivalent value in rupees.

(15) If $\frac{x}{y} = 2 : 3$ then $\frac{x^2}{3} \div y^2 = ?$

(a) $\frac{4}{9}$ (b) $\frac{4}{3}$ (c) $\frac{16}{9}$ (d) $\frac{4}{27}$ (e) $\frac{16}{27}$

(16) Two numbers are in the ratio $m : n$. The first number is (?) per cent of the second number.

(a) $\frac{m}{m+n} \times 100$ (b) $\frac{m}{m-n} \times 100$ (c) $\frac{n}{m} \times 100$ (d) $\frac{m}{n} \times 100$ (e) None

(17) A man divides his property so that ratio of his son's share to his wife's and the ratio of the wife's share to his daughter are both 3 : 1. If the daughter gets Rs 10,000 less than the son, then the total worth of his property is:

(a) Rs 25,000 (b) Rs 16,250 (c) Rs 65,000 (d) Rs 27,350 (e) None

Answers

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (a)
10. (b)	**11.** (a)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (b)	

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E-2 A started a bunsiness with a capital of Rs 10,000 and 4 months later, B joined him with a capital of Rs 5,000. What is the share of A in the total profit of Rs 2,000 at the end of the year?

S-2 $$\frac{\text{Profit of } A}{\text{Profit of } B} = \frac{\text{MEI of } A}{\text{MEI of } B} = \frac{\text{Amount} \times \text{No. of months}}{\text{Amount} \times \text{No. of months}}$$

$$= \frac{10,000 \times 12}{5000 \times 8} = \frac{3}{1}$$

$\therefore$ Profit share of $A = \frac{3}{3+1} \times 2000 =$ **Rs 1,500**

E-3 In a business, A, B, and C invested Rs 380, Rs 400 and Rs 420 respectively. Divide a net profit of Rs 180 among the partners.

S-3 A's profit : B's profit : C's profit = A's investment : B's investment : C's investment
= 380 : 400 : 420 = 19 : 20 : 21

Profit share of $A = \frac{19}{60} \times 180 =$ **Rs 57**

Profit share of $B = \frac{20}{60} \times 180 =$ **Rs 60**

Profit share of $C = \frac{21}{60} \times 180 =$ **Rs 63.**

E-4 A, B and C enter into a partnership. 'A' contributes Rs 320 for 4 months, 'B' contributes Rs 510 for 3 months, and 'C' contributes Rs 270 for 5 months. If the total profit is Rs 208, find the profit share of the partners.

S-4 A's profit : B's profit : C's profit = MEI of A : MEI of B : MEI of C
= 320 × 4:510 × 3:270 × 5
= 1280 : 1530 : 1350
= 128 : 153 : 135

$\therefore$ Profit of $A = \frac{128}{(128+153+135)} \times 208 = \frac{128}{416} \times 208 =$ **Rs 64**

Profit of $B = \frac{153}{416} \times 208 =$ **Rs 76.50**

Profit of $C = \frac{135}{416} \times 208 =$ **Rs 67.50**.

E-5 A, B and C enter into partnership with a total of Rs 8,200. A's capital is Rs 1,000 more than B's and Rs 2,000 less than C's. What is B's share of the year's profit of Rs 2,460?

S-5 Given $A = B + 1{,}000 = C - 2{,}000$

$\Rightarrow$ $C = B + 3{,}000$

$\therefore$ $A + B + C = (B + 1{,}000) + B + (B + 3{,}000) =$ Rs 8,200 (given)

$\therefore$ $3B + 4{,}000 = 8{,}200$

$B =$ Rs 1,400

$\therefore$ Share of profit of $B = \frac{1,400}{8,200} \times 2,460 =$ **Rs 420**.

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Let the total time be 12 months

$\therefore$ A's profit : B's profit : C's profit = MEI of A : MEI of B : MEI of C

$$= \frac{1}{6}\times\left(\frac{1}{6}\times 12\right):\frac{1}{3}\times\left(\frac{1}{3}\times 12\right):\frac{1}{2}\times 12$$

$$= \frac{1}{3}:\frac{4}{3}:6$$

$$= 1 : 4 : 18$$

$\therefore$ Share of $A = \dfrac{1}{1+4+18} \times 2300 =$ **Rs 100.**

REGULAR PROBLEMS

(1) The annual income of A is 10% less than that of B whose income is 20% more than that of C. If the monthly income of C is Rs 200, find the total annual income of A, B, and C together.
(a) Rs 7,046 (b) Rs 7,772 (c) Rs 6,872 (d) Rs 7,872 (e) None of these

(2) A sum of money is to be divided among A, B and C in the ratio 2 : 3 : 7. If the total share of A and B together is Rs 1,500 less than C, what is A's share in it? **(NIC, '80)**
(a) Rs 1,000 (b) Rs 1,500 (c) Rs 2,000
(d) Data insufficient (e) None of these

Hint: $7x - (2x + 3x) = 1{,}500 \quad \therefore\ x = 750 \quad \therefore\ A = 2x = 1{,}500.$

(3) Nirmal and Kapil started a business investing Rs 9,000 and Rs 12,000 respectively. After 6 months, Kapil withdrew half of his investment. If after a year, the total profit was Rs 4,600, what was Kapil's share in it?
(a) Rs 2,000 (b) Rs 2,600 (c) Rs 1900 (d) Rs 2,300 (e) None of these

(4) The ratio between the ages of Amar and Kabir at present is 2 : 3. After three years, the ratio of their ages will be 3 : 4. Find the present age of Amar in years.
(a) 6 (b) 9 (c) 15
(d) Data insufficient (e) None of these

Hint: $\dfrac{\text{Amar's age}}{\text{Kabir's age}} = \dfrac{2x-3}{3x-3} = \dfrac{3}{4}.$

(5) Rs 750 is distributed among A, B and C such that A's share : B's share = 2 : 3 and B's share : C's share = 6 : 5. The share of A is,
(a) Rs 150 (b) Rs 175 (c) Rs 200 (d) Rs 250 (e) None of these

(6) The cost of a black and white TV and a colour TV are in 3 : 8 and total price of both is Rs 12,100. The difference in their prices is
(a) Rs 6,600 (b) Rs 6,050 (c) Rs 5,500 (d) Rs 5,100 (e) Rs 5,000

Hint: $\dfrac{8x+3x}{8x-3x} = \dfrac{12{,}100}{?} \Rightarrow ? = \dfrac{5}{11}\times 12{,}100 = 5{,}500$

(7) A profit of Rs 450 is divided between two partners, one of whom has contributed Rs 1,200 for 5 months and the other Rs 750 for 4 months. How much amount the second partner received?
(a) Rs 300 (b) Rs 425 (c) Rs 150 (d) Rs 175 (e) None of these

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9 MIXTURES

9.1 DEFINITION

Mixtures are generally of two types. When two different **ingredients** are mixed together, it is known as **simple mixture**, e.g. a mixture of water and milk; water and pure spirit.

When two or more simple mixtures (made of same ingredients of same or different proportions) are mixed together to form another mixture, it is known as a **compound mixture**.

9.2 ALLIGATION RULE

Alligation literally means "linking". The **alligation rule** states that,

'When different quantities of same or different ingredients of different cost (value) are mixed together to produce a mixture of a mean cost (value), the ratio of their quantities are inversely proportional to the differences in their cost from the mean cost (value)'

$$\frac{\text{Quantity of smaller cost ingredient}}{\text{Quantity of larger cost ingredient}} = \frac{\text{Larger cost} - \text{Mean cost}}{\text{Mean cost} - \text{Smaller cost}}$$

Let C_1 = cost price of 1st ingredient

C_2 = cost price of 2nd ingredient

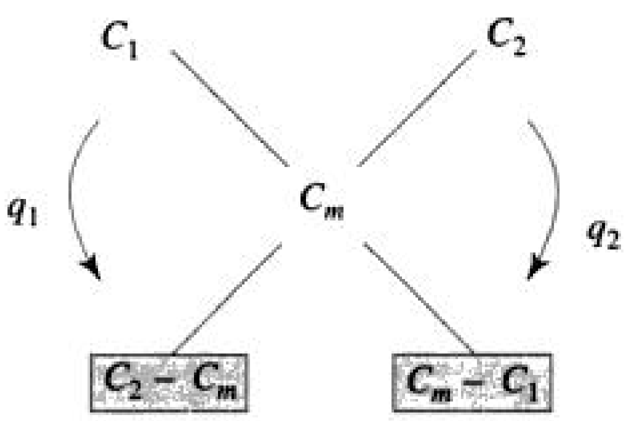
Now, these two ingredients are mixed to produce a mixture such that

C_m = cost price of the mixture (mean price)

then, the ratio of the quantity of two ingredients needed to produce this mixture is given by:

$$\frac{\text{quantity of 1}^{\text{st}}\text{ ingredient}}{\text{quantity of 2}^{\text{nd}}\text{ ingredient}} = \frac{C_2 - C_m}{C_m - C_1}$$

Diagram Representation



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$$= \frac{\left(\frac{a}{a+b}\right)P + \left(\frac{x}{x+y}\right)Q}{\left(\frac{b}{a+b}\right)P + \left(\frac{y}{x+y}\right)Q}$$ (Using Rule-1)

Similarly, in **percentage form,**
if ingredient A is $m\%$ in the P kg of first mixture
and ingredient A is $n\%$ in the Q kg of second mixture,
then, in the final mixture, containing these two mixtures,

$$\text{the new \% of } A = \frac{(\text{quantity of } A \text{ in first mixture} + \text{in second mixture})}{\text{amount of mixture}} \times 100\%$$

$$= \frac{(m\% \text{ of } P + n\% \text{ of } Q)}{(P+Q)} \times 100\%$$ (Using Rule-2)

Example: In two alloys, the ratio of copper and zinc are 3 : 4 and 5 : 8. If 14 kg of first alloy and 26 kg of second alloy are mixed, then find the ratio of copper and zinc in the new alloy.

Solution:

	Alloy 1	Alloy 2	New Alloy
Copper —	$\frac{3}{7} \times 14$	$\frac{5}{13} \times 26$	$\frac{3}{7} \times 14 + \frac{5}{13} \times 26$
Zinc —	$\frac{4}{7} \times 14$	$\frac{8}{13} \times 26$	$\frac{4}{7} \times 14 + \frac{8}{13} \times 26$

The required ratio of copper and zinc in the new alloy

$$= \frac{\text{Quantity of copper}}{\text{Quantity of Zinc}} = \frac{\frac{3}{7} \times 14 + \frac{5}{13} \times 26}{\frac{4}{7} \times 14 + \frac{8}{13} \times 26} = \frac{6+10}{8+16} = \frac{2}{3}$$

Rule 5*

When two mixtures are mixed, then the ratio of the quantities of each mixture in the final (new) mixture can be found by applying Alligation Rule to the **parts of same ingredient** in the two mixtures and also in the final mixture.

Example: Two vessels contain spirit and water mixed respectively in the ratios 3 : 1 and 5 : 3. Find the ratio in which these are to be mixed to get a new mixture in which the ratio of spirit to water is 2 : 1.

