Mon. Not. R. astr. Soc. (1989) **237**, 1127–1146

Biased clustering in the cold dark matter cosmogony

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Accepted 1988 November 15. Received 1988 November 15; in original form September 6

Summary. We have used the Press–Schechter approximation to calculate the abundance and large-scale clustering of dark haloes in the CDM cosmogony. Applying these results to quasars at $z=3$–4, we find a wide range of haloes which are both sufficiently massive and sufficiently numerous to host quasars, and the prediction for the clustering strength is correspondingly uncertain. Current estimates of quasar clustering require that the parent haloes lie at the most massive end of the allowed range, with velocity dispersions in the range 550–900 km s^{-1}, and that the quasar luminosity be quite tightly correlated with the parent halo properties. Another application is to the diffuse X-ray background. If this is generated by discrete sources associated with dark haloes then the upper limits on the X-ray angular correlation function constrain the halo masses to be $<10^{12}\, h^{-1}\, M_\odot$. We have considered two simple models for galaxy formation. In the first, which has some attractions for disc galaxies, we identify galaxies with dark haloes forming at the present epoch. We find that haloes with abundance like that of L_* galaxies have large-scale clustering which is nearly unbiased, and the smaller haloes are antibiased — quite a negative feature. In the second, the galaxies we see today are assumed to be a fossil remnant of haloes which formed earlier. The stellar and dark particle velocity dispersions are assumed to be equal, and the luminosity for each halo is adjusted to give a tight and universal L–V relation like that observed. We identify an epoch when the comoving number density of haloes with a given luminosity is maximized, and we calculate how this number density is biased by long wavelength modes. The results here are more encouraging: All of the 'galaxies' in this model are positively biased. The enhancement of the light-to-mass ratios for rich clusters is substantial, though dependent on the normalization. For a low normalization the bias is strong enough to reconcile virial estimates of M/L for clusters with $\Omega=1$.

1 Introduction

In this paper we calculate the clustering properties of the dark matter condensations ('haloes') produced in the cold dark matter scenario, and apply the results to galaxies and other astronomical objects which might plausibly be identified with some subset of these haloes. We use the analytic formalism derived by Press & Schechter (1974) to calculate the number density of dark haloes as a function of mass and redshift. While approximating the haloes as a two-parameter (M, z) family is a major oversimplification, these parameters are sufficient to determine gross properties such as binding energies, pressures, ages, etc., which we suspect are important in determining which haloes contain luminous objects.

In the cold dark matter ('CDM') model the post-z_{eq} Gaussian random density field has a very pink spectrum, with the result that relatively long wavelength modes have a substantial influence on much smaller scales. Long wavelength perturbations not only move mass around, but also modulate the properties of non-linear condensations, inducing biasing in the clustering of such objects. We will approximate the effect of long waves simply as a constant shift in the background density, or equivalently, as shifts in the effective local Ω. More rigorously, one would allow for the anisotropy of expansion introduced by the long waves (see Lacey 1988), but as we are ignoring the shapes of the perturbations we shall only consider the 'monopole' interaction. Some justification for ignoring the external tidal field comes from the smallness of the tidally induced spins found in N-body haloes, and from the success of the Press–Schechter model (and variants thereof) in reproducing the multiplicity functions seen in these numerical experiments.

With this approximation the effect of long wavelength perturbations is simply to modulate the collapse times of non-linear objects. This leads to a simple expression of biasing in terms of the mass distribution function $n(M, a) \equiv \partial N(>M, a)/\partial M$, where $N(>M, a)$ is the comoving number density of haloes of mass $>M$ when the universe has an expansion factor a:

$$n'(M, a) = n(M, a') \quad \text{with} \quad a' = a(1 - \Delta_{\mathrm{B}}/\Delta_{\mathrm{obj}}). \tag{1}$$

Here Δ_{obj} is the initial density contrast of the objects and Δ_{B} is the amplitude of the background perturbation. This is the 'peak background split' discussed by Bardeen *et al.* (1986), and states that, with our approximations, the non-linear condensations in a perturbed region are identical to those seen in an unperturbed region, but at a slightly different time. In this manner, we can calculate the linear response to a linear background perturbation Δ_{B}, and this can be expressed in terms of a bias parameter b. For objects which have large bias, we can also calculate the non-linear response, while still remaining in the regime where $\Delta_{\mathrm{B}} \lesssim 1$. This can be used to estimate the enhancement of the number of galaxies per unit mass in rich clusters, with which it is interesting to compare the observed cluster light-to-mass ratios.

While it is fairly straightforward to calculate the instantaneous clustering bias for dark matter haloes, the application to astronomical objects we actually observe is fraught with uncertainty of interpretation.

Probably the cleanest application is to the clustering of rich clusters. The density contrast of a virialized cluster is $\Delta = 2V_{\mathrm{circ}}^2/(HR)^2 = 200(V_{\mathrm{circ}}/1500\ \mathrm{km\ s^{-1}})^2(R/R_{\mathrm{Abell}})^{-2}$. Thus it seems quite reasonable to identify Abell clusters with recently collapsed dark matter condensations which, according to the simple spherical collapse model should have density contrasts of about 200. With this identification, a very strong amplification of the clustering of Abell clusters is predicted (Kaiser 1984). This still falls far short of the estimates of Hauser & Peebles (1973) and of Bahcall & Soneira (1983), though is similar to the estimate of Sutherland (1988), who has attempted to correct for apparent non-spatial clustering.

There are other applications where an analogous calculation may be appropriate. It may be that quasars can be identified with some class of dark matter halo. If so, we can predict the clustering properties of quasars by finding the objects on the M, z plane at the observed redshift which have the appropriate number density. We will explore this in Section 3.2. A major difficulty in making such predictions is that the appropriate number density of haloes may, for a number of reasons, be many orders of magnitude greater than the actual number density of quasars observed. We find that the resulting bias prediction is very uncertain, but at least we can place upper bounds on the clustering strength for these objects.

Another application where the instantaneous bias calculation may be usefully applied is the problem of the 1–3 keV diffuse X-ray background. If this is produced by discrete sources then the study of arcmin-scale fluctuations (Hamilton & Helfand 1987) and limits on the angular correlation function (Barcons & Fabian 1989) place constraints on the number density and small-scale clustering of such sources. It is interesting to ask whether there are a class of objects in the CDM hierarchy which satisfy these constraints, and, if so, what are their properties. We will address this question in Section 3.3, where we find that these observations place tight constraints on the masses of discrete objects in CDM, if these are to make the X-ray background.

We next consider the clustering of galaxies. Naively, one might simply explore the M, z plane, find objects with number densities and masses (or velocity dispersions) commensurate with real galaxies and read off the bias factor or estimate the enhancement in the number density of galaxies per unit mass in clusters, etc. There are good reasons, however, to be suspicious of such results. One can certainly identify objects whose abundance would have been quite strongly enhanced in overdense regions at that epoch, but only because the analogous objects in underdense regions were due to collapse slightly later. It is therefore possible to contrive a 'theory for galaxy formation' which gives a substantial bias either by invoking some hypothetical feedback to switch off galaxy formation, or by postulating that the efficiency of star formation is a step function of the halo properties, with the position of the transition finely tuned to coincide with the edge of the Gaussian distribution. It is not difficult to see what is required here; on galactic scales where $n \simeq -2$ lines of constant ν are parallel to contours of constant pressure, so if we set a carefully tuned pressure threshold we can trim off the high-ν objects and get a strong clustering bias. Here we wish to consider less radical possible solutions to the problem.

A useful way to illustrate the problem we are faced with here is to consider the space–time diagram for the virialized haloes (selected at a given density contrast): for a two dimensional space, this would resemble a forest of trees. At the uppermost levels (high redshift) we have thin branches representing sub-galactic objects. These merge together into progressively larger objects, and the clusters at the present epoch would be the thickest trunks at ground level. A theory for galaxy formation would tell us how to assign luminosity to these branches. We will explore two alternative models for galaxy formation: in the first (Section 4.1) we calculate the bias that would arise if galaxy formation is a steady ongoing process, and if, as suggested by numerical experiments, the properties of even the dense inner parts of haloes are determined by the binding energy of the most recently collapsed material. This might be appropriate for the formation of disc galaxies, and in this case one can apply the instantaneous bias results fairly directly.

