# ResKD: Residual-Guided Knowledge Distillation

Xuewei Li\*, Songyuan Li\*, Bourahla Omar, Fei Wu, and Xi Li

Abstract—Knowledge distillation, aimed at transferring the knowledge from a heavy teacher network to a lightweight student network, has emerged as a promising technique for compressing neural networks. However, due to the capacity gap between the heavy teacher and the lightweight student, there still exists a significant performance gap between them. In this paper, we see knowledge distillation in a fresh light, using the knowledge gap, or the residual, between a teacher and a student as guidance to train a much more lightweight student, called a res-student. We combine the student and the res-student into a new student, where the res-student rectifies the errors of the former student. Such a residual-guided process can be repeated until the user strikes the balance between accuracy and cost. At inference time, we propose a sample-adaptive strategy to decide which res-students are not necessary for each sample, which can save computational cost. Experimental results show that we achieve competitive performance with 18.04%, 23.14%, 53.59%, and 53.80% of the teachers' computational costs on the CIFAR-10, CIFAR-100, Tiny-ImageNet, and ImageNet datasets. Finally, we do thorough theoretical and empirical analysis for our method.

 ${\it Index Terms} \hbox{---} Knowledge \quad Distillation, \quad Residual, \quad Sample-Adaptive.}$ 

## I. INTRODUCTION

S deep learning goes deeper, state-of-the-art neural networks [1]-[4] have obtained better and better performance, and yet demand more and more computational resources. While models with large capacity can achieve high accuracy, they are impractical for resource-limited devices such as embedded systems. To this end, researchers have studied cost-effective networks [5]-[8] and efficient training strategies [9]-[14]. Knowledge distillation (KD) [15] has emerged as a compression technique where an analogy of the teacher-student relationship is drawn to explain the idea that the knowledge of a powerful yet heavy teacher network can be distilled into a lightweight student network by adding a loss term that encourages the student to mimic the teacher.

Due to the capacity gap between a heavy teacher and a lightweight student, there is still a significant performance gap between them. Existing KD methods have made efforts to modify the loss term to improve the student's performance [9], [16]–[19]. However, the discrepancy between a teacher and a student can be considered as knowledge, and it remains relatively unexplored.

In this paper, we see knowledge distillation in a fresh light, using the knowledge gap between a teacher and a student as guidance. First, we train a student network  $S_0$  from a teacher T as usual. Then, we train a much more lightweight network, named a res-student, to learn the knowledge gap,

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or the *residual*, between the teacher T and the student  $S_0$ . The combination of the student  $S_0$  and the res-student  $R_1$  becomes a new student  $S_1$ , where  $R_1$  corrects the errors of  $S_0$ . Similarly, an even more lightweight res-student  $R_2$  can be used to learn the knowledge gap between T and  $S_1$  to build a new student  $S_2$ . Such a residual-guided process can be repeated until a final student  $S_n$  is obtained. We call our framework residual-guided knowledge distillation (ResKD). The idea is akin to approximating functions by a polynomial. In a series expansion of a function, a higher-order polynomial would be a better approximation but would demand more computations. Similarly, a higher-order ResKD-style student would be closer to the teacher but would be more expensive. Users can control the total capacity of the final student network  $S_n$  by setting hyper-parameters for termination.

In addition, the knowledge gap mentioned above is different from sample to sample. For instance, given an  $S_2$  student, we observed that, for some images,  $S_1$  (i.e.,  $S_0 + R_1$ ) or even  $S_0$  itself has highly confident scores, while  $R_2$  or  $R_1 + R_2$  has little contributions. It is unnecessary to use all the resstudents for each sample at inference time. Thus, we introduce a sample-adaptive strategy for the inference phase. For each sample, if the confidence of  $S_i$  is high enough, we truncate the unnecessary res-students to save computational cost.

We do experiments on several standard benchmarks. The experimental results show that we achieve competitive performance with 18.04%, 23.14%, 53.59%, and 53.80% of the teacher's FLOPs for CIFAR-10, CIFAR-100, Tiny-ImageNet, and ImageNet respectively. Also, we apply our ResKD framework to different KD methods and show that the framework is generic to knowledge distillation methods. Finally, we analyze the effectiveness of this idea and use informativeness [20] to visualize the bridging gap process.

Our contributions in this paper are summarized as follows:

- We design a residual-guided learning method that uses a series of res-students to bridge the gap between a student network and a teacher network.
- We introduce a sample-adaptive strategy at inference time to make our framework adaptive to different samples to save the cost of additional res-student networks.
- We evaluate our method on different datasets and perform detailed experiments that showcase the importance of each part of the framework.

# II. RELATED WORK

# A. Knowledge Distillation

We categorize knowledge distillation methods in terms of the number of stages. Traditionally, knowledge distillation is a two-stage method, in which a teacher network is trained first, and then a student network is trained under the guidance of the teacher network. Bucilă et al. [21] pioneered the idea of transferring the knowledge from a cumbersome model to a small model. Hinton et al. [15] popularized this idea by the concept of knowledge distillation (KD), in which a student neural network is trained with the benefit of the soft targets provided by teacher networks. Compared to traditional one-hot labels, the output from a teacher network contains more information about the fine-grained distribution of data, which helps the student achieve better performance. Recently, many works have focused on improving the information propagation way or putting strictness to the distillation process via optimization [16], [22]–[27] to teach the student better. For example, Peng et al. [16] proposed that a student network should not only focus on mimicking from a teacher at an instance level, but also imitating the embedding space of a teacher so that the student can possess intra-class compactness and inter-class separability. In addition, the effect of different teachers is also researched [28]-[30]. For example, Sau et al. [29] proposed an approach to simulate the effect of multiple teachers by injecting noise to the training data and perturbing the logit outputs of a teacher. In such a way, the perturbed outputs not only simulate the setting of multiple teachers but also result in noise in the softmax layer, thus regularizing the distillation loss. With the help of many teachers, the student is improved a lot. Kang et al. [30] used Neural Architecture Search (NAS) to acquire knowledge for both the architecture and the parameters of the student network from different teachers. Besides the classic image classification task, KD can also be used in many other different fields, such as face recognition [31], visual question answering [32], video tasks [33], [34] etc.

