Generalizing Quantum Mechanics
for Quantum Spacetime[1]

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ABSTRACT

Familiar textbook quantum mechanics assumes a fixed background spacetime to define states on spacelike surfaces and their unitary evolution between them. Quantum theory has changed as our conceptions of space and time have evolved. But quantum mechanics needs to be generalized further for quantum gravity where spacetime geometry is fluctuating and without definite value. This paper reviews a fully four-dimensional, sum-over-histories, generalized quantum mechanics of cosmological spacetime geometry. This generalization is constructed within the framework of generalized quantum theory. This is a minimal set of principles for quantum theory abstracted from the modern quantum mechanics of closed systems, most generally the universe. In this generalization, states of fields on spacelike surfaces and their unitary evolution are emergent properties appropriate when spacetime geometry behaves approximately classically. The principles of generalized quantum theory allow for the further generalization that would be necessary were spacetime not fundamental. Emergent spacetime phenomena are discussed in general and illustrated with the example of the classical spacetime geometries with large spacelike surfaces that emerge from the 'no-boundary' wave function of the universe. These must be Lorentzian with one, and only one, time direction. The essay concludes by raising the question of whether quantum mechanics itself is emergent.

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1 Introduction

Does quantum mechanics apply to spacetime? This is the question the organizers asked me to address. It is an old issue. The renowned Belgian physicist Léon Rosenfeld wrote one of the first papers on quantum gravity [1], but late in his career came to the conclusion that the quantization of the gravitational field would be meaningless[1] [3, 4]. Today, there are probably more colleagues of the opinion that quantum theory needs to be replaced than there are who think that it doesn't apply to spacetime. But in the end this is an experimental question as Rosenfeld stressed .

This lecture will answer the question as follows: *Quantum mechanics can be applied to spacetime provided that the usual textbook formulation of quantum theory is suitably generalized.* A generalization is necessary because, in one way or another, the usual formulations rely on a fixed spacetime geometry to define states on spacelike surfaces and the time in which they evolve unitarily one surface to another. But in a quantum theory of gravity, spacetime geometry is generally fluctuating and without definite value. The usual formulations are emergent from a more general perspective when geometry is approximately classical and can supply the requisite fixed notions of space and time.

A framework for investigating generalizations of usual quantum mechanics can be abstracted from the modern quantum mechanics of closed systems [5, 6, 7] which enables quantum mechanics to be applied to cosmology. The resulting framework — generalized quantum theory [8, 9, 10] — defines a broad class of generalizations of usual quantum mechanics.

A generalized quantum theory of a physical system (most generally the universe) is built on three elements which can be very crudely characterized as follows:

- The possible fine-grained descriptions of the system.

- The coarse-grained descriptions constructed from the fine-grained ones.

- A measure of the quantum interference between different coarse-grained descriptions incorporating the principle of superposition

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[1]Rosenfeld considered the example of classical geometry curved by the expected value of the stress-energy of quantum fields. Some of the difficulties with this proposal, including experimental inconsistencies, are discussed by Page and Geilker [2].
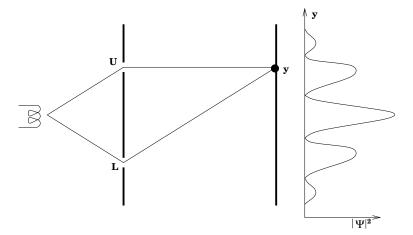
Figure 1: The two-slit experiment. An electron gun at left emits an electron traveling towards a screen with two slits, U and L, its progress in space recapitulating its evolution in time. The electron is detected at a further screen in a small interval Δ about the position y. It is not possible to assign probabilites to the alternative histories of the electron in which it went through the upper slit U on the way to y, or through the lower slit L on the way to y because of the quantum interference between these two histories.

We will define these elements more precisely in Section 6, explain how they are used to predict probabilities, and provide examples. But, in the meantime, the two-slit experiment shown in Figure 1 provides an immediate, concrete illustration.

A set of possible fine-grained descriptions of an electron moving through the two-slit apparatus are its Feynman paths in time (histories) from the source to the detecting screen. One coarse-grained description is by which slit the electron went through on its way to detection in an interval Δ about a position y on the screen at a later time. Amplitudes $\psi_U(y)$ and $\psi_L(y)$ for the two coarse-grained histories where the electron goes through the upper or lower slit and arrives at a point y on the screen can be computed as a sum over paths in the usual way (Section 4). The natural measure of interference between these two histories is the overlap of these two amplitudes integrated over the interval Δ in which the electron is detected. In this way usual quantum mechanics is a special case of generalized quantum theory.

Probabilities cannot be assigned to the two coarse-grained histories il-

2

lustrated in Figure 1 because they interfere. The probability to arrive at y should be the sum of the probabilities to go by way of the upper or lower slit. But in quantum theory, probabilities are squares of amplitudes and

$$|\psi_U(y) + \psi_L(y)|^2 \neq |\psi_U(y)|^2 + |\psi_L(y)|^2 . \tag{1.1}$$

Probabilities can only be predicted for sets of alternative coarse-grained histories for which the quantum interference is negligible between every pair of coarse-grained histories in the set (decoherence).

Usual quantum mechanics is not the only way of implementing the three elements of generalized quantum theory. Section 7 sketches a sum-over-histories generalized quantum theory of spacetime. The fine-grained histories are the set of four-dimensional cosmological spacetimes with matter fields on them. A coarse graining is a partition of this set into (diffeomorphism invariant) classes. A natural measure of interference is described. This is a fully four-dimensional quantum theory without an equivalent 3+1 formulation in terms of states on spacelike surfaces and their unitary evolution between them. Rather, the usual 3+1 formulation is emergent for those situations, and for those coarse grainings, where spacetime geometry behaves approximately classically. The intent of this development is not to propose a new quantum theory of gravity. This essentially low energy theory suffers from the usual ultraviolet difficulties. Rather, it is to employ this theory as a model to discuss how quantum mechanics can be generalized to deal with quantum geometry.

