Lens Distortion Correction Using Ideal Image Coordinates

Article *in* IEEE Transactions on Consumer Electronics · September 2009

DOI: 10.1109/TCE.2009.5278053 · Source: IEEE Xplore

Lens Distortion Correction Using Ideal Image Coordinates

Junhee Park, Seong-Chan Byun, and Byung-Uk Lee, *Member*, IEEE

Abstract — *This paper proposes a fast and simple mapping method for lens distortion correction. Typical correction methods use a distortion model defined on distorted coordinates. They need inverse mapping for distortion correction. Inverse mapping of distortion equations is not trivial; approximation must be taken for real time applications. We propose a distortion model defined on ideal undistorted coordinates, so that we can reduce computation time and maintain the high accuracy. We verify accuracy and efficiency of the proposed method from experiments*[1].

Index Terms — lens distortion, camera calibration, radial distortion, image warping

I. INTRODUCTION

Lens distortions are long-known phenomena that prohibit use of a simple pinhole camera model in most camera applications. Even though the distortions are hard to correct, they do not influence quality of the image intensity. However they have significant impact on image geometry. Geometric distortion of an image can be categorized into radial distortion, decentering distortion, thin prism distortion, and other types of distortion. Radial distortion is prominent when taking pictures using wide-angle lenses. Radial distortion displaces image points inward or outward from the ideal position. Negative radial displacement is named as pincushion distortion, whereas positive radial displacement is known as barrel distortion, as shown in Fig. 1. The camera aperture placed between a lens and an image sensor introduces pincushion distortion. Otherwise, barrel distortion occurs when the aperture is outside of the lens [1].

Correction of lens distortion has been studied in the context of the camera calibration. Tsai [2] utilized known corresponding points in 3D space to recover distortion parameters. Weng [3] employed calibration objects to extract distortion parameters. These methods need 3D information and are based on distorted image coordinates to calculate distortion parameters. Zhang [4] used planar pattern taken from different orientations. This method does not need 3D information; instead it needs multiple images from different orientations. Nonmetric lens distortion correction methods do not require known 3D data either; they use geometric invariants such as lines or vanishing points [5]. Recently, nonparametric radial distortion models have been proposed in [6], [7]. Reference [7] reports accurate modeling of distortion. They adopted quartic polynomial to model the distortion; therefore it has five degrees of freedom, while most of radial distortion models have two degrees of freedom.

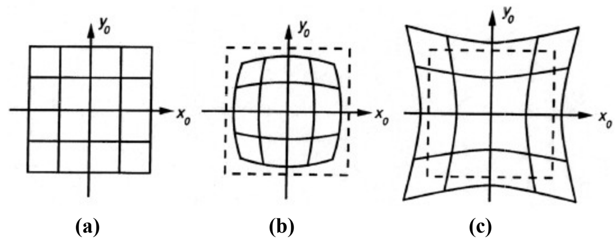


Fig. 1. Radial distortion. (a) Ideal image with no distortion, (b) Barrel Distortion, (c) Pincushion Distortion.

Radial distortion is noticeable for cameras embedded in cellular phones, since they employ wide angle lenses. The resources for cell phone cameras are limited, therefore a simple method is required to correct for the distortion.

Theoretically radial distortion is symmetric to the optical axis. We assume that the principal point, where the optical axis intersects the image plane, is the center of radial distortion [8]. Then we can model distortion parameters using the distance from the distortion center of a distorted image.

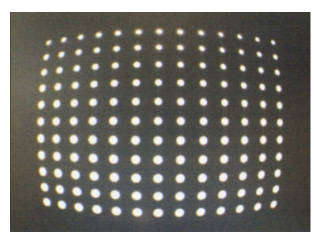


Fig. 2. An example of a distortion image captured with wide angle lens.

Distorted feature data points are extracted in a captured image shown in Fig. 2, and matched to the ideal undistorted points. Distortion coefficients are calculated using a distortion model.

