Multidimensional finite quantum gravity

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We advance a class of unitary higher derivative theories of gravity that realize an ultraviolet completion of Einstein general relativity in any dimension. This range of theories is marked by an entire function, which averts extra degrees of freedom (including poltergeists) and improves the high energy behavior of the loop amplitudes. It is proved that only one-loop divergences survive and the theory can be made super-renormalizable regardless of the spacetime dimension. Moreover, using the Pauli-Villars regularization procedure introduced by Diaz-Troost-van Nieuwenhuizen-van Proeyen (DTPN) and applied to Einstein's gravity by Anselmi, we are able to remove the divergences also at one-loop, making the theory completely finite in any dimension as expected by Anselmi and Asorey-Lopez-Shapiro.

Introduction — The microscopic structure of the universe is thoroughly compatible with quantum field theory and the standard model of particle physics based on renormalizability and perturbative theory. In this paper we assume the latter ones as the guiding principles for all fundamental interactions and we seek a new theory of gravity that can encompass such features. It is erroneously thought that general relativity and quantum mechanics are not compatible, but there is nothing inconsistent between them. Just like for the Fermi theory of weak interactions, quantum Einstein's gravity is perfectly consistent, solid and calculable, it is only non-renormalizable. At short distances, higher order operators in the Lagrangian become decisive suggesting that we need an ultraviolet completion of Einstein's gravity. The question is: what are the relevant operators? The answer lies in a "New Classical Theory of Gravity" that is free of singularities at classical level and renormalizable or finite at quantum level. We believe that the two objectives are interdependent. Therefore, the aim of this work is to extend *classical* Einstein-Hilbert theory to make gravity compatible with the above guiding principles (renormalization and perturbative theory) in the "quantum field theory framework". We begin with a new unitary higher derivative theory for gravity in a multidimensional spacetime [1–8] (see also [9–18] and [19–22] for nonlocal infrared extended theories of gravity), and we show that the quantum theory can be made super-renormalizable because only one-loop divergences survive. Moreover, these one-loop divergences can be removed by introducing Pauli-Villars determinant regulators (in the number of $2D \times 3$, where D is the spacetime dimension) in the Diaz-Troost-van Nieuwenhuizen-van Proeyen formalism (DTPN) [23] which explicitly preserves covariance. Action, measure, and regularization procedure are BRS (Becchi-Rouet-Stora) invariant [24–26]. We end up with a completely finite theory (all the beta functions can consistently annulled) and the outcome is a completely "finite quantum field theory of gravity" in any dimension. This work is a completion of the previous work on polynomial [27] and non-polynomial [1–7], super-renormalizable quantum gravity applying Anselmi's scheme [26].

New gravity — The aim of this section is to define a "new theory of gravity" in a D-dimensional spacetime assuming the following consistency requirements.

1. Unitarity. A general theory is well defined if "tachyons" and "ghosts" are absent, in which case the corresponding propagator has only first poles with real masses (no tachyons) and with positive residues (no ghosts.)

2. Super-renormalizability or finiteness. This hypothesis makes consistent the theory at quantum level in analogy with all the known fundamental interactions.

3. Lorentz invariance. This is a symmetry of nature well tested experimentally beyond the Planck mass.

4. Last but not least, the energy conditions can not be violated on the matter side, but they can be violated on the gravity side because of the higher-derivative operators in the classical theory. This property is crucial to avoid the "singularities" that plague almost all the solutions of Einstein's gravity [28–36].

The minimal action for gravity, which will prove compatible with the above properties, reads as follows[1],

$$S_{\mathrm{g}} = -\int d^D x \sqrt{-g}\, 2\kappa_D^{-2}\Big(R + G_{\mu\nu}\, \frac{e^{H(-\Box_\Lambda)}-1}{\Box}\, R^{\mu\nu}\Big), \quad (1)$$

where $z \equiv -\Box_\Lambda \equiv -\Box/\Lambda^2$ and $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the covariant d'Alembertian operator. Λ is an invariant fundamental mass scale and $G_{\mu\nu}$ is the Einstein's tensor.

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[1] *Definitions* — The metric tensor $g_{\mu\nu}$ has signature $(+ - \cdots -)$ and the curvature tensors are defined as follows: $R^\mu_{\nu\rho\sigma} = -\partial_\sigma \Gamma^\mu_{\nu\rho} + \ldots,\ R_{\mu\nu} = R^\rho_{\mu\rho\nu},\ R = g^{\mu\nu}R_{\mu\nu}$

The entire function $V^{-1}(z) \equiv \exp H(z)$ ($H(z)$ is in turn an entire function) is non-polynomial and satisfies the following general properties [7]:

(i). $V^{-1}(z)$ is real and positive on the real axis and it has no zeroes on the whole complex plane $|z| < +\infty$. This requirement assures the absence of extra gauge-invariant poles other than the transverse massless physical graviton pole. A maximal extension of the theory (1) compatible with unitarity has been published in [37].

(ii). $|V^{-1}(z)|$ has the same asymptotic behavior along the real axis at $\pm\infty$.

(iii). we define $2N + 4 = D_{\rm odd} + 1$ in odd dimension and $2N + 4 = D_{\rm even}$ in even dimension to avoid fractional powers of the D'Alembertian operator. Therefore, it exists $\Theta > 0$ such that

$$\lim_{|z| \to +\infty} |V^{-1}(z)| \to |z|^{\gamma+N+1},$$
$$\gamma \geqslant D_{\rm even}/2 \;\; {\rm and} \;\; \gamma \geqslant (D_{\rm odd} - 1)/2, \qquad (2)$$

for the argument of z in the following conical regions $C = \{z \,|\; -\Theta < \arg z < +\Theta, \; \pi - \Theta < \arg z < \pi + \Theta\}$, for $0 < \Theta < \pi/2$. The last condition is necessary in order to achieve the highest convergence of the theory in the ultraviolet regime. The necessary asymptotic behavior is imposed not only on the real axis, but also on the conic regions that surround it. In an Euclidean spacetime, the condition (ii) is not strictly necessary if (iii) applies.

