# HoloScope: Topology-and-Spike Aware Fraud Detection

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#### ABSTRACT

As online fraudsters invest more resources, including purchasing large pools of fake user accounts and dedicated IPs, fraudulent attacks become less obvious and their detection becomes increasingly challenging. Existing approaches such as average degree maximization suffer from the bias of including more nodes than necessary, resulting in lower accuracy and increased need for manual verification. Hence, we propose HoloScope, which uses information from graph topology and temporal spikes to more accurately detect groups of fraudulent users. In terms of graph topology, we introduce "contrast suspiciousness," a dynamic weighting approach, which allows us to more accurately detect fraudulent blocks, particularly low-density blocks. In terms of temporal spikes, HoloScope takes into account the sudden bursts and drops of fraudsters' attacking patterns. In addition, we provide theoretical bounds for how much this increases the time cost needed for fraudsters to conduct adversarial attacks. Additionally, from the perspective of ratings, HoloScope incorporates the deviation of rating scores in order to catch fraudsters more accurately. Moreover, HoloScope has a concise framework and sub-quadratic time complexity, making the algorithm reproducible and scalable. Extensive experiments showed that HoloScope achieved significant accuracy improvements on synthetic and real data, compared with state-of-the-art fraud detection methods.

#### **KEYWORDS**

Graph Mining, Fraud Detection, Burst and Drop

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# 1 INTRODUCTION

Online fraud has become an increasingly serious problem due to the high profit it offers to fraudsters, which can be as much as \$5 million from 300 million fake "views" per day, according to a report of Methbot [20] on Dec 2016. Meanwhile, to avoid detection, fraudsters can manipulate their geolocation, internet providers, and IP address, via large IP pools (852,992 dedicated IPs). Suppose a fraudster has a accounts or IPs, and wants to rate or click b objects (e.g., products) at least 200 times, as required by their customers.

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The edge density of the fraudulent block is then  $200 \cdot b/(a \cdot b) = 200/a$ . We thus see that with enough user accounts or IPs, the fraudsters can serve as many customers as they need while keeping density low. This presents a difficult challenge for most existing fraud detection methods.

Current dense block detection methods [4, 26, 27] maximize the arithmetic or geometric average degree. We use "fraudulent density" to indicate the edge density that fraudsters create for the target objects. However, those methods have a bias of including more nodes than necessary, especially as the fraudulent density decreases, which we verified empirically. This bias results in low precision, which then requires intensive manual work to verify each user. Fraudar [11] proposed an edge weighting scheme based on inverse logarithm of objects' degrees to reduce this bias, which was inspired by IDF [23, 28]. However, their weighting scheme is fixed globally and affects both suspicious and normal edges, lowering the precision of Fraudar, which can be seen from results on semi-real (with injected labels) and real data (see Fig. 1).

Accurately detecting fraudulent blocks of lower density requires aggregating more sources of information. Consider the attribute of the creation time of edges: fraudulent attacks tend to be concentrated in time, e.g., fraudsters may surge to retweet a message, creating one or more sudden bursts of activity, followed by sudden drops after the attack is complete. Sudden bursts and drops have not been directly considered together in previous work.

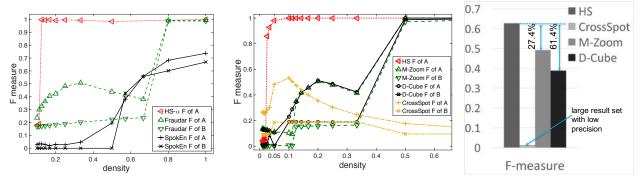
Tensor based methods [12, 26, 27] incorporate edge attributes into a multi-way tensor formulation, e.g., IPs, rating scores and time. However, those methods rely on time-binning to incorporate temporal information, and then treat time bins independently, which loses information about bursts and drops, which our approach captures.

Therefore, we propose HoloScope, which combines suspicious signals from graph topology, temporal bursts and drops, and rating deviation. Our graph topology-based weighting scheme dynamically reweights objects according to our beliefs about which users are suspicious. Temporally, HoloScope detects suspicious spikes of bursts and drops, which increases the time cost needed for fraudsters to conduct an attack. In terms of rating deviation, our approach takes into account how much difference there is between an object's ratings given by suspicious users and non-suspicious users.

In summary, our contributions are:

- Unification of signals: we make holistic use of several signals, namely connectivity (i.e., topology), temporal bursts and drops, and rating deviation in a systematic way.
- Theoretical analysis of fraudsters' obstruction: we show that if the fraudsters use less than an upper bound of time for an attack, they will cause a suspicious drop or burst. In other words, HoloScope obstructs fraudsters by increasing the time they need to perform an attack.

<sup>\*</sup>The work was done when Shenghua Liu was a visiting researcher at Carnegie Mellon University



(a) HS- $\alpha$  using topology information outper-(b) HS using holistic attributes provides clear (c) HS achieves the best F-measure, on real forms competitors improvement, and performs the best. data from Sina Weibo

Figure 1: (a) and (b) show experimental results on BeerAdvocate dataset. HS- $\alpha$  and HS are both our methods, where the former only uses topology information. We increase the # of injected fraudsters from 200 to 2000 for HS- $\alpha$ , and to 20000 for HS. The corresponding density is shown on the horizontal axis. (c) shows accuracy results on Sina Weibo, with ground truth labels.

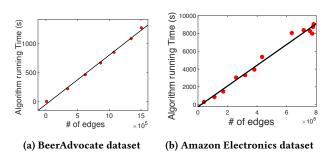


Figure 2: HoloScope (HS) runs in near-linear time.

- Effectiveness: we achieved higher accuracy than the competitors on semi-real and real datasets. In fact, HoloScope using only topology information (HS-α) outperformed the graph-based competitors (see Fig. 1a), while HoloScope (HS) using all attributes achieved further improvement, and outperformed the tensor-based competitors (see Fig. 1b and 1c).
- Scalability: HoloScope runs in subquadratic time in the number of nodes, under reasonable assumptions. Fig. 2 shows that its running time increases near-linearly with the number of edges.

