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# The amplitude of mass fluctuations in the Universe

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**ABSTRACT**

We investigate how the rms linear fluctuation in the mass distribution on scales of 8 $h^{-1}$ Mpc[1] (denoted by $\sigma_8$) is constrained by the masses and abundances of rich clusters of galaxies. The derived value of $\sigma_8$ is almost independent of the shape of the fluctuation spectrum, but depends strongly on the cosmological density parameter. We find $\sigma_8 \approx 0.52$–$0.62$ for a critical density universe, and $\sigma_8 \approx 1.25$–$1.58$ for a spatially flat universe with $\Omega_0 = 0.2$. Our results conflict with the high amplitude inferred for an $\Omega = 1$ cold dark matter universe from the *COBE* anisotropy measurements and advocated on other grounds by Couchman & Carlberg. However, our estimates of $\sigma_8$ are consistent with alternative models which provide a good fit to the observed shape of galaxy correlations on scales up to 50 $h^{-1}$ Mpc and match on to the *COBE* results on larger scales.

**Key words:** galaxies: clustering – dark matter – large-scale structure of Universe.

## 1 INTRODUCTION

The relative distribution of mass and light in the Universe remains an unresolved problem of central importance for the interpretation of cosmological data. It has long seemed clear that the observed random velocities within groups and clusters of galaxies imply a total mass per galaxy which is only a small fraction (10–30 per cent) of the critical value needed to close the Universe (e.g. Faber & Gallagher 1979; Trimble 1987). This observation led to the idea that, if the Universe does indeed have the critical density, then the dominant mass component should be less clustered than the observed galaxy distribution. In such a universe, galaxy formation must be biased in favour of the overdense regions which collapse to make groups and clusters (Davis et al. 1985; Bardeen 1986; Kaiser 1986). However, some recent simulations of galaxy formation and galaxy clustering have suggested that this reasoning may be flawed, and that the low dynamical estimates of the mean mass density of the Universe could be due to systematic biases in galaxy *velocities* relative to those of the unseen dark matter component, rather than to biases in galaxy *positions* (Carlberg, Couchman & Thomas 1990; Couchman & Carlberg 1992). This result is particularly interesting in the light of the recent *COBE* measurement of fluctuations in the microwave background radiation (Smoot et al. 1992), since the observed anisotropies are compatible with the standard cold dark matter (CDM) model if the rms fluctuation in mass on cluster scales is about equal to that in light. The strongly biased CDM model favoured by Davis et al. appears to be excluded by the *COBE* data at about the $2.5\sigma$ level (Efstathiou, Bond & White 1992; Wright et al. 1992).

In a previous paper we used numerical simulations of a CDM universe to argue that the observed abundance of rich clusters of galaxies is inconsistent with such an 'unbiased' normalization of the fluctuation amplitude (Frenk et al. 1990). In agreement with this result, Evrard (1989) derived an rms linear fluctuation amplitude within 8 $h^{-1}$ Mpc spheres, $\sigma_8 \sim 0.5$–$0.7$, by considering the abundance of high-velocity dispersion clusters in the standard CDM model, while Henry & Arnaud (1991) used the abundance of clusters as a function of X-ray temperature to infer $\sigma_8 = 0.59 \pm 0.02$ for scale-free models of $\Omega = 1$ universes. Lilje (1992) applied the same argument with similar results for the standard CDM model. Bond & Myers (1992), on the other hand, have argued that cluster properties imply a significantly higher fluctuation amplitude. The differences between these estimates come from differences in how observational data are interpreted to estimate the abundance of clusters above a given mass or X-ray temperature, and in how these abundances are related to the predictions of the Press–Schechter (1974) formalism. Similarly, recent papers by Oukbir & Blanchard (1992) and Bahcall & Cen (1992) argue that cluster abundances can be fitted by an unbiased

[1] We write the Hubble constant as $H_0 = 100\ h\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

normalization, $\sigma_8 \approx 1$, in a low-density universe, $\Omega_0 \sim 0.2$, either with or without a cosmological constant, whereas we find below that a significantly higher normalization is required.