An explicit example of $\exp H(z)$ that satisfies the properties (i)-(iii) can be easily constructed [7],

$$V^{-1}(z) \equiv e^{H(z)} = e^{\frac{1}{2}\left[\Gamma\left(0, p^2(z)\right)+\gamma_E\right]} |p(z)|$$
$$= \underbrace{e^{\frac{\gamma_E}{2}} |p(z)|}_{V_\infty^{-1}(z)} + \underbrace{\left(e^{\frac{1}{2}\Gamma\left(0, p^2(z)\right)} - 1\right) e^{\frac{\gamma_E}{2}} |p(z)|}_{V^{-1}(z)-V_\infty^{-1}(z) \,\sim\, e^{-F(z)} \,,\; F(z)>0}, \quad (3)$$

where $p(z)$ is a polynomial of degree $\gamma + N + 1$ such that $p(0) = 0$ and ${\rm Re}(p^2(z)) > 0$. In (3) $\gamma_E = 0.577216$ is the Euler's constant and $\Gamma(a, z) = \int_z^{+\infty} t^{a-1} e^{-t} dt$ is the incomplete gamma function. If we choose $p(z) = z^{\gamma+N+1}$ the Θ angle defining the cone C of (iii) turns out to be $\Theta = \pi/(4\gamma + 4N + 4)$. For $p(z) = z^{\gamma+N+1} \equiv z^n$ ($n \equiv \gamma + N + 1$), the function $F(z)$ in (3) is well approximated, at least along the real axis, by $F(z) \propto z^m$ with $m \in \mathbb{N}$, $m \gtrsim n$. Two examples are: $F(z) \approx 2|z|^5$ for $n = 4$ or $F(z) \approx 2z^{12}$ for $n = 10$. A crucial property of the form factor for the convergence of the theory is that

$$\lim_{z \to +\infty} V(z)^{-1} := V_\infty^{-1}(z) = e^{\frac{\gamma_E}{2}} |z|^{\gamma+N+1} \quad {\rm and}$$

$$\lim_{z \to +\infty} \left(\frac{V(z)^{-1}}{e^{\frac{\gamma_E}{2}} |z|^{\gamma+N+1}} - 1 \right) z^n = 0 \;\; \forall n \in \mathbb{N}. \qquad (4)$$

Gauss-Bonnet operator — In $D > 4$ the Gauss-Bonnet invariant is not a total derivative; however, at the same time it does not affects the propagator around flat spacetime and it can only contribute to the interaction vertexes. In the theory (1) it does not even affect the divergent part of the one-loop effective action, but it gives a contribution to the finite parts of the one-loop diagrams.

Propagator — Splitting the spacetime metric in the flat Minkowski background and the fluctuation $h_{\mu\nu}$ defined by $g_{\mu\nu} = \eta_{\mu\nu} + \kappa_D h_{\mu\nu}$, we can expand the action (1) to the second order in $h_{\mu\nu}$. The result of this expansion together with the usual harmonic gauge fixing term reads [38]: $\mathcal{L}_{\rm lin} + \mathcal{L}_{\rm GF} = h^{\mu\nu} \mathcal{O}_{\mu\nu,\rho\sigma} h^{\rho\sigma}/2$, where the operator $\mathcal{O}$ is made of two terms, one coming from the linearization of (1) and the other from the following gauge-fixing term, $\mathcal{L}_{\rm GF} = \xi^{-1} \partial^\nu h_{\nu\mu} \omega(-\Box_\Lambda) \partial_\rho h^{\rho\mu}$ ($\omega(-\Box_\Lambda)$ is a weight functional [7, 39, 40].) The d'Alembertian operator in $\mathcal{L}_{\rm lin}$ and in the functional weight ω must be conceived on the flat spacetime. Inverting the operator $\mathcal{O}$ [38], we find the two-point function in the harmonic gauge ($\partial^\mu h_{\mu\nu} = 0$),

$$\mathcal{O}^{-1} = \frac{V(k^2/\Lambda^2)}{k^2} \left(P^{(2)} - \frac{P^{(0)}}{D-2} \right) + \frac{\xi(2P^{(1)} + \bar{P}^{(0)})}{2k^2 \, \omega(k^2/\Lambda^2)}. \, (5)$$

The tensorial indexes for the operator $\mathcal{O}^{-1}$ and the projectors $\{P^{(0)}, P^{(2)}, P^{(1)}, \bar{P}^{(0)}\}$ have been omitted. The above projectors are defined by [38, 41]: $P^{(2)}_{\mu\nu,\rho\sigma}(k) = (\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho})/2 - \theta_{\mu\nu}\theta_{\rho\sigma}/(D-1)$, $P^{(1)}_{\mu\nu,\rho\sigma}(k) = (\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho})/2$, $P^{(0)}_{\mu\nu,\rho\sigma}(k) = \theta_{\mu\nu}\theta_{\rho\sigma}/(D-1)$, $\bar{P}^{(0)}_{\mu\nu,\rho\sigma}(k) = \omega_{\mu\nu}\omega_{\rho\sigma}$, $\theta_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu/k^2$ and $\omega_{\mu\nu} = k_\mu k_\nu/k^2$.

Power counting and super-renormalizability — Let us then examine the ultraviolet behavior of the quantum theory. According to the property (iii) in the high energy regime, the propagator in the momentum space and the leading interaction vertex are schematically given by

$$\mathcal{O}^{-1}(k) \sim \frac{1}{k^{2\gamma+2N+4}} \quad {\rm in \; the \; ultraviolet}, \qquad (6)$$

$$\mathcal{V}^{(n+2)} \sim h^n \Box_\eta h \frac{V^{-1}(-\Box_\Lambda)}{\Box} \Box_\eta h \; \to \; h^n \Box_\eta h \Box_\eta^{\gamma+N} \Box_\eta h \,,$$

where $\Box_\eta := \eta^{\mu\nu} \partial_\mu \partial_\nu$. In (6) the indices for the gravitational fluctuations $h_{\mu\nu}$ are omitted. From (6), the upper bound to the superficial degree of divergence in a spacetime of "even" or "odd" dimension reads

$$\omega(G)_{\rm even} = D_{\rm even} - 2\gamma(L - 1), \qquad (7)$$
$$\omega(G)_{\rm odd} = D_{\rm odd} - (2\gamma + 1)(L - 1). \qquad (8)$$

In (7) and (8) we used the topological relation between vertexes V, internal lines I and number of loops L: $I = V + L - 1$. Thus, if $\gamma > D_{\rm even}/2$ or $\gamma > (D_{\rm odd} - 1)/2$, only 1-loop divergences survive in this theory. Therefore, the theory can be made super-renormalizable by introducing in the classical action local operators of mass

dimensionality up to M^D [1–6, 10–14]. Once more let us reiterate what we mean by "renormalizable theory". A theory is renormalizable *iff* all the divergent contributions to the effective action are proportional to operators already present in the classical theory.

Quantum gravity — In the previous sections we showed unitary around flat stacetime and power-counting super-renormalizability. In this section we quantize the theory defined by (1) in the "path-integral formulation". We use the background field method and we focus our attention on the DTPN-Pauli-Villars regularization [23], which preserves covariance: the regularized Lagrangian, the measure and the regularization procedure turn out to be BRS-invariant (quantum symmetry which involves the graviton and the ghost fields after gauge fixing.) This procedure permits to remove the one-loop divergences without modifying the classical Lagrangian (1). Since multi-loop amplitudes are convergent, we get a completely finite theory of quantum gravity (all beta functions vanish). By imposing appropriate and consistent conditions on the Pauli-Villars fields, we will be able to remove all the divergences between the maximal ones having the form of a cosmological constant and the logarithmic ones proportional to $R_{\mu\nu\rho\sigma}^{D/2}$.

