Quantum Cosmology:

Problems for the 21st Century*

James B. Hartle[†] (February 7, 2008)

Abstract

Two fundamental laws are needed for prediction in the universe: (1) a basic dynamical law and (2) a law for the cosmological initial condition. Quantum cosmology is the area of basic research concerned with the search for a theory of the initial cosmological state. The issues involved in this search are presented in the form of eight problems.

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[†]Institute for Theoretical Physics, University of California, Santa Barbara, CA, 93108; e-mail address: hartle@itp.ucsb.edu

I. WHAT ARE THE FUNDAMENTAL LAWS?

Physics, like other sciences, is concerned with explaining and predicting the regularities of specific physical systems. Stars, the solar system, high-temperature superconductors, fluid flows, atoms, and nuclei are just some of the many examples. Beyond particular systems, however, physics aims at finding laws that predict the regularities exhibited universally by all physical systems — without exception, without qualification, and without approximation. These are the fundamental laws of physics. This essay is concerned with the fundamental law for the initial condition of the universe.

Ideas for the nature of the fundamental laws have varied as new realms of phenomena have been explored experimentally. However, until recently, all of the various ideas for fundamental laws have had one feature in common: They were proposals for dynamical laws — laws that predicted regularities in time. The laws of Newtonian mechanics, electrodynamics, general relativity, and quantum theory all have this character.

The Schrödinger equation is an example of fundamental dynamical law:

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi . {(1.1)}$$

A fundamental theory of dynamics supplies the Hilbert space and the Hamiltonian operator H. However, a differential equation like (1.1) makes no predictions by itself. To solve (1.1), an initial condition — the state vector at one moment — must also be given. The Schrödinger equation then expresses the regularities in time that emerge from this initial state.

Where do the boundary conditions necessary to solve dynamical laws come from? In most of physics we study the evolution of subsystems of the universe and determine the boundary conditions by observation or experimental preparation. If we are interested in the evolution of the electromagnetic field in a room and observe no incoming radiation, we solve Maxwell's equations with no incoming radiation boundary conditions. To predict the probability for the decay of an atom prepared in an excited state, we solve the Schrödinger equation with that excited state as an initial condition at the time of preparation, and so on. Boundary conditions for the evolution of subsystems are obtained from *observations* of the rest of the universe outside the subsystem of interest.

Cosmology, however, presents us with an essentially different problem. The dynamical laws governing the evolution of the universe — the classical Einstein equation, for instance — require boundary conditions to yield solutions. But in cosmology, by definition, there is no "rest of the universe" to pass their specification off to. The cosmological boundary condition must be one of the fundamental laws of physics.

The inference is inescapable from the physics of the last sixty years that the fundamental laws are all quantum mechanical. If that is assumed, a theory of the initial condition is a theory of the universe's initial quantum state. The search for a fundamental theory of this initial cosmological quantum state is the aim of that area of basic research which has come to be called quantum cosmology.

A view thus emerges that there are two fundamental laws of physics:

- A theory of the basic dynamics,
- A theory of the initial condition of the universe.

Were the universe governed by the Schrödinger equation (1.1), the basic theory of dynamics would specify the Hamiltonian H; a theory of the initial condition would be a law for the initial quantum state.

The search for the fundamental dynamical law has been seriously under way since the time of Newton. Classical mechanics, Newtonian gravity, electrodynamics, special relativity, general relativity, quantum mechanics, quantum electrodynamics, quantum chromodynamics, electro-weak theory, grand unified theories, and superstring theories are but some of the important milestones in this search. By contrast, the search for a theory of the initial condition of the universe has been seriously under way for only a few decades. Why this difference? The answer lies in the empirical locality of the fundamental interactions on scales above the Planck length ($\sim 10^{-33}$ cm), or put differently, the empirical fact that the fundamental interactions may be effectively described by a local quantum field theory on these scales. Assuming locality, the Hamiltonian of the whole universe can be deduced from experiments on familiar, laboratory, scales. However, typical ideas for the initial quantum state of the universe are non-local. They imply regularities in space that emerge mostly on large, cosmological scales. For example, the temperature of the cosmic microwave background is the same across the sky to one part in 10^5 , a distance which corresponded to 10^{20} km at the time the radiation was emitted. It is only the recent progress in observational cosmology that has given us a picture of the universe on large enough scales of both space and time that is sufficiently detailed to confront with the predictions of a theory of the initial state of the universe.

II. QUANTUM COSMOLOGY AND THE EVERYDAY

Can those not interested in regularities on cosmological scales safely ignore the initial condition of the universe? Not if they seek a fundamental explanation of a number of its features we ordinarily take for granted. In this section we offer a few examples.

