Finite N Heterotic Matrix Models and Discrete Light Cone Quantiztion

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The finite N version of Matrix theory describes M-theory and superstrings in so-called discretized light cone quantization (DLCQ). Its role has been explained for M-theory in 11 dimensions and for type IIA theory. We show novelties which arise by generalizing the ideas to heterotic strings. The states which arise in O(N) theories with odd N we interpret as fields which are antiperiodic in the longitudinal direction. Consistency of these ideas provides a new evidence for the conjecture that finite N models describe sectors with a given longitudinal momentum.

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1. Introduction

According to ref's [2,3] Matrix Theory is meaningful even for a finite size of the matrices and describe a particular sector of states with a given value of longitudinal momentum in the so called Discretized Light Cone Quantization (DLCQ). In DLCQ [4-6], the lightlike direction x^- is compactified and thus the points with the same value of x^i , i = 1, ... 9 and x^+ (the light cone time) and with x^- differing by an integer multiple of $2\pi R$ are identified.

Because of this compactification, the component P^+ of momentum which is conjugate to x^- must have quantized values $P^+ = N/R$. Since this component is conserved, the sectors of the Hilbert space with a given value of N are mutually decoupled. The physics of the sector of M-theory with $P^+ = N/R$ is described by the U(N) supersymmetric quantum mechanics [1]. The same is true not only for M-theory, but also for its toroidal compactifications like the IIA strings.

The low energy limit of M(atrix) theory usually allows a spacetime interpretation and can be approximated by a quantum field theory. The fields in such a field theory are periodic in x^- with period R. We will present a similar picture for heterotic strings where some of the fields are antiperiodic in x^- .

2. Short review of matrix strings

After a compactification of X^1 to a circle, the compact coordinate is represented by the covariant derivative with respect to a new spatial dimension σ , dual to X^1 . The original 0+1 dimensional model [1] describing M-theory in 11 dimensions in DLCQ becomes a 1+1 dimensional model describing M-theory on a circle which is known to be equivalent to type IIA strings. Each string can have longitudinal momentum $P^+ = N/R$ and in the limit of zero coupling (i.e. zero radius of X^1) it is approximated by matrices $X^i(\sigma)$ which are diagonalizable for each σ and periodic up to a cyclic permutation [7] of length N. In [8] such solutions were explained using the notion of the moduli space of the corresponding supersymmetric Yang Mills theory.

Paper [9] showed that the matrix strings reproduce the lowest order interactions with a correct scaling law $\lambda \approx R^{3/2}$. It is also expected that the matrix strings are able to give the higher order contact interaction terms which are necessary to fill the entire moduli space of Riemann surfaces in the light cone gauge calculations.

The idea of supersymmetric Yang Mills theories living on the dual torus can be continued up to T^3 compactification of M-theory. The central charges are described by fluxes and base space momenta: the momentum is $\int P^i$, the transverse membrane charges are $\int F^{ij}$ and the longitudinal membrane charges Z^{i-} are identified with momenta in the base space. Note that in the case of matrix strings, Z^{1-} corresponds to strings' $L_0 - \tilde{L}_0$.

3. Short review of heterotic matrix models

The heterotic matrix models were pioneered in [11] as the mechanics of D0-branes in type IA theory. In [12] the new 16 vector fermionic degrees freedom corresponding to strings joining D0-branes and D8-branes were found to be the source of the E_8 symmetry. In [13] the model [11] was exhibited as an orbifold of the original model [1]. This study showed that the model describes M-theory with one boundary (and thus one E_8 group) and contains also open membranes. Possible membrane topologies were further investigated in [14] using the technology of area preserving diffeomorphisms. In [10] toroidal compactifications of heterotic strings were first formulated via matrix models and the origin of various sectors and GSO projections was explained.

Let us briefly discuss some technical matters. The original model [1] describes M-theory with X^- compactified to a circle of radius R. We can find a Z_2 symmetry [13] reversing sign of X^1 and multiplying spinors by γ^1 . But this reversing must be accompanied by transposition of all the matrices which can be visualized by reversing of the orientation of membrane.

Restricting the matrices containing the fundamental degrees of freedom to be invariant under this operation, we restrict them to be real (or pure imaginary) matrices and the original symmetry U(N) is reduced to O(N): we have no condition for the determinant which will be important for GSO projections and sectors – and also we must allow both even and odd values of N. For consistency, we must also add 16 fermionic vector degrees of freedom which are the source of E_8 symmetry. These extra fields can be understood as a twisted sector of an underlying theory generating the degrees of freedom of M(atrix) theory and in lower dimensions their origin can be connected with two-cycles of K3 manifolds shrunk to zero size.

