New Signatures and Sources for the Detection of WIMP Dark Matter in the Solar System

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Abstract. I first outline new results on the angular modulation of WIMP dark matter scattering on targets in terrestrial laboratories, based on our uncertainties of the WIMP halo distribution, I then outline an exciting new result which indicates that for the high end of allowed SUSY WIMP scattering cross sections there exists a new distribution of WIMP dark matter in our solar system which could produce a dramatically different signal from halo WIMP dark matter in terrestrial detectors.

1. Introduction

If our galactic halo is dominated by WIMPs, searching for signatures which allow a WIMP signal to be extracted from other backgrounds in terrestrial detectors will be of increasing importance, especially as the predicted signal levels in detectors decrease. Here I outline two different ongoing research programs I am involved in which bear on this issue. The first involves the first detailed examination of the directionality of WIMP scattering in terrestrial detectors for a variety of models of the WIMP distribution in the galactic halo. We have developed a formalism which allows the differential scattering rate over energy and angle to be determined from any incident distribution, including anisotropic ones. We then explore not only the predicted distributions, but also the ability, given a fixed number of events, to unambiguously demonstrate evidence for underlying directionality in the events. The second project uncovers the existence of a new WIMP distribution in the solar system. Due to the onset of chaos in solar system orbits induced by Jupiter, some WIMPs trapped in the solar system due to scattering in the Sun can move to orbits which no longer intersect the Sun. We estimate the space density of such WIMP orbits near the Earth, and find it can be considerable for realistic SUSY parameters at the upper end of the allowed range of scattering cross sections.

2. Angular Anisotropy and WIMP signatures

It has been recognized for some time that while the annual modulation of the WIMP scattering rate in detectors, due to the Earth's motion about the Sun is small, possible angular anisotropies can be large (i.e. [1]). However estimates that were performed were based on the simple assumption of a spherical isothermal sphere model for the galactic dark matter halo. Morever, while forward-backward assymmetries can be very large, this measure can be misleading. Sometimes a large anisotropy simply reflects a very small number of events, so that poisson uncertainties can dramatically reduce the significance of any of such effect.

We have developed a formalism [2] which allows the incorporation of aniostropies in the underlying WIMP distribution, due either to a non-spherical isothermal distribution, or perhaps due to non-isothermal flows, for example such as those predicted by Sikivie [3].The rate in a detector depends upon the density ρ_o of WIMPs near the Earth, and the velocity distribution $f(v)$ of WIMPs in the Galactic halo near the Earth. As a function of the energy deposited, Q, direct-detection experiments measure the number of events per day per kilogram of detector material. Qualitatively, this event rate is simply $R \sim n\sigma <v>/m_n$, where the WIMP number density is $n = \frac{\rho_o}{m_\chi}$. σ is the elastic-scattering cross section, $<v>$ is the average speed of the WIMP relative to the target, and we divide the detector mass M_{det} by the target nucleus mass m_n, to get the number of target nuclei.

More accurately, one should take into account the fact that the WIMPs move in the halo with velocities determined by $f(\vec{v})$, that the differential cross section depends upon $f(\vec{v})$ through a form factor $\frac{d\sigma}{d|q|^2} \propto F^2(Q)$, and that detectors have a threshold energy E_T, below which they are insensitive to WIMP-nuclear recoils. In addition, the Earth moves through the Galactic halo and this motion should be taken into account via $f(\vec{v})$.

Consider a WIMP of mass m_χ moving with velocity $\vec{v} = cos\alpha\hat{x} + sin\alpha sin\beta\hat{y} + sin\alpha cos\beta\hat{z}$ in the laboratory frame. This WIMP scatters off of a nucleus of mass m_n which recoils with energy $E_n = \frac{m_n m_\chi^2}{(m_n+m_\chi)^2}v^2(1-\mu)$, where $\mu = cos\theta^*$ and θ^* is the WIMP scattering angle in the center-of-mass frame. The velocity $\vec{u}$ of the nuclear recoil makes an angle γ relative to the x axis: $cos\gamma = \frac{\hat{x}\cdot\vec{u}}{|\vec{u}|} = (\frac{1-\mu}{2})^{\frac{1}{2}}cos\alpha - (\frac{1+\mu}{2})^{\frac{1}{2}}cos\xi sin\alpha$, where $-\sqrt{\frac{1+\mu}{2}}sin\xi = \frac{\vec{u}\cdot(\hat{x}\times\vec{v})}{|\vec{u}\cdot(\hat{x}\times\vec{v})|}$.