• Isolated Subsystems

In one way we use a very weak theory of the initial condition every day. Many subsystems of the universe, in the laboratory and elsewhere, are approximately isolated for periods of time and can can be approximately described by solving the Schrödinger equation for the subsystem alone. In effect, we assume that for the purposes of making predictions about the subsystem, the wave function of the universe can be approximated by

$$\Psi(q^i, Q^A, t) \approx \psi(q^i, t)\Phi(Q^A, t) \tag{2.1}$$

where q^i and Q^A are coördinates referring to subsystem and the rest of the universe respectively and ψ and Φ evolve *separately* under the Schrödinger equation. But what are the grounds for such an approximation? They do not lie in the nature of the Hamiltonian because that generally specifies interactions between all the coördinates. Rather the existence of isolated subsystems is a property of the quantum state. In discussing isolated

subsystems, we are making weak quantum cosmological assumptions about the nature of this initial state.

• The Quasiclassical Realm

Classical deterministic laws govern a wide range of phenomena in the universe over a broad span of time, place, and scale. This quasiclassical realm is one of the most immediate facts of our experience. But indeterminacy and distributed probabilities are the characteristics of a quantum mechanical universe. Classical deterministic dynamics can be but an approximation to the unitary evolution of the Schrödinger equation and the reduction of the state vector. To what do we owe the validity of this approximation? In part it arises from a coarse-grained description with positions and momenta specified to accuracies well above the limitations of the uncertainty principle for instance. But coarse graining is not enough; there must also be some restriction on the initial state. Ehrenfest's theorem is a simple illustration of why. For the motion of a particle in one dimension, Ehrenfest's theorem relates the acceleration of the expected position to the expected value of the force:

$$m \frac{d^2\langle x \rangle}{dt^2} = -\left\langle \frac{\partial V(x)}{\partial x} \right\rangle . \tag{2.2}$$

This is generally true, but for *certain* states, typically narrow wave packets, the right hand side may be replaced by the force evaluated at the expected position to a good approximation resulting in the deterministic classical equation of motion

$$m \frac{d^2 \langle x \rangle}{dt^2} \approx -\frac{\partial V(\langle x \rangle)}{\partial x} . \tag{2.3}$$

Just as only certain states lead to classical behavior in this simple model, so also only certain cosmological initial conditions will lead to the quasiclassical realm of familiar experience.* That too is a feature of the universe that must ultimately be traced to the initial condition.

• Homogeneity of the Thermodynamic Arrow of Time.

Isolated systems evolve towards equilibrium. That is a consequence of statistics. But in this universe presently isolated systems are mostly evolving towards equilibrium in the *same* direction of time. That is the homogeneity of the thermodynamic arrow of time. This is not a fact which can be explained by statistics or a property of the Hamiltonian alone for that is approximately time-reversal invariant. The homogeneity of the thermodynamic arrow of time follows from a fundamental law of the initial condition which mandates that the progenitors of today's isolated systems were all far from equilibrium in the early universe. As Boltzmann put it: "The second law of thermodynamics can be proved from the [time-reversible] mechanical theory if one assumes that the present state of the universe ... started to evolve from an improbable state" [2].

• History.

The reconstruction of history is useful for understanding the present in science as well as in human affairs. For example, we can best understand the character of biological species by understanding their evolution. We can best explain the present large scale distribution of

^{*}For a more quantitative discussion see, e.g. [1].

galaxies by understanding how galaxies arose from tiny density fluctuations present shortly after the big bang. Such examples could be easily multiplied.

In physics, the reconstruction of history means using the fundamental laws to calculate the probabilities of alternative past events assuming the values of present records. Classically, present records *alone* are enough to calculate those probabilities by using them as the starting point for running the deterministic classical equations of motion backward in time. To reconstruct history in quantum mechanics, however, requires a theory of the initial condition in addition to present records.

The source of this difference between classical and quantum mechanics can be traced to the arrow of time in usual quantum theory. Quantum mechanics treats the future differently from the past. To be sure, the Schrödinger equation (1.1) can be run backwards in time as well as forwards. But the Schrödinger equation is not the only law of evolution in quantum theory. In the usual story, when a measurement is made, the wave function is "reduced" by the action of the projection operator P representing the outcome of the measurement, and then renormalized. This is a "second law of evolution":

$$\Psi \to \frac{P\Psi}{||P\Psi||} \ . \tag{2.4}$$

The evolution of the Schrödinger equation forwards in time is interrupted by (2.4) on a measurement. While the Schrödinger equation can be run backwards in time, the law (2.4) cannot, and that is a simple way of seeing the arrow of time in usual quantum mechanics. The same kind of arrow of time persists in more general quantum theories of closed systems where (2.4) is effectively used in in the construction of the probabilities of histories which are not necessarily of the outcomes of measurements.

