Unitarity and Causality in Generalized Quantum Mechanics for Non-Chronal Spacetimes

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Abstract

Spacetime must be foliable by spacelike surfaces for the quantum mechanics of matter fields to be formulated in terms of a unitarily evolving state vector defined on spacelike surfaces. When a spacetime cannot be foliated by spacelike surfaces, as in the case of spacetimes with closed timelike curves, a more general formulation of quantum mechanics is required. In such generalizations the transition matrix between alternatives in regions of spacetime where states can be defined may be non-unitary. This paper describes a generalized quantum mechanics whose probabilities consistently obey the rules of probability theory even in the presence of such non-unitarity. The usual notion of state on a spacelike surface is lost in this generalization and familiar notions of causality are modified. There is no signaling outside the light cone, no non-conservation of energy, no "Everett phones", and probabilities of present events do not depend on particular alternatives of the future. However, the generalization is acausal in the sense that the existence of non-chronal regions of spacetime in the future can affect the probabilities of alternatives today. The detectability of non-unitary evolution and violations

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of causality in measurement situations are briefly considered. The evolution of information in non-chronal spacetimes is described.

I. INTRODUCTION

Conventional formulations of the quantum mechanics of matter fields in a curved background spacetime require that this spacetime be foliable by a family of spacelike surfaces. A family of spacelike surfaces is needed just to define a state of the matter fields on a spacelike surface and the progress of this state into the future by either unitary evolution between spacelike surfaces or by "state vector reduction" on them. However, not all spacetimes admit a foliation by spacelike surfaces. For example, spacetimes with closed timelike curves, such as would be produced by the motion of wormhole mouths, permit no foliating family of spacelike surfaces [1]. The quantum mechanics of matter fields in spacetimes with such non-chronal regions therefore cannot be formulated in terms of the evolution of states on spacelike surfaces. Rather, a more general formulation of quantum mechanics is required. Generalizations based on the ideas of quantum computation have been described by Deutsch [2] and generalizations based on the algebraic approach to field theory have been discussed by Yurtsever [3]. Here, we pursue another class of generalizations based on the sum-overhistories formulation of quantum theory. Generalizations of this kind have previously been discussed by Klinkhammer and Thorne [4], Friedman, Simon, and Papastamatiou [5] and the author [6]. Specifically, we explore the notions of unitarity and causality and the connections between them in this class of generalizations.

Feynman's sum-over-histories formulation of quantum mechanics is a natural route to a generalized quantum mechanics of matter fields in spacetimes with non-chronal regions because, with it, quantum mechanics may be cast into a fully spacetime form that does not employ a notion of state on a foliating family of spacelike surfaces [4–6]. For example, in the sum-over-histories formulation, quantum dynamics is expressed, not through a differential equation, but rather by giving the amplitude for a fine-grained field history — a four-dimensional field configuration, $\phi(x)$. In Feynman's prescription this amplitude is proportional to

$$\exp(iS[\phi(x)]/\hbar) \tag{1.1}$$

where S is the action functional for the field. Quantum dynamics can be defined in this way even when spacetime contains non-chronal regions. The alternatives potentially assigned probabilities by quantum theory can also be described four-dimensionally as partitions, or coarse grainings, of the fine-grained field histories into classes. For instance, the four-dimensional field histories could be partitioned by the values of the field configurations $\phi(\mathbf{x})$ on a spacelike surface σ . The amplitudes for such alternatives define state functionals on σ in familiar quantum theories formulated in terms of states on spacelike surfaces. However, even in non-chronal regions, where there are no foliating families of spacelike surfaces, we can still define meaningful coarse-grainings of four-dimensional field configurations. For example, we could partition the field histories by the value of a field averaged over a region of spacetime deep inside a wormhole throat. A decoherence functional defining the interference between such individual alternatives may be defined and the probabilities for decohering sets of alternatives calculated. In this way the quantum theory of fields may be put into fully four-dimensional form free from the need of a foliating family of spacelike surfaces [6–9].

If the non-chronal regions of spacetime are bounded, then the spacetime contains initial and final regions before and after the non-chronal one in which familiar alternatives of the spatial field configurations can be defined on spacelike surfaces (Figure 1). Transition probabilities between such alternatives are of interest. Transition amplitudes between a definite spatial field configuration, $\phi'(\mathbf{x})$, on an initial spacelike surface σ' and a configuration $\phi''(\mathbf{x})$ on a final surface σ'' are given by a sum-over-histories expression of the form

$$\langle \phi''(\mathbf{x}), \sigma'' | \phi'(\mathbf{x}), \sigma' \rangle = \int_{[\phi', \phi'']} \delta \phi \exp(iS[\phi(x)]/\hbar) .$$
 (1.2)

The sum is over four-dimensional field configurations between σ' and σ'' that match the prescribed spatial configurations on those surfaces. By such methods, for example, an S-matrix for scattering through spacetime regions with closed timelike curves could be defined and calculated.

When spacetime can be foliated by a family of spacelike surfaces, eq. (1.2) coincides with the unitary evolution operator generated by the Hamiltonian for the family. That is because, as Dirac [10] and Feynman [11] showed, when two spacelike surfaces are close together, the matrix elements of the operator effecting unitary evolution between them is proportional to $\exp(iS)$ where S is the action of the classical field history interpolating between the two. Explicitly in the case of two constant-time surfaces in Minkowski space

$$\exp\left\{iS\left[\phi''(\mathbf{x}), t''; \phi'(\mathbf{x}), t'\right]/\hbar\right\} \propto \langle\phi''(\mathbf{x})| \exp\left[-iH(t'' - t')/\hbar\right] |\phi'(\mathbf{x})\rangle . \tag{1.3}$$

However, in the absence of a connection like (1.3), or even a well defined meaning for its right hand side, there is no particular reason to expect a construction like (1.2) to yield a unitary transition matrix.¹ Calculations by Klinkhammer and Thorne [4] in non-relativistic quantum mechanics first suggested that the evolution defined by (1.2) might be non-unitary. General results of Friedman, Papastamation and Simon [5,12] in field theory, and explicit examples of Boulware [13] and Politzer [14], show the following: The scattering matrix constructed from the sum-over-histories (1.2) is unitary for free field theories in spacetimes with closed timelike curves, but not, in general for interacting theories, order by order in perturbation theory.² This paper discusses the implications of this non-unitarity.

Even were spacetime foliable by spacelike surfaces it would still be difficult to reconcile non-unitary evolution with the notion of state on a spacelike surface. The reasons, stated clearly by T. Jacobson [17], are reviewed in Section II. However, a generalized quantum mechanics neither requires, nor does it always permit, a notion of "state on a spacelike surface". In Section III we spell out enough details of the generalized sum-over-histories quantum mechanics sketched above to show how it consistently incorporates non-unitary evolution represented by the transition matrix (1.2) without employing a notion of state. The price for this generalization is not only the absence of a notion of state on a spacelike surface,

¹A conclusion also reached by Deutsch [2] from the point of view of quantum computation.

²Goldwirth, Perry and Piran [15] concluded that even free theories were non-unitary. This was corrected in [16], where some of the results of Klinkhammer and Thorne [4] for free theories are included.

but also a violation of causality that is discussed in Section IV. There is no violation of causality in the sense that signals propagate outside the light cone. Neither do probabilities in the present depend on specific alternatives in the future. However, the existence of future non-chronal regions of spacetime will influence probabilities in the present. A theory of the future geometry of spacetime, as well as of the initial condition of the closed system and the geometry up to the present, is thus required for present prediction.

The theory of laboratory scattering measurements in the presence of non-chronal regions is developed in Section V and used to give a preliminary discussion of how violations of unitarity and causality might be detected. In Section VI we introduce a notion of spacetime information that is not tied to a notion of state on a spacelike surface and shows how it evolves in a spacetime with non-chronal regions. Section VII shows that various anomalies such as non-conservation of energy, signaling faster than light, and communication between non-interfering branches that exist in some other generalizations of quantum mechanics are absent from this one.

II. NON-UNITARITY AND THE QUANTUM MECHANICS OF STATES

In its simplest interpretations, non-unitary evolution of a quantum state defined on spacelike surfaces is either inconsistent, or, as shown by T. Jacobson [17], dependent on the choice of spacelike surfaces. This section briefly reviews these arguments.

We consider a fixed background spacetime containing a bounded, non-chronal region NC, as shown in Figure 1. Consider an initial state $|\psi(\sigma')\rangle$ on a spacelike surface σ' before NC. Suppose its evolution to a spacelike surface σ'' after NC is given by a non-unitary evolution operator X:

$$|\psi(\sigma'')\rangle = X|\psi(\sigma')\rangle$$
 (2.1)

³We shall be more precise about the meanings of "before" and "after" in Section III.

We now consider the calculation of the probabilities of an exhaustive set of exclusive alternatives on this spacelike surface represented by a set of (Schrödinger-picture) projection operators satisfying

$$\sum_{\alpha} P_{\alpha} = 1 \quad , \quad P_{\alpha} P_{\beta} = \delta_{\alpha\beta} P_{\beta} . \tag{2.2}$$

What rule should be used to calculate these probabilities?

The usual prescription for the probability of the alternative corresponding to P_{α} on σ is

$$p(\alpha; \sigma) = \parallel P_{\alpha} | \psi(\sigma) \rangle \parallel^2 . \tag{2.3}$$

If $|\psi(\sigma')\rangle$ is normalized so that

$$\sum_{\alpha} p(\alpha; \sigma') = 1 , \qquad (2.4)$$

then (2.1) will imply for the probabilities of the same alternatives on the later surface

$$\sum_{\alpha} p(\alpha; \sigma'') = \left\langle \psi(\sigma') | X^{\dagger} X | \psi(\sigma) \right\rangle \neq 1.$$
 (2.5)

When X is not unitary, probability is not conserved and the prescription (2.3) for assigning probabilities is thus inconsistent.