Recently, some KD methods have been proposed to have less or more than two stages. For one thing, KD can be a one-stage strategy [9], [35]. Zhang et al. [9] proposed that a pool of untrained student networks with the same network structure can be used to simultaneously learn the target task together instead of the traditional two-stage knowledge distillation strategy. For another, a line of research [36], [37] focuses on KD methods with more than two stages. In [36], several Teacher Assistant networks are used to transfer the knowledge from a teacher more softly and effectively. The teacher propagates its knowledge to the assistant networks first and then the assistant networks propagate its knowledge to the student network. In [37], the student network has the same architecture as the teacher network at the beginning. The last block of the student network is replaced by a block that having a simple architecture and training the last block in the first step. Next, the penultimate block is replaced similarly and the last two blocks are trained. In this style, all blocks are trained after several stages and a simple student network is achieved

In this paper, we mainly focus on how to use the gap between the teacher and the student as knowledge. We use a series of lightweight networks, named res-students, to learn the gap in a multi-stage manner.

## B. Ensemble Methods

Ensemble methods, which have been studied extensively for improving model performance [38]–[40], are strategies to

TABLE I NOTATIONS

T	A teacher network or its logits
$S_0$	A classic student network or its logits
$S_i, i > 0$	A ResKD student network or its logits
$R_i$	A res-student network or its logits
$\Delta_0$	The gap between $S_0$ and $T$
$\Delta_i$ , $i > 0$	The gap between $S_i$ and $T_i$
$D_t$	A training set
$D_{v}$	A validation set
$\mathcal{L}_{ ext{KD}}$	The knowledge distillation loss function
$\mathcal{L}_{T-S}$	The loss function used between logits of $T$ and $S$
$\sigma(\cdot)$	The Softmax function
$\mathbf{f_j}$	Feature maps

combine models by averaging, majority voting or something else, which means several models having the same status are used to improving final performance. Ensembles of models perform at least as well as each of its ensemble members [41]. There are several lines of research of ensemble methods: introducing different regularization, reducing training time [42] and saving test time [21].

For combining knowledge distillation and the ensemble idea, Lan et al. [43] proposed to aggregate the logits of several homogeneous student models to become an ensemble teacher and then to distill the knowledge from the teacher. On the contrary, we first carry on knowledge distillation to build a student and res-students and then combine them. Also, the roles that the student and res-students play are not the same. The student acquires the knowledge from the teacher, while the res-students acquire the knowledge from knowledge gaps. Furthermore, we introduce a sample-adaptive strategy to decide which res-students to use at inference time.

# III. RESIDUAL-GUIDED KNOWLEDGE DISTILLATION

In this section, we present our residual-guided knowledge distillation framework (ResKD). First, we briefly introduce the background. Second, we describe our main idea of residual-guided learning. Next, we propose a sample-adaptive strategy at inference time. Finally, we put all the things together to build a whole framework. For convenience, Table I summarizes the notations we use in this section.

# A. Background and Notations

We first describe a general formulation of knowledge distillation, and then introduce the residual-guided knowledge distillation in this formulation.

We define a teacher network T as a function

$$T = f(\mathbf{x}, w_T, \alpha_T),\tag{1}$$

where  $\mathbf{x}$  is an input image,  $\mathbf{w}_T$  is the weights of T, and  $\alpha_T$  is its architecture parameters. Similarly,  $S = f(\mathbf{x}, w_S, \alpha_S)$  denotes a student network. For convenience, a network and its logits are used interchangeably. The goal of knowledge distillation is to learn  $w_S$  so that the results of the student S are as close as possible to the teacher T.

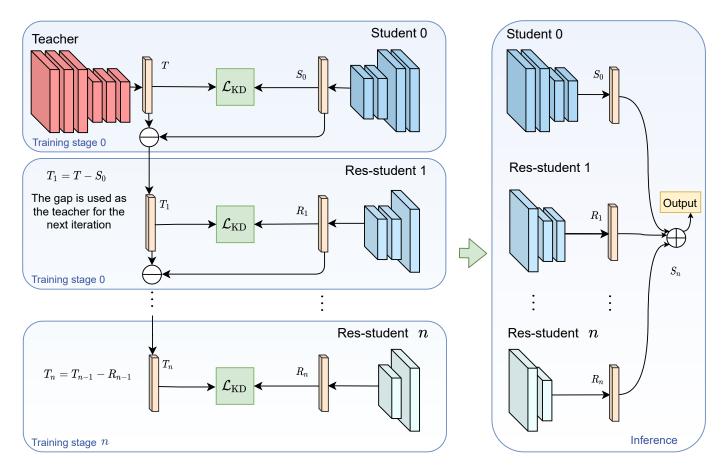


Fig. 1. (Best viewed in color) The main idea of the ResKD framework. Training stage 0 is just the same as traditional KD: a student network  $S_0$  is trained to learn from a teacher T. In stage 1, a res-student  $R_1$  is trained to learn from a teacher  $T_1$  which is the residual between T and  $S_0$ . Similarly, in stage i, a res-student  $R_i$  is trained from a teacher  $T_i$  until  $R_n$  is obtained.  $S_0$  and all the res-students are combined to build a new student  $S_n$ .

Classic knowledge distillation from a teacher is done in two steps. First, the student architecture  $\alpha_S$  is determined. Then,  $w_S$  is optimized by

$$\underset{w_S}{\operatorname{argmin}} \sum_{i=1}^{N} \mathcal{L}_{KD} \left( f(\mathbf{x}^{(i)}, w_S, \alpha_S), f(\mathbf{x}^{(i)}, w_T, \alpha_T) \right), \quad (2)$$

where  $\mathbf{x}^{(j)}$  is an input image in a dataset with N training samples and  $\mathcal{L}_{\mathrm{KD}}$  is defined as

$$\mathcal{L}_{KD}(S, y^{(j)}, T, \tau, t) = \tau \cdot t^2 \cdot \mathcal{L}_{T-S}(\sigma(\frac{S}{t}), \sigma(\frac{T}{t}))$$

$$+ (1 - \tau) \cdot \mathcal{L}_{CE}(\sigma(S), y^{(j)}),$$
(3)

where  $y^{(j)}$  is the label of the input image  $\mathbf{x}^{(j)}$ ,  $\sigma(\cdot)$  is the softmax function,  $\mathcal{L}_{CE}$  is the conventional cross-entropy loss function,  $\tau$  and t are scalar hyper-parameters, and  $\mathcal{L}_{T-S}$  is the loss between the student's logits and the teacher's logits, e.g., the Kullback–Leibler divergence.