The alternative we will explore in Section 4.2 is to assume that the process of galaxy formation has largely terminated (and so this may perhaps be relevant to the formation of ellipticals and spheroids) and requires somewhat different calculational techniques. We will assume that some stars form in all of the branches of the tree according to some formula $L(M, z)$, and that the present day galaxies are fossil remnants of this earlier star forming activity. For this type of model one will obtain a positive bias if $\partial L/\partial z > 0$. We shall argue that if the stellar and dark matter velocity dispersions are equal then the small scatter and apparent universality of the L–V relation do indeed imply a positive bias for galaxies in this model and we make quantitative predictions for the bias as a function of the number density of galaxies. While this model is empirically motivated and is relatively 'natural' in that no sharp threshold for galaxy formation is assumed, the physical cause of the bias remains uncertain. More unsettling still is the way that the inferred $L(M, z)$ function seems contrived to compress the intrinsically broad $M-V$ relation into a much narrower $L-V$ relation. Possible alternatives and modifications are discussed in Section 4.3.

2 Press–Schechter formalism

Press & Schechter (1974) derived an analytic approximation for the comoving number density of haloes in the mass range M to $M+dM$ at a given redshift:

$$n(M, z, t)\, dM = \frac{2\rho_0}{\sqrt{2\pi}} \frac{t(1+z)}{M} \left(\frac{-1}{\sigma^2} \frac{\partial \sigma}{\partial M} \right) \exp\left(-\frac{t^2(1+z)^2}{2\sigma^2} \right) dM, \tag{2}$$

where $\sigma(M)$ is the rms density fluctuation of the initial Gaussian density field when filtered on a mass scale M, and t is the threshold linear theory overdensity at which regions turnaround and collapse. We adopt $t=1.68$ as applicable for simple spherical collapse, and use $\sigma(M)$ for CDM as calculated by Bond & Efstathiou (1984). The only free parameters in this theory are the normalization, which is conventionally specified as σ_8, the linear theory rms mass fluctuation in spheres of radius 8 h^{-1} Mpc, and the Hubble parameter $H=100\ h$ km s^{-1} Mpc^{-1} which determines the fundamental length scale in the problem. We shall use the spectrum appropriate for $h=0.5$.

This mass function may be derived from a very simple picture for the formation of non-linear condensations: regions which are initially overdense by a fraction $(\Delta\rho/\rho)_i$ turn around and form virialized objects with a final density contrast $(\Delta\rho/\rho)_f \simeq 200$ after the universe has expanded by a factor $(a_f/a_i) \simeq 1.68/(\Delta\rho/\rho)_i$. This approximation enables the final distribution of non-linear objects to be calculated from the statistics of the initial Gaussian density field. It is remarkable that this approximation derived from consideration of spherically symmetric perturbations seems to describe the N-body results quite well; $\Delta\rho/\rho \simeq 200$ does indeed seem to delineate the transition from infall to quasi-equilibrium, and the mass spectrum seems to agree well with results of group finding algorithms applied to numerical experiments (Efstathiou *et al.* 1988). Even better results can be obtained from a somewhat different model (Cole, Kaiser & Efstathiou 1988) which incorporates the same ingredients in a rather different way. However, the Press–Schechter formula is sufficient for our present purposes.

Before we can make concrete predictions for the abundance of haloes, we must determine the normalization parameter. Since it seems reasonable to identify rich clusters with recently formed systems, we may estimate σ_8 by comparing the number density of rich clusters as a function of velocity dispersion with the present number density of haloes of the same velocity dispersion. We have done this by first determining the mean and scatter in velocity dispersion for each richness class using the Struble & Rood catalogue (1987), (see Fig. 1), and then converting this richness distribution to number density as a function of $V_{\rm circ}$ using the normalization of Bahcall & Soneira (1983). For the present epoch haloes we have $V_{\rm circ} = 1.6 \times 10^{-2}(M/h^{-1} M_\odot)^{1/3}$ km s^{-1} from the virial theorem, with an assumed overdensity of 200. Fig. 2 shows the rich cluster data and the present day Press–Schechter distribution of haloes for two choices of normalization; $\sigma_8 = 0.4$ and 0.6, which seem to span the range compatible with the cluster data. The lower value, $\sigma_8 = 0.4$, is comparable with the normalization adopted in the extensive N-body simulations of Efstathiou and collaborators, while $\sigma_8 = 0.6$ is the value advocated by Bardeen *et al.*, from the statistics of peaks in Gaussian random fields. We will use these values throughout to illustrate the uncertainty arising from the uncertain normalization.

It is also of interest to compare the present day distribution of haloes with the observed

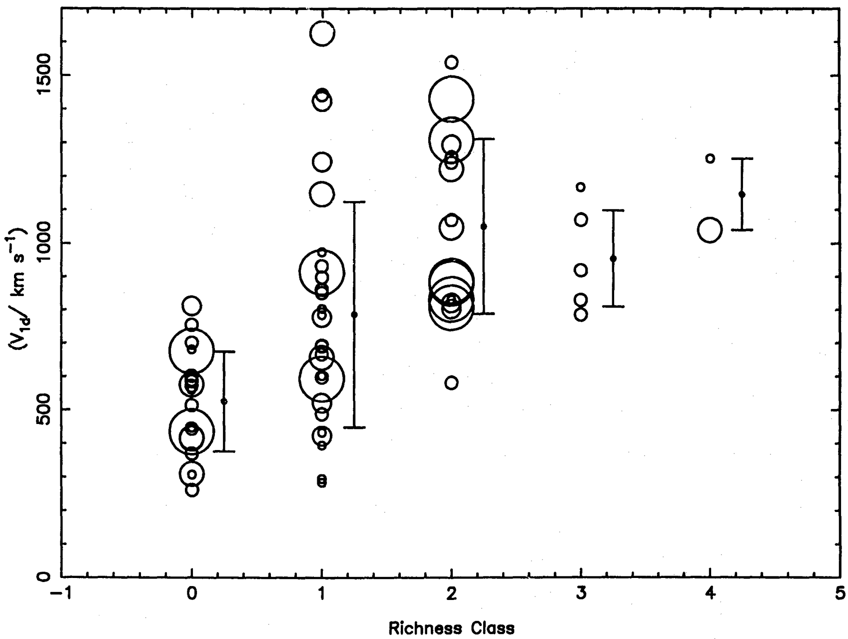

Figure 1. Velocity dispersion versus richness. The line-of-sight velocity dispersions of Abell clusters taken from the Struble and Rood catalogue (1987) for richness classes 0 to 4. Larger symbols have been used for clusters for which more galaxy redshifts have been obtained.

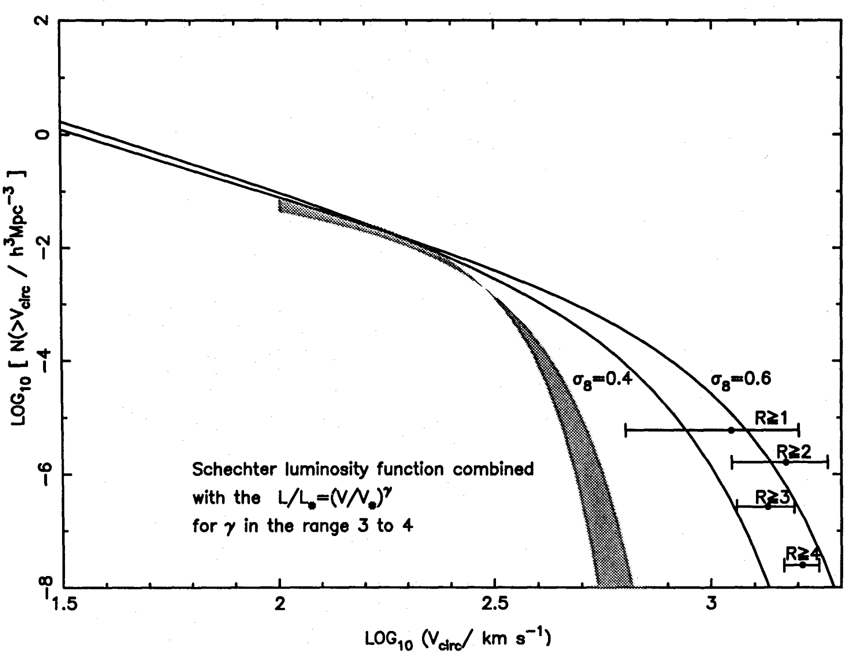

Figure 2. A comparison of the present day spectrum of dark matter haloes as predicted by the Press–Schechter formalism, for normalizations of $\sigma_8 = 0.4$ and 0.6, with the Abell cluster data taken from Fig. 1, and the present day galaxy distribution, estimated as stated in the text.

galaxy distribution. In order to do this we must determine what halo velocity dispersion to associate with a galaxy of a given luminosity L. We have assumed $V_{\rm stars} = V_{\rm halo}$, and then related the galaxy velocity dispersion to luminosity using a Tully–Fisher/Faber–Jackson type law,

$$\frac{L}{L_*} = \left(\frac{V}{V_*}\right)^{\gamma}, \qquad (3)$$

with $V_* = 300$ km s^{-1}, and γ in the range of 3–4, which spans the range for the relationship appropriate for optical and infrared luminosities. We have then calculated their number density using the Schechter function fit to the empirical luminosity function:

$$\Phi(L)\,dL = \Phi_*\left(\frac{L}{L_*}\right)^{\alpha} \exp\left(-\frac{L}{L_*}\right)\frac{dL}{L_*}, \tag{4}$$

where $\Phi_* = 1.2\times10^{-2}\ h^3$ Mpc^{-3}, $\alpha = -1.25$ and $L_* = 1.0\times10^{10}\ h^{-2}\ L_\odot$ (Kirshner *et al.* 1983).

