PRACTICE TEST - THREE

ANSWER KEY

PAPER - 1

PART-I (PHYSICS)

1.	(2)	2.	(5)	3.	(3)	4.	(7)	5.	(2)	6.	(6)	7.	(1)
8.	(4)	9.	(ABC)	10.	(AB)	11.	(ABC)	12.	(ABCD)	13.	(AD)	14.	(AC)
15.	(ABD)	16.	(D)	17.	(CD)	18.	(ABCD)						

19. (A) p,q,t (B) p,r,s (C) q,t (D) r

20. (A) p,q,t (B) r,s (C) p,q,t (D) q

PART-II (CHEMISTRY)

21.	(4)	22.	(6)	23.	(4)	24.	(5)	25.	(3)	26.	(4)	27.	(8)
28.	(4)	29.	(B)	30.	(ACD)	31.	(ABC)	32.	(ABCD)	33.	(ABC)	34.	(ABCD)
35.	(ABCD)	36.	(ABC)	37.	(AC)	38.	(ABCD)						

39. (A) p,q,t (B) p,q,r (C) p,r,s (D) p,q,r,s,t

40. (A) s,t (B) p, r (C) q (D) p, r

PART-III (MATHEMATICS)

41.	(7)	42.	(7)	43.	(2)	44.	(1)	45.	(5)	46.	(4)	47.	(1)
48.	(7)	49.	(BD)	50.	(AD)	51.	(ABC)	52.	(AB)	53.	(BC)	54.	(BC)
55.	(BCD)	56.	(ABD)	57.	(ABC)	58.	(ABC)						

59. (A) q,t,s (B) q (C) p, q, t (D) p

60. (A) s (B) q (C) p (D) q

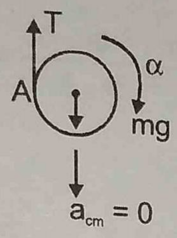
PRACTICE TEST - ONE
HINTS & SOLUTIONS

PAPER - 1
PART-I (PHYSICS)

1. $a_A = a = \alpha \cdot R$ ...(i)

$T - mg = 0$ ...(ii)

$T.R = \frac{mR^2}{2}.\alpha$ ....(iii)



$\therefore \quad g = \frac{a}{2}$

2. Area perpendicular to the light = 1 x cos60^0

Energy falling on the surface=intesity x perp.area =3 x 1 x cos60^0 = $\frac{3}{2}$ Watt

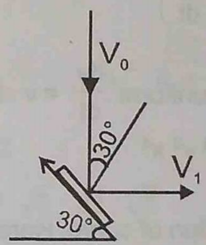
Momentum carried by the light per sec = 3/(2c) = 5 x 10^{-9}

3. $V_0 \sin 30° = V_1 \cos 30°$ (i)

$eV_0 \cos 30° = V_1 \sin 30°$ (ii)

Dividing (i) & (ii)

$\frac{1}{e} \tan 30° = \frac{1}{\tan 30°}$ $\qquad \frac{1}{e} = 3$



4. $W = \int \vec{F}.d\vec{r}$

$= \int (ydx + xydy) \qquad \because \quad 2x = 3y$

$= \int_0^2 \left(\frac{3}{2}ydy + \frac{3}{2}y^2dy\right) \qquad \therefore \quad dx = \frac{3}{2}dy = \left[\frac{3}{4}y^2 + \frac{y^3}{2}\right]_0^2 = 7$ Joule.

5. A ray entering through surface A and travelling along the inner side of the rod will be reflected by the outer side with the smallest angle θ, at which the reflected ray is tangent to the inner side.

$\theta \geq \theta_{critical} = \sin^{-1}\left(\frac{1}{n}\right)$

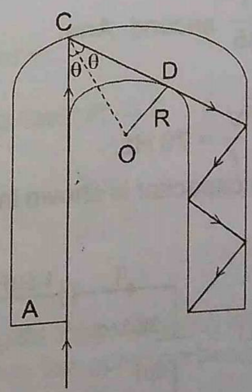
$\sin\theta \geq$

but $\sin\theta = \frac{R}{R+d}$

$\Rightarrow \quad \frac{R}{R+d} \geq \frac{1}{3/2}$

$\frac{R+d}{R} \leq \frac{3}{2} \quad ; \quad \frac{d}{R} \leq \frac{1}{2}$

$\frac{R}{d} \geq 2 \quad ; \quad \left(\frac{R}{d}\right)_{min} = 2$



6. $W_{earth} = mg_e h \qquad = m\frac{4}{3}G d\pi R.h$

$W_{planet} = m.\frac{4}{3}Gd\pi\frac{R}{4}.2h \qquad \Rightarrow \qquad \frac{W_{earth}}{W_{planet}} = 2$

$W_{planet} = \frac{W}{2} = \frac{3W}{6}$.

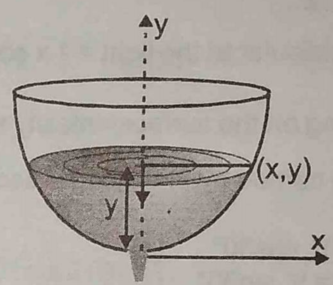
7. $V_0 = \frac{h}{e}f - \frac{\phi}{e}$

if f = 0

$V_0 = -\frac{\phi}{e} = -1V$

$\Rightarrow \phi = 1eV$

8. A_1 – area of orifice



From equation of continuity

$\sqrt{2gy}\ A_1 = (\pi x^2)\left(-\frac{dy}{dt}\right)$

For time scale to be uniform $\frac{dy}{dt}$ = v should be constant

$\therefore \qquad 2gy.A_1^{\,2} = \pi^2 v^2 x^4$

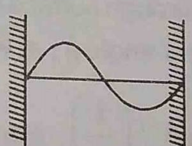
$y = ax^4$, is the equation of curve. Here $a = \frac{\pi^2 v^2}{2gA_1^2}$

9. Charge is distributed over the surface of conductor in such a way that net field due to this charge and outside charge q is zero inside. Field due to only q is non-zero.

10. Speed of wave in wire $V = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell}A \times \frac{1}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell\rho}}$

Maximum time period means minimum frequency; that means fundamental mode.

$f = \frac{V}{\lambda} = \frac{V}{2\ell}$



$\ell = \lambda$

$\therefore \qquad T = \frac{2\ell}{V} = 2\ell\sqrt{\frac{\ell\rho}{Y\Delta\ell}} = \frac{1}{35}$ second **Ans.**

$\therefore$ (f = 35 Hz)

and; frequency of first overtone $= \frac{V}{\ell} = 70$ Hz.

11. In the initial state, charge on each capacitor is shown in figure-1.

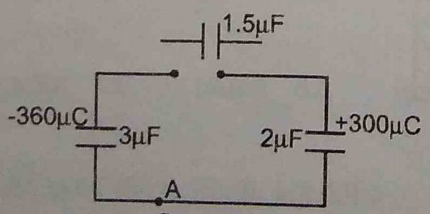


figure-1

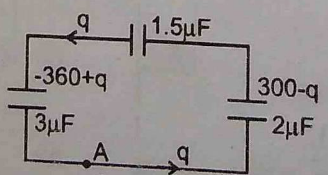
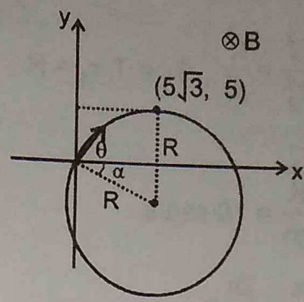


figure-2

Let charge q flow anticlockwise in the circuit before it achieves steady state as shown in figure-2.
Applying KVL to figure 2.
$\therefore$ final charge on 1.5 μF capacitor is q = 180 μC and final charge on 2 μF capacitor is 300 – q = 120 μC.

12. $R^2 - (R-5)^2 = (5\sqrt{3})^2$



$R^2 - R^2\,(R-5)^2 = (5\sqrt{3})^2$

$R^2 - R^2 - 25 + 10\,R = 75$

$R = 10$ m

$\sin\alpha = \dfrac{1}{2}$, a = 30° , θ = 90 – α = 60°

$$\frac{mv}{qB} = R \Rightarrow v = \frac{RqB}{m} = \frac{10\times10^{-6}\times10}{5\times10^{-5}} = 2 \text{ m/s}$$

13. **For case–1 :**

$$\rho gh = \frac{1}{2}\rho v_1^2 \quad \Rightarrow \quad v_1 = \sqrt{2gh} \quad \Rightarrow \quad t_1 = \sqrt{\frac{10h}{g}}$$

For case–2 :

$$\rho g2h + 2\rho gh = \frac{1}{2}z\rho v_2^2 \Rightarrow \quad v_2 = \sqrt{4gh} \quad \Rightarrow \quad t_2 = \sqrt{\frac{6h}{g}}$$

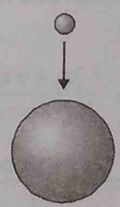
For case–3 :

$$\rho g2h + 2\rho g2h + 3\rho gh = \frac{1}{2}3\rho v_3^2 \Rightarrow \quad v_3 = \sqrt{6gh} \quad \Rightarrow \quad t_3 = \sqrt{\frac{2h}{g}}$$

$x_1 = v_1 t_1 = \sqrt{20}$ $\qquad$ $x_2 = \sqrt{24}$ $\qquad$ $x_3 = \sqrt{12}$

$$x_1 : x_2 : x_3 = \sqrt{20} : \sqrt{24} : \sqrt{12} = \sqrt{10} : \sqrt{12} : \sqrt{6} = \sqrt{5} : \sqrt{6} : \sqrt{3}.$$

14. (A) It will fall because mg is acting on it towards the centre of planet and initial velocity is zero. It'll move in straight line.