The optimized  $w_S$  from Eq. (2) makes the predictions obtained from  $S = f(\mathbf{x}, w_S, \alpha_S)$  as close to those obtained by  $T = f(\mathbf{x}, w_T, \alpha_T)$  as possible, but we still observe a difference  $\Delta$  between the teacher's and student's logits:

$$\Delta = T - S,\tag{4}$$

which is what we propose to reduce as follows.

# B. Residual-Guided Learning

We propose a framework distilling knowledge in a residual-guided fashion: we use the difference, or the *residual*, between a teacher and a student as guidance. Given a teacher network, we first train a student network by classic knowledge distillation. Then, we introduce another student, called a res-student, to learn their residual. After that, we combine the original student and the res-student to form a new student. Such a process can be repeated to add more res-students to further bridge the performance gap, as shown in Fig. 1.

Formally, let the logits of the teacher be T, the logits of the first student be  $S_0$ , and the logits of the res-student networks be  $\{R_i\}_{i=1}^n$ . We train  $S_0$  and  $\{R_i\}_{i=1}^n$  in stages. In stage 0,  $S_0$  is trained in a classic knowledge distillation manner with T as the teacher network. The gap can be observed by  $\Delta_0 = T - S_0$ . In stage 1, the gap  $\Delta_0$  is defined as a new teacher, i.e.,  $T_1 = \Delta_0$ , to train a res-student  $R_1$ . Once we obtain  $R_1$ , we can define a new student  $S_1 = S_0 + R_1$ . Similarly, in each following stage, we use the gap from the current stage  $\Delta_i = T_i - R_i$  as the next stage teacher network  $T_{i+1}$ . We distill a res-student  $R_{i+1}$  from  $T_{i+1}$  and obtain the next student  $S_{i+1} = S_0 + R_1 + \cdots + R_{i+1}$ . Finally, we use the student network and res-student networks trained in all n+1 stages. We feed an input image to all networks

Fig. 2. The overview of our NAS-assisted architecture. We use the same setting as that of STACNAS [44]. In addition, we use KD loss in both search and fine-tuning stages.

independently and sum their logits to obtain the final logits:

$$S_n = S_0 + \sum_{i=1}^n R_i. (5)$$

**NAS-assisted architecture.** When we use a res-student to bridge the gap to the teacher, it is better to use NAS to get the architecture of the res-student network instead of a handcrafted one. We also use the knowledge distillation loss function in the search stage when using NAS, which is reflected by adding  $\alpha_S$  as an optimizable parameter in Eq. (2):

$$\underset{w_S, \alpha_S}{\operatorname{argmin}} \sum_{j=1}^{N} \mathcal{L}_{KD} \left( f(\mathbf{x}^{(j)}, w_S, \alpha_S), f(\mathbf{x}^{(j)}, w_T, \alpha_T) \right). \tag{6}$$

We use the same search space as that of STACNAS [44]. A STACNAS-style neural network consists of a series of building blocks called cells. We define a set of candidate operations *O* to be used inside these cells, and from which we will select the best operations for the architecture.

The construction of cells and the whole network can be described as a directed acyclic graph (DAG) as shown in Fig. 2. A cell contains a sequence of N nodes  $\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_N$  each of which is a stack of feature maps. These nodes are connected by directed edges. An edge (i, j) represents some operation  $o_k^{(i,j)}(\cdot) \in O$  that transforms  $\mathbf{f}_i$  to  $\mathbf{f}_j$ . In a certain cell, each node is computed as a weighted sum of all its predecessors:

$$\mathbf{f}_j = \sum_{i < j} \sum_{o_k \in O} \alpha_k^{(i,j)} o_k^{(i,j)} (\mathbf{f}_i), \tag{7}$$

where  $\alpha_k^{(i,j)}$  is the weight for the operation  $o_k^{(i,j)}$  applied to  $\mathbf{f}_i$  when calculating the node  $\mathbf{f}_j$ . We use the same training strategy as that of STACNAS [44] to obtain the final suitable network.

**Termination condition.** When a ResKD student  $S_i$  is close enough to the teacher T, we can stop the residual-guided process. Here, we define a concept "Energy" which helps to measure the difference between a student and a teacher.

The Energy of a network  $S_i$  for a certain sample  $\mathbf{x}^{(j)}$  is

Energy(
$$S_i$$
, { $\mathbf{x}^{(j)}$ }) =  $\|\sigma(S_i(\mathbf{x}^{(j)}))\|_2^2$ , (8)

Algorithm 1 ResKD training

**Input:** Training set  $D_t = \{(\mathbf{x}^{(j)}, y^{(j)})\}_{j=1}^N$  and teacher network  $T(\mathbf{x}) = f(\mathbf{x}, w_T, \alpha_T)$ .

**Output:**  $S_0, R_1, R_2, \dots, R_n$  and a threshold TH<sub>energy</sub>. Sample a validation set  $D_v = \{(\mathbf{x}^{(j)}, y^{(j)})\}_{j=1}^{N_v}$  from the training set  $D_t$  uniformly.

Train  $S_0$ ;  $T_1 = \Delta_0 = T - S_0$ ; i = 0.

repeat

i = i + 1.

 $R_i = KD(T_i)$ .

 $T_{i+1} = T_i - R_i$ ;  $S_i = S_{i-1} + R_i$ .

**until** Energy( $S_i, D_v$ ) > 90% · Energy( $T, D_v$ )

n = i

 $TH_{\text{energy}} = \text{Energy}(S_n, D_v)$ 

**return** networks  $S_0, R_1, \dots, R_n$  and a threshold  $TH_{energy}$ .

where  $\sigma(\cdot)$  is the softmax function, and the Energy of a network  $S_i$  on a certain dataset D is

Energy(
$$S_i, D$$
) =  $\frac{1}{N} \cdot \sum_{i=1}^{N} \|\sigma(S_i(\mathbf{x}^{(j)}))\|_2^2$ . (9)

In this way, the Energy of a network represents its overall confidence on a dataset. When the Energy of  $S_i$  has reached a comparable level of the Teacher's Energy (e.g., 90%), we can set n = i and finish the residual-guided learning process.