In addition, in Microblog, Sina Weibo<sup>1</sup> data, HoloScope achieved higher F-measure than the competitors in detecting the ground truth labels, with high precision and recall. The code of HoloScope is open-sourced for reproducibility <sup>2</sup>.

#### 2 RELATED WORKS

Most existing works study anomaly detection based on the density of blocks within adjacency matrices [13, 22], or multi-way tensors [26, 27]. [8] and CoreScope [25] proposed to use Shingling and K-core algorithms respectively to detect anomalous dense block in

Table 1: Comparison between HoloScope and other fraud detection algorithms.

	Fraudar [11]	SpokEn [22]	CopyCatch [3]	CrossSpot [12]	BP-based methods [1, 21]	M-Zoom/D-Cube [26, 27]	HoloScope
scalability	✓.	✓	✓	✓.	<b>√</b>	✓.	<b>√</b>
camouflage	<b>√</b>		?	$\checkmark$		<b>√</b>	<b>√</b>
hy-community			?	?			<b>✓</b>
spike-aware			?				✓

huge graphs. Taking into account the suspiciousness of each edge or node in a real life graph potentially allows for more accurate detection. Fraudar [11] proposed to weight edges' suspiciousness by the inverse logarithm of objects' indegrees, to discount popular objects. [2] found that the degrees in a large community follow a power law distribution, forming hyperbolic structures. This suggests penalizing high degree objects to avoid unnecessarily detecting the dense core of hyperbolic community.

In addition to topological density, EdgeCentric [24] studied the distribution of rating scores to find the anomaly. In terms of temporal attribute, the identification of burst period has been studied in [15]. A recent work, Sleep Beauty (SB) [14] more intuitively defined the awakening time for a paper's citation burst for burst period. [31] clustered the temporal patterns of text phrases and hash tags in Twitter. Meanwhile, [9, 10] modeled the time stamped rating scores with Bayesian model and autoregression model respectively for anomalous behavior detection. Even though [6, 16, 30] have used bursty patterns to detect review spam, a sudden drop in temporal spikes has not been considered yet.

However, aggregating suspiciousness signals from different attributes is challenging. [5] proposed RRF (Reciprocal Rank Fusion) scores for combining different rank lists in information retrieval. However, RRF applies to ranks, not suspiciousness scores.

 $<sup>^{\</sup>rm 1}$  The largest Microblog service in China, http://www.weibo.com

<sup>&</sup>lt;sup>2</sup>https://github.com/shenghua-liu/HoloScope

CrossSpot [12], a tensor-based algorithm, estimated the suspiciousness of a block using a Poisson model. However, it did not take into account the difference between popular and unpopular objects. Moreover, although CrossSpot, M-Zoom [26] and D-Cube [27] can consider edge attributes like rating time and scores via a multi-way tensor approach, they require a time-binning approach. When time is split into bins, attacks which create bursts and drops may not stand out clearly after time-binning. The problem of choosing bin widths for histograms was studied by Sturges [29] assuming an approximately normal distribution, and Freedman-Diaconis [7] based on statistical dispersion. However, the binning approaches were proposed for the time series of a single object, which is not clear for different kinds of objects in a real life graph, namely, popular products and unpopular products should use different bin sizes.

Belief propagation [21] is another common approach for fraud detection which can incorporate some specific edge attributes, like sentiment [1]. However, its robustness against adversaries which try to hide themselves is not well understood.

CopyCatch [3] detected lockstep behavior by maximizing the number of edges in blocks constrained within time windows. However, this approach ignores the distribution of edge creation times within the window, and does not capture bursts and drops directly.

Finally, we summarize our competitors compared to our Holo-Scope in Table 1. We use "hy-community" to indicate whether the method can avoid detecting the naturally-formed hyperbolic topology which is unnecessary for fraud detection. HoloScope is the only one which considers temporal spikes (sudden bursts and drops and multiple bursts) and hyperbolic topology, in a unified suspiciousness framework.

# 3 PROPOSED APPROACH

The definition of our problem is as follows.

PROBLEM 1 (INFORMAL DEFINITION). Given quadruplets (user, object, timstamp, #stars), where timestamp is the time that a user rates an object, and #stars is the categorical rating scores.

- **Find** a group of suspicious users, and suspicious objects or its rank list with suspiciousness scores,
- to optimize the metric under the common knowledge of suspiciousness from topology, rating time and scores.

To make the problem more general, *timestamps* and *#stars* are optional. For example, in Twitter, we have (*user*, *object*, *timestamp*) triples, where *user* retweets a message *object* at *timestamp*. In a static following network, we have pairs (*user*, *object*), with *user* following *object*.

As discussed in previous sections, our metric should capture the following basic traits:

Trait 1 (Engagement). Fraudsters engage as much firepower as possible to boost customers' objects, i.e., suspicious objects.

Trait 2 (Less Involvement). Suspicious objects seldom attract non-fraudulent users to connect with them.

Trait 3 (Spikes: Bursts and Drops). Fraudulent attacks are concentrated in time, sometimes over multiple waves of attacks, creating bursts of activity. Conversely, the end of an attack corresponds to sudden drops in activity.

Trait 4 (Rating Deviation). High-rating objects seldom attract extremely low ratings from normal users. Conversely, low-rating objects seldom attract extremely high ratings from normal users.

Thus we will show in the following sections, that our proposed metric can make holistic use of several signals, namely topology, temporal spikes, and rating deviation, to locate suspicious users and objects satisfying the above traits. That is the reason we name our method as HoloScope (HS).

