Unattainability of the Trans-Planckian regime in Nonlocal Quantum Gravity

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Based on the ultraviolet asymptotic freedom of nonlocal gravity, we show that the trans-Planckian energy regime is unattainable in any viable laboratory experiment. As physical implications, it turns out that the violation of micro-causality, typical of nonlocal field theories, can never be detected in any laboratory experiment, while the asymptotic freedom of the theory easily solves the so called trans-Planckian cosmological problem.

In order to have a (super-)renormalizable or finite gravitational theory in the perturbative quantum field theory framework, we are forced to introduce a new nonlocal [1–7] or higher derivative action principle for gravity [8–13], including quadratic gravity [14], which has been proved to be unitary in [15]. In this paper we will focus on nonlocal gravity [3–5], but most of the results can be extended to the case of other higher-derivative theories [8, 9]. Nonlocal gravity (NLG) is a well defined theory at classical as well as at quantum level. Indeed, all the classical solutions of the Einstein-Hilbert theory (EH) are solutions of NLG too [16], and, most importantly, the stability analysis of such solutions in NLG is in one to one correspondence with the same analysis in the local EH theory [17–19]. In particular, it has been shown that the Minkowski spacetime is stable, under any Strongly Asymptotically Flat (SAF) initial data set satisfying a Global Smallness Assumption (GSA), exactly like in EH theory [17, 18]. At quantum level, the theory is tree-level indistinguishable from EH, namely all the $n-$points scattering amplitudes are the same of the EH theory [20], but it turns out to be super-renormalizable or even finite at higher orders in the loop expansion [2, 3, 5]. It deserves to be noticed the two fold feature of the non locality: on one side it avoids the propagation of ghost-like degrees of freedom at any perturbative order in the loop expansion [21], on the other side it improves the ultraviolet behaviour of the theory. Finally, the macroscopic causality (based on the Shapiro's time delay) is satisfied [22] because, as mentioned above, the tree-level scattering amplitudes of NLG are the same of the EH theory.

In this paper we introduce the new concept of experimentally *unattainability* of the trans-Planckian regime as a consequence of the asymptotic freedom of nonlocal quantum gravity [3–5] at energies above $E_{\rm NL} = \ell^{-1}$, where ℓ is the nonlocality lenght-scale[1]. As a consequence of such asymptotic freedom, we show that is impossible to accelerate particles in the laboratory at energies above $E_{\rm NL}$. Indeed, at such high energies interactions are very suppressed and the particles decouple from any device that could accelerate them.

As related issues, we address the problem of causality violation in nonlocal theories, and the cosmological trans-Planckian problem.

Regarding the causality violation, which is typically expected in nonlocal theories, it turns out to be not measurable experimentally in nonlocal gravity because in order to measure such an effect we should be able to probe a time scale $\Delta t \lesssim \ell$, which is equivalent to probe the spacetime at a length scale $\Delta x \lesssim \ell$. Therefore, one needs wave packets tighter than ℓ, containing frequencies higher than $\ell^{-1} = E_{\rm NL}$. However, we have anticipated above that particles can not be accelerated to such an high energy, thus, such wave-packets can not be produced. Hence, we can conclude that causality violations can not be measured in laboratory experiments.

For what concerns the cosmological trans-Planckian problem [23], nothing seems to prevent trans-Planckian energies to be attained in the early universe, namely at the inflationary age. However, particles are non-interacting at such energies because of the asymptotic freedom of the theory. Therefore, the equations of motion for the cosmological perturbations can not be significantly affected by the nonlinear interaction terms, namely we do not expect sensible non-Gaussianities in the spectrum of primordial perturbations as a consequence of the well defined perturbative expansion in an asymptotically free theory. Moreover, higher derivative nonlocal operators do not affect the linearized theory in the ultraviolet regime because they do not introduce extra perturbative degrees of freedom. We conclude that the ultraviolet asymptotic freedom of nonlocal gravity easily overcomes the so called cosmological Trans-Planckian problem [23].

In order to support the statements above we start proving the asymptotic freedom of Nonlocal Gravity closely following the analysis in [27] for higher derivative gravity and higher derivative gauge theory. The minimal action for nonlocal gravity reads,

$$S_{\rm NL} = -\frac{2}{\kappa^2}\int d^4x\sqrt{|g|}\left(R + G_{\mu\nu}\frac{e^{H(\Box)}-1}{\Box}R^{\mu\nu}\right) , \quad (1)$$

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[1] For the sake of simplicity, hereafter we identify ℓ with the Planck length $\ell_P = \sqrt{\hbar G/c^3}$. However, all our results will be valid as far as $\ell_P \lesssim \ell$.

where $\kappa^2 = 32\pi G_N$, $G_{\mu\nu}$ is the Einstein's tensor, and $\exp H(z)$ is an entire analytic function, asymptotically polynomial for $|z| \to \infty$, that is properly constructed in order to avoid nonlocal divergences at quantum level (the reader can find more details in [2–5]).

