Susskind's Challenge to the Hartle-Hawking No-Boundary Proposal and Possible Resolutions *

Don N. Page †
Institute for Theoretical Physics
Department of Physics, University of Alberta
Room 238 CEB, 11322 – 89 Avenue
Edmonton, Alberta, Canada T6G 2G7

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Abstract

Given the observed cosmic acceleration, Leonard Susskind has presented the following argument against the Hartle-Hawking no-boundary proposal for the quantum state of the universe: It should most likely lead to a nearly empty large de Sitter universe, rather than to early rapid inflation. Even if one adds the condition of observers, they are most likely to form by quantum fluctuations in de Sitter and therefore not see the structure that we observe. Here I present my own amplified version of this argument and consider possible resolutions, one of which seems to imply that inflation expands the universe to be larger than $10^{10^{10^{122}}}$ Mpc.

^{*}Alberta-Thy-07-06, hep-th/0610199

[†]Internet address: don@phys.ualberta.ca

Introduction

Our part (or subuniverse [1] or bubble universe [2, 3] or pocket universe [4]) of the entire universe (or multiverse [5, 6, 7, 8, 9, 10, 11, 12] or metauniverse [13] or omnium [14] or megaverse [15] or holocosm [16]) is observed to be highly special in a way that does not seem to be implied purely by the known dynamical laws. For example, it is seen to be very large on the Planck scale, with low large-scale curvature, and with approximate homogeneity and isotropy of the matter distribution on the largest scales that we can see today. It especially seems to have a very high degree of order in the early universe that has enabled entropy to increase, as described by the second law of thermodynamics [17, 18, 19].

Two leading proposals for special quantum states of the universe are the Hartle-Hawking 'no-boundary' proposal [20, 21, 22, 23, 24, 25, 26, 27, 28, 29] and the 'tunneling' proposal of Vilenkin, Linde, and others [30, 31, 32, 33, 34, 35]. In toy models incorporating presumed approximations for these proposals, both of these models have seemed to lead to low-entropy early universes and so might explain the second law of thermodynamics. If a suitable inflaton is present in the effective low-energy dynamical theory, and if sufficient inflation occurs, both proposals have seemed to lead to approximate homogeneity and isotropy today.

Here I shall focus on the Hartle-Hawking no-boundary proposal, in which the wavefunction of the universe (expressed as a functional of the 3-dimensional geometry and matter field configuration on a spatial 3-surface that in some sense represents the universe at any one moment of time) is given by a path integral over all compact Euclidean 4-dimensional geometries and matter fields that have the 3-dimensional geometry and matter field configuration as its only boundary. (Because of this one boundary of the 4-geometries, where the wavefunction is evaluated, one might say that the proposal would be better named the Hartle-Hawking 'one-boundary' proposal [28], but here I shall continue to use the usual nomenclature.) This model is certainly incomplete for various technical reasons [36, 37, 38, 28], but in simple toy models, it seems to predict several special features of the observed universe [21, 22, 39, 40, 23, 41, 26, 28]: Lorentzian signature, large size, near-critical density, low anisotropies, inhomogeneities starting in ground states to fit cosmic microwave background radiation (CMB) data, and entropy starting low to explain the second law of thermodynamics.

However, Leonard Susskind [42] (cf. [43, 44, 45]) has argued that the cosmological constant or quintessence or dark energy that is the source of the present observations of the cosmic acceleration [46, 47, 48, 49, 50, 51, 52] would give a large Euclidean 4-hemisphere as an extremum of the Hartle-Hawking path integral that would apparently swamp the extremum from rapid early inflation. Therefore, to very high probability, the present universe should be very nearly empty de Sitter spacetime, which is certainly not what we observe.

This argument is a variant of Vilenkin's old objection [53] that the no-boundary proposal favors a small amount of inflation, whereas the tunneling wavefunction favors a large amount. Other papers that have attacked the Hartle-Hawking wavefunction include [54, 55, 56]. However, Susskind was the first to impress upon me

the challenge to the Hartle-Hawking no-boundary proposal from the recent cosmic acceleration.

Of course, it may be pointed out that most of de Sitter spacetime would not have observers and so would not be observed at all, so just the fact that such an unobserved universe dominates the path integral is not necessarily contrary to what we do observe. To make observations, we are restricted to the parts of the universe which have observers. One should not just take the bare probabilities for various configurations (such as empty de Sitter spacetime in comparison with a spacetime that might arise from a period of rapid early inflation). Rather, one should consider conditional probabilities of what observers would see, conditional upon their existence [13, 57, 29].

However, the bare probability of an empty de Sitter spacetime forming by a large 4-hemisphere extremum of the Hartle-Hawking path integral dominates so strongly over that of a spacetime with an early period of rapid inflation that even when one includes the factor of the tiny conditional probability for an observer to appear by a vacuum fluctuation in empty de Sitter, the joint probability for that fluctuation in de Sitter dominates over the probability to form an inflationary universe and thereafter observers by the usual evolutionary means. Therefore, the argument goes, almost all observers will be formed by fluctuations in nearly empty de Sitter, rather than by the processes that we think occurred in our apparently inflationary universe.

The problem then is that almost all of these fluctuation-observers will not see any significant ordered structures around them, such as the ordered large-scale universe we observe. Thus our actual observations would be highly atypical in this no-boundary wavefunction, counting as strong observational evidence against this theory (if the calculation of these probabilities has indeed been done correctly). As Dyson, Kleban, and Susskind put it in a more general challenge to theories with a cosmological constant [43], "The danger is that there are too many possibilities which are anthropically acceptable, but not like our universe." See [58, 59, 60, 61, 62, 63] for further descriptions of this general problem.

