"Just So" Neutrino Oscillations Are Back

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Recent evidence for oscillations of atmospheric neutrinos at Super-Kamiokande suggest, in the simplest see-saw interpretation, neutrino masses such that 'just so' vacuum oscillations can explain the solar neutrino deficit. Super-K solar neutrino data provide preliminary support for this interpretation. We describe how the just-so signal—an energy dependent seasonal variation of the event rate, might be detected within the coming years and provide general arguments constraining the sign of the variation. The expected variation at radiochemical detectors may be below present sensitivity, but a significant modulation in the ^{7}Be signal could shed light on the physics of the solar core—including a direct measure of the solar core temperature.

A decade ago we argued [1] that seasonal modulation of the solar-neutrino signal seen in various detectors could be a unique signature of 'just-so' vacuum neutrino oscillations [2], with the typical solar-neutrino oscillation length comparable to the Earth-Sun distance. The resulting oscillations could provide the large suppression needed to explain the ^{37}Cl data [3,1]. Since then, much attention has been paid to matter-enhanced (MSW) oscillations, for which large suppressions of the signal is obtained for a broad range of masses, and with which virtually any combination of suppressed signals at different detectors is explicable. In the intervening decade, solar neutrinos have been studied at many facilities, yet a definite resolution to the puzzle has not been identified. We argue that the just-so scenario, although it requires finely tuned masses, is both more easily testable and more readily falsifiable.

The Super-Kamiokande Collaboration has announced a remarkable result [4]. Evidence for vacuum oscillations between muon neutrinos and another species (not ν_e) was reported at a highly significant level and with a nearly maximum mixing angle. The required squared-mass difference is $\Delta m^2 \equiv |m_1^2 - m_2^2| \simeq 0.005$ eV^2. With the plausible hypothesis that the oscillation involves ν_μ and ν_τ, and that the mass difference is dominated by the tau neutrino mass, this suggests $m_{\nu_\tau} \approx .07$ eV.

There are several reasons to reconsider just-so solar neutrino oscillations in light of this result. While we have no direct empirical information on the neutrino mass spectrum, an elegant yet simple origin for the Majorana masses of light neutrinos is the so-called see-saw mechanism [5], which yields:

$$m_{\nu_i} \approx m_{q_i,l_i}^2/M_X, \qquad (1)$$

where m_{q_i,l_i} are the masses of the associated up-type quark or charged lepton in family i, and M_X is a heavy mass scale associated with the symmetry breaking responsible for neutrino masses. Taking the quark masses

to be relevant, we expect $m(\nu_\mu) \approx .07 \times (1.5/175)^2 \approx 7 \times 10^{-6}$ eV, and a much smaller $m(\nu_e)$. These neutrino masses are just right to provide a just-so explanation of the solar-neutrino flux.

Given our lack of understanding the origin of even the charged lepton masses, one might find the above reasoning suspect. It has been argued, for example, that maximal vacuum oscillations with a much smaller wavelength, resulting in a uniform halving of the observed neutrino signal, might resolve the solar neutrino problem, especially if there is still some systematic error in the ^{37}Cl data [6]. On the other hand and rather remarkably, the Super-K collaboration reports tentative evidence, based on their measurements of the shape of the recoil electron spectrum, for just-so oscillations with masses provisionally in the range $6 \leq \Delta m^2 \leq 60 \times 10^{-11}$ eV^2 [7].

If the just-so solution is correct, must we wait for confirmation from observations of neutral-current events at SNO or for unambiguous results from Borexino? We argue that this may not be necessary because of the unique signature of vacuum oscillations: an energy and time dependent variation of the signal as a function of the distance to the source. In our analysis, we re-examine the seasonal variation of both the overall flux (*i.e.*, integrated over energy) and the recoil electron spectrum for the mass regions now of interest. (Note: before the most recent Super-K result several other groups explored various just-so signatures(i.e. [8,9,10])).

