SOME SPECULATIONS ABOUT BLACK HOLE ENTROPY IN STRING THEORY

By Leonard Susskind¹

Physics Department

Stanford University

Stanford, CA 94305-4060

and

Department of Physics and Astronomy Rutgers University Piscataway, NJ 08855-0849

ABSTRACT. The classical Bekenstein entropy of a black hole is argued to arise from configurations of strings with ends which are frozen on the horizon. Quantum corrections to this entropy are probably finite unlike the case in quantum field theory. Finally it is speculated that all black holes are single string states. The level density of strings is of the right order of magnitude to reproduce the Bekenstein entropy.

I. Introduction

There are some puzzles concerning the entropy of black holes which I would like to consider from the point of view of string theory. First of all the meaning of the Bekenstein entropy

$$(1.1) S_B = \frac{1}{4} \frac{\text{Area}}{4G\hbar}$$

has always been mysterious. Entropy, as generally understood, has to do with the counting of configurations of some set of degrees of freedom. What the degrees of freedom of the horizon are and why they give entropy of order $1/\hbar$ has remained obscure.²

The second puzzle concerns the higher order quantum corrections to the entropy. G 'tHooft has emphasized[1] that conventional quantum fields contribute an ultraviolet divergence to S which blows up at the horizon. This is despite the fact that no curvature or other invariant signal becomes large. The ultraviolet divergent entropy is proportional to the area and although down by a factor of \hbar from S_B it is infinitely larger.

¹susskind@dormouse.stanford.edu

²Hereafter \hbar will be set to 1

It has not been properly appreciated except by 't Hooft that this ultraviolet divergence of S is the same problem as Hawking's information paradox and that any theory which naturally produces a finite entropy will also solve this problem. From the perspective of a distant observer nothing ever reaches the horizon. Instead all matter settles into layers which eternally sink toward it. If the entropy of matter near the horizon is infinite, indefinite amounts of information can be stored arbitrarily close by. This information can not be emitted until the horizon shrinks to quantum mechanical proportions and perhaps not even then. By contrast, a theory in which the information storage capacity is finite has no choice but to reemit information as the horizon shrinks. In a previous paper[2] I showed that in string theory, from the outside observers vantage point, the substance of infalling strings not only never reaches the horizon but never entirely sinks past the stretched horizon. Therefore it seems appropriate to ask whether string theory leads to a finite entropy.

The last puzzle concerns the connection between the spectrum of black holes and that of unperturbed strings. In both cases that level density increases rapidly with mass. Furthermore, most of the spectrum of strings must actually be black holes since they lie within their Schwarzschild radii. Nevertheless I do not know of any speculation that the two spectra may really be the same.[3] In fact at first sight such a suggestion seems nonsensical. The level density of black holes grows like $\exp 4\pi M^2$ while that of strings is exponential in the first power of the mass. We shall see that this argument is wrong and that the two spectra, when properly interpreted, could easily be the same.

II. Thermodynamics in Rindler Space

I will begin by outlining the procedures involved in constructing the thermodynamics of Rindler space. The euclidean continuation is ordinary flat space in cylindrical polar coordinates

(2.1)
$$ds^2 = r^2 d\theta^2 + dr^2 + dx_{\perp}^2$$

where the angular variable θ is the euclidean time, r is the radical variable (not to be confused with the Schwarzschild coordinate) and X_{\perp} is the 2 space parallel to the horizon. The horizon itself is the surface r=0.

The Rindler hamiltonian H_R is the generator of θ -rotations.

$$(2.2) H_R = \frac{\partial}{\partial \theta}$$

The partition function from which thermodynamics is derived is

$$(2.3) Z = Tr \exp\{-\beta H_R\}$$

For physical applications β should be set equal to 2π . For the purposes of thermodynamic analysis it must be left as a free variable. Note that varying β is equivalent to introducing a conical singularity at the horizon with an angular deficit $2\pi - \beta$.

The free energy $F(\beta)$ is given by

$$(2.4) F = \frac{1}{\beta} \log Z$$

and the entropy by

$$(2.5) S = \frac{\partial F}{\partial T_R}$$

where the Rindler temperature is $T_R = 1/\beta$. Eq. 2.5 is the reason we need to be able to vary β away from 2π .

Consider first the thermodynamics of a free scalar field. The partition function can be carried out as a sum over first quantized closed path particle trajectories in a well known manner. The only new ingredient is the conical singularity at r = 0.

It is easy to see that paths which do not wind around r=0 contribute no β dependence to the free energy. This means that they can be dropped when calculating the entropy. Thus we find that the entropy is the first order variation with respect to β of the sum of paths which wind one or more times around the horizon. This principle extends to interacting field theories described in terms of networks of paths forming Feynman diagrams. Only those networks which topologically encircle r=0 contribute to entropy. It is clear that the entropy divergences found in conventional quantum field theory are due to very small loops near r=0.