Solution: Let us consider the parts of spirit in both the given mixtures and also in the new mixture.

$$\text{The parts of spirit in the first mixture} = \frac{3}{3+1} = \frac{3}{4}$$

$$\text{The parts of spirit in the second mixture} = \frac{5}{5+3} = \frac{5}{8}$$

$$\text{The parts of spirit in the new mixture} = \frac{2}{2+1} = \frac{2}{3}$$

*Note the difference in the application of Rule-4 and Rule-5. As the **Rule-4** is used to find the quantity of **each ingredient** in the final mixture, but the **Rule-5** is to find the quantity of **each mixture** in the final mixture.

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Solved Examples

E-1 In what proportion should one variety of oil at Rs 9.50 per kg be mixed with another at Rs 10 per kg to get a mixture worth Rs 9.60 per kg?

S-1 By alligation rule [9.2] and using the formula, we get

$$\frac{q_1}{q_2} = \frac{c_2 - C_m}{C_m - c_1} = \frac{10 - 9.6}{9.6 - 9.5} = \frac{4}{1}$$

$\therefore$ Two varieties of oil are mixed in the ratio **4 : 1**.

E-2 In what ratio must 25% alcohol be mixed with 50% alcohol to get a mixture of 40% alcohol strength?

S-2 Here two same ingredients (i.e. alcohol), but of different strength are mixed. Hence, by the alligation rule,

$$\frac{q_{25\%}}{q_{50\%}} = \frac{50 - 40}{40 - 25} = \frac{10}{15} = \frac{2}{3}$$

$\therefore$ 25% alcohol is mixed with 50% alcohol in **2 : 3**.

E-3 A merchant lent out Rs 1,000 in two parts, one at 8% and the other at 10% interest. The yearly average comes out to be 9.2%. Find the amount lent in two parts.

S-3 $\frac{\text{Quantity lent at } 8\%}{\text{Quantity lent at } 10\%} = \frac{10 - 9.2}{9.2 - 8} = \frac{0.8}{1.2}$, by alligation rule

$$= \frac{2}{3}$$

$\therefore$ Dividing Rs 1,000 in $\frac{2}{3}$ as,

Quantity of money lent at 8% $= \frac{2}{2+3} \times 1000 =$ Rs 400.

Quantity of money lent at 10% $= \frac{3}{2+3} \times 1000 =$ **Rs 600**.

E-4 Pure milk costs Rs 3.60 per litre. A milkman adds water to 25 litres of pure milk and sells the mixture at Rs 3 per litre. How many litres of water does he add? **(UDC, '82)**

S-4 $\frac{q_{\text{pure milk}}}{q_{\text{water}}} = \frac{3.00 - 0}{3.6 - 3.0} = \frac{3}{0.6} = \frac{\mathbf{5}}{\mathbf{1}}$ (Since cost of water = 0)

For every 5 litres of milk, he adds 1 litre of water,
For every 25 litres of milk, he adds 5 litres of water.

E-5 How many kg of salt at 42 paise per kg must a man mix with 25 kg of salt at 24 paise per kg so that he may, on selling the mixture at 40 paise per kg gain 25% on the outlay? **(IITCE, '89)**

S-5 $\frac{q_1}{q_2} = \frac{c_2 - C_m}{C_m - c_1}$, by Alligation Rule,

where c_1, c_2, and C_m are always **cost price** of the ingredients and mixture.

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10 CHAIN RULE

10.1 DEFINITION

When quantities of different kinds are connected to one another so that we know how much of one quantity is equivalent to a given quantity of a second, etc. we can determine how much of the last kind is equivalent to a given quantity of first kind by the *chain rule*.

10.2 DIRECT PROPORTION AND INDIRECT PROPORTION

If increase or decrease of a quantity Q_1 causes increase or decrease of another quantity Q_2 in the same extent, then, Q_1 is directly proportional to $Q_2 \Rightarrow Q_1 \propto Q_2$

I. Number of persons $\propto$ Amount of work done, i.e. more persons, more work
II. Number of days $\propto$ Amount of work, i.e. more days, more work
III. Working rate $\propto$ Amount of work, i.e. more working rate, more work
IV. Efficiency of man $\propto$ Amount of work, i.e. more efficiency of man, more work

Combining I, II, III, and IV, (Man $\times$ days $\times$ Workrate $\times$ Efficiency) $\propto$ Amount of work. If increase of a quantity Q_1 causes decrease of a quantity Q_2, in the same extent, then,

Q_1 is indirectly proportional to $Q_2 \Rightarrow Q_1 \propto \dfrac{1}{Q_2}$

V. Number of men $\propto \dfrac{1}{\text{No. of days}}$, i.e. more the men, less the no. of days required

10.2.1 Important Formula

(a) $$\frac{\text{Man}_1 \times \text{Days}_1 \times \text{Work rate}_1}{\text{Amount of work done}_1} = \frac{\text{Man}_2 \times \text{Days}_2 \times \text{Work rate}_2}{\text{Amount of work done}_2}$$

Remember, *"Man days" required per unit work is always same*. In fact, Man $\times$ Days specify the volume of job or work.

(b) If in place of men there are engines burning coal for certain number of hours, then, the above equation changes to

$$\frac{\text{No. of Engine}_1 \times \text{Hours}_1 \times \text{Consumption Rate}_1}{\text{Amount of coal burnt}_1} = \frac{\text{Engine}_2 \times \text{Hours}_2 \times \text{Consumption Rate}_2}{\text{Amount of coal burnt}_2}$$

because, here the job of engine is to burn the coal.

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Example: Mukesh can do $\frac{2}{7}$th of an work in 1 day. In how many days can he complete the same work?

Solution: Time of completion by Mukesh alone = alone time = $\dfrac{1}{\text{1 day's work}}$

$$= \frac{1}{2/7}$$

$$= \frac{7}{2} \text{ days} = 3\frac{1}{2} \text{ days}$$

Therefore, Mukesh can complete the job alone in $3\frac{1}{2}$ days.

Concept 5

When more than one person are working on the same piece of work, then their combined 1 day's work = sum of 1 day's work by each person. i.e., If A, B and C are three persons working on a job, then $(A + B + C)$'s 1 day's work = A's 1 day work + B's 1 day work + C's 1 day work.

Example: A person 'P' can alone do a work in 10 days and 'Q' can do it in 15 days. What amount of work is done by P and Q together in one day?

Solution: $(P + Q)$'s 1 day work = P's 1 day work + Q's 1 day work. Now, using concept (3),

$$\text{1 day's work} = \frac{1}{\text{alone time}}$$

We can find

$$(P + Q)\text{'s 1 day work} = \left(\frac{1}{10} + \frac{1}{15}\right)\text{th part of total work.}$$

$$= \frac{3 + 2}{30} = \frac{1}{6}\text{th part of total work.}$$

Corollary

A's 1 day work = $(A + B + C)$'s 1 day work − B's 1 day work − C's 1 day work

Similarly, B's 1 day work = $(A + B + C)$'s 1 day work − A's 1 day work − C's 1 day work and so on.

Concept 6

It is the application of concept (4) for more than one person.

The reciprocal of combined 1 day's work gives the time for completion by the persons working together.

i.e., $$\text{time for completion} = \frac{1}{\text{combined 1 day's work}}$$

It implies that

if three persons, say, A, B and C are working together on a job, then

$$\text{time for completion by them} = \frac{1}{(A + B + C)\text{'s 1 day's work}}$$

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E-4 Singvi and Ravi can do a job alone in 10 days and 12 days respectively. Singvi starts the work and after 6 days Ravi also joins to finish the work together. For how many days Ravi actually worked on the job?

S-4 Let the work be finished in T days, then using the concept (8),

$$\frac{\text{no. of days Singvi worked}}{\text{alone time}} + \frac{\text{no. of days Ravi worked}}{\text{alone time}} = 1$$

$$\Rightarrow \qquad \frac{T}{10} + \frac{T-6}{12} = 1$$

$$\Rightarrow \qquad T = \frac{90}{11}$$

$\therefore$ Ravi worked for T – 6 i.e. $\frac{90}{11} - 6 = 2\frac{2}{11}$ days.

E-5 A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each take separately to do the same work?

S-5 Since, $(A + B) + (B + C) + (C + A) = 2(A + B + C)$

So, using concept (5), we get,

2 $(A + B + C)$'s 1 day work = $(A + B)$'s 1 day work + $(B + C)$'s 1 day work + $(C + A)$'s 1 day work

$$= \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$$

$$= \frac{1}{5}$$

$\Rightarrow$ $(A + B + C)$'s 1 day work $= \frac{1}{5 \times 2} = \frac{1}{10}$ th part of work

Now, to find out alone time for A, we may write

$$A = (A + B + C) - (B + C)$$

Therefore, A's 1 day work = $(A + B + C)$'s 1 day work – $(B + C)$'s 1 day work

$$= \frac{1}{10} - \frac{1}{15}$$

$$= \frac{1}{30} \text{ th part of work}$$

So, Alone time for $A = \frac{1}{1/30}$ = **30 days**.

Similarly, B's 1 day work $= \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$

Alone time for $B = \frac{1}{1/20}$ = 20 days.

and C's 1 day work $= \frac{1}{10} - \frac{1}{12} = \frac{1}{60}$

(6) Sundar can copy 80 pages in 20 hours, Sundar and Purbasa can copy 135 pages in 27 hours. How many pages Purbasa can copy in 20 hours?

(a) $\frac{55}{7}$ (b) 55 (c) 27 (d) 20 (e) 35

(7) Tapas works twice as much as Mihir. If both of them finish the work in 12 days, Tapas alone can do it in:

(a) 20 days (b) 24 days (c) 18 days (d) 36 days (e) 20 days

(8) Xavier can do a job in 40 days. He worked on it for 5 days and then Paes finished it in 21 days. In how many days Xavier and Paes can finish the work? **(Bank PO, '93)**

(a) 10 (b) 15 (c) 20 (d) $\frac{840}{61}$ (e) 12

Hint: To find the number of days required to finish a work, one day's work of each person is to be found out first. One day's job of Paes is to be found out with the given data as one day job of Xavier $= \frac{1}{40}$ (known)

(9) Mary has 'm' minutes of home work in each of her 's' subjects. In one hour she completes what part of her home work?

(a) $\frac{m}{s}$ (b) $\frac{ms}{60}$ (c) $\frac{60}{ms}$ (d) $\frac{1}{ms}$ (e) $\frac{s}{m}$

(10) Gouri can knit a pair of socks in 3 days. Gita can knit the same in 6 days. If they are knitting together, in how many days will they knit two pairs of socks?

(a) 4 (b) 2 (c) $4\frac{1}{2}$ (d) 3 (e) 6

(11) Sun, Mon and Don can do a work in 6 days. Sun and Don can do it in 10 days. In how many days will Mon alone do the work?

(a) 20 (b) $3\frac{3}{4}$ (c) 15 (d) 4 (e) 16

Hint: Refer the text

(12) One man can paint a house in 'r' days and another man can do it in 't' hours. If they can together do it in 'd' days, then 'd' is given by:

(a) $\frac{1}{r}+\frac{1}{t}$ (b) $\frac{rt}{r+t}$ (c) $\frac{24rt}{r+t}$ (d) $\frac{rt}{24(r+t)}$ (e) $\frac{rt}{t+24r}$

(13) Dinesh and Dipu can design an application software in 16 hours and 12 hours respectively. Dinesh joins Dipu 4 hours before completing the design, Dipu had started the design work alone. Find how many days Dipu has worked alone? **(IA, '93)**

(a) 5 (b) $\frac{5}{24}$ (c) 7 (d) $\frac{2}{7}$ (e) 9

(14) John can do a piece of work in 15 days. If he is joined by Jill who is 50% more efficient, in what time will John and Jill together finish the work?

