THE ASTROPHYSICAL JOURNAL, 286:38–41, 1984 November 1

ANGULAR MOMENTUM GROWTH IN PROTOGALAXIES

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Received 1984 April 9; accepted 1984 May 22

ABSTRACT

I expand a 1970 analysis by Doroshkevich which shows that the angular momentum of galaxies grew to first order (in proportion to t) during the linear phases of protogalactic evolution. This result is confirmed in N-body simulations of the formation of structure. The well-known 1969 study of Peebles found growth at second order only (in proportion to $t^{5/3}$) because its analysis was restricted to spherical regions. In such regions growth occurs purely as a result of convective effects on the bounding surface; the material *initially* within a spherical volume gains no angular momentum in second-order perturbation theory. These considerations do not affect estimates of the *total* angular momentum acquired by a galaxy in the gravitational instability picture.

Subject headings: galaxies: formation — galaxies: internal motions — numerical methods

I. INTRODUCTION

Fifteen years ago the origin of the angular momentum of galaxies was the subject of considerable debate (e.g., Peebles 1969; Oort 1970; Harrison 1971). Since that time it has gradually come to be accepted that this angular momentum can be explained by some variant of Hoyle's (1949) idea that protogalaxies are spun up by the tidal fields of their neighbors. The alternative hypothesis that galactic spins are the relics of primordial vortical motions is now in some disfavor. (See the reviews by Jones 1976, Gott 1977, and Efstathiou and Silk 1983). The first detailed calculation of the acquisition of angular momentum in the early stages of protogalactic evolution was made by Peebles (1969). Peebles used linear theory to find the growth rate of the spin angular momentum contained within a comoving spherical region of the expanding universe. He found that the angular momentum in such regions grows only to second order in the perturbation expansion and in proportion to $t^{5/3}$ for an Einstein–de Sitter universe. His result led to the widespread notion that this same behavior should apply in general to the early evolution of protogalaxies (e.g., Peebles 1971; Binney and Silk 1979; Efstathiou and Jones 1979; Peebles 1980; Efstathiou and Silk 1983). This is not, in fact, the case. As was pointed out by Doroshkevich (1970), the angular momentum of a protogalaxy normally grows at first order (in proportion to t for a flat universe) and Peebles's result is a consequence of the symmetry he imposed. Doroshkevich gives few details of his calculations which have been quite unjustifiably neglected in more recent reviews and research articles. It thus seems worthwhile to present them in greater detail, to explore the conditions for their validity and the reasons for their disagreement with Peebles's results, and to check their predictions against numerical simulations.

In § II, I go through the perturbation theory treatment of angular momentum acquisition and display the physical significance of the first-order growth. In § III, I reanalyze the problem discussed by Peebles (1969) and show that the second-order growth he found is a result of the convective transfer of angular momentum across the boundary of his spherical region. In § IV, I use some large N-body simulations to verify Doroshkevich's result.

II. FIRST-ORDER THEORY

The growth of spin in a protogalaxy is most easily analyzed using the formulation of linear perturbation theory described by Zel'dovich (1970). Let us consider the growth of fluctuations in an expanding Friedmann universe filled with pressure-free matter (dust). The local overdensity, defined by $\delta(\boldsymbol{r}, t) = \rho(\boldsymbol{r}, t)/\rho_0(t) - 1$, where ρ_0 is the mean density, can then be written as a separable function of time and of a comoving coordinate $\boldsymbol{x}$ related to $\boldsymbol{r}$ by $\boldsymbol{r} = a\boldsymbol{x}$, where $a(t)$ is the cosmological expansion factor. Thus $\delta(\boldsymbol{x}, t) = b(t)\delta_0(\boldsymbol{x})$. In a flat universe (the case of most interest) b is proportional to a, but it is convenient to retain a separate notation so that b can be considered as the small parameter in a perturbation expansion. Because δ is separable, so also is the peculiar gravitational potential, $\Phi(\boldsymbol{x}, t) = (b/a)\Phi_0(\boldsymbol{x})$. The peculiar acceleration is proportional to the spatial gradient of this function and so (by direct integration in time) are the peculiar velocity and displacement vectors. Thus we can write the trajectory of a dust particle in linear theory as

$$\boldsymbol{r}(\boldsymbol{q}, t) = a\boldsymbol{x}(\boldsymbol{q}, t) = a(t)[\boldsymbol{q} - b(t)\nabla\varphi(\boldsymbol{q})] , \tag{1}$$

where $\boldsymbol{q}$ is a Lagrangian coordinate defined as the $\boldsymbol{x}$ position of the particle at $t \rightarrow 0$ and $\varphi(\boldsymbol{q})$ is a potential with the units of area which is proportional to $\Phi_0(\boldsymbol{q})$.

