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Naturalness and the Landscape

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Abstract

In this paper I review some arguments of Douglas and myself concerning the concept of naturalness in string theory. I explain why the usual argument for low energy supersymmetry do not apply in this context. An incorrect argument in [7] is corrected. The statistical properties of the Landscape of vacua are strongly biased against low energy supersymmetry.

1 Naturalness

In attempting to go beyond the standard model, theorists have been guided by the principle of technical naturalness: Quantities such as the Higgs mass should not require a high degree of fine-tuning. The well known problem arises because radiative loop effects can sometimes produce huge corrections that are much larger than the quantities themselves. Delicately adjusting underlying constants, to make the Higgs mass thirty orders of magnitude smaller than the expected corrections, is generally deemed unreasonable as well as unnatural.

Technicolor–a beautiful solution to the problem, and low energy supersymmetry–an ugly solution, have been invoked to eliminate the fine tunings of the Higgs potential, but neither cure seems much better than the disease. In both cases the additional structure ruins the surprisingly elegant way in which the standard model explains the absence of neutral flavor changing currents, proton decay and large neutrino masses. To avoid the new disasters that supersymmetry makes possible, several otherwise unmotivated symmetries have to be invoked. Nevertheless, the minimal supersymmetric standard model is by far the most popular hypothesis of particle physicists. This is so despite the fact that no one has ever found a way to stabilize the cosmological constant, at least not in a way that is consistent with the observed bounds on the scale of supersymmetry breaking.

During the last couple of years an entirely new paradigm has emerged from the ashes of a more traditional view of string theory. The basis of the new paradigm is the stupendous Landscape of sting theory vacua [1][2][3][4]– especially the non-supersymmetric vacua [5]. These vacua appear to be so numerous that the word *Discrtuum* [1] is used to describe the spectrum of possible values of the cosmological constant. Combining this richness with the principles of inflationary cosmology, one is led to a universe of extraordinary diversity. Some features of our usual physical laws are almost certainly contingent on the local environment.

If the Landscape and the Discretuum are real, the idea of naturalness must be replaced with something more appropriate. I will adopt the following tentative replacement: First eliminate all vacua which do not allow intelligent life to evolve. Here we need to make some guesses. I'll guess that life cannot exist in the cores of stars, cold interstellar dust clouds or on planets rich in silicon but poor in carbon. I'll also guess that black holes, red giants and pulsars are not intelligent.

Next scan the remaining fraction of vacua for various properties. If the property in question is common among these "anthropically acceptable" vacua then the property is natural. By common I mean that some non-negligible fraction of the vacua have the required property. If however, the property is very rare, even among this restricted class, then it should be deemed unnatural. Of course there is no guarantee that we are not exceptional, even among the small fraction of anthropically acceptable environments. It is in the nature of statistical arguments that rare exceptions can and do occur.

Michael Douglas has advocated essentially the same definition although he prefers to avoid the use of the word anthropic wherever possible, and substitute "phenomenologically acceptable". We have both attempted to address the following question: Are the vacua with anthropically small enough cosmological constants and Higgs masses, numerically dominated by low energy supersymmetry or by supersymmetry breaking at very high energy scales [8][7]? In other words is low energy supersymmetry breaking natural? My conclusion–I won't attempt to speak for Douglas–is that the most numerous "acceptable vacua" do *not* have low energy supersymmetry. Phenomenological supersymmetry appears to be unnatural.

As Douglas has emphasized, the numerical abundance of one kind of vacuum over another may not be the whole story. History also counts. There may be cosmological reasons why some vacua are produced more abundantly than others, even if they are less numerous in the Landscape. We will return to this later. But at the moment, without a better knowledge of cosmological dynamics it seems that there is a bias against low energy supersymmetry.

The purpose of this paper is to put all these ideas together, in one place, in a way that is accessible to non-experts.

2 Why a Landscape and a Discretuum

The essential reason that a typical Calabi Yau compacification has an enormous Landscape of vacua is the relatively large number of degrees of freedom describing the compacification. The term Landscape, or energy Landscape, is in wide use in a number of fields of physics. The simplest example that I know of is the $3N$ dimensional configuration space of a large molecule made of N atoms. Such systems can be incredibly rich in metastable ground states. Generally the number of classically stable configurations grows exponentially with N. For 500 atoms the number can easily be e^{500} or bigger.