(a) 10 days (b) 6 days (c) 18 days
(d) data insufficient (e) 8 days

12 PIPES AND CISTERNS

The problems often asked on pipes and cisterns have the similarity with those of time and work. Hence, the approach to solve the problems became quite easy if one has a good command on the problems of time and work. In pipes and cisterns, the following basic parameters are often asked to be calculated:

(a) Time—It means time for **filling** or **emptying** pipes that have been actually kept open

(b) Work— It means the part of the cistern to be filled or emptied or the part of cistern actually filled or emptied. In actual practice, while solving the problems on pipes and cisterns, the amount or part of cistern assigned for filling or emptying (is always) is a unit, unless otherwise specified.

If a **filling** pipe can fill a cistern **alone** in f_1 min., then **Alone Fill Time** for that filling pipe = f_1

If an **emptying** pipe can empty a cistern **alone** in e_1 min., then

Alone Emptying Time for that emptying pipe = $-e_1$

(–)ve is used for all cases of emptying work.

When a cistern is filled completely, amount of work done (filling) = 1

When a cistern full of water is emptied completely. Amount of **Fill Work** = Amount of **Empty Work**

$\therefore$ Amount of **Fill Work – Empty Work** = 0

12.1 BASIC CONCEPTS

The concepts used to solve problems on time and work can also be applied to the problems on pipes and cistern; because here the work refers to the work of filling or emptying the cistern and 'persons' have been replaced by the fill pipes/empty pipes/leaks that are doing the work.

(A) For one Pipe

Concept A-1

$$\text{Fill or empty work done} = \frac{\text{time of work}}{\text{alone time of the pipe}}$$

Concept A-2

$$\text{Fill or empty work done in 1 unit}^{*} \text{ time} = \frac{1}{\text{alone time}}$$

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(2) A cistern normally takes 4 hours to be filled by a tap but because of a leak, takes 2 hours more. In how many hours will the leak empty a full cistern?
(a) 10 h (b) 14 h (c) 6 h (d) 12 h (e) 8 h

Hint: Use $E = \dfrac{f(f+x)}{x} = \dfrac{4(4+2)}{2} = 12$.

(3) A cistern is normally filled by a tap in 5 hours, but suddenly a leak develops and it empties the full cistern in 30 hours. With the leak, the cistern is filled in
(a) 8 h (b) 6 h (c) 10 h (d) $7\frac{1}{2}$h (e) 20 h

Hint: Use $E = \dfrac{f(f+x)}{x}$ $\therefore$ $30 = \dfrac{5(5+x)}{x}$ $\therefore$ $x = 1$h our more $\therefore$ cistern is now filled in 5 + 1 = 6 hours.

(4) A cistern is filled by two taps in a hours and b hours respectively and is then emptied by a tap in c hours. If all three taps are open, the cistern is filled in F hours. The relation between F, a, b, and c are given by,

(a) $F = (a + b) - c$ (b) $F = \dfrac{a \times b}{c}$

(c) $\dfrac{1}{F} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}$ (d) $F = \dfrac{1}{\frac{1}{a} + \frac{1}{b} - \frac{1}{c}}$

(e) $F = \dfrac{ab + bc}{ac}$.

(5) A tap can fill a swimming pool in h hours. What part of the pool is filled in y hours?

(a) yh (b) $\dfrac{h}{y}$ (c) $\dfrac{y}{h}$ (d) $y + h$ (e) $h - y$

(6) From a leaking tap 'a' drop come out in 'b' min. If there are 'c' drops in a litre, then in how many hours. One litre of water will be wasted?

(a) $\dfrac{60a}{bc}$ (b) $\dfrac{a}{bc}$ (c) $\dfrac{abc}{60}$ (d) $\dfrac{bc}{60a}$ (e) $\dfrac{bc}{a}$

(7) Two taps can fill a tank in 12 and 18 min. respectively. Both are kept open for 2 min. and the first is turned off. In how many min. more will the tank be filled?
(a) 15 min. (b) 20 min. (c) 11 min. (d) 13 min. (e) 8 min.

(8) Filling pipe, if opened alone, takes 5 min. to fill a cistern. Suddenly, during the course of filling, the waste pipe (which is of similar size and flow as of fill pipe) is opened for 2 min., then the cistern will be filled in

(a) $3\frac{1}{7}$min. (b) $3\frac{1}{3}$min. (c) 5 min. (d) 3 min. (e) 7 min.

(9) Two filling pipes can fill a cistern in 10 and 12 min. respectively and when the waste pipe is open, they can together fill it in 15 min. The waste pipe can empty the full cistern in

(a) 15 min. (b) 37 min. (c) $8\frac{4}{7}$ min. (d) $7\frac{1}{2}$ min. (e) 8 min.

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(4) Rakesh on selling a lamp for Rs 450 bears a loss of 20%. To earn a profit of 20%, he should sell the article for: **(Railway ASM, '01)**
(a) Rs 675 (b) Rs 600 (c) Rs 625 (d) Rs 680 (e) Rs 650

(5) A trader purchased a bag of rice containing 70 kg for Rs 175. He sold it at the rate of Rs 2.75 per kg. Find the profit or loss%.
(a) 10% gain (b) 10% loss (c) 12.5% gain (d) 12.5% loss (e) None of these

(6) A man sold his book for Rs 891, thereby gaining $\frac{1}{10}$ of its cost price. The cost price is: **(RBI, '97)**
(a) Rs 850 (b) Rs 810 (c) Rs 851 (d) Rs 840 (e) None

(7) 400 mangoes were bought at Rs 125 per hundred and were sold at a profit of Rs 100. The selling price per dozen is:
(a) Rs 15 (b) Rs 1.5 (c) Rs 18 (d) Rs 10 (e) Rs 22

Hint: Find the selling price for 400 mangoes in first step

(8) A man sells an article at a loss of 10%. If he had received Rs 9 more, he would have gained $12\frac{1}{2}$%. The cost price of the article is:
(a) Rs 40 (b) Rs 36 (c) Rs 32 (d) Rs 48 (e) None

Hint: Use $\dfrac{\text{Difference in sale price}}{\text{related \% difference of gain or loss}} = \text{cost price}$

(9) John buys a telephone handset for Rs 600 from Kolkata and sells it at Ranchi at a gain of 25%. If his overhead expenses are 5% of the selling price, then he sold the telephone handset for
(a) Rs 1000 (b) Rs 780 (c) Rs 750 (d) Rs 800 (e) Rs 720

(10) A person marks his goods 20% higher than cost price and allows a discount of 5%. The percentage profit is:
(a) 15 (b) 20 (c) 5 (d) 14 (e) None

(11) A shopkeeper earns a profit of 12% on selling a book at 10% discount on the printed price. The ratio of the cost price and the printed price of the book is: **(Bank PO, '96)**
(a) 45 : 56 (b) 50 : 61 (c) 99 : 125 (d) 36 : 79 (e) 63 : 92

Tips: Assume printed price = Rs 100

(12) What is a single discount equivalent to a discount series of 10% and 20%?
(a) 25 (b) 30 (c) 28 (d) 15 (e) 27

(13) A dishonest shopkeeper professes to sell potatoes at the cost price, but he weighs 950 gm, instead of one kg. What is the percentage of profit? **(RRB Guwahati, '97)**
(a) 50 (b) 5 (c) 6.5 (d) 5.26 (e) 6

Tips: Since the shopkeeper sells at the cost price so his profit is only due to underweighing

(14) A shopkeeper by selling 44 calculators, earns a profit of equal to the selling price of 11 calculators. His profit percentage is: **(NABARD, '98)**
(a) $33\frac{1}{3}$ (b) 20 (c) 25 (d) 30 (e) 40

Hint: Refer solved problems

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14 SIMPLE INTEREST

14.1 DEFINITION

If a person A borrows some money from another person B for a certain period, then after that specified period, the borrower has to return the money borrowed as well as some additional money. This additional money that borrower has to pay is called **interest**. The actually borrowed money by A is called **principal** (SUM). The principal and the interest together is called **amount**. The interest that the borrower has to pay for every 100 rupees borrowed each year is known as **rate per cent per annum**. It is denoted as $R\%$ per annum $= \frac{R}{100}$.

The time for which the borrowed money has been used is called the **time**. It is denoted as T years. The interest is directly proportional to the principal, the rate and the time for which the borrowed sum is used.

If the interest on a certain sum borrowed for a certain period is reckoned uniformly, then it is called **Simple Interest** and denoted as *S.I.*

$\therefore$ **Simple Interest** $(S.I.) = P \times \frac{R}{100} \times T$,

where P = Principal or the sum borrowed

R = Rate per cent per annum

T = Number of years for which the borrowed money has been used.

Now, simple interest + principal = amount

$\Rightarrow$ Simple interest = amount – principal

If amount is denoted by 'A', then

$$\boxed{\begin{array}{l}\text{Simple interest} = A - P = \dfrac{PRT}{100} \\ \text{(SI)}\end{array}}\ * \qquad \textbf{(1)}$$

14.2 EFFECT OF CHANGE OF P, R AND T ON SIMPLE INTEREST

Since, Simple interest $= \frac{PRT}{100}$

---

* **Special Memory Clips** for use of relation (1)

(1) SI $= \frac{PRT}{100}$ can be rearranged as per requirement of the problem as $P = \frac{100 \times SI}{RT}$, $R = \frac{100 \times SI}{PT}$ and $T = \frac{100 \times SI}{RT}$, so, in each case we find the numerator as $100 \times$ SI and the denominator as product of other two.

(2) Two different cases can be compared by using the relation as $\frac{A_1 - P_1}{A_2 - P_2} = \frac{P_1 R_1 T_1}{P_2 R_2 T_2}$

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$$A = 100\left(1+\frac{3\times 2}{100}\right) = 100\times\frac{106}{100} = \textbf{Rs 106}.$$

(ii) $R = 6$ % per annum, $T = 4$ months $= \frac{4}{12}$ years.

$$\therefore\quad A = 500\left(1+\frac{6\times 4}{100\times 12}\right) = 500\times\frac{102}{100} = \textbf{510}.$$

(iii) $T = 150$ days $= \frac{150}{365}$ year

$$\therefore\quad A = 400\left(1+\frac{3.65\times 150}{100\times 365}\right) = 400\times\frac{203}{200} = \textbf{Rs 406}.$$

(iv) Using the formula,

$$\boxed{\frac{A}{S.I.} = \frac{100}{RT}+1}$$ *[Refer 14.3]*

$$\frac{540}{108} = \left(\frac{100}{5\times T}+1\right) \Rightarrow T = \textbf{5 years}.$$

(v) $T = 2\frac{2}{5}$ year $= \frac{12}{5}$ years, A = 1,120, R = 5%, using the formula,

$$\frac{A}{S.I.} = \frac{100}{RT}+1;\quad \frac{1{,}120}{S.I.} = \frac{100}{5\times\frac{12}{5}}+1$$

$$\Rightarrow\quad \frac{1{,}120}{S.I.} = \frac{112}{12}\quad\Rightarrow\quad S.I. = \textbf{Rs 120}.$$

E-3 On what sum of money lent out at 9% per annum simple interest for 6 years does the simple interest amount to Rs 810? **(RBI '82)**

S-3 [Refer 14.1] Using the formula

$$P = \frac{100\times S.I.}{R\times T} = \frac{100\times 810}{9\times 6} = \textbf{Rs 1,500}.$$

E-4 At what rate of interest per annum will a sum double itself in 8 years? **(Bank PO '86)**

S-4 [Refer 14.4], Using the formula,
$R\times T = 100\times(N-1)$; Here $T = 8$ yrs. $N = 2$ times R =?

$$\Rightarrow\quad R\times 8 = 100\times(2-1)\qquad\Rightarrow\quad R = \frac{100}{8} = \mathbf{12\frac{1}{2}}\,\%.$$

E-5 A sum when reckoned at simple interest $2\frac{1}{2}$% per annum amounts to Rs 630 after 2 years. Find the sum.