The Lagrangian (1) can be regularized with a set of complex vectors $W_{i\mu}$, a set of real vectors $Z_{i\mu}$, and a set of real tensor $T_{i\mu\nu}$. In the background field method the ghosts are regularized by the complex and the real vectors, while the metric is regularized by the real tensors. However, this is just a convention because in general W, Z and T altogether regularize the entire range of divergences [24]. The regularized Lagrangian is made of six operators,

$$\mathcal{L} = \mathcal{L}_\text{g} + \mathcal{L}_\text{GF} + \mathcal{L}_\text{GH} + \sum_{i=1}^{n_t} \mathcal{L}_{t_i} + \sum_{i=1}^{n_v} \mathcal{L}_{v_i} + \sum_{i=1}^{n_z} \mathcal{L}_{z_i}. \quad (9)$$

Since we are interested in the divergent contributions to the one-loop effective action, the property (3) allows us to focus just on the ultraviolet limit of (1), namely

$$\mathcal{L}_\text{g} \sim -2\kappa_D^{-2}\sqrt{-g}\, G_{\mu\nu} \frac{e^{\frac{\gamma_E}{2}}\, p(-\Box_\Lambda)}{\Box} R^{\mu\nu}. \quad (10)$$

The general polynomial involved in the divergent contributions to the one-loop effective action is

$$p_{\gamma+\text{N}+1}(z) = a_\text{N}\, z^{\gamma+\text{N}+1} + \cdots + a_{\text{N}-\frac{D}{2}}\, z^{\gamma+\text{N}+1-\frac{D}{2}}. \quad (11)$$

The other operators coming from (3) can only contribute to the finite part because any correction to the leading ultraviolet limit is exponentially suppressed as shown by (4). n_v is the number of W-vector fields, n_z is the number of Z-vector fields and n_t is the number of T-tensor fields, while

$$\mathcal{L}_{v_i} = \sqrt{-g}\Big[\bar{W}_{i\mu}(\Box + M_{v_i}^2)W_i^\mu$$
$$+\bar{W}_{i\mu}(\nabla^\mu\nabla_\nu - 2\beta_g\nabla_\nu\nabla^\mu)W_i^\nu\Big], \quad (12)$$

$$\mathcal{L}_{z_i} = \sqrt{-g}\Big[-\frac{1}{\alpha_g}g^{\mu\nu}\, Z_{i\mu}(\Box + M_{z_i}^2)(\Box + M_{z_i}^2)_\Lambda^{\text{N}+\gamma}\, Z_{i\nu}$$
$$-\frac{1}{\alpha_g}Z_{i\mu}\,(\gamma_g\nabla^\mu\nabla^\nu - \nabla^\nu\nabla^\mu)\Box_\Lambda^{\text{N}+\gamma}\, Z_{i\nu}\Big], \quad (13)$$

$$\mathcal{L}_{t_i} = \frac{2}{\kappa_D^2}\sqrt{-g}\Big[-\frac{e^{\frac{\gamma_E}{2}}}{4}T_i^{\mu\nu}(\Box + M_{t_i}^2)p(-\Box - M_{t_i}^2)T_{i\mu\nu}$$
$$+TR\nabla^{2\text{N}+2\gamma+2}T + \cdots + T(R\nabla^{D-4}R)\nabla^{2\gamma}T\Big]. \quad (14)$$

The constants α_g, β_g and γ_g and more details about the action for the tensor fields $T_{i\mu\nu}$ will be given shortly. In the background field method the metric $g_{\mu\nu}$ splits in a background portion $\bar{g}_{\mu\nu}$ and in a quantum fluctuation $h_{\mu\nu}$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \quad (15)$$

The non minimal couplings in the Lagrangians (12), (13) and (14) are chosen in order to look similar to $\mathcal{L}_\text{GH}$ and $\mathcal{L}_\text{g} + \mathcal{L}_\text{GF}$, respectively, when both of them are expanded around the background $\bar{g}_{\mu\nu}$. The gauge fixing and FP-ghost actions are

$$S_\text{GF} = \int d^D x\sqrt{-\bar{g}}\,\chi_\mu\, C^{\mu\nu}\,\chi_\nu\,, \quad \chi_\mu = \bar{\nabla}_\sigma h_\mu^\sigma - \beta_g\bar{\nabla}_\mu h\,,$$

$$C^{\mu\nu} = -\frac{1}{\alpha_g}\left(\bar{g}^{\mu\nu}\Box + \gamma_g\bar{\nabla}^\mu\bar{\nabla}^\nu - \bar{\nabla}^\nu\bar{\nabla}^\mu\right)\Box_\Lambda^{\text{N}+\gamma}\,,$$

$$S_\text{GH} = \int d^D x\sqrt{-\bar{g}}\left[\bar{C}_\alpha\, M_\beta^\alpha\, C^\beta + b_\alpha C^{\alpha\beta}b_\beta\right]\,,$$

$$M_\beta^\alpha = \Box\delta_\beta^\alpha + \bar{\nabla}^\alpha\bar{\nabla}_\beta - 2\beta_g\bar{\nabla}_\beta\bar{\nabla}^\alpha. \quad (16)$$

In (16) we used a gauge fixing with weight function $C^{\mu\nu}$ [27]. $\alpha_g, \beta_g, \gamma_g$ are dimensionless gauge fixing parameters, but the beta-functions are independent from them (see proof in [27].) For $\beta_g = 1/2$ and $\gamma_g = 1$, the Lagrangians for the Pauli-Villars fields (12) and (13) and the Lagrangians for the ghosts $\bar{C}, C, b$ simplify to

$$\mathcal{L}_C = \sqrt{-\bar{g}}\Big[\bar{C}_\mu\Box C_i^\mu - R^\mu{}_\nu\bar{C}_\mu C^\nu\Big], \quad (17)$$

$$\mathcal{L}_{v_i} = \sqrt{-\bar{g}}\Big[\bar{W}_{i\mu}(\Box + M_{v_i}^2)W_i^\mu - R^\mu{}_\nu\bar{W}_{i\mu}W_i^\nu\Big],$$

$$\mathcal{L}_b = \frac{\sqrt{-\bar{g}}}{-\alpha_g}\Big[b_\mu\,\Lambda^2\Box_\Lambda^{\text{N}'+1}b^\mu - b_\mu\, R^{\mu\nu}\Box_\Lambda^{\text{N}'}\, b_\nu\Big], \quad (18)$$

$$\mathcal{L}_{z_i} = \frac{\sqrt{-\bar{g}}}{-\alpha_g}\Big[Z_{i\mu}\,\Lambda^2(\Box + M_{z_i}^2)_\Lambda^{\text{N}'+1}Z_i^\mu - Z_{i\mu}\, R^{\mu\nu}\Box_\Lambda^{\text{N}'}\, Z_{i\nu}\Big],$$

where $\text{N}' = \text{N}+\gamma$. If we introduce new variables rescaling the fields $X \in \{\bar{W}, W, Z, \bar{C}, C, b\}$ by $X \to (-g)^{-1/4}X$, we see that the mass terms for the Pauli-Villars fields give contribution to the propagator and to extra vertexes that are proportional to the mass square elevated to some integer power. The latter vertexes are present neither in the graviton, nor in the ghosts fields; however, they do not originate divergences, if the Pauli-Villars regularization conditions that will tackle later in the paper (conditions (23) and (38)) are satisfied. It follows that, in the redefined variables, $\mathcal{L}_{v_i}$ regularizes $\mathcal{L}_C$ and $\mathcal{L}_{z_i}$ regularizes

$\mathcal{L}_b$. A similar analysis can be carried out for the graviton and the tensors T.

