INFORMATION LOSS AND ANOMALOUS SCATTERING*

Amanda Peet^{†‡}, Leonard Susskind and Lárus Thorlacius[§]

Physics Department
Stanford University, Stanford, CA 94305-4060

ABSTRACT

The approach of 't Hooft to the puzzles of black hole evaporation can be applied to a simpler system with analogous features. The system is 1+1 dimensional electrodynamics in a linear dilaton background. Analogues of black holes, Hawking radiation and evaporation exist in this system. In perturbation theory there appears to be an information paradox but this gets resolved in the full quantum theory and there exists an exact S-matrix, which is fully unitary and information conserving. 't Hooft's method gives the leading terms in a systematic approximation to the exact result.

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[†] Supported in part by Stanford University Physics Department Fellowship Fund

[‡] peet@slacvm.bitnet

[§] larus@dormouse.stanford.edu

1. Introduction

It is a controversial issue whether or not quantum coherence can be maintained during the formation and subsequent evaporation of a black hole. At one end of the spectrum of opinion is Hawking's suggestion that this process indicates a new level of unpredictability introduced into quantum mechanics by gravity [1]. Another proposal, which is also radical from the point of view of quantum mechanics, is that information about the initial quantum state of the system is carried by a Planck scale stable remnant [2,3]. Perhaps the most conservative position has been advocated by 't Hooft who argues that this process should be thought of as a conventional scattering event in which the black hole is an intermediate state somewhat analogous to a complex intermediate nucleus formed in a nuclear collision [4]. 't Hooft has taken some tentative steps toward an S-matrix description of such events but the precise meaning of the resulting S-matrix remains unclear. We feel that this approach deserves attention and should be explored. It may indeed provide a resolution of the above paradox, or else one would like to see this logical possibility ruled out.

In order to avoid some of the formidable technical obstacles posed by quantum gravity in 3+1 dimensions one can instead consider black hole evolution in 1+1dimensions. Of course this simplified setting does not capture all the physics of real black holes but it does contain an information paradox analogous to the one originally posed by Hawking. We begin in section 2 by outlining the arguments leading to 't Hooft's S-matrix in 1+1 dimensions. For this discussion we use a simple model recently proposed by Callan et al. [5] and subsequently discussed in [6-13]. It turns out that one obtains some exact expressions where approximations had to be made in the higher dimensional theory. The physical interpretation of our S-matrix is nevertheless every bit as obscure as 't Hooft's. The main purpose of this paper is to clarify some of the issues involved by considering a simpler system, which shares many features with two-dimensional black holes, but can be solved explicitly. The system in question is the 1+1 dimensional Schwinger model with the unusual feature that the electrodynamic coupling strength depends on position. It varies from vanishing coupling at one end of space to infinite coupling at the other. The two ends correspond to spatial infinity (weak coupling) and the deep interior of the black hole (strong coupling). This is also the appropriate coupling dependence to describe s-wave fermion scattering off a 3+1 dimensional extreme magnetic dilaton black hole[14]. A similar model arose in the analysis of monopole catalysis in [15,16]. The methods we use in this paper may find application in that context also.

In section 3 we describe the analogy between black hole physics and 1 + 1 dimensional electrodynamics. The question of the existence of a unitary S-matrix

is shown to be similar in the two cases. In section 4 we set up and solve the classical equations for the formation of an object called a "charge-hole" by an incoming electric charge. We then discuss the electromagnetic analogue of Hawking radiation. In this section no attempt is made to include back-reaction on the electromagnetic field of the charge-hole due to the emitted radiation. Section 5 is devoted to describing 't Hooft's method as applied to our model and an expression is derived for an S-matrix . Section 6 uses the method of bosonization to account for back-reaction and gives an exact expression for the single-particle elastic S-matrix between one-fermion states. Then we construct the generalization to arbitrary states and show that the exact S-matrix is a generalization of 't Hooft's, with well-defined procedures for extracting amplitudes in Fock space. Finally, in section 7, the information problem is briefly discussed for the electrodynamic and gravitational systems.

2. 't Hooft's S-matrix for 1+1 dimensional gravity

In this section we will repeat 't Hooft's argument for the form of the quantum S-matrix for black hole physics in a simplified 1+1 dimensional context. We will make no attempt in this section to clarify or

interpret 't Hooft's theory. The reader is advised to skim this section lightly and return to it after reading the subsequent material.

