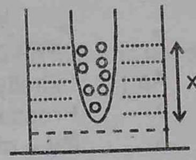
HINTS & SOLUTIONS
PRACTICE TEST - THREE

PART-A (PHYSICS)

1. At equilibrium position

$Ax\rho g = Mg \qquad M = Ax\rho$ ——(1)

If tube displaced by distance y vertically downward then net force on tube



$A(x+y)\rho g - Mg = Ma$

$a = \frac{A\rho g}{M} y = \frac{Mg}{xM} y \quad \Rightarrow \quad a = (g/x)y$

Time period $\quad T = 2\pi\sqrt{x/g}$

2. $e = \frac{BdA}{dt}$

$= \frac{Bd}{dt}(\pi r^2) \qquad = B2\pi r\frac{dr}{dt}$

3. Δp - Implse = Ft = 3mgt

5. here $y = bt - Ct^2$

$V = \frac{dy}{dt} = b - 2ct$

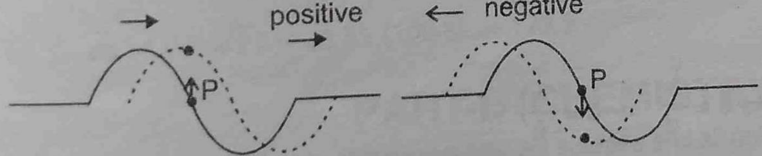
$a = \frac{dV}{dt} = -2C$ = acceleration due gravity.

6. The magnitude of displacement in the given time interval = $\frac{a}{2}$

Time taken by the particle to cover a distance $\frac{a}{2}$ starting from rest = $\frac{T}{6}$

Hence the magnitude of average velocity over given time interval is = $\frac{a/2}{T/6} = \frac{3a}{T}$

8.



9. By Lenz' law, direction of magnetic force will be down.

10. $MP = -\left(\frac{f_0}{f_e}\right)$ to increase the magnifying we will decrease f_e.

So, we will use eyepiece of high power.

11. $W = U_f - U_i = m\left[-\frac{GM}{R} + \frac{3}{2}\frac{GM}{R}\right] = \frac{1}{2} \quad \frac{GMm}{R}$

$\Rightarrow \quad \frac{GMm}{2R} = \frac{mgR^2}{2R} \Rightarrow \quad mgR/2 = gR$

13. $I \propto A^2 \qquad \therefore \frac{I_1}{I_2} = \frac{2^2}{3^2} = 4/9$

14. $q = \int_0^{t=1} I\,dt = 2$ coulombs

15. Work done by atmosphere = $P_{atm}\,\Delta V$

$= P_{atm}\dfrac{V}{2}$ ....(i)

As ; Initially gas in container is in thermodynamic equilibrium with its surroundings.

$\therefore$ Pressure inside cylinder = P_{atm}

& $PV = nRT$

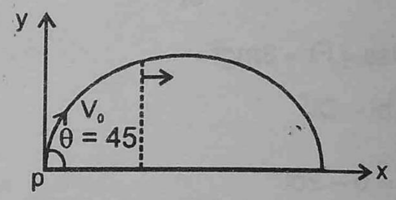
$\Rightarrow P_{atm}V = nRT$ or $V = \dfrac{nRT}{P_{atm}}$

Putting in (1), $W = \dfrac{nRT}{2}$

16. $x = v_0 \cos 45° \times t = \dfrac{v_0 t}{\sqrt{2}}$

$\tau = mgx = \dfrac{mgv_0 t}{\sqrt{2}} = \dfrac{dL}{dt}$

$\Rightarrow L = \dfrac{mgv_0}{\sqrt{2}}\displaystyle\int_0^{v_0/g} t\,dt = \dfrac{mv_0^3}{2\sqrt{2}\,g}$



17. After $t = \dfrac{T}{4}$, the particle will be at mean position

$v = v_{max} = \omega A$

18. $\Delta Q = mS\Delta T$

Since in boiling $\Delta T = 0$, $S = \infty$

20. $e = (\vec{v} \times \vec{B}).\ell$

$e = [\hat{i} \times (3\hat{i} + 4\hat{j} + 5\hat{k})].5\hat{j}$ $\Rightarrow e = 25$ volt

21. For small angles $\sin\theta \approx \theta$.

$d \sin\theta = d\theta = \lambda$ $\Rightarrow \theta = \dfrac{\lambda}{d}$ $\Rightarrow d = \dfrac{\lambda}{\theta}$

Since d is constant

$\dfrac{\lambda_1}{\theta_1} = \dfrac{\lambda_2}{\theta_2}$ $\therefore \lambda_2 =$ 720 nm