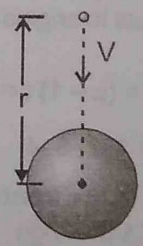


(C) Time of fall can be found by two methods :

I Method : By energy conservation

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 - \frac{GMm}{R} \quad -(1)$$

using this we get V = f(r). Now use

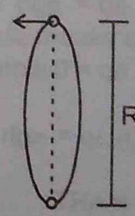


$$V = -\frac{dr}{dt} \quad \Rightarrow f(r) = -\frac{dr}{dt}$$

$$\Rightarrow \int_R^{R'} \frac{dr}{f(r)} = -\int_0^t dt \quad ; \quad R' = \text{radius of the planet.}$$

In the final expression (or in the beginning itself) $R' \to 0$ $\quad \{\because R >> R'\}$

you will get $\quad t = \dfrac{T}{4\sqrt{2}}$



Here $\dfrac{GMm}{R^2} = m\left(\dfrac{2\pi}{T}\right)^2 R$

Note : This method is longer. If a student gets idea of solving the question only by this method then it is better to leave this question because it will consume more time.

II Method : Kepler's Law : $T^2 \propto r^3$.

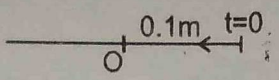
Assume that the satellite moves in elliptical path with maximum and minimum distances from centre as R and R'.

$\because R >> R'$ $\therefore$ velocity at R is very small ($\simeq 0$). When it reaches R' then it touches the surface of the planet. This motion (from R to R') is almost, same as given in the question.

Now $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$, $T_1 = T, r_1 = R$; $r_2 = \frac{R+R'}{2} \simeq \frac{R}{2}$ $\therefore T_2 = \frac{T}{4\sqrt{2}}$

15. $\omega = \sqrt{\frac{K}{m}} = 10$ rad/s

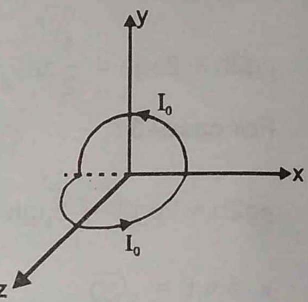
$T = \frac{2\pi}{\omega} = \frac{2\pi}{10}$ s



Maximum speed will be at the natural length of the spring $T/4 = \frac{2\pi}{10\times4} = \frac{\pi}{20}$s.

Time taken to cover 0.1 m is $\frac{T}{4} = \frac{\pi}{20}$ s. Time taken to cover $\frac{1}{2}$ × 0.1m is $\frac{T}{4}\times\frac{2}{3} = \frac{2\pi}{10\times4}\times\frac{2}{3} = \frac{\pi}{30}$s

16. Portion in x–y plane will not experience any net force due to x–component of magnetic field. Only its z–component will be effective. So, current in the loop is as shown.



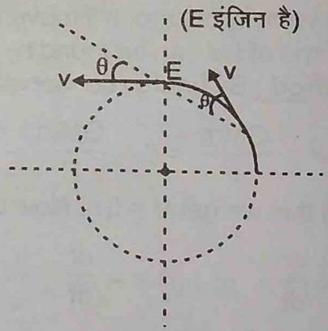
$\Rightarrow \quad \vec{M} = \frac{\pi R^2}{2} I_0(\hat{k}+\hat{j})$

net force on the portion is in x-z plane = $I_0 2R\hat{i} \times \frac{M_0 I_0}{2\pi R}(\hat{i}+\hat{k}) = -\frac{\mu_0 I_0^{\,2}}{\pi}\hat{j}$

Net torque = $\vec{M}\times\vec{B} = \frac{\mu_0 I_0^{\,2} R}{4}(\hat{i}+\hat{j}-\hat{k})$

Net magnetic field at the origin = $\frac{\mu_0 I_0}{4R}(\hat{j}+\hat{k}) + \frac{\mu_0 I_0}{2\pi R}(\hat{i}+\hat{k})$.

E is Engine
(E इंजिन है)



17. $f_{obs} = \frac{f\,[v_s + v\cos\theta]}{[v_s + v\cos\theta]} = f$

$\lambda_{obs} = \frac{v_s + v\cos\theta}{f}$

For any observer in train frequency observed is equal to original frequency but observed wavelength is more.

18. $\Delta x = (\mu - 1)\,t = n\lambda \quad \Rightarrow \quad t = \frac{n\lambda}{(\mu-1)}$

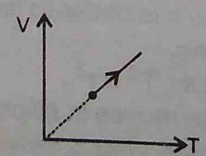
$n = 1, 2, 3, 4 \quad \Rightarrow \quad t = 1.18, 2.36, 3.54$ and $4.72\ \mu$m

19. (p) (i) two points in same horizontal level will have same pressure.
(ii) $\Delta p = \rho gh$
(q) (i) $\Delta p = \rho a\ell \neq 0$ along vertical
(ii) $\Delta p = \rho gh \neq 0$ along vertical.
(r) pressure at every point is zero
(ii) $\Delta p = 0$ along horizontal
(s) (i) $\Delta p = \rho gh + \rho ah$ gh along vertical

(t) (i) $\Delta p = \pi gh$ along vertical
(ii) $\Delta p = \frac{1}{2}\rho\omega^2 r^2$ along horizontal.

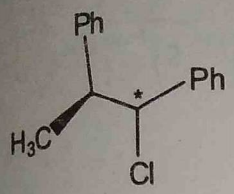
20. PV = nRT

$U = \frac{f}{2}nRT$

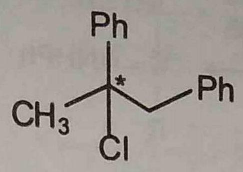


Isobaric Process

PART-II (CHEMISTRY)

21. 

(Pair of diasteromers)

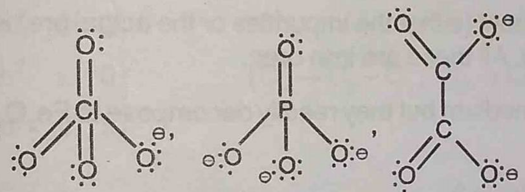


(Pair of enantiomers)

22. X has 3 stereocentres with similar ends. Hence, 6 stereoisomers. Y= $CH_3-\overset{*}{C}H(D)-CHO$ has one sterteocentre, hence 2 stereoisomers. Z= $CH_3-CH(D)-CH=NOH$ has 2 stereocentres, hence 4 stereoisomers.

23. (1) BF_6^{3-} does not exist as there is no vacant d-orbital in B and hence it cannot exceed its covalency beyond four.
(2) OF_4 does not exist as oxygen cannot exceed its covalency as it does not have d-orbital.
(3) NCl_5 does not exist as there is no d-orbital with N.
(4) HFO_4 does not exist as there is no vacant d-orbital in F and hence it cannot exceed its covalency.

