D-branes and Fat Black Holes

JUAN MALDACENA[*]

Department of Physics
Princeton University , Princeton, NJ 08544

LEONARD SUSSKIND[†]

Department of Physics
Stanford University, Stanford, CA 94305-4060

The application of D-brane methods to large black holes whose Schwarzschild radius is larger than the compactification scale is problematic. Callan and Maldacena have suggested that despite apparent problems of strong interactions when the number of branes becomes large, the open string degrees of freedom may remain very dilute due to the growth of the horizon area which they claim grows more rapidly than the average number of open strings. Such a picture of a dilute weakly coupled string system conflicts with the picture of a dense string-soup that saturates the bound of one string per planck area. A more careful analysis shows that Callan and Maldacena were not fully consistent in their estimates. In the form that their model was studied it can not be used to extrapolate to large mass without being in conflict with the Hawking Bekenstein entropy formula. A somewhat modified model can reproduce the correct entropy formula. In this "improved model" the number of string bits on the horizon scales like the entropy in agreement with earlier speculations of Susskind.

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[*] malda@puhep1.princeton.edu
[†] susskind@dormouse.stanford.edu

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1. The Problem

D-Brane methods [1] have made it possible to compute the degeneracy of the ground states of systems which have the same quantum numbers of certain BPS extreme black holes [2]. The results in every case agree with the Hawking Bekenstein entropy, thus supporting the principle that black holes satisfy the rules of standard quantum mechanics and statistical mechanics. Furthermore, the development of a large horizon for such objects confirms the idea that the dimensionality of the space of states of a region of space never exceeds the exponential of the area measured in planck units [5]. The D-brane methods are somewhat indirect and follow a logic first outlined by Sen [6]. The system one is interested in is really a gravitationally collapsed strongly interacting system. However, using the adiabatic invariance of the number and mass of BPS states, one can slowly turn off the gravitational coupling without changing the degeneracy. The number of ground states can then be computed in the limit of zero coupling where the D-brane technology is sufficient. This method of counting is very powerful for BPS states but does not really give a picture of the degrees of freedom of the real gravitationally collapsed system. Thus, to understand the excited states, their decays and issues of information transfer, a more direct understanding of the collapsed black hole and its degrees of freedom may be needed.

However in [7] the low lying non-BPS spectrum was calculated in the naive weakly coupled theory and found to agree with the classical result. This led to the proposal [7] that the validity of the weakly coupled D-brane picture might extend all the way to the region of collapsed black holes. Naively this seems unlikely because one would expect that crowding a large number of branes together would lead to a strongly coupled mess even if the coupling constant is small. To argue that this might not happen ref. [7] made the observation that gravitation causes the distribution of branes and strings to swell up and form a macroscopic horizon with radius R_e. It was further claimed that this effect dilutes the strings so much that they become weakly interacting. If this is true it is very difficult to understand the connection between these stringy degrees of freedom and the dense planckian distribution of open strings that was derived in ref. [8] . We will see in what follows that the estimates of [7] are not correct and that a corrected version of the model leads to a condensate of strings consistent with ref. [8] . However, the success of the corrected model suggests that for certain purposes the very low energy modes are weakly interacting with the dense soup.

We study the same five dimensional black hole solution of type IIB string theory com-

pactified on $T^4 \times S^1$ that was considered in [7]. For notations and conventions we refer the reader to that reference. The system in question consists of a collection of 1-branes and 5-branes wrapped around the compact space. The 1-branes are wound around the S^1 which has radius R. It will be useful to visualize the other four compact dimensions as smaller than R. However R itself is to be regarded as strictly finite and fixed throughout. The number of one and five branes is Q_1 and Q_5. In addition there are massless open strings carrying Kaluza Klein momentum $P = N/R$ in the direction of the S^1. These open strings each connect a 1-brane to a 5-brane. They can be indexed by a pair of integers $[a, \alpha]$ which refer to which 1-brane and 5-brane the string ends on. There are all together $4Q_1Q_5$ types of such massless fermionic strings and a similar number of bosonic strings[‡]. The charges Q_1, Q_5, N are assumed to be integers and are related by

$$c_p N = c_1 Q_1 = c_5 Q_5 = R_e{}^2 \tag{1.1}$$

where c_p, c_1, c_5 are constants defined in [7] and R_e is the Schwarzschild radius of the extremal, BPS state with those quantum numbers.