In practice, we calculate Energy on a validation set  $D_{\nu}$  whose data are uniformly sampled from the training set D. The overall training process is shown in Algorithm 1.

# C. Sample-Adaptive Inference

When we finish the residual-guided learning process, we have  $S_n = S_0 + \sum_{i=1}^n R_i$ , where  $R_i$  is supposed to bridge the gap  $\Delta_{i-1}$  between  $S_{i-1}$  and T. However, for each sample,  $\Delta_{i-1}$  is also different. For instance, if the sample is easy to recognize,  $\Delta_0$  can be subtle, and if the sample is difficult, even  $\Delta_2$  can be considerable. In other words, for an easy sample,  $S_0$  is enough, but for a difficult sample, we should use more res-students.

Based on the above observation, we propose an adaptive strategy for each sample at inference time. Similar to the Energy idea in Section III-B, we uniformly sample a validation set  $D_v$  from the training set D, and calculate an Energy threshold for the final ResKD student  $S_n$ :

$$TH_{energy} = Energy(S_n, D_v).$$
 (10)

When the Energy of  $S_i$  with a sample  $\mathbf{x}^{(j)}$ , i.e., Energy( $S_i$ , { $\mathbf{x}^{(j)}$ }), is higher than TH<sub>energy</sub>, we set the rest resstudents  $R_{i+1}, \dots, R_n$  aside. As a result, given a sample  $\mathbf{x}^{(j)}$ , we define  $S_n^{(i)}$  as the student network it uses at inference time:

$$S_n^{(i)}(\mathbf{x}^{(j)}) = S_L(\mathbf{x}^{(j)}), \ 0 \le L \le n,$$
 (11)

where we use  $S_L$  instead of  $S_n$  for the sample  $\mathbf{x}^{(j)}$ . For example, let  $S_n = S_3$ . Given a sample  $\mathbf{x}^{(j)}$ , if Energy $(S_1, \mathbf{x}^{(j)}) > TH_{\text{energy}}$ , then L = 0, which means that we only use  $S_0$  and  $R_1$ , and set  $R_2$  and  $R_3$  aside. The whole process is shown in Algorithm 2.

# **Algorithm 2** Sample-adaptive inference

```
Input: a test sample \mathbf{x}^{(j)}, networks S_0, R_1, R_2, \cdots, R_n and a threshold \mathrm{TH}_{\mathrm{energy}}. Output: S_L and the logits of S_L for the test sample \mathbf{x}^{(j)}. calculate S_0(\mathbf{x}^{(j)}); E_0 = \mathrm{Energy}(S_0, \{\mathbf{x}^{(j)}\}). i = 0. while \mathrm{TH}_{\mathrm{energy}} \geq E_i and i < n. do calculate R_i(\mathbf{x}^{(j)}); i = i + 1 E_i = E_{i-1} + \mathrm{Energy}(R_i, \{\mathbf{x}^{(j)})\}. end while L = i. return S_L and the logits of S_L for the test sample \mathbf{x}^{(j)}.
```

## D. ResKD: The Whole Framework

When faced with a knowledge distillation problem, our first step is to train a student network  $S_0$ , which can be handcrafted or searched, under the guidance of the teacher T. Next, we start to use our residual-guided learning strategy to find resstudents. We train the res-student  $R_i$  in a KD manner under the guidance of  $T_i$ . Such a residual-guided process can be repeated until  $S_i$  has achieved the Energy comparable to T. At inference time, we apply our sample-adaptive strategy.

## IV. EXPERIMENTS

In this section, we first introduce the datasets and protocols we use in Section IV-A. Next, we do ablation studies in Section IV-B to evaluate the effectiveness of our strategies. Finally, we evaluate our framework on several datasets in Section IV-C.

## A. Datasets and Protocols

The CIFAR-10 (CIFAR-100) dataset consists of 50k training images and 10k testing images in 10 (100) classes. We use a weight decay of 0.0001 and a momentum of 0.9. Our models are trained with a mini-batch size of 128. We start with a learning rate of 0.1, divide it by 10 at 150 and 200 epochs, and terminate training at 250 epochs. We follow the simple data augmentation in [1] for training: 4 pixels are padded on each side, and a  $32 \times 32$  crop is randomly sampled from the padded image or its horizontal flip for training. For testing, we only evaluate the single view of the original  $32 \times 32$  image.

The **ImageNet** consists of 1000 classes. Our models are trained on the 1.28 million training images, and evaluated on the 50k validation images. Images are randomly resized and a 224×224 crop is randomly sampled from the resized image or its horizontal flip for training. For testing, we scale the short side of images to 224, and a 224×224 center crop is sampled from the scaled image. When training ResNet series networks, We start with a learning rate of 0.1, divide it by 10 at 30, 60 and 80 epochs, and terminate training at 90 epochs. We use a weight decay of 0.0001 and a momentum of 0.9. When dealing with the network given by neural architecture search, we use a similar setting as [44]: We use a weight decay of 0.0003 and a momentum of 0.9. The power annealing learning rate schedule is used to control the learning rate from 0.1 to 0.0001 in 63

#### TABLE II

COMPARISON WITH OTHER ENSEMBLE METHODS ON CIFAR-10/100. RES110 IS THE RESNET-110 TEACHER NETWORK. RES20 (SGD) IS A RESNET-20 TRAINED WITH SGD, AND RES20 (KD) IS A RESNET-20 TRAINED BY KD TO LEARN FROM THE TEACHER. 'Ensemble', 'Trained together' AND 'Ours' SHARE THE SAME ARCHITECTURE WHERE  $S_0$  IS RESNET-20 AND  $R_1$  IS RESNET-14. 'ENSEMBLE' MEANS THAT RESNET-20 AND RESNET-14 ARE TRAINED INDEPENDENTLY AND THE SUM OF THEIR LOGITS IS USED FOR EVALUATION. 'TRAINED TOGETHER' MEANS THAT THE COMBINATION OF RESNET-20 AND RESNET-14 IS TRAINED TOGETHER.

