# Quantum Gravitational Collapse

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#### Abstract

We apply the recent results in Loop Quantum Cosmology and in the resolution of Black Hole singularity to the gravitational collapse of a star. We study the dynamic of the space time in the interior of the Schwarzschild radius. In particular in our simple model we obtain the evolution of the matter inside the star and of the gravity outside the region where the matter is present. The boundary condition identify an unique time inside and outside the region where the matter is present. We consider a star during the collapse in the particular case in which inside the collapsing star we take null pressure, homogeneity and isotropy. The space-time outside the matter is homogeneous and anisotropic. We show that the space time is singularity free and that we can extend dynamically the space-time beyond the classical singularity.

### Introduction

We consider the gravitational collapse in the particular case the spatial section of space time is subdivided in two parts, a first part with matter and a second part without. Classically we consider a dust star of null pressure and we report the known results for the gravitational collapse to produce a classical singularity. In the quantum theory we use the recent results obtained for the Schwarzschild solution inside the horizon [3] to describe the quantum theory outside the region which contains the matter. In fact outside the matter and inside the horizon we consider a general two dimensional minisuperspace with space sections of topology  $\mathbf{R} \times \mathbf{S}^2$  which is know as Kantowski-Sachs space time [5]. The Schwarzschild space time inside the horizon is a particular representative of this class of metrics. On the other side inside the matter we take the Friedmann space time with positive curvature. In this paper we use the same non Schrödinger procedure of quantization used in the previous paper [2] and in the work of V. Husain and O. Winkler on quantum cosmology and introduced by Halvorson [8] and also by A. Ashtekar, S. Fairhust and J. Willis [6]. In particular in this paper we resolve the Hamiltonian constraint inside and outside the region where the matter is located. We resolve the Hamiltonian constraint inside the matter without specifing the matter type but in the case of dust matter we match the wave function inside and outside the matter obtaining a temporal coordinate for all the space time that is connected with the star radius. The wave function " $\psi(\tilde{a}, \phi, a)$ " that resolves the Hamiltonian constrain is interpreted as the wave function for the matter field " $\phi$ " (which represents the dust matter) and the gravitational field "a" at the time "a": "a" is proportional to the radius of the star. Than we consider the particular case of a scalar field and we quantize the complete Hamiltonian constraint using the same quantization procedure [2], [6], [5] also for the matter field. In this case we cannot match the Schwarzschild space time outside the matter because for the scalar field the pressure is not equal to zero. On the other side in this case we can match with the Vaydia space time as in the paper [7].

The main result is that the space time is singularity free. In fact using the method [10] we can define the inverse volume density and the operator  $\frac{1}{a^6}$  in terms of the holonomy analog and the

volume itself and we show that these quantities are always finite and upper bounded. The operator  $\frac{1}{a^6}$  is proportional to  $\mathcal{R}_{\mu\nu\rho\sigma}\,\mathcal{R}^{\mu\nu\rho\sigma}$  in the case of the Schwarzschild metric and is divergent in the classical theory producing the singularity. From the point of view of the matter region we show that the classical singular quantity "1/a" is finite in the quantum theory. Another important result is that using also the result in [10] we can obtain the Hamiltonian constraint in terms of the volume and at the quantum level we have a discrete equation depending on three parameters: the matter field " $\phi$ " (dust matter), the gravitational field " $\tilde{a}$ " and the time "a". This equation says that we can extend dynamically the space time beyond the classical singularity.

The paper is organized as follows: in the first section we resume the classical collapse inside and outside the horizon in the case of dust matter. In the second section we introduce the Hamiltonian constraint for gravity inside the horizon in the two region inside and outside the matter. In the third section we quantize the system using the non Schrödinger procedure of quantization [8] and [6]. In this section we show that the inverse volume operator is singularity free in quantum gravity and also that the Hamiltonian constraint acts like a difference operator as in loop quantum cosmology. In the fourth section we quantize an homogeneous and isotropic scalar field using also in this case the analog of loop quantization. In this case we cannot match with the Kantowski-Sachs Space-Time because the pressure is not equal to zero.

# 1 Classical Theory

### 1.1 Classical Gravitational Collapse

The space-time outside the matter but inside the horizon is an homogeneous, anisotropic space-time with spatial section of topology  $\mathbf{R} \times \mathbf{S}^2$ , this is the Kantowski-Sachs Space-Time that we have studied in [3]. The metric in this case is

$$ds^{2} = -\tilde{N}(t)dt^{2} + \tilde{a}^{2}(t)dr^{2} + b^{2}(t)(\sin^{2}\theta d\phi^{2} + d\theta^{2}). \tag{1}$$

On the other side we assume that inside the matter the space-time is homogeneous, isotropic and with null pressure, so the metric must to be the Friedmann solution

$$ds^{2} = -N(t)dt^{2} + a^{2}(t)\left[d\chi^{2} + \sin^{2}\chi(\sin^{2}\theta d\phi^{2} + d\theta^{2})\right].$$
 (2)

Usually in the classical theory the much simple example is the gravitational collapse of a spherically symmetric, perfect fluid star of zero pressure and uniform density inside the region where the matter is localized and the Schwarzschild solution outside the matter. The metric (1) reduces to the Schwarzschild solution for the particular case  $\tilde{a}^2 = \frac{2MG_n}{b} - 1$  where M is the ADM mass of the black hole. In this particular case we can match the two space time regions and in particular we can match the two dimensional spherical surface inside and the two dimensional surface outside the matter. We will extend the match also at the quantum level.

#### 1.2 The Hamiltonian Constraint

The Diff-constraints for the metrics in (1) and (2) are identically satisfied. The Hamiltonian constraint outside the region where is localized the matter is calculated in [3] and it is

$$H_{out} = \frac{G_N |\tilde{a}| p_{\tilde{a}}^2}{2R b^2} - \frac{G_N p_{\tilde{a}} p_b \operatorname{sgn}(\tilde{a})}{Rb} - \frac{R}{2G_N} |\tilde{a}|; \tag{3}$$

the Hamiltonian constraint inside the matter is

$$H_{in} = -\left(\frac{p_a^2}{8|a|} + 2|a|\right) + \frac{16\pi G_N}{3} H_{\phi}(a),\tag{4}$$

where  $H_{\phi}(a)$  is the hamiltonian constraint of the dust (first case), in this case we can match with the Schwarzschild solution or of an isotropic and homogeneous scalar matter (second case) but in this case we cannot match with the Schwarzschild solution. The volume of the space section inside and outside the matter are respectively

$$V_{in} = \int_0^{\chi_0} d\chi \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \, h_{in}^{1/2} = 2\pi \left( \chi_0 - \sin(\chi_0) \, \cos(\chi_0) \right) |a|^3 \equiv \mathcal{V}(\chi_0) \, |a|^3$$