Consider the following action for 1+1 dimensional dilaton gravity [5]:

$$I = \int d^2x \left[e^{-2\phi} \left(R + 4(\nabla \phi)^2 + 4\lambda^2 \right) - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right]. \tag{2.1}$$

This theory has received considerable attention as a toy model for black hole physics [5-13] and we will be brief here. We use the conformal gauge

$$g_{++} = g_{--} = 0, \quad g_{+-} = -\frac{1}{2}e^{2\rho} \ .$$
 (2.2)

The linear dilaton vacuum is given in light-cone "Kruskal" coordinates by

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 x^+ x^- , \qquad (2.3)$$

and the classical static black hole solution is

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 x^+ x^- + \frac{M}{\lambda}$$
 (2.4)

Let us consider a geometry describing infalling massless matter, in the form of

a shock wave, with energy-momentum tensor

$$T_{++}^{f} = \frac{M}{\lambda x_{0}^{+}} \delta(x^{+} - x_{0}^{+}) \tag{2.5}$$

where x_0^+ is the coordinate of the null trajectory of the shock and M is the total energy carried by it. The gravitational and dilaton fields are constructed by patching together the vacuum solution for $x^+ < x_0^+$ and a black hole solution with mass M for $x^+ > x_0^+$. In order to keep the dilaton and metric continuous at $x^+ = x_0^+$, it is necessary to translate the black hole solution along the x^- axis by $-\frac{M}{\lambda^3 x_0^+}$. The full solution is

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 x^+ x^- - \frac{M}{\lambda x_0^+} (x^+ - x_0^+) \theta(x^+ - x_0^+) . \tag{2.6}$$

If further energy δM , in the form of another incoming shock-wave, is added to the black hole, the result is simply another shift $-\frac{M}{\lambda^3 x_1^+}$ in x^- on the null trajectory $x^+ = x_1^+$. This is illustrated in figure 1. In four dimensions the corresponding coordinate transformation across the shock front is more complicated [4]. However, near the horizon and for $\delta M \ll M$ it can be approximated by a simple shift.*

For a continuous incoming flux $T_{++}(x^+)$ the solution is

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 x^+ x^- - \int_0^{x^+} dx_0^+ T_{++}(x_0^+) (x^+ - x_0^+)$$

$$= -\lambda^2 x^+ x^- - P_+(x^+) \left[x^+ - \frac{1}{P_+(x^+)} \int_0^{x^+} dx_0^+ x_0^+ T_{++}(x_0^+) \right],$$
(2.7)

where $P_+(x^+) = \int_0^{x^+} dx_0^+ T_{++}(x_0^+)$ is the total incoming Kruskal momentum conjugate to x^+ . From this expression it is clear that the final black hole geometry is indistinguishable at the classical level from a black hole formed by a single incoming shock wave carrying energy $\bar{M} = \lambda \int_0^\infty dx_0^+ x_0^+ T_{++}(x_0^+)$ in along $\bar{x}_0^+ = \frac{\bar{M}}{\lambda P_+(\infty)}$.

In [4] 't Hooft argues that such coordinate shifts influence the quantum vacuum of the matter fields. In particular, infalling matter will induce a unitary

[★] That a uniform shift is the full answer in two dimensions has also been noted by E. Verlinde and H. Verlinde [17].

transformation on the outgoing modes,

$$U = \exp(i\delta x^- P_-) , \qquad (2.8)$$

where $P_{-} = \int_{-\infty}^{0} dx_{0}^{-} T_{--}(x_{0}^{-})$ generates x^{-} translations of the Kruskal coordinates and $\delta x^{-} = P_{+}(\infty)$ is the coordinate shift calculated above. Written in a more symmetric form 't Hooft's S-matrix is

$$S = \exp(\frac{i}{\lambda^2} P_+ P_-) \ . \tag{2.9}$$

The proper interpretation of this expression is elusive. It should be pointed out that (2.9) cannot be the final answer. Indeed, the final state obtained in this way does not reflect any properties of the initial state except the total incoming Kruskal momentum, so this S-matrix cannot keep track of the full structure of quantum states.

In section 5 a similar line of reasoning will lead to an analogous expression for an S-matrix (with the same shortcomings) in our 1+1 dimensional electrodynamics. In section 6 we go on to derive a fully unitary S-matrix and show how the 't Hooft-like result is the leading term in a systematic expansion.