We have now all the conditions to define the partition function with the right functional measure compatible with BRS invariance [26],

$$Z = \int \mu(g,\bar{g}) \prod_{\mu \leqslant \nu} \mathcal{D}g_{\mu\nu} \prod_{\nu} \mathcal{D}\bar{C}_{\nu} \prod_{\mu} \mathcal{D}C^{\mu} \prod_{\mu} \mathcal{D}b_{\mu}$$

$$\prod_{i=1}^{n_v} \left[\prod_{\nu} \mathcal{D}'\bar{W}_{i\nu} \prod_{\mu} \mathcal{D}'W_i^{\mu} \right] \prod_{i=1}^{n_z} \prod_{\mu} \mathcal{D}'Z_{i\mu} \prod_{j=1}^{n_t} \prod_{\mu \leqslant \nu} \mathcal{D}'T_{j\mu\nu}$$

$$e^{i \int d^D x \left[\mathcal{L}_g + \mathcal{L}_{GF} + \mathcal{L}_{GH} + \sum_{i=1}^{n_v} \mathcal{L}_{v_i} + \sum_{i=1}^{n_z} \mathcal{L}_{z_i} + \sum_{i=1}^{n_t} \mathcal{L}_{t_i} \right]}. \quad (19)$$

We can evaluate the functional integral and express the partition function as a product of determinants, namely[2]

$$Z = e^{iS_g(\bar{g}_{\mu\nu})} \left\{ \det \left[\frac{\delta^2(S_g + S_{GF})}{\delta h_{\mu\nu} \delta h_{\rho\sigma}} \right] \right\}^{-\frac{1}{2}} (\det M_\beta^\alpha)$$

$$(\det C^{\alpha\beta})^{\frac{1}{2}} \prod_{i=1}^{n_z} \left[\det \left(\underbrace{\frac{\delta S_{z_i}^2}{\delta Z_{i\mu} \delta Z_{i\nu}}}_{\mathcal{O}_{z_i}} \right) \right]^{\frac{c_{z_i}}{2}} \quad (21)$$

$$\prod_{i=1}^{n_v} \left[\det \left(\underbrace{\frac{\delta S_{v_i}^2}{\delta \bar{W}_{i\mu} \delta W_{i\nu}}}_{\mathcal{O}_{v_i}} \right) \right]^{\frac{c_{v_i}}{2}} \prod_{i=1}^{n_t} \left[\det \left(\underbrace{\frac{\delta S_{t_i}^2}{\delta T_{i\mu\nu} \delta T_{i\rho\sigma}}}_{\mathcal{O}_{t_i}} \right) \right]^{\frac{c_{t_i}}{2}},$$

where c_{v_i}, c_{z_i} and c_{t_i} come from the definition of the functional integration on the Pauli-Villars fields W, Z, T. Furthermore, we impose on them the following regularizations conditions [26]

$$\boxed{\sum_{j=1}^{n} c_{u_j} = c_u} \quad (22)$$

$$\boxed{\sum_{j=1}^{n} c_{u_j} (M_j^2)^q = 0, \quad 0 < q \leqslant \left[\frac{D}{2} \right]} \quad (23)$$

in which $[x]$ is the integer part of the number x. The index $u = v, z, t$ and the constants $c_v = -2$, $c_z = -1$, $c_t = 1$. To calculate the one-loop effective action we need to expand the action plus the gauge-fixing term to the second oder in the quantum fluctuation $h_{\mu\nu}$

$$H_{\mu\nu,\rho\sigma} = \frac{\delta^2 S_g}{\delta h_{\rho\sigma} \delta h_{\mu\nu}}\bigg|_{h=0} + \frac{\delta \chi_\delta}{\delta h_{\rho\sigma}} C^{\delta\tau} \frac{\delta \chi_\tau}{\delta h_{\mu\nu}}\bigg|_{h=0}. \quad (24)$$

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[2] The DTNP formalism is based on a particular definition of the functional integration on the Pauli-Villars regulators. If χ_j is a bosonic Pauli-Villars field we have

$$\int \mathcal{D}'\chi_j \, e^{\chi_j^T A \chi_j} = (\det A)^{\frac{c_j}{2}} \quad \text{(formal gaussian integral)}, \quad (20)$$

where A is a generic χ_j-independent infinite matrix and ^{T} denotes the transposition, while c_j is a coefficient associated with the Pauli-Villars field χ_j.

The explicit calculation of H goes behind the scope of this paper and here we only offer the tensorial structure in terms of the curvature tensor of the background metric and its covariant derivatives. Since we are only interesting in the divergent contributions to the one-loop effective action, from (10) the *minimal* operator $H_{\mu\nu,\rho\sigma}$ consists only of the terms coming from the asymptotic limit of the form factor, namely

$$H = \Box^{N+\gamma+2} + (\bar{R} + \dots) \cdot \bar{\nabla}^{2N+2\gamma+2}$$
$$+ (\bar{\nabla}\bar{R} + \dots) \cdot \bar{\nabla}^{2N+2\gamma+1} + (\bar{R}^2 + \dots)\bar{\nabla}^{2N+2\gamma}$$
$$+ (\bar{\nabla}\bar{R}^2 + \dots) \cdot \bar{\nabla}^{2N+2\gamma-1} + (\bar{\nabla}^2\bar{R}^2 + \dots) \cdot \bar{\nabla}^{2N+2\gamma-2}$$
$$+ \dots + (\bar{R}\,\bar{\nabla}^{D-4}\bar{R} + \dots) \cdot \bar{\nabla}^{2N+2\gamma+4-D}, \quad (25)$$

where $\bar{\nabla}^Y \equiv \bar{\nabla}_{\mu_1} \dots \bar{\nabla}_{\mu_Y}$. In (25) we explicitly showed how the coefficients depend on the curvature tensors in a compact notation. The dots inside the round brackets indicate operators with less derivatives of the background metric tensor. These operators come from lower powers in the polynomial $p(z)$. For example, the last coefficient in (25) reads,

$$\bar{R}\,\bar{\nabla}^{D-4}\bar{R} + \dots = \bar{R}\,\bar{\nabla}^{D-4}\bar{R} + \bar{R}\,\bar{\nabla}^{D-6}\bar{R}$$
$$+ \dots + \bar{R}^2 + \bar{R} + \text{const}. \quad (26)$$

However, from here on we assume the polynomial in (11) to be proportional to $z^{\gamma+N+1}$ to simplify our analysis.