22. $C_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega_c t$

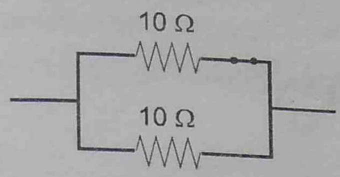
$= A_c \sin\omega_c t + \dfrac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$

$- \dfrac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$

23. Angular momentum = $\dfrac{nh}{2\pi}$

i.e. same for all

24. If $V_A > V_B$



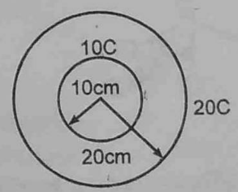
25 The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

26. First excitation energy $= RhC\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = RhC\,\frac{3}{4}$

$\therefore \frac{3}{4} RhC = V$ e.v. $\quad \therefore RhC = \frac{4V}{3}$ e.v.

27. Image velocity (w.r.t. mirror) $= -m \times$ object velocity (w.r.t. mirror)
Here $m = 1$.

28. potential difference due to inner 10C charge



$= K\,10\left(\frac{1}{\cdot 1}-\frac{1}{\cdot 2}\right) = 9 \times 10^{10}\ (5) = 45 \times 10^{10} = 4.5 \times 10^{11}V$

potential difference due to outer charge = 0
$\therefore$ p.d. $= 4.5 \times 10^{11}V$

29.
$$qV = \frac{1}{2}mv^2$$
$$qvB = \frac{mv^2}{r}$$
$$\frac{q}{m} = \frac{v^2}{2V}$$
$$r = \frac{mv}{qB} = \frac{v}{B} \times \frac{2V}{v^2} = \frac{2V}{Bv} = 12cm$$

30. $\phi = \vec{E}\cdot\vec{ds} = (i + \sqrt{2}\,j + \sqrt{3}\,k).\ (100\ k) = 173.2$

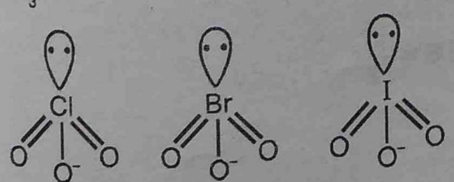
PART-B (CHEMISTRY)

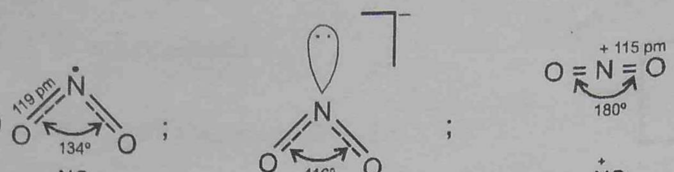
31. All the monosaccharide are reducing sugar but sucrose and all polysaccharide are non reducing sugar.

32. $XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

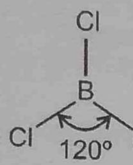
33. (1) The bond angle decreases with decrease in electronegativities and increases in size of central atom. Hence the correct order of O–X–O bond angle is $ClO_3^- > BrO_3^- > IO_3^-$.

	X–O (pm)	O–X–O (Angle)
ClO_3^-	149	107°
BrO_3^-	165	104°
IO_3^-	181	100°

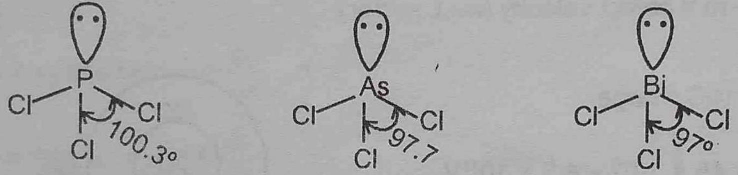


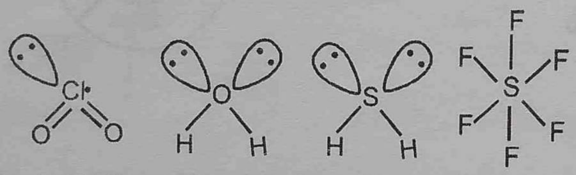
(2) 

NO_2 ; NO_2^- ; $\overset{+}{N}O_2$

(3)  (trigonal planar)

In chlorides of group 15th elements the central atoms have sp^3 hybridisation and thus have pyramidal shapes with one lone pair of electrons. The bond angle decreases down the group with decreasing electronegativity.



(4) 

(≈ 117°) (104.5°) (92°) (90°)

34. Due to less dipole moment and less strain

35. $P_2O_3 + 3H_2O + 3H_2O \rightarrow 2H_3PO_3$

110 gm 164 gm.