3. The electrodynamic analogy

Consider 1+1 dimensional quantum electrodynamics coupled to a background dilaton field ϕ . The gauge invariant action is

$$I = \int d^2x \left[i\overline{\psi}\gamma^{\mu}(\partial_{\mu} + iA_{\mu})\psi - \frac{1}{4}e^{-2\phi(x)}F_{\mu\nu}F^{\mu\nu} \right]. \tag{3.1}$$

The dilaton field is a static non-dynamical background and its only role in our model is to define a position-dependent coupling constant,

$$g^2(x) = e^{2\phi(x)} . (3.2)$$

We will choose a particular dilaton background motivated by the "linear dilaton vacuum" of 1+1 dimensional gravity,

$$\phi(x) = -x^1 \ , \tag{3.3}$$

where x^1 is the space-like coordinate in Minkowski space. By analogy with the black hole case we shall consider the region $x^1 \to +\infty$ as asymptotic exterior

space. In this region the coupling $g^2(\phi)$ vanishes exponentially and free fermions can propagate. The region $x^1 \to -\infty$, where the coupling diverges, is analogous to the infinite throat deep in the interior of certain extreme magnetically charged black holes [18]. The question we want to address is whether or not quantum information is ever lost to an observer at $x^1 \to +\infty$. More specifically: is the S-matrix for the asymptotic states at $x^1 \to +\infty$ unitary?

Consider the Penrose diagram in figure 2 for flat 1+1 dimensional space-time with a linear dilaton background. An incoming particle originating on \mathcal{I}_R^- can either propagate to \mathcal{I}_R^+ , thereby escaping the region of strong coupling, or it can continue propagating toward \mathcal{I}_L^+ , in which case it is "lost" to the outside observer. The unitarity of the S-matrix will therefore in general require asymptotic states to be defined on both \mathcal{I}_L^\pm and \mathcal{I}_R^\pm .

In both linear dilaton electrodynamics and 1+1 dimensional dilaton gravity, left- and right-moving modes of matter fields are uncoupled at the classical level and in perturbation theory. In dilaton gravity this is apparent in the conformal gauge (2.2) where the matter fields f_i satisfy free wave equations. Incoming (left-moving) perturbations experience no scattering and the same is true of right-moving perturbations. In linear dilaton electrodynamics the analogous gauge choice is light-cone gauge $A_- = 0$ (or $A_+ = 0$), where the Dirac equation,

$$\gamma^{\mu} (\partial_{\mu} + iA_{\mu}) \psi = 0 , \qquad (3.4)$$

separates into a pair of uncoupled equations,

$$\partial_{-}\psi_{L} = 0 ,$$

$$(\partial_{+} + iA_{+}) \psi_{R} = 0 .$$

$$(3.5)$$

The left-moving component appears to be completely decoupled (or the right-moving component in $A_+ = 0$ gauge). In perturbation theory the asymptotic final states will have particles on both \mathcal{I}_L^+ and \mathcal{I}_R^+ and it seems that information is inevitably lost to an observer at $x^1 \to +\infty$.

In both theories, non-perturbative effects associated with quantum anomalies invalidate the above reasoning. In dilaton gravity, the conformal anomaly is responsible for the emission of right-moving Hawking radiation when a left-moving particle creates a black hole [19,5]. In linear dilaton electrodynamics the axial anomaly causes a very similar phenomenon, in which an outgoing current discharges the field caused by an incoming charged particle, and in this case one can show that the outgoing radiation carries all the initial quantum information.

4. Charge hole physics

4.1. Classical solution

Let us begin with classical 1+1 dimensional electromagnetism. Maxwell's equations take the form

$$\partial_{\mu} \left(\frac{F^{\mu\nu}}{q^2(x)} \right) = j^{\nu} . \tag{4.1}$$

The source-free equations are

$$\partial_{\mu} \left(\frac{F^{\mu\nu}}{g^2(x)} \right) = 0 . \tag{4.2}$$

In two space-time dimensions the field strength tensor only has one independent component, *

$$F^{\mu\nu} = F\epsilon^{\mu\nu} \,\,\,\,(4.3)$$

and we see from (4.2) that $\frac{F}{g^2}$ is constant. Thus the general source-free solution is described in terms of one free parameter q,

$$F^{\mu\nu} = q g^{2}(x) \epsilon^{\mu\nu}$$

$$= q e^{-2x^{1}} \epsilon^{\mu\nu}$$

$$= q e^{(x^{-}-x^{+})} \epsilon^{\mu\nu} ,$$
(4.4)

where we have introduced the light-cone coordinates $x^{\pm} = x^0 \pm x^1$.

