

# CDM and Baryons as Distinct Fluids in a Linear Approximation for the Growth of Structure

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A single fluid approximation which treats perturbations in baryons and dark matter as equal has sometimes been used to calculate the growth of linear matter density perturbations in the Universe. We demonstrate that properly accounting for the separate growth of baryon and dark matter fluctuations can change some predictions of structure formation in the linear domain in a way that can alter conclusions about the consistency between predictions and observations for  $\Lambda$ CDM models versus modified gravity scenarios. Our results may also be useful for 21cm tomography constraints on alternative cosmological models for the formation of large scale structure.

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## I. INTRODUCTION

Since the discovery that the universe is apparently dark energy-dominated causing an observed acceleration ([1, 2], also inferred indirectly on the basis of other observational constraints, i.e. see, for example [3]), a vast expenditure of effort has been made towards possible explanations of the acceleration. The standard paradigm of cold dark matter with a cosmological constant ( $\Lambda$ CDM) in the cosmological framework of general relativity (GR) accommodates all experimental evidence, and remains the simplest and most economical cosmological model consistent with the data. Though the  $\Lambda$ CDM cosmology fits all the present data, issues such as the hierarchy and coincidence problems remain which highlight the issue of how the acceleration can be realized in a fully consistent theoretical framework. This has led to a consideration of alternative cosmological models, including models in which gravity varies away from GR on large scales. As observations become increasingly precise, the  $\Lambda$ CDM picture will be put through ever more rigorous tests in the effort to constrain new physics. It is important therefore to have accurate theoretical frameworks by which to judge whether observations may indicate a discrepancy with the predictions of the standard model.

By now a standard way to constrain various cosmological alternatives is via an exploration of the growth of linear matter perturbations for various redshifts. These perturbations have been parameterized via a growth index (see for example [4–9]), which has a specific value for  $\Lambda$ CDM (to first order in deviations from a purely CDM dominated universe, where the deviation is due to a cosmological constant, this index has been estimated to be simply 6/11).

The growth is typically found in the following manner.

One defines a matter overdensity given in  $k$ -space by  $\delta \equiv \delta\rho/\rho$  with  $\rho$  being the background matter density. Using the standard Einstein equations, one finds the dynamical equation typically called the growth equation, for a single component matter field in a matter dominated universe

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \rho \delta = 0 \quad (1)$$

where the overdot is a derivative with respect to coordinate time and we have dropped the  $k$  index. This relation is given in the synchronous gauge where it holds on all scales (for gauge related issues see for example [10–17]). From here one can define a function  $g \equiv d\ln\delta/d\ln a \equiv \Omega_m(a)^\gamma$  which leads to the equation

$$g' + g^2 + g \left( \frac{\dot{H}}{H^2} + 2 \right) = \frac{3}{2} \Omega_m \quad (2)$$

where the prime denotes a derivative with respect to the natural log of the scale factor. The function  $g$  can be identified as the growth factor for matter density perturbations. The solution to this for a flat universe with a dark energy equation of state  $w_\Lambda$  is given, to first order in the expansion parameter  $1 - \Omega_\Lambda$  by

$$\gamma = \frac{3(w_\Lambda - 1)}{6w_\Lambda - 5} \quad (3)$$

This relation reduces to  $\gamma = 6/11$  for the case of  $\Lambda$ CDM. The growth factor  $g$  defined in (2) is affected *only* by CDM overdensities. In our paper we will consider the effect of considering both CDM and baryonic perturbations on the growth factor.

Therefore we focus on two facts:

- The matter content of the universe is not solely composed of cold dark matter, as the baryonic content is roughly one-fifth that of dark matter [18]. If one writes separate growth equations for dark matter and baryonic matter densities, each contain a source term involving the gravitational potential

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which is a function of the full matter content, including both baryons and dark matter (for example, the third term in Eq.(1) arises from the Poisson equation for the gravitational potential).

- Because baryonic matter has a non-zero sound speed, dark matter and baryonic matter perturbations obey different dynamical equations [19–21].

