

Solutions (Practice Test - One)

15. for $t = 0$

$$x = A \sin \phi \dots\dots\dots (1)$$

$$v = A\omega \cos \phi \dots\dots\dots (2)$$

$$a = -A\omega^2 \sin \phi \dots\dots\dots (3)$$

By (1) and (3)

$$\omega = \sqrt{-\frac{a}{x}} = \sqrt{-\frac{32 \text{ m/s}^2}{-0.08 \text{ m}}} = 20 \text{ rad/s}$$

By (1) and (2)

$$\frac{x}{v} = \frac{\tan \phi}{\omega} \Rightarrow \tan \phi = 1$$

$$\text{or } \phi = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}$$

$$\text{At } t = 0, x \text{ is negative hence } \phi = \frac{5\pi}{4}$$

$$\text{from (1), } A = \frac{x}{\sin \phi} = 0.08 \times \sqrt{2} \text{ m} = 11.3 \text{ cm}$$

16. Consider a ring of radius x and thickness dx .

$$\text{Equivalent current in this ring} = \frac{\omega}{2\pi} \times \text{charge on ring} = \frac{\omega}{2\pi} \times (2\pi x dx) \frac{Q}{\pi R^2}$$

$$dB (\text{due to this ring}) = \frac{\mu_0}{2x} \left(\frac{\omega}{2\pi} \frac{2xQ}{R^2} dx \right) \quad \therefore B = \int_0^R \frac{\mu_0 \omega}{2\pi} \frac{Q}{R^2} dx = \frac{\mu_0 \omega Q}{2\pi R^2} \cdot R = \frac{\mu_0 \omega Q}{2\pi R}.$$

$$17. \lambda' = \frac{V - V_s}{f} = \frac{332 - 32}{1000} = 0.3 \text{ m}$$

$$f = f \frac{(V + V_0)}{V - V_s} = 1000 \times \frac{332 + 64}{332 - 32} = 1320 \text{ Hz}$$

$$\lambda'' = \frac{V - V_0}{f'} = 0.2 \text{ m.}$$

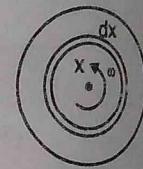
$$18. d \sin \theta = \lambda$$

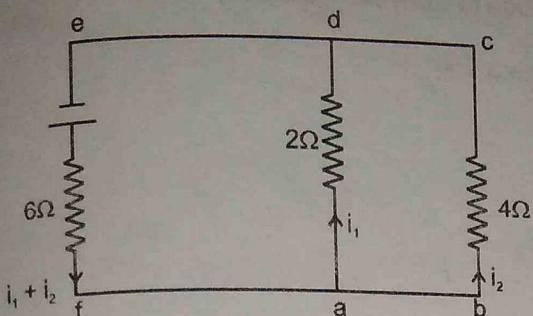
For first bright fringe. (There will be one bright fringe on both sides of central bright fringe and both are called first bright fringe.)

$$\sin \theta = \frac{\lambda}{d} \leq 1 \Rightarrow d \geq \lambda.$$

If $d = 2\lambda$, two maxima will be formed.19. (A) When switch S_1 is closed and S_2 is open $5 \times 10^{-7} \text{ A}$ current will flow, d to a.

$$(B) I = \frac{\epsilon}{R} = \frac{d\phi/dt}{R} = \frac{A(dB/dt)}{R} = \frac{10 \times 10^{-3} \times 2 \times 1 \times 10^{-4}}{6} = 3.33 \times 10^{-7} \text{ A, d to a}$$

(C) When both S_1 and S_2 are either open or closed; current through ad is zero.(D) When both S_1 and S_2 are closed then equivalent circuit will like shown below.



By applying KVL in both loops we get $i_1 = 9.09 \times 10^{-8} \text{ A}$.

20. (A) $T = 2\pi\sqrt{\frac{m}{k}}$ $m \uparrow$ $T \uparrow$

$$E = \frac{1}{2}kA^2$$

(B) $E = \frac{1}{2}kA^2$ $A \uparrow$ $T \uparrow$

(C) $T = 2\pi\sqrt{\frac{m}{k}}$ $k \uparrow$ $T \downarrow$

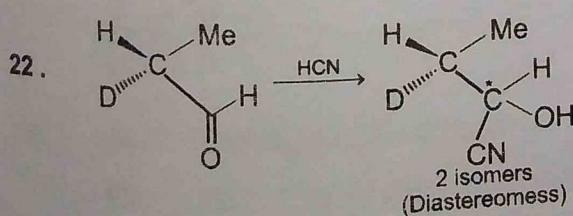
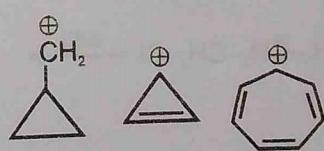
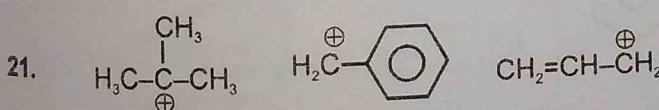
$$E = \frac{1}{2}kA^2$$
 $k \uparrow$ $E \uparrow$

(D) $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$ $k_{eq} \uparrow$ $T \downarrow$

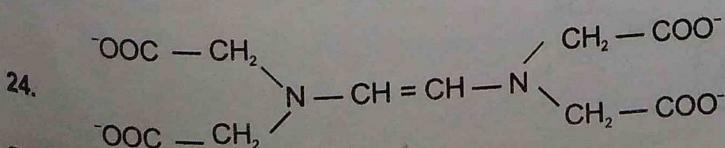
$$k_{eq} = 2k$$

$$E = \frac{1}{2}k_{eq}A^2$$
 $k_{eq} \uparrow$ $T \uparrow$

PART-II (CHEMISTRY)



23. $O=C=C=O$ (4σ and 4π bonds)
Sum = $4 + 4 = 8$



Solutions (Practice Test - One)

25. 2 vacant orbitals per molecule participate in hybridisation (1 from each B atom).

$$\therefore \text{total number of vacant orbitals} = \frac{5600}{28 \times 100} \times 2 \times N_A = 4 \times N_A.$$

So, X = 4.

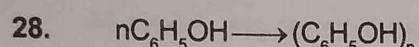
26. $E = 13.6 \left(\frac{2^2}{n_1^2} \right) + KE_1 \quad \text{and} \quad E = 13.6 \left(\frac{3^2}{n_2^2} \right) + KE_2$

Given : $KE_1 - KE_2 = \pm 2.55 = \pm 13.6 \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$

on solving 2 equations,

$$| n_2 - n_1 | = 8 \text{ or } 2$$

27. a, b, d, f



$$i = 1 + \left(\frac{1}{n} - 1 \right) \beta$$

$$\beta = 100\%, \quad \therefore \beta = 1$$

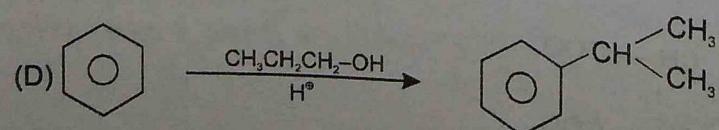
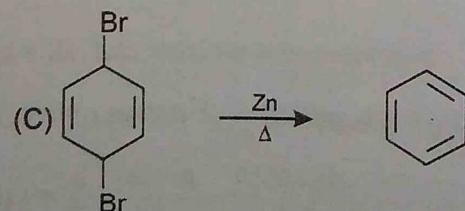
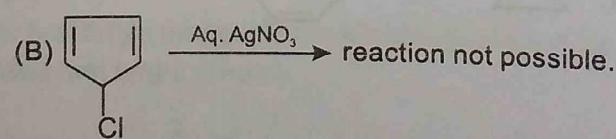
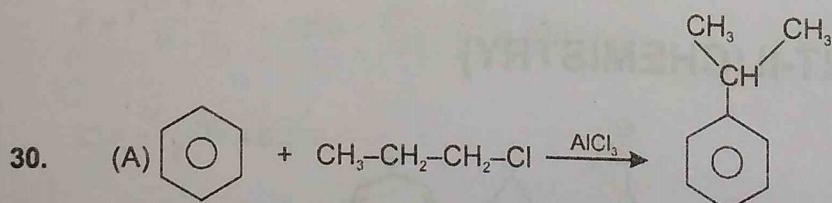
$$\Delta T_f = \frac{1000 \times K_f \times w_1}{m_1 \times w_2} \times i$$

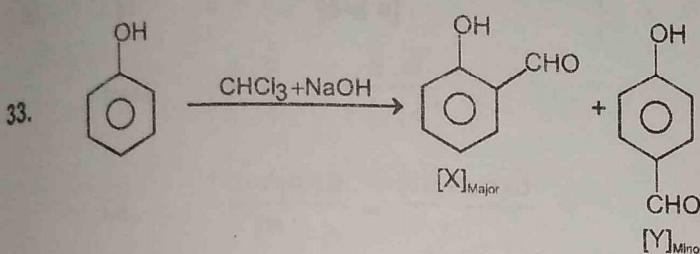
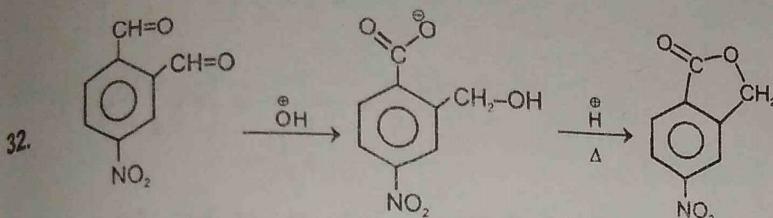
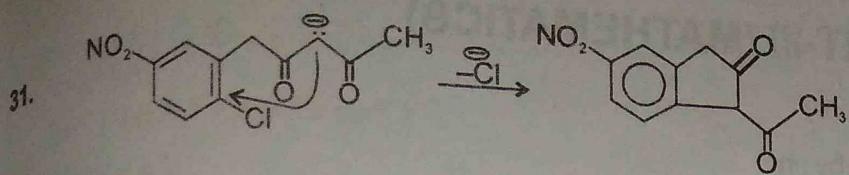
$$0.93 = \frac{1000 \times 1.86 \times 9.4 \times i}{94 \times 100}$$

$$i = \frac{1}{2}$$

$$n = 2$$

29. LiAlH₄ is strong reducing agent than the NaBH₄. NaBH₄ cannot reduce ester functional group.





34. $^{24}\text{Cr} = [\text{Ar}]^{18} 3\text{d}^5 4\text{s}^1$; $^{25}\text{Mn} = [\text{Ar}]^{18} 3\text{d}^5 4\text{s}^2$

$^{25}\text{Cu} = [\text{Ar}]^{18} 3\text{d}^{10} 4\text{s}^1$; $^{30}\text{Zn} = [\text{Ar}]^{18} 3\text{d}^{10} 4\text{s}^2$.

37. $2\text{A} \rightarrow \text{Product}$

$$\frac{1}{[\text{A}]_t} - \frac{1}{[\text{A}]_0} = kt$$

$$\frac{1}{[\text{A}]_t} = \frac{1}{[\text{A}]_0} + kt$$



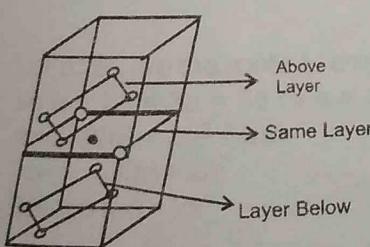
Slope (दाल) = k
Intercept (अंतःखण्ड) = $\frac{1}{[\text{A}]_0}$

38. Fcc can be viewed in two following ways -

(i) Planes along the faces (and parallel to it) of the unit cell.

⇒ Each atom touches 4 in same layer, 4 in layer above and 4 in layer below it.

(ii) Planes along closest packed spheres → each atom touches 6 atom in same layer, 3 in layer above and 3 in layer below it.



39. (A) $\text{CuFeS}_2 ; \text{Cu}_2\text{S} + 2\text{Cu}_2\text{O} \longrightarrow 6\text{ Cu} + \text{SO}_2 ; \text{Cu}_2\text{O} + \text{C} \longrightarrow 2\text{ Cu} + \text{CO}$

(B) $\text{PbS} ; \text{PbS} + 2\text{PbO} \longrightarrow 3\text{Pb} + \text{SO}_2 ; \text{PbO} + \text{C} \longrightarrow \text{Pb} + \text{CO} ; \text{PbO} + \text{CO} \rightarrow \text{Pb} + \text{CO}_2$

(C) Ag_2S ; Cyanide process, leaching with alkali metal cyanide followed by displacement with zinc dust.

(D) $\text{CuCO}_3 \cdot \text{Cu}(\text{OH})_2$; Calcination $\longrightarrow \text{CuO} + \text{C} \longrightarrow \text{Cu} + \text{CO}$

PART-III (MATHEMATICS)

41. Let $\vec{d} = x(\vec{b} \times \vec{c}) + y(\vec{c} \times \vec{a}) + z(\vec{a} \times \vec{b})$

Take dot product with $\vec{a}, \vec{b}, \vec{c}$ one by one

$$\vec{a} \cdot \vec{d} = x[\vec{a} \vec{b} \vec{c}] + 0 + 0 \therefore x = \frac{\vec{a} \cdot \vec{d}}{[\vec{a} \vec{b} \vec{c}]}$$

$$\text{Similarly } y = \frac{\vec{b} \cdot \vec{d}}{[\vec{a} \vec{b} \vec{c}]}, z = \frac{\vec{c} \cdot \vec{d}}{[\vec{a} \vec{b} \vec{c}]}$$

$$\therefore \vec{d} = \frac{(\vec{a} \cdot \vec{b})(\vec{b} \times \vec{c}) + (\vec{b} \cdot \vec{d})(\vec{c} \times \vec{a}) + (\vec{c} \cdot \vec{d})(\vec{a} \times \vec{b})}{[\vec{a} \vec{b} \vec{c}]} \quad \therefore \left| \frac{(\vec{a} \cdot \vec{d})(\vec{b} \times \vec{c}) + (\vec{b} \cdot \vec{d})(\vec{c} \times \vec{a}) + (\vec{c} \cdot \vec{d})(\vec{a} \times \vec{b})}{[\vec{a} \vec{b} \vec{c}]} \right| = 1$$

42. $p = \sin(px - y) \Rightarrow px - y = \sin^{-1} p \dots \dots \dots \text{(i)}$

differentiate w.r.t. $x \Rightarrow x \frac{dp}{dx} = \frac{1}{\sqrt{1-p^2}} \frac{dp}{dx}$

$$\Rightarrow \frac{dp}{dx} = 0 \text{ or } \frac{1}{\sqrt{1-p^2}} = x \Rightarrow p = \frac{\sqrt{x^2 - 1}}{x} \text{ put in equation (i)}$$

$$y = \sqrt{x^2 - 1} - \sin^{-1} \frac{\sqrt{x^2 - 1}}{x}$$

$$a = 1, b = 1, c = 1$$

43. Now $\left(\frac{\cos \alpha}{\sin \alpha} + \frac{\cos(\alpha + \beta)}{\sin(\alpha + \beta)} \right) \left(\frac{\cos \beta}{\sin \beta} - \frac{3 \cos(2\alpha + \beta)}{\sin(2\alpha + \beta)} \right)$

$$= \frac{\sin(2\alpha + \beta)}{\sin \alpha \sin(\alpha + \beta)} \left(\frac{\cos \beta}{\sin \beta} - \frac{3 \cos(2\alpha + \beta)}{3 \sin \beta} \right) = \frac{3 \sin \beta}{\sin \alpha \sin(\alpha + \beta)} \frac{\cos \beta - \cos(2\alpha + \beta)}{\sin \beta} = \frac{3(2 \sin(\alpha + \beta) \sin \alpha)}{\sin \alpha \sin(\alpha + \beta)} = 6$$