To express this in a slightly different way, if A are the conditions for observations, and if B are the conditions for ordered observations, we want a theory giving the conditional probability P(B|A) not too many orders of magnitude smaller than unity, since we see B. But if the no-boundary quantum state produces A mostly by de Sitter fluctuations, it seems that it gives $P(B|A) \ll 1$.

The general nature of this objection was forcefully expressed by Eddington 75 years ago [64]: "The *crude* assertion would be that (unless we admit something which is not chance in the architecture of the universe) it is practically certain that at any assigned date the universe will be almost in the state of maximum disorganization. The *amended* assertion is that (unless we admit something which is not chance in the architecture of the universe) it is practically certain that a universe containing mathematical physicists will at any assigned date be in the state of maximum disorganization which is not inconsistent with the existence of such creatures. I think it is quite clear that neither the original nor the amended version applies. We are thus driven to admit anti-chance; and apparently the best

thing we can do with it is to sweep it up into a heap at the beginning of time."

In Eddington's language, Susskind's challenge is that the Hartle-Hawking noboundary proposal seems to lead to pure chance (the high-entropy nearly-empty de Sitter spacetime), whereas to meet the challenge, we need to show instead that somehow in the very early universe (near, if not at, the "beginning of time") it actually leads to anti-chance, something far from a maximal entropy state.

Of course, another possibility is simply that the Hartle-Hawking no-boundary proposal is wrong. Hawking himself admitted this possibility [65] (cf. also [29]): "I'd like to emphasize that this idea that time and space should be finite without boundary is just a *proposal*: it cannot be deduced from some other principle. Like any other scientific theory, it may initially be put forward for aesthetic or metaphysical reasons, but the real test is whether it makes predictions that agree with observation." Susskind is making the argument that its predictions do not agree with observation.

Numerical Illustrations

Let us make a numerical illustration of this problem. For simplicity and concreteness, let us take $\Omega_{\Lambda}=0.72\pm0.04$ from the third-year WMAP results of [50] and $H_0=72\pm8$ km/s/Mpc from the Hubble Space Telescope key project [66], and let us drop the error uncertainties. In Planck units, $\hbar=c=G=1$, this gives $H_0\approx 1.3\times 10^{-61}$ and $\Lambda=3\Omega_{\lambda}H_0^2\approx 3.4\times 10^{-122}$, which would give a Euclidean 4-hemisphere of radius $a_{\rm dS}=\sqrt{3/\Lambda}\approx 9.4\times 10^{60}$ and a Euclidean action of $S_E({\rm de~Sitter})=-\pi a_{\rm dS}^2/2\approx -1.4\times 10^{122}$. This extremum of the Hartle-Hawking path integral would thus give an unnormalized bare probability of

$$P_{\text{bare}}(\text{de Sitter}) = e^{-2S_{E}(\text{dS})} = e^{\pi a_{dS}^{2}} = e^{3\pi/\Lambda} \sim e^{10^{122.44}}.$$
 (1)

Now we need to ask, given this de Sitter spacetime, what is the probability of having an observer or observation. I do not know what the minimum requirement for an observation is, but it certainly seems sufficient to have a human brain in the right state for a sufficient time. If we assume that a human brain of minimum mass, say, 1 kg, can make an observation in a very short time if it is in the right state, then the minimum requirement would be for the brain to fluctuate into existence in a region of size, say, r = 30 cm, that is separate from the antimatter that would also exist during the vacuum fluctuation. This gives a dimensionless action of (cf. [67])

$$S_E(\text{brief brain}) \sim \frac{Er}{\hbar c} \sim \frac{(1 \text{ kg})(3 \times 10^8 \text{ m/s})(0.3 \text{ m})}{10^{-34} \text{ J} \cdot \text{s}} \sim 10^{42}.$$
 (2)

This will then give a conditional probability of each brain state, in some region of de Sitter spacetime just large enough to contain the brain, given that that spacetime exists, of about $e^{-2S_E(\text{brain})} \sim e^{-10^{42}}$. This should be multiplied by the number of orthogonal brain states within the region that would correspond to an observation,

and by the number of spacetime regions where the observer can fluctuate into existence and make an observation, in order to give P(brain state|de Sitter). However, for simplicity let us assume that the product of these numbers is much less than $e^{10^{42}}$ and so does not much change the upper exponent (42).

A more conservative assumption [68] would be that a human observer requires a 1 kg brain to last a time long enough for neural signals to travel across it, say 0.1 second. The dimensionless action for this is

$$S_E(\text{medium brain}) \sim \frac{E\Delta t}{\hbar} \sim \frac{(1 \text{ kg})(3 \times 10^8 \text{ m/s})^2 (0.1 \text{ s})}{10^{-34} \text{ J} \cdot \text{s}} \sim 10^{50}.$$
 (3)

An even more conservative assumption would be that the brain should be a thermal fluctuation into a real existence from the de Sitter temperature $T_{\rm dS} = 1/(2\pi a_{\rm dS}) = 1.70 \times 10^{-62}$, which gives a Boltzmann probability factor of $\exp\left(-2S_E(\log \text{brain})\right)$ with

$$S_E(\text{long brain}) = \frac{\pi a_{\text{dS}} E}{\hbar c} \approx 1.4 \times 10^{69}.$$
 (4)

Effectively this assumption was used in [59] to calculate the time for a 'Boltzmann brain' (BB) [69, 11, 70] to appear in the local viewpoint considered there, $t_{BB} \sim \exp(2S_E(\log \text{brain}))$; here I shall suggest that it might be more realistic to use the smaller brief brain (bb) time $t_{bb} \sim \exp(2S_E(\text{brief brain})) \sim e^{10^{42}}$ in that viewpoint. In [68] I used $S_E(\text{medium brain}) \sim 10^{50}$ to estimate that if the de Sitter spacetime