Comparing these distributions (Fig. 2) we see that the number densities of DM haloes and real galaxies agree quite well, though only over a small range of velocity dispersion. How the bright end cut-off in the galaxy luminosity function comes about is not fully understood. It may be associated with the inability of gas to cool in high temperature haloes (Rees & Ostriker 1977). Empirically it seems that above some mass the theoretical curve should really be associated with the number density of groups and clusters of galaxies rather than individual galaxies. At the low velocity end there are increasingly more haloes than galaxies of the same velocity dispersion and so the faint end slope is too steep (Schaeffer & Silk 1985). This is also seen in N-body CDM calculations (Frenk *et al.*, 1988).

3 Instantaneous clustering bias

3.1 CLUSTERING OF HALOES

From the mass function [equation (2)] describing the evolving distribution of haloes we can determine the instantaneous bias parameter $b(M, z)$ for haloes of mass M at redshift z by applying the peak-background split and considering the effect of adding a small growing background perturbation $\Delta_B \equiv \Delta_0/(1+z)$ to some region. In this region the threshold for collapse will be reduced to $t = 1.68 - \Delta_B$, and consequently the number of haloes of mass M at redshift z will be increased by a factor $n(M, z, 1.68-\Delta_B)/n(M, z, 1.68)$. Thus the bias parameter $b(M, z)$, which is the ratio of the Eulerian perturbation in number density to the present density perturbation is given by:

$$b(M, z) = \frac{1}{(1+z)} - \frac{1}{(1+z)} \times \left.\frac{\partial \ln n(M, z, t)}{\partial t}\right|_{t=1.68}, \tag{5}$$

or using equation (2),

$$b(M, z) = \frac{1}{(1+z)} - \frac{1}{1.68(1+z)} + \frac{1.68(1+z)}{\sigma^2(M)}. \tag{6}$$

Contours of this bias parameter along with contours of the number density of haloes with mass in the range M to $2M$ present at redshift z are shown in Fig. 3(a) and (b), for normalizations of $\sigma_8 = 0.4$ and 0.6. We see that the bias is strongest for rare massive haloes for which $\sigma(M)/(1+z)$ is low; the 'high-v' objects, the borderline unbiased objects are the one-sigma haloes, for which $\sigma(M)/(1+z) = 1.68$, and smaller haloes are actually antibiased.

3.2 QUASARS

Various estimates are now available for the clustering of quasars, and even some indication of the evolution of the clustering. From Fig. 3(a) and (b), we can read off the clustering strength for haloes with abundances like quasars. We take as a rough datum the estimate of Efstathiou

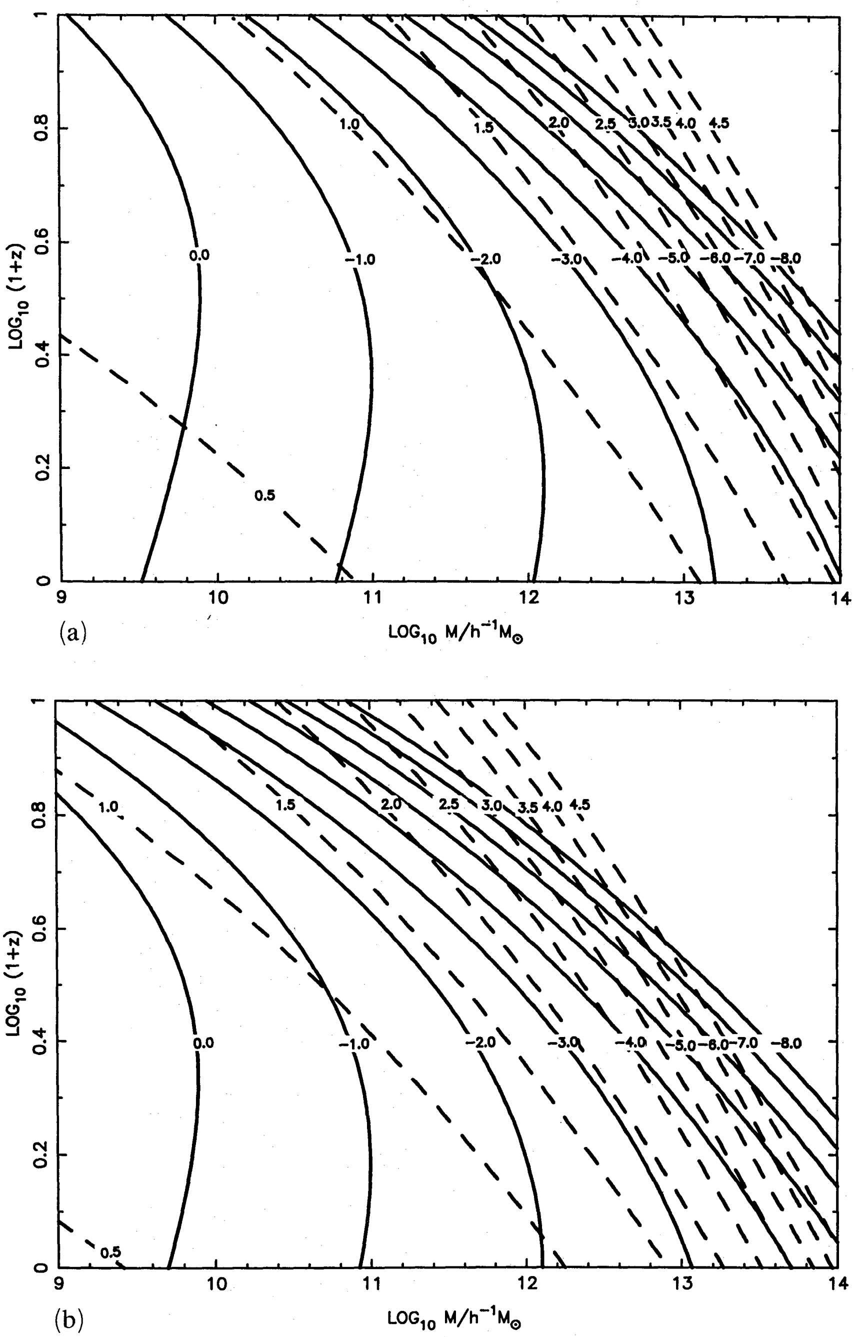

Figure 3. For normalizations (a) $\sigma_8 = 0.4$, and (b) $\sigma_8 = 0.6$ contours are shown of $\log_{10}[N(M \to 2M, z)/h^3 \text{ Mpc}^{-3}]$, where $N(M \to 2M, z)$ is the number density of haloes with mass in the range M to $2M$ present at redshift z, and of the bias parameter b of equation (6).

& Rees (1988), that the number density of quasars with luminosity $L \geq 2.5 \times 10^{39}\ h^{-2}$ J s^{-1} at $3 < z < 4$ is $N_Q \sim 1.5 \times 10^{-8}\ h^3$ Mpc^3. For normalization $\sigma_8 = 0.4$ we find that the mass of the haloes with this number density is $M = 5 \times 10^{12}\ h^{-1}$ Mpc, and the bias factor is $b = 4$–4.5. For $\sigma_8 = 0.6$ the appropriate values are $M = 2.2 \times 10^{13}\ h^{-1}$ Mpc and $b = 3.5$–4. With this identification we would be led to conclude that the clustering strength for these quasars would be very strongly amplified relative to the present epoch mass clustering, and would be intermediate between bright galaxies and rich clusters. This seems in good accord with the current estimates Iovino & Shaver (1988) and Boyle *et al.* (1988).

There is good reason to suspect, however, that this simple one-to-one correspondence between haloes of a given mass and observed quasars is likely to give an overestimate of the clustering strength. The problem is that the appropriate abundance of parent haloes may, for one reason or another, be very much greater than the observed abundance, and the clustering of this larger parent population, of which we see only a small random sample, would be much reduced.