Here we explore the consequences of properly incorporating both of these effects, and quantify and compare differences in the perturbative densities when the full set of baryonic plus dark matter equations are solved versus the case when baryons are ignored. One of the central purposes of this effort is to compare the relative differences between results of these two approaches compared to the differences obtained when different cosmological models are explored, in order to determine the sensitivity to cosmological model dependence versus the need to properly account for baryons.

To explore these effects we perform calculations under both the standard  $\Lambda$ CDM scenario and a modified gravity model, for which we choose the DGP [22] model. We compare bias factors, total matter density perturbations and the growth factor in these to cosmological scenarios to explore the sensitivity to not including baryonic perturbations. We find that an accurate treatment of baryonic fluctuations will alter quantities like the bias factor and the total matter density fluctuation even in the linear regime in a way that can exceed the change induced in the quantities by varying the background cosmology. On the other hand we find that the growth factor, defined as  $\frac{d \ln \delta}{d \ln a}$  is relatively insensitive to inclusion or non-inclusion of baryonic dynamics over linear scales. It is on the other hand very sensitive to the background cosmological model.

In Section II we briefly present the formalism for the full set of dynamical equations. In Section III we present our results, and finally in Sec. IV we conclude with a brief discussion of their implications.

## II. CALCULATION

The full set of coupled linear differential equations for the growth of perturbations in dark matter ( $\delta_c$ ) and baryons ( $\delta_b$ ) along with radiation ( $\delta_{rad}$ ) is

$$\begin{aligned} \ddot{\delta}_c + 2H(z)\dot{\delta}_c &= \frac{3}{2}H^2(f_c\delta_c + f_b\delta_b + f_r\delta_{rad}) \\ \ddot{\delta}_b + 2H(z)\dot{\delta}_b + c_s^2k^2 &= \frac{3}{2}H^2(f_c\delta_c + f_b\delta_b + f_r\delta_{rad}) \\ \delta_{rad} &= \frac{4}{3}\delta_b \end{aligned} \quad (4)$$

$$\begin{aligned} f_c &= \Omega_c(z) & f_b &= \Omega_b(z) \\ f_r &= \Omega_{rad}(z), 0 & (z > z_{dec}, z < z_{dec}) \end{aligned} \quad (5)$$

where  $\Omega_c$ ,  $\Omega_b$  and  $\Omega_{rad}$  are CDM, baryons and radiation energy density at a given epoch. We define  $c_s = \left(\frac{\delta P}{\delta \rho}\right)_S$  [20], where the subscript  $S$  stands indicates that  $c_s$  is defined at constant entropy,  $S$ . At low redshifts  $c_s$  is primarily due to baryonic pressure and at high-redshift (pre-decoupling redshifts), the contribution is mainly from radiation pressure. At high redshifts we use Eqns.(4) and (5) to solve for the baryonic and dark matter perturbations. We evolve the perturbations in each Fourier mode  $k$ , starting from the epoch of horizon entry for that particular mode. Therefore our boundary values for  $\delta_b$  and  $\delta_c$ , corresponding to a mode  $k$ , are set at the epoch of horizon entry,  $z_{ent}$ , such that

$$\begin{aligned} A &= 3\delta_{COBE} \\ k_0 &= \frac{a(z_{eq})H(z_{ent})}{c} \\ \delta_{c,b}(z_{ent}, k) &= A \left(\frac{k}{k_0}\right)^{\frac{n_s}{2}} (k < k_{eq}) \\ &= A \left(\frac{k}{k_0}\right)^{\frac{n_s-4}{2}} (k > k_{eq}) \\ \dot{\delta}_{c,b}(z_{ent}, k) &= \delta_{c,b}(z_{ent}, k)H(z_{ent}) (z_{ent} > z_{eq}) \\ &= \delta_{c,b}(z_{ent}, k)a(z_{ent})H(z_{ent}) (z_{ent} < z_{eq}) \end{aligned} \quad (6)$$

In Eq.(6), modes that enters the horizon at the epoch of matter and radiation equality ( $z_{eq}$ ) are denoted by  $k_0$ . Fluctuations are normalized by using the temperature fluctuation over angular scales of 7 degrees at the surface of last scattering measured by the COBE mission which is denoted by  $\delta_{COBE}$  [23]. The spectral index of the primordial fluctuations (coming from very early times) is set by  $n_s$  over all Fourier modes.

