The Census Taker's Hat

Leonard Susskind *

School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 130-722, Korea,

and

Department of Physics, Stanford University Stanford, CA 94305-4060, USA

Abstract

If the observable universe really is a hologram, then of what sort? Is it rich enough to keep track of an eternally inflating multiverse? What physical and mathematical principles underlie it? Is the hologram a lower dimensional quantum field theory, and if so, how many dimensions are explicit, and how many "emerge?" Does the Holographic description provide clues for defining a probability measure on the Landscape?

The purpose of this lecture is first, to briefly review a proposal for a holographic cosmology by Freivogel, Sekino, Susskind, and Yeh (FSSY), and then to develop a physical interpretation in terms of a "Cosmic Census Taker:" an idea introduced in reference [1]. The mathematical structure—a hybrid of the Wheeler DeWitt formalism and holography—is a boundary "Liouville" field theory, whose UV/IR duality is closely related to the time evolution of the Census Taker's observations. That time evolution is represented by the renormalization-group flow of the Liouville theory.

Although quite general, the Census Taker idea was originally introduced in [1], for the purpose of counting bubbles that collide with the Census Taker's bubble. The "Persistence of Memory" phenomenon discovered by Garriga, Guth, and Vilenkin, has a natural RG interpretation, as does slow roll inflation. The RG flow and the related C-theorem are closely connected with generalized entropy bounds.

^{*}Lecture given at "String Phenomenology and Cosmology Workshop," KIAS and Yukawa Institute, September 24-28, 2007

1 Introduction

Of all the "String Inspired" cosmological scenarios, only one seems to me to have an element of inevitability to it. The facts and principles that drive it are as follows:

- Observational evidence supports the existence of a period of slow-roll inflation during which the universe exponentially expanded by a factor no less than e^{50} . The universe grew to a size which is at least 1,000 times larger (in volume) than the portion which is observable.
- A small residual vacuum energy of order $10^{-123}M_p^4$ remained at the end of inflation and now dominates the energy density of the universe. If this situation persists, then not only is the universe at least 1,000 larger than what can be seen; it is 1,000 larger than what can **ever** be seen [2].
- String Theory apparently gives rise to an immense Landscape of de Sitter vacua [3][5][4][6] with a very dense "discretuum" of vacuum energies. None of these vacua are absolutely stable: each can decay to vacua with smaller cosmological constant.
- Black Hole (or Observer) Complementarity, [7] [8] [9], and the Holographic Principle, [10] have been confirmed by string theory, at least in a certain wide class of backgrounds [11][12][13]. The implication is twofold. On the one hand, observer complementarity requires the identification of a causal patch; conventional quantum mechanics only makes sense within such a patch. The Holographic Principle requires that a region of space be described by boundary degrees of freedom whose number does not exceed the area, measured in Planck Units.
- Inflation, if it lasts long enough, has a tendency [14] to populate the Landscape with a great diversity of nucleated "pocket universes."

The first two items imply that all of observable cosmology consisted of a roll from one value of the vacuum energy (probably no bigger than $10^{-14}M_p^4$), to its final current value. How and why the universe began with such an unnatural energy density is not explained by any standard theory, but the Landscape suggests the following guess: At some point in the remote past the universe occupied a point on the Landscape with a much higher vacuum energy, perhaps of order one in Planck units. Rolling, unimpeded, to a vacuum

energy of 10^{-14} without getting stuck in a local minimum is unlikely. (Think of rolling a bowling ball from the top of Mount Everest to sea level.) It is far more likely that the universe would get stuck in many minima, and have to tunnel [15] multiple times, before arriving at the very small vacuum energy required by conventional slow-roll inflation. We will not dwell on Anthropic issues in this paper, but I would point out that a long period of conventional inflation appears to be required for structure formation [16]. The argument is similar to the well-known Weinberg argument concerning the cosmological constant.

These considerations strongly suggest that the period of conventional slow-roll inflation was preceded by a tunneling event from a previous neighboring vacuum. In other words, the observed universe evolved by a sudden bubble nucleation from an "Ancestor" vacuum, once removed on the Landscape. It seems obvious that one of the next big questions for cosmology will be to find the theoretical and observational tools to confirm or refute the past existence of an Ancestor, and to find out as much as we can about it. If we are lucky and the amount of slow-roll inflation that followed bubble nucleation is as small as observational evidence allows, then we have a chance of seeing features of the Ancestor imprinted on the sky [16]. The two smoking guns would be:

- Negative spatial curvature: bubble nucleation leads to a negatively curved, infinite,
 FRW universe.
- Tensor modes in the CMB, but only in the lowest harmonics. Although the vacuum energy subsequent to tunneling (during conventional slow-roll inflation) was almost certainly too small to create observable tensor modes, the cosmological constant in the Ancestor was probably much larger. During the Ancestor epoch, large tensor fluctuations would be created by rapid inflation. A tail (diminishing rapidly with l) of those fluctuations could be visible if the number of slow-roll e-foldings is minimal.

If the observational evidence for an Ancestor is weak, so is the current theoretical framework. To many of us, eternal inflation, bubble nucleation, and a multiverse, seem all but inevitable, but it is also true that they have inspired what Bjorken¹ has called "the most extravagant extrapolation in the history of physics." Eternal inflation leads to an uncontrolled infinity of "pocket universes" which we have no good idea how to regulate—the inevitable has led to the preposterous. In my opinion, this situation reflects serious confusion, and perhaps even a crisis.

¹James Bjorken, private communication.

Eternal inflation is not the only extravagance that we have had to tame in recent decades. I have in mind the fact that a naive but very compelling interpretation of black holes seemed, at one time, to imply that a black hole can absorb an infinite amount of information behind its horizon [17]. By feeding a black hole with coherent energy at the same rate that it evaporates, it would seem that an infinity of bits could be lost to the observable world.

I believe these two crises may be related. In both cases the infinities result from "cutting across horizons" and attempting to describe global space-like surfaces with *independent* degrees of freedom at each location. The cure is to focus attention on a single causal region, and to describe it by a Holographic set of degrees of freedom [7] [18].

In FSSY [19] the authors described one such holographic framework—call it holography in a hat—based on mathematical ideas that have become familiar from String Theory. At the same time, Shenker and collaborators [1] have developed an intuitive "gedanken observational" approach based on a fictitious observer called the "Census Taker." My purpose in this lecture is to explain the close connection between these ideas.

2 The Census Bureau

Let us begin with a precise definition of a causal patch. Start with a cosmological spacetime and assume that a future causal boundary exists. For example, in flat Minkowski space the future causal boundary consists of \mathcal{I}^+ (future light like infinity) and a single point, time-like-infinity. For a (non-eternal) Schwarzschild black hole, the future causal boundary has an additional component: the singularity.

A causal patch is defined in terms of a point $\underline{\mathbf{a}}$ on the future causal boundary. I'll call that point the "Census Bureau²." The causal patch is, by definition, the causal past of the Census Bureau, bounded by its past light cone. For Minkowski space, one usually picks the Census Bureau to be time-like-infinity. In that case the causal patch is all of Minkowski space as seen in figure 1.

²This term originated during a discussion between myself and Steve Shenker in a Palo Alto Cafe. Neither of us will admit to having coined it first, but it wasn't me.

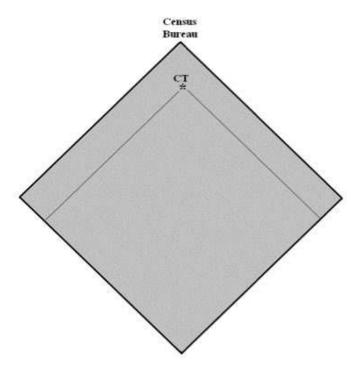


Figure 1: Conformal diagram for ordinary flat Minkowski space. The causal patch associated with the "Census Bureau" is the entire space-time. A Census Taker and his past light-cone are also shown.

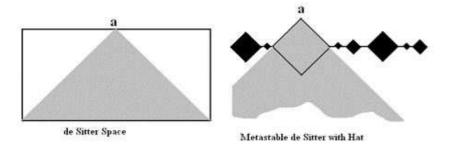


Figure 2: Conformal diagrams for eternal and metastable de Sitter space. The grey areas are causal patches associated with the points **a**. In the metastable case the causal patch is associated with the tip of a hat.

In the case of the Schwarzschild geometry, $\underline{\mathbf{a}}$ can again be chosen to be time-like-infinity, in which case the causal patch is everything outside the horizon of the black hole. There is no clear reason why one can't choose $\underline{\mathbf{a}}$ to be on the singularity, but it would lead to obvious difficulties.

The term "Census Taker" was introduced [1]) to denote an observer, at a point inside a causal patch, who looks back into the past and collects data. He can count galaxies, other observers, hydrogen atoms, colliding bubble-universes, civilizations, or anything else within his own causal past. As time elapses the Census Taker sees more and more of the causal patch. Eventually all Census takers within the causal patch arrive at the Census Bureau where they can compare data.

De Sitter space has the well known causal structure as shown in figure 2. In this case all points at future infinity are equivalent: the Census Bureau can be located at any of them. However, String Theory and other considerations suggest that de Sitter minima are never stable. After a series of tunneling events they eventually end in terminal vacua with exactly zero or negative cosmological constant. The entire distant future of de Sitter space is replaced by a fractal of terminal bubbles.

Decay to negative cosmological constant always leads to a singular crunch. Barring governmental stupidity, this seems an unlikely place for a Census Bureau. The disadvantages (or advantages) of locating a government agency at a crunch are the same as at a black hole singularity.

Terminal vacua with zero cosmological constant—seem more promising; the bubble then evolves to an open, negatively curved, FRW geometry, bounded by a "hat" [19]. The Census Bureau is at the tip of the hat.

In the case of the black hole, the degrees of freedom beyond the horizon, i.e., outside the causal patch, are redundant descriptions of degrees of freedom within the patch: they should not be double-counted. We assume that the same is true of the causal patch of a hat. In both cases the conventional rules of quantum mechanics are expected to apply **only** within the causal patch. Furthermore the rules should respect the Holographic Principle.

The reader may wonder about the relationship between hatted terminal geometries, and observational cosmology with a non-zero cosmological constant. There are two answers: the first is that for many purposes, the current cosmological constant is so small that it can be set to zero. Later we will argue that the conformal field theory description of the approximate hat which results from non-zero cosmological constant is an ultraviolet cut-off version of the type of field theory that describes a hat.

The second answer was emphasized by Shenker, et al. [1] who argued that because our present de Sitter vacuum will eventually decay, a Census Taker can look back into our current vacuum from a point at or near the tip of a hat, and gather information. In principle the Census Taker can peek back, not only into the Ancestor vacuum (our vacuum in this case), but also into bubble collisions with other vacua of the Landscape. Much of this paper is about the gathering of information as the Census Taker's time progresses, and how it is encoded in the renormalization-group (RG) flow of a holographic field theory.

3 Open FRW and Euclidean ADS

The classical space-time in the interior of a Coleman De Luccia bubble, has the form of an open infinite FRW universe, Let \mathcal{H}_3 represent a hyperbolic geometry with constant negative curvature.

$$d\mathcal{H}_3^2 = dR^2 + \sinh^2 R \ d\Omega_2^2. \tag{3.1}$$

The metric of open FRW is

$$ds^{2} = -dt^{2} + a(t)^{2} d\mathcal{H}_{3}^{2}, \tag{3.2}$$

or in terms of conformal time T (defined by dT = dt/a(t))

$$ds^{2} = a(T)^{2}(-dT^{2} + d\mathcal{H}_{3}^{2})$$
(3.3)

Note that in (3.1) the radial coordinate R is a hyperbolic angle and that the symmetry of the spatial sections is the non-compact group O(3,1). This O(3,1) symmetry plays a central role in what follows.

If the vacuum energy in the bubble is zero, i.e., no cosmological constant, then the future boundary of the FRW region is a hat. The scale factor a(t) then has the early and late-time behaviors

$$a(T) \sim t \sim De^T.$$
 (3.4)

_

For early time when $T \to -\infty$ the constant D is conveniently chosen to be the Hubble scale of the Ancestor, H^{-1} .

$$a(T) = H^{-1}e^{T} \qquad (T \to -\infty)$$
(3.5)

At late time it is always larger. In the simplest thin-wall case D is given by the Ancestor Hubble-length at all times.

In figure 3, a conformal diagram of FRW is illustrated, with surfaces of constant T and R shown in red and blue. The green region represents the de Sitter Ancestor vacuum. Figure 4 shows the Census Taker, as he approaches the tip of the hat, looking back along his past light cone.

Part of the inspiration for FSSY was the geometry of the spatial slices of constant T. Each slice, taken by itself, is a three dimensional, negatively curved, hyperbolic plane. It is very familiar to relativists and string theorists, being identical to 3-D Euclidean *anti de Sitter space*. The best way that I know of for becoming familiar with the hyperbolic plane is to study Escher's drawing "Limit Circle IV." It is both a drawing of Euclidean ADS and also a fixed-time slice of open FRW. In figure 5, the green circle is the intersection of Census Takers past light cone with the time-slice. As the Census Taker advances in time, the green circle moves out, ever closer to the boundary.

A fact (to be explained later) which will play a leading role in what follows, concerns the Census Taker's angular resolution, i.e., his ability to discern small angular variation. If the time at which the CT looks back is called T_{CT} , then the smallest angle he can resolve is

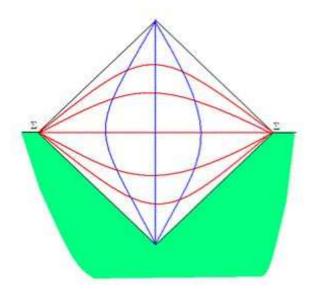


Figure 3: A Conformal diagram for the FRW universe created by bubble nucleation from an "Ancestor" metastable vacuum. The Ancestor vacuum is shown in green. The red and blue curves are surfaces of constant T and R. The two-sphere at spatial infinity is indicated by Σ .

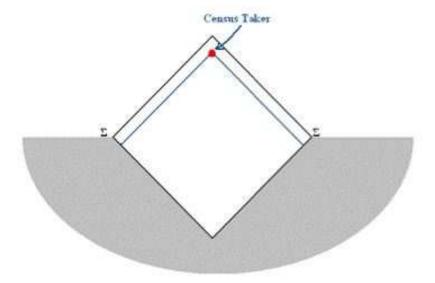


Figure 4: The Census Taker is indicated by the red dot. The blue lines represent his past light-cone.



Figure 5: Escher's drawing of the Hyperbolic Plane, which represents Euclidean anti de Sitter space or a spatial slice of open FRW. The green circle shows the intersection of the Census Taker's past light-cone, which moves toward the boundary with Census-Taker-time.

of order $\exp(-T_{CT})$. It is as if the CT were looking deeper and deeper into the ultraviolet structure of a quantum field theory on Σ .

The boundary of anti de Sitter space plays a key role in the ADS/CFT correspondence, where it represents the extreme ultraviolet degrees of freedom of the boundary theory. The corresponding boundary in the FRW geometry is labeled Σ and consists of the intersection of the hat \mathcal{I}^+ , with the space-like future boundary of de Sitter space. From within the interior of the bubble, Σ represents **space-like infinity.** It is the obvious surface for a holographic description. As one might expect, the O(3,1) symmetry which acts on the time-slices, also has the action of two dimensional conformal transformations on Σ . Whatever the Census Taker sees, it is very natural for him to classify his observations under the conformal group. Thus, the apparatus of (Euclidean) conformal field theory, such as operator dimensions, and correlation functions, should play a leading role in organizing his data.