The situation here is quite different to the treatment of Abell clusters. In that case the selection criteria refer directly to the scale and density appropriate for virializing structures. The mass-to-light ratios for clusters are roughly constant, so it seems reasonable to assume that we are seeing a fairly complete and representative sample of the most massive haloes. Contrast this happy situation with that pertaining to quasars. Here the scale of the active nuclear regions is many orders of magnitude smaller than the parent haloes, so it would be somewhat remarkable if there was a very tight correlation between halo mass and quasar luminosity. Whether it is the case that the observed quasars are predominantly formed in intrinsically massive haloes with run-of-the-mill light-to-mass ratios, or in a tiny ultra-luminous subset of much more numerous lower mass haloes depends on the unknown form of the distribution of luminosities at fixed mass. It seems at least plausible that the latter is the case, and the clustering strength will therefore be much less. Also, if the lifetime of the quasar activity is less than the time for which the population must be maintained, this will also cause the number of parent haloes to further exceed the number of observed quasars.

The uncertainty in these predictions can be bounded only by placing constraints on the required mass and lifetimes of quasars. Following Efstathiou & Rees (1988), we write the halo mass as

$$M_{\text{halo}} \simeq 1.25 \times 10^{11} \left(\frac{\varepsilon}{0.1}\right)^{-1} \left(\frac{F}{0.01}\right)^{-1} \left(\frac{t_{\text{Q}}}{2.5 \times 10^{8}\, h^{-1}\, \text{yr}}\right) h^{-3}\, M_{\odot}. \tag{7}$$

Here ε is the fraction of the black hole mass which is converted to radiation, F is the ratio of the black hole mass to the halo mass and t_{Q} is the lifetime of the quasar which appears divided by the time between $z=4$ and 3, which is the time for which the quasar population must be maintained.

There is enormous uncertainty in the appropriate values for these quantities. Comparing with the masses obtained above for the haloes with $N_{\text{halo}} = N_{\text{Q}}$, (and assuming $t_{\text{Q}} = 2.5 \times 10^{8}$ h^{-1} yr for the moment) we see that these haloes could house sufficiently luminous quasars even if εF were as low as $5.6 \times 10^{-6}\ h^{-2}$ (for this and for the results below we will assume $\sigma_8 = 0.6$). On the other hand, if we were to assume $\varepsilon = 0.1$ and $F = 0.01$ then the required mass is only $1.25 \times 10^{11}\ h^{-3}\ M_{\odot}$; the number density is a million times larger and the clustering bias would be only $b \simeq 1$. While one might baulk at the rather high efficiency implied by $F = 0.01$ — if $\Omega_{\text{baryon}} \simeq 0.1$, this would require that about 10 per cent of the baryons find their way into the central engine — one should bear in mind that this is not supposed to be a representative number but rather that appropriate for a tiny fraction $\sim 10^{-6}$ of the most efficient haloes.

A further complication which may reduce the clustering still further is the possibility that the quasars burn for less than $2.5 \times 10^{8}\ h^{-1}$ yr. If the quasars burn at the Eddington limit, then this would imply $t_{\text{Q}} = 4 \times 10^{8}\ \varepsilon$ yr; the halo mass (still assuming $F = 0.01$) would be about another order of magnitude smaller, and the clustering amplitude would fall to $b = 0.75$. The number of parent haloes is increased in this case by about a factor of 10, so the fraction of haloes which undergo quasar activity at some time in their life is similar to that obtained above, so again we would be dealing with an extreme one-in-a-million subsample.

In summary, we have found that in CDM there is a wide range of haloes which are both sufficiently abundant and sufficiently massive to host quasar activity. The prediction for the clustering of these objects is correspondingly vague. Proponents of this theory would not be embarrassed if the clustering estimates were revised downwards. The present estimates are only compatible with CDM if there is a rather small scatter in quasar luminosities at fixed halo mass. A testable consequence is the resulting link between halo velocity dispersions and clustering strength: if the clustering is indeed large, then the hosts must be massive, with halo rotation velocities around 900 km s^{-1} (550 km s^{-1} for $\sigma_8=0.4$). This is similar to the value for poor clusters, perhaps suggesting a similarity with present day cooling flows. At the other extreme, the rotation velocities would be around 250 km s^{-1}, similar to bright galaxies today.

3.3 X-RAY BACKGROUND

Hamilton & Helfand (1987) have analysed the arcmin-scale fluctuations of the 1–3 keV X-ray background using *Einstein* IPC data. They have shown that if this background is produced by a population of randomly positioned point sources, then in order not to produce fluctuations in intensity larger than the observational limits on arcmin-scales, the number density of such sources must be greater than 5000 deg^{-2}. When one takes into account possible spatial clustering of sources, this constraint becomes $N \geq 5000(1+J)\ \mathrm{deg}^{-2}$, where J is the integral over the spatial correlation function of the sources after smoothing with a filter of one arcmin corresponding to the point spread function of the *Einstein* observatory IPC:

$$J=\frac{N}{\Delta r_c}(2\pi)\left(\frac{180}{\pi}\right)^2\int_{s=-\infty}^{\infty}\int_{\theta=0}^{\infty}\xi(\Delta r)\exp(-\theta^2/4\theta_{\mathrm{IPC}}^2)\,\theta d\theta ds. \tag{8}$$

Here N is the number of sources per square degree at a mean comoving distance r_c, Δr_c is the comoving depth over which they are spread, s is their line-of-sight comoving separation, θ their angular separation and $\xi(\Delta r)$ is their spatial correlation function at comoving separation Δr, and $\Delta r^2=s^2+(r_c\theta)^2$.

Another constraint comes from the analysis of Barcons & Fabian (1989) who have estimated the angular correlation function $w(\phi)$ from the same data, and concluded that $w(\phi)<2.5\times10^{-3}$ (at 95 per cent confidence) on scales of 1–5 arcmin. This requires that the smoothed projected correlation function of the sources satisfy,

$$\frac{1}{\Delta r_c}\frac{(\pi)}{\theta_{\mathrm{IPC}}^2(2\pi)^2}\int_{s=-\infty}^{\infty}\int_{\psi=0}^{2\pi}\int_{\theta=0}^{\infty}\xi(\Delta r)\exp(-\theta^2/4\theta_{\mathrm{IPC}}^2)\,\theta\, d\theta\, d\psi\, ds\leq 2.5\times10^{-3}, \tag{9}$$

where ψ is a polar angle and $\Delta r^2=s^2+r_c^2(\theta^2+\phi^2+2\phi\theta\cos\psi)$.

It is interesting to ask whether there are any haloes in the two parameter M, z distribution whose number density and clustering properties satisfy these two constraints. In the linear regime the correlation function $\xi(\Delta r)$ of haloes of mass M at redshift z is simply the bias parameter of equation (6) squared multiplied by the spatial correlation function ξ_ρ of the underlying CDM density distribution. Thus it is possible to calculate directly the region in the M, z plane within which point sources associated with haloes in the mass range $M\rightarrow 2M$ and redshift range $(1+z)\rightarrow 2(1+z)$ could produce the whole of the 1–3 keV X-ray background without violating the constraints on arcmin-scale fluctuations. Fig. 4(a) and (b) shows, for haloes in the redshift range $(1+z)\rightarrow 2(1+z)$ and mass in the range $M\rightarrow 2M$, contours of number density of objects per square degree. Also shown are curves representing the Hamilton & Helfand and Barcons & Fabian constraints when linear theory clustering is taken into account. The weakest limit applies if the background is generated at $z\simeq 1$, in which case