44. $\tan^{-1} \left(\frac{p-q}{1+pq} \right) = \tan^{-1} p - \tan^{-1} q \text{ (since } p, q > 0 \text{)} \dots \dots \text{(1)}$

$$\text{now } pr < -1 \text{ and } p > 0 \text{ so } r < 0$$

$$\text{Let } r = -t \quad \tan^{-1} \left(\frac{r-p}{1+rp} \right) = -\tan^{-1} \left(\frac{t+p}{1+tp} \right) \quad rp < -1$$

$$tp > 1$$

$$-[-\pi + \tan^{-1} t + \tan^{-1} p]$$

$$\pi - \tan^{-1} p + \tan^{-1} r \quad \dots \dots \text{(2)}$$

$$\tan^{-1} \left(\frac{q-r}{1+qr} \right) \quad (\text{since } qr > -1, qt < 1) \quad r < 0, \quad r = -t$$

$$= \tan^{-1} \left(\frac{q+t}{1-qt} \right) = \tan^{-1} q + \tan^{-1} t \quad qt < 1$$

$$= \tan^{-1} q - \tan^{-1} r \quad \dots \dots \text{(3)}$$

from (1), (2) and (3)

$$n = 2$$

45. $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + 1$

$$f'(x) = 1 + 3\tan^2 x \sec^2 x$$

$$f'\left(\frac{\pi}{4}\right) = 1 + 3 \times 2 = 7$$

$$g'\left(\frac{\pi}{4} + 1\right) = \frac{1}{f'\left(\frac{\pi}{4}\right)} = \frac{1}{7}$$

46. Let $\vec{a} = \lambda \vec{b} + \mu \vec{c}$

then $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{\vec{a} \cdot \vec{d}}{|\vec{a}| |\vec{d}|}$

i.e. $\frac{(\lambda \vec{b} + \mu \vec{c}) \cdot \vec{b}}{|\vec{b}|} = \frac{(\lambda \vec{b} + \mu \vec{c}) \cdot \vec{d}}{|\vec{d}|}$

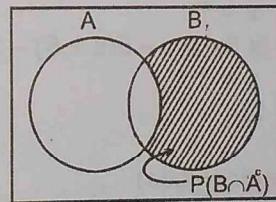
i.e. $\frac{[\lambda(2\hat{i} + \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})] \cdot (2\hat{i} + \hat{j})}{\sqrt{5}} = \frac{[\lambda(2\hat{i} + \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})] \cdot (\hat{j} + 2\hat{k})}{\sqrt{5}}$

i.e. $\lambda(4+1) + \mu(2-1) = \lambda(1) + \mu(-1+2)$ i.e. $4\lambda = 0$ i.e. $\lambda = 0$

$\therefore \hat{a} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

47. $P(A) = \frac{1}{4}, P(A \cup B) = \frac{1}{2}$

$$P\left(\frac{B}{A^C}\right) = \frac{P(B \cap A^C)}{P(A^C)}$$



$$= \frac{P(A \cup B) - P(A)}{1 - P(A)} = \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}.$$

48. Let $\alpha, 2\alpha$ are the roots of equation

$$\text{so } \alpha + 2\alpha = 3\alpha = 3a \Rightarrow \alpha = a$$

$$\text{and } \alpha(2\alpha) = 2\alpha^2 = f(a)$$

$$\Rightarrow f(a) = 2a^2$$

$$\text{Hence } f(x) = 2x^2$$

49. $x + y + z - 1 = 0$

$$4x + y - 2z + 2 = 0$$

∴ direction ratios of the line are $\langle -3, 6, -3 \rangle$

$$\text{i.e. } \langle 1, -2, 1 \rangle$$

$$\text{Let } z = k, \text{ then } x = k - 1, y = 2 - 2k$$

i.e. $(k-1, 2-2k, k)$ is any point on the line

∴ $(-1, 2, 0), (0, 0, 1)$ and $\left(-\frac{1}{2}, 1, \frac{1}{2}\right)$ are points on the line

∴ (A), (B) and (C) are correct options

Solutions (Practice Test - One)

50. $1 + |\cos x| + \cos^2 x \dots$

$$\begin{aligned} &= \frac{1}{1 - |\cos x|} \Rightarrow 8^{\frac{1}{1 - |\cos x|}} = 4^3 \\ &\Rightarrow 2^{\frac{3}{1 - |\cos x|}} = 2^6 \Rightarrow \frac{3}{1 - |\cos x|} = 6 \Rightarrow 1 - |\cos x| = \frac{1}{2} \\ &|\cos x| = \frac{1}{2} \Rightarrow \cos x = \pm \frac{1}{2} \end{aligned}$$

$\cos x = \frac{1}{2}$ will give least positive value of x

$$x = \frac{\pi}{3}$$

51. All AAAAAA BBBB D EEF can be arranged in $\frac{12!}{5! 3! 2!}$

Between the gaps C can be arranged in ${}^{13}C_3$ ways

$$\text{Total ways} = {}^{13}C_3 \times \frac{12!}{5! \times 3! \times 2!}$$

Number of ways = without considering separation of C – in which all C's are together – in which exactly

$$\text{two C's are together} = \frac{15!}{5!(3!)^2 2!} - \frac{13!}{5! 3! 2!} - \frac{12!}{5! 3!} {}^{13}C_2$$

52. Coordinates of O are (5, 3) and radius = 2

Equation of tangent at A(7, 3) is $7x + 3y - 5(x + 7) - 3(y + 3) + 30 = 0$

i.e. $2x - 14 = 0$ i.e. $x = 7$

Equation of tangent at B(5, 1) is $5x + y - 5(x + 5) - 3(y + 1) + 30 = 0$

i.e. $-2y + 2 = 0$ i.e. $y = 1$

∴ coordinate of C are (7, 1)

∴ area of OACB = 4

Equation of AB is $x - y = 4$ (radical axis)

Equation of the smallest circle is

$$(x - 7)(x - 5) + (y - 3)(y - 1) = 0$$

$$\text{i.e. } x^2 + y^2 - 12x - 4y + 38 = 0$$

53. L.R. = $\frac{2b^2}{a} = \frac{2\cos^2 \alpha}{\cot \alpha} = \frac{1}{2}$

$$\Rightarrow \sin 2\alpha = \frac{1}{2} \Rightarrow 2\alpha = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \alpha = \frac{\pi}{12}, \frac{5\pi}{12}$$

54. $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{s(s-a)}{\Delta} + \frac{s(s-b)}{\Delta} + \frac{s(s-c)}{\Delta}$

$$= \frac{s}{\Delta} [3s - (a + b + c)] = \frac{s[3s - 2s]}{\Delta} = \frac{s^2}{\Delta}$$

$$= \left(\frac{a+b+c}{2}\right)^2 \times \frac{4R}{abc} = \frac{(a+b+c)^2 R}{abc}$$

$$[\because \Delta = \frac{abc}{4R}] \quad \text{and} \quad \frac{s^2}{\Delta} = \frac{\Delta^2}{r^2 \Delta} = \frac{\Delta^2}{r^2}$$

55. Since $\frac{e}{2}$ and $\frac{e'}{2}$ are eccentricities of a hyperbola and its conjugate

$$\therefore \frac{4}{e^2} + \frac{4}{e'^2} = 1 \quad \text{i.e.} \quad 4 = \frac{e^2 e'^2}{e^2 + e'^2}$$

line passing through the points $(e, 0)$ and $(0, e')$
 $e'x + ey - ee' = 0$

it is tangent to the circle $x^2 + y^2 = r^2$

$$\therefore \frac{ee'}{\sqrt{e^2 + e'^2}} = r \quad \therefore \quad r^2 = \frac{e^2 e'^2}{e^2 + e'^2} = 4 \quad \therefore \quad r = 2$$

56. $(B^T AB)^T = B^T A^T (B^T)^T = B^T A^T B$
 $= B^T AB$ iff A is symmetric
 $\therefore B^T AB$ is symmetric iff A is symmetric

57. $\log_{\sin x} \frac{|x|}{x} \Rightarrow \sin x \in (0, 1) \text{ and } x \in (0, \infty)$

$$\therefore x \in \bigcup_{n \in W} \left(2n\pi, 2n\pi + \frac{\pi}{2} \right) \cup \left(2n\pi + \frac{\pi}{2}, (2n+1)\pi \right) \text{ and } y \in \{0\}$$

exactly

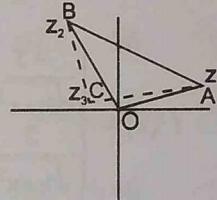
58. $|z_2 + iz_1| = |z_1| + |z_2| \Rightarrow \operatorname{Arg}(iz_1) = \operatorname{Arg}(z_2)$

$$\Rightarrow \operatorname{Arg}(z_2) - \operatorname{Arg}(z_1) = \frac{\pi}{2}$$

$$\text{Let } z_3 = \frac{z_2 - iz_1}{1-i}$$

$$\begin{aligned} (1-i)z_3 &= z_2 - iz_1 \\ \Rightarrow (z_3 - z_2) &= i(z_3 - z_1) \\ \Rightarrow (z_2 - z_3) &= i(z_1 - z_3) \end{aligned}$$

$$\therefore \angle ACB = \frac{\pi}{2} \text{ and } |z_2 - z_3| = |z_1 - z_3| \Rightarrow AC = BC$$



$$\therefore AB^2 = AC^2 + BC^2 \Rightarrow AC = \frac{5}{\sqrt{2}}. \quad (\because AB = 5)$$

$$\therefore \Delta ABC = \frac{1}{2} AC \cdot BC = \frac{AC^2}{2} = \frac{25}{4} \text{ square unit}$$

59. We have, $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{12}$

$$(A) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{1}{2}$$

$$(B) \quad P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)} = \frac{2}{3}$$

$$(C) \quad P\left(\frac{B}{A' \cap B'}\right) = \frac{P(B \cap (A' \cap B'))}{P(A' \cap B')} = \frac{P(\emptyset)}{P(A' \cap B')} = 0$$

$$(D) \quad P\left(\frac{A'}{B}\right) = P(A') = \frac{2}{3}$$

Solutions (Practice Test - One)

60. P. Equation of the chord whose mid point is $(0, 3)$ is

$$\frac{3y}{25} - 1 = \frac{9}{25} - 1 \quad \text{i.e.} \quad y = 3$$

if intersects the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

$$\text{at } \frac{x^2}{16} = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow x = \pm \frac{16}{5}$$

$$\therefore \text{length of the chord} = \frac{32}{5}$$

$$\text{thus } \frac{4k}{5} = \frac{32}{5} \quad \therefore k = 8$$

$$\text{Q. } f(x) = \cos^{-1}(4x^3 - 3x) = \cos^{-1}(\cos 3\theta) = \begin{cases} 3\theta & , 0 < \theta < \frac{\pi}{3} \\ 2\pi - 3\theta & , \frac{\pi}{3} < \theta < \frac{\pi}{2} \end{cases}$$

$$y = \begin{cases} 3\cos^{-1}x & , \frac{1}{2} < x < 1 \\ 2\pi - 3\cos^{-1}x & , 0 < x < \frac{1}{2} \end{cases}$$

$$\therefore f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}} & , \frac{1}{2} < x < 1 \\ \frac{3}{\sqrt{1-x^2}} & , 0 < x < \frac{1}{2} \end{cases}$$

$$a = \lim_{x \rightarrow \frac{1}{2}^+} f'(x) = -2\sqrt{3}$$

$$b = \lim_{x \rightarrow \frac{1}{2}^-} f'(x) = 2\sqrt{3} \quad \therefore a + b + 3 = 3$$

$$\text{R. } \log(-2x) = 2 \log(x+1)$$

$$-2x > 0 \Rightarrow x < 0 \quad \dots\dots(i)$$

$$x+1 > 0 \Rightarrow x > -1 \quad \dots\dots(ii)$$

from (i) & (ii), we get $x \in (-1, 0)$

$$\therefore -2x = (x+1)^2 \Rightarrow x^2 + 4x + 1 = 0 \Rightarrow x = \frac{-4 \pm 2\sqrt{3}}{2}$$

so only one solution lies in $(-1, 0)$

$$\text{S. Since } (2, 3) \text{ lies on } ax + by - 5 = 0$$

$$\therefore 2a + 3b - 5 = 0$$

$$\text{Slope of } ax + by - 5 = 0$$

$$\Rightarrow \left(-\frac{a}{b} \right) = -1 \quad \text{i.e.} \quad a = b$$

$$\therefore a = 1, b = 1$$

$$\therefore |a+b| = 2$$

PRACTICE TEST - ONE

ANSWER KEY

PAPER - 2

PART-I (PHYSICS)

- | | | | | | | | | | | | | | |
|-----|------|-----|------|-----|--------|-----|------|-----|--------|-----|------|-----|-------|
| 1. | (1) | 2. | (1) | 3. | (4) | 4. | (8) | 5. | (7) | 6. | (2) | 7. | 4 |
| 8. | (3) | 9. | (CD) | 10. | (ABCD) | 11. | (AC) | 12. | (BC) | 13. | (BC) | 14. | (BCD) |
| 15. | (BD) | 16. | (CD) | 17. | (A) | 18. | (B) | 19. | (ABCD) | 20. | (AD) | | |

PART-II (CHEMISTRY)

- | | | | | | | | | | | | | | |
|-----|------|-----|------|-----|------|-----|-------|-----|-------|-----|--------|-----|-------|
| 21. | (6) | 22. | (3) | 23. | (4) | 24. | (8) | 25. | (5) | 26. | (2) | 27. | (4) |
| 28. | (5) | 29. | (AD) | 30. | (BC) | 31. | (ACD) | 32. | (ABC) | 33. | (ABCD) | 34. | (ABC) |
| 35. | (BD) | 36. | (AC) | 37. | (B) | 38. | (D) | 39. | (B) | 40. | (A) | | |

PART-III (MATHEMATICS)

- | | | | | | | | | | | | | | |
|-----|------|-----|------|-----|-------|-----|------|-----|------|-----|------|-----|------|
| 41. | (4) | 42. | (3) | 43. | (4) | 44. | (8) | 45. | (1) | 46. | (3) | 47. | (8) |
| 48. | (8) | 49. | (AB) | 50. | (ABC) | 51. | (AC) | 52. | (AB) | 53. | (AD) | 54. | (AB) |
| 55. | (BD) | 56. | (AB) | 57. | (B) | 58. | (A) | 59. | (D) | 60. | (A) | | |

PRACTICE TEST - ONE

HINTS & SOLUTIONS

PAPER - 2

PART-I (PHYSICS)

1. For the first refraction image will be formed on object it self.
Now refraction at water air surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0 \Rightarrow v = 9 \text{ cm}$$

$$2. \omega = \sqrt{\frac{\mu g}{R}} = 1 \text{ rad/sec}$$

$$3. \frac{\Delta Q}{\Delta t} = K_A A \frac{dT}{dx}$$

$\frac{dT}{dx}$ in conductor A = slope of graph = $\frac{4}{3}$; $\frac{dT}{dx}$ in conductor B = slope of graph = 1

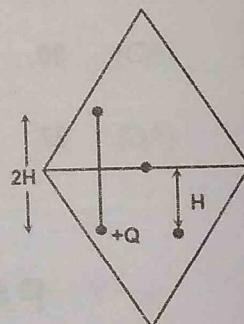
Since both conductors are connected in series, same heat current will flow in A and B.

$$120 \times \frac{4}{3} = 160$$

4. If we place another similar cone on this one, net flux = $\frac{Q}{\epsilon_0}$

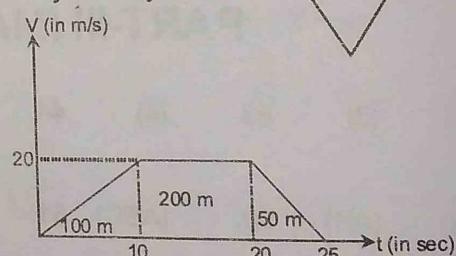
$$\Rightarrow \text{Flux with the upper cone} = \frac{Q}{\epsilon_0} - \frac{3Q}{5\epsilon_0} = \frac{2Q}{5\epsilon_0} = \frac{8Q}{20\epsilon_0}$$

This must also be the flux associated with the lower cone when the charge is raised through a height $2H$ because of symmetry.