We will refer to the classical object described by (4.4) as a "charge-hole". It corresponds to a static black hole in dilaton gravity. The parameter q which replaces the mass of a black hole is of course the charge carried by the charge hole. The analog of the gravitational collapse solution (2.6) is a charge hole formed by an incoming charged particle. Let the trajectory be $x^+ = x_0^+$, where x_0^+ is a constant. The resulting field is given by

$$F^{\mu\nu} = q \,\theta(x^+ - x_0^+) \,e^{x^- - x^+} \,\epsilon^{\mu\nu} \,\,. \tag{4.5}$$

From Maxwell's equations (4.1) we see that the field in (4.5) corresponds to a current

$$j_{+} = q \,\delta(x^{+} - x_{0}^{+}) , \qquad (4.6)$$

The charge hole vector potential is easily computed in the light-cone gauge $A_{-}=0$.

^{*} Our conventions are $\epsilon^{01} = +1$ and metric signature (-, +).

It is given by

$$A_{+}(x) = -\frac{q}{2} \left[\theta \left(x^{+} - x_{0}^{+} \right) e^{(x^{-} - x^{+})} + \alpha(x^{+}) \right] , \qquad (4.7)$$

where $\alpha(x^+)$ is arbitrary.

4.2. Analogue of Hawking Radiation

It has been remarked that Hawking radiation can be viewed as pair production near the event horizon with one particle escaping to infinity and its partner falling into the black hole. This phenomenon also occurs in the field of a charge hole, where one member of the pair is attracted and the other is repelled. The radiation is in the form of charged particles and, much as in the black hole case, it persists indefinitely unless back-reaction on the charge-hole is accounted for. Apparently, an outside observer only detects the outgoing particles and must use a density matrix description of the evaporation process.

The Hawking effect appears in the quantum theory of matter in the curved, but classical, geometry of a black hole. Let us therefore consider the behavior of the quantized fermion field in the background of a charge-hole. The gauge field has an effect on the fermion system through the axial anomaly. The most efficient way to account for the anomaly is to bosonize the fermion field. We therefore begin by reviewing the standard bosonization rules.

One makes the following identifications between fermion variables and composite operators of a real boson field Z:

$$\overline{\psi}\gamma^{\mu}\psi = j^{\mu} \leftrightarrow \frac{1}{\sqrt{\pi}} \epsilon^{\mu\nu} \partial_{\nu} Z ,$$

$$\psi_{L} \leftrightarrow : \exp(i\sqrt{4\pi}Z_{L}) : ,$$

$$\psi_{R} \leftrightarrow : \exp(i\sqrt{4\pi}Z_{R}) : ,$$

$$(4.8)$$

where we have divided Z into left- and right-moving parts,

$$Z_{L,R} = \frac{1}{2} \left[Z \mp \int_{x^1}^{\infty} dx^1 (\partial_0 Z) \right] .$$
 (4.9)

Written in terms of the bosonic field the action (3.1) becomes

$$I = \int d^2x \left[-\frac{1}{2} \partial^{\mu} Z \partial_{\mu} Z - \frac{1}{\sqrt{4\pi}} \epsilon^{\mu\nu} F_{\mu\nu} Z - \frac{1}{4q^2(x)} F^{\mu\nu} F_{\mu\nu} \right] . \tag{4.10}$$

The equation of motion for Z is

$$\nabla^2 Z = \frac{1}{\sqrt{4\pi}} \epsilon^{\mu\nu} F_{\mu\nu} , \qquad (4.11)$$

which in the background of (4.5) becomes

$$\partial_{+}\partial_{-}Z = \frac{q}{2\sqrt{4\pi}}\theta(x^{+} - x_{0}^{+})e^{x^{-} - x^{+}}.$$
 (4.12)

The solution with appropriate boundary conditions corresponding to no incoming radiation is

$$Z = -\frac{q}{2\sqrt{4\pi}} \left[e^{(x^- - x^+)} - e^{(x^- - x_0^+)} \right] \theta \left(x^+ - x_0^+ \right) . \tag{4.13}$$

To examine the outgoing radiation we go to the limit $x^+ \to +\infty$

$$Z \to \frac{q}{2\sqrt{4\pi}} e^{(x^- - x_0^+)}$$
 (4.14)

Using (4.8) we see that an outgoing flux of charge is produced

$$j_{-} = \frac{q}{4\pi} e^{x^{-}} e^{-x_{0}^{+}} . {(4.15)}$$

This flux is the analogue of the outgoing Hawking radiation which is produced by a gravitational collapse. According to (4.15) the radiation persists forever, eventually radiating an infinite charge, just as the black hole radiates an infinite mass unless back-reaction is accounted for.