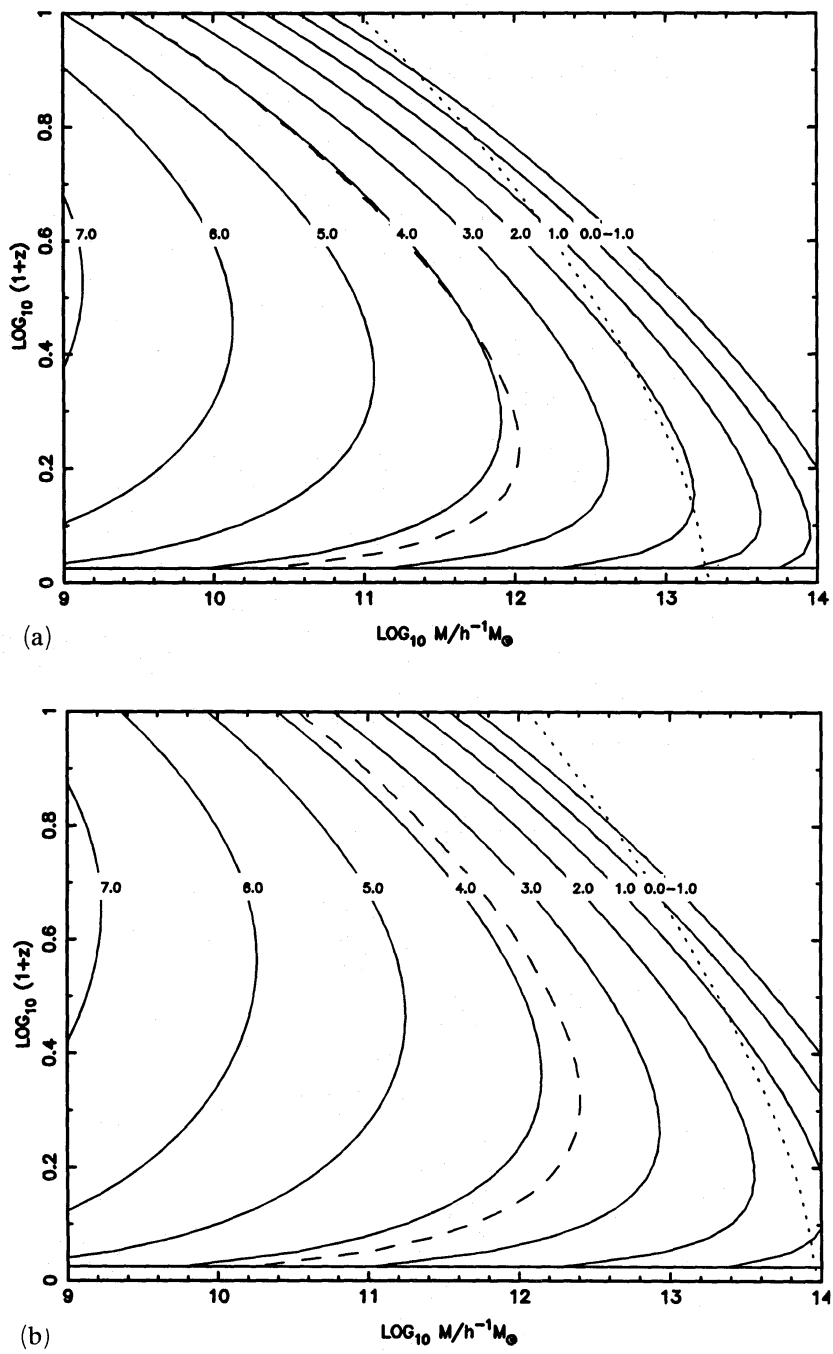

Figure 4. For normalizations (a) $\sigma_8 = 0.4$, and (b) $\sigma_8 = 0.6$ contours are shown of $\log_{10}[N(M \to 2M), (1+z) \to 2(1+z)]/h^3\,\mathrm{Mpc}^{-3}$, where $[N(M \to 2M, (1+z) \to 2(1+z)]$ is the number of haloes per square degree with mass in the range M to $2M$ and in the range $(1+z)$ to $2(1+z)$. Also plotted are the two curves to the right of which haloes are either too rare or too clustered, to be compatible with the hypothesis that they are the sites inhabited by point sources producing the whole of the 1–3 keV X-ray background, and the limits on small-scale fluctuations and correlations of the X-ray background.

the upper limit from the Hamilton & Helfand constraint $M_{\mathrm{halo}} \lesssim 12^{12}\ h^{-1}\ M_{\odot}$; quite similar to the masses of bright galaxy haloes. If the generation redshift is $z \simeq 5$, then the upper mass limit falls by about an order of magnitude. The current estimates of the angular correlation function give much weaker limits on the halo masses.

4 Galaxy clustering

We now consider two highly idealized models for the formation of galaxies. In the first we shall assume that galaxy formation is an ongoing process, with the luminosity tied to the properties of the most currently virializing material, so the clustering can be calculated by taking a slice through the distribution of haloes at the present epoch. This model is possibly relevant for disc galaxies. In the second we assume that galaxy formation on the whole has finished — what we see today are fossil remnants of the past history of the halo dendogram — and we calculate the biasing implied by identifying the stellar and dark matter velocity dispersions and making the 'galaxies' respect a universal and tight L–V relation. This model may be applicable to ellipticals and spheroids.

4.1 LATE GALAXY FORMATION

A popular picture for formation of disc galaxies which has evolved over the years (notable contributions being the works of White & Rees 1978; Fall & Efstathiou 1980; Gunn 1982) can be outlined as follows. Collapsing perturbations violently relax to form dark matter haloes with density profiles $\rho(r) \propto r^{-2}$, while tidal fields induce angular momentum in these haloes, with spin parameters $\lambda \simeq 0.05$–0.1. Gas falling in is shock heated to the halo virial temperature, and adopts a hydrostatic equilibrium configuration with a density run like that of the dark halo. At later times gas from some radius within the currently virializing radius can cool in a Hubble time. This gas contracts quasistatically until rotationally supported, producing an approximately self gravitating rotationally supported disc with $V_{\rm rot} \simeq V_{\rm halo}$. It is encouraging that for a halo with $V_{\rm rot}$ equal to that of our galaxy and with $\Omega_{\rm baryon} \simeq 0.1$, in accord with standard nucleosynthesis, the radius at which $t_{\rm cool} = t_{\rm dyn}$ is ~ 100 kpc, so after collapse by a factor $\simeq 1/\lambda$ this might plausibly produce a disc of reasonable dimensions. Somewhat less encouraging is the L–V relation predicted here. The mass–velocity relation is $M \propto V^3$, since the objects have the same density at their virializing radius, but a smaller fraction of the gas can cool in the hotter haloes, so the predicted log L–log V slope is always less than 3. Approximating the cooling time as $t_{\rm cool} \propto T^{\beta} \rho^{-1}$ one predicts $L \propto V^{(3-\beta)}$, or roughly $L \propto V^2$ for galactic temperatures. This is a shallower slope than the 'canonical' $L \propto V^4$, though not so far from the slope of $\simeq 2.7$ which is more relevant for optical luminosity (Tully & Fisher 1977). Perhaps it is unreasonable to expect the theory to predict such details with great accuracy, and a positive point is that this type of model would at least predict that the galaxies form a one-parameter family; a property not automatically shared by other models.

In this picture, galaxy formation is an ongoing process; the number density of haloes with a given rotation velocity is only slowly varying, and in any one of these haloes the radius for which $t_{\rm cool} = t_H$, and therefore the mass of cooled gas, is an increasing function of time. If so it would seem reasonable to associate the present distribution of haloes with the galaxy distribution. If this is the case then the expected bias for these objects is simply given by equation (6) with $z = 0$. In Fig. 5 we have plotted the bias as a function of cumulative number density. We have chosen number density rather than velocity dispersion, for example, as this seems to be more robustly determined by the model. We see that for rare objects such as rich clusters which are on the steeply rising part of the mass function, Fig. 2, a strong positive bias is predicted. However, for objects with number densities $n \simeq 10^{-2}\, h^3\, \mathrm{Mpc}^{-3}$, as is typical of ordinary bright galaxies, no bias or even an anti-bias would be predicted unless a very low normalization were to be adopted.

We consider this prediction of such a low bias to be quite a negative feature of this otherwise physically well motivated scenario for galaxy formation. The bias parameter derived here

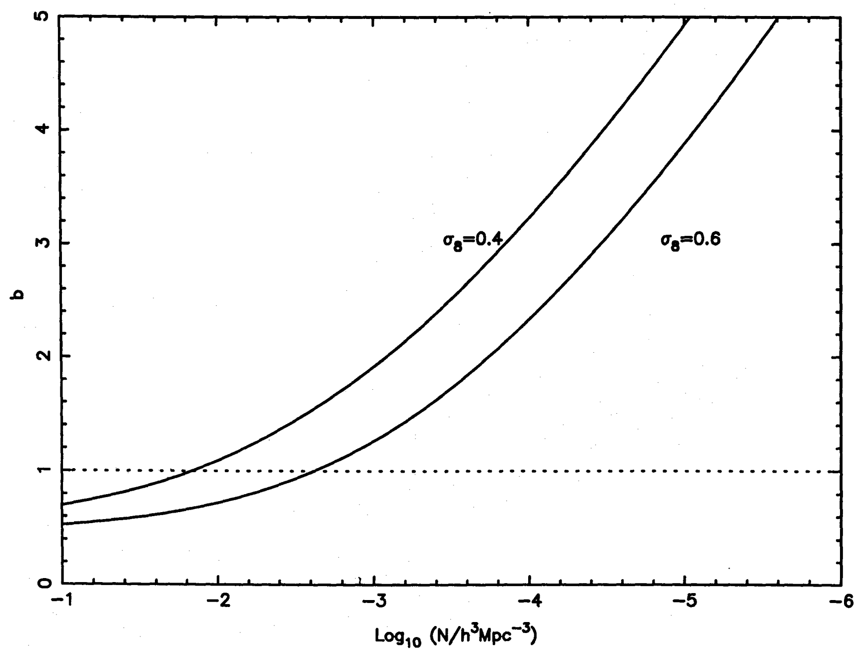

Figure 5. The bias parameter b as a function of halo cumulative number density, for the present day distribution of haloes.

applies directly only to large-scale structure. The predicted bias is considerably less than that inferred from a comparison of optical angular dipoles and peculiar velocity estimates (Kaiser & Lahav 1988) which indicate $b \simeq 2$ for optical galaxies. With a late formation it is difficult to extrapolate into the regime of currently non-linear clustering. None the less, if one takes seriously the idea that the supply of gas for star formation is regulated by the condition that $t_{\rm cool} = t_H$, then on quite general grounds it is easy to see that even in the non-linear regime one expects to obtain a net anti-bias for this gas supply, and therefore, most naturally, for the net abundance of luminous stars. The problem is simply that in regions which are overdense on large scales, the characteristic virial temperatures of haloes will surely be enhanced, while the density of the haloes at virialization is just a multiple of the global background density, and is therefore expected to be unchanged, hence the ratio $t_{\rm cool}/t_H$ will be increased.