Of course, all explanations of solar neutrino deficit are subject to a small seasonal variation of the overall event rate due to the eccentricity of Earth's orbit. Additional effects characteristic of just-so solutions are energy dependent and can average out when the signal is averaged over energies. Thus, the *rate* measurements at Super-K and at the radiochemical detectors, which have failed to detect seasonal variations, cannot exclude just-so oscillations. Depending on the neutrino mass we shall show that just-so oscillations can lead to seasonal variations

in the *spectrum* of recoil energies [1]. These variations can be radically energy dependent and are potentially detectable at Super-K. In cases where the energy dependence of the seasonal variation is weak, the integrated event rate can vary seasonally over 5% in addition to its expected variation due to the earth's eccentricity. This effect should soon be detectable. The sign of the variation may be telling as well. Lower masses lead to uniformly larger seasonal variations than expected in the absence of oscillations, while larger masses can result in variations which cancel out the variation due to the earth's eccentricity (although, as we derive below, this possibility is disfavored by the Super-K spectral information).

Our neutrino mass model, motivated by Super-K data, involves three left-handed mass eigenstates. Mixing between the flavor eigenstates is governed by three angles θ_1 θ_2, and θ_3. We assume the heaviest mass eigenstate, that playing a role in atmospheric neutrino oscillations, to have a mass of .07 eV. Oscillations between a solar electron neutrino and this state, if present, will undergo many oscillations on their way to the detector sun and average out to yield an essentially energy-independent suppression. Oscillations between electron and muon neutrinos, however, are assumed to be just-so. Thus we take

$$ m_1 \sim 0, \quad m_2 \sim 10^{-5} \text{ eV}, \quad m_3 \sim 0.1 \text{ eV}. $$

Adopting the conventional KM parameterization, we write the flavor eigenstates as:

$$ \nu_e \equiv c_2 c_3 \nu_1 + c_2 s_3 \nu_2 + s_2 e^{-i\delta} \nu_3, $$
$$ \nu_\mu \equiv $$
$$ -(c_1 s_3 + s_1 s_2 c_3 e^{i\delta})\nu_1 + (c_1 c_3 - s_1 s_2 s_3 e^{i\delta})\nu_2 + s_1 c_2 \nu_3. $$

from which we obtain:

$$ P(\nu_e \to \nu_e)\big|_{\text{solar}} = $$
$$ 1 - \frac{\sin^2 2\theta_2}{2} - \cos^4 \theta_2 \sin^2 2\theta_3 \sin^2 (m_2^2 R/4E). $$

For atmospheric oscillations, for which m_2 is too small to play much of a role, we obtain:

$$ P(\nu_\mu \to \nu_\mu)\big|_{\text{atmospheric}} = $$
$$ 1 - 4\sin^2 \theta_1 \cos^2 \theta_2 \sin^2 (m_3^2 R/4E) \begin{cases} \sin^2 \theta_2 & (\nu_e) \\ \cos^2 \theta_2 \cos^2 \theta_1 & (\nu_\tau) \end{cases} $$

Note that the CP-violating phase δ plays no role. Super-K (and CHOOZ) suggest that $\nu_\mu \to \nu_e$ is small for atmospheric neutrinos. In the limit $\theta_2 = 0$:

$$ P\big|_{\text{solar}} = 1 - \sin^2 2\theta_3 \sin^2 (m_2^2 R/4E), $$
$$ P\big|_{\text{atmospheric}} = 1 - \sin^2 2\theta_1 \sin^2 (m_3^2 R/4E). $$

We use the general form here, but take $\sin^2 2\theta_2 < 0.1$ to assure that atmospheric muon neutrinos oscillate mostly into ν_τ.

We can readily determine the resulting energy-dependent seasonal suppression of the neutrino signal at Super-K as a function of neutrino mass. Our numerical routine uses the B neutrino spectrum from Bahcall and Pinnsoneault [11] and standard neutrino-electron scattering cross sections. We assume a sharp detector threshold of 6.5 MeV and integrate over the Super-K electron energy resolution function. Predicted rates are multiplied by a normalization factor of ~ 0.9 to account for detector efficiency, deadtime, etc., thereby ensuring that the observed rate of 13.5 events/day corresponds, as reported, to $\sim 47\%$ of its SSM value [11].