5. 't Hooft-type S-matrix for linear dilaton electrodynamics

In this section we will derive an approximate expression for the S-matrix. The arguments parallel 't Hooft 's construction for black hole physics as in section 2.

Let us consider the theory in the gauge $A_{-}=0$. The vector potential describing the field of an infalling charge is given by (4.7). The right-moving field ψ_{R} satisfies

$$\left(\partial_{+} + iA_{+}\right)\psi_{R} = 0 , \qquad (5.1)$$

with the general solution

$$\psi_R = \exp[iS(x)] \chi_R , \qquad (5.2)$$

where $S(x) = \int_{-\infty}^{x_+} dx^+ A_+$ and χ_R is a free field. Thus the effect of the gauge field is to multiply the outgoing fermion field by a position-dependent phase factor

 $e^{iS(x)}$. Inserting the charge hole vector potential (4.7) gives

$$S(x) = \frac{q}{2}\theta(x^{+} - x_{0}^{+}) \left[e^{(x^{-} - x^{+})} - e^{(x^{-} - x_{0}^{+})} \right] - \frac{q}{2} \int \alpha(x^{+}) . \tag{5.3}$$

The second term is an arbitrary constant c times -q. To compute the S-matrix we consider the limit $x^+ \to +\infty$, where

$$S(x) \to -\frac{q}{2} \left[e^{(x^- - x_0^+)} + c \right]$$
 (5.4)

Thus the effect of the charge hole gauge field on the fermion system is a canonical transformation which multiplies ψ_R by a phase

$$\psi_R(x^-) \to \exp[-i\frac{q}{2}(e^{x^--x_0^+}+c)]\,\psi_R .$$
(5.5)

The transformation (5.5) is a unitary transformation equivalent to the action of the unitary operator

$$U = \exp\left[i \int dx^{-} S(x^{-}) j_{R}(x^{-})\right]$$

= $\exp\left[-i \int dx^{-} \frac{q}{2} \left(e^{(x^{-} - x_{0}^{+})} + c\right) j_{R}(x^{-})\right],$ (5.6)

where

$$j_R = \psi_R^{\dagger} \psi_R \ . \tag{5.7}$$

Let us next suppose that instead of a single delta-function the incoming charge is described by a continuous classical flux $j_L(x^-)$. The resulting unitary operator is easily computed to be

$$U = \exp\left[-\frac{i}{2} \int dx_0^+ dx^- j_L(x_0^+) \left(e^{x^- - x_0^+} + c\right) j_R(x^-)\right]. \tag{5.8}$$

At this point $j_L(x^+)$ is the classical incoming current and $j_R(x^-)$ is the quantum operator $\psi_R^{\dagger}\psi_R$. The symmetry of the expression, however, suggests that j_L and j_R can be treated on an equal footing as operators in the incoming and outgoing Fock spaces.

The S-matrix (5.8) is quite similar to 't Hooft's gravitational S-matrix (2.9). In particular, it cannot be a fully correct description of the scattering any more

than (2.9) is. To see this, consider an incoming current $j_L(x^+)$. According to (5.8) the resulting final state is given by

$$U|0\rangle = \exp\left[-\frac{i}{2} \int dx^{-} \left(Ae^{x^{-}} + Bc\right) j_{R}(x^{-})\right]|0\rangle$$
, (5.9)

where A and B are two moments of $j_L(x^+)$

$$A = \int dx^{+} j_{L}(x^{+})e^{-x^{+}}$$

$$B = \int dx^{+} j_{L}(x^{+}) .$$
(5.10)

Evidently, the final state depends on only two parameters describing the incident particles. There is clearly no way that such a final state can keep track of the full complexity of the incident state and thus (5.8) cannot define a unitary S-matrix in the Fock spaces of in and out particles.