Architecture	Acc. (%)	#Params (M)	MFLOPs
Teacher Res110	94.19 / 72.44	1.728 / 1.734	255 / 255
Students Res20 (SGD) Res20 (KD)	91.80 / 68.82 93.06 / 70.09	0.270 / 0.276 0.270 / 0.276	41 / 41 41 / 41
Res20-14 ensemble trained together Ours	93.17 / 70.41 93.12 / 70.85 <b>93.27 / 71.96</b>	0.442 / 0.454 0.442 / 0.454 0.442 / 0.454	68 / 68 68 / 68 68 / 68

epochs. Our models are trained with a mini-batch size of 128. For simplicity, we use the architecture proposed by [44] for ImageNet.

The **Tiny-ImageNet** consists of 200 classes. Our models are trained on the 100k training images, and evaluated on the 10k validation images. Images are randomly resized and a 224 × 224 crop is randomly sampled from the resized image or its horizontal flip for training. For testing, we resize the original images to 224×224 and evaluate them. When training ResNet series networks, We start with a learning rate of 0.1, divide it by 10 at 40, 80 and 120 epochs, and terminate training at 150 epochs. We use a weight decay of 0.0001 and momentum of 0.9. When dealing with the network given by neural architecture search, we use the same setting as we do on ImageNet. Our models are trained with a mini-batch size of 128.

When it comes to the termination condition of residual-guided learning, we set comparable Energy as 90% of T's Energy. In the following sections, when it is not mentioned, we use  $L_2$  loss function for the  $\mathcal{L}_{T-S}$  in Eq. (3). "KD" in some table (e.g., Table IV) means using this loss function, too.

We do all experiments on four NVIDIA GTX 1080 Ti GPU cards.

# B. Ablation Study

In this part, we do ablation studies on CIFAR-10/100. We don't use the termination condition of our framework as described in Section III as we can see how every residual-guided training stage would contribute to the whole framework. Also, without loss of generality, we don't use NAS in ablation studies.

1) Residual-guided learning.: First, we validate the effect of a res-student. We use ResNet-110 as the teacher T, ResNet-20 as the classic KD student  $S_0$ , and ResNet-14 as the resstudent  $R_1$ . As illustrated in Table II, our ResKD student  $S_1 = S_0 + R_1$  outperforms  $S_0$ , which means a res-student can correct the errors of a classic KD student.

#### TABLE III

Multi-stage residual-guided learning on CIFAR-10/100. ResX-Y-Z means we use ResNet-X as  $S_0$ , ResNet-Y as  $R_1$ , and ResNet-Z as  $R_2$ . NAS14 denotes a network searched by NAS with comparable parameters to ResNet-14. Multi-stage residual-guided learning can further bridge the gap between the teacher and the student. **Bold**: The best results out of students. Underline: The second best results out of students.

Architecture	Acc. (%)	#Params (M)	MFLOPs
Teacher Res110	94.19 / 72.44	1.728 / 1.734	255 / 255
Students Res20 (SGD) Res20 (KD)	91.80 / 68.82 93.06 / 70.09	0.270 / 0.276 0.270 / 0.276	41 / 41 41 / 41
ResKD S <sub>1</sub> Res20-14 Res20-20	93.27 / 71.96 93.35 / 72.22	0.442 / 0.454 0.539 / 0.551	68 / 68 82 / 82
ResKD S <sub>2</sub> Res20-14-8 Res20-14-14	93.30 / 72.18 <b>93.68</b> / <b>72.42</b>	0.518 / 0.535 0.615 / 0.632	80 / 80 94 / 94

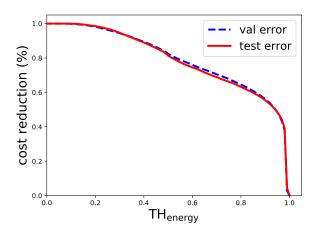


Fig. 3. The relationship between the cost reduction of  $R_1$  and  $TH_{\rm energy}$  on both validation set and test set on CIFAR-100. T is ResNet-110,  $S_0$  is ResNet-20, and  $R_1$  is ResNet-14. The result indicates that it is reasonable to calculate  $TH_{\rm energy}$  on a validation set for the sample-adaptive strategy.

However, since we obtain a ResKD student by combining two networks, it is arguable whether the effect of the resstudent comes from our residual-guided learning. It could be attributed to additional capacity. Thus, it is necessary to compare a ResKD student with other types of combinations, such as an ensemble network, to show the effect of our method. As illustrated in Table II, our method also outperforms the "ensemble" and the "trained together" approaches, which means that using res-students is more effective than using an ensemble.

We can repeat residual-guided learning to further bridge the gap between the student and the teacher. As illustrated in Table III, we train another res-student  $R_2$  to build  $S_2$ , and the performance of  $S_2$  is better than  $S_1$ .

2) Sample-adaptive inference.: To adopt the sample-adaptive strategy, we need to calculate a threshold  $TH_{energy}$ . As described in Section III-C, we calculate  $TH_{energy}$  on a validation set. Here, we justify our choice.

#### TABLE IV

EFFECT OF SAMPLE-ADAPTIVE INFERENCE AND KD METHODS ON CIFAR-10/100. T is ResNet-110,  $S_0$  is ResNet-20, and  $R_1$  is ResNet-14.  $S_1$  means using both  $S_0$  and  $R_1$ . NAS14 denotes a network searched by NAS with comparable parameters to ResNet-14. Using sample-adaptive inference, the performance would edge down but the computational cost would be saved considerably. Our ResKD framework can be applied to different KD methods. **Bold**: The best results out of students. Underline: The second best results out of students.

Architecture	Optimizer	SA	Acc. (%)	#Params (M)	MFLOPs
Teacher Res110	SGD		94.19 / 72.44	1.728 / 1.734	255 / 255
Students Res20 Res20 Res20	KD DML RKD		93.06 / 70.09 92.53 / 68.96 92.45 / 68.88	0.270 / 0.276 0.270 / 0.276 0.270 / 0.276	41 / 41 41 / 41 41 / 41
ResKD students Res20-14 Res20-14 Res20-14 Res20-14 Res20-14	KD KD DML DML RKD RKD	✓ ✓ ✓	93.27 / 71.96 93.27 / 71.94 92.88 / 70.30 92.88 / 70.29 93.17 / 71.52 93.17 / 71.52	0.442 / 0.454 0.442 / 0.454 0.442 / 0.454 0.442 / 0.454 0.442 / 0.454 0.442 / 0.454	68 / 68 44 / 52 68 / 68 50 / 58 68 / 68 48 / 55

## TABLE V

The effect of different KD loss functions  $\mathcal{L}_{T-S}$  on CIFAR-10. ResX-Y-Z means we use ResNet-X as  $S_0$ , ResNet-Y as  $R_1$ , and ResNet-Z as  $R_2$ . As a result, our framework is generic to  $\mathcal{L}_{T-S}$ . **Bold**: The best results out of students. <u>Underline</u>: The second best results out of students.