# 3.1 HoloScope metric

To give a formal definition of our metric, we describe the quadruplets (user, object, timstamp, #stars) as a bipartite and directed graph  $\mathcal{G} = \{U, V, E\}$ , which U is the source node set, V is the sink node set, and connections E is the directed edges from U to V. Generally, graph  $\mathcal{G}$  is a multigraph, i.e., multiple edges can be present between two nodes. Multiple edges mean that a user can repeatedly comment or rate on the same product at a different time, as common in practice. Users can also retweet message multiple times in the Microblog  $Sina\ Weibo$ . Each edge can be associated with rating scores (#stars), and timestamps, for which the data structure is introduced in Subsection 3.1.2.

Our HoloScope metric detects fraud from three perspectives: topology connection, timestamps, and rating score. To easily understand the framework, we first introduce the HoloScope in a perspective of topology connection. Afterwards, we show how we aggregate the other two perspectives into the HoloScope. We first view  $\mathcal G$  as a weighted adjacency matrix  $\mathbb M$ , with the number of multiple edges (i.e., edge frequency) as matrix elements.

Our goal is to find lockstep behavior of a group of suspicious source nodes  $A \subset U$  who act on a group of sink nodes  $B \subset V$ . Based on Trait 1, the total engagement of source nodes A to sink nodes B can be basically measured via density measures. There are many density measures, such as arithmetic and geometric average degree. Our HoloScope metric allows for any such measure. However, as the average degree metrics have a bias toward including too many nodes, we use a measure denoted by D(A, B) as the basis of the HoloScope, defined as:

$$D(A,B) = \frac{\sum_{v_i \in B} f_A(v_i)}{|A| + |B|}$$
 (1)

where  $f_A(v_i)$  is the total edge frequency from source nodes A to a sink node  $v_i$ .  $f_A(v_i)$  can also be viewed as an engagement from A to  $v_i$ , or A's lockstep on  $v_i$ , which is defined as

$$f_A(v_i) = \sum_{(u_j, v_i) \in E \land u_j \in A} \sigma_{ji} \cdot e_{ji}$$
 (2)

where constant  $\sigma ji$  are the global suspiciousness on an edge, and  $e_{ji}$  is the element of adjacency matrix  $\mathbb{M}$ , i.e., the edge frequency between a node pair  $(u_j, v_i)$ . The edge frequency  $e_{ji}$  becomes a binary in a simple graph. The global suspiciousness as a prior can come from the degree, and the extra knowledge on fraudsters, such as duplicated review sentences and unusual behaving time.

To maximize D(A, B), the suspicious source nodes A and the suspicious sink nodes B are mutually dependent. Therefore, we introduce *contrast suspiciousness* in an informal definition:

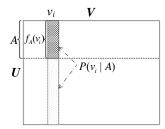


Figure 3: An intuitive view of our definitions in the Holo-Scope.

Definition 3.1 (contrast suspiciousness). The contrast suspiciousness denoted as  $P(v_i \in B|A)$  is defined as the conditional likelihood of a sink node  $v_i$  that belongs to B (the suspicious object set), given the suspicious source nodes A.

A visualization of the contrast suspiciousness is given in Fig. 3. The intuitive idea behind contrast suspiciousness is that in the most case, we need to judge the suspiciousness of objects by currently chosen suspicious users A, e.g., an object is more suspicious if very few users not in A are connected to it (see Trait 2); the sudden burst of an object is mainly caused by A (see Trait 3); or the rating scores from A to an object are quite different from other users (see Trait 4). Therefore, such suspiciousness makes use of the contrasts between users in A and users not in A or the whole set.

Finally, instead of maximizing D(A, B), we maximize the following expectation of suspiciousness D(A, B) over the probabilities  $P(v_i \in B|A)$ :

$$\max_{A} HS(A) := \mathbb{E} [D(A, B)]$$

$$= \frac{1}{|A| + \sum_{k \in V} P(v_k | A)} \sum_{i \in V} f_A(v_i) P(v_i | A)$$
 (3)

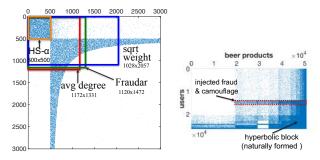
where for simplicity we write  $P(v_i|A)$  to mean  $P(v_i \in B|A)$ .  $1 - P(v_i|A)$  is the probability of  $v_i$  being a normal sink node. We dynamically calculate the contrast suspiciousness for all the objects, after every choice of source nodes A.

Using this overall framework for our proposed metric HS(A), we next show how to satisfy the remaining Traits. To do this, we define contrast suspiciousness  $P(v_i|A)$  in a way that takes into account various edge attributes. This will allow greater accuracy particularly for detecting low-density blocks.

3.1.1 HS- $\alpha$ : Less involvement from others. Based on Trait 2, a sink node should be more suspicious if it only attracts connections from the suspicious source nodes A, and less from other nodes. Mathematically, we capture this by defining

$$P(v_i|A) \propto q(\alpha_i)$$
, where  $\alpha_i = \frac{f_A(v_i)}{f_U(v_i)}$  (4)

where  $f_U(v_i)$  is the weighted indegree of sink node  $v_i$ . Similar to  $f_A(v_i)$ , the edges are weighted by global suspiciousness.  $\alpha_i$  measures the involvement ratio of A in the activity of sink node  $v_i$ . The scaling function  $q(\cdot)$  is our belief about how this ratio relates to suspiciousness, and we choose the exponential form  $q(x) = b^{x-1}$ , where base b > 1.



(a) HS- $\alpha$  finds the exact dense (b) Hyperbolic community in block in the synthetic data BeerAdvocate data

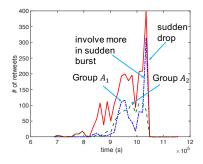
Figure 4: (a) The synthetic data consists of hyperbolic and rectangular blocks, with volume density around 0.84 and 0.60 respectively. The camouflage is randomly biased to columns of high degree. (b) A real data of naturally-formed hyperbolic community, and injected dense block. The injection is  $2000 \times 200$  with biased camouflage.