S-5 [Refer 14.3],

$$A = P\left[1+\frac{RT}{100}\right]\quad A = 630,\ P = ?$$

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(7) A sum of money doubles itself in 10 years at simple interest. In how many years would it treble itself? **(Bank PO '78)**

(a) 10 (b) 15 (c) 20 (d) 25 (e) None of these

(8) At what rate per cent (simple interest) will a sum of money double itself in 16 years?

(a) 50% (b) 5% (c) $6\frac{1}{4}$% (d) 10% (e) 20%

(9) On retirement, a person gets 1.53 lakhs of his provident fund which he invests in a scheme at 20% p.a. His monthly income from this scheme will be

(a) Rs 2,450 (b) Rs 2,500 (c) Rs 2, 550 (d) Rs 2,600 (e) Rs 2,700

(10) Find out the capital required to earn a monthly interest of Rs 600 at 6% simple interest.

(a) Rs 1.0 lakhs (b) Rs 1.2 lakhs (c) Rs 1.1 lakhs
(d) Rs 1.3 lakhs (e) None of these

(11) A sum of money lent out at simple interest amounts to Rs 2,520 in 2 years and Rs 2,700 in 5 years. Find the rate % p.a.

(a) 3% (b) 4% (c) $2\frac{1}{2}$% (d) 5% (e) None of these

(12) A sum of money doubles itself in 7 yrs. In how many years it becomes four fold?

(a) 10 yrs. (b) 35 yrs. (c) 14 yrs. (d) 21 yrs. (e) 28 yrs.

Hint: Use, $\frac{RT_1}{RT_2} = \frac{N_1 - 1}{N_2 - 1}$ $\quad\therefore\quad \frac{7}{T_2} = \frac{2-1}{4-1}$ $\quad\therefore\quad T_2 = 21$.

(13) I gave some money at simple interest and at the end of 10 yrs, got back twice the sum. The rate % p.a. was

(a) 2% (b) 4% (c) 5% (d) 10% (e) 20%

Hint: Use $RT = (N - 1) \times 100$.

(14) A man pays 40 times the annual rent to purchase a building. The rate % p.a. he derives from his investment is

(a) 25% (b) 2.5% (c) 5% (d) 40% (e) 4%

Tips: Rate% p.a = $\frac{100}{\text{No. of times the annual rent}}$%.

(15) If Re 1 produces Rs 9 in 60 years at simple interest, the rate % p.a. is

(a) $13\frac{1}{3}$% (b) 14% (c) 15% (d) $12\frac{1}{2}$% (e) None of these

Hint: If investment (P) = 1 Re $S.I.$ = Rs 9.

(16) If Re 1 becomes Rs 9 in 60 years at simple interest, the rate % p.a. is

(a) 15% (b) $12\frac{1}{2}$% (c) $13\frac{1}{3}$% (d) 12 % (e) None of these

Hint: If investment (P) = Re 1 then $S.I = 9 - 1$ = Rs 8.

(17) If x is the simple interest on y and y is the simple interest on z, the rate % and the time being the same in both cases, what is the relation between x, y and z? **(MBA '80)**

(a) $x^2 = yz$ (b) $y^2 = xz$ (c) $z^2 = xy$ (d) $xyz = 1$ (e) $x = 2y + z$

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15.4 SPECIAL CASES

Case I

When interest is NOT **compounded yearly**
Amount is given by

$$A = P\left(1 + \frac{R}{n \times 100}\right)^{nt}$$

where, n = number of conversions (or compounding) per year, Refer 15.2.

Example: Compute the compound interest on Rs 2,000 for 3 years at 10% per annum, when compounded half-yearly.

Solution: Here, Principal (P) = Rs 2,000
rate (R) = 10% per annum
time (t) = 3 years
no. of conversions per year (n) = 2, since interest is compounded half-yearly.
Using the relation

$$A = P\left(1 + \frac{R}{n \times 100}\right)^{nt}$$

$$\Rightarrow \quad A = 2{,}000\left(1 + \frac{10}{2 \times 100}\right)^{2\times 3}$$

$$= 2{,}000 \times \left(\frac{21}{20}\right)^{6}$$

$$= \text{Rs } 2{,}680$$

$\therefore$ Compound Interest = 2,680 – 2,000 = Rs 680

Case II

When rate % is **NOT same for every year** and interest is **compounded yearly**
The basic formula can be re-written as

$$A = P\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right) \ldots \text{ up to '}t\text{' times,}$$

where R is the rate % for every year
but, if rate % is **not same for every year**, then

$$A = P\left(1 + \frac{R_1}{100}\right)^{t_1} \times \left(1 + \frac{R_2}{100}\right)^{t_2} \ldots \text{ and so on.}$$

where, R_1 = Rate % p.a. for t_1 years
R_2 = Rate % p.a. for t_2 years
and so on

Example: Find the amount of Rs 4,000 for 5 years compounded annually, the rate of interest being 10% for the first 3 years and 20% for the next 2 years.

Solution: Here, principal = Rs 4,000

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$\Rightarrow$ time on steamer $= 8 \div \frac{4}{5} = 8 \times \frac{5}{4} = 10$ hrs.

$\therefore$ time on rail $= 13 - 10 = 3$ hours

$\therefore$ distance by rail $= 3 \times 60 = 180$ km

(31) The distance from P to Q is 25 km; 6 km of which are uphills; 12 km downhill and the rest 7 km in level ground. The time in which a man would go from P to Q and back again, supposing his speed uphill is 2 km, downhill 3 km and on the level ground $3\frac{1}{2}$ km per hour, is:

(a) 9 hours (b) 10 hours (c) 15 hours (d) 18 hours (e) 19 hours

(32) A hare pursued by a greyhound, is 50 of her own leaps ahead of him. While the hare takes 4 leaps, the greyhound takes 3 leaps. In one leap, the hare goes $1\frac{3}{4}$ metres and the greyhound $2\frac{3}{4}$ metres. In how many leaps will the greyhound overtake the hare?

(a) 210 leaps (b) 180 leaps (c) 200 leaps (d) 224 leaps (e) 315 leaps.

Answers

1. (d)	**2.** (c)	**3.** (b)	**4.** (e)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (d)
10. (d)	**11.** (c)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (c)	**17.** (d)	**18.** (c)
19. (b)	**20.** (a)	**21.** (d)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (c)	**27.** (a)
28. (e)	**29.** (d)	**30.** (c)	**31.** (e)	**32.** (a)				

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19

BOATS AND STREAMS

19.1 INTRODUCTION

The following terms are frequently used in the problems on boats and streams:

(a) **Still Water:** It implies that the speed of water in the river is zero.

(b) **Stream Water:** It implies that the water in the river is moving.

19.2 SPEED OF MAN (BOAT) AND STREAM

Direction →

Man's rate of rowing (or speed of boat) in **still** water = x km/h.

← UPSTREAM ROWING | DOWNSTREAM ROWING →

Speed of current (or stream) = y km/h.

Direction →

Let, the man's rate of rowing (or speed of boat) in **still** water = x km/h
and the speed of stream (or current) = y km/h

(a) **Downstream** (With the stream) **Rowing**:
It indicates that the stream **favours** the man's rowing (or boating).
i.e. direction of rowing and direction of flow (stream) is same.
∴ Man's rate of rowing downstream = **DOWNSTREAM RATE** = $x + y$

(b) **Upstream** (Against the stream) **Rowing**:
It indicates that the stream flows against the man's rowing (or boating)
i.e. direction of rowing and direction of stream (current) are opposite.
∴ Man's rate of rowing upstream = **UPSTREAM RATE** = $x - y$

19.3 IMPORTANT FORMULAE

Assumption:

Man's rate of rowing in still water = x
Speed of stream (current) = y

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Solved Examples

E-1 In one kilometer race, A beats B by 36 metres or 9 seconds. Find A 's time over the course.

S-1 Here A is the winner and B is the loser.
Using the formula, 20.2

$$\frac{\text{Winner's time}}{\text{Loser's distance}} = \frac{\text{beat time + start time}}{\text{beat distance + start distance}}$$

$$\Rightarrow \quad \frac{A\text{'s time}}{1{,}000-36} = \frac{9+0}{36+0}$$

$$\Rightarrow \quad A\text{'s time} = \frac{9}{36} \times 964 \text{ s} = 241 \text{ s}.$$

A's time over the course is **241 s**.

E-2 A runs $1\frac{1}{3}$ as fast as B. If A gives B a start of 30 metres, how far must be the winning post, so that the race ends in a dead heat?

S-2 Assuming L = distance of the winning post such that the race ends in a dead heat, i.e. both the participants A and B reach the winning post at the same time.
$\therefore$ time taken by A = time taken by B

$$\Rightarrow \quad \frac{L}{\frac{4}{3}V} = \frac{L-30}{V} \qquad \left(\text{Since } t = \frac{d}{V}\right)$$

$$\Rightarrow \quad L = 120$$

$\therefore$ length of race (distance) of winning post is **120 m**.

E-3 P runs a kilometer in 4 min. and Q in 4 min. 10 seconds. How many metres start can P give Q in a kilometer race so that the race may end in a dead heat?

S-3 P runs a kilometer in 4 min. (= 240 seconds)
Q runs a kilometer in 4 min. 10 s (= 250 seconds)
$\therefore$ P can beat Q by 10 seconds
But if P gives Q a start of 10 seconds or x metres so that the race may end in a dead heat, i.e, beat time = 0, beat distance = 0.
Then, using the formula 20.2

$$\frac{\text{Loser's time}}{\text{Winner's distance}} = \frac{\text{beat time + start time}}{\text{beat distance + start distance}}$$

$$\Rightarrow \quad \frac{250}{1{,}000} = \frac{0+10}{0+x} \quad \Rightarrow \quad x = 40$$

Hence if P gives Q a start of **40 metres** in a race of one kilometer, the race will end in a dead heat.

E-4 P can run a kilometer in 4 min. 50 seconds and Q in 5 min. By what distance can P beat Q?

S-4 Here, P is the winner and Q is the loser. Using the formula,

$$\frac{\text{Loser's time}}{\text{Winner's distance}} = \frac{\text{beat time + start time}}{\text{beat distance + start distance}}$$

$$\Rightarrow \quad \frac{300}{1000} = \frac{10+0}{\text{beat distance}+0} \quad \Rightarrow \quad \text{beat distance} = \frac{100}{3} \text{ metre}$$

$\therefore$ P beats Q by $\mathbf{33\frac{1}{3}}$ metres in a kilometer race.

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Hint: Race ends in dead heat, i.e. time taken by J and K are same.