Preliminary analyses of the Super-K spectrum, and the rates at other solar neutrino experiments, lead to a variety of estimates for parameters that best explain the data [12,13]: Spectral measurements by Super-K suggest (assuming $\theta_2 = 0$) $\Delta m^2 \simeq 4 \times 10^{-10}$ eV^2 [12]. A global analysis of event rates at all neutrino detectors, combined with the Super-K spectrum, suggests (again with $\theta_2 = 0$) $\Delta m^2 \simeq 7 \times 10^{-11}$ eV^2 [13]. Our analysis spanned both these estimates as well as the smaller value suggested by the see-saw mechanism: $2 \leq \Delta m^2 \leq 60 \times 10^{-11}$ eV^2, with mixing angles chosen to fit the observed rate of 13.5 events/day within uncertainties.

The predicted seasonal dependence of the integrated Super-K signal as a function of Δm^2 can be understood as follows: For small mass differences the oscillation length far exceeds the Earth-Sun distance. As Δm^2 increases, oscillations kick in and the survival probability of an electron neutrino decreases monotonically, so that as the Earth-Sun distance increases from perihelion to aphelion the ν_e flux at the detector decreases via the inverse-square law *and* through the effect of just-so oscillations. For $2 \leq \Delta m^2 \leq 6 \times 10^{-11}$eV^2, the seasonal variation of the signal significantly exceeds its no-oscillation value.

For $\Delta m^2 \simeq 6 \times 10^{-11}$eV^2, the mean oscillation probability at Earth is largest and the electron neutrino flux reaches a minimum. For slightly larger values, oscillations tend to *increase* the ν_e flux as the Earth-Sun distance increases. For this case the just-so and inverse-square effects oppose one another and the overall event rate can even increase. As Δm^2 is further increased, the seasonal effect oscillates in sign. At the same time, the seasonal effect becomes energy dependent so that the signals in different energy bins at Super-K would differ dramatically from one another. For even larger Δm^2, the just-so signal washes out due to averaging over the different neutrino energies: the seasonal variation becomes conventional and energy independent.

We have explored these effects quantitatively using several different numerical analyses. To maximize statistics and sensitivity to seasonal variation, the relevant quantity to consider is not the day of the year, but rather the Earth-Sun distance, which is probed twice over the course of the year. Seasonal variation should be plotted as a function of "days since perihelion".

Figure 1 shows the ratio R of the predicted number of Super-K events within 90 days of perihelion to those

in the rest of the year, versus Δm^2 and with maximal oscillations. The lower part of the figure displays the predicted overall event rate at Super-K versus Δm^2. The effects outlined earlier are evident. For small Δm^2 the seasonal variation is enhanced compared to the no-oscillation case. For larger values, the seasonal variation oscillates in and out of phase with the $1/r^2$ variation. Moreover, the shift in the sign of the seasonal variation (where the curve crosses the no-oscillation point) occurs precisely at the maxima and minima of the event rate curve, validating the picture discussed above.

so effect canceling the seasonal variation and the event rate increasing with Δm^2, and hence decreasing with neutrino energy. This behavior is evident in fig.(3), where there is virtually no seasonal variation and the event rate drops precipitously at high energies. However, observations at Super-K suggest an upturn in the neutrino signal at high energies. Thus, these values of Δm^2 are disfavored. *The just-so oscillations indicated by Super-K data suggest that the seasonal variation of the overall event rate is in phase (or at least, not out of phase) with the inverse square variation* (see also [10]).
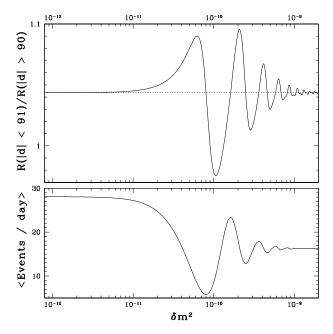

FIG. 1. Upper: Seasonal variation predicted for the solar neutrino signal in Super-K for maximal just-so oscillations (no oscillations—dashed curve) versus neutrino Δm^2. The number of events within 90 days of perihelion is divided by the number of events more than 90 days removed from perihelion. Lower: Overall event rate vs. Δm^2

Figures 2, 3, and 4 show the time variation of the differential and integrated event rates at Super-K for three different values of Δm^2. At the left, we display the temporal variation of the predicted neutrino signal at various energies between 6 and 14 MeV, with oscillation parameters as specified. At the right, for the same parameters, we display the temporal variation of the overall event rates. We also indicate the expected variation in the signal in the absence of oscillations.