We should point out that our model is extreme in that the halo properties at quite high density contrasts where the gas can currently cool are locked to the properties at larger radii where the new material is currently falling in. This assumption is motivated by N-body experiments, which lead one to attribute the non-linear profiles of clusters to relaxation (Frenk *et al.* 1988). An alternative view is that the profiles are determined by the initial spectrum (e.g. Hoffman & Shaham 1985), which the dense inner parts of haloes reflecting the binding energy of material which virialized at some earlier epoch, and which has remained dynamically intact since then. If this picture is closer to reality then other calculational techniques are called for.

4.2 EARLY GALAXY FORMATION

In this section we will take a rather different approach. This is designed to model the biasing that might develop if galaxy formation has effectively finished. Rather than assume some *ad hoc* threshold for galaxy formation, we let the luminosity generated in each branch of the space–time halo dendogram be a continuous function of the halo properties, with this function chosen so that the 'galaxies' respect the observed relation between luminosity and velocity dispersion. Specifically, we assume that $V_{\rm stars} = V_{\rm halo}$ and that each halo is assigned a luminosity which is proportional to the 4th power of the stellar velocity dispersion. This rule is empirically motivated, and relatively 'natural' in that no *ad hoc* threshold is introduced.

At first sight it might seem that the empirical L–V relations would be compatible with an unbiased 'mass-traces-light' model. After all, it is well known that in the CDM model, the spectral index is $n \simeq -2$ on galactic scales, and this results in a mean mass velocity relation $M_* \propto V_*^4$. Thus if we simply assumed that mass converts to light with equal efficiency everywhere then this would produce the observed L–V relation, and the clustering of stellar luminosity would of course be unbiased. While it is true that the mean log L–log V relation is acceptable, on closer inspection a couple of problems emerge. We will discuss these in some detail. The failure of the unbiased model on these grounds is instructive for two reasons: it shows that, under certain assumptions at least, some kind of clustering bias seems inevitable, and it also reveals just what those assumptions are. While they are reasonable, they are not sacrosanct, and in Section 4.3 we will consider relaxing them.

The first problem concerns the scatter in the mass–velocity relation. If we simply take the positive half of a Gaussian for the distribution of collapse redshifts at a given mass, and convert this to the distribution of the absolute magnitude estimator $M = 2.5 \log V^4$, this gives a predicted scatter of about 2.2 mag. This is something of an overestimate since it includes perturbations which have yet to collapse. Truncating the distribution helps somewhat, but the revised estimate of about 1 mag is still considerably larger than that observed, and this presumably includes a substantial component from instrumental errors, etc. It would be possible to avoid this problem if, as envisaged by Faber (1982), the different morphological types are stratified along lines of constant ν, possibly because of systematic dependence of the angular momentum on ν. (Here ν denotes the initial amplitude of the perturbation in units of the rms.) It is important to realize that with this type of stratification, the various galaxy types would still be unbiased tracers of the mass, as in the monopole approximation for the effect of long wavelength modes at least, there is no modulation of the spins, or indeed of any parameters depending on the local shape of the initial perturbation field. It now seems unlikely that the correlation between ν and angular momentum is sufficiently strong to effect the required stratification. Perhaps there is some other 'shape' determinant for morphology — even if this were the case, the model would still fall foul of a second, less escapable problem.

The second problem concerns the effect of long wavelength density perturbations. These will perturb the rotation velocities but not the luminosities (by assumption), so that in regions which are now overdense we would expect to see less luminous galaxies at a given velocity dispersion. This would cause the apparent Hubble constant to depend on the local density of the galaxies used as distance estimators, and would play havoc with studies of departures from pure Hubble flow. To get an idea of the strength of this effect consider a 'top-hat' perturbation of radius R, at distance r from the observer and with overdensity $\Delta_B \ll 1$. If the mean L–V relation has slope 4, then the fractional error in the distance is

$$\frac{\Delta r}{r} = \frac{\Delta_B}{1.68(1+z_f)}, \tag{10}$$

if we assume that the haloes containing the galaxies collapse, on average, at redshift z_f. On the other hand, the real peculiar velocity due to a spherical perturbation of radius R and amplitude Δ_B is $v_{\rm pec} = (1/3)\Delta_B HR$, so the spurious 'peculiar velocity' $H\Delta r$ would overwhelm the true peculiar velocities for perturbations at distances $r > 1.68(1+z_f)R/3$. Thus we see that in this hypothetical unbiased model, such systematic offsets in the L–V relations would greatly exceed any real peculiar velocities, but this is apparently not seen.

Setting $L = V^4$ exactly obliviates both problems and also implies a bias. A straightforward way to see this is to consider an individual object and image subjecting this to a background perturbation Δ_B. From the virial theorem and the assumptions about virialization, $L = V^4$ implies $L = M^{4/3}(1+z_c)^2$. Thus an earlier collapse time leads to a higher L and therefore to a net luminosity bias

$$L \rightarrow L' = (1 + \Delta_B/\Delta_{obj})^2 L. \tag{11}$$

A great advantage of the crude analytical approach adopted here is that one can see explicitly where the bias comes from, namely the highly non-trivial assumed dependence of L/M on collapse redshift. However empirically motivated and 'natural' the identification $L \equiv V^4$, this theory still lacks a physical explanation. All we have shown is that assuming both $V_{halo} = V_{stars}$ and $L \propto M_{halo}$ looks unacceptable. In what follows we shall explore the consequences of retaining the former assumption, and therefore making variations in L/M reproduce a tight and universal L–V relation.

One can use the Press–Schechter formalism to quantify the biasing. Fig. 6 shows, as a function of redshift, the number density of haloes $N(\geq V_{circ})$ with circular velocities greater than some value V_{circ}, and therefore with luminosities greater than $L_* \ (V_{halo}/V_*)^4$. Each member of this family of curves rises rapidly as the hierarchical growth proceeds, reaches a broad maximum and then gradually falls as these systems merge to form systems with higher velocity dispersions. The fact that these curves have broad maxima is consistent with the notion that we are dealing with a population of haloes with roughly constant comoving number density which are gradually accreting smaller haloes and doing so in such a way as to grow flat rotation curves.

In this theory of galaxy formation, haloes of a given velocity dispersion are associated with galaxies of the same velocity dispersion. It therefore follows directly that there is a fairly well defined era of galaxy formation, where the number density of haloes with velocity dispersions typical of galaxies peaks. In Fig. 6, the normalization is $\sigma_8 = 0.6$, and we see that haloes with circular velocity $V_* = 300$ km s^{-1} reach their peak number density of 8×10^{-3} h^3 Mpc^{-3} at a redshift of $z \simeq 1$. This agrees reasonably well with the present number density of galaxies of

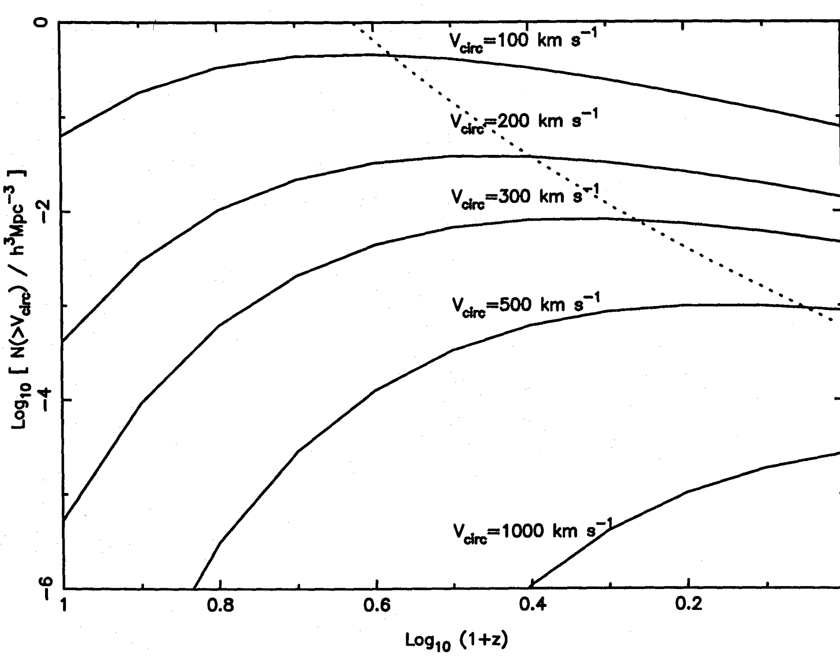

Figure 6. The cumulative number density of haloes with circular velocities greater than V_{circ} as a function of $(1+z)$.