In this paper, we investigate this problem for spatially flat universes both with and without a cosmological constant. We use new computer simulations to test the Press–Schechter scheme in the mass range of interest, and we show analytically why $\sigma_8$ is insensitive to the shape of the fluctuation spectrum, provided that it is estimated using clusters in the appropriate mass range. We also discuss how these results relate to observations of clustering on large scales and to the recent *COBE* anisotropy measurements, expanding on the discussion by Efstathiou, Bond & White (1992).

## 2 CALCULATION OF CLUSTER ABUNDANCES

The theory of Press & Schechter (1974) predicts the abundance of objects as a function of mass in universes where structure grows from random phase initial fluctuations. It gives surprisingly good fits to the mass functions derived directly from *N*-body simulations of hierarchical clustering (Efstathiou & Rees 1988; Efstathiou et al. 1988; Bond et al. 1991; White & Frenk 1991). Below, we present additional tests of this formalism in the regime in which we will need it.

We define a mass parameter, $m_A$, by writing the mass of a rich cluster contained in a sphere of the Abell radius, $r_A = 1.5\ h^{-1}$ Mpc, as $M_A = 6.9 \times 10^{14} m_A\ h^{-1}\ M_\odot$. In units of the mean matter density of the Universe, the average density within this sphere is $\bar{\delta}(r_A) = 178\, m_A/\Omega_0$, where $\Omega_0$ is the usual density parameter. A spherical perturbation in an Einstein–de Sitter universe, assumed to virialize at twice the age and half the radius of its maximum expansion state, has $\bar{\delta} \approx 178$ at virialization (Gunn & Gott 1972). This overdensity surface is often assumed to delimit the region within which the total virialized mass of an object should be calculated. In a low-density, spatially flat universe $[\Omega_0 < 1$ and $\Lambda = 3H_0^2(1-\Omega_0)]$ we find that a generalization of the argument of Gunn & Gott leads to $\bar{\delta} \approx 178\, \Omega_0^{-0.6}$ for this bounding surface. (This is an approximation to results obtained using the equations of Peebles 1984 and Lahav et al. 1991.) In the neighbourhood of the bounding surface, numerical simulations (e.g. Frenk et al. 1988; Kauffman, in preparation) show that mean overdensity depends on radius approximately as $\bar{\delta} \propto r^{-2}$. Thus we can estimate the virial radius, defined by $\bar{\delta}(r_v) = 178\, \Omega_0^{-0.6}$, as $r_v = m_A^{1/2} \Omega_0^{-0.2} r_A$. The cluster mass contained within $r_v$ is thus

$$M_v = 6.9 \times 10^{14}\, m_A^{3/2}\, \Omega_0^{-0.2}\, h^{-1}\, M_\odot.$$

The characteristic length-scale of the mass fluctuations which give rise to clusters is related to the radius of the sphere which contains the mass, $M_v$, in the unperturbed background universe:

$$r_L = (178\, \Omega_0^{-0.6})^{1/3}\, r_v = 1.05\, \Omega_0^{-0.4}\, m_A^{1/2}\, r_8, \tag{1}$$

where $r_8 = 8\ h^{-1}$ Mpc. The measured rms fluctuation in *galaxy* numbers within spheres of radius $r_8$ is close to unity (Davis & Peebles 1983). Because of this, the rms *linear* fluctuation in *mass*, $\sigma_8$, within a sphere of radius $r_8$, is often taken as a measure of the amplitude of density fluctuations. The inverse of $\sigma_8$ is usually called the biasing parameter, *b*. Thus $\sigma_8 = 1/b = 1$ corresponds to an 'unbiased' normalization, whereas $\sigma_8 \approx 0.4$ corresponds to the highly biased normalization favoured by Davis et al. (1985) for their standard flat CDM model. The close coincidence between $r_L$ and $r_8$ (equation 1) for objects with the masses of rich clusters of galaxies is the reason why our conclusions about biasing are almost independent of the *shape* of the initial fluctuation spectrum.