In order to prove the asymptotic freedom of the theory (1), we recast the action (1) in the following form, that can also include the cosmological constant [10],

$$S = \int d^4x\sqrt{-g}\Big[\omega_{-2} - \omega_{-1}R + \sum_{n=0}^{\infty}\Big(\omega_n^{(0)} R\Box^n R + \omega_n^{(2)} R_{\mu\nu}\Box^n R^{\mu\nu}\Big)\Big]\,. \quad (2)$$

We have a trivial running of the couplings

$$\alpha_i \in \Big\{\omega_{-2}, \omega_{-1}, \omega_1^{(0)}, \omega_2^{(0)}\Big\}\,, \quad (3)$$

namely

$$\alpha_i = \alpha_{i,0} + \beta_i t\,, \quad (4)$$

where the β_i are the beta functions and $t = \log \mu/\mu_0$, having set $\mu_0 \equiv E_{\rm NL}$. We can expand the action in powers of the graviton field around the Minkowski spacetime, finding

$$\begin{aligned} S \;=\; \int d^4x\Big[&\omega_{-2}\left(1+h+h^2+h^3+O(h^4)\right) \\ &-\omega_{-1}(h\Box h + h^2\Box h + O(h^4)) \\ &+\sum_{n=0}^{\infty}\omega_n^{(0)}\left(h\Box^{n+2}h + h^2\Box^{n+2}h + O(h^4)\right) \\ &+\sum_{n=0}^{\infty}\omega_n^{(2)}\left(h\Box^{n+2}h + h^2\Box^{n+2}h + O(h^4)\right)\Big], \end{aligned} \quad (5)$$

where we missed the tensorial structure and all the indexes in favour of the explicit structure of the vertexes, which do matter in proving the asymptotic freedom. For the sake of simplicity, we study a class of theories asymptotically monomial in order to avoid the running of the cosmological constant and of the Newton constant. Hence, by the following rescaling of the graviton field,

$$h_{\mu\nu} \to \alpha_2(t)^{-1/2}\, h_{\mu\nu} = \omega_0^{(2)}(t)^{-1/2} h_{\mu\nu} \equiv f(t)\, h_{\mu\nu}\,, \quad (6)$$

where we have defined

$$f^2 = \frac{f_0^2}{1+f_0^2\beta_2 t}\,, \quad (7)$$

the action (5) turns into

$$\begin{aligned} S = \int d^Dx\Big[&\omega_{-2}(1+fh+f^2h^2+f^3h^3+O(f^4h^4)) \\ &-\omega_{-1}(f^2h\Box h + f^3h^2\Box h + O(f^4h^4)) \\ &+\sum_{n=0}^{\infty}\omega_n^{(0)}\left(f^2h\Box^{n+2}h + f^3h^2\Box^{n+2}h + O(f^4h^4)\right) \\ &+\sum_{n=0}^{\infty}\omega_n^{(2)}\left(f^2h\Box^{n+2}h + f^3h^2\Box^{n+2}h + O(f^4h^4)\right)\Big]. \end{aligned} \quad (8)$$

At any fixed order in the number of derivatives, the leading nonlinear terms in (8) are the ones involving only two gravitons, so that

$$\begin{aligned} S = \int d^4x\Big\{&\omega_{-2}(1+fh+f^2h^2) - \omega_{-1}f^2h\Box h \\ &+\sum_{n=0}^{\infty}\Big[\omega_n^{(0)}f^2h\Box^{n+2}h + \omega_n^{(2)}f^2h\Box^{n+2}h\Big] + O(f^3h^3)\Big\}. \end{aligned} \quad (9)$$

From (9) it is evident that, in the ultraviolet regime all the interactions become negligible, so that the theory is asymptotically free. We also emphasize that the asymptotic freedom in the coupling $f(t)$ ensures the validity of the perturbation theory in f.