The boundary conditions for  $\delta_{c,b}$  described by Eq.(6) come from the following argument. Given a primordial power spectra of shape  $P_i(k) \propto k^{n_s}$ , fluctuations grow during the radiation dominated epoch such that  $\delta \propto a^2$ . Therefore when a given mode enters the horizon, its power is described by  $P_{ent}(k) \propto a^2 P_i(k) \sim k^{n_s-4}$  if  $z_{ent} > z_{eq}$  [24], and we replace  $\delta_{c,b}(z_{ent}, k) \propto \sqrt{P_{ent}(k)}$ . Note that if  $n_s \approx 1$ , then  $k^3 P_{ent}(k)$  is constant at horizon crossing. Similarly, we find that,  $P_{ent}(k) \propto k^{n_s}$  corresponding to the modes which enter the horizon after matter-radiation equality.

The calculation of  $c_s$  involves matter and radiation temperatures along with their fluctuations. We calculate the matter temperature,  $T_{mat}(z)$ , at a given epoch  $z$  using the following equation [25].

$$\begin{aligned} \frac{d\bar{T}_{mat}}{dt} &= 2H(z)\bar{T}_{mat} \\ &+ \frac{x_e(t)}{t_\gamma} (\bar{T}_\gamma - \bar{T}_{mat}) a^{-4} \end{aligned} \quad (7)$$

$$(8)$$

The radiation temperature,  $T(z)$ , at a given epoch  $z$  is estimated by the standard relation  $T(z) = T_0(1+z)$ , with

$T_0$  given by the present CMB temperature [26]. Fluctuations in the matter temperature after mechanical decoupling ( $z > 1100$ ),  $\delta_T$ , are calculated using

$$\frac{d\delta_T}{dt} = \frac{2}{3} \frac{d\delta_b}{dt} - \frac{x_e(t)}{t_\gamma} a^{-4} \frac{\bar{T}_\gamma}{T} \delta_T \quad (9)$$

$\delta_T$  thus computed is in turn used for the calculation of the sound speed post recombination, and therefore the modified baryonic growth equation becomes [27]

$$\begin{aligned} \ddot{\delta}_b + 2H(z)\dot{\delta}_b &= \frac{3}{2}H^2(f_c\delta_c + f_b\delta_b) \\ &- \frac{k^2}{a^2} \frac{k_B T}{\mu} (\delta_b + \delta_T) \end{aligned} \quad (10)$$

For scales which enter the horizon before the epoch of recombination,  $z_{rec}$ , matter temperature fluctuations are described by  $\delta_T = \delta_{T_\gamma}$  at  $z = z_{ent}$ .

As we would like to examine growth not only in the standard cosmology, but in a modified gravity scenario as well, we will now discuss how the calculation needs to be altered. In the DGP scenario, the CDM and baryon perturbation equations are modified such that in the source term on the right hand side of Eq.(4), the factor  $H(z)^2$  is replaced by  $H_{DGP}^2 g(a, k)$ . In this case  $H_{DGP}(z)$  is the modified background expansion rate and  $g(a, k)$  is the factor by which Newton's constant gets modified under the new gravity scenario.

We use [28] and [29] to construct  $H_{DGP}(z)$  and  $g(a, k)$ . Using these modifications and Eqns.(4-5), we calculate the growth of perturbations in the DGP theory up to a scale corresponding to  $k < 0.05 \text{ Mpc}^{-1}$ . We chose to restrict ourselves to these scales in order to avoid complications due to non-linear PPF parameters as described in [28]. The matter temperature perturbations in Eqns.(7-10) also get modified by replacing  $H(z)$  with  $H_{DGP}(z)$ .