5. Accelerated of Lift

$t = 0 \text{ to } t = 10$	2 m/s^2
$t = 10 \text{ to } t = 20$	zero
$t = 20 \text{ to } t = 25$	-4 m/s^2
$V - t = \text{curve}$	



6. $\tan \theta = \frac{a}{g} = \frac{h_2 - h_1}{h_2 \tan 45^\circ + h_1 \tan 45^\circ} = \frac{4 \text{ cm}}{20 \text{ cm}} \Rightarrow a = 2 \text{ m/s}^2$

7. Let Δl be the magnitude of contraction due to cooling by $T_1 - T_2 = 100^\circ\text{C}$

$$\therefore \frac{\Delta l}{l} = \alpha (T_1 - T_2) \quad \dots (1)$$

From condition of equilibrium

$$\frac{YA}{l} \Delta l = mg \quad \dots (2)$$

From equation (1) and (2) we get

$$YA \alpha (T_1 - T_2) = mg$$

$$m = \frac{YA \alpha (T_1 - T_2)}{g}. \text{ Solving } m = 4 \text{ kg Ans.}$$

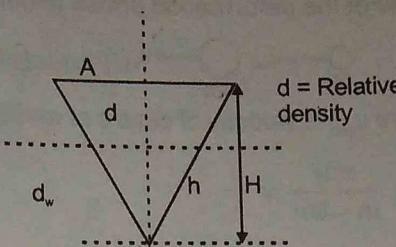


8. For floating condition buoyant force = weight

$$\frac{1}{3} A \cdot \left(\frac{h}{H}\right)^2 h \cdot d_w \cdot g = \frac{1}{3} (AH) d_w \cdot d \cdot g$$

$$h = H(d)^{1/3} = 4 \left(\frac{27}{64}\right)^{1/3} \quad h = 3 \text{ m}$$

Ans. 3



$$\vec{E}_C = \frac{\rho \vec{OC}}{3\epsilon_0} + \frac{-\rho \vec{BC}}{3\epsilon_0} + \frac{Kq}{AC^3} \vec{AC}$$

$$\vec{E}_C = \frac{\rho \vec{OB}}{3\epsilon_0} + \frac{Kq}{AC^3} \vec{AC}$$

Which is dependent on C hence field is non uniform inside cavity.

$$\vec{E}_A = 0 + \frac{\rho \vec{OA}}{3\epsilon_0} + \frac{-\rho(R/2)^3}{3\epsilon_0 R^2} \cdot \vec{BA}$$

$$\vec{E}_A = \frac{\rho \vec{OA}}{3\epsilon_0} - \frac{\rho R}{24\epsilon_0 R} \cdot \vec{BA} = \frac{\rho}{3\epsilon_0} \frac{R}{2} \vec{OA} - \frac{\rho}{24\epsilon_0} R \cdot \vec{OA}$$

$$\vec{E}_A = \left(\frac{\rho R}{6\epsilon_0} - \frac{\rho R}{24\epsilon_0 R} \right) \vec{OA} = \frac{\rho}{8\epsilon_0} R \cdot \vec{OA} = \frac{\rho 2 \vec{OA}}{8\epsilon_0} = \frac{\rho \vec{OA}}{4\epsilon_0}$$

Similarly for \vec{E}_B Hence C and D

10. Molecular wt. = $16 M_0$
mass = $2 m_0$

- Molecular wt. = M_0
mass = m_0

$$n_A = \frac{n_0}{8}$$

$$n_B = n_0$$

$$(A) \text{ K.E./atom} = \frac{f}{2} k.T. = \frac{f}{2} k.T. \text{ for both the gases.}$$

$$(B) C_{rms_A} = \sqrt{\frac{3RT}{16M_0}}, \quad C_{rms_B} = \sqrt{\frac{3RT}{M_0}}, \quad (C_{rms})_B = 4(C_{rms})_A$$

$$(C) (P)_A = \frac{(n_0/8)RT}{V}, \quad (P_B) = \frac{n_0 RT}{V}, \quad (P_B) = 8(P_A)$$

$$(D) n_B = 8 n_A$$

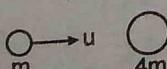
11. The H-type atom is in the third excited state i.e. $n = 4$.

$$\text{Energy corresponding to wave length } \frac{6200}{51} \text{ nm} = \frac{12400 \times 51}{62000} = 10.2 \text{ eV}$$

This is the $E_2 - E_1$ for H and $E_4 - E_2$ for He^+ .

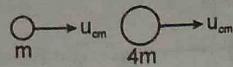
we get $z = 2$ for $4 \rightarrow 2$ radiation
Hence the atom is Helium ion.

- Ans. He^+ ,
(b) Let u be the speed of neutron before collision



Solutions (Practice Test - One)

At end of the deformation phase (when the kinetic energy of (neutron + He⁺) system is least)



Where u_{cm} is velocity of centre of mass. From conservation of momentum

$$u_{cm} = \frac{mu}{m+4m} = \frac{u}{5}$$

$$\text{The loss of kinetic energy} = \frac{1}{2} mu^2 - \frac{1}{2} m \left(\frac{u}{5}\right)^2 - \frac{1}{2} 4m \left(\frac{u}{5}\right)^2 = \frac{4}{5} \left(\frac{1}{2} mu^2\right)$$

If K is the kinetic energy of electron then the maximum loss in K.E. of system is

$$\frac{4}{5} K = 51 \text{ eV} \quad \text{or} \quad K = \frac{51 \times 5}{4} = 63.75 \text{ eV}$$

$$K_{\min} = \frac{255}{4} = 63.75 \text{ eV}$$

12. (A) initial acceleration of any point on circumference of ring is $g \cos \theta$
 (B) initial acceleration of centre of ring is $g \sin \theta \cos \theta$

$$(C) \tan \theta = \frac{b-a}{h}$$

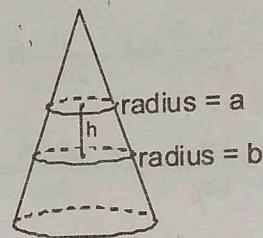
$$h = (b-a) \cot \theta$$

$$mg(b-a) \cot \theta = \frac{1}{2} k(2\pi b - 2\pi a)^2$$

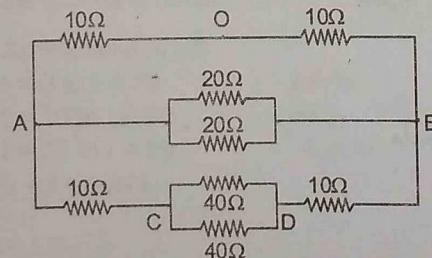
$$mg(b-a) \cot \theta = \frac{1}{2} 4\pi^2 k(b-a)^2$$

$$b-a = \frac{mg \cot \theta}{2\pi^2 k}$$

- (D) at the moment maximum vertical displacement acceleration of centre of ring is upward.



13. Effective circuit



14. Initially potential and kinetic both energies zero and from conservation of mechanical energy total energy of the two objects zero

Further, decrease in P.E. = increase in K.E.

$$\frac{G(m)(4m)}{r} = \frac{1}{2} \mu v_r^2$$

$$v_r = \sqrt{\frac{10Gm}{r}} \Rightarrow \text{Total K.E.} = \frac{G(m)(4m)}{r} = \frac{4Gm^2}{r}$$

15. Smaller the least count greater the precision.

$$\text{Mean absolute error } |\Delta x| = \frac{\sum |x - x_i|}{N}$$

16. The process is isothermal compression.

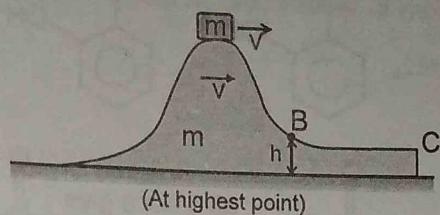
17. Let 'u' be the required minimum velocity. By momentum conservation :
 $mu = (m + m)v \Rightarrow v = u/2$.

Energy equation :

$$\frac{1}{2} mu^2 = \frac{1}{2} (2m)v^2 + mgh.$$

Substituting $v = u/2$:

$$u = 2\sqrt{gh}$$



18. By work-energy theorem on the system :

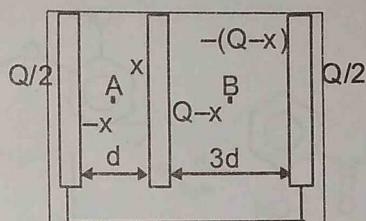
$$-(\mu mg)(BE) - mgh = -\frac{1}{4} mu^2 = -\frac{1}{4} m (4gh) \Rightarrow BE = \frac{H-h}{\mu}$$

19. Field inside the conductor is zero

20. Since P.D between left and right plate is zero.

$$\frac{x}{A \epsilon_0} d = \frac{Q-x}{3A \epsilon_0} \times 3D$$

$$x = Q/2$$



PART-II (CHEMISTRY)

- | | |
|--|---|
| 21. CO_3^{2-} | : 1 or 2 donor oxygen atoms. |
| $\text{NH}_2-\text{CH}_2-\text{COO}^-$ | : 1 nitrogen and 1 oxygen donor atoms. |
| EDTA | : 5 or 6 donor atoms (N and O). |
| $\text{C}_2\text{O}_4^{2-}$ | : 2 donor oxygen atoms. |
| SO_4^{2-} | : 1 or 2 donor oxygen atoms. |
| $\text{CH}_3\text{COCHCOCH}_3^-$ | : 2 donor oxygen atoms. |
| $\text{CH}_3\text{C}\equiv\text{N}$ | : 1 donor nitrogen atom. |
| SCN^- | : Either 1 nitrogen atom or 1 sulphur atom. |
| DMG | : 2 donor nitrogen atoms. |

22. Complex containing more number of 5 & 6 membered rings is more stable.

23. Given, $P_1 = P$, $V_1 = V$, $T_1 = T$

$$P_2 = P_1, V_2 = V - \frac{100/21}{100}V, T_2 = T ; \quad P \times V = P_2 \times \left(V - \frac{100/21}{100}V \right)$$

$$P_2 = \frac{21}{20}P ; \quad \Delta P = \frac{1}{20}P. \quad \% \text{ increase} = \frac{1}{20} \frac{P}{P} \times 100 = 5\% \text{ Ans.}$$

24. Since in acidic medium, A^{n+} is oxidized to AO_3^- , the change in oxidation state from $(+5)$ to $(+n) = 5 - n$

\therefore Total number of electrons that have been given out during oxidation of 2.68×10^{-3} moles of A^{n+}
 $= 2.68 \times 10^{-3} \times (5 - n)$

Thus the number of electrons added to reduce 1.61×10^{-3} moles of MnO_4^- to Mn^{2+} , i.e.,

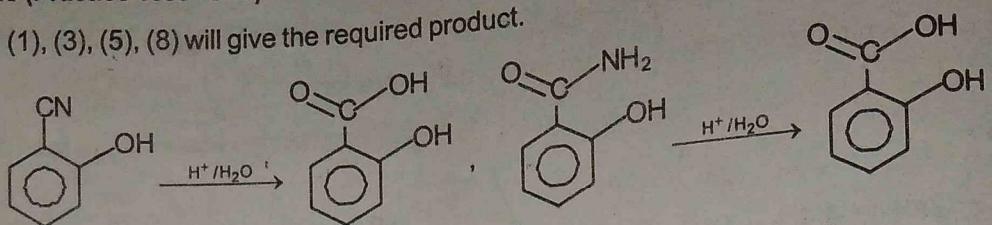
$$(+5) \text{ to } (+2) = 1.61 \times 10^{-3} \times 5$$

$$1.61 \times 10^{-3} \times 5 = 2.68 \times 10^{-3} \times (5 - n)$$

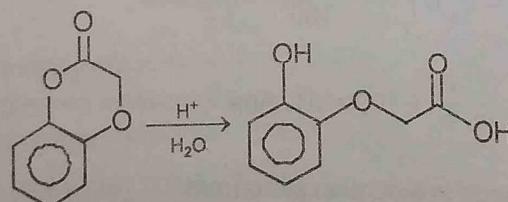
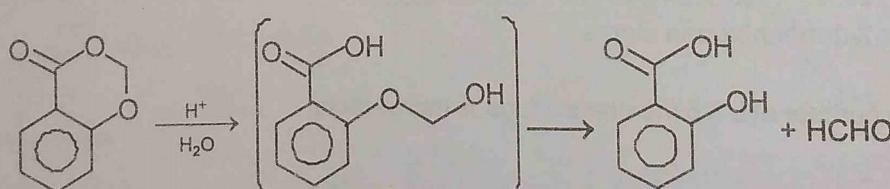
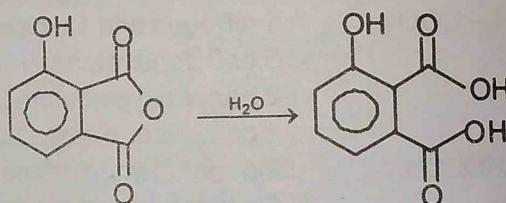
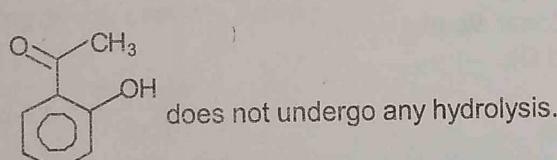
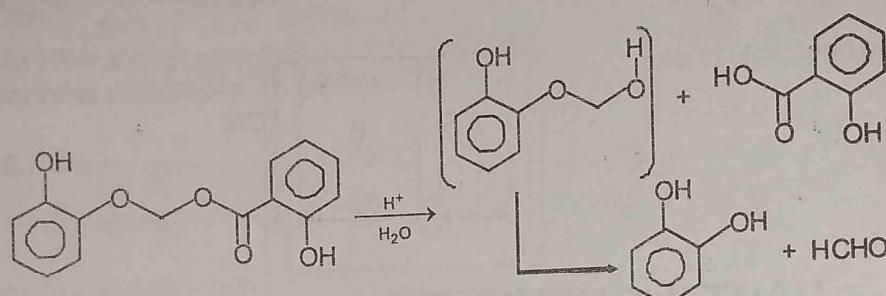
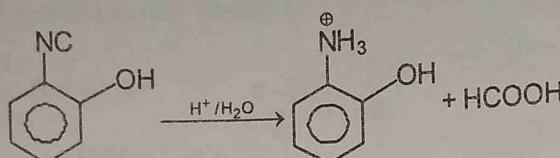
$$5 - n = \frac{1.61 \times 5}{2.68} \quad \text{or} \quad = 5 \frac{8.05}{2.68} \approx 2$$

Solutions (Practice Test - One)

27. (1), (3), (5), (8) will give the required product.