In complicated situations, such as multiple bubble collisions, Σ requires a precise definition. The asymptotic light-cone \mathcal{I}^+ (which is, of course, the limit of the Census Takers past light cone), can be thought of as being formed from a collection of light-like generators. Each generator, at one end, runs into the tip of the hat, while the other end eventually enters the bulk space-time. The set of points where the generators enter the bulk define Σ .

4 The Holographic Wheeler DeWitt Equation

Supposedly, String Theory is a quantum theory of gravity, and indeed it has proved to be a remarkably powerful one, but only in certain special backgrounds. As effective as it is in describing scattering amplitudes in flat (supersymmetric) space-time, and black holes in anti-de Sitter space, it is an inflexible tool which at present is close to useless for formulating a mathematical framework for cosmology. What is it that is so special about flat and ADS space that allows a rigorous formulation of quantum gravity, and why are cosmological backgrounds so difficult?

The problem is frequently blamed on *time-dependence*. But time-dependent deformations of anti de Sitter space or Matrix Theory are easy to describe. Something else is the culprit. There is one important difference between the usual String Theory backgrounds and more interesting cosmological backgrounds. Asymptotically-flat and anti de

Sitter backgrounds have a property that I will call *asymptotic coldness*. Asymptotic coldness means that the boundary conditions require the energy density to go to zero at the asymptotic boundary of space³. Similarly, the fluctuations in geometry tend to zero. This condition is embodied in the statement that all physical disturbances are composed of normalizable modes. Asymptotic coldness is obviously important to defining an S-matrix in flat space-time, and plays an equally important role in defining the observables of anti de Sitter space.

But in cosmology, asymptotic coldness is never the case. Closed universes have no asymptotic boundary, and homogeneous infinite universes have matter, energy, and geometric variation out to spatial infinity; under the circumstances an S-matrix cannot be formulated. String theory at present is ill equipped to deal with *asymptotically warm* geometries. To put it another way, there is a conflict between a homogeneous cosmology, and the Holographic Principle which requires an isolated, cold, boundary.

The traditional approach to quantum cosmology—the Wheeler-DeWitt equation—is the opposite of string theory; it is very flexible from the point of view of background dependence—it doesn't require any definite boundary condition, it can be formulated for a closed universe, a flat or open FRW universe, de Sitter space, or for that matter, flat and anti de Sitter space spacetime—but it is not a consistent quantum theory of gravity. It is based on an obsolete approach—local quantum field theory—that fails to address the problems that String Theory and the Holographic Principle were designed to solve: the huge overcounting of degrees of freedom implicit in a local field theory.

FSSY suggested a way out of dilemma: synthesize the Wheeler-DeWitt philosophy with the Holographic Principle to construct a Holographic Wheeler-DeWitt theory. We will begin with a review of the basics of conventional WDW; For a more complete treatment, especially of infinite cosmologies, see [24].

The ten equations of General Relativity take the form

$$\frac{\delta}{\delta g_{\mu\nu}}I = 0 \tag{4.1}$$

³Note that asymptotic coldness refers only to conditions at spatial infinity. A violation of asymptotic coldness does not imply that the temperature remains finite as the time goes to infinity, although even this is a problem in geometries that contain de Sitter boundary conditions. Hats are somewhat better in that they become cold at late time.

where I is the Einstein action for gravity coupled to matter. The canonical formulation of General Relativity makes use of a time-space split [23]. The six space-space components are more or less conventional equations of motion, but the four equations involving the time index have the form of constraints. These four equations are written,

$$H^{\mu}(x) = 0. \tag{4.2}$$

They involve the space-space components of the metric g_{nm} , the matter fields Φ , and their conjugate momenta. The time component $H^0(x)$, is a local Hamiltonian which "pushes time forward" at the spatial point x. More generally, if integrated with a test function,

$$\int d^3x \ f(x) \ H^0(x) \tag{4.3}$$

it generates infinitesimal transformations of the form

$$t \to t + f(x). \tag{4.4}$$

Under certain conditions H^0 can be integrated over space in order to give a global Hamiltonian description. Since H^0 involves second space derivatives of g_{nm} , it is necessary to integrate by parts in order to bring the Hamiltonian to the conventional form containing only first derivatives. In that case the ADM equations can be written as

$$\int d^3x \ H = E. \tag{4.5}$$

The Hamiltonian density H has a conventional structure, quadratic in canonical momenta, and the energy E is given by a Gaussian surface integral over spatial infinity. The conditions which allow us to go from (4.2) to (4.5) are satisfied in asymptotically cold flat-spacetime, as well as in anti de Sitter space; in both cases global Hamiltonian formulations exist. Indeed, in anti de Sitter space the Hamiltonian of the Holographic boundary description is identified with the ADM Energy, but, as we noted, cosmology, at least in its usual forms, is never asymptotically cold. The only recourse for a canonical description, is the local form of the equations (4.2).

When we pass from classical gravity to its quantum counterpart, the usual generalization of the canonical equations (4.2) become the Wheeler DeWitt equations,

$$H^{\mu}|\Psi\rangle = 0 \tag{4.6}$$

where the state vector $|\Psi\rangle$ is represented by a wave functional that depends only on the space components of the metric g_{mn} , and the matter fields Φ .

The first three equations

$$H^m|\Psi\rangle = 0 \quad (m = 1, 2, 3.)$$
 (4.7)

have the interpretation that the wave function is invariant under spatial diffeomorphisms,

$$x^n \to x^n + f^n(x^m) \tag{4.8}$$

In other words $\Psi(g_{mn}, \Phi)$ is a function of spatial invariants. These equations are usually deemed to be the easy Wheeler-DeWitt equations.

The difficult equation is the time component

$$H^0|\Psi\rangle = 0. (4.9)$$

It represents invariance under local, spatially varying, time translations. Not only is equation (4.9) difficult to solve; it is difficult to even formulate: the expression for H^0 is riddled with factor ordering ambiguities. Nevertheless, as long as the equations are not pushed into extreme quantum environments, they can be useful.

4.1 Wheeler DeWitt and the Emergence of Time

Asymptotically cold backgrounds come equipped with a global concept of time. But in the more interesting asymptotically warm case, time is an approximate, derived, concept [24, 25], which emerges from the solutions to the Wheeler-DeWitt equation. The perturbative method for solving (4.9) that was outlined in [24], can be adapted to the case of negative spatial curvature. We begin by decomposing the spatial metric into a constant curvature background, and fluctuations. Since we will focus on open FRW cosmology, the spatial curvature is negative, the space metric having the form,

$$ds^{2} = a^{2} \left(dR^{2} + \sinh^{2} R \left(d\theta^{2} + \sin^{2} \theta d\phi^{2} \right) \right) + a^{2} h_{mn} dx^{m} dx^{n}$$
(4.10)

In (4.10) a is the usual FRW scale factor and the x's are (R, θ, ϕ) .

The first approximation, in which all fluctuations are ignored, is usually called the minisuperspace approximation, but it really should be seen as a first step in a semiclassical expansion. In lowest order, the Wheeler DeWitt wave function depends only on the scale factor a. To carry out the leading approximation in open FRW it is necessary to introduce an infrared regulator which can be done by bounding the value of R,

$$R < R_0 \ (R_0 >> 1).$$
 (4.11)

Lets also define the total dimensionless coordinate-volume within the cutoff region, to be V_0 .

$$V_0 = 4\pi \int dR \sinh^2 R \approx \frac{1}{2} \pi e^{2R_0}.$$
 (4.12)

The first (mini-superspace) approximation is described by the action,

$$L = \frac{-aV_0\dot{a}^2 - V_0a}{2} \tag{4.13}$$

Defining P to be the momentum conjugate to the scale factor a,

$$P = -aV_0\dot{a} \tag{4.14}$$

the Hamiltonian H^0 is given by 4 ,

$$H^{0} = \frac{1}{2V_{0}} P \frac{1}{a} P + \frac{1}{2} V_{0} a \tag{4.15}$$

Finally, using $P = -i\partial_a$, the first approximation to the Wheeler-DeWitt equation becomes,

$$-\partial_a \frac{1}{a} \partial_a \Psi - V_0^2 a \Psi = 0. \tag{4.16}$$

The equation has the two solutions,

$$\Psi = \exp(\pm iV_0 a^2),\tag{4.17}$$

corresponding to expanding and contraction universes; to see which is which we use (4.14). The expanding solution, labeled Ψ_0 is

$$\Psi_0 = \exp(-iV_0 a^2). \tag{4.18}$$

From now on we will only consider this branch.

 $^{^4}$ The factor ordering in the first term is ambiguous. I have chosen the simplest Hermitian factor ordering

There is something funny about (4.18). Multiplying V_0 by a^2 seems like an odd operation. V_0a^3 is the proper volume, but what is V_0a^2 ? The answer in flat space is that it is junk, but in hyperbolic space its just the proper area of the boundary at R_0 . One sees from the metric (3.3) that the coordinate volume V_0 , and the coordinate area A_0 , of the boundary at R_0 , are (asymptotically) equal to one another, to within a factor of 2.

$$A_0 = 2V_0. (4.19)$$

Thus the expression in the exponent in (4.18) is $-\frac{1}{2}iA$, where A is the proper area of the boundary at R_0 .

$$\Psi_0 = \exp(-2iA). \tag{4.20}$$

This is a suggestive indication of a Wheeler-DeWitt boundary-holography of open FRW.

To go beyond the mini-superspace approximation one writes the wave function as a product of Ψ_0 , and a second factor $\psi(a, h, \Phi)$ that depends on the fluctuations.

$$\Psi(a, h, \Phi) = \Psi_0 \quad \psi(a, h, \Phi) = \exp(-iV_0 a^2) \quad \psi(a, h, \Phi). \tag{4.21}$$

By integrating the Wheeler-DeWitt equation over space, and substituting (4.21), an equation for ψ can be obtained.

$$i\partial_a \psi + \frac{1}{aV_0} \partial_a \frac{1}{a} \partial_a \psi = H_m \psi \tag{4.22}$$

In this equation H_m has the form of a conventional Hamiltonian (quadratic in the momenta) for both matter and metric fluctuations.

In the limit of large scale factor the term $\frac{1}{aV_0}\partial_a \frac{1}{a}\partial_a \psi$ becomes negligible and (4.22) takes the form of a Schrodinger equation.

$$i\partial_a \psi = H_m \psi \tag{4.23}$$

Evidently the role of a is not as a conventional observable, but a parameter representing the unfolding of cosmic time. One does not calculate its probability, but instead constrains it—perhaps with a delta function or a Lagrange multiplier. As Banks has emphasized [25], in this limit, and maybe only in this limit, the wave function ψ has a conventional interpretation as a probability amplitude.

4.2 Holographic WDW

All of this brings us to the central question of this lecture: what form does the correct holographic theory take in asymptotically warm cosmological backgrounds? The answer suggested in FSSY was a holographic version of the Wheeler-DeWitt theory, living on the space-like boundary Σ .

As we have described it, the Wheeler-DeWitt theory is a throwback to an older view of quantum gravity based on the existence of bulk, space-filling degrees of freedom. It has become clear that this is a tremendous overestimate of the capacity of space to contain quantum information. The correct (holographic) counting of degrees of freedom is in terms of the area of the boundary of space [10]. In the present case of open FRW, the special role of the boundary is played by the surface Σ at $R = \infty$.

Just as in the ADS/CFT correspondence [13], it is useful to define a regulated boundary, Σ_0 , at $R = R_0$. In principle R_0 can depend on angular location on Ω_2 . In fact later we will discuss invariance under gauge transformations of the form

$$R \to R + f(\Omega_2).$$
 (4.24)

(the notation $f(\Omega_2)$). indicating that f is a function of location on Σ_0 .)

The conjecture of FSSY is that the correct Holographic description of open FRW is a Wheeler-DeWitt equation, but one in which the degrees of freedom are at the boundary of space, i.e., on Σ , instead of being distributed throughout the bulk.

Thus we assume the existence of a set of boundary fields, that include a two dimensional spatial metric on Σ_0 . The induced spatial geometry of the boundary can always be described in the conformal gauge in terms of a Liouville field $U(\Omega_2)$.

$$ds^{2} = e^{2U(\Omega_{2})}e^{2R_{0}(\Omega_{2})}d\Omega_{2}^{2}$$
(4.25)

U may be decomposed into a homogeneous term U_0 , and a fluctuation; obviously the homogeneous term can be identified with the FRW scale factor by

$$e^{U_0} = a. (4.26)$$

In section 8 we will give a more detailed definition of the Liouville degree of freedom.

In addition we postulate a collection of boundary "matter" fields. The boundary matter fields, y, are not the limits of the usual bulk fields Φ , but are analogous to the boundary gauge fields in the ADS/CFT correspondence. In this paper we will not speculate on the detailed form of these boundary matter fields.

4.3 The Wave Function

In addition to U and y, we assume a local Hamiltonian $H(x^i)$ that depends only on the boundary degrees of freedom (the notation x^i refers to coordinates of the boundary Σ), and a wave function $\Psi(U, y)$,

$$\Psi(U,y) = e^{-\frac{1}{2}S + iW}. (4.27)$$

At every point of Σ , Ψ satisfies

$$H(x^i) \ \Psi(U, y) = 0$$
 (4.28)

In equation (4.27), S(U, y) and W(U, y) are real functionals of the boundary fields. For reasons that will become clear, we will call S the action. However, S should not in any way be confused with the four-dimensional Einstein action.

The local Hamiltonian $H(x^i)$, and the imaginary term W in the exponent, play important roles in determining the expectation values of canonical momenta, as well as the relation between scale factor and ordinary time. In this paper H and W will play secondary roles.

We make the following three assumptions about S and W:

- Both S and W are invariant under conformal transformations of Σ . This follows from the symmetry of the background geometry: open FRW.
- The leading (non-derivative) term in the regulated form of W is $-\frac{1}{2}A$ where A is the proper area of Σ_0 ,

$$W = -\frac{1}{2} \int_{\Sigma} e^{2R_0} e^{2U} + \dots {4.29}$$

This follows from (4.20).

• S and W have the form of **local** two dimensional Euclidean actions on Σ . In other words they are integrals, over Σ , of densities that involve U, y, and their derivatives with respect to x^i .

The first of these conditions is just a restatement of the symmetry of the Coleman De Luccia instanton. Later we will see that this symmetry is spontaneously broken by a number of effects, including the extremely interesting "Persistence of Memory" discovered in by Garriga, Guth and Vilenkin [20].

The second condition follows from the bulk analysis described earlier in equation (4.20). It allows us to make an educated guess about the dependence of the local Hamiltonian $H(x^i)$ on U. A simple form that reproduces 4.20 is

$$H(x) = \frac{1}{2}e^{-2U}\pi_U^2 - 2e^{2U} + \dots {4.30}$$

where π_U is the momentum conjugate to U. It is easily seen that the solution to the equation $H\Psi = 0$ has the form (4.20).