In [68] I used S_E (medium brain) $\sim 10^{50}$ to estimate that if the de Sitter spacetime lasts longer than about $10^{50}t_0 \sim 10^{60}$ years, then the spacetime 4-volume would be so large that one would expect many observers to fluctuate into existence in it, rather than having just a very low probability per de Sitter spacetime. An even more severe problem seems to occur if the spacetime decays probabilistically with a half-life greater than about 20 billion years, since then the expectation value of the 4-volume per comoving 3-volume would diverge in the future and produce an infinite number of fluctuation observers or Boltzmann brains [58, 61, 63]. However, here let us assume either that the de Sitter spacetime will not last so long, or else that there are other ways of circumventing these problems, such as the ones suggested in [59, 60, 62].

Combining the unnormalized bare probability for nearly empty de Sitter with the most conservative conditional probability for an observation within such a de Sitter spacetime gives the unnormalized probability of an observation from the Euclidean de Sitter extremum as

$$P_{\text{observation,unnormalized}}(\text{de Sitter}) \sim \exp\left(+10^{122.44} - 10^{69.4}\right).$$
 (5)

In comparison, let us calculate the probability of forming an observer through an inflationary universe. In this case observers can presumably develop through normal Lorentzian evolution (with paths in the path integral having real Lorentzian action or purely imaginary Euclidean action during the Lorentzian part of the evolution) after one has Lorentzian inflation, so there will not be the huge suppression factor of about $e^{-10^{42}}$, $e^{-10^{50}}$, or $e^{-10^{69.4}}$ that occurs in empty de Sitter. This by itself

certainly makes it sound as if more observers ought to be produced by inflation than by empty de Sitter. However, in the Euclidean path integral of the Hartle-Hawking no-boundary proposal, one also needs to compare the bare probability of producing the inflationary universe, which seems to be much, much less than the bare probability of producing a large de Sitter spacetime directly by the large Euclidean 4-dimensional hemisphere extremum.

Although the details are unimportant for the qualitative result of the argument, for concreteness let us consider inflation driven by a single scalar field ϕ with potential $V(\phi)$. In the Hartle-Hawking no-boundary path integral, inflation can start by an extremum of the action that has a nearly-round Euclidean small 4-dimensional hemisphere with nearly constant scalar field value ϕ_0 , radius squared $a_0^2 \approx 3/[8\pi V(\phi_0)]$, and Euclidean action $S_E(\text{inflation}) \approx -\pi a_0^2/2 \approx 3/[16V(\phi_0)]$.

In the account of Liddle and Lyth [71], who use the reduced Planck mass $M_{\rm Pl} = (8\pi G/\hbar c)^{-1/2} = 1/\sqrt{8\pi} \approx 0.20$ in terms of the usual Planck units $\hbar = c = G = 1$, the magnitude of the scalar density perturbations from inflation is given by

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 = \frac{1}{24\pi^2 M_{\text{Pl}}^4} \frac{V}{\epsilon} = \frac{8V}{3\epsilon},\tag{6}$$

where

$$\epsilon \equiv \epsilon(\phi) = \frac{M_{\rm Pl}^2}{2} \left(\frac{V'}{V}\right)^2 \equiv \frac{1}{16\pi} \left(\frac{1}{V} \frac{dV}{d\phi}\right)^2$$
(7)

is one of the slow-roll parameters [71] that I am assuming is much less than unity, and everything is to be evaluated at horizon exit for wavenumber k = aH.

The Liddle-Lyth quantity $\mathcal{P}_{\mathcal{R}}(k)$ seems to be the same quantity called $\Delta_{\mathcal{R}}(k)$, the amplitude of curvature perturbations, in the WMAP analysis [50], which at a wavenumber of $k = 0.002/\mathrm{Mpc}$ is given in terms of the amplitude of density fluctuations, A, as $29.5 \times 10^{-10} A$. Table 5 of the 3-year WMAP data [50] gives $A \approx 0.8$, so I shall take that as a representative value below.

Now if for simplicity and concreteness we suppose that the inflaton potential has a power-law form with exponent α , say $V = \lambda M_{\rm Pl}^{4-\alpha} \phi^{\alpha}$, then the slow-roll parameter is $\epsilon = (1/2)\alpha^2 M_{\rm Pl}^2/\phi^2 \approx \alpha/(4N)$, where N is the number of e-folds of inflation from that value of ϕ to the end of inflation at $\phi_{\rm end} \approx \alpha M_{\rm Pl}/\sqrt{2}$, assuming that $\phi \gg \phi_{\rm end}$.

In terms of these quantities, if the nearly-round Euclidean 4-hemisphere has the nearly-constant scalar field value ϕ_0 (which can be interpreted to be very nearly the initial value for the Lorentzian inflation that is the analytic continuation of the Euclidean 4-hemisphere), and if ϕ_0 is taken as a lower limit for the value that causes the horizon exit at what is now the fiducial wavenumber of k = 0.002/Mpc, then the Euclidean action of the 4-hemisphere is, roughly,

$$S_E \stackrel{>}{\sim} -\frac{1}{2\epsilon\Delta_R} \approx -\frac{2N}{\alpha\Delta_R} = -6.78 \times 10^8 \frac{N}{\alpha A} \approx -2 \times 10^{10},$$
 (8)

where for the very last number I have taken $\alpha = 2$ for the $\frac{1}{2}m^2\phi^2$ potential that does seem to fit the WMAP data [50] better than the $\lambda\phi^4$ potential with $\alpha = 4$,

and I have used the value N=50 from [71] and the value A=0.8 from [50]. This result also agrees well with the result of using $m=7.5\times 10^{-6}=1.5\times 10^{-6}$ and $\phi_0=\phi_*=14M_{\rm Pl}=2.8$ for the $\frac{1}{2}m^2\phi^2$ potential from the example on page 252 of [71]. Very, very crudely, when ϕ_0 is not much larger than the value of ϕ giving $k=0.002/{\rm Mpc}$, then this Euclidean action goes as the inverse square of a typical galactic peculiar velocity (in units of the speed of light, of course), multiplied by N that is very roughly a logarithm of the ratio of some energy in the range of the Planck energy to some energy in the range of atomic energies.