We will normalize R so that the period of x^- is always $2\pi R$. In the case of heterotic strings, the graviton appears first in N=2 models, so we have the identity $P^+=N/2R$. The fields whose states appear already in N=1 models are antiperiodic in x^- so their P^+ is an odd multiple of 1/2R. (Of course, we could keep the condition $P^+=N/R$ but then the period in x^- would be equal to πR .) The O(2N) model reduces to the U(N) model far from the domain wall; both models describe states with $P^+=N/R$.

Let us discuss the simplest (free) models N=1 describing states with one quantum $P^+=1/2R$. For the [1] model, we have 1×1 matrices P^i, X^i, θ^i and the P^- is given by $R\cdot P_i^2/2$. The 16 fermions θ ensure the $2^{16/2}=256$ -fold degeneracy of the state, reproducing 128 bosonic and 128 fermionic physical states of supergravity. For the [11,12,13] model, only $X^2 \dots X^9, P^2 \dots P^9$ and half θ 's survive, but we have extra $\chi^1 \dots \chi^{16}$. The energy of massless states is given by

$$P^{-} = \frac{(P_i)^2}{2P^{+}} = R \cdot P_i^2 \tag{3.1}$$

but the degeneracy from θ 's is now only $2^{8/2} = 16$ and there is an extra 256-fold degeneracy from χ 's. But not all 256 states are physical. The gauge group is O(1) which has only one nontrivial element T, changing the sign of the 1-dimensional vectors. It affects only the vectors χ , so it anticommutes with them. As a result, T acts as the GSO operator, selecting only 128 from 256. So finally, the O(1) model describes only 16.128 states which is the SO(16) spinor part of the gauge supermultiplet.

For O(2) symmetric system, we expect emergence of the graviton supermultiplet and the 120 part (SO(16) adjoint) of the gauge supermultiplet. For higher values of N, the situation will be similar to that of N = 1 and N = 2 for odd and even N, respectively.

Thus the shift in x^- by $2\pi R$ is equivalent to the rotation by 2π in the SO(16) subgroups of the gauge groups E_8 , under which 120 is even and 128 odd. As was explained in [10], AP and PA sectors of the $E_8 \times E_8$ heterotic string appears for odd N models while AA and PP sectors for even N models.

We can also explain the nature of GSO projections for finite N. There must be two GSO projections acting at the χ fields. One GSO operator anticommutes with all the χ degrees of freedom. This operator is given by the global transformation given by the -1 matrix from the O(N) gauge group and therefore even for finite N, only even states survive the condition

of invariance under the gauge group. In the case of the quantum mechanics with one domain wall, this is the only projection we need.

But for the heterotic strings with two E_8 factors we need one extra GSO operator anticommuting with half of χ 's only. This projector is not available for finite N models but is only result of the large N limit, see [10].

The situation here is similar to the level-matching conditions which are also not available [2] for finite N. Among the string states for N=1, for instance, we find even the states which do not obey the conventional level-matching conditions and the second GSO projection. However, in the large N limit such unmatched states decouple. The similarity between the second GSO projection and the level-matching conditions is not an accident, of course. In AP and PA sectors, fulfilling of level-matching conditions automatically implies the GSO projection (whose GSO operator anticommutes with the A half-integral fermions). In AA sector the level-matching condition implies the GSO projection whose operator anticommutes with all the χ fermions which is connected with the fact that for even N where states of AA sector appear, -1 matrix belongs even to SO(N).

4. Broken E_8 symmetry

Now we see why E_8 symmetry is not manifest in the heterotic matrix models. Using the spacetime interpretation, the E_8 is not manifest because it is broken by DLCQ procedures to SO(16): going around x^- circle is equivalent to a nontrivial transformation in E_8 (which is however trivial for SO(16) subgroup).

The situation is similar to the nonmanifest Lorentz generators. These generators are also not manifest because of the DLCQ machinery which selects priviliged lightlike directions.

In both cases, the 128 generators of E_8 or the nontrivial Lorentz generators mix the states with different values of P^+ . But also in both cases we can reconstruct the symmetry when the period R goes to infinity while keeping the P^+ fixed (since we study states with particular finite values of P^+). Because of the fixed P^+ , large R limit means large N limit and thus in the large N limit we reconstruct whole E_8 symmetry.

For a possible SO(32) symmetry, we could expect that the states (120,1) and (1,120) of the $SO(16) \times SO(16)$ group appear together with gravitons for even N and the states (16,16) for odd N. The interpretation will be completely analogous in this case.

5. Conclusion

In this paper we described new evidence for the conjecture that finite N matrix models describe a sector with a given value of longitudinal momentum in DLCQ: we explained how this works for the heterotic matrix models. The new ingredient is the antiperiodicity of some fields in the lightlike direction.

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