24. $Fe_4^{III}[Fe^{II}(CN)_6]_3$

25. 4 structures each : 

26. Given binding energy of 2nd excited state = 24 eV

$\Rightarrow \quad 1.51\,Z^2 = 24 \quad \Rightarrow \quad Z^2 = 16 \quad \Rightarrow \quad Z = 4$

27. $$AgSCN \rightleftharpoons \underset{S}{Ag^+} + \underset{(S+0.1)}{SCN^-}$$

$$E_{cell} = E^{\circ}_{Ag^+/Ag} - \frac{0.0591}{1}\log\frac{1}{[Ag^+]}$$

$$0.45 = 0.8 + 0.0591\log[Ag^+]$$

$$[Ag^+] = 1\times10^{-6}$$

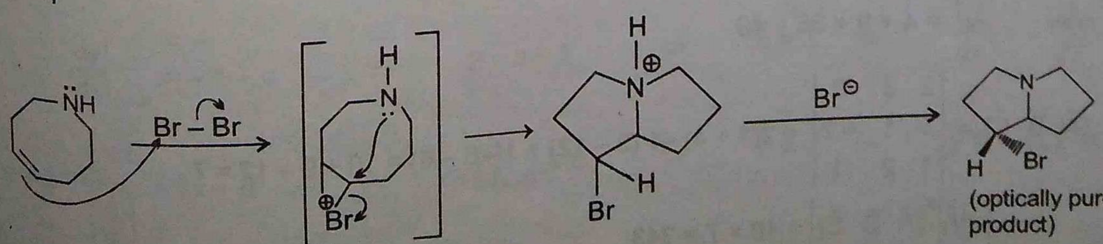
$$\therefore Ksp = 1\times10^{-6}\times10^{-1} = 1\times10^{-7} \quad \therefore Ans = 1-(-7) = 8$$

28. $$\Delta T_B = k_b \cdot m$$

$$0.1 = 0.5\times\frac{2.4}{m\times100}\times1000$$

$m = 120\ g = 12n + 2n + 16n = 30n$

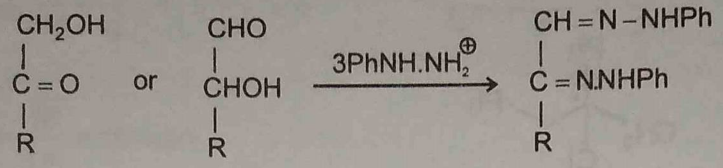
$n = 4$

29. 

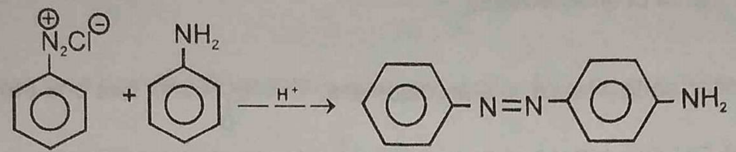
+ HBr

(optically pure product)

30. Osazone formation involves following reaction :



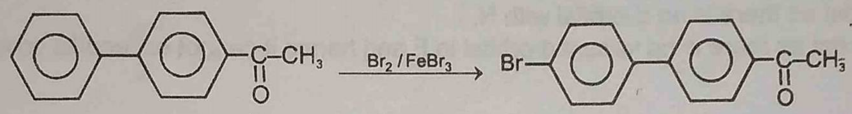
31.



Product shows geometrical isomerism
Product shows colour due to extended conjugation

32. Peptide linkage (–CO–NH–) has all the above properties.

33.



35. Ores in which one of the component (either the impurities or the actual ore) is magnetic can be concentrate by magnetic separation method. All these are iron ores.

36. $[FeO_4]^{2-}$ are formed in alkaline medium but they readily decompose to Fe_2O_3 and O_2. Here oxidation state o iron is +6.

37. For second order reactions, order is 2.

40. (A) For reversible process $\Delta S_{universe} = 0$ and for adiabatic process $\Delta S_{system} = 0$
(B) For reversible process $\Delta S_{universe} = 0$, and in vaporisation entropy of system increases
(C) Number of gaseous moles decreases. So, entropy of system decreases and N_2 is more stable. S process is spontaneous. For spontaneous process $\Delta S_{universe} > 0$
(D) Number of gaseous moles increases. So, entropy of system increases.

PART-III (MATHEMATICS)

41. $9^x - 3^x + 1 = \left(3^x - \frac{1}{2}\right)^2 + \frac{3}{4} \ge \frac{3}{4}$

Now $\forall\, x \in (-\infty, 1)$

Range of $9^x - 3^x + 1$ is $\left[\frac{3}{4}, 7\right)$ $\Rightarrow [9^x - 3^x + 1] \in \{0,1,2,3,4,5,6\}$

42. $\vec{V} = \vec{A} \times \left((\vec{A}.\vec{B})\vec{A} - (\vec{A}.\vec{A})\vec{B}\right)\vec{C} = \left(\underbrace{\vec{A} \times (\vec{A}.\vec{B})\vec{A}}_{\text{zero}} - (\vec{A}.\vec{A})\vec{A} \times \vec{B}\right).\vec{C} = -|\vec{A}|^2 [\vec{A}\ \vec{B}\ \vec{C}]$

now $|\vec{A}|^2 = 4 + 9 + 36 = 49$

$$[\vec{A}\ \vec{B}\ \vec{C}] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix} = 2(1+4) - 1(3-12) + 1(-6-6) = 10 + 9 - 12 = 7$$

$\therefore \left| -|\vec{A}|^2 [\vec{A}\ \vec{B}\ \vec{C}] \right| = 49 \times 7 = 343$

43. $$\frac{{}^nC_4\left(2^{1/4}\right)^{n-4}\left(3^{-1/4}\right)^n}{{}^nC_4\left(3^{-1/4}\right)^{n-4}\left(2^{-1/4}\right)^n}=\frac{\sqrt6}{1} \quad ; \quad \frac{2^{\frac{n-4}{4}}\cdot 3^{-1}}{3^{-\frac{n-4}{4}}\cdot 2^{1}}=\frac{\sqrt2\sqrt3}{1}$$

$$6^{\frac{n-4}{4}-1-\frac12}=1 \quad ; \quad \frac{n-4}{4}-1-\frac12=0$$

$n=10$

44. $R_1\times a,\ R_2\times b,\ R_3\times c$

$$f(x)=\frac{1}{abc}\begin{vmatrix}a^3+ax & a^2b & a^2c\\ ab^2 & b^3+bx & b^2c\\ ac^2 & bc^2 & c^3+cx\end{vmatrix}=\begin{vmatrix}a^2+x & a^2 & a^2\\ b^2 & b^2+x & b^2\\ c^2 & c^2 & c^2+x\end{vmatrix}$$ (Taking a,b,c common from C_1, C_2, C_3)

$$R_1\to R_1+R_2+R_3=(a^2+b^2+c^2+x)\begin{vmatrix}1 & 1 & 1\\ b^2 & b^2+x & b^2\\ c^2 & c^2 & c^2+x\end{vmatrix}$$

$$=(a^2+b^2+c^2+x)\begin{vmatrix}1 & 0 & 0\\ b^2 & x & 0\\ c^2 & 0 & x\end{vmatrix}$$ $(C_2\to C_2-C_1,\ C_3\to C_3-C_1)$

$=x^2(a^2+b^2+c^2+x)$

$f(x)=x^3+(a^2+b^2+c^2)x^2$

$f'(x)=3x^2+2x(a^2+b^2+c^2)$

$=x(3x+2(a^2+b^2+c^2))\le 0$

$$\Rightarrow x\in\left[-\frac23(a^2+b^2+c^2),0\right]$$

45. Case-I $\dfrac{M_1M_2W_1}{M_2W_1}$ ${}^4C_1\cdot{}^4C_1=16$

Case-II ${}^4C_2\cdot{}^4C_1=24$

Case-III ${}^4C_3\cdot{}^5C_2=40$

total $80=16\lambda$

46. $$\frac{d^2y}{dx^2}=x^{-3/2}$$

$$\frac{dy}{dx}=\frac{x^{-1/2}}{-\frac12}+c$$

$$\because \left.\frac{dy}{dx}\right|_{x=4}=-1+c \Rightarrow 2=c-1 \Rightarrow c=3 \Rightarrow \frac{dy}{dx}=-\frac{2}{\sqrt x}+3$$

$$y=-2\cdot\frac{x^{1/2}}{\left(\frac12\right)}+3x+d \quad \because y|_{x=0}=d \Rightarrow 0=d \Rightarrow y=-4\sqrt x+3x$$

47. $$\ell = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}.\sqrt{1+\frac{1}{\sqrt{x}}}}} = 1$$

$$m = \lim_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x} = \lim_{x\to\infty} \frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x}.\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{x+\sqrt{x}\sqrt{\left(1+\frac{1}{\sqrt{x}}\right)}}+\sqrt{x}} = \lim_{x\to\infty} \frac{\sqrt{x}.\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{x}.\sqrt{1+\frac{1}{\sqrt{x}}\sqrt{\left(1+\frac{1}{\sqrt{x}}\right)}}+1} = \frac{1}{2} \quad \therefore \quad 2(\ell - m) = 1$$

48. $P(t_1^2,\ 2t_1)$ $\qquad$ $Q(t_2^2,\ 2t_2)$

slope of $PQ = \frac{2}{t_1+t_2} = 1 \Rightarrow t_1 + t_2 = 2$

Let the point of intersection of normal is (h,k)

$h = (t_1^2 + t_2^2 + t_1t_2 + 2),$ $\qquad$ $k = -t_1t_2(t_1 + t_2)$

$h = (t_1 + t_2)^2 - t_1t_2 + 2$ $\qquad$ $k = -2t_1t_2$

$h = 4 - t_1t_2 + 2$ $\qquad$ $t_1t_2 = -\frac{k}{2}$

$h = 6 - t_1t_2$

$\therefore\ h = 6 + \frac{k}{2} \quad \Rightarrow 2h - k = 12$

$\therefore\ 2x - y = 12 \quad \therefore\ \alpha = 2,\ \ell = 12$ $\quad ; \quad$ $\frac{\alpha+\ell}{2} = 7$

49. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$x = \tan\theta$ $\qquad$ $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$y = \sin^{-1}(\cos 2\theta)$ $\qquad$ $-\pi < 2\theta < \pi$

$y = \sin^{-1}\sin\left(\frac{\pi}{2} - 2\theta\right)$ $\qquad$ $-\frac{\pi}{2} < \frac{\pi}{2} - 2\theta < \frac{3\pi}{2}$

$-\frac{\pi}{2} < \frac{\pi}{2} - 2\theta \le \frac{\pi}{2} \Rightarrow -\pi < -2\theta \le 0 \Rightarrow 0 \le \theta < \frac{\pi}{2}$ $\quad ; \quad$ $y = \frac{\pi}{2} - 2\theta$

$y = \frac{\pi}{2} - 2\tan^{-1}x \qquad \Rightarrow x \ge 0$

$\frac{\pi}{2} < \frac{\pi}{2} - 2\theta < \frac{3\pi}{2} \Rightarrow x < 0$ $\quad ; \quad$ $y = \pi - \left(\frac{\pi}{2} - 2\theta\right) = \frac{\pi}{2} + 2\tan^{-1}x \qquad \Rightarrow x < 0$

$y = \frac{\pi}{2} + 2\tan^{-1}x,\quad x < 0$ $\quad ; \quad$ $y' = \left[\begin{array}{ll} \frac{-2}{1+x^2} & x > 0 \\ \frac{2}{1+x^2} & x < 0 \end{array}\right.$ $\quad ; \quad$ $\left.\frac{dy}{dx}\right|_{=1} = -1$

50. $5\cos2\theta + 1 + \cos\theta + 1 = 0$
$5(2\cos^2\theta - 1) + \cos\theta + 2 = 0$
$10\cos^2\theta + \cos\theta - 3 = 0$

$\cos\theta = \dfrac{-1\pm 11}{20} = \dfrac{1}{2}, -\dfrac{3}{5}$ $\quad\therefore\quad \theta = \dfrac{\pi}{3},\ \pi - \cos^{-1}\dfrac{3}{5}$

51. Option (A) If $\tan^{-1}x \ge 0 \quad \Rightarrow x \ge 0$
$\tan|\tan^{-1}x| = \tan(\tan^{-1}x) = x$
If $\tan^{-1}x \le 0 \quad \Rightarrow x \le 0$
$\tan|\tan^{-1}x| = \tan(-\tan^{-1}x) = -\tan(\tan^{-1}x) = -x$
$\therefore\ \tan|\tan^{-1}x| = |x|$
Option (B)
$y = \cot|\cot^{-1}x| = x \qquad \cot^{-1}x > 0$
Option (C)
$\tan^{-1}|\tan x| \ne |x|$
Option (D)
$\sin|\sin^{-1}x| = |x|$
$\therefore$ A,B,C are False

53. $xy = 1 \Rightarrow y = \dfrac{1}{x} \quad \Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{x^2}$

$\therefore$ slope of normal $= x^2 = -\dfrac{a}{b} > 0$

$\Rightarrow \dfrac{a}{b} < 0 \quad \Rightarrow$ either $a < 0, b > 0$ or $a > 0, b < 0$.

54. $ax^2 - bx + \dfrac{1}{2a} = a\left(x^2 - \dfrac{b}{a}x + \dfrac{b^2}{4a^2}\right) + \dfrac{1}{2a} - \dfrac{b^2}{4a} = a\left(x - \dfrac{b}{2a}\right)^2 + \dfrac{2-b^2}{4a}$

$\therefore \quad k = \dfrac{b}{2a}$ and $y_0 = \dfrac{2-b^2}{4a}$

thus $\dfrac{b}{2a} = 2.\dfrac{2-b^2}{4a}$ $\quad$ i.e $\quad b^2 + b - 2 = 0$ $\quad$ i.e $\quad b = -2, 1$

55. lines $L_1 = \dfrac{x+3}{-4} = \dfrac{y-6}{3} = \dfrac{z}{2}$ and $L_2 = \dfrac{x-2}{-4} = \dfrac{y+1}{1} = \dfrac{z-6}{1}$
Let shortest distance = d

$$d = \left|\frac{(\vec{a}_1 - \vec{a}_2).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$\vec{a}_1 = -3\hat{i} + 6\hat{j}$; $\quad \vec{a}_2 = 2\hat{i} - \hat{j} + 6\hat{k}$

$\vec{b}_1 = -4\hat{i} + 3\hat{j} + 2\hat{k}$; $\quad \vec{b}_2 = -4\hat{i} + \hat{j} + \hat{k}$

$$d = \frac{|(-5\hat{i} + 7\hat{j} - 6\hat{k}).(\hat{i} - 4\hat{j} + 8\hat{k})|}{|\hat{i} - 4\hat{j} + 8\hat{k}|} = 9$$

56. $3A \cdot 3A^T = \begin{bmatrix} -1 & 2 & x \\ -2 & 1 & -2 \\ -2 & -2 & y \end{bmatrix} \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ x & -2 & y \end{bmatrix}$

$$= \begin{bmatrix} 1+4+x^2 & 2+2-2x & 2-4+xy \\ 2+2-2x & 4+1+4 & 4-2-2y \\ 2-4+xy & 4-2-2y & 4+4+y^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\therefore \quad 5 + x^2 = 9, \quad 4 - 2x = 0, \quad 4 - 2 - 2y = 0 \quad xy = 2$

i.e. $\quad x = 2, y = 1$

57. The given system of linear equations will have a non-trivial solution if

$$\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

Expanding the determinant along C_1 we get

$\lambda(-\cos^2\alpha - \sin^2\alpha) - (-\sin\alpha\cos\alpha - \sin\alpha\cos\alpha) - (\sin^2\alpha - \cos^2\alpha) = 0$

$\Rightarrow \quad -\lambda + \sin 2\alpha + \cos 2\alpha = 0$

$\Rightarrow \quad \lambda = \sin 2\alpha + \cos 2\alpha = \sqrt{2}\sin(\pi/4 + 2\alpha)$

$\Rightarrow \quad -\sqrt{2} \le \lambda \le \sqrt{2}$

58. $f(x) = \begin{cases} \dfrac{2}{1+x}, & -1 < x < 0 \\ 1, & x = 0 \\ \dfrac{1}{1+x}, & 0 < x < 1 \\ 0, & 1 \le x < 2 \end{cases}$

$\lim\limits_{x\to 0^-} f(x) = 2$; $\lim\limits_{x\to 0^+} f(x) = 1$; $\lim\limits_{x\to 1^-} f(x) = \dfrac{1}{2}$; $\lim\limits_{x\to 1^+} f(x) = 0$

59. (A) equation of normal at (t^2, t) is $y - t = -2t(x - t^2)$

it passes through (c, 0) then $t^2 = c - \dfrac{1}{2} > 0$

$c > \dfrac{1}{2}$

(B) $\quad z^3 + az^2 + bz + c = 0$

$|z| = 1$

$|z_1 z_2 z_3| = |-c|$

$\Rightarrow \quad |z_1|\,|z_2|\,|z_3| = |c|$

$\Rightarrow \quad |c| = 1$

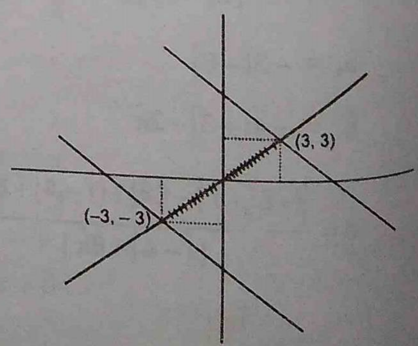
(C) $\quad$ Line $y = x$ cuts the line $|x + y| = 6$ at $(-3, -3)$ and $(3, 3)$

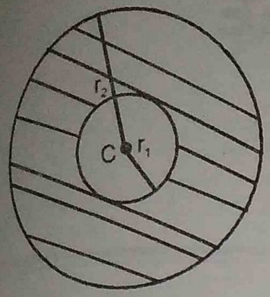
then $-3 < a < 3$

$0 \le |a| < 3$

$[|a|] = 0, 1, 2$

(D)





C(1, 0)

$r_1 = \sqrt{8}$ $r_2 = 4$

point ([a + 1], [a]) lies inside line circle $x^2 + y^2 - 2x - 15 = 0$ and out side the circle $x^2 + y^2 - 2x - 7 = 0$

$4 < [a]^2 < 8$ using $[a + 1] = [a] + 1$

Number of values of a = 0

60. (A) $f(x) = \sin\left(2\pi x + \left\{\frac{x}{2}\right\}\right) = \sin 2\pi x \cos\left\{\frac{x}{2}\right\} + \cos 2\pi x \sin\left\{\frac{x}{2}\right\}$