The input parameters of our calculation are  $n_s$ ,  $\delta_{T_\gamma}$ ,  $\Omega_{b,c}$ ,  $h$ ,  $\Omega_k$ ,  $\Omega_{tot}$  and  $z_{eq}$ . Additionally, the epoch of decoupling,  $z_{dec}$ , is determined such that the photon mean-free path is larger than Hubble distance, or  $\lambda_\gamma = \frac{1}{n_e \sigma_T} \sim cH^{-1}$ . For simplicity, we set  $\Omega_\nu = 0$ ,  $\frac{dn_s}{dnk} = 0$  and allow a sharp drop in optical depth of photons at the epoch of mechanical decoupling. In the next section we will describe our results using WMAP9 values [26], along with additional DGP fits.

### III. RESULTS

In Figure 1 we represent the background expansion rate with respect to redshift,  $H(z)$ , in different cosmological cases. We consider fCDM, fDGP (flat  $\Lambda$ CDM and flat DGP), and oDGP (open DGP) models. For cosmological parameters we use  $\Omega_c h^2 = 0.12$ ,  $\Omega_b h^2 = 0.023$ ,  $h = 0.69$  from WMAP9, and apply those to both fCDM and fDGP models. For the oDGP models we use  $\Omega_c h^2 = 0.099$ ,  $\Omega_b h^2 = 0.023$ ,  $\Omega_k = 0.03$  and  $h = 0.76$  [29], and for the

fDGP model we use  $\Omega_c h^2 = 0.12$ ,  $\Omega_b h^2 = 0.023$ ,  $h = 0.69$ . Eqn.(4-5) of [29] are incorporated to compute the modified expansion rate  $H_{DGP}(z)$ . We have plotted up to  $z \sim 1$  to highlight the effect at low redshift, where one would expect modifications of gravity designed to mimic dark energy to be most relevant.

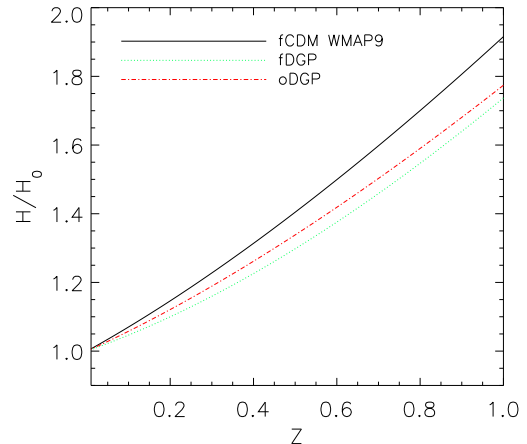


FIG. 1: Background expansion rates ( $H/H_0$ ) as a function of redshift based on different fCDM (flat  $\Lambda$ CDM), fDGP (flat DGP) and oDGP (open DGP) cosmologies. Relevant cosmological parameters are described in the text.

In Figure 2 we present the bias, defined as  $b(z, k) = \frac{\delta_b}{\delta_c}$ , with respect to redshift. We choose three length scales corresponding to Fourier modes  $k = 0.005$ ,  $k = 0.01$  and  $k = 0.05$  in units of  $\text{Mpc}^{-1}$ , where  $b(z, k)$  is represented in those regimes respectively by solid, dotted and dashed lines. We choose fCDM, fDGP and oDGP cosmologies described by the same cosmological parameters as in Figure 1. For a single fluid model the bias is unity by construction. From Figure 2 it is evident that in all scales explored, the difference in bias between the DGP and CDM models is less than their deviation from unity. This deviation (with respect to unit bias in a single-fluid model) increases both with redshift and diminishing scale. For low redshift ( $z \sim 0.2$ ) and  $k = 0.01 \text{ Mpc}^{-1}$ , the difference between the bias calculated from fCDM and fDGP models (using WMAP9 parameters) is about 0.04% with a difference of 4.2% in the background expansion rate. The bias becomes close to 1% near  $z \sim 1$ , while the difference in bias between fCDM and fDGP (using WMAP9 parameters) is only up to 0.02% for  $k = 0.05 \text{ Mpc}^{-1}$ .