$\% \ P_2O_3$ in H_3PO_3 is $= \frac{110 \times 100}{164} = 67.07\%$

Hence (1)

36. In lanthanides (At no of elements 58 to 71) the electronic configuration of three outermost shells are $(n-2)f^{1-14}$, $(n-1)s^2p^6d^{0\text{ to }1}ns^2$.

37. $$\frac{E_a}{T_1} = \frac{E_a^c}{T_2}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{E_a^c}{E_a} = \frac{75}{100} = 0.75, \quad \frac{300}{T_1} = 0.75 \Rightarrow T_1 = \frac{300}{0.75} = 400\text{ K} = 127^\circ C$$

38. $C + O_2 \longrightarrow CO_2$

$\frac{1}{2}$ mol $\frac{1}{2}$ mol

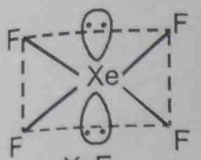
$$\Delta H = \frac{94}{2}\text{ Kcal} \times \frac{50}{100} = \frac{47}{2}\text{ Kcal} = q \Rightarrow n_{H_2O} \times 9.6 = \frac{47}{2}$$

$$n_{H_2O} = \frac{47}{2 \times 9.6} = 2.42\text{ mole}$$

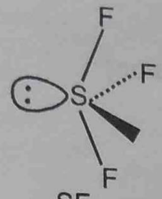
39. Carbocation (1) is antiaromatic and hence is least stable. Carbocation (2), (3) and (4) are all secondary but (2) and (3) are aromatic. Further since (3) is more strained than (2), therefore, (2) is the most stable carbocation.

40. The alkyl groups (CH_3) are activating groups CH_3 group meta to – OH group have additive function.

41.



XeF_4



SF_4

42. $\lambda_e = \dfrac{h}{\sqrt{2m_eKE_e}} = \dfrac{h}{\sqrt{2\times 1/1837 m_p \times 16E}}$, $\lambda_p = \dfrac{h}{\sqrt{2m_pKE_p}} = \dfrac{h}{\sqrt{2m_p \times 4E}}$.

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha KE_\alpha}} = \frac{h}{\sqrt{2\times 4m_p \times E}}$$

$\therefore \quad \lambda_e > \lambda_p = \lambda_\alpha$.

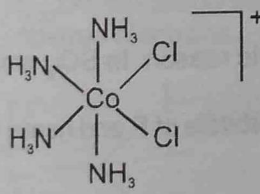
43. CaH_2 - ionic hydride as it exists as Ca^{2+} and H^-.
NH_3 - containing one lone pair of electrons is called electron rich covalent hydride.
CH_4 - containing no lone pair of electrons is called electron precise covalent hydride.
B_2H_6 - has too few electrons for writing its Lewis dot structure is called electron deficient covalent hydride.
$TiH_{1.73}$ - hydrogen atoms entangled in to the voids of the lattice of transition elements.
It is therefore, called as interstitial or metallic hydride.

44. Sucrose will have minimum value at Δp and thus

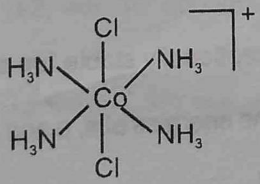
$\Delta P = P^o - P_S$

or $P_S = P^o - \Delta P$ is maximum

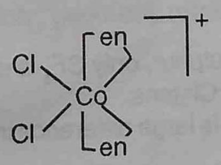
45.



cis–



trans–



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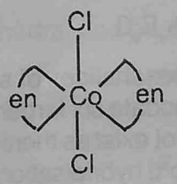
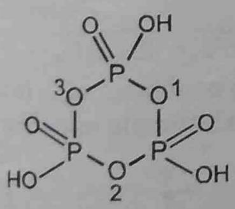


trans–

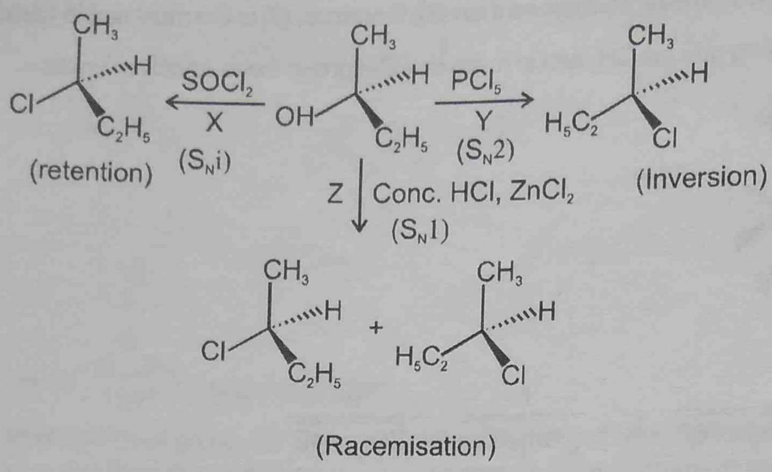
46.