According to the Press–Schechter ansatz, the fraction of the mass in virialized objects of mass greater than $M_v$ is twice the fraction of the mass in the linear density field with overdensity greater than 1.68 when smoothed with a top-hat filter of radius $r_L$. This ansatz derives from the fact that a spherical perturbation in an Einstein–de Sitter universe virializes (according to the above definition) when its linear overdensity, extrapolated to the epoch under consideration, reaches the critical value, $\delta_c = 1.68$. The corresponding number for low-density flat universes differs negligibly from 1.68 for the values of $\Omega_0$ of interest to us, and so we adopt $\delta_c = 1.68$ irrespective of $\Omega_0$. At high redshift the mean overdensity within a randomly placed sphere is normally distributed with an rms value determined by the power spectrum of density fluctuations. We denote the present, linearly extrapolated, values of this rms by $\Delta(r_L)$; by definition $\Delta(r_8) = \sigma_8$. For $r_L$ near $r_8$ we can approximate:

$$\Delta(r_L) \approx \sigma_8 (r_8/r_L)^\gamma = \sigma_8 (1.05\, \Omega_0^{-0.4}\, m_A^{1/2})^{-\gamma}, \tag{2}$$

where the index $\gamma$ measures the local slope of the fluctuation spectrum, and is given by $\gamma = 0.68 + 0.4\Gamma$ for the family of fluctuation spectra discussed by Efstathiou, Bond & White (1992). $\Gamma = \Omega_0 h$ for CDM universes but can take other values in modifications of this model, such as models with a mixture of hot and cold dark matter (Davis, Summers & Schlegel 1992) or low-density flat CDM models. The index, $\gamma$, in equation (2) was estimated by fitting values of $\Delta(r_L)$ obtained by integration over the appropriate linear power spectra (equation 7 of Efstathiou, Bond & White). Equation (2) holds for $r_L$ within a factor of two or so of $r_8$, and for $0.15 \lesssim \Gamma \lesssim 0.5$.

The Press–Schechter formula for the abundance of objects with mass $M_A$ is

$$n(M_A)\, dM_A = \frac{-3\delta_c}{(2\pi r_L^2)^{3/2} \Delta^2} \frac{d\Delta}{dM_A} \exp(-\delta_c^2/2\Delta^2)\, dM_A. \tag{3}$$

If we approximate $\Delta$ using equation (2), this expression simplifies and can be used to calculate the mean number of rich clusters in a sphere of radius $r_8$, with mass exceeding $M_A$,

$$N_{M_A} \equiv \frac{4\pi}{3} r_8^3\, n(>M_A) \approx \frac{2}{\sqrt{\pi}} \left(\frac{\delta_c}{\sqrt{2}\sigma_8}\right)^{3/\gamma} \int_{y_{\min}}^{\infty} y^{-3/\gamma} \exp(-y^2)\, dy, \tag{4}$$

$$y_{\min} = \frac{\delta_c}{\sqrt{2}\sigma_8} (1.1\, m_A\, \Omega_0^{-0.8})^{\gamma/2}.$$

The quantity $N_{M_A}$ is directly measurable and is independent of *h*. For massive clusters (i.e. $y_{\min} > 1$), the value of $\sigma_8$ determined from equation (4) depends mainly on the combination $m_A\, \Omega_0^{-0.8}$, with only a weak dependence on the shape of the power spectrum through the index, $\gamma$. Since $N_{M_A} \ll 1$, equation (4) determines the combination $(m_A\, \Omega_0^{-0.8})^{\gamma/2}/\sigma_8$

accurately even if the cluster abundance is uncertain. Furthermore, since $\gamma/2 \approx 0.4$, relatively large uncertainties in $m_A$ can be tolerated without destroying the precision of the value of $\sigma_8$ derived for any chosen value of $\Omega_0$. This insensitivity to observationally uncertain parameters is the greatest virtue of this technique for estimating the amplitude of mass fluctuations.

It is easy to improve on equation (4) by substituting numerically determined values of $\Delta(r_L)$ and its derivative into equation (3), and integrating the result numerically. In Fig. 1, we show how well the cluster abundances obtained in this way agree with those found in *N*-body simulations. For this test, we carried out two sets of three simulations, each simulation containing a million particles. These follow the evolution of clustering within a cubic region 300 $h^{-1}$ Mpc on a side. Our numerical techniques are identical to those discussed by Frenk et al. (1990). In the new models the linear force resolution is about 0.08 $h^{-1}$ Mpc, sufficient to resolve the formation of the cores of rich clusters. One set of models followed an Einstein–de Sitter universe with $\Gamma = 0.5$ (corresponding to 'standard' CDM for $h = 0.5$), while the other followed a low-density flat universe with $\Gamma = 0.2$. Both sets were stopped when the linear density fluctuation amplitude reached $\sigma_8 = 1$, at which time $\Omega_0$ was 0.2 in the low-density case. Clusters were found in the simulations by locating high-density regions with a 'friends-of-friends' group finder (e.g. Davis et al. 1985), and then getting masses from the particle count within spheres of comoving radius $r_A$ and $r_A/3$ centred