The bias can be directly related to the total density fluctuation  $\delta = f_c \delta_c + f_b \delta_b$  such that  $\delta = (f_c + b(z, k)f_b) \delta_c$ , where  $b(z, k)$  represents the bias at a given epoch and a given scale  $k$ . In Figure 3 we display  $\delta_{tot}(z, k)$  as a function of redshift in the linear regime,  $k = 0.005 \text{ Mpc}^{-1}$ . The same normalization was used for all cosmologies, set by Eq.(6). We use dotted lines to refer to the single fluid (s-f) models and solid lines for the baryon+CDM fluid models. We note that at low redshift, the difference in the oDGP and fCDM models is

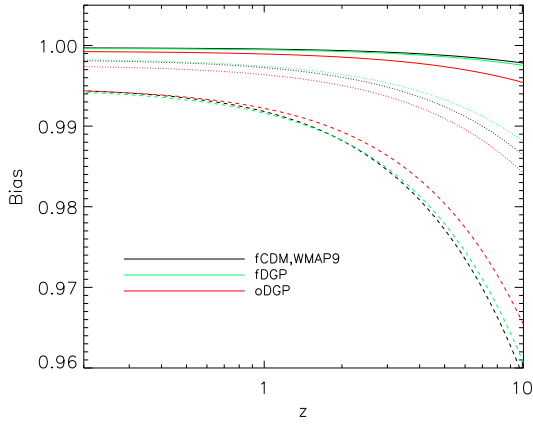


FIG. 2: The bias as a function of redshift in different cosmologies for scales corresponding to  $k = 0.005$ (solid lines),  $k = 0.01$ (dotted lines) and  $k = 0.05$ (dashed lines) in units of  $\text{Mpc}^{-1}$ .

comparable to that between the fCDM, s-f and two fluid models. This is an intriguing conclusion which suggests the importance of the two-fluid treatment in order to correctly use structure formation observations to constrain cosmological models. The significance of  $\delta_{tot}$  is that it is a scale and cosmology dependent quantity. Observationally, future weak lensing surveys can estimate  $\delta_{tot}$  but the interpretation of observational data must be made by properly incorporating bias on all scales of interest.

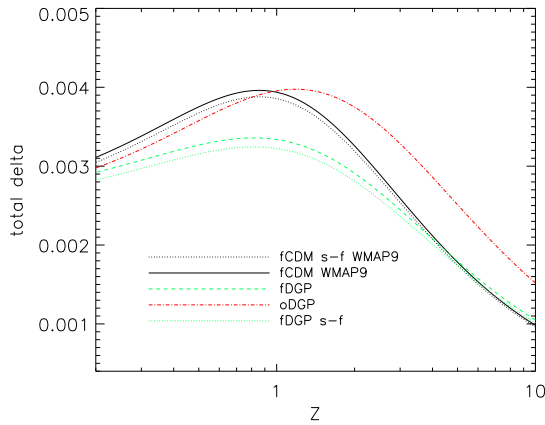


FIG. 3: The total matter fluctuation  $\delta = f_c \delta_c + f_b \delta_b$  is shown as a function of redshift under different cosmologies at a scale corresponding to  $k = 0.005 \text{Mpc}^{-1}$ . Solid lines indicate two-fluid models and dotted lines represent single fluid (s-f) models.

In Figure 4, we plot growth factor *only* due to CDM  $g_c = \frac{d \ln \delta_c}{d \ln a}$  (solid lines) with respect to redshift for a chosen scale corresponding to  $k = 0.005 \text{Mpc}^{-1}$ . Note that we over plot (dashed line) the parameterization  $\Omega_m^{\frac{6}{11}}$  [30] which agrees quite well with the numerical estimates for fCDM. We therefore conclude that  $g_c$  is roughly scale-

independent, but is sensitive to the background cosmology.