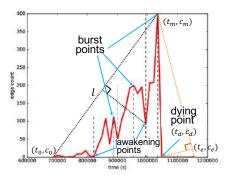
As previous work showed, large communities form hyperbolic structures, which is generated in our synthetic data (see the lower-right block in Fig. 4a), and also exists in real BeerAdvocate data (see Fig. 4b). For clarity, our HoloScope method are denoted as HS- $\alpha$  when it is only applied on a connection graph. The results of the synthetic data show that HS- $\alpha$  detected the exact dense rectangular block (b=128), while the other competitors included a lot of non-suspicious nodes from the core part of hyperbolic community resulting in low accuracy. In the beer review data from the BeerAdvocate website, testing on different fraudulent density (see Fig. 1a), our HS- $\alpha$  remained at high accuracy, while the other methods' accuracy drops quickly when the density drops below 70%.

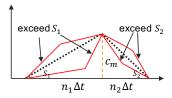
The main idea is that HS- $\alpha$  can do better because it dynamically adjusts the weights for sink nodes, penalizing those sink nodes that also have many connections from other source nodes not in A. In contrast, although Fraudar proposed to penalize popular sink nodes based their indegrees, these penalties also scaled down the weights of suspicious edges. The Fraudar (green box) only improved the unweighted "average degree" method (red box) by a very limited amount. Moreover, with a heavier penalty, the "sqrt weight" method (blue box) achieved better accuracy on source nodes but worse accuracy on sink nodes, since those methods used globally fixed weights, and the weights of suspicious were penalized as well. Hence the hyperbolic structure pushes those methods to include more nodes from its core part.

In summary, our HS- $\alpha$  using dynamic contrast suspiciousness can improve the accuracy of fraud detection in 'noisy' graphs (containing hyperbolic communities), even with low fraudulent density.

3.1.2 Temporal bursts and drops. Timestamps for edge creation are commonly available in most real settings. If two subgroups of Microblog users have the same number of retweets to a message, can we say they have the same suspiciousness? As an example shown in Fig. 5a, the red line is the time series (histogram of time bins) of the total retweets of a message in Microblog, Sina Weibo.







(a) Group  $A_1$  is more suspicious than  $A_2$ , due to (b) Detection of temporal bursts and drops the sudden burst and drop

(c) Proof the time cost obstruction

Figure 5: (a) and (b) are the time series (histogram) of a real message being retweeted in Microblog, Sina Weibo. The horizontal axis is the seconds after 2013 - 11 - 1. (c) illustrates our proof of time cost obstruction.

The blue dotted line and green dashed line are the retweeting time series respectively from user groups  $A_1$  and  $A_2$ . The two series have the same area under the time series curves, i.e., the same number of retweets. However, considering that fraudsters tend to surge to retweet a message to reduce the time cost, the surge should create one or more sudden bursts, along with sudden drops. Therefore, the suspiciousness of user groups  $A_1$  and  $A_2$  become quite different even though they have the same number of retweets, which cannot be detected solely based on connections in the graph. Thus we include the temporal attribute into our HoloScope framework for defining contrast suspiciousness.

Denote the list of timestamps of edges connected to a sink node v as  $T_v$ . To simplify notation, we use T without subscript when talking about a single given sink node v. Let  $\mathcal{T} = \{(t_0, c_0), (t_1, c_1), \cdots, (t_e, c_e)\}$  as the *time series* of T, i.e., the histogram of T. The count  $c_i$  is the number of timestamps in the time bin  $[t_i - \Delta t/2, t_i + \Delta t/2)$ , with bin size  $\Delta t$ . The bin size of histogram is calculated according to the maximum of Sturges criteria and the robust Freedman-Diaconis' criteria as mentioned in related works. It is worth noticing that the HoloScope can *tune* different bin sizes for different sink nodes, e.g., popular objects need fine-grained bins to explore detailed patterns. Hence, the HoloScope is more flexible than tensor based methods, which use a globally fixed bin size. Moreover, the HoloScope can update the time series at a low cost when T is increasing.

To consider the burst and drop patterns described in Trait 3, we need to decide the start point of a burst and the end point of a drop in time series  $\mathcal{T}$ . Let the burst point be  $(t_m, c_m)$ , having the maximum value  $c_m$ . According to the definition in previous work "Sleeping Beauty", we use an auxiliary straight line from the beginning to the burst point to decide the start point, named the awakening point of the burst. Fig. 5b shows the time series  $\mathcal{T}$  (red polygonal line) of a message from Sina Weibo, the auxiliary straight line l (black dotted line) from the lower left point  $(t_0, c_0)$  to upper right point  $(t_m, c_m)$ , and the awakening point for the maximum point  $(t_m, c_m)$ , which is defined as the point along the time series  $\mathcal{T}$  which maximizes the distance to l. As the dotted line perpendicular

to l suggests in this figure, the awakening point  $(t_a, c_a)$  satisfies

$$t_{a} = \underset{(c,t) \in \mathcal{T}, t < t_{m}}{\arg \max} \frac{|(c_{m} - c_{0})t - (t_{m} - t_{0})c + t_{m}c_{0} - c_{m}t_{0}|}{\sqrt{(c_{m} - c_{0})^{2} + (t_{m} - t_{0})^{2}}}$$
(5)

Finding the awakening point for one burst is not enough, as multiple bursts may be present. Thus, sub-burst points and the associated awakening points should be considered. We then propose a recursive algorithm *MultiBurst* in Alg. 1 for such a purpose.

#### Algorithm 1 MultiBurst algorithm.

**Input** Time series  $\mathcal{T}$  of sink node v, beginning index i, end index j **Output** A list of awakening-burst point pairs,

 $s_{am}$ : slope of the line passing through each point pair,  $\Delta c$ : altitude difference of each point pair.