We estimate distortion parameters to obtain a corrected image. Then we need a mapping from the undistorted coordinates to the distorted position to obtain the image intensity at the corrected position. However, most of available distortion models employ mapping from the distorted position to the ideal undistorted coordinates [2], [5]. Therefore they require time consuming inverse mapping [9], or inaccurate approximations [10]. Moreover inverse mappings are not practical for real time applications. To

[1] This work was supported in part by LG Electronics, the Ministry of Knowledge Economy (MKE), Korea Industrial Technology Foundation (KOTEF) through the Human Resource Training Project for Strategic Technology, and the Acceleration Research Program of the Ministry of Education, Science and Technology of Korea and the Korea Science and Engineering Foundation.

Junhee Park and Byung-Uk Lee are with Department of Electronics Engineering, Ewha W. University, Seoul, Korea (e-mail: corelpark@gmail.com, bulee@ewha.ac.kr).

Seong-Chan Byun is with LG Electronics Institute of Technology, Seoul, Korea (e-mail: schbyun@lge.com).

Contributed Paper
Manuscript received June 10, 2009

eradicate the inverse mapping, we propose to adopt a distortion model on undistorted coordinates. We verify that the proposed model is accurate and satisfactory for mobile phone applications.

This paper is organized as follows. Section II and III describe distortion models and inverse mapping. We present distortion model based on ideal undistorted coordinates in Section IV. Experimental results and conclusions are described in Section V and VI.

II. DISTORTION MODEL

In this section we describe radial distortion models. Let (X_d, Y_d) be distorted image coordinates, (X_u, Y_u) be ideal undistorted image position, and (D_x, D_y) be distortion defined by the following equation (1). Distortion is obtained by subtracting distorted image position from the ideal.

$$\begin{bmatrix} D_x \\ D_y \end{bmatrix} = \begin{bmatrix} X_u \\ Y_u \end{bmatrix} - \begin{bmatrix} X_d \\ Y_d \end{bmatrix} \tag{1}$$

A. Typical Distortion Model [2],[8],[10]

The most popular radial model is even-order polynomial model. Lens distortion is modeled by (2) using distortion coefficient κ_i. Assume that the principal point is (0, 0). Note that distortion is represented in terms of distorted image coordinates (X_d, Y_d). There are many algorithms to solve for the distortion parameters κ_i. Since this equation is linear in κ_i's they can be calculated using pseudoinverse.

$$\begin{bmatrix} D_x \\ D_y \end{bmatrix} = \begin{bmatrix} X_d \\ Y_d \end{bmatrix} \left[R_d^2 \kappa_1 + R_d^4 \kappa_2 + R_d^6 \kappa_3 + \cdots \right],\; R_d = \sqrt{{X_d}^2 + {Y_d}^2} \tag{2}$$

Higher order terms are insignificant in most cases, therefore only two terms are used in many applications. We can find distortion (D_x, D_y) and compensate for it using two distortion coefficients κ_1, and κ_2 of (3).

$$\begin{bmatrix} X_u \\ Y_u \end{bmatrix} = \begin{bmatrix} X_d \\ Y_d \end{bmatrix} \left[1 + R_d^2 \kappa_1 + R_d^4 \kappa_2 \right] \tag{3}$$

If the number of pattern features is N, each data point gives two equations, therefore we have $2N$ equations. Since there are two unknowns, and the equations are linear, we can obtain a minimum mean square solution using pseudoinverse.

B. Division model [11]-[13]

Fitzgibbon [11] proposed the division model for ease of calculation in homogeneous coordinates, and showed that the model is as accurate as the traditional model (3). The typical distortion model multiplies a correction term, however distortion can be modeled using division distortion coefficients κ_{d1}, and κ_{d2} by (4).

$$\begin{bmatrix} X_u \\ Y_u \end{bmatrix} = \begin{bmatrix} X_d \\ Y_d \end{bmatrix} / \left[1 + R_d^2 \kappa_{d_1} + R_d^4 \kappa_{d_2} \right] \tag{4}$$

III. INVERSE MAPPING

Using a distortion model and distortion parameters, we can map distorted points to undistorted position. The mapping from (X_d, Y_d) to (X_u, Y_u) is illustrated in Fig. 3. When we calculate a corrected image, we need to obtain image intensities at ideal pixel grid position (i, j), therefore we need a mapping from the ideal to the distorted position. This requires inverse mapping of (3), as illustrated in Fig. 4.