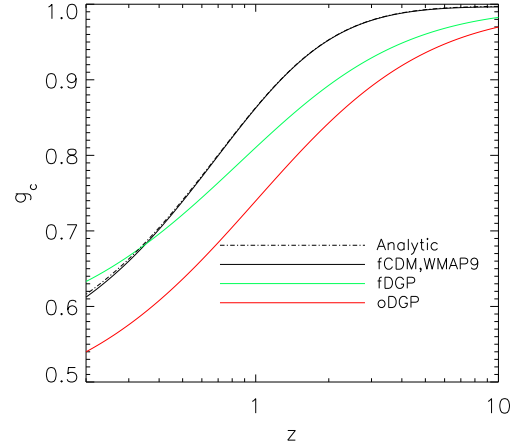


FIG. 4: The growth factor due to *only* CDM described by  $g_c = \frac{d \ln \delta_c}{d \ln a}$  is plotted as a function of redshift for different fCDM (flat  $\Lambda$ CDM), fDGP (flat DGP) and oDGP (open DGP) cosmologies. This is done at a single scale  $k = 0.005 \text{Mpc}^{-1}$ . Two component models are presented in solid lines and the dash-dotted line referred as analytic indicates  $\Omega_m(z)^\gamma$  such that  $\gamma = \frac{6}{11}$  for fCDM using WMAP9 parameters.

We can also consider the evolution of the total growth factor due to CDM and baryons as  $g_{tot} = \frac{d \ln \delta_{tot}}{d \ln a}$  with respect to redshift at a scale of  $k = 0.005 \text{Mpc}^{-1}$ , to check to see if there is any difference in using this value instead of the CDM growth factor. We define  $\delta_{tot} = \frac{\delta_c f_c + \delta_b f_b}{f_c + f_b}$ . We find that both parameters  $g_c$  and  $g_{tot}$  are weakly dependent on the selection of scale  $k$ , with  $g_{tot}$  to be comparatively more rigid over a range of various  $k$  values. However in general  $g_c \sim g_{tot}$  so there is no significant handicap in using  $g_c$  to constrain cosmological models.

#### IV. DISCUSSION AND FUTURE DIRECTIONS

In this work we have examined the impact on calculations of the growth of structure through the use of a simple single matter fluid approximation vs. a model that correctly incorporates baryons and cold dark matter in a two component analysis. We have then compared the difference in matter perturbations in the standard  $\Lambda$ CDM cosmology and modified DGP gravity.

Among the quantities we discussed in our paper, the growth factor  $g = \frac{d \ln \delta}{d \ln a}$  is measured observationally from galaxy redshift surveys [31]. We found the growth factor,  $g$  to be rigid with variation in  $k$  under both  $\Lambda$ CDM and DGP cosmologies, and largely independent of baryonic dynamics.

Using weak lensing to measure the total  $\delta = f_c \delta_c + f_b \delta_b$  is a way to get a handle on total matter density fluctuations but this quantity is sensitive to baryonic dynamics



(3) and depends on the relevant scale of structure formation *even in the linear regime*. In addition, we find that at low-redshift ( $0.7 < z < 1.5$ ) the effect of baryonic dynamics can be comparable to that introduced by modifying the underlying cosmology. We show in Figure 3 that for  $z \sim 1$ , modifications due to two component model is comparable to that due to modifying the cosmology. Therefore accurate inclusion of the baryonic dynamics is required for interpreting observations associated with this quantity

It is also worth noting various studies of the growth of large scale structures in different modified gravity contexts have been performed using a single growth equation (for a sample see [32–41]). Preliminary results, to be described in a future work suggest that the use of a proper two component fluid formalism can significantly weaken the ability to distinguish between cosmological predictions in such models.

Finally, another area of cosmology which has attracted

a great deal of interest recently, and relies on accurate calculations of the evolution of density perturbations, is the signal arising from the 21cm spin-flip transition of neutral hydrogen (for recent reviews, see [42–44]). One may study, for example, the perturbations of the brightness temperature of the CMB over a large redshift range in the so-called Dark Ages, which rely on the density perturbations of hydrogen. These perturbations are seeded by dark matter, and therefore a precise calculation merits the inclusion of the full baryonic plus dark matter system. We are currently using the formalism we have described here to investigate how it will impact upon conclusions one may draw from the use of such observations.

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