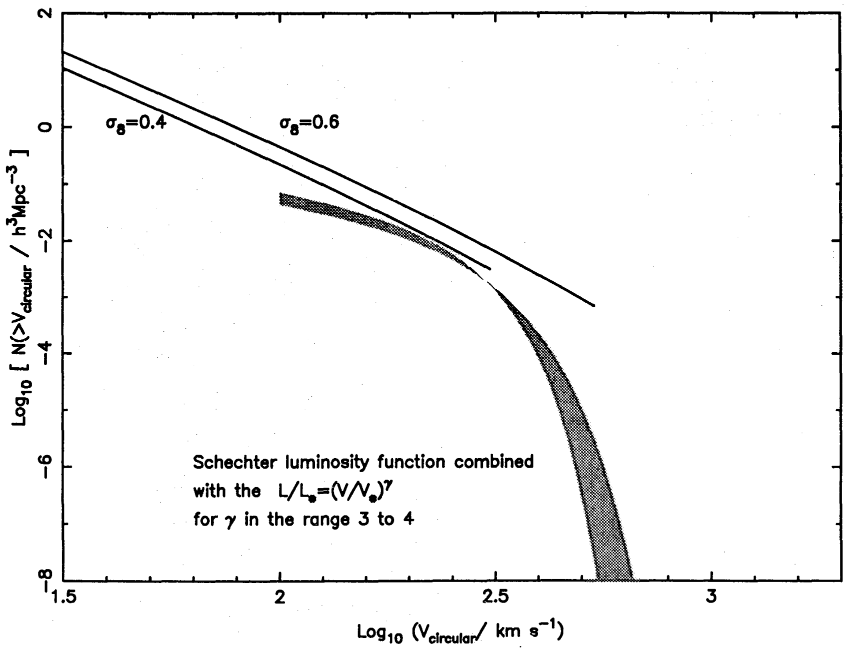

Figure 7. A comparison of the peak number density of haloes with circular velocity greater than $V_{\rm circ}$ with the present day galaxy distribution.

that velocity dispersion. Furthermore the mass and radius for such a condensation at redshift $z=1$ are $M=2.2\times10^{12}\ h^{-1}\ M_\odot$ and $r=100\ h^{-1}$ kpc, respectively, which seem reasonable for the formation of a typical L_* galaxy, if the baryons can dissipate their binding energy and collapse by about a factor ~ 10 in radius.

To make a more detailed comparison, we have plotted in Fig. 7 the distribution of present day galaxies using the Schechter luminosity function, as in Fig. 2, and curves showing $N_{\rm max}(\geq V_{\rm circ})$, the maximum number density of haloes with velocity dispersion greater than some $V_{\rm circ}$. These curves terminate at the velocity dispersion of the haloes which are just reaching their peak number density at the present epoch. Above this velocity the maximum number density of haloes is simply equal to the present number density, as plotted in Fig. 2. Again, we see a discrepancy at the low velocity end of the spectrum, somewhat worse even than the present epoch Press–Schechter distribution. Even if merging occurs, the faint end slope cannot be reduced below that of the present day halo distribution. While this theory also fails to reproduce the detailed form of the galaxy distribution, the overall number density of galaxies with velocity dispersions of around 300 km s^{-1} is reasonable.

Having made this tentative identification of the galaxy distribution and the distribution of the CDM haloes we are in a position to address the question of biasing. If we consider adding a background perturbation $\Delta_{\rm B}=\Delta_0/(1+z)$ to some region, then the effect is to cause each overdense region to collapse earlier, so the curves of Fig. 6 become displaced to the left; $(1+z)'=(1+z)(1+\Delta_{\rm B}/1.68)$, but also relabelled; $V'=V(1+\Delta_{\rm B}/1.68)^{1/2}$, since an earlier collapse time will lead to a higher velocity dispersion. This causes the peak number per unit mass to have a strong non-linear dependence on the magnitude of the background perturbation, which is well fitted by a simple exponential $N_{\rm max}(\geq V)\propto\exp(\gamma(V)\Delta_0)$. On large scales, where Δ_0 is small, this converts to a present day bias given by $b=1+\gamma$. Fig. 8 shows how this bias parameter b depends on both the velocity dispersion V and normalization σ_8, and indicates that in this model the rarer more luminous galaxies are more strongly clustered. A qualitative difference between this model and that considered in the previous section is that here one finds that galaxies of all luminosities are positively biased.

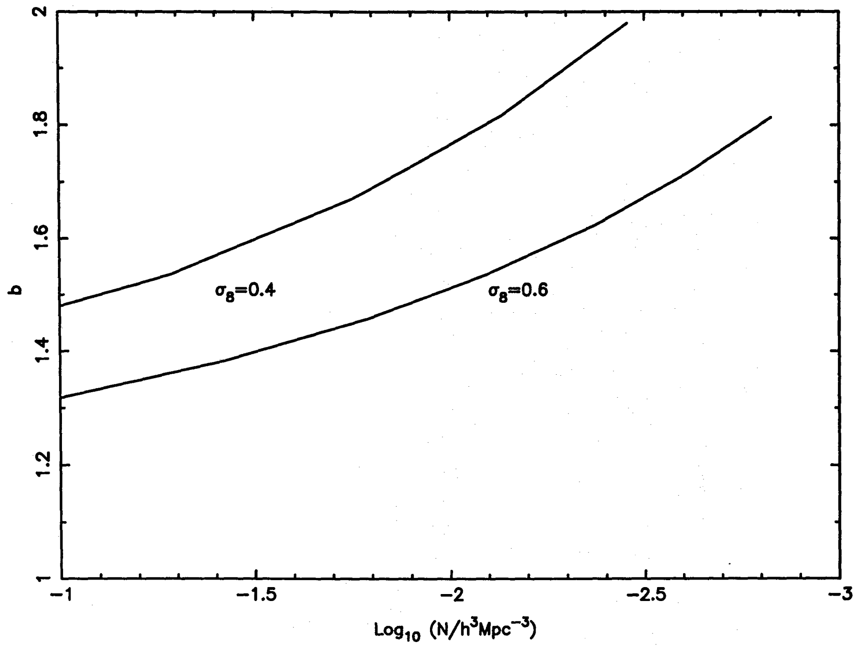

Figure 8. The bias parameter b as a function of halo cumulative number density.

In this model galaxies form before clusters collapse, so we can also estimate the amount by which the L/M for clusters will be enhanced over the global value. We have chosen this statistic, rather than say the enhancement of the number of galaxies per unit mass, as the net luminosity is presumably insensitive to merging. If we identify as clusters the recently virialized systems whose mean initial overdensity when smoothed on a cluster mass scale equals 1.68, then the luminosity function in the clusters will be modified from the global value $\Phi(L)$, equation (4) to $\Phi_{\rm clus}(L) \propto \exp[\gamma(L)]\Phi(L)$. Integrating over this new luminosity function using the computed values of γ, we find the enhancement of L/M in the clusters is about 2.2 for $\sigma_8 = 0.6$ and 4.2 for $\sigma_8 = 0.4$, with a slow variation with the lower luminosity limit.

Thus we see that in this model, the predicted bias for large-scale clustering is substantial, and is similar, though slightly less than that determined from analysis of dipoles of galaxy counts (Kaiser & Lahav 1988). The enhancement of L/M in clusters is also large. For the low normalization at least, the bias is strong enough to reconcile the estimates of cluster mass-to-light ratios with closure density. For the high normalization the bias is weaker, but this model is not necessarily excluded as this effect may be complemented by dynamical friction which could be efficient at further segregating light and mass in the non-linear environment provided by the cluster and its sub-units (Hoffman *et al.* 1982; Barnes 1983; Evrard 1986, 1987).

4.3 VARIATIONS ON A THEME

We have argued that the joint assumptions $V_{\rm stars} = V_{\rm halo}$ and $L \propto M$ have unpleasant consequences for the scatter and universality of the luminosity–velocity dispersion relations. The simple model we have explored in Section 4.2 keeps the former assumption and puts all the 'galaxies' exactly on a tight and universal L–V relation by fiat. The observed scatter, while small, is not zero, nor is the relation for real galaxies known to be absolutely universal — all we know is that spurious 'peculiar velocities' do not seem to be very much larger than the real velocities we expect from the gravitational influence of observed large-scale clustering. There is therefore possibility to relax the simple assumptions made hitherto, with important consequences for the clustering bias.