($-\text{C}\equiv\text{N}$, $-\text{C}(=\text{O})\text{NH}_2$, $-\text{C}(=\text{O})\text{N}(\text{CH}_3)_2$; all groups give $-\text{C}(=\text{O})\text{OH}$ on acid hydrolysis)

28. Compound (ii), (v), (vi), (vii), (viii), give CO_2 gas on heating.30. (B) Aluminium hydroxide is a +ve sol, so - ve ions are effective in coagulation.
(C) Cellulose solution is an example of macromolecular colloid.

31. (A) Average velocity = 0, gas molecules are moving randomly
 (B) All molecules do not have same speed there is maxwell distribution of molecular speeds.
 (C) For an open container, $n_1 T_1 = n_2 T_2$; where n_1 and n_2 are number of mole of gas in the container at temperatures T_1 and T_2 .

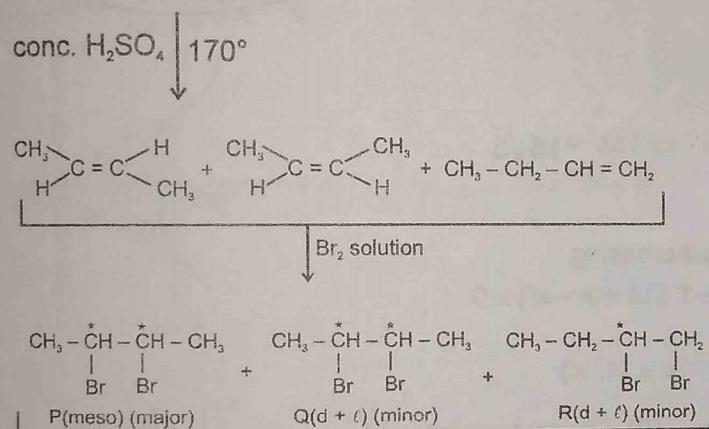
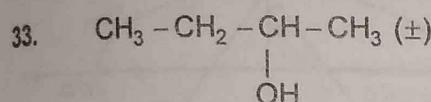
$$\text{So, } n_2 = \frac{300}{400} \cdot n_1$$

Hence fraction of gas that goes out = $1/4$ of the original sample.

$$(D) Z = \frac{V_m, \text{real}}{V_m, \text{ideal}} = \frac{V_m, \text{real}}{(22.4L)} \text{ (at STP)}$$

$$\text{So, } V_m \text{ real} < 22.4 \text{ L}$$

32. (A) Tollen's reagent does not oxidise C = C
(B) Perkin reaction
(C) E-1cB Mechanism
(D) KMnO_4 oxidises both alkene and aldehyde



34. P & Q are tautomers similarly P & R are tautomers.
Q is more stable than R due to intramolecular hydrogen bonding

36. (i) $\text{Bi} \downarrow$ (black) (ii) $\text{Ag}_2\text{CrO}_4 \downarrow$ (red) (iii) $\text{Bi}_2\text{S}_3 \downarrow$ (black)
 (iv) $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3 \downarrow$ (Blue)

38. P must be $\begin{array}{c} \text{Ph}-\text{C}=\text{CH}-\text{Ph} \\ | \\ \text{CH}_3 \end{array}$

39. ΔH is positive as vaporisation is endothermic process.
 ΔS is positive as liquid is changing to gas. ΔG is positive as process is non-spontaneous.

- $$40. \quad \Delta G^\circ = -2.303 RT \log K.$$

$$-5.76 \times 1000 = -2.303 \times \frac{25}{3} \times 300 \log K; \quad \log K = 1$$

$$K_p = 10 \text{ atm.} \quad K_p = P_{CO_2} = 10 \text{ atm.}$$

PART-III (MATHEMATICS)

41. Equation of ellipse is

$$\frac{x^2}{4} + \frac{y^2}{1} = 1 \quad \dots(i)$$

Given that equation of altitude is $x = 0$ from A that means one side parallel to x-axis and given that length of the side is λ so triangle is equilateral.

$$\text{Equation of AB will be } y = \sqrt{3}x + 1 \quad \dots(ii)$$

Solving (i) and (ii)

$$x = 0, -\frac{8\sqrt{3}}{13}$$

$$y = -\frac{24}{13} + 1 = -\frac{11}{13}, 1$$

$$\text{Coordinate of B} = \left(-\frac{8\sqrt{3}}{13}, -\frac{11}{13} \right)$$

$$\text{So, } \lambda = \sqrt{\frac{64 \times 3}{169} + \frac{24 \times 24}{169}} = \frac{16\sqrt{3}}{13} \Rightarrow 13\lambda = 16\sqrt{3}$$

42. $\because f''(x) > 0 \Rightarrow f'(x)$ is increasing

$$\text{Now } g'(x) = (2x - 1)[f'(x^2 - x - 10) - f'(14 + x - x^2)] \geq 0$$

$$\text{Case-1 : } x \geq \frac{1}{2} \text{ and } x \geq 4 \Rightarrow x \in [4, \infty)$$

$$\text{Case -2 } 2x - 1 \leq 0 \text{ and } f'(x^2 - x - 10) \leq f'(14 + x - x^2)$$

$$\Rightarrow x \leq \frac{1}{2} \text{ and } -3 \leq x \leq 4 \Rightarrow x \in \left[-3, \frac{1}{2} \right]$$

43. If common ratio is 1 numbers are 111, 222, ..., 999

If common ratio is $\frac{1}{2}$ (or $\frac{1}{2}$) numbers are 124, 248 (421, 842)

If common ratio is $\frac{1}{3}$ (or $\frac{1}{3}$) numbers are 139 (931)

If common ratio is $\frac{3}{2}$ (or $\frac{2}{3}$) numbers are 469 (964)

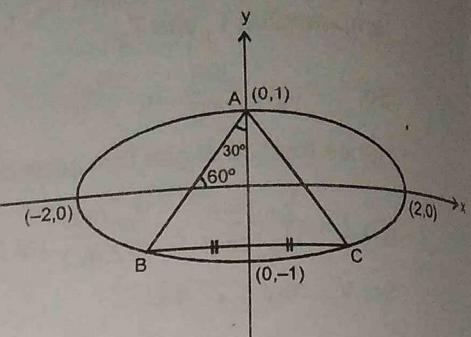
Hence total = 17

44. Total number of parts = number of point of intersection + 2

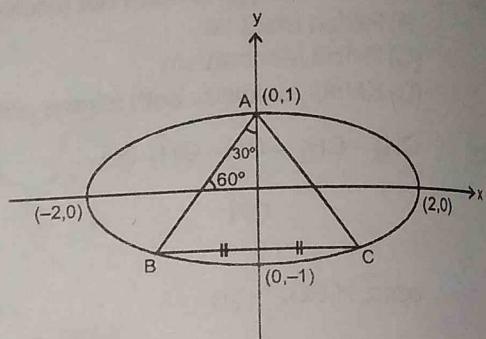
$$= 2 \cdot {}^n C_2 + 2$$

$$f(x) = n^2 - n + 2$$

$$f(3) = 8$$



45.



46.

47.

48.

49.

50.

45. $|\overline{AB}|^2 + |\overline{CD}|^2 = 3^2 + 11^2 = 130 = 7^2 + 9^2 = |\overline{BC}|^2 + |\overline{DA}|^2$
 $\overline{AB} + \overline{BC} + \overline{CD} + \overline{DA} = 0$ (As it will form a closed loop)

$$|\overline{DA}|^2 = |(\overline{AB} + \overline{BC} + \overline{CD})|^2$$

$$|\overline{DA}|^2 = |\overline{AB}|^2 + |\overline{BC}|^2 + |\overline{CD}|^2 + 2(\overline{AB} \cdot \overline{BC} + \overline{BC} \cdot \overline{CD} + \overline{CD} \cdot \overline{AB})$$

$$|\overline{DA}|^2 = |\overline{AB}|^2 + |\overline{BC}|^2 + |\overline{CD}|^2 + 2(\overline{BC}^2 + \overline{AB} \cdot \overline{BC} + \overline{BC} \cdot \overline{CD} + \overline{CD} \cdot \overline{AB})$$

$$|\overline{DA}|^2 + |\overline{BC}|^2 = |\overline{AB}|^2 + |\overline{CD}|^2 + 2[(\overline{AB} + \overline{BC}) \cdot (\overline{BC} + \overline{CD})]$$

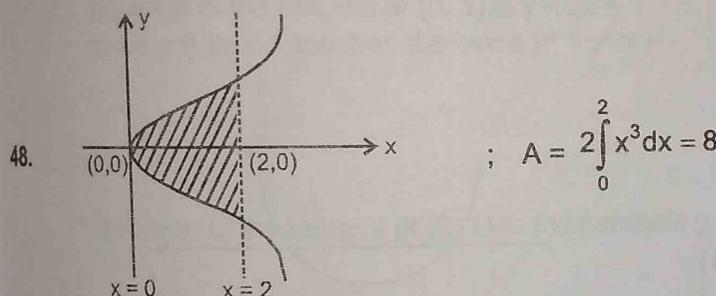
$$\overline{AC} \cdot \overline{BD} = 0$$

46. $I = (I - 3A)(I - \alpha A)$
 $\Rightarrow I = I - \alpha A - 3A + 3\alpha A^2 \Rightarrow I = I + (2\alpha - 3)A \Rightarrow 2\alpha = 3$

47. $\log_2(a+b)(c+d) \geq 4 \Rightarrow (a+b)(c+d) \geq 16$
AM \geq GM

$$\frac{a+b+c+d}{2} \geq \sqrt{(a+b)(c+d)} \geq 4$$

$$\Rightarrow a+b+c+d \geq 8$$



49. consider $\frac{2|x|}{|x-1|} - |x| = \frac{|x|^2}{|x-1|}$
i.e. $2 - |x-1| = |x|, x \neq 0$

case-I $x < 0$, then

$$2 + x - 1 = -x \quad \text{i.e.} \quad x = -\frac{1}{2}$$

case-II $0 \leq x < 1$

$$2 + x - 1 = x \text{ (not possible)}$$

case-III $x \geq 1$

$$2 - x + 1 = x \quad \text{i.e.} \quad x = \frac{3}{2}$$

50. (A) ${}^{10+3-1}C_{3-1} = {}^{12}C_2 = \text{total number of terms}$

(B) coefficient of $a^8 b^4 c = \frac{10!}{8! 1! 1!} = 90$

(C) the term $a^4 b^5 c^3 \rightarrow \text{does not exist}$

(D) coefficient of $a^4 b^5 c^1 = \frac{10!}{4! 5!} \text{ etc.}$

Solutions (Practice Test - One)

51. $6a^2 + (7b - c)a - (3b^2 - 4bc + c^2) = 0$
 $\Rightarrow 12a + 7b - c = \pm \sqrt{121b^2 + 25c^2 - 110bc}$

$\Rightarrow 12a + 7b - c = \pm (11b - 5c)$

$\Rightarrow (3a - b + c) = 0 \text{ or } (2a + 3b - c) = 0$

$\Rightarrow (3, -1)$ or $(-2, -3)$ lies on the line $ax + by + c = 0$

52. Centre of circle is $\left(\frac{p}{2}, 0\right)$

Radius = p

Equation of circle is $\left(x - \frac{p}{2}\right)^2 + y^2 = p^2$... (1)

$y^2 = 2px$... (2)

$\Rightarrow x^2 + px = \frac{3p^2}{4} \Rightarrow 4x^2 + 4px - 3p^2 = 0$

$\Rightarrow x = \frac{p}{2} \quad \& \quad x = -\frac{3p}{2}$

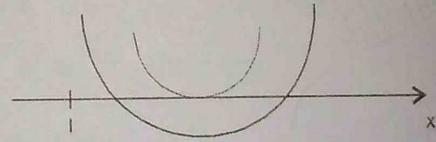
$y^2 = p^2 \quad \& \quad y^2 = -ve$

$\Rightarrow \left(\frac{p}{2}, p\right) \text{ and } \left(\frac{p}{2}, -p\right)$

53. $f(x) = 2x^2 + \lambda x - (\lambda + 1) = 0$

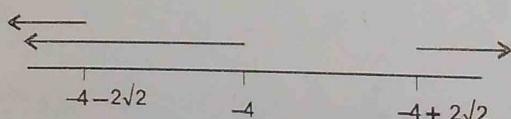
$D = \lambda^2 + 8(\lambda + 1) \geq 0$

$\lambda \in [-\infty, -4 - 2\sqrt{2}] \cup [-4 + 2\sqrt{2}, \infty]$ (1)



$\frac{-b}{2a} = \frac{-\lambda}{4} > 1 \Rightarrow \lambda < -4$ (2)

$f(1) = 2 + \lambda - (\lambda + 1) > 0, \forall \lambda \in \mathbb{R}$



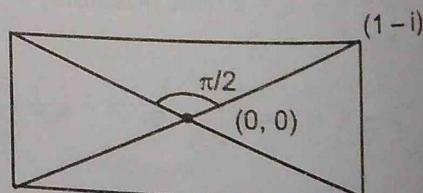
So,

$\lambda \in (-\infty, -4 - 2\sqrt{2}]$

54. Vertices are $(1 - i)e^{\pm i\pi/2}$

$(1 - i)(\pm i)$

So vertices are $1 + i$ & $-1 - i$



55. we have $\cos^{-1} \frac{y}{\sqrt{1+y^2}} = \tan^{-1} \frac{1}{y}$

$\sin^{-1} \frac{3}{\sqrt{10}} = \tan^{-1} 3$

$\therefore \text{given equation} \equiv \tan^{-1} x + \tan^{-1} \frac{1}{y} = \tan^{-1} 3$

$$\Rightarrow \tan^{-1} \frac{1}{y} = \tan^{-1} \frac{3-x}{1+3x} \quad \Rightarrow \tan^{-1} \frac{1}{y} = \tan^{-1} \frac{3-x}{1+3x} \quad \Rightarrow y = \frac{1+3x}{3-x}$$

$\therefore x, y \in \mathbb{N} \therefore x = 1, 2$
 $x=1 \Rightarrow y=2 ; x=2 \Rightarrow y=7$

56. $\because 1+a.a > 0, \forall a \in \mathbb{R} \therefore R_1$ is reflexive
 Let $(a, b) \in R_1$ s.t. $1+ab > 0 \Rightarrow 1+ba > 0$ ($\because ab = ba$)
 $\Rightarrow (b, a) \in R_1 \Rightarrow R_1$ is symmetric

Now $\left(1, \frac{1}{2}\right) \in R_1$ & $\left(\frac{1}{2}, -1\right) \in R_1$ as $1 + (-1) \frac{1}{2} > 0$ & $1 - \frac{1}{2} 1 > 0$

but $(1, -1) \notin R_1$ as $1 - 1.1 = 0$ not > 0

$\therefore R_1$ is not transitive

Sol. 57 to 58.