A common expectation is that spacetime is itself emergent from something more fundamental. In that case a generalization of usual quantum mechanics will surely be needed and generalized quantum theory can provide a framework for discovering it (Section 8). Emergence in quantum theory is discussed generally in Section 9. Section 10 describes the emergence of Lorentz signatured classical spacetimes from the no-boundary quantum state of the universe.

Section 11 concludes with some thoughts about whether quantum mechanics itself could be emergent from something deeper. But before starting on the path of extending quantum theory so far we first offer some remarks on where it is today in Section 2.
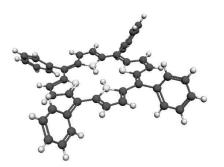
2   Quantum Mechanics Today

Three features of quantum theory are striking from the present perspective: its success, its rejection by some of our deepest thinkers, and the absence of compelling alternatives.

Quantum mechanics must be counted as one of the most successful of all physical theories. Within the framework it provides, a truly vast range of phenomena can be understood and that understanding is confirmed by precision experiment. We perhaps have little evidence for peculiarly quantum phenomena on large and even familiar scales, but there is *no* evidence that all the phenomena we do see, from the smallest scales to the largest of the universe, cannot be described in quantum mechanical terms and explained by quantum mechanical laws. Indeed, the frontier to which quantum interference is confirmed experimentally is advancing to ever larger, more 'macroscopic' systems[2]. The textbook electron two-slit experiment shown schematically in Fig. 1 has been realized in the laboratory [12]. Interference has been confirmed for the biomolecule tetraphenylporphyrin ($C_{44}H_{30}N_4$) and the flurofullerine ($C_{60}F_{48}$) in analogous experiments [13] (Figure 2). Experiments with superconducting squids have demonstrated the coherent superposition of macroscopic currents [14, 15, 16]. In particular, the experiment of Friedman, *et al.* [16] exhibited the coherent superposition of two circulating currents whose magnetic moments were of order $10^{10}\mu_B$ (where $\mu_B = e\hbar/2m_e c$ is the Bohr magneton). Experiments under development will extend the boundary further [17]. Experiments of increasing ingenuity and sophistication have extended the regime in which quantum mechanics has been tested. No limit to its validity has yet emerged.

Even while acknowledging its undoubted empirical success, many of our greatest minds have rejected quantum mechanics as a framework for fundamental theory. Among the pioneers, the names of Einstein, Schrödinger, DeBroglie, and Bohm stand out in this regard. Among our distinguished contemporaries, Adler, Leggett, Penrose, and 't Hooft could probably be counted in this category. Much of this thought has in common the intuition that quantum mechanics is an effective approximation of a more fundamental theory built on a notion of reality closer to that classical physics.

Remarkably, despite eighty years of unease with its basic premises, and despite having been tested only in a limited, largely microscopic, domain,

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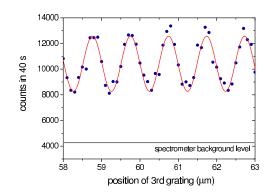[2]For an insightful and lucid review see [11].

Figure 2: Interference of Biomolecules. The molecule tetraphenylporphyrin ($C_{44}H_{30}N_4$) is shown at left. Its quantum interference fringes in a Talbot-Lau interferometer are shown at right from experiment carried out in Anton Zeilinger's group (Hackermüller, *et al.* [13].)

no fully satisfactory alternative to quantum theory has emerged. By fully satisfactory we mean not only consistent with existing experiment, but also incorporating other seemingly secure parts of modern physics such as special relativity, field theory, and the standard model of elementary particle interactions. As Steve Weinberg summarized the situation, "It is striking that it has not so far been possible to find a logically consistent theory that is close to quantum mechanics other than quantum mechanics itself" [18]. Alternatives to quantum theory meeting the above criteria would be of great interest if only to guide experiment.

There are several directions under investigation today which aim at a theory from which quantum mechanics would be emergent. Neither space nor the author's competence permit an extensive discussion of these ideas. But we can mention some of the more important ones.[3]

Bohmian mechanics [20] in its most representative form is a deterministic but highly non-classical theory of particle dynamics whose statistical predictions largely coincide with quantum theory [21]. Fundamental noise [22] or spontaneous dynamical collapse of the wave function [23, 24] are the underlying ideas of another class of model theories whose predictions are distinguishable from those of quantum theory, in principle. Steve Adler has proposed a statistical mechanics of deterministic matrix models from which

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[3]The references to these ideas are obviously not exhaustive, nor are they necessarily current. Rather, they are to typical sources. For an encyclopedic survey of different interpretations and alternatives to quantum mechanics, see [19].

quantum mechanics is emergent [25]. Gerard 't Hooft has a different set of ideas for a determinism beneath quantum mechanism that are explained in his article in this volume [26]. Roger Penrose has championed a role for gravity in state vector reduction [27, 28]. This has not yet developed into a detailed alternative theory, but has suggested experimental situations in which the decay of quantum superpositions could be observed [28, 17].

In the face of an increasing domain of confirmed predictions of quantum theory and the absence as yet of compelling alternatives, it seems natural to extend quantum theory as far as it will go — to the largest scales of the universe and the smallest of quantum gravity. That is the course we shall follow in this paper. But as mentioned in the introduction, usual quantum theory must be generalized to apply to cosmology and quantum spacetime. We amplify on the reasons in the next section.

3   Spacetime and Quantum Theory

Usual, textbook quantum theory incorporates definite assumptions about the nature of space and time. These assumptions are readily evident in the two laws of evolution for the quantum state Ψ. The Schrödinger equation describes its unitary evolution between measurements.

$$ i\hbar \, \frac{\partial \Psi}{\partial t} = H\Psi \; . \tag{3.1} $$

At the time of an ideal measurement, the state is projected on the outcome and renormalized

$$ \Psi \to \frac{P\Psi}{\|P\Psi\|} \; . \tag{3.2} $$

The Schrödinger equation (3.1) assumes a fixed notion of time. In the non-relativistic theory, t is the absolute time of Newtonian mechanics. In the flat spacetime of special relativity, it is the time of any Lorentz frame. Thus, there are many times but results obtained in different Lorentz frames, are unitarily equivalent.