The highly nontrivial assumption is the third item—the locality of the action. As a rule quantum field theory wave functions are not local in this sense. That the action S is local is far from obvious. In our opinion it is the strongest (meaning the weakest) of our assumptions and the one most in need of confirmation. At present our best evidence for the locality is the discrete tower of correlators, including a transverse, traceless, dimension-two correlation function, described in the next section. In principle, much more information can be obtained from bulk multi-point functions, continued to Σ . For example, correlation functions of h_{ij} would allow us to study the operator product expansion of the energy-momentum tensor.

As I said, the assumption that S is local is a very strong one, but I mean it in a rather weak sense. One of the main points of this lecture is that there is a natural RG flow in cosmology (see section 6). By locality I mean only that S is in the basin of attraction of a local field theory. If it is true, locality would imply that the measure

$$\Psi^*\Psi = e^{-S} \tag{4.31}$$

has the form of a local two dimensional Euclidean field theory with action S, and that the Census Taker's observations could be organized not only by conformal invariance but by conformal field theory.

5 Data

The conjectured locality of the action S is based on data calculated by FSSY. The background geometry studied in [19] was the Minkowski continuation of a thin-wall Coleman De Luccia instanton, describing transitions from the Ancestor vacuum to a hatted vacuum. For a number of reasons such a background cannot be a realistic description of cosmology. First of all, there is a form of spontaneous breaking of the O(3,1) symmetry that Garriga, Guth, and Vilenkin call "The Persistence of Memory." In section 8 we will see that these type of effects are "dual" to effects expected in the theory of RG-flows.

More importantly, we do not live in a universe with zero cosmological constant. Observational cosmology has come close to ruling out vanishing cosmological constant, but also theoretical considerations rule it out; in the Landscape of String Theory the only vacua with exactly vanishing cosmological constant are supersymmetric. Nevertheless, hatted geometries are interesting in that they are the simplest versions of asymptotically warm geometries.

In FSSY, correlation functions were computed in the thin-wall, Euclidean, Coleman De Luccia instanton, and then continued to Minkowski signature. The more general situation, including the possibility of slow-roll inflation after tunneling, is presently under investigation with Ben Freivogel, Yasuhiro Sekino, and Chen Pin Yeh. Here we will mostly confine ourselves to the thin-wall case.

We begin by reviewing some facts about three-dimensional hyperbolic space and the solutions of its massless Laplace equation. An important distinction is between normalizable modes (NM) and non-normalizable modes (NNM); a scalar minimally coupled field χ is sufficient to illustrate the important points.

The norm in hyperbolic space is defined in the obvious way:

$$\langle \chi | \chi \rangle = \int dR d\Omega_2 \ \chi^2 \ \sinh^2 R$$
 (5.1)

In flat space, fields that tend to a constant at infinity are on the edge on normalizability. With the help of the delta function, the concept of normalizability can be generalized to continuum-normalizability, and the constant "zero mode" is included in the spectrum of the wave operator, but in hyperbolic space the normalization integral (5.1) is exponentially divergent for constant χ . The condition for normalizability is that $\chi \to 0$ at least as fast as e^{-R} . The constant mode is therefore non-normalizable.

Normalizable and non-normalizable modes have very different roles in the conventional ADS/CFT correspondence. NM are dynamical excitations with finite energy and can be produced by events internal to the anti de Sitter space. By contrast NNM cannot be excited dynamically. Shifting the value of a NNM is equivalent to changing the boundary conditions from the bulk point of view, or changing the Lagrangian from the boundary perspective. But, as we will see, in the cosmological framework of FSSY, asymptotic warmness blurs this distinction.

5.1 Scalars

Correlation functions of massless (minimally coupled) scalars, χ , depend on time and on the **dimensionless** geodesic distance between points on \mathcal{H}_3 . In the limit in which the points tend to the holographic boundary Σ at $R \to \infty$, the geodesic distance between points 1 and 2 is given by,

$$l = R_1 + R_2 + \log(1 - \cos\alpha) \tag{5.2}$$

where α is the angular distance on Ω_2 between 1 and 2. It follows on O(3,1) symmetry grounds that the correlation function $\langle \chi(1)\chi(2)\rangle$ has the form,

$$\langle \chi(1)\chi(2)\rangle = G(T_1, T_2, l_{1,2})$$

= $G\{T_1, T_2, (R_1 + R_2 + \log(1 - \cos\alpha))\}.$ (5.3)

Before discussing the data on the Coleman De Luccia background, let us consider the form of correlation functions for scalar fields in anti de Sitter space. We work in units in which the radius of the anti de Sitter space is 1. By symmetry, the correlation function can only depend on l, the proper distance between points. The large-distance behavior of the two-point function has the form

$$\langle \chi(1)\chi(2)\rangle \sim \frac{e^{-(\Delta-1)l}}{\sinh l}.$$
 (5.4)

In anti de Sitter space the dimension Δ is related to the mass of χ by

$$\Delta(\Delta - 2) = m^2. (5.5)$$

We will be interested in the limit in which the two points 1 and 2 approach the boundary at $R \to \infty$. Using (5.2) gives

$$\langle \chi(1)\chi(2)\rangle \sim e^{-\Delta R_1}e^{-\Delta R_2}(1-\cos\alpha)^{-\Delta}.$$
 (5.6)

It is well known that the "infrared cutoff" R, in anti de Sitter space, is equivalent to an ultraviolet cutoff in the boundary Holographic description [13]. The exponential factors, $\exp(-\Delta R)$ in (5.6) correspond to cutoff dependent wave function renormalization factors and are normally stripped off when defining boundary correlators. The remaining factor, $(1 - \cos \alpha)^{-\Delta}$ is the conformally covariant correlation function of a boundary field of dimension Δ .

In FSSY it was claimed that in the Coleman De Luccia background, the correlation function contains two terms, one of which was associated with NM and the other with NNM. A third term was found, but ignored on the basis that it was negligible when continued to the boundary. In fact the third term has an interesting significance that we will come back to, but first we will review the terms studied in FSSY.

In [19] the correlation function was expressed as a sum of two contour integrals on the k plane–k being an eigenvalue of the Laplacian on \mathcal{H}_3 . The integral involves a certain reflection coefficient $\mathcal{R}(k)$ for a Schrodinger equation, derived from the wave equation on the Coleman De Luccia instanton. The contour integral is

$$e^{-(T_1+T_2)} \oint_C \frac{dk}{2i} \mathcal{R}(k) e^{-ik(T_1+T_2)} \frac{\left(e^{-ikR} - e^{-ikR-2\pi k}\right)}{2\sinh R \sinh k\pi}$$
 (5.7)

The contours of integration are shown in Figure 6. The integrand has poles at all imaginary values of k, with a double pole at k = i. In addition there may be other singularities in the lower half plane. FSSY studied only the terms coming from the upper contour labeled \mathbf{a} in the figure. It contains two terms related to NM and NNM respectively.

The normalizable contribution, G_1 , is an infinite sum, each term having the form (5.6) with T-dependent coefficients. For late times,

$$G_{1} = \sum_{\Delta=2}^{\infty} G_{\Delta} e^{(\Delta-2)(T_{1}+T_{2})} \frac{e^{-(\Delta-1)l}}{\sinh l}$$

$$\to \sum_{\Delta=2}^{\infty} G_{\Delta} e^{(\Delta-2)(T_{1}+T_{2})} e^{-\Delta R_{1}} e^{-\Delta R_{2}} (1-\cos\alpha)^{-\Delta}$$
(5.8)

where Δ takes on integer values from 2 to ∞ , and G_{Δ} are a series of constants which depend on the detailed CDL solution.

The connection with conformal field theory correlators is obvious; equation (5.8) is a sum of correlation functions for fields of definite dimension Δ , but with coefficients which

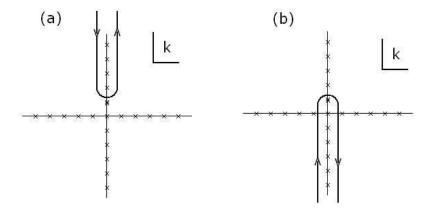


Figure 6: Contours of integration for the two contributions G_1 , G_2 .

depend on the time T. (It should be emphasized that the dimensions Δ in the present context are not related to bulk four dimensional masses by (5.5).) Note that the sum in (5.8) begins at $\Delta = 2$, implying that every term falls at least as fast as $\exp(-2R)$ with respect to either argument. Thus every term is normalizable.

Let us now extrapolate (5.8) to the surface Σ . Σ can be reached in two ways—the first being to go out along a constant T surface to $R = \infty$. Each term in the correlator has a definite R dependence which identifies its dimension.

Another way to get to Σ is to first pass to light-like infinity, \mathcal{I}^+ , and then slide down the hat, along a light-like generator, until reaching Σ . For this purpose it is useful to define light-cone coordinates, $T^{\pm} = T \pm R$.

$$G_1 = e^{-(T_1^+ + T_2^+)} \sum_{\Lambda} G_{\Lambda} e^{(\Lambda - 1)(T_1^- + T_2^-)} (1 - \cos \alpha)^{-\Lambda}$$
 (5.9)

We note that apart from the overall factor $e^{-(T_1^+ + T_2^+)}$, G_1 depends only on T^- , and therefore tends to a finite limit on \mathcal{I}^+ . If we strip that factor off, then the remaining expression consists of a sum over CFT correlators, each proportional to a fixed power of e^{T^-} . In the limit $(T^- \to -\infty)$ in which we pass to Σ , each term of fixed dimension tends to zero as $e^{(\Delta-1)(T_1^- + T_2^-)}$ with the dimension-2 term dominating the others.

The second term in the scalar correlation function discussed by FSSY consists of a single term,

$$G_2 = \frac{e^l}{\sinh l} (T_1 + T_2 + l) \to \left\{ T_1^+ + T_2^+ + \log(1 - \cos \alpha) \right\}$$
 (5.10)

The contribution (5.10) does not have the form of a correlator of a conformal field of definite dimension. To understand its significance, consider a canonical massless scalar field in two dimensions. On a two sphere the correlation function is ultraviolet divergent and has the form

$$\log\left\{\kappa^2(1-\cos\alpha)\right\} \tag{5.11}$$

where κ is the ultraviolet regulator momentum. If the regulator momentum varies with location on the sphere–for example in the case of a lattice regulator with a variable lattice spacing–formula (5.11) is replaced by

$$\log\left\{ (1 - \cos\alpha) \right\} + \log\kappa_1 + \log\kappa_2 \tag{5.12}$$

Evidently if we identify the UV cutoff κ with T^+ ,

$$\log \kappa = T^+ \tag{5.13}$$

the expressions in (5.10) and (5.12) are identical. The relation (5.13) is one of the central themes of this paper, that as we will see, relates RG flow to the observations of the Census Taker.

That the UV cutoff of the 2D boundary theory depends on R is very familiar from the UV/IR connection [13] in anti de Sitter space. In that case the T coordinate is absent and the log of the cutoff momentum in the conformal field theory would just be R. The additional time dependent contribution in (5.13) will become clear later when we discuss the Liouville field.

The logarithmic ultraviolet divergence in the correlator is a signal that massless 2D scalars are ill defined; the well-defined quantities being derivatives of the field. When calculating correlators of derivatives, the cutoff dependence disappears. Thus for practical purposes, the only relevant term in (5.12) is $\log (1 - \cos \alpha)$.

The existence of a dimension-zero scalar field on Σ is a surprise. It is obviously associated with bulk field-modes which don't go zero for large R. Such modes are non-normalizable on the hyperbolic plane, and are usually not included among the dynamical variables in anti de Sitter space.

In String Theory the only massless scalars in the hatted vacua would be moduli, which are expected to be "fixed" in the Ancestor. For that reason FSSY considered the effect of adding a four-dimensional mass term, $\mu\chi^2$, in the Ancestor vacuum. The result on the boundary scalar was to shift its dimension from $\Delta = 0$ to $\Delta = \mu$ (for small μ less than the Ancestor Hubble constant the corresponding mode stays non-normalizable). However the correlation function was not similar to those in G_1 , each term of which had a dependence on T^- . The dimension μ term depends only on T^+ :

$$G_2 \to e^{-\mu T_1^+} e^{-\mu T_2^+} \log (1 - \cos \alpha)^{-\mu}$$
 (5.14)

The two terms, (5.9) and (5.14) depend on different combinations of the coordinates, T^+ and T^- . It seems odd that there is one and only one term that depends solely on T^+ and all the rest depend on T^- . In fact the only reason is that FSSY ignored an entire tower of higher dimension terms, coming from the contour **b** that, like (5.14), depend only on T^+ . From now on we will group all terms independent of T^- into the single expression G_2 :

$$G_2 = \sum_{\Delta'} G_{\Delta'} e^{-(\Delta')(T_1^+ + T_2^+)} (1 - \cos \alpha)^{-\Delta'}$$
 (5.15)

The Δ' include μ , the positive integers and whatever other poles appear for ik < 1. In the case $\mu = 0$, the leading term in G_2 is (5.10).

We will return to the two terms G_1 and G_2 in section 6.5.

5.2 Metric Fluctuations

To prove that there is a local field theory on Σ , the most important test is the existence of an energy-momentum tensor. In the ADS/CFT correspondence, the boundary energy-momentum tensor is intimately related to the bulk metric fluctuations. We assume a similar connection between bulk and boundary fields in the present context. In FSSY, metrical fluctuations were studied in a particular gauge which we will call the **Spatially Transverse-Traceless** (STT) gauge. The coordinates of region I can be divided into FRW time, T, and space x^m where m = 1, 2, 3. The STT gauge for metric fluctuations is defined by

$$\nabla^m h_{mn} = 0$$

$$h_m^m = 0 (5.16)$$

In the second of equations (5.16), the index is raised with the aid of the background metric (3.3). The main benefit of the STT gauge is that metric fluctuations satisfy minimally coupled, massless, scalar equations, and the correlation functions are similar to G_1 and G_2 . However the index structure is rather involved. We define the correlator,

$$\langle h_{\nu}^{\mu} h_{\tau}^{\sigma} \rangle = G \begin{Bmatrix} \mu \sigma \\ \nu \tau \end{Bmatrix}$$

$$= G_1 \begin{Bmatrix} \mu \sigma \\ \nu \tau \end{Bmatrix} + G_2 \begin{Bmatrix} \mu \sigma \\ \nu \tau \end{Bmatrix}. \tag{5.17}$$

The complicated index structure of G was worked out in detail in FSSY. In this paper we quote only the results of interest–in particular those involving elements of $G \begin{Bmatrix} \mu \sigma \\ \nu \tau \end{Bmatrix}$ in which all indecies lie in the two-sphere Ω_2 . Thus we consider the correlation function $G_1 \begin{Bmatrix} ik \\ jl \end{Bmatrix}$.

As in the scalar case, G_1 consists of an infinite sum of correlators, each corresponding to a field of dimension $\Delta = 2, 3, 4, ...$ The asymptotic T and R dependence of the terms is identical to the scalar case, and the first term has $\Delta = 2$. This is particularly interesting because it is the dimension of the energy-momentum tensor of a two-dimensional boundary conformal field theory. Once again this term is also time-independent.