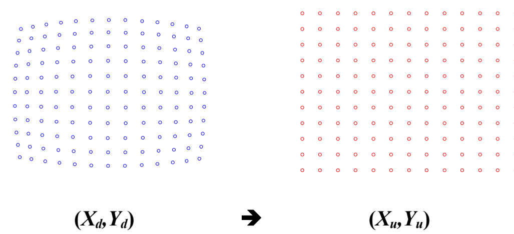


Fig. 3. Mapping from distortion coordinates (X_d, Y_d) to ideal coordinates (X_u, Y_u).

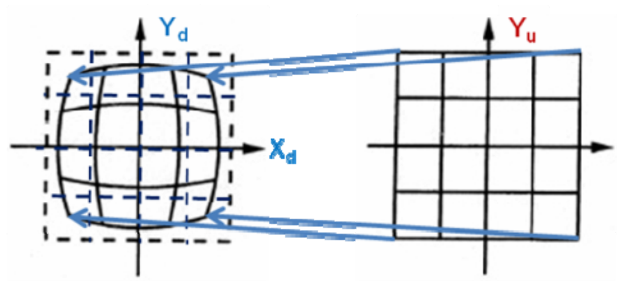


Fig. 4. Inverse mapping from image grid to distorted position.

In other words, we find a mapping from undistorted integer pixel position (i, j) to distorted position $(X_d(i,j), Y_d(i,j))$ and find the pixel value using interpolation, because the distorted position does not fall on integer grid pixel position. Since the distortion modeling equation is of order four, it is not trivial to find inverse mapping for each pixel. Therefore approximation methods for inverse mapping have been proposed [10]. For inverse mapping, it ignores error terms higher than 4^{th} order and approximates using the square term of R_d as in (5).

$$\begin{bmatrix} X_u \\ Y_u \end{bmatrix} = \begin{bmatrix} X_d \\ Y_d \end{bmatrix} \left[1 + \kappa R_d^2 \right] \tag{5}$$

We can rewrite (5) using R_d and R_u.

$$R_u = R_d \left(1 + \kappa R_d^2 \right) \Leftrightarrow \kappa R_d^3 + R_d - R_u = 0,\; R_u = \sqrt{{X_u}^2 + {Y_u}^2} \tag{6}$$

Assume that $\kappa R_d^2 << 1$, then, we can reduce the cubic polynomial equation to a 2nd order polynomial as in (7).

$$R_u \approx \frac{R_d}{1-\kappa R_d^2} \Leftrightarrow R_d = \frac{-1+\sqrt{1+4\kappa R_u^2}}{2\kappa R_u} \tag{7}$$

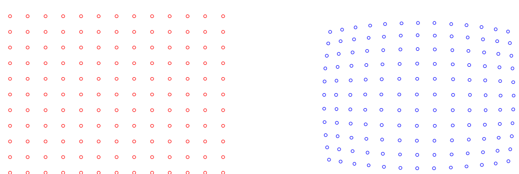
Using (7), image warping can be found by (8) [10].

$$X_d = \frac{R_d}{R_u} X_u,\ Y_d = \frac{R_d}{R_u} Y_u \tag{8}$$

By applying this method for warping, correction for distortion can be implemented without using higher order terms. However, it shows higher error than model (3) or (4) due to the approximation.

Gribbon [14] iteratively solved the mapping equation instead of inverse mapping. The relationships are precalculated and placed in a LUT.

IV. DISTORTION MODEL USING IDEAL COORDINATES



(X_u, Y_u) ➔ (X_d, Y_d)

Fig. 5. Mapping from ideal coordinates (X_u, Y_u) to distortion coordinates (X_d, Y_d).