How then does one calculate the probabilities of past events assuming present records in quantum mechanics? The simple answer is that one works forwards in time from the initial state. Evolving forwards using (1.1) and (2.4) one calculates the joint probabilities of both alternative events of interest in the past and the alternative values of the present records that follow them. From these one calculates the conditional probabilities of the past events given our particular present records in the usual way.

This process involves the initial state in an essential way. Strictly speaking, therefore, one cannot make any statements about the past without a theory of the universe's initial condition.

• Phenomenology of the Initial Condition.

While the above four everyday features of the universe are fundamentally traceable to the universe's initial quantum state, there is a large set of initial states that would give rise to them. Put differently, the existence of isolated subsystems together with the applicability of classical physics, the second law of thermodynamics, and the possibility of historical explanation are not strong constraints on the initial quantum state. Neither are the observations of large scale features of the universe such as its approximate homogeneity and isotropy or

[†]There are generalizations of quantum theory without an arrow of time in which the asymmetry of the usual theory may be understood as a difference between initial and final conditions, e.g. [3]. We shall not consider these here.

the fluctuations in the cosmic background radiation. The data are meager and the Hilbert space of the observable universe is vast.

It would be possible to investigate quantum cosmology phenomenologically by asking for the constraints present observations place on the initial state of the universe. A density matrix ρ is the way quantum mechanics represents the statistical distribution of states with associated probabilities that would be inferred. To investigate the initial condition phenomenologically is therefore to ask for the density matrices consistent with observed features of the universe.

The observed features of the universe may not uniquely fix an initial condition but one should not exaggerate their weakness. The density matrix $\rho = I/Tr(I)$, where I is the unit matrix, is the unique representation of complete ignorance of the initial condition (i.e. no condition at all). But it also corresponds to infinite temperature in equilibrium $(\rho \propto \exp(-H/kT))$ — an initial condition whose implication of infinite temperature today is obviously inconsistent with present observations.

The entropy $S/k = -Tr(\rho \log \rho)$ is a measure of the missing information about the initial state in a density matrix ρ . Most of the entropy in the matter in the visible universe is in the cosmic background radiation, a number of order $S/k \approx 10^{80}$. As Penrose [4] has stressed, this is a large number, but infinitesimally small compared to the maximum possible value of $S/k \approx 10^{120}$ if all that matter composed a black hole.

This essay, however, is not concerned with phenomenology. Rather, it is concerned with the fundamental law of the initial condition. We shall therefore assume that the universe has a initial state $|\Psi\rangle$ and discuss the issues involved in a search for the principles which determine it.

III. PROBLEMS

Enumerating issues is one way of summarizing the present status of an area of science, and motivating future research. Certainly, setting problems is a more pleasant task than solving them, and quantum cosmology is such a young field that it is easier to summarize problems than to survey accomplishments. It is in this spirit that the author offers the following eight problems in quantum cosmology:

• Problem 1: What Principle Determines the Initial Condition of the Universe?

The evidence of the observations is that the universe was simpler earlier than it is now — more homogeneous, more isotropic, with matter more nearly in thermal equilibrium. This is evidence for a simple, discoverable initial condition of the universe. But what principles determine that initial state?

The most developed proposal for a principle determining the initial condition is the "no-boundary" wave function of Stephen Hawking and his associates [5]. The idea is that the initial condition of a closed universe is the cosmological analog of a ground state. This does not mean the lowest eigenstate of some Hamiltonian. Intuitively, the total energy of a closed universe is zero for there is no place outside from which to measure it. Correspondingly the Hamiltonian vanishes.

But the lowest eigenstate of a Hamiltonian is not the only way to find the ground state even in the elementary case of a particle moving in a potential V(x). In that case, the ground state wave function may be expressed directly as a sum over Euclidean paths, $x(\tau)$:

$$\psi_0(y) = \sum_{\text{paths } x(\tau)} \exp(-I[x(\tau)]/\hbar)$$
(3.1)

where $I = \int d\tau [m\dot{x}^2/2 + V(x)]$ is the Euclidean action. The sum is over paths $x(\tau)$ that have the argument of the wave function, y, as one end point, and a configuration of minimum action in the infinite past as another. Verify it for the harmonic oscillator for example!