$$V_{out} = \int_0^R dr \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \, h_{out}^{1/2} = 4\pi R |\tilde{a}| b^2$$
(5)

where  $\chi_0$  define the boundary of the star of radius  $R_0 = |a|\chi_0$  and where R is a cut-off on the space radial coordinate. We can work also with radial densities because the model is homogeneous and all the following results remain identical. In another way, the spatial homogeneity enables us to fix a linear radial cell  $\mathcal{L}_r$  and restricts all integrations to this cell [4]. We have two canonical pairs outside the matter, one is given by  $\tilde{a} \equiv x_{\tilde{a}}$  and  $p_{\tilde{a}}$ , the other is given by  $b \equiv x_b$  and  $p_b$  for which the Poisson brackets are  $\{x_{\tilde{a}}, p_{\tilde{a}}\} = 1$  and  $\{x_b, p_b\} = 1$ . Inside the matter we have two canonical couple  $x_a$  and  $p_a$  with  $\{x_a, p_a\} = 8\pi G_N$  and the canonical couple for the matter that for the scalar field consists of  $\phi$  and  $p_{\phi}$ . From now on we consider  $x_a, x_b, x_{\phi}, x_{\tilde{a}} \in \mathbb{R}$  and we will introduce the modulus of  $x_a, x_b, x_{\phi}$  and  $x_{\tilde{a}}$  where it is necessary.

## 1.3 Classical Theory Inside the Matter

We study the gravity sector and the matter sector. In the two cases we follow [2], [8], [4] and [6]. We introduce an algebra of classical observable and we write the quantities of physical interest in terms of these variables.

#### Gravity Sector

We start with the gravity sector and as in Loop Quantum Gravity we use the fundamental variables  $x_a, U_{\gamma_a}$  and the volume  $V_{in} = \mathcal{V}(\chi_0) |x_a|^3$ , where

$$U_{\gamma_a}(p_a) \equiv \exp\left(\frac{i\gamma_a}{L_a} p_a\right),\tag{6}$$

the parameter  $\gamma_a$  is a real and  $L_a$  fixes the unit of length [6]. This variable can be seen as the momentum analog of the holonomy variable of loop quantum gravity. We have also that

$$\{x_a, U_{\gamma_a}\} = i \frac{8\pi G_N \gamma_a}{L_a} U_{\gamma_a},$$

$$U_{\gamma_a}^{-1} \{V_{in}^n, U_{\gamma_a}\} = i \frac{24\pi G_N \gamma_a}{L_a} n |x_a|^{3n-1} \operatorname{sgn}(x_a) \mathcal{V}^n(\chi_0).$$
(7)

From this relations we can construct the following quantity that we will use extensively

$$\frac{\operatorname{sgn}(x_a)}{\sqrt{|x_a|}} = -\frac{2L_a i}{8\pi G_N \gamma_a \mathcal{V}^{1/6}(\chi_0)} U_{\gamma_a}^{-1} \{V_{in}^{\frac{1}{6}}, U_{\gamma_a}\}. \tag{8}$$

We are interested to the quantity  $\frac{1}{|x_a|}$  because classically this quantity diverge because it is proportional to the Ricci invariant.

We are also interested to the Hamiltonian constraint and to the dynamics of the minisuperspace model.

#### Matter Sector (first case)

The first possibility is to take the dust matter with zero pressure, in this case the super momentum is zero and the Hamiltonian constraint [9] is very simple (see the appendix)

$$H_{\phi} = |p_{\phi}|. \tag{9}$$

The canonical pair is  $(\phi, p_{\phi})$  with Posson bracket is  $\{\phi, p_{\phi}\} = 1$ ; as in the gravity sector and in analogy with loop quantum gravity we take as fundamental variables  $\phi$  and  $U_{\gamma_{\phi}}$ , where

$$U_{\gamma_{\phi}}(p_{\phi}) \equiv \exp\left(\frac{i\gamma_{\phi} \, 8\pi G_{N}}{L_{\phi}} \, p_{\phi}\right),$$

$$\{x_{\phi}, U_{\gamma_{\phi}}\} = i\frac{\gamma_{\phi} \, 8\pi G_{N}}{L_{\phi}^{2}} \, U_{\gamma_{\phi}}$$

$$(10)$$

### Matter Sector (second case)

Another possibility is to take a scalar, isotropic and homogeneous scalar field with  $H_{\phi}$  of the constraint equation (4) equal to

$$H_{\phi} = \frac{1}{2|a|^3} \Big[ p_{\phi}^2 + a^6 U(\phi) \Big]. \tag{11}$$

Also in this case the canonical pair is  $(\phi, p_{\phi})$  with Posson bracket  $\{\phi, p_{\phi}\} = 1$ ; as fundamental variables we take  $\phi$  and  $U_{\gamma_{\phi}}$ , where

$$U_{\gamma_{\phi}}(p_{\phi}) \equiv \exp\left(\frac{i\gamma_{\phi}\sqrt{8\pi G_{N}}}{L_{\phi}^{2}}p_{\phi}\right),$$

$$\{x_{\phi}, U_{\gamma_{\phi}}\} = i\frac{\gamma_{\phi}\sqrt{8\pi G_{N}}}{L_{\phi}^{2}}U_{\gamma_{\phi}}$$
(12)

In the two cases using the introduced quantities we can quantize the matter sector as the gravity sector following [6].

## 1.4 Classical Theory Outside the Matter

The classical theory outside the matter but always inside the horizon has been described in reference [3]; we recall very rapidly the Hamiltonian constraint and the volume. The Hamiltonian constraint is

$$H_{c} = \frac{G_{N} |\tilde{a}| p_{\tilde{a}}^{2}}{2R b^{2}} - \frac{G_{N} p_{a} p_{b} \operatorname{sgn}(\tilde{a})}{R b} - \frac{R}{2G_{N}} |\tilde{a}|,$$

$$V = 4\pi R |x_{\tilde{a}}| |x_{b}|^{2}.$$
(13)

where R has been defined previously. The fundamental variables are  $x_{\tilde{a}}, x_b$  and

$$U_{\gamma_{\bar{a}}}(p_{\bar{a}}) \equiv \exp\left(\frac{8\pi G_N \gamma_{\bar{a}}}{L_{\bar{a}}^2} i p_{\bar{a}}\right),$$

$$U_{\gamma_b}(p_b) \equiv \exp\left(\frac{8\pi G_N \gamma_b}{L_b} i p_b\right),$$
(14)