Consider a function, $V(\phi)$, of N variables, ϕ_i, defined in a hypercube of linear size L. Suppose the function is bounded above and below and that the scale on which the function varies is some fraction of L. For definiteness let the scale of variation be $L/10$. Now consider V as a function of one variable ϕ_1, while keeping all others fixed. As ϕ_1 is varied from 0 to L, V oscillates about 10 times so that there are 10 local minima. If we trace out those minima as $\phi_2, \phi_3, ..., \phi_N$ are varied, they sweep out surfaces of co-dimension 1.

Now do the same for each ϕ_i in turn. Any intersection of N hypersurfaces defines a local minimum. The number of local minima is obviously 10^N. Thus exponentially large numbers of minima are not unusual in high dimensional spaces. Moreover they are likely to be evenly distributed over the N dimensional hypercube. In other words the number of local minima is an extensive quantity, proportional to the volume of moduli space.

Furthermore, if the range of variation of V is V_0 then the typical spacing of levels will be of order

$$\epsilon \sim V_0 10^{-N}. \tag{2.1}$$

The primary degrees of freedom in a string compactification are moduli and fluxes. The moduli and fluxes are commonly treated quite differently. The moduli are continuous variables while the fluxes are quantized. However fluxes can also be treated as scalars with a potential of the form

$$V(f) = c_1 f^2 + c_2 \cos f. \tag{2.2}$$

For large c_2 the potential has evenly spaced minima and the energy at each minima grows quadratically with f as expected of a flux.

The particular fluxes that interest us are the 3-form RR and NS fluxes that can be distributed over the 3-cycles of the Calabi Yau manifold. The number of independent fluxes is therefore related to the number of 3-cycles in the 6-dimensional Calabi Yau space. This number can be several hundred. In addition the moduli are also numerous and in the hundreds. All told the number of degrees of freedom in a Calabi Yau compacification can be as big as 1,000 or bigger. Thus it is expected that the number of metastable vacua for a given Calabi Yau compacification could be 10^{1000} and the spacing between energy levels (cosmological constants) of order $10^{-1000} M_p^4$. Of the 10^{1000} vacua roughly 10^{880} would be expected to have cosmological constants in the anthropic range.

The fluxes, being integer valued (with some appropriate normalization) form a hypercubic lattice [1] with a uniform density of points. We will use this later. The moduli on the other hand live on a moduli space which is not flat. It has a metric and a potential energy function $V(\phi)$. The potential implicitly depends on the discrete values of the fluxes. For fixed values of the fluxes, there are expected to be a large number of local minima due to the high dimensionality of the moduli space. For simplicity I will assume that the minima are more or less evenly distributed over the moduli space with a fixed density. In this way counting of vacua is replaced by integrating over a region of moduli space [3].

3 The Cosmological Constant

Long ago Weinberg estimated an anthropic bound on the cosmological constant. He calculated that if $\lambda > 10^{-119} M_p^4$ galaxies could not have formed[6]. Remarkably Weinberg's estimate is not far from the observed value of the cosmological constant. I will not be making a big mistake if I refer to both the anthropic range and the measured value of the cosmological constant as λ.

I will call the fraction of vacua, with a certain property, *the probability for that property.* We want to know the fraction of anthropically acceptable vacua that have low, versus high, energy supersymmetry breaking. Toward that end we define a conditional probability

$$P(M_{susy}|\lambda)$$

for the supersymmetry breaking scale to be msy, given that the cosmological constant is λ.

It is a little more intuitive to discuss a related but different quantity: the probability that the cosmological constant is λ, given that the supersymmetry breaking scale is M_{susy}:

$$P(\lambda|M_{susy}).$$

$P(M_{susy}|\lambda)$ and $P(\lambda|M_{susy})$ are related by Bayes' theorem. If we define $P(\lambda)$ and $P(M_{susy})$ to be the unconditional probabilities for given values of their arguments then Bayes' theorem states

$$P(M_{susy}|\lambda) = P(\lambda|M_{susy}) \frac{P(M_{susy})}{P(\lambda)}. \tag{3.1}$$

A naive but incorrect argument for $P(\lambda|M_{susy})$ was used in [7]. It states that $P(\lambda|M_{susy})$ is proportional to the degree of fine tuning needed to keep radiative corrections from ruining an initial fine tuning of λ.

$$P(\lambda|M_{susy}) \sim \frac{\lambda}{M_{susy}^4}. \tag{3.2}$$

If correct this would provide a bias toward low energy supersymmetry breaking, which could however be offset by the factor $P(M_{susy})$ in 3.1. As we will see, this factor is given by

$$P(M_{susy}) \sim M_{susy}^{n-1} \tag{3.3}$$

where n is the number of independent order parameters that can break supersymmetry . We will return to this after discussing the correct version of 3.2 given by Douglas [8].