This construction of a ground state wave function generalizes to closed universes. For definiteness suppose, for a moment, that the basic variables of the fundamental dynamical theory are the geometry of four-dimensional spacetime \mathcal{G} , represented by metrics on manifolds, together with matter fields such as the quark, lepton, gluon, and Higgs fields which we generically denote by $\phi(x)$. The arguments of cosmological wave functions are these basic variables restricted to spacelike surfaces, specifically the three-geometries of these surfaces, ${}^{3}\mathcal{G}$, and the field configurations on these surfaces, $\chi(\mathbf{x})$. The "no-boundary" wave function is of the form

$$\Psi_0[{}^3\mathcal{G}, \chi(\mathbf{x})] = \sum_{\mathcal{G}, \phi(x) \in C} \exp(-I[\mathcal{G}, \phi(x)]/\hbar)$$
(3.2)

where $I[\mathcal{G}, \phi(\mathbf{x})]$ is the action for gravitation and matter. The "no boundary" wave function is specified by giving the class C of four geometries \mathcal{G} , and matter fields $\phi(x)$ summed over in (3.2). So that the construction is analogous to (3.1), these geometries \mathcal{G} should have Euclidean (signature ++++) and have one boundary at which they match the three-geometry where the wave function is evaluated. The matter fields must similarly match their boundary value. The defining requirement is that the \mathcal{G} 's have no *other* boundary, whence the name "no-boundary" proposal.

Nothing goes on in a typical ground state in a fixed background spacetime. In field theory, the ground state is the time-translation invariant vacuum! However, this is not the context of the quantum cosmology of closed universes. Spacetime geometry is not fixed and there is therefore no notion of time-translation. Interesting histories therefore can happen; and the attractive nature of gravity makes things happen even in this cosmological analog of the ground state. In particular, initial, small, quantum, ground state fluctuations from homogeneity and isotropy that are predicted by this initial condition can grow by gravitational attraction to produce all the complexity in the universe that we see today.

This prescription for the "no-boundary" wave function is not complete. The reason is that the action $I[\mathcal{G}, \phi(x)]$ for gravitation coupled to matter is unbounded below. Were the sum in (3.2) extended over real, Euclidean geometries and fields, it would diverge! Rather, the sum must be taken over a class C of complex geometries and fields. A complex contour of summation is, in fact, essential for the "no-boundary" wave function to predict the nearly classical behavior of geometry we observe in the present epoch. But many different complex convergent contours are possibly available and correspondingly there are many different "no-boundary" wave functions. These do not differ in their semi-classical predictions; but we still lack a complete principle for fixing this wave function of the universe.

The "no-boundary" idea has been described in terms of an effective theory of dynamics in which spacetime and matter fields are treated as fundamental variables. If spacetime is not fundamental, as in string theory or non-perturbative quantum gravity, then extending the idea to such theories becomes an important question. The essentially topological nature of the idea gives some hope that such an extension is possible.

The "no-boundary" wave function is not the only idea for a theory of the initial condition. Other notable candidates are the "spontaneous nucleation from nothing wave function" [6], and the ideas associated with the "eternally self-reproducing inflationary universe" [7]. Space does not permit a review of these and other theories, and the similarities and differences in their predictions. Discriminating between these and other ideas that may arise is certainly a problem for the 21st century.

• Problem 2: How Can Quantum Gravity be Formulated for Cosmology?

Gravity governs the evolution of the universe on the largest scales of space and time. That fact alone is enough to show that a quantum theory of gravity is required for a quantum theory of cosmology. Were the behavior of the universe on present cosmological scales all that was of interest, then a low energy approximation to quantum gravity would be adequate. Indeed most of the exploration of quantum cosmology has been carried out in such a low energy approximation assuming spacetime geometry and quantum fields are the basic variables with Einstein's theory coupled to matter as the basic action. Any divergences that arise are truncated in one way or another.

It is a reasonable expectation that low-energy, large scale, features of the universe, such as the galaxy-galaxy correlation function, are insensitive to the nature of quantum gravity on very small scales. But in quantum cosmology we aim not only at an explanation of such large scale features, but also at a theory of the initial condition adequate to describe the probabilistic details of the earliest moments of the universe. The inevitability of an initial singularity in classical Einstein cosmologies strongly suggests that the earliest moments of the universe will exhibit curvatures of spacetime characterized by the Planck length

$$\ell \equiv (\hbar G/c^3)^{1/2} \approx 10^{-33} \text{cm}$$
 (3.3)

— the only combination of the three fundamental constants governing relativity, quantum mechanics, and gravity that has the dimensions of length. By making similar combinations with the right dimensions we can exhibit the Planck scales of energy and time. The universe at the epochs characterized by these scales will therefore depend on the detailed form of the fundamental quantum dynamical law for gravity.

There are a number of candidates for a finite, manageable quantum theory of gravity, notably superstring theory and non-perturbative canonical quantum gravity. However, neither of these theories is ready for application to quantum cosmology. String theory, for instance, exists in a practical sense as a set of rules for classical backgrounds and quantum perturbations away from them. Developing such theories to the point where they can be used for the non-perturbative quantum dynamics of closed cosmologies is thus an important problem.

The problem to be faced is not merely one of technique. Both of the approaches mentioned, and others as well, hint that spacetime geometry may not be a basic dynamical variable. If that is true, it becomes a conceptual issue just how to frame cosmological questions in the variables of the fundamental dynamical theory.