As in the previews paper [3], we will use the following relations to quantize the sistem:

$$\begin{aligned}
\{x_{\tilde{a}}, U_{\gamma_{\tilde{a}}}(p_{\tilde{a}})\} &= 8\pi G_N \frac{i \gamma_{\tilde{a}}}{L_{\tilde{a}}^2} U_{\gamma_{\tilde{a}}}(p_{\tilde{a}}), \\
\{x_b, U_{\gamma_b}(p_b)\} &= 8\pi G_N \frac{i \gamma_b}{L_b} U_{\gamma_b}(p_b), \\
U_{\gamma_{\tilde{a}}}^{-1} \{V^m, U_{\gamma_{\tilde{a}}}\} &= (4\pi R|x_b|^2)^m m |x_{\tilde{a}}|^{m-1} i \gamma_{\tilde{a}} \frac{8\pi G_N}{L_{\tilde{a}}^2} \mathrm{sgn}(x_{\tilde{a}}), \\
U_{\gamma_b}^{-1} \{V^n, U_{\gamma_b}\} &= (4\pi R|x_{\tilde{a}}|)^n 2n |x_b|^{2n-1} i \gamma_b \frac{8\pi G_N}{L_b} \mathrm{sgn}(x_b).
\end{aligned} \tag{15}$$

With the quantities (15) we construct the Hamiltonian constraint and the classically singular observable.

# 2 Quantum Theory

We construct the quantum theory in analogy with the procedure used in loop quantum gravity and in particular following [2]. First of all for any canonical couple inside and outside the matter we must take an algebra of classical functions that is represented as quantum configuration operators. For any couple of canonical variable we choose the algebra generated by the function

$$W(\lambda) = e^{i\lambda x/L} \tag{16}$$

where  $\lambda \in \mathbb{R}$ . The algebra consists of all function of the form

$$f(x) = \sum_{j=1}^{n} c_j e^{i\lambda_j x/L}$$
(17)

where  $c_j \in \mathbb{C}$  and their limits with respect to the supremum norm and with x we indicate  $x_a$  or  $x_b$ . This algebra is the algebra of almost periodic function over  $\mathbb{R}$   $(AP(\mathbb{R}))$ . The algebra  $(AP(\mathbb{R}))$  is isomorphic to  $C(\bar{\mathbb{R}}_{Bohr})$  that is the algebra of continuous functions on the Bohr-compactification of  $\mathbb{R}$ . This suggests to take inside and outside the matter region the Hilbert space  $L_2(\bar{\mathbb{R}}_{Bohr}^2, d\mu_0)$ , where  $d\mu_0$  is the Haar measure on  $\bar{\mathbb{R}}_{Bohr}^2$ .

Inside the Matter (first case)

We recall that inside the matter the canonical couples are the matter canonical couple  $(\phi, p_{\phi})$  and the gravity canonical couple  $(a, p_a)$ . The complete Hilbert space inside the matter is the tensor product

$$|\lambda_{a}\rangle \otimes |\lambda_{\phi}\rangle \equiv |e^{i\lambda_{a} x_{a}/L_{a}}\rangle \otimes |e^{i\lambda_{\phi} x_{\phi}/L_{\phi}}\rangle,$$

$$\langle \mu_{a}|\lambda_{a}\rangle = \delta_{\mu_{a},\lambda_{a}} \qquad \langle \mu_{\phi}|\lambda_{\phi}\rangle = \delta_{\mu_{\phi},\lambda_{\phi}}.$$
(18)

The action of the configuration operators  $\widehat{W}_a(\lambda_a)$  and  $\widehat{W}_\phi(\lambda_\phi)$  on the base is definited by

$$\widehat{W}_{a}(\lambda_{a})|\mu_{a}\rangle = e^{i\lambda_{a}\hat{x}_{a}/L_{a}}|\mu_{a}\rangle = e^{i\lambda_{a}\mu_{a}}|\mu_{a}\rangle,$$

$$\widehat{W}_{\phi}(\lambda_{\phi})|\mu_{\phi}\rangle = e^{i\lambda_{\phi}\hat{x}_{\phi}/L_{\phi}}|\mu_{\phi}\rangle = e^{i\lambda_{\phi}\mu_{\phi}}|\mu_{\phi}\rangle.$$
(19)

Those operators are weakly continuous in  $\lambda_a$ ,  $\lambda_b$  and this imply the existence of self-adjoint operators  $\hat{x}_a$  and  $\hat{x}_b$ , acting on the basis states according to

$$\hat{x}_a |\mu_a\rangle = \mu_a L_a |\mu_a\rangle, 
\hat{x}_\phi |\mu_\phi\rangle = \mu_\phi L_\phi |\mu_\phi\rangle.$$
(20)

Now we introduce the operators corresponding to the classical momentum functions  $U_{\gamma_a}$  and  $U_{\gamma_{\phi}}$  of (6) and (12). We define the action of  $\hat{U}_{\gamma_a}$  and  $\hat{U}_{\gamma_{\phi}}$  on the basis states using the definitions (24) and using a quantum analog of the Poisson brackets between  $x_a$  and  $U_{\gamma_a}$  and  $v_{\phi}$  and  $v_{\phi}$ 

$$\hat{U}_{\gamma_a}|\mu_a\rangle = |\mu_a - \gamma_a\rangle \qquad \hat{U}_{\gamma_\phi}|\mu_\phi\rangle = |\mu_\phi - \gamma_\phi\rangle, 
\left[\hat{x}_a, \hat{U}_{\gamma_a}\right] = -\gamma_a L_a \hat{U}_{\gamma_a} \qquad \left[\hat{x}_\phi, \hat{U}_{\gamma_\phi}\right] = -\gamma_\phi L_\phi \hat{U}_{\gamma_\phi}.$$
(21)

### Inside the Matter (second case)

In this second case we consider the scalar matter inside the horizon. The canonical couples are the matter canonical couple  $(\phi, p_{\phi})$  and the same gravity canonical couple  $(a, p_a)$  as in the case of dust matter. The Hilbert space is

$$|\lambda_{a}\rangle \otimes |\lambda_{\phi}\rangle \equiv |e^{i\lambda_{a} x_{a}/L_{a}}\rangle \otimes |e^{i\lambda_{\phi}\sqrt{8\pi G_{N}}x_{\phi}}\rangle,$$
  

$$\langle \mu_{a}|\lambda_{a}\rangle = \delta_{\mu_{a},\lambda_{a}} \qquad \langle \mu_{\phi}|\lambda_{\phi}\rangle = \delta_{\mu_{\phi},\lambda_{\phi}}.$$
(22)

The action of the configuration operators  $\widehat{W}_a(\lambda_a)$  and  $\widehat{W}_{\phi}(\lambda_{\phi})$  on the base is definited by

$$\widehat{W}_{a}(\lambda_{a})|\mu_{a}\rangle = e^{i\lambda_{a}\widehat{x}_{a}/L_{a}}|\mu_{a}\rangle = e^{i\lambda_{a}\mu_{a}}|\mu_{a}\rangle,$$

$$\widehat{W}_{\phi}(\lambda_{\phi})|\mu_{\phi}\rangle = e^{i\lambda_{\phi}\sqrt{8\pi G_{N}}\widehat{x}_{\phi}}|\mu_{\phi}\rangle = e^{i\lambda_{\phi}\mu_{\phi}}|\mu_{\phi}\rangle.$$
(23)