The projection in the second law of evolution (3.2) is in Hilbert space. But in field theory or particle mechanics, the Hilbert space is constructed from configurations of fields or position in physical *space*. In that sense it is the state on a spacelike surface that is projected (3.2).

A Short History of Spacetime and Quantum Theory

Newtonian Physics	Fixed 3-d space and a single universal time t.	**Non-relativistic Quantum Theory:** The Schrödinger equation $$i\hbar(\partial\Psi/\partial t) = H\Psi$$ holds between measurements in the Newtonian time t.
Special Relativity	Fixed flat, 4-d spacetime with many diifferent timelike directions.	**Relativistic Quantum Field Theory:** Choose a Lorentz frame with time t. Then (between measurements) $$i\hbar(\partial\Psi/\partial t) = H\Psi\,.$$ The results are unitarily equivalent to those from any other choice of Lorentz frame
General Relativity	Fixed, but curved spacetime geometry	**Quantum Field Theory in Curved Spacetime:** Choose a foliating famliy of spacelike surfaces labeled by t. Then (between measurements) $$i\hbar(\partial\Psi/\partial t) = H\Psi\,.$$ But the results are *not* generally unitarily equivalent to other choices.
Quantum Gravity	Geometry is *not* fixed, but rather a quantum variable	**The Problem of Time:** What replaces the Schrödinger equation when there is no fixed notion of time(s)?
M-theory, Loop quantum gravity, Posets, etc.	Spacetime is not even a fundamental variable	**?**

Because quantum theory incorporates notions of space and time, it has changed as our ideas of space and time have evolved. The accompanying table briefly summarizes this co-evolution. It is possible to view this evolution as a process of increasing generalization of the concepts in the usual theory. Certainly the two laws of evolution (3.1) and (3.2) have to be generalized somehow if spacetime geometry is not fixed. One such generalization is offered in this paper, but there have been many other ideas [29]. And if spacetime geometry is emergent from some yet more fundamental description, we can certainly expect that a further generalization — free of any reference to spacetime — will be needed to describe that emergence. The rest of this article is concerned with these generalizations.

4   The Quantum Mechanics of Closed Systems

This section reviews, very briefly, the elements of the modern quantum mechanics of closed systems[4] aimed at a quantum mechanics for cosmology. To keep the present discussion manageable we focus on a simple model universe of particles moving in a very large box (say $\gtrsim$ 20,000 Mpc in linear dimension). Everything is contained within the box, in particular galaxies, stars, planets, observers and observed (if any), measured subsystems, and the apparatus that measures them.

We assume a fixed background spacetime supplying well-defined notions of time. The usual apparatus of Hilbert space, states, operators, Feynman paths, etc. can then be employed in a quantum description of the contents of the box. The essential theoretical inputs to the process of prediction are the Hamiltonian H and the initial quantum state $|\Psi\rangle$ (the 'wave function of the universe'). These are assumed to be fixed and given.

The most general objective of a quantum theory for the box is the prediction of the probabilities of exhaustive sets of coarse-grained alternative time histories of the particles in the closed system. For instance, we might be interested in the probabilities of an alternative set of histories describing the progress of the Earth around the Sun. Histories of interest here are typically very coarse-grained for at least three reasons: They deal with the position of the Earth's center-of-mass and not with the positions of all the particles

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[4]See, *e.g.* [5, 6, 7] for by now classic expositions at length or [30] for a shorter summary.

in the universe. The center-of-mass position is not specified to arbitrary accuracy, but to the error we might observe it. The center-of-mass position is not specified at all times, but typically at a series of times.

But, as described in the Introduction, not every set of alternative histories that may be described can be assigned consistent probabilities because of quantum interference. Any quantum theory must therefore not only specify the sets of alternative coarse-grained histories, but also give a rule identifying which sets of histories can be consistently assigned probabilities as well as what those probabilities are. In the quantum mechanics of closed systems, that rule is simple: probabilities can be assigned to just those sets of histories for which the quantum interference between its members is negligible as a consequence of the Hamiltonian H and the initial state $|\Psi\rangle$. We now make this specific for our model universe of particles in a box.

Three elements specify this quantum theory. To facilitate later discussion, we give these in a spacetime sum-over-histories formulation.

1. *Fine-grained histories*: The most refined description of the particles from the initial time $t = 0$ to a suitably large final time $t = T$ gives their position at all times in between, *i.e.* their Feynman paths. We denote these simply by $x(t)$.

2. *Coarse-graining*: The general notion of coarse-graining is a partition of the fine-grained paths into an exhaustive set of mutually exclusive classes $\{c_\alpha\}, \alpha = 1, 2, \cdots$. For instance, we might partition the fine-grained histories of the center-of-mass of the Earth by which of an exhaustive and exclusive set of position intervals $\{\Delta_\alpha\}, \alpha = 1, 2, \cdots$ the center-of-mass passes through at a series of times $t_1, \cdots t_n$. Each coarse-grained history consists of the bundle of fine-grained paths that pass through a specified sequence of intervals at the series of times. Each coarse-grained history specifies an orbit where the center-of-mass position is localized to a certain accuracy at a sequence of times.