• Problem 3: What is the Generalization of Quantum Mechanics Necessary for Quantum Gravity and Quantum Cosmology?

A generalization of usual quantum mechanics is needed for quantum gravity. That is because usual quantum mechanics relies in essential ways on a fixed, background spacetime geometry, in particular, to specify the notion of time that enters centrally into the formalism. This reliance on a fixed notion of time shows up in any of the various ways of formulating usual quantum theory — the idea of a state at a moment of time, the preferred role of time in the Schrödinger equation, the inner product at a moment of time, the reduction of the state vector at a moment of time, the commutation of fields at spacelike separated points, the equal time commutators of coördinates and momenta, etc., etc.

But in quantum gravity, spacetime geometry is not fixed, rather it is a quantum dynamical variable, fluctuating and generally without definite value. A generalization of usual quantum theory that does not require a fixed spacetime geometry, but to which the usual theory is a good approximation in situations when the geometry is approximately fixed, is therefore needed for quantum gravity and quantum cosmology. What, therefore, do we mean more generally by a quantum mechanical theory?

The most general objective of any quantum theory is the prediction of the probabilities of alternative, coarse-grained histories of the universe as a single, closed quantum mechanical system. For example, one might be interested in predicting the probabilities of the set of possible orbits of the earth around the sun. Any orbit is possible, but a Keplerian ellipse has overwhelming probability. Such histories are said to be coarse-grained because they do not specify the coördinates of every particle in the universe, but only those of the center of mass of the earth and sun, and these only crudely and not at every time.

However, the characteristic feature of a quantum mechanical theory is that consistent probabilities cannot be assigned to every set of alternative histories because of quantum mechanical interference. Nowhere is this more clearly illustrated than in the famous two-slit experiment shown in Figure 1. Electrons can proceed from an electron gun at left towards detection at a point y on a screen along one of two possible histories — the history passing through the upper slit, A, and the history passing through the lower slit, B. In the usual story, probabilities cannot be assigned to these two histories if we have not measured which slit the electron passed through. It would be inconsistent to do so because the the probability to arrive at y would not be the sum of the probabilities to arrive there going through the upper slit and lower slit:

$$p(y) \neq p_A(y) + p_B(y) \tag{3.4}$$

because of quantum mechanical interference. In quantum mechanics probabilities are squares of amplitudes and, of course,

$$|\psi_A(y) + \psi_B(y)|^2 \neq |\psi_A(y)|^2 + |\psi_B(y)|^2$$
 (3.5)

A necessary consistency condition would not be satisfied.

A rule is thus needed in quantum theory to specify which sets of alternative histories may be assigned probabilities and which may not. In the usual, "Copenhagen" formulations of quantum mechanics presented in textbooks, probabilities can be assigned to the histories of alternatives of a subsystem that were "measured" by an "observer". But such formulations

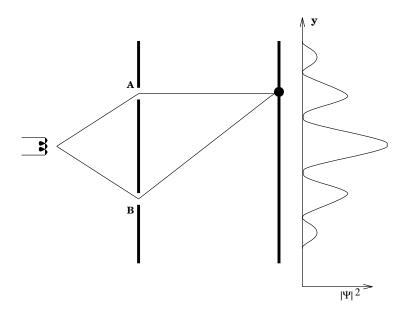


FIG. 1. The Two-Slit Experiment

are not general enough for quantum cosmology which seeks to describe the early universe where there were neither measurements nor observers present.

In the more general quantum mechanics of closed systems[‡] that rule is simple: probabilities can be assigned to just those sets of alternative histories for which there is vanishing interference between the individual members as a consequence of the system's initial state $|\Psi\rangle$. To make this quantitative we need the measure of this interference.

When there is a well-defined fixed notion of time, sequences of alternative sets of events at a series of times define a set of alternative histories. An individual history in such a set is a particular series of events, say $\alpha \equiv (\alpha_1, \alpha_2, \dots, \alpha_n)$ at times $t_1 < t_2 < \dots < t_n$. In usual quantum mechanics such a history is represented by a corresponding chain of (Heisenberg-picture) operators,

$$C_{\alpha} \equiv P_{\alpha_n}^n(t_n) \cdots P_{\alpha_2}^2(t_2) P_{\alpha_1}^1(t_1) , \qquad (3.6)$$

time ordered from right to left. The application of the C_{α} to the initial state vector $|\Psi\rangle$ gives the branch state vector

$$C_{\alpha}|\Psi\rangle$$
 (3.7)

corresponding to the history. Interference vanishes in a set of alternative histories when the branch state vectors corresponding to the different histories are mutually orthogonal. Sets of alternative histories with vanishing interference are said to *decohere*. The probabilities $p(\alpha)$ of the individual histories in a decoherent set are the squared lengths of the branch state vectors

[‡] For expositions see [8,9].

$$p(\alpha) = ||C_{\alpha}|\Psi\rangle||^2 . \tag{3.8}$$

Decoherence insures the consistency of these probabilities.