There are several domains of the parameter space that should be distinguished. The first involves the limit in which the coupling constant is small enough that all interactions can be ignored. In this case the Schwarzschild radius is smaller than the string scale, $l_s = \sqrt{\alpha'}$. In this situation, reliable state counting can be done for both the BPS and non-BPS states. However it is only the BPS spectrum which can be continued to more interesting regions with any rigor. In the second region the Schwarzschild radius is much bigger than than l_s but much smaller than R, the size of S^1. This is the region of "black strings". There is some reason to think that the naive D-brane spectrum may describe the spectrum of non-BPS states in this region [9]. However ref. [7] is interested in the much more interesting but problematic region in which the the Schwarzschild radius is much bigger than any other scale in the problem, in particular the compactification scale R. This domain of parameters we will call *fat* black holes. One of the claims of [7] is that in the fat limit the strings become infinitely dilute on the horizon so that a weak coupling method may be sufficient to understand not only the extreme states but also the low lying excitations.

[‡] In fact there is also a number of (1,1) and (5,5) strings which are necessary to ensure that we are at a minimum of the potential (D-flatness conditions). They are determined by the (1,5) and (5,1) strings and are not independent degrees of freedom (for large Q_1, Q_5).

Let us review the argument of [7] for the microscopic entropy of the BPS state. The momentum P along the X^5 is a sum over the modes of the massless open strings, each of which carries an integer momentum in units of $1/R$.

$$P = \frac{1}{R}\sum_i i n_i = \frac{N}{R} \tag{1.2}$$

where n_i is the number of open string quanta in the ith mode. The counting is done with the aid of a partition function

$$Z = \sum d(N)q^N = \left[\prod_{n=1,\infty} \frac{1+q^n}{1-q^n}\right]^{4Q_1Q_5} \tag{1.3}$$

In this expression $d(N)$ is the degeneracy of the BPS state with charges (Q_1, Q_5, N). For large N ($N \gg Q_1 Q_5$) we can safely argue that $d(N)$ is of order $\exp(2\pi\sqrt{Q_1 Q_5 N})$. However, we shall now see that this is incorrect in the limit in which the three charges grow in fixed proportion. For simplicity we will take all three charges to be equal to N and calculate the asymptotic value of $d(N)$. We use the usual trick of isolating the power series coefficients of Z by contour integration

$$d(N) \approx \int dq \frac{Z(q)}{q^{N+1}} \tag{1.4}$$

The contour of integration surrounds the origin. A straight forward saddle point method can be used to estimate $d(N)$. Define $Z = \exp Q_1 Q_5 U(q) = \exp N^2 U(q)$. The integral has the form

$$d(N) \approx \int \frac{dq}{q} \exp[N^2 U(q) - N \log(q)] \tag{1.5}$$

The function $U(q)$ behaves like $8q$ near $q = 0$. The saddle occurs at $q = \frac{1}{8N}$ for $N \to \infty$ and we find

$$d(N) \to \exp[N + N \log 8N] \tag{1.6}$$

Thus the entropy behaves like $N \log N$. This is much smaller than the Bekenstein Hawking value which scales like $N^{3/2}$. There is a simple explanation of what is happening. Consider a 1+1 dimensional gas of free left moving massless radiation consisting of N^2 species of bosons with average energy N/R. An effective temperature T is introduced.

This temperature is the effective temperature of the left movers and should not be confused with the Hawking temperature which is related to the effective temperature of the right movers [7]. Assume that the entropy, energy and number of quanta are extensive. Then, on dimensional grounds, the entropy and number of particles will be proportional to one another (this ignores a mild logarithmic infrared divergence in the number of quanta that we will return to). Up to numerical factors the energy and entropy satisfy

$$E = N^2 R T^2 = \frac{N}{R} \tag{1.7}$$

$$S = N^2 R T \tag{1.8}$$

Eliminating the temperature T gives

$$S = N^{3/2}$$

which is consistent with the results of [7]. But now calculating the temperature

$$T = \frac{1}{R N^{1/2}}$$

we notice that as N grows T is driven to zero. At some point the wavelength $1/T$ of a typical quantum exceeds R and the assumption of extensivity fails. In fact it is straight forward to show that the characteristic thermal length scale $RN^{1/2}$ is nothing but the Schwarzschild radius R_e. In other words extensivity fails when the black hole becomes fat.