3. *Measure of Interference*: Branch state vectors $|\Psi_\alpha\rangle$ can be defined for each coarse-grained history in a partition of the fine-grained histories into classes $\{c_\alpha\}$ as follows

$$\langle x|\Psi_\alpha\rangle = \int_{c_\alpha} \delta x \ \exp(iS[x(t)]/\hbar) \langle x'|\Psi\rangle. \qquad (4.1)$$

9

Here, $S[x(t)]$ is the action for the Hamiltonian H. The integral is over all paths starting at x' at $t = 0$, ending at x at $t = T$, and contained in the class c_α. This includes an integral over x'. (For those preferring the Heisenberg picture, this is equivalently

$$|\Psi_\alpha\rangle = e^{-iHT/\hbar} P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1) |\Psi\rangle \tag{4.2}$$

when the class consists of restrictions to position intervals at a series of times and the P's are the projection operators representing them.)

The measure of quantum interference between two coarse-grained histories is the overlap of their branch state vectors

$$D(\alpha', \alpha) \equiv \langle \Psi_{\alpha'} | \Psi_\alpha \rangle. \tag{4.3}$$

This is called the *decoherence functional*.

When the interference between each pair of histories in a coarse-grained set is negligible

$$\langle \Psi_\alpha | \Psi_\beta \rangle \approx 0 \text{ all } \alpha \neq \beta, \tag{4.4}$$

the set of histories is said to *decohere*[5]. The probability of an individual history in a decoherent set is

$$p(\alpha) = \| |\Psi_\alpha\rangle \|^2. \tag{4.5}$$

The decoherence condition (4.3) is a sufficient condition for the probabilities (4.4) to be consistent with the rules of probability theory. Specifically, the p's obey the sum rules

$$p(\bar{\alpha}) \approx \sum_{\alpha \in \bar{\alpha}} p(\alpha) \tag{4.6}$$

where $\{\bar{c}_{\bar{\alpha}}\}$ is any coarse-graining of the set $\{c_\alpha\}$, *i.e.* a further partition into coarser classes. It was the failure of such a sum rule that prevented consistent probabilities from being assigned to the two histories previously discussed in the two-slit experiment (Figure 1). That set of histories does not decohere.

Decoherence of familiar quasiclassical variables is widespread in the universe. Imagine, for instance, a dust grain in a superposition of two positions, a multimeter apart, deep in intergalactic space. The 10^{11} cosmic background

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[5]This is the *medium* decoherence condition. For a discussion of other conditions, see, *e.g.* [31, 32, 33].

10

photons that scatter off the dust grain every second dissipate the phase coherence between the branches corresponding to the two locations on the time scale of about a nanosecond [34].

Measurements and observers play no fundamental role in this generalization of usual quantum theory. The probabilities of measured outcomes can, of course, be computed and are given to an excellent approximation by the usual story.[6] But, in a set of histories where they decohere, probabilities can be assigned to the position of the Moon when it is not being observed and to the values of density fluctuations in the early universe when there were neither measurements taking place nor observers to carry them out.

5 Quantum Theory in 3+1 Form

The quantum theory of the model universe in a box in the previous section is in fully 4-dimensional spacetime form. The fine-grained histories are paths in spacetime, the coarse-grainings were partitions of these, and the measure of interference was constructed by spacetime path integrals. No mention was made of states on spacelike surfaces or their unitary evolution.

However, as originally shown by Feynman [35, 36], this spacetime formulation is equivalent to the familiar 3+1 formulation in terms of states on spacelike surfaces and their unitary evolution through a foliating family of such surfaces. This section briefly sketches that equivalence emphasizing properties of spacetime and the fine-grained histories that are necessary for it to hold.

The key observation is illustrated in Figure 3. Sums-over-histories that are single-valued in time can be factored across constant time surfaces. A formula expressing this idea is

$$\int_{[A,B]} \delta x \, e^{iS[x(t)]/\hbar} = \int dx \, \psi_B^*(x,t)\psi_A(x,t) \,. \tag{5.1}$$

The sum on the left is over all paths from A at $t = 0$ to B at $t = T$. The amplitude $\psi_A(x,t)$ is the sum of $\exp\{iS[x(t)]\}$ over all paths from A at $t = 0$ to x at a time t between 0 and T. The amplitude $\psi_B(x,t)$ is similarly constructed from the paths between x at t to B at T.

The wave function $\psi_A(x,t)$ defines a state on constant time surfaces. Unitary evolution by the Schrödinger equation follows from its path integral

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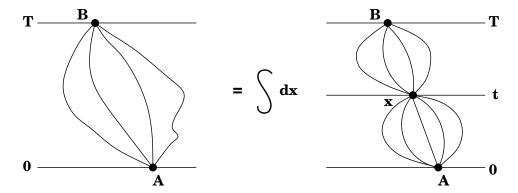
[6]See, *e.g.* [8], Section II.10.

Figure 3: The origin of states on a spacelike surface. These spacetime diagrams are a schematic representation of Eq. (5.1). The amplitude for a particle to pass from point A at time $t = 0$ to a point B at $t = T$ is a sum over all paths connecting them weighted by $\exp(iS[x(t)])$. That sum can be factored across an intermediate constant time surface as shown at right into product of a sum from A to x on the surface and a sum from x to B followed by a sum over all x. The sums in the product define states on the surface of constant time at t. The integral over x defines the inner product between such states, and the path integral construction guarantees their unitary evolution in t. Such factorization is possible only if the paths are single valued functions of time.

construction.[7] The inner product between states defining a Hilbert space is specified by (5.1). In this way, the familiar 3+1 formulation of quantum mechanics is recovered from its spacetime form.

The equivalence represented in (5.1) relies on several special assumptions about the nature of spacetime and the fine-grained histories. In particular, it requires[8]:

- A fixed Lorentzian spacetime geometry to define timelike and spacelike directions.

- A foliating family of spacelike surfaces through which states can evolve.

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[7]Reduction of the state vector (3.2) also follows from the path integral construction [37] when histories are coarse-grained by intervals of position at various times.

[8]The usual 3+1 formulation is also restricted to coarse-grained histories specified by alternatives at definite moments of time. More general spacetime coarse-grainings that are defined by quantities that extend over time can be used in the spacetime formulation. (See, *e.g.* [38] and references therein.) Spacetime alternatives are the only ones available in a diffeomorphism invariant quantum graviity.