Those operators are weakly continuous in  $\lambda_a$ ,  $\lambda_b$  and this imply the existence of self-adjoint operators  $\hat{x}_a$  and  $\hat{x}_b$ , acting on the basis states according to

$$\hat{x}_a |\mu_a\rangle = \mu_a L_a |\mu_a\rangle,$$

$$\hat{x}_\phi |\mu_\phi\rangle = \frac{\mu_\phi}{\sqrt{8\pi G_N}} |\mu_\phi\rangle.$$
(24)

The operators corresponding to the classical momentum functions are  $U_{\gamma_a}$  and  $U_{\gamma_{\phi}}$  and the action on the basis states is

$$\hat{U}_{\gamma_a}|\mu_a\rangle = |\mu_a - \gamma_a\rangle \qquad \hat{U}_{\gamma_\phi}|\mu_\phi\rangle = |\mu_\phi - \gamma_\phi\rangle, 
\left[\hat{x}_a, \hat{U}_{\gamma_a}\right] = -\gamma_a L_a \hat{U}_{\gamma_a} \qquad \left[\hat{x}_\phi, \hat{U}_{\gamma_\phi}\right] = -\frac{\gamma_\phi}{\sqrt{8\pi G_N}} \hat{U}_{\gamma_\phi}. \tag{25}$$

### Outside the Matter

Outside the matter the Hilbert space [3] is generated by the basis

$$|\lambda_{\tilde{a}}\rangle \otimes |\lambda_b\rangle \equiv |e^{i\lambda_{\tilde{a}}x_{\tilde{a}}}\rangle \otimes |e^{i\lambda_bx_b/L_b}\rangle,$$
  

$$\langle \mu_{\tilde{a}}|\lambda_{\tilde{a}}\rangle = \delta_{\mu_{\tilde{a}},\lambda_{\tilde{a}}} \qquad \langle \mu_b|\lambda_b\rangle = \delta_{\mu_b,\lambda_b}.$$
(26)

The self-adjoint operators  $\hat{x}_{\tilde{a}}$  and  $\hat{x}_b$  act on the basis states according to

$$\hat{x}_{\tilde{a}}|\mu_{\tilde{a}}\rangle = \mu_{\tilde{a}}|\mu_{\tilde{a}}\rangle,$$

$$\hat{x}_{b}|\mu_{b}\rangle = L_{b}\mu_{b}|\mu_{b}\rangle.$$
(27)

The analog of the classical momentum functions  $U_{\gamma_a}$  and  $U_{\gamma_b}$  of (14) are  $\hat{U}_{\gamma_a}$  and  $\hat{U}_{\gamma_b}$ . Those operators act on the basis states

$$\hat{U}_{\gamma_{\bar{a}}}|\mu_{\tilde{a}}\rangle = |\mu_{\tilde{a}} - \gamma_{\tilde{a}}\rangle \qquad \hat{U}_{\gamma_b}|\mu_b\rangle = |\mu_b - \gamma_b\rangle, 
\left[\hat{x}_{\tilde{a}}, \hat{U}_{\gamma_{\tilde{a}}}\right] = -\gamma_{\tilde{a}}\hat{U}_{\gamma_{\tilde{a}}} \qquad \left[\hat{x}_b, \hat{U}_{\gamma_b}\right] = -\gamma_b\hat{U}_{\gamma_b}.$$
(28)

Using the standard quantization procedure  $[,] \to i\hbar\{,\}$ , and using the first of the two equations of (7) and the second of (12) we obtain

$$L \equiv L_{\tilde{a}} = L_b = L_a = L_{\phi} = \sqrt{8\pi G_n \hbar}.$$
 (29)

## 2.1 Singularity Resolution in Quantum Theory

We resume in this section the singularity resolution of the gravitational collapse in loop quantum gravity. In particular we report the regular spectrum of the operator  $1/\det(E)$  outside the matter which is connected with the Schwarzschild singularity and the operator 1/a which is connected with the singularity inside the matter. We can observe also that the singularity inside the matter is analogues to the cosmological singularity.

We start with the space-time outside the matter; using equation (5) and in particular  $V = 4\pi R|x_{\tilde{a}}||x_b|^2$  we obtain the following spectrum of the volume operator

$$\hat{V}|\mu_{\tilde{a}},\mu_{b}\rangle = 4\pi R |\hat{x}_{\tilde{a}}| |\hat{x}_{b}|^{2} |\mu_{\tilde{a}},\mu_{b}\rangle = 4\pi R L_{b}^{2} |\mu_{\tilde{a}}| |\mu_{b}|^{2} |\mu_{\tilde{a}},\mu_{b}\rangle. \tag{30}$$

Now we show that the operator  $\frac{1}{\det(E)} = \frac{1}{\sqrt{h}} \sim \frac{1}{|\tilde{a}| |b|^2}$  does not diverge in the quantum theory and so we don't have any singularity.

Using extensively the relation (8) we can define the classical and so the quantum operator  $1/\det(E)$  (we promote the Poisson Brackets to commutators) [3]. The spectrum of this operator is

$$\frac{\widehat{1}}{\det(\mathbf{E})}|\mu_{\tilde{a}},\mu_{b}\rangle = \frac{2^{6} 3^{15}}{L^{2}} |\mu_{\tilde{a}}|^{5} |\mu_{b}|^{6} [|\mu_{b}-1|^{\frac{1}{2}} - |\mu_{b}|^{\frac{1}{2}}]^{12} ||\mu_{\tilde{a}}-1|^{\frac{1}{3}} - |\mu_{\tilde{a}}|^{\frac{1}{3}}|^{9} [|\mu_{b}-1|^{\frac{2}{3}} - |\mu_{b}|^{\frac{2}{3}}]^{6}, (31)$$

and we can see that it is upper bounded.