After isolating the dimension-two term and stripping off the factors $\exp(-2R)$, the resulting correlator is called $G_1\left\{\begin{smallmatrix}ik\\jl\end{smallmatrix}\right\}|_{\Delta=2}$. The calculations of FSSY revealed that this term is two-dimensionally traceless, and transverse.

$$G_{1} \begin{Bmatrix} ik \\ il \end{Bmatrix} |_{\Delta=2} = G_{1} \begin{Bmatrix} ik \\ jk \end{Bmatrix} |_{\Delta=2} = 0$$

$$\nabla_{i} G_{1} \begin{Bmatrix} ik \\ jl \end{Bmatrix} |_{\Delta=2} = 0.$$
(5.18)

Equation (5.18) is the clue that, when combined with the dimension-2 behavior of $G_1 \begin{Bmatrix} ik \\ il \end{Bmatrix}|_{\Delta=2}$, hints at a local theory on Σ . It insures that it has the precise form of a two-point function for an energy-momentum tensor in a conformal field theory. The only ambiguity is the numerical coefficient connecting $G_1 \begin{Bmatrix} ik \\ jl \end{Bmatrix}|_{\Delta=2}$ with $\langle T_j^i T_l^k \rangle$. We will return to this coefficient momentarily.

The existence of a transverse, traceless, dimension-two operator is a necessary condition for the boundary theory on Σ to be local: at the moment it is our main evidence. But there is certainly more that can be learned by computing multipoint functions. For example, from the three-point function $\langle hhh \rangle$ it should be possible verify the operator product expansion and the Virasoro algebra for the energy-momentum tensor.

Dimensional analysis allows us to estimate the missing coefficient connecting the metric fluctuations with T_j^i , and at the same time determine the central charge c. In [19] we found c to be of order the horizon entropy of the Ancestor vacuum. We repeat the argument here:

Assume that the (bulk) metric fluctuation h has canonical normalization, i.e., it has bulk mass dimension 1 and a canonical kinetic term. Either dimensional analysis or explicit calculation of the two point function $\langle hh \rangle$ shows that it is proportional to square of the Ancestor Hubble constant.

$$\langle hh \rangle \sim H^2.$$
 (5.19)

Knowing that the three point function $\langle hhh \rangle$ must contain a factor of the gravitational coupling (Planck Length) l_p , it can also be estimated by dimensional analysis.

$$\langle hhh \rangle \sim l_p H^4.$$
 (5.20)

Now assume that the 2D energy-momentum tensor is proportional to the boundary dimensiontwo part of h, i.e., the part that varies like e^{-2R} . Schematically,

$$T = qh (5.21)$$

with q being a numerical constant. It follows that

$$\langle TT \rangle \sim q^2 H^2$$

 $\langle TTT \rangle \sim q^3 l_p H^4.$ (5.22)

Lastly, we use the fact that the ratio of the two and three point functions is parametrically independent of l_p and H because it is controlled by the classical algebra of diffeomorphisms: [T, T] = T. Putting these elements together we find,

$$\langle TT \rangle \sim \frac{1}{l_p^2 H^2}$$
 (5.23)

Since we already know that the correlation function has the correct form, including the short distance singularity, we can assume that the right hand side of (5.23) also gives the central charge. It can be written in the rather suggestive form:

$$c \sim Area/G \quad (G = l_n^2) \tag{5.24}$$

where Area refers to the horizon of the Ancestor vacuum. In other words, the central charge of the hypothetical CFT is proportional to the horizon entropy of the Ancestor.

5.3 Dimension Zero Term

The term $G_2 \begin{Bmatrix} ik \\ jl \end{Bmatrix}$ begins with a term, which like its scalar counterpart, has a non-vanishing limit on Σ . It is expressed in terms of a standard 2D bi-tensor $t \begin{Bmatrix} ik \\ jl \end{Bmatrix}$ which is traceless and transverse in the two dimensional sense. If the correlation function were given just by $t \begin{Bmatrix} ik \\ jl \end{Bmatrix}$, it would be a pure gauge artifact. One can see this by considering the linearized expression for the 2D curvature-scalar C,

$$C = \nabla_i \nabla_j h^{ij} - 2\nabla^i \nabla_i Tr h. \tag{5.25}$$

The 2D curvature associated with a traceless transverse fluctuation vanishes, and since $t \begin{Bmatrix} ik \\ jl \end{Bmatrix}$ by itself is traceless-transverse with respect to both points, it would be pure gauge if it appeared by itself.

However, the actual correlation function $G_2 \begin{Bmatrix} ik \\ jl \end{Bmatrix}$ is given by

$$G_2 \begin{Bmatrix} ik \\ il \end{Bmatrix} = t \begin{Bmatrix} ik \\ il \end{Bmatrix} \{ R_1 + T_1 + R_2 + T_2 + \log(1 - \cos\alpha) \}$$
 (5.26)

The linear terms in R+T, being proportional to $t \begin{Bmatrix} ik \\ jl \end{Bmatrix}$ are pure gauge, but the finite term

$$t \begin{Bmatrix} ik \\ jl \end{Bmatrix} \log(1 - \cos \alpha) \tag{5.27}$$

gives rise to a non-trivial 2D curvature-curvature correlation function of the form

$$\langle CC\rangle = (1 - \cos \alpha)^{-2}. \tag{5.28}$$

One difference between the metric fluctuation h, and the scalar field χ , is that we cannot add a mass term for h in the Ancestor vacuum to shift its dimension.

Finally, as in the scalar case, there is a tower of higher dimension terms in the tensor correlator, $G_2 \begin{Bmatrix} ik \\ jl \end{Bmatrix}$ that only depend on T^+ .

The existence of a zero dimensional term in $G_2\left\{\substack{ik\\jl}\right\}$, which remains finite in the limit $R\to\infty$ indicates that fluctuations in the boundary geometry–fluctuations which are due to the asymptotic warmness–cannot be ignored. One might expect that in some way these fluctuations are connected with the field U that we encountered in the Holographic version of the Wheeler-DeWitt equation. In the next section we will elaborate on this connection.

That is the data about correlation functions on the boundary sphere Σ that form the basis for our conjecture that there exists a local holographic boundary description of the

open FRW universe. There are a number of related puzzles that this data raises: First, how does time emerge from a Euclidean QFT? The bulk coordinate R can be identified with scale size just as in ADS/CFT but the origin of time requires a new mechanism.

The second puzzle concerns the number of degrees of freedom in the boundary theory. The fact that the central charge is the entropy of the Ancestor suggests that there are only enough degrees of freedom to describe the false vacuum and not the much large number needed for the open FRW universe at late time.

6 Liouville Theory

6.1 Breaking Free of the STT Gauge

The existence of a Liouville sector describing metrical fluctuations on Σ seems dictated by both the Holographic Wheeler-DeWitt theory and from the data of the previous section. It is clear that the Liouville field is somehow connected with the non-normalizable metric fluctuations whose correlations are contained in (5.26), although the connection is somewhat obscured by the choice of gauge in [19]. In the STT gauge the fluctuations h are traceless, but not transverse (in the 2D sense). From the viewpoint of 2D geometry they are not pure gauge as can be seen from the fact that the 2D curvature correlation does not vanish. One might be tempted to identify the Liouville mode with the zero-dimension piece of (5.26). To do so would of course require a coordinate transformation on Ω_2 in order to bring the fluctuation h_i^j to the "conformal" form $\tilde{h}\delta_i^j$.

This identification may be useful but it is not consistent with the Wheeler-DeWitt philosophy. The Liouville field U that appears in the Wheeler-DeWitt wave function is not tied to any specific spatial gauge. Indeed, the wave function is required to be invariant under gauge transformations,

$$x^{\mu} \to x^{\mu} + f^{\mu}(x) \tag{6.1}$$

under which the metric transforms:

$$g_{\mu\nu} \to g_{\mu\nu} + \nabla_{\mu} f_{\nu} + \nabla_{\nu} f_{\mu}. \tag{6.2}$$

Let's consider the effect of such transformations on the boundary limit of h_{ij} . The components of f along the directions in Σ induce 2D coordinate transformation under

which h transforms conventionally. Invariance under these transformations merely mean that the action S must be a function of 2D invariants.

Invariance under the shifts f^R and f^T are more interesting. In particular the combination $f^+ = f^R + f^T$ generates non-trivial transformations of the boundary metric h_{ij} . An easy calculation shows that,

$$h_i^j \to h_i^j + f^+(\Omega_2)\delta_i^j$$
. (6.3)

In other words, shift transformations f^+ , induce Weyl re-scalings of the boundary metric. This prompts us to modify the definition of the Liouville field from

$$U = T + \tilde{h} \tag{6.4}$$

to

$$U = T + \tilde{h} + f^{+}. \tag{6.5}$$

One might wonder about the meaning of an equation such as (6.5). The left side of of the equation is supposed to be a dynamical field on Σ , but the right side contains an arbitrary function f^+ . The point is that in the Wheeler DeWitt formalism the wave function must be invariant under shifts, but in the original analysis of FSSY a specific gauge was chosen. Thus, in order to render the wave function gauge invariant, one must allow the shift f^+ to be an integration variable, giving it the status of a dynamical field.

A similar example is familiar from ordinary gauge theories. The analog of the Wheeler-DeWitt gauge-free formalism would be the unfixed theory in which one integrates over the time component of the vector potential. The analog of the STT gauge would be the Coulomb gauge. To go from one to the other we would perform the gauge transformation

$$A_0 \to A_0 + \partial_0 \phi. \tag{6.6}$$

Integrating over the gauge function ϕ in the path integral would restore the gauge invariance that was given up by fixing Coulomb gauge.

Returning to the Liouville field, since both \tilde{h} and f are linearized fluctuation variables, we see that the classical part of U is still the FRW conformal time.

One important point: because the effect of the shift f^+ is restricted to the trace of h, it does not influence the traceless-transverse (dimension-two) part of the metric fluctuation, and the original identification of the 2D energy-momentum tensor is unaffected.

Finally, invariance under the shift f^- is trivial in this order, at least for the thin wall geometry. The reason is that in the background geometry, the area does not vary along the T^- direction.

Given that the boundary theory is local, and includes a boundary metric, it is constrained by the rules of two-dimensional quantum gravity laid down long ago by Polyakov [28]. Let us review those rules for the case of a conformal "matter" field theory coupled to a Liouville field. Two-dimensional coordinate invariance implies that the central charge of the Liouville sector cancels the central charge of all other fields. We have argued in [19] (and in section 5) that the central charge of the matter sector is of order the horizon area of the Ancestor vacuum, measured in Planck Units. It is obvious from the 4-dimensional bulk viewpoint that the semiclassical analysis that we have relied on, only makes sense when the Hubble radius is much larger than the Planck scale. Thus we we take the central charge of matter to satisfy c >> 1. As a consequence, the central charge of the Liouville sector, c_L , must be large and negative. Unsurprisingly, the negative value of c is the origin of the emergence of time.

The formal development of Liouville theory begins by defining two metrics on Ω_2 . The first is what I will call the reference metric \hat{g}_{ij} . Apart from an appropriate degree smoothness, and the assumption of Euclidean signature, the reference metric is arbitrary but fixed. In particular it is not integrated over in the path integral. Moreover, physical observables must be independent of \hat{g}_{ij}

The other metric is the "real" metric denoted by g_{ij} . The purpose of the reference metric is merely to implement a degree of gauge fixing. Thus one assumes that the real metric has the form,

$$g_{ij} = e^{2U}\hat{g}_{ij}. (6.7)$$

The real metric-that is to say U-is a dynamical variable to be integrated over.

For positive c_L the Liouville Lagrangian is

$$L_L = \frac{Q^2 \sqrt{\hat{g}}}{8\pi} \left\{ \hat{\nabla} U \hat{\nabla} U + \hat{R} u \right\}$$
 (6.8)

where \hat{R} , $\hat{\nabla}$, all refer to the sphere Ω_2 , with metric \hat{g} . The constant Q is related to the central charge c_L by

$$Q^2 = \frac{c_L}{6} \tag{6.9}$$

The two dimensional cosmological constant has been set to zero for the moment, but it will return to play a surprising role. For future reference we note that the cosmological term, had we included it, would have had the form,

$$L_{cc} = \sqrt{\hat{g}} \lambda e^{2U}. ag{6.10}$$

It is useful to define a field $\phi = 4QU$ in order to bring the kinetic term to canonical form. One finds,

$$L_L = \frac{\sqrt{\hat{g}}}{8\pi} \left\{ \hat{\nabla}\phi \hat{\nabla}\phi + Q\hat{R}\phi \right\}$$
 (6.11)

and, had we included a cosmological term, it would be

$$L_{cc} = \sqrt{\hat{g}\lambda} \exp\frac{\phi}{2Q}.$$
 (6.12)

By comparison with the case of positive c_L , very little is rigorously understood about Liouville theory with negative central charge. In this paper we will make a huge leap of faith that may well come back to haunt us: we assume that the theory can be defined by analytic continuation from positive c_L . To that end we note that the only place that the central charge enters (6.11) and (6.15) is through the constants Q and γ , both of which become imaginary when c_L becomes negative. Let us define,

$$Q = i\mathcal{Q}. \tag{6.13}$$

Equations (6.11) and (6.15) become,

$$L_{L} = \frac{\sqrt{\hat{g}}}{8\pi} \left\{ \hat{\nabla}\phi \hat{\nabla}\phi + i\mathcal{Q}\hat{R}\phi \right\}$$

$$= \frac{\sqrt{\hat{g}}}{8\pi} \left\{ \hat{\nabla}\phi \hat{\nabla}\phi + 2i\mathcal{Q}\phi \right\}$$
(6.14)

(where we have used $\hat{R} = 2$), and

$$L_{cc} = \sqrt{\hat{g}}\lambda \exp\frac{-i\phi}{2\mathcal{Q}}.$$
(6.15)

Let us come now to the role of λ . First of all λ has nothing to do with the fourdimensional cosmological constant, either in the FRW patch or the Ancestor vacuum. Furthermore it is not a constant in the action of the boundary theory. Its proper role is as a **Lagrange multiplier** that serves to specify the time T, or more exactly, the global scale factor. The procedure is motivated by the Wheeler-DeWitt procedure of identifying the scale factor with time. In the present case of the thin-wall limit, we identify $\exp 2U$ with $\exp 2T$. Thus we insert a δ function in the path integral,

$$\delta\left(\int\sqrt{\hat{g}}(e^{2U}-e^{2T})\right) = \int dz \exp iz \left(\int\sqrt{\hat{g}}(e^{2U}-e^{2T})\right)$$
(6.16)

The path integral (which now includes an integration over the imaginary 2D cosmological constant z) involves the action

$$L_L + L_{cc} = \frac{\sqrt{\hat{g}}}{8\pi} \left\{ \hat{\nabla}\phi \hat{\nabla}\phi + 2i\mathcal{Q}\phi + 8\pi iz \exp\frac{-i\phi}{2\mathcal{Q}} - 8\pi ize^{2T} \right\}$$
(6.17)

There is a saddle point when the potential

$$V = 2i\mathcal{Q}\phi + 8\pi iz \exp\frac{-i\phi}{2\mathcal{Q}} - 8\pi ize^{2T}$$
(6.18)

is stationary; this occurs at,

$$\exp \frac{-i\phi}{2Q} = e^{2T}$$

$$z = i\frac{Q^2}{8\pi}e^{-2T}$$
(6.19)

or in terms of the original variables,

$$e^{2U} = e^{2T}$$

$$\lambda = \frac{Q^2}{8\pi}e^{-2T}$$
(6.20)

Once λ has been determined by (6.20), the Liouville theory with that value of λ determines expectation values of the remaining variables as functions of the time. Thus, as we mentioned earlier, the cosmological constant is not a constant of the theory but rather a parameter that we scan in order to vary the cosmic time.