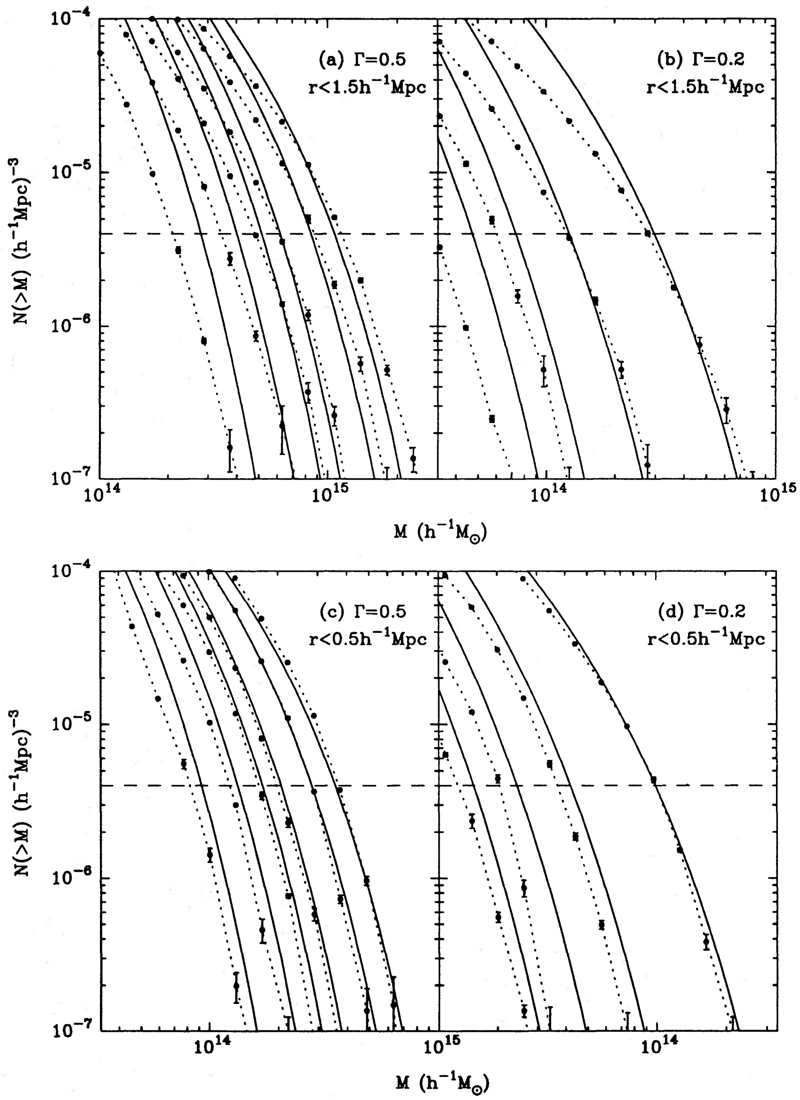


**Figure 1.** Abundances of rich clusters for various amplitudes of the mass fluctuation spectrum plotted as a function of cluster mass. Mass is measured within a radius of 1.5 $h^{-1}$ Mpc in panels (a) and (b), and within 0.5 $h^{-1}$ Mpc in panels (c) and (d). The filled symbols show the results obtained by averaging the abundances in three cosmological *N*-body simulations; error bars denote one standard deviation of the mean. The solid lines are theoretical abundances computed by integrating equation (3) for the appropriate power spectrum. Panels (a) and (c) give results for the standard CDM model, i.e. an Einstein–de Sitter universe with a $\Gamma = h = 0.5$ power spectrum (Efstathiou et al. 1992, equation 7). Panels (b) and (d) are for a spatially flat low-density universe ($\Omega_0 = 0.2$ at the final output time) with a $\Gamma = 0.2$ power spectrum. The curves plotted are for linear amplitudes of (from left to right) $\sigma_8 = 0.4, 0.5, 0.59, 0.67, 0.83$ and 1.0 in panels (a) and (c), and $\sigma_8 = 0.4, 0.5, 0.67$ and 1.0 in panels (b) and (d). The horizontal dashed line indicates an abundance of $4 \times 10^{-6}$ $h^3$ Mpc$^{-3}$, which is about half the mean space density of Abell $R \geq 1$ clusters.

