What Connects Different Interpretations of Quantum Mechanics?*

James B. Hartle[†] Department of Physics, University of California Santa Barbara, California 93106-9530

Abstract

We investigate the idea that different interpretations of quantum mechanics can be seen as restrictions of the consistent (or decoherent) histories quantum mechanics of closed systems to particular classes of histories, together with the approximations and descriptions of these histories that the restrictions permit.

1 Introduction

A list of some thirteen different interpretations of quantum mechanics was discussed at the concluding session of this conference¹. The very length of this list invites the questions: What are the relationships between these interpretations?; To what uses may they be put?; and Is it possible to objectively settle on one?. This brief article offers some personal reflections on these questions.

The defining thread connecting interpretations of quantum theory is their agreement on the probabilities for the outcomes of measurements, at least to an excellent approximation. Some formulations may provide probabilities for further kinds of alternatives such as the position of the Moon when it is

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[†]hartle@physics.ucsb.edu

¹See the contribution of A. Elitzur to this volume.

not receiving attention from observers, or the values of density fluctuations in the very early universe when there were no observers around. However, a formulation that does not reproduce the standard textbook answers for the probabilities of *measurements* it is not an interpretation of *quantum mechanics*. Rather, it is a different theory. Such alternatives to quantum theory are of great interest but not the subject of this essay.

The idea explored here is that a number of different interpretations of quantum mechanics can be connected through the consistent (or decoherent) quantum mechanics of a closed system. Specifically, a number of interpretations can be seen as restrictions of consistent histories quantum theory to particular kinds of sets of alternative histories² together with the approximations and special descriptions of the sets that these restrictions permit. This essay examines three cases where this connection can be made and gives brief discussions of the utility of the restrictions involved.

2 The Quantum Mechanics of Closed Systems

We begin with a very brief review of the quantum mechanics of a closed system, most generally the universe as a whole³. To simplify the discussion we neglect quantum gravity and assume a fixed background spacetime geometry. The familiar apparatus of Hilbert space, states, and operators may then be employed to formulate the quantum mechanics of the closed system⁴. As a simple model, we can think of a large, isolated box of N non-relativistic particles. Dynamics can be specified in terms of particle positions \vec{x}_i and momenta \vec{p}_i by a Hamiltonian

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m_i} + V(\vec{x}_i). \tag{2.1}$$

Both observers and observed, if any, are contained inside. This is evidently not the most general description of a closed system but it will suffice to

² "Frameworks" in the terminology of Griffiths.

³For details see the classic expositions in [1, 2, 3].

⁴For the generalizations that may be otherwise required, see [4]. We view the quantum mechanics of closed systems as an extension and completion of the Everett formulation and therefore do not count that as a separate interpretation.

illustrate some of the connections between interpretations that we describe later.

We take the closed system to be described by a quantum state $|\Psi\rangle$. The most general objective of quantum theory is the prediction from H and $|\Psi\rangle$ of the probabilities of the individual members of a set of coarse-grained alternative histories of the closed system. A history is described by giving a sequence of alternatives $(\alpha_1, \dots \alpha_n)$ at a series of times t_1, \dots, t_n . Alternatives at a moment of time t_k are represented by an exhaustive set of orthogonal, Heisenberg picture, projection operators $\{P_{\alpha_k}^k(t_k)\}$ and a history of alternatives is represented by the corresponding chain of projections called a class operator

$$C_{\alpha} = P_{\alpha_n}^k(t_n) \cdots P_{\alpha_1}^1(t_1). \tag{2.2}$$

(On the left side of (2.2) we have abbreviated the whole chain $(\alpha_1, \dots, \alpha_n)$ by a single index α). For example, if we are interested in a history of the Earth moving around the Sun, the P's might be projections onto exclusive ranges of the center of mass position of the Earth at a sequence of times. This set of histories is coarse-grained because alternatives are not specified at every time but only at some times, because the center of mass position is not specified exactly but only in certain ranges, and because not every variable describing the universe is specified but only the center of mass of the Earth.