The generalization of (2.3)

$$p(\alpha; \sigma) = \frac{\parallel P_{\alpha} | \psi(\sigma) \rangle \parallel^2}{\parallel | \psi(\sigma) \rangle \parallel^2}$$
 (2.6)

suggests itself as a way of maintaining the requirement that the probabilities of an exhaustive set of alternatives sum to unity. However, Jacobson [17] has shown that this rule is not covariant with respect to the choice of spacelike surfaces. Consider a set of alternatives $\{P_{\alpha}(R)\}$ that distinguish only properties of fields on a spacelike surface that are restricted to a region R that is spacelike separated from a non-chronal region NC. For example, an exhaustive set of ranges of the average of a field over R defines one such set of alternatives. Since R is spacelike to NC it may be considered either as a part of a spacelike surface σ' that is before NC or as part of a spacelike surface σ'' that is after NC (Figure 2). According to (2.6) and (2.1), the probabilities for the alternatives $\{\alpha\}$ evaluated on σ' would be

$$p(\alpha; \sigma') = \frac{\langle \psi(\sigma') | P_{\alpha}(R) | \psi(\sigma') \rangle}{\langle \psi(\sigma') | \psi(\sigma') \rangle} , \qquad (2.7)$$

while on σ'' they would be

$$p(\alpha; \sigma'') = \frac{\left\langle \psi(\sigma') | X^{\dagger} P_{\alpha}(R) X | \psi(\sigma') \right\rangle}{\left\langle \psi(\sigma') | X^{\dagger} X | \psi(\sigma') \right\rangle} . \tag{2.8}$$

These must be equal since the alternatives are the same.

A state on σ' that is an eigenvector of the field configuration $\phi(\mathbf{x})$ with $\mathbf{x}\epsilon R$, evolves into a state on σ'' that is also an eigenvector of $\phi(\mathbf{x})$, with $\mathbf{x}\epsilon R$, having the same eigenvalue. Thus,

$$[X, P_{\alpha}(R)] = 0. \tag{2.9}$$

Were X unitary, (2.9) would imply the equality of the numerators in (2.7) and (2.8) and of the denominators. However, when X is non-unitary the expressions (2.7) and (2.8) cannot be equal for all states $|\psi(\sigma)\rangle$. Non-unitary evolution therefore implies that the probabilities for the alternatives $\{P_{\alpha}(R)\}$ are different on σ' and σ'' . Quantum mechanics defined by the rule (2.6) is not covariant with respect to the choice of spacelike surfaces unless the evolution is unitary. This is the essence of Jacobson's argument.

Thus a non-unitary transition matrix, say constructed by a sum-over-histories as in (1.2), cannot be used to construct a quantum mechanics in which probabilities are computed from a "state of the system on a spacelike surface" using either of the prescriptions (2.1) or (2.6) if we insist on covariance with respect to the choice of spacelike surfaces. In the next Section we shall show how a generalized quantum mechanics that avoids this problem can be constructed incorporating such non-unitary transition matrices. Such generalizations will not, of course, admit a notion of "state on a spacelike surface" in any of the senses discussed in this Section.

III. GENERALIZED QUANTUM MECHANICS

In this Section we will spell out more concretely some details of generalized quantum theories that consistently incorporate non-unitary evolution. We have described the principles of generalized quantum mechanics elsewhere [6,9] and do not review them here. The sum-over-histories quantum mechanics for non-chronal spacetimes that was sketched in the Introduction is one example of a generalized quantum mechanics incorporating non-unitary evolution. However, it turns out that with respect to alternatives defined on spacelike surfaces where they exist, and the transitions between them, a more general discussion can be given which is largely independent of the specific mechanism of non-unitarity [for example that of (1.2)]. We shall exhibit this general framework.

We are concerned most generally with the quantum mechanics of a closed system containing both observers and observed, measuring apparatus and measured subsystems. In the present investigation, the closed system is an interacting quantum field theory in a fixed given, background spacetime geometry. To keep the notation manageable we shall consider a single, scalar field, $\phi(x)$. We shall assume that spacetime outside a bounded region NC is foliable by spacelike surfaces (see Figure 3). Thus we can identify an initial region $\mathcal{IN}(NC)$ outside of NC, no point of which can be reached from any point of NC by a timelike curve that is future pointing outside of NC. $\mathcal{IN}(NC)$ is foliable by spacelike surfaces. Similarly we can define a final region $\mathcal{FN}(NC)$ [generally overlapping $\mathcal{IN}(NC)$] that is foliable by spacelike surfaces. We will loosely refer to $\mathcal{IN}(NC)$ as "before" NC and $\mathcal{FN}(NC)$ as "after" NC. Later we can consider the case of several disjoint non-chronal regions.

The most general objective of a quantum theory is the prediction of the probabilities of the individual histories in an exhaustive set of alternative, coarse-grained histories of the closed system. As mentioned above, we shall restrict attention in this paper to sets of histories consisting of alternatives defined on spacelike surfaces foliating the initial and final regions. This has the advantage that the usual apparatus of operators on Hilbert space may be used to describe these alternatives.⁴

In the Schrödinger picture, an exhaustive and exclusive set of alternatives defined on a

⁴For discussion of more general classes of spacetime alternatives see, e.g. [6,7,18]

spacelike surface corresponds to a set of projection operators $\{P_{\alpha}\}$ satisfying (2.2). The P_{α} , for example, might be projections onto ranges of values a field averaged over a spatial region R in the surface. Specifying (generally different) sets of alternatives $\{P_{\alpha_1}^1\}$, $\{P_{\alpha_2}^2\}$, ..., $\{P_{\alpha_n}^n\}$ on a sequence of non-intersecting spacelike surfaces $\sigma_1, \dots, \sigma_n$ defines a set of coarse-grained alternative histories for the system. A particular history corresponds to a particular sequence of alternatives $\alpha_1, \dots, \alpha_n$, that we shall often abreviate by a single index, viz: $\alpha = (\alpha_1, \dots, \alpha_n)$. The exhaustive set of histories consists of all possible sequences $\{\alpha\}$. The histories are coarse grained because not all information is specified that could be specified. Alternatives are not specified at each and every time, and the alternatives that are specified do not correspond to a complete set of states unless all the P's are one-dimensional.

A quantum theory of a closed system does not assign probabilities to every set of coarsegrained histories of a closed system. In the two-slit experiment, for example, we cannot assign probabilities to the alternative histories in which the electron went through one slit or the other and arrived at a definite point on the detecting screen. It would be inconsistent to do so because, as a consequence of quantum mechanical interference, these probabilities would not correctly sum to the probability to arrive at the designated point on the screen. The quantum mechanics of closed systems assigns probabilities only to the members of sets of alternative, coarse-grained histories for which there is negligible interference between the individual histories in the set as a consequence of the system's dynamics and boundary conditions [19–21]. Such sets of histories are said to decohere. In a generalized quantum theory, the interference between histories in a set is measured by a decoherence functional incorporating information about the system's dynamics and initial condition. The decoherence functional, $D(\alpha', \alpha)$, is a complex function of pairs of histories satisfying certain general conditions that we shall describe below. The set decoheres if $D(\alpha', \alpha)$ is sufficiently small for all pairs of different histories in the set $\{\alpha\}$. When that is the case, the probabilities of the individual histories $p(\alpha)$ are the diagonal elements of $D(\alpha', \alpha)$. The rule both for when probabilities may be assigned to a set of coarse-grained histories and what these probabilities are may thus be summarized by the fundamental formula:

$$D(\alpha', \alpha) \approx \delta_{\alpha'\alpha} p(\alpha)$$
 (3.1)

When spacetime is completely foliable by spacelike surfaces, the decoherence functional of familiar Hamiltonian quantum mechanics is given by

$$D(\alpha', \alpha) = Tr \Big[P_{\alpha'_{n}}^{n} U(\sigma_{n}, \sigma_{n-1}) P_{\alpha'_{n-1}}^{n-1} U(\sigma_{n-1}, \sigma_{n-2})$$

$$\cdots P_{\alpha'_{1}}^{1} U(\sigma_{1}, \sigma_{0}) \rho U(\sigma_{0}, \sigma_{1}) P_{\alpha_{1}}^{1} \cdots U(\sigma_{n-2}, \sigma_{n-1}) P_{\alpha_{n-1}}^{n-1} U(\sigma_{n-1}, \sigma_{n}) P_{\alpha_{n}}^{n} \Big]$$
(3.2)

where ρ is the density matrix describing the initial condition of the system of fields on an initial spacelike surface, σ_0 , and $U(\sigma'', \sigma')$ is the unitary evolution operator between spacelike surfaces σ' and σ'' .

Generalizing the form of the decoherence functional (3.2) generalizes Hamiltonian quantum mechanics. A wide class of generalizations called generalized quantum theories [6,9] have decoherence functionals that (i) are Hermitian: $D(\alpha, \alpha') = D^*(\alpha', \alpha)$, (ii) are normalized: $\Sigma_{\alpha\alpha'}D(\alpha, \alpha') = 1$, (iii) have positive diagonal elements: $D(\alpha, \alpha) \geq 0$, and, most importantly, (iv) obey the principle of superposition in the following sense: A coarse graining of the set $\{\alpha\}$ means a partition of that set into a new set of (generally larger) exhaustive and exclusive classes, $\{\bar{\alpha}\}$. A decoherence functional satisfies the principle of superposition when

$$D(\bar{\alpha}', \bar{\alpha}) = \sum_{\alpha' \in \bar{\alpha}'} \sum_{\alpha \in \bar{\alpha}} D(\alpha', \alpha)$$
(3.3)

for all coarse grainings $\{\bar{\alpha}\}$ of $\{\alpha\}$. When a set of histories decoheres, and probabilities are assigned according to the fundamental formula (3.1), the numbers $p(\alpha)$ lie between 0 and 1 and satisfy the most general form of the probability sum rules

$$p(\bar{\alpha}) = \sum_{\alpha \in \bar{\alpha}} p(\alpha) \ . \tag{3.4}$$

The decoherence functional of Hamiltonian quantum mechanics, (3.2), is easily seen to satisfy (i) — (iii), and satisfies *sums* of the projections in the fine-grained set.