It is now clear how to define $\mathcal{L}_{t_i}$ in (14) to regularize the gravitational Lagrangian,

$$\mathcal{L}_{t_i} = \sqrt{-g}\, T_i^{\mu\nu} \left(H_{\mu\nu,\rho\sigma}(g_{\mu\nu}) \Big|_{\Box \to \Box + M_{t_i}^2} \right) T_i^{\rho\sigma}, \quad (27)$$

where we replaced the background metric $\bar{g}_{\mu\nu}$ with $g_{\mu\nu}$ and all tensors are now defined by the metric $g_{\mu\nu}$. In the background field method the Lagrangians for W and T have also to be expanded using (15). However, they are already quadratic in the Pauli-Villars fields and we can simply substitute $g_{\mu\nu}$ with $\bar{g}_{\mu\nu}$ in $\mathcal{L}_{v_i}$, $\mathcal{L}_{z_i}$ and $\mathcal{L}_{t_i}$. The field redefinition $X \to (-g)^{-1/4}X$ also applies to the fields T. For the tensors T as well as for the vectors Z there are extra vertexes proportional to the mass, which are absent for the graviton and the ghost b. However, such operators do not give divergent contributions to the one-loop amplitudes because of the set of conditions (23) and (38). The similarities between $\mathcal{L}_{GH}$ and $\mathcal{L}_{v_i} + \mathcal{L}_{z_i}$, and $\mathcal{L}_g + \mathcal{L}_{GF}$ and $\mathcal{L}_{t_i}$ are now evident since $\sum_i \mathcal{L}_{t_i}$ regularizes $\mathcal{L}_g + \mathcal{L}_{GF}$, while $\sum_i (\mathcal{L}_{v_i} + \mathcal{L}_{z_i})$ regularizes $\mathcal{L}_{GH}$.

We can now move on to calculate the one-loop effective action [27],

$$\Gamma^{(1)} = -i \log Z = S_g(\bar{g}_{\mu\nu})$$

$$+ \frac{i}{2} \log \det(H) - i \sum_{j=1}^{n_t} \frac{c_{t_j}}{2} \log \det(\mathcal{O}_{t_j})$$

$$- i \log \det(M) - i \sum_{j=1}^{n_v} \frac{c_{v_j}}{2} \log \det(\mathcal{O}_{v_j})$$

$$- \frac{i}{2} \log \det(C) - i \sum_{j=1}^{n_z} \frac{c_{z_j}}{2} \log \det(\mathcal{O}_{z_j}). \quad (28)$$

Given a general operator $\mathcal{O} = \mathcal{O}_0 + \mathcal{O}_I$, where $\mathcal{O}_0$ refers to the free part and $\mathcal{O}_I$ refers to the interaction part,

$$\log \det \mathcal{O} = \text{Tr} \log \mathcal{O} = c \, \text{Tr}[\log(1 + \mathcal{O}_0^{-1} \mathcal{O}_I)] \qquad (29)$$

$$= c \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} \int d^D x_1 \ldots d^D x_n \, \mathcal{O}_0^{-1}(x_1 - x_2) \mathcal{O}_I(x_2)$$

$$\mathcal{O}_0^{-1}(x_2 - x_3)\mathcal{O}_I(x_3) \ldots \ldots \mathcal{O}_0^{-1}(x_n - x_1)\mathcal{O}_I(x_1),$$

where $c = $ constant. In the background field method we define the fields $f_{\mu\nu}$ related to the background metric by $\bar{g}_{\mu\nu} = \eta_{\mu\nu} + f_{\mu\nu}$. $\mathcal{O}_0^{-1}$ ($\mathcal{O}_I$) is the propagator (interaction vertex) around flat spacetime for any field circulating in the loop diagram: the graviton $h_{\mu\nu}$, the tensors $T_{i\mu\nu}$, the ghosts $\bar{C}_\nu, C^\mu, b_\mu$ or the vectors $W_{i\mu}, \bar{W}_{i\nu}, Z_{i\mu}$.

Let us now elaborate on the extra vertexes englobing the mass of the Z vectors or T tensors when we expand the background metric around the flat spacetime. For the case of the vectors Z, let us expand the action to the linear order in the fluctuation f (a similar analysis applies to the tensors T),

$$\mathcal{L}_{z_i} = Z_{i\mu} \Lambda^2 \Big[(\Box_0 + M_{z_i}^2)_\Lambda^{N'+1} \qquad (30)$$

$$+ \sum_{k=1}^{N'+1} (\Box_0 + M_{z_i}^2)^{k-1} \delta\Box (\Box_0 + M_{z_i}^2)^{N'+1-k} \Big] Z_i^\mu$$

$$- Z_{i\mu} R^{\mu\nu} \Box_\Lambda^{N'} Z_{i\nu}$$

$$= Z_{i\mu} \Lambda^2 \Big[(\Box_0 + M_{z_i}^2)_\Lambda^{N'+1} + V_1(\partial^{2N'+2}, f)$$

$$+ \underbrace{V_2(\partial^{2N'}, M_{z_i}^2, f)}_{V_2(M_{z_i}^2, \ldots, M_{z_i}^{2N'})} + O(M_{z_i}^4) \Big] Z_i^\mu - Z_{i\mu} \underbrace{R^{\mu\nu}\Box_\Lambda^{N'}}_{V_R} Z_{i\nu} \,,$$

where $\Box_0 = \eta^{\mu\nu}\partial_\mu\partial_\nu$. The vertexes V_2 are present only for the Pauli-Villars fields, while the vertexes V_1 are present for the b ghost, too. However, the contribution of V_2 is zero because of the sets of constraints (23) and (38) (see the Appendix for more technical details[3].)

