Comment on "Consistent Sets Yield Contrary Inferences in Quantum Theory"

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In [1], Kent correctly points out that in consistent histories quantum theory it is possible, given initial and final states, to construct two different consistent families of histories (what Kent calls consistent sets), in each of which there is a proposition that can be inferred with probability one, and such that the projectors representing these two propositions are mutually orthogonal. In response we stress that, according to the rules of consistent history reasoning two such propositions are not contrary in the usual logical sense [2]: namely, that one can infer that if one is true then the other is false, and both could be false. No single consistent family contains both propositions, together with the initial and final states, and hence the propositions cannot be logically compared.

Let us say projectors P and Q are "perpendicular", to employ a term with no logical connotation, if PQ=0 and $P \neq I-Q$. If two "perpendicular" projectors occur in the same consistent family, they represent mutually exclusive events such that if the probability of one is 1 (true), that of the other is 0 (false), and thus the events corresponding to these projectors are "contrary" in the logical sense. However, the requirement stated in italics is absolutely essential, for it is a basic rule of reasoning in the consistent history formalism that all inferences must be carried out in a single consistent family [3,4].

As an example [5], suppose a particle can be in one of three non-overlapping boxes, for which the corresponding orthogonal states are $|A\rangle, |B\rangle, |C\rangle$, and assume the dynamics is trivial: if the particle starts in one box, it stays there. Let

$$|\Phi\rangle = (|A\rangle + |B\rangle + |C\rangle)/\sqrt{3}, |\Psi\rangle = (|A\rangle + |B\rangle - |C\rangle)/\sqrt{3},$$
 (1)

be initial and final states at times t_0 and t_2 . There is a consistent family of histories \mathcal{A} including these initial and final states, and, at a time t_1 between t_0 and t_2 , the projectors $A = |A\rangle\langle A|$ and $\tilde{A} = I - A$, which correspond to the particle being or not being in box A. Application of the consistent history formalism yields $\Pr(A|\Phi,\Psi)=1$; that is, using family \mathcal{A} , it is certain that the particle at time t_1 is in box A. There is also a second consistent family \mathcal{B} in which $B = |B\rangle\langle B|$ and $\tilde{B} = I - B$ are possible intermediate states at time t_1 , and using this family one finds $\Pr(B|\Phi,\Psi)=1$.

The projectors A and B are "perpendicular" in the sense defined above. However, there is no consistent family which includes Φ at t_0 and Ψ at t_2 together with both

A and B at t_1 . Thus inferences that concern both A and B, such as that the events are contrary, cannot be drawn from the initial and final data. (For additional remarks, see App. A of [6].) Kent employs the term "contrary" for the relationship between projectors that we have been calling "perpendicular", whether or not they occur in the same consistent family. But importing a term from classical logic into quantum theory must be done with some care in order to avoid confusion, as illustrated by the example just discussed.

By contrast, if a projector P occurs in a consistent family, I-P necessarily occurs in the same family, according to the rules in [4]. Hence, one can say that P and I-P are "contradictory" in the sense of classical logic [2]. There are no initial and final states for which projectors P and I-P can have probability one in different consistent families, because the probability assigned to any event on the basis of given data is independent of the consistent family in which it occurs. Kent [1] expresses concern that the consistent history formalism thereby treats contradictory and perpendicular projectors in different ways. However, we see no necessary reason for symmetry, because, while a pair P and I - P can always be associated with the logical notion of "contradictory", perpendicular projectors need not be associated with the logical concept of "contrary", as in the examples Kent is concerned with. Alternative, more restrictive, formulations of consistent histories quantum mechanics, as in [7], should be judged as theories on their own merits and ultimately by comparison with experiment.

Consistent histories quantum theory is logically consistent, consistent with experiment, consistent with the usual quantum predictions for measurements, and applicable to the most general physical systems. It may not be the only theory with these properties, but in our opinion, it is the most promising among present possibilities.

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