It should be noted that the existence of the saddle point (6.20) is peculiar to the case of negative c.

6.2 Liouville, Renormalization, and Correlation Functions

6.3 Preliminaries

There are two preliminary discussions that will help us understand the application of Liouville Theory to cosmic holography. The first is about the ADS/CFT connection between the bulk coordinate R, and renormalization-group-running of the boundary field theory. There are three important length scales in every quantum field theory. The first is the "low energy scale;" in the present case the low energy scale is the radius of the sphere which we will call L.

The second important length is the "bare" cutoff scale—where the underlying theory is prescribed. Call it a. The bare input is a collection of degrees of freedom, and an action coupling them. In a lattice gauge theory the degrees of freedom are site and link variables, and the couplings are nearest neighbor to insure locality⁵ In a ferromagnet they are spins situated on the sites of a crystal lattice.

The previous two scales have obvious physical meaning but the third scale is arbitrary: a sliding scale called the renormalization or reference scale. We denote it by δ . The reference scale is assumed to be much smaller than L and much larger than a, but otherwise it is arbitrary. It helps to keep a concrete model in mind. Instead of a regular lattice, introduce a "dust" of points with average spacing a. It is not essential that a be uniform on the sphere. Thus the spacing of dust points is a function of position, $a(\Omega_2)$. The degrees of freedom on the dust-points, and their nearest-neighbor couplings, will be left implicit.

Next we introduce a second dust at larger spacing, δ . The δ -dust provides the reference scale. It is well known that for length scales greater than δ , the bare theory on the a-dust can be replaced by a renormalized theory defined on the δ -dust. The renormalized theory will typically be more complicated, containing second, third, and n^{th} neighbor couplings.

Generally, the dimensionless form of the renormalized theory will depend on δ in just such a way that physics at longer scales is exactly the same as it was in the original theory. The dimensionless parameters will flow as the reference scale is changed.

If there is an infrared fixed-point, and if the bare theory is in the basin of attraction of the fixed-point, then as δ becomes much larger than a, the dimensionless parameters

⁵Nearest neighbor is common but not absolutely essential. However this subtlety is not important for us.

will run to their fixed-point values. In that case the continuum limit $(a \to 0)$ will be a conformal field theory with SO(3,1) invariance.

Similar things hold in the theory of bulk anti de Sitter space, although in that case the discussion of the bare scale is less relevant—one might as well take the continuum limit $a \to 0$ from the start. In the boundary field theory the infrared scale is provided by the spherical boundary of ADS. From the bulk viewpoint the boundary is at infinite proper distance, at $R = \infty$. However, the time for a signal to reach the boundary and be reflected back to the bulk is finite. In that respect anti de Sitter space behaves like a finite cavity, requiring specific boundary conditions. To be definite, the bulk theory is infrared regulated by replacing Σ with a reference-boundary Σ_0 , at finite R. Specifying the boundary conditions on Σ_0 is equivalent to specifying the field theory parameters at scale δ . In parallel with the field theory discussion, the cutoff R, can vary with angular position: $R = R_0(\Omega_2)$. We can now state the UV/IR connection by the simple identification,

$$\delta(\Omega_2) = e^{-R_0(\Omega_2)} \tag{6.21}$$

A useful slogan is that "Motion along the R direction is the same as renormalization-group flow."

Now to the second preliminary–some observations about Lioville Theory. Again it is helpful to have a concrete model. Liouville theory is closely connected with the theory of dense, planar, "fishnet" diagrams [26] such as those which appear in large N gauge theories, and matrix models [27][29][30]. The fishnet plays the role of the bare lattice in the previous discussion, but now it's dynamical—we sum over all fishnet diagrams, assuming only that the spacing (on the sphere) is everwhere much smaller than the sphere size, L. As before, we call the angular spacing between neighboring points on the sphere, $a(\Omega)$.

Each fishnet defines a metric on the sphere. Let $d\alpha$ be a small angular interval (measured in radians). The fishnet-metric is defined by⁶

$$ds^2 = \frac{d\alpha^2}{a(\Omega)^2} \tag{6.22}$$

As before we introduce a reference scale δ . It can also be a fishnet, but now it is fixed, its vertices nailed down, not to be integrated over. We continue to assume that δ satisfies

⁶Strictly speaking there is no need to introduce a discrete fishnet lattice at the scale δ . It is sufficient to just define a continuous function $\delta(\Omega_2)$, and from it define a reference metric by (6.22).

the inequalities, $a(\Omega) \ll \delta(\Omega) \ll L$, but otherwise it is arbitrary. The δ -metric is defined by

$$ds_{\delta}^2 = \frac{d\alpha^2}{\delta(\Omega)^2} \tag{6.23}$$

We can now define the Liouville field U. All it is is the ratio of the reference and fishnet scales:

$$e^U \equiv \delta/a \tag{6.24}$$

Using (6.24) together with $\delta = e^{-R}$, and $ds = \frac{d\alpha}{a}$, we see that U is also given by the relation,

$$ds = d\alpha \ e^{(R_0 + U)}. \tag{6.25}$$

In (6.25) both R_0 and U are functions of location on Ω_2 , but only U is dynamical, i.e., to be integrated over.

6.4 Liouville in the Hat

With that in mind, we return to cosmic holography, and consider the metric on the regulated spatial boundary of FRW, Σ_0 . In the absence of fluctuations it is

$$ds^2 = e^{2R_0}e^{2T}d^2\Omega_2.$$

In general relativity it is natural to allow both R_0 and T to vary over the sphere, so that

$$ds^2 = e^{2R_0(\Omega_2)}e^{2T(\Omega_2)}d^2\Omega_2 (6.26)$$

The parallel between $(6.25 \text{ and } (6.26) \text{ is obvious. Exactly as we might have expected from the Wheeler-DeWitt interpretation, the Liouville field, <math>U$, may be identified with time T, at least when both are large.

$$U \approx T \tag{6.27}$$

To summarize, Let's list a number of correspondences:

$$\begin{array}{cccc} \delta & \leftrightarrow & \mu^{-1} \leftrightarrow e^R \\ \lambda & \leftrightarrow & e^{-T} \end{array}$$

$$a \leftrightarrow e^{-T^+} = e^{-(T+R)} \tag{6.28}$$

One other point about Liouville Theory: the density of vertices of a fishnet is normally varied by changing the weight assigned to vertices. When the fishnet is a Feynman diagram the weight is a coupling constant g. It is well known that the coupling constant and Liouville cosmological constant are alternate descriptions of the same thing. Either can be used to vary the average vertex density-increasing it either by increasing g or decreasing g. The very dense fishnets correspond to large g and therefore large FRW time, whereas very sparse diagrams dominate the early Planckian era.

6.5 Proactive and Reactive Objects in Quantum Field Theory

There are two kinds of objects in Wilsonian renormalization that correspond quite closely to the terms G_1 and G_2 that we have found in the section 5. I don't know if there is a term for the distinction, but I will call them "proactive and "reactive. Proactive objects are not quantities that we directly measure; they are objects which go into the definition of the theory. The best example is the exact Wilsonian action, defined at a specific reference scale. The form of proactive quanities depends on that reference scale, and so does the value of their matrix elements; indeed their form, varies with δ in such a way as to keep the physics fixed at longer distances.

By contrast, reactive objects are observables whose value does not depend at all on the reference scale. They do depend on the "bare" cutoff scale a through wave function renormalization constants, which typically tend to zero as $a \to 0$. The wave function renormalization constants are usually stripped off when defining a quantum field but we will find it more illuminating to keep them.

The distinction between these two kinds of objects is subtle, and is perhaps best expressed in Polchinski's version of the exact Wilsonian renormalization group [32]. In that scheme, at every scale there is a renormalized description in terms of local defining fields $\phi(x)$, but the proactive action grows increasingly complicated as the reference scale is lowered.

Consider the exact effective action defined at reference scale δ . It is given by an infinite expansion of the form

$$L_W(\delta) = \sum_{\Delta=2}^{\infty} g_{\Delta} \mathcal{O}_{\Delta} \tag{6.29}$$

Where \mathcal{O}_{Δ} are a set of operators of dimension Δ , and g_{Δ} are dimensional coupling constants. The renormalization flow is expressed in terms of the dimensionless coupling constants,

$$\tilde{g}_{\Delta} = g_{\Delta} \delta^{(2-\Delta)}. \tag{6.30}$$

The \tilde{g} satisfy RG equations,

$$\frac{d\tilde{g}}{d\log\delta} = -\beta(\tilde{g}),\tag{6.31}$$

and at a fixed point they are constant. Thus the dimensional constants g_{Δ} in the Lagrangian will grow with δ . Normalizing them at the bare scale a, in the fixed-point case we get,

$$g_{\Delta} = g_a \left\{ \frac{\delta}{a} \right\}^{(\Delta - 2)} \tag{6.32}$$

$$L_W(\delta) = \sum_{\Delta=2}^{\infty} \mathcal{O}_{\Delta} \left\{ \frac{\delta}{a} \right\}^{(\Delta-2)}.$$
 (6.33)

Now consider the two point function of the effective action, $\langle L_W(\delta)L_W(\delta)\rangle$, evaluated at distance scale $L >> \delta$

$$\langle L_W(\delta)L_W(\delta)\rangle = \sum_{\Delta=2}^{\infty} \langle \mathcal{O}_{\Delta}\mathcal{O}_{\Delta}\rangle \left(\frac{\delta}{a}\right)^{2(\Delta-2)}$$
(6.34)

Suppose the theory is defined on a sphere of radius L and we are interested in the correlator $\langle L_W(\delta)L_W(\delta)\rangle$ between points separated by angle α . The factor $\langle \mathcal{O}_{\Delta}\mathcal{O}_{\Delta}\rangle$ is the two-point function of a field of dimension Δ , in a theory on the sphere of size L, with an ultraviolet cutoff at the reference scale δ . Accordingly it has the form

$$\langle \mathcal{O}_{\Delta} \mathcal{O}_{\Delta} \rangle = \left(\frac{\delta}{L}\right)^{2\Delta} (1 - \cos \alpha)^{-\Delta}$$
 (6.35)

where the two factors of $\left(\frac{\delta}{L}\right)^{\Delta}$ are the ultraviolet-sensitive wave function renormalization constants. The final result is

$$\langle L_W(\delta)L_W(\delta)\rangle = \sum_{\Delta=2}^{\infty} C_{\Delta} \left(\frac{\delta}{a}\right)^{2(\Delta-2)} \left(\frac{\delta}{L}\right)^{2\Delta} (1 - \cos\alpha)^{-\Delta}$$
 (6.36)

Note the odd dependence of (6.36) on the arbitrary reference scale δ . That dependence is typical of proactive quantities.

Now consider a reactive quantity such as a fundamental field, a derivative of such a field, or a local product of fields and derivatives. Their matrix elements at distance scale L will be independent of the reference scale (although it will depend of the bare cutoff a) and be of order,

$$\langle \phi \phi \rangle \sim \left(\frac{a}{L}\right)^{2\Delta_{\phi}}$$
 (6.37)

where Δ_{ϕ} is the operator dimension of ϕ . Thus we see two distinct behaviors for the scaling of correlation functions:

$$\left(\frac{\delta}{a}\right)^{2(\Delta-2)} \left(\frac{\delta}{L}\right)^{2\Delta}$$
 proactive (6.38)

and

$$\left(\frac{a}{L}\right)^{2\Delta_{\phi}}$$
 reactive (6.39)

The formulas are more complicated away from a fixed point but the principles are the same.

We note that the effective action is not the only proactive object. The energy-momentum tensor, and various currents computed from the effective action, will also be proactive. As we will see these two behaviors–proactive and reactive–exactly correspond to the dependence in (5.9) and (5.10).

Now we are finally ready to complete the discussion about the relation between the correlators of Section 5 and proactive/reactive operators. Begin by noting that in ADS/CFT, the minimally coupled massless (bulk) scalar is the dilaton, and its associated boundary field is the Lagrangian density. It may seem puzzling that in the present case, an entire infinite tower of operators seems to replace, what in ADS/CFT is a single operator. In the case of the metric fluctuations a similar tower replaces the energy-momentum tensor. The puzzle may be stated another way. The FRW geometry consists of an infinite number of Euclidean ADS time slices. At what time (or what 2D cosmological constant) should we evaluate the boundary limits of the metric fluctuations, in order to define the energy-momentum tensor? As we will see, a parallel ambiguity exists in Liouville theory.

Return now, to the three scales of Liouville Theory: the infrared scale L, the reference scale δ , and the fishnet scale a, with $L >> \delta >> a$. It is natural to assume that the basic theory is defined at the bare fishnet scale a by some collection of degrees of freedom at each lattice site, and also specific nearest-neighbor couplings—the latter insuring locality. Now imagine a Wilsonian integration of all degrees of freedom on scales between the fishnet

scale and the reference scale, including the fishnet structure itself. The result will be a proactive effective action of the type we described in equation (6.29). Moreover the correlation function of L_{eff} will have the form (6.36). But now, making the identifications (6.21) and

$$\frac{\delta}{a} = e^U = e^T, \tag{6.40}$$

we see that equation (6.38) for proactive scaling becomes (for each operator in the product)

$$e^{(\Delta-2)T}e^{-\Delta R} \tag{6.41}$$

This is in precise agreement with the coefficients in the expansion (5.8). Similarly the reactive scaling (6.39) is $e^{-\Delta T^+}$, in agreement with the properties of G_2 .

It is not obvious to me exactly why the bulk fields should correspond to proactive and reactive boundary fields in the way that they do. I might point out the solutions to the wave equation in the bulk are generally sums of two types of modes,

$$\chi_{-} \rightarrow g_{-}(\Omega_{2}) F_{-}(T^{-}) e^{-2T}$$

$$\chi_{+} \rightarrow g_{+}(\Omega_{2}) F_{+}(T^{+}).$$
(6.42)

Evidently the two types of solutions couple to objects that are reactive and proactive under the RG flow.

What happens to the proactive objects if we approach Σ by sending $T^+ \to \infty$ and $T_- \to -\infty$? In this limit only the dimension-two term survives: exactly what we would expect if the matter action ran toward a fixed point. All of the same things hold true for the tensor fluctuations. Before the limit $T^- \to -\infty$, the energy-momentum tensor consists of an infinite number of higher dimension operators but in the limit, all tend to zero except for the dimension two term.

It should be observed that the higher dimension contributions to $G_1 \begin{Bmatrix} ik \\ jl \end{Bmatrix}$ are not transverse in the two-dimensional sense. This is to be expected: before the limit is taken, the Liouville field does not decouple from the matter field, and the matter energy-momentum is not separately conserved. But if the matter theory is at a fixed point, i.e., scale invariant, the Liouville and matter do decouple and the matter energy-momentum should be conserved. Thus, in the limit in which the dimension-two term dominates, it should be (and is) transverse-traceless.

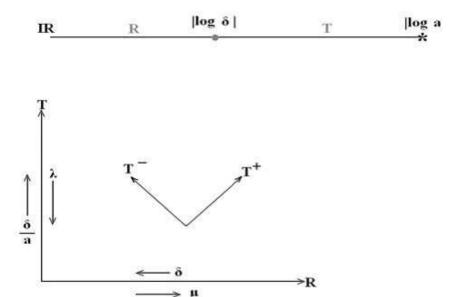


Figure 7: The Wilson line of scales, and the two dimensional R, T plane.