$\therefore$ use $t = \dfrac{J\text{'s distance}}{J\text{'s speed}} = \dfrac{K\text{'s distance}}{K\text{'s speed}}$

$\therefore$ $\dfrac{x}{\frac{11}{8}k} = \dfrac{x-150}{k}$ $\therefore$ x = Length of race = 550 metres.

(10) In a 500 metres race, A beats B by 50 metres or 8 seconds. Find A's time over the course.
(a) 2 minutes (b) 1 minutes 12 seconds
(c) 2 minutes 35 seconds (d) 1 minutes

(11) In a 300 metre race, A beats B by 5 seconds or 15 metres. A's time over the course is
(a) 95 s (b) 100 s (c) 85 s (d) 90 s (e) 105 s

(12) X can run 20 metres while Y runs 25 metres. In a kilometre race, Y beats X by
(a) 150 metres (b) 200 metres (c) 250 metres
(d) 175 metres (e) 300 metres

(13) In a 500 metres race, the ratio of speeds of two contestants A and B is 3 : 4. A has a start of 140 metres, then **(MBA '80)**
(a) B wins by 20 metres (b) A wins by 20 metres
(c) B wins by 40 metres (d) B wins by 25 metres
(e) A wins by 40 metres

(14) If A can run a kilometre in $3\frac{1}{6}$ minutes and B in $3\frac{1}{3}$ minutes, by what distance can A beat B?
(a) 100 metres (b) 75 metres (c) 60 metres
(d) 50 metres (e) 125 metres

Answers

1. (d) **2.** (d) **3.** (d) **4.** (b) **5.** (c) **6.** (b) **7.** (c) **8.** (b) **9.** (d)
10. (b) **11.** (a) **12.** (b) **13.** (b) **14.** (d)

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(7) The hands of a clock coincide after every 64 minutes of correct time. How much is the clock fast or slow in 24 hours.

Hint: See **E-8**

(8) An astronomical clock has its dial divided into 24 divisions instead of 12, and the small hand goes round in 24 hours, the large hand going round once every hour. The 24th hour is noon. Find when the hands are at right angles between 24 and 1.

Hint: Required time $T = \frac{4}{23}(H \times 30 + A)$ minutes past H

and $\frac{4}{23}(360 + H \times 30 - A)$ minutes past H

Here $H = 0$, because initial position of hour hand is at 24.

Also $A° > H \times 30$ since $A = 90°$.

(9) A man goes out in between 5 p.m. and 6 p.m. When he comes back in between 6 p.m. and 7 p.m., he observes that the two hands of a clock have interchanged their position. Find when the man did go out.

Hint: Let the man goes out x minutes past H, we use

$$x = \frac{720 + 780H}{143} \text{ minutes past } H$$

Here $H = 5$ p.m. $\therefore$ $x = \frac{720 + 780 \times 5}{143}$ minutes past 5

Answers

1. $27\frac{3}{11}$ min past 5

2. $54\frac{6}{11}$ min past 4

3. $49\frac{1}{11}$ min past 9

4. $43\frac{7}{11}$ min past 5, $10\frac{10}{11}$ min past 5

5. $54\frac{6}{11}$ min past 1

6. $11\frac{109}{121}$ min slow

7. $32\frac{8}{11}$ min fast

8. $15\frac{15}{23}$ min past 24 and $46\frac{22}{23}$ min. past 24

9. $32\frac{4}{13}$ min past 5

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(38) The difference between the circumference and the diameter of a circle is 60 cms. The area of the circle is

(a) 425 cm^2 (b) 616 cm^2 (c) 154 cm^2 (d) 308 cm^2 (e) 512 cm^2

(39) When the perimeter and area of a square are numerically equal, then the numerical value of its side is

(a) 2 (b) 4 (c) 16 (d) 8 (e) 1

(40) If the ratio of areas of two circles is 16 : 25, the ratio of their circumference is

(a) 25 : 16 (b) 5 : 4 (c) 4 : 5 (d) 3 : 5 (e) 3 : 4

Hint: Use $\sqrt{\dfrac{A_1}{A_2}} = \dfrac{c_1}{c_2}$ Where c_1 and c_2 are circumferences of two circles.

(41) If the ratio of circumference of two circles is 4 : 9, the ratio of their area is

(a) 2 : 3 (b) 9 : 4 (c) 16 : 81 (d) 3 : 5 (e) 4 : 9

(42) If it costs Rs y to whitewash the 4 walls of a room, how much it would cost if all the dimensions were to be 5 times more?

(a) Rs $5y$ (b) Rs $25y$ (c) Rs $25y^2$ (d) Rs $20y$ (e) $5y^2$

Hint: If all dimensions of 4 walls of room becomes K times more then area become K^2 times more.

(43) It costs Rs 0.3 per square centimetre to carpet a room. The cost of carpeting a room $\dfrac{x}{100}$ metres long and half as broad is

(a) Rs $\dfrac{3x^2}{100}$ (b) Rs $\dfrac{3}{20}x^2$ (c) Rs $\dfrac{3x}{50}$

(d) Rs $\dfrac{9x^2}{100}$ (e) Rs $\dfrac{3x^2}{200}$

(44) A picture 30″ × 20″ has a frame $2\dfrac{1''}{2}$ wide. The area of the picture is approximately how many times the area of the frame?

(a) 4 (b) $2\dfrac{1}{2}$ (c) 2 (d) 5 (e) $4\dfrac{1}{2}$

(45) A rectangular grass plot in a drawing room measures 4′ × 3′. How many 2 inch square tiles would have to be put around its outside edge to completely frame the plot?

(a) 88 (b) 84 (c) 432 (d) 864 (e) 216

(46) By how much % approximately is the diagonal of a square less than two of its sides combined?

(a) 30 (b) 40 (c) 29 (d) 39 (e) Data insufficient

Hint: % less = $\dfrac{2a - a\sqrt{2}}{2a} \times 100\%$.

47) A typist uses a paper 12″ by 5″ length wise and leaves a margin of 1″ at the top and the bottom and a margin of $\dfrac{1''}{2}$ on either side. What fractional part of the paper is available to him for typing?

(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{5}{7}$ (e) $\dfrac{3}{4}$

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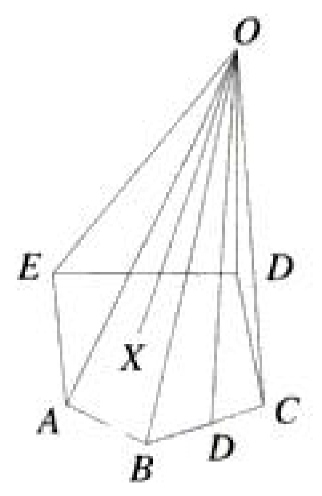
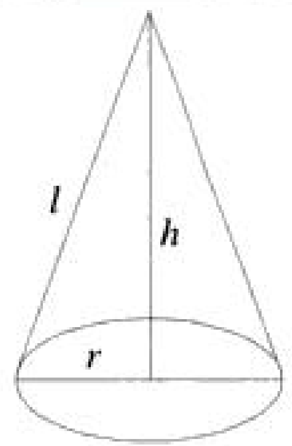
Name of Solid Figure	Figure	Shape of Curved Surface/Base	Surface Area: Curved (C)	Surface Area: Base (B)	Surface Area: Total (C + 2B)	Volume (V)
Pyramid	 *AOB*, *BOC*, *COD*, *DOE* and *EOA* are side (curved) faces *D* is the mid point of any edge of base *OD* is the slant height = *l* *OX* is the perpendicuiar dropped on base from common vertex *O* *OX* is the altitude = *h*	**Surface** Triangle **Base** Polygon	$= \frac{1}{2} \times P \times l$ where *P* is the base perimeter *l* is the slant height	Area of polygon		$\frac{1}{3} \times$ (base area) $\times$ altitude
Cone (Right Circular)	 *r* is the radius of base *h* is the altitude *l* is the slant height	**Base** Circle	$\pi r l$ where $l = \sqrt{h^2 + r^2}$	πr^2	$\pi r (l + r)$	$\frac{1}{3}$ (base area) $\times$ altitude $= \frac{1}{3} \pi r^2 \times h$

(*Contd.*)

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Rule

If the denominator is more by 'K' times the numerator, then the ratio or given (fraction) is less by $\frac{1}{K}$
Similarly, if the denominator is less by K times the numerator, then the given ratio (fraction) is more by $\frac{1}{K}$.

24.3 PERCENTAGE

Ratios or proportions with the base 100 are known as **percantages** (%). For example, a ratio $\frac{x}{y}$ becomes $\frac{x}{y} \times 100$ in percentage form, and it denotes-what part of y is x if y is 100.

Percentage is a very useful tool for comparison in the analysis of data. For example, the ratio of girls to total students in school P and school Q are $\frac{1174}{2333}$, and $\frac{665}{1406}$.

This can however be better comprehended in a percentage form, which is for,

$$\text{School A} = \frac{1174}{2333} \times 100 = 50.3 \text{ per cent}$$

$$\text{School B} = \frac{665}{1406} \times 100 = 47.3 \text{ per cent}$$

This reveals that the girl ratio is little over 50% in school A i.e. no. of girls is more than the no. of boys, on the other hand, school B with 47.3% girl ratio has a predominance of boys.

24.3.1 Techniques to Interpret the Data Involving the Percentage

P(1) *To find by how much per cent X is more or less than Y (or over Y) or compared to Y.*

$$\text{Required percentage} = \frac{\text{value of } X - \text{value of } Y}{\text{value of } Y} \times 100$$

base value for comparison

So, the denominator part contains the value with which the comparison is made.

NB: than Y/over Y/compared to Y means the base of comparison is value of Y and hence, the value of Y will be in the denominator.

P(2) *To find the percentage change in any value in year M compared to that value in year K (or over year K)*

Compared to year K (or over year K) means the base of comparison is the value in year K.

base year

$$\Rightarrow \quad \text{Required \% change } (x) = \frac{\text{value (year } M) - \text{value (year } K)}{\text{Value (Year } K)} \times 100$$

only base year in denominator

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E-8 In which year did Kerala contribute approximately one-fifth in the total number of students in that year?

(a) 1995 (b) 1996 (c) 1997 (d) 1998 (e) 1999

S-8 *(c)*

Contributions of Kerala in different years

$$1995 \longrightarrow \frac{1035}{5308} \approx \frac{103}{530} \approx \frac{1}{5}$$

$$1996 \longrightarrow \frac{940}{5395} \approx \frac{94}{540} \approx \frac{1}{6}$$

$$1997 \longrightarrow \frac{1200}{5955} \approx \frac{1200}{6000} = \frac{1}{5} \quad \left(\text{closest to } \frac{1}{5}\right)$$

$$1998 \longrightarrow \frac{1400}{5895} \approx \frac{14}{59} \approx \frac{1}{4}$$

$$1999 \longrightarrow \frac{1500}{6435} \approx \frac{15}{64} \approx \frac{1}{4}$$

Tips: Do not try to calculate the above ratios, rather do approximation and mental calculation to check which ratio is closest to $\frac{1}{5}$.

E-9 In the case of which state/U.T. was there an increase and decrease respectively in number of students in alternate years?

(a) West Bengal (b) Kerala (c) Maharashtra (d) Karnataka (e) Delhi

S-9 *(a)*

E-10 From which state/U.T. did least number of students go abroad over the years among the given states/ U.T.?