The assumptions leading to equation (1) are valid provided $\langle\delta^2\rangle \ll 1$. This condition is satisfied over the period preceding the collapse of a protogalaxy only if the initial field of density fluctuations has a coherence length of protogalactic scale. In general, this will not be the case, and equation (1) will cease to describe the detailed evolution of the matter distribution long before galaxies form. Indeed for a scale-free fluctuation distribution (e.g., white noise), equation (1) may never provide a valid description. We may avoid this formal problem by applying the equation not to the actual density field but to a smoother field obtained from it by convolution with a window function of protogalactic scale. This procedure assumes that small nonlinear structures have negligible influence on the gravitational evolution of larger scale quasilinear fluctuations. The assumption that nonlinear mode-mode coupling can be neglected is necessary for any simple description of the linear phases of protogalactic evolution. While it appears physically reasonable, it must ultimately be justified either by a deeper theoretical analysis or by direct comparison with numerical experiments. I shall implicitly adopt this assumption in the analysis I present below.

The spin angular momentum of the material which makes up a protogalaxy may be written as

$$J(t) = \int_{V_L} [r(q, t) - \bar{r}(t)] \times v(q, t)\rho_0 a^3 d^3q \, ,$$

where V_L is the Lagrangian volume which initially contained the matter and $\bar{r} = V_L^{-1} \int_{V_L} r d^3q$ is its center of mass at time t. In comoving coordinates this may be written

$$J(t) = \rho_0 a^5 \int_{V_L} (x - \bar{x}) \times \dot{x} d^3q \quad \text{(exact)} \, . \tag{2}$$

This expression involves no approximation; however, since the leading term in $\dot{x}$ is first order and is parallel to the first-order displacement it can be written correctly to second order as

$$J(t) = \rho_0 a^5 \int_{V_L} (q - \bar{q}) \times \dot{x} d^3q \quad \text{(2d order)} \tag{3}$$

The first-order part of this is

$$J(t) = -\rho_0 a^5 \dot{b} \int_{V_L} (q - \bar{q}) \times \nabla\varphi d^3q \quad \text{(1st order)} \, . \tag{4}$$

Converting this to an integral over the surface Σ_{V_L} of V_L, I find

$$J(t) = -\rho_0 a^5 \dot{b} \int_{\Sigma_{V_L}} \varphi(q)(q - \bar{q}) \times dS \, . \tag{5}$$

Thus J vanishes to first order if Σ_{V_L} is spherical or is an equipotential of φ, but in general it has a finite first-order part. Doroshkevich (1970) notes that this result was first pointed out by Zel'dovich.

We can get further insight into the meaning of the integrals in equations (4) and (5) if we assume that φ can be adequately represented in V_L by the first three terms of a Taylor series about $\bar{q}$;

$$\varphi(q) = \varphi(\bar{q}) + (q_i - \bar{q}_i) \left.\frac{\partial\varphi}{\partial q_i}\right|_{q} + \frac{1}{2}(q_i - \bar{q}_i) \left.\frac{\partial^2\varphi}{\partial q_i \partial q_j}\right|_{q} (q_j - \bar{q}_j) \, .$$