# If j - i < 2 then return

 $(t_m, c_m)$  = point of maximum altitude between indices i and j.

 $(t_a, c_a)$  = the awakening point as Eq (5) between indices i and j.

 $\Delta c_{am} = c_m - c_a$ , and  $s_{am} = \frac{\Delta c_{am}}{(t_m - t_a)}$ 

Append  $\{(t_a, c_a), (t_m, c_m)\}$ ,  $s_{am}$ , and  $\Delta c_{am}$  into the output.

 $MultiBurst(\mathcal{T}, i, a-1)$ 

k = Find the first local min position from indices m + 1 to j  $MultiBurst(\mathcal{T}, k, j)$ 

After finding awakening and burst points, the contrast suspiciousness of burst awareness satisfies  $P(v_i|A) \propto q(\varphi_i)$ , where  $\varphi_i$  is the involvement ratio of source nodes in A in multiple bursts. Let the collection of timestamps from A to sink node  $v_i$  be  $T_A$ . Then,

$$\varphi_i = \frac{\Phi(T_A)}{\Phi(T_U)}, \text{ and } \Phi(T) = \sum_{(t_a, t_m)} \Delta c_{am} \cdot s_{am} \sum_{t \in T} \mathbf{1}(t \in [t_a, t_m])$$
 (6)

where  $s_{am}$  is the slope from the output of MultiBurst algorithm. Here  $s_{am}$  is used as a weight based on how steep the current burst is. This definition of suspiciousness satisfies Trait 3. It is worth noticing that the MultiBurst algorithm only needs to be executed once. With the preprocessed awakening and burst points, the contrast suspiciousness of edges connected to v has  $O(d_v)$  complexity, where  $d_v$  is the degree of sink node v. Hence the complexity for overall sink nodes are O(|E|).

In fact, sudden drops are also a prominent pattern of fraudulent behavior as described in Trait 3, since after creating the attack is complete, fraudsters usually stop their activity sharply. To make use of the suspicious pattern of a sudden drop, we define the *dying point* as the end of a drop. As Fig. 5b suggests, another auxiliary straight line is drawn from the highest point  $(t_m, c_m)$  to the last point  $(t_e, c_e)$ . The dying point  $(t_d, c_d)$  can be found by maximizing the distance to this straight line. Thus we can discover the "sudden drop" by the absolute slope value  $s_{bd} = \frac{(c_m - c_d)}{(t_d - t_m)}$  between the burst point and the dying point. Since there may be several drops in a fluctuated time series  $\mathcal{T}$ , we choose the drop with the maximum fall. To find the maximum fall, we also need a recursive algorithm, similar to Alg. 1:

1) Find a maximum point  $(t_m, c_m)$ , and the corresponding dying point  $(t_d, c_d)$  by definition; 2) Calculate the current drop slope  $s_{bd}$ , and the drop fall  $\Delta c_{bd} = c_m - c_d$ ; 3) Recursively find drop slope and drop fall for the left and right parts of  $\mathcal{T}$ , i.e.,  $t < t_m$  and  $t \geq t_d$  respectively.

As a result, the algorithm returns the maximum drop fall  $\Delta c_{bd}$ , and its drop slope  $s_{bd}$ , which it has found recursively. Finally, we use the weighted drop slope  $\Delta c_{bd} \cdot s_{bd}$  as a global suspiciousness in equation (2), to measure the drop suspiciousness. Each edge connected to the sink node v is assigned the same drop suspiciousness. We use a logarithm scale for smoothing those edge weights.

With this approach to detect bursts and drops, we now show that this provides a provable time obstruction for fraudsters.

Theorem 3.2. Let N be the number of edges that fraudsters want to create for an object. If the fraudsters use time less than  $\tau \geq \sqrt{\frac{2N\Delta t(S_1+S_1)}{S_1\cdot S_1}}$ , then they will be tracked by a suspicious burst or drop, where  $\Delta t$  is the size of time bins, and  $S_1$  and  $S_2$  are the slopes of normal rise and decline respectively.

PROOF. The most efficient way to create N edges is to have one burst and one drop, otherwise more time is needed. As shown in Fig. 5c, in order to minimize the slope, every point in the time series should in line with the two auxiliary straight lines to the highest point  $c_m$ , separately from the awakening and dying points. Otherwise, the slopes will exceed the normal values  $S_1$  and  $S_2$ . Hence we only consider the triangle with the auxiliary lines as its two edges. It is worth noticing that a trapezoid whose legs have the same slopes as the triangle's edges cannot have a shorter time cost. Then

$$\frac{c_m}{n_1\Delta t}=S_1, \qquad \frac{c_m}{n_2\Delta t}=S_2, \qquad (n_1+n_2)\cdot c_m=2N'.$$

Here  $n_1$  and  $n_2$  are the number of time bins before and after the burst. N' is the total number of rating edges, and  $N' \geq N$  consider some edges from normal users. Thus, solving the above equations, we have

$$\tau = (n_1 + n_2)\Delta t = \sqrt{\frac{2N'\Delta t(S_1 + S_2)}{S_1 \cdot S_2}} \ge \sqrt{\frac{2N\Delta t(S_1 + S_2)}{S_1 \cdot S_2}}$$

We also have the height of burst,  $c_m \ge \sqrt{\frac{2N\Delta t S_1 S_2}{S_1 + S_2}}$ . Thus, the maximum height of time series  $\mathcal{T}$  cannot be larger by far than that of a normal sink node. That is the reason that we use the weighted  $\varphi_i$  in equation (6) and weighted drop slope in equation (2).