Before discussing how to fix the model for fat black holes we would like to discuss two other difficulties that we will see are related. The first involves the density of open strings that forms on the horizon. These open strings are strikingly similar to the objects discussed by one of us [8]. These open strings are expected to become very dense on the horizon with ≈ 1 open string per planck area. However [7] estimates the density of strings and find that it goes to zero as the mass and charge get large. To see why, return to eq. (1.2). Since the i's are all positive integers it is evident that the total number of strings is bounded by N. Furthermore the horizon area grows like $N^{3/2}$. Thus the number of strings per unit area goes to zero at least as fast as $N^{-\frac{1}{2}}$. One may also say that the number of strings per unit entropy is going to zero. This hypothetical behavior is to be contrasted with the situation

in which extensivity prevails. In this case the number of quanta scales in the same way as the entropy. For example in the case of fermions the average total number of quanta at temperature $1/\beta$ is given by the integral

$$n_f = R \int d\omega \frac{1}{e^{\beta\omega} + 1} \approx RT \tag{1.9}$$

For bosons the corresponding integral is mildly divergent

$$n_b = R \int d\omega \frac{1}{e^{\beta\omega} - 1} \approx RT \log(RT) \tag{1.10}$$

Note that the logarithm comes from quanta with wave length longer than $1/T$ but these infrared quanta do not contribute significantly to either the energy or the entropy.

The second point involves the mass gap to the first excited state above the BPS state. We can excite the system by adding a pair of oppositely moving open stings in the lowest mode. Since the lowest mode has energy $1/R$ this gives a gap of order

$$\delta M \approx 1/R \tag{1.11}$$

This is very strange in the limit in which the largest dimension of the black hole is the Schwarzschild radius. A much more plausible value would involve some negative power of R_e. It is surprising that ref. [7] was able to reproduce the very low temperature thermodynamics with a theory that produces much too large a gap.

2. The Fix

It is important to understand that there is nothing wrong with the value of the BPS entropy reported in [7]. There are more rigorous ways to obtain it by using supersymmetry and analytic continuation from a region of parameters where the D-brane model is reliable [2]. It is the use of the naive version of the model in a domain of parameters where it does not apply which is at fault. In what follows we will suggest a modification of the model which simultaneously fixes all three problems discussed in section 1. The model is based on an observation of Das and Mathur [12]. Let us begin with an analogy from elementary quantum mechanics. Consider a circular loop of wire of unit radius whose center is at the

origin of the r, θ plane. A bead of unit mass moves on the wire and for obvious reasons the angular momentum of the bead is quantized in integer multiples of $\hbar$. The energy spectrum is given by

$$E = \frac{l^2}{2} \tag{2.1}$$

for all integer l. Now consider a wire which is wrapped n times around the same circle. Eq. (2.1) still gives the energy levels if we allow l to be an integer multiple of $1/n$. The system simulates fractional angular momentum. The real physical system of wire plus bead must, of course, have integer angular momentum but the energy spectrum may be expressed in terms of a "psuedo-angular momentum" which is not the true generator of spatial rotations but rather the generator of rotations of the bead relative to the physical wire.

Next let us consider a set of Q_1 1-branes wrapped on S^1, ignoring for the time being, the 5-branes. In a very naive way we may distinguish the various ways the branes interconnect. For example they may connect up so as to form one long brane of total length $R' = RQ_1$. At the opposite extreme they might form Q_1 disconnected loops. The spectra of open strings is different in each case. For the latter case the open strings behave like Q_1 species of 1 dimensional particles, each with energy spectrum given by integer multiples of $1/R$. In the former case they behave more like a single species of 1 dimensional particle living on a space of length $Q_1 R$. The result [12] is a spectrum of single particle energies given by integer multiples of $\frac{1}{Q_1 R}$. In other words the system simulates a spectrum of fractional charges. For consistency the total charge must add up to an integer multiple of $1/R$ but it can do so by adding up fractional charges. Note that in this case, as opposed to the bead and wire example, the branes by themselves cannot carry any momentum since they are invariant under boosts along directions parallel to the branes.