L.C.M. of fundamental periods of the functions involved is 2

$$f(x + 1) = \sin\left(2\pi(x+1) + \left\{\frac{x+1}{2}\right\}\right)$$

$$= \sin\left(2\pi x + \left\{\frac{x+1}{2}\right\}\right) \neq f(x)$$

(B) $\vec{u} \cdot \vec{n} = 0,\ \vec{v} \cdot \vec{n} = 0 \Rightarrow \vec{n} = \lambda\,(\vec{u} \times \vec{v})$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = i\,(0) - j(0) + k\,(-2) = -2k$$

$\vec{n} = \hat{k}$ since n is unit vector

$|\vec{w} \cdot \vec{n}| = |(\hat{i} + 2\hat{j} + 3\hat{k}) \cdot \hat{k}| = 3$

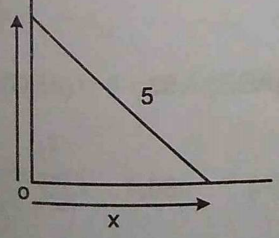
(C) Since f(x) is bijective,

$\therefore$ f(0) = 0 or 2 but f(0) = 0 $\Rightarrow$ c = 0 (which is not true)

$\therefore$ f(0) = 2

$\therefore$ f(2) = 0

(D)



$y = \sqrt{25 - x^2}$

$$\frac{dy}{dt} = \frac{-x}{\sqrt{25 - x^2}}\frac{dx}{dt} = \frac{-4}{\sqrt{25 - 16}} \cdot 2 = -\frac{8}{3}$$

$\therefore$ the ladder slides down at the rate of $\frac{8}{3}$ cm/s $\therefore$ $\lambda = 3$

PRACTICE TEST - THREE

ANSWER KEY

PAPER - 2

PART-I (PHYSICS)

1.	(8)	2.	(2)	3.	(6)	4.	(4)	5.	(5)	6.	(6)	7.	(2)
8.	(6)	9.	(ABD)	10.	(ACD)	11.	(AD)	12.	(BC)	13.	(ACD)	14.	(ACD)
15.	(ABC)	16.	(BD)	17.	(C)	18.	(D)	19.	(A)	20.	(B)		

PART-II (CHEMISTRY)

21.	(6)	22.	(5)	23.	(0)	24.	(5)	25.	(1)	26.	(7)	27.	(6)
28.	(1)	29.	(AB)	30.	(ABD)	31.	(BD)	32.	(ABC)	33.	(ABC)	34.	(BCD)
35.	(AC)	36.	(ABCD)	37.	(C)	38.	(B)	39.	(C)	40.	(B)		

PART-III (MATHEMATICS)

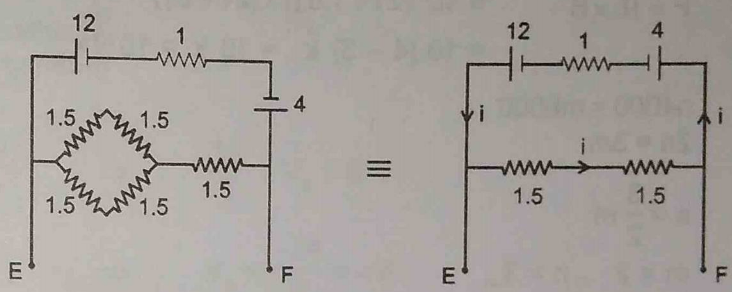
41.	(1)	42.	(1)	43.	(3)	44.	(8)	45.	(8)	46.	(0)	47.	(4)
48.	(2)	49.	(AC)	50.	(ABC)	51.	(ABD)	52.	(ABD)	53.	(ABC)	54.	(BC)
55.	(ABCD)	56.	(BD)	57.	(B)	58.	(A)	59.	(A)	60.	(C)		

PRACTICE TEST - THREE
HINTS & SOLUTIONS
PAPER - 2

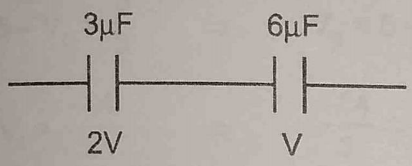
PART-I (PHYSICS)

1. $W = QE\dfrac{2R}{\pi} = 8J.$

2. $i = \dfrac{12-4}{1+1.5+1.5} = \dfrac{8}{4} = 2A$



$V_{EF} = 3 \times 2 = 6V$



$2V + V = 6$

$V = 2$

3. $W_{earth} = mg_e h \qquad = m\dfrac{4}{3}Gd\pi R.h$

$W_{planet} = m.\dfrac{4}{3}Gd\pi\dfrac{R}{4}.2h \quad \Rightarrow \quad \dfrac{W_{earth}}{W_{planet}} = 2$

$W_{planet} = \dfrac{W}{2} = \dfrac{3W}{6}.$

4. $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) \Rightarrow -\dfrac{df}{f^2} = \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) dn$ (differentiating both sides)

$\Rightarrow -\dfrac{df}{f}(n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) dn \Rightarrow -df = f\dfrac{dn}{n-1}$

but $dn = n_V - n_R \qquad \therefore\ -df = \dfrac{n_V - n_R}{n-1} f \Rightarrow df = -wf \Rightarrow f_v - f_R = -wf$

$= -0.4 \times 10 = 4$ cm **Ans.: $f_R - f_v = 4$ mm**

5. $g_h = g.\dfrac{R^2}{(R+d)^2} \quad h = d \quad \Rightarrow g_d = g_0\left(1 - \dfrac{d}{R}\right) \quad \Rightarrow \dfrac{R^2}{(R+d)^2} = 1 - \dfrac{d}{R}$

$R^2.R = (R+d)^2(R-d) \quad \Rightarrow R^3 = (R^2 + d^2 + 2dR)(R-d)$

$R^3 = R^3 + Rd^2 + 2dR^2 - dR^2 - d^3 - 2d^2R$

$0 = -d^2R + dR^2 - d^3$

$d^2 + d.R - R^2 = 0$

$d = \left(\dfrac{\sqrt{5}-1}{2}\right)R$

6. $\Delta t = \dfrac{10\text{km}}{2\times10^8} - \dfrac{10\text{km}}{2.1\times10^8}$

$= \dfrac{10\times10^3}{10^8}\left[\dfrac{2.1-2}{4.2}\right] = \dfrac{1}{10^4}\times\dfrac{1}{(42)}$

$f = 42 \times 10^4 = 420 \text{ Khz} = 60 \times \text{Khz} \quad \Rightarrow \quad X = 42$

7. Since the wire is kept in the uniform field it can be replaced by the straight wire connecting 'O' and 'P'.

Its $\quad \vec{L} = (2\hat{i}+1.5\hat{j}), \quad \vec{B} = (2\hat{i}+2\hat{j})$

$\vec{L} = (2\hat{i}+1.5\hat{j}), \quad \vec{B} = (2\hat{i}+2\hat{j})$

$\vec{F} = I\vec{L}\times\vec{B} \quad = 10\ (2\hat{i}+1.5\hat{j})\times(2\hat{i}+2\hat{j})$

$= 10\ [4-3]\ \hat{k} \ = 10\ \hat{k} \ = 10\text{N}\hat{k}$

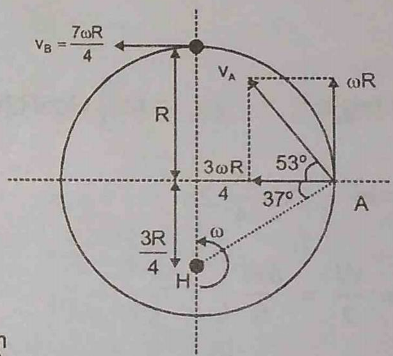
8. $n4000 = m6000$

$2n = 3m$

$n = \dfrac{3}{2}m$

$m = 2 \quad , n = 3$

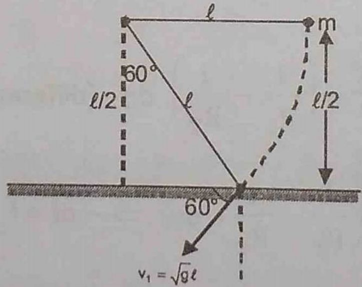
$y = \dfrac{2.6000\times2\times10^{-10}}{1\times10^{-3}} = 2.4 \text{ mm}$



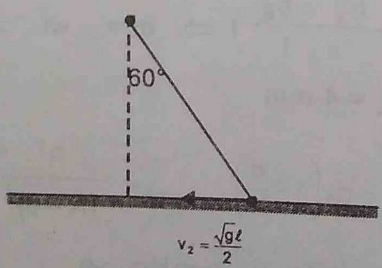
9. $v_A = \dfrac{\omega R}{4}\sqrt{9+16} = \dfrac{5\omega R}{4} = \omega r \Rightarrow r = \dfrac{5R}{4}$