Suppose we consider a spacetime and a single non-chronal region NC and restrict attention to alternatives defined on spacelike surfaces either entirely in the region $\mathcal{IN}(NC)$

"before" NC or in the region $\mathcal{FN}(NC)$ "after" it. Suppose the evolution between a spacelike surface σ_- before NC and a spacelike surface σ_+ after NC is not described by a unitary matrix U, but by a non-unitary matrix X_S . The decoherence functional (3.2) with U replaced by X_S no longer satisfies the general requirements (i) — (iv). However, the following generalization does satisfy them:

$$D(\alpha', \alpha) = N \ Tr \Big[P_{\alpha'_{n}}^{n} U(\sigma_{n}, \sigma_{n-1}) \cdots P_{\alpha'_{k+1}}^{k+1} U(\sigma_{k+1}, \sigma_{+}) X_{S} U(\sigma_{-}, \sigma_{k}) P_{\alpha'_{k}}^{k} \cdots U(\sigma_{2}, \sigma_{1}) P_{\alpha'_{1}}^{1}$$

$$U(\sigma_{1}, \sigma_{0}) \ \rho U(\sigma_{0}, \sigma_{1}) P_{\alpha_{1}}^{1} U(\sigma_{1}, \sigma_{2}) \cdots P_{\alpha_{k}}^{k} U(\sigma_{k}, \sigma_{-}) X_{S}^{\dagger} U(\sigma_{+}, \sigma_{k+1}) P_{\alpha_{k+1}}^{k+1} \cdots U(\sigma_{n-1}, \sigma_{n}) P_{\alpha_{n}}^{n} \Big]$$
 (3.5)

where

$$N^{-1} = Tr\left(X\rho X^{\dagger}\right) \tag{3.6}$$

and $\sigma_1, \dots, \sigma_k$ lie before σ_- in $\mathcal{IN}(NC)$ while $\sigma_{k+1}, \dots, \sigma_n$ lie after σ_+ in $\mathcal{FN}(NC)$.

The expression (3.5) may be simplified by introducing a kind of Heisenberg picture with operators

$$P_{\alpha_i}^i(\sigma_i) = U^{-1}(\sigma_i, \sigma_0) P_{\alpha_i}^i U(\sigma_i, \sigma_0) \quad , \quad \sigma < \sigma_-$$
(3.7a)

$$P_{\alpha_i}^i(\sigma_i) = U^{-1}(\sigma_i, \sigma_f) P_{\alpha_i}^i U(\sigma_i, \sigma_f) \quad , \quad \sigma > \sigma_+$$
(3.7b)

and

$$X = U^{-1}(\sigma_+, \sigma_f) X_S U(\sigma_-, \sigma_0) . \tag{3.8}$$

where σ_f is a final surface in the far future. Then (3.5) is

$$D(\alpha',\alpha) = N \operatorname{Tr} \left[P_{\alpha'_{n}}^{n}(\sigma_{n}) \cdots P_{\alpha'_{k+1}}^{k+1}(\sigma_{k+1}) X P_{\alpha'_{k}}^{k}(\sigma_{k}) \right]$$

$$\cdots P_{\alpha'_{1}}^{1}(\sigma_{1}) \rho P_{\alpha_{1}}^{1}(\sigma_{1}) \cdots P_{\alpha_{k}}^{k}(\sigma_{k}) X^{\dagger} P_{\alpha_{k+1}}^{k+1}(\sigma_{k+1}) \cdots P_{\alpha_{n}}^{n}(\sigma_{n}) \right].$$

$$(3.9)$$

The expression can be written even more compactly if we introduce the notation

$$C_{\alpha} = P_{\alpha_k}^k(\sigma_k) \cdots P_{\alpha_1}^1(\sigma_1) \tag{3.10}$$

for a chain of projections on spacelike surfaces before σ_{-} , and

$$C_{\beta} = P_{\beta_n}^n(\sigma_n) \cdots P_{\beta_{k+1}}^{k+1}(\sigma_{k+1})$$
 (3.11)

for a chain on spacelike surfaces after σ_+ . Then

$$D(\beta', \alpha'; \beta, \alpha) = \frac{Tr\left(C_{\beta'}XC_{\alpha'}\rho C_{\alpha}^{\dagger}X^{\dagger}C_{\beta}^{\dagger}\right)}{Tr\left(X\rho X^{\dagger}\right)}.$$
(3.12)

The decoherence functional (3.12) defines a quantum mechanics that reduces to the usual one (3.2) when the evolution is unitary, but generalizes it when it is not. It consistently assigns probabilities to decoherent sets of histories. There is no issue of the violation of a probability sum rule like (2.5) here. All probability sum rules (3.4) are satisfied as a consequence of decoherence including the elementary requirement that the probabilities of an exhaustive set of alternatives sum to 1. Neither is there hypersurface dependence of local probabilities as with Jacobson's rule (2.6). From (3.12) it follows that the probability of a set of alternatives $P_{\alpha}(R)$ that distinguish only field values on a local piece of a spacelike surface R that is everywhere spacelike separated from the non-chronal region NC are

$$p(\alpha, \sigma') = N \operatorname{Tr} \left[X P_{\alpha}(R) \rho P_{\alpha}(R) X^{\dagger} \right]$$
(3.13)

when R is considered part of a spacelike surface σ' to the before NC, and given by

$$p(\alpha; \sigma'') = N \operatorname{Tr} \left[P_{\alpha}(R) X \rho X^{\dagger} P_{\alpha}(R) \right]$$
(3.14)

when R is considered part of a spacelike surface after NC. However, since $P_{\alpha}(R)$ and X commute [cf]. Eq. (2.8), eqs (3.13) and (3.14) are equivalent. The rule (2.6) includes or does not include the non-unitary evolution operator X depending on which surface is chosen. By contrast the rules (3.13) and (3.14) both include an X. The order of the X with respect to projection operator representing the alternative in R is different depending on whether R is considered a part of σ' or σ'' , but that order is immaterial since the operators commute. The generalized quantum mechanics defined by the decoherence functional (3.12) is thus consistent with elementary requirements. In the following we shall explore its consequences.

IV. CAUSALITY

The past influences the future but the future does not influence the past; that is the essence of causality. A fixed spacetime geometry whose causal structure defines "future" and "past" as needed just to ask whether a theory is consistent with causality or not. A fixed background spacetime has been assumed for the field theories that are the concern of this paper, but the future and past cannot be unambiguously distinguished for points inside non-chronal regions connected by closed timelike curves. However, we can ask whether the probabilities of a set of alternatives defined entirely outside such regions are independent of the geometry of spacetime to their future. It is straightforward to see that the generalized quantum mechanics of matter fields described in the previous Section is not causal in this sense if the evolution through non-chronal regions is not unitary.

Suppose that spacetime contains a single non-chronal region that is to our future and we are concerned with the probabilities of a chain of alternatives C_{α} all occurring before the non-chronal region. If these alternatives decohere, then their probabilities $p(\alpha)$ are given, according to (3.1) and (3.12) by

$$p(\alpha) = N \ Tr(X C_{\alpha} \rho C_{\alpha}^{\dagger} X^{\dagger}) \tag{4.1}$$

where X describes the evolution through the non-chronal region and $N^{-1} = Tr(X\rho X^{\dagger})$. Were X unitary, the cyclic property of the trace could then be used to show

$$p(\alpha) = Tr(C_{\alpha}\rho C_{\alpha}^{\dagger}) . \tag{4.2}$$

Eq. (4.2) could then be written out in the Schrödinger picture using (3.5). Since only $U(\sigma, \sigma_0)$ for values of σ less than the last σ_n occur in the chain C_{α} , there is no dependence on the geometry of spacetime to the future of the surface σ_n , whether or not it contains non-chronal regions. In this sense, unitary evolution leads to causality.

If X is not unitary then the probabilities defined by (4.1) depend on the future geometry of spacetime. Experiments could, in principle, detect the existence of non-chronal regions in

our future by testing whether present data is better fit by (4.2) or (4.1) with the appropriate X. We shall return to some simple considerations of such experiments in Section V.

Another way of seeing that information about the future is required to calculate present probabilities is to write $\rho_f = X^{\dagger}X$ and use the cyclic property of the trace to reorganize (4.1) as

$$p(\alpha) = N \ Tr(\rho_f C_\alpha \rho C_\alpha^\dagger) \tag{4.3}$$

where now $N^{-1} = Tr(\rho_f \rho)$. Eq. (4.3) is the formula for the probabilities of a generalized quantum mechanics with both an initial condition ρ and a final condition ρ_f . Such generalizations were discussed in [19] and [22] for the quantum mechanics of closed systems. Information about *both* the future and the past is required to make predictions in the present. In the example under discussion, that information concerns the failure of unitarity in the future arising from non-chronal regions of spacetime.

The notion of state of the system on a spacelike surface provides the most familiar expression of causality in usual quantum mechanics. From a knowledge of the state in the present, all future probabilities may be predicted. Thus the present determines the future. We next show that the generalized quantum mechanics under discussion does not contain such a notion of state.

When the probability formula is the usual (4.2), it is straightforward reformulate it in terms of states on spacelike surfaces. Let σ denote the spacelike surface defining the present, let C_{α} denote a history of alternatives that have already happened, and C_{β} a history of future alternatives whose probabilities we wish to predict. The conditional probability for the future alternatives given the past ones is,

$$p(\beta|\alpha) = p(\beta,\alpha)/p(\alpha)$$
 (4.4)

If the joint probabilities on the right hand side of (4.4) are given by (4.2), then $p(\beta|\alpha)$ can be written in terms of an effective density matrix ρ_{eff} defined on σ as

$$p(\beta|\alpha) = Tr[C_{\beta}\rho_{\text{eff}}(\sigma)C_{\beta}^{\dagger}] , \qquad (4.5)$$

where

$$\rho_{\text{eff}}(\sigma) = \frac{C_{\alpha}\rho C_{\alpha}^{\dagger}}{Tr(C_{\alpha}\rho C_{\alpha}^{\dagger})} \ . \tag{4.6}$$

The density matrix $\rho_{\text{eff}}(\sigma)$ is the usual notion of state on a spacelike surface. As σ advances, $\rho_{\text{eff}}(\sigma)$ is constant in time in this Heisenberg picture until the time of a new alternative is reached at which point it is "reduced" by the addition of a new projection to the chain C_{α} . The conditional probabilities of future decoherent alternatives continue to be given by (4.5) with the new $\rho_{\text{eff}}(\sigma)$.