Now let us return to the case of both 1 and 5 branes. By suppressing reference to the four compact directions orthogonal to x^5 we may think of the 5 branes as another kind of 1 brane wrapped on S^1. The 5-branes may also be connected to form a single multiply wound brane or several singly wound branes. Let us consider the spectrum of (1,5) type strings (strings which connect a 1-brane to a five-brane) when both the 1 and 5 branes each form a single long brane. The 1-brane has total length $Q_1 R$ and the 5-brane has length $Q_5 R$. A given open string can be indexed by a pair of indecies $[a, \alpha]$ labelling which loop of 1-brane and 5-brane it ends on. As a simple example choose $Q_1 = 2$ and $Q_5 = 3$. Now start with

the $[1, 1]$ string which connects the first loop of 1-brane to the first loop of 5-brane. Let us transport this string around the S^1. When it comes back to the starting point it is a $[2, 2]$ string. Transport it again and it becomes a $[1, 3]$ string. It must be cycled 6 times before returning to the $[1, 1]$ configuration. It follows that such a string has a spectrum of a single species living on a circle of size $6R$. More generally, if Q_1 and Q_5 are relatively prime the system simulates a single species on a circle of size $Q_1 Q_5 R$. If the $Q's$ are not relatively prime the situation is slightly more complicated but the result is the same. For example suppose $Q_1 = Q_5 = Q$, again assume the 5 and 1-branes each form a single long brane, then a string will return to its original configuration after cycling around Q times. This time the system simulates Q species living on a circle of length Q. But it is also possible to remove one loop from either the 1 or 5 brane and allow it to form a separate disconnected loop. In this case we have a system consisting of a brane of length QR, one of length $(Q - 1)R$ and a short loop of length R. Since Q and $Q - 1$ are relatively prime the open strings which connect them live on an effective brane of length $Q(Q - 1)R$. Thus there is always a way of hooking up branes so that the effective length is of order $Q_1 Q_5 R$. In fact we will argue that this type of configurations give the largest entropy, and will therefore be dominant.

It can also be seen from the original derivation of the black hole entropy by Vafa and Strominger [2], that the system should have low energy modes with energy of order $1/RQ_1 Q_5$. In this derivation the degrees of freedom that carry the momentum were described by a superconformal field theory on the orbifold $(T^4)^{Q_1 Q_5}/S(Q_1 Q_5)$. A careful analysis of this theory shows that the low energy modes are present. This again corresponds to a particular way of wrapping the branes, since the ground state degeneracy corresponds to that of a *single* wound string (in the U-dual picture) with winding and momentum $(n, m) = (Q_1, Q_5)$ [4].

If, on the other hand, the system consists of singly wound 1 and 5 branes then there are effectively $Q_1 Q_5$ species of open strings living on a circle of size R. This is the case analyzed in ref. [7] .

We can easily see that this model gives the correct value for the extremal entropy. Let us consider the case where Q_1 and Q_5 are relatively prime. As in [7] the open strings have 4 bosonic and 4 fermionic degrees of freedom and carry total momentum N/R. This time the quantization length is $R' = Q_1 Q_5 R$ and the momentum is quantized in units of $(Q_1 Q_5 R)^{-1}$. Thus instead of being at level N the system is at level $N' = NQ_1 Q_5$. In place of the

original Q_1Q_5 species we now have a single specie. To compute the extremal entropy we may introduce a partition function analogous to eq.(1.3) with the appropriate replacements. The result is

$$S = 2\pi\sqrt{N'} = 2\pi\sqrt{NQ_1Q_5} \tag{2.2}$$

A qualitative argument can be given by taking the $Q's$ to be of order N and assuming extensivity. Again suppressing numerical constants

$$E = N/R = R'T^2 = N^2RT^2 \tag{2.3}$$

$$S = R'T = N^2RT \tag{2.4}$$

The equations are exactly the same as eqs. (1.7) and (1.8) and the effective temperature is unchanged. However this time the condition for extensivity is that

$$R' > T^{-1}$$

which is easily satisfied. The picture is somewhat reminiscent of that proposed in [11] although the details differ. Next consider the energy gap. Obviously the gap is given by replacing R by Q_1Q_5R in eq. (1.11)

$$\delta M = \frac{1}{(R')} = \frac{1}{(Q_1Q_5R)} \tag{2.5}$$

Using formulae in ref. [7] one finds that the gap scales like

$$\delta M = \frac{G_N}{R_e^4} \tag{2.6}$$

where G_N and R_e are the Newton constant and Schwarzshild radius.