Then if I use this estimate for a lower bound on the Euclidean action for the 4-hemisphere, the unnormalized bare probability for inflation becomes

$$P_{\text{bare}}(\text{inflation}) = e^{-2S_{\text{E}}(\text{inflation})} = e^{\pi a_0^2} = e^{3/[8V(\phi_0)]} \lesssim e^{10^{10.6}}.$$
 (9)

If ϕ_0 were taken to be larger, which is certainly consistent with the observations that only place a minimum value on the number of e-folds of inflation, then $P_{\text{bare}}(\text{inflation})$ would be smaller, asymptotically approaching unity as $V(\phi_0)$ approaches infinity with the ever-rising potential.

If inflation does occur, then one would expect the conditional probability of observers to be of the order of unity (not suppressed by a Euclidean fluctuation action), so one would get

$$P_{\text{observation,unnormalized}}(\text{inflation}) \lesssim e^{10^{10.6}}.$$
 (10)

This is much less than $P_{\text{observation,unnormalized}}$ (de Sitter), so if we normalize be dividing by the total unnormalized probability for observations, we get that the normalized probability for an observation to occur in an inflationary solution (rather than from a fluctuation in nearly empty de Sitter) would be

$$P_{\text{observation}}(\text{inflation}) \stackrel{<}{\sim} \exp\left(-10^{122.44} + 10^{69.4} + 10^{10.6}\right).$$
 (11)

In fact, if one just asks for the normalized probability of an ordered observation, that would much more likely occur from a fluctuation in the large nearly-empty de Sitter than in an inflationary universe, so

$$P_{\rm observation}({\rm order}) \sim {{\rm number\ of\ brain\ states\ with\ ordered\ observations}\over {\rm number\ of\ brain\ states\ with\ any\ observations}}.$$
 (12)

This would still be expected to be a fraction much less than unity, depending on how ordered the observation is required to be. (For example, one might expect that the fraction of observations with, say, 1000 ordered bits of information would be of the order of 2^{-1000}). Therefore, according to these probabilities given by this approximate calculation in a toy minisuperspace-plus-homogeneous-inflaton model, it would be very improbable for an observation chosen randomly from the predictions of the model to have the order that we see in our actual observations. That is, our actual observations would be highly atypically ordered according to this model, and this fact counts as strong observational evidence against the model.

The conclusion of Susskind's argument [42], which I have expanded in my own words here, is that the Hartle-Hawking no-boundary proposal for the quantum state of the universe is inconsistent with our observations.

I indeed take this as a very serious objection to the no-boundary proposal, for which I do have not seen or thought of a rebuttal that I would regard as completely satisfactory. However, since this proposal has in the past (at least in highly approximate toy models) seemed to provide solutions for a number of deep cosmic mysteries (perhaps foremost the explanation of the very low entropy of the very early universe necessary to explain the second law of thermodynamics), I am loathe to give it up. Therefore, I would like to regard Susskind's objection not so much as a no-go theorem but more as a challenge (to discover either how to save the Hartle-Hawking proposal or how to replace it).

Possible Resolutions

In this spirit, let me consider various possible resolutions to Susskind's challenge to the Hartle-Hawking no-boundary proposal, though readily admitting that none I have thought of yet seems to be completely satisfactory.

(1) The first conceivable resolution to Susskind's challenge is that for some unknown reason observers can't form from fluctuations in nearly-empty de Sitter spacetime (or for some reason they have probabilities suppressed enormously much more greatly than that calculated above for a brain to last 0.1 seconds).

A separate motivation for this possibility is the calculation of [68] that if our current accelerating universe lasts longer than about $10^{50}t_0 \sim 10^{111} \sim 10^{60}$ years into the future, the comoving 4-volume corresponding to the Solar System, say, would have far more observers produced by vacuum fluctuations than are likely to exist from ordinary life on Earth over the entire history of the Solar System. The results of [58, 61, 63] imply that this problem could arise from a quantum half-life as low as 20 billion years (rather than an end to the universe at a definite time that could be as long as $\sim 10^{60}$ years in the future [68]). Therefore, even if we exclude nearly-empty de Sitter spacetimes formed in the Hartle-Hawking path integral, it would seem that our ordered observations would be highly atypical even within what we think is happening in our part of the universe, if it lasts long enough.

Of course, one possibility is that our part of the universe does not last this long (at least while expanding exponentially at roughly the present rate) [72, 73, 74, 75, 76, 77, 78, 79, 80, 68, 58, 61, 63]. Perhaps the current slow cosmic acceleration is not caused by a cosmological constant (or by a scalar field at a positive minimum of its potential, which is effectively essentially the same thing if the tunneling rate out from this minimum is negligible in 10^{60} years). Perhaps instead it is due to a scalar field that is slowly sliding down a potential with a slope [72, 73, 74, 75, 76, 77, 78, 79, 80, 68] that is very small but which is sufficient for the potential to go negative and lead to a big crunch within 10^{60} years. (The observational evidence presented in [68] only gave a lower limit of about 26 billion years in the future, assuming that the potential is not convex. Improved measurements of the w parameter are expected

to give a gradual improvement of this lower limit, but it seems totally unrealistic to expect the observational lower limit to be raised to 10^{60} years within the near future, by which of course I mean within some humanly accessible time scale $\ll 10^{60}$ years.)