Finally, we can sum all the higher derivative terms in (9) to reconstruct the analytic form factor for the kinetic operator of the graviton field. So far we obtain the following free asymptotic action in the ultraviolet regime

$$\begin{aligned} S_{\rm NL}^{(2)} = -\frac{2}{\kappa^2}\int d^Dx\Big[&(\sqrt{-g}R)^{(2)} + G_{\mu\nu}^{(1)}\frac{e^{H(\Box)}-1}{\Box}R^{(1)\mu\nu} \\ &+\omega_0^{(0)}(t)(R^{(1)})^2 + \omega_0^{(2)}(t)R_{\mu\nu}^{(1)}R^{(1)\mu\nu}\Big]\,, \end{aligned} \quad (10)$$

where the labels (2) and (1) refer to quadratic and linear expansion in the graviton. The two operatorsn the second line do not affect unitarity, because the initial conditions (or classical value) of the renormalization group equations for $\omega_0^{(0)}(t)$ and $\omega_0^{(2)}(t)$ are $\omega_0^{(0)}(t=0)=0$ and $\omega_0^{(2)}(t=0)=0$. Therefore, $\omega_0^{(0)}(t)$ and $\omega_0^{(2)}(t)$ can be absorbed in the perturbative finite logarithmic parts of the quantum effective action and/or are cancelled by a proper choice of the renormalization scale. In the latter case we require the minimal subtraction scheme in which μ is selected such that $1+\beta\exp -H(k)\,k^2\,\log k^2/\mu^2$ does not introduce any other real pole. In a purely gravitational theory this is automatically satisfied $\forall\mu$ because of the k^2 momentum power in front of the log.

For completenss, we write the linearized equations of motion for the graviton field h as given by the action (10), that read

$$e^{H(\Box)}\Box\, h_{\mu\nu} = 0\,. \quad (11)$$

We stress that equations (11) have the same solutions of EH theory, because $\exp H(\Box)$ is an invertible operator with no zeros in the whole complex plane at finite energy.

The non locality and super-renormalizability of the gravitational field, including the consequent ultraviolet asymptotic freedom, can be extended to the whole standard model of particle physics (SM), see for instance [28]. As a consequence, it is impossible to accelerate any particle to trans-Plankian energies in any laboratory experiment, since all the fundamental interactions become negligible above the energy scale $E_{\rm NL}$. Hence, we conclude that it is impossible to attain and probe the trans-Planckian regime in the laboratory.

In force of the asymptotic freedom of the theory, we are now ready to discuss the causality violation at the nonlocality scale ℓ typical of nonlocal field theories. In particular, we will show how the unattainability of trans-Planckian energies makes the causality violation undetectable in any laboratory experiment.

To explain how causality violations emerge in nonlocal theories, we consider a toy model consisting of a nonlocal scalar field in Minkowski spacetime coupled to an external source J. The Lagrangian density of the scalar field reads [28–31]

$$\mathcal{L}_\phi = -\frac{1}{2}\phi\, e^{H(-\sigma\Box)}\left(\Box + m^2\right)\phi + \phi\, J\,, \tag{12}$$

where σ is a parameter with dimensions $[\sigma] = -2$ that fixes the nonlocality scale $\ell = \sqrt{\sigma}$. The function $\exp[H(z)]$ must be entire, i.e., analytic with no poles at finite z, so that the unitarity of the theory is guaranteed [21]. Moreover, it must be $\exp[H(z)] \to \infty$ for $|z| \to \infty$. According to (12), the equation of motion of the scalar field is:

$$e^{H(-\sigma\Box)}\left(\Box + m^2\right)\phi(x) = J(x)\,, \tag{13}$$

and its solution reads:

$$\phi(x) = \phi_0(x) + \int d^4y\; G_R(x-y)\, J(y)\,, \tag{14}$$

where $G_R(x-y)$ is the Green function satisfying the following equation,

$$e^{H(-\sigma\Box)}\left(\Box + m^2\right)G_R(x) = \delta^{(4)}(x)\,. \tag{15}$$

The solution of (15) can be easily written in the Fourier space, namely

$$G_R(x) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-\left[H(\sigma k^2)+ikx\right]}}{m^2-k^2} = e^{-H(-\sigma\Box)}G^0_R(x), \tag{16}$$

where $G^0_R(x)$ is the retarded Green function of the local Klein-Gordon theory,

$$G^0_R(x) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ikx}}{m^2-k^2}, \tag{17}$$

with boundary condition $G_R(x) = 0$ for $x^0 < 0$.

Replacing (16) in (14), on can recast (14) as follows,

$$\phi(x) = \phi_0(x) + \int d^4y\; G^0_R(x-y)\, e^{-H(-\sigma\Box_y)}\, J(y)\,, \tag{18}$$

and $\Box_y$ is calculated deriving with respect to y. Note that the support of the function $e^{-H(-\sigma\Box)}J$ in (18) will be different from that of J, i.e., the source J is smeared by the action of the entire function $\exp\left[H(-\sigma\Box)\right]$.