One possibility is to stick with $V_{\rm stars}=V_{\rm halo}$, but adopt a more general form for $L(M, z)$. For instance, one might entertain models such as,

$$L=M^{\alpha}(1+z_{\rm c})^{\beta}, \tag{12}$$

in which α and β are constants (Kaiser 1988). The model of Section 4.2 is the special case $\alpha=4/3$, $\beta=2$. With this model we can still arrange that the mean L–V relation has the form $L\propto V_{\rm halo}^4$. If we approximate the initial power spectrum as a power-law with spectral index $n=-2$, as appropriate on the mass scale of galaxies, then this constraint requires

$$\alpha=(\beta+6)/6. \tag{13}$$

The model has one free parameter, β. For the unbiased model ($\beta=0$) we argued that the scatter and systematic shifts in the L–V relation were unacceptable. The model of Section 4.2 has $\beta=2$, and amply satisfies any observational constraint of this kind. The scatter about the relation is proportional to $|\beta-2|$, so we could probably live with β as large as 3. These 'high-β' models are particularly interesting since the clustering bias parameter b is proportional to β. Moreover, the enhancement of L/M in a cluster is, as we have seen, a strongly non-linear function of b, so with $\beta\simeq 3$ one would reconcile the virial estimates with $\Omega=1$ even with a larger normalization.

The high-β models also have very intriguing consequences for large-scale streaming studies. Just as in the unbiased model the prediction was that in overdense regions the galaxies at a given L would have higher rotation velocities, if $\beta>2$ the converse applies. This has the effect of appearing to enhance the scale of peculiar motions (Kaiser 1988). If we are falling towards an attractor then we will underestimate the distances to galaxies in this overdense region. Consequently, the attractor itself will appear to be moving in the same direction as us, even if, in fact, it is at rest. If there is any problem with the amplitude or scale of large-scale motions — though the evidence at present (e.g. Kaiser & Lahav 1988) for any such problem seems slight — then the beneficial effects of the high-β model are twofold: not only does this model permit a higher normalization while still giving substantial enhancement of L/M in clusters, but it also predicts that the derived 'peculiar velocities' will be augmented by spurious systematic effects, and that the apparent coherence length for the streaming motions be enhanced.

So far we have said nothing about the physical mechanism that, in these models, adjusts the efficiency of star formation to maintain an acceptable L–V relation. The L/M dependence on mass is quite weak; what these models require is that an earlier collapse time means more stars, but why? An earlier collapse will increase the density, the virial temperature, and the pressure. One piece of physics that many would suspect plays an important role is the collisional cooling of the gas. Higher density certainly tends to increase the cooling rate, but the increase in temperature nearly balances this, so the ratio of cooling time to dynamical time is hardly altered by a long wave swell — contours of $t_{\rm cool}/t_{\rm dyn}$ are roughly parallel to lines of constant mass in the M, z plane — so this physics is not the biasing agent (though the cooling criterion may well be important in setting the high luminosity cut-off in the luminosity function). More promising is the ideal that one can form more stars per unit mass of gas in a deeper potential wells. Such a dependence has been suggested by Larson (1974), and the idea has more recently been revived by Dekel & Silk (1986) in the context of CDM. Such a dependence might plausibly arise if supernovae are efficient at expelling gas from shallow potential wells. For this regulation to be important for ordinary bright galaxies requires more energy and/or better coupling to the gas than assumed by Dekel & Silk, but much less than assumed by proponents of the explosive scenario for galaxy formation (Ostriker & Cowie 1981; Ikeuchi 1981). A related possibility is that the increased pressure might modify the initial stellar mass function by reducing the Jeans length. Ashman & Carr (1988) have suggested that such an effect might result in *high* mass-to-light ratios in high pressure systems. While a very large increase in pressure might suppress the formation of moderate mass stars, for realistic pressures it seems more likely that one would suppress predominantly the higher mass supernovae progenitors, so, if these act as regulators we would expect *lower* mass-to-light ratio at high pressure, and therefore a positive bias. Yet another possibility is that speeding up the collapse allows more stars to form before the supernovae go off.

Finally, one might consider relaxing the $V_{\rm stars} = V_{\rm halo}$ assumption. The fluctuations in light-to-mass ratio that seem required if we keep to this appear, in retrospect, somewhat contrived. In the model of Section 4.2 there has to be some kind of conspiracy to put the galaxies born in haloes with a broad $M-V$ relation back on a tight $L-V$ relation. It is almost as if galaxies had been carefully designed to give astronomers good distances! Perhaps what this is telling us is that the $L-V$ relations have even less connection to the parent halo properties than we had hitherto assumed. There is some observational constraint on how much the ratio of stellar to dark velocities can vary, but given that with an empirical $L \propto V^4$ relation, a 20 per cent variation in V would give a factor 2 change in L, there seems to be no real observational support for the strict equality we have assumed. It is easy to see how one can construct a model for galaxy formation with stronger bias than those considered above. One could, for instance, hypothesize that $V_{\rm stars}$ is a function of $V_{\rm halo}$, with $V_{\rm stars}/V_{\rm halo}$ an increasing function of $V_{\rm halo}$ and let $L = V_{\rm stars}^4$ as before. This 'theory' automatically satisfies the observed $L-V$ relations. The calculation of the biasing proceeds much as before; long waves modulate $V_{\rm halo}$, but this now converts to amplified fluctuations in $V_{\rm stars}$ and hence in light-to-mass ratio. These considerations lead us into a dangerous area of largely untestable speculation. The one qualitative prediction that is perhaps worth mentioning is that postulating $V_{\rm stars}/V_{\rm halo}$ to be an increasing function of $V_{\rm halo}$ would have a beneficial effect on the faint end slope of the luminosity function.

5 Conclusions

The Press–Schechter distribution, while a rather rough approximation compared to N-body experiments, provides a very useful medium for considering the biasing of astrophysical objects. We consider the lack of exactitude in our treatment of the gravitational clustering of dark matter to be the least of our problems. We are a long way from an *a priori* theory for the formation of galaxies and other luminous tracers; the best one can reasonably hope to do is to construct more or less physically reasonable rules for assigning luminosity to haloes. The predicted clustering bias then plays an important role in testing these hypotheses. If we are inclined to believe that the universe has critical density then the apparent bias of galaxy formation towards dense environments surely provides an important clue to help us unravel the physics. Crudely speaking, we are using the modulation of the initial conditions for structure formation by longer wavelength modes to provide us with a series of 'control experiments'.

For high-z quasars we found that abundance estimates and physical constraints on the masses of the parent haloes still allowed a wide mass range. Including the additional constraint of the strong clustering seen narrows down the allowed range to the most massive end; in CDM theory, quasars are predicted to reside in the analogue, at $z \simeq 3-4$, of the most rich clusters today. These host clusters should have velocity dispersions like poor clusters at the present epoch. Interestingly, the clustering estimates rule out the possibility that we are seeing a small ultra-luminous subset of some large parent halo population. This tells us that the quasar properties are quite tightly linked to the much larger scale halo properties.

Making a simple one-to-one correspondence between CDM haloes and the putative sources for the diffuse X-ray background we found that the limits on the fine scale granularity of the XRB constrain the parent haloes to be no more massive than galactic haloes.

The situation regarding galaxy formation is much less clear. (Though perhaps this reflects only our relative lack of knowledge about quasars and the source of the X-ray background which allows us, at present, to entertain very simple models.) We have looked at two very idealized models for galaxy formation. At one extreme, we associated haloes with galaxies by assuming that the stellar velocity dispersion at radii ~ 10 kpc is identical to that of the most recently virialized dark matter at radii of several hundred kpc. We found the bias for the present epoch haloes with abundance like that of bright galaxies to be too small. In the second model we explored, galaxy formation was assumed to have essentially finished by the present. The velocity of stars and dark particles were identified, and luminosities assigned according to a strict equality between L and V^4. This model, with its continuous dependence of luminosity on halo properties seems to us to be at least a small advance towards a potentially realistic theory from previous models which invoked a very sharp threshold to separate 'galaxies' from 'failed galaxies'. The biasing predicted in this model seems much more promising. For suitable normalization, at least, we found that the predicted enhancement of light-to-mass ratios in clusters and other dense environments is sufficient to reconcile the hypothesis $\Omega = 1$ with virial analysis.

Acknowledgments

NK received support from NSERC and the Candian Institute for Advanced Research. SMC acknowledges a SERC studentship and the hospitality of CITA.

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