If the probability formula is (4.1) or (4.3) then it is *not* possible to construct a ρ_{eff} on a spacelike surface from which alone future probabilities can be predicted.⁵ Additional information about the existence of future non-chronal regions summarized by ρ_f in (4.2) is required. There is thus no notion of the state of the system on a spacelike surface in this generalized quantum mechanics.

The existence of non-unitary evolution in the future not only acausally affects the probabilities of present alternatives, it also affects their decoherence. Consider, for example, a set of histories defined by alternatives $\{P_{\alpha}(\sigma)\}$ a single spacelike surface that is before any non-chronal region. The decoherence functional according to (3.13) is

$$D(\alpha', \alpha) = N \operatorname{Tr}[X P_{\alpha'}(\sigma) \rho P_{\alpha}(\sigma) X^{\dagger}] . \tag{4.7}$$

Were X unitary, any set of alternatives automatically decoheres because of the cyclic property of the trace. If X is non-unitary then only certain sets of P_{α} will decohere. Decoherence is therefore acausally affected by the spacetime geometry of the future. However, typical mechanisms of decoherence that involve the rapid dispersal of phase information among ignored variables that interact with those of interest operate essentially locally in time.⁶

⁵This is evident from the formulae (4.1) and (4.3) and has been widely discussed in quantum mechanics in various contexts. See [23,24,21] for recent examples.

⁶See, for example, the discussion in [21] and the references therein.

Such mechanisms may be essentially unaffected by non-unitary evolution in the future. We may, for example, continue to expect the decoherence of alternatives that define the present quasiclassical domain of familiar experience even in the presence of a modest number of future non-chronal regions.

As we have seen, generalized quantum mechanics with non-unitary evolution violates causality because information about the future is required to calculate the probabilities in the present. However, it is important to stress that it is not information about the specific alternatives that occur in the future that is required. Rather it is information about the future geometry of spacetime that enters as a fixed input in this quantum field theory in curved spacetime. Independence of present probabilities on specific alternatives that occur in the future is guaranteed by the probability sum rules that follow from decoherence. If $\{\alpha\}$ denotes a set of alternatives accessible to us and $\{\beta\}$ another set in the future such that $\{\alpha,\beta\}$ decoheres, then the probability of alternative α can be calculated two ways. First it can be calculated directly from (4.1). Second, it can be calculated by saying that one of the alternatives $\{\beta\}$ occurs in the future and summing $p(\alpha,\beta)$ over the unknown values of $\{\beta\}$. These two calculations agree because the alternatives $\{\alpha,\beta\}$ decohere, [cf. (3.4)]

$$p(\alpha) \approx \sum_{\beta} p(\alpha, \beta)$$
 (4.8)

For example⁷, suppose a non-chronal region of spacetime exists in the future but is contained inside an impenetrable box with a door. Observers in the future have the alternatives of opening the door to let fields propagate in this region or leaving it closed and preventing fields from interacting with it. Present probabilities are affected by the existence of such a region, whether the door is opened or not, but unaffected by the specific decision the future observers take.

In a quantum mechanics based on the decoherence of coarse-grained sets of alternative histories, probability sum rules like (4.8) hold in much wider circumstances than those

⁷Suggested to the author by J. Friedman

described above. For example, the probabilities of alternatives $\{\alpha\}$ in the present are independent of specific unknown alternatives $\{\beta\}$ whether these are in the future, past or spacelike separated from $\{\alpha\}$ provided the joint set of alternatives decoheres. Even in a quantum theory of gravity where spacetime geometry is a dynamical variable, and we could envision observers in the future deciding whether to create non-chronal regions or not, similar results would be expected to hold. Thus we could not determine by present observations whether observers in the future will decide to build Morris, Thorne, Yurtsever-type time machines.

V. TESTING NON-UNITARY EVOLUTION AND CAUSALITY VIOLATION

How might the non-unitary evolution and causality violation of the present generalized quantum mechanics of non-chronal spacetimes be tested in the laboratory? This section offers a preliminary discussion.

We begin with a simple model of scattering through a non-chronal region of spacetime. More specifically, we imagine that a small non-chronal region of spacetime has been located and that we direct particle beams so as to interact in that region, measuring their asymptotic states by apparatus that does not itself interact with the non-chronal region to a good approximation. The incoming particles are prepared in pure initial states and final pure states are detected by the apparatus. This is certainly not the most general measurement situation that can be envisioned but gives a simple illustration of the effects of non-unitarity.⁸

We suppose the Hilbert space of the closed system factors into a tensor product, $\mathcal{H}_s \otimes \mathcal{H}_r$ of a Hilbert space \mathcal{H}_s describing the scattering particles and a Hilbert space \mathcal{H}_r describing the rest, including the apparatus for preparation and detection. We consider an initial state

⁸A more general measurement theory may be exhibited along the lines described in Ref. [6], Section II.10. It is subject to limitations of the character described in Ref. [7] regarding the influence of the measuring apparatus on the probabilities of the measured alternatives.

 ρ at time t_1 corresponding to the preparation of the experiment as described above. We consider the alternatives for the scattering particles in which they are prepared and detected in members of complete sets of states $\{|\alpha,t\rangle\}$ in \mathcal{H}_s at various times (e.g., wave packets with approximately definite momentum). These alternatives are represented as a set of projection operators

$$S_{\alpha}(t) = |\alpha, t\rangle\langle\alpha, t| \otimes I_r \tag{5.1}$$

in the Heisenberg-like picture represented in Section III.

The histories describing the scattering process are represented by the chains $S_{\beta}(t_2)S_{\alpha}(t_1)$ where t_1 and t_2 are the initial and final times of the scattering process. The decoherence functional for these histories is then [cf. (3.9)]

$$D(\beta', \alpha'; \beta \alpha) = N \operatorname{Tr} \left[S_{\beta'}(t_2) X S_{\alpha'}(t_1) \rho S_{\alpha}(t_1) X^{\dagger} S_{\beta}(t_2) \right]$$
(5.2)

where N is given by (3.6).

The defining assumption of the model is that, with an initial ρ appropriate to the experimental set-up described above, the non-unitary evolution operator X effectively acts only on \mathcal{H}_s . Noting that the decoherence of the alternatives β at time t_2 is automatic because of the cyclic property of the trace, we may then write

$$D(\beta', \alpha'; \beta, \alpha) = \delta_{\beta'\beta} N \langle \beta, t_2 | X | \alpha', t_1 \rangle \langle \alpha', t_1 | Sp(\rho) | \alpha, t_1 \rangle \langle \alpha, t_1 | X^{\dagger} | \beta, t_2 \rangle$$
(5.3)

where $Sp(\rho)$ is the operator on \mathcal{H}_s that is the trace of ρ over \mathcal{H}_r and X is the restriction of the non-unitary evolution to \mathcal{H}_s .

The decoherence of the measured alternatives α at the initial time t_1 is not automatic. However, in this model of a measurement we assume that the interaction of the particles with the apparatus effects the decoherence of the alternative initial states of the particles, say, by correlation with an independent, persistent record of the their initial state (as in Ref. [6], Section II.10). Effectively we assume

$$\langle \alpha', t_1 | Sp(\rho) | \alpha, t_1 \rangle \propto \delta_{\alpha'\alpha} .$$
 (5.4)

The joint probabilities for this now decoherent set of histories are

$$p(\beta, \alpha) = N |\langle \beta, t_2 | X | \alpha, t_1 \rangle|^2 \langle \alpha, t_1 | Sp(\rho) | \alpha, t_1 \rangle . \tag{5.5}$$

In scattering experiments it is not so much the joint probability $p(\beta, \alpha)$ of both initial and final states that is of interest, but rather the conditional probability $p(\beta|\alpha)$ of a final state, β , given an initial one, α . This is constructed in the standard way [cf. (4.4)] with the result

$$p(\beta|\alpha) = \frac{|\langle \beta, t_2 | X | \alpha, t_1 \rangle|^2}{\sum_{\beta} |\langle \beta, t_2 | X | \alpha, t_1 \rangle|^2} .$$
 (5.6)

All reference to the external apparatus has canceled from this effective expression for conditional probabilities. Except for the normalizing denominator, (5.6) is the usual expression the probability of a scattering process. Indeed, were X unitary, the denominator would be unity. The net effect of the non-unitarity of X has simply been to normalize the usual quantity $|\langle \beta t_2 | X | \alpha t_1 \rangle|^2$ so that the probability sum rule $\Sigma_{\beta} p(\beta | \alpha) = 1$ is satisfied.

Were small size non-chronal regions widespread in spacetime, the difference between the probabilities predicted by (5.6) and a standard formula for the same scattering in flat spacetime would be both a means of detecting such non-chronal regions and verifying the non-unitarity of evolution through them. In the absence of estimates of the sizes and density of non-chronal regions and of the X's which describe the evolution through them, we cannot provide estimates of the effect of non-unitary evolution here. It is through (5.6), however, that such effects would be calculated [25].

In a similar way, we could estimate the acausal effects of future non-chronal regions on present experiments. We would compare standard flat space formulae for probabilities with those computed from formulae like (3.13) with a non-unitary X describing the effect of non-chronal regions in the future on present measurement situations. We would then be led to formulae like (5.3) or (5.6) for the probabilities of the measured subsystem but with the non-unitary X to the future of the projections describing the measurement. The measurement situation would have a chance of detecting a departure from strict causality

only if there were a significant probability that the subsystem under study interacted with a non-chronal region subsequent to the time of the experiment. If non-chronal regions are sparse in the future history of spacetime, then we might expect this probability to be very, very small and the resulting violation of causality negligible. If, however, there were a roiling sea of non-chronal regions near a generic final singularity, then the probabilities for causality violation might be more interesting. The present generalized quantum mechanics provides a way of estimating them.