Interference is thus measured by the decoherence functional:

$$D(\alpha', \alpha) = \left\langle \Psi | C_{\alpha'}^{\dagger} C_{\alpha} | \Psi \right\rangle \tag{3.9}$$

which becomes the central element in the theory. The condition of decoherence and the resulting probabilities may be expressed by the single formula

$$D(\alpha', \alpha) = \delta_{\alpha'\alpha} p(\alpha) . \tag{3.10}$$

The sets of possible coarse-grained histories, their decoherence functional, and (3.10) are the minimal elements of a quantum theory. A broad framework for quantum theories built on these elements, called generalized quantum mechanics, can be formulated in terms of decoherence functionals obeying general principles of Hermiticity, normalization, positivity, and the principle of superposition [10,11].

Histories represented by strings of projections at definite moments of time (3.6) and a decoherence functional (3.9) are the way that usual quantum theory implements the principles of generalized quantum theory. But there are many other ways, and among them are possibilities for generalizing usual quantum mechanics so that it works in the absence of a fixed spacetime geometry. Generalized sum-over-histories quantum theories have been discussed that put quantum theory into fully spacetime form with four-dimensional notions of histories, coarse grainings, and decoherence [11]. But the principles of generalized quantum mechanics are only a minimal set of requirements for quantum theory. What further principles determine the correct quantum mechanics for quantum gravity and quantum cosmology?

• Problem 4: What are the Definite Predictions of the Initial Condition for the Universe on Large Scales?

Extracting the predictions of a theory of the initial condition and comparing them with observations is a central problem in quantum cosmology. Predictions take the form of probabilities for present observations. The theory stands or falls on those predictions with probabilities sufficiently close to one (or zero) being observed (or not observed). These are called the *definite* predictions of the theory. We expect few of them. A simple, comprehensible, discoverable theory of the initial condition cannot predict all the complexity observed in the present universe with probability near one [12]. Rather, most predictions, as of the stock market, the weather, or the number of moons of Jupiter, will have more distributed probabilities based on the initial condition alone. (The vast majority will be nearly uniformly distributed which is no prediction at all.) In quantum cosmology one must search among the possible predictions for those which are predicted with probability near one. Interestingly, definite predictions may occur on all scales. For the purposes of simplicity we have divided the problem of what are the definite predictions of a theory of the initial condition into problems concerning regularities on cosmological, familiar, and microscopic scales.

Quantum cosmologists expect that a number of the general large scale features of the universe will be definite predictions of a theory of its initial condition. These include an approximately classical cosmological spacetime geometry after the Planck epoch, the approximate homogeneity and isotropy of the geometry and matter on scales above several hundred

megaparsecs[§], the approximate spatial flatness of the universe (or what is the same thing its vast age in Planck units), the initial spectrum of quantum fluctuations which grew to become the galaxies, a sufficiently long inflationary epoch, and the cosmological abundances of the matter and radiation species.

The probabilities for these features of the universe arising from various theories of the initial condition have been explored in highly simplified models valid only in limited regions of the configuration space of possible present universes. The output of some of these calculations, such as the prediction of the spectrum of initial quantum fluctuations [13], are among the most successful achievements of quantum cosmology. But much more needs to be done to extend these calculations to the whole of configuration space with greater accuracy, generality and a precise quantum mechanical interpretation. That is a practical and immediate problem for quantum cosmology.

• Problem 5: What are the Definite Predictions of the Initial Condition for Features of the Universe on Familiar Scales?

We may treat this problem briefly because the obvious features of the universe on familiar scales that are traceable to the initial condition have been discussed qualitatively in Section II. However, those qualitative conclusions raise quantitative questions:

What are the coarse-grained variables defining a quasiclassical realm governed by deterministic laws and how are these variables related to the principle that determines the initial condition? How refined a quasiclassical description of the universe is possible before decoherence is lost or determinism is overwhelmed by quantum noise? How far in space and time can a quasiclassical description be extended? How do the phenomenological equations of motion that exhibit the determinism of the quasiclassical realm follow from the fundamental dynamical law, an initial condition, and an appropriate coarse-grained description? What is the connection of the coarse graining used to define a quasiclassical realm with that which is necessary to exhibit a second law of thermodynamics? How far out of equilibrium is the early universe in this coarse graining?

In short, a theory of the initial condition presents the challenge of defining quantitatively those features of the universe on familiar scales which are traceable, in part, to the nature of the initial condition.