Architecture	Optimizer	$\mathcal{L}_{KD}$	Acc. (%)	#Params (M)	MFLOPs
Teacher					
Res110	SGD	-	94.19	1.728	255
Students					
Res20	SGD	-	91.80	0.270	41
Res20	KD	$L_2$	93.06	0.270	41
Res20	KD	KL	92.76	0.270	41
Res20	DML	$L_2$	92.53	0.270	41
Res20	RKD	$L_2$	92.45	0.270	41
ResKD students					
Res20-14	KD	$L_2$	93.27	0.442	68
Res20-14	KD	KL	93.50	0.442	<del>68</del>
Res20-20	KD	$L_2$	93.35	0.539	82
Res20-20	KD	KL	93.57	0.539	82
Res20-20-20	KD	$L_2$	93.95	0.809	123
Res20-20-20	KD	KL	93.93	0.809	123
Res20-14	DML	$L_2$	92.88	0.442	68
Res20-14	RKD	$L_2$	93.17	0.442	68

We use ResNet-110 as the teacher network, ResNet-20 as  $S_0$  and ResNet-14 as  $R_1$ . As shown in Fig. 3, the relationship between cost reduction and TH<sub>energy</sub> is very similar. Thus, it is reasonable to calculate TH<sub>energy</sub> on a validation set.

We carry on experiments to show the effect of the sampleadaptive strategy. As shown in Table IV, the performance of models adopting sample-adaptive strategy would edge down but the computational cost would be much lower.

- 3) Different loss functions.: To show that our residual-guided learning is generic to different  $\mathcal{L}_{T-S}$ , we carry on experiments using Kullback–Leibler divergence and  $L_2$  distance loss function on CIFAR-10. As illustrated in Table V, both Kullback–Leibler divergence and  $L_2$  distance loss function work well with our ResKD framework.
- 4) Different KD methods.: To show that our framework is generic to knowledge distillation methods, we also apply our

TABLE VI

PERFORMANCE ON CIFAR-10 / CIFAR-100. T is ResNet-110,  $S_0$  is a ResNet-20,  $R_1$  is a res-student whose architecture is searched by NAS.  $S_1$  means using both  $S_0$  and  $R_1$ . 'SA' means using our sample-adaptive strategy at inference time. Energy is the metric we mention in Section III-B. **Bold**: The best results out of students. Underline: The second best results out of students.

Architecture	Optimizer	SA	Acc. (%)	#Params (M)	MFLOPs	MFLOPs proportion to $T$ (%)	Energy
Teacher Res110	SGD		94.19 / 72.44	1.728 / 1.734	255 / 255	100 / 100	0.9993 / 0.9894
$Students$ $S_0$	KD		93.06 / 70.09	0.270 / 0.276	41 / 41	16.08 / 16.08	0.9923 / 0.8380
ResKD students S <sub>1</sub> S <sub>1</sub>	KD KD	<b>√</b>	93.92 / 74.06 93.92 / 74.06	0.453 / 0.485 0.453 / 0.485	79 / 81 46 / 59	30.98 / 31.76 18.04 / 23.14	0.9958 / 0.9204

#### TABLE VII

PERFORMANCE ON TINY-IMAGENET / IMAGENET. T IS RESNET-50,  $S_0$  IS A RESNET-18,  $R_1$  IS A RES-STUDENT WHOSE ARCHITECTURE IS SEARCHED BY NAS.  $S_1$  MEANS USING BOTH  $S_0$  AND  $S_1$ . 'SA' MEANS USING OUR SAMPLE-ADAPTIVE STRATEGY AT INFERENCE TIME. ENERGY IS THE METRIC WE MENTION IN SECTION III-B. **BOLD**: THE BEST RESULTS OUT OF STUDENTS. UNDERLINE: THE SECOND BEST RESULTS OUT OF STUDENTS.

Architecture	Optimizer	SA	Acc. (%)	#Params (M)	MFLOPs	MFLOPs proportion to $T$ (%)	Energy
Teacher Res50	SGD		70.42 / 75.50	23.92 / 25.56	4117 / 4119	100 / 100	0.9963 / 0.7782
$Students$ $S_0$	KD		67.90 / <u>70.02</u>	11.28 / 11.69	1821 / 1821	44.23 / 44.22	0.9805 / 0.6575
ResKD students S <sub>1</sub> S <sub>1</sub>	KD KD	<b>√</b>	68.98 / 71.37 68.99 / 71.37	16.38 / 17.40 16.38 / 17.40	2453 / 2454 2206 / 2216	59.58 / 59.58 53.59 / 53.80	0.9919 / 0.8335

residual-guided learning to deep mutual learning (DML) [9] and relational knowledge distillation (RKD) [27]. As shown in Table IV and Table V, our framework achieves consistent results on different KD methods.

# C. ResKD: The Whole Framework

We use our whole framework on CIFAR-10, CIFAR-100, Tiny-ImageNet and ImageNet. As illustrated in Table VI and Table VII, ResKD achieves consistent results on these datasets. Our ResKD student  $S_1 = S_0 + R_1$  where  $R_1$ 's architecture is searched by NAS outperforms  $S_0$  in both accuracy and energy, which means a res-student can also correct the errors of a classic KD student on all four datasets.

# V. ANALYSIS: WHY RESIDUAL-GUIDED LEARNING WORKS

In this section, we try to shed some light on why and how our residual-guided learning helps the training process.