- Fine-grained histories that are single-valued in the time labeling the spacelike surfaces in the foliating family.

As an illustrative example where the equivalence does not hold, consider quantum field theory in a fixed background spacetime with closed timelike curves (CTCs) such as those that can occur in wormhole spacetimes [39]. The fine-grained histories are four-dimensional field configurations that are single-valued on spacetime. But there is no foliating family of spacelike surfaces with which to define the Hamiltonian evolution of a quantum state. Thus, there is no usual 3+1 formulation of the quantum mechanics of fields in spacetimes with CTCs.

However, there is a four-dimensional sum-over-histories formulation of field theory in spacetimes with CTCs [40, 41, 42]. The resulting theory has some unattractive properties such as acausality and non-unitarity. But it does illustrate how closely usual quantum theory incorporates particular assumptions about spacetime, and also how these requirements can be relaxed in a suitable generalization of the usual theory.

6   Generalized Quantum Theory

In generalizing usual quantum mechanics to deal with quantum spacetime, some of its features will have to be left behind and others retained. What are the minimal essential features that characterize a quantum mechanical theory? The generalized quantum theory framework [8, 30, 10] provides one answer to this question. Just three elements abstracted from the quantum mechanics of closed systems in Section 4 define a generalized quantum theory.

- *Fine-grained Histories*: The sets of alternative fine-grained histories of the closed system which are the most refined descriptions of it physically possible.

- *Coarse-grained Histories*: These are partitions of a set of fine-grained histories into an exhaustive set of exclusive classes $\{c_\alpha\}, \alpha = 1, 2 \cdots$. Each class is a coarse-grained history.

- *Decoherence Functional*: A measure of quantum interference $D(\alpha, \alpha')$ between pairs of histories in a coarse-grained set, meeting the following conditions:

i. Hermiticity: $D(\alpha, \alpha') = D^*(\alpha', \alpha)$

ii. Positivity: $D(\alpha, \alpha) \geq 0$

iii. Normalization: $\Sigma_{\alpha\alpha'} D(\alpha, \alpha') = 1$

iv. Principle of superposition: If $\{\bar{c}_{\bar{\alpha}}\}$ is a further coarse-graining of $\{c_\alpha\}$, then

$$\bar{D}(\bar{\alpha}, \bar{\alpha}') = \sum_{\substack{\alpha \in \bar{\alpha} \\ \alpha' \in \bar{\alpha}'}} D(\alpha, \alpha')$$

Probabilities $p(\alpha)$ are assigned to sets of coarse-grained histories when they decohere according to the basic relation

$$D(\alpha, \alpha') \approx \delta_{\alpha\alpha'} \, p(\alpha) \, . \tag{6.1}$$

These $p(\alpha)$ satisfy the basic requirements for probabilities as a consequence of i)–iv) above. In particular, they satisfy the sum rule

$$p(\bar{\alpha}) = \sum_{\alpha \in \bar{\alpha}} p(\alpha) \tag{6.2}$$

as a consequence of i)–iv) and decoherence. For instance, the probabilities of an exhaustive set of alternatives always sum to 1.

The sum-over-histories formulation of usual quantum mechanics given in Section 4 is a particular example of a generalized quantum theory. The decoherence functional (4.1) satisfies the requirements i)–iv). But its particular form is not the only way of constructing a decoherence functional. Therein lies the possibility of generalization.

7 A Quantum Theory of Spacetime Geometry

The low energy, effective theory of quantum gravity is a quantum version of general relativity with a spacetime metric $g_{\alpha\beta}(x)$ coupled to matter fields. Of course, the divergences of this effective theory have to be regulated to extract predictions from it.[9] These predictions can therefore be expected

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[9]Perhaps, most naturally by discrete approximations to geometry such as the Regge calculus (see, *e.g.* [43, 44])

to be accurate only for limited coarse-grainings and certain states. But this effective theory does supply an instructive model for generalizations of quantum theory that can accommodate quantum spacetime. This generalization is sketched in this section.

The key idea is that the fine-grained histories do not have to represent evolution *in* spacetime. Rather they can be histories *of* spacetime. For this discussion we take these histories to be spatially closed cosmological four-geometries represented by metrics $g_{\alpha\beta}(x)$ on a fixed manifold $M = \mathbf{R} \times M^3$ where M^3 is a closed 3-manifold. For simplicity, we restrict attention to a single scalar matter field $\phi(x)$.

The three ingredients of a generalized quantum theory for spacetime geometry are then as follows:

- *Fine-grained Histories*: A fine-grained history is defined by a four-dimensional metric and matter field configuration on M.

- *Coarse-grainings*: The allowed coarse-grainings are partitions of the metrics and matter fields into four-dimensional *diffeomorphism invariant* classes $\{c_\alpha\}$.

- *Decoherence Functional*: A decoherence functional constructed on sum-over-history principles analogous to that described for usual quantum theory in Section 4. Schematically, branch state vectors $|\Psi_\alpha\rangle$ can be constructed for each coarse-grained history by summing over the metrics and fields in the corresponding class c_α of fine-grained histories, *viz.*

$$|\Psi_\alpha\rangle = \int_{c_\alpha} \delta g \delta \phi \, \exp\{iS[g,\phi]/\hbar\} \, |\Psi\rangle \,. \tag{7.1}$$

A decoherence functional satisfying the requirements of Section 6 is

$$D(\alpha', \alpha) = \langle \Psi_{\alpha'} | \Psi_\alpha \rangle \,. \tag{7.2}$$

Here, $S[g, \phi]$ is the action for general relativity coupled to the field $\phi(x)$, and $|\Psi\rangle$ is the initial cosmological state. The construction is only schematic because we did not spell out how the functional integrals are defined or regulated, nor did we specify the product between states that is implicit in both (7.1) and (7.2). These details can be made specific in models [9, 45, 46], but they will not be needed for the subsequent discussion.