According to the structure of cyclic metaphosphoric acid, $(HPO_3)_3$, three are there P – O – P bonds.

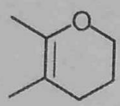
47. $2r = \dfrac{\sqrt{3}\,a}{2} \Rightarrow a = \dfrac{2(2r)}{\sqrt{3}} = \dfrac{2\times 1.73}{1.73} = 2Å = 200$ pm

48.

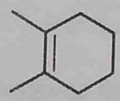


Total Products = 2

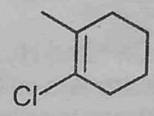
49.



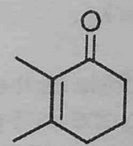
+m effect of oxygen



Hyperconjugative effects



–I effect of Cl



–M effect of >C=O

50. A atoms = $8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

B atoms = $12 \times \frac{1}{4} = 3$

C atoms = 1

So formula = A_4B_3C

51. (1) Amongst hexahalides of sulphur, only SF_6 is exceptionally stable for steric reason. In SCl_6, smaller S cannot accomodate six larger Cl^- ions.

(2) PH_5 does not exist as there is large difference in the energies of s, p and d orbitals of P and hence it does not undergo sp^3d hybridisation.

(3) NCl_5 does not exist as there is no d-orbital with N.

(4) BF_6^{3-} does not exist as there is no vacant d-orbital in B and hence it cannot exceed its covalency beyond four.

(5) Oxygen cannot exceed its covalency as it does not have d-orbital.

52. We need to obtain E° for the couple

(3) $Au^{3+}(aq) + 2e^- \longrightarrow Au^+(aq)$

from the values of E° for the couples

(1) $Au^+(aq) + e^- \longrightarrow Au(s)$ $E° = 1.69V$

(2) $Au^{3+}(aq) + 3e^- \longrightarrow Au(s)$ $E_2° = 1.40V$

We see that (3) = (2) – (1), therefore

$\Delta_rG_3° = \Delta_rG_1° - \Delta_rG_2°$

$-n_3FE_3° = -n_1FE_1° - n_2FE_2°$

Solving $E_3°$ we obtain

$$E_3^0 = \frac{n_2E_2^0 - n_1E_1^0}{n_3} = \frac{(3)\times(1.40V)-(1)\times(1.69V)}{2} = 1.26\ V$$

Then $R : Au^{3+}(aq) + 2e^- \longrightarrow Au^{+}(aq)$ $E_R^\circ = 1.26V$

$L : 2Fe^{3+}(aq) + 2e^- \longrightarrow 2Fe^{2+}(aq)$ $E_L^\circ = 0.77V$

$R - L : 2Fe^{2+}(aq) + Au^{3+}(aq) + \longrightarrow 2Fe^{3+}(aq) + Au^{+}(aq)$

$E^\circ = E_R^\circ - E_L^\circ = (1.26V) - (0.77V) = +0.49V$

$2.303\ RT \log K = nFE^\circ$

$$\log K = \frac{nFE^\circ}{2.303RT}$$

$$\log K = \frac{2 \times 0.49}{0.059}$$

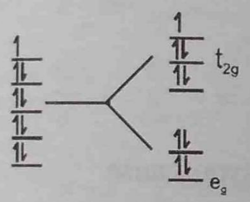
$K = 4 \times 10^{16}$

53. M–C π bond is formed by the donation of a pair of electrons from a filled d-orbital of metal into the vacant antibonding π^* orbital of carbon monoxide.

54. $k = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{150.5}\ \text{min}^{-1}$; $t = \dfrac{2.303}{k} \log \dfrac{100}{100-40} = \dfrac{2.303 \times 150.5}{0693} \log \dfrac{100}{60} = 111$ minutes

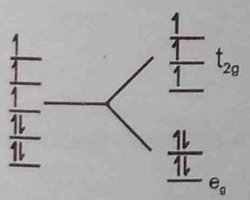
55. As we move down the group of alkaline earth metal the size of atom increases so I[st] ionisation enthalpy decreases.

56. (1) $_{27}Co$, oxidation state (–1) with $3d^{10}$ configuration, geometry tetrahedral and diamagnetic in nature.
(2) $_{27}Co$, oxidation state (0) with $3d^9$ configuration; geometry tetrahedral and paramagnetic with one unpaired electron.



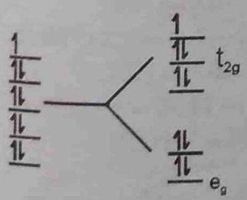
$\mu = \sqrt{1(1+2)} = \sqrt{3} = 1.73$ B.M.