We propose a distortion modeling method using ideal undistorted coordinates, instead of using distorted coordinates, so that we can reduce the error of approximation and computational complexity at the same time. Let the distorted coefficient on ideal coordinates be κ_{u1}, and κ_{u2}. Then distortion modeling is defined as (8).

$$\begin{bmatrix} X_d \\ Y_d \end{bmatrix} = \begin{bmatrix} X_u \\ Y_u \end{bmatrix} \left[1 + R_u^2 \kappa_{u_1} + R_u^4 \kappa_{u_2}\right] \tag{8}$$

Note that the distortion parameters are defined on undistorted coordinates, while they are defined on distorted coordinates in (3).

Also using the division model (9), we can compensate for distortion without inverse mapping. Let the distorted division coefficient be κ_{ud1}, and κ_{ud2}. Then division distortion modeling is (9).

$$\begin{bmatrix} X_d \\ Y_d \end{bmatrix} = \begin{bmatrix} X_u \\ Y_u \end{bmatrix} / \left[1 + R_u^2 \kappa_{ud_1} + R_u^4 \kappa_{ud_2}\right] \tag{9}$$

The computation time can be reduced and approximation for inverse mapping is eradicated using these models, resulting in higher accuracy in distortion correction.

V. EXPERIMENTAL RESULTS

We use a PC web camera with 2.5mm Lens of 150˚ FOV (field of view). The resolution of image is 640 by 480 pixels, and the fiducial point array is 13 by 11.

A. *Detecting feature point*

We use 2D Gaussian circles as fiducials, and apply correlation to detect the position. The 2D Gaussian shape reduces mismatching, and gives reliable estimation in noisy observations.

B. *Establishing ideal coordinate*

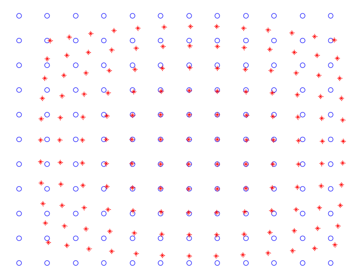


Fig. 6. Examples of ideal coordinates (o) and distorted coordinates (*).

We need to find the ideal positions from distorted coordinates. First we assume that the principal point p_0 is the image center, and then we recover ideal image grid from eight fiducial points nearest to the principal point. We plot the undistorted pattern points (o) in Fig. 6.

C. *Optimization*

To minimize the error between the compensated and ideal positions, we optimize distortion parameters **P** using the Nelder-Mead simplex method [15]. The distortion parameters to be estimated consist of distortion coefficients κ_1, κ_2, principal point X_0, Y_0, and rotation angle θ. The Nelder-Mead simplex method uses multidimensional unconstrained nonlinear minimization. We can find the parameters minimizing RMS (root mean square) error between the ideal and distortion compensated positions. The objective function is given in (10). Position vector $\mathbf{C}_i$ represents compensated points, $\mathbf{U}_i$ denotes undistorted ideal points, and N is the number of points.

$$\arg\min_{\mathbf{P}} \quad e = \frac{1}{N}\sum_i \sqrt{(\mathbf{C}_i - \mathbf{U}_i)^2},\ \mathbf{C}_i = \begin{bmatrix} X_{ci} \\ Y_{ci} \end{bmatrix},\ \mathbf{U}_i = \begin{bmatrix} X_{ui} \\ Y_{ui} \end{bmatrix} \tag{10}$$

D. Correction error and mapping time

Distortion correction error is summarized in Table I. The RMS error between distortion coordinates and the ideal coordinates is 11.8 pixels, and maximum error is 52.5 pixels before correction. After compensation using distortion parameters, the RMS error is decreased to about 1 pixel, and maximum error is less than 5 pixels. The accuracy of the proposed methods, (8) and (9), shows comparable performance as the conventional methods defined on distorted coordinates (3) and (4). However the simplified inverse mapping method using only the second order term, (5), shows much larger maximum error.