Now I want to study black hole entropy using an analogous method. Consider the euclidean continuation of the Schwarzschild metric in which time is periodic. The geometry near r=0 is identical to Rindler space. Far from the horizon 2.1 is modified to

$$ds^{2} = 16m^{2}G^{2}d\theta^{2} + dr^{2}$$
+ transverse metric
$$= -dt^{2} + dr^{2} + \text{transverse}$$
(2.6)

where

$$(2.7) t = -4imG\theta$$

is the Schwarzshild time coordinate.

The ordinary energy (mass) of a black hole is conjugate to Schwarzshild time and the conventional temperature is defined in terms of this energy. The energy and Schwarzshild temperature are given in terms of the mass by

(2.8)
$$S = 4\pi M^2 G$$
$$Ts = \frac{1}{8\pi MG}$$

In transforming energy from Schwarzshild to Rindler coordinates care must be taken to insure that the Rindler energy E_R is conjugate to $i\theta$. We assume M is conjugate to t. Thus

(2.9)
$$[E_R(M), i\theta] = i$$
$$[M, t] = i$$

Using 2.7 and the usual properties of commitators gives

$$(2.10) E_R = 2M^2G$$

It is interesting that both entropy and Rindler energy are extensive functions of the horizon area $(A = 16\pi M^2 G^2)$.

(2.11)
$$E_R = \frac{A}{8\pi G} = 4\pi 2M^2 G$$
$$S = \frac{A}{4G} = 4\pi M^2 G$$

To find the Rindler temperature we use the first law of thermodynamics

$$(2.12) dE_R = T_R dS$$

which gives

$$(2.13) T_R = \frac{1}{2\pi}$$

To compute from first principles directly in Rindler coordinates we must calculate the free energy of a euclidean black hole with an angle deficit of $(2\pi - \beta)$. Again this introduces a conical singularity at the horizon. The lowest order contribution to F is order $1/\hbar$ and is given terms of the classical action.

(2.14)
$$F = \frac{1}{\beta} \operatorname{Action}$$

The action consists of three distinct terms. The first is proportional to the integrated Ricci scalar which vanishes by virtue of the Einstein equations. The second involves the extrinsic curvature at the boundary at large r. This term is proportional to β and does not contribute to the entropy. The third contribution is due to the curvature delta function at the conical singularity. It is proportional to the horizon area and to the deficit angle $2\pi - \beta$. Explicit calculation gives

(2.15)
$$F = \frac{\beta - 2\pi}{\beta} \frac{\text{Area}}{8\pi G}$$

and

(2.16)
$$S = \frac{\partial F}{\partial T_R} = \frac{\text{Area}}{4G}$$

III. Black Hole Entropy in String Theory

Evaluating the partition function in a conical background provides a general framework for calculating black hole entropy including the classical Bekenstein term and quantum corrections from both matter and gravitational field. To apply it to string theory requires formulating the theory in backgrounds with small but arbitrary angle deficits. This has not been done. I will therefore restrict my remarks to certain general features. The free energy is given as a sum over world sheet configurations of arbitrary genus. By an argument similar to that used in field theory, the only nonvanishing contributions to the entropy come from world sheets which in some way wrap around or touch the singularity at the horizon. For example a torus can surround the horizon as in fig(1) This configuration describes the contribution to the entropy of a free closed string. This is seen by slicing the figure at some fixed euclidean time such as $\theta = 0$ (See fig 2) Another configuration in which the horizon intersects the torus is shown in fig 3 In this case the instantaneous configurations involve strings with their ends frozen on the horizon as in fig 4 Such configurations must be included in the space of states of the black hole.

The genus k surfaces contribute with a coefficient G^{k-1} . Evidently the Bekenstein entropy corresponds to a genus zero surface with the topology of a sphere as in fig 5 These surfaces describe the evolution of a single string with ends on the horizon. In Minkowski space the endpoints of the string can not move because of the infinite time dilation at the horizon but the rest of the string is free to wiggle. This solves the puzzle of the origin of S_B in String theory.

One might wonder why the zero genus contributions do not vanish as they usually do in string theory. The reason involves both the conical singularity at the origin and the fact that the deficit angle at infinity does not vanish. The presence of these genus zero contributions very near the horizon is almost certainly related to a similar effect found by Atick and Witten[4] in high temperature string theory and large $N\ QCD$.