6. Exact S-matrix for linear dilaton electrodynamics

6.1. One-particle S-matrix

Using the bosonization rules of section 4, the action for linear dilaton electrodynamics can be written

$$I = \int d^2x \left[-\frac{1}{2} \partial_{\mu} Z \partial^{\mu} Z - \frac{1}{\sqrt{4\pi}} Z \epsilon^{\mu\nu} F_{\mu\nu} - \frac{1}{4g^2(x)} F^{\mu\nu} F_{\mu\nu} \right] . \tag{6.1}$$

The vector potential can be integrated out to give the following effective action for the boson field Z:

$$I = \int d^2x \left[-\frac{1}{2} \partial_\mu Z \partial^\mu Z - \frac{g^2(x)}{2\pi} Z^2 \right] . \tag{6.2}$$

This procedure is analogous to that used in [10] to make local the conformal anomaly term in dilaton gravity.

The Z field now has a mass which increases indefinitely in the negative x^1 -direction. Thus it is evident that any finite-energy configuration must be totally reflected. An observer at $x^1 \to +\infty$ will recover all information. This fact is not at all apparent in the original fermionic formulation. Nevertheless, one can construct a unitary S-matrix for fermions. We will first illustrate this by computing the amplitude for a single fermion to be elastically reflected.

An initial state of definite energy is described on \mathcal{I}_R^- by

$$|in\rangle = \int dx^{+} e^{-ip_{+}x^{+}} \psi_{L}(x^{+}) |0\rangle ,$$
 (6.3)

where $|0\rangle$ is the in-vacuum. Using the bosonization prescription (4.8) this can be written as

$$|in\rangle = \int dx_0^+ e^{-ip_+ x_0^+} : e^{i\sqrt{4\pi}Z_L(x_0^+)} : |0\rangle .$$
 (6.4)

From the boson point of view this is a linear superposition of coherent states

$$: e^{i\sqrt{4\pi}Z_L(x_0^+)} : |0\rangle$$
 (6.5)

Each such coherent state is identified with a classical configuration $Z_C(x)$ and evolves in time into another coherent state according to the classical equations of motion. The initial configuration corresponding to (6.5) is a left-moving step function

$$Z_C = \sqrt{\pi}\theta(x^+ - x_0^+) \ . \tag{6.6}$$

Note that the charge carried by a configuration is given by

$$Q = \int_{-\infty}^{+\infty} dx \, j^0 = \int_{-\infty}^{+\infty} dx \, \frac{1}{\sqrt{\pi}} \frac{\partial Z}{\partial x} = \frac{1}{\sqrt{\pi}} \left[Z_C(+\infty) - Z_C(-\infty) \right] \,. \tag{6.7}$$

Thus the net incoming charge is proportional to the height of the step function.

The incoming state has the form (6.6) on \mathcal{I}_R^- i.e. at $x^- \to -\infty$. To find the subsequent evolution we need to solve the classical equations for Z_C ,

$$\partial_{+}\partial_{-}Z_{C} = -\frac{1}{4\pi}g^{2}(x)Z_{C} = -\frac{1}{4\pi}e^{(x^{-}-x^{+})}Z_{C}$$
, (6.8)

subject to the boundary conditions (6.6) at \mathcal{I}_R^- . Note that we do not need to impose boundary conditions on \mathcal{I}_L^- because the mass term in (6.8) diverges there forcing Z to vanish. The appropriate solution can for example be found by using a a coordinate system which turns (6.8) into a Klein-Gordon equation with a uniform tachyonic mass [14]. It is given by

$$Z_C = \sqrt{\pi} \,\theta(x^+ - x_0^+) \,J_0\left[\frac{1}{\sqrt{\pi}} e^{\frac{1}{2}x^-} \sqrt{e^{-x_0^+} - e^{-x^+}}\right] \,. \tag{6.9}$$

It is instructive to examine (6.9) on a series of time slices, showing how the field evolves. This is illustrated in figure 3. We see that the point charge continues to

penetrate toward $x^1 \to -\infty$ but becomes more and more tightly screened as time evolves. Asymptotically it becomes totally screened. A reflected charge of equal magnitude moves off to the right towards the asymptotic weak coupling region and it is followed by a series of pairs with ever higher frequency but lower charge. The degenerate left-moving "blip" is an artefact of having arbitrarily high-energy components in the localized state $\psi(x_0)|0\rangle$. The actual initial state (6.3) is a superposition of such localized states and has finite energy.