The class operators C_{α} defined in (2.2) permit the construction of branch state vectors

$$|\Psi_{\alpha}\rangle = C_{\alpha}|\Psi\rangle \tag{2.3}$$

for each history in the coarse-grained set. A set of histories *decoheres* when there is negligible mutual interference between all the branch state vectors:

$$\langle \Psi_{\alpha} | \Psi_{\alpha'} \rangle \approx 0, \ \alpha \neq \alpha'.$$
 (2.4)

The joint probability p_{α} for all the events in a history α is

$$p_{\alpha} = \| |\Psi_{\alpha}\rangle \|^2 = \|C_{\alpha}|\Psi\rangle\|^2 \tag{2.5}$$

when the set of histories decoheres. Decoherence ensures the validity of the probability sum rules which are among the defining properties of probability.

The above discussion is brief, certainly oversimplified in some respects, but sufficient we hope for understanding the remarks which follow. The key point for the ensuing discussion is the following: Decoherent histories quantum mechanics predicts probabilities for many different sets of alternative histories which are *complimentary* in the following sense: Each set is part of a complete quantum description of the system, but there is no fine-grained decoherent set histories of which all the decoherent sets are coarse-grainings. A set of histories coarse-grained by the Earth's center of mass momentum is an example of a set which (if decoherent) would be complementary to the set coarse-grained by the Earth's center of mass position.

Given H and $|\Psi\rangle$, it is in principle possible to calculate all decoherent sets. Among these is the quasiclassical realm of everyday experience, coarse-grained by the variables of classical physics, and exhibiting classical patterns correlation in time summarized approximately by classical equations of motion. As human observers we focus almost entirely on coarse-grainings of this quasiclassical realm. However, quantum theory does not distinguish the quasiclassical realm from other decoherent sets except by properties such as its classicality.

The picture of quantum reality which emerges from the quantum mechanics of closed systems is very different from the reality of classical physics involving, as it does, many *complementary* descriptions of the universe that are mutually incompatible. Restricting the allowed sets of histories by some principle⁵ typically yields a description of reality that is closer in character to the familiar classical one. We will see that in the cases to be discussed.

3 Three Case Studies

This section considers the idea offered in the Introduction for three different interpretations of quantum theory.

3.1 Copenhagen Quantum Mechanics

The Copenhagen quantum mechanics found in most textbooks is concerned with the probabilities of histories of the outcomes of measurements carried out by observers. The subsystem being observed is described by a Hilbert space \mathcal{H}_s . Dynamics is specified by a Hamiltonian h acting on \mathcal{H}_s when the subsystem is isolated. Initially the subsystem is assumed to be a state $|\psi\rangle$ in \mathcal{H}_s . The outcomes of a measurement carried out at time t_k can be described by a set of orthogonal, Heisenberg picture, projection operators $\{s_{\alpha_k}^k(t_k)\}$, $\alpha_k = 1, 2, \cdots$ analogous to the P's described in Section II. The probabilities

⁵A set selection principle in terminology of Dowker and Kent [5].

for a history of ideal measurements (ones that disturb the subsystem as little as possible) at times t_1, \dots, t_n are given by the analog of (2.5).

$$p_{\alpha} = \left\| s_{\alpha_n}^n(t_n) \cdots s_{\alpha_1}^1(t_1) |\psi\rangle \right\|^2. \tag{3.1}$$

Consistency is not an issue for these probabilities. Probabilities for a coarsergrained history need not be the sum of the probabilities of finer-grained histories consistent with it. Finer and coarser grained measurements correspond to different interactions of the subsystem with an external apparatus. Sets of histories describing alternative measurements do not have to decohere.

Copenhagen quantum mechanics is an approximation to the quantum mechanics of closed systems that is appropriate for histories of measurement situations when the decoherence of alternatives that register the outcomes of the measurements can be idealized as exact. We sketch only the essential features of a demonstration which are essentially the same as many measurement models. For details see, e.g. [6], Section II.10.