on the group barycentres. The smaller radius corresponds better to the region within which masses can be estimated accurately for real clusters. We obtain theoretical predictions for the mass within $r_A/3$ by assuming that it is a third of the mass within $r_A$.

In Fig. 1, the abundances found in the simulations are compared directly with the theory described above. In the case of the $\Omega=1$ models (Figs 1a and c), the predicted cluster abundances form a one-parameter family, so that the sequence of output times can be taken to represent different fluctuation amplitudes for a region of the present Universe $200r_A$ on a side. In low-density universes with power spectra of the same shape and identical Hubble constants, the abundances of clusters depend on two parameters, $\Omega$ and $\sigma_8$. In Figs 1(b) and (d), we have plotted the abundances of clusters above a given mass (within comoving radius, $r_A$) at different epochs in a universe which at present has $\Omega_0=0.2$ and $\sigma_8=1$. The theoretical curves shown have been calculated accordingly from the Press–Schechter formalism.

The discrepancies between the theoretical and *N*-body results at the first output time shown in Fig. 1 probably arise from lack of evolution in the simulations; these have expanded from their initial conditions by factors of only 1.69 ($\Gamma=0.5$) and 1.29 ($\Gamma=0.2$) at this time. At later times and at low abundances (i.e. for massive clusters) the Press–Schechter predictions agree well with our *N*-body results. However, for more abundant, less massive clusters, the theory predicts larger masses at a given abundance than are found in the simulations. The effect is quite substantial for masses within $r_A$, but is weak when masses are measured within $r_A/3$. It occurs because $r_A$ is considerably larger than $r_v$ for low-mass clusters, and our assumption that $M(r_A)=(r_A/r_v)M_v$ leads to a significant overestimate of the mass in collapsed objects in such cases. In practice we will be comparing theory with observation at an abundance of $4\times10^{-6}\ h^3\ \mathrm{Mpc}^{-3}$, corresponding to $N_{M_A}=0.009$. Fig. 1 shows that theory and simulation agree extremely well at this abundance level. The abundances we infer from our simulations also agree very well with those which Bahcall & Cen (1992) derive from their own *N*-body work.

## 3 APPLICATION TO REAL CLUSTERS

The abundance of Abell clusters has been measured both as a function of Abell richness and as a function of X-ray temperature. Results from various studies of Abell samples are in good agreement and suggest that clusters with an Abell richness $R\geq1$ have an abundance of about $8\times10^{-6}\ h^3\ \mathrm{Mpc}^{-3}$ (Bahcall & Soneira 1983; Scaramella et al. 1991; Efstathiou et al. 1992). Catalogues of cluster velocity dispersions have been compiled by Frenk et al. (1990), Zabludoff, Huchra & Geller (1990) and Girardi et al. (1993). For clusters with $R\geq1$, these studies find *median* velocity dispersions of between 806 and 850 km s$^{-1}$. As Frenk et al. note, these values are almost certainly overestimates of the true median because the sample of clusters with velocity dispersions is biased towards richer objects, and because velocity dispersions are biased upwards by contamination effects. Nevertheless, we adopt 850 km s$^{-1}$ as a conservative estimate of the true median dispersion of $R\geq1$ clusters. If a spherically symmetric population of galaxies with density profile $\rho\propto r^{-\varepsilon}$ is in equilibrium within a spherical 'isothermal' potential well, the mass within radius $r$ can be estimated as

$$M(r)=\frac{(\varepsilon-2\beta)}{[\varepsilon-\beta(\varepsilon-1)]}\frac{\varepsilon\sigma^2 r}{G}, \tag{5}$$