4 Douglas' Argument

In what follows I will assume that supersymmetry is broken at a scale well below the Planck or string scale. By this I do not necessarily mean the usual low energy supersymmetry scale. I only mean low enough to justify a supergravity description. The context is low energy type 2b string theory compactified on a Calabi Yau manifold with 3-form fluxes threading various 3-cycles. Fortunately we don't need to know very much about the mathematics to determine the relevant features of the various probability distributions.

The (complex) 3-form fluxes are labelled G and they can thread any three cycle in the Calabi Yau space. The Calabi Yau space itself comes equipped with a holomorphic 3-form that depends on the shape moduli. It is labelled Ω.

The starting point for Douglas' argument is the classical superpotential induced by a flux background:

$$W = \int_{cy} G \wedge \Omega \tag{4.1}$$

where the integral is over the compact manifold. The only important feature of the superpotential is that it is linear in the fluxes.

The $N = 1$ supergravity expression for the potential on moduli space is

$$V = e^K (F_a F^a - 3|W|^2) + D^2 \tag{4.2}$$

Where K is the Kahler potential, the $F's$ are the F-term supersymmetry breaking order parameters

$$F_a = \frac{\partial W}{\partial \phi_a} + \frac{\partial K}{\partial \phi_a} W \tag{4.3}$$

and D^2 is the sum of squares of the D-term order parameters. Identifying the potential with the cosmological constant We can rewrite 4.2 in the form

$$V = \lambda_0 + F^2 + D^2.. \tag{4.4}$$

where

$$\lambda_0 = -3e^K |W|^2 \tag{4.5}$$

and $F^2 + D^2$ is the sum of the squares of the supersymmetry breaking order parameters. The condition that the cosmological constant be in the anthropically allowed range is schematically expressed as

$$\lambda_0 + F^2 + D^2 = 0 \pm \lambda. \tag{4.6}$$

The condition that the cosmological constant be almost exactly 0 is

$$\lambda_0 + D^2 \sim 0. \tag{4.7}$$

Let's first consider the spectrum of λ_0 for particular values of the moduli, The spectrum is rich because the background take on 10^{1000} values. From 4.1 we see that W is linear in the fluxes. Since the fluxes form an evenly spaced lattice, if W and the fluxes were real variables we would expect the probability for a given value of W to be flat and featureless near the origin. But as Douglas has emphasized, W is a complex variable. In other words a given value of λ_0 defines a circle in the complex plane. Therefore it is $|W|^2$ or λ_0 which has a uniform featureless probability distribution. This is true for any value of the moduli. Of course, over a range from zero to the string scale the probability will vary but near zero it is neither singular nor zero. Defining $P(\lambda_0|\phi)$ to be the probability for a given λ_0 at the point ϕ in the moduli space we find that it is not strongly dependent on λ_0. Since λ_0 varies over a range $(-M_p^4, 0)$

$$P(\lambda_0|M_{susy}) \sim \frac{1}{M_p^4}. \tag{4.8}$$

Precisely the same can be said for the probability that the cosmological constant has value λ given the value of M_{susy}.

$$P(\lambda|M_{susy}) \sim \frac{1}{M_p^4}. \tag{4.9}$$

In order to implement equation 3.1 we need to know how the unconditional probability for M_{susy}^2 varies. In a model with only a single D term it is natural to assume that it is uniform near $M_{susy} = 0$. But, as remarked in both [7] and [8], the probability becomes biased toward the largest values of M_{susy} if there is more than a single supersymmetry breaking order parameter. Since each F-term is a complex variable and each D-term is a real variable, the total number of supersymmetry breaking parameters is

$$N_{sb} = 2N_f + N_d. \tag{4.10}$$

Moreover the terms can be normalized so that the total supersymmetry breaking can be expressed in terms of a sum of squares.

$$M_{susy}^2 = \sqrt{\sum D^2 + \sum F^2}. \tag{4.11}$$

In this case for small M_{susy} we expect

$$P(M_{susy}) \sim M_{susy}^{2N_f+N_d-1}. \tag{4.12}$$

We then find that 3.1 depends on M_{susy} according to

$$P(M_{susy}|\lambda) = \frac{\lambda}{M_p^4}\left(\frac{M_{susy}}{M_p}\right)^{2N_f+N_d-1} \tag{4.13}$$

thus pushing the probability distribution even further toward high energy supersymmetry breaking. Evidently there is no statistical advantage in making the supersymmetry breaking scale low, at least as far as the cosmological constant is concerned.