However, the mapping time of the proposed method shows dramatic improvement from the conventional models; the proposed method (8) is the fastest. Inverse mapping takes much longer time to solve for inverse mapping of quartic polynomials in conventional methods. We verify that the proposed method reduces inverse mapping time noticeably while maintaining the same level of accuracy for a wide angle lens. The mapping time is calculated for feature grids using Matlab on Core2 CPU 6600. The mapping time is summarized in Table II.

The distortion parameters are provided in Table III and experimental results are shown in Fig. 7.

TABLE I
DISTORTION CORRECTION ERROR

Distortion Parameters			ERROR (pixels)	
			RMS	MAX
Before correction			11.8	52.5
After correction with nonlinear optimization	(3)	$\kappa_1, \kappa_2, X_0, Y_0, \theta$	1.2	4.7
	(4)	$\kappa_{d1}, \kappa_{d2}, X_0, Y_0, \theta$	1.1	4.4
	(5)	κ (2nd order polynomial approximation)	1.5	8.5
	(8)	$\kappa_{u1}, \kappa_{u2}, X_0, Y_0, \theta$	1.1	4.7
	(9)	$\kappa_{ud1}, \kappa_{ud2}, X_0, Y_0, \theta$	1.2	4.9

TABLE II
EXECUTION TIME

Distortion model		Mapping time
(3)	κ_1, κ_2	496.1 ms
(4)	κ_{d1}, κ_{d2}	502.9 ms
(5)	κ (2nd order polynomial approximation)	0.02 ms
(8)	κ_{u1}, κ_{u2}	0.02 ms
(9)	$\kappa_{ud1}, \kappa_{ud2}$	0.07 ms

TABLE III
DISTORTION PARAMETERS

		κ_1	κ_2	X_0	Y_0	θ
After correction with nonlinear optimization	(3)	κ_1=1.2x10^{-6}	κ_2=1.2x10^{-11}	328.2	249.1	-0.2°
	(4)	κ_{d1}=-1.3x10^{-6}	κ_{d2}=-7.1x10^{-12}	328.1	249.5	-0.2
	(5)	κ =2.0x10^{-6}		329.5	249.9	-0.004°
	(8)	κ_{u1}=-1.4x10^{-6}	κ_{u2}=9.6x10^{-13}	328.2	249.3	-0.2°
	(9)	κ_{ud1}=1.4x10^{-6}	κ_{ud2}=1.5x10^{-12}	328.4	248.8	-0.2°

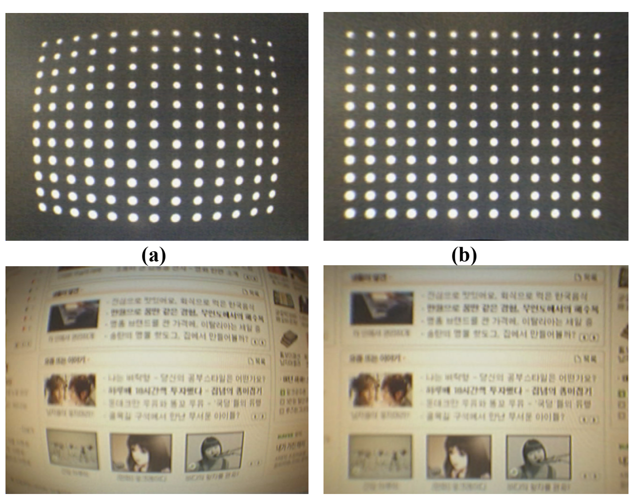


Fig. 7. Experimental results. (a) distorted feature image, (b) compensated image of (a) using model (8), (c) real image (d) compensated image of (c) using model (8).

VI. CONCLUSION

We propose a camera lens distortion correction method that is modeled on ideal coordinates. Our experiments verify that the proposed method is more efficient and faster than conventional inverse mapping methods. The proposed model do not need approximation and inverse mapping, therefore it does not increase the error while reducing the computation time.