where $\sigma$ is the line-of-sight velocity dispersion, and the tangential and radial velocity dispersions, $\sigma_t$ and $\sigma_r$, are related by $\sigma_t^2=(1-\beta)\sigma_r^2$, and are assumed to be independent of radius. $\beta=0$ for an isotropic velocity distribution, and $\beta=1$ for purely radial orbits. Studies of the cluster–galaxy cross-correlation function show that $\varepsilon\approx2.2$ over the radius range of interest (Seldner & Peebles 1977; Lilje & Efstathiou 1988). For this value, the dependence of the mass estimate on $\beta$ is weak for plausible values of $\beta$. Simulations of cluster formation suggest that the velocity dispersions of clusters should be radially biased (Evrard 1990), but we conservatively adopt $\beta=0$, which will cause us to overestimate the cluster masses. Our median velocity dispersion then corresponds to masses of $5.5\times10^{14}\ h^{-1}\ \mathrm{M}_\odot$ within the Abell radius, and $1.8\times10^{14}\ h^{-1}\ \mathrm{M}_\odot$ within $r_A/3$. We will therefore assume that the abundance of clusters with mass exceeding these values is $4\times10^{-6}\ h^3\ \mathrm{Mpc}^{-3}$.

The abundance of clusters as a function of X-ray temperature has been estimated by Edge et al. (1990) and by Henry & Arnaud (1991). From their cumulative temperature functions we find that clusters with temperatures exceeding $T$ have abundance $4\times10^{-6}\ h^3\ \mathrm{Mpc}^{-3}$ for $kT=3.5$ and 3.7 keV, respectively, At a radius of 0.5 $h^{-1}$ Mpc, the density of the X-ray emitting gas is observed to be falling off approximately as $\rho\propto r^{-2}$ in most clusters (Jones & Forman 1984). Assuming that the gas is isothermal, the mass within this radius is given by

$$M(r)=2\,\frac{kTr}{\mu m_p G}, \tag{6}$$

where $\mu$ is the mean molecular weight of the gas. For a temperature $kT=3.6$ keV, this gives a total mass within $r_A/3$ of $1.4\times10^{14}\ h^{-1}\ \mathrm{M}_\odot$. The estimated mass within $r_A$ is plausibly three times greater than this, i.e. $4.2\times10^{14}\ h^{-1}\ \mathrm{M}_\odot$, but there is no available temperature information at such large radii which would allow a mass to be estimated from X-ray data directly.

Our standard abundance of $4\times10^{-6}\ h^3\ \mathrm{Mpc}^{-3}$ is shown by the horizontal dashed lines in Fig. 1. In an Einstein–de Sitter universe with $\Gamma=0.5$, the cluster masses estimated above correspond to this abundance for $\sigma_8\approx0.52$–$0.62$, the lower value corresponding to the X-ray mass and the upper value to the mass determined from velocity dispersions. The dependence of this result on the shape of the power spectrum is extremely weak. For example, reducing $\Gamma$ to 0.2 changes the derived range of $\sigma_8$ to 0.54–0.63. Hence we conclude that *unbiased* Gaussian density fluctuations in a universe with $\Omega=1$ are inconsistent with observation.

In a low-density, flat universe the cluster masses which correspond to our standard abundance are somewhat smaller than the observed masses, even for $\sigma_8=1$. For a model with $\Gamma=\Omega_0=0.2$, equation (4) gives $\sigma_8\approx1.25$–$1.58$. The mass distribution in such a universe must therefore be somewhat *more* clustered than the galaxies in order to provide sufficiently massive rich clusters.

## 4 DISCUSSION

Our analysis has shown that the masses of rich clusters of galaxies provide a sensitive measurement of the amplitude of linear fluctuations in *mass* on the standard scale of 8 $h^{-1}$ Mpc where *galaxy* fluctuations have near unit amplitude. This measurement is almost independent of the shape of the fluctuation spectrum and depends primarily on the Gaussian nature of the initial density field and on the mean cosmological density. Our results confirm our earlier numerical work (Frenk et al. 1990), as well as the analytical work of Evrard (1989), Henry & Arnaud (1991) and Lilje (1992), all of whom used methods similar to that of this paper. For a spatially flat universe, we conclude from the results of Fig. 1 and equation (4) that $\sigma_8 \approx 0.57\ \Omega_0^{-0.56}$.