The RG flow is usually thought of in terms of a single independent flow-parameter. In some versions it's the logarithm of the bare cutoff scale, and in other formulations it's the log of the renormalization scale. In the conventional ADS/CFT framework, R can play either role. One can imagine a bare cutoff at some large R_0 or one can push the bare cutoff to infinity and think of R as a running renormalization scale.

However, for our purposes, it is better to keep track of both scales. One can either think of a one-dimensional (logarithmic) axis—we can call it the "Wilson line"—extending from the infrared scale to the fishnet scale a, or a two dimensional R, T plane. In either case the effective action as a function of two independent variables. Figure 7 shows a sketch of the Wilson line and the two dimensional plane representing the two directions R, and T.

The two independent parameters can be chosen to be a and δ , or equivalently R and T.

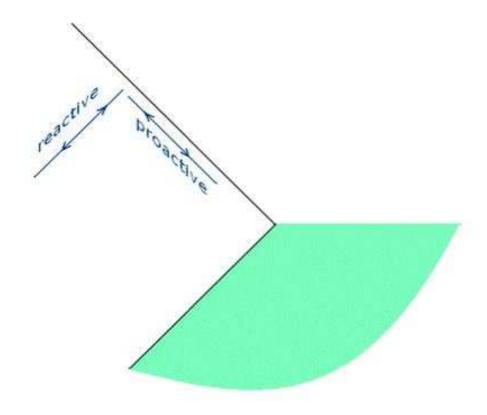


Figure 8: Proactive and reactive quantities scale with the two light-like directions T^- and T^+

.

Yet another choice is to work in momentum space. The reference energy-scale is usually called μ .

$$\mu = e^R \tag{6.43}$$

And in the case of negative central charge, the two-dimensional cosmological constant λ can replace T.

In this light, it is extremely interesting that the distinction between proactive and reactive scaling, corresponds to motion along the two light-like directions T^- and T^+ as depicted in figure 8.

It is important to understand that the duality between FRW cosmology and Liouville 2D gravity does not only involve the continuum fixed point theory. As long as T is finite

the theory has some memory of the bare theory. It's only in the limit $T \to \infty$ that the theory flows to the fixed point and loses memory of the bare details. We will come back to this point in section 8 when I discuss the Garriga, Guth, Vilenkin [20] "Persistence of Memory" phenomenon.

6.6 Boussovian Bounds and The c-Theorem

When a conformal matter theory is coupled to a Liouville field, the two sectors decouple, except for the constraint that the total central charge vanish,

$$c + c_L = 0, (6.44)$$

where c and c_L refer to matter and Liouville respectively. Moreover the matter central charge is constant since a conformal theory is, by definition, at a fixed-point. From now on when I speak of the central charge I will be referring to the matter sector only.

There is one caveat to this rule that c is constant at a fixed point: it applies straightforwardly as long as the reference scale is much smaller than the infrared scale, i.e., the size of the sphere. However, the finite (coordinate) size of the boundary sphere provides an infrared cutoff that is similar to the confinement scale in a confining gauge theory. As the reference scale becomes comparable to the total sphere, the theory runs out of lower energy degrees of freedom and, with some definition, the c-function will go to zero.

The central charge is a measure of the number of degrees of freedom in an area-cell of size e^{-2T^+} . Naively, the holographic principle would say it is the area in such a cell in Planck units. However, as we have seen in the past, this is not always the case [21] [22].

Recall that motion up and down the T^- axis, at fixed T^+ , corresponds to the usual RG flow of the Wilsonian action (keeping the bare scale fixed and letting the reference scale vary). Thus, at a fixed point, the area along a fixed T^+ line should be constant. In the thin wall limit the area is given by

$$A = e^{2T} \sinh^2 R. \tag{6.45}$$

As long as R >> 1 the area is indeed constant for fixed T^+ . However as we approach $R \sim 1$ the area quickly tends to zero, consistent with the remarks above.

More generally, away from fixed points the Zamolodchikov c-theorem requires c to decrease with increasing reference-scale δ . This seems to suggest that the area must

monotonically decrease as T^- increases, with T^+ held fixed, which, as we will see raises a paradox; the area is not monotonic beyond the thin-wall approximation.

The study of how area varies along light sheets—"Boussology"—has a long and celebrated history which I will assume you are familiar with [22]. Rather than deal with the equations that determine how area varies I will draw some Bousso diagrams and tell you the conclusions. First the thin-wall case: Figure 9 shows the FRW patch of a thin-wall Coleman De Luccia nucleation. In fact it is the forward light-cone of a point in flat Minkowski space. The red line is a light-sheet of constant T^+ . The Bousso wedges indicate the light-like directions along which the area decreases. The entire geometry consists of a single region in which all wedges point toward the origin—the vertical side of the triangle—at R=0. The area is maximum at "enter": it is almost constant from "enter" to a and then b, but by c it starts to significantly decrease. All of this is in accord with the a-theorem.

Next, in figure 10 some more interesting cosmological examples are shown. On the left is pure radiation dominated FRW. Unlike the thin-wall case it consists of two regions separated by the bold black line. In this case the area *increases* from "enter" to a, reaching a maximum at b and then decreasing very imperceptibly to c. At c it starts a quick decent to zero. This looks dangerous.

On the right side of figure 10 the blue region is a patch with a finite vacuum energy. It is intended to model an era of slow-roll inflation. For simplicity, at reheating (horizontal edge of the blue region) I have attached it to flat space-time. More realistically one might attach it to the radiation dominated case but the result would be the same.

In this slow-roll case the area starts an exponential increase at "enter" and again reaches a maximum at b. Beyond that the behavior is the same as for thin-wall. In fact this behavior of an increase of area, followed by a decrease is very generic. It would seem to violate the c-theorem.

The point, however, is that the identification of c with area is wrong. The number of degrees of freedom of a system is a direct measure of the maximum entropy of that system. The right measure of c, for example at the point a, is not the area of a, but rather the maximum entropy that can pass through the light-sheet from a to the origin at R = 0. According to Bousso, that is given by the maximum entropy on the interval between a and

Thin-Wall

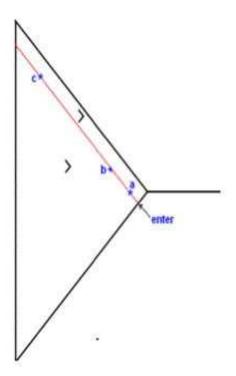


Figure 9: Bousso diagram for the FRW geometry resulting from a thin-wall CDL bubble. The entire FRW geometry consists of a single region in which the contracting light-sheets all point in the same directions. The light-like red line is a surface of constant T^+

.

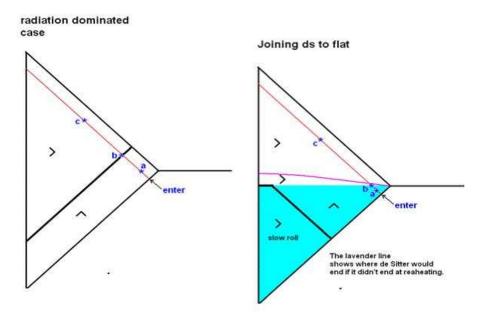


Figure 10: Bousso diagrams for radiation dominated and slow-roll universes. The slow-roll case has been simplified by attaching a de Sitter region to a patch of flat space-time. The blue area represents the inflating region. The light-like red line is a surface of constant T^+ .

b, plus the maximum entropy between b and the origin. Calling the areas of the various points A(a), A(b), A(c) the maximum entropy of the light-sheet bounded by a is given by,

$$S_{max}(a) = (A(b) - A(a)) + (A(b)). (6.46)$$

It is maximized at "enter" and monotonically decreases to to zero at R=0. Thus, identifying $c=S_{max}$ restores the monotonicity of the central charge.

It's interesting to compare the naive behavior, c = A with the correct formula $c = S_{max}$ in the slow-roll inflationary case. The naive formula would have c exponentially increasing from "enter" to b. The correct formula has it exponentially decreasing toward its (inflated) value at b.

7 Scaling and the Census Taker

7.1 Moments

Now we come to the heart of the matter: the intimate connection between the scaling behavior of two-dimensional quantum field theory and the observations of a Census Taker as he moves toward the Census Bureau. In order to better understand the connection between the cutoff-scale and T^+ , let's return to the similar connection between cutoff and the coordinate R in anti de Sitter space. We normalize the ADS radius of curvature to be 1; with that normalization, the Planck area is given by 1/c where c is the central charge.

Consider the proper distance between points 1 and 2 given by (5.2). The relation $l = R_1 + R_2 + \log(1 - \cos\alpha)$ is approximate, valid when l and $R_{1,2}$ are all large. When $l \sim 1$ or equivalently, when

$$\alpha^2 \sim e^{-(R_1 + R_2)} \tag{7.1}$$

equation (5.2) breaks down. For angles smaller than (7.1) the distance in anti de Sitter space behaves like

$$l \sim e^R \alpha.$$
 (7.2)

Thus a typical correlation function will behave as a power of $(1 - \cos \alpha)$ down to angular distances of order (7.1) and then fall quickly to zero.

The angular cutoff in anti de Sitter space has a simple meaning. The solid angle corresponding to the cutoff is of order e^{-2R} while the area of the regulated boundary is

 e^{+2R} . Thus, metrically, the cutoff area is of order unity. This means that in Planck units, the cutoff area is the central charge c of the boundary conformal field theory.

Now consider the cutoff angle implied by (5.12) and (5.13). By an argument parallel to the one above, the cutoff angle becomes

$$\alpha^2 \sim e^{-(T_1^+ + T_2^+)} \tag{7.3}$$

Once again this corresponds to a proper area on Σ_0 (the regulated boundary) which is time-independent, and in Planck units, of order the central charge.

Consider the Census Taker looking back from some late time T_{CT} . For convenience we place the CT at R=0. His backward light-cone is the surface

$$T + R = T_{CT}. (7.4)$$

The CT can never quite see Σ . Instead he sees the regulated surfaces corresponding to a fixed proper cutoff (figure 4). The later the CT observes, the smaller the angular structure that he can resolve on the boundary. This is another example of the UV/IR connection, this time in a cosmological setting.

Let's consider a specific example of a possible observation. The massless scalar field χ of section 5 has an asymptotic limit on Σ that defines the dimension zero field $\chi(\Omega)$. Moments of χ can be defined by integrating it with spherical harmonics,

$$\chi_{lm} = \int \chi(\Omega) Y_{lm}(\Omega) d^2\Omega \tag{7.5}$$

It is worth recalling that in anti de Sitter space the corresponding moments would all vanish because the normalizable modes of χ all vanish exponentially as $R \to \infty$. The possibility of non-vanishing moments is due entirely to the asymptotic warmness of open FRW.

We can easily calculate ⁷ the mean square value of χ_{lm} (It is independent of m).

$$\langle \chi_l^2 \rangle = \int \log(1 - \cos \alpha) P_l(\cos \alpha) \sim \frac{1}{l(l+1)}$$
 (7.6)

⁷I am indebted to Ben Freivogel for explaining equation (7.6) to me.

It is evident that at a fixed Census Taker time T_{CT} , the angular resolution is limited by (7.3). Correspondingly, the largest moment that the CT can resolve corresponds to

$$l_{max} = e^{T_{CT}} (7.7)$$

Thus we arrive at the following picture: the Census Taker can look back toward Σ but at any given time his angular resolution is limited by (7.3) and (7.7). As time goes on more and more moments come into view. Once they are measured they are frozen and cannot change. In other words the moments evolve from being unknown quantum variables, with a gaussian probability distribution, to classical boundary conditions that explicitly break rotation symmetry (and therefore conformal symmetry). One sees from (7.6) that the symmetry breaking is dominated by the low moments.

But I doubt that this phenomenon ever occurs in an undiluted form. Realistically speaking, we don't expect massless scalars in the non-supersymmetric Ancestor. In Section 5 we discussed the effect of a small mass term, in the ancestor vacuum, on the correlation functions of χ . The result of such a mass term is a shift of the leading dimension⁸ from 0 to μ . This has an effect on the moments. The correlation function becomes

$$e^{-\mu T_1^+} e^{-\mu T_2^+} (1 - \cos \alpha)^{-\mu}$$
 (7.8)

and the moments take the form

$$\langle \chi_l^2 \rangle = e^{-2\mu T_{CT}} \int (1 - \cos \alpha)^{\mu} P_l(\cos \alpha)$$
 (7.9)

The functional form of the l dependence changes a bit, favoring higher l, but more importantly, the observable effects decrease like $e^{-2\mu T_{CT}}$. Thus as T_{CT} advances, the asymmetry on the sky decreases exponentially with conformal time. Equivalently it decreases as a power of proper time along the CT's world-line.

7.2 Homogeneity Breakdown

Homogeneity in an infinite FRW universe is generally taken for granted, but before questioning homogeneity we should know exactly what it means. Consider some three-dimensional

⁸In the de Sitter/CFT correspondence [34], the dimension of a massive scalar becomes complex when the mass exceeds the Hubble scale. In our case the dimension remains real for all μ . I thank Yasuhiro Sekino for this observation.

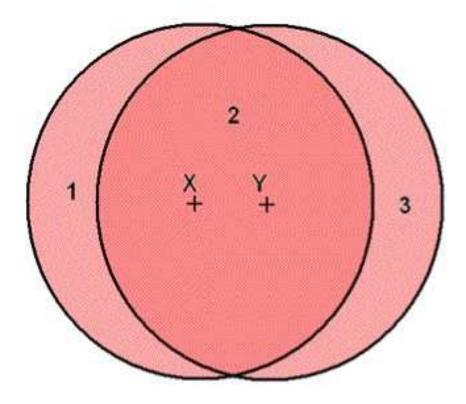


Figure 11: Two large spheres centered at X and Y.

scalar quantity such as energy density, temperature, or the scalar field χ . Obviously the universe is not uniform on small scales, so in order to define homogeneity in a useful way we need to average χ over some suitable volume. Thus at each point X of space, we integrate χ over a sphere of radius r and then divide by the volume of the sphere. For a mathematically exact notion of homogeneity the size of the sphere must tend to infinity. The definition of the average of χ at the point X is

$$\overline{\chi(X)} = \lim_{r \to \infty} \frac{\int \chi d^3 x}{V_r} \tag{7.10}$$

Now pick a second point Y and construct $\overline{\chi(Y)}$. The difference $\overline{\chi(X)} - \overline{\chi(Y)}$ should go to zero as $r \to \infty$ if space is homogeneous. But as the spheres grow larger than the distance between X and Y, they eventually almost completely overlap. In figure 11 we see that the

difference between $\overline{\chi(X)}$ and $\overline{\chi(y)}$ is due to the two thin crescent-shaped regions, 1 and 3. It seems evident that the overwhelming bulk of the contributions to $\overline{\chi(X)}$, $\overline{\chi(Y)}$ come from the central region 3, which occupies almost the whole figure. The conclusion seems to be that the averages, if they exist at all, must be independent of position. Homogeneity while true, is a triviality.

