SU-ITP 99/9
NSF-ITP-10-99

Puzzles and Paradoxes About Holography

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Abstract

The holographic connection between large N Super Yang Mills theory and gravity in anti deSitter space requires some very paradoxical behaviors of the SYM theory in the limit that the curvature of the AdS geometry becomes small. The paradoxes can be resolved by assuming a very rich collection of hidden degrees of freedom of the SYM theory which store information but give rise to no local energy density. These degrees of freedom are needed to make possible sudden apparently acausal energy momentum flows. Such behavior would be impossible in classical field theory as a consequence of the positivity of the energy density . We believe these degrees of freedom are the key to understanding the holographic coding of detailed bulk information.

1 Introduction and Review

That there is some sort of holographic [1, 2] correspondence between maximally supersymmetric $SU(N)$ Yang Mills theory and supergravity or string theory on $AdS_5 \times S^5$ [3, 4, 5] has been established beyond reasonable doubt. In this paper we will assume the correspondence in the strongest sense, namely, that SYM theory and IIB string theory are equivalent for all values of N and gauge coupling constant.

Equivalence between theories in different dimensions immediately raises questions about how detailed bulk information in one theory can be completely coded in lower dimensional degrees of freedom. Despite the large amount of evidence that we have for the AdS/CFT correspondence, there is not yet any direct translation of the configurations of one theory to the other. We will see that the existence of such a dictionary requires behavior for the large N limit of SYM theory which seems very unusual and even unphysical, but we will argue that it is neither.

The parameters of the SYM theory are the gauge coupling constant g and the number of colors N. These are related to the radius of curvature R of the $AdS_5 \times S^5$, the string length scale l_s, and the string coupling g_s by

$$\begin{aligned} g_s &= g^2 \\ R &= l_s (N g^2)^{1/4} \ . \end{aligned} \tag{1.1}$$

Throughout we will neglect constants of order one.

The features of the correspondence that are most relevant to our discussion are the following:

1. Certain local gauge invariant SYM operators correspond to bulk supergravity fields evaluated at the boundary of the AdS space. That is, the expectation value of the SYM operater is determined from the boundary value of the field, as explained more fully in section 3. For example the energy momentum tensor $T_{\mu\nu}$ of the SYM theory corresponds to the metric perturbation $h_{\mu\nu}$. Similarly $F_{\mu\nu}F^{\mu\nu}$ corresponds to the dilaton field ϕ.

2. The ultraviolet-infrared connection [6] relates the short wave length ultraviolet modes of SYM theory to the supergravity modes near the boundary of AdS space. To be more precise let us introduce "cavity coordinates" in which the AdS metric ds^2 is written in terms of a dimensionless metric dS^2 and the radius of curvature R

$$\begin{aligned} ds^2 &= R^2 dS^2 \\ dS^2 &= \left(\frac{1+r^2}{1-r^2}\right)^2 dt^2 - \left(\frac{2}{1-r^2}\right)^2 (dr^2 + r^2 d\Omega) \ . \end{aligned} \tag{1.2}$$

The coordinates t, r are dimensionless. The SYM theory will be thought of as living on the dimensionless unit sphere Ω times the dimensionless time t. All quantities such as energy, distance, and time in the SYM theory are regarded as dimensionless. To relate them to corresponding bulk quantities the conversion factor is R. Thus for example, a time interval δt corresponds to a proper interval $R\delta t$ in the bulk theory. Similarly an energy E_{SYM} in the SYM theory is related to the bulk energy by $E_{\mathrm{SYM}} = E_{\mathrm{bulk}} R$.

The UV–IR connection states that supergravity degrees of freedom at $1 - r = \delta$ correspond to SYM degrees of freedom with wavelength $\sim \delta$. The UV–IR connection is at the heart of the holographic requirement that the number of degrees of freedom should be of order the area of the boundary measured in Planck units. It also suggests that physical systems near the center of the AdS space ($r = 0$) should be described by modes of the SYM theory of the longest wavelength, that is the homogeneous modes on Ω.