The errors on this estimate are difficult to quantify, since they are almost entirely due to systematic uncertainties in the mass estimates for rich clusters of galaxies. The masses determined from optical observations are likely to be high for at least four reasons: (i) velocity dispersions tend to be overestimated because of contamination by non-cluster galaxies; (ii) radial anisotropies, if present, imply smaller masses for a given observed velocity dispersion than the isotropic model we have assumed; (iii) velocity measurements are generally concentrated much nearer the centre than the radii at which we estimate masses; in the few clusters where sufficient data are available, the observed dispersion falls with radius; and (iv) the sample of clusters with measured velocity dispersions is biased towards richer and presumably more massive objects. It is thus encouraging that the characteristic mass determined from X-ray observations is similar to, though slightly smaller than, that found from velocity dispersions. The X-ray masses may themselves be overestimated if the gas temperature in clusters falls significantly with radius; they could also be underestimated if bulk motions contribute significantly to support of the gas (Evrard 1990), or if the gas has strong, small-scale inhomogeneities (Walsh & Miralda-Escudé 1993). On the whole, these arguments suggest that our estimates of $\sigma_8$ will be biased high. This is supported by the fact that both Bahcall & Cen (1992) and Oukbir & Blanchard (1992) advocate smaller values of $\sigma_8$ than we do for low-density universes. The differences can be traced to Bahcall & Cen's use of cluster masses about 40 per cent smaller than ours, and to Oukbir & Blanchard's models substantially underpredicting the temperature of clusters at our fiducial abundance.

Thus it seems that the value $\sigma_8 = 1.25$ advocated by Couchman & Carlberg (1992) for a critical density CDM model is strongly excluded. If this value were correct, the median gas temperature for $R \geq 1$ Abell clusters would be $\sim 13$ keV and the median velocity dispersion would be $\sim 1400$ km s$^{-1}$. These values are well outside the ranges allowed by observation. It is unlikely that X-ray temperatures and galaxy velocity dispersions could be significantly lower than their equilibrium values in the cores of rich clusters, since the dynamical times there are less than a tenth of the Hubble time. While various 'velocity biases' might reduce the global rms relative velocities of galaxy pairs below those predicted by the cosmic virial theorem (Davis & Peebles 1983), we see no effect that could produce the large reductions in temperature and velocity dispersions which are needed in cluster cores if the data are to be compatible with a critical-density universe with $\sigma_8 \gtrsim 1$.

Our determination of $\sigma_8$ may be compared with the amplitude of mass fluctuations on larger scales inferred from the *COBE* anisotropy measurements. If we regard these as fixing the overall amplitude of the fluctuations, then, for any assumed shape of the initial fluctuation spectrum, we can calculate the expected amplitude at 8 $h^{-1}$ Mpc, $\sigma_8$. Comparison of this extrapolation with our measurement of $\sigma_8$ is a test of the assumed spectral shape. For example, if the *COBE* anisotropies arise from primordial scale-invariant scalar perturbations in a 'standard' CDM universe with $\Omega = 1$ and $h = 0.5$, then we would expect to measure $\sigma_8 = 1.1 \pm 0.2$ (Wright et al. 1992; Efstathiou et al. 1992). This number is about twice the value we infer for an $\Omega_0 = 1$ universe from the abundances of rich clusters. The standard CDM model is thus rejected at about the $2.5\sigma$ confidence level unless the *COBE* fluctuations have been overestimated, or they have some other origin such as large-scale gravitational waves.

A similar comparison can be made with the mass fluctuation amplitude inferred from the peculiar velocities of galaxies. In this case the difference in characteristic scales is smaller, since peculiar velocities probe scales of order 15–50 $h^{-1}$ Mpc. For the class of models we are considering here, the estimate of $\sigma_8$ from the abundance of clusters is lower than the values derived by comparing peculiar velocity fields with the density field of *IRAS* galaxies (Kaiser et al. 1991). The discrepancy is marginal, however, given the published uncertainties in the *IRAS* measurement. Finally, as discussed by Efstathiou et al. (1992), the shape of the observed galaxy correlation function on large scales is most simply interpreted as constraining the *shape* of the fluctuation spectrum, independently of its amplitude. The implied spectral shape, our determination of $\sigma_8$, and the large-scale fluctuation amplitude measured by *COBE* are mutually consistent for spatially flat universes with a wide range of values of $\Omega_0$.

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