3.1.3 Rating deviation and aggregation. We now consider edges with categorical attributes such as rating scores, text contents, etc. For each sink node  $v_i$ , we use the KL-divergence  $\kappa_i$  between the distributions separately from the suspicious source nodes A and the other nodes, i.e.,  $U \setminus A$ . We use  $U \setminus A$  for KL-divergence instead of the whole source nodes U, in order to avoid the trivial case where most of the rating scores are from A. The rating deviation  $\kappa_i$  is scaled into [0,1] by the maximum value before being passed into function  $q(\cdot)$  to compute contrast suspiciousness. The neutral scores can be ignored in the KL-divergence for the purpose of detecting fraudulent boosting or defamation. Moreover, rating deviation is meaningful when both A and  $U \setminus A$  have the comparable numbers of ratings. Thus, we weighted  $\kappa_i$  by a balance factor,  $\min\{f_A(v_i)/f_{U\setminus A}(v_i), f_{U\setminus A}(v_i), f_{U\setminus A}(v_i), f_{U\setminus A}(v_i)\}$ .

To make holistic use of different signals, i.e., topology, temporal spikes, and rating deviation, we need a way to aggregate those signals together. We have tried to use RRF (Reciprocal Rank Fusion) scores from Information Retrieval, and wrapped the scores with and without scaling function q(x). Compared to RRF score, we found that a natural way of joint probability by multiplying those signals together:

$$P(v_i|A) = b^{\alpha_i + \varphi_i + \kappa_i - 3},\tag{7}$$

was the most effective way to aggregate. In a joint probability, we can consider the absolute suspicious value of each signal, as opposite to the only use of ranking order. Moreover, being wrapped with q(x), the signal values cannot be canceled out by multiplying a very small value from other signals. A concrete example is that a suspicious spike can still keep a high suspiciousness score by multiplying a very small score from low fraudulent density.

Moreover, HoloScope dynamically updates the contrast suspiciousness  $P(v_i|A)$ . Thus the sink nodes being added with camouflage will have a very low contrast suspiciousness, with respect to the suspicious source nodes A. This offers HoloScope the resistance to camouflage.

#### 3.2 Algorithm

Before designing the full algorithm for large scale datasets, we firstly introduce the most important sub-procedure *GreedyShaving* in Alg. 2.

At the beginning, this greedy shaving procedure starts with an initial set  $A_0 \subset U$  as input. It then greedily deletes source nodes from A, according to users' scores S:

$$S(u_j \in A) = \sum_{v_i: (u_j, v_i) \in E} \sigma j i \cdot e_{ji} \cdot P(v_i|A),$$

which can be interpreted as how many suspicious nodes that user  $u_j$  is involved in. So the user is less suspicious if he has a smaller score, with respect to the current contrast suspiciousness  $\mathcal{P}$ , where we use  $\mathcal{P}$  to denote a vector of contrast suspiciousness of all sink nodes. We build a priority tree to help us efficiently find the user with minimum score. The priority tree updates the users' scores and maintains the new minimum as the priorities change. With removing source nodes A, the contrast suspiciousness  $\mathcal{P}$  change, in which we then update users' scores  $\mathcal{S}$ . The algorithm keeps reducing A until it is empty. The best  $A^*$  maximizing objective HS and  $P(v|A^*)$  are returned at the end.

#### Algorithm 2 GreedyShaving Procedure.

Given bipartite multigraph  $\mathcal{G}(U,V,E)$ , initial source nodes  $A_0\subset U$ .

Initialize:  $A=A_0$   $\mathcal{P}=$  calculate contrast suspiciousness given  $A_0$  S= calculate suspiciousness scores of source nodes A. MT= build priority tree of A with scores S.

while A is not empty  $\mathbf{do}$  u= pop the source node of the minimum score from MT.  $A=A\setminus u$ , delete u from A.

Update  $\mathcal{P}$  with respect to new source nodes A.

Update MT with respect to new  $\mathcal{P}$ . HS= estimate objective as Equation (3).

end while

return  $A^*$  that maximizes objective  $HS(A^*)$  and  $P(v|A^*)$ ,  $v\in V$ .

Since awakening and burst points have been already calculated for each sink node as an initial step before the *GreedyShaving* procedure, the calculation of the contrast suspicious P(v|A) for a sink node v only needs O(|A|) time. With source node j as the j-th one removed from  $A_0$  by the *GreedyShaving* procedure,  $|A_0| = m_0$ , and the out degree as  $d_i$ , the complexity is

$$\sum_{j=2,\cdots,m_0} O(d_j \cdot (j-1) \cdot \log m_0) = O(m_0|E_0|\log m_0)$$
 (8)

where  $E_0$  is the set of edges connected to source nodes  $A_0$ .

With the *GreedyShaving* procedure, our scalable algorithm can be designed so as to generate candidate suspicious source node sets. In our implementation, we use singular vector decomposition (SVD) for our algorithm. Each top singular vector gives a spectral view of high connectivity communities. However, those singular vectors are not associated with suspiciousness scores. Thus combined with the top singular vectors, our fast greedy algorithm is given in Alg. 3.

Theorem 3.3 (Algorithm complexity). In the graph  $\mathcal{G}(U,V,E)$ , given |V| = O(|U|) and  $|E| = O(|U|^{\epsilon_0})$ , the complexity of Fast Greedy algorithm is subquadratic, i.e.,  $o(|U|^2)$  in little-o notation, if the size of truncated user set  $|\tilde{U}^{(k)}| \leq |U|^{1/\epsilon}$ , where  $\epsilon > \max\{1.5, \frac{2}{3-\epsilon_0}\}$ .