15

A few remarks about the coarse-grained histories may be helpful. To every physical assertion that can be made about the geometry of the universe and the fields within, there corresponds a diffeomorphism invariant partition of the fine-grained histories into the class where the assertion is true and the class where it is false. The notion of coarse-grained history described above therefore supplies the most general notion of alternative describable in spacetime form. Among these we do not expect to find local alternatives because there is no diffeomorphism invariant notion of locality. In particular, we do not expect to find alternatives specified at a moment of time. We do expect to find alternatives referring to the kind of relational observables discussed in [47] and the references therein. We also expect to find observables referring to global properties of the universe such as the maximum size achieved over the history of its expansion.

This generalized quantum mechanics of spacetime geometry is in fully spacetime form with alternatives described by partitions of four-dimensional histories and a decoherence functional defined by sums over those histories. It is analogous to the spacetime formulation of usual quantum theory reviewed in Section 4.

However, unlike the theory in Section 4, we cannot expect an equivalent 3+1 formulation, of the kind described in Section 5, expressed in terms of states on spacelike surfaces and their unitary evolution between these surfaces. The fine-grained histories are not 'single-valued' in any geometrically defined variable labeling a spacelike surface. They therefore cannot be factored across a spacelike surface as in (5.1). More precisely, there is no geometrical variable that picks out a unique spacelike surface in all geometries.[10]

Even without a unitary evolution of states the generalized quantum theory is fully predictive because it assigns probabilities to the most general sets of coarse-grained alternative histories described in spacetime terms when these are decoherent.

How then is usual quantum theory used every day, with its unitarily evolving states, connected to this generalized quantum theory that is free from them? The answer is that usual quantum theory is an approximation to the more general framework that is appropriate for those coarse-grainings and initial state $|\Psi\rangle$ for which spacetime behaves classically. One equation

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[10]Spacelike surfaces labeled by the trace of the extrinsic curvature K foliate certain classes of classical spacetimes obeying the Einstein equation [48]. However, there is no reason to require that non-classical histories be foliable in this way. It is easy to construct geometries where surfaces of a given K occur arbitrarily often.

will show the origin of this relation. Suppose we have a coarse-graining that distinguishes between fine-grained geometries only by their behavior on scales well above the Planck scale. Then, for suitable states $|\Psi\rangle$ we expect that the integral over metrics in (7.2) can be well approximated semiclassically by the method of steepest descents. Suppose further for simplicity that only a single classical geometry with metric $\hat{g}_{\alpha\beta}$ dominates the semiclassical approximation. Then, (7.2) becomes

$$|\Psi_\alpha\rangle \approx \int_{\hat{c}_\alpha} \delta\phi \, \exp\{iS[\hat{g}, \phi]/\hbar\} \, |\Psi\rangle \tag{7.3}$$

where $\hat{c}_\alpha$ is the coarse-graining of $\phi(x)$ arising from c_α and the restriction of $g_{\alpha\beta}(x)$ to $\hat{g}_{\alpha\beta}(x)$. Eq. (7.3) effectively defines a quantum theory of the field $\phi(x)$ in the fixed background spacetime with the geometry specified by $\hat{g}_{\alpha\beta}(x)$. This is familiar territory. Field histories are single valued on spacetime. Sums-over-fields can thus be factored across spacelike surfaces in the geometry $\hat{g}$ as in (5.1) to define field states on spacelike surfaces, their unitary evolution, and their Hilbert space product. Usual quantum theory is thus recovered when spacetime behaves classically and provides the fixed spacetime geometry on which usual quantum theory relies.

From this perspective, familiar quantum theory and its unitary evolution of states is an effective approximation to a more general sum-over-histories formulation of quantum theory. The approximation is appropriate for those coarse-grainings and initial states in which spacetime geometry behaves classically.

8   Beyond Spacetime

The generalized quantum theory of spacetime sketched in the previous section assumed that geometry was a fundamental variable — part of the description of the fine-grained histories. But on almost every frontier in quantum gravity one finds the idea that continuum geometry is not fundamental, but will be replaced by something more fundamental. This is true for string theory [49], loop quantum gravity [50], and the causal set program [51, 52] although space does not permit a review of these speculations.

Can generalized quantum theory serve as a framework for theories where spacetime is emergent rather than fundamental? Certainly we cannot expect to have a notion of 'history'. But we can expect some fine-grained description,

or a family of equivalent ones, and that is enough. A generalized quantum theory needs

- The possible fine-grained descriptions of the system.

- The coarse-grained descriptions constructed from the fine-grained ones.

- A measure of quantum interference between different coarse-grained descriptions respecting conditions i)–iv) in Section VI.

Generalized quantum theory requires neither space nor time and can therefore serve as the basis for a quantum theory in which spacetime is emergent.

9   Emergence/Excess Baggage

The word 'emergent' appears in a number of places in the previous discussion. It probably has many meanings. This section aims at a more precise understanding of what is meant by the term in this essay.

Suppose we have a quantum theory defined by certain sets of fine-grained histories, coarse-grainings, and a decoherence functional. Let's call this the *fundamental* theory. It may happen that the decoherence and probabilities of limited kinds of sets of coarse-grained histories are given approximately by a second, *effective* theory. The two theories are related in the following way:

- Every fine-grained history of the effective theory is a coarse-grained history of the fundamental theory.

- The decoherence functionals approximately agree on a limited class of sets of coarse-grained histories.

$$D^{\text{fund}}(\alpha', \alpha) \approx D^{\text{eff}}(\alpha', \alpha).$$

(9.1)

  On the right, α' and α refer to the fine-grained histories of the effective theory. On the left, they refer to the corresponding coarse-grained histories of the fundamental theory.