Since no point of the parabola is below x-axis

$$\therefore a^2 - 4 \leq 0$$

\therefore maximum value of a is = 2

equation of the parabola, when a = 2 is $y = x^2 + 2x + 1$

it intersects y-axis at (0, 1)

equation of the tangent at (0, 1) is $y = 2x + 1$

since $y = 2x + 1$ touches the circle $x^2 + y^2 = r^2$

$$\therefore r = \frac{1}{\sqrt{5}}$$

Equation of the tangent at (0, 1) to the parabola $y = x^2 + ax + 1$ is $\frac{y+1}{2} = \frac{a}{2}(x+0) + 1$

$$\text{i.e. } ax - y + 1 = 0 \quad \therefore r = \frac{1}{\sqrt{a^2 + 1}}$$

radius is maximum when $a = 0$

\therefore equation of the tangent is $y = 1 \quad \therefore$ slope of the tangent is 0

59. Let $0 < \alpha < \beta < 1$, and α, β are the roots of

$$f(x) = x^3 - 3x + k = 0 \Rightarrow f(\alpha) = f(\beta) = 0$$

\Rightarrow f(x) satisfies (Rolle's Theorem)

$$\Rightarrow f'(c) = 0$$

$$\Rightarrow 3c^2 = 3$$

$$\Rightarrow c = \pm 1$$

but c must be lies between α & β .

Hence $k \in \emptyset$

60. Let $f(x) = \tan^{-1} x$

then for some $\alpha \in (x, y)$, $f'(\alpha) = \frac{\tan^{-1} y - \tan^{-1} x}{y - x}$ (LMVT)

$$\Rightarrow \left| \frac{1}{1+\alpha^2} \right| = \left| \frac{\tan^{-1} x - \tan^{-1} y}{x-y} \right| \quad \left(\left| \frac{1}{1+\alpha^2} \right| \leq 1 \right)$$

$$\Rightarrow |\tan^{-1} x - \tan^{-1} y| \leq |x - y|$$

PRACTICE TEST - TWO

ANSWER KEY

PAPER - 1

PART-I (PHYSICS)

- | | | | | | | | | | | | | | |
|-----|-------------------------------------|-----|---|-----|-------|-----|-------|-----|------|-----|------|-----|------|
| 1. | (8) | 2. | (4) | 3. | (3) | 4. | (3) | 5. | (1) | 6. | (4) | 7. | (3) |
| 8. | (9) | 9. | (AC) | 10. | (ACD) | 11. | (BC) | 12. | (BD) | 13. | (CD) | 14. | (AB) |
| 15. | (ABC) | 16. | (ACD) | 17. | (AD) | 18. | (ABD) | | | | | | |
| 19. | (A) q,t (B) p,r,s (C) p,r,s (D) p,t | 20. | (A) p,q,t (B) p,q,s (C) p,q,t (D) p,q,s | | | | | | | | | | |

PART-II (CHEMISTRY)

- | | | | | | | | | | | | | | |
|-----|-------------------------|-----|----------------------------------|-----|--------|-----|------|-----|------|-----|-----|-----|------|
| 21. | (1) | 22. | (5) | 23. | (4) | 24. | (6) | 25. | (3) | 26. | (9) | 27. | (1) |
| 28. | (5) | 29. | (AD) | 30. | (ABCD) | 31. | (B) | 32. | (CD) | 33. | (D) | 34. | (AD) |
| 35. | (BD) | 36. | (BD) | 37. | (ABCD) | 38. | (BC) | | | | | | |
| 39. | (a) r (b) q (c) p (d) s | 40. | (A) p (B) p, q (C) q, r, s (D) r | | | | | | | | | | |

PART-III (MATHEMATICS)

- | | | | | | | | | | | | | | |
|-----|-------------------------|-----|-------------------------|-----|-------|-----|--------|-----|------|-----|-------|-----|------|
| 41. | (4) | 42. | (4) | 43. | (1) | 44. | (8) | 45. | (8) | 46. | (5) | 47. | (9) |
| 48. | (2) | 49. | (ABC) | 50. | (AC) | 51. | (ABCD) | | | 52. | (BCD) | 53. | (AB) |
| 54. | (BC) | 55. | (ABD) | 56. | (ABC) | 57. | (AC) | 58. | (BD) | | | | |
| 59. | (A) r (B) p (C) t (D) q | 60. | (A) s (B) p (C) q (D) r | | | | | | | | | | |

PRACTICE TEST - TWO

HINTS & SOLUTIONS

PAPER - 1

PART-I (PHYSICS)

$$1. \int_0^v v dv = \int_0^x (8 - 2x) dx \Rightarrow \frac{v^2}{2} = 8x - x^2 \Rightarrow v^2 = 16x - 2x^2$$

At B, $v = 0$ so, $x = 8$ Hence, AB = 8

(3)

$$2. \frac{\lambda_n}{2} = 6 \text{ cm} \Rightarrow \frac{2L}{2n} = 6 \Rightarrow n = \frac{L}{6}$$

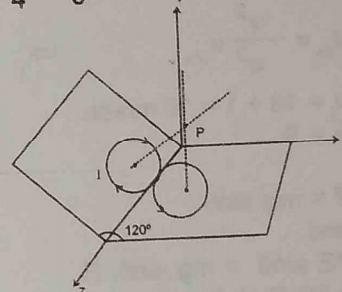
(AB)

$$3. \frac{\lambda_{n+1}}{2} = 4 \text{ cm} \Rightarrow \frac{2L}{2(n+1)} = 4 \Rightarrow n+1 = \frac{L}{4} \Rightarrow \frac{L}{4} - \frac{L}{6} = 1 \Rightarrow L = 12 \text{ cm}$$

$$3. B_1 = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + 3R^2)^{3/2}} = \frac{\mu_0 I}{16R}$$

$$B^2 = B_1^2 + B_1^2 + 2B_1 B_1 \cos 120^\circ$$

$$\Rightarrow B = B_1 = \frac{\mu_0 I}{16R} = \frac{3\mu_0 I}{48R}$$



$$(1) 4. (10 \times 11 - 10 \times 6) \times 10^{-4} \times 2T = \Delta W$$

$$50 \times 10^{-4} \times 2T = 3 \times 10^{-4} \Rightarrow T = \frac{3}{100} = 3 \times 10^{-2} \text{ N/m.}$$

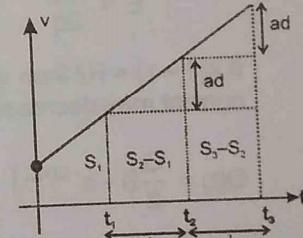
(AD)

5. From figure

$$(S_3 - S_2) - (S_2 - S_1) = ad.d$$

$$S_1 + S_3 - 2S_2 = ad^2$$

$$a = \frac{S_3 + S_1 - 2\sqrt{S_1 S_3}}{d^2} = \frac{(\sqrt{S_3} - \sqrt{S_1})^2}{d^2}$$

∴ Ans. $n = 1$ 

6. Let acceleration of wedge be 'b' and 2kg block with respect to wedge be 'a'.

Using newton's law for system

$$210 = 9b + 2(b - a \cos 45^\circ) \quad \dots(i)$$

F.B.D. of 2kg block :

$$2g \sin 45^\circ = 2(b \cos 45^\circ - a) \quad \dots(ii)$$

$$N - 2g \cos 45^\circ = b \sin 45^\circ \quad \dots(iii)$$

After solving equation (i) and (ii) we have $b = 20 \text{ m/s}^2$

$$a = \frac{10}{\sqrt{2}} \text{ m/s}^2$$

time taken by m to reach the top

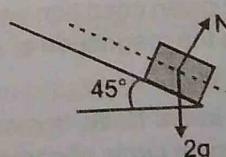
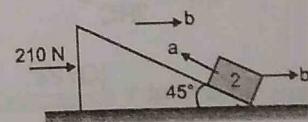
$$10\sqrt{2} = \frac{1}{2} \cdot \frac{10}{\sqrt{2}} t^2 \Rightarrow t = 2 \text{ s}$$

distance moved by wedge.

$$\therefore x = \frac{1}{2} b t^2 = \frac{1}{2} \cdot 20 \cdot (2)^2 = 40 \text{ m}$$

$$x = 40 \text{ m} = 4 \text{ Dm}$$

Ans.

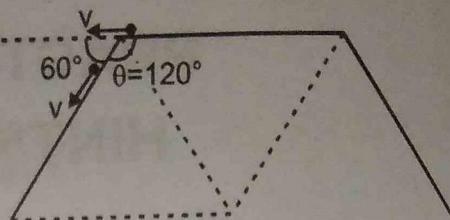


7. Impulse = | change in linear momentum |

$$= |\vec{P}_f - \vec{P}_i| = \sqrt{(mv)^2 + (mv)^2 + 2(mv)^2 \cos(\pi - \theta)}$$

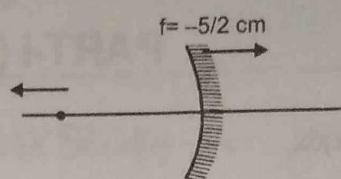
$$\text{Impulse} = 2mv \cos \frac{\theta}{2} = 2mv \cos 60^\circ$$

$$I = mv = 3 \text{ kg m/s}$$



$$8. \frac{1}{f} = \frac{1}{f_m} - \frac{2}{f_t} = \frac{-1}{5} - \frac{2}{10} = \frac{-4}{10}$$

$$f = -\frac{5}{2} \text{ cm}$$



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$v' = -\frac{2}{5} + \frac{3}{10} = -\frac{1}{10}$$

$$v = -10 \text{ cm}$$

$$v_{lm} = -\frac{v^2}{u^2} v_{om} \Rightarrow v_i - 1 = -\left(\frac{-10}{-10/3}\right)^2 (-1 - 1)$$

$$v_i = 18 + 1 = 19 \text{ m/sec.}$$

$$\alpha = 9.$$

$$9. f = mg \sin \theta$$

and

$$PE \sin \theta = mg \sin \theta \cdot R$$

$$q 2R E \sin \theta = mg \sin \theta R$$

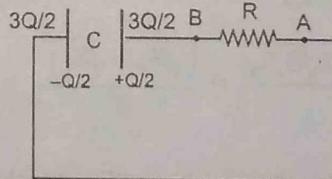
$$E = \frac{mg}{2q}$$

Ans. (A) & (C)

10. If $x(1 - \ell) < R\ell$ then $v_A > v_B$ if x is increased then current may decrease, become zero or may increase.

$$11. Q(t) = \frac{Q}{2} \left(1 - e^{-t/RC}\right)$$

$$i(t) = \frac{Q}{2RC} e^{-t/RC} \quad (\text{from B to A})$$



$$\Delta \text{heat} = \frac{(Q/2)^2}{2C}$$

12. Under the given condition particle shall start from rest and final achieve a steady state terminal velocity. Therefore the magnitude of acceleration will decrease to zero and speed shall increase from zero to a constant value.

13. All points in the body, in plane perpendicular to the axis of rotation revolve in concentric circles. All points lying on circle of same radius have same speed (and also same magnitude of acceleration) but different directions of velocity (also different directions of acceleration). Hence there cannot be two points in the given plane with same velocity or with same acceleration. As mentioned above, points lying on circle of same radius have same speed. Angular speed of body at any instant w.r.t. any point on body is same by definition.

14. When ball is immersed in liquid, weight will decrease due to buoyancy force. The loss in the weight will be increase in the weight of the tank. Hence $W_1 > W_3$ and $W_2 < W_4$. Also

$$W_1 + W_2 = W_3 + W_4$$

15. (A) The upper part of graph shows that acceleration is positive and becomes zero but just after this instant, the displacement cannot become negative suddenly. So, (A) is wrong.
 (B) Displacement is not positive when velocity is negative. So, (B) is wrong.
 (C) Velocity cannot increase if acceleration is negative. So, (C) is wrong.
 (D) If velocity is negative then displacement will also be negative. So, (D) is correct.

$$17. i = -\frac{k_0 x^2}{L^2} A \frac{dT}{dx} \Rightarrow \int_{T_H}^T dT = -\frac{iL^2}{k_0 A} \int_1^x \frac{dx}{x^2} \Rightarrow T - T_H = \frac{iL^2}{k_0 A} \left[\frac{1}{x} \right]_1^x$$

$$18. \frac{W}{4\pi a^2} = \left(\frac{N}{\Delta t} \right) \frac{hc}{\lambda} \times \frac{1}{s} \quad \& \quad eV = \left(\frac{hc}{\lambda} - \phi \right)$$

$$19. (A) \frac{1}{f} = \underbrace{\left(\frac{n_\ell}{n_s} - 1 \right)}_{-} \underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2} \right)}_{+}$$

$f = -ve$

$$P = \frac{1}{f} = -ve$$

q,t

$$(B) \frac{1}{f} = \underbrace{\left(\frac{n_\ell}{n_s} - 1 \right)}_{+} \underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2} \right)}_{+}$$

$f = +ve$

$$P = \frac{1}{f} = +ve$$

p,r,s

$$(C) \frac{1}{f} = \underbrace{\left(\frac{n_\ell}{n_s} - 1 \right)}_{-} \underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2} \right)}_{-}$$

$$f = +, \quad P = \frac{1}{f} = +ve$$

p,r,s

$$(D) f = \frac{R}{2} \quad f = -$$

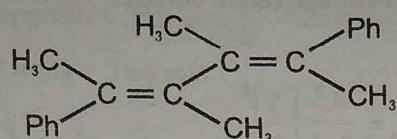
$$P = -\frac{1}{f} = +$$

p,t

20. (A) Both charge will move towards each other so electric potential energy will decrease. Centre of mass will be at rest all the time.
 (B) Charge 2C will move towards right and dipole move towards left. Centre of mass will be at rest all the time.
 (C) Dipoles will attract each other. So, kinetic energy will increase and electric potential energy will decrease.
 (D) Electric force on dipole will be in right direction. So, kinetic energy will increase and electric potential energy will decrease.

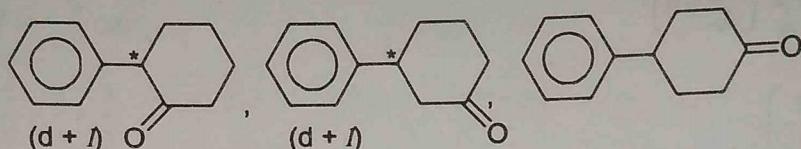
PART-II (CHEMISTRY)

21. Only one product (Single stereoisomers)



(Trans-Trans)

22.