It is very interesting that this same result can also be obtained from a thermodynamic argument [3]. The authors in [3] argue that thermodynamics of near extremal black holes will only break down when the temperature is so low that the specific heat is of order unity. For a five dimensional black hole this happens at a temperature $T \approx \frac{G_N}{R_e^4}$. In order for the

black hole to be able to radiate at such low temperature the gap should be of order eq. (2.6). The very long length scale

$$R_g = \frac{R_e{}^4}{G_N} \qquad (2.7)$$

associated with the gap has no obvious analog for a Schwarzschild black hole.

For the four dimensional black holes studied in [13] the same arguments lead to a mass gap $\delta M = 1/RQ_2Q_5Q_6$. This agrees with the point at which the classical thermal approximation breaks down, which is $\delta M \sim \frac{G_N}{R_e^3}$ in four dimensions.

Having gotten a reasonable behavior for the mass gap we may hope to reproduce the correct behavior of the near-extreme non-BPS entropy. The arguments of [7] can once again be applied to the fixed model. As in [7] there will be a contribution to the near extremal entropy from right movers. If the right movers carry momentum N_R/R then by an argument identical to that preceding eq.(2.2) we find an incremental entropy

$$\delta S = 2\pi\sqrt{N_R'} = 2\pi\sqrt{N_R Q_1 Q_5} \qquad (2.8)$$

This accounts for only a third of the contribution needed in order to agree with the entropy obtained from the classical black hole solution. However as explained in [10] the classical solutions indicate that an additional contribution from antibranes must be present. If we naively apply U-duality which interchanges N,Q_1 and Q_5 we can account for the other 2/3 of the incremental entropy. This argument clearly needs to be tightened.

Finally, we may also borrow the argument concerning the evaporation process of near extreme black holes from [7]. Instead of Q_1Q_5 species of open strings we have only a single specie. The right moving temperature is again identified as the Hawking temperature and is unchanged from [7].The factor Q_1Q_5 appearing in eq(4.6) of [7] now appears as a volume factor for the gas of particles.

Our last point concerns the relation between the D-brane picture and the arguments given in refs.[5] and [8]. In [8] arguments were made to show that the degrees of freedom that give rise to horizon entropy are open strings attached to the horizon. It was also argued in [5] that no matter how small the string coupling g may be, when an area-density of $1/g^2$ (in string units) is achieved, interactions become important. This "saturation density" defines both the planck density and the density of open strings on the horizon of any black hole.

As we have seen, whenever extensivity prevails, as it does in the "fixed" model, the entropy and number of quanta scale the same way. Thus, apart from the very infrared quanta, the density of open strings on the puffed up horizon will always be of order one per planck unit of area in agreement with [8].

As a consequence of the high density of string it is generally unlikely that weak coupling methods, including the use of D-branes, can be used to quantitatively study the interaction of a black hole with infalling matter. However, there may be an exception range of parameters which is amendable to weak coupling approximations although we can offer no clear proof of this. The success of the model in the very long wave length region suggests that the infrared quanta with wave length larger than R_e but smaller than than the very long length scale R_g may behave like goldstone bosons and decouple from the dense horizon soup. In this respect we might recall an entirely different, but apparently analogous, physical situation: electrons and nuclei in a metal. The low energy thermodynamics can be fairly well reproduced by considering the electrons to be free, though there are lots of charges present. In that case we have a better understanding as to which physical questions can be answered by regarding the electrons as free and which require taking into account the interactions. Hopefully, further studies of these models will provide a similar understanding for the fat black hole case.

What is the justification for picking the configurations in which the 1 and 5 branes each form single long branes and ignoring other configurations? The answer lies in the entropy of each configuration. As we have seen, the entropy of the configuration in which the branes form a maximal number of short loops of size R is only of order $N \log N$ while the entropy of the state with maximally long connected components is of order $N^{\frac{3}{2}}$. In fact, the configurations with the maximum entropy are always those in which the branes behave as if they were a single long brane.

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