VI. INFORMATION

In usual quantum mechanics, the state of a system on a spacelike surface σ is as complete a description of that system as it is possible to give. When the state is given, the missing information about the system is zero. When it is only possible to give probabilities for an ensemble of possible states, the system is described by a (Schrödinger-picture) density matrix $\rho_{\text{eff}}(\sigma)$. The missing information is then given more generally by

$$S(\sigma) = -Tr[\rho_{\text{eff}}(\sigma)\log\rho_{\text{eff}}(\sigma)]$$
(6.1)

which vanishes when the probability is unity for a single state and zero for all orthogonal ones, *i.e.*, when ρ is pure. This missing information is conserved under unitary evolution.

The generalized quantum mechanics of matter fields in non-chronal spacetimes that is under discussion in this paper does not generally permit a notion of "state on a spacelike surface" as discussed in Section IV. How then do we define the information available about these fields on a spacelike surface? How does this information evolve? This Section is devoted to some answers to these questions.

As in the discussion of Sections III and IV, we begin by considering a spacetime with a single non-chronal region. We will consider two spacelike surfaces σ' and σ'' the first of which is *before* the non-chronal region and the second *after* (Figure 1). On these surfaces we consider *decoherent* sets of alternatives represented by sets of orthogonal, commuting

projection operators $\{P'_{\beta}\}$ and $\{P''_{\beta}\}$ respectively. The probabilities of these alternatives are given by the diagonal elements of the decoherence functional, specifically [cf. (3.13) and (3.14)]

$$p\left(\alpha; \sigma', \{P'_{\beta}\}\right) = N \operatorname{Tr}\left(X P'_{\alpha} \rho P'_{\alpha} X^{\dagger}\right) , \qquad (6.2a)$$

$$p\left(\alpha; \sigma'', \left\{P_{\beta}''\right\}\right) = N \operatorname{Tr}\left(P_{\alpha}'' X \rho X^{\dagger} P_{\alpha}''\right) . \tag{6.2b}$$

The assumed decoherence of the alternatives allows the probabilities (6.2) to be written in other useful ways. When (3.1) is satisfied

$$p(\alpha) \equiv D(\alpha, \alpha) \approx \sum_{\alpha'} D(\alpha', \alpha) \approx \sum_{\alpha'} D(\alpha, \alpha')$$
 (6.3)

yielding the expressions

$$p\left(\alpha; \sigma', \left\{P'_{\beta}\right\}\right) \approx N Tr\left(P'_{\alpha}\rho X^{\dagger}X\right) \approx N Tr\left(P'_{\alpha}X^{\dagger}X\rho\right)$$
 (6.4a)

$$p\left(\alpha; \sigma'', \left\{P''_{\beta}\right\}\right) \approx N Tr\left(P''_{\alpha}X\rho X^{\dagger}\right)$$
 (6.4b)

These relations may be written more symmetrically as

$$p\left(\alpha; \sigma', \left\{P'_{\beta}\right\}\right) \approx Tr\left(P'_{\alpha}\hat{\rho}\right) ,$$
 (6.5a)

$$p\left(\alpha; \sigma'', \left\{P_{\beta}''\right\}\right) \approx Tr\left(P_{\alpha}''\bar{\rho}\right) ,$$
 (6.5b)

$$Tr\left(\rho[X^{\dagger}X,\rho]\right) \approx 0 ,$$
 (6.5c)

where $\hat{\rho}$ and $\bar{\rho}$ are the density matrices

$$\hat{\rho} = \{\rho, X^{\dagger}X\}/Tr(X\rho X^{\dagger}) , \qquad (6.6)$$

$$\bar{\rho} = X \rho X^{\dagger} / Tr(X \rho X^{\dagger}) , \qquad (6.7)$$

 $\{\cdot,\cdot\}$ denoting an anticommutator. These relations will be helpful in what follows.

With these preliminaries we turn to a definition of the missing information on a spacelike surface σ . First, generalizing a construction of Jaynes [26,21], we define, $S(\sigma; \{P_{\beta}\})$, the missing information on σ relative to a set of alternatives $\{P_{\beta}\}$ whose probabilities are $\{p_{\beta}\}$. This is the maximum of the entropy functional

$$S(\tilde{\rho}) = -Tr\left(\tilde{\rho}\log\tilde{\rho}\right) \tag{6.8}$$

over all density matrices $\tilde{\rho}$ that reproduce the probabilities p_{α} through $Tr(P_{\alpha}\tilde{\rho}) = p_{\alpha}$. Of course, it is possible to lose information about a system by asking stupid questions. To obtain a good measure of the missing information on a spacelike surface, $S(\sigma)$, we, therefore, minimize $S(\sigma; \{P_{\beta}\})$ over all decohering sets of alternatives $\{P_{\beta}\}$. Explicitly then,

$$S(\sigma) = \min_{\substack{\text{decoherent} \\ \{P_{\alpha}\}}} \left\{ \max_{\substack{\tilde{\rho} \text{ with} \\ Tr(P_{\alpha}\tilde{\rho}) = p_{\alpha}}} S(\tilde{\rho}) \right\} . \tag{6.9}$$

If we assume for a moment that the projections $\{P_{\alpha}\}$ are all onto finite dimensional subspaces of the Hilbert space \mathcal{H} , then the density matrix that maximizes $\mathcal{S}(\tilde{\rho})$ subject to the probability constraint is a standard construction

$$\tilde{\rho}_{\text{max}} = \sum_{\alpha} \left[p(\alpha; \sigma, \{P_{\beta}\}) P_{\alpha} / Tr(P_{\alpha}) \right] , \qquad (6.10)$$

and

$$S(\sigma; \{P_{\beta}\}) = S(\tilde{\rho}_{\text{max}}) = \sum_{\alpha} p(\alpha; \sigma, \{P_{\beta}\}) \log \left[p(\alpha; \sigma, \{P_{\beta}\}) / Tr(P_{\alpha}) \right] . \tag{6.11}$$

The limit of $\{P_{\alpha}\}$ with infinite trace can then be considered.

Eq. (6.9) defines the missing information a spacelike surface σ and holds whether the probabilities $\{p_{\alpha}\}$ for the alternatives $\{P_{\alpha}\}$ follow from a notion of state on the spacelike surface, as in usual quantum mechanics, or from a generalized quantum mechanics decoherence functional as in (3.13). It is instructive to see how the familiar result (6.1), is a consequence of this definition when the probabilities p_{α} are given by a density matrix $\rho_{\text{eff}}(\sigma)$ through

$$p_{\alpha} = Tr \left[P_{\alpha} \rho_{\text{eff}}(\sigma) \right] . \tag{6.12}$$

To see this note that from (6.10), that when (6.12) holds

$$-Tr\left[\tilde{\rho}_{\max}\log\tilde{\rho}_{\max}\right] = -Tr\left[\rho_{\text{eff}}(\sigma)\log\tilde{\rho}_{\max}\right] . \tag{6.13}$$

The inequality⁹

$$-Tr\left(\rho_1\log\rho_2\right) \ge -Tr\left(\rho_1\log\rho_1\right) , \qquad (6.14)$$

which holds for any two density matrices ρ_1 and ρ_2 , then implies

$$S(\alpha; \{P_{\alpha}\}) \equiv \mathcal{S}(\tilde{\rho}_{\text{max}}) \ge \mathcal{S}[\rho_{\text{eff}}(\sigma)]$$
 (6.15)

The lower bound is reached by choosing the projections $\{P_{\alpha}\}$ to be onto a basis in which $\rho_{\text{eff}}(\sigma)$ is diagonal. In this connection the $Tr(P_{\alpha})$ term in (6.11) which contributes $+\Sigma_{\alpha}p(\alpha)\log Tr(P_{\alpha})$ to the missing information is important. It is that term which favors choosing P_{α} which are as refined (low dimensional) as possible. In usual quantum mechanics, the decoherence functional for alternatives defined on a single spacelike surface is $D(\alpha', \alpha) = Tr[P_{\alpha'}\rho_{\text{eff}}(\sigma)P_{\alpha}]$, so that any set of alternatives $\{P_{\alpha}\}$ decoheres because of the cyclic property of the trace. When the minimization of $S(\sigma; \{P_{\alpha}\})$ over decoherent alternatives is carried out in the definition (6.9), we therefore find

$$S(\sigma) = \min_{\{P_{\beta}\}} S(\sigma; \{P_{\beta}\}) = -Tr\left[\rho_{\text{eff}}(\sigma)\log\rho_{\text{eff}}(\sigma)\right]$$
(6.16)

— the familiar and anticipated result.

We now return to the question of the relation between the missing information $S(\sigma')$ and $S(\sigma'')$ on two spacelike surfaces before and after a non-chronal region in the generalized quantum theories of Section III. Eq. (3.9) shows that the decoherence functional for alternatives after the last non-chronal region is the same as that of usual quantum mechanics with the effective density matrix $\bar{\rho}$ defined in (6.7). In particular, the cyclic property of the trace shows that alternatives confined to a single surface after all non-chronal regions always decohere. We therefore have from the argument in the preceding paragraph

$$S(\sigma'') = S(\bar{\rho}) = S\left(\frac{X\rho X^{\dagger}}{Tr(X\rho X^{\dagger})}\right). \tag{6.17}$$

 $^{^9\}mathrm{For}$ a convenient proof, see [27] or the discussion in the Appendix.

The missing information (6.17) bears no special relation to the missing information in ρ that would be calculated in usual quantum mechanics according to (6.1). Indeed, one can show that, for any ρ , there is always an operator X that will give $S(\bar{\rho})$ any value from zero up to its maximum. That fact, however, is of no special interest in generalized quantum mechanics. Both X and ρ are needed in the computation of all probabilities and there are thus no alternatives for which $S(\sigma)$ is given by (3.1).