Using the background field method and the Pauli-Villars regularization, the main divergent integrals con-

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[3] We can put the determinant for the Z fields in the following form,

$$\det\Big((\Box_0 + M_{z_i}^2)_\Lambda^{N'+1} + V_1 + V_2(M_{z_i}^2, \ldots, M_{z_i}^{2N'}) + V_R\Big)^{\frac{c_{z_j}}{2}}$$

$$= \det\Big((\Box_0 + M_{z_i}^2)_\Lambda^{N'+1} + V_1 + V_R\Big)^{\frac{c_{z_j}}{2}}$$
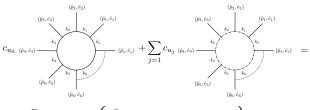
$$\cdot \underbrace{\det\left(1 + \frac{V_2(M_{z_i}^2, \ldots, M_{z_i}^{2N'}) + V_R}{(\Box_0 + M_{z_i}^2)_\Lambda^{N'+1} + V_1 + V_R}\right)^{\frac{c_{z_j}}{2}}}_{\lim_{M_{z_i} \to +\infty}(\ldots)=1}. \qquad (31)$$

Taking the limit $M_{z_i} \to +\infty$ only the first factor allows the correct normalization to regularize the b ghost's action, while the second factor tends to one.

tributing to the one-loop effective action have the following form,

$$\int \frac{d^D k}{(2\pi)^D} {\sum_{j=0}}' c_{u_j} \left\{ \prod_{i=1}^s \frac{1}{[(k+p_i)^2 + M_{u_j}^2]^n} \right\} P(k)_{2sn}. \qquad (32)$$

$P_{2sn}(k)$ is a polynomial function of degree $2ns$ in the momentum k (generally it also relies on the external momenta $\bar{p}_a$), $p_i = \sum_{a=1}^i \bar{p}_a$, and $u = v, z, t$ labels the three different Pauli-Villars fields. The positive integer n is: $n = \gamma + N + 2$ for $h_{\mu\nu}$ and $T_{i\mu\nu}$, $n = \gamma + N + 1$ for b_μ and $Z_{i\mu}$, while $n = 1$ for $C, \bar{C}$ and $W, \bar{W}$. The ${\sum}'$ is from $j = 0$ to $j = n_{u_j}$ and it also includes the massless fields with the convention $M_{u_0} = 0$. $c_{v_0} = 2$ for the ghosts $C, \bar{C}$ loops, $c_{z_0} = 1$ for the ghost b loops, and $c_{t_0} = -1$ for the case of the graviton loops. We can write, as usual,

$$\prod_{i=1}^s \frac{1}{[(k+p_i)^2 + M_{u_j}^2]^n} =$$

$$= c \int_0^1 \left(\prod_{i=1}^s x_i^{n-1} dx_i \right) \delta\left(1 - \sum_{i=1}^s x_i\right) \frac{1}{[k'^2 + M_{u_j}'^2]^{ns}},$$

$$k' = k + \sum_{i=1}^s x_i p_i \,,$$

$$M_{u_j}'^2 = M_{u_j}^2 - \left(\sum_{i=1}^s x_i p_i\right)^2 + \sum_{i=1}^s p_i^2 x_i \,. \qquad (33)$$

where c = constant. In (32), we move outside the convergent integral in x_i and we replace k' with k

$$\int \frac{d^D k}{(2\pi)^D} {\sum_{j=0}}' c_{u_j} \frac{P'(k, p_i, x_i)_{2sn}}{(k^2 + M_{u_j}'^2)^{ns}} \,. \qquad (34)$$

Using Lorentz invariance and missing the argument x_i, we replace the polynomial $P'(k, p_i, x_i)_{2ns}$ with a polynomial of degree $n \times s$ in k^2, namely $P''(k^2, p_i)_{ns}$. In the intermediate steps we integrate (34) in $|k|$ from zero to a cut-off Λ_c and then we take the limit $\Lambda_c \to \infty$. Therefore the integral (34) reduces to

$$\int_0^{\Lambda_c} \frac{d^D k}{(2\pi)^D} {\sum_{j=0}}' c_{u_j} \frac{P''(k^2, p_i)_{ns}}{(k^2 + M_{u_j}'^2)^{ns}} \,. \qquad (35)$$

We can decompose the polynomial $P''(k^2, p_i)_{ns}$ in a product of external and internal momenta only to obtain the divergent contributions,

$$P''(k^2, p_i)_{ns} = \sum_{\ell=0}^{[D/2]} \alpha_\ell(p_i) k^{2ns-2\ell} =$$

$$k^{2ns}\alpha_0 + k^{2ns-2}\alpha_1(p_i) + k^{2ns-4}\alpha_2(p_i) + \ldots . \qquad (36)$$

By changing variables to $y = |k|^2/M_{u_j}'^2$ and neglecting multiplicative constants common to all the fields, the integral (35) turns out to be,

$$\mathcal{I} = \sideset{}{'}\sum_{j=0} c_{u_j} \int_0^{\frac{\Lambda_c^2}{M_{u_j}'^2}} dy\, y^{\frac{D-2}{2}} \frac{M_{u_j}'^D}{(1+y)^{ns}} \Big[\alpha_0\, y^{ns} + \qquad (37)$$

$$+ \frac{\alpha_1(p_i)}{M_{u_j}'^2} y^{ns-1} + \frac{\alpha_2(p_i)}{M_{u_j}'^4} y^{ns-2} + \frac{\alpha_3(p_i)}{M_{u_j}'^6} y^{ns-3} + \dots \Big]$$

$$= \sideset{}{'}\sum_{j=0} c_{u_j} \int_0^{\frac{\Lambda_c^2}{M_{u_j}'^2}} dy\, y^{\frac{D-2}{2}} \frac{M_{u_j}'^D}{(1+y)^{ns}} \sum_{\ell=0}^{[D/2]} \frac{\alpha_\ell(p_i)}{M_{u_j}'^{2\ell}} y^{ns-\ell}.$$

The integral is straightforward and the result for general D is given in the appendix. Finally, the conditions we must add to (22) and (23) for the integrals (37) (or (41)) to vanish are

$$\boxed{\sum_j c_{u_j} (M_{u_j}^2)^p \left(\log \frac{M_{u_j}^2}{\mu_u^2} \right)^{\epsilon_D} = 0} \qquad (38)$$

where $\epsilon_D = 1$ and $p = 0, 1, 2, \dots, D/2$ for D even, while $\epsilon_D = 0$ and $p = 1/2, 3/2, \dots, D/2$ for D odd. In this way the theory is fully regularized. The price to pay to make the theory finite is the introduction of three arbitrary constants μ_u [26].

Finite quantum gravity in even and odd dimension — Using the results of the above section we can now show that all the beta-functions are zero and the theory is finite in any dimension. Suppose the masses of the Pauli-Villars fields are taken to be finite, for now, and consider the terms of the one-loop effective action that are expected to diverge when one lets the Pauli-Villars masses go to infinity. This terms in even dimension can only be proportional to

$$\Lambda_u^D = \sum_{j=1} c_{u_j} M_{u_j}^D \log \frac{M_{u_j}^2}{\mu_u},$$

$$\Lambda_u^{D-2} = \sum_{j=1} c_{u_j} M_{u_j}^{D-2} \log \frac{M_{u_j}^2}{\mu_u},$$

$$\Lambda_u^{D-4} = \sum_{j=1} c_{u_j} M_{u_j}^{D-4} \log \frac{M_{u_j}^2}{\mu_u},$$

$$\Lambda_u^{D-6} = \sum_{j=1} c_{u_j} M_{u_j}^{D-6} \log \frac{M_{u_j}^2}{\mu_u},$$

$$\dots$$
$$\dots$$

$$\Lambda_u^2 = \sum_{j=1} c_{u_j} M_{u_j}^2 \log \frac{M_{u_j}^2}{\mu_u},$$

$$\log \frac{\Lambda_u^2}{\mu_u} = \sum_{j=1} c_{u_j} \log \frac{M_{u_j}^2}{\mu_u}. \qquad (39)$$