The results validate our *a priori* reasoning and provide some new insights. For smaller values of Δm^2, as in fig.(2), the integrated and differential seasonal variations are similar—as expected because none of the detected neutrinos have completed one full oscillation. In this case, the overall seasonal variation can exceed its no-oscillation value by an additional 5-6%.

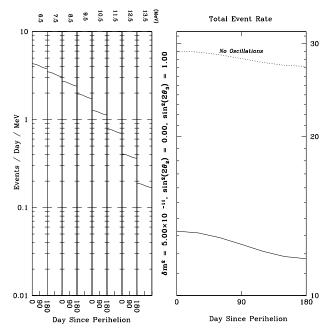For intermediate values of Δm^2, fig.(1) shows the just-

FIG. 2. Predicted seasonal variation for the solar neutrino signal in Super-K as a result of just-so oscillations, for $\Delta m^2 = 5 \times 10^{-11}$ eV^2, and mixing angle chosen to reproduce the Super-K observed event rate (within quoted uncertainties). On the left is shown the variation of the number of events/day/MeV predicted at various electron energies. On the right is the variation of the total event rate. (events/day)(dashed curve = no-oscillations)

At the higher end of the mass range, as in fig.(4), the difference in seasonal modulation at different energies is noticeable. This is true even when the integrated event rate variation does not differ significantly from that expected for no oscillations. This unique and unambiguous signature of just-so oscillations means that measurements of the seasonal variation of the differential spectrum can be of great utility when combined those of the overall event rate. Note that if the Super-K energy resolution ($\sigma \approx 0.58\sqrt{E}$ MeV) could be sharpened, the above effect would be dramatically enhanced!

Note (i.e. see [1]) that the seasonal variation in both the Cl and Ga signals can be less than 10% for just-so oscillations because of the integration over energies inherent in chemical detectors (this effect is most striking

for the Cl detector). While the Ga signal is expected to have a larger seasonal variation than Cl, it may be small enough to have been missed.
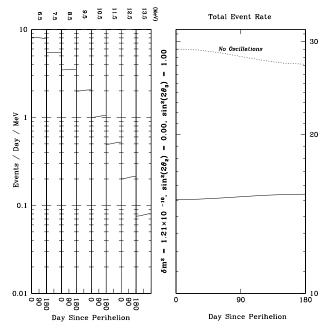

FIG. 3. same as figure (2), for $\Delta m^2 = 1.2 \times 10^{-10}$ eV^2, unfavored as described in the text
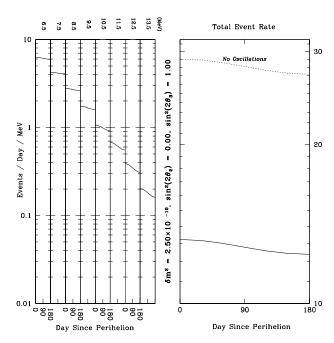

FIG. 4. same as figure (2), for $\Delta m^2 = 2.5 \times 10^{-10}$ eV^2.

Finally, a larger annual variation is expected in a detector such as Borexino, which is strongly sensitive to the 0.86 MeV ^{7}Be neutrino line. Seasonal variations as large as 40% can be possible at this energy [1,9]. Perhaps even more exciting is the fortuitous result [14] that the *phase* of time-dependent oscillations in the Be neutrino signal, combined with the amplitude of the oscillations, can give a direct measurement of the central solar core temperature. While the possibility of utilizing oscillations for this purpose exists only over a limited range of masses and mixing angles in the just-so range, it would be remarkable if nature allowed such a measurement.

The just-so solution of the solar neutrino problem regained credibility as a result of recent Super-K data. Furthermore, the technique that Super-K used to provide its strongest evidence for atmospheric neutrino oscillations—a time and energy dependent modulation of the neutrino signal—may be exploited to resolve one of the longest standing puzzles in particle astrophysics.

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