However, it would seem to require extraordinary fine tuning to have a potential with a small enough nonzero slope to be consistent with our observations and yet allow the universe to slide into oblivion. Having a minimum of the potential of low enough value to give the observed cosmic expansion might be explained by the anthropic principle (restricting attention to probabilities conditional upon observers) [81, 82], but there does not seem to be any obvious similar argument why the current very low value (in Planck units) of the potential should also be accompanied by a very low nonzero value of the slope. It would seem much more likely that one were at a minimum of a potential than that one were at a low value of a potential that also has a gradual slope extending far enough to lead to negative values for the potential (and hence an eventual big crunch for the universe). Therefore, one is tempted to look for other resolutions of the problem posed by the possibility of the production of observers by vacuum fluctuations.

One possibility for this is that observers require nonzero globally conserved quantities that are almost entirely absent in nearly-empty de Sitter spacetime. However, this would seem to require that observers must extend over the entire space, or else one could simply have a fluctuation in which the required value of the conserved quantity appeared in the smaller region where the observer is, and then the complementary region would have the negative of this quantity, so that the total quantity over the entire space remains zero [68]. It seems rather implausible to propose that as observers, each of us extends over all of space, though one might note that similarly counter-intuitive things seem to occur in the representation of a bulk gravitational quantum state on the conformal boundary in the AdS/CFT correspondence.

Therefore, I cannot rule out the possibility that nearly-empty de Sitter space cannot produce observers by vacuum fluctuations, but it does seem rather far-fetched to me to suppose that it cannot.

(2) A second conceivable resolution of Susskind's challenge is that the Euclidean action of the inflationary universe can be made very large and negative by connecting it by a thin bridge or tube or thread to a large Euclidean de Sitter 4-sphere, thereby making its Euclidean action even more negative than that of pure Euclidean de Sitter without inflation [83, 29]. However, then the question would be that if this were possible, why not also have the nearly-empty de Sitter itself also connected by a bridge to another large 4-sphere to reduce its action as well and keep it more negative than that of the inflationary universe? Furthermore, if one allowed one bridge to another 4-space of negative action, what prevents there from having an arbitrarily large number of bridges connecting to an arbitrarily large number of 4-spaces of negative action, thereby making the Euclidean action unbounded below? This would then seem to make the theory degenerate into nonsense.

So for this second possibility to be valid, it would seem that there must be some unknown principle that allows an inflationary universe to be connected to a large Euclidean de Sitter 4-sphere, but not for the nearly-empty de Sitter 4-hemisphere to be similarly connected to something else to reduce its action similarly.

One proposal that might be sufficient to rule out the catastrophe of an arbitrary number of bridges connecting some space in the path integral to an arbitrary number of 4-spaces of negative action would be that one should approximate the Euclidean path integral by a sum only over actual extrema of the action, real or complex Euclidean solutions of the Einstein equations coupled to the matter fields [28]. It would seem likely that this would allow either of the two extrema discussed above, the large 4-dimensional hemisphere (which analytically extends into the Lorentzian regime as nearly-empty de Sitter spacetime) and the Euclidean inflationary solution (with its tiny approximately round 4-dimensional hemisphere followed by its analytical extension to a Lorentzian inflationary universe), but perhaps not solutions with an arbitrary number of bridges connecting different large Euclidean regions with large negative action. (When one considers the amplitude for observers, one could still take just complex classical solutions, but now slightly inhomogeneous ones that end up with the perturbed final 3-space having an observer configuration as a fluctuation that would raise the real part of the Euclidean action in the de Sitter case but mainly just give an imaginary correction to the Euclidean action in the inflationary case.)

(3) This leads to the third conceivable resolution, which is that there is an actual extremum connecting the inflationary solution to a large negative-action Euclidean de Sitter, but none connecting two Euclidean de Sitter spaces. Then there would be a (probably complex) Euclidean solution of the field equations with huge negative action (from the Euclidean de Sitter part) and yet having a part that analytically continues to a Lorentzian inflationary universe that can explain our observations, without there being a solution with even more negative Euclidean action (say from two Euclidean de Sitter solutions somehow connected together to make a new solution with more negative action). Such an inflationary solution with a huge negative action would almost necessarily be inhomogeneous, which might make it difficult to discover. There is no evidence that I am aware of that strongly suggests its existence, but then there is none I know that would rule it out either.

This suggested resolution has the advantage that in principle one could look for an explicit realization, though it might be difficult. The main trouble that I see with it is that it seems somewhat implausible to me that an inflationary solution would have an extension (a modified solution that includes a much larger complex Euclidean region, not just an extension of the same solution), still as a solution of the field equations, that includes a region giving huge negative action, if one cannot do the same for the Euclidean de Sitter solution. And even if one somehow succeeded in finding such an extension of the inflationary solution, it might be difficult to prove that there is no analogous extension of the Euclidean de Sitter solution. Therefore, at present I would regard this conceivable resolution as quite speculative, though it would be very exciting if one could find an actual complex solution of the character outlined above (with the appropriate nonsingular Euclidean boundary conditions of the no-boundary proposal). Such an actual mathematical solution would be of

greater scientific value than many of the speculative proposals I am desperately tossing out for consideration in this paper.