The missing information, $S(\sigma')$ on a surface σ' before a non-chronal region is not as easily calculable as $S(\sigma'')$ because the strictures of decoherence are non-trivial. However, it is possible to show that $S(\sigma')$ cannot be less than $S(\sigma'')$. The demonstration proceeds along the lines of the evaluation of $S(\sigma)$ for usual quantum mechanics as derived above, but requires a more general inequality than (6.14). Specifically, it requires the inequality

$$-Tr(A\log \rho_2) \ge -Tr(A\log A) \tag{6.18}$$

valid for any density matrix ρ_2 and linear operator A of the form

$$A = B\rho_1 B^{-1} (6.19)$$

for some B and density matrix ρ_1 , such that the left-hand-side of (6.18) is real and positive. The condition (6.19) is enough to allow the definition of $\log A$. This and the derivation of (6.18) are discussed in the Appendix.

We can apply this inequality to obtain a lower bound on $S(\sigma'; \{P_{\beta}\})$ as given by (6.11) by noting that, according to (6.4a), the probabilities for decohering alternatives $\{P_{\alpha}\}$ on σ' may be written

$$p(\alpha; \sigma', \{P_{\beta}\}) = Tr(P_{\alpha}A)$$
(6.20)

where A is the operator

$$A = \frac{X^{\dagger} X \rho}{Tr\left(X^{\dagger} X \rho\right)} \ . \tag{6.21}$$

The operator A is not a density matrix. (It is not generally even Hermitian.) However, it is of the form (6.19) with $B = (X^{\dagger})^{-1}$ and $\rho_1 = \bar{\rho} = X \rho X^{\dagger} / Tr(X \rho X^{\dagger})$. It is an elementary calculation in a basis in which $\tilde{\rho}_{\text{max}}$ of (6.10) is diagonal to verify

$$-Tr\left(A\log\tilde{\rho}_{\max}\right) = -Tr\left(\tilde{\rho}_{\max}\log\tilde{\rho}_{\max}\right) \tag{6.22}$$

We may use this and (6.18) with $\rho_2 = \tilde{\rho}_{max}$ to derive

$$S(\sigma; \{P_{\beta}\}) \equiv S(\tilde{\rho}_{\max}) \ge -Tr(A\log A). \tag{6.23}$$

However, the cyclic property of the trace together with the definition of $\log A$ (see Appendix) imply that $Tr(A \log A) = Tr(\bar{\rho} \log \bar{\rho})$ which is $S(\sigma'')$ [cf. (6.17)]. When the minimum of the left-hand side of (6.23) is taken over all decoherent sets $\{P_{\beta}\}$ on σ' , we therefore have

$$S(\sigma') \ge S(\sigma'') = S(\bar{\rho}) = S\left(\frac{X\rho X^{\dagger}}{Tr\left(X\rho X^{\dagger}\right)}\right)$$
 (6.24)

Thus, information can be gained but not lost in evolving from a spacelike surface before a non-chronal region to a spacelike surface after all such regions.

The inequality (6.24) becomes an equality in the special case that $X^{\dagger}X$ commutes with ρ . Then the projections onto a basis in which ρ and $X^{\dagger}X$ are simultaneously diagonal are decoherent and give $\tilde{\rho}_{\text{max}} = \bar{\rho}$. If $X^{\dagger}X$ and ρ do not commute, then there certainly is no set of one-dimensional projections that are decoherent, so one would conjecture that the bound provided by $S(\bar{\rho})$ is not realized. The determination of an optimum lower bound on $S(\sigma')$ then becomes an interesting question.

The possibility of information gain in moving from one spacelike surface is not surprising from the point of view of the requirements of decoherence. On a spacelike surface σ'' to the future of all non-chronal regions any set of alternatives $\{P_{\beta}\}$ is decoherent. On a surface σ' to the past of some non-chronal regions only certain sets of alternatives will decohere. There are thus more questions with which to extract information about the quantum system on σ'' than on σ' and a corresponding decrease in missing information is to be expected.

We have so far considered the case of a spacetime with a single non-chronal region, but a general spacetime may have many. When spacelike surfaces may be passed between such regions, we may consider the missing information on each and the evolution of the missing information from surface to surface. In general, we do not expect any particular relation between the values of $S(\sigma)$ as we pass from one surface to a later one. It may increase or it may decrease depending on the requirements of decoherence. However, on each surface the missing information must satisfy the inequality (6.24) where σ'' is to the future of all non-chronal regions. On such a surface the strictures of decoherence are least and the information available about the system the most.

As we discussed in the Introduction, generalized quantum mechanics permits quantum theory to be formulated in fully four-dimensional form that does not rely on a notion of state on a spacelike surface. In such a theory complete information about a system is not necessarily available on every spacelike surface. Rather the appropriate notion of information is itself four-dimensional as described in this Section. It may be necessary to search in many regions of spacetime to recover complete information about a system [28].

VII. NO NON-CONSERVATION OF ENERGY, SIGNALING FASTER THAN LIGHT, OR EVERETT PHONES

Generalized quantum mechanics is a modest generalization of familiar quantum theory that retains the principle of the linear superposition of amplitudes in the form (3.3). Generalized quantum theories, such as the one under discussion, may have a notion of a Heisenberg-like state that specifies the initial and final conditions, but will not always permit a notion of an evolving state on a spacelike surface. Various other generalizations of quantum mechanics have been proposed that retain the notion of a state on a spacelike surface but abandon or modify the principle of superposition in some way. Recent examples, are the work of Banks, Peskin, and Susskind [29] and Srednicki [30] in which pure density matrices evolve into mixed ones, and Weinberg's non-linear quantum mechanics [31]. The generalization of Banks, Peskin, and Susskind suffers from energy non-conservation while that of Weinberg can permit communication faster than light and communication with alternative branches of the universe in situations that have been called the "Everett phone" by Polchinski [32].

The above generalizations of quantum mechanics are non-linear because they incorporate a non-linear law of evolution for states on a spacelike surface. The generalized quantum mechanics for non-chronal spacetimes under discussion in this paper cannot be characterized as linear or non-linear in this way because it does not generally permit a notion of state on a spacelike surface much less a discussion of the law for its evolution. This generalization does respect the linear principle of superposition in the sense of (3.3). However, because of the normalization factor (3.6) probabilities are not quadratically related to a pure state vector describing the initial condition as they would be in usual quantum mechanics. For this reason it is prudent to examine the present generalized quantum mechanics for energy non-conservation, signaling faster than light, and Everett phones. In this Section we shall show that decoherence prohibits all of these anomalies. Our arguments apply to all generalized quantum theories although we shall describe them here for the particular case of the generalized quantum mechanics of fields in non-chronal spacetimes.

Energy Conservation

When spacetime geometry is time-dependent we do not expect conservation of the total energy of matter fields moving in it, even classically. Where there are compact non-chronal regions of spacetime, the geometry will certainly be time dependent. However, we can still analyse the question of energy conservation in those regions where spacetime is locally time independent. Specifically, consider a region of spacetime that is foliable by spacelike surfaces labeled by a coördinate t such that $\partial/\partial t$ is a Killing vector which asymptotically corresponds to a time-translation in some Lorentz frame. We can then define the energy-momentum four-vector of the matter fields on a spacelike surface of constant t as

$$P^{\alpha}(t) = \int_{t} d\Sigma^{\beta} T_{\beta}^{\alpha} \tag{7.1}$$

where T^{α}_{β} is the stress-energy of the matter fields and $d\Sigma^{\beta}$ is an element of the surface of constant t. In particular $P^{t} \equiv H$ is the total energy of the matter fields and the corresponding quantum mechanical operator is the generator of translations in t. The total energy is conserved between surfaces of constant t because $\partial/\partial t$ is a Killing vector, which means that

the operator H is independent of time.

Whether energy is conserved quantum mechanically is a question of the probabilities for the correlation of the values of H on two different surfaces of constant t. A specific example of the calculation of such probabilities will illustrate all the features of the general case. Consider a single non-chronal region as discussed in Section III, and suppose that before the non-chronal region there is a region of spacetime with a time-translation symmetry in the sense discussed above. Let $\{P_{\alpha}^{H}(t)\}$ denote a set of projections onto an exhaustive set of ranges $\{\Delta_{\alpha}\}$ of the total energy in matter fields, H, in the Heisenberg-like picture specified by (3.7a). Since H is independent of t before the non-chronal region, the projections $\{P_{\alpha}^{H}(t)\}$ are also.

Now consider a set of histories which contain the projections $\{P_{\alpha}^{H}(t)\}$ at two different times t_1 and t_2 in the region of time-translation symmetry. The chain of projections before σ_{-} [cf. (3.10)] would have the form

$$C_{\alpha} = C_{\alpha_c}^c P_{\alpha_2}^H(t_2) C_{\alpha_b}^b P_{\alpha_1}^H(t_1) C_{\alpha_a}^a \tag{7.2}$$

where $C_{\alpha_a}^a$, $C_{\alpha_b}^b$, and $C_{\alpha_c}^c$ are themselves chains of projections. Suppose this set of alternative histories decoheres. The joint probabilities for the individual histories may be calculated from (3.1) and (3.12). Conservation of energy would mean

$$p(\beta, \alpha_c, \alpha_2, \alpha_b, \alpha_1, \alpha_a) \propto \delta_{\alpha_1 \alpha_2} \tag{7.3}$$

for any choice of the other alternatives α_a , α_b , α_c and β . Eq. (7.3) is not a consequence of any operator identity since $C_{\alpha_b}^b$ in (7.2) need not commute with H. However, it is a consequence of decoherence.¹⁰ Decoherence guarantees the consistency of probability sum rules. Thus, in particular

¹⁰The argument appears to be part of the lore of consistent histories. The author learned it from R.W. Griffiths. For a more detailed discussion including a consideration of approximate decoherence see [33].

$$p(\alpha_2, \alpha_1) = \sum_{\beta, \alpha_c, \alpha_b, \alpha_a} p(\beta, \alpha_c, \alpha_2, \alpha_b, \alpha_1, \alpha_a) . \tag{7.4}$$

However, the history which consists just of alternative values of the energy at time t_1 and t_2 is represented by the chain

$$P_{\alpha_2}^H(t_2)P_{\alpha_1}^H(t_1) \propto \delta_{\alpha_2\alpha_1} . \tag{7.5}$$

This chain vanishes unless $\alpha_1 = \alpha_2$ because the projections onto the values of a conserved quantity are independent of time and projections for different alternatives are orthogonal. Thus, $p(\alpha_2, \alpha_1) \propto \delta_{\alpha_1 \alpha_2}$. Since the right-hand side of (7.4) is a sum of positive numbers, (7.3) follows also.