We consider a closed system with a Hilbert space $\mathcal{H}_s \otimes \mathcal{H}_r$ where \mathcal{H}_s is the Hilbert space of the measured subsystem and the \mathcal{H}_r is the Hilbert space of the rest of the universe including any measuring apparatus and observers. We assume an initial state of the form $|\Psi\rangle = |\psi\rangle \otimes |\Phi_r\rangle$ and consider a sequence of measurements at a series of times t_1, \dots, t_n . Measured alternatives of the subsystem are described by projection operators whose Schrödinger picture representatives have the form $S_{\alpha_k}^k = s_{\alpha_k}^k \otimes I_r$. In a typical measurement situation, an alternative such as $S_{\alpha_k}^k$ becomes correlated with an alternative of the apparatus and in particular with persistent records of the measurements. The orthogonality and persistence of these records guarantees the decoherence of the histories of measured outcomes. If the usual assumption is made that the measurement interaction disturbs the subsystem as little as possible (ideal measurement), then

$$p_{\alpha} = \left\| S_{\alpha_n}^n(t) \cdots S_{\alpha_1}^1(t_1) |\Psi\rangle \right\|_{\mathcal{H}}^2 \approx \left\| s_{\alpha_n}^n(t_n) \cdots s_{\alpha_1}^1(t_1) |\psi\rangle \right\|_{\mathcal{H}_r}^2. \tag{3.2}$$

Thus Copenhagen quantum mechanics is recovered as a restriction of, and approximation to, the quantum mechanics of closed systems. The second equality in (3.2) is not exact but true to an excellent approximation in realistic measurement situations — typically far beyond the accuracy which the probabilities can be checked or the physical situation modeled.

The utility of the approximate quantum mechanics of measured subsystems is evident. It is a truly excellent approximation for every laboratory

experiment which has tested the principles of quantum theory. Further, the calculations of the approximate Copenhagen probabilities utilizing just the Hilbert space of the measured subsystem will generally be vastly simpler than in the Hilbert space of the universe. These advantages, however, should not obscure the utility of embedding the Copenhagen quantum mechanics in the more general quantum mechanics of closed systems for understanding measurements (as above) and calculating just how good an approximation it is.

3.2 Bohm Theory

To summarize the features of Bohm theory [7] that are relevant to the present discussion, it is convenient to restrict attention to the closed system consisting of N, non-relativistic particles in a box discussed in Section II. An initial wave function $\Psi(\vec{x}_1, \dots, \vec{x}_N, 0)$ is given. The particles in the box move on trajectories $\vec{x}_i(t)$ that obey two deterministic equations. The first is the Schrödinger equation for Ψ :

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi. \tag{3.3}$$

Then, writing $\Psi = R \exp(iS)$ with R and S real, the second equation is the deterministic equation for the $\vec{x}_i(t)$

$$m_i \frac{d\vec{x}_i}{dt} = \vec{\nabla}_{\vec{x}_i} S(\vec{x}_1, \dots, \vec{x}_N). \tag{3.4}$$

The initial wave function is the initial condition for (3.3). The theory becomes a statistical theory with the assumption that the *initial values* of the \vec{x}_i are distributed according to the probability density on configuration space

$$\wp(\vec{x}_1, \dots, \vec{x}_N, 0) = |\Psi(\vec{x}_1, \dots, \vec{x}_N, 0)|^2, \text{ at the initial time } 0.$$
 (3.5)

Once this initial probability distribution is fixed, the probability of any later alternatives is fixed by the deterministic equation (3.4).

A coarse-grained Bohmian history $\alpha \equiv (\alpha_n, \dots, \alpha_1)$ is defined by a sequence of ranges $\{\Delta_{\alpha_k}^k\}$ of the \vec{x}_i at a series of times $t_1, \dots t_n$ and consists of the set of Bohmian trajectories $\vec{x}_i(t)$ that cross those ranges at the specified times.

The predictions of Bohm theory and the quantum mechanics of closed systems can be compared for sets of alternative histories coarse-grained by ranges of the position \vec{x}_i at different times as above. Generally different probabilities are predicted for the same set of histories [8]. This difference arises as follows: Bohm histories are deterministic. That means that the probability that the particles traverse a series of regions of configuration space at a sequence of times is the same as the probability of the initial values of \vec{x}_i that evolve to those trajectories under the equations of motion (3.3) and (3.4). The probability of a Bohm trajectory can therefore be represented as

$$p_{\alpha}^{(BM)} = \|B_{\alpha}|\Psi\rangle\|^2 \tag{3.6}$$

where B_{α} is a projection onto the appropriate initial conditions.