(4) A fourth conceivable resolution, rather going in the opposite direction from that of the previous one, is that Euclidean de Sitter is not an allowed extremum of the path integral with a cosmological constant. It would seem likely that even if one attempts to make the Hartle-Hawking path integral manageable by restricting the sum to extrema, one might need to restrict the sum only to a certain subset of all complex extrema.

For example, it was found in one calculation [21] for a 3-dimensional sphere of size smaller than the equatorial 3-sphere of the 4-sphere solution for the chosen value of the cosmological constant, that even though there were two classical solutions (one in which the 3-sphere boundary bounded less than half of the 4-sphere, and the other in which it bounded more than half), only the solution with the 3-sphere bounding less than half of the 4-sphere contributed to a preferred contour integral for the path integral (and even though the other solution had lower Euclidean action).

It also might be expected that one could have complex Euclidean solutions that wind around various singularities [84, 28], and that the real part of the action could be made arbitrarily negative by winding around in the appropriate direction. In this case it would not seem to make sense to include the solutions with arbitrarily negative action, so one might need to make some restriction on the number of times the complex solution could wind around various singularities. However, it is not clear to me what the correct procedure would be to accomplish this.

Nevertheless, it might turn out that somehow the correct procedure, once found, would rule out using the Euclidean de Sitter extremum but would still allow the inflationary solution. Again, at the moment this remains pure speculation, and it is hard to see why something so simple as the Euclidean de Sitter 4-hemisphere (and its analytic continuation into the Lorentzian regime) would be excluded.

- (5) A related fifth conceivable resolution is that even if Euclidean de Sitter is an allowed solution that would contribute to the Hartle-Hawking path integral with a true cosmological constant, it (or a similar large 4-space) is not an allowed solution with the actual quintessence or dark energy that drives the currently observed cosmic acceleration. It is hard to see how quintessence or dark energy would not give a Euclidean solution if a cosmological constant does, but I do not have a rigorous proof against this, so I am therefore listing it as one of the conceivable possibilities.
- (6) A sixth possibility is that whatever resolves the problem of the infinite measure in inflation might also in some way solve the problem raised by Susskind (though I certainly don't see why this would necessarily occur). Inflation, particularly eternal inflation [85, 86, 87, 88], seems to be able to lead to an arbitrarily large universe, with an arbitrarily large number of observers, which makes it problematic how to calculate the probability of various observed features by taking the ratio of the numbers of the corresponding infinite sets of observers [89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 59, 60, 99]. It is conceivable that the resolution of this dilemma might also regulate the huge bare probability ascribed to the nearly-empty de Sitter spacetime in the Hartle-Hawking path integral. However,

it is also possible that the two problems are rather separate, so that a solution to one will not immediately give a solution to the other.

(7) A seventh possible resolution of Susskind's challenge is that the integral over the initial value ϕ_0 of the scalar field ϕ , being infinite if the ϕ has an infinite range, will dominate over the huge but finite value of $P_{\text{bare}}(\text{de Sitter}) \sim e^{10^{122.44}}$. This is the same type of argument that was used in [100] to say that the Hartle-Hawking no-boundary proposal leads to the prediction that the flatness parameter has unit probability to be $\Omega = 1$ (in a minisuperspace model that did not include inhomogeneous modes that would realistically be expected to give some cosmic variance about $\Omega = 1$ in the observed part of the universe), despite the fact that the no-boundary wavefunction peaks at the minimum possible value of inflation.

Although we did not refer to the tunneling wavefunction in this paper [100] on the flatness of the universe, our argument for the infinite measure from the integral over ϕ_0 would also answer the challenge of Vilenkin [53] and others [54, 55, 56] that the no-boundary proposal favors a small amount of inflation, whereas the tunneling wavefunction favors a large amount. Our argument would imply that even if there is a huge bare probability (i.e., before normalization) for a small amount of inflation in the no-boundary proposal, if one includes an infinite range for the initial value ϕ_0 of the inflaton field, that gives an infinite measure, which of course dominates over the large but finite measure or bare probability for a small amount of inflation.

In the case of Susskind's challenge from the huge negative action of a large Euclidean de Sitter solution, the problem is quantitatively more acute, since the action of a large Euclidean de Sitter solution is even enormously much more negative than that of the smallest theoretical amount of inflation ($\phi_0 \sim 1$, much smaller than the observational lower limit for at least roughly 50 e-folds of inflation). However, this huge negative action is still finite, so qualitatively the solution of an infinite range for ϕ_0 can still work just as it did in the previous case (assuming that it did there).

It is amusing to consider the quantitative implications of this proposed resolution of Susskind's challenge. If one imagined that ϕ really has only a finite range but attempted to make that range so large, say up to ϕ_{max} , that the integral over $d\phi_0$ dominates over $P_{\text{bare}}(\text{de Sitter}) \sim \mathrm{e}^{10^{122.44}}$, we would need $\phi_{\text{max}} \gtrsim e^{10^{122.44}}$. Then if $V(\phi)$ rises asymptotically as some power of ϕ , the amount of inflation as ϕ undergoes slow roll from near ϕ_{max} to near unity (at the end of inflation) is exponential in a power of ϕ_{max} . If this power is an exponent that is of the order of magnitude of unity, then after inflation, the size of the universe will be at least of the crude order of $10^{10^{10^{122}}}$ (meaning that the logarithm of the logarithm of the logarithm of the size will be at least roughly 122). Of course, it is hard to imagine why ϕ would have a finite range if its range extended up to at least roughly $10^{10^{122}}$, so it seems much more likely that ϕ would then simply have an infinite range. In that case, the probability would then be unity that the universe would expand larger than any fixed finite size. This is effectively almost the same as saying that the universe will have an infinite amount of expansion, though strictly speaking "infinite" here should be taken to mean just "arbitrarily large."