3. The existence of a flat space limit [7, 8]. This limit involves $N \to \infty$ but is not the 't Hooft limit in which $g^2 N$ and the energy is kept fixed. The flat space limit is given by

$$\begin{array}{lcl} g^2 & \to & \text{fixed} \\ N & \to & \infty \ . \end{array} \tag{1.3}$$

In addition, all energy scales are kept fixed in string units. This means that the dimensionless SYM energy scales like R, or using eq. (1.1)

$$E_{\mathrm{SYM}} \to N^{1/4} \ . \tag{1.4}$$

As argued in [7, 8] an S-matrix can be defined in this limit in terms of SYM correlation functions.[1] We will outline the construction but the reader is referred to the references for details.

A bulk massless particle of energy k is is described by a SYM excitation of energy ω given by

$$\omega = Rk = (Ng^2)^{1/4} l_s k \ . \tag{1.5}$$

We require the particles to collide within a space-time region called the "lab". The lab is centered at $t = r = 0$ and has a large but fixed size L in the bulk theory.[2] In terms of dimensionless coordinates the lab dimensions are

$$\delta t \sim \delta r \sim L/R \ . \tag{1.6}$$

Since L is fixed in string units

$$\delta t \sim \delta r \sim (Ng^2)^{-1/4} \ . \tag{1.7}$$

Creation and annihilation operators for emitting and absorbing particles at the AdS boundary can be defined. In order that the particles pass through

[1] See refs. [9] for further developments.

[2] For generic wavepackets the analysis in refs. [7, 8] shows that they grow as $N^{1/8}$ due to geometric optics effects, but for simplicity we imagine special packets chosen to intersect in a volume of order N^0. The size and duration of any collision process is determined by the external energies and so is of order N^0.

the lab they must be emitted at time $t_{\rm in} \approx -\pi/2$. The collision process lasts for a fixed time in string units which means a dimensionless time of order $N^{-1/4}$. Thus in the dimensionless time of the SYM theory the duration of the collision becomes negligible as $N \to \infty$. The outgoing particles will will arrive at the boundary at time $t_{\rm out} \approx \pi/2$. The graviton creation operator corresponding to this situation are given by

$$A_{\rm in} = \int dt\, d^3x\, T_{\mu\nu}(x,t) e^{i\omega t} G(x - x_0, t + \pi/2) \tag{1.8}$$

where the integration is over the boundary and x_0 is the point from which the graviton is emitted. Annihilation operators $A_{\rm out}$ at $t = \pi/2$ are defined in a similar manner. In order to make sure that the particles pass through the origin the functions G in eq. (1.8) must not be too sharply peaked at x_0. We refer the reader to [7, 8] for a discussion of this point.

Let us consider a state involving a packet of gravitons emitted at $t = -\pi/2$ from the boundary at point x_0, in a uniform state on the S^5. The packet is very well concentrated in the dimensionless coordinates t, r, Ω. In order to translate this into the SYM theory we need to compute the gravitational field of such a source. Fortunately this has been done in [10]. The result is an AdS generalization of the Aichelburg Sexl metric and like that metric, it is described by a shock wave that propagates in the bulk with the particle. The thickness of the shock in dimensionless coordinates tends to zero with the spread of the packet. The intersection of the shock wave with the boundary forms a 2-sphere or shell which expands away from the point x_0 with the speed of light. Both in front of the shell and behind it $\langle T_{\mu\nu} \rangle$ vanishes. The shell expands to its maximum size at $t = 0$ and then contracts to the antipodal point at $t = +\pi/2$. Thus the expectation value of the SYM energy momentum tensor has its support on such a moving shell and is zero everywhere else.

The description given above is somewhat surprising in view of the UV–IR connection. We might have expected that in the SYM description the energy would be transferred from the short wave length modes of the field theory to long wave lengths as the graviton moves toward $r = 0$. In this event the sharp features of the shell should have dissipated. However the energy stays concentrated in a thin shell who's thickness tends to zero with N. This in itself is somewhat puzzling.

