Anthropic Reasoning and Quantum Cosmology*

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Prediction in quantum cosmology requires a specification of the universe's quantum dynamics and its quantum state. We expect only a few general features of the universe to be predicted with probabilities near unity conditioned on the dynamics and quantum state alone. Most useful predictions are of conditional probabilities that assume additional information beyond the dynamics and quantum state. Anthropic reasoning utilizes probabilities conditioned on 'us'. This paper discusses the utility, limitations, and theoretical uncertainty involved in using such probabilities. The predictions resulting from various levels of ignorance of the quantum state are discussed.

I. INTRODUCTION

If the universe is a quantum mechanical system, then it has a quantum state. This state provides the initial condition for cosmology. A theory of this state is an essential part of any final theory summarizing the regularities exhibited universally by all physical systems and is the objective of the subject of quantum cosmology. This essay is concerned with the role the state of the universe plays in anthropic reasoning — the process of explaining features of our universe from our existence in it [1]. The thesis will be that anthropic reasoning in a quantum mechanical context depends crucially on assumptions about the universe's quantum state.

II. A MODEL QUANTUM UNIVERSE

Every prediction in a quantum mechanical universe depends on its state if only very weakly. Quantum mechanics predicts probabilities for alternative possibilities, most generally the probabilities for alternative histories of the universe. The computation of these probabilities requires both a theory of the quantum state as well as the theory of the dynamics specifying its evolution.

To make this idea concrete while keeping the discussion manageable, we consider a model quantum universe. The details of this model are not essential to the subsequent discussion of anthropic reasoning but help to fix the notation for probabilities and provide a specific

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example of what they mean. Particles and fields move in a large, perhaps expanding box, say presently 20,000 Mpc on a side. Quantum gravity is neglected — an excellent approximation for accessible alternatives in our universe later than 10^{-43} s from the big bang. Spacetime geometry is thus fixed with a well defined notion of time and the usual quantum apparatus of Hilbert space, states, and their unitary evolution governed by a Hamiltonian can be applied¹.

The Hamiltonian H and the state $|\Psi\rangle$ in the Heisenberg picture are the assumed theoretical inputs to the prediction of quantum mechanical probabilities. Alternative possibilities at one moment of time t can be reduced to yes/no alternatives represented by an exhaustive set of orthogonal projection operators $\{P_{\alpha}(t)\}$, $\alpha = 1, 2, \cdots$ in this Heisenberg picture. The operators representing the same alternatives at different times are connected by

$$P_{\alpha}(t) = e^{iHt/\hbar} P_{\alpha}(0) e^{-iHt/\hbar}. \tag{2.1}$$

For instance, the P's could be projections onto an exhaustive set of exclusive ranges of the center-of-mass position of the Earth labeled by α . The probabilities $p(\alpha)$ that the Earth is located in one or another of these regions at time t is

$$p(\alpha|H, \Psi) = ||P_{\alpha}(t)|\Psi\rangle||^{2}. \tag{2.2}$$

The probabilities for the Earth's location at a different time is given by the same formula with different P's computed from the Hamiltonian by (2.1). The notation $p(\alpha|H, \Psi)$ departs from usual conventions (e.g. [2]) to indicate explicitly that all probabilities are conditioned on the theory of the Hamiltonian H and quantum state $|\Psi\rangle$.

Most generally quantum theory predicts the probabilities of sequences of alternatives at a series of times — that is *histories*. An example is a sequence of ranges of center of mass position of the Earth at a series of times giving a coarse-grained description of its orbit. Sequences of sets of alternatives $\{P_{\alpha_k}^k(t_k)\}$ at a series of times t_k , $k=1,\cdots,n$ specify a set of alternative histories of the model universe. An individual history α in the set corresponds to a particular sequence of alternatives $\alpha \equiv (\alpha_1, \alpha_2, \cdots, \alpha_n)$ and is represented by the corresponding chain of projection operators C_{α}

$$C_{\alpha} \equiv P_{\alpha_n}^n(t_n) \cdots P_{\alpha_1}^1(t_1) \quad , \quad \alpha \equiv (\alpha_1, \cdots, \alpha_n) .$$
 (2.3)

The probabilities of the histories in the set are given by

$$p(\alpha|H,\Psi) \equiv p(\alpha_n, \dots, \alpha_1|H, \Psi) = ||C_\alpha|\Psi\rangle||^2$$
(2.4)

provided the set decoheres, i.e. provided the branch state vectors $C_{\alpha}|\Psi\rangle$ are mutually orthogonal. Decoherence ensures the consistency of the probabilities (2.4) with the usual rules of probability theory².