Obvious extensions of this argument show that, in general, decoherence guarantees the conservation of energy in regimes of spacetime that possess a time-translation symmetry in the sense described above.

No Signaling Faster than Light

The meaning of "signaling faster than light" requires careful definition. It is not simply a matter of being able to infer the probabilities of alternatives in one spacetime region from alternatives in a spacelike separated region. The EPRB situation in which two spins prepared in a singlet state move into separate spacelike regions is an example. From a determination of the spin direction of one it is possible to infer the spin of the other. That, however, is not a signal sent faster than light. The determination only exploits a correlation present in the initial state. To investigate whether it is possible to signal faster than light we should investigate whether from alternatives in one spacetime region one can infer the probabilities of another system in another spacelike separated region when the two systems were *initially uncorrelated* and remained spacelike separated at all subsequent times.

A classic example in the EPRB situation is the question of whether carrying out a measurement on one spin can influence the probabilities of the second. From a closed system point of view there is a third system — the apparatus — which is initially uncorrelated with either spin and which subsequently becomes correlated with one of them with a certain probability while the remaining spin remains always spacelike separated. As we shall show

below it is straightforward to show from the causality of field theory that spacelike separated alternatives remain uncorrelated if they were initially uncorrelated and there is no signaling faster than light.

When spacetime possesses non-chronal regions, correlations arise not only from an initial condition but also from future non-chronal regions. We remarked in Section IV that the formula (4.3) for the probabilities of alternatives to the past of all non-chronal regions was like that of quantum mechanics with a final as well as an initial condition. Systems that are now spacelike separated may be in causal contact both in the past and future and correlations arising from both initial and final conditions could exist in the present between them. We should count neither as signaling faster than light. To meaningfully consider the possibility of signaling faster than light, we should consider two systems that are uncorrelated both with respect to the initial condition of the system and also any conditions that may exist in the future.

The situation may be illustrated with a simple example. Consider two spacelike separated regions R_1 and R_2 whose pasts intersect the initial surface in disjoint regions D_1 and D_2 . Suppose that the fields in D_1 and D_2 are initially uncorrelated. This means that the initial state can be written as $\rho_1 \otimes \rho_2$ where the fields in D_1 act only on ρ_1 and the fields in D_2 act only on ρ_2 .

Similarly, suppose that there are no non-chronal regions to the past of either R_1 or R_2 and no non-chronal regions in the common future of both D_1 and D_2 . In particular, this means that there are no common non-chronal regions in the future of both R_1 and R_2 and ensures that there are no correlations between fields in these regions by virtue of future non-chronal regions.

In a causal field theory the field operators in a spacetime region R are related by the Heisenberg equations of motion only to operators in the future and past of that region. The non-unitary evolution X thus factors into a part X_1 acting only on ρ_1 , referring to the non-chronal regions in the future of D_1 , and a similar X_2 acting on ρ_2 . The joint probability of a decoherent set of alternatives $P_{\alpha_1}(R_1)$ referring to region R_1 and another $P_{\alpha_2}(R_2)$ referring

to R_2 then also factors

$$p(\alpha_1, \alpha_2) = N_1 T r_1 \left[X_1^{\dagger} X_1 P_{\alpha_1}(R_1) \rho_1 \right] N_2 T r_2 \left[X_2^{\dagger} X_2 P_{\alpha_2}(R_2) \rho_2 \right] . \tag{7.6}$$

where $N_1 = Tr_1(X_1\rho_1X_1^{\dagger})$ and $N_2 = Tr_2(X_2\rho_2X_2^{\dagger})$. The causality of field theory together with the absence of initial and final correlations thus implies the factorization of joint probabilities of alternatives in spacelike separated regions

$$p(\alpha_1, \alpha_2) = p(\alpha_1) p(\alpha_2) . (7.7)$$

The probabilities of alternatives in spacelike separated regions are thus independent, and signaling faster than light is not possible.

The fact that it is not possible to signal faster than light in the absence of built in correlations does not mean that it would not be interesting to investigate exploiting the correlations between spacelike separated regions provided by non-chronal regions in their common future.

Everett Phones

In his analysis of Weinberg's non-linear quantum mechanics, Polchinski [32] has given an example of a kind of "communication" between different branches of the wave function that he dubbed the "Everett phone". Specifically, if briefly, he considers sets of histories of a spin $-\frac{1}{2}$ ion in a Stern-Gerlach apparatus and a "macroscopic observer". At time t_1 , the z-component of the spin is determined by the splitting of the Stern-Gerlach beams. At time t_2 , if the spin was up, no action is taken by the observer. If the spin was down, it is either left alone or flipped with some probability. At time t_3 , if the spin was up at time t_1 , the z-component of the spin is again determined. In Weinberg's non-linear quantum mechanics, the probability of the measurement of the spin at time t_3 , in the branch where the spin was up at time t_1 , depends on whether the observer did or did not flip the spin in the alternative branch where the spin was down at time t_1 and the measurement at t_3 does not occur. That is the "Everett phone". In the language of the quantum mechanics of closed systems this is simply an inconsistent set of histories.

These histories of the closed system spin and observer are represented by a sequence of three branch dependent sets of projections at the times t_1, t_2 , and t_3 . They are branch dependent because, whether the alternatives {flip, no flip} or the trivial unit projection are used at time t_2 depends on the specific alternatives {up, down} at time t_1 . Similarly the sets of projections used at t_3 depend on the specific alternatives at time t_1 . Let { $P_{\uparrow}(t), P_{\downarrow}(t)$ } be the projections representing whether the spin is up or down at time t. Let { $P_{f}(t_2), (P_{\bar{f}}(t_2))$ } represent the alternatives that the spin was flipped or not flipped. The four histories in the set described by Polchinski would be represented by the chains

$$P_{\uparrow}(t_3)I(t_2)P_{\uparrow}(t_1) \quad , \quad I(t_3)P_f(t_2)P_{\downarrow}(t_1) \quad ,$$

$$P_{\downarrow}(t_3)I(t_2)P_{\uparrow}(t_1) \quad , \quad I(t_3)P_{\bar{f}}(t_2)P_{\downarrow}(t_1) \quad ,$$

$$(7.8)$$

where trivial unit projections have been included for clarity and vanishing chains have been omitted.

Branch dependence is not an obstacle to defining the decoherence of a set of histories [20,21,34]. If the above set decohered, the probabilities of the individual histories would be given by

$$p(\alpha) = N Tr \left(X C_{\alpha} \rho C_{\alpha}^{\dagger} X^{\dagger} \right) \tag{7.9}$$

where C_{α} , $\alpha = 1, 2, 3, 4$, is one of the four chains in (7.8). The probabilities of histories in which the spin is up or down at t_3 are independent of whether the spin was flipped or not flipped at t_2 simply because the corresponding chains contain neither the projection P_f nor $P_{\bar{f}}$.

The above is a specific example of a general situation. Consider a decoherent set of branch dependent histories $\{\alpha\}$. Partition this set of histories into the class consisting of a single history α and the class $\neg \alpha$ consisting of all other histories. That partition is a coarse graining of the set $\{\alpha\}$ and so is also decoherent. In the coarser-grained set, the probability of α remains $p(\alpha)$. The probability of $\neg \alpha$ is

$$p(\neg \alpha) = \sum_{\beta \neq \alpha} p(\beta) . \tag{7.10}$$

Thus both $p(\alpha)$ and $p(\neg \alpha)$ are manifestly independent of alternatives in the other branches. There are no "Everett phones" in generalized quantum mechanics. Decoherence guarantees the independence of individual branches.

VIII. CONCLUSION

The familiar quantum mechanics of unitarily evolving states on spacelike surfaces depends centrally on the existence of a fixed background spacetime geometry with a well-defined causal structure that is foliable by spacelike surfaces on which the states can be defined. When spacetime geometry is not fixed, as in quantum gravity, or when it is fixed but not foliable by spacelike surfaces, some modification of familiar quantum theory seems inevitable. Generalized quantum theory provides a broad framework for constructing extensions of familiar quantum theory that can apply when spacetime is not fixed [9] or when it is fixed but not foliable by spacelike surfaces. Such theories are unlikely to permit a notion of a unitarily evolving state on a spacelike surface or possess familiar notions of causality.

In this paper we have discussed a generalized sum-over-histories quantum mechanics for matter fields in background spacetimes with non-chronal regions. The geometry is fixed and given once and for all time. The matter fields do not modify it. Alternatives are defined four-dimensionally as partitions of spacetime field configurations — a notion general enough to describe alternatives in the non-chronal regions which are not foliable by spacelike surfaces. Transition amplitudes between alternatives on spacelike surfaces outside the non-chronal regions are defined by sums of $\exp[i(\arctan)]$ over intermediate field configurations. The non-unitarity of such transition amplitudes can be incorporated into generalized quantum theory through an appropriately defined notion of decoherence. All probability sum rules are satisfied for decoherent alternatives because decoherence implies them.

This generalized quantum theory of fields in spacetimes with non-chronal regions does not display a number of familiar features of quantum theory in flat background spacetime. Most importantly the theory cannot be reformulated in terms of states on spacelike surfaces. That is not surprising since the spacetime itself does not possess a foliating family of spacelike surfaces. Lost with the notion of state is the familiar idea of causality in the sense that the entire four-dimensional spacetime geometry past, present, and future must be known to establish the decoherence and predict the probabilities of alternatives in the present.

Fundamentally spacetime geometry is not fixed but variable quantum mechanically. Quantum fluctuations in spacetime geometry are central to a discussion of non-chronal regions because it is only through the intervention of quantum gravity that spacetimes with non-chronal regions could ever evolve [35,36]. The present generalized quantum mechanics of matter fields in a fixed background spacetime is thus only a model or an approximation to a more general quantum mechanics including the gravitational field. It serves to illustrate, however, how much of the structure of familiar quantum mechanics is tied to assumptions concerning the character of spacetime geometry and what departures from this structure we may expect in a generalized quantum mechanics of geometry as well as matter fields.