One final point concerns the role of destabilizing radiative corrections. In fact they play no role. Radiative corrections will shift each vacuum by amounts that depend on M_{susy}. They may shuffle the very dense spectrum somewhat but nothing important changes.

5 The Gauge Hierarchy

The original motivation for low energy supersymmetry was the fact that the Higgs mass is some 17 orders of magnitude smaller than the fundamental Planck scale. Supersymmetry eliminates the quadratic self mass divergences that potentially destabilize the hierarchy and require un-natural fine tuning. Once this was realized, the popularity of low energy supersymmetry sky rocketed.

The Higgs fine-tuning problem is quite similar to that of the cosmological constant. In both cases the expected loop corrections to a dimensional quantity are many orders of magnitude larger than the quantity itself. In both cases unbroken supersymmetry would eliminate those divergences. And in both cases there are tight anthropic constraints that require the quantity to be not much bigger than its empirical value. Therefore it is obviously important to know if low energy supersymmetry is statistically favored by the existence of the gauge hierarchy.

In both [7] and [8], the claim is made that the smallness of the Higgs mass scale favors low energy supersymmetry by two inverse powers of M_{susy}. More precisely, the probability that the Higgs mass scale is m_h given that the supersymmetry breaking scale is M_{susy} was assumed to be

$$P(m_h|M_{susy}) = \frac{m_h^2}{M_{susy}^2}. \tag{5.1}$$

This is precisely analogous to the incorrect assumption 3.2. However it can be justified if we make the following assumption: The Higgs mass is small before supersymmetry breaking. Supersymmetry breaking and radiative corrections will tend to correct the Higgs squared-mass by an amount of order M_{susy}^2. The probability that it will end up in the allowed range is then given by 5.1.

However it is well known that the Higgs mass is not automatically small in the supersymmetric limit. This fact is the origin of the so called μ problem. There is a perfectly good supersymmetric mass term that can lift the Higgs mass as high as you like. Super-phenomenology deals with this in the same way that it deals with all unwanted effects: invoke a discrete symmetry to forbid the offending operator.

If we don't introduce such discrete symmetries then the masses of the two Higgs bosons required by super-phenomenology are

$$m_h^2 = \mu^2 \pm B \tag{5.2}$$

where B is a soft, supersymmetry breaking, off-diagonal term in the Higgs mass matrix. The usual assumption is that both μ and B are small so that no fine tuning is needed to keep m_h small. However we have seen a similar situation in which there was a strong statistical bias in favor of such apparent fine tuning.

The various probability functions that replace $P(\lambda|M_{susy})$, $P(M_{susy}$, $P(\lambda)$ and $P(M_{susy}|\lambda)$ are $P(m_h^2|M_{susy})$, $P(M_{susy})$, $P(m_h^2)$ and $P(M_{susy}|m_h^2)$. In this case the role of the supersymmetry breaking scale is played by the off-diagonal element B. If we assume that μ tends to zero in the limit of vanishing fluxes then the situation is completely parallel to that in the last section; there is no statistical preference for low energy supersymmetry.

6 Conclusions

Without further input, a prediction that supersymmetry will not be seen at the TEV scale seems warranted. A very interesting alternative that combines some of the best features of supersymmetry with high scale breaking was recently given in [9]. The proposal of [9] requires the fermionic super-partners to remain light (TEV scale) while lifting the scalars to the unification scale. Keeping the fermions light requires approximate discrete symmetries. One of the next things to determine will be how generic approximate discrete symmetries are on the landscape.

If it turns out that low energy supersymmetry is a feature of TEV physics then we will have to conclude that other considerations outweigh the counting of vacua on the Landscape. As emphasized by Douglas, these considerations could be cosmological. Perhaps certain regions of the Landscape are easier to populate than others. For example, valleys with high walls surrounding them on every side would be more difficult to reach than valleys protected by lower walls. One could imagine that such high walls always surround vacua with high-scale supersymmetry breaking, but that some low scale vacua have low barriers. Another possibility is that the dominant configuration feeding the other regions of the Landscape is the one that inflates the fastest. Some vacua might be closer to this configuration than others. In this case they would have a large advantage. But in the absence of additional knowledge, low energy supersymmetry seems very unlikely.

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