The other divergent contributions are absent because the properties (22) and (23) of the Pauli-Villars fields have been enforced. The effective regularized Lagrangian including only divergent contributions reads

$$\boxed{\mathcal{L}_{\text{eff}} = -\sqrt{|g|}\, 2\kappa_D^{-2} \left(R + G_{\mu\nu} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{\mu\nu} \right)}$$

$$+ \sum_u \sum_{j=1} c_{u_j} M_{u_j}^D \log \frac{M_{u_j}^2}{\mu_u}\, a_u \sqrt{|g|}$$

$$+ \sum_u \sum_{j=1} c_{u_j} M_{u_j}^{D-2} \log \frac{M_{u_j}^2}{\mu_u}\, b_u \sqrt{|g|} R$$

$$+ \sum_u \sum_{j=1} c_{u_j} M_{u_j}^{D-4} \log \frac{M_{u_j}^2}{\mu_u} \sqrt{|g|}(c_u^{(1)} R^2 + c_u^{(2)} R_{\mu\nu}^2$$

$$+ c_u^{(3)} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma})$$

$$+ \dots\dots$$

$$+ \sum_u \sum_{j=1} c_{u_j} \log \frac{M_{u_j}^2}{\mu_u} \sqrt{-g} \Big(d_u^{(1)} R_{\dots}^{\text{N}+2}$$

$$+ d_u^{(2)} R_{\dots}^{\text{N}-1} \nabla R_{\dots} \nabla R_{\dots} + d_u^{(3)} R_{\dots} \Box^{\text{N}} R_{\dots} + \dots \Big). \qquad (40)$$

When the conditions in (38) are applied, all the above operators from the second row onwards vanish and the classical Lagrangian (1), highlighted with a box in (40), turns out to be completely regularized.

Explicit evaluation of the integrals (32) — We hereby explicitly calculate the integral (37) in a multidimensional spacetime:

$$\mathcal{I} = \sideset{}{'}\sum_{j=0} c_{u_j} M_{u_j}'^D \sum_{\ell=0}^{[D/2]} \frac{\alpha_\ell(p_i)}{M_{u_j}'^{2\ell}} e^{-\frac{1}{2} i\pi(D - 2\ell + 2ns)}$$

$$\mathrm{B}_{-(\Lambda_c^2/M_{u_j}'^2)^2} \left(\frac{D}{2} - \ell + ns, 1 - ns \right),$$

where $B_z(a,b) = \int_0^z t^{a-1}(1-t)^{b-1} dt$. Expanding the result for large values of the cut-off Λ_c we find

$$\mathcal{I} = \sideset{}{'}\sum_{j=0} c_{u_j} M_{u_j}'^D \sum_{\ell=0}^{[D/2]} \frac{\alpha_\ell(p_i)}{M_{u_j}'^{2\ell}} \times$$

$$\frac{1}{\Gamma(ns)} \Bigg[\Gamma\left(\ell - \frac{D}{2} \right) \Gamma\left(\frac{D}{2} - \ell + ns \right)$$

$$+ \left(\frac{\Lambda_c^2}{M_{u_j}'^2} \right)^{D-2\ell} \left(\frac{2\Gamma(ns)}{D - 2\ell} + \frac{2\Gamma(ns+1)}{(-D + 2\ell + 2)\left(\frac{\Lambda_c^2}{M_{u_j}'^2} \right)^2} \right.$$

$$+ \frac{\Gamma(ns+2)}{(d - 2(\ell+2))\left(\frac{\Lambda_c^2}{M_{u_j}'^2} \right)^4} + \frac{\Gamma(ns+3)}{(6(\ell+3) - 3D)\left(\frac{\Lambda_c^2}{M_{u_j}'^2} \right)^6}$$

$$\left. + O\left(\frac{M_{u_j}'^2}{\Lambda_c^2} \right)^4 \right) \Bigg],$$

where again $[x]$ is the integer part of the number x.

Finally, we explicitly evaluate the sum on the integer ℓ, namely

$$
\begin{aligned}
\mathcal{I} = \sideset{}{'}\sum_{j=0} c_{u_j} M_{u_j}^{\prime D} &\left\{ \alpha_0 \left[a_D^{(1)} \left(\frac{\Lambda_{\rm c}}{M_{u_j}^{\prime}} \right)^D \right. \right. \\
&+ a_{D-2}^{(1)} \left(\frac{\Lambda_{\rm c}}{M_{u_j}^{\prime}} \right)^{D-2} + \cdots + a_1^{(1)} \log \left(\frac{\Lambda_{\rm c}}{M_{u_j}^{\prime}} \right)^2 + a_0^{(1)} \right] \\
&+ \frac{\alpha_1(p_i)}{M_{u_j}^{\prime 2}} \left[a_{D-2}^{(2)} \left(\frac{\Lambda_{\rm c}}{M_{u_j}^{\prime}} \right)^{D-2} + a_{D-4}^{(2)} \left(\frac{\Lambda_{\rm c}}{M_{u_j}^{\prime}} \right)^{D-4} \right. \\
&\left. + \cdots + a_1^{(2)} \log \left(\frac{\Lambda_{\rm c}}{M_{u_j}^{\prime}} \right)^2 + a_0^{(2)} \right] + \ldots \\
&\left. + \frac{\alpha_{[D/2]}(p_i)}{M_{u_j}^{\prime D}} \left[a_1^{([D/2])} \log \left(\frac{\Lambda_{\rm c}}{M_{u_j}^{\prime}} \right)^2 + a_0^{([D/2])} \right] \right\}. \quad (41)
\end{aligned}
$$

In this last expression (41), $a_i^{(j)}$ are numerical constants depending on n and/or s, while the logarithmic contributions must be understood only in even dimension.

For the vertexes V_2 in (30) (analog vertexes are present for the T tensors), the polynomial in (32) reads

$P(k)_{2sn-2}$ because it has at least two less derivatives and the result in (41) will be proportional to at least one power of $M_{z_j}^2$ or $M_{t_j}^2$ ($j \geqslant 1$.) Therefore, we will only need the relation (23) to make the divergent integrals vanish. This paragraph makes clear why we do not have to worry about the extra vertexes, although there is no similar contribution coming from the ghost b (or the graviton h for the case of the tensors T.)