We next turn to the question of the finiteness of the higher genus quantum corrections. In the absence of precise tools for quantizing strings on singular spaces I can only quote some circumstantial evidence for finiteness. However, before doing so we must take care of a trivial divergence associated with the infinite volume at spatial infinity. If the black hole is in thermal equilibrium with its environment, the nonvanishing temperature will cause a volume divergence in all extensive thermodynamical variables. This can be easily overcome by passing to the limit of infinite black hole mass. The resulting geometry is flat Rindler space with a deficit angle. In Rindler space-times of dimension greater than two the thermodynamical variables are infrared finite. The entropy per unit area is therefore well defined.

As I have mentioned in sect 2 the ordinary field theoretic divergences in entropy arise from paths of vanishing length which encircle the horizon. In string theory the analogue of a path of given proper time is a torus with complex modular parameter τ . It is well known that the integration region over τ which could potentially cause ultraviolet divergences should be excised because it corresponds to an infinite overcounting of geometrically identical tori. There is no mechanism for generating ultraviolet divergences if the relevant loop integrals are anything like other string amplitudes.

The finiteness of string theoretic loops is due to the extreme paucity of degrees of freedom at short distances. This lack of short distance structure is seen in several ways.

1) Atick and Witten have shown[4] that string theory behaves more softly at high temperatures than any possible continuum quantum field theory. Roughly

speaking the high temperature thermodynamics is consistent with a lattice theory in which the spacing is finite. Similar evidence comes from the work of Klebanov and Susskind[5] who show that exact string amplitudes can be derived from a spacetime lattice theory with non vanishing spacing.

- 2) High energy scattering amplitudes at fixed angle are the traditional method of uncovering short distance structure. Gross and Mende[6] have shown that such amplitudes vanish like gaussian functions of momentum transfer.
- 3) It appears to be impossible to force the dimensions of compact space dimensions to be smaller than a certain size of order ℓ_s .[7]
- 4) Following the progress of a string falling toward a horizon, an external observer fails to see the object Lorentz contract.[2] There appears to be a minimum longitudinal size that strings can occupy. Furthermore for non vanishing coupling there is a bound to the number of strings that can pass through a small region without inducing violent interactions.

All these facts point to a common conclusion. When we attempt to localize strings or parts of strings in distances much smaller than ℓ_s we discover a complete lack of local degrees of freedom. This strongly suggests that higher genus contributions to black hole entropy is finite and that to an external observer, indefinite quantities of information can not collect arbitrarily near the event horizon. What is desperately needed is a computation to confirm this.

IV. Black Holes as Single String States

I turn now to a radically different way to estimate the entropy of a black hole. When strings fall on to a horizon an external observer sees them spread out and eventually fill the stretched horizon.[2] One can regard this phenomenon as a melting of strings as they encounter Hagedorn temperature conditions[4] at a distance $\sim \ell_s$ from the event horizon. The entropy of single string states is so large that strings on the horizon will tend to form a single string when the Hagedorn temperature is approached. The implication is that all black hole states are in one to one correspondence with single string states.

Now it has been observed in the past that the high mass-low angular momentum states of string theory must be black holes since they lie within their Schwarzshild radii. I would like to make the heretical suggestion that the spectrum of black holes and the spectrum of single string states are identical.[3] Furthermore this provides us with a direct way to estimate the number of levels and therefore the entropy.

Before we can actually compare the spectra we must deal with somewhat trivial but numerically very important effect. The classical gravitational field outside the stretched horizon is not a low order effect. We must imagine removing the effects of this field before we try to compare low order string theory with black hole physics. The main effect which must be accounted for is the large red shift of clock rates that takes place between the stringy stretched horizon and an observer at infinity. In other words a rescaling of all energy levels of the black hole should be done.

The stretched horizon is the place where the local Unruh temperature becomes hot enough for stringy effects to become important, i.e. the Hagedom temperature.

This means a distance $\sim \ell_s = (\alpha^1)^{-\frac{1}{2}}$ from the event horizon at r = 0. At this distance, the proper time of a fiducial observer is given by

$$\tau = i\theta \ell_s$$

where $i\theta$ is Rindler time. All quantities with units of energy should be rescaled by dividing the corresponding Rindler quantities by ℓ_s . The resulting quantities would also be appropriate for an observer at infinity if the effects of the classical gravitational field could be removed.

Thus we define the stretched horizon energy and temperature of the black hole to be

$$E(S.H.) = \frac{2M^2G}{\ell_s}$$

$$(4.2) T(S.H.) = 1/2\pi\ell_s$$

Another way to think of the relation between E(S.H.) and M is that the long range field outside the stretched horizon renormalizes the mass from its "bare" value E(SH) to its renormalized value M. Therefore I would like to suggest that a black hole of mass M should be identified with a string state of mass E(S.H.).