2 History of a Collision

Paradoxes become apparent when we consider the collision of two packets. The packets are emitted from points x_1, x_2 at time $t = -\pi/2$. The first thing to notice is that from the viewpoint of the boundary SYM theory the behavior of the system after $t = 0$ becomes infinitely sensitive to the details of the the emission process. As an example suppose x_1 and x_2 are separated by an angle of 90 degrees on the 3-sphere Ω. If the two particles are emitted at the same time they will reach $r = 0$ simultaneously and collide. But suppose the emission processes are separated by time $\epsilon \ll 1$. In this case the arrivals will be separated by a time *in string units* of order $\epsilon(Ng^2)^{1/4}$. This means that if $\epsilon > (Ng^2)^{-1/4}$ the particles will miss each other and pass essentially unscattered. On the other hand if $\epsilon < (Ng^2)^{-1/4}$ a collision will take place leading to a very different final state. In other words shifting the parameters of the emission process by tiny amounts will lead to large differences in the outcome.

Consider a process in which two packets of fixed energy in string units are emitted from diametrically opposing points in such a way that they pass through the lab. In the SYM theory the process starts out as a pair of expanding thin shells of energy. At time $t = 0$ the shells meet. Now from what was said above, one might expect the evolution just after $t = 0$ to be supersensitive to to the initial conditions. However this is not true. In fact the two shells just pass through one another without any apparent interaction. The reason is that as $R \to \infty$, points near the boundary are so far (again in string units) from the sources that the gravitational field equations linearize. The energy-momentum continues to be concentrated on the thin shells which are now contracting toward the points antipodal from where they originated. Not only is $\langle T_{\mu\nu} \rangle$ zero everywhere off the shells but so are all other SYM fields that correspond to classical supergravity fields.

We now come to a critical question. The vanishing of $\langle T_{\mu\nu} \rangle$ in classical field theory would indicate that the local state of the system is vacuum-like. In other words all fields or functionals of fields supported off the shells should have their vacuum values. As we shall see, this can not be true in the quantum theory. We will find that the after the shells pass through each other the region between them must be excited away from the local vacuum configuration despite the fact that the expectation value of the energy density (as well as the value of every SYM field which corresponds to classical supergravity) vanishes.

A simple illustrative example is the case where the two packets are prepared so as to collide head-on at $t = r = 0$. Assume the particles have a fixed energy which is much larger than the 10-dimensional Planck mass. In this case they will form a 10-dimensional Schwarzschild black hole. Not all the energy will go into the black hole but much of it will continue to propagate as gravitational bremsstrahlung. According to the assumption of a flat space limit [7, 8] the percentage of energy trapped in the black hole can be calculated in the flat space limit [11]. The black hole quickly becomes spherically symmetric and then decays by Hawking evaporation. The entire history of the black hole lasts a fixed time in string units and therefore a dimensionless time which tends to zero like $N^{-1/4}$.

How does the creation and evaporation of the black hole affect the metric and other supergravity fields at the AdS boundary? The answer is that it doesn't, at least at first. In fact the supergravity fields do not respond until light has had a chance to propagate from $r = 0$ to the boundary. The evaporating black hole sends out a spherically symmetric signal which arrives at the boundary at $t = \pi/2$. The arrival of the signal is very sudden, occupying a time $\delta t \sim (Ng^2)^{-1/4}$. At this time the entire boundary suddenly "lights up" with a spherically symmetric distribution of energy which in total equals the mass of the black hole. In other words a fraction of the energy originally stored in the collapsing shells very quickly flows and is redistributed into a homogeneous component. We will call this phenomenon "light-up."