Now we study the singularity problem from the matter point of view and so we report the famous and important result of loop quantum cosmology because inside the matter, as said in the first section, space-time is exactly a Friedmann cosmological model with k=1, the operator that we study is the quantum analog of  $1/|x_a|$  which we define in quantum theory using the quantization of the classical Poisson bracket (7). In this way we obtain (for  $\gamma_a = 1$ ) the operator

$$\widehat{\frac{1}{|x_a|}} = \frac{1}{2\pi l_p^{2l_0^{\frac{2}{3}}}} \left( \hat{U}_{\gamma_a}^{-1} \left[ \hat{V}_{in}^{\frac{1}{6}}, \hat{U}_{\gamma_a} \right] \right)^2.$$
(32)

Using the volume operator  $\widehat{V}_{in} = \mathcal{V}(\chi_0) |\widehat{x}_a|^3$  and the eigenvalue equation

$$\widehat{V}_{in}|\mu_a,\mu_\phi\rangle = \mathcal{V}(\chi_0)|\mu_a|^3|\mu_a,\mu_\phi\rangle \tag{33}$$

we obtain the action of the classical singular operator on the basis states

$$\frac{\widehat{1}}{|x_a|} |\mu_a, \mu_\phi\rangle = \sqrt{\frac{2}{\pi l_p^2}} \left( |\mu_a|^{\frac{1}{2}} - |\mu_a - 1|^{\frac{1}{2}} \right)^2 |\mu_a, \mu_\phi\rangle.$$
(34)

We can see that the spectrum is bounded from below and so there is no singularity in the quantum theory also also in presence of matter.

### 2.2 The Hamiltonian Constraint in Quantum Theory

Now we want to resolve the Hamiltonian constraint in all the space time inside the horizon of the black hole in the case of dust matter. In fact only in this case we can match the region inside the matter with the Kantowski-Sachs space time outside.

The solutions of the Hamiltonian constraint are in the  $C^*$  space that is the dual of the dense subspace C of the kinematical space H. A generic element of this space for our system which consists of two parts, a region where is the matter localized and an exterior region, is of the form

$$\langle \psi | = \sum_{\mu} \psi(\alpha, \beta) \langle \alpha, \beta |.$$
 (35)

where the variables " $\alpha$ " and " $\beta$ " are the eigenvalues of the operators " $\hat{x}_a$ " and " $\hat{x}_{\phi}$ " inside the matter and " $x_b$ " and " $x_{\bar{a}}$ " outside the matter. The constraint equation  $\hat{H}|\psi\rangle = 0$  is now interpreted as an equation in the dual space  $\langle \psi | \hat{H}^{\dagger} = 0$ ; from this equation we can derive a relation for the coefficients  $\psi(\alpha, \beta)$ . The Hamiltonian constraint that we must impose to define the physical space is

$$\hat{H} = \begin{cases} \hat{H}_{in} & \text{inside the matter} \\ \hat{H}_{out} & \text{outside the matter} \end{cases}$$
 (36)

Where  $\widehat{H}_{in}$  and  $\widehat{H}_{out}$  are define

$$H_{in} = -\left(\frac{p_a^2}{8} \frac{1}{|x_a|} + \frac{2}{\mathcal{V}^{1/3}(\chi_0)} V^{1/3}\right) + \frac{16\pi G_N}{3} H_{\phi}(a),$$

$$H_{out} = \frac{G_N p_{\tilde{a}}^2}{2R} \frac{|x_{\tilde{a}}|}{x_b^2} - \frac{G_N p_{\tilde{a}} p_b}{R} \frac{\operatorname{sgn}(x_b) \operatorname{sgn}(x_{\tilde{a}})}{|x_b|} - \frac{R}{2G_N} |x_{\tilde{a}}|$$
(37)

Now we quantize this Hamiltonian constraint. As we know, the operators  $p_{\tilde{a}}$ ,  $p_b$ ,  $p_a$  and  $p_{\phi}$  don't exist in our quantum representation and so we choose the following alternative representation for the operators  $p_{\tilde{a}}^2$ ,  $p_{\tilde{a}} p_b$ ,  $p_a^2$  and  $p_{\phi}$ . The first two operators were obtained in reference [3] starting from the classical relations

$$p_{a}^{2} = \frac{L_{a}^{4}}{(8\pi G_{N})^{2}} \lim_{\gamma_{a} \to 0} \left( \frac{2 - U_{\gamma_{a}} - U_{\gamma_{a}}^{-1}}{\gamma_{a}^{2}} \right),$$

$$p_{a} p_{b} = \frac{L_{a}^{2} L_{b}}{2(8\pi G_{N})^{2}} \lim_{\gamma_{a}, \gamma_{b} \to 0} \left[ \left( \frac{U_{\gamma_{a}} + U_{\gamma_{b}} - U_{\gamma_{a}} U_{\gamma_{b}} - 1}{\gamma_{a} \gamma_{b}} \right) + \left( \frac{U_{\gamma_{a}}^{-1} + U_{\gamma_{b}}^{-1} - U_{\gamma_{a}}^{-1} U_{\gamma_{b}}^{-1} - 1}{\gamma_{a} \gamma_{b}} \right) \right],$$
(38)

the other two operators can be define using the classical expressions

$$p_a^2 = \frac{L_a^2}{\gamma_a} \lim_{\gamma_a \to 0} \left( \frac{2 - U_{\gamma_a} - U_{\gamma_a}^{-1}}{\gamma_a^2} \right),$$

$$p_\phi = \frac{L_\phi^4 L_b}{8\pi G_N} \lim_{\gamma_\phi, \to 0} \left( \frac{2 - U_{\gamma_\phi} - U_{\gamma_\phi}^{-1}}{\gamma_\phi^2} \right). \tag{39}$$

(we have a physical interpretation setting  $\gamma_{\tilde{a}} = \gamma_{\phi} = \gamma_{a} = \gamma_{b} = l_{F}/L_{phys}$ , where  $L_{Phys}$  is the characteristic size of the system and  $l_{F}$  is a fundamental length scale).

Now we are ready to construct the correct operators  $\widehat{H}_{in}$  and  $\widehat{H}_{out}$  in terms of the space volume and the momentum analog of the holonomy variable of loop quantum gravity. From the paper [3] we report

$$\hat{H}_{out} = \frac{1}{32\pi^{2}G_{N}R^{2}\gamma_{a}^{2}\gamma_{b}^{4}} \left[ 2 - \hat{U}_{\tilde{a}} - \hat{U}_{\tilde{a}}^{-1} \right] \left( \hat{U}_{b}^{-1} \left[ \hat{V}^{\frac{1}{4}}, \hat{U}_{b} \right] \right)^{4} \\
+ \frac{3^{6}}{2^{11}\pi^{5}R^{4}L^{4}G_{N}\gamma_{b}^{5}\gamma_{a}^{7}} \left[ \left( \frac{\hat{U}_{\tilde{a}} + \hat{U}_{b} - \hat{U}_{\tilde{a}}\hat{U}_{b} - 1}{2} \right) + \left( \frac{\hat{U}_{\tilde{a}}^{-1} + \hat{U}_{b}^{-1} - \hat{U}_{\tilde{a}}^{-1}\hat{U}_{b}^{-1} - 1}{2} \right) \right] \\
- \left( \hat{U}_{b}^{-1} \left[ \hat{V}^{\frac{1}{4}}, \hat{U}_{b} \right] \right)^{4} \left( \hat{U}_{a}^{-1} \left[ \hat{V}^{\frac{1}{3}}, \hat{U}_{a} \right] \right)^{3} \left( \hat{U}_{b}^{-1} \left[ \hat{V}^{\frac{1}{3}}, \hat{U}_{b} \right] \right)^{3} + \\
- \frac{1}{8\pi G_{N}L^{2}\gamma_{b}^{2}} \left( \hat{U}_{b}^{-1} \left[ \hat{V}^{\frac{1}{2}}, \hat{U}_{b} \right] \right)^{2}. \tag{40}$$