(3) $_{27}Co$, oxidation state (+2) with $3d^7$ configuration ; geometry tetrahedral and paramagnetic with three unpaired electrons.

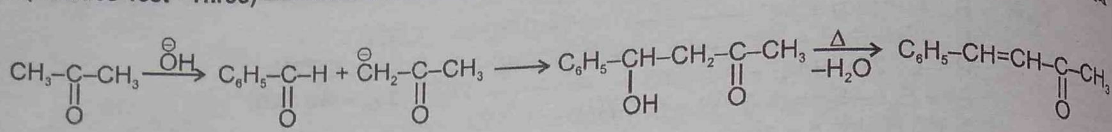


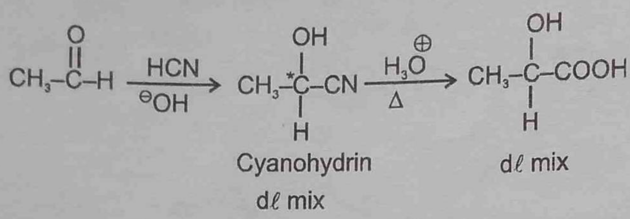
$\mu = \sqrt{3(3+2)} = \sqrt{15} = 3.87$ B.M.

(4) $_{27}Co$, oxidation state (0) with $3d^9$ configuration ; geometry is tetrahedral and paramagnetic in nature with one unpaired electron.



$\mu = \sqrt{1(1+2)} = \sqrt{3} = 1.73$ B.M.

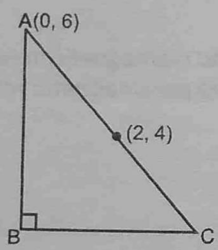
57. 

58. 

59. Adsorption is an exothermic process ($\Delta H = -$ ve). However entropy of the system also decreases ($\Delta S_{sys} = -$ve). Hence adsorption is thermodynamically move favourable at low temperature as $\Delta G = \Delta H - T\Delta S$.

60. Leaving group ability $\propto$ Stability of anion.

PART-C (MATHEMATICS)

61. 

As B is orthocenter so Δ is right angle so circumcenter will be the mid point of hypotenuse
So C(4, 8)

62. Let mid point be (h, k)
then $hx + ky = h^2 + k^2$
now by homogenization of $x^2 = a(x + y)$

$$x^2 = a(x+y)\left(\frac{hx+ky}{h^2+k^2}\right) \qquad \text{.... (i)}$$

now (i) subtend 90° at origin
$\therefore$ $(h^2 + k^2) - ah - ak = 0$
locus (h, k), $x^2 + y^2 = ax + ay$
by comparing a = 2

63. Let point is $(r\cos\theta, r\sin\theta)$ whose distance from origin is r.
It will lie on curve.
$2r^2\cos^2\theta + 5r^2\cos\theta\sin\theta + 2r^2\sin^2\theta = 1$

$$2r^2 + \frac{5r^2}{2}\sin 2\theta = 1$$

$$4r^2 + 5r^2\sin 2\theta = 2$$

$$r^2 = \frac{2}{4 + 5\sin 2\theta}$$

$$r^2 = \frac{2}{9} \text{ [minimum when } \sin 2\theta = 1]$$

$$r = \frac{\sqrt{2}}{3}$$

64. $1 + 2x + 3x^2 + \ldots\ldots 3n.\, x^{3n-1} = \frac{d}{dx}[x + x^2 + \ldots\ldots + x^{3n}]$

$= \frac{d}{dx}\left[\frac{x - x^{3n+1}}{1-x}\right] = \frac{(1-x)[1-(3n+1).x^{3n})] + (x - x^{3n+1})}{(1-x)^2}$

put $x = \omega$

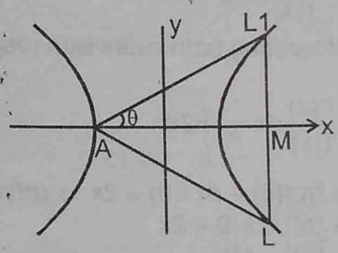
$1 + 2\omega + 3\omega^2 + \ldots\ldots + 3n\omega^{3n-1} = \frac{(1-\omega)(1-3n-1)+(\omega-\omega)}{(1-\omega)^2} = -\frac{3n}{1-\omega} = \frac{3n(\omega^2-1)}{(\omega-1)(\omega^2-1)} = 3n(\omega^2 - 1)$