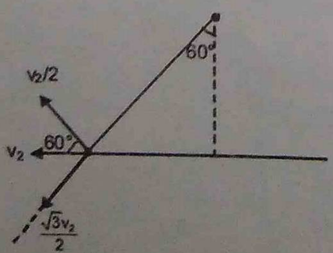
$\vec{v}_B = -\dfrac{7\omega R}{4}\hat{i}$

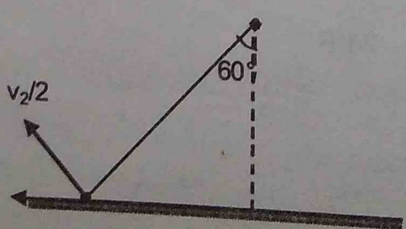
11. Just before string gets taught



Just after string gets taught

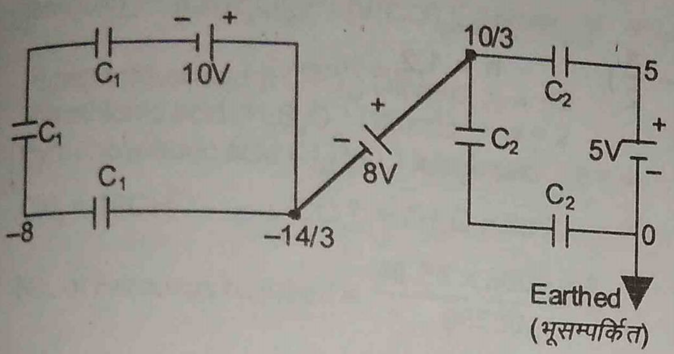






Maximum height achieved during sub sequence motion :

$$h_{max} = \frac{\left(\frac{v_2}{2}\right)^2}{2g} = \frac{v_2^2}{8g} = \frac{1}{8g}\frac{g\ell}{4} = \frac{\ell}{32} \qquad \Rightarrow \Delta E = \frac{mg\ell}{2} - \frac{mg\ell}{32} = \frac{15mg\ell}{32}$$

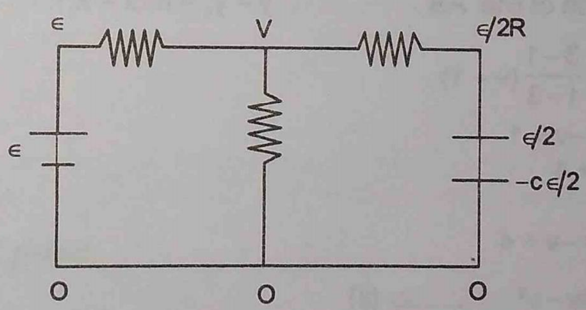
12. 

$$5 - V_C = \frac{5}{3} \quad \Rightarrow \quad V_C = 5 - \frac{5}{3} = \frac{10}{3} \quad \Rightarrow \quad \frac{10}{3} - V_B = 8$$

$$V_B = \frac{-14}{3} \quad \Rightarrow \quad \frac{-14}{3} - V_A = \frac{10}{3} \quad \Rightarrow \quad V_A = \frac{-24}{3} = -8$$

13. If we displace the electron slightly toward x direction, it will thrown away toward right. So eql. is unstable along x direction.
If we displace the electron slightly towards y direction, No extra force will act. So eql. is neutral along y axis
If we displace the electron toward z direction, it will be attracted and try to come to eql. positron. So eql. is stable along z direction.

14. $$\frac{V}{R} + \frac{V-\varepsilon}{R} + \frac{V - \frac{\varepsilon}{2k}}{2R} = 0$$

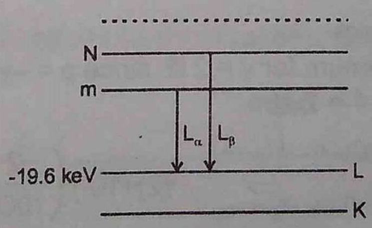


$$2v + 2v - 2\varepsilon + V - \frac{\varepsilon}{2k} = 0$$

$$5V = 2\varepsilon + \frac{\varepsilon}{2k}$$

$$\frac{2\varepsilon + \frac{\varepsilon}{2k}}{5} = \frac{9\varepsilon}{20}$$

15. $$\lambda_{min.} = \frac{12400}{V_0} Å$$



$$\frac{12400}{20{,}000} = .62 \text{ Å}$$

16. Field due to current carrying conductor is perpendicular to the plane directed inward . So by fleming rigtht hand rule there will be an urge of flow of current through xy. but NO current flow due toincomplete circuit with the rails. current flow tendency y to x

∴ Electron density will be more at and y compare to x

18. $y_{net} = 4\text{ mm }[\sin(4\pi(\sec^{-1})t + \frac{\pi}{6}]\cos\{2\pi(m^{-1})x + \frac{\pi}{6}\}$

$= (4mm)\cos(2\pi x + \frac{\pi}{6})\sin\{4\pi t + \frac{\pi}{6}\}$

position of node

$= 2\pi x + \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots\ldots\ldots (2n - \frac{\pi}{2})$ $\quad n = 1,2,\ldots\ldots\ldots$

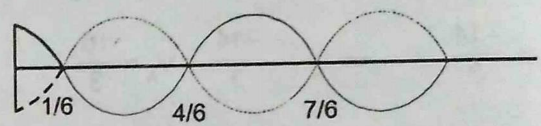
$x = \frac{\frac{\pi}{3}}{2\pi}, \frac{\frac{4\pi}{3}}{2\pi}, \ldots\ldots\ldots\ldots \frac{(2n-1)\pi/2 - \pi/6}{2\pi}$

$x = \frac{1}{6}, \frac{4}{6}, \frac{7}{6}, \ldots\ldots\ldots\ldots \frac{(3n-2)}{6}n$

position of antinode

$2\pi x + \frac{\pi}{6} = 2n\frac{\pi}{2}$

$\Rightarrow x = (6n-1)\frac{1}{12} = \frac{5}{12}, \frac{11}{12}, \frac{17}{12} \ldots\ldots\ldots$



20. $pv = nRT$

$(3 \times 10^5)\left(\frac{1}{1000}\right) = n\frac{25}{3}(300) \Rightarrow (300)\left(\frac{3}{25}\right) = 300\,n$

$\Rightarrow \quad n = \frac{3}{25}$ mole $\quad \ldots\ldots\ldots\text{(i)}$

Equation of line AB $\quad y - y_1 = m(x - x_1)$

$p - 3 = \frac{3-1}{1-3}(v-1)$

$p - 3 = -v + 1$

$p = -v + 4$

$\frac{nRT}{v} = -v + 4$

$nRT = 4v - v^2 \quad \ldots\ldots\ldots\text{(ii)}$

Hence $\frac{5}{2}nRT = \left(\frac{5}{2}\right)(4v - v^2)$

$U = \left(\frac{5}{2}\right)(4 - v)\,v$

Hence (A) is correct

(ii) T is maximum for v = 2 lit. since $p = -v + 4$

$\therefore \quad p = -2 + 4 = 2$ atm.

$pv = nRT \quad \Rightarrow \quad (2)(10^5)\left(\frac{2}{1000}\right) = 1(T_{max})$

$T_{max} = 400$ K **Ans. (B)**

Hence, $U_{max} = nC_v T_{max}$

$= n\frac{5R}{2}T_{max} = (nR)\left(\frac{5}{2}\right)(400) = 1000$ J. **Ans.**

PART-II (CHEMISTRY)

21. $C.F.S.E = [-0.4\, n_{t_{2g}} + 0.6 n_{e_g}]\, \Delta_0 + np$

$[Cr(H_2O)_6]Cl_2$ is $t_{2g}^{1,1,1} eg^{1,0} = -0.6\Delta_0 = -\frac{6}{10}\Delta_0$

22. $[Mn(CN)_6]^{3-}$, $[Cr(H_2O)_6]^{3+}$, $[V(CO)_6]$, $[Ni(NH_3)_6]^{2+}$, $[Cu(NH_3)_4]^{2+}$ are paramagnetic.

23. Phosphorous acid (H_3PO_3) Dibasic $\therefore x = 2$
Tetrathionic acid ($H_2S_4O_6$) Dibasic $\therefore y = 2$
Pyrophosphoric acid ($H_4P_2O_7$) tetrabasic $\therefore z = 4$

24. $CH_4 + 10OH^- \longrightarrow CO_3^{2-} + 7H_2O + 8e^-$

No. of Faradays required = $\frac{48.25 \times 3600 \times 1}{96500}$

Hence mol. of CH_4 required = $\frac{1}{8} \times \frac{48.25 \times 3600 \times 1}{96500}$. Volume of CH_4 = 5.04 L.