On the other side the operator  $\widehat{H}_{in}$  is

$$\widehat{H}_{in} = -\left[\frac{1}{2\pi^2 \mathcal{V}(\chi_0)\gamma_a^4} \left(2 - \hat{U}_a - \hat{U}_a^{-1}\right) \left(\hat{U}_b^{-1} \left[\hat{V}^{\frac{1}{6}}, \hat{U}_b\right]\right)^2 + \frac{2}{\mathcal{V}(\chi_0)^{1/3}} \hat{V}^{\frac{1}{3}}\right] + \frac{16\pi G_N}{3} \widehat{H}_{\phi}(x_a) \quad (41)$$

Now we resolve the Hamiltonian constraint. As we said at the beginning of this section the solutions of the Hamiltonian constraint is obtained in the  $C^*$ . A generic element of this space for the space-time inside the matter is

$$\langle \psi | = \sum_{\mu_{\tilde{a}}, \mu_b} \psi(\mu_{\tilde{a}}, \mu_b) \langle \mu_{\tilde{a}}, \mu_b |. \tag{42}$$

In [3] we obtained an equation for the coefficients  $\psi(\mu_{\tilde{a}}, \mu_b)$ 

$$[2\alpha(\mu_{\tilde{a}},\mu_{b}) - 2\beta(\mu_{\tilde{a}},\mu_{b}) + \gamma(\mu_{\tilde{a}},\mu_{b})] \psi(\mu_{\tilde{a}},\mu_{b}) - [\alpha(\mu_{\tilde{a}} + \gamma_{\tilde{a}},\mu_{b}) - \beta(\mu_{\tilde{a}} + \gamma_{\tilde{a}},\mu_{b})] \psi(\mu_{\tilde{a}} + \gamma_{\tilde{a}},\mu_{b}) + \\ - [\alpha(\mu_{\tilde{a}} - \gamma_{\tilde{a}},\mu_{b}) + \beta(\mu_{\tilde{a}} - \gamma_{\tilde{a}},\mu_{b})] \psi(\mu_{\tilde{a}} - \gamma_{\tilde{a}},\mu_{b}) + \beta(\mu_{\tilde{a}},\mu_{b} + \gamma_{b}) \psi(\mu_{\tilde{a}},\mu_{b} + \gamma_{b}) + \\ - \beta(\mu_{\tilde{a}},\mu_{b} - \gamma_{b}) \psi(\mu_{\tilde{a}},\mu_{b} - \gamma_{b}) + \beta(\mu_{\tilde{a}} + \gamma_{\tilde{a}},\mu_{b} + \gamma_{b}) \psi(\mu_{\tilde{a}} + \gamma_{\tilde{a}},\mu_{b} + \gamma_{b}) \\ - \beta(\mu_{\tilde{a}} - \gamma_{\tilde{a}},\mu_{b} - \gamma_{b}) \psi(\mu_{\tilde{a}} - \gamma_{\tilde{a}},\mu_{b} - \gamma_{b}) = 0$$

$$(43)$$

where the function  $\alpha, \beta, \gamma$ , always following [3], are

$$\alpha(\mu_{\tilde{a}}, \mu_{b}) = \frac{L^{2}}{8\pi^{2}RG_{N}\gamma_{b}^{4}\gamma_{a}^{2}} \left( |\mu_{\tilde{a}}|^{\frac{1}{4}}|\mu_{b} - \gamma_{b}|^{\frac{1}{2}} - |\mu_{\tilde{a}}|^{\frac{1}{4}}|\mu_{b}|^{\frac{1}{2}} \right)^{2},$$

$$\beta(\mu_{\tilde{a}}, \mu_{b}) = -\frac{L^{2}}{2(8\pi)^{2}G_{N}R\gamma_{b}^{5}\gamma_{a}^{7}} \left( |\mu_{\tilde{a}}|^{\frac{1}{4}}|\mu_{b} - \gamma_{b}|^{\frac{1}{2}} - |\mu_{\tilde{a}}|^{\frac{1}{4}}|\mu_{b}|^{\frac{1}{2}} \right)^{4}$$

$$\left( |\mu_{\tilde{a}} - \gamma_{a}|^{\frac{1}{3}}|\mu_{b}|^{\frac{2}{3}} - |\mu_{b}|^{\frac{1}{3}}|\mu_{b}|^{\frac{2}{3}} \right)^{3}$$

$$\left( |\mu_{\tilde{a}}|^{\frac{1}{3}}|\mu_{b} - \gamma_{b}|^{\frac{2}{3}} - |\mu_{\tilde{a}}|^{\frac{1}{3}}|\mu_{b}|^{\frac{2}{3}} \right)^{3},$$

$$\gamma(\mu_{\tilde{a}}, \mu_{b}) = \frac{R}{2G_{N}\gamma_{b}^{2}} \left( |\mu_{\tilde{a}}|^{\frac{1}{2}}|\mu_{b} - \gamma_{b}| - |\mu_{\tilde{a}}|^{\frac{1}{2}}|\mu_{b}| \right)^{2}. \tag{44}$$

Now we evaluate the action of  $\hat{H}_{in}$  on the states  $|\mu_a, \mu_{\phi}\rangle$ 

$$\hat{H}_{in} |\mu_{a}, \mu_{\phi}\rangle = -\frac{L}{2\gamma_{a}^{4}} \left( |\mu_{a} - \gamma_{a}|^{\frac{1}{2}} - |\mu_{a}|^{\frac{1}{2}} \right)^{2} \left( 2 |\mu_{a}, \mu_{\phi}\rangle - |\mu_{a} - \gamma_{a}, \mu_{\phi}\rangle - |\mu_{a} + \gamma_{a}, \mu_{\phi}\rangle \right) 
-2L|\mu_{a}| |\mu_{a}, \mu_{\phi}\rangle + \frac{16\pi G_{N}}{3} \hat{H}_{\phi}(x_{a})|\mu_{a}, \mu_{\phi}\rangle$$
(45)