(a) Kerala (b) Maharashtra (c) Delhi (d) West Bengal (e) Karnataka

S-10 *(d)*

State/U.T.	*No. of studetns going abroad*
Maharashtra	4308
Kerala	6075
Karnataka	3655
West Bengal	2625 ⟶ Least
Delhi	8100
Andhra Pradesh	4255

Directions (11-15): Study the following table and answer the questions given below it.

Annual Income of Five Schools

Figures in '00 rupees

Sources of income	*School*				
	A	*B*	*C*	*D*	*E*
Tuition Fee	120	60	210	90	120
Term Fee	24	12	45	24	30
Donations	54	21	60	51	60
Grants	60	54	120	42	55
Miscellaneous	12	3	15	3	15
Total	270	150	450	210	280

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E-40 How many districts have same $\frac{SMT}{MT}$ ratio?

(a) One (b) Two (c) Three (d) Four (e) None

S-40 *(b)* Since $\frac{SMT}{MT}$ <<1, so, calculate $\frac{MT}{SMT}$ and check for how many districts the value of $\frac{MT}{SMT}$ are same (Refer S-36).

Two districts (C & E) have same $\frac{SMT}{MT}$ ratio.

Directions (41-45): Study the following table and answer the questions below it.

Percentage Share of Major Countries in Global Producion of Natural Rubber

Figures in percentage

Year	Countries				
	A	B	C	D	Others
1960	4	27	38	16	15
1970	7	25	39	13	16
1980	9	23	38	14	16
1990	8	28	36	13	13
2000	11	22	34	15	18

E-41 In which country, the percentage share of global production remains most steady during 1960-2000?

(a) A (b) B (c) C (d) D (e) others

S-41 *(d)*

Here standard deviation need not be calculated, but difference of maximum value and minimum value will serve.

	A	B	C	D	others
Difference	11-4	28-22	39-34	16-13	18-13
	7	6	5	3	5

$\downarrow$ minimum deviation, most steady

Hence, the percentage share is most steady in D.

E-42 If the global production of rubber in 1980 was 144 thousand tonnes then production in country D was (in tonnes)

(a) 21×10^5 (b) 175000 (c) 11000 (d) 2×10^4 (e) 18300

S-42 *(d)*

The production of rubber in D in 1980 = 14% of 144 thousand tonnes

$= 0.14 \times 144 \times 10^3$ tonnes $= 2 \times 10^4$ tonnes

Tips: Do not try to calculate 0.14×144, because $(14)^2 = 196$

Therefore, $14 \times 144 \approx 2000 \Rightarrow 0.14 \times 144 = 20$

E-43 If the global production of rubber in 2000 is double that of 1970, then the percentage change in production at B during the period is

(a) –12 (b) $-42\frac{2}{11}$ (c) + 76 (d) + 19 (e) + 12

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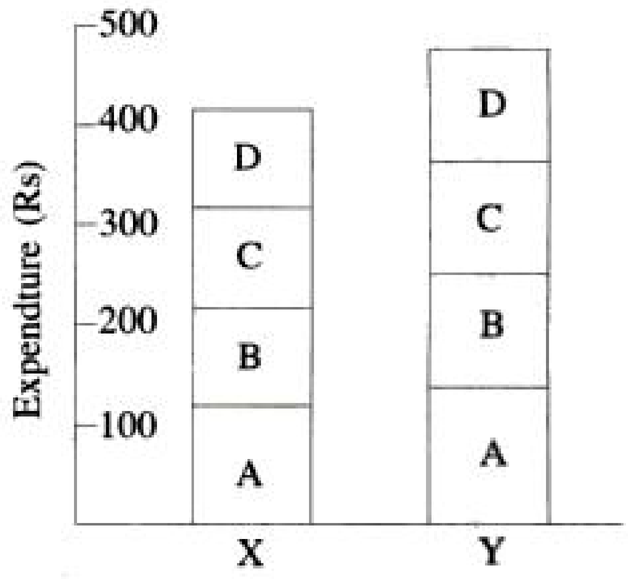
(b) Sub-divided Bar Chart or Component Bar Chart A sub-divided bar chart is used to represent various parts or sub-classes of total magnitude of the given variable.
Sub-divided bar chart showing the monthly expenditure of two persons *X* & *Y*



A – expenditure on food

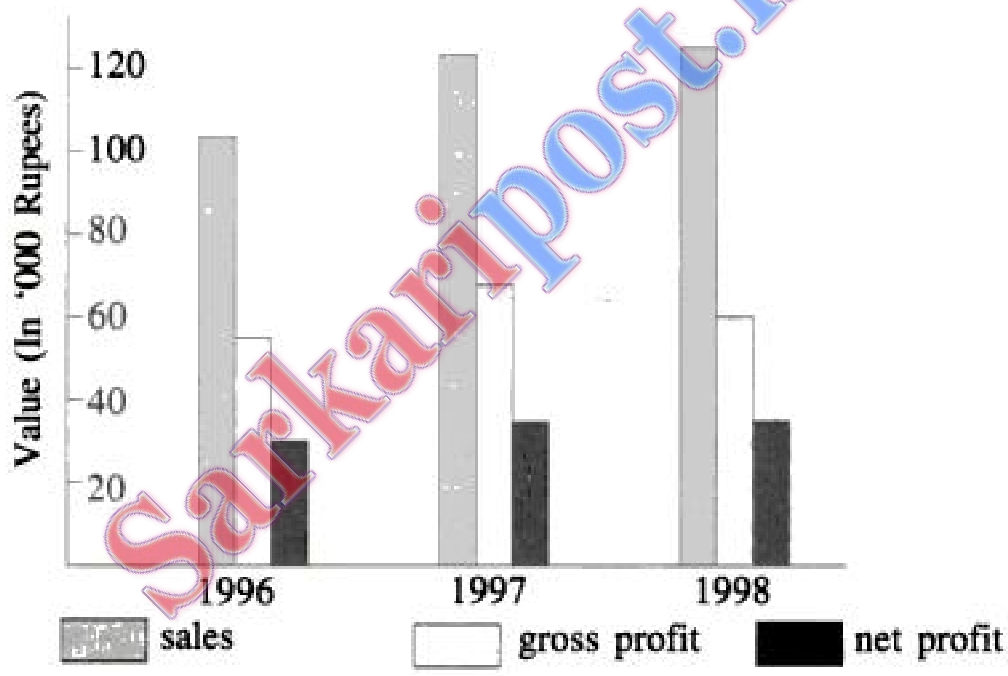
B – expenditure on education

C – expenditure on clothing

D – Miscellaneous expenditure

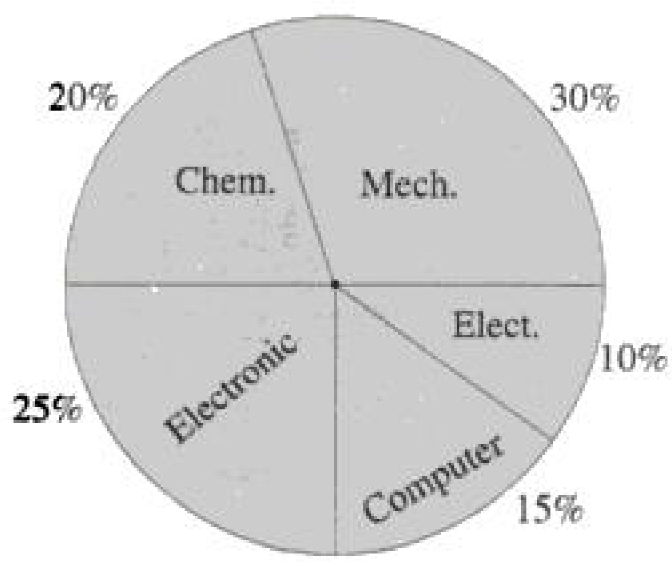
(c) Multiple Bar Charts In this type, two or more bars are constructed adjoining each other, to represent either different components of a total or to show miltiple variables.

In the adjoining bar chart three parameters of a company *X* have been represented for the years 1996, 1997, 1998



(ii) Pie Charts

In this method of data representation, the total quantity is distributed over a total angle of 360° which is one complete circle or pie. Unlike the bar charts, here the data can be plotted with respect to only one parameter.



Total Engg. Students = 800

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From (B), $x - y = 4 \Rightarrow \frac{x}{y}$ *cannot be found out*

$\Rightarrow$ *(B) is insufficient.*]

62. What is the value of $p/2$?

(a) $2p - q = 10$

(b) $p + q = 8$ *Ans.* **(e)**

[Justification: *Since two unknowns are there and two distinct equations are given, p/2 can be found out with (A) and (B) together.*]

63. Is y greater than x?

(a) $3x + 2y = 11$

(b) $x - y = 2$ *Ans.* **(b)**

[Justification: *(A) does not indicate whether y is greater or smaller than x so, (A) is not sufficient. (B) indicates that* $x - y$ *is a (+) ve quantity* $\Rightarrow$ *y is smaller than x.* $\Rightarrow$ *(B) alone is sufficient.*]

64. If $y = 3$, what is the value of $y - x$?

(a) $y + x = 6$

(b) $x = 3$ *Ans.* **(e)**

[Justification: *The question asks about the value of an expression* $y - x$ *which involves two unknowns and hence two distinct equations are needed. It is to be noted here that the question itself has one equation as* $y = 3$. *Hence only one more equation is required. Therefore, either (A) or (B) alone is sufficient.*]

65. What are the dimensions of a certain rectangle?

(a) The perimeter of the rectangle is 68

(b) The diagonal of the rectangle is 26 *Ans.* **(e)**

[Justification: *(A) gives that* $2x + 2y = 68$ *and*
(B) gives $x^2 + y^2 = 26^2$. *x, y are length and breadth.*
Therefore, two unknowns can be found out from two distinct equations.
Therefore (A) and (B) together are sufficient.]

66. Is $p > q$?

(a) $p^2 > q^2$

(b) $p - q > 0$ *Ans.* **(b)**

[Justification: *(A) does not imply whether p or q is (+) ve or (–) ve because the square of any number is always positive. Therefore, if* $p^2 = 9$ *(say) and* $q^2 = 4$ *then p may be + 3 or –3 and q may be +2 or –2. Therefore, comparison of p and q is not possible.*
(B) states that $p > q \Rightarrow$ *(B) alone is sufficient.*]

Type-2 (Three Statement Types)

Directions: In each of the following questions, a question is asked followed by three statements. While answering the question, you may or may not require the data provided in all the statements. You have to read the question and the three statements and then decide upon whether the question can be answered with any one or two of the statements or all the three statements are required to answer the question. The answer number bearing the statements which are required to answer the question is your answer.

67. What is the difference between the present ages of Kaberi and Dipa?

(I) The ratio between Kaberi's present age and her age after eight years is 4 : 5.

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What will come in place of (d)?

(1) 95 (2) 51

(3) 99 (4) 49

(5) None of these

13. 7 4 5 9 20 52.5

3 (a) (b) (c) (d) (e)

What will come in place of (c)?

(1) 4.5 (2) 2

(3) 6 (4) 7

(5) None of these

14. 3 10 32 111 460 2315

2 (a) (b) (c) (d) (e)

What will come in place of (b)?

(1) 29 (2) 30

(3) 26 (4) 28

(5) None of these

15. 5 8 6 10 7 12

7 (a) (b) (c) (d) (e)

What will come in place of (c)?

(1) 14 (2) 16

(3) 9 (4) 11

(5) None of these

Directions (16-20): In each of the following questions, a question if followed by information given in three statements. You have to study the question alongwith the statements and decide the information given in which of the statement (s) is necessary to answer the question.