The asymptotic out-state on \mathcal{I}_R^+ is obtained by taking the limit $x^+ \to +\infty$ in (6.9)

$$Z_C \to \sqrt{\pi} J_0 \left[\frac{1}{\sqrt{\pi}} e^{\frac{1}{2}(x^- - x_0^+)} \right]$$
 (6.10)

The corresponding coherent quantum state is given by

$$: \exp\left[i \int dx^{-} 2\partial_{-} Z_{C}(x^{-}, x_{0}^{+}) Z_{R}(x^{-})\right] : |0\rangle . \tag{6.11}$$

Thus the final state is

$$\int dx_0^+ e^{-ip_+ x_0^+} : \exp\left[i \int dx^- 2\partial_- Z_C(x^-, x_0^+) Z_R(x^-)\right] : |0\rangle . \tag{6.12}$$

The elastic scattering amplitude is the overlap of this state with an outgoing fermion,

$$\int dx_0^- e^{iq_- x_0^-} \langle 0| : \exp[-i\sqrt{4\pi}Z_R(x_0^-)] : . \tag{6.13}$$

A standard coherent state calculation yields an amplitude,

$$A(q_{-}, p_{+}) = \int dx_{0}^{+} dx_{0}^{-} e^{i(q_{-}x_{0}^{-} - p_{+}x_{0}^{+})} \exp\left(\int \frac{dv}{v + i\epsilon} J_{0}\left[\frac{1}{\sqrt{\pi}} e^{\frac{1}{2}(v + x_{0}^{-} - x_{0}^{+})}\right]\right)$$

$$= 2\pi \delta(p_{+} - q_{-}) \int dx e^{-ip_{+}x} \exp\left(\int \frac{dv}{v + i\epsilon} J_{0}\left[\frac{1}{\sqrt{\pi}} e^{\frac{1}{2}(v - x)}\right]\right).$$
(6.14)

The $i\epsilon$ prescription takes care of the ultra-violet divergences but, as it stands, this expression is still infra-red divergent. This is because we have used a simple logarithm for the boson propagator in the coherent state calculation, whereas a more careful evaluation, using a regularized propagator, would give a finite result. An alternative, if somewhat crude, subtraction procedure is simply to subtract from the Bessel function in the v integral in (6.14) a step function $\theta(v_0 - v)$, which cancels the $v \to -\infty$ infrared divergence. The dependence on the subtraction point, v_0 , can be absorbed into the overall normalization of the amplitude, which we have not kept track of here. If desired, the normalization can be determined by the physical requirement that the probability for elastic reflection of a fermion approaches unity as the energy tends to zero.

6.2. The full S-matrix

Now we want to construct the full operator S-matrix for the scattering of arbitrary fermion states. The best way to achieve this is to first obtain the exact S-matrix for bosons and then appeal to the equivalence between the Hilbert spaces of the bosons and fermions to read off the fermion S-matrix. The boson amplitudes are easy to obtain because (6.2) defines a free field theory. Let us start with the LSZ-reduced expression for a one-particle S-matrix element for bosons, which is obtained by sandwiching the operator

$$S_{1\to 1} = i \int d^2x_1 \, d^2x_2 \, Z_R(x_1^-) \, \overrightarrow{\nabla}_1^2 G(x_1, x_2) \, \overleftarrow{\nabla}_2^2 \, Z_L(x_2^+)$$
 (6.15)

between asymptotic single boson Fock states. By using the coordinate system in which the equation of motion for Z becomes a tachyonic Klein-Gordon equation, and demanding that the propagator vanishes in the strong coupling region, one is led to

$$G(x_1, x_2) = \sqrt{\pi} J_0 \left[\frac{1}{\sqrt{\pi}} \sqrt{|e^{-x_1^+} - e^{-x_2^+}| |e^{x_1^-} - e^{x_2^-}|} \right].$$
 (6.16)

After inserting this propagator into (6.15) and some integrations, we find

$$S_{1\to 1} = i \int dx_1^- dx_2^+ \,\partial_- Z_R(x_1^-) \,J_0\left[\frac{1}{\sqrt{\pi}} e^{\frac{1}{2}(x_1^- - x_2^+)}\right] \,\partial_+ Z_L(x_2^+) \ . \tag{6.17}$$