# A. Theoretical Analysis

Our residual-guided learning is based on the gap between a teacher T and a student S. We should use a suitable metric to measure the gap between T and S. Inspired by [20], we measure the informativeness of training examples by analyzing their resulting gradients since the training data contribute to optimization via gradients. The gap of informativeness

#### TABLE VIII

GI of different student networks to the teacher network on CIFAR-10. T is ResNet-110, and  $S_0$  is ResNet-20. ResX-Y-Z means that we use ResNet-X as  $S_0$ , ResNet-Y as  $R_1$ , and ResNet-Z as  $R_2$ . Bold: The best results out of students.

Architecture	Acc. (%)	$L_2$ to T	$GI/C_1$ to T
Teacher			
Res110	94.19	-	-
Student			
Res20	93.06	0.90	1.73
ResKD networks	,		
Res20-8	93.03	0.82	1.64
Res20-14	93.27	0.74	1.53
Res20-20	93.35	0.61	1.37
Res20-20-8	93.55	0.49	1.15
Res20-20-14	93.84	0.43	1.03
Res20-20-20	93.95	0.37	0.89

(gap\_info, GI) between S and T for a training example  $\mathbf{x}^{(j)}$  at an iteration t is defined as:

$$GI(\mathbf{x}^{(j)}, S, T, t) = \|\nabla_{\theta_t} \mathcal{L}(S(\mathbf{x}^{(j)}), T(\mathbf{x}^{(j)}))\|_2,$$
 (12)

where  $\nabla_{\theta_t}$  denotes the gradients of S's parameters  $\theta$  at the iteration t. However, computing this  $L_2$ -norm directly is expensive. Instead, we could estimate the upper bound  $\widehat{GI}$  for GI.

Following the work of [45] and without loss of generality, we use a multi-layer perceptron (MLP) as the model in our analysis. Let  $\theta^{(l)} \in \mathcal{R}^{M_l \times M_{l-1}}$  be the weight matrix for layer l and  $\sigma^{(l)}(\cdot)$  be a Lipschitz continuous activation function, and then we have:

$$\mathbf{a}^{(0)} = \mathbf{x}^{(j)},$$

$$h^{(l)} = \theta^{(l)} \mathbf{a}^{(l-1)},$$

$$\mathbf{a}^{(l)} = \sigma^{(l)} (h^{(l)}),$$

$$f(\mathbf{x}^{(j)}, \theta) = \mathbf{a}^{(L)}.$$
(13)

where  $\mathbf{a}^{(l)}$  denotes the feature maps after layer l and  $\mathbf{x}^{(j)}$  is a certain sample. We define:

$$\Sigma_{l}'(h^{(l)}) = \operatorname{diag}(\sigma^{'(l)}(h_{1}^{(l)}), \sigma^{'(l)}(h_{2}^{(l)}) \cdots, \sigma^{'(l)}(h_{M_{l}}^{(l)})). \tag{14}$$

$$\Pi^{(l)} = (\prod_{i=1}^{L-1} \Sigma_i'(h^{(i)}) \theta_{i+1}^T) \Sigma_L'(h^{(l)}). \tag{15}$$

 $\mathcal{L}$  is the loss function  $\mathcal{L}(S(\mathbf{x}^{(i)}), T(\mathbf{x}^{(i)}))$  in Eq. (12). The GI<sup>(l)</sup> is the informativeness of the parameters in layer l and it can be expressed as:

$$GI^{(l)} = \|(\Pi^{(l)} \nabla_{\mathbf{a}^{(L)}} \mathcal{L}) (\mathbf{a}^{(l-1)})^T \|_2$$

$$\leq \|\Pi^{(l)} \|_2 \|(\mathbf{a}^{(l-1)})^T \|_2 \|\nabla_{\mathbf{a}^{(L)}} \mathcal{L}\|_2.$$
(16)

Various weight initialization [46] and activation normalization techniques [47], [48] uniformise the activations across samples. As a result, the variation of the gradient norm is mostly captured by the gradient of the loss function with respect to the pre-activation outputs of the last layer of our neural network. Consequently, we can derive the following upper bound to the gradient norm of all the parameters. Suppose that  $C_{max}$  is a constant:

$$GI \le C_{max} \|\nabla_{\mathbf{a}^{(L)}} \mathcal{L}\|_2. \tag{17}$$

Based on Eq. (17), we set:

$$\widehat{GI} = C_{max} \|\nabla_{\mathbf{g}(L)} \mathcal{L}\|_2. \tag{18}$$

For our method, we use  $L_2$  distance loss function to measure the difference between current student network and the guidance of current teacher:

$$\mathcal{L} = \|S - T\|_2^2. \tag{19}$$

We calculate the first derivative of our loss function:

$$\nabla_{\mathbf{a}^{(L)}} \mathcal{L} = 2 \cdot ||S - T||_2. \tag{20}$$

According to Eq. (18) and Eq. (20), we set  $C_1 = 2C_{max}$ , and we define  $S_0^*/R_i^*$  is the result that the  $S_0/R_i$  has been optimized well:

$$\widehat{GI}_{S_0} = C_1 ||S_0 - T||_2,$$

$$\widehat{GI}_{S_1} = C_1 ||R_1 - (T - S_0^*)||_2,$$

$$\widehat{GI}_{S_2} = C_1 ||R_2 - (T - S_0^* - R_1^*)||_2,$$

$$\cdots.$$
(21)

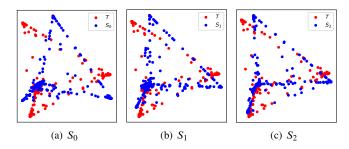


Fig. 4. The visualization of 2D PCA representation of ResKD student's logits using different number of res-students on CIFAR-10. The red dots in (a), (b) and (c) are the distribution of T, ResNet-110, and they are the same one. (a) the distribution of  $S_0$ . (b) the distribution of  $S_1$ . (c) the distribution of  $S_2$ .