We can estimate the entropy of a black hole by counting the levels of fundamental strings. The number of states at mass level r = E satisfies

(4.3)
$$\log N(E) \sim E\ell_s$$

Using 4.2 we find a black hole of mass M has a level density satisfying

(4.4)
$$\log N(M) \sim M^2 G$$

which says that the entropy is of order S_B . It is therefore not inconsistent to suppose that a correspondence exists between black holes and fundamental string states.

Vafa has pointed out that this correspondence may extend to extreme charged black holes. In this case the mass renormalization due to the gravitational field energy exactly cancels the electromagnetic energy so that string states of mass M should be compared with black holes of mass M. If, for example, we consider charge arising from winding modes then the minimum mass of a string of charge Q is proportional to Q. This corresponds exactly to the extremal black hole.

If the view of black hole entropy in this section is correct then there can be little doubt that the quantum corrections to S_B are finite. These quantum corrections would result from the finite shifting of levels due to higher genus world sheets.

Finally I would like to mention an observable effect of the string theory on black hole evaporation. In the usual picture the final evaporation process takes place at planckian temperatures. The last radiated particles would carry energy of order the Planck mass.

To best appreciate the difference that string theory makes, it is helpful to pretend that the string coupling $g^2 = G/\ell_s^2$ is extremely small so that the Planck and string

scales are extremely well separated. Let us consider the radius of an average excited string state of mass M, ignoring all higher order effects including the long range gravitational field. Ignoring quantum fluctuations of the string the mean radius of the average configuration of mass M can be shown to be

$$(4.5) R_{ST} = \sqrt{M\ell_s^3}$$

Comparing this with the Schwarzschild radius, $R_{SCH} \sim MG = Mg^2\ell_s^2$, we see the two are equal when

$$(4.6) M = M_a = (\ell_s g^4)^{-1}$$

Thus for $g \ll 1$ there is a large range of masses for which the conventional string configuration is larger than R_{SCH} and no black hole behavior should occur.

On the otherhand an evaporating black hole should behave conventionally until the red shift factor at the stretched horizon is of order unity. At this point there is no large red shift factor and strings should behave like strings. The mass at this point is

$$(4.7) M = M_b = \frac{1}{g^2 \ell_s}$$

Several things happen at this point. The first is that the area of the black hole horizon has become equal to ℓ_s^2 . Second, the Hawking temperature has reached the Hagedorn temperature. Third, the conventional mass M and the stretched horizon energy E(S.H.) become equal. Finally at this point the Bekenstein entropy $\sim M^2G^2$ becomes of order the string entropy $M\ell_s$.

Between M_a and M_b we have two descriptions, ie. string and black hole, both of which should apply but which disagree.

The resolution of this inconsistency is that in this region there are two configurations. The first is metastable and describes a conventional string with radius larger than R_{SCH} . The second, with larger entropy is stable and consists of a black hole with the string gravitationally collapsed to the stretched horizon.

Below mass M_b the string entropy exceeds the black hole entropy so that the black hole becomes thermodynamically unstable. Accordingly when a black hole reaches the Hagedorn temperature it "inflates" to a string whose size exceeds that of a black hole by the factor

$$\frac{R_{ST}}{R_{SCH}} \sim \frac{1}{g}$$

Thereafter it decays like a weakly coupled string. In particular the momenta of the emitted particles never exceeds the Hagedorn temperature.

Acknowledgments.

This paper was written while the author was visiting Rutgers University. He is grateful not only for the hospitality, but for many exciting discussions with the members of the theory group and other visitors. In particular C. Vata pointed out

the significance of extremal charged states in string theory. The observations about the final stages of evaporation arose in a conversation with Steve Shenker about possible experimental signatures. As always discussions with Tom Banks, Nathan Seiberg and Mike Douglas were very valuable and fun. Helpful discussions with Ed Witten and Curt Callan are also appreciated.

References

- 1. G. 't Hooft, Private Communication.
- 2. L. Susskind, Strings, Black Holes and Lorentz Contraction, Stanford University preprint SU-ITP-93-21 hep-th/9308139.
- 3. 't Hooft has had the long standing belief that black holes are the extrapolation of elementary particles to high mass. See for example G. 't Hooft, Nucl. Phys. **B 335**, 138.
- 4. J. Attick and E. Witten, Nucl. Phys. **B 310**, 291.
- 5. J. Polchinski, Comm. Math Phys. 104, 37.
- 6. I. Klebanov and L. Susskind, Continuum strings from discrete field theories, Nucl Phys. B 309, 175.
- 7. D.J. Gross and P. Mende, Nucl Phys. **B 303**, 407.
- 8. K. Huang and S. Weinberg, Phys REv Lett **25**, 895, S. Fubini and G. Veneziano, Nuovo cimento 64 A 1969 1640.