The solution of the Hamiltonian constraint is in the dual space of the dense subspace of the kinematical space  $\mathcal{H}$  and so a generic element of this space, as we said at the beginning of this section, is  $\langle \psi | = \sum_{\mu_a,\mu_\phi} \psi(\mu_a,\mu_\phi) \langle \mu_a,\mu_\phi |$ . The equation for the coefficients  $\psi(\mu_a,\mu_\phi)$  is

$$\alpha(\mu_{a})\psi(\mu_{a},\mu_{\phi}) + \beta(\mu_{a} + \gamma_{a})\psi(\mu_{a} + \gamma_{a},\mu_{\phi}) + \beta(\mu_{a} - \gamma_{a})\psi(\mu_{a} - \gamma_{a},\mu_{\phi}) = -\frac{16\pi G_{N}}{3}\hat{H}_{\phi}(a)\psi(\mu_{a},\mu_{\phi}),$$
(46)

where the functions of the eigenvalues  $\alpha(\mu_a)$  and  $\beta(\mu_a)$  are

$$\alpha(\mu_a) = -\frac{L}{\gamma_a^4} \left( |\mu_a - \gamma_a|^{\frac{1}{2}} - |\mu_a|^{\frac{1}{2}} \right)^2 - 2L|\mu_a|$$

$$\beta(\mu_a) = \frac{L}{2\gamma_a^4} \left( |\mu_a - \gamma_a|^{\frac{1}{2}} - |\mu_a|^{\frac{1}{2}} \right)^2$$
(47)

and 
$$\hat{H}_{\phi}(a)\psi(\mu_a,\mu_{\phi}) = \sum_{\mu'_a,\mu'_{\phi}} \psi(\mu'_a,\mu'_{\phi})\langle \mu'_a,\mu'_{\phi}|\hat{H}_{\phi}(a)|\mu_a,\mu_{\phi}\rangle.$$

At this point we have the wave solution inside and outside the matter for the gravitational collapse and we can give an interpretation for the equation (43) and (46). Both equations are difference equations and the physical states are combinations of a countable number of components of the form  $\psi(\mu + n\gamma, \nu + m\delta)|\mu + n\gamma, \nu + \delta\rangle$  ( $\gamma, \delta \sim l_P/L_{Phys} \sim 1$ ;  $\mu = \mu_a$  or  $\mu = \mu_{\tilde{a}}$  and  $\gamma = \gamma_a$  or  $\gamma = \gamma_{\tilde{a}}$ ,  $\nu = \mu_{\phi}$  or  $\nu = \mu_b$  inside or outside the matter). Outside the matter any component corresponds to a particular value of the volume; so we can interpret  $\psi(\mu_{\tilde{a}} + \gamma_{\tilde{a}}, \mu_b + \gamma_b)$  as the function of the Black Hole inside the horizon at the time  $\mu_b + \gamma_b$ , if we interprete b as the time and  $\tilde{a}$  as the space partial observable that defines the quantum fluctuations around the Schwarzschild solution. In the same way, inside the matter, we can interpret the function  $\psi(\mu_a + \gamma_a, \mu_{\phi} + \gamma_{\phi})$  as the wave function of the matter at the time  $\mu_a + \gamma_a$ . In the next section we impose the boundary condition on the area operator from inside and from outside the matter view and we obtain a single time coordinate.

## 2.3 Boundary Condition and Time Arrow

At this point we can impose the boundary condition inspired from the classical condition on the inside and outside area of the star. In particular we impose that the operator area spectrum which define the surface of the star from the point of view of the inside region is identically to the area operator spectrum from the point of view of the region outside the matter. The operator area are

$$\widehat{A}_{in} = 4\pi \widehat{|x_a|}^2 \sin(\chi_0),$$

$$\widehat{A}_{out} = 4\pi \widehat{|x_b|}^2.$$
(48)

The spectrum of the two operators can be obtained from the bases (22) and (26)

$$\widehat{A}_{in}|\mu_a,\mu_\phi\rangle = 4\pi \widehat{|x_a|}^2 \sin^2(\chi_0) |\mu_a,\mu_\phi\rangle = 4\pi |\mu_a|^2 \sin^2(\chi_0) |\mu_a,\mu_\phi\rangle,$$

$$\widehat{A}_{out}|\mu_{\tilde{a}},\mu_b\rangle = 4\pi \widehat{|x_b|}^2 |\mu_{\tilde{a}},\mu_b\rangle = 4\pi |\mu_b|^2 |\mu_{\tilde{a}},\mu_b\rangle.$$
(49)

At this point we identify the inside and outside spectrum and we obtain a relation between the inside and outside eigenvalues  $\mu_a$  and  $\mu_b$ 

$$|\mu_a|^2 \sin^2(\chi_0) = |\mu_b|^2. \tag{50}$$

From this relation we can see that  $\mu_a \sim \mu_b$  and so if we define "a" and "b" as time coordinates for the inside and outside space-time the boundary condition imply that we have only one time coordinate in all space-time.

## 2.4 Quantum Dust Matter

In this subsection we quantize the simply Hamiltonian for the dust matter. In particular we follow the quantization program of reference [6] and apply the non unitary equivalent quantization to the isotropic and homogeneous dust matter.

We give the form of the Hamiltonian constraint in comoving coordinates. In this case the energy tensor is homogeneous and so it is independent from the spatial section coordinates. In comoving coordinates the supermomentum is automatically zero and the classical hamiltonian becomes

$$H_{\phi} = |p_{\phi}|. \tag{51}$$

As p is positive as shown in [9], we can take  $H = p_{\phi}$ . Now using the classical variables (10), we can obtain the quantum operator  $H_{\phi}$ 

$$\widehat{H}_{\phi} = \widehat{p}_{\phi} = \frac{L_{\phi}}{G_N} \left( \frac{\widehat{U}_{\phi} - \widehat{U}_{\phi}^{-1}}{2i\gamma_{\phi}} \right). \tag{52}$$

The action of this operator on the state  $|\mu_a, \mu_{\phi}\rangle$  is

$$\widehat{H}_{\phi}|\mu_{a},\mu_{\phi}\rangle = \frac{L_{\phi}}{2i\gamma_{\phi}G_{N}}\Big(|\mu_{a},\mu_{\phi}-\gamma_{\phi}\rangle - |\mu_{a},\mu_{\phi}+\gamma_{\phi}\rangle\Big)$$
(53)

## 2.5 Quantum Scalar Field

In this section we quantize the scalar field in the same way of the dust matter sector and of the gravity sector; in particular we follow the quantization program of reference [6] and apply the non unitary equivalent quantization to an isotropic and homogeneous scalar field.