Equation (4) then gives

$$\begin{aligned} J_i(t) &= -a^2 \dot{b} \epsilon_{ijk} \left.\frac{\partial^2\varphi}{\partial q_j \partial q_l}\right|_{q} \int_{V_L} (q_l - \bar{q}_l)(q_k - \bar{q}_k)\rho_0 a^3 d^3q \, , \\ &= -a^2 \dot{b} \epsilon_{ijk} T_{jl} I_{lk} \, , \end{aligned} \tag{6}$$

where $\mathbf{T}$ is proportional both to the tidal tensor and to the local deformation tensor at $\bar{q}$, while $\mathbf{I}$ is the inertia tensor of the matter in V_L. This tensor product is of the usual form employed to calculate the torque on an extended body in a tidal field. J vanishes if and only if $\mathbf{T}$ and $\mathbf{I}$ have the same principal axes. Equation (6) shows that angular momentum arises in first order because the first-order tidal field couples to the zeroth-order quadrupole moment of the irregular boundary of the protogalaxy. Because the inertia tensor depends only on the shape of the protogalaxy itself whereas the tidal tensor depends in addition on the disposition of neighboring perturbations, the principal axes of $\mathbf{T}$ and $\mathbf{I}$ should not in general coincide. Unfortunately, it is difficult to prove this rigorously because of the uncertainty as to how Σ_{V_L} should be specified in order to encompass the matter which ends up in a collapsing protogalaxy. This problem can, however, be investigated by direct numerical simulation (see below).

III. ANGULAR MOMENTUM OF SPHERICAL REGIONS

It is interesting to study the problem discussed by Peebles (1969) and to elucidate its relationship to the analysis of the last section. As an intermediate step we analyze the special case when the *Lagrangian* volume V_L is a sphere; the angular momentum then vanishes to first order. Following the methods of Peebles (1969) we can evaluate the second-order expression for J (eq. [3]) by looking at its time derivative,

$$\dot{J} = \frac{2\dot{a}}{a} J + \rho_0 a^5 \int_{S_L} q \times \ddot{x} d^3q \, , \tag{7}$$

where the center of the sphere has been assumed to coincide with the origin. The full nonlinear equation of motion for $x(q, t)$ is

$$\ddot{x} + 2\frac{\dot{a}}{a}\dot{x} = \frac{1}{a^2}\nabla_x \psi \, , \tag{8}$$

where $\psi[x(q), t]$ is the exact peculiar potential and the gradient is taken with respect to x. To lowest order we have

$$\psi(q, t) = -\frac{d}{dt}(a^2\dot{b})\varphi(q) \, . \tag{9}$$

Substituting for $\ddot{x}$ in equation (7) we find

$$\dot{J} = \rho_0 a^3 \int_{S_L} q \times \nabla_x \psi d^3q \, . \tag{10}$$

Now to second order

$$\begin{aligned} \frac{\partial\psi}{\partial x_i} &= \frac{\partial q_j}{\partial x_i}\frac{\partial\psi}{\partial q_j} \\ &= \frac{\partial\psi}{\partial q_i} + b\frac{\partial^2\varphi}{\partial q_i \partial q_j}\frac{\partial\psi}{\partial q_j} \\ &= \frac{\partial\psi}{\partial q_i} - b\frac{d}{dt}(a^2\dot{b})\frac{\partial^2\varphi}{\partial q_i \partial q_j}\frac{\partial\varphi}{\partial q_j} \\ &= \frac{\partial}{\partial q_i}\left[\psi - \frac{1}{2}b\frac{d}{dt}(a^2\dot{b})|\nabla_q\varphi|^2\right] , \end{aligned}$$

where I have used the first-order approximation for ψ from equation (9). Thus $\nabla_x \psi$ can be expressed to second order as the gradient of a scalar function with respect to q. As a result the integral in equation (10) can be converted into a surface integral

$$\dot{J} = \rho_0 a^3 \int_{\Sigma_{S_L}} \left[\psi - \frac{1}{2}b\frac{d}{dt}(a^2\dot{b})|\nabla\varphi|^2\right] q \times dS = 0 \, . \tag{11}$$

Thus the matter which was *initially* in a spherical volume of comoving space gains no angular momentum in second order perturbation theory.