65. $AM = a + ae$

$ML' = \frac{b^2}{a}$

$Q = 30^o$

$\frac{L'M}{AM} = \frac{1}{\sqrt{3}}$



$\Rightarrow \frac{\left(\frac{b^2}{a}\right)}{a+ae} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{b^2}{a^2} = \frac{e}{\sqrt{3}} = e^2 - 1 \qquad \Rightarrow \sqrt{3}\,e^2 - e - \sqrt{3} = 0 \qquad e = \frac{(\sqrt{3}+1)}{\sqrt{3}}$

66. $L_1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \qquad A\,(3, 8, 3)\ \vec{n}_1 = (3, -1, 1)$

$L_2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} \qquad B(-3, -7, 6)\vec{n}_2\,(-3, 2, 4)$

Shortest distance $= \frac{\overrightarrow{AB}.(\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|} = \frac{\begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{(-4-2)^2 + (12+3)^2 + (6-3)^2}} = 3\sqrt{30}$

67. $f(x + y) = f\left(\frac{xy}{4}\right) \qquad \ldots\ldots.(i)$

putting ; $y = 0$

$f(x) = f(0) \qquad \ldots\ldots\ldots(ii)$

putting $x = -4$, $f(-4) = f(0)$

$\therefore \quad f(0) = -4$

putting $x = 2011$ in equation (ii)

$\therefore \quad f(2011) = f(0) = -4$

68. $\lim\limits_{x\to 0^-} \frac{2\left[1-\left(1-\frac{7x}{256}\right)^{1/8}\right]}{2\left[\left(1+\frac{5x}{32}\right)^{1/5}-1\right]} = \lim\limits_{x\to 0^-} \frac{1-\left(1-\frac{1}{8}.\frac{7x}{256}+\ldots\ldots\right)}{\left(1+\frac{1}{5}.\frac{5x}{32}+\ldots\ldots\right)-1} = \frac{\frac{1}{8}.\frac{7}{256}}{\frac{1}{5}.\frac{5}{32}} = \frac{7}{64}$

69. Given that $f(x + y) = f(x).f(y)$ all $x \in R$
Putting $x = y = 0$ in (1), we get $f(0)\{f(0) - 1\} = 0 \Rightarrow f(0) = 0$ or $f(0) = 1$
If $f(0) = 0$, then $f(x) = f(x + 0) = f(x).f(x) = 0$ for all $x \in R$

Which is not true (given $f(x) \neq 0$)
So, $f(0) = 1$

$$\therefore\ f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$$

$$= f(x)\lim_{h\to 0}\frac{f(h)-1}{h} = f(x)\lim_{h\to 0}\frac{f(x)-f(0)}{h-0} \qquad (\because\ f(0) = 1)$$

$$= f(x)f'(0) = 2f(x) \qquad (\because\ f'(0) = 1)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2$$

Integrating both sides with respect to x and taking limit 0 to x

$$\int_0^x \frac{f'(x)}{f(x)}dx = \int_0^x 2dx$$

$\Rightarrow \ell n\, f(x) - \ell n\, f(0) = 2x \Rightarrow \ell n f(x) - \ell n 1 = 2x$
$\Rightarrow \ell n f(x) - 0 = 2x$
$\therefore\ f(x) = e^{2x}$

Clearly $f(x)$ is everywhere continuous and differentiable

70. $y = f(x)$ passes through (2, 0) and (0, 1)
$0 = f(2)$ and $f(0) = 1$

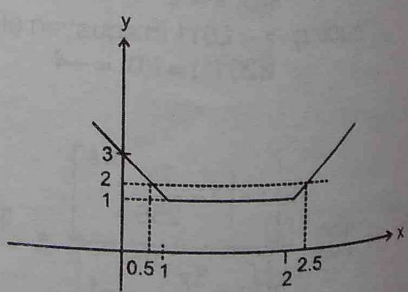
$$\int_0^2 f(x)\,dx = \frac{3}{4} \text{ given}$$

$$\text{Now} \int_0^2 xf'(x)\,dx = [xf(x)]_0^2 - \int_0^2 f(x)\,dx = 2f(2) - \int_0^2 f(x)\,dx = -\frac{3}{4}$$

71. $\log_{1/4}[\log_2(x + 2)] > 0$ is defined if
$\log_2(x + 2) > 0$ and $x + 2 > 0$
$\Rightarrow$ $x + 2 > 1$ and $x > -2$
$\Rightarrow$ $x > -1$
given logarithm is defined if $x > -1$
Now $\log_{1/4}[\log_2(x + 2)] > 0$
$\Rightarrow$ $\log_2(x + 2) < 1$
$x + 2 < 2$
$x < 0$
so $x \in (-1, 0)$ .....(1)
$|x - 1| + |x - 2| < 2$

so $x \in \left(\frac{1}{2}, \frac{5}{2}\right)$ .....(2)

Intersection of (1) and (2) is empty set.