For a free field theory the full S-matrix is obtained by exponentiating the single particle expression

$$S = \exp\left[i \int dx_1^- dx_2^+ \,\partial_- Z_R(x_1^-) \,J_0\left[\frac{1}{\sqrt{\pi}} e^{\frac{1}{2}(x_1^- - x_2^+)}\right] \,\partial_+ Z_L(x_2^+)\right] \,. \tag{6.18}$$

Exactly the same operator expression can now be used to compute S-matrix elements in the fermion basis. For example, the single-particle matrix element (6.14) is given by

$$\int dx^{+} dx^{-} e^{i(q_{-}x^{-} - p_{+}x^{+})} \langle 0 | : e^{-i\sqrt{4\pi}Z_{R}(x^{-})} : S : e^{+i\sqrt{4\pi}Z_{L}(x^{+})} : |0\rangle . \tag{6.19}$$

The general expression (6.18) can be written directly in fermion language by using

the fermion-boson correspondence,

$$\frac{1}{\sqrt{\pi}} \epsilon^{\mu\nu} \partial_{\nu} Z = j^{\nu} , \qquad (6.20)$$

giving

$$S = \exp\left(i\pi \int dx^{+} dx^{-} j_{R}(x^{-}) J_{0}\left[\frac{1}{\sqrt{\pi}} e^{\frac{1}{2}(x^{-} - x^{+})}\right] j_{L}(x^{+})\right).$$
 (6.21)

Evidently the exact S-matrix is of the form advocated by 't Hooft but with a more complicated kernel than (5.9). In fact, the correspondence can be seen directly by expanding the Bessel function in a power series in $e^{x^--x^+}$. The first two terms of the expansion pick up the moments in $(5.9)^*$. The full series expansion involves all the moments making it possible for unitarity to be restored.

The meaning of the higher terms in the series expansion can be given a graphical interpretation. Each successive power of $e^{x^--x^+}$ corresponds to a closed loop of fermions in the gauge field propagator, which enters into the calculation of the phase shift of the outgoing fermions.

7. Information retrieval

Having established the existence of a unitary S-matrix for linear dilaton electrodynamics, it is interesting to ask how the information in a complex initial state is radiated back.

For example, suppose an initial state of given total charge Q described by a coherent state with some modulations on the Z-field. Now consider boosting the configuration to higher energy so that the information carrying modulations are squeezed into a smaller volume. At extremely high energy it will become indistinguishable from a step function whatever its initial profile. However, boosting a configuration cannot change its information content. How, then, does the final state remember the incident structure?

The answer is in the very high-frequency exponentially attenuating tail in figure 3. In the limit of infinite boost, the tail extends to $x^1 \to -\infty$, and because of the increasing frequency in this region it carries infinite energy. In a finite energy configuration, the tail is bounded. The details of the initial configuration are coded in the details of the high-frequency low-amplitude tail. In other words, an

^{*} In fact there is a factor of two discrepancy between the coefficients in (5.9) and (6.21). This factor can be traced to the asymmetric treatment of incoming and outgoing currents in section 5 and does not appear in a more symmetric calculation

energetic collection of low charge fermion pairs trails the main bulk of the outgoing charged radiation and information about all the details of the boosted initial state are coded into modulations on that tail.

We do not know to what extent the mechanism for information retrieval carries over to two-dimensional gravity, let alone the real world. Obviously we cannot expect the information in a black hole to be radiated in a late tail of high energy quanta since most of the energy of the black hole will already have been radiated. Note, however, that in the analogy between two-dimensional gravity and linear dilaton electrodynamics, gravitational energy is replaced by electric charge. The information carrying tail in linear dilaton electrodynamic carries very little charge which should perhaps be interpreted in gravity as information escaping from a black hole remnant in a long tail of very soft radiation, containing a large number of quanta. Since the coding of the information into long wavelength quanta would have to be a very slow process [1,2] such a proposal would probably suffer from the drawbacks of stable remnant theories. We hope to return to these points.

Another point worth noting is that the unitarity of the S-matrix depends on the field content of the theory. For example, if two species of fermions were coupled to the electromagnetic field the difference of their charge densities would not be expelled from the strongly coupled region. In this case one linear combination of the bosonizing fields would carry information to $x^1 \to -\infty$ where it would be lost to an outside observer. Perhaps information can only be conserved in some theories.

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FIGURE CAPTIONS

- 1) The effect of an infalling shock wave on a black hole geometry. The event horizon shifts outward.
- 2) "Penrose diagram" for a charge-hole.
- 3) Evolution in time of the bosonizing field, for an incoming fermion.