Peebles (1969) showed that the total spin angular momentum in a spherical *Eulerian* volume grows to second order. In view of the result just obtained, this must reflect surface effects which convect angular momentum across the boundary. This can be shown explicitly as follows. Let us evaluate equation (3) over the Lagrangian volume which corresponds to the matter contained in the Eulerian sphere at time t. This volume differs from the corresponding Lagrangian sphere S_L by an amount which grows at first order. We may thus write (3) to sufficient accuracy as

$$J(t) = \rho_0 a^5 \int_{S_L} (\boldsymbol{q} - \bar{\boldsymbol{q}}) \times \dot{x} d^3 q + \rho_0 a^5 \int_{\Sigma_{S_L}} (\boldsymbol{q} - \bar{\boldsymbol{q}}) \times \dot{x} b \nabla\varphi \cdot d\boldsymbol{S} ,$$

where to first order

$$\bar{\boldsymbol{q}}(t) = \frac{1}{V_L} \int_{\Sigma_{S_L}} \boldsymbol{q} b \nabla\varphi \cdot d\boldsymbol{S} .$$

Making this substitution, using our result (11) and retaining only second-order terms,

$$J(t) = -\rho_0 a^5 b\dot{b} \int_{\Sigma_{S_L}} \boldsymbol{q} \times (\nabla\varphi - \overline{\nabla\varphi})\nabla\varphi \cdot d\boldsymbol{S} . \quad (12)$$

This result can be written more suggestively as

$$J(t) = -\frac{b}{\dot{b}} \int_{\Sigma_{S_E}} \boldsymbol{r} \times (\boldsymbol{v} - \bar{\boldsymbol{v}})\rho \boldsymbol{v} \cdot d\boldsymbol{S}' , \quad (13)$$

where the integral is now taken over the surface of the Eulerian sphere. This shows that the Eulerian sphere gains angular momentum purely as a result of convective transport across its boundary and not as a result of torques acting on the matter interior to it. The connection with Peebles's analysis can be made by converting equation (12) into a volume integral

$$J(t) = -\rho_0 a^5 b\dot{b} \int_{S_L} \{(\nabla^2\varphi)\boldsymbol{q} \times (\nabla\varphi - \overline{\nabla\varphi}) + \boldsymbol{q} \times [(\nabla\varphi \cdot \nabla)\nabla\varphi]\} d^3 q .$$

The second term in the integrand can be converted back to a surface integral which vanishes on Σ_{S_L}. This leaves

$$J(t) = -\rho_0 a^5 b\dot{b} \int_{S_L} \nabla^2\varphi \boldsymbol{q} \times (\nabla\varphi - \overline{\nabla\varphi}) d^3 q . \quad (14)$$

Using $\delta(\boldsymbol{q}) = b\nabla^2\varphi$ and converting from Lagrangian back to Eulerian variables, this is equal at second order to

$$J(t) = \rho_0 a^2 \int_{S_E} \delta(\boldsymbol{x})\boldsymbol{x} \times (\dot{\boldsymbol{x}} - \bar{\dot{\boldsymbol{x}}}) d^3 x , \quad (15)$$

which is the Eulerian expression which Peebles used to calculate $\boldsymbol{J}$.

IV. EXPERIMENTAL VERIFICATION

I have attempted to verify the results of § II in two 32,768 particle N-body experiments which are part of an ongoing collaborative program to study clustering in an expanding universe (Frenk, White, and Davis 1983; White, Frenk, and Davis 1983; Efstathiou *et al.* 1984). Both simulations used a particle-mesh method on a 64^3 grid to calculate forces and advance the particles. One experiment was designed to study a neutrino-dominated universe (White, Frenk, and Davis 1983). Its density field began with an rms δ of 22% and a large coherence length. It was allowed to expand by a factor of 20. The other experiment began with a Poisson distribution of particles within the computational volume and was allowed to expand by a factor of 32. The evolution of these models can be taken to represent the formation of structure in the "pancake" and hierarchical clustering pictures, respectively. In both, the background universe was taken to be flat and initial velocities were set so that only the growing mode was present. In the last time frame of each experiment I identified clusters by linking particles with separations smaller than 0.4 of the mean interparticle spacing, and then joining all "friends of friends." This procedure resulted in clusters with mean δ values in the range 50–400. Finally I calculated the angular momentum at earlier times of those particles which ended up in each of these clusters.