72. Let $A(8,-2), B(2,-2), C(8,6)$

$AB = \sqrt{36+0} = 6$

$BC = \sqrt{36+64} = 10$

$CA = \sqrt{0+64} = 8$

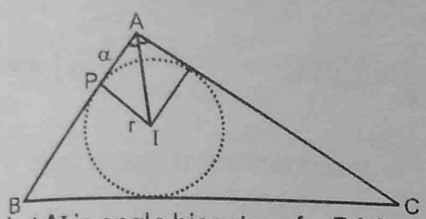
So not a right angle

73. Put $x = 1$

$2^{20} = a_0 + a_1 + a_2 + \ldots\ldots\ldots + a_{40}$

where $a_0 = 1$ and $a_{40} = (-2)^{20}$

$\therefore\quad a_1 + a_2 + \ldots\ldots\ldots + a_{39} = 2^{20} - a_0 - a_{40} = 2^{20} - 1 - 2^{20} = -1$

74. 

Let AI is angle bisector of $\angle BAC$

$\frac{r}{\alpha} = \tan\frac{A}{2} \qquad \therefore \qquad \alpha = r\cot\frac{A}{2}$

Similarly $\beta = r\cot\frac{B}{2}$; $\gamma = r\cot\frac{C}{2}$

In a ΔABC, we have the identity

$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} \qquad \therefore\ \frac{\alpha}{r} + \frac{\beta}{r} + \frac{\gamma}{r} = \frac{\alpha}{r}\cdot\frac{\beta}{r}\cdot\frac{\gamma}{r} \Rightarrow \frac{1}{r}(\alpha+\beta+\gamma) = \frac{1}{r^3}\alpha\beta\gamma$

$\Rightarrow \qquad r = \sqrt{\frac{\alpha\beta\gamma}{\alpha+\beta+\gamma}}$

75. Since PQ is perpendicular to the plane and direction ratio of PQ are $\alpha-\alpha', \beta-\beta', \gamma-\gamma'$ hence equation of plane can be written as

$$(\alpha-\alpha')x + (\beta-\beta')y + (\gamma-\gamma')z = 0$$

also plane passes through $\left(\frac{\alpha+\alpha'}{2}, \frac{\beta+\beta'}{2}, \frac{\gamma+\gamma'}{2}\right)$

hence $\quad (\alpha-\alpha')\left(\frac{\alpha+\alpha'}{2}\right) + (\beta-\beta')\left(\frac{\beta+\beta'}{2}\right) + (\gamma-\gamma')\left(\frac{\gamma+\gamma'}{2}\right) = 0$

$\Rightarrow \qquad \alpha^2 + \beta^2 + \gamma^2 = (\alpha')^2 + (\beta')^2 + (\gamma')^2$

76. Using $\tan\theta = \cot\theta - 2\cot 2\theta$

we get $E = (\cot\theta - 2\cot 2\theta) + 2(\cot 2\theta - 2\cot 4\theta) + 4(\cot 4\theta - 2\cot 8\theta) + \ldots\ldots$

$\ldots + 2^{14}(\cot 2^{14}\theta - 2\cot 2^{15}\theta) + 2^{15}\cot 2^{15}\theta = \cot\theta$

77. Given series $= \tan^{-1}\frac{(x+1)-x}{1+x(x+1)} + \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)} + \ldots\ldots$ n terms

$= \tan^{-1}(x+1) - \tan^{-1}x + \tan^{-1}(x+2) - \tan^{-1}(x+1) + \ldots\ldots$ n terms

$= \tan^{-1}(x+n) - \tan^{-1}x$

78. Let

$S = 1 + 3y + 5y^2 + 7y^3 + \ldots\ldots\infty$

$yS = \quad y + 3y^2 + 5y^3 + \ldots\ldots\infty$

$(1-y)S = 1 + 2y + 2y^2 + 2y^3 + \ldots\ldots\infty$

$$(1-y)S = 1 + \frac{2y}{1-y} = \frac{1+y}{1-y}$$

$$S = \frac{(1+y)}{(1-y)^2}$$

Putting $y = 1 - \frac{1}{x}$

$$S = \frac{1+1-\frac{1}{x}}{\left(\frac{1}{x}\right)^2} = 2x^2 - x$$

79. $x + y + z \le n$

$x + y + z + t = n$

when $t \ge 0$

no. of solutions

${}^{n+4-1}C_{4-1} = {}^{n+3}C_3$

80. Greatest value of f is $\frac{\pi}{2}$ and least value is 0.

81. $5^4 = 625$

82. Time taken by boat $= \frac{300}{x}$ hours

petrol consumed $= \left(2 + \frac{x^2}{600}\right)\frac{300}{x}$ liter

expenses on travelling

$$E = 200 \times \frac{300}{x} + \left(2 + \frac{x^2}{600}\right)\frac{3000}{x} = \frac{60000}{x} + \frac{6000}{x} + 5x = \frac{66000}{x} + 5x$$

$\frac{dE}{dx} = \frac{-66000}{x^2} + 5 < 0$ for all $[25, 60]$. Most economical speed is 60 kmph.