To use either (2.2) or (2.4) to make predictions, a theory of both H and $|\Psi\rangle$ is needed. No state; no predictions.

¹ For a more detailed discussion of this model in the notation used here, see [2]. For a quantum framework when spacetime geometry is not fixed, see *e.g.* [3].

² For a short introduction to decoherence see [2] or any of the classic expositions of decoherent (consistent) histories quantum theory [4, 5, 6].

III. WHAT IS PREDICTED?

"If you know the wave function of the universe, why aren't you rich?" This question was once put to me by my colleague Murray Gell-Mann. The answer is that there are unlikely to be any alternatives relevant to making money that are predicted as sure bets conditioned just on the Hamiltonian and quantum state alone. A probability $p(\text{rise}|H, \Psi)$ for the stock market to rise tomorrow could be predicted from H and $|\Psi\rangle$ through (2.2) in principle. But it seems likely that the result would be a useless $p(\text{rise}|H, \Psi) \approx 1/2$ conditioned just on the 'no boundary' wave function [7] and M-theory.

It's plausible that this is the generic situation. To be manageable and discoverable, the theories of dynamics and the quantum state must be short — describable we in terms of a few fundamental equations and the explanations of the symbols they contain. It's therefore unlikely that H and $|\Psi\rangle$ contain enough information to determine most of the interesting complexity of the present universe with significant probability [8, 9]. We hope that the Hamiltonian and the quantum state are sufficient conditions to predict certain large scale features of the universe with significant probability. Approximately classical spacetime, the number of large spatial dimensions, the approximate homogeneity and isotropy on scales above several hundred Mpc, and the spectrum of density fluctuations that were the input to inflation are some examples of these. But even a simple feature like the time the Sun will rise tomorrow will not be usefully predicted by our present theories of dynamics and the quantum state alone.

The time of sunrise does become predictable with high probability if a few previous positions and orientations of the Earth in its orbit are supplied in addition to H and $|\Psi\rangle$. That is a particular case of a *conditional probability* of the form

$$p(\alpha|\beta, H, \Psi) = \frac{p(\alpha, \beta|H, \Psi)}{p(\beta|H, \Psi)}$$
(3.1)

for alternatives α (e.g. the times of sunrise) given $H, |\Psi\rangle$ and further alternatives β (e.g. a few earlier positions and orientations of the Earth). The joint probabilities on the right hand side of (3.1) are computed using (2.4) as described in Section II.

Conditioning probabilities on specific information can weaken their dependence on H and $|\Psi\rangle$ but does not eliminate it. That is because any specific information available to us as human observers (like a few positions of the Earth) is but a small part of that needed to specify the state of the universe. The P_{β} used to define the joint probabilities in (3.1) by (2.4) therefore spans a very large subspace of Hilbert space. As a consequence $P_{\beta}|\Psi\rangle$ depends strongly on $|\Psi\rangle$. For example, to extrapolate present data on the Earth to its position 24 hours from now requires that the probability be high that it moves on a classical orbit in that time and that the probability be low that it is destroyed by a neutron star now racing across the galaxy at near light speed. Both of these probabilities depend crucially, if weakly, on the nature of the quantum state [10].

Many useful predictions in physics are of conditional probabilities of the kind discussed in this section. We next turn to the question of whether we should be part of the conditions.

IV. ANTHROPIC REASONING — LESS IS MORE

A. Anthropic Probabilities

In calculating the conditional probabilities for predicting some of *our* observations given others, there can be no objection of principle to including a description of 'us' as part of the conditions,

$$p(\alpha|\beta, \text{`us'}, H, \Psi)$$
. (4.1)

Drawing inferences using such probabilities is called *anthropic reasoning*. The motivation is the idea is that probabilities for certain features of the universe might be sensitive to this inclusion.