Figure 1 illustrates the growth of angular momentum in these models. For the members of each cluster, I took $J(t)$ and divided it by $a^{3/2}J(t_i)$, where t_i is the initial time. If growth obeys the theory of § II this quantity should remain equal to unity. For each simulation the figure shows the average of its logarithm as a function of time for clusters with more than 100 members. (The largest clusters in each simulation had 2–3 × 10^3 members.) In addition, the evolution of four "typical" clusters is shown for each simulation. The theoretical prediction is followed closely at small expansion factors. However, as a increases, the density contrast of clusters becomes large and angular momentum transfer to them ceases

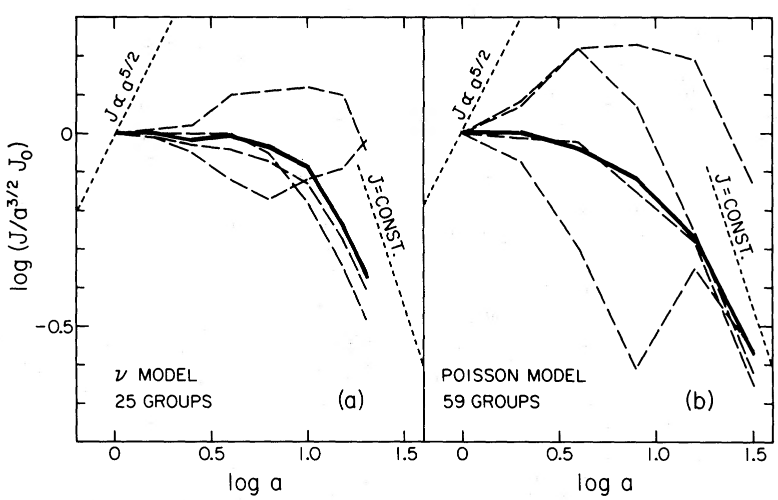

FIG. 1.—Angular momentum normalized to the value predicted by linear theory is shown as a function of expansion factor for the particles which end up in groups of more than 100 members in two N-body experiments. Fig. 1*a* is a simulation of a neutrino-dominated universe, while in Fig. 1*b* the particles were initially distributed uniformly at random. Solid lines show the mean for all rich groups in each model while dashed lines show the behavior of "typical" individual groups. The dotted lines show the relationships $J \propto a^{5/2}$ which would be obeyed if angular momentum grew at second order in perturbation theory, and J = const which is expected at late times.

to be effective. In addition, nonlinear effects cause the behavior of J to vary considerably from cluster to cluster at later times. These variations are larger in the Poisson model than in the neutrino model, presumably because nonlinear structure occurs on a much wider range of scales in the former case. In his own numerical models using ~ 100 particles Peebles (1971) found similar results and noted that they were discrepant with the $J \propto t^{5/3}$ law predicted in his earlier paper; he suggested that nonlinear effects might explain the discrepancy. Efstathiou and Jones (1979) also used a similar procedure to analyze the growth of angular momentum in a series of 1,000 body simulations of clustering from Poisson initial conditions. Their results are difficult to interpret because they did not start with pure growing mode fluctuations. Although they claim to find consistency with $J \propto t^{5/3}$, the relation $J \propto t$ appears an equally acceptable fit to their data.

The above results confirm Doroshkevich's contention that the material now seen in galaxies probably gained its angular momentum by first-order tidal torquing in the early universe [$J(t) \propto t$]. However, the total spin acquired depends critically on how this process was terminated. The final spin can be estimated correctly *to order of magnitude* by the usual dimensional arguments based on the tidal influence of neighbors near the epoch of turnaround (e.g., Thuan and Gott 1977) or by Peebles's (1969) estimate of the rms spin of spherical regions. However, the quantitative distribution of final spin values can only be obtained by examining suitable N-body experiments. Our series of simulations will be employed for this purpose elsewhere.

I wish to thank my colleagues at the Institute for Theoretical Physics for lively discussions. I also thank the Director and his staff for their warm hospitality during my stay. George Efstathiou, Carlos Frenk, and Marc Davis kindly allowed me to use our N-body simulations to test my results. This work was supported in part by the National Science Foundation under grant PHY 77-27084, supplemented by funds from the National Aeronautics and Space Administration.

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