The WIMP distribution function $f(v,\alpha,\beta)$ determines the event rate per nucleon in the detector: $dR = f(v,\alpha,\beta)\,v^3 dv\,dcos\alpha\,d\beta\frac{d\sigma}{d\mu}d\mu\frac{d\xi}{2\pi}$, where $0 < v < \infty, -1 < cos\alpha < 1, 0 < \beta < 2\pi, -1 < \mu < 1$, and $0 < \xi < 2\pi$. Assume that the nucleon-WIMP scattering has following form: $\frac{d\sigma}{d\mu} = \frac{\sigma_o}{2}F^2(Q)$ with form factor suppression. The rate of events in which the nucleus recoils with energy Q is then given by $\frac{dR}{dQ} = \frac{\sigma_o\rho_\chi}{2m_r^2 m_\chi}F^2(Q)\int_{v_{min}}^{\infty} v\,dv\int d\Omega_{\alpha,\beta}\,f(v,\alpha,\beta)$, where $v_{min}^2 = \frac{(m_\chi+m_n)^2 Q}{2m_\chi^2 m_n}$ is the minimum WIMP velocity that can produce a nuclear recoil of energy Q.

We then derived the following equation for the angular differential cross section.

$$\frac{dR}{d\Omega_{\gamma,\phi}} = \frac{\sigma_o\rho_\chi}{\pi m_n m_\chi}\int_{v_{min}}^{\infty} v^3\,dv F^2(Q(v,J))\int d\Omega_{\alpha,\beta}\,f(v,\alpha,\beta)\,J(\alpha,\beta;\gamma,\phi)\,\Theta(J) \tag{1}$$

Here the function $J(\alpha,\beta;\gamma,\phi)$ takes care of the arbitrary dependence on the detector position and has the following property.

$$J(\alpha, \beta; \gamma, \phi) \equiv [cos\gamma\,cos\alpha \,+\, sin\gamma\,sin\alpha\,cos(\phi - \beta)] \int d\Omega_{\alpha,\beta} J(\alpha, \beta; \gamma, \phi)\,\Theta(J(\alpha, \beta; \gamma, \phi)) \,=\, \pi$$

Note that the equation above holds for any (γ, ϕ) (and $\Theta(J)$ is the usual step function).

Equation (1) was originally obtained by the series of Jacobians evaluated for successive transformations of angles from the incident direction to the outgoing direction of the recoiled target nucleus. The end result is actually independent of the intermediate two body collision channels and gives a simple geometric identity. When we follow the track of recoiled particles, not the scattered one as in traditional fixed target experiments, we have this simple relation between the incident angle and the outgoing events.

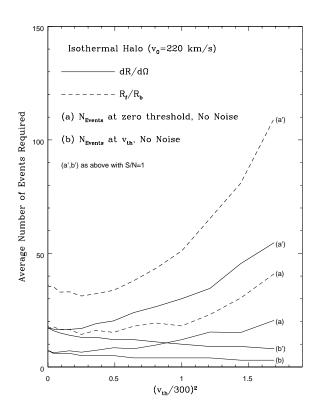We have also derived the event rate as a function of both deposited energy and the outgoing angle.

$$\frac{d\,R}{dQ\,d\Omega_{\gamma,\phi}} = \frac{m_n\sigma_o\rho_\chi}{8\pi\,m_r^4 m_\chi}\,Q\,F^2(Q)\int d\Omega_{\alpha,\beta}\,f(v(Q,J),\alpha,\beta)\,\frac{\Theta(J(\alpha,\beta;\gamma,\phi))}{J^3} \qquad (2)$$

With these three event rate formulae, we have evaluated the differential cross sections and event rates for arbitrary forms of the WIMP halo distribution function. Our results [2] indicate that depending upon the actual distribution function, only 15-30 events will be required over the course of a year in order to unambiguously differentiate a non-isotropic distribution from a flat background, with a signal to noise ratio of unity, independent of the halo distribution, unless it is co-rotating. Displayed in Figure 1, for example, are the estimates, as a function of detector threshold, for the number of events required to differentiate isothermal halo events from a flat background, with and without noise. For each set of curves, the lower one gives the number of events needed above the threshold, while the upper curve gives the equivalent total number of events required for zero threshold to yield the required number above the stated threshold. Also shown are the number of events required if only the forward/backward ratio and not the full distribution is measured. With a far greater number of events, features of the underlying WIMP distribution could be inferred.