When two theories are related in this way we can say that the effective theory is *emergent* from the fundamental theory. Loosely we can say that

the restrictions, and the concepts that characterize them, are emergent. It should be emphasized that an approximate equality like (9.1) can be expected to hold, not just as a consequence of the particular dynamics incorporated into decoherence functionals, but also only for particular states.
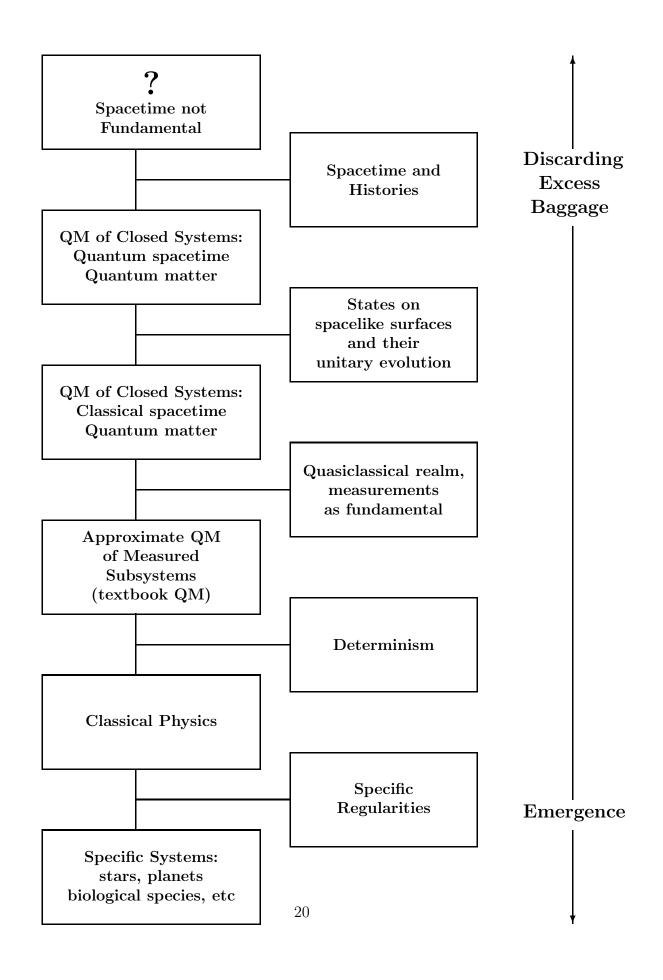
Several examples of emergence in this sense have been considered in this essay: There is the possible emergence of a generalized quantum theory of spacetime geometry from a theory in which spacetime is not fundamental. There is the emergence of a 3+1 quantum theory of fields in a fixed background geometry from a four-dimensional generalized quantum theory in which geometry is a quantum variable. There is the emergence of the approximate quantum mechanics of measured subsystems (textbook quantum theory) from the quantum mechanics of the universe. And there is the emergence of classical physics from quantum phyaics.

Instead of looking at an effective theory as a restriction of a more fundamental one, we may look at the fundamental theory as a generalization of the effective one. That perspective is important because generalization is a way of searching for more comprehensive theories of nature. In passing from the specific to the more general some ideas have to be discarded. They are often ideas that were once perceived to be general because of our special place in the universe and the limited range of our experience. But, in fact, they arise from special situations in a more general theory. They are 'excess baggage' that has to be discarded to reach a more comprehensive theory [53]. Emergence and excess baggage are two ways of looking at the same thing.

Physics is replete with examples of emergence and excess baggage ranging from Earth-centered theories of the solar system to quantum electrodynamics. The chart on the next page helps understand the stages of emergence and generalization in quantum mechanics discussed in this essay provided it is not taken too rigidly or without qualification.

The chart can be read in two ways: Reading from the bottom up, the boxes on the left describe a path of generalization — from the specific to the general. Starting from the regularities of specific systems such as the planetary orbits, we move up to the general laws of classical physics, to textbook quantum theory, through various stages of assumptions about spacetime, to a yet unknown theory where spacetime is not fundamental. The excess baggage that must be jettisoned at each stage to reach a more general perspective is indicated in the middle tower of boxes.

Reading from the top down the chart tells a story of emergence. Each box on the left stands in the relation of an effective theory to the one before

```
┌─────────────────────────┐
│           ?             │                                                    ↑
│      Spacetime not      │
│       Fundamental       │                                          Discarding
└─────────────────────────┘         ┌─────────────────────────┐        Excess
            │                        │      Spacetime and      │        Baggage
            ├────────────────────────│        Histories        │
            │                        └─────────────────────────┘
┌─────────────────────────┐
│   QM of Closed Systems:  │
│     Quantum spacetime    │
│      Quantum matter      │
└─────────────────────────┘         ┌─────────────────────────┐
            │                        │        States on        │
            │                        │    spacelike surfaces   │
            ├────────────────────────│        and their        │
            │                        │     unitary evolution   │
┌─────────────────────────┐         └─────────────────────────┘
│   QM of Closed Systems:  │
│    Classical spacetime   │
│      Quantum matter      │
└─────────────────────────┘         ┌─────────────────────────┐
            │                        │   Quasiclassical realm,  │
            │                        │       measurements       │
            ├────────────────────────│      as fundamental      │
            │                        └─────────────────────────┘
┌─────────────────────────┐
│      Approximate QM      │
│       of Measured        │
│        Subsystems        │
│      (textbook QM)       │
└─────────────────────────┘         ┌─────────────────────────┐
            │                        │                          │
            ├────────────────────────│       Determinism        │
            │                        │                          │
┌─────────────────────────┐         └─────────────────────────┘
│                          │
│     Classical Physics    │
│                          │
└─────────────────────────┘         ┌─────────────────────────┐
            │                        │        Specific          │
            ├────────────────────────│      Regularities        │    Emergence
            │                        └─────────────────────────┘
┌─────────────────────────┐
│    Specific Systems:     │
│     stars, planets       │                                                    ↓
│  biological species, etc │
└─────────────────────────┘
              20
```

it. The middle boxes now describe phenomena that are emergent at each stage.