This behavior seems extremely bizarre. The instantaneous rearrangement of energy appears to violate causality. However this may not be so. To better understand it we will give describe an analogous example involving a sudden flow of electric charge. Consider an example in which initially there is a concentration of charge in some region R_1. At time $t = 0$ the charge is found to disappear from R_1 and reappear at R_2 which is outside the forward light cone of R_1. To see how this can happen, imagine a wire connecting R_1 and R_2. The wire is full of electrons and positive ions so that it is electrically neutral. Now we prearrange observers at each point of the wire so that at $t = 0$ they move each electron slightly toward R_2. The result is a sudden appearance of charge at the ends with no charge density ever occurring anywhere else. Now if there was already a charge at R_1 it would be cancelled by the new charge at that point. The net result would be a sudden redistribution of charge.

Two ingredients are necessary for such behavior. The first is that the current vector j^μ be spacelike. Since the charge density on the wire is always zero it is clear that the current is purely in the spacelike direction. This also means that in some frame the charge density was negative. This of course is not a difficulty since charge density can be either positive or negative.

The other ingredient is *prearrangement*. The physical conditions along the wire must include agents with synchronized clocks that instructed in advance to act simultaneously.

The sudden flow of energy requires the same two ingredients. In order that the energy is rearranged so suddenly the flux of energy $\langle T^{0i} \rangle$ must be much larger than the energy density itself. This means that the energy density can be made negative by a Lorentz boost. However, unlike electric charge, energy is not allowed to be negative in SYM theory. Since in classical SYM theory the energy density is positive this kind of flow of energy is absolutely forbidden in the classical theory. However quantum theory allows local negative energy densities as long as they are (over)compensated by nearby positive energy density [12]. In the next section we will analyze this in more detail and show that the bounds as in ref. [12] are consistent with the behavior required of the SYM theory.

A second puzzle concerns locality. Let us suppose that before light-up the region between the separating shells is physically indistinguishable from the vacuum of the SYM theory. By this we mean that all expectation values of functionals of fields in this region are identical to their vacuum values. Then the light-up is impossible. To see this we consider a point (x, t) on the boundary just after light-up. The point is not near the points where the shells are localized. The SYM Heisenberg equations of motion can be used to express the energy density at this point in terms of local fields at a time just before light-up. Furthermore, causality requires that the only fields that can be involved are in the region between the shells where we have assumed vacuum conditions. It follows that the energy density at (x, t) must be the same as for the vacuum, that is, zero.

The resolution of these paradoxes, assuming the correspondence is really as strong as we believe, must be that the region between the shells at time $0 < t < \pi/2$ must not be vacuum-like even though the expectation value of the energy-momentum tensor vanishes. Thus we are forced to postulate that in a region in which $\langle T_{\mu\nu} \rangle = \langle F^{\mu\nu} F_{\mu\nu} \rangle = \ldots = 0$, the vacuum is excited to a non-vacuum-like local state which provides the precursor for the later event that we called light-up. The precursor fields must play the role of the prearranged agents which simultaneously move charges in the electric example. Furthermore there must be a very rich manifold of such precursor configurations. To see this suppose we change the initial emission parameters by a small amount of order $N^{-1/4}$. As we have seen this can lead to a very large change in the results of the collision. For example such a change can cause an increase of the impact parameter so that a peripheral grazing collision results. In this case the particles may get deflected through a small angle. Again, the news of the collision does not arrive at the boundary until $t = \pi/2$. As before the energy must suddenly rearrange but this time the result is not a spherically symmetric component but a new pair of localized small shells at shifted positions. Also as before, the information must be locally stored in a configuration with vanishing expectation value for the energy momentum tensor. Evidently all the local physical processes that can take place in the lab are coded in precursor configurations.

3 Principal, Interest, and Causality

In this section we analyze further the necessary negative energy density and prearrangement in the SYM theory.

The gravitational field after the collision can be regarded as a superposition of the spherically symmetric black hole field with an outgoing pulse of gravitational radiation. Similarly the gauge theory is in an approximate product state. The black hole corresponds to a static positive $\langle T^{00} \rangle$, so the negative $\langle T^{00}\rangle$ and seeming acausality must come from the outgoing pulse. Thus they should be seen already in the free field approximation to the bulk fields.