3. A New Solar System WIMP Distribution?

WIMP dark matter can scatter elastically in the Sun, be gravitationally captured, and eventually settle in the Solar core and annihilate. Recently however, we [4, 5] have found that perturbations due to the planets, combined with the non-Coulomb nature of the gravitational potential inside the Sun, imply that WIMPS which are gravitationally captured by scattering in surface layers of the Sun can evolve chaotically into orbits whose trajectories no longer intersect the Sun. For orbits having a semi-major axis smaller than $1/2$ of Jupiter's orbit, WIMPS can persist in the solar system for billions of years. There can thus be a new, previously unanticipated, distribution of WIMPs which intersect the Earth's orbit. For WIMPs which might be detected in the next generation of underground detectors, this solar system distribution could be significant, providing a complementary signal to that of galactic halo dark matter.

We focus on the sub-population of WIMPs which scatter on a nucleus located near the surface of the Sun, and thereby loose just enough energy to stay in Earth-crossing orbits. These will
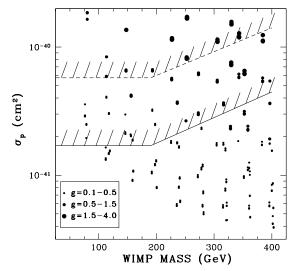
Figure 1. Number of Events needed to distinguish halo events arising from an isothermal distribution (with $v_o = 220 \mathrm{kms}^{-1}$) from a flat noise background

Figure 2. Solar capture factor $g_{\mathrm{tot}}^{(-10)}$ as a function of WIMP mass and cross section. Hatched lines give existing upper limits assuming a local halo density of $0.3 \mathrm{GeVcm}^{-3}$ (lower) or $0.1 \mathrm{GeVcm}^{-3}$ (upper).

be susceptible to small gravitational perturbations by the planets. We are interested then in the differential capture rate, per energy, and per angular momentum, of WIMPs in the Sun, and in particular only in the fraction of WIMPs which have angular momenta in a small range $[J_S - \epsilon, J_S]$ where J_S is the angular momentum for a WIMP exactly grazing the Sun. Using the scattering cross section of WIMPs on nuclei given above, one can, after considerable computation (see [5]), derive the rate with which WIMPs scatter on nuclei with atomic number A to end up into bound solar orbits with semi-major axis between $[a, a + da]$ (corresponding to $[\alpha, \alpha + d\alpha]$ with $\alpha \equiv G_N M_{Sun}/a$), and with specific angular momentum $J \geq J_{\min}$.

$$\left. \frac{d\dot{N}_A}{d\alpha} \right|_{J \geq J_{\min}} \simeq \frac{n_X}{v_o} \int_{r \geq r_{\min}} d^3\mathbf{x}\, n_A(\mathbf{x})\, \sigma_A \left(1 - \frac{J_{\min}^2}{r^2\, v_{\mathrm{esc}}^2(r)} \right)^{1/2} K_A(r, \alpha) . \tag{3}$$

Here, the minimum radius $r_{\min}$ (impact parameter) is defined in terms of the minimum angular

momentum $J_{\min}$ by $r_{\min} v_{\rm esc}(r_{\min}) \equiv J_{\min}$, where $v_{\rm esc}(r_{\min})$ is the escape velocity at $r_{\min}$, $n_{A,X}$ is the density of atomic targets A, and WIMPs X, respectively, v_o is the rms circular velocity of WIMPs in the Galaxy, and the "capture function" $K_A(r, \alpha)$ involves an integral over the WIMP local phase space distribution at the Sun, weighted over the scattering form factor.

The Sun scattering events create a population of solar-system bound WIMPS, moving (for $a \sim 1\,{\rm AU}$) on very elliptic orbits which traverse the Sun over and over again. For the values of WIMP-nuclei cross sections we shall be mostly interested in here (corresponding to effective WIMP-proton cross sections in the range $4 \times 10^{-42} - 4 \times 10^{-41}\,{\rm cm}^2$), the mean opacity of the Sun for orbits with small impact parameters is in the range $10^{-4} - 10^{-3}$. This means that after $10^3 - 10^4$ orbits these WIMPs will undergo a second scattering event in the Sun, and end up in its core where they will ultimately annihilate.

The only way to save some of these WIMPs is to consider a fraction of WIMPs which have impact parameters $r_{\min}$ in a small range near the radius of the Sun R_S. Focussing on such a subpopulation of WIMPs has two advantages: (i) they traverse a small fraction of the mass of the Sun and therefore their lifetime on such grazing orbits is greatly increased, and, *more importantly* (ii) during this time, gravitational perturbations due to the planets can build up and push them on orbits which no longer cross the Sun.