10 Emergence of Signature

Classical spacetime has Lorentz signature. At each point it is possible to choose one timelike direction and three orthogonal spacelike ones. There are no physical spacetimes with zero timelike directions or with *two* timelike directions. But is such a seemingly basic property fundamental, or is it rather, emergent from a quantum theory of spacetime which allows for all possible signatures? This section sketches a simple model where that happens.

Classical behavior requires particular states [54]. Let's consider the possible classical behaviors of cosmological geometry assuming the 'no-boundary' quantum state of the universe [55] in a theory with only gravity and a cosmological constant Λ. The no-boundary wave function is given by a sum-over-geometries of the schematic form

$$\Psi\left[h\right] = \int_e \delta g\, e^{-I\left[g\right]/\hbar} . \tag{10.1}$$

For simplicity, we consider a fixed manifold[11] M. The key requirement is that it be compact with one boundary for the argument of the wave function and no other boundary. The functional $I\left[g\right]$ is the Euclidean action for metric defining the geometry on M. The sum is over a complex contour $\mathcal{C}$ of g's that have finite action and match the three-metric h on the boundary that is the argument of Ψ.

Quantum theory predicts classical behavior when it predicts high probability for histories exhibiting the correlations in time implied by classical deterministic laws [58, 54]. The state Ψ is an input to the process of predicting those probabilities as described in Section 7. However, plausibly the output for the predicted classical spacetimes in this model are the extrema of the action in (10.1). We will assume this (see [9] for some justification). Further, to keep the discussion manageable, we will restrict it to the *real* extrema. These are the real tunneling geometries discussed in a much wider context in [59].

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[11]Even the notion of manifold may be emergent in a more general theory of certain complexes [56, 57].
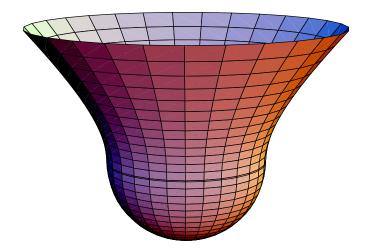
Figure 4: The emergence of the Lorentz signature $(-, +, +, +)$ of spacetime. The semiclassical geometry describing a classical spacetime which becomes large according to the 'no-boundary' proposal for the universe's quantum state. The model is pure gravity and a cosmological constant. Purely Euclidean geometries $(+, +, +, +)$ or purely Lorentzian geometries are not allowed as described in the text. What is allowed is the real tunneling geometry illustrated above consisting of half a Euclidean four-sphere joined smoothly onto an expanding Lorentzian de Sitter space at the moment of maximum contraction. This can be described as the nucleation of classical Lorentz signatured spacetime. There is no similar nucleation of a classical geometry with signature $(-, -, +, +)$ because it could not match the Euclidean one across a spacelike surface.

Let us ask for the semiclassical geometries which become large, *i.e.* contain symmetric three surfaces with size much larger than $(1/\Lambda)^{1/2}$. There are none with Euclidean signature. The purely Euclidean extremum is the round four-sphere with linear size $(1/\Lambda)^{1/2}$ and contains no symmetric three surfaces with larger size. There are none with purely Lorentzian signature either because these cannot be regular on M. There are, however, tunneling solutions of the kind illustrated in Figure 4 in which half of a Euclidean four-sphere is matched to expanding DeSitter space across a surface of vanishing extrinsic curvature.

Could a spacetime with two time and two space directions be nucleated in this way? The answer is 'no' because the geometry on a surface could not have the three spacelike directions necessary to match onto the half of a four-sphere.

22

Thus, in this very simple model, with many assumptions, if we live in a large universe it must have one time and three space dimensions. The Lorentzian signature of classical spacetime is an emergent property from an underlying theory not committed to this signature.

11   Beyond Quantum Theory

The path of generalization in the previous sections began with the textbook quantum mechanics of measurement outcomes in a fixed spacetime and ended in a quantum theory where neither measurements nor spacetime are fundamental. In this journey, the principles of generalized quantum theory are preserved, in particular the idea of quantum interference and the linearity inherent in the principle of superposition. But the end of this path is strikingly different from its beginning.

The founders of quantum theory thought that the indeterminacy of quantum theory "reflected the unavoidable interference in measurement dictated by the magnitude of the quantum of the action" (Bohr). But what then is the origin of quantum indeterminacy in a closed quantum universe which is never measured? Why enforce the principle of superposition in a framework for prediction of the universe which has but a single quantum state? In short, the endpoint of this journey of generalization forces us to ask John Wheeler's famous question, "How come the quantum?" [60].

Could quantum theory itself be an emergent effective theory? Many have thought so (Section 2). Extending quantum mechanics until it breaks could be one route to finding out. 'Traveler, there are no paths, paths are made by walking.'

12   Conclusion

Does quantum mechanics apply to spacetime? The answer is 'yes' provided that its familiar textbook formulation is suitably generalized. It must be generalized in two directions. First, to a quantum mechanics of closed systems, free from a fundamental role for measurements and observers and therefore applicable to cosmology. Second, it must be generalized so that it is free from any assumption of a fixed spacetime geometry and therefore applicable when spacetime geometry is a quantum variable.

Generalized quantum theory built on the pillars of fine-grained histories, coarse-graining, and decoherence provides a framework for investigating such generalizations. The fully, four-dimensional sum-over-histories effective quantum theory of spacetime geometry sketched in Section 7 is one example. In such fully four-dimensional generalizations of the usual theory, we cannot expect to recover an equivalent 3+1 formulation in terms of the unitary evolution of states on spacelike surfaces. There is no fixed notion of spacelike surface. Rather, the usual 3+1 formulation emerges as an effective approximation to the more general story for those coarse grainings and initial states in which spacetime geometry behaves classically.

If spacetime geometry is not fundamental, quantum mechanics will need further generalization and generalized quantum theory provides one framework for exploring that.

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