The utility of anthropic reasoning depends on how sensitive probabilities like (4.1) are to the inclusion of 'us'. To make this concrete, consider the probabilities for a hypothetical cosmological parameter we will call Λ . We will assume that H and $|\Psi\rangle$ imply that Λ is constant over the visible universe, but only supply probabilities for the various constant values it might take through (2.4). We seek to compare $p(\Lambda|H,\Psi)$ with $p(\Lambda|\text{'us'},H,\Psi)$. In principle, both are calculable from (2.4) and (3.1). Figure 1 shows three possible ways they might be related:

- $p(\Lambda|H, \Psi)$ is peaked around one value as in Fig. 1(a). The parameter Λ is determined either by H or $|\Psi\rangle$, or by both.³ Anthropic reasoning is not necessary; the parameter is already determined by fundamental physics.
- $p(\Lambda|H,\Psi)$ is distributed and $p(\Lambda|\text{'us'}, H, \Psi)$ is also distributed as in Fig. 1(b). Anthropic reasoning is inconclusive. One might as well measure the value of Λ and use this as a condition for making further predictions⁴ *i.e.* work with probabilities of the form $p(\alpha|\Lambda, H, \Psi)$.
- $p(\Lambda|H, \Psi)$ is distributed but $p(\Lambda|\text{'us'}, H, \Psi)$ is peaked. Anthropic reasoning helps to explain the value of Λ .

The important point to emphasize is that a theoretical hypothesis for H and $|\Psi\rangle$ is needed to carry out anthropic reasoning. Put differently, a theoretical context is needed to decide whether a parameter like Λ can vary, and to find out how it varies, before using anthropic reasoning to restrict its range. The Hamiltonian and quantum state provide this context. In the Section V we will consider the situation where the state is imperfectly known.

B. Less is More

While there can be no objections of principle to including 'us' as a condition for the probabilities of our observations, there are formidable obstacles of practice:

• We are complex physical systems requiring an extensive environment and a long evolutionary history whose description in terms of the fundamental variables of H and $|\Psi\rangle$ may be uncertain, long, and complicated.

³ As, for example, in the as yet inconclusive discussions of baby universes [12].

⁴ As stressed by Hawking and Hertog [13].

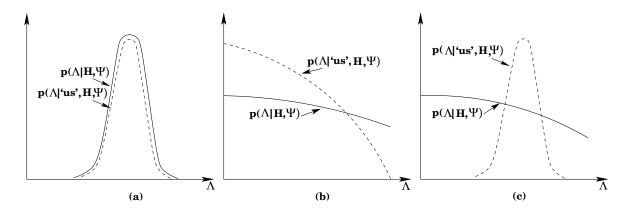


FIG. 1: Some possible behaviors for probabilities for the value of a cosmological parameter Λ with and without the condition 'us' are illustrated. In the situation illustrated in (a) the value of Λ is fixed by H and $|\Psi\rangle$ and anthropic reasoning is not needed. In (b) anthropic probabilities are distributed so that anthropic reasoning is useless in fixing Λ . Anthropic reasoning is useful in the situation (c).

• The complexity of the description of a condition including 'us' may make the calculation of the probabilities long or impossible as a practical matter.

In practice, therefore, anthropic probabilities (4.1) can only be estimated or guessed. Theoretical uncertainty in the results is thereby introduced.

The objectivity striven for in physics consists, at least in part, in using probabilities that are not too sensitive to 'us'. We would not have science if anthropic probabilities for observation depended significantly on which individual human being was part of the conditions. The existence of schizophrenic delusions shows that this is possible so that the notion of 'us' should be restricted to exclude such cases.

For these reasons it is prudent to condition probabilities, not on a detailed description of 'us', but on the weakest condition consistent with 'us' that plausibly provides useful results like those illustrated in Fig. 1c. A short list of conditions of roughly decreasing complexity might include:

- human beings;
- carbon-based life;
- information gathering and utilizing systems (IGUSes);
- at least one galaxy;
- a universe older than 5 Gyr;
- no condition at all.