REFERENCES

[1] E. Hecht, and A. Zajac, *Optics*, Addison-Wesley, 2002, pp. 184-186
[2] Tsai, R., “A versatile camera calibration technique for high-accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses,” *IEEE J. Robotics and Automation*, vol. 3, no. 4, pp. 323-344, Aug. 1987.
[3] J. Weng, P. Cohen, and M. Herniou, “Camera calibration with distortion models and accuracy evaluation,” *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 14, no. 10, pp. 965-980, Oct. 1992.
[4] Z. Zhang, “A flexible new technique for camera calibration,” *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 22, no. 11, pp. 1330-1334, Nov. 2000.
[5] Moumen Ahmed and Aly Farag, “Nonmetric calibration of camera lens distortion: differential methods and robust estimation,” *IEEE Trans. Image Processing*, vol. 14, no. 8, pp. 1215-1230, Aug. 2005.
[6] R. Hartley, and S. B. Kang, “Parameter-free radial distortion correction with center of distortion estimation,” *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 29, no. 8, pp. 1309-1321, Aug. 2007.
[7] Seok-Han Lee, Sang-Keun Lee, and Jong-Soo Choi, "Correction of radial distortion using a planar checkerboard pattern and its Image," *IEEE Trans. Consumer Electronics*, vol. 55, no. 1, pp. 27-33, Feb. 2009.
[8] R. I. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, 2nd Ed. Cambridge: CUP, 2003, pp. 189-193.
[9] Shishir Shah and J. K. Aggarwal, “Intrinsic parameter calibration procedure for a (high-distortion) fish-eye lens camera with distortion

model and accuracy estimation," *Pattern Recognition*, vol. 29, no. 11, pp. 1775-1788, Nov. 1996.

[10] Wonpil Yu, "An embedded camera lens distortion correction method for mobile computing applications," *IEEE Trans. Consumer Electronics*, vol. 49, no. 4, pp. 894-901, Nov. 2003.

[11] Fitzgibbon, A.W., "Simultaneous linear estimation of multiple view geometry and lens distortion," *IEEE Conf. Computer Vision and Pattern Recognition*, vol. 1, pp. 125-132, 2001.

[12] John Mallon and Paul F. Whelan, "Precise radial un-distortion of images," *IEEE Conf. Pattern Recognition*, vol. 1, pp. 18-21, Aug. 2004.

[13] SriRam Thrithala and Marc Pollefeys, "The radial trifocal tensor: a tool for calibration the radial distortion of wide-angle cameras," *IEEE Conf. Computer Vision and Pattern Recognition*, vol. 1, pp. 321-328, 2005.

[14] K. T. Gribbon, C. T. Johnston and D. G. Bailey, "A Real-time FPGA Implementation of a Barrel Distortion Correction Algorithm with Bilinear Interpolation," *Proceedings of Image and Vision Computing New Zealand Conference*, pp. 408-413, Nov. 2003.

[15] J.A. Nelder and R. Mead, "A simplex method for function minimization," *Computer Journal*, vol. 7, no. 4, pp. 308-313, Jan. 1965.

Junhee Park received the B.S. and the M.S. degree in Electronic Engineering from Ewha W. University in 2001 and 2003, respectively. She has worked at EtoM Solution and Korea Electric Test Institute as an engineer from 2003 to 2006. She is currently in a Ph.D. program at Ewha W. University. Her research interests include image processing and computer vision.

Seong-Chan Byun received the B.S. and the M.S. degree in Computer Science from Yonsei University in 1997 and 1999, respectively. He is with Advanced Research Institute, LG Electronics ARI since 2000, where he developed imaging applications for mobile cameras. He is interested in image and video processing for mobile device.

Byung-Uk Lee received the B.S. degree in Electronic Engineering from Seoul National University in 1979, and the M.S. degree in Electrical Science from Korea Advanced Institute of Science and Technology in 1981, and Ph.D. in Electrical Engineering from Stanford University in 1991. He has worked for Daewoo Electronics Co. from 1981 to 1985 and from 1991 to 1995. He joined the Department of Electronics Engineering, Ewha W. University in 1995, where he is currently a Professor. His research interests include image processing and computer vision.