PROOF. The FastGreedy algorithm executes GreedyShaving in a constant iterations.  $A_0$  is assigned to  $\tilde{U}^{(k)}$  in every GreedyShaving procedure. Then  $m_0 = |A_0| = |\tilde{U}^{(k)}|$ . In the adjacency matrix  $\mathbb{M}$  of the graph, we consider the submatrix  $\mathbb{M}_0$  with  $A_0$  as rows and V as columns. If the fraudulent dense block is in submatrix  $\mathbb{M}_0$ , then we assume that the block has at most  $O(m_0)$  columns. Excluding the dense block, the remaining part of  $\mathbb{M}_0$  is assumed to have the same density with the whole matrix  $\mathbb{M}$ . Therefore, the total number of edges in  $\mathbb{M}_0$  is

$$O(|E_0|) = O(m_0^2 + \frac{m_0 \cdot |E|}{|U|}) = O(|U|^{2/\epsilon} + |U|^{1/\epsilon - 1}|E|)$$

Then based on equation (8), the algorithm complexity is

$$O(m_0|E_0|\log m_0) = O((|U|^{3/\epsilon} + |U|^{2/\epsilon - 1 + \epsilon_0})\log |U|)$$

Therefore, if  $\epsilon > \max\{1.5, \frac{2}{3-\epsilon_0}\}$ , then the complexity is subquadratic  $o(|U|^2)$ .

In real life graph,  $\epsilon_0 \leq 1.6$ , so if  $\epsilon > 1.5$  the complexity of FastGreedy algorithm is subquadratic. Therefore, without loss of performance and efficiency, we can limit  $|\tilde{U}^{(k)}| \leq |U|^{1/1.6}$  for truncating an ordered U in the FastGreedy algorithm for a large dataset.

In FastGreedy algorithm for HS- $\alpha$ , SVD on adjacency matrix  $\mathbb M$  is used to generate initial blocks for the GreedyShaving procedure. Although we can still use SVD on  $\mathbb M$  for HS with holistic attributes, yet considering attributes of timestamps and rating scores may bring more benefits. Observing that not every combination of # of stars, timestamps and product ids has a value in a multi-way tenor representation, we can only choose every existing triplets (object, timestamp, #stars) as one column, and user as rows, to form a new matrix. The above transformation is called the matricization of a tensor, which outputs a new matrix. With proper time bins, e.g., one hour or day, and re-clustering of #stars, the flattening matrix becomes more dense and contains more attribute information. Thus we use such a flattening matrix with each column weighted by the sudden-drop suspiciousness for our FastGreedy algorithm.

#### Algorithm 3 FastGreedy Algorithm for Fraud detection.

Given bipartite multigraph  $\mathcal{G}(U,V,E)$ .  $\mathbb{L}=$  get first several left singular vectors for all  $L^{(k)}\in\mathbb{L}$  do Rank source nodes U decreasingly on  $L^{(k)}$   $\tilde{U}^{(k)}=$  truncate  $u\in U$  when  $L^{(k)}_u\leq \frac{1}{\sqrt{|U|}}$  GreedyShaving with initial  $\tilde{U}^{(k)}$ . end for return the best  $A^*$  with maximized objective  $HS(A^*)$ , and the rank of  $v\in V$  by  $f_{A^*}(v)\cdot P(v|A^*)$ .

# 4 EXPERIMENTS

**Table 2: Data Statistics** 

Data Name	#nodes	#edges	time span	
BeerAdvocate [18]	26.5K x 50.8K	1.07M	Jan 08 - Nov 11	
Yelp	686K x 85.3K	2.68M	Oct 04 - Jul 16	
Amazon Phone & Acc [17]	2.26M x 329K	3.45M	Jan 07 - Jul 14	
Amazon Electronics [17]	4.20M x 476K	7.82M	Dec 98 - Jul 14	
Amazon Grocery [17]	763K x 165K	1.29M	Jan 07 - Jul 14	
Amazon mix category [19]	1.08M x 726K	2.72M	Jan 04 - Jun 06	

In the experiments, we only consider the significant multiple bursts for fluctuated time series of sink nodes. We keep those awakening-burst point pairs with the altitude difference  $\Delta c$  at least 50% of the largest altitude difference in the time series. Table 2 gives the statistics of our six datasets which are publicly available for academic research  $^3$ . Our extensive experiments showed that the performance was insensitive to scaling base b, and became very stable when larger than 32. Hence we choose b=32 in the following experiments.

 $<sup>^3 \</sup>mbox{Yelp dataset}$  is from https://www.yelp.com/dataset\_challenge

		source nodes				sink nodes				
Data Name	metrics*	M-Zoom	D-Cube	CrossSpot	HS	M-Zoom	D-Cube	CrossSpot	HS	
	auc	0.7280	0.7353	0.2259	0.9758	0.6221	0.6454	0.1295	0.9945	
BeerAdvocate	F≥90%	0.5000	0.5000	-	0.0333	0.5000	0.5000	-	0.0333	
	auc	0.9019	0.9137	0.9916	0.9925	0.9709	0.8863	0.0415	0.9950	
Yelp	F≥90%	0.2500	0.2000	0.0200	0.0143	0.0250	1.0000	-	0.0100	
Amazon	auc	0.9246	0.8042	0.0169	0.9691	0.9279	0.8810	0.0515	0.9823	
Phone & Acc	F≥90%	0.1667	0.5000	-	$0.0200^{\dagger}$	0.1429	0.1000	-	$0.0200^{\dagger}$	
Amazon	auc	0.9141	0.9117	0.0009	0.9250	0.9142	0.7868	0.0301	0.9385	
Electronics	F≥90%	0.2000	0.1250	-	0.1000	0.1000	0.5000	-	0.1250	
Amazon	auc	0.8998	0.8428	0.0058	0.9250	0.8756	0.8241	0.0200	0.9621	
Grocery	F≥90%	0.1667	0.5000	-	0.1000	0.1250	0.2500	-	0.1000	
Amazon	auc	0.9001	0.8490	0.5747	0.9922	0.9937	0.9346	0.0157	0.9950	
mix category	F≥90%	0.2500	0.5000	$0.2000^{\dagger}$	0.0167	0.0100	0.2000	-	0.0100	

Table 3: Experimental results on real data with injected labels

# 4.1 Evaluation on different injection density

In the experiments, we mimic the fraudsters' behaviors and randomly choose 200 objects that has no more than 100 indegree as the fraudsters' customers, since less popular objects are more likely to buy fake ratings. On the other hand, the fraudulent accounts can come from the hijacked user accounts. Thus we can uniformly sample out a number of users from the whole user set as fraudsters. To test on different fraudulent density, the number of sampled fraudsters ranges from 200 to 20000. Those fraudsters as a whole randomly rate each of the 200 products for 200 times, and also create some biased camouflage on other products. As a results, the fraudulent density ranges from 1.0 to 0.01 for testing. The rating time was generated for each fraudulent edge: first randomly choosing a start time between the earliest and the latest creation time of the existing edges; and then plus a randomly and biased time interval from the intervals of exiting creation time, to mimic the surge of fraudsters' attacks. Besides, a high rating score, e.g. 4 or 4.5, is randomly chosen for each fraudulent edge<sup>4</sup>.