To better understand this fact, we consider the case in which J is an impulsive source centered at some point $P = (\tau, \vec{q})$ of the spacetime, namely

$$J(y) = g\,\delta^4(y-P) = g\,\delta(y^0-\tau)\delta^3(\vec{y}-\vec{q}), \tag{19}$$

where g is a dimensionless coupling constant. As an example, in [29] it has been studied the case in which $H(-\sigma\Box_y) = \sigma^2\,\Box^2$, and it has been shown that the generalized function $e^{-H(-\sigma\Box_y)}\,J(y)$ has a support of size $\sim \sigma^2 = \ell^4$ around P. Therefore, the impulsive source (19) is smeared out by the action of the nonlocal form factor into an effective source of finite support $\sim \ell^4$. Since the local retarded Green function is such that $G^0_R(x-y) \propto \theta(x^0-y^0)$, the integral in (18) is nonzero also for $\tau - \ell < x^0 < \tau$. Therefore, the scalar field is affected by the source before the same is turned on at the time τ. This examples shows in a clear fashion how causality is violated in nonlocal theories.

In general, the causality violation is due to the fact that the support Ω_ℓ of the effective source $e^{-H}J$ differs from the support Ω of the source J. Indeed, Ω_ℓ is obtained from Ω deforming its frontier $F(\Omega)$ by a displacement of order ℓ, so that ℓ defines the scale of the causality violation. If the source J is localized in a region of width $\Delta \gg \ell$, there will be no substantial difference between Ω_ℓ and Ω, so that the causality violation will be irrelevant. On the other hand, if Ω has a width $\Delta \lesssim \ell$ (for instance, in the example of the impulsive source (19) we have $\Delta = 0$) the difference between Ω_ℓ and Ω will be relevant. Thus, in order to produce a detectable causality violation, the source J must be localized in a region of width $\Delta \lesssim \ell$.

Since the source J represents the interaction of the scalar field ϕ with other particles, J will be a function of other fields, e.g. $J = e\bar{\psi}\psi$ or $J = \varphi^4$, where ψ and φ are a spinor and a scalar field respectively. Therefore, in order for J to have a support of width $\Delta \lesssim \ell$, the fields ψ and φ in the given example, must be localized in a region of width $\Delta \lesssim \ell$, so that they must be arranged in wave-packets of width $\Delta \lesssim \ell$, containing frequencies $k^0 \gtrsim \ell^{-1}$. Hence, to measure the causality violation, we need particles of Trans-Planckian energies $E \gtrsim E_{\rm NL}$. However, we have already pointed out that the ultraviolet asymptotic freedom of the theory prevents us to produce particles of such an high energy in the laboratory. Therefore, we conclude that causality violations of nonlocal theories cannot be detected in the laboratory.

Finally, we would like to expand on the cosmological trans-Planckian problem [23, 24] in super-renormalizable nonlocal quantum gravity. In order to get nearly Gaussian scale-invariant primordial perturbations, it is essential to set the initial state for the perturbations to be the Bunch-Davis state. The latter one coincides with the Minkowski solution in the infinite past when it is generically accepted that the perturbative modes are deep inside the horizon. Therefore, sufficiently back in the past, the equations for perturbations reduces to the ones in Minkowski spacetime. However, the cosmological metric is only conformal to the Minkowski metric, hence, the wave lengths λ of the perturbations will be sub-Planckian at some instant in the past, namely $\lambda \lesssim \ell_{\rm P}$ for $\tau \lesssim t_{\rm P}$ ($\lambda \sim a(\tau)$). From the effective field theory point of view, it is hard to explain why higher derivative operators, at linear and nonlinear level in the perturbations, do not affect the observed spectrum when $\lambda \simeq O(1/M_{\rm P})$. Indeed, operators like[2]

$$\mathcal{R}\frac{\Box^k}{M_{\rm P}^{2k}}\mathcal{R}\,, \tag{20}$$

will be of the same order of magnitude as R at the Planck energy scale. This puzzle has a simple solution in nonlocal gravity [25], because the theory is asymptotically free, so that the contribution of interactions and higher order terms as (20) to the dynamics of cosmological perturbations go to zero in the infinite past, as proved above in this paper (see formulas (5)-(10)). Moreover, the full nonlocal kinetic term describes the propagation of the same degrees of freedom of the local theory as evident from (11). This latter feature is characteristic of the nonlocal theory (1). The same argument applies to all the other fundamental interactions whether we extend the standard model couple to gravity to a complete super-renormalizable theory [26].

In conclusion, we have shown that in nonlocal quantum gravity particles are asymptotically free in the ultraviolet regime, therefore, the trans-Planckian regime is unattainable in laboratory experiments. As physical implications, the scale at which causality violation are expected can not be tested in laboratory experiments, and the cosmological trans-Planckian problem actually does not occur in nonlocal quantum gravity.

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