25. Mole atoms of X = 0.75 × 4 = 3
Mole atoms of O = 2 × 2 = 4
Hence the product is X_3O_4 (g)
Initial moles of gaseous reactants, $n_1 = 2$ (oxygen only)
Final moles of gaseous product, $n_2 = 1$ (X_3O_4)

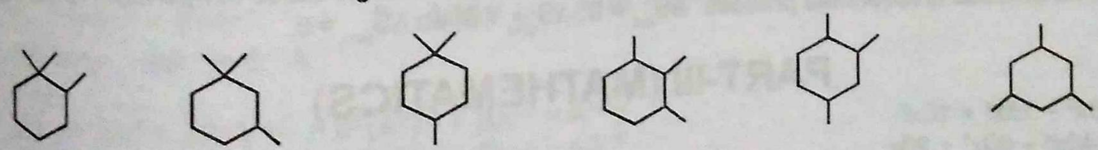
Hence, $\frac{P_2}{P_1} = \frac{n_2T_2}{n_1T_1} = \frac{1 \times 600}{2 \times 300} = 1$　　or　　$P_2 : P_1 : 1 : 1$

26. $KMnO_4 + H^+ + Fe_{0.9}O \longrightarrow Mn^{2+} + Fe^{3+}$

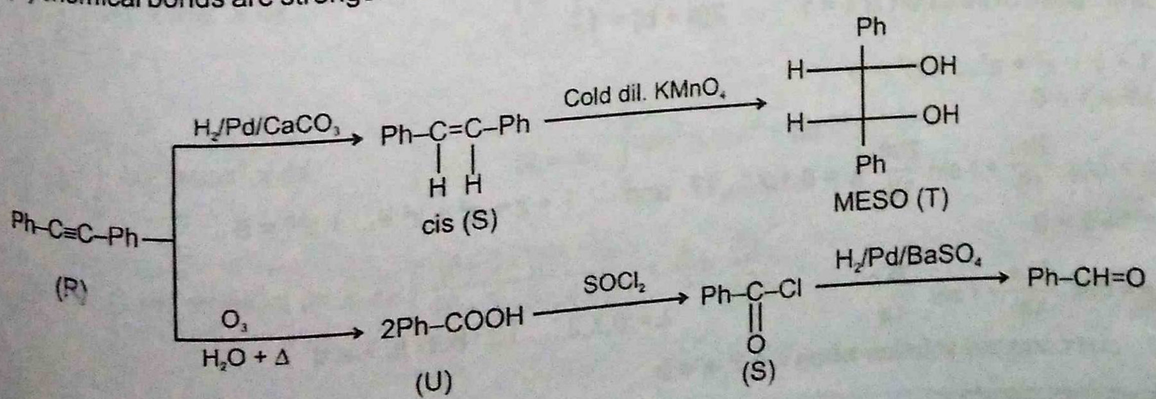
vf = 5　　$vf = 0.9\left(3 - \frac{20}{9}\right)$
$= 0.7$

$\therefore \frac{n_{KMnO_4}}{n_{Fe_{0.9}O}} = \frac{0.7}{5}$

$\therefore n_{KMnO_4}$ required = $\frac{0.7}{5} \times 50 = 7$ moles

27. 

30. (A) ΔH = –ve for adsorption
(B) fact
(D) chemical bonds are stronger than vander waal's forces so chemical adsorption is more exothermic.

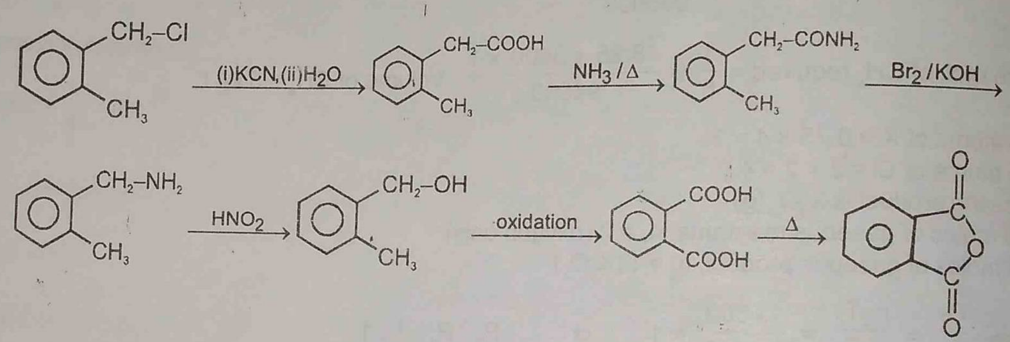
32. 

34. $CH_3-CH_2-CH_2-MgBr \xrightarrow[(ii)\ H_2O]{(i)\ CO_2} CH_3-CH_2-CH_3-COOH$

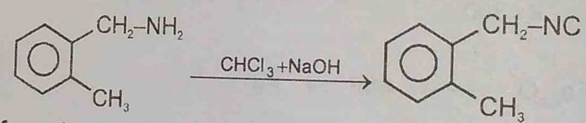
35. π-bond are formed by parallel overlaping of same orbitals. So, correct options are :
(A) d_{xy} and p_y along x—axis (C) d_{xy} and p_x along y—axis

36. (A) $2Cu^{2+} + 5I^- \longrightarrow 2CuI\downarrow \text{(white)} + I_3^-$; white precipitate in intensely brown solution.
$Pb^{2+} + 2I^- \longrightarrow PbI_2\downarrow \text{(yellow)}$
(B) $BaCO_3 + 2HCl \longrightarrow BaCl_2 + CO_2\uparrow + H_2O$
$BaSO_4 + HCl \longrightarrow$ No reaction
(C) $SO_3^{2-} + Pb^{2+} \longrightarrow PbSO_3\downarrow \text{(white)}$
$S^{2-} + Pb^{2+} \longrightarrow PbS\downarrow \text{(black)}$
(D) $Bi^{3+} + NO_3^- + H_2O \longrightarrow BiO(NO_3)\downarrow \text{(white)} + 2H^+$
$Ag^+ + NO_3^- + H_2O \longrightarrow$ No precipitation

37.



38. Carbyl amine reaction



39. As can be understood from the question. The area enclosed in the cyclic process in S Vs T diagram is heat In ABCDEFGA, one cycle is clockwise while the other is anticlockwise. But both have equal area.
So heat = 0. $\because$ $\Delta U = 0$ $\Rightarrow$ $W = 0$.

40. (A) BC and DA are adiabatic process. So ΔS_{sys} as well as $\Delta S_{surr} = 0$
(B)During AB process, the entropy of system is increasing at constant temperature. This is an isothermal expansion.
(C) The process BC is adiabatic ($\because \Delta S = 0$) and it is compression because temperature is increasing.
(D) The process is reversible process, $\Delta S_{sys} = 0$, $\Delta S_{univ} = 0$, so $\Delta S_{surr} = 0$

PART-III (MATHEMATICS)

41. $y = 8x^5 - 15x^4 + 10x^2$
$y' = 40x^4 - 60x^3 + 20x$
$= 20x(2x^3 - 3x^2 + 1)$
$= 20x(x-1)^2(2x+1)$
Local maxima occurs at $x = -1/2$
and point of inflexion is $x = 1$ $\therefore$ $2(a + b) = 1$

42. $1 + z + z^2 + z^3 .... + z^{17} = 0$
$z^{18} - 1 = 0$

$z = \cos\frac{2k\pi}{18} + i\sin\frac{2k\pi}{18}$ $k = 0,1,2,...,17$ and $1 + z + z^2 + z^3 + ... + z^{13} = 0$

$z^{14} - 1 = 0$

$z = \cos\frac{2\lambda\pi}{14} + i\sin\frac{2\lambda\pi}{14}$ $\lambda = 0,1,2,....,13$ but $k, \lambda \neq 0$

$\therefore$ one common solution when $\lambda = 7$, $k = 9$

43. $I = (I - 3A)(I - \alpha A)$

$\Rightarrow \quad I = I - \alpha A - 3A + 3\alpha A^2$

$\Rightarrow \quad I = I + (2\alpha - 3)A$

$\Rightarrow \quad 2\alpha = 3$

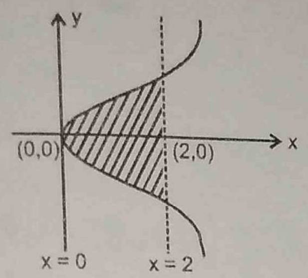
44. $\log_2(a+b)(c+d) \geq 4 \Rightarrow (a+b)(c+d) \geq 16$

AM $\geq$ GM

$$\frac{a+b+c+d}{2} \geq \sqrt{(a+b)(c+d)} \geq 4$$

$\Rightarrow a+b+c+d \geq 8$

45. $A = 2\int_0^2 x^3 dx = 8$



46. $AB = A + B \Rightarrow B = AB - A = A(B - I) \Rightarrow \det(B) = \det(A)\det(B - I) = 0 \Rightarrow \det(B) = 0$