23. Refer Notes.

25. IO_6^{5-} , XeO_6^{4-} , N_3^-

26. $(E_{n_2 \rightarrow 3})_{\text{Li}^{2+}} = (E_{3 \rightarrow \infty})_{\text{He}^{\ddagger}} + \text{KE}_{\text{electron}}$ [Here $(E_{3 \rightarrow \infty})_{\text{He}^{\ddagger}} = \text{KE}_{\text{electron}}$]

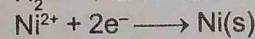
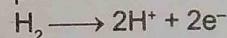
$$\therefore 13.6 (3)^2 \left(\frac{1}{3^2} - \frac{1}{n_2^2} \right) = 13.6 (2)^2 \left[\frac{1}{3^2} - \frac{1}{\infty^2} \right] \times 2$$

On solving, we get $n_2 = 9$

27. $E_{\text{cell}} = E_{\text{cell}}^{\circ} \frac{-0.0591}{2} \log \frac{[\text{H}^+]^2}{[\text{Ni}^{2+}][\text{H}_2]}$

$$\Rightarrow E_{\text{cell}} - E_{\text{cell}}^{\circ} = \frac{-0.0591}{1} \log [\text{H}^+] = 0.0591 \times \text{pH}$$

$$\Rightarrow \text{pH} = 1$$

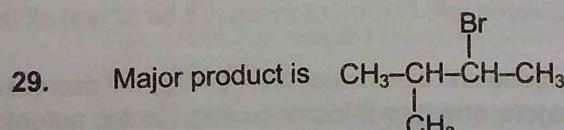


28. $\Delta T_f = i K_f m$ $i = 1 + (n - 1) \alpha$

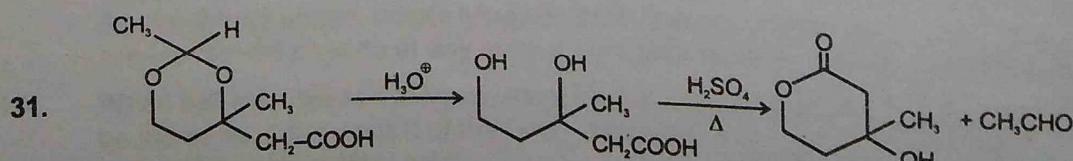
$$0.558 = i \times 1.86 \times 0.1 \quad 3 = 1 + (n - 1).1$$

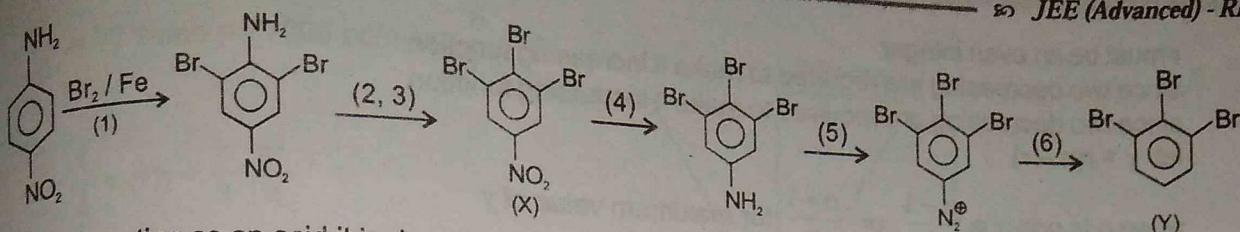
$$i = 3$$

$$[\text{Co}(\text{NH}_3)_5\text{Cl}].\text{Cl}_2 \Rightarrow x = 5$$

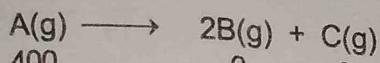


It has one chiral centre. So, Major product is mixture of two enantiomers.





Quinol is acting as an acid it is donating H^+ ions and also as a reducing agent due to $Ag^+ \rightarrow Ag$.



$$t=0 \quad 400 \quad 0 \quad 0$$

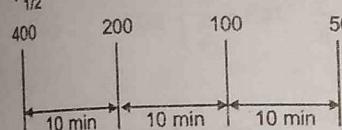
$$t=20 \text{ min} \quad 400-p \quad 2p \quad p$$

$$\text{Given } 400 - p + 2p + p = 1000$$

$$400 + 2p = 1000$$

$$p = 300 \text{ mm} ; k = \frac{1}{20} \ln \frac{400}{400-300} = \frac{1}{20} \ln 4 ; k = \frac{\ln 2}{10} \text{ min}^{-1}$$

$$T_{1/2} = 10 \text{ min} ; \text{Value of } k = 0.0693 \text{ min}^{-1}$$



After 30 min Partial Pressure of A is 50 mm,

After 30 min Partial Pressure of B is 700 mm

After 30 min Partial Pressure of C is 350 mm , After 30 min total pressure become 1100 mm

39. (A) Bauxite is leached with NaOH (concentrated) to form soluble $Na[Al(OH)_4]$ complex and insoluble impurities are filtered off.
- (B) Carbonate and hydroxide ores are heated in absence of air below their melting point to convert in to their oxides in reverberatory furnace. This is called calcination. So magnesite, $MgCO_3$ is subjected to calcination.
- (C) This method is commonly used for the concentration of the low grade sulphide ores like galena, PbS (ore of Pb) ; copper iron pyrites $Cu_2S \cdot Fe_2S_3$ or $CuFeS_2$ (ore of copper) ; zinc blende, ZnS (ore of zinc) etc., and is based on the fact that gangue and ore particles have different degree of wettability with water and pine oil; the gangue particles are preferentially wetted by water while the ore particles are wetted by oil.
- (D) Chromite ore ($FeO \cdot Cr_2O_3$) having magnetic properties is separated from non-magnetic silicious impurities by magnetic separator.

PART-III (MATHEMATICS)

41. x can be 2,3,4,5,6. The number of ways in which sum of 2,3,4,5,6 can occur is given by the coefficients of x^2, x^3, x^4, x^5, x^6 in $(3x + 2x^2 + x^3)^6$ $((x + 2x^2 + 3x^3)^6 = 3x^2 + 8x^3 + 14x^4 + 8x^5 + 3x^6$
This shows that sum that occurs most often is 4.

42. Equation of tangent is $y = 2x \pm \sqrt{4a^2 + b^2}$
this is normal to the circle $x^2 + y^2 + 4x + 1 = 0$
this tangent passes through $(-2, 0)$
- $$\Rightarrow 0 = -4 \pm \sqrt{4a^2 + b^2} \Rightarrow 4a^2 + b^2 = 16.$$
- Using AM \geq GM, we
- $$\frac{4a^2 + b^2}{2} \geq \sqrt{4a^2 \cdot b^2} \Rightarrow ab \leq 4$$

$$43. \tan \theta = \frac{2\tan^2 \theta - \frac{1}{3}\tan \theta}{1 + \frac{2}{3}\tan^3 \theta} \Rightarrow \tan \theta = 0 \text{ or } \frac{\frac{2\tan \theta - 1}{3}}{1 + \frac{2}{3}\tan^3 \theta} = 1$$

$$\tan^3 \theta - 3\tan \theta + 2 = 0 \Rightarrow \tan \theta = 1 \text{ or } -2$$

$$\alpha = 0, \beta = 1, \gamma = -2$$

Solutions (Practice Test - Two)

44. r must be an even integer
 since two decreasing are required to make it increasing function
 since two decreasing are required to make it increasing function
 let $y = r(n - r)$

when n is odd $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ for maximum value of y

when n is even $r = \frac{n}{2}$ for maximum value of y

\therefore maximum (y) = $\frac{n^2 - 1}{4}$ when n is odd, $\frac{n^2}{4}$ when n is even

$$\alpha = 4, \beta = 4$$

45. Number of terms in each group 1, 3, 7, 15,.....

last term in each group 1, 4, 11, 26,..... a_n

$$\text{Now } S = 1 + 4 + 11 + 26 + \dots + a_n$$

$$S = 1 + 4 + \dots + a_n$$

-

$$\Rightarrow a_n = 1 + 3 + 7 + 15 \dots, n \text{ term}$$

$$a_n = 1 + 3 + 7 + 15 \dots, n \text{ term}$$

$$a_n = 1 + 3 + 7 \dots, n^{\text{th}} \text{ term}$$

-

$$n^{\text{th}} \text{ term} = 1 + 2 + 4 + 8 + \dots = (2^n - 1)$$

$$\text{So } a_n = \sum_{n=1}^n (2^n - 1) = 2(2^n - 1) - n$$

So last term of n^{th} group = $2(2^n - 1) - n$

last term of $(n-1)^{\text{th}}$ group = $2[2^{n-1} - 1] - (n-1)$

Sum of the numbers in the n^{th} group

$$= \left\{ 1 + 2 + 3 + \dots + 2(2^n - 1) - n \right\} - \left\{ 1 + 2 + 3 + \dots + 2(2^{n-1} - 1) - (n-1) \right\}$$

$$= \frac{(2^{n+1} - n - 2)(2^{n+1} - n - 1) - \{2^n - (n-1)\}(2^n - n)}{2}$$

$$= 3 \cdot 2^{2n-1} - (2n+5) 2^{n-1} + (n+1)$$

$$a+b = 08$$

46. $(x-1)(x-\alpha_1)(x-\alpha_2) \dots (x-\alpha_8) = x^8 - 1$

$$\therefore (2-\alpha_1)(2-\alpha_2) \dots (2-\alpha_8) = 2^8 - 1$$

Now since $2-\alpha_1$ and $2-\alpha_8$ are conjugates of each other

$$\therefore |2-\alpha_1| = |2-\alpha_8|$$

similarly

$$|2-\alpha_2| = |2-\alpha_7|, \quad |2-\alpha_3| = |2-\alpha_6| \quad \text{and} \quad |2-\alpha_4| = |2-\alpha_5|$$

$$\therefore |(2-\alpha_1)(2-\alpha_3)(2-\alpha_5)(2-\alpha_7)| = \sqrt{2^8 - 1} = \sqrt{511}$$

47. $n(S) = \text{Total number of numbers} = 5 \times {}^5C_4 \times 4! = 5(5!)$

five digit numbers divisible by 6 are formed by using the numbers 0, 1, 2, 4 & 5 or 1, 2, 3, 4 & 5

$$\therefore \text{number of such numbers} = n(E) = 2(4!) + 2 \times 3 \times 3! + 4 \times 3! = 108$$

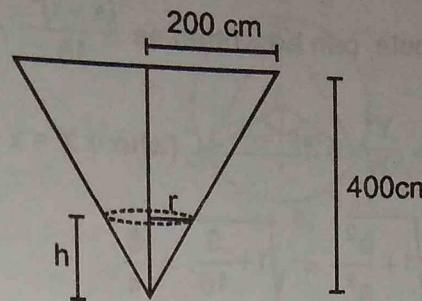
$$\therefore P(E) = 0.18$$

48. $\frac{dv}{dt} = 77 \text{ l/min} = 77000 \text{ cc/min.}, \frac{h}{r} = 2$

$$V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} \frac{h^3}{4}$$

$$\therefore \frac{dv}{dt} = \frac{\pi h^2}{4} \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{77000 \times 4 \times 7}{22 \times 70 \times 70} = 20 \text{ cm/min}$$



49. (A) $\sin\left(\tan^{-1} 3 + \tan^{-1} \frac{1}{3}\right) = \sin \frac{\pi}{2} = 1$

(B) $\cos\left(\frac{\pi}{2} - \sin^{-1} \frac{3}{4}\right) = \cos\left(\cos^{-1} \frac{3}{4}\right) = \frac{3}{4}$

(C) $\sin\left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1} \frac{\sqrt{63}}{8} = \theta$

so $\sin \theta = \frac{\sqrt{63}}{8}$ if $\cos \theta = \frac{1}{8}$

we have $\cos \frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \frac{3}{4}$

$$\sin \frac{\theta}{4} = \sqrt{\frac{1-\cos \frac{\theta}{2}}{2}} = \frac{1}{2\sqrt{2}}$$

Now $\log_2 \sin\left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8}\right) = \log_2 \frac{1}{2\sqrt{2}} = -\frac{3}{2}$

(D) $\cos^{-1} \frac{\sqrt{5}}{3} = \theta$

$$\cos \theta = \frac{\sqrt{5}}{3}$$

$\therefore \tan \frac{\theta}{2} = \frac{3 - \sqrt{5}}{2}$ which is irrational

50. $(\vec{a} - \vec{b}) \times [(\vec{b} + \vec{a}) \times (2\vec{a} + \vec{b})] = \vec{b} + \vec{a}$

$$\Rightarrow (\vec{a} - \vec{b}) \cdot (2\vec{a} + \vec{b}) \cdot (\vec{b} + \vec{a}) - \{(\vec{a} - \vec{b}) \cdot (\vec{b} + \vec{a})\} (2\vec{a} + \vec{b}) = \vec{b} + \vec{a}$$

$$\Rightarrow (2 - \vec{a} \cdot \vec{b} - 1)(\vec{b} + \vec{a}) = \vec{b} + \vec{a} \quad \Rightarrow \quad \text{either } \vec{b} + \vec{a} = \vec{0} \text{ or } 1 - \vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow \text{either } \vec{b} = -\vec{a} \text{ or } \vec{a} \cdot \vec{b} = 0 \quad \Rightarrow \quad \text{either } \theta = \pi \text{ or } \theta = \frac{\pi}{2}$$

51. Given hyperbola can be written as $\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1 \quad \text{where } X = x - 1, Y = y - 1$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$$

$$\text{Directrices are } X = \pm \frac{a}{e}$$

$$\Rightarrow x - 1 = \pm \frac{16}{5} \quad \Rightarrow \quad x = \frac{21}{5} \text{ and } x = -\frac{11}{5}$$

$$\text{Length of Latus rectum} = \frac{2b^2}{a} = \frac{9}{2} \text{ and focii are } \Rightarrow X = \pm ae, Y = 0 \Rightarrow (6, 1) \text{ and } (-4, 1)$$

52.

$$x + 3y + 2z = 6 \quad \dots \text{(i)}$$

$$x + \lambda y + 2z = 7 \quad \dots \text{(ii)}$$

$$x + 3y + 2z = \mu \quad \dots \text{(iii)}$$

(A) If $\lambda = 2$, then D = 0, therefore unique solution is not possible

(B) If $\lambda = 4$, $\mu = 6$

$$x + 3y = 6 - 2z$$

$$x + 4y = 7 - 2z$$

$$\therefore y = 1 \text{ and } x = 3 - 2z$$

substituting in equation (iii)

$$3 - 2z + 3 + 2z = 6 \text{ is satisfied}$$

\therefore infinite solutions

(C) $\lambda = 5, \mu = 7$

consider equation (ii) and (iii)

$$x + 5y = 7 - 2z$$

$$x + 3y = 7 - 2z$$

$$\therefore y = 0, x = 7 - 2z \text{ are solution}$$

sub. in (i)

$$7 - 2z + 2z = 6 \quad \text{does not satisfy}$$

\therefore no solution

(D) if $\lambda = 3, \mu = 5$

then equation (i) and (ii) have no solution

\therefore no solution

53. Given $x^2 = ay$ & $y - 2x = 1$

$$\therefore x^2 = a(2x + 1)$$

$$\Rightarrow x^2 - 2ax - a = 0$$

$$x_1 + x_2 = 2a, x_1 x_2 = -a$$

$$\therefore \sqrt{40} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow 40 = (x_1 - x_2)^2 + \left(\frac{x_1^2}{a} - \frac{x_2^2}{a} \right)^2 \Rightarrow 40 = (x_1 - x_2)^2 \left[1 + \frac{(x_1 + x_2)^2}{a^2} \right]$$

$$\Rightarrow 40 = ((x_1 + x_2)^2 - 4x_1 x_2) \left[\frac{4a^2}{a^2} + 1 \right] \Rightarrow 40 = 5(4a^2 + 4a)$$

$$\Rightarrow a^2 + a - 2 = 0$$

$$\Rightarrow a = -2, 1$$

Solutions (Practice Test - Two)