16. What is the cost of flooring the rectangular hall?
    I. Length and the breadth of the hall are in the respective ratio 3 : 2.
    II. Length of the hall is 48 metres and cost of flooring is Rs 850/- per square metre.
    III. Perimeter of the hall is 160 metres and cost of flooring is Rs 850/- per square metre.
    (1) Only I and II
    (2) Only I and III
    (3) Only III
    (4) Only I and either II or III
    (5) Any two of the three
17. What is the rate of interest p.c.p.a.?
    I. An amount doubles itself in 5 years on simple interest.
    II. Difference between the compound interest and the simple interest earned on a certain amount in two years is Rs 400.
    III. Simple interest earned per annum is Rs 2,000.
    (1) Only I
    (2) Only II and III
    (3) Any two of the three
    (4) All I, II and III
    (5) Only I or only II and III
18. What is the two digit number?
    I. The difference between the two digit number and the number formed by interchanging the digits is 27.
    II. The difference between the two digits is 3.
    III. The digit at unit's place is less than that at ten's place by 3.
    (1) Only I and II
    (2) Only I and either II or III
    (3) Only I and III
    (4) All I, II and III
    (5) Even with all the three statements answer cannot be given.
19. What is the present age of Subir?
    I. The present age of Subir is half to that of his father.
    II. After 5 years, the ratio's of Subir's age to that of his father's age will be 6 : 11.
    III. Subir is 5 years younger than his brother.
    (1) Only I and II
    (2) Only I and III
    (3) Only II and III
    (4) All I, II and III
    (5) Even with all the three statements answer cannot be given.
20. In how many days can 10 women finish a work?
    I. 10 men can complete the work in 6 days.
    II. 10 men and 10 women together can complete the work in $3\frac{3}{7}$ days
    III. If 10 men work for 3 days and thereafter 10 women replace them, the remaining work is completed in 4 days.
    (1) Only I and II
    (2) Any two of the three
    (3) Only I and III

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35. What is the capacity of a cylindrical tank?
   (I) Radius of the base is half of its height which is 28 metres.
   (II) Area of the base is 616 sq. metres and its height is 28 metres.

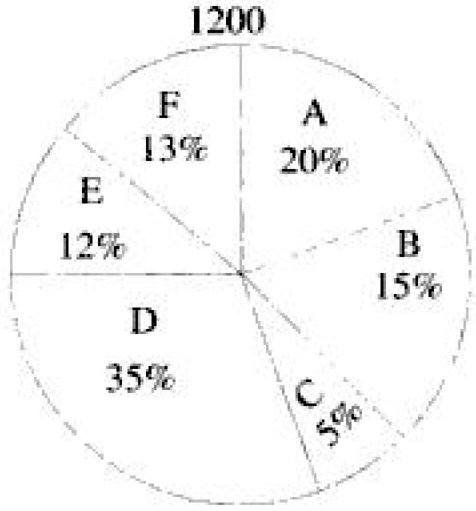
Directions (36–40): Study the following information to answer the given questions.

Percentage of students in various courses (*A, B, C, D, E, F*) and percentage of girls out of these.

Total students : 1200

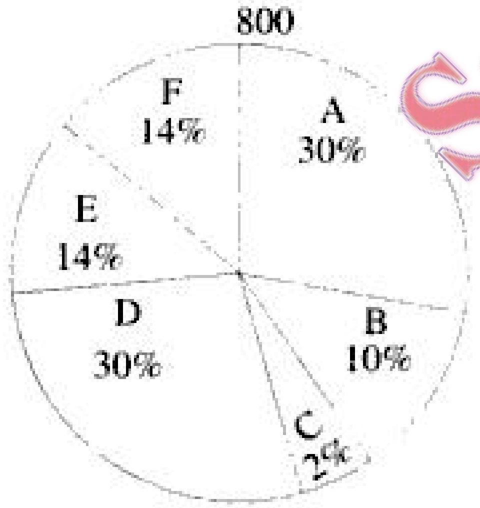
(800 girls + 400 boys)

Percentage students in various courses.



Total Girls : 800

Percentage of Girls in courses



36. For course *D* what is the respective ratio of boys and girls?
   (1) 3 : 4 (2) 4 : 5
   (3) 3 : 5 (4) 5 : 6
   (5) None of these
37. For which pair of courses is the number of boys the same?
   (1) *E* & *F* (2) *A* & *D*
   (3) *C* & *F* (4) *B* & *D*
   (5) None of these
38. For course *E*, the number of girls is how much per cent more than the boys course *E*?
   (1) 250 (2) 350
   (3) 150 (4) 80
   (5) None of these
39. For which course is the number of boys the minimum?
   (1) *E* (2) *F*
   (3) *C* (4) *A*
   (5) None of these
40. How many girls are in course *C*?
   (1) 44 (2) 16
   (3) 40 (4) 160
   (5) None of these

Directions (41–45): In each of these questions a number series is given. Only one number is **wrong** each series. You have to find out the **wrong** number.

41. 10 15 24 35 54 75 100
   (1) 35 (2) 75
   (3) 24 (4) 15
   (5) 54
42. 1 3 4 7 11 18 27 47
   (1) 4 (2) 11
   (3) 18 (4) 7
   (5) 27
43. 3 2 3 6 12 37.5 115.5
   (1) 37.5 (2) 3
   (3) 6 (4) 2
   (5) 12
44. 2 8 32 148 765 4626 32431
   (1) 765 (2) 148
   (3) 8 (4) 32
   (5) 4626
45. 2 3 11 38 102 229 443
   (1) 11 (2) 229
   (3) 102 (4) 38
   (5) 3

Directions (46–50): In each of the following questions two equations are given. You have to solve them and

Give answer (1) if $p < q$;
Give answer (2) if $p > q$;
Give answer (3) if $p \leq q$
Give answer (4) if $p \geq q$
Give answer (5) if $p = q$

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Answers

1. (2)	**2.** (4)	**3.** (3)	**4.** (2)	**17.** (1)	**18.** (1)	**19.** (1)	**20.** (4)
5. (2)	**6.** (2)	**7.** (4)	**8.** (1)	**21.** (1)	**22.** (3)	**23.** (4)	**24.** (3)
9. (2)	**10.** (4)	**11.** (3)	**12.** (4)	**25.** (4)	**26.** (2)	**27.** (1)	**28.** (3)
13. (3)	**14.** (2)	**15.** (3)	**16.** (1)	**29.** (1)	**30.** (3)		

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28. What is the rate of interest p.c.p.a.?
    I. The amount doubles itself in 10 years.
    II. The simple interest accrued in 5 years in Rs 5,000.
29. In how many days can *B* alone complete the work?
    I. *B* and *C* together can complete the work in 8 days.
    II. *A* and *B* together can complete the work in 12 days.
30. Want is the mother's present age?
    I. Present ages of Mother and child are in the ratio of 5 : 1 respectively.
    II. Four years hence the ratio of the ages of mother and child will be 17 : 5 respectively.

Directions (31–35): Study the following table carefully to answer these questions:

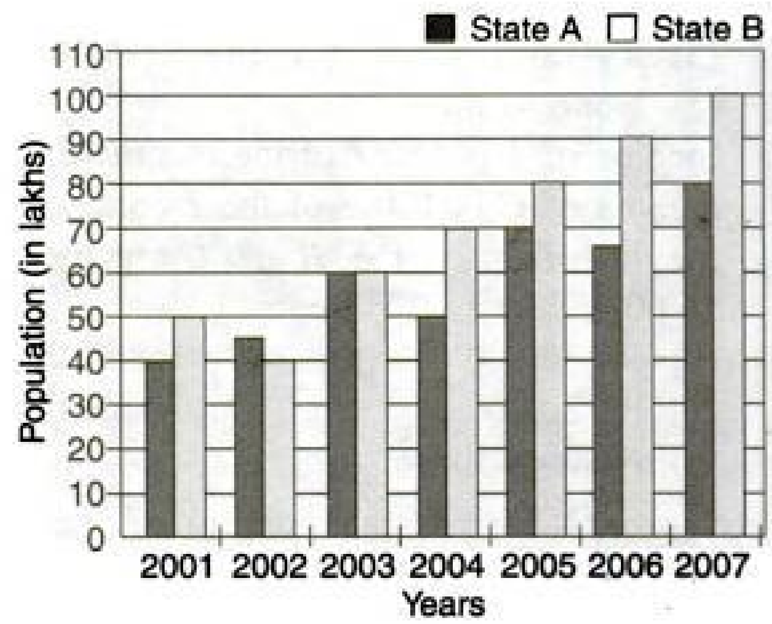
Table Showing Number (in lakhs) of Instruments Manufactured by Six Companies Over the Years

Year	*Company*					
	A	*B*	*C*	*D*	*E*	*F*
2002	45	35	48	42	50	49
2003	40	32	52	46	48	45
2004	48	36	50	43	56	48
2005	49	37	45	48	52	44
2006	46	30	55	50	54	50
2007	52	38	47	40	51	52

31. What is the average number of instruments manufactured by Company *C* for all the given years?
    (1) 49,00,000 (2) 49,50,000
    (3) 48,50,000 (4) 48,00,000
    (5) None of these
32. No. of instrument manufactured by Company *E* in 2004 is **approximately** what per cent of the total no. of instruments manufactured by all the companies together in 2004?
    (1) 23 (2) 25
    (3) 20 (4) 16
    (5) 18
33. Total number of instruments manufactured by Company *A* is **approximately** what per cent of the total no. of instruments manufactured by Company *F* for all the given years together?
    (1) 97 (2) 87
    (3) 92 (4) 90
    (5) 85
34. No. of instruments manufactured by Company *B* in 2005 is **approximately** what per cent of the total no. of instruments manufactured by Company *B* in all the years together?
    (1) 20 (2) 16
    (3) 14 (4) 18
    (5) 22
35. What is the ratio between the total no. of instruments manufactured by all the companies together in 2007 and that in 2006 respectively?
    (1) 56 : 57 (2) 57 : 56
    (3) 29 : 28 (4) 28 : 29
    (5) None of these

Directions (36–40): Study the following graph carefully to answer the questions:

Population of Two States (in lakhs) Over the Years



36. Population of State *B* in 2002 is what per cent of the total population of State *B* in all the years together? (Rounded off to two digits after decimal)
    (1) 8.26 (2) 7.26
    (3) 8.32 (4) 7.82
    (5) None of these

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Percentage of Marks Obtained by Six Students in Different Subjects

Student	*Subject*					
	History	*Geography*	*Mathematics*	*Science*	*Economics*	*English*
	Maximum Marks					
	50	*50*	*100*	*75*	*100*	*120*
A	86	82	77	72	69	66
B	80	74	73	66	76	84
C	68	92	89	78	75	72
D	76	84	83	81	59	60
E	72	66	82	87	62	78
F	84	64	93	63	81	54

46. What are the average marks obtained by all students in Geography?
(1) 41.5 (2) 42.5
(3) 38.5 (4) 39.5
(5) None of these

47. Marks obtained by *E* in Economics are **approximately** what percent of the total marks obtained by him in all subjects together?
(1) 12 (2) 8
(3) 10 (4) 22
(5) 17

48. What is the overall percentage of marks obtained by all the students in English?
(1) 83 (2) 69
(3) 71 (4) 81
(5) None of these

49. What are total marks obtained by *D* in History, Mathematics, Science and English together?
(1) 256.25 (2) 275.55
(3) 235.75 (4) 253.75
(5) None of these

50. What are the total marks obtained by *B* in all subjects together?
(1) 376.3 (2) 373.6
(3) 376.6 (4) 373.3
(5) None of these

Answers

1. (1)	**2.** (5)	**3.** (2)	**4.** (4)
5. (3)	**6.** (5)	**7.** (1)	**8.** (4)
9. (2)	**10.** (3)	**11.** (1)	**12.** (4)
13. (4)	**14.** (1)	**15.** (5)	**16.** (2)
17. (5)	**18.** (2)	**19.** (3)	**20.** (4)
21. (4)	**22.** (3)	**23.** (5)	**24.** (3)
25. (1)	**26.** (3)	**27.** (5)	**28.** (1)
29. (4)	**30.** (5)	**31.** (2)	**32.** (3)
33. (1)	**34.** (4)	**35.** (1)	**36.** (5)
37. (3)	**38.** (1)	**39.** (2)	**40.** (4)
41. (1)	**42.** (3)	**43.** (4)	**44.** (4)
45. (2)	**46.** (3)	**47.** (5)	**48.** (2)
49. (4)	**50.** (1)		

SET 21

PUNJAB NATIONAL BANK (MT), 2009

Directions (1–10): What should come in place of the question mark (?) in the following questions?