The main problem with this proposed resolution of Susskind's challenge is that it generally requires that the inflaton be allowed to be so large that its potential gives energy densities far in excess of the Planck density (unless the potential levels off below the Planck value, which is a distinct possibility, though perhaps one that would be considered to be fine tuned). Then one might suppose that the inflaton should be cut off at the Planck density. However, even while admitting that we do not yet know what should happen at the Planck density, one might say [101] that this cut off is ad hoc, so we cannot be sure that the proposed solution, with ϕ allowed to be infinitely large (or at least as large as $P_{\text{bare}}(\text{de Sitter}) \sim e^{10^{122.44}}$), is not qualitatively valid. In other words, even though it may be doubtful that it is really right, we cannot be sure it is wrong either.

A related problem is that if the inflaton comes from some modulus or other field in superstring/M theory, there are conjectures that in the presence of gravity the volume of the moduli space is finite (see, e.g., [102]). If so, the integral over the allowed range of the inflaton field would give a finite answer that almost certainly not compensate for P_{bare} (de Sitter). This means that it may be hard to combine this proposed solution to Susskind's challenge with superstring/M theory. However, this is just an unproved conjecture [103], and the KKLT construction [104] suggests that it may well be wrong.

(8) An eighth possible resolution of Susskind's challenge is that the inflationary component of the wavefunction expands to such an utterly enormous size that it produces more ordered observers than the nearly empty de Sitter spacetime does of disordered observers through vacuum fluctuations, even when one includes the huge bare probability of the nearly empty de Sitter spacetime. For this resolution to work, one would need to restrict the 4-volume of the de Sitter spacetime (e.g., by something that prevented it from lasting too long and expanding too many times, perhaps the same thing that might prevent too many observers from occurring by vacuum fluctuations in the future of our subuniverse [68, 58, 59, 60, 61, 62, 63]) so that it produces a strictly finite number of observers, and then allow the inflationary universe to expand so much more that it produces more observers, even after including the ratio of their bare probabilities that seems to weight the nearly-empty de Sitter spacetime by such a large factor relative to the inflationary solution.

In the case of a minisuperspace comparison between the tunneling and the noboundary quantum states [101], for suitable potentials (including the simple massive scalar inflaton), even deterministic slow roll without stochastic inflation can produce enough volume from a large enough ϕ_0 to compensate for the higher bare probability of a small ϕ_0 (and a resulting amount of inflation too small to be consistent with observations), without having to go to ϕ_0 so high that one exceeds the Planck density. However, for the same minisuperspace idea to save the no-boundary proposal in comparison with the Euclidean de Sitter extremum, it appears that for most reasonable potentials that rise indefinitely with the inflaton field, one would need to allow the initial energy density to exceed the Planck value.

However, if one goes to stochastic or eternal inflation [85, 86, 87, 88], it appears to allow the universe to inflate to arbitrarily large size even without the potential

ever exceeding the Planck value (though for an ever-rising potential it does seem that the stochastic evolution pictured for the scalar field would be required to be rather finely tuned to avoid ever exceeding the Planck energy density; this problem would not arise if the potential instead has a maximum value below the Planck value [105]). In this case one could always get enough spatial volume, and hence number of observers, in the inflationary solution to compensate for the enormous unnormalized bare probability of the Euclidean de Sitter spacetime, assuming that the latter somehow is not similarly allowed to inflate by a sufficient amount to produce its own larger number of observers by vacuum fluctuations.

One might think that including the processes of stochastic inflation would take one outside the zero-loop approximation advocated in [28] to avoid some of the infinities of the path integral. However, one might conjecture that the effects of stochastic inflation could arise from taking into account complex inhomogeneous classical solutions of the field equations (extrema of the action). It would be very interesting to see whether this indeed is the case.

Another way to get an arbitrarily large amount of inflation is to suppose that the inflaton potential has a rather flat maximum, and that inflation starts at the top of this hill [30, 31, 27]. In this case one could get homogeneous complex classical extrema with arbitrarily large amounts of approximately real Lorentzian inflation, expanding the universe to arbitrarily large size. However, one could object that the de Sitter-like extrema corresponding to the currently observed cosmic acceleration can also expand the universe to arbitrarily large size in the distant future, so it is not obvious why the arbitrarily large size from rapid early inflation would dominate over the arbitrarily large size from the slow late inflation.

Again it seems that we must imagine that for some reason the large nearly-empty de Sitter solution cannot do something that the rapid-inflation solution can. In the first proposed resolution above, it was the formation of observers that was proposed to be denied the nearly-empty de Sitter solution. In the second suggestion it was supposed that the nearly-empty de Sitter solution cannot be attached by a bridge to another large 4-sphere to make its Euclidean action enormously more negative as it was proposed could happen to the inflationary solution. In the third speculation, it was supposed that the nearly-empty de Sitter solution cannot be combined with another space to give an actual extremum with greatly reduced action, even though it was conjectured that this might be able to be done for the inflationary solution. In the fourth suggestion, it was proposed that Euclidean de Sitter is not actually an allowed extremum for the Hartle-Hawking path integral, whereas the inflationary solution supposedly is. In the fifth idea, it was suggested that Euclidean de Sitter might not be a solution at all for whatever it actually is that is driving the currently observed cosmic acceleration. In the more vague sixth proposal, the solution to the infinite measure problem is supposed to reduce the naïve de Sitter bare probability much more than that of inflation. In the seventh proposed solution to Susskind's challenge, it is the arbitrarily large range of ϕ_0 that the inflationary solutions have that the de Sitter solution does not have. (Here it perhaps is most easy to see the distinction between the two solutions, which is why I am perhaps most attracted to this possibility.) Finally, in the eighth possibility, it is proposed that the de Sitter space cannot expand large enough to produce arbitrarily many observers (by vacuum fluctuations), even though the inflationary universe can (though in this case by the ordinary evolutionary process that we believe occurred in our observed subuniverse).