The correspondence between the bulk fields and boundary operators is of the form [4, 5]

$$\left\langle \exp \int_{S^3\times R} h_{\mu\nu} T^{\mu\nu} \right\rangle = Z_g \ . \tag{3.1}$$

The bulk partition function Z_g is evaluated with the boundary condition

$$\gamma_{\mu\nu} \ \equiv \ g_{\mu\nu} - \overline{g}_{\mu\nu} \ \to \ (1-r)^{-2} R^2 h_{\mu\nu} \ , \tag{3.2}$$

with $\overline{g}_{\mu\nu}$ being the unperturbed metric (1.2). Varying both sides of (3.1) with respect to h, the variation of Z_g is a surface term and so

$$T_{\mu\nu} \sim \frac{R}{G} \lim_{r\to 1} (1-r)^{-2} \gamma_{\mu\nu} \ , \tag{3.3}$$

where divergent terms in the limit are dropped. This is the rough form, a more precise result being given in ref. [13]. It holds as an operator relation [14, 15], at least in the classical limit.

Consider now a linearized supergravity solution of frequency ω. The relation (3.3) implies that the energy density in the SYM theory is also oscillatory, and so is negative as often as positive. Intuitively this should not happen, as a state whose space-time averaged energy density is zero should be the vacuum. Indeed, as explained in ref. [12], a "loan" in the form of negative energy density must not only be repaid, but repaid with interest. To see this from the gravity point of view we must include the first correction to the linearized solution. The oscillating solution has energy density, producing a second order correction to the metric. The angle-averaged correction to γ_{tt} is $GE(1-r)^2/R$ where E is the energy in SYM units. Evaluating the energy of the linearized solution,

$$E \sim \frac{1}{G} \int d^4x \sqrt{-g}\, \partial_t \gamma_{\mu\nu} \partial^t \gamma^{\mu\nu} = O(\omega^2 \gamma^2 / GR) \ , \tag{3.4}$$

gives the "interest"

$$\mathcal{I} = T^{(2)}_{\mu\nu} = O(\omega^2 \gamma^2_{\mu\nu} / GR) \ . \tag{3.5}$$

This is to be compared with the "principal" (3.3),

$$\mathcal{P} = T^{(1)}_{\mu\nu} = O(R \gamma_{\mu\nu} / G) \ . \tag{3.6}$$

The ratio

$$\frac{\mathcal{I}}{\mathcal{P}^2} = O(\omega^2 G / R^3) = O(\omega^2 G_{10} / R^8) = O(\omega^2 / N^2) \tag{3.7}$$

is independent of the size of the perturbation. We will see that this result is quite consistent with general principles of quantum field theory. The fact that is decreases with the number N^2 of quantum fields fits with the fact that negative energy density is a quantum effect. In particular, assume that for a single quantum field the ratio i/p^2 is independent of the perturbation. Then

$$\frac{\mathcal{I}}{\mathcal{P}^2} = \frac{N^2 i}{(N^2 p)^2} \propto N^{-2} \ . \tag{3.8}$$

Now let us see in more detail that the behavior we have found in the SYM theory can be reproduced in a theory of N^2 *free* scalar fields ϕ_{ij}. Consider the "squeezed" state

$$|\psi\rangle = \exp\biggl[\frac{1}{2}\int d^3\mathbf{k}\, d^3\mathbf{k}'\, f(\mathbf{k},\mathbf{k}') a^\dagger_{ij}(\mathbf{k}) a^\dagger_{ji}(\mathbf{k}')\biggr]|0\rangle \ . \tag{3.9}$$