We remember that the scalar field Hamiltonian constraint operator from (11) is

$$\widehat{H}_{\phi} = \frac{\widehat{p}_{\phi}^2}{2} \frac{\widehat{1}}{|x_a|^3} + \widehat{U(\phi)} |\widehat{x}_a|^3.$$

$$(54)$$

Now using the relation (12), the Poisson brackets (15) and the scalar field analog of the momentum operator (38), we can write the operator (54) in terms of  $V_{in}$ ,  $U_{\gamma_a}$  and the scalar field couple  $(\phi, U_{\gamma_{\phi}})$ ,

$$\widehat{H}_{\phi} = \left[ \frac{1}{2G_N l_p^2 \pi^2 \mathcal{V}(\chi_0) \gamma_a^6 \gamma_\phi^2} \left( 2 - \hat{U}_{\gamma_{\phi}} - \hat{U}_{\gamma_{\phi}}^{-1} \right) \left( \hat{U}_{\gamma_a}^{-1} \left[ \hat{V}_{in}^{\frac{1}{6}}, \hat{U}_{\gamma_a} \right] \right)^6 + \frac{1}{\mathcal{V}(\chi_0)} \widehat{U}(\phi) \, \hat{V}_{in} \right]. \tag{55}$$

At this point we can apply the operator (54) on the basis  $|\mu_a, \mu_\phi\rangle$  and obtain

$$\widehat{H}_{\phi}|\mu_{a},\mu_{\phi}\rangle = \frac{L_{a}^{3}}{2G_{N}l_{p}^{2}\pi^{2}\mathcal{V}(\chi_{0})\gamma_{a}^{6}\gamma_{\phi}^{2}} \left(|\mu_{a}-\gamma_{a}|^{\frac{1}{2}}-|\mu_{a}|^{\frac{1}{2}}\right)^{6} \left(2|\mu_{a},\mu_{\phi}\rangle-|\mu_{a},\mu_{\phi}-\gamma_{\phi}\rangle-|\mu_{a},\mu_{\phi}+\gamma_{\phi}\rangle\right) + L_{a}^{3}|\mu_{a}|^{3}U(\mu_{\phi})|\mu_{a},\mu_{\phi}\rangle.$$
(56)

Using this relation we obtain the complete quantum solution of the Hamiltonian constraint inside the matter

$$\alpha(\mu_{a}, \mu_{\phi})\psi(\mu_{a}, \mu_{\phi}) + \beta(\mu_{a} + \gamma_{a})\psi(\mu_{a} + \gamma_{a}, \mu_{\phi}) + \beta(\mu_{a} - \gamma_{a})\psi(\mu_{a} - \gamma_{a}, \mu_{\phi}) + \gamma(\mu_{a})\psi(\mu_{a}, \mu_{\phi} + \gamma_{\phi}) + \gamma(\mu_{a})\psi(\mu_{a}, \mu_{\phi} + \gamma_{\phi}) = 0$$
(57)

where the functions  $\alpha(\mu_a, \mu_\phi)$ ,  $\beta(\mu_a)$  and  $\gamma(\mu_a)$  are

$$\alpha(\mu_{a}, \mu_{\phi}) = -\frac{L}{\gamma_{a}^{4}} \left( |\mu_{a} - \gamma_{a}|^{\frac{1}{2}} - |\mu_{a}|^{\frac{1}{2}} \right)^{2} - 2L|\mu_{a}|$$

$$+ \frac{16l^{3}}{3l_{p}^{2}\pi\gamma_{a}^{6}\gamma_{\phi}^{2}} \left( |\mu_{a} - \gamma_{a}|^{\frac{1}{2}} - |\mu_{a}|^{\frac{1}{2}} \right)^{6} + \frac{16\pi G_{N}L^{3}}{3} |\mu_{a}|^{3}U(\mu_{\phi})$$

$$\beta(\mu_{a}) = \frac{L}{2\gamma_{a}^{4}} \left( |\mu_{a} - \gamma_{a}|^{\frac{1}{2}} - |\mu_{a}|^{\frac{1}{2}} \right)^{2}$$

$$\gamma(\mu_{a}) = -\frac{8L^{3}}{3l_{p}^{2}\pi\gamma_{a}^{6}\gamma_{\phi}^{2}} \left( |\mu_{a} - \gamma_{a}|^{\frac{1}{2}} - |\mu_{a}|^{\frac{1}{2}} \right)^{6}$$
(58)

In this case we cannot mach the solution inside the matter with the Schwarzschild solution outside the matter. In fact for the scalar matter the pressure it is not equal to zero as for the Schwarzschild solution or the Kantowski-Sachs symmetric solution. We can however match the scalar matter solution with the Vaydia solution.

## Conclusions

In this work we have applied the non Schrödinger quantization procedure used in the previous papers [2], [3] onthe black hole singularity and in the work of V. Husain and O. Winkler on quantum cosmology. This quantum formalism was introduced by Halvorson [8] and also by A. Ashtekar, S. Fairhust and J. Willis [6] in a very clear mathematical form. In this paper we have studied the gravitational collapse inside the horizon or, in other words, when all the matter has passed the horizon of the black hole. In this particular configuration we have subdivided the space-time inside the horizon in two regions, one where the dust matter is localized and the other where the space time is the Kantowski-Sachs space-time. We have studied also the quantum theory when the matter is an homogeneous, isotropic scalar field but in this case the pressure is not zero and does not give a matching with the space time outside the matter.

The principal results of our model are the following.

The first one is the absence of singularity in the space-time region where the matter is localized. In fact in the particular case of dust matter but also in the case of scalar matter the model is analogous to a cosmological model where the singularity is absent in the quantum theory [4]. On the other side the space-time outside the matter, which is a Kantowski-Sachs space time [5] with space topology  $\mathbf{R} \times \mathbf{S}^2$  and which contains the Schwarzschild metric inside the horizon as a particular classical solution, is singularity free. The other interesting point is that the space-time can be extended beyond the singularity. This is correct for the region which contains the matter and the vacuum region.

The other interesting point is the quantum match between the two regions. When we impose the classical matching between the area of the 2-sphere  $\mathbf{S}_2$  inside and outside the matter at the quantum level we obtain the interesting result that only a temporal coordinate survive in all the space time. In fact we start with two coordinates inside the matter "a" and " $\phi$ " that we interpret as time (a) and dust matter ( $\phi$ ) and two coordinates outside the matter, but inside the horizon, that are the time coordinate "b" and the gravitational coordinate " $\tilde{a}$ ". When we match the area spectrum of the 2-sphere inside the matter  $b^2(t)(\sin^2\theta d\phi^2 + d\theta^2)$  and the area spectrum of the 2-sphere outside the matter  $a^2(t)\sin^2\chi_0(\sin^2\theta d\phi^2 + d\theta^2)$  we obtain only one time coordinate. This relation is the quantum version of the classical matching  $a\sin\chi_0 = b$ : for any eigenvalue  $\mu_a$  of the operator " $\hat{a}$ " and for any eigenvalue  $\mu_b$  of " $\hat{b}$ " we have  $|\mu_a|\sin\chi_0 = |\mu_b|$ .

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