Thus we have at least an eight-fold way of potential solutions to save the Hartle-Hawking no-boundary proposal (and what it might explain, such as the mysterious arrow of time) from Susskind's challenge. As one can see from the discussion above, I am not too happy with any of them, but at the moment I would guess that the seventh, with the infinite measure from the integration over an infinite range of possible initial values of the inflaton scalar field ϕ , seems the least unattractive.

Other Possibilities

Since it is not certain whether any of these eight proposals (or others I have not yet thought of or that other people might propose) really give a satisfactory resolution of Susskind's challenge, let us now turn to the possibility that the Hartle-Hawking no-boundary proposal is wrong and that one should turn to another proposal for the quantum state of the universe. Here I shall just examine the tunneling proposals of Vilenkin, Linde, and others [30, 31, 32, 33, 34, 35].

For the present purposes, the main difference from the Hartle-Hawking proposal will be taken to be the sign of the Euclidean action for at least the homogeneous isotropic complex Euclidean FRW solutions like Euclidean de Sitter and FRW inflation [31, 30]. (It seems problematic to take the opposite sign for inhomogeneous and/or anisotropic perturbations without leading to some instabilities, and it is not clear how to give a sharp distinction between the modes that are supposed to have the reversed sign of the action and the modes that are supposed to retain the usual sign of the action, but for this paper I shall generally leave aside this and related problems [106, 107, 108, 109, 110, 111]. Vilenkin has emphasized [30] that this criticism does not seem to apply to his tunneling proposal, which does not simply have the reversed sign of the Euclidean action for all modes, but here I shall just focus on the homogeneous mode, for which Vilenkin's proposal effectively does have the opposite sign.)

In this case the Euclidean de Sitter solution would give

$$P_{\text{bare}}(\text{de Sitter}) = e^{+2S_{E}(\text{dS})} = e^{-\pi a_{\text{dS}}^{2}} = e^{-3\pi/\Lambda} \sim e^{-10^{122.44}}.$$
 (13)

Assuming that a vacuum fluctuation producing an observer has the usual sign of the Euclidean action, one would then get

$$P_{\text{observation,unnormalized}}(\text{de Sitter}) \sim \exp\left(-10^{122.44} - 10^{69.4}\right).$$
 (14)

These bare probabilities could then be compared with the inflationary probabilities

$$P_{\text{bare}}(\text{inflation}) = e^{+2S_{\text{E}}(\text{inflation})} = e^{-\pi a_0^2} = e^{-3/[8V(\phi_0)]} \gtrsim e^{-10^{10.6}}$$
 (15)

and

$$P_{\text{observation,unnormalized}}(\text{inflation}) \gtrsim e^{-10^{10.6}}.$$
 (16)

This dominates the corresponding $P_{\text{observation,unnormalized}}$ (de Sitter), so if we again normalize be dividing by the total unnormalized probability for observations, for the tunneling wavefunction we now get that the normalized probability for an observation to occur in an inflationary solution would be

$$P_{\text{observation}}(\text{inflation}) \sim 1.$$
 (17)

Thus the tunneling wavefunction would be consistent with our ordered observations in this way (at least if one could solve the other problems associated with it).

It is a bit disconcerting that the controversy between the no-boundary and tunneling wavefunctions [106, 107, 108, 109, 110, 111] has not yet been resolved. In terms of the numbers above, they give probabilities of large empty de Sitter spacetimes that differ by a factor of more than $10^{10^{12^2}}$, which is the ten thousand million million millionth power of a googolplex! However, even this might pale beside the uncertainties of whether the various infinite factors discussed above should be included (particularly that of the integration over an infinite range of the initial value ϕ_0 of the inflaton).

One argument [85, 86, 87, 88, 90] is that at very late times, where the volume of space has grown so large that that is where almost all observers are expected to be, eternal inflation leads to the same predictions for all of the various proposed wavefunctions. This picture is now being explored in the context of the string landscape [56, 104, 45, 112, 113, 114, 115, 116, 15, 117, 118, 119, 120, 102], with one of the recent ideas being that the probabilities of the various string vacua depends not only on the various actions but also on the decay rates [98, 121, 60]. Whether these ideas can be cast into the form of the Hartle-Hawking no-boundary proposal or are consistent with it remains to be fully explored. On the other hand, the canonical classical measure [122, 123] gives an ambiguous probability for inflation [124]. Gibbons and Turok have recently shown [125] that the divergence in the canonical measure is removed if one identifies universes which are so flat they cannot be observationally distinguished by observers like us, living in the late universe but with access to only a finite portion of space in the past. With this identification of very flat universes, the canonical measure becomes finite, and it gives an exponentially small probability for a large number of inflationary e-foldings. However, [124] also implies that one can alternatively choose other cutoffs in which the probability of inflation is large [126]. These examples show that it is certainly not the case that all choices of measure (or initial conditions or wavefunctions) lead to the same predictions, so one would really like to know what the quantum state is and what measure it predicts for observations.

In summary, Susskind has raised a serious challenge to the Hartle-Hawking noboundary proposal for the quantum state of the universe. There are several potential resolutions of this challenge, but it is not yet clear whether any of them is satisfactory. If no resolutions can be found, the challenge leaves us with the mystery of what the quantum state might be to be consistent with our observations.

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