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APPENDIX:

In this Appendix we shall provide derivation and discussion of a few results concerning the entropy functional used in Section VI. We consider operators A that can be represented as

$$A = B\rho_1 B^{-1} \tag{A1}$$

for some density matrix ρ_1 . We then define

$$\log A = B \log \rho_1 B^{-1} \tag{A2}$$

where $\log \rho_1$ is the Hermitian matrix that is diagonal in the basis in which ρ_1 is diagonal with diagonal elements $\log p_i$ if p_i are the eigenvalues of ρ . We may then construct the entropy functional of A and note that

$$S(A) = -Tr(A\log A) = -Tr(B\rho_1 \log \rho_1 B^{-1})$$
$$= -Tr(\rho_1 \log \rho_1) . \tag{A3}$$

The inequality (6.18)

$$-Tr(A\log \rho_2) \ge -Tr(A\log A) , \qquad (A4)$$

where ρ_2 is any density matrix such that the left-hand side is real and positive, may be proved by extremizing the functional $-Tr(A \log C)$ over all operators C with Tr(C) = 1 (and thus over density matrices in particular). Using a Lagrange multiplier μ to enforce the constraint Tr(C) = 1, the condition for an extremum is

$$\delta_C \left[-Tr(A\log C) - \mu Tr(C) \right] = 0 . \tag{A5}$$

This yields

$$AC^{-1} = \mu . (A6)$$

Taking Tr(C) = 1 into account, this can only be satisfied by

$$C = A. (A7)$$

since Tr(A) = 1 from (A1). Since C = A is the unique extremum of $-Tr(A \log C)$ among all operators with unit trace, it will also be an extremum among operators C such that $-Tr(A \log C)$ is real and positive because, from (A3), $-Tr(A \log A)$ is real and positive. To complete the demonstration of the inequality (A4) it remains only to show that the extremum is a minimum. However, it is straightforward to exhibit density matrices ρ_2

for which $-Tr(A \log \rho_2)$ is greater than $-Tr(A \log A)$ by using the basis $|i\rangle$ in which ρ_2 is diagonal with eigenvalues λ_1 to write

$$-Tr(A\log \rho_2) = \sum_{i} (-\log \lambda_i) \langle i|A|i\rangle.$$
 (A8)

If the dimension of the Hilbert space is a finite number N, choose $\lambda_i = 1/N$. Then, since Tr(A) = 1 the left-hand side of (A8) is just $\log N$. This is the maximum value of $\mathcal{S}(\rho)$ on density matrices and therefore greater than $-Tr(A \log A)$ which, according to (A3), is $\mathcal{S}(\rho_1)$.

The reader worried about the assumption of a finite dimensional Hilbert space in the last argument can consider the following slightly more special argument: If there is a basis in which $\langle i|A|i\rangle$ is real and positive for a single basis vector $|i\rangle$, then we can choose $\lambda_i=0$ for that vector to obtain a positive, infinite value of $-Tr(A\log\rho_2)$ from (A8). The operator A given by (6.21) certainly has such a basis, for example the basis in which ρ is diagonal. Then

$$\langle i|A|i\rangle = Np_i\langle i|XX^{\dagger}|i\rangle$$
 (A9)

where p_i are the eigenvalues of ρ . The right-hand side of (A9) is clearly real and positive.

Thus, the extremum C = A is a minimum and the bound (A4) is established.

REFERENCES

- M. Morris, K.S. Thorne, and U. Yurtsver, Phys. Rev. Lett. 61, 1446, (1988);
 F. Echevverria, G. Klinkhammer, and K.S. Thorne, Phys. Rev. D44, 1077, (1991);
 J.L. Friedman and M. Morris, Phys. Rev. Lett. 66, 401, (1991); and J. Friedman,
 M. Morris, I.D. Novikov, F. Echeverria, G. Klinkhammer, K.S. Thorne, and U. Yurtsever, Phys. Rev. D42, 1915, (1990).
- [2] D. Deutsch, Phys. Rev. **D44**, 3197, (1991).
- [3] U. Yurtsever, Algebraic Approach to Quantum Field Theory on Non-Globally-Hyperbolic Spacetimes. Preprint UCSBTH-92-43.
- [4] G. Klinkhammer and K.S. Thorne, unpublished 1990 manuscript.
- [5] J.L. Friedman, N.J. Papastamatiou, and J.Z. Simon, *Phys. Rev.* **D46**, 4456, (1992).
- [6] J.B. Hartle, The Quantum Mechanics of Cosmology, in Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics, ed. by S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg, World Scientific, Singapore (1991) pp. 65-157.
- [7] J.B. Hartle, Phys. Rev. **D44**, 3173, (1991).
- [8] J.B. Hartle, in Vistas in Astronomy 37, 569, (1993); and in Topics on Quantum Gravity and Beyond (Essays in Honor of Louis Witten on His Retirement) ed. by F. Mansouri and J.J. Scanio, World Scientific, Singapore (1993); and to appear in the Proceedings of the IVth Summer Meeting on the Quantum Mechanics of Fundamental Systems, Centro de Estudios Científicos de Santiago, Santiago, Chile, December 26-30, 1991.
- [9] J.B. Hartle, Spacetime Quantum Mechanics and the Quantum Mechanics of Spacetime in Proceedings of the 1992 Les Houches Summer School Gravitation and Quantizations. Preprint UCSBTH92-21.

- [10] P.A.M. Dirac, Phys. Zeit. Sowjetunion 3, 64, (1933).
- [11] R.P. Feynman, Rev. Mod. Phys. 20, 267, (1948).
- [12] J.L. Friedman, N.J. Papastamatiou, and J.Z. Simon, Phys. Rev. **D46**, 4441, (1992).
- [13] D. Boulware, Phys. Rev. **D46**, 4421, (1992).
- [14] H.D. Politzer, Phys. Rev. **D46**, 4470, (1992).
- [15] D. Goldwirth, M. Perry, and T. Piran, Gen. Rel. Grav. 25, 7, (1993).
- [16] D. Goldwirth, M. Perry, T. Piran, and K.S. Thorne (preprint).
- [17] T. Jacobson, in Conceptual Problems of Quantum Gravity, ed. by A. Ashtekar and J. Stachel, Birkhäuser, Boston, (1991).
- [18] N. Yamada and S. Takagi Prog. Theor. Phys. 85, 985, (1991); ibid. 87, 77, (1992).
- [19] R. Griffiths, J. Stat. Phys. 36, 219, (1984).
- [20] R. Omnès, J. Stat. Phys. 53, 893, (1988); ibid 53, 933, (1988); ibid 53, 957, (1988);
 ibid 57, 357, (1989); Rev. Mod. Phys. 64, 339, (1992).
- [21] M. Gell-Mann and J.B. Hartle in Complexity, Entropy, and the Physics of Information, SFI Studies in the Sciences of Complexity, Vol. VIII, ed. by W. Zurek, Addison Wesley, Reading or in Proceedings of the 3rd International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology ed. by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura, Physical Society of Japan, Tokyo (1990).
- [22] M. Gell-Mann and J.B. Hartle, in Proceedings of the NATO Workshop on the Physical Origins of Time Assymmetry, Mazagon, Spain, September 30-October4, 1991 ed. by J. Halliwell, J. Perez-Mercader, and W. Zurek, Cambridge University Press, Cambridge (1993).
- [23] W. Unruh in New Techniques and Ideas in Quantum Measurement Theory, ed. by

- D.M. Greenberger, Ann. N.Y. Acad. Sci. 480, New York Academy of Science, New York (1986).
- [24] Y. Aharonov and L. Vaidman, J. Phys. A 24, 2315, (1991).
- [25] S. Rosenburg (to be published).
- [26] R.D. Rosenkrantz, ed. E.T. Jaynes: Papers on Probability, Statistics, and Statistical Physics, D. Reidel, Dordrecht, 1983.
- [27] D. Ruelle, Statistical Mechanics, Benjamin, New York (1969).
- [28] J.B. Hartle (to be published)
- [29] T. Banks, L. Susskind, and M. Peskin, Nucl. Phys. **B244**, 125, (1984).
- [30] M. Srednicki, Is Purity Eternal? Preprint UCSBTH-92-22.
- [31] S. Weinberg, Ann. Phys. (N.Y.) **194**, 336, (1989); Phys. Rev. Lett. **62**, 485, (1989).
- [32] J. Polchinski, Phys. Rev. Lett. 66, 397, (1991).
- [33] R. Laflamme and J.B. Hartle (unpublished).
- [34] M. Gell-Mann and J.B. Hartle, Phys. Rev. **D47**, 3345, (1993).
- [35] G. Klinkhammer, Phys. Rev. D 43, 2542, (1991).
- [36] R.M. Wald and U. Yurtsever, Phys. Rev. D 44, 403, (1991).

FIGURES

- FIG. 1. A compact non-chronal region of spacetime NC with spacelike surfaces σ' and σ'' before and after. Alternatives may be defined on these spacelike surfaces, but the transition matrix between them defined by a sum over intermediate field configurations is not necessarily unitary if the field is interacting.
- FIG. 2. A local piece of a spacelike surface R that is spacelike separated from a non-chronal region NC. R may be regarded either as lying on a spacelike surface σ' before NC or as lying on a spacelike surface σ'' after NC. If quantum mechanics is to be consistently formulated in terms of states on spacelike surfaces, then a prescription must be given for whether to compute the probabilities of alternatives confined to R with σ' or σ'' if the evolution through NC is not unitary for the results are not the same.
- FIG. 3. A spacetime with a single non-chronal region NC. Before NC there is an initial region that can be foliated by spacelike surfaces some of which are illustrated. Afterwards there is a similar final region. The text describes a generalized quantum mechanics for computing the probabilities of decoherent histories of alternatives defined on these surfaces even when the evolution through NC is non-unitary.