Conclusions — In this paper we explicitly showed that a class of unitary non-polynomial theories of gravity with asymptotic polynomial behavior, that has already been proved super-renormalizable (only one-loop divergences survive), is actually completely regularized even at one-loop order using the Pauli-Villars regularization procedure introduced by Diaz-Troost-van Nieuwenhuizen-van Proeyen (DTPN) and applied to Einstein's gravity by Anselmi. The regularization mechanism is diagrammatically shown in Fig.1. Multi-loops amplitudes are finite because of the higher derivative scaling of the theory in the ultraviolet regime. Since there are no divergent contributions, all the beta functions are zero. It turns out that the theory is completely finite to any order in the loop expansion.
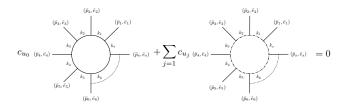

FIG. 1: Diagrammatic representation of the one-loop cancellation due to the Pauli-Villars fields. The fields $h_{\mu\nu}$, $C^\mu, \bar{C}_\nu$ and b_α circulate in the loop on the left, while the fields $T_{i\mu\nu}$, $W_i^\mu, \bar{W}_{i\nu}$ and $Z_{i\mu}$ circulate in the loop on the right. u_0 and u_j are defined in the text.

[1] L. Modesto, Phys. Rev. D **86**, 044005 (2012) [arXiv:1107.2403 [hep-th]].
[2] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Phys. Rev. Lett. **108**, 031101 (2012) [arXiv:1110.5249 [gr-qc]]; T. Biswas, T. Koivisto and A. Mazumdar, arXiv:1302.0532 [gr-qc]; T. Biswas, A. Conroy, A. S. Koshelev and A. Mazumdar, Class. Quant. Grav. **31**, 015022 (2013) [arXiv:1308.2319 [hep-th]].
[3] L. Modesto, Astron. Rev. 8.2 (2013) 4-33 [arXiv:1202.3151 [hep-th]]; L. Modesto, arXiv:1202.0008 [hep-th]; L. Modesto, arXiv:1206.2648 [hep-th].
[4] S. Alexander, A. Marciano and L. Modesto, Phys. Rev. D **85**, 124030 (2012) [arXiv:1202.1824 [hep-th]].
[5] F. Briscese, A. Marciano, L. Modesto and E. N. Saridakis, Phys. Rev. D **87**, 083507 (2013) [arXiv:1212.3611 [hep-th]].
[6] N. V. Krasnikov, Theor. Math. Phys. **73**, 1184 (1987) [Teor. Mat. Fiz. **73**, 235 (1987)].
[7] E. T. Tomboulis [hep-th/9702146v1].
[8] J. Khoury, Phys. Rev. D **76**, 123513 (2007) [hep-th/0612052].
[9] G. Calcagni, M. Montobbio and G. Nardelli, Phys. Lett. B 662, 285 (2008) [arXiv:0712.2237 [hep-th]].
[10] G. V. Efimov, "Nonlocal Interactions" [in Russian], Nauka, Moskow (1977).
[11] G. V. Efimov, review paper [in Russian].
[12] V. A. Alebastrov, G. V. Efimov, V. A. Alebastrov and G. V. Efimov, Commun. Math. Phys. **31**, 1 (1973).
[13] V. A. Alebastrov and G. V. Efimov, Commun. Math. Phys. **38**, 11 (1974).
[14] G. V. Efimov, Theor. Math. Phys. **128**, 1169 (2001) [Teor. Mat. Fiz. **128**, 395 (2001)].
[15] J. W. Moffat, Phys. Rev. D **41**, 1177 (1990).
[16] D. Evens, J. W. Moffat, G. Kleppe and R. P. Woodard, Phys. Rev. D **43**, 499 (1991).
[17] J.W. Moffat, Eur. Phys. J. Plus 126, 43 (2011) [arXiv:1008.2482 [gr-qc]].
[18] N.J. Cornish, N. J. Cornish, Mod. Phys. Lett. A **7**, 1895 (1992); N. J. Cornish, Mod. Phys. Lett. A **7**, 631 (1992).
[19] S. Deser and R. P. Woodard, Phys. Rev. Lett. **99**, 111301 (2007) [arXiv:0706.2151 [astro-ph]].
[20] S. Deser and R. P. Woodard, JCAP **1311**, 036 (2013) [arXiv:1307.6639 [astro-ph.CO]].
[21] L. Modesto and S. Tsujikawa, Phys. Lett. B **727**, 48 (2013) [arXiv:1307.6968 [hep-th]].
[22] M. Jaccard, M. Maggiore and E. Mitsou, Phys. Rev. D **88**, 044033 (2013) [arXiv:1305.3034 [hep-th]]; S. Foffa, M. Maggiore and E. Mitsou, arXiv:1311.3421 [hep-th].
[23] A. Diaz, W. Troost, P. van Nieuwenhuizen and A. Van

Proeyen, Int. J. Mod. Phys. A **4**, 3959 (1989).

[24] D. Anselmi, Phys. Rev. D **45**, 4473 (1992).

[25] D. Anselmi, Phys. Rev. D **48**, 680 (1993).

[26] D. Anselmi, Phys. Rev. D **48**, 5751 (1993) [hep-th/9307014].

[27] M. Asorey, J. L. Lopez and I. L. Shapiro, Int. J. Mod. Phys. A **12**, 5711 (1997) [hep-th/9610006].

[28] L. Modesto, J. W. Moffat and P. Nicolini, Phys. Lett. B **695**, 397 (2011) [arXiv:1010.0680 [gr-qc]].

[29] G. Calcagni, L. Modesto and P. Nicolini, arXiv:1306.5332 [gr-qc].

[30] C. Bambi, D. Malafarina and L. Modesto, arXiv:1306.1668 [gr-qc].

[31] C. Bambi, D. Malafarina and L. Modesto, Phys. Rev. D **88**, 044009 (2013) [arXiv:1305.4790 [gr-qc]].

[32] A. S. Koshelev, Class. Quant. Grav. **30**, 155001 (2013) [arXiv:1302.2140 [astro-ph.CO]].

[33] T. Biswas, A. S. Koshelev, A. Mazumdar and S. Y. .Vernov, JCAP **1208**, 024 (2012) [arXiv:1206.6374 [astro-ph.CO]].

[34] A. S. Koshelev and S. Y. .Vernov, Phys. Part. Nucl. **43**, 666 (2012) [arXiv:1202.1289 [hep-th]].

[35] A. S. Koshelev, Rom. J. Phys. **57**, 894 (2012) [arXiv:1112.6410 [hep-th]].

[36] S. Y. .Vernov, Phys. Part. Nucl. **43** (2012) 694 [arXiv:1202.1172 [astro-ph.CO]].

[37] F. Briscese, L. Modesto and S. Tsujikawa, Phys. Rev. D **89**, 024029 (2014) [arXiv:1308.1413 [hep-th]].

[38] A. Accioly, A. Azeredo and H. Mukai, J. Math. Phys. **43**, 473 (2002); F. d. O. Salles and I. L. Shapiro, arXiv:1401.4583 [hep-th].

[39] K. S. Stelle, Phys. Rev. D **16**, 953 (1977).

[40] I. L. Buchbinder, Sergei D. Odintsov, I. L. Shapiro, "Effective action in quantum gravity", IOP Publishing Ltd 1992.

[41] P. Van Nieuwenhuizen, Nuclear Physics B 60 478-492 (1973).