83. $$f(\alpha) = \int_0^1 \frac{x^{\cos\alpha} - 1}{\ell nx}dx$$

$$\frac{df}{d\alpha} = \int_0^1 (-\sin\alpha)\cdot x^{\cos\alpha}.dx = -\left(\sin\alpha.\frac{x^{\cos\alpha+1}}{\cos\alpha+1}\right)_0^1 = \frac{-\sin\alpha}{1+\cos\alpha}$$

Integrating $f(\alpha) = \ell n\,(1 + \cos\alpha) + c$

$f\left(\frac{\pi}{2}\right) = 0 \quad \Rightarrow \quad c = 0 \quad \therefore \quad f(\alpha) = \ln(1 + \cos\alpha)$

84. $$\lim_{x\to 0}\frac{(1+x)^{1/x} - e + \frac{ex}{2}}{11x^2} \qquad \left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\text{ रूप}\right)$$

By expansion $= \lim_{x\to 0}\frac{e\left(1 - \frac{x}{2} + \frac{11}{24}x^2 + ...\right) - e + \frac{ex}{2}}{11x^2} = \frac{e}{24}$

85. $f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \ldots\ldots + \frac{f^n(1)}{n!} = 1 + \frac{n}{1!} + \frac{n(n-1)}{2!} + \ldots. + \frac{n!}{n!}$

$= {}^nC_0 + {}^nC_1 + {}^nC_1 + \ldots\ldots + {}^nC_n = 2^n$

86. $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$

integrating factor $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y} \Rightarrow x.e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} e^{\tan^{-1}y}.dy$

$= \tan^{-1}y.\, e^{\tan^{-1}y} - e^{\tan^{-1}y} + c = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$

87. $P[E_1] = \frac{3}{6} = \frac{1}{2}$ $\qquad P(E_2) = \frac{3}{6} = \frac{1}{2}$

P_1 (ball drawn from first box is white) $= \frac{4}{9}$

P_2 (ball draw from 2[nd] box is white) $= \frac{5}{9}$

$\therefore$ By Bayer's theorm probability of ball draw from first box $= \frac{\frac{1}{2}.\frac{4}{9}}{\frac{1}{2}.\frac{4}{9} + \frac{1}{2}.\frac{5}{9}} = \frac{4}{9}$

88. $AA' = I$ for orthogonal matrix

$$\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix} \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 4b^2+c^2 & 2b^2-c^2 & -2b^2+c^2 \\ 2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ 2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad b = \pm\frac{1}{\sqrt{6}},\ a = \pm\frac{1}{\sqrt{2}},\ c = \pm\frac{1}{\sqrt{3}}$

89. Let $f(x) = \sin x - \frac{\pi}{2} + 1$

$\Rightarrow \quad f(0) = 0 - \frac{\pi}{2} + 1 = 1 - \frac{\pi}{2} < 0$ and $f\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{2} + 1 = 2 - \frac{\pi}{2} > 0$

$f(\pi) = 0 - \frac{\pi}{2} + 1 = 1 - \frac{\pi}{2} < 0$ and $f\left(\frac{3\pi}{2}\right) = -1 - \frac{\pi}{2} + 1 = -\frac{\pi}{2} < 0$

since $f(0)$ and $f\left(\frac{\pi}{2}\right)$ are opposite in sign therefore a real roots of $f(x) = 0$ lies between 0 and $\frac{\pi}{2}$.

Similarly $f(x) = 0$ has a root in the interval $\left(\frac{\pi}{2}, \pi\right)$

90. $I_{11} = \int_0^1 (1-x^5)^{11}dx \quad = \int_0^1 (1-x^5)(1-x^5)^{10}dx = I_{10} - \int_0^1 x^5(1-x^5)^{10}dx = I_{10} + \int_0^1 \frac{x}{5}(1-x^5)^{10}\ (-5x^4)\,dx$

Integrating by parts. $\quad I_{11} = I_{10} + \left(\frac{x}{5}\frac{(1-x^5)^{11}}{11}\right)_0^{11} - \frac{1}{55}\int_0^1 (1-x^5)^{11}dx \quad = I_{10} - \frac{I_{11}}{55}$

$I_{11} = (1 + \frac{1}{55})\, I_{11} - I_{10} \quad ; \quad \frac{I_{10}}{I_{11}} = \frac{56}{55}$