• Problem 6: What are the Definite Predictions of the Initial Condition on Microscopic Scales

Our understanding of the world on microscopic scales above that set by the Planck length is summarized by the effective field theories which govern phenomena on these scales — for example, the standard model of elementary particle physics. However the forms of these effective field theories may be only distantly related to the form of the fundamental dynamical law. An analogous situation at a different scale may help explain why: The form of the Navier-Stokes equation which governs the dynamics of much of the quasiclassical realm is not easily guessed from the Lagrangian of the standard model of particle physics. In particular, the Navier-Stokes equation incorporates dissipation and depends on constitutive relations between density, pressure, temperature, viscosity, etc. — relations not contained

[§] The megaparsec is a convenient unit for cosmology. One megaparsec is 3.3 million light years. The size of the visible universe is several thousand megaparsecs.

in the Lagrangian of the standard model.

Of course, we understand qualitatively the relation between the laws of the standard model and the Navier-Stokes equation. The Navier-Stokes equation applies, not generally, not exactly, but only approximately in particular circumstances. It is *effective* equation with a limited range of approximate validity. In quantum mechanics particular circumstances are represented by the quantum state and a coarse-grained description. It is the quantum state whose special properties allow a classical approximation, set up the conditions for dissipation, determine the constituents, and allow for the local equilibrium from which the constitutive relations follow.

But the standard model itself may be only an effective approximation to a more fundamental dynamical law such as heterotic superstring theory or non-perturbative quantum gravity. We may therefore restate the problem of the definite predictions of the initial condition as follows: What features of the effective dynamical laws that govern the elementary particle system at accessible energy scales are traceable to the cosmological initial condition and what to the fundamental dynamical law? For instance, what is the origin of the locality of the effective interactions in a theory of the quantum state that is intrinsically non-local?

The investigations of the effects of wormholes by Hawking, Coleman, Giddings and Strominger, and others indicate just how strong the effect of the initial condition on the effective interactions could be (for a review see, e.g. [14]). Suppose that the sum over geometries defining the "no-boundary" wave function in (3.2) includes a sum over wormhole geometries — four dimensional geometries with many "handles" rather like a teacup has a handle. Suppose that the Planck scale (3.3) is the characteristic size of these wormholes in the geometries that contribute the most to the sum. Fields propagating in such geometries can go down a wormhole and emerge from one. On the much larger scales accessible to us, we would see the effect of Planck scale wormholes as local interactions which create and destroy particles. The net effect is to add to any local Lagrangian an infinite series of local interactions with coupling constants that are not fixed once and for all by the fundamental dynamical law or even by a renormalization procedure, but rather vary probabilistically with a distribution determined by the initial condition. If the distribution was sharp (as was hoped for the cosmological constant) then the couplings would be predicted.

A similar decoupling between the observed coupling constants and the basic Lagrangian would hold if the initial condition predicted domains of space much larger than our visible universe in which breaking of the symmetries of the fundamental dynamical law occurred in different ways in different places leading to a differing effective theories in different domains. The form of the effective theory governing our domain would then be only a probabilistic prediction of the fundamental dynamical law.

It has proved difficult to push such ideas very far, but their lesson is clear. The form and couplings of the effective interactions at accessible scales may be probabilistically distributed in a way which depends on the initial condition. Finding these distributions and how sharp they are is therefore an important problem in quantum cosmology.

• Problem 7: What Does Quantum Cosmology Predict for IGUSes?

Most of predictions of the initial condition that we have considered so far are described in terms of alternatives of the quasiclassical realm. But there are many sets of decohering histories of the universe arising from a theory of its initial condition and dynamics that have nothing to do with the usual quasiclassical realm. These sets may be quantum mechanically incompatible with each other and with the usual quasiclassical realm in the sense that pairs of them cannot be combined into a common decohering set. Such incompatible sets are not contradictory; rather they are complementary ways of viewing the unfolding of the initial condition into alternative histories. The quantum mechanics of closed systems does not distinguish between such incompatible sets of alternative histories except by properties such as their classicality. All are in principle available for the process of prediction.

Yet, as observers, we describe the universe almost exclusively in terms of the familiar variables of classical physics. What is the reason for this narrow focus in the face of all the other non-quasiclassical decohering sets of alternative histories? Some see this disparity between the possibilities allowed by quantum theory and the possibilities utilized by us as grounds for augmenting quantum mechanics by a further fundamental principle that would single out one decohering set of histories from all others [15]. That is an interesting line of thought, but another is to seek an explanation within the existing quantum mechanics of closed systems.