Fig. 1b shows the results of HoloScope HS on the BeerAdvocate data. When the fraudulent density decreases from the right to the left along the horizontal axis, HS can keep as high F-measure on accuracy as more than 80% before reaching 0.025 in density, better by far than the competitors. Since HS returns suspiciousness scores for sink nodes, we measure their accuracy using AUC (the area under the ROC curve), where ROC stands for receiver operating characteristic. For the detection of suspicious objects, HS achieved more than 0.95 in AUC for all the testing injection density, with a majority of tests reaching to 1.0.

In order to give a comparison on all six data sets with different injection density, we propose to use the two metrics: a low-case "auc" and the lowest *detection* density, described in the notes of Table 3. The table reports the fraud detection results of our Holo-Scope (HS) and competitors on the six datasets. Since the accuracy curve stops at 0.01 (the minimum testing density); and we add zero accuracy at zero density, the ideal value of auc is 0.995. The auc on source and sink nodes are reported separately. As the table

suggests, our HS achieved the best auc among the competitors, and even reached the ideal auc in two cases. Since the HS outputs the suspiciousness scores for sink nodes, we used the area under the upper-case AUC (similar to F-measure) accuracy curve along all testing density. Although it is sometimes unfair to compare F-measure with AUC, since the smallest value of AUC is around 0.5, yet the high auc values can indicate the high F-measure values on our top suspicious list.

Furthermore, we compare the lowest *detection* density in Table 3. The better a method is, the lower density it should be able to detect well. As we can see, HS has the smallest detection density in most cases, which can be as small as  $^{200}/_{14000} = 0.0143$  on source nodes, and reached the minimum testing density of 0.01 on sink nodes. That means we can detect fraudsters in high accuracy even if they use 14 thousand accounts to create  $200 \times 200$  fake edges for 200 objects. The fraudulent objects can also be detected accurately, even if 20 thousand fraudsters are hired to create 200 fake edges for each object.

#### 4.2 Evaluation on Sina Weibo with real labels

We also did experiments on a large real dataset from Sina Weibo, which has 2.75 million users, 8.08 million messages, and 50.1 million edges in Dec 2013. The user names and ids, and message ids are from the online system. Thus we can check their existence status in the system to evaluate the experiments. If the messages or the users were deleted from the system, we treat them as the basis for identifying suspicious users and messages. Since it is impossible to check all of the users and messages, we firstly collected a candidate set, which is the union of the output sets from the HS and the baseline methods. The real labels are from the candidate set by checking the status whether they still exists in Sina Weibo (checked in Feb. 2017). We used a program on the API service of Sina Weibo to check the candidate user and message id lists, finally resulting in 3957 labeled users and 1615 labeled messages.

The experimental results in Fig. 1c show that HS achieved high F-measure on accuracy, which detected 3781 labeled users higher than M-Zoom's 1963 labeled users. The F-measure of HS improved

<sup>\*</sup> we use the two metrics: the area under the curve (abbrev as low-case "auc") of the accuracy curve as drawn in Fig. 1b, and the lowest detection density that the method can detect in high accuracy (> 90%).

<sup>†</sup> one of the above fraudulent density was not detected in high accuracy.

 $<sup>^{4}</sup>$  the injection code is also open-sourced for reproducibility

about 30% and 60%, compared with M-Zoom and D-Cube respectively. CrossSpot biased to include a large amount of users in their detection results, which detected more than 100 labeled users but with extremely low precision, i.e., less than 1%. That is the reason CrossSpot got the lowest F-measure, which is less than 1.5%. For labeled messages, the HS achieved around 0.8 in AUC, while M-Zoom and D-Cube got lower recall, and CrossSpot still suffered very low F-measure with higher recall. Therefore, our HoloScope outperformed the competitors in real-labeled data as well.

# 4.3 Scalability

To verify the complexity, we choose two representative datasets: BeerAdvocate data which has the highest volume density, and Amazon Electronics which has the most edges. We truncated the two datasets according to different time ranges, i.e., from the past 3 months, 6 months, or several years to the last day, so that the generated data size increases. Our algorithm is implemented in Python. As shown in Fig. 2, the running time of our algorithm increases almost linearly with the number of the edges.

# 5 CONCLUSION

We proposed a fraud detection method, HoloScope, on a bipartite graph which can have timestamps and rating scores. HoloScope has the following advantages: 1) **Unification of signals:** we make holistic use of several signals, namely topology, temporal spikes, and rating deviation in our suspiciousness framework in a systematic way. 2) **Theoretical analysis of fraudsters' obstruction:** we showed that if the fraudsters use less than an upper bound of time to rate an object, they will cause a suspicious drop or burst. In other words, our HoloScope can obstruct fraudsters and increases their time cost. 3) **Effectiveness:** we achieved higher accuracy on both semi-real and real datasets than the competitors, achieving good accuracy even when the fraudulent density is low. 4) **Scalability:** while HoloScope needs to dynamically update the suspiciousness of objects, the algorithm is sub-quadratic in the number of nodes, under reasonable assumptions.

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