Then

$$\langle\psi|T_{\mu\nu}|\psi\rangle = \frac{N^2}{2}\Bigl(\partial_\mu\partial'_\nu - \frac{1}{2}\eta_{\mu\nu}\partial^\rho\partial'_\rho\Bigr)F(\mathbf{x},t,\mathbf{x}',t')|_{\mathbf{x}=\mathbf{x}',t=t'} + O(f^2N^2) \tag{3.10}$$

where

$$F(\mathbf{x},t,\mathbf{x}',t') = \int \frac{d^3\mathbf{k}\, d^3\mathbf{k}'}{(2\omega_k 2\omega_{k'})^{1/2}}\, f(\mathbf{k},\mathbf{k}') \exp(i\mathbf{k}\cdot\mathbf{x} + i\mathbf{k}'\cdot\mathbf{x}' - i\omega_k t - i\omega_{k'}t') \ . \tag{3.11}$$

This effect of order f comes from contraction of two lowering operators in $T_{\mu\nu}$ with two raising operators in $|\psi\rangle$ or vice versa with $\langle\psi|$; this is the principal $\mathcal{P}$. The interest $\mathcal{I}$ appears at order f^2, from terms in $T_{\mu\nu}$ with one raising and one lowering operator, and has a positive average.

This free field example displays the following features that were found to be necessary in the SYM case:

- The principal has nonzero frequency $\omega_k + \omega_{k'}$ and so averages to zero.
- The ratio $\mathcal{I}/\mathcal{P}^2$ is independent of the strength f of the perturbation, and scales as N^{-2}.
- *The principal is acausal.* The phase velocity

  $$\frac{\omega_k + \omega_{k'}}{|\mathbf{k}+\mathbf{k}'|} = \frac{|\mathbf{k}| + |\mathbf{k}'|}{|\mathbf{k}+\mathbf{k}'|} \geq 1 \tag{3.12}$$

  is superluminal. Moreover this relation is homogeneous so the group velocity is the same. Thus the principal can suddenly become nonzero at a spacetime point even if it vanishes in and on the entire past light cone.

Thus the properties required of the SYM theory, while unfamiliar, are consistent with quantum field theory; they are not even associated with large 't Hooft parameter.

In this free field example we can analyze the violation of causality. Of course there must be some departure from vacuum in the past light-cone of any point where the principal is nonvanishing. How is this to be detected? It is implausible that the interest is the key, as it is too small in magnitude and in number of degrees of freedom. To see the answer, note that the squeezed state consists of correlated pairs of particles, and the principal arises only when these are coincident. If they are approaching one another in separated packets, the principal vanishes. To detect the disturbance we need a *nonlocal* but $U(N)$ invariant observable such as

$$\langle\psi|\phi_{ij}(\mathbf{x},t)\phi_{ji}(\mathbf{x}',t)|\psi\rangle = F(\mathbf{x},t,\mathbf{x}',t')|_{t=t'} \ . \tag{3.13}$$

Moreover this precursor fields behaves causally, as F satisfies a massless wave equation on both the unprimed and primed coordinates.

Presumably similar nonlocal precursor fields are need in the SYM case, though it will be harder to construct them explcitly in a gauge invariant way. It should be noted that local fields evolve into these nonlocal fields under the equations of motion. Thus the operator $T_{\mu\nu}(\mathbf{x},t)$ is in a sense its own precursor, but becomes nonlocal when expanded in terms of the local fields at time t'. This is consistent with the formalism of refs. [14, 15].

4 Final Remarks

We have shown that the AdS/CFT correspondence in the strong form that we have assumed requires sudden rearrangements of energy density in the SYM theory which appear to defy causality. However on closer inspection they do not if certain precursor fields exist in regions of vanishing energy density. These precursors are responsible for storing the detailed information of what is taking place deep in the interior of the AdS space. In particular they contain all the local bulk information in the flat space limit. The usual SYM quantities such as $T_{\mu\nu}$ which are connected with classical supergravity fields are merely the very large distance infrared emissions (at the given time t), which contain very little detailed information.

5 Acknowledgements

We would like to thank V. Balasubramanian, G. Horowitz, N. Itzhaki, A. Peet, M. Peskin, S. Ross and S. Shenker for helpful conversations. This work was supported in part by NSF grants PHY94-07194, PHY97-22022, and PHY98-70115.

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