1. $\sqrt{571536} \div 42 \times ? = 5850$
   (1) 420 (2) 240
   (3) 315 (4) 325
   (5) None of these
2. $(34)^{56} \times (34)^{-53} = ?$
   (1) 39304 (2) 1156
   (3) 170504 (4) 102
   (5) None of these
3. $378.35 + 478 \div 12.5 = ?$
   (1) 508.268 (2) 416.59
   (3) 425.28 (4) 68.508
   (5) None of these
4. $(550\% \text{ of } 250) \div 275 = ?$
   (1) 15 (2) 1.5
   (3) 0.5 (4) 25
   (5) None of these
5. $334.41 + 47.26 + 1.25 + 5 + 0.66 = ?$
   (1) 411.24 (2) 396.15
   (3) 388.58 (4) 376.85
   (5) None of these
6. $74844 \div ? = 54 \times 63$
   (1) 34 (2) 42
   (3) 22 (4) 54
   (5) None of these
7. $(21.35)^2 + (12.25)^2 = ?$
   (1) 171.4125 (2) 605.885
   (3) 604.085 (4) 463.8125
   (5) None of these
8. $124 + 56 \times 1.5 - 12 = ?$
   (1) –1890 (2) 252
   (3) 230 (4) 196
   (5) None of these
9. $\sqrt[3]{1092727} = ?$
   (1) 108 (2) 99
   (3) 97 (4) 107
   (5) None of these
10. $(46351 - 36418 - 4505) \div ? = 1357$
    (1) 4 (2) 6
    (3) 3 (4) 2
    (5) None of these

Directions (11–15): What should come in place of the question mark (?) in the following number series?

11. 124 228 436 ? 1684 3348
    (1) 844 (2) 852
    (3) 872 (4) 834
    (5) None of these
12. 1108 1117 1142 1191 ? 1481
    (1) 1312 (2) 1300
    (3) 1272 (4) 1204
    (5) None of these
13. 25 30 70 260 1280 ?
    (1) 6400 (2) 7680
    (3) 6380 (4) 7660
    (5) None of these
14. 8484 4248 2112 1074 513 ?
    (1) 201 (2) 280.5
    (3) 256.5 (4) 171
    (5) None of these
15. 154 162 226 ? 954 1954
    (1) 242 (2) 554
    (3) 442 (4) 642
    (5) None of these
16. The number obtained by interchanging the two digits of a two-digit number is less than the original number by 18. The sum of the two digits of the number is 16. What is the original number?
    (1) 97 (2) 87
    (3) 79
    (4) cannot be determined
    (5) None of these
17. By how much is $\frac{4}{5}$ of 1150 less than $\frac{5}{6}$ of 1248?
    (1) 140 (2) 115
    (3) 125 (4) 120
    (5) None of these
18. In how many different ways can the letters of the word 'OPERATE' be arranged?
    (1) 5040 (2) 720

(1) $\frac{5}{8}$ (2) $\frac{3}{5}$

(3) $\frac{4}{5}$ (4) $\frac{5}{6}$

(5) None of these

31. Farah got married 8 years ago. Today her age is $1\frac{2}{7}$ times her age at the time of her marriage. At present her daughter's age is one-sixth of her age. What was her daughter's age 3 years ago?
(1) 6 years (2) 4 years
(3) 3 years
(4) Cannot be determined
(5) None of these

32. If $25a + 25b = 115$, then what is the average of a and b?
(1) 4.6 (2) 2.5
(3) 4.5 (4) 3.4
(5) None of these

33. On a test consisting of 250 questions, Jassi answered 40% of the first 125 questions correctly. What per cent of the other 125 questions does she need to answer correctly for her grade on the entire exam to be 60%?
(1) 75 (2) 80
(3) 60
(4) Cannot be determined
(5) None of these

34. Swapnil, Akash and Vinay begin to jog around a circular stadium. They complete their revolutions in 36 seconds, 48 seconds and 42 seconds respectively. After how many seconds will they be together at the starting point?
(1) 504 seconds (2) 940 seconds
(3) 1008 seconds (4) 470 seconds
(5) None of these

35. *A*, *B*, *C*, *D* and *E* are five consecutive odd numbers. The sum of *A* and *C* is 146. What is the value of *E*?
(1) 75 (2) 81
(3) 71 (4) 79
(5) None of these

Directions (36–40): Each question below is followed by two statements *A* and *B*. You are to determine whether the data given in the statement is sufficient to answer the question. You should use the data and your knowledge of Mathematics to choose between the possible answers.

Give answer (1) if the statement *A* alone is sufficient to answer the question, but the statement *B* alone is not sufficient.

Give answer (2) if the statement *B* alone is sufficient to answer the question, but the statement *A* alone is not sufficient.

Give answer (3) if both statements *A* and *B* together are needed to answer the question.

Give answer (4) if either the statement *A* alone or the statement *B* alone is sufficient to answer the question.

Give answer (5) if you cannot get the answer from the statements *A* and *B* together, but need even more data.

36. In how many years can a simple interest of Rs 6,570 be obtained on an amount of Rs 36,500?
A. The rate of simple interest is 6 p.c.p.a.
B. The difference between the simple interest and the compound interest is Rs 402.084.

37. What is a three-digit number?
A. Two-fifths of the number is half of 204.
B. 20% of the number is 51.

38. What is the number of teachers in a school?
A. Each teacher takes at least three lectures in a day.
B. There are 45 lectures in a week.

39. What is Raveena's age?
A. Raveena is half as old as Karishma.
B. Raveena's age is $\frac{3}{5}$ of her mother's age who is 45 years old.

40. What is the area of a rectangular plot?
A. The length of the plot is 375 metres.
B. The length of the plot is thrice its breadth.

41. The average of 5 positive integers is 436. The average of the first two numbers is 344 and the average of the last two numbers is 554. What is the third number?

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35. Manisha invests an amount of Rs 39,300 for 4 years at the rate of 4 pcpa. What amount of approximate compound interest will she obtain at the end of 4 years?
(1) Rs 6,675 (2) Rs 6,650
(3) Rs 6,288 (4) Rs 6,356
(5) Rs 6,450

Directions (36–40): Study the following table carefully and answer the questions given below it.

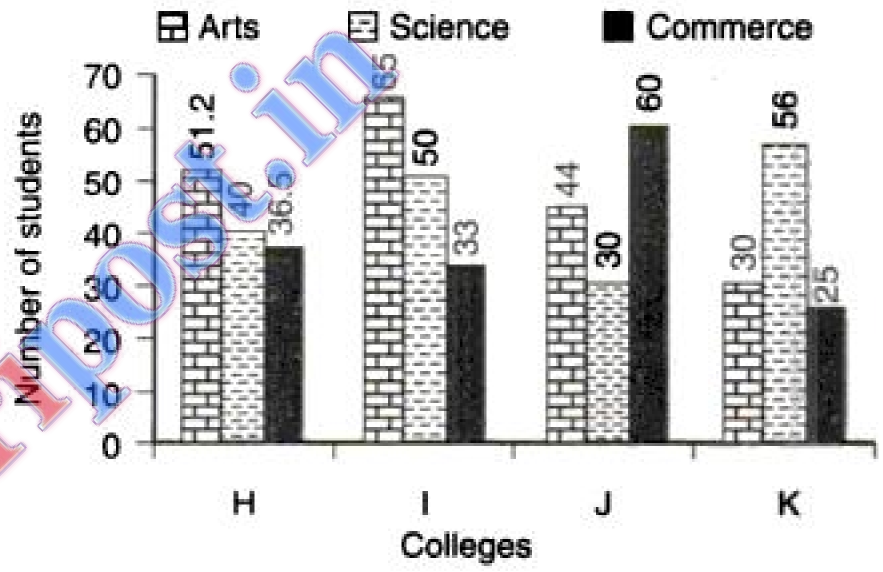
Price per kilogram (in Rs) at which 5 different farmers sell 6 different products

Product	*Farmers* P	Q	R	S	T
Rice	20	15	20.5	24	22
Wheat	18	16	15	15.5	20
Jowar	16.5	15	18	20	15
Bajra	15	14	14.5	13.5	12
Maize	13	14	12.5	12	14.5
Sugarcane	10	8	11.5	10.5	8

36. What is the difference between the average price of per kg of Maize sold by all the farmers and the average price of per kg of Sugarcane sold by all the farmers?
(1) Rs 3 (2) Rs 4.5
(3) Rs 4.2 (4) Rs 3.6
(5) None of these
37. What is the ratio of the total price (per kg) of all the products sold by *Q* to that sold by *R*?
(1) 41 : 43 (2) 43 : 48
(3) 43 : 46 (4) 41 : 48
(5) None of these
38. If farmer *Q* sells 14 kg of Wheat, 21 kg of Jowar and 30 kgs of Sugarcane, what will be his total earning?
(1) Rs 699 (2) Rs 779
(3) Rs 650 (4) Rs 786
(5) None of these
39. If farmer *P* sells 50 kgs. of Rice and farmer *S* sells 60 kg of Wheat, what is the difference in amount by either of the farmers in selling their products?
(1) Rs 70 (2) Rs 65
(3) Rs 80 (4) Rs 85
(5) None of these
40. If each farmer sells 100 kg of Jowar and 50 kg of Bajra, which farmer will earn the most?
(1) P (2) R
(3) S (4) Q
(5) T

Directions (41–45): Study the following graph carefully and answer the questions given below it.

Number of Students studying in Various Colleges from Various Faculties (Number in thousands)



41. What is the difference between the total number of students studying in college *H* and that studying in college *K*?
(1) 16100 (2) 15800
(3) 16300 (4) 16700
(5) None of these
42. What is the total number of students studying in all the colleges together?
(1) 520900 (2) 520700
(3) 610200 (4) 510800
(5) None of these
43. What is the ratio of the students from the faculty of Science from colleges *H* and *I* together to the students from the same faculty from colleges *J* and *K* together?
(1) 43 : 45 (2) 41 : 43
(3) 45 : 43 (4) 43 : 41
(5) None of these

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