This is correct in flat space, but surprisingly it can break down in hyperbolic space ⁹. The reason is quite simple: despite appearances the volume of regions 1 and 3 grow just as rapidly as the volume of 2. The ratio of the volumes is of order

$$\frac{V_1}{V_2} = \frac{V_3}{V_2} \sim \frac{l}{R_{curvature}} \tag{7.11}$$

and remains finite as $r \to \infty$.

To be more precise we observe that

$$\overline{\chi(X)} = \frac{\int_{1} \chi + \int_{2} \chi}{V_{1} + V_{2}}
\overline{\chi(Y)} = \frac{\int_{3} \chi + \int_{2} \chi}{V_{3} + V_{2}}$$
(7.12)

and that the difference $\overline{\chi(X)} - \overline{\chi(Y)}$ is given by

$$\overline{\chi(X)} - \overline{\chi(Y)} = \frac{\int_1 \chi}{V_1 + V_2} - \frac{\int_1 \chi}{V_1 + V_2}$$
 (7.13)

which, in the limit $r \to \infty$ is easily seen to be proportional to the dipole-moment of the boundary theory,

$$\overline{\chi(X)} - \overline{\chi(Y)} = l \int \chi(\Omega) \cos \theta d^2 \Omega = l \chi_{1,0}, \qquad (7.14)$$

where l is the distance between X and Y.

Since, as we have already seen for the case $\mu=0$, the mean square fluctuation in the moments does not go to zero with distance, it is also true that average value of $|\overline{\chi(X)}-\overline{\chi(Y)}|^2$ will be nonzero. In fact it grows with separation.

However there is no reason to believe that a dimension zero scalar exists. Moduli, for example, are expected to be massive in the Ancestor, and this shifts the dimension of the

⁹L.S. is grateful to Larry Guth for explaining this phenomenon, and to Alan Guth for emphasizing its importance in cosmology.

corresponding boundary field. In the case in which the field χ has dimension μ , the effect (non-zero rms average of moments) persists in a somewhat diluted form. If a renormalized field is defined by stripping off the wave function normalization constants, $\exp(-\mu T^+)$, the squared moments still have finite expectation values and break the symmetry. However, from an observational point of view there does not seem to be any reason to remove these factors. Thus it seems that as the Census Taker time tends to infinity, the observable asymmetry will decrease like $\exp(-2\mu T_{CT})$.

8 Bubble Collisions and Other Matters

The Census Taker idea originated with attempts to provide a measure on the Landscape. By looking back toward Σ , the Census Taker can see into bubbles of other vacua—bubbles that in the past collided with his hatted vacuum. By counting the bubbles of each type on the sky, he can try to define a measure on the Landscape. Whether or not this can be done, it is important to our program to understand the representation of bubble collisions in the language of the boundary holographic field theory.

Long ago, Guth and Weinberg [35] recognized that a single isolated bubble is infinitely unlikely, and that a typical "pocket universe" will consist of a cluster of an unbounded number of colliding bubbles, although if the nucleation rate is small the collisions will in some sense be rare. To see why such bubble clusters form it is sufficient to recognize why a single bubble is infinitely improbable. In figure 12 the main point is illustrated by drawing a time-like trajectory that approaches Σ from within the Ancestor vacuum. The trajectory has infinite proper length, and assuming that there is a uniform nucleation rate, a second bubble will eventually swallow the trajectory and collide with the original bubble. Repeating this process will produce an infinite bubble cluster.

More recently Garriga, Guth, and Vilenkin [20] have argued that the multiple bubble collisions must spontaneously break the SO(3,1) symmetry of a single bubble, and in the process render the (pocket) universe inhomogeneous and anisotropic. The breaking of symmetry in [20] was described, not as spontaneous breaking, but as explicit breaking due to initial conditions. However, spontaneous symmetry breaking is nothing but the memory of a temporary explicit symmetry breaking, if the memory does not fade with time. For example, a small magnetic field in the very remote past will determine the

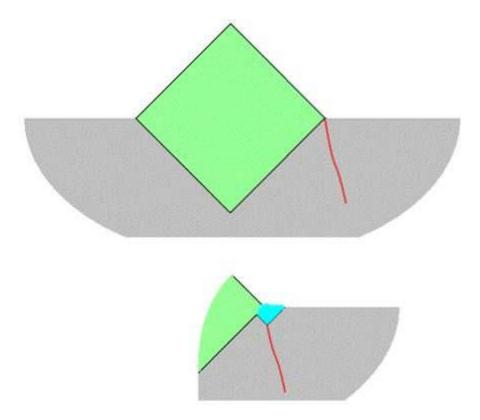


Figure 12: The top figure represents a single nucleated bubble. The red trajectory is a time-like curve of infinite length approaching Σ . Because there is a constant nucleation rate along the curve, it is inevitable that a second bubble will nucleate as in the lower figure. The two bubbles will collide.

direction of an infinite ferromagnet for all future time. Spontaneous symmetry breaking is "The Persistence of Memory."

The actual observability of bubble collisions depends on the amount of slow-roll inflation that took place after tunneling. Much more than 60 e-foldings would probably wipe out any signal, but our interest in this paper is conceptual. We will take the viewpoint that anything within the past light-cone of the Census Taker is in principle observable.

In the last section we saw that perturbative infrared effects are capable of breaking the SO(3,1) symmetry, and it is an interesting question what the relation between these two mechanisms is. The production of a new bubble would seem to be a non-perturbative effect that adds to the perturbative symmetry breaking effects of the previous section. Whether it adds distinctly new effects that are absent in perturbation theory is not obvious and may depend on the specific nature of the collision. Let us classify the possibilities.

8.1 Collisions with Identical Vacua

The simplest situation is if the true-vacuum bubble collides with another identical bubble, the two bubbles coalescing to form a single bubble, as in the top of figure 13.

The surface Σ is defined by starting at the tip of the hat and tracking back along light-like trajectories until they end—in this case at a false vacuum labeled \mathbf{F} . The collision is parameterized by the space-like separation between nucleation points. Particles produced at the collision of the bubbles just add to the particles that were produced by ordinary FRW evolution. The main effect of such a collision is to create a very distorted boundary geometry, if the nucleation points are far apart. When they are close the double nucleation blends in smoothly with the single bubble. These kind of collisions seem to be no different than the perturbative disturbances caused by the non-normalizable mode of the metric fluctuation. Garriga, Guth, and Vilenkin, compute that the typical observer will see multipole moments on the sky, but as we've seen, similar multipole moments can also occur perturbatively.

In the bottom half of figure 13 we see another type of collision in which the colliding bubbles correspond to two different true vacua: red (r) and blue (b). But in this case red

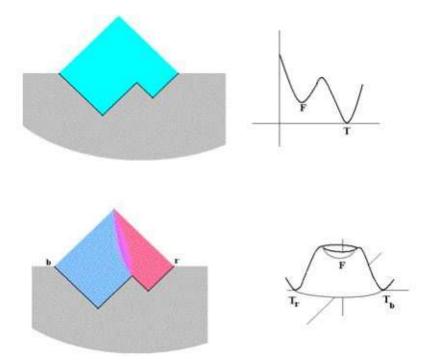


Figure 13: In the top figure two identical bubbles collide. This would be the only type of collision in a simple landscape with two discrete minima—one of positive energy and one of zero energy. In the lower figure a more complicated situation is depicted. In this case the false vacuum F can decay to two different true vacua, "red" and "blue," each with vanishing energy. The two true vacua are connected by a flat direction, but CDL instantons only lead to the red and blue points.

and blue are on the same moduli-space, so that they are connected by a flat direction¹⁰. Both vacua are included within the hat. In the bulk space-time they bleed into each other, so that as one traverses a space-like surface, blue gradually blends into purple and then red.

On the other hand the surface Σ is sharply divided into blue and red regions, as if by a one dimensional domain wall. This seems to be a new phenomenon that does not occur in perturbation theory about either vacuum.

As an example, consider a case in which a red vacuum-nucleation occurs first, and then much later a blue vacuum bubble nucleates. In that case the blue patch on the boundary will be very small and The Census Taker will see it occupying tiny angle on the sky. How does the boundary field theorist interpret it? The best description is probably as a small blue instanton in a red vacuum. In both the bulk and boundary theory this is an exponentially suppressed, non-perturbative effect.

However, in a conformal field theory the size of an instanton is a modulus that must be integrated over. As the instanton grows the blue region engulfs more and more of the boundary. Eventually the configuration evolves to a blue 2D vacuum, with a tiny red instanton. One can also think of the two configurations as the observations of two different Census Takers at a large separation from one another. Which one of them is at the center, is obviously ambiguous.

The same ambiguous separation into dominant vacuum, and small instanton, can be seen another way. The nucleation sites of the two bubbles are separated by a space-like interval. There is no invariant meaning to say that one occurs before the other. A element of the de Sitter symmetry group can interchange which bubble nucleates early and which nucleates later.

Nevertheless, a given Census Taker will see a definite pattern on the sky. One can always define the CT to be at the center of things, and integrate over the relative size of the blue and red regions. Or one can keep the size of the regions fixed–equal for example–and integrate over the location of the CT.

From both the boundary field theory, and the bubble nucleation viewpoints, the probability for any finite number of red-blue patches is zero. Small red instantons will be

¹⁰I assume that there is no symmetry along the flat direction, and that there are only two tunneling paths from the false vacuum, one to red, and one to blue.

sprinkled on every blue patch and vice versa, until the boundary becomes a fractal. The fractal dimensions are closely connected to operator dimensions in the boundary theory. Moreover, exactly the same pattern is expected from multiple bubble collisions.

But the Census taker has a finite angular resolution. He cannot see angular features smaller than $\delta \alpha \sim \exp(-T_{CT})$. Thus he will see a finite sprinkling of red and blue dust on the sky. At T_{CT} increases, the UV cutoff scale tends to zero and the CT sees a homogeneous "purple" fixed-point theory.

The red and blue patches are reminiscent of the Ising spin system (coupled to a Liouville field). As in that case, it makes sense to average over small patches and define a continuous "color field" ranging from intense blue to intense red. It is interesting to ask whether Σ would look isotropic, or whether there will be finite multipole moments of the renormalized color color field (as in the case of the χ field). The calculations of Garriga, Guth, and Vilenkin suggest that multipole moments would be seen. But unless for some reason there is a field of exactly zero dimension, the observational signal should fade with Census Taker time.

There are other types of collisions that seem to be fundamentally different from the previous. Let us consider a model landscape with three vacua—two false, B and W (Black and White); and one true vacuum T. Let the vacuum energy of B be bigger than that of W, and also assume that the decays $B \to W$, $B \to T$, and $W \to T$ are all possible. Let us also start in the Black vacuum and consider a transition to the True vacuum. The result will be a hat bounded by Σ .

However, if a bubble of W forms, it may collide with the T bubble as in figure 14. The W bubble does not end in a hat but rather, on a space-like surface. By contrast, the true vacuum bubble does end in a hat. The surface Σ is defined as always, by following the light-like generators of the hat backward until they enter the bulk-either Black or White-as in figure 14.

In this case a portion of the boundary Σ butts up against B, while another portion abuts W. In some ways this situation is similar to the previous case where the boundary was separated into red and blue regions, but there is no analogue of the gradual bleeding of vacua in the bulk. In the previous case the Census Taker could smoothly pass from red to blue. But in the current example, the CT would have to crash through a domain wall in order to pass from T to W. Typically this happens extremely fast, long before the CT

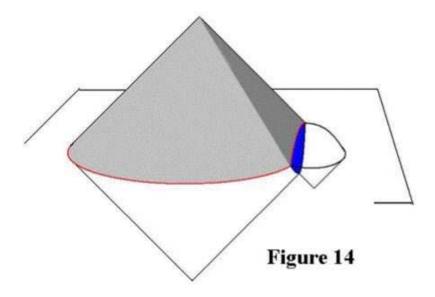


Figure 14: A bubble of True vacuum forms in the Black false vacuum and then collides with a bubble of White vacuum. The true vacuum is bounded by a hat but the White vacuum terminates in a space-like surface. Some generators of the hat intersect the Black vacuum and some intersect the White. Thus Σ , shown as the red curve, is composed of two regions.

could do any observation. In fact if we define Census Takers by the condition that they eventually reach the Census Bureau, then they simply never enter W.

From the field theory point of view this example leads to a paradox. Naively, it seems that once a W patch forms on Σ , a B region cannot form inside it. A constraint of this type on field configurations would obviously violate the rules of quantum field theory; topologically (on a sphere) there is no difference between a small W patch in a B background, and a small B patch in a W background. Thus field configurations must exist in which a W region has smaller Black spots inside it. There is no way consistent with locality and unitarity to forbid bits of B in regions of W.

Fortunately the same conclusion is reached from the bulk point of view. The rules of tunneling transitions require that if the transition $B \to W$ is possible, so must be the transition $W \to B$, although the probability for the latter would be smaller (by a large density of states ratio). Thus one must expect B to invade regions of W.

As the Census Taker time advances he will see smaller and smaller spots of each type. If one assumes that there are no operators of dimension zero, then the pattern should fade into a homogeneous average grey, although under the conditions I described it will be almost White.

The natural interpretation is that the boundary field theory has two phases of different free energy, the B free energy being larger than that of W. The dominant configuration would be the ones of lower free energy with occasional fluctuations to higher free energy.

8.2 The Persistence of Memory

The "Persistence of Memory" reported in [20] had nothing to do with whether or not the Census Taker's sees a fading signal: Garriga, Guth, and Vilenkin were not speaking about Census Taker time at all. They were referring to the fact that no matter how long after the start of eternal inflation a bubble nucleates, it will remember the symmetry breaking imposed by the initial conditions; not whether the signal fades with T_{CT} . Returning to figure 12, one might ask why no bubble formed along the red trajectory in the infinitely remote past. The authors of [20] argue that eternal inflation does not make sense without an initial condition specifying a past surface on which no bubbles had yet formed. That surface invariably breaks the O(3,1) symmetry and distinguishes a "preferred Census Taker" who is at rest in the frame of the initial surface. He alone sees an isotropic

sky whereas all the other Census Takers see non-zero anisotropy. What's more the effect persists no matter how late the nucleation takes place.

As before, when the Census Taker's time advances, the asymmetry should become diluted if there are no dimension zero operators, but the existence of a preferred Census Taker at finite time makes this symmetry breaking seem different than what we have discussed up to now.

Let us consider how this phenomenon fits together with the RG flow discussed earlier¹¹. Begin by considering the behavior for finite δ in the limit of small a. It is reasonable to suppose that in integrating out the many scales between a and δ , the theory would run to a fixed point. Now recall that this is the limit of very large T. If in fact the theory has run to a fixed point it will be conformally invariant. Thus we expect that the symmetry O(3,1) will be unbroken at very late time.

On the other hand consider the situation of δ/a near 1. The reference and bare scales are very close and very few degrees of freedom have been integrated out. There is no reason why the effective action should be near a fixed point. The implication is that at very early time (recall, $\delta/a = e^T$) the physics on a fixed time slice will not be conformally invariant. Near the beginning of an RG flow the effective action is strongly dependent on the bare theory. The implication of a breakdown of conformal symmetry is that there is no symmetry between Census Takers at different locations in space. In such situations the center of the (deformed) anti de Sitter space is, indeed, special.

