Trouble For Remnants

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ABSTRACT

An argument is presented for the inconsistency of black hole remnants which store the information which falls into black holes. Unlike previous arguments it is not concerned with a possible divergence in the rate of pair production. It is argued that the existence of remnants in the thermal atmosphere of Rindler space will drive the renormalized Newton constant to zero.

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It has been argued that information which falls into a black hole eventually ends up in a Planck scale remnant [1]. In order for these remnants to keep track of all the possible states that can form an arbitrarily large black hole, the number of distinct species of remnant must be infinite. From the point of view of entropy, one must suppose that black holes have an infinite additional entropy above and beyond the usual Bekenstein, Hawking entropy. The infinite addition must be thought of as mass independent if it is not to modify black hole thermodynamics. We will call this term S_R and refer to it as the internal entropy of the black hole or remnant.

The idea of remnants apparently leads to an obvious thermodynamic catastrophe. Any thermodynamic system such as the sun would find it entropically infinitely favorable to turn all its energy into zero temperature remnants. Various mechanisms including pair production could create the remnants. One particular source of worry is that the divergent number of species could lead to infinite rates for such production but even finite rates could be catastrophic if they allowed the remnant to come to equilibrium on a time scale of order the age of the universe.

To avoid such disasters, remnant theorists propose various mechanisms for enormously suppressing the rate of such production [2]. According to this view, true ultimate thermal equilibrium would be remnant dominated but it would also be unattainable in any reasonable amount of time. Argument has gone back and forth over the consistency of such suppression mechanisms [3].

In this note I will present an argument for the inconsistency of remnants which is not based on infinite production rates. We will assume that remnants are small objects of about the Planck size and Planck mass. There are theories of remnants in which the objects are seen from the outside as small but in which the interior of the remnant expands to some enormous size [2]. For our purposes it is only important that when seen from outside, the remnant is small. We also assume that the gravitational force at asymptotic distances is governed by a finite value of the Newton constant G.

We begin by recalling a low energy theorem due to the author and Uglum [4], concerning the entropy of Rindler space. This entropy may be computed in terms of a path integral in a conical euclidian space-time. If the deficit angle of the cone is $(2\pi - \beta)$ and the partition function is $Z(\beta)$ then the entropy is given by

$$S = -\beta^2 \,\partial_\beta \left[\frac{\log Z(\beta)}{\beta} \right] \tag{1}$$

On the other hand the path integral for the conical geometry may be calculated from a knowledge of the effective gravitational action. As shown in [4] the result only depends on the fully renormalized Newtonian coupling constant G. The low energy theorem states that the total entropy per unit area is given by the Bekenstein, Hawking formula

$$\frac{S}{A} = \frac{1}{4G} \tag{2}$$

Radiative corrections to the coupling constant must be in one to one correspondence with contributions to the entropy in a consistent theory. Furthermore, the entropy used in eq.(2) is the full entropy in exact thermal equilibrium described by the Unruh density matrix [5]

$$\rho_u = \frac{\exp(-2\pi H_r)}{Z} \tag{3}$$

where H_r is the Rindler Hamiltonian.

As is well known, the thermal state describes a thermal atmosphere of particles that are seen by fiducial observers (FIDOS) at rest with respect to the accelerated Rindler coordinates [6]. At a proper distance ρ from the horizon the local FIDO sees a proper temperature $T(\rho)$.

$$T(\rho) = \frac{1}{2\pi\rho} \tag{4}$$

The thermal atmosphere will contain all particle species in equilibrium at that local temperature. At places where the temperature is smaller than the mass of the ith particle type, the density of that species will be of order

$$N_i \sim \exp[-2\pi\rho M_i] \tag{5}$$

It is important to note that it does not matter what mechanisms may or may not exist to quickly bring these particles to equilibrium. The low energy theorem relates G to the exact equilibrium value of the entropy.

Now let us consider the presence of the hypothetical remnants in the thermal atmosphere. Assuming that their mass is the Planck mass (any other mass can be substituted with no change in the argument) the density of any given remnant species is

$$N_i \sim \exp[-2\pi\rho M_p] \tag{6}$$

Since the number of distinct species is $\exp S_R$ the total remnant density is

$$N_R \sim \exp[S_R - 2\pi\rho M_p] \tag{7}$$

Eq.(7) only makes sense if N_R is less than the close packing density which for definiteness we take to be the Planck density.

Now let us assume S_R becomes astronomically large. It will be entropically favorable for the density to become as large as possible out to distances satisfying

$$S_R - 2\pi\rho M_p = 0 (8)$$

or

$$\rho = \frac{S_R}{M_n} \tag{9}$$

Thus, when $S_R \to \infty$ the entire Rindler space fills up with remnants out to arbitrarily large distances, even to where the acceleration is negligible. This certainly looks like a theory which is out of control.

Since each remnant has an internal entropy S_R , the total entropy per unit volume in the region $\rho < \frac{S_R}{M_p}$ is

$$\sigma \sim S_R M_p^{\ 3} \tag{10}$$

Finally, integrating over ρ , one finds an entropy per unit area given by

$$\frac{S}{A} \sim S_R^2 M_p^2 \tag{11}$$

It is interesting to contrast this behavior with the effects of ordinary black holes in the thermal atmosphere. By an ordinary black hole I mean one with the usual Bekenstein, Hawking entropy. Thus consider the probability for a black hole to materialize in the thermal atmosphere at a distance ρ from the Rindler horizon. Let the black hole have mass M and entropy $S_{B.H.} = 4\pi M^2 G$. As long as the black hole is significantly smaller than ρ then the probability is of order

$$P = \exp[-2\pi\rho M + 4\pi M^2 G] \tag{12}$$

Obviously, the only black holes which can be studied in this way are those with Schwarz-schild radii significantly smaller than ρ . This means the mass should always satisfy $2MG < \rho$.

If we integrate (12) over M from $M = M_p$ to $MG = \alpha \rho$ with $\alpha < .5$ we find the integral is dominated by the lower end of the integration, which gives

$$P = \exp{-2\pi\rho M_p} \tag{13}$$

As we should expect, black holes are exponentially suppressed in the thermal atmosphere even though their entropy is enormous. Remnants, by contrast, would overrun the system.

Thus, while it was our intention to add a large additive constant to the entropy we instead find in eq.(11) an even bigger area dependent term. Evidently as $S_R \to \infty$ the entropy per unit area will be driven to infinity and with it the inverse of G at asymptotic distances. I can see no way to avoid this without giving up remnants.

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