With proper optimization for  $R_1$ , we have the best  $R_1^*$ . The best  $R_1^*$  is better than other value of  $R_1$  include that  $R_1$  always equals 0, so we have the upper bound of  $\widehat{GI}_{S_1}$  in residual-guided knowledge distillation.

$$\widehat{GI}_{S_1} \le C_1 \|0 - (T - S_0^*)\|_2$$

$$\le \widehat{GI}_{S_0}.$$
(22)

We rewrite Eq. (22) and get similar conclusion of other equations, and set  $\Delta_i^{(\mathrm{GI})} \geq 0$ :

$$\widehat{GI}_{S_0} = \widehat{GI}_{S_1} + \Delta_0^{(GI)}, 
\widehat{GI}_{S_1} = \widehat{GI}_{S_2} + \Delta_1^{(GI)}, 
\dots, 
\widehat{GI}_{S_{n-1}} = \widehat{GI}_{S_n} + \Delta_{n-1}^{(GI)}.$$
(23)

We add all the equations in Eq. (23):

$$\widehat{GI}_{S_0} = \widehat{GI}_{S_n} + \sum_{i=0}^{n-1} \Delta_i^{(GI)}.$$
(24)

 $R_i^*$  is the best result of optimizing  $R_i$  to approach  $T-S_0-\sum_{j=1}^{i-1}R_j$ , so we suppose that when we have optimized  $R_i^*$ ,  $\widehat{\mathrm{GI}}_{s_i}$  has changed to  $k_i(T-S_0-\sum_{j=1}^{i-1}R_j)$  and  $k_i\in(0,1)$ . In this constraint,  $\widehat{\mathrm{GI}}_{s_i}$  can be rewritten as:

$$\Delta_i^{(GI)} = k_i \cdot \widehat{GI}_{S_{i-1}}, k_i \in (0, 1).$$
 (25)

When the  $R_i$  becomes stronger, the  $k_i$  becomes larger and the network  $R_i$  bridge the gap better. Also the  $\widehat{GI}_{S_i}$  can be rewritten as:

$$\widehat{GI}_{S_i} = (\prod_{j=1}^{i} (1 - k_j))(T - S_0^*),$$

$$k_i \in (0, 1).$$
(26)

We can learn that the final performance of  $S_i$  is depended on the expression ability of each network in  $S_0, R_1, \dots, R_n$ . In Section IV-B, we will show some results that how the choice of a certain res-student network and the number of res-student networks affect the final performance.

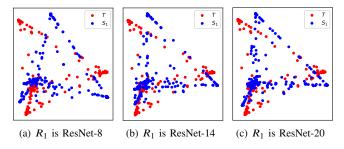


Fig. 5. The visualization of 2D PCA representation of  $S_1$ 's logits using different  $R_1$  on CIFAR-10. The red dots in (a), (b) and (c) are the distribution of T, ResNet-110, and they are the same one. The blue dots are the distributions of different  $S_1$  using different  $R_1$ .  $S_0$  is ResNet-20.

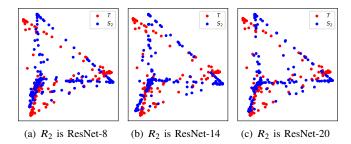


Fig. 6. The visualization of 2D PCA representation of  $S_2$ 's logits using different  $R_2$  on CIFAR-10. The red dots in (a), (b) and (c) are the distribution of T, ResNet-110, and they are the same one. The blue dots are the distributions of  $S_2$  using different  $R_2$ .  $S_0$  and  $R_1$  is ResNet-20.

# B. Empirical Analysis

In this part, first we show the GI of different students to validate our theoretical analysis on CIFAR-10. Next, we show the distribution of logits of different students directly to validate our residual-guided knowledge distillation.

**gap\_info (GI) observation.** We empirically verify whether the expression ability of the student network and res-student networks will influence the GI. The average GI and the average logits distribution samples that anyone of our residual-guided model predicts correctly and  $S_0$  gives the wrong prediction are showed. We use ResNet-110 as the teacher network, and ResNet-20 as the  $S_0$  on CIFAR-10. When we focus on  $S_0$ , we set ResNet-20 as  $S_1$  and observe how different res-student network  $S_1$  affects the final performance. The expression of Res $S_1$ - $S_2$ - $S_1$ - $S_2$ - $S_2$ - $S_3$ - $S_3$ - $S_3$ - $S_4$ - $S_3$ - $S_4$ - $S_5$ -

Firstly, we discuss the influence when the number of resstudent networks used increases. The results are shown in Table VIII. When we use the first res-student network  $R_1$  to bridge the gap between T and  $S_0$ , the output of  $S_0 + R_1$  is more similar to the output of T (the average second norm to T is less) and the GI is smaller than the one of  $S_0$ . Similarly, when we use  $R_2$ , the output of  $S_0 + R_1 + R_2$  is more likely to T and the GI continues decreasing. When we use ResNet-20 for  $S_0$ ,  $R_1$  and  $R_2$ , the GI is about the half of  $S_0$ .

Next, we discuss the difference among different latest resstudent networks. In Table VIII, the different number of resstudent networks can cause different results. When we use ResNet-8 / 14 / 20 to for  $R_1$ , the "GI/ $C_1$  to T" and " $L_2$  to T" decrease, and when we use ResNet-20, the value is the

smallest. The phenomenon is similar when we use ResNet-8 / 14 / 20 to for  $R_2$  and fix  $S_0$  and  $R_1$ . We can learn that the stronger  $R_1/R_2$  is, the more similar  $S_1/S_2$  is with the T.

**Logits visualization.** We show the 2D PCA representation of different  $S_i$ 's logits on CIFAR-10. In each figure, the red points are the same one and indicate the distribution of T's logits.

In Fig. 4, the blue points indicate the distributions of  $S_0$ 's,  $S_1$ 's and  $S_2$ 's logits. We use ResNet-20 for  $S_0$ ,  $R_1$  and  $R_2$  in this figure. We can learn that with the number of res-students increasing, the distribution of  $S_i$ 's logits is more and more similar with the distribution of T's.

In Fig. 5 (Fig. 6), The blue points indicate the distribution of  $S_1$  ( $S_2$ ). We use ResNet-20 / 14 / 8 for  $R_1$  ( $R_2$ ), and  $S_0$  is ResNet-20. In Fig. 6,  $R_1$  is ResNet-20. We can learn that when res-students  $R_1$  ( $R_2$ ) is stronger, the distribution of  $S_1$  ( $S_2$ ) is more similar with the distribution of T.

# VI. CONCLUSION

We have studied an under-explored yet important field in knowledge distillation of neural networks. We have shown that using res-students to bridge the gap between student and teacher is a key to improve the quality of knowledge distillation. We propose our residual-guided learning and sample-adaptive inference to realize this idea. We also validate the effectiveness of our approach in various datasets and studied its properties both empirically and theoretically.

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