Human beings, bacteria, and certain computers, are examples of information gathering and utilizing systems (IGUSes). Roughly, an IGUS is a subsystem of the universe that makes observations and thus acquires information, makes predictions on the basis of that information using some approximation (typically very crude) to the quantum mechanical laws of nature, and exhibits behavior based on these predictions.** To explain why IGUSes are exhibited by the universe, or why they behave the way they do, or to answer questions like "Why do we utilize quasiclassical variables?", one must seek to understand how IGUSes evolved as physical systems within the universe. In quantum cosmology that means examining the probabilities of a set of histories that define alternative evolutionary tracks. For IGUSes that can be characterized in terms of alternatives of the usual quasiclassical realm, it is a plausible conjecture that they evolved to focus on the usual quasiclassical alternatives because these present enough regularity over time to permit prediction by relatively simple models (schemata). This would be one kind of explanation of why we utilize the usual quasiclassical realm. However, we should not pretend that we are close to being able to carry out a calculation of the relevant probabilities or even likely to be in the early 21st century!

But what of sets of histories that are completely unrelated to the usual quasiclassical realm? Might some of these exhibit IGUSes with high probability that make predictions in terms of variables very different from the familiar quasiclassical ones? Or is the usual quasiclassical realm somehow distinguished with respect to exhibiting IGUSes? To answer such questions one would need a general characterization of IGUSes that is applicable to all kinds of histories — not just quasiclassical ones — and an ability to calculate the probabilities of various courses of the IGUSes' evolution. Such questions, while quite beyond our power to answer in the present, illustrate the range of predictions in principle possible in a quantum universe from a fundamental theory of dynamics and the initial condition of the universe.

^{**}IGUSes are complex adaptive systems in the context of quantum mechanics. For more on the general characterization of complex adaptive systems see [16].

IV. UNIFICATION

The universal laws that govern the regularities of every physical system are one goal of physics. A fundamental dynamical law is one objective. Quantum cosmology is concerned with the equally necessary fundamental law specifying the initial condition of the universe.

Historically, many of the advances towards the fundamental laws have had in common that some idea that was previously thought to be universal was subsequently seen to be only a feature of our special place in the universe and the limited range of our experience. With more data, the idea was seen to be a true physical fact, but one which is a special situation in a yet more general theory. The idea was a kind of "excess baggage" which had to be jettisoned to reach a more general, more comprehensive, and more fundamental perspective [17].

It is not difficult to cite examples of such excess theoretical baggage in the history of physics: the idea that the earth was the center of the universe, the idea of Newtonian absolute time, the idea that the increase entropy was a basic dynamical law, the idea that spacetime geometry is fixed, the idea of a classical world separate from quantum mechanics, etc. etc. Further, and more importantly for the present discussion, one can cite examples concerning the nature of the fundamental laws themselves: the idea that thermodynamics was separate from mechanics, the idea that electricity was separate from magnetism, and more recently the idea that there were separate weak and electromagnetic interactions. These seemingly distinct theories were eventually unified. Today, extrapolations of the standard model of the electro-weak and strong interactions suggest a unified theory of these forces characterized by an energy scale a little below the Planck scale.

Examples such as those just cited have led some physicists to speculate that the existing separation between the dynamical laws for the gravitational and other forces is also an example of excess baggage arising from the limitations of present experiments to energies well below the Planck scale, and to search for a unified fundamental law for dynamics of all the forces. Secure in the faith that fundamental laws are mathematically simple, heterotic superstring theory or its extensions have been the inspiring results.

However, such a unified dynamical law does not really deserve the common designation of "a theory of everything" or a "final theory". Quantum cosmology offers a further opportunity for unification beyond dynamical laws. Could it be that the apparent division of the fundamental laws into a law for dynamics and a law for an initial quantum state is a kind of excess baggage similar to those described above? Gell-Mann [18] has stressed that there is already an element of unification in ideas such as the "no-boundary" proposal. In (3.2) the same action that determines fundamental dynamics also determines the quantum state of the universe. Despite this connection, the "no-boundary" proposal is a separate principle specifying one wave function out of many possible ones. Thus we have an eighth problem for quantum cosmology:

• Problem 8: Is there a Fundamental Principle that would Single Out Both a Unified Dynamical Law and a Unique Initial Quantum State for the Universe? Could that same Principle Single Out the Form of Quantum Mechanics from Among Those Presented by Generalized Quantum Theory?

In such a unification of the law of dynamics, the cosmological boundary condition, and the principles of quantum mechanics, we would, at last, have a truly unified fundamental law of physics governing the universe as a whole and everything within it. That is truly a worthy problem for physics in the twenty-first century!

V. FURTHER READING

Ref [19] is Scientific American article introducing quantum cosmology. An accessible but more advanced introductory review is [20]. That article contains a nearly exhaustive list of references at the time and a guide to the literature. For an introduction to the quantum mechanics of closed systems, see e.g. [8]. For an exposition of the applications of quantum mechanics to cosmology see [11].

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