The probabilities of the *same* set of histories would be calculated in decoherent histories quantum theory from [cf. (2.5)]

$$p_{\alpha}^{(\mathrm{DH})} = \|C_{\alpha}|\Psi\rangle\|^2 \tag{3.7}$$

provided the set is decoherent. Here the C_{α} are chains of projections like (2.2) It is a simple observation is that a chain of projections like (2.2) is not generally a projection and that therefore $p_{\alpha}^{(\text{BM})}$ will not agree generally with $p_{\alpha}^{(\text{DH})}$. (See [8] for examples and further discussion.)

Another way of seeing the difference is to note that in Bohm theory the wave function always evolves by the Schrödinger equation — unitary evolution. But the action of a chain of projections C_{α} on the initial state can be described as unitary evolution interrupted by the action of the projections (reduction).

Only in the case of histories with alternatives at a single time are the predictions of Bohm theory and the quantum mechanics of closed systems guaranteed to agree. Then the C_{α} are projections. But this is an important case because it leads to the conclusion that Bohm theory and the quantum mechanics of closed systems agree on the probabilities of the outcome of measurements.

One characteristic of a measurement situation which seems generally agreed upon is that the results of a measurement are recorded — at least for a time. A history C_{α} of measurement outcomes is recorded in a set of alternatives $\{R_{\alpha}\}$ at a time later than the last alternative in C_{α} if the values of the R_{α} are correlated with the outcomes of the measurements described by the C_{α} . The R_{α} 's are projections even if the C_{α} 's are not. Bohm theory and

the quantum theory of closed systems will therefore agree on the probabilities of these records.

Bohm theory can therefore be seen as a restriction of the quantum theory of closed systems to alternatives describing the records of measurements (in the \vec{x} 's) together with the description of these outcomes in terms of deterministic trajectories obeying (3.3) and (3.4). An advantage of Bohm theory (that is, of this restriction) that we believe would be claimed by its proponents is the clear specification of one set of histories (of the \vec{x} 's) as preferred over others. A potential disadvantage is that these histories, although deterministic, may not be classical even in situations where the correlations of classical physics in time are predicted with high probability by the quantum mechanics of closed systems [9]. Thus, for example, even when a classical past is retrodicted from present records from the quantum mechanics of closed systems, Bohm theory may predict a non-classical one depending on the nature of the initial condition [8].

3.3 Sum-Over-Histories

The starting point for a sum-over-histories formulation of quantum mechanics is the specification of one set of fine-grained histories. For the model universe of non-relativistic particles in a box, these are the particle paths $\vec{x}_i(t), i=1,\cdots,N$. The allowed coarse-grainings are partitions of this set of fine-grained histories into an exhaustive set of exclusive classes. For example, the paths could be partitioned by how they traverse a set of regions of configuration space $\{\Delta_{\alpha_k}\}$, $\alpha_k=1,2,\cdots$ at sequence of times $t_k, k=1,\cdots,n$. The class operators C_{α} are specified by giving their matrix elements as sums over the fine-grained paths in the coarse-grained class labeled by α . Denoting a point in the 3N-dimensional configuration space by x, this sum is

$$\langle x''|C_{\alpha}|x'\rangle = \int_{\alpha} \delta x \, e^{iS[x(t)]/\hbar}.$$
 (3.8)

Here, S[x(t)] is the action functional and the sum is over all fine-grained histories in the class labeled by α . For instance, in the partition by sequences of sets of regions at a series of times, a coarse-grained history α is labeled by the regions $(\alpha_1, \dots, \alpha_n)$ crossed at the sequence of times and the sum in (3.8) defining the class operator is over paths that cross these regions. The construction of probabilities is then as described in Section II.