47. $\sqrt{x+iy} = \frac{a+ib}{c+id} \Rightarrow (x + iy) = \left(\frac{a+ib}{c+id}\right)^2 \Rightarrow |x + iy| = \left|\frac{a+ib}{c+id}\right|^2$

$$\Rightarrow \sqrt{x^2+y^2} = \frac{a^2+b^2}{c^2+d^2} = 2 \Rightarrow x^2 + y^2 = 4$$

48. $p = \sin(px - y) \Rightarrow px - y = \sin^{-1}p$ ....(i)

differentiate w.r.t. x

$\Rightarrow x\frac{dp}{dx} = \frac{1}{\sqrt{1-p^2}}\frac{dp}{dx} \qquad \Rightarrow \frac{dp}{dx} = 0$ or $\frac{1}{\sqrt{1-p^2}} = x \Rightarrow p = \frac{\sqrt{x^2-1}}{x}$ put in equation

$y = \sqrt{x^2-1} - \sin^{-1}\frac{\sqrt{x^2-1}}{x}$. $\quad a = 1, b = 1, c = 1$

49. $f(1) = 0, f(1^-) = 0, f(1^+) = 1 - 1 = 0$

$f(-1) = 0, f(-1^-) = 4 - 1 = 3, f(-1^+) = 1$

Thus f(x) is continuous at $x = 1$ but not at $x = -1$

50. Given $B^T = B$ and $A^T = A$ and $AB = BA$

$\Rightarrow ABA^{-1} = BAA^{-1} \quad \Rightarrow \quad B = ABA^{-1} \Rightarrow \quad A^{-1}B = A^{-1}ABA^{-1} \Rightarrow \quad A^{-1}B = BA^{-1}$

Similarly $AB^{-1} = B^{-1}A$

Now

(A) $(A^{-1}B)^T = B^T(A^{-1})^T = B^T(A^T)^{-1} = BA^{-1} = A^{-1}B$

(B) $(AB^{-1})^T = (B^{-1})^T A^T = (B^T)^{-1} A^T = B^{-1}A = AB^{-1}$

(C) $(A^{-1}B^{-1})^T = (B^{-1})^T (A^{-1})^T = (B^T)^{-1} (A^T)^{-1} = B^{-1}A^{-1} = A^{-1}B^{-1}$

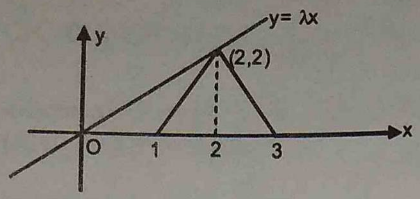
51. $I = \int_0^{2\pi} x\sin^6 x\cos^4 x\,dx \quad ; \quad I = \int_0^{2\pi} (2\pi - x)\sin^6 x\cos^4 x\,dx$

Add

$2I = 2\pi\int_0^{2\pi} \sin^6 x\cos^4 x\,dx \quad ; \quad 2I = 4\pi\int_0^{\pi} \sin^6 x\cos^4 x\,dx$

$$I = 4\pi\int_0^{\pi/2} \sin^6 x\cos^4 x\,dx = 4\pi\left(\frac{(5\times3\times1)(3\times1)}{10\times8\times6\times4\times2}\right)\times\frac{\pi}{2} = \frac{3\pi^2}{128}$$

52.



53. $-\dfrac{b}{2a} = \dfrac{3}{2}$

$b = -3a$ ...(1)

$c = 1$

$\alpha + 2 = 3$

$\alpha = 1 \quad f(x) = \lambda(x-1)(x-2)$

$1 = \lambda(2)$

$\lambda = \dfrac{1}{2}$

$f(x) = \dfrac{1}{2}(x-1)(x-2)$

54. $\dfrac{2^{3x}+3^{3x}}{2^{2x}.3^{x}+3^{2x}.2^{x}} = \dfrac{7}{6} \Rightarrow \dfrac{a^3+b^3}{a^2b+ab^2} = \dfrac{7}{6}$

$\dfrac{a^2+b^2-ab}{ab} = \dfrac{7}{6}$

$\dfrac{a}{b} + \dfrac{b}{a} = \dfrac{13}{6}$

$t + \dfrac{1}{t} = \dfrac{13}{6}$

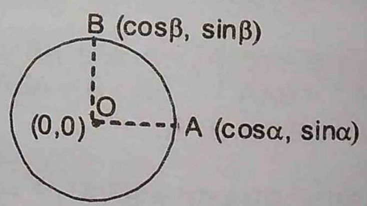
$6t^2 - 13t + 6 = 0$

$6t^2 - 9t - 4t + 6 = 0$

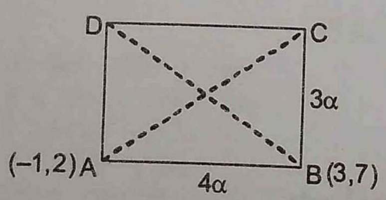
$3t(2t-3) - 2(2t-3) = 0$

$t = \dfrac{3}{2}, \dfrac{2}{3} \Rightarrow \left(\dfrac{2}{3}\right)^x = \left(\dfrac{2}{3}\right) \text{ or } \left(\dfrac{2}{3}\right)^{-1} \Rightarrow x = -1, 1$

55. For AOB to be a right angled triangle $(\alpha - \beta) = \pm\dfrac{\pi}{2}$

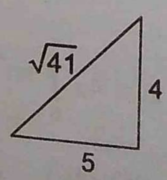


56.



$\Rightarrow 4\alpha = \sqrt{41}$ $\quad\Rightarrow 3\alpha = \dfrac{3\sqrt{41}}{4}$

$m_{AB} = \dfrac{5}{4} \Rightarrow m_{BC} = -\dfrac{4}{5}$



$\Rightarrow \sin\theta = \dfrac{4}{\sqrt{41}}, \cos\theta = -\dfrac{5}{\sqrt{41}}$

$$C \equiv \left[3 \pm \frac{3\sqrt{41}}{4}\left(-\frac{5}{\sqrt{41}}\right), 7 \pm \frac{3\sqrt{41}}{4}\left(\frac{4}{\sqrt{41}}\right)\right] \equiv \left[-\frac{3}{4}, 10\right] \text{ or } \left[\frac{27}{4}, 4\right]$$

mid point of diagonal can be = $\left(\frac{23}{8}, 3\right)$ or $\left(\frac{-7}{8}, 6\right)$

$$[d] = \left[\frac{\sqrt{1105}}{8}\right] = 4 \text{ or } [d] = \left[\sqrt{\frac{2353}{64}}\right] = \left[\sqrt{36.76}\right] = 6$$

Sol. 57 to 58.

$y = mx - 2m - m^3$

$12 = 15m - 2m - m^3$

$m^3 - 13m + 12 = 0$

$(m-1)(m-3)(m+4) = 0$

$m = 1, 3, -4$

$\therefore$ $P(1, -2)$, $Q(9, -6)$, $R(16, 8)$

Equation of circle circumscribing the triangle PQR will be $x^2 + y^2 - 17x - 6y = 0$

Centroid of ΔPQR is $\left(\frac{26}{3}, 0\right)$

Sol. 59 to 60.

Let $E_1 \rightarrow$ die A is used ; $E_2 \rightarrow$ die B is used

$C \rightarrow$ red face appears in any throw.

$$P(E_1) = \frac{1}{2} = P(E_2)$$

$$P(C/E_1) = \frac{^4C_1}{^6C_1} = \frac{2}{3}, \quad P(C/E_2) = \frac{^2C_1}{^6C_1} = \frac{1}{3}$$

$$P(C) = P(E_1)\,P(C/E_1) + P(E_2).\,P(C/E_2) = \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$$

Let D = Red face appears in III throw

E = Red face appears in first two throws

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(E/E_1) = \frac{2}{3} \cdot \frac{2}{3} = \left(\frac{2}{3}\right)^2, \quad P(D/EE_1) = \frac{2}{3}$$

$$P(E/E_2) = \frac{1}{3} \times \frac{1}{3} = \left(\frac{1}{3}\right)^2$$

$$P(D/EE_2) = \frac{1}{3}$$

$$\therefore \quad P(D/E) = \frac{P(E_1)\,P(E/E_1)P(D/EE_1) + P(E_2).P(E/E_2)P(D/EE_2)}{P(E_1).P(E/E_1) + P(E_2).P(E/E_2)}$$

$$= \frac{\frac{1}{2}\left(\frac{2}{3}\right)^2 \times \frac{2}{3} + \frac{1}{2}\left(\frac{1}{3}\right)^2 \times \frac{1}{3}}{\frac{1}{2} \times \left(\frac{2}{3}\right)^2 + \frac{1}{2} \times \left(\frac{1}{3}\right)^2} = \frac{3}{5}$$