59. (1) Clearly

$$\frac{5}{3} < \alpha < \frac{7}{2}$$

(2) $\therefore [\alpha] = 1, 2, 3$
 \because origin & $(\alpha^2, \alpha + 1)$ lie on the same side of both lines
 $\therefore 3\alpha^2 - (\alpha + 1) + 1 > 0 \text{ & } \alpha^2 + 2(\alpha + 1) - 5 < 0$

$$\Rightarrow a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty \right) \text{ & } \alpha \in (-3, 0) \cup \left(\frac{1}{3}, 1 \right)$$

$$\therefore \alpha \in (-3, 0) \cup \left(\frac{1}{3}, 1 \right)$$

$$\therefore [\alpha] = -3, -2, -1, 0$$

(3) Required circle is $(x - 1)^2 + (y - 1)^2 + \lambda(3x - y - 2) = 0$ it passes through $(1, -1)$

$$\therefore 0 + 4 + \lambda(3 + 1 - 2) = 0$$

$$\lambda = -\frac{4}{2} = -2$$

$$\begin{aligned} \therefore (x - 1)^2 + (y - 1)^2 - 2(3x - y - 2) &= 0 \\ \Rightarrow x^2 + y^2 - 2x - 2y + 2 - 6x + 2y + 4 &= 0 \\ \Rightarrow x^2 + y^2 - 8x + 6 &= 0 \end{aligned}$$

$$\therefore \text{Centre is } (4, 0) = (a, b)$$

(4) Required locus is

$$\begin{aligned} (x - 1)^2 + (y - 2)^2 &= (x + 2)^2 + (y + 1)^2 \\ \Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 &= x^2 + 4x + 4 + y^2 + 2y + 1 \\ \Rightarrow 6x + 6y &= 0 \\ \Rightarrow x + y &= 0 \end{aligned}$$

$$\therefore \boxed{a - 2b = -1}$$

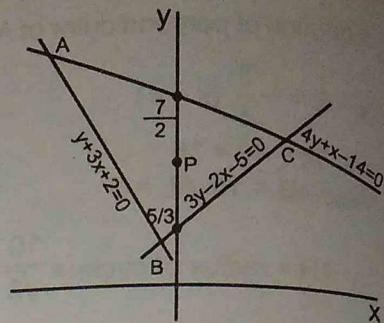
60. (1) obvious

(2) $\frac{z_3 - z_1}{z_3 - z_2}$ is purely Imaginary. Triangle is right angled with right angle at z_3 .

(3) Obtuse angled triangle

$$(4) |z_3 - z_1| = |z_3 - z_2|$$

and $\frac{z_3 - z_1}{z_3 - z_2}$ is purely Imaginary, So triangle is right angled and isosceles.



PRACTICE TEST - TWO

ANSWER KEY

PAPER - 2

PART-I (PHYSICS)

- | | | | | | | | | | | | | | |
|-----|------|-----|------|-----|-------|-----|------|-----|------|-----|------|-----|--------|
| 1. | (5) | 2. | (7) | 3. | (9) | 4. | (7) | 5. | (1) | 6. | (5) | 7. | (4) |
| 8. | (2) | 9. | (AB) | 10. | (ABC) | 11. | (AC) | 12. | (AC) | 13. | (BC) | 14. | (ABCD) |
| 15. | (BC) | 16. | (BD) | 17. | (B) | 18. | (C) | 19. | (B) | 20. | (C) | | |

PART-II (CHEMISTRY)

- | | | | | | | | | | | | | | |
|-----|-------|-----|-------|-----|------|-----|-------|-----|--------|-----|-----|-----|-------|
| 21. | (2) | 22. | (0) | 23. | (3) | 24. | (6) | 25. | (5) | 26. | (2) | 27. | (4) |
| 28. | (8) | 29. | (BCD) | 30. | (AC) | 31. | (ABC) | 32. | (ABCD) | 33. | (D) | 34. | (ABC) |
| 35. | (ABD) | 36. | (ABC) | 37. | (B) | 38. | (B) | 39. | (C) | 40. | (C) | | |

PART-III (MATHEMATICS)

- | | | | | | | | | | | | | | |
|-----|------|-----|-------|-----|-------|-----|------|-----|------|-----|-------|-----|------|
| 41. | (0) | 42. | (4) | 43. | (2) | 44. | (3) | 45. | (5) | 46. | (0) | 47. | (8) |
| 48. | (8) | 49. | (AB) | 50. | (ACD) | 51. | (BC) | 52. | (AB) | 53. | (ABC) | 54. | (AB) |
| 55. | (AD) | 56. | (ABD) | 57. | (A) | 58. | (B) | 59. | (A) | 60. | (B) | | |

PRACTICE TEST - TWO

HINTS & SOLUTIONS

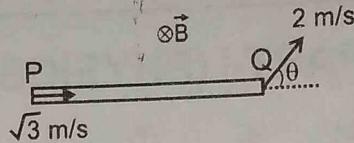
PAPER - 2

PART-I (PHYSICS)

1. $2\cos\theta = \sqrt{3}$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$



$$\omega_{QP} = \frac{2\sin\theta}{\ell} = 1. \text{ Induced emf} = \frac{1}{2}B\omega^2\ell = \frac{1}{2}B(1)(1) = 1/2$$

2. In an adiabatic expansion,
 $TV^{\gamma-1} = \text{constant}$

$$T_0 V^{\gamma-1} = T \left(\frac{V}{5}\right)^{\gamma-1} \quad \gamma = 1 + \frac{2}{5}$$

$$\therefore T = (273) \cdot (5)^{2/5} \quad \langle (KE)_{\text{rotational}} \rangle = kT = 1.38 \times 10^{-23} \times 273 \times (5)^{2/5} = 7 \times 10^{-21} \text{ J (approx.)}$$

3. Side of cube = a

$$Mg = \frac{3}{4} a^3 \rho g. \quad \text{If side of cavity is } b.$$

$$\text{New mass of cube} = (a^3 - b^3) \frac{3}{4} \rho + b^3 \frac{6}{4} \rho = (a^3 + b^3) \frac{3}{4} \rho$$

$$a^3 \rho g = (a^3 + b^3) \frac{3}{4} \rho g \quad \Rightarrow \quad b^3 = \frac{a^3}{3}, \quad b = \left[\frac{1}{3}\right]^{\frac{1}{3}} a \quad \Rightarrow \quad b = \left[\frac{4}{9} \frac{M}{\rho}\right]^{\frac{1}{3}}$$

4. Given that $m_1 - m_2 = 6$ unit ... (1)
 equation of motion

$$\frac{Gm_1m_2}{r^2} = m_1\omega^2 r_1 = m_1\omega^2 \frac{m_2 r}{m_1 + m_2}$$

$$\therefore m_1 + m_2 = \frac{\omega^2 r^3}{G} = 8 \text{ unit} \quad \dots (2)$$

$$\text{by (1) and } \frac{m_1}{m_2} = 7$$

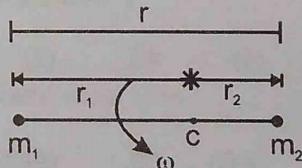
5. Consider the system gas+cylinder+piston.

From the conservation of energy,

heat given to the system + work done by the atmospheric pressure on the system = $\Delta U + \Delta KE$ of the system

$$Q - p_a A \Delta \ell = \frac{5}{2} n R \Delta T + \Delta KE \text{ of the sys}$$

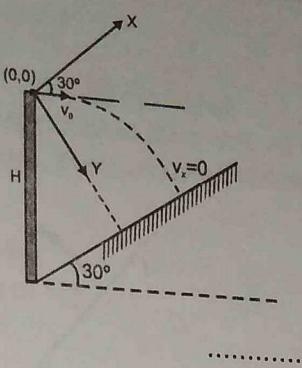
$$20 \times 4.2 - 10^5 \times 10 \times 10^{-4} \times 4.2 \times 10^{-2} = \frac{5}{2} \times 2 \times 2 \times 1.8 \times 4.2 + \Delta KE \text{ of the system}$$



$$\begin{aligned} a_x &= -g \sin 30^\circ \\ a_y &= g \cos 30^\circ \\ u_x &= V_0 \cos 30^\circ \\ u_y &= V_0 \sin 30^\circ \\ v_x &= u_x + a_x t \end{aligned}$$

$$0 = \frac{V_0 \sqrt{3}}{2} - \frac{g}{2} t$$

$$t = \frac{V_0 \sqrt{3}}{g}$$



.....(i)

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$(H \cos 30^\circ - 0) = V_0 \sin 30^\circ t + \frac{1}{2} g \cos 30^\circ t^2 \quad \dots \dots \dots \text{(ii)}$$

From (i) &

$$\Rightarrow \frac{2gH}{V_0^2} = 5.$$

$$\begin{aligned} 7. \quad Q &= [M(Ra^{226}) - M(Rn^{222}) - M(He^4)] \times 931 \\ &= (226.025406 - 222.017574 - 4.002603) u \times 931 \\ &= 0.005229 u \times 931 \frac{\text{MeV}}{\text{u}} \end{aligned}$$

$$Q = 4.87 \text{ MeV}$$

$$\begin{aligned} 8. \quad V_0 &= V_c - R\omega \\ &= 4 - (.2) \times 10 = 2 \text{ m/s} \end{aligned}$$

10. The maximum compression in spring for shown situation is $\frac{m_2 F}{2(m_1 + m_2)K}$.

Since $(m_1 + m_2)K$ is same for all situations ; the compression is maximum for spring 3 and least for spring 2.

$$11. \quad \text{Use } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

13. (A) Current will be same for each section
 (B) E \propto V_d , so $E_B > E_A > E_C$

$$J \propto E \Rightarrow E \propto \frac{1}{A}$$

$$(C) \phi = \vec{E} \cdot \vec{A} = \frac{\vec{J}}{\sigma} \cdot \vec{A}. \quad \text{So, } \phi = i$$

as 'i' is same for each section, so ϕ is also same
 (D) current flows from high potential to low potential so $V_A > V_B > V_C$.

$$14. \quad \text{The potential gradient} = \frac{10}{(500+500)} \times \frac{500}{10} = 0.5 \text{ Vm}^{-1}$$

Since potential difference across

$$R_1 = 0.5 \times 2 = 1 \text{ V}$$

Since potential difference across

$$R_2 = 0.5 \times 8 = 4 \text{ V}$$

$$E = 1 + 4 = 5 \text{ V}$$

Also since in meter bridge the balancing length = 20 cm.

$$\frac{R_1}{20} = \frac{1}{1}$$

15. $F = \frac{YA}{L} x$

So, $k = \frac{YA}{L}$ and $W = \frac{1}{2} \frac{YAx^2}{L}$.

18. (17 to 18)

$N + T = mg \sin 30^\circ$

$$\tau_{\text{Net}} = 0, mg \sin 30^\circ \frac{L}{4} = T \frac{3L}{4}$$

$$T = \frac{mg}{6}$$

$$f = mg \cos 30^\circ = \frac{\sqrt{3}mg}{2}$$

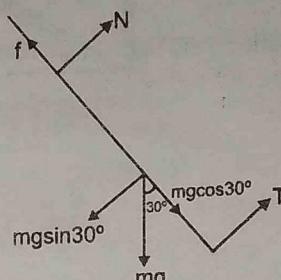
19. $\lambda = 4m$ and तथा $f = 500 \text{ Hz}$.

$$\therefore V = f\lambda = 200 \text{ m/s}$$

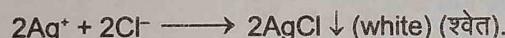
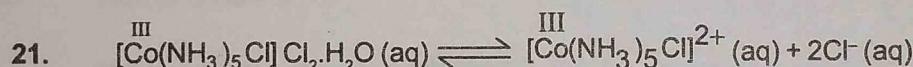
$$\therefore V = \sqrt{\frac{T}{\mu}} \quad \therefore T = \mu v^2 = (0.1) \times (200)^2 = 400 \text{ N}$$

20. Since integral number of waves shall cross a point in 5 seconds, therefore power transmitted in 5 seconds is

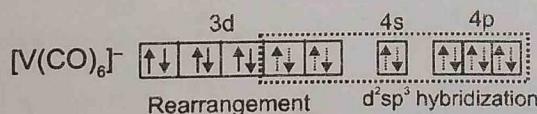
$$= \langle P \rangle \times 5 = 2\pi^2 f^2 A^2 \mu v \times 5$$



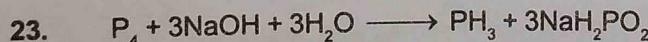
PART-II (CHEMISTRY)



22. The Vandium is in -1 oxidation state and CO is a strong field ligand so compels for the pairing of electrons. Thus the complex has $d^2 sp^3$ hybridisation and is diamagnetic.



$$\mu_{\text{BM}} = \sqrt{n(n+2)} = 0 \text{ as there is no unpaired electrons.}$$



24. $\lambda^\infty_{\text{K}^+} = 96.50$

$$\therefore \text{Ionic mobility } \mu^\infty_{\text{K}^+} = \frac{\lambda^\infty_{\text{K}^+}}{F} = \frac{96.50}{96500} = 10^{-3} \text{ cm}^2 \text{ sec}^{-1} \text{ volt}^{-1}$$

$$\text{Potential gradient applied} = \frac{6.0}{10} = 0.6 \text{ volt cm}^{-1}$$

$$\text{Ionic mobility } (\mu) = \left(\frac{\text{speed of ion}}{\text{potential gradient}} \right)$$

$$\text{So speed of } (\text{K}^+) = (10^{-3} \times 0.6) = 6 \times 10^{-4} \text{ cm/sec.}$$

$$\therefore \text{Distance travelled in 2 hours 46 minutes and 40 seconds}$$

$$d = 6 \text{ cm.}$$

25. Total pressure is = 80 cm of Hg
 V. P. of H_2O is = 20 cm of Hg

\therefore Pressure of H_2 gas = $80 - 20 = 60$ cm of Hg

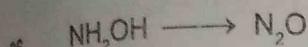
Now if the volume of the piston is doubled then V.P. of water won't change since there is water in the container to maintain the equilibrium. Pressure of H_2 will become half

$$\Rightarrow P_{H_2} = 30 \text{ cm of Hg}$$

V. P. of water = 20 cm of Hg

$$\text{Total pressure} = 20 + 30 = 50 \text{ cm of Hg.}$$

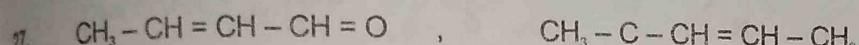
Therefore answer = $50/10 = 5$.



(-1) (+1) oxidation number of nitrogen.

\therefore Vf = change in oxidation number of nitrogen = 2.

Organic



28. \therefore Total number of stereoisomers = $2^{n-1} = 8$.