The crucial point to realize is the following. If we consider a WIMP orbit with a generic impact parameter $r_{\min} << R_S$, it will undergo a large perihelion precession $\Delta\omega \sim 2\pi$ per orbit, i.e. $\dot\omega \sim n$, because the potential $U(r)$ *within the Sun* is modified compared to the exterior $1/r$ potential leading to the absence of perihelion motion. In other words, the trajectory of the WIMP will generically be a fast advancing *rosette*. When the rosette motion is fast, planetary perturbations do not induce any secular evolution in the semi-major axis and in the eccentricity of the WIMP orbit. Such WIMPs will end up in the core of the Sun.

A new situation arises for WIMP orbits which graze the Sun, because these orbits feel essentially a $1/r$ potential due to the Sun, so that their rosette motion will be very slow. One can then show that planetary perturbations will in this case alter the eccentricity of orbits so that such orbits will no longer intersect the Sun.

We can finally estimate the density of WIMPs which diffuse out to solar-bound orbits and which can survive to the present time by simply integrating our differential capture rate over all trajectories $J > J_{min} \equiv G_{min}$ which end up out of the Sun, suitably averaged over the initial WIMP distribution. The rate (per $\alpha = G_N m_S/a$) of solar capture of WIMPs which subsequently survive out of the Sun to stay within the inner solar system then depends on the A-dependent combination: $g_A \equiv \frac{f_A}{m_A}\sigma_A K_A^s$, where f_A is the fraction (by mass) of element A in the Sun, and K_A^s is the Sun-surface value of the capture function mentioned above. Note that the A-dependence is entirely contained in g_A with dimensions [cross section]/[mass]. The total capture rate is then dependent on $\sum_A g_A = g_{\rm tot}$. From the point of view of experiment, however, the differential rate, dR/dQ, per keV per kg per day, of scattering events in a laboratory sample made of element A is what counts. Comparing this differential rate due to our new WIMP population with that due to the predicted galactic halo population we find a ratio which reaches the maximum,

$$\rho(Q) \equiv \frac{(dR/dQ)^{\rm new}}{(dR/dQ)^{\rm standard}} = 1.1\, g_{\rm tot}^{(-10)}, \tag{4}$$

for energy transfers Q smaller than $Q_E = 2(m_X/(m_X + m_A))^2 m_A v_E^2$, where $g_{\text{tot}}^{(-10)}$ is the value of g_{tot} when cross sections are expressed in terms of $10^{-10} GeV^{-2}$. For a target of Germanium (the present material of choice in several ongoing cryogenic WIMP detection experiments), Q_E is in the keV range. If $g_{\text{tot}}^{(-10)} \sim 1$ this yields a 100 % increase of the differential event rate below the value $Q = Q_E \sim$ keV which is typical for the energy deposit due to the new WIMP population, whose characteristic velocity $\sim v_E$ is smaller than that of galactic halo WIMPs.

This is our central result. We expect that this is in fact an underestimate of the actual contribution due to WIMPs trapped in the solar system, because we have incorporated in our calculations only the average long-range perturbing effects of the planets. In any case, the relevance of this result depends completely on the actual value of $g_{\text{tot}}^{(-10)}$. Interestingly, if one explores the allowed parameter space of neutralinos in the Minimal Supersymmetric Standard Model, shown for example in Figure 2, for SUSY parameter $\mu > 0$, we find values of $g_{\text{tot}}^{(-10)}$ in excess of 1 for the range of parameters which involve a remnant WIMP density in excess of about 2% of the closure density, and whose scattering rate remains below the sensitivity of current detectors.

Could such a new WIMP population be detectable? Existing detectors tend to lose sensitivity in the range of a few keV. However, these results motivate pushing hard in this direction. In the first place, the new population will have a strongly anisotropic velocity distribution. Not only will this greatly help distinguish it from backgrounds, but a comparison of the annual modulation of any signal from this distribution with the higher energy signal from a halo WIMP distribution would be striking. If neutralinos exist in the range detectable at the next generation of detectors, this new WIMP population must exist at a sizeable level. Finally, we note that the indirect neutrino signature of such WIMPs which might subsequently be captured by the Earth, and annihilate, could be dramatic. The new population has a characteristic velocity which more closely matches the escape velocity from the Earth than does the background halo population. As a result, resonant capture off elements such as Iron in the Earth could be greatly enhanced.

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