Sum-over-histories quantum theory is evidently a restriction of the quantum mechanics of closed systems described in Section II. All the possible sets of projection operators that might occur in the construction of a set of alternative histories like (2.1) are restricted to projections on ranges of position. The predictions of the restricted sets agree because of identities that express sums-over-histories in terms of operators. For instance,

$$\int_{[x'',\Delta_n,\cdots,\Delta_1,x']} dx \, e^{iS[x(t)]/\hbar} = \left\langle x'' \left| P_{\Delta_n}^n(t_n) \cdots P_{\Delta_1}^1(t_1) \right| x' \right\rangle \tag{3.9}$$

where the sum on the left hand side is over all paths that start at x' pass through the regions $(\Delta_1, \dots, \Delta_n)$ at times t_1, \dots, t_n and end at x' [10].

The sum-over-histories formulation of quantum theory is not usually discussed as a different *interpretation* of quantum mechanics. But it can be [11, 12] because, like Bohm theory, it specifies a fundamental set of variables. In effect, it posits a set selection principle. To the extent that the quasiclassical realm in which we operate as human observers can be described as a coarse graining of configuration space [13] no predictive power is lost in making this restriction. However, the restriction is not so strong as to narrow the range of available sets just to the quasiclassical realm.

There is some loss in convenience with a sum-over-histories formulation because quantities like the momentum of a particle must be described in spacetime terms — by time of flight for example [11]. But there is also potential gain. A sum-over-histories restriction provides a head start in the characterization of classicality and the explanation of its origin (see e.g. [14]). A sum-over-histories formulation of quantum mechanics is the natural framework for investigating generalizations of quantum mechanics that are necessary to describe spacetime alternatives extended over time (e.g. [15]) and those which may be needed for a quantum theory of gravity [16, 4].

4 Is There One Interpretation of Quantum Mechanics?

It would be interesting to investigate how many different interpretations of quantum theory can be seen as restrictions of the quantum mechanics of closed systems together with the approximations and particular descriptions of histories that these restrictions permit. That would be at least one way of connecting different interpretations and a common basis for discussing their assumptions, advantages, motivations, and limitations.

It would be equally interesting to identify interpretations of quantum mechanics which cannot be viewed as restrictions of the quantum mechanics of closed systems for some fundamental reason. (And not simply because they lack the coherence to decide.) Consistent histories quantum mechanics is logically consistent, consistent with experiment as far as is known, consistent with textbook predictions for measurements, and applicable to the most general physical systems. However, it may not be the only theory with these properties. Investigations of interpretations that do not fit within its umbrella framework may lead in different directions.

Can we distinguish between the different interpretations that are restrictions of the quantum mechanics of closed systems? Not by experiment or observations. By assumption the different interpretations agree on the predictions for measurement to excellent approximations. It seems unlikely to this author that we can settle on one interpretation by argument and discussion. (There is some empirical evidence for this conclusion.) There are too many individually held opinions on the objectives to be met by the restrictions. But neither does there seem to be a compelling need to settle among interpretations that are restrictions of a common quantum mechanics of closed systems

We may be able to distinguish interpretations by their utility and/or their promise as starting points for generalizations or alternatives to quantum theory. For instance, Copenhagen quantum mechanics is inadequate for cosmology. In cosmology there is no fundamental division of the closed system into two parts, one of which measures the other. Measurements and observers cannot be fundamental in a theory that seeks to describe the early universe where neither existed. In a quantum world there are generally no variables that behave classically in all circumstances. As another example, sum-over-histories quantum theory may be a productive route to generalizing usual quantum theory to incorporate the dynamical spacetime geometry of quantum gravity [4].

Many years ago, when an instructor at Princeton, I discussed my first effort in understanding quantum mechanics [17] with Eugene Wigner. At the conclusion of the discussion I asked him whether I should publish my results. Wigner explained that there were some subjects — and the interpretation of quantum mechanics was one of these — that one couldn't learn about by reading books or attending lectures. One just had to work through them

by oneself. And usually, if people took the trouble to do this and reached a conclusion, they published a paper. "So", he said, "why shouldn't you?" Maybe that is another reason there are so many interpretations of quantum theory.

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