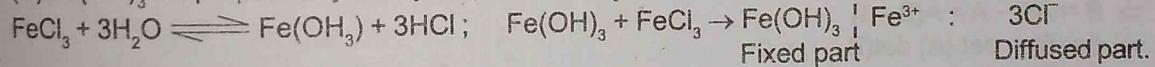
29. (A) Burns only in pure dioxygen gas.

(D) Availability of electrons for metallic bonding \downarrow .

30. (A) $\Delta G = \Delta H - T\Delta S < 0$ as $\Delta S < 0$ so ΔH has to be negative

(B) micelles formation will take place above T_k and above CMC

(C) $Fe(OH)_3$ sol prepared by the hydrolysis of $FeCl_3$ solution adsorbs Fe^{3+} and this is positively charged.



Positive charge on colloidal sol is due to adsorption of Fe^{3+} ion (common ion between $Fe(OH)_3$ and $FeCl_3$).

(D) Fe^{3+} ions will have greater flocculating power so smaller flocculating value.

31. $\frac{r_{\text{vapours}}}{r_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{\text{vapours}}}}$; ; $\frac{4}{3} = \sqrt{\frac{32}{M}}$

$$M = 18 \quad \text{Atomic weight} = \frac{18}{3} = 6$$

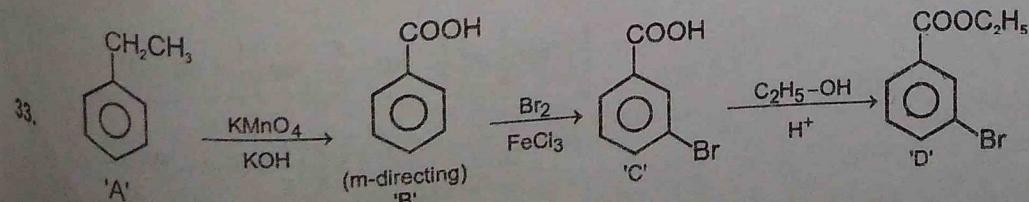
$$V.D. = 9 \quad \text{Density} = \frac{18}{22.4} = 0.8035 \text{ g/lit.}$$

$$PV = 2RT$$

$$PV = Z \frac{W}{V} RT$$

$$PM = ZdRT$$

$$Z = \frac{PM}{dRT} = \frac{18P}{dRT}$$



Solutions (Practice Test - Two)

34. Picric acid, Aspirin and Squaric acid liberate CO₂ gas of reaction with Sodium bicarbonate.

35. N_2O_5 Solid has NO_2^+ and NO_3^- ions

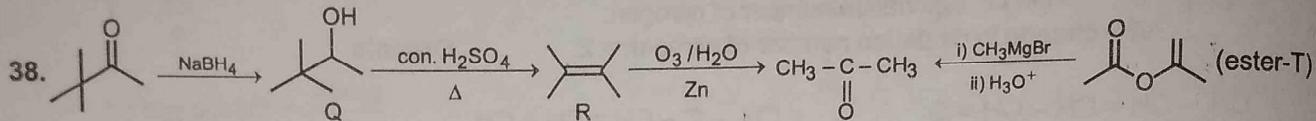
PCl_5 solid has PCl_4^+ and PCl_5^+ ions

XeF_6 solid has XeF_5^+ and F^- ions

ICl_3 solid exist as dimer I_2Cl_6 .

ICl_3 solid exist as dimer I_2Cl_6 .

36. Gives prussian blue precipitate of $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$ with Fe^{3+} .



39. At half neutralisation $\text{pH} = \text{pK}_b$

40. For complete neutralisation of WA and SB

$$pH = 7 + \frac{1}{2} pK_a + \frac{1}{2} \log C$$

where $C = \frac{0.1}{2}$ of CH_3COONa

PART-III (MATHEMATICS)

- $$41. \quad AB = A + B$$

$$\Rightarrow B = AB - A = A(B - I)$$

$$\Rightarrow \det(B) = \det(A) \det(B - I) = 0$$

$$\Rightarrow \det(B) = 0$$

$$42. \quad \sqrt{x+iy} = \frac{a+ib}{c+id} \Rightarrow (x+iy) = \left(\frac{a+ib}{c+id} \right)^2 \Rightarrow |x+iy| = \left| \frac{a+ib}{c+id} \right|^2$$

$$\Rightarrow \sqrt{x^2 + y^2} = \frac{a^2 + b^2}{c^2 + d^2} = 2 \Rightarrow x^2 + y^2 = 4$$

differentiate w.r.t. x

$$\Rightarrow x \frac{dp}{dx} = \frac{1}{\sqrt{1-p^2}} \frac{dp}{dx}$$

$$\Rightarrow \frac{dp}{dx} = 0 \text{ or } \frac{1}{\sqrt{1-p^2}} = x \Rightarrow p = \frac{\sqrt{x^2-1}}{x} \text{ put in equation}$$

$$y = \sqrt{x^2 - 1} - \sin^{-1} \frac{\sqrt{x^2 - 1}}{x}$$

$$a = 1, b = 1, c = 1$$

Given $\vec{r} = x\hat{i} + y\hat{j}$ (in a plane)

$$\Rightarrow \vec{r} \cdot (10\hat{j} - 8\hat{i} - \vec{r}) = 40$$

$$\text{Gives } x^2 + y^2 + 8x - 10y + 40 = 0$$

It is circle $(-4, 5)$ and radius = 1

$$p_1 = \max \{(x+2)^2 + (y-3)^2\}; p_2 = \min \{(x+2)^2 + (y-3)^2\}$$

(x, y) is any point on the circle.

$$p_2 = (2\sqrt{2} - 1); p_1 = (2\sqrt{2} + 1)$$

$$p_1^2 + p_2^2 = 18 \text{ (even)} \Rightarrow \text{tangents are drawn.}$$

$$\text{Slope of AB, } \left(\frac{dy}{dx} \right)_{(2, 2)} = -2$$

$$\text{Equation of AB, } 2x + y = 6, B = (3, 0)$$

$$\overline{OA} = 2\hat{i} + 2\hat{j}, \overline{OB} = 3\hat{i}, \overline{AB} = \overline{OB} - \overline{OA}$$

$$\overline{AB} = \hat{i} - 2\hat{j}, \overline{AB} \cdot \overline{OB} = (\hat{i} - 2\hat{j}) \cdot (3\hat{i}) = 3 \quad \text{Ans. 3}$$

45. Let centre be $C(h, k)$

$$\therefore CP \perp AB$$

$$\Rightarrow \frac{2-k}{\frac{3}{2}-h} = \frac{3}{4}$$

$$\Rightarrow 6h - 8k = -7 \quad \dots \text{(i)}$$

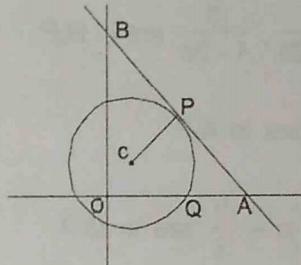
$$\therefore CP = CQ$$

$$\Rightarrow \left(h - \frac{3}{2} \right)^2 + (k - 2)^2 = \left(h - \frac{3}{2} \right)^2 + k^2$$

$\Rightarrow k = 1$ put in (i), we get

$$6h = 1 \Rightarrow h = \frac{1}{6}$$

$$\therefore \text{radius}(r) = CQ = \sqrt{\left(\frac{1}{6} - \frac{3}{2} \right)^2 + 1} = \sqrt{\left(\frac{1-9}{6} \right)^2 + 1} = \frac{5}{3} \quad \therefore 3r = 5$$



$$46. \int \frac{3x^2 + 2x}{x^6 + 2x^5 + x^4 + 2x^3 + 2x^2 + 5} dx$$

$$= \int \frac{3x^2 + 2x}{(x^3 + x^2 + 1)^2 + 4} dx = \frac{1}{2} \tan^{-1} \left(\frac{x^3 + x^2 + 1}{2} \right) + C$$

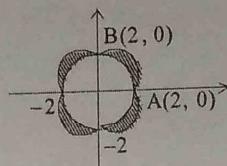
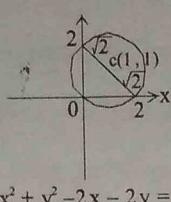
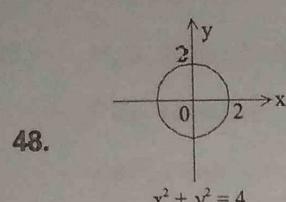
$$F(x) = \frac{1}{2} \tan^{-1} \left(\frac{x^3 + x^2 + 1}{2} \right) + C$$

$$\therefore F(1) - F(0) = \frac{1}{2} \left(\tan^{-1} \frac{3}{2} - \tan^{-1} \frac{1}{2} \right) = \frac{1}{2} \tan^{-1} \frac{4}{7}$$

$$\therefore 0 < F(1) - F(0) < \frac{1}{2} \cdot \frac{\pi}{2} < 1$$

Solutions (Practice Test - Two)

47. The triangle has circumcentre at origin and its orthocentre lying on the circumcircle.



48.

$$\text{required area} = 4 \text{ (area in 1st quadrant)} = 4 \left[\frac{\pi(\sqrt{2})^2}{2} - \left(\frac{\pi \times 2^2}{4} - \frac{1}{2} \times 2 \times 2 \right) \right] = 4(\pi - \pi + 2) = 8$$

49. $\tan A + \tan B + \tan C = \tan A \tan B \tan C = 3 \tan B = 6 \tan A$

$\therefore 3 \tan A + \tan C = 6 \tan A$

$\tan C = 3 \tan A$

$\therefore \tan A + 2 \tan A + 3 \tan A = \tan A \cdot 2 \tan A \cdot 3 \tan A \quad \text{i.e.} \quad \tan A = \tan^3 A$

i.e. $\tan^2 A = 1 \quad \text{i.e.} \quad \tan A = 1 \quad \therefore A = \frac{\pi}{4}$

$\therefore \tan A \tan B = 2 \tan^2 A = 2$

50. (A) $\frac{a}{1-2a}, \frac{b}{1-2b}, \frac{c}{1-2c}$ are in H.P. $\Leftrightarrow \frac{1-2a}{a}, \frac{1-2b}{b}, \frac{1-2c}{c}$ are in A.P.

$\Leftrightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. $\Leftrightarrow a, b, c$ are in H.P.

(B) $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ are in G.P. $\Rightarrow \ln \left(a - \frac{b}{2} \right), \ln \frac{b}{2}, \ln \left(c - \frac{b}{2} \right)$ are in A.P.

(C) $c - \frac{b}{2}, \frac{b}{2}, a - \frac{b}{2}$ are in G.P.

(D) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. $\Rightarrow e^{1/a}, e^{1/b}, e^{1/c}$ are in G.P.

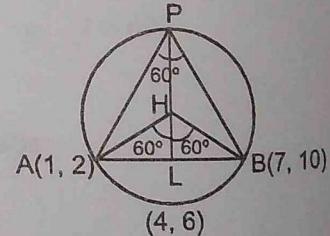
51. Equation of perpendicular of AB is

$$y - 6 = -\frac{3}{4}(x - 4)$$

$$\Rightarrow 3x + 4y = 36$$

$$\text{Also } AB = 10, AL = 5$$

$$\therefore AH = \text{radius of circle} = \frac{10}{\sqrt{3}} = AL \cosec 60^\circ$$



52. $P \equiv (2t^2, 4t)$

$SP = 4$ (given)

$$\Rightarrow 2 + 2t^2 = 4 \Rightarrow t = \pm 1 \Rightarrow P \equiv (2, 4) \text{ or } (2, -4)$$

$$\text{Given } f(1) = 2 \text{ and } f(x+y) = f(x) \cdot f(y) \Rightarrow a_1 = f(1) = 2 \text{ and } a_2 = f(2) = 4$$

53. Given $r = |\cos \theta|$

$$\Rightarrow r^2 = |\cos \theta| \Rightarrow x^2 + y^2 = |x| \Rightarrow x^2 + y^2 = \mp x$$

$$\Rightarrow x^2 + y^2 \mp x = 0 \Rightarrow x^2 + y^2 + x = 0,$$

$$x^2 + y^2 - x = 0$$

Centre $\left(-\frac{1}{2}, 0\right)$

Centre $\left(\frac{1}{2}, 0\right)$

Radius $= \frac{1}{2}$

Radius $\frac{1}{2}$

\therefore They are touching each other at origin.

Given $\log_x a, a^{x/2}, \log_b x$ are in GP. So, $a^x = \log_x a \cdot \log_b x$

54. $\Rightarrow a^x = (\log_b a) \Rightarrow x = \log_a (\log_b a)$

$$\Rightarrow x = \frac{\log\left(\frac{\log a}{\log b}\right)}{\log a} \Rightarrow x = \frac{\log(\log a) - \log(\log b)}{\log a}$$

55. $\therefore \sin B = \frac{b \sin A}{a}$ and $A < \frac{\pi}{2}$

when $b \sin A = a$, $\sin B = 1$, $B = \frac{\pi}{2}$ (possible)

when $\sin B < 1$

$$\frac{b \sin A}{a} < 1$$

$$b \sin A < a$$

If $b < a$, then one triangle possible

If $b > a$, then two triangle possible

56. Given function can be re-defined as

$$y > 0 \Rightarrow y = \frac{1}{2}x ; x \geq 0$$

$$y < 0 \Rightarrow y = \frac{1}{10}x ; x < 0$$

A.P.

$$\therefore y = \begin{cases} \frac{x}{2}, & x \geq 0 \\ \frac{x}{10}, & x < 0 \end{cases}$$

Also. $f(0+h) = f(0-h) = f(0) = 0$

$\Rightarrow f(x)$ is continuous at $x = 0$ but it is not diff. at $x = 0$ as $Rf'(0) = \frac{1}{2}$ & $Lf'(0) = \frac{1}{10}$

57. The diagonals are $\vec{d}_1 = 3\hat{a} - 2\hat{b} + 2\hat{c} + (-\hat{a} - 2\hat{c}) = 2\hat{a} - 2\hat{b}$

$$\vec{d}_2 = 3\hat{a} - 2\hat{b} + 2\hat{c} - (-\hat{a} - 2\hat{c}) = 4\hat{a} - 2\hat{b} + 4\hat{c}$$

$$\text{Angle between them} = \cos^{-1} \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1| \cdot |\vec{d}_2|} = \cos^{-1} \left(\frac{8+4}{2\sqrt{2}(6)} \right) = \cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

58. $\hat{a} \times \hat{b} = \hat{c} \Rightarrow \hat{c} \cdot \hat{a} \times \hat{b} = \hat{c} \cdot \hat{c} = 1$

$$\Rightarrow [\hat{a} \hat{b} \hat{c}] = 1$$

$$\Rightarrow \hat{a} \cdot (\hat{b} \times \hat{c}) + \hat{b} \cdot (\hat{c} \times \hat{a}) + \hat{c} \cdot (\hat{a} \times \hat{b}) = 3$$

59. $\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} \sin^{-1} [\tan h] = \lim_{h \rightarrow 0} \sin^{-1} (0) = 0$

60. $\lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} \sin^{-1} [\tan (-h)] = \lim_{h \rightarrow 0} \sin^{-1} (-1) = -\frac{\pi}{2}$