Shenker and I suggest that the GGV boundary condition at the onset of eternal inflation is the same thing as the initial condition on the RG flow. In other words, varying the GGV boundary condition is no different from varying the bare fishnet theory.

Is it possible to tune the bare action so that the theory starts out at the fixed point? If this were so, it would be an initial condition that allowed exact conformal invariance for all time. Of course it would involve an infinite amount of fine tuning and is probably not reasonable. But there may be reasons to doubt that it is possible altogether, even though in a conventional lattice theory it is possible.

The difficulty is that the bare and renormalized theories are fundamentally different. The bare theory is defined on a variable fishnet whose connectivity is part of the dynamical

¹¹These observations are based on work with Steve Shenker.

degrees of freedom. The renormalized theory is defined on the fixed reference lattice. The average properties of the underlying dynamical fishnet are replaced by conventional fields on the reference lattice. Under these circumstances it is hard to imagine what it would mean to tune the bare theory to an exact fixed point.

The example of the previous subsection involving two false vacua, B and W, raises some interesting questions. First imagine starting with GGV boundary conditions such that, on some past space-like surface, the vacuum is pure Black and that a bubble of true vacuum nucleates in that environment. Naively the boundary is mostly black. That means that *in the boundary theory* the free energy of Black must be lower than that of White.

But we argued earlier that white instantons will eventually fill Σ with an almost white, very light grey color, exactly as if the initial GGV condition were White. That means that White must have the lower potential energy. What then is the meaning of the early dominance of B from the 2D field theory viewpoint?

The point is that it is possible for two rather different bare actions to be in the same broad basin of attraction and flow to the same fixed point. The case of Black GGV conditions corresponds to a bare starting point (in the space of couplings) where the potential of B is lower than W. During the course of the flow to the fixed point the potential changes so that at the fixed-point W has the lower energy.

On the other hand, White GGV initial conditions corresponds to starting the flow at a different bare point—perhaps closer to the fixed point—where the potential of W is lower.

This picture suggests a powerful principle. Start with the space of two-dimensional actions, which is broad enough to contain a very large Landscape of 2D theories. With enough fields and couplings the space could probably contain everything. As Wilson explained [31], the space divides itself into basins of attraction. Each initial state of the universe is described either as a GGV initial condition, or as a bare starting point for an RG flow. The endpoints of these flows correspond to the possible final states-the hats—that the Census Taker can end up in.

We have not exhausted all the kinds of collisions that can occur—in particular collisions with singular, negative cosmological constant vacua. A particularly thorny situation results if there is a BPS domain wall between the negative and zero CC bubbles, then as

shown by Freivogel, Horowitz, and Shenker [36] the entire hat may disappear in a catastrophic crunch. A possible interpretation is that the catastrophe is due to the existence of a relevant operator which destabilizes the fixed point. These and other issues will be taken up in a paper with Shenker.

8.3 A Remark about Supersymmetry

Most likely, the only 4D vacua with exactly vanishing cosmological constant are supersymmetric. Does that mean that the boundary theory on Σ is also supersymmetric? The answer is no: correlators on Σ are largely determined by the properties of the non-supersymmetric Ancestor vacuum. For example, the gravitino will be massive in the Ancestor, and the methods of [19] would give different dimensions, Δ , for the graviton and gravitino fluctuations.

In fact Σ is contiguous with the de Sitter Ancestor and has every reason to strongly feel the supersymmetry breaking. It is the region close to the tip of the hat where the physics should be dominated by the properties of the supersymmetric terminal vacuum. If one looks at the expansion (5.9), it is clear that the tip of the hat is dominated by the asymptotically high dimensional terms. Thus we expect supersymmetry to manifests itself asymptotically, in the spectrum and operator products of high dimensional operators.

8.4 Flattened Hats and other Tragedies

In a broad sense this paper is about phenomenology: the Census Taker could be us. If we lived in an ideal thin-wall hat we would see, spread across the sky, correlation functions of a holographic quantum field theory. We could measure the dimensions of operators both by the time dependence of the received signals, and their angular dependence. Bubble collisions would appear as patches resembling instantons.

Unfortunately (or perhaps fortunately) we are insulated from these effects by two forms of inflation—the slow-roll inflation that took place shortly after bubble nucleation—and the current accelerated expansion of the universe. The latter means that we don't live in a true hatted geometry. Rather we live in a flattened hat, at least if we ignore the final decay to a terminal vacuum.

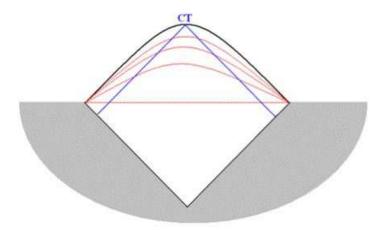


Figure 15: If a CDL bubble leads to a vacuum with a small positive cosmological constant, the hat is replaced by a rounded space-like surface. The result is that no Census Taker can look back to Σ .

The Penrose diagram in figure 15 shows an an Ancestor, with large vacuum energy, decaying to a vacuum with a very small cosmological constant. The important new feature is that the hat is replaced by a space-like future infinity. Consider the Census Taker's final observations as he arrives at the flattened hat. It is obvious from figure 18 that he cannot look back to Σ . His past light cone is at a finite value of T^+ . Thus for each time-slice T, there is a maximum radial variable $R = R_0(T)$ within his ken, no matter how long he waits. In other words there is an unavoidable ultraviolet cutoff. It is completely evident that a final de Sitter bubble must be described by a theory with no continuum limit; in other words not only a non-local theory, but one with no ultraviolet completion.

This suggests that de Sitter Space may have an intrinsic imprecision. Indeed, as Seiberg has emphasized, the idea of a metastable vacuum is imprecise, even in condensed matter physics where they are common¹².

The more tragic fact is that all of the memory of a past bubble may, for observational purposes, be erased by the slow-roll inflation that took place shortly after the Coleman De Luccia tunneling event—unless it lasted for the minimum permitted number of e-foldings [16]. In principle the effects are imprinted on the sky, but in an exponentially diluted form.

8.5 Note About W

What I have described is only half the story: the half involving S, the real part of $\log \Psi$. The other half involves W, the phase of the wave function. Knowing S is enough to compute the expectation values of the fields y at a given value of time, or, strictly speaking, at a value of the scale factor. Scanning over scale factors can be done by varying the Liouville cosmological constant.

However, quantum mechanics cannot be complete without the phase of the wave function. In particular, the values of conjugate momenta requires knowledge of W. The same is true for products of fields at different times.

9 Some Conclusions

I've given some circumstantial evidence that there is a duality between cosmology on the Landscape, and two-dimensional conformal field theory, with a Liouville field. The

¹²I am grateful to Nathan Seiberg for discussions on this point.

data that supports the theory are the computations done in [19], but more importantly, a compelling physical picture accompanies the data. The most pertinent observation is that the sky is a two-sphere. Moreover, it is covered with interesting observable correlations; in principle we can look back through the surface of last scattering and observe these correlations at any past time. The Liouville field, or alternately, the Liouville cosmological constant (not to be confused with the four-dimensional cosmological constant) is the dual of that time, along the observers backward light-cone.

As we view the deep sky from increasingly late times, we can in principle see greater angular detail on Σ_0 , i.e., spatial "almost-infinity". The increasing angular resolution defines a renormalization group flow that begins with some bare action, and ends at an infrared fixed point. The starting point of the flow is equivalent to the boundary condition, whose memory, Garriga, Guth, and Vilenkin have argued, is persistent. The phenomena of symmetry breaking—the picking out of a special Census Taker at the center of things—by the GGV's initial condition, is equivalent to the breaking of conformal symmetry, at the start of a RG flow. As in anti de Sitter space, breaking conformal symmetry makes the center of ADS (at R=0) special.

Our considerations are based on the Holographic Principle but with a new twist. The asymptotic warmness of space requires a field in order to represent geometric fluctuations at infinity. By now we are used to one or more spatial directions emerging holographically, but this new Liouville degree of freedom creates a new emergence—of time.

Examples of cosmic phenomena that can be simply interpreted as two-dimensional field theory phenomena are the bubble collisions of Guth and Weinberg [35], which appear as instantons on the two-dimensional sky; the fading of the initial conditions with Census Taker time is connected with the spectrum of conformal dimensions—in particular the lack of dimension zero scalars; and ordinary slow-roll inflation corresponds to an exponential decrease in the Zamolodchikov c-function [33]. It will be interesting to try to interpret CMB fluctuations in this language but this has not been studied—at least that I know of.

10 Warning to All Census Takers

Memo from the Director

Keep in mind that when you look back toward Σ , what you see will be influenced not

only by conditions on the regulated boundary, but also by the gravitational field between you and Σ_0 . Angular separations detected by you, must be corrected for nearby gravitational distortions such as lensing and gravitational waves. Please make all corrections before reporting your data.

11 Acknowledgments

I am enormously grateful to my collaborators, Ben Freivogel, Yasuhiro Sekino, Chen-Pin Yeh, Steve Shenker, and Alex Maloney. This lecture represents an ongoing discussion with them. Any (positive) reference to it should be accompanied by references to [1] and [19].

Discussions with Larry Guth and Alan Guth were important to my understanding of the breaking of symmetries in Hyperbolic space.

I would also like to acknowledge helpful conversations with Raphael Bousso, another believer in the causal patch; Simeon Hellerman; and Matt Kleban for various insights; and finally, Nathan Seiberg for explaining the intrinsic imprecision of metastable states.

Any conceptual or computational errors in the paper are of course my own.

References

- [1] A. Maloney, S. Shenker, L. Susskind, unpublished
- [2] N. Kaloper, M. Kleban and L. Sorbo, "Observational implications of cosmological event horizons," Phys. Lett. B **600**, 7 (2004) [arXiv:astro-ph/0406099]; M. Kleban, "Inflation unloaded," [arXiv:hep-th/0412055].
- [3] Raphael Bousso, Joseph Polchinski, Quantization of Four-form Fluxes and Dynamical Neutralization of the Cosmological Constant, hep-th/0004134, JHEP 0006 (2000) 006
- [4] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi: De Sitter vacua in string theory. Phys. Rev. D 68, 046005 (2003), hep-th/0301240.
- [5] L. Susskind, The Anthropic Landscape of String Theory [arXiv:hep-th/0302219]

- [6] Michael R. Douglas, The statistics of string/M theory vacua, JHEP 0305 (2003) 046, hep-th/0303194
- [7] L. Susskind, L. Thorlacius, J. Uglum, The Stretched Horizon and Black Hole Complementarity, Phys.Rev. D48 (1993) 3743-3761, arXiv:hep-th/9306069
- [8] Y. Kiem, Herman L. Verlinde, Erik P. Verlinde Quantum horizons and complementarity. *Trieste 1994, High energy physics and cosmology* 586-606.
- [9] Tom Banks, Willy Fischler M theory observables for cosmological space-times. hep-th/0102077
- [10] G. 't Hooft, Dimensional Reduction in Quantum Gravity, Dimensional Reduction in Quantum Gravity, gr-qc/9310026
 L. Susskind, The World as a Hologram, J.Math.Phys. 36 (1995) 6377-6396, hep-th/9409089
- [11] Juan Maldacena, The Large N Limit Of Superconformal Field Theories And Supergravity, Int.J.Theor.Phys.38:1113-1133,1999, hep-th/9711200
- [12] Edward Witten, Anti-de Sitter space and holography. Adv.Theor.Math.Phys.2:253-291,1998, hep-th/9802150
- [13] Susskind, L., and E. Witten, 1998, The holographic bound in Anti-de Sitter space, hep-th/9805114.
- [14] J.R. Gott, Nature 295, 304 (1982); A.H. Guth and E. Weinberg, Nucl. Phys. B212, 321 (1983); P.J. Steinhardt, in The Very Early Universe, ed. by G.W. Gibbons, S.W.Hawking and S.T.C. Siklos (Cambridge University Press, Cambridge, 1983); J.R. Gott and T.S. Statler, Phys. Lett. 136B, 157 (1984); M. Bucher, A.S. Linde, A D 1986 Eternal chaotic inflation, Mod. Phys. Lett. A 1, 81-5; Linde, A D 1986 Eternally existing selfreproducing chaotic inflationary universe, Phys.
- [15] S. R. Coleman and F. De Luccia, "Gravitational Effects On And Of Vacuum Decay," Phys. Rev. D 21, 3305 (1980).
- [16] Ben Freivogel, Matthew Kleban, Maria Rodriguez Martinez, Leonard Susskind, Observational Consequences of a Landscape, JHEP 0603 (2006) 039 , arXiv:hep-th/0505232

- [17] S. W. Hawking, Comm Math. Phys. 43 (1975), 199, Phys. Rev. D14 (1976), 2460.
- [18] Raphael Bousso, Ben Freivogel, I-Sheng Yang, Eternal Inflation: The Inside Story, Phys.Rev. D74 (2006) 103516 arXiv:hep-th/0606114
- [19] Ben Freivogel, Yasuhiro Sekino, Leonard Susskind, Chen-Pin Yeh, "A Holographic Framework for Eternal Inflation," Phys.Rev. D74 (2006) hep-th/0606204
- [20] Jaume Garriga, Alan H. Guth, Alexander Vilenkin, Eternal inflation, bubble collisions, and the persistence of memory, arXiv:hep-th/0612242
- [21] Fischler, W., and L. Susskind, 1998, Holography and cosmology, hep-th/9806039.
- [22] R. Bousso, The holographic principle, Rev. Mod. Phys. 74 (2002) 825 [arXiv:hep-th/0203101]
- [23] R. Arnowitt, S. Deser and C. W. Misner, Gravitation: an introduction to current research, Louis Witten ed. (Wiley 1962), Chapter 7, pp 227-265, [arXiv:gr-qc/0405109].
- [24] Tom Banks , W. Fischler , Leonard Susskind, Quantum Cosmology In (2+1)-Dimensions And (3+1)-Dimensions. Nucl. Phys. B262:159,1985.
- [25] Tom Banks, T C P, Quantum Gravity, the Cosmological Constant and All That... Nucl. Phys. B249:332,1985.
- [26] A. Kraemmer, H.B. Nielsen and L. Susskind Nucl. Phys. B28 (1971), p. 34.
- [27] G 't Hooft, Nucl.Phys.B72:461,1974
- [28] Alexander M. Polyakov, Quantum Geometry of Bosonic Strings. Phys.Lett.B103:207-210,1981.
- [29] Michael R. Douglas, Stephen H. Shenker, Nucl. Phys. B335:635,1990.
- [30] David J. Gross, Alexander A. Migdal, Phys.Rev.Lett.64:127,1990.
- [31] K. G. Wilson, Rev.Mod.Phys.47:773,1975
- [32] J. Polchinski, Nucl.Phys.B231:269-295,1984
- [33] A.B Zamolodchikov, "Irreversibility' Of The Flux Of The Renormalization Group In A 2-D Field Theory, JETP Lett.43:730-732,1986.

- [34] Andrew Strominger, The dS/CFT Correspondence, arXiv:hep-th/0106113, Journal-ref: JHEP 0110 (2001) 034
- [35] Alan H. Guth , Erick J. Weinberg, Could the Universe Have Recovered from a Slow First Order Phase Transition? Published in Nucl.Phys.B212:321,1983.
- [36] Ben Freivogel, Gary T. Horowitz, Stephen Shenker, Colliding with a Crunching Bubble, arXiv:hep-th/0703146