For example, the probabilities used to bound the cosmological constant Λ [1, 14] make use of the fourth and fifth on this list under the assumption that including earlier ones will not much affect the anthropically-allowed range for Λ . To move down in the above list of conditions is to move in the direction of increasing theoretical certainty and decreasing computational complexity. With anthropic reasoning, less is more.

V. IGNORANCE IS NOT BLISS

The quantum state of a single isolated subsystem generally cannot be determined from a measurement carried out on it. That is because the outcomes of measurements are distributed probabilistically and the outcome of a single trial does not determine the distribution. Neither can the state be determined from a series of measurements because measurements disturb the state of the subsystem. The Hamiltonian can not be inferred from a sequence of measurements on one subsystem for similar reasons. In the same way, we can not generally determine either the Hamiltonian or the quantum state of the universe from our observations of it. Rather these two parts of a final theory are theoretical proposals, inferred from partial data to be sure, but incorporating theoretical assumptions of simplicity, beauty, coherence, mathematical precision, etc. To test these proposals we search among the conditional probabilities they imply for predictions of observations yet to be made with probabilities very near one. When such predictions occur we count it a success of the theory, when they do not we reject it and propose another.

Do we need a theory of the quantum state? To analyze this question, let us consider various degrees of theoretical uncertainty about it.

A. Total Ignorance

In the model cosmology in a box of Section II, theoretical uncertainty about the quantum state can be represented by a density matrix ρ that specifies probabilities for its eigenstates to be $|\Psi\rangle$. Total ignorance of the quantum state is represented by a ρ proportional to the unit matrix. To illustrate this and the subsequent discussion, assume for the moment that the dimension of the Hilbert space is very large but finite. Then total ignorance of the quantum state is represented by

$$\rho_{\text{tot. ign.}} = \frac{I}{Tr(I)} \tag{5.1}$$

which assigns equal probability any to any member of any complete set of orthogonal states. The density matrix (5.1) predicts thermal equilibrium, infinite temperature, infinitely large field fluctuations, and maximum entropy [9]. In short, its predictions are inconsistent with observations. This is a more precise way of saying that every useful prediction depends in some way on a theory of the quantum state. Ignorance is not bliss.

B. What We Know

A more refined approach to avoiding theories of the quantum state is to assume that it is unknown except for reproducing our present observations of the universe. The relevant density matrix is

$$\rho_{\rm obs} = \frac{P_{\rm obs}}{Tr(P_{\rm obs})} \tag{5.2}$$

where $P_{\rm obs}$ is the projection on our current observations — "what we know". 'Observations' in this context mean what we directly observe and record here on Earth and not the inferences we draw from this data about the larger universe. That is because those inferences are

based on assumptions about the very quantum state that (5.2) aims to ignore. For instance, we observed nebulae long before we understood what they were or where they are. The inference that the nebulae are distant clusters of stars and gas relies on assumptions about how the universe is structured on very large scales that are in effect weak assumptions on the quantum state.

Even if we made the overly generous assumption that we had somehow directly observed and recorded every detail of the volume 1 km above the surface of the Earth, say at a 1 mm resolution, that is still a tiny fraction ($\sim 10^{-60}$) of the volume inside the present cosmological horizon. The projection operator $P_{\rm obs}$ therefore defines a very large subspace of Hilbert space. We can expect that the entropy of the density matrix (5.2) will therefore be near maximal, close to that of (5.1), and its predictions similarly inconsistent with further observations.

In the context of anthropic reasoning, these results show that conditioning probabilities on 'us' alone is not enough to make useful predictions. Rather, a theory of H and $|\Psi\rangle$ are needed in addition as described in the previous section.

VI. A FINAL THEORY

Let us hope that one day we will have a unified theory based on a *principle* that will specify *both* quantum dynamics (H) and a unique quantum state of the universe $(|\Psi\rangle)$. That would truly